LOUGHBOROUGH UNIVERSITY OF TECHNOLOGY

AN INVESTIGATION INTO FACTORS AFFECTING WELDING PENETRATION

BY

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September 1982.

Supervisor: Mr. B.A. Moore.

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INTRODUCTION

This project, which was carried out in collaboration with the Department of Engineering Production, involves exploring the effect on weld penetration of a variety of changing circumstances, represented by seven factors each being explored at three levels. Four response measurements (each associated with weld penetration) were made. As the full factorial experiment is excessively large, involving 2,187 different treatment combinations, it was proposed that initially a 1/9 - replicate of the 3^7 factorial experiment be designed, possibly duplicated to test for repeatability and to provide an independent error measurement. A suitable design was constructed, the fractional factorial being arranged in blocks with each block providing one day's work.

The Engineering Production Department was to do the actual welding using the proposed design in order to obtain data for the subsequent analysis. Some preliminary experimentation was, in fact, carried out but the full planned experiment could not be carried out in the allotted time because of repeated machine breakdown.

The preliminary experimentation, combined with previous work in this area, provided information on the likely range of each response over the design space, and gave some guidance on the relative importance of the factors. It was therefore suggested that data should be simulated, using the design put forward, in as realistic a manner as possible. The data should then be analysed in the same way as genuine data, so that the same procedure

could still be followed when the real data became available.

The project report has been divided into four chapters. In Chapter 1 a project description is given from the Engineering viewpoint. A precise description of all the factors and responses involved in adaptive controlled welding has been provided. Current welding techniques are considered and reasons given for the need to automate welding and to use robots. Some associated work is also given in this chapter, together with some consideration of the role played by a carefully designed experiment.

Chapter 2 is basically giving the background theory for the use of hypersquares in confounding part of some higher order interaction. The ideas of working in the notation of congruences and using this for confounding and selection of the fractional replicate are introduced in this chapter, together with the associated notion of aliasing, as a background to the construction of the proposed design.

In Chapter 3, the ideas of confounding, choice of fractional replicate and aliasing structures introduced in Chapter 2 have been used to construct a design for arranging a $1/9 \times 3^7$ factorial experiment in 9 blocks of 27 units. The defining contrasts, the aliasing structures of the chosen confounded effects with the defining contrasts and with some other factorial effects have been given. (The full aliasing structure is too extensive to be listed here). The actual design for the first 3 days of the 9-day scheme has been provided in this chapter together with the form of the analysis of variance. The remaining part of the design appears in the appendix.

Chapter 4 gives the analysis of the simulated data for each

of the four response measurements. The method used to simulate data for each response has been explained. The simulation was made to yield results that would conform with guidance obtained from the Engineering Production Department. A brief discussion on the GLIM package, used to accomplish the simulation and also to carry out the ANOVA, is given. Although GLIM-3 is not primarily oriented towards analysis of variance, it is capable of handling a complex design such as that proposed, and it was felt that the modelling capability would be useful at the next stage of the exercise. (Glim - 4, available shortly, will have an improved ANOVA capability). Some discussion of the modelling aspect is given with the conclusions, together with suggestions on some form of transformation which might be necessary in the case of real data if residuals were found to be correlated.

CHAPTER 1

ADAPTIVE CONTROLLED WELDING

1.1 INTRODUCTION.

In this chapter, an attempt is made to explain what is involved in the type of welding being considered and also to indicate those factors responsible and the way in which they affect the weld bead geometry.[1]

A brief description of current welding techniques together with associated work is given. The reason is also given for the need to carefully design an experiment to check which of the factors and their interactions are more "important" and the extent to which they are involved in those models describing each response of the weld bead geometry. An explanation as to why automated welding procedure should be employed is also provided.

1.2 COMPONENTS INVOLVED AND THE BACKGROUND.

The need to fabricate metal plates to form enclosed volumes occurs in many industries. Enclosed box sections provide a rigid structure with low material weight. Such a combination is useful in vehicle manufacture particularly earth moving machinery, shipbuilding and the construction of Railway rolling stock. These enclosed forms present a problem to the welding Engineer as full penetration welding through the entire thickness of the plate is required whilst access is restricted to the outer side of the component.

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1. *Terms underlined in this chapter are explained in the glossary of terms - Appendix A1.*

The research considered is directed towards developing automated techniques for single sided full penetration welding with particular emphasis on Railway carriage and locomotive constructions. The Bogies, underframes and transoms of these vehicles are typical of other enclosed components to be found throughout industry.

Bogies, (Fig. 1.1), are highly stressed components, so the need for rigidity. However they are relatively unsprung and their weight must be minimized in order to reduce track damage.

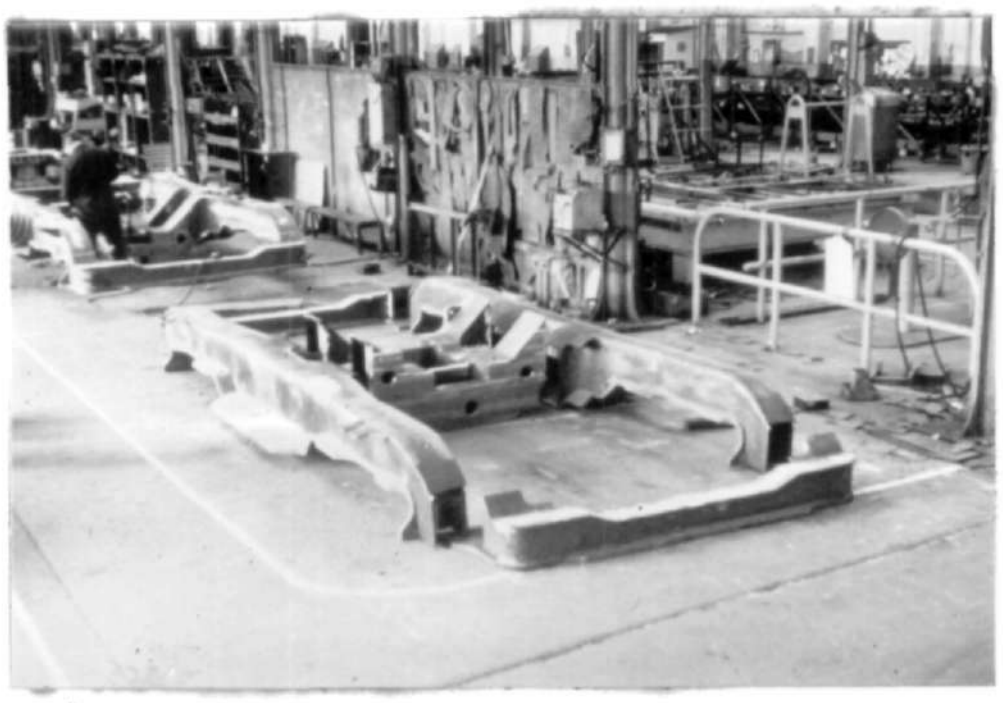


Fig. 1.1 Bogie Frame under construction.

Similarly, carriage underframes (Fig. 1.2), which extend the entire length of the vehicle must be as light as possible to improve the energy efficiency of the transport system.



Fig. 1.2 An Electric Locomotive Assembly showing the Carriage Underframes.

This carriage underframe is then fixed on the assembled Bogie shown in Fig. 1.3 to make up a complete Electric locomotive or a complete coach as can be seen from Fig. 1.4 and Fig. 1.5.

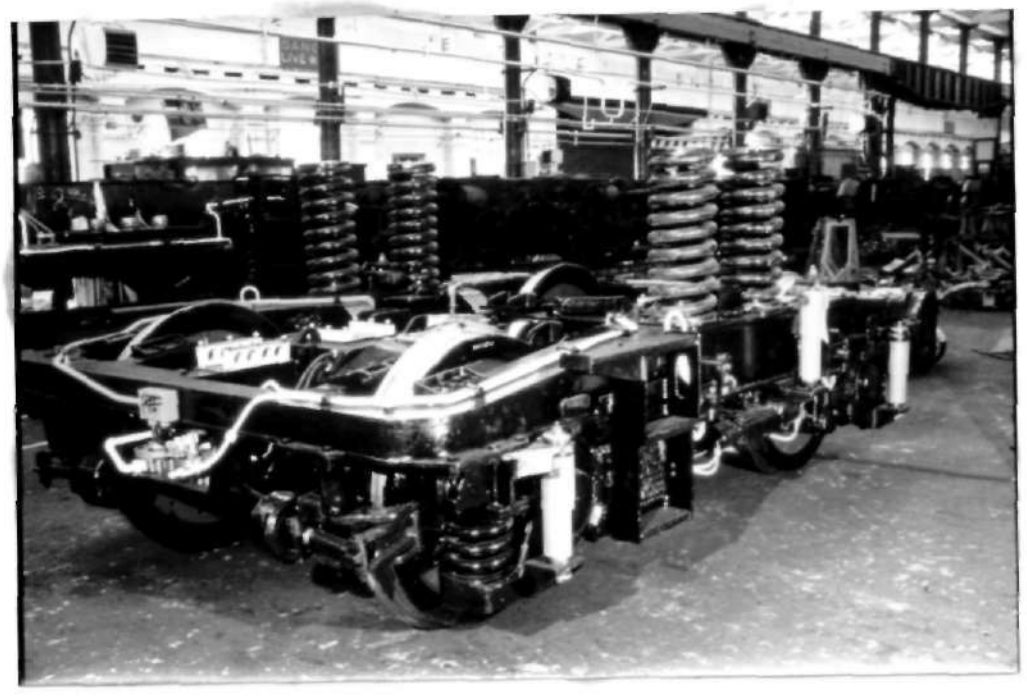


Fig. 1.3 A Completely Assembled Bogie Frame on Wheels.

It can be noted also that there are two types of Bogies shown here for the electric locomotive (Fig. 1.4) and the coach (Fig. 1.5).



Fig. 1.4 An Electric Locomotive showing its Bogies and Carriage Underframe.

Carriage underframes too differ for both the locomotive above and for the sleeper coach below.



Fig. 1.5 A Sleeper Coach showing its kind of Bogies and Carriage Underframe.

1.3 CURRENT WELDING TECHNIQUES.

Several manual welding techniques can be used to produce full penetration single sided welds, however each has its own limitations. The main difficulty is in achieving full penetration without producing a hole in the component and several weld backing techniques are employed to prevent this.

Typical of these is the method adopted by "Metro Camell", where a steel bar is placed inside the box section along the joint line, prior to assembly (Fig. 1.6). This increases the Effective Thickness of the plate so overcoming the need for a full penetration weld. But this solution, on the other hand, incurs additional material costs for the backing bar and a corresponding gain in weight thereby exerting more pressure on the tracks.

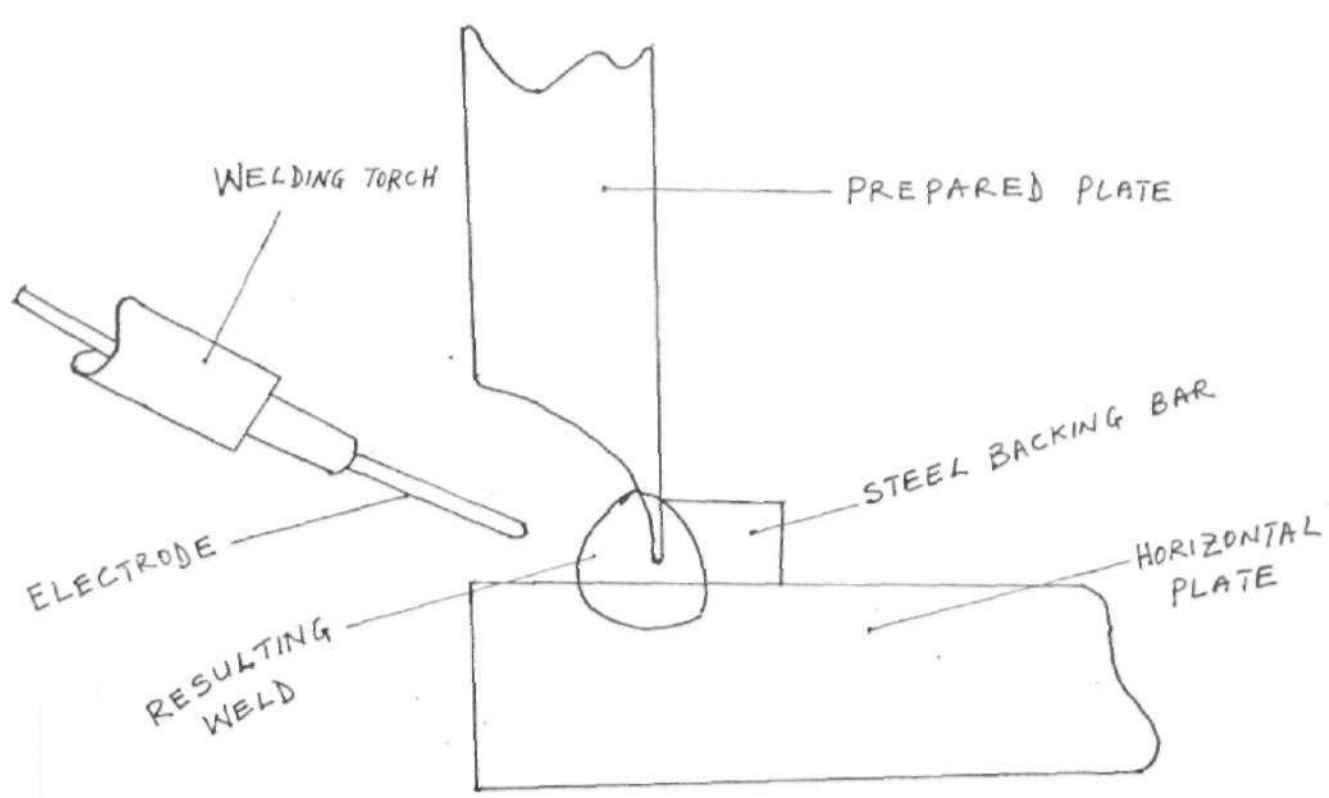


Fig. 1.6 Single Sided Welding Using a Steel Backing Bar.

A flux backing technique (Fig. 1.7) has been developed by British Rail Engineering Limited (Derby locomotive works). In this, a refractory powder flux is placed inside the box section so preventing the molten weld metal falling through the joint. This method of flux backing incurs additional costs for the consumable flux and increased labour costs associated with positioning and removal of the flux powder.

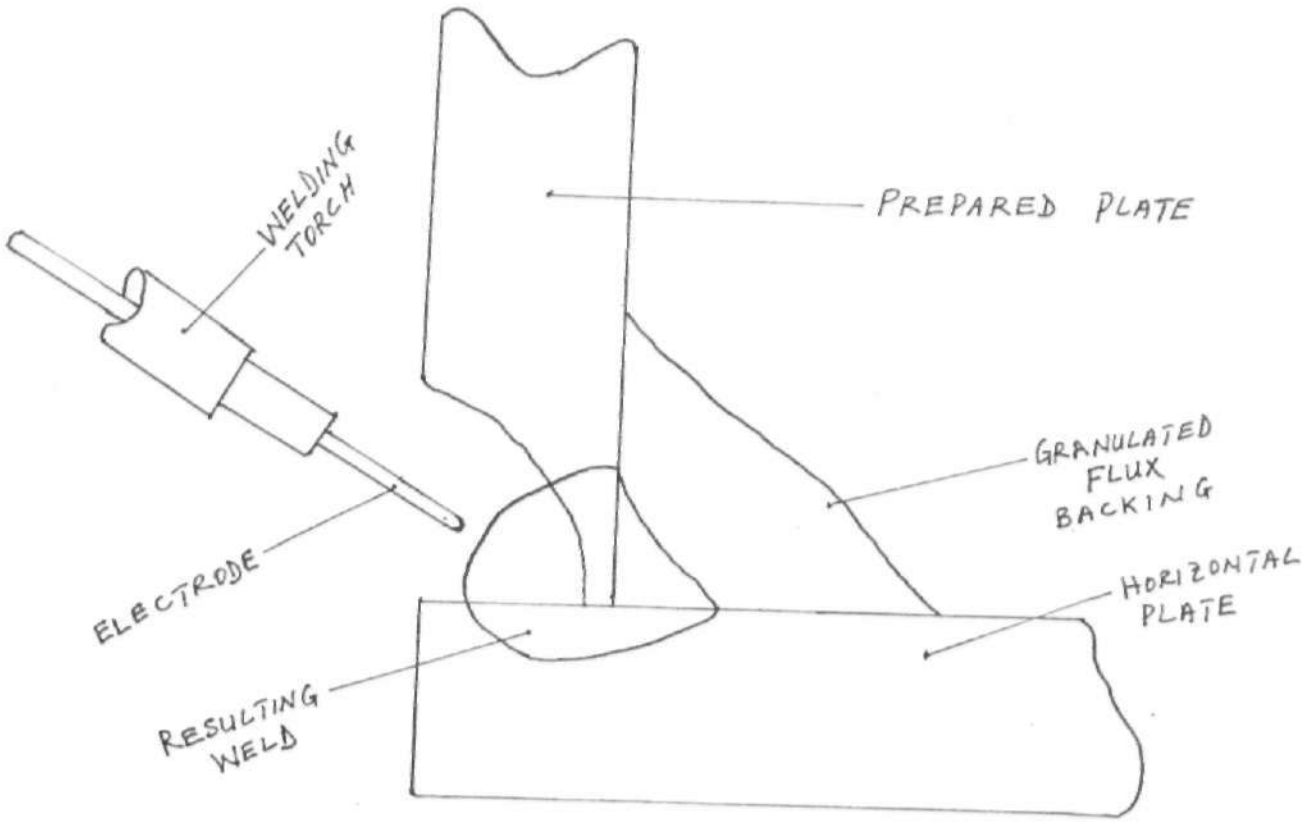


Fig. 1.7 Single Sided Welding Using a Flux Backing.

Several other weld backing techniques are available including copper backing, the use of refractory glass fibre and asbestos tapes, however the components' shape often prevents the use of such devices.

A novel welidng technique has been developed by British Rail Engineering Limited (Crewe locomotive works). This is based on a

flux cored wire, self-shielding arc process. Welding is achieved by melting the metal in an arc struck between a continuous comsumable filler metal electrode and the work piece (Fig. 1.8). Oxidation of the molten metal is prevented by a flux contained within the tubular electrode, the arc burns a hole through the plate at the joint and as the electrode travels along the joint metal from the front of the hole, melts and flows to the back of the hole under capillary action, solidifying to produce a full penetration weld.

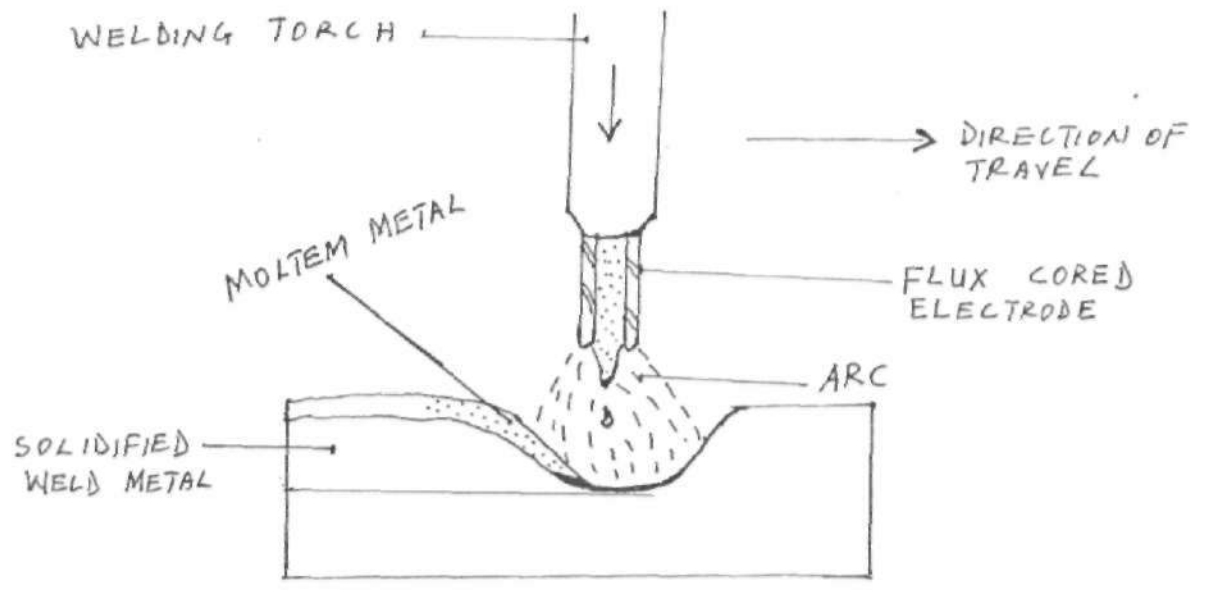


Fig. 1.8 Process Diagram → Flux-cored Arc Welding.

This keyhole technique requires no weld backing and forms the subject for this initial research effort. The welding process described is however manual, that is it relies on the skill of the welder using tactile, visual and auditory information from the process to regulate the welding conditions to achieve a high quality weld.

1.4 AUTOMATIC WELDING AND USE OF ROBOTS.

Because of the limitations mentioned in the manual welding techniques described in the above section, it is desirable to automate the welding of locomotive Bogies and Underframes currently manually welded for the following reasons:

i) Significant cost savings can be achieved.

ii) Quality can be improved.

iii) Production rates may be increased.

iv) Highly skilled welders are difficult to come by.

v) The work is unpleasant due to the fumes, dirt, noise and heat emitted from the arc; the safety clothing necessary imposes severe restrictions on movement; and furthermore the work is physically demanding.

Early or first generation robots in use in factories are both basic and relatively few in number and flux cored arc welding remains a predominantly manual process. This may be explained by the skill the human operator brings to the task:

a) Inspection - The work piece is recognised and inspected before and after welding.

b) Work piece movement - The welder can load and orientate the work piece to the best position for welding.

c) Torch Manipulation - The welder is a portable and compact torch manipulator able to align himself to the work piece and weld in various positions.

d) Process Control - The welder carries out the welding programme and provides feedback control of the process using information provided by his senses of vision, hearing, touch and smell.

The first generation robots currently commercially available are able to follow pre-programmed paths along which they perform preset welding sequences, but are unaware of their surroundings and so are unable to adapt to changes in them. This has serious disadvantages when applied to arc welding in that the accuracy of the work pieces is limited by the tolerances of the machines used to produce them. The weld seam, material thickness and relative positions of the plates will change between successive work pieces but the robot will not adapt to these as the manual welder does.

Intelligent automation is achieved by the current robots being developed which have some decision making capability based on various sensing devices. These include tactile sensors, noncontact displacement transducers and vision provided by television cameras or photodiode arrays. Such devices are able to provide microcomputers with information to enable decisions to be made to regulate the performance of the robot adaptively. Adaptive control is required if quality welding is to be achieved.

1.5 PROCESS MODELLING.

As a first step towards adaptive control of the key hole flux cored welding technique it is necessary to relate the process inputs to the outputs. The following seven factors have been identified as having a significant effect on weld bead geometry:

i) Travel speed (U)

ii) Arc Current (I)

iii) Arc Voltage (V)

iv) Root Face Thickness (T)

v) Root Gap (G)

vi) Travel Angle (A)

vii) Electrode Extension (Stickout) (E). [1]

See Figures 1.9, 1.10 and 1.11 for diagrams describing the above seven factors. Following initial tests to define the boundaries to the seven factors which produce a satisfactory weld, the levels of the experiment were chosen.

These levels were as distinct as possible within the boundaries to ensure no overlap between levels and to cover a wide enough range to give reasonably general results.

Four responses were identified (Fig. 1.12) as Effective Bead Width (w), Under Bead Throst Thickness (hu), Under Bead Minimum Leg Length (ℓu) and Maximum Penetration (p), which provide a means of controlling the weld bead geometry. The welded boxes were sectioned through each trial and etched to enable the weld bead geometry to be measured. A microscope with a precision slide was used to measure each response accurately. These responses are not of equal importance on the weld quality and are initially considered separately. The most important of the responses is Penetration followed in that order of importance by Under Bead Minimum Leg Length, Under Bead Throat Thickness and Effective Bead Width.

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1. *Explanation to the seven factors is given in the glossary of terms: Appendix A1.*

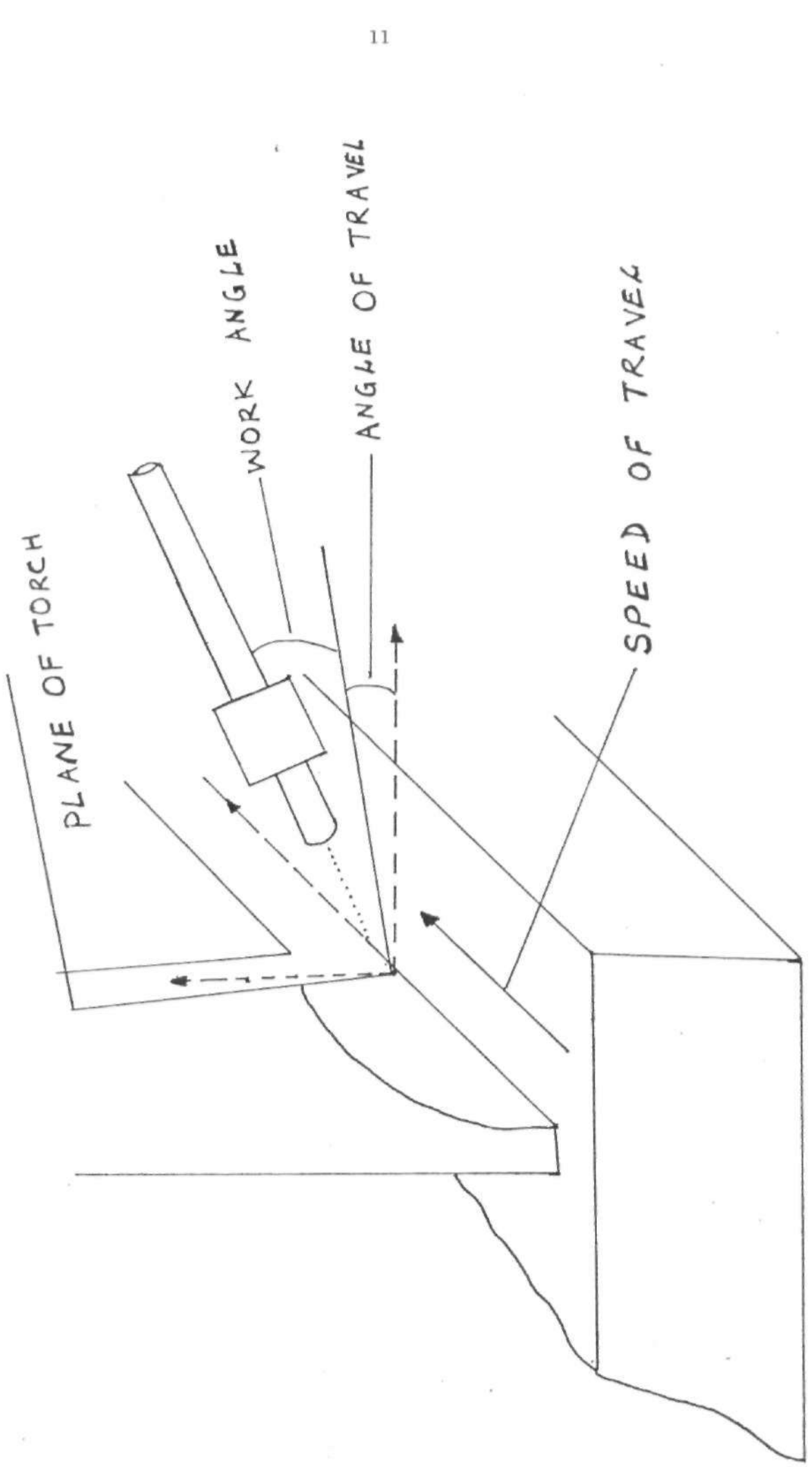


Fig. 1.9 Diagram showing Travel Speed (U) and Travel Angle (A).

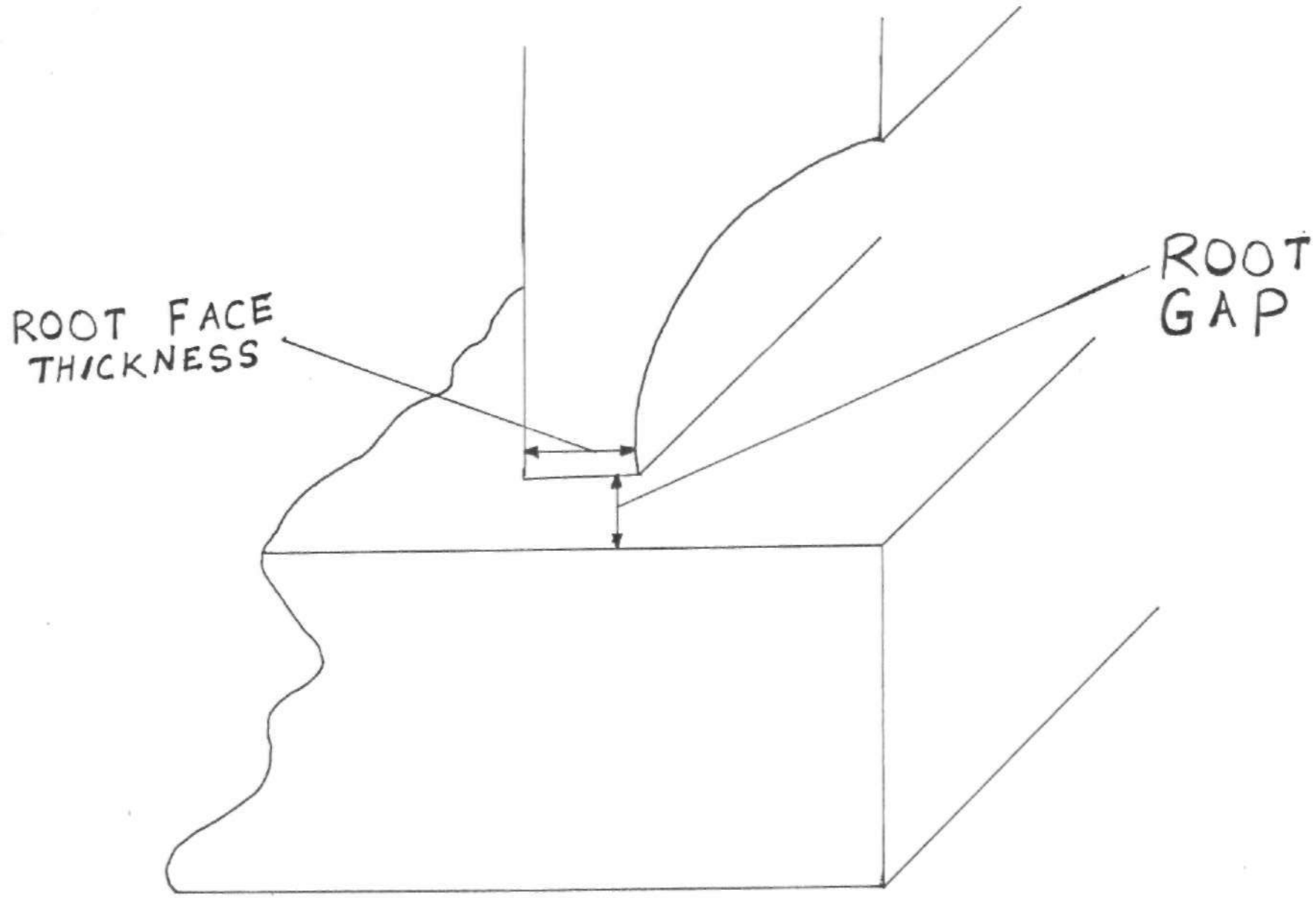


Fig. 1.10 Diagram showing Root Face Thickness (T) and Root Gap (G).

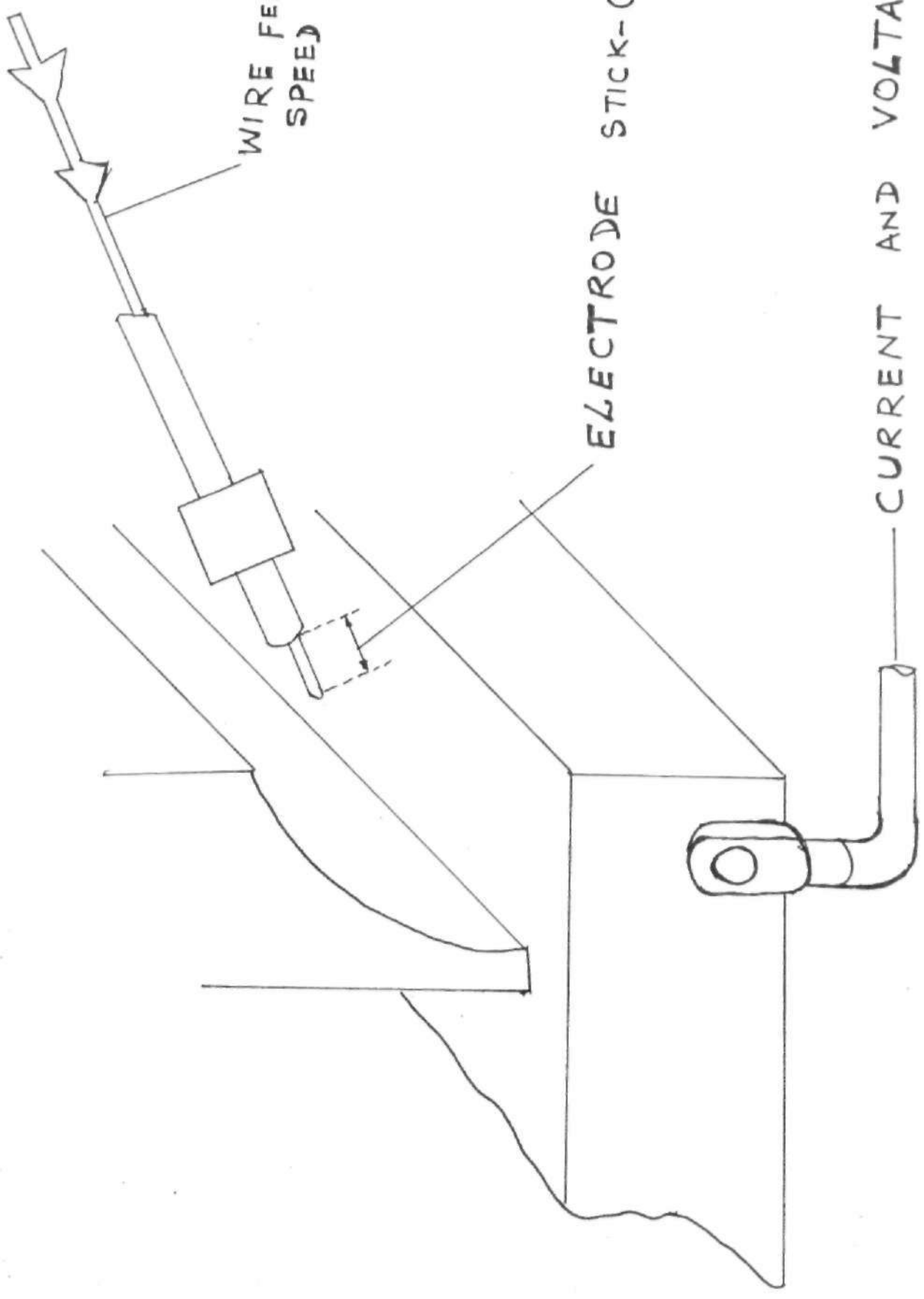


Fig. 1.11 Diagram showing Electrode Stick-out (E), Current (I) and Voltage (V).

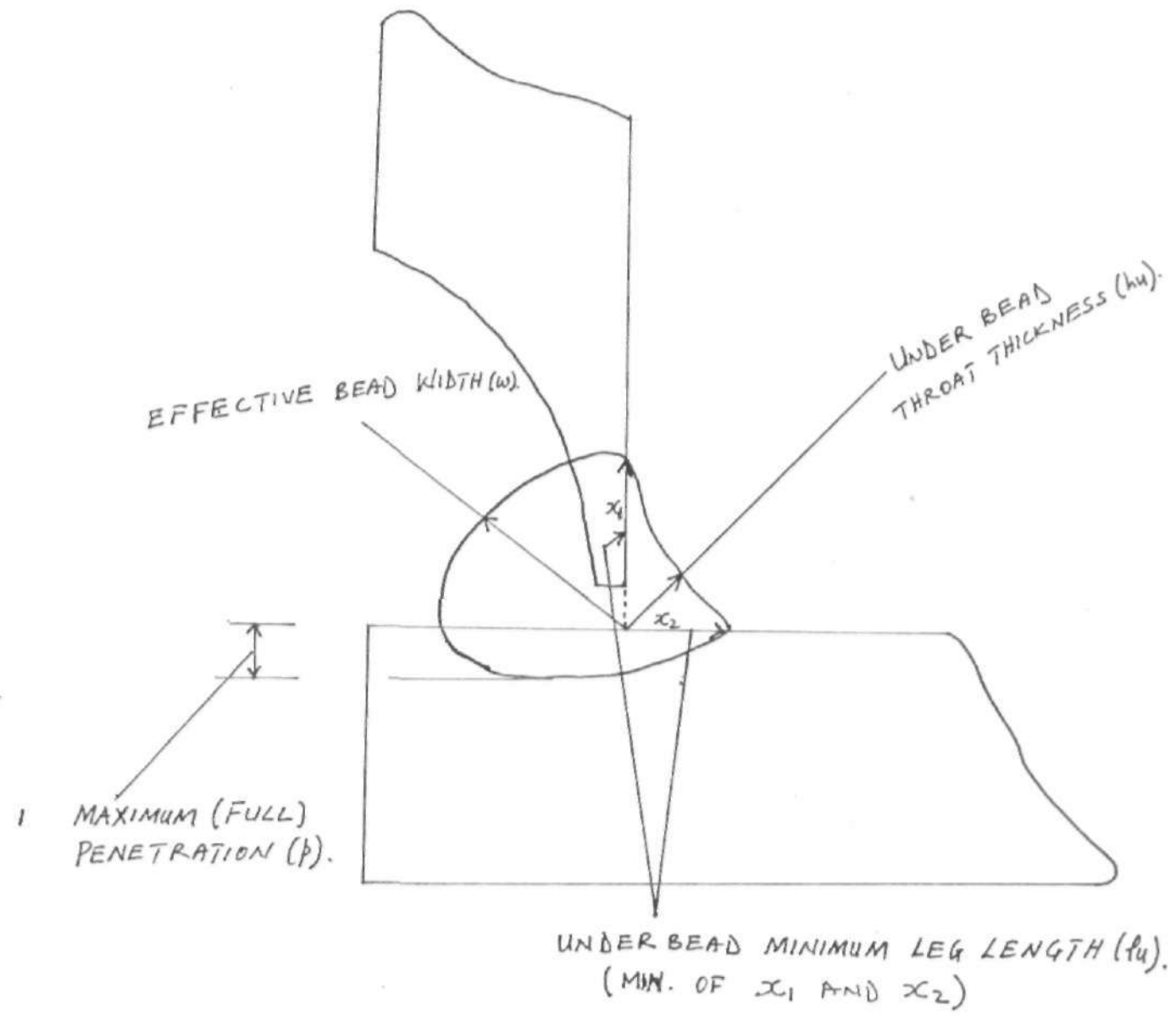


Fig. 1.12 Response Measurements - Forming the Weld Bead Geometry.

During production welding, it may not be possible to monitor all the factors directly. A television camera will collect data corresponding to the keyhole size and shape which will be a function of the factors. It is proposed to analyse the effect of the factors on the keyhole information at a later date in order to establish the best method of control. Meanwhile, the expected effects of increasing each of the factors on the responses are given as:

i) Travel speed (U) - Increase in Travel Speed will significantly reduce penetration, decrease Under bead minimum leg length, Under bead throat thickness and Effective bead width.

ii) Arc Current (I) - Increase in this will significantly increase effectivebead width, will also increase penetration, Under bead leg length and throat thickness (but only slightly).

iii) Arc Voltage (V) - Increase in Arc Voltage significantly increases penetration, slightly increases Under bead leg length and throat thickness but slightly reduces effective bead width.

iv) Root Face Thickness - Increase in this has no effect on penetration, increases effective bead width, decreases Under bead minimum leg length and Under bead throat thickness.

v) Root Gap (G) - Increase in root gap will significantly increase Under bead leg length and Under bead throat thickness but will reduce effective bead width significantly and has no effect on penetration.

vi) Travel Angle (A) - An optimum travel angle is thought to occur towards which penetration increases to a maximum, Under bead leg length and Under bead throat thickness increase, but effective bead width reduces. Away from the maximum (either direction) the opposite will occur.

vii) Electrode Extension (E) - As this increases, penetration will reduce, (stickout) effective bead width will increase, Under bead throat thickness and Under bead leg length will also reduce.

1.6 ASSOCIATED WORK.

Research work in adaptive welding is growing through the diversity of materials, the numbers of different joints that can be made and the variety of processes that can make them limit the applicability of the results as yet no underlying body of knowledge relevant to all processes or materials has been assimilated.

To control the welding process it is necessary to know the relation between the procedure inputs such as arc voltage, current, wire feed rate, torch travel speed, torch position, work piece fit up and thickness and the output such as the weld bead geometry. Doherty et al. have derived some equations describing the submerged

arc welding and flux cored arc welding processes. The latter relates torch travel speed, wire feed speed, voltage and contact tip to work piece distance to the resulting weld bead geometry. These experiments were restricted to the flat position, using butt joints without edge preparation.

Rider has developed an adaptive control technique using visual information to determine the molten weld pool width. Control algorithms were based on the empirical relationships developed by Boughton which relates travel speed, welding current and weld pool width to the weld penetration. This research was restricted to tungsten inert gas welding of stainless steel.

The work of Scott and Brandt was directed towards using a tactile sensor to detect the position and width of a seam in horizontal plates so enabling movement of the welding torch to be controlled. This work was restricted to open butt joints in horizontal plates using tungsten inert gas welding with a filler wire.

1.7 NEED FOR EXPERIMENTAL DESIGN.

The above authors use formulae, either theoretically or experimentally derived, to summarize relationships between input variables (the factors) and output variables (the responses). Having decided to relate a defined set of welding variables to a defined set of geometric dimensions, the investigator has little option but to adopt a statistical approach to the design of the experiments that will generate his data. The decision to use factorial designs was based on an early investigation (at the

welding Institute) which showed that weld variables interacted with each other in such a manner that anything but the most carefully designed set of experiments could give misleading results.

To design a factorial experiment it is necessary to define the experimental variables clearly and to decide on a number of levels for each factor. In this particular research seven factors were explored each at three levels leading to a 3^7 factorial design with 3^7 = 2,187 experiments to be done. Because of the time and costs that are involved in doing 2187 experiments, it has been decided that it is not practicable to do a full replicate of a 3^7 factorial design and so a fractional replicate has been considered and this forms the basis of the subject matter in the next two chapters of this project.

The analysis of variance is carried out to establish for any output parameters (in this case, some geometric dimension of the weld bead) the relative importance of the factors which formed the controlled variables of the experimental set.

CHAPTER 2

THE ROLE OF HYPERSQUARES IN CONFOUNDING AND FRACTIONAL REPLICATION IN 3^N DESIGNS

2.1 INTRODUCTION.

In this chapter we consider fractional replication in the 3^n system and explain briefly how this and blocking can often be achieved through hypersquares by confounding only some part of a chosen factorial effect. These ideas are developed further by introducing a notation of working in terms of congruences which is useful when fractional 3^n experiments are considered.

2.2 THE USE OF HYPERSQUARES.

The randomized-blocks design may be considered as a one-dimensional blocking method to control heterogeneity of the experimental units. Latin Squares can be used in a similar way for two-dimensional blocking. Here each treatment occurs once in each row and once in each column, with the restriction that the numbers of rows, columns, and treatments must be the same. The Graeco-Latin Square designs simply carry one step further the process of stratifying the treatments as shown in Table 2.1.

Table 2.1.

$A\alpha$	$C\beta$	$B\gamma$
$B\beta$	$A\gamma$	$C\alpha$
$C\gamma$	$B\alpha$	$A\beta$

In this type of design, both the Latin and the Greek letters occur once in each row and once in each column, and each Latin letter occurs once with each Greek letter. In this manner, row, column, Latin letters and Greek letters effects can be estimated independently, and the sum of squares in the analysis of variance partitioned accordingly.

This concept of using a pair of orthogonal Latin Squares to form a Graeco-Latin square is extended to the construction of hypersquares. For a square of size $n \times n$ and $n > 3$, a hypersquare can be constructed by superimposing 3 or up to (n-1) orthogonal Latin Squares if a complete set of (n-1) orthogonal Latin Squares is available. The maximum number of factors which can be handled in a $n \times n$ square is (n+1), and the assumption of no interaction is always necessary.

Hypersquares have a weakness that if (n+1) factors are considered, since the sum of squares corresponding to each factor takes (n-1) degrees of freedom, and $(n+1)(n-1) = n^2 - 1$, it will not be possible to compute the error sum of squares for comparisons in the final analysis of variance.

2.3 THE I AND J COMPONENTS OF INTERACTION.

Consider a 3^2 factorial arrangement with factors A and B each at 3 levels a_o, a_1, a_2 and b_o, b_1, b_2 respectively. In Table 2.2 below, (i,j) denotes the response with factor A at level a_i and factor B at level b_j.

Table 2.2.

		B		
		b_o	b_1	b_2
A	a_o	(0,0)	(0,1)	(0,2)
	a_1	(1,0)	(1,1)	(1,2)
	a_2	(2,0)	(2,1)	(2,2)

The comparisons among rows:

$$(A)_{i=0} = (0\ 0) + (0\ 1) + (0\ 2)$$

$$(A)_{i=1} = (1\ 0) + (1\ 1) + (1\ 2)$$

$$(A)_{i=2} = (2\ 0) + (2\ 1) + (2\ 2)$$

give the main effect of A (on 2 degrees of freedom).

The comparisons among columns:

$$(B)_{j=0} = (0\ 0) + (1\ 0) + (2\ 0)$$

$$(B)_{j=1} = (0\ 1) + (1\ 1) + (2\ 1)$$

$$(B)_{j=2} = (0\ 2) + (1\ 2) + (2\ 2)$$

give the main effect of B (on 2 degrees of freedom).

The AB interaction sum of squares on 4 degrees of freedom (d.f) can be partitioned into either

i) 4 components with a single degree of freedom each,

or ii) 2 components with 2 d.f. each.

The second partitioning is very useful for constructing fractional factorial designs with factors at 3 levels. It makes use of the basic properties of a 3 × 3 Graeco-Latin Square. Superimposing a Graeco-Latin Square on Table 2.2 gives Table 2.3 below.

Table 2.3.

		B		
		b_o	b_1	b_2
A	a_o	$I_2\ J_1$ (0,0)	$I_2\ J_2$ (0,1)	$I_3\ J_3$ (0,2)
	a_1	$I_2\ J_3$ (1,0)	$I_3\ J_1$ (1,1)	$I_1\ J_2$ (1,2)
	a_2	$I_3\ J_2$ (2,0)	$I_1\ J_3$ (2,1)	$I_2\ J_1$ (2,2)

The analysis of variance is of the form

Source of Variation		d.f.
Between rows	(A)	2
Between columns	(B)	2
AB interaction (on 4 d.f.) — Between levels of I		2
AB interaction (on 4 d.f.) — Between levels of J		2

Hence the AB interaction is split into two independent components each on 2 d.f. written as AB(I) and AB(J) and these are called the "I" and "J" components of the AB interaction.

2.4 CONFOUNDING.

Suppose it is desired to arrange the 9 treatment combinations in Table 2.3 in 3 blocks of 3 units each, we may choose to confound either the I or the J component of the AB interaction if we are to estimate the main effects A and B.

If we choose to confound the I-component of AB with blocks, then we end up with the following arrangement:

Block 1 : (0,0) (1,2) (2,1)

Block 2 : (0,1) (1,0) (2,2)

Block 3 : (0,2) (1,1) (2,0)

The 2 d.f. confounded with blocks here correspond to the 2 d.f. for the I-component of AB interaction and differences between AB(I) correspond to differences between blocks. That is, the AB(I) term has been confounded with blocks.

2.5 EXPRESSING IN TERMS OF CONGRUENCES.

Consider the 3^2 experiment in Table 2.2. This factorial experiment can be arranged in 3 blocks of 3 units using the orthogonal Latin Square shown in Table 2.4 below.

Table 2.4.

A \ B	b_o	b_1	b_2
a_o	$A\alpha$ (0,0)	$C\beta$ (0,1)	$B\gamma$ (0,2)
a_1	$B\beta$ (1,0)	$A\gamma$ (1,1)	$C\alpha$ (1,2)
a_2	$C\gamma$ (2,0)	$B\alpha$ (2,1)	$A\beta$ (2,2)

Let the I-totals be the totals corresponding to A, B and C and the J-totals be the totals corresponding to α, β and γ. Then the 3^2 experiment can be arranged in blocks of 3 units using either the I-totals:

$$I_o = 0\,0 + 1\,1 + 2\,2$$

$$I_1 = 1\,0 + 2\,1 + 0\,2$$

$$I_2 = 2\,0 + 0\,1 + 1\,2$$

or the J-totals:

$$J_o = 0\,0 + 1\,2 + 2\,1$$

$$J_1 = 0\,1 + 1\,0 + 2\,2$$

$$J_2 = 0\,2 + 1\,1 + 2\,0 \ .$$

The AB(I) interaction is given by 2 d.f. between the I-totals and the AB(J) interaction is given by 2 d.f. between the J-totals.

So confounding in a 3^2 experiment is therefore a case of using a Latin Square with blocks occupying the treatment positions and the two factors occupying the rows and columns respectively.

Let a treatment combination be represented by ℓ_1, ℓ_2 where ℓ_1 is the level of factor A and ℓ_2 the level of factor B, so that 0 0 represents the first level of factor A and the first level of factor B respectively.

The three I-totals are obtained by

$$\ell_1 + 2\ell_2 \equiv 0 \quad \text{(modulo 3)}$$

$$\ell_1 + 2\ell_2 \equiv 1 \quad \text{(modulo 3)}$$

$$\ell_1 + 2\ell_2 \equiv 2 \quad \text{(modulo 3)}$$

respectively, working in terms of modulo 3 (mod.3), and the

three J-totals are obtained by

$$\ell_1 + \ell_2 \equiv 0 \pmod 3$$

$$\ell_1 + \ell_2 \equiv 1 \pmod 3$$

$$\ell_1 + \ell_2 \equiv 2 \pmod 3$$

respectively.

Therefore we can write AB(I) as AB^2, and AB(J) as AB, where the powers attached to the letters are the coefficients of ℓ_1 and ℓ_2. The main effects of A and B are given by examining the 3 totals for which

$$\ell_1 \equiv 0, 1, 2 \pmod 3$$

and $$\ell_2 \equiv 0, 1, 2 \pmod 3$$

respectively.

Thus the "between treatments" sum of squares (S.S) on 8 d.f. (in the 3^2 experiment) has been split into 4 orthogonal components A, B, AB^2 and AB each on 2 d.f.

It is observed that the treatment combinations specified by the equation

$$2\ell_1 + 2\ell_2 \equiv 0, 1, 2 \pmod 3$$

for A^2B^2 are identical to those for AB; and the treatment combinations specified by

$$2\ell_1 + \ell_2 \equiv 0, 1, 2 \pmod 3$$

for A^2B are the same as those for AB^2.

Clearly, therefore, $AB = A^2B^2$ and $AB^2 = A^2B$. For convenience, the power of the first digit is usually taken to be unity. The rule

now for multiplying together two effects is that the power of each letter is reduced modulo 3 if necessary and the power of the first digit made unity, for example

$$A^2B = (A^2B)^2 = A^4B^2 = AB^2 \quad .$$

2.6 EXTENSIONS AND GENERALIZATIONS.

For the 3^3 experiment, a treatment combination is given by ℓ_1, ℓ_2, ℓ_3 and the 27 treatment combinations can be arranged in 3 blocks of 9 for instance by confounding one component of the ABC interaction. The 8 d.f. for the ABC interaction can be split into 4 components each on 2 d.f. and any one of these may be chosen to be confounded with blocks. This will enable all main effects and all 2-factor interactions to be estimated. The main effects and components of interactions are given as follows, where all functions are reduced modulo 3 when necessary:

$$\begin{aligned}
A &: \ell_1 = 0, 1, 2 \\
B &: \ell_2 = 0, 1, 2 \\
C &: \ell_3 = 0, 1, 2 \\
AB(I) = AB^2 &: \ell_1 + 2\ell_2 = 0, 1, 2 \\
AB(J) = AB &: \ell_1 + \ell_2 = 0, 1, 2 \\
AC(I) = AC^2 &: \ell_1 + 2\ell_3 = 0, 1, 2 \\
AC(J) = AC &: \ell_1 + \ell_3 = 0, 1, 2 \\
BC(I) = BC^2 &: \ell_2 + 2\ell_3 = 0, 1, 2 \\
BC(J) = BC &: \ell_2 + \ell_3 = 0, 1, 2 \\
ABC(W) = AB^2C^2 &: \ell_1 + 2\ell_2 + 2\ell_3 = 0, 1, 2 \\
ABC(X) = AB^2C &: \ell_1 + 2\ell_2 + \ell_3 = 0, 1, 2 \\
ABC(Y) = ABC^2 &: \ell_1 + \ell_2 + 2\ell_3 = 0, 1, 2 \\
ABC(Z) = ABC &: \ell_1 + \ell_2 + \ell_3 = 0, 1, 2.
\end{aligned}$$

This method can easily be extended to more than 3 factors. For example in the case of 4 factors, it may be desired to arrange the $3^4 = 81$ treatments in 3 blocks of 27. The ABCD interaction on 16 d.f. could be split into eight components, each on 2 d.f. and any one of these confounded with blocks. Alternatively, the 81 treatments could be placed in 9 blocks of 9 units. Here the 8 d.f. confounded with blocks are obtained from the 4 components of interaction.

2.7 GENERAL RULE.

If pairs of d.f. corresponding to say, X and Y are confounded with blocks, then so are the pairs of d.f. corresponding to XY and XY^2. For example, if we choose to confound AB^2C^2 and BC^2D, then consequently, we also confound

$$AB^2C^2 \times BC^2D = AB^3C^4D = ACD$$

and

$$AB^2C^2 \times (BC^2D)^2 = AB^4C^6D^2 = ABD^2$$

(when indices are reduced modulo 3), each on 2 d.f.

For these effects chosen to be confounded, the 9 blocks would now be given by the solution to

$$\ell_1 + 2\ell_2 + 2\ell_3 \equiv 0, 1, 2 \quad (\text{mod}.3)$$

$$\ell_2 + 2\ell_3 + \ell_4 \equiv 0, 1, 2 \quad (\text{mod}.3)$$

with each pair of equations, (and there are 9 such pairs) giving one block.

The block containing treatment combination 0 0 0 0 is called the Principal Block. The other blocks can then be obtained

immediately from this by "adding" each treatment combination in the Principal Block to any treatment combination which satisfies the equations above and which has not already been used, with the levels being reduced modulo 3 where necessary.

The above process may be extended to the general 3^n system. In the general case, we have $(3^n - 1)/2$ factorial effects, each on 2 d.f. The partitioning of an interaction into components of 2 d.f. by this process is simply a convenient mathematical device to extract orthogonal components of an interaction. These components have no concrete physical interpretation. Testing each of these separately for significance is generally not meaningful.

The combinatorial properties of the 3^n system can be generalized to the general p^n with p as a prime number. The treatment combinations are denoted by $(\ell_1, \ell_2, \ldots, \ell_n)$. The ℓ_i's are factorial levels having values from 0 to (p-1). All numbers are reduced modulo p. The $(p^n - 1)$ degrees of freedom may be partitioned into $(p^n - 1)/(p - 1)$ sets of $(p - 1)$ degrees of freedom. Each set of $(p - 1)$ degrees of freedom is given by the comparisons among the p sets of p^{n-1} treatment combinations.

2.8 FRACTIONAL REPLICATION IN THE 3^n SYSTEM.

The basic ideas of constructing a fractional replicate of a 3^n arrangement can be illustrated by the use of a simple example.

Suppose we require a 1/3 - replicate of a 3^5 experiment, it is natural to select as the defining contrast one of the 16 components of the highest order interaction each on 2 d.f. One of these

components could be ABCDE for instance, and will be confounded to give the 1/3 - replicate. This then is the defining contrast and the aliases of effect A say are given by

$$A \times ABCDE \quad \text{and} \quad A \times (ABCDE)^2$$

i.e. $A \equiv AB^2C^2D^2E^2 \equiv BCDE.$

In a similar way, aliases of other effects are obtained. The defining contrasts are used to obtain the aliases system. The aliases of an effect or interaction are obtained by multiplying the effect or interaction with each term in the defining contrasts. e.g.

$$A \times ABCDE \quad \text{and} \quad A \times (ABCDE)^2$$

i.e. $A^2BCDE \rightarrow (A^2BCDE)^2 = AB^2C^2D^2E^2$

and $A^3B^2C^2D^2E^2 \rightarrow B^2C^2D^2E^2 \rightarrow (B^2C^2D^2E^2)^2 \rightarrow BCDE$

$$AB \times ABCDE \rightarrow A^2B^2CDE \rightarrow ABC^2D^2E^2$$

$$AB \times (ABCDE)^2 \rightarrow A^3B^3C^2D^2E^2 \rightarrow (C^2D^2E^2)^2 \rightarrow CDE \quad .$$

From there, it can easily be seen that all main effects will be aliased with 3 - factor interactions and above, while 2 - factor interactions will be aliased with components of 3 - factor interactions and above. If it is assumed that the 3 - factor interactions and above are negligible, this design provides estimates of all main effects and all 2 - factor interactions, whilst the highest order interaction provides degrees of freedom for residual.

The design is constructed of those treatment combinations satisfying

$$\ell_1 + \ell_2 + \ell_3 + \ell_4 + \ell_5 \equiv 0, 1, 2 \pmod{3}$$

with the 81 treatments corresponding to the 0 equation giving a suitable 1/3 - replicate. If we wished to arrange this 1/3 - replicate in 9 blocks of 9 treatments, the necessary 8 d.f. to be confounded could come from the 4 components of interaction. If we confound ABC^2 and ADE^2 then we shall also be confounding

$$(A^2BC^2DE^2)^2 \rightarrow AB^2CD^2E \quad \text{and} \quad A^3BC^2D^2E^4 \rightarrow BC^2D^2E \ .$$

The aliases of these confounded components can now be obtained as before. Thus, the 81 treatments from the 1/3 - replicate are placed in 9 blocks so as to satisfy the 9 pairs of equations

$$\ell_1 + \ell_2 + 2\ell_3 \equiv 0, 1, 2 \pmod{3}$$

$$\ell_1 + \ell_4 + 2\ell_5 \equiv 0, 1, 2 \pmod{3} \ .$$

These ideas contained in this section form the basis of the theory for the proposed design given in the next chapter.

CHAPTER 3

PROPOSED DESIGN FOR ARRANGING A 1/9 x 3^7 FACTORIAL EXPERIMENT IN 9 BLOCKS OF 27

3.1 INTRODUCTION.

In this chapter, 7 factors which are for the time being labelled A, B, C, D, E, F and G are each explored at 3 levels, with 4 types of response measurements.

The chosen 1/9 - replicate of a 3^7 factorial experiment enables all main effects and all 2-factor interactions to be determined. The 1/9 - replicate provides 243 treatments which are divided into 9 blocks of 27 treatment combinations, each set of 27 treatment combinations providing one day's work which is accepteble to the Engineering Production Department as being practicable within the range of experiments to be done in one day. The proposed scheme is designed to provide the 27 treatment combinations for each day (block). The structure of aliasing and the form of analysis of variance are given.

3.2 THEORY BEHIND ARRANGEMENTS OF 1/9 x 3^7 FACTORIAL IN 9 BLOCKS OF 27.

By using the notation and rules introduced in Chapter 2, fractional replicates of experiments with several factors each at 3 levels can be constructed. For fractional replicates in 3^n factorial, we need to choose one defining contrast if we require a 1/3 - replicate and two defining contrasts if we require a 1/9 - replicate.

Some component(s) of higher order interaction(s) could then be used as defining contrast(s) and so confounded to give the required fractional replicate.

Suppose we then want to arrange the chosen $1/9 \times 3^7 = 243$ treatments in 9 blocks of 27, we require 8 d.f. to be confounded with blocks. To get these 8 d.f., we require 4 components of some higher order interactions each on 2 d.f. to be confounded with blocks. The general rule for confounding in a 3^n system is used to develop the proposed design. That is, if pairs of d.f. corresponding say to X and Y are confounded with blocks, then so are the pairs of d.f. corresponding to XY and XY^2.

Applying this rule and for the 8 d.f. required to be confounded with blocks we choose two higher order interactions $\underline{AB^2F^2G}$ and $\underline{BCDF}$ as confounded effects each on 2 d.f. and consequently, $AB^2F^2G \times BCDF = \underline{ACDG}$ and $AB^2F^2G \times (BCDF)^2 = \underline{ABC^2D^2F^2G}$ will also be confounded each on 2 d.f. giving us the required 8 d.f. to be confounded with blocks. Also to obtain the 1/9 - replicate of this factorial experiment, we need to choose 2 higher order factor interactions as defining contrasts. So we choose $\underline{ABCD^2E}$ and $\underline{CD^2E^2F^2G^2}$ as defining contrasts.[1]

Here too,

$$ABCD^2E \times CD^2E^2F^2G^2 = \underline{ABC^2DF^2G^2}$$

and

$$ABCD^2E \times (CD^2E^2F^2G^2)^2 = \underline{ABE^2FG}$$

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1. *The choice of defining contrasts and confounded effects is made in such a way as to avoid confounding "important" effects when aliasing structures are considered.*

will also be confounded with blocks; and the indices of the factorial effects are reduced modulo 3.

3.3 THE ALIASING STRUCTURES.

Because we have taken a fractional replicate instead of a full replicate, the problem of aliasing arises whereby factorial effects are unable to be determined individually but rather in sets. The factorial effects are aliased not only with the defining contrasts but also with the confounded effects as well. For example, consider the aliasing for the chosen defining contrasts and the confounded effects:

$$AB^2F^2G \times ABCD^2E = A^2CD^2EF^2G = \underline{AC^2DE^2FG^2}$$

$$AB^2F^2G \times (ABCD^2E)^2 = \underline{BC^2DE^2F^2G}$$

and

$$AB^2F^2G \times CD^2E^2F^2G^2 = \underline{AB^2CD^2E^2F}$$

$$AB^2F^2G \times (CD^2E^2F^2G^2)^2 = \underline{AB^2C^2DEG^2}$$

noting that here too the indices of the factorial effects are reduced modulo 3 and the convention of making the first digit raised to power of unity is adopted.

Similarly, each of the remaining 3 factorial effects confounded with blocks is aliased with a further 8 effect in the following way:

For BCDF we have

$$BCDF \times ABCD^2E = \underline{AB^2C^2EF}$$

$$BCDF \times (ABCD^2E)^2 = A^2D^2E^2F = \underline{ADEF^2}$$

$$BCDF \times CD^2E^2F^2G^2 = \underline{BC^2E^2G^2}$$

$$BCDF \times (CD^2E^2F^2G^2)^2 = \underline{BD^2EF^2G}$$

$$BCDF \times ABC^2DF^2G^2 = \underline{AB^2D^2G^2}$$

$$BCDF \times (ABC^2DF^2G^2)^2 = A^2C^2F^2G = \underline{ACFG^2}$$

$$BCDF \times ABE^2FG = \underline{AB^2CDE^2F^2G}$$

$$BCDF \times (ABE^2FG)^2 = A^2CDEG^2 = \underline{AC^2D^2E^2G}$$

ACDG will be aliased with

$$ACDG \times ABCD^2E = A^2BC^2EG = \underline{AB^2CE^2G^2}$$

$$ACDG \times (ABCD^2E)^2 = B^2D^2E^2G = \underline{BDEG^2}$$

$$ACDG \times CD^2E^2F^2G^2 = \underline{AC^2E^2F^2}$$

$$ACDG \times (CD^2E^2F^2G^2)^2 = \underline{AD^2EFG^2}$$

$$ACDG \times ABC^2DF^2G^2 = A^2BD^2F^2 = \underline{AB^2DF}$$

$$ACDG \times (ABC^2DF^2G^2)^2 = B^2C^2FG^2 = \underline{BCF^2G}$$

$$ACDG \times ABE^2FG = A^2BCDE^2FG^2 = \underline{AB^2C^2D^2EF^2G}$$

$$ACDG \times (ABE^2FG)^2 = B^2CDEF = \underline{BC^2D^2E^2F}$$

and $ABC^2D^2F^2G$ will be aliased with

$$ABC^2D^2F^2G \times ABCD^2E = A^2B^2DEF^2G = \underline{ABD^2E^2FG^2}$$

$$ABC^2D^2F^2G \times (ABCD^2E)^2 = \underline{CE^2F^2G}$$

$$ABC^2D^2F^2G \times CD^2E^2F^2G^2 = \underline{ABDE^2F}$$

$$ABC^2D^2F^2G \times (CD^2E^2F^2G^2)^2 = \underline{ABCEG^2}$$

$$ABC^2D^2F^2G \times ABC^2DF^2G^2 = A^2B^2CF = \underline{ABC^2F^2}$$

$$ABC^2D^2F^2G \times (ABC^2DF^2G^2)^2 = \underline{DG^2}$$

$$ABC^2D^2F^2G \times ABE^2FG = A^2B^2C^2D^2E^2G^2 = \underline{ABCDEG}$$

$$ABC^2D^2F^2G \times (ABE^2FG)^2 = C^2D^2EF = \underline{CDE^2F^2}$$

Other factorial effects have a similar aliasing structure.

For example the main effects will be aliased in the form:

$$A \times ABCD^2E = A^2BCD^2E = \underline{AB^2C^2DE^2}$$

$$A \times (ABCD^2E)^2 = B^2C^2DE^2 = \underline{BCD^2E}$$

$$A \times CD^2E^2F^2G^2 = \underline{ACD^2E^2F^2G^2}$$

$$A \times (CD^2E^2F^2G^2)^2 = \underline{AC^2DEFG}$$

$$A \times ABC^2DF^2G^2 = A^2BC^2DF^2G^2 = \underline{AB^2CD^2FG}$$

$$A \times (ABC^2DF^2G^2)^2 = B^2CD^2FG = \underline{BC^2DF^2G^2}$$

$$A \times ABE^2FG = A^2BE^2FG = \underline{AB^2EF^2G^2}$$

$$A \times (ABE^2FG)^2 = B^2EF^2G^2 = \underline{BE^2FG}$$

From this we can see that all the main effects will be aliased with 4-factor interactions and above.

Now consider the aliasing of some 2-factor interactions:

$$AB \times ABCD^2E = A^2B^2CD^2E = \underline{ABC^2DE^2}$$

$$AB \times (ABCD^2E)^2 = C^2DE^2 = \underline{CD^2E}$$

$$AB \times CD^2E^2F^2G^2 = \underline{ABCD^2E^2F^2G^2}$$

$$AB \times (CD^2E^2F^2G^2)^2 = \underline{ABC^2DEFG}$$

$$AB \times ABC^2DF^2G^2 = A^2B^2C^2DF^2G^2 = \underline{ABCD^2FG}$$

$$AB \times (ABC^2DF^2G^2)^2 = \underline{CD^2FG}$$

$$AB \times ABE^2FG = A^2B^2E^2FG = \underline{ABEF^2G^2}$$

$$AB \times (ABE^2FG)^2 = \underline{EF^2G^2}$$

In this manner, all 2-factor interactions will be aliased with 3-factor interactions and above.

The aliasing of some 3-factor interactions is given by the form:

$$ABC \times ABCD^2E = A^2B^2C^2D^2E = \underline{ABCDE^2}$$

$$ABC \times (ABCD^2E)^2 = \underline{DE^2}$$

$$ABC \times CD^2E^2F^2G^2 = \underline{ABC^2D^2E^2F^2G^2}$$

$$ABC \times (CD^2E^2F^2G^2)^2 = \underline{ABDEFG}$$

$$ABC \times ABC^2DF^2G^2 = A^2B^2DF^2G^2 = \underline{ABD^2FG}$$

$$ABC \times (ABC^2DF^2G^2)^2 = C^2D^2FG = \underline{CDF^2G^2}$$

$$ABC \times ABE^2FG = A^2B^2CE^2FG = \underline{ABC^2EF^2G^2}$$

$$ABC \times (ABE^2FG)^2 = \underline{CEF^2G^2}\ .^{1}$$

Not full aliasing structures are given since there are over

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1. *The underlined factor interactions are those aliased with the given factorial effects.*

500 of these with similar forms as the ones given making it unnecessary to give all of them. So with this choice of confounded effects and defining contrasts together with the above aliasing structures, if it is assumed that the 3-factor interactions and above are negligible, then the proposed design will enable all the main effects and all the 2-factor interactions to be determined.[1] The 3-factor and higher order interactions will then be used as error/residual for comparisons in the final analysis of variance.

3.4 USE OF CONGRUENCE METHOD FOR BLOCKING AND SELECTION OF THE 1/9 - REPLICATE.

To obtain the blocks of treatments, consider:

(i) The chosen defining contrasts $ABCD^2E$, $CD^2E^2F^2G^2$.

(ii) The chosen confounded effects AB^2F^2G, BCDF.

We then solve the 4 congruence equations for the linear combinations of levels of treatment effects corresponding to the 4 chosen effects above by

$$\ell_1 + \ell_2 + \ell_3 + 2\ell_4 + \ell_5 \equiv 0, 1, 2 \pmod 3$$

$$\ell_3 + 2\ell_4 + 2\ell_5 + 2\ell_6 + 2\ell_7 \equiv 0, 1, 2 \pmod 3$$

$$\ell_1 + 2\ell_2 + 2\ell_6 + \ell_7 \equiv 0, 1, 2 \pmod 3$$

$$\ell_2 + \ell_3 + \ell_4 + \ell_6 \equiv 0, 1, 2 \pmod 3$$

where $\ell_1, \ell_2, \ldots, \ell_7$ are the levels corresponding to factors A, B, C, D, E, F and G respectively, and the coefficients of the

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1. *The Engineering Production Department is interested only in the main effects and two factor interactions, so the choice of defining contrasts and confounded effects is made in such a way that we do not loose any information on these.*

above linear treatment combinations correspond to the factor powers or indices of the chosen effects; that is the first two equations correspond to the linear combinations of the levels of the two chosen defining contrasts and the last two equations correspond to those for the two chosen confounded effects.

The 3 levels of each factor are given by 0, 1 and 2. The $3 \times 3 \times 3 \times 3$ linear combinations of congruence equations above give 81 sets of 27, from which we select 9. The selection of the 9 blocks is given by the solutions to

$$\ell_1 + \ell_2 + \ell_3 + 2\ell_4 + \ell_5 \equiv 0 \pmod{3}$$

$$\ell_3 + 2\ell_4 + 2\ell_5 + 2\ell_6 + 2\ell_7 \equiv 0 \pmod{3}$$

$$\ell_1 + 2\ell_2 + 2\ell_6 + \ell_7 \equiv 0, 1, 2 \pmod{3}$$

$$\ell_2 + \ell_3 + \ell_4 + \ell_6 \equiv 0, 1, 2 \pmod{3}$$

giving the 9 blocks of 27 treatment combinations each, with the first pair of equations giving the 1/9 - replicate selected.

The Principal Block (Block 1) contains the treatment 0 0 0 0 0 0 0, that is all the 7 factors set at their first level, so that solving the congruence equations with 0 on the right hand side (r.h.s.) leads to the 27 treatment combinations in the Principal Block. Once the Principal Block is determined, the other blocks are then obtained by finding a treatment combination not already used which belongs to the same 1/9 - replicate and "adding" (mod.3) to the treatments in the Principal Block. At the bottom of each day's experiments there are the set of 4

congruence equations which each of the 27 treatment combinations for that day satisfy in order to obtain the complete orthogonal design that has been proposed.

3.5 THE DESIGN MATRIX.

The design matrix showing combinations of levels for the 7 factors is given below for the first 3 days of the 9-day scheme.[1]

DAY 1 : (PRINCIPAL BLOCK)

Experiment	A ℓ_1	B ℓ_2	C ℓ_3	D ℓ_4	E ℓ_5	F ℓ_6	G ℓ_7
1	0	0	0	0	0	0	0
2	0	1	1	2	0	2	0
3	0	2	2	1	0	1	0
4	1	0	1	0	1	2	1
5	2	0	2	0	2	1	2
6	1	1	2	2	1	1	1
7	1	2	0	1	1	0	1
8	2	1	0	2	2	0	2
9	2	2	1	1	2	2	2
10	1	0	1	1	2	1	0
11	2	0	2	2	1	2	0
12	1	1	2	0	2	0	0
13	1	2	0	2	2	2	0
14	2	0	2	1	0	0	1
15	0	0	0	1	1	2	2
16	2	1	0	0	0	2	1
17	2	2	1	2	0	1	1
18	0	1	1	0	1	1	2
19	0	2	2	2	1	0	2
20	2	1	0	1	1	1	0
21	2	2	1	0	1	0	0
22	0	0	0	2	2	1	1
23	1	0	1	2	0	0	2
24	0	1	1	1	2	0	1
25	0	2	2	0	2	2	1
26	1	1	2	1	0	2	2
27	1	2	0	0	0	1	2

$$\ell_1 + \ell_2 + \ell_3 + 2\ell_4 + \ell_5 \equiv 0 \pmod{3}$$

$$\ell_3 + 2\ell_4 + 2\ell_5 + 2\ell_6 + 2\ell_7 \equiv 0 \pmod{3}$$

$$\ell_1 + 2\ell_2 + 2\ell_6 + \ell_7 \equiv 0 \pmod{3}$$

$$\ell_2 + \ell_3 + \ell_4 + \ell_6 \equiv 0 \pmod{3}$$

DAY 2 : (BLOCK 2)

Experiment	A ℓ_1	B ℓ_2	C ℓ_3	D ℓ_4	E ℓ_5	F ℓ_6	G ℓ_7
1	0	0	2	2	0	0	0
2	0	1	0	1	0	2	0
3	0	2	1	0	0	1	0
4	1	0	0	2	1	2	1
5	2	0	1	2	2	1	2
6	1	1	1	1	1	1	1
7	1	2	2	0	1	0	1
8	2	1	2	1	2	0	2
9	2	2	0	0	2	2	2
10	1	0	0	0	2	1	0
11	2	0	1	1	1	2	0
12	1	1	1	2	2	0	0
13	1	2	2	1	2	2	0
14	2	0	1	0	0	0	1
15	0	0	2	0	1	2	2
16	2	1	2	2	0	2	1
17	2	2	0	1	0	1	1
18	0	1	0	2	1	1	2
19	0	2	1	1	1	0	2
20	2	1	2	0	1	1	0
21	2	2	0	2	1	0	0
22	0	0	2	1	2	1	1
23	1	0	0	1	0	0	2
24	0	1	0	0	2	0	1
25	0	2	1	2	2	2	1
26	1	1	1	0	0	2	2
27	1	2	2	2	0	1	2

$$\ell_1 + \ell_2 + \ell_3 + 2\ell_4 + \ell_5 \equiv 0 \pmod 3$$

$$\ell_3 + 2\ell_4 + 2\ell_5 + 2\ell_6 + 2\ell_7 \equiv 0 \pmod 3$$

$$\ell_1 + 2\ell_2 + 2\ell_6 + \ell_7 \equiv 0 \pmod 3$$

$$\ell_2 + \ell_3 + \ell_4 + \ell_6 \equiv 1 \pmod 3$$

DAY 3 : (BLOCK 3)

Experiment	A ℓ_1	B ℓ_2	C ℓ_3	D ℓ_4	E ℓ_5	F ℓ_6	G ℓ_7
1	0	0	1	1	0	0	0
2	0	1	2	0	0	2	0
3	0	2	0	2	0	1	0
4	1	0	2	1	1	2	1
5	2	0	0	1	2	1	2
6	1	1	0	0	1	1	1
7	1	2	1	2	1	0	1
8	2	1	1	0	2	0	2
9	2	2	2	2	2	2	2
10	1	0	2	2	2	1	0
11	2	0	0	0	1	2	0
12	1	1	0	1	2	0	0
13	1	2	1	0	2	2	0
14	2	0	0	2	0	0	1
15	0	0	1	2	1	2	2
16	2	1	1	1	0	2	1
17	2	2	2	0	0	1	1
18	0	1	2	1	1	1	2
19	0	2	0	0	1	0	2
20	2	1	1	2	1	1	0
21	2	2	2	1	1	0	0
22	0	0	1	0	2	1	1
23	1	0	2	0	0	0	2
24	0	1	2	2	2	0	1
25	0	2	0	1	2	2	1
26	1	1	0	2	0	2	2
27	1	2	1	1	0	1	2

$$\ell_1 + \ell_2 + \ell_3 + 2\ell_4 + \ell_5 \equiv 0 \pmod 3$$

$$\ell_3 + 2\ell_4 + 2\ell_5 + 2\ell_6 + 2\ell_7 \equiv 0 \pmod 3$$

$$\ell_1 + 2\ell_2 + 2\ell_6 + \ell_7 \equiv 0 \pmod 3$$

$$\ell_2 + \ell_3 + \ell_4 + \ell_6 \equiv 2 \pmod 3$$

3.6 THE FORM OF ANALYSIS OF VARIANCE.

For the $1/9 \times 3^7$ experiment arranged in 9 blocks of 27, and for the chosen confounded effects, the form of analysis of variance is:

Source of Variation		s.s.	d.f.	M.S.
Between Blocks (and confounded effects)			8	
7 main effects	A		2	
	B		2	
	C		2	
	.		.	
	.		.	
	.		.	
	.		.	
	.		.	
	G		2	
$7C_2 = 21 \rightarrow$ 2-factor interactions	AB		4	
	AC		4	
	.		.	
	.		.	
	.		.	
	FG		4	
Error/Residual (available 3-factor interactions and above)			136	
TOTAL			242	

It is proposed that this fractional replicate be repeated, to test for repeatability and to check assumptions made about the 3-factor interactions and above. In order to avoid systematic combinations arising as a result of the way in which the entire design matrix was constructed, it is suggested that treatment combinations within each block be randomized in such a way that the order of experiments within each block is taken at random. This randomization is also advisable as two 1/9 - replicates are proposed and one such randomization is given in Appendix A2 using random number tables.

CHAPTER 4

ANALYSIS, CONCLUSIONS AND SUGGESTIONS

4.1 INTRODUCTION.

In this chapter, analysis of variance is carried out on simulated data and some attention is given to the problem of modelling the response measurements. The reason for simulating data being that the Engineering Production Department which had planned to do the actual welding to obtain data for analysis was unable to do so as a result of machine breakdown, making it impossible to get the actual data before December 1982. It was therefore decided that data simulation was necessary to show the steps which would have been taken for the analysis if the actual data had been available. The simulated data was also used to explain the facilities provided by the GLIM package for such analysis.

The analysis was carried out on each of the four responses separately and the computer output of such analysis using the GLIM package has been provided in the chapter for the first response y_1. Some conclusions were drawn based on the outcome of the analysis on the simulated data. A brief discussion on modelling using the results obtained from ANOVA is also given. Some suggestions on the sort of transformation that would be required if the residuals were found to be correlated is made. Also, in this chapter, the actual factors U, I, V, T, G, A and E mentioned in Chapter 1 were used to replace A, B, C, D, E, F and G respectively.

4.2 SIMULATION AND USE OF THE GLIM PACKAGE.

The GLIM package has a facility for data simulation and this facility was used to simulate data for each response separately. The simulation was based on the guide lines obtained from the Engineering Production Department. For example, the 3 levels of each factor were measured accurately. These measured levels, low, middle and high for each of the factors are given in Table 4.1.

Table 4.1.

Factors	Unit	Low	Middle	High
Angle of Travel (A)	Degree	50	60	70
Current (I)	A	300	335	370
Electrode Stickout (E)	mm	10	20	30
Root Face Thickness (T)	mm	0.5	1.5	2.5
Root Gap (G)	mm	0	0.7	1.5
Speed of Travel (U)	m/min	0.3	0.5	0.7
Voltage (V)	V	20	25	30

For the use of the GLIM package in simulation, these levels of the factors were each coded about zero giving -1, 0, 1 for low, middle and high respectively.

The expected ranges of each of the four responses based upon preliminary experimentation were given as shown in Table 4.2.

Table 4.2.

Responses		Unit	Range
Effective Bead Width	(y_4)	mm	4.0 → 12.0
Maximum Penetration	(y_3)	mm	1.0 → 6.0
Under Bead Minimum Leg Length	(y_1)	mm	0.5 → 8.0
Under Bead Throat Thickness	(y_2)	mm	0.5 → 6.0

To simulate data for each response within the given ranges, linear models were constructed using the information given in Chapter 1 about the effect of each factor on each response. Those factors believed to have significant effects on the responses were included in the model used to simulate data for that response, and some interactions were also involved in such models. The choice of interactions included in each model was made in such a way that for each model, the value of the response at the lower level of all the factors in the model will give the lower limit of the range and the value considered at the upper level will give the upper limit of the range for that particular response. The constants in the model being the average values of the given ranges. The choice of coefficients of the factors in each model were based on the available information given as to the believed relative importance of the factors.

The model used to simulate data for each response is shown just above the simulated data for each response on the computer output provided.[1] From these outputs, the sums of squares are

---

1. *The GLIM output for response y_1 is given in this chapter as an example of steps taken to construct the ANOVA table, while outputs for the remaining 3 responses are given in Appendix A3.*

computed by differencing the deviances. For example the sums of squares (S.S.) for the main effects are computed by subtracting the resulting deviance when they are fitted from the total deviance. For instance S.S. for U = Total deviance - Deviance when U is fitted.

Similarly, S.S. for an interaction say UV is obtained by S.S. for UV = Total deviance - Deviance when U.V is fitted - S.S. for U - S.S. for V. So that all the sums of squares of the factorial effects and their corresponding degrees of freedom are computed by this sort of differencing. Similar models are used for each response with differences only in the coefficients of the factors included in each model.

4.3 THE COMPUTER OUTPUT FOR RESPONSE (y_1): UNDER BEAD MINIMUM LEG LENGTH.

```
PRIMOS Version 18.3b/LUCC
SYSB   (6) LOGGED IN 17 SEP 1382  11:44
USERS 12   RESPONSE RATING *****
OK, A 'MA AWEE>AAI
OK, OPEN NEWDSIGN 4,3
OK, GLIM
 GLIM 3.12 (C)1977 ROYAL STATISTICAL SOCIETY, LONDON


> $C      THE ANALYSIS OF VARIANCE ON SIMULATED DATA FOR THE RESPONSE
>         UNDER BEAD MINIMUM LEG LENGTH FOR THE GIVEN RANGE (0.5 - 8.
>



  $UNITS 243
> $DAT U$DIN 8$
> $DAT I$DIN 8$
> $DAT V$DIN 8$
> $DAT T$DIN 8$
> $DAT G$DIN 8$
> $DAT A$DIN 8$
> $DAT E$DIN 8$
> $FAC U 3 I 3 V 3 T 3 G 3 A 3 E 3 BLOCKS 9
> $CAL BLOCKS=%GL(9,27)
> $CAL EE=%ND(%LR(0))*.1
> $CAL Y1=EE+(U-2)*.5+(G-2)*1+(T-2)*.5+(U-2)*(V-2)*.5+
>           (I-2)*(T-2)*.5+4.25+(G-2)*(A-2)*.75


> $PR Y1$
    4.022    2.549    2.142    4.163    6.192    4.686    4.411    5.035
    6.398    5.343    5.275    3.945    7.001    4.993    4.276    2.118
    4.850    3.093    4.380    4.234    3.975    5.146    4.000    4.104
    4.091    2.311    2.219    3.099    2.597    1.934    4.301    5.685
    4.314    3.242    5.635    4.907    5.266    4.688    5.052    6.053
    4.583    3.226    3.835    3.071    4.727    3.781    4.698    5.338
    4.247    3.822    3.934    6.422    2.065    4.383    3.664   0.8741
    4.362    4.415    5.109    3.767    5.210    4.536    7.975    5.191
    4.034    4.497    5.228    3.901    3.686    2.950    3.230    3.303
    3.246    4.980    5.282    4.764    3.797    3.994    5.938    3.104
    3.246    3.815    5.168    4.350    5.450    2.473    6.560    5.983
    3.508    2.626    4.611    4.689    2.490    3.353    3.229    5.438
    3.205    3.590    3.535    5.929    4.513    5.319    4.397    5.268
    2.529    3.219    5.356    4.192    4.176    3.166    4.109    6.035
    4.322    4.780    5.561    3.650    4.018    3.323    6.966    3.945
    1.594    5.216    6.095    4.144    3.854    4.810    2.402    5.671
    5.094    3.300    4.176    3.774    1.419    4.584    4.282    4.808
    4.796    4.985    3.352    4.773    3.549    3.348    5.303    4.812
    4.370    5.418    3.192    4.253    6.216    4.281    3.497    6.498
    1.022    2.284    2.423    3.664    4.711    5.652    3.846    3.300
    4.543    6.123    2.735    1.461    3.208    3.284    4.048    4.336
    4.308    4.790    6.890    5.267    5.271    4.339    6.427    3.968
    3.827    3.122    4.760    3.315    4.895    4.781    4.756    5.938
    4.915    3.521    5.419    2.302    2.797    1.763    4.879    2.034
    3.814    6.113    5.316    4.143    7.138    4.504    6.058    5.242
    4.290    4.523    3.670    3.321    3.968    2.627    3.289    3.672
    5.247    4.290    5.404    3.795    5.188    3.999    3.995    2.502
    4.212    3.235    4.283    4.513    3.425    5.506    6.435    2.765
    5.482    2.453    5.546    3.082    3.032    5.278    4.327    4.110
    3.739    5.333    3.249    6.637    5.761    2.511    4.269    3.120
    3.158    4.844    4.264
>
```

```
  $YVAR V1
> $ACC 8
>

  $FIT$
  CYCLE  DEVIANCE      DF
    1     357.6       242

> $PR %DV$
    357.61444
> $FIT U$
  CYCLE  DEVIANCE      DF
    1     317.2       240

> $PR %DV$
    317.19537
> $FIT I$
  CYCLE  DEVIANCE      DF
    1     357.6       240

> $PR %DV$
    357.57141
> $FIT V$
  CYCLE  DEVIANCE      DF
    1     357.6       240

> $PR %DV$
    357.60205
> $FIT T$
  CYCLE  DEVIANCE      DF
    1     318.6       240

> $PR %DV$
    318.56580
> $FIT G$
  CYCLE  DEVIANCE      DF
    1     196.3       240

> $PR %DV$
    196.26984
> $FIT A$
  CYCLE  DEVIANCE      DF
    1     357.6       240

> $PR %DV$
    357.60925
> $FIT E$
  CYCLE  DEVIANCE      DF
    1     357.6       240

> $PR %DV$
    357.61432
>
```

```
  $FIT U. I$
  CYCLE   DEVIANCE      DF
     1     317.1       234

> $PR %DV$
    317.13995
> $FIT U. V$
  CYCLE   DEVIANCE      DF
     1     289.8       234

> $PR %DV$
    289.82031
> $FIT U. T$
  CYCLE   DEVIANCE      DF
     1     278.1       234

> $PR %DV$
    278.10809
> $FIT U. G$
  CYCLE   DEVIANCE      DF
     1     155.8       234

> $PR %DV$
    155.77155
> $FIT U. A$
  CYCLE   DEVIANCE      DF
     1     317.1       234

> $PR %DV$
    317.13904
> $FIT U. E$
  CYCLE   DEVIANCE      DF
     1     317.2       234

> $PR %DV$
    317.16711
>

  $FIT I. V$
  CYCLE   DEVIANCE      DF
     1     357.5       234

> $PR %DV$
    357.46014
> $FIT I. T$
  CYCLE   DEVIANCE      DF
     1     291.9       234

> $PR %DV$
    291.92993
> $FIT I. G$
  CYCLE   DEVIANCE      DF
     1     196.2       234

> $PR %DV$
    196.18985
```

```
> $FIT I.A$
 CYCLE  DEVIANCE      DF
    1    357.5       234

> $PR %DV$
   357.53888
> $FIT I.E$
 CYCLE  DEVIANCE      DF
    1    357.5       234

> $PR %DV$
   357.54224
> $FIT V.T$
 CYCLE  DEVIANCE      DF
    1    318.4       234

> $PR %DV$
   318.42926
> $FIT V.G$
 CYCLE  DEVIANCE      DF
    1    196.2       234

> $PR %DV$
   196.21387
> $FIT V.A$
 CYCLE  DEVIANCE      DF
    1    357.6       234

> $PR %DV$
   357.57086
> $FIT V.E$
 CYCLE  DEVIANCE      DF
    1    357.6       234

> $PR %DV$
   357.56256
>

 $FIT T.G$
 CYCLE  DEVIANCE      DF
    1    157.2       234

> $PR %DV$
   157.17249
> $FIT T.A$
 CYCLE  DEVIANCE      DF
    1    318.5       234

> $PR %DV$
   318.49097
> $FIT T.E$
 CYCLE  DEVIANCE      DF
    1    318.5       234

> $PR %DV$
   318.52539
```



```
> $FIT G. A$
  CYCLE   DEVIANCE       DF
     1     135. 6        234

> $PR %DV$
    135. 56229
> $FIT G. E$
  CYCLE   DEVIANCE       DF
     1     196. 2        234

> $PR %DV$
    196. 24710
> $FIT A. E$
  CYCLE   DEVIANCE       DF
     1     357. 6        234

> $PR %DV$
    357. 58636
> $FIT BLOCKS$
  CYCLE   DEVIANCE       DF
     1     357. 6        234

> $PR %DV$
    357. 59698
> $FIT U+I+V+T+G+A+E+U. I+U. V+U. T+U. G+U. A+U. E+I. V+I. T+I. G+I. A+I. E+
>       V. T+V. G+V. A+V. E+T. G+T. A+T. E+G. A+G. E+A. E$
  CYCLE   DEVIANCE       DF
     1     1. 245        144

> $PR %DV$
    1. 2449601
> $STOP$
OK, LO
AAI7543  (6) LOGGED OUT 17 SEP 1982  12:20
TIME USED=  0:36   2:54   0:04
```

4.4 ANALYSIS OF VARIANCE FOR EACH OF THE RESPONSES.

(i) UNDER BEAD MINIMUM LEG LENGTH (y_1).

Source of Variation	S.S.	d.f.	M.S.	
Between Blocks	0.01746	8	0.0021825	
U	40.41907	2	20.209535	*
I	0.04303	2	0.021515	
V	0.01239	2	0.006195	
T	39.04864	2	19.52432	
G	161.3446	2	80.6723	**
A	0.00519	2	0.002595	
E	0.00012	2	0.00006	
UI	0.01239	4	0.0030975	
UV	27.36267	4	6.8406675	*
UT	0.03864	4	0.00966	
UG	0.07922	4	0.019805	
UA	0.05114	4	0.012785	
UE	0.02814	4	0.007035	
IV	0.09888	4	0.02472	
IT	26.57284	4	6.64821	*
IG	0.03696	4	0.00924	
IA	0.02734	4	0.006835	
IE	0.02905	4	0.0072625	
VT	0.12415	4	0.0310375	
VG	0.04358	4	0.010895	
VA	0.02600	4	0.0065	
VE	0.03937	4	0.0098425	
TG	0.04871	4	0.0121775	
TA	0.06964	4	0.01741	
TE	0.04029	4	0.0100725	
GA	60.70236	4	15.17559	*
GE	0.02262	4	0.005655	
AE	0.02277	4	0.0056925	
Residual	1.22718	136	0.0090234	
TOTAL	357.61444	242		

(ii) UNDER BEAD THROAT THICKNESS (y_2).

Source of Variation	S.S.	d.f.	M.S.	
Between Blocks	0.01752	8	0.00219	
U	40.41907	2	20.209535	*
I	0.04309	2	0.021545	
V	0.01245	2	0.006225	
T	39.04871	2	19.524355	*
G	161.34465	2	80.672325	**
A	0.00519	2	0.002595	
E	0.00019	2	0.000095	
UI	0.01233	4	0.0030825	
UV	6.95462	4	1.738655	*
UT	0.03860	4	0.00965	
UG	0.079201	4	0.0198003	
UA	0.05115	4	0.0127875	
UE	0.02813	4	0.0070325	
IV	0.09882	4	0.024705	
IT	6.56055	4	1.6401375	*
IG	0.03690	4	0.009225	
IA	0.02728	4	0.00682	
IE	0.02899	4	0.0072475	
VT	0.12405	4	0.0310125	
VG	0.04351	4	0.0108775	
VA	0.02600	4	0.0065	
VE	0.03930	4	0.009825	
TG	0.048636	4	0.012159	
TA	0.06961	4	0.0174025	
TE	0.04021	4	0.0100525	
GA	6.747224	4	1.686806	*
GE	0.02252	4	0.00563	
AE	0.02276	4	0.00569	
Residual	1.227919	136	0.0090288	
TOTAL	263.21918	242		

(iii) MAXIMUM PENETRATION (y_3).

Source of Variation	S.S.	d.f.	M.S.	
Between Blocks	0.01745	8	0.0021813	
U	90.9985	2	45.49925	*
I	0.04303	2	0.021515	
V	159.6177	2	79.80885	**
T	0.01691	2	0.008455	
G	0.00311	2	0.001555	
A	0.00519	2	0.002595	
E	0.00012	2	0.00006	
UI	0.01239	4	0.0030975	
UV	0.046698	4	0.0116745	
UT	6.81137	4	1.7028425	*
UG	0.0792	4	0.0198	
UA	0.05115	4	0.0127875	
UE	0.02817	4	0.0070425	
IV	0.09891	4	0.0247275	
IT	0.0285	4	0.007125	
IG	0.03693	4	0.0092325	
IA	6.36108	4	1.59027	
IE	0.02905	4	0.0072625	
VT	0.12415	4	0.0310375	
VG	7.44984	4	1.86246	*
VA	0.02600	4	0.0065	
VE	0.03939	4	0.0098475	
TG	0.0487	4	0.012175	
TA	0.06963	4	0.0174075	
TE	0.04034	4	0.010085	
GA	0.01965	4	0.0049125	
GE	0.02259	4	0.0056475	
AE	0.02283	4	0.0057075	
Residual	1.227092	136	0.0090227	
TOTAL	273.37567	242		

(iv) EFFECTIVE BEAD WIDTH (y_4)

Source of Variation	S.S.	d.f.	M.S.	
Between Blocks	0.01746	8	0.0021825	
U	90.99854	2	45.49927	*
I	0.04303	2	0.021515	
V	0.01239	2	0.006195	
T	0.01691	2	0.008455	
G	252.30496	2	126.15246	**
A	0.00519	2	0.002595	
E	0.00012	2	0.00006	
UI	0.01239	4	0.0030975	
UV	0.04663	4	0.0116575	
UT	0.03863	4	0.0096575	
UG	0.07919	4	0.0197975	
UA	27.08837	4	6.7720925	*
UE	0.02814	4	0.007035	
IV	0.09894	4	0.024735	
IT	60.1250	4	15.03125	*
IG	0.03696	4	0.00924	
IA	0.02734	4	0.006835	
IE	0.02906	4	0.007265	
VT	0.12414	4	0.031035	
VG	62.76245	4	15.690613	*
VA	0.02600	4	0.0065	
VE	0.03937	4	0.0098425	
TG	0.04871	4	0.0121775	
TA	0.06964	4	0.01741	
TE	0.04028	4	0.01007	
GA	0.01965	4	0.0049125	
GE	0.02262	4	0.005655	
AE	0.02277	4	0.0056925	
Residual	1.22723	136	0.0090238	
TOTAL	495.41211	242		

4.5 OBSERVATIONS FROM THE ANALYSIS OF VARIANCE.

By inspection from the ANOVA tables, those effects marked (*) are those with significant influence on the respective responses and those marked with double asterisks are those with yet greater influence on the corresponding responses. In the actual data, these effects would then be used in the construction of individual models for the responses. But for the simulated data, this was expected because those significant effects correspond exactly to the effects we fed in the models for data simulation. Also, those models with the same coefficient for some factors tend to give the same results for such factors in the ANOVA, and this too is expected since similar models were used in each case with differences only in the coefficients of certain factors.

This however, would not be the case if actual data were available in which case the outcome of results from ANOVA would be useful in model construction. The results for blocks also tend to be almost the same in all the analysis showing no significant blocking effect in each case. This too is not surprising since blocks were not specifically included in the models used for simulation. In actual data, the test for the significance of blocking will be necessary in order to check for any day to day variation and their effects on weld penetration. The residuals, as expected produce almost the same results in all the four responses as well. This is due to the fact that the same standard error was used in each model for simulation. The outcome of the results obtained from ANOVA if actual data had been used would be important toofor comparisons and a check for correlations amongst

them and a suggestion on some form of transformation is indicated in the conclusion of this chapter.

4.6 A BRIEF DISCUSSION ON MODELLING.

If actual data from welding were available, the outcome of the results from ANOVA with respect to factors found to have significant influence on weld penetration would then be put forward for modelling.

In this case of using simulated data, there has been no need to write out any models for the individual responses since these would correspond exactly to the ones put forward for simulation.

The GLIM package, though not necessarily the best to be used for ANOVA, was designed principally for the purposes of model fitting and would prove very useful in this role in the situation when actual data was available. Just as an illustration on how useful this package would be for the purpose of fitting generalised linear models, consider those factors found significant from the ANOVA for the response y_1. After choosing these significant effects, we are interested in obtaining their coefficients in the fitted linear model. For this response, the fitted model would be of the form

$$y_1 = a + b_{11}U + b_{12}T + b_{13}G + b_{14}UV + b_{15}IT + b_{16}GA$$

corresponding to those factors found from ANOVA to be signifcant. The coefficients b_{11}, b_{12}, b_{13}, b_{14}, b_{15} and b_{16} obtained from the estimates shown for each of the corresponding factors in the

```
 GLIM 3.12 (C)1977 ROYAL STATISTICAL SOCIETY, LONDON


> $C      AN EXAMPLE ON MODELLING USING SIMULATED DATA FOR THE
>         RESPONSE Y1: UNDERBEAD MINIMUM LEG LENGTH.
>

  $UNITS 243
> $DAT U$DIN 8$
> $DAT I$DIN 8$
> $DAT V$DIN 8$
> $DAT T$DIN 8$
> $DAT G$DIN 8$
> $DAT A$DIN 8$
> $DAT E$DIN 8$
> $CAL EE=%ND(%LR(0))*.1
> $CAL Y1=EE+(U-2)*.5+(G-2)*1+(T-2)*.5+(U-2)*(V-2)*.5+
>              (I-2)*(T-2)*.5+4.25+(G-2)*(A-2)*.75


> $PR Y1$
   4.022   2.549   2.142   4.163   6.192   4.686   4.411   5.035
   6.398   5.343   5.275   3.945   7.001   4.993   4.276   2.118
   4.850   3.093   4.380   4.234   3.975   5.146   4.000   4.104
   4.091   2.311   2.219   3.099   2.597   1.934   4.301   5.685
   4.314   3.242   5.635   4.907   5.266   4.688   5.052   6.053
   4.583   3.226   3.835   3.071   4.727   3.781   4.698   5.338
   4.247   3.822   3.934   6.422   2.065   4.383   3.664   0.8741
   4.362   4.415   5.109   3.767   5.210   4.536   7.975   5.191
   4.034   4.497   5.228   3.901   3.686   2.950   3.230   3.303
   3.246   4.980   5.282   4.764   3.797   3.994   5.938   3.104
   3.246   3.815   5.168   4.350   5.450   2.473   6.560   5.983
   3.508   2.626   4.611   4.689   2.490   3.353   3.229   5.438
   3.205   3.590   3.535   5.929   4.513   5.319   4.397   5.268
   2.529   3.219   5.356   4.192   4.176   3.166   4.109   6.035
   4.322   4.780   5.561   3.650   4.018   3.323   6.966   3.945
   1.594   5.216   6.095   4.144   3.854   4.810   2.402   5.671
   5.094   3.300   4.176   3.774   1.419   4.584   4.282   4.808
   4.796   4.985   3.352   4.773   3.549   3.348   5.303   4.812
   4.370   5.418   3.192   4.253   6.216   4.281   3.497   6.498
   1.022   2.284   2.423   3.664   4.711   5.652   3.846   3.380
   4.543   6.123   2.735   1.461   3.208   3.284   4.048   4.336
   4.308   4.790   6.890   5.267   5.271   4.339   6.427   3.968
   3.827   3.122   4.760   3.315   4.895   4.781   4.756   5.938
   4.915   3.521   5.419   2.302   2.797   1.763   4.879   2.034
   3.814   6.113   5.316   4.143   7.138   4.504   6.058   5.242
   4.290   4.523   3.670   3.321   3.968   2.627   3.289   3.672
   5.247   4.290   5.404   3.795   5.188   3.999   3.995   2.582
   4.212   3.235   4.283   4.513   3.425   5.506   6.435   2.765
   5.482   2.453   5.546   3.082   3.032   5.278   4.327   4.110
   3.739   5.333   3.249   6.637   5.761   2.511   4.269   3.120
   3.150   4.844   4.264
>
```

```
>
  $YVAR Y1
>
  $CAL UV=(U-2)*(V-2)
> $CAL IT=(I-2)*(T-2)
> $CAL GA=(G-2)*(A-2)
>
  $FIT U+T+G+UV+IT+GA$
  CYCLE  DEVIANCE      DF
    1     2.252       236

>$DIS E$
        ESTIMATE       S.E.        PARAMETER
     1  0.2720      0.2732E-01   %GM
     2  0.4994      0.7676E-02   U
     3  0.4909      0.7676E-02   T
     4  0.9980      0.7676E-02   G
     5  0.5029      0.9401E-02   UV
     6  0.4960      0.9401E-02   IT
     7  0.7496      0.9401E-02   GA
   SCALE PARAMETER TAKEN AS   0.9544E-02
>
  $STOP$
OK. LOGOUT
AA17543  (5) LOGGED OUT 22 SEP 1982  13:22
TIME USED=  0:13   0:15   0:11
```

computer output below are very similar to the coefficients of the corresponding effects fed in the model for simulation. In the actual case when the model is not known beforehand, these would then be used as estimates of the coefficients of those effect found from ANOVA to be significant for fitting the model for the response being considered.

4.7 CONCLUSIONS AND SUGGESTIONS.

The same standard error was used in each model for data simulation and not surprisingly, the results for the residuals in ANOVA tend to be almost the same for each response. This, however, may not be the case if actual data were available.

In the case of data obtained from actual welding test pieces, the residuals obtained from model fitting will be examined as part of the process of assessing the adequacy of the fit. For a satisfactory model, the residuals should be independently and normally distributed with a constant variance for each response. If the residuals show evidence of correlation, then it is possible that some factor omitted from the model is responsible, and we might consider whether guidance on this can be provided by suitable transformation. For example, a transformation in the direction of maximum variance could be made using Principal Components Analysis. Statistical packages other than GLIM are available for dealing with situations involving Principal Components Analysis.

Other people who have been working on the same sort of Analysis on factors affecting weld penetration, including Doherty, suggest the use of log variables in modelling. The reasons for

this are not completely clear, but this practice is often followed to ensure closer conformity to model requirements.

The design put forward for this project has been arranged on a sequential basis to allow for extension to a 1/3-replicate if this appears to be necessary following analysis of the first stage experiment using a 1/9-replicate. The extension to a 1/3-replicate also depends on the practical feasibility.

Alternatively, it was suggested that the 1/9-replicate be duplicated, to check for repeatability and the design has been arranged to cope with this too.

If the Engineers involved in the practical work are concerned only to discover which of the 7 factors involved have a real effect on weld penetration, these could easily be obtained from the ANOVA, and modelling might not be necessary. However, if they are really concerned with a predictive model as well, i.e. predicting penetration for the given levels of the 7 factors or simply trying to obtain a better understanding of the process, then both the Analysis of Variance and modelling become essential. The former will be used to indicate which of the factors have significant influence on the responses. These are then used in the latter for predicting penetration for the given levels of the 7 factors.

Because of the large number of factors involved, a double precision was used in the execution of GLIM package to obtain values for these factors in ANOVA.

APPENDIX A1

THE GLOSSARY OF TERMS

Effective Bead Width : The width of the weld bead measured from the point of backface plate intersection to the exposed bead contour along a line 45^{o} to the upper surface of the lower plate.

Electrode Extension (Stickout) : This is the length of electrode between the torch contact tip and the welding Arc; which is manipulated by the welder by changing the torch to work distance during welding.

Maximum Penetration : The maximum depth of weld metal penetration into the lower plate measured perpendicularly from the upper surface of the lower plate.

Travel Angle : The angle of the electrode with respect to the joint in the direction of motion measured from the vertical, given the seam is in the horizontal position.

Underbead Minimum Leg Length : This is the lesser of the deposited metal thickness measured normal to the weld seam along the backface of the work piece.

Underbead Throat Thickness : Deposited metal height from a point of backface plate intersection to the underbead contour and bisecting the angle of intersection.

Weld Bead Geometry : The four responses considered, that is Effective bead width, Maximum penetration, Underbead minimum leg length, and Underbead throat thickness together form what is called Weld Bead geometry.

APPENDIX A2

DESIGN MATRIX

DAY 4 : (BLOCK 4)

Experiment	A ℓ_1	B ℓ_2	C ℓ_3	D ℓ_4	E ℓ_5	F ℓ_6	G ℓ_7
1	1	1	0	0	1	2	0
2	1	2	1	2	1	1	0
3	1	0	2	1	1	0	0
4	2	1	1	0	2	1	1
5	0	1	2	0	0	0	2
6	2	2	2	2	2	0	1
7	2	0	0	1	2	2	1
8	0	2	0	2	0	2	2
9	0	0	1	1	0	1	2
10	2	1	1	1	0	0	0
11	0	1	2	2	2	1	0
12	2	2	2	0	0	2	0
13	2	0	0	2	0	1	0
14	0	1	2	1	1	2	1
15	1	1	0	1	2	1	2
16	0	2	0	0	1	1	1
17	0	0	1	2	1	0	1
18	1	2	1	0	2	0	2
19	1	0	2	2	2	2	2
20	0	2	0	1	2	0	0
21	0	0	1	0	2	2	0
22	1	1	0	2	0	0	1
23	2	1	1	2	1	2	2
24	1	2	1	1	0	2	1
25	1	0	2	0	0	1	1
26	2	2	2	1	1	1	2
27	2	0	0	0	1	0	2

$$\ell_1 + \ell_2 + \ell_3 + 2\ell_4 + \ell_5 \equiv 0 \pmod{3}$$

$$\ell_3 + 2\ell_4 + 2\ell_5 + 2\ell_6 + 2\ell_7 \equiv 0 \pmod{3}$$

$$\ell_1 + 2\ell_2 + 2\ell_6 + \ell_7 \equiv 1 \pmod{3}$$

$$\ell_2 + \ell_3 + \ell_4 + \ell_6 \equiv 0 \pmod{3}$$

DAY 5 : (BLOCK 5)

Experiment	A ℓ_1	B ℓ_2	C ℓ_3	D ℓ_4	E ℓ_5	F ℓ_6	G ℓ_7
1	0	0	0	1	1	0	1
2	0	1	1	0	1	2	1
3	0	2	2	2	1	1	1
4	1	0	1	1	2	2	2
5	2	0	2	1	0	1	0
6	1	1	2	0	2	1	2
7	1	2	0	2	2	0	2
8	2	1	0	0	0	0	0
9	2	2	1	2	0	2	0
10	1	0	1	2	0	1	1
11	2	0	2	0	2	2	1
12	1	1	2	1	0	0	1
13	1	2	0	0	0	2	1
14	2	0	2	2	1	0	2
15	0	0	0	2	2	2	0
16	2	1	0	1	1	2	2
17	2	2	1	0	1	1	2
18	0	1	1	1	2	1	0
19	0	2	2	0	2	0	0
20	2	1	0	2	2	1	1
21	2	2	1	1	2	0	1
22	0	0	0	0	0	1	2
23	1	0	1	0	1	0	0
24	0	1	1	2	0	0	2
25	0	2	2	1	0	2	2
26	1	1	2	2	1	2	0
27	1	2	0	1	1	1	0

$$\ell_1 + \ell_2 + \ell_3 + 2\ell_4 + \ell_5 \equiv 0 \pmod{3}$$

$$\ell_3 + 2\ell_4 + 2\ell_5 + 2\ell_6 + 2\ell_7 \equiv 0 \pmod{3}$$

$$\ell_1 + 2\ell_2 + 2\ell_6 + \ell_7 \equiv 1 \pmod{3}$$

$$\ell_2 + \ell_3 + \ell_4 + \ell_6 \equiv 1 \pmod{3}$$

DAY 6 : (BLOCK 6)

Experiment	A ℓ_1	B ℓ_2	C ℓ_3	D ℓ_4	E ℓ_5	F ℓ_6	G ℓ_7
1	0	1	0	0	2	1	0
2	0	2	1	2	2	0	0
3	0	0	2	1	2	2	0
4	1	1	1	0	0	0	1
5	2	1	2	0	1	2	2
6	1	2	2	2	0	2	1
7	1	0	0	1	0	1	1
8	2	2	0	2	1	1	2
9	2	0	1	1	1	0	2
10	1	1	1	1	1	2	0
11	2	1	2	2	0	0	0
12	1	2	2	0	1	1	0
13	1	0	0	2	1	0	0
14	2	1	2	1	2	1	1
15	0	1	0	1	0	0	2
16	2	2	0	0	2	0	1
17	2	0	1	2	2	2	1
18	0	2	1	0	0	2	2
19	0	0	2	2	0	1	2
20	2	2	0	1	0	2	0
21	2	0	1	0	0	1	0
22	0	1	0	2	1	2	1
23	1	1	1	2	2	1	2
24	0	2	1	1	1	1	1
25	0	0	2	0	1	0	1
26	1	2	2	1	2	0	2
27	1	0	0	0	2	2	2

$$\ell_1 + \ell_2 + \ell_3 + 2\ell_4 + \ell_5 \equiv 0 \pmod 3$$

$$\ell_3 + 2\ell_4 + 2\ell_5 + 2\ell_6 + 2\ell_7 \equiv 0 \pmod 3$$

$$\ell_1 + 2\ell_2 + 2\ell_6 + \ell_7 \equiv 1 \pmod 3$$

$$\ell_2 + \ell_3 + \ell_4 + \ell_6 \equiv 2 \pmod 3$$

DAY 7 : (BLOCK 7)

Experiment	A ℓ_1	B ℓ_2	C ℓ_3	D ℓ_4	E ℓ_5	F ℓ_6	G ℓ_7
1	0	2	1	0	0	0	1
2	0	0	2	2	0	2	1
3	0	1	0	1	0	1	1
4	1	2	2	0	1	2	2
5	2	2	0	0	2	1	0
6	1	0	0	2	1	1	2
7	1	1	1	1	1	0	2
8	2	0	1	2	2	0	0
9	2	1	2	1	2	2	0
10	1	2	2	1	2	1	1
11	2	2	0	2	1	2	1
12	1	0	0	0	2	0	1
13	1	1	1	2	2	2	1
14	2	2	0	1	0	0	2
15	0	2	1	1	1	2	0
16	2	0	1	0	0	2	2
17	2	1	2	2	0	1	2
18	0	0	2	0	1	1	0
19	0	1	0	2	1	0	0
20	2	0	1	1	1	1	1
21	2	1	2	0	1	0	1
22	0	2	1	2	2	1	2
23	1	2	2	2	0	0	0
24	0	0	2	1	2	0	2
25	0	1	0	0	2	2	2
26	1	0	0	1	0	2	0
27	1	1	1	0	0	1	0

$$\ell_1 + \ell_2 + \ell_3 + 2\ell_4 + \ell_5 \equiv 0 \pmod{3}$$

$$\ell_3 + 2\ell_4 + 2\ell_5 + 2\ell_6 + 2\ell_7 \equiv 0 \pmod{3}$$

$$\ell_1 + 2\ell_2 + 2\ell_6 + \ell_7 \equiv 2 \pmod{3}$$

$$\ell_2 + \ell_3 + \ell_4 + \ell_6 \equiv 0 \pmod{3}$$

DAY 8 : (BLOCK 8)

Experiment	A ℓ_1	B ℓ_2	C ℓ_3	D ℓ_4	E ℓ_5	F ℓ_6	G ℓ_7
1	0	1	2	0	0	1	1
2	0	2	0	2	0	0	1
3	0	0	1	1	0	2	1
4	1	1	0	0	1	0	2
5	2	1	1	0	2	2	0
6	1	2	1	2	1	2	2
7	1	0	2	1	1	1	2
8	2	2	2	2	2	1	0
9	2	0	0	1	2	0	0
10	1	1	0	1	2	2	1
11	2	1	1	2	1	0	1
12	1	2	1	0	2	1	1
13	1	0	2	2	2	0	1
14	2	1	1	1	0	1	2
15	0	1	2	1	1	0	0
16	2	2	2	0	0	0	2
17	2	0	0	2	0	2	2
18	0	2	0	0	1	2	0
19	0	0	1	2	1	1	0
20	2	2	2	1	1	2	1
21	2	0	0	0	1	1	1
22	0	1	2	2	2	2	2
23	1	1	0	2	0	1	0
24	0	2	0	1	2	1	2
25	0	0	1	0	2	0	2
26	1	2	1	1	0	0	0
27	1	0	2	0	0	2	0

$$\ell_1 + \ell_2 + \ell_3 + 2\ell_4 + \ell_5 \equiv 0 \pmod 3$$

$$\ell_3 + 2\ell_4 + 2\ell_5 + 2\ell_6 + 2\ell_7 \equiv 0 \pmod 3$$

$$\ell_1 + 2\ell_2 + 2\ell_6 + \ell_7 \equiv 2 \pmod 3$$

$$\ell_2 + \ell_3 + \ell_4 + \ell_6 \equiv 1 \pmod 3$$

DAY 9 : (BLOCK 9)

Experiment	A ℓ_1	B ℓ_2	C ℓ_3	D ℓ_4	E ℓ_5	F ℓ_6	G ℓ_7
1	0	0	0	1	1	1	0
2	0	1	1	0	1	0	0
3	0	2	2	2	1	2	0
4	1	0	1	1	2	0	1
5	2	0	2	1	0	2	2
6	1	1	2	0	2	2	1
7	1	2	0	2	2	1	1
8	2	1	0	0	0	1	2
9	2	2	1	2	0	0	2
10	1	0	1	2	0	2	0
11	2	0	2	0	2	0	0
12	1	1	2	1	0	1	0
13	1	2	0	0	0	0	0
14	2	0	2	2	1	1	1
15	0	0	0	2	2	0	2
16	2	1	0	1	1	0	1
17	2	2	1	0	1	2	1
18	0	1	1	1	2	2	2
19	0	2	2	0	2	1	2
20	2	1	0	2	2	2	0
21	2	2	1	1	2	1	0
22	0	0	0	0	0	2	1
23	1	0	1	0	1	1	2
24	0	1	1	2	0	1	1
25	0	2	2	1	0	0	1
26	1	1	2	2	1	0	2
27	1	2	0	1	1	2	2

$$\ell_1 + \ell_2 + \ell_3 + 2\ell_4 + \ell_5 \equiv 0 \pmod 3$$

$$\ell_3 + 2\ell_4 + 2\ell_5 + 2\ell_6 + 2\ell_7 \equiv 0 \pmod 3$$

$$\ell_1 + 2\ell_2 + 2\ell_6 + \ell_7 \equiv 2 \pmod 3$$

$$\ell_2 + \ell_3 + \ell_4 + \ell_6 \equiv 2 \pmod 3$$

To repeat the entire experiments, i.e. to obtain the 2nd - replicate of $1/9 \times 3^7$ factorial, we randomize the experiments within each of the 9 blocks in the 1st rep. using random number tables. The order of the experiments to be done in this rep. together with numbers corresponding to those experiments in the 1st replicate are shown below after randomization.

ORDER	DAY 1	DAY 2	DAY 3	DAY 4	DAY 5	DAY 6	DAY 7	DAY 8	DAY 9
1	3	26	7	21	16	9	17	13	14
2	10	2	10	8	4	10	6	11	24
3	20	21	8	26	5	14	20	26	1
4	18	14	27	3	9	21	26	22	11
5	13	18	3	5	22	25	27	7	8
6	26	19	16	12	15	11	4	6	15
7	1	25	4	18	19	7	19	5	5
8	21	4	23	24	8	4	14	21	22
9	16	13	14	10	13	1	22	19	20
10	14	23	18	15	2	3	12	24	23
11	19	27	5	20	26	12	13	9	3
12	2	20	11	17	6	17	5	23	21
13	9	1	25	11	3	16	2	25	26
14	11	9	24	22	17	5	21	2	12
15	12	7	17	6	12	22	11	10	27
16	25	6	19	25	7	8	24	1	4
17	5	3	15	1	21	18	18	20	7
18	6	5	6	27	20	6	1	27	25
19	4	16	9	7	14	13	8	12	6
20	27	17	12	2	10	20	25	14	18
21	17	22	22	9	27	2	7	4	13
22	23	24	1	13	24	24	3	16	2
23	24	15	2	14	23	15	23	3	19
24	22	8	20	4	11	26	10	18	10
25	7	12	21	19	1	19	15	8	17
26	8	11	13	16	26	23	9	15	9
27	15	10	26	23	18	27	16	17	16

APPENDIX A3 : COMPUTER OUTPUT.

```
PRIMOS Version 18.3b/LUCC
SYSB   (8) LOGGED IN 17 SEP 1982  16:16
USERS 17   RESPONSE RATING *****
OK, A 'MA AWEE>AAI
OK, OPEN NEWDSIGN 4,3
OK, GLIM
 GLIM 3.12 (C)1977 ROYAL STATISTICAL SOCIETY, LONDON



> $C      ANALYSIS OF VARIANCE ON SIMULATED DATA FOR THE RESPONSE(Y2
>       UNDERBEAD THROAT THICKNESS FOR THE GIVEN RANGE (0.5 - 6.0).
>


  $UNITS 243
> $DAT U$DIN 8$
> $DAT I$DIN 8$
> $DAT V$DIN 8$
> $DAT T$DIN 8$
> $DAT G$DIN 8$
> $DAT A$DIN 8$
> $DAT E$DIN 8$
> $FAC U 3 I 3 V 3 T 3 G 3 A 3 E 3 BLOCKS 9
> $CAL BLOCKS=%GL(9,27)
> $CAL EE=%ND(%LR(0))*.1
> $CAL Y2=EE+(U-2)*.5+(G-2)*1+(T-2)*.5+(U-2)*(V-2)*.25+
>             (I-2)*(T-2)*.25+3.25+(G-2)*(A-2)*.25
>

$PR Y2$
  2.022   2.049   1.392   2.913   4.692   3.686   3.411   4.785
  4.898   4.343   4.275   3.445   5.251   3.243   3.026   1.868
  3.600   2.093   3.380   3.484   3.225   4.146   2.750   3.604
  3.091   1.811   1.469   2.099   1.847   1.184   3.551   4.935
  3.314   2.492   4.885   3.907   4.016   3.688   4.552   4.553
  2.833   2.226   3.085   2.321   3.477   2.781   3.448   4.338
  3.497   2.322   3.184   4.672   1.565   3.133   2.164  0.6241
  2.862   3.415   4.359   2.767   3.960   4.036   5.975   4.441
  3.034   3.997   3.978   2.901   2.936   2.450   2.230   2.553
  2.246   3.980   4.032   3.514   2.047   3.744   4.188   2.604
  2.246   2.815   3.918   3.350   4.450   1.223   5.560   4.733
  2.508   1.626   3.111   3.939   1.990   2.853   2.479   4.438
  2.205   2.840   3.285   4.679   3.763   3.569   2.897   4.268
  2.029   1.969   4.106   3.192   2.926   2.166   3.109   4.535
  3.072   3.780   4.811   2.400   3.268   2.573   4.966   2.445
  1.344   4.216   4.595   3.394   3.104   3.810   2.402   4.921
  4.594   1.800   2.926   2.274   1.169   3.584   3.282   3.558
  4.046   3.735   1.852   3.523   2.799   2.348   4.303   3.812
  3.370   3.668   2.442   3.503   4.966   2.531   3.497   5.248
 0.7717   1.784   2.173   2.414   3.461   4.652   2.846   2.300
  4.043   4.373   1.485   1.461   1.958   2.534   3.548   3.586
  3.308   4.540   5.140   4.267   4.271   3.589   4.927   2.718
  2.827   2.372   3.510   2.315   3.645   3.781   3.506   4.688
  3.165   3.271   3.669   1.802   1.797   1.013   2.879   1.534
  2.814   4.613   4.066   3.143   5.638   4.254   4.558   4.242
  3.540   4.273   2.670   2.571   2.468   2.627   2.289   2.922
  3.997   3.290   4.154   2.795   3.938   3.249   2.495   1.752
  2.962   2.235   3.283   4.013   2.675   4.006   5.185   2.015
  3.732   2.203   4.546   2.082   1.782   4.278   3.827   3.360
  2.989   3.833   2.749   5.387   4.761   1.511   3.019   2.120
  1.900   3.844   3.264
>
```

```
$YVAR Y2
> $ACC 8
>

$FIT$
 CYCLE  DEVIANCE      DF
    1    263.2       242

> $PR %DV$
   263.21918
> $FIT BLOCKS$
 CYCLE  DEVIANCE      DF
    1    263.2       234

> $PR %DV$
   263.20166
> $FIT U$
 CYCLE  DEVIANCE      DF
    1    222.8       240

> $PR %DV$
   222.80011
> $FIT I$
 CYCLE  DEVIANCE      DF
    1    263.2       240

> $PR %DV$
   263.17609
> $FIT V$
 CYCLE  DEVIANCE      DF
    1    263.2       240

> $PR %DV$
   263.20673
> $FIT T$
 CYCLE  DEVIANCE      DF
    1    224.2       240

> $PR %DV$
   224.17047
> $FIT G$
 CYCLE  DEVIANCE      DF
    1    101.9       240

> $PR %DV$
   101.87453
> $FIT A$
 CYCLE  DEVIANCE      DF
    1    263.2       240

> $PR %DV$
   263.21399
> $FIT E$
 CYCLE  DEVIANCE      DF
    1    263.2       240

> $PR %DV$
   263.21899
>
```

```
$FIT U.I$
CYCLE  DEVIANCE      DF
   1     222.7      234

> $PR %DV$
   222.74469
> $FIT U.V$
CYCLE  DEVIANCE      DF
   1     215.8      234

> $PR %DV$
   215.83304
> $FIT U.T$
CYCLE  DEVIANCE      DF
   1     183.7      234

> $PR %DV$
   183.71280
> $FIT U.G$
CYCLE  DEVIANCE      DF
   1     61.38      234

> $PR %DV$
   61.376259
> $FIT U.A$
CYCLE  DEVIANCE      DF
   1     222.7      234

> $PR %DV$
   222.74377
> $FIT U.E$
CYCLE  DEVIANCE      DF
   1     222.8      234

> $PR %DV$
   222.77179
> $FIT I.V$
CYCLE  DEVIANCE      DF
   1     263.1      234

> $PR %DV$
   263.06482
> $FIT I.T$
CYCLE  DEVIANCE      DF
   1     217.6      234

> $PR %DV$
   217.56683
> $FIT I.G$
CYCLE  DEVIANCE      DF
   1     101.8      234

> $PR %DV$
   101.79454
>
```

```
$FIT  I.A$
CYCLE  DEVIANCE      DF
   1     263.1       234

> $PR %DV$
    263.14362
> $FIT I.E$
CYCLE  DEVIANCE      DF
   1     263.1       234

> $PR %DV$
    263.14691
> $FIT V.T$
CYCLE  DEVIANCE      DF
   1     224.0       234

> $PR %DV$
    224.03397
> $FIT V.G$
CYCLE  DEVIANCE      DF
   1     101.8       234

> $PR %DV$
    101.81857
> $FIT V.A$
CYCLE  DEVIANCE      DF
   1     263.2       234

> $PR %DV$
    263.17554
> $FIT V.E$
CYCLE  DEVIANCE      DF
   1     263.2       234

> $PR %DV$
    263.16724
> $FIT T.G$
CYCLE  DEVIANCE      DF
   1     62.78       234

> $PR %DV$
    62.777184
> $FIT T.A$
CYCLE  DEVIANCE      DF
   1     224.1       234

> $PR %DV$
    224.09567
> $FIT T.E$
CYCLE  DEVIANCE      DF
   1     224.1       234

> $PR %DV$
    224.13007
>
```

```
  $FIT G.A$
  CYCLE  DEVIANCE      DF
     1    95.12       234

> $PR %DV$
    95.122116
> $FIT G.E$
  CYCLE  DEVIANCE      DF
     1    101.9       234

> $PR %DV$
    101.85182
> $FIT A.E$
  CYCLE  DEVIANCE      DF
     1    263.2       234

> $PR %DV$
    263.19104
>

  $FIT U+I+V+T+G+A+E+U.I+U.V+U.T+U.G+U.A+U.E+I.V+I.T+I.G+I.A+I.E+ .
>       V.T+V.G+V.A+V.E+T.G+T.A+T.ET"+G.A+G.E+A.E$
  CYCLE  DEVIANCE      DF
     1    1.245       144

> $PR %DV$
    1.2449596
>

  $STOP$
OK, LOGOUT
AAI7543  (8) LOGGED OUT 17 SEP 1982  16:45
TIME USED=  0:30   2:56   0:08
```

```
PRIMOS Version 18.36/LUCC
SYSB   (6) LOGGED IN 17 SEP 1982  16:46
USERS 15   RESPONSE RATING **
OK, A 'MA AWEE>AAI'
OK, OPEN NEWDSIGN 4,3
OK, GLIM
 GLIM 3.12 (C)1977 ROYAL STATISTICAL SOCIETY, LONDON


> $C      ANALYSIS OF VARIANCE ON SIMULATED DATA FOR THE RESPONSE(Y3)
>        MAXIMUM PENETRATION FOR THE GIVEN RANGE (1.0 - 6.0).
>


  $UNITS 243
> $DAT U$DIN 8$
> $DAT I$DIN 8$
> $DAT V$DIN 8$
> $DAT T$DIN 8$
> $DAT G$DIN 8$
> $DAT A$DIN 8$
> $DAT E$DIN 8$
> $FAC U 3 I 3 V 3 T 3 G 3 A 3 E 3 BLOCKS 9
> $CAL BLOCKS=%GL(9,27)
> $CAL EE=%ND(%LR(0))*.1
> $CAL Y3=EE+(U-2)*.75+(V-2)*1+(U-2)*(T-2)*.25+(G-3**2)*(V-2)*.25+
>           3.5+(I-2)*(A-2)*.25
>

  $PR Y3$
    2.522    2.549    3.392    3.163    5.192    4.436    2.411    3.285
    4.398    3.593    5.275    4.695    2.501    5.243    1.526    3.368
    4.600    2.843    3.380    3.234    3.975    1.146    3.750    2.854
    4.591    4.061    2.719    3.599    2.097    3.184    2.301    4.435
    3.564    4.242    5.635    2.907    2.266    3.938    3.552    5.053
    4.333    3.726    5.085    3.321    1.477    2.531    4.948    3.338
    3.997    2.822    1.684    2.672    3.565    4.383    3.164    3.624
    1.862    4.415    2.859    2.517    3.210    4.036    5.975    4.691
    2.534    2.247    3.978    3.901    2.186    4.200    4.730    3.803
    1.746    4.230    5.032    3.014    4.297    3.744    1.688    2.854
    3.496    2.565    3.418    4.850    4.200    3.723    5.560    2.733
    2.008    2.626    4.361    3.689    4.990    3.853    3.729    2.438
    1.955    2.590    3.285    4.429    1.263    2.569    2.647    4.518
    3.779    4.219    5.356    3.192    1.926    2.916    3.359    3.285
    5.072    4.780    2.061    3.400    4.768    3.573    4.966    4.195
    3.094    5.716    1.095    3.144    4.104    2.810    3.902    3.171
    4.094    2.300    3.676    2.274    3.669    4.334    2.532    1.808
    2.046    3.735    3.352    5.023    4.549    2.848    3.553    4.562
    3.620    5.168    4.442    2.753    5.466    2.281    2.497    4.248
    3.272    3.284    3.673    3.914    1.461    3.402    2.846    4.300
    4.543    2.123    2.985    2.961    1.958    4.784    2.548    2.586
    3.558    4.540    5.390    4.767    3.771    2.339    3.427    3.218
    3.077    3.872    5.260    4.065    1.645    4.281    5.006    2.688
    3.915    4.271    1.669    2.302    3.547    3.763    1.379    2.534
    2.564    4.113    3.816    4.393    5.638    3.254    2.308    4.492
    3.540    5.023    4.170    3.821    4.468    3.627    2.289    2.422
    5.497    3.040    3.654    2.795    1.438    3.249    3.245    4.002
    1.712    2.985    3.783    3.763    4.675    4.756    2.435    3.265
    4.232    3.203    5.546    4.082    2.532    5.528    1.327    3.110
    4.239    2.583    4.249    3.387    4.261    2.011    3.519    2.370
    3.400    4.594    2.764
```

```
—/
    $YVAR Y3
> $ACC 8
>

    $FIT$
    CYCLE  DEVIANCE      DF
       1    273.4       242

> $PR %DV$
    273.37567
> $FIT BLOCKS$
    CYCLE  DEVIANCE      DF
       1    273.4       234

> $PR %DV$
    273.35822
> $FIT U$
    CYCLE  DEVIANCE      DF
       1    182.4       240

> $PR %DV$
    182.37717
> $FIT I$
    CYCLE  DEVIANCE      DF
       1    273.3       240

> $PR %DV$
    273.33264
> $FIT V$
    CYCLE  DEVIANCE      DF
       1    113.8       240

> $PR %DV$
    113.75797
> $FIT T$
    CYCLE  DEVIANCE      DF
       1    273.4       240

> $PR %DV$
    273.35876
> $FIT G$
    CYCLE  DEVIANCE      DF
       1    273.4       240

> $PR %DV$
    273.37256
> $FIT Ä$
    CYCLE  DEVIANCE      DF
       1    273.4       240

> $PR %DV$
    273.37048
> $FIT E$
    CYCLE  DEVIANCE      DF
       1    273.4       240

> $PR %DV$
    273.37555
>
```

$FIT U. I$

```
 CYCLE  DEVIANCE      DF
    1    182.3       234

> $PR %DV$
   182.32175
> $FIT U.V$
 CYCLE  DEVIANCE      DF
    1    22.71       234

> $PR %DV$
   22.712772
> $FIT U.T$
 CYCLE  DEVIANCE      DF
    1    175.5       234

> $PR %DV$
   175.54889
> $FIT U.G$
 CYCLE  DEVIANCE      DF
    1    182.3       234

> $PR %DV$
   182.29486
> $FIT U.A$
 CYCLE  DEVIANCE      DF
    1    182.3       234

> $PR %DV$
   182.32083
> $FIT U.E$
 CYCLE  DEVIANCE      DF
    1    182.3       234

> $PR %DV$
   182.34888
>

 $FIT I.V$
 CYCLE  DEVIANCE      DF
    1    113.6       234

> $PR %DV$
   113.61603
> $FIT I.T$
 CYCLE  DEVIANCE      DF
    1    273.3       234

> $PR %DV$
   273.28723
> $FIT I.G$
 CYCLE  DEVIANCE      DF
    1    273.3       234

> $PR %DV$
   273.29260
> $FIT I.A$
 CYCLE  DEVIANCE      DF
    1    267.8       234

> $PR %DV$
   266.96637
```

```
> $FIT I.E$
  CYCLE   DEVIANCE       DF
     1     273.3        234

> $PR %DV$
    273.30347
> $FIT V.T$
  CYCLE   DEVIANCE       DF
     1     113.6        234

> $PR %DV$
    113.61691
> $FIT V.G$
  CYCLE   DEVIANCE       DF
     1     106.3        234

> $PR %DV$
    106.30502
> $FIT V.A$
  CYCLE   DEVIANCE       DF
     1     113.7        234

> $PR %DV$
    113.72678
> $FIT V.E$
  CYCLE   DEVIANCE       DF
     1     113.7        234

> $PR %DV$
    113.71846
>

  $FIT T.G$
  CYCLE   DEVIANCE       DF
     1     273.3        234

> $PR %DV$
    273.30695
> $FIT T.A$
  CYCLE   DEVIANCE       DF
     1     273.3        234

> $PR %DV$
    273.28394
> $FIT T.E$
  CYCLE   DEVIANCE       DF
     1     273.3        234

> $PR %DV$
    273.31830
```

```
> $FIT G. A$
  CYCLE  DEVIANCE      DF
     1     273.3       234

> $PR %DV$
    273.34772
> $FIT G. E$
  CYCLE  DEVIANCE      DF
     1     273.3       234

> $PR %DV$
    273.34985
> $FIT A. E$
  CYCLE  DEVIANCE      DF
     1     273.3       234

> $PR %DV$
    273.34753
>



  $FIT U+I+V+T+G+A+E+U. I+U. V+U. T+U. G+U. A+U. E+I. V+I. T+I. G+I. A+I. E+
>       V. T+V. G+V. A+V. E+T. G+T. A+T. E+G. A+G. E+A. E$
  CYCLE  DEVIANCE      DF
     1     1.245       144

> $PR %DV$
    1.2449601
>

  $STOP$
OK, LOGOUT
AAI7543  (6) LOGGED OUT 17 SEP 1982  17:14
TIME USED=  0:28   2:53   0:08
```

```
PRIMOS Version 18.3b/LUCC
SYSB   (4) LOGGED IN 20 SEP 1982  12:15
USERS 9   RESPONSE RATING *****
OK, A 'MA AWEE>AAI
OK, OPEN NEWDSIGN 4.3
OK, GLIM
 GLIM 3.12 (C)1977 ROYAL STATISTICAL SOCIETY, LONDON


> $C     ANALYSIS OF VARIANCE FOR THE RESPONSE (Y4),
>       EFFECTIVE BEAD WIDTH FOR THE RANGE (4.0 - 12.0).
>



  $UNITS 243
> $DAT U$DIN 8$
> $DAT I$DIN 8$
> $DAT V$DIN 8$
> $DAT T$DIN 8$
> $DAT G$DIN 8$
> $DAT A$DIN 8$
> $DAT E$DIN 8$
> $FAC U 3 I 3 V 3 T 3 G 3 A 3 E 3 BLOCKS 9
> $CAL BLOCKS=%GL(9,27)
> $CAL EE=%ND(%LR(0))*.1
> $CAL Y4=EE+(U-2)*.75+(G-2)*1.25+(U-2)*(A-2)*.5+
>             (G-2)*(V-2)*.75+8.0+(T-2)*(I-2)*.75
>

$PR Y4$
  8.022    5.549    5.142    8.663   11.44    7.936    8.161    8.785
 10.40     9.343    8.525    9.945    9.251   6.243    6.776    8.868
  8.350    7.093    8.630    8.734    7.725   6.896    6.000    9.104
  8.091    5.811    6.719    5.099    6.347   5.434    7.301    9.185
  8.064    7.242   10.39     8.907    9.266   9.188    9.302   10.05
  7.833    7.476    7.085    8.071    7.227   7.781    8.698    9.088
  9.247    7.322    8.184    8.672    6.815   6.983    6.664    4.624
  7.612    8.165    9.109    8.017    8.710   9.536   11.97     9.191
  9.784    8.497    8.728    6.901    5.936   7.950    5.980    7.303
  6.996    8.480    8.282    9.264    6.547   9.744    7.188    7.604
  6.746    8.065    8.668    8.100   10.20    5.722   11.06     9.733
  7.008    5.876    7.111    9.189    6.490   7.603    6.729    8.688
  6.455    6.840    8.535    9.179    8.263   8.569    7.397    9.268
  6.779    6.719    8.856    8.942    7.676   6.666    7.859    9.285
  6.822   10.03     9.211    7.900    8.768   6.073   11.97     5.945
  6.844    7.466    6.595    9.144    8.104   8.560    8.902    9.171
  9.594    7.550    8.676    6.274    4.669   7.834    8.032    7.808
  9.546    8.735    6.602    9.273    6.799   7.598    9.553    8.312
  8.120    6.168    7.192    7.253   10.72    7.531    7.997    9.748
  4.772    4.534    8.673    8.164    6.711   9.152    7.346    8.550
 10.04     9.373    5.985    2.961    6.708   7.284    8.298    7.336
  8.058    8.540   11.14    10.02    10.02    9.089    9.177    7.718
  6.827    8.872    6.760    8.065    7.895   8.781    8.256    9.438
  6.665    9.771    7.169    7.302    6.797   5.263    7.879    5.534
  8.064   10.61     8.816    7.893   11.39    8.754    8.558    8.242
  8.540    9.273    7.420    7.821    5.468   8.127    6.039    6.422
  9.247    9.540    8.654    7.545    7.688   9.749    6.745    6.752
  7.212    7.735    7.533    9.263    7.175  10.01     9.435    8.265
  7.732    5.953   11.05     5.832    6.782   8.028    7.327    8.110
  8.489    7.833    8.499    9.887   10.01    7.011    8.769    5.870
  5.900    8.094    8.014
```

```
>
  $Y ""YVAR Y4
> $ACC 8
> $FIT$
  CYCLE  DEVIANCE      DF
     1    495.4       242

> $PR %DV$
    495.41211
> $FIT BLOCKS$
  CYCLE  DEVIANCE      DF
     1    495.4       234

> $PR %DV$
    495.39465
> $FIT U$
  CYCLE  DEVIANCE      DF
     1    404.4       240

> $PR %DV$
    404.41357
> $FIT I$
  CYCLE  DEVIANCE      DF
     1    495.4       240

> $PR %DV$
    495.26908
> $FIT V$
  CYCLE  DEVIANCE      DF
     1    495.4       240

> $PR %DV$
    495.39972
> $FIT T$
  CYCLE  DEVIANCE      DF
     1    495.4       240

> $PR %DV$
    495.39520
> $FIT G$
  CYCLE  DEVIANCE      DF
     1    243.1       240

> $PR %DV$
    243.10715
> $FIT A$
  CYCLE  DEVIANCE      DF
     1    495.4       240

> $PR %DV$
    495.40692
> $FIT E$
  CYCLE  DEVIANCE      DF
     1    495.4       240

> $PR %DV$
    495.41199
>
```

```
$FIT U.I$
CYCLE  DEVIANCE      DF
   1     404.4       234

> $PR %DV$
    404.35815
> $FIT U.V$
CYCLE  DEVIANCE      DF
   1     404.4       234

> $PR %DV$
  404.35455
> $FIT U.T$
CYCLE  DEVIANCE      DF
   1     404.4       234

> $PR %DV$
    404.35803
> $FIT U.G$
CYCLE  DEVIANCE      DF
   1     152.0       234

> $PRV" %DV$
    152.02942
> $FIT U.A$
CYCLE  DEVIANCE      DF
   1     377.3       234

> $PR %DV$
    377.32001
> $FIT U.E$
CYCLE  DEVIANCE      DF
   1     404.4       234

> $PR %DV$
    404.38531
> $FIT I.V$
CYCLE  DEVIANCE      DF
   1     495.3       234

> $PR %DV$
    495.25775
> $FIT I.T$
CYCLE  DEVIANCE      DF
   1     435.2       234

> $PR %DV$
    435.22717
> $FIT I.G$
CYCLE  DEVIANCE      DF
   1     243.0       234

> $PR %DV$
    243.02716
> $FIT I.A$
CYCLE  DEVIANCE      DF
   1     495.3       234

> $PR %DV$
    495.33655
> $FIT I.E$
CYCLE  DEVIANCE      DF
   1     495.3       234

> $PR %DV$
    495.33990
>
```

```
  $FIT V.T$
  CYCLE  DEVIANCE      DF
     1    495.3       234

> $PR %DV$
    495.25867
> $FIT V.G$
  CYCLE  DEVIANCE      DF
     1    180.3       234

> $PR %DV$
    180.33231
> $FIT V.A$
  CYCLE  DEVIANCE      DF
     1    495.4       234

> $PR %DV$
    495.36853
> $FIT V.E$
  CYCLE  DEVIANCE      DF
     1    495.4       234

> $PR %DV$
    495.36023
> $FIT T.G$
  CYCLE  DEVIANCE      DF
     1    243.0       234

> $PR %DV$
    243.04153
> $FIT T.A$
  CYCLE  DEVIANCE      DF
     1    495.3       234

> $PR %DV$
    495.32037
> $FIT T.E$
  CYCLE  DEVIANCE      DF
     1    495.4       234

> $PR %DV$
    495.35480
> $FIT G.A$
  CYCLE  DEVIANCE      DF
     1    243.1       234

> $PR %DV$
    243.08231
> $FIT G.E$
  CYCLE  DEVIANCE      DF
     1    243.1       234

> $PR %DV$
    243.08441
>
```

```
$FIT A. E$
CYCLE  DEVIANCE      DF
   1    495. 4      234

> $PR %DV$
  495. 38403

$FIT U+I+V+T+G+A+E+U. ". I+U. V+U. T+U. G+U. A+U. E+I. V+I. T+I. G+I. A+I. E+
>      V. T+V. G+V. A+V. E+T. G+T. A+T. E+G. A+G. E+A. E$



CYCLE  DEVIANCE      DF
   1    1. 245      144

> $PR %DV$
  1. 2449617


$STOP$
OK. LOGOUT
AA17542  (4) LOGGED OUT 20 SEP 1982. 13:42
TIME USED=  0:26   2:55   0:02
```

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