PROBLEMS AND PROBLEM SOLVING STRATEGIES FOR SENIOR SECONDARY SCHOOL MATHEMATICS

BY

CALEB B0LAJI

(AN INDEPENDENT STUDY)

A PROJECT SUBMITTED TO THE POSTGRADUATE SCHOOL, AHMADU BELLO UNIVERSITY, ZARlA, IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER IN EDUCATION (M.ED. MATHEMATICS EDUCATION)

DEPARTMENT OF EDUCATION
FACULTY OF EDUCATION
AHMADU BELLO UNIVERSITY
ZARIA

MARCH, 1984

DECLARATION

I hereby declare that this project has been written by me and that it is a record of my own research work. It has not been presented in any previous application for a higher degree. All quotations are indicated and the sources of information are specifically acknowledge by means of references.

Caleb Bolaji

Date_____

APPROVAL SHEET

This project (Problems and Problem-Solving Strategies for Senior Secondary School Mathematics) has been read and approved as meeting the required standard for the award of Masters degree (M.Ed.) in Curriculum and Instruction (Mathematics Education).

Supervisor

Internal Examiner

External Examiner

Date

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Problems and Problem Solving Strategies

for Senior Secondary School Mathematics.

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DEDICATION

To my late father and mother who were unable to reap the fruits of their labour, and to my wife Olajumoke for her endurance and support through this great hardship and loneliness on the successful completion of this course.

ABSTRACT

"PROBLEMS AND PROBLEM SOLVING STRATEGIES
FOR SENIOR SECONDARY SCHOOL MATHEMATICS"

The purpose of this project is to develop problems and problem-solving strategics for Senior Secondary School Mathematics and to sugqest the methods for teaching probl solving at the Secondary School level.

The study begins by reviewing previous work on:- problem-solving methods; how to set up a problem; easy and difficult problems; and explores factors that affect problem solving.

In the first part, procedures on strategies of problem-solving:- identified pattern in teaching problem solving, presentation, attention, observation and exploration, classification, were then discussed and further exploration and generalizations were enumerated.

In the second part, problems of "Novelty" were presented. The term "Novelty" problem could be generalized to include abstract mathematical problems, or puzzles, curiosities and recreational problems, requiring creativity or originality.

These Novelty problems will hopefully shed more light on the creative aspect of mathematics learning at the senior secondary school levels., Because of the importance of problem-solving in mathematics examinations, a collection of secondary school mathematics is presented.

Finally, some suggestions for further research were made to test the suitability of each item in the unit. There is a need for further investigations to ascertain whether or not certain, or all items are appropriate to Nigerian students. Thirdly, a number of areas related to the general issue of the items in Nigeria would provide rewarding research experience and supply useful information.

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CHAPTER I

INTRODUCTION

Nigeria has undergone great changes in the spheres of culture, politics and education in the past two decades. These changes have affected every Nigerian in one way or the other. In addition to these changes or the reason for these changes is the constant importation of foreign culture, politics and education. These have surely have adverse effects on the people.

Amidst these changes is the advancement of the different disciplines mathematics included. As these areas of knowledge were caught up in the changes, the results were some upheavals and harphazardness in the development and understanding of the different disciplines. Furthermore, the disciplines witnessed many uncoordinated projects, abrupt changes and other ills. These upheavals affected the progress and advancement of mathematics in the nation.

With this background in mind, the question that remains is "What is happening in Secondary School Mathematics Curriculum?" The senior secondary school Mathematics Curriculum looks very much like a hurried work. It appears to have been constructed with the

students who will do no more mathematics beyond "O" level primarily in mind; and the two main objectives in teaching of mathematics at the secondary level (to develop precise, logical and abstract thinking, to provide necessary mathematical background for further education) seem to have been largely ignored.

Journals dealing with mathematics and the teaching of mathematics devote considerable space to problems and their solutions. The emphasis in this electronic age should be, not on performing algorithms in a rote manner, but on developing and using algorithms to solve problems.

Also, after the topics in algebraic processes, one could expect the syllabus to introduce "selection of pertinent data from a problem with logic and if possible, identification of problems with sufficient/insufficient data, for example, to establish that 2 = 1, given that x and y be any two nonzero real number such that x = y. ∴ The inclusion of the problem solving would make the syllabus discriminary and it would be possible to find those students who can continue with mathematics at a higher level. The syllabus as it is now, leaves a wide gap between the secondary school work and future learning in mathematics.

It is the view of the researcher that we are approaching a crisis stage in school mathematics in Nigeria. Policy makers in education are not confronting the deepest problems facing society. With advancement in the socio-economic and technological fields, the life of the individual is becoming more and more complex, fraught with a number of problems which the individual and society have to face in the near future. The responsibility of the school becomes increasingly important, to develop scientific attitudes in students so that they may solve their problems independently for better adjustment in the future complex society.

Apart from all these mentioned above, there are at least three major and urgent problems faced in our daily lives: In mathematics education as related to our daily lives:

(1) School mathematics is generally not keeping pace with the changing needs for mathematical ability dictated by developing technologies.

(2) Most students are not studying sufficient school mathematics to prepare them for their futures as workers, consumers or citizens.

(3) There is a growing shortage of qualified mathematics teachers in secondary school classrooms.

This is one of the reasons why the researcher feels that this topic is relevant.

Since the national policy on education has determined that mathematics should become one of the core courses in all secondary schools, mathematicians, and mathematics-educators are faced with serious problems of curriculum reorganisation and adoption, some of which are:

(a) In the revised levels, some new content has been introduced, some old content has been deleted, and some contents have been placed on different levels.

(b) New methods and approaches such as better problem-solving techniques which will accelerate the teaching-learning processes in our secondary schools have also been built into the new model.

(c) Attempts have been made to simplify these methods into smaller units for easy understanding, and application by both students and teachers.

A major problem, as the researcher sees it, is the fact that students are ill-equipped from secondary schools to handle mathematical problems. It is the contention of the researcher that mathematics is poorly taught, if it is taught at all. Some of our outstanding mathematicians have worked closely with secondary school teachers in finding alternative methods of instruction and discussing

what should be included in the mathematics programme. The Benin (1977) mathematics conference and the subsequent curricula that came out of it were cases in point in this direction. However, this innovation did not take root, and was not given the opportunity to prove its usefulness or otherwise before it was unduly abandoned due to criticism from the public. Therefore these problems and problem solving strategies are an attempt to simplify and, integrate mathematics with societal life as mentioned earlier.

Aside from the introduction of new contents and the proposals for a better organisation of mathematics curricula, the dominant themes of the revisionists have been more emphasis on understanding, student involvement, and discovery learning. Both understanding and discovery call for more thinking. More thinking implies more use of problem solving techniques. One major result of this project emphasis is that more teachers should encourage students to propose some of their own problems, thereby reducing the use of proformulated ones, and at the same time encouraging creativity... a precondition for successful living in our modern technological age.

1.2 Justification of the Study

As there is an urgent need from the Nigerian economic, social and political sectors to meet the demand for skilled manpower, it is essential that money

and time are not wasted on incomplete education, a common feature of educational wastage. Therefore, the products of a secondary school system must be trained to comprehend and think intelligently, analytically and comprehensively about the world of which Nigeria is an integral part. Time spent in a secondary school should not be wasted in memorising mathematical formulae merely for the purpose of gaining a certificate. A well-planned out programme in mathematics as in other subjects such as chemistry, physics, biology and economics - can play a major role in contributing to the general education that the young Nigeria needs. In Nigeria, therefore, the syllabus and approach to mathematics must be designed so that the content is meaningful and useful even to those students who will receive no more formal education after the end of the senior (three-years) secondary school. Willoughby (1967) suggested that the knowledge acquired should have continued significance for the students no matter what they do after leaving the secondary school. Also Falokun (1978) states that the syllabus and approach to mathematics have to bridge the gap between the academic world of the secondary school and life in school, rather than have the sharp cleavage which at present renders the school work unreal and makes its products unable to contribute effectively to the development of the country.

The purpose of this study, therefore, is to develop problems and problem solving strategies for implementation in the senior secondary school mathematics. This will not only facilitate its adoption into the new senior three-years level, but also remove some limitations that have been a hindrance to the learning of mathematics in Nigeria.

Very often, in a young person's life (if not in everybody's), involvement in problem situations leads to frustration and sometimes this may result in "wishful" thoughts, or cynicism. One really sometimes wishes that life would be constant and that the few problems therein could be done over and over again in exactly the same way, or a short-cut... a magic word could be invented for solving problems. If this were so knowing how to solve problems would not be so compelling.

Frequently the problem with senior secondary school mathematics is that there is too little problem solving. The problems the researcher refers to here are open-ended, for which the students do not have ready algorithms. Such problems require the students to employ a variety of problem solving strategies and to engage in divergent-higher level thinking. Furthermore, senior secondary school students tend to love a puzzles; and when they have

successfully solved an interesting problem, they usually are motivated to become more involved in the related mathematics. Puzzles are interesting because we regard them as games and games are fun. It is the author's contention that mathematics by puzzles can lead students to appreciate and enjoy the study of mathematics.

The justification for teaching students how to form and solve problems that may require qualitative, and analytical thinking is crystal clear to many educators. In whatever circumstances they are, in their jobs, or in some institutions of learning, students will have to be able to solve problems that very often reflect real life situations. In fact, to be able to do this is the pay-off of their education. It is not an exaggeration to say that: "...there are few, if any kinds of instruction that potentially have more values, than the considerable effort teachers ought to expand in teaching their students how to solve problems (NCTM, 1967: 233).

A relevant question that could be considered at this point is: should mathematics courses contain more problems? As noted earlier, a study of the learning-process reveals that the only way an individual can learn

how to solve problems is by solving them. Polya (1962) illustrates this by the learning of swimming. If one wants to learn how to swim, (or play tennis), one must practice.

At present, in our secondary school, mathematics teachers have not seen the importance of teaching problem-solving, either because of the ignorance about the process of learning, or from sheer neglect. Part of the blame, however, rests with the facts that most examples and problems presented to Nigerian students, up to recent times, have been foreign to them.

Our students, therefore, must study the process of problem-solving not as an end in itself; but in order that will later apply the generalizations, which in turn will enable them to transfer their problem-solving ability to new problems as they come. Woodruff (NCTM, 1967: 233) states:

> ... in face of what is known about the relative absence of transfer of learning in most school subjects, it is the height of folly to expect students to develop problem solving skills as an incidental learning unless considerable time and attention is devoted directly to it, in which case it ceases to be incidental ...

Therefore, the usefulness of this work and any subsequent work on it is that, the problems and problem solving strategies highlighted in this paper are likely to generate debates among mathematics educators and such a debate may lead to cooperative research programme among mathematicians and mathematics educators. If such cooperative research programmes are carried,out, we have a chance of making substantial progress in increasing our understanding of how mathematics curricula can be improved for the solution of future problems. This paper will highlight problems and problem solving strategies for senior secondary school and will also offer suggestions.

ORGANIZATION AND SUMMARY OF THE STUDY

CHAPTER ORGANIZATION

This paper has been divided into six chapters: Chapter I is the introduction, a formal statement of the problem, and justification of the study. Chapter II is the survey of literature. It includes

(1) Psychological Research on Problem Solving; and

(2) Research on Problem Solving Strategies, and

(3) A Summary of other literature helpful in the research procedures. These studies represent the background and point of departure of the present study.

Chapter III is problem solving teaching strategies; identified pattern in teaching problem solving such as Presentation, Attention, Observation and Exploration, Classification, Further Exploration and Generalizations, this aspect is discussed in detail.

Chapter IV is the problems of Novelty, and classification of the Novelty problems onto objectives, contents and activities/materials were discussed in details. The last part consist of topics in the 1979 syllabus and how they fit into the stated objectives, while the remaining section consist of samples of problems of novelty.

Chapter V is a selection of over hundred Exercises and Questions from WASC "O" level mathematics.

Chapter VI conclusion and Recommendations as well as comments on further necessary research in this area.

Summary

This study is centred on the problems and problem solving strategies for senior secondary school.

The study reviews past work on problem solving strategies as well as problems. The study seems necessary to fill an important gap in mathematics syllabus at a crucial time for application of the expected results.

Therefore, the point of view in this study is that enough problems and problem solving strategies should be set out in the school mathematics syllabus for the provision of teaching/learning for the senior secondary students in Nigeria.

It is hoped that this project will give planners a guide, and the opportunity for discussions on the feasibility of the problems and problem solving strategies for senior secondary school. The solution to the problems and problem solving strategies included in turn should enrich the content of the mathematics courses offered.

Definition of Terms

Definitions

A few terms will be given special meanings for purposes of this study, slightly different or more sharply focussed than the general usage of the terms in other writings on education. Before proceeding any further, it is necessary to discuss these special shades of meaning.

Problem Solving

The word "Problem Solving" occurs in many different professions and disciplines and has many different meanings. Trouble shooting, for example, is a form of problem-solving;

creating new ideas or inventing new products or techniques is another. However, problem solving in mathematics is more specific. To this study, activities classified as problem solving in mathematics include:

(a) Solving simple word problems that appear in standard textbooks,

(b) Solving nonroutine problems or puzzles,

(c) Applying mathematics to problems of the "real" work, and

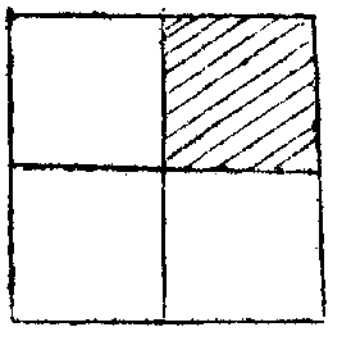
(d) Creating and testing mathematical conjectures that may lead to new fields of study.

Novelty Problem

The term "Novelty problem" could be generalized to include abstract mathematical problems or puzzles, recreational problems, and problems requiring creativity or originality, so the novelty problem will shed more light on the creative aspect of mathematics learning-teaching at the senior secondary school levels.
As an example; a farmer has four sons and owns a square piece of land. He decides to retire and take a SQUARE piece of land in one corner, equal to one-quarter the area of the original square. The remaining L-shape portion is to be divided equally among the four Sons

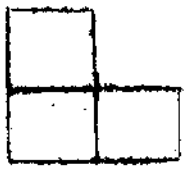
in plots of the same size and shape. How is this done?

Solution



Stages

1. What is the problem?
2. This is to be divided by 4
3. Subdivide to make it and divides by 4.
4. (a) Try squares.



Therefore 12 small squares

$\frac{12}{4}$ = 3 each to get 3 squares.

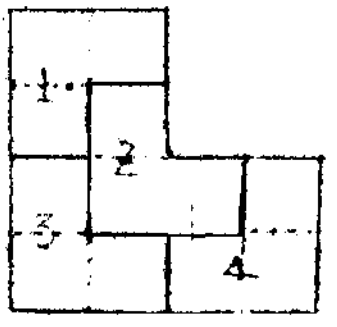
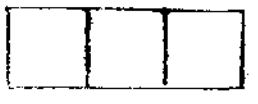
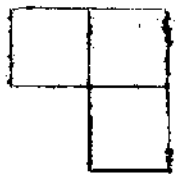


fig (iii)

Only two ways (a)



(b)



4 Trials (a) fails

(b) succeeds fig.(iii)

The student who wishes to become an expert in problem solving must solve problems. There are no two ways about it. Looking for a magic formula which will make you overnight will only lead to disappointment and frustration... solving problems appears as the most cardinal mathematical activity. "The solution of problems is the most characteristic and peculiar sort of voluntary thinking". Wrote the renowned American psychologist William James.

Question, Exercise and Problem

Thus a question requires recall, an exercise provides drill and practice, and a problem requires careful thought and synthesis of knowledge.

CHAPTER II

REVIEW OF RELATED LITERATURE

This review of literature will cover the psychological Research on problem solving, research on problem solving strategies and concluding remark.

2.1 Psychological Research on Problem-Solving

The Gestalt psychologists such as Kohler (1925) and Wertheimer (1945) described problem-solving as a matter of integrating previously learned responses. Kohler et al. (1925) mentioned that true problem-solving was insightful, meaning that the organization occured relatively sudden (the "aha" experience) and that the organisation of responses was both enduring and readily generalizable. But Shepard (1964) states that:

> Human subjects who are certainly capable of problem-solving and thought do not abandon this faculty when they enter the learning laboratory. On the contrary, in order to deal with the welter of unfamiliar material with which they are suddenly beset in the usual rate learning task, human subjects will resort to any strategy of heuristic device at their disposal.

Kohler (1925) observed that his apes needed to have previously learned the relevant responses before they could put these responses together into a complete problem-solving situation. Thus, the apes

who fit the two sticks together to reach a banana beyond the reach of either stick, had first to learn that a banana outside the case can be brought in with a stick, Kohler concluded that when the ape fit the two sticks into one, more or less by chance, the solution to the problem occured swiftly, and experience in solving similar problems may either help or harm.

Harlow (1959) conducted a series of experiments on monkeys and human subjects of low mental abilities. He concluded that: "Generalizing broadly to human behaviour, we hold that original learning within an area is difficult and frustrating, but after mastery, learning within the same becomes simple and effortless".

Lester (1977) states that out of frustration over our inability to deal successfully with the problem-solving dilemma, we have recently turned to the psychology literature for assistance. Lester further states some of the most significant advances in our understanding of the learning and teaching of mathematics have resulted from the efforts of educational psychologists. In support of this view, Davis (1960) states that the nature of the problem-solving process and factors related to learning and

teaching problem-solving have been given considerable attention by psychologists. In view of this, he maintains reviews of psychology research on problem solving begin with a statement like, "Research in human problem-solving has a well-earned reputation for being the most chaotic of all identifiable categories of human learning."

According to Lester (1977), a variety of tasks have been used in problem solving research and the tasks found in the research literature include puzzle problems, anagram problems, concepts. Identification problems, arithmetic computation problems, and standard textbook story problems as well as many other types. Lester states that problem solving research has been conducted by experimenters with very different positions on the nature of problem-solving. Also behaviorists, developmental psychologists, gestaltists, and proponents of information processing approaches have all contributed to the research literature on problem solving.

Among them, Simon (1976) stated how contemporary information processing approaches could change our current notions about thinking and learning. Some suggestions seem relevant for teaching mathematical problem-solving: viz;

> An important component of problem solving skill lies in being able to recognize salient problem features rapidly... and the skills of searching for effective problem representations are probably learnable and teachable skills...

Another prominent psychologist, Maier (1970) found certain features significantly affecting the difficulty of a problem as:

"Misleading, incorrect solutions,

__ requirement for generating an unusal response,

__ difficulty in locating surmountable obstacles,

__ type of demands made on idea-getting versus idea-evaluating processes,

__ motivation factors and degree of stress."

Also Brownell (1942: 11) wrote a classic paper of problem-solving that includes a section on "Practical Suggestions for Development Ability in Problem-Solving", and stated that: "When a learning situation is intended to be a problem, the relationships necessary to its solution should be well within the child's understanding and identifiable by him with reasonable effort."

In review of the educational literature dealing with problem solving, the psychologists and educators have studied problem-solving in a variety of contexts in reviews and discussions of these researches (Cronbach, 1954; Bichler, 1975; DeCecco, 1968; and McDonald, 1965) make general conclusions that:

> Problem solving is impeded when the problem solver is anxious, threatened, or confused as to the nature of the problem and its solution specific procedures for solving problems tend to improve an individual's ability to solve similar problems but tend to inhabit resolution of problems that require a "new" approach... when an individual understands a principle and has had an opportunity to practice its recognition and employment in a variety of situations, then that individual is able to 'transfer' knowledge of the principle in subsequent problem situations, (Gagne et al. 1975: 12).

Robinson (1977: 22) distinguishes the difference between an exercise and a problem that:

> An exercise is a verbal rentetion of a mathematical sentences, or a verbal description of a situation of which some specific process applies; while problem requiring creativity, originality are descriptions of situations for which no routine process has been previously learned.

Reys and Krulik (1980) state that whoever develops or chooses problems must know something about "good" problems. When textbook authors, or teachers develop or select a problem, they must consider these its levels of interest and difficulty as they relate to the learner.

Although a few studies have investigated what types of problems students prefer, no relationship of these preference to problem-solving success was

found (Travers, 1967; Cohen, 1977). If textbooks continue to fail to present problems that challenge pupils and that provide teachers with meaningful instructional materials for developing problem-solving skills, then teachers must accept the challenge in which Wheatley (1977: 36) supports the view that:

> Students need to be faced with problems in which the approach is not apparent, and encouraged to generate many alternative approaches, testing each hypothesis generated, ... students will come to understand the nature of problem-solving.

Research has focused, in part, what makes problems difficult or easy. Cromer, (1971); Jerman, (1972); Loftus, (1974); and Meyer, (1976) have studied the difficulty level of problems by using such techniques as factor analysis, in which a computer manipulates data on students' answers until the data cluster into a number of groupings, or by ascertaining the relationship of the data from any one factor to total success. Cromer, (1971) classified such factors as mathematical, reading, reasoning, and process variables. Beardslee and Jerman (1973) explored the difficulty level of a large number variance in difficulty level for the computational problems considered. The studies suggest that

factors of mathematical achievement accounted for the most variance according to (Jerman 1972; Sagiv, 1978; Zaleushi, 1978 and Webb, 1979). These studies suggest that a problem is more difficult to solve when it is wordy, requiring many complex, operational steps and techniques to arrive at a solution.

So far we have discussed many advantages and disadvantages in various methods of teaching mathematics to student. It will be noted that the review so far tend to be in favour of the problem-solving approach, which is the major thrust of this research. The researcher is convinced that an innovative design and teaching strategies of problem-solving for secondary school students will greatly enhance the teaching learning processes in mathematics education in our schools.

Other research conducted by Ammon (1973) and Bolduce (1970) showed that manipulative materials, pictures, diagrams, and similar aids enhance the probability of a problem being understood and solved correctly. But Robinson (1973) noted that there is no relationship between the use of materials and problem-solving success, on the other hand, Suydam and Higgins (1977) support that the use of concrete and pictorial aids facilitate mathematics education studies of secondary school children.

Some studies have considered the structure of problems at early grade levels (Steffe and Johnson, 1971). Nelson, (1977) attempted to structure problems that are "good" for young children who are not sophisticated in the use of mathematical symbols. Nelson further stated that the physical structure of a problem can make one more difficult to solve than others, and requirements of one task can influence young children's choice of procedure in attempting another task when the same apparatus is involved.

The National Council of Supervisors of Mathematics (1977) asserts that "learning to solve problems is the principal reason for studying mathematics", and the National Council of Teacher of Mathematics (1980) recommended that "problem-solving be the focus of school mathematics in the 1980s". The impact of these documents and the 1980 NCTM Yearbook on problem-solving may well insure that problem-solving will be the "theme of the 1980's (in Krulik, 1980) in school mathematics".

There are parallel developments at the university level. Halmos (1980) "advocates more emphasis on the problem approach to teaching". A subcommittee of the Mathematical Association of America's Committee on the teaching of undergraduate mathematics (1980) is now surveying problem solving courses and plans

a report "making recommendations regarding the place *of* problem-solving in the curriculum and ways to teach it", (p.22*0.

The emphasis in recent mathematics research and in the NCTM I98O Yearbook indicate that there will foo a major stress on heuristics, both in research studies and in classroom instruction Suydam (I98O), in a research synthesis, writes that "research evidence otrongly enhanced by teaching students to use a wid© variety of strategies on heuristics, both general and specific". But Lester (I980) provides a broad and somewhat more restrained overview of the problem-solving literature© In both cases, the bulk of the research cited is very tentative. Correlations between the use of heuristics and competent problem-solving performance have been obtained (Kantowski. 1977)i and results in a variety of experiments suggest that problem instruction via heuristics will enhance sttidents[1] problem-solving performance (Lucas, 1972; Smith, 1973; Goldberg, 1975)-

Gorman (1967) analyzed research on word problems conducted between 1925 and I965 and identified 37 studies that were of acceptable quality. Riedesel {I969) lists several procedures for improving problem-solving based on research* Kilpatrick **(1969)**

reports on the problem-solving research from 1964 to 1969 have similar implications to the present research.

Several researchers have examined the structure of problems and the effect of reading skills upon the solution of problems. Burns and Yonally (1964) found that pupils made significantly higher scores on problems in which the numerical data were in proper solution order.

A number of other factors contribute to problem-solving achievement. Studies of Stevenson (1925) and Corle (1958) reveal that pupils often give little attention to the actual problems; instead, they almost randomly manipulate numbers. Smith and Hedders (1954) found that experimental materials and commercial textbook materials, were at a reading difficulty higher than that prescribed by reading formula analysis. Several researchers have identified factors associated with high achievement in problem-solving. According to Klausmeier, 1961: 148 states some traits as:

> Intelligence, Computational ability, ability to analyze problems, arithmetic vocabulary, ability to use quantitative relationships that are social in nature, ability to note irrelevant detail, and knowledge of arithmetical concepts.

Problem setting has also been found to be important to problem-solving achievement (Bowman, 1932). His

findings are mixed. However the generalizations that problems should be of interest to the children and relevant to lives are justifiable conclusions.

2.2 Research on Problem-solving Strategies

Moser (1980) expresses a concern about the use of manipulative materials. He found that children in the primary grades used simpler and less efficient processes (such as counting) when cubes or other objects were available. When materials were removed and numbers made larger, the children used more sophisticated strategies to solve addition and subtraction problems.

Marcucci (1980) analyzed and integrated the findings from 33 studies conducted since 1950, that methods of teaching problem solving more classified into four general types: viz;

1. Modelling - a method characterized by the use of visual aids, manipulative materials or physical models to illustrate the conditions or relationship in the problem.

2. Systematic - a method emphasizing a prescriptive approach to problem solving.

3. Guided discovery - a method where appropriate questioning strategies are used to guide a student to discover the solution of the problem.

4. Heuristic - a method stressing the teaching of general problem-solving skills such as drawing diagrams or simplifying a problem by using smaller numbers.

The heuristic type of problem-solving instruction was not effective, according to the data Marcucci reanalyzed. The systematic approach produced a slightly positive effect, while the modeling and guided discovery methods were about as effective as the problem-solving instruction received by a central group, Sowder et al (1977) also recommended teaching problem-solving strategies as a result of their analysis of research.

Although Putt (1979) found no significant differences in the problem-solving performance of fifth-grade groups taught or not taught heuristics, the way in which students went about solving problems changed as they were taught heuristics. There was a marked difference in the way they viewed the problem solving process, in the type of questions they asked for understanding a problem, in the strategies they used for solving problems, and even in the way they named these strategies.

Bredie (1980) reported sixth graders could learn heuristics from programmed materials, while Metwali (1979) found that giving few problems plus discussion between students and teacher was more effective than working many problems. In addition to finding that training on heuristics increased

problem-solving performed. Herrington (1980) noted that the traditional textbook approach of presenting a step-by-step procedure for problem-solving appeared insufficient. Students profited significantly more from an approach involving logical steps for organising information before applying appropriate computational procedures.

Zweng et al. (1979) tried to find out how to help children solve problems by interviewing 162 pupils in grades 3 through 6. When a child had difficulty deciding what to do, the interviewer provided:

(a) a leading questions,

(b) a modification of the statement of the problem, or

(c) a directive to use manipulative materials or drawings.

The leading question of most help focused the child's attention on the action, on part-whole relationships, or on the size of the answer.

Carpenter et al. (1980) strongly encouraged teachers to have children analyze a problem and devise a plan. Children should identify the relevant data in a problem and decide whether it is sufficient to solve the problem. They should apply a process or relationships in the problem, rather than solely applying some algorithm to whatever numbers are given.

In support, Carpenter et al. (1980b) stated that:

> Students do not learn problem-solving strategies as isolated skills to be used only in special problem-solving lessons... Problem solving should be an integral part of all instruction. If students are to learn that problem solving is an important mathematical activity, they need to see their teachers using basic problem solving strategies when attacking a new problem...

Zweng et al. (1979) also notice children's difficulty with multistep problems. They reported that by sixth grade most students could solve most single-step problems without help. Multistep problems were far more difficult, and there was no discernible increase in performance between grades 3 and 6. Rate problems were even more difficult than multistep problems for all except sixth graders.

Lack of reading skill is frequently proposed as an important ingredient contributing to poor problem-solving ability. Evidence from many studies has indicated that a correlation exists between reading ability and problem solving scores (Lee, 1980), although some previous research indicated that it might account for only a small proportion of the variance (Kinfong and Holtan, 1977). These was also evidence that instruction on reading skills can lead to higher problem-solving scores (Muraski,

1979). Also Lutes (1926) compared diagrammatic-graphic procedures with nondirected experience in solving many heterogeneous problems, but there was no conclusion in the experiment.

Furthermore, several interview studies have shown that even when students were taught formal procedures for solving problems they actually used their own devices first and then put their results in the form expected by the teacher (Burch 1949; Corle, 1958).

Bichler (1976: 86) offers the following general suggestions on how to teach students to solve problems. Teachers should:

> make students aware of how they are solving problems, make sure that students have the necessary prerequisies for working a problem, encourage an atmosphere conducive to free inquiry and the expression of ideas, set an example as an intuitive thinker, and keep in mind that students have individual differences in problem-solving ability.

Although evidence is inconclusive on the best methods of teaching learning mathematics there are indications that students improve in their ability to solve mathematical problems when they are made aware of both general, and task-specific problem solving principles, and when they have opportunity

to practice using these principles. However, many research in secondary school mathematics teaching indicate that very little time is spent on problem-solving and more time is devoted to discussing a method than solving the problems.

It is the author's contention that the above findings relevant as they might be to this research, are by no means conclusive. This research is intended to combine various strategies in the teaching-learning of mathematics through the puzzles/game approach. This method, it is hoped will involve both the teacher and the learner as joint ventures in our educational enterprise.

Polya in his classic work How to Solve it (1954) sets forth principles that can be used in solving-problems, namely; understanding the problem, devising a plan, carrying out the plan, and looking back. Also Polya (1962: 30) states that:

> The first rule of teaching is to know what you are supposed to teach, and teacher wishing to impart the right attitude of mind toward problems to his students should have acquired that attitude himself.

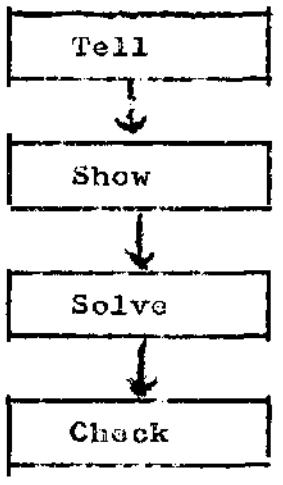
Helping students learn problem-solving is an important goal of school mathematics and it has not been widely advocated in Nigeria. This is why is of special

significance to the study because lends credence to the importance and implications to the present research.

Kilpatrick (1969) make the following observation with reference to Polya's principles:

> Modern interest in heuristic the study of the methods and rules of discovery and invention is due principally to Polya (1957, 1962, 1965), who has set forth principles for problem-solving which he postulates, correspond to mental actions...

Polya's theoretical steps could be translated into classroom activities as indicated below, and this comparison will give:



a guide for a model to be presented in the following chapter. Some researchers have adapted Polya's steps and have used them (or variations of them) in developing instructional sequences for teaching problem-solving.

From the above literature review so far, it is then possible to examine and identify problems and problem solving strategies which will add to the learning and teaching mathematics. In the next chapter, the author will identify the problem solving strategies what are associated with mathematics teaching in Nigeria and offer suggestions for better teaching/learning in mathematics.

CHAPTER III

PROBLEM SOLVING TEACHING STRATEGIES

This study was developed from the author's previous experience with Diploma II:, "Problem Solving in Mathematics Education", and B.Ed/B.Sc. II, ME200: "Strategies of Secondary School Mathematics Teaching" at Ahmadu Bello University, Zaria.

This chapter deals with problem solving teaching strategies for students, preservice and inservice teachers - which will serve as an additional unit in the syllabus of mathematics for senior secondary school. The procedures on strategies of problem solving: Identified pattern in teaching problem solving, presentation, attention, observation, and exploration, classification will be discussed and further exploration and generalizations will be enumerated.

Problem-solving situations are frequently manufactured superficially by the teacher or by commercial dealers in educational materials and they may misrepresent the mathematical problems encountered in the student's world. Therefore, materials should be provided which honestly require mathematical skills which the students can presently employ in problem-solving situations.

If teachers are to develop the problem solving capabilities of students, it is necessary to prepare

these teachers in this crucial art and skill of problem-solving. The preservice teacher is young, full of enthusiasm, and has not as yet adopted the erroneous belief that teaching specific algorithms to solve specific problems is the only answer to teaching problem solving.

Fundamental to a preservice programme for teachers of mathematics is the clarification of the meaning of the terms Problem and Problem-solving. Many teachers use the words Question, Exercise and Problem interchangeably, but there are differences. As an example, let's look at the following:

What is the product of 8 and 7? or

What is 8 x 7?

Is this a question? An exercise? A Problem? or is it all three? It depends!

Thus a question requires recall, an exercise provides drill and practice, and a problem requires careful thought and synthesis of knowledge. In support, Krulik and Rudnick (1982) state:

> What is a problem at one time to one person may be an exercise or even a question at some later stage of what person's mathematical development. Furthermore, what is a problem for one person can easily be an exercise or a question for someone else.

A preservice program should also be explicitly stated and carefully demonstrated to the students that problem-solving is a process. As a process, problem-solving is the means by which an individual uses previously acquired knowledge and understanding to satisfy the demands of an unfamiliar situation. The student must synthesize what he or she has learned and apply it to the new and different situation. A preservice program should take future teachers through the same kinds of activities and experiences that they will require of their own students over years, hence, like any other skill, problem-solving requires of practice. The teacher has to learn or invent the games himself/herself.

There are two components of the issue of problem-solving for the classroom teacher. One is the practical side. What is my rationale for the choices I am making in the classroom? What do I do with the students to promote problem-solving? The other is the theoretical side. There is no separation between these two aspects of responsible and effective classroom practice. Theory without practice is useless; practical application without a theoretical base is irresponsible (Burns, 1982). Teachers must have opportunities to solve problems as a regular and

on-going part of their duty experience. For teachers to produce students that will be problem-solvers they need to blend the two together.

Teachers need models of problem-solvers who are active learners to emulate, seekers who are willing to plunge into new situations, not always knowing the answer or what the out-come might be. It's important that teachers focus on the thinking involved, not just the answer. Since much of our curriculum is answer-oriented. This Project , therefore, provides us with the perfect opportunity to make the curriculum process-oriented. Teachers must be willing to learn along with their students, and to share their own striving with the students when some problems are difficult, so that students can see the struggle along with the teacher. These were the observations made by the researcher during the teaching of problem-solving to Diploma II of Mathematics-Education, Ahmadu Bello University, Zaria 1981/82 Session.

Very often teachers pretend to know all the answers to the problems. This makes them, in the eyes of the students, to possess a monopoly of knowledge which to the researcher, is erronous. The researcher is advocating an integrated approach in problem-solving whereby students and teachers see a problem

together, define and design how to tackle it together. Thus the students become learners and teachers as well. Our rationale is that when students are given the opportunity to teacher and learn, they can therefore proceed to learn on their own since one of the aims of education, is teaching children how to learn.

The researcher also noted that teacher-training programs in mathematics education that have a weakness in common, namely, concentration on skills at the expense of thinking. This is true for facets of the preservice education of mathematics teachers, and for mathematical content and for pedagogy.

Cognitive mathematical behaviour can be classified into three broad categories or levels. The first involves memorization of facts, definitions, rules and procedures. At this level a student is judged successful if he remembers, what he has read from books or taught by his teachers, and can meaningfully reproduce them in the same context. For example, a student who has been shown a proof that the sum of the measures of the angles of a triangle is 180° can recall, reproduce, and understand the proof, but he may not have the ability to use this information to find the sum of the measures of the angles of a quadrilateral.

The mental activity of generalizing or transferring knowledge from one learning context to another, similar one is the second level of mathematical cognitive behavior. For example, the person who operates at this level of thinking should understand the operational algorithms of base ten and also be able to understand these algorithms in some other bases, particularly when the student has access to the appropriate basic knowledge which we hope will enable him generate new knowledge.

The third level of mathematical thinking involves reorganising and restructuring the variables in a problem so that new relationships which facilitate the finding of a solution can emerge. Avital and Shettleworth, (1968) describe this as the "Open search" level of mathematical thinking and experience that is missing from so many teacher education programs for prospective teachers of mathematics in Nigeria.

Krulik (1982) shows a flowchart of the problem solving process (see figure 1) the boldly outlined command boxes indicate the procedures that are basic to the problem solving process. In our classroom this set of rules occupies a central position. Reference is constantly made to it in our attempt to

Fig. 1 **A Flow Chart of the Problem-Solving Process**



Adapted from Krulik, Stephen and Rudnick, Jesse *Problem Solving: a Handbook for teachers* Allyn and Bacon, Inc. Boston, Mass. 1980

help the preservice teachers become good problem-solvers. The same flowchart will later serve as the basis for the teachers' own teaching guide.

For most preservice teachers, the most difficult and important part of the problem-solving approach is to "select a strategy". A wide variety of problem solving strategies must be identified and illustrated by the use of many problem situations. Some problems and solution strategies just naturally fit together. Indeed, more than one strategy may be necessary to solve a particular problem. The researcher maintain's that preservice teachers should be encouraged to find multiple solution strategies as shown below.

Strategies for Problem-Solving

1. Pattern Recognition
2. Working backwards
3. Guess and test
4. Simulation or experimentation
5. Reduction
6. Exhaustive listing
7. Logical deduction
8. Representing data
   (a) Graph
   (b) Equation
   (c) Algebraic expression

(d) Table

(e) Chart

(f) Diagram

To support this Rednick, (1982) states that:

> It is important to continually emphasize that the goal of problem solving lies in the solution process, not necessarily in the answer itself.

This is more so because, a student can very possibly guess the correct answer to a problem; but what use is the answer to him if he does not know or understand the process?

Generally speaking, in the teaching of problem solving, there seems to be a pattern that takes the form of the following strategies:

> "Presentation, attention, observation and exploration, classification, further exploration and generalisation", (NCTM, 1959).

Several lists of stages for solving problems have been proposed (Suydam, 1980), many of which are based on Polya's (1973). "How To Solve It". The specific list used is not as important as the fact that students be equipped with some general frame-work they can apply in problem solving situations.

The teaching of solving almost all types of problems fall under the above mentioned patterns. The stages will be considered briefly:

3.10 Identified pattern in teaching problem solving.

a. Presentation

In preparing for the development of thinking process, the factors which require the closest attention are how the problem situation is chosen and how it is presented to the students.

b. Attention

The aim of the presentation stage is to arouse interest and thus further stimulate the netural curiosity of the students. At this stage the teacher should be ready and patient to meet all sorts of responses. In such a situation the teacher can make students respond faster by raising some questions which will help them think alright. What should always be encouraged is the use of questions, thus promoting an enquiring attitude.

c. Observation and Exploration

Very often the first attempt at a solution is not satisfactory, but certainly sensible results can always be obtained after comparing the several trails the student has made, which however could neither be proved or disproved by him or anybody else . . . then what happens? the next step below should followed?

... Classification

In trying to organise his data into a recognisable pattern, the interested student could employ several methods, one of which is graphing. That is done by tabulating the different dimensions with their respective in different columns. Thus the student is able to plot the graph which will lead him to a better understanding of the solution of the problem. Students, therefore, should be encouraged to tabulate and graph their results in an orderly manner, because a study of successive trials might even reveal mistakes in calculations, which could not have been discovered in any other way.

Further Exploration

Periodically progress apparently comes to a temporary halt, when the student re-examines his data, and reflecting upon other possible ways of collecting the facts, seeks and considers alternative solutions. It is here that he begins to ponders the possibilities of other outcomes if, for example, the initial conditions were changed, or he may relate his solution to similar problems previously solved.

.. Generalizations

Through generalizations, the student is enabled to apply his knowledge of the solution of a problem

to another similar problem. It is the teacher's duty to see that his students do not have compartmentalised techniques of solving problems. However, the extent of a student's generalization may depend upon his acquired knowledge and insight to solve problems.

Key Ideas in Teaching Problem Solving

This section is not intended to be an entirely new insight in the teaching of problem-solving, but it is devoted to sorting out the important aspects concerned; at least the ones considered important in the opinion of the researcher on the strength of his design.

Experience in the daily life of an individual can bear out the fact that one may have less problems than he encounters sometimes if he sets his priorities right.

Whereas various stages from both the teacher's and the student's point of view were considered in the process of solving a problem in the last section, here attention is focused on the teacher alone and his specific duty of teaching problem solving. Johnson and Rising (1967), enumerated the key ideas as follows:

(i) The teacher should provide students with the right atmosphere conductive for learning how to solve problems.

(ii) The teacher should establish a high level of motivation for his students and maintain the same.

(iii) The teacher should devise means of helping students to understand a given problem situation.

(iv) The teacher should encourage flexibility and variety in problem solving approach.

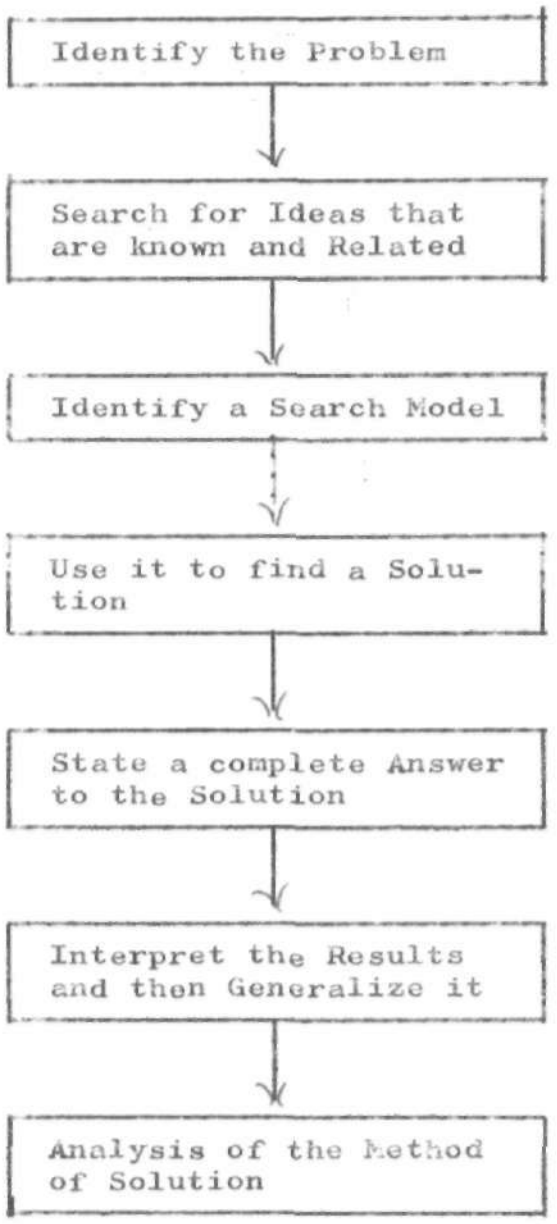
(v) The teacher should guide the students on how to construct a "search model". Consider a particular case used to make a simple construction as indicated below by Earp. (1970).

(vi) Above all, the teacher must be protected and tolerate in his approach to teach problem-solving techniques.

(vii) Coach the students in asking useful questions and giving suggestions.

(viii) The teacher should make the students aware of the fact that the method of solution is more important than the solution.

(ix) The success in solving problems is in the solving of problems, so the teacher should provide adequate practice at frequent intervals.



A flowchart showing the pattern in solving a problem. (Figure 3.2). Earp, (1970).

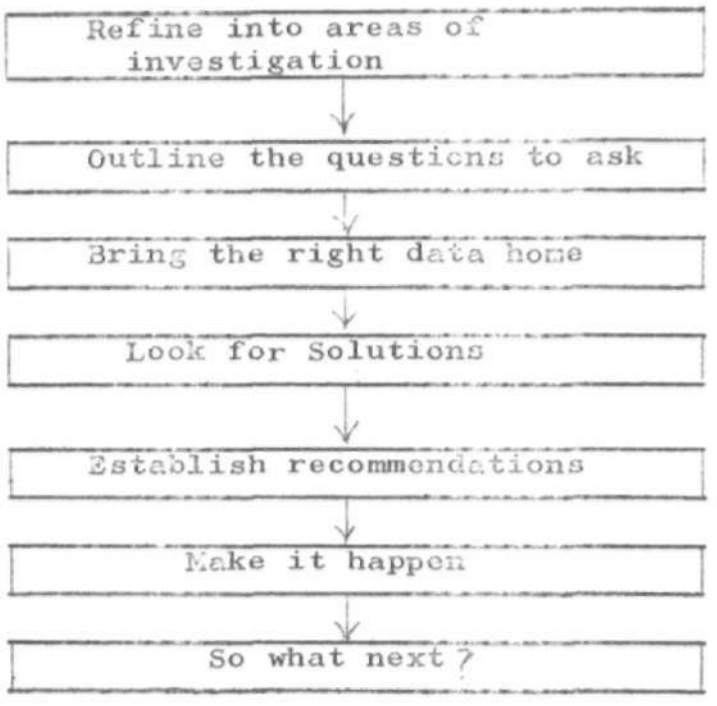
No doubt the task of the teacher in teaching problem solving is a difficult one. The emphasis should be on motivating students, on engineering in them a spirit of inquiry, or promoting an attitude conductive to analytical thinking. A bare answer to a problem is no longer acceptable - what is important is the development of a solution and an analysis and generalization of the same. The researcher hopes that through this analytical method, students will more readily and easily recongize and understand the binding effects of basic mathematical concepts; and having accomplished this, the structure of this important and core subject should be within easy grasp of the average senior secondary school students.

Providing problem-solving experiences in the classroom means finding suitable problems of novelty to present to the students. Teachers need to introduce the problems, to allow the students the time they need to explore, and then summarize their results with them, drawing on all of their ideas to lead them towards the appropriate and realistic conclusions. The teacher's focus needs to be on stretching the students' thinking. The point of the researcher is not that this emphasis is to replace concerns about mathematics skills, where the focus

may be on accurate results, but rather, that both aspects of the mathematics program are essential, for all students, at all grade levels.

It is useful for purpose of instruction and sometimes for purpose of research of this to consider the problem solving process as consisting of several steps. A number of different multi-step models have been proposed in the literature review in chapter II of this project. The number of steps ranges from four to seven or even more, and their names and descriptions also vary, but all the models have been proposed as attempts to refine and better define the complex process of problem solving strategies.

With all these various models and considering the student concern, the researcher adopted and adapted problem solving models of Polya (1961) and Earp (1970) and comes up with a new model as indicated below that will serve as a guidance for problem solvers.



A flourchart showing stages in solving a problem.(fig.iv).

So far, these are various strategies for problem solving. In the next chapter, author will classify novelty problems in terms of objectives, contents, and activities. And objectives of novelty problems will be classified into the new (1979) syllabus which comprises the following topics:

(i) Number and Numeration
(ii) Algebraic Processes
(iii) Measuration
(iv) Plane Geometry
(v) Trigonometry
(vi) Statistics and,
(vii) Probability.

Finally, samples of novelty problems will be presented.

CHAPTER IV

PROBLEMS OF NOVELTY

This section of the paper consist of novelty problems, classification of problems of Novelty in terms of objectives, contents and activities and these objectives will be classified into the new (1979) syllabus which comprises the following topics:

(i) Number and Numeration

(ii) Algebraic Processes

(iii) Measuration

(iv) Plane Geometry

(v) Trigonometry

(vi) Statistics and,

(vii) Probability.

The term "Novelty Problem" could be generalized to include abstract mathematical problems or puzzles, recreational problems, and problems requiring creativity or orginality, so these Novelty problems will shed more light on the creative aspect of mathematics learning-teaching at the senior secondary school levels.

The emphasis in novelty problems are on process and not product. There is no concentration in the early levels on the "right" answer. The emphasis is on analyzing the data, determining the facts, posing

questions, and so on.

The sequence goes from readiness, to strategies, to analyzis of word problems, to brain storming. The subsection on brainstorming is an attempt to encourage the students to approach problems of novelty in a creative manner. The following observations are placed on a tabular form (Table 4.1) that problems of novelty can be classified in terms of objectives, contents and activities just as the new (1979) syllabus for mathematics. The intention is to point out that there is no one way to solve problems. There are many methods and procedures, and students are encouraged to explore as many as possible. Under the subsection on strategies, the intent is not so much that students be experts in each strategy as it is that students, by their study of the various strategies will develop a memory bank to full on when they are faced with a problem to be solved. This was indicated in chapter III of this project. Also Kinsella (1970) indicated that solving novelty problems is the specific achievement of intelligence; intelligence is the specific gift of man. The ability to go round an obstacle, to undertake an indirect course where no direct course presents itself, raises the clever animal above the dull one, raises man far above the most clever of animals, and men of talent above their fellow men.

TABLE 4.1

THE PROPOSED DESIGN OF STUDY: SENIOR SECONDARY LEVEL

TOPIC	OBJECTIVES/AIMS	CONTENTS	ACTIVITIES/MATERIALS
Novelty Problems	Students will be able to: 1. Solve problems by seeing patterns and relationships. 2. Learn effective ways of solving problems by drawing correct inferences through inductive and deductive reasoning. 3. Read critically in order to improve their thinking skills. 4. Express themselves clearly, by the proper use of quantifiers' language. 5. Recognize the major components of the premises and the conclusion of a good argument or word problem. 6. Learn through common sense and example, the importance of determining the truth value of a premise and the validity of an argument.	Readiness (a) Opportunities for communication in quantitative language. (b) Recognition of number in everyday use. (c) Dramatization of number situations. (d) Defines and solves problems using manipulative materials in a variety of situations. Strategies (a) Number patterns to solve problems. (b) Identification of the question being asked in a simple word problem. (c) Concrete or pictorial representation to solve simple word problems.	"Think" Problems The problems in this category, consisting of riddles, puzzles, and word problems, were designed to help students solve simple problems and analyze the reasoning they used to reach their solutions. For example: when does 7 plus 6 equal 1? Critical Reading Problems were designed to help students practice critical reading: listing the facts, working directly from what is given and determining exactly wh what the question asks:- For example: Is the following reasoning correct?

Novelty Problems	OBJECTIVES/AIMS	CONTENTS	ACTIVITIES/ MATERIALS
	7. Use the techniques of set theory to solve problems. 8. Use diagrams, and charts as aids to novelty problems. 9. Construct their own logic problems. 10. Use opportunities for communication in quantitative language. 11. Select pertinent data from a problem. 12. Determine the existence of a problem situation and identify relevant information. 13. Extend the use of guess and verify. 14. Analyse problem situations and develop alternative solutions.	(d) Guess-and-verify strategy (e) Organised list to solve problems. (f) Charts, diagrams, tables, to aid in the solution of problem. Analysis of Word Problems (a) Introduce reviewing a problem and its solutions. (b) Selection of pertinent data from a problem. (c) Identification of problems with sufficient/ insufficient data. (d) Identification of explicit/implicit information. Brain Storming (a) Defines and solves problems using manipulative materials in a variety of situations. (b) Analyzes and interprets puzzles and brain teasers.	"Our national soccer team won the championship, since I am on the team, it follows that I played very well". Sample problems are in the remaining part of this chapter.

NOVELTY PROBLEMS	OBJECTIVES/AIMS	CONTENTS	ACTIVITIES/MATERIAL
		(c) Reads and discusses a number of problem statement, defines the problem situations, and identifies data relevant to solving the problem. (d) Solve problems using manipulative material such as toys and blocks. (e) Determines possible methods of solution of a variety of verbal and concrete problems. (f) Examines both concrete and explores alternative solutions. (g) Discuss problems methods to solve real-life situation problems.	

So far, the objectives (Table 4.1) mentioned above will be classified into the following topics of the senior secondary mathematics syllabus (1979), and see the suitability of each item.

(i) Number and Numeration

(a) Solve problems by seeing patterns and relationship.

(b) Read critically in order to improve their thinking skills.

(c) Use the techniques of set theory to solve problems.

(ii) Algebraic Processes

(a) Select pertinent data from a problem.

(b) Express themselves clearly, by the proper use of quatifiers language.

(c) Recognize the major components of the premises and the conclusion of a good argument or word problem.

(d) Learn through common sense and example, the importance of determining the truth value of a premise and validility of an argument.

(iii) Measuration

(a) Select pertinent data from a problem.

(b) Analyse problem situations and develop alternative solutions.

(iv) Plane Geometry and Trigonometry

(a) Learn effective ways of solving problems by drawing correct inferences through inductive and deductive reasoning.

(b) Learn through common sense and example, the importance of determining the truth value of a premise and the validility of an argument.

(c) Use diagrams, and charts as aids to novelty problems.

(d) Construct their own logic problems.

(e) Use opportunities for communication in quantitative language.

(v) Statistics and Probability

(a) Use diagrams, and charts as aids to problem solving.

(b) Construct their own logic problems.

(c) Select pertinent data from a problem.

(d) Extend the use of guess and verify.

(e) Analyse problem situations and develop alternative solutions.

If education fails to contribute to the development of the intelligence, it is obviously incomplete, states Polya (1962). Yet intelligence is essentially the ability to solve problems: everyday problems, personal problems, social problems, scientific problems, puzzles, and all sorts of problems. The student develops his intelligence by using it, he learns to do problems by doing them. Then, which kind of problems should a senior secondary school student do in order to develop his ability to solve problems? The remaining section of this chapter will be a selection of novelty problems for secondary schools and general remarks.

A SELECTION OF NOVELTY PROBLEMS
(FOR SECONDARY SCHOOLS)

1. Try to establish that 2=1, let x and y be any two nonzero real numbers such that x = y.

2. A square Cake of uniform height is iced evenly on its top and on its four sides. Describe how to cut the cake into five pieces (vertical cuts only) such that each piece has the same amount cake and the same amount of icing.

3. A farmer has four sons and owns a square piece of land. He decides to retire and take a square piece of land in one corner, equal to one-quarter the area of the original square. The remaining L-Shaped portion is to be divided equally among the four sons in plots of the same size and same shape. How is this done?

4.

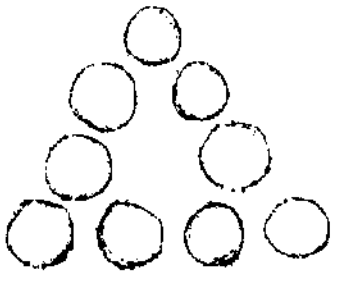


figure 4.1

Average the digits 1 to 9 in the triangle shown above so that the sum of each side is 17. (Note: one digit does not appear more than once). Is there another "sidesum" (other than 17) using the digits 1 to 9? What are all the possible sidesums?

5. Musa had six sticks of matches. He told his son to use the match sticks to form a triangle in a square such that two heads of matches must be meeting at the same point. Form the figure.

6. In the diagram shown below, determine the size of the shaded area.

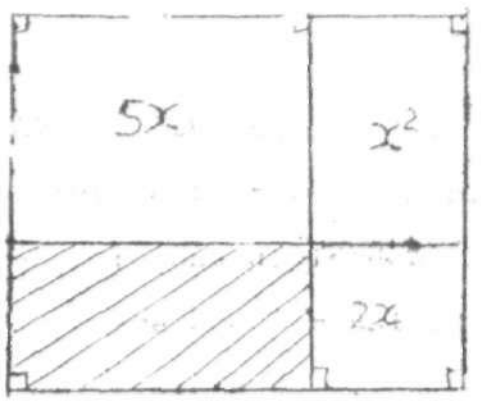


figure 4.2

7. Given 1, 2, 3, 4, 5, 6, 7, 8 and 9, complete the table on the next page so that when you add along the row or column or diagonal your answer is always 15.

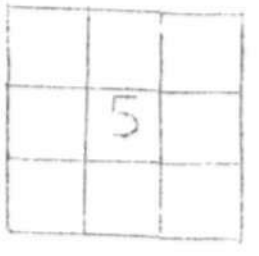


figure 4.3

8. A construction company in Zaria stocked six barrels as shown below. Each barrel was numbered, numbers being 1, 2, 3, 4, 5 and 6. Can you re-stock the barrels so that the totals of the numbers in each line of three barrels are the same?

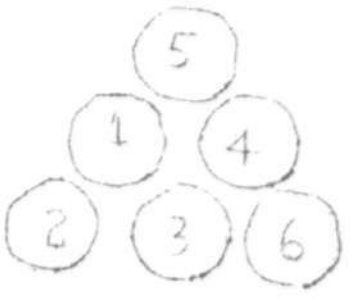


figure 4.4

9. Use each numbers 1 through 7 only once. Arrange them so each row adds to 12.

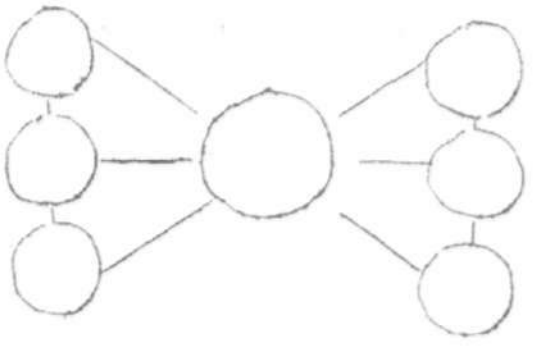
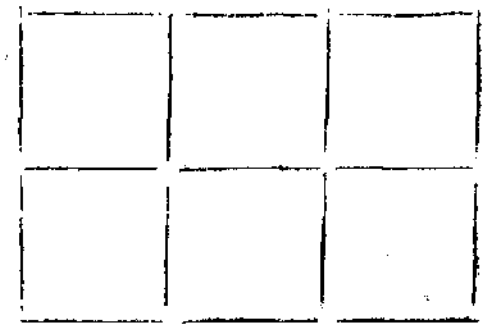


figure 4.5

10. $12^2 = 144$ If you write both numbers backwards you get the true statement $21^2 = 441$. Find other examples of this behaviour.

11. A child described what he saw to the mother like this: There were two ducks in front of a duck, two ducks behind a duck, and one duck in the middle. The question is: How many ducks were there?

12. Make a shape like the one below using match sticks can you take any 5 matches and leave only 3 squares (of the same size)? figure 4.6



13. Hannatu is 8 years old and 1.4 metres tall. If she continutes to grow by age. What will be her height when she is 24 years old?

14. At a social gathering at Demonstration Secondary School meant to welcome from one students of the school, we have 21 boys and 20 girls. If the sitting arrangement is done

alternatively in one row starting with a boy. Who will be at the centre a boy or a girl?

15. Rebecca and Jane has the same age and Rebecca is 1.5 metres in height. What is Jane's age?

16. At Ojo's birthday party, his mother said that his age is a prime number, his father argued that his age is an even number. Their little son Ojo who is a mathematics student discovered that his parents were correct. What is Ojo's age?

17. Without picking up your pencil, what is the least number of straight lines are required in order to pass through each collection of points so each point is touched once, and only once?



figure 4.7

18. A man has a goat, some yams and a lion. He wants to cross the river with them and the only boat could carry him and one of the items only. Now if he leaves the goat with yams, the

goat will eat the yams and if he leaves the lion with the goat the lion will eat the goat. So how would this man cross with these items so that the goat will not eat the yams and the lion will not eat the goat?

19. When does 7 plus 6 equal 1?

20. What part of your body are you unable to touch with both hands?

21. Take three oranges from four. How many do you have?

22.

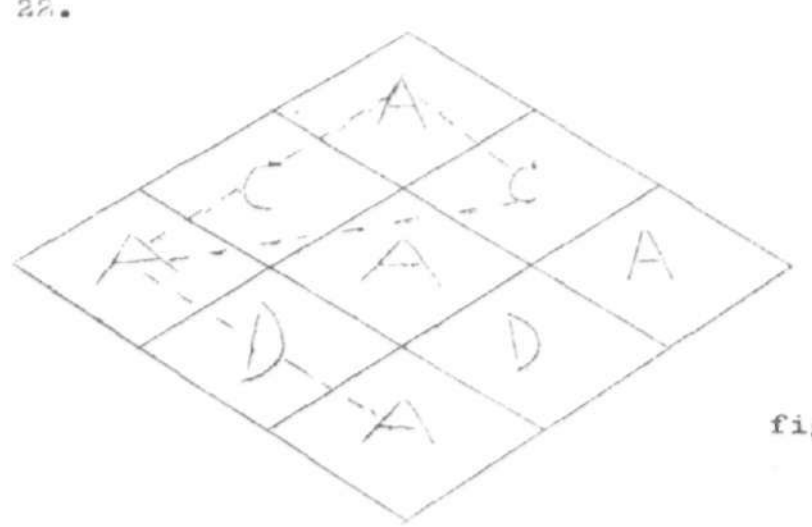


figure 4.8

Above is a magic square formed with letters from the word "Acada". The dotted lines show how the word can be formed using 2 different ways. How many different ways are there altogether? (Always start at the top). Attempt to form a similar magic square with the word "Joseph". Can you explain your result?

23. In how many ways can you spell the word "MATHEMATICS" starting from the top and working down through the following array? See the figure below.



figure 4.9

24. In a secondary school in Zaria, there were 30 students and 30 lockers. Each year for students lined up in alphabetical order and performed the following strange ritual: The first student opened every locker. The second student went to every second locker and closed it. The third student went to every third locker and changed it. (i.e. if the locker was open, he closed it, if it was closed, he opened it). In a similar manner, the fourth, fifith, sixth, . . . lockers, which lockers were open? (Also try for 100 lockers).

25. Complete the sequence 1, 1, 2, 3, 5, 8, 13, ... into three places. (Note: give the next 3 terms of the sequence).

26. Complete the square below so that all rows, columns, and diagonals have the same sum.

$3\frac{1}{2}$		
		$-2\frac{1}{2}$

figure 4.10

27. In order to encourage his son in the study of algebra, a father promised to pay his son eight naira for every problem solved correctly and to fine him five naira for each incorrect solution. After 26 problems neither owed anything to the other. How many problems did the boy solve correctly?

28. When Mr. and Mrs. Bello took the airplane from Kaduna to Lagos, they had together 94 kilograms of bagggage. He paid ₦1.50 and she paid ₦2.00 for excess weight. If Mr. Bello made the trip

by himself with the combined baggage of both of them, he would have to pay ₦13.50. How many kilograms of baggage can one person take along without charge?

29. A father who has three sons leaves them 1600 cows. They will states that the eldest shall have 200 cows more than the second, and the second shall have 100 cows more than the youngest. Find the share of each (Euler).

30. Mallam Musa leaves four sons, who share his property in the following manner:

    The first takes half of the fortune, minus 3000 naira.

    The second takes a third, minus 1000 naira.

    The third exactly one fourth of the property.

    The fourth takes 600 naira and the fifth part of the property.

What was the whole fortune, and how much did each son received? (Euler).

31. Mallam Ojo leaves at his death several children, who share his property in the following manner:

    The first receives a hundred naira and the tenth part of what remains.

The second receives two hundred naira and the tenth part of what remains.
The third takes three hundred naira and the tenth part of what remains.
The fourth takes four hundred naira and the tenth part of what remains,
and so on.

Now it is found at the end that the property has been divided equally among all the children. Find how much the total property was, how many children there were, and how much each received? (Euler).

32. Draw the figure in the next page without lifting your pencil from the paper, you are allowed to cross the same segment with your pencil more than once; but it can be done without any crossings.

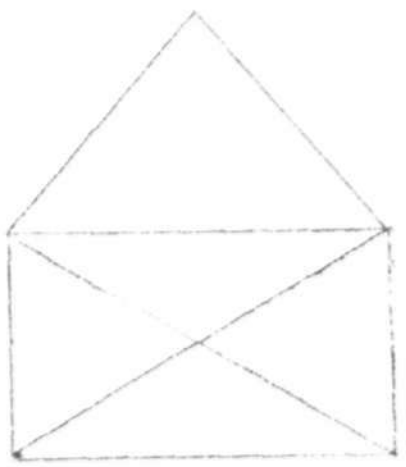


figure 4.11

33. Three persons play together. In the first game, the first loses to each of the other two as much money as each of them has. In the next, the second person loses to each of the other two as much money as they have already. Lastly, in the third game, the first and second person gain each from the third as much money as they had before. They then leave off and find that they have all anequal sum, namely, 24 naira each. Find how much money each had at the start. (Euler).

34. A farmer has hens and rabbits. These animals have 50 heads and 140 feet. How many hens and how many rabbits has the farmer?

35. Jumoke is 12 years old and 1.4 meters tall. What will be her height when she is 30 years old? If her growth is progressive with her age.

36. Find two numbers whose sum is 11 and whose product is 24.

37. A rectangle is inscribed in a quadrant of a circle as shown below. Given that the length of segment OA is 5cm and that of segment AP is 1cm, find the length of the diagonal AC.

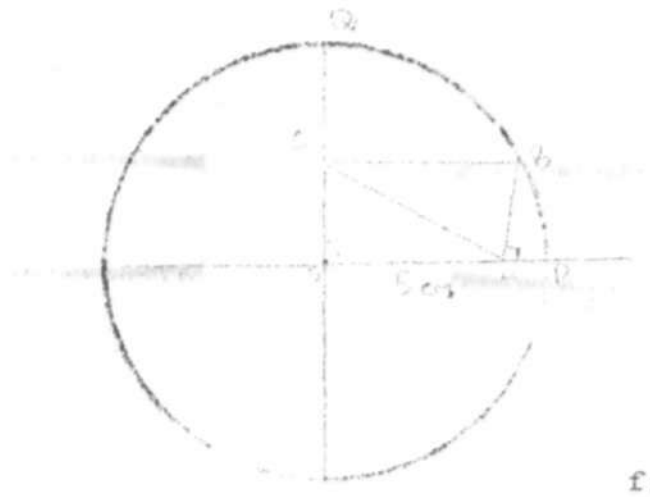


figure 4.12

38. When Ali solved a problem he got an incorrect answer of 16. In the last step, Ali took the square root of the number rather than squaring it. What was the correct answer?

39. 5 cats can catch 5 lizards in 5 minutes. At this rate, how many cats are needed to catch 100 lizards in 100 minutes?

40. Replace the letters with sing digits.

(a)
$$\begin{array}{r} \text{FORTY} \\ +\ \text{TEN} \\ \text{TEN} \\ \hline \text{SIXTY} \end{array}$$

(b)
$$\begin{array}{r} \text{ZZZZ} \\ +\ \text{YYYY} \\ \text{ZZZZ} \\ \hline \text{YXXXZ} \end{array}$$

(c)
$$\begin{array}{r} \text{SEND} \\ +\ \text{MORE} \\ \hline \text{MONEY} \end{array}$$

(d)
$$\begin{array}{r} \text{SAVE} \\ +\ \text{MORE} \\ \hline \text{MONEY} \end{array}$$

41. How many prime number are there between 1 and 100?

42. A drawer contains red and green socks. You want a pair of matched socks. If you reach into the drawer without looking at the socks, what is the least number of socks you would have to have out to be certain of a matched pair?

43. S, H, and E are different digits. Replace the letters with numbers to make the statement true. $(HE)^2 = SHE$.

44. Musa whose age is five years weighs 20 kilogram. What will be his weight when he becomes 15 years old? Progressively.

45. How many triangles can you find in this star?

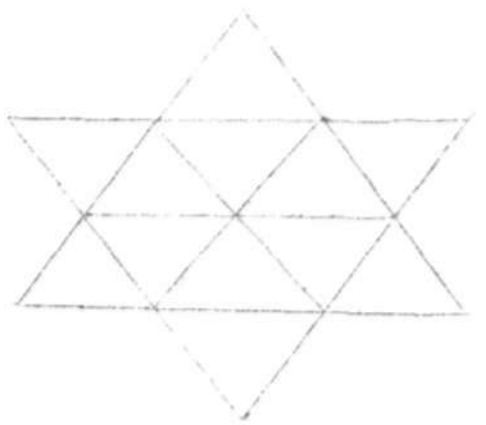


figure 4.13

46. 17 and its reverse, 71 are both primes. Name three other 2 digit primary numbers whose digits, when reversed, produce prime numbers.

47. How many zeros are found in order at the end of 40! (40 x 39 x 38 x ... x 1)

48. How many squares are on a 8 x 8 checkerboard?

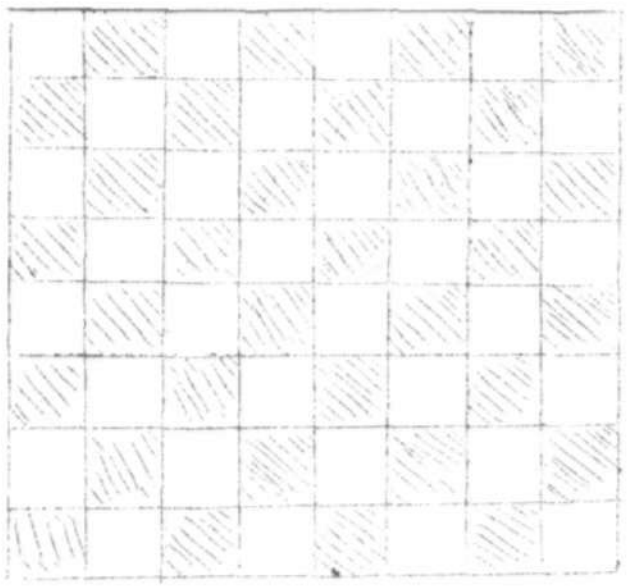


figure 4.14

49. The product of two whole numbers is one million. No digit of either number is a zero. What are the numbers?

50. Ali and Sule live at opposite ends of the same street. Ali had to deliver a parcel at Sule's home, Sule one at Ali's home. They started at the same moment; each walked at constant speed and returned home immediately after leaving the parcel at its destination. They met the first time face to face, coming from

opposite directions, at the distance of a kilometers from Ali's home and the second time at the distance of 6 kilometers from Sule's home.

(i) How long is the street?

(ii) If a = 300. and b = 400 who walks faster?

51. There is a number (an integer) which yields a square if you add it to 100 and another square if you add it to 168. Which number is it?

52. Ahmadu's stamp collection consists of three books. Two tenths of his stamps are in the first book, several seventhe in the several book, and these are 303 stamps in the third book. How many stamps has Ahmadu? Is the condition sufficient to determine the unknown?

53. The area and the perimeter of a right-angled Triangle being given, to find the Hypothenuse. (Newton)

54. Having given the sides of any parallelogram and one of the Diagonals, to find the other Diagonal.

1. General Notes

The Teaching - Learning Strategy of the proposed chapter 3 can adopt any of the following teaching methodologies provided they can lead to meaningful learning of the concepts by students.

(a) Lecture,

(b) Demonstration,

(c) The equiry approach,

(d) Simulation and games,

(e) Discussion,

(f) Individualized instructions,

(g) Group method,

(h) Any other appropriate method.

2. The examination in novelty problems should be designed generally to test students in the ways listed by Days et al. (1979) to:

(i) Recall mathematical facts

(ii) Synthesize the facts with a view to make accurate description and explanation of the problem.

(iii) Interpret and make sense of mathematics in novelty problems.

(iv) Apply the knowledge gained to the solution of natural and man-made problems.

Finally, Kalejaiye (1981) concluded that it is only through a conceptually based programme which this project aims at accomplishing that problems of novelty can come from a variety of sources. Many can be developed from the textbook.

Content being studied while others can involve model situations from life outside the classroom or through mathematical curiosities and recreational puzzles.

So far, the researcher has discussed novelty problems, how the stated objectives (Table 4.1) fits into the new (1979) syllabus and a selection of novelty problems were presented. In the next chapter, the author will discuss questions-exercises and about one hundred questions-exercises will be presented.

CHAPTER V

QUESTIONS-EXERCISES FROM THE SECONDARY SCHOOL SYLLABUS

The new (1979) syllabus is a compromise between the old "traditional mathematics" and "modern mathematics". The objectives, the contents, and activities for each year of every level of education are clearly spelt out, and are in line with the National Policy on Education. This is intended to simplify the work of the teacher, thereby facilitating effective teaching. The new (1979) syllabus comprises the following topics:

(i) Number and Numeration

(ii) Algebraic Processes

(iii) Measuration

(iv) Plane Geometry

(v) Trigonometry

(vi) Statistics and

(vii) Probability.

Since an exercise provides drill and practice and a question requires recall and while problems of novelty requires careful thought and synthesis of knowledge. To complete, an exercise, one must first determine the applicable algorithm, then perform the relevant mathematical manipulation. Both steps are necessary; neither alone is sufficient. Similarly, novelty problem has two aspects; (a) the process set of behaviours or activities that direct the search for the

solution, and (b) the product, or the actual solution. Both the process and the product are essential components of the problem solving experience which have been lacking in the school mathematics. And, since the main objectives in the teaching of mathematics at the secondary level (to develop necessary mathematical background for further education) but inclusion of problems and problem solving strategies will serve the same purpose in developing precise, logical and abstract thinking in the students.

From the above listed topics, the author extracted about one hundred questions-exercises and their answers at Appendix II of this project.

A SELECTION OF OVER HUNDRED QUESTIONS-EXERCISES FROM WASC "O" LEVEL MATHEMATICS

1. Compute the mean mark and the standard deviation of the following marks scored by 12 students in a mathematics test:

   50, 53, 49, 54, 56, 51, 48, 50, 57, 51, 56, 52.

2. 15 students in a class never play football, 18 never play handball and 20 never play hockey. 2 play all three games, 8 play just two of the three games and 15 play just one of the three games.

   (i) How many students in the class do not play football, handball or hockey?

(ii) How many students are there in the class?

(iii) What is the greatest possible number of students that can play just hockey?

3. A club has 50 members who play either basketball or football or both; 30 of them play basketball while 35 members play football. How many member play both games?

4. In a physical science class of 34, 24, students offer chemistry, 10 take both chemistry and mathematics, 11 take both chemistry and physics, and 12 take both mathematics and physics. Twenty offer physics and 16 take mathematics. How many students take all three subjects?

5. (a) Find the inverse of $f(x) = 3x - 6$

(b) Draw the graph of $y = x^2 (4 - x)$ for values of x from -1 to 4. Estimate from your graph the greatest value of y.

What is the value of y when.

(i) $x = -\frac{1}{2}$

(ii) $x = 1\frac{1}{2}$

Find the values of x corresponding to $y = 3$.

6. (a) A triangle ABC has sides AB = 5cm, BC = 3cm, and AC = 7cm.

(i) What is the measure of angle ABC?

(ii) A perpendicular from B to $\overline{AC}$ meets $\overline{AC}$ at D. Find the length of this perpendicular.

(b) A ladder 8m long leans against a vertical wall and is inclined at 31° to the horizontal. How far up the wall does the ladder reach?

7. ABCD is a parallelogram in which the coordinates of A and B are (0, 0) and (3, 3) respectively. Calculate the coordinates of vertices C and D if the diagonals of the parallelograms intersect at the point (4, 2).

8. Find the equations of the lines which make an angle and with $\overline{OX}$ such that tan = ¾, and of length 2 units from the origin.

9. Determine the equation of the line through the point (6, -1) which is perpendicular to the line $y - 3x - 1 = 0$.

10. Find the equation of the line that passes through the point (3, 1) and parallel to $2x + 5y - 4 = 0$.

11. In figure 5.1 A, B and C are the centres of three touching circles. AB = 14cm, BC = 10cm, and AC = 18cm. Find the radius of each circle.

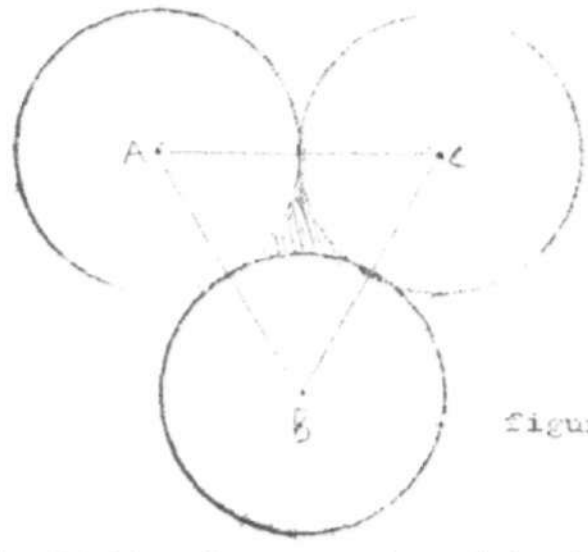


Figure 5.1

12. (a) Eight spheres each raduis 1cm are fitted into a hollow cube of edge 4cm. Find the volume of the unoccupied space inside metal cylinder.

(b) Calculate the area of a sphere whose diameter is 7cm.

13. Solve the simultaneous equations.

(i) $2x + 3y = 1$

$3x^2 + 7xy + 4y^2 = 0$

(ii) $2xy - x^2 = 24$

$2y^2 - xy = 30$

14. (a) In a single throw of 2 dice, find the probability of obtaining a total score of (i) 9 (ii) 10.

(b) There are three groups of cards in a closed bag: 3 green and 1 black, 2 green, and 2 black, and 1 green and 3 black cards.

One card is drawn at random from each group. Find the probability that the three cards eventually selected are one green and two black ones.

15. (a) (i) A "fair" coin (one that is properly minted and that cannot stand or edge when tossed) is tossed once and then tossed twice.

(ii) A die is thrown once.

For (i) and (ii) state all possible outcomes in each case.

(b) Four coins are tossed simultaneously. Write down a list of the possible outcomes.

16. Find the equation that fits this graph below.

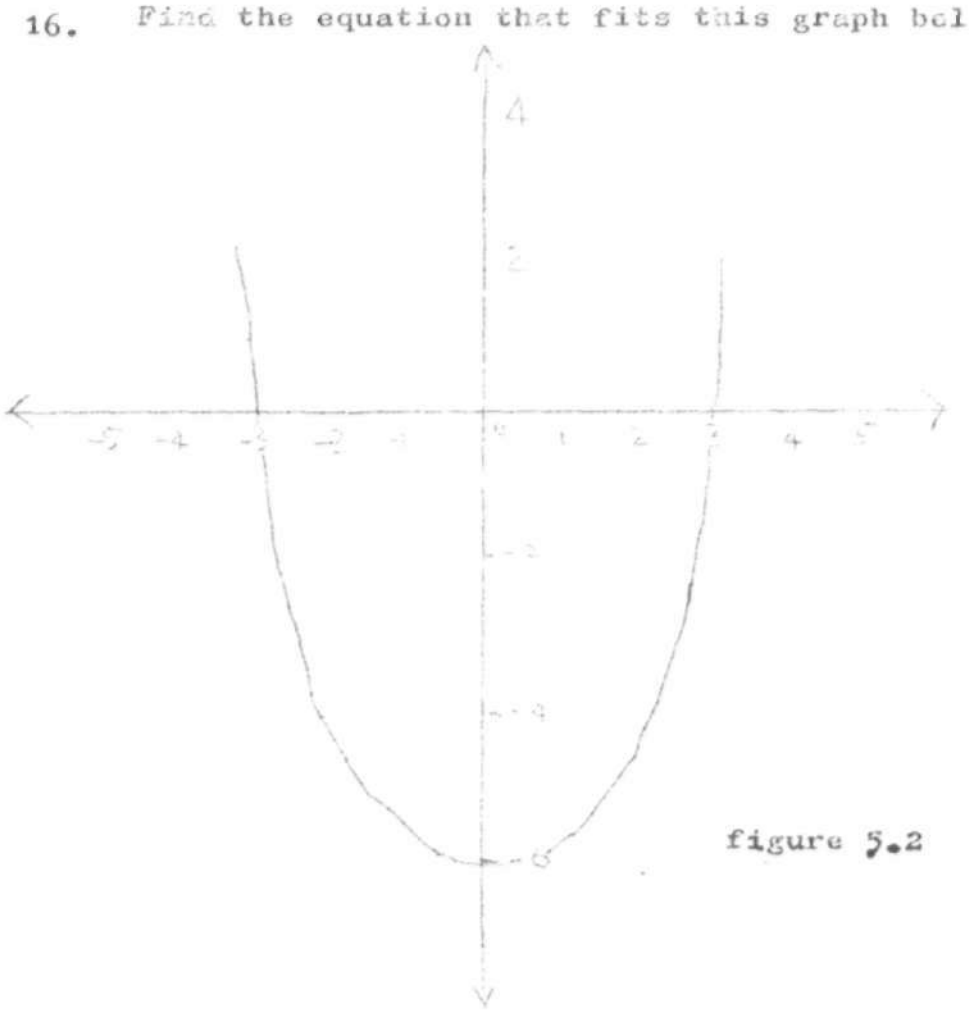


figure 5.2

17. (a) If $\frac{1}{3}x^{\frac{1}{3}} = 1$ find x

(b) Simplify $\frac{(-x^3y^6)^{\frac{1}{3}}}{(xy^4)^{\frac{1}{4}}}$

(c) Given $\frac{x + y}{x + 3y} = \frac{2}{3}$. What is x:y?

18. Find the average of the three areas marked I, II and III below. The large square has an area of $9m^2$. O is the centre of the square and P and Q are mid points of sides.

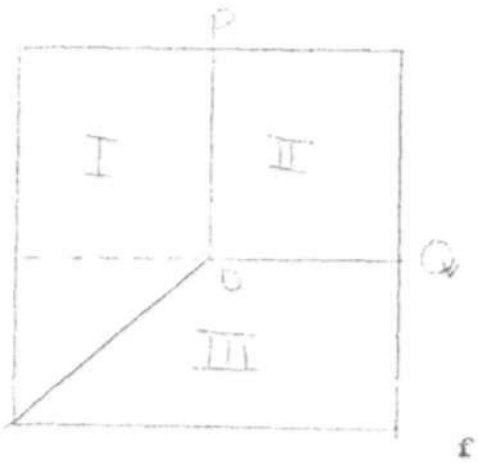


figure 5.3

19.

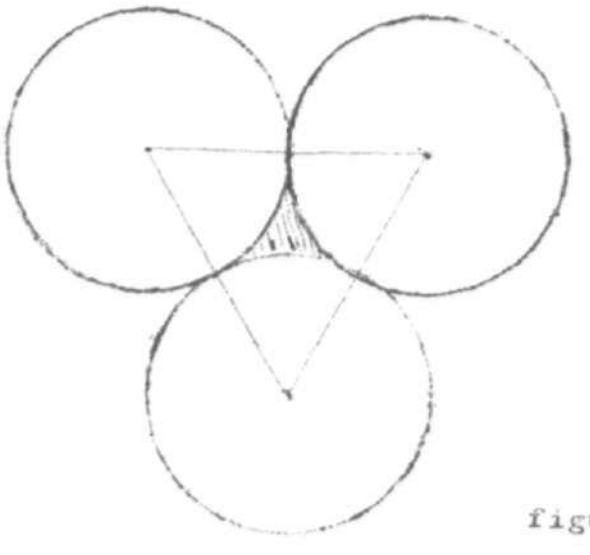


figure 5.4

The diagram above shows three circles of radius r, touching each other. The centres are joined to make a triangle. What is the area of the shaded portion.

20. If $n^{4/3} = 1296$ find n

21. Solve for x; $\dfrac{\frac{1}{x}}{\frac{1}{a} - \frac{1}{b}} = 1$

22. A fuel tank contains $6\frac{1}{2}m^3$ of kerosine. What is this volume in litres?

23. A piece of paper of the shape in the next page is to be folded to form an envelope. The central portion is a rectangle of sides 6cm x 4cm and the mid-points rectangle of sides A, B, C and D. The length of the altitudes of the triangular portion are shown. Find the total surface area (both sides) of the piece of paper.

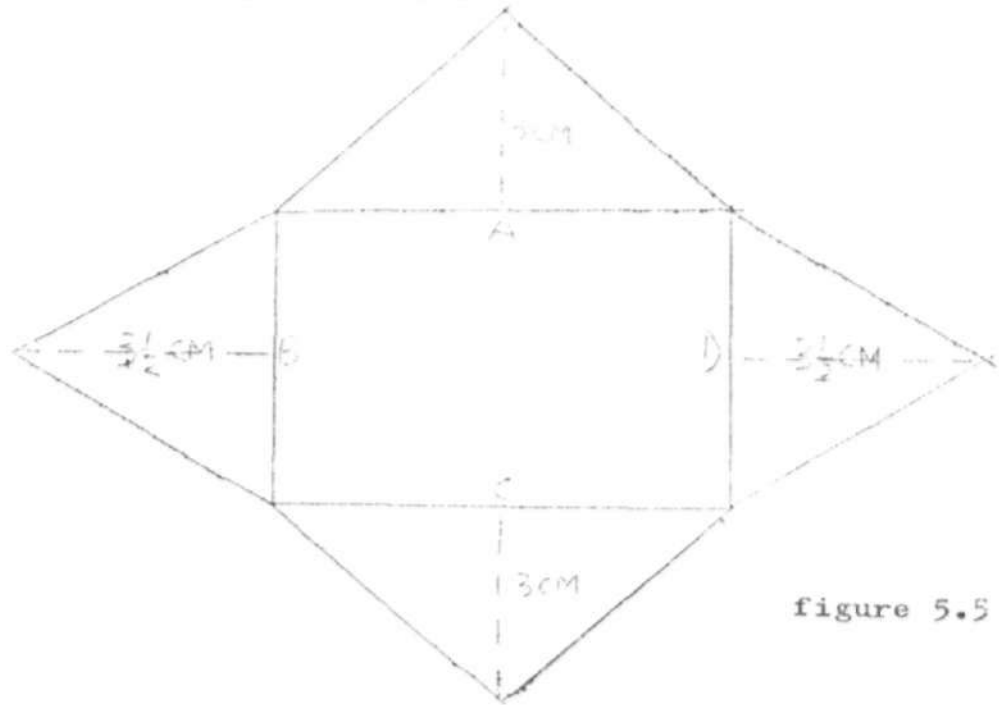


figure 5.5

24. If 2^2 is raised to the power 2^2, what is the result.

25. A man left his money to his four sons to be divided in the ratio 2:3:4:6 if the largest share was ₦240, what was the total amount of money left to the sons?

26. Olu's watch gains m minute per hour and Bisi's watch loses n minute per day. They get their watches together on time at 9.00 a.m. What difference in seconds will be between the watches at noon?

27. Simplify $\frac{-14a^5b^8c^3}{49a^3b^2c^2}$

28. The figure shown three squares, one inside the other what is the shaded area?

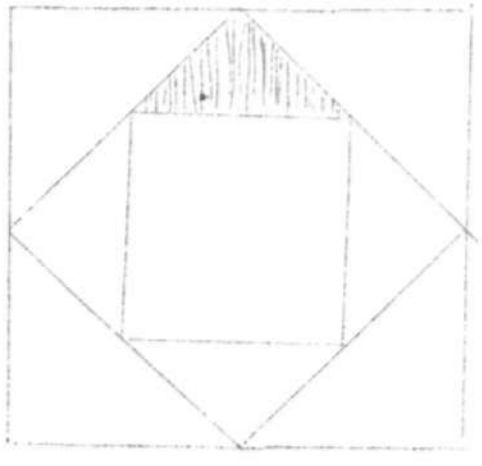


figure 5.6

29. The figure shows a pattern of squares arranged such that each vertex of an inner square is the mid-point of a side an outer square The outer

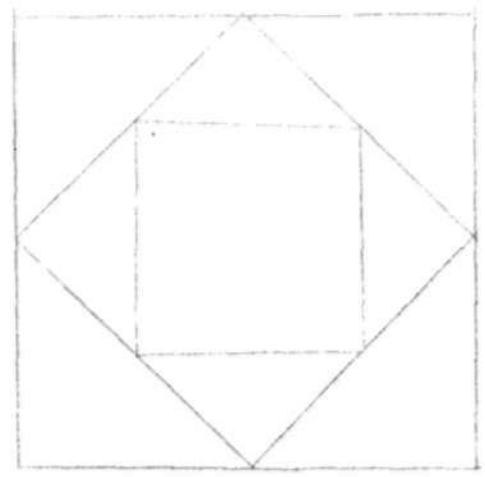


figure 5.7

most square has sides of length 6cm. Find in cm^2, the area of inner most square.

30. Hassan invested ₦100 in a business for 3 months. Ali invested ₦40 for 6 months, and Musa invested ₦60 for 2 months. Hassan receives ₦15 of the ₦33 profit. How much of the profit does Musa receive? The profit varies as the amount invested and the time for which the investment is made.

31 A student paid 60% of his school fees. There was then balance of ₦10.50 to pay. What were the total fees?

32. Segun bought a peugeot car for ₦6400.00. After one year he reinsured it for 60% of its original value and paid a premium which was nominally 8½% of the sum insured. Calculate the premium actually paid if there was a no-claims bonus of 40%

33. The employees of a factory are expected to contribute to the National Provident Fund in proportion to their monthly salaries. If the factory manager who is on a monthly salary of ₦300.00 contributes ₦120.00, what will a clerk on a salary of ₦150.00 contribute to the Fund.

34. Ade bought a motorcycle for ₦570.00 and sold it for ₦600.00 while his friend Ojo bought a car for ₦3700.00 and sold it for ₦4000.00. Which of them made the greater profit per cent?

35. (i) Simplify $\sqrt{12} - \sqrt{75} + \sqrt{27}$

(ii) Rationalize the denominator $\frac{5\sqrt{3}}{3+2\sqrt{5}}$

36. A man earns ₦2500.00 per annum, out of which he spends 15% on his children's education. How much has be left for other expenses?

37. Use calculator to evaluate:

(i) $\frac{1980 \times 0.754}{31.82\pi}$

where $\pi = 3.142$

(ii) $\frac{T}{p} = \frac{1980 \times 0.754}{31.82}$

Use the result in (i) to deduce the value of p, correct to three significant figures.

38. A number consists of three digits with the digit 4 in the middle. The sum of the digits is 18. If the first and the third digits are interchanged, the original number is decreased by 198. Find the number.

39. Using a rule and pair of compasses only, construct a parallelogram XYZ such that XY = 6.4cm, YZ = 4.7cm and angle XYZ = 120°.

40. Musa, a motor mechanic, bought the following spare parts from the wholesale department of New Nigeria Motors Limited: Six pockets of Bosch spark plugs at ₦7.00 per packet of ten, four tubes of gasket cement at ₦2.20 per tube, twenty packets of generator brushes at ₦1.95 per packet of two, one set of valves consisting of four inlet and four exhaust valves at ₦2.20 per valve, three water pump repair kits at ₦10.45 each, five carburettor repair kets at ₦15.5 each, three sets of overhauling gaskets at ₦12.20 a set. Prepare the bill, allowing a discount of 5% for cash payment.

41. Simplify $\frac{(1-(4)(-5) \times (-2)(-3)}{(-6)(-2) - (-7)(-3)}$

42. The populations of two states are in the ratio 3:5. The states share ₦26 million in ratio of their populations. What will be the share of the state with the smaller population?

43. The sum of money ₦30000 is deposited into a bank. If the interest at the end of 4 years is ₦3600. Find the rate at which the bank pays the simple interest per annum.

44. Expand $(2x+1)^3$

(ii) Factorize $x^2 + x-6$

(iii) Solve the equation $3x + \frac{1}{x} = 4$

45. If $x = 3$ is a root of the equation $x^2 + kx + 3 = 0$, Find the value of k.

(ii) Given $V = \frac{1}{3}\pi r^2 h$, find r when $V = b\pi$ and $h = 2$

46. Simplify

(i) $(225^{\frac{1}{2}} + 85^{o}) \times 256^{-\frac{1}{4}}$

(ii) $\frac{2p}{q-2} \times \frac{2^{-1}p^{-1}}{q2}$

47. (a) Add 1011_{two}, 101_{two} and 1110_{two}, leaving your answer in base two.

(b) If $142_n = 47_{ten}$, find n.

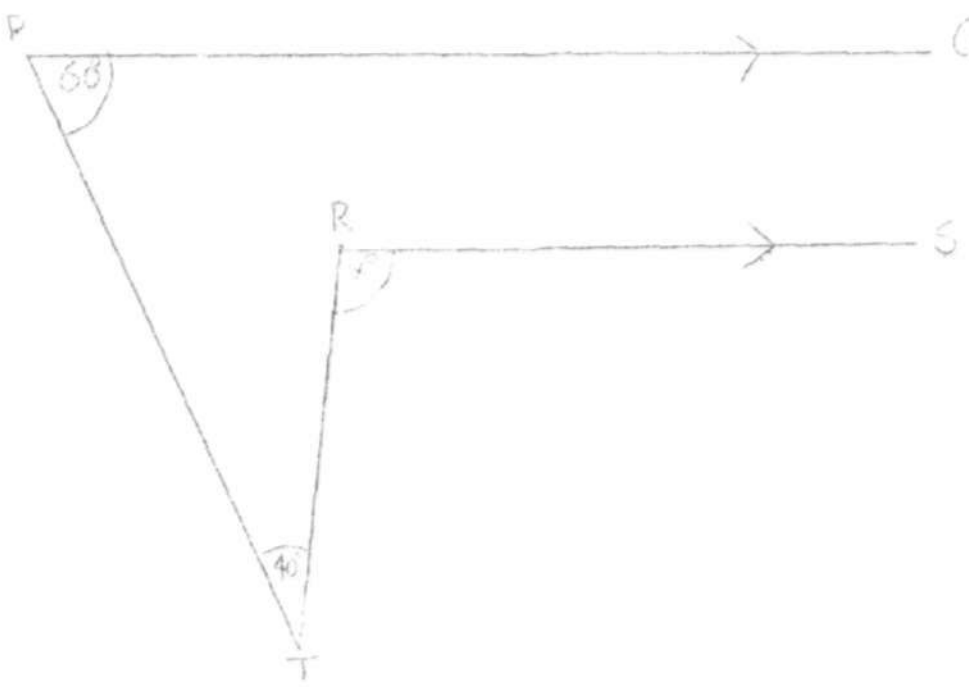
48. Find a two figures number knowing that the sum of its figures is 10 and that the number is 36 greater than the number with reversed figures.

49. A bag of corn can feed 250 chickens for 30 days. How long would the same bag feed 100 chickens?

50. Solve the equation $125^{(x + 3)} = 5$

51. Find the value of n such that $3^n = \frac{1}{9^2}$

52. In the figure 5.8, PQIIRS, angle QPT = 60^o, angle PTR = 40^o, and angle TRS = x.



Calculate the value of x. figure 5.8

53. In an examination Ali scored 72% in mathematics, x% in physics and 58% in chemistry. If his mean mark in the three subjects was 60%, calculate x.

54. One interior angle of a regular polygon is 140°. How many sides has the polygon?

55. Sule, Bala and Musa share a certain sum of money in the ratio 4½:6:10½ respectively. If Musa has ₦1.20 more than Bala, find how much Sule receives.

56. A book cost a retailer ₦1.52. He fixed the selling price at 25% higher than the cost price and then allowed a discount of 10% to schools. How much did a school pay per book?

57. In a group of 85 students 6 students were absent, in another group of 74 students, 4 students were absent on the same day. In which group was the percentage of attendance was higher?

58. At the beginning of the 1981/82 school year, a man invested ₦2000.00 in a bank at 12½% compound interest. If he withdrew ₦650.00 at the end of each school year to pay his son's school fees, how much had he left at the beginning of the 1982/83 year?

59. A man firing at a target can score from 2 to 7 points. The following table shows the results obtained when he fired 20 shots.

Score	2	3	4	5	6	7
Frequency	2	4	5	3	4	2

(a) State the modal score

(b) Estimate the median

(c) Calculate the mean

60. A motorist who did 30000 kilometers in a certain year observed that his car consumed fuel at the rate of 1 litre to 25 kilometers, while petrol cost 15½ kobo per litre. He spent ₦5.00 on oil per month, and ₦30.00 on his vehicle licence and ₦200.00 on insurance for that year. Calculate, correct to the nearest kobo, the average expenditure on his car.

(i) Per kilometre

(ii) Per week.

61 The following are the marks scored by 50 candidates who took part in a mathematics examination.

93	70	35	83	60	61	73	44	90
64	88	56	52	61	43	57	40	58
95	71	52	25	86	79	35	73	44
82	67	63	48	78	65	98	28	72
68	50	46	62	35	70	41	63	73

Using class interrals of 21-30, 31-40, ...

(a) form the frequency table;

(b) calculate the mean mark of this distribution;

(c) draw the cumulative frequency curve and use it to find:

(i) the median

(ii) the interquantile range.

62 Solve the simultaneous equations

$$2x + 3y = -4$$

$$3x - 2y = 7$$

63. (i) Evaluate (¼ of 38) ÷ $(2\frac{1}{3} - \frac{3}{4})$

(ii) Find the solution of the equation

$$\frac{2}{3}(x+5) = \frac{1}{4}(5x-3)$$

(iii) Factorize $6x^2 + 11xy + 4y^2$

64. The adjacent sides of a parallelogram are 4cm and 6cm. The distance between the longer parallel sides is 2.5cm. Calculate the area of the parallelogram.

65. The pie chart represents the percentage scores of a student in his end-of-term examinations.

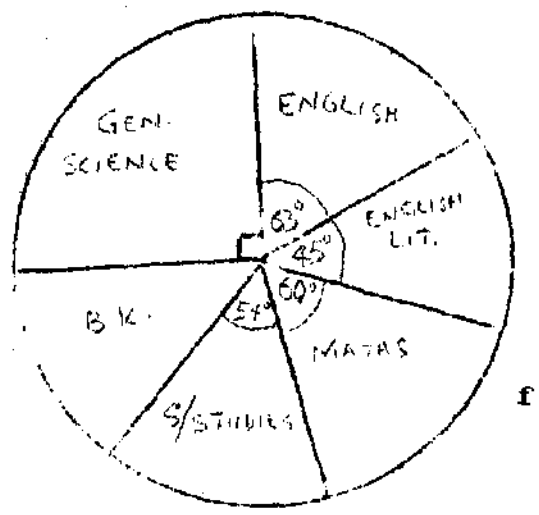


figure 5.9

(i) If the student scored 42% in English, what was his score in Bible knowledge?

(ii) If the pass mark was 50%, in how many subjects did the student pass?

(iii) In which subject was his score equal to his mean score for the six subjects?

66. Evaluate $10.33^2 - 9.67^2$

(ii) If $30 = 5.477$, find $(5.477 \times 3)^2$

(iii) Express 1740,000m in kilometres, leaving your answer in standard form.

67. In the figure 5.10, O is the centre of the circle of radius 5cm and PQ is a chord of length 6cm.

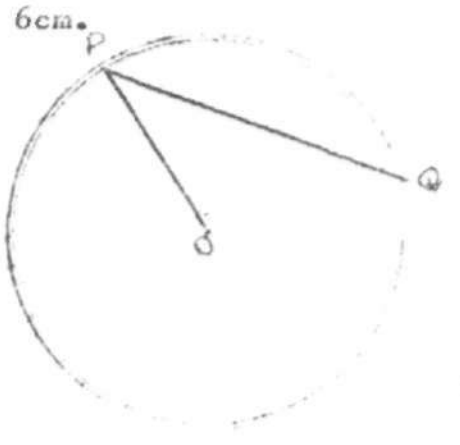


figure 5.10

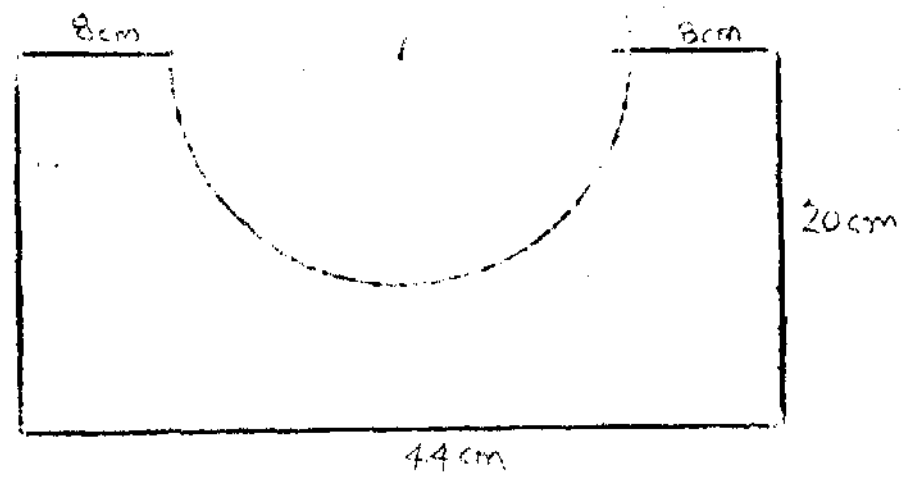
What is the distance of O from PQ?

68. A snake is a zoo was found to increase in length at a rate of 15cm per month. Give the length of the snake x months after attaining a length of 5.5m?

69. The length and width of a rectangle are pcm and qcm respectively. What is the perimeter of the rectangle if $p - q = 7$ and $p + 3q = 11$?

70. In the figure 5.11 $|PQ| = 12cm$, $|PS| = 8cm$ and the angles marked are equal.

72. The figure 5.13, (not drawn to scale) shows a rectangular piece of cardboard with a semi-circular piece removed.



Calculate the perimeter of the figure.5.13 (Take $\pi = \frac{22}{7}$).

73. If x is positive, for what range of values of x is $4 + 3x < 10$?

(ii) If $P > q$ and p and q are the values of x which satisfy the equation $(2x-1)(3x-1) = 0$. Evaluate $(3p + q)$.

74. A field is twice as long as its broad. How many yards of fencing are needed to go right round the field, if it is 156 yards.

75. Luka is twice as old as Bisi. Funke is 3 years younger than Luka and the sum of their age is 32. Find the ages of Luka, Bisi and Funke.

76. Simplify

$$\frac{a - \frac{1}{1 - a}}{1+a}$$

77. In triangle ABC, angle A = 3 angle B and angle C = angle B + 30°. Find the number of degrees in each angle.

78. How many minutes do we have in a leap year?

79. If a piece of work takes t days before 5 men can finish it, how long will it takes 3 men if they all work at the same rate (write the equation of relation).

80. What is the sum of 24 seven, 31 four, and 83 five in base Ten?

81. If a goat is tied with a rope which is 13 metres long, what is the area within the reach of the goat if there is nothing to disturb the goat from moving.

82. The perimeter of a right angled triangle is 90cm, and the hypothensis is 39cm long. Find the length of other 2 sides.

83. An open box is bent up from a sheet of thin metal (x+2) on long and (x-1) on wide, by

cutting out a square from one side 1cm at each corner, and then folding along the dotted lines. Obtain a formula for the volume of the box.

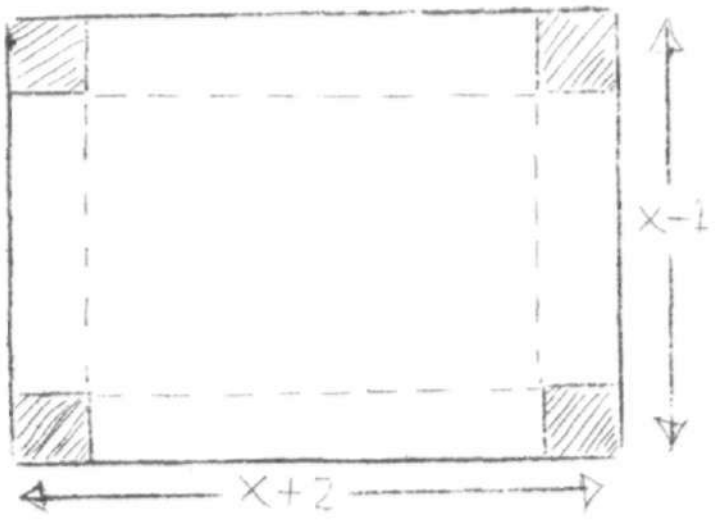


figure 5.14

84. The area of a rectangular field is $336m^2$. If its length is 12m, find its breadth. The corner of the field wants to fence it with wire ring, how many bundles will he buy if there are 10m in a bundle?

85. Is it true that when $x = 2$ is substituted in the expression $(x-2)^2 + (x+2)^2 - 2(x^2+4)$; the expression becomes zero?

86. If $x:y = 3:5$, find the value of

$$\frac{3x^2 + 5xy - y^2}{2x^3 + 3xy + y^2}$$

87. (a) The product of two numbers whose sum is 70 is 1200. Find them.

(b) Factorize $x^2 - 4xy + 3y^2 - 2y - 1$

88. Assuming that $\log 10^2 = 0.3010$, $\log 10^3 = 0.4771$ and $\log 10^7 = 0.8451$.

Evaluate

(i) $\log 10^{35}$

(ii) $\log 10\ 14$

89. Draw two straight lines $\overline{AOB}$, $\overline{COD}$ intersecting at O. On $\overline{OA}$, mark a point Q such that OQ = 2.5cm. Join Q to C and D. If $\overline{OD}$ and $\overline{OC}$ are equal. What do have to say of the figure formed.

90. From the top of a building 50m high, the angle of depression of a car is 55°. Find the distance of the car from the foot of the building.

91. A retired army general was paid off by the army council after his 25 years of service. This was how his benefit was calculated. 100% of his annual salary at age of retirement plus 15% of same salary for the number of years he served the nation and was given ₦14,858 as benefit. What was his annual salary at age of retirement?

92. (a) Bako buy pencils at 54k a dozen and selling at 5 for 33k, a trader made 94½k profit. How many pencils did he buy?

(b) A refrigerator can be purchased for ₦126 or by 24 monthly instalments of ₦5.50. How much is saved by paying cash?

93. (a) A room 5m long and 3m wide has a stained border 6m wide all round it next to the walls. What is the cost of covering the rest of the floor with matting at the rate of 50 a square metre?

(b) Two mats 3m long and $1\frac{1}{3}$m wide are placed on a floor 4m long and $3\frac{1}{3}$m wide. What part of the floor has no mat on it?

94. (i) Mallam Bako sells goods for ₦274.05 and thus makes a profit of ₦20.30. What percentage profit does he make: (a) on the cost price? (b) on the selling price?

(ii) A dealer bought ¾ tonne of sugar at ₦18 for 50kg and sold it at 42k a kilogram. What was his percentage profit?

95. (a) Find the cost of 1800 articles at 59½k each, deducing 3k in each complete naira as discount for cash.

(b) A trader sold sugar at 60k for a 2 kilogram bag and made ₦56 profit on a tonne. At what average rate per kilogram did he buy it?

96. (a) A woman went to market with ₦5.00. She bought some fish weighing 29 kilograms at 38k a kilogram ¾ kilogram of butter at ₦1.60 a kilogram, and 9 eggs at 48k a dozen. She then paid a bill for ₦1.24½ and put 75k in the Post office savings Bank. How much had she left?

(b) 30 cases, each containing 24 tins of fruit, were bought for a store. By selling the tins at 20k each, a profit of ₦21.60 was made. What was the cost per tin to the dealer?

97. (i) A book seller bought some books at 64k each and on equal number at 90k. The dealer books cost ₦6.50 more than the cheaper.

(a) How many were bought of each kind?

(b) What was the total cost of the books?

(ii) A tobacco dealer bought 22,500 cigarettes at ₦9.00 per 1,000. If 1,500 were not fit for sale and he sold the rest at 20 for 19k, what profit or loss did he make?

98. (a) The area of a square plot is $\frac{1}{16}$ hectare. Find the length of the fencing going all round it.

(b) An isosceles triangle has equal sides 5cm long and a height of 3cm. Calculate the base and find the area.

99. (a) A rectangle is twice as long as it is broad, and the perimeter is 30cm. Find the area by calculation.

(b) A room is 4.75m long, 3.25 wide and 4m high. Find the area of a window if it occupies 7½ percent of the total wall space.

100. (i) A cylinder has a base of radius 3½cm and a height of 6cm. Find:

(a) the curved surface;

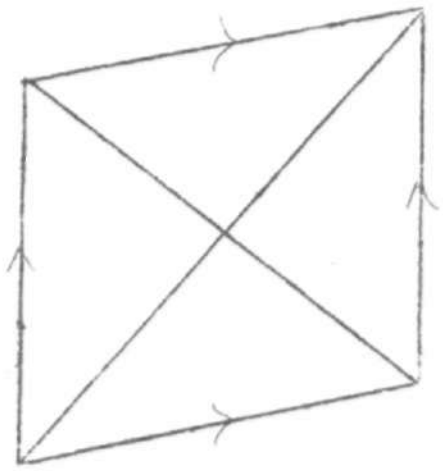
(b) the total surface;

(c) the volume.

(ii) Find the amount of iron in a rod 1.2m long if the cross section is a circle of radius 1cm.

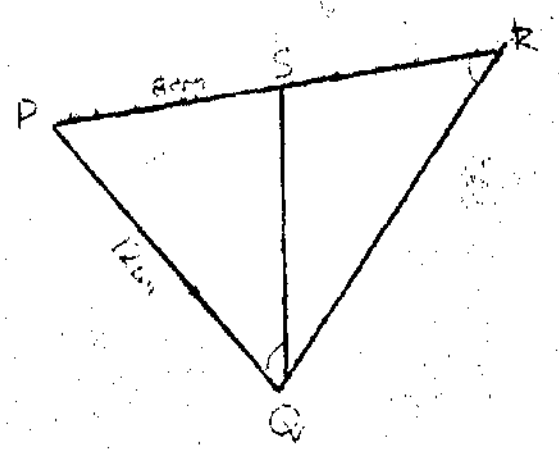
101. The following are the heights in metres of a group of ten girls in a class: 1.35, 1.25. 1.35, 1.40, 1.35, 1.50, 1.45, 1.50, 1.35, and 1.20

102. The figure 5.15 shows a rhombus of side 5cm.



If one diagonal is 6cm long, calculate the length of the other diagonal.

Finally; the author was able to distinguish the difference between an exercise, a question and a problem. A selection of about one hundred questions and exercises were presented. The point of view in this study is that enough problems and problem solving strategies should be set out in the school mathematics syllabus for the provision of teaching/learning for the senior secondary school students in Nigeria. In the next chapter covers the recommendations, summary, suggestions for further research and final remarks.



What is the value of IPRI? figure 5.11

71. The diagram shows a water tank in the form of a cuboid with the dimensions as shown.

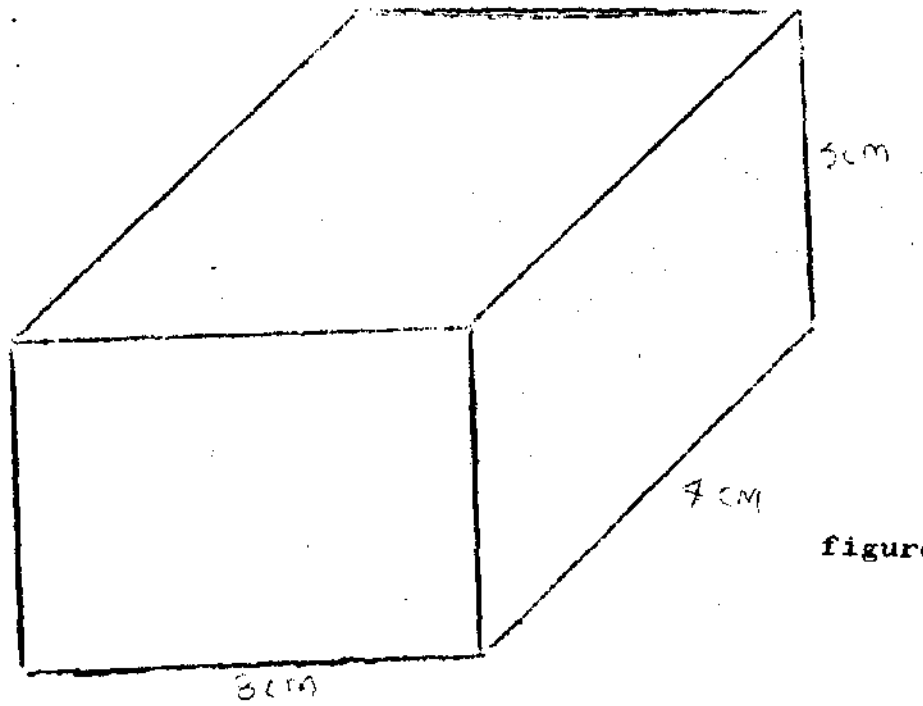


figure 5.12

Find the amount of water it will contain.

(1000cm^3 = 1 litre).

CHAPTER VI

THE PLANNED PROGRAMME: PROBLEMS ENVISAGED RECOMMENDATIONS AND CONCLUDING REMARKS

One outstanding problem in the design of a new programme of study outlined in chapters III and IV is the high degree of uncertainty arising from its implementation. Some of the areas of uncertainty are discussed below.

1. A major problem encountered was the selection and organisation of the importance of problems for solving that will make up a comprehensive programme of study and the development of a clear outline of the logic of the proposals based on the working concepts and procedures. It was feared that certain problems may be emphasized or some important ones may be over-looked.

2. Until other supporting studies are undertaken to test the effectiveness of this proposed programme, some questions listed below may remain unanswered.

   (a) Is the planned units/programme comprehensive enought to give adequate coverage in a full three year senior secondary course?

(b) Would students, who may not be able to move into university, have acquired sufficient skills and knowledge to contribute effectively to Nigeria's manpower needs? In other words, does this unit really constitute the fundamental mathematical knowledge a graduate of a senior secondary school ought to have?

3. The teacher variables is another factor that can give rise to programming difficulty. Teachers differ widely in their training in mathematics and in their perception of mathematics. Therefore, they need to be provided with problem-solving, teaching methods by specialists.

4. Students with varying backgrounds in related subjects will have varied knowledge hence their readiness and abilities to the learning of the new concepts introduced may differ markedly. What kinds of ideas and concepts can be grasped by students at certain ages? Experience has shown that any problem-solving or problem can be introduced at any stage in the school course provided that it conforms with Jerome Bruner's (1966 p. 86).

statement that "the foundation of any subject can be taught to anybody at any age in some intellectually honest form." So what Nigerian mathematics teachers need to do is to give a logical presentation to the students of whatever concept or idea there is to be introduced in problem solving.

6.1 Recommendations for Future Use

No matter how well conceived a unit is, and no matter how suitable in scope and content it appears, what normally ensures its success is how it is taught. In effect, the success of the new design will depend largely upon the conscientiousness of the teachers.

The recommendations listed below will help ease some potential problems in planning a scheme of study.

1. If any new programme is to be extensively adopted throughout the country, there must be large scale inservice training programmes for some secondary school mathematics teachers. This inservice should be uniformly spaced, and continuous until the teachers are reasonably confortable with the programme.

2. The use of a teachers' guide and aid would help in the success of the new unit because these can provide important background information in approaching and organising of material and the sequencing of units, especially helping teachers who are teaching it for the first time.

3. Any unit or programme of study should be a standard against which the teacher can make his/her own judgements and measure the progress of his/her students. Careful thought would, therefore, have to be given on the methods employed in introducing of a new unit. The introduction of a common programme of study to serve the new educational policy may help teachers re-examine their position especially in the reappraisal of methods. Kasperson (1967) indicated that "strategies permit the better trained and more capable on educational path". Therefore, to obtain maximum results from any programme of study it is imperative for the teacher to choose the best materials, examples and illustrations in developing the concepts indicated by the programme.

6.2 Summary and Concluding Remarks

Mathematics in Nigeria in the future may encounter new and varied educational problems but the degree to which these problems are solved will depend on the nature of educational researchers and changes in the policy of education. It will be worthwhile if educational planners provide reasons for any proposed change.

Mathematics must also be prepared to answer questions about the nature of problem-solving and why it is worthwhile to teach it.

It is this researcher's contention that the proposed problem-solving unit of study for the Nigerian senior secondary schools should be based on current and perceived societal issues and problems. The primary responsibility of mathematics is to equip the learners with knowledge, skills, attitudes and values needed to face a challenging and dynamic society. Students need also to be equipped with the interdisciplinary information system in order that they will be able to view Nigeria's external linkages against the background of the "developed world" on one hand and the so called "developing world" on the other. The unequal economic, political and social links between these two worlds is an important element in problem-solving units for Nigeria.

The national economic, social and political objectives embodied in the Nigerian Constitution (national integretion, unity and self-reliance, equity, and democratic commonwealth) are objectives whose pursuit have either direct or indirect problem-solving components. For example, problem-solving is in a position to provide policy-makers with the essential data base, and recommendations for tackling some of our internal social and economic problems. The programme is also meant for students of senior secondary schools who will either have the opportunity for furthering their education or who may opt for the employment sector.

However, it is hoped that with the adoption of the 6-3-3-4 tier system, educational planners will critically examine the problems facing teachers of a new syllabus in order to enable them to plan intelligently and make the "new" idea a valuable experience for themselves and their students.

Recommendations

(a) Problem-solving must be the focus of school mathematics in the future.

(b) Mathematics teachers should create classroom environments in which problem-solving can flourish. The researcher suggests that

greater emphasis should be placed on problem-solving methods, in both teacher training institutions and in secondary schools, "a teacher who had no personal experience of creative work can scarcely expect to be able to inspire, lead, help, or even recognize the creative activity of his students" (Polya, 1962: 209).

Advanced teacher training institutions, and colleges of education in general, and the university in particular might upgrade the quality of teachers' output by introducing courses that emphasize the "whats-hows and the whys aspects of mathematics learning. So far there has been some progress made in this direction at Ahmadu Bello University, Zaria. (B.Ed./B.Sc. II ME 200; "Strategies of Secondary School Mathematics Teaching", and Diploma II:, "Problem-Solving in Mathematics Education," Ahmadu Bello University, Zaria.)

(c) Mathematics programmes must take full advantage of computer machines and calculators as tools at all levels of problem-solving. Learning how to use the calculators effectively is an assert for students. It will also enable the students

solve the problem faster depending on their solid grasp of the problem solving techniques.

(d) Much ado has been made about standard versus nonstandard, routine versus nonroutine, textbook versus nontextbook problems. The procedures involved in solving problems are the same. A problem-solving process is what we should be teaching learners, not just types of problems. Teachers have a ready source with which to teach problem-solving - the standard textbooks should be considered for all schools on problem-solving.

(e) More mathematics study must be required for all students and a flexible curriculum with a greater range of options should be designed to accomodate the diverse needs of the students population.

6.3 SUGGESTIONS FOR FURTHER RESEARCH

1. Although the Novelty problems have been tested as samples of Diploma II Mathematics - Education's students, further research is needed to test the suitability of each item in the unit.
2. There is a need for further investigations to ascertain whether or not certain, or all items are appropriate to Nigerian students.

3. Thirdly, a number of area related to the general issue of the items in Nigeria would provide rewarding research experience and supply useful information.

Final Remarks

It is hoped that the present work will help to fulfil some of the greatest needs of our present day technological advancement in Nigeria.

Young men and women should learn to use their minds to:

(i) Sustain for mankind the numerous benefits that accrue from advanced technology; and

(ii) to continue researching the sciences for solutions to new unsolved problems in such fields as medicine, physics, chemistry and the social sciences.

The supportive role of mathematics in both of these areas of human endeavour is well documented (Growth of Mathematical Ideas; NCTM, 1959). Mathematics in its role as servant of the sciences, provides the technologist and the research worker with a concise and precise working language, and in its role as Queen of Sciences it gives birth to and

nourishes creative thinking. In numerous ways, mathematics offers unique opportunities for creative and orginal thinking, forming and solving original problems, establishing theorems with original proofs, discovering relationships and stating them in one's own words, are all but the starting experiences in creative thinking.

REFERENCES

Alexander, T. (1982), "Scope and Sequence for a Problem Solving Curriculum", Arithematic Teacher, Vol. 29, No. 6, pp. 62 - 65.

Alkali, A.M., (1976) "The Effectiveness of Teaching Problem solving by the Discovery and Expository method, An Experimental analysis" unpublished B.A. Project, BUK Kano,

Alexander, V.E., (1960), "Seventh Graders Ability to Solve problems", School Science and Mathematics Vol. 60, No. 4, pp. 603 - 606.

Avital, S.M. and Sara, J.S. (1968), "Objectives for Mathematics learning: Some Ideas for the Teacher" The Ontario Institute for Studies in Education.

Baget, D.D. et al. (1982), "Games and Puzzles for Teaching and Learning Mathematics", Unpublished B.Sc. (Ed.) Project Ahmadu Bello University, Zaria

Bana, J.P. and Doyal N. (1977), "Some Effects of Distractions in Nonverbal Mathematical Problems", Alberta Journal of Educational Research, Vol. 23 (December, 1977) pp. 268-279.

_____ (1978), "Distractors in Nonverbal Mathematical Problems", Journal for Research in Mathematics Education. Vol. 9 (January 1978) pp. 56 - 61.

Balow, I.H. (1964), Reading and Computation ability as determinants of Problem Solving", The Arithematic Teacher, Vol. II, pp. 11 - 22.

Biehler, R.F. (1974), Psychology Applied to Teaching 2nd ed., Boston, Houghton Miffin.

Bourgeois, R. and Doyal, N. (1977), "Young Children's Behavior in Solving Division Problems", Alberta Journal of Educational Research, Vol. 22 (September, 1977) pp. 178 - 185.

Bowman, H.L. (1932), "The Relation of Reporte Preference in Problem Solving", Journal of Educational Psychology, Vol. 23, pp. 266 - 276.

Bello, Ahmadu Bello University, (1972) Designs TEST: Department of Education, ABU, ZARIA.

Brodie, J.W.B., "The mediated Acquisition and Application of Heuristic Problem Solving Strategies in a defined Word Problem Task", unpublished Ph.D. dissertation, the Catholic University of America, 1980.

Brownell, W.A. (1942), "Problem Solving", In the Psychology of Learning," Forty-First Yearbook of the National Society for the Study of Education pp. 415 - 443.

Bryant, S.J. et al. (1965), Non-Routine Problems in Algebra, Geometry and Trigonometry, New York, McGraw-Hill Book Company.

Bulter, W.F.L. and Banks, J.F. (9170), The Teaching of Secondary Mathematics, McGraw-Hill Book Company London.

Burch, R.L. (1947), "An Evaluation of Analytic Testing in Arithmetic Problem Solving. Duke University.

Burns, P.C. and Yonally, I.L. (1964), "Does the order of Presentation of numerical data in multisteps Arithmetic Problems affect their difficulty", School Science and Mathematics, Vol. 64, pp. 267 - 270.

Buswell, G.T. and Kersh, B.Y. (9156), "Patterns of Thinking in Solving Problems", University of California Education, Vol. 12, No. 2, pp. 165-182.

Carpenter, T.P. et al. (1980), "Solving verbal problems Results and Implications from National Assessment", Arithematic Teacher, Vol. 28 pp. 8 - 13.

_____, "The Effect of Problem Structure on First - Grader's Initial Solution Processess for Simpler Addition and Subtraction Problems", In Suydam M.N. (Ed) (1980) Yearbook, Reston Virgina NCTM (9180) Journal for Research in Mathematics Education.

Chase, C.I. (1982), "The Position of Certain Variables in the Prediction of problem solving", In Arithmetic Teacher Journal of Education Research Vol. 54, pp(9-14) pp. 9 - 14.

Corle, C.G. (1958), "Thought Processes of Sixth-Grade Pupils while Solving Verbal Problems in Arithmetic", In Arithmetic Teacher, Vol. 5, pp. 103 - 200.

Cronbach, L.J. (1954), Educational Psychology, New York, Harcourt Brace Joranorich.

Dauda, B.C. (1974), "Problem Solving in the Teaching of Secondary School Method", unpublished B.Sc.(Ed.) Project Ahmadu Bello University, Zaria.

Daily, B.W., (1975) "Ability of High School Pupils to select Essential Data in Solving Problems", contributions to Education, No 190, New York. Bureau publications, Columbia University.

Davis, G.A. (1966), "Current Status of Research and Theory in Human Problem-Solving", Psychological BulletinVol. 3 66 pp. 36 - 54.

_____ (1973), "Psychology of Problem Solving: Theory and Practice. New York, Basicbooks.

Days, H. et al (1979), "Problem Structure, Cognitive level, and Problem Solving performance", Journal for Research in Mathematics Education, Vol. 10, No. 2, pp. 135 - 146.

Earp, N.W. (1970), "Procedures for Teaching Reading in Mathematics", Arithmetic Teacher, Vol. 7 (November, 1970), pp. 575 - 579.

Evan, M.M. (1982), "Problem Solving for the Junior Highs School", Arithmetic Teacher, Vol. 29, No. 6.

Falohun, C.C. (1981) "Concept Formation in Algebraic Equation and Problem solving among Form five Students in Oyo State of Nigeria", unpublished Ph.D. Dissertation University of Ibadan, June, 1981.

Fennema, E. (1974), "Mathematics Learning and the Sexes", A Review Journal for Research in Mathematics Education, Vol. 5, (May, 1974), pp. 126 - 139.

Federal Ministry of Education (1979), Senior Secondary School Mathematics Curriculum, March 1979, Lagos.

Federal Government Press, (1977), National Policy on Education Revised 1981, p.16.

Flender, F.O. (1978), Reflections on a Problem Solving Study" International Journal of Mathematical Education in Science and Technology, Vol. 9 (February, 1978).

Gagne, R.L., and Smith, E.C. Jr. (1962), "A Study of the Effects of Verbalization on Problem Solving", Journal of Experimental Psychology, Vol. 63, pp. 12 - 18.

Getzals, J.W. (9164), "Creative Thinking, Problem Solving and Instruction", In the Theories of Learning and Instruction: Sixty-third Yearbook of the National Society of Education, University of Chicago Press, 1964.

Guilford, J.P. et al. (1957), "A Revised Structure of intellect", Reports from the Psychological Laboratory, No. 19, Sarkeley and Los Angeles, University of California Press, 1957.

Gorman, C.J.A. (1980), "Critical Analysis of Research on Written Problems in Elementary School Mathematics", unpublished Ph.D. thesis, University of Pitsburgh, 1967. In Arithmetic Teacher, Vol. 28, No. 2, (October, 1980), p. 8 - 13.

Henney, L. (1971), "Improving Mathematics Verbal Problem Solving Ability through Reading Instruction," Arithmetic Teacher, Vol. 18 (April, 1971) pp. 223 - 229.

Herrington, J.P. (1980), "Experimental Analysis of Cognitive Style and Information Processing Variables in Mathematical Problem Solving", unpublished Ph.D. Dissertation, Southern Illinois University at Edwardsville, 1980.

Hunt, E.B. (1962), Concept of Learning: An Information Processing Problem, New York, Wiley.

Inhelder, B. and Piaget, J. (1958), The Growth of Logical Thinking from Childhood to Adolescence, New York: Basic books.

Jerman, M.E. (1973), "Individualized Instruction in Problem Solving in Elementary School Mathematics", Journal for Research in Mathematics, Vol. 4, No. 1, pp. 6 - 19.

Johnson, D.A. and Rising, G.R. (1967), Guidelines for Teaching Mathematics, Wadsworth Publishing Company Inc. California.

Kabora, Y.M. (1979), "Factors affecting performance in Problem solving test of Primary, Qur'anic and Islamiyya School Pupils in Kano", unpublished B.A. (Ed.) Project, Bayero University Kano, 1979.

Kalejaiye, A.O. (1979), "An Analysis of the Performance of Bilingual Children in Solving Verbal Problems in Mathematics", unpublished Ph.D. Dissertation University of Ibadan, 1979.

_____, (1981) "An Investigation of the factors Related to Verbal Problem-Solving Abilities among Nigerian Primary School Pupils", unpublished personal Research project, 1981.

_____, (1981), "The Role of Language in Solving Problems in Mathematics in the Primary School", unpublished Personal Research Project, 1981.

_____, (1980), Improving the Problem Solving Performances of Primary and Secondary School Pupils", unpublished personal Research Project 1980.

Kantowski, M.G. (1977), "Processes Involved in Mathematical Problem Solving", Journal for Research in Mathematics Education, 1977. Vol. 18, pp. 163 - 183.

Kilpatrick, J. (1969), "Problem Solving in Mathematics", Review of Educational Research, (1969) Vol. 39, pp. 523 - 534.

_____, (1978), "Research on Problem Solving in Mathematics", School Science and Mathematics, Vol. 78 (March, 1978), pp. 189 - 192.

Kinsella, J.D. and Boyd, D.H. (1977), "A Search for Reading Difficulties Among Erred Word-Problems", Journal for Research in Mathematics Education, Vol. 8 (May, 1977).

Kinsella, J.J. (1970), "Problem Solving" NCTM. 1970. The Teaching of Secondary School Mathematics Thirty-Third Yearbook, NCTM.

Klausmeier, H.J. and Loughli, L.J. (9161), "Behaviours During Problem Solving Among Children of Low Average, and High Intelligence", Journal of Educational Psychology, 1961, Vol. 52, pp. 148-152.

Kleinmuntz, B. (1966), Problem-Solving: Research Method, and Theory, 2nd ed. John Wiley, Inc. New York, 1966.

Kohler, W. (1925), The Mentality of Apes, New York. Harcourt Brace.

Krulik, S. and Rudnick, J. (1982), Arithmetic Teacher Vol. 29, No. 6 (February 1982) pp. 42 - 44.

Langley, K.R., (1972), "An Information Processing Analysis of Problem Solving behavior", unpublished Ph.D. dissertation Carnegie Institute of Technology Pittsburgh Pa 1961. In Mathematics Teacher Vol. 6, No. 32 (December, 1972).

LeBlanc, J.F. (1977), "You can Teach Problem Solvings", Arithmetic Teacher, Vol. 25 (November, 1977) pp. 16 - 20.

Lee, W.L. (1979), "A Comparison of Verbal Problem Solving in Arithmetic of Learning Disabled and Non-learning Disabled Seventh Grade Males", unpublished Ph.D. Dissertation, University of Kansas, 1979.

Lester, F.K. (1977), "Ideas About Problem Solving: A Look at Some psychological Research", Arithmetic Teacher, Vol. 2 (November, 1977), pp. 12 - 15.

Luce, R.D. et al. (1963), Handbook of Mathematical Psychology Vols. 1 and II, New York, Wiley.

Luchins, A.S. (1942), "Mechanization in Problem Solving, The effect of Einstellung", Educational Psychology, New York.

Lubos, O.S. (9126), "An Evaluation of three Techniques for Improving Ability to Solve Arithmetic Problems": A Study in the Psychology of Problem Solving University of IOWA, 1926.

Lyda, W.J. (1947), "Direct Practical Experiences in Mathematics and Success in Solving Realistic Verbal Reasoning Problems in Arithmetic", Mathematics Teacher, No. 40, (April, 1947) pp. 32 - 45.

Maiser, N.R.F. (1970), Problem Solving and Creativity in Individual Groups, Belmont, California, Brooks Cole, 1970.

Marcuca, E.G., (1980), "A Meta-Analysis of Research on Methods of Teaching Mathematical Problem Solving", unpublished Ph.D. Dissertation, University of Iowa, 1980. In Arithmetic Teacher Vol. 24 (February, 1982).

MacKinnon, D.W. (1962), "The Nature of Creative Talent", American Psychologist, Vol. 7, (July, 1962).

Macworth, N.H. (1965), "Originality" American Psychologist, Vol. 8, (April, 1965).

McDonald, F.J. (1965), Educational Psychology, 2nd ed. Belmont, California: Wadsworth, 1965.

Metwali, S.M., (1979) "Expressed Interest and Extent of Practice as they Affect Outcomes of Instructions in Problem Solving", unpublished Ph.D. Dissertation. The Ohio State University, 1979.

Meyer, R.A. (1978), "Mathematical Problem Solving Performance and Intellectual Abilities of Fourth-Grade Children", Journal for Research in Mathematics Education, 1978 Vol. 9, No. 5, pp. 334 - 348.

Miller, G.A. (1962), "Some Psychological Studies of Grammer," American Psychologist, 1962, Vol. 17, pp. 748 - 762.

Muranmi, V.S. (1982) "A Study of the Effect of Explicit Reading Instruction on Reading Performance in Mathematics and on Problem-Solving Abilities of Sixth Graders," unpublished Ph.D. Dissertation Michigan State University, 1978. In Arithmetic Teacher, 1982, Vol..29, No. 6, pp. 56 - 61.

MCTM: (1980), "National Council of Supervisors of Mathematics", An Agenda for Action, Reston Virginia. The Council, 1980.

_____ (1969), "Hints for Problem Solving,: Topics in Mathematics for Elementary School Teachers Series, Book 17, Washington D.C. (1969).

_____ (1963), Enrichment Mathematics for High School Twenty-eighth Yearbook, Washington D.C., 1963.

_____ (1967), The Learning of Mathematics: Its Theory and Practice. Twenty-First Yearbook, Washington D.C., 1967.

_____ (1980), Problem Solving in School Mathematics NCTM, Inc. 1980.

_____ (1959), The Growth of Mathematical Ideas - Twenty-fourth Yearbook, Washington D.C., 1959.

Newell, A. et al (1958), "Elements of a theory of Human Problems Solving", Psychology of Education 1958, Vol. 5, pp. 151 - 166.

Newwll, A. and Semon, H.A. (9172), Human Problem Solving Englewood Cliffs, N.J. Prentice-Hall, 1972.

Nilson, N. (1971), Problem Solving Method in Artificial Intelligence. New York, McGraw Hall, 1971.

Parnes, S.J. and Arnold, M. (1959), "Effect of Brainstorming Instruction on Creative Problem Solving by Trained and untrained subjects", Journal of Educational Psychology Vol. 50, (August, 1959) pp. 171 - 176.

Polya, G. (1957), How to Solve It. 2nd ed. Garden City, New York, Doubleday, 1957.

_____ (1968), Mathematical Discovery: On Understanding, Learning and Teaching Problem-Solving, New York, Wiley, 1962, (Vol. 1), 1968 (Vol. 2.).

Putt, I.J. (1980), "An Exploratory Investigation of Two Methods of Instruction in Mathematical Problem Solving at the Fifth Gra delevel", unpublished Ph.D. Dissertation, Indiana University 1978. In Suydan M.N. (1980). Yearbook of NCTM.

Riddesel, C.A. (1969), "Problem Solving: Some Suggestions from Research", Arithmetic Teacher 1969, Vol. 16, pp. 54 - 58.

Richard, J.S. (9180), Research in Mathematics Education NCTM, Inc. 1980, pp. 286 - 323.

Robinson, E. (1977), "On the Uniqueness of Problems In Mathematics", Arithmetic Teacher Vol. 25 (November, 1977) pp. 22 - 26.

Shepard, R. (1963), "Review of E.A. Feigenbaum and J. Feldman (Eds). Computers and Thought; New York McGraw-Hill, 1963.

Skemp, R.R. (1977), The Psychology of Learning Mathematics Renguain Books Limited, Middle Sex, England.

Sowder, L.J. et al (1979), "A review of Research on Solving Routine Problems in Pre-College Mathematics", Dekalb, Illinois University, 1979 In Arithmetic Teacher, 1980, Vol. 11, pp. 26-34.

Smith, M.J. and Heddens, J.W. (1964), "The readibility of Experimental Mathematics Materials", Arithmetic Teacher, 1964, Vol. 11, pp. 939-944.

Stevenson, P.R. (1925), "Difficulties in Problem Solving", Journal of Educational Research, 1925 Vol. 11, pp. 95 - 103.

Stilwell, M.E. (1967), "The Development and Analysis of a Category System for Systematic observation of Teacher-Pupil Interaction During Geometry Problem-Solving Activity", Arithmetic Teacher Vol. 8 (1967).

Suydam, M. (1980), "Untangling Clues from Research on Problem Solving", In S. Krulik (Ed), Problem Solving in School Mathematics, 1980, Yearbook, of NCTM Reston Va. The Council, 1980.

Suydam, M.N. and Weaver J.F. (1977), "Research on Problem Solving: Implications for Elementary School Classrooms", Arithmetic Teacher Vol. 25 (November, 1977) pp. 56 - 61.

Vos, K.E. (1976), "The Effects of Three Instructional Strategies on Problem Solving Behaviors in Secondary School Mathematics", Journal For Research in Mathematics Education, Vol. 7 (November, 1976) pp. 264 - 275.

Webb, N.L. (1979), "Processes, Conceptual Knowledge, and Mathematical Problem Solving Ability", Journal for Research in Mathematics Education Vol. 10 (March, 1979), pp. 82 - 93.

Wertheimer, M. (1945), Productive Thinking, New York, Harper and Row, 1945.

Wheatley, G.H. (1977), "The Right Hemisphere's Role in Problem Solving" Arithmetic Teacher, Vol. 25 (November, 1977), pp. 36 - 39.

Wickelgren, W. (1974), How to Solve Problems. Elements of Theory of Problems and Problem Solving. San Francisco, W.H. Freeman and Co. 1974.

Willoughby, S.S. (1967), Contemporary Teaching of Secondary School Mathematics. John Wiley, Inc. New York, 1967.

Yates, D.S. (1976), "Magic Triangles and a Teacher's Discovery", Arithmetic Teacher Vol. 23, (May, 1976) pp. 351 - 354.

Zweng, M.J. et al. (1980), "Children's Strategies of Solving Verbal Problems", August 1979, (ERIC Ed. 178, 379) In Arithmetic Teacher Vol. 40 (June, 1980), pp. 36 - 40.

APPENDIX I

SOLUTIONS TO THE SELECTION OF NOVELTY PROBLEMS FOR SECONDARY SCHOOLS

1. If $x = y$, the $x^2 - xy$

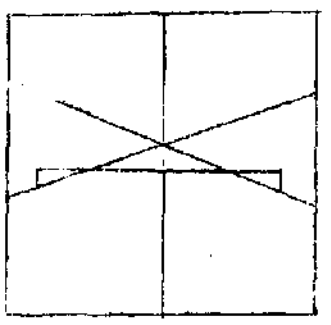
$\therefore x^2 - y^2 = xy - y^2$ and $(x+y)(x-y) = y(x-y)$

Hence, $x + y = y$

Since $x = y$, then by substitution $2y = y$

Since y is a nonzero real number, if follows that $2 = 1$!

2. An intuitive geometric approach could be tried, An algebraic approach could be tried also. But, finally, the fifth piece is really two "pieces joined" at a single point, not quite what the a minor adjustment trading congruent triangles yields, thereby a satisfactory solution as stated below will come.

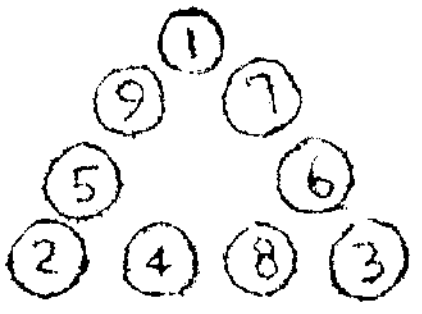


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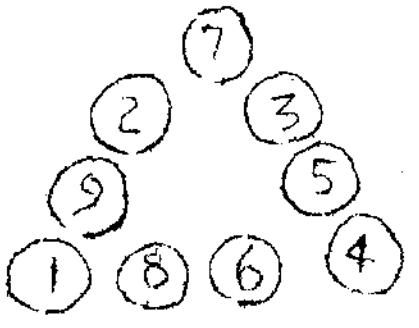
see page 14 Chapter I

4. First observe that the sum of the numbers 1 to 9 is 45 and the sum of the three sides is 3.17 or 51. Thus, the sum of the corner number continue.

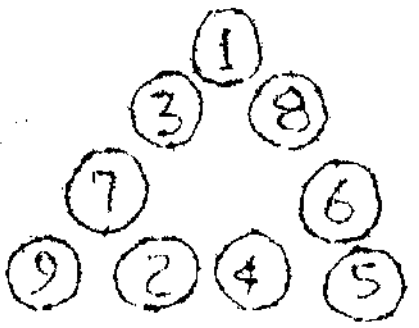
(i)



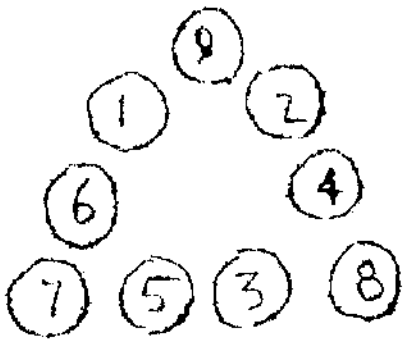
(ii)



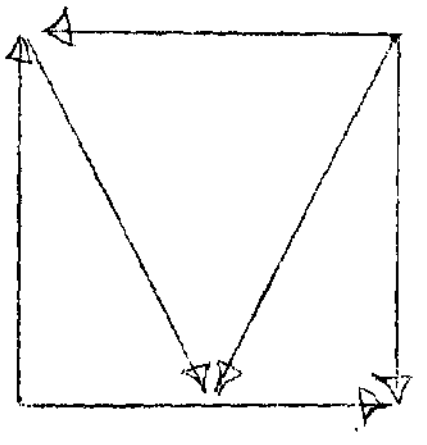
(iii)



(iv)



5.



6.

. 10 Square units

7.

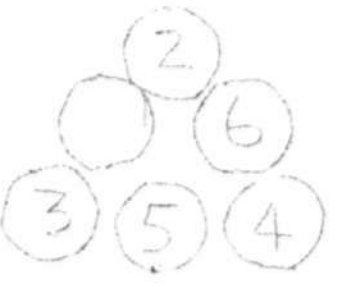
(i)

8	1	6
3	5	7
4	9	2

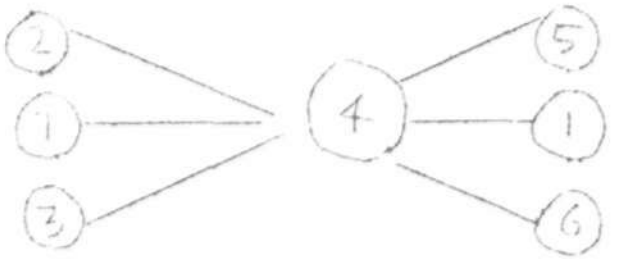
(ii)

6	7	2
1	5	9
8	3	4

8.



9.



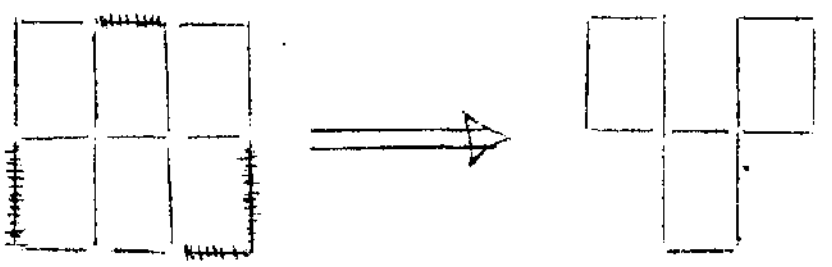
10. $13^2 = 169$ and $31^2 = 961$

$12^2 = 144$ and $21^2 = 441$

$01^2 = 001$ and $10^2 = 100$

11. There were three ducks.

12. All together there are 17 match sticks making the 6 boxes. Removing only 5 match-sticks you leave only s quare is possible.

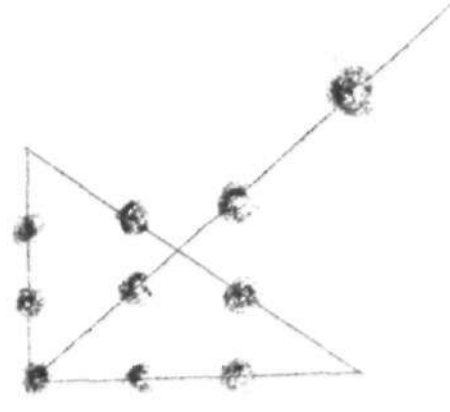


13. No solution. Since one's age is not proportional to his height.

14. Middle number - $41+1 = \frac{42}{2} = 21$. Since the middle number is add and are boys:. the middle person is a boy.

15. No solution.

16. Ojo is 2 years old.

17. 

Note: The order does not matter

18. No solution.

19. When you are looking at a clock.

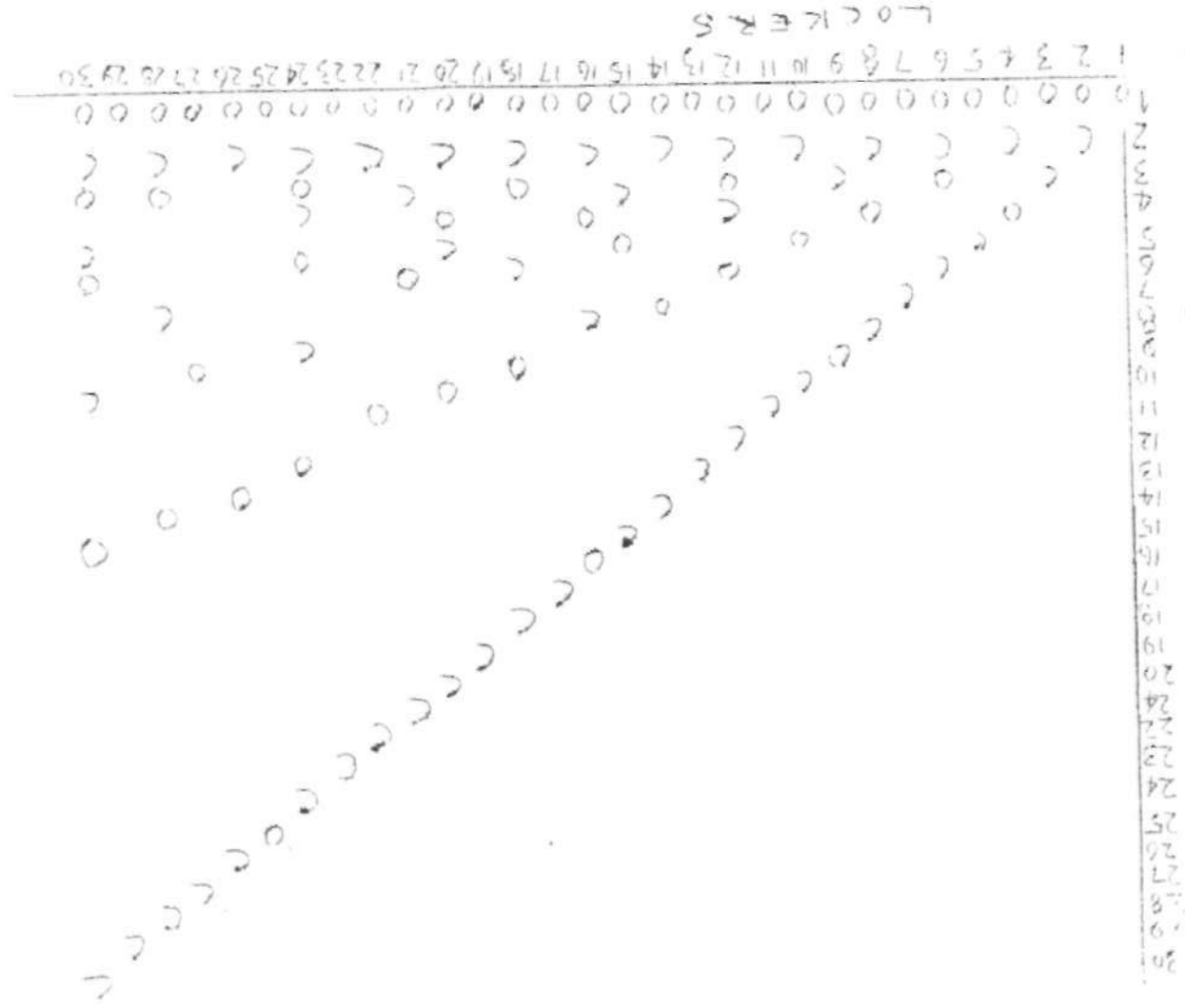
Suprisingly, you may have little difficult with this problem. You may reasoned that if you add the two numbers, the result would just increase to 13, but if you arranged the number in a cycle, rather than in a number line, you could obtain the correct answer.

20. Your elbow or back of one hand.

21. Three.

22. 6 ways. A magic square cannot be formed with a word with an even numbers of letters.

23. In 252 ways. Below is the solution using pascal's Triangle.


STUDENTS
LOCKERS

25. 1, 1, 2, 3, 5, 8, 13, 21, 34, and 55 such that each term is the sum of the two terms which immediately procede it; thus 5+8 = 13, and 8+13 = 21 and so on. The sequence is named after Leonard Fibonacci of pisg, born 1175 and has many special properties.

26.

$2\frac{1}{2}$	$-2\frac{1}{2}$	$1\frac{1}{2}$
$-1\frac{1}{2}$	$\frac{1}{2}$	$2\frac{1}{2}$
$-\frac{1}{2}$	$4\frac{1}{2}$	$-2\frac{1}{2}$

27. 10

10 x ₦8 = ₦80

- 16 x ₦5 = ₦80

0

Simultanuous equations

$5w = 26$ 1 x 5

$8 - 5w = 0$ 2

$5r + 5w = 26$ x 5 3

\+ $8r - 5w = 0$ 4

$13r = 130$

$r = 10$

Substitute r in equation 1

10 + w = 26

w = 26 - 10

w = 16

28. Mr. L kilogram, Mrs. w kilogram, x kilogram are free: $1+w = 94$, $\frac{1-x}{1.5} = \frac{w-x}{2} = \frac{94-x}{13.5}$; $x = 40$.

29. 700, 500, x = 400 from $x + (x+100) + (x+300) = 1,600$.

30 Each son receives 3000.

31. If the share of each child is x, and the whole fortune y, the shares of the children are:

1st $x = 100 + \frac{y-100}{10}$.

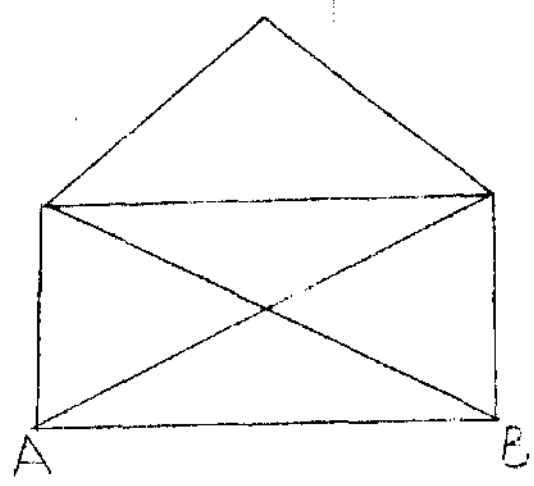
2nd $x = 200 + \frac{y-200}{10}$

3rd 3rd $x = 300 + \frac{y-2x-300}{10}$ and so on.

The difference of any two consective right hand side is $100-\frac{x+100}{10}$

If this difference equals 0 (because all the shares are equal) x = 900, then (from the first equation) y = 8100, there were 9 children.

32.



Other paths are also possible, but all must start and end and A and B (or B and A).

33. Let the three players initially have the amount x, y and z, respectively; it will be advantageous to consider x+y+z = s. (S = 72), we have to consider the amounts owned by the at four different instant; any two consecutive instants are separated by a game; the total sum owned by the three is always S:

1st	2nd	3rd
x	y	z
2x-s	2y	2z
4z-2s	4y-s	4z
8x-4s = 24	8y-2s = 24	8z-s = 24

Hence x = 3a, y = 21 z = 12

34. Let y represents the nos. of rabbits then the nos of hens 50-y.

If one rabbit has 4 legs then y rabbits will have 4xy legs.

Also if one hen has 2 legs then (50-y) hens will have 2x(50-y) legs.

Since there are 140 legs altogether then the statement will be reduced to algebraic form.

$4y+2(50-y) = 140$

Solve for y

$4y+100-2y = 100$

$2y = 40$

$y = 20$

Substitute for the value of y = rabbits that will be 20 rabbits. Hens (50-y) = 50-20 = 30

Hens = 30.

35. No solution.

36. . . 3 and 8

37. .. AC = 6cm since AC = OB = radius

= AO + AP

= 5cm + 1cm

(Incidently - we can now also find OC

$Q = \sqrt{6^2 - 5^2} = \sqrt{11}$.

38. $256^2 = 65536$

39. 5 cats are needed. 5 cats catch 1 lizard/minute (5 lizards in 5 minutes)

Hencein 100 minutes these 5 cats would catch 100 lizards.

40.

(a)	(b)	(c)	(d)
29786	9999	9567	9683
850	+ 1111		
+ 850	8888	+ 1085	+ 1053
31486	19998	10052	10736

41. 30

42. 4

43. $(25)^2 = 25 \times 25 = 625$

44. No solution.

45. 20 12 of side 1 unit
6 of side 2 units
2 of side 3 units.

46. 13 and 31; 19 and 91

47. We first note that the number of 0's at the end of a number is the number of factors of 10 it has; since $10 = 5.2$, we need only ask how many 5's and 2's appear as factors of 40! Infact, since many more 2's and 5's will appear in the product, the answer (and our subgoal) will be the number of 5's that appear as factors of the first 40 integers, namely twenty-four.

48. 204 squares.

49. 15625 x 64 (i.e. 56×2^6)

50. (Stanford 1959) we use the notation

u for Ali's speed

v for Sule's speed

t_1 for the time (counted from the start) the boys meet the first time,

t_2 for the time they meet the second time.

d the desired distance of the two houses.

then $ut_1 = a$, $ut_2 = d+b$

$vt_1 = d-a$ $vt_2 = 2d-b$

(i) By expressing $\frac{u}{v}$ in two different ways, we obtain $\frac{a}{d-a} = \frac{d+b}{2d-b}$

Hence, after discarding the vanishing root, we find $d = 3a-b$.

(ii) Of course Ali Numericall: $\frac{u}{v} = \frac{3}{2}$

51. $100+x = y^2$, $168 + x = 7^2$

Subtracting we obtain

$$(2-y)(2+y) = 68$$

Since $68 = 2^2 17$ can be decomposed into a product of two factors in just three ways:

$$68 = 1.68 = 2.34 = 4.17$$

and y and z must be both odd or both even, there is just one solution:

$2 - y = 2$, $2+y = 34$, $2 = 18$, $Y = 16$, $x = 156$.

52. (Stanford 1957) Ahmadu has x stamps of which if sevenths are in the second book; x and y are positive integers;

$$\frac{2x}{10} \times \frac{yx}{7} \times 303 = x$$

and hence

$$x = \frac{3.5.7.101}{28 - 5y}$$

The denominator on the right-hand side must be positive and odd since it must divide the numerator which is odd. This leaves three possibilities: 1, 3, and 5, and only the last one is suitable: $y = 5$ and $x = 35$ 35 are uniquely determined.

53. Let

A, 2s, a, b, c stand for

area, perimeter, hypothenuse, remaing sides, respectively, A and s are given, a, b, and c known.

To solve the system

$$a+b+c = 2s, \quad bc = 2A; \quad a^2 = b^2 + c^2$$

express $(b+c)^2$ in two different ways:

$$(25-a)^2 = a^2+4A$$

$$a = 5 - \frac{A}{5}$$

54. If a and b are the lengths of two nonparallel sides, and c and d the lengths of the two diagonals, then $2(a^2+b^2) = c^2+d^2$

In fact, the diagonals dissect the parallelogram into four triangles: Apply the law of cofines to two neighboring triangles.

APPENDIX II

ANSWERS TO THE SELECTION OF ONE HUNDRED QUESTIONS-EXERCISES FROM W.A.S.C "O" LEVEL MATHEMATICS

1. 54.4, 3.1 or (3.2)
2. (i) 5
   (ii) 30
   (iii) 8
3. 15 members of the club play both basketball and football.
4. 5 students offer all three subjects.
5. when $x = -0.5$ $y = 1.7$
   when $x = 1.5$ $y = 5.5$
   when $y = 3$, $x = -0.6$, 1.0, or 3.9.
6. (a) (i) $ABC = 120^{\circ}$
   (ii) 1.856cm
   (b) 4.12m. the ladder reaches 4.12m up to the wall.
7. (2, 4), (5, 1)
8. $3x + 4y + 10 = 0$ and $3x + 4y - 10 = 0$.
9. $3y + x - 3 = 0$.
10. $5y + x - 11 = 0$.
11. Circle centre A, 11cm
    Circle Centre B, 3cm
    Circle Centre C, 7cm
12. (a) 30.49 cm^3
    (b) 454 cm^3
13. (i) $x = -1$; $y = 1$, $x = -4$, $y = 3$
    (ii) $x = \pm 4$, $y = \pm 5$
14. (a) (i) $\frac{1}{5}$ (ii) $\frac{1}{12}$
    (b) $\frac{13}{32}$

15. (a) (i) (HH), (HY), (TH), (TT)

(ii) All possible outcomes will be (1, 2, 3, 4, 5, 6) for brevity.

(b) If H is heads, and T is tails. There will be the possible outcomes.

16. $y = x^2-x-6$

17. (a) $x = 27$

(b) $-5x$

(c) 3:1

18. $3\pi^2$

19. $\sqrt{3r^2 - \frac{1}{2}\pi r^2}$

20. 216

21. $\frac{ab}{b-a}$

22. 64,000 litres

23. $112cm^2$

24. 256

25. N600

26. $10m + 3n$

27. $-\frac{28a^2b^6c}{7}$

28. $\frac{4}{25}$ of the area of the largest square.

29. 9

30. N6.00

31. N26.25

32. ₦212.16

33. ₦22.50

34. OJO

35. (i) 0 (ii) $\frac{15\sqrt{3} - 10\sqrt{15}}{-11}$

36. ₦2125.00

37. (i) 17.2 (ii) 0.582

38. 046

39. ₦241.21

40. $13\frac{2}{9}$

41. 975,000,0

42. 3%

43. (i) $8x^3+12x^2+6x+1$; (ii) $(x-2)(x+3)$;
(iii) $(3x-1)(x-1)x = \frac{1}{3}$ or $x = 1$
$x = 2$ or $x = 3$

44. (i) -4 (ii) 3

45. (i) 4 (ii) 1

46. (a) 101101 (b) n = 5

47. 2

48. 35 days

49. $-2\frac{2}{3}$ or $\frac{8}{3}$

50. -2

51. 200^o

52. 50

53. ₦4m

54. ₦1.20

55. ₦1.71

56. 50 A

57. ₦74.22

58. (a) 4; (b) 4; (c) 4.45

59. (i) 2 kobo (to the nearst kobo).

(ii) ₦9.15

60. (a)

Marks	21-30	31-40	41-50	51-60	61-70	71-80
f	2	5	7	9	11	8

81-90	91-100
5	3

(b) Mean = 61.7 + 0.1 (c)

(c) (i) 57.5 + 0.5,

(ii) 44.5 69.5 + 0.1

61. x = 1, y = -2

62. (3x + 4y) (2x + y)

63. 15cm^2

64. (i) 40%, (ii) two, (iii) Mathematics

65. (i) 13.20, (ii) 270, (iii) 1.74 x 10^{3}km

66. 4cm

67. (0.15 + 5.5x)m

68. 14cm

69. 10cm

70. 96000 litres

71. 144cm

72. $1\frac{2}{6}$

73. 660 yards of fencing are required

74. Luka 14 years; Bisi 7 years; Funke 11 years.

75. -1

76. $\angle A = 90^{\circ}$; $\angle B = 30^{\circ}$; $\angle C = 60^{\circ}$

77. 92/040 minutes in a leap year.

78. $2\frac{2}{3}$ days

79. 43 base 10

80. $531.14m^2$ or $53m^2$

81. AB + BC = 51cm

82. $x^2 - 3xcm^3$

83. 20m; 8 bundles

84. Yes

85. $\frac{77}{124}$

86. (a) 30, 40 (b) (x-3y-1)(x-y-1)

87. 1.5154; 1.1461

88. An Isocelse triangle

89. 25m

90. ₦5128

91. (a) 45 articles (b) ₦6

92. (a) ₦3.32 (b) $5\frac{1}{3}$ sq. m.

93. (i) (a) ₦8 or 8% (b) ₦$7\frac{11}{27}$ or $7\frac{11}{27}$%

(ii) Gain $16\frac{2}{3}$%

94. (a) ₦1,038.87

(b) 24.4k

95. (a) 69½k (b) 17k

96. (i) (a) 25 books

(b) ₦38.50

(ii) ₦3 loss

97. (a) 100m

(b) 8cm; 12 sq cm.

98. (a) 50 sq. cm.

(b) 4.8 sq. cm.

99. (i) (a) 132 sq. cm.

(b) 209 sq. cm.

(c) 231 cub. cm.

(ii) $377\frac{1}{7}$ cub. cm.

100. 1.37cm

101. 3cm.