A STUDY ON MAGNETOHYDRODYNAMIC NATURAL AND MIXED CONVECTION FLOWS IN VERTICAL CONCENTRIC ANNULI FILLED WITH POROUS MATERIAL

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A STUDY ON MAGNETOHYDRODYNAMIC(MHD) NATURAL AND MIXED CONVECTION FLOWS IN VERTICAL CONCENTRIC ANNULI FILLED WITH POROUS MATERIAL

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DEPARTMENT OF MATHEMATICS,
FACULTY OF SCIENCE,
AHMADU BELLO UNIVERSITY, ZARIA,
NIGERIA.

MAY, 2016.

DECLARATION

I declare that the work in this dissertationentitled "A STUDY ON MAGNETOHYDRODYNAMIC(MHD)NATURAL AND MIXED CONVECTION FLOWS IN VERTICAL CONCENTRIC ANNULI FILLED WITH POROUS MATERIAL" has been carried out by me in the Department of Mathematics. The information derived from the literature has been duly acknowledged in the text and a list of references provided. No part of this dissertation was previously presented for another degree or diploma at this or any other Institution.

_____ _____

Raphael Ahemba CHIA Date
Name of Student

CERTIFICATION

This dissertationentitled “A STUDY ON MAGNETOHYDRODYNAMIC(MHD) NATURAL AND MIXED CONVECTION FLOWS IN VERTICAL CONCENTRIC ANNULI FILLED WITH POROUS MATERIAL” byRAPHAEL AHEMBA CHIA, meets the regulation governing the award of the degree of Masters of Science of the Ahmadu Bello University, Zaria and is approved for its contribution to knowledge and literary presentation.

_____ _____

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DEDICATION

To my late strict father, Mr. Ugbadi Patrick Chia, whose examples I cherish and to my late step mother, Mrs. Ugbadi Juliana Nyiekula.Both died at the time of this research.

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Finally, all is nonsense, absolute nonsense, if I fail to give sincere thanks to **Almighty God** who is my life and everything. To Him, I return the honor which this work would bring to me.

ABSTRACT

This dissertationconsiders steady fully developed free and mixed convective flows of a viscous, incompressible, electrically conducting fluid in vertical concentric annuli, formed by two infinite and vertical concentric cylinders, filled with porous material having variable porosity in the presence of radial magnetic field. Unified exact solutions are derived by taking into account an isothermal or isoflux thermal boundary condition at the outer surface of inner cylinder. The solutions obtained are graphically represented and the effects of various controlling parameters such as Hartmann number, Darcy number, ratio of viscosity and the gap between the cylinders on the flow formation are discussed. The major results reveal that velocity of the fluid in the first problem is higher in case of isothermal heating of outer surface of inner cylinder compared to constant heat flux heating when the gap between cylinders is less or equal to radius of inner cylinders while reverse phenomena occur when the gap between cylinders is greater than radius of inner cylinder. In the second problem, velocity is approximately the same for both isothermal and isoflux thermal boundary conditions.

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NOMENCLATURE AND GREEK LETTERS

Nomenclature

T Dimensionless temperature

T_w' constant temperature

a Dimensional radius of inner cylinder

b Dimensional radius of outer cylinder

u' dimensional velocity of the fluid

g acceleration due to gravity

K_o constant permeability of porous medium at outer surface of inner cylinder

T' Dimensional temperature of the fluid

T_w constant temperature

T_∞ temperature of inner surface of outer cylinder

q' constant heat flux

k^* thermal conductivity

r' Dimensional radial distance

r Dimensionless radial distance

M Hartmann number

D_{a_o} Darcy number

u Dimensionless velocity of the fluid

g acceleration due to gravity

P Dimensionless pressure

Q Mass flux

Greek letters

μ coefficient of viscosity

$\boldsymbol{\nu}$ kinematic viscosity of fluid

μ_e magnetic permeability

μ_f fluid viscosity

μ_{eff} effective viscosity (μ_{eff} / ε)

σ electrical conductivity

β coefficient volume expansion

λ gap between the cylinders

γ ratio of viscosity

ε porosity of the porous medium

ρ density of fluid

τ_1 Skin friction at the outer surface of inner cylinder

τ_λ Skin friction at the inner surface of outer cylinder

CHAPTER ONE

GENERAL INTRODUCTION

1.1 Introduction

To appreciate the importance of fluid dynamics in life demands little more than just a glance around us. In general, life as we know would not exist if there are no fluids and the behavior they exhibit. The water and air we respectively drink and breathe are fluids. In addition, our body fluids are mostly water based. Essential to our healthy living is the proper movement of these fluids within our bodies. In a more practical setting, like in our transportation systems, recreation, entertainment (sound from radio speakers for example) and our sleep (water beds), fluids greatly influence our comfort. It is clear to see from this that engineers need a clear knowledge of fluid behavior to handle many systems of their encounter. The study of electrically conducting fluid (e.g. liquid metals) called magnetohydrodynamics (MHD) flow is found in numerous pieces of literature. This is due to its many areas of industrial applications in geophysics and engineering to determine the desired convective flow in cases like the design of MHD power generators, cooling of nuclear reactors and changing of metals solidification processes. Natural convection flow in a vertical channel is vital in many transport processes both in nature and engineering applications. Examples of flow and heat transfer include natural circulation in geothermal reservoirs, porous insulators, solar power collectors, spreading of pollutants and so on. Similarly, combined natural and forced convection flow known as mixed convection flow in a vertical channel has many applications both in nature and engineering in many transport processes such as in cooling of electronic devices

by fans, solar collectors exposed to wind currents, cooling of nuclear reactors during emergency shutdown and heat exchangers placed in a low-velocity environment. Mixed convection flow occurs either when the impact of external forces in natural convection or buoyancy forces in forced convection is significant. The combination of natural and forced convection is noticeable, especially when the forced flow velocity is low and/or the temperature gradient is large. Convection flow through porous media has also received much attention. The process is vital to many engineering and scientific applications. These include; separation of oil from sand by steam, porous heat exchangers, geological flows and movement of nutrient in mammalian tissues. When these kinds of flows are slowed down, are modeled by the use of Darcy's law $\Delta p = \frac{\mu u' L}{K}$ (where these quantities are as defined in the nomenclatures). This expression relates pressure gradient Δp spanning the length L of the medium to fluid velocity u', averaged over a large scale. The study of transport processes in annular geometry has also become an interesting area of research due to its wide range of applications, such as in cooling of turbine rotors and high speed gas bearings, electrical devices cooling system, heat exchangers and in drilling operation of oil and gas wells. Several researchers such as Singh et al (1997), Singh and Singh (2012) have investigated natural convection flows in vertical concentric annuli under different phenomena.It is in this background that our choice of this study is articulated.

1.2 Statement of the problem

Singh et al (1997) studied natural convection in vertical concentric annuli in the presence of a radial magnetic field a physical situation of heating or cooling the inner cylinder either isothermally or at constant heat flux. This problem however, is limited in application particularly in situations where the gap between the cylinders is filled with a material of varying porosity and the fluid transport process is actuated not only by the buoyancy forces but also by pressure forces. It is significant therefore, to stretch the work of Singh et al (1997) to capture these situations. Hence, this research endeavors to investigate the impact of variable porosity of the porous material trapped between the cylinders. Also, the influences of channel gap, magnetic field and pressure gradient as well as ratio of viscous forces on flow formation in natural convection in a vertical concentric annuli filled with porous material of variable porosity in the presence of a radial magnetic field are investigated.

1.3 Aim and objectives of the study

The overall aim of this research is to study natural and mixed convection flows in a vertical concentric annuli filled with porous material of varying porosity in presence of a radial magnetic field.

The objectives to attain the set aim are to:

(i) examine the flow behaviour of the fluid with variable porosity (D_{a_o}) of the porous material trapped between the cylinders

(ii) examine the effect of the gap between the cylinders (λ) on the flow formation

(iii) investigate the effect of Hartmann number (M) on flow formation

(iv) investigate fluid flow behavior with the ratio of viscosity (γ)

(v) consider the impact of pressure gradient (P) on the fluid flow in the annuli.

1.4 Research methodology

To attain the above set objectives, a survey of the existing works on natural and mixed convection flow in annuli under different phenomena is conducted and stretched to capture newly identified physical situations where the previous works are limited. The mathematical problems of "natural and mixed convection flow in a vertical concentric annuli filled with porous material of variable porosity in the presence of a radial magnetic field, which is nonlinear,were solved analytically using method of undetermined coefficients after transformation to linearity. The skin friction and mass flux are obtained in analytical form. Using a computer package, MATLAB 7.5 (2007b), the results obtained were presented graphically and in numerical values. The graphs and numerical values are analyzed in order to ascertain the impact of each of the controlling parameters on the flow.

1.5 Organization of the dissertation

This dissertation is grouped into five chapters with references and appendices. Chapter one comprises basically the general introduction of the dissertation. Literature review and the methodology employed in the research together with the solutions of the problemsare contained in chapters two and three respectively. While

chapter four deals with the results and discussions. Chapter five presents summary and conclusion. It is then, followed by references and appendices.

1.6 Basic Definitions

Annulus: is the area bounded by two concentric cylinders.

Boussinesq approximation: is the assumption that the flow is taken under sufficiently small difference in temperature and density.

Convection: is the transfer of heat from one place to another by the movement of a heated fluid.

Darcy Number: relative effect of permeability of a medium verses cross-sectional area. It is denoted by $D_a = \frac{K_o}{d^2}$

Dimensionless quantity: is a quantity without an associated physical dimension.

Fluid: is any substance that deforms continuously when subjected to a shear stress, no matter how small.

Forced convection: is a mechanism or form of heat transport in which fluid motion is generated by an external source (like suction device fan and pump).

Hartmann Number: is the ratio of electromagnetic force to the viscous force. It is denoted by $H_a = BL\sqrt{\frac{\sigma}{\mu}}$

Magnetic Field: is the magnetic effect of electric currents and magnetic material

Mass flux: is the rate of mass flow per unit area.

MHD: implies Magnetohydrodynamics which is the study of the magnetic properties of moving conductive fluids.

Mixed convection: is the type of heat transport caused by both natural and force convections.

Natural convection: is a mechanism or form of heat transport in which fluid motion is generated only by density differences in fluid cause by temperature gradients.

Porous: admitting the passage of fluid through pores of a solid surface.

Pressure gradient: is the change in atmospheric pressure per unit of horizontal distance in the direction which it changes most rapidly.

Skin friction: is the shear stress that occurs between the fluid and the solid surface of boundaries.

Viscosity: is a fluid property through which a fluid offers resistance to shear stresses.

1.7 Basic Hydrodynamic Equations

Continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho q) = 0$$

where, q is velocity of the flow.

If ρ is constant then, the equation reduces to

$$\nabla.q = 0$$

(ii) Momentum equation(Navier-Stokes Equation)

$$\rho\left[\frac{\partial q}{\partial t}+(q.\nabla)q\right]=\rho F-\nabla p+\mu\nabla^2 q$$

Where p is the specific thermodynamic work and F an eternal field

(iii) Energy equation

$$\rho C_\rho \frac{DT}{Dt}=k\nabla^2 T+\mu\Phi_\upsilon$$

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

The studies of boundary value problems related to the fluid flow through porous media have attracted the attention of scientists and engineers due to its numerous applications in civil engineering for simulation of the ground water flow and in chemical engineering for analyzing the real mechanism of oil spillage in the earth's ground among others. In addition, the fluid and heat transfer processes through porous media have been extensively studied in the past, due to its practical utility in nuclear waste disposal, solid matrix heat exchanger, thermal insulation and so on. The annulus represents a common geometry employed in a variety of heat transfer systems ranging from simple heat exchangers to the most complicated nuclear reactors. Since the flow and heat transfer in a cylindrical annular configuration contain all the essential physics that are common to all confined natural convective flows, a complete understanding of the flow in such geometry is very essential. As a result, several authors have made studies as regard the natural convective flows in annuli under different physical situations. This chapter therefore, presents some of the literature that set the knowledge through which the present study emerged. The survey is classified as follows:

2.2 Natural convection

The study of fluid flow phenomena in annular geometry is largely used in the field of heat exchangers. A typical application is the gas cooled nuclear reactors in which the

cylindrical fissionable fuel elements are placed axially in vertical coolants channel within the graphite moderators and the cooling gas is flowing along the channel parallel to the fuel elements (Nield and Bejan, 2006). The practical applications of flow in annular space can also be found for example in drilling operation of oil and gas wells and the design of magnetohydrodynamic power generators. Some related works to the fully developed laminar natural convection flows in vertical annuli include the works of El-Shaarawi and Sarhan(1981), Joshi(1987) and El-Shaarawi and Al-Nimb (1990). El-Shaarawi and Sarhan(1981) reported the fully developed free convection flows in vertical annuli with isothermal and adiabatic boundaries. Joshi(1987), presented the fully developed free convection flows in vertical annuli by considering two isothermal boundaries, the inner of which is kept at a higher temperature than the outer one. In the work of El-Shaarawi and Al-Nimb(1990), the effect of four fundamental boundary conditions by obtaining the corresponding fundamental solutions was investigated. These boundary conditions are obtained by combining each of the two conditions of having one boundary maintained at uniform heat flux or at uniform wall temperature with each of the conditions while the opposite boundary is kept isothermal at the inlet fluid temperature or adiabatic. The survey on the related literature also indicate that Lee and Kuo(1998), Mahmud and Fraser (2002),Leong and Lai (2006) as well as Jha*et al.*(2011) have studied natural convection in annuli considering different physical phenomena.

In addition, a numerical study was conducted by Venkatachalappa*et al.*(2001) to appreciate the effect of rotation on the axisymmetric flow driven by buoyancy in an annulus cavity formed by two concentric vertical cylinders which rotate about their

axis with different angular velocities. The inner and outer side walls are kept isothermally at different temperatures while the horizontal top and bottom wall are adiabatic.

Other inspiring works carried out on annular cavity when filled with porous materials include Jha (2005) who presented a study on free convection flow in an annulus porous medium. Before Jha(2005), Char and Lee (1998) already reported a numerical solution on natural convection in a porous medium saturated with cold water under density inversion within a vertical annulus. The numerical results indicate that the inversion parameter and radius ratio influence significantly, the flow structure and heat transfer rate in the annulus. Sankar*et al.*(2011)on the other hand, investigated natural convection flow in a vertical annulus filled with fluid saturated porous medium when the inner wall is discretely heated, while the outer one is kept isothermally at a lower temperature. The top and bottom walls and the unheated portions of the inner wall are maintained adiabatic. A year later, Asmaa (2012) conducted an investigation to find the effect of free convection on the developing forced laminar upward and downward flow in a vertical annulus when the inner is heated uniformly and the motionless outer cylinder is kept adiabatically insulated.

2.3Magnetohydrodynamic(MHD)

Magnetohydrodynamic(MHD)phenomenon has received considerable attention during the last two decades due to its importance from the energy generation point of view, and one may envisage MHD generators for power generation. MHD pumps are already in use in chemical energy technology for pumping electrically conducting

fluids at some of the atomic energy centers. Besides these applications, when the fluid is electrically conducting, the natural convection flow is appreciably influenced by an imposed magnetic field. The MHD flow problem in an annulus was first presented by Globe (1959). He considered fully developed laminar MHD flow in an annular channel. Jain and Mehta (1962) extended the problem to capture the situation when fluid is injected and sucked through the on the walls. An exact solution of electrically conducting viscous incompressible flow in an annulus with porous wall under an external radial magnetic field was obtained by Nandi (1973). Antimiror and Kolyshkin (1984) studied the unsteady magneto hydrodynamic flow in an annular channel with radial magnetic field while Singh *et al.*(1997) studied the effect of mixed kind of thermal boundary conditions on the free convective flow of an electrically conducting in annuli in the presence of a radial but muted induced magnetic fields. Later on, Singh and Singh (2011) stretched the problem to account for the induced magnetic field.

Conversely, MHD flows in porous media have received wide coverage in the development of energy generation systems; the application to paper, foodstuffs, personal care products, textile coating and suspension solutions industries; and the interest in astrophysical and geophysical fluid dynamics. Tan and Masuoka (2005), Khan *et al*.(2011) and Salah *et al*.(2011) have obtained exact solutions for the velocity field of a magnetic fluid corresponding to Stokes's first problem in a porous space under the influence of an applied magnetic field. Similar to Stokes's problems, Khan *et al.*(2007 and 2008) solved the modified Darcy's law for transient MHD flow in a porous medium of various boundary condition subjected to an external magnetic

field. In a related work, through analytical means, Khan *et al*. (2006) obtained the velocity field for an axisymmetric and fully developed MHD flow within a porous space in a circular pipe based on the modified Darcy's law. Srinivas and Muthuraj (2010) studied the MHD flow in a vertical wary porous medium with slip-flow boundary conditions in the presence of a temperature-dependent heat source. Recently, Jha*et al.* (2015) examined an exact solution of a fully developed MHD natural convection flow in a vertical annular micro channel.

2.4 Mixed convection

Many investigations have been carried out on mixed convection flow due to its usefulness in cooling of electronic devices by fans, solar collectors exposed to wind currents, cooling of nuclear reactors during emergency shutdown and heat exchangers placed in a low-velocity environment. To cite just but a few works in this direction, Voicu*et al.* (2007) studied numerically the steady state, simultaneously developing laminar mixed convection in a vertical double pipe heat exchanger when the flow is upward and parallel. The numerical results showed that when Richardson number is greater or equal to one, there is a reverse flow in the warm fluid. Before their report, Akhilesh*et al.* (2001) studied mixed convection flow in a vertical annulus filled with a porous material. They gave consideration to a situation when the outer surface of inner cylinder is heated isothermally and at isoflux with the gap between the two cylinders being less, equal and greater than the radius of the inner cylinder. The study on mixed convection flow in a vertical annulus filled with porous materials with variable porosity when one of the cylinders is heated isothermally and the other isoflux was presented by Kaurangini and Jha (2008). A year later, they

reconsidered the same problem when the porous materials trapped between the two cylinders are of constant porosity and with isothermally heated wall of the inner cylinder (Kaurangini and Jha, 2009). Again, Kaurangini and Jha (2010) presented results on the combined effect of Natural and forced convection flow in a vertical annulus filled with porous materials of constant porosity when the inner surface of the outer cylinder is either heated isothermally or constant heat flux. Other exclusive research was carried out by. Besides, Mohammed *et al.* (2009) investigated numerically, the steady state, laminar, axi-symmetric, mixed convection heat transfer in the annulus between two concentric vertical cylinders using porous inserts. The inner cylinder is at constant heat flux and the outer one insulated. One of their findings explained that, in general, the heat transfer is enhanced by the presence of porous layers of high thermal conductivity ratios. Esmail and Mokheimer (2014) investigated why flow reversal occurs in buoyancy-aided flows in mixed convection in vertical concentric annuli. While Jha and Aina (2015) were electrified by the knowledge of Avci and Aydin (2009) to examine the fully developed mixed convective heat transfer of a Newtonian fluid in vertical micro-porous-annulus formed by two concentric micro-tubes.

On the other hand, Barletta *et al.* (2008) presented a study on mixed convection with heating effects in a vertical porous annulus with a radial and varying magnetic field. Some of the related works are those of Barletta *et al.* (2009), Mozayyeni and Rahimi(2012) and Malvandi*et al.* (2015).

The survey of the related literatures enabled us to conveniently claim that the present study is new and has never been reported in literature.

CHAPTER THREE

MATHEMATICAL ANALYSIS

3.1 Introduction

This chapter is devoted to derivation of the momentum and energy equations of the flow formation under discussion. The non-dimensional form of the derived equations is obtained and transformed into a linear system of differential equations.

3.2 Natural Convection Flow in Vertical Concentric Annuli Filled with Porous Material Having Variable Porosity in Presence of Radial Magnetic Field.

The main objective in this section is to explore the impact of variable porosity of a material on natural convection flow in vertical concentric annuli filled with porous material in presence of radial magnetic field.

3.2.1 Mathematical Description

A steady laminar fully developed free convective flow of an electrically conducting fluid in vertical concentric annuli filled with porous material having variable porosity(see figure 3.1)

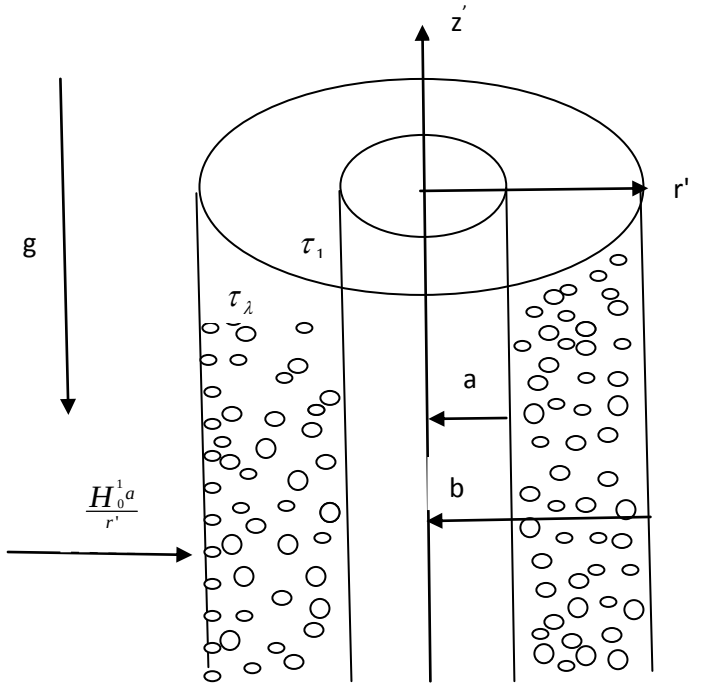

Fig 3.1:The configuration of problem 3.2.

is consideredin the presence of a radial magnetic field. The z' andr'-axes are taken along the axis of the co-axial cylinders in the vertical upwards and radial directions respectively. Let *a* and*b* be the radius of inner and outer cylinders respectively. Also, the magnetic field is directed radially outward and is of the form $H_0' a/r'$. We also consider the physical situation when the inner cylinder is heated or cooled either isothermally or at a constant heat flux so that its temperature (temperature of outer surface of the inner cylinder) is different from the ambient temperature T_∞'. Besides, in order to capture the variable porosity of the porous medium, it is assumed that $K = K_0 r'^2$. The flow is depending only on r' since it is fully developed and the cylinders are of infinite length. Thus, invoking the usual Boussinesq approximation,

the mathematical description capturing the present physical situation can be expressed as:

$$\frac{\mu_{eff}}{\mu_f}\left[\frac{d^2u'}{r'^2}+\frac{1}{r'}\frac{du'}{dr'}\right]-\left(\mu_e H_o' a\sqrt{\sigma/\mu}\right)^2\frac{u'}{r'^2}-\frac{au'}{K_o r'^2}+\frac{\rho g\beta\left(T'-T_\infty'\right)}{\mu}=0 \quad (3.1)$$

$$\frac{d^2T'}{dr'^2}+\frac{1}{r'}\frac{dT'}{dr'}=0 \quad (3.2)$$

Subject to the following boundary conditions:

$$u'=0;\begin{cases} T'=T_w' \quad or \quad \dfrac{dT'}{dr'}=\dfrac{q'}{k^*}, \quad at \quad r'=a \\ T'=T_\infty' \quad at \quad r'=b \end{cases} \quad (3.3)$$

Where u' is the velocity of the fluid, ρ and g, the fluid density and the acceleration due to gravity respectively, and μ the coefficient of viscosity. Also, μ_e , μ_f and μ_{eff} are respectively the magnetic permeability, fluid viscosity and effective viscosity. Moreover, σ is the electrical conductivity, T' the temperature of the fluid, β the coefficient volume expansion, r' and K_0 are the radius of the cylindrical annulus and constant permeability of the porous medium, respectively. Besides, a and b are respectively the radii of the inner and outer cylinders, while T_w' is constant temperature of the inner cylinder.

3.3 Mixed Convection Flow in Vertical Concentric Annuli Filled with Porous Material Having Variable Porosity in Presence of Radial Magnetic Field.

Here, the major objective is to investigate the influence of variable porosity and radial magnetic field on mixed convection flow in vertical concentric annuli with porous materials of different porosity.

3.3.1 Mathematical Description

Consider the steady laminar fully developed mixed convective flow of an electrically conducting fluid in open-ended vertical concentric channel of infinite length. The z' and r'-axes are respectively taken in the vertical upwards direction and radial direction.

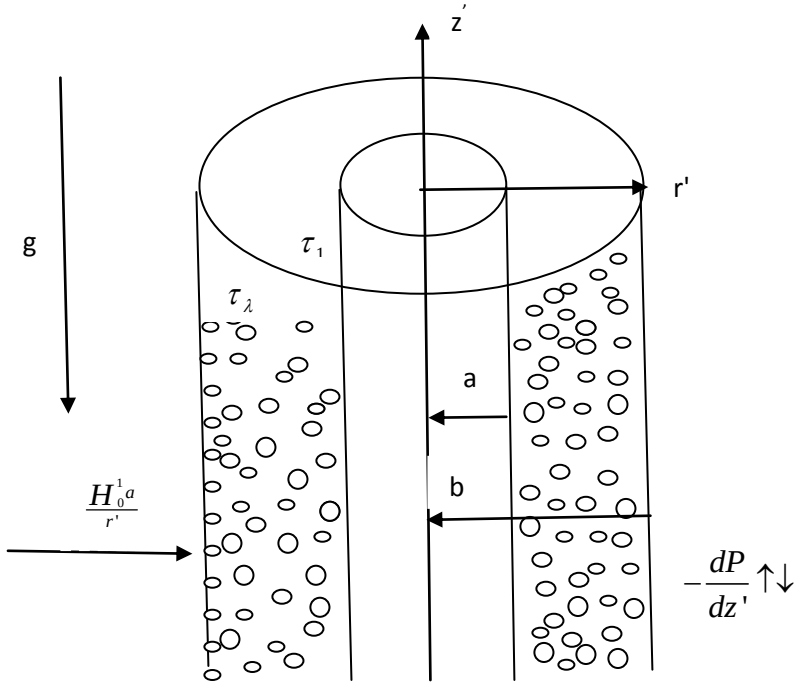

Fig 3.2: The configuration of problem 3.3

Let a and$b$ be the radius of inner and outer cylinders respectively (see figure 3.2 above). The magnetic field is directed radial outward and is of the form $\frac{H_0 a}{r}$. It is assumed that the magnetic Reynolds number is very small, which corresponds to negligibly induced magnetic field compared to the externally applied one. Also, we consider the physical situation when the inner cylinder is heated or cooled either isothermally or at a constant heat flux so that its temperature (temperature of outer surface of the inner cylinder) is different from the ambient temperature T_∞'. The fluid velocity u along z' is depending only on r' since it is fully developed and the cylinders are of infinite length. Thus, employing Boussinesq approximation, the dimensional basic equations for the problem considered are presented as:

$$\frac{\mu_{eff}}{\mu_f}\left[\frac{d^2u'}{r'^2}+\frac{1}{r'}\frac{du'}{dr'}\right]-\left(\mu_e H_o' a\sqrt{\sigma/\mu}\right)^2\frac{u'}{r'^2}-\frac{a^2u'}{k_o r'^2}+\frac{\rho g\beta\left(T'-T_\infty'\right)}{\mu}-\frac{1}{\rho}\frac{dP}{dz'}=0 \quad (3.4)$$

$$\frac{d^2T'}{dr'^2}+\frac{1}{r'}\frac{dT'}{dr'}=0 \quad (3.5)$$

Subject to the following boundary conditions:

$$u=0;\quad \begin{cases} T'=T_w' \quad or \quad \dfrac{dT'}{dr'}=\dfrac{q'}{k^*}, & at\ r'=a \\ T'=T_\infty' & at\ r'=b \end{cases} \quad (3.6)$$

Where $\frac{dP}{dz'}$ is constant derived from the hydrodynamic pressure $P = p + \rho g z'$ which depends on z' only (since the flow condition is fully developed), while the other parameters are as defined in section 3.2.1 above.

3.4 Non-Dimensionalization

Dimensionless analysis employs the fundamental units of dimensions. i.e. Mass(m), Length(l) and Time(t). Its usefulness is found in experimental works, for it gives a direction to factors that have significant impact on the studied phenomena. Also, it is commonly used to establish the relationships between several variables. Based on the understanding of the present problems, let the following non-dimensional parameters be defined as:

$$\left.\begin{aligned} u &= \frac{\mu u'}{\rho g \beta a^2 \Delta T} \; or \frac{\nu u'}{g \beta a^2 \Delta T}, \qquad r = \frac{r'}{a}, \qquad \lambda = \frac{b}{a}, \qquad P = -\frac{dP}{dz'} \Big/ \left(\rho g \beta \Delta T\right) \\ T &= \frac{T' - T'_{\infty}}{\Delta T}, \qquad D_{a_o} = \frac{D_a}{r'^2} = \frac{K_o}{a^2 r'^2}, \qquad \gamma = \frac{\mu_{eff}}{\mu_f}, \qquad M = \mu_e H'_o a \sqrt{\frac{\sigma}{\mu}} \end{aligned}\right\} \quad (3.7)$$

where, $D_a = \frac{K_o}{a^2}$, $\Delta T = T'_w - T'_{\infty}$ or $\frac{q'a}{k^*}$ according as inner cylinder is maintained at constant temperature T'_W or constant heat flux q'. k^* is thermal conductivity. Now, employing the non-dimensional parameters in equation (3.7), the controlling equations (3.1) and (3.2) become non-dimensional as shown below:

$$\gamma\left[\frac{\mathrm{d}^2u}{dr^2}+\frac{1}{r}\frac{du}{dr}\right]-\left(M^2+\frac{1}{D_{a_o}}\right)\frac{u}{r^2}+T=0 \quad (3.8)$$

$$\frac{d^2T}{dr^2}+\frac{1}{r}\frac{dT}{dr}=0 \quad (3.9)$$

with the following boundary conditions indicating the isothermal and constant heat flux cases of the inner cylinder:

$$u=0; \quad \begin{cases} T=1 \ \ or \ \ \dfrac{dT}{dr}=-1, & at\ r=1 \\ T=0, & at\ r=\lambda \end{cases} \quad (3.10)$$

The boundary conditions for the temperature field at the inner cylinder can be combined to obtain a unified solution for both cases. Thus, equation (3.11) below is modeled after equation (3.10)

$$u=0; \quad \begin{cases} \mathrm{A}\dfrac{\mathrm{dT}}{\mathrm{dr}}+BT=C, & at\ r=1 \\ T=0, & at\ r=\lambda \end{cases} \quad (3.11)$$

By assigning appropriate values to *A*,*B* and *C* give the desired case.

Also, the equations (3.4) and (3.5) using the appropriate non-dimensional parameters given in equation (3.7) become dimensionless and presented as:

$$\gamma\left[\frac{\mathrm{d}^2u}{dr^2}+\frac{1}{r}\frac{du}{dr}\right]-\left(M^2+\frac{1}{D_{a_o}}\right)\frac{u}{r^2}+T+P=0 \quad (3.12)$$

$$\frac{d^2T}{dr^2}+\frac{1}{r}\frac{dT}{dr}=0 \quad (3.9)$$

Subject to the boundary conditions given in equation (3.11)

3.5 Transformation for Linear Second Order Differential Equations

The equations (3.9) and (3.12) can be transformed into a second order linear differential equation with constant coefficients by the use of the substitution:

let $r=e^z$

$\Rightarrow z=\ln(r)$

Taking the first and second derivatives of u and T with respect to r give

$$\frac{du}{dr}=\frac{du}{dz}\frac{dz}{dr}=\frac{1}{r}\frac{du}{dz} \quad (3.13)$$

$$\frac{d^2u}{dr^2}=\frac{1}{r^2}\frac{d^2u}{dz^2}-\frac{1}{r^2}\frac{du}{dz} \quad (3.14)$$

And

$$\frac{dT}{dr}=\frac{dT}{dz}\frac{dz}{dr}=\frac{1}{r}\frac{dT}{dz} \quad (3.15)$$

$$\frac{d^2T}{dr^2}=\frac{1}{r^2}\frac{d^2T}{dz^2}-\frac{1}{r^2}\frac{dT}{dz} \quad (3.16)$$

Substituting equations (3.13) – (3.16) appropriately in Equations (3.8), (3.9) and (3.12) render the latter set of equations in linear form as follows:

$$\gamma \frac{d^2u}{dz^2} - \left(M^2 + \frac{1}{D_{a_o}} \right) u = -Te^{2z} \tag{3.17}$$

$$\frac{d^2T}{dz^2} = 0 \tag{3.18}$$

$$\gamma \frac{d^2u}{dz} - \left(M^2 + \frac{1}{D_{a_o}} \right) u = -(T + P)e^{2z} \tag{3.19}$$

3.6 Solution to Problem3.2

The solution of the energy equation (3.18) using the boundary conditions given in equation (3.11) is:

$$T = \frac{C}{C_1} \log\left(\frac{r}{\lambda} \right) \tag{3.20}$$

Substituting equation (3.20) into the momentum equation (3.17) and simplifying gives

$$\gamma \frac{d^2u}{dz^2} - \left(M^2 + \frac{1}{D_{a_o}} \right) u = -e^{2z} \frac{C}{C_1} \left(z - \ln(\lambda) \right) \tag{3.21}$$

Applying the method of undetermined coefficients, the above equation (3.21) is solved within the confine of equation (3.11). Below is the result obtained:

$$u = C_3 r^k + C_4 r^{-k} + \frac{C}{\gamma C_1} r^2 \left(\frac{\log\left(\frac{r}{\lambda}\right)}{C_1} + \frac{4}{C_2^2} \right) \tag{3.22}$$

where,

$$k = \sqrt{\frac{1}{\gamma}\left(M^2 + \frac{1}{D_{a_0}}\right)}, \qquad C_1 = A - B log\lambda, \qquad C_2 = \frac{M^2}{\gamma} + \frac{1}{\gamma D_{a_0}} - 4,$$

$$C_3 = -\frac{C}{\gamma C_1 (1 - \lambda^{-2k})}\left[\frac{\lambda^{-2k} log\lambda}{C_2} - 4\left(\frac{\lambda^{-2k} - \lambda^{2-k}}{C_2^2}\right)\right], C_4$$

$$= \frac{C}{\gamma C_1 (1 - \lambda^{-2k})}\left[\frac{log\lambda}{C_2} - 4\left(\frac{1 - \lambda^{2-k}}{C_2^2}\right)\right].$$

3.6.1 Skin Friction of Problem 3.2

Another physical quantity associated with the flow considered is the skin friction (τ) which is the friction between a moving fluid and its bounding surfaces.The skin frictions at the outer and inner surfaces of the inner and outer cylinders respectively are obtained by taking the first derivative of the velocity (u) given in equation (3.22) with respect to r as:

$$\tau_1 = \left.\frac{du}{dr}\right|_{r=1} = kC_3 - kC_4 + \frac{C}{\gamma C_1}\left[\frac{1 - 2\log(\lambda)}{C_2} + \frac{8}{C_2^2}\right] \tag{3.23}$$

$$\tau_\lambda = \left.\frac{du}{dr}\right|_{r=\lambda} = kC_3\lambda^{k-1} - kC_4\lambda^{-k-1} + \frac{\lambda C}{\gamma C_1}\left[\frac{1}{C_2} + \frac{8}{C_2^2}\right] \tag{3.24}$$

Besides, the amount of fluid being dragged by the flow through the annuli called the mass flux is also a physical quantity of interest to this study. It is handy by taking adefinite integral of equation (3.22) with respect to r as shown below:

$$Q = \int_1^{\lambda} 2\pi\, rudr$$
$$= 2\pi\left[\frac{C_3}{k+2}\left(\lambda^{k+2}-1\right)+\frac{C_4}{2-k}\left(\lambda^{2-1}-1\right)+\frac{C}{\gamma C_1}\left(\frac{\log(\lambda)}{4C_2}-\frac{\lambda^4-1}{16C_2}+\frac{\lambda^4-1}{C_2^2}\right)\right] \qquad (3.25)$$

3.7 Solution to Problem 3.3

Equation (3.19) in expanded form using equation (3.20) can be presented as:

$$\gamma\frac{d^2u}{dz}-\left(M^2+\frac{1}{D_{a_o}}\right)u=-\left[\frac{C}{C_1}\left(z-\ln(\lambda)\right)+P\right]e^{2z} \qquad (3.26)$$

The solution of the above equation (3.26) is obtained by employing undetermined coefficient method and equation (3.11) religiously. The equation (3.27) below is the solution obtained

$$u = C_3 r^k + C_4 r^{-k} + r^2\left[\frac{C}{\gamma C_1}\left(\frac{\log\left(\frac{r}{\lambda}\right)}{C_2}+\frac{4}{C_2^1}\right)+\frac{P}{\gamma C_2}\right] \qquad (3.27)$$

The expression for dimensionless pressure gradient can be calculated by using the condition

$$\int_1^{\lambda} ru(r)dr = \frac{1}{2}\left(\lambda^2-1\right)$$

where,

$$k = \sqrt{\frac{1}{\gamma}\left(M^2 + \frac{1}{D_{a_0}}\right)}, \qquad C_1 = A - B log\lambda, \qquad C_2 = \frac{M^2}{\gamma} + \frac{1}{\gamma D_{a_0}} - 4,$$

$$C_3 = -\frac{C}{\gamma C_1(1-\lambda^{-2k})}\left[\frac{\lambda^{-2k} log\lambda}{C_2} - 4\left(\frac{\lambda^{-2k} - \lambda^{2-k}}{C_2^2}\right)\right] - \frac{P(\lambda^{-2k} - \lambda^{2-k})}{\gamma C_2(1-\lambda^{-2k})}$$

$$C_4 = \frac{C}{\gamma C_1(1-\lambda^{-2k})}\left[\frac{log\lambda}{C_2} - 4\left(\frac{1-\lambda^{2-k}}{C_2^2}\right)\right] + \frac{P(1-\lambda^{2-k})}{\gamma C_2(1-\lambda^{-2k})}$$

$$C_5 = \frac{(1-\lambda^{2-k})^2}{\gamma C_2(2-k)(1-\lambda^{-2k})} + \frac{(\lambda^{k+2}-1)(\lambda^{-2k} - \lambda^{2-k})}{\gamma C_2(k+2)(1-\lambda^{-2k})} + \frac{\lambda^4 - 1}{4\gamma C_2}$$

$$P = \frac{1-\lambda^2}{2C_5} + \frac{C(1-\lambda^{k+2})}{C_5\gamma C_1(k+2)(1-\lambda^{-2k})}\left[\frac{\lambda^{-2k} log\lambda}{C_2} - 4\left(\frac{\lambda^{-2k} - \lambda^{2-k}}{C_2^2}\right)\right]$$
$$+ \frac{(\lambda^{2-k}-1)}{C_5\gamma C_1(2-k)(1-\lambda^{-2k})}\left[\frac{log\lambda}{C_2} - 4\left(\frac{1-\lambda^{2-k}}{C_2^2}\right)\right]$$
$$+ \frac{C}{C_5\gamma C_1}\left(\frac{\log\lambda}{4C_2} - \frac{\lambda^4-1}{16C_2} + \frac{\lambda^4-1}{C_2^2}\right)$$

3.7.1 Skin Friction of Problem 3.3

The skin frictions at the outer surface of the inner cylinder τ_1 and the inner surface of the outer cylinder τ_λ are obtained by differentiating equation (3.27) once with respect to r. Thus,

$$\tau_1 = \left.\frac{du}{dr}\right|_{r=1} = kC_3 - kC_4 + \frac{C}{\gamma C_1}\left[\frac{1-2\log(\lambda)}{C_2} + \frac{8}{C_2^2}\right] - \frac{2P}{C_2} \qquad (3.28)$$

$$\tau_{\lambda} = \left.\frac{du}{dr}\right|_{r=\lambda} = kC_3\lambda^{k-1} - kC_4\lambda^{-k-1} + \frac{\lambda C}{\gamma C_1}\left[\frac{1}{C_2} + \frac{8}{C_2^2}\right] - \frac{2P\lambda}{C_2} \tag{3.29}$$

It is also important to consider the mass flux of the fluid (Q) through the annuli. It is captured by integrating equation (4.9) with respect to r and the result is as shown below:

$$Q = \int_1^{\lambda} 2\pi\, rudr$$

$$= 2\pi\left[\frac{C_3}{k+2}\left(\lambda^{k+2} - 1\right) + \frac{C_4}{2-k}\left(\lambda^{2-k} - 1\right) + \frac{C}{\gamma C_1}\left(\frac{\log(\lambda)}{4C_2} - \frac{\lambda^4 - 1}{16C_2} + \frac{\lambda^4 - 1}{C_2^2}\right)\right.$$

$$\left. - \frac{P}{4C_2}\left(\lambda^4 - 1\right)\right] \tag{3.30}$$

3.8: Validation of the Methods

In order to validate the accuracy of the methods adopted, we seek to find out the solution obtained by Singh *et al.* (1997), which should be in perfect agreement with our solution in equation(3.22) for problem 3.2at large value of the variable porosity and equal values of viscosity. By letting $D_{a_0} = 10^4$ and $\gamma = 1$ inequation(3.22), the solution for velocity distribution given by Singh *et al.*(1997) is obtained. Similarly, by setting $P = 0$ in equation(3.27), the expression for the solution of problem 3.2 given in equation(3.22) is obtained.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 Introduction

This chapter presents the results obtained from the previous chapter in the form of line graphs and tables that permit easy understanding of the discussion of such results.

4.2 Results and Discussion of Problem 3.2

Natural convection in vertical concentric annuli filled with porous material having variable porosity in the presence of radial magnetic field with constant heat flux and isothermally heated cylinders is investigated. In order to see the effect of controlling parameters such as Hartmann number (M), Darcy number (D_{a0}), gap between inner and outer cylinders (λ) and ratio of viscosity (γ) on the flow, we have plotted the line graphs for velocity and presented in figures 4.1 – 4.6. The values of skin-friction and mass flux have also been calculated in Tables 1-2. The present parametric study has been performed over reasonable range of $10^{-3} \leq D_{a_0} \leq 10^{-1}$, $0.5 \leq \lambda \leq 1.5$ and $5 \leq M \leq 15$ while the gap between the concentric cylinders (λ) are taken as 1.8, 2.0, 3.0, and 4.0. The results presented here are limited to velocity field since temperature field has already been extensively discussed by the Singh *et al.* (1997).

Figures 4.1 and 4.2 present the velocity distribution for isothermal and constant heat flux cases for different values of Darcy number $\left(D_{a_0}\right)$ and gap between inner and outer cylinders (λ). It is observed from these figures that velocity of the fluid

increases when Darcy number (D_{a_0}) and the gap between the cylinders increase for both isothermal and constant heat flux cases. In addition, when $\lambda \leq 2$, the velocity is higher for isothermal case compare to the constant heat flux case while the reverse trend occurs when $\lambda > 2$.

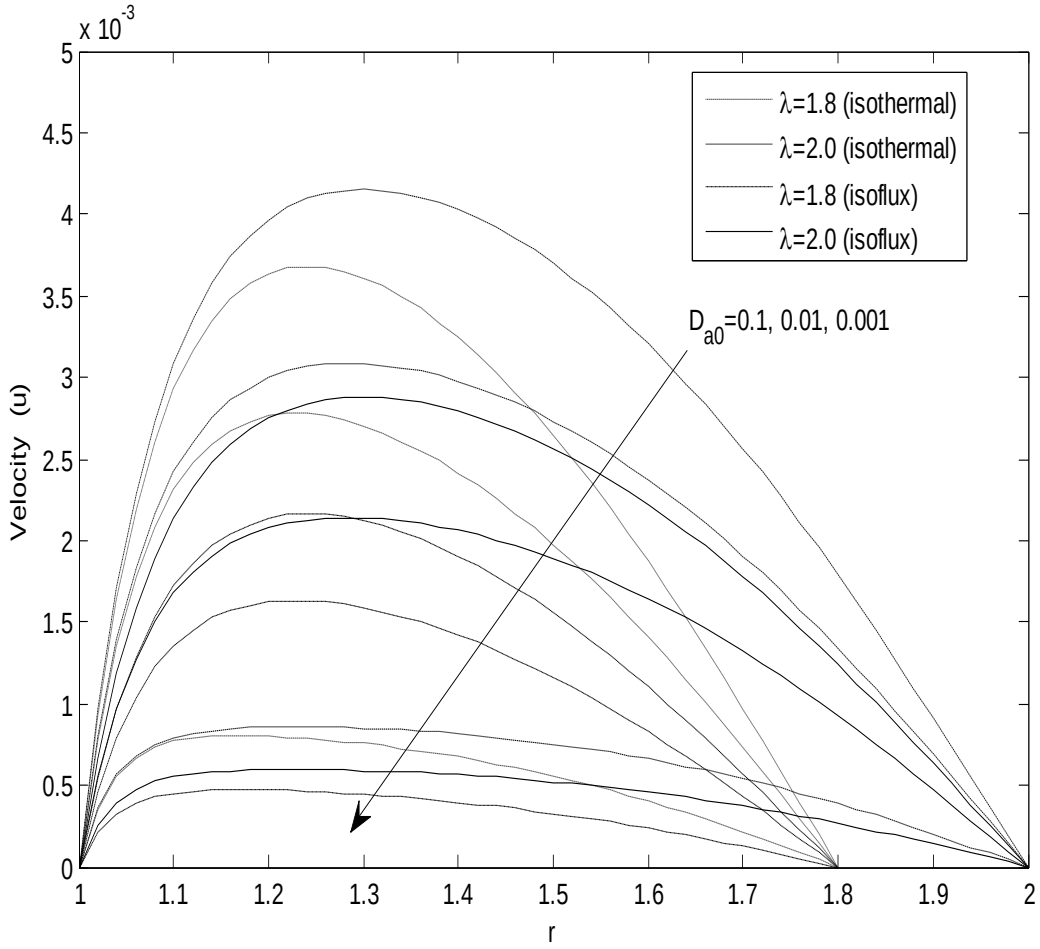

Fig 4.1: Velocity Distribution for different values of D_{a_o} (M =15.0, γ= 1.5, λ = 1.8 and2)

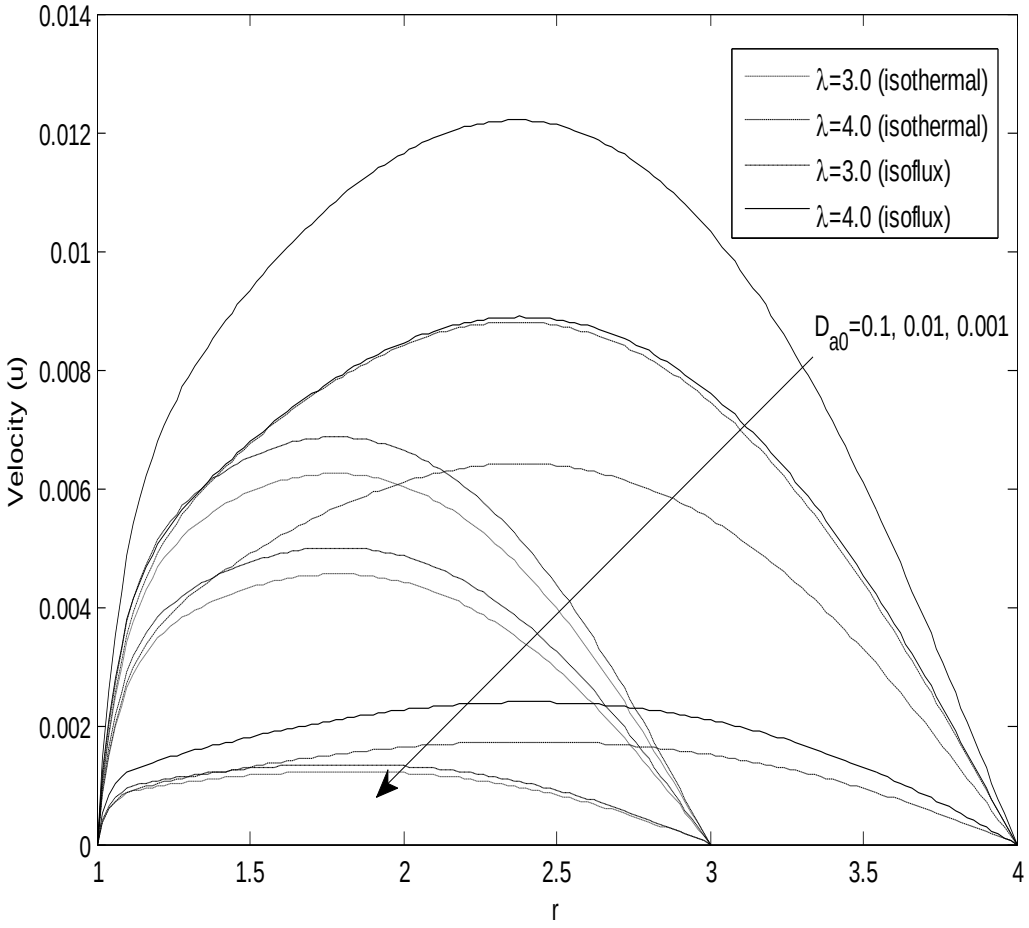

Fig 4.2: Velocity Distribution for different values of D_{a_0}

($M = 15.0$**,** $\gamma = 1.5$**,** $\lambda = 3$ **and 4)**

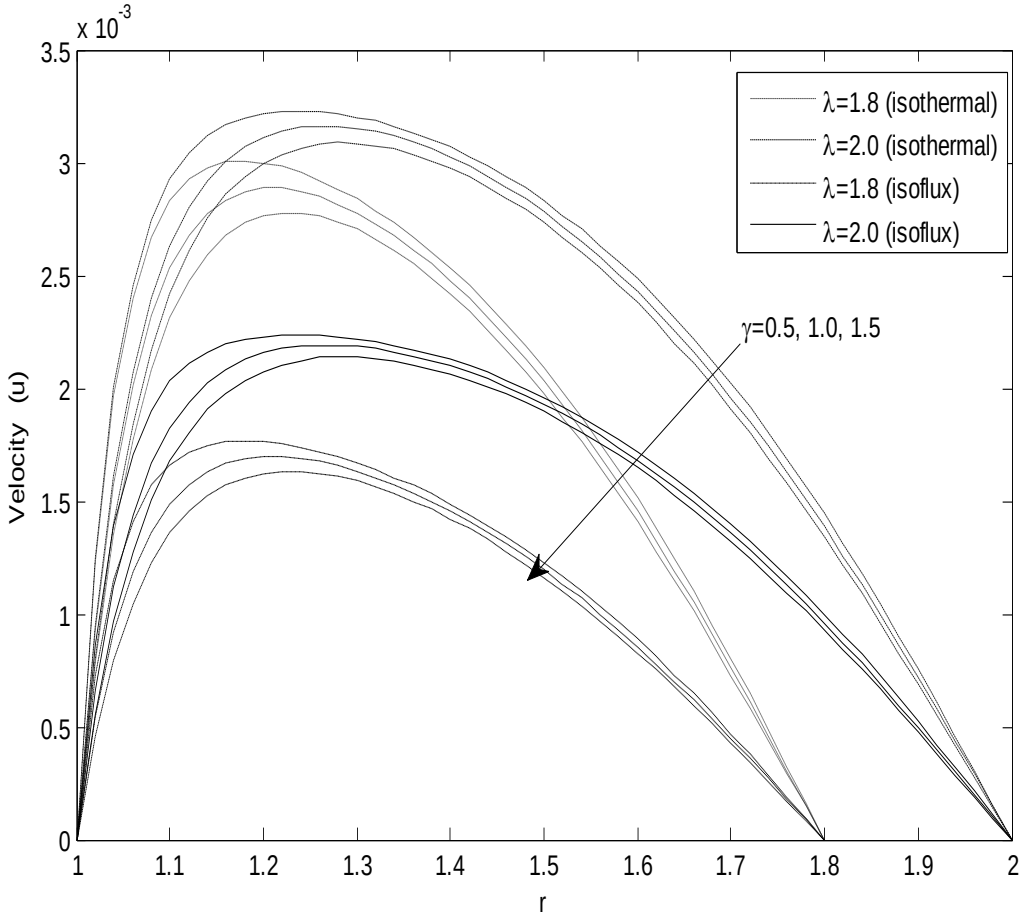

Fig 4.3: Velocity Distribution for different values γ
($M = 15.0, D_{a_0} = 0.01, \lambda = 1.8$ and 2)

Figures 4.3 and 4.4 show the distribution of velocity for both isothermal and constant heat flux cases for different values of the ratio of viscosity (γ) and gap between inner and outer cylinders (λ). As in figures 4.1 and 4.2, it is clear from Figures 4.3 and 4.4 that, velocity of the fluid increases when the gap between the cylinders increases.

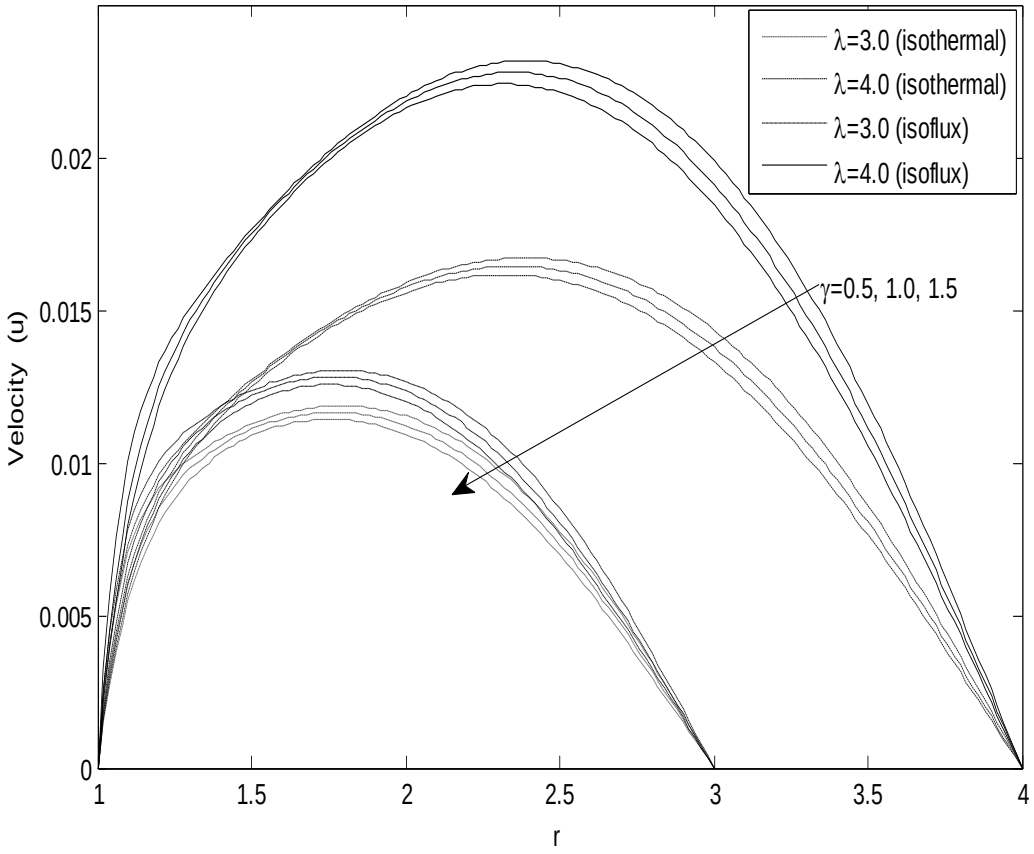

Fig 4.4: Velocity Distribution for different values of γ
($M = 15.0, D_{a_0} = 0.01, \lambda = 3.0$ and 4)

It is also observed that when $\lambda \leq 2$, the velocity is more for isothermal case as compare to the constant heat flux case while the reverse situation occurs when $\lambda > 2$. In addition, the influence of ratio of viscosity (γ) is to decrease the velocity field for both cases of the thermal heating.

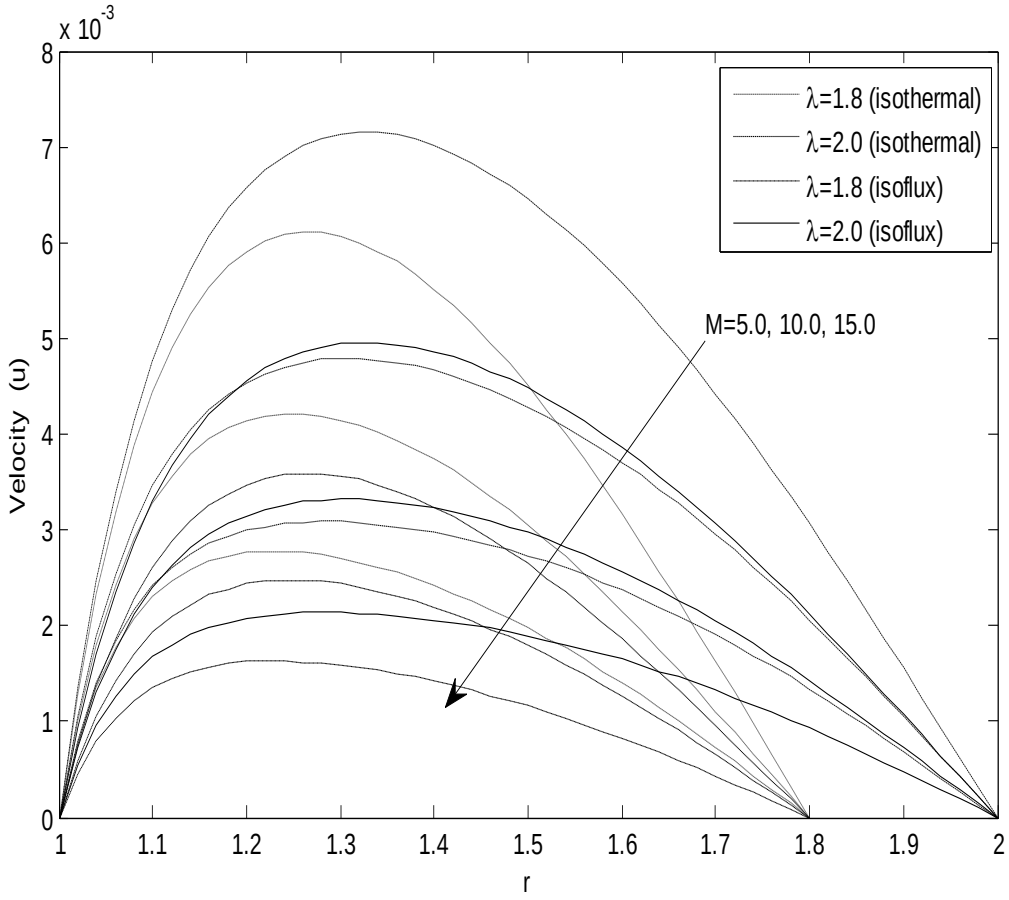

Fig 4.5: Velocity Distribution for different values of M
($\gamma = 1.5$, $D_{a_0} = 0.01$, $\lambda = 1.8$ **and 2**)

The velocity distributions for isothermal and constant heat flux cases for different values of radial magnetic field(M) and gap between inner and outer cylinders (λ) are presented in Figures 4.5 and 4.6, respectively. It is found from these Figures that velocity of the fluid increases when the gap between the cylinders increases as explained above.

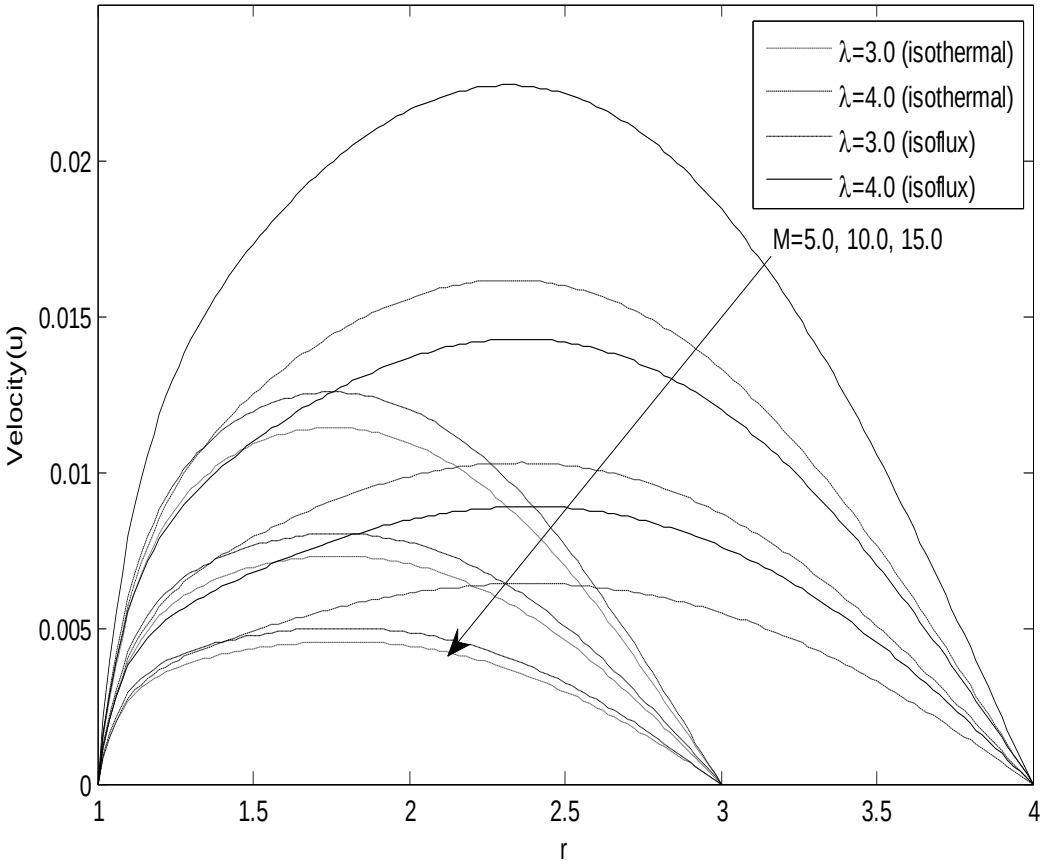

Fig 4.6: Velocity Distribution for different values of M ($\gamma = 1.5$, $D_{a_0} = 0.01$, $\lambda = 3$ and 4)

As earlier noted,when $\lambda \leq 2$, the velocity is higher for isothermal case than the constant heat flux case while the reverse trend occurs when $\lambda > 2$. Also, the impact of increasing radial magnetic field is to reduce the velocity field for both isothermal and constant heat flux cases.

The numerical values for skin-friction τ_1 (at the outer surface of inner cylinder), τ_λ (at the inner surface of outer cylinder) and mass flux (Q) for both isothermal heating and constant heat flux cases at the outer surface of the inner cylinder are presented in Tables 1and 2 at fixed value of $M = 5.0$.

Table1. Numerical values for skin friction and mass flux, when $\lambda \leq 2$ and $M = 5.0$

			isothermal case			Constant heat flux		
$\lambda D_{a_0} \gamma$			$\tau_1 \tau_\lambda$	Q		$\tau_1 \tau_\lambda$	Q	
1.8	0.1	0.5	0.2315	0.0551	0.0921	0.1360	0.0324	0.0541
		1.0	0.1531	0.0436	0.0715	0.0900	0.0256	0.0420
		1.5	0.1170	0.0360	0.0584	0.0688	0.0212	0.0343
	0.01	0.5	0.1270	0.0193	0.0327	0.0746	0.0113	0.0192
		1.0	0.0888	0.0175	0.0296	0.0522	0.0103	0.0174
		1.5	0.0714	0.0161	0.0270	0.0420	0.0095	0.0159
	0.001	0.5	0.0444	0.0027	0.0045	0.0261	0.0016	0.0026
		1.0	0.0314	0.0026	0.0044	0.0185	0.0016	0.0026
		1.5	0.0257	0.0026	0.0043	0.0151	0.0015	0.0025
2.0	0.1	0.5	0.2435	0.0530	0.1464	0.1688	0.0367	0.1015
		1.0	0.1661	0.0434	0.1194	0.1151	0.0301	0.0828
		1.5	0.1298	0.0369	0.1009	0.0899	0.0256	0.0699
	0.01	0.5	0.1297	0.0182	0.0493	0.0899	0.0126	0.0342
		1.0	0.0918	0.0166	0.0456	0.0636	0.0115	0.0316
		1.5	0.0747	0.0154	0.0426	0.0518	0.0107	0.0295
	0.001	0.5	0.0447	0.0026	0.0066	0.0310	0.0018	0.0046
		1.0	0.0317	0.0025	0.0065	0.0220	0.0017	0.0045
		1.5	0.0260	0.0024	0.0064	0.0180	0.0017	0.0044

Table 2. Numerical values for skin friction and mass flux, when $\lambda > 2$ and $M = 5.0$

			isothermal case			Constant heat flux		
$\lambda D_{a_0} \gamma$			$\tau_1 \tau_\lambda$	Q		$\tau_1 \tau_\lambda$	Q	
3.0	0.1	0.5	0.2693	0.0508	0.6630	0.2958	0.0558	0.7284
		1.0	0.1966	0.0434	0.5946	0.2160	0.0477	0.6532
		1.5	0.1626	0.0386	0.5408	0.1786	0.0424	0.5942
	0.01	0.5	0.1353	0.0172	0.2045	0.1486	0.0189	0.2246
		1.0	0.0981	0.0157	0.1963	0.1078	0.0173	0.2156
		1.5	0.0816	0.0147	0.1893	0.0896	0.0161	0.2080
	0.001	0.5	0.0452	0.0024	0.0261	0.0497	0.0027	0.0287
		1.0	0.0323	0.0024	0.0259	0.0355	0.0026	0.0284
		1.5	0.0266	0.0023	0.0257	0.0292	0.0025	0.0282
4.0	0.1	0.5	0.2786	0.0537	1.7883	0.3862	0.0744	2.4791
		1.0	0.2085	0.0460	1.6409	0.2890	0.0638	2.2747
		1.5	0.1765	0.0411	1.5242	0.2447	0.0570	2.1129
	0.01	0.5	0.1372	0.0182	0.5415	0.1903	0.0252	0.7506
		1.0	0.1004	0.0166	0.5238	0.1391	0.0230	0.7261
		1.5	0.0841	0.0155	0.5086	0.1165	0.0215	0.7051
	0.001	0.5	0.0454	0.0026	0.0685	0.0630	0.0036	0.0949
		1.0	0.0325	0.0025	0.0681	0.0451	0.0035	0.0943
		1.5	0.0268	0.0024	0.0677	0.0371	0.0034	0.0938

It can be clearly seen from these Tables that, the skin friction and mass flux increase with increase in Darcy number (D_{a_0}), while increasing the ratio of viscosity (γ) causes the skin friction and mass flux to decline for both isothermal heating and constant heat flux cases. Also, it is found that increasing the gap between the inner and outer cylinders (λ),promotes the skin friction and mass flux. Furthermore, from comparative study of tables 1 and 2, it is deduced that when the gap between cylinders is less or equal to radius of inner cylinder, the skin friction and mass flux arehigher for isothermal heating of the inner cylinder than constant heat flux case while the result reverses when gap between cylinders is greater than radius of inner cylinder. Additionally, the increase or decrease in τ_1 is higher than τ_λ when $\lambda \leq 2$ while the reverse phenomena occurs when $\lambda > 2$

4.3 Results and Discussion of Problem 3.3

MHD mixed convection flow in vertical concentric annuli filled with porous material having variable porosity with constant heat flux and isothermally heated cylinders is investigated. As in section 4.2, the effects of the controlling parameters such as Hartmann number(M), Darcy number (D_{a0}), gap parameter (λ) , ratio of viscosity $(\gamma)$and pressure gradient (P)on the flow is numerically presented in figures 4.7 – 4.13 and tables 3 – 8, using$10^{-3} \leq D_{a_o} \leq 10^{-1}$, $0.5 \leq \gamma \leq 1.5$ and $5 \leq M \leq 15$, while $\lambda$is taken as 1.8, 2.0, 3.0, and 4.0.

The plot of velocity profiles for both isothermal and constant heat flux cases for different values of each of these parameters are presented in figures 5.7 to 5.13. Generally, it is found from the figures that velocity of the fluid increases when the

gap between the cylinders increases. Also, the velocity of the fluid when the outer surface of the inner cylinder is heated isothermally is approximately equal to that of isoflux heating of the outer surface of the inner cylinder as shown in figure 5.13.

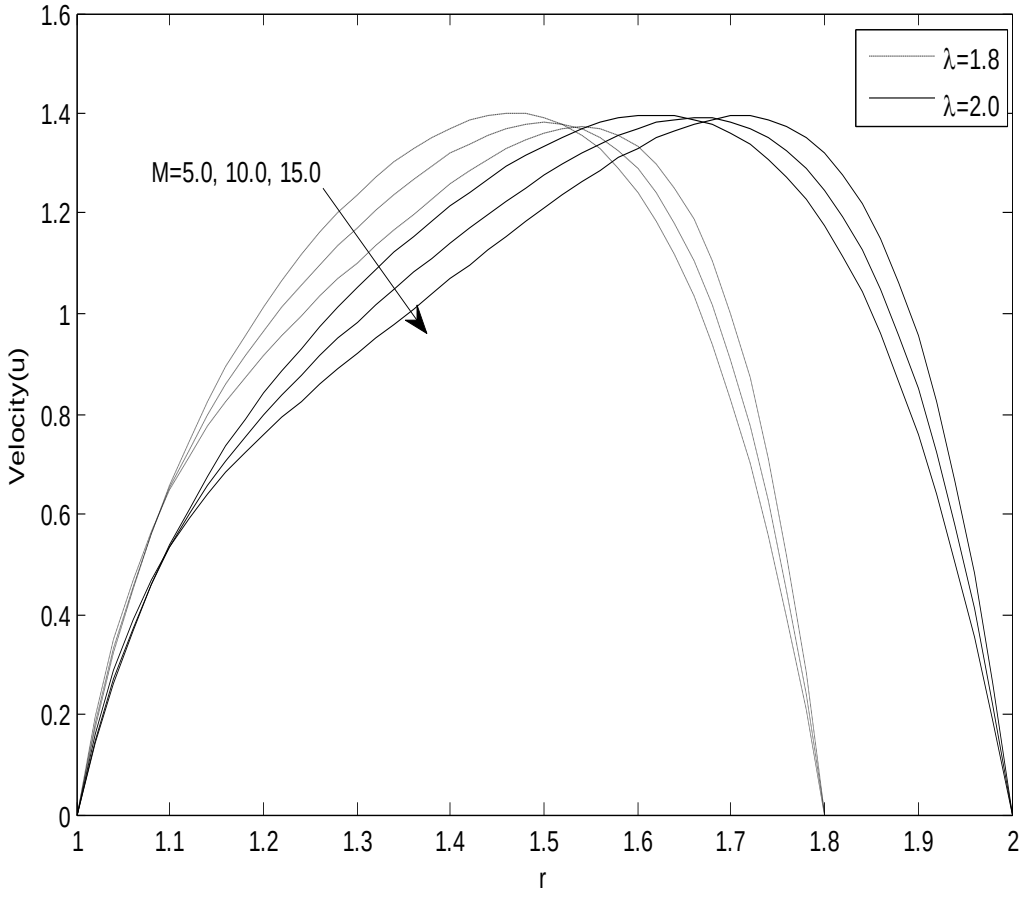

Fig 5.7a: Velocity profiles for different M for isothermal case ($\gamma = 15$, $D_{a_0} = 0.01$, $\boldsymbol{\lambda = 1.8}$ and 2)

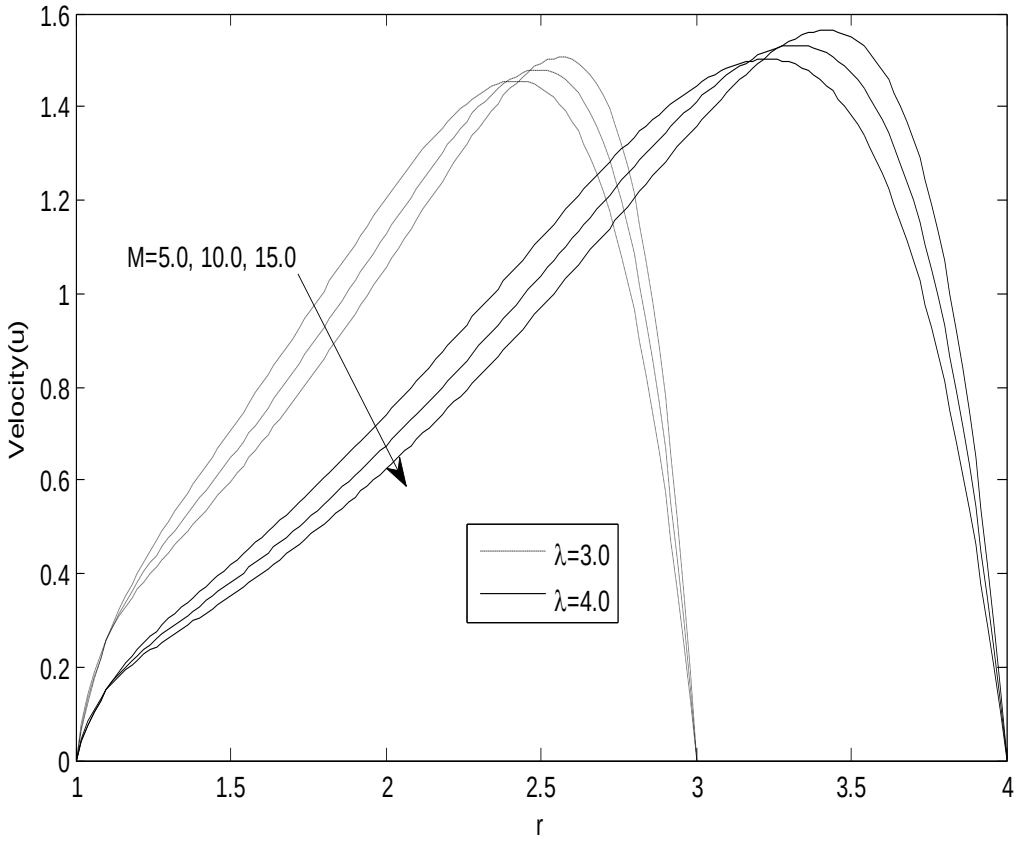

Fig 5.7b: Velocity Profiles for different M for isothermal case ($\gamma = 15$, $D_{a_0} = 0.01$, $\lambda = 3.0$ and 4.0)

The plot of velocity profiles against r for both isothermal and constant heat flux cases for different Hartmann number (M) and gap between inner and outer cylinders (λ) are presented in Figures 5.7 and 5.8, respectively. Clearly, the impact of growing Hartmann number (M) is to decrease the velocity ofthe fluid for both isothermal and constant heat flux (isoflux) cases.

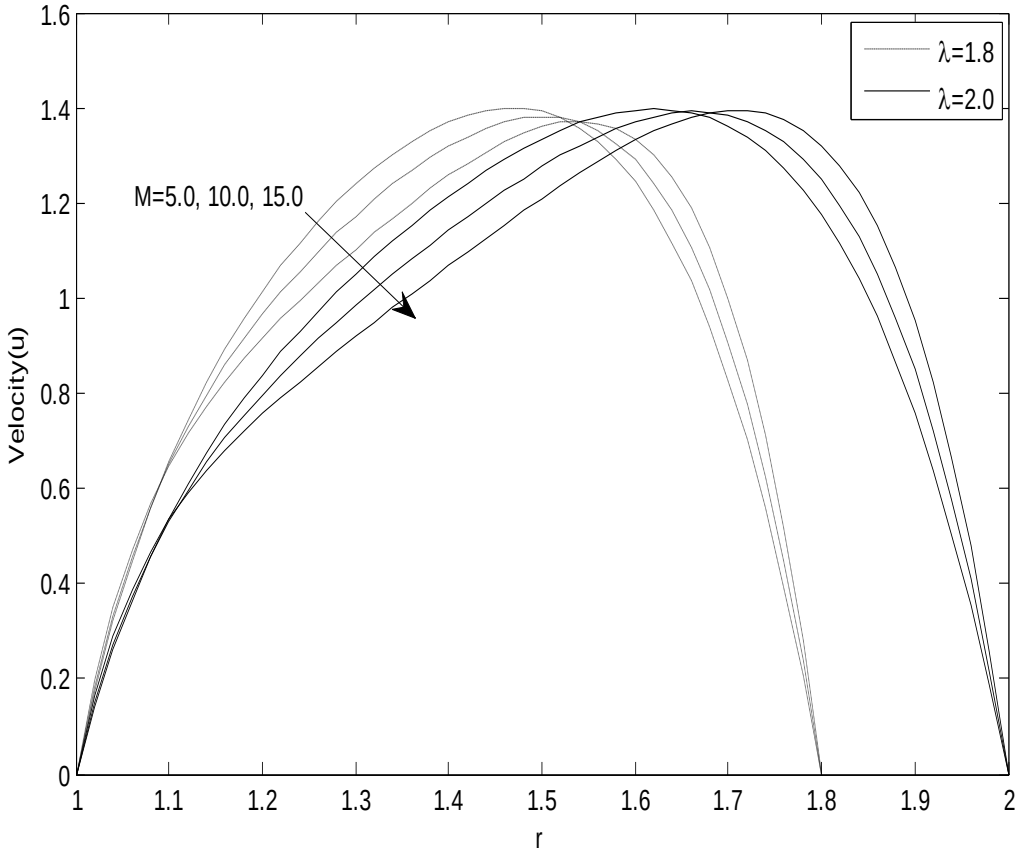

Fig 5.8a: Velocity Profiles for different M for constant heat flux case ($\gamma = 15$, $D_{a_0} = 0.01$, $\boldsymbol{\lambda = 1.8}$ and 2)

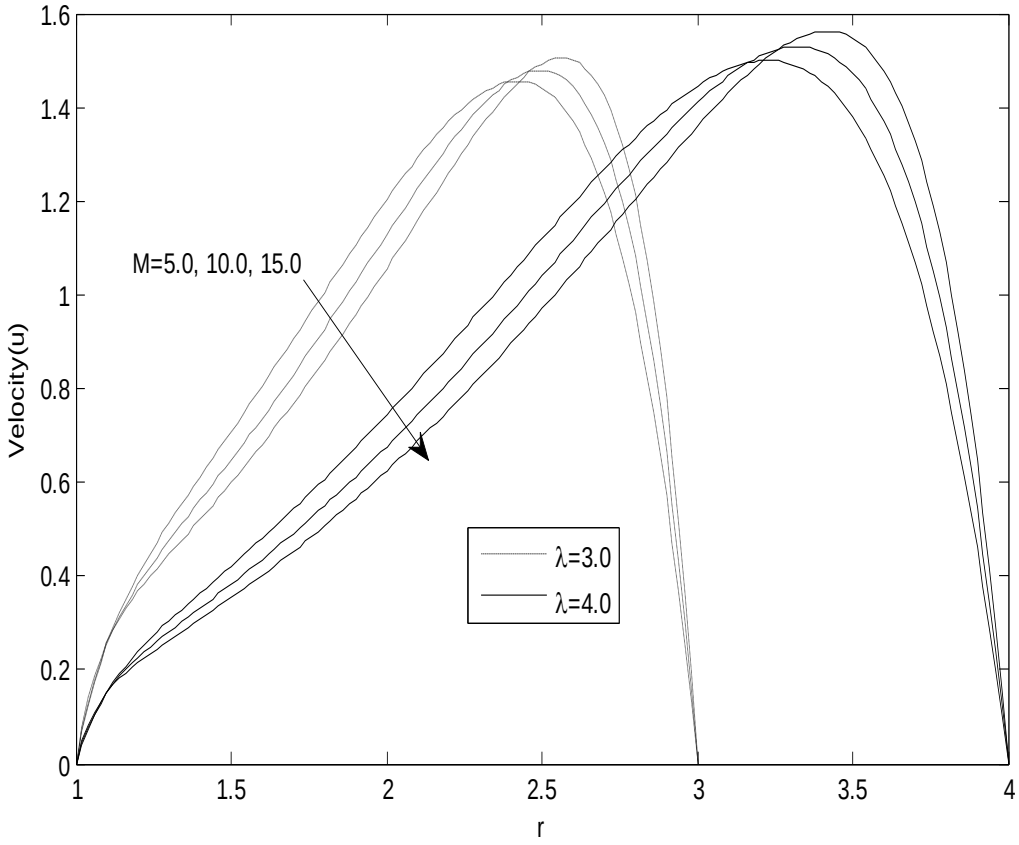

Fig 5.8b: Velocity Profiles for different M for constant heat flux case ($\gamma = 15$, $D_{a_0} = 0.01$, $\boldsymbol{\lambda = 3.0}$ and $\boldsymbol{4.0}$)

Figures 5.9 and 5.10 present thevelocity profiles for isothermal and constant heat flux cases respectively, for different values of Darcy number(D_{a_0}) and gap between inner and outer cylinders (λ). It is interesting to note that the effect of increasing Darcy number D_{a_0} is to promote and reduce the velocity of the fluid at the outer surface of the inner cylinder and at the inner surface of the outer cylinder respectively. The increasing nature can be explained by the fact that for large Darcy number, the permeability of the medium is large, as a result less resistance is offered by the medium on the fluid flow.

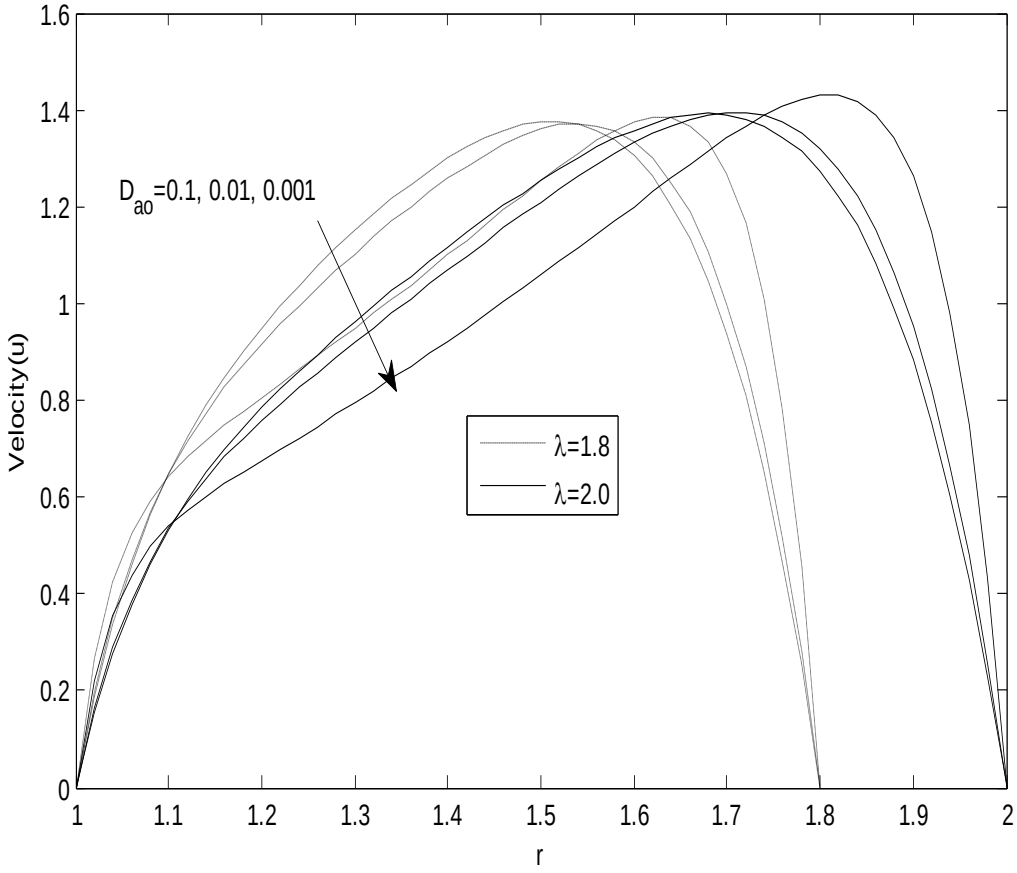

Fig 5.9a: Velocity Profiles for different D_{a_o} for isothermal case ($\gamma = 1.5$, $M = 15$, $\boldsymbol{\lambda = 1.8}$ and 2)

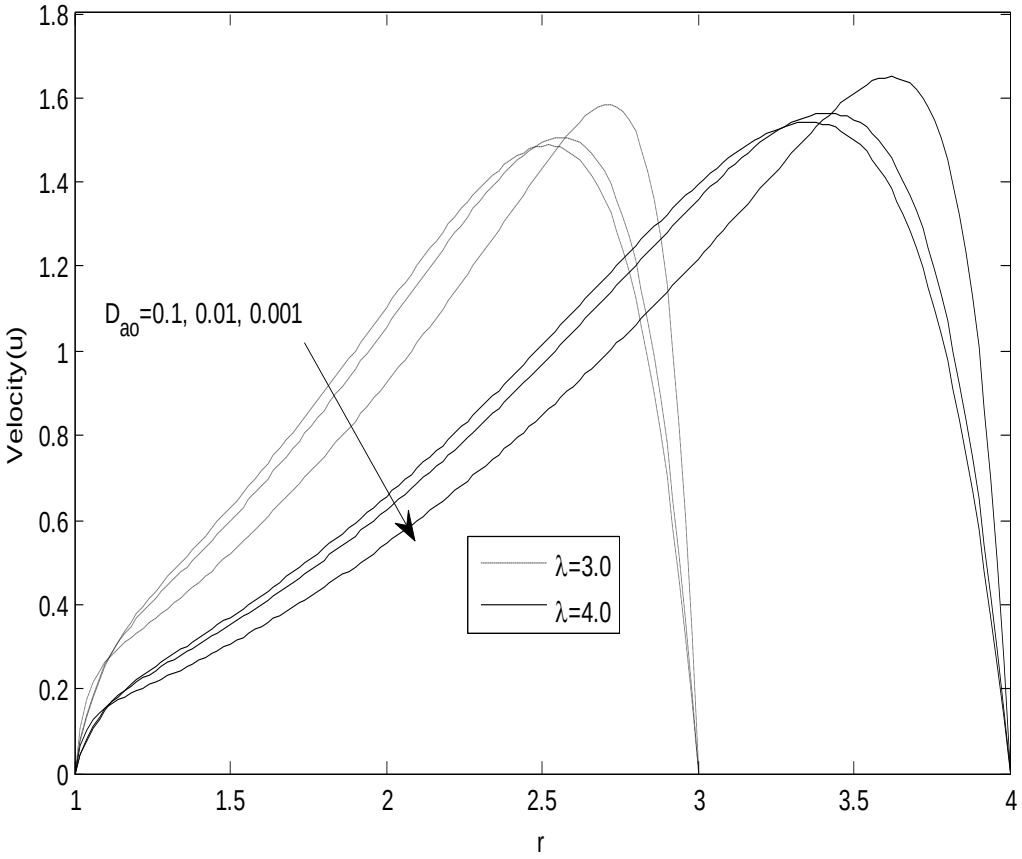

Fig 5.9b: Velocity Profiles for different D_{a_0} for isothermal case ($\gamma = 1.5$, $M = 15$, $\boldsymbol{\lambda = 3.0}$ and $\mathbf{4.0}$)

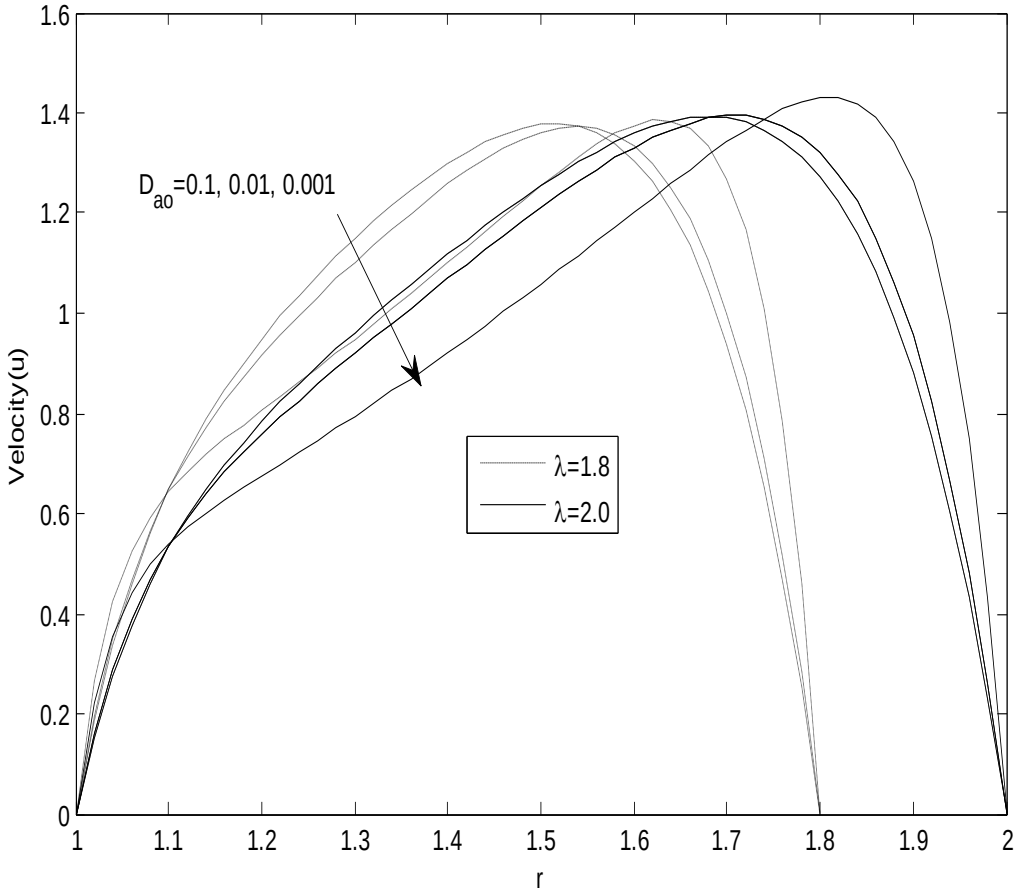

Fig 5.10a: Velocity Profiles for different D_{a_o} for constant heat flux case ($\gamma = 1.5$, $M = 15$, $\lambda = 1.8$ and 2)

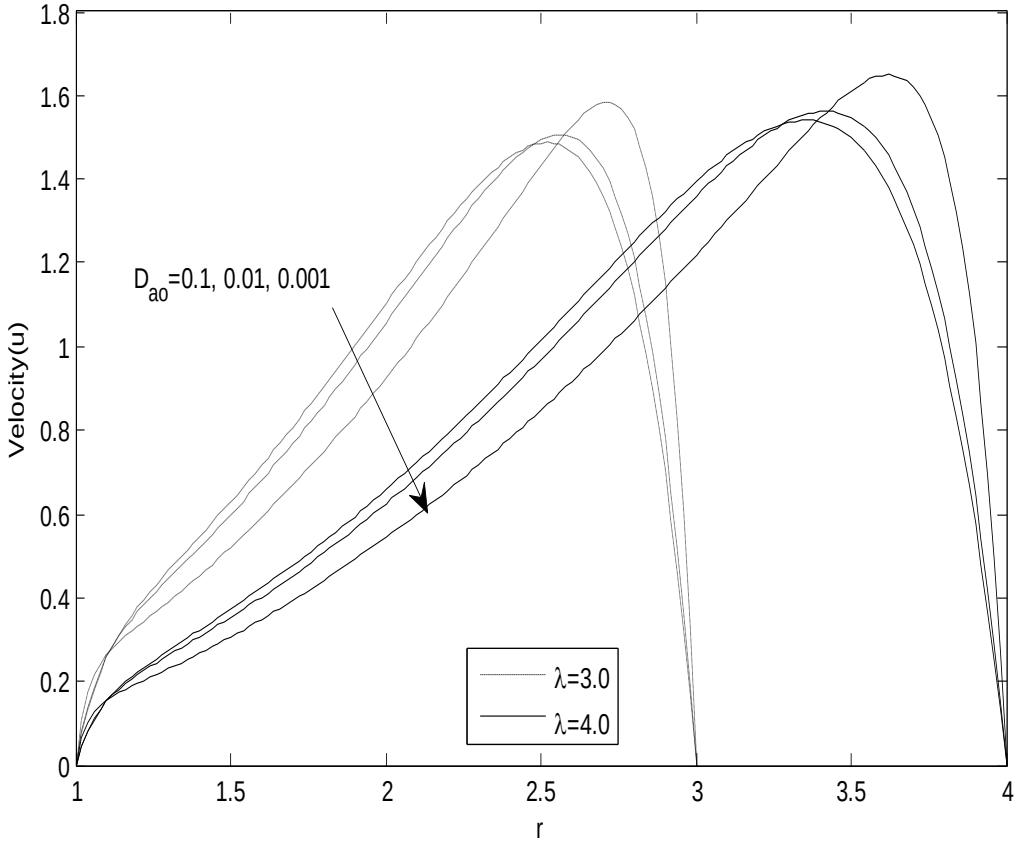

Fig 5.10b: Velocity Profiles for different D_{a_o} for constant heat flux case ($\gamma = 1.5$, $M = 15$, $\lambda = 3.0$ and 4.0)

Figures 5.11 and 5.12 show the plot of velocity profiles for isothermal and constant heat flux cases respectively, for different values of the ratio of viscosity (γ) and gap between inner and outer cylinders(λ). It is observed from Figures 5.11 and 5.12 that increasing the ratio of viscosity (γ) increases the velocity of the fluid at the outer surface of the inner cylinder. At the inner surface of the outer cylinder, velocity increases with decrease in the ratio of viscosity (γ). It is important to note that, the different flow behavior of the fluid at the surfaces of the cylinders as seen in figures 5.7-5.13 is an attempt to obey the law of conservation of mass in the annuli.

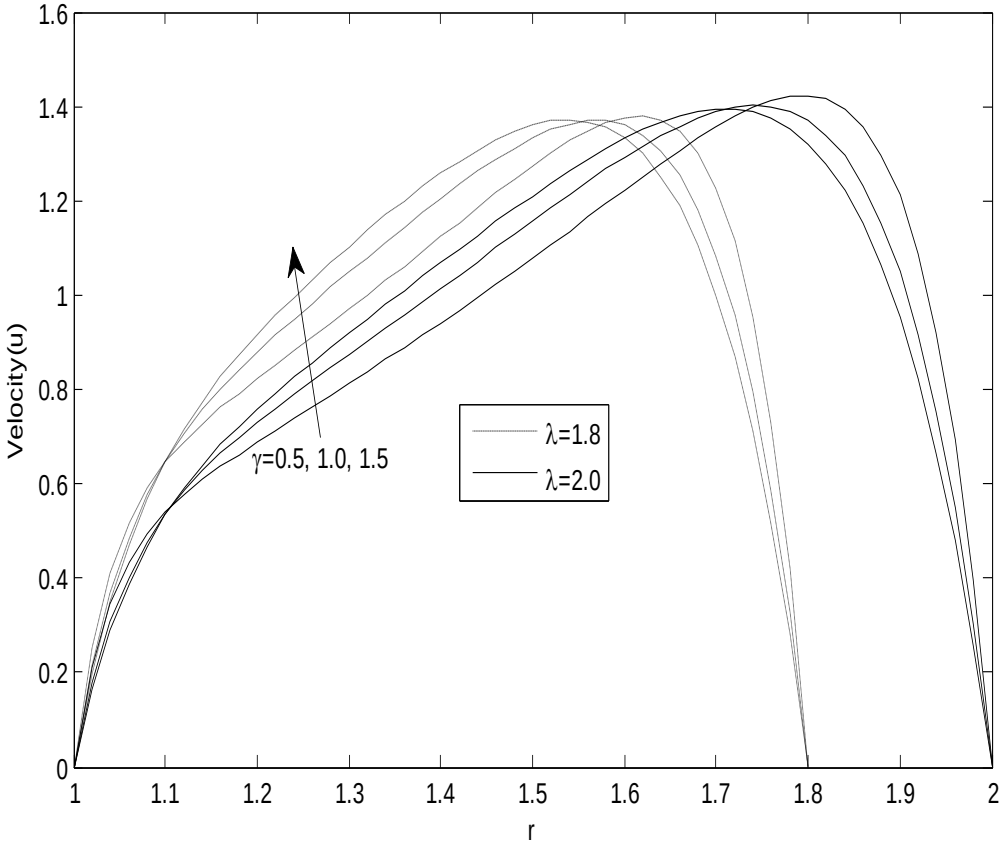

Fig 5.11a: Velocity Profiles for different γ for isothermal case ($D_{a_0} = 0.1$, $M = 15$, $\lambda = 1.8$ and 2)

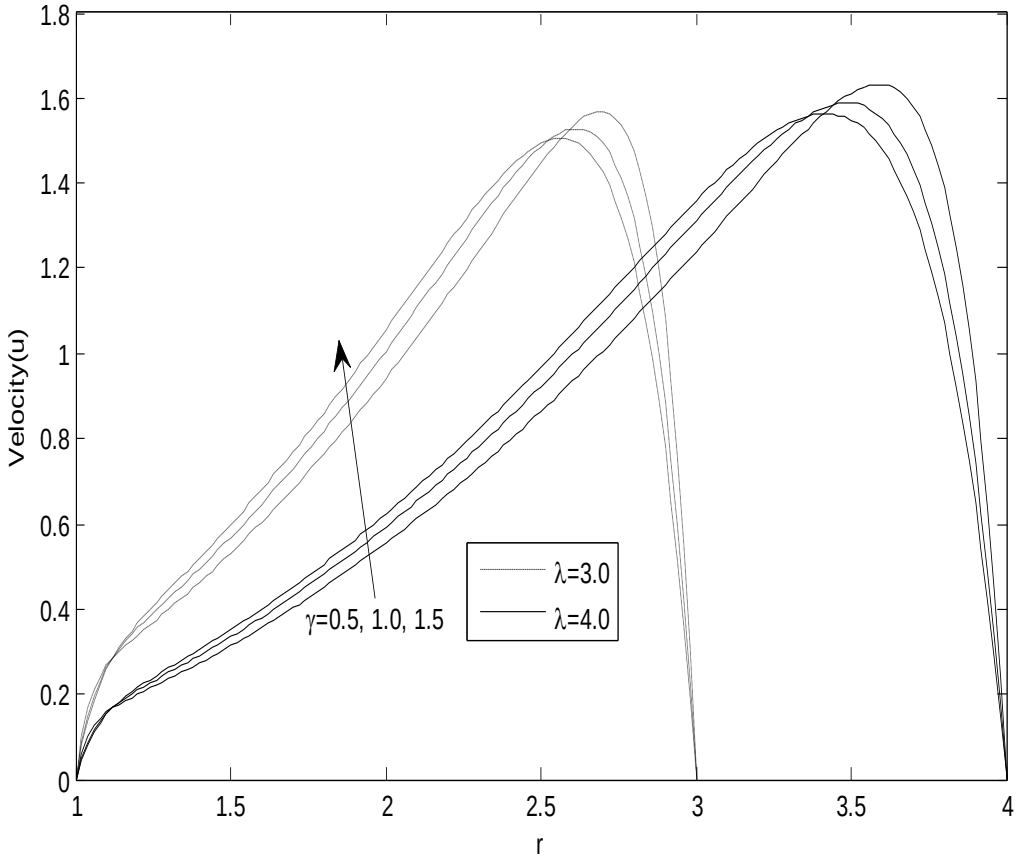

Fig 5.11b: Velocity Profiles for different γ for isothermal case ($D_{a_0} = 0.01$, $M = 15$, $\lambda = 3.0$ and 4.0)

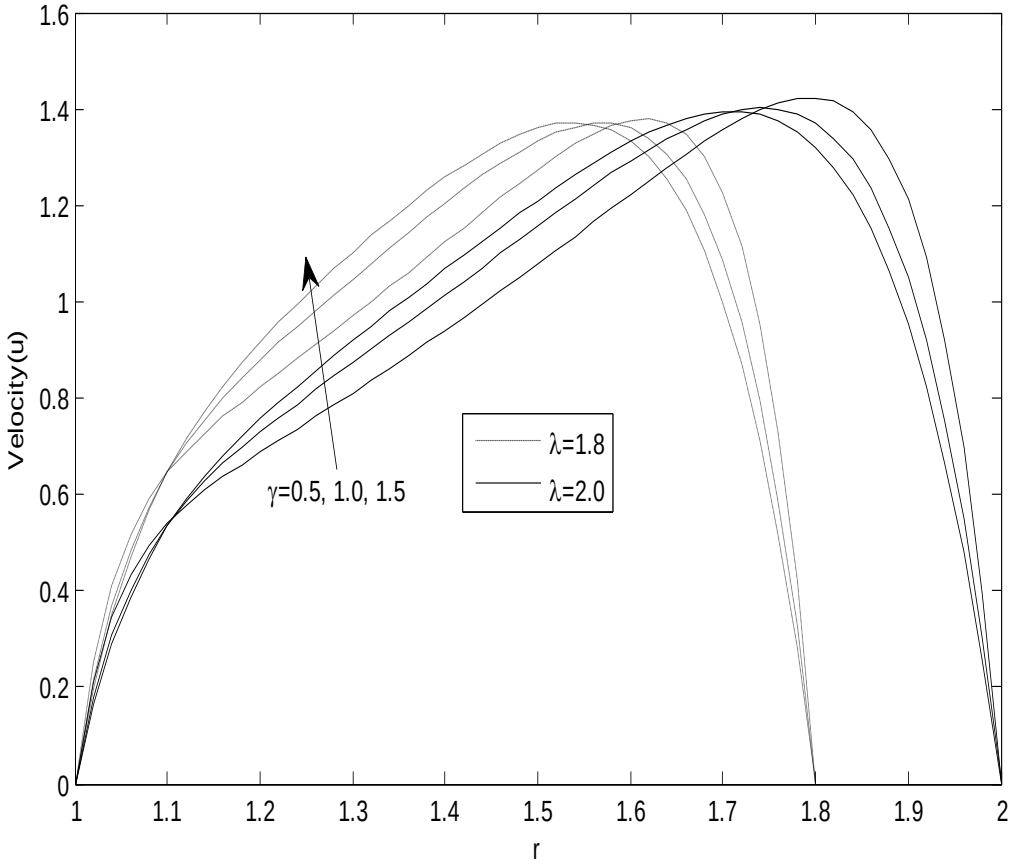

Fig 5.12a: Velocity Profiles for different γ for constant heat flux case ($D_{a_0} = 0.01$, $M = 15$, $\boldsymbol{\lambda = 1.8}$and 2)

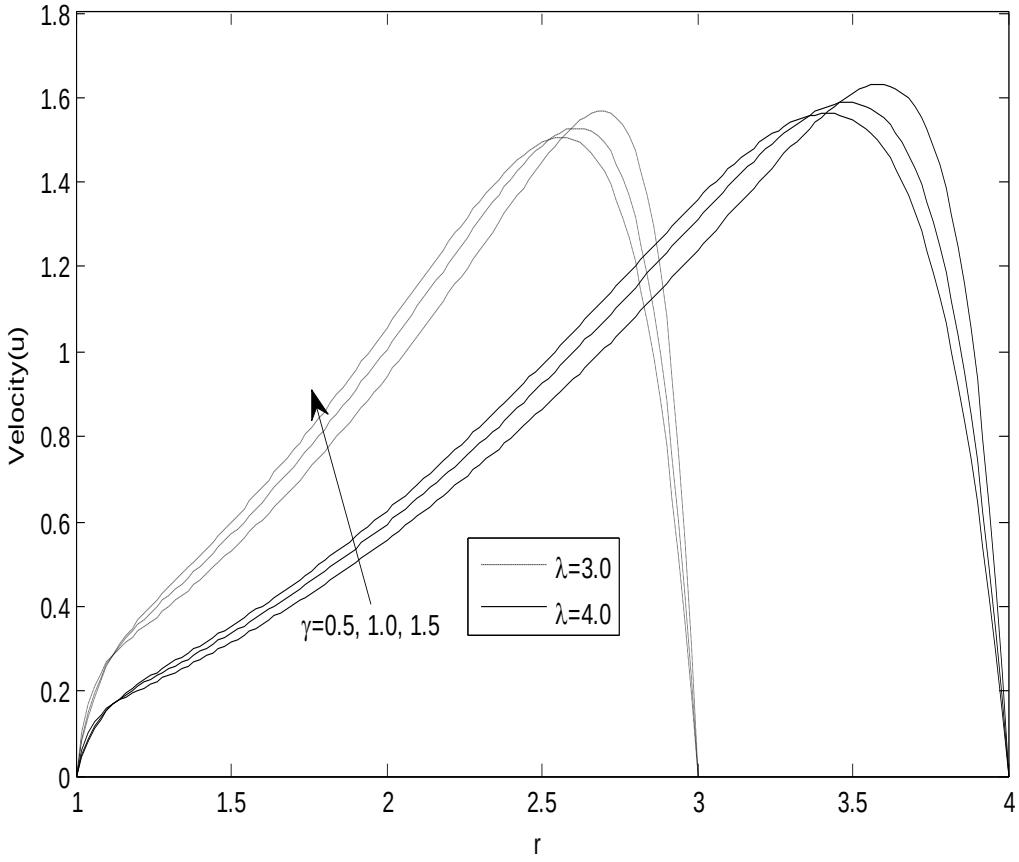

Fig 5.12b: Velocity Profiles for different γ for constant heat flux case ($D_{a_0} = 0.01$, $M = 15$, $\lambda = 3.0$ and 4.0)

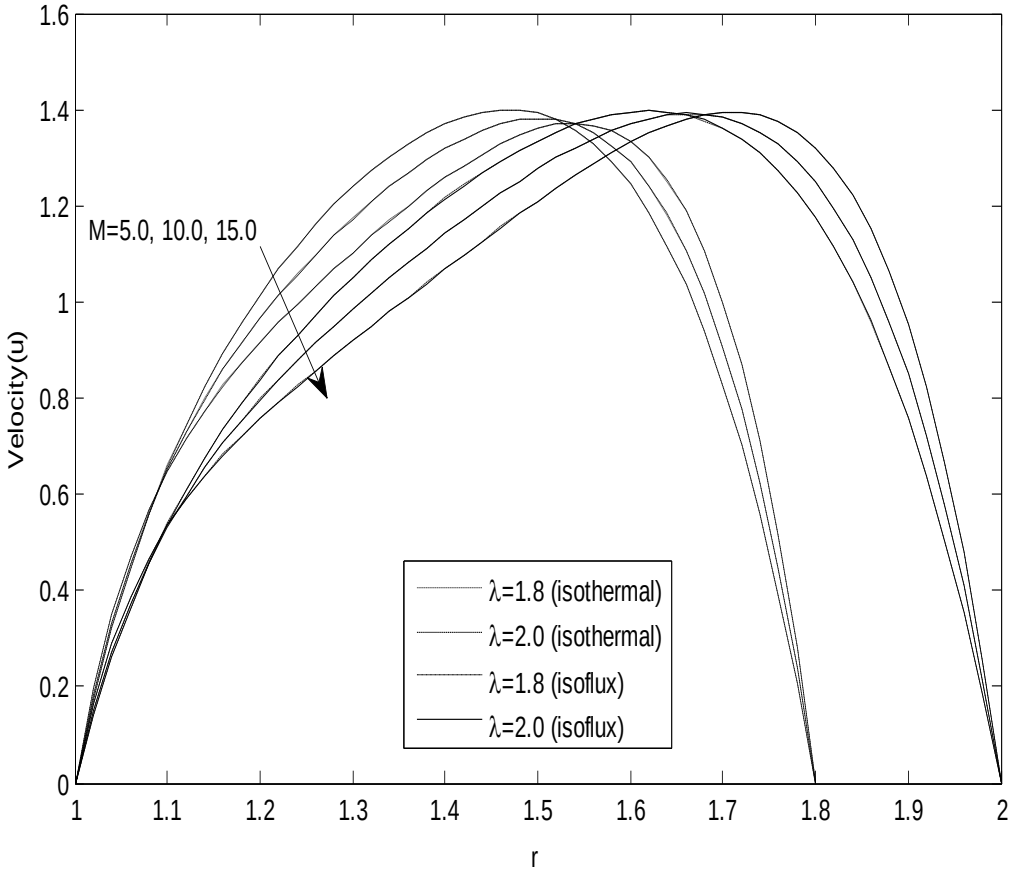

Fig 4.13a: Velocity Profiles for different values of M
($\gamma = 1.5, D_{a_0} = 0.01, \lambda = 1.8$ **and 2)**

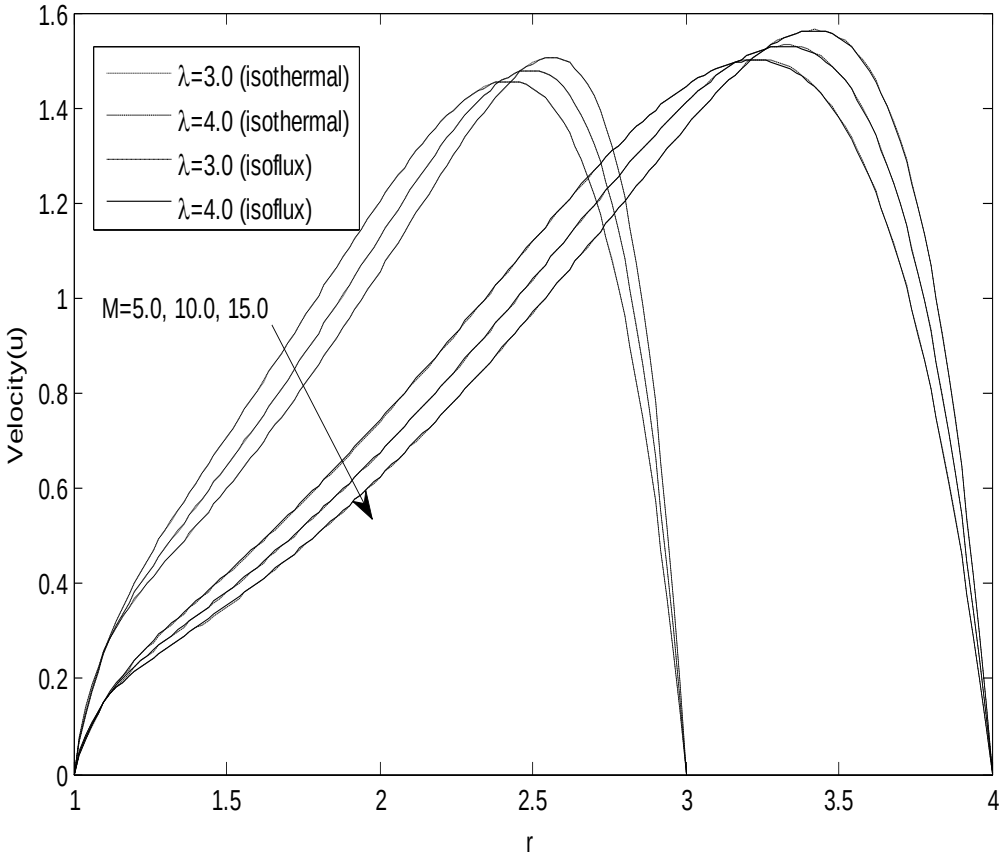

Fig 4.13b: Velocity Profiles for different values of M
($\gamma = 1.5$, $D_{a_0} = 0.01$, $\lambda = 3$ and 4)

Table 3. Numerical values for skin friction and pressure gradient,

When $\lambda \leq 2$, $M = 5.0$

			isothermal case			constant heat flux		
λ	D_{a_0}	γ	τ_1	τ_λ	P	τ_1	τ_λ	P
1.8	0.1	0.5	9.2909	10.4072	29.9835	9.2450	10.4410	30.1473
		1.0	8.7921	8.8414	39.7217	8.7655	8.8606	39.8896
		1.5	8.6485	8.2491	49.2151	8.6297	8.2626	49.3848
	0.01	0.5	11.7143	16.2000	80.0608	11.6842	16.2231	80.2149
		1.0	10.0073	12.5358	91.7606	9.9879	12.5504	91.9201
		1.5	9.4016	11.0076	102.3069	9.3870	11.0185	102.4694
	0.001	0.5	24.8966	40.9518	537.7627	24.8848	40.9614	537.9040
		1.0	18.7641	29.7564	562.2742	18.7560	29.7630	562.4188
		1.5	16.0967	24.8186	581.9483	16.0902	24.8238	582.0952
2.0	0.1	0.5	7.5450	9.1822	23.7064	7.5057	9.2101	23.8211
		1.0	7.1081	7.5744	30.1601	7.0844	7.5907	30.2789
		1.5	6.9988	6.9463	36.3338	6.9817	6.9580	36.4544
	0.01	0.5	9.6911	14.8481	66.0361	9.6667	14.8665	66.1427
		1.0	8.1722	11.3029	74.3980	8.1561	11.3146	74.5090
		1.5	7.6245	9.7914	81.6882	7.6121	9.8002	81.8018
	0.001	0.5	20.9586	38.2914	452.5447	20.9493	38.2988	452.6415
		1.0	15.7383	27.7217	471.4424	15.7319	27.7267	471.5418
		1.5	13.4598	23.0511	486.4535	13.4547	23.0551	486.5546

The numerical values for skin-friction τ_1 (at the outer surface of inner cylinder), τ_λ (at the inner surface of outer cylinder), and pressure gradient (P) for both isothermal heating and constant heat flux cases are presented in tables 3 and 4 at fixed value of $M = 5.0$. It is observed that the skin friction and pressure gradient (P) decrease with increase in Darcy number (D_{a_0}) and decrease in ratio of viscosity (γ) for both isothermal and constant heat flux cases. It is clear that the effect of increasing pressure gradient parameter is to reduce skin friction. In comparison, when $\lambda \leq 2$, τ_1 is higher for isothermal case than constant heat flux case, τ_λ on the other hand is higher for constant heat flux case than isothermal case.

Table 4. Numerical values for skin friction and pressure gradient

When$\lambda > 2$, $M = 5.0$

λ	$D_{a_0}\gamma$		isothermal case			Constant heat flux		
			τ_1	τ_λ	P	τ_1	τ_λ	P
3.0	0.1	0.5	3.6204	6.2340	10.6834	3.6380	6.2225	10.2447
		1.0	3.3775	4.8592	12.7137	3.3893	4.8520	12.6834
		1.5	3.3609	4.2746	14.4931	3.3700	4.2692	14.4617
	0.01	0.5	4.7300	10.7057	31.7296	4.7396	10.6988	31.7039
		1.0	3.9034	7.9677	34.9364	3.9102	7.9630	34.9093
		1.5	3.5908	6.7627	37.5487	3.5962	6.7591	37.5206
	0.001	0.5	10.3889	28.4078	223.8200	10.3923	28.4052	223.7970
		1.0	7.7616	20.4634	231.9986	7.7640	20.4616	231.9751
		1.5	6.6101	16.9454	238.3952	6.6120	16.9439	238.3711
4.0	0.1	0.5	2.2011	4.7776	6.1214	2.2794	4.7264	6.0281
		1.0	2.0355	3.6850	7.2021	2.0906	3.6521	7.1017
		1.5	2.0343	3.2083	8.1146	2.0785	3.1833	8.0098
	0.01	0.5	2.8155	8.3253	18.4953	2.8565	8.2951	18.4122
		1.0	2.3111	6.1758	20.2952	2.3403	6.1555	20.2071
		1.5	2.1172	5.2254	21.7423	2.1411	5.2095	21.6507
	0.001	0.5	6.1173	22.2351	131.3857	6.1314	22.2236	131.3118
		1.0	4.5662	16.0046	136.0802	4.5762	15.9966	136.0042
		1.5	3.8859	13.2447	139.7423	3.8941	13.2383	139.6647

The reverse trend is observed when $\lambda > 2$. Also, it is found from the tables that with the increase in the gap between inner and outer cylinders (λ), the skin friction decreases. Also, pressure gradient is reduced with increase in λ.Tables 5 to 8 show that the skin friction exhibits higher increasing tendency in the mixed convection situation of flow as compared to that of the natural convection method.

Table 5.Comparison of natural and mixed convections for isothermal case when $\lambda \le 2, M = 5.0$

λ	$D_{a_0}\gamma$		Natural Convection		Mixed Convection	
			τ_1	τ_λ	τ_1	τ_λ
1.8	0.1	0.5	0.2315	0.0551	9.2909	10.4072
		1.0	0.1531	0.0436	8.7921	8.8414
		1.5	0.1170	0.0360	8.6485	8.2491
	0.01	0.5	0.1270	0.0193	11.7143	16.2000
		1.0	0.0888	0.0175	10.0073	12.5358
		1.5	0.0714	0.0161	9.4016	11.0076
	0.001	0.5	0.0444	0.0027	24.8966	40.9518
		1.0	0.0314	0.0026	18.7641	29.7564
		1.5	0.0257	0.0026	16.0967	24.8186
2.0	0.1	0.5	0.2435	0.0530	7.5450	9.1822
		1.0	0.1661	0.0434	7.1081	7.5744
		1.5	0.1298	0.0369	6.9988	6.9463
	0.01	0.5	0.1297	0.0182	9.6911	14.8481
		1.0	0.0918	0.0166	8.1722	11.3029
		1.5	0.0747	0.0154	7.6245	9.7914
	0.001	0.5	0.0447	0.0026	20.9586	38.2914
		1.0	0.0317	0.0025	15.7383	27.7217
		1.5	0.0260	0.0024	13.4598	23.0511

Table 6.Comparison of natural and mixed convections for isothermal case when $\lambda > 2, M = 5.0$

$\lambda D_{a_0} \gamma$			Natural Convection		Mixed Convection	
			τ_1	τ_λ	τ_1	τ_λ
3.0	0.1	0.5	0.2693	0.0508	3.6204	6.2340
		1.0	0.1966	0.0434	3.3775	4.8592
		1.5	0.1626	0.0386	3.3609	4.2746
	0.01	0.5	0.1353	0.0172	4.7300	10.7057
		1.0	0.0981	0.0157	3.9034	7.9677
		1.5	0.0816	0.0147	3.5908	6.7627
	0.001	0.5	0.0452	0.0024	10.3889	28.4078
		1.0	0.0323	0.0024	7.7616	20.4634
		1.5	0.0266	0.0023	6.6101	16.9454
4.0	0.1	0.5	0.2786	0.0537	2.2011	4.7776
		1.0	0.2085	0.0460	2.0355	3.6850
		1.5	0.1765	0.0411	2.0343	3.2083
	0.01	0.5	0.1372	0.0182	2.8155	8.3253
		1.0	0.1004	0.0166	2.3111	6.1758
		1.5	0.0841	0.0155	2.1172	5.2254
	0.001	0.5	0.0454	0.0026	6.1173	22.2351
		1.0	0.0325	0.0025	4.5662	16.0046
		1.5	0.0268	0.0024	3.8859	13.2447

Table 7.Comparison of natural and mixed convections for constant heat flux case when $\lambda \le 2, M = 5.0$,

			Natural Convection		Mixed Convection	
λ	D_{a_0}	γ	τ_1	τ_λ	τ_1	τ_λ
1.8	0.1	0.5	0.1360	0.0324	9.2450	10.4410
		1.0	0.0900	0.0256	8.7655	8.8606
		1.5	0.0688	0.0212	8.6297	8.2626
	0.01	0.5	0.0746	0.0113	11.6842	16.2231
		1.0	0.0522	0.0103	9.9879	12.5504
		1.5	0.0420	0.0095	9.3870	11.0185
	0.001	0.5	0.0261	0.0016	24.8848	40.9614
		1.0	0.0185	0.0016	18.7560	29.7630
		1.5	0.0151	0.0015	16.0902	24.8238
2.0	0.1	0.5	0.1688	0.0367	7.5057	9.2101
		1.0	0.1151	0.0301	7.0844	7.5907
		1.5	0.0899	0.0256	6.9817	6.9580
	0.01	0.5	0.0899	0.0126	9.6667	14.8665
		1.0	0.0636	0.0115	8.1561	11.3146
		1.5	0.0518	0.0107	7.6121	9.8002
	0.001	0.5	0.0310	0.0018	20.9493	38.2988
		1.0	0.0220	0.0017	15.7319	27.7267
		1.5	0.0180	0.0017	13.4547	23.0551

Table 8.Comparison of natural and mixed convections for constant heat flux case when $\lambda > 2$, $M = 5.0$

λ	D_{a_0}	γ	Natural Convection τ_1	τ_λ	Mixed Convection τ_1	τ_λ
3.0	0.1	0.5	0.2958	0.0558	3.6380	6.2225
		1.0	0.2160	0.0477	3.3893	4.8520
		1.5	0.1786	0.0424	3.3700	4.2692
	0.01	0.5	0.1486	0.0189	4.7396	10.6988
		1.0	0.1078	0.0173	3.9102	7.9630
		1.5	0.0896	0.0161	3.5962	6.7591
	0.001	0.5	0.0497	0.0027	10.3923	28.4052
		1.0	0.0355	0.0026	7.7640	20.4616
		1.5	0.0292	0.0025	6.6120	16.9439
4.0	0.1	0.5	0.3862	0.0744	2.2794	4.7264
		1.0	0.2890	0.0638	2.0906	3.6521
		1.5	0.2447	0.0570	2.0785	3.1833
	0.01	0.5	0.1903	0.0252	2.8565	8.2951
		1.0	0.1391	0.0230	2.3403	6.1555
		1.5	0.1165	0.0215	2.1411	5.2095
	0.001	0.5	0.0630	0.0036	6.1314	22.2236
		1.0	0.0451	0.0035	4.5762	15.9966
		1.5	0.0371	0.0034	3.8941	13.2383

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATION

5.1 Summary

This dissertation investigatesnatural and mixed convection flows invertical concentric annuli filled with porous material having variable porosity in the presence of radial magnetic field with constant heat flux and isothermally heated cylinders.The energy and momentum equations for these problems were modeled after the Boussinesq approximation and are expressed in dimensionless form using suitable non-dimensional parameters. The non-dimensional equations,which are nonlinear, were transformed into a system of linear second order ordinary differential equations.Unified exact solutions are derived by taking into consideration an isothermal or isoflux thermal boundary condition at the outer surface of the inner cylinder. The solutions obtained are graphically represented and the effect of various controlling parameters such as Hartmann number (M), Darcy number (D_{a_0}), ratio of viscosity (γ) and the gap between the cylinders (λ) on each flow formation is discussed. Two cases are considered, natural and mixed convection flows in vertical concentric annuli filled with porous material having variable porosity in presence of radial magnetic field.

5.2 Conclusion

The steady laminar fully developed free and mixed convective flows of an electrically conducting fluid in open-ended vertical concentric annuli of infinite

length filled with porous material having variable porosity in the presence of a radial magnetic field are investigated.Unified exact solutions are derived by taking into consideration an isothermal or isoflux thermal boundary condition at the outer surface of inner cylinder. The solutions obtained are graphically represented and the effects of various controlling parameters such as Hartmann number (M), Darcy number (D_{a_0}), ratio of viscosity (γ) and the gap between the cylinders (λ) on the flow formation are discussed. Results indicate that the effect of increasing Darcy number (D_{a_0})and the gap between the cylinders (λ) is to increase the fluid velocity,skin friction and mass flux.Besides, velocity, skin friction and mass flux have decreasing tendency as the Hartmann number (M) and ratio of viscosity (γ) increase. Additionally, the skin friction can be reduced by applying more pressure to the fluid flow.

Comparatively,it is found from problem 3.2 that, when the gap between cylinders is less or equal to radius of the inner cylinder, the velocity, skin friction and mass flux are higher for isothermal heating of the inner cylinder than constant heat flux case, while the reverse process is achieved when the gap between cylinders is greater thanradius of the inner cylinder.In problem 3.3, the velocity for isothermal heating of the inner cylinder is approximately equal tothat of constant heat flux case.However, when the gap between cylinders is less or equal to theradius of the inner cylinder,τ_1 is higher for isothermal case than constant heat flux case, while τ_λ is higher for constant heat flux case than isothermal case. The reverse phenomena occurwhen the gap between cylinders is greater than radius of the inner cylinder. In addition, the

velocity and skin friction have higher increasing tendency in problem 3.3 than that of problem 3.2.

Results from this dissertation will be useful inengineering and scientific applications such as separation of oil from sand by steam, porous heat exchangers, geological flows, drilling operation of oil and gas wells and the design of magnetohydrodynamic power generators.

5.3 Recommendation

We have presented a study on natural and mixed convection flows in vertical concentric annuli filled with porous materials of variable porosity in presence of radial magnetic field. The study of transport processes in annular geometry is important in many engineering and aerodynamic applications such as in drilling operation of oil and gas wells and the design of magnetohydrodynamic power generators.Therefore, further research should be carried out to cover the unsteady/transient aspects of the present research work. The unsteady/transient flow formation can be analyzed only by numerical techniques (Finite difference, Finite element etc.). The results presented in this dissertation can be used to verify the result obtained by numerical techniques at large value of time (steady state situation).

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APPENDICES

APPENDIX I

NATURAL CONVECTION FLOW IN VERTICAL CONCENTRIC ANNULI FILLED WITH POROUS MATERIAL OF VARIABLE POROSITY UNDER A RADIAL MAGNETIC FIELD.

VELOCITY

```
[r] = (1.0 : 0.02 : 2.0);

a =1.0;

b = 0.0;

c = -1.0;

m = 0.5;

v = 1.5;

d = 0.1;

L = 2.0;

x1 = r./L;

x2 = log(x1);

c1 = a - (b.*log(L));

c2 = ((m.*m)./v) + (1./(v.*d)) - 4;
```

```
x3 = (x2./c2) + (4./(c2.*c2));

x4 = c./(v.*c1);

x5 = x4.*r.*r.*x3;

k = sqrt((1./v).*((m.*m) + (1./d)));

x6 = ((L.^(-2.*k)) - (L.^(2-k)))./(c2.*c2);

x7 = ((L.^(-2.*k)).*log(L))./c2;

x8 = 1.- (L.^(-2.*k));

c3 = -x4.*(1./x8).*(x7 - (4.*x6));

x9 = (1.- (L.^(2-k)))./(c2.*c2);

x10 = (log(L))./c2;

c4 = x4.*(1./x8).*(x10 - (4.*x9));

u = (c3.*r.^k) + (c4.*r.^(-k)) + x5;

hold on

Xlabel ({'r'...

    ,'Fig.1.Velocity distribution for different values of M when λ ≤ 2 ( γ = 1.5, D_{a_0} = 0.1 ),'})...

,Ylabel('Velocity  (U)');

plot(r,u,'.k','linestyle','-')
```

```
hold off

   % propertyeditor('on')



SKIN FRICTION

[r]=(1.0:0.2:3.0);

a=0.0;

b=1.0;

c=1.0;

m=0.5;

v=0.5;

d=0.1;

L=3.0;

c1=a-(b.*log(L));

c2=((m.*m)./v)+(1./(v.*d))-4;

x3=((1.-2.*log(L))./c2)+(8./(c2.*c2));

x4=c./(v.*c1);

x5=x4.*x3;
```

```
k=sqrt((1./v).*((m.*m)+(1./d)));

x6=((L.^(-2.*k))-(L.^(2-k))./(c2.*c2);

x7=((L.^(-2.*k)).*log(L))./c2;

x8=1.-(L.^(-2.*k));

c3=-x4.*(1./x8).*(x7-(4.*x6));

x9=(1.-(L.^(2-k)))./(c2.*c2);

x10=(log(L))./c2;

c4=x4.*(1./x8).*(x10-(4.*x9));

T1=(k.*c3)-(k.*c4)+x5

x11=(L./c2)+((8.*L)./(c2.*c2));

x12=x4.*x11;

T2=-((k.*c3.*(L.^(k-1)))-(k.*c4.*(L.^(-k-1)))+x12)
```

MASS FLUX

```
[r]=(1.0:0.2:3.0);

a=0.0;

b=1.0;
```

```
c=1.0;

m=0.5;

v=0.5;

d=0.1;

L=3.0;

c1=a-(b.*log(L));

c2=((m.*m)./v)+(1./(v.*d))-4;

x3=((1.-2.*log(L))./c2)+(8./(c2.*c2));

x4=c./(v.*c1);

x5=x4.*x3;

k=sqrt((1./v).*((m.*m)+(1./d)));

x6=((L.^(-2.*k))-(L.^(2-k)))./(c2.*c2);

x7=((L.^(-2.*k)).*log(L))./c2;

x8=1.-(L.^(-2.*k));

c3=-x4.*(1./x8).*(x7-(4.*x6));

x9=(1.-(L.^(2-k)))./(c2.*c2);

x10=(log(L))./c2;
```

```
c4=x4.*(1./x8).*(x10-(4.*x9));

T1=(k.*c3)-(k.*c4)+x5

x11=(L./c2)+((8.*L)./(c2.*c2));

x12=x4.*x11;

T2=-((k.*c3.*(L.^(k-1)))-(k.*c4.*(L.^(-k-1)))+x12)

x13=k+2;

x14=2-k;

x15=(c3.*((L.^x13)-1))./x13;

x16=(c4.*((L.^x14)-1))./x14;

x17=(L.^4)-1;

x18=(log(L)./(4.*c2))-(x17./(16.*c2))+(x17./(c2.*c2));

q=2.*3.1428.*(x15+x16+(x4.*x18))

%hold on;

  % plot(r,T2 );
```

APPENDIX II

MHD NATURAL CONVECTION FLOW IN VERTICAL CONCENTRIC ANNULI FILLED WITH POROUS MATERIAL OF VARIABLE POROSITY UNDER A RADIAL MAGNETIC FIELD.

VELOCITY

```
[r] = (1.0 : 0.02 : 2.0);

a = 1.0;

b = 0.0;

c = -1.0;

m = 5.0;

v = 1.5;

d = 0.1;

L = 2.0;

x1 = r./L;

x2 = log(x1);

c1 = a - (b.*log(L));

c2 = ((m.*m)./v) + (1./(v.*d)) - 4;

k = sqrt((1./v).*((m.*m) + (1./d)));
```

```
xx1 = (-1 + L.^(k+2)).*((L.^(-2.*k)) - (L.^(2.-k)));

xx2 = c2.*(k+2).*(1.- (L.^(-2.*k)));

xx3 = xx1./xx2;

xx4 = (1.-(L.^(2.-k))).*(1.-(L.^(2.-k)));

xx5 = c2.*(2.-k).*(1.- (L.^(-2.*k)));

xx6 = xx4./xx5;

xx7 = (-1+(L.^4))./(4.*c2);

c3 = (1./v).*(xx3 + xx6 + xx7);

x3 = (1.- (L.^2))./(2.*c3);

x4 = c./(v.*c1);

xx8 = (1.-(L.^(k+2)))./(k+2);

x5 = 1./(1.-L.^(-2.*k));

x6 = ((L.^(-2.*k)).*log(L))./c2;

x7 = ((L.^(-2.*k)) - (L.^(2.-k)))./(c2.*c2);

c4 = (1./c3).*x4.*xx8.*x5.*(x6 - (4.*x7));

xx9 = (-1 + L.^(2.-k))./(2.-k);

x8 = (log(L))./c2;
```

```
x9 = (1.- L.^(2.-k))./(c2.*c2);

c5 = (1./c3).*x4.*xx9.*x5.*(x8 - (4.*x9));

x10 = (log(L)./(4.*c2)) - ((-1+L.^4)./(16.*c2)) + ((-1+(L.^4))./(c2.*c2));

p = x3 + c4 + c5 + ((1./c3).*x4.*x10)

x11 = (x2./c2) + (4./(c2.*c2));

x12 = r.*r.*((x4.*x11) - (p./(v.*c2)));

cc4 = -x4.*x5.*(x6 - (4.*x7)) - ((p./(v.*c2)).*x5.*((L.^(-2.*k)) - (L.^(2-k))));

cc5 = x4.*x5.*(x8 - (4.*x9)) + ((p./(v.*c2)).*x5.*(1.-L.^(2.-k)));

u = (cc4.*r.^k) + (cc5.*r.^(-k)) + x12

hold on;

plot(r,u,'k','linestyle','-');

hold off;

propertyeditor;

colormap gray
```

PRESSURE GRADIENT, SKIN FRICTION AND MASS FLUX

```
[r] = (1.0 : 0.02 : 1.8);

a = 0.0;

b = 1.0;

c = 1.0;

m = 5.0;

v = 1.0;

d = 0.01;

L = 1.8;

x1 = r./L;

x2 = log(x1);

c1 = a - (b.*log(L));

c2 = ((m.*m)./v) + (1./(v.*d)) - 4;

k = sqrt((1./v).*((m.*m) + (1./d)));

xx1 = (-1+L.^(k+2)).*((L.^(-2.*k)) - (L.^(2.-k)));

xx2 = c2.*(1.- (L.^(-2.*k))).*(k+2);

xx3 = xx1./xx2;
```

```
xx4 = (1.- (L.^(2-k))).*(1.- (L.^(2-k)));

xx5 = c2.*(1.- (L.^(-2.*k))).*(2-k);

xx6 = xx4./xx5;

xx7 = ((L.^4) -1)./(4.*c2);

c3 = (1./v).*(-xx3 - xx6 - xx7);

x3 = ((L.^(-2.*k)) - (L.^(2-k)))./(c2.*c2);

x4 = ((L.^(-2.*k)).*log(L))./c2;

x5 = c./(v.*c1);

x6 = 1.-(L.^(-2.*k));

c4 = x5.*(1./x6).*(x4 - (4.*x3));

xx8 = ((L.^(k+2))-1)./(k+2);

x7 = (1.-(L.^(2-k)))./(c2.*c2);

x8 = (log(L))./c2;

c5 = x5.*(1./x6).*(x8 - (4.*x7));

xx9 = (1- (L.^(2-k)))./(2-k);

x9 = (log(L)./(4.*c2)) - (((L.^4)-1)./(16.*c2)) + (((L.^4)-1)./(c2.*c2));

x10 = ((L.^2)-1)./(2.*c3);
```

```
P = x10 - (x5.*(1./c3).*x9) + ((1./c3).*xx9.*c5) + ((1./c3).*xx8.*c4)

x11 = (1./v).*p.*xx7;

cc4 = -c4 - ((p./(v.*c2.*x6)).*((L.^(-2.*k)) - (L.^(2-k))));

cc5 = c5 + ((p.*(1.- (L.^(2-k)))).(v.*c2.*x6));

x12 = ((1.- 2.*log(L))./c2) + (8./(c2.*c2));

T1 = (k.*cc4) - (k.*cc5) + (x5.*x12) - (2.*p)./(v.*c2)

x13 = (L./c2) + ((8.*L)./(c2.*c2));

x14 = x5.*x13;

x15 = (2.*p.*L)./(v.*c2);

T2 = -((k.*cc4.*(L.^(k-1))) - (k.*cc5.*(L.^(-k-1))) + x14 - x15)

Q = 2.*3.1428.*((cc4.*xx8) - (cc5.*xx9) + (x5.*x9) - x11)
```