NUCLEAR RADII CALCULATIONS USING THE HARTREE-FOCK FORMALISM AND DROPLET MODEL.

BY

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DECLARATION

It is hereby declared that :

1. The report presented in this thesis is the result of research carried out by the author.

2. This thesis has not been submitted in whatever form to any other institution, organization or body, for the award of any degree.

3. All inclusions from the work of others have been duly acknowledged.

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CERTIFICATION

This thesis entitled - NUCLEAR RADII CALCULATIONS USING THE HARTREE-FOCK FORMALISM AND THE DROPLET MODEL by MUSA BABAR DANJAJI meets the regulations governing the award of the degree of MASTER OF SCIENCE of Ahmadu Bello University, and is approved for its contribution to knowledge and literary presentation

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ABSTRACT

The Droplet Model (DM) by Myers and Swaitecki is reviewed.

The Hartree-Fock (HF) formalism is here used to test the accuracy of the DM by calculating the radii of the proton and neutron distributions in the nuclei. The test is simplified by considering only nuclei with magic proton and neutron numbers. This choice helps to eliminate shell and pairing effects, which are not included in the DM but are automatically taken account of in the Hartree-Fock formalism.

The calculations of these radii by the DM is done by substituting values of the coefficients in the DM into the expressions for the radii. These coefficients are obtained by using the least-square fit method on some known nuclei masses in the mass expression of the DM.

The same radii are again calculated by the HF method using a phenomenological four parameter velocity dependent Yukawa potential.

The nuclear matter radius constant and the nuclear compressibility indicated by this HF phenomenological interaction are compared with that of the DM. It is found that the values for these two parameters differ for the two methods. These two parameters led to different radii for the DM and HF calculations. The DM radii are systematically smaller than the HF calculations for the nuclei considered.

In an effort to improve the DM results, inclusion of

higher - order terms into the DM is proposed. This inclusion however leads to the question of convergence of the DM expansion in powers of A^{w}

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ABBREVIATIONS.

LDM	Liquid drop model
DM	Droplet model
HF	Hartree-Fock
TF	Thomas - Fermi
A	Atomic - mass
I	Asymmetry
N	Number of neutrons
Z	Number of protons
ρ_n	Neutron density
ρ_z	Proton density
ρ_o	Density of standard nuclear matter
$\sum_n$	Surface for the neutron charge distribution
$\sum_z$	Surface for the proton charge distribution
t	Neutron skin thickness
τ	Dimensionless neutron skin thickness
n	Normal coordinate across neighboring surfaces
$\sum$	Mean surface of the system (nucleaus)
ε	The deviation of the nuclear density ρ from its nuclear matter value ρ_c.
$\tilde{\varepsilon}$	Density non uniformity function.
δ	Local asymmetry.
$\tilde{\delta}$	The neutron excess nonuniformity function
a_1	Volume energy coefficient
a_2	Surface energy coefficient
a_3	Curvature correction coeffient

Abbreviation (Contd.)

J	Symmetry energy coefficient
K	Compressibility coefficient
L	Density-Symmetry coefficient
N	Symmetry anharmonicity coefficient
Q	Effective surface stiffness coefficient
C_1	Coulomb energy coefficient
C_2	Volume redistribution coefficient
C_3	Diffuseness correction coefficient
C_4	Exchange correction coefficient
C_5	Surface redistribution coefficient
r_o	Nuclear radius constant.

CHAPTER 1

INTRODUCTION

One of the main objectives of the study of nuclear physics is the understanding of the structure of nuclei. This expression is used to include all aspects of the motion of intranuclear nucleons: their paths in space, their momenta, the correlations between them, the energies binding them to each other and of course the radii of the distributions of the components (Protons and neutrons) in nuclei. Mathematically, the complete description of nuclear structure is contained in the correct total wave function of the nucleus. There are formidable difficulties in the experimental or theoretical determination of the behavior of all the degrees of freedom of a dynamical system of such complexity as a moderately heavy nucleus. Both our present inadequate knowledge and the general need for systematization drive us to look for conceptual or mathematical models. From a nuclear model, it must be possible, in a systematic way and without prohibitively lengthy calculation to predict various observable properties of the nuclides. The test of the useful ness or realism of any model is the extent to which its predictions are confirmed by experiment.

The study of nuclei has mainly been marked by two different approaches. The microscopic approach, which is only interested in individual particles in the nucleus and the macroscopic approach which deals with the nucleons on collective basis. The relationship between these two approaches

is that between a less general one and a more general one. The macroscopic approach is the less general one and contains the microscopic one as an average. There has been attempts at different times by different authors to bridge the gap between these two approaches. Two of such authors are (Myers and Swiatecki (1969))

Myers and Swiatecki worked on the refinement of the Liquid Drop Model (LDM) which was first proposed by Weizsacker (1935). The LDM is a macroscopic model since it deals with nucleons in the nucleus on collective basis. The work of Myer and Swiatecki on the LDM paved the way for a new type of model- the Droplet Model (DM) which will be referred to in this work as standard Droplet model.

The present work is similar to that of Myers and Swiatecki. The aim of this work is to introduce some systematic refinement that was not included in the standard Droplet Model and to compare the calculations of the radii distributions of neutrons and protons of some nuclei with that calculated for the same nuclei from the Hartree-Fock formalism.

The refinements in the DM are associated with effects arising from the smallness of nuclei. These refinements may be regarded as additional terms in an expansion of nuclear properties in terms of the fundamental dimensionless ratio characterizing nuclei, namely the ratio of the interparticle spacing to the nuclear radius. This ratio is given approximately by $A^{-1/3}$ where A is the mass number (number of protons plus number of neutrons) of nuclei. In introducing these refinements

to the LDM a step is beingtaken in the direction of the microscopic theories in the following sense: one may regard effects of order A as belonging to the domain of nuclear matter studies, effects of order $A^{2/3}$ as belonging to the LDM and effects of order A^0 as belonging to a microscopic theory where individual particles (consisting of one A^{th} part of the system) dominate the approach. This leaves a gap associated with effects of order $A^{1/3}$ and this work is mainly concerned with filling of this gap.

One of the assumptions in the LDM that prompted its refinement, is the assumption that the nucleus was incompressible. This particular assumption cannot be true, since a natural phenomenon like spontaneous fission requires the nucleus to be deformed (i.e deviation from sphericity) before it can take place and deformation of the nucleus is caused by the compressibility of it. The DM in its refinement concluded that there must be a finite compressibility of the nucleus.

The other assumption of the liquid drop model was that the neutron and proton densities are uniform in the bulk. In the DM these densities are no longer constant in the bulk, since the nucleus is now compressible and compression of the nucleus can now cause dilatations of the densities.

Another assumption of the LDM is that the neutron and proton densities were the same and equal to the bulk value ρ_o (the density of standard nuclear matter). The LDM fails to take into account that heavy nuclei depends on excess neutrons for their stability, in other words, in heavy nuclei

the number of neutrons (N) must be greater than the number of protons (Z). As a result the neutron and proton densities are no longer constant, since the densities are directly proportional to the number of nucleons; instead the DM now introduced a separate values for them ρ_n and ρ_z which are the neutron and proton densities respectively. The DM even went a step further to recognise the possibility of the creation of a neutron skin in the presence of a neutron excess.

The LDM also assumed that the proton and neutron radii are the same and equal to the radius of the nucleus ($R = r_o A^{1/3}$). In the DM separate values R_n and R_z are introduced, which are the neutron and proton radii respectively. These radii are now determined from the competition between the surface tension which tends to decrease the radius and the Coulomb repulsion which tends to increase it.

In the LDM the neutron and proton densities falls abruptly to zero on a sharp surface. The LDM treated the surface of a nucleus from a classical point of view. The DM now treats the surface from a quantum mechanical argument which results to the conclusion that the surface of the nucleus is no longer sharp but diffuse and that the densities now go to zero in diffuse surfaces. Since there are now separate densities for the neutrons and protons (ρ_n and ρ_z) which now deviates from that corresponding to standard nuclear matter value (ρ_o), the DM is now able to introduce separate effective sharp surfaces for both protons and neutrons distributions. These surfaces are represented by $\sum_n$ for neutrons and $\sum_z$ for protons.

The DM also found it convenient to introduce a mean effective sharp surface Σ and a (thin) neutron skin thickness t. The neutron skin thickness is simply the outward normal distance from the surface Σ_z to the (neighboring) surface Σ_n. This neutron skin is now a function of position on these surfaces. The mean surface Σ is introduced to represent the mean location of the surfaces Σ_n and Σ_z. The mean is being approximately weighted with the surface values of the densities ρ_n^s and ρ_z^s. The weighting is done such that the excess of neutrons in the density distribution between Σ and Σ_n (assuming that Σ_n is outside Σ) is the same as the deficiency of protons between Σ_z and Σ (that is the number of protons that would be needed to extend the proton density from Σ_z to Σ). This means that if n denotes a normal coordinate across neighboring surfaces Σ_n and Σ_z then the position of Σ is given by

$$n = \frac{n_n \rho_n^s + n_z \rho_z^s}{\rho_n^s + \rho_z^s}$$

where n_n and n_z are the values of n locating Σ_n and Σ_z.

The DM also found it convenient to introduce two new parameters (ε and δ), which take care of the deviation of the density from their nuclear matter value and the nuclear composition at any point in the nucleus respectively

$$\varepsilon = -\frac{1}{3}\frac{\rho - \rho_o}{\rho_o} \quad \text{and} \quad \delta = \frac{\rho_n - \rho_z}{\rho_n + \rho_z}$$

These two new parameters also have averages $\bar{\varepsilon}$ and $\bar{\delta}$ (numbers) and functions of position $\tilde{\varepsilon}$ and $\tilde{\delta}$. The averages are taken over the volume enclosed by the mean surface Σ. (This implies that the densities ρ_n and ρ_z originally defined only

with the respective surfaces $\sum_n$ and $\sum_z$, are imagined to be extrapolated when necessary to the mean surface $\sum$). The parameter $\bar{\varepsilon}$ now represents the shift in the total nuclear density in the interior of the nucleus away from the value for symmetric infinite nuclear matter. The two contributions to the shift are so called (i) the density effect

(ii) the dilatational effect of the Coulomb repulsion.

The Droplet Model which forms the basis of the macroscopic part of modern mass formula express the energy in terms of three components: They are the volume energy term, a surface energy term and a Coulomb energy term. In this project the total energy of the nucleus consist of the potential energy only, the kinetic is not treated since the work is interested only in equilibrium state. The volume energy is assumed to be an integral over the volume inside the surface $\sum$ of an energy density e which is a function of the densities ρ_n and ρ_z (or equivelently $\bar{\varepsilon}$ and $\bar{\delta}$). The surface energy is assumed to be an integral over the surface $\sum$ of a surface energy density σ which is a function of the conditions prevailing at the surface, like the curvature nature of the surface due to its diffuseness and the neutron skin. The Coulomb energy is due to the only known long range force in the nucleus. This force is the repulsive force between protons. It counterbalances the other forces which are mainly attractive. The attractive forces are of the order of the dimension of the nucleus, in other words they are of short ranges since the dimension of the nucleus is about 10^{-13} cm. The other characterisic of these attractive forces are that they are charge independent,

that is the force between a proton and another proton is the same as that between a neutron and another neutron. It is also the same as that between a neutron and a proton ($F_{n-n} = F_{p-p} = F_{n-p}$). They are also charge symmetric ($F_{n-n} = F_{p-p}$). The forces are spin dependent and a non central component. The Coulomb energy is now defined as a double integral over the inside of the surface $\sum_z$ of the protron densities ρ_{z1} at position r_1 and ρ_{z2} at position r_2 divided by the distance between these two ($|r_1 - r_2|$) proton distribution. A probability of approach function P(1,2) is also introduced in the Coulomb energy term to take care of the anticorrelation of protons in identical spin states due to the Pauli exclusion principle.

The Droplet Model of saturating, two component system (such as nuclei) results from expansions of properties such as energy in powers of three small qunatities (ϵ, δ and $\tau = t/r_o$). The smallness of the non-uniformities of the densities (ρ_n, ρ_z) enables one to make use of Taylor series expansions in powers of these deviations from constancy. This Taylor series expansion of the energy now gives expressions for the new coefficients, which are actually the corrections (microscopic terms) introduced by the DM due to the refinement of the LDM. These coefficients turned out to be eleven, two of which may be eliminated. Four of the nine parameters are the standard adjustable parameters of the Liquid drop model and five are new coefficients specifying various properties of nuclear systems (nuclear compressibility and the curvature correction to the surface tension are two examples)

In this work two new additional terms are being introduced to the nine that are already in the Standard Droplet Model. These two new terms are higher order terms in the Taylor series expansion for the volume and surface energy that were first neglected in the standard droplet model of Myers and Swiatecki. These two new terms lead to a new expression for the coefficient of the nuclear compressibility quite different from the one in the standard droplet model. Minimizing the DM energy with respect to density variation leads to equations in closed form specifying the separate neutron and proton radii and the densities non uniformities in terms of the expression of the new compressibility coefficient.

All the coefficients of the mass formula ... can be determined by fitting directly to the mass data. However, in practice the coefficients cannot be determined unambigously in this way. It turns out that different careful fits of the semi empirical mass formula to the mass data give different results for the various macroscopic (or liquid drop terms) and microscopic terms (terms due to correction in the LDM by the DM) mainly because of different ways of handling the microscopic terms, which arise from shell-model and pairing effect. The uncertainities would be of no importance if the modern semi empirical mass formula were being used merely for interpolation since the available mass data are confined to the region close to the stability line, but they can become serious when the formula is used for its real purpose of extrapolating to exotic nuclei. This problem is important

because it is the possibility of extrapolating from known to unknown nuclei that provides the motivation for the continued interest in the mass formula.

One conceivable way in which this ambiguity in the extrapolation from the known nuclei could be reduced would be to carry out a single particle calculation by the Hartree-Fock method. This is a microscopic model which takes shell and pairing effects into account automatically and self-conn consistently.

However the need for a new model especially the Droplet model lies in the fact that the HF calculations are very tediu and complicated calculations, more over times taken for most of these calculations are so large, that computers always give time limit for them, but the droplet model is a simple statistical model and its calculations are straight forward, which can mostly be solved by statistical methods. The different masspformula fits for the DM that are currently available could then be assessed by comparing their values for the DM parameters with Hartree-Fock values. The more a given set of parameters is consistent with a good Hartree-Fock effective interaction the greater will be our confidence in it.

In this work all the coefficients are first calculated by fitting experimental masses with the DM mass formula. To make a comparison possible, the coefficients were again calculated using a Hartree-Fock method for a phenomenological effective interaction, chosen in the form of a four-parameter

velocity dependent Yukawa potential. It should be noted that the Hartree-Fock method is not treated in detail in this work but only used for comparison sake, since it is a microscopic method as against the DM which is a semi-microscopic method having a statistical approach.

The parameters calculated from the mass fit are then substituted into the expression for the sharp radii R_n and R_z to obtain values for them. The ones calculated from the Hartree-Fock method are again substituted into the expression of R_n and R_z to obtain new values for the sharp radii for the same nucleus. Other values for the same radii (R_n and R_z) are also given from direct HF calculation. The values given by direct HF calculation and that given by DM parameters are then compared.

The present work has, been performed within the frem-work of a general programme aimed at using the Hartree-Fock method as a test of the accuracy of the DM.

By this comparison a more valuable extrapolation into the region far away from the stability curve can be guaranteed with some definite accuracy.

CHAPTER 2

THE LIQUID DROP MODEL (LDM)

Although this chapter is devoted to the LDM it is pertinent to discuss a few points about the shell model, since it forms the guideline that led to the formulation of the LDM by Bohr (1937).

The shell model took a bold step towards the study of nuclear properties by, trying to study the behaviour of each individual nucleon . In contrast with the situation of atoms, the nucleus contains no massive central body which can act as a force centre, because of this, the assumption is that each nucleon experiences a central attractive force which can be ascribed to the average effect of all the other (A-1) nucleons in the nucleus. By this assumption each nucleon behaves as though it were moving independently in a central field which is described by a short range potential well, secondly this potential is assumed to be the same for all values of ℓ (azimutal quantum number).

The main assumption behind the design of the shell model is that the interaction between indivudual nucleons has to be weak. That is to say in this assumed central potential, each nucleon is imagined to be capable of describing an orbit of well defined energy and angular momentum, in a manner analogous to the behaviour of atomic electrons. This condition implies that each nucleon can describe at least several revolutions without being disturbed or scattered by other nucleons. The assumed

"mean free path" between collisions therefore has to be at least several times the nuclear radius.

One could be tempted to say that this weak interaction assumption is quite contrary to the well demonstrated strong interaction between nucleons, as seen in scattering experiments (Ferguson and Walker, 1940), i.e. all incident nuclear particles are scattered or captured. The mean free path for an incident nucleon is therefore short compared with the nuclear radius.

This weak - interaction paradox was resolved by Weisskopf (1951) by introducing the Pauli exclusion principle. According to him, among individual nucleons within a nucleus in its ground level, or a level having small excitation energies, the expected strong interaction may be present but unable to manifest itself, because all the quantum states into which the incident nucleon might be scattered are already occupied. Contrariwise an incident nucleon can be scattered or captured into a previously unoccupied and highly excited quantum state. Thus it is possible to accept the model of weak interaction between the consituent nucleons within a nucleus at low excitation energies, without denying the inherently strong character of the interaction between free nucleons.

The success of the shell model lies mainly in its ability to give a nearly correct energy sequence for nucleon states having different values of ℓ. It is also able to account for the then known "magic numbers"

up to O^{16} where the first P shell closes. By assuming strong spin-orbit forces (i.e introducing jj coupling) for indivudual nucleons a sequence of independent- particle states emerges which matches the experimentally known "magic numbers"

The LDM is a complete antithesis to the shell model. As stated ealier it is a macroscopic approach to the study of nuclear properties in that it treats the nucleons in the nucleus collectivelly rather than individually as in the shell model.

The underlying assumption in the liquid drop model is that a nucleus can be likened to a drop of liquid with a specific surface tension. The constituent nucleons are pictured as the molecules in the liquid. As a result the nucleus has a constant density independent of size. The property of constant density possessed by nuclei prompts an analogy with the physics of an incompressible fluid. Hence the first assumption of the LDM is that the nucleus is incompressible.

According to the LDM the energy of the nucleus can be considered as an integral over all space of an energy density, which is a functional of particles densities. An important subclass of such systems is that of saturating systems. Such system are characterized by the fact that in the limit of a very large ensemble of particles the nuclear density tends to uniform values over the bulk of the system and the

total energy has a stable minimum when considered as a function of the uniform densities. A condition for a system of interacting nucleons to show saturation is that the interparticle attraction have certain properties, in particular they should not be too attractive at short distances or high densities.

Therefore the LDM presumes the followings :-

(i) that the nucleus is a saturating system

(ii) that the neutron and proton densities are constant

(iii) The range of interparticle forces is negligible compared to the dimensions of the system, i.e. the nuclear forces have a short range character and are effective only between nearest neigbours.

(iv) and that the nucleus has a very sharp boundary i.e. the radius of the neutron and proton (densities) are equal ($R_n = R_z$).

With these assumptions the semi-empirical mass formula was derived by Weizsacker (1935) which consists of a volume term, surface term and a Coulomb term.

The volume, surface and Coulomb energy of the semi-emperical mass formula can be derived if we consider the dynamics of the system. This has led to the formulation of different kinds of nuclear potentials.

The semi-empirical mass formula of Weizsacker (1935) is then written as

$$M_{Z,A} = 1.008142\ Z + 1.008982\ (A-Z) - a_1 A + a_2 A^{2/3} + a_3 Z^2 A^{-1/3} + a_4 (Z-A/2)^2 A^{-1} + \delta\ a_5 A^{-1/2} \text{ (amu)} \quad \ldots 2.1$$

$\left[1.008142\ Z + 1.008982\ (A-Z) \right]$ is the mass of the constituent parts of the nucleus a

a_1 is the volume energy coefficient

a_2 is the surface energy coefficient

a_3 is the coulomb energy coefficient

a_4 is the symmetry energy coefficient.

a_5 is the pairing coefficient.

δ gives the effect of pairing on binding energy.

$$\delta = \begin{cases} +1 & \text{for even-even nuclei} \\ 0 & \text{for odd - A nuclei} \\ -1 & \text{for odd - odd nuclei} \end{cases}$$

CHAPTER 3
DROPLET MODEL (DM)

3.1 Qualitative Consideration Of The (DM)

The droplet model (DM) was introduced in order to effect a systematic refinement of the liquid drop model (LDM) originated by Von Weizäcker (1935). Futher developments in the refinement of the LDM can be found in works of Bethe (1936), Bohr and Wheeler (1936), Feenberg (1947), and Green (1954)

The DM as the LDM has a macroscopic approach to the explanation of the properties of the nucleus. This approach is only concerned with the collective degrees of freedom which is quite contrary to the shell model which is a microscopic approach and it's mainly concerned with the individual-particles. The microscopic theory is the more general one and contains the macroscopic one as an average in other words the nuclear properties such as masses or density distributions can be considered as being made up of a smooth macroscopic part and an oscillating microscopic part. The present discussion of the DM is similar to that of Myers and Swiatecki.

The role of the LDM in the understanding of the macroscopic aspects of nuclear properties is that it is an approximate solution of the nuclear many-body problem. For saturating leptodermous - (having a thin skin) systems like nuclei, the LDM potential energy may be thought of simply as a first-order description in terms of two small expansion parameters; the ratio of the surface diffuseness to the size

of the system (proportional to $A^{-1/3}$), and the square of the relative neutron excess I^2, where I equals (N-Z)/A. The DM in its uniform improvement of the LDM carries the leptodermous expansion to one higher order so as to include mass formula terms in $A^{1/3}$, $I^2 A^{2/3}$ and $I^4 A$. In analysing the situation one can consider effects of order A as belonging to the domain of nuclear matter studies, effects of order $A^{2/3}$ characterising the LDM, and effects of order A^0 as belonging to a microscopic theory where individual particles (consisting of one A^{th} part of the system) dominate the approach as in the shell model. This leaves a gap associated other effects of order $A^{1/3}$ and the DM is concerned with filling this gap. One can then conclude that the DM makes a step in the direction of the shell model in that it associates itself with the effect arising from the smallness of the nuclei.

Regardless of the fact that the energies associated with terms of order $A^{1/3}$ are several times as large as typical A^0 terms, such as shell effects, the refinement of the $A^{1/3}$ categories are not associated with prominant experimental features because of their smooth dependence on the neutron and proton numbers. Nevertheless the DM has also got some experimental backing in that there are now experimental evidence for some of its refinement to the LDM such as finer details of the nuclear charge distribution, the isotope effect in nuclear radii, curvature corrections to the nuclear surface tension and the presence of a neutron skin.

The LDM in its formulation of the nuclear mass, assumed that the nucleus is incompressible and the range of inter-particle forces is negligible compared with the dimensions of the system. If this asumption is to be accepted then deformed nuclei should not exist and natural process such spontaneous fission in large nuclei ($Z \leq 82$) should not occur. Nuclei are actually deformed espectially when they undergo sportaneous fission. The Coulomb energy tend to expand the nucleus while the surface energy tend to contract it. If the nucleus is always spherical as assumed by the LDM then we expect every nuclei to be stable, since a sphere is the most stable shape with minimum surface area and hence minimum surface energy. But in spantaneous fission the nucleus is actually compressed by the surface tension, which result in deviation from sphericity (deformation is produced) and hence an increase in the surface area together with the surface energy, a condition which now favours instability of the nucleus. In its refinement the DM made provision for this finite compressibility of the nucleus by the surface tension.

The other assumption of the LDM is that the charges are uniformly distributed in the nucleus. This assumption has been proved experimentally not to be true. Hahn et al (1956) measured the angular distribution of charged nuclear particles in heavy elements such as gold by using a 153 - and 183 - Mev electrons on such targets. One of the principal results of this electron scattering experiment is the conclusion that nuclei do not have a uniform charge distribution but that it is fairly flat inside (a small slope inside) and a well defined

slope at the edge.

Now that we have concluded that the nucleus has got finite compressibility, we should expect the bulk properties especially that of uniform distribution of the charge densities to change. If the nucleus is to be treated as a compressible liquid and then subjected to an increased pressure i.e compression the nucleus will surely contract. A change in bulk thus occurs and the bulk strain is defined by

$$\text{bulk strain} = \frac{\text{change in volume}}{\text{original volume}}$$

The bulk stress on the nucleus is the increased force per unit area, the compressibility K of the nucleus is then defined as

$$K = \frac{\text{bulk Stress}}{\text{bulk strain}} = - V\frac{dP}{dV}$$

i.e If the original volume of the nucleus is V, the small change in volume may be denoted by - dV when the pressure (compression) increases by a small amount dP, the minus indicates that the volume decreases. For the LDM we expect the value of K to be zero but the DM has a provision for this coefficient as a result the DM is able to give finer details of the nuclear distribution.

Hofstader (1955), from his electron scattering experiment, came to the conclusion that nuclei do not have sharp edges as in the LDM, but rather that there is an appreciable region over which the nucleon density falls away. In other words the surface of the nucleus is actually diffuse., hence

there must be curvature correction to the nuclear surface tension and it is given by the DM.

The size and density of a nucleus are determined mainly by the nuclear forces. Since nuclear forces are charge-independent, the LDM then assumes that neutrons and protons must have the same spatial distribution and hence the same radii $R_N = R_z$ where R_n and R_z are the radius of neutron and Proton distribution respectively. But the nuclear forces are not the only forces present in the nucleus The electrostatic force is also present as a relatively small correction to the nuclear forces. Because of these repulsive electrostatic (Coulomb) forces between protons, we may expect the neutrons to have a slightly greater density near the center of the nucleus. It is also possible for the neutron distribution to have a radius slightly larger than that of the protons due to exclusion-principle effects. It can now be concluded that both neutrons and protons, each occupy a simply connected region and that the distributions overlap in a certain part of these regions. The surfaces may be characterized by R_n and R_z satisfying

$$\rho_n (R_n) = 0 \quad \text{and} \quad \rho_z (R_z) = 0$$

where ρ_n and ρ_z are the neutron and proton densities distribution. The DM in its refinement now gives separate expressions for R_n and R_z .

The exclusion principle always dictates eqaul number of neutrons and protons for stability but when the number of neutrons are more than the number of protons, the exclusion principle leads to an increase in energy. This increase in

energy is known as the symmetry energy. The energy is increased because the (n - z) extra neutrons (neutron excess) form a layer of "skin", known as neutron skin around the surface. The interaction at the surface is only between neutron-neutron, while as the bulk consists of interactions between neutron-neutron, proton-proton and neutron-proton. Any nuclear model must give a correction for this extra binding due to the neutron skin, just as the DM will do.

There exist in nature nuclei with the same number of protons (z) but different masses. J. J. Thomson (1913) demonstrated the existence of chemically identical atoms with different masses by the deflection of ionized atoms. Such nuclei are called isotopes. Two isotopes have slightly different radii. Variations in the nuclear radius can be found spectroscopically by a study of the isotope shifts in atomic spectra. It is well known that in the case of hydrogen the spectral lines of isotopes are slightly displaced. Observation of this isotope shift was used by Urey, Brickwedde and Murphy (1971) to discover deuterium. Schuler, Kopfermann, and others (1931), showed also that heavy nuclei give isotope shift. The DM made provision for this isotope effect on nuclear radii.

For light nuclei the Coulomb repulsion is well defined as a result most of the protons spend a large part of their time at the surface and this causes a central depression in the densities. One can then expect a proton skin only for light nuclei. Correction for this central depression by Coulomb repulsion must be provided for in the DM.

The two fundamental refinements which characterize the DM are associated with the finiteness of the thickness of the nuclear surface and finiteness of the nuclear compressibility. Thus unlike the case in the LDM the density distribution of neutrons and protons are no longer considered to be constant inside a sharp boundary and zero outside but are approximately constant in the bulk of the nucleus and instead of a sharp boundary there is now a diffuse surface region in which the densities decrease smoothly to zero.

The DM in its refinement first defined the degrees of freedom which consists of two sets (against that of the LDM which is infinite). The first is associated with specifying the shape of the (now diffuse) boundary. The second is concerned with specifying the (small) density non uniformities in the bulk. The DM then went a step further in redefining the degrees of freedom in terms of some variables. The variables are the density distributions ρ_z and ρ_n of the Z protons and N neutrons which will be characterized by two diffuse surfaces which are the neutron and proton radii (R_n and R_z). The neutron radius is defined as that radius which will contain all the N neutrons if the bulk neutron density were extrapolated outward. The effective proton radius R_z is defined in an analogous way. An effective nuclear radius R can be defined by extrapolating the bulk nuclear density ρ (equal to $\rho_n + \rho_z$) outward until it contains all A particles.

The DM introduces two new parameters which are defined in terms of its degrees of freedom. These two new parameters characterizes the density non-uniformities and the diffuse

surface. The first parameter is defined as

$$\varepsilon = -\frac{1}{3}\,\rho\,\frac{\rho - \rho_0}{\rho_\bullet} \qquad \ldots\ldots\ldots\ldots\ldots \quad (3.1)$$

Equation 3.1 specifies the deviation of the nuclear density ρ from its nuclear matter value $\rho_\bullet$ (or a uniform central density that corresponds to the LDM) The $-\frac{1}{3}$ appears in connection with the conventional definition of the compressibility coefficient. The other parameter is that of the local assymetry δ defined as

$$\delta = \frac{\rho_n - \rho_z}{\rho_n + \rho_z} \qquad \ldots\ldots\ldots \quad (3.2)$$

Equation 3.2 specifies the nuclear composition at any point in the nucleus, it is analogous to the total asymmetry I defined as

$$I = \frac{N - Z}{N + Z} \qquad \ldots\ldots\ldots \quad (3.3)$$

The definition of ε and δ from equations 3.1 and 3.2 applies only to the bulk of the nucleus. In the surface they are defined as the outward extrapolation of their bulk behaviour, since most of the variables in their definitions diffuse near the surface. The parameters ε and δ are actually dimensionless.

The DM further states, that since the bulk densities diffuse at the surface they can also have average values. The average value of the neutron density is denoted by $\bar{\rho}_n$ and that of the proton is $\bar{\rho}_z$. The difference between their average and their final values is a small quantity. One can define deviations in density from the average values by

$$\tilde{\rho}_n = \rho_n - \bar{\rho}_n \quad \text{and} \quad \tilde{\rho}_z = \rho_z - \bar{\rho}_z$$

By inspection of equation (3.1) and (3.2), it is obvious that the dimensionaless parameters ϵ and δ can also be expressed in terms of their average values properties. The average values of ϵ and δ are defined as

$$\bar{\epsilon} = -\frac{1}{3}\frac{\bar{\rho} - \rho_u}{\rho_o} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.4)$$

and

$$\bar{\delta} = \frac{\bar{\rho}_n - \bar{\rho}_z}{\bar{\rho}_n + \bar{\rho}_z} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.5)$$

The DM also defined a parameter for the neutron excess called the neutron skin (t). The quantity t is defined as

$$t = R_n - R_z$$

in other word it is the thickness of the effective neutron skin, and it is an important variable in discussing surface properties. It is related to the average dimensionless parameter $\bar{\delta}$ by the expression

$$\tau = \frac{2}{3}(1 - \bar{\delta})A^{1/3} \quad \ldots\ldots\ldots\ldots \quad (3.6)$$

where τ is the demensionless skin thickness and is equal to t/r_o. r_o is the nuclear radius constant. From equation (3.6), the skin thickness becomes larger as the mass number A(= z + n) increases:

The derivation of equation (3.6) is shown in appendix I.

In order to enhance our understanding of the differences between the DM and the LDM, let us consider the variation in their degrees of freedom by examining the diagrams below:

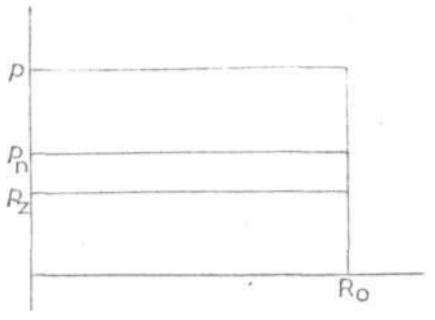


Fig. 3-1 Behaviour of the LDM degrees of freedom as a function of radial distance.

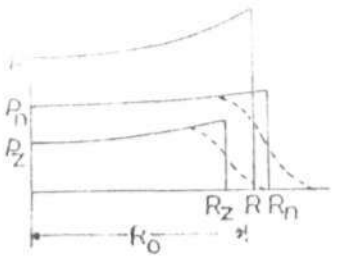


Fig. 3-2 Behaviour of the DM degrees of freedo as a function of radial distance.

Figure 3.1 shows the schematic LDM nuclear desities ρ_n, ρ_z and ρ (where $\rho = \rho_o$, the density distribution for nuclear matter) as a function of the radial distance. There is no spatial variation of the bulk densities in the LDM and the location of all the surfaces is at the nuclear matter radius R_o.

Figure 3.2 shows a schematic DM nuclear densities as a function of the radial distance. The solid lines show the bulk functions ρ_n, ρ_z and ρ extrapolated outward to the effective sharp radii R_n, R_z and R, the LDM radius R_o is also shown for the sake of comparison. The dashed lines serve to indicate that the surfaces are really diffused.

The change between Figure 3.2 and Figure 3.1 may be thought of as consisting of two parts. The first part is due to the separate compressions or dilatations of the uniform neutron and proton distributions to the new radii. The second is

the redistribution of the densities from their average value $\bar{\rho}_n$, $\bar{\rho}_z$, and $\bar{\rho}$ to their final values of ρ_n, ρ_z, and ρ.

The LDM raius R_o corresponds to a uniform central density ρ_o. They are defined as

$$R_o = r_o A^{1/3} \quad \ldots\ldots \quad 3.7$$

and

$$\rho_o = \left(\frac{4}{3}\pi\, r_o^3\right)^{-1} \quad \ldots\ldots \quad 3.8$$

For the DM the radii and average densities are related by

$$\frac{4}{3}\pi\, R_n^3\, \bar{\rho}_n = N \quad \ldots\ldots \quad (3.9\ a)$$

$$\frac{4}{3}\pi\, R_z^3\, \bar{\rho}_z = Z \quad \ldots\ldots \quad (3.9\ b)$$

$$\frac{4}{3}\pi\, R^3\, \rho = A \quad \ldots\ldots \quad (3.9\ c)$$

From equations 3.1 and 3.2 the pair of dimensionless functions ϵ and δ are related to the densities by

$$\rho/\rho_o = 1 - 3\,\epsilon \quad \ldots\ldots \quad (3.10\ a)$$

$$\rho_n/\rho = \frac{1}{2}(1 + \delta) \quad \ldots\ldots \quad (3.10\ b)$$

$$\rho_z/\rho = \frac{1}{2}(1 - \delta) \quad \ldots\ldots \quad (3.10\ c)$$

Substituting for ρ from 3.10a into 3.10b

$$\rho_n = \frac{1}{2}\rho_o\,(1 - 3\epsilon)\,(1 + \delta)$$

$$\rho_n = \frac{1}{2}\,\rho_o\left[1 + \delta - 3\epsilon - 3\epsilon\delta\right]$$

ϵ and δ are very small number hence $\epsilon\delta$ would be very small, therefore to first order approximation the above equation reduces to

$$\rho_n = \frac{1}{2}\rho_o(1 - 3\epsilon + \delta) \quad \ldots\ldots \quad (3.11\ a)$$

using the same argument for ρ_z it expression becomes.

$$\rho_z = \frac{1}{2}\,\rho_o(1 - 3\epsilon - \delta) \quad \ldots\ldots \quad (3.11\ b)$$

The radii R, R_n and R_z can also be written in terms of the

average values of the dimensionless parameters ϵ and δ as

$$R = r_o A^{1/3} (1 + \bar{\epsilon}) \quad \ldots\ldots \qquad (3.12a)$$

Equation 3.12a is derived by substituting (3.10a) and (3.8) in to (3.9) i.e.

$$R^3 = \frac{3}{4\pi} \frac{A}{\rho} = \frac{3}{4\pi} \frac{A}{\rho_o(1-3\bar{\epsilon})} = \frac{3}{4\pi} \times \frac{4\pi r_o^3}{3} \times \frac{A}{1-3\epsilon}$$

$$R^3 = Ar_o^2 \frac{1}{1-3\epsilon} = Ar_o^3(1-3\epsilon)^{-1}$$

$$R = r_o A^{1/3}(1 - 3\epsilon)^{-1/3}$$

Using the binomial expansion and neglecting higher terms then R reduces to

$$R = r_o A^{1/3}(1 + \bar{\epsilon}) \text{ as in (3.12a).}$$

The expression for R_n is derived by substituting 3.11a and 3.8 into 3.9a

$$\frac{4}{3}\pi R_n^3 \bar{\rho}_n = N$$

$$R_n^3 = \frac{3}{4\pi} \frac{N}{\bar{\rho}_n} = \frac{3}{4\pi} \times \frac{N}{\rho_o} \times 2\left[1 - 3\bar{\epsilon} + \bar{\delta}\right]^{-1}$$

$$R_n^3 = \frac{3}{4\pi} \times \frac{4\pi}{3} \times 2Nr_o^3 \times \left[1 - 3\bar{\epsilon} + \bar{\delta}\right]^{-1}$$

$$R_n = r_o (2N)^{1/3}\left[1 - 3\bar{\epsilon} + \bar{\delta}\right]^{-1/3}$$

Using binomial expansion and neglecting higher terms then the above expression can be written as

$$R_n = r_o(2N)^{1/3}\left[1 + \bar{\epsilon} - \frac{1}{3}\bar{\delta}\right] \quad \ldots\ldots \qquad (3.12b)$$

Using the same argument as above the expression for R_z can be written as

$$R_z = r_o(2z)^{1/3}(1 + \bar{\epsilon} + \frac{1}{3}\bar{\delta}) \quad \ldots\ldots \qquad (3.12c)$$

Equations (3.1) to (3.12c) are given by Myers and Switecki (1969).

3.2 Energy Expression:

The total energy of a system consists of the potential and kinetic energies.

Our own system is the nucleus and in this project we are only interested in the potential energy of the system. Hence the Hamiltonian for the nucleus in this work is just the potential energy. The potential energy of the nucleus consists of a volume energy term, a surface energy term, and the electrostatic energy term. The volume energy depends on the bulk density distribution. The surface energy depends on the surface condition and includes a dependence on the local curvature of the effective surface. The electrostatic energy depends only on the proton distribution.

The energy of the system is now a function of the new parameters introduced by the DM (ϵ, δ and τ). In other words the energy is a function of the new degrees of freedom.

Let us now consider the various terms (volume, surface, Coulombic energies) that make up the total energy of the system.

3.2.1 Volume Energy Term

The nucleus can be considered as consisting of mainly two different components (neutrons and protons). The volume energy is then given as an approximation of the integral over all space of energy density which is a functional of the particle densities. The volume energy expression may then be written as

$$E_V = \int_{\text{volume}} e\ \rho\ d\tau$$

where e ρ is the energy density which consists of the binding energy per particle e times the total density $\rho(\rho_z + \rho_n)$.

$d\tau = r^2 \sin\theta \, d\theta \, d\phi \, dr$ (r is the spatial variation of the density distributions in the nucleus) is an element of the total volume of the nucleus over which the integration is taking place.

This volume energy which is the major contribution to the total energy, stems from the mutual interactions of the nucleons under the infleunce of nuclear forces. Interaction between nucleons are essentially confined to just those between nearest neighbors, for if the binding were infleunced by those nucleons lying beyond the nearest neighbors its strength would increase as the total number of nucleons increases and the volume energy term would be proportional to A^2 rather than to A. Since the number of neighbors each nucleon has is independent of the overall size of the system, e the binding energy per particle is then independent of A. This is cleary a characteristic of any system in which the range of interaction between particles is small compared with the dimensions of the system.

3.2.2 Surface Energy Term:

This energy depends on the surface effect. Von Weizsacker (1935) pointed out that the surface effect consists of a classical and a "quantum mechanical" part.

Classically we may assume the nucleus to have a sharp boundary (as in LDM) at a certain distance R from the center.

Those particles which are nearer the boundary at a certain distance R from the center than the range of the nuclear forces a, will not contribute their full share to the binding energy. This leads to an increase of the total energy by an amount of the relative a/R.

In quantum theory, the boundary can never be sharp, (DM), because this would mean an infinite derivative of all particle wave functions and consequently an infinite Kinetic energy. The surface layer must be spread out over a certain region of the order of magnitude of one particle wave length. The narrower the surface region, the larger will be the additional kinetic energy. The broader the region, the more will the total energy be reduced in magnitude. Therefore there will be an optimum breadth of the surface layer.

Wick (1934) has first pointed out that the nuclear binding energy will be reduced, especially for small nuclei, by the existence of a surface of the nucleus because particles at the surface interact, in the average, only with half as many other particles as do particles in the interior of the nucleus. Therefore the particles at the surface will contribute less to the total binding energy when compared with the particles in the interior since binding is only completed by a particle when it is fully surrounded by other particles. The situation is quite analogous to the surface tensions of liquids.

The surface of the nucleus is an ill-defined quantity when its edge is diffuse. However, it seems appropriate to take the sphere of radius R as the reference surface for the

surface energy. It can then be assumed that the surface energy is proportional to the area of this reference surface.

Feenberg (1947) pointed out that the surface energy should contain a symmetry energy contribution, since the surface energy is a consequence of the failure of nuclear saturation at the edge of the nucleus. This symmetry energy is actually due to the neutron excess (N-Z) or "neutron skin". In light nuclei there is a tendency for the number of neutrons and protons to be equal, as in $_6C^{12}$, $_7N^{14}$ and $_8O^{16}$. This is because the (n-P) force can dominate the (n-n) and (P-P) forces. Heavy nuclei must always contain appreciably more neutrons than protons, because by attempting to build a heavy nucleus out of equal numbers of protons and neutrons will result to a violently unstable nucleus. This is due to the fact that the large disruptive Coulomb energy could not be overcome by the available (nP), (nn), and (PP) attractive forces, hence it is necessary to introduce a neutron excess to provide enough total actractive force to dominate adequately the Coulomb repulsion. It must be noted at this point that due to the increasing neutron excess with growing A the root mean square radius $< R_n^2 >^{1/2}$ of the neutron distribution exceeds the charge (proton) distribution radius $< R_z^2 >^{1/2}$.

The surface energy is then defined as an integral over the surface $\int$ of a surface tension coefficient σ. The expression is written as

$$E_s = \int_{\text{surface}} \sigma \, ds$$

ds is an element of the surface area.

the Heisenberg force (due to exchange of charge), the Bartlett force (exchange of spin direction), and the Majorana force (exchange of position due to exchange charge and spin).

The exchange energy correction expression is written

$$E_{exchange} = -\frac{3}{4}\left(\frac{3}{\pi}\right)^{-1/3} e^2 \int_{R_z} \rho^{4/3}\, d\tau \quad \text{(Myers and Swiatecki (1969))}$$

e is the protonic charge,

ρ_z is the proton charge distribution.

Now that exchange correction is accounted for the Coulomb energy can now be written as

$$E_{Coulomb} = \frac{1}{2} e^2 \int_{R_z} \frac{P_{12}\, \rho_{z(1)}\, \rho_{z(2)}}{r_{12}}\, d\tau_1\, d\tau_2$$

P_{12} is the relative probability for approach of two protons

r_{12} is the relative position of the two protons.

The other correction is due to the diffuseness of the nuclear surface. Much discussion has been made about this diffuse surface in section 3.2.2. The energy expression for the diffusenss correction is given as

$$E_{difuseness} = -4\pi Ze \int_{-\infty}^{\infty} \delta\rho\, \cap\, d\, \cap$$

where $\cap$ is the outward normal to the surface $\delta\rho$ is the difference between the diffuse and the sharp charge densities

$$\delta\rho = \rho(n)_{difuse} - \rho(n)_{sharp}$$

The coulomb energy is the other energy that plays a major part in the stability of the nucleus because it has an opposite role to that of the surface energy. It always tends to increase the volume of the nucleus due to its repulsive effect between the protons.

The total Hamiltonian for the nucleus is written as

$$E = \int_{volume} \varepsilon\, \rho\, d\tau + \int_{surface} \sigma\, ds + E\ coulomb \quad \ldots\ldots \quad (3.13a)$$

3.2.4 Taylor Series Expansion Of The Energy.

The expressions for the new coefficients introduced by the DM are found by doing a Taylor series expansion of the energy of the system which is now a function of the parameters ϵ, δ and τ.

The expansion parameters in the treatment of DM are considered to be the ratios of the surface and coulomb energies to the bulk (or volume) energy. The smallness of the former parameter means a small surface - to - volume ratio. The implication of this is a large system in the sense that the characteristic spatial dimensions of the system should be large compared with the thickness of the diffuse surface region (or with the range of the interparticle forces or with the inter particle spacing). In the nuclear case the ratio of the inter particle spacing to the nuclear radius is of the order of $A^{-1/3}$ which may be considered as our expansion parameter.

From the volume energy term, the bulk energy per particle e depends on the neutron and proton densities, i.e. the energy e is now a function of ε and δ which can be written as

$$e = e(\varepsilon, \delta) \quad \ldots\ldots\ldots \qquad (3.13b)$$

By doing a Taylor series expansion of equation 3.13b. The energy per particle e becomes

$$e = -a_1 + J\delta^2 + \frac{1}{2}K\varepsilon^2 - L\varepsilon\delta^2 + \frac{1}{2}M\delta^4 + \left[\frac{1}{6}B\varepsilon^3\right] + \ldots (3.13c)$$

The derivation of equation (3.13c) is shown in appendix II.

By virtue of the symmetry between neutrons and protons this energy must be a symmetric function of δ, therefore the

expansion has been carried out in powers of ε and δ^2 (rather than δ). On the other hand there is no term linear in ε in equation (3.13c) since the energy per particle e is stationary with respect to deviations of the density from its standard nuclear matter value ρ_o.

Except for the term in parenthesis, equation (3.13c) is the standard version of the Droplet model. (3.13c) comprises of all the terms that are necessary in extending the nuclear mass formula. The coefficient a_1 is the binding energy per particle of nuclear matter. J is the volume symmetry energy coefficient. This are the only two coefficients given by the LDM. The other coefficients are the new coefficients introduced by the DM as a correction to the LDM. The quantity K is the nuclear compressibility coefficient. L. gives the density dependence of the symmetry energy and M is the coefficient which specifies the deviation of the symmetry energy from a quadratic dependence on δ. The last term in equation 3.13c is new and it is the major contribution of my thesis. Its effect will be discussed in the chapter containing the conclusion.

The surface tension coefficient σ is a function of the conditions at the surface, especially the neutron and proton densities or the values of ε and δ on the surface, of the neutron skin thickness and the surface curvature (equal to 2/R for a sphere). The surface tension coefficient σ can now be written as $\sigma(\tau, \delta_s)$.

The Taylor series expansion for the surface energy is given by

$$4\Pi\, r_o^2\, \sigma = a_2 + H t^2 + 2P\tau\delta_s - G\delta_s^2 + a_3 R^{-1} + (D\, \varepsilon^2) \quad \ldots(3.14d)$$

The derivation of equation (3.14d) is shown in appendix II.

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a_2 is the normal surface energy coefficient given by IDM. It has dimensions of an energy. The terms H, P and G are the new coefficients which specify the dependence of the surface energy coefficient on the neutron skin thickness t, the value of the bulk asymmetry δ extrapolated to the mean surface δ_s. The fifth term is the correction for the curvature of the surface, the curvature being proportional to $R^{-1}(r_o/R)$. The term in the parenthesis is the new term introduced but the former ones are those of the standard droplet model. The effect of the new term will be discussed in the conclusive part of this project.

Before the total energy can be written, we have to take into consideration the fact that the energy is functional of the density variables $\tilde{\varepsilon}$ and $\tilde{\delta}$ and a function of R_n and R_p, as a result it is convenient to divide the functions ε and δ into two parts.

$$\left.\begin{aligned} \varepsilon &= \bar{\varepsilon} + \tilde{\varepsilon} \\ \delta &= \bar{\delta} + \tilde{\delta} \end{aligned}\right\} \quad \ldots\ldots \quad (3.14e)$$

The first term on the right hand side of equation (3.14e) (a number) is the average value of ε or δ and the second part (a function of position) is the deviation from the average. The total energy which is now a function of $\bar{\varepsilon}$, $\bar{\delta}$ and a functional of $\tilde{\varepsilon}$, $\tilde{\delta}$ is written as

$$E(\bar{\varepsilon},\ \bar{\delta},\ \tilde{\varepsilon},\ \tilde{\delta},\ \int_n,\ \int_p,\ \int) = \left[-a_1 + J\bar{\delta}^2 + \frac{1}{2}K\bar{\varepsilon}^2 - L\bar{\varepsilon}\bar{\delta}^2 + \frac{1}{2}M\bar{\delta}^4\right]A$$

$$+ \int_{R_o} \left[J\bar{\delta}^2 + \frac{1}{2}K\bar{\epsilon}^2 \ldots\ldots \right] \rho_o \, d\tau$$

$$+ a_2 A^{2/3}(1 + 2\bar{\epsilon}) + \left[H\tau^2 + 2P\tau(\bar{\delta} + \bar{\delta}_s) - G(\bar{\delta} + \bar{\delta}_s)^2 \right] A^{2/3} + a_3 A^{1/3} + \frac{3e^2}{5r_o} \frac{Z^2}{A^{1/3}}\left(1 - \bar{\epsilon} - \frac{\bar{\delta}}{3} + \frac{I}{3}\right)$$

$$+ \frac{1}{2}e\rho_o \int_{R_o} \frac{Ze}{R_o}\left[\frac{3}{2} - \frac{1}{2}\left(\frac{r}{R_o} \right)^2 \right] (-3\bar{\epsilon} - \bar{\delta})$$

$$- C_3 \frac{Z^2}{A} - C_4 \frac{Z^{4/3}}{A^{1/3}} \qquad \ldots\ldots\ldots\ldots \quad (3.15)$$

Equation (3.15) is the basic energy equation for the DM specialized to spherical shape. It contains two variables $\bar{\epsilon}, \bar{\delta}$ two function variables $\tilde{\epsilon}$, $\tilde{\delta}$, two constitution parameters A and I (or N and Z) and eleven coefficients r_o, a_1, a_2, a_3, G, H, J, K, L, M and P.

3.2.5 : Minimization Of The Energy:

Every system tends to have minimum energy for its stability for this system (the nucleus) to be stable equation (3.15) has to be minimum. For this energy to be minimum, we have to determine $\bar{\epsilon}$, $\bar{\delta}$ and $\tilde{\epsilon}$, $\tilde{\delta}$ that will make the energy stationary by demanding that the energy change ∂E vanish for all small particle-preserving variation.

The values of $\bar{\epsilon}$, $\bar{\delta}$ and $\tilde{\epsilon}$, $\tilde{\delta}$ that make the energy stationary are called the equilibrium values. The equilibrium value of $\bar{\delta}$ does not depend on G and the two other surface energy coefficients H and P do not appear separately but only together with J in the combination of Q, where Q is defined as

$$Q = H/(1 - \frac{2}{3}\frac{P}{J}) \qquad \ldots\ldots\ldots \quad (3.16)$$

The equilibrium values of $\bar{\varepsilon}$, $\bar{\delta}$ and $\tilde{\varepsilon}, \tilde{\delta}$ are given by

$$\delta = \left[I + \frac{9e^2}{40r_o} \frac{Z^2}{A^{5/3}}\right] \Big/ \left[1 + \frac{9}{4}\frac{J}{Q} A^{-1/3}\right] \quad \ldots\ldots (3.17)$$

and

$$\varepsilon = \frac{1}{K}\left(-2a_2 A^{-1/3} + L\delta^2 + \frac{3e^2}{5r_o} \frac{Z^2}{A^{4/3}}\right) \quad \ldots\ldots (3.18).$$

Derivation of equ (3.17) and (3.18) are shown in appendix III. Thus for each nculeus of interest, the sharp radii values, R_N and R_Z can be obtained by substituting for ε and δ in the radii expressions below

$$R_N = r_o A^{1/3}\left[1 + \varepsilon + \frac{1}{3}(I - \delta)\right] \quad \ldots\ldots (3.19).$$

$$R_Z = r_o A^{1/3}\left[1 + \varepsilon - \frac{1}{3}(I - \delta)\right] \quad \ldots\ldots (3.20)$$

Now that we know the equilibrium value for δ , we can then obtain an expression for the neutron skin and discuss it in detail. The new expression for the neutron skin in terms of the new coefficients is

$$t = \frac{3}{2} r_o \frac{J}{Q} I \left(1 - \frac{9}{4}\frac{J}{Q} A^{-1/3} + \ldots\ldots\right) \quad \ldots\ldots (3.21a)$$

The formation of a neutron skin results from two opposing effects; the neutron excess which tends to produce a neutron skin and the electric repulsion (proportional to $Z^2/A^{4/3}$) tends to reduce its thickness. The former is proportional to symmetry energy coefficient. The restoring force limiting the extent of a neutron skin is provided by the coefficient Q. Q can be said to be an effective stiffness against the pulling apart of the neutron and proton surfaces, when A becomes too large the thickness (t) becomes independent of A, tending to the semi-infinite limit

$$t = \frac{3}{2} r_o \, J/Q \, I \qquad \ldots\ldots\ldots \quad (3.21b).$$

In the case of a vanishing effective stiffness Q the bulk asymmetry tend to zero and all the excess neutrons move into the surface (since there is no resistance against forming a neutron skin). A nucleus must be very large before the skin thickness attains its limiting form, this is proportional to the factor 9/4 $(J/Q)A^{-1/3}$ which must be small although it is not always small for typical values of A.

The poor convergence of equation (3.21a) is due to the size of the coefficient 9/4 (J/Q) because it depends on the relative sizes of the two physical properties of nuclei described by J and Q. The symmetry energy coefficient J is made up from three contributions; one due to the interaction energy between like particles, one due to kinetic energy of the neutrons and protons and the other one due to interaction energy between unlike particles. The coefficient H to which Q is proportional depends only on unlike particles.

The energy expression at equilibrium is then written as

$$\begin{aligned} E = {} & (-a_1 + J\delta^2 + \tfrac{1}{2}K\epsilon^2 - L\epsilon\delta^2 + \tfrac{1}{2}M\delta^4)A \\ & + a_2(1 + 2\epsilon)A^{2/3} + Q\tau^2 A^{2/3} + a_3 A^{1/3} \\ & + C_1 \frac{Z^2}{A^{1/3}}\left(1 - \epsilon + \tfrac{1}{2}\tau A^{-1/3}\right) - C_2 \, Z^2 \, A^{1/3} - C_3 \frac{Z^2}{A} \\ & - C_4 \, Z^{4/3}/A^{1/3} \qquad \ldots\ldots\ldots\ldots \quad (3.22) \end{aligned}$$

Equation (3.22) shows the various physical effect associated with the refinement introduced by the DM. The first two lines correspond to volume and surface energies, the last two lines are associated with the Coulomb energy. Part of

the first line are changes to the volume energy associated with the adjustments of the neutron and proton radii. The second line contains various corrections to the surface energy and the last term in the second line being the curvature correction. In the third line the term in the bracket corrects the Coulomb energy for a changed proton radius, the second term in the third line corrects for charge redistribution (and is composed of a decrease in Coulomb energy and an increase in volume energy), the third term in this line is the Coulomb diffuseness correction, and the last line, the Coulomb exchange correction.

The leading terms (with δ replaced by I) are $- a_1 A + JI^2A + a_2A^{2/3} + C_1Z^2/A^{1/3}$ and correspond to the LDM approximation.

CHAPTER 4

THE HARTREE AND HARTREE-FOCK FORMALISM

4.1 The Hartree Theory

This theory consists in solving the Schrodinger equation for a system of N fermions moving independently in spherically symmetric potential. The equation is

$$H\psi_T = E_T \psi_T \quad \ldots\ldots \quad 4.1$$

where H is the Hamiltonian defined as

$$H = \sum_i (T_i + V_i) + \tfrac{1}{2} \sum_i \sum_j V_{ij}(\bar{r}_i \bar{r}_i)$$

T_i is the kinetic energy

V_i is some external potential V_{ij} is a two-body interaction potential. For example in the case of the atom

$$V_i = \frac{Ze^2}{r_i} \quad \text{and} \quad V_{ij} = \frac{e^2}{|\bar{r}_i - \bar{r}_j|}$$

The eigenvalue E_T is the total energy of the atom and the eigenfunction ψ_T is the function describing the motion of all its N fermions.

The Hartree method consists in finding the best wave-function ψ_T of product form

$\psi_T = \psi(\bar{r}_1)\ \psi(\bar{r}_2)\ \ldots\ldots\ \psi(\bar{r}_N)$ for the N-particle wave-function

Then $(\psi_T, H\psi_T) \equiv \int \psi_T^* \quad H \quad \psi_T \, d\tau$

$$= \sum_i \int \psi_i^* (r_i)(T_i + V_i)\psi_i(r_i) d\bar{r}_i$$

$$+ \tfrac{1}{2} \sum_i \sum_{i+j} \iint \psi_i^* (r_i)\psi_j^*(r_j) V_{ij} \psi_i(r_j) d\bar{r}_i \, d\bar{r}_j$$

i and j referring to the various particles.

The total energy $(\psi_T, H\psi_T)$ is then varied with (ψ_T, ψ_T) fixed,

i.e.

$$\int \delta\psi_T^* \, H\psi_T \, d\tau + \int \psi_T^* \, H \, \delta\psi_T \, d\tau = 0$$

under the condition

$$\int \delta\psi_T^* \psi_T \, d\tau + \int \psi_T^* \, \delta\psi_T = 0$$

This is a constrained extreme, and the two equations can be combined by the introduction of a Lagrange multiplier, Thus

$$\int \delta\psi_T^* (H\psi_T - \lambda\psi_T) \, d\tau = 0 \text{ for all } \delta\psi_T^*$$

If this quantity is to vanish for all $\delta\psi_T^*$, the quantity multiplying $\delta\psi^*$ in the integrand must vanish, i.e.

$$H\psi_T = \lambda\psi_T$$

which is just the Schrodinger equation. From the corresponding equation on $\delta\psi_T$ we obtain the complex conjugate of this. The variation of $\delta\psi^*$ produces

$$(T_i + V_i)\psi_i(r_i) + \sum_{j \, i} \left| \int \psi_j^*(r_j) V_{ij} \psi_i(r_j) \, d\bar{r}_j \right| \psi_i(x_i) = \lambda_i \psi_i(x_i) \quad \ldots 4.2$$

The λ_i is the Lagrange multiplier associated with the i^{th} variation. Let us rename it ε_i i.e. $\lambda_i \equiv \varepsilon_i$, since it clearly has the significance of the single particle energy of the i^{th} particle. The total energy of the atom can now be written as

$$E_T = \sum_{i=1} \varepsilon_i \quad \ldots 4.3$$

Equation 4.2 is called the Hartree equation for self-consistent field.

The Hartree-Fock Theory

The Hartree-Fock equation is just the Hartree equation generalized to include exchange. In the formulation of the Hartree equation some approximations were made, two of such are, the use of symmetric wave-

function to describe fermi particles and the neglect of correlation between the particles. These two approximations no longer hold in the HF theory.

For a system of non-interacting particles the wavefunction that normally describe their behaviour is an antisymmetric wavefunction. This is due to the fact that the antisymmetric wavefunction takes care of the Pauli-exclusion principle automatically. For a system of N non-interacting particles their antisymmetric eigenfunction can be written as a slater determinant.

$$\psi_T = \frac{1}{\sqrt{N!}} \begin{vmatrix} \psi_\alpha(1)\psi_\alpha(2) & \cdots & \psi_\alpha(N) \\ \psi_\beta(1)\psi_\beta(2) & \cdots & \psi_\beta(N) \\ \vdots & & \\ \psi_\nu(1)\psi_\nu(2) & \cdots & \psi_\nu(N) \end{vmatrix} \quad \ldots 4.4$$

where $\alpha, \beta \ldots \nu$ are single quantum number each designating a particular set of the four quantum numbers required to specify the state of a particle.

Since the value of this determinant consists of sums and differences of N! terms comprised of products of one-particle wavefunctions, such as $\psi_\alpha(1), \psi_\beta(2) \ldots \psi_\nu(N)$ and similar terms in which the particle labels are permuted, ψ_T is simply a linear combination of degenerate wavefunctions. The constant $1/\sqrt{N!}$ is such that ψ_T will be normalized if each of the products of one-particle wavefunctions is normalized. Because a determinant vanishes if any two rows are identical it is apparent that ψ_T has the required property of vanishing if more than one particle is in the same state (e.g. if $\beta = \alpha$).

Normally non-interacting particles in a system arrange themselves in such a way as to minimize the total energy of the system. In this arrangement there is a tenedency for all the particles to be in the same quantum state with n=1, ℓ =0 orbital. But the Hartree-Fock theory now takes care of this problem by the use of a anti-symmetric wavefunction and the introduction of an exchange term.

The Hartree equations are obtained by varying one-particle wave-functions in such a way as to make the energy an extreme. The Hartree-Fock equations are also derived in the same manner, which can now be written as

$$\frac{-h^2}{2m}\psi_j(r_j) + \frac{Ze^2}{r}\psi_j(r_j) + \sum_{i=1}^{N}\int \psi_i^*(r_i)V(r_i,r_j)\psi_i(r_i)dr_i\ \psi_j(r_j)$$
$$- \sum_{i=1}^{N}\int \psi_i^*(r_i)V(r_i,r_j)\ \psi_i(r_j)\psi_j(r_i)dr_i = \varepsilon_j\psi_j(r_j) \quad \ldots 4.5$$

The $\psi_i^{'s}$ are assumed to depend on spin as well as coordinates, and the integration dr_i include summation over spin, so that the exchange terms, the last ones on the left hand side of equation (4.5) automatically vanish unless the functions ψ_i and ψ_j corresponds to spins in the same direction. $V(r_i,r_j)$ is the two-body interaction potential or the phenomenolocal interaction. The third term in equation (4.5) is called the single particle potential. It is clearly the potential energy acting on the particle at position r_i due to interaction between the other particles. The last term is the exchange term, it actually takes care of correlation between the particles. It is the Hartree Correction,

which clearly involves a deficit of one particle in calculating the potential acting on the particle under consideration. the physical meaning for this is the fact that the particle does not act on itself and the charge deficiency is once more one unit. This charge is generally rather localized around the particle under consideration.

In general the HF equations can be reqarged as ordinary Schroedinger equations for the motion of Fermi-particles, each particle moving in a slightly different potential field, which is computed by electrostatics from the particles of the system, corrected by the removal of an exchange charge, equal in magnitude to one particle whose motion is being investigated.

The HF equations furnish the best set of one-particle wavefunctions for use in a self consistent approximation to the problem of the motion of nucleons in the nucleus. The HF that yeilds a single-particle potential in which the nucleons move. The single potential is obtained through an iterative procedure, in which a first approximation is chosen for the wavefunction, the Hamiltonian is then calculated, and solved for new wavefunction. The iteration must clearly be continued until the new wavefunction reproduce the Hamiltonian from which they were derived than self consistency has been achieved. Once the self-consistent Hamiltonian is known, one can readily calculate the total energy and the density distrituions in the HF approximation.

The underlying aim in nuclear HF calculations is to reproduce self-consistently from an assumed basic phenomenological interaction the properties of known nuclei, such as energies, radii, spectra, magic numbers etc. If and when known nuclei can be understood in this self-consistent manner, one can then attempt to extrapolate to less well-known regions of the periodic table, such as neutron-rich or super heavy nuclei.

CHAPTER 5

COMPARISON OF RESULTS AND CONCLUSION

5.1 Comparison Of Results:

All the coefficients J.K,L, a_2 and Q appearing in equation (3.17) and (3.18) are obtained by fitting experimental masses with the DM mass formula. To make a comparison possible, these coefficients are calculated using a Hartree-Fock method for a phenomenological effective interaction chosen in the form of a four-parameter velocity dependent Yukawa Potential. The interaction is written as

$$V = -2V_{\ell,u} \frac{e^{-r/r_n}}{r/r_n} \left[1 - \left(\frac{P}{b}\right)^2\right] \quad \ldots 5.1$$

The factor 2 takes care of spin dependence of the particles V_ℓ,V_u, r_n and b are the four adjustable parameters in the interaction.

V_ℓ and V_u are the stength of the interaction between like partciles (neutron-neutron, protom proton) and unlike unlike particles (neutron-Proton) respectively. r_n is the range of the Yukawa force r is the distance between the particles P, is the magnitude of the relative momentum of the particles.

b is the critical value of the relative momentum at which the attractive force (whose strength decreases with increasing relative momentum) vanishes and beyond which the force becomes repulsive Hartree-Fock distribution always show fluctuations, especially close to the origin so that it is not immediately clear how to extract unambiguos values

for R_p and R_n. This problem was solved by fitting a simple two-parameter Fermi function written as

$$\rho(r) = \frac{\rho_o}{1+\exp\left[\frac{r-c}{a_o}\right]} \quad \ldots\ldots\ldots \quad (5.2)$$

where C is the "half density" radius

a_o surface thickness parameter

Some of the coefficients like J, K, L and r_o are calculated by first considering an infinite nuclear matter since these parameters are properties of the bulk of the system. By infinite nuclear matter, we mean a system which is isotropic i.e. there being no preferred direction. There can be no spin-orbit term and define surface for such a system because density is constant. The calculation is then repeated for a semi-infinite nuclear matter to obtain a_2 and Q. The coefficients a_2 and Q are properties at the surface of the nucleus. The semi- infinite nuclear matter consists of a medium in three directions (x,y,z). In two of its directions (x,y) it is of infinite extent and constant density, but has a well defined surface perpendicular to the Z - axis, so that the density falls to zero for large positive values of Z, while for large negative values of Z the local properties of the medium approach those of ordinary infinite nuclear matter.

The DM parameters obtained for this interaction

in the HF calculation and the one obtained for the mass data fitting are given in Table 5.1 below :

TABLE 5.1

DROPLET-MODEL PARAMETERS.

	HF	MASS - DATA
a_2	15.986 Mev	15.96 Mev
a_2	20.760 "	20.69 "
Q	17.04 "	17 "
K	240 "	240 "
L	123.53 "	100 "
M	0 "	0 "
r_o	1.14 fm	1.18 fm

The sharp radii R_N and R_z are then obtained by substituting the values for the above parameters of the HF fitting into their expressions for a given nucleus which can then be compared with the value given directly by the HF calculation on the same nucleus using the same force. The possibility of making such a comparison is of considerable interest in view of the discrepancy noted by Pearson (1980) between the value of the charge radius constant r_o given by the DM and value given by the HF method when they are fitted to the data (charge distribution and masses). The values quoted for the two DM mass formula of Myers (1976) and Von Groote et al (1976) are r_o = 1.18 and 1.167 fm respectively. As for the HF method the large number of phenomenologically successful effective interactions (like Gogny force, Skyrme- Osray

Qualitatively, inspection of equations (3.20) and (3.18) indeed suggest that it should be possible to change r_o while maintaining a fit to the sharp radii R_z, provided K is changed: By so doing the binding energies are little affected by the choice of K (Goote, 1979). However quantitatively this is impossible. To see this, let us consider the case of ^{208}pb, for which $R_z A^{-1/3}$ is 1.13 fm experimentally (Myers, 1977). Thus in order to bring r_o into accord with its HF value we require that the correction factor in equation (3.20) be effectively equal to one. Since $\bar{\delta}$ is essentially smaller than one for heavy nuclei it follows that $\bar{\varepsilon}$ would have to be positive. But these two parameters $\bar{\delta}$ and $\bar{\varepsilon}$ are negative from the atomic and nuclear datas of Myers (1976) and Griite et al (1976) and for no reasonable change of these parameters can this sign be reversed.

Thus no matter how K is varied there is no hope of reducing the DM value of r_o to its HF value. In fact there is this fundamental difference between the two, that in the HF case the bulk density of ^{208}pb corresponds to a dilatation of infinite nuclear matter ($\bar{\varepsilon} \geq 0$) while according to the DM there is a compression ($\bar{\varepsilon} < 0$) (Pearson, 1979).

As concrete examples for the comparison we chose doubly magic nuclei. This is to eliminate shell effect which are not included in the DM, whereas they are

automatically included in the HF method. The values of R_z and R_N so obtained are listed table 5.2 below:

TABLE 5.2

NUCLEUS		DM		HF	
		R_z (fm)	R_N (fm)	R_z (fm)	R_N (fm)
40	C_a 20	3.611	3.5998	3.874	3.826
48	C_a 28	3.791	3.9990	4.001	4.206
78	C_a 50	4.263	4.5860	4.356	4.814
48	N_i 20	4.021	3.901	4.271	4.011
56	N_i 28	4.172	4.160	4.394	4.267
75	N_i 50	4.563	4.763	4.659	4.889
90	Z_r 50	4.879	4.961	5.042	5.155
100	S_n 50	5.138	5.122	5.372	5.281
132	S_n 82	5.121	5.172	5.635	5.798
208	P_b 126	6.557	6.710	6.718	6.811
260	P_b 184	7.053	7.370	6.069	7.134

5.2 Discussion and Conclusion.

The results in Table 4.2 show equivalent sharp radii of a number of nuclei that have been calculated by the DM, using parameters that have been calculated by the HF method with a given effective force (Yukawa force). They are found to be systematically smaller than the sharp radii given by direct HF calculation on the nucleus in question with the same force. This can be as a result of nuclear matter being squeezed by the surface tension

much more in the DM than in the HF method. A similar observation has been made ealier by Pearson (1982) but using a different effective interaction (Gogny force). According to Pearson (1982) it was not clear whether this phenomenon was not model dependent, hence in this work the calculation is repeated using a different interaction (Yukawa potential). The DM fit requires a larger value of r_o in order to compensate for the tendency of the DM to give smaller radii. But this bears a strong correlation with the compressibility K.

In order to reduce the discrepany in r_o without necessarily changing K, higher order terms in taylor series expansion for the energy are now introduced. These are the terms in parenthesis in equation (3.13c) and (3.13d).

The inclusion $D\varepsilon^2$ in equation (3.13d) is considered as a kind of reducing K thus

$$AK' \rightarrow AK + 4\Pi r_o^2 A^{2/3} D \quad \ldots\ldots\ldots \quad (5.6)$$

$$K' = K + 4\Pi r_o^2 A^{-1/3} D \quad \ldots\ldots\ldots \quad (5.7)$$

This new value K' is used to determine a new equilibrium value for ε, so that on minimizing the energy, one obtains

$$\varepsilon \overset{\sim}{=} \varepsilon_o \left(1 - \frac{B}{2K'} \varepsilon_o \right) \quad \ldots\ldots\ldots\ldots \quad (5.8)$$

with $\varepsilon_o = \frac{1}{K'} \left(- 2a_2 A^{-1/3} + L\bar{\delta}^2 + \frac{3e^2 Z^2}{5r_o A^{-1/3}} \right)$

On refitting the nuclear masses, we found B = 141 Mev and D = - 38 Mev fm^{-2}

For lighter nuclei a very serious problem will arise, since in the expression for the new ε there is a singularity

occuring at A ≅ 7 for the present value of D. Conceivably our value for D is too large and is negative, so that the inclusion of this term cannot possibly remove the anomaly. Further more the existence of the singularity in ε even if it is located at A-values for which one does not normally use the DM, raises serious question as to the convergence of the expansion in powers of $A^{-1/3}$.

To conclude it was found that there are certain systematic differences between the HF method and the DM in so far as as nuclear radii are concerned. Inclusion of higher-order terms in the DM expansion helps in no way and infact reveals serious problems concerning the convergence of the DM expansion.

Further comparisons of these results could be enhanced by considering the Thomes-Fermi (TF) model. This is a semiclassical model, which is simpler than the HF method yet more rigorous than the DM. This is however left for the future. The TF approximation in nuclei has some how been discussed by Bhaduri (1977) and by Grammaticos and Voros (1979; 1980).

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APPENDIX I

The Derivation Of The Dimensionless Neutron Skin Thickness τ.

The effective neutron skin thickness is defined as

$$t = R_n - R_z \quad \ldots\ldots\ldots \quad \text{(i)}$$

Its relation to $\bar{\delta}$ is derived by defining the volume of the effective neutron skin as

$$\frac{4}{3}\pi\left(R_n^3 - R_z^3\right) \quad \text{------} \quad \text{(ii)}$$

From equation (3.12b)

$$R_n = r_o(2N)^{1/3}\left[1 + \bar{\varepsilon} - \frac{1}{3}\bar{\delta}\right]$$

and from (3.12c)

$$R_z = r_o\,(2\,z)^{1/3}\left(1 + \bar{\varepsilon} + \frac{1}{3}\bar{\delta}\right)$$

Equation (ii) can be rewritten as

$$\frac{4}{3}\pi(R_n^3 - R_z^3) = \frac{4}{3}\pi\left(R_n - R_z\right)\left(R_n^2 + R_n R_z + R_z^2\right)$$

$$\frac{4}{3}\pi\left(R_n^3 - R_z^3\right) = \frac{4}{3}\pi t\left(R_n^2 + R_n R_z + R_z^2\right) \ldots\ldots \quad \text{(iii)}$$

But

$$R_n^2 = \left[r_o\,(2N)^{1/3}\left[1 + \bar{\varepsilon} - \frac{1}{3}\bar{\delta}\right]\right]^2$$

$$R_n^2 = r_o^2(2N)^{2/3}\left[1 + \bar{\varepsilon} - \frac{1}{3}\bar{\delta}\right]^2$$

For nuclei having the same number of protons as neutrons

$2N = A$

$$R_n^2 = r_p^2\,(A)^{2/3}\left[1 + \bar{\varepsilon} - \frac{1}{3}\bar{\delta}\right]^2$$

Doing a binomial expansion

$$R_n^2 = r_o^2\,(A)^{2/3}\left[1 + 2\bar{\varepsilon} - \frac{2}{3}\delta - \frac{2}{3}\bar{\varepsilon}\bar{\delta} + \bar{\varepsilon}^2 + \frac{1}{9}\bar{\delta}^2\right]$$

$$\frac{R_u^3}{R^3} = \frac{3N}{4\Pi\,\bar{\rho}_n} \times \frac{4\Pi\rho}{3A} = \frac{N/A}{\bar{\rho}_n/\bar{\rho}}$$

$$\frac{R_n^3}{R^3} = \frac{3Z}{4\Pi\,\bar{\rho}_z} \times \frac{4\Pi\rho}{3A} = \frac{Z/A}{\bar{\rho}_z/\bar{\rho}}$$

$$\frac{R_n^3}{R^3} - \frac{R_z^3}{R^3} = \frac{N/A}{\bar{\rho}_n/\bar{\rho}} - \frac{Z/A}{\bar{\rho}_z/\bar{\rho}}$$

therefore

$$\frac{3t}{R} \simeq \frac{R_n^3}{R^3} - \frac{R_z^3}{R^3}$$

$$\frac{3t}{R} = \frac{N/A}{\bar{\rho}_n/\bar{\rho}} - \frac{Z/A}{\bar{\rho}_z/\bar{\rho}}$$

$$\frac{3t}{R} = \frac{1+I}{1+\bar{\delta}} - \frac{1-I}{1-\bar{\delta}} \quad \ldots\ldots (v)$$

where $I = \frac{N-Z}{A}$

and $\bar{\delta} = \frac{2\bar{\rho}_n}{\rho} - 1$, or $\bar{\delta} = 1 - \frac{2\bar{\rho}_z}{\rho}$

Solving equation (v)

$$\frac{3t}{R} = \frac{(1+I)(1-\bar{\delta}) - (1-I)(1+\bar{\delta})}{(1+\bar{\delta})(1-\bar{\delta})} = \frac{2(1-\bar{\delta})}{(1-\bar{\delta})(1+\bar{\delta})}$$

$$\frac{3t}{R} = \frac{2}{(1+\bar{\delta})} = 2(1+\bar{\delta})^{-1} \simeq 2(1-\bar{\delta})$$

$$\frac{t}{R} = \frac{2}{3}(1-\bar{\delta})$$

$$\frac{r}{r_c A^{1/3}} \simeq \frac{2}{3}(1-\bar{\delta}) \quad \text{where } R = r_o A^{1/3}$$

$$\frac{t}{r_o} \simeq \frac{2}{3}(1-\bar{\delta})A^{1/3} \quad \ldots\ldots (vi)$$

The dimentionless thickness is defined as

$$\tau = t/r_o$$

Hence equation (vi) becomes

$$\tau \simeq \frac{2}{3}(1 - \bar{\delta})A^{1/3} \quad . \quad . \quad . \quad . \quad (3.6)$$

APPENDIX II

The Taylor Series Expansion Of The Energy Per Particle (e)

$$e(\varepsilon, \delta) = e(_0) + \frac{\partial e}{\partial \varepsilon}\varepsilon + \frac{\partial e}{\partial \delta}\delta + \left[\frac{1}{2!}\frac{\partial^2 e}{\partial \varepsilon^2}\right]_{a_o} \varepsilon^2$$

$$+ \frac{1}{2!}\frac{\partial^2 e}{\partial \varepsilon \partial \delta}\varepsilon\,\delta + \left[\frac{1}{2!}\frac{\partial^2 e}{\partial \delta^2}\right]_{\delta,o} \delta^2$$

$$+ \frac{1}{3!}\frac{\partial^3 e}{\partial \delta \partial \varepsilon^2}\delta\varepsilon^2 + \frac{1}{3!}\frac{\partial^3 e}{\partial \delta^2 \partial \varepsilon}\delta^2\varepsilon$$

$$+ \left[\frac{1}{3!}\frac{\partial^3 e}{\partial \varepsilon^3}\varepsilon^3\right]_{\varepsilon=o} + \left[\frac{1}{4!}\frac{\partial^4 e}{\partial \delta^4}\delta^4\right]_{\delta=o}$$

This expansion can again be written as

$$e(\varepsilon,\delta) = -a_1 + J\delta^2 + \frac{1}{2}K\varepsilon^2 - L\varepsilon\delta^2 + \frac{1}{2}M\delta^4 + \frac{1}{6}B\varepsilon^3$$

where

$\frac{1}{2!}\frac{\partial^2 e}{\partial \varepsilon^2} = \frac{1}{2}K$, K is the nuclear compressibility

$\frac{1}{2!}\frac{\partial^2 e}{\partial \delta^2} = J$, J is the volume symmetry coefficient.

$\frac{1}{3!}\frac{\partial^3 e}{\partial \delta^2 \partial \varepsilon} = -L$; L is the density-symmetry coefficient.

$\frac{1}{4!}\frac{\partial^4 e}{\partial \delta^4} = \frac{1}{2}M$, M is the symmetry on harmonicity coefficient.

$\frac{1}{3!}\frac{\partial^3 e}{\partial \varepsilon^3} = \frac{1}{6}B$ B is related to the nuclear compressibility.

The energy is a symmetry function of δ by virtue of the symmetry between neutrons and protons, therefore the expansion has been carried out in powers of ε and δ^2 (rather than δ).

On the other hand there is no term linear in ϵ in the equation, since the energy per particle e is stationary with deviation of the density from its standard matter value ρ_o.

The Taylor Series Expansion of The Surface Energy.

$$4\pi r_o^2 \sigma(\tau, \delta_s, \epsilon_s) = \sigma(_0) + \frac{\partial\sigma}{\partial\tau}\tau + \frac{\partial\sigma}{\partial\delta_s}\delta_s + \frac{\partial\sigma}{\partial\epsilon}\epsilon_s + \frac{1}{2!}\frac{\partial^2\sigma}{\partial\tau^2}\tau^2 + \frac{1}{2!}\frac{\partial^2\sigma}{\partial\delta_s^2}\delta_s^2 + \frac{1}{2!}\frac{\partial^2\sigma}{\partial\tau\partial\delta_s}\tau\delta_s + \frac{1}{2!}\frac{\partial^2\sigma}{\partial\epsilon_s^2}\epsilon_s^2$$

where

$$\frac{1}{2!}\frac{\partial^2\sigma}{\partial\tau^2} = H$$

$$\frac{1}{2!}\frac{\partial^2\sigma}{\partial\delta_s^2} = -G$$

$$\frac{1}{2!}\frac{\partial^2\sigma}{\partial\tau\partial\delta_s} = 2P$$

$$\frac{1}{2!}\frac{\partial^2\sigma}{\partial\epsilon_s^2} = D\epsilon^2$$

H, P, G specify the dependence of the surface energy on τ, the neutron skin thickness and on δ_s, the bulk value of the bulk asymmetry (δ) extrapolated to the mean surface I at r = R. D is the nuclear compressibility.

APPENDIX III

Derivation Of The Expressions For Both ε And δ.

The energy of the nucleus as a function of $\bar{\varepsilon}$ and $\bar{\delta}$ and is a functional of $\tilde{\varepsilon}$ and $\tilde{\delta}$ is written as

$$E(\bar{\varepsilon},\bar{\delta};\tilde{\varepsilon},\tilde{\delta}) = \tfrac{1}{2}K\bar{\varepsilon}^2A - L\bar{\varepsilon}\bar{\delta}^2A + a_2A^{2/3}(1+2\bar{\varepsilon}) - C_1\frac{Z^2}{A^{1/3}}\bar{\varepsilon}$$

$$+ A\left[J\bar{\delta}^2 - L\bar{\varepsilon}\bar{\delta}^2 + \tfrac{1}{2}M\bar{\delta}^4\right] + A^{2/3}\left[H\tau^2 + 2P\tau(\bar{\delta}+\tilde{\delta}_s)\right] - G(\bar{\delta}+\tilde{\delta}_s)^2A^{2/3} - C_1\frac{Z^2}{A^{1/3}}\frac{\bar{\delta}}{3}$$

$$+ \frac{1}{2}\iiint_{R_o}\left[K\tilde{\varepsilon}^2 - 3ev\,\tilde{\varepsilon}\right]\rho_o$$

$$+ \iiint_{R_o}\left[J\tilde{\delta}^2 - \frac{ev}{2}\rho_o\,\tilde{\delta}\right]$$

$$- a_1A + a_3A^{1/3} + C_1\frac{Z^2}{A^{1/3}}\left(1+\frac{I}{3}\right) - C_3\frac{Z^2}{A} - C_4\frac{Z^{4/3}}{A^{1/3}} \quad \ldots\ldots \quad (3.15)$$

The condition for stability of the nucleus is that the energy (equation (3.15)) of it should be minimum. Minimization of the energy is equivalent to the first differential of the energy being zero. That is,

$$\partial E(\bar{\varepsilon},\ \bar{\delta};\ \tilde{\varepsilon},\ \tilde{\delta}) = \left[\frac{\partial E}{\partial\bar{\varepsilon}}\ ,\ \frac{\partial E}{\partial\bar{\delta}}\ ;\ \frac{\partial E}{\partial\tilde{\varepsilon}}\ ,\ \frac{\partial E}{\partial\tilde{\delta}}\right] = 0\ .$$

Differentiating line (1) with respect to $\bar{\varepsilon}$

$$\frac{\partial E}{\partial\bar{\varepsilon}} = K\bar{\varepsilon}A - L\,\bar{\delta}^2A + 2a_2A^{2/3} - C_1\frac{Z^2}{A^{1/3}}$$

Differentiating line (2) with respect to $\bar{\delta}$

$$\frac{\partial E}{\partial\bar{\delta}} = A\left[2J\bar{\delta} - 2L\bar{\varepsilon}\bar{\delta} + 2M\bar{\delta}^3\right] - 2G(\bar{\delta}+\tilde{\delta}_s)A^{2/3} - \frac{C_1Z}{3A^{1/3}} + 2P\tau A^{2/3} \quad \ldots\ldots \quad (i)$$

In differentiating the remaining part of line (2) and noting that the dimentionless skin thickness τ is related to $\bar{\delta}$ by $\tau = \frac{2}{3}(I-\bar{\delta})A^{1/3} = \frac{2I}{3}A^{1/3} - \frac{2A^{1/3}}{3}\bar{\delta}$

$$\frac{\partial\tau}{\partial\bar{\delta}} = \frac{-2A^{1/3}}{3}$$

The coefficients of $\partial\tilde{\varepsilon}$ and $\partial\tilde{\delta}$ may be set equal to zero directly, but particle conservation must be ensured for the variations associated with $\partial\tilde{\varepsilon}$ and $\partial\tilde{\delta}$. Thus instead of setting the coefficients of $\partial\tilde{\delta}$ equal to zero we write

$$K\tilde{\varepsilon} - \frac{3}{2}ev = \text{constant}$$

$$2J\tilde{\delta} - \frac{1}{2}ev = \text{constant}$$

where the constants are Langrange multipliers which can be determined by taking the average of each equation. Since the averages of $\tilde{\varepsilon}$ and $\tilde{\delta}$ are zero the constants are found to be equal to

$$-\frac{3}{2}e\bar{v} \text{ and } -\frac{1}{2}e\bar{v}$$

The coefficient of $\partial\bar{\delta}$ from equation (i) is given as

$$\left[2J\bar{\delta} - 2L\bar{\varepsilon}\bar{\delta} + 2M\bar{\delta}^3\right]A + \left[\left[2H\tau + 2P\left(\delta + \tilde{\delta}_s\right)\right]\frac{d\tau}{d\bar{\delta}}\right.$$

$$\left. - 2G\left(\bar{\delta} + \tilde{\delta}_s\right) + 2P\tau\right]A^{2/3} - C_1\frac{Z}{3A^{1/3}}$$

From equation (3.6)

$$\tau = \frac{2}{3}\left(1 - \bar{\delta}\right)A^{1/3}$$

therefore $\frac{d\tau}{d\bar{\delta}} = -\frac{2}{3}A^{1/3}$

Substituting for $\frac{d\tau}{d\bar{\delta}}$ into the coefficient of $\partial\bar{\delta}$

$$\left[2J\bar{\delta} - 2L\bar{\varepsilon}\bar{\delta} + 2M\bar{\delta}^3\right]A + \left[-\left[2H\tau + 2P\left(\bar{\delta} + \tilde{\delta}_s\right)\right]\frac{2}{3}A^{1/3}\right.$$

$$\left. - 2G\left(\bar{\delta} + \tilde{\delta}_s\right) + 2P\tau\right]A^{2/3} = C_1\frac{Z^2}{3A^{1/3}}$$

which can be rewritten as

$$\left[2J\bar{\delta} - \frac{4}{3}H\tau - \frac{4}{3}P\left(\bar{\delta} + \tilde{\delta}_S\right)\right]A - \frac{1}{3}C_1\frac{Z^2}{A^{1/3}} \qquad \ldots\ldots \text{(ii)}$$

$$+\left[2P\tau - 2G\left(\bar{\delta} + \tilde{\delta}_S\right)\right]A^{2/3} - 2LA\bar{\varepsilon}\bar{\delta} + 2MA\bar{\delta}^3$$

In taking variations around stationary values of the energy, the equations determining the optimum values of the variables need be of lower order terms than the energy equation itself. Consequently the second line of equation (ii) is neglected because of higher order terms.

The first term in the second line is smaller by $A^{-1/3}$ the second term by $\bar{\varepsilon}$ and the last term by $\tilde{\delta}^2$, than the corresponding terms in the first line.

The vanishing coefficinet of $\partial\bar{\delta}$ can then be written as

$$2J\bar{\delta} - \frac{4}{3}H\tau - \frac{4}{3}P\left(\bar{\delta} + \tilde{\delta}_S\right) - \frac{1}{3}C_1\frac{Z^2}{A^{4/3}} = 0 \qquad \ldots\ldots \text{(iii)}$$

The vanishing coefficient of $\partial\bar{\varepsilon}$ in equation (i) gives

$$K\bar{\varepsilon} + 2a_2A^{-1/3} - L\bar{\delta}^2 - C_1\frac{Z^2}{A^{4/3}} = 0 \qquad \ldots\ldots\ldots \text{(iv)}$$

From (iv)

$$\bar{\varepsilon} = \frac{1}{K}\left(-2a_2A^{-1/3} + L\delta^2 + \frac{3e^2}{5r_o}\frac{Z}{A^{4/3}}\right)$$

which gives equation (3.18).

Equation (iii) can be rewritten as

$$\left(2J - \frac{4}{3}P\right)\bar{\delta} - \frac{4}{3}H\tau - \frac{4}{3}P\bar{\delta}_S - \frac{1}{3}C_1\frac{Z^2}{A^{4/3}} = 0$$

$$\left(2J - \frac{4}{3}P\right)\bar{\delta} = \frac{4}{3}H\tau + \frac{4}{3}P\bar{\delta}_S + \frac{1}{3}C_1\frac{Z^2}{A^{4/3}}$$

But $\tau = \frac{2}{3}\left(I - \bar{\delta}\right)A^{1/3}$

$$\left(2J - \frac{4}{3}P\right)\bar{\delta} - \frac{4}{3}H\tau = \frac{4}{3}P\bar{\delta}_S + \frac{1}{3}C_1\frac{Z^2}{A^{4/3}}$$

$$\left(2J - \frac{4}{3}P\right)\bar{\delta} - \frac{4}{3}H\left(\frac{2}{3}(I-\bar{\delta})\right)A^{1/3} = \frac{4}{3}P\bar{\delta}_s + \frac{1}{3}C_1\frac{Z^2}{A^{4/3}}$$

$$\left(2J - \frac{4}{3}P + \frac{8HA^{1/3}}{9}\right)\bar{\delta} = \frac{8HIA^{1/3}}{9} = \frac{4}{3}P\bar{\delta}_s + \frac{1}{3}C_1\frac{Z^2}{A^{4/3}}$$

$$\left(2J - \frac{4}{3}P + \frac{8HA^{1/3}}{9}\right)\bar{\delta} = \frac{8HIA^{1/3}}{9} + \frac{4}{3}P\bar{\delta}_s + \frac{1}{3}C_1\frac{Z^2}{A^{4/3}}$$

Multiply throughout by $\left(\frac{9}{8HA^{1/3}}\right)$

$$\left(\frac{9J}{4HA^{1/3}} - \frac{3P}{2HA^{1/3}} + 1\right)\bar{\delta} = I + \frac{3P\bar{\delta}_s}{2HA^{1/3}} + \frac{3}{8}\frac{C_1 Z^2}{HA^{5/3}}$$

$$\bar{\delta} = \frac{I + \frac{3P\bar{\delta}_s}{2HA^{1/3}} + \frac{3}{8}\frac{C_1 Z^2}{HA^{5/3}}}{1 + \frac{9J}{4HA^{1/3}} + \frac{3P}{2HA^{1/3}}}$$

But $\bar{\delta}_s = -\frac{C_1}{6J}\frac{Z^2}{A^{4/3}}$

$$\bar{\delta} = \frac{I + \frac{3}{8}\frac{C_1 Z}{A^{5/3}H} + \frac{3}{2}\frac{P}{H}A^{-1/3}\left(-\frac{C_1 Z^2}{6JA^{4/3}}\right)}{1 + \frac{9}{4}\frac{J}{H}A^{-1/3} - \frac{3}{2}\frac{P}{H}A^{-1/3}}$$

$$\bar{\delta} = \frac{I + \frac{3}{8}\frac{C_1}{H}\frac{Z^2}{A^{5/3}} - \frac{1}{4}\frac{Z^2 PC_1}{JHA^{5/3}}}{1 + \frac{9}{4}\frac{J}{H}A^{-1/3} - \frac{3}{2}\frac{P}{H}A^{-1/3}}$$

$$\bar{\delta} = \frac{I + \left[1 - \frac{2}{3}\frac{P}{J}\right]\frac{3C_1 Z^2}{HA^{5/3}}}{1 + \left[1 - \frac{2}{3}\frac{P}{J}\right]\frac{9}{4}\frac{JA^{-1/3}}{H}}$$

$$\bar{\delta} = \frac{I + \left[\frac{1 - \frac{2}{3}\frac{P}{J}}{H}\right]\frac{3C_1 Z^2}{8A^{5/3}}}{1 + \left[\frac{\left(1 - \frac{2}{3}\frac{P}{J}\right)}{H}\right]\frac{9J}{4}A^{-1/3}}$$

The coefficient Q is given as

$$Q = \frac{H}{1 - \frac{2}{3}\frac{P}{J}}$$

Substituting for the term in the bracket in the expression for $\bar{\delta}$ above

$$\bar{\delta} = \frac{I + \frac{3}{8}\frac{C_1}{Q} Z^2 A^{5/3}}{1 + \frac{9}{4}\frac{J}{Q} A^{-1/3}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3.17)$$