LEVEL OF GEOMETRICAL THINKING OF NIGERIAN SENIOR SECONDARY SCHOOL STUDENTS

BY

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DECLARATION

I, Maruf, O. Ibrahim, do solemnly declare that this thesis entitled "Level of Geometrical thinking of Nigerian Senior Secondary Students" has been written by me and it is a record of my own research work. It has never been presented in any previous thesis. All quotations are indicated by quotation marks or indentations and the sources of literature are duly acknowledged in the references.

Maruf, O. Ibrahim

CERTIFICATION

The undigned do hereby certify that they have read, and recommended the Faculty Board of Education, their acceptance of this thesntitled "Level of Geometrical Thinking of Nigeria Senior Secory Students" submitted by Maruf, Olasunkkanmi Ibrahim in pial fulfilment of the requirements for the award of degree ofster of Education in Mathematics Education.

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This research work is dedicated to all those who have contributed in one way or the other to my educational success.

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ABSTRACT

The purpose of this study was to identify level of geometrical thinking of Nigerian Senior Secondary School Students using modified Van Hiele's Theory.

Van Hiele's theory stated that there are five sequential levels of geometrical thinking and that no learner can attain a higher level until the preceding lower level has being mastered.

For this study, the country was clustered into four regions, and a state with its capital was randomly drawn from each region. In all, there was a total of 504,320 students of SS3 for the year 1997 (SSCE 1997 Statistics), out of which 188 students were randomly drawn based on Roscoe's (1975) recommendation for sample size.

Four hypotheses were formulated to test whether or not there exist significant relationship between students achievement in lower level test and higher level test, to test the existence or non existence of significant difference din performance by gender and science or art students. Also tested was existence or non-existence of significant difference in students performance in the five geometrical topics treated.

Findings of the study revealed that:

- Significant relationship existed between achievement of students in lower level test and higher level test.
- There was significant difference in achievement between male and female students in the two tests.
- There was significant difference in achievement between science and art students in the two tests.

- Finally, the study shows that the students operate at lower level of thinking and they understand task on parallelogram better than they do for properties and tangents to circle; similarity and congruency; triangle, and circle and cyclic quadrilaterals.

The justification for this study lies on the fact that the test item adopted for the study would serve as a model for diagnosing learning difficulty students face in geometry and appropriate measure of remediation can be taken by placing them at their appropriate levels of thinking as recommended by this study.

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DEFINITION OF TERMS

For the purpose of this study the following terms are defined as below.

Geometry is that aspect of mathematics which deals with the study of shapes.

Geometrical Thinking is the process through which a student solves geometrical problems.

Geometrical Understanding is the ability to response correctly to questions on geometry.

Van Hiele's Theory: This is a proposition formulated by Van Hiele in which he stated that there exist five sequential levels of geometrical thinking.

Modified Van Hiele's Theory: This is the grouping of the five levels of geometrical thinking into two classes viz lower level and higher level.

ABBREVIATION OF TERMS

1. PG = Parallelograms
2. TG = Triangles
3. SC = Similarity and Congruency
4. CC = Circles and Cyclic Quardrilateral
5. PT = Properties and Tangent to Circles
6. LLT = Lower Level Test
7. HLT = Higher Level Test
8. CLT = Combined Level Test

CHAPTER ONE

1.0. THE PROBLEM

1.1. INTRODUCTION

The role of mathematics in scientific and technological development has enhanced the importance of learning and teaching of the subject. According to Iransoro (1987), the use of modern science, which is the desire of most countries for the development of egalitarian and technological societies and other desirable aspirations compel us to accord increasing weight to the teaching and learning of mathematics. Howe (1992), remarked that there is general belief and acceptance that mathematical methods are central solution of all kinds of problem. Howe (1992), further pointed out that parents seek for appropriate grades for their children, employers seek candidates with the requisite mathematical skills, social scientists and physical scientists seek for ready made mathematical models, while politicians and planners seek instant solution based on mathematical models. There is hardly any field where mathematics is not useful. Farmers, hunters, housewives and many others use mathematics in their daily life activities.

In Nigeria, priority is accorded to the learning and teaching of mathematics in the school curriculum (NPE, 1991). The National Policy on Education emphasizes the need to equip students with knowledge that will enable them to effectively live in a modern age of science and technology. In spite of the importance of this subject, over the years, unprecedented number of students fail the subject especially in the senior secondary school examination conducted by West African Examination Council. The Chief Examiner's Report by WAEC (1993) shows that students performance in the subject is too poor. He stated that the 1992 senior secondary school certificate examination result showed that mathematics recorded the lowest grades of failures when compared with other subjects.

Public symposia, deliberation at workshops, slogans like war against poor achievement in mathematics, negative reports from the examiners on the mathematics results of GCE, SSCE, JAMB, IJMB, and many examinations of the like are pointers to show that something is wrong with the learning and teaching of the subject.

Whether mathematics is difficult or not, from experience, derived from teaching, the researcher notes that most students certainly regard it as difficult. If secondary school student are asked to sum up their view of the subject, many would describe it in terms of one of the three "ds": dull, difficult and disliked. This group of students devise various means of avoiding mathematics lesson.

Unfortunately; geometry, the most practical and visualisable aspect of mathematics is the area of mathematics that many students find difficult (Ale, 1989). Lassa (1986) asserted that when examining the performance of the students based on the various aspect of mathematics viz. number system, measurement, algebraic processes, geometry and statistics ... Geometry was the least understood, followed by statistics and measurement.

There are other evidences in the literature which show that support Lassa's (1986) assertion that geometry is the branch of mathematics many students dislike most. Megarvey (1981) reported that deductive proofs have the most powerful tool of mathematical thinking, and that the deductive proof approach to the development of concept is however, not pedagogically sound, especially at the secondary school level. Roffer (1981), stated that when first year students of the Oregon University were asked to list mathematical topics they liked best and topics they liked least, in the pre-university classes, the topic that most of the students disliked was the high school geometry.

Similarly, Korau (1995), Inekwe (1996) and the Chief Examiner's Report for WAEC/SSCE examination (1995, 1996) have shown that performance of students in mathematics generally and geometry in particular is lower than other aspects of mathematics such as statistics, algebra, etc.

The general low interest and poor performance in geometry is giving educators, researchers and the public great concerns. Lassa (198[illegible]), Ale (1986). described that situation of mathematics education in Nigeria as being in a sorry state. Iransoro (1987) reported that various reports on school mathematics in Nigeria indicate that the students level of geometrical thinking is low, and that few researches have been carried out in geometry. To the best knowledge of this researcher, effort have been made to solve the problem. Angel and his associates (1991), Kankia (1995), Tuokwa (1996) have done work in the area and made a number of recommendations on how to resolve the problem yet the problem persist. It is the belief of the present researcher that more researches on geometrical understanding of SS3 Nigerian students would in no small way alleviate the problem of phobia of geometry and mathematics.

There is no gain saying that some geometry contents is too abstract; but then the causes of students poor performance could be traced to carefree and nonchalant attitude on the part of the students and negligence on the part of their teachers (Iuekwe, 1984). However, Van Hiele (1959), Iransoro (1987) and Angel, et. al. (1991) attributed the cause of students poor performance in geometry to lack of coordination between level of teachers' teaching and students level of understanding, the stand which the present researcher holds.

Van Hiele (1959), proposed a theory for the development of geometrical thinking that identified five levels of thinking, ordered sequentially so that students can move from one level of thinking to the next as their capability increased.

Van Hiele (1959) maintained that the levels are suitable for assessing students levels of geometrical thinking and that the levels form a hierarchy so that a student can only attain a higher level, if he or she has mastered materials at the lower level. According to him, the first level "level 0" deals with recognition of geometrical shapes; the second level (level 1) deals with analysis of properties that form geometrical shapes, the third level (level 2) deals with classification of the properties, the fourth level (level 3) deals with deductive organization of the properties of the geometrical shapes. While the fifth level (level 4) deals with abstract interpretation of geometrical properties. Some researches have shown that the fifth level of Van Hiele's theory is above the scope of secondary school students (Angel et. al., 1991, Iranoro 1987, May-Berry 1983). The present researcher agrees with these fact and therefore used the first four levels of the theory to identify the level average Nigerians senior secondary school students attained. This study, it is believed, would create and develop students interest in the learning of the subject, geometry cum mathematics, while teachers and other educators would gain a focus as to the aspect of the subject to give more attention in their teaching. Identifying the level of geometrical thinking many of the students attained would nveriably expose their deficient levels of thinking, so that effort can be geared towards remedying the problem areas.

1.2 Statement of the Problem

There are lapses in coordination between the teaching and learning of mathematics in general and geometry in particular (Obioma, 1985; Iransoro, 1987; Ezike 1989). For sometime now, the failure rate of students in SSCE mathematics examination is generally high (WAEC, 1996; ALIO, 1997; Anekwe, 1997, and geometry is reported to be the most difficult aspect of it (Iransoro, 1987; Korsu, 1995; Inekwe, 1996). Many researchers attributed the cause of this high rate of failure to among others negative attitude of the students towards the subject, method of teaching, difficulty associated with the specialized language of the subject, students lack of necessary skills for the problem solving process and many more (Ozoro, 1973; Adeniyi, 1988; Osibodu, 1988 and Alio, 1997). Very few researchers made mention of lack of coordination between levels of teachers teaching and levels of students learning as the major factor contributing to students mass failure and generally high rate of failure in mathematics and geometry specifically (Van Hiele, 1959; Mayberry 1983; Iransoro, 1987; Angel et al, 1991; Inekwe 1996).

The researcher is of the view that lack of coordination between teaching and learning of geometry is the major factor causing students failure in geometry and hitherto mathematics. Hence the problem of this research lie in identifying the level of geometrical thinking students in Nigerian Senior Secondary School have attained, so that teachers teaching level can tally with students understanding level. It is the belief of this researcher that, by this study, the problem of high rate of students failure in mathematics may be resolved.

1.3. Research Questions

This study is aimed at answering the following research question:

1. What level of geometrical thinking do average Nigerian Senior Secondary School students possess?
2. How does gender affect levels of geometrical thinking of Nigerian Senior Secondary School students?
3. In what ways do subject grouping affect levels of geometrical thinking of Nigerian Senior Secondary School students?
4. How do students perform in the different topics in Geometry?

1.4. Research Hypotheses

The following null hypotheses were tested, based on:

1. HO 1: There is no significant relationship between students performance at the lower level test (LLT) and their performance at the higher level test (HLT), on a test of geometry.

2. HO 2(a): There is no significant difference between the means scores of male students' and their female counterparts at the lower level test (LLT).
3. HO 2(b): There is no significant difference between the mean scores of male students' and their female couterparts at the higher level test (HLT).
4. HO 3(a): There is no significant difference between the mean scores of Science students and those of Art students performance at the lower level test (LLT).
5. HO 3(b): There is no significant difference between the mean scores of Science students and those of the Art students at the higher level test (HLT).
6. HO 4(a): There is no significant difference between the mean scores of students for five topics at the lower level test (LLT).
7. HO 4(b): There is no significant difference between the mean scores of students in Van Hiele's five Higher level test (HLT).

1.5. Purpose of the Study

Nigeria may not develop unless efforts are geared towards having large number of indigenous scientists and mathematicians, to do this, there is need to expedite move to check the rate of failure of students in mathematics and geometry its basic tool. This study is a move towards improving the teaching and learning of geometry. Therefore the following purposes of the study are the basic steps to attain the move.

1. To identify level of geometrical thinking of Nigerian Senior Secondary School students.
2. To determine whether or not gender difference has effect on levels of geometrical thinking of the students.
3. to establish whether differences in subjects grouping (science and art) affect levels of geometrical thinking of the students.
4. to determine whether levels of geometrical thinking of the students differ for the various geometric topics under study.

1.6. Basic Assumptions.

The following assumptions have been made in this study.

1. That the test items developed for this study measure the first four levels of geometrical thinking of Senior Secondary School students.
2. That all the subjects used in this study possess all the prerequisite knowledge for the task prepared.
3. That all the subjects used in this study are exposed to good quality and adequate mathematics teachers, healthy academic environment and effective teaching.

1.7. Justification of the Study

Many countries in the world today are competing for supremacy, and the countries at the rear back of the race are looked down upon as third world or developing countries of which Nigeria is one, Huwe (1989).

There is no means by which mathematics in general and geometry in particular can be divorced from these developmental credos of a nation, Sherad (1981) defined geometry as the basic mathematics skills students need to learn in order to function effectively in the three dimensional world, NPE (1991), emphasised the need to equip students with knowledge of mathematics so as to enable them live effectively in modern age of science and technology. The improvement of teaching and learning of mathematics and geometry will enhance availability of indigenous experts who will be able to solve Nigerian problems.

Judging from the importance of mathematics to technological development of a country, the lapses in coordination between teachers' teaching level and students understanding level is greatly hampering the full realisation of National Development objectives. Therefore, identifying level of geometrical thinking of Nigerian Senior Secondary School students, the potential labour force of the country, would in no small amount alleviate the problem of mass failure of students in the subject and hence make the subject interesting and exciting to the students. Here in lies one of the justification of this study. Also, it has been noticed that there are few researches on this subject, especially as they relates to the overall senior secondary school students in Nigeria. Thus this study would justifiably provide a useful contribution to researches in this area.

Again, this research would also lead to an improved teaching techniques of geometry and mathematics and thereafter enhance the realisation and actualisation of the technological development of the country. Hence the multiplicity of the justification for this research effort.

1.8. Delimitations

The main theme of this study is the identification of geometrical thinking levels of Nigerian Senior Secondary School students. Therefore, the following delimitations are imposed on the study:

1. The study is delimited to the year three students of the Senior Secondary Schools (SS3) in Nigeria.
2. Although Van Hiele's theory consist of five levels of geometrical thinking, this study considers the first four levels and classified them into two parts, the lower levels and the higher levels.
3. The study also considered the fact that variables which affect learning are many and can not be exhausted. Therefore, variables such as gender, subject grouping and performances in the different topics are variables considered in this study.
4. The study is delimited to plane geometry taught at year 3 Senior Secondary School class (SS3).

1.9 Summary

This chapter has provided an explanation to the problem of the study. It shows that the study is focussed on the use of Van Hiele's theory to identify level of geometrical thinking among Senior Secondary School students in Nigeria. The chapter also highlighted the aims and objectives of the study.

CHAPTER TWO

REVIEW OF LITERATURE

2.1 Introduction

In recent years, researches have and are still being carried out in difference parts of the country to determine the factors responsible for the poor performance of Secondary School students in Mathematics. Iransoro (1987), Ale (1989), Bolaji (1995), attributed two major factors which include teachers' approach and poor background of the students. However, Iransoro (1987) and Angel et al (1991) linked the problem to lack of coordination between teachers' teaching and students understanding. This study is in line with this view, as it agrees that the problem of poor performance in mathematics by Nigerian senior secondary school students could be due to lack of coordination between teachers teaching techniques and students learning styles. Similar situation occurred in U.S.A. and some other countries. Hart's (1981) study indicates that students overcome the coordination problem through the use of Van Hiele's theory. Hence, this study too is to use Van Hiele's theory to identify levels of geometrical thinking of Senior Secondary School students in Nigeria.

This chapter is therefore divided into six major topics as follows:

i. Conceptual over-view of geometry.

ii. Uses of geometry.

iii. Teaching of geometry.

iv. Spatial visualization and gender difference in school geometry.

v. The Van Hiele's levels of geometrical thinking

vi. Studies related to statistical analysis.

2.2 Conceptual Overview

The origin of geometry lie in the practical problem faced by the earliest civilization. There was the need to mark out boundaries, construct canals, build houses and temples and many other activities that require measurement of physical space. Penrose (1979) stated that one of the most fruitful sources of mathematical (geometry) intuition is physical space and this provide us with the basic concept of Euclidean geometry.

Here Penrose failed to realise that man's effort to determine the true law's of the space or three dimensional figure led to the concept of Euclidean geometry. It is noteworthy to know that continuous thought and criticism on Euclidean geometry suggested several alternative aspect of geometry often referred to as non-Euclidean geometry. among these are analytic geometry, algebraic geometry, coordinate geometry, descriptive geometry, differential geometry, and projective geometry. Butler (1970), in his study on geometry had remarked that:

"the modern point of view concerning geometry is that, it is a creation of man;s intellect, its distinctive structure being delineated by the basic postulates, the underlined elements, the dimensionality of space of orientation, and the techniques of investigation".

Putting together the views of Penrose and Butler on geometry, one would realise that geometry can be divided into two types or groups viz: the ancient or the Euclidean geometry and the modern or the non-Euclidean geometry. The former emphasised the study of physical space and physical figures, while the latter lay more emphasis on idealization of shapes such as points, lines, surfaces and solid or the combination of these geometrical magnitude as we have in the geometrical contents of secondary school mathematics syllabus.

It is generally assumed that non-Euclidean is the beam of the day in the modern world, Kruteski (1976), Hoffer (1981), Osafehinti (1990) and Obioma (1991). The concern of mathematics educators is what aspect of the geometry should be reflected in our secondary school curriculum. Ulrich (1973) said "that the mathematician whose specially is algebra sees geometry as a vehicle for algebraic expression, if the mathematician specialty is vectors, he believes that vector should be used to interpret geometry".

This is to say that theory oriented mathematician views geometry as an excellent example of group. Hoffman (1973) in his write up, maintained that geometry should be developed as part of the mathematics course and that vectors, transformation and coordinate method should receive some attention.

The views of Ulrich and Hoffman are indeed conflicting. while Ulrich views the various aspect of geometry as an entity of their own, Hoffman feels that the various aspect of geometry should not be separated, rather they should be merged together as a single subject with mathematics. From the researcher's experience as a teacher of mathematics, geometry is a threat and a stumbling block to the achievement of most Senior Secondary School students in mathematics. therefore. if geometry is considered as a subject of its own in the school curriculum, may be a great step would have been taken to resolve the dilemma in Secondary School mathematics. Falayajo and osafehinti (1990), reported that greater emphasis was placed on geometry, however, the impact did not reflect in students' performance in mathematics. This could be as a result of manner by which the teaching of geometry is being handled in the country. Iransoro (1987) describes the teaching and learning of geometry in Nigeria Secondary Schools in respect of a statement credited to Moulten (1974) that:

"A great deal of attention is paid to computation of areas, perimeter and volume with the idea that students need to be able to make these computations in order to buy land, etc. A large amount of time is also devoted to categorizing objects into classes such as isosceles, scalene, convex, closed, acute and the like, without any time having been spent on a study of the significance of the categorizing properties. And a lot of effort is put into various ruler compass constructions at a time when the philosophical justifications is unknown to the students."

Combing geometry with other aspect of mathematics as the case is in Nigeria, would give little or no time for the moral justification for teaching and learning of geometry. This study, therefore, is of the view that if geometry is given recognition as a subject, then a great role would have been taken to solve the problem of mass failure of students in mathematic. For by so doing, there would be room to expose the students to the importance and uses of geometry.

2.3. Uses of Geometry

Most often than not, geometry has been viewed as just the study of such terms as rectangle, line segment, angles, circles, congruences, logic and proofs. Very few view it as a great untapped source of ideas, processes and attitude. Schneli (1954) said that no study is better adopted than geometry to discipline the minds of the young. It is within their grasp, it interests, excites, tasks, and stores the mind; not only stores it with useful knowledge, but furnishes it with valuable habits. The memory is aided, fresh, interest awaken, and the whole mind invigorated by the generalization of geometrical truth.

In his own view, Niven (1979), stated that:

> "Certainly there can be no doubt about the importance of geometry in its supporting role not only in other part of mathematics but also such areas as engineering, architecture, physics, astronomy etc".

From the above it can be deduced that the importance of teaching geometry has been to develop functional thinking in students and as well as to facilitate easy transfer of training from geometry to non geometrical situation, as (Astardor (1954), posited in the following remark. Geometry achieves its highest possibilities if, in addition to its direct and practical usefulness, it can establish a power to think clearly in geometry situations and to use the same discrimination in non-geometric situation if it can develop an attitude of mind which tends always to analyses situations, to understand their interrelationships, to question hasty conclusions, to express clearly, precisely and accurately non-geometric as well as geometric ideas. to crown up the usefulness of geometry, Schnell (1954) has this to say:

> "Much attention has been given to the problem of transferring techniques of reasoning employed in geometry to everyday non-geometric situations and to developing a critical attitude of mind on the part of the pupils".

With these reviews on usefulness of geometry, it can briefly be summarised that, geometry increases students functional thinking and facilitates smooth transfer of ideas, processes and concept from geometric situations to non-geometric situations.

2.3.2 Transfer of Training in Geometry

There have been heated debates on the transfer ability of knowledge from one subject to another. Transfer of training can be defined as the influence which an improvement or change in one mental function has upon later performance or influence of learning upon other mental function. Researchers far back to the early century have been in disagreement as to whether or not learning a particular subject could influence the understanding of another subject. Betz (1938 said that:

> "I do not think that any subject transfers automatically, and in every case. The real problem of transfer is a problem of organising training that it will carry over in the minds of students in other fields. There is a method of teaching a subject so that it will transfer, and there are other methods of teaching the subject so that the transfer impact will be very minimal. Mathematics as a subject can not be described as sure to transfer. All depend upon the way in which the subject is handled".

Wheeler (1935), had the same view as Betz, he said that "No transfer will occur unless the material as learned in connection with the field to transfer is desired". Also, thorndike (1963), in his study on transfer of training maintained that the sort of logical thinking done in geometry does not transfer automatically to situations outside of geometry. However Butler (1970) was in disagreement with the view that transferability of learning is not automatic. To him, teaching for transfer is the highest goal in mathematics learning.

These early writers did not take cognisance of the fact that there are three type of transfer viz: positive transfer, negative transfer and zero transfer. Positive transfer occurs when the previous performance benefits the performance on the subsequent task, while negative transfer occurs when previous (performance destructs performance on the second task, while negative transfer occurs when previous (performance destructs performance on the second task, and zero transfer occurs when the previous performance has no effect on the performance of the second task.

Bolaji (1995), in his study, found that problem-solving in mathematics is the main instructional procedure that gives room for transfer of learning automatically. According to him, problem-solving is a form of transfer of learning in which experience in one task influences performance on another task. Dececco and Crawford (1980) are of the view that most experimental researches depict transferability of learning because positive transfer occur when experimental group is superiors in performance to the control group, negative transfer takes place when the control group is superiors in performance to the experimental group, zero transfer occur when both groups perform at about the same level.

In essence, from the literatures reviewed above, instructional methodology is the basic determinant of transferability of a subject. This study is of the view that if geometry is taught properly so that students' level of understanding is in the same line with the teachers' teaching level, then the students would find it easy to transfer ideas, processes and concept gained in geometry to other non-geometric situations. The big question however, is how can these students be taught in geometry to facilitate better understanding and smooth transfer of knowledge from geometry to non-geometric situation.

2.4. Teaching of Geometry

Write-ups and researches have shown that great deal is known today on general characteristics of human learning (Robinson, 19); Decacco and Crawford, 1988; Bolaji, 1995; Inekwe, 1996; etc). How current learning interacts with memory of the past learning; how teaching and learning of geometry specifically can be sufficiently carried out to facilitate greater achievement in understanding cum-interaction of memory and learning of the subject is a major concern of mathematics educators.

Meanwhile, the use of different methods of teaching to suit varying situations have been advocated by educationists. These methods include expository, guided discovery and problem solving. However, researches have shown that teachers play prominent role in students' learning. Jahun (1991), states that:

The question of whether mathematics is taught properly or not depend on (i) teacher's familiarity with the mathematics syllabus, (ii) ability to choose instructional strategies that will bring about the realization of the stated objectives and (iii) efforts to bring forth life experiences that can bring the concept of the topics being taught close to the students.

Ezike (1989), in his own write-up, submitted that at every stage the teacher of mathematics is confronted with three basic questions namely (i) helping the students to develop understanding and mastery of new concepts, principles, relationships and skills (ii) helping them to maintain understanding and skill already attained, and (iii) helping them to score maximum transfer of learning to their social environment.

In order to emphasis the role of teachers in the teaching of geometry and mathematics. Howard (1941) asserts that:

> "In mathematics at all events, there is one thing only of primary importance, that a teacher should make a honest attempt to understand the subject he teaches as well as he can and should expand the truth to his pupils to the limits of their patience and capability."

Even though most of the literatures reviewed here, discussed more about mathematics than geometry. This is so because it is difficult to separate teaching of geometry from that of other mathematics, since there is no separate geometry syllabus in Nigerian secondary school. However, the researchers points of view show that teaching of geometry should be more student oriented so that teachers could consider the level of at which their average students are. This is in line with the focus of this study.

2.5. Spatial Vizualization and Gender Differences in School Geometry

In Nigeria, it has been an established fact that students performance in mathematics in general and geometry in particular is very poor. Ale (1989) says "mathematics education in this country is in a sorry situation. There has been so much concern and outcry from many quarters about the poor performance of students in mathematics". Ale (1989) went further to say that "some female learners complain that mathematics is a masculine subject while biology is feminine and there is no need for female to study mathematics. Bolaji (1995), was also of the view that gender related differences exist in student cognitive and attitudinal behaviour. Glennon and Callahan (1986) stated that:

> "evidence would suggest to the teacher, that boys achieve higher than girls on test dealing with mathematical reasoning while the girls will achieve higher than boys on test on computational ability."

All these write-ups indicate that mathematical or geometrical cast of mind is based on spatial vizualization and verbal logical thought. Baltisa (1990) examined gender difference in spatial thought and verbal logical thought. He came up with the report that male students and female students differed in spatial vizualization, but they do not in logical reasoning ability. Fennema and Carpenter (1981) found that male students significantly out performed females students in the areas of geometry and measurement. Tyler (1965) had the same view. He maintained that girls in general do not seem to do well as boys in tasks involving judging or manipulating spatial figures. Usiskin (1983), however, disagree with the assertion that boys perform better than girls in geometry, he maintained that boys and girls have equal ability in learning geometry and geometrical proofs. This study is to investigate whether there exist gender difference in spatial visualisation.

2.6. The Van Hieles Level:

Many students commit errors when answering question out of sheer carelessness or due to lack of the basic facts with which to answer the question. for instance, a student who multiplies 34 by 3 to obtain 92 can be said to be careless about principle of place value, while a student who add 1/2 to 1/3 to get 2/5 can be said to lack mastery or understanding of the basic facts on addition of fraction. Whatever the pattern of error, it is essential to identify where the student are in error so that a meaningful learning can take place. Rising (1977) remarked that "the marks numerals, and erasures that students have put on their papers may tell a lot more that whether the answer is correct,... whatever the reason for an error pattern our job is to identify the behavioral objective lacking".

In common usage, behavioural objectives refer to intended change to be brought about in a learner. Bloom (1956) and Kiathwohl (1964) identified three domains of behavioural objectives vizs - cognitive domain; affective domain; and the psychomotor domain. cognitive domain has to do with reasoning or thinking; affective deals with interest while psychomotor deals with skills.

Out of the three behavioural objectives, cognitive domain, the intellectual classroom behaviour is receiving greater attention of researchers. Bloom (1956) classified this domain into five hierarchical levels - knowledge, comprehension, application, analysis and synthesis. Van Hiele (1959), however, identified also five levels of cognitive development in geometry and these are referred to as Van Hiele's levels of geometrical thinking. the first level (level 0) is concern with identification of geometrical objects. At this level the student identifies a figure by its appearance as a whole. Burgar and Shamghnessy (1985) described this level as visualization level, to them, description of figure is purely visual and if a learner at this level is asked why a figure is rectangle, he may say that it is called rectangle because it is like window or door. Hoffer (1981), however, did not agree with the chronological order of the level, he maintained that the first level should be called level "1" instead of level "0". Mayberry (1983) referred to this level as the basic level or level of recognition. To him, students at this level, learn vocabulary and recognizes a shape as a whole.

The second level deals with analysis. At this level the student is alert to the parts of figures and how they are related. For example, a student at this level will identify a square by recognising that all the sides are congruent, each angle is a right angle, the diagonals are perpendicular to each other and so on. More identification of collections of properties is attained, however, well organisation ability may be lacking. for example a kite here may be regarded as parallelogram.

The third level is concern with ordering. here properties and classification assume an organisation. That one property follows from another can be partially digested. The student order properties logically and begins to appreciate the role of general definitions. At this stage, the student can confidently call a square a special rectangle or special rhombus.

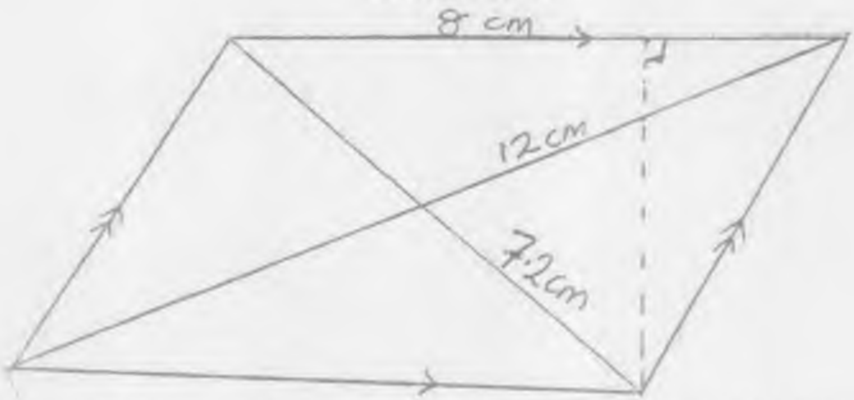
The fourth level, the formal deduction level, is all about deductive organisation. The student here understands the significance of deduction and the role of postulates, theorems and proofs.

The fifth level deals with rigor, here geometry is seen as an abstract system where point or object can be interpreted in any way consistent with the axioms of the system.

Many authors have shown that the fifth level of Van Hiele's levels of geometrical thinking is above the scope of secondary school students (Rising, 1997; Iransoro, 1987; Angle et al, 1991; etc). The general syllabus for senior secondary school mathematics shows that the geometrical aspect of the syllabus covered only the first four levels of the Van Hiele's levels. Hoover (1973), however, did not agree with Van Hiele's classification of geometrical thinking levels. To him, geometrical thinking levels are better classified into lower thinking level and higher thinking level The lower level according to Hoover (1973), should cover exercise involving memory, that is, recognition and analysis; while the higher level involve synthesis, that is, formal deduction and informal deduction.

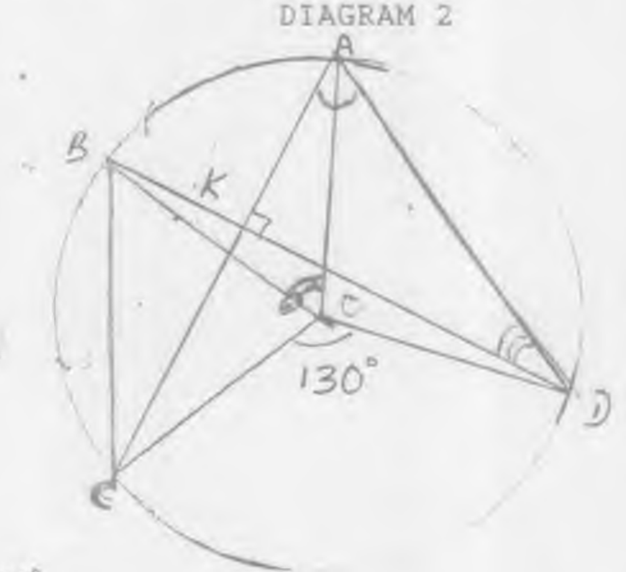
It is obvious that whatever classification is given to levels of geometrical thinking, Van Hiele's structure can not be ruled out. In Nigeria, most examination bodies for Senior Secondary School mathematics concentrate on both lower and higher levels of geometrical thinking aspect of mathematics. S.S.C.E. (1996), shows that students of senior secondary school in Nigeria are evaluated at both lower and higher levels of geometrical thinking. Question such as construct a parallelogram ABCD with /AB/ = 8cm, /AC/ = 12cm, and /BD/ = 7.2cm; and construct perpendicular from B to CD; are structured tests of ability of the students at a lower level of geometrical thinking. This is because the question demand the students thinking level of recognition of the shape parallelogram, and as well as thinking level of analysis of the shape to determine perpendicular from B to CD.

DIAGRAM 1



Also question such as draw circle ABCD with centre O. AC and BD intersect at right angles at K. Angle COD is 130: calculate angles (i) DAC, (ii) ADB, and (iii) AOB; is a question that demands the factual manipulative competence of the students at a higher level of geometrical thinking. This is so because the question expect the students to make formal and or informal deduction of cycle theorems which states that, the angle which an arc of a circle subtends at the centre is twice that which it subtend at any point on the remaining part of the circumference, and (ii) sum of angles of triangle is 180^0.

DIAGRAM 2



(i) DAC = $\frac{130^0}{2}$ = 65^0 ... Theorem on cycle

(ii) ADB = $90^0-65^0=25^0$ .. Theorem on Triangle

(iii) AOB = 2xADB= 50^0 Theorem on cycle

It can be noted from the questions above that no student can function adequately on a higher level unless the student has had adequate experience on the lower level. Therefore, it is obvious to note that no two people who are reasoning on two different levels can understand each other. Hence, a learner has to be on the same level with the teaching of the teacher for a meaningful knowledge to be acquired.

Van Hiele's theory has been found to be valid in places like Russia and United States of America, (Hart, 1981; Usiskin, 1982), while Iransoro (1987), proved it to be valid in a state in Nigeria. Therefore, the present study is to adopt the theory to identify level of geometrical thinking of senior secondary school students in Nigeria generally.

2.7. Studies Related to Statistical Analysis

The first statistical tool adopted in this study is the procedure for determining the validity of the instrument used for data collection. Here experts views on purposefulness of the test items were scored and ranked; and Kendall's coefficient of concordance test was used to determine the validity index.

Kendall's coefficient test is given by

$$Wc = \frac{12\Sigma d^2}{m^2(n^3-n) - (m\Sigma(t^3-t)}$$ where Wc is the coefficient

of concordance with ties in rank; d is the difference between observed rank total for each expert view and the expected rank total for each expert view; m is the number of sets of test items, n is the number of experts; and t is number of ties is the rank of the expert view for each set of test items.

The second statistical techniques adopted in the study is Pearson Product Moment Correlation (PPMR) for test of reliability of the test scores. Here the formulation for PPMR is given by

$$r = \frac{\sum XY - \frac{\sum X \sum Y}{N}}{\sqrt{\left[\sum X^2 - \frac{(\sum X)^2}{N}\right]\left[\sum Y^2 - \frac{(\sum Y)^2}{N}\right]}}$$

where r is the coefficient of correlation, x is the split-half scores obtained by the student in the combined level tests (CLT½); y is the second split-half scores obtained by the students in the combined level tests (CLT½) and N is half total number of students.

Other data analyses include both the descriptive and inferential aspects of statistical tools. At the descriptive level, the analyses involve summarization and organisation of data in form of frequency distributions. The purpose is to study the characteristic of the distributions.

The mean, standard deviation, student (t) test, and analysis of variance (f) test were used to compare the variability and the consistency of the various level groups according to some variables of interest such as gender, subject grouping and topics.

2.8. Summary of Related Literature

The survey first highlighted conceptual overview of geometry. It was noted that geometry originated from the practical problem faced by the earliest civilization. The literature reviewed highlighted the place of geometry in secondary school curriculum.

The importance of geometry also came into focus in which it was stated that geometry develop functional thinking and facilitates transfer of training from geometry to non-geometric situation.

In the chapter, it was noted that emphasis is often given to computational aspect of mathematics and that geometry an integral part of mathematics (as the case is in Nigeria) require conceptual and structural instructions. Therefore in the teaching of geometry and mathematics, a guiding principle have to be followed in-toto.

Spatial visualization and gender differences was also reviewed. This showed that geometry and mathematics generally require cast of mind on spatial thought and verbal logical thought. However literature showed that females did not perform as males in activities that involve spatial visualization.

Also the Van-Hiele levels and its implication for students was given. The researcher viewed that Nigeria secondary school mathematics teachers should use the theory for meaningful teaching and learning to take place. Lastly, the statistical techniques used in the study were discussed.

CHAPTER THREE

RESEARCH METHODS

3.1. Introduction

This chapter describes the design, method and procedures employed in the conduct of this research. The population, sample and sampling techniques were jointly discussed, while data gathering instrument and procedures as well as the analysis method were equally treated.

3.2. Research Method and Design

In an attempt to answer the research questions and also for testing the hypotheses, the research method used the survey type. The choice of the survey method was dictated by the nature of the research problem being investigated. In other words, since this is a kind of research covering a wide expanse of land with research subject numbering several thousands, it becomes convenient to use the survey method. The design of the research however, is dictated by the nature of the research variables and attributes.

Two geometrical thinking level tests were used as the main instrument for the study. These are Lower Level Test (LLT) and the Higher Level Test (HLT); the two tests, however, were combined to form Combined Level Test (CLT) for the purpose of data analysis. The research was designed to elicit information on effect of gender difference and subject grouping on levels of geometrical thinking of the students and as well to verify if there exist relationship between students thinking at the lower level and the higher level. Also students performance in the various topics were looked into.

Symbolically, the design was expressed as follow:

LLT ----> HLT-------->CLT

Table 3.1 Shows the data code use in the research design.

Table 3.1 Research Design and Data Code

Variables/ Attributes	Lower Level Test (LLT)	Higher Level Test (HLT)	Combined Level Test (CLT)
A. Gender			
Male (M)	LLT1	HLT1	CLT1
Female (F)	LLT2	HLT2	CLT2
B. Subject Groups			
Science (S)	LLT3	HLT3	CLT3
Art (A)	LLT4	HLT4	CLT4
C. Location			
North (NT)	LLT5	HLT5	CLT5
Middle Belt	LLT6	HLT6	CLT6
East	LLT7	HLT7	CLT7
West	LLT8	HLT8	CLT8
D. Topics			
Parallelogram (PG)	LLT9	HLT9	CLT9
Triangles(TG)	LLT10	HLT10	CLT10
Similarity & Congruency (CC)	LLT11	HLT11	CLT11
Circles & Cyclic Quad (CC)	LLT12	HLT12	CLT12
Properties & Tangent to circle (PT)	LLT13	HLT13	CLT13

The lower level test covers the first three levels of geometrical thinking as propounded by Van hiele 1959, while the higher level test takes care of the fourth level of Van Hiele's geometrical thinking. The two tests combined shall form the major basis for which analysis of the study will be carried out at (95%) ninety-five percent confidence level.

3.3. The Population

The population of this study are the 1996/97 year three Senior Secondary School Students in Nigeria. Efforts made to get the exact population size for the year running from Federal Ministry of education and Federal office of statistics was to no avail. However, in view of the fact that all the year three students of Senior Secondary School Nigeria are expected to enroll for senior secondary school examination (SSCE), the population size for the study was derived from the SSCE 1996 enrollment figure to be 504,320 students (SSCE statistics 1997).

3.4. The Sample

The year three students of Senior Secondary School in Nigeria are the respondents for this study. The choice of the students was based on the fact that at the year three class of Senior Secondary Schools, it is believed that the plane geometry aspect of mathematics syllabus would have been adequately treated.

Therefore, a representative sample of the students from the population was drawn using Roscoe's suggestion (1975) which states that for an ex-post-factor research such as this, a researcher could use a sample size of 30 or more.

In order to have a manageable sample size based on Roscoe's recommendation therefore, a sample size of 188 was adopted for this research. Fourty seven (47) subjects were randomly selected from each Centre.

3.5. Sampling Procedures/Techniques

For the purpose of this study, the country was clustered into four regions viz the North, Middle Belt, West and the East. Table 3.2 shows the regions and their states.

Table 3.2 Regions and their States

North	Middle Belt	West	East
Adamawa	Benue	Delta	Abia
Bauchi	Kogi	Edo	Akwa Ibom
Borno	Kwara	Ekiti	Anambra
Gombe	Nassarawa	Lagos	Bayelsa
Jigawa	Niger	Ogun	C/River
Kaduna	Plateau	Ondo	Ebonyi
Kano		Osun	Enugu
Katsina		Oyo	Imo
Kebbi			River
Sokoto			
Taraba			
Yobe			
Zamfara			

All the states in each clustered region were numbered alphabetically and a state was randomly picked from each region.

However, in order to have a good control of the research work, the capital towns of the selected states were used as the research centres.

Two schools stratified into science based and art based schools were randomly selected from each centre. In all, 8 schools were covered in the study, while school type - mixed sex or single sex school was also taken into consideration.

Table 3.3 shows the sample distribution on the bases of school name, location, type and subject group.

Table 3.3

Name of School	Location		SCIENCE		ART	
	Centre	State	Male	Female	Male	Female
Dalla GGSS	Kano	Kano	0	12	0	12
Rumfa College	Kano		13	0	10	0
Mariam B.S.S	Minna	Niger	0	23	0	0
G.S.S	Minna		0	0	24	0
Ikolaba Grm.Sch	Ibadan	Oyo	7	6	5	5
B.Philip H/Sch.	Ibadan		6	6	6	6
Ibeku H.Sch.	Umuahia	Abia	6	7	6	5
Afara Tech.Sch	Umuahia		6	6	6	5

Kano State with its capital Kano represented the North. Niger state - Minna its capital stands for Middle Belt; Ibadan in Oyo State was picked to represent the West and Umuahia in Abia state represented Eastern region in the study. As shown in the Table 3.3 above, fourty seven (47) students were selected from each centre while the break down of the sample into subject grouping and gender shows that Kano has twenty five (25) science students and twenty two (22) art students; twenty three (23) of the students are male and twenty four (24) are female. In Minna twenty three (23) students are from science group and twenty four (24) of them from art class; twenty four (24) are male students twenty three (23) are female students. In Ibadan twenty five (25) students are from science class, twenty two (22) students from art class; twenty four (24) of the students from this centre are males, while twenty three (23) of them are females. In Umuahia however, twenty five (25) students are from science class and twenty two (22) of them are from art class, twenty four (24) of the students from the centre are males while twenty three (23) are females. School types used in Ibadan and Umuahia are mixed-sex schools while those used in Kano and Minna are the single-sex schools.

3.6. Instrumentation

The purpose of this study is to identify the level of geometrical thinking of Nigerian Senior Secondary School students. Therefore, the instrument used are two geometrical thinking level tests developed by the researcher.

The tests questions are set to conform with a Van Hiele's (1959) geometrical thinking structures. The items are modified into two level structures, as against Van Hiele's five level structures and they are within the scope of Senior Secondary school Mathematics Syllabus. Also the questions are similar to questions set by Iransoro (1987), Angel et al (1991), Cannon et al (1990) and SSCE (1996).

The instruments are made up of two tests Viz-Lower Level test (LLT) and the Higher Level Test (HLT). The lower level test contains twenty (20) test items while the higher level test contains five test items. The two tests covered five geometrical topics parallelograms (PG), triangle (TG); Similarity and

Congruency (SC); Circles and Cyclic quadrilaterals (CC); and properties and tangent to circles (PT). The questions are set evenly across the topics for the levels as shown in Table 3.3.

Table 3.4: Topics Distribution of test item in LLT and HLT

Topics	LLT	HLT
Parallelogram	4	1
Triangles	4	1
Circle and cyclic Quad.	4	1
Similarity and Congruency	4	1
Properties and Tangent to Circles	4	1
Total	20	5

Also the lower level test has items measuring students' ability to recognise or visualise geometrical shape; identify or analyse the properties of the shape; and make abstract definition of the geometrical shape. The higher level test, on the other hand, has one test item for each topic that measures students ability to make deductive reasoning in postulates, theorems and proofs. Table 3.5 gives detail on distribution of test items per topic and per objective.

Table 3:5: Distribution of Items per topic and Per Objective

Contents Topics	Objective LLT Visualisation	Analyses	Abstraction	HLT Deduction	Total
Parallelogram	2	1	1	1	5
Triangles	1	1	2	1	5
Circle and Cyclic Quad.	1	1	2	1	5
Similarity and Congruency	2	1	1	1	5
Properties and Tangent Circle	2	1	1	1	5
Total	8	5	7	5	25

Table 3.5 explain the specification of contents and objectives.

3.7. Administration of the Instrument

The tests were administered to randomly selected students of year three class of senior Secondary Schools (SS 3 students). For the four sampled states, the states ministry of education were first contacted on the researchers' intention to conduct the study in the states. Two schools either science, art or both were thereafter randomly selected from list of schools in the state capital. The Principal and Mathematics tutors of the selected schools were also informed of the aims and objectives of the study. On permission by the school authority, the services of two mathematics tutors were employed to assist the researcher in sample selection (using the class registers) and also in administering the tests.

The lower level test in each of the centre lasted for forty-five (45) minutes on the average. The higher level test, however, lasted for 90 minutes on the average in all the centres. The variation in time for the tests was due to their nature. The first was objective while the second was essay. In order to assure the respondents of the confidentiality of the study, their names or school registration number was not required. Only their sex, subject group (i.e. science or art) researcher's code number and state where school is located are required for the purpose of analysis.

3.8. Scoring the Tests

Each of the test contains five topic areas as earlier mention, and each topic area contains four (4) questions for lower level test and one (1) question for the higher level test. A question correctly answered in the lower level test was awarded two and half (2) marks, so that for a topic area ten (10) marks was the highest obtainable mark, while for the five (5) topic areas, the highest obtainable mark was fifty (50) marks. In essence, the highest mark a student can obtain in the lower level test was 50 marks.

In the higher level test, however, a fully answered question was awarded ten (10) mark for each topic areas, while partial answers are marked strictly in compliance with marking scheme developed by the researcher. In all, the highest mark obtainable in the higher level test was 50 marks also. The combined marks for the two tests can not exceed 100 marks.

3.9 The Pilot Study

The purpose of the pilot study was to validate the lower level test (LLT) and the higher level test (HLT). This was done by determining the reliability and efficiency of the two instruments in identifying level of geometrical thinking of Senior Secondary School students in Nigeria. The pilot study was also meant to serve as a model and an insight to the main study. Demonstration Secondary School, A.B.U., Zaria was selected from the mixed sex schools in Zaria for the pilot study. Two pairs of parallel tests were developed and administered on the 9th of December, 1996 when the students were writing their first term examination. The choice of the period was to enable the students put in their best in the test.

Fifty (50) students were **randomly** selected from the arm of SS3 class for the pilot study. Students from both science and art classes were drawn on equal proportion using their class registers, twenty-six (26) of the students are female while twenty four (24) are male students. Table 3.5 shows students distribution in terms of subject grouping and gender for the pilot study.

Table 3.6: Students Distribution for the Pilot Study

Subject Groups	Science	Art	Total
Male	8	16	24
Female	17	9	26
Total	25	25	50

The test which contains twenty-five (25) test items (both major and sub-numbered test items) for lower level test and higher level test were administered to the students simultaneously. It took them average time of one hour forty minutes (1hr 40mins) for the first test and one hour thirty minutes (1hr 30 mins) for the second parallel test. Fourty five minutes for the lower level test and between one and half hours to one hour twenty minutes for the higher level test.

Both students and staff members of the schools, especially the principal and the mathematics teachers all accorded the researcher their full cooperation.

The result of the pilot study shows that there was no significant difference between the pair of parallel tests. As Jahun (1988) rightly said that two or more tests are said to be parallel if their variances are not statistically significantly different. To establish the significance of the differences (if any) between the pair of parallel tests, t-test was employed.

3.10. Reliability and Validity of the Instrument

According to Adewumi and Ogunlade (1991), validity refers to the degree to which a test measures what it purports to measure. Borg and Gall (1983) however, described validity as a quantity that indicates the magnitude of relationship between the test scores and the criterion measure. Four experts in the field of mathematics education were asked to comment on the validity of the test item and their comment was in terms of whether each topic measure exactly what it intends to measure or if it slightly deviate or if there is strong deviation from what it suppose to measure or if it is ambigious. The first view was ranked 4, the second view ranked 3, the third ranked 2 and the last ranked 0.

Table 3.7: Ranks of Expert's View on Validity of Test for the Five Topics

Expert (n)	Topics (M) 1	2	3	4	5	Observed (Fo)	Expected (Fe)	Fo-Fe	$(Fo-Fe)^2$
1.	3	4	3	4	3	17	12.5	4.5	20.25
2.	4	2	4	3	4	17	12.5	4.5	20.25
3.	1	1	1	2	1	6	12.5	-6.5	42.25
4.	2	3	2	1	2	10	12.5	-2.5	6.25
TOTAL	10	10	10	10	10	50	50		89.0

$$Fe \frac{MN(N+1)}{2N} = \frac{5X4\ (4+1)}{2x4} = 12.5$$

Coefficient of Concordance

$$(W) = \frac{12\ d^2}{^2(N^3 - N)} = \frac{12x89}{25\ (60)} = \frac{10}{15} = 0.67$$

The validity coefficient for the two tests combined is 0.67 which was found to be significant at a = 0.05.

Reliability of test was defined by Adewumi and Ogunisade (1991) as the consistency or the stability of test scores over time and under varying conditions. They went further to describe reliability as a measure of that gives precise and consistent resulting scores.

The reliability of these instruments for this study was evaluated by computing the reliability coefficient using split-half method. Here the test items are pooled and splitted into odd and even number items Appendix E and F. The scores obtained from the odd and even numbered items are correlated and the reliability of the test was determined using the Spearman Brown's formula:

Reliability Coefficient (rtt) = $\frac{2r_{ii}}{1+r_{ii}}$ where r_{ii} is the reliability of one half of the test. Therefore, reliability coefficients for LLT and HLT are thus:

LLT (rtt) = 0.77 HLT (rtt) = 0.79. These coefficients were found to be significant at $\alpha = 0.05$.

3.11. Methods of Data Analyses

The scores obtained in the two tests are the main data for the study and the data were analysed using the following statistical techniques.

1. Frequency tables, mean and standard deviation were used in this study. These techniques were used to describe the data.
2. Pearson Product Moment Correlation coefficient techniques was also used to compute coefficient of reliability of the scores obtained by the instrument.

$$r = \frac{N\Sigma XY - \Sigma X \Sigma Y}{\sqrt{[N\Sigma X^2 - (\Sigma X)^2][N\Sigma Y^2 - (\Sigma Y)^2]}}$$

3. The Kendall's coefficient of concordance was also used to establish the validity estimates.

$$W = \frac{12\ d^2}{m^2(n^3-n)-M\ (t^3-t)}$$

4. The t-test statistics were employed to determine the existence or absence of significant differences between the mean scores for science students and art students or between female students and the male students.
5. ANOVA (F-test statistics) was also used in determining the existence or absence of significant differences in the mean scores for the various topic areas.

3.12. Summary

This chapter highlighted the design of the study as being a repeated measure design, the population, sample and sampling techniques were also discussed. Based on the analyses of the result of the pilot study, the instrument was seen to be a satisfactory device for the study.

CHAPTER FOUR

4.0. DATA PRESENTATION AND ANALYSIS

4.1 Introduction

In this chapter attempts have been made to present and analyse the data collected for this study. Descriptive statistics was particularly used to present the data in the form of frequency distributions. Also statistics such as percentages, means, and standard deviations were computed; while inferential statistics were used to test the hypotheses stated.

4.2 Presentation of the Data

A total number of 188 subjects (respondents) were served with the two thinking levels tests LLT and HLT in the four centres. There was no mortality for all the respondents fully participated in the exercise.

For the purpose of simplicity and clarification, the data presentation and analyses was done region by region and later all states combined.

ABIA STATE (EASTERN REGION): A total number of 47 students were randomly choosen for the tests which took place at Ibeku High School Umuahia and Afara Technical College. Out of these students, 23 or 49% are male students, 24 or 51% are females students, 25 or 53% are from science classes, while 22 or 47% are from art classes.

For the LLT, the mean and standard deviation of the marks obtained by the students are 15.9 and 5.5 respectively. for the HLT, the mean and standard deviation of the marks are 1.7 and 2.9 respectively, while for the combine tests (CLT) the mean and standard deviation of the scores are 17.6 and 6.4 respectively. These results as can be seen in Tables 4.1 and 4.2 depict poor performance in all the test as most of the scores clustered around 0-29 marks. However, performance in LLT was considerably better than the performance in HLT. With these, one can assume that the students are operating at the lower level of geometrical thinking, since 100% of them scored less than 40 marks.

Table 4.1: Frequency Distribution for Eastern Region Students Performance in LLT

Marks	Frequency	Cumulative Frequency
0-4	2	2
5-9	15	17
10-14	9	26
15-19	9	35
20-24	11	46
25-29	1	47
30-34	0	47
35-39	0	47
40-44	0	47
45-49	0	47

Table 4.2: Frequency Distribution for Eastern Region Students Performance in HLT

Marks	Frequency	Cumulative Frequency
0-4	38	38
5-9	8	46
10-14	1	47
15-19	0	47
20-24	0	47
25-29	0	47
30-34	0	47
35-39	0	47
40-44	0	47
45-49	0	47

NIGER STATE (MIDDLE BELT): A total number of 47 students were also randomly choosen from this region and the test took place at Maryam Babangida Girls College and Government Secondary School, Minna. Twenty four (24) or 51% of the students are males; 23 or 49% are female students, 24 or 51% are science students while 23 or 49% are art students.

The mean and standard deviation for students performances in the lower level test (LLT) are 13.7 and 6.5 respectively; while the performance in HLT shows mean and standard deviation of 1.9 and 2.8 respectively; and in the combine level tests (CLT), the mean and standard deviation of the students' performance are respectively 15.5 and 7.4. Tables 4.3 and 4.4 shows the frequency distribution for students performances in LLT and HLT. Also students performance here coincidentally clustered around 0-29 just as it was in the Earth.

Table 4.3: Frequency Distribution for Middle Belt Students' Performance in LLT

Marks	Frequency	Cumulative Frequency
0-4	2	2
5-9	15	17
10-14	9	26
15-19	11	35
20-24	1	46
25-29	0	47
30-34	0	47
35-39	0	47
40-44	0	47
45-49	0	47

Table 4.4: Frequency Distribution for Middle Belt Students' Performance in HLT

Marks	Frequency	Cumulative Frequency
0-4	38	38
5-9	8	46
10-14	1	47
15-19	0	47
20-24	0	47
25-29	0	47
30-34	0	47
35-39	0	47
40-44	0	47
45-49	0	47

These results show that the students performance in all the tests was nothing to write home about. The results show fairly better performance in LLT than HLT, hence one can conclude that the students operate at the lower level of geometrical thinking, since 100% of the students scored in the range of 40 to 0 mark.

KANO STATE (NORTHERN REGION): A total number of 47 students were equally (randomly) choose for the tests and these took place at Rumfa College and Dalla Government Girls Secondary School, Kano. Out of these students, 24 or 51% are males, 23 or 49% are females, 128 or 60% are from science classes while 19 or 40% are from art classes.

The mean and standard deviation of students performance in the lower level test (LLT) are 15.5 and 7.6 respectively, while the mean and standard deviation in HLT are 4.9 and 11.2 respectively; and for the combine level test (CLT) the mean and standard deviation are 20.4 and 14.3 respectively. Tables 4.5 and 4.6 show the frequency distribution for students performances in the tests.

Table 4.5: Frequency Distribution for Northern Region Students' Performance in LLT

Marks	Frequency	Cumulative Frequency
0-4	4	11
5-9	6	38
10-14	11	45
15-19	15	47
20-24	4	47
25-29	5	
30-34	2	
35-39	0	
40-44	0	
45-49	0	

Table 4.6: Frequency Distribution for Northern Region Students' Performance in HLT

Marks	Frequency	Cumulative Frequency
0-4	38	40
5-9	0	41
10-14	2	44
15-19	1	47
20-24	1	47
25-29	0	
30-34	2	
35-39	2	
40-44	1	
45-49	0	

Although the results in this state/region show remarkably fair performance than the results of the other states/regions treated (which could be attributed to government concern for education by creating science school board), however, the results still show that the students level of geometrical thinking is in the lower level, since 89.4% of the students scored between the range of 40 to 0 mark.

OYO STATE (WESTERN REGION): Also a total number of 47 students were randomly choose as in other regions. 23 or 49% of these students are males, 24 or 51% are females, 28 or 60% are from science classes while 19 or 40% are from art classes.

Students performance for this region is almost a replicate of students performance in other regions. The mean and standard deviation of the students performance in the lower level test (LLT) are 15.9 and 4.9 respectively while in the higher level test (HLT) the mean and standard deviation are respectively 1.8 and 3.5; and for the two tests combined (CLT) the mean and standard deviation are 17.7 and 7.4 respectively. These results too, show fair performance in the lower level test than that of the higher level test. Tables 4.7, and 4.8 give clearer picture of the results 97.9% of the students operate in the lower level of geometrical thinking.

Table 4.7: Frequency Distribution for Western Region Students' Performance in LLT

Marks	Frequency	Cumulative Frequency
0-4	1	1
5-9	3	4
10-14	15	19
15-19	16	35
20-24	11	46
25-29	0	46
30-34	1	47
35-39	0	47
40-44	0	47
45-49	0	47

Table 4.8: Frequency Distribution for Western Region Students' Performance in HLT

Marks	Frequency	Cumulative Frequency
0-4	41	41
5-9	4	45
10-14	1	46
15-19	1	47
20-24	0	47
25-29	0	47
30-34	0	47
35-39	0	47
40-44	0	47
45-49	0	47

NIGERIA (ALL REGIONS): Overall total number of 188 students were randomly selected for the study. Out of these, 94 or 50% are male students while 94 or 50% are female students; and 105 or 56% are from science classes while 83 or 44% were selected from art classes. Tables 4.9, 4.10 and 4.11 show break down of the students performance in LLT, HLT and CLT respectively for all the regions.

Table 4.9: Frequency Distribution for (All the Regions) Students' Performance in LLT

Marks	Frequency	Cumulative Frequency
0-4	7	7
5-9	29	36
10-14	49	85
15-19	56	141
20-24	35	176
25-29	9	185
30-34	3	188
35-39	0	188
40-44	0	188
45-49	0	188

Table 4.10: Frequency Distribution for (All the Regions) Students' Performance in HLT

Marks	Frequency	Cumulative Frequency
0-4	157	157
5-9	17	174
10-14	6	180
15-19	2	182
20-24	1	183
25-29	0	183
30-34	2	185
35-39	2	187
40-44	1	188
45-49	0	188

Table 4.11: Frequency Distribution for (All the Regions) Students' Performance in CLT

Marks	Frequency	Cumulative Frequency
0-9	37 (19.7)	37 (19.7%)
10-19	95 (50.5)	132 (70.2%)
20-29	42 (50.5)	174 (92.6%)
30-39	08 (22.3)	182 (96.8%)
40-49	05 (02.7)	187 (99.4%)
50-59	00 (0.0)	188 (99.4%)
60-69	01 (0.5)	188 (100%)
70-79	00 (0.0)	188 (100%)
80-89	00 (0.0)	188 (100%)
90-99	00 (0.0)	188 (100%)
Total	188	

The results for all the regions show that the mean and standard deviation for LLT, HLRT and CLT are 13.61.

4.3. THE HYPOTHESES TESTING

Chapter One, a number of hypotheses were formulated with thr view of anwsering the questions raised. In this section attempts were made to test the hypotheses at 0.05 level level significance.

Hypothesis 1: There is no significant relationships between students performance at the lower level test (LLT) and their performance at the higher level test (HLT). This hypothesis was tested using Pearson Product Moment correlation (PPMR) and the result showed that there was significant relationship between students performance in the lower level test and their performance in the higher level test. The hypothesis was rejected because the result of the test gave a correlation coefficient of 0.40 while the critical value at x-level 0.05 and 187 degree of freedom is 0.195. This shows that the calculated coefficient is more than the critical value, therefore the hypothesis was rejected. Table 4.12 gives a summary of the correlation analysis for LLT and HLT.

Table 4.12. Summary of PPMR for Lower Level Test (LLT) and Higher Level Test (HLT)

	X	SD	N	DF	CalR	Crit R
LLT	13.61	5.33	188	187	0.40	0.195
HLT	2.37	5.94	188			

Hypothesis rejected

Table 4.12 shows that there exist very low positive correlation between students performance at the lower level test and their performance at the higher level.

Hypothesis 2(a): There is no significant difference between the mean scores of male students and their female counterparts in the lower level test (LLT). This hypothesis was tested using t-test statistic and the result of the analysis shows that there exist significant difference in the mean scores of male students and the mean scores of female students in the lower level test.

Table 4.13: Summary of T-Test computation By Gender in Low Level Test

GENDER	X	SD	N	DF	T (Cal)	T(Crit)
MALE	15.93	6.01	94	186	26.64	1.96
FEMALE	14.52	6.37	94			

Significant difference, T(Calc) > 1.96.

The analysis shown in Table 4.13 indicated that male students performed better than female students in the lower level test.

Hypothesis 2(b): There is no significant difference between the mean scores of male students and their female counterparts in the higher level test (HLT). This hypothesis was also tested by the use of t-test analysis and the result shows that there exist significant difference between the mean scores of male students and those female student. Table 4.14 is a summary of the t-test analysis for gender in the higher level test.

Table 4.14 Summary of T-Test Computation by Gender in Higher Level Test

GENDER	X	SD	N	T(cal)	DF	T(Crit)
MALE	3.56	7.80	94	38.36	186	1.96
FEMALE	1.56	4.21	94			

Significant difference, T(Calc) > 1.96.

Table 4.14 shows that male students supercede female students in performance in the Higher Level Test.

Hypothesis 3(a): This hypothesis was tested by the use of t-test analysis and the result shows that there exists significant difference between the mean scores of Science students and those of Art Students. Table 4.14 is a summary of the t-test computation for subject groups.

Table 4.15 Summary of T-Test Computation by subject Groups in the Lower Level Test (LLT)

SUBJECT GROUP	$\bar{X}$	SD	N	T(cal)	DF	T(Crit)
SCIENCE	16.01	6.36	105	33.44	186	1.96
ART	14.23	5.92	83			

From Table 4.15, the average score for Science students superseded the average score for Art students, and based on the t-test analysis, one can conclude that the science students performed better than Art students in the lower level test.

Hypothesis 3(b): The result of the test of this hypothesis, using t-test analysis, shows that Science students performed better than the Art Students in the higher level test. Table 4.16 is a summary of T-test computation by subject groups in the higher level test.

Table 4.16 Summary of T-Test Computation By Subject Groups in the Higher Level Test (HLT)

SUBJECT GROUPS	$\bar{X}$	SD	N	T(cal)	DF	T(Crit)
SCIENCE	2.92	6.69	105	15.05	186	1.96
ART	2.11	5.86	83			

From this table, the result of the t-test computation shows that there exist significant difference in the mean score of Science Student and that of the Art counterpart. The hypothesis was rejected because the calculated t-test is greater than the critical t-test from statistical table. There is no significant difference in the mean scores of students for the five topics in the combined level test (CLT).

Hypothesis 4: This hypothesis was tested using analysis of variance (ANOVA) and the results show that there exist significant difference in the student performance in various topics for the two tests combined Table 4.17 is a Summary of ANOVA computation for the topic in the combined test.

Table 4.17: Summary of ANOVA Computation for the Topics in the Combined Tests

SOURCE OF VARIATION	SS	DFD	MS	F(Calc)	F-CRITICAL
Between	922.12	4	230.53	27.83	2.37
Within	7743.78	935	8.28		
Total	8665.90	939			

From the result shown in Table 4.17 there exist significant difference between the mean scores of the students in the five topics under study. Since the F (Calculated) which equals 27.83 is greater than F critical which equals 2.37. Therefore the hypothesis is rejected, thus there is statistical significant difference in the performance of the students in the combined level test (CLT) for topic areas.

4.4. Discussions

It could be recalled that the objective of this study was to identify level of geometrical thinking of Nigerian senior secondary school students adopting a modified van Hieles theory. Test items that conform with Van Hieles theory was administered to 188 students of senior secondary school across the nation.

Generally, the students achievement in the two tests was very poor especially the higher level test. However, in the lower level test, the grand mean scores was 13.61 out of a total score of 50 while in the higher level test a grand mean scores of 2.37 out of total score of 50. For the two tests combined, the grand mean scores was 17.78 over a total score of 100, only 6 students out of 188 students met the cut off points of 41 and above while the remaining 182 or 96% of the students scored 40 and below. For a student to be at a higher level of geometrical thinking he or she is expected to score at least 40 marks out of 100. So by implication only 6 or 3.2% of the students attained the higher level thinking in geometry.

Analysis of the data also revealed that the students performance in the five geometrical topics differ significantly. The students show better understanding of parallelogram in which the grand mean score was 5.33 over a total score of 20 in the combine test (CLT). The other geometric topics treated in the study had the following means: trigonometry is 3.46; similarity and congruency 3.38; circles and cyclic quadrilaterals 3.35; and properties and tangents to circles 2.26.

The result of the analysis associated with the first null hypothesis, (H01) shows that there was significant relationship in students performance in the lower level test (LLT) and their performance in the higher level test (HLT). It is not surprising to have this result because the grand mean scores of students performance in the lower level test was 13.61 while that of the higher level test was 2.37.

Furthermore, only 6 or 3.2% of the total sample of students that participated in the test scored 41 over hundred (100) and above. This was an indication that the majority of the students operate at the lower level of geometrical thinking. This also conforms with Van Hiele's theory that says that any student that operate at a higher level must master skill at the lower level, the reverse, however, do not hold.

The second null hypothesis (H02) was intended to find out if gender difference could affect the performance of the students in geometrical thinking tests. The t-test results in Table 4.17 and 4.18 indicated that the performance of male students and that of female student generally differs significantly. The one tailed t-tests show that male students performed better than the female students, these results also agrees with the findings of Osafehinti (1983) and Aina (1986) who reported that male students are consistently superior in mathematics achievement test to their female counterparts.

The analyses of the data for testing the third hypotheses Ho (3a) and Ho(3b) indicated that there exist significant difference between the performance or achievement of Science and Art students of senior secondary three classes in geometrical thinking level tests. These results conforms with the general believe that Science students always perform better in mathematics than the Art students and that art students always perform better in English than the Science students (Odili, 1990; Eniayeju 1997 etc).

The last null hypothesis (H04) was formulated to find out if the students have preference or rather performed better in any particular topic area treated in the study or not. It was found that the students achievement in the five topics differs significantly. Majority of the students tested performed better in parallelogram than they did in the other topics.

4.5. Summary

From the results of the data analysis for this study, it was observed that the distribution of scores related to students achievement in geometrical thinking level revealed a very low and poor performance.

Various statistical techniques such as percentages, means and standard deviation were used. Also PPMR, t-test statistic and analysis of variance (ANOVA) were used as well to test the various hypothesis of the study.

Four main hypotheses were tested which led to number of positions as explained above. In essence, the level of geometrical thinking of Nigerian senior secondary school students was identified to be the lower level.

CHAPTER FIVE

5.0. SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 Introduction

In this chapter attempts are made to summarise the results of the findings of this study. Conclusion were drawn with respect to some observations made in the study. Also the nature of the distributions of scores on each of the two levels (LLT and HLT) were interpreted. The mean and standard deviations of various groups were compared while the reliability and validity indices were further discussed.

5.2. Summary

This research aimed at identifying level of geometrical thinking Nigerian senior secondary school students attain. Therefore, two level tests were designed as the main instrument for the study. Also the study tried to establish whether gender differences and subject grouping affect the students achievement in geometrical thinking test. In all, four major hypotheses were formulated and tested. A total sample of one hundred and eighty eight (188) students randomly drawn from eight (8) schools across the nation were used in the study. Various statistical tools such as percentages, means, and standard deviations were used. Also, PPMR, t-test, and analysis of variance (ANOVA) were used PPMR, to test the existence or non-existence of significant relationship or differences in the hypotheses formulated for the study.

At the 5% level of significance, the results of tests of the hypotheses are summarized thus:

HO1 that significant relationship existed between students' achievement in the lower level test (LLT) and their achievement in the higher level test (HLT).

HO2(a) There are significant differences between the mean scores of male students achievement in the lower geometrical thinking levels test (LLT) and the mean scores of female students achievement in the Lower geometrical thinking levels test (LLT).

HO2(b) There are significant differences between the mean scores of male students achievement in the Higher geometrical thinking levels test (HLT) and the mean scores of female students achievement in the Higher geometrical thinking levels test (HLT).

HO3(a) Significant difference exists between the mean scores of Science students achievement and that of their Art counterparts in the lower level test (LLT).

HO3(b) Significant difference exists between the mean scores of Art students achievement and that of their Science counterparts in the higher level test (HLT).

HO4: There exists significant differences in the mean scores of students achievement in the five topic areas treated in the study.

5.3. Limitations of the Study

The limitation of the study are as follows:

1. Final year senior secondary school students were used for the study.

2. School type (Federal Government Schools, Private Schools or State own schools) could influence students achievement due to differences in funding and administration. Therefore, schools are generalised in this study.

3. All respondents for this study were supervised by research assistants under the supervision of the researcher.

5.4. Conclusion

Based on the findings of this study one of the major conclusion of this study is that students' achievement in geometrical thinking level test was very low. It can be concluded that Nigerian students operate at the lower level of geometrical thinking. Also the study shows that male students' level of geometrical thinking supersede female students level of geometrical thinking.

In conformation to the general believe that science students are mathematically inclined while art students inclined towards English language. This study shows that there is significant difference in the geometrical thinking abilities of science students to those of art students.

Considering the achievement of students from the four regions, one can conclude that students across the country operate in equal level of geometripal thinking.

Also from the study, students' achievement in various topics show that the students' understanding of properties and tangents to circle was very low, while parallelogram receives better understanding by the students.

5.3. Recommendations

In the light of the findings of this study, the following recommendations for further research are made.

1. Based on the outcome of the study it was discovered that the students achievement was very poor across the regions. It is therefore recommended that more research studies should be conducted in order to find more viable solutions to the poor achievements in geometrical thinking. Such findings should be made available to school teachers who are the coordinator of teaching and learning of mathematics.
2. The result of this study shows that students operate at the lower level of geometrical thinking. It is therefore recommended that the mathematics teachers should teach at the pace of the students so that meaningful learning can take place.

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APPENDIX A

LOWER LEVEL TEST

INSTRUCTION

(i) This test intends to measure your understanding of geometry, hence answer All Questions honestly and to the best of your ability.

(ii) Tick [ ] or write the appropriate answer at the right column. Use back page if more space is required for answer and indicate the section and question number.

PART ONE: STUDENTS ~~PERSONALITY~~ Bio Data

(1) Sex: Male [ ] Female [ ]

(2) Subject Group: Science [ ] Art [ ]

(3) State: Niger [ ] Abia [ ] Kano [ ] or Oyo [ ]

(4) Code Number:

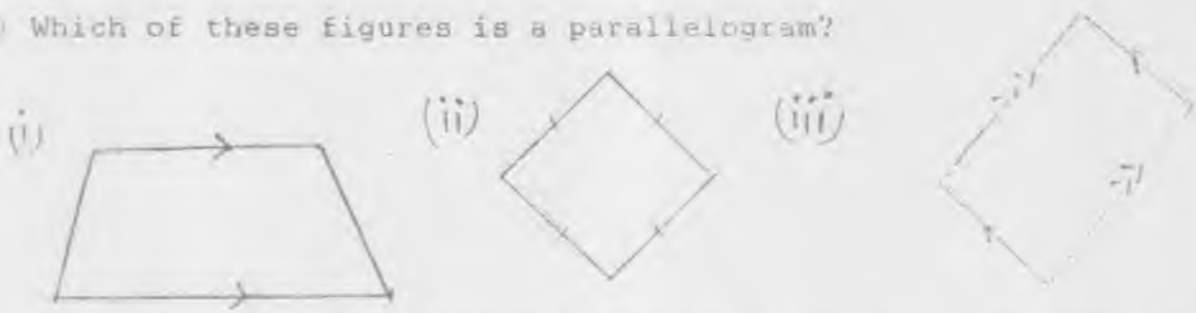
....

PART TWO: LOWER LEVEL TEST IN PLANE GEOMETRY

Section A: Parallelograms

Questions.

(1a) Which of these figures is a parallelogram?



(b) Name any two geometrical shapes that can be called parallelogram

(2) Write three properties of a parallelogram

(3) Briefly define a parallelogram

Section B: Triangles

Questions

(1) Which of the figures is an Isosceles triangles?

(2) Name any two properties of an Isosceles triangle.

(3a) An Isosceles triangles has no equal angle True or False

(3b) What is an Isosceles triangle?

Section C: Similarity and Congruency

Questions

(1) From the diagram below complete the statement: triangle ABC is congruent to

(2) If two plane figures are similar, are their areas equal: (Yes or No) Explain

(3a) Which of the following statement is true?
(i) All squares are congruent
(ii) All squares are similar

(3b) Define Congruent Triangle

Section D: Circle and Cyclic Quadrilaterals

Questions.

(1) Which of the figures below is a circle?

(b) Name two objects that have the shape of a circle

(2) Mention one property about cyclic quadrilateral

(3) Briefly define cyclic quadrilateral.

Section E: Properties and Tangent to Circles

Questions:

(1a) Which of the following shaded figures describes segment of a circle?

(b) Draw two figures to show difference between a tangent and chord to circles.

(2) What name is given to a straight line that joins two points on circumference of a circle and passes through the centre of the circle?

(3) Briefly define a tangent to a circle.

APPENDIX B

HIGHER LEVEL TEST

INSTRUCTION

(i) This test intends to measure your understanding of geometry, hence answer All questions honestly and to the best of your ability.

(ii) Tick [ ] or write the appropriate answer at the right column. Use back page if more space is required for answer and indicate the section and question number.

PART ONE: STUDENT ~~PERSONALITY~~ — Bio data

(1) Sex: Male [ ] Female [ ]

(2) Subject Group: Science [ ] Art [ ]

(3) State: Niger [ ] Abia [ ] Kano [ ] or Oyo [ ]

(4) Code Number:

....

PART TWO: HIGHER LEVEL TEST

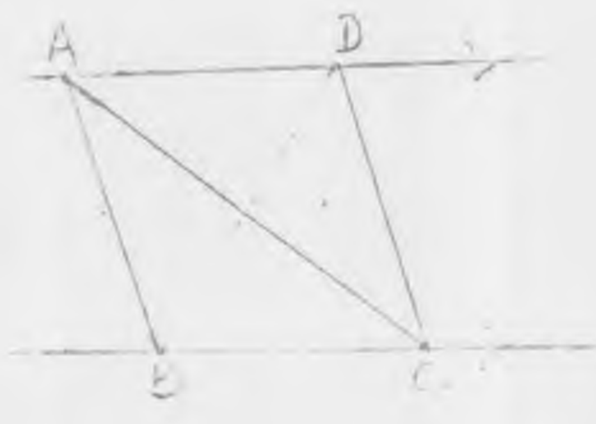
Questions

(1) Show that the opposite angles of a parallelogram are equal.

(2) Prove that sum of angles of a triangle is two right angles.

(3) In diagram C below ABC and DBG are two triangles having the same base and between the same parallel lines. Prove that their areas are equal.

Diagram C:

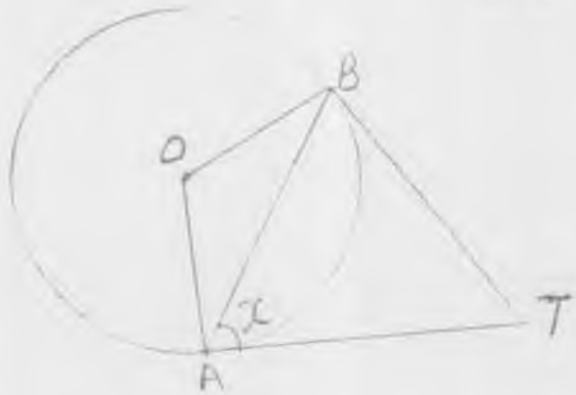


(4) In diagram D below O is the centre of the circle A, B, C, are points on the circumference prove that angle BOC = 2 X angle BAC

Diagram D:

(5) From diagram E below, TA is the tangent to the circle centre O, AB is a chord, if BAT = X show that BOA = 2X and Show that TA = TB

Diagram E:



APPENDIX C

ANSWERS TO LOWER LEVEL TEST

Section A:

(1a) iii
(1b) Square, rectangle, rhombus

(2) (i) Pairs of opposite sides are equal (ii) Pairs of opposite angles are equal (iii) Pairs of opposite sides are parallel and (iv) Diagonal bisect one another.

(3) (a) Parallelogram is a quadrilateral with pairs of opposite sides parallel and equal.

Section B:

(1) (a)

(2) (i) It has two equal sides (ii) It has two equal angles

(3) (a) False (b) It is a triangle that has two equal sides and two equal angles.

Section C:

(1) Triangle EDF

(2) No: Similar figures are alike but differs in magnitude

(3a) (ii)

(b) Congruent triangles are triangles that have two equal sides and one equal (included) angles or three equal sides.

Section D:

(1) (a) 1

(b) Ring, Tyres, Top surface of cylinder, cover of round tin, bowl etc.

(2) Opposite angle are supplementary

(3) Cyclic quadrilateral is a quadrilateral inscribed in a circle

Section E:

(1) (a) (ii)

(b)
Drawing
(2) Diameter

Tangent

(3) Tangent to circle is straight line that touches a point on circumference of a circle such that it is always perpendicular to the radius of the circle.

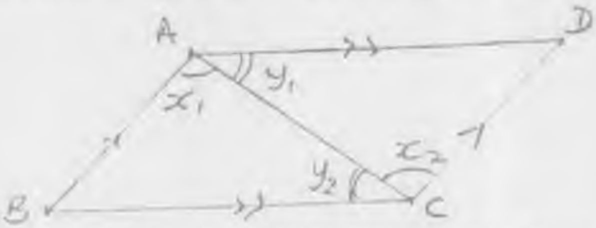
APPENDIX D:

ANSWERS TO HIGHER LEVEL TEST

(1) Construction: Draw diagonal AC

Proof: In triangles ABC and CDA, X1 = X2 ....
(alternate angles of parallel lines AB and DC)
Similarly Y1 = Y2 .... (alternate angles of parallel lines and AD and BC

AC is common therefore triangle ABC is congruent to CDA hence angle B = angle A = angle C.

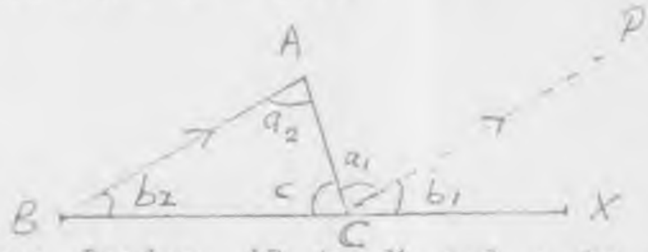


(2) Construction: Produce BC to X and draw CP parallel to BA
Proof: a1 = a2......(alternate angles of parallel lines)

Similarly b1 = b2....(corresponding angles)

c + a1 + b1 = 180....(angle in a straight line)

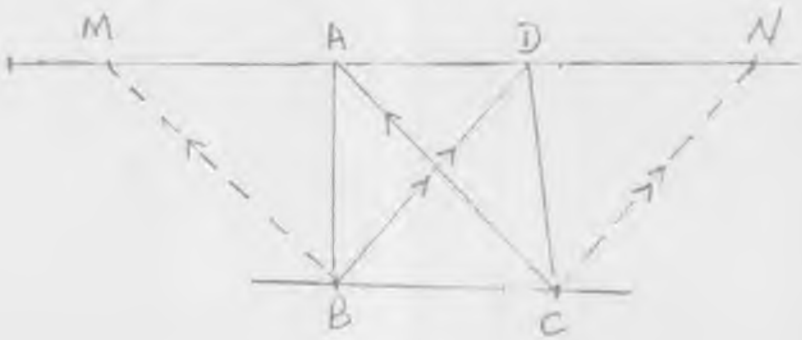
Therefore ACB + A + B = 180.



(3) Construction: Produce AD to M and n to left and right respectively such that MB AC and BD CN

Proof ABC = ½ BMAC.... (diagonal bisector of
BDC = ½ BDNC ......(diagonal bisector of

But BMAC = BDNC Therefore ABC = BDC.



APPENDIX E

SCORES IN LLT AND HLT FOR GENDER AND SUBJECT

GENDER								SUBJECT							
L L T				H L T				L L T				H L T			
MALE		FEMALE		MALE		FEMALE		SCIENCE		ART		SCIENCE		ART	
x	f	x	f	x	f	x	f	x	f	x	f	x	f	x	f
0	1	1	1	0	64	0	65	3	1	0	1	0	68	0	61
2	1	2	1	2	5	1	2	4	1	1	1	1	1	1	1
6	4	3	2	4	1	2	11	6	7	2	2	2	7	2	9
7	2	4	1	4	3	4	6	7	3	3	1	3	1	4	1
8	2	5	1	5	2	5	1	8	2	5	1	4	8	5	1
9	3	6	6	6	6	6	3	9	4	6	3	5	2	6	1
10	4	7	3	8	5	7	2	10	5	7	2	6	4	7	1
11	6	8	4	10	3	8	2	11	6	8	4	7	1	8	3
12	6	9	4	12	1	10	1	12	5	9	3	8	4	12	1
14	11	10	3	14	1	36	1	13	4	10	2	10	4	14	1
15	4	11	4	16	1			14	7	11	4	16	1	18	1
16	7	12	5	18	1			15	6	12	6	22	1	30	1
17	8	13	5	22	1			16	3	13	1	30	1	36	1
18	3	14	5	30	2			17	7	14	9	36	1		
19	8	15	7	36	1			18	5	15	5	40	1		
20	7	16	6	40	1			19	3	16	12				
21	3	17	6					20	9	17	5				
22	3	18	5					21	7	18	3				
23	3	19	4					22	4	19	9				
24	3	20	4					23	3	20	2				
25	2	21	5					24	3	21	1				
26	1	22	4					25	4	22	1				
28	1	23	2					26	1	23	2				
29	2	24	1					28	1	24	1				
30	1	25	2					29	2	29	1				
33	1							30	1	31	1				
35	2							31	1						
N 94		94		. 94		94		105		83		105		83	
X 15.93		14.52		3.56		1.56		16.01		14.23		2.92		2.11	
SD 6.01		6.37		7.60		4.21		6.36		5.92		6.69		5.86	
t-test (calc)		26.64				38.36				33.44				15.05	

APPENDIX E

SPLIT HALF CORRELATION FOR LLT

X	Y	X-X	Y-Y	$(X-X)^2$	$(Y-Y)^2$	(X-X)(Y-Y)	X	Y	X-X	Y-Y	$(X-X)^2$	$(Y-Y)^2$	(X-X)(Y-Y)
10	10	-5	-6	25	36	30	14	14	-1	-2	001	4	2
10	13	-5	-3	25	9	-15	11	13	-4	-3	016	9	12
24	16	9	0	81	0	0	04	06	-11	-10	121	100	110
21	21	6	5	36	25	30	00	01	-15	-15	225	225	225
06	14	-9	-2	81	4	18	09	12	-06	-4	016	16	24
12	29	-3	13	09	169	-39	06	12	-09	-4	081	16	36
11	08	-4	-8	16	64	32	10	12	-05	-4	025	16	20
25	19	10	3	100	9	30	20	21	05	5	25	25	25
06	14	-9	-2	81	4	18	09	06	-06	-6	036	36	36
18	20	3	4	08	16	12	17	17	02	1	004	1	02
19	17	4	1	16	1	4	15	19	00	3	000	09	00
14	20	-1	4	01	16	-4	03	06	-12	-6	144	36	72
22	23	7	7	49	49	49	19	24	04	8	016	64	32
12	15	-3	-1	09	1	3	20	31	05	15	025	225	75
17	19	2	3	04	9	6	14	15	-01	-1	001	1	01
18	25	3	9	09	81	27	17	16	02	0	004	00	00
08	21	-1	5	49	25	-35	13	10	-02	-6	004	36	12
23	12	8	-6	64	36	-42	14	14	-01	-2	001	4	2
12	15	-3	-1	09	1	3	08	13	-07	-3	049	9	21
21	15	6	-1	36	1	-6	18	17	03	1	009	1	3
10	17	-5	1	25	1	-5	20	23	05	7	025	49	35
12	18	-3	2	09	4	-6	12	18	-03	2	009	4	-6
16	12	1	-4	01	16	-4	14	11	-01	-5	001	25	5
06	07	-9	-9	81	81	81	11	07	-04	-9	016	81	36
02	06	-13	-10	169	100	131	16	14	01	-2	001	4	-2
03	-12	-10	144	100	120	20	23	05	7	025	049	49	35
05	18	-8	100	64	80	14	17	-01	1	001	1	1	-1
12	17	-3	-9	09	81	27	21	06	4	036	16	24	5
26	29	11	4	121	16	44	19	14	04	016	04	16	-8
19	21	4	5	16	25	20	18	17	02	1	009	1	3
11	17	4	-9	16	81	36	22	20	07	4	049	16	28
14	15	-1	-7	01	49	7	12	17	03	1	009	1	-3
11	09	-4	-7	16	49	28	20	30	05	1	001	1	1
08	15	-7	-1	49	1	7	20	30	05	14	025	196	70
20	08	5	-8	25	64	-40	15	16	00	0	000	0	0
15	22	0	6	00	36	0	16	16	01	0	001	0	0
19	23	4	7	16	49	28	22	21	07	5	049	25	35
17	19	2	3	04	9	06	13	16	-02	0	004	0	0
22	22	7	6	49	36	42	10	16	-05	0	004	0	0
07	11	-8	-5	64	25	40	-	-	-	-	-	-	-

11	11	-4	-5	16	25	20	1415	1518	3446	3465	21310
09	18	-6	2	36	4	-12	TOTAL	TOTAL			
06	18	-9	-2	81	4	18					
16	09	1	-7	01	49	-7					
11	24	-4	8	16	64	-32					
24	22	9	6	81	36	54					
15	21	0	5	00	25	0					
25	19	10	3	100	9	30					
16	29	1	13	01	169	13					
20	25	14	9	196	81	126					
28	31	13	15	169	225	195					
16	19	1	3	01	9	3					
15	14	0	-2	00	4	0					
14	18	-1	2	01	4	-2					
15	13	0	-3	00	9	0					

$$\bar{X} = \frac{1415}{94} = 15.05 = 15$$

$$\bar{Y} = \frac{1518}{94} = 16.15 = 16$$

$$r_1 = \frac{2131}{\sqrt{3448 \times 3465}} = \frac{2131}{3456.6} = 0.82$$

APPENDIX G

SPLIT HALF CORRELATION FOR BLC

X	Y	X-X	Y-Y	$(X-X)^2$	$(Y-Y)^2$	(X-X)(Y-Y)	X	Y	X-X	Y-Y	$(X-X)^2$	$(Y-Y)^2$	(X-X)(Y-Y)
0	0	-2	-3	4	9	6	0	2	-2	-1	4	1	2
0	0	-2	-3	4	9	6	8	2	6	-1	36	1	-6
0	0	-2	-3	4	9	6	0	0	-2	-3	4	9	6
0	0	-2	-3	4	9	6	0	0	-2	-3	4	9	6
0	2	0	-1	0	1	0	0	0	-2	-3	4	9	6
0	0	-2	-3	4	9	6	8	0	6	-3	36	9	-18
10	0	8	-3	64	9	24	2	18	0	15	0	225	0
0	0	-2	-3	4	9	6	8	4	6	1	36	1	6
0	0	-2	-3	4	9	6	14	8	12	5	144	25	60
4	6	2	3	4	9	6	0	2	-2	-1	4	1	2
0	0	-2	-3	4	9	6	2	0	0	-3	0	9	0
0	0	-2	-3	4	9	6	0	0	-2	-3	4	9	6
0	6	-2	3	4	9	-6	0	0	-2	-3	4	9	6
5	0	3	-3	9	9	-9	0	0	-2	-3	4	9	6
0	0	-2	-3	4	9	6	0	0	-2	-3	4	9	6
0	1	-2	-2	4	4	4	0	0	-2	-3	4	9	6
6	8	-4	5	16	25	20	0	0	-2	-3	4	9	6
0	8	-4	5	4	25	-10	0	0	-2	-3	4	9	6
8	4	6	1	36	1	6	0	0	-2	-3	4	9	6
1	2	-1	-1	1	1	1	0	4	-2	1	4	1	-2
4	2	2	-1	4	1	-2	2	0	0	-3	0	9	0
2	6	0	3	0	9	0	0	0	-2	-3	4	9	6
0	2	-2	-1	4	1	2	3	2	1	-1	1	1	-1
0	0	-2	-3	4	9	6	2	2	0	-1	0	1	0
0	0	-2	-3	4	9	6	2	4	0	1	0	1	0
10	0	8	-3	64	9	24	0	0	-2	-3	4	9	6
0	0	-2	-3	4	9	6	0	0	-2	-3	4	9	6
0	8	-2	3	4	9	6	0	0	-2	-3	4	9	6
0	0	-2	-3	4	9	6	0	0	-2	-3	4	9	6
0	0	-2	-3	4	9	6	2	0	0	-3	0	9	0
0	40	-2	37	4	1369	-74	0	0	-2	-3	4	9	6
0	0	-2	-3	4	9	6	0	0	-2	-3	4	9	6
0	0	-2	-3	4	9	6	0	0	-2	-3	4	9	6
0	0	-2	-3	4	9	6	0	4	-2	1	4	1	-2
0	0	-2	-3	4	9	6	0	0	-2	-3	4	9	6
22	36	20	33	400	1089	660	0	5	-2	2	4	4	-4
30	30	28	27	784	729	756	0	0	-2	-3	4	9	6
0	8	-2	-3	4	9	6	0	0	-2	-3	4	9	6
0	0	-2	-3	4	9	6			5	12	3 9	9 81	27
0	0	-2	-3	4	9	6	4	6	2	3	4	9	6

16	36	24	33	196	1089	462	0	7	-2	4	4	16	-8	
0	0	-2	-3	4	9	6	10	8	8	-3	64	9	24	
1	0	-2	-3	4	9	6	0	0	-2	-3	4	9	6	
1	0	-2	-3	4	9	6	0	0	-2	-3	4	9	6	
0	0	-2	-3	4	9	6	4	7	2	1	4	16	8	
	0	-2	-3	4	9	6								
1	0	-2	-3	4	9	6	194	278	TOTAL		2176	5314		2249
0	0	-2	-3	4	9	6								
0	0	-2	-3	4	9									
0	0	-2	-3	4	9	6								

$X = \Sigma \Sigma = 3$

$$R_i = \frac{2249}{\sqrt{2176 \times 5314}} = 0.659$$

APPENDIX E

SCORES IN LLT AND HLT FOR GENDER AND SUBJECT

	GENDER								SUBJECT							
	L L T				H L T				L L T				H L T			
	MALE		FEMALE		MALE		FEMALE		SCIENCE		ART		SCIENCE		ART	
	x	f	x	f	x	f	x	f	x	f	x	f	x	f	x	f
	0	1	1	1	0	64	0	65	3	1	0	1	0	68	0	61
	3	1	2	1	2	5	1	2	4	1	1	1	1	1	1	1
	6	4	3	2	4	1	2	11	6	7	2	2	2	7	2	9
	7	2	4	1	4	3	4	6	7	3	3	1	3	1	4	1
	8	2	5	1	5	2	5	1	8	2	5	1	4	8	5	1
	9	3	6	6	6	6	6	3	9	4	6	3	5	2	6	1
	10	4	7	3	8	5	7	2	10	5	7	2	6	4	7	1
	11	6	8	4	10	3	8	2	11	6	8	4	7	1	8	3
	12	6	9	4	12	1	10	1	12	5	9	3	8	4	12	1
	14	11	10	3	14	1	36	1	13	4	10	2	10	4	14	1
	15	4	11	4	16	1			14	7	11	4	16	1	18	1
	16	7	12	5	18	1			15	6	12	6	22	1	30	1
	17	6	13	5	22	1			16	3	13	1	30	1	36	1
	18	3	14	5	30	2			17	7	14	9	36	1		
	19	9	15	7	36	1			18	5	15	5	40	1		
	20	7	16	6	40	1			19	3	16	10				
	21	3	17	6					20	9	17	5				
	22	3	18	5					21	7	18	3				
	23	3	19	4					22	4	19	9				
	24	3	20	4					23	2	20	2				
	25	2	21	5					24	3	21	1				
	26	1	22	4					25	4	22	3				
	28	1	23	2					26	1	23	2				
	29	2	24	1					28	1	24	1				
	30	1	25	2					29	2	29	1				
	29	1							30	1	31	1				
	31	2							31	1						
N	94		94		94		94		105		83		105		83	
X	15.93		14.52		3.56		1.56		16.01		14.23		2.92		2.11	
SD	6.01		6.37		7.80		4.21		6.36		5.92		6.69		5.86	
t-test (calc)	26.64				38.36				33.44				15.05			