# RELIABILITY ANALYSIS OF REINFORCED CONCRETE BRIDGE DECKS SUBJECT TO FATIGUE

**BY**

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**(M.Sc/ENG/31106/2012-2013)**

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**ZARIA, NIGERIA.**

**JANUARY, 2016**

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**A DISSERTATION SUBMITTED TO THE SCHOOL OF POSTGRADUATE STUDIES,**
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**ZARIA-NIGERIA**

**JANUARY, 2016**

## DECLARATION

I hereby declare that the works in this thesis titled "**Reliability Analysis of Reinforced Concrete Bridge Decks Subject to Fatigue"** was carried out by me in the Department of Civil Engineering, under the supervisions of Dr. A. Ocholi and Dr. J.K. Mohammed. All sources of information which are not originally mine have been acknowledged in the reference.

_____ _____

Thomas Unogwu Osumeje Date

## CERTIFICATION

This thesis titled "**Reliability Analysis of Reinforced Concrete Bridge Decks Subject to Fatigue"** by **Thomas Unogwu Osumeje** meets the regulations governing the award of the degree in masters of Science in Civil Engineering, Ahmadu Bello University, and is approved for its contribution to knowledge and literary presentation.

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## DEDICATION

I dedicate this work to the almighty God for sustenance and grace to carry out this work successfully and also to my late brother James Osumeje.

## ACKNOWLEDGEMENTS

First and foremost, all praise and honour be to the almighty God for his guidance and endeavours through life.

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## ABSTRACT

In this thesis the reliability analysis of reinforced concrete bridge decks subjected to high cycle fatigue was undertaken. A simply supported (single span) reinforced concrete bridge deck was specifically used for the investigation. The statistical models of capacity loss were derived. The uncertainties in structural resistance and the applied loadings were fully accommodated using probabilistic method. The limit state function(Failure Modes) for the flexural capacity of the deck slab, shear capacity of the deck beam and flexural capacity of the deck beam girder was developed and evaluated using the First Order Reliability Method(FORM). The entire process was implemented through a developed MATLAB program "dc_fatigue.m". Analysis was carried out on all the failure modes considered by varying the geometrical and material properties of the system and their respective safety indices were determined. Failure due to shear in the deck beam gave the least safety index range of 6.06 to -2.11 at 3cycles/min for 10years and 40years respectively indicating the most critical section, when compared to the flexural failure with a safety index range value of 7.59 to 0.25 at 3cycles/min for 10years and 40years respectively. It was generally observed that the reliability index increased as the depth of section and concrete grade increased. Also, the coefficient of variation due to the concrete strength and depth of section decreased with increase in reliability index, from a safety index range of 7.61 to -0.47 at 5% covariance and from 3.47 to -0.23 at 20% covariance under stress load of 3cycles/min. This trend is expected as quality control plays a very important role in achieving this level of safety in the system. Thus, for a single span bridge of 15m and less under high level of traffic load, a deck beam depth of 1200mm, deck slab of 250mm and concrete strength class of 30-35 are adequate for design and construction.

## TABLE OF CONTENTS


| Content | | | Page |
|---|---|---|---|
| Title page | | ii | |
| Declaration | | | iii |
| Certification | | | iv |
| Dedication | | | v |
| Acknowledgement | | | vi |
| Abstract | | | vii |
| Table of Contents | | | viii |
| List of Figures | | | xi |
| List of Tables | | | xv |
| List of Appendices | | | xv |
| List of Symbols/Abbreviation | | | xvii |
| **CHAPTER ONE** | | | |
| **1.0** | **INTRODUCTION** | | |
| 1.1 | Preamble | | 1 |
| 1.2 | Problem Statement | | 3 |
| 1.3 | Justification of Study | | 4 |
| 1.4 | Aim and Objectives | | 6 |
| 1.4.1 | Aim | 6 | |
| 1.4.2 | Objectives | 6 | |
| 1.5 | Scope and Limitation | | 7 |
| 1.5.1 | Scope | 7 | |
| 1.5.2 | Limitation | | 7 |

## CHAPTER TWO


**2.0 LITERATURE REVIEW**

2.1 Preamble 8

2.2 Basis of Fatigue 10

2.3 Classification of details in concrete structural component 11

2.4 Fatigue 12

2.4.1 Fatigue of concrete 12

2.4.2 Fatigue of reinforcement 13

2.4.3 Fatigue of reinforced concrete 14

2.5 Fatigue failure of reinforced concrete structures 14

2.5.1 Shear and Bond failures 15

2.6 Structural Reliability 16

2.7 Limit State Principles 16

2.7.1 Failure Events and Basic Random Variables 18

2.7.2 Reliability Index 20

2.7.3 Hasofer-Lind Reliability Index 21


## CHAPTER THREE


**3.0 RESEARCH METHODOLOGY**

3.1 Estimation of parameters for reliability of bridge deck 23

3.2 Bridge Model 23

3.3 Structural Material 24

3.4 Load Combination 24

3.4.1 Dead Load Effect 25

3.4.2 Live Load Effect 25

3.5 Structural Resistance Models 28

3.6 Generation of Limit State Function 29

3.6.1 Failure of the Slab 30

3.6.1.1 Applied Dead Load 30
3.6.2 Failure Model for Beams 31
3.6.2.1 Interior Beams32
3.6.2.1.1 Limit State Function for Shear in Interior Beam 33
3.6.2.2 Limit State Function for Moment Flexure in Interior Beam 34
3.7 Computer Analysis Procedure 36
3.7.1 MATLAB Program 37

**CHAPTER FOUR**

**4.0 RESULT DISCUSSION**

4.1 Failure due to moment flexure in Slab (Failure Mode One) 39
4.1.1 Effect of deck slab depth variation on the Safety Index 39
4.1.2 Effect of Coefficient of Variation due to concrete strength under bending on the Safety Index with respect to the stress cycle 40
4.1.3 Effect of Coefficient of Variation due to slab depth under bending on the Safety Index with respect to the stress cycle 42
4.2 Failure due to shear in deck beam(Failure Mode Two) 44
4.2.1 Effect of beam depth variation on the Safety Index with respect shear stress under stress cycle 44
4.2.2 Effect of concrete strength variation on the Safety Index with respect to stress cycle46
4.2.3 Effect of coefficient of variation due to concrete strength under shear on the safety index with respect to the stress cycle. 47
4.2.4 Effect of coefficient of variation due to beam depth under shear on the safety index with respect to the stress cycle 49
4.3 Failure due to Moment Flexure (Failure Mode Three) 50
4.3.1. Effect of beam depth variation on the Safety Index with respect

to flexure under stress Cycle 51

4.3.2 Effect of concrete strength variation under bending on the Safety Index with respect to stress cycle 52

4.3.3 Effect of coefficient of variation due to concrete strength under flexure on the Safety Index with respect to the stress cycle 54

4.3.4 Effect of coefficient of variation due to beam depth under flexure on the Safety Index with respect to the stress cycle 56

**CHAPTER FIVE**

**5.0 CONCLUSION AND RECOMMENDATION**

5.1 Conclusion 59

5.2 Recommendation 59

**REFERENCES** 60

## LIST OF FIGURES

Figure 2.1: Possible shear failure modes in beams with stirrups 15

Figure 2.2: Probability density function of Load, Resistance and Safety Margin 22

Figure 2.3: Reliability Index as shortest distance to origin 22

Figure 2.4: Hasofer-Lind reliability index 22

Figure 3.1: Notional Lane Arrangement on the Carriage Way26

Figure 3.2:Traffic Load in Bridge (load model 1, LM1)27

Figure 3.3:Portion of slab supported by two interior beams30

Figure 3.4:Beam cross sectional dimension 32

Figure 3.5:Arrangement of LM1 for Critical Shear 33

Figure 3.6:Arrangement of point of maximum bending on LM1 35

Figure 3.7: Flow Chart for MATLAB Implementation of FORM 37

Figure 4.1:Safety Index Versus Stress Cycle (per min) at Slab Deck

Depth of 200mm 39

Figure 4.2:Safety Index Versus Stress Cycle (per min) at Slab Deck

Depth of 250mm 39

Figure 4.3:Safety Index Versus Stress Cycle (per min) at Slab Deck

Depth of 300mm 40

Figure 4.4:Safety index Versus Stress Cycle at 5% COV due to Concrete 41

Figure 4.5:Safety index Versus Stress Cycle at 10% COV due to Concrete 41

Figure 4.6:Safety index Versus Stress Cycle at 20% COV due to Concrete 42

Figure 4.7:Safety Index Versus Stress Cycle at 5% COV due to Slab Depth 42

Figure 4.8:Safety Index Versus Stress Cycle at 10% COV due to Slab Depth 43

Figure 4.9:Safety Index Versus Stress Cycle at 20% COV due to Slab Depth 43

Figure 4.10:Safety Index Versus Stress Cycle (per min) at Beam Depth

of 1000mm 44

Figure 4.11:Safety Index Versus Stress Cycle (per min) at Beam Depth of 1100mm 44

Figure 4.12:Safety Index Versus Stress Cycle (per min) at Beam Depth of 100mm 45

Figure 4.13:Safety Index Versus Stress Cycle (per min) at $f_{cu}$=25N/mm$^2$ 46

Figure 4.14:Safety Index Versus Stress Cycle (per min) at $f_{cu}$=30N/mm$^2$ 46

Figure 4.15:Safety Index Versus Stress Cycle (per min) at $f_{cu}$=35N/mm$^2$ 46

Figure 4.16:Safety Index Versus Stress Cycle (per min) at $f_{cu}$=40N/mm$^2$ 47

Figure 4.17:Safety Index Versus Stress Cycle (per min) at 5% COV due to Concrete 47

Figure 4.18:Safety Index Versus Stress Cycle (per min) at10% COV due to Concrete 48

Figure 4.19:Safety Index Versus Stress Cycle (per min) at20% COV due to Concrete 48

Figure 4.20:Safety Index Versus Stress Cycle (per min) at 5% COV due to Beam Depth 49

Figure 4.21:Safety Index Versus Stress Cycle (per min) at 10% COV due to Beam Depth 49

Figure 4.22:Safety Index Versus Stress Cycle (per min) at 20% COV due to Beam Depth 50

Figure 4.23:Safety Index Versus Stress Cycle (per min) at Beam Depth =1000mm 51

Figure 4.24Safety Index Versus Stress Cycle (per min) at Beam

Depth =1100mm 51

Figure 4.25:Safety Index Versus Stress Cycle (per min) at Beam Depth =1200mm 52

Figure 4.26:Safety Index Versus Stress Cycle (per min) at $f_{cu}$=25N/mm$^2$ 53

Figure 4.27:Safety Index Versus Stress Cycle (per min) at $f_{cu}$=30N/mm$^2$ 53

Figure 4.28:Safety Index Versus Stress Cycle (per min) at $f_{cu}$=35N/mm$^2$ 53

Figure 4.29:Safety Index Versus Stress Cycle (per min) at $f_{cu}$=40N/mm$^2$ 54

Figure 4.30:Safety Index Versus Stress Cycle (per min) at 5% COV due to Concrete 55

Figure 4.31:Safety Index Versus Stress Cycle (per min) at 10% COV due to Concrete 55

Figure 4.32:Safety Index Versus Stress Cycle (per min) at 20% COV due to Concrete 55

Figure 4.33:Safety Index Versus Stress Cycle (per min) at 5% COV due to Depth 57

Figure 4.34:Safety Index Versus Stress Cycle (per min) at 10% COV due to Depth 57

Figure 4.35:Safety Index Versus Stress Cycle (per min) at 20% COV due to Depth 57

## LIST OF TABLES


| | | |
|---|---|---|
| Table 3.1: | Characteristic value for Load Model 1(LM1) | 27 |
| Table 3.2: | Statistical Model of the Basic Design Variables in Slab | 29 |
| Table 3.3: | Statistical Model of the Basic Design Variables in Beam | 32 |

## LIST OF APPENDICES


Appendix I: MATLAB Program 64

Appendix II: Safety Index chart 112

Appendix III: Eurocode Design criteria for Reinforced Concrete deck 122

## LIST OF SYMBOL/ABBREVIATION

| | |
|---|---|
| A | Cross-Sectional Area |
| $A_s$ | Cross-Sectional Area of Steel |
| $A_s$ | Critical shear for live load |
| $b$ | Width of Section |
| d | Effective Depth |
| CDF | Cumulative Distribution Function |
| COV | Coefficient of Variation |
| COVQ | Coefficient of variation of load |
| COVR | Coefficient of variation of resistance |
| $F$ | Frabrication |
| $f_{yd}$ | Steel Strength |
| $f_{cd}$ | Concrete Compressive Strength |
| $f_{\mathbf{x}}(x)$ | Joint Probability Density Function of the Random Variables $\boldsymbol{X}$ |
| $G_{asph}$ | Dead Load due to Asphalt |
| $G_{conc}$ | Dead Load due to Concrete |
| $G_k$ | Dead Load |
| G(x) | Limit State Function equation |
| h | Depth of Section |
| L | length of member section |
| LM | Load Model |
| $M$ | Material Properties |
| $M_{dl}$ | Dead Load Moment |
| $M_{Ed}$ | Total Applied Moment |
| $M_{LL}$ | Live Load Moment |

$M_{max}$ Maximum Bending Moment

$M_{qk}$ Moment due to Dynamic Live Load

$M_{Qk}$ Moment due to Static Live Load

$M_{Rd}$ Design Resistance or Capacity

$n_i$ Maximum number of Notional Lanes

$P$ Structural Modelling

$P_f$ Probability of Failure

Q Represents the load Effect

$Q_k$ Static Live Load

$Q_k$ Live Load Moment

$R$ Represents the Resistance

$R$ Resistance

$R_n$ Nominal Resistance

$V_{dt}$ Dead load Shear

$V_{Ed}$ Applied Shear

$V_{Rd}$ Ultimate Shear Capacity

$V_{ll}$ Live load Shear

$w$ Carriage Way Width

$w_r$ Maximum Width of remaining Area of Carriage Way

$Z_R$ Reduced Random Variable for resistance

$Z_Q$ Reduced Variable for load

$\alpha_{Qi}$ and $\alpha_{qi}$Load Adjustment Factors

$\beta$ Safety Index

$\delta y$ Deflection change

$q_k$ Dynamic Live Load

$\sigma_R$ The Standard Deviation of Resistance

$\sigma_Q$ The Standard Deviation of load

$\mu_r$ The Mean Resistance,

$\mu_Q$ The Mean Load

$\emptyset_c$ Uncertainty Coefficient due to Concrete

$\emptyset_{asph}$ Uncertainty Coefficient due to Asphalt

$\emptyset_{Qk}$ Uncertainty Coefficient due to Live Load

$\emptyset_R$ Uncertainty Resistance Model Coefficient

$\Phi^{-1}$ Inverse standard normal distribution function

# CHAPTER ONE

## INTRODUCTION

### 1.1 Preamble

Researches on fatigue behaviour of concrete materials started at the end of the 19$^{th}$ century, due to failure in many concrete structures caused by fatigue rupture of the concrete (Rteil et.al, 2011). Results of experimental and theoretical studies of the fatigue properties of plain concrete, reinforcing bars, prestressing tendons, and also of structural concrete members, have been accumulating steadily over the past thirty years.

The assessment of existing structures will become a more frequent task for engineers in the near future due to the increasing age of existing infrastructure. These may be due to reasons such as(Jansen, 1996); Change in intended use of the structure, new regulations with higher load requirements for the structure, indications of ongoing deterioration in the structure, unusual incidents during use (e.g. vehicle impact, fire, earthquakes), inadequate serviceability, discovery of design or construction errors.

The purpose of reliability analysis in this research is to verify the overall stability and establishment of action effects, i.e. the distribution of internal forces and moments. In turn, this will enable the calculation of stresses, strains, curvature, rotation and displacements. In certain complex structures, the type of analysis used (e.g. finite-element analysis) will yield internal stresses and strains and displacements directly.

To carry out the analysis, both the geometry and the behaviour of the structure will need to be idealized. Commonly, the structure is idealized by considering it as made up of elements depicted.

Some of the causes in variation of uncertainty in structural analysis and design by Svensson (1997) include: the material properties, structural properties of components, load variation, parameter estimations and model errors.

Even if Svensson's analysis is sufficiently accurate, it is necessary to note that there is a fundamental difference among the causes described. The first three items are actual random variables and they constitute the casualty of the investigated phenomenon. On the contrary, the fourth and fifth items are caused by a lack of knowledge of the true parameters involved (usually due the limited amount of experimental data) or of the fatigue damage mechanism under variable-amplitude loading or of the structural analysis of the component investigated. These last items cause an additional uncertainty margin to evaluation of the actual variability of the fatigue phenomena.

According to this judgement, the first group implies a problem in estimating a random variable, while the second group relates to the evaluation of variance and proper confidence interval on the obtained estimation. Thus, an accurate assessment of fatigue strength under service loading should include both correct estimation of each statistical parameter involved and sound evaluation of the confidence interval of each estimated quantity, particularly when the overall probability of failure is concerned.

Design of reinforced concrete bridges is normally done on the basis of structural analysis. The purpose of the analysis is to find a distribution of sectional forces which fulfils limit state criteria and is suitable for design. In the past, structural analyses were often done with simplified models, for example two-dimensional (2D) equivalent beam or frame models. Such a model is not able to describe the distribution of forces in transverse direction.

Therefore a design according to a 2D equivalent model will not give a true linear elastic distribution, even though the design might fulfil requirements in ultimate limit state (ULS) after sufficient plastic redistribution(Karin et.al,2010).

Current practice for the design of reinforced concrete bridges is based on a linear elastic structural analysis in which a suitable distribution of sectional forces is sought. Such an analysis is today required to account for all the structural response, implying a demand for three-dimensional (3D) models capable of describing the force distribution in longitudinal and transverse directions.

### 1.2 Problem Statement

Deterioration of reinforced concrete bridge decks has been a major problem in the country. Both Federal Ministry of Works and Federal Roads Maintenance Agency records show that as at the year 2008, there were 1,696 Bridges that had been completed from year to year and were in use on the Federal Highways nationwide. However,as at date some of these bridges are requiring emergency intervention and some have even been shut to traffic. According to the data of the "national bridge inventory" obtained from the U.S. Department of Transportation, it is estimated that deficiencies occur mostly in the decks in more than half of the bridges in United States (Mabsout et.al 2004).

It is not only an economic problem but also risk to commuters who traverse these bridges. Some of the deficiencies may include; slight damages to the deck surfaces, spalling of top concrete surface and the decrease in serviceability of the deck which may lead to danger to the public. As such there is the need to understand the behaviour of bridge deck under service conditions and develop a reliable way to check the serviceability of the deck.

In the Nigeria, most of the bridge decks are constructed as reinforced concrete slabs supported by precast prestressed beam girders. Such decks have traditionally been designed using the "empirical strip method", based on a conventional beam theory, which assumed that the slab is continuous over fixed supports. As a result, the top part of the slab is reinforced with steel bars to resist the negative moments, and the bottom part of the slab is reinforced with steel bars to resist the positive moments. It is believed that the decks designed by empirical method are more resistant to deterioration due to fewer sources of corrosion (fewer steel rebars). This empirical method has been adopted in the current AASHTO LRFD code (2010). In bridges, there are usually a large number of cycles of significant live load, and fatigue will almost always precede fracture. Therefore, controlling fatigue is practically more important than controlling fracture because this can result in increased displacement of the reinforcing bar relative to the concrete, known as slip (Rteil et.al, 2011).

Therefore, in this thesis, a reliability based analysis with a probabilistic view would be carried out to check the uncertainties in structural resistance and applied loads. Limit state function for flexural capacity of deck slab and beam as well as shear capacity in deck beam were developed and evaluated using FORM. The entire process was implemented through a developed MATLAB program.

### 1.3 Justification of the Study

In the past, fatigue failure has been primarily regarded as a design problem associated with civil engineering structures such as highways bridges, car parks and rail bridges which are subjected to reasonably well defined man-induced repeated loads. However, serious

fatigue problems can also arise as the result of complex, naturally occurring, ill-defined patterns of repeated loads. For example, fatigue problems in concrete structures have arisen recently in relation to off-shore and marine construction(Karin et.al, 2010).

In RC slab damage process, microcracks occur and gradually develop, and at the final stage, large cracks occur and slab displacement increases sharply. Mid-span deflection caused by fatigue is thought to be a possible indicator for the degree of slab damage under the influence of overall progress of slab cracking.The fatigue lives of flexural members are typically a function of the stress range induced in the reinforcement (Braimah et.al,2006). It is practically hard to exactly interpret the loading histories of bridge deck.

The failure can occur even if the maximum stress is below the ordinary strength of the material. Some materials have a certain stress limit e.g. steel, which means that the stress variation below a certain level can be repeated infinitely many times without fatigue failure. Fatigue failure is characterized by fracture in a localized area of a structure which is exposed to cyclic loading (Karin et.al, 2010).

A single lane of a Class A highway bridge experiences an average daily truck traffic of over 1000, or 27 million trucks over a 75 year design life (CSA A23.3, 2010).Bridge decks directly sustain these repeated moving wheel loads and may be susceptible to fatigue damage (El-Ragaby et al. 2007). Fatigue life and fatigue strength of reinforced concrete elements areinfluenced by many factors including the material properties of the concrete and reinforcement, reinforcement ratio, transverse reinforcement, minimum and maximum values of repeated loading, range and rate of loading as well as environmental factors such as temperature and humidity. The response of a member subjected to fatigue loading is

affected by both the strength of the materials and the interaction between the concrete and reinforcement (Higgins et al. 2006). It is generally agreed that the most important factor influencing fatigue behaviour is the applied load range,or rather the induced stress range in each component.

The results of this research can be usefully employed in practice only when appropriate fatigue design formats have been developed which provide adequate, but not excessive, margins of safety against fatigue failure. The safety margins and the associated design safety coefficients would allow realistically for inherent variability in loads, in the fatigue properties of the materials, and in the performance of members and systems under high cyclic loads.

## 1.4 Aim and Objectives

### 1.4.1 Aim

The aim of this research is to assess the fatigue reliability of a typical RC bridge deck under limit state condition.

### 1.4.2 Objectives

The objectives of the study include;

i. Identify the possible fatigue modes of failure of the bridge deck.

ii. Develop a performance function for each mode of failure.

iii. Establish the statistical models for each random variables.

iv. Develop a MATLAB program to evaluate the reliability functions, and compute probabilities of failure and safety indices.

v. Carry out sensitivity analysis using the developed program.

## 1.5 Scope and Limitation

### 1.5.1 Scope

The research project was to treat fatigue failure in reinforced concrete bridge deck. It is devoted to the testing of the time-dependent safety of RC bridge deck using statistical tools**MATLAB** and **FORM**.In order to simplify and emphasize the fatigue analysis, resistance models and load values together with information on their stochastic parameters have been developed, evaluated and implemented in accordance with the provisions of the Eurocode system.

### 1.5.2 Limitations

This work is limited to numerical analysis and not laboratory work was carried out.

# CHAPTER TWO

## LITERATURE REVIEW

### 2.1 Preamble

A large number of concrete bridge structures throughout the country are now being considered for repair or replacement. Short span bridges comprise a major portion of the national bridge inventory. According to data from the Federal Ministry of Works and Federal Road Maintenance Agency, there were 1,705 Bridges that had been completed and were in use along the Federal Highways nationwide as at the year 2010. However, as at date more than 20 of these bridges are said to be structurally deficient or functionally obsolete and some have even been shut to traffic.Alternatively, bridges and box culverts can be constructed from precast segments to minimize construction time and ensure a quality concrete product. Post-tensioning is often used in order to reduce member dimensions, create watertight sections, limit cracking and control deflection. Recently, the use of FRP reinforcement in concrete slab bridges has been studied at the University of Missouri in Rolla and implemented in the widening and rehabilitation of the Southview Bridge (Fico et al. 2006 and Galati et al. 2004).

During the life cycle of a bridge, variable stress range cycles due to multiple random dynamic loads might lead to fatigue damage accumulations at structure's details. Such damages might develop into micro cracks and lead to serious structure failures in

components or for the whole structure. A systematic scheme for fatigue life prediction and reliability evaluation is still challenging despite the extensive progress on the modeling of vehicle–bridge dynamic interactions and fatigue damage accumulation rules(Deng et.al, 2010)

The coupling forces were proven to be significantly affected by the vehicle speed and road roughness conditions and resulted in significant effects on the dynamic responses of short span bridges

The road surface conditions progressively deteriorate and lead to an increase of fatigue damage accumulation rate. Therefore, the nonlinearities of the fatigue damage accumulations would be neglected and might lead to a large underestimation of fatigue damages, which are usually ignored from many research scopes(Zhang et.al, 2012)

Variable stress ranges from dynamic vehicle loads can induce fatigue damage accumulations at certain bridge components and accelerate the road surface deterioration in bridges' life cycle. The interactions of the road surface deterioration and random dynamic vehicle loads might accelerate the fatigue damage accumulations and lead to serious fatigue failures when such damages increase to a certain limit (Wei et.al, 2013).

Bridges do not usually fail due to inadequate load capacity, except when an overweight truck is illegally driven onto an old bridge with very low load rating. When bridge superstructures "fail," it is usually because of excessive deterioration by corrosion and/or fatigue cracking rather than inadequate load capacity. Although most deterioration can be attributed to lack of proper maintenance, there are choices made in design that also can have an impact on service life. Yet the design process for bridges is focused primarily on

load capacity rather than durability. Only aspects of fatigue and fracture that are relevant to design or assessment of bridge deck and superstructure components are discussed.

The problems that occur are confined to older bridges. These problems are, for the most part, relatively minor and can be corrected with inexpensive retrofits. The problems are even easier to avoid in new designs. Therefore, because there are some fatigue problems with older bridges, one should not get the impression that there are ongoing fatigue problems with modern bridges designed by the present fatigue-design specifications.

In bridges, there are usually a large number of cycles of significant live load, and fatigue will almost always precede fracture. Therefore, controlling fatigue is practically more important than controlling fracture.

The fatigue and fracture design procedures outlined in this chapter are included in the American Association of State Highway and Transportation Officials (AASHTO, 2012) specifications for bridges. Some of the bridges built before the mid-1970s (when the present fatigue-design specifications were adopted) may be susceptible to fatigue cracking.

There are valuable lessons that can be learned from the problems that these bridges experienced, and several examples will be used in this chapter to illustrate various points. The histories of fatigue cracking should not create the false impression that there is an inherent fatigue problem with steel bridges.

## 2.2Basis of Fatigue

Fatigue is a phenomenon where a material loses its original strength due to cyclic loading with successive damage development. Fatigue in concrete depends on the load amplitude

and the number of cycles as well as on the stress level. For steel, the amplitude and number of cycles cause the fatigue.

The failure can occur even if the maximum stress is below the ordinary strength of the material. Some materials have a certain stress limit e.g. steel, which means that the stress variation below a certain level can be repeated infinitely many times without fatigue failure. Fatigue failure is characterized by fracture in a localized area of a structure which is exposed to cyclic loading.

When a structure fails due to fatigue loading, the structure has reached its fatigue life. There are two types of fatigue loading that can result in different failure characteristics. They are called Low-cycle fatigue and High-cycle fatigue. Low-cycle fatigue means that the load is applied at high stress levels for a relatively low number of cycles, while the High-cycles fatigue corresponds to a large number cycles at lower stresses.

### 2.3Classification of Details in Concrete Structural Components

Concrete structures are typically less sensitive to fatigue than welded steel and aluminium structures. However, fatigue may govern the design when impact loading is involved, such as pavement, bridge decks, and rail ties. Also, as the age of concrete girders in service increases, and as the applied stress ranges increase with increasing strength of concrete, the concern for fatigue in concrete structural members has also increased.

The fatigue strength of straight, unwelded reinforcing bars and prestressing strand can be described in terms of the categories for steel details described with the Category B,*S*–*N* curve.

ACI Committee 215 report recommends that half of the stress range for straight bars be used, i.e., 69 MPa(69N/mm$^2$) for the worst-case minimum stress. Equating this recommendation to the *S–N* curves for steel details, bent reinforcement may be treated as a Category D detail. If prestressed members are designed with sufficient precompression that the section remains uncracked, there is not likely to be any problem with fatigue. This is because the entire section is resisting the load ranges and the stress range in the prestressing strand is minimal. Similarly, for unbonded prestressed members, the stress ranges will be very small. However, there is reason to be concerned for bonded prestressing at cracked sections because the stress range increases locally. The concern for cracked sections is even greater if corrosion is involved. The pitting from corrosive attack can dramatically lower the fatigue strength of reinforcement (Adamu et.al 2014).

**2.4Fatigue**

A single lane of a Class A highway bridge experiences an average daily truck traffic of over 1000, or 27 million trucks over a 75 year design life (CSA A23 2010). Bridge decks directly sustain these repeated moving wheel loads and may be susceptible to fatigue damage (El-Ragaby et al. 2007a). Fatigue life and fatigue strength of reinforced concrete elements are influenced by many factors including the material properties of the concrete and reinforcement, reinforcement ratio, transverse reinforcement, minimum and maximum values of repeated loading, range and rate of loading as well as environmental factors such as temperature and humidity.The response of a member subjected to fatigue loading is affected by both the strength of the materials and the interaction between the concrete and reinforcement (El-Ragaby et al. 2007b, Higgins et al. 2006). It is generally agreed that the

most important factor influencing fatigue behaviour is the applied load range, or rather the induced stress range in each component.

Fatigue of composite materials has been studied for many decades, although the majority of the work done has focused on aerospace, marine and transportation applications and is not generally applicable to the construction field. Only a limited number of studies are available on the fatigue behaviour of reinforcement in concrete structures for civil engineering applications.

### 2.4.1 Fatigue of Concrete

Fatigue in concrete was recognized rather late, in comparison to steel. Concrete is a non-homogenous material and its fatigue resistance is influenced by many different factors e.g. moisture content, cement/water ratio and load effects such as load frequency and maximum load level. During the hardening period air bubbles and micro-cracks are formed. The micro-cracks appear due to thermal strain, which is caused by temperature variations. When the micro-cracks propagate the fatigue process starts, which is a progressive process. At the beginning of the loading the propagation of the micro-cracks is rather slow. As loading continues the micro-cracks will proceed propagate and lead to macro-cracks, which may grow further. The macro-cracks determine the remaining fatigue life caused by stress until failure occurs.

### 2.4.2 Fatigue of Reinforcement

Reinforcement bars can have many different types of surfaces; they can be plain, have ribs or be indented. The purpose of having ribbed or indented bars is to increase the mechanical interaction between the steel and the concrete. Ribbed and indented bars give increased

stress concentrations in comparison to plain bars; these stress concentrations reduce the resistance to fatigue and therefore the shaping of the ribs is important, for example, the transition between the bars and the rib has to be smooth. Differently from concrete the reinforcing steel has a stress limit; this means that the stress variation below a certain level can be repeated infinitely many times without causing any fatigue damage. This is only possible if the material shows plastic behaviour. There are many different parameters that determine the fatigue life of the reinforcement. Some parameters that affect the fatigue life are e.g. the stress variation, the surface of the bar and the nominal area/dimension of the bar. Which one that is governing is hard to determine and researchers have come to diverging conclusions regarding which parameter that affects the fatigue strength more than others. In general it is likely to be a combination of several factors.

### 2.4.3 Fatigue of Reinforced Concrete

Bending fatigue tests on concrete beams simulate service conditions more closely than axial bar tests since the interactive effects at the reinforcement-to-concrete interface can be considered. Cyclic frequencies are generally limited by high ranges of deflection and the necessity to avoid local heating due to friction at crack locations in the concrete; high frequency testing can generate a significant amount of heat at the concrete-bar interface and increasing the cyclic frequency can decrease the observed fatigue life. It is also necessary to make assumptions about the load carrying contribution of the concrete and the accuracy of the location of the reinforcement. Repeated loading can also have a detrimental effect on the bond between reinforcing bars and the surrounding

concrete. Factors affecting bond strength under cyclic loads include the concrete compressive strength, cover, bar size, anchorage length, surface conditions, reinforcement properties, confinement, strain range and rate of loading. The primary mechanism governing bond failure in conventionally reinforced concrete members is the progressive crushing of the concrete in front of the lugs. This can result in increased displacement of the reinforcing bar relative to the concrete, known as slip

(Rteil et al. 2011).

## 2.5Fatigue Failure in Reinforced Concrete Structures

Due to the fact that structures are becoming more slender, the traffic volume is increasing, the axel loads are larger, and the traffic speed limits are higher; the interest of fatigue in concrete structures has increased during the last few years. Concrete fatigue is mainly a problem of offshore structures, railway sleepers and bridges because these types of structures are often exposed to repeated loading. This project is focused on reinforced concrete bridge deck. With increased axel loads the condition for our bridges have changed and many existing bridges are nowadays required to carry larger loads than what they were originally designed for.

### 2.5.1 Shear and Bond Failures

The fatigue resistance for these cases is, relatively to the static resistance, sometimes very low, about 40-60%; and therefore it is very important to consider this in design. The shear fatigue failure is highly dependent on if the beam is provided with shear reinforcement, or not. In total the fatigue shear resistance is higher with shear reinforcement than without, but it is dependent on the different types of fatigue loading.

Beams without shear reinforcement have two different modes in which shear fatigue failure can occur. Either the beam can fail when a diagonal crack has propagated across the entire section, or by crushing in the concrete in the compressive zone above the shear crack, Figure 2.1b. When the beam is provided with shear reinforcement it can fail in four different ways. They are fatigue in the shear reinforcement, fatigue in the longitudinal reinforcement where it is crossed by a shear crack, fatigue in the compressed concrete above the shear crack and fatigue in the compressed concrete in the web, see Figure 2.1a

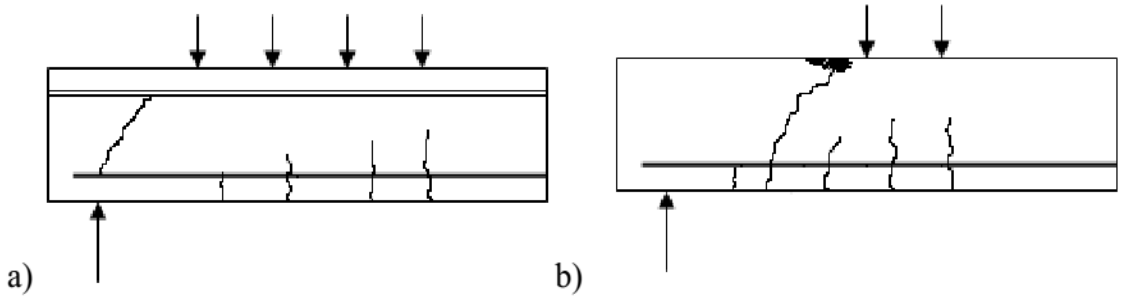



Figure 2.1 Possible shear fatigue failure modes in beams without stirrups: a)excessive development of diagonal cracking b) fatigue of concrete incompression above the shear crack.

**2.6Structural Reliability**

As a result of the variation in uncertainties related to material strengths and other characteristics, loads imposed on the structure, and even the analysis methods used for evaluation and design, it is impossible to achieve absolute safety of the structure. Indeed, loads and load carrying capacities are not perfectly known quantities, they are random variables. Therefore, structural reliability analysis requires the probabilistic modelling of

these uncertainties and it provides the method for quantification of the probability that the structure does not satisfy the performance criteria.

By taking common structural design a step backwards, closer to its origin, higher utilisation is achievable for the structure. This backward step is achieved by using reliability theory, which is the basic tool for code calibration. Reliability analysis is a powerful tool that can improve and nuance the understanding of a structure and increases the utilisation of both load carrying capacities and service life for existing structures.

### 2.7Limit State Principles

All comprehensive design specification are written to establish an acceptable level of safety. There are many methods of attempting to provide safety and the method inherent in many modern bridge design specifications, including the LRFD specification, Ontario Highway Bridge Design Code, Eurocode and the AASHTO specification is probability-based reliability analysis. The method for treating safety issues in modern specifications is the establishment of "limit states" to define groups of events or circumstances that could cause a structure to be unserviceable for its original intent.

The LRFD and Eurocode specifications are written in a probability-based limit state format requiring examination of some, or all of the limit states defined below for each design component of a bridge.

i. **Ultimate Limit States (ULS)**: are mostly related to the loss of load carrying capacity. When an Ultimate Limit State (ULS) is exceeded, a catastrophic failure of the structure occurs, such as collapse or loss of operability. ULSs can be the

formation of a plastic hinge, crushing of concrete, buckling or loss of stability. These are the limit state considered in a reliability-based design code.

ii. **The serviceability limit state**: deals with restrictions on stress, deformation, and crack width under regular service conditions. These provisions are intended to ensure the bridge performs acceptably during its design life.

iii. **The fatigue and fracture limit state**: deals with restrictions on stress range excursions. These provision are intended to limit crack growth under repetitive loads to prevent fracture during the design life of the bridge.

iv. **The strength limit state**: is intended to ensure that strength and stability, both local and global, are provided to resist the statistically significant load combinations that a bridge will experience in its design life. Extensive distress and structural damage may occur under strength limit state conditions, but overall structural integrity is expected to be maintained.

v. **The extreme event limit state**: is intended to ensure the structural survival of a bridge during a major earth quake, or when collided by a vessel, vehicle or ice flow, or where the foundation is subject to scour that would accompany a flood of extreme recurrence, usually considered to be 500years. These provisions deals with circumstances considered to be unique occurrences whose return period is significantly greater than the design life of the bridge. The joint probability of these events is extremely low, and, therefore, they are specified to be applied separately. Under these extreme conditions, the structure is expected to undergo considerable inelastic deformation by which locked-in force effects due to temperature effects, creep, shrinkage, and settlement will be relieved.

### 2.7.1Failure Events and Basic Random Variables

In reliability analysis of technical systems and components the main problem is to evaluate the probability of failure corresponding to a specified reference period. However, also other non-failure states of the considered component or system may be of interest, such as excessive damage, unavailability, etc. In general any state, which may be associated with consequences in terms of costs, loss of lives and impact to the environment are of interest. In the following we will not differentiate between these different types of states but for simplicity refer to all these as being failure events, however, bearing in mind that also non-failure states may be considered in the same manner. It is convenient to describe failure events in terms of functional relations, which if they are fulfilled define that the considered event will occur. A failure event may be described by a functional relation, the limit state function g(x) in the following way:

$$\boldsymbol{F = \{g(x) \leq 0\}} \qquad (2.8)$$

Where the components of the vector ***x***are realisations of the so-called basic random variables

**X** representing all the relevant uncertainties influencing the probability of failure. In Equation (2.8) the failure event ***F***is simply defined as the set of realisation of the function $g(\boldsymbol{x})$, which is zero or negative.

As already mentioned, other events other than failure may be of interest in reliability analysis and e.g. in reliability updating problems also events of the following form are highly relevant: $\mathbf{I=\{}\boldsymbol{h}\mathbf{(}\boldsymbol{x}\mathbf{)=0\}}$ (2.9)

Having defined the failure event the probability of failure may be determined by the following integral:

$$\boldsymbol{P_f = \int_{g(x)\leq 0}^{1} f_x(x)\,dx} \quad (2.10)$$

where $f_{\mathbf{x}}(x)$ is the joint probability density function of the random variables $\boldsymbol{X}$. This integral is, however, non-trivial to solve and numerical approximations are expedient.

The acceptability criteria are often based on engineering judgment (arbitrary decision). For instance, consider a beam that fails if the moment due to the loads exceeds the moment carrying capacity. Then the corresponding limit state function can be written as follows:

$$g = g(x_1, x_2, x_3, \ldots\ldots, x_n) = R - Q \quad (2.11)$$

where R represents the resistance (moment carrying capacity), Q represents the load effect (total moment applied) and $x_i$ represent the random variables of load and resistance such as dead load, live load, length, depth, etc. The limit state function represents the boundary beyond which the structure no longer functions. The probability of failure, $P_f$, is equal to the probability that the undesired performance will occur. Mathematically, this can be expressed in terms of the limit state function as:

$$P_f = P(R\text{-}Q < 0) = P(g < 0) \quad (2.12)$$

If both R and Q are continuous random variables, then each has a probability density function (PDF) such as shown in Figure 2.3. Furthermore, R-Q is also a random variable with its own PDF. This is also shown in Figure 2.3. The probability of failure corresponds to the shaded area in Figure 2.3. Specifically the probability of failure is:

$$P_f = \int_{-\infty}^{+\infty} F_R(x_i) f_Q(x_i)\, dx_i \quad (2.13)$$

where $F_R(x)$ is the CDF of resistance R and $f_Q(x)$ is the PDF of the load Q.

Because there are often multiple random variables that determine R and Q, the evaluation of equation (2.14) cannot be calculated as this would require complex and time consuming numerical techniques. Moreover, there is often insufficient data to fully define the basic variables needed for this numerical procedure in order to obtain acceptable accuracy. Therefore, it is convenient to measure structural safety in terms of a reliability index.

### 2.7.2Reliability Index

A formal definition of the reliability index is that it represents the shortest distance from the origin of standard space (reduced variable space) to the limit state line $g(Z_R, Z_Q) = 0$, in the reduced variables space, as shown in Figure 2.4, where $Z_R$ is the reduced random variable for resistance and $Z_Q$ is the reduced variable for load. The reduced form of a random variable, *X*, is given by:

$$Z_x = \frac{X-\mu_x}{\sigma_x} \quad (2.14)$$

There are various procedures available for calculation of β. These procedures vary with regard to accuracy and required input data.

The reliability index, β, is related to the probability of failure, $P_f$, by:

$$\beta = -\Phi^{-1}(P_f) \quad (2.15)$$

where $\Phi^{-1}$ is the inverse standard normal distribution function.

### 2.7.3Hasofer-Lind Reliability Index

To overcome the invariance problem of the FOSM method, Hasofer and Lind (1974) proposed a modified reliability index formulation, the Advanced First Order Second Moment reliability moment (AFSOM). In this method, the limit state function is evaluated at a point known as the “design point” instead of the mean values. The design point is located on failure surface, g = 0, and since this point is a priori unknown, an iteration

technique must be used to solve for the reliability index. As it was done in the FOSM method, the Hasofer-Lind method consists by first transforming each of the random variables into standard normal space. As before, the Hasofer-Lind reliability index is defined as the shortest distance from the origin of the reduced variable space to the limit state function or failure surface g =0 as presented in Figure (2.3)

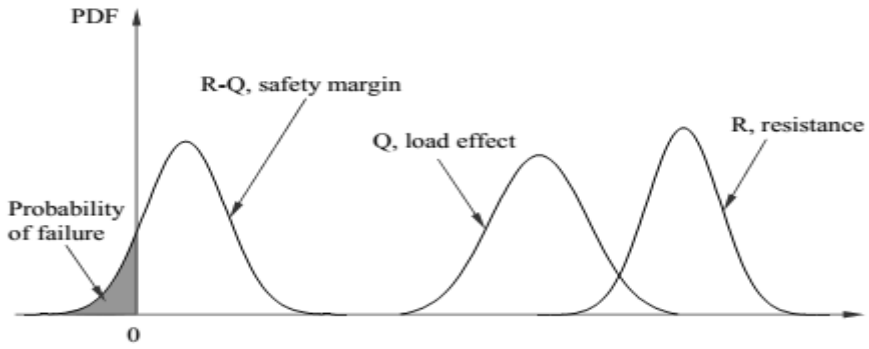



Figure (2.2) Probability Density Function of load, resistance, and safety margin(David, 2010)

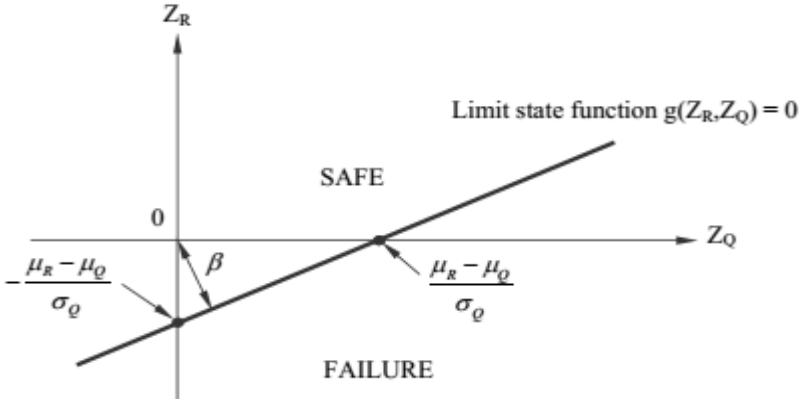



Figure 2.3.Reliability index as shortest distance to origin (David, 2010)

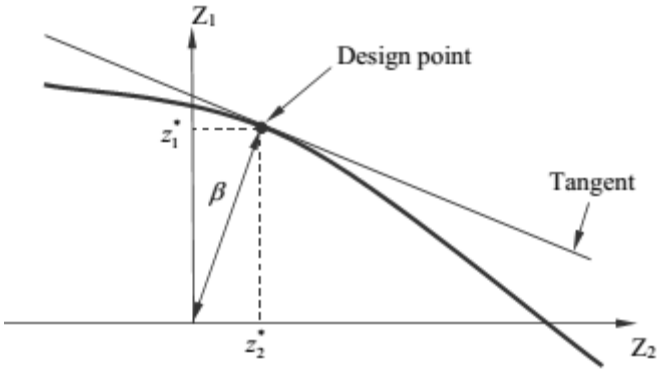



Figure 2.4.Hasofer-Lind reliability index(David, 2010)

# CHAPTER THREE

## METHODOLOGY

### 3.1Estimation of Parameters for Reliability of Bridge Decks

The load component of highway bridges can be divided into several groups, such as dead, live load, (static and dynamic), environmental loads (temperature, wind, earthquake, earth pressure, ice) and other loads (collision, braking load). Load components are treated as random variables, their variation is described by a cumulative distribution function (CDF), a mean value and a coefficient of variation(Handbook 4,2005).

The basic load combination for highway bridges considered in this study is the combination of dead load, live load and dynamic load.

### 3.2 Bridge Model

The simply supported bridge is made of two equal spans of 15.0m each which covers an effective length of 30.0m is located in an open area. It is characterized by an open cross

section composed of seven (7number) precast reinforced concrete longitudinal beams set at constant spacing of 1.70m.

The upper flanges of the precast longitudinal beams are duly connected to a 0.20m deep in-situ deck slab cast on a 0.05m thick precast concrete slab formwork. Giving a deck slab thickness of 0.25m. The superstructure is integrated with the sub structure via bearing pads. The foundation for the bridge consists of cast in-situ reinforced concrete piles with pile caps.

The total width of the bridge is 11.0m. The carriage way is 7.3m wide and has a walkway on each side of 1.5m wide.

### 3.3Structural Materials

Materials chosen according to Eurocode 1 part 1 and Eurocode 2part 2 (2006) the strength class of structural normal weight of concrete is C40/C50, reinforced steel is B500C.

Density of reinforced concrete $\gamma$=25.0Kn/$m^3$

Density of Asphalt concrete=23.5Kn/$m^3$

Weight of parapet wall=0.5Kn/m

### 3.4Load Combinations

Actions on the bridge are determined according to Eurocode 1, parts 1, 2 and 5(2006) and load combinations derived from Eurocode 0 (2006).

Structures are usually designed to withstand all the actions or loads to which they are subjected to during their intended service life. Loads on structures are classified into Dead loads, Live loads and Environmental loads.

Dead loads are due to the selfweight of the structural elements and include permanent equipment's and installations attached to the structure. They may be considered as constant throughout the life of the structure except after major renovations or upgrading. The live loads (Static and Dynamic) are usually modelled as time-dependent random quantities (Nathan T, 2014). Environmental loads are due to extreme environmental events such as large earth quakes, earth pressure and severe weather conditions. Load components are treated as random variable, their variation is described by a cumulative distribution function (CDF), a mean value and a coefficient of variation.

### 3.4.1 Dead Load Effects

Dead load $G_k$ is the load due to weight of the structure itself and non-structural elements permanently connected to the bridge. Because of the difference in variation it is recommended (Nowak 1993) to consider the following components of $G_k$.

$G_1$= weight of factory made elements (steel, precast concrete)

$G_2$= weight of cast in place concrete

$G_3$= weight of wearing surface (asphalt)

$G_4$= weight of miscellaneous weight (e.g parapet wall, railing, luminaries)

All components of dead load are typically treated as normal random variables. Usually it is assumed that the total dead load remain constant throughout the life of the structure.

### 3.4.2 Live Load Effects

The live load effect for highway bridges include all forms of load effects produced by vehicular movement on the bridges. It basically consist of a static and dynamic component. Generally, this effect depends on many parameters (Nowak 2000) such as span length, vehicle weight, axle loads, axle configuration, position of vehicle on the bridge (transverse and longitudinal), number of vehicles on the bridge (multiple presence), and stiffness of structural members.

According to Eurocode 1, "Action on structures, part 2: general actions- traffic loads on bridges", traffic loads should be applied on the carriage way, longitudinal and transversely in the most adverse position, according to the influence surface, in order to maximize the considered load effects. The first operation to be performed consists of determining the width ***w*** of the carriageway and the number of notional lanes.

The carriage way width ***w*** depends, first of all, on whether the walkways are accessible to vehicular traffic or not. In the present case, the walkways are raised 150mm above the carriageway, therefore $w$=7.30m.

As $w$> 6.0m, each notional lane is 3.0m wide and a maximum number of notional lanes $n_i$ which can be considered is given by the equation 3.1.

$n_i = Lnt[w/3m] = Lnt[7.3/3] = 2.43 \approx 2.0$ ( 3.1)

And the maximum width of remaining area $w_r$ is:

$w_r = w - n_i \times 3.0 = 7.3\text{m} - 2\text{x}3.0\text{m} = 1.3\text{m}$ ( 3.2)

The arrangements is thus illustrated in Figure 3.1

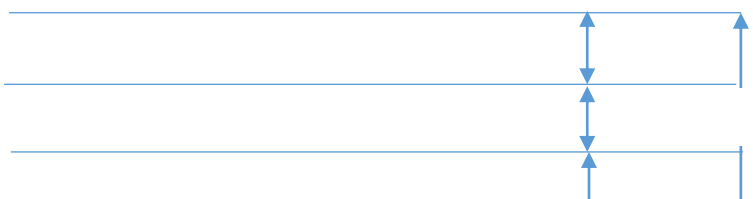


Figure 3.1. Notional lane arrangement on the carriage way

Eurocode 1, part 2, calls for four separable calculation models. In this case, only Load Models, LM1 and LM4 (pedestrian traffic) are relevant. Load Model 2 (LM2) concerns local verifications, and Load Model 3 corresponding to special vehicles (LM3) are not considered, as the bridge is not concerned with special vehicle transit.

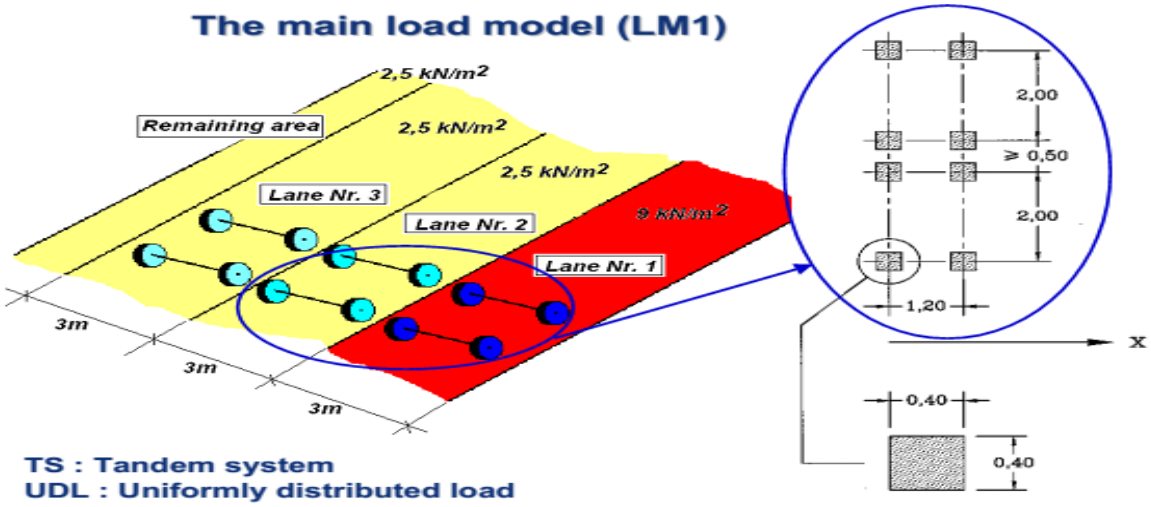



Figure 3.2.Traffic Load on Bridge(Load Model One) Eurocode 2(2006)

On the $i^{th}$notional lane, the main load model LM1 provides for a tandem system of axles weighing $\alpha_{Qi}Q_{ik}$, accompanied by a uniformly distributed load $\alpha_{qi}\, q_{ik}$, were $\alpha_{Qi}$ and $\alpha_{qi}$ are the adjustment factors. For bridges spanning less than 25m the code allows for a value of $\alpha_Q$=0.8 which will be used in this study. The values of $\alpha_{ik}$and $q_{ik}$ calculated including the dynamic amplification coefficient are given in table 2

Table 3.1 Characteristic values for Load Model 1(LM1)

| Notional lane | $Q_k$(Kn) | $q_k$ (Kn/m$^2$) |
|---|---|---|
| Lane 1(Nr1) | 300 | 9.0 |
| Lane 2 (Nr2) | 200 | 2.5 |
| Remaining area | 0 | 2.5 |

With respect to the crowd loading, Eurocode 1-part2 prescribed a nominal value of 5.0Kn/m$^2$, and a combination value of 3.0Kn/m$^2$.

LM1 and LM4 loads must be distributed in the most unfavourable way(both transversely and longitudinally) for the effect under consideration, bearing in mind, however, that a single lane cannot hold more than one tandem system and that the tandem system, must be considered in full, that is to say with all the four wheels.

### 3.5Structural Resistance Models

The capacity of a bridge is a function of the strength of its components and connections. The strength R is a random variable having uncertainties that fall into the three categories(Tabsh et.al,1991): material properties, M, which includes material strength, modulus of elasticity, cracking stress and chemical composition; fabrication F, including geometry, dimensions and section modulus: and structural modelling, P, reflecting assumptions and approximate analysis methods, material tests, simulations, observations of existing structures and engineering judgement are used to model the mean and coefficient of variation for *M*, *F*, and *P*.

The resistance *R* or a structural element is usually modelled as a product of the nominal resistance $R_n$ (as determined from a code rule) and the three mentioned parameters above,

material properties *M* (accounting for the uncertainty in the material properties), fabrication *F* (accounting for the uncertainty in the geometry of the element) and P(accounting for the uncertainty in the approximate method of analysis and idealized models of material behaviour).

$R=R_nMFP$(3.3)

The statistical parameters of load and resistance are now determined for each bridge configuration. Assuming that the load and resistance are lognormal random variables, the formula for reliability index can be expressed in terms of the given data (μR, μQ, COVR, COVQ) as follows:

$$\beta = \frac{Ln\left(\frac{\mu_R}{\mu_Q}\sqrt{\frac{cov_Q^2+1}{cov_R^2+1}}\right)}{\sqrt{Ln\left((cov_R^2+1)(cov_Q^2+1)\right)}} \quad (3.4)$$

where μ*R* is the mean resistance, μ*Q* is the mean load, COVR is the coefficient of variation of resistance, and COVQ is the coefficient of variation of load.

$$\beta = \frac{\mu_R-\mu_Q}{\sqrt{\sigma_R^2+\sigma_Q^2}} \quad (3.5)$$

where $\mu_R$is the mean resistance, μ*Q* is the mean of load, $\sigma_R$is the standard deviation of resistance, and $\sigma_Q$ is the standard deviation of load.

$$\sigma Ln2X = Ln(COV_X^2 + 1) \quad (3.6)$$

$$\mu_{Ln(x)} = Ln(\mu_x) - \frac{1}{2}\sigma_{Ln(x)}^2 \quad (3.7)$$

where μLn(X) is the mean value of Ln(X), and σLn(X) is the standard deviation of Ln(X).

**3.6Generation of Limit State Function for Compound Reliability Analysis**

Table 3.2 Statistical Model of the Basic Design Variables

| S/No | Design Variables | Notation | Unit | Distribution Model | Mean and COV |
|---|---|---|---|---|---|
| 1 | Resistance Model Uncertainty | $\Phi_R$ | - | Normal | (1.0 ; 0.05) |
| 2 | Concrete Load Model Uncertainty | $\Phi_c$ | - | Normal | (1.0 ; 0.05) |
| 3 | Asphalt Load Model Uncertainty | $\Phi_{asph}$ | - | Normal | (1.0 ; 0.05) |
| 4 | Traffic Load Model Uncertainty | $\Phi_Q$ | - | Normal | (1.0 ; 0.05) |
| 5 | Concrete Compressive Strength | $f_{cd}$ | $N/mm^2$ | Lognormal | (30 ; 0.05) |
| 6 | Steel Strength | $f_{yd}$ | $N/mm^2$ | Lognormal | (500 ; 0.05) |
| 7 | Area of Steel | $A_s$ | $mm^2$ | | Nominal |
| 8 | Width of Slab | $b$ | mm | Normal | (1000 ; 0.05) |
| 9 | In-situ Slab Thickness | $h$ | mm | Normal | (250 ; 0.05) |
| 10 | Effective Depth | $d$ | mm | Normal | (219 ; 0.05) |

### 3.6.1 Failure of the Slab

For the slab the critical mode of failure is flexure. The bridge deck is supported on eight precast rectangular beams spaced at 1.7m centres. Considering a portion of the deck slab supported on any two interior beams as shown in Figure 3.2.

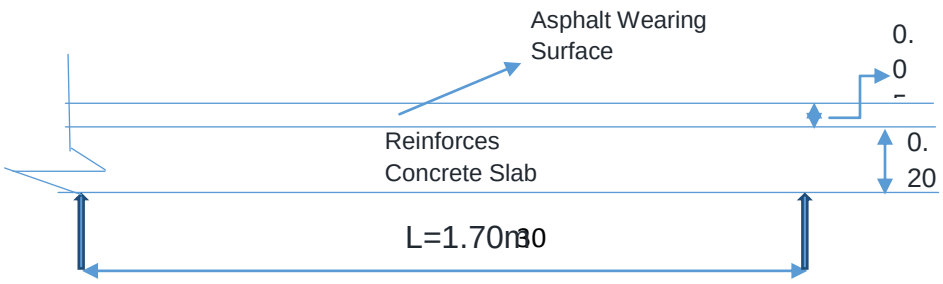


Figure 3.3. Portion of slab supported by two interior beams

### 3.6.1.1 Applied Dead Load

$G_k = G_{conc} + G_{asph}$ (3.8)

Were $G_{conc}$ =cross section area of the slab x density of reinforced concrete

$G_{asph}$ =cross sectional area of the slab x density of asphalt concrete

The dead load moment $M_{dl}$ is:

$M_{dl} = G_k l^2/8$ (3.9)

$M_{dl} = 0.125\{(0.25 \text{ x } 1 \text{ x } 25\emptyset_c) + (0.04 \text{ x } 1 \text{ x } 23.5\emptyset_{asph})\} \text{ x } 1.7^2$

$M_{dl} = 0.3613(6.25\emptyset_c + 0.94\emptyset_{asph})$

$M_{dl} = 2.26\emptyset_c + 0.34\emptyset_{asph}$ (3.10)

The live load moment $M_{dl}$ on the slab is based on a single wheel truck of load model 1(LM1) placed in the centre of the slab which produces a concentrated load pf $Q_k$ =150 Kn between two beams plus a uniformly distributed load $q_k$ =9.0Kn/m$^2$ over the slab.

The live load moment $M_{ll}$per meter width of slab is

$M_{ll} = \alpha_Q(Q_k l/32) + \alpha_q(q_k l^2/8)$ (3.11)

$M_{ll} = \emptyset_{Qk}\{(0.03125 \text{ x } 0.8 \text{ x } 150 \text{ x } 1.7) + (0.125 \text{ x } 0.8 \text{ x } 9.0 \text{ x } 1.7^2)\}$

$M_{ll}$=8.98$\emptyset_{Qk}$(3.12)

Total applied moment, $M_{Ed}$ is

$M_{Ed}$ = $M_{dl}$+$M_{ll}$(3.13)

$M_{Ed}$=2.26$\emptyset_{c}$+0.34$\emptyset_{asph}$+8.98$\emptyset_{Qk}$(3.14)

Design resistance or capacity of the slab from the code, $M_{Rd}$ is;

$M_{Rd}$=$\emptyset_R A_s f_{yd}$[d–($A_s f_{yd}$/2$b f_{cd}$)] (3.15)

Therefore the limit state equation is

$G(x_1)$=$M_{Rd}$ - $M_{Ed}$(3.16)

$G(x_1)$=$\emptyset_R Asfyd$[d – (Asfyd /2bfcd)] –$\left(2.26\emptyset_c + 0.34\emptyset_{asph} + 8.98\emptyset_{Qk}\right)$(3.17)

### 3.6.2 Failure Mode for Beams

The beams can fail by either moment/flexure or by shear (i.e it has two main failure modes). The interior beam would to be considered since it carries the most load.

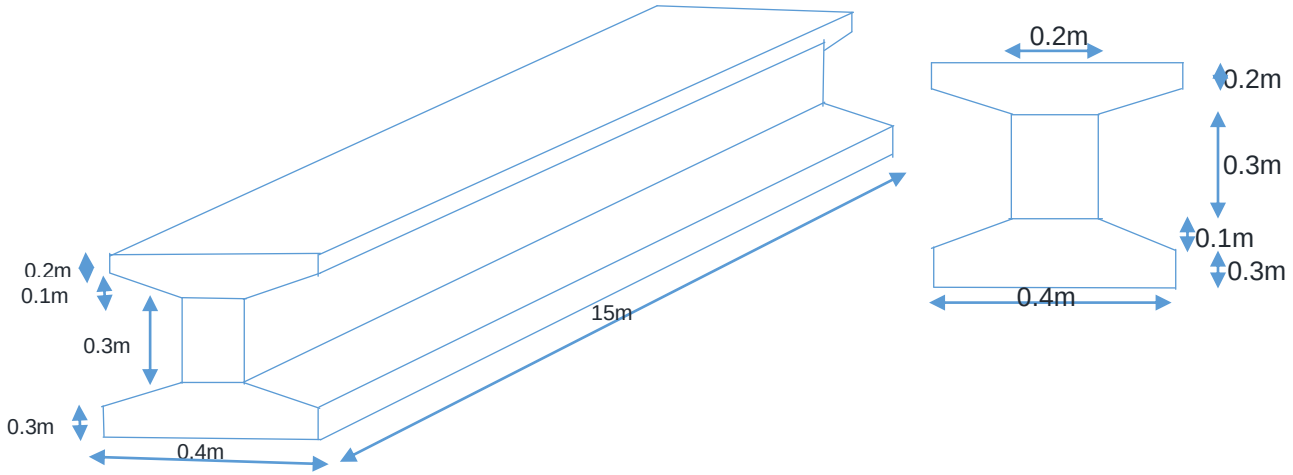


Figure 3.4.Beam Cross Sectional Dimensions

Area of Precast Beam= $A_1 + 2A_2 + A_3 + A_4$

$$=[(0.4 \text{ x } 0.2) + 2(\frac{1}{2}(0.4 + 0.2)\text{x } 0.1) + (0.2 \text{ x } 0.3) + (0.4 \text{ x } 0.3)]$$

$$=0.08 + 0.06 + 0.06 + 0.12$$

$$A = 0.32m^2$$

### 3.6.2.1 Interior Beams

Table 3.3 Statistical Model of the Basic Design Variables for Beam

| S/No | Design Variables | Notation | Unit | Distribution Model | Mean and COV |
|---|---|---|---|---|---|
| 1 | Resistance Model Uncertainty | $\Phi_R$ | - | Normal | (1.0 ; 0.05) |
| 2 | Concrete Load Model Uncertainty | $\Phi_c$ | - | Normal | (1.0 ; 0.05) |
| 3 | Asphalt Load Model Uncertainty | $\Phi_{asph}$ | - | Normal | (1.0 ; 0.05) |
| 4 | Traffic Load Model Uncertainty | $\Phi_Q$ | - | Normal | (1.0 ; 0.05) |
| 5 | Concrete Compressive Strength | $f_{cd}$ | $N/mm^2$ | Lognormal | (30 ; 0.05) |
| 6 | Steel Strength | $f_{yd}$ | $N/mm^2$ | Lognormal | (500 ; 0.05) |
| 7 | Area of Steel | $A_s$ | $mm^2$ | | Nominal |
| 8 | Width of Beam | $b$ | mm | Normal | (400 ; 0.05) |
| 9 | Beam Height | $h$ | mm | Normal | (1000 ; 0.05) |
| 10 | Effective Depth | $d$ | mm | Normal | (937 ; 0.05) |

#### 3.6.2.1.1 Limit State Function for Shear for Interior Beam

The limit state function for shear in the interior beams is function of shear capacity and shear load demand on the beam.

The ultimate shear capacity of the beam $V_{Rd}$when $cot\theta$=2.5 given in EN1992-2 as;

$V_{Rd}=0.138f_{ck}(1-f_{ck}/250)$ (3.18)

The applied shear $V_{Ed}$is a function of the dead load shear which includes the weight of asphalt concrete slab and beam together with the live load shear (LM1). This implies that;

$G_k=G_{slab+beam}+G_{asph}$(3.19)

$G_k$ = (0.25 x 1.0 + 0.32) x 25$\emptyset_c$+ 0.04 x 1.0 x 23.5$\emptyset_{asph}$

$G_k=(14.25\emptyset_c+0.94\emptyset_{asph})$Kn/m (3.20)

The dead load shear is therefore:

$V_{dt}=G_k l/2=0.5(14.25\emptyset_c + 0.94\emptyset_{asph})15=107\emptyset_c + 14.1\ \emptyset_{asph}$(3.21)

To obtain the critical live load shear, the LM1 is placed with the point load over the support as shown in Figure 3.4.

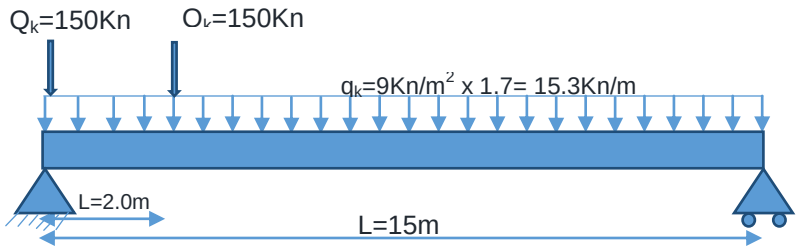



Figure 3.5. Arrangement of LM1 for critical shear

Critical shear for live load $V_u$ is;

$V_u = \frac{Q_k(2L-2)}{L} + \frac{Q_k L}{2}$(3.22)

$V_u = \left(\frac{150(2\ x\ 15-2)}{15} + \frac{15.3\ x\ 15}{2}\right) \emptyset_Q$=394.8$\emptyset_Q$(3.23)

Applied shear stress $V_{Ed}$ is;

$V_{Ed} = [\frac{V_{dl}+V_u}{bd}]$ (3.24)

$V_{Ed} = \left(\frac{107\emptyset_c + 14.1\emptyset_{asph} + 394.8\emptyset_Q}{bd}\right)$ (3.25)

And the limit state function;

$G(x_2)=V_{Rd}-V_{Ed}$ (3.26)

$G(x_2)=0.138 f_{ck}\left(1-\frac{f_{ck}}{250}\right) - [\frac{107\emptyset_Q + 14.1\emptyset_{asph} + 394.8\emptyset_Q}{bd}]$(3.27)

### 3.6.2.2Limit State Function for Moment (Flexure) in Interior Beam

The moment capacity for the beam is similar to that of the slab except for dimensional differences.

$M_{Rd} = \emptyset_R A_{s(t)} f_{yk}[d - \frac{A_{s(t)} f_{yk}}{2b f_{ck}}$ (3.28)

The resultant moment due to dead load is;

$M_{dl}= G_k l^2\ M_{dl} = \frac{G_k l^2}{8} = 0.125x15^2(14.25\Phi_c + 0.94\Phi_{asph})$ (3.29)

$M_{dl}$ =401$\emptyset_C$ + 26.4$\emptyset_{asph}$(3.30)

The resultant moment due to live load

$M_{ll}$= [ $M_{Qk}$ + $M_{qk}$] (3.31)

$$M_{qk} = \frac{\alpha_{qk}(1.7\ x\ q_k)L^2}{8} = 0.125\text{x}0.8\text{x}15.3\text{x}15^2 = 344.3\emptyset_Q \quad (3.32)$$

To get $M_{Qk}$ from LM1, the maximum bending moment occurs in the section on which one of the two tandem axles is applied at a distance from the beam mid-point equal to half that between the aforesaid axle and the resultant of the moving load train is shown in the Figure 3.6.

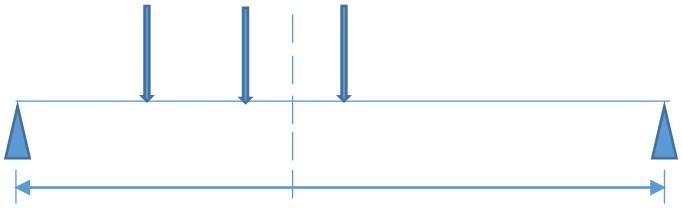


Figure 3.6 Arrangement of point of maximum bending of LM1

Therefore, the maximum bending moment on the beam would occur in the section and is given by

$$M_{max} = M_{(s)} = \frac{R}{4}[l - 2\ x\ C + \frac{C^2}{L}] \quad (3.33)$$

So, by applying the resultant of the pair of tandem axles corresponding to the midline, we obtain a constant bending moment between the concentrated loads as;

$$M'_{max} = \frac{R}{4}(L - 2C) \quad (3.34)$$

for LM1 $C = \frac{1.2}{2} = 0.6m$ and L=16m

Where R=($Q_k$ + $Q_k$)=150 + 150=300KN

Therefore; $M_{max} = \frac{\alpha_{Qk}}{4} R(L - 2C) = \left(\frac{0.8\ x\ 300}{4}(15 - \frac{2x1.2}{2}\right) \emptyset_Q$(3.35)

$M_{max}$=$828\emptyset_Q$

$M_{LL} = M_{Qk} + M_{qk} = |3.44.3 + 828|\emptyset_Q = 1172.3\emptyset_Q$ (3.36)

$M_{Ed} = M_{dl} + M_{LL} = 401\emptyset_c + 26.4\emptyset_{asph} + 1172.3\emptyset_Q$(3.37)

Therefore the limit state function is;

$G(x) = M_{Rd} - M_{Ed}$(3.38)

$G(x_3) = A_{st} f_{yk} \left(d - \frac{A_{st} f_{yk}}{2b f_{ck}}\right) \emptyset_R - \left(401\emptyset_c + 26.4\emptyset_{asph} + 1172.3\emptyset_Q\right)$(3.39)

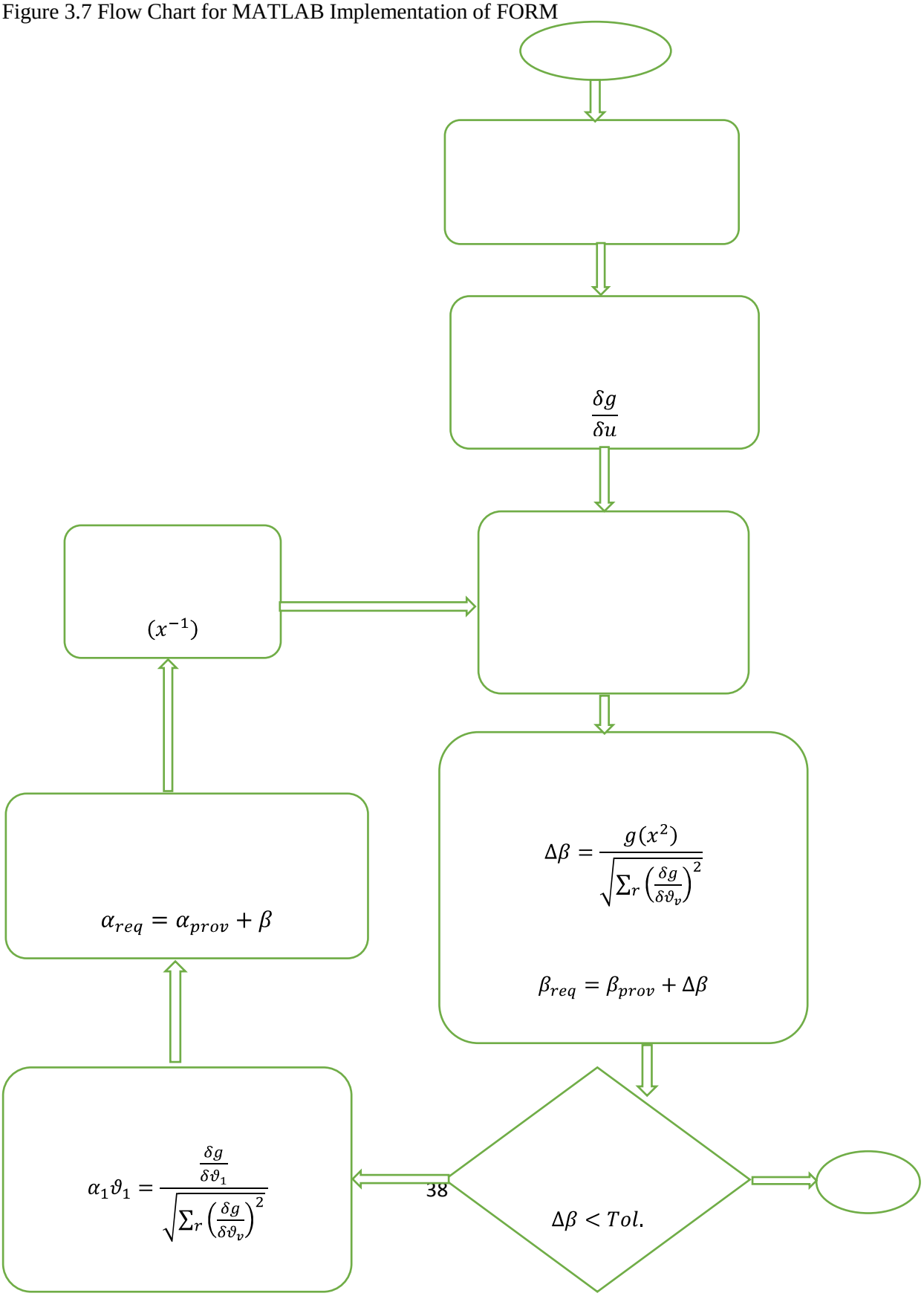
### 3.7Computer Analysis Procedure

A First Order Reliability Analysis was carried out using MATLAB program on a simply supported RC bridge deck and the safety indices under fatigue failure modes of flexure and shear were carried out in accordance with the Eurocode specifications by varying the following parameters;

1. Variation of slab (concrete and asphalt) thickness(200-300mm).
2. The coefficient of variation with respect to the depth of slab section (0.05-0.20).
3. The coefficient of variation with respect to the concrete strength (0.05-0.20).
4. Variation of the depth of beam girder with respect to shear and moment flexure (1000-1200m).
5. The coefficient of variation with respect to the depth of beam girder under both shear and flexure (0.05-0.20).

6. The coefficient of variation with respect to the concrete strength of beam girder under both shear and flexure (0.05-0.20).

Figure 3.7 Flow Chart for MATLAB Implementation of FORM



### 3.7.1 MATLAB PROGRAM

The MATLAB procedure used in this research for the calculation of the safety index requires the user to enter in the following;

i. First choose the failure mode
ii. Enter the concrete compressive strength
iii. The steel yield strength
iv. Moment coefficient due to Concrete
v. Moment coefficient due to Asphalt
vi. Moment coefficient due to Live Load
vii. The depth of section(slab deck/beam girder)
viii. Area of reinforcing steel
ix. Steel diameter
x. Maximum number of load cycle repetition
xi. Wheel Load Frequency (per min)
xii. Number of years after bridge is open to traffic.

The program then computes the safety index and probability of failure. The results obtained are tabulated in the appendix and plots shown in Chapter Four.

# CHAPTER FOUR

## RESULTS AND DISCUSSION

### 4.1 Failure due to Moment Flexure in Slab

#### 4.1.1 Effect of Deck Slab Depth Variation on the Safety Index

The variation of slab depth from 200-300mm, for a stress range of 3-15 load cycles/min whose strength class of concrete and steel as C30 and S500 respectively were adopted from Eurocode 2, 2004 was carried out using MATLAB program dc_fatigue.m (see appendix I) and the reliability indices for the failure mode obtained were tabulated (see appendix II) and thus represented in the graphs Figures 4.1-4.3.

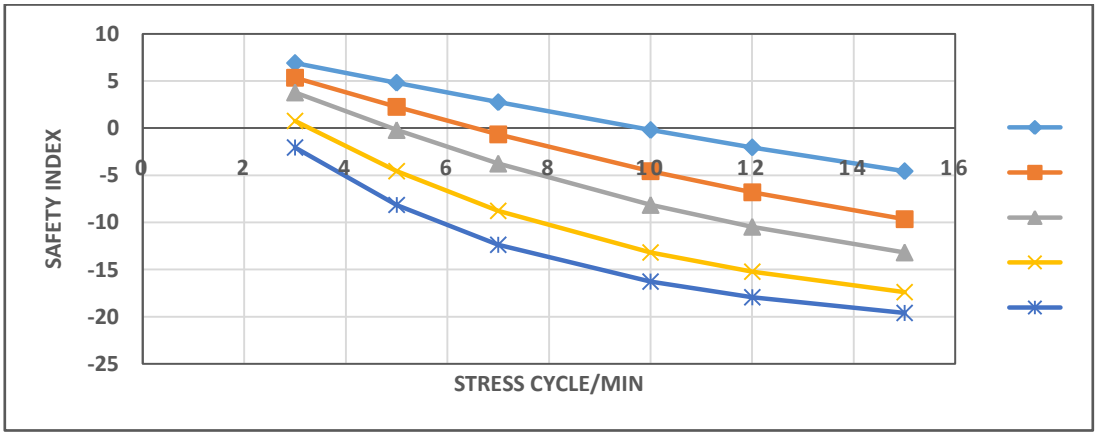



Fig. 4.1 Safety Index Versus Stress Cycle (per min) at Slab Deck Depth of 200mm

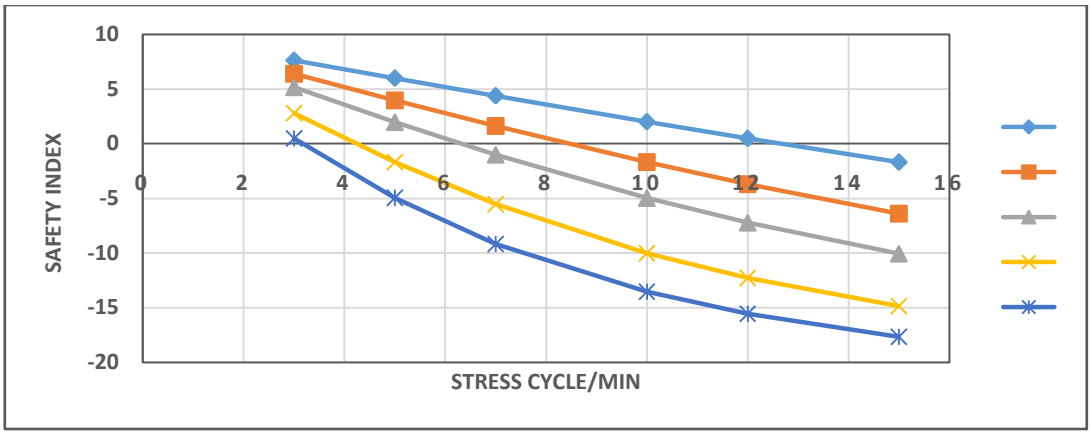



Fig. 4.2 Safety Index Versus Stress Cycle (per min) at Slab Deck Depth of 250mm

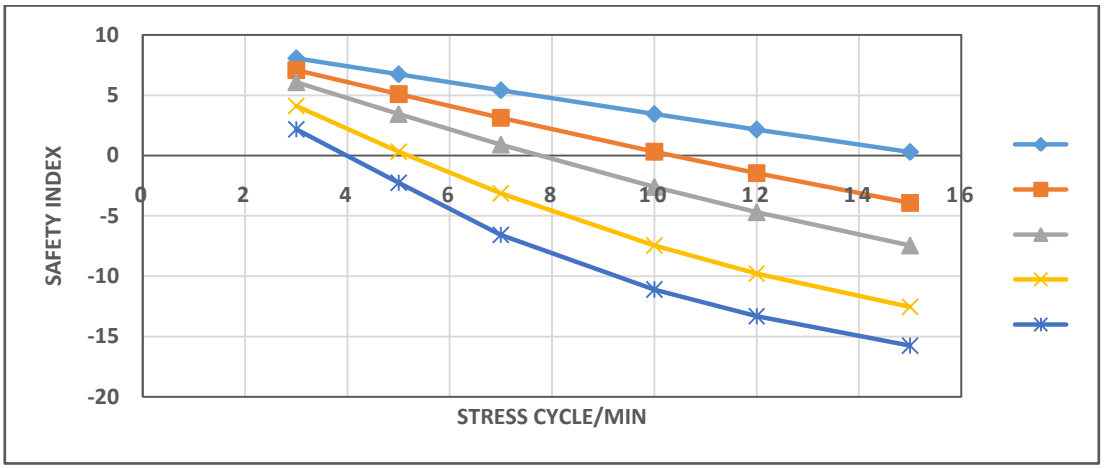



Fig. 4.3 Safety Index Versus Stress Cycle (per min) at Slab Deck Depth of 300mm

The variation of safety indices with slab depth for failure due to moment flexure were caused by forces derived from loading the member at a frequency range rate of 3-15cycles per minute. It was noted that as the load rate increases repeatedly over the years, the safety index reduce. This trend is expected because as the load rate increases under repeated level of stress cycles, the moment also increases as such the safety index reduces while the probability of failure increases to the point of fatigue failure.

At 10years after member is exposed to traffic, the safety index(i.e probability measure of the structural capacity)drops from the value of 6.91 at 3cycles/min to -4.57 at

15cycles/min, 7.61 at 3cycles/min to -1.69 at 15cycles/min and 8.06 at 3cycles/min to 0.29 at 15cycles/min at depths of 200, 250 and 300mm respectively.The PLOT also showed that at 10years the deck slab was more resistant to moment flexure than at the other period of time.

### 4.1.2 Effect of Coefficient of Variation due to Concrete Strength under Flexure on the Safety Index with respect to the Stress Cycle

Coefficient of variation is a very useful tool that indicates magnitude of dispersion of data about a central value (Ang et.al,1974). Melchersin 1987 assigned certain values of coefficient of variation to correspond to Excellent, average and poor quality for batch in in-situ concrete compressive strength. The value of coefficient of variation used throughout in this study for concrete strength is 0.05 which indicates an excellent quality control, 0.10 indicates average and 0.2 indicate poor quality control and the effect of this on the safety index being observed. The safety index is found to increase with decrease in the coefficient of variation of concrete. The table of result for this analysis is given (see appendix II) for Figures (4.4-4.6).

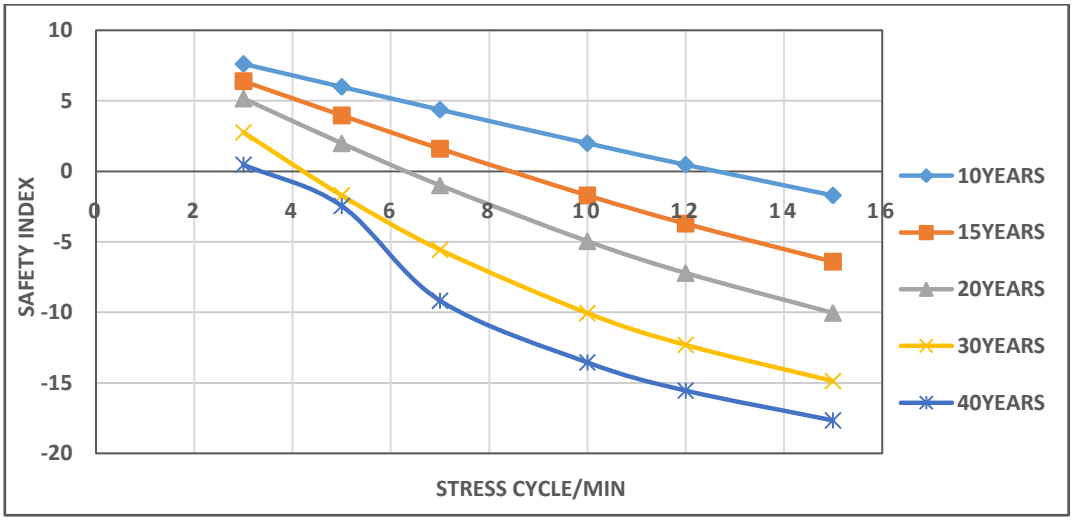



Figure 4.4 Safety index Versus Stress Cycle at 5% COV due to Concrete

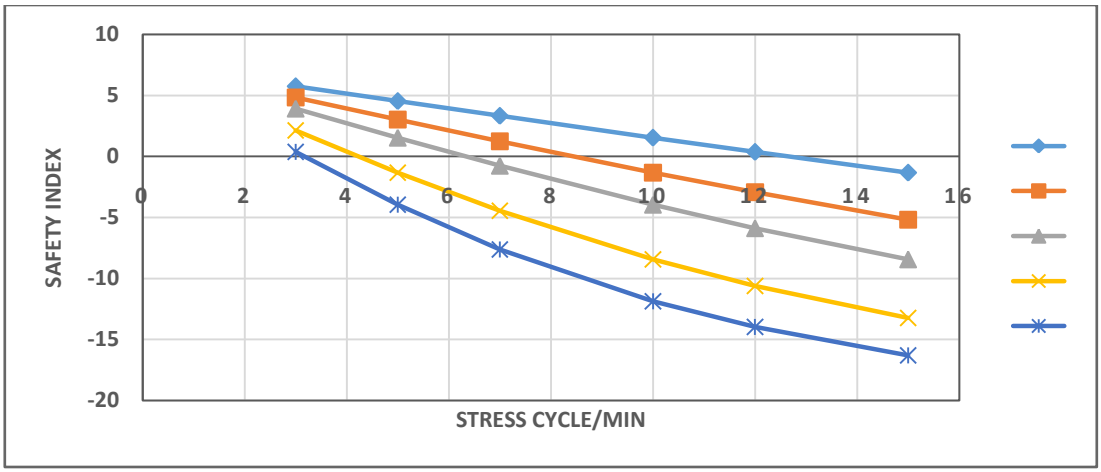



Figure 4.5Safety index Versus Stress Cycle at 10% COV dueConcrete

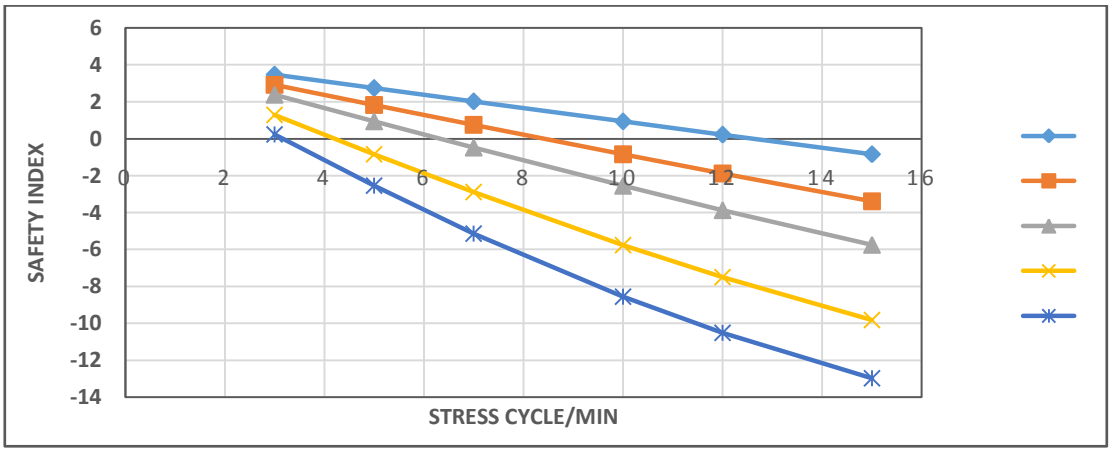



Figure 4.6 Safety index VersusStress Cycle at 20% COV due Concrete

When the fatigue initiation time was at 3 cycles/min the slab capacity was at 7.61, 6.39, 5.18, 2.78, and 0.47 for 10, 15, 20, 30 and 40 years respectively at strength covariance of 5% and at 12cycle/min and above, the safety index dropped from 0.47 to -15.54.As the strength covariance increase up to 20%, the safety index gradually dropped to 3.47, 2.92, 1.29 and 0.23 for stress level of 3cycles/min from10-40 years respectively.

It can be observed that the reliability index decrease with increase in concrete strength and vice versa. This can be explained by the likely decrease in ultimate strain for higher grade concrete (Mosley et.al,2007) thereby increasing the tendency for compression failure before tension failure and complete fatigue failure afterwards.

### 4.1.3 Effect of Coefficient of Variationdue to Slab Depth under Flexure on the Safety Index with respect to the Stress Cycle

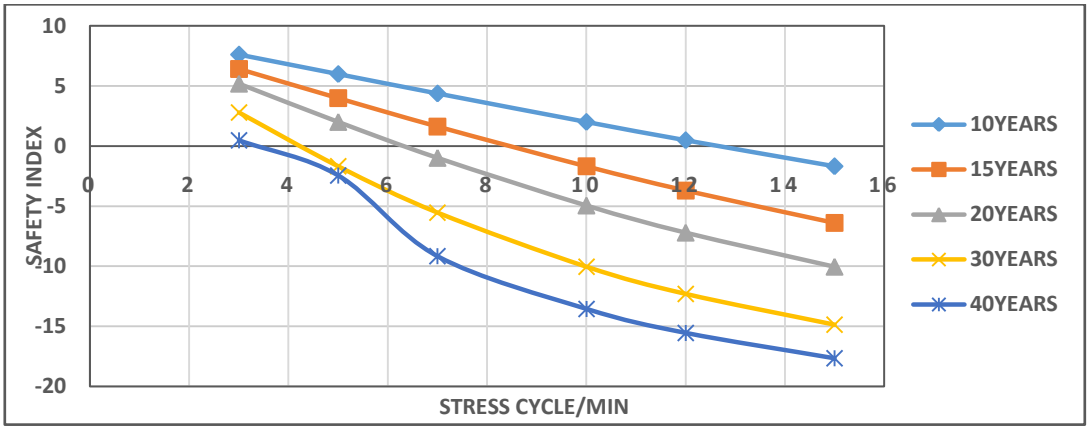



Figure 4.7 Safety Index Versus Stress Cycleat 5% COV due to Slab Depth

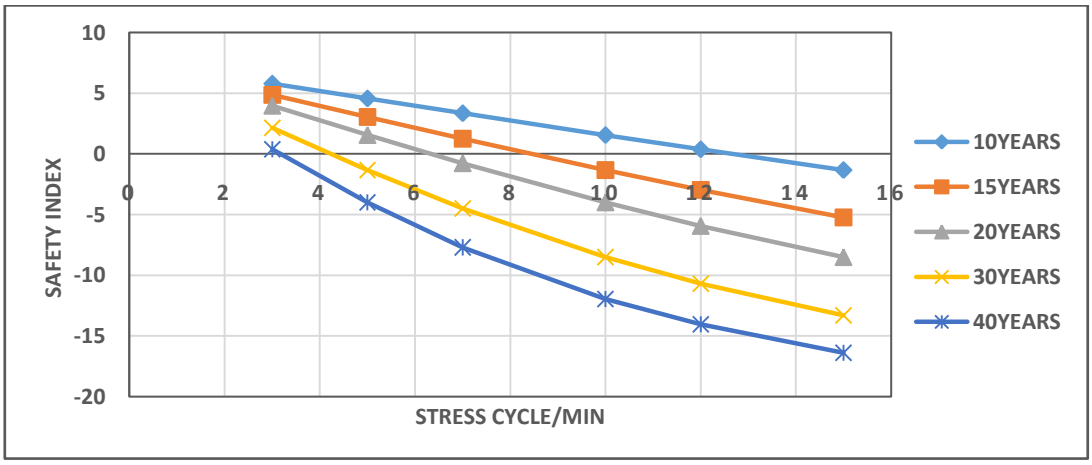



Figure 4.8 Safety Index Versus Stress Cycle at 10% COV due to Slab Depth

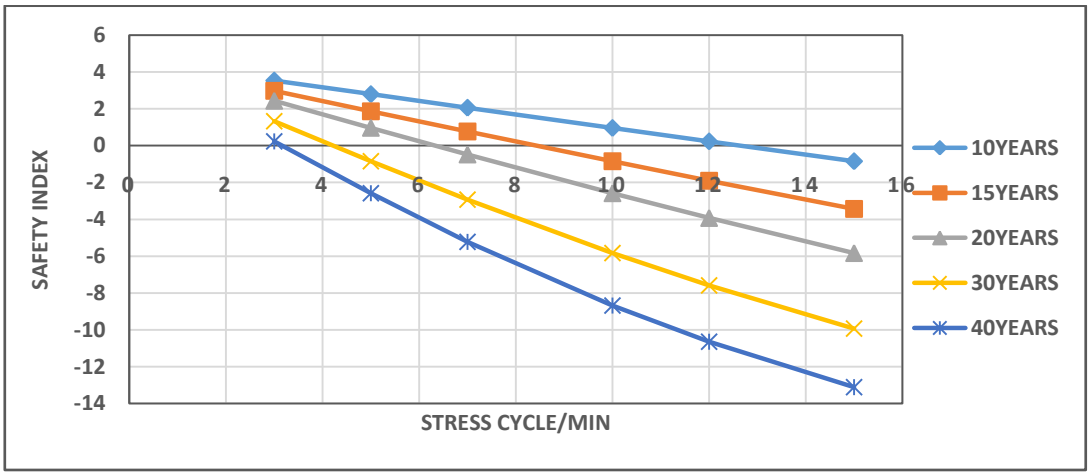


Figure 4.9 Safety Index Versus Stress Cycle at 20% COV due to Slab Depth

As can be observed from the plots, the capacity loss of a structural member under fatigue stress decrease linearly with the increase in stress cycle over time.

At 10years after member is exposed to traffic, the safety index drops from the value of 6.91 at 3cycles/min to -4.57 at 15cycles/min, 7.61 at 3cycles/min to -1.69 at 15cycles/min and 8.06 at 3cycles/min to 0.29 at 15cycles/min at depths of 200, 250 and 300mm respectively.

It can be deduced that from the figures above that the safety index increased with a decrease in COV and vice versa.At 15 cycles/min and above there was ultimate fatigue failure as all the safety index values gave a negative response to stress action

### 4.2.0 Failure due to Shear in Deck Beam

The effective depth of concrete girder (beam) was varied from 1000-1200mm for a period of 10-40 years when the deck is open to traffic. The strength class of both concrete and steel of C40/50 and S500 were obtained from Eurocode 2, 2006. A reliability analysis was carried out using the MATLAB program “dc_fatigue.m” (appendix I) and the safety indices were tabulated (appendix II) and represented inFigures 4.10-4.12

### 4.2.1 Effect of Beam Depth Variation on the Safety Index with respect to Shear Stress under Stress Cycle

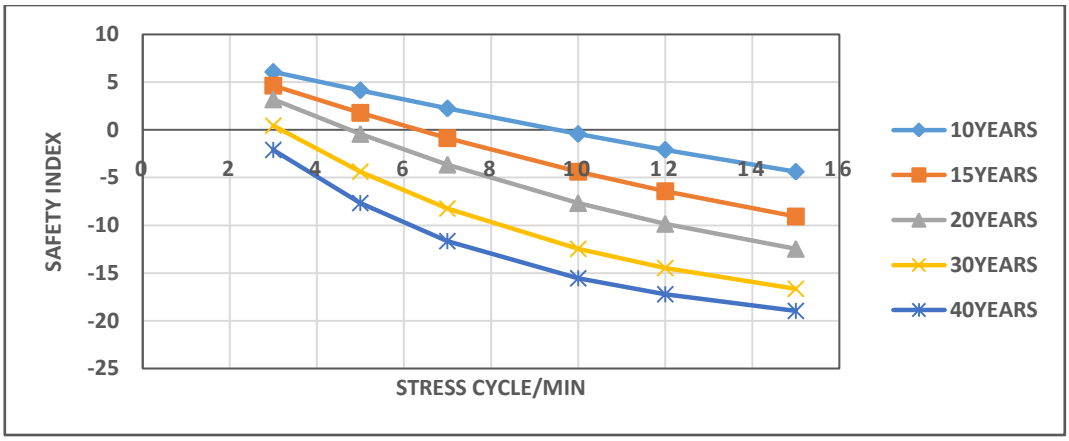



Figure 4.10 Safety Index Versus Stress Cycle (per min) at Beam Depth of 1000mm

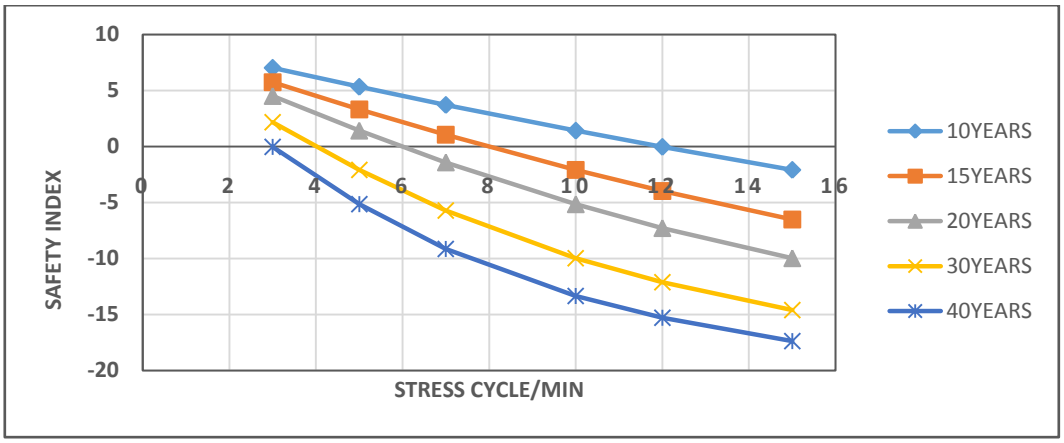



Figure 4.11 Safety Index VersusStress Cycle (per min) at Beam Depth of 1100mm

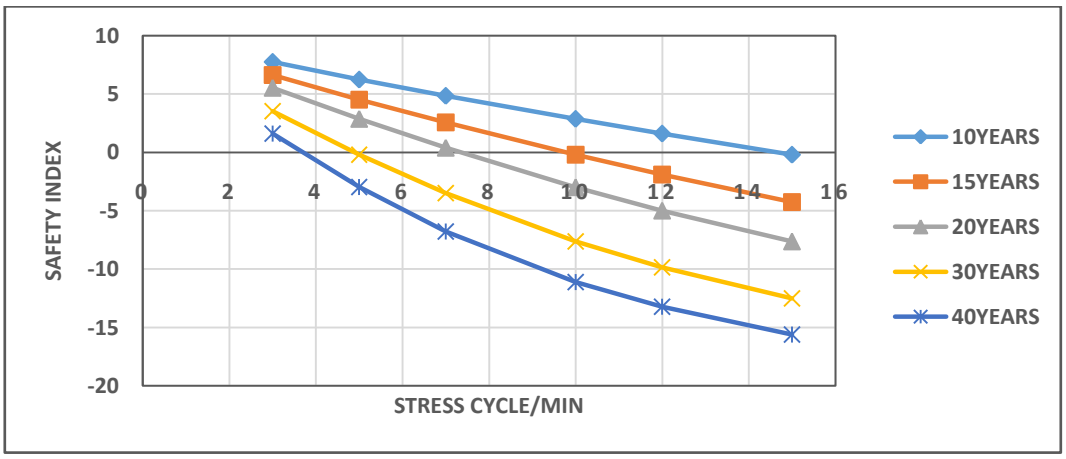



Figure 4.12 Safety Index VersusStress Cycle (per min) at Beam Depth of 1200mm

As the stress cycle increased, the reliability displayed a nonlinear reduction. At a stress level of 7cycles/min, the respective reliability indices are 4.85, 2.56, 0.39, -3.49 and -6.79, after 10, 15, 20, 30 and 40 years respectively. Therefore, the capacity loss of member subjected to fatigue also depends on the value of the stress range over time.

At 10cycles/min the safety index was -0.44, 1.42 and 2.88 for 1000mm, 1100mm and 1200mm respectively after 10years exposure to traffic.Above 12cycles/min there was negative increase in safety all through i.e it has reached it ultimate fatigue life.The safety index increased with an increase in the depth of concrete.

### 4.2.2 Effect of Concrete Strength Variation on the Safety Index with respect to Stress Cycle

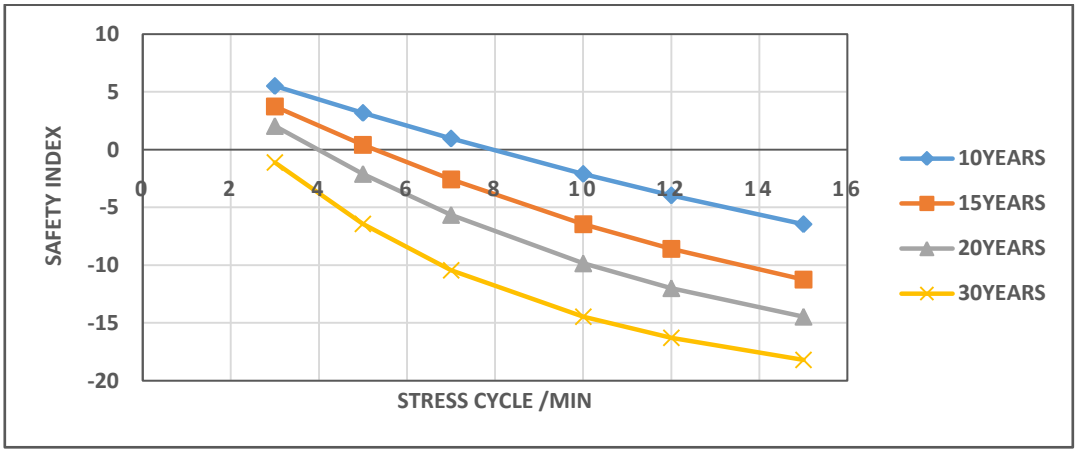


Figure 4.13 Safety Index Versus Stress Cycle (per min) at $f_{cu}$=25N/mm$^2$

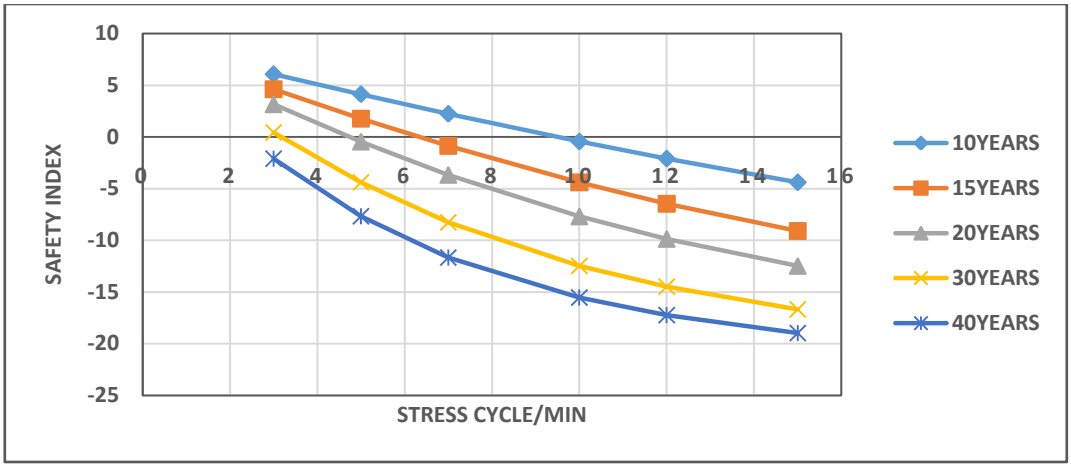



Figure 4.14 Safety Index Versus Stress Cycle (per min) at $f_{cu}$=30N/mm$^2$

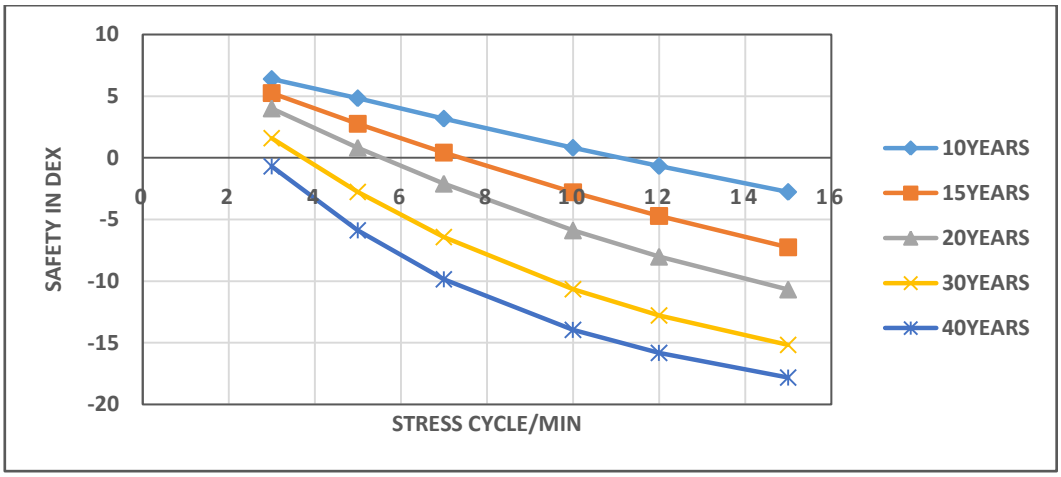



Figure 4.15 Safety Index Versus Stress Cycle (per min) at $f_{cu}$=35N/mm$^2$

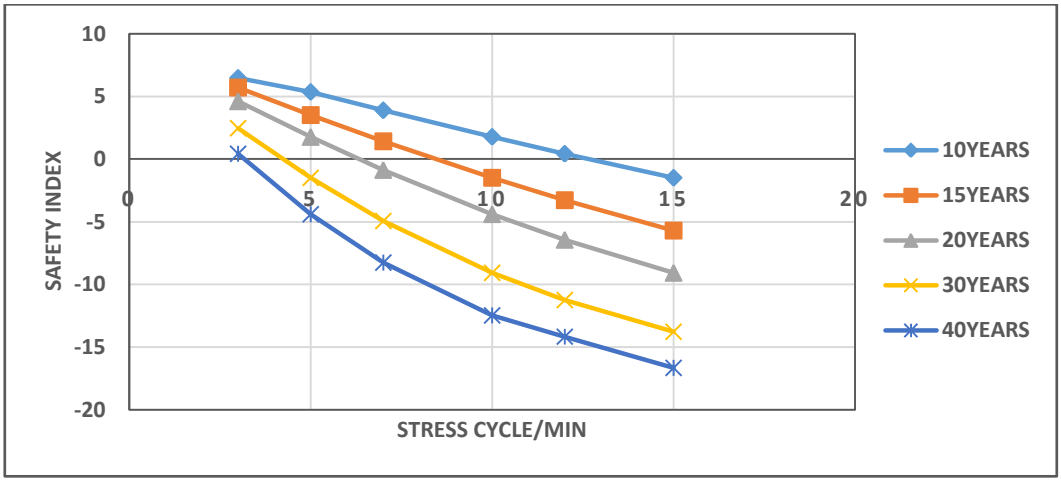


Figure 4.16 Safety Index Versus Stress Cycle (per min) at $f_{cu}$=40N/mm$^2$

The lower the concrete grade the higher the probability of failure the lower the safety index. At a stress level of 3 cycles/min, the beam gave an increase in safety index for grades 25, 30, 35 and 40 as -1.12, 0.42, 1.57 and 2.46 after 30 years exposure to traffic. As the concrete strength class is increased, the safety index also increase due to increase in compressive strength and also reduces by a corresponding increase in stress level with respect to the concrete strength.

### 4.2.3 Effect of Coefficient of Variationof Concrete on the Safety Index with respect to Stress Cycle

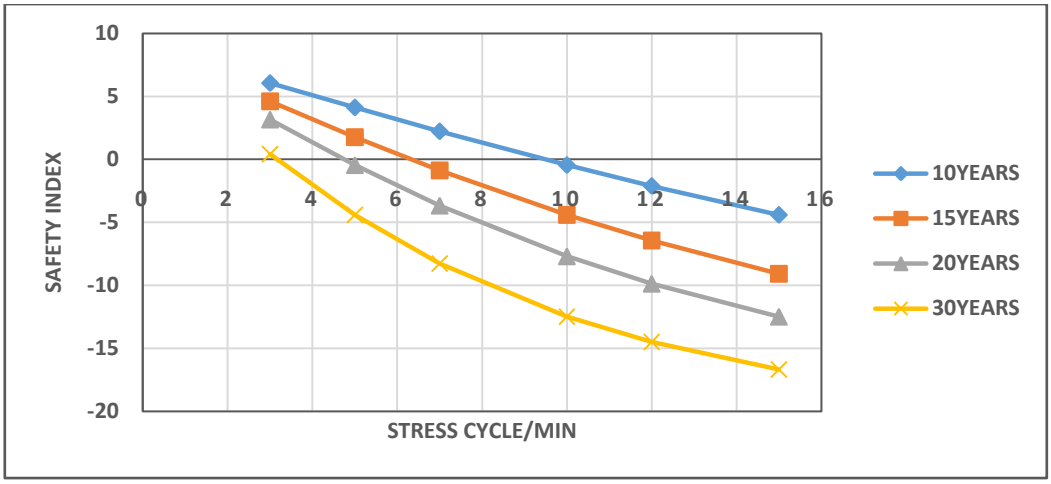



Figure 4.17 Safety Index Versus Stress Cycle (per min) at 5% COV due to Concrete

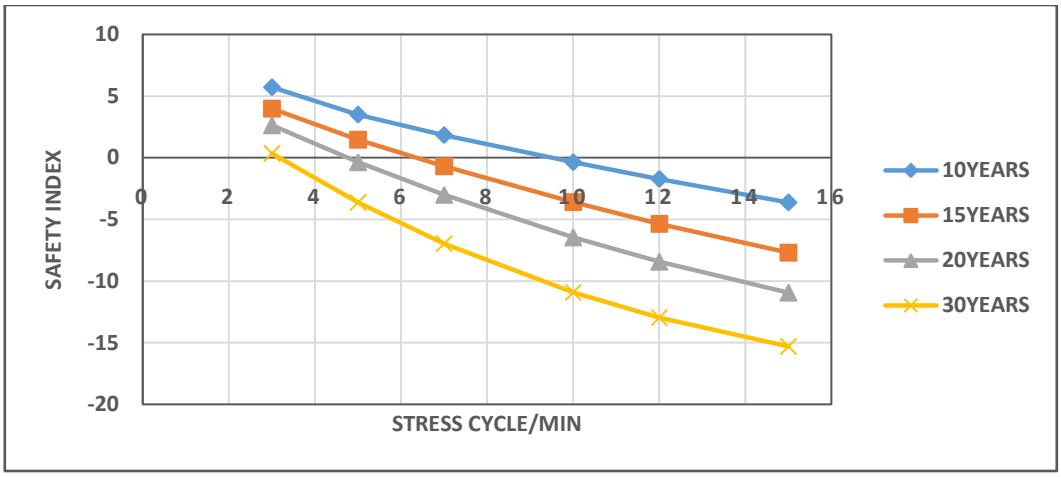



Figure 4.18 Safety Index Versus Stress Cycle (per min) at10% COV due to Concrete

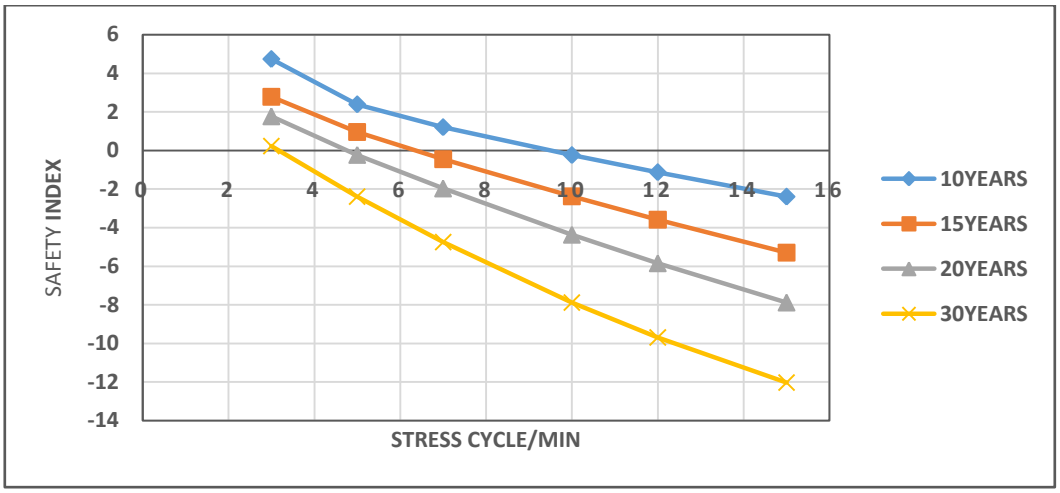



Figure 4.19 Safety Index Versus Stress Cycle (per min) at 20% COV dueTo Concrete

It can be deduced from the Figures (4.17-4.19) that as the safety index increases under condition of stress action the COV decreases and vice versa.At 3 cycles/min there was appositive decrease in safety index from 6.10, 4.61, 3.17, and 0.42 for a period of 10, 15, 20 and 30 years respectively.At 12cycles/min and above, there was a negative level of safety all through the years indicating complete(ultimate) fatigue failure of the beam under shear. It is very necessary to take note of the coefficient of variation of concrete when carrying out the reliability analysis because the condition of quality control on site is a very

important parameter on the analysis as this will be in the input data that affects the concrete strength.

### 4.2.4 Effect of Coefficient of Variationof Beam Depth on the Safety Index with respect to Stress Cycle

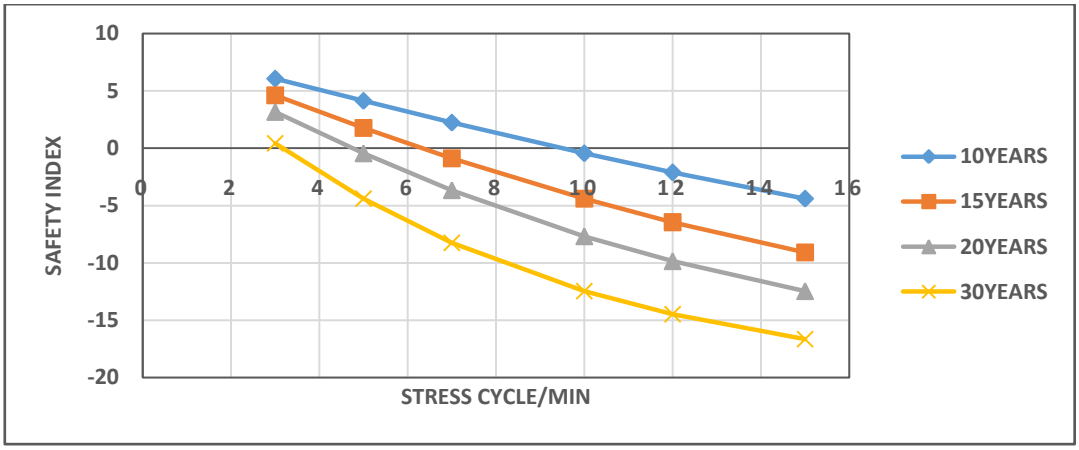



Figure 4.20 Safety Index Versus Stress Cycle (per min) at 5% COV due to Beam Depth

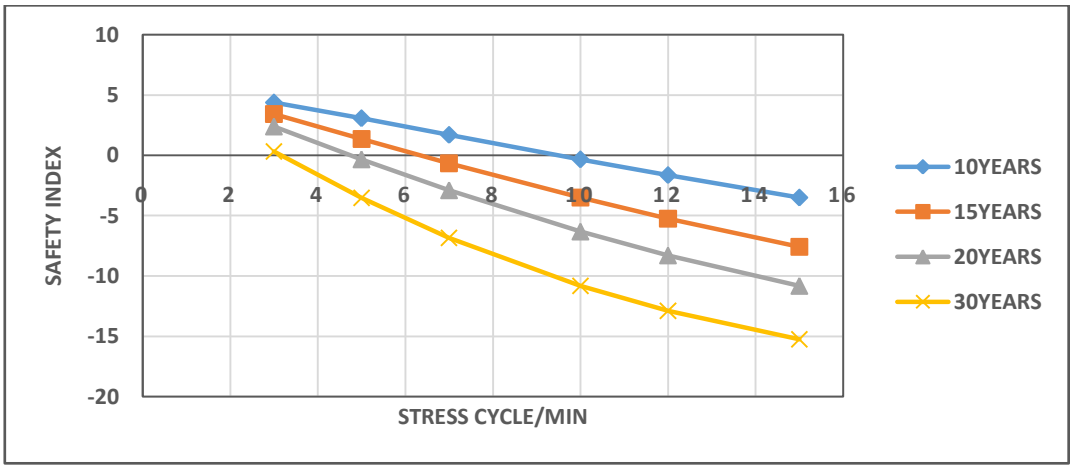



Figure 4.21 Safety Index Versus Stress Cycle (per min) at 10%COV due to Beam Depth

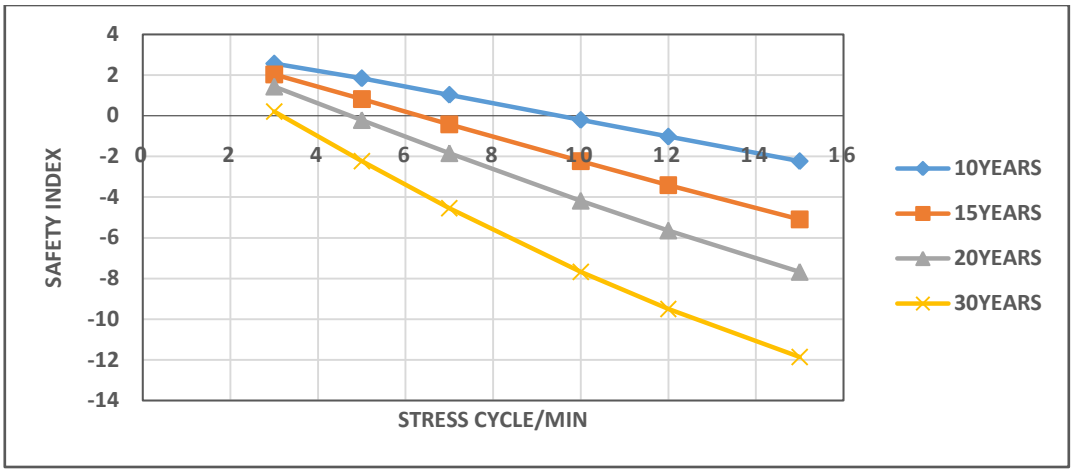



Figure 4.22 Safety Index Versus Stress Cycle (per min) at 20% COVdue to Beam Depth

The failure in the beam due to shear stress caused by axial forces derived from equation (3.27) and represented in the Figures 4.20-4.22 gave an increase in value of the safety index as the COV due to depth decreases.At high coefficient of variation of load action (say 20%), the reliability index for the beam dropped from 2.56 after 10years at 3cycles/min(when load action covariance was 20%) to -16.7 after 40years at 15cycles/min. Likewise, the reliability index dropped from 4.38 and 2.56 after 10years at 3cycles/min (at load action covariance of 10% and 5%) to -15.5 and -11.9 respectively after 40years at 15cycles/min.Under high stress action of load, shear failure start to occur from the critical section near the support and diagonal cracks begins to develop as a result of fatigue.

### 4.3 Failure due to Flexure

The depth of beam was varied from 1000-1200mm alongside load cycle of 3-15 stress cycle/min for a period of 10-40years and a reliability analysis was carried out using MATLAB program “dc_fatigue.m” for fatigue failure under moment condition(see appendix I). Concrete and steel grades of 30N/mm$^2$ and 500N/mm$^2$from Eurocode 2, 2006

were adopted respectively. The safety indices for the failure mode under bending are tabulated (see appendix II) and then graphically represented in the Figures 4.23-4.25.

### 4.3.1 Effect of Beam Depth Variation on the Safety Index with respect to Bending Moment under Stress Cycle

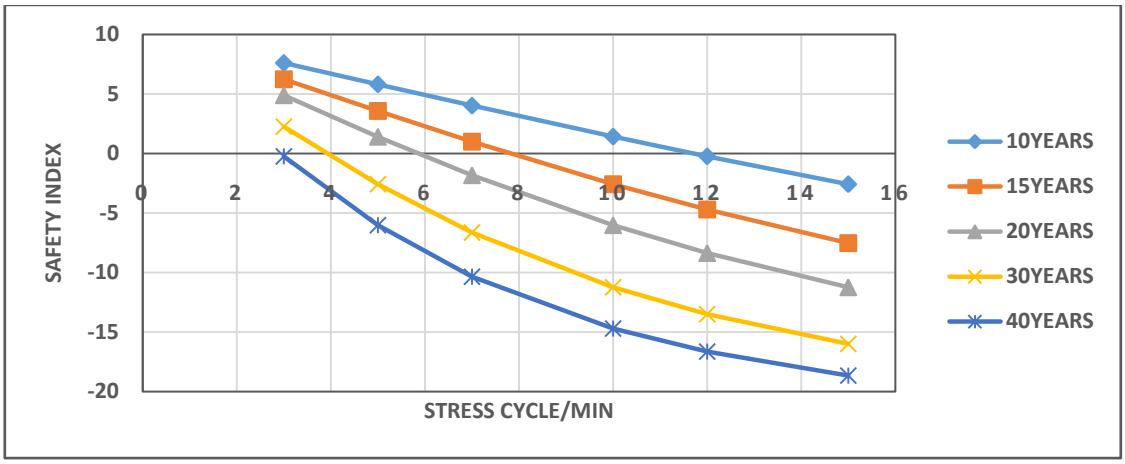



Figure 4.23 Safety Index Versus Stress Cycle (per min) at Beam Depth =1000mm

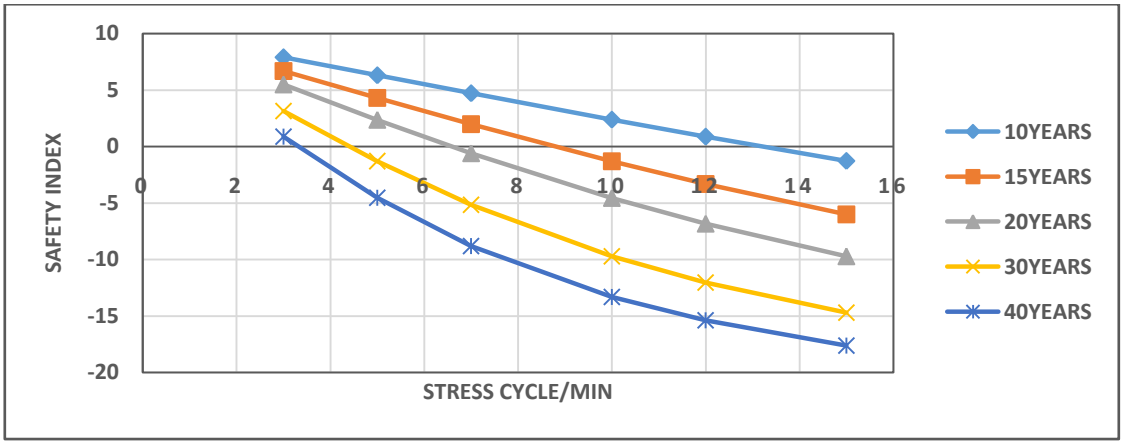



Figure 4.24 Safety Index Versus Stress Cycle (per min) at Beam Depth =1100mm

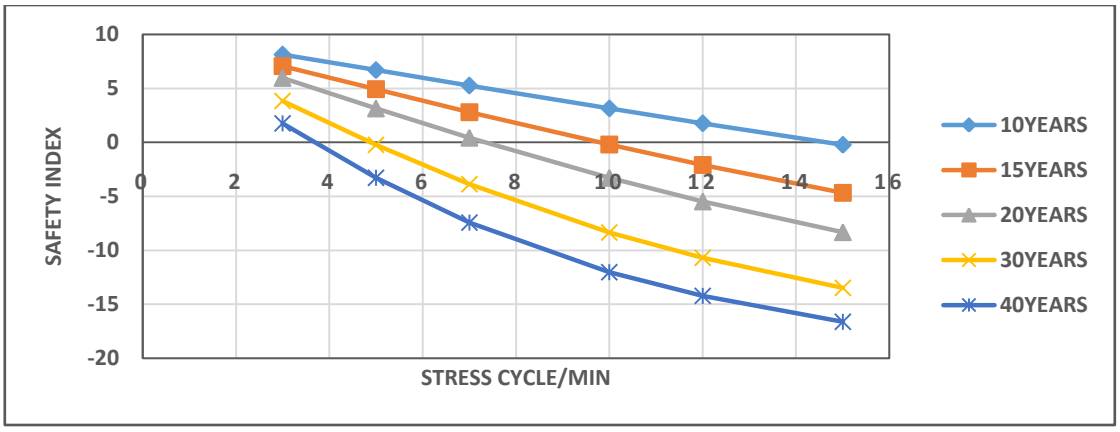



Figure 4.25 Safety Index Versus Stress Cycle (per min) at Beam Depth =1200mm

FromFigures 4.23-4.25it was deduced that the safety index increased as the beam depth increased and vice versa. Also at 7cycles/min and below the reliability index value is 4.01 to -0.25, 4.69 to 0.86 and 5.25 to 1.76 for beam depths of 1.0m, 1.1m and 1.2m respectively after 10 and 40years to traffic. At 12cycles and above there was a negative decrease in safety index indicating total fatigue failure.

At 12cycles/mins it is safe for depths of 1100 and 1200mm after 10years, this is because the compression zone of the beam is safer from failure as the tension zone(most critical zone) are purely under bending stress due to its concrete properties (Bulavs et.al, 2005).

Most beams lose their design strength and durability as the load exceeds its design capacity. The fatigue failure zone are initiated by the cracks from tension zone and extended to the compression zone before reaching its ultimate fatigue life. These cracks usually starts from the bottom of the applied load which indicates flexural failure (Marco et.al, 2005).

### 4.3.2 Effect of Concrete Strength under Bending on the Safety Index with respect to the Stress Cycle

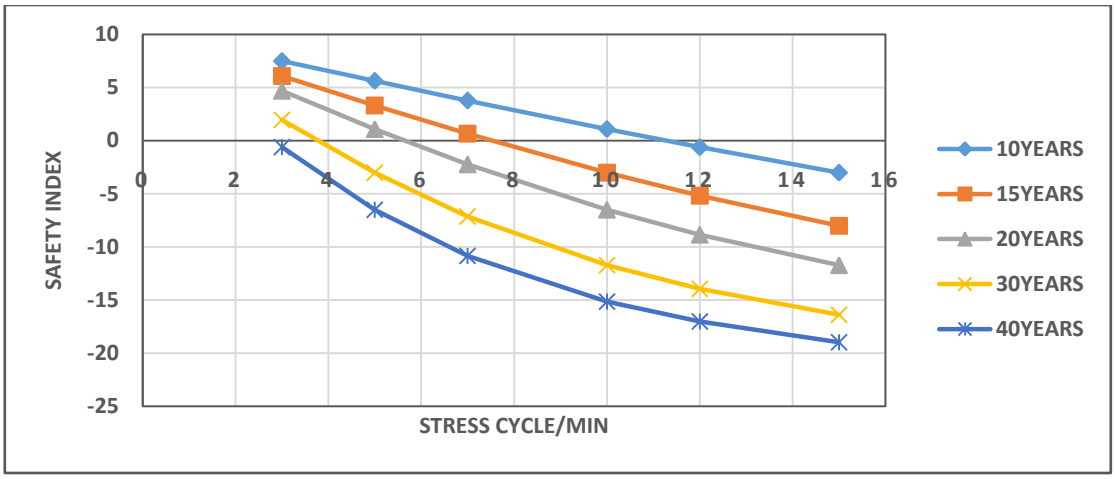



Figure 4.26 Safety Index Versus Stress Cycle (per min) at $f_{cu}$=25N/mm$^2$

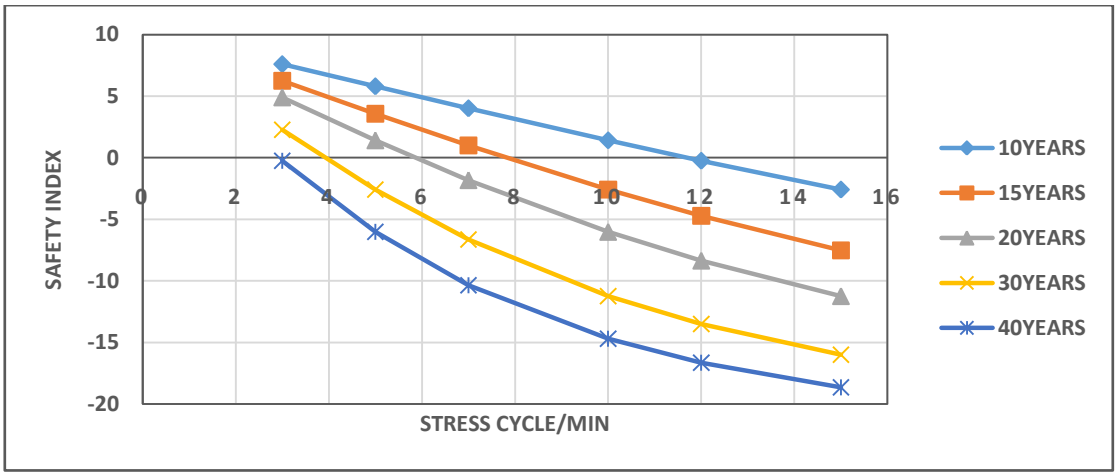



Figure 4.27 Safety Index Versus Stress Cycle (per min) at $f_{cu}$=30N/mm$^2$

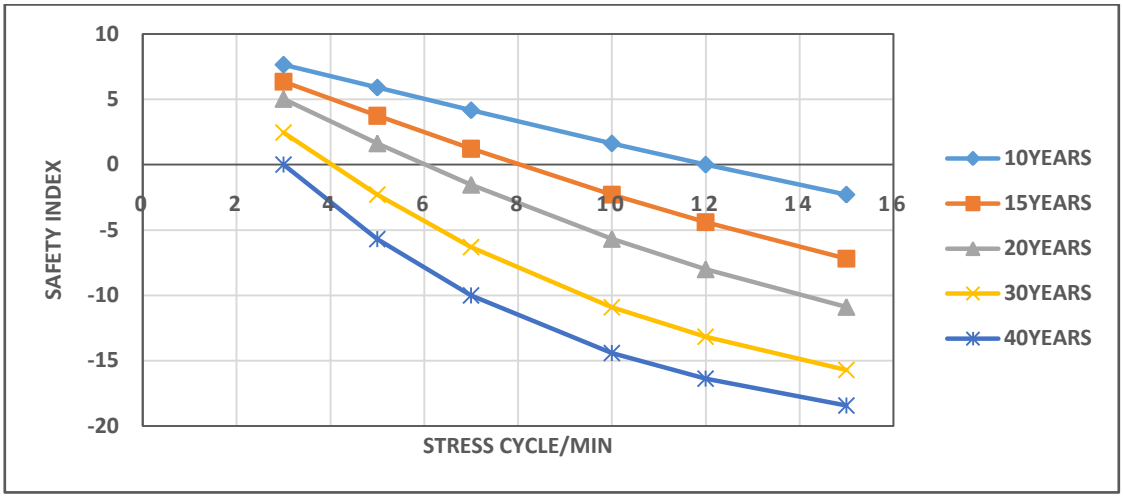



Figure 4.28 Safety Index Versus Stress Cycle (per min) at $f_{cu}$=35N/mm$^2$

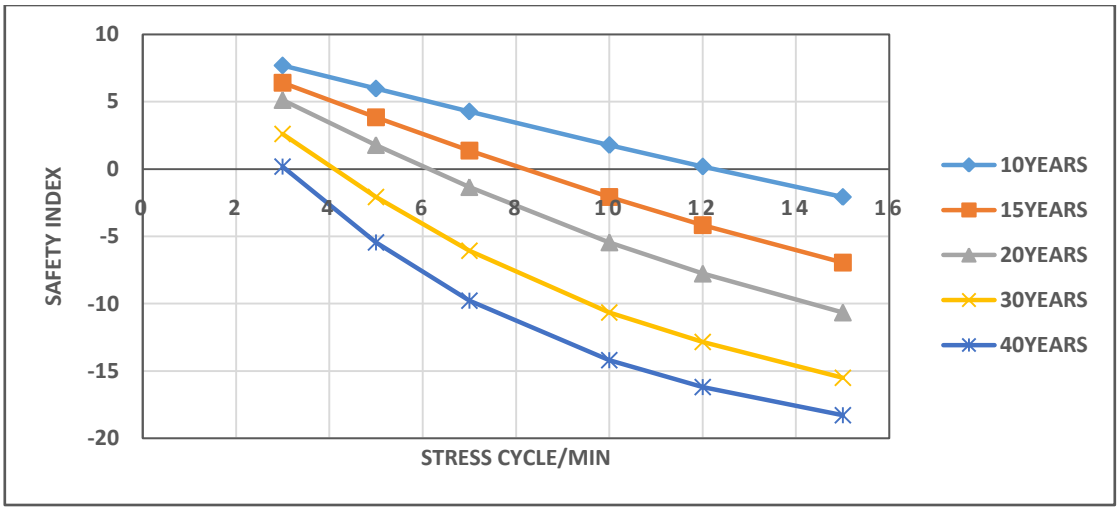



Figure 4.29 Safety Index Versus Stress Cycle (per min) at $f_{cu}$=40N/mm$^2$

Figures 4.23-4.26 show the variation of safety index with stress cycle for concrete grades 25, 30, 35 and 40 respectively. It was noted that as the strength of concrete increased the safety index slightly increases as well under the same level of stress cycle. At a stress level of 5cycles/min and below, the beam was safe under bending for up to 20years and at 40 years there was ultimate fatigue failure as the beam failed under all levels of stress applied.

At a stress level of 10cycles/min after 10 years to 15cycles/min after 40 years, the safety index dropped from 1.08 to -19, 1.41 to -18.7, 1.63 to -18.4 and 1.78 to -18.3 for concrete grades of 25, 30, 35 and 40 respectively

### 4.3.3 Effect of Coefficient of Variationof Concrete Strength under Bending on the Safety Index with respect to the Stress Cycle

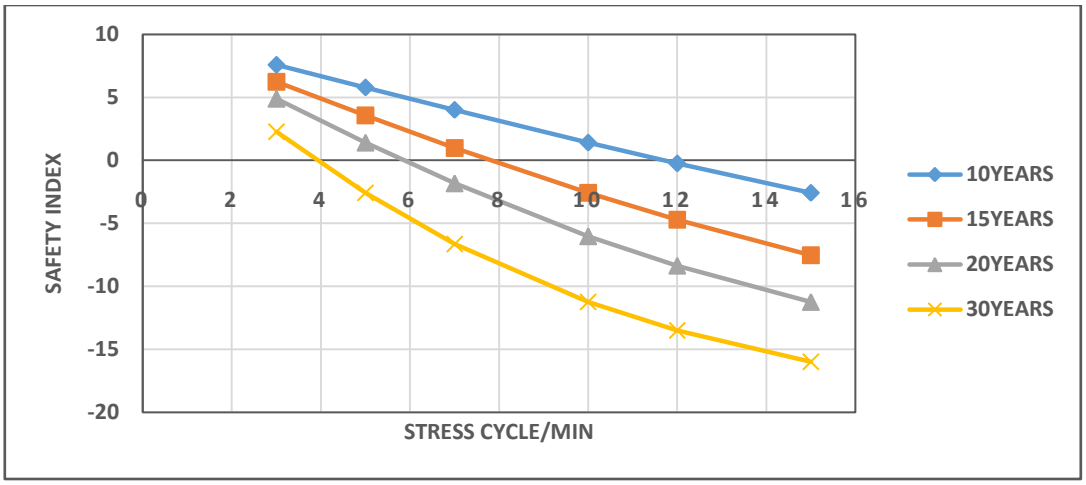



Figure 4.30 Safety Index Versus Stress Cycle (per min) at 5% COV due to Concrete

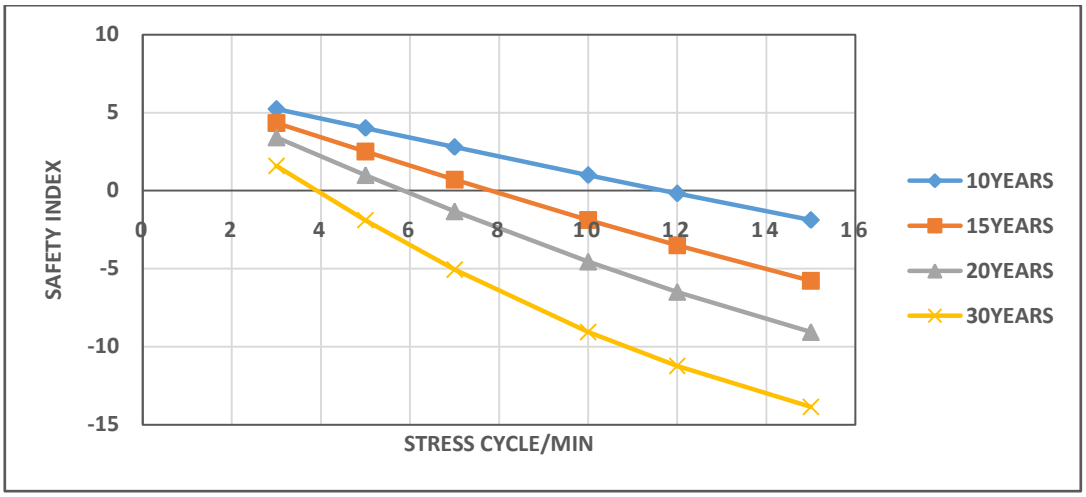



Figure 4.31 Safety Index Versus Stress Cycle (per min) at 10% COV due to Concrete

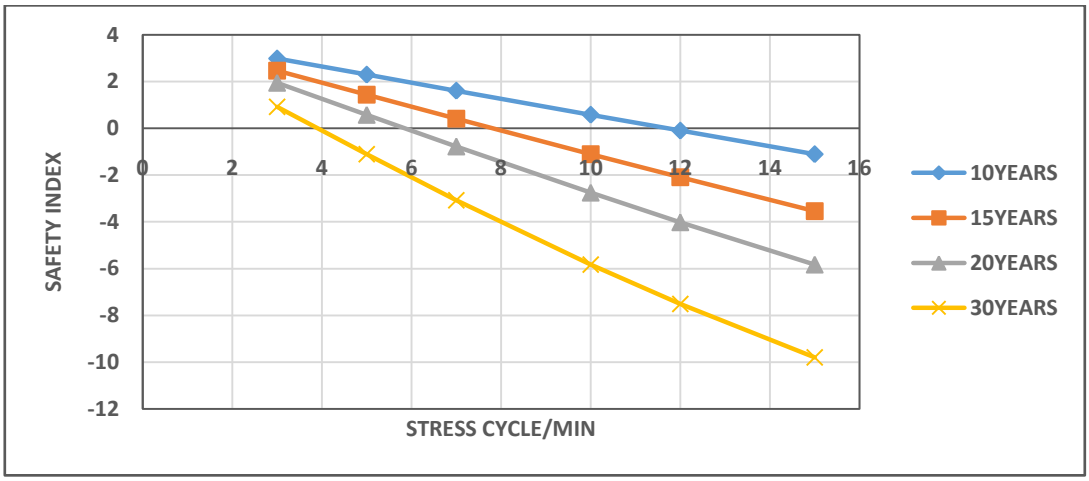



Figure 4.32 Safety Index Versus Stress Cycle (per min) at 20% COV due to Concrete

The Figures 4.30-4.32show the variation of safety index with stress cycle for coefficient of variation due to concrete grades. It was observed that at COV=0.05 the concrete was strongest due to it higher safety index value as compared to COV =0.10 and COV=0.20. At a stress level of 3 cycles/min and below, the beam was safe all through the year and at 12 cycles and above there was fatigue failure all through.At a stress level of 10cycles/min after 10 years to 15cycles/min after 40 years, the safety index dropped from 1.08 to -19, 1.41 to -18.7, 1.63 to -18.4 and 1.78 to -18.3 for concrete grades of 25, 30, 35 and 40 respectively For concrete grades of 30 and 35 on different projects (Mohamed , 2012) found out that the compressive strength is well represented by the log-normal distribution with a COV of 0.2 and 0.22 respectively. Also for higher grade of 40 and above one can use additives to achieve the higher concrete strength so that the COV be around 0.15-0.17 which is reasonable. On the other hand, for a grade 30 and 35 concrete, the values are between the lower and higher grade thus higher variations are needed to reach required grade with higher COV which revealed that for any type of project, one is required to increase the quality control procedure to reach the required concrete grades.

### 4.3.4 Effect of Coefficient of Variationof Beam Depth under Bending on the Safety Index with respect to the Stress Cycle

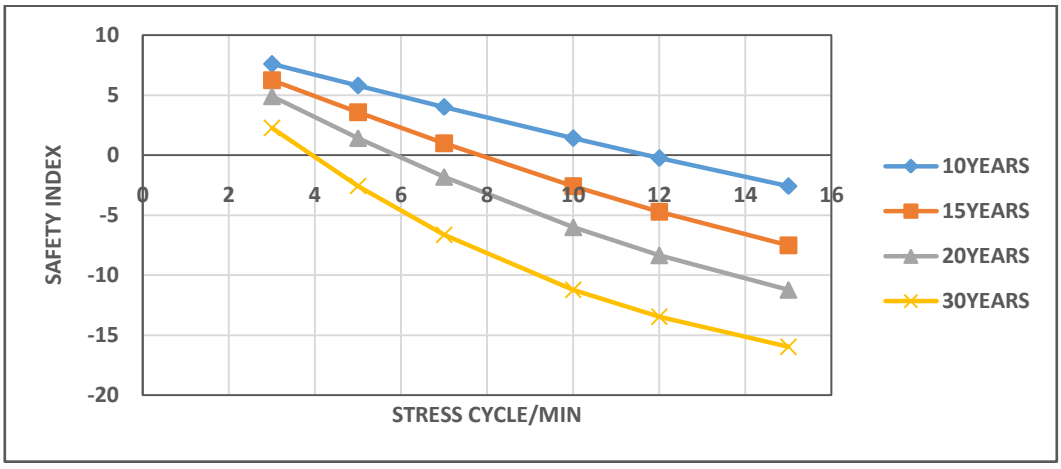



Figure 4.33 Safety Index Versus Stress Cycle (per min) at 5% COV due to Depth

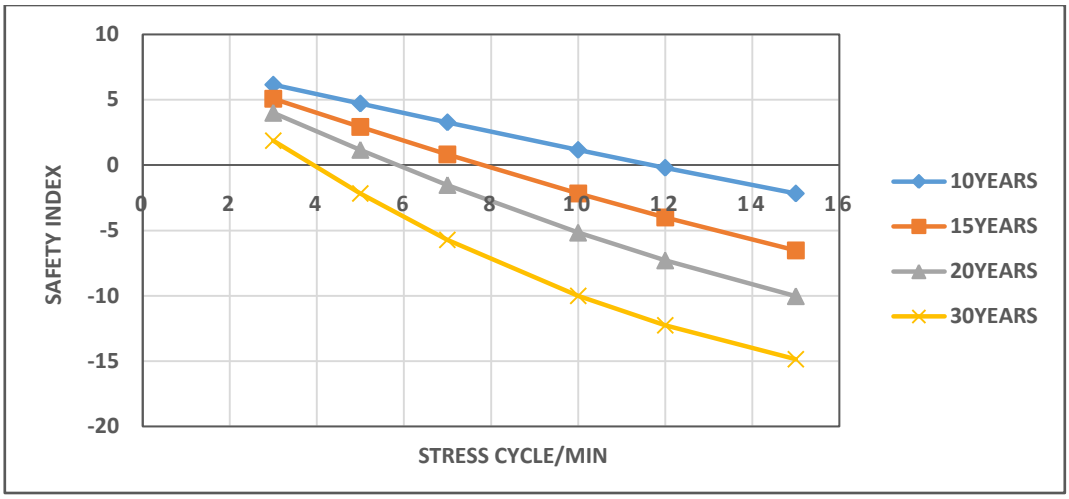



Figure 4.34 Safety Index Versus Stress Cycle (per min) at 10% COV due to Depth

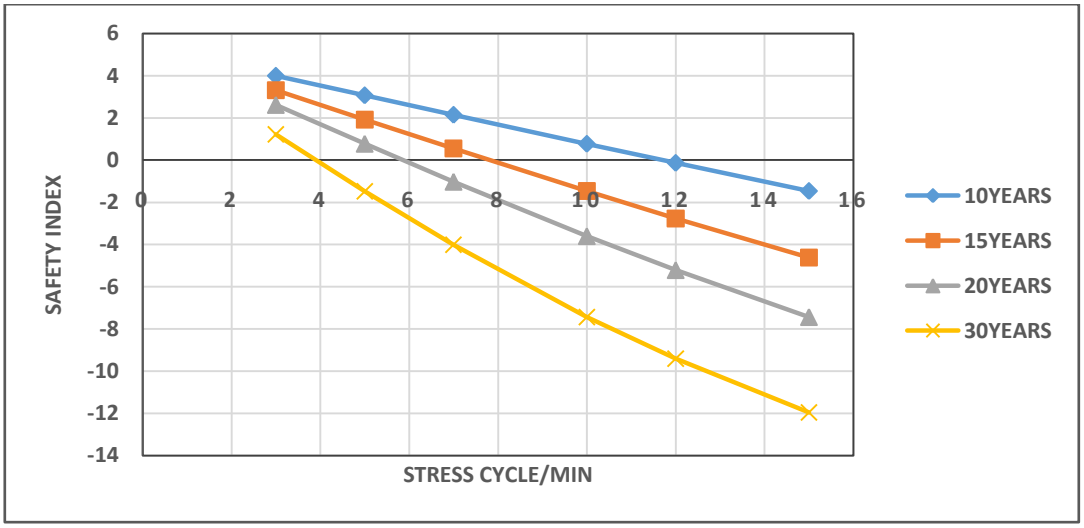



Figure 4.35 Safety Index Versus Stress Cycle (per min) at 20% COV due to Depth

For Figures 4.33-4.35it can be observed that the higher the COV the lower the safety index and vice versa.At a stress level of 10cycles/min after 10 years to 15cycles/min after 40 years, the safety index dropped from 1.08 to -19, 1.41 to -18.7, 1.63 to -18.4 and 1.78 to -18.3 for concrete grades of 25, 30, 35 and 40 respectively At 5 cycles/min the beam was safe under bending as all the safety index values were in the positive zone. At 12

cycles/min and above, the beam reached its ultimate fatigue life under effect of bending stress as all the safety index values were negative.

# CHAPTER FIVE

# CONCLUSION AND RECOMMENDATIONS

## 5.1 CONCLUSIONS

From the reliability analysis carried out on RC bridge deck under fatigue stress and failure modes of shear and flexure conditions, the following conclusions can be reached.

1. The failure due to shear stress has the most critical effect as it gave the lowest safety index range of values of 6.06 to -2.11 at 3cycles/min for 10years and 40years respectively when compared to flexural stress with a safety index value of 7.59 to -0.25 at 3cycles/min for 10years and 40years.
2. The safety index was found to increase as the depth of section increases under repeated level of stress.
3. The safety index was found to increase with decrease in the coefficient of variation of the concrete and depth of section from a range of 7.61 to -0.47 at 5% covariance and from 3.47 to -0.23 at 20% covariance under stress level of 3 cycles/min.

## 5.2RECOMMENDATIONS

The following recommendations are made;

1. For a single span bridge of 15m and less under high level of traffic load, a deck beam depth of 1200mm, deck slab of 250mm and concrete strength class of 30-35 are adequate for design and construction

2. The quality control of concrete should be adequate to increase durability and reduce crack effect under fatigue stress action.

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Federal Highway Administration, 200 New Jersey Avenue, SE Washington D.C 20590, (Retrieved October 21, 2015)fromhttp://www.fhwa.**dot**.gov/**bridge**/britab.cfm

## APPENDIX I

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
%
%             THIS IS A MATLAB PROGRAM FOR RELIABILITY ANALYSIS
%
%                     FOR REINFORCED CONCRETE BRIDGE DECK
%
%                          SUBJECTED TO FATIGUE
%
%
%
%                            THE MAIN PROGRAM
%
%
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
%  CODE DEVELOPED BY: THOMAS U. OSUMEJE
%
%  MAJOR SUPERVISOR                     MINOR SUPERVISOR
%  DR. A. OCHOLI                        DR. M. J. KAURA
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%
clear; clc; close all
fid12 = fopen('RESULT_12.m','w');
t = clock;
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%
disp('-------------------------------------------------------
');
disp('   1:  FAILURE MODE 1: Flexural Failure of Slab
');
disp('   2:  FAILURE MODE 2: Shear Failure of Interior Beams
');
```

```
disp('   3:  FAILURE MODE 3: Flexural Failure of Interior Beams
');
disp('   0:  Exit
');
disp('
');
analysistype = input('   CHOOSE FAILURE MODE OPTION FROM ABOVE
');
switch analysistype
%
case 0 % ---- EXIT
%
   disp('           ');
   disp('  Bye, bye.');
   disp('           ');
%
case 1 % ----FAILURE MODE 1: FLEXURAL FAILURE OF THE SLAB
%
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
disp('  ')
disp('    This is a MATLAB program for  ');
disp('    Probabilistic Analysis of reinforced ');
disp('    concrete bridge deck subjected to high frequency fatigue  ');
disp('    Code developed by Thomas Unogwu Osumeje');
disp('    Under the supervision of Dr. Amana Ocholi and Dr M.J. Kaura ');
disp('  ')
%
disp('   The live load moment on the slab is based on a single wheel')
disp('   truck load model 1, placed in the centre of the slab')
disp('   ')
fck = input('   Input the concrete compressive strength');
disp('    ')
fyk = input('   Input the steel yield strength');
disp('    ')
cons11 = input('   Input slab moment coefficient due to concrete');
disp('    ')
cons1 = cons11*1000000.0;
cons21 = input('   Input slab moment coefficient due to ashpalt');
disp('    ')
cons2 = cons21*1000000.0;
cons31 = input('   Input slab moment coefficient due to live load');
disp('    ')
cons3 = cons31*1000000.0;
disp('   ')
h = input('   Input the depth of the slab in mm');
%
AS = input('   Input Area of Reinforcing Steel in mm^2');
%
steel_diameter = input('   Input Steel Diameter in mm');
%
c_cover = 20.0;
d = h - c_cover - (steel_diameter/2.0) ;
maximum_rept = input('   maximum number of load repetition');
%
```

```
wheel_frequency = input('Input wheel frequency (Number of vehicles per
minutes');
%
number_of_years = input('Input number of years after openining to
traffic');
%
actual_rept = wheel_frequency*number_of_years*360.0*24.0*60.0;
Damage_factor = maximum_rept/actual_rept;
%
fyd = fyk*Damage_factor ;fcd = fck*Damage_factor;
if(Damage_factor > 1.0)
    Damage_Facor = 1;
end
[moder,b,modec,modeash,modeqk]...
          = thomas_input_mode1;
disp('     ')
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
beta_mode_1 =
thomas_slab_flexure(moder,AS,fyd,d,b,fcd,cons1,cons2,cons3,...
    modec,modeash,modeqk);
pf1 = thomas_probability_of_failure(beta_mode_1);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
disp('this is the safety index for failure mode 1')
disp('    ')
disp(beta_mode_1)
disp('    ')
disp('this is the probability of failure for failure mode 1')
disp('    ')
disp(pf1)
%
case 2 % ----FAILURE MODE 2: SHEAR FAILURE OF INTERIOR BEAMS
%
disp('   ')
disp('    This is a MATLAB program for
');
disp('    Probabilistic Analysis of reinforced
');
disp('    concrete bridge deck subjected to high frequency fatigue
');
disp('    Code developed by Thomas U. Osumeje
');
disp('    Under the supervision of Dr. Amana Ocholi & Dr. M.J. Kaura
');
disp('   ')
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
%  The LM1 is placed with the point load over the support and 2m away
%  from the support in order to obtain the critical shear due to live
%  load
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
```

```
disp('    The live load moment on the slab is based on a single wheel')
disp('    truck load model 1, placed in the centre of the slab')
disp('    ')
fck = input('    Input the concrete compressive strength');
disp('     ')
fyk = input('    Input the steel yield strength');
disp('     ')
cons11 = input('    Input interior beam shear coefficient due to
concrete');
disp('     ')
cons4 = cons11*1000.0;
cons21 = input('    Input interior beam shear coefficient due to
ashpalt');
disp('     ')
cons5 = cons21*1000.0;
cons31 = input('    Input interior beam shear coefficient due to live
load');
disp('     ')
cons6 = cons31*1000.0;
disp('    ')
h = input('    Input the depth of the beam in mm');
%
AS = input('    Input Area of Reinforcing Steel in mm^2');
%
steel_diameter = input('    Input Steel Diameter in mm');
%
c_cover = 50.0;
d = h - c_cover - (steel_diameter/2.0) ;
%
maximum_rept = input('    maximum number of load repetion');
%
wheel_frequency = input('Input wheel frequency (Number of vehicles per
minutes');
%
number_of_years = input('Input number of years after openining to
traffic');
%
actual_rept = wheel_frequency*number_of_years*360.0*24.0*60.0;
Damage_factor = maximum_rept/actual_rept;
%
fyd = fyk*Damage_factor ;fcd = fck*Damage_factor;
if(Damage_factor > 1.0)
    Damage_Facor = 1;
end
[moder,b,modec,modeash,modeqk]...
         = thomas_input_mode2;
disp('     ')
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
beta_mode_2 = thomas_beam_shear(moder,d,b,fcd,cons4,cons5,cons6,...
    modec,modeash,modeqk);
pf2 = thomas_probability_of_failure(beta_mode_2);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
disp('this is the safety index for failure mode 2')
disp('    ')
disp(beta_mode_2)
```

```
disp('   ')
disp('this is the probability of failure for failure mode 2')
disp('   ')
disp(pf2)
%
case 3 % ----FAILURE MODE 3: FLEXURAL MOMENT FAILURE OF INTERIOR BEAMS
%
disp('  ')
disp('    This is a MATLAB program for
');
disp('    Probabilistic Analysis of reinforced
');
disp('    concrete bridge deck subjected to high frequency fatigue
');
disp('    Code developed by Thomas U. Osumeje
');
disp('    Under the supervision of Dr. Amana Ocholi & Dr. M.J. Kaura
');
disp('  ')
%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
%  The LM1 is placed with the point load over the support and 2m away
%  from the support in order to obtain the critical shear due to live
%  load
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
disp('   The live load moment on the slab is based on a single wheel')
disp('   truck load model 1, placed in the centre of the slab')
disp('   ')
fck = input('   Input the concrete compressive strength');
disp('    ')
fyk = input('   Input the steel yield strength');
disp('    ')
cons11 = input('   Input interior beam moment coefficient due to
concrete');
disp('    ')
cons7 = cons11*1000000.0;
cons21 = input('   Input interior beam moment coefficient due to
ashpalt');
disp('    ')
cons8 = cons21*1000000.0;
cons31 = input('   Input interior beam moment coefficient due to live
load');
disp('    ')
cons9 = cons31*1000000.0;
disp('   ')
h = input('   Input the depth of the beam in mm');
%
AS = input('   Input Area of Reinforcing Steel in mm^2');
%
steel_diameter = input('Input Steel Diameter in mm');
%
```

```
c_cover = 50.0;
hbeam = 1000.0;
d = h - c_cover - (steel_diameter/2.0) ;
%
maximum_rept = input('   maximum number of load repetion');
%
wheel_frequency = input('Input wheel frequency (Number of vehicles per
minutes');
%
number_of_years = input('Input number of years after openining to
traffic');
%
actual_rept = wheel_frequency*number_of_years*360.0*24.0*60.0;
Damage_factor = maximum_rept/actual_rept;
%
fyd = fyk*Damage_factor ;fcd = fck*Damage_factor;
if(Damage_factor > 1.0)
    Damage_Facor = 1;
end
[moder,b,modec,modeash,modeqk]...
         = thomas_input_mode3;
     disp('    ')
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
beta_mode_3 = thomas_beam_flexure(moder,AS,...
    fyd,d,b,fcd,cons7,cons8,cons9,modec,modeash,modeqk);
pf3 = thomas_probability_of_failure(beta_mode_3);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
disp('this is the safety index for failure mode 3')
disp('   ')
disp(beta_mode_3)
disp('   ')
disp('this is the probability of failure for failure mode 3')
disp('   ')
disp(pf3)
%
otherwise% ------------------------------------------------
   disp('    ');
   disp('  You entered an invalid choice.');
end% -----

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
%       TRANSFORMATION OF NON-NORMAL DISTRIBUTION
%       TO EQUIVALENT NORMAL DISTRIBUTION
%
%        RACKWITZ FIESSLER TRANSFORMATION
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
function [DISTYPE1,DISTYPE2,DISTYPE3,DISTYPE4,DISTYPE5, ...
    DISTYPE6,DISTYPE7,DISTYPE8,DISTYPE9] = fatigue_distribution_mode1
%
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
% DISTYPE1 for strength model uncertainty
% DISTYPE2 for cross sectional area;
% DISTYPE3 for steel strength;
% DISTYPE4 for depth;
% DISTYPE5 for width;
% DISTYPE6 for concrete strength;
% DISTYPE7 for concrete weight model uncertainty;
% DISTYPE8 for asphalt weight model uncertainty;
% DISTYPE9 for traffic load model uncertainty;
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
% INCLUDED DISTRIBUTION MODELS
%
% TYPE 1 FOR: Normal DISTRIBUTION
% TYPE 2 FOR: LOGNORMAL DISTRIBUTION
% TYPE 3 FOR: GUMBEL DISTRIBUTION
% TYPE 4 FOR: WEIBUL DISTRIBUTION
% TYPE 5 FOR: FRECHET DISTRIBUTION
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
DISTYPE1 = 1; DISTYPE2 = 1;
DISTYPE3 = 1; DISTYPE4 = 1;
DISTYPE5 = 1; DISTYPE6 = 1;
DISTYPE7 = 1; DISTYPE8 = 1;
DISTYPE9 = 1;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
%       TRANSFORMATION OF NON-NORMAL DISTRIBUTION
%       TO EQUIVALENT NORMAL DISTRIBUTION
%
%        RACKWITZ FIESSLER TRANSFORMATION
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
function [DISTYPE1,DISTYPE2,DISTYPE3,DISTYPE4,DISTYPE5, ...
    DISTYPE6,DISTYPE7] = fatigue_distribution_mode2
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
% DISTYPE1 for strength model uncertainty
% DISTYPE2 for depth;
% DISTYPE3 for width;
% DISTYPE4 for concrete strength;
% DISTYPE5 for concrete weight model uncertainty;
% DISTYPE6 for asphalt weight model uncertainty;
% DISTYPE7 for traffic load model uncertainty;
```

```
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
% INCLUDED DISTRIBUTION MODELS
%
% TYPE 1 FOR: Normal DISTRIBUTION
% TYPE 2 FOR: LOGNORMAL DISTRIBUTION
% TYPE 3 FOR: GUMBEL DISTRIBUTION
% TYPE 4 FOR: WEIBUL DISTRIBUTION
% TYPE 5 FOR: FRECHET DISTRIBUTION
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
DISTYPE1 = 1; DISTYPE2 = 1;
DISTYPE3 = 1; DISTYPE4 = 1;
DISTYPE5 = 1; DISTYPE6 = 1;
DISTYPE7 = 1;


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
%       TRANSFORMATION OF NON-NORMAL DISTRIBUTION
%       TO EQUIVALENT NORMAL DISTRIBUTION
%
%        RACKWITZ FIESSLER TRANSFORMATION
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
function [DISTYPE1,DISTYPE2,DISTYPE3,DISTYPE4,DISTYPE5, ...
    DISTYPE6,DISTYPE7,DISTYPE8,DISTYPE9] = fatigue_distribution_mode3
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
% DISTYPE1 = for strength model uncertainty
% DISTYPE2 = for cross sectional area;
% DISTYPE3 = for steel strength;
% DISTYPE4 = for depth;
% DISTYPE5 = for width;
% DISTYPE6 = for concrete strength;
% DISTYPE7 = for concrete weight model uncertainty;
% DISTYPE8 = for asphalt weight model uncertainty;
% DISTYPE9 = for traffic load model uncertainty;
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
% INCLUDED DISTRIBUTION MODELS
%
% TYPE 1 FOR: Normal DISTRIBUTION
% TYPE 2 FOR: LOGNORMAL DISTRIBUTION
% TYPE 3 FOR: GUMBEL DISTRIBUTION
% TYPE 4 FOR: WEIBUL DISTRIBUTION
```

```
% TYPE 5 FOR: FRECHET DISTRIBUTION
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
DISTYPE1 = 1; DISTYPE2 = 1;
DISTYPE3 = 1; DISTYPE4 = 1;
DISTYPE5 = 1; DISTYPE6 = 1;
DISTYPE7 = 1; DISTYPE8 = 1;
DISTYPE9 = 1;


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
function [mean, std] = frechet(rfmu,rmfx)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%
%
%           SUBROUTINE FOR RACKWITZ FIESSLER TRANSFORMATION
%
%       TRANSFORMATION OF GUMBEL VARIABLES TO EQUIVALENT NORMAL          %
%                              VARIABLES
%
%
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%
mean1 = rfmu;
stan1 = rmfx;
%
X = mean1;
%
AG = 2.33/((stan1/mean1)^(0.6770001));
cooler = 1-(1/AG);
cooler2 = gamma(cooler);
xx = X;
XU = mean1/cooler2;
cdf  = exp(-(XU/xx)^AG);
pdf  = cdf*(AG/XU)*(XU/xx)^(AG+1);
cdfx = cdf;
%
Z    = invnorm(cdfx);
pdfz = exp((-Z^2)/2.0D0)/sqrt(6.2833185307D0);
std = pdfz/pdf;
mean = X-rmfx*Z;
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


%
function  xx = gamma(yy)
if(yy<=0)
    xx = 0;
```

```
elseif(yy > 0) && (yy < 1.0)
    xx = gmh(yy);
elseif(yy >= 1.0)
    xxx = yy - 1.0;
    xx = gmh(xxx);
end


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
function [mean, std] = gumbel(rfmu,rmfx)
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%
%
%           SUBROUTINE FOR RACKWITZ FIESSLER TRANSFORMATION
%
%       TRANSFORMATION OF GUMBEL VARIABLES TO EQUIVALENT NORMAL          %
%                             VARIABLES
%
%
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%
mean1 = rfmu;
stan1 = rmfx;
%
X = mean1;
%
AG = 3.14159264/sqrt(6D0)/stan1;
XU = mean1 - 0.57722/AG;
XT = exp(-AG*(X-XU));
cdf  = exp(-XT);
pdf  = AG*XT*cdf;
cdfx = cdf;
%
Z    = invnorm(cdfx);
pdfz = exp((-Z^2)/2.0D0)/sqrt(6.2833185307D0);
std = pdfz/pdf;
mean = X-rmfx*Z;
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
function xx = gmh(y)
%
xx = 1.0 + y*(-0.577191652D0 + ...
     y*(0.9878205891D0 + y*(-0.897056937D0 +...
     y*(0.918206857D0 + y*(0.765704078D0 +...
     y*(0.482199394D0 + y*(-0.193527818D0 +...
     y*(0.035868343D0)))))))));
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
function Z = invnorm(cdfx)
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%
%
%         SUBROUTINE TO EVALUATE INVERSE NORMAL CUMMULATIVE
%
%                   DISTRIBUTION FUNCTION (CDF)
%
%
%
% Uses Hasting Polynomial 26.2.23 of 'Handbook of Mathematical
%
% Functions 1972. M. Abramcwitz and T. M. Stegun Ang 55
%
% National Bureau of Standards US Department of Commerce
%
%
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
if(cdfx<0.5)
    T = sqrt(log(1.0/cdfx^2));
else
    T = sqrt(log(1.0/(1.0-cdfx)^2));
end
Y = (0.001308*T+0.189269)*(T+1.432788)*T+1.0;
Y = Y-((0.010328*T+0.802853)*T+2.515517)/Y;
if(cdfx<0.5)
    Z = -Y;
else
    Z = Y;
end
end
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
function[mean,std] = lognormal(rfmu,rmfx)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%
% SUBROUTINE FOR RACKWITZ FIESSLER TRANSFORMATION OF LOGNORMAL
%       DISTRIBUTION TO EQUIVALENT NORMAL DISTRIBUTION
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%
x0 = rfmu-rmfx;
x = rfmu;
om = sqrt(1.0+(rmfx/(rfmu-x0))^2);
std = (x-x0)*sqrt(2.0*log(om));
```

```
mean = x+(x-x0)*log((rfmu-x0)/(x-x0)/om);
end
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
function [mean, std] = weibull(rfmu,rmfx)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%
%
%           SUBROUTINE FOR RACKWITZ FIESSLER TRANSFORMATION
%
%       TRANSFORMATION OF GUMBEL VARIABLES TO EQUIVALENT NORMAL           %
%                               VARIABLES
%
%
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%
mean1 = rfmu;
stan1 = rmfx;
%
A = ((stan1/mean1)^2)+1;
X = mean1;
XX = X;
%
AG = ((A-1)^(-1/1.835))-0.5;
%
DEL = -0.5;
AG = AG+DEL;
fool1 = 1+(1/AG);
fool11 = gamma(fool1);
ET = mean1/fool11;
TMP = exp(-(XX/ET)^AG);
cdf = 1-TMP;
pdf = (AG/ET)*((XX/ET)^(AG-1))*TMP;
cdfx = cdf;
%
Z    = invnorm(cdfx);
pdfz = exp((-Z^2)/2.0D0)/sqrt(6.2833185307D0);
std = pdfz/pdf;
mean = X-rmfx*Z;
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
function [moder,b,modec,modeash,modeqk]...
         = fatigue_input_mode1
```

```
moder = 1.0;
%AS = 300.0;
b = 1000.0;
modec = 1.0;
modeash = 1.00;
modeqk = 1.0;


%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
function [moder,b,modec,modeash,modeqk]...
         = fatigue_input_mode2
moder = 1.0;
%AS = 7000.00;
steel_diameter = 25.0;
c_cover = 50.0;
hbeam = 1000.0;
d = hbeam - c_cover - (steel_diameter/2.0) ;
b = 400.0;
modec = 1.0;
modeash = 1.00;
modeqk = 1.0;


%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
function [moder,b,modec,modeash,modeqk]...
         = fatigue_input_mode3
moder = 1.0;
%AS = 7000.00;
steel_diameter = 25.0;
c_cover = 50.0;
hbeam = 1000.0;
d = hbeam - c_cover - (steel_diameter/2.0) ;
b = 400.0;
modec = 1.0;
modeash = 1.00;
modeqk = 1.0;


%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
%
% cov1 for strength model uncertainti
% cov2 for depth;
% cov3 for width;
% cov4 for concrete strength;
% cov5 for concrete weight model uncertainty;
% cov6 for ashpalt weight model uncertainty;
% cov7 for traffic load model uncertainty;
%
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
function [cov1,cov2,cov3,cov4,cov5,cov6,cov7,cov8,cov9] =
fatigue_covariance_mode1
%
cov1 = 0.05;
cov2 = 0.05;
cov3 = 0.05;
cov4 = 0.05;
cov5 = 0.05;
cov6 = 0.05;
cov7 = 0.05;
cov8 = 0.05;
cov9 = 0.05;
end


%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
%
% cov1 for strength model uncertainty
% cov2 for depth;
% cov3 for width;
% cov4 for concrete strength;
% cov5 for concrete weight model uncertainty;
% cov6 for asphalt weight model uncertainty;
% cov7 for traffic load model uncertainty;
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
function [cov1,cov2,cov3,cov4,cov5,cov6,cov7] = fatigue_covariance_mode2
%
cov1 = 0.05;
cov2 = 0.05;
cov3 = 0.05;
cov4 = 0.05;
cov5 = 0.05;
cov6 = 0.05;
cov7 = 0.05;
end

%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
%
% cov1 for strength model uncertainty
% cov2 for depth;
% cov3 for width;
% cov4 for concrete strength;
% cov5 for concrete weight model uncertainty;
% cov6 for asphalt weight model uncertainty;
% cov7 for traffic load model uncertainty;
```

```
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
function [cov1,cov2,cov3,cov4,cov5,cov6,cov7,cov8,cov9] =
fatigue_covariance_mode3
%
cov1 = 0.05;
cov2 = 0.05;
cov3 = 0.05;
cov4 = 0.05;
cov5 = 0.05;
cov6 = 0.05;
cov7 = 0.05;
cov8 = 0.05;
cov9 = 0.05;
end

function pf = fatigue_probability_of_failure(beta)
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%   REFERENCE: Melchers R. E. Structural Reliability analysis
%              and prediction. 1999. Ellis Horwood Limited
%              John Wlley and Sons. pp 372 to 374
%
% COMPLEMENTARY STANDARD NORMAL ROUTINE
%
if(beta <= 0)
    pf = 1.0;
elseif(beta > 0) && (beta <= 0.01)
    pf = 0.4960;
elseif(beta > 0.01) && (beta <= 0.02)
    pf = 0.4920;
elseif(beta > 0.02) && (beta <= 0.03)
    pf = 0.4880;
elseif(beta > 0.03) && (beta <= 0.04)
    pf = 0.4841;
elseif(beta > 0.04) && (beta <= 0.05)
    pf = 0.4801;
elseif(beta > 0.05) && (beta <= 0.06)
    pf = 0.4761;
elseif(beta > 0.06) && (beta <= 0.07)
    pf = 0.4721;
elseif(beta > 0.07) && (beta <= 0.08)
    pf = 0.4681;
elseif(beta > 0.08) && (beta <= 0.09)
    pf = 0.4642;
elseif(beta > 0.09) && (beta <= 0.10)
    pf = 0.4602;
elseif(beta > 0.10) && (beta <= 0.11)
    pf = 0.4562;
elseif(beta > 0.11) && (beta <= 0.12)
    pf = 0.4522;
elseif(beta > 0.12) && (beta <= 0.13)
    pf = 0.4483;
elseif(beta > 0.13) && (beta <= 0.14)
```

```
    pf = 0.4443;
elseif(beta > 0.14) && (beta <= 0.15)
    pf = 0.4404;
elseif(beta > 0.15) && (beta <= 0.16)
    pf = 0.4364;
elseif(beta > 0.16) && (beta <= 0.17)
    pf = 0.4325;
elseif(beta > 0.17) && (beta <= 0.18)
    pf = 0.4286;
elseif(beta > 0.18) && (beta <= 0.19)
    pf = 0.4247;
elseif(beta > 0.19) && (beta <= 0.20)
    pf = 0.4207;
elseif(beta > 0.20) && (beta <= 0.21)
    pf = 0.4168;
elseif(beta > 0.21) && (beta <= 0.22)
    pf = 0.4129;
elseif(beta > 0.22) && (beta <= 0.23)
    pf = 0.4091;
elseif(beta > 0.23) && (beta <= 0.24)
    pf = 0.4052;
elseif(beta > 0.24) && (beta <= 0.25)
    pf = 0.4013;
elseif(beta > 0.25) && (beta <= 0.26)
    pf = 0.3974;
elseif(beta > 0.26) && (beta <= 0.27)
    pf = 0.3936;
elseif(beta > 0.27) && (beta <= 0.28)
    pf = 0.3897;
elseif(beta > 0.28) && (beta <= 0.29)
    pf = 0.3859;
elseif(beta > 0.29) && (beta <= 0.30)
    pf = 0.3821;
elseif(beta > 0.30) && (beta <= 0.31)
    pf = 0.3783;
elseif(beta > 0.31) && (beta <= 0.32)
    pf = 0.3745;
elseif(beta > 0.32) && (beta <= 0.33)
    pf = 0.3707;
elseif(beta > 0.33) && (beta <= 0.34)
    pf = 0.3669;
elseif(beta > 0.34) && (beta <= 0.35)
    pf = 0.3632;
elseif(beta > 0.35) && (beta <= 0.36)
    pf = 0.3594;
elseif(beta > 0.36) && (beta <= 0.37)
    pf = 0.3557;
elseif(beta > 0.37) && (beta <= 0.38)
    pf = 0.3520;
elseif(beta > 0.38) && (beta <= 0.39)
    pf = 0.3483;
elseif(beta > 0.39) && (beta <= 0.40)
    pf = 0.3446;
elseif(beta > 0.40) && (beta <= 0.41)
    pf = 0.3409;
elseif(beta > 0.41) && (beta <= 0.42)
    pf = 0.3372;
```

```
elseif(beta > 0.42) && (beta <= 0.43)
    pf = 0.3338;
elseif(beta > 0.43) && (beta <= 0.44)
    pf = 0.3300;
elseif(beta > 0.44) && (beta <= 0.45)
    pf = 0.3264;
elseif(beta > 0.45) && (beta <= 0.46)
    pf = 0.3228;
elseif(beta > 0.46) && (beta <= 0.47)
    pf = 0.3192;
elseif(beta > 0.47) && (beta <= 0.48)
    pf = 0.3156;
elseif(beta > 0.48) && (beta <= 0.49)
    pf = 0.3121;
elseif(beta > 0.49) && (beta <= 0.50)
    pf = 0.3085;
elseif(beta > 0.50) && (beta <= 0.51)
    pf = 0.3050;
elseif(beta > 0.51) && (beta <= 0.52)
    pf = 0.3015;
elseif(beta > 0.52) && (beta <= 0.53)
    pf = 0.2981;
elseif(beta > 0.53) && (beta <= 0.54)
    pf = 0.2948;
elseif(beta > 0.54) && (beta <= 0.55)
    pf = 0.2912;
elseif(beta > 0.55) && (beta <= 0.56)
    pf = 0.2877;
elseif(beta > 0.56) && (beta <= 0.57)
    pf = 0.2843;
elseif(beta > 0.57) && (beta <= 0.58)
    pf = 0.2810;
elseif(beta > 0.58) && (beta <= 0.59)
    pf = 0.2776;
elseif(beta > 0.59) && (beta <= 0.60)
    pf = 0.2743;
elseif(beta > 0.60) && (beta <= 0.61)
    pf = 0.2709;
elseif(beta > 0.61) && (beta <= 0.62)
    pf = 0.2676;
elseif(beta > 0.62) && (beta <= 0.63)
    pf = 0.2644;
elseif(beta > 0.63) && (beta <= 0.64)
    pf = 0.2611;
elseif(beta > 0.64) && (beta <= 0.65)
    pf = 0.2579;
elseif(beta > 0.65) && (beta <= 0.66)
    pf = 0.2546;
elseif(beta > 0.66) && (beta <= 0.67)
    pf = 0.2514;
elseif(beta > 0.67) && (beta <= 0.68)
    pf = 0.2483;
elseif(beta > 0.68) && (beta <= 0.69)
    pf = 0.2451;
elseif(beta > 0.69) && (beta <= 0.70)
    pf = 0.2420;
elseif(beta > 0.70) && (beta <= 0.71)
```

```
    pf = 0.2389;
elseif(beta > 0.71) && (beta <= 0.72)
    pf = 0.2358;
elseif(beta > 0.72) && (beta <= 0.73)
    pf = 0.2327;
elseif(beta > 0.73) && (beta <= 0.74)
    pf = 0.2297;
elseif(beta > 0.74) && (beta <= 0.75)
    pf = 0.2266;
elseif(beta > 0.75) && (beta <= 0.76)
    pf = 0.2236;
elseif(beta > 0.76) && (beta <= 0.77)
    pf = 0.2207;
elseif(beta > 0.77) && (beta <= 0.78)
    pf = 0.2177;
elseif(beta > 0.78) && (beta <= 0.79)
    pf = 0.2148;
elseif(beta > 0.79) && (beta <= 0.80)
    pf = 0.2119;
elseif(beta > 0.80) && (beta <= 0.81)
    pf = 0.2090;
elseif(beta > 0.81) && (beta <= 0.82)
    pf = 0.2061;
elseif(beta > 0.82) && (beta <= 0.83)
    pf = 0.2033;
elseif(beta > 0.83) && (beta <= 0.84)
    pf = 0.2005;
elseif(beta > 0.84) && (beta <= 0.85)
    pf = 0.1977;
elseif(beta > 0.85) && (beta <= 0.86)
    pf = 0.1949;
elseif(beta > 0.86) && (beta <= 0.87)
    pf = 0.1922;
elseif(beta > 0.87) && (beta <= 0.88)
    pf = 0.1894;
elseif(beta > 0.88) && (beta <= 0.89)
    pf = 0.1867;
elseif(beta > 0.89) && (beta <= 0.90)
    pf = 0.1841;
elseif(beta > 0.90) && (beta <= 0.91)
    pf = 0.1814;
elseif(beta > 0.91) && (beta <= 0.92)
    pf = 0.1788;
elseif(beta > 0.92) && (beta <= 0.93)
    pf = 0.1762;
elseif(beta > 0.93) && (beta <= 0.94)
    pf = 0.1736;
elseif(beta > 0.94) && (beta <= 0.95)
    pf = 0.1711;
elseif(beta > 0.95) && (beta <= 0.96)
    pf = 0.1685;
elseif(beta > 0.96) && (beta <= 0.97)
    pf = 0.1660;
elseif(beta > 0.97) && (beta <= 0.98)
    pf = 0.1635;
elseif(beta > 0.98) && (beta <= 0.99)
    pf = 0.1611;
```

```
elseif(beta > 0.99) && (beta <= 1.00)
    pf = 0.1587;
elseif(beta > 1.00) && (beta <= 1.01)
    pf = 0.1563;
elseif(beta > 1.01) && (beta <= 1.02)
    pf = 0.1539;
elseif(beta > 1.02) && (beta <= 1.03)
    pf = 0.1515;
elseif(beta > 1.03) && (beta <= 1.04)
    pf = 0.1492;
elseif(beta > 1.04) && (beta <= 1.05)
    pf = 0.1469;
elseif(beta > 1.05) && (beta <= 1.06)
    pf = 0.1446;
elseif(beta > 1.06) && (beta <= 1.07)
    pf = 0.1423;
elseif(beta > 1.07) && (beta <= 1.08)
    pf = 0.1401;
elseif(beta > 1.08) && (beta <= 1.09)
    pf = 0.1379;
elseif(beta > 1.09) && (beta <= 1.10)
    pf = 0.1357;
elseif(beta > 1.10) && (beta <= 1.11)
    pf = 0.1335;
elseif(beta > 1.11) && (beta <= 1.12)
    pf = 0.1314;
elseif(beta > 1.12) && (beta <= 1.13)
    pf = 0.1292;
elseif(beta > 1.13) && (beta <= 1.14)
    pf = 0.1271;
elseif(beta > 1.14) && (beta <= 1.15)
    pf = 0.1251;
elseif(beta > 1.15) && (beta <= 1.16)
    pf = 0.1230;
elseif(beta > 1.16) && (beta <= 1.17)
    pf = 0.1210;
elseif(beta > 1.17) && (beta <= 1.18)
    pf = 0.1190;
elseif(beta > 1.18) && (beta <= 1.19)
    pf = 0.1170;
elseif(beta > 1.19) && (beta <= 1.20)
    pf = 0.1151;
elseif(beta > 1.20) && (beta <= 1.21)
    pf = 0.1131;
elseif(beta > 1.21) && (beta <= 1.22)
    pf = 0.1112;
elseif(beta > 1.22) && (beta <= 1.23)
    pf = 0.1094;
elseif(beta > 1.23) && (beta <= 1.24)
    pf = 0.1075;
elseif(beta > 1.24) && (beta <= 1.25)
    pf = 0.1057;
elseif(beta > 1.25) && (beta <= 1.26)
    pf = 0.1038;
elseif(beta > 1.26) && (beta <= 1.27)
    pf = 0.1020;
elseif(beta > 1.27) && (beta <= 1.28)
```

```
    pf = 0.1003;
elseif(beta > 1.28) && (beta <= 1.29)
    pf = 0.9853e-01;
elseif(beta > 1.29) && (beta <= 1.30)
    pf = 0.968e-01;
elseif(beta > 1.30) && (beta <= 1.31)
    pf = 0.9510e-01;
elseif(beta > 1.31) && (beta <= 1.32)
    pf = 0.9342e-01;
elseif(beta > 1.32) && (beta <= 1.33)
    pf = 0.9176e-01;
elseif(beta > 1.33) && (beta <= 1.34)
    pf = 0.9013e-01;
elseif(beta > 1.34) && (beta <= 1.35)
    pf = 0.8851e-01;
elseif(beta > 1.35) && (beta <= 1.36)
    pf = 0.8692e-01;
elseif(beta > 1.36) && (beta <= 1.37)
    pf = 0.8535e-01;
elseif(beta > 1.37) && (beta <= 1.38)
    pf = 0.8380e-01;
elseif(beta > 1.38) && (beta <= 1.39)
    pf = 0.8227e-01;
elseif(beta > 1.39) && (beta <= 1.40)
    pf = 0.8076e-01;
elseif(beta > 1.40) && (beta <= 1.41)
    pf = 0.7927e-01;
elseif(beta > 1.41) && (beta <= 1.42)
    pf = 0.7781e-01;
elseif(beta > 1.42) && (beta <= 1.43)
    pf = 0.7636e-01;
elseif(beta > 1.43) && (beta <= 1.44)
    pf = 0.7494e-01;
elseif(beta > 1.44) && (beta <= 1.45)
    pf = 0.7353e-01;
elseif(beta > 1.45) && (beta <= 1.46)
    pf = 0.7216e-01;
elseif(beta > 1.46) && (beta <= 1.47)
    pf = 0.7078e-01;
elseif(beta > 1.47) && (beta <= 1.48)
    pf = 0.6944e-01;
elseif(beta > 1.48) && (beta <= 1.49)
    pf = 0.6811e-01;
elseif(beta > 1.49) && (beta <= 1.50)
    pf = 0.6681e-01;
elseif(beta > 1.50) && (beta <= 1.51)
    pf = 0.6552e-01;
elseif(beta > 1.51) && (beta <= 1.52)
    pf = 0.6426e-01;
elseif(beta > 1.52) && (beta <= 1.53)
    pf = 0.6301e-01;
elseif(beta > 1.53) && (beta <= 1.54)
    pf = 0.6178e-01;
elseif(beta > 1.54) && (beta <= 1.55)
    pf = 0.6057e-01;
elseif(beta > 1.55) && (beta <= 1.56)
    pf = 0.5938e-01;
```

```
elseif(beta > 1.56) && (beta <= 1.57)
    pf = 0.5821e-01;
elseif(beta > 1.57) && (beta <= 1.58)
    pf = 0.5706e-01;
elseif(beta > 1.58) && (beta <= 1.59)
    pf = 0.5592e-01;
elseif(beta > 1.59) && (beta <= 1.60)
    pf = 0.5480e-01;
elseif(beta > 1.60) && (beta <= 1.61)
    pf = 0.5370e-01;
elseif(beta > 1.61) && (beta <= 1.62)
    pf = 0.5282e-01;
elseif(beta > 1.62) && (beta <= 1.63)
    pf = 0.5155e-01;
elseif(beta > 1.63) && (beta <= 1.64)
    pf = 0.5050e-01;
elseif(beta > 1.64) && (beta <= 1.65)
    pf = 0.4947e-01;
elseif(beta > 1.65) && (beta <= 1.66)
    pf = 0.4846e-01;
elseif(beta > 1.66) && (beta <= 1.67)
    pf = 0.4746e-01;
elseif(beta > 1.67) && (beta <= 1.68)
    pf = 0.4648e-01;
elseif(beta > 1.68) && (beta <= 1.69)
    pf = 0.4552e-01;
elseif(beta > 1.69) && (beta <= 1.70)
    pf = 0.4457e-01;
elseif(beta > 1.70) && (beta <= 1.71)
    pf = 0.4363e-01;
elseif(beta > 1.71) && (beta <= 1.72)
    pf = 0.4272e-01;
elseif(beta > 1.72) && (beta <= 1.73)
    pf = 0.4182e-01;
elseif(beta > 1.73) && (beta <= 1.74)
    pf = 0.4093e-01;
elseif(beta > 1.74) && (beta <= 1.75)
    pf = 0.4006e-01;
elseif(beta > 1.75) && (beta <= 1.76)
    pf = 0.3921e-01;
elseif(beta > 1.76) && (beta <= 1.77)
    pf = 0.3836e-01;
elseif(beta > 1.77) && (beta <= 1.78)
    pf = 0.3754e-01;
elseif(beta > 1.78) && (beta <= 1.79)
    pf = 0.3673e-01;
elseif(beta > 1.79) && (beta <= 1.80)
    pf = 0.3593e-01;
elseif(beta > 1.80) && (beta <= 1.81)
    pf = 0.3515e-01;
elseif(beta > 1.81) && (beta <= 1.82)
    pf = 0.3438e-01;
elseif(beta > 1.82) && (beta <= 1.83)
    pf = 0.3363e-01;
elseif(beta > 1.83) && (beta <= 1.84)
    pf = 0.3289e-01;
elseif(beta > 1.84) && (beta <= 1.85)
```

```
    pf = 0.3216e-01;
elseif(beta > 1.85) && (beta <= 1.86)
    pf = 0.3144e-01;
elseif(beta > 1.86) && (beta <= 1.87)
    pf = 0.3074e-01;
elseif(beta > 1.87) && (beta <= 1.88)
    pf = 0.3005e-01;
elseif(beta > 1.88) && (beta <= 1.89)
    pf = 0.2938e-01;
elseif(beta > 1.89) && (beta <= 1.90)
    pf = 0.2872e-01;
elseif(beta > 1.90) && (beta <= 1.91)
    pf = 0.2807e-01;
elseif(beta > 1.91) && (beta <= 1.92)
    pf = 0.2743e-01;
elseif(beta > 1.92) && (beta <= 1.93)
    pf = 0.2680e-01;
elseif(beta > 1.93) && (beta <= 1.94)
    pf = 0.2619e-01;
elseif(beta > 1.94) && (beta <= 1.95)
    pf = 0.2553e-01;
elseif(beta > 1.95) && (beta <= 1.96)
    pf = 0.2500e-01;
elseif(beta > 1.96) && (beta <= 1.97)
    pf = 0.2442e-01;
elseif(beta > 1.97) && (beta <= 1.98)
    pf = 0.2385e-01;
elseif(beta > 1.98) && (beta <= 1.99)
    pf = 0.2330e-01;
elseif(beta > 1.99) && (beta <= 2.00)
    pf = 0.2275e-01;
elseif(beta > 2.00) && (beta <= 2.01)
    pf = 0.2222e-01;
elseif(beta > 2.01) && (beta <= 2.02)
    pf = 0.2169e-01;
elseif(beta > 2.02) && (beta <= 2.03)
    pf = 0.2118e-01;
elseif(beta > 2.03) && (beta <= 2.04)
    pf = 0.2068e-01;
elseif(beta > 2.04) && (beta <= 2.05)
    pf = 0.2018e-01;
elseif(beta > 2.05) && (beta <= 2.06)
    pf = 0.1970e-01;
elseif(beta > 2.06) && (beta <= 2.07)
    pf = 0.1923e-01;
elseif(beta > 2.07) && (beta <= 2.08)
    pf = 0.1876e-01;
elseif(beta > 2.08) && (beta <= 2.09)
    pf = 0.1831e-01;
elseif(beta > 2.09) && (beta <= 2.10)
    pf = 0.1788e-01;
elseif(beta > 2.10) && (beta <= 2.11)
    pf = 0.1743e-01;
elseif(beta > 2.11) && (beta <= 2.12)
    pf = 0.1700e-01;
elseif(beta > 2.12) && (beta <= 2.13)
    pf = 0.1659e-01;
```

```
elseif(beta > 2.13) && (beta <= 2.14)
    pf = 0.1618e-01;
elseif(beta > 2.14) && (beta <= 2.15)
    pf = 0.1578e-01;
elseif(beta > 2.15) && (beta <= 2.16)
    pf = 0.1539e-01;
elseif(beta > 2.16) && (beta <= 2.17)
    pf = 0.1500e-01;
elseif(beta > 2.17) && (beta <= 2.18)
    pf = 0.1483e-01;
elseif(beta > 2.18) && (beta <= 2.19)
    pf = 0.1426e-01;
elseif(beta > 2.19) && (beta <= 2.20)
    pf = 0.1390e-01;
elseif(beta > 2.20) && (beta <= 2.21)
    pf = 0.1355e-01;
elseif(beta > 2.21) && (beta <= 2.22)
    pf = 0.1321e-01;
elseif(beta > 2.22) && (beta <= 2.23)
    pf = 0.1287e-01;
elseif(beta > 2.23) && (beta <= 2.24)
    pf = 0.1253e-01;
elseif(beta > 2.24) && (beta <= 2.25)
    pf = 0.1222e-01;
elseif(beta > 2.25) && (beta <= 2.26)
    pf = 0.11191e-01;
elseif(beta > 2.26) && (beta <= 2.27)
    pf = 0.1160e-01;
elseif(beta > 2.27) && (beta <= 2.28)
    pf = 0.1130e-01;
elseif(beta > 2.28) && (beta <= 2.29)
    pf = 0.1101e-01;
elseif(beta > 2.29) && (beta <= 2.30)
    pf = 0.1072e-01;
elseif(beta > 2.30) && (beta <= 2.31)
    pf = 0.1044e-01;
elseif(beta > 2.31) && (beta <= 2.32)
    pf = 0.1017e-01;
elseif(beta > 2.32) && (beta <= 2.33)
    pf = 0.9903e-02;
elseif(beta > 2.33) && (beta <= 2.34)
    pf = 0.9642e-02;
elseif(beta > 2.34) && (beta <= 2.35)
    pf = 0.9387e-02;
elseif(beta > 2.35) && (beta <= 2.36)
    pf = 0.9138e-02;
elseif(beta > 2.36) && (beta <= 2.37)
    pf = 0.8894e-02;
elseif(beta > 2.37) && (beta <= 2.38)
    pf = 0.8657e-02;
elseif(beta > 2.38) && (beta <= 2.39)
    pf = 0.8424e-02;
elseif(beta > 2.39) && (beta <= 2.40)
    pf = 0.8198e-02;
elseif(beta > 2.40) && (beta <= 2.41)
    pf = 0.7976e-02;
elseif(beta > 2.41) && (beta <= 2.42)
```

```
    pf = 0.7760e-02;
elseif(beta > 2.42) && (beta <= 2.43)
    pf = 0.7550e-02;
elseif(beta > 2.43) && (beta <= 2.44)
    pf = 0.7344e-02;
elseif(beta > 2.44) && (beta <= 2.45)
    pf = 0.7143e-02;
elseif(beta > 2.45) && (beta <= 2.46)
    pf = 0.6947e-02;
elseif(beta > 2.46) && (beta <= 2.47)
    pf = 0.6758e-02;
elseif(beta > 2.47) && (beta <= 2.48)
    pf = 0.6589e-02;
elseif(beta > 2.48) && (beta <= 2.49)
    pf = 0.6387e-02;
elseif(beta > 2.49) && (beta <= 2.50)
    pf = 0.6210e-02;
elseif(beta > 2.50) && (beta <= 2.51)
    pf = 0.6037e-02;
elseif(beta > 2.51) && (beta <= 2.52)
    pf = 0.5868e-02;
elseif(beta > 2.52) && (beta <= 2.53)
    pf = 0.5703e-02;
elseif(beta > 2.53) && (beta <= 2.54)
    pf = 0.5543e-02;
elseif(beta > 2.54) && (beta <= 2.55)
    pf = 0.5386e-02;
elseif(beta > 2.55) && (beta <= 2.56)
    pf = 0.5234e-02;
elseif(beta > 2.56) && (beta <= 2.57)
    pf = 0.5085e-02;
elseif(beta > 2.57) && (beta <= 2.58)
    pf = 0.4940e-02;
elseif(beta > 2.58) && (beta <= 2.59)
    pf = 0.4799e-02;
elseif(beta > 2.59) && (beta <= 2.60)
    pf = 0.4661e-02;
elseif(beta > 2.60) && (beta <= 2.61)
    pf = 0.4527e-02;
elseif(beta > 2.61) && (beta <= 2.62)
    pf = 0.4397e-02;
elseif(beta > 2.62) && (beta <= 2.63)
    pf = 0.4269e-02;
elseif(beta > 2.63) && (beta <= 2.64)
    pf = 0.4145e-02;
elseif(beta > 2.64) && (beta <= 2.65)
    pf = 0.4025e-02;
elseif(beta > 2.65) && (beta <= 2.66)
    pf = 0.3907e-02;
elseif(beta > 2.66) && (beta <= 2.67)
    pf = 0.3793e-02;
elseif(beta > 2.67) && (beta <= 2.68)
    pf = 0.3681e-02;
elseif(beta > 2.68) && (beta <= 2.69)
    pf = 0.3573e-02;
elseif(beta > 2.69) && (beta <= 2.70)
    pf = 0.3467e-02;
```

```
elseif(beta > 2.70) && (beta <= 2.71)
    pf = 0.3364e-02;
elseif(beta > 2.71) && (beta <= 2.72)
    pf = 0.3264e-02;
elseif(beta > 2.72) && (beta <= 2.73)
    pf = 0.3167e-02;
elseif(beta > 2.73) && (beta <= 2.74)
    pf = 0.3072e-02;
elseif(beta > 2.74) && (beta <= 2.75)
    pf = 0.2980e-02;
elseif(beta > 2.75) && (beta <= 2.76)
    pf = 0.2890e-02;
elseif(beta > 2.76) && (beta <= 2.77)
    pf = 0.2803e-02;
elseif(beta > 2.77) && (beta <= 2.78)
    pf = 0.2718e-02;
elseif(beta > 2.78) && (beta <= 2.79)
    pf = 0.2635e-02;
elseif(beta > 2.79) && (beta <= 2.80)
    pf = 0.2555e-02;
elseif(beta > 2.80) && (beta <= 2.81)
    pf = 0.2477e-02;
elseif(beta > 2.81) && (beta <= 2.82)
    pf = 0.2401e-02;
elseif(beta > 2.82) && (beta <= 2.83)
    pf = 0.2327e-02;
elseif(beta > 2.83) && (beta <= 2.84)
    pf = 0.2256e-02;
elseif(beta > 2.84) && (beta <= 2.85)
    pf = 0.2286e-02;
elseif(beta > 2.85) && (beta <= 2.86)
    pf = 0.2118e-02;
elseif(beta > 2.86) && (beta <= 2.87)
    pf = 0.2052e-02;
elseif(beta > 2.87) && (beta <= 2.88)
    pf = 0.1988e-02;
elseif(beta > 2.88) && (beta <= 2.89)
    pf = 0.1926e-02;
elseif(beta > 2.89) && (beta <= 2.90)
    pf = 0.1866e-02;
elseif(beta > 2.90) && (beta <= 2.91)
    pf = 0.1807e-02;
elseif(beta > 2.91) && (beta <= 2.92)
    pf = 0.1750e-02;
elseif(beta > 2.92) && (beta <= 2.93)
    pf = 0.1695e-02;
elseif(beta > 2.93) && (beta <= 2.94)
    pf = 0.1641e-02;
elseif(beta > 2.94) && (beta <= 2.95)
    pf = 0.1589e-02;
elseif(beta > 2.95) && (beta <= 2.96)
    pf = 0.1538e-02;
elseif(beta > 2.96) && (beta <= 2.97)
    pf = 0.1489e-02;
elseif(beta > 2.97) && (beta <= 2.98)
    pf = 0.1441e-02;
elseif(beta > 2.98) && (beta <= 2.99)
```

```
    pf = 0.1395e-02;
elseif(beta > 2.99) && (beta <= 3.00)
    pf = 0.1350e-02;
elseif(beta > 3.00) && (beta <= 3.01)
    pf = 0.1306e-02;
elseif(beta > 3.01) && (beta <= 3.02)
    pf = 0.14264e-02;
elseif(beta > 3.02) && (beta <= 3.03)
    pf = 0.1223e-02;
elseif(beta > 3.03) && (beta <= 3.04)
    pf = 0.1183e-02;
elseif(beta > 3.04) && (beta <= 3.05)
    pf = 0.1144e-02;
elseif(beta > 3.05) && (beta <= 3.06)
    pf = 0.1107e-02;
elseif(beta > 3.06) && (beta <= 3.07)
    pf = 0.1070e-02;
elseif(beta > 3.07) && (beta <= 3.08)
    pf = 0.1035e-02;
elseif(beta > 3.08) && (beta <= 3.09)
    pf = 0.1001e-02;
elseif(beta > 3.09) && (beta <= 3.10)
    pf = 0.9676e-03;
elseif(beta > 3.10) && (beta <= 3.11)
    pf = 0.9354e-03;
elseif(beta > 3.11) && (beta <= 3.12)
    pf = 0.9042e-03;
elseif(beta > 3.12) && (beta <= 3.13)
    pf = 0.8740e-03;
elseif(beta > 3.13) && (beta <= 3.14)
    pf = 0.8447e-03;
elseif(beta > 3.14) && (beta <= 3.15)
    pf = 0.8163e-03;
elseif(beta > 3.15) && (beta <= 3.16)
    pf = 0.7888e-03;
elseif(beta > 3.16) && (beta <= 3.17)
    pf = 0.7622e-03;
elseif(beta > 3.17) && (beta <= 3.18)
    pf = 0.7363e-03;
elseif(beta > 3.18) && (beta <= 3.19)
    pf = 0.7113e-03;
elseif(beta > 3.09) && (beta <= 3.20)
    pf = 0.6871e-03;
elseif(beta > 3.20) && (beta <= 3.21)
    pf = 0.6636e-03;
elseif(beta > 3.21) && (beta <= 3.22)
    pf = 0.6409e-03;
elseif(beta > 3.22) && (beta <= 3.23)
    pf = 0.6189e-03;
elseif(beta > 3.23) && (beta <= 3.24)
    pf = 0.5976e-03;
elseif(beta > 3.24) && (beta <= 3.25)
    pf = 0.5770e-03;
elseif(beta > 3.25) && (beta <= 3.26)
    pf = 0.5570e-03;
elseif(beta > 3.26) && (beta <= 3.27)
    pf = 0.5377e-03;
```

```
elseif(beta > 3.27) && (beta <= 3.28)
    pf = 0.5190e-03;
elseif(beta > 3.28) && (beta <= 3.29)
    pf = 0.5009e-03;
elseif(beta > 3.29) && (beta <= 3.30)
    pf = 0.4834e-03;
elseif(beta > 3.30) && (beta <= 3.31)
    pf = 0.4664e-03;
elseif(beta > 3.31) && (beta <= 3.32)
    pf = 0.4500e-03;
elseif(beta > 3.32) && (beta <= 3.33)
    pf = 0.4342e-03;
elseif(beta > 3.33) && (beta <= 3.34)
    pf = 0.4189e-03;
elseif(beta > 3.34) && (beta <= 3.35)
    pf = 0.4040e-03;
elseif(beta > 3.35) && (beta <= 3.36)
    pf = 0.3897e-03;
elseif(beta > 3.36) && (beta <= 3.37)
    pf = 0.3758e-03;
elseif(beta > 3.37) && (beta <= 3.38)
    pf = 0.3624e-03;
elseif(beta > 3.38) && (beta <= 3.39)
    pf = 0.3494e-03;
elseif(beta > 3.39) && (beta <= 3.40)
    pf = 0.3369e-03;
elseif(beta > 3.40) && (beta <= 3.41)
    pf = 0.3248e-03;
elseif(beta > 3.41) && (beta <= 3.42)
    pf = 0.3131e-03;
elseif(beta > 3.42) && (beta <= 3.43)
    pf = 0.3017e-03;
elseif(beta > 3.43) && (beta <= 3.44)
    pf = 0.2908e-03;
elseif(beta > 3.44) && (beta <= 3.45)
    pf = 0.2802e-03;
elseif(beta > 3.45) && (beta <= 3.46)
    pf = 0.2700e-03;
elseif(beta > 3.46) && (beta <= 3.47)
    pf = 0.2602e-03;
elseif(beta > 3.47) && (beta <= 3.48)
    pf = 0.2507e-03;
elseif(beta > 3.48) && (beta <= 3.49)
    pf = 0.2415e-03;
elseif(beta > 3.49) && (beta <= 3.50)
    pf = 0.2326e-03;
elseif(beta > 3.50) && (beta <= 3.51)
    pf = 0.2240e-03;
elseif(beta > 3.51) && (beta <= 3.52)
    pf = 0.2157e-03;
elseif(beta > 3.52) && (beta <= 3.53)
    pf = 0.2077e-03;
elseif(beta > 3.53) && (beta <= 3.54)
    pf = 0.2000e-03;
elseif(beta > 3.54) && (beta <= 3.55)
    pf = 0.1926e-03;
elseif(beta > 3.55) && (beta <= 3.56)
```

```
    pf = 0.1854e-03;
elseif(beta > 3.56) && (beta <= 3.57)
    pf = 0.1784e-03;
elseif(beta > 3.57) && (beta <= 3.58)
    pf = 0.1717e-03;
elseif(beta > 3.58) && (beta <= 3.59)
    pf = 0.1653e-03;
elseif(beta > 3.59) && (beta <= 3.60)
    pf = 0.1591e-03;
elseif(beta > 3.60) && (beta <= 3.61)
    pf = 0.1531e-03;
elseif(beta > 3.61) && (beta <= 3.62)
    pf = 0.1473e-03;
elseif(beta > 3.62) && (beta <= 3.63)
    pf = 0.1417e-03;
elseif(beta > 3.63) && (beta <= 3.64)
    pf = 0.1363e-03;
elseif(beta > 3.64) && (beta <= 3.65)
    pf = 0.1311e-03;
elseif(beta > 3.65) && (beta <= 3.66)
    pf = 0.1261e-03;
elseif(beta > 3.66) && (beta <= 3.67)
    pf = 0.1212e-03;
elseif(beta > 3.67) && (beta <= 3.68)
    pf = 0.1166e-03;
elseif(beta > 3.68) && (beta <= 3.69)
    pf = 0.1121e-03;
elseif(beta > 3.69) && (beta <= 3.70)
    pf = 0.1077e-03;
elseif(beta > 3.70) && (beta <= 3.71)
    pf = 0.1036e-03;
elseif(beta > 3.71) && (beta <= 3.72)
    pf = 0.9956e-04;
elseif(beta > 3.72) && (beta <= 3.73)
    pf = 0.9569e-04;
elseif(beta > 3.73) && (beta <= 3.74)
    pf = 0.9196e-04;
elseif(beta > 3.74) && (beta <= 3.75)
    pf = 0.8837e-04;
elseif(beta > 3.75) && (beta <= 3.76)
    pf = 0.8491e-04;
elseif(beta > 3.76) && (beta <= 3.77)
    pf = 0.8157e-04;
elseif(beta > 3.77) && (beta <= 3.78)
    pf = 0.7836e-04;
elseif(beta > 3.78) && (beta <= 3.79)
    pf = 0.7527e-04;
elseif(beta > 3.79) && (beta <= 3.80)
    pf = 0.7230e-04;
elseif(beta > 3.80) && (beta <= 3.81)
    pf = 0.6943e-04;
elseif(beta > 3.81) && (beta <= 3.82)
    pf = 0.6667e-04;
elseif(beta > 3.82) && (beta <= 3.83)
    pf = 0.6402e-04;
elseif(beta > 3.83) && (beta <= 3.84)
    pf = 0.6147e-04;
```

```
elseif(beta > 3.84) && (beta <= 3.85)
    pf = 0.5901e-04;
elseif(beta > 3.85) && (beta <= 3.86)
    pf = 0.5664e-04;
elseif(beta > 3.86) && (beta <= 3.87)
    pf = 0.5437e-04;
elseif(beta > 3.87) && (beta <= 3.88)
    pf = 0.5218e-04;
elseif(beta > 3.88) && (beta <= 3.89)
    pf = 0.5007e-04;
elseif(beta > 3.89) && (beta <= 3.90)
    pf = 0.4804e-04;
elseif(beta > 3.90) && (beta <= 3.91)
    pf = 0.4610e-04;
elseif(beta > 3.91) && (beta <= 3.92)
    pf = 0.4422e-04;
elseif(beta > 3.92) && (beta <= 3.93)
    pf = 0.4242e-04;
elseif(beta > 3.93) && (beta <= 3.94)
    pf = 0.4069e-04;
elseif(beta > 3.94) && (beta <= 3.95)
    pf = 0.3902e-04;
elseif(beta > 3.95) && (beta <= 3.96)
    pf = 0.3742e-04;
elseif(beta > 3.96) && (beta <= 3.97)
    pf = 0.3588e-04;
elseif(beta > 3.97) && (beta <= 3.98)
    pf = 0.3441e-04;
elseif(beta > 3.98) && (beta <= 3.99)
    pf = 0.3298e-04;
elseif(beta > 3.99) && (beta <= 4.00)
    pf = 0.3162e-04;
elseif(beta > 4.00) && (beta <= 4.05)
    pf = 0.2557e-04;
elseif(beta > 4.05) && (beta <= 4.10)
    pf = 0.2062e-04;
elseif(beta > 4.10) && (beta <= 4.15)
    pf = 0.1659e-04;
elseif(beta > 4.15) && (beta <= 4.20)
    pf = 0.1332e-04;
elseif(beta > 4.20) && (beta <= 4.25)
    pf = 0.1067e-04;
elseif(beta > 4.25) && (beta <= 4.30)
    pf = 0.8524e-05;
elseif(beta > 4.30) && (beta <= 4.35)
    pf = 0.6794e-05;
elseif(beta > 4.35) && (beta <= 4.40)
    pf = 0.5402e-05;
elseif(beta > 4.40) && (beta <= 4.45)
    pf = 0.4285e-05;
elseif(beta > 4.45) && (beta <= 4.50)
    pf = 0.3391e-05;
elseif(beta > 4.50) && (beta <= 4.55)
    pf = 0.2677e-05;
elseif(beta > 4.55) && (beta <= 4.60)
    pf = 0.2108e-05;
elseif(beta > 4.60) && (beta <= 4.65)
```

```
    pf = 0.1656e-05;
elseif(beta > 4.65) && (beta <= 4.70)
    pf = 0.1298e-05;
elseif(beta > 4.70) && (beta <= 4.75)
    pf = 0.1015e-05;
elseif(beta > 4.75) && (beta <= 4.80)
    pf = 0.7914e-06;
elseif(beta > 4.80) && (beta <= 4.85)
    pf = 0.6158e-06;
elseif(beta > 4.85) && (beta <= 4.90)
    pf = 0.4780e-06;
elseif(beta > 4.90) && (beta <= 4.95)
    pf = 0.3701e-06;
elseif(beta > 4.95) && (beta <= 5.00)
    pf = 0.2859e-06;
elseif(beta > 5.00) && (beta <= 5.05)
    pf = 0.2203e-06;
elseif(beta > 5.05) && (beta <= 5.10)
    pf = 0.1694e-06;
elseif(beta > 5.10) && (beta <= 5.15)
    pf = 0.1299e-06;
elseif(beta > 5.15) && (beta <= 5.20)
    pf = 0.9935e-07;
elseif(beta > 5.20) && (beta <= 5.25)
    pf = 0.7582e-07;
elseif(beta > 5.25) && (beta <= 5.30)
    pf = 0.5772e-07;
elseif(beta > 5.30) && (beta <= 5.35)
    pf = 0.4384e-07;
elseif(beta > 5.35) && (beta <= 5.40)
    pf = 0.3321e-07;
elseif(beta > 5.40) && (beta <= 5.45)
    pf = 0.2510e-07;
elseif(beta > 5.45) && (beta <= 5.50)
    pf = 0.1892e-07;
elseif(beta > 5.50) && (beta <= 5.55)
    pf = 0.1423e-07;
elseif(beta > 5.55) && (beta <= 5.60)
    pf = 0.1067e-07;
elseif(beta > 5.60) && (beta <= 5.65)
    pf = 0.7985e-08;
elseif(beta > 5.65) && (beta <= 5.70)
    pf = 0.5959e-08;
elseif(beta > 5.70) && (beta <= 5.75)
    pf = 0.4436e-08;
elseif(beta > 5.75) && (beta <= 5.80)
    pf = 0.3293e-08;
elseif(beta > 5.80) && (beta <= 5.85)
    pf = 0.2438e-08;
elseif(beta > 5.85) && (beta <= 5.90)
    pf = 0.1800e-08;
elseif(beta > 5.90) && (beta <= 5.95)
    pf = 0.1325e-08;
elseif(beta > 5.95) && (beta <= 6.00)
    pf = 0.9716e-09;
elseif(beta > 6.00) && (beta <= 6.10)
    pf = 0.5220e-09;
```

```
elseif(beta > 6.10) && (beta <= 6.20)
    pf = 0.2778e-09;
elseif(beta > 6.20) && (beta <= 6.30)
    pf = 0.1483e-09;
elseif(beta > 6.30) && (beta <= 6.40)
    pf = 0.7636e-10;
elseif(beta > 6.40) && (beta <= 6.50)
    pf = 0.3945e-10;
elseif(beta > 6.50) && (beta <= 6.60)
    pf = 0.2010e-10;
elseif(beta > 6.60) && (beta <= 6.70)
    pf = 0.1023e-10;
elseif(beta > 6.70) && (beta <= 6.80)
    pf = 0.5130e-11;
elseif(beta > 6.80) && (beta <= 6.90)
    pf = 0.2549e-11;
elseif(beta > 6.90) && (beta <= 7.00)
    pf = 0.1254e-11;
elseif(beta > 7.00) && (beta <= 7.10)
    pf = 0.6107e-12;
elseif(beta > 7.10) && (beta <= 7.20)
    pf = 0.2946e-12;
elseif(beta > 7.20) && (beta <= 7.30)
    pf = 0.1407e-12;
elseif(beta > 7.30) && (beta <= 7.40)
    pf = 0.6654e-13;
elseif(beta > 7.40) && (beta <= 7.50)
    pf = 0.3116e-13;
elseif(beta > 7.50) && (beta <= 7.70)
    pf = 0.1445e-13;
elseif(beta > 7.60) && (beta <= 7.80)
    pf = 0.6636e-14;
elseif(beta > 7.70) && (beta <= 7.80)
    pf = 0.3017e-14
elseif(beta > 7.80) && (beta <= 7.90)
    pf = 0.1359e-14;
elseif(beta > 7.90) && (beta <= 8.00)
    pf = 0.6056e-15;
elseif(beta > 8.00) && (beta <= 8.10)
    pf = 0.2673e-15;
elseif(beta > 8.10) && (beta <= 8.20)
    pf = 0.1169e-15;
elseif(beta > 8.20) && (beta <= 8.30)
    pf = 0.5058e-16;
elseif(beta > 8.30) && (beta <= 8.40)
    pf = 0.2167e-16;
elseif(beta > 8.40) && (beta <= 8.50)
    pf = 0.9197e-17;
elseif(beta > 8.50) && (beta <= 8.60)
    pf = 0.3864e-17;
elseif(beta > 8.60) && (beta <= 8.70)
    pf = 0.1608e-17;
elseif(beta > 8.70) && (beta <= 8.80)
    pf = 0.6623e-18;
elseif(beta > 8.80) && (beta <= 8.90)
    pf = 0.2701e-18;
elseif(beta > 8.90) && (beta <= 9.00)
```

```
    pf = 0.1091e-18;
elseif(beta > 9.00) && (beta <= 9.10)
    pf = 0.4363e-19;
elseif(beta > 9.10) && (beta <= 9.20)
    pf = 0.1728e-19;
elseif(beta > 9.20) && (beta <= 9.30)
    pf = 0.6773e-20;
elseif(beta > 9.30) && (beta <= 9.40)
    pf = 0.2629e-20;
elseif(beta > 9.40) && (beta <= 9.50)
    pf = 0.1011e-20;
elseif(beta > 9.50) && (beta <= 9.60)
    pf = 0.3847e-21;
elseif(beta > 9.60) && (beta <= 9.70)
    pf = 0.1450e-21;
elseif(beta > 9.70) && (beta <= 9.80)
    pf = 0.5408e-22;
elseif(beta > 9.80) && (beta <= 9.90)
    pf = 0.1998e-22;
elseif(beta > 9.90)
    pf = 0.1998e-22;
end
if(pf <= 0)
    pf = 0;
end


%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
function beta_mode_1 = Fatigue_slab_flexure(moder,AS,fyd,...
    d,b,fcd,cons1,cons2,cons3,...
    modec,modeash,modeqk)
%
[cov1,cov2,cov3,cov4,cov5,cov6,cov7,cov8,cov9] =
fatigue_covariance_mode1;
%
cv(1) = cov1; cv(2) = cov2; cv(3) = cov3; cv(4) = cov4;
cv(5) = cov5; cv(6) = cov6; cv(7) = cov7; cv(8) = cov8;
cv(9) = cov9;
%
[DISTYPE1,DISTYPE2,DISTYPE3,DISTYPE4,DISTYPE5, ...
    DISTYPE6,DISTYPE7,DISTYPE8,DISTYPE9] = distribution_mode1;
%
rfmu(1) = moder; rmfx1 = moder*cv(1);
rfmu(2) = AS; rmfx2 = AS*cv(2);
rfmu(3) = fyd; rmfx3 = fyd*cv(3);
rfmu(4) = d; rmfx4 = d*cv(4);
rfmu(5) = b; rmfx5 = b*cv(5);
rfmu(6) = fcd; rmfx6 = fcd*cv(6);
rfmu(7) = modec; rmfx7 = modec*cv(7);
rfmu(8) = modeash; rmfx8 = modeash*cv(8);
rfmu(9) = modeqk; rmfx9 = modeqk*cv(9);
%
if(DISTYPE1 == 1)
%
```

```
% DOING NORMAL
%
    mean(1) = rfmu(1);
    sd(1) = rmfx1;
elseif(DISTYPE1 == 2)
%
% DOING LOGNORMAL TRANSFORMATION
%
 [mean(1) sd(1)] = lognormal(rfmu(1),rmfx1);
elseif(DISTYPE1 == 3)
%
% DOING GUMBEL TRANSFORMATION
%
 [mean(1) sd(1)] = gumbel(rfmu(1),rmfx1);
elseif(DISTYPE1 == 4)
%
% DOING WEIBULL TRANSFORMATION
%
 [mean(1) sd(1)] = weibull(rfmu(1),rmfx1);
elseif(DISTYPE1 == 5)
%
% DOING FRECHET TRANSFORMATION
%
 [mean(1) sd(1)] = frechet(rfmu(1),rmfx1);
end
if(DISTYPE2 == 1)
%
% DOING NORMAL
%
    mean(2) = rfmu(2);
    sd(2) = rmfx2;
elseif(DISTYPE2 == 2)
%
% DOING LOGNORMAL TRANSFORMATION
%
 [mean(2) sd(2)] = lognormal(rfmu(2),rmfx2);
elseif(DISTYPE2 == 3)
%
% DOING GUMBEL TRANSFORMATION
%
 [mean(2) sd(2)] = gumbel(rfmu(2),rmfx2);
elseif(DISTYPE2 == 4)
%
% DOING WEIBULL TRANSFORMATION
%
 [mean(2) sd(2)] = weibull(rfmu(2),rmfx2);
elseif(DISTYPE2 == 5)
%
% DOING FRECHET TRANSFORMATION
%
 [mean(2) sd(2)] = frechet(rfmu(2),rmfx2);
end
if(DISTYPE3 == 1)
%
% DOING NORMAL
%
    mean(3) = rfmu(3);
```

```
    sd(3) = rmfx3;
elseif(DISTYPE3 == 2)
%
% DOING LOGNORMAL TRANSFORMATION
%
 [mean(3) sd(3)] = lognormal(rfmu(3),rmfx3);
elseif(DISTYPE3 == 3)
%
% DOING GUMBEL TRANSFORMATION
%
 [mean(3) sd(3)] = gumbel(rfmu(3),rmfx3);
elseif(DISTYPE3 == 4)
%
% DOING WEIBULL TRANSFORMATION
%
 [mean(3) sd(3)] = weibull(rfmu(3),rmfx3);
elseif(DISTYPE3 == 5)
%
% DOING FRECHET TRANSFORMATION
%
 [mean(3) sd(3)] = frechet(rfmu(3),rmfx3);
end
if(DISTYPE4 == 1)
%
% DOING NORMAL
%
    mean(4) = rfmu(4);
    sd(4) = rmfx4;
elseif(DISTYPE4 == 2)
%
% DOING LOGNORMAL TRANSFORMATION
%
 [mean(4) sd(4)] = lognormal(rfmu(4),rmfx4);
elseif(DISTYPE4 == 3)
%
% DOING GUMBEL TRANSFORMATION
%
 [mean(4) sd(4)] = gumbel(rfmu(4),rmfx4);
elseif(DISTYPE4 == 4)
%
% DOING WEIBULL TRANSFORMATION
%
 [mean(4) sd(4)] = weibull(rfmu(4),rmfx4);
elseif(DISTYPE4 == 5)
%
% DOING FRECHET TRANSFORMATION
%
 [mean(4) sd(4)] = frechet(rfmu(4),rmfx4);
end
if(DISTYPE5 == 1)
%
% DOING NORMAL
%
    mean(5) = rfmu(5);
    sd(5) = rmfx5;
elseif(DISTYPE5 == 2)
%
```

```
% DOING LOGNORMAL TRANSFORMATION
%
 [mean(5) sd(5)] = lognormal(rfmu(5),rmfx5);
elseif(DISTYPE5 == 3)
%
% DOING GUMBEL TRANSFORMATION
%
 [mean(5) sd(5)] = gumbel(rfmu(5),rmfx5);
elseif(DISTYPE5 == 4)
%
% DOING WEIBULL TRANSFORMATION
%
 [mean(5) sd(5)] = weibull(rfmu(5),rmfx5);
elseif(DISTYPE5 == 5)
%
% DOING FRECHET TRANSFORMATION
%
 [mean(5) sd(5)] = frechet(rfmu(5),rmfx5);
end
if(DISTYPE6 == 1)
%
% DOING NORMAL
%
    mean(6) = rfmu(6);
    sd(6) = rmfx6;
elseif(DISTYPE6 == 2)
%
% DOING LOGNORMAL TRANSFORMATION
%
 [mean(6) sd(6)] = lognormal(rfmu(6),rmfx6);
elseif(DISTYPE6 == 3)
%
% DOING GUMBEL TRANSFORMATION
%
 [mean(6) sd(6)] = gumbel(rfmu(6),rmfx6);
elseif(DISTYPE6 == 4)
%
% DOING WEIBULL TRANSFORMATION
%
 [mean(6) sd(6)] = weibull(rfmu(6),rmfx6);
elseif(DISTYPE6 == 5)
%
% DOING FRECHET TRANSFORMATION
%
 [mean(6) sd(6)] = frechet(rfmu(6),rmfx6);
end
if(DISTYPE7 == 1)
%
% DOING NORMAL
%
    mean(7) = rfmu(7);
    sd(7) = rmfx7;
elseif(DISTYPE7 == 2)
%
% DOING LOGNORMAL TRANSFORMATION
%
 [mean(7) sd(7)] = lognormal(rfmu(7),rmfx7);
```

```
elseif(DISTYPE5 == 3)
%
% DOING GUMBEL TRANSFORMATION
%
 [mean(7) sd(7)] = gumbel(rfmu(7),rmfx7);
elseif(DISTYPE7 == 4)
%
% DOING WEIBULL TRANSFORMATION
%
 [mean(7) sd(7)] = weibull(rfmu(7),rmfx7);
elseif(DISTYPE7 == 7)
%
% DOING FRECHET TRANSFORMATION
%
 [mean(7) sd(7)] = frechet(rfmu(7),rmfx7);
end
if(DISTYPE8 == 1)
%
% DOING NORMAL
%
    mean(8) = rfmu(8);
    sd(8) = rmfx8;
elseif(DISTYPE8 == 2)
%
% DOING LOGNORMAL TRANSFORMATION
%
 [mean(8) sd(8)] = lognormal(rfmu(8),rmfx8);
elseif(DISTYPE8 == 3)
%
% DOING GUMBEL TRANSFORMATION
%
 [mean(8) sd(8)] = gumbel(rfmu(8),rmfx8);
elseif(DISTYPE8 == 4)
%
% DOING WEIBULL TRANSFORMATION
%
 [mean(8) sd(8)] = weibull(rfmu(8),rmfx8);
elseif(DISTYPE8 == 5)
%
% DOING FRECHET TRANSFORMATION
%
 [mean(8) sd(8)] = frechet(rfmu(8),rmfx8);
end
if(DISTYPE9 == 1)
%
% DOING NORMAL
%
    mean(9) = rfmu(9);
    sd(9) = rmfx9;
elseif(DISTYPE9 == 2)
%
% DOING LOGNORMAL TRANSFORMATION
%
 [mean(9) sd(9)] = lognormal(rfmu(9),rmfx9);
elseif(DISTYPE9 == 3)
%
% DOING GUMBEL TRANSFORMATION
```

```
%
 [mean(9) sd(9)] = gumbel(rfmu(9),rmfx9);
elseif(DISTYPE9 == 4)
%
% DOING WEIBULL TRANSFORMATION
%
 [mean(9) sd(9)] = weibull(rfmu(9),rmfx9);
elseif(DISTYPE9 == 5)
%
% DOING FRECHET TRANSFORMATION
%
 [mean(9) sd(9)] = frechet(rfmu(9),rmfx9);
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
sev1 = sd(1); sev2 = sd(2); sev3 = sd(3); sev4 = sd(4);
sev5 = sd(5); sev6 = sd(6); sev7 = sd(7); sev8 = sd(8);
sev9 = sd(9);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
dev1 = mean(1); dev2 = mean(2); dev3 = mean(3); dev4 = mean(4);
dev5 = mean(5); dev6 = mean(6); dev7 = mean(7); dev8 = mean(8);
dev9 = mean(9);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
part1 = (dev2*dev3*dev4) - (((dev2^2)*(dev3^2))/(2.0*dev5*dev6));
part2 = (dev1*dev3*dev4) - ((dev1*(2.0*dev2)*(dev3^2))/(2.0*dev5*dev6));
part3 = (dev1*dev2*dev4) - ((2.0*dev1*(dev2^2)*(dev3))/(2.0*dev5*dev6));
part4 =  dev1*dev2*dev3;
part5 = ((dev1*(dev2^2)*(dev3^2))/(2.0*(dev5^2)*dev6));
part6 = ((dev1*(dev2^2)*(dev3^2))/(2.0*dev5*(dev6^2)));
part7 = -cons1;
part8 = -cons2;
part9 = -cons3;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
deno = sqrt(((part1^2)*(sev1^2)) + ((part2^2)*(sev2^2)) + ...
    ((part3^2)*(sev3^2)) + ((part4^2)*(sev4^2)) + ...
    ((part5^2)*(sev5^2)) + ((part6^2)*(sev6^2)) + ...
    ((part7^2)*(sev7^2)) + ((part8^2)*(sev8^2)) + ...
    ((part9^2)*(sev9^2)));
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
nume = (dev1*dev2*dev3*dev4) - ((dev1*(dev2^2)*(dev3^2))/(2.0*dev5*dev6))
...
```

```
    - cons1*dev7 - cons2*dev8 - cons3*dev9;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
if(deno <= 0)
    beta_mode_1 = 1000;
else
beta_mode_1 = nume/deno;
end
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%

%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
function beta_mode_2 =
Fatigue_beam_shear(moder,d,b,fcd,cons4,cons5,cons6,...
    modec,modeash,modeqk)
%
[cov1,cov2,cov3,cov4,cov5,cov6,cov7] = fatigue_covariance_mode2;
%
cv(1) = cov1; cv(2) = cov2; cv(3) = cov3; cv(4) = cov4;
cv(5) = cov5; cv(6) = cov6; cv(7) = cov7;
%
[DISTYPE1,DISTYPE2,DISTYPE3,DISTYPE4,DISTYPE5, ...
    DISTYPE6,DISTYPE7] = distribution_mode2;
%
rfmu(1) = moder; rmfx1 = moder*cv(1);
rfmu(2) = d; rmfx2 = d*cv(2);
rfmu(3) = b; rmfx3 = b*cv(3);
rfmu(4) = fcd; rmfx4 = fcd*cv(4);
rfmu(5) = modec; rmfx5 = modec*cv(5);
rfmu(6) = modeash; rmfx6 = modeash*cv(6);
rfmu(7) = modeqk; rmfx7 = modeqk*cv(7);
%
if(DISTYPE1 == 1)
%
% DOING NORMAL
%
    mean(1) = rfmu(1);
    sd(1) = rmfx1;
elseif(DISTYPE1 == 2)
%
% DOING LOGNORMAL TRANSFORMATION
%
 [mean(1) sd(1)] = lognormal(rfmu(1),rmfx1);
elseif(DISTYPE1 == 3)
%
% DOING GUMBEL TRANSFORMATION
%
 [mean(1) sd(1)] = gumbel(rfmu(1),rmfx1);
elseif(DISTYPE1 == 4)
%
```

```
% DOING WEIBULL TRANSFORMATION
%
 [mean(1) sd(1)] = weibull(rfmu(1),rmfx1);
elseif(DISTYPE1 == 5)
%
% DOING FRECHET TRANSFORMATION
%
 [mean(1) sd(1)] = frechet(rfmu(1),rmfx1);
end
if(DISTYPE2 == 1)
%
% DOING NORMAL
%
    mean(2) = rfmu(2);
    sd(2) = rmfx2;
elseif(DISTYPE2 == 2)
%
% DOING LOGNORMAL TRANSFORMATION
%
 [mean(2) sd(2)] = lognormal(rfmu(2),rmfx2);
elseif(DISTYPE2 == 3)
%
% DOING GUMBEL TRANSFORMATION
%
 [mean(2) sd(2)] = gumbel(rfmu(2),rmfx2);
elseif(DISTYPE2 == 4)
%
% DOING WEIBULL TRANSFORMATION
%
 [mean(2) sd(2)] = weibull(rfmu(2),rmfx2);
elseif(DISTYPE2 == 5)
%
% DOING FRECHET TRANSFORMATION
%
 [mean(2) sd(2)] = frechet(rfmu(2),rmfx2);
end
if(DISTYPE3 == 1)
%
% DOING NORMAL
%
    mean(3) = rfmu(3);
    sd(3) = rmfx3;
elseif(DISTYPE3 == 2)
%
% DOING LOGNORMAL TRANSFORMATION
%
 [mean(3) sd(3)] = lognormal(rfmu(3),rmfx3);
elseif(DISTYPE3 == 3)
%
% DOING GUMBEL TRANSFORMATION
%
 [mean(3) sd(3)] = gumbel(rfmu(3),rmfx3);
elseif(DISTYPE3 == 4)
%
% DOING WEIBULL TRANSFORMATION
%
 [mean(3) sd(3)] = weibull(rfmu(3),rmfx3);
```

```
elseif(DISTYPE3 == 5)
%
% DOING FRECHET TRANSFORMATION
%
 [mean(3) sd(3)] = frechet(rfmu(3),rmfx3);
end
if(DISTYPE4 == 1)
%
% DOING NORMAL
%
    mean(4) = rfmu(4);
    sd(4) = rmfx4;
elseif(DISTYPE4 == 2)
%
% DOING LOGNORMAL TRANSFORMATION
%
 [mean(4) sd(4)] = lognormal(rfmu(4),rmfx4);
elseif(DISTYPE4 == 3)
%
% DOING GUMBEL TRANSFORMATION
%
 [mean(4) sd(4)] = gumbel(rfmu(4),rmfx4);
elseif(DISTYPE4 == 4)
%
% DOING WEIBULL TRANSFORMATION
%
 [mean(4) sd(4)] = weibull(rfmu(4),rmfx4);
elseif(DISTYPE4 == 5)
%
% DOING FRECHET TRANSFORMATION
%
 [mean(4) sd(4)] = frechet(rfmu(4),rmfx4);
end
if(DISTYPE5 == 1)
%
% DOING NORMAL
%
    mean(5) = rfmu(5);
    sd(5) = rmfx5;
elseif(DISTYPE5 == 2)
%
% DOING LOGNORMAL TRANSFORMATION
%
 [mean(5) sd(5)] = lognormal(rfmu(5),rmfx5);
elseif(DISTYPE5 == 3)
%
% DOING GUMBEL TRANSFORMATION
%
 [mean(5) sd(5)] = gumbel(rfmu(5),rmfx5);
elseif(DISTYPE5 == 4)
%
% DOING WEIBULL TRANSFORMATION
%
 [mean(5) sd(5)] = weibull(rfmu(5),rmfx5);
elseif(DISTYPE5 == 5)
%
% DOING FRECHET TRANSFORMATION
```

```
%
 [mean(5) sd(5)] = frechet(rfmu(5),rmfx5);
end
if(DISTYPE6 == 1)
%
% DOING NORMAL
%
    mean(6) = rfmu(6);
    sd(6) = rmfx6;
elseif(DISTYPE6 == 2)
%
% DOING LOGNORMAL TRANSFORMATION
%
 [mean(6) sd(6)] = lognormal(rfmu(6),rmfx6);
elseif(DISTYPE6 == 3)
%
% DOING GUMBEL TRANSFORMATION
%
 [mean(6) sd(6)] = gumbel(rfmu(6),rmfx6);
elseif(DISTYPE6 == 4)
%
% DOING WEIBULL TRANSFORMATION
%
 [mean(6) sd(6)] = weibull(rfmu(6),rmfx6);
elseif(DISTYPE6 == 5)
%
% DOING FRECHET TRANSFORMATION
%
 [mean(6) sd(6)] = frechet(rfmu(6),rmfx6);
end
if(DISTYPE7 == 1)
%
% DOING NORMAL
%
    mean(7) = rfmu(7);
    sd(7) = rmfx7;
elseif(DISTYPE7 == 2)
%
% DOING LOGNORMAL TRANSFORMATION
%
 [mean(7) sd(7)] = lognormal(rfmu(7),rmfx7);
elseif(DISTYPE5 == 3)
%
% DOING GUMBEL TRANSFORMATION
%
 [mean(7) sd(7)] = gumbel(rfmu(7),rmfx7);
elseif(DISTYPE7 == 4)
%
% DOING WEIBULL TRANSFORMATION
%
 [mean(7) sd(7)] = weibull(rfmu(7),rmfx7);
elseif(DISTYPE7 == 7)
%
% DOING FRECHET TRANSFORMATION
%
 [mean(7) sd(7)] = frechet(rfmu(7),rmfx7);
end
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
sev1 = sd(1); sev2 = sd(2); sev3 = sd(3); sev4 = sd(4);
sev5 = sd(5); sev6 = sd(6); sev7 = sd(7);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
dev1 = mean(1); dev2 = mean(2); dev3 = mean(3); dev4 = mean(4);
dev5 = mean(5); dev6 = mean(6); dev7 = mean(7);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
part1 = (0.138*dev4*dev3*dev2) - (0.138*dev3*dev2*(dev4^2)/250.0);
part2 = (0.138*dev1*dev4*dev3) - (0.138*dev1*dev3*(dev4^2)/250.0);
part3 = (0.138*dev1*dev4*dev2) - (0.138*dev1*dev2*(dev4^2)/250.0);
part4 = (0.138*dev1*dev3*dev2) - (0.276*dev1*dev3*dev2*dev4/250.0);
part5 = -cons4;
part6 = -cons5;
part7 = -cons6;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
deno = sqrt(((part1^2)*(sev1^2)) + ((part2^2)*(sev2^2)) + ...
    ((part3^2)*(sev3^2)) + ((part4^2)*(sev4^2)) + ...
    ((part5^2)*(sev5^2)) + ((part6^2)*(sev6^2)) + ...
    ((part7^2)*(sev7^2)));
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
nume = (0.138*dev4*dev3*dev2*dev1) -
((0.138*dev3*dev2*(dev4^2)*dev1)/250.0)...
    - cons4*dev5 - cons5*dev6 - cons6*dev7;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
if(deno <= 0)
    beta_mode_2 = 1000;
else
beta_mode_2 = nume/deno;
end
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%

%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
```

```
%
function beta_mode_3 = Fatigue_beam_flexure(moder,AS,...
    fyd,d,b,fcd,cons7,cons8,cons9,modec,modeash,modeqk)
%
[cov1,cov2,cov3,cov4,cov5,cov6,cov7,cov8,cov9] =
fatigue_covariance_mode3;
%
cv(1) = cov1; cv(2) = cov2; cv(3) = cov3; cv(4) = cov4;
cv(5) = cov5; cv(6) = cov6; cv(7) = cov7; cv(8) = cov8;
cv(9) = cov9;
%
[DISTYPE1,DISTYPE2,DISTYPE3,DISTYPE4,DISTYPE5, ...
    DISTYPE6,DISTYPE7,DISTYPE8,DISTYPE9] = distribution_mode3;
%
rfmu(1) = moder; rmfx1 = moder*cv(1);
rfmu(2) = AS; rmfx2 = AS*cv(2);
rfmu(3) = fyd; rmfx3 = fyd*cv(3);
rfmu(4) = d; rmfx4 = d*cv(4);
rfmu(5) = b; rmfx5 = b*cv(5);
rfmu(6) = fcd; rmfx6 = fcd*cv(6);
rfmu(7) = modec; rmfx7 = modec*cv(7);
rfmu(8) = modeash; rmfx8 = modeash*cv(8);
rfmu(9) = modeqk; rmfx9 = modeqk*cv(9);
%
if(DISTYPE1 == 1)
%
% DOING NORMAL
%
    mean(1) = rfmu(1);
    sd(1) = rmfx1;
elseif(DISTYPE1 == 2)
%
% DOING LOGNORMAL TRANSFORMATION
%
 [mean(1) sd(1)] = lognormal(rfmu(1),rmfx1);
elseif(DISTYPE1 == 3)
%
% DOING GUMBEL TRANSFORMATION
%
 [mean(1) sd(1)] = gumbel(rfmu(1),rmfx1);
elseif(DISTYPE1 == 4)
%
% DOING WEIBULL TRANSFORMATION
%
 [mean(1) sd(1)] = weibull(rfmu(1),rmfx1);
elseif(DISTYPE1 == 5)
%
% DOING FRECHET TRANSFORMATION
%
 [mean(1) sd(1)] = frechet(rfmu(1),rmfx1);
end
if(DISTYPE2 == 1)
%
% DOING NORMAL
%
    mean(2) = rfmu(2);
    sd(2) = rmfx2;
```

```
elseif(DISTYPE2 == 2)
%
% DOING LOGNORMAL TRANSFORMATION
%
 [mean(2) sd(2)] = lognormal(rfmu(2),rmfx2);
elseif(DISTYPE2 == 3)
%
% DOING GUMBEL TRANSFORMATION
%
 [mean(2) sd(2)] = gumbel(rfmu(2),rmfx2);
elseif(DISTYPE2 == 4)
%
% DOING WEIBULL TRANSFORMATION
%
 [mean(2) sd(2)] = weibull(rfmu(2),rmfx2);
elseif(DISTYPE2 == 5)
%
% DOING FRECHET TRANSFORMATION
%
 [mean(2) sd(2)] = frechet(rfmu(2),rmfx2);
end
if(DISTYPE3 == 1)
%
% DOING NORMAL
%
    mean(3) = rfmu(3);
    sd(3) = rmfx3;
elseif(DISTYPE3 == 2)
%
% DOING LOGNORMAL TRANSFORMATION
%
 [mean(3) sd(3)] = lognormal(rfmu(3),rmfx3);
elseif(DISTYPE3 == 3)
%
% DOING GUMBEL TRANSFORMATION
%
 [mean(3) sd(3)] = gumbel(rfmu(3),rmfx3);
elseif(DISTYPE3 == 4)
%
% DOING WEIBULL TRANSFORMATION
%
 [mean(3) sd(3)] = weibull(rfmu(3),rmfx3);
elseif(DISTYPE3 == 5)
%
% DOING FRECHET TRANSFORMATION
%
 [mean(3) sd(3)] = frechet(rfmu(3),rmfx3);
end
if(DISTYPE4 == 1)
%
% DOING NORMAL
%
    mean(4) = rfmu(4);
    sd(4) = rmfx4;
elseif(DISTYPE4 == 2)
%
% DOING LOGNORMAL TRANSFORMATION
```

```
%
 [mean(4) sd(4)] = lognormal(rfmu(4),rmfx4);
elseif(DISTYPE4 == 3)
%
% DOING GUMBEL TRANSFORMATION
%
 [mean(4) sd(4)] = gumbel(rfmu(4),rmfx4);
elseif(DISTYPE4 == 4)
%
% DOING WEIBULL TRANSFORMATION
%
 [mean(4) sd(4)] = weibull(rfmu(4),rmfx4);
elseif(DISTYPE4 == 5)
%
% DOING FRECHET TRANSFORMATION
%
 [mean(4) sd(4)] = frechet(rfmu(4),rmfx4);
end
if(DISTYPE5 == 1)
%
% DOING NORMAL
%
    mean(5) = rfmu(5);
    sd(5) = rmfx5;
elseif(DISTYPE5 == 2)
%
% DOING LOGNORMAL TRANSFORMATION
%
 [mean(5) sd(5)] = lognormal(rfmu(5),rmfx5);
elseif(DISTYPE5 == 3)
%
% DOING GUMBEL TRANSFORMATION
%
 [mean(5) sd(5)] = gumbel(rfmu(5),rmfx5);
elseif(DISTYPE5 == 4)
%
% DOING WEIBULL TRANSFORMATION
%
 [mean(5) sd(5)] = weibull(rfmu(5),rmfx5);
elseif(DISTYPE5 == 5)
%
% DOING FRECHET TRANSFORMATION
%
 [mean(5) sd(5)] = frechet(rfmu(5),rmfx5);
end
if(DISTYPE6 == 1)
%
% DOING NORMAL
%
    mean(6) = rfmu(6);
    sd(6) = rmfx6;
elseif(DISTYPE6 == 2)
%
% DOING LOGNORMAL TRANSFORMATION
%
 [mean(6) sd(6)] = lognormal(rfmu(6),rmfx6);
elseif(DISTYPE6 == 3)
```

```
%
% DOING GUMBEL TRANSFORMATION
%
 [mean(6) sd(6)] = gumbel(rfmu(6),rmfx6);
elseif(DISTYPE6 == 4)
%
% DOING WEIBULL TRANSFORMATION
%
 [mean(6) sd(6)] = weibull(rfmu(6),rmfx6);
elseif(DISTYPE6 == 5)
%
% DOING FRECHET TRANSFORMATION
%
 [mean(6) sd(6)] = frechet(rfmu(6),rmfx6);
end
if(DISTYPE7 == 1)
%
% DOING NORMAL
%
    mean(7) = rfmu(7);
    sd(7) = rmfx7;
elseif(DISTYPE7 == 2)
%
% DOING LOGNORMAL TRANSFORMATION
%
 [mean(7) sd(7)] = lognormal(rfmu(7),rmfx7);
elseif(DISTYPE5 == 3)
%
% DOING GUMBEL TRANSFORMATION
%
 [mean(7) sd(7)] = gumbel(rfmu(7),rmfx7);
elseif(DISTYPE7 == 4)
%
% DOING WEIBULL TRANSFORMATION
%
 [mean(7) sd(7)] = weibull(rfmu(7),rmfx7);
elseif(DISTYPE7 == 7)
%
% DOING FRECHET TRANSFORMATION
%
 [mean(7) sd(7)] = frechet(rfmu(7),rmfx7);
end
if(DISTYPE8 == 1)
%
% DOING NORMAL
%
    mean(8) = rfmu(8);
    sd(8) = rmfx8;
elseif(DISTYPE8 == 2)
%
% DOING LOGNORMAL TRANSFORMATION
%
 [mean(8) sd(8)] = lognormal(rfmu(8),rmfx8);
elseif(DISTYPE8 == 3)
%
% DOING GUMBEL TRANSFORMATION
%
```

```
 [mean(8) sd(8)] = gumbel(rfmu(8),rmfx8);
elseif(DISTYPE8 == 4)
%
% DOING WEIBULL TRANSFORMATION
%
 [mean(8) sd(8)] = weibull(rfmu(8),rmfx8);
elseif(DISTYPE8 == 5)
%
% DOING FRECHET TRANSFORMATION
%
 [mean(8) sd(8)] = frechet(rfmu(8),rmfx8);
end
if(DISTYPE9 == 1)
%
% DOING NORMAL
%
    mean(9) = rfmu(9);
    sd(9) = rmfx9;
elseif(DISTYPE9 == 2)
%
% DOING LOGNORMAL TRANSFORMATION
%
 [mean(9) sd(9)] = lognormal(rfmu(9),rmfx9);
elseif(DISTYPE9 == 3)
%
% DOING GUMBEL TRANSFORMATION
%
 [mean(9) sd(9)] = gumbel(rfmu(9),rmfx9);
elseif(DISTYPE9 == 4)
%
% DOING WEIBULL TRANSFORMATION
%
 [mean(9) sd(9)] = weibull(rfmu(9),rmfx9);
elseif(DISTYPE9 == 5)
%
% DOING FRECHET TRANSFORMATION
%
 [mean(9) sd(9)] = frechet(rfmu(9),rmfx9);
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
sev1 = sd(1); sev2 = sd(2); sev3 = sd(3); sev4 = sd(4);
sev5 = sd(5); sev6 = sd(6); sev7 = sd(7); sev8 = sd(8);
sev9 = sd(9);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
dev1 = mean(1); dev2 = mean(2); dev3 = mean(3); dev4 = mean(4);
dev5 = mean(5); dev6 = mean(6); dev7 = mean(7); dev8 = mean(8);
dev9 = mean(9);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
part1 = (dev2*dev3*dev4) - (((dev2^2)*(dev3^2))/(2.0*dev5*dev6));
part2 = (dev1*dev3*dev4) - ((dev1*(2.0*dev2)*(dev3^2))/(2.0*dev5*dev6));
part3 = (dev1*dev2*dev4) - ((2.0*dev1*(dev2^2)*(dev3))/(2.0*dev5*dev6));
part4 =  dev1*dev2*dev3;
part5 = ((dev1*(dev2^2)*(dev3^2))/(2.0*(dev5^2)*dev6));
part6 = ((dev1*(dev2^2)*(dev3^2))/(2.0*dev5*(dev6^2)));
part7 = -cons7;
part8 = -cons8;
part9 = -cons9;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
deno = sqrt(((part1^2)*(sev1^2)) + ((part2^2)*(sev2^2)) + ...
    ((part3^2)*(sev3^2)) + ((part4^2)*(sev4^2)) + ...
    ((part5^2)*(sev5^2)) + ((part6^2)*(sev6^2)) + ...
    ((part7^2)*(sev7^2)) + ((part8^2)*(sev8^2)) + ...
    ((part9^2)*(sev9^2)));
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
nume = (dev1*dev2*dev3*dev4) - ((dev1*(dev2^2)*(dev3^2))/(2.0*dev5*dev6))
...
    - cons7*dev7 - cons8*dev8 - cons9*dev9;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
if(deno <= 0)
    beta_mode_3 = 1000;
else
beta_mode_3 = nume/deno;
end
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
```

**APPENDIX II**

**SAFTEY INDICES FOR THE PLOT**
**FAILURE MODE ONE**

**Variation ofSafety Index to the Stress Cycle/min with respect to Slab Depth**

| | H=200mm | | | | |
|---|---|---|---|---|---|
| | | | | | |
| STRESS CYCLE/MIN | 10YEARS | 15YEARS | 20YEARS | 30YEARS | 40YEARS |
| 3 | 6.9075 | 5.3319 | 3.77715 | 0.7555 | -2.0447 |
| 5 | 4.8093 | 2.2416 | -0.2055 | -4.5733 | -8.1447 |
| 7 | 2.7473 | -0.676 | -3.7624 | -8.7658 | -12.3623 |
| 10 | -0.2055 | -4.5733 | -8.1447 | -13.1822 | -16.2871 |
| 12 | -2.0447 | -6.8114 | -10.4593 | -15.216 | -17.9396 |
| 15 | -4.5733 | -9.6434 | -13.1822 | -17.3844 | -19.6121 |
| | | | | | |
| | | | | | |

| | H=250mm | | | | |
|---|---|---|---|---|---|
| | | | | | |
| STRESS CYCLE/MIN | 10YEARS | 15YEARS | 20YEARS | 30YEARS | 40YEARS |
| 3 | 7.6134 | 6.3934 | 5.1757 | 2.7774 | 0.4727 |
| 5 | 5.9869 | 3.968 | 1.9965 | -1.6977 | -4.9524 |

| | | | | | |
|---|---|---|---|---|---|
| 7 | 4.369 | 1.6105 | -0.9911 | -5.5455 | -9.1672 |
| 10 | 1.9965 | -1.6977 | -4.9524 | -10.0427 | -13.5481 |
| 12 | 0.4727 | -3.7079 | -7.2088 | -12.3035 | -15.5446 |
| 15 | -1.6977 | -6.3987 | -10.0427 | -14.8634 | -17.6594 |
| | | | | | |

| | H=300mm | | | | |
|---|---|---|---|---|---|
| | | | | | |
| STRESS CYCLE/MIN | 10YEARS | 15YEARS | 20YEARS | 30YEARS | 40YEARS |
| 3 | 8.0577 | 7.0642 | 6.0692 | 4.0921 | 2.1585 |
| 5 | 6.7325 | 5.0772 | 3.4413 | 0.2954 | -2.2599 |
| 7 | 5.4073 | 3.118 | 0.9071 | -3.1429 | -6.5858 |
| 10 | 3.4413 | 0.2954 | -2.5998 | -7.4543 | -11.0962 |
| 12 | 2.1585 | -1.4765 | -4.6959 | -9.7715 | -13.2996 |
| 15 | 0.2954 | -3.9339 | -7.4543 | -12.5366 | -15.7433 |
| | | | | | |

**Variationof COVofSlabdue to Concrete Strength**

| | | H=250mm | COV=0.05 | | |
|---|---|---|---|---|---|
| | | | | | |
| STRESS CYCLE/MIN | 10YEARS | 15YEARS | 20YEARS | 30YEARS | 40YEARS |
| 3 | 7.6134 | 6.3934 | 5.1757 | 2.7774 | 0.4727 |
| 5 | 5.9869 | 3.968 | 1.9965 | -1.6977 | -2.9524 |
| 7 | 4.369 | 1.6105 | -0.9911 | -5.5455 | -9.1672 |

| | | | | | |
|---|---|---|---|---|---|
| 10 | 1.9965 | -1.6977 | -4.9524 | -10.0427 | -13.5481 |
| 12 | 0.4727 | -3.7079 | -7.2088 | -12.3035 | -15.5446 |
| 15 | -1.6977 | -6.3987 | -10.0427 | -14.8634 | -17.6594 |

| | | H=250mm | COV=0.10 | | |
|---|---|---|---|---|---|
| | | | | | |
| STRESS CYCLE/MIN | 10YEARS | 15YEARS | 20YEARS | 30YEARS | 40YEARS |
| 3 | 5.7315 | 4.8244 | 3.9181 | 2.1211 | 0.3651 |
| 5 | 4.522 | 3.016 | 1.5301 | -1.329 | -3.9728 |
| 7 | 3.316 | 1.2366 | -0.7723 | -4.4709 | -7.6499 |
| 10 | 1.5301 | -1.329 | -3.9728 | -8.4584 | -11.8814 |
| 12 | 0.3651 | -2.9449 | -5.9 | -10.63 | -13.9784 |
| 15 | -1.329 | -5.1979 | -8.4584 | -13.2502 | -16.3243 |

| | | H=250mm | COV=0.20 | | |
|---|---|---|---|---|---|
| | | | | | |
| STRESS CYCLE/MIN | 10YEARS | 15YEARS | 20YEARS | 30YEARS | 40YEARS |
| 3 | 3.4669 | 2.9239 | 2.3811 | 1.2989 | 0.2259 |
| 5 | 2.7429 | 1.8392 | 0.9399 | -0.8325 | -2.5507 |
| 7 | 2.0197 | 0.7609 | -0.4816 | -2.886 | -5.1405 |
| 10 | 0.9399 | -0.8325 | -2.5507 | -5.7518 | -8.5666 |
| 12 | 0.2259 | -1.8712 | -3.873 | -7.4901 | -10.5204 |
| 15 | -0.8325 | -3.3832 | -5.7518 | -9.8185 | -12.9789 |

**VARIATION OF COV. OF SLAB DUE TO DEPTH OF SECTION**

| | | H=250mm | COV=0.05 | | |
|---|---|---|---|---|---|
| | | | | | |
| STRESS CYCLE/MIN | 10YEARS | 15YEARS | 20YEARS | 30YEARS | 40YEARS |
| 3 | 7.6134 | 6.3934 | 5.1757 | 2.7774 | 0.4727 |
| 5 | 5.9869 | 3.968 | 1.9965 | -1.6977 | -0.9524 |
| 7 | 4.369 | 1.6105 | -0.9911 | -5.5455 | -9.1672 |
| 10 | 1.9965 | -1.6977 | -4.9524 | -10.0427 | -13.5481 |
| 12 | 0.4727 | -3.7079 | -7.2088 | -12.3035 | -15.5446 |
| 15 | -1.6977 | -6.3987 | -10.0427 | -14.8634 | -17.6594 |

| | | H=250mm | COV=0.10 | | |
|---|---|---|---|---|---|
| | | | | | |
| STRESS CYCLE/MIN | 10YEARS | 15YEARS | 20YEARS | 30YEARS | 40YEARS |
| 3 | 5.7871 | 4.8709 | 3.9556 | 2.1409 | 0.3684 |
| 5 | 4.5655 | 3.0445 | 1.5443 | -1.3405 | -4.0044 |
| 7 | 3.3475 | 1.248 | -0.7791 | -4.5059 | -7.7016 |
| 10 | 1.5443 | -1.3405 | -4.0044 | -8.5132 | -11.9425 |
| 12 | 0.3684 | -2.9692 | -5.9435 | -10.69 | -14.0378 |
| 15 | -1.3405 | -5.2373 | -8.5132 | -13.3106 | -16.377 |

| | | H=250mm | COV=0.20 | | |
|---|---|---|---|---|---|
| | | | | | |
| STRESS CYCLE/MIN | 10YEARS | 15YEARS | 20YEARS | 30YEARS | 40YEARS |
| 3 | 3.5292 | 2.9763 | 2.4236 | 1.3219 | 0.2298 |
| 5 | 2.792 | 1.8719 | 0.9565 | -0.8468 | -2.5931 |

| | | | | | |
|---|---|---|---|---|---|
| 7 | 2.0557 | 0.7743 | -0.49 | -2.9336 | -5.2201 |
| 10 | 0.9565 | -0.8468 | -2.5931 | -5.8391 | -8.6824 |
| 12 | 0.2298 | -1.9028 | -3.9353 | -7.5964 | -10.6485 |
| 15 | -0.8468 | -3.4383 | -5.8391 | -9.943 | -13.1127 |

**FAILURE MODE TWO**

**VARIATION IN DEPTH OF SECTION**

| | | H=1000mm | | | |
|---|---|---|---|---|---|
| | | | | | |
| STRESS CYCLE/MIN | 10YEARS | 15YEARS | 20YEARS | 30YEARS | 40YEARS |
| 3 | 6.0619 | 4.6132 | 3.1671 | 0.4232 | -2.1057 |
| 5 | 4.126 | 1.7697 | -0.4448 | -4.399 | -7.6835 |
| 7 | 2.23 | -0.8696 | -3.6616 | -8.2625 | -11.676 |
| 10 | -0.4448 | -4.399 | -7.6835 | -12.471 | -15.5488 |
| 12 | -2.1057 | -6.4491 | -9.856 | -14.4743 | -17.2352 |
| 15 | -4.399 | -9.0855 | -12.471 | -16.6646 | -18.9791 |

| | | H=1100mm | | | |
|---|---|---|---|---|---|
| | | | | | |
| STRESS CYCLE/MIN | 10YEARS | 15YEARS | 20YEARS | 30YEARS | 40YEARS |
| 3 | 7.0274 | 5.748 | 4.5094 | 2.1658 | -0.0341 |
| 5 | 5.3288 | 3.3187 | 1.4167 | -2.0861 | -5.1521 |
| 7 | 3.7111 | 1.048 | -1.4193 | -5.711 | -9.1368 |
| 10 | 1.4167 | -2.0861 | -5.1521 | -9.97 | -13.3358 |
| 12 | -0.0341 | -3.98 | -7.281 | -12.1346 | -15.2797 |

| | | | | | |
|---|---|---|---|---|---|
| 15 | -2.0861 | -6.5158 | -9.97 | -14.6139 | -17.3652 |

| | | H=1200mm | | | |
|---|---|---|---|---|---|
| | | | | | |
| STRESS CYCLE/MIN | 10YEARS | 15YEARS | 20YEARS | 30YEARS | 40YEARS |
| 3 | 7.7554 | 6.6076 | 5.5333 | 3.5204 | 1.6161 |
| 5 | 6.2417 | 4.5105 | 2.875 | -0.1919 | -2.9754 |
| 7 | 4.8472 | 2.5563 | 0.3998 | -3.4955 | -6.7877 |
| 10 | 2.875 | -0.1919 | -2.9754 | -7.6182 | -11.1102 |
| 12 | 1.6161 | -1.898 | -4.9811 | -9.8376 | -13.2359 |
| 15 | -0.1919 | -4.2524 | -7.6182 | -12.4984 | -15.6111 |

## VARIATION OF CONCRETE STRENGTH

| | | Fcu=25 | | |
|---|---|---|---|---|
| | | | | |
| STRESS CYCLE/MIN | 10YEARS | 15YEARS | 20YEARS | 30YEARS |
| 3 | 5.4958 | 3.7399 | 2.0452 | -1.1215 |
| 5 | 3.1671 | 0.4232 | -2.1057 | -6.4491 |
| 7 | 0.9556 | -2.5837 | -5.6581 | -10.4365 |
| 10 | -2.1057 | -6.4491 | -9.856 | -14.47 |
| 12 | -3.9598 | -8.5982 | -12.0026 | -16.2886 |
| 15 | -6.4491 | -11.2511 | -14.4743 | -18.1999 |

| | | Fcu=30 | | | |
|---|---|---|---|---|---|
| | | | | | |

| STRESS CYCLE/MIN | 10YEARS | 15YEARS | 20YEARS | 30YEARS | 40YEARS |
|---|---|---|---|---|---|
| 3 | 6.0619 | 4.6132 | 3.1671 | 0.4232 | -2.1057 |
| 5 | 4.126 | 1.7697 | -0.4448 | -4.399 | -7.6835 |
| 7 | 2.23 | -0.8696 | -3.6616 | -8.2625 | -11.676 |
| 10 | -0.4448 | -4.399 | -7.6835 | -12.471 | -15.5488 |
| 12 | -2.1057 | -6.4491 | -9.856 | -14.4743 | -17.2352 |
| 15 | -4.399 | -9.0855 | -12.471 | -16.6646 | -18.9791 |

| | | Fcu=35 | | | |
|---|---|---|---|---|---|
| | | | | | |
| STRESS CYCLE/MIN | 10YEARS | 15YEARS | 20YEARS | 30YEARS | 40YEARS |
| 3 | 6.3897 | 5.2442 | 3.9878 | 1.5742 | -0.6883 |
| 5 | 4.8232 | 2.7628 | 0.8026 | -2.7856 | -5.888 |
| 7 | 3.1671 | 0.4232 | -2.1057 | -6.4491 | -9.856 |
| 10 | 0.8026 | -2.7856 | -5.888 | -10.676 | -13.9545 |
| 12 | -0.6883 | -4.7068 | -8.0174 | -12.7906 | -15.8242 |
| 15 | -2.7856 | -7.2544 | -10.676 | -15.1857 | -17.8123 |

| | | Fcu=40 | | | |
|---|---|---|---|---|---|
| | | | | | |
| STRESS CYCLE/MIN | 10YEARS | 15YEARS | 20YEARS | 30YEARS | 40YEARS |
| 3 | 6.4668 | 5.7136 | 4.6132 | 2.4622 | 0.4232 |
| 5 | 5.3492 | 3.5242 | 1.7697 | -1.4949 | -4.399 |
| 7 | 3.8843 | 1.4283 | -0.8696 | -4.9343 | -8.2625 |

| | | | | | |
|---|---|---|---|---|---|
| 10 | 1.7697 | -1.4949 | -4.399 | -9.0855 | -12.471 |
| 12 | 0.4232 | -3.2827 | -6.4491 | -11.2511 | -14.1743 |
| 15 | -1.4949 | -5.7087 | -9.0855 | -13.784 | -16.6646 |

**VARIATION OF COV DUE TO CONCRETE STRENGTH**

| | | COV=0.05 | | |
|---|---|---|---|---|
| | | | | |
| STRESS CYCLE/MIN | 10YEARS | 15YEARS | 20YEARS | 30YEARS |
| 3 | 6.0619 | 4.6132 | 3.1671 | 0.4232 |
| 5 | 4.126 | 1.7697 | -0.4448 | -4.399 |
| 7 | 2.23 | -0.8696 | -3.6616 | -8.2625 |
| 10 | -0.4448 | -4.399 | -7.6835 | -12.471 |
| 12 | -2.1057 | -6.4491 | -9.856 | -14.4743 |
| 15 | -4.399 | -9.0855 | -12.471 | -16.6646 |

| | | COV=0.10 | | |
|---|---|---|---|---|
| | | | | |
| STRESS CYCLE/MIN | 10YEARS | 15YEARS | 20YEARS | 30YEARS |
| 3 | 5.7102 | 3.965 | 2.6319 | 0.3448 |
| 5 | 3.4944 | 1.4499 | -0.362 | -3.6178 |
| 7 | 1.8333 | -0.7078 | -3.0012 | -6.968 |
| 10 | -0.362 | -3.6178 | -6.4508 | -10.9323 |
| 12 | -1.717 | -5.3673 | -8.4242 | -12.9669 |
| 15 | -3.6178 | -7.7138 | -10.9323 | -15.3151 |

| | | COV=0.20 | | |
|---|---|---|---|---|
| | | | | |
| STRESS CYCLE/MIN | 10YEARS | 15YEARS | 20YEARS | 30YEARS |
| 3 | 4.7409 | 2.7726 | 1.7594 | 0.2252 |
| 5 | 2.3939 | 0.9529 | -0.2361 | -2.3883 |
| 7 | 1.2097 | -0.4617 | -1.9735 | -4.7464 |
| 10 | -0.2361 | -2.3883 | -4.3685 | -7.8771 |
| 12 | -1.1223 | -3.5947 | -5.8444 | -9.6926 |
| 15 | -2.3883 | -5.3022 | -7.8771 | -12.0268 |

**VARIATION OF COV DUE TO DEPTH OF SECTION**

| | | COV=0.05 | | |
|---|---|---|---|---|
| | | | | |
| STRESS CYCLE/MIN | 10YEARS | 15YEARS | 20YEARS | 30YEARS |
| 3 | 6.0619 | 4.6132 | 3.1671 | 0.4232 |
| 5 | 4.126 | 1.7697 | -0.4448 | -4.399 |
| 7 | 2.23 | -0.8696 | -3.6616 | -8.2625 |
| 10 | -0.4448 | -4.399 | -7.6835 | -12.471 |
| 12 | -2.1057 | -6.4491 | -9.856 | -14.4743 |
| 15 | -4.399 | -9.0855 | -12.471 | -16.6646 |
| | | | | |

| | | COV=0.10 | | |
|---|---|---|---|---|
| | | | | |
| STRESS CYCLE/MIN | 10YEARS | 15YEARS | 20YEARS | 30YEARS |
| 3 | 4.3768 | 3.4125 | 2.378 | 0.3249 |

| | | | | |
|---|---|---|---|---|
| 5 | 3.0693 | 1.3443 | -0.3437 | -3.511 |
| 7 | 1.6877 | -0.6742 | -2.9038 | -6.843 |
| 10 | -0.3437 | -3.511 | -6.3258 | -10.8259 |
| 12 | -1.6485 | -5.2453 | -8.3026 | -12.8783 |
| 15 | -3.511 | -7.5899 | -10.8259 | -15.2501 |

| | | | | |
|---|---|---|---|---|
| | | COV=0.20 | | |
| | | | | |
| STRESS CYCLE/MIN | 10YEARS | 15YEARS | 20YEARS | 30YEARS |
| 3 | 2.5636 | 2.0353 | 1.4353 | 0.1998 |
| 5 | 1.8386 | 0.8194 | -0.2127 | -2.2422 |
| 7 | 1.0255 | -0.4184 | -1.8423 | -4.5555 |
| 10 | -0.2127 | -2.2422 | -4.1813 | -7.6825 |
| 12 | -1.0326 | -3.4189 | -5.6472 | -9.511 |
| 15 | -2.2422 | -5.1072 | -7.6825 | -11.8715 |

**FAILURE MODE THREE**

**VARIATION OF DEPTH OF SECTION**

| | | h=1000mm | | | |
|---|---|---|---|---|---|
| | | | | | |
| STRESS CYCLE/MIN | 10YEARS | 15YEARS | 20YEARS | 30YEARS | 40YEARS |
| 3 | 7.5872 | 6.2383 | 4.8943 | 2.2601 | -0.2483 |
| 5 | 5.7892 | 3.5653 | 1.4072 | -2.5835 | -6.026 |
| 7 | 4.006 | 0.9867 | -1.8264 | -6.6448 | -10.3627 |
| 10 | 1.4072 | -2.5835 | -6.026 | -11.2452 | -14.7151 |

| | | | | | |
|---|---|---|---|---|---|
| 12 | -0.2483 | -4.7187 | -8.3656 | -13.4947 | -16.6469 |
| 15 | -2.5835 | -7.5302 | -11.2452 | -15.9912 | -18.6609 |

| | | h=1100mm | | | |
|---|---|---|---|---|---|
| | | | | | |
| STRESS CYCLE/MIN | 10YEARS | 15YEARS | 20YEARS | 30YEARS | 40YEARS |
| 3 | 7.8839 | 6.6874 | 5.4922 | 3.1332 | 0.8574 |
| 5 | 6.2885 | 4.3052 | 2.3632 | -1.2966 | -4.5512 |
| 7 | 4.6995 | 1.9822 | -0.5941 | -5.1478 | -8.8188 |
| 10 | 2.3632 | -1.2966 | -4.5512 | -9.7138 | -13.3286 |
| 12 | 0.8574 | -3.3029 | -6.8278 | -12.0392 | -15.4105 |
| 15 | -1.2966 | -6.0083 | -9.7138 | -14.6982 | -17.6338 |

| | | h=1200mm | | | |
|---|---|---|---|---|---|
| | | | | | |
| STRESS CYCLE/MIN | 10YEARS | 15YEARS | 20YEARS | 30YEARS | 40YEARS |
| 3 | 8.1169 | 7.0429 | 5.968 | 3.8367 | 1.761 |
| 5 | 6.6844 | 4.8977 | 3.1369 | -0.2276 | -3.2897 |
| 7 | 5.2537 | 2.7898 | 0.424 | -3.8598 | -7.4377 |
| 10 | 3.1369 | -0.2276 | -3.2897 | -8.3299 | -12.0222 |
| 12 | 1.761 | -2.1061 | -5.4816 | -10.6883 | -14.2173 |
| 15 | -0.2276 | -4.6874 | -8.3299 | -13.4606 | -16.6184 |

**VARIATION OF CONCRETE STRENGTH**

| | | fcu=25 | | | |
|---|---|---|---|---|---|
| | | | | | |
| STRESS CYCLE/MIN | 10YEARS | 15YEARS | 20YEARS | 30YEARS | 40YEARS |
| 3 | 7.4811 | 6.0812 | 4.6879 | 1.9641 | -0.6175 |
| 5 | 5.6155 | 3.3123 | 1.0848 | -3.0069 | -6.4996 |
| 7 | 3.7682 | 0.6518 | -2.2339 | -7.1234 | -10.8421 |
| 10 | 1.0848 | -3.0069 | -6.4996 | -11.7174 | -15.1304 |
| 12 | -0.6175 | -5.1778 | -8.8508 | -13.9351 | -17.0116 |
| 15 | -3.0069 | -8.0135 | -11.7174 | -16.3746 | -18.9594 |

| | | fcu=30 | | | |
|---|---|---|---|---|---|
| | | | | | |
| STRESS CYCLE/MIN | 10YEARS | 15YEARS | 20YEARS | 30YEARS | 40YEARS |
| 3 | 7.5872 | 6.2383 | 4.8943 | 2.2601 | -0.2483 |
| 5 | 5.7892 | 3.5653 | 1.4072 | -2.5835 | -6.026 |
| 7 | 4.006 | 0.9867 | -1.8264 | -6.6448 | -10.3627 |
| 10 | 1.4072 | -2.5835 | -6.026 | -11.2452 | -14.7151 |
| 12 | -0.2483 | -4.7187 | -8.3656 | -13.4947 | -16.6469 |
| 15 | -2.5835 | -7.5302 | -11.2452 | -15.9912 | -18.6609 |

| | | fcu=35 | | | |
|---|---|---|---|---|---|

| | | | | | |
|---|---|---|---|---|---|
| STRESS CYCLE/MIN | 10YEARS | 15YEARS | 20YEARS | 30YEARS | 40YEARS |
| 3 | 7.647 | 6.3343 | 5.0257 | 2.4562 | 0.0016 |
| 5 | 5.8972 | 3.7302 | 1.6226 | -2.2927 | -5.6948 |
| 7 | 4.16 | 1.2113 | -1.5477 | -6.3092 | -10.0211 |
| 10 | 1.6226 | -2.2927 | -5.6948 | -10.9076 | -14.4142 |
| 12 | 0.0016 | -4.3998 | -8.0227 | -13.177 | -16.3811 |
| 15 | -2.2927 | -7.1899 | -10.9076 | -15.7124 | -18.4422 |

| | | fcu=40 | | | |
|---|---|---|---|---|---|
| | | | | | |
| STRESS CYCLE/MIN | 10YEARS | 15YEARS | 20YEARS | 30YEARS | 40YEARS |
| 3 | 7.6841 | 6.3984 | 5.116 | 2.5952 | 0.1815 |
| 5 | 5.9701 | 3.8457 | 1.7762 | -2.0812 | -5.4509 |
| 7 | 4.2673 | 1.3718 | -1.3457 | -6.0616 | -9.7661 |
| 10 | 1.7762 | -2.0812 | -5.4509 | -10.6549 | -14.1868 |
| 12 | 0.1815 | -4.166 | -7.7684 | -12.8378 | -16.1793 |
| 15 | -2.0812 | -6.9382 | -10.6549 | -15.501 | -18.2756 |

## VARIATION OF COV DUE TO CONCRETE STRENGTH

| | | COV=0.05 | | |
|---|---|---|---|---|
| | | | | |
| STRESS CYCLE/MIN | 10YEARS | 15YEARS | 20YEARS | 30YEARS |
| 3 | 7.5872 | 6.2383 | 4.8943 | 2.2601 |
| 5 | 5.7892 | 3.5653 | 1.4072 | -2.5835 |

| | | | | |
|---|---|---|---|---|
| 7 | 4.006 | 0.9867 | -1.8264 | -6.6448 |
| 10 | 1.4072 | -2.5835 | -6.026 | -11.2452 |
| 12 | -0.2483 | -4.7187 | -8.3656 | -13.4947 |
| 15 | -2.5835 | -7.5302 | -11.2452 | -15.9912 |

| | | COV=0.10 | | |
|---|---|---|---|---|
| | | | | |
| STRESS CYCLE/MIN | 10YEARS | 15YEARS | 20YEARS | 30YEARS |
| 3 | 5.2415 | 4.3239 | 3.4079 | 1.5933 |
| 5 | 4.0182 | 2.4966 | 0.997 | -1.8849 |
| 7 | 2.7996 | 0.7009 | -1.3241 | -5.047 |
| 10 | 0.997 | -1.8849 | -4.5459 | -9.0535 |
| 12 | -0.1779 | -3.5118 | -6.4837 | -11.2324 |
| 15 | -1.8849 | -5.7779 | -9.0535 | -13.8579 |

| | | COV=0.20 | | |
|---|---|---|---|---|
| | | | | |
| STRESS CYCLE/MIN | 10YEARS | 15YEARS | 20YEARS | 30YEARS |
| 3 | 2.9813 | 2.4645 | 1.9479 | 0.9181 |
| 5 | 2.2922 | 1.4323 | 0.5764 | -1.1125 |
| 7 | 1.604 | 0.4059 | -0.7779 | -3.0758 |
| 10 | 0.5764 | -1.1125 | -2.7546 | -5.8351 |
| 12 | -0.1036 | -2.1045 | -4.0232 | -7.5229 |

| | | | | |
|---|---|---|---|---|
| 15 | -1.1125 | -3.5527 | -5.8351 | -9.8087 |

## VARIATION OF COV DUE TO DEPTH OF SECTION

| | | COV=0.05 | | |
|---|---|---|---|---|
| | | | | |
| STRESS CYCLE/MIN | 10YEARS | 15YEARS | 20YEARS | 30YEARS |
| 3 | 7.5872 | 6.2383 | 4.8943 | 2.2601 |
| 5 | 5.7892 | 3.5653 | 1.4072 | -2.5835 |
| 7 | 4.006 | 0.9867 | -1.8264 | -6.6448 |
| 10 | 1.4072 | -2.5835 | -6.026 | -11.2452 |
| 12 | -0.2483 | -4.7187 | -8.3656 | -13.4947 |
| 15 | -2.5835 | -7.5302 | -11.2452 | -15.9912 |

| | | COV=0.10 | | |
|---|---|---|---|---|
| | | | | |
| STRESS CYCLE/MIN | 10YEARS | 15YEARS | 20YEARS | 30YEARS |
| 3 | 6.1544 | 5.0712 | 3.9905 | 1.8576 |
| 5 | 4.7104 | 2.9176 | 1.1603 | -2.1704 |
| 7 | 3.2741 | 0.815 | -1.5282 | -5.7231 |

| | | | | |
|---|---|---|---|---|
| 10 | 1.1603 | -2.1704 | -5.1685 | -10.0276 |
| 12 | -0.2062 | -4.0136 | -7.2938 | -12.2658 |
| 15 | -2.1704 | -6.5258 | -10.0276 | -14.8644 |

| | | COV=0.20 | | |
|---|---|---|---|---|
| | | | | |
| STRESS CYCLE/MIN | 10YEARS | 15YEARS | 20YEARS | 30YEARS |
| 3 | 3.9993 | 3.3034 | 2.6082 | 1.2254 |
| 5 | 3.0715 | 1.915 | 0.7683 | -1.471 |
| 7 | 2.1458 | 0.5408 | -1.0304 | -4.0141 |
| 10 | 0.7683 | -1.471 | -3.6034 | -7.4344 |
| 12 | -0.1377 | -2.7656 | -5.2109 | -9.4203 |
| 15 | -1.471 | -4.6193 | -7.4344 | -11.9631 |

**APPENDIX III**

**Reinforced Concrete Bridge Design Criteria to Eurocode & UK National Annex**

**Index**

1. Introduction
2. Serviceability Limit State
3. Ultimate Limit State
4. Shear

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## 1. Introduction

Both the Serviceability and Ultimate Limit States need to be considered.
Serviceability Limit State ensures that crack widths do not exceed specified values, and also ensures that concrete and reinforcement stresses are maintained below a safe limit.
Ultimate Limit State ensures that the structure will not collapse.

## 2. Serviceability Limit State

i) Crack Control
Cracks in concrete can be caused by :

- corrosion of the reinforcement which causes the concrete to spall
- thermal movements, particularly cooling from heat of hydration (called early thermal cracking)
- structural actions such as bending, shear or torsion

Corrosion of reinforcement is controlled by use of suitable concrete grades and providing adequate cover to the reinforcement.
Cracks due to thermal movements are controlled by providing minimum nominal steel area

and restricting the maximum bar spacing. Ciria Publication C660 is used to calculate the minimum steel area and bar spacing to control early thermal cracking.

The width of shear cracks is controlled by ultimate strength calculations.

Crack widths caused by bending and tension may be calculated using Clause 7.3.4, alternatively

Clause 7.3.3 may be used to limit the stress in the bar for a given bar diameter, or a given bar spacing, from

Tables 7.2N & 7.3N.

For calculating the crack width only the quasi-permanent load combination is used.

$\psi_2 = 0$ for traffic actions and $\psi_2 = 0.5$ for thermal actions consequently only thermal actions (variable action) are considered in combination with the permanent actions.

However only secondary effects of the temperature difference need to be considered; the primary self-equilibrating stresses may be ignored.

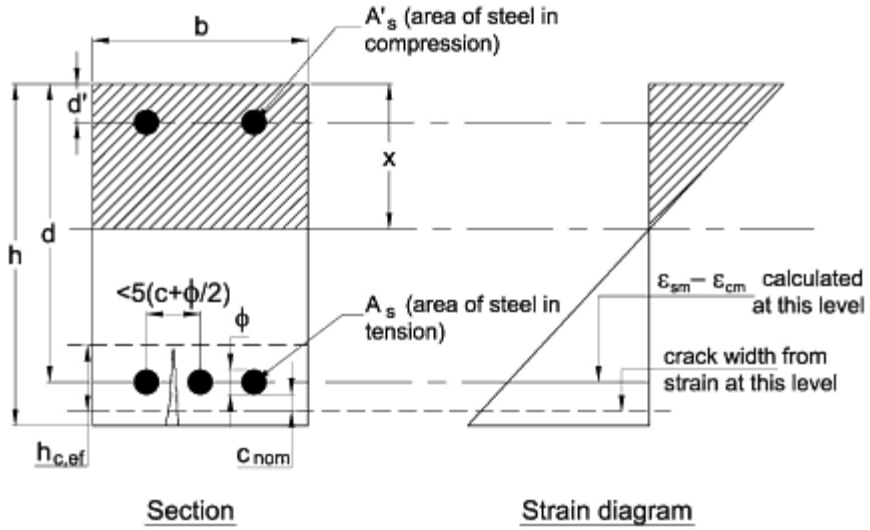
Crack widths may be calculated using Clause 7.3.4. The crack width ($w_k$) is a function of the spacing between the cracks ($s_{r,max}$) and the strain in the reinforcement ($\varepsilon_{sm}$) and concrete ($\varepsilon_{cm}$):

$w_k = s_{r,max}(\varepsilon_{sm} - \varepsilon_{cm})$ ... ....Equation (7.8)

where $s_{r,max} = k_3c + k_1k_2k_4\varphi/\rho_{p,eff}$ .... ...Equation (7.11)

and $(\varepsilon_{sm} - \varepsilon_{cm}) = \{\sigma_s - [k_t f_{ct,eff}(1 + \alpha_e\rho_{p,eff})/\rho_{p,eff}]\}/E_s \geq 0.6\sigma_s/E_s$ .... ...Equation (7.9)

Using a rectangular section as an example:



We first need to determine the position of the neutral axis:

$E_s = 200kN/mm^2$ (clause 3.2.7(4))

The short-term value of $E_{c,eff}$ is obtained from the value of $E_{cm}$ in Table 3.1 which is appropriate to the live load portion of the moment ($M_{st}$). The permanent load portion of the moment ($M_{qp}$) has a long-term value of $E_{c,eff}$ to take account of the effects of creep. The modified value of $E_{c,eff}$ used for the crack width calculation is an intermediate value between the short and long term values.

Modified $E_{c,eff} = E_{cm}(M_{st} + M_{qp}) / [M_{st} + M_{qp}(1 + \varphi)]$ where $\varphi$ is the value of the creep coefficient $\varphi(\infty,t_0)$.

$\alpha_{eff}$ = Modular ratio = $E_s / E_{c,eff}$.

$\rho_{p,eff} = A_s / bd$ and $\rho'_{p,eff} = A'_s / bd$

Then

$x/d = \sqrt{\{[\alpha_{eff}\rho_{p,eff}+(\alpha_{eff}-1)\rho'_{p,eff}]^2+2[\alpha_{eff}\rho_{p,eff}+(\alpha_{eff}-1)\rho'_{p,eff}d'/d]\}}-[\alpha_{eff}\rho_{p,eff}+(\alpha_{eff}-1)\rho'_{p,eff}]$

Second Moment of Area of Cracked Section $I_c = bx^3/3 + (\alpha_{eff}-1)A_s'(x-d')^2 + \alpha_{eff}A_s(d-x)^2$

Hence reinforcement stress $\sigma_s = \alpha_{eff}M_{Ed}(d-x)/I_c$ where $M_{Ed} = M_{qp}+M_{st}$

$k_t$ is obtained by interpolation between 0.4 for long-term and 0.6 for short-term loadings:

$k_t = 0.4 + (0.2M_{st}/M_{Ed})$

$f_{ct,eff} = f_{ctm}$ (from Table 3.1)

$\alpha_e = \alpha_{eff} = E_s / E_{c,eff}$

$\rho_{p,eff} = (A_s + \xi_1 A'_p)/A_{c,eff}$ where $A'_p = 0$

$A_{c,eff} = bh_{c,ef}$ where $h_{c,ef}$ is the lesser of 2.5(h-d), (h-x)/3 or h/2

$(\varepsilon_{sm} - \varepsilon_{cm})$ can now be evaluated from Equation (7.9)

$s_{r,max}$ can also be evaluated from Equation (7.11) with:

$k_1 = 0.8$ (for high bond reinforcement)

$k_2 = 0.5$ (for bending)

$k_3 = 3.4$ and $k_4 = 0.425$ (from UK National Annex to EN 1992-1-1)

The Crack Width $w_k$ can now be calculated using Equation (7.8) in clause 7.3.4(1)

ii) Stress Limits

The methods given in the code for checking crack widths assumes a linear elastic behaviour. It is therefore necessary to check the stress limits to ensure this assumption is

correct. in particular that the strains caused by transient loads will not become permanent. The section properties are determined by considering whether the concrete in the tension zone is cracked or not. This is done by assuming initially that the section is uncracked whereby the second moment of area of the section $I_c$ is for the full concrete section ($bh^3/12$ for a rectangular section).

The maximum tensile stress in the concrete is obtained from $M_{Ed}\ y / I_c$ and compared with the tensile strength of the concrete $f_{ctm}$ from Table 3.1.

If the tensile stress is greater than $f_{ctm}$ then the section will be considered cracked and the stresses calculated using a cracked elastic section analysis similar to that used to determine the crack width described above.

All load combinations need to be checked to ensure that the stress limits are not exceeded. As the maximum design variable loading can occur at any time during the life of the bridge then, for the cracked section, it is usual to check the stresses at first opening and also after all the creep has occurred at the end of the life of the bridge. This is done by using $E_{c,eff} = E_{cm}$ for both the short and long-term loading when the bridge is first opened, and using the modified value of $E_{c,eff}$, as described in the crack control calculation above, for the full life of the bridge.

The stress limits given in the code are:

$k_3 f_{yk} = 0.8 f_{yk}$ for reinforcement in tension (BS EN 1992-1-1:2004 Clause 7.2.(5)).

$k_1 f_{ck} = 0.6 f_{ck}$ for concrete in compression (BS EN 1992-2:2005 Clause 7.2.(102)).

## 3. Ultimate Limit State

To determine the moment of resistance of a member at failure by limit state analysis the following assumptions are made:

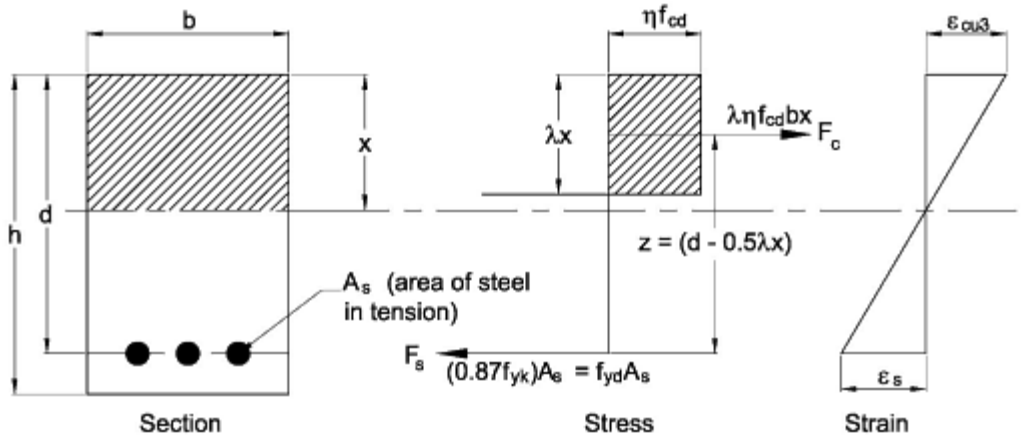
a. The distribution of strain across any section is linear. This means that plane sections before bending remain plane after bending, and the strain at any point is proportional to its distance from the neutral axis.
b. The resistance of concrete in tension is ignored.

c. The relationship between the stress and strain in the reinforcement is as shown in Figure 3.8 of EN 1992-1-1 with $\gamma_s = 1.15$.
d. The relationship between the stress and strain in the concrete is as shown in Figure 3.4 of EN 1992-1-1 with $f_{cd} = \alpha_{cc} f_{ck}/\gamma_c$ ( $\alpha_{cc} = 0.85$, $\gamma_c = 1.5$). The distribution of stress in the concrete at failure may be represented by a uniform stress $f_{av} = \eta f_{cd}$ acting over the depth of the compression zone $\lambda X$ where X is the depth to the neutral axis.

Clause 3.1.7(3) of EN 1992-1-1 allows a rectangular concrete stress distribution to be assumed as shown below. The value of $\lambda = 0.8$ for $f_{ck} \leq 50$ MPa, and $\lambda = 0.8-(f_{ck}-50)/400$ for $50 < f_{ck} \leq 90$ MPa.

The value of the uniform stress $f_{av} = \eta f_{cd}$ where $\eta = 1.0$ for $f_{ck} \leq 50$ MPa, and $\eta = 1.0-(f_{ck}-50)/200$ for $50 < f_{ck} \leq 90$ MPa.

For the singly reinforced rectangular section (assuming the reinforcement yields):




Taking moments about the centre of compression for the tensile force

**$M_u = f_{yd} A_s z = 0.87 f_{yk} A_s z$**

The depth to the neutral axis depends upon the reinforcement provided and is obtained by equating the forces:

$F_s = F_c \rightarrow 0.87 f_{yk} A_s = 0.85 \lambda \eta f_{ck} bx/1.5$

Rearranging and dividing both sides by d we get:

$(x/d) = (1.53 f_{yk} A_s) / (\lambda \eta f_{ck} bd)$

Lever arm $z = (d - 0.5\lambda x)$ so substituting for x we get:

**$z = (1 - [0.77f_{yk}A_s] / [\eta f_{ck}bd])d$**

To meet the criteria for assuming the reinforcement yields then the strain in the steel $\varepsilon_s$ needs to be greater than $\varepsilon_{s,yield.}$

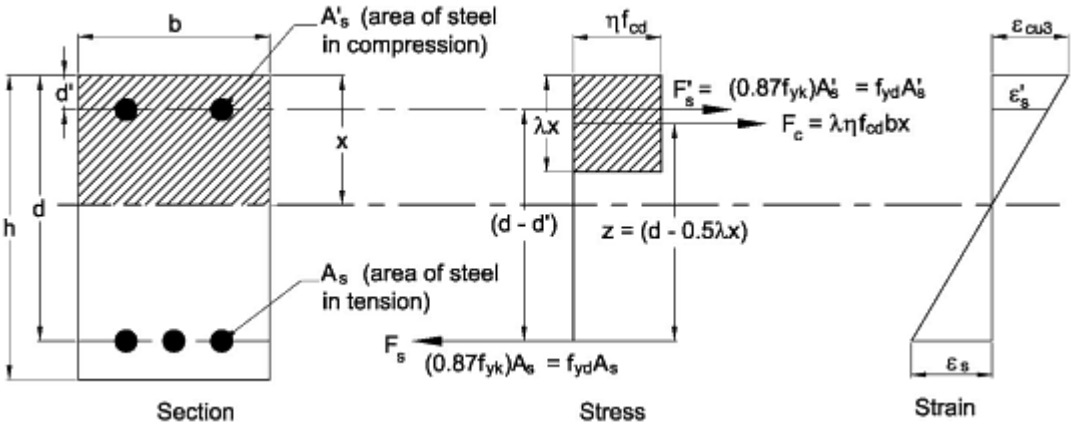
From the strain diagram $\varepsilon_s = \varepsilon_{cu3}(d-x)/x$ which has to be greater than $0.87f_{yk}/E_s$

This re-arranges to **$x/d \leq 1 / \{[0.87f_{yk}/(E_s\varepsilon_{cu3})]+1\}$** ... Equation(A)

If x/d does not meet this condition then the options are:

1. calculate the actual force in the reinforcement using the strain compatibility method and hence the moment of resistance, or
2. increase d, or
3. increase the concrete strength or add compression reinforcement.

For the doubly reinforced rectangular section:




Equating the tensile and compressive forces

$F_s = F_c + F'_s \rightarrow 0.87f_{yk}A_s = 0.85\lambda\eta f_{ck}bx/1.5 + 0.87f_{yk}A'_s$

$0.87f_{yk}(A_s - A'_s) = 0.57\lambda\eta f_{ck}bx$ ... Equation(B)

If compression reinforcement needs to added as for option (3) in the singly reinforced section above then the procedure is:

i. reduce x in equation (A) so that it satisfies the equation

ii. provide compression reinforcement $A'_s$ , by re-arranging equation (B)

$A'_s = A_s - 0.65\lambda\eta f_{ck}bx/f_{yk}$

If both $A_s$ and $A'_s$ are known then equation (B) can be re-arranged to determine x:

$x = 1.53f_{yk}(A_s - A'_s) / (\lambda\eta f_{ck}b)$

To meet the criteria for assuming the reinforcement yields then the tension reinforcement can be checked using equation (A).

To ensure the compression reinforcement yields then the strain in the steel $\varepsilon'_s$ needs to be greater than $\varepsilon_{s,yield.}$

From the strain diagram $\varepsilon'_s = \varepsilon_{cu3}(1-d'/x)$ which has to be greater than $0.87f_{yk}/E_s$

This re-arranges to $x/d' \leq 1 / \{1 - [0.87f_{yk}/(E_s\varepsilon_{cu3})]\}$.

If either of these checks fails then the ultimate moment of resistance needs to be determined using the strain compatibility method.

## 4. Shear

a) Members Without Shear Reinforcement

Tests have been carried out on beams with and without shear reinforcement. The results for members without shear reinforcement show a correlation between the concrete strength, the amount of longitudinal tension reinforcement and the depth of the section.

The shear strength ($V_{Rd,c}$) of a section **without** shear reinforcement is give by:

$\mathbf{V_{Rd,c} = [C_{Rd,c}k(100\rho_1 f_{ck})^{1/3} + k_1\sigma_{cp}]b_w d}$ (BS EN 1992-2 Clause 6.2.2(101))

with a minimum value of $\mathbf{V_{Rd,c} = (v_{min} + k_1\sigma_{cp})b_w d}$

where:

$C_{Rd,c} = 0.18/\gamma_c$ (UK NA to BS EN 1992-2 Clause 6.2.2(101))

$k = 1 + \sqrt{(200/d)} \leq 2.0$

$\rho_1 = A_{sl} / b_w d \leq 0.02$

$A_{sl}$ = Area of longitudinal tensile reinforcement at effective depth d

$b_w$ = smallest width of section in the tension zone

$f_{ck}$ = concrete cylinder strength (limited to C50 ~ see UK NA to BS EN 1992-2 Clause 3.1.2(102)P)

$k_1$ = 0.15 (UK NA to EN 1992-2 Clause 6.2.2(101))

$\sigma_{cp} = N_{Ed} / A_c$ (N/mm$^2$)

$N_{Ed}$ = Axial force from any applied loading

$A_c$ = Gross concrete area

$v_{min} = 0.035k^{3/2}f_{ck}^{1/2}$

Note: For sections close to supports the UK National Annex to BS EN 1992-2 Clause 6.2.2(101) allows the value of $C_{Rd,c}$ to be increased to $(0.18/\gamma_c)\bullet(2d/a)$ provided that the shear force $V_{Ed}$ is not multiplied by β (see EN 1992-1-1 Clause 6.2.2 (6)) and the longitudinal reinforcement is fully anchored at the support, where 'a' is the distance from the edge of the support (or centre of bearing where flexible bearings are used) to the position at which the shear resistance is considered.

It is convenient not to put shear links in concrete slabs and if $V_{Rd,c}$ is found to be insufficient then it is usually more efficient to provide additional longitudinal tension reinforcement.

Beams, however, have to be provided with shear links even if $V_{Rd,c}$ is found to be adequate. BS EN 1992-1-1 Clause 9.2.2(5) says:

the minimum shear reinfrocement ratio $\rho_{w,min} = A_{sw} / (s.b_w.\sin\alpha) = (0.08\sqrt{f_{ck}}) / f_{yk}$

Clause 9.2.2(6) says the spacing (s) of the shear links should not exceed $s_{l,max} = 0.75d(1 + \cot\alpha)$

where:

$A_{sw}$ = area of shear reinforcement in length s (one link with 2 legs = 2 x leg area)

s = spacing of links measured along the axis of the member

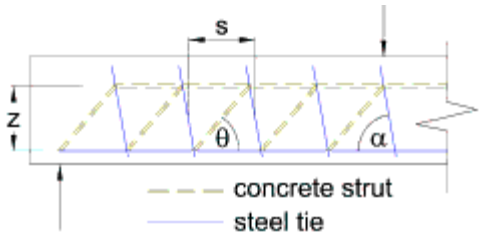
$b_w$ = breadth of the web of the menber

α = the angle between the link and the longitudinal axis of the member

$f_{yk}$ = characteristic yield strength of the reinforcement

b) <u>Members With Shear Reinforcement</u>

Clause 6.2.3 of BS EN 1992-1-1 uses a truss model to evaluate the shear resistance of concrete members with shear reinforcement. The concrete resists the compressive forces whilst the reinforcement resists the tensile forces.




$\theta$ = the angle between the concrete compression strut and the longitudinal axis of the member. Values of $\theta$ between $45^o$ and $21.8^o$ ($1 \leq \cot\theta \leq 2.5$) may be used.

$\alpha$ = the angle between the link and the longitudinal axis of the member. Values of $\alpha$ between $45^o$ and $90^o$ may be used.

By considering vertical equilibrium on a section parallel to the line of the concrete strut it can be shown that the design shear force which can be resisted by the yielding shear reinforcement is:

$V_{Rd,s} = A_{sw}zf_{ywd}(\cot\theta+\cot\alpha)\sin\alpha/s$ ..... [Clause 6.2.3(4) equation (6.13)]

By considering vertical equilibrium on a section perpendicular to the line of the concrete strut it can be shown that the design shear force limited by compression in the concrete is:

$V_{Rd,max} = \alpha_{cw}b_wz\nu_1f_{cd}(\cot\theta+\cot\alpha)/(1+\cot^2\theta)$ ..... [Clause 6.2.3(4) equation (6.14)]

where:

$f_{ywd}$ = design yield strength of the shear reinforcement

$\alpha_{cw}$ = 1.0 for non-prestressed structures

$\nu_1 = 0.6(1-f_{ck}/250)(1-0.5\cos\alpha)$ ..... [UK NA to 1992-2 clause 6.2.3(103)]

Note: If stress of shear reinforcement is below 80% of $f_{yk}$ then $\nu_1 = 0.54(1-0.5\cos\alpha)$

c) <u>Additional Longitudinal Reinforcement</u>

Clause 6.2.3(107) of BS EN 1992-2 requires an additional tensile force $\Delta F_{td} = 0.5V_{Ed}(\cot\theta-\cot\alpha)$ to be resisted by the longitudinal reinforcement due to the force developed from the truss analogy.

The total force in the longitudinal steel due to bending and the truss force = $M_{Ed}/z + \Delta F_{td}$. The total force should not be greater than $M_{Ed,max}/z$ where $M_{Ed,max}$is the maximum moment along the beam.