RELIABILITY-BASED ANALYSIS OF COMPOSITE SOLID SLENDER TIMBER COLUMNS WITH ALUMINIUM LAMINATES USING SELECTED NIGERIAN TIMBER SPECIES

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ZARIA, NIGERIA

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A DISSERTATION SUBMITTED TO THE SCHOOL OF POSTGRADUATE STUDIES, AHMADU BELLO UNIVERSITY, ZARIA, IN PARTIAL FULFILLMENT FOR THE AWARD OF MASTER OF SCIENCE IN CIVIL ENGINEERING

DEPARTMENT OF CIVIL ENGINEERING

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AHMADU BELLO UNIVERSITY,

ZARIA, NIGERIA

March, 2018

DECLARATION

I declare that the work in this Dissertationentitled "**RELIABILITY-BASED ANALYSIS OF COMPOSITE SOLID SLENDER TIMBER COLUMNS WITH ALUMINIUM LAMINATES USING SELECTED NIGERIAN TIMBER SPECIES"** has been carried out by me in the department of Civil Engineering, Ahmadu Bello University, Zaria. No part of the Dissertation has been previously presented for another degree or diploma at this or any other Institution. All quotations are indicated and the sources of the information are appropriately acknowledged by means of references.

Samuel Nagu-Num MANGUT _____ _____

Signature Date

CERTIFICATION

This Dissertation entitled **''RELIABILITY-BASED ANALYSIS OF COMPOSITE SOLID SLENDER TIMBER COLUMNS WITH ALUMINIUM LAMINATES USING SELECTED NIGERIAN TIMBER SPECIES''** by Samuel Nagu-NumMANGUTmeets the regulations governing award of the degree of Master of Scienceof Civil Engineering of Ahmadu Bello University, Zaria and is approved for its Contribution to knowledge and literary presentation.

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DEDICATION

Dedicated to my father, Audu Mangut, and in loving memory of my late mother,Zibiah Alti Mangut,who share in the burdens of my academic pursuits.

ACKNOWLEDGEMENTS

All thanks to God Most High, the father of our Lord Jesus Christ, the giver of wisdom, knowledge and understanding for his innumerable gifts to me through the course of life's journey.

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ABSTRACT

Four timber species (*Strombosia pustulata*, *Macrocarpa biquertii*, *Entandrophragma cylindricum* and *Nauclea diderichii*) were laminated with aluminium sheets of varying thicknesses.Axial and flexural loads were applied to the composite columns to determine their behaviour under such loadings. Finite element analysis was carried out in Abaqus/CAE 6.10 (2010) and the reliability analysis was carried out using the First Order Reliability Analysis (FORM) in MATLAB (2007) on the laminated timber columns. The results for finite element analysis show the effect of the thickness of Aluminium laminates on the strength capacity of timber columns whereby deflection reduced from 3.28×10^{-2}mm for timber column without laminate to 2.96×10^{-2}mm with the introduction of 20mm aluminium laminate in the x-axis. Similarly, the deflection reduced from 5.18×10^{-2}mm to 1.18×10^{-2}mm in the y-axis, and 5.45mm to 1.093mm in the z-axis for the timber (*Strombosia pustulata* specie) column that is fixed-free end restraint condition. For the reliability analysis, bending, buckling and flexural buckling failure modes were considered. The results for compression mode of failure show that the column is safewith safety index values of 4.3 and 9.68 for load parameters and dead-to-live load parameters respectively without laminates. The study shows that the most critical failure mode for the column is flexural buckling and hence isnot safe for imposed loads that are greater than 20kN but made safe with the use of aluminium laminate of 8mm.

TABLE OF CONTENTS

TITLE	PAGE
Cover	
Title Page	i
Declaration	ii
Certification	iii
Dedication	iv
Acknowledgements	v
Abstract	vi
Table of contents	vii
List of Figures	xii
List of Tables	xv
List of Appendices	xvi

CHAPTER ONE: INTRODUCTION

1.1 General 1

1.2 Problem Statement and Justification of Study 4

1.3 Aim and Objectives 4

1.3.1 Aim 4

1.3.2 Objectives 4

1.4 Scope and Limitations of Study 5

1.4.1 Scope of Study 5

1.4.2 Limitations of Study 6

CHAPTER TWO: LITERATURE REVIEW

2.1 Timber Columns 7

2.1.1 Strength classes of timber 8

2.1.2 Strength capacity of solid timber columns 9

2.1.3 Eccentricity of loading and end restraint on timber columns 10

2.1.4 Material properties of timbers 10

2.1.5 Factors affecting strength (mechanical properties) of solid timber 15

2.1.6 Failure modes of timber columns 18

2.2 Aluminium laminates 18

2.2.1 Properties of aluminium 18

2.2.2 Structural applications of aluminium 19

2.2.3 Structural design of aluminium laminates 20

2.2.4 Structural details of aluminium 21

2.2.5 Mechanical properties of aluminium 21

2.3 Lamination Technique of Wood 22

2.4 Buckling Strength of Columns 23

2.5 Effect of End Restraint on Columns 24

2.6 Aluminium Laminated Timber Strengths and Load Action on Composite Sections 24

2.6.1 Compositecolumn strength 24

2.6.2 Load models on a composite column 25

2.7 Reliability of Structures 30

2.7.1 Basic failure modes in a structure 31

2.7.2 Uncertainties in reliability assessment 31

2.7.3 The limit state principle 32

2.7.4 First order reliability methods 32

2.8 Finite Element Analysis 35

2.8.1 Computation of displacements and stress resultants 36

2.8.2 Computation of stresses and strains 38

2.8.3 Finite element analysis with abaqus/CAE 39

CHAPTER THREE: MATERIALS AND METHODS

3.1 Materials 42

3.1.1 Composite timber and aluminium column 42

3.1.2 Axially loaded timber columns 43

3.1.3 MATLAB R2011b 43

3.1.4 Abaqus 44

3.2 Methods 44

3.2.1 General description 44

3.2.2 Finite element method 45

3.2.3 Modelling of composite timber with aluminium laminate 45

3.2.4 Timber properties 46

3.2.5 Aluminium properties 46

3.2.6 Displacement of composite columns subjected to axial loads 47

3.2.7 Modelling in Abaqus/CAE 47

3.2.8 Reliability assessment 52

3.3 Limit State Structural Design Parameters 53

3.4 Limit State Equations for Reliability Analysis 57

3.5 Stochastic Models of the Basic Variables 59

CHAPTER FOUR: DISCUSSION OF RESULTS

4.1 Finite Element Analysis Results 62

4.2 Reliability Analysis Results 82

4.2.1 Results for Compression Mode of Failure 82

4.2.2 Results for bending mode of failure 84

4.2.3 Results for flexural buckling mode of failure 86

CHAPTER FIVE: CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion 89

5.2 Recommendations 90

REFERENCES 92

LIST OF FIGURES

FIGURE		PAGE
2.1	Sectional Properties of Laminate Timber Column for Bending Failure Analysis	27
2.2	Failure Functions in the Variable Space	33
3.1	Sequence Stages for Achieving the Composite Cross Section	42
3.2	Typical Grain Direction of a Timber Column	43
3.3	Creating an Abaqus Native Part	48
3.4	Timber Column before Property Assignment and After Property Assignment	49
3.5	Assembly of Timber Column and Aluminium Laminate	49
3.6	Interaction between Timber Column andAluminium Laminate	50
3.7	Load Application and Boundary Conditions	50
3.8	Seeded and Meshed Timber/Aluminium Composite Column	51
3.9	Visualization of Abaqus/CAE Result	52
3.10	Pitched Portal Frame	54
3.11	Forces and Moments Diagrams on the Pitched Portal Frame	54
3.12	Plan of Portal Frame	55
3.13	Horizontal Reactions	55

4.1 Member-End Force for Fixed-Fixed Timber Column with No Aluminium laminates 62

4.2 Member-End Force for Fixed-Fixed Timber Column with 4mm Aluminium laminates 62

4.3 Member-End Force for Fixed-Fixed Timber Column with 6mm Aluminium laminates 63

4.4 Member-End Force for Fixed-Fixed Timber Column with 8mm Aluminium laminates 63

4.5 Member-end force for fixed-free timber column with no laminate 64

4.6 Member-end force for fixed-free timber column with 4mm laminates 64

4.7 Member-end force for fixed-free timber column with 6mm laminate 65

4.8 Member-End Force of Fixed-free Timber Column with 8mm Aluminium laminate 65

4.9 Member-End Force of pinned-pinned Timber Column with No Aluminium laminate 66

4.10 Member-End Force of pinned-pinned Timber Column with 4mm Aluminium laminate 66

4.11 Member-End Force of pinned-pinned Timber Column with 6mm Aluminium laminate 67

4.12 Member-End Force of pinned-pinned Timber Column with

	8mm Aluminium laminate	67
4.13	Stress in Fixed-Fixed Timber Column with No Aluminium laminates	68
4.14	Stress in fixed-fixed timber column with 4mm Aluminium laminate	68
4.15	Stress in fixed-fixed timber column with 6mm Aluminium laminate	69
4.16	Stress in fixed-fixed timber column with 8mm Aluminium laminate	69
4.17	Stress in fixed-free timber column with no laminate	70
4.18	Stress in fixed-free timber column with 4mm Aluminium laminate	70
4.19:	Stress in fixed-free timber column with 6mm Aluminium laminate	71
4.20	Stress in fixed-free timber column with 8mm Aluminium laminate	71
4.21	Stress in pinned-pinned timber column with no laminate	71
4.22	Stress in pinned-pinned timber column with 4mm Aluminium laminate	72
4.23	Stress in pinned-pinned timber column with 6mm Aluminium laminate	73
4.24:	Stress in pinned-pinned timber column with 8mm aluminium laminate	73
4.25	Deflection in X-Axis versus Laminate Thickness for *Strombosia pustulata*	74
4.26	Deflection in Y-Axis versus Laminate Thickness for *Strombosia pustulata*	75
4.27	Deformation in Z-Axis versus Laminate Thickness for *Strombosia pustulata*	75
4.28	Stress Column versus Laminate Thicknesses for *Strombosia pustulata*	75

4.29 Deflection in X-Axis versus Laminate Thickness for *Macrocarpa bequertii* 76

4.30 Deflection in Y-Axis versus Laminate Thickness for *Macrocarpa bequertii* 77

4.31 Deformation in Z-Axis versus Laminate Thickness for *Macrocarpa bequertii* 77

4.32 Stress in Composite Column versus Laminate Thickness for *Macrocarpa bequertii* 78

4.33 Deflection in X-axis versus Laminate Thickness for *Nauclea diderrichii* 78

4.34 Deflection in Y-axis versus Laminate thickness for *Nauclea diderrichii* 78

4.35 Deformation in Z-Axis versus Laminate Thickness for *Nauclea diderrichii* 79

4.36 Stress in Composite Column versus Laminate Thickness for *Nauclea diderrichii* 79

4.37 Deflection in X-axis versus Laminate Thickness for *Entandrophragma cylindricum* 79

4.38 Deflection in Y-axis versus Laminate Thicknessfor *Entandrophragma cylindricum* 80

4.39 Deformation in Z-axis versus Laminate Thicknessfor *Entandrophragma cylindricum* 80

4.40 Stress in Composite Column versus Laminate Thickness for *Entandrophragma cylindricum* 80

4.41 Comparison of displacement of four timber species 81

4.42 Safety index versus Laminate thickness at various loads for *Strombosia pustulata* 82

4.43 Safety index versus Laminate thickness at varying load ratios for

Strombosia pustulata 83

4.44 Safety index versus Laminate thickness for test species at design parameters 83

4.45 Safety Index versus Laminate Thickness at Various Loads for *Strombosia Pustulata* 84

4.46 Safety index versus Laminate thickness at various Load ratios for *Strombosia pustulata* 85

4.47 Safety index versus Laminate thickness at various column heights for *Strombosia pustulata* 86

4.48 Safety index versus Laminate thickness four timber species using Standard design parameters 86

4.49 Safety index versus Laminate thickness at various loads for *Strombosia pustulata* 87

4.50 Safety index versus Laminate thickness at dead-to-live loads ratios for *Strombosia pustulata* 87

4.51 Safety Index versus Laminate Thickness with Varied Column Height 87

LIST OF TABLES

TABLE		PAGE
2.1	ResultsofCharacteristicValuesofOtherMaterialPropertiesBasedon18%Mc	13
2.2	SummaryofResults ofPhysical andMechanicalPropertiesofSelectedTimbers	14
3.1	Stochastic Parameters for Compression Failure Mode	60
3.2	Stochastic Parameters for Bending Failure Mode	60
3.3	Stochastic Parameters for Flexural Failure Mode	61

LIST OF APPENDICES

APPENDIX		PAGE
A	PROGRAM LISTING	96
B1	Deflection of *Strombosia* in X-Axis for the Three End-Restraint Conditions	102
B2	Deflection of *Strombosia* in Y-Axis for the Three End-Restraint Conditions	102
B3	Deflection of *Strombosia* in Z-Axis for the Three End-Restraint Conditions	102
B4	Stress in *Strombosia* for the Three End-Restraint Conditions	103
B5	Deflection of *Macrocarpa* in X-Axis for the Three End-Restraint Conditions	103
B6	Deflection of *Macrocarpa* in Y-Axis for the Three End-Restraint Conditions	103
B7	Deflection of *Macrocarpa* in Z-Axis for the Three End-Restraint Conditions	104
B8	Stress in *Macrocarpa* for the Three End-Restraint Conditions	104
B9	Deflection of *Nauclea* in X-Axis for the Three End-Restraint Conditions	104
B10	Deflection of *Nauclea* in Y-Axis for the Three End-Restraint Conditions	105
B11	Deflection of *Nauclea* in Z-Axis for the Three End-Restraint Conditions	105

B12 Stress in *Nauclea* for the Three End-Restraint Conditions 105

B13 Deflection of *Entandophragma* in X-Axis for the Three End-Restraint Conditions 106

B14 Deflection of *Entandophragma* in Y-Axis for the Three End-Restraint Conditions 106

B15 Deflection of *Entandophragma* in Z-Axis for the Three `End-Restraint Conditions 106

B16 Stress in *Entandophragma* for Fixed-Free, Pinned-Pinnedand Fixed-Fixed 107

CHAPTER ONE

INTRODUCTION

1.1 General

Timber, as one of the materials used in construction, is a sustainable resource (Porteous and Kermani, 2007). It is one of the few natural and renewable construction materials that existand is used for construction, carpentry and upholstery. Also, timber is an organic material and thus is subject to deterioration with time (Robert, 2010). These limitations are basically related to the importance of trees in the ecosystem and how felling trees could exacerbate the problems of ozone layer depletion. Trees take very long time to be fully grown and mature, hence optimum use of timber will ensure adequate balance in logging and replacement. Timber from well-managed forests is one of the most sustainable resources available and it is one of the oldest known materials used in construction. It has a very high strength-to-weight ratio, it is capable of transferring both tension and compression forces, and is naturally suitable as a flexural member. Timber is a material that is used for a variety of structural forms such as beams, colums, trusses, girders, and is also used in building systems such as piles, deck members, railway sleepers and in formwork for concrete (Porteous and Kermani, 2007).

Extensive research over the past few decades has resulted in comprehensive information on material properties of timber and its reconstituted and engineered products and their effects on structural design and service performance (Gentile, 2000; Mamman, 2014; Mohammed, 2014; Abubakar and Nabade 2013b; Avent, 1985; Forest Product laboratory, 1999). Centuries of experience of use of timber in buildings has shown the safe methods of construction, connection details and design limitations.

There are a number of inherent characteristics that make timber an ideal construction material. These include its high strength-to-weight ratio, its impressive record for durability and performance and good insulating properties against heat and sound. Timber also benefits from its natural growth characteristics such as grain patterns, colours and its availability in many species, sizes and shapes that make it a remarkably versatile and an aesthetically pleasing material. Timber can easily be shaped and connected using nails, screws, bolts and dowels or adhesively bonded together. Timber structures can be highly durable when properly treated, detailed and built. They can easily be reshaped or altered, and if damaged they can be repaired (Porteous and Kermani, 2007; Hollaway and Cadei, 2002).

The limitations in maximum cross-sectional dimensions and lengths of solid sawn timbers, due to available log sizes and natural defects, are overcome by the recent developments in composite and engineered wood products (APA, 2009; Zahn and Rammer, 1995). Finger jointing and various lamination techniques have enabled timbers (elements and systems) of uniform and high quality in any shape, form and size to be constructed; being only limited by the manufacturing and/or transportation boundaries. Solid timber is rapidly becoming scarce and expensive due to logging and the long period of time it takes for most trees to grow to maturity.Aluminium, on the other hand, is a lightweight and durable metal. It is an abundant element found in the earth crust. It is the third most abundant element and the most abundant metal in existence. It is silvery in appearance when freshly cut, is a good conductor of heat and electricity and is easily shaped by moulding and extruding. The extraction process of aluminium is electrolysis (Grjotheim and Welch, 1988) which enables large amounts of the metal to be available for construction purposes.Aluminium has two main advantages when compared with other metals. Firstly, it has a low density, about one third that of iron and copper.

Secondly, although it reacts rapidly with the oxygen in air, it forms a thin tough and impervious oxide layer which resists further oxidation. This removes the need for surface protection coatings such as those required with other metals, in particular with iron.

There are many different timbers in the market that range in price, characteristics and strength. Timber is an excellent choice for any sort of woodwork but good quality timber with minimum flaws comes with a bit extra cost due to the reasons above. Hence, there is need to maximally use timber for efficiency in construction and to reinforce the slender timber columns withaluminium laminates to increase the stiffness and sectional properties of the slender columns.

Traditionally, in Nigeria, timbers have been used to a significant extent in construction purposes and particularly building constructions. Over 80% of the timber and timber products in Nigeria are utilized for different purposes (Ohagwu and Ugwuishiwu, 2011). The major uses of timber in building construction are roofing members, doors, frames, and staircases. They are also used for scaffolding and shuttering during construction. In road construction, large quantities of woods are also used for frameworks, pilling materials, road signboards, temporary shades, and road paving plants in temporary construction site. They are also used at petroleum exploitation sites (Ohagwu and Ugwuishiwu, 2011). However, the structural strength of solid timbers needs to be increased in order to sustain more structural loads. This can be done by using aluminium laminates to increase its strength and durability by increasing the stiffness and sectional properties of the timber. This is the work presented in this study and including the application of probability in order to obtain its optimum resistance strength in analysis and design.

1.2 Statement of Problem and Justification of Study

Trees are of immense importance in the ecosystem and felling of trees could exacerbate the problems of ozone layers depletion (Akimbo and Lawrence, 1996). The limitations in maximum cross-sectional dimensions and lengths of solid sawn timbers, due to available log sizes and natural defects make the use of solid timber to be limited to small construction and structural works. Solid timber is rapidly becoming expensive due to excessive logging and the long period of time it takes for most trees to grow to maturity.

Timber is a highly sustainable construction material that has high-strength-to weight ratio as compared to other construction materials, the environmental impact of processing and milling timber for construction purpose is at a low level due to the low energy use and low level off pollution associated with the manufacturing of timber. Timber has very great aesthetic properties owing to its natural growth characteristics such as grain patterns. Also, timber structures can be highly durable when properly treated, detailed and built. It is for these reasons that it becomes necessary to study the behaviour of timber laminated with aluminium laminates.

1.3 Aim and Objectives

1.3.1 Aim

The aim of this work is to carry out reliability-based evaluation of the structural performance of composite solid slender timber columns of some selected Nigerian timber species with aluminium laminates.

1.3.2 Objectives

The objectives are to:

i. Generate the limit state equations for columns according to Eurocode 5 (2004) requirements for reliability analysis,

ii. Develop a program for reliability analysis in MATLAB R2011b (2011) for the composite columns,

iii. Carry out finite element analysis, using Abaqus, of the composite columns under axial loads for different end restraint conditions,

iv. Check the structural performance of the columns considering the properties of some selected Nigerian timber species under various failure criteria, and

v. Study the effects of aluminium laminate thickness on the load carrying capacity of the columns.

1.4 Scope and Limitations of Study

1.4.1 Scope of study

This study is based on the modification of the strength of solid timber columns with the introduction of aluminium laminates. The strength variation of the timber is assessed to determine the optimum strength with varying aluminium laminate thickness. A finite element analysis of the four timber species laminated with aluminium sheets have been carried out in Abaqus CAE 6.10 (2010) to determine the effect of the laminate on the structural capacity of solid timber columns. Also, reliability analysis of solid timber columns laminated with aluminium sheet of varying thickness were carried out considering certain target reliability indices values. The reliability processes were carried out considering three failure modes which are bending, buckling and flexure. MATLAB R2011b (2011) was used to run the First Order Reliability Method analyses incorporating programs that were designed for the three failure modes. The selected Nigerian timbers used in this study are *StrombosiaPustulata, MacrocarpaBequerti, Nauclea Diderrichii* and *Entandrophragma Cylindricum.*

1.4.2 Limitations of study

This study is limited to the use of four selected Nigerian timbers which are *StrombosiaPustulata, MacrocarpaBequerti, Nauclea Diderrichii* and *Entandrophragma Cylindricum*with their local names as Itako, Oporoporo, Opepe and Ijebu respectively. The timbers are used as slender columns which are laminated with varying aluminium laminate thickness and end-restraint condition viz-a-viz fixed-free, fixed-fixed and pinned-pinned end conditions.

CHAPTER TWO

LITERATURE REVIEW

2.1 Timber Columns

Timber columns are structural members that are subjected to axial loadings and in some situations, combined axial and bending stresses (as in the case of beam-columns). Timber columns function mainly as compression members (Malhotra, 1980) and have their uses as columns supporting floors, beams and girders, struts in roof trusses and bridge girders, round poles serving as bridge piers, piles for supporting foundations, bracings for columns, and as the framework for wooden stud walls. Columns which are slender (having slenderness ratio greater than 160) fail by buckling either before or after the elastic limit has been reached depending on the proportions of the column. The behaviour and design of timber columns has been the subject of research for many years.

Leicester and Pearson (2012) established that the concepts of the creep buckling theory were effective in timber columns. Thus the behaviour of timber columns subject to compressive stresses would conform to the Euler buckling stresses. The availability of long duration measurements of column strength under various deflection modes were used to obtain their findings through the stated guidelines in the Australian Standard AS1720.1 (1988).

Pearson (1954) investigated the effects of species, slenderness ratio, eccentricity of load and orientation of growth rings on the strength of solid timber columns. Over 400 specimens were tested, including more than 250 eccentrically loaded columns. Slenderness (L/d) ranged from 5 to 50 with eccentricity to depth ratio of 0 to 0.5. Based on the test results, a modification of Jezek's formula was proposed for calculating the maximum strength of eccentrically loaded columns.

2.1.1 Strength classes of timber

The design properties of timber are determined non-destructively through visual strength grading criteria or by machine strength grading via measurements such as the following (Porteous and Kermani, 2007, Robert 2010): flatwise bending stiffness, using a three-point loading system; density, using x-rays or gamma rays techniques; and modulus of elasticity, by means of resonant vibrations (dynamic response) using one or a combination of these methods. Timber has great strength properties coupled with its fire resistance and long durations when properly treated (BS EN 1995-1-2:2004, Mohammed, 2014). In tests, woods such as the spruce of Douglas fir fail at a tensile stress well in the excess of 100N/mm^2. The same woods as larger elements fail in compression at 40N/mm^2 to 50N/mm^2. NCP 2(1973) gives the strength grading of different timbers found in Nigeria. The requirements for strength grading of other applicable solid timbers are detailed in the following standards: BS EN 14081-1 (2005), BS EN 14081-2 (2005) and EN 338 (2007), which also gives the value of deformation modification factors, k_{def}. Most European Union countries have their own long-established visual grading rules and as such guidance for visual strength grading of softwoods and hardwoods is provided in the following British Standards: BS 4978 (1996) and BS 5756 (1997)

The concept of grouping timber into strength classes was introduced into the United Kingdom with BS 5268-2 in 1984. BS EN 338:2003 defines a total of 18 strength classes: 12 for softwoods – C14, C16, C18, C20, C22, C24, C27, C30, C35, C40, C45 and C50; and six for hardwoods – D30, D35, D40, D50, D60 and D70. The letters C and D refer to coniferous species (C classes) or deciduous species (D classes), and the number in each strength class refers to its 'characteristic bending strength' in N/mm^2 units; for example, C40 timber has a characteristic bending strength of 40 N/mm^2. It

ranges from the weakest grade of softwood, C14, to the highest grade of hardwood, D70, often used in Europe.

The Nigerian code of practice for use of timber (NCP 2, 1973) for construction has specified and designated seven (7) strength groups: N1, N2, N3, N4, N5, N6 and N7, with N1 depicting strength class for timbers that have the greatest structural properties and N7 having timbers that have the least strength properties.

2.1.2 Strength capacity of solid timber columns

Olatunji and Longworth (1983) studied the ultimate strength of timber columns. Their study shows that timber has an average yield stress of 48MPa which is also the ultimate strength using an elasto-plastic stress-strain curve. In the experiments, they found out that the value of ratio of yield strain to ultimate strain obtained was 0.864. The average values in tension were 62MPa for ultimate stress, 0.0032mm/mm for ultimate strain and 20,800MPa for the modulus of elasticity.

Abubakar and Nabade (2013b) carried out experiments to determine the physical and mechanical properties of some Nigerian timber species. Their results show that Nigerian timbers have average compressive strengths of 29.58N/mm^2 for *Strombosia Pastulata*, 20.82N/mm^2 for *Macrocarpa biquertii*, 27.18N/mm^2 for *Nauclea Diderrichii* and 24.16N/mm^2 for *Entandrophragma Cyclindricum*. Recommendations based on the findings made say that *Macrocarpa* should not be used for high bearing properties.

Malhotra (1988) investigated the strength of solid and built-up timber compression members. The efficiencies of various types of built-up timber columns with different spacing of connectors were compared and it was realized that the efficiencies of the built-up columns first declined with increase in slenderness and then increased from a slenderness of 80.

2.1.3 Eccentricity of loading and end restraint on timber columns

For a centrally loaded timber column, the Euler-Engesser tangent modulus buckling load is regarded as the load the ideal column can carry without too large a deflection and it's taken as the basis for the design of centrally loaded columns (Malhotra and Manzur, 1970).Eccentrically loaded timber columns are analyzed by the secant formula, modified secant formula and Perry-Robertson formula. A critical column load is determined at which increase of lateral deflections takes place without any increase of load.

Malhotra (1984) tested 560 timber columns of two cross-sectional dimensions, 38mmx 89mm and 64mmx 89mm, for values of eccentricities of loads. All columns were tested with pinned-end condition. He observed that an eccentricity has smaller effect on the strength of long columns than that of short and intermediate columns.

2.1.4 Material properties of timbers

2.1.4.1 Physical properties

Physical properties are the quantity characteristics of wood and its behaviour to external influences other than applied forces (Abubakar and Nabade, 2013b). Solid timbers are timbers that have not been engineered to meet up with particular structural and architectural requirements. They are timbers that depend on their natural strength capacities to serve the purpose for which they are used. The physical properties of timber affect these structural properties that are needed. Density and moisture content are important physical characteristics of timber that affect its strength. The value of density is an indicator of mechanical properties (Porteous and Kermani, 2010).

2.1.4.2 Mechanical properties

Section 3 of BS EN 1995-1-1:2004 deals with the material properties and defines the strength and stiffness parameters, stress–strain relations and gives values for

modification factors for strength and deformation under various service classes and/or load duration classes. The characteristic values are defined as the population 5th-percentile values obtained from the results of tests with duration of approximately 5 min at the equilibrium moisture content of the test pieces relating to a temperature of 20^0C and a relative humidity of 65%.

In addition to providing characteristic strength and stiffness properties and density values for each strength class (and the rules for allocation of timber populations, i.e. combinations of species, source and grade, to the classes), BS EN 338:2003 and EN 408 (2004) list the equations that form the relations between some of the characteristic values for properties other than bending strength, mean modulus of elasticity in bending and density.

The relationships between the characteristic strength and stiffness properties as stated in Eurocode 5 (2004) and EN384 (2004) are given as follows:

i. Tensile strength parallel (0) to grain, $f_{t,0,k} = 0.6\ f_{m,k}$
ii. Compression strength parallel (0) to grain, $f_{c,0,k} = 5(\ f_{m,k})^{0.45}$
iii. Shear strength, $f_{v,k}$= minimum of {3.8 and $0.2(\ f_{m,k})^{0.8}$}
iv. Tensile strength perpendicular (90) to grain, $f_{t,90,k}$ = minimum of {0.6 and $0.0015\rho_k$}
v. Compression strength perpendicular (90) to grain,

$$f_{c,90,k} = 0.007\rho_k \text{ for softwoods}$$

$$f_{c,90,k} = 0.015\rho_k \text{ for hardwoods}$$

vi. Modulus of elasticity parallel (0) to grain,

$$E_{0.05} = 0.67E_{0,\ mean} \text{ for softwoods}$$

$$E_{0.05} = 0.84E_{0,\ mean} \text{ for hardwoods}$$

vii. Mean modulus of elasticity perpendicular (90) to grain,

$$E_{90,\,mean} = E_{0,\,mean}/30 \text{ for softwoods}$$

$$E_{90,\,mean} = E_{0,\,mean}/15 \text{ for hardwoods}$$

viii. Mean shear modulus, $G_{mean} = E_{0,\,mean}/16$.

Timber has good mechanical properties which are: elastic properties (moduli of elasticity *E*, moduli of rigidity *G*, Poisson's ratios), hardness, tensile strength parallel to grain,compressive strength parallel to grain, modulus of rupture sustained by a compression parallel-to-grain, compressive stress perpendicular to grain, shear strength parallel to grain. The studies carried out by Abubakar and Nabade (2013a) and Abubakar and Nabade (2013b) show a summary of the mechanical and physical properties of some selected timber species found in Nigeria. They have been reproduced here as Tables 2.1 and 2.2.

TABLE2.1
ResultsofCharacteristicValuesofOtherMaterialPropertiesBasedon18%MC (Abubakar and Nabade, 2013a)

MaterialProperties	TimberSpe			
	Strombosiapustulata	*Macrocarpabequertii*	*Naucleadiderrichii*	*Entandrophragmacyclindricum*
TensionParallel $f_{t,0,k}$(N/mm2)	31.18	14.28	25.84	19.87
Tension Perpendicular$f_{t,90,k}$(N/mm2)	0.6	0.4	0.6	0.6
Compression Parallel$f_{c,0,k}$ (N/mm2)	29.58	20.82	27.18	24.16
Compression Perpendicular$f_{c,90,k}$(N/mm2)	8.63	2.18	10.46	7.35
ShearStrength$f_{v,k}$(N/mm2)	4.0	3.0	4.0	4.0
5%MOEParallel $E_{0,05}$(kN/mm2)	9.18	4.95	8.42	7.36
MeanMOEPerpendicular $E_{90,mean}$((KN/mm2)	0.73	0.25	0.67	0.58
MeanShearModulusG_{mean} (kN/mm2)	0.68	0.46	0.63	0.55
MeanDensity ρ_{mean} *(kg/m^3)*	690	373	838	588

TABLE 2.2SummaryofResults ofPhysical andMechanicalPropertiesofSelectedTimbers (Abubakar and Nabade, 2013a)

TimberSpecies	*Strombosiapustulata*	*Macrocarpabiquertii*	*Naucleadiderrichii*	*Entandrophragmacyclindricum*
No.ofspecimensforbending&MOE/MC	40/15	40/15	40/15	40/15
MeasuredMC(%)	23.27	24.81	23.22	20.06
Char.Density@ measuredMC (kg/m^3)	590.60	321.31	716.04	494.65
Adj.Char.Densityto 18%MC (kg/m^3)	575	311	697	490
3pt.Char.Bendingstrength (N/mm^2)	50.6	25.7	42.1	40.0
4pt.Char.Bendingstrength (N/mm^2)	44.9	19.0	36.4	31.1
Adj.3pt.Char.Bendingstrengthto18%MC (N/mm^2)	59.91	32.20	49.73	42.51
Adj.4pt.Char.Bendingstrengthto18%MC (N/mm^2)	51.97	23.80	43.06	33.12
3pt.Char.MOE (kN/mm^2)	12.50	6.88	11.11	9.92
4pt.Char.MOE(kN/mm^2)	10.00	6.67	9.28	8.50
Adj.3pt.Char.MOEto18%MC	13.471	7.410	12.001	10.219
Adj.4pt.Char.MOEto18%MC	10.925	7.392	10.029	8.357
Tensionparallel to grain (kN)	31.18	14.28	25.84	19.87
Tensionperp. To grain (kN)	0.6	0.4	0.6	0.6
Compressionparallel to grain(kN/mm^2)	29.58	20.82	27.18	24.16
Compressionperp. Grain(kN/mm^2)	8.63	2.18	10.46	7.35
Shearstrength (kN/mm^2)	4.0	3.0	4.0	4.0
5%MOEparallel(kN/mm^2)	9.18	4.95	8.42	7.36
MeanMOEperp.(kN/mm^2)	0.73	0.25	0.67	0.58
Meanshearmodulus(kN/mm^2)	0.68	0.46	0.63	0.55
MeasuredMeandensity (kg/m^3)	690	373	838	588

2.1.5 Factors affecting strength (mechanical properties) of solid timber

Timber is an organic material and thus is easily affected by many natural factors including water, insect and fungal attacks, which subject timber to deteriorate with time. These factors influence the strength of timber and hence alter the durability and design life of timber. These causes are well taken care of by the proper conversion of lumber to timber, careful drying and adequate preservative treatment of the timber with appropriate preservative measure. Many factors must be considered when designing and constructing wood buildings, including structural, insulation, moisture, and sound control. Some of these factors are considered in the following subsections.

2.1.5.1 Heartwood and sapwood

The wood laid on late in the life of a tree is softer, lighter, weaker, and more even-textured than that produced earlier. It follows that in a large log the sapwood, because of the time in the life of the tree when it was grown, may be inferior in hardness, strength, and toughness to equally sound heartwood from the same log.

2.1.5.2 Weightand specific gravity

The crushing strength parallel to the grain, fibre stress at elastic limit in bending, and shearing strength along the grain of wood vary in direct proportion to the weight of dry wood per unit of volume when green. Other strength values follow different laws. The hardness varies in a slightly greater ratio than the square of the density. The modulus of rupture in bending lies between the first power and the square of the density. This, of course, is true only in case the greater weight is due to increase in the amount of wood substance.

2.1.5.3 Cross grain

Cross grain is a very common defect in timber. One form of it is produced in lumber by the method of sawing and has no reference to the natural arrangement of the wood elements. Thus if the plane of the saw is not approximately parallel to the axis of the log the grain of the lumber cut is not parallel to the edges and is termed diagonal. This is likely to occur where the logs have considerable taper, and in this case may be produced if sawed parallel to the axis of growth instead of parallel to the growth rings.

Lumber and timber with diagonal grain is always weaker than straight-grained material, the extent of the defect varying with the degree of the angle the fibres make with the axis of the stick. In the vicinity of large knots the grain is likely to be cross. The defect is most serious where wood is subjected to flexure, as in beams.

2.1.5.4 Knots

Knots are portions of branches included in the wood of the stem or larger branch. Knots materially affect checking and warping, ease in working, and cleavability of timber. They are defects which weaken timber and depreciate its value for structural purposes where strength is an important consideration. The weakening effect is much more serious where timber is subjected to bending and tension than where under compression. The extent to which knots affect the strength of a beam depends upon their position, size, number, direction of fibre, and condition. A knot on the upper side is compressed, while one on the lower side is subjected to tension.

Extensive experiments by the U.S. Forest Service (2010) indicate the following effects of knots on structural timbers:

(1) Knots do not materially influence the stiffness of structural timber.

(2) Only defects of the most serious character affect the elastic limit of beams. Stiffness and elastic strength are more dependent upon the quality of the wood fibre than upon defects in the beam.

(3) The effect of knots is to reduce the difference between the fibre stress at elastic limit and the modulus of rupture of beams. The breaking strength is very susceptible to defects.

(4) Sound knots do not weaken wood when subject to compression parallel to the grain.

2.1.5.5 Moisture content

The general effect of the water content upon the wood substance is to render it softer and more pliable. A similar effect of common observation is in the softening action of water on rawhide, paper, or cloth. Within certain limits the greater the water content the greater its softening effect.

Drying produces a decided increase in the strength of wood, particularly in small specimens. The greatest increase due to drying is in the ultimate crushing strength and strength at elastic limit in endwise compression; these are followed by the modulus of rupture, and stress at elastic limit in cross-bending, while the modulus of elasticity is least affected. For air-dry wood the ratios are considerably lower, particularly in the case of the ultimate strength and the elastic limit. Stiffness (within the elastic limit), while following a similar law, is less affected. In the case of shear parallel to the grain, the general effect of drying is to increase the strength, but this is often offset by small splits and checks caused by shrinkage.

2.1.6 Failure modes of timber columns

Timber, like other structural materials fail under excessive loads. The determination of these failure loads would lead to a safe design of structures which would adequately transmit applied loads on timber structures. Such modes of failure are in wide array but for the sake of this study, the failure modes of timber columns are considered which are:

i. Material failure (crushing)
ii. Elastic buckling (Euler)
iii. Inelastic buckling (combination of buckling and material failure)

2.2 Aluminium Laminates

Laminates are structural elements that are wrapped around main structural components for the purpose of protection against adverse conditions that can undermine the structural capacity of a structure. Laminates also add to the strength of the structure by increasing the stiffness and the geometrical properties of the structural elements/composites with which they jointly form composites.Aluminium's extrudability makes aluminium available in various intricate sections and shapes thus enhancing flexibility in design. Some properties of aluminium make it on the disadvantage side for construction. These necessitate greater details in design to cater for such properties. These properties are: low modulus of elasticity which is one-third that of steel, low mechanical properties hence greater fatigue, low melting point and higher coefficient of expansivity which demands wider expansion joints and electrolytic attack from other materials including steel and copper.

2.2.1 Properties of Aluminium

Themainpropertiesofaluminium(Mazzolani,1985b,1998, 1999,2003, 2004) which make it possible for aluminium to be used as a structural element are:

i. Lightness: the unit mass of aluminium is $\gamma = 2700kg/m^3$, which is equal to one-third that of steel;

ii. Corrosion resistance: the exposed surface of aluminium combines with oxygen to form a thin inert aluminium oxide (Al_2O_3) film which blocks further oxidation, contrary to steel which must always be corrosion protected in any kind of environment.

Aluminium is combined with other elements and metals to improve on its structural properties.

Fromthepointofviewofmechanicalresistance,aluminiumalloysseriesformabigfamily ofmaterials,wheretheelasticlimit widelyvariesfrom30Nmm^{-2} (pure aluminium)to500Nmm^{-2} (AlZnMgCualloy)andthe ultimateelongationinmanycases,butnotalways,liesinasuitableoratleastacceptablerangefors tructuralapplications.

Themainmechanicalproperties of aluminiumare:elasticlimitoryieldstress,ultimatestrength,Young'smodulus (E): 70,000N/mm^2, ultimate elongation (ε), specific weight (γ):27,000N/m^3, thermal elongation coefficient (α): 23x10^{-6} per ^{o}C and Poisson Ratio (ν): 0.3 (Mazzolani, 2008, BS EN 1999:2007).

2.2.2 Structural applications of aluminium

The structuralapplicationswhichfit structural purposesin the fieldofcivil engineeringarethefollowing:

a. Long-span roofsystemsinwhichliveloadsaresmallcomparedwithdeadloads.

b. Structures located ininaccessible placesfar from thefabrication shop,for whichtransport economy and ease oferection are ofextreme importance.
c. Structures situated incorrosive or humid environments such as swimming pool roofs, river bridges(Mazzolani &Mele, 1997; Mazzolani, 2001b)
d. Structures having moving parts, such as sewage plant, crane bridges (Mazzolani, 1985a) and moving bridges, where lightnessmeaneconomy of power under service.
e. Structuresforspecialpurposes forwhichmaintenanceoperationsareparticularly difficult and mustbelimited, asincaseof masts, lightingtowers,antennastower.

2.2.3 Structural design of aluminiumlaminates

The basic principles for structural design with aluminium are similar to those for design with steel. Consideration needs to be made for the reduced elastic modulus of aluminium, which means reduced buckling strength and stiffness. In aluminium the strength of welds and the adjacent heat-affected-zone are significantly less than in the base metal. Design codes generally address the issue of weaker welded properties by designing all of the structure for this reduced strength.

A formula for the buckling strength of a plate panel in compression is given by Bleich and Ramsey (1951) and BS EN 1999: 2007 as:

$$\sigma_c = \frac{\pi^2 E\eta}{12(1-\nu)}\left(\frac{t}{s}\right)^2 \qquad (2.1)$$

Where:

σ_c = critical plastic buckling strength,

E = Elastic modulus,

ν = Poisson ratio,

η = a modulus factor dependant on the aspect ratio and the shape of the stress-strain curve,

t = thickness of plate,

s = width of plate, and

K = a factor that depends on the aspect ratio, the condition of support at the edges, and the condition of loading on the edges.

2.2.4 Structural details of aluminium

Many of the same structural problems are posed in aluminium structure as in steel structure for details such as intersections of structural members or avoidance of discontinuities. In many cases the solution to the problem, the structural detail selected, will be the same in aluminium as in steel. The design of structural timber is carried out based on the procedures outlined in BS EN 1999: 2007.

Opportunities in aluminium, particularly for the ease with which unique structural shapes can be extruded, lead to structural details that are unique to aluminium structure. The result is generally lighter structure at a reduced total cost. However, such details often have discontinuities for which detailed stress analysis, including fatigue analysis should be performed, and fatigue testing of such details is needed.

2.2.5 Mechanical properties of aluminium

The elastic modulus of aluminium is about one-third that of steel, as is the density. These properties vary slightly between alloys and are necessary for structural design. Unlike many steel alloys, aluminium alloys do not have a defined yield point. Rather, there is a gradual increase in strain rate with added stress. The yield strength, or proof stress, is defined by a 0.2 percent offset of the engineering stress-strain curve from testing of a

tensile specimen.Aluminium has an elastic modulus 1/3 that of steel, but its density is also 1/3 that of steel.

2.3 Lamination Technique of Wood

Decher et al (2012) and Choi et al (2008) carried out a research on the effect of fibre reinforced polymer lammelae on the load bearing capacity of timber beams. Wood planks were interposed with Glass Fibre Reinforced Lammelae (GFRP) which possess physic-chemical properties similar to wood and has complementary mechanic properties to wood. It was discovered that the application of modern GFRP system with relatively simple techniques gives added value to timber structural and architectural points of view. Good results were obtained in terms of strength and stability requirements.

Timber fibres have axial orientation (isotropic) which makes them bear axial loads efficiently. But for slender timber columns, there is need for increasing load bearing capacity against buckling.

The procedure of obtaining such a hybrid cross section follows some steps as given by Decher et al (2012) and presented thus:

a) Wood surface preparation by clearing away dirt or de-fibrated zones resulted from the cutting off process, ensuring a healthy surface

b) Impregnation of wood surface with polyester resin, which constitutes the primer, and the start of gel formation;

c) Application of the second layer of resin and of successively impregnated glass thread strips until the desired glue thickness strip is reached;

d) Positioning the aluminium strip

e) Finally, the whole packet is pressed, for obtaining the optimal contact between materials.

This packet forms a module with different strength properties, superior to wood. For a certain structural element this module will repeat until reaching the necessary strength, without extremely increasing of cross-section`s dimensions.

2.4 Buckling Strength of Columns

Long columns fail by buckling at stress levels that are below the elastic limit of the column material. Buckling in long, slender columns is due to eccentricities in loading and irregularities in the column material whereby the column gives way (fail) in the weaker axis.

The basic formula for determining the strength of a column in compression was developed by Euler. This formula is gives the strength of a timber in resisting buckling considering material and geometrical properties and is generally expressed in the form:

$$\sigma_{cr} = \frac{\pi^2 EI}{\left(\frac{kl}{r}\right)^2} \qquad (2.2)$$

Where:

σ_{cr} = elastic critical buckling stress,

E = Elastic modulus,

l = the length of the column,

r = the radius of gyration of the column, equal to $\sqrt{(I/A)}$

I = lowest moment of inertia of the cross section of the column,

A = cross sectional area,

k = a factor on column length dependent on end conditions

2.5 Effects of End Restraint on Columns

The end restraints of a column significantly influence the load-carrying capacity of the column (Malhotra, 1982). Generally, a column held in position and restrained against rotation at both its ends (fixed ends) is much stronger than a pinned-end column of same length and cross-section. The effect of column end restraints is recognized in the design procedure by the introduction of effective length concept. The slenderness ratio of a column is computed on the basis of its effective length rather than on unbraced length. The effective length is taken as the distance between the points of inflection on the buckled column.

Series of tests on timber columns with fixed ends have shown that the strength of fixed-end columns is much higher than that of corresponding pinned end columns. In the case of some intermediate and large slenderness values, this strength ratio was observed to be more than two to one.

2.6 Aluminium Laminated Timber Strengths and Load Actions on Composite Sections

In a composite column member consisting of different structural materials, any load applied on the column would be distributed evenly based on the sectional properties (area under action and length) and the material properties (stiffness and strength) of the various components. Similarly, the resistance to action of loads offered by the materials will be based on the modular ratio of both materials constituting the column.

2.6.1 Composite column strength

The strain in timber is affected by the inclusion of aluminium laminates (Mosley et al, 1999). The overall strain of the composite members is given by

$$\varepsilon_c = \varepsilon_T + \varepsilon_A \qquad (2.3)$$

$$\varepsilon_c = \frac{f_T}{E_T} + \frac{f_A}{E_A} \quad (2.4)$$

Where ε_T is strain of timber, f_T is stress in timber component of area A_T and E_T is Young's modulus of timber and ε_T, f_T and E_T have same meanings for aluminium component of area A_A.

The strength of the composite column is a function of the modulus of elasticity and the area (moment of inertia) of the composite materials. These materials add up their strengths in such a way as to withstand the critical loads applied on them which is given by the expression (Mosley et al, 1999, Oyenuga, 1998):

$$N = 0.35 f_T A_T + 0.70 f_A A_A \quad (2.5)$$

2.6.2 Load models on a composite column

Heterogeneous bars under direct stress (compression failure mode)

For the composite timber-aluminium column that is subjected to compression stresses, the materials will be strained by equal amounts.

The timber of cross sectional area, A_T, and young's modulus E_T, the resulting stress being f_T, and the aluminium having corresponding values of A_T, E_A and f_A. If the composite column is under a load, P, the initial strain, x, is given as

$$x = \frac{f_A}{E_A} \quad (2.6a)$$

$$x = \frac{f_T}{E_T} \quad (2.6b)$$

And the total load is given as

$$P = A_A f_A + A_T f_T \quad (2.7)$$

$A_A f_A$and $A_T f_T$, being the loads carried by each of the constituent material.

From equation 2.6a and equation 2.6b, $f_A = E_A x = \frac{E_A}{E_T} f_T$

Therefore,

$$P = f_T \left(A_T + \frac{A_A E_A}{E_T} \right) \tag{2.8}$$

Hence, the stress in the timber section will be

$$f_T = \frac{P}{A_T \left(1 + \frac{A_A E_A}{A_T E_T} \right)} \tag{2.9}$$

Conversely, the stress in the aluminium section will be

$$f_A = \frac{P}{A_A \left(1 + \frac{A_T E_T}{A_A E_A} \right)} \tag{2.10}$$

Hence, the total compressive stress acting on the column will be

$$\sigma_{total} = \frac{P}{A_T \left(1 + \frac{A_A E_A}{A_T E_T} \right)} + \frac{P}{A_A \left(1 + \frac{A_T E_T}{A_A E_A} \right)} \tag{2.11}$$

Heterogeneous bars under bending stress (bending failure mode)

The composite timber-aluminium column would behave as one in resisting bending induced in it as a result of bending moments. The two materials are rigidly connected as shown in Figure 2.1 and thus the strains in the two materials are same due to bending stresses at a section.

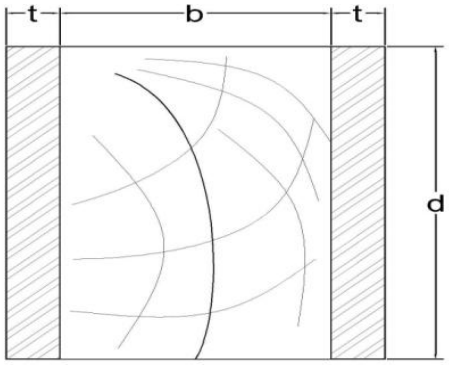


Figure 2.1 Sectional Properties of Laminated Timber Column for Bending Failure Analysis

The aluminium gives a higher modulus of elasticity than the timber. Considering the column which comprises of timber of breadth, b, sandwiched between two sheets of aluminium of thickness, t, both having a depth d. Let the stresses in timber and aluminium, f_T and f_T, be at a distance y from the neutral axis in the timber and aluminium respectively. Since the strain in the two materials is the same,

$$\frac{f_T}{E_T} = \frac{f_A}{E_A} \tag{2.12}$$

Therefore,

$$\frac{f_T}{f_A} = \frac{E_T}{E_A} = m \tag{2.13}$$

Where, E_T is the modulus of elasticity of timber E_A is the modulus of elasticity of aluminium and *m* is modular ratio. Hence, $f_A = mf_T$

Let M_T and M_A be the moments of resistance of the timber and the aluminium respectively. Then

$$M_T = f_T . z_T = f_T \times \frac{bd^2}{6} \tag{2.14}$$

and

$$M_A = f_A . z_A = f_A \times \left(\frac{td^2}{6} \times 2\right) \quad (2.15)$$

Since $f_A = mf_T$

$$M_A = mf_T \times \left(\frac{td^2}{3}\right) \quad (2.16)$$

Total moment of resistance $M = M_T + M_A$

Therefore,

$$M = f_T \times \frac{bd^2}{6} + mf_T \times \left(\frac{td^2}{3}\right) \quad (2.17)$$

Which implies that

$$M = f_T \times \frac{(b + 2mt)d^2}{6} \quad (2.18)$$

For the composite timber-aluminium column that is subjected to bending stresses, the maximum stress in the composite section is given by

$$\sigma_{max} = \frac{My_{composite}}{I_{composite}} \quad (2.19)$$

The total stress in the composite column based on individual stresses in each material

$$\sigma_{total} = \sigma_T + \sigma_A \quad (2.20)$$

Where

σ_T is stress in timber member and σ_A is the stress in the aluminium component.

Since there is an interaction between the two materials, the bending stresses will be distributed in the ratio of the flexural rigidity of both materials. In such case, the moment of inertia of the two materials joined together is given by:

$$I_{composite} = I_{timber} + I_{aluminium} \tag{2.21}$$

Substituting into equation 2.19, the applied bending stress becomes

$$\sigma = \frac{My_{composite}}{I_{timber} + I_{aluminium}} \tag{2.22}$$

And since timber and aluminium have different modulus of elasticity, the stresses in compound column will be distributed based on the modulus as expressed below:

Bending stress in aluminium,

$$\sigma = \frac{My_{composite}E_A}{E_T I_T + E_A I_A} \tag{2.23}$$

Bending stress in timber,

$$\sigma = \frac{My_{composite}E_T}{E_T I_T + E_A I_A} \tag{2.24}$$

Hence, the total applied bending stress on the composite column is given as:

$$S = \frac{My_{composite}}{E_T I_T + E_A I_A}(E_A + E_T) \tag{2.25}$$

The applied moment on the composite column is given by:

$$\mathrm{M_{applied}} = \frac{wL^2}{8} \tag{2.26}$$

Where L is the column length and w is applied load given as $q_k(1.35\alpha + 1.5)$

2.7 Reliability of Structures

Reliability of structural systems can be defined as the probability that a structure under consideration has a proper performance throughout its lifetime (Sørensen, 2004; Faber and Sørensen, 2002). The aim of structural reliability assessment is to quantify the reliability of structures under consideration of all uncertainties associated with the formulation of the failure criteria of the structure (Kohler, 2007; Kohler and Svensson, 2002). Reliability methods are used to estimate the probability of failure. The information of the models which the reliability analyses are based on is generally not complete. Thus, the estimated reliability is considered as a nominal measure of the reliability and not as an absolute number. However, if the reliability is estimated for a number of structures using the same level of information and the same mathematical models, then useful comparisons can be made on the reliability level of these structures.

The reliability estimated as a measure of the safety of a structure can be used in a decision (e.g. design) process. In order to be able to estimate the reliability using probabilistic concepts it is necessary to introduce stochastic variables and/or stochastic processes/fields and to introduce failure and non-failure behaviour of the structure under consideration.

Sørensen (2004) outlined the main steps in a reliability analysis which are:

1. Select a target reliability level.
2. Identify the significant failure modes of the structure.
3. Decompose the failure modes in series systems of parallel systems of single components (only needed if the failure modes consist of more than one component).

4. Formulate failure functions (limit state functions) corresponding to each component in the failure modes.
5. Identify the stochastic variables and the deterministic parameters in the failure functions. Further specify the distribution types and statistical parameters for the stochastic variables and the dependencies between them.
6. Estimate the reliability of each failure mode.
7. In a design process change the design if the reliabilities do not meet the target reliabilities. In a reliability analysis the reliability is compared with the target reliability.
8. Evaluate the reliability result by performing sensitivity analyses.

2.7.1 Basic failure modes in a structure

In a reliability analysis of a column, the typical failure modes are: buckling, fatigue, crushing and shear at dowel points. These failure modes are the limit states which guide the design of structures be it deterministic or stochastic designs (Mohammed, 2014, Mamman, 2014). These limit states are classified under the ultimate limit states which corresponds with maximum load carrying capacity, conditional limit states which corresponds with the load-carrying capacity if a local part of the structure has failed and the serviceability limit state that are related to the normal use of the structure.

2.7.2 Uncertainties in reliability assessment

In structural reliability assessment, the set of basic variables of a problem in general is constituted of both random and deterministic variables for the geometry, material properties and load characteristics (Kohler and Faber, 2004). Reliability analysis models the basic variables which characterize the behaviour of a structure. These variables (Eurocode 5) are loads, strengths, materials and dimensions and they could be dependent

or independent. The uncertainty models are grouped under physical uncertainty, measurement uncertainty, statistical uncertainty and model uncertainty (Sorensen, 2004, JCSS, 2006).

2.7.3 The limit state principle

The performance of an engineering structure depends on the type and magnitude of the applied load and the structural strength and stiffness. Whether the performance is considered satisfactory depends on the requirements which must be satisfied (Kohler, 2007). Among others, the reliability of structure is dependent on its resistance against collapse, limitation of damages or of deflections, or other criteria.

It is convenient to describe failure events in terms of functional relations, which if they are fulfilled, define that the failure event **F** will occur:

$$f = \{g \leq 0\} \qquad (2.27)$$

Where g is a limit state function, the components of the vector **x** are basic random variables **X** representing all relevant uncertainties influencing the problem at hand. The failure event **F** is defined as the set of realisations of the limit state function g, which are zero or negative.

2.7.4 First Order Reliability Method

The First Order Reliability Method (FORM) is a level II (reliability index method) analysis for solving probability of failure where uncertain parameters are modelled by the mean values and the standard deviations, and by the correlation coefficients between stochastic variables (Sorensen, 2004; Afolayan, 2005). The stochastic variables are implicitly assumed to be normally distributed. The level II methods are calibrated by modelling the joint distribution functions of uncertain quantities (JCSS, 2006).

FORM involves the use of stochastic variables and models, where the stochastic variables are denoted g(x) = (x_1,... , x_n). The n stochastic variables could model physical uncertainty, model uncertainty or statistical uncertainties. For the joint density function for the stochastic variable g(x), the elements in the vector of expected values and the covariance vector are

$\mu_i = E[X_i]$, i,j = 1, ..., n

$C_i = Cov[X_i,\ X_j]$, i,j = 1, ..., n

The variance of the variable $X_i$is $\sigma_i^2 = C_{ij}$ and the coefficient of correlation between X_i and X_j is defined by

$$\rho_i = \frac{C_{ij}}{\sigma_i \sigma_i}, \qquad i,j = 1, \dots, n \tag{2.28}$$

The application of FORM gives the state of the structure; whether the structure is in a safe state or in a failure state. The basic variable space is divided, by the failure state (limit state surface), into two sets: the safe set ω_s and the failure set ω_f as shown in Figure 2.2.

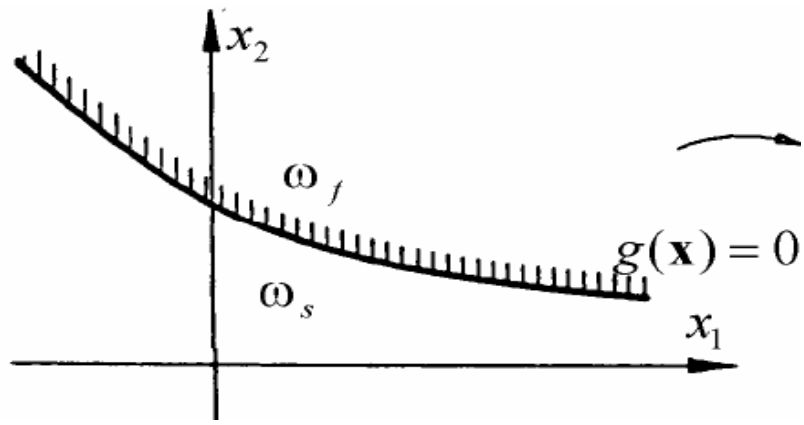


Figure 2.2: Failure Functions in the Variable Space (Sørensen, 2004a)

The failure surface is expressed by the equation:

$$g(x) = g(x_1, \dots, x_n) = 0 \tag{2.29}$$

Where g(x) is the failure function

The failure function is such that positive values of g correspond to safe states and negative values correspond to failure state.

$$g(x) = \begin{cases} > 0, & x \in \omega_s \\ \leq 0, & x \in \omega_f \end{cases} \tag{2.30}$$

When x is replaced with X in the failure function, the safety margin M is obtained.

$$M = g(x) \tag{2.31}$$

M is a stochastic variable and the probability of failure P_f of the component is

$$P_f = P(M \leq 0) = P(g(x) \leq 0) = \int_{g(x) \leq 0} f_x(x) dx \tag{2.32}$$

If R is the resistance and S the effect of actions, the performance function g is given as (BS EN 1990: 2002; Ditlevson and Madsen, 2005; Melchers, 1999):

$$g = R - S \tag{2.33}$$

R, S and g are random variables.

If the performance function, g, is normally distributed, the expected value μ_g and standard deviation σ_g can be expressed as:

$$\beta = \frac{\mu_g}{\sigma_g} \tag{2.34}$$

$\beta$is the Hasofer & Lind reliability index and it is defined as the smallest distance from the origin *O* in the *u*-space to the failure surface $g(x) = 0$.

The equation can be expressed to tally with the performance function

$$\mu_g - \beta\sigma_g = 0 \tag{2.35}$$

And

$$P_f = P(g(x) \leq 0) = P\left(\mu_g - \beta\sigma_g \leq 0\right) = P\left(\cup \leq -\frac{\mu_g}{\sigma_g}\right) = \Phi(-\beta) \qquad (2.37)$$

Φ is the standard normal distribution function and $\cup$ is a standard normally distributed variable with expected value zero and unit standard deviation ($\mu_g = 0,\ \sigma_g = 1$).

For a linear failure function, M, if the stochastic variables P and S are independent, then the index becomes:

$$\beta = \frac{\mu_R - \mu_S}{\sigma_R - \sigma_S} \qquad (2.38)$$

For other distributions of g, β is only a conventional measure of the reliability $P_s = (1 - P_f)$.

If the reliability analysis is a with non-linear failure function, then the safety margin $M = g(x)$ is not normally distributed. The safety margin is thus linearised to obtain an estimate of the reliability index with the point corresponding to the expected point.

$$M \cong g(\mu_x) + \sum_{i=1}^{n} \frac{\partial g}{\partial x_i} \qquad (2.39)$$

2.8 Finite Element Analysis

The Finite Element Analysis (FEA) is a numerical method for solving problems of engineering and mathematical physics. It is useful for problems with complicated geometries, loadings, and material properties where analytical solutions cannot be obtained. It is used to obtain approximate solutions of boundary value problems which often represent a physical structure. The finite element method is a numerical technique for finding approximate solutions to boundary value problems for partial differential

equations (Szabo and Babuska, 1991). It subdivides a large problem into smaller, simpler parts called finite elements. The finite elements are then assembled into larger (global) systems of equations that model the entire problem (Abubakar, 2014).

Finite elements analyses have found very appreciable applications in numerical modelling of structures or components to be built. The results conform well to those obtained in actual laboratory tests and modelling. It is possible to model load actions from impact (blast) loads, wind loads, vibration (earthquake) loads etc. The practical use of finite element method is based matrix algebra and the use of electronic computer as the iterations are normally large. This is because it is only in matrix form that the complete solution process can be expressed in a compact and elegant manner.

2.8.1 Computation of Displacements and Stress Resultants

Considering a column of constant cross-section loaded at a point x along the length of the column, the reaction at the left support is given by (Szabo and Babuska, 1991):

$$F_1 = \left(AE\frac{du}{dx}\right)_{x=-l/2} \tag{2.40}$$

Where A is the cross-section area, E is the stiffness of the column and $\frac{du}{dx}$ is the component of displacement

Using one finite element, the solution will be of the form

$$u_{FE} = \sum_{i=0}^{n} \mathrm{a_i}\Phi_{i+1}(\xi) \tag{2.41}$$

Where $\Phi_{i+1}(\xi)$(i = 1, 2,...) are the internal shape functions.

If the constrained stiffness matrix is perfectly diagonal, the finite element solution becomes:

$$u_{FE} = \frac{F_0 l}{2AE \sum_{i=0}^{n} \Phi_{i+1}(\xi)} (2.42)$$

The displacement components of an element under action of loads are thus given by the following:

$$u_x = \sum_{i=0}^{n} \mathrm{a_i} \Phi_i(x, y) \quad (2.43)$$

$$u_y = \sum_{i=0}^{n} \mathrm{a_{i+1}} \Phi_i(x, y) \quad (2.44)$$

Where $\Phi_i(x, y)$ are the basis functions defined by a domain

The basis functions $\Phi_i(x, y)$ are related to the shape functions by

$$N_I(\xi, \eta) = \Phi_i \left(Q_x^{(e)}(\xi, \eta), Q_y^{(e)}(\xi, \eta) \right) \quad (2.45)$$

Where I is the number of shape function, related to i and e. For element e, we have

$$u_x = \sum_{I=1}^{n^{(e)}} a_I \, N_I(\xi, \eta) \quad (2.46)$$

$$u_y = \sum_{I=1}^{n^{(e)}} a_{I+n^{(e)}} \, N_I(\xi, \eta) \quad (2.47)$$

Where $n^{(e)}$ is the number of elemental shape function which is dependent upon the polynomial degree. The displacement components u_x, u_y are computed from equation 2.41. If the global coordinates (x, y) are given, then the corresponding coordinates (ξ, η)

are computed from equation 2.45. The computation of potential functions is analogous to the computation of displacement components.

2.8.2 Computation of stresses and strains

The stress values are usually computed directly by computing the strains from the displacement components and the stresses from the appropriate stress-strain relationships. The stresses are computed from the equation and gives

$$\begin{bmatrix} \sigma_x^{(u)} & \tau_{xy}^{(u)} \\ \tau_{xy}^{(u)} & \sigma_{xy}^{(u)} \end{bmatrix} \begin{Bmatrix} n_x \\ n_x \end{Bmatrix} - \sigma \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} n_x \\ n_x \end{Bmatrix} = 0 \tag{2.48}$$

The eigenvalues are usually denoted by σ_1, σ_2. In the case of plane strain, the third principal stress is zero. In the case of plane strain, the third principal stress is computed from the stress-strain relationship, using the condition $\varepsilon_z = 0$. For example, when the material is isotropic, then

$$\sigma_3 = \sigma_z = v\big(\sigma_x + \sigma_y\big) = v(\sigma_1 + \sigma_2) \tag{2.49}$$

The normalized eigenvectors of equation 2.49 are unit vectors which determine the direction of the corresponding principal stress. In two-dimensional analysis, σ_3 is normal to the x-y plane.

The equivalent stress, also called von Mises stress, is defined as follows:

$$\sigma_3 = \sqrt{\frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]} \tag{2.50}$$

One of the most frequently used criteria for predicting the onset of yielding in ductile materials is the von Mises criterion which states the material begins to yield when the equivalent stress reaches a critical value.

2.8.3 Finite Element Analysis with Abaqus/CAE

Abaqus/CAE is a software suite for finite element analysis (numerical analysis) used for modelling and analysis of mechanical and structural components and assemblies. Abaqus is capable of both pre and post processing (Beeman, 2014). The Abaqus standard provides analysis technology to solve traditional implicit finite element analysis including static, dynamic and thermal analysis, all powered with the widest range of contact and nonlinear material options. The analysis procedures carried out in Abaqus fall into categories which are modelled, simulated and analyzed in modules (Dassault Systemes, 2010) viz-a-viz: part, property, assembly, step, interaction, load, mesh, job and visualization. These modules are discussed below.

i. Part module: this module provides the necessary tools for creating, editing and managing the building blocks of and Abaqus model which is a feature that captures design intent and contains the geometry information as well as a set of rules that govern the behaviour of the geometry.

ii. Property module: this module specifies the properties of a part or part region by creating a section and assigning it to the part.

iii. Assembly module: the assembly module is a module for creating and modifying the assembly of parts. The model consists of only one assemble, which is composed of instances of parts from the model.

iv. Step module: the step module is used to create analysis steps (sequences), specify output requests, specify adaptive meshing and specify analysis control. The analysis step sequence provides a convenient way to capture changes in the loading and boundary conditions of the model, changes in interactions of model parts, and the removal or additions of part. It also allows change in the analysis procedure, the data output and various controls.

v. Interaction module: this module is a step-dependent module where all objects/procedures must be assigned to a step in which the analyses are active. The module allows one to define and name regions of the model to which the interactions should be applied.

vi. Load module: the load module is also a step-dependent module whose procedures are directed by specifying the analysis steps in which they will active. The load module contains load cases whichare sets of loads (concentrated loads, uniform loads, thermal loads etc) and boundary conditions used to define a particular loading condition.

vii. Mesh module: is the module which allows generation of meshes on parts and assemblies within Abaqus/CAE. Various levels of automation and controls are available for creating a mesh that meets the needs of analysis. The process of assigning mesh attributes to the model such as seeds, mesh techniques and element types is feature base. As such it is possible to modify the parameters that define a part or an assembly, and the mesh attribute specified within the mesh module will generate automatically.

viii. Job module: this is the module used to analyze models that have gone through the mentioned tasks involved in defining the model. The job model is used to create a job, to submit it for analysis and to monitor it progress.

ix. The Visualization module provides graphical display of finite element models and results. It obtains model and result information from the output database. Information placed in the output database can be controlled by modifying output requests in the Step module. Model and results can be viewed by producing any of the following plots: Undeformed shape which displays the initial shape or the base state of model, deformed shape which displays the

shape of the model according to the values of a nodal variable such as displacement, Contours whichdisplays the values of an analysis variable such as stress or strain at a specified step and frame of the analysis. The Visualization module represents the values as customized coloured lines, coloured bands, or coloured faces on your model.

CHAPTER THREE

MATERIALS AND METHODS

3.1 Materials

3.1.1 Composite timber and aluminiumcolumn

The column considered in this work is composite column section of timber and aluminium laminate assembled by gluing the elements (timber-aluminium) together to function as a single unit shown in Figure 3.1. With these sections, the design rules are formulated on the assumption that no slip will arise between the elements of the section at any of the joint position as addressed in EN 1995, section 9. That is to say that the glue will take the strength of the timber and aluminium to join them as though they were one single element.

The introduction of structural elements that control the features and mechanical properties of the timber which is orthotropic and may contain some defects which undermine its structural integrity complements and increase the strength of the timber becomes necessary.Application of aluminium sheets would support high stress that will overcome the mechanical properties of wood.

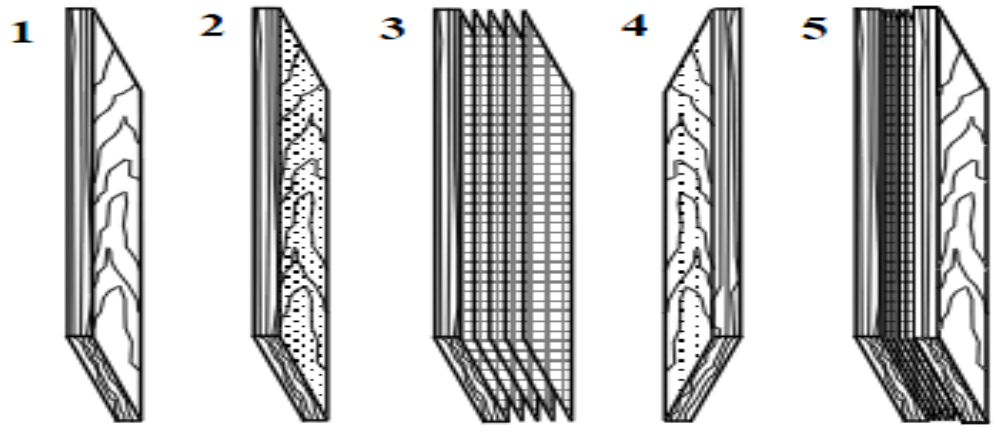


Figure 3.1: Sequence Stages for Achieving the Composite Cross Section (Decher, 2012)

3.1.2 Axially loaded timber columns

The concept of the design of models in this study adopts the idealization that the timber used is axially loaded, homogenous and perfectly straight as the current practice for the design of timber columns is based on the concept of pure, axially loaded columns.However, columns in actual structures such as frames and trusses have some unavoidable bending moments along the member due to various factors such as nominal eccentricity of axial load, initial crookedness in the column length, non-homogeneity of the material, fabrication tolerances, etc.

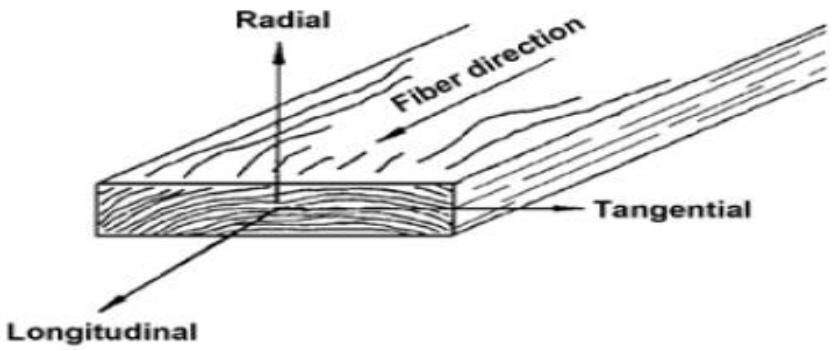


Figure 3.2: Typical Grain Direction of a Timber Column (Kretschmann, 2010)

3.1.3 MATLAB R2011b

MATLAB (MATrix LABoratory) is a fourth-generation high-level programming language and interactive environment for numerical computation, visualization and programming that is developed by MathWorks. It allows matrix manipulations; plotting of functions and data; implementation of algorithms; creation of user interfaces; interfacing with programs written in other languages, including C, C++, Java, and FORTRAN; analyze data; develop algorithms; and create models and applications. It has numerous built-in commands and math functions that help you in mathematical

calculations, generating plots and performing numerical methods. It can be run both under interactive sessions and as a batch job.

Computer programs were developed in MATLAB programming language. MATLAB R2011b software was used to run the reliability analysis of the composite timber-aluminium columns. The programs were setup in six directories as follows:

1. Main program directory
2. Safety directory
3. Distribution model setup directory
4. Coefficient of variation directory
5. Distribution transformation directory
6. Probability of failure directory

3.1.4 ABAQUS CAE 6.10

The ABAQUS CAE version 6.10 software was used for the simulations reported in this study. Abaqus is a software suite for finite element analysis (numerical analysis) used for modelling and analysis of mechanical and structural components and assemblies. Abaqus is capable of both pre and post processing. It is divided into modules which make a step by step procedure in modelling any structure.

3.2 Methods

3.2.1 General description

The timber columns used were of rectangular cross sections and dead loads were established according to Eurocode 1 (EN 1991:1-1, 2004). The modification factor takes into account the duration of load effect and moisture content and its value is considered as constant and equal to k_{mod} = 0:60, as described in EN 1995:1-1 (2004).

The timber was analyzed first without aluminium laminates to determine the behaviour of the solid timber under load with different end restraint conditions and then with the aluminium laminates to determine the effect of aluminium on the stiffness of timber in withstanding compressive stress and buckling.

The solid timber column section is axially loaded, subjected to accommodate compression forces which result in reaction forces, displacements in the three axes of the column and the Von mises stresses acting along the length of the timber column.

3.2.2 Finite Element Method

The ABAQUS CAE version 6.10 (2010) software was used for the simulations reported in this study. The step-by-step procedures are divided into modules which make a step by step procedure in modelling any structure. The modules are divided into part, property, assembly, interaction, step, load, mesh, job and visualization which give the final output of results in various forms that can easily be read and interpreted.

The elements used, modelling details and analysis techniques are described in the sections following. The simulation models consist of the solid timber column with aluminium laminates.

3.2.3 Modelling of composite timber with aluminium laminate

Composite timber with aluminium laminates were modelled into Abaqus/CAE 6.10 to simulate the behaviour of solid slender timber that is laminated with varying thicknesses of aluminium. Buckling mode of failure on the composite timber was considered. The interaction between the timber and aluminium was set at rough to avoid slip with the timber-aluminium interface. Three boundary conditions (fixed-free, pinned-pined, fixed-fixed) applicable to columns were modelled into the software to determine the behaviour of the column in such end restraints.

3.2.4 Timber properties

Geometrical properties

Sectional area, A = 300mm x 100mm

Length, L = 3500mm

Material properties

Poisson ratio, υ = 0.3

Modulus of elasticity

Strombosia pustulata (Itako), E = 9.18 kN/mm^2

Nauclea diderichii (Opepe), E = 4.95 kN/mm^2

Macrocarpa bequertii (Oporoporo), E = 8.42 kN/mm^2

Entandrophragma cylindricum (Ijebu), E = 7.36 kN/mm^2

3.2.5 Aluminium properties

Geometrical properties

The aluminium was used as laminates with a constant width of 75mm and varying thicknesses of 4mm, 6mm, 8mm, 10mm, 12mm, 14mm, 16mm, 18mm and 20mm.

Length, L=3500mm

Material properties (EN 1999.1:2007)

Modulus of Elasticity, E = 70kN/mm^2

Density, γ = 2700kg/m^3 = 2700 x 10^{7}kN/mm^3

Poisson ratio, υ = 0.3

Loads and boundary conditions

The composite column was axially loaded. The magnitude of the load applied was based on the axial compressive strengths of the four species of the timber used. Loads and boundary conditions must be applied to the rectangular geometry in order to accurately model a column with the boundary conditions considered in the study. Anaxialload of 15kN/mm was applied on the column which cuts across the compressive strength capacity of the four timber species. The end constrains which are represented by the reference point's six degrees of freedom (U1, U2, U3, UR1, UR2, and UR3) were used. The boundary conditions are: fixed-fixed, fixed-free and pinned-pinned. The fixed (Encastre) boundary condition applied to the base of the column restrains movement in the U1, U2, U3, UR1, UR2, and UR3 direction.

3.2.6 Displacement of composite columns subjected to axial loads

Oneofthemajordifferencesbetweenanisotropicandacompositecolumn isthatinthelatter case the effect of shear deformations must be taken into account (Larsen, 2001), while for the composite column, the laminates take care of shear deformations and torsion deformation. In torsion, thesheardeformationsmayplayanimportantrole,whichisrepresentedbythewarpingshear stiffness ($S_{\omega\omega}$).

The analysis of the composite columns are classified under stresses, displacement and reaction forces for timber without laminates and subsequently, timber is laminated with aluminium sheets of varying thicknesses of 4mm to 20mm in 2mm increment to check the response of the column with varying laminate thickness.

3.2.7 Modelling in Abacus/CAE (2010)

The following are the procedures that were followed in modelling the Aluminium laminated timber for analysis:

(a) Part creationaluminium

Two parts were created in this stage i.e. the part for the timber column and aluminium laminate. The geometrical property of the timber used was 100mm x 300mm x 3500mm to create the column and that of the laminate was 300mm breadth by 3500mm long with the thickness being varied. A rectangular end section was first created and then extruded to obtain the column as represented in Figure3.3

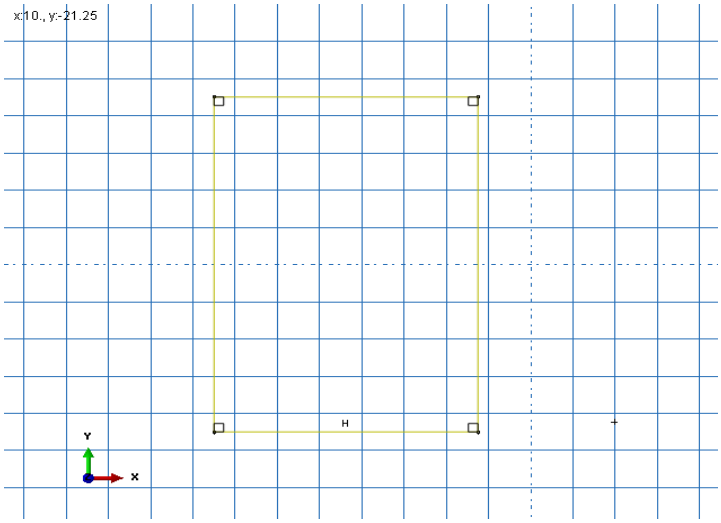


Figure 3.3: Creating an Abaqus Native Part

(b) Property creation and assignment

The properties of both timber and aluminium were created in separate dialog boxes and later assigned to the two different parts representing them. The properties entail densities, modulus of elasticity, poison ratio and yield strength of the timber and aluminium.

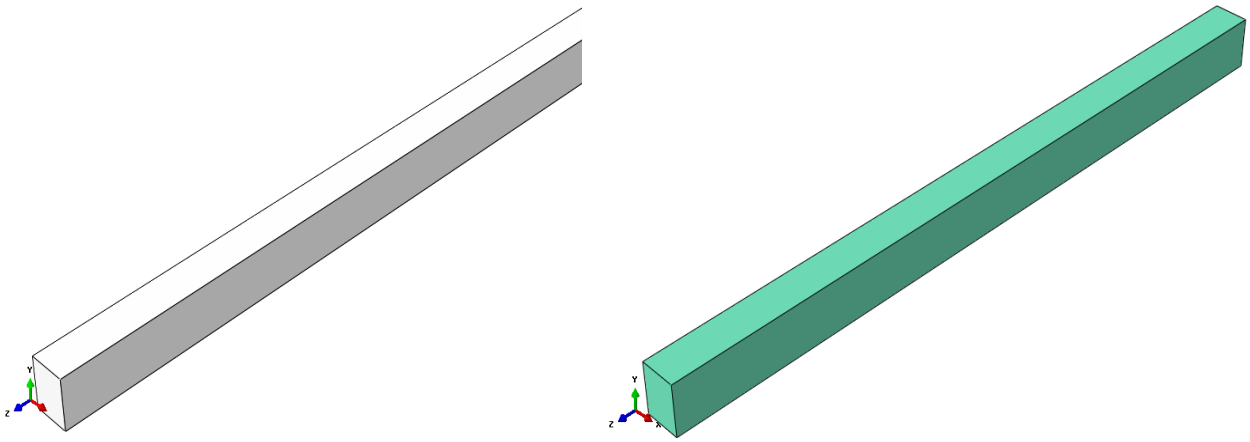


Figure 3.4: Timber Column before Property Assignment and After Property Assignment

(c) Assembly of parts

At this stage the timber column is merged with the aluminium laminate to form the composite column as shown in Figure 3.5. The procedure is carried out by calling up the various parts and translating their instances to the position for which they are required to be at.

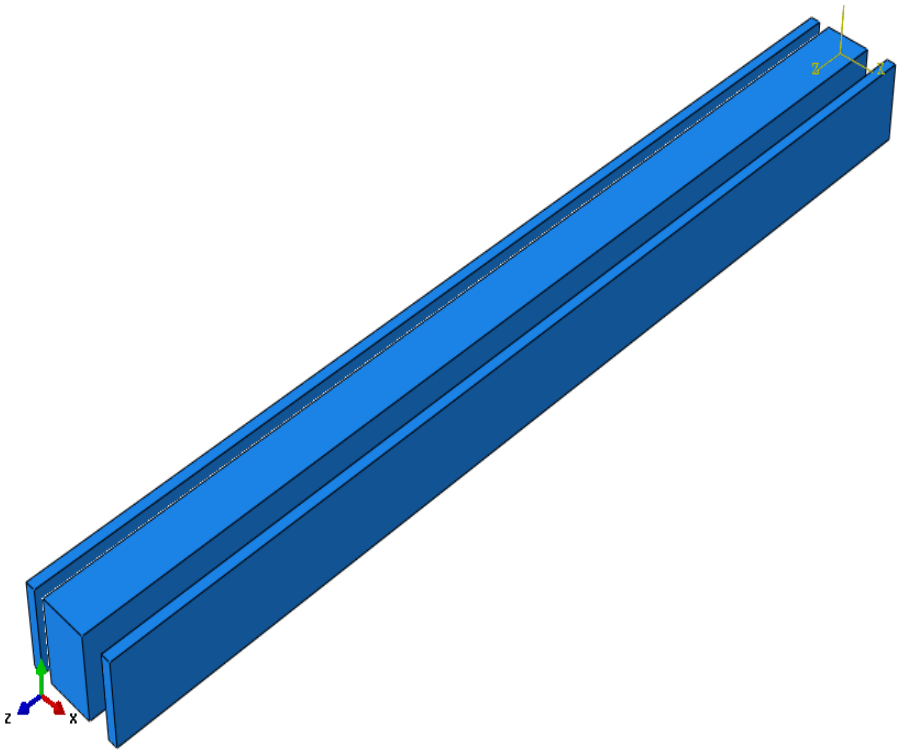


Figure 3.5: Assembly of Timber Column and Aluminium Laminate

(d) Step creation

Two analysis steps were created: the initial step which is the default analysis step in all analysis and the step 1 which runs all other analysis. The number of iterations for the analysis, the result output formats and requirements were also set at this stage. Many steps can be set up based on analysis and results required.

(e) Interactions

The interactions between the timber column and the aluminium laminates were created. The interaction was set to surface-to-surface contact as shown in Figure 3.6. A tangential behaviour rough friction formulation was chosen to ensure no slip occurs between the surfaces in contact.

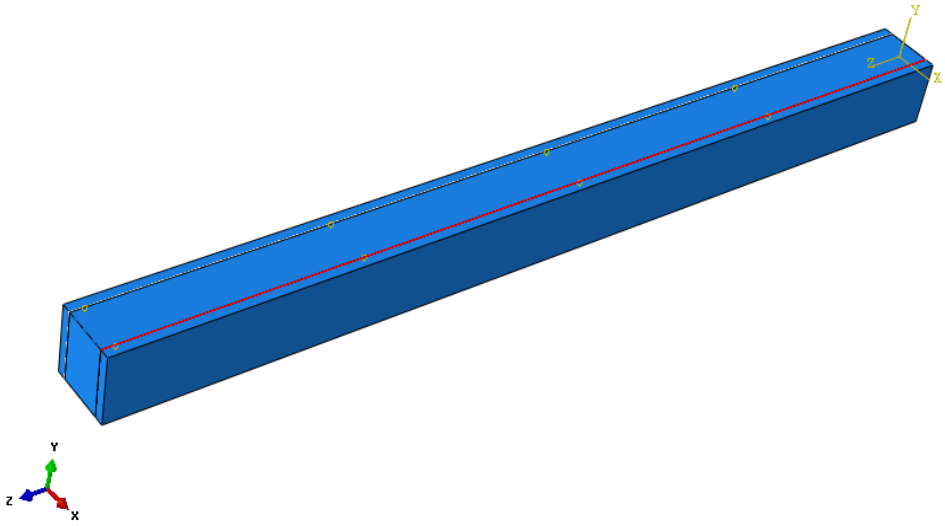


Figure 3.6: Interaction between Timber Column andAluminium Laminate

(f) Application of load and boundary conditions

The loads were applied as axial loads on the column as shown in Figure 3.7. The modelled loads were applied based on the timber characteristic properties of compression strength parallel to their grains. At this stage, the boundary conditions were also applied for the different end restraints considered.

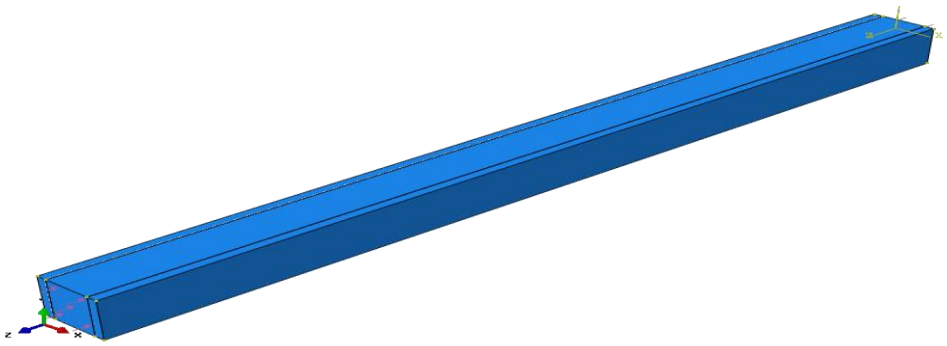


Figure 3.7: Load Application and Boundary Conditions

(g) Meshing Of Model

Meshing of the composite column was carried out on individual parts. The parts were first seeded by breaking them into element of 75mm to make the parts into elements and nodes for easy and refined analysis.

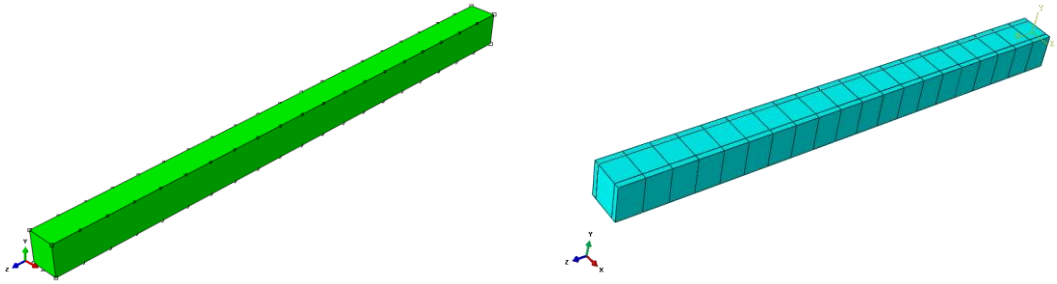


Figure 3.8: Seeded and Meshed Timber/Aluminium Composite Column

(h) Creating, editing, and manipulating job module

At this stage, the meshed model was finally analyzed by creating a job name and a general description of the model and then writing all the input parameters for simulation and analysis. It is at this stage that the model is submitted for analysis.

(i) Visualization of result

The visualization of the result was done to obtain values for deformation of the composite column as well as reaction forces and stress in the column. Figure 3.9 gives a sample of results.

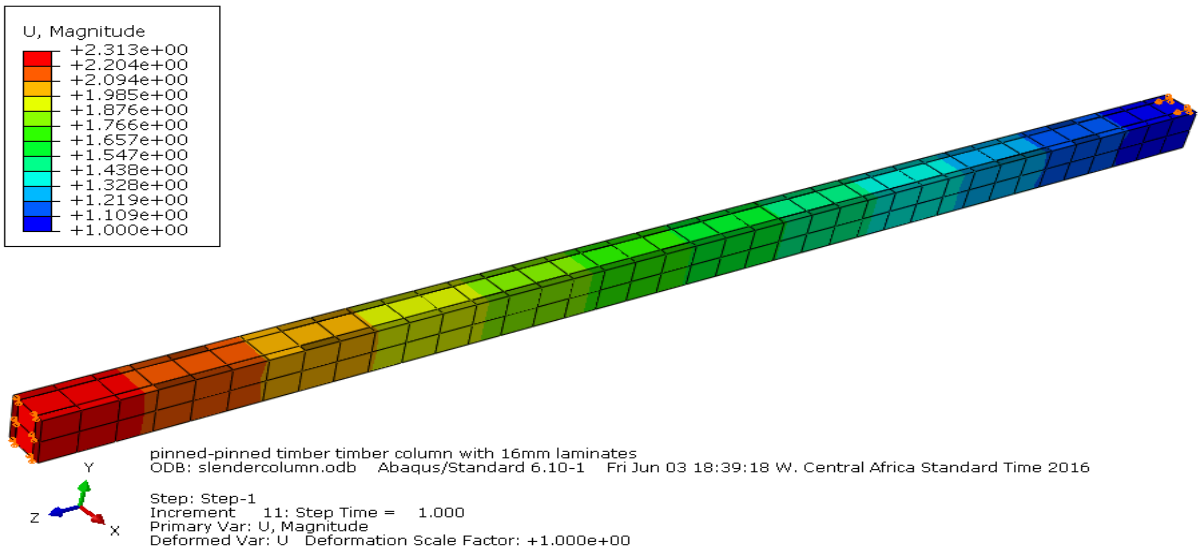


Figure3.9: Visualization of Abaqus/CAE Result

3.2.8 Reliability assessment

The reliability-based analyses were carried out with a computer program written in MATLAB. The computer program performs the reliability analysis of axially loaded solid slender columns. The values used (basic variables) in both the load models and resistance models are properties obtained from Abubakar and Nabade (2013a) and other relevant literature.Three failure modes were considered as follows: compression criterion (failure mode I), bending criterion (failure mode II) and flexural-buckling (failure mode III).

The structural reliability theory is concerned with the rational treatment of the uncertainties encountered in design, assessment, inspection and maintenance planning.The aim of structural reliability assessment is to quantify the reliability of structure under the consideration of all uncertainties associated with the formulation of a failure criterion.

Reliability is basically the complement of the failure probability ($1 - P_f$) that is the probability that a structure will not fail. A failure criterion is expressed through the limit

state function g(x), which is formulated by means of models based on physical understanding and empirical data where parameter such as material properties and load characteristics may be uncertain.

3.3 Limit State Structural Design Parameters

The column used in the reliability analysis is considered as an axially loaded solid rectangular column with aluminium laminates at the weaker axis of buckling.

i. Column geometry: the column is considered rectangular in section with varying thickness of aluminium laminates. The length of the timber is also considered for compression, bending, combination of both compression and bending, and buckling.

ii. For the load model, the factors of safety are give as:

$\gamma_q = 1.5$for imposed load

$\gamma_g = 1.35$for dead load

iii. The timber column is considered a structural member in a dwelling (EN 14081-1) and it is considered as permanent structure. Hence, the strength modification, k_{mod} is 0.6 (EN 1995: 2004).

iv. The properties of timber that were obtained from Abubakar and Nabade (2013a) were used for the basic variables whereas the statistical parameters and distribution models were obtained from Benu and Sule (2012).

v. The properties of aluminium used in the design were obtained from Mazzolani (2008)

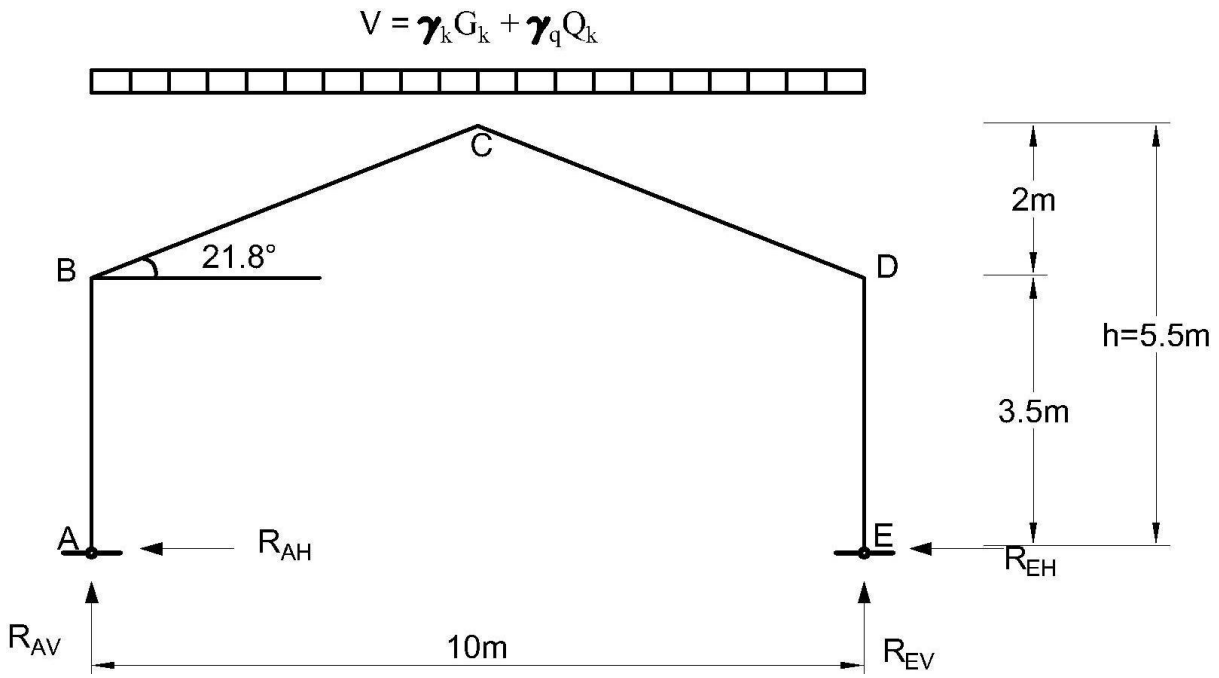


Figure3.10: Pitched Portal Frame (Trahair et al, 1977)

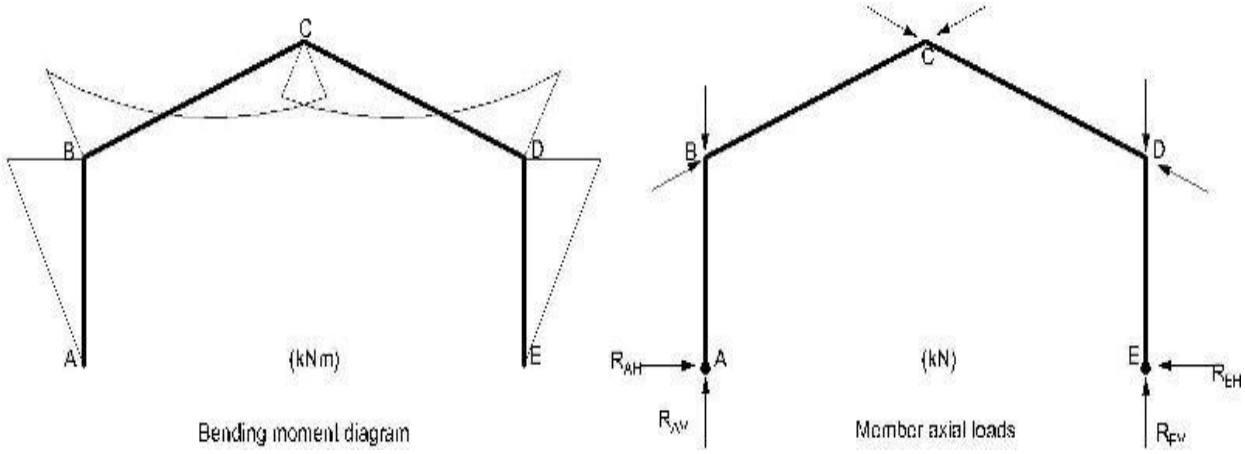


Figure3.11: Forces and Moments Diagrams on the Pitched Portal Frame (Trahair et al, 1977)

Length of roof slope:

From Pythagoras: $x^2 = 5^2 + 2^2$

$$x = \sqrt{25 + 4}$$

$$x = \sqrt{29}$$

$$x = 5.385\ m$$

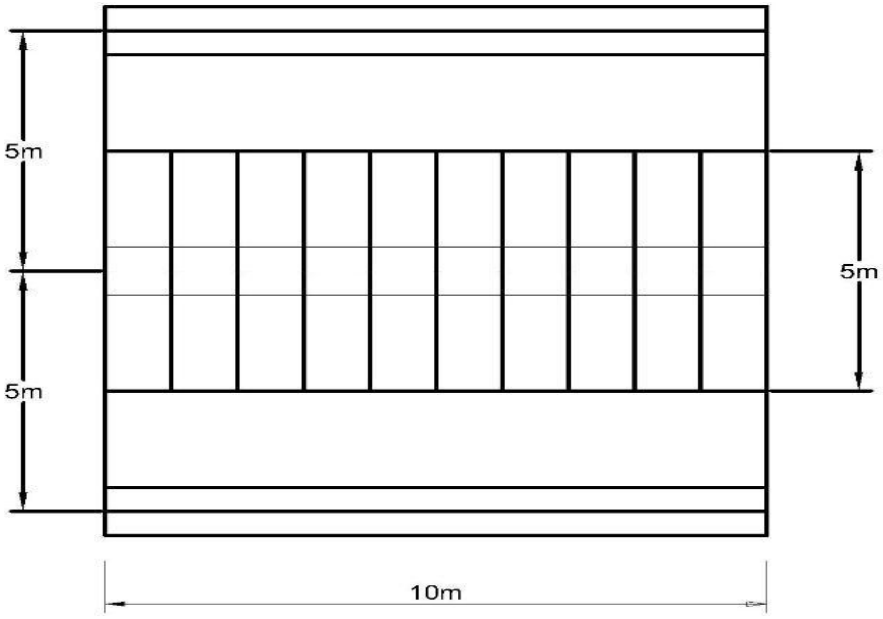


Figure 3.12: Plan of Portal Frame (Mamman, 2014)

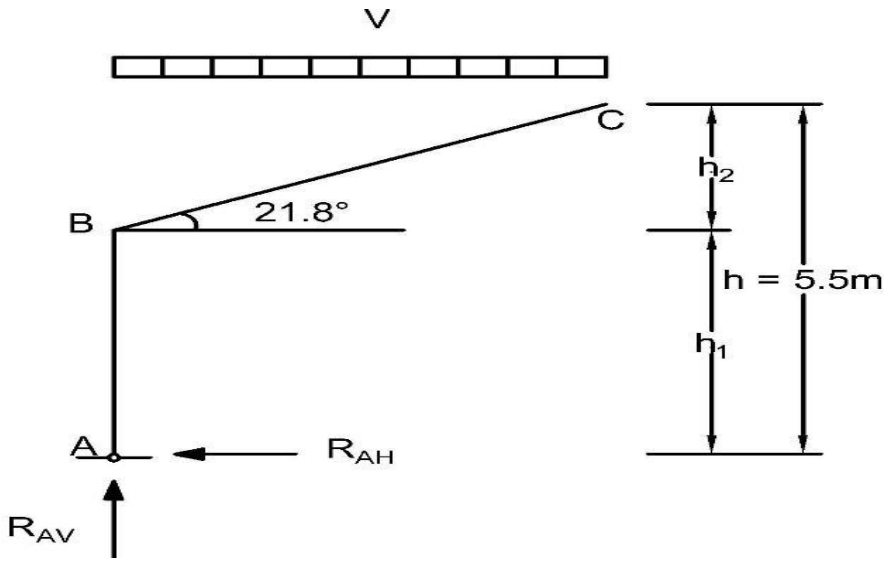


Figure 3.13: Horizontal Reactions (Mamman, 2014)

$$\sum M_c : R_{AH}(h_1 + h_2) + R_{AV}.\frac{l}{2} = V_1.\frac{l}{2}.\frac{l}{4} \tag{3.1}$$

$$R_{AH}h + R_{AV}.\frac{l}{2} = V_1.\frac{l^2}{8} \tag{3.2}$$

$$R_{AH} = \frac{V_1.\frac{l^2}{8} - R_{AV}.\frac{l}{2}}{h} \tag{3.3}$$

$$\sum H : R_{AH} + R_{EH} \tag{3.4}$$

$$R_{AH} = |R_{EH}| \quad (3.5)$$

Vertical reactions are given as:

$$R_{EV} = V_1 . \frac{l}{2} \quad (3.6)$$

$$\sum H : R_{AV} + R_{EV} = V_1 \quad (3.7)$$

$$R_{AV} = Vl - R_{EV} \quad (3.8)$$

$$V = \gamma_g G_k + \gamma_q Q_k \quad (3.9)$$

Dead load is G_k is obtained as follows:

Roof load measured on slope:

i.	Sheeting	=	0.10kN/m^2
ii.	Insulation	=	0.12kN/m^2
iii.	Purlins	=	0.03kN/m^2
iv.	Truss	=	0.13kN/m^2
	Total dead load		0.41kN/m^2

Wall:

Sheeting/ insulation board = 0.35kN/m^2

Loading:

Roof dead load = $0.41 \times 5 \times 5.385\ kN$ = 11.04kN

Wall dead load = $0.35 \times 3.55 \times 5\ kN$ = 6.125kN

Column self weight = $0.3 \times 0.1 \times 3.5 \times 5.6\ kN$ = 0.588kN

Total dead load = $11.04 \times 6.125 \times 0.588$

= 17.753kN

Imposed load Q_k = 0.5kN/m^2 (EN 1991-1-1:2002)

Q_k = $0.5 \times 5.5 \times 5$ =13.37kN

Load ratio = $\frac{G_k}{Q_k}$ = $\frac{17.753}{13.75}$ = 1.29

Design load, V = $(\gamma_g G_k + \gamma_q Q_k)$

V = $(1.35 \times 17.753 + 1.5 \times 13.75)$

$$= \quad 31.09\ kN$$

3.4 Limit State Equations for Reliability Analysis

Compression failure mode

The design resistances given in Eurocode 5 (2004) and Eurocode 9 (2007) have been merged in equation 2.9 using the modular ratio. Design resistance for composite column

$$f_{c,0,d} = \frac{k_{mod} f_{c,0,k} + \alpha f_{c,0,k}}{\gamma} \tag{3.10}$$

The applied stress on the composite column

$$\sigma_{c,0,d} = P\left(\frac{1}{A_T\left(1 + \frac{A_A E_A}{A_T E_T}\right)} + \frac{1}{A_A\left(1 + \frac{A_T E_T}{A_A E_A}\right)}\right) \tag{3.11}$$

Where K_{mod} is a modification factor taking into account the effect of the duration of load and moisture content,

$\gamma$is the partial factor for a material property,

A_T is area of timber section and A_T is area of aluminium,

E_T is young's modulus of timber and E_T young's modulus of aluminium,

P is the design load which is in the form, $Q_K(\gamma_g \alpha + \gamma_q)$ where Qk is the live load and α is the dead load to live load ratio.

From the performance function $G_{(x)} = R - S$,

$$G_{(x)} = \frac{k_{mod} f_{c,0,k} + \alpha f_{c,0,k}}{\gamma} - Q_K(\gamma_g \alpha + \gamma_q)\left(\frac{1}{A_T\left(1 + \frac{A_A E_A}{A_T E_T}\right)} + \frac{1}{A_A\left(1 + \frac{A_T E_T}{A_A E_A}\right)}\right) \tag{3.12}$$

Bending failure mode

The design resistance for bending and the applied bending stress are obtained in equations 2.9 and 2.15 respectively.

Bending moment of resistance,

$$f_{m,0,d} = k_{mod} f_T \times \frac{(b + 2mt)d^2}{6} \tag{3.13}$$

Applied bending moment on composite column,

$$M = 0.125 Q_K (1.35\alpha + 1.5) L^2 \tag{3.14}$$

From the performance function $G_{(x)} = R - S$,

$$G_{(x)} = \left(k_{mod} f_T \times \frac{(b + 2mt)d^2}{6} \right) - 0.125 Q_K (1.35\alpha + 1.5) L^2 \tag{3.15}$$

Flexure buckling failure mode

There is the tendency for the column to buckle in bending due to axial load subjected on it. EN 1995:2004 gives the flexural buckling of timber to satisfy the interactive formula in the following equation,

$$\left(\frac{\sigma_{m,d}}{k_{crit} f_{m,d}} \right)^2 + \frac{\sigma_{c,0,d}}{k_{c,z} f_{c,0,d}} \leq 1 \tag{3.16}$$

From the performance function $G_{(x)} = R - S$,

$$G_{(X)} = 1 - \left(\left(\frac{\sigma_{m,d}}{k_{crit} f_{m,d}} \right)^2 + \frac{\sigma_{c,0,d}}{k_{c,z} f_{c,0,d}} \right) \tag{3.17}$$

Where:

$\sigma_{m,d}$is the bending stress, $\sigma_{c,0,d}$ is the design compressive stress parallel to grain,$f_{m,d}$ is the bending strength parallel to grain, $f_{c,0,d}$ is the design compressive strength parallel to grain.

$K_{c,z}$ is the column instability factor given as:

$$k_{c,z} = \frac{1}{k_z + \sqrt{k^2{}_z - \lambda^2{}_{rel,z}}}$$

Where: $k_z = 0.5\left(1 + \beta_c\left(\lambda_{rel,z} - 0.3\right) + \lambda^2{}_{rel,z}\right)$ and β_c is a factor for members within a define limit and is 0.2 for solid timber.

The relative slenderness ratio $\lambda_{rel,z} = \frac{\lambda_z}{\pi}\sqrt{\frac{f_{c,0,k}}{E_{0.05}}}$

$\lambda_z$is the slenderness ratio in the z-axis and is the fifth percentile value of the modulus of elasticity parallel to the grain.

3.5 Stochastic Models of the Basic Variables

The values of the basic variables as used in the designed programs are presented in Tables 3.1 and 3.2.

Table3.1: Stochastic Parameters for Compression Failure Mode (Abubakar and Nabade 2013a, Benu and Sule 2012)

S/NO	VARIABLES	MEANING	DISTRIBUTION	MEAN	COVARIANCE	SD
1	$f_{c,0,k}$(itako)	Characteristic compressive strength parallel to grain	lognormal	29.58 N/mm^2	0.13	3.84
2	$f_{c,0,k}$(Oporoporo)		lognormal	20.82 N/mm^2	0.13	2.71
3	$f_{c,0,k}$(opepe)		lognormal	27.18 N/mm^2	0.13	3.53
4	$f_{c,0,k}$(ijebu)		lognormal	24.16 N/mm^2	0.13	3.14
5	$f_{c,0,k}$(aluminium)		lognormal	100 N/mm^2	0.13	13
6	Qk	Imposed load	Gumbel	15kN	0.32	4.8
7	B	Width of column	Normal	300mm	0.47	141
8	H	Thickness of column	Normal	100mm	0.47	47
9	E_{itako}	Modulus of elasticity	Lognormal	9.18 kN/mm^2	0.21	1.93
10	$E_{Oporoporo}$		lognormal	4.95 kN/mm^2	0.21	1.04
11	E_{opepe}		Lognormal	8.42 kN/mm^2	0.21	1.77
12	E_{ijebu}		Lognormal	7.36 kN/mm^2	0.21	1.55
13	$E_{Aluminium}$		lognormal	70 kN/mm^2	0.21	14.7

Table 3.2: Stochastic Parameters for Bending Failure Mode (Abubakar and Nabade 2013a, Benu and Sule 2012)

S/NO	VARIABLES	MEANING	DISTRIBUTION	MEAN	COVARIANCE	SD
1	$f_{c,90,k}$(itako)	Characteristic compressive strength perpendicular to grain	Lognormal	51.97 N/mm^2	0.13	6.76
2	$f_{c,90,k}$(Oporoporo)		Lognormal	32.30 N/mm^2	0.13	4.2
3	$f_{c,90,k}$(opepe)		Lognormal	23.80 N/mm^2	0.13	3.1
4	$f_{c,90,k}$(ijebu)		Lognormal	33.12 N/mm^2	0.13	4.3
5	$f_{c,90,k}$(Aluminium)		Lognormal	100 N/mm^2	0.13	13
6	Qk	Imposed load	Gumbel	15kN	0.32	4.8
7	B	Width of column	Normal	300mm	0.47	141
8	H	Thickness of column	Normal	100mm	0.47	47
9	L	Column height	Normal	3500mm	0.28	980
10	E_{itako}	Modulus of elasticity	Lognormal	10.0 kN/mm^2	0.21	1.93
11	$E_{Oporoporo}$		Lognormal	6.67 kN/mm^2	0.21	1.04
12	E_{opepe}		Lognormal	9.28 kN/mm^2	0.21	1.77
13	E_{ijebu}		Lognormal	8.50 kN/mm^2	0.21	1.55
14	$E_{Aluminium}$		lognormal	70 kN/mm^2	0.21	14.7

Table3.3: Stochastic Parameters for Flexural Failure Mode (Abubakar and Nabade 2013a, Benu and Sule 2012)

S/NO	VARIABLES	MEANING	DISTRIBUTION	MEAN	COVARIANCE	SD
1	$f_{c,0,k}$(itako)	Characteristic compressive strength parallel to grain	lognormal	29.58 N/mm^2	0.13	3.84
2	$f_{c,0,k}$(Oporoporo)		lognormal	20.82 N/mm^2	0.13	2.71
3	$f_{c,0,k}$(opepe)		lognormal	27.18 N/mm^2	0.13	3.53
4	$f_{c,0,k}$(ijebu)		lognormal	24.16 N/mm^2	0.13	3.14
5	$f_{c,0,k}$(Aluminium)		lognormal	100 N/mm^2	0.13	13
1	$f_{c,90,k}$(itako)	Characteristic compressive strength perpendicular to grain	Lognormal	51.97 N/mm^2	0.13	6.76
2	$f_{c,90,k}$(Oporoporo)		Lognormal	32.30 N/mm^2	0.13	4.2
3	$f_{c,90,k}$(opepe)		Lognormal	23.80 N/mm^2	0.13	3.1
4	$f_{c,90,k}$(ijebu)		Lognormal	33.12 N/mm^2	0.13	4.3
5	$f_{c,90,k}$(Aluminium)		Lognormal	100 N/mm^2	0.13	13
6	Qk	Imposed load	Gumbel	15kN	0.32	4.8
7	B	Width of column	Normal	300mm	0.47	141
8	H	Thickness of column	Normal	100mm	0.47	47
9	L	Column height	Normal	3500mm	0.28	980
10	E_{itako}	Modulus of elasticity	Lognormal	10.0 kN/mm^2	0.21	1.93
11	$E_{Oporoporo}$		Lognormal	6.67 kN/mm^2	0.21	1.04
12	E_{opepe}		Lognormal	9.28 kN/mm^2	0.21	1.77
13	E_{ijebu}		Lognormal	8.50 kN/mm^2	0.21	1.55
14	$E_{Aluminium}$		lognormal	70 kN/mm^2	0.21	14.7

CHAPTER FOUR

DISCUSSION OFRESULTS

4.1 FINITE ELEMENT ANALYSIS RESULTS

Using the input data prepared as given in Chapter three, runs of the Abaqus CAE (2010) software gave member-end forces, stresses and displacements in the composite columns. The results for member-end forces are as shown in Figures 4.1 to 4.12 for the different end-restraint conditions.

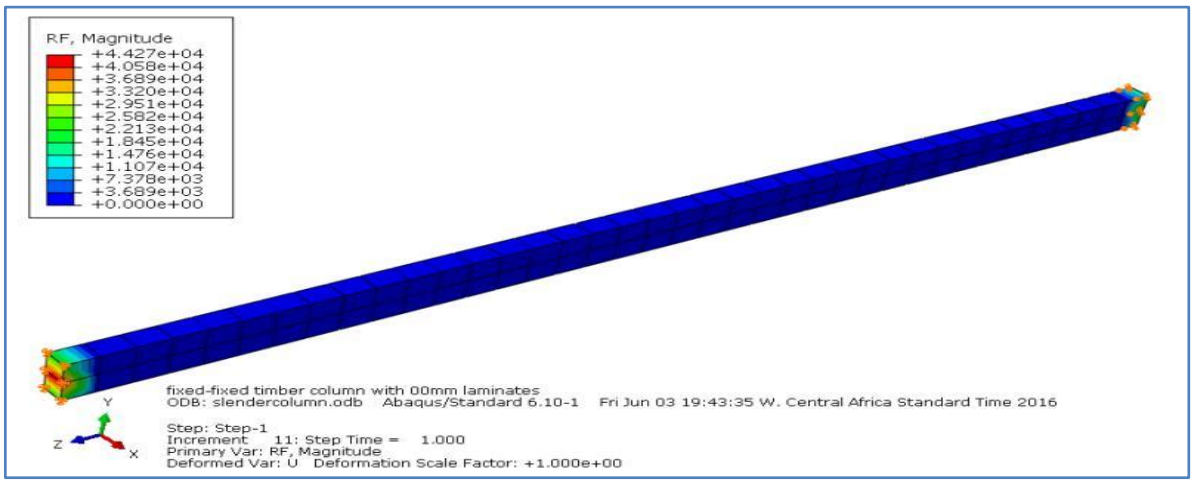


Figure 4.1: Member-End Force for Fixed-Fixed Timber Column with No Aluminium laminates

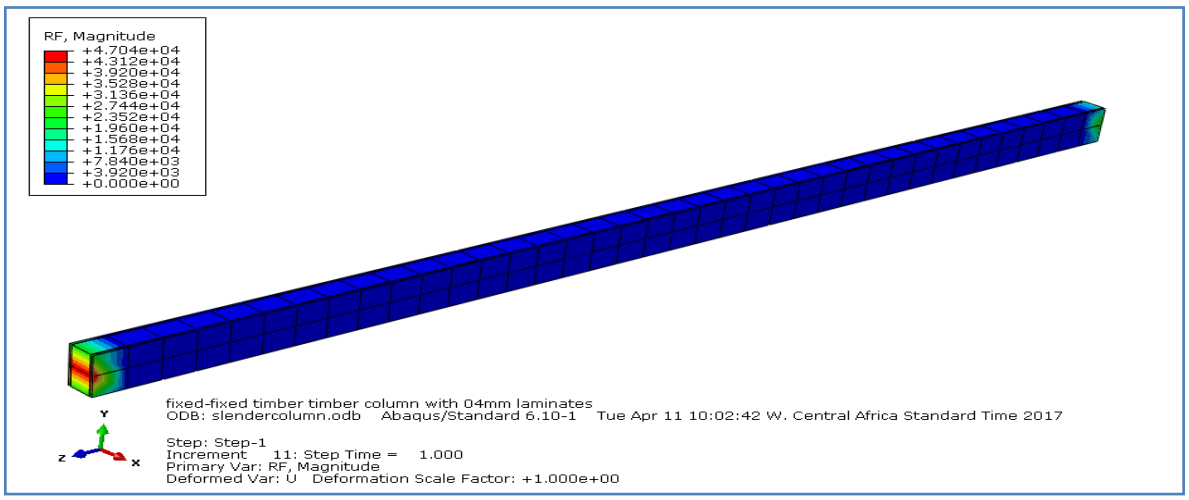


Figure 4.2: Member-End Force for Fixed-Fixed Timber Column with 4mm Aluminium laminate

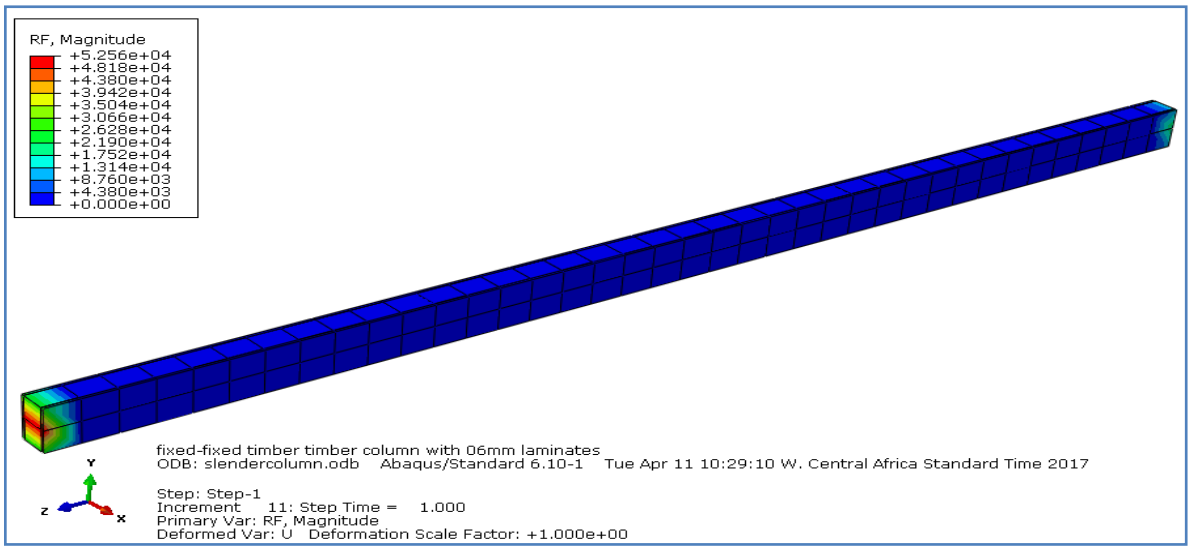


Figure 4.3: Member-End Force for Fixed-Fixed Timber Column with 6mm Aluminium laminate

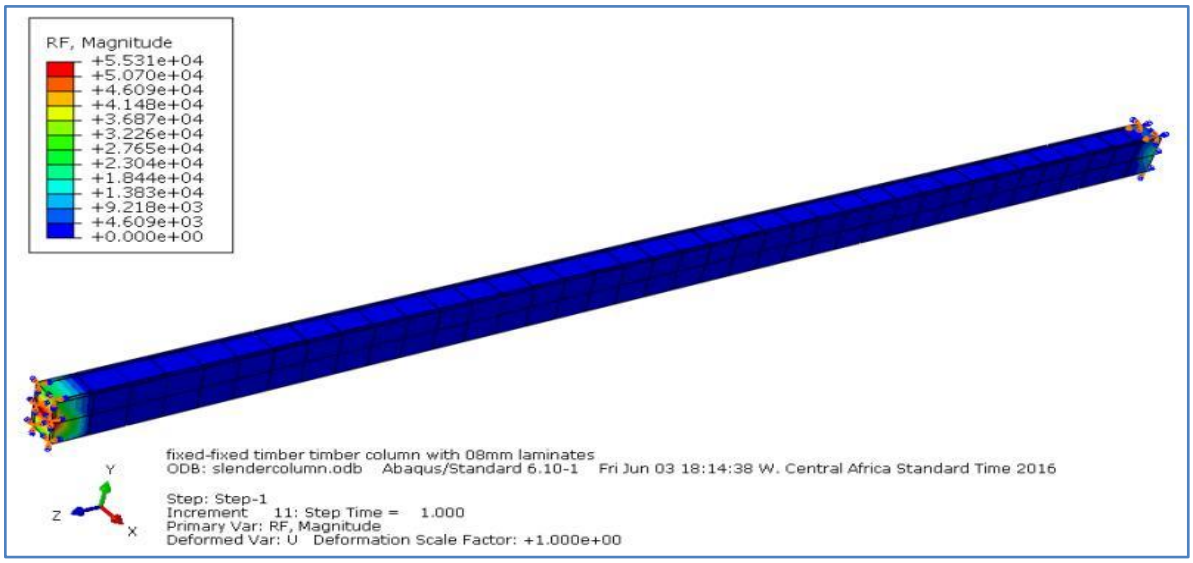


Figure 4.4: Member-End Force for Fixed-Fixed Timber Column with 8mm Aluminium laminate

From Figures 4.1 to 4.4, it can be seen that for the fixed-fixed ended composite columns, as the thickness of the aluminium laminate increases from 0 to 8mm there is corresponding increase in the member-end forces from 4.427×10^{4}N for the column

without laminate to 4.704×10^4N for 4mm laminates, 5.256×10^4N for 6mm and 5.531×10^4N for 8mm laminates.

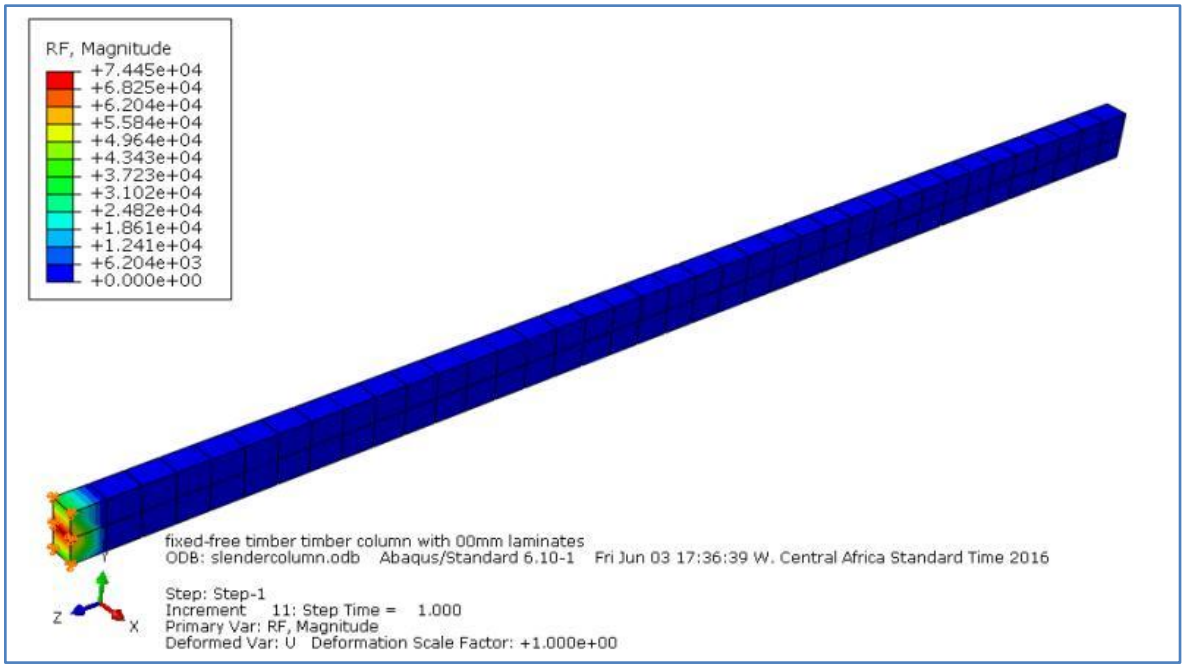


Figure 4.5: Member-end force for fixed-free timber column with no laminate

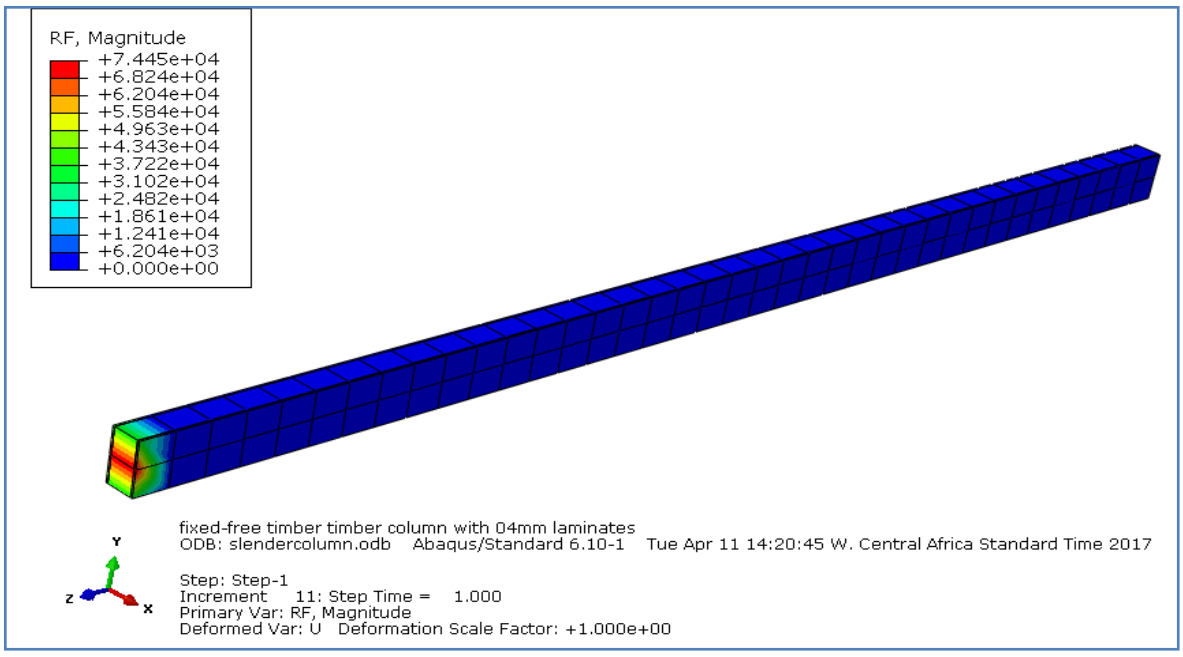


Figure 4.6: Member-end force for fixed-free timber column with 4mm laminate

From Figures 4.5 to 4.8, it can be observed that there is an insignificant change in the member-end forces whereby the timber column with no laminate has value of 7.445×10^4N, those with 4mm, 6mm and 8mm laminates have corresponding member-end forces of 7.445×10^4 N, 7.442×10^4N and 7.437×10^4N respectively.

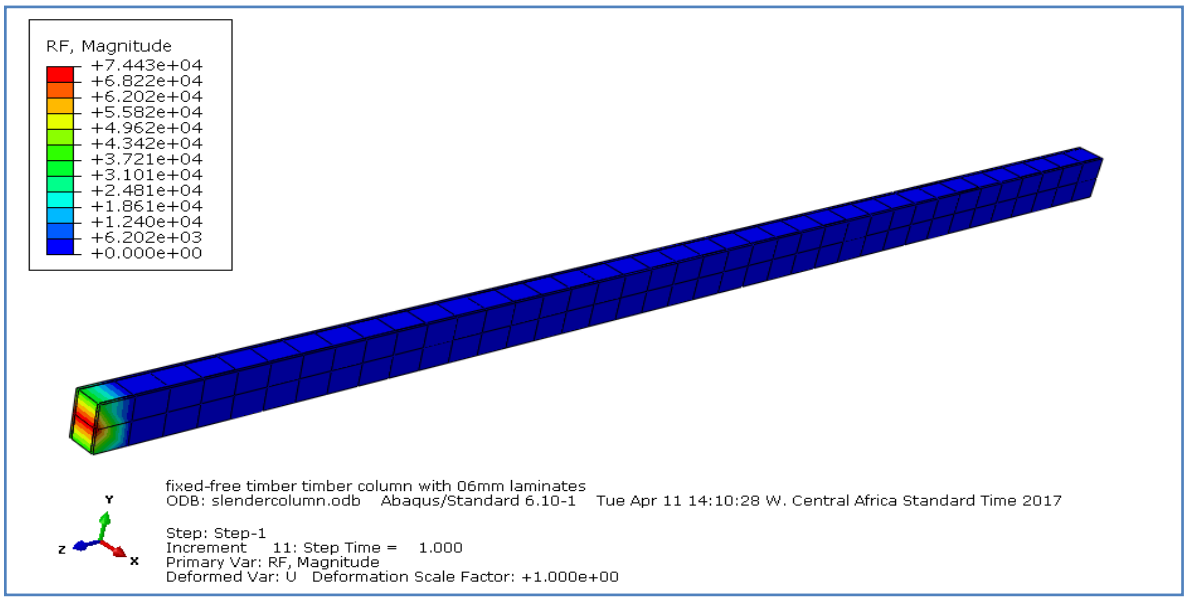


Figure 4.7: Member-end force for fixed-free timber column with 6mm laminate

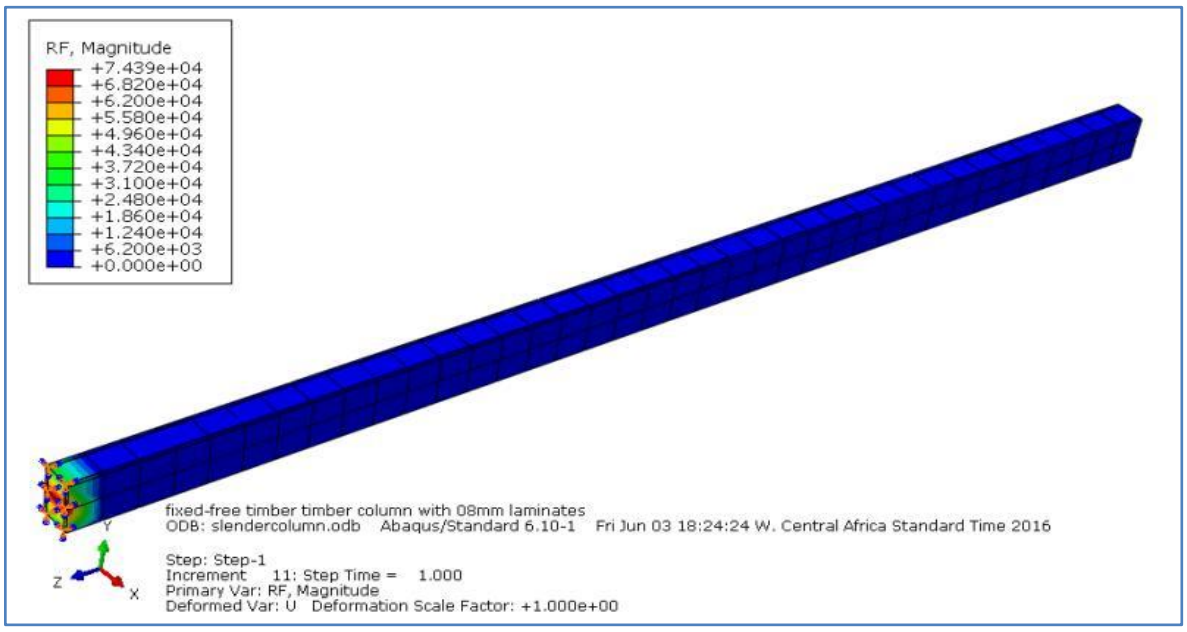


Figure 4.8: Member-End Force of Fixed-free Timber Column with 8mm Aluminium laminate

In the case of pinned-pinned end-restraint condition, as seen in Figures 4.9 to 4.12, almost all the aluminium laminates gave the same values of 7.449×10^{4}N.

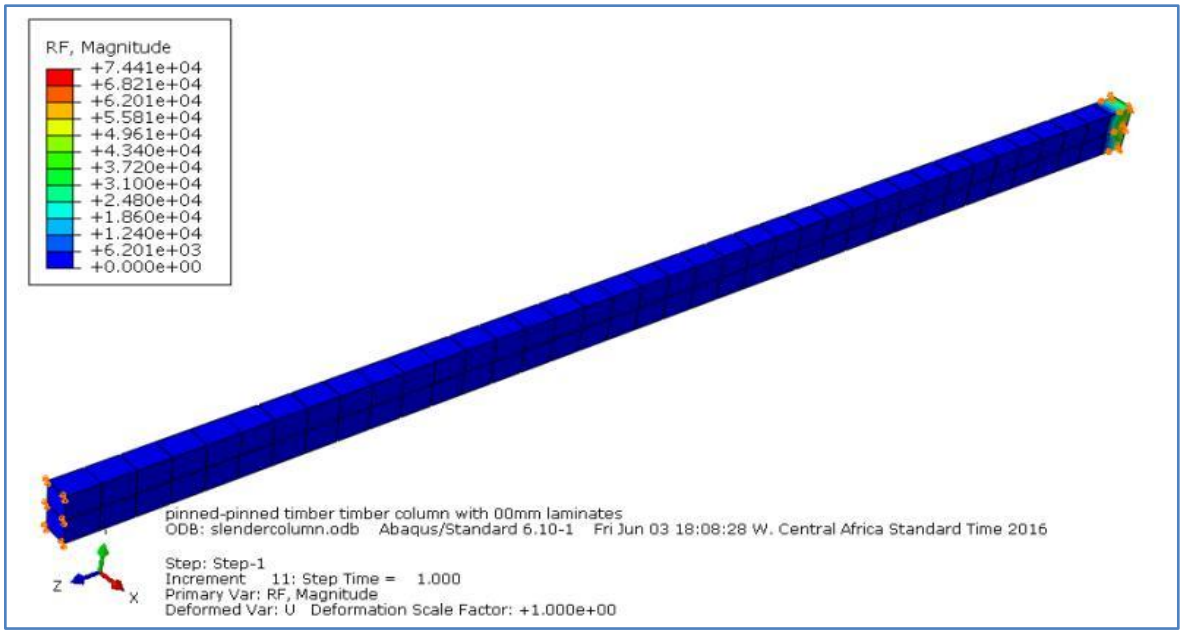


Figure 4.9: Member-End Force of pinned-pinned Timber Column with No Aluminium laminate

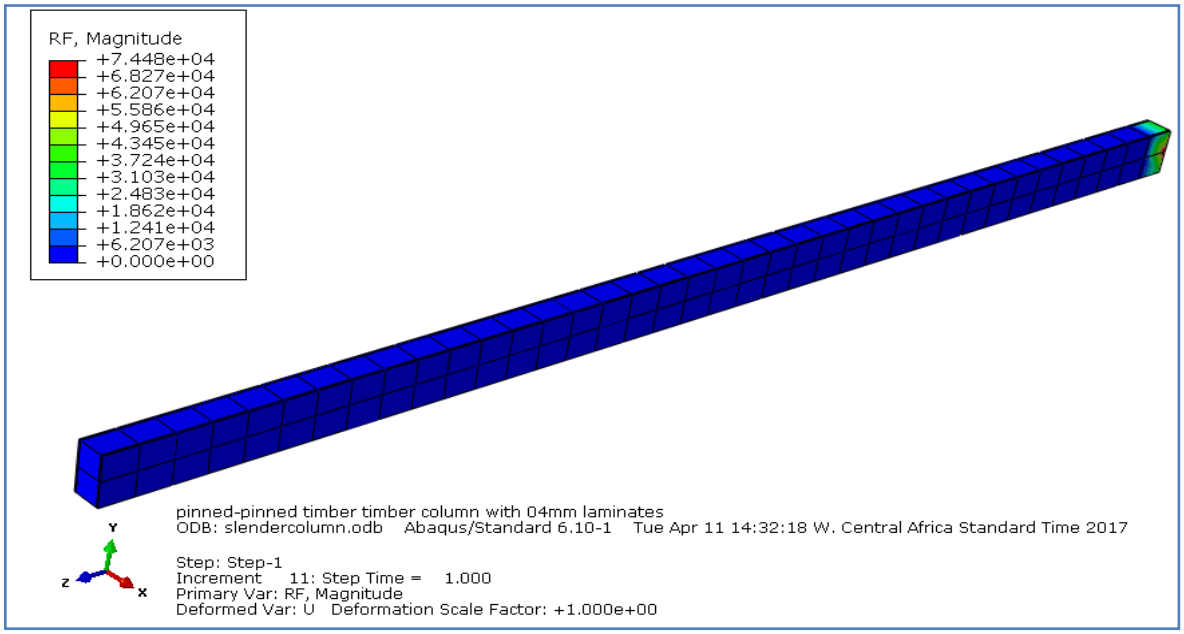


Figure 4.10: Member-End Force of pinned-pinned Timber Column with 4mm Aluminium laminate

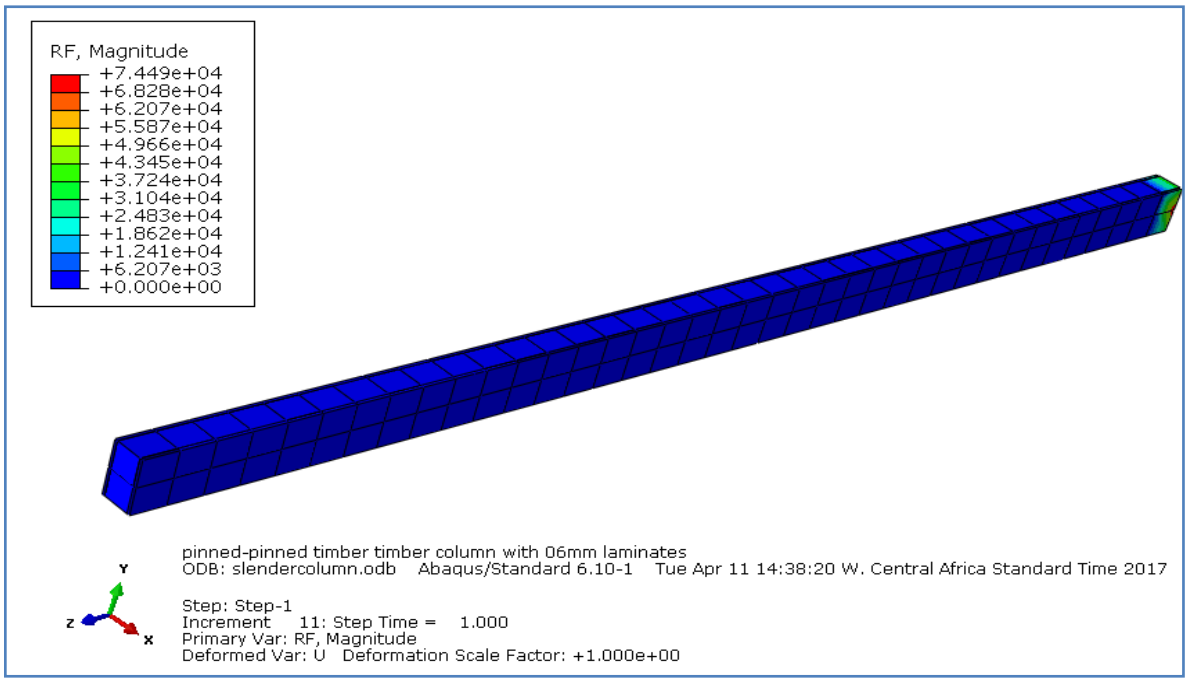


Figure 4.11: Member-End Force of pinned-pinned Timber Column with 6mm Aluminium laminate

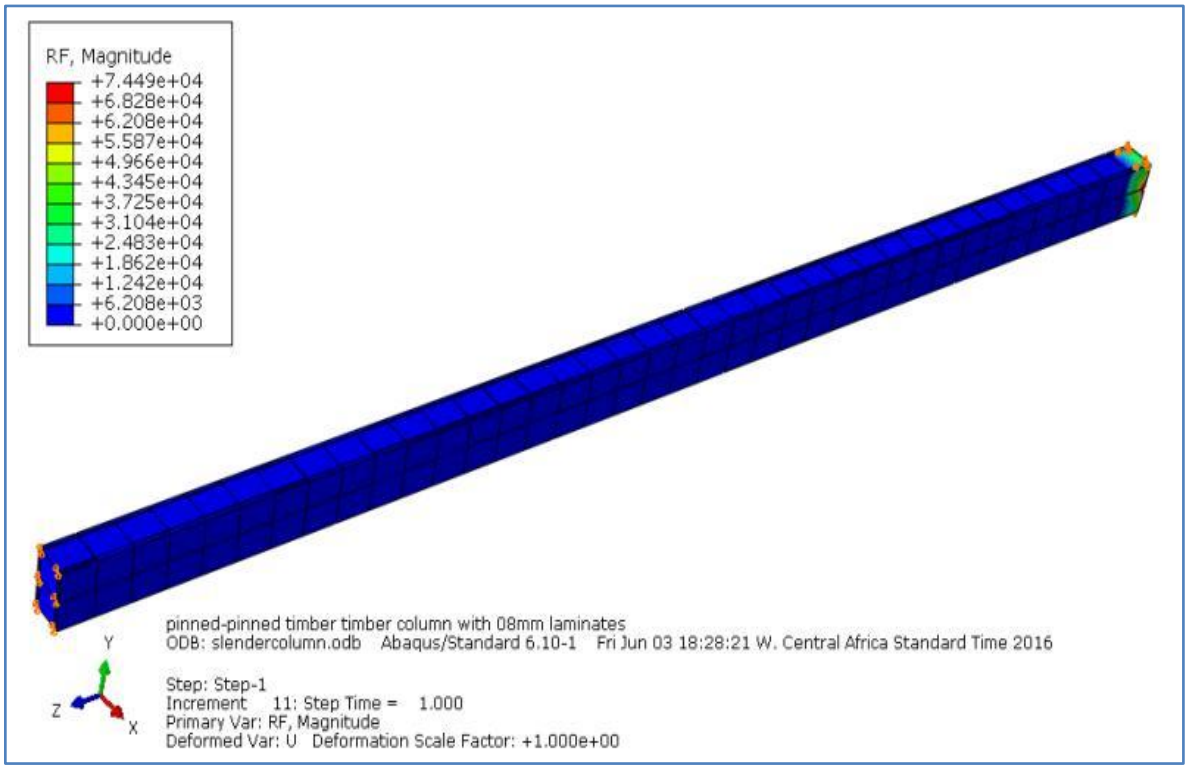


Figure 4.12: Member-End Force of pinned-pinned Timber Column with 8mm Aluminium laminate

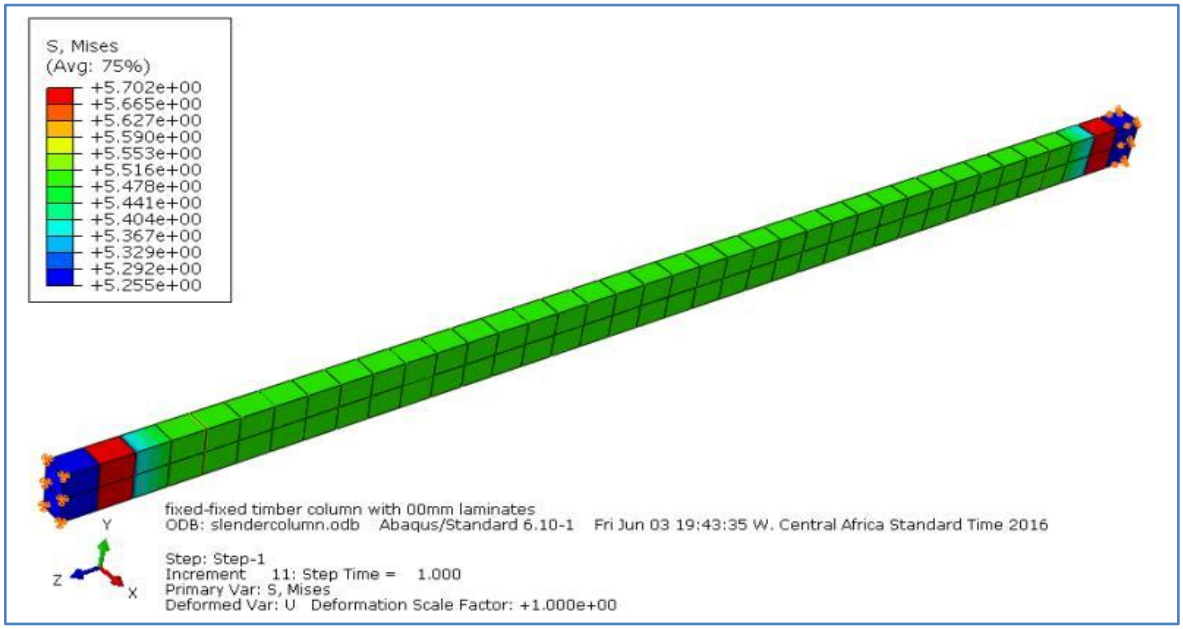


Figure 4.13: Stress in Fixed-Fixed Timber Column with No Aluminium laminates

From Figure 4.13 to 4.16, the Von Mises stresses in the composite timber column with fixed-fixed end-restraint condition gave values of 5.70N/mm^2 for the timber column without laminates, 29N/mm^2,29.3 N/mm^2 and 29.3N/mm^2 for the composite column with 4mm, 6mm and 8mm laminates respectively.

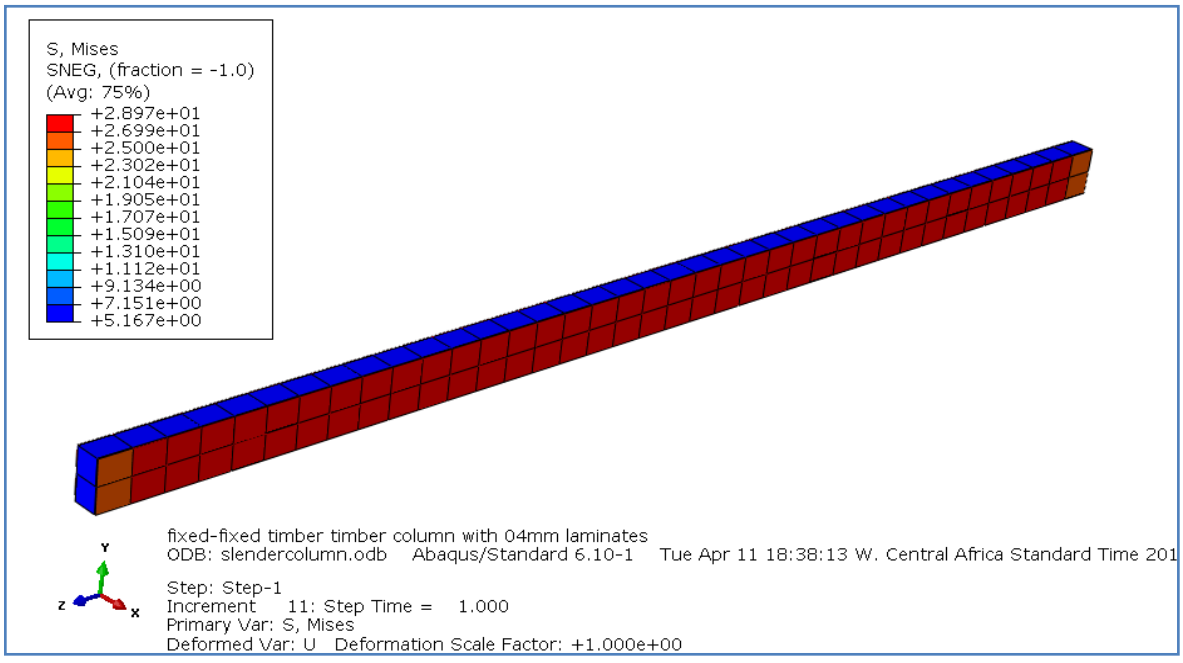


Figure 4.14: Stress in fixed-fixed timber column with 4mm aluminium laminate

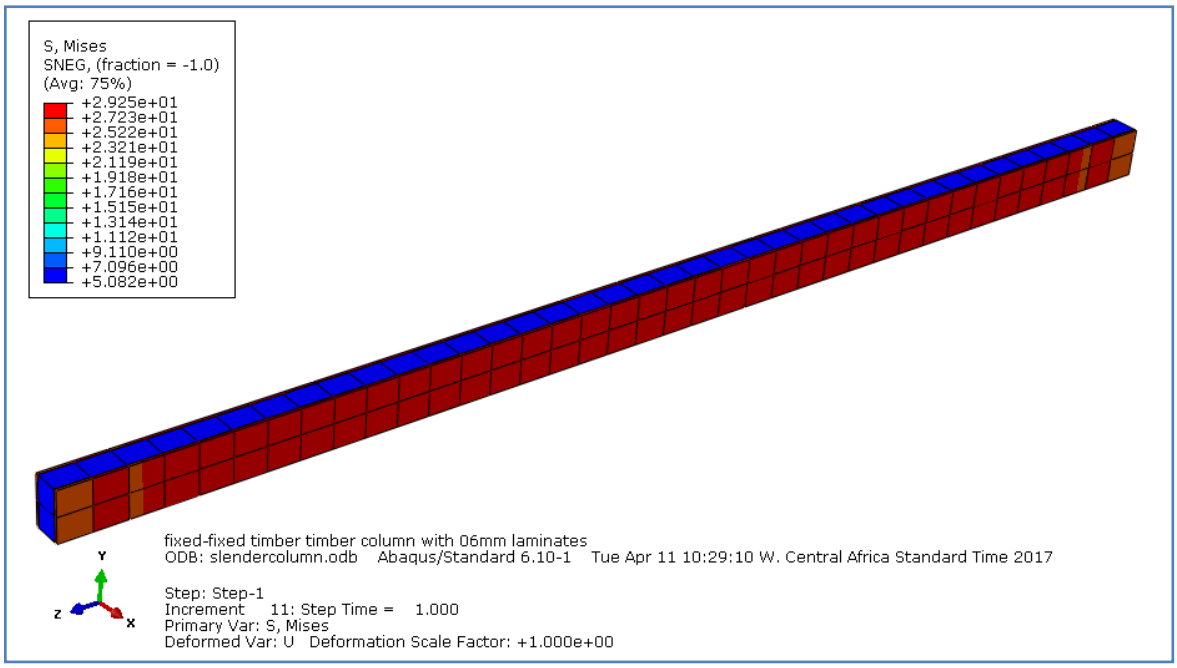


Figure 4.15: Stress in fixed-fixed timber column with 6mm aluminium laminate

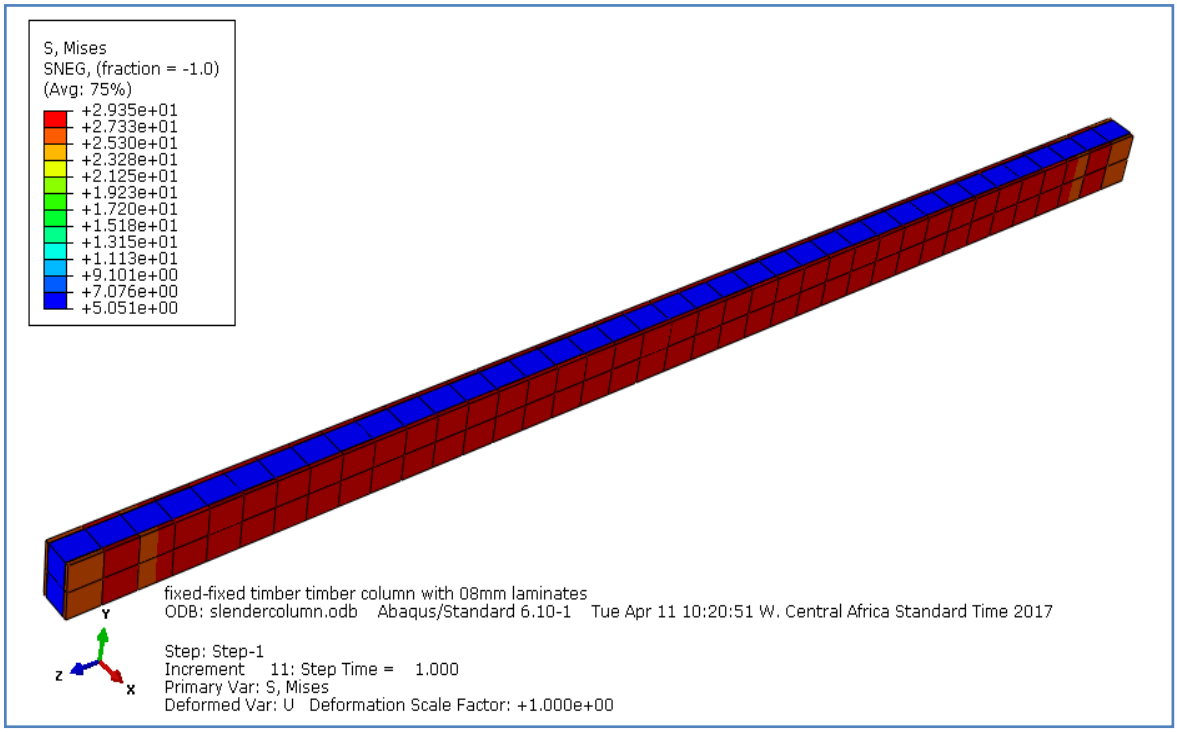


Figure 4.16: Stress in fixed-fixed timber column with 8mm aluminium laminate

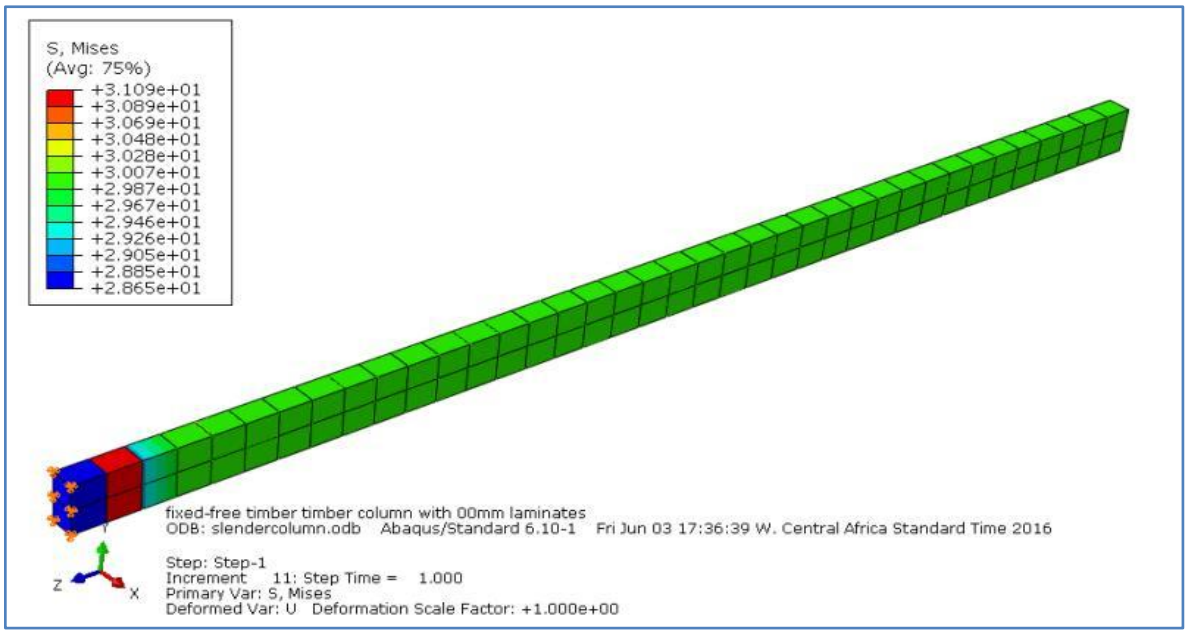


Figure 4.17: Stress in fixed-free timber column with no laminate

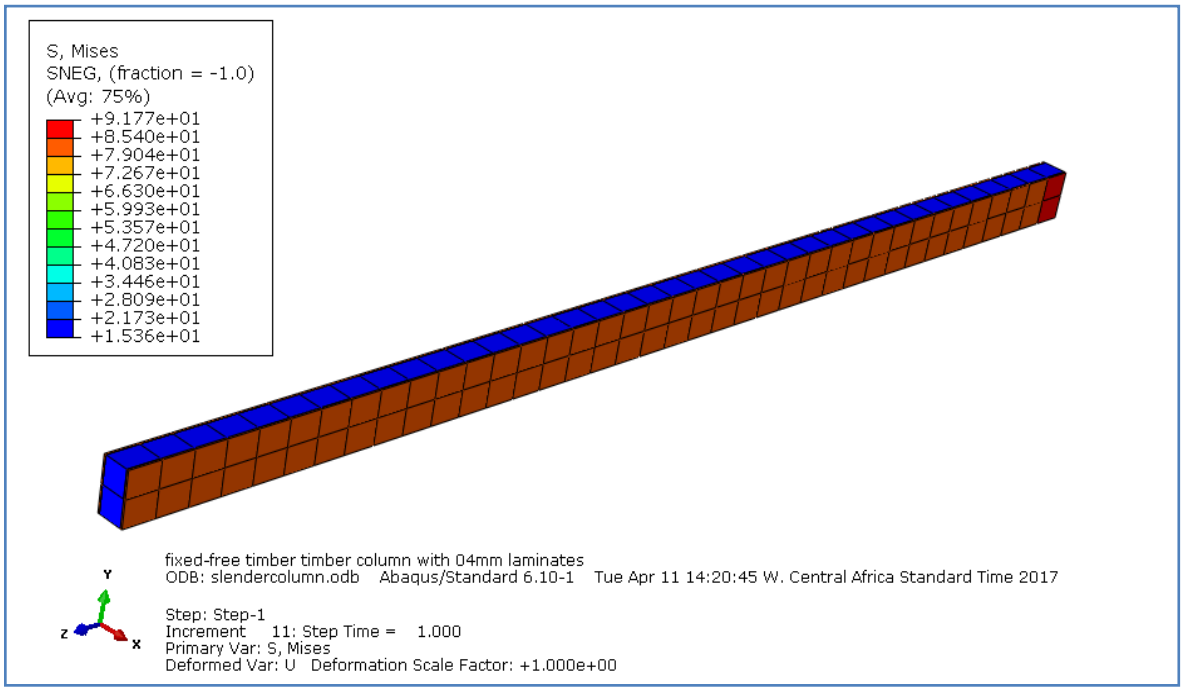


Figure 4.18: Stress in fixed-free timber column with 4mm aluminium laminate

In Figures 4.17 to 4.20, it can be observed that there is an increase from 31.1N/mm^2 for the timber column without laminates to a stress of 91.8N/mm^2 for the timber column with 4mm laminate which then decreases to 80.2N/mm^2 for 6mm laminates and further decrease to 72.4N/mm^2 for 8mm laminates which is also seen as the peak illustrated in

figures 4.40, 4.44, 4.48 and 4.52. This is due to the stress redistribution to the aluminium laminates.

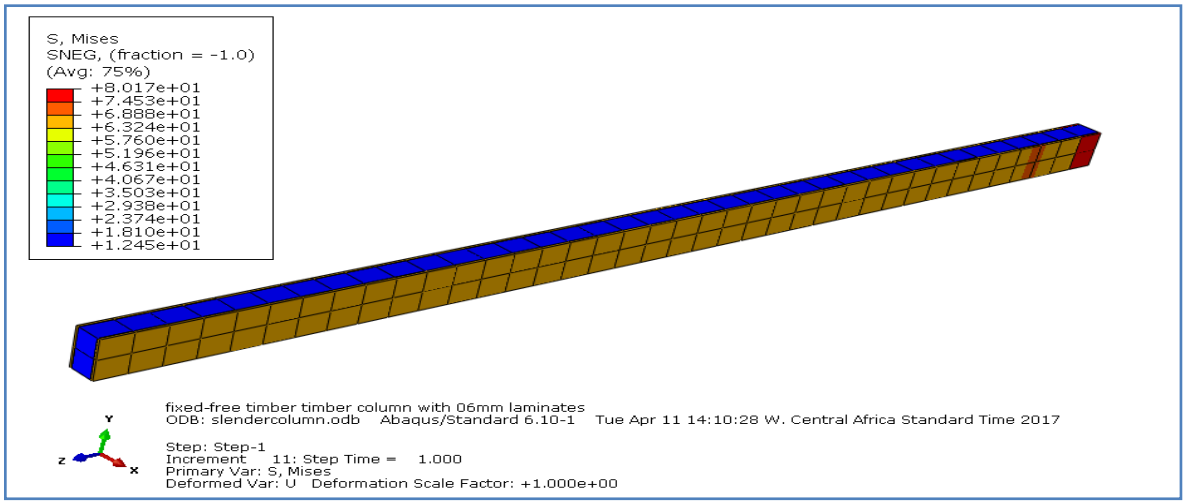


Figure 4.19: Stress in fixed-free timber column with 6mm aluminium laminate

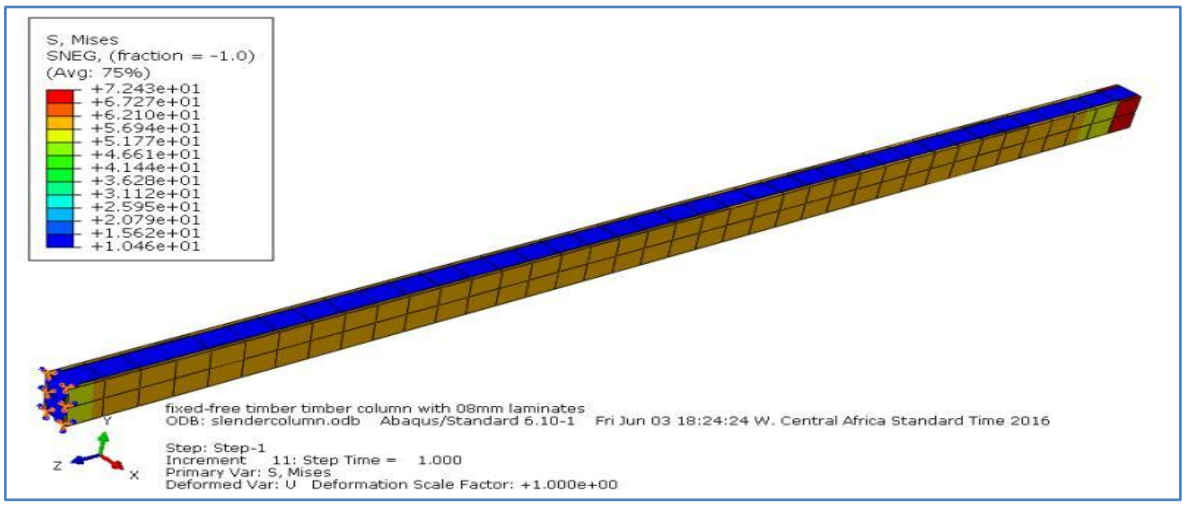


Figure 4.20: Stress in fixed-free timber column with 8mm aluminium laminate

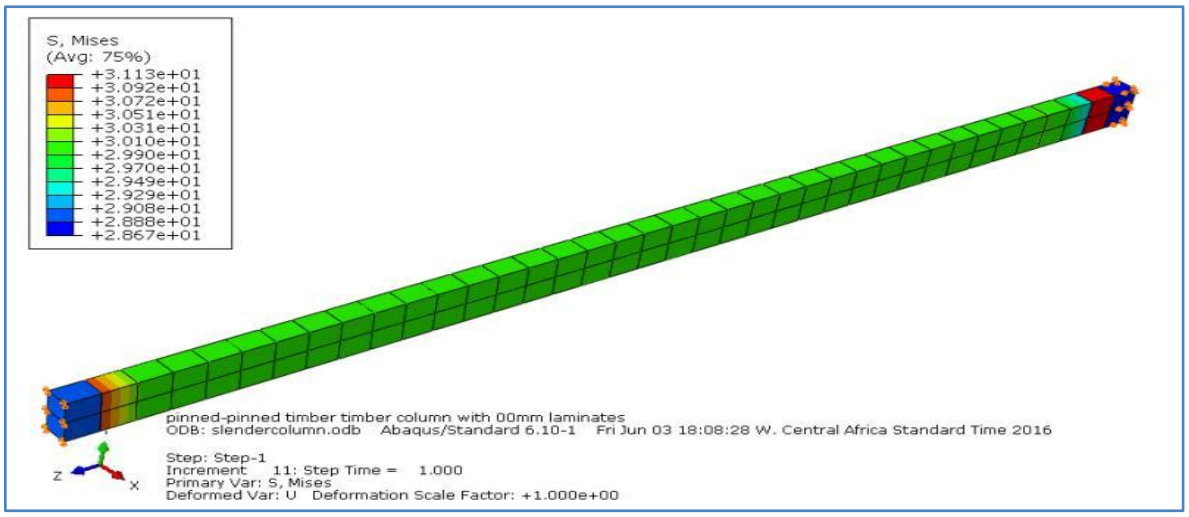


Figure 4.21: Stress in pinned-pinned timber column with no laminate

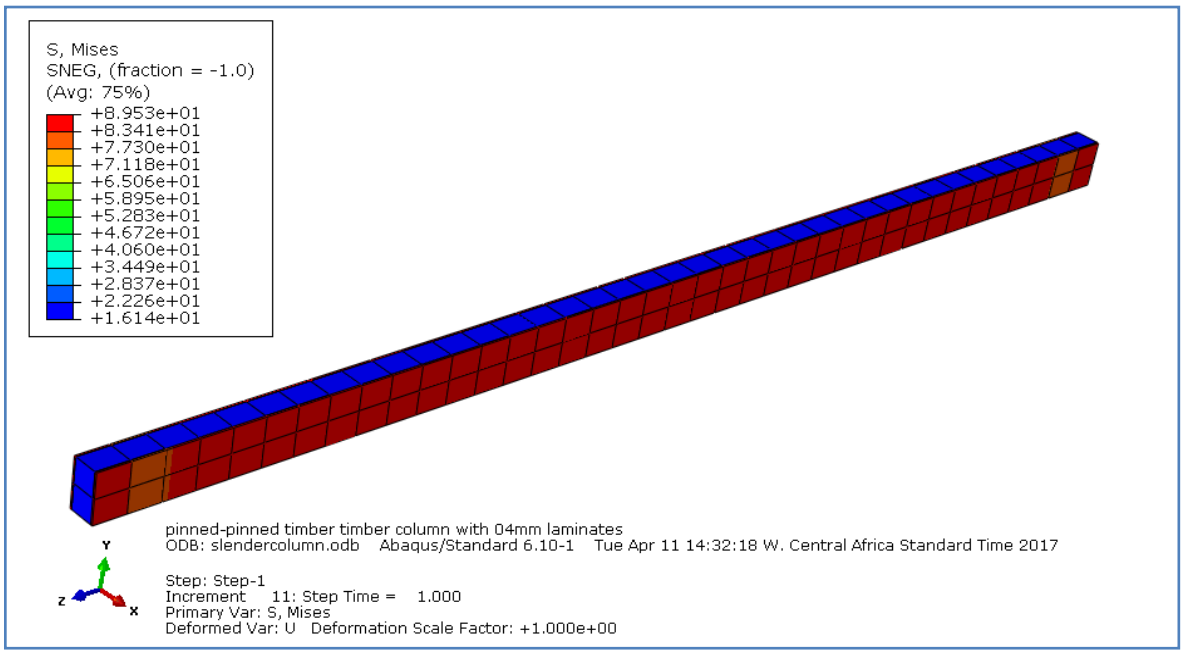


Figure 4.22: Stress in pinned-pinned timber column with 4mm aluminium laminate

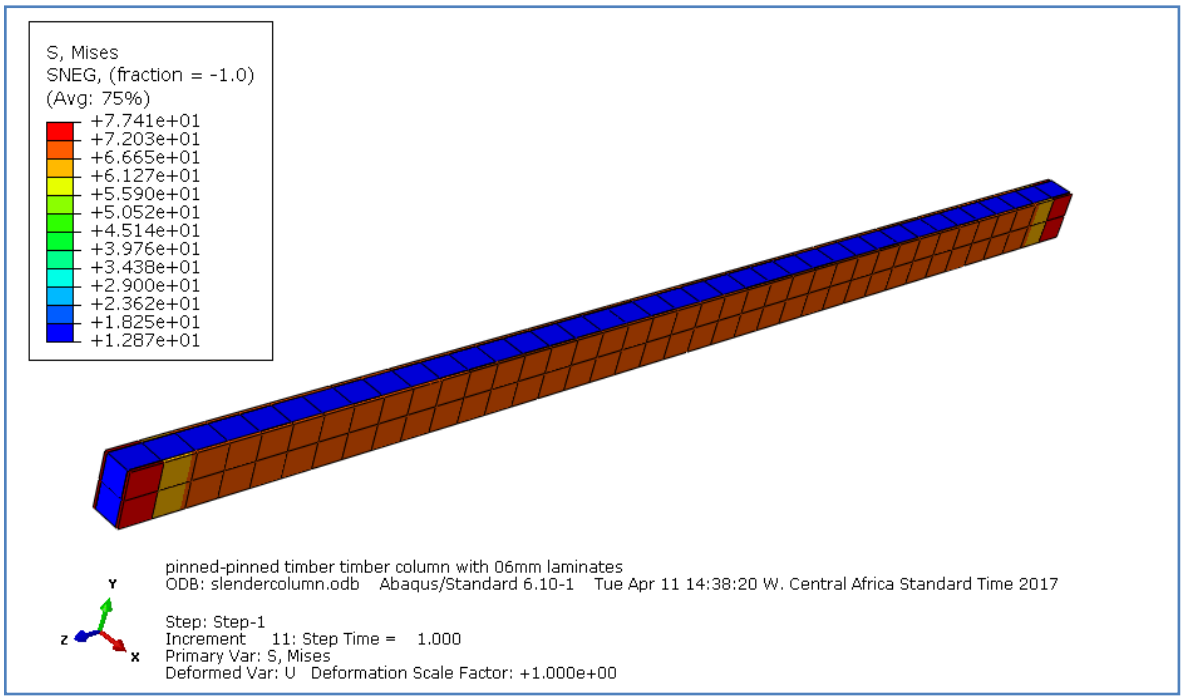


Figure 4.23: Stress in pinned-pinned timber column with 6mm aluminium laminate

Figures 4.21 to 4.24 show the effect of the change of laminate thickness on the stress in the composite timber column for pinned-pinned end-restraint condition. This can be seen in the change from 31.1N/mm^2 for timber column with no laminate to 89.5N/mm^2 and subsequently 77.4N/mm^2 and 69N/mm^2 for column with laminate thickness of 4mm, 6mm and 8mm respectively.

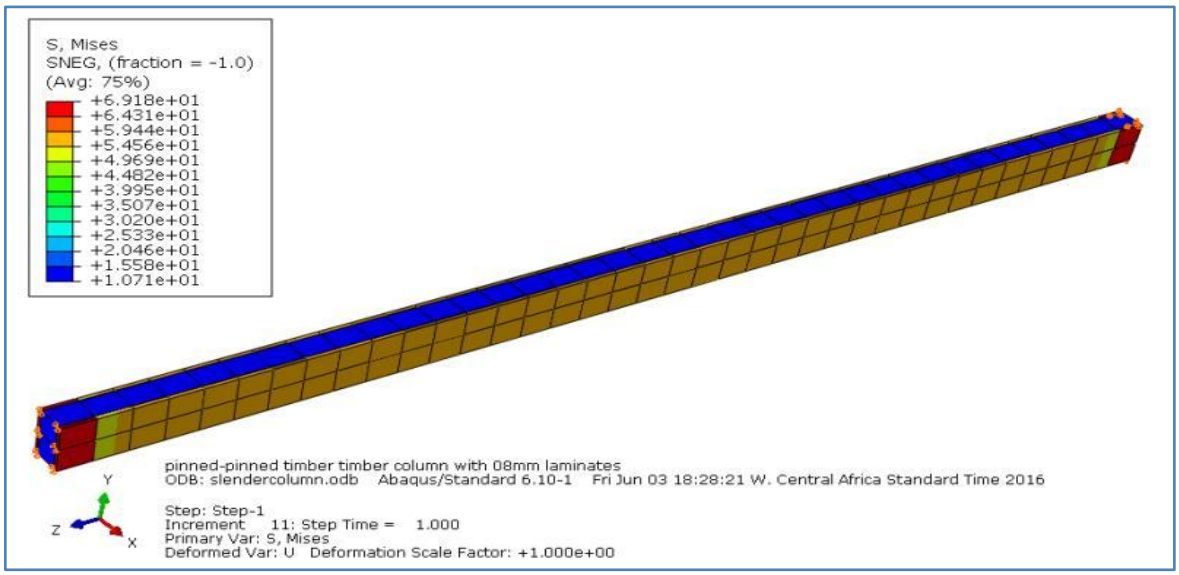


Figure 4.24: Stress in pinned-pinned timber column with 8mm aluminium laminate

Figures 4.25 to 4.50 show the behaviour of the laminated columns under the effect of increase in laminate thickness and end restraint condition for the four timber species used in this study. It can be observed that the deflection in all the three axes of the columns reduced with an increase in laminate thickness. This is due to the increase in section and hence an increase in stiffness and material properties of the composite column which ultimately increases the strength of the column.

It will also be observed that the end-restraint on the column affected the mode and amount of buckling. This is seen in the graphs that for fixed-fixed end condition, the amount of displacement is far less than that of the pinned-pinned and that of the fixed-free end condition. All these are related to the effective buckling length of the column which is least for fixed-fixed end conditions. Another observation is the fact that the deflection for the fixed end support shows little deviation from the initial values with increase in laminate thickness.

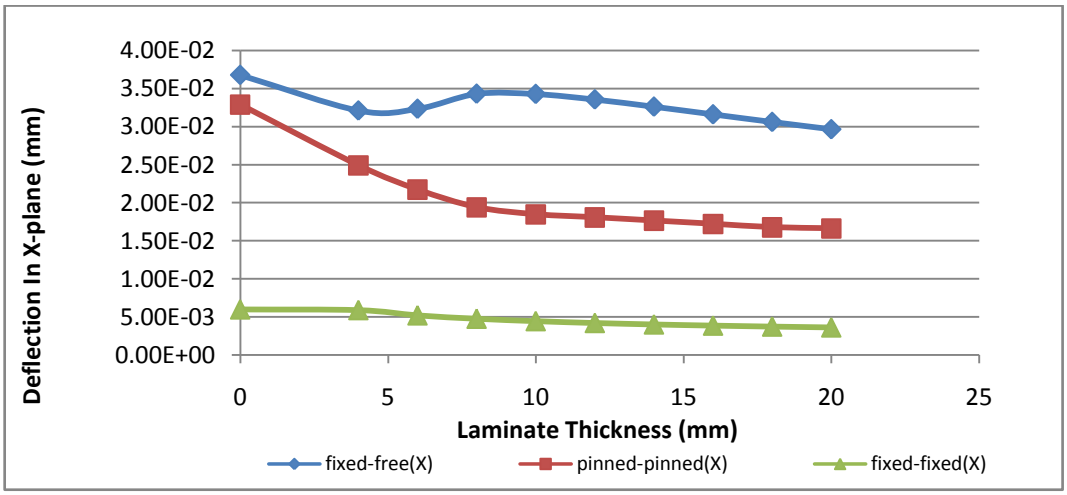


Figure 4.25: Deflection in x-axis versus Laminate Thickness for *Strombosia pustulata*

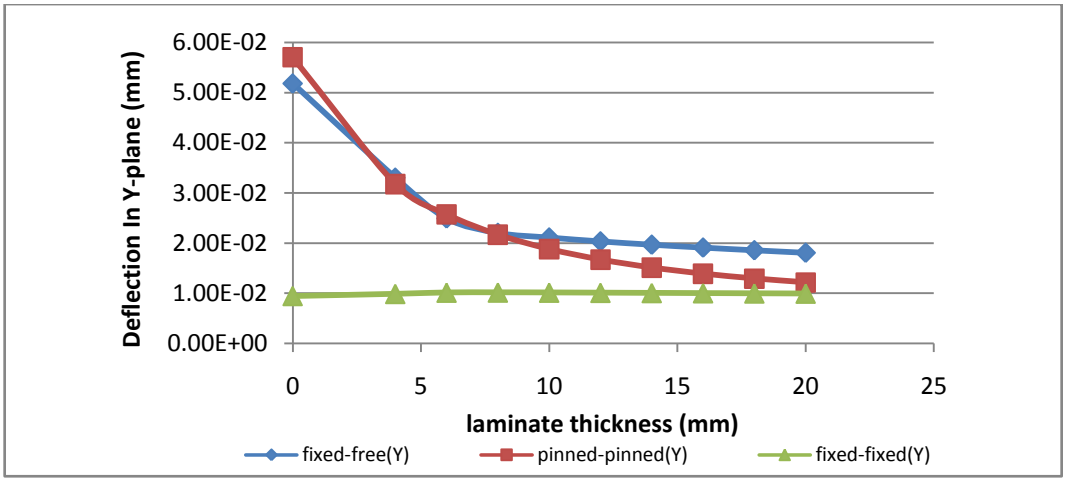


Figure 4.26: Deflection in y-axis versus Laminate Thickness for *Strombosia pustulata*

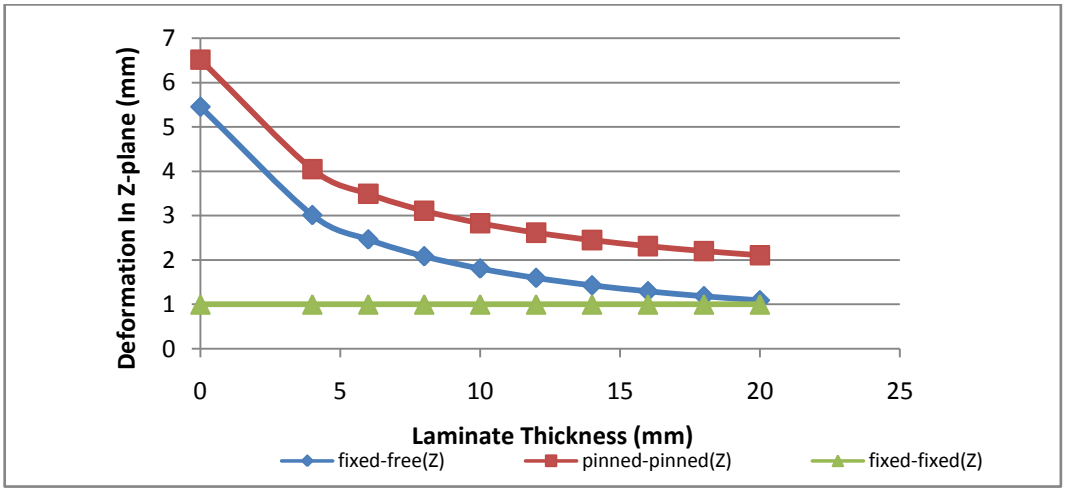


Figure 4.27: Deformation in z-axis versus Laminate Thickness for *Strombosia pustulata*

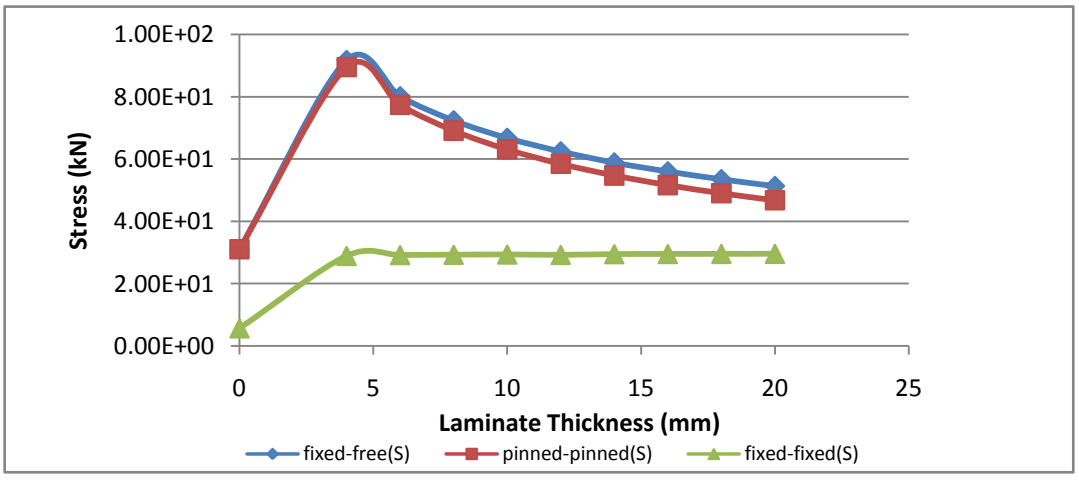


Figure 4.28: Stress Column versus Laminate Thicknesses for *Strombosia pustulata*

Figures 4.25, 4.26, 4.27 and 4.28show the displacement of the timber column in the x-axis, y-axis and z-axis respectively, whereby the displacement reduced from 3.28×10^{-2}mm to 2.96×10^{-2}mm in the x-axis while in the y-axis, the displacement reduced from 5.18×10^{-2}mm to 1.18×10^{-2}mm and 5.45mm to 1.093mm in the z-axis for the timber (*Strombosia Pustulata* specie) column that is fixed at one end and free at the other end. The same charts show the result of displacement in x-axis from 3.2×10^{-2}mm to 1.66×10^{-2}mm, 5.07×10^{-2}mm to 1.22×10^{-2}mm in y-axis and 6.511mm to 2.108mm in the z-axis for pinned-pinned end restraint condition.

The value of displacement of the timber column for the fixed-fixed end condition shows a slight variation as can be observed from the charts. There is a little change in displacement from 5.99×10^{-3}mm to 3.63×10^{-3}mm in the x-axis, and from 9.49×10^{-3}mm to 9.97×10^{-3}mm in the y-axis, while that of the z-axis stands at 1mm throughout the variation of laminate thickness without any significant change in displacement. This is due to the fact that the fixed-fixed end condition gives rigid connection to the column and thus there is little or no movement in any of the three axes of displacement.

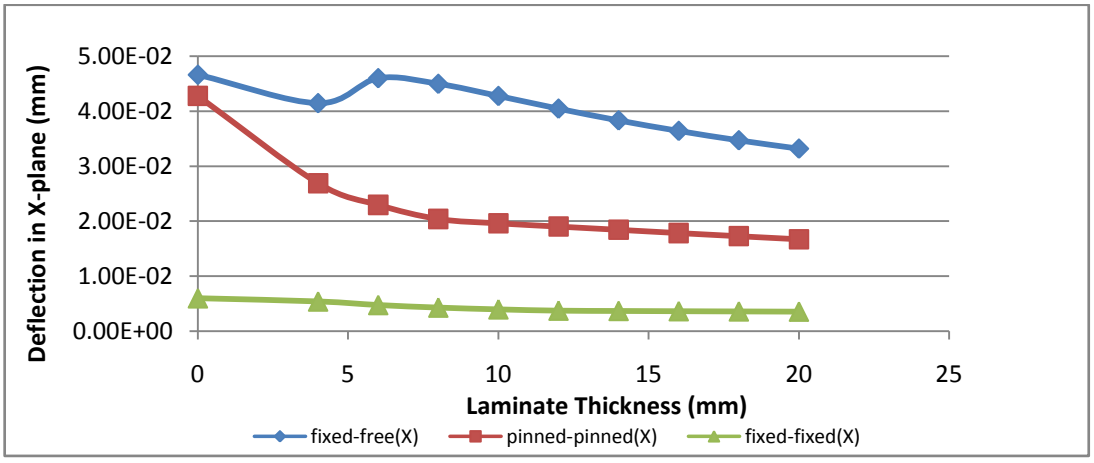


Figure 4.29: Deflection in x-axis versus Laminate Thickness for *Macrocarpa bequertii*

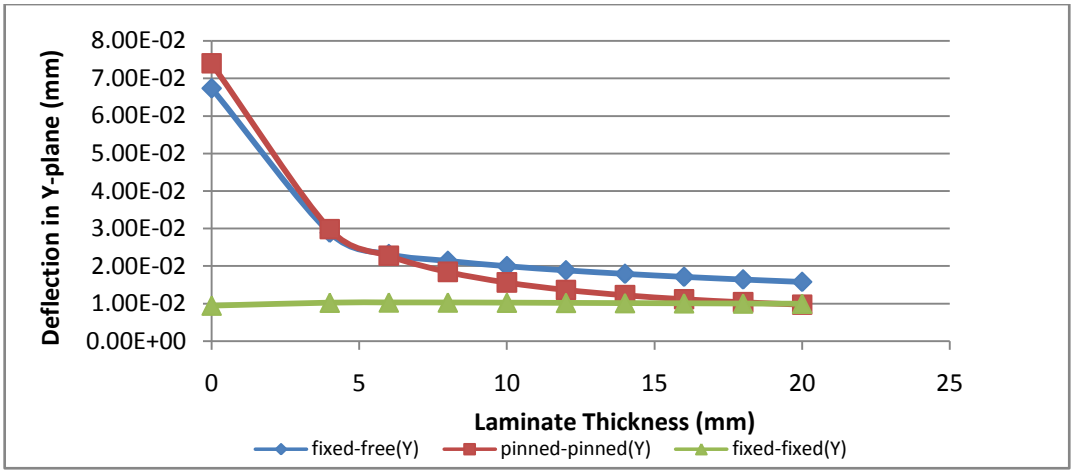


Figure 4.30: Deflection in y-axis versus Laminate Thickness for *Macrocarpa bequertii*

Figure 4.28 shows the stress behaviour of the column when loaded in various end conditions for *Strombosia pustulata* timber species. The stress increases from an initial value of 3.11×10^{1} N/mm^2 for timber without laminate to 9.18×10^{1} N/mm^2 and then reduced gradually to 5.14×10^{1} N/mm^2 with progressive laminate thickness for a fixed-free end restraint. In the case of the pinned-pinned end restraint, there was an increase from 3.11×10^{1} N/mm^2 to 8.95×10^{1} N/mm^2 and then finally reduced to 4.68×10^{1} N/mm^2. The stress pattern for fixed-fixed end restraint shows an increase from 5.7 N/mm^2 to 2.9×10 N/mm^2 where it remained at the constant value of 2.9×10 N/mm^2.

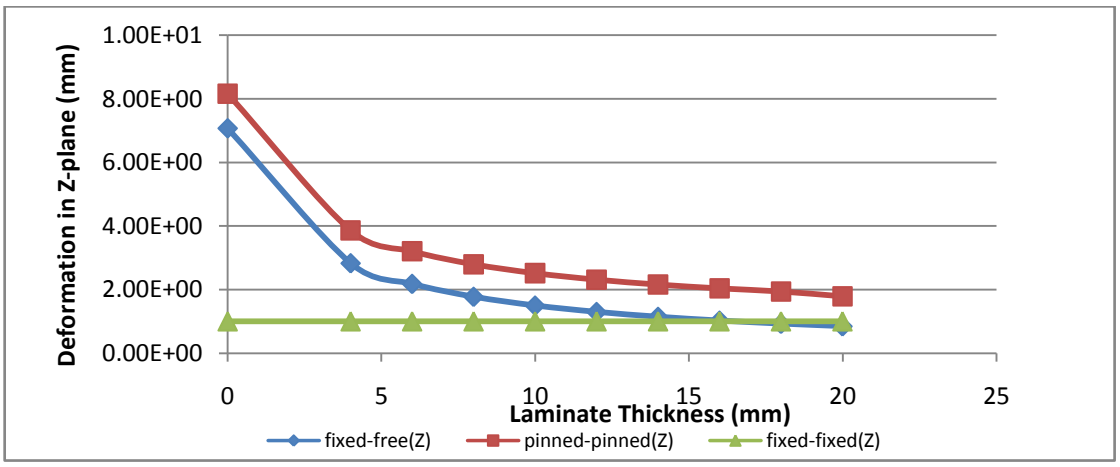


Figure 4.31: Deformation in z-axis versus Laminate Thickness for *Macrocarpa bequertii*

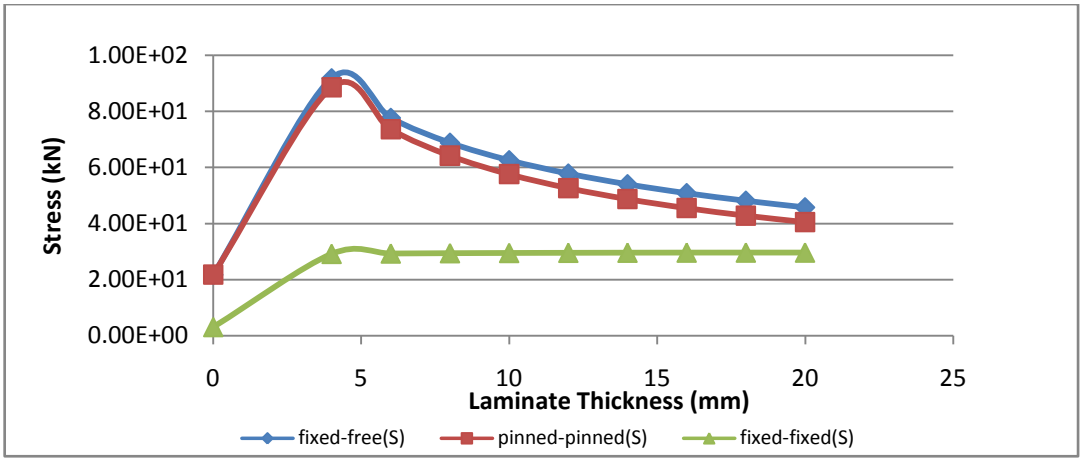


Figure 4.32: Stress in Composite Column versus Laminate Thickness for *Macrocarpa bequertii*

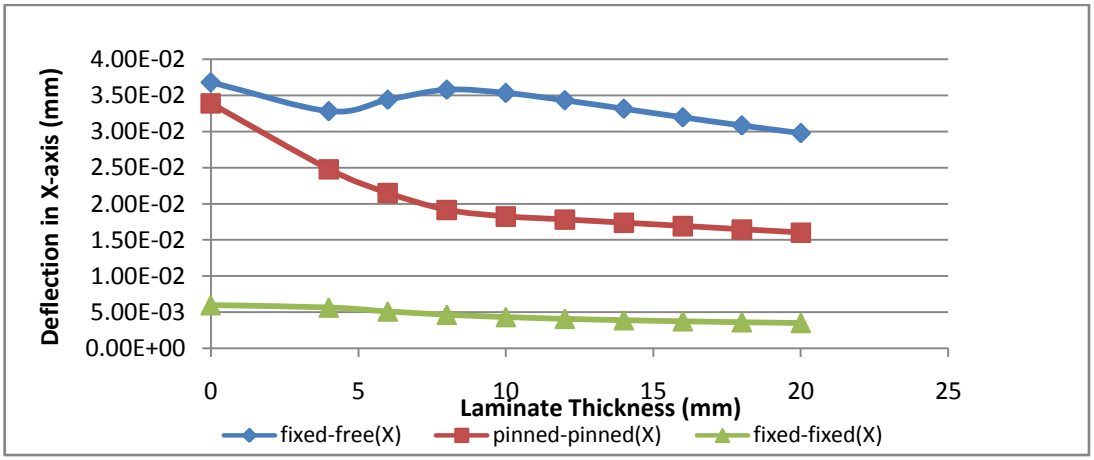


Figure 4.33: Deflection in x-axis versus Laminate Thickness for *Nauclea diderrichii*

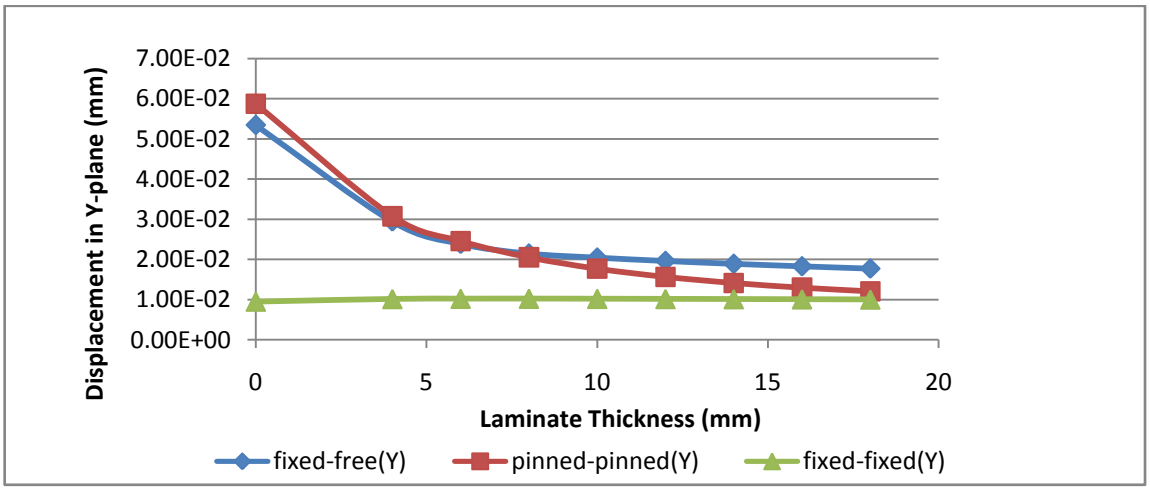


Figure 4.34: Deflection in y-axis versus Laminate thickness for *Nauclea diderrichii*

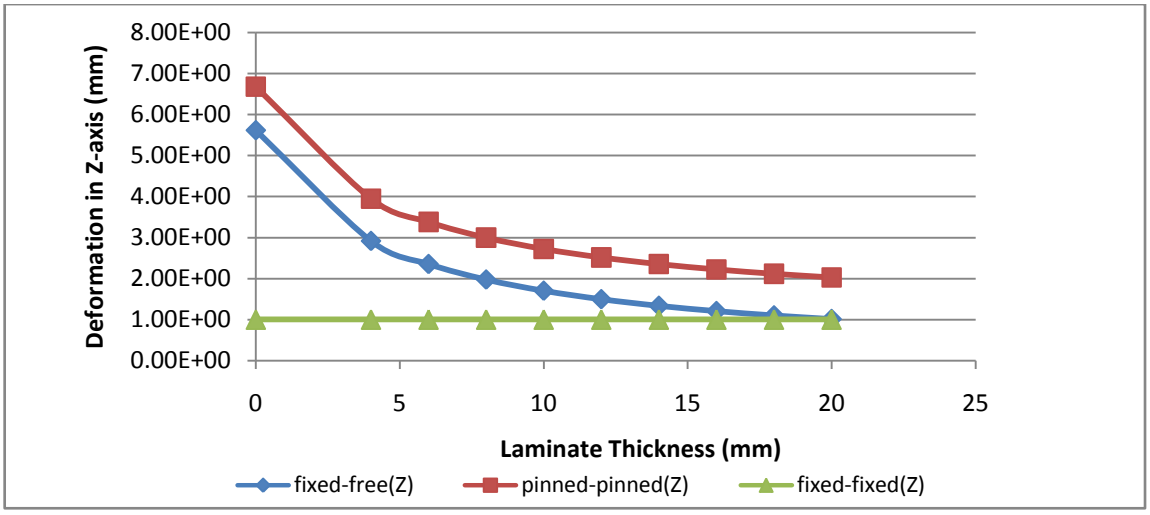


Figure 4.35: Deformation in z-axis versus Laminate Thickness for *Nauclea diderrichii*

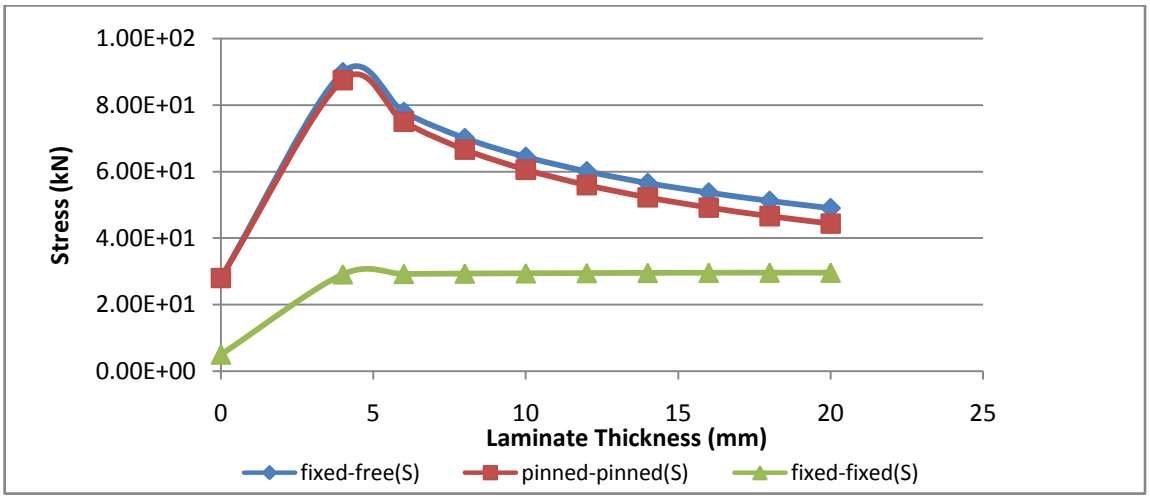


Figure 4.36: Stress in Composite Column versus Laminate Thickness for *Nauclea diderrichii*

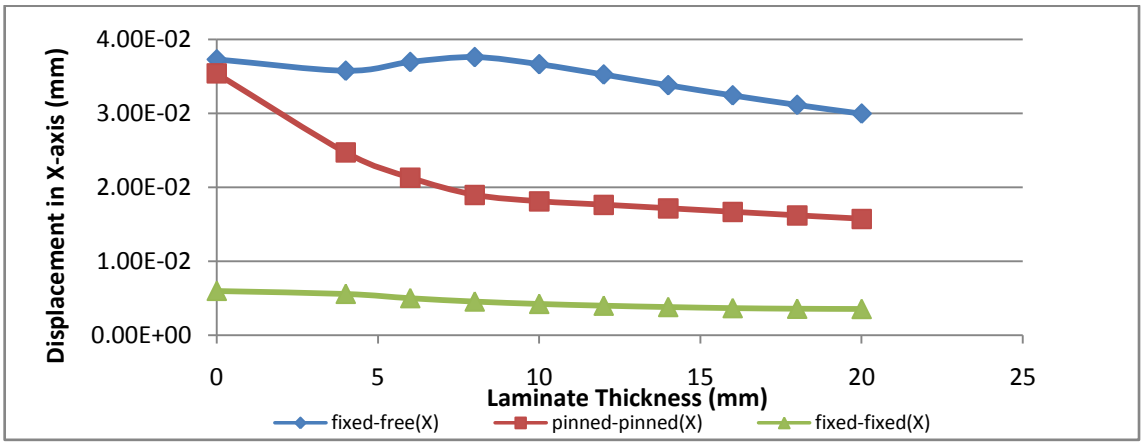


Figure 4.37: Deflection in x-axis versus Laminate Thickness for *Entandrophragma cylindricum*

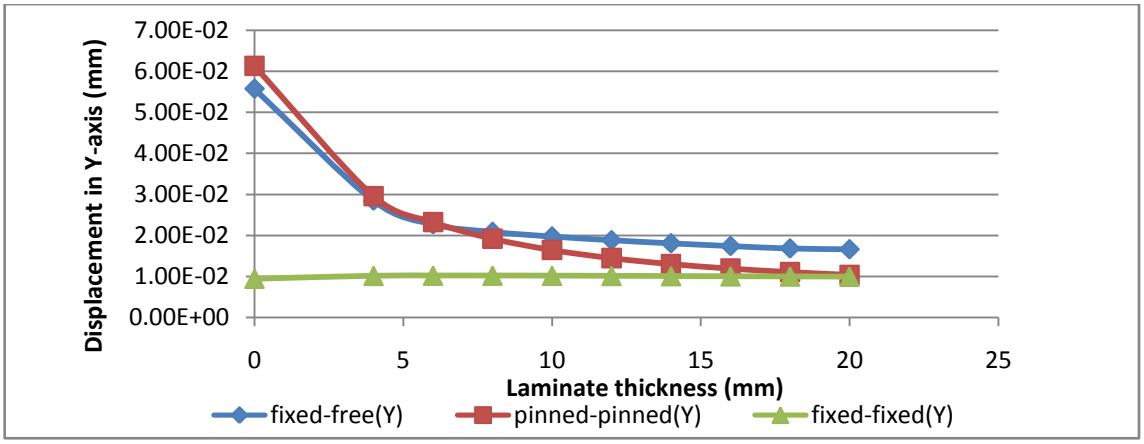


Figure 4.38: Deflection in y-axis versus Laminate Thicknessfor *Entandrophragma cylindricum*

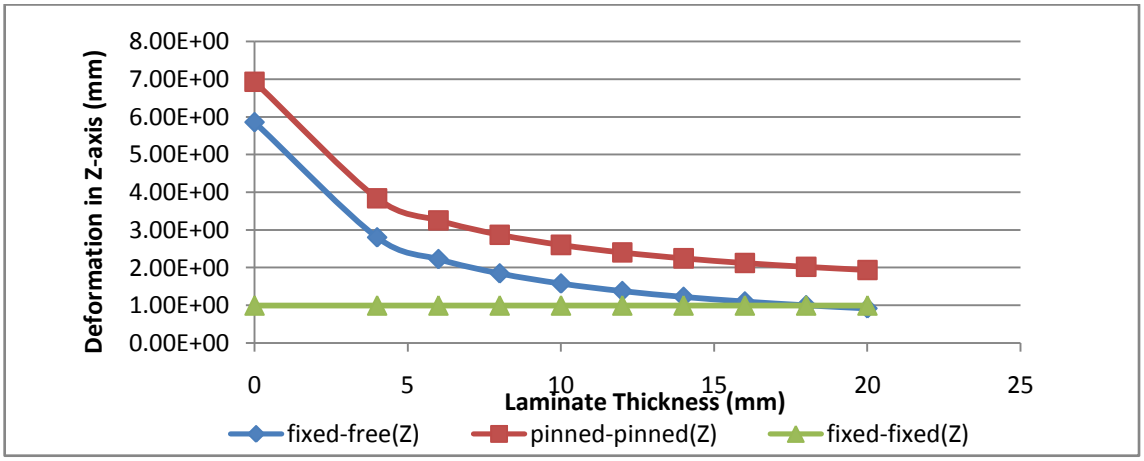


Figure 4.39: Deformation in z-axis versus Laminate Thicknessfor *Entandrophragma cylindricum*

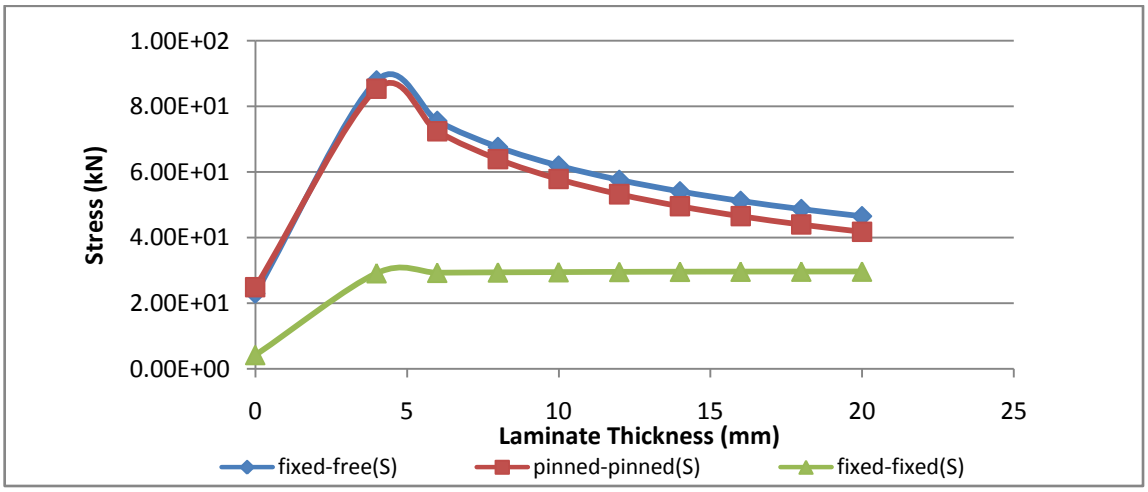


Figure 4.40: Stress in Composite Column versus Laminate Thickness for *Entandrophragma cylindricum*

The stress behaviour as shown in Figures 4.28, 4.31, 4.35, 4.40 shows a sharp increase in the Von mises stress from 30N/mm^2 for just the timber column without laminates to 90N/mm^2 after which there is a steady decrease in stress with further variation of the laminate thickness.This implies that the laminated column is capable of carrying loads which are far above the characteristic strength of timber columns without laminates.

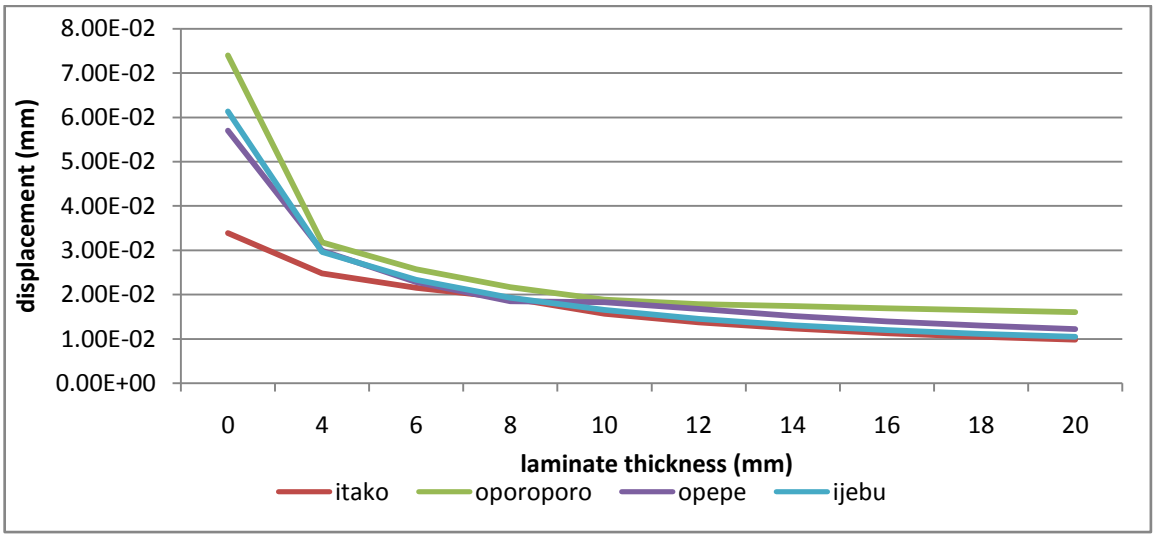


Figure 4.41: Comparison of displacement of four timber species

In the case of the effect of the strength of the timber specie on the deflection and stress pattern of the composite columns, a comparison in the four timber species shows that with increase in timber strength, there is a reduction in deflection in all the three axes as shown in figure 4.41. The deflections in the x-axis for the fixed-free end-restraint condition show that *Strombosia pustulata*, which has a modulus of elasticity of 9.18 kN/mm^2, undergoes a deflection of 3.68×10^{-2}mm, *Macrocarpa biquerti* which has a modulus of elasticity of 4.95kN/mm^2 undergoes a deflection of 4.67×10^{-2}mm, *Nauclea diderichii* with a modulus of elasticity of 8.42kN/mm^2 undergoes a deflection of 3.68×10^{-2}mm whereas *Entandrophragma cylindricum* with a modulus of elasticity of

7.36 kN/mm^2 undergoes a deflection of 3.73×10^{-2}mm. This shows that the strength of timber species plays a vital role in the load carrying capacity of the timber column.

4.2 Reliability AnalysisResults

The results for the reliability analysis are presented in Figures 4.42 to 4.51 and all results show a general improvement in the safety (reliability) indices of the column with increase in laminate thickness. The performance of the structure shows varied safety indices for the three modes of failure and for the design parameters that were inputted into the limit state functions.

4.2.1 Results for compression mode of failure

The results for the compression mode of failure show that the column is very safe in compression with all varied parameters having safety indices well above the target safety index of 3.8. Figure 4.42 shows the effects of varying the imposed loads on the safety of the column whereby the safety index increases steadily from 17.4 without laminate to 29.6 with 20mm laminate for *Strombosia pustulata*when an imposed load of 10kN is applied to the column. It can be observed that the increase in imposed load has a great impact on the safety of the column of 17.4 for the imposed load of 10kN to 4.3 for the imposed load of 30kN for the timber column without laminate.

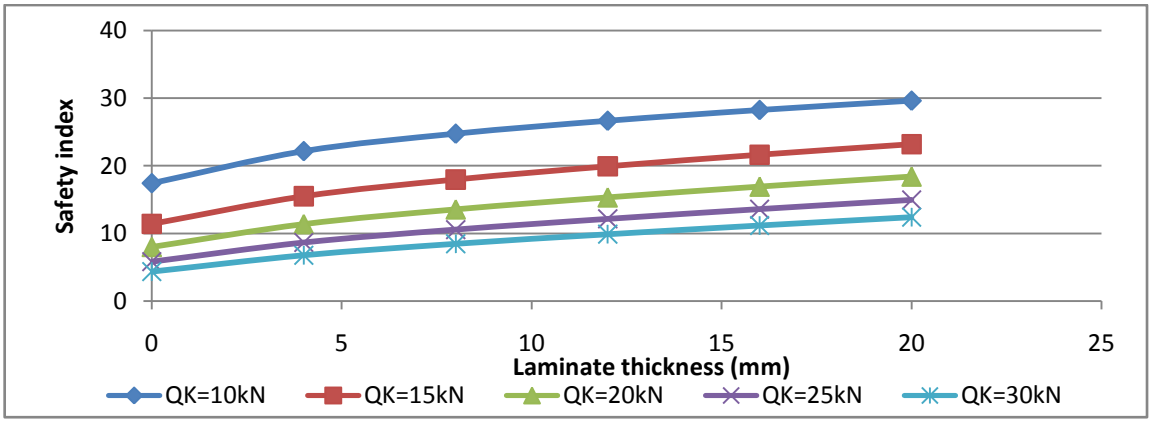


Figure 4.42: Safety index versus Laminate thickness at various loads for *Strombosia pustulata*

The effect of varying load ratios on the safety of the column is seen in figure 4.43 where, for a laminate thickness of 8 mm the safety index is 22.1, 23.7, 25.6, 27.6 and 29.7 for load ratios of 1.0, 0.8, 0.6, 0.4 and 0.2 respectively.

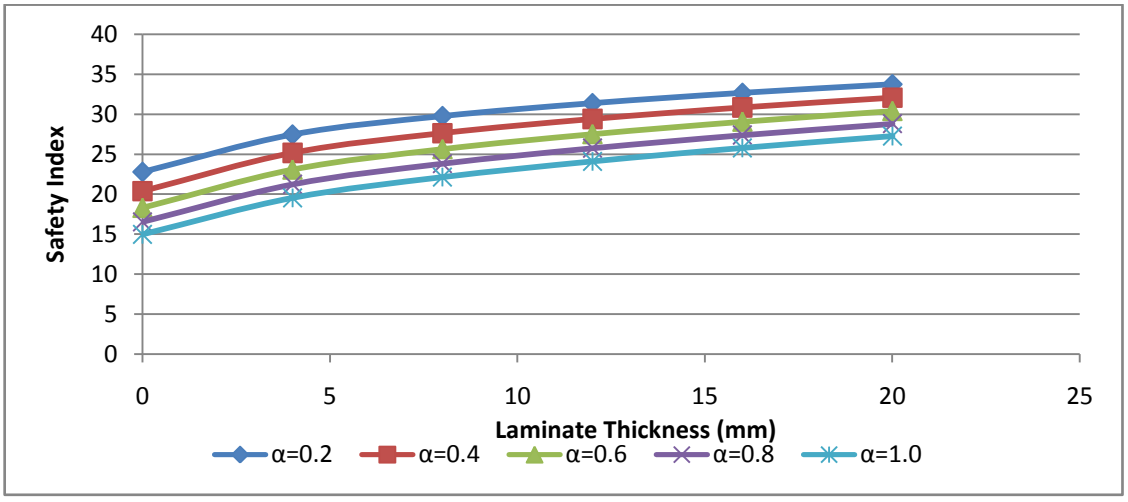


Figure 4.43: Safety index versus Laminate thickness for dead-to-live ratios for *Strombosia pustulata*

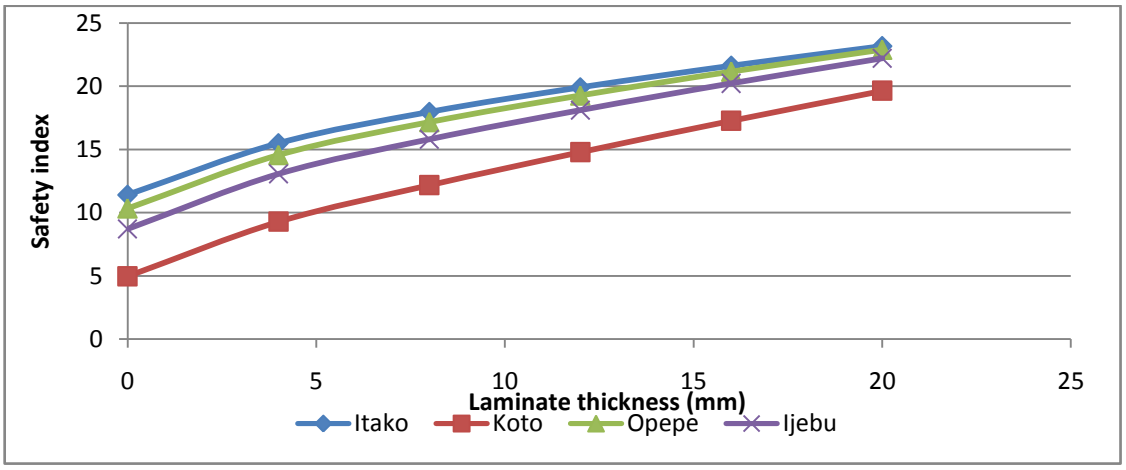


Figure 4.44: Safety index versus Laminate thickness for test species

Figure 4.44 shows the effects of the strength of the timber species on the safety indices of the column using all the standard design parameters of imposed load, column height, section of the timber column and thickness of aluminium laminates. It will be observed that varying the imposed loads has a greater effect on the safety index with a change in safety indices from 4.3 to 17.4 for the timber column without laminates and from 12.4 to

29.6 for the column with laminate thickness of 20mm. The effect of the timber compressive strength is also evident in Figure 4.44 with the big disparity in the safety indices for the four timber species. It can be observed that itako timber specie gave a large safety index of 11.4 without laminate and increased steadily to a safety index 23.2 with laminate of 20mm thickness.

4.2.2 Results for bending mode of failure.

The results of the safety indices for the bending criterion of failure for the laminated timber columns are shown in Figures 4.45 to 4.48 with the varied parameters being imposed load, load ratios, column height and timber bending strength.

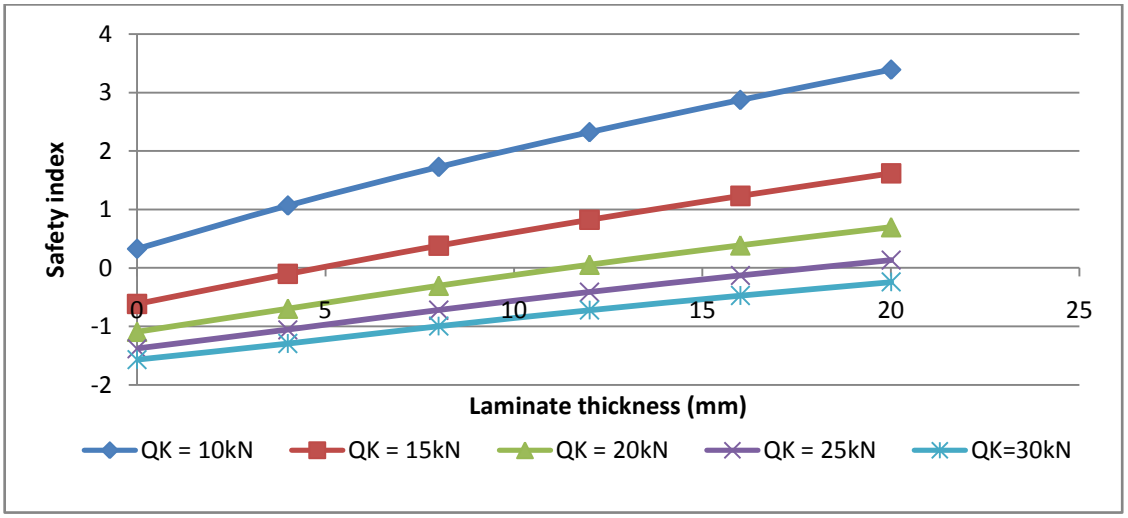


Figure 4.45: Safety Index versus Laminate Thickness for Various Loads for *Strombosia Pustulata*

Figure 4.45 shows the effects of load variation on the safety of timber columns in bending. It can be observed that the timber column is not safe for all imposed loads except for the applied load of 10kN where the safety index is 0.32 for column without laminate and 3.38 for laminate with thickness of 20mm.

Figure 4.46gives the effect of varying load ratios on the safety of the column under bending loads. It can be observed that the column is not safe for laminate thickness less than 8mm with the values of the least safety index of 0.027.

Figure 4.47 shows the effect thatvarying of height of column will have on the safety of the column. The column with height of 2.5 miswithin the safe zone with safety index of 0.14 without laminate and safety indices of 0.3, 1.5, 2.0, 2.5 and 3.0 for laminate thickness of 4 mm, 8 mm, 12 mm, 16 mm and 20 mm respectively.For heights greater than 2.5 m, the inclusion of laminate to the column is necessary to make the columns safe. It is also observed in Figure 4.47 that the change in length gave the most critical state in the change of the safety indices of the column while variations in load ratios have the least impact on the column safety index. As such the design of the column should be more centred on the column height than other parameters in bending failure mode.

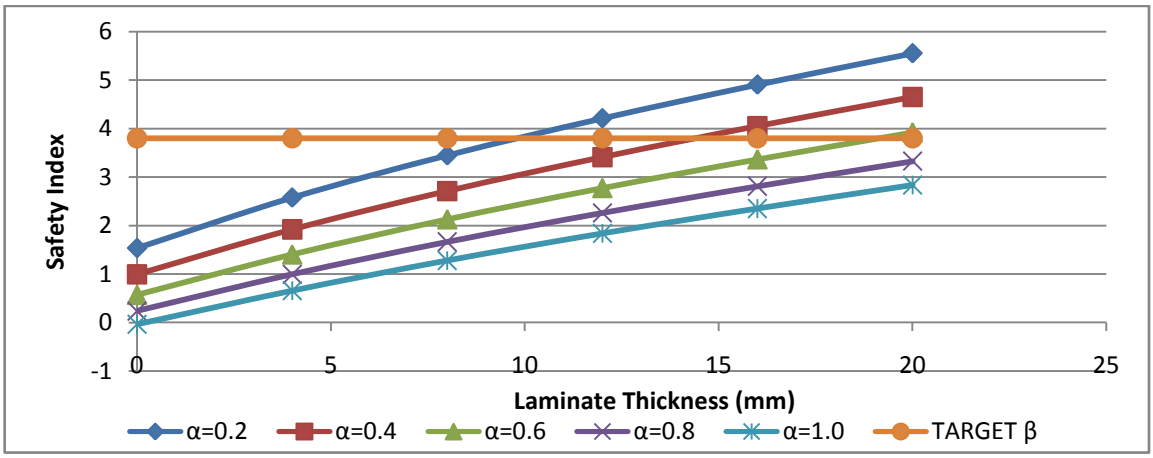


Figure 4.46: Safety index versus Laminate thickness for various dead-to-live ratios for *Strombosia pustulata*

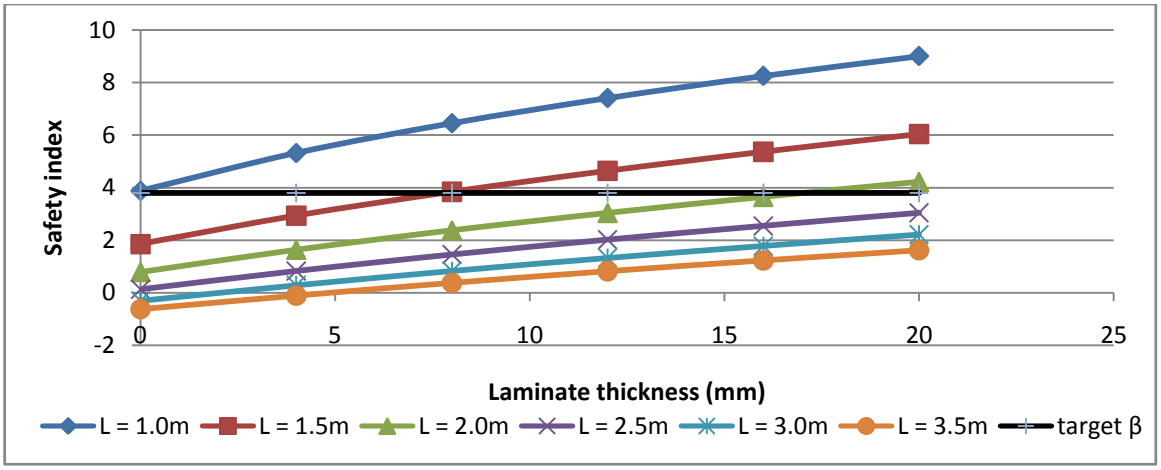


Figure 4.47: Safety index versus Laminate thickness for various column heights for *Strombosia pustulata*

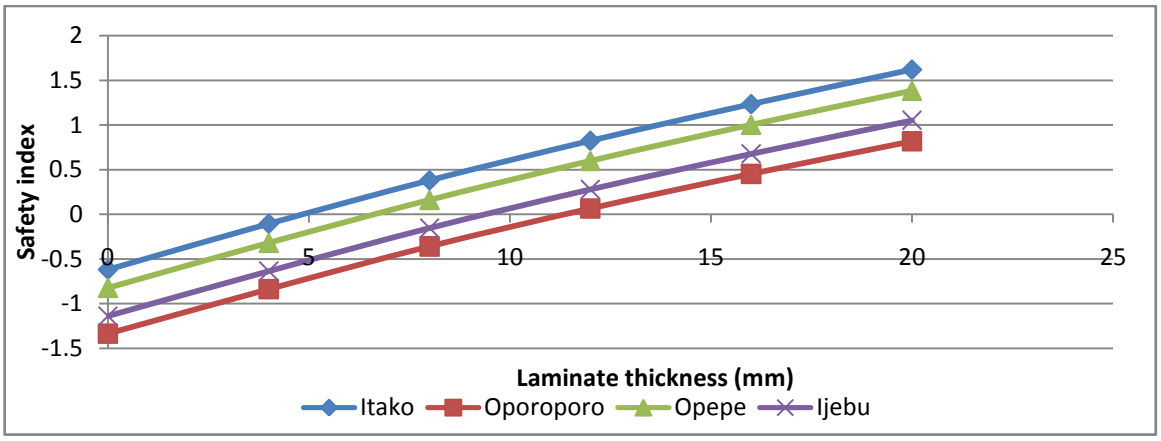


Figure 4.48: Safety index versus Laminate thickness four timber species using standard design parameters

Figure 4.48 shows the safety indices for the four timber species under the same design parameters of column height, load and geometric section. It can be observed that itako is safest of the four timber species with safety index of 0.37 with a laminate of 8 mm thickness at which point the other timber species are in the failure zone. Hence, a laminates of 12 mm thickness are needed for the columns to be safe.

4.2.3 Results for flexural buckling mode of failure

Figures 4.49 to 4.51 show the safety indices of the timber column for the flexural buckling mode of failure. The plots show that varying loads and column height are the critical parameters that affect the safety of the column in this mode of failure.

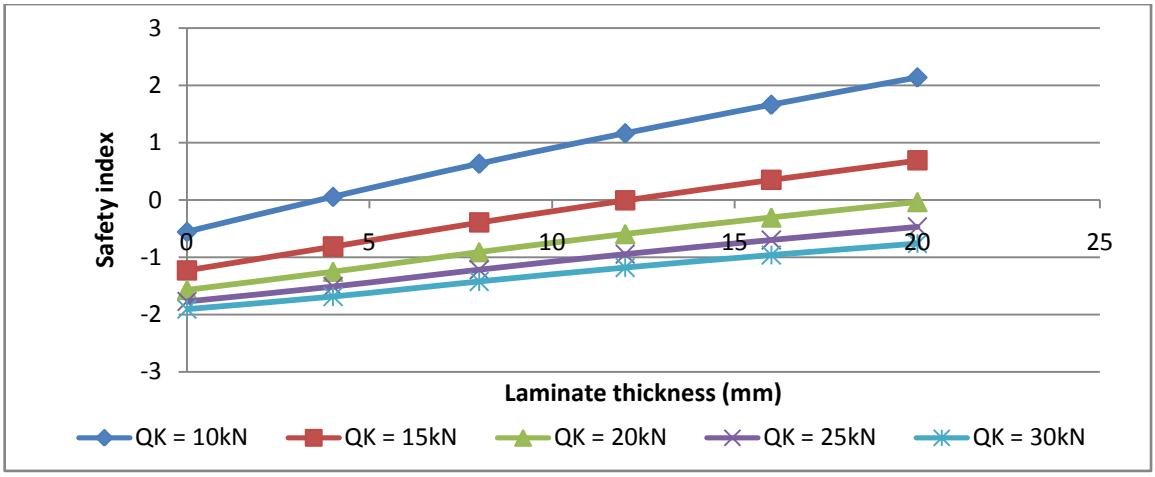


Figure 4.49: Safety index versus Laminate thickness for various loads for *Strombosia pustulata*

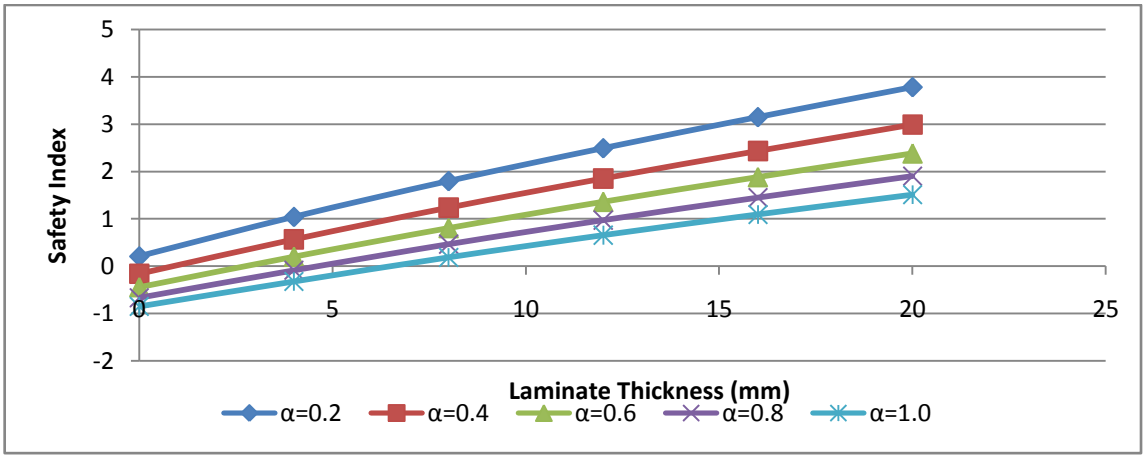


Figure 4.50: Safety index versus Laminate thickness for dead-to-live loads ratios for *Strombosia pustulata*

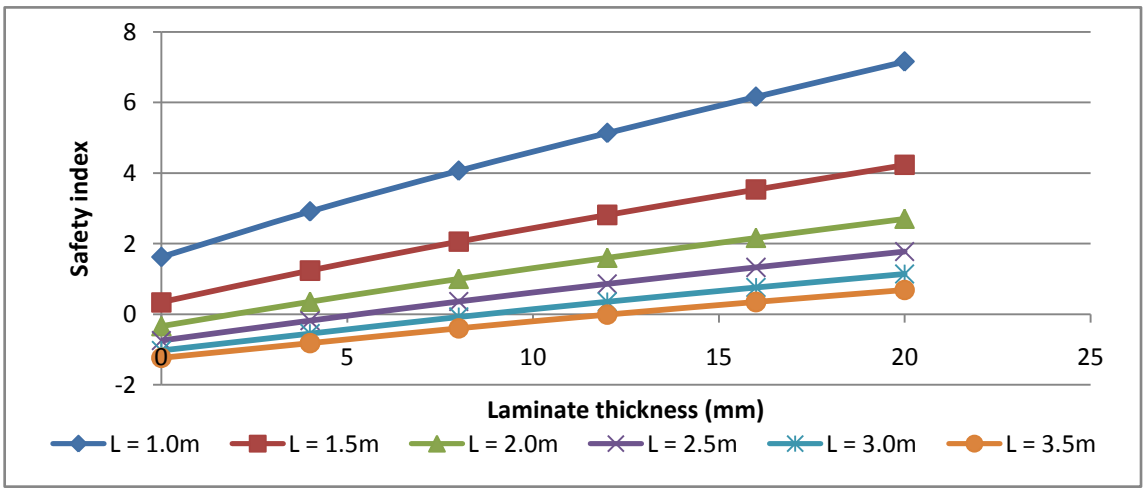


Figure 4.51: Safety Index versus Laminate Thickness with Varied Column Height

Figure 4.49 shows the result of varying imposed loads on the safety of the laminated timber columns. It will be observed that the column becomes unsafe for imposed loads of 20kN, 25kN and 30kN at all laminate thickness. The column is safe for 15kN when the laminate of 12mm thickness is used and is safe for all values of 10kN with the highest safety index being 2.14 for laminate of 20mm thickness.

Figure 4.50 shows the result of load ratios on the safety of the composite column where the column is safe when a laminate thickness of 12mm is used for the column.

Figure 4.51 shows the safety indices based on the height of column for flexural buckling. The result shows that that the column is safe for heights of 1m and 1.5m. For greater heights of columns, laminates of 4 mm thickness and 8 mm thickness are to bring columns of heights of 2 m and 2.5 m respectively to the safe zone while laminate of 12 mm thickness is needed to make the columns of heights 3 m and 3.5 m safe.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

5.1 CONCLUSION

In this research work, Finite Element Analysis and Reliability-based analyses of solid slender timber columns were carried out to evaluate the strength of four timber species (*Strombosia pustulata, Macrocarpa biquertii, Entandrophragma cylindricum and Nauclea diderichii*)laminated with aluminium sheets to determine the effect of the laminates on the load carrying capacity of the composite column. The analysis of laminated slender timber columns with varying aluminium laminate thickness were carried out considering the effects of load variation, timber strength and column height on the safety of the timber columns.

From the findings of the studies, the following conclusions are made:

i. The type of end restraint condition affects the load carrying capacity of the timber columns. The displacement in the three axes of displacement of the columns is least for the fixed-fixed restraint condition.

ii. The higher the strength of the timber, the higher the value of safety index and the lower the displacement of the timber and hence the greater the load carrying capacity of the timber.

iii. Of the modes of failure considered, the most critical failure mode is the flexural buckling mode of failure with safety indices as low as -1.23 for column height of 3.5 m as against what is obtainable for bending mode of failure which is -0.61 for the same column height of 3.5 m.

iv. The variation in imposed loads affects the safety indices of the laminated timber columns.

v. The results of the study show that for a timber column, the effect of change in imposed loads had an effect on the safety indices of columns for both compression and bending but is more critical in bending.

vi. The effect of variation of load ratio is more profound on the safety index of columns in bending while the effect of change in load ratio showed little effect on the overall change of safety index.

vii. It is observed in the charts that the aluminium laminate greatly increased the strength of the column and hence gave a favourable increase in the safety indices for all the failure criteria and on the strength of timber.

viii. The strength of timber column is increased significantly with the increase in laminate thickness. The results of the finite element analysis show that for deflection, there is little change in deflection from the point of application of the aluminium laminates of 8 mm to 10 mm thickness. This value is coincidental with the point of deflection in the x-axis where there seem to be a change in the weaker axis of buckling.

5.2 RECOMMENDATIONS

Based on the findings in this study, the following recommendations are made

i. In the design of columns, the critical mode of failure to be considered and designed for, is the flexural buckling mode of failure where the geometrical properties such as height, thickness and depth of the column,

ii. The nature of load variations during the design life should be considered especially in structures where the functions of the structures can be altered for other purposes,

iii. Designs for columns using fixed-fixed end restraint conditions are safe. However, for aluminium laminate of 8 – 10 mm thickness are recommended for use with timber columns of 3.5 m height, 300 mm depth and 100 mm thickness,

iv. Thus, it will be recommended that for a design of this nature, 8-10mm laminate thickness be used along with a pinned ended connection for reduced stress, and

v. The impact of the 8mm and 10mm laminate is coincidental for both finite element analysis and for reliability analysis whereby most if the failure criteria gave a safe column for most of the analysis variable considered, hence the use of finite element analysis and reliability analysis of structures are veritable tools in the safe design of structures.

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APPENDICES

APPENCIX A: PROGRAM LISTING

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% THIS IS A MATLAB PROGRAM FOR THE STRUCTURAL RELIABILITY
ANALYSIS
% OF
%
%
% NAME: MANGUT SAMUEL NAGU-NUM
% REG. NO: P13EGCV8022
%
% MAJOR SUPERVISOR: PROF. I. ABUBAKAR
% MINOR SUPERVISOR: Dr. A. OCHOLI
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% ROUTINE FOR DATA INPUT
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
disp('   ')
clc
[cov1,cov2,cov3,cov4,cov5]...
    = covariance1bending;
disp('    ')
QK = input(' INPUT THE IMPOSED LOAD ON COLUMN IN kN/m   ');
disp('    ')
k = input('  INPUT KMOD OF TIMBER  ');
disp('    ')
L = input(' INPUT THE HEIGHT OF COLUMN IN m   ');
disp('    ')
B = input('   INPUT TIMBER MODULUS OF ELASTICITY IN N/mm   ');
disp('    ')
D = input('   INPUT ALUMINIUM MODULUS OF ELASTICITY IN N/mm   ');
disp('    ')
ALPHA = input('   Input dead to live load ratio   ');
disp('    ')
gammag = 1.35; gammaq = 1.5; A = 1.3;
FT = input('   INPUT TIMBER BENDING STRENGTH N/mm^2   ');
disp('    ')
FA = input('INPUT BENDING STRENGTH OF ALUMINIUM  ');
disp('    ')
BT = input('   INPUT THICKNESS OF THE TIMBER COLUMN IN mm   ');
disp('    ')
BA = input('   INPUT THICKNESS OF THE ALUMINIUM LAMINATE IN mm   ');
disp('    ')
disp('    ')
```

```
DC = input(' INPUT DEPTH OF COMPOSITE COLUMN  ');
disp('    ')
IT = (BT*DC^3)/12;
disp('    ')
IA = (BA*DC^3)/12;
disp('    ')
H = DC/2;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% ROUTINE FOR THE FIRST ORDER RELIABILITY ANALYSIS
%
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
beta_mode_1 =
mangut_bending(QK,gammag,ALPHA,gammaq,H,L,A,FTb,FA,D,k,B,...
     IT,IA,cov1,cov2,cov3,cov4,cov5);
pf = probability_of_failure(beta_mode_1);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
disp('The safety index')
disp('   ')
disp(beta_mode_1)
disp('   ')
disp('The probability of failure')
disp('   ')
disp(pf)
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% THIS IS A MATLAB PROGRAM FOR THE STRUCTURAL RELIABILITY ANALYSIS
% OF COMPOSITE TIMBER COLUMN WITH ALUMINIUM LAMINATE
%
% NAME: MANGUT SAMUEL NAGU-NUM
% REG. NO: P13EGCV8022
%
% MAJOR SUPERVISOR: Prof. I. ABUBAKAR
% MINOR SUPERVISOR: Dr. A. OCHOLI
%        #
%
         #
%######################################################################
#
%
%
disp('   ')
clc
[cov1,cov2,cov3,cov4,cov5,cov6,cov7]...
  = covariance1flex;
disp('    ')
QK = input(' INPUT THE IMPOSED LOAD ON COMPOSITE COLUMN IN N   ');
disp('    ')
ALPHA = input('   INPUT DEAD TO LIVE LOAD RATIO   ');
disp('    ')
Bb = input(' INPUT TIMBER COMPRESSION MODULUS OF ELASTICITY IN N/mm^2 ');
disp('    ')
Bc = input(' INPUT TIMBER BENDING MODULUS OF ELASTICITY IN N/mm^2 ');
disp('    ')
D = input(' INPUT ALUMINIUM YOUNGS MODULUS OF ELASTICITY IN N/mm^2   ');
disp('    ')
k = input(' INPUT KMOD OF TIMBER ');
disp('     ')
FTc = input('   INPUT TIMBER COMPRESSIVE STRENGTH N/mm^2   ');
FTb = input('   INPUT TIMBER BENDING STRENGTH N/mm^2   ');
disp('    ')
FA = input('  INPUT ALUMINIUM COMPRESSIVE STRENGTH N/mm^2   ');
disp('    ')
BA = input('   INPUT THICKNESS OF ALUMINIUM LAMINATE IN mm   ');
disp('     ')
BT = input('   INPUT THICKNESS OF TIMBER   ');
disp('     ')
DC = input('  INPUT DEPTH OF COMPOSITE COLUMN  ');
disp('    ')
%X = input('   INPUT AREA OF TIMBER IN mm^2   ');
```

```
X = BT*DC;
disp('    ')
%%m = 0;
disp('    ')
%n = 150;
Y = BA*DC;
disp('    ')
L = input('  INPUT THE HEIGHT OF COLUMN IN m   ');
disp('    ')
B = input('  INPUT TIMBER MODULUS OF ELASTICITY IN N/mm   ');
disp('    ')
gammag = 1.35; gammaq = 1.5; A = 1.3;
disp('    ')
BT = input('  INPUT THICKNESS OF THE TIMBER COLUMN IN mm   ');
disp('    ')
IT = (BT*DC^3)/12;
disp('    ')
IA = (BA*DC^3)/12;
disp('    ')
H = DC/2;
%
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% ROUTINE FOR THE FIRST ORDER RELIABILITY ANALYSIS
% THE RELIABILITY ANALYSIS IS EXECUTED THROUGH A SPECIAL
% MATLAB FUNCTION CALLED 'mangut_compression'
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
beta_mode_1 = mangut_flexure(k,FTc,FTb,H,FA,D,Bb,Bc,X,Y,QK,ALPHA,gammag,...
        gammaq,L,IT,IA,cov1,cov2,cov3,cov4,cov5,cov6,cov7);
pf = probability_of_failure(beta_mode_1);
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
disp('The safety index')
disp('   ')
disp(beta_mode_1)
disp('   ')
disp('The probability of failure')
disp('   ')
disp(pf)
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% THIS IS A MATLAB PROGRAM FOR THE STRUCTURAL RELIABILITY
ANALYSIS
% OF COMPOSITE TIMBER COLUMN WITH ALUMINIUM LAMINATE
%
% NAME: MANGUT SAMUEL NAGU-NUM
% REG. NO: P13EGCV8022
%
% MAJOR SUPERVISOR: Prof. I. ABUBAKAR
% MINOR SUPERVISOR: Dr. A. OCHOLI
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% ROUTINE FOR DATA INPUT
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
disp('  ')
clc
[cov1,cov2,cov3,cov4,cov5]...
  = covariance1;
disp('   ')
QK = input(' INPUT THE IMPOSED LOAD ON COMPOSITE COLUMN IN N   ');
disp('   ')
ALPHA = input('   INPUT DEAD TO LIVE LOAD RATIO   ');
disp('   ')
B = input(' INPUT TIMBER YOUNGS MODULUS OF ELASTICITY IN N/mm^2   ');
disp('   ')
D = input(' INPUT ALUMINIUM YOUNGS MODULUS OF ELASTICITY IN
N/mm^2   ');
disp('   ')
gammag = 1.35; gammaq = 1.5; A = 1.3;
k = input(' INPUT KMOD OF TIMBER ');
disp('    ')
FT = input('   INPUT TIMBER COMPRESSIVE STRENGTH N/mm^2   ');
disp('   ')
FA = input('   INPUT ALUMINIUM COMPRESSIVE STRENGTH N/mm^2   ');
disp('   ')
X = input('   INPUT AREA OF TIMBER IN mm^2   ');
disp('   ')
m = input('   INPUT THICKNESS OF ALUMINIUM LAMINATE IN mm   ');
%%m = 0;
disp('   ')
n = 300;
Y = m*n;
%o = input('  input strain in timber  ');
%p = input('  input strain in Aluminium  ');
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% ROUTINE FOR THE FIRST ORDER RELIABILITY ANALYSIS
% THE RELIABILITY ANALYSIS IS EXECUTED THROUGH A SPECIAL
% MATLAB FUNCTION CALLED 'mangut_compression'
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
beta_mode_1 = mangut_compression(QK,gammag,ALPHA,gammaq,k,A,B,D,FT,...
    FA,X,Y,cov1,cov2,cov3,cov4,cov5);
pf = probability_of_failure(beta_mode_1);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
disp('The safety index')
disp('   ')
disp(beta_mode_1)
disp('   ')
disp('The probability of failure')
disp('   ')
disp(pf)
```

APPENDIX B:

Table B1: Deflection of *Strombosia* in x-axis for the Three End-Restraint Conditions

THICKNESS (mm)	fixed-free (mm)	pinned-pinned(mm)	fixed-fixed (mm)
0	3.68E-02	3.29E-02	5.99E-03
4	3.21E-02	2.49E-02	5.89E-03
6	3.23E-02	2.17E-02	5.21E-03
8	3.43E-02	1.94E-02	4.77E-03
10	3.43E-02	1.85E-02	4.45E-03
12	3.36E-02	1.81E-02	4.21E-03
14	3.26E-02	1.77E-02	4.02E-03
16	3.16E-02	1.72E-02	3.87E-03
18	3.06E-02	1.68E-02	3.74E-03
20	2.96E-02	1.66E-02	3.63E-03

Table B2: Deflection of *Strombosia*in y-axis for the Three End-Restraint Conditions

THICKNESS(mm)	fixed-free(mm)	pinned-pinned(mm)	fixed-fixed (mm)
0	5.18E-02	5.70E-02	9.49E-03
4	3.30E-02	3.18E-02	9.90E-03
6	2.49E-02	2.57E-02	1.02E-02
8	2.20E-02	2.17E-02	1.02E-02
10	2.11E-02	1.88E-02	1.02E-02
12	2.03E-02	1.67E-02	1.01E-02
14	1.97E-02	1.51E-02	1.01E-02
16	1.91E-02	1.39E-02	1.00E-02
18	1.86E-02	1.30E-02	1.00E-02
20	1.81E-02	1.22E-02	9.97E-03

Table B3: Deflection of *Strombosia* in z-axis for the Three End-Restraint Conditions

THICKNESS(mm)	fixed-free(mm)	pinned-pinned(mm)	fixed-fixed (mm)
0	5.45	6.511	1
4	3.012	4.044	1
6	2.464	3.49	1
8	2.085	3.108	1
10	1.809	2.829	1
12	1.598	2.616	1
14	1.431	2.449	1
16	1.297	2.313	1
18	1.186	2.202	1
20	1.093	2.108	1

Table B4: Stress in *Strombosia*for the Three End-Restraint Conditions

THICKNESS(mm)	fixed-free(kN)	pinned-pinned(kN)	fixed-fixed (kN)
0	3.11E+01	3.11E+01	5.70E+00
4	9.18E+01	8.95E+01	2.90E+01
6	8.02E+01	7.74E+01	2.93E+01
8	7.24E+01	6.92E+01	2.94E+01
10	6.68E+01	6.32E+01	2.94E+01
12	6.25E+01	5.85E+01	2.93E+01
14	5.89E+01	5.48E+01	2.96E+01
16	5.61E+01	5.17E+01	2.96E+01
18	5.36E+01	4.91E+01	2.96E+01
20	5.14E+01	4.68E+01	2.96E+01

Table B5: Deflection of *Macrocarpa* in x-axisfor the Three End-Restraint Conditions

THICKNESS(mm)	fixed-free(mm)	pinned-pinned(mm)	fixed-fixed(mm)
0	4.67E-02	4.28E-02	5.99E-03
4	4.15E-02	2.69E-02	5.41E-03
6	4.60E-02	2.30E-02	4.76E-03
8	4.50E-02	2.04E-02	4.30E-03
10	4.28E-02	1.96E-02	3.98E-03
12	4.05E-02	1.90E-02	3.75E-03
14	3.83E-02	1.84E-02	3.69E-03
16	3.64E-02	1.78E-02	3.65E-03
18	3.47E-02	1.73E-02	3.61E-03
20	3.32E-02	1.67E-02	3.57E-03

Table B6: Deflection of *Macrocarpa* in x-axisfor the Three End-Restraint Conditions

THICKNESS(mm)	fixed-free(mm)	pinned-pinned(mm)	fixed-fixed(mm)
0	6.73E-02	7.40E-02	9.49E-03
4	2.89E-02	2.99E-02	1.03E-02
6	2.32E-02	2.28E-02	1.03E-02
8	2.14E-02	1.85E-02	1.03E-02
10	2.00E-02	1.57E-02	1.03E-02
12	1.89E-02	1.37E-02	1.02E-02
14	1.80E-02	1.23E-02	1.02E-02
16	1.72E-02	1.12E-02	1.01E-02
18	1.65E-02	1.04E-02	1.00E-02
20	1.58E-02	9.77E-03	9.99E-03

Table B7: Deflection of *Macrocarpa* in x-axisfor the Three End-Restraint Conditions

THICKNESS(mm)	fixed-free(mm)	pinned-pinned(mm)	fixed-fixed (mm)
0	7.07E+00	8.159	1
4	2.829	3.861	1
6	2.18	3.205	1
8	1.774	2.795	1
10	1.497	2.515	1
12	1.296	2.312	1
14	1.143	2.158	1
16	1.023	2.037	1
18	9.26E-01	1.939	1
20	8.46E-01	1.787	1

Table B8: Stress in *Macrocarpa*for the Three End-Restraint Conditions

THICKNESS(mm)	fixed-free(kN)	pinned-pinned(kN)	fixed-fixed(kN)
0	2.18E+01	2.18E+01	3.08E+00
4	9.18E+01	8.86E+01	2.92E+01
6	7.77E+01	7.36E+01	2.93E+01
8	6.88E+01	6.42E+01	2.94E+01
10	6.26E+01	5.76E+01	2.95E+01
12	5.79E+01	5.26E+01	2.96E+01
14	5.40E+01	4.87E+01	2.96E+01
16	5.09E+01	4.55E+01	2.96E+01
18	4.82E+01	4.28E+01	2.96E+01
20	4.58E+01	4.05E+01	2.96E+01

Table B9: Deflection of *Nauclea* in x-axis for the Three End-Restraint Conditions

THICKNESS(mm)	fixed-free(mm)	pinned-pinned(mm)	fixed-fixed(mm)
0	3.68E-02	3.39E-02	5.99E-03
4	3.28E-02	2.48E-02	5.66E-03
6	3.44E-02	2.15E-02	5.12E-03
8	3.58E-02	1.92E-02	4.67E-03
10	3.53E-02	1.83E-02	4.35E-03
12	3.43E-02	1.78E-02	4.11E-03
14	3.31E-02	1.74E-02	3.92E-03
16	3.20E-02	1.69E-02	3.77E-03
18	3.08E-02	1.65E-02	3.65E-03
20	2.98E-02	1.60E-02	3.54E-03

Table B10: Deflection of *Nauclea* in y-axis for the Three End-Restraint Conditions

THICKNESS(mm)	fixed-free(mm)	pinned-pinned(mm)	fixed-fixed(mm)
0	5.34E-02	5.87E-02	9.49E-03
4	2.94E-02	3.07E-02	1.01E-02
6	2.39E-02	2.46E-02	1.02E-02
8	2.15E-02	2.05E-02	1.02E-02
10	2.05E-02	1.77E-02	1.02E-02
12	1.96E-02	1.57E-02	1.01E-02
14	1.89E-02	1.41E-02	1.01E-02
16	1.83E-02	1.30E-02	1.01E-02
18	1.77E-02	1.21E-02	1.00E-02
20	1.72E-02	1.13E-02	9.97E-03

Table B11: Deflection of *Nauclea* in z-axis for the Three End-Restraint Conditions

THICKNESS(mm)	fixed-free(mm)	pinned-pinned(mm)	fixed-fixed(mm)
0	5.62E+00	6.68	1
4	2.92E+00	3.946	1
6	2.352	3.378	1
8	1.973	2.995	1
10	1.7	2.719	1
12	1.494	2.512	1
14	1.333	2.35	1
16	1.204	2.22	1
18	1.098	2.113	1
20	1.01	2.024	1

Table B12: Stress in *Nauclea*for the Three End-Restraint Conditions

THICKNESS(mm)	fixed-free(kN)	pinned-pinned(kN)	fixed-fixed(kN)
0	2.80E+01	2.80E+01	4.98E+00
4	8.99E+01	8.75E+01	2.91E+01
6	7.79E+01	7.50E+01	2.93E+01
8	7.01E+01	6.66E+01	2.94E+01
10	6.44E+01	6.06E+01	2.95E+01
12	6.01E+01	5.60E+01	2.95E+01
14	5.65E+01	5.23E+01	2.96E+01
16	5.37E+01	4.92E+01	2.96E+01
18	5.12E+01	4.66E+01	2.96E+01
20	4.90E+01	4.44E+01	2.96E+01

Table B13: Deflection of *Entandophragma* in x-axis for the Three End-Restraint Conditions

THICKNESS (mm)	fixed-free (mm)	pinned-pinned (mm)	fixed-fixed (mm)
0	3.73E-02	3.54E-02	5.99E-03
4	3.58E-02	2.47E-02	5.58E-03
6	3.69E-02	2.13E-02	5.00E-03
8	3.76E-02	1.90E-02	4.55E-03
10	3.66E-02	1.81E-02	4.23E-03
12	3.53E-02	1.76E-02	3.99E-03
14	3.38E-02	1.72E-02	3.80E-03
16	3.24E-02	1.67E-02	3.66E-03
18	3.12E-02	1.62E-02	3.57E-03
20	3.00E-02	1.57E-02	3.54E-03

Table B14: Deflection of *Entandophragma* in y-axis for the Three End-Restraint Conditions

THICKNESS(mm)	fixed-free (mm)	pinned-pinned (mm)	fixed-fixed (mm)
0	5.58E-02	6.13E-02	9.49E-03
4	2.85E-02	2.96E-02	1.02E-02
6	2.27E-02	2.33E-02	1.02E-02
8	2.09E-02	1.93E-02	1.02E-02
10	1.97E-02	1.65E-02	1.02E-02
12	1.88E-02	1.45E-02	1.02E-02
14	1.81E-02	1.31E-02	1.01E-02
16	1.74E-02	1.20E-02	1.01E-02
18	1.68E-02	1.11E-02	1.00E-02
20	1.66E-02	1.04E-02	9.98E-03

Table B15: Deflection of *Entandophragma* in x-axis for the Three End-Restraint Conditions

THICKNESS (mm)	fixed-free(Z) (mm)	pinned-pinned(Z) (mm)	fixed-fixed(Z) (mm)
0	5.86E+00	6.93	1
4	2.81E+00	3.838	1
6	2.23	3.254	1
8	1.851	2.872	1
10	1.583	2.601	1
12	1.383	2.4	1
14	1.229	2.245	1
16	1.106	2.121	1
18	1.006	2.02	1
20	9.23E-01	1.936	1

Table B16: Stress in *Entandophragma* for Fixed-Free, Pinned-Pinned and Fixed-Fixed

THICKNESS (mm)	fixed-free (kN)	pinned-pinned (kN)	fixed-fixed (kN)
0	2.29E+01	2.49E+01	4.24
4	8.79E+01	8.54E+01	2.92E+01
6	7.56E+01	7.24E+01	2.93E+01
8	6.76E+01	6.39E+01	2.94E+01
10	6.19E+01	5.78E+01	2.95E+01
12	5.76E+01	5.32E+01	2.95E+01
14	5.41E+01	4.96E+01	2.96E+01
16	5.12E+01	4.65E+01	2.96E+01
18	4.87E+01	4.40E+01	2.96E+01
20	4.65E+01	4.18E+01	2.97E+01