STRESS DISTRIBUTION OF SHORT REINFORCED CONCRETE COLUMNS LOADED UNDER SMALL ECCENTRICITIES

BY

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November, **1997**

DECLARATION

I hereby declare that this Thesis has been entirely composed by me and that it is a record of my research work. It has not been accepted in any previous publication for the award of higher degree.

Where quotations are made from previous publications, the sources of information are duly acknowledged by means of reference.

NASIR AHMED
November, 1997.

CERTIFICATION

We hereby recommend that this Thesis, prepared under our supervision by AHMED, NASIR entitled: "STRESS DISTRIBUTION OF SHORT REINFORCED CONCRETE COLUMNS LOADED UNDER SMALL ECCENTRICITIES" be accepted as partial fulfilment of the requirements for the award of Master of Science degree in Civil Engineering (Structures) of Ahmadu Bello University, Zaria and is approved for its contribution to knowledge and literary presentation.

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ABSTRACT

The stress distribution of short Reinforced Concrete Columns was investigated via a study of their Load-strain relationship. The Principal variables under investigation were Hoop type and Hoop spacing.

A total of twenty columns were test
middle points of the columns, Modes of failure and ultimate loads were noted. The results show that strains are greater at the two ends are minimum in the middle of the columns. Also, the presence of closely spaced ties show an enhancement of ductility whereas the two Hoop types considered (Diamond shaped Hoops and Star-shaped Hoops) provide an increment of the same order to the load carrying capacity.

SYMBOLS AND NOTATIONS

f'_c	The Cube strength of Concrete
ε_{cm}	The Maximum extreme fibre compressive strain attained
p''	The volumetric ratio of transverse reinforcement to the confined concrete
d	The effective depth of the column Cross-section
ε_{ce}	The Compressive strain at which the maximum stress is just attained
ε_{cf}	The strain at which the stress has fallen down to 85% of maximum stress in the curve proposed by Soliman and Yu (17)
E_c	The modulus of elasticity for concrete.
ε_{50u}	The unconfined concrete strain at 50% of maximum stress in the Kent and Park curve.
ε_{50c}	The confined concrete strain at which the stress has fallen down to 50% of maximum stress in Kent and Park curve.
ε_{20c}	The confined concrete strain at which the stress has fallen down to 20% of maximum stress in Kent and Park curve.
f_c	The compressive stress in concrete.
ε_c	The compressive strain in concrete

z	A parameter used in determining the stress at any given strain in the Kent and Park Stress -strain curve
b''	Breadth of the confined concrete cross-section
ε_{50h}	The difference in strain at 50% of maximum stress between the confined and Unconfined concrete.
b	Breath of the specimen.
K	The ratio by which the strength of concrete increased as a result of confinement.
f''_c	The maximum stress achieved in confined concrete = $Kf'c$
f_y	The yield strength of steel.

CHAPTER ONE

INTRODUCTION

1.1 General

The purpose of any structural member is to transmit forces, and it is of the greatest importance for the designer to know the manner in which the force is to be transmitted in each member. This is because it may be that the mode of internal distribution of the force is such that failure of material of which the element is made may occur at some point or points in the element. Failure may mean that the material of the element undergoes such elastic changes of dimension that the member cannot be considered to serve its designed purpose or it yields.

The column is most often associated with the image of the post and lintel: a vertical member of regular section subjected to axial compression. Actually, the column in the modern reinforced concrete frame may be subjected to reversible axial force, bending, torsion and shear. Furthermore, the column may be non-prismatic and have irregular cross-section perforated by opening for conduits running along and across its axis.

A short column is one in which the length effect or deflection response under load is very small. According to a 1970 survey of the ACI-ASCE column committee (1), 90 percent of all braced column and 40 percent of all unbraced column can be designed as short columns. Braced columns are those where shear walls, diagonal bracing and so on, largely prevent relative lateral movement of the two column ends.
For many reasons, columns used in practice behave differently from ideal columns. The ideal column is perfectly straight, its material is perfectly homogeneous, and it

is perfectly axially loaded. Such a column would fail by direct crushing of the material in a manner similar to that of a short block, and would have a uniform compressive stress over its cross-section. But the ideal column does not exist. The columns that we actually use are never perfectly homogeneous, they contain unavoidable flaws of manufacture, however small and they are rarely, if ever, perfectly axially loaded. This departure from the ideal accounts for the behaviour exhibited by real columns.

Hudson (2) in his work on reinforced concrete columns observed that the location of failure was random but in most cases was near one end of the column. Richart and Staehle (3), in their report confirmed this, where they noted that the greater number of failures occurred in the upper third of the column shaft. This suggests a concentration of stresses at the top and bottom of a column shaft when it is fully loaded.

It is the aim of this research work to investigate the variability of stress along the length of a short reinforced concrete column with a view to determining whether the failure pattern described above could be the result of stress concentration at the two ends of the column. The effect of tie type and tie spacing on the magnitude of stress development in a column under loading is also investigated.

To achieve this, an experimental study of short columns loaded under small eccentricities is carries out. Two parameters namely tie type and tie spacing, are taken as variables while the reinforcement bar diameter of 6mm for longitudinal steel and 4mm for ties and concrete grade of 30 are kept constant through out the investigation. Five columns are tested for each aspect of the study.

Unfortunately, there is no practical method for directly measuring stress. Strains are therefore measured at the desired points with the aid of strain gauges and the stresses inferred from the observed strain values based on the fact that the two quantities are strictly proportional.

1.2 Aim

To propound a relationship between the stress distribution of a short column loaded under small eccentricities and its height.

1.3 OBJECTIVES

- To provide an insight into the nature of stress distribution for short reinforced concrete columns.
- To investigate the effects of hoop arrangement and spacing on the stress distribution for short reinforced concrete columns.

1.4 SCOPE AND LIMITATIONS

The work would be limited to short reinforced concrete columns. Columns of size 100 x 100 x 750mm will be loaded to failure under compression. These dimensions are limited by the requirements of the preliminary calculations that the ultimate loads will lie within the 100 - ton maximum capacity of the available testing machine and at the same time be fairly large enough to simulate prototype sizes as much as possible.

Although there are several variables that may affect the results of the tests, it is not possible to consider all of them in one test programme.

1.5 Justification

The ability to predict the behaviour of a structural element is the first pre-requisite to a good design. It enables us to predict the deformation of a loaded structure, its adequacy under service loads and its failure mode and capacity at over loads up to collapse. The latter information being the best measure of the safety factor against catastrophic damage. Because of the considerable investment in concrete structures, there is need to systematically study their behaviour with a view to learning not only their existing conditions but their probable conditions in years to come.

The estimation of column stresses at points of failure would assist in conceiving and choosing the best structural form for a particular application. In addition, it would provide information needed in proper selection of material strengths and physical dimensions. This thesis lays the foundation upon which further investigation in this area can be based.

CHAPTER TWO
LITERATURE REVIEW

2.1 Confinement of concrete

Concrete is confined by restraining it in directions at right angles to the direction of the applied stress. Confinement in reinforced concrete columns is provided in the form of hoops, spirals or lateral ties. They enable the concrete and the main longitudinal bars resist the axial load more effectively.

An analytical study of the confining action of a rectangular tie in a uniaxially loaded column indicate that for a typical rectangular tie, the confining force is concentrated near column corners, with a large portion near the centre of each column face being unrestrained. This rather inefficient distribution of confining stress, provides an explanation for the relative ineffectiveness of lateral ties when compared with spirals. In circular spirals axial hoop tension provides confining pressure around the circumference. This pressure is, however, only present at the corners in the case of rectangular ties. The unsupported length of the hoops only contributes when the bar diameter is thick enough to prevent buckling at high stresses. At low levels of stress in concrete, the transverse reinforcement is hardly stressed; hence the concrete is unconfined. The concrete becomes confined when at stress approaching the uniaxial strength, the transverse strains become very high because of progressive internal cracking and the concrete bears out against the transverse reinforcement, which then applies a confining reaction to the concrete. Thus, the transverse reinforcement provides passive confinement.

Lateral confinement of concrete columns creates a triaxial state of compression. As noted by Burdette and Hilsdorf (8), considerable work has been done with regard to the behaviour of concrete subjected to triaxial compression and the results which are generally consistent indicate that both the strength and ductility of such columns are significantly increased due to the lateral confinement.

With regard to strength increase due to the presence of lateral ties, however, the results of several investigators vary widely. This variation is most clearly seen by comparing the results of the tests by Bertero and Felippa (4), with those of Roy and Sozen (5).

For approximate volumetric ratios of lateral reinforcement of 0.015 and 0.025, Bertero and Felippa (4), obtained strength increases of 13% and 26% respectively. While for a lateral reinforcement ratio of 0.02, Roy and Sozen (5), obtained no strength increase.

The results of tests by Stockl (6) and Szulczynski and Sozen (7), fall, in general between the extreme values cited.

Fig 2.1 show load-strain curves from concrete prisms tested by Bertero and Felippa (4), which contained various amounts of Square ties. The effect of the different transverse steel contents on the ductility is quite appreciable, but the effect on strength is much smaller.

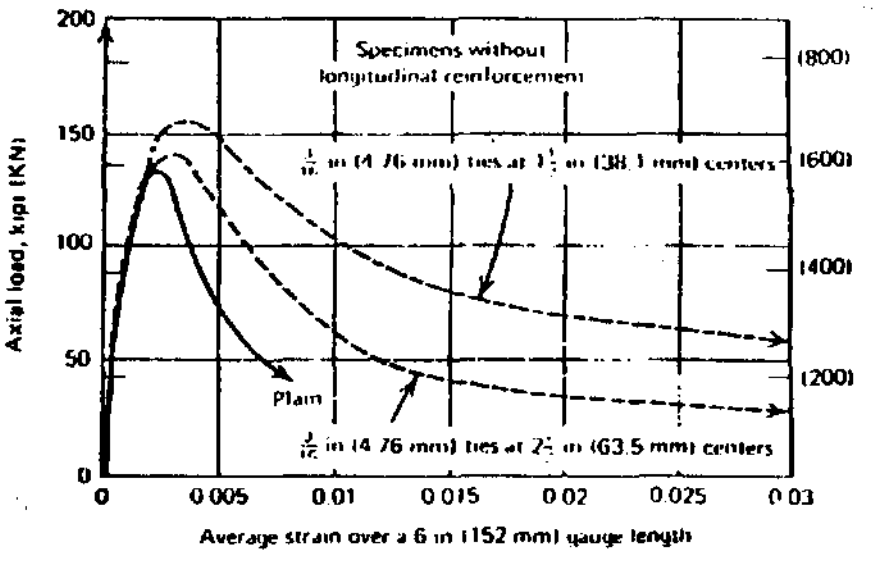


Fig. 2.1: Axial load-strain curves for 108mm Square Concrete Prisms with Various Contents of Square Ties tested by Bartero and Felippa. (4)

It was also noted by Burdette and Hilsdorf (8), that lateral confinement has no measurable effect on behaviour at loads less than about 80% to 90% of the unconfined strength of the concrete. At this stage of loading, the lateral strains begins to increase, indicating the occurrence of significant vertical micro-cracking. The lateral restraint, which developed as a result of the micro-cracking, inhibited to some extent further micro-cracking and local failure in the specimen.

As noted by Sargin et al (9), It is generally agreed that lateral reinforcement increases ductility, but this conclusion has so far remained largely qualitative. There does not seem to be any agreement even about the parameters to be considered.

The volumetric ratio of lateral reinforcement alone or in combination with tie spacing, quality of steel and the grade of confined concrete. Also the effects of strain gradient and the relative areas of core and cover have not been considered in depth.

2.2 Failure Patterns

Hudson,(2) observed that lateral ties exert a strong influence on the mode of failure of axially loaded members. He noted that for columns with ties between the extremities (irrespective of spacing) failure occurred by spalling of the cover concrete between ties, followed immediately by buckling of the reinforcement. The location of failure was random but in most cases near one end of the column. Axially loaded columns with no intermediate ties failed by vertical splitting along or close to the centre line. Of some interest was his observation that final failure occurred at a load roughly 20% greater than the load at which the split appeared to be complete, whereas those specimens containing intermediate ties gave virtually no warning of impending collapse.

2.3 Spacing of Lateral Ties.

It is known from column studies that for the lateral binders to be effective in their action, they should be spaced at a distance not greater than twelve times the smallest longitudinal bar diameter. An increase in the spacing of binders decrease considerably, the confining effect of the binders. This is why spacing of lateral ties is specified in various codes of practice. It was found by Iyengar et al (10), that the influence of binders is negligible for spacing equal to or greater than the lateral dimension of the test Specimen.

King, (11) in his investigation on the effect of lateral reinforcement in reinforced concrete columns established that normal binders, if at close enough pitch, have a material effect on the strength of reinforced concrete columns, and that such effect is a function of their yield load, their pitch and the strength of the longitudinal steel. He also posited that the effect of the main steel can be apparently increased to well above it's yield load if binders are sufficient. Only very closely spaced ties would increase column capacity beyond the load at which the shell spalls locally and the bars would buckle. The main advantage of closely spaced ties is that ductility or toughness is increased. Figure 2.2 illustrates the effect of spacing on efficiency of confinement. The concrete is confined by arching of the concrete between the transverse bars and if the spacing is large it is evident that a large volume of the concrete cannot be confined and may spall way.

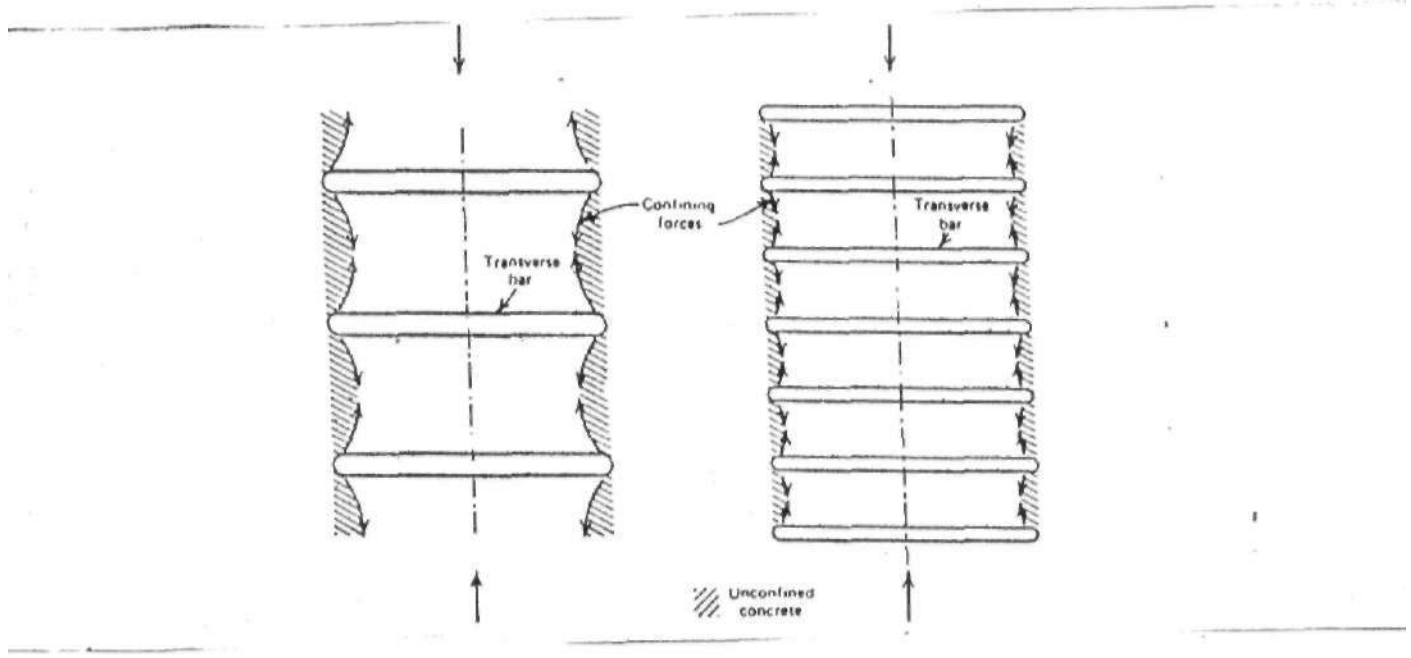


Fig. 2.2: Effect of Spacing of Transverse Steel on Efficiency of Confinement

2.4 Effects of Creep and Shrinkage

2.4.1 Creep

Until relatively recently, it was generally assumed that creep effects are beneficial in relieving over stressed sections by allowing a redistribution of load to less highly stressed regions. On the other hand, shrinkage was regarded as an inevitable fact that had to be tolerated. Creep and shrinkage of concrete have a strong influence on the stresses in the steel and the concrete of an axially loaded reinforced concrete column at service load, tending to increase the longitudinal steel stress and to reduce the concrete stress.

Mauch , (12) observed that creep can significantly decrease the capacity of hinged reinforced concrete columns, independent of whether the total creep strain or only a part of it depends on the time at which the column is first loaded. Similar though some what smaller, reductions are expected for restrained concrete columns.

He also noted that the most significant reductions in carrying capacity due to creep are 40 to 50% for columns with slenderness ratio L/D =25 to 40. Smaller reduction occur for shorter columns. For very short columns the only effect of creep is a small reduction of the concrete strength.

The creep deformation of concrete under constant axial compressive stress is illustrated in fig. 2.3 As the figures reveals, the creep proceeds at a decreasing rate with time. If the load is removed, the elastic strain is immediately recovered. However, this recovered elastic strain is less than the initial elastic strain because the elastic modulus increases with age. The elastic recovery is followed by a creep recovery which is a small proportion of the total creep strain.

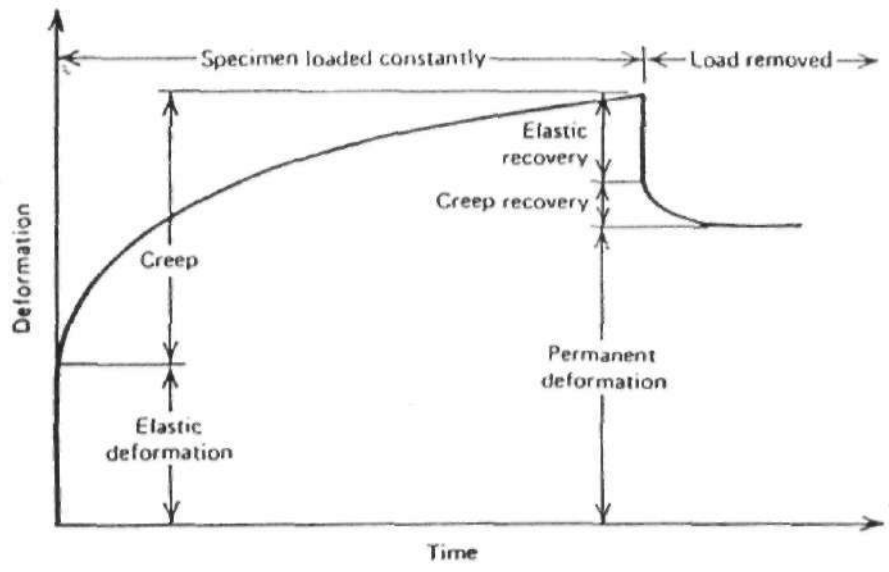


Fig. 2.3: Typical Creep Curve for concrete with constant axial compressive stress.

2.4.2 Shrinkage

As concrete loses moisture by evaporation, it shrinks. Since moisture is never uniformly withdrawn throughout the concrete, the differential moisture changes causes differential shrinkage tendencies and internal stresses. Stresses due to this is one of the reasons for insisting on moist covering conditions The larger the ratio of surface area to member cross-section the largerwill be the resulting shrinkage. Therefore, large specimens shrink much less than small ones.

Mauch (12) observed that shrinkage generally had a minor effect for symmetrically reinforced columns he investigated. In such cases shrinkage does not directly contribute to time dependent curvature. It will however assist creep in redistributing

stresses from concrete to steel and thus causing yield in the reinforcing steel, which in turn decreases the stiffness of the column.

With increasing heights of buildings, the time - dependent shortening of columns and shear walls becomes of much greater importance due to the cumulative nature of creep, shrinkage and temperature related deformation. It was noted (13) that an 80- story high concrete column, for example, might be expected to shorten by about 4.6cm due to creep and 6cm due to shrinkage.

A curve showing the increase in shrinkage strain with time appears in Fig. 2.4. The shrinkage occurs at a decreasing rate with time. The final shrinkage strains vary greatly, being generally in the range 0.0002 to 0.0006 but sometime as much as 0.0010.

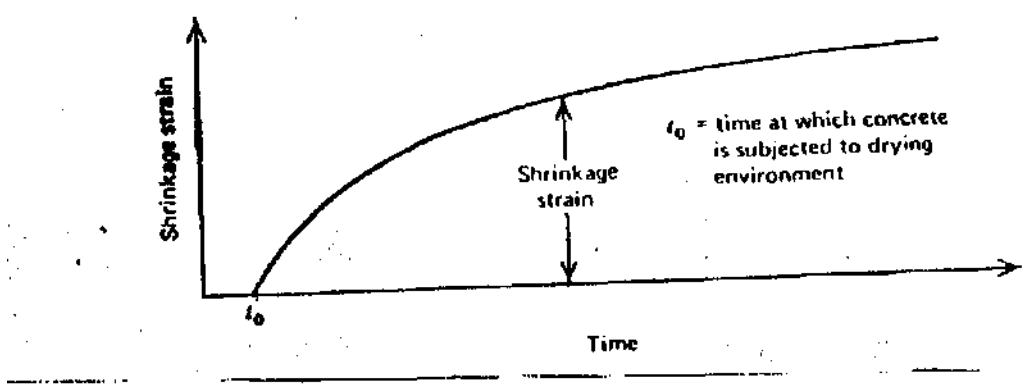


Fig. 2.4: Typical shrinkage curve for concrete.

2.5 Review of stress-strain relationship for concrete confined by Rectangular Hoops proposed by past Researchers.

Various investigators as shown below have proposed stress - strain relationships for concrete confined by rectangular hoops. Some of the proposed curves are shown in Fig. 2.5.

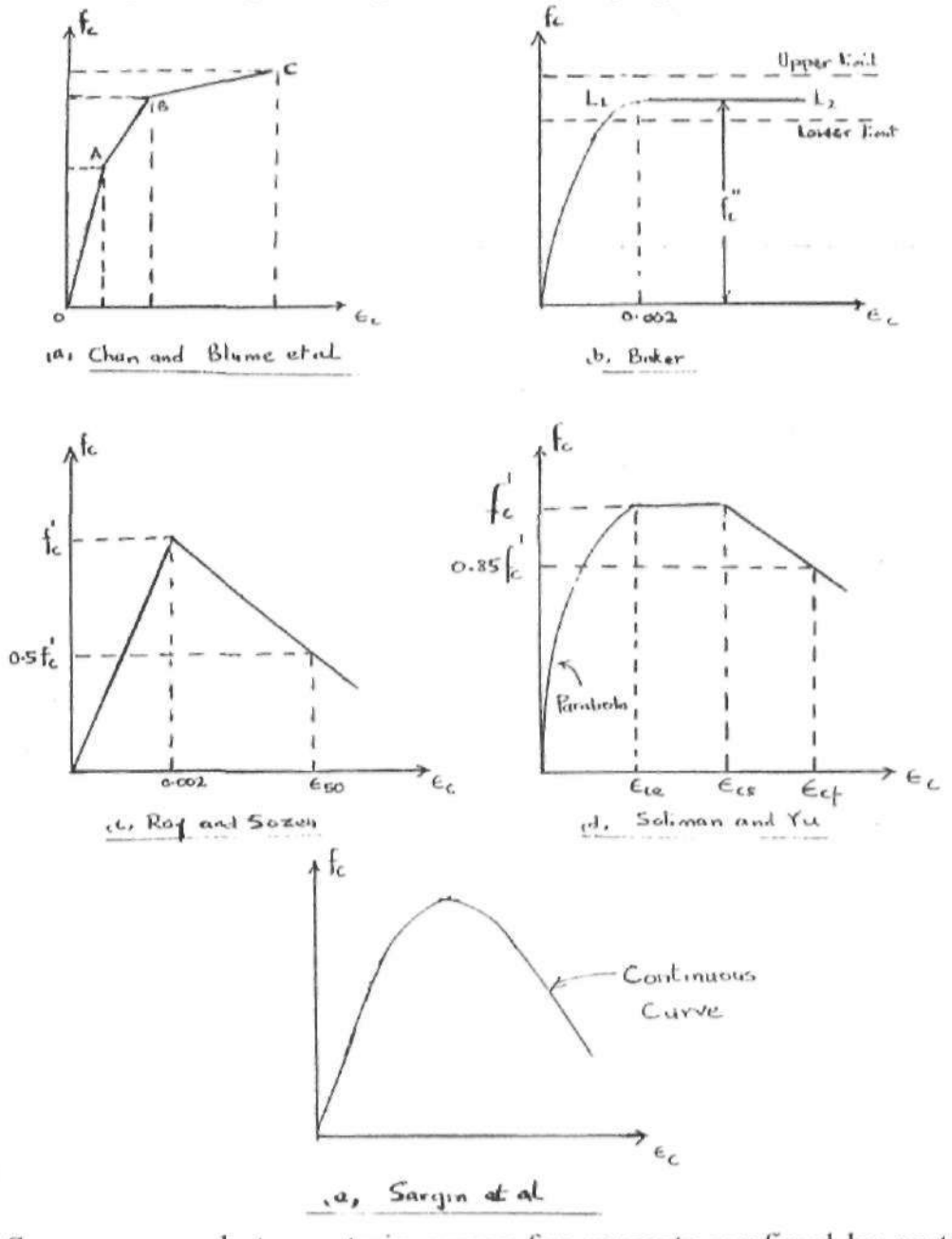


Fig .2.5: Some proposed stress-strain curves for concrete confined by rectangular hoops

Chan, (14) Studied the effects of spacing and diameter of transverse steel and proposed a linear curve (fig.2.5a) in which OAB approximates the curve for Unconfined concrete. BC depends on the transverse reinforcement. Blume et al, (15) also adopted a linear curve in which OA approximated the curve for unconfined concrete up to 0.85 f'_c and ABC (atimes replaced by a single straight line) depended on the content and yield stress of the transverse confinement.

Baker and Amarakone (16), recommended a parabola up to a maximum stress dependent on the strain gradient across the section, then a horizontal branch to a strain dependent on the strain gradient and the transverse steel content. They proposed the maximum compressive strain to be :

$$\varepsilon_{cm} = 0.0015 \left[1 + 150\rho'' + (0.7 + 10\rho'') \, d/kd \right] \leq 0.00 \qquad (2.1)$$

Where

ρ'' = Volumetric ratio of transverse reinforcement to bound concrete

d = effective depth of section.

Kd = neutral axis depth

The maximum strain was limited to 0.01.

Roy and Sozen (5), suggested a curve in which the ascending branch is a straight line from the origin to a point with the stress f'_c and a strain of 0.002. The falling branch is also a straight line having a strain at $0.5f'_c$ which was linearly related to the transverse steel content.

Soliman and Yu (17): Their curve consisted of a parabola and two straight lines with stresses and strains at the critical points related to transverse steel content and spacing and the confined area. The parabola is up to a stress of f'_c and strain of ε_{cc}, followed by a flat region at f'_c but with the strain increasing to ε_{cs} and a final falling section which includes the point $(0.8f'_c, \varepsilon_{cf})$, up to the point of failure ε_{ce}, which is the compressive strain at which the maximum stress is just attained and is given as:

$$\varepsilon_{ce} = \frac{2f'_c}{E_c} \qquad (2.2)$$

ε_{ce} and ε_{cf} depend on the spacing of binders whereas E_c is the modulus of elasticity of concrete. The maximum compressive strain was given as

$$\varepsilon_{cu} = 0.003\ (1 + 0.8q'') \qquad (2.3)$$

for which the quantity q'' is a measure of the effectiveness of transverse reinforcement.

$$q'' = (1.4\frac{A_b}{A_c} - 0.45)\quad \frac{As''(S_o - S)}{A''_s + 0.0028BS^2} \qquad (2.4)$$

Where:

A_b = Area of bound concrete under compression

A_c = Total area of concrete under compression

A''_s = Cross-sectional area of transverse steel.

S_o = Maximum longitudinal spacing after which transverse reinforcement is not effective in confining concrete.

S = Actual longitudinal spacing of transverse reinforcement.

B = b'', the breadth of confined concrete or d'', effective depth of confined concrete whichever is greater.

The maximum compressive strain ε_{cu} was found to be closer to experimental results than that of Baker (16).

Sargin et al, (9) proposed a general equation that gives a continuos stress-strain curve related to the content, spacing and yield strength of the transverse steel, the strain gradient across the section, and concrete strength.

Kent and Park, (18) proposed the stress-strain curve shown in fig. 2.6 for concrete confined by rectangular hoops.

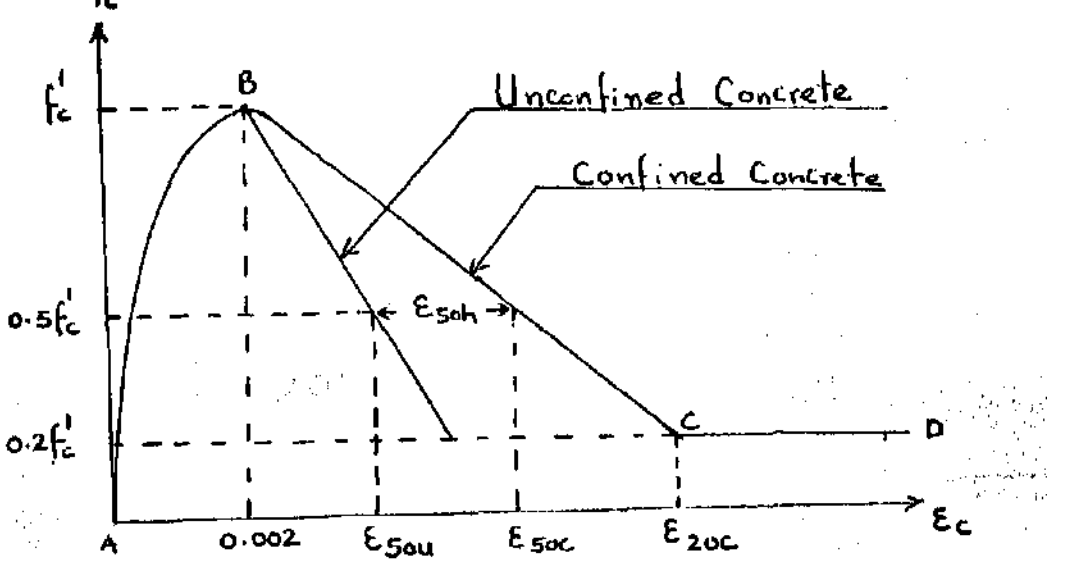


Fig. 2.6: Kent and Park curve for concrete confined by rectangular hoops.

The characteristics of the suggested curve are as follows:

Region AB: This is the ascending part of the curve and is represented by a second degree parabola, as in the case for curves proposed by Baker and Amarakone (16), Sargin et al (9) and Soliman and Yu (17). It is assumed that the confining steel has no effect on the shape of this part of the curve or the strain at maximum stress. At the peak B, it is assumed that the maximum stress reached by the confined concrete is the cylinder strength f'_c and the strain to be 0.002. The stress at any point is given by:

$$f_c = f'_c \left[\frac{2\varepsilon_c}{0.002} - \left(\frac{\varepsilon_c}{0.002} \right)^2 \right] \qquad (2.5)$$

Region BC: This is a linear falling branch similar to that suggested by Roy and Sozen (5). The parameter Z defines its slope and it is specified by the strain when the stress has fallen to $0.5f'_c$ as obtained from experimental evidence. For the unconfined concrete, this strain is given as:

$$\varepsilon_{50u} = \frac{3 + 0.002\, f'_c}{f'_c - 1000} \qquad (2.6)$$

Where f'_c = concrete cylinder strength in psi (1 psi = 0.00689N/mm^2).

For the confined concrete, the strain ε_{50u} is given by the unconfined strain ε_{50c} plus the additional ductility due to rectangular hoops, ε_{50h}, that is,

$$\varepsilon_{50c} = \varepsilon_{50u} + \varepsilon_{50h} \tag{2.7}$$

Where

$$\varepsilon_{50h} = \frac{3}{4}\rho''_s\sqrt{\frac{b''}{S_h}} \tag{2.8}$$

The confined concrete stress is given by

$$f'_c = f'_c\ [1\text{-}z\ (\varepsilon_c - 0.002)] \tag{2.9}$$

Where

$$z = \frac{0.5}{\varepsilon_{50u} + \varepsilon_{50h} - 0.002} \tag{2.10}$$

Region CD: This region accounts for the ability of concrete to sustain some stresses at very large strains. The confined concrete is assumed to sustain a stress of $0.2f'_c$ from ε_{20c} to the point of rupture. No such region exists in unconfined concrete. For the region CD where $\varepsilon_c \geq \varepsilon_{20c}$, we have

$$f'_c = 0.2\ f'_c \tag{2.11}$$

Studies of the influence of rectangular steel hoops on the stress-strain curve given by equations (2.5) to (2.11) when f'_c is 27.6 N/mm^2 and the ratio $S_h/b'' = 0.5$, shows a great improvement (additional ductility) in the falling branch behaviour for small contents of rectangular hoops. However, the improvement becomes progressively less significant as more hoops are added.

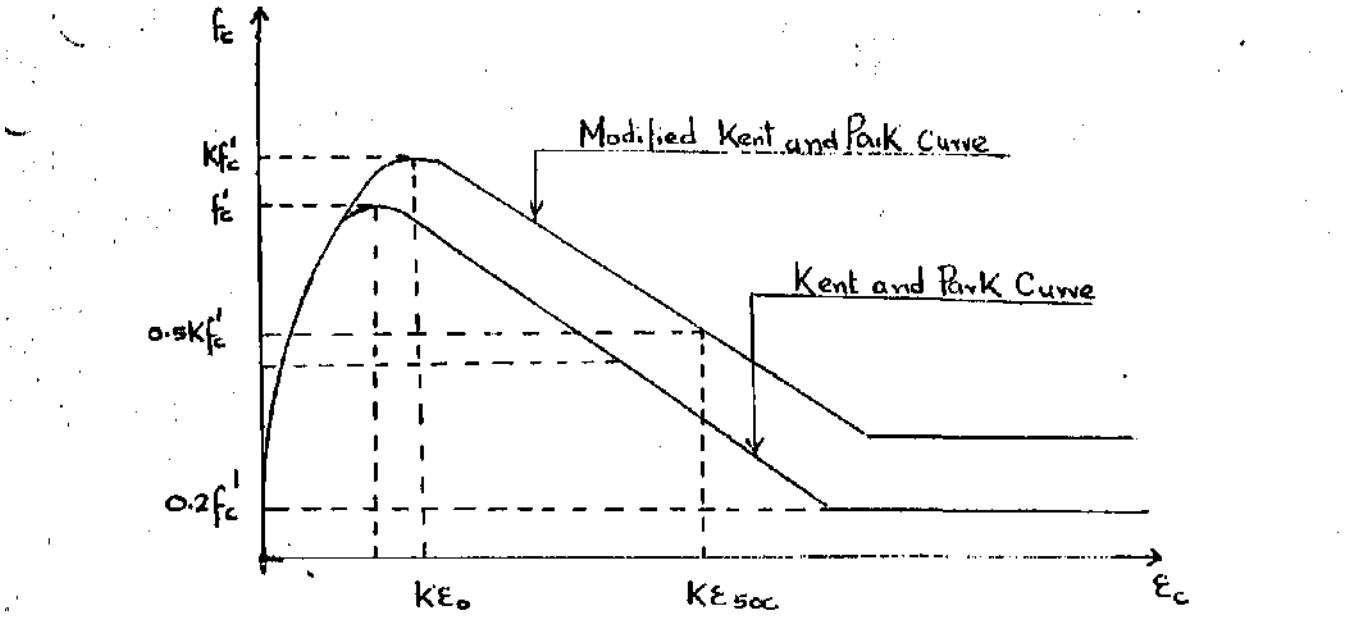


Fig. 2.7: The Modified Kent and Park curve.

The equations were derived from results from specimens with variables in the following ranges: S_h/b = 0.35 to 2.0, p_s = 0.35 to 2.4% and f'_c = 20.7 to 27.6 N/mm^2.

The modified Kent and Park stress-strain curve: Scott et al (19) discovered that the Kent and Park stress-strain curve (fig. 2.6) did not take into consideration the increase in concrete strength due to confinement. To redress this shortcoming, they proposed that the cylinder stress be multiplied by a factor K given by:

$$K = 1 + \rho'' \frac{f_y}{f'_c} \tag{2.12}$$

Thus, for $\varepsilon_c \leqslant 0.0002K$

$$f_c = K f'_c \left[\frac{2\varepsilon_c}{0.002} - \left(\frac{\varepsilon_c}{0.002} \right)^2 \right] \tag{2.13}$$

with $f'c$ and fy in Mpa (N/mm^2) and for $\varepsilon_c \geqslant 0.002K$

$$f_c = Kf'_c [\, 1 - z\,(\varepsilon_c - 0.002K)] \tag{2.14}$$

where

$$z = \frac{0.5}{\frac{3 + 0.29\, f'_c}{145 f'_c - 1000} + \frac{3}{4}\rho'' \sqrt{\frac{b''}{s}} - 0.002K} \tag{2.15}$$

The authors also suggested that the maximum concrete compressive strain at ultimate strength be given by :

$$\varepsilon_{cu} = 0.004 + 0.9\, \rho'' \frac{f_y}{300} \tag{2.16}$$

with f_y in MPa.

The modified curve was found to be in agreement with experimental results than the original Kent and Park curve.

CHAPTER THREE

EXPERIMENTAL PROGRAMME

3.1 General

The test programme was divided into two. The first part deals with the influence of stirrup type on the stresses developed in a short reinforced concrete column, while the second part investigates the effect of spacing of ties on the stresses. Twenty (20) short columns of size 100x100x750mm were tested.

In the first part, two arrangements namely the diamond-shaped over lapping hoops and the star-shaped hoops were chosen and numbered A1 and A3 respectively. For each arrangement, five columns with identical properties and labelled A11 to A15 and A31 to A35 respectively were prepared and tested to ensure consistency.

In the second part of the test, the spacing of ties which was 80mm in the first part of the experiment was now reduced to half (40mm) at the top and bottom of the columns where maximum stresses are expected. Other factors like stirrup type, reinforcement size, and concrete grade (30) were kept constant. As in the first part, the two stirrup arrangements (i.e. diamond overlapping and star-shaped hoops) with the reduced spacing were numbered B1 and B3. Five columns with identical properties and labelled B11 to B15 and B31 to B35 respectively were again casted and tested for each arrangement. Table 3. 1 shows the details.

TABLE 3.1: GENERAL DETAILS OF TEST SPECIMENS

CONSTANTS:

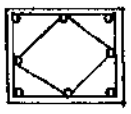
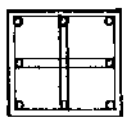
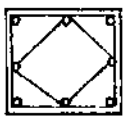
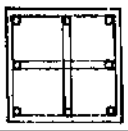
Dimension of specimens = 100x100x750mm

Concrete cover to reinforcement = 10mm

Size of main reinforcement = 6mm

Size of Hoop reinforcement = 4mm

Design concrete strength = 30N/mm^2

SPECIMEN LABEL	ARRANGEMENT	LONGITUDINAL STEEL			TRANSVERSE STEEL	
		No. of Bars	Diameter(m m)	Area (mm^2)	Diameter (mm)	Spacing (mm)
A1		8	6	226.19	4	80
A3		8	6	226.19	4	80
B1		8	6	226.19	4	40 top and Bottom only
B3		8	6	226.19	4	40 Top and Bottom only

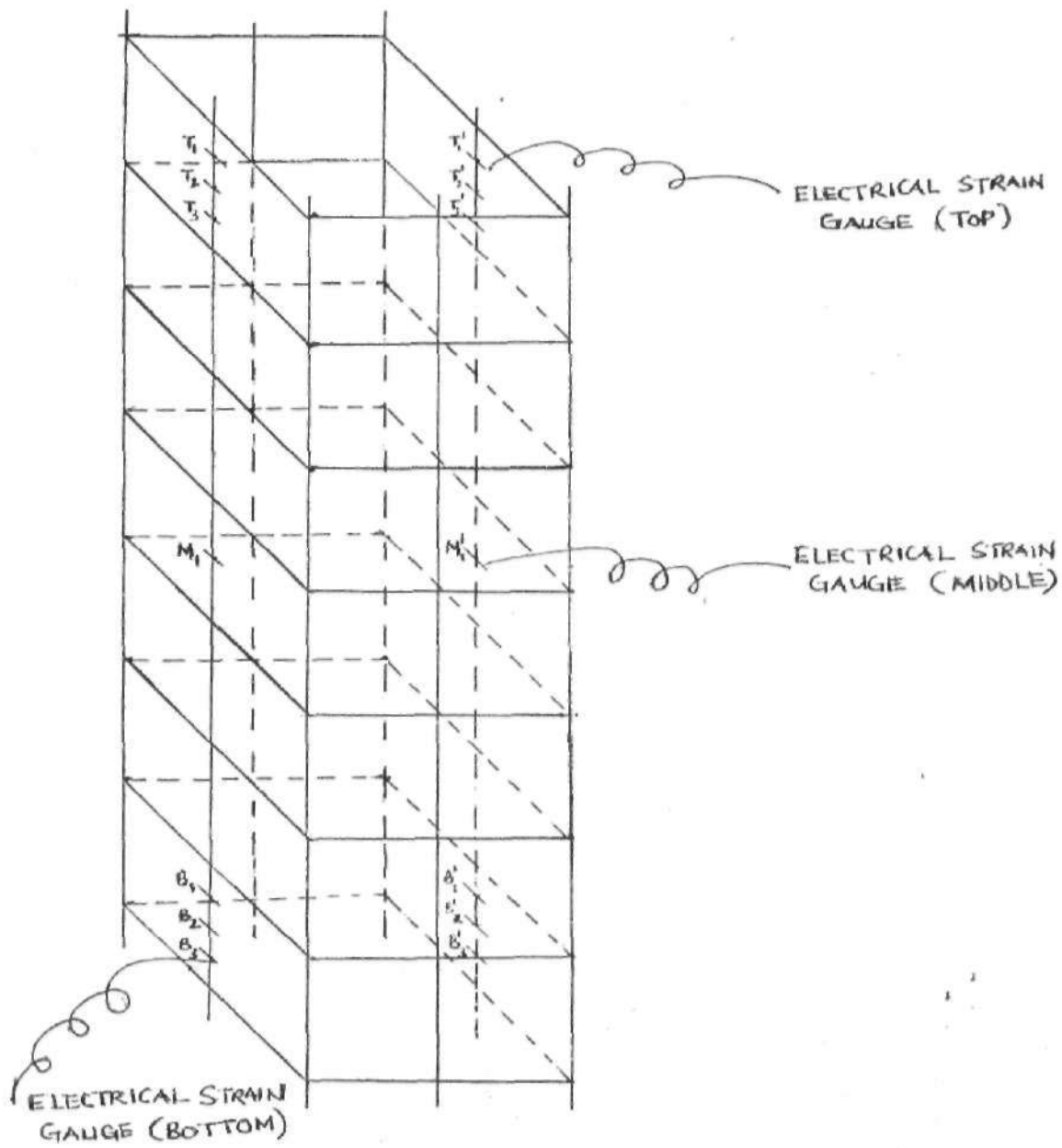


Fig. 3.1: Strain Gauge Positions for a Typical Column.

To identify columns and the embedded strain gauges after casting, the strain gauge wires were provided with masking tape tags. Each tag indicated the column label and the position and number of the strain gauge the wires are attached to. Thus $A13T_1$, represents the third columns in the A1 series (A13) with the wires attached to the first strain gauge at the top (T_1), on one face of the column. $A13T_1'$ on the other hand represents the same column as above (A13), but with the wires attached to the first strain gauge on the opposite face (T_1'). Figure 3.1 gives details of all strain positions for a typical column.

It was not possible to include all variables that may likely affect the results of the test in this investigation because of the limitations of time and finance. This informed the limitation of the study to only two factors namely stirrup type and spacing.

3.2 Materials

All materials used were handled and stored carefully in order to avoid damage, deterioration, or contamination before being utilised.

3.2.1 Aggregates

River sand free from silt and coagulated lumps is used as fine aggregate. The coarse aggregate consisted of crushed rock free from fine particles with a maximum size of 8mm. The stones were sieved to the required sizes before utilization.

3.2.2 Cement

Ordinary Portland cement, the "Ashaka" brand manufactured in Gombe state is used. It was obtained from the same source and used in order of delivery.

3.2.3 Reinforcement

Mild steel bars are used as reinforcements. 6mm bars were used as longitudinal reinforcement while the hoop reinforcement consist of 4mm bars. All reinforcements were kept free from loose rust, grease and similar material that may impair bond with concrete. The bars were also tested to check their strength. For each diameter used, four samples were tested. Results of the tensile test are shown in Tables 3.2(a) and 3.2 (b) below.

TABLE 3.2: RESULTS OF TENSILE TEST FOR STEEL

(a)

Bar identify (main bar)	Diameter (mm)	Gage length (mm)	Yield Load (kN)	Failure Load (kN)	A_s (mm^2)	f_y (N/mm^2)
1	6	30	7.8	10.1	28.274	275.87
2	6	30	7.8	10.0	28.274	272.34
3	6	30	7.8	9.8	28.274	272.34
4	6	30	7.8	9.8	28.274	258.19

Average f_y (yield stress) = 269.69 N/mm^2

(b)

IDENTITY (MAIN BAR)	DIAMETER (mm)	Gage Length (mm)	Yield Load (kN)	Failure Load (kN)	A_S (mm^2)	f_y (N/mm^2)
1	4	20	5.0	6.4	12.566	397.89
2	4	20	5.2	6.1	12.566	413.82
3	4	20	5.1	6.1	12.566	405.86
4	4	20	5.1	6.2	12.566	405.86

Average f_y (yield stress) = 405.86 N/mm^2

3.3 Instrumentation

3.3.1 Strain gauge

Deformations are measured with TML temperature compensated strain gauge, type PL - 60 -11, with a gauge factor of 2.14, gauge resistance of 120 $\pm$ 0.3 ohms and an accuracy of within 1.8 x $10^{-6}/^{0}$ C approximately. It is to be noted that these are the only types available to us in view of the acute scarcity of the items in the Market. All PL gauges are efficient within an operational temperature range of - 200^{0}C to 80^{0}C. Fourteen strain gauges are used per column as indicated in Fig. 3.1

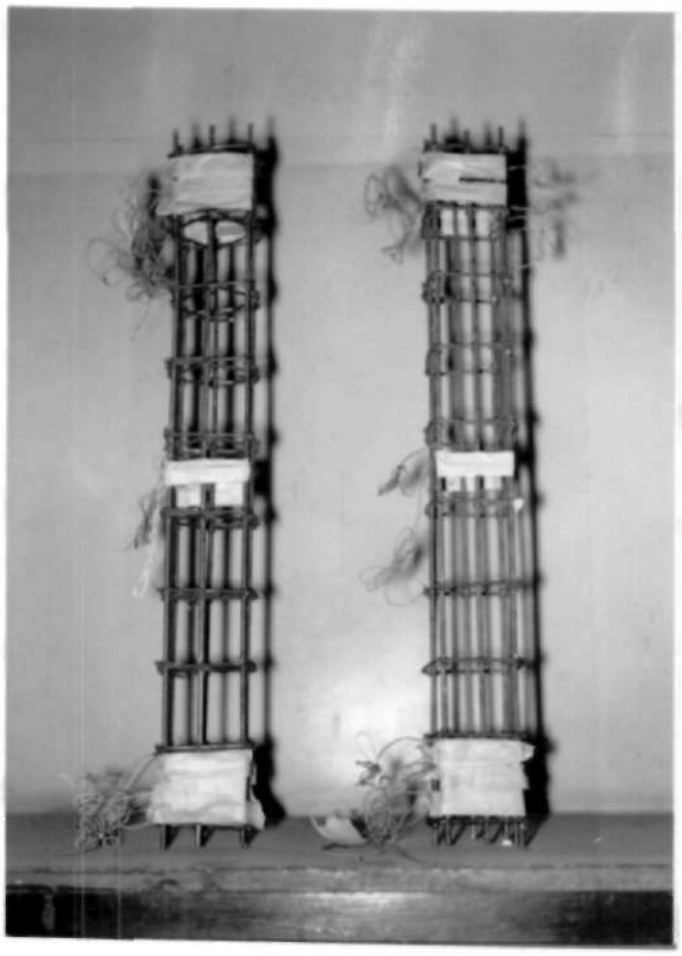


PLATE 1: Arrangement of Strain gauges.

The column surfaces are thoroughly cleaned before the strain gauges are fixed to them. This is to allow for proper contact. A two component epoxy resin/hardner mix called Araldite is used as binder. The binder is applied in thin layer free from air bubbles and fully cured. Plate 1 shows the arrangement of the reinforcement cage with the PL - strain gauges.

3.4 Preparation of specimen

The composition by weight of the ingredients required to achieve the desired strength of 30 N/mm^2 for each batch of 5 columns and 3 cubes are as follows:

Portland Cement	-	18.78kg	Fine aggregate	-	34.59kg
Coarse aggregate	-	64.27kg	Water	-	9.96kg

The columns are cast and vibrated in horizontal position in a partitioned plywood mould. Plate (2). This to ensure uniformity of strength and increased productivity. Casting is done for five columns and for each columns casted, three cubes are prepared from the batch of concrete.

The mould was oiled a day before casting in order to allow for easy removal of specimen. Crushed stone aggregates about 10mm thick are used as biscuits at the bottom and sides of each reinforcement cage to provide the required cover of 10mm.

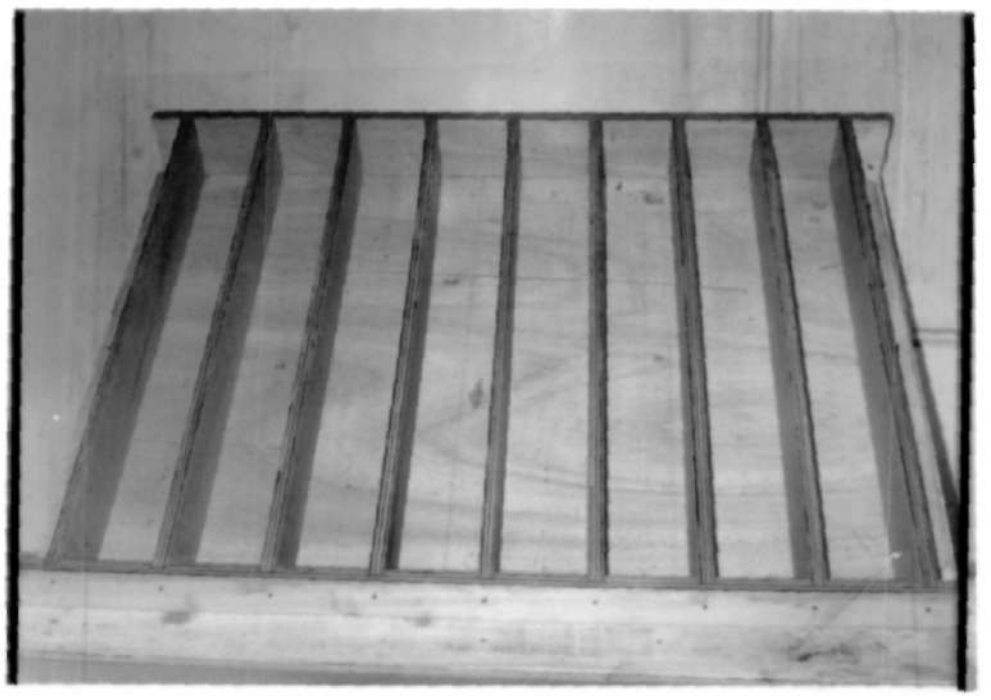


PLATE 2: Plywood Mould used for casting columns horizontally.

All reinforcement cages are placed centrally in the formwork before placing the concrete using a scoop and small hand trowels. The compaction is by means of a vibrating table. The casted columns, together with the concrete cubes were left for 24 hours on the vibrating table and then demoulded. The columns are cured for 28 days using wetted jute bags while the concrete cubes are cured in an immersion tank for the same period. At the end of 28days the columns are painted ready for testing. The age of the specimen at the time of testing ranges from 30 to 40 days.

Testing of the columns to failure is carried out using the Avery universal Testing Machine while that of the cubes is by the Denison Compression Testing Machine.

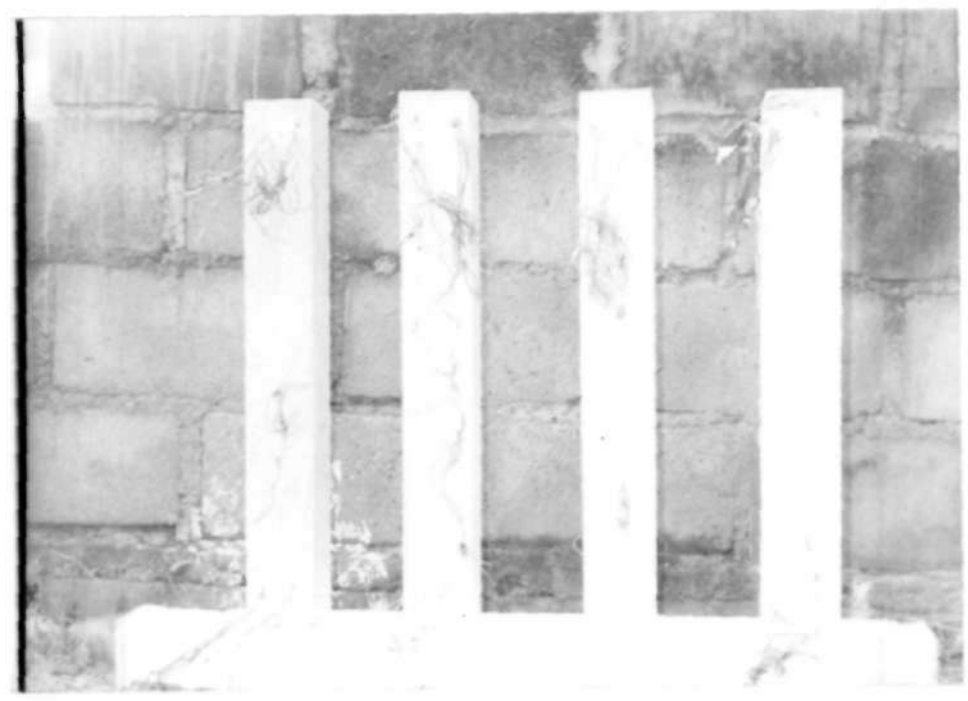


PLATE 3: Painted columns ready for Testing.

3.5 Testing

The principal variables under investigation are the ties type and spacing. Columns are loaded and the following are observed:

1. Strain development at various points along the length of the columns.
2. Modes of failure
3. Ultimate loads.

These observations enabled the study of load-strain relationship of the columns and the effects of tie type and spacing on the ultimate loads and strains.

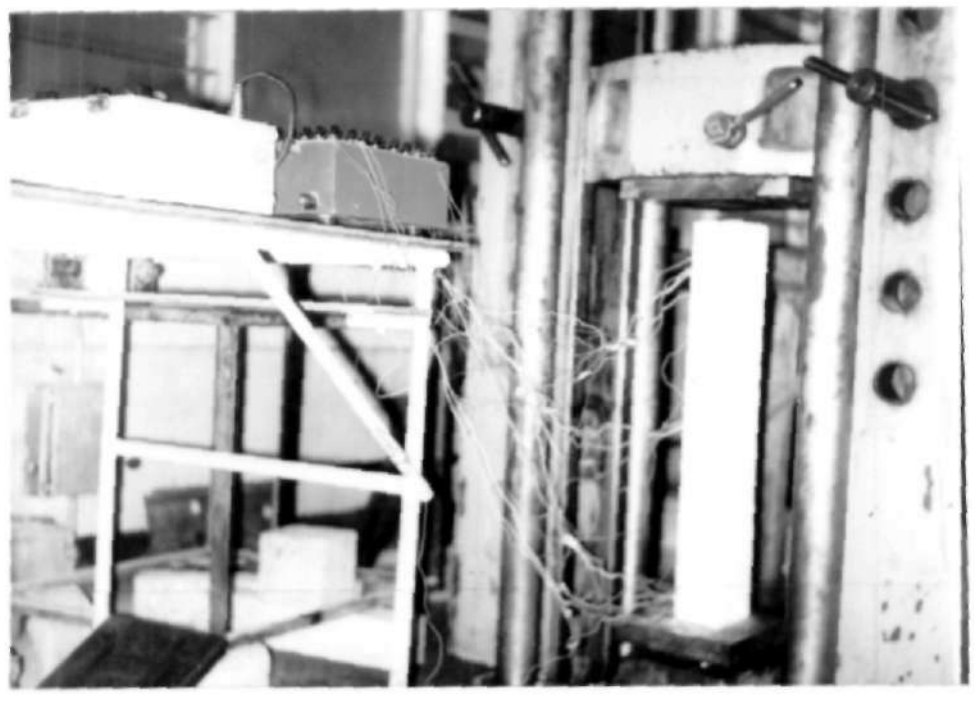


PLATE 4: Column positioned on Testing Machine before loading.

The Universal testing machine was used to test all columns. The separation between the grip holders of the machine were lowered and fixed at a position where the clearance was just enough to accommodate the 750mm long specimen. Before testing, the resistance of all the electrical strain gauges was checked. The control cubes were tested with the Denison compression testing machine. After placing the specimen on the machine, the terminals of all miniature stranded cables soldered to the fourteen strain gauges were connected to the TML static strain meter and all readings were adjusted to give a satisfactory balance. Plate 5 shows the arrangement of a typical column after test. One of the strain gauges is used as a dummy gauge through out the test, so that any dimensional change caused by temperature variations is compensated.

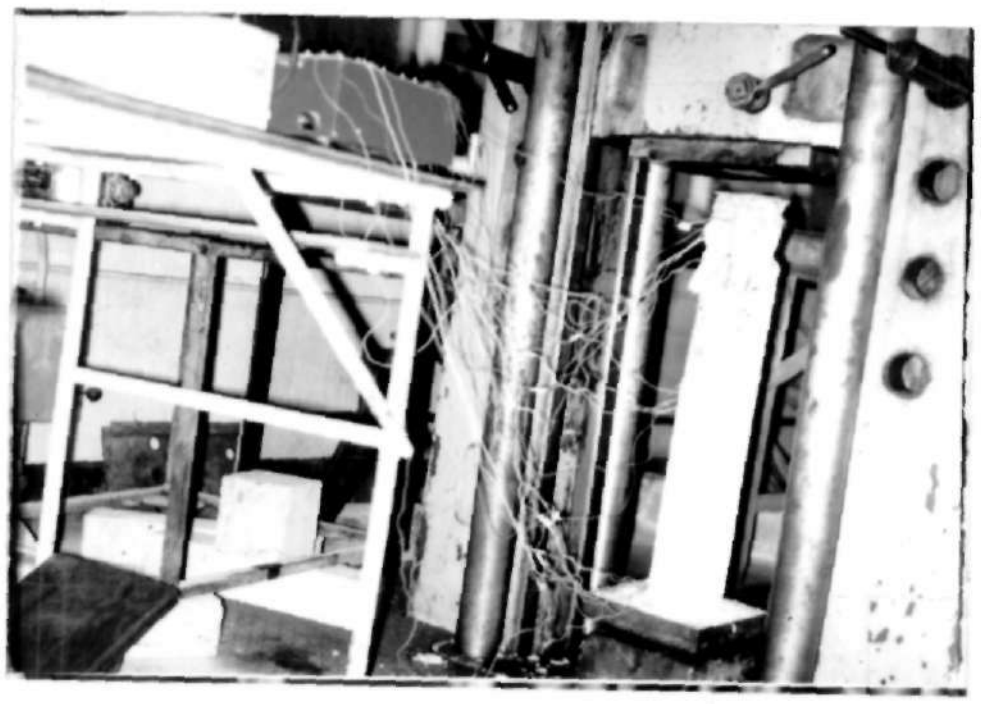


PLATE 5: A Typical column after failure on the testing machine

After careful positioning, the zero readings of all the strain gauges were recorded. The columns are then loaded at increments of 50kN until a maximum of 150kN calculated for the ultimate capacity of the column, after which an increment of 25kN is applied until failure. This is done in order to have proper observation of strain behaviour near the failure load. Tables 3.4 - 3.7 and Table 3.8 are results of the strain values and the ultimate load respectively for the various classes of tests carried on the columns.

3.5.1 Concrete strength

Concrete cubes of 150x150x150mm in size are crushed to determine the compressive strength of the concrete. They were tested after a minimum curing period of 28 days using a Denison testing machine.

Results are shown in Table 3. 3 (a) - (d).

TABLE 3.3 RESULTS OF CONCRETE CUBE STRENGTH TEST

FIRST BATCH:- Design strength = 30 N/mm^2

(a)

CUBE No.	WEIGHT (Kg)	CRUSHING LOAD (kN)	STRENGTH (N/mm^2)
C1	8.35	700	31.11
C2	8.15	670	29.78
C3	8.25	670	29.78

Average strength = 30.22 N/mm^2

SECOND BATCH:- Design strength = 30 N/mm^2.

(b)

CUBE No.	WEIGHT (Kg)	CRUSHING LOAD (kN)	STRENGTH (N/mm^2)
C1	8.35	650	28.89
C2	8.45	660	29.33
C3	8.20	660	29.33

Average strength = 29.18 N/mm^2

THIRD BATCH:- Design strength = 30 N/mm^2.

(c)

CUBE No.	WEIGHT (Kg)	CRUSHING LOAD (kN)	STRENGTH (N/mm^2)
C1	8.30	650	28.89
C2	8.35	700	31.11
C3	8.20	660	29.33

Average strength = 29.78 N/mm^2

FOURTH BATCH:- Design strength = 30 N/mm^2.

(d)

CUBE No.	WEIGHT (Kg)	CRUSHING LOAD (kN)	STRENGTH (N/mm^2)
C1	7.85	660	29.33
C2	7.95	660	29.33
C3	8.25	685	30.44

Average strength = 29.70 N/mm^2

A consideration of results of the batches shows a consistency in the weight of the tested cubes which is an indication of a fair use of materials constituents for concrete. The calculated average strength of all the batches also fall below the targeted strength of 30N/mm^2.

3.5.2 Column Deformation

Strain measurements were made at each stage of loading and the maximum values recorded at 225kN for each series are shown in Tables 3.4 - 3.7. Details of results for each stage of loading are given in Appendix A.

Shrinkage and Creep effects were neglected. Compressive stresses were assumed to be zero before the application of loads and perfect bound is assumed between concrete and steel.

TABLE 3.4: Strain values for A1 series columns at 225kN

Height of Strain gauges (mm)	Column Strain Values					
	A11	A12	A13	A14	A15	Ave. (A11-A15)
33	-	.000560	.000950	.000440	.000565	.000629
51	.000430	.000520	.000495	.000335	.000195	.000395
73	.000365	.000410	000550	.000305	.000050	.000336
375	.000325	.000255	.000655	.000300	.000070	.000321
677	.001440	.000450	.001940	.001470	.000770	.001232
699	.000985	.001980	-	.001020	000.140	.00103
717	-	.002270	-	.001900	.000600	.00159

TABLE 3.5: Strain values for A3 series columns at 225kN

Height of Strain gauges (mm)	Column Strain Values					
	A31	A32	A33	A34	A35	Ave. (A31-A35)
33	.000400	.000320	.000680	.000260	.000290	.000390
51	.000600	.000090	.000325	.000500	.000200	.000343
73	.000180	.000710	000445	.000150	.000485	.000286
375	.000080	.000070	.000195	.000050	.000375	.000154
677	.001530	.000155	.000700	.000165	.000990	.000708
699	.000910	.000200	.001190	.000110	-	.00603
717	.002070	.000065	.000340	.000400	.000210	.000617

TABLE 3.6: Strain values for B1 series columns at 225kN

Height of Strain gauges (mm)	Column Strain Values					
	B11	B12	B13	B14	B15	Ave. (B11-B15)
33	.0003320	.000470	.000230	.000550	.000480	.000410
51	.000290	.000340	.000780	-	.000440	.000463
73	.000190	.000385	000280	.000560	.000195	.000322
375	.000320	.000145	.000170	.000365	.000200	.000240
677	.000435	.001080	.000320	.000265	.000160	.000452
699	.000690	.000650	.000415	.000450	.000415	.000524
717	.000590	.000840	.000820	.000435	.000170	.000571

TABLE 3.7: Strain values for B3 series columns at 225KN

Height of Strain gauges (mm)	Column Strain Values					
	B31	B32	B33	B34	B35	Ave. (B31-B35)
33	.000800	.000520	.000410	.000660	-	.000598
51	.000470	.000220	.000455	.000180	.000235	.000312
73	.000280	.000250	000280	.000300	.000615	.000345
375	.000220	.000280	.000480	.000130	.000180	.000258
677	.001175	.000650	.001045	.001180	.000670	.000944
699	.001075	.002100	.002750	.001055	.001340	.00166
717	.001030	.001095	.001355	-	-	.00116

3.5.3 Ultimate load capacities of columns

Results of Ultimate load carried by the columns tested in this investigation are presented in Table 3.8. the results are arranged according to the various series to show the contribution of the two ties types investigated to the ultimate bearing capacities of the columns. Although the average ultimate loads for all the series are of the same order, columns of series A1 and B1 (Diamond -shaped overlapping ties) have slightly higher average ultimate load capacity than those of the A3 and B3 series (Star-shaped ties).

TABLE 3.8: ULTIMATE LOAD RESULTS

S/No	COLUMN		ULTIMATE LOAD (kN)	AVERAGE ULTIMATE LOAD (kN)
	SERIES	MARK		
	A1			
1		A11	275	
2		A12	276	
3		A13	250	272
4		A14	285	
5		A15	274	
	A3			
6		A31	272	
7		A32	274	
8		A33	225	263
9		A34	300	
10		A35	242	
	B1			
11		B11	265	
12		B12	264	
13		B13	290	278
14		B14	275	
15		B15	295	
	B3			
16		B31	265	
17		B32	256	
18		B33	250	245
19		B34	246	
20		B35	209	

The followings are constant for all columns:

- Size of specimens: 100x100x750mm.
- Longitudinal Bars: 8No. 6mm Diameter bars.
- Lateral ties: 4mm Diameter bars.

CHAPTER FOUR

RESULTS ANALYSIS AND DISCUSSION

4.1 Observations

There was little variation in temperature during the test because no changes were observed. This, and the temperature compensation of the strain gauges helped to minimize apparent strain caused by temperature changes.

Cracks appeared at about 180kN which is 66% of the average failure load of 272kN in the A1 series, 145kN or 55% of the average failure load of 263kN in the A3 series, 181kN or 65% of the failure load of 278kN in the B1 series and 118kn or 48% of the failure load of 245kn in the B3 series. The low value of 48% is due to column B34 which started cracking at an usual load value of 50kN. This is far lower than the calculated failure load of about 150kN and in fact, the lowest value for which any of the columns tested showed its first cracks. These percentages are generally indicative of the fact that stirrups arrangement in the A1 and B1 Series (Diamond-shaped over lapping hoops) gives a shorter warning period before failure when compared with the arrangement in the A3 and B3 series (star-shaped hoops)

4.2 Failure Modes

The general failure mode observed was mainly diagonal shearing failure of the concrete and a buckling failure of the column steel between ties. Column failure is mostly at the top or bottom except for B31 and B32 which failed at both ends .This is shown in plate 6.

Failure in most cases is accompanied with spalling of concrete cover on all sides. In columns where tie spacing was reduced at top and bottom, failure mostly occurred below the closely-spaced ties as shown in plate 7.

PLATE 6: Failure of columns at both ends.

Buckling in between ties was also lesser at the heavily bound ends. It was observed that the A1 and B1 series (Diamond shaped hoops) sustained higher loads before failure than the A3 and B3 series (star-shaped hoops).

Tie failure was recorded in only a few cases. This is attributable to the method used in fixing the ties, which was by hooking them through an angle of 180° and then adequately securing them to the main bars with wire.

PLATE 7: Failure of columns with reduced spacing of Ties.

4.3 Discussion of results

4.3.1 Variability of stresses along a column

Fig. 4.1 below, shows average strain values plotted against height of columns. The average strain values are for the five columns tested for each series at a load of 225kN. Details of similar results for the individual columns are given in Appendix B.

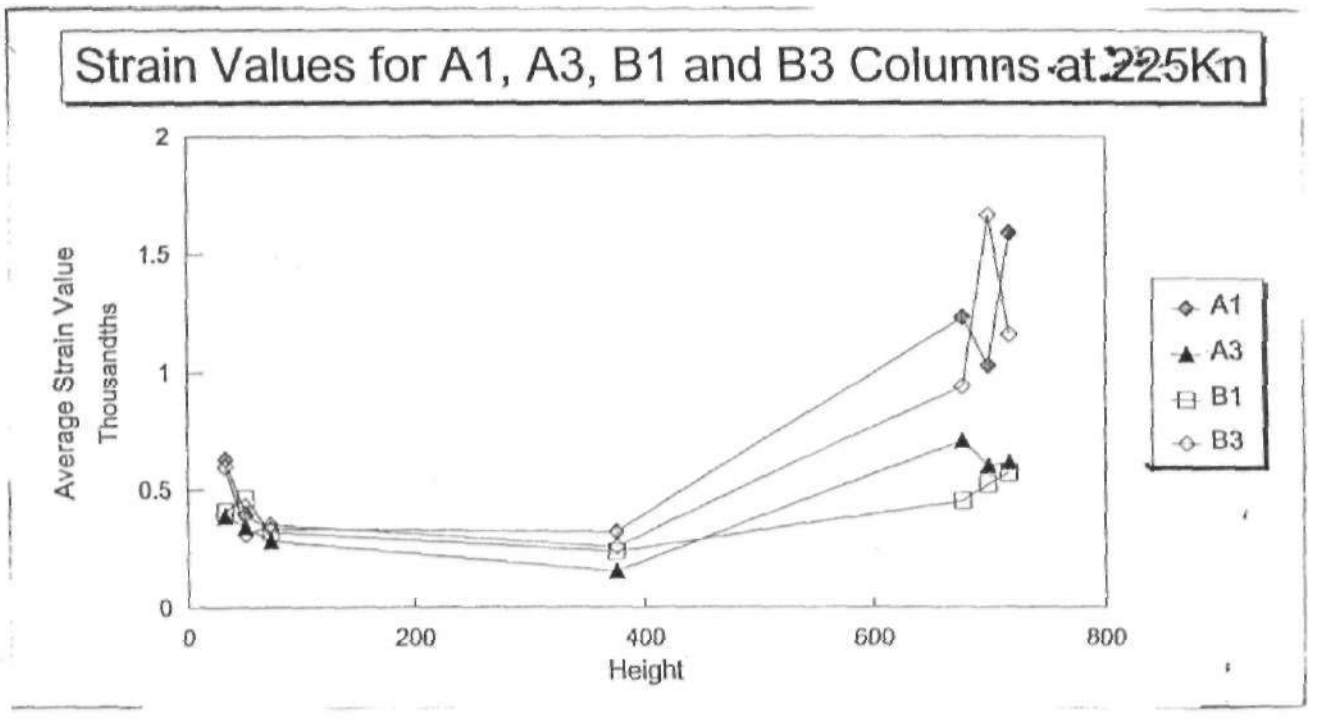


Fig. 4.1: Graph of Average strain values Against Height of columns for various series.

An observation of all the graphs clearly shows that strains are greater at the two ends and are minimum in the middle of the columns. The strains developed at the top are however in turn, larger than those developed at the bottom. This disparity could perhaps be due to the nature of the end support which is fixed at the top and hinged at the bottom and probably due to the eccentricity of loading.

The observed pattern of the strains and thus stresses, confirms the expectations that since failure is usually located at the two ends of a column, stresses are greater there.

The second aspect of our study is to investigate the effect of spacing on stress development at the two ends of the column. Whereas, as graphs of A1 series and B1 series of Fig. 4.1 show, reducing the spacing of hoops produced reduced strain development in the diamond-shaped over lapping hoops, the effect was not similar in the star-shaped hoops as shown by graphs of A3 and B3 series.

Considering the graph in Fig.4.1, simple linear equations can be generated for the lines between the points, 73mm to 375mm and 375 to 677mm, on the horizontal axis.

The equation will have a gradient/Intercept form as follows:-

$$\varepsilon = Hm + C \qquad (4.1)$$

Where:

ε = Strain

H = Height of column

$\mathbf{m}$ = gradient of line

$\mathbf{C}$ = Constant.

Generally, the equation of a line having two points on the graph shown in fig 4.1, will be given by:

$$\frac{\varepsilon - \varepsilon_1}{H - H_1} = m = \frac{\varepsilon_2 - \varepsilon_1}{H_2 - H_1} \qquad (4.2)$$

Thus, for the B1 series, (Fig. 4.1) the equation of the line between the points (73, 0.000322) and (375, 0.000240) will be $\varepsilon = -2.72 \times 10^{-7} H + 3.02 \times 10^{-4}$ and that of the line between the points (375, 0.000240) and (677,0.000452) will be

$\varepsilon = 7.020 \times 10^{-7} H - 2.32 \times 10^{-5}$.

Similar equations can be generated for all the series and the results are shown in Table 4.1 below.

TABLE 4.1: Equations relating strain and Height for various column series.

Column Series	Region I (73mm - 375mm)	Region II (375mm - 677mm)
B1	$\varepsilon = -2.72 \times 10^{-7} H + 3.02 \times 10^{-4}$	$\varepsilon = 7.02 \times 10^{-7} H - 2.32 \times 10^{-5}$
B3	$\varepsilon = -2.88 \times 10^{-7} H + 3.24 \times 10^{-4}$	$\varepsilon = 2.27 \times 10^{-6} H - 5.94 \times 10^{-4}$
A1	$\varepsilon = -4.97 \times 10^{-8} H + 3.40 \times 10^{-4}$	$\varepsilon = 3.012 \times 10^{-6} H - 1.45 \times 10^{-3}$
A3	$\varepsilon = -4.37 \times 10^{-7} H + 3.18 \times 10^{-4}$	$\varepsilon = 1.83 \times 10^{-6} H - 5.34 \times 10^{-4}$

The above equations will enable the prediction of strain values between the points 73mm to 375mm and 375mm to 677mm which constitute the major portion of the column shaft.

Based on the above results also, stress, σ, can be taken as εE, where E is the modulus of elasticity.

Therefore, stress will be given as:

$$\sigma = E(Hm + C) \qquad (4.3)$$

4.3.2. Load-carrying capacity

For the purpose of this study, two parameters namely, stirrup- type and stirrup spacing were taken as variables while characteristic strength and reinforcement sizes were kept constant.

Blume et al, (15), proposed that the strength of core of a rectangular column with closely spaced ties can be taken as:

$$f_c = 0.85f'_c + (4.1\ A_{sb}\ f_{yt}) / a.s \qquad (4.4)$$

The above equation does not include the effect of longitudinal reinforcement and refers only to laterally tied rectangular columns. To enable a comparison of theoretical values using equation (4.4) above, and experimental values, Chiwuzie, (21), modified the above equation by Blume et al, (15) as:

$$P_u = 0.85\ f'_c\ A_h + A_s f_y + (2.05m\ A_k\ A_{sb}\ f_{yt}) / a.s \qquad (4.5)$$

Where:

P_u = Ultimate strength of a column

A_k = Cross - sectional area of column core.

A_s = Area of compressive reinforcement.

f_y = Yield strength of longitudinal reinforcement.

f'_c = Compressive strength of concrete

A_b = Total area of concrete

m = Number of legs in one link of hoop.

A_{sb} = Area of cross-section of a leg of hoop reinforcement

a = The longer side of a rectangular concrete are enclosed by the hoop

S = Centre to centre spacing of hoops

f_{yt} = Yield stress for the hoop

Applying the modified Blume et al equation, the calculated ultimate load $[P_u\ (BNC)]$ and observed average ultimate load are shown in Table 4.2.

TABLE 4.2: Experimental and Theoretical ultimate loads

A1 Series Columns		A3 Series Columns	
Average Experimental P_u (kN)	Calculated P_u (BNC) kN	Average Experimental P_u (kN)	Calculated P_u (BNC) kN
272.00	327.24	263.00	297.49

It is seen from the above Table that for both the observed and calculated ultimate loads, the A1 series columns (Diamond -shaped hoops) are able to carry more load than the A3 series columns (Star-shaped hoops).

There is also a marked difference between the observed experimental ultimate load in both series and the calculated ultimate load using the modified equation of Blume et al (equation 4.5). The loads given by the modified equation are higher that the observed ultimate loads. This suggests that the modified equation does not adequately estimate the ultimate load capacities of the columns. There is need therefore to carry out further column tests with a view to proposing a further modification of equation 4.5

4.3.3 Ductility

Ductility is an important consideration in the behaviour of structures under loading. It is important to ensure that in the extreme event of a structure being loaded to failure, it will behave in a ductile manner. This means ensuring that the structure will not fail in a brittle manner without warning but will be capable of large deformations at near maximum load carrying capacity. Ductility assures gradual failure, allows internal force redistribution, and high energy absorption in the case of dynamic loading.

To ensure ductile behaviour special attention has to be given to details such as confinement of compressive concrete, longitudinal steel contents and anchorage of reinforcement amongst others. Taking confinement as a measure of ductility and making a comparison between A1 series (Diamond-shaped hoops) and A3 series (Star-shaped hoops), it is clear from fig 4.1, that for the same spacing of hoops (80mm), larger strains (0.001232 at a height of 677mm in A1 series for instance) and thus stresses are developed in A1 series than in the A3 series. (0.000708 at 677mm).

This means columns with the star-shaped hoops develops less stresses and are thus more ductile and tougher under loading than columns with the diamond shaped hoops. This results also tallies with the observation made in section 4.1 that the star-shaped hoops gives a longer warning period before failure when compared with the Diamond-shaped hoops. Furthermore, comparing the strain development in the A1 series and B1 series especially at the top, also shows that there is an enhancement of ductility due to increase in the number of ties as has been noted by many investigators such as: Burdette and Hilsdorf (8) and Sargin et al (9).

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

This work is on the load-stress relationship of short reinforced concrete columns with the aim of studying the effects of tie type and tie spacing on ultimate loads and stresses. Also investigated is their effect on the ductility of confined concrete. In order to achieve this, small-size columns of sizes 100x100x750mm were tested. The main reinforcement bars used are 6mm in diameter while the hoop reinforcement are 4mm diameter bars with a concrete grade of 30. Based on the values of the test results, the following conclusions are reached.

1. Compressive stresses in short reinforced concrete columns are concentrated at the two ends of the column where failure occurred and are minimum in the middle.
2. The presence of closely spaced ties causes an appreciable increase in column ductility.
3. Columns with the diamond-shaped hoops are able to carry higher loads than columns with the star-shaped hoops, although, the difference is very small. In fact, it can be said that the two hoop types provide load carrying capacity assistance of the same order.
4. For a given tie spacing and percentage of lateral reinforcement, columns with the star-shaped hoops develop less stresses and are thus more ductile and tougher under loading than columns with the diamond shaped hoops. The presence of the cross-bar arrangement in the star-shaped hoops is probably

responsible for this, because of the added flexural rigidity it provides the hoops.

5.2 Recommendations.

Columns receive sustained loading as opposed to the relatively rapid testing of specimens conducted in this work. To understand their behaviour at service load levels, long term column tests with suitable strain gauges that can withstand the long duration need to be carried out. Secondly, creep and shrinkage effect on the long term behaviour of columns need be studied.

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APPENDIX A: DETAIL OF STRAIN VALUES FOR EACH STAGE OF LOADING

B1 SERIES

B11

LOAD (kN)	Strains ($x10^{-6}$)												
	T1	T2	T3	T1'	T2'	T3'	M	M'	B1	B2	B3	B1'	B2'	B3'
50	-	-	10	160	110	70	90	70	10	40	10	-	-	10
100	-	-	10	260	250	210	120	130	40	90	20	-	-	40
150	-	-	20	280	350	380	160	200	70	130	150	-	-	90
175	-	-	30	320	420	480	180	230	100	170	210	-	-	130
200	-	-	90	410	530	590	220	270	170	230	320	-	-	180
225	-	-	170	590	690	700	360	280	190	290	390	-	-	250
250	-	-	220	700	810	770	430	290	220	320	440	-	-	320

B12

LOAD (kN)	Strains ($x10^{-6}$)													
	T1	T2	T3	T1'	T2'	T3'	M	M'	B1	B2	B3	B1'	B2'	B3'
50	70	10	140	110	100	30	10	20	10	60	30	10	10	10
100	80	10	150	810	300	180	10	40	60	70	80	80	30	60
150	90	20	160	1030	470	500	20	50	130	110	210	130	80	130
175	100	70	200	1160	530	710	30	100	160	150	300	210	120	180
200	120	140	550	1250	680	1020	40	140	210	220	410	310	190	270
225	210	360	850	1470	940	1310	50	240	310	390	550	460	290	390
250	310	540	960	1610	1240	1810	90	250	430	500	680	580	330	460

B13

LOAD (kN)	Strains ($x10^{-6}$)													
	T1	T2	T3	T1'	T2'	T3'	M	M'	B1	B2	B3	B1'	B2'	B3'
50	190	30	10	70	20	40	20	-	30	-	90	60	40	-
100	380	80	20	220	90	90	40	-	50	-	90	110	110	-
150	580	150	70	340	230	170	80	-	70	-	100	220	250	-
175	580	200	110	410	290	220	100	-	80	-	120	270	360	-
200	1050	260	190	520	390	290	130	-	90	-	140	330	540	-
225	1000	350	280	640	480	360	170	-	130	-	230	430	780	-
250	910	460	370	740	570	420	210	-	170	-	350	490	1060	-

B14

LOAD (kN)	Strains ($x10^{-6}$)													
	T1	T2	T3	T1'	T2'	T3'	M	M'	B1	B2	B3	B1'	B2'	B3'
50	40	50	10	50	80	90	20	120	90	-	270	350	-	80
100	100	30	20	150	180	180	30	180	140	-	320	420	-	140
150	170	90	30	220	270	290	40	270	180	-	420	520	-	200
175	250	180	50	280	350	330	70	330	210	-	520	600	-	230
200	310	240	90	320	440	350	100	410	250	-	650	660	-	240
225	470	350	130	400	550	400	170	560	330	-	820	790	-	280
250	570	420	160	470	620	460	230	660	400	-	970	910	-	330

B3 SERIES

B31

LOAD (kN)	Strains ($x10^{-6}$)													
	T1	T2	T3	T1'	T2'	T3'	M	M'	B1	B2	B3	B1'	B2'	B3'
50	100	10	10	20	60	200	40	-	10	20	30	50	30	50
100	200	20	10	160	230	350	60	-	20	50	120	70	70	290
150	520	40	20	360	480	530	80	-	600	180	330	80	140	460
175	950	80	60	550	690	720	120	-	100	320	460	90	230	600
200	1410	230	210	830	1030	1070	160	-	260	430	590	130	320	710
225	1130	770	380	930	1380	1970	220	-	350	500	720	210	440	880
250	1420	1370	470	1040	1810	2510	280	-	630	590	1050	460	710	1280

B32 -

LOAD (kN)	Strains ($x10^{-6}$)													
	T1	T2	T3	T1'	T2'	T3'	M	M'	B1	B2	B3	B1'	B2'	B3'
50	30	-	10	20	90	-	10	10	10	50	20	-	-	40
100	70	-	50	60	600	-	20	60	20	60	60	-	-	90
150	160	-	90	310	1360	-	30	140	40	80	80	-	-	250
175	390	-	340	550	1790	-	40	220	60	100	100	-	-	400
200	730	-	510	820	2100	-	80	310	70	190	140	-	-	600
225	1130	-	650	1060	2100	-	150	410	250	220	220	-	-	820
250	1080	-	880	1260	2270	-	280	530	610	500	390	-	-	1240

B33

LOAD (kN)	Strains ($x10^{-6}$)													
	T1	T2	T3	T1'	T2'	T3'	M	M'	B1	B2	B3	B1'	B2'	B3'
50	90	-	10	70	10	20	-	30	10	20	-	10	40	60
100	280	-	60	120	240	90	-	60	20	40	-	20	130	210
150	1010	-	600	210	630	230	-	90	20	70	-	40	290	330
175	1460	-	890	410	1010	240	-	110	50	90	-	50	370	380
200	1630	-	1210	520	1300	280	-	130	70	110	-	80	450	420
225	1900	-	1690	810	2750	400	-	480	250	330	-	310	580	410
250	2190	-	-	990	3350	570	-	470	350	590	-	380	380	40

B34

LOAD (kN)	Strains ($x10^{-6}$)													
	T1	T2	T3	T1'	T2'	T3'	M	M'	B1	B2	B3	B1'	B2'	B3'
50	-	10	-	-	10	100	10	10	10	-	80	10	10	30
100	-	80	-	-	220	300	20	20	20	-	210	30	20	100
150	-	180	-	-	840	710	50	50	80	-	310	110	80	270
175	-	400	-	-	1100	840	80	50	100	-	420	170	110	390
200	-	540	-	-	1380	950	120	50	160	-	660	260	120	490
225	-	730	-	-	1380	1180	190	70	250	-	750	350	180	570
250	-	-	-	-	-	1200	80	790	420	-	830	390	140	350

A1 SERIES

A11

LOAD (kN)	Strains ($x10^{-6}$)													
	T1	T2	T3	T1'	T2'	T3'	M	M'	B1	B2	B3	B1'	B2'	B3'
50	-	10	-	-	140	70	40	120	60	20	-	50	-	-
100	-	20	-	-	500	130	70	230	90	50	-	90	-	-
150	-	70	-	-	1120	330	100	320	150	120	-	170	-	-
175	-	140	-	-	1470	680	130	350	150	190	-	210	-	-
200	-	160	-	-	1750	960	170	390	300	280	-	270	-	-
225	-	170	-	-	1800	1440	240	410	350	430	-	380	-	-
250	-	390	-	-	1890	1700	310	430	380	560	-	580	-	-

A12

LOAD (kN)	Strains ($x10^{-6}$)													
	T1	T2	T3	T1'	T2'	T3'	M	M'	B1	B2	B3	B1'	B2'	B3'
50	180	-	30	440	540	50	40	70	90	300	50	30	40	70
100	190	-	80	1210	9440	100	120	150	140	370	100	80	100	140
150	220	-	130	1210	930	160	140	220	200	430	200	130	180	190
175	240	-	150	1510	1350	200	180	250	240	460	280	210	230	220
200	250	-	230	2320	1880	240	210	310	370	550	510	240	360	370
225	320	-	320	4220	1980	760	170	340	480	610	680	340	430	440
250	860	-	170	4720	3980	1120	190	240	540	600	1140	1580	590	550

A13

LOAD (kN)	Strains ($x10^{-6}$)													
	T1	T2	T3	T1'	T2'	T3'	M	M'	B1	B2	B3	B1'	B2'	B3'
50	-	-	90	-	-	690	50	50	30	60	40	20	30	220
100	-	-	100	-	-	710	90	160	50	110	80	50	40	310
150	-	-	120	-	-	720	170	330	160	240	130	110	90	410
175	-	-	380	-	-	880	280	660	400	490	230	260	170	770
200	-	-	520	-	-	1180	330	780	530	580	270	330	210	990
225	-	-	1180	-	-	2700	390	920	650	690	390	450	300	1510
250	-	-	-	-	-	-	-	-	-	-	-	-	-	-

A14

LOAD (kN)	Strains ($x10^{-6}$)													
	T1	T2	T3	T1'	T2'	T3'	M	M'	B1	B2	B3	B1'	B2'	B3'
50	-	-	-	1180	470	1080	30	220	120	370	220	70	80	150
100	-	-	-	1240	500	1190	50	260	170	390	260	80	100	180
150	-	-	-	1380	600	1860	80	310	220	410	330	80	110	220
175	-	-	-	1540	680	1500	90	340	260	410	380	80	120	220
200	-	-	-	1680	820	1450	110	400	300	450	440	210	130	260
225	-	-	-	1900	1020	1470	120	480	380	500	550	230	170	330
250	-	-	-	2270	1330	2220	140	600	460	580	700	260	210	410

A3 SERIES

A31

LOAD (kN)	Strains ($x10^{-6}$)												
	T1	T2	T3	T1'	T2'	T3'	M	M'	B1	B2	B3	B1'	B2'	B3'
50	-	-	-	50	80	270	10	10	10	1180	90	180	20	-
100	-	-	-	190	310	510	20	30	20	1150	170	200	30	-
150	-	-	-	960	450	1190	30	50	50	1140	240	210	30	-
175	-	-	-	1490	620	1180	50	60	60	1140	260	200	30	-
200	-	-	-	1690	780	1160	70	60	70	1130	300	180	40	-
225	-	-	-	2070	910	1530	80	80	170	1150	400	190	50	-
250	-	-	-	2230	-	2390	140	70	280	1160	480	200	70	-

A32

LOAD (kN)	Strains ($x10^{-6}$)													
	T1	T2	T3	T1'	T2'	T3'	M	M'	B1	B2	B3	B1'	B2'	B3'
50	10	-	10	30	20	80	10	-	-	-	10	90	10	70
100	20	-	10	30	40	90	10	-	-	-	70	110	10	130
150	40	-	10	20	90	130	30	-	-	-	140	130	20	190
175	50	-	20	20	120	160	40	-	-	-	170	140	30	250
200	70	-	30	30	150	210	50	-	-	-	210	160	60	280
225	90	-	30	40	200	280	70	-	-	-	250	170	90	390
250	100	-	30	50	250	390	70	-	-	-	280	200	120	230

A33

LOAD (kN)	Strains ($x10^{-6}$)													
	T1	T2	T3	T1'	T2'	T3'	M	M'	B1	B2	B3	B1'	B2'	B3'
50	100	180	280	50	80	-	30	50	190	240	90	20	10	970
100	210	260	660	20	1280	-	80	50	200	230	80	30	40	960
150	350	340	660	20	1330	-	140	60	220	210	140	90	120	980
175	430	430	580	20	1370	-	200	70	240	180	180	130	150	1070
200	520	570	600	30	1500	-	260	80	270	270	180	190	170	1160
225	620	780	700	60	1600	-	300	90	380	240	180	510	410	1180
250	-	-	-	-	-	-	-	-	-	-	-	-	-	-

A34

LOAD (kN)	Strains ($x10^{-6}$)													
	T1	T2	T3	T1'	T2'	T3'	M	M'	B1	B2	B3	B1'	B2'	B3'
50	160	70	50	-	20	50	10	30	-	180	120	10	-	-
100	330	100	80	-	40	100	10	40	-	460	130	110	-	-
150	410	100	110	-	50	100	30	40	-	450	170	130	-	-
175	430	110	140	-	60	110	40	40	-	600	200	140	-	-
200	420	120	150	-	70	120	50	40	-	520	230	140	-	-
225	400	140	170	-	80	160	60	40	-	500	260	150	-	-
250	370	140	180	-	100	180	50	50	-	480	260	160	-	-

A15, A35, B15 AND B35 SERIES

A15

LOAD (kN)	Strains ($x10^{-6}$)												
	T1	T2	T3	T1'	T2'	T3'	M	M'	B1	B2	B3	B1'	B2'	B3'
50	150	-	40	60	30	-	-	30	10	10	120	20	10	60
100	440	-	80	70	30	-	-	40	20	20	180	30	10	100
150	660	-	130	90	50	-	-	70	20	60	570	50	10	170
175	850	-	180	100	100	-	-	60	20	80	680	50	30	210
200	1010	-	250	100	120	-	-	70	30	160	770	50	40	260
225	1090	-	770	110	140	-	-	70	40	330	780	60	60	350
250	1400	-	1420	130	150	-	-	90	280	640	700	130	100	520

A35

LOAD (kN)	Strains ($x10^{-6}$)												
	T1	T2	T3	T1'	T2'	T3'	M	M'	B1	B2	B3	B1'	B2'	B3'
50	-	-	-	60	-	40	20	20	30	30	40	80	-	-
100	-	-	-	100	-	140	130	30	70	80	120	90	-	-
150	-	-	-	120	-	450	270	50	180	140	180	80	-	-
175	-	-	-	140	-	520	410	70	240	180	210	110	-	-
200	-	-	-	140	-	570	470	90	370	210	250	110	-	-
225	-	-	-	210	-	990	630	120	770	200	290	200	-	-
250	-	-	-	-	-	1410	360	1350	600	330	210	-	-	-

B15

LOAD (kN)	Strains ($x10^{-6}$)												
	T1	T2	T3	T1'	T2'	T3'	M	M'	B1	B2	B3	B1'	B2'	B3'
50	40	10	40	10	60	-	30	-	40	20	10	20	30	40
100	60	10	50	30	130	-	50	-	50	50	80	50	70	130
150	80	60	70	100	230	-	90	-	80	120	190	130	190	270
175	90	130	90	130	300	-	110	-	70	140	260	170	280	350
200	100	220	120	170	390	-	140	-	60	180	320	240	430	440
225	130	320	160	210	510	-	200	-	70	230	410	320	650	550
250	180	420	220	270	630	-	250	-	80	240	460	380	890	630

B35

LOAD (kN)	Strains ($x10^{-6}$)													
	T1	T2	T3	T1'	T2'	T3'	-	M'	B1	B2	B3	B1'	B2'	B3'
50	-	-	80	-	10	10	-	70	130	60	-	410	20	-
100	-	-	140	-	210	150	-	80	180	90	-	820	70	-
150	-	-	310	-	670	470	-	100	260	110	-	910	120	-
175	-	-	500	-	1050	590	-	150	270	200	-	920	170	-
200	-	-	640	-	1340	700	-	180	350	270	-	880	200	-
225	-	-	450	-	1710	390	-	630	-	110	-	1100	80	-
250	-	-	-	-			-				-			-

Appendix B: Plots of Strain Values against Height for all Columns

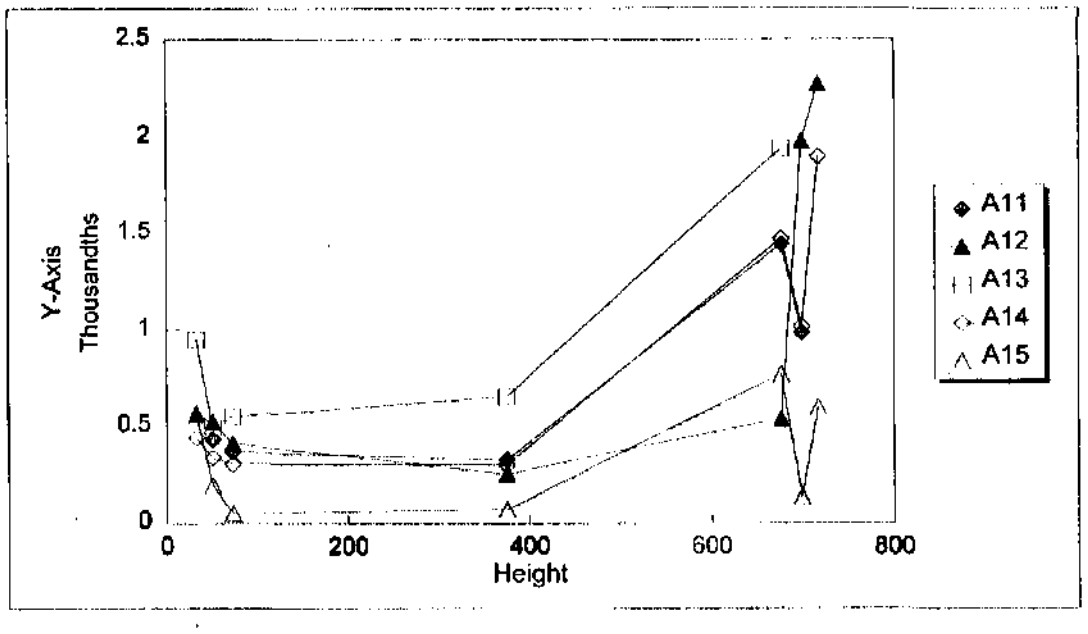

Fig. B1: A1 Column series

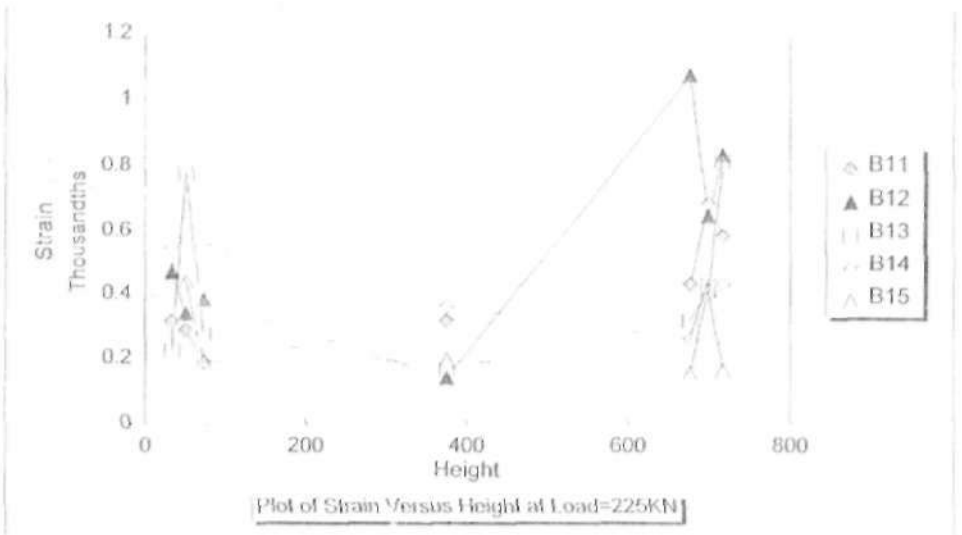

Fig B3: B1 Column series

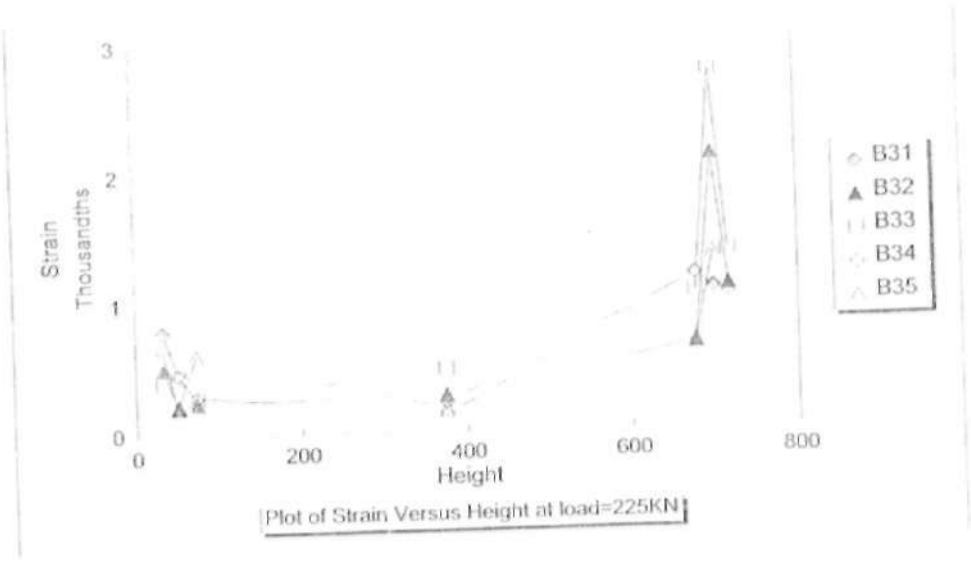

Fig. B4: B3 Column series