ANALYSIS OF INTRA-URBAN BUS TRAVEL TIMES

BY

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DECLARATION

I hereby declare that this dissertation has been composed by myself and that it is a record of my own research work. It has not been accepted in any previous application for a higher degree. All sources of information are specifically acknowledged by means of references.

E.E. Chiejina

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April 1982.

TO MY FAMILY AND UKPONU STEVE IN APPRECIATION
OF THEIR HELP AND SUPPORT

ABSTRACT

Link travel times of scheduled, fixed-route intra-urban bus transit have been studied with the following primary objectives:

- To choose a most appropriate candidate distribution model from several potentially possible probability distribution models of travel times.
- To construct simple predictive models of travel times, in terms of variables related to system characteristics, for a specific site.
- To develop simple mathematical models of stop dwell times of buses.

Building upon a review of literature, the study presents an overview of the relative appropriateness of some alternative probability models of travel times. The distributions studied are the Normal, Log-Normal, Gamma, and Beta distributions. In particular, the potentials of Log-Normal distribution model are explored based on its practical and statistical properties, estimability of the parameters, and the stochastic nature of travel times. Kolmogorov-Smirnov goodness-of-fit test and the decision rule for the 5% significance level confirms the appropriateness of Log-Normal model, relative to observed data, as an underlying probability model for day-to-day stochastic distributions of travel

time. Data collected on a bus line run by Kaduna State Transport Authority in metropolitan Kaduna are used to estimate and evaluate the models. Simple mathematical models to predict the parameters of the Log-Normal model as a function of system characteristics are constructed.

Only link length and number of roundabouts have been found to be statistically significant; surprisingly, number of intersections is statistically insignificant as a factor. Because examination shows that the explanatory variables in the proposed models are not collinear, the models derived do not have biased coefficients. Also, simple analytical models of stop dwell times of buses are formulated.

The major findings, in the light of the peculiar set of present day conditions, are that bus total dead times at stops are a random variable following a Gamma distribution and boarding/alighting times are linear functions of number of boarding/alighting passengers.

Estimation results provide tentative and reasonable support for the conceptual and theoretical bases of the models. Also, by collating models predictions with observed values, it is demonstrated that a workable theoretical and empirical construct exists for characterizing

link travel times. The proposed model is conceptually and operationally simple and should provide intra-urban bus transit operators or planners with an analytical framework to address the subject of link travel times in particular and bus transit planning in general.

The study concludes with implications of results ; and directions for further research are suggested.

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NOTATION

The following symbols are used in this dissertation:

$a_o, a_1, b_o, b_1, \beta_o, \beta_1$ = Calibration constants determined by regression equation;

$\mathbf{a}$ = Location parameter of probability model, threshold;

$\mathbf{C}$ = Number of coefficients estimated in a regression equation;

$cov(,)$ = covariance;

D_n^{α}, D_n = Kolmogorov-Smirnov Statistic, Observed maximum absolute difference between sample and theoretical cumulative distributions for significance test;

D_a, D_b = Dead times for alighting and boarding events;

D_{ab} = Dead times for simultaneous alighting and boarding;

$E(X)$ = Expected value of random variable X;

$E_Y(.)$ = Expectation with respect to Y;

$E(X|Y)$ = Expectation of X given Y;

F = Significance ratio for regression, ANOVA;

$F(x)$ = Cumulative distribution function;

$F_G(.)$ = Gamma cumulative distribution function

$F_{LN}(.), F_N(.)$ = Cumulative distributions of Log-Normal, Standard Normal random variables;

$F_{\chi^2}(.)$	= Chi-Square cumulative distribution function;
$f(x)$	= Probability density function of random variable X;
G	= Geometric mean;
H	= Bus headway;
H_o	= Hypothesis under test, Null hypothesis;
H_1	= Alternative hypothesis;
K	= Shape parameter of Gamma distribution;
$K_o, K_1, K_2, \ldots K_n$	= Calibration constants determined by regression equation;
L	= Likelihood function;
L	= Link length;
$\ln$	= Natural logarithm;
M_1'	= First moment about the origin, mean (sample moment);
M_2'	= Second moment about the origin(sample moment);
M_r'	= rth moment about the origin(sample moment);
M_2	= Second central moment, variance (sample moment);

M_3 = Third moment about the mean;

M_r = rth central moment (sample moment);

n,N = Sample size ;

n_a = Number of alighting passengers;

n_b = Number of boarding passengers ;

N_{in} = Number of intersections;

N_{rd} = Number of roundabouts;

q = Quantile;

q,r = Beta shape parameters ;

Q = Number of arriving passengers;

R = Correlation coefficient for regression equation, ANOVA;

$\bar{R}^2$ = Coefficient of determination corrected for degrees of freedom;

S = Sample standard deviation;

S_a = Stop dwell time for alighting process;

S_b = Stop dwell time for boarding process;

S_{ab} = Stop dwell time for simultaneous alighting and boarding;

S(x) = Sample cumulative distribution function;

t = t-value for statistical test;

T = Statistic (mean);

T_a, T_b = Alighting, Boarding times;

T_{md} = Median of link travel time;

T_{me} = Mean of link travel time;

$var(X)$ = Variance of random variable X;

$var_y(.)$ = Variance with respect to Y;

$var(X|Y)$ = Variance of X given Y;

X = Random variable;

$\bar{X}$ = Arithmetic mean of sample values X_i;

x_i = Sample value;

x_m = Median value of random variable X;

$\overset{o}{x}$ = Modal value of random variable X;

$x_{(r)}$ = rth order statistic;

x_q = Quantile of order q;

z = Standard Normal value;

z_q = Standard Normal quantile of order q;

α = Significance level for statistical tests; Probability of type I error;

$B(q,r)$ = Beta function;

$B_x(q,r)$ = Incomplete Beta function;

$\Gamma(K)$ = Gamma function of parameter K;

λ = Scale parameter of Gamma distribution, Exponential distribution;

$\bar{\lambda}_p$ = Passenger mean arrival rate;

$\hat{\lambda}_w$ = Overall weighted parameter for both alighting and boarding processes;

$\hat{\lambda}_{wa}$ = Weighted parameter for alighting process;

$\hat{\lambda}_{wb}$ = Weighted parameter for boarding process;

$I_x(q,r)$ = Incomplete Beta function ratio;

Π = Product function;

γ = Proportion of arriving passengers that board a bus that arrives at a stop;

$\varepsilon, \varepsilon_i \varepsilon_j$ = Independent identically distributed random error for regression;

μ = Normal location parameter (mean);

Ψ = Digamma function;

ψ' = Trigamma function;

$\psi(\hat{\delta})$ = Function of $\hat{\delta}$;

θ = Parameter;

$\hat{\theta}$ = Maximum likelihood estimate of θ;

ν, d.f. = Degrees of freedom;

σ = Normal scale parameter(Standard deviation);

$\sum$ = Summation;

χ^2 = Chi-square;

$\frac{\partial}{\partial\theta}$ = Derivative of a function with respect to θ;

$\int d\theta$ = Integral of a function with respect to θ; and

$\int dx$ = Integral of a function with respect to x.

SUBSCRIPTS

a	= Alighting passengers;
b	= Boarding passengers;
G	= Gamma distribution;
i	= Number index;
in	= Intersection;
j	= Number index;
LN	= Log-Normal distribution
md	= median;
me	= Mean;
N	= Normal distribution;
p	= Passenger;
q	= Quantile order;
r	= Moment index;
rd	= Roundabout;
w	= Overall weighting for boarding and alighting events;
wa	= Weighting for alighting event;
wb	= Weighting for boarding event; and
χ^2	= Chi-square.

ABBREVIATIONS

The following abbreviations are used in this dissertation:

ANOVA	Analysis of Variance;
BLUE	Best Linear Unbiased Estimate;
CBD	Central Business District;
CDF	Cumulative Distribution Function;
CV	Coefficient of Variation;
K-S	Kolmogorov-Smirnov;
KSTA	Kaduna State Transport Authority;
LSE	Least Square Estimate;
LF	Likelihood Function;
ML	Maximum Likelihood
MLE	Maximum Likelihood Estimate;
NAG	Numerical Algorithm Group;
SEE	Standard Error of Estimate;
SPSS	Statistical Package for the Social Sciences; and
TMO	Two Man Operated.

CHAPTER 1

INTRODUCTION

This study focuses on scheduled, fixed-route intra-urban bus transit, but bus travel times on links are given particular emphasis. The aspects of bus travel times studied are link travel times and stop dwell times.

Bus transit, however, possesses some characteristic features which tend to distinguish it from other forms of public transportation. It, for example, seldom operates on exclusive rights-of-way and its operation is highly dependent on physical characteristics of the route, traffic volumes, passenger loads, and even weather.

Conceivably, separation between homes and work, and similarity in most working hours through the whole spectrum of jobs have resulted in movement of people between homes and work each day at approximately the same times in the day and considerable volumes of traffic along relatively narrow corridors. In effect, heavy concentration of trips both in terms of time and space are the typical problems of commuting traffic: congestion, delays, frustration, and so forth. Highly reliable transit service, therefore, is of common interest and

benefit to passengers and operators, and could almost always attract auto users to public transit.

Travel time is an important variable in explaining travel impedance, run choice, mode choice, and so forth. It is also a level of service measure most clearly susceptible to variations. To transit planners or operators, knowledge of travel time is essential in determining appropriate schedule time for bus trips, layover or slack time, and effective systems' operations and management. To transit users, travel time is vitally important vis-a-vis on-time destination achievements. In this light, it is important to transit planners or operators to have basic tools for analysing service performance.

However, the performance of fixed route public transport is hardly a novel subject. Over the years, particularly in developed countries such as the United States of America and Britain, various aspects of the conventional fixed route service have been extensively researched.

This study attempts to analyse bus travel time distributions and their affecting factors, and to construct simple predictive models of travel time, based on system characteristics, for a specific site.

A subsidiary aim is to examine the characteristics of bus stop dwell times and to estimate models for stop dwell times.

The analysis is done in the context of a real-life bus transit system run by Kaduna State Transport Authority (KSTA) in metropolitan Kaduna.

1.1 STUDY METHODOLOGY

1.1.1 FIELD OBSERVATIONS

The data set collected on a bus line run by KSTA in metropolitan Kaduna were used to calibrate and illustrate the models. Kaduna is the capital of Kaduna State in Northern Nigeria, has about 2 million inhabitants and consists largely of high density residential areas. The KSTA is the only agency that provides scheduled, fixed-route intra-city bus service in the state.

The bus route (Route One) studied traverses a major traffic corridor connecting the Central Business District (CBD) of the city with Kawo and Kakuri suburbs. The round trip-length of the route (Kawo-Kakuri-Kawo) is approximately 30.9-km. The route is sufficiently long with a dual carriageway and contains a number of round-abouts and minor intersections and no traffic lights.

Route One is worked by Two-Man-Operated (TMO), two-door, 52-seater Steyr buses. The two doors are provided with a transmission interlock system which prevents the doors from being opened while the bus is in motion. Passengers board through the rear door and exit through the front door. The fare structure is graduated and the conductor collects fares and issues ticket manually while the bus is moving.

Data were obtained by on-board survey. Observers riding the buses, equipped with stop-watches and electronic digital watches, recorded simultaneously the bus dwell times at each bus stop (excluding stop events at termini), the boarding/alighting times and the number of patrons served, dead times between the instant of bus stopping and the first passenger boarding or alighting (i.e., pre-boarding/alighting dead times). The dead times between the instant of last passenger boarding/alighting and the instant of bus departure (i.e., post-boarding/alighting dead times) were also recorded.

Link travel times, that is,travel times between two bus stops were recorded. The KSTA furnished information on the spaces between some bus stops. The number of roundabouts and number of intersections were observed directly. After a preliminary survey, the observations

lasted for five consecutive days (Monday through Friday) and mostly during the morning peak periods, in June 1981. Altogether, 30 observations of travel times on each of the 34 links were made. A larger data base would have been desired but resource limitation was a major constraint.

1.1.2 BUS TRAVEL TIMES ON LINKS

The conventional fixed-route systems provide one-dimensional corridor service and they have regular stops while also operating on schedule. Herein, a fixed bus route is assumed to consist of links which can be defined as segments connected by node pairs. Thus, a bus route is defined simply as an ordered sequence of links connecting pairs of bus stops. Links possess locational and spatial attributes such as length, capacity, and direction. Nodes may be regarded as access points to a transit system. For a bus system, the nodes correspond to bus stops.

It is assumed that individual measures of travel time over a given link, represent a random sample from a positive random variable which itself is a real-valued function defined on a sample space.

The first step is to identify the desired characteristics of a link travel time distribution, and then search for a model which could embrace as many of the desired characteristics as possible, and be mathematically convenient manually, or with a computer

Before calibration of models, the observational data collected on different weekdays (Monday through Friday) and mostly during the morning peak periods were checked against any statistical differences in means of travel times using analysis of variance (ANOVA). The analysis was inevitable as data had been collected during different weekdays. Moreover, the analysis could reflect variations in bus performance which might be related to daily trends as well as form a basis for aggregation of data if the differences in means of travel times among the different weekdays were not statistically significant, or stratification of data, if the means were statistically significant. However, the results of computer program utilized, SPSS (Statistical Package for the Social Sciences) subprogram ANOVA, showed that the means of travel times among the different weekdays were not significantly different statistically for all the 34 links studied.

Two strategies for modelling of travel times were evaluated, namely: 1) distribution modelling technique; and 2) stepwise regression technique. In the latter, models were constructed recursively through successive addition of explanatory variables that meet certain statistical criteria. Independent variables were entered only if they had a minimum F-value of 4.0 and Tolerance of 0.05 using SPSS subprogram REGRESSION. Likely impacts on travel times of basic predictor variables, namely, link length, number of roundabouts, and number of intersections along a fixed route were examined and evaluated. It was felt that the use of a probability distribution model to capture a large spatial and temporal variability in travel times on each link could permit a more thorough evaluation of information for accurate analysis and predictions, as well as provide a better and more comprehensive picture of travel time distributions.

By reviewing the available literature the relative appropriateness of several potentially possible probability distribution models of travel time was evaluated. The distribution models studied included the Normal, Log-Normal, Gamma, and Beta distributions. In particular,

the potentials of Log-Normal model were explored. Sequel to analysis of variance, the parameters of the Log-Normal model were estimated for each of the 34 links by the method of maximum likelihood. Log-Normal distribution model was considered in this investigation on the following grounds: 1) the model is highly flexible, positively skewed, and limited to positive values of a random variable of which travel times are assumed to be one; 2) it is computationally tractable and related to Normal distribution via logarithmic transformation; 3) the model is a result of a series of multiplicative effects, but the logarithms of travel time variable become Normally distributed and the effects become additive; and 4) the Log-Normal model is an appropriate model when the value of an observed variable is a random proportion of the previously observed value. In addition, it was felt that the relationship of Log-Normal model with Normal model could permit predictions of its parameters (which are Normally distributed) as a function of system characteristics without serious violations of some underlying assumptions of regression analysis.

A nonparametric test developed by Kolmogorov and Smirnov (Kalmogorov-Smirnov test, or K-S test) was used

to test the null hypothesis that the empirical data and Log-Normal distribution are not statistically different at 5% level of significance. To apply the K-S test, the empirical cumulative distribution function (sample cumulative frequency distribution) as well as the hypothesized cumulative distribution function were evaluated for each distinct observation. The K-S statistic D_n measures the maximum absolute deviation of the empirical distribution function from the theoretical distribution function. The acceptance or rejection of null hypothesis was tested by comparing D_n to tabulated critical values D_n^{α} at various significance levels α and values of n (number of samples).

The K-S goodness-of-fit test was preferred to the more traditional Chi-square (χ^2) goodness-of-fit test after careful study of their relative merits. In essence, the K-S test is exact even for small samples, conceptually and operationally simple, and it is generally considered to be more powerful than the χ^2 test (Conover, 1971).

The parameters of the Log-Normal distribution were modelled in terms of independent variables related to system characteristics. It is believed that if it is

possible to obtain a reasonably good fit from the predictor variables then a tool has been derived to predict the distribution of travel times for a specific site.

In order to ease the required computations, a computer program was written and used along with SPSS subprograms and NAG (Numerical Algorithm Group) routine. The NAG routine S15ABF was used to compute the Normal cumulative distribution function.

1.1.3 STOP DWELL TIMES

The time which a bus spends at a stop while picking up and letting off passengers constitutes a delay to patrons already on board and extra operating costs to the transit agency. According to previous studies, the dwell times of buses at stops depend on a number of factors including the number of passengers getting on or getting off, fare structure, fare collection and ticket-issuing system, type of bus in use and mode of operation.

Because intending boarding passengers seem to lack knowledge of bus schedule, often inquire about bus destination, and drivers sometimes kill time because of patron inadequacies, bus dead times and stop dwell times were passengers/drivers dependent and were essentially

random.

In order to increase the accuracy of model calibration, data on bus dwell times at stops were decomposed into pre-boarding/alighting dead times, post-boarding /alighting dead times, and pure boarding/alighting times and the number of boarding/alighting passengers. The selected approach allowed separate models of the distributions of dwell time components to be constructed and the relative impacts, on entire dwell time, of different dwell time components assessed. For the purpose of comparing empirical frequency distributions with theoretical probability density functions, frequency diagrams of dead times were constructed. On the basis of a visual examination of the frequency diagrams, the Exponential and Gamma distributions were evaluated as possible theoretical distributions to fit the data. A more objective evaluation of the adequacy of the fit was achieved by conducting a K-S test and a decision rule for 5% significance level. The boarding/alighting times were regressed on the number of boarding/alighting patrons. The regression constants and regression coefficients were checked for statistical significance or lack of significance. Where a regression constant was insignificantly different from zero, OPTION 19 of

the SPSS subprogram REGRESSION was used to "force regression through origin".

1.1.4 PASSENGER ARRIVALS AND HEADWAY DISTRIBUTION

It is assumed that passenger arrivals at every bus stop along a fixed-route are of the same pattern and follow a stationary Poisson process - the arrival rate being constant over a time period. In other words, passengers arrive randomly and independently at a bus stop. The assumption of stationary Poisson distribution of arrival process is elegant where bus services are unreliable, or frequent such as during rush-hours when headways are short. When bus services are sufficiently infrequent or reliable with schedules published and adhered to, passengers take advantage of the schedules and correlate their arrivals with bus schedules. Such a situation results in passengers' choice of arrival time to minimize their waiting times.

As data on bus headways and number of arriving passengers were not gathered, expressions derived for the mean and variance of the number of arriving passengers and number of boarders were not calibrated.

CHAPTER 2

LITERATURE REVIEW

2.1 PREVIOUS TREATMENTS: BUS TRAVEL TIMES ON LINKS

Several studies have addressed the subject of travel time distributions. The distribution of link travel time has been described as bell shaped but skewed to the right. Such studies have been carried out by Welding (1957), Kulash (1971), Newell (1974), dePirey (1971), Polus (1975), Jenkins (1976), and Turnquist (1979).

Welding in a study of the effects of close-interval service observed that bus travel time distribution on a given link was close to a Normal distribution with the curve truncated at + 1.5 standard deviations to the low side and + 2.5 standard deviations to the high side.

Kulash, in the analysis of a highly reliable service, used a simulation model wherein interarrival times of vehicles were allowed to follow an Exponential distribution. However, this is the extreme case of purely random vehicle arrival times.

dePirey created a simulation model of a bus route and assumed bus journey times to be drawn from a Normal distribution which was truncated at + 1.0 standard deviation on the low side and +2.0 standard deviations

on the high side to avoid extremely short or long travel times.

Newell's work examined the dynamic instability for service around a closed loop, travel times being assumed to be uniformly distributed.

However, the approaches of Kulash, dePirey, and Newell were descriptive and they simply assumed distributions which appeared to have the desired shapes.

Polus's approach was to determine a distribution with the proper characteristics. His result was a Beta distribution with parameters T and R. The form of the distribution was:

$$f(y) = \frac{1}{Bc^{T-1}} (y)^{R-1} \cdot (c-y)^{T-R-1}$$

$$0 \leq y \leq c \quad (2.1)$$

where y is travel time variable and B is the Beta function:

$$B = \frac{\Gamma(R)\Gamma(T-R)}{\Gamma(T)} \quad (2.2)$$

and $\Gamma(.)$ is a Gamma function.

The shape of the distribution was determined by assuming a relationship between the two parameters:

$$T = 1.5R \quad (2.3)$$

c, the upper limit of the distribution is defined as

$$c = 1.5\ m(y) \quad (2.4)$$

where m(y) is the mean travel time, and the factor 1.5 serving to stretch the distribution over the appropriate time scale, since outside of the unit interval were less than 1 percent of the observed travel time. This gave the distribution its overall form. Polus's work involved a study of transportation system reliability and his use of this distribution was to predict reliabilities, which were reciprocal of standard deviations of travel times, from values that the Beta parameters would take on in specific situations.

Jenkins discussed the use of a Gamma distribution in a particular simulation study. The form of the distribution was:

$$f(T_1) = 158(T_1-0.76)\exp[-3.6(T_1-0.76)] \qquad (2.5)$$

where T_1 is the travel time in minutes between two points. This was based on a total of 120 observations of travel times with a calculated mean of 1.3min., and calculated standard deviation of 0.4min. The calibrated distribution was precisely a shifted Gamma distribution. Jenkins's example came closest to providing a usable descriptor of travel times; however, it provided only a distribution for a particular link.

More recently, Turnquist proposed and calibrated a shifted Gamma distribution of travel times. The shift which accounted for minimum travel time was evaluated as the quotient of the length of a link and the speed limit in effect. The computed shift was subtracted from each observed travel time so that delay times remained. At this stage, the Gamma distribution was fitted to the data. In effect, a Gamma distribution of delay times was obtained.

The above review shows that a more general descriptive model which can be easily applied to any link in a system is needed and requiring only the minimum of data acquisition without serious sacrifice of the accuracy of estimation. It is hoped that the present effort would fill the vacuum.

Based on a literature search, the distribution of travel times is likely to be bell shaped, but skewed with the longer tail extending toward the longer travel times. In other words, there is a finite probability of getting very large values of travel time such as when a bus is delayed by traffic or other incidents; however, the longer tail could permit exceptionally long times in rare instances. Ideally, the tail for the short time should also be truncated, since there is

a minimum reasonable travel time between two points depending on the distance between the points, bus operating speed, bus efficiency, route conditions and so forth.

Conceivably, a set of probability distribution models likely to possess some or all the desired characteristics would include the Gamma distribution, the Beta distribution, and the Log-Normal distribution. However, these distributions could be either used directly or shifted to account for minimum travel time.

While wishing to have a distribution with the greatest chance of reproducing observed distributions of travel times, it is clearly of little or no use at all to build models based on precise mathematical theories and techniques when the resulting models are too complex and time consuming to be economically worth using.

The next chapters describe the derivation of such models, data collection, and calibration of models.

2.2 PREVIOUS ATTEMPT TO MODEL STOP DWELL TIMES

Several previous studies have investigated the dwell times of buses at stops as passengers board and alight from buses.

Jordan and Turnquist (1979) calibrated a regression model of stop dwell times for pure boarding process.

The model was of the form:

$$S_b = \beta_o + \beta_1 n_b \qquad (2.6)$$

where S_b is dwell time for pure boarding process, β_o and β_1 are regression constants and n_b is the number of boarders.

Boardman and Kraft (1970) similarly calibrated stop dwell times for pure boarding and alighting processes and they are also of the form:

$$S_b = b_o + b_1 n_b \qquad (2.7)$$

$$S_a = a_o + a_1 n_a \qquad (2.8)$$

where S_b and S_a are dwell times for pure boarding and alighting respectively, b_o, b_1, a_o, and a_1 are regression constants, n_b, and n_a are the number of boarding and alighting passengers respectively.

Cundill and Watts (1973) have calibrated models of the form shown in (2.7) and (2.8) based on British experience.

These models imply that bus dead time is a constant and that stop dwell time increases linearly with the number of boarding and alighting passengers. These models are based on Two-Man-Operated (TMO), two-door buses.

However, where bus dead times seem to be passengers/drivers dependent, it is very likely that dead times

would be a random variable rather than a constant. In relation to observations in Kaduna, it is felt that a reasonable model of bus dwell times would be obtained by assessing the distributions of dwell time components (dead times and boarding/alighting times) separately.

CHAPTER 3

CANDIDATE MODELS OF BUS TRAVEL TIMES ON LINKS AND GOODNESS-OF-FIT TESTS.

This chapter describes and introduces the development of the properties of some continuous, univariate probability distributions suitable for modelling bus travel times on a link. A complete description of a random variable would be accomplished by specifying its probability distribution and the associated parameters. Empirical and inferential methods for determining probability distributions are described. Naturally, questions pertaining to determination of the parameters, such as the mean value μ and variance σ^2 of a Normal model, and the choice of specific distributions are of interest. Based on sampled observational data, information about the probability distribution may be inferred, and its parameters estimated statistically. The validity of an assumed probability distribution, may be verified or disproved statistically, based on finite samples of a population, by GOODNESS-OF-FIT tests, including specifically the CHI-SQUARE (χ^2) and the KOLMOGOROV-SMIRNOV (K-S) test.

In addition, the regression model used in analysing

the parameters of the distribution as well as the boarding and alighting times of passengers is presented. Statistics for checking the reasonableness of regression parameters and the overall significance of a regression model, based on computer subprograms, are presented and discussed.

3.1 PROBABILITY DISTRIBUTIONS

3.1.1 THE NORMAL(GAUSSIAN) DISTRIBUTION

This is perhaps the best-known and most widely used probability distribution. The Normal or Gaussian distribution is continuous and unlimited in range. Thus, a random variable that has the Normal distribution can assume positive and negative values. The distribution is symmetrical about its mean, and its mode, mean, and median are essentially equal. The equality of mode, mean, and median is a characteristic feature of a probability density function that is symmetric and unimodal.

The Normal distribution has often been found to be a most appropriate model for processes resulting from ADDITIVE effects. However, because random variables

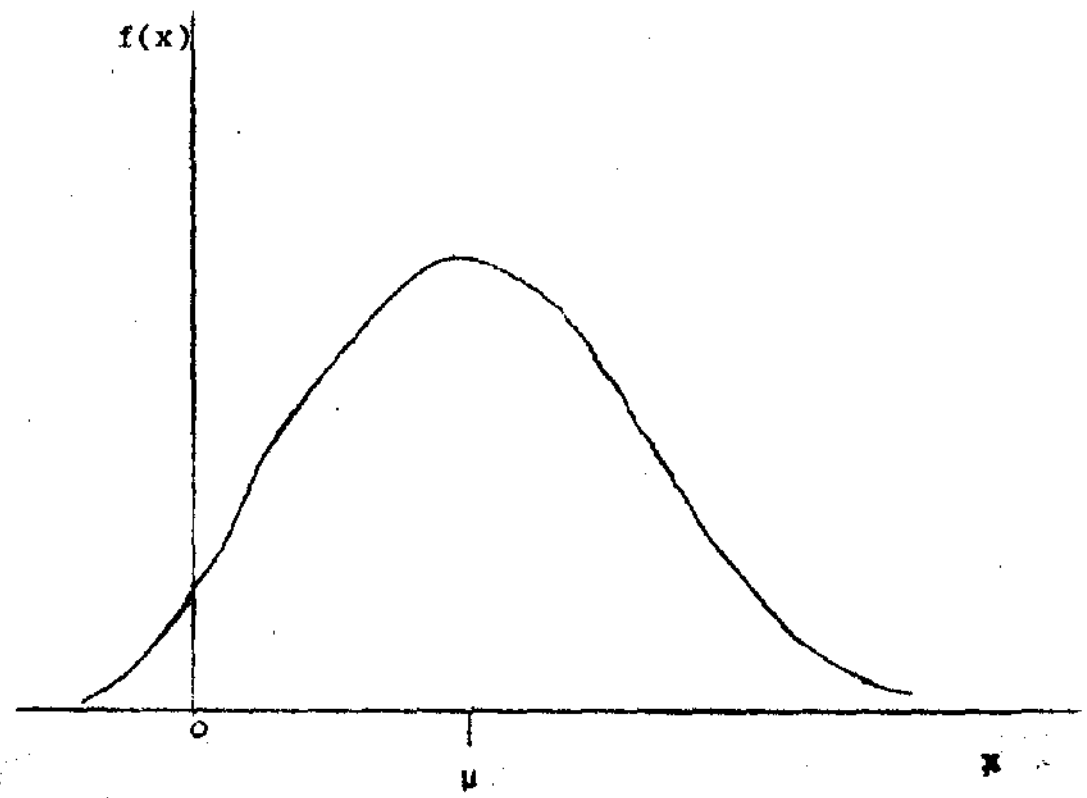


Figure 3.1 Normal Probability Density Function

in engineering work are typically positive and their frequency distributions are, in most instances, skewed to the right, the Normal model should be applied with prudence.

The Normal family of densities is defined as:

$$f(x) = \begin{cases} \frac{1}{\sigma\sqrt{2\Pi}} \exp[-\frac{1}{2}(\frac{x-\mu}{\sigma})^2] & -\infty < x < \infty \\ 0 & \text{otherwise} \end{cases} \qquad (3.1)$$

$\sigma > 0, \quad \mu < \infty$

μ and σ are LOCATION and SCALE parameters respectively.

The Normal cumulative Distribution Function (CDF) is:

$$F(X) = \frac{1}{\sqrt{2\Pi}} \int_{-\infty}^{X} \frac{1}{\sigma} \exp[-\tfrac{1}{2}(\frac{x-\mu}{\sigma})^2] \, dx \qquad (3.2)$$

The expected value (mean) and variance of a Normally distributed random variable X are:

$$E(X) = \mu \qquad (3.3)$$

$$Var(X) = \sigma^2 \qquad (3.4)$$

Thus, μ and σ which are the parameters of a Normal distribution are the mean and standard deviation respectively of the variate.

The coefficient of variation (CV), is thus:

$$CV = \frac{\sigma}{\mu} \qquad (3.5)$$

As alluded to earlier, the Normal model is unlimited in range. Since travel time values are essentially positive and their frequency distributions are almost always positively skewed, the Normal model should be used cautiously. Nevertheless, Welding (1957) in a study of the effects of close interval service observed that bus travel time distribution on a given link was close to a Normal distribution with the curve truncated at + 1.5 standard deviations to the low side and + 2.5 standard

deviations to the high side. Also dePirey (1971) created a simulation model of a bus route and assumed bus journey times to be drawn from a Normal population which was truncated at + 1.0 standard deviation on the low side and + 2.0 standard deviations on the high side to avoid extremely short or long travel times.

3.1.2 THE LOGARITHMIC NORMAL (LOG-NORMAL) DISTRIBUTION

If a random variable X is Log-Normally distributed, its natural Logarithm lnX, is Normally distributed. In effect, the parameters μ and σ refer to the expected value and standard deviation, respectively, of the related Normal distribution. Basically, there are two types of Log-Normal model: the two-parameter Log-Normal model and the three-parameter Log-Normal model.

(1) The Two-Parameter Log-Normal Distribution

The probability density function of a two-parameter Log-Normally distributed random variable X is expressed as:

$$f(x) = \begin{cases} \dfrac{1}{\sigma x\sqrt{2\Pi}} \exp\left[-\tfrac{1}{2}\left(\dfrac{\ln x - \mu}{\sigma}\right)^2\right] & 0 < x < \infty \\ 0 & \text{otherwise} \end{cases} \qquad (3.6)$$

$$-\infty < \mu < \infty, \quad \sigma > 0$$

where μ and σ are the parameters. σ is a SHAPE parameter.

Its cumulative distribution function (CDF) is:

$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_0^x \frac{1}{x} \exp[-\tfrac{1}{2}(\frac{\ln x - \mu}{\sigma})^2]dx \qquad (3.7)$$

It has been shown that the following relationship holds (Bury, 1975):

$$F_{LN}(x) = F_N(Z = \frac{\ln x - \mu}{\sigma}) \qquad (3.8)$$

where $F_{LN}(x)$ = CDF of a Log-Normal random variable with parameters μ and σ evaluated at x,

$F_N(.)$ = CDF of the standard Normal random variable [i.e.,N(O,1)].

The density values of the Log-Normal and Normal models are related by:

$$f_{LN}(x) = \frac{1}{\sigma x} f_N(\frac{\ln x - \mu}{\sigma}) = \frac{1}{x} f_N(\ln x) \qquad (3.9)$$

So that Log-Normal density ordinates can also be evaluated by using standard Normal tables.

Since lnX, is Normally distributed with mean μ and variance σ^2, that is, $\ln X \sim N(\mu\ \sigma^2)$ then:

$$\mu = E(\ln X) \qquad (3.10)$$

$$\sigma^2 = Var(\ln X) \qquad (3.11)$$

The mean and variance of a Log-Normally distributed

random variable with parameters μ and σ are:

$$E(X) = \exp(\mu + \tfrac{1}{2}\sigma^2) \qquad (3.12)$$

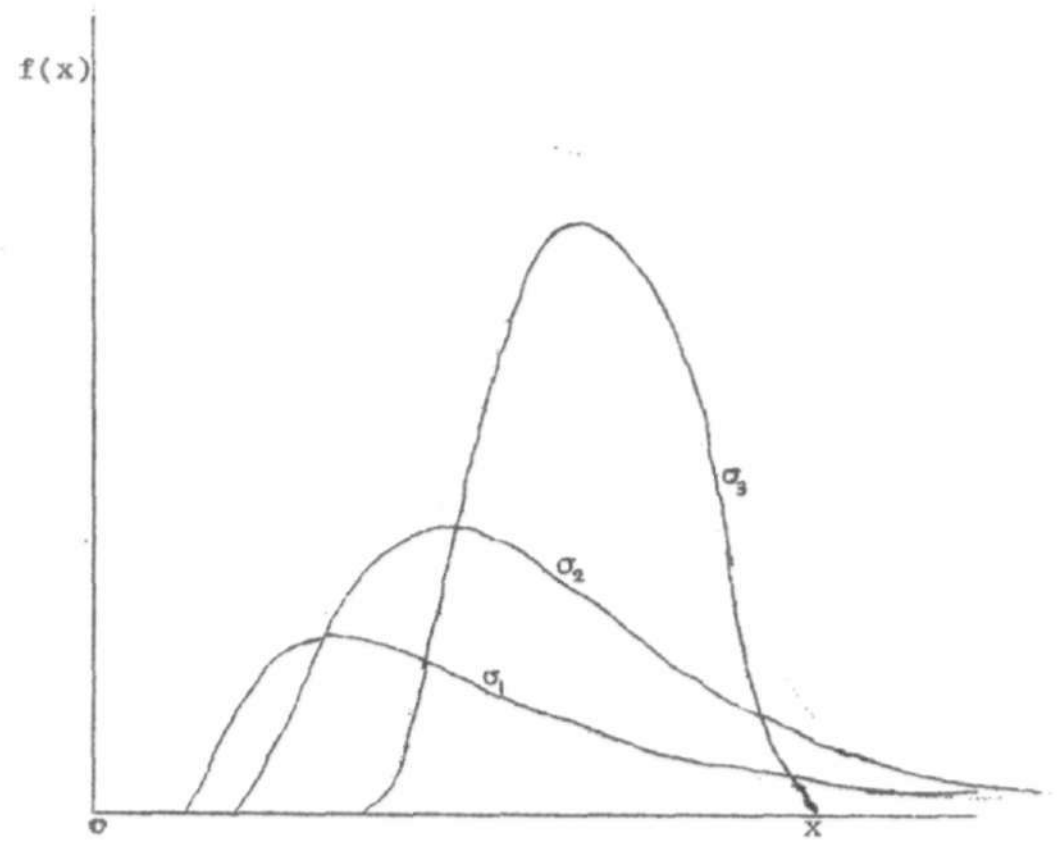


Figure 3.2 Log-Normal Density Functions

$$Var(X) = [E(X)]^2 [\exp(\sigma^2) - 1] \qquad (3.13)$$

The mode and median are expressed as:

$$\overset{o}{x} = \exp(\mu-\sigma^2) \qquad (3.14)$$

$$x_m = \exp(\mu) \qquad (3.15)$$

The coefficient of variation (CV) is thus:

$$CV = \sqrt{Var(X)}/E(X) = [\exp(\sigma^2)-1]^{\frac{1}{2}} \qquad (3.16)$$

(2) The Three-Parameter Log-Normal(Shifted Log-Normal) Distribution

The Log-Normal distribution shifted by an amount a is:

$$f(x) = \begin{cases} \dfrac{1}{(x-a)\sigma\sqrt{2\pi}} \exp\left[-\tfrac{1}{2}\left(\dfrac{\ln(x-a)-\mu}{\sigma}\right)^2\right] & -\infty<a<x \\ 0 & \text{otherwise} \end{cases} \qquad (3.17)$$

$$\sigma > 0,\ \mu<\infty\ ,\ x > a$$

where a, μ, and σ are the parameters. The LOCATION parameter a is often termed THRESHOLD value of the model.

The mean and variance of the shifted Log-Normal model are:

$$E(X) = a + \exp(\mu + \sigma^2/2) \qquad (3.18)$$

$$Var(x) = [E(X) - a]^2.[\exp(\sigma^2) -1]$$

$$= \exp[\mu+\sigma^2/2]^2.[\exp(\sigma^2)-1] \qquad (3.19)$$

The mode is given by:

$$\overset{o}{x} = a + \exp(\mu-\sigma^2) \qquad (3.20)$$

and median is:

$$x_m = a + \exp(\mu) \qquad (3.21)$$

The coefficient of variation can be expressed as:

$$CV = \frac{\exp[\mu+\sigma^2/2][\exp(\sigma^2)-1]^{\frac{1}{2}}}{a + \exp(\mu+\sigma^2/2)} \qquad (3.22)$$

The Log-Normal distribution is continuous, postively skewed, and limited to positive values of a random variable. It is computationally tractable and related to Normal distribution via logarithmic transformation. Its SHAPE parameter σ makes the model highly flexible. The degree of skewness increases with increasing value of σ . It is usually more peaked than the Normal distribution. The Log-Normal distribution can be derived as the model for a process whose value results from the MULTIPLICATIVE effects. It is also an appropriate model when the value of an observed variable is a random proportion of previously observed value.

The Log-Normal distribution is suitable for modelling a random variable, such as travel time, whose frequency distributions are skewed to the right.

3.1.3 THE GAMMA DISTRIBUTION

The Gamma distribution is continuous, unimodal, and takes on positive values only. The Gamma model can be thought of as the sum of K independent identically distributed Exponential random variables each with same parameter λ - mean rate. However, as K increases the Gamma distribution approximates to Normal distribution (by Central Limit Theorem). There are two types of Gamma distribution: the two-parameter Gamma distribution and the three-parameter Gamma distribution.

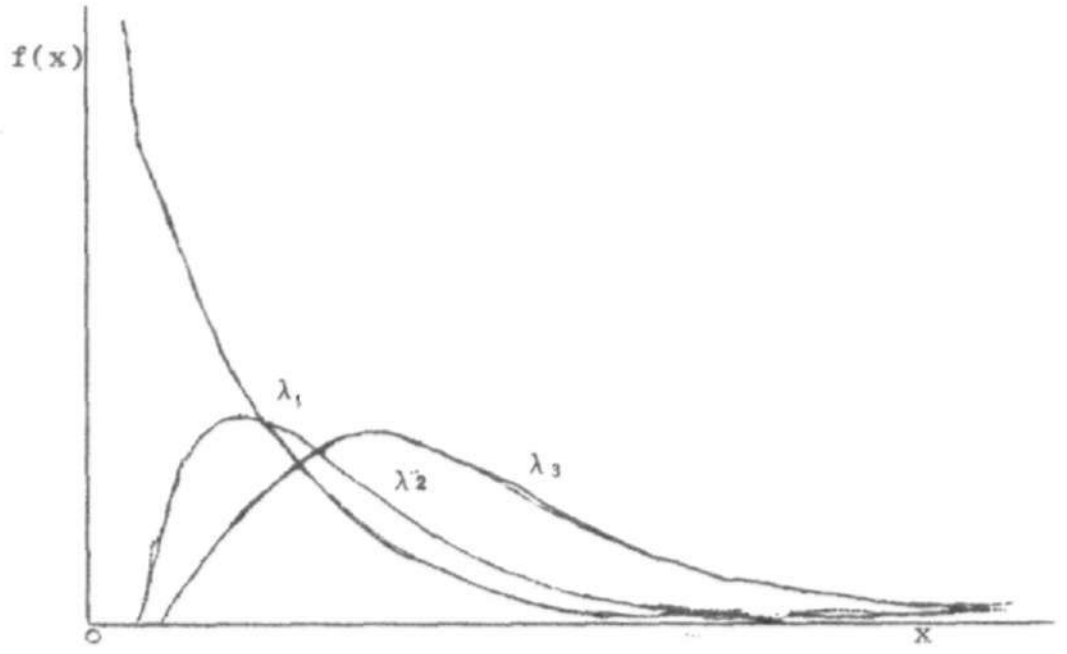


Figure 3.3 Gamma Probability Density Functions

(1) The Two-Parameter Gamma Distribution

The probability density function of a random variable X having a Gamma distribution is of the form:

$$f(x) = \begin{cases} \frac{\lambda^K}{\Gamma(K)} x^{K-1} e^{-\lambda x} & x > c \\ 0 & \text{otherwise} \end{cases} \qquad (3.23)$$

$$\lambda > c, \quad K > c$$

with SCALE parameter λ and SHAPE parameter K.

The Gamma function

$\Gamma_x(K) = \int_0^\infty x^{K-1} e^{-x} dx$ is equal to the factorial function $(K-1)!$ when K is a positive integer.

The Gamma cumulative distribution function (CDF) is:

$$F(x) = \frac{1}{\Gamma(K)} \int_0^x \lambda(\lambda x)^{K-1} e^{-\lambda x} dx$$

However, the Incomplete Gamma function ratio which is the Gamma CDF is:

$$F(x) = \frac{\Gamma_{\lambda x}(K)}{\Gamma(K)}$$

Hence the value of Gamma CDF can be obtained directly from tables of Incomplete Gamma function ratio.

The expected value and variance are given by:

$$E(X) = K/\lambda \qquad (3.25)$$

$$Var(X) = K/\lambda^2 \qquad (3.26)$$

The mode is:

$$\overset{o}{x} = (K-1)/\lambda \qquad (3.27)$$

Its coefficient of variation CV is:

$$CV = 1/\sqrt{K} \qquad (3.28)$$

(2) The Three-Parameter Gamma (Shifted Gamma) Distribution

The shifted Gamma distribution has a density function of the form:

$$f(x) = \begin{cases} \frac{\lambda^K}{\Gamma(K)}(x-a)^{K-1}e^{-\lambda(x-a)} & x>a \\ 0 & \text{otherwise} \end{cases} \qquad (3.29)$$

$\lambda>0$, $K>0$

a, K, and λ are the parameters. It can be seen that a is a LOCATION parameter.

The expected value and variance of a shifted Gamma distribution are given by:

$$E(X) = a + K/\lambda \qquad (3.30)$$

$$Var(X) = K/\lambda^2 \qquad (3.31)$$

From equation (3.31) it can be seen that the variance of a two-parameter Gamma model is equal to that of a three-parameter model.

The mode is:

$$\overset{o}{x} = a + (K-1)/\lambda \qquad (3.32)$$

and the coefficient of variation is:

$$CV = (\sqrt{K}/\lambda)/a+K/\lambda \qquad (3.33)$$

The Gamma model permits representation of a wide diversity of distributional shapes over the interval from zero or some arbitrary origin to infinity. The Gamma model is skewed to the right, hence it is suitable for characterizing a random variable whose values are essentially positive and frequency distributions positively skewed.

Jenkins (1976), in a particular simulation study, discussed the use of Gamma distribution. Travel times on a particular link were modelled by a shifted Gamma distribution. Turnquist (1979) proposed and calibrated a shifted Gamma model of travel times. The shift was computed as the link length divided by the speed limit.

3.1.4 THE BETA DISTRIBUTION

The Beta distribution is continuous. It can describe a wide variety of empirical distributions through variations in its parameters q and r. The density function

of the Beta distribution, depending on the parameters q and r, will have different shapes. It can be symmetrical about its mean value (when q=r), or it can be skewed to the right (when $q < r$), or to the left (when $q > r$). High flexibility of shape recommends the Beta model whenever a random variable X is bounded, say between finite limits a and b.

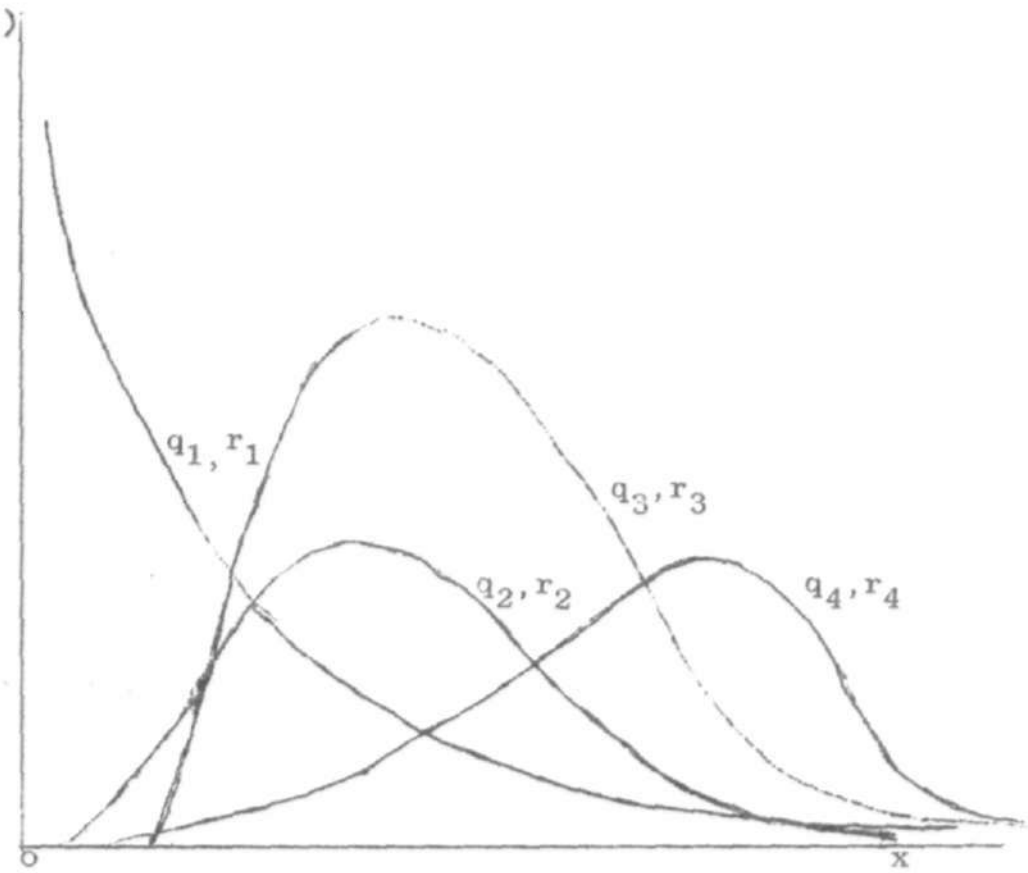


Figure 3.4 Beta Distributions

(1) The Two-parameter Beta distribution

The density function of a random variable that takes values in the unit interval (0,1) is

$$f(x) = \begin{cases} \frac{1}{B(q,r)} x^{q-1}(1-x)^{r-1} & 0<x<1 \\ 0 & \text{Otherwise} \end{cases} \qquad (3.34)$$

$q>0, r>0$

where q and r are SHAPE parameters, yielding the symmetry relation:

$$f(x;q,r) = f(1-x;r,q) \qquad (3.35)$$

and B(q,r) is the BETA FUNCTION, where:

$$B(q,r) = \int_0^1 x^{q-1}(1-x)^{r-1}\, dx$$

which is related to the Gamma function as follows:

$$B(q,r) = \Gamma(q)\Gamma(r)/\Gamma(q+r) \qquad (3.36)$$

and $\Gamma(.)$ is a Gamma function.

The Beta cumulative distribution function (CDF) is:

$$F(x) = \frac{1}{B(q,r)} \int_0^x x^{q-1}(1-x)^{r-1}\, dx \qquad (3.37)$$

However, the Beta CDF equation (3.37) can be expressed as incomplete Beta function ratio:

$$F(x) = \frac{B_x(q,r)}{B(q,r)} \qquad (3.38)$$

Hence, the Beta CDF can be obtained from tables of Incomplete Beta function ratio.

It can also be shown that:

$$F(x;q,r) = 1-F(1-x;\ r,q) \qquad (3.39)$$

The mean and variance of a Beta distribution are;

$$E(X) = q/(q + r) \qquad (3.40)$$

$$Var(X) = qr/[(q+r)^2(q+r+1)] \qquad (3.41)$$

The mode is:

$$\overset{o}{x} = (q-1)/(q+r-2) \qquad (3.42)$$

The median is obtained from:

$$\frac{B_{x_{0,5}}(q,r)}{B(q,r)} = 0.5 \qquad (3.43)$$

(2) The Four-Parameter Beta distribution

The density function of a Beta distribution with limits a and b is:

$$f(x) = \begin{cases} \frac{1}{B(q,r)} \frac{(x-a)^{q-1}\ (b-x)^{r-1}}{(b-a)^{q\ r-1}} & a<x<b \\ 0 & \text{Otherwise} \end{cases} \qquad (3.44)$$

$0<a<b$, $q>0$, $r>0$

q,r, a, and b are parameters. a is in fact a LOCATION parameter

The mean and variance are expressed as

$$E(X) = a + \frac{q}{q+r}(b-a) \qquad (3.45)$$

$$Var(X) = \frac{qr}{(q+r)^2(q+r+1)}(b-a)^2 \qquad (3.46)$$

The mode is thus:

$$\overset{o}{x} = a + \frac{(q-1)}{q+r+2}(b-a)^2 \qquad (3.47)$$

The Incomplete Beta function ratio is usually given in tables for $q \geq r$. But for $q < r$ the ratio is:

$$\frac{B_u(q,r)}{B(q,r)} = 1 - \frac{B_{1-u}(r,q)}{B(r,q)} \qquad (3.48)$$

where:

$$u = \frac{x-a}{b-a}$$

The Beta distribution model is very flexible and can be applied to different skewness situations. However, the estimation of Beta model parameters is quite problematic, Nevertheless, Polus (1975) in a study of transportation system reliability calibrated a Beta distribution. The upper limit of the Beta distribution was obtained by multiplying the mean travel time by a factor 1.5 which serves to stretch the distribution over the appropriate time scale, since outside of the unit interval were less than 1 percent of

the observed travel times. The shape was determined by assuming a relationship between the two shape parameters T and R:

$$T = 1.5R$$

However, the first and second moments approximated satisfactorily the corresponding moments of the observed travel times, but the shape of the probability density function did not conform well to the shape of observed histograms.

Thus far, some special properties of some continuous univariate probability distribution functions which are suitable for modelling link travel times have been presented and described.

3.2 REGRESSION ANALYSIS

In regression analysis the relationship, represented by a mathematical model, between one specific variable, the DEPENDENT VARIABLE, and one or more other variables called the INDEPENDENT(EXPLANATORY) VARIABLES, together with a set of relevant assumptions is studied. The independent or explanatory variables may be random or deterministic. The REGRESSION FUNCTION involves a set of unknown parameters.

When the function is linear in the PARAMETERS, but not necessarily in the explanatory variables, then the regression model is said to be linear; otherwise, the model is classified as non-linear. In some cases, however, non-linear regression problems may be converted to linear ones by appropriate transformation of the original variables.

The reasons for regression analysis are twofold. The first is to obtain a DESCRIPTION of the relationship between the variables as an indicator of possible causality. The second reason is to PREDICT the value of the dependent variable from a set of values of independent variables. However, regression analysis has the disadvantage of being too restrictive.

3.2.1 SIMPLE LINEAR REGRESSION

The simplest form of regression model is the SIMPLE LINEAR REGRESSION which is of the form:

$$Y = K_o + K_1 X$$

where Y is the dependent variable, X is the independent variable, and K_o and K_1 are parameters (regression coefficients) whose values are to be estimated.

If the regression is required to pass through the origin (i.e., no constant term, $K_o=0$) the model becomes:

$$Y = K_1 X \qquad (3.50)$$

3.2.2 MULTIPLE LINEAR REGRESSION

In regression analysis, it is aasumed that the effects of independent variables are ADDITIVE. Multiple linear regression is obtained when two or more variables are operating jointly. It is of the form:

$$y = K_o + X_1 + K_2X_2 + K_3X_3 + \ldots + K_n X_n \qquad (3.51)$$

where Y is the dependent variable, $X_1, X_2, X_3, \ldots, X_n$ are the independent variables. K_o, K_1, K_2, K_3, ..., K_n are the unknown parameters. In effect, K_1, K_2, K_3, ..., K_n are called PARTIAL REGRESSION COEFFICIENTS, and K_o is the REGRESSION CONSTANT which represents that portion of the value of dependent variable not explained by the independent variables.

The magnitude of a partial regression coefficient may be given intuitive interpretation in the following way. The magnitude of a partial regression coefficient represents the expected change in dependent variable Y, with a change of one unit in independent variable X when the other independent variables are held constant.

For a regression through the origin, then:

$$Y = K_1X_1 + K_2X_2 + K_3X_3 + \ldots + K_nX_n \qquad (3.52)$$

with unknown parameters $K_1, K_2, K_3, \ldots, K_n$.

3.3 PARAMETER STRUCTURE

A complete description of a random variable would be accomplished by specifying its probability distribution as well as the associated values of its parameters. The univariate, continuous distributions addressed in the previous chapter feature basically three types of parameter: 1) LOCATION parameter (denoted by a, or μ in the case of Normal model); 2) SCALE parameter (denoted by λ); and 3) SHAPE parameter (denoted by K, or σ in the case of Log-Normal model). A LOCATION parameter a or μ , "locates" the probability density function on the measurement axis of x without altering the distribution appearance. In other words, the location parameter a or μ relates the density function to the origin of the measurement scale x with the result that a or μ and x are dimensionally the same. The relation between a or μ and x is of the form (x - a), or $(x - \mu)$; otherwise, a or μ is not a location parameter.

A SCALE parameter λ is a factor that "scales" the

measurement variable x. In a distribution model, a scale parameter is always associated with x in the form λx (or x/λ). In effect, a scale parameter relates to the units of measurement on x. A scale parameter always influences the appearance of a density function by affecting the dispersion of x. Except for symmetrical distributions, a scale parameter also affects the value of certain centrality measures of x such as the expected value.

A SHAPE parameter K, affects the basic "shape" of a probability density function. However, the specific effect of K on the shape of a density function depends on the density function in question. A shape parameter K is not associated with x in any standard form, and it neither relates to the origin nor to the scale of measurement on x. A shape parameter K, is an intrinsic distinguishing property of a density function; besides, it is usually the most difficult parameter to estimate and often the most important.

3.4 PARAMETER ESTIMATION TECHNIQUES

The techniques of deriving probabilistic information and of estimating parameter values from sampled

empirical data are embodied in the methods of STATISTICAL INFERENCE in which information obtained from sampled data is used to make generalization about the population from which the samples were drawn. A set of sample values on which basis parameters can be estimated is usually assumed to constitute a RANDOM SAMPLE, implying that successive sample values are independent and the underlying population remains the same from one sample value to another.

Classical methods of estimation of parameters are POINT and INTERVAL estimations. While point estimation is concerned with the computation of a single value, from an empirical data set, to represent the parameter of the underlying population, interval estimation goes further to establish a statement of confidence in the estimated quantity, resulting in the determination of an interval that contains the parameter value with a prescribed level of confidence. The common methods of point estimations are the METHOD OF MOMENTS and the METHOD OF MAXIMUM LIKELIHOOD. Another, but less commonly used method is the METHOD OF QUANTILES.

Generally, a known parameter value and an estimate thereof, based on observational data, will be differentiated by placing a circumflex (^) above the maximum likelihood

estiamte, or (^{v}) above the moment estimate, or ($^{\sim}$) above the quantile estimate.

3.4.1 THE METHOD OF MOMENTS

The method of moments evaluates a parameter via estimating the corresponding sample moments, usually the mean and variance, and higher moments if necessary, of the variate. Moment estimates are often simple to compute, however, these estimates are usually biased and relatively inefficient.

The sample mean and variance can be written as:

$$\bar{x} = M_1' = \frac{1}{n} \sum_{i=1}^{n} x_i \qquad (3.53)$$

$$S^2 = M_2 = \frac{1}{n} \sum_{i=1}^{n} (x_1 - \bar{x})^2 \qquad (3.54)$$

The third moment about the mean is thus:

$$M_3 = \frac{1}{n} \sum_{i=1}^{n} (x_1 - \bar{x})^3 \qquad (3.55)$$

In general the r^{th} moment about origin is:

$$M_r' = \frac{1}{n} \sum_{i=1}^{n} x_i^r \qquad (3.56)$$

and about mean $\bar{x}$ is

$$M_r = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^r \qquad (3.57)$$

3.4.2 THE METHOD OF MAXIMUM LIKELIHOOD

The method of maximum likelihood derives the point estimator of the parameter directly. The maximum likelihood estimates (MLE) of a parameter possess optimal variance properties. In particular, for large sample size, the maximum likelihood estimator is often considered the "best" estimate in that it has the minimum variance (asymtotically) (Hoel, 1962).

Consider a random variable X with density function $f(x;\theta)$ in which θ is the parameter. On the basis of the sample values $x_1,\ldots, x_n$, the likelihood of obtaining a particular sample value x_i can be assumed to be proportional to the value of the probability density function evaluated at x_i. Then, assuming random sampling, the LIKELIHOOD FUNCTION (LF) of observing the set $x_i, \ldots, x_n$ is:

$$L(x_1,\ldots,x_n;\theta) = f(x_i;\theta)f(x_2;\theta)\ldots f(x_n;\theta) \quad (3.58)$$

The MAXIMUM LIKELIHOOD ESTIMATOR $\hat{\theta}$ is then the value of θ that maximizes the likelihood function $L(x_i,\ldots, x_n;\theta)$. This estimator may be obtained by differentiating $L(x_i,\ldots, x_n;\theta)$ with respect to θ and setting the derivative equal to zero, giving usually an absolute maximum (Hoel, 1962); that is, $\hat{\theta}$ is obtained as a solution to the following equation:

$$\frac{\partial L(x_i, \ldots, x_n;\theta)}{\partial\theta} = 0 \quad (3.59)$$

It is frequently more convenient, because of the multiplicative nature of the likelihood function, to maximize the logarithm of the likelihood function instead; that is:

$$\frac{\partial \text{Log } L(x_1, \ldots, x_n; \theta)}{\partial \theta} = 0 \qquad (3.60)$$

The solution for $\hat{\theta}$ from each of the two equations above is the same.

For density functions with two or more parameters, the likelihood function becomes:

$$L(x_i, \ldots, x_n; \theta_1, \ldots, \theta_m) = \prod_{i=1}^{n} f(x_i; \theta_1, \ldots \theta_m) \qquad (3.61)$$

where $\theta_1, \ldots, \theta_m$ are the m parameters to be estimated. In this case, the maximum likelihood estimators would be obtained from the solution to the following set of simultaneous equations:

$$\frac{\partial \text{Log } L(x_1, \ldots, x_n; \theta_1, \ldots, \theta_m)}{\partial \theta_j} = 0; \qquad (3.62)$$

$$j=1, \ldots, m.$$

3.4.3 THE METHOD OF QUANTILES

Sometimes, the method of quantiles provides a simple and rapid method of estimation of a parameter. This method,

though convenient, is relatively inefficient. The method of quantiles is suitable when trading efficiency of estimation for computational convenience. The method of quantiles also applies directly to censored samples provided the chosen order statistic is available (Bury,1975).

Consider a distribution model having a single parameter θ. The expression:

$$F(x_q ; \theta) = q \qquad (3.63)$$

defines the quantile x_q, implying that 100q percent of all possible values of the random variable X lie below x_q. Suppose that the r^{th} observation from an ordered sample of size n is chosen so that its proportion r/n is equal to or just greater than q (i.e., $r/n \geq q$). With $x_{(r)}$ replacing x_q, the equation above may then serve as an estimating equation for the parameter θ, and the order q may be chosen to minimize the variance of the corresponding estimator $\tilde{\theta}$. That is:

$$F(x_{(r)}; \theta) = q \qquad (3.64)$$

where q remains to be chosen

For density functions with two or more parameters $(\theta_1, \ldots, \theta_m)$ then m values of q and the corresponding SAMPLE approximations of x_q give a set of m simultaneous equations of the form shown above (3.64) for determining the m estimators of the parameter components θ_i.

3.5 GOODNESS-OF-FIT TESTS

The validity of an assumed probability distribution may be verified or disproved statistically, in the light of available data, by GOODNESS-OF-FIT tests. Two such tests frequently used are the CHI-SQUARE (χ^2) and the KOLMOGOROV-SMIRNOV (K-S) tests. These tests, however, depend on a specified significance level α, the choice of which is arbitrary, and may not provide absolute information on the validity of a specific distribution. Conceivably, a distribution that is acceptable at one significance level may be unacceptable at another significance level.

Usually, the null hypothesis H_o, is that the available data and a specified theoretical distribution are not statistically different. The null hypothesis H_o, is NOT REJECTED if the probability of obtaining the observed event is greater than the chosen significance level α; otherwise H_o is REJECTED.

The goodness-of-fit tests sometimes called the SIGNIFICANCE TESTS are one-sided tests in that these tests check whether observational data disagree 'SIGNIFICANTLY' with the given hypothesis H_o.

Although the χ^2 and K-S tests are dependent on a specified significance level, the choice of which is largely

a subjective matter, these tests remain useful for delineating the relative degree of validity of the different distributions.

3.5.1 CHI-SQUARE (χ^2) TEST

The χ^2 goodness-of-fit test is applicable to composite hypothesis (i.e, when parameters need to be estimated from data) and to discrete distributions. The χ^2 test procedure is based on limiting process, hence it is required that the expected frequency and number of class intervals should not be too small (at least five). The χ^2 test is designed for nominal type data; it utilizes only the number of observations in intervals rather than the observations themselves. Thus, since the test requires grouping of data, sample information is lost.

3.5.2 KOLMOGOROV-SMIRNOV (K-S) TEST

The K-S goodness-of-fit test makes use of data in ordinal form. It applies only to continuous data as well as small samples. It is a non-parametric test. The K-S test is computationally simple and does not require grouping of data. To apply the K-S test, the empirical

cumulative distribution function (sample cumulative frequency distribution) as well as the hypothesized cumulative distribution must be evaluated for each distinct observation. The K-S statistic D_n measures the maximum absolute deviation of the empirical distribution function from the theoretical distribution function. If the observed statistic D_n is less than the tabulated critical value D_n^{α} (n = number of samples) the proposed distribution is acceptable at the prescribed significance level α; otherwise, the hypothesized distribution would be rejected.

The K-S test is preferred to the χ^2 test because the K-S test is exact even for small samples and it is generally considered to be more powerful than the χ^2 test (Conover, 1971).

3.6 PARAMETER ESTIMATION OF REGRESSION MODELS

The method used to determine values of the parameters of a regression model is called the METHOD OF LEAST SQUARES. The method, based on a given set of data, aims at minimizing the sums of squares of the differences between the observed values of the dependent variable and the values predicted by the regression equation for that set of data.

Consider the following random model

$$Y_i = K_o + K_1X_{1i} + K_2X_{2i} + \ldots + K_nX_{ni} + \epsilon_i \quad i=1,2,\ldots,n.$$

ϵ_i are assumed to be independent and Normally distributed random variable (error) with mean 0 and variance σ^2. That is, the classical least-square assumptions are:

$$E(\epsilon_i) = 0 \qquad 1 \leq i \leq n$$

$$Var(\epsilon_i) = \sigma^2 \qquad 1 \leq i \leq n$$

$$Cov(\epsilon_i, \epsilon_j) = 0 \qquad 1 \leq i \leq j \leq n$$

The method of least-squares calculates the unknown regression parameters $K_o, K_1, K_2, \ldots, K_n$ by minimizing

$$\sum_{i=1}^{n} \epsilon_i$$

The estimates $\hat{K}_o, \hat{K}_1, \hat{K}_2, \ldots, \hat{K}_n$ are in fact Best Linear Unbiased Estimates (BLUE). That is, among unbiased linear estimates, the least-square estimates have minimum variance (from Gauss-Markov Theorem). For a more complete discussion of the least-squares regression method and mathematical details, see Draper and Smith (1966).

A further technique often employed in multiple linear regression analysis is STEPWISE REGRESSION in which regression models are constructed recursively through successive addition of independent variable from a specified set of possible independent variables. Sometimes, in stepwise regression, inclusion of independent

variables is controlled by certain statistical criteria.

Statistical validity of regression models can be assessed by considering standard statistical tests. Standard computer packages, such as SPSS (Statistical Package for the Social Sciences) and NAG (Numerical Algorithm Group) routine are available for regression abalysis and their output includes statistics for such tests.

3.7 SPSS SUBPROGRAM REGRESSION

Output from SPSS subprogram REGRESSION includes the following: 1) regression coefficients and their standard errors. 2) t-values or F-values for testing whether individual coefficients are significantly different from zero. The t-value is the ratio of a regression coefficient to its standard error; t-values can be tested against the Student's t-distribution with degrees of freedom the same as for the residual sum of squares. The F-values for testing the significance of individual regression coefficients are equal to the squares of the t-values (i.e., $F = t^2$). The overall significance of the regression model can be determined from the F-value for the Analysis of Variance; the F-value can be tested against the F-distribution with degrees of freedom equal to those attributable to the regression and those of the residual

term. 3) Multiple correlation coefficient, R, measures the correlation between the dependent variable and the set of independent variables and takes a value between zero and unity. 4) Coefficient of determination, R^2, (square of multiple correlation coefficient) measures the proportion of variance in the dependent variable explained jointly by the independent variables included in the regression equation. It is also a measure of goodness-of-fit of the regression. It takes a value between zero and unity. Thus, a high value of R^2 indicates a good fit, and a low value a poor fit. Generally, the value of R^2 increases with increasing number of independent variables. 5) Adjusted coefficient of determination, $\bar{R}^2$, accounts for the number of coefficients estimated (independent variables and constant, if used) in relation to the number of observations. As the value of R^2 must increase with increase in the number of independent variables it becomes tempting to choose a regression overloaded with independent variables. Therefore, $\bar{R}^2$ serves to compromise a good fit and the number of independent variables, in which case further addition of variables to a regression causes $\bar{R}^2$ to decrease. The SPSS computes $\bar{R}^2$ from the relation

$$\bar{R}^2 = R^2 - \left(\frac{C - 1}{N - C}\right)(1 - R^2)$$

where C is the number of coefficients estimated (independent variables and constant, if used), N is the number of observations. The value of $\bar{R}^2$ need not necessarily be positive. 6) Standard Error of Estimate, SEE, measures the standard deviation of the residuals, that is, the standard deviation about the regression line established. 7) Coefficient of variability, CV, measures the standard deviation of Y given X expressed as a percentage of the mean of the dependent variable. In other words, CV is the ratio of SEE to the mean of the dependent variable and it is unitless. 8) ELASTICITY is the percentage change in Y divided by the percentage change in X_i, where the change is measured from the mean of each variable.

CHAPTER 4

FIELD OBSERVATIONS

This chapter focuses on data acquisition. A number of tasks were involved in the collection of data. Having defined the data required for this study, the collection of field data had to be done, by on-board survey, using a bus line run by Kaduna State Transport Authority (KSTA) in metropolitan Kaduna. A brief description of the study site is given.

4.1 STUDY SITE AND DATA SET

Metropolitan Kaduna is the capital of Kaduna State in Northern Nigeria, has a population of about 2 million and consists largely of high density residential areas. The KSTA is the only agency that provides scheduled, fixed-route intra-city bus service in the state.

The bus route (Route One) studied traverses a major traffic corridor connecting the Central Business District (CBD) of the city with Kawo and Kakuri surburbs. The round trip-length of the route (Kawo-Kakuri-Kawo) is approximately 30.9-km. The route is sufficiently long with a dual carriageway and contains a number of roundabouts and minor intersections, and no traffic lights.

Route One is worked by Two-Man-Operated (TMO), two-door, 52-seater Steyr buses. The two doors are provided with a transmission interlock system which prevents the doors from being opened while the bus is in motion. Passengers board through the rear door and exit through the front door. The fare structure is graduated and the conductor collects fares and issues ticket manually while the bus is moving.

Data were collected by observers riding the buses, and equipped with stop-watches and electronic digital watches. The data included the following: bus dwell times at each bus stop (excluding stop events at termini), boarding/alighting times and the number of boarding/alighting passengers, dead times between the instant of bus stopping and the first passenger boarding or alighting (i.e., pre-boarding/alighting dead times), dead times between the instant of last passenger boarding/alighting and the instant of bus departure (i.e. post-boarding/alighting dead times). Also recorded were link travel times, that is, travel times between two bus stops.

The KSTA furnished information on the spaces between some bus stops. The number of roundabouts and number of intersections were observed directly.

After a preliminary survey, the observations lasted for five consecutive weekdays and mostly during the morning peak period, in June 1981. It is felt that survey from Monday through Friday could reflect daily trends and also reduce the possibility of atypical conditions. Altogether, 50 observations of travel times on each of the 34 links were made. A larger data base would have been desired but resource limitation was a major constraint.

Having obtained the field data, the travel times were checked for any statistical differences in the means of travel times among the different weekdays using analysis of variance (ANOVA). The analysis was deemed necessary as data had been collected during different weekdays within the morning peak periods. Also, the analysis could form a basis for aggregation of data if the differences in means of travel times among the different weekdays were statistically insignificant or decomposition of data if the means were significantly different. Incidentally, the results of the computer program utilized, SPSS subprogram ANOVA, showed that the means of travel times among the different weekdays were insignificantly different statistically for all the 34 links studied. On this basis, the data for each link were aggregated resulting in a larger data base for subsequent analyses.

CHAPTER 5

CALIBRATION OF THE MODEL: BUS TRAVEL TIMES ON LINKS

This chapter dwells on the calibration of mathematical models for bus travel times on links. Two strategies for modelling of travel times were evaluated, namely: 1) distribution modelling technique; and 2) stepwise regression technique. In the latter, models were constructed recursively through successive addition of explanatory variables that meet certain statistical criteria. Likely impacts of basic variables, namely, link length, number of roundabouts, and number of intersections, on travel times were examined and evaluated. However, before constructing the models, the field data were checked against statistical differences in means of travel times among the different weekdays using Analysis of Variance (ANOVA). The means for different weekdays were found not to be significantly different statistically.

5.1 REGRESSION MODELLING APPROACH

The basic hypothesis underlying this approach is that travel time can be explained by independent variables related to system characteristics. The major

advantages of regression models are that they are usually very simple to develop and use, and reflect actual conditions. However, they have the disadvantage of being too restrictive and because they are empirical in nature, they fail to establish causal relationship between the dependent and independent variables.

The following explanatory variables were examined:

LENGTH (L) It is considered that over relatively long links, buses may be exposed to more external factors which affect their travel times. Besides, under favourable conditions, one would expect that bus travel times on a link will increase with the length of the link.

NUMBER OF ROUNDABOUTS (N_{rd}) Roundabouts, besides easing flow of traffic are also major sources of traffic delays and other incidents. Vehicles on approaching a roundabout may slow down or stop and travel time may increase. It is felt that this variable would influence bus travel time.

NUMBER OF INTERSECTIONS (N_{it}) Intersections existing along a link could be sources of bottleneck, interfering cross-traffic, and other disruptions. It is believed that this variable would exert some influence on bus travel time.

An additional variable such as traffic volumes should have been considered. However, its inclusion was not feasible for reasons such as resources required and difficulties in acquiring the data, and the deceptive nature of this performance measure. When congestion is high, for example, very low volumes of traffic may be observed. Also, very low volumes may ensue during adverse weather conditions when vehicles are compelled to slow down. Similarly, low traffic volumes will be observed under very favourable traffic conditions.

Other variables which could affect bus travel times are weather conditions, driving conditions, drivers' behaviour, road surface, vehicle efficiency, and so forth. However, these variables cannot easily be quantified. In any case, it is unreasonable to have a regression equation stuffed with too many independent variables. The objective is to obtain the simplest possible equation that adequately describes a relationship. Hence, if it is possible to obtain a reasonably good fit from the listed predictor variables, the model can be considered adequate.

The available predictor variables were sifted using stepwise regression analysis to seek out those which have strong statistical association with the

dependent variable, and non-collinearity with other explanatory variables. Two centrality measures of the dependent variables: the mean (T_{me}) and the median (T_{md}) were used. The latter was considered because for skewed distributions, median is normally considered as an appropriate statistic for central tendency. Besides, the median is not sensitive to a small number of extreme observations as is the mean. Though for transit planning purposes, the median may not be a very useful parameter, it still enables a transit operator to gain insight into the amount of time at which vehicles travel as faster as they travel slower.

The results of the analysis using SPSS Subprogram REGRESSION are presented below.

5.1.1 RESULTS: REGRESSION TECHNIQUE

The stepwise regression method based on the criteria that variables with a minimum F-value of 4.0 and Tolerance of 0.05 are eligible for inclusion at any step was used. The results obtained are as follows:

Model (5.1)

$$T_{me} = \underset{(5.0698)}{22.3411} + \underset{(9.0344)}{63.4407}\, L \qquad (5.1a)$$

F-values = 19.419, 49.310

$R = 0.8039$, $R^2 = 0.6462$, $\bar{R}^2 = 0.6331$

Overall Regression F-value = 49.310

SEE = 10.7866sec.

SEE = Standard Error of Estimate or Standard deviation of residuals.

$\bar{R}^2$ = the corrected coefficient of determination which accounts for the number of coefficients estimated (independent variables and constant if used) in relation to the number of observations.

NB: The value in parentheses under each estimated coefficient is the standard error of that estimate.

$$T_{me} = \underset{(4.5966)}{21.6023} + \underset{(4.1397)}{10.9271}N_{rd} + \underset{(8.3452)}{59.0265}\,L \qquad (5.1b)$$

F-values = 22.087, 6.967, 50.029

$R = 0.8491$, $R^2 = 0.7210$, $\bar{R}^2 = 0.6995$

Overall Regression F-value = 33.588

SEE = 9.7617sec.

Model (5.2)

$$T_{md} = \underset{(4.7015)}{21.4900} + \underset{(8.3779)}{61.8368}\,L \qquad (5.2a)$$

F-values = 20.893, 54.478

$R = 0.8177$, $R^2 = 0.6686$, $\bar{R}^2 = 0.6564$

Overall Regression F-value = 54.478

SEE = 10.0028sec.

$$T_{md} = \underset{(4.2083)}{20.7733} + \underset{(3.7900)}{10.6011 N_{rd}} + \underset{(7.6402)}{57.5543\ L} \qquad (5.2b)$$

F-values = 24.366, 7.824, 56.748

$R = 0.8633$, $R^2 = 0.7453$, $\bar{R}^2 = 0.7257$

Overall Regression F-value = 38.035

SEE = 8.9370sec.

The results show that impacts of link length and number of roundabouts are statistically significant. Incidentally, the number of intersections is found to be statistically insignificant. Furthermore, examination of correlation matrix indicates that the explanatory variables are not correlated, implying that models derived from them do not have biased coefficients. The regression constants and partial correlation coefficients have the correct sign and their magnitudes are reasonable. In addition, the regression constants are statistically significant. Thus, the significance of the regression constants may reflect driving conditions, interactions with the general traffic stream, or interference delays and other such incidents. This is not surprising in the

case of urban bus transit operating on non-priority routes, or in mixed traffic as buses share rights-of-way and interact with other vehicles. However, the statistical insignificance of the number of intersections may be due to a true lack of significance, or it may reflect limitations in the data set.

Model (5.2) which represents the median of link travel time is a slightly better equation statistically than model (5.1). The standard errors of estimate (SEE) of the models are smaller than the corresponding mean link travel time models', implying that predictions based on the median models are slightly more reliable and fraught with less errors. This finding is consistent with the concept and exposition that for an asymmetric distribution, the median is less sensitive to a small number of extreme observations than the mean. However, for transit planning purposes, the mean is a more useful statistic, in which case model (5.1) is more useful.

Attempts to model the variance of link travel time resulted in low values of R^2 (coefficient of determination), implying that the independent variables did not have much explanatory power. Therefore, the average value of variance is selected as the best

predictor and it is computed to be 141.623sec.

At this point, it may serve to caution that though the dependent variables are highly correlated with existing explanatory variables, tne models will still yield inconsistent predictions where policy changes or circumstances alter existing cørrelations between the dependent variables and the explanatory variables in existing models,

5.2 DISTRIBUTION MODELLING APPROACH

Day-to-day stochastic distributions of bus travel time over links were evaluated using a Log-Normal distribution model, Individual measures of travel time were assumed to be a random sample from a positive random variable. It is felt that the use of a probability distribution model to capture a large spatial and temporal variability in travel times on each link could permit a more thorough evaluation of information for accurate analysis and predictions. Furthermore, the approach gives a better and more comprehensive picture of the distribution of observed phenomena (see Figure 5.1). Log-Normal model was considered in this investigation for reasons alluded to earlier. In addition, it is felt that the relationship of Log-Normal model with Normal model could permit predictions of its parameters (which are Normally distributed) as a function of system characteristics without

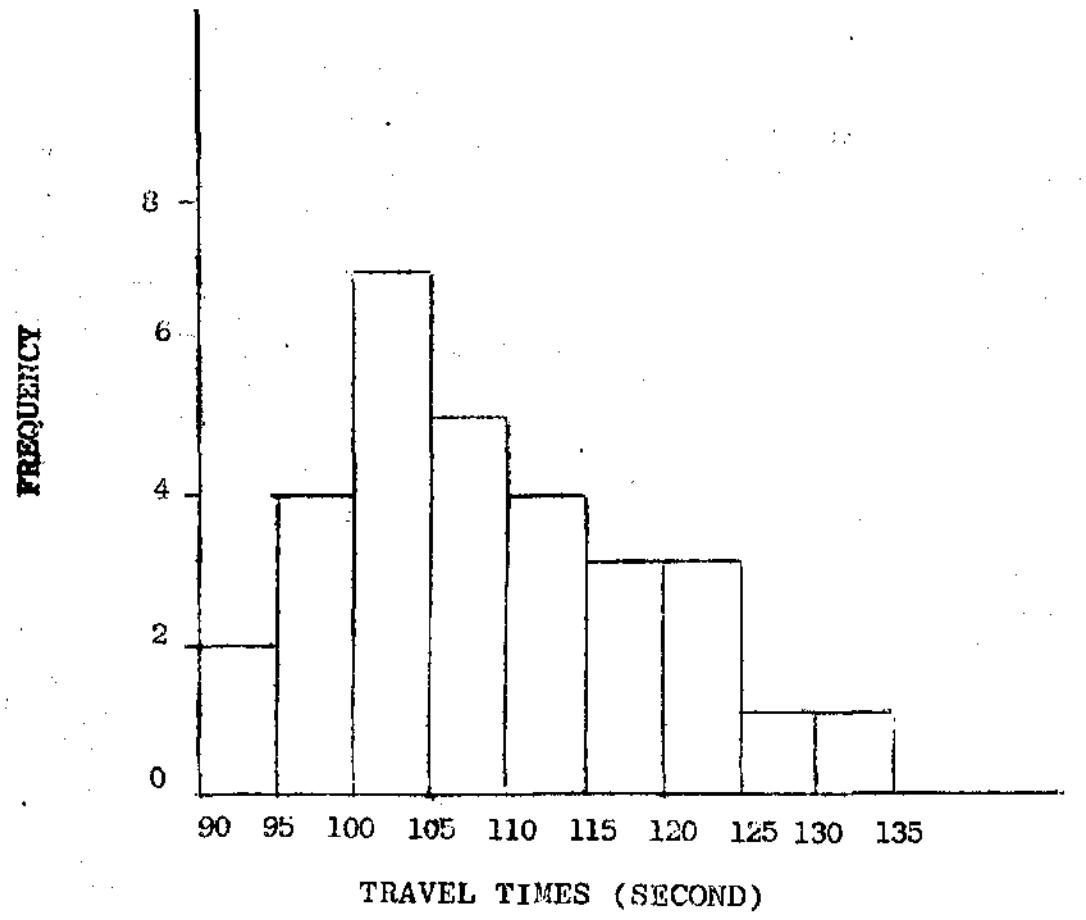


Figure 5.1 Observed Histogram of Bus Travel Time between Stadium and Station 1.40-Km.

gross violation of some underlying assumptions of regression analysis.

The parameters of the Log-Normal distribution were estimated for each of the 34 links by the method of maximum likelihood. In each, the K-S test was performed to evaluate the appropriateness of the candidate distribution model, relative to observed data, at the 5% significance level. (A FORTRAN IV computer program written for this analysis and the output can be found in Appendix B. The NAG routine was used to compute the Normal cumulative distribution function CDF).

Having obtained the maximum likelihood estimators of the Log-Normal distribution, an attempt was made to predict the parameters in terms of explanatory variables related to system characteristics. However, the last five links had to be discarded because of lack of knowledge of the link attributes particularly link length, leaving a total of 29 links for parameter predictions based on regression analysis.

The results of the effort are presented below.

5.2.1 RESULTS: DISTRIBUTION FITTING TECHNIQUE

The results of the analysis show that the null hypothesis that the empirical data and Log-Normal distribution model are not statistically different could not be rejected at 5% level of significance using Kolmogorov-Smirnov test. Therefore, the Log-Normal model, relative to data from Kaduna, is an underlying probability distribution model for link travel times.

Efforts to predict the parameters of the Log-Normal distribution as a function of system characteristics resulted in the following regression models.

Model 5.3

$$\mu = 3.3781 + 1.0883\ L \qquad (5.3a)$$
$$(0.1042) \quad (0.1856)$$

F-values = 1051.511, 34.371

$R = 0.7484$, $R^2 = 0.5601$, $\bar{R}^2 = 0.5438$

Overall Regression F-value = 34.371

SEE = 0.2216

NB: The figure in parentheses under each estimated coefficient represents the standard error of that estimate. $\bar{R}^2$ is the corrected coefficient of determination which accounts for the number of coefficients estimated (independent variables and constant if used) in relation to the number of observations.

$$\mu = \underset{(0.9850)}{3.3656} + \underset{(0.8871)}{0.1842N_{rd}} + \underset{(0.1788)}{1.0139}\ L \qquad (5.3b)$$

F-values = 1167.483, 4.313, 32.146

R = 0.7891, R^2=0.6266, $\bar{R}^2$ = 0.5936

Overall Regression F-value = 21.450

SEE = 0.2092

Model 5.4

$$\mu = \underset{(0.8028)}{4.4457} + \underset{(0.9679)}{0.6865}\ \ln(L) \qquad (5.4a)$$

F-values = 3066.901, 50.317

R = 0.8067, R^2 = 0.6508, $\bar{R}^2$ = 0.6379

Overall Regression F-value = 50.317

SEE = 0.1975

$$\mu = \underset{(0.7895)}{4.3638} + \underset{(0.7515)}{0.1987N_{rd}} + \underset{(0.8865)}{0.6498}\ \ln(L) \qquad (5.4b)$$

F-values = 3054.775, 6.992, 53.728

R = 0.8514, R^2 = 0.7248, $\bar{R}^2$ = 0.7036

Overall Regression F-value = 34.237

SEE = 0.1786

Model (5.5)

$$\exp(\mu) = \underset{(4.8242)}{21.2421} + \underset{(8.6041)}{63.6168}\ L \qquad (5.5a)$$

F-values = 19.355, 54.668

$R = 0.8182$, $R^2 = 0.6694$, $\bar{R}^2 = 0.6572$

Overall Regression F-value = 54.668

SEE = 10.2729sec.

$$\text{Exp}(\mu) = \underset{(4.3934)}{20.5480} + \underset{(3.9567)}{10.2664}\,\bar{x}_{rd} + \underset{(7.9762)}{59.4695}\,L \qquad (5.5b)$$

F-values = 21.874, 6.733, 55.589

$R = 0.8587$, $R^2 = 0.7374$, $\bar{R}^2 = 0.7172$

Overall Regression F-value = 36.504

SEE = 9.3301 sec.

Model (5.6)

$$1/\sigma^2 = \underset{(5.1222)}{71.0469}\,L \qquad (5.6)$$

F-value = 192.391

$R = 0.9343$, $R^2 = 0.8730$, $\bar{R}^2 = 0.8730$

Overall Regression F-value = 192.391

SEE = 15.4792

The results, thus, demonstrate that the parameters of the Log-Normal distribution can be predicted based on stepwise regression. On the basis that only variables with a minimum F-value of 4.0 and Tolerance of 0.05 are

eligible for inclusion at any step, link length and number of roundaboutswere found to have a statistically significant effect. Surprisingly, the number of intersections was found to be statistically insignificant as a factor. Inspection of regression models for μ[models (5.3 to 5.5)] reveals that the regression constant is highly significant. This may be attributable to the effects of untested factors such as driving conditions, interactions with the general traffic stream, or interference delays and other such incidents. Also, this is not surprising in the case of bus transit operating on non-priority routes or mixed traffic as buses share rights-of-way and interact with other vehicles. However, the statistical insignificance of the number of intersections may be due to a true lack of significance, or it may reflect limitations in the data set. Consideration of correlation matrix indicates that the independent variables are not collinear, implying that the regression coefficients and their sign are not irrational.

It should be noted that model (5.5) was derived based on an equivalent expression for median of Log-Normal distribution, see equation (3.15). Additionally, model (5.6) demonstrates that the parameter σ^2 of Log-Normal distribution is a function of link length.

However, the SEE of model (5,6) is fairly high indicating that predictions based on the model are less accurate and fraught with errors. Thus, the major weakness of the model lies in the prediction of σ. Perhaps these models for parameter estimation of link travel time distributions would have benefited from better and more accurate evaluation if the effects of additional variables such as traffic volumes, bus interactions with vehicle stream, and so forth had been extensively evaluated or explicitely accounted for.

However, since the parameters μ and σ of the Log-Normal distribution are directly related in the form (and reproduced here for convenience),

$$\sigma^2 = \sum (\ln x_i - \mu)^2 \qquad i = 1, \ldots, n \qquad (A.2.2b)$$

where x_i = individual measures of travel time; the parameter μ can be predicted from the models above and σ derived from the expression above. Though this approach may not be helpful at planning stage as x_i must be obtained from trial runs. By selecting models (5.4b) and (5.5b on statistical grounds, as the best pair for predicting μ, and using model (5,6) to predict σ. or adopting the expression (A,2,2b) to derive σ (see Tables B,3.1 to B,3.3), the mean and variance of link travel times were

computed from the equations (see Appendix B, Tables B.4.1 to B.4.4 for the computer program and the output) reproduced here for convenience:

$$E(X) = \exp(\mu + \tfrac{1}{2}\sigma^2) \qquad (3.12)$$

$$Var(X) = [E(X)]^2 [\exp(\sigma^2) - 1] \qquad (3.13)$$

5.3 SCATTERGRAM: MODELS PREDICTIONS VERSUS OBSERVED MEAN AND VARIANCE OF LINK TRAVEL TIMES

For the purpose of comparing model predictions with observed values of mean and variance of the distribution of link travel times, scatterplot diagrams are illustrated using SPSS subprogram SCATTERGRAM and REGRESSION.

Figures 5.2 and 5.3 show plots of estimated versus observed mean and variance of link travel times respectively using the maximum likelihood estimators. A line at 45° indicating "estimated =observed" is shown for comparison. The output of SPSS subprogram SCATTERGRAM and REGRESSION shows that the correlation coefficient, R, between the estimated mean and the observed mean of link travel times is about 1(R=0.9999). This indicates that the mean of link travel times is reproduced very accurately by the Log-Normal distribution model. Also, the correlation coefficient between estimated and observed

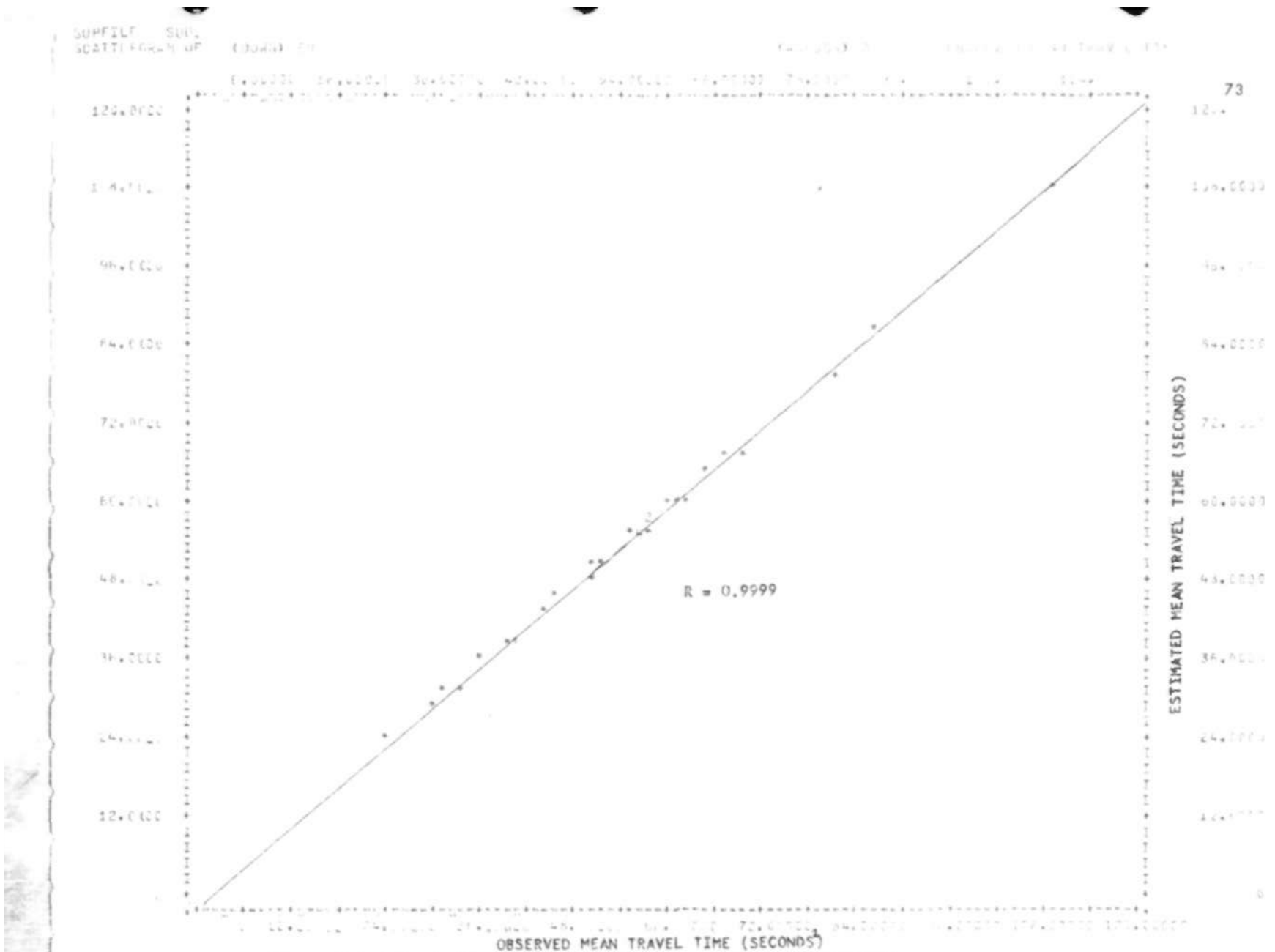


Figure 5.2 Estimated Versus Observed Mean Travel Time on Links Using Maximum Likelihood Estimators of Log-Normal Model

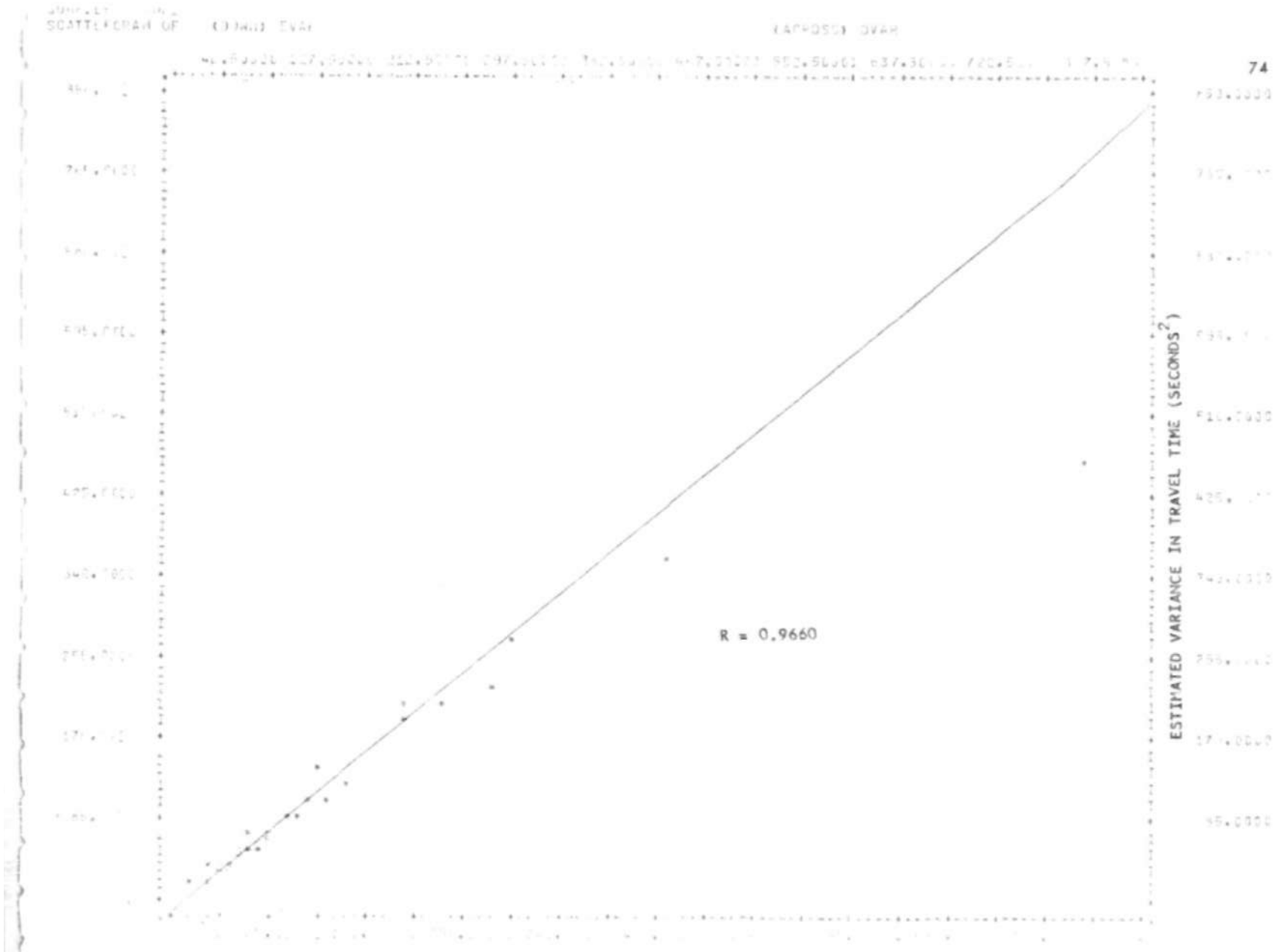


Figure 5.3 Estimated Versus Observed Variance in Travel Time on Links Using Maximum Likelihood Estimators of Log-Normal Model

variance of link travel times is 0.96602. This analysis, therefore, serves to add further credibility to Log-Normal model as the underlying model of link travel times.

Figures 5.4 and 5.5 illustrate the predicted versus the observed mean and variance of link travel times respectively when model (5.4b) is used to compute μ and σ derived from the equation (A.2.2b). The correlation coefficient between the model predictions and observed mean is 0.8679. The predicted and observed variance of link travel times have a correlation coefficient of 0.6740.

Figure 5.6 shows the relationship between predicted and observed values of the mean of the distribution of link travel times using model (5.5b) and expression (A.2.2b) to predict μ and derive σ respectively. The correlation between the model values and the observed values is 0.8605. The corresponding results on the variance of the distribution of link travel times are illustrated in Figure 5.7. The results are good with a correlation coefficient of 0.6994.

Figure 5.8 illustrates the comparison for mean of link travel time when models (5.4b) and (5.6) are used. The correlation coefficient between model predictions and observed mean is 0.8542.

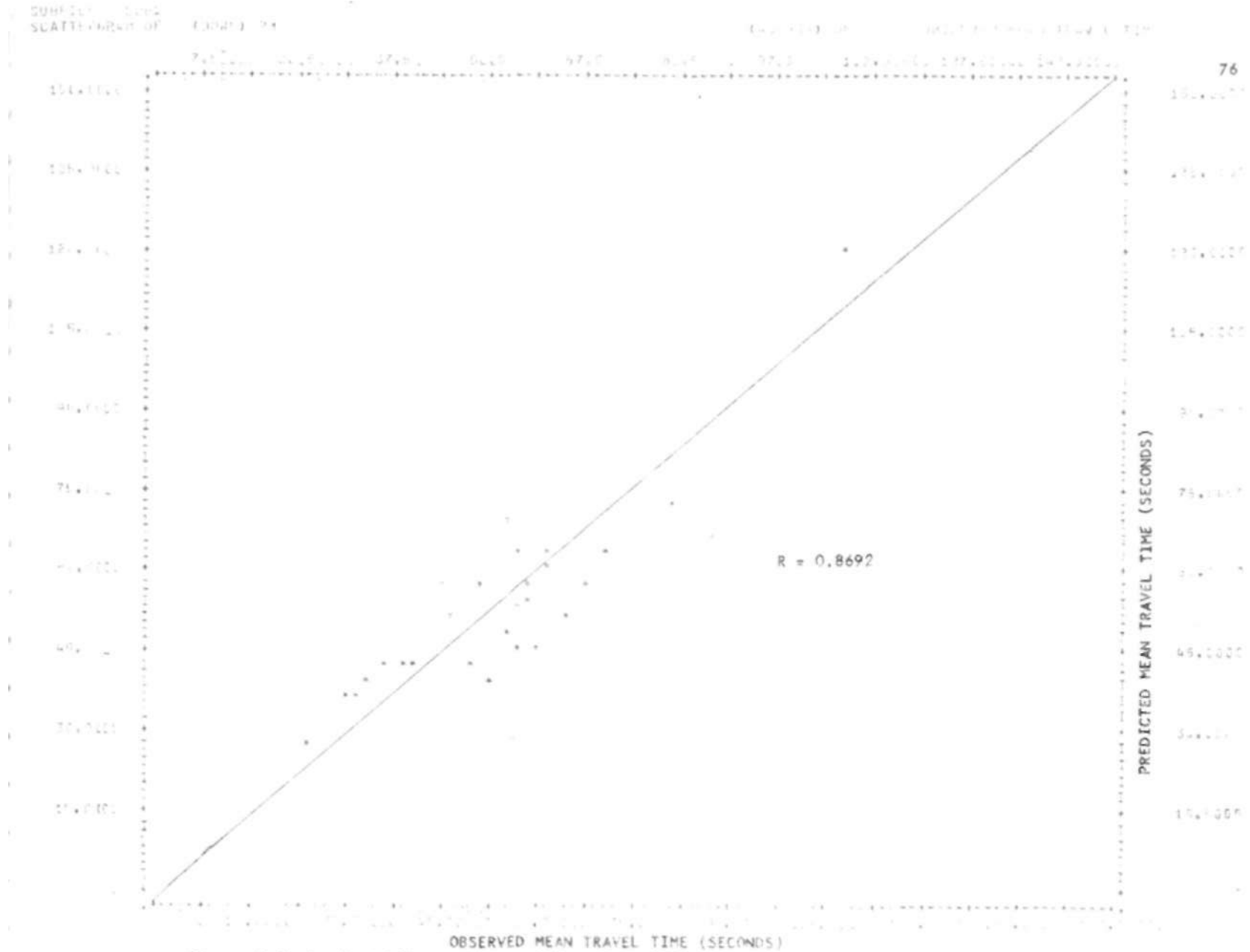


Figure 5.4 Predicted Versus Observed Mean Travel Time on Links Using Model (5.4b) and Equation (A.7.2b)

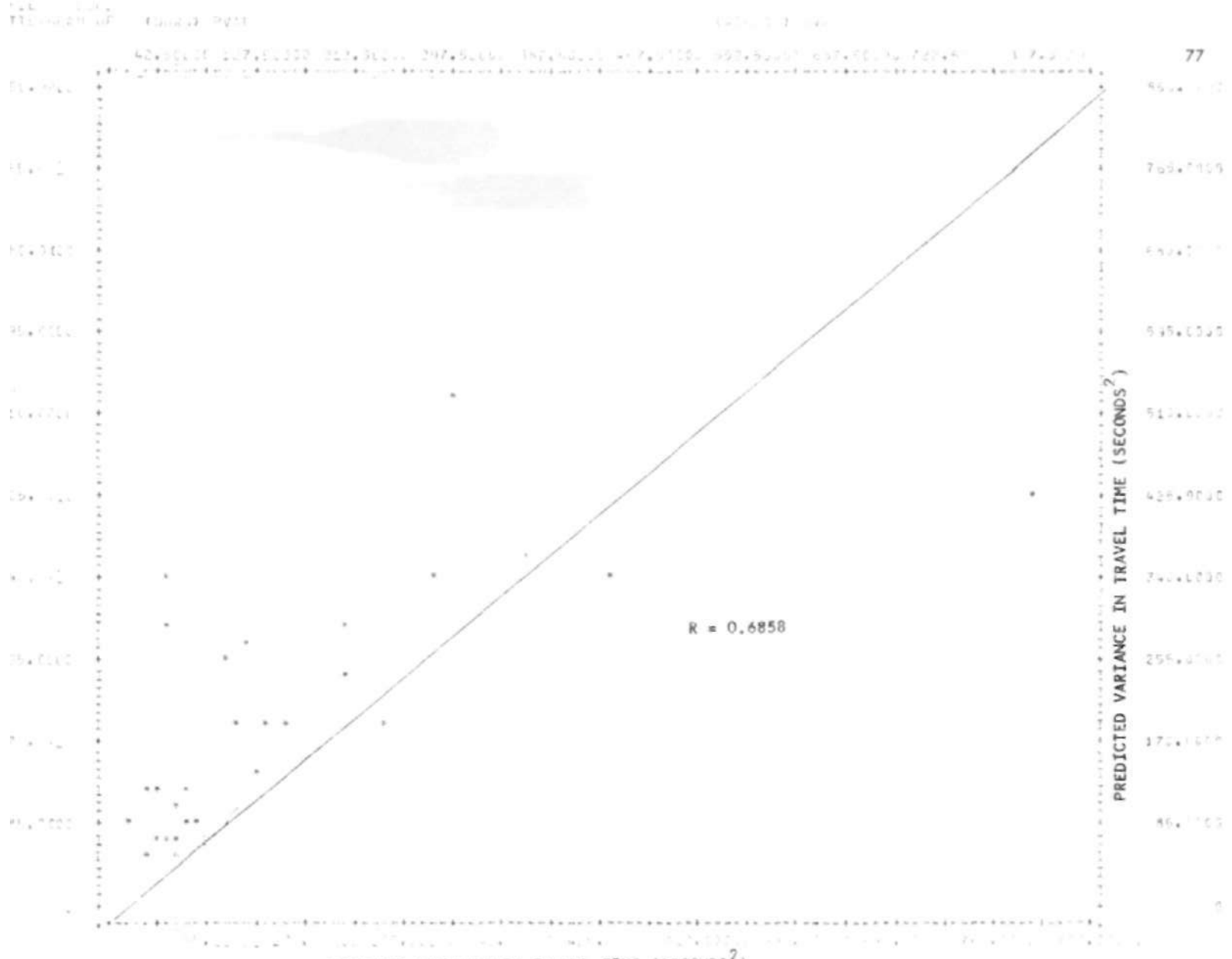


Figure 5.5 Predicted Versus Observed Variance in Travel Time on Links Using Model (5.4b) and Equation (A.2.7b)

The comparison of the variance of link travel time is shown in Figure 5.9. The correlation coefficient between predicted values and observed values of variance is 0.3969.

The results of mean of link travel time are shown in Figure 5.10 when model (5.5b) for μ and model (5.6) for σ^2 are used. The correlation coefficient between the mean value from the models and observed values is 0.8483. Figure 5.11 shows the comparison of variance of link travel time. The correlation coefficient between model values and observed values of variance of link travel time is 0.3275.

The above analyses provide an overall indication of the goodness-of-fit for the entire procedure. In general, while the predictions of the mean of link travel times are quite satisfactory, the predicted values of variance of link travel time are unsatisfactory. This is the result of poor fit of the parameter σ. It is precisely in this area that further research is welcomed. Perhaps, these models for parameter estimation of link travel time distributions would have benefited from better and more accurate evaluation if the effects of additional variables such as traffic valumes, bus

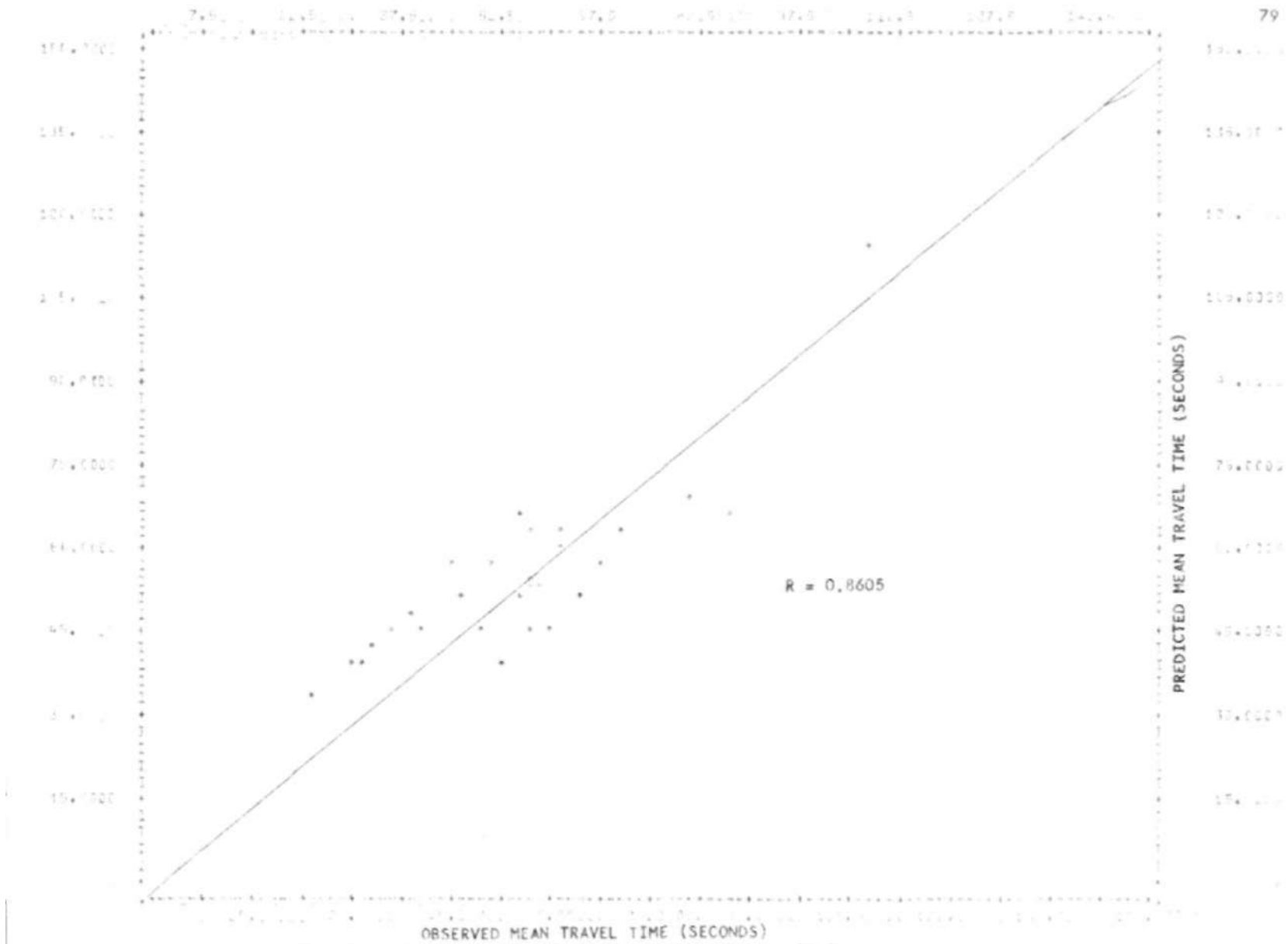


Figure 5.6 Predicted Versus Observed Mean Travel time on Links Using Model (5.5b) and Equation (A.7.2b)

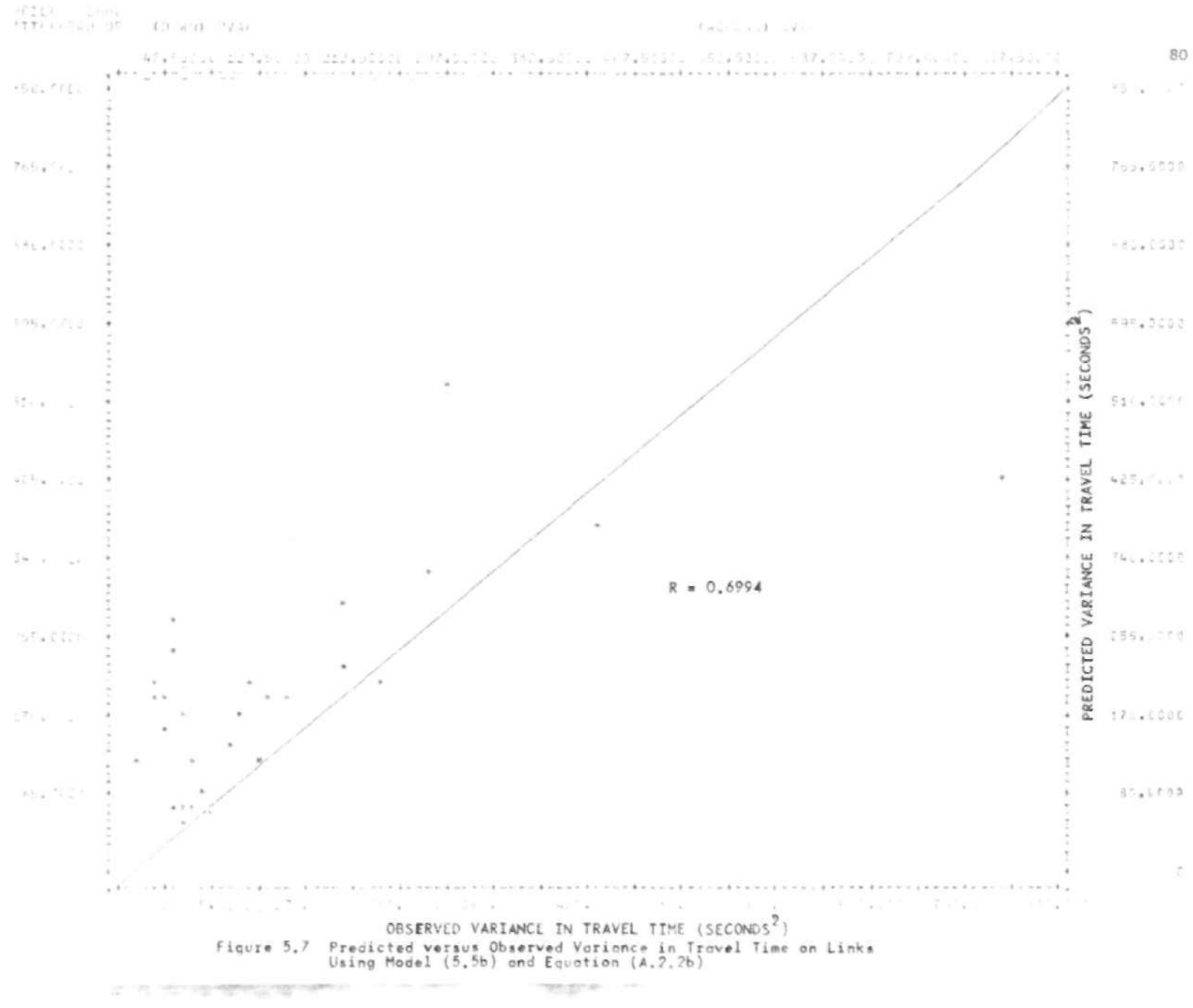


Figure 5.7 Predicted versus Observed Variance in Travel Time on Links Using Model (5.5b) and Equation (A.2.2b)

interactions with the general vehicle stream, driving conditions, and so forth had been extensively evaluated, or explicitely accounted for.

Although, the models, like other models, have some shortcomings or imperfections, predictions thereof are sufficiently accurate. Nevertheless, the Log-Normal model, relative to observed links in Kaduna metropolis, is deemed apposite. In addition, the procedure and the resulting models are efficient, simple, and could provide intra-urban bus transit operators or planners with an analytical tool to address the subject of link travel times on urban routes.

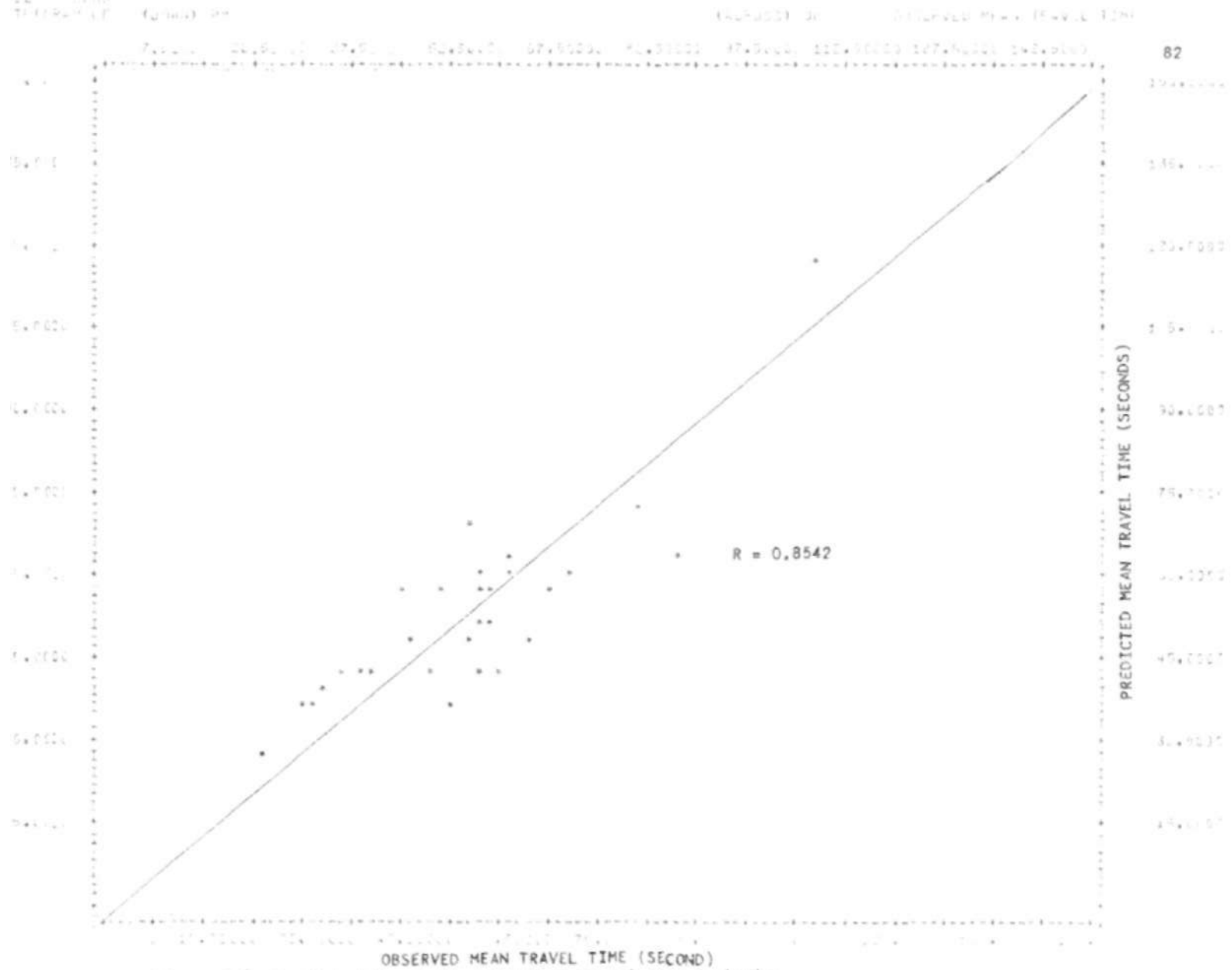


Figure 5.8 Predicted Versus Observed Mean Travel Time on Links Using Models (5.4b) and (5.6)

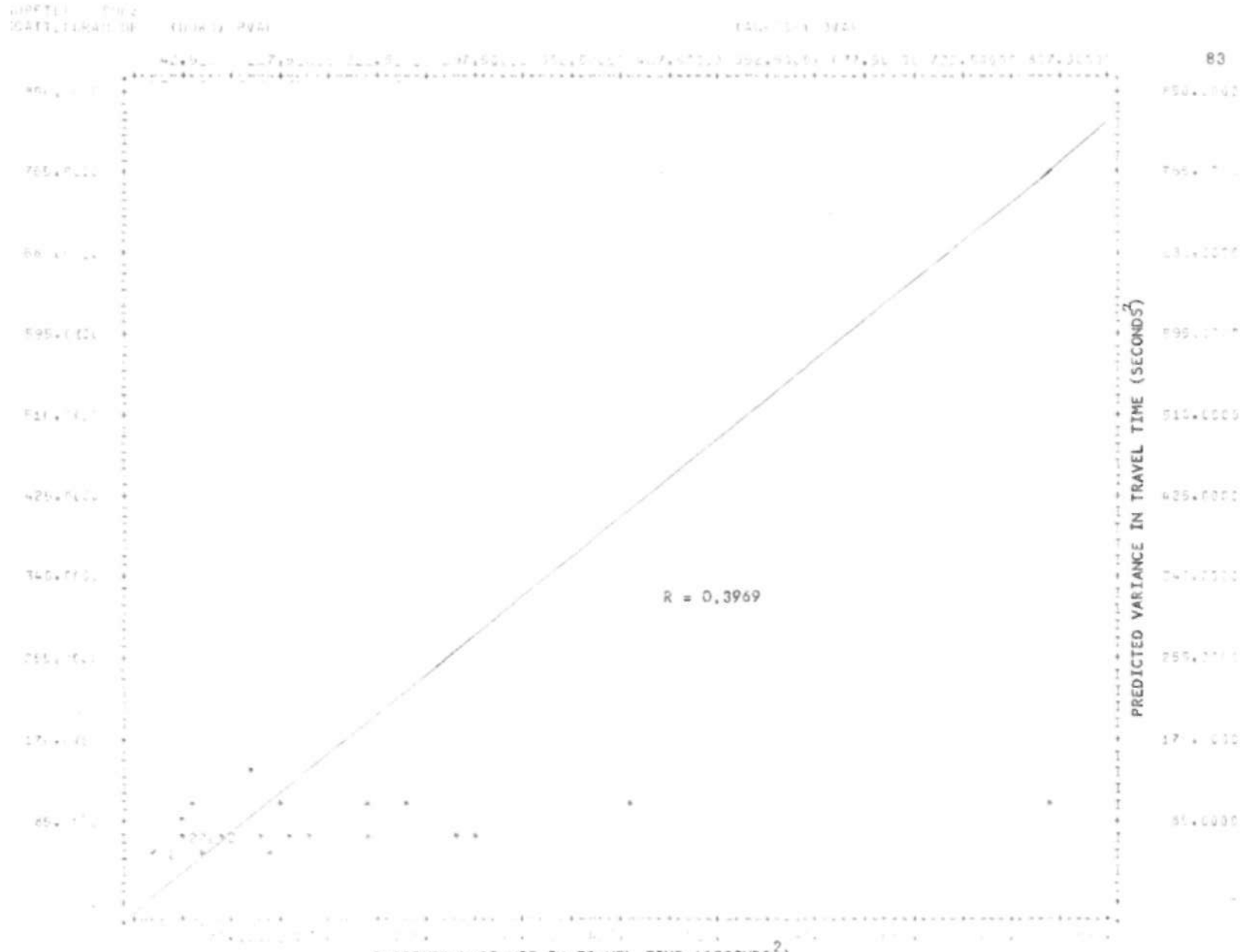


Figure 5.9 Predicted Versus Observed Variance in Travel Time on Links Using Models (5.4b) and (5.6)

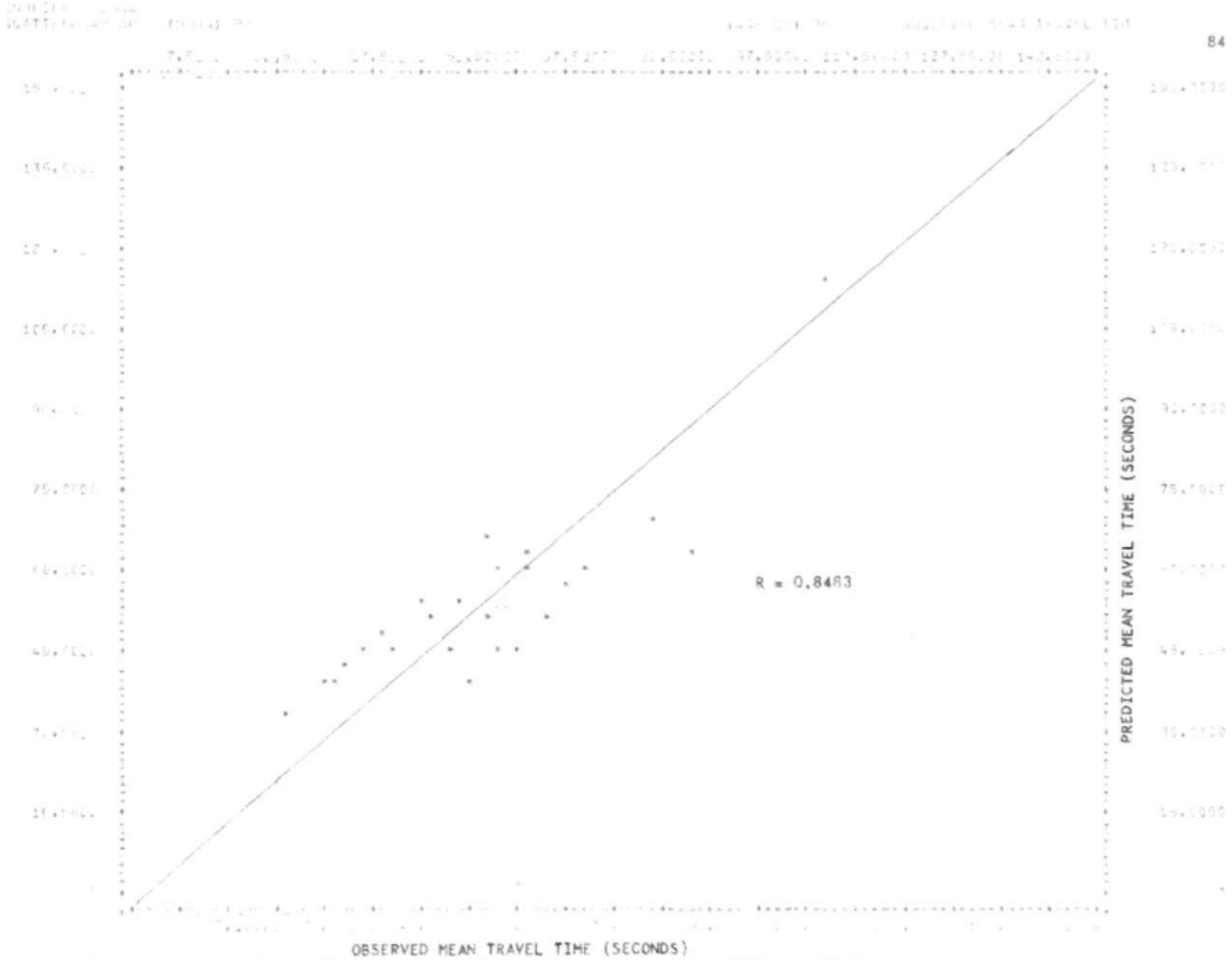


Figure 5.10 Predicted Versus Observed Mean Travel Time on Links Using Models (5.5b) and (5.6)

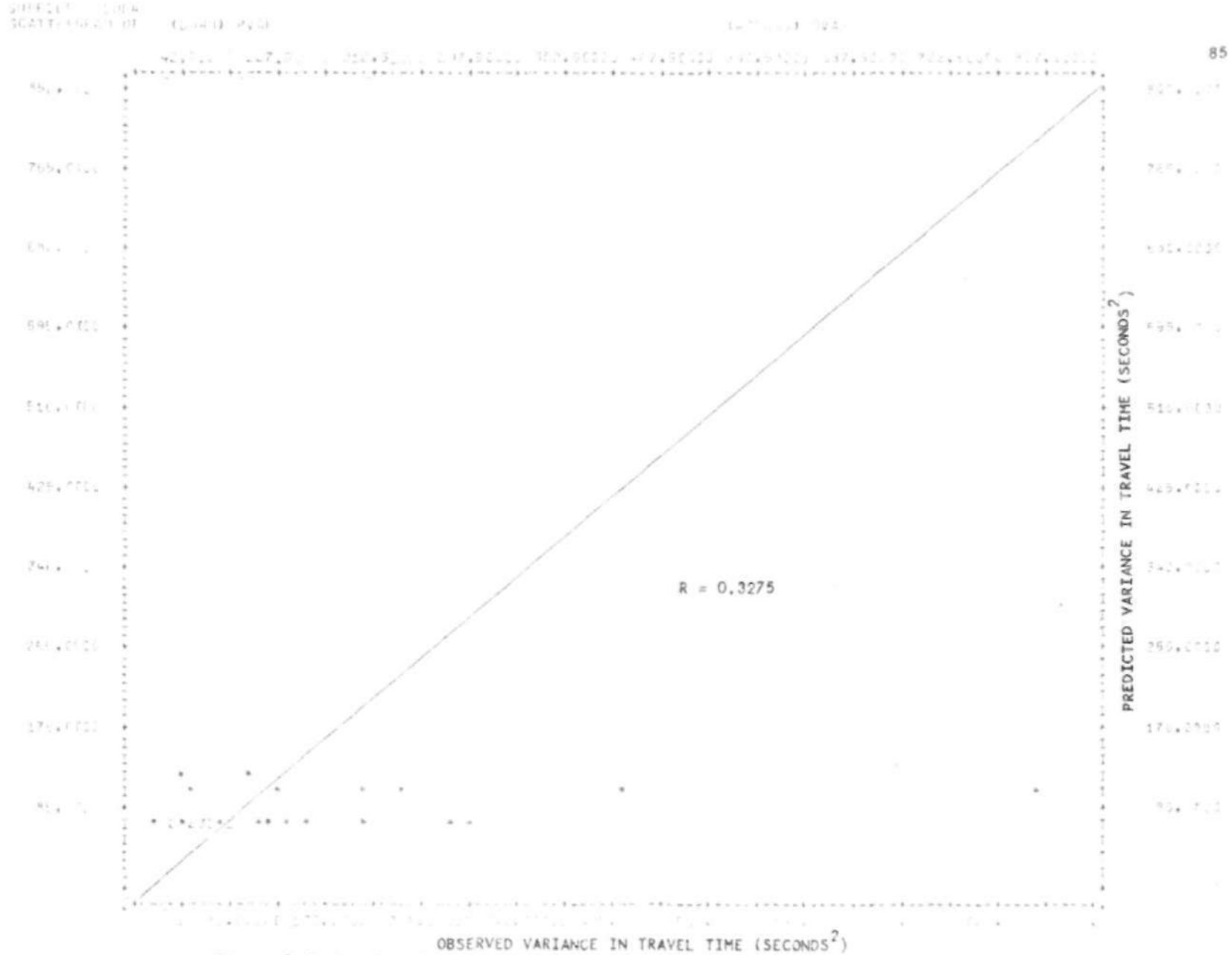


Figure 5.11 Predicted Versus Observed Variance in Travel Time on Links Using Models (5.5b) and (5.6)

CHAPTER 6

CALIBRATION OF THE MODEL: STOP DWELL TIMES

Calibration of mathematical models for dwell times of buses at stops is the subject of this chapter. Data collected on Route One, which is worked by TMO two-door 52-seater Steyr, formed a basis for formulation of models. Based on the peculiar set of present-day conditions, components of dwell times, namely, dead times and boarding/alighting times were studied separately to assess their relative impacts on dwell times as well as improve on the accuracy of predictions. As the existing ridership was at very low levels (about 3 passengers per stop), 193 observations were available for calibration of models of bus stop dwell times.

6.1 DEAD TIMES

It is likely that a major source of dwell time variability is fluctuation in dead times prior to and after boarding/alighting events and variations in boarding/alighting times. Equally likely to be significant are number of passengers served and characteristics of passengers.

Data on bus dead times were stratified into

pre-boarding/alighting dead times and post-boarding/alighting dead times. For the purpose of comparing empirical frequency distributions with theoretical probability density functions , the corresponding FREQUENCY DIAGRAMS were constructed. After visual and careful examination of the frequency diagrams which are highly skewed to the right (see Figures 6.1 to 6.4), the Exponential and Gamma distribution models were evaluated as possible theoretical distributions to fit the data.

The parameters of the hypothesized distributions have been estimated for pre-and post-boarding/alighting dead times by the method of maximum likelihood. Using Kolmogorov-Smirnov test, the null hypotheses that these candidate distributions and the empirical data are not significantly different could not be rejected at 5% level of significance. This may be due to a true lack of significance, or it may reflect limitations in the data set. Conceptually, however, each of the distributions appears to be a valid model of dead times, the Gamma model less estimates the observed distributions of pre-boarding/alighting, or post-boarding/alighting dead times. The estimated shape parameters $\hat{K}$, of the Gamma distributions are pointedly Exponential distributions.

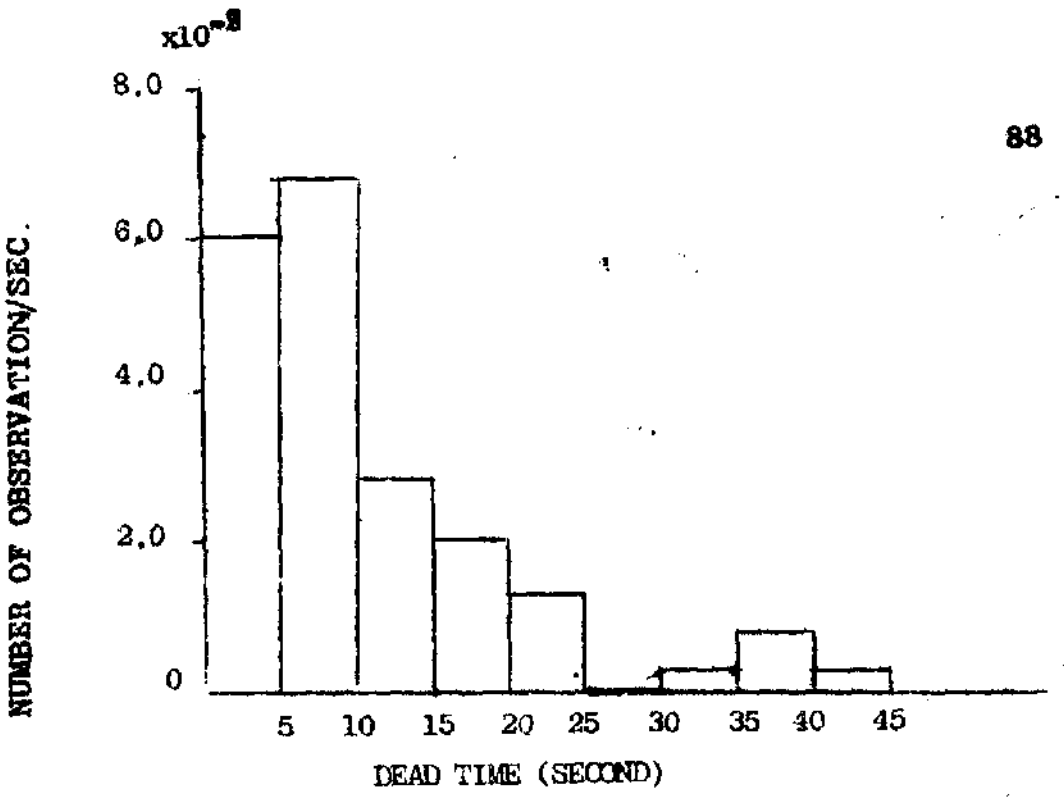


Figure 6.1 Observed Frequency Diagram of Pre-boarding Dead Times

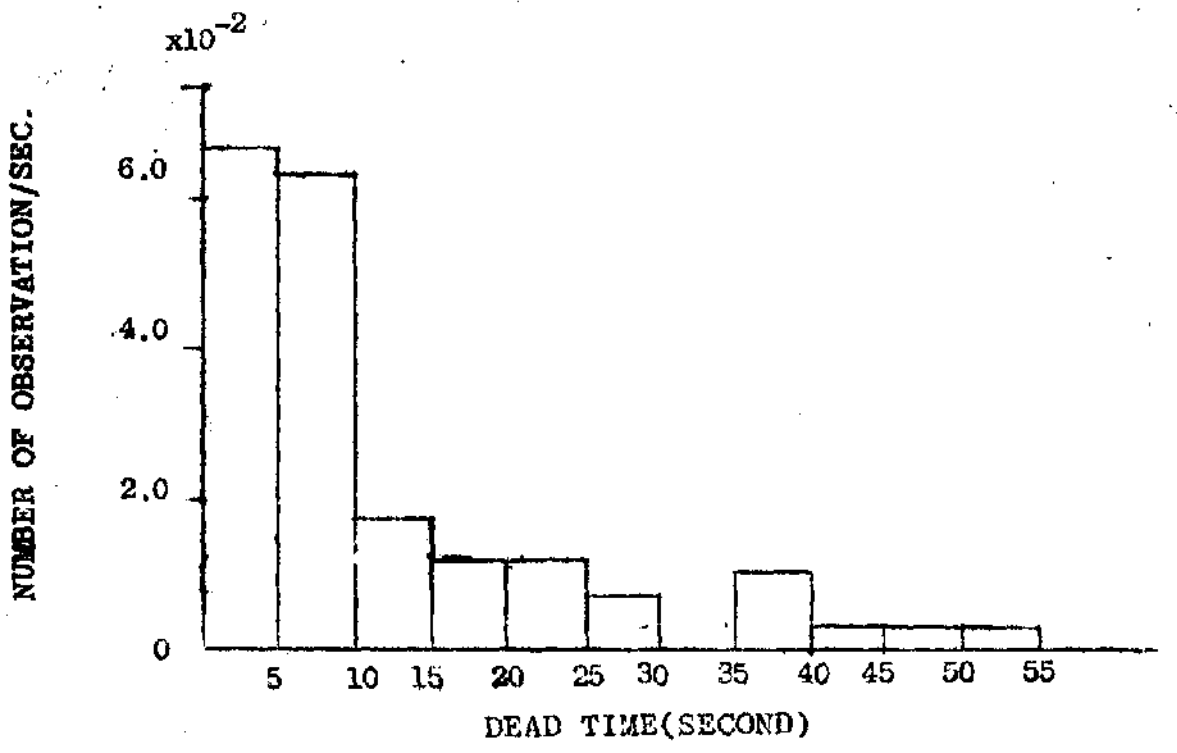


Figure 6.2 Observed Frequency Diagram of Post-boarding Dead Times.

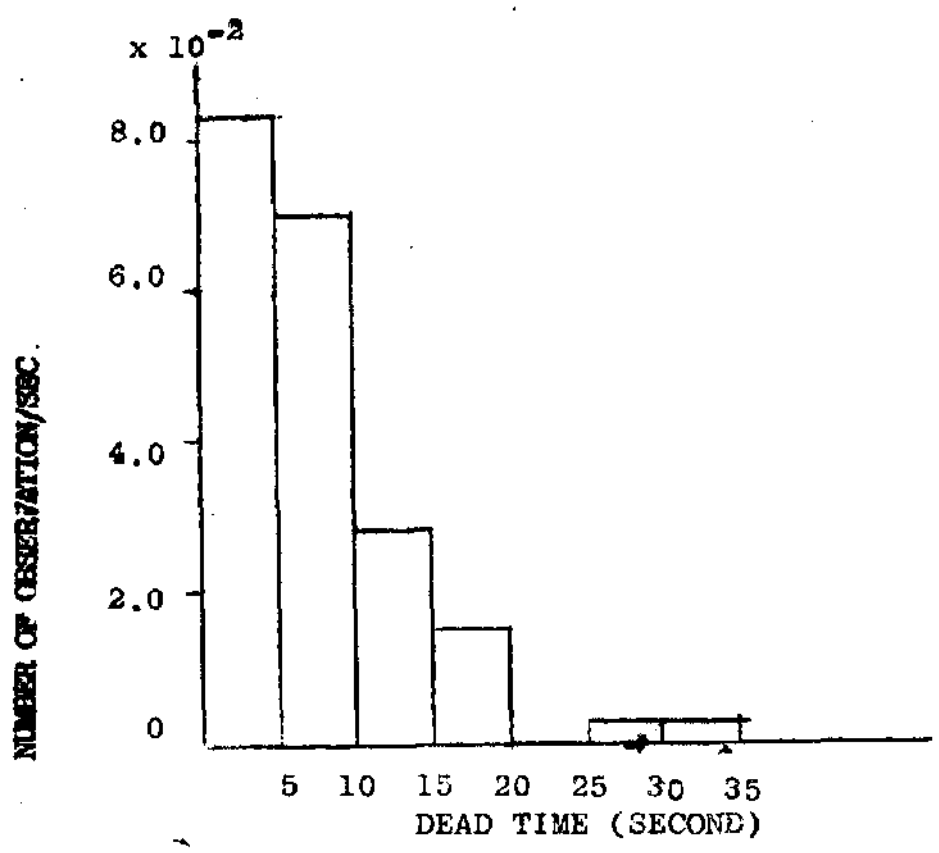


Figure 6.3 Observed Frequency Diagram of Pre-alighting Dead Time

x10^{-2}

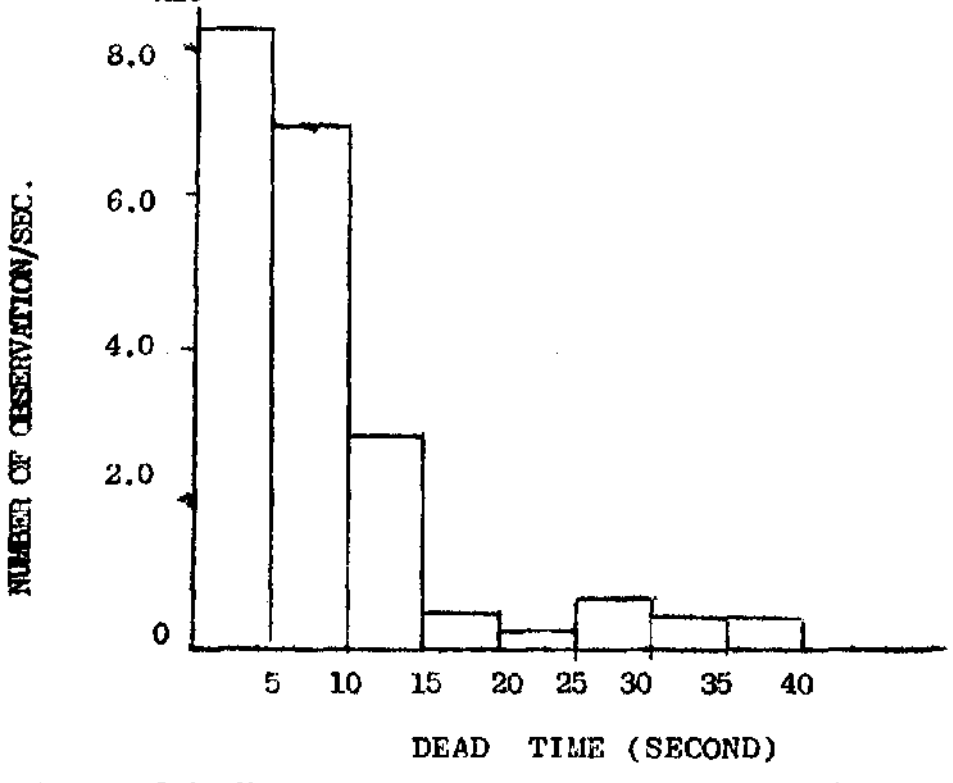


Figure:6.4 Observed Frequency Diagram of Post-Alighting Dead Time

Though the pre-/post-boarding or alighting dead time distributions are highly skewed to the right, the longer tail of the distributions could permit unusually long times in rare instances. Incidentally, bus dead times were passengers/drivers dependent as intending boarding passengers seem to lack knowledge of bus schedule, often inquire about bus destination, and drivers sometimes kill time because of patron inadequacies.

Since one might argue that pre-boarding/alighting dead times being elapsed time between the instant of bus stopping and the instant of first passenger boarding or alighting (and the post-boarding/alighting dead times being the delay time between the instant of last passenger boarding or alighting and the instant of bus departure) are essentially random, in such a case the Exponential distributions become apposite models. Therefore, the Exponential distributions are chosen for further analysis.

Thus far, the maximum likelihood estimates of the parameters of the Exponential models are specific to each of the boarding or alighting processes. The Exponential pre-boarding dead times and post-boarding dead times have respective means of 10.477 sec and 11.617 sec (mean rates $\hat{\lambda}$ are 0.09545/sec and 0.08608/sec respectively).

Therefore, a probability that Exponential pre-boarding dead times are smaller than Exponential post-boarding dead times is sought. By conditioning on post-boarding dead time, the probability is computed as 0.5258 (see Appendix C.4) This analysis was repeated for alighting process. The probability that Exponential pre-alighting dead times (with mean 6.859 sec) are less than Exponential post-alighting dead times (with mean 7.604sec) is also found to be 0.5258. In other words, both events are equiprobable and the probability of occurence of each event is approximately 50% which has been shown in Appendix C.4. Therefore, it is reasonable to treat the pre-boarding and post-boarding dead times (or pre-alighting and post-alighting dead times) as independent and identically distributed Exponential random variables.

It is now determined whether a single value of $\hat{\lambda}$ could be used to characterize the Exponential dead times for pre-boarding and post-boarding processes A weighted average of $\hat{\lambda}$ yields a value of $\hat{\lambda}_{wb}$ = 0.09076/sec for boarding event; the $\hat{\lambda}$ values were weighted by the number of observations in each case. This value falls within the 95% confidence interval on $\hat{\lambda}$ (see Appendix C.2) for both cases.

A K-S test was performed testing the hypothesis that the pre-and post-boarding dead times were Exponentially distributed with a weighted parameter $\hat{\lambda}_{wb}$= 0.09076/sec. At 5% level of significance, the null hypothesis could not be rejected. Therefore, it was concluded that this value could be used for both pre-/post-boarding dead time distributions. Since pre-andpost-boarding dead times could be treated as independent and identically distributed Exponential random variables having a common mean rate $\hat{\lambda}_{wb}$ = 0.09076/sec, their sum has a Gamma distribution with parameters 2 and 0.09076/sec.

A similar analysis was performed for pre-and post alighting dead times. The weighted average of $\hat{\lambda}$ was 0.13865/sec which falls within the 95% confidence level on $\hat{\lambda}$ for both cases. The null hypothesis that the Exponential model with a parameter $\hat{\lambda}_{wa}$ = 0.13865/sec and empirical distribution of pre-/post-alighting dead time are not statistically different could not be rejected. By treating the pre-alighting and post-alighting dead times as independent, identically distributed Exponential random variables, it follows that their sum has a Gamma distribution with parameters 2 and 0.13865/sec.

Further analysis was performed to establish whether a common scale parameter $\hat{\lambda}$ could characterize dead times for both boarding and alighting events. An overall weighted average was evaluated to be $\hat{\lambda}_w$= 0.12134/ sec.,which falls within the 95% confidence interval on $\hat{\lambda}$ for alighting process only. However, at 5% significance level the null hypothesis that Exponential distribution with a common parameter $\hat{\lambda}_w$ = 0.12134/sec.,and the empirically observed pre-/post-boarding or alighting dead time distributions are not statistically different could not be rejected . Regarding pre-/post-boarding dead times as well as pre-/post-alighting dead times as independent identically distributed Exponential random variables with a common parameter $\hat{\lambda}_w$ = 0.12134/ sec., the sum of each event follows a Gamma distribution with parameters 2 and 0.12134/sec. In other words, the Gamma distribution with a shape parameter K = 2 and a scale parameter $\hat{\lambda}_w$ = 0.12134/sec is the underlying probability model for total dead times of bus at any stop.

6.2 BOARDING AND ALIGHTING TIMES

With respect to pure boarding process, boarding times T_b, were found to be a linear function of the number

of boarders n_b. A linear regression model, using SPSS subprogram REGRESSION, was found to be:

$$E(T_b|n_b) = \underset{(0.9069}{0.0264} + \underset{(0.2081)}{3.8782n_b} \qquad (6.1)$$

F - values = 0.0085, 347.460

R = 0.9257, R^2 = 0.8570, $\bar{R}^2$ = 0.8545

Overall regression F-value = 347.460

SEE = 5.2445sec.

Consideration of the results indicates that coefficient associated with boarding passenger variables is highly significant, but the regression constant is not significantly different from zero. Both of these coefficients, however, have the correct sign. Therefore, a regression through origin is a suitable model. Using OPTION 19 of SPSS subprogram REGRESSION to "force regression through origin" resulted in the following simpler model:

$$E(T_b|n_b) = \underset{(0.1540}{3.8823n_b} \qquad (6.2)$$

F - value = 635.451

R = 0.9566, R^2 = 0.9150, $\bar{R}^2$ = 0.9150

Overall regression F-value = 635.451

SEE = 5.1999sec.

This model shows that the expected boarding time per passenger is approximately 4.0sec. The correlation coefficient R, between estimates derived from the model and observed values is 0.9566 and the coefficient of determination R^2, indicates that about 92% of the variation in boarding time is explained by linear regression on the boarding passenger variable. Although the boarding time is a linear function of number of boarding passengers, very few observations for large values of boarding passengers were recorded. However, the range of observations was between 1 and 21 passengers, with the bulk of observations lying between 1 and 3 passengers. Since very few observations for large values of boarders n_b were embodied in the calibration of model, no guarantee of linearity in that region can be offered. It might be that the presence of a large number of boarding passengers could change the boarding rate by affecting passengers behaviour and inter-boarding time of passengers.

The same model structure, with different parameters was developed for alighting process. The regression model for alighting time T_a given number of alighting passengers n_a was:

$$E(T_a|n_a) = \underset{(0.6617)}{0.2074} + \underset{(0.1462)}{2.9847n_a} \qquad (6.3)$$

F - values = 0.09824, 416.880

$R = 0.8946$, $R^2 = 0.8003$, $\bar{R}^2 = 0.7984$

Overall regression F-value = 416.880

SEE = 5.1758sec.

Because the regression constant was insignificantly different from zero, this model was rejected in favour of a regression equation through the origin. The resulting predictive model was

$$E(T_a|n_a) = \underset{(0.1106)}{3.0145n_a} \qquad (6.4$$

F - value = 743.129

Overall regression F-value = 743.129

$R = 0.9361$, $R^2 = 0.8762$, $\bar{R}^2 = 0.8762$

SEE = 5.1536sec.

The implication of this predictive equation was that the alighting rate was about 3.0 seconds per passenger and that approximately 88% of the observed variance of alighting time was accounted for by the model. However, between 1 and 27 passengers alighted at a stop with a majority of alighting passengers n_a lying between 1 and 5. As the latter was predominant in the model

calibrated and very few observations for large values of n_a were recorded, no guarantee of linearity in that region can be assured. The alighting rate could as well alter in the light of a large number of alighting passengers by affecting the inter-alighting time of passengers.

6.3 STOP DWELL TIMES.

For pure boarding, or pure alighting process and in the case of TMO two-door buses, the stop dwell times could reasonably be taken as the sum of bus dead times and boarding/alighting times.

(1) STOP DWELL TIME FOR BOARDING EVENT (S_b)

$$S_b = D_b + T_b \qquad (6.5)$$

where D_b = dead time for boarding process. From previous analysis

$$D_b \sim \Gamma(2,\ 0.09076)$$

$$T_b = 3.8823n_b$$

(2) STOP DWELL TIME FOR ALIGHTING EVENT (S_a)

$$S_a = D_a + T_a \qquad (6.6)$$

where D_a = dead time for alighting process. Based on the

foregoing analysis

$$D_a \sim \Gamma(2, 0.13865)$$

$$T_a = 3.0145 n_a$$

(3) STOP DWELL TIME FOR BOARDING/ALIGHTING EVENTS(S_{ab})

It would be reasonable to treat the stop dwell time of bus for simultaneous boarding and alighting as the slower of the two processes; that is,

$$S_{ab} = \max(D_a + T_a, D_b + T_b). \qquad (6.7)$$

CHAPTER 7

PASSENGER ARRIVALS AND HEADWAY DISTRIBUTION

Conceivably, separation between homes and work, and similarity in most working hours through the whole spectrum of jobs have resulted in movement of people between homes and work each day at about the same times in the day. However, it is assumed that passenger arrivals at every bus stop along a fixed-route are of the same pattern and follow a stationary Poisson process - the arrival rate being constant over a time period. That is, passengers arrive randomly and independently at a bus stop. The assumption of stationary Poisson distribution of arrival process is elegant where bus services are unreliable, or frequent such as during rush-hours when headways are short. When bus services are sufficently infrequent or reliable with schedules published and adhered to, passengers take advantage of the schedule and correlate their arrivals with bus schedules. Such a situation results in passengers' choice of arrival time to minimize their waiting times. We shall discuss the random passenger arrival at bus stops.

Since passengers' arrival pattern is assumed to be

a stationary Poisson process, the mean and variance of the number of arriving passengers are equal. Suppose X and Y are two random variables; the mean and variance of one variable may depend on the value of the other variable. From the concepts of conditional expectations and conditional variance (Ang and Tang,1975).

$$E(X) = E_Y[E(X|Y)]$$

$$Var(X) = E_Y[Var(X|Y)] + Var_Y[E(X|Y)]$$

The implication of the latter equation is that the total (unconditional) variance of a random variable is equal to the sum of the expected value of its conditional variance and the variance of its conditional expectation. The subscript Y on E and Var emphasizes that expectation or variance is with respect to Y. It should be noted that whereas $E(X|Y=y)$ is a constant, $E(X|Y)$ is a random variable whose mean is given by the expression above.

Suppose the passenger mean arrival rate is $\bar{\lambda}_p$, the expected number of arrivals Q given headway H is given by

$$E(Q|H) = \bar{\lambda}_p H \qquad (7.1)$$

with variance

$$Var(Q|H) = \bar{\lambda}_p H \qquad (7.2)$$

The unconditional expectation and variance of the number of arriving passengers are thus

$$E(Q) = E[E(Q|H)] = E(\bar{\lambda}_p H) = \bar{\lambda}_p E(H) \qquad (7.3)$$

$$\begin{aligned} Var(Q) &= E[Var(Q|H)] + Var[E(Q|H)] \\ &= E(\bar{\lambda}_p H) + Var(\bar{\lambda}_p H) \\ &= \bar{\lambda} p E(H) + \bar{\lambda}_p^2 \, Var(H) \end{aligned} \qquad (7.4)$$

When a passenger arrives at a bus stop, he joins a queue of potential riders awaiting pick up. If γ represents a proportion of arriving passengers that board a bus that arrives at a stop, then the expected number of boarders is

$$E(n_b) = \gamma \, \bar{\lambda}_p E(H) \qquad (7.5)$$

with variance

$$Var(n_b) = \gamma \, \bar{\lambda}_p E(H) + \gamma \bar{\lambda}^2 Var(H) \qquad (7.6)$$

However, since bus headway data were not collected these equations were not calibrated.

CHAPTER 8

SUMMARY AND DISCUSSION OF RESULTS

Having completed the analyses, it is now appropriate to consider what conclusions may be drawn from this study. The major findings are summarized herein.

This study has addressed scheduled, fixed-route intra-urban bus transit service with particular emphasis on travel times of buses on links. The analysis was done in the context of a real-life bus system run by Kaduna State Transport Authority (KSTA) in metropolitan Kaduna. Data collected from the field studies were used to estimate the models.

Prior to calibration of link travel time models, observational data collected on different weekdays and mostly during the morning peak periods were checked against any statistical differences in means of travel times using analysis of variance (ANOVA). The analysis was considered necessary as data had been collected during different weekdays within the morning Peak periods. Moreover, the analysis could reflect variations in bus performance which might be related to daily trends as well as form a basis for aggregation of data if the differences in means of travel times among the different weekdays were not statistically significant,

or stratification of data if the means were significantly different. Incidentally, the results of the computer program used, SPSS subprogram ANOVA, indicated that the means of travel times among the different weekdays were not significantly different statistically for all the 34 links studied. In other words, the null hypothesis that the population means are equal could not be rejected and the differences among means can be attributed to chance. In the absence of clear statistical underpinning concerning the differences in means of link travel times among the different weekdays, the data for each link were treated as samples of the same population. In this light, the data for each link were aggregated resulting in a larger data base for subsequent analyses.

Two strategies for modelling link travel times were used based on distribution modelling technique and stepwise regression technique in which models were constructed recursively through successive addition of independent variables that meet certain statistical criteria. Independent variables were entered only if they had a minimum F-value of 4.0 and Tolerance of 0.05. The basic hypothesis underlying the regression approach is that link travel time can be explained by independent variables related to system characteristics. The major advantages of regression models are that they are usually very simple to develop and use, and reflect actual conditions. However, they have the disadvantage

of being too restrictive. While stepwise regression technique was used to calibrate successfully the mean of link travel time, calibration of variance of link travel resulted in very low values of coefficient of determination R^2, namely, the independent variables used did not have much explanatory power. Consequently, distribution modelling approach was adopted. Using a statistical distribution to model travel time takes better advantage of the total data set and provides the analyst with a more comprehensive picture of the distributions of observed travel time.

The Log-Normal probability model was considered in this investigation on the following grounds: 1) the model is highly flexible, positively skewed, and limited to positive values of a random variable of which travel time is one; 2) it is computationally tractable and related to Normal distribution via logarithmic transformation; 3) the model is a result of a series of multiplicative effects, but the logarithms of travel time variable become Normally distributed and the effects become additive; and 4) the Log-Normal model is an appropriate probability model when the value of an observed variable is a random proportion of the previously observed value.

Statistical analysis of the data for each of the 34 links confirmed the appropriateness of Log-Normal distributions as the underlying probability model for day-to-day stochastic distributions of travel times. The Kolmogorov-Smirnov statistics and the decision rule for the 5% significance level were adopted for goodness-of-fit test. An attempt was made to predict the parameters of Log-Normal distribution in terms of explanatory variables related to system characteristics. The study findings indicate that only link length and number of roundabouts were statistically significant. Surprisingly, number of intersection is statistically insignificant as a factor; this may be due to a true lack of significance, or it may reflect limitations in the data set. Detailed examiniation showed that the explanatory variables in the proposed models were not collinear, implying that the models derived did not have biased coefficients. In essence, the parameter μ of the Log-Normal distribution was evaluated as a function of link length and number of roundabouts, while the parameter σ was a function of link length.

It is to be expected that variables such as traffic volumes, general vehichle stream, drivers' behaviour,

driving conditions, bus efficiency, weather, and so forth which were not considered in this study also affect travel time. However, the model for parameter estimation σ of Log-Normal distribution is particularly weak. While it might be useful to pursue a more extensive examination of these factors, the indication from the results here is that a substantial portion of the variation in link travel times is already accounted for in the model. Consequently, methods for utilizing, or to explicitely account for these additional variables, in order to improve on the prediction of the parameter σ in particular, require further study. Conceivably, computer simulation models based on the findings of this study could permit refinement of, or improvement on the existing model for σ as well as enable the transit operator experiment with different policy options before actually trying the preferred ones in the field.

Simple mathematical models of stop dwell times of buses were also formulated. Observations showed that ridership level was low (about 3 passengers per stop) and passengers lack knowledge of bus schedule. Intending boarding passengers often inquire about bus destination, and drivers sometimes kill time because of patron

inadequacies and these result in overly dead times and stop dwell times. In essence, bus dead times were passengers/drivers dependent. The major findings, in the light of the peculiar set of present-day conditions, were that bus total dead times at stops were a random variable following a Gamma distribution and that boarding /alighting times were linear functions of number of boarding/alighting passengers.

It may serve to caution that although the dependent variables are highly correlated with existing explanatory variables, the models will still yield inconsistent predictions where policy changes or circumstances alter existing correlations between dependent variables and explanatory variables in existing models.

Estimation results provide tentative but reasonable support for the conceptual and theoretical bases of the models. Predictions from models compare very favourably with observed values. The proposed model is relatively simple and should provide intra-urban bus transit operators or planners with an analytical framework to address the subject of link travel time in particular and bus transit planning in general.

While it is recognised that major gaps in our knowledge of bus transit travel times and impacts still remain, it is hoped that ideas and suggestions contained in this dissertation fill a small gap in our knowledge of these matters and may act as a motivation for further investigation.

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APPENDIX A

A.1 ESTIMATION OF NORMAL DISTRIBUTION PARAMETERS

A.1.1 MAXIMUM LIKELIHOOD ESTIMATES

Suppose a random variable X is Normally distributed; that is,

$$f(x;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\tfrac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$$

The likelihood function of a sample of size n is

$$L(x_i;\mu,\sigma) = \prod_{i=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\tfrac{1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2\right]$$

$$= (2\pi)^{-n/2}\,\sigma^{-n} \exp\left[-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2\right]$$

The log-likelihood function is

$$\ln L = -\frac{n}{2}\ln 2\pi - n\ln\sigma - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2$$

To maximize the likelihood function we have

$$\frac{\partial \ln L}{\partial \mu} = \frac{1}{\hat{\sigma}^2}\sum_{i=1}^{n}(x_i-\hat{\mu}) = 0 \qquad (A.1.1)$$

$$\frac{\partial \ln L}{\partial \sigma} = -\frac{n}{\hat{\sigma}^2} + \frac{1}{\hat{\sigma}^3}\sum_{i=1}^{n}(x_i-\hat{\mu})^2 = 0 \qquad (A.1.2)$$

Since $\hat{\sigma}^2 \neq 0$, the solution to these two simultaneous equations yields

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad (A.1.3)$$

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\hat{\mu})^2 \qquad (A.1.4)$$

These are the maximum likelihood estimates of the parameters μ and σ of a Normally distributed sample of size n.

A.1.2 MOMENT ESTIMATES

The moment estimators of a Normal distribution are

$$E(X) = \tilde{\mu} = \frac{1}{n}\sum_{i=1}^{n} xi \qquad (A.1.5)$$

$$\text{Var}(X) = \tilde{\sigma}^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\tilde{\mu})^2 \qquad (A.1.6)$$

A.2 ESTIMATION OF LOG-NORMAL DISTRIBUTION PARAMETERS

A.2.1 MAXIMUM LIKELIHOOD ESTIMATES (MLE): TWO-PARAMETER CASE

The Log-Normal probability density function is:

$$f(x_i;\mu,\sigma) = \frac{1}{x\sigma\sqrt{2\Pi}}\exp\left[-\tfrac{1}{2}\left(\frac{\ln x-\mu}{\sigma}\right)^2\right]$$

The likelihood function of a Log-Normal sample of size n is:

$$L(x_i;\mu,\sigma) = \prod_{i=1}^{n}\left(\frac{1}{x_i\sigma\sqrt{2\Pi}}\exp\left[-\tfrac{1}{2}\left(\frac{\ln x_i-\mu}{\sigma}\right)^2\right]\right)$$

$$= \sigma^{-n}(2\Pi)^{n/2}\prod_{i=1}^{n}x_i^{-1}\exp\left[-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(\ln x_i-\mu)^2\right]$$

To maximize the likelihood function we have

$$\frac{\partial \ln L}{\partial \mu} = \frac{1}{\hat{\sigma}^2} \sum_{i=1}^{n} (\ln x_i - \hat{\mu}) = 0 \qquad \text{(A.2.1a)}$$

$$\frac{\partial \ln L}{\partial \sigma} = -\frac{n}{\hat{\sigma}} + \frac{1}{\hat{\sigma}^3} \sum_{i=1}^{n} (\ln x_i - \hat{\mu})^2 = 0 \qquad \text{(A.2.1b)}$$

As $\hat{\sigma} \neq 0$, the solution to these two simultaneous equations yields:

$$\hat{\mu} = \frac{1}{n} \sum_{i=1}^{n} \ln x_i \qquad \text{(A.2.2a)}$$

$$\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} (\ln x_i - \hat{\mu})^2 \qquad \text{(A.2.2b)}$$

These are the maximum likelihood estimators of μ and σ^2

A.2.2 MOMENT ESTIMATES: TWO-PARAMETER CASE

The moment estimators of μ and σ^2 are:

$$\hat{\mu} = 2\ln M_1' - \tfrac{1}{2}\ln M_2' \qquad \text{(A.2.3a)}$$

$$\hat{\sigma} = \ln M_2' - 2\ln M_1' \qquad \text{(A.2.3b)}$$

Where M_1' and M_2' are sample moments (first moment about origin and second moment about origin respectively) in terms of X_i. That is,

$$M_1' = \frac{1}{n} \sum_{i=1}^{n} X_i$$

$$M_2' = \frac{1}{n} \sum_{i=1}^{n} X_i^2$$

For small samples, these estimators are biased. Since they are consistent however, they are asymptotically unbiased (Bury 1975). Generally, moment estimates are inefficient for all but very small values of σ^2; consequently, the Maximum Likelihood (ML) estimators are generally preferred even though they necessitate a logarithmic data transformation for their evaluation (Bury, 1975)

A.2.3. MAXIMUM LIKELIHOOD ESTIMATES: THREE-PARAMETER CASE

The three-parameter Log-Normal model is:

$$f(x;a,\mu,\sigma) = \frac{1}{(x-a)\sigma\sqrt{2\pi}} \exp(-\tfrac{1}{2}(\frac{\ln(x-a)-\mu}{\sigma})^2)$$

When the amount of shift, a, (location parameter) also called the threshold value is known, the transformed data $x^1 = (x-a)$ can be applied to the two-parameter case.

When a is unknown, parameter estimation is difficult. The estimation of a is particularly problematic.

Suppose $x_1, \ldots, x_n$ are observed sample values which are Log-Normally distributed. The likelihood function is

$$L(x_i;a,\mu,\sigma) = \prod_{i=1}^{n} \left(\frac{1}{\sigma(x_i-a)\sqrt{2\pi}} \exp\left[-\tfrac{1}{2}\left(\frac{\ln(x_i-a)-\mu}{\sigma}\right)^2\right]\right)$$

$$= (\sigma\sqrt{2\pi})^{-n} \prod_{i=1}^{n} (x_i-a)^{-1} \exp\left(-\frac{1}{2\sigma^2} \sum_{i=1}^{n} [\ln(x_i-a)-\mu]\right)$$

The log-likelihood function of all observations becomes

$$\ln L(x_i;a,\mu,\sigma) = -n\ln\sigma - \ln(\sqrt{2\pi}) - \sum_{i=1}^{n}\ln(x_i-a) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}[\ln(x_i-a)-\mu]^2$$

To maximize the likelihood function we obtain, for complete sample, the MLE equations as follows:

$$\frac{\partial \ln L}{\partial \mu} = \frac{1}{\hat{\sigma}^2}\sum_{i=1}^{n}[\ln(x_i-\hat{a})-\hat{\mu}] = 0 \qquad \text{(A.2.4a)}$$

$$\frac{\partial \ln L}{\partial \sigma} = -\frac{n}{\hat{\sigma}} + \frac{1}{\hat{\sigma}^3}\sum_{i=1}^{n}[\ln(x_i-\hat{a})-\hat{\mu}]^2 = 0 \qquad \text{(A.2.4b)}$$

$$\frac{\partial \ln L}{\partial a} = \sum_{i=1}^{n}\frac{1}{x_i-\hat{a}} + \frac{1}{\hat{\sigma}^2}\sum_{i=1}^{n}\frac{\ln(x_i-\hat{a})-\mu}{x_i-a} = 0$$

$$= \sum_{i=1}^{n}\frac{1}{x_i-\hat{a}} + \frac{1}{\hat{\sigma}^2}\sum_{i=1}^{n}\frac{\ln(x_i-\hat{a})}{x_i-\hat{a}} - \frac{1}{\hat{\sigma}^2}\sum_{i=1}^{n}\frac{\hat{\mu}}{x_i-\hat{a}} = 0$$

$$= (\hat{\sigma}^2 - \hat{\mu})\sum_{i=1}^{n}\frac{1}{x_i-\hat{a}} + \sum_{i=1}^{n}\frac{\ln(x_i-\hat{a})}{x_i-\hat{a}} = 0 \qquad \text{(A.2.4c)}$$

From the first two equations (A.2.4a) & (A.2.4b)

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n}\ln(x_i-\hat{a}) \qquad \text{(A.2.5a)}$$

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}[\ln(x_i-\hat{a})-\hat{\mu}]^2 \qquad \text{(A.2.5b)}$$

Eliminating $\hat{\mu}$ and $\hat{\theta}^2$ from equations (A.2.4a-c) above (Cohen Jr., 1951) gives

$$f(\hat{a}) = \sum_{i=1}^{n} \frac{1}{x_i-\hat{a}} \left(\frac{1}{n} \sum_{i=1}^{n} \left[\ln(x_i-\hat{a}) - \frac{1}{n} \sum_{i=1}^{n} \ln(x_i-\hat{a}) \right]^2 - \frac{1}{n} \sum_{i=1}^{n} \ln(x_i-\hat{a}) \right)$$

$$+ \sum_{i=1}^{n} \frac{\ln(x_i-\hat{a})}{x_i-\hat{a}} = 0 \qquad \text{(A.2.5c)}$$

This equation must be inverted numerically for $\hat{a}$. Thus, the function $f(\hat{a})$ is evaluated from (A.2.5c) for a number of trial values for $\hat{a}$. Interpolation for $f(\hat{a}) = 0$ gives $\hat{a}$.

It is easily seen that the three-parameter likelihood function tends to infinity as the first-order statistic $x_{(1)}$ approaches $\hat{a}$ (as it must when the sample size n becomes large)(Hill, 1963). The corresponding estimates of (μ,σ) is $(-\infty, +\infty)$; so that maximum likelihood estimation is strictly not applicable.

On the other hand, in practical estimation problems it is usually found (Harter & Moore, 1966) that the FORMAL solution of the maximum likelihood (ML) equations corresponds to a LOCAL maximum of the likelihood function (LF), resulting in reasonable estimates of (a,μ,σ). When an estimate $\hat{a}$ cannot be obtained in the permissible range $\hat{a} \leq x_{(1)}$, the MLE of a is taken to be $\hat{a} = x_{(1)}$.

For subsequent ML estimation of (μ,σ), the smallest observation $x_{(1)}$ must then be censored from the sample (Harter & Moore, 1966); thus the ML equations remain finite.

A.2.4 QUANTILE ESTIMATES: THREE-PARAMETER CASE:

The method of quantiles can be employed to estimate (a,μ,σ) by means of three suitably chosen sample order statistics, corresponding to quantiles x_q, $x_{0.5}$ and x_{1-q}. Because of the close relation of the Log-Normal with Normal models, the parameters (a,μ,σ) can be estimated from three equations obtainable from the expression:

$$z_q = \frac{\ln(x_q-a) - \mu}{\sigma} \qquad \text{(A.2.6)}$$

where z_q is standard Normal quantile (i.e., q100th percentile for a standard Normal variable) and x_q is the corresponding quantile calculated from data.
The three estimating equations (from three corresponding quantiles x_q, $x_{0.5}$ and x_{1-q}) are

$$\tilde{\sigma}z_q = \ln(x_q-\tilde{a}) - \tilde{\mu} \qquad \text{(A.2.7a)}$$

$$0 = \ln(x_{0.5}-\tilde{a}) - \tilde{\mu} \qquad \text{(A.2.7b)}$$

$$-\tilde{\sigma}z_q = \ln(x_{1-q}-\tilde{a}) - \tilde{\mu} \qquad \text{(A.2.7c)}$$

(For any two symmertric quantiles q and 1-q, then $z_{1-q} = -z_q$).

Reasonable estimates are obtained when $q=0.05$ (Aitchison and Brown, 1957). The direct solution of these equations gives the estimates:

$$\tilde{\sigma} = \frac{1}{Zq} \ln \left[\frac{x_{1-q} - x_{0.5}}{x_{0.5} - x_q}\right] \qquad \text{(A.2.8a)}$$

$$\tilde{\mu} = \ln\left[\frac{x_q - x_{0.5}}{\exp(\tilde{\sigma} Zq) - 1}\right] \qquad \text{(A.2.8b)}$$

$$\tilde{a} = x_{0.5} - \exp(\tilde{\mu}) \qquad \text{(A.2.8c)}$$

No iteraction is required to evaluate the quantile estimates (A.2.8a - c), but the efficiencies of (A.2.8a-c) can be expected to be lower then those of corresponding MLEs.

A.3 ESTIMATION OF GAMMA DISTRIBUTION PARAMETERS

A.3.1 MAXIMUM LIKELIHOOD ESTIMATES: TWO-PARAMETER CASE

A two-parameter Gamma model is

$$f(x;\lambda,k) = \frac{\lambda}{\Gamma(k)}(\lambda x)^{K-1} \exp(-\lambda x)$$

The likelihood function of a Gamma sample of size n is:

$$L(x;\lambda,K) = [\Gamma(K)]^{-n} \lambda^{nK} \prod_{i=1}^{n} x_i^{K-1} \exp(-\lambda \sum_{i=1}^{n} x_i)$$

The corresponding log-likelihood function is

$$\ln L = -n \ln \Gamma(K) + nK \ln\lambda + (K-1) \ln \prod_{i=1}^{n} x_i - \lambda \sum_{i=1}^{n} x_i$$

To maximize the log-likelihood function we obtain

$$\frac{\partial \ln L}{\partial \lambda} = \frac{n\hat{K}}{\hat{\lambda}} - \sum_{i=1}^{n} x_i = 0 \qquad \text{(A.3.1a)}$$

$$\frac{\partial \ln L}{\partial K} = -n \frac{d}{dK} \ln\Gamma(\hat{K}) + n\ln\hat{\lambda} + \ln \prod_{i=1}^{n} x_i = 0$$

$$= -n\,\Psi(\hat{K}) + n\ln\hat{\lambda} + n\ln \prod_{i=1}^{n} x_i^{1/n} = 0$$

$$= -n\Psi(\hat{K}) + n\ln\hat{\lambda} + n\ln G = 0 \qquad \text{(A.3.1b)}$$

Solution of the two simultaneous equations gives the MLEs as:

$$\hat{K}/\hat{\lambda} = \frac{1}{n} \sum_{i=1}^{n} x_i = \bar{x} \qquad \text{(A.3.2a)}$$

$$-\ln G = \ln\hat{\lambda} - \Psi(\hat{K}) \qquad \text{(A.3.2b)}$$

where $G = \prod_{i=1}^{n} x_i^{1/n}$ = GEOMETRIC MEAN OF sample x

$\Psi(\hat{K}) = \frac{d}{dk}\ln\Gamma(\hat{K})$ = DIGAMMA FUNCTION (values of $\Psi(\hat{K})$ can be obtained from tables)

Combining the ML equations gives

$$\ln \frac{\bar{x}}{G} = \ln\hat{K} - \Psi(\hat{K}) = g(\hat{K}) \qquad \text{(A.3.3)}$$

The MLE $\hat{K}$ can be obtained from (A.3.3) by inverse interpolation for a given sample value of $\ln(\bar{x}/G)$ (see Abramowitz, 1964)

Alternatively, the following highly accurate approximation (Greenwood and Durand, 1960) can be used

to obtain $\hat{K}$ directly:

$$\hat{K} = (0.5001 + 0.1649g - 0.0544g^2)g^{-1}, \quad 0 \leq g \leq 0.577 \qquad (A.3.4)$$

$$K = (17.80 + 11.97g + g^2)^{-1}.(8.899 + 9.060g + 0.9775g^2)g^{-1}$$

$$0.577 \leq g \leq 17 \qquad (A.3.5)$$

where $g = \ln(\bar{x}/G)$. The error of either formula is less than 0.01% (Greenwood and Durand, 1960)

A.3.2 MOMENT ESTIMATES: TWO-PARAMETER CASE

The method of moments may be used to estimate the parameters of a Gamma model as follows

$$E(X) = \overset{v}{K}/\overset{v}{\lambda} = \frac{1}{n}\sum_{i=1}^{n} x_i = \bar{X} \qquad (A.3.6a)$$

$$Var(X) = \overset{v}{K}/\overset{v}{\lambda}^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2 \qquad (A.3.6b)$$

Solving the two equations simultaneously yields

$$\overset{v}{K} = \frac{\bar{x}^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2/(n-1)} \qquad (A.3.7a)$$

$$\overset{v}{\lambda} = \frac{\bar{x}}{\sum_{i=1}^{n}(x_i-\bar{x})^2/(n-1)} \qquad (A.3.7b)$$

Alternatively the moment estimators can be derived as follows:

$$M_1' = \frac{1}{n}\sum_{i=1}^{n} x_i = \overset{v}{K}/\overset{v}{\lambda} \qquad (A.3.8a)$$

$$M_2' = \frac{1}{n} \sum_{i=1}^{n} x_i^2 = \overset{v}{K}(\overset{v}{K}+1)/\overset{v}{\lambda}^2 \qquad \text{(A.3.8b)}$$

Solving simultaneously, thus

$$\overset{v}{K} = \frac{(M_1')^2}{M_2'-(M_1')^2} \qquad \text{(A.3.9a)}$$

$$\overset{v}{\lambda} = \frac{M_1'}{M_2' - (M_1')^2} \qquad \text{(A.3.9b)}$$

A.3.3 MAXIMUM LIKELIHOOD ESTIMATES: THREE-PARAMETER CASE

The three-parameter Gamma model is

$$f(x;a,\lambda,K) = \frac{\lambda^K}{\Gamma(K)} (x-a)^{K-1} e^{-\lambda(x-a)}$$

when the shift a is known, data on the three-parameter variable X, can be transformed to $x' = (x-a)$, and two-parameter method can be applied. Inferences are then conditional on the known value a. When a is unknown, its value must be estimated from data on X.

For REGULARITY CONDITIONS to hold in the neighbourhood of the true parameter values, then $K > 2$ (Bury, 1975). The ML equations are, therefore, only applicable when $K > 2$.

The likelihood function of a sample of independent Gamma observations is

$$L(x_i;a,\lambda,K) = [\Gamma(K)]^{-n} \lambda^{nK} \prod_{i=1}^{n} (x_i-a)^{K-1} \exp(-\lambda \sum_{i=1}^{n} (x_i-a))$$

The corresponding log-likelihood function is

$$\ln L = -n\ln\Gamma(K) + nK\ln\lambda + (K-1)\ln \prod_{i=1}^{n}(x_i-a) - \lambda\sum_{i=1}^{n}(x_i-a)$$

To maximize the likelihood function, then

$$\frac{\partial \ln L}{\partial \lambda} = \frac{n\hat{K}}{\hat{\lambda}} - \sum_{i=1}^{n}(x_i-\hat{a}) = 0$$

$$= \frac{n\hat{K}}{\hat{\lambda}} - \sum_{i=1}^{n} x_i + n\hat{a} = 0$$

$$= \frac{\hat{K}}{\hat{\lambda}} - \bar{x} + \hat{a} = 0 \qquad \text{(A.3.10a)}$$

$$\frac{\partial \ln L}{\partial K} = n\frac{\partial}{\partial K}\Gamma(\hat{K}) + n\ln\hat{\lambda} + \ln\prod_{i=1}^{n}(x_i-\hat{a}) = 0$$

$$= -n\Psi(\hat{K}) + n\ln\hat{\lambda} + \sum_{i=1}^{n}\ln(x_i-\hat{a}) = 0 \qquad \text{(A.3.10b)}$$

$$\frac{\partial \ln L}{\partial a} = -(\hat{K}-1)\frac{1}{\sum_{i=1}^{n}(x_i-\hat{a})} + \hat{\lambda}n = 0$$

$$= \frac{1}{n}\sum_{i=1}^{n}(x_i-\hat{a})^{-1} - \hat{\lambda}(\hat{K}-1)^{-1} = 0 \qquad \text{(A.3.10c)}$$

Substituting the first equation into the second and third gives two expressions in terms of $\hat{a}$ and $\hat{K}$:

$$\hat{K} = \frac{1}{n}(\bar{x}-\hat{a})(\hat{K}-1)\sum_{i=1}^{n}(x_i-\hat{a})^{-1} \qquad \text{(A.3.11a)}$$

$$\Psi(\hat{K}) - \ln\hat{K} = \frac{1}{n}\sum_{i=1}^{n}\ln(x_i-\hat{a}) - \ln(\bar{x}-\hat{a}) \qquad \text{(A.3.11b)}$$

where $\Psi(\hat{K})$ = DIGAMMA FUNCTION.

These equations may be solved simultaneously for $\hat{a}$, $\hat{K}$ provided $\hat{K} > 2$ (Harter and Moore, 1965).

When $1 < k \leq 2$, the likelihood function is monotone decreasing with a, so that $\hat{a} = 0$. The remaining parameters (K,λ) can then be estimated by two-parameter methods.

When $K \leq 1$, the likelihood function is monotone increasing with a so that $\hat{a} = x_{(1)}$. The data on X then can be transformed by $[x-x_{(1)}]$ to two-parameter data. However, $x_{(1)}$ itself must be censored from the sample, to obtain finite terms in the two-parameter likelihood equation (A.3.2b)

The ML equations for censored sample of the variable X are somewhat complicated and must be solved numerically by iteration (Harter and Moore,1965).

A.3.4 MOMENT ESTIMATES

Moment estimators of the parameters a, λ, and K are obtained by equating sample moments to corresponding population moments. The moment estimates are, therefore,

$$\hat{a} = \bar{x} - \frac{2(M_2)^2}{M_3} \qquad \text{(A.3.12a)}$$

$$\overset{v}{\lambda} = \frac{M_3}{2M_2} \qquad \text{(A.3.12b)}$$

$$\overset{v}{K} = \frac{4(M_2)^3}{(M_3)^2} \qquad \text{(A.3.12c)}$$

where $\bar{x} = \overset{v}{a} + \overset{v}{K}/\overset{v}{\lambda} = \frac{1}{n}\sum_{i=1}^{n} x_i$

$$M_2 = \overset{v}{K}/\overset{v}{\lambda}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2$$

$$M_3 = 2\overset{v}{K}/\overset{v}{\lambda}^3 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^3$$

A.4 ESTIMATION OF BETA DISTRIBUTION PARAMETERS

A.4.1 MAXIMUM LIKELIHOOD ESTIMATES: TWO-PARAMETER CASE

The ML equations, for the shape parameters, of a sample x of n independent observations from a Beta distribution are

$$\Psi(\overset{\Lambda}{q}) - \Psi(\overset{\Lambda}{q} + \overset{\Lambda}{r}) = \ln \prod_{i=1}^{n} x_i^{1/n} = \ln G_1 \qquad \text{(A.4.1a)}$$

$$\Psi(\overset{\Lambda}{r}) - \Psi(\overset{\Lambda}{q} + \overset{\Lambda}{r}) = \ln \prod_{i=1}^{n} (1-x_i)^{1/n} = \ln G_2 \qquad \text{(A.4.1b)}$$

where $\Psi(.)$ is the tabulated DIGAMMA FUNCTION (see Abramowitz, 1964). Solution to these equations are by trial and error.

A.4.2 MOMENT ESTIMATES: TWO-PARAMETER CASE

Moment estimators for the shape parameters q and r are obtained by equating the first two Beta moments about origin to corresponding sample moments:

$$M_1' = q/(q+r) \qquad \text{(A.4.2a)}$$

$$M_2' = q(q+r)/[(q+r)(q+r+1)] \qquad \text{(A.4.2b)}$$

Solving (A.4.2a) and (A.4.2b) simultaneously gives the moment estimators:

$$\overset{v}{q} = \frac{(M_1')^2 - M_1'M_2'}{M_2' - (M_1')^2} \qquad \text{(A.4.3a)}$$

$$\overset{v}{r} = \frac{M_1' - M_2'}{M_2' - (M_2')^2} - \overset{v}{q} \qquad \text{(A.4.3b)}$$

where $M_1' = \frac{1}{n}\sum_{i=1}^{n} x_i = \bar{x}$

$$M_2' = \frac{1}{n}\sum_{i=1}^{n} x_i^2$$

NB The Kth moment of X, about origin, of a two-parameter Beta model is:

$$E(X^K) = M_K' = \frac{\Gamma(q+r)\Gamma(q+K)}{\Gamma(q)\Gamma(q+r+K)} \qquad \text{(A.4.5)}$$

The third and fourth central moments of a two-parameter Beta model are:

$$M_3 = 2qr(r-q)/[(q+r)^3(q+r+1)(q+r+2)] \qquad \text{(A.4.6)}$$

$$M_4 = \frac{3qr[2(q-r)^2 + qr(q+r+2)]}{(q+r)^4(q+r+1)(q+r+2)(q+r+3)} \qquad (A.4.7)$$

A.4.3 MAXIMUM LIKELIHOOD ESTIMATES: FOUR-PARAMETER CASE

The maximum likelihood estimates of a four-parameter Beta model are extremely difficult to derive. However, the method of moments can be employed to estimate the parameters fairly accurately.

A.4.4 MOMENT ESTIMATES: FOUR-PARAMETER CASE

In order to estimate the four parameters q,r,a, and b the first four Beta moments are equated to corresponding sample moments. The sample size n should be large if estimates with a reasonably small sampling variance are to be obtained (Bury, 1975). Thus

$$M_1' = a + \frac{q}{q+r}(b-a) \qquad (A.4.8)$$

$$M_2 = \frac{qr}{(q+r)^2(q+r+1)}(b-a)^2 \qquad (A.4.9)$$

$$M_3 = \frac{2qr(r-q)}{(q+r)^3(q+r+1)(q+r+2)}(b-a)^3 \qquad (A.4.10)$$

$$M_4 = \frac{3qr[2(q-r)^2+qr(q+r+2)]}{(q+r)^4(q+r+1)(q+r+2)(q+r+3)}(b-a)^4 \qquad (A.4.11)$$

where

$$M_1' = \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \ , \quad M_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2$$

$$M_3 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^3, \quad M_4 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^4$$

APPENDIX B

FITTING STATISTICAL DISTRIBUTIONS TO OBSERVED LINK TRAVEL TIMES

NULL HYPOTHESIS H_0, ALTERNATIVE HYPOTHESIS H_1

H_0: Distribution function of the observed random variable is Log-Normal.

H_1: Distribution function of the observed random variable is other than Log-Normal.

K-S Statistic D and Decision rule for 5% Significance level:

$D_n^{\alpha} = D_{30}^{.05} = 0.132$ From Statistical Tables.

($\alpha = 0.05$ $n = 30$)

$D_n = \max|S(X) - F(X)|$

where $S(X)$ = empirical CDF and $F(X)$ = theoretical CDF and D_n = observed maximum K-S Statistic.

REJECT H_0 if $D_n > D_n^{\alpha}$ (i.e., If $D_n > 0.132$); otherwise, DO NOT REJECT H_0.

MAXIMUM LIKELIHOOD ESTIMATES (MLE) OF LOG-NORMAL MODEL

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} \ln x_i \qquad \text{(A.2.2a)}$$

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n} (\ln x_i - \hat{\mu})^2 \qquad \text{(A.2.2b)}$$

```
C
C     ******************************************************************
C     THIS PROGRAM ANALYSES THE GOODNESS-OF-FIT OF LOG-NORMAL MODEL
C     RELATIVE TO OBSERVED BUS TRAVEL TIMES ON LINKS IN KADUNA
C     METROPOLIS.THE BUS TRANSIT SERVICE IS RUN BY KADUNA STATE
C     TRANSPORT AUTHORITY.NAG ROUTINE S15ABF IS USED TO COMPUTE
C     THE NORMAL CUMULATIVE DISTRIBUTION FUNCTION.THE PARAMETERS
C     OF THE LOG-NORMAL MODEL ARE ESTIMATED BY THE METHOD OF MAXIMUM
C     LIKELIHOOD.

C     NOTATION USED IN THIS PROGRAM:
C              AMEW    LOG-NORMAL PARAMETER MU
C              CUM     CUMULATIVE DISTRIBUTION
C              IFREQ   FREQUENCY
C              ZIGMA2  LOG-NORMAL PARAMETER SIGMA
C              D       KOLMOGOROV-SMIRNOV STATISTIC
C              F(X)    THEORETICAL CUMULATIVE DISTRIBUTION FUNCTION
C              S(X)    EMPIRICAL CUMULATIVE DISTRIBUTION FUNCTION
C              K-S     KOLMOGOROV-SMIRNOV
C              Y       NORMAL CUMULATIVE DITRIBUTION FUNCTION
C              Z       NORMAL STANDARD VALUE
C     ******************************************************************
      PROGRAM LOG(LOGDATA,OUTF,TAPE5=LOGDATA,TAPE6=OUTF)
      REAL S15ABF
      DIMENSION TIME(1020),ANLOG(1020),Z(1020),CUM(1020),IFREQ(1020),
     1AMEW(34),ZIGMA2(34),Y(1020),D(1020)
      K=1
   40 READ(5,101,END=999)TIME(K),IFREQ(K),CUM(K)
      K=K+1
      GO TO 40
  999 DO 102 I=1,K
  102 ANLOG(I)=ALOG(TIME(I))
  101 FORMAT(29X,F3.0,8X,I1,24X,F5.1/)
      J1=1
      J2=1
      DO 105 JJ=1,34
      SUM=0.0
      KOUNT1=0
```

```
116 CONTINUE
    DO 118 II=1,K
118 D(II)=ABS(CUM(II)-Y(II))
    KOUNT1=30
    KL=0
    DO 119 L1=1,K
    IF(KOUNT1.EQ.30) GO TO 123
    GO TO 124
123 WRITE(6,121)
    KL=KL+1
    WRITE(6,122)AMEW(KL),ZIGMA2(KL)
    WRITE(6,125)
    KOUNT1=0
124 KOUNT1=KOUNT1+IFREQ(L1)
    WRITE(6,120)TIME(L1),IFREQ(L1),CUM(L1),Z(L1),Y(L1),D(L1)
119 CONTINUE
120 FORMAT(15X,F5.1,4X,I1,5X,F6.4,1X,F9.4,1X,F9.4,1X,F9.4)
121 FORMAT(1H1,/////,25X,*FITTING LOG-NORMAL DISTRIBUTION TO*/26X,*OB
   1ERVED LINK TRAVEL TIMES*//28X,*PARAMETERS OF LOG-NORMAL MODEL*)
122 FORMAT(/26X,* MU    =*,F9.4,*   ;  *,*SIGMA    =*,F9.4)
125 FORMAT(/16X,*TIME  FREQ.  RELATIVE      Z        NORMAL     (K-S)*/
   115X,*(SEC.)*,8X,*CUM.FREQ.*,14X,*CDF*/31X,*S(X)*,15X,*F(X)*,
   28X,*D*/)
    STOP
    END
```

TABLE B.1.1 LINK 1
FITTING LOG-NORMAL DISTRIBUTION TO
OBSERVED LINK TRAVEL TIMES

PARAMETERS OF LOG-NORMAL MODEL

MU = 4.1539 ; SIGMA = .2112

TIME (SEC.)	FREQ.	RELATIVE CUM.FREQ. S(X)	Z	NORMAL CDF F(X)	(K-S) D
40.0	1	.0330	-2.2021	.0138	.0192
48.0	1	.0670	-1.3386	.0903	.0233
49.0	2	.1330	-1.2410	.1073	.0257
51.0	1	.1670	-1.0515	.1465	.0205
52.0	1	.2000	-.9596	.1586	.0314
54.0	1	.2330	-.7808	.2175	.0155
55.0	1	.2670	-.6939	.2439	.0231
56.0	1	.3000	-.6086	.2714	.0286
57.0	1	.3330	-.5248	.2999	.0331
59.0	2	.4000	-.3614	.3589	.0411
60.0	1	.4330	-.2818	.3890	.0440
61.0	1	.4670	-.2035	.4193	.0477
62.0	1	.5000	-.1266	.4496	.0504
63.0	1	.5330	-.0508	.4798	.0532
64.0	2	.6000	.0238	.5095	.0905
68.0	1	.6330	.3109	.6221	.0109
69.0	1	.6670	.3801	.6480	.0190
70.0	1	.7000	.4482	.6730	.0270
71.0	1	.7330	.5154	.6969	.0361
73.0	1	.7670	.6459	.7412	.0258
76.0	1	.8000	.8377	.7989	.0011
77.0	1	.8330	.8996	.8158	.0172
80.0	1	.8670	1.0806	.8601	.0069
84.0	1	.9000	1.3117	.9052	.0052
87.0	1	.9330	1.4779	.9303	.0027
93.0	1	.9670	1.7937	.9636	.0034
103.0	1	1.0000	2.2774	.9886	.0114

TABLE B.1.2 LINK 2
FITTING LOG-NORMAL DISTRIBUTION TO
OBSERVED LINK TRAVEL TIMES

PARAMETERS OF LOG-NORMAL MODEL

MU = 3.6914 ; SIGMA = .1709

TIME (SEC.)	FREQ.	RELATIVE CUM.FREQ. S(X)	Z	NORMAL CDF F(X)	(K-S) D
30.0	1	.0330	-1.6979	.0448	.0118
31.0	1	.0670	-1.5060	.0660	.0010
32.0	2	.1330	-1.3203	.0934	.0396
33.0	1	.1670	-1.1402	.1271	.0399
34.0	1	.2000	-.9656	.1671	.0329
35.0	3	.3000	-.7960	.2130	.0870
36.0	1	.3330	-.6311	.2640	.0690
37.0	1	.3670	-.4708	.3189	.0481
38.0	3	.4670	-.3148	.3765	.0905
39.0	1	.5000	-.1628	.4353	.0647
40.0	1	.5330	-.0147	.4941	.0389
41.0	2	.6000	.1298	.5516	.0484
42.0	1	.6330	.2708	.6067	.0263
43.0	1	.6670	.4085	.6585	.0085
44.0	1	.7000	.5430	.7064	.0064
46.0	1	.7330	.8030	.7890	.0560
47.0	3	.8330	.9289	.8235	.0095
48.0	1	.8670	1.0521	.8536	.0134
49.0	1	.9000	1.1727	.8795	.0205
50.0	1	.9330	1.2909	.9016	.0314
52.0	1	.9670	1.5204	.9358	.0312
61.0	1	1.0000	2.4544	.9929	.0071

TABLE B.1.3 LINK 3
FITTING LOG-NORMAL DISTRIBUTION TO
OBSERVED LINK TRAVEL TIMES

PARAMETERS OF LOG-NORMAL MODEL

MU = 3.8005 ; SIGMA = .2390

TIME (SEC.)	FREQ.	RELATIVE CUM.FREQ. S(X)	Z	NORMAL CDF F(X)	(K-S) D
30.0	1	.0330	-1.6706	.0474	.0144
32.0	1	.0670	-1.4006	.0807	.0137
34.0	3	.1670	-1.1469	.1257	.0413
35.0	1	.2000	-1.0257	.1525	.0475
37.0	1	.2330	-.7932	.2138	.0192
39.0	1	.2670	-.5729	.2834	.0164
40.0	3	.3670	-.4670	.3203	.0467
41.0	2	.4330	-.3637	.3580	.0750
42.0	1	.4670	-.2629	.3963	.0707
43.0	1	.5000	-.1644	.4347	.0653
44.0	2	.5670	-.0682	.4728	.0942
45.0	1	.6000	.0258	.5103	.0897
46.0	1	.6330	.1177	.5469	.0861
47.0	1	.6670	.2077	.5823	.0847
48.0	1	.7000	.2958	.6163	.0837
49.0	1	.7330	.3821	.6488	.0842
50.0	1	.7670	.4666	.6796	.0874
51.0	1	.8000	.5494	.7087	.0913
57.0	1	.8330	1.0148	.8449	.0119
58.0	1	.8670	1.0876	.8616	.0054
59.0	1	.9000	1.1591	.8768	.0232
67.0	1	.9330	1.6911	.9546	.0216
69.0	1	.9670	1.8141	.9652	.0018
87.0	1	1.0000	2.7839	.9973	.0027

TABLE B.1.4 LINK 4
FITTING LOG-NORMAL DISTRIBUTION TO
OBSERVED LINK TRAVEL TIMES

PARAMETERS OF LOG-NORMAL MODEL

MU = 4.0252 ; SIGMA = .1398

TIME (SEC.)	FREQ.	RELATIVE CUM.FREQ. S(X)	Z	NORMAL CDF F(X)	(K-S) D
41.0	1	.0330	-2.2284	.0129	.0201
43.0	1	.0570	-1.8879	.0295	.0375
46.0	1	.1000	-1.4456	.0799	.0201
47.0	1	.1330	-1.2518	.1053	.0277
48.0	1	.1670	-1.1012	.1354	.0316
49.0	1	.2000	-.9538	.1701	.0299
50.0	2	.2670	-.8093	.2192	.0478
51.0	1	.3000	-.6677	.2522	.0478
52.0	1	.3330	-.5289	.2985	.0345
53.0	1	.3670	-.3926	.3473	.0197
54.0	1	.4000	-.2590	.3978	.0022
55.0	1	.4330	-.1278	.4492	.0162
56.0	2	.5000	.0011	.5004	.0004
57.0	1	.5330	.1277	.5508	.0178
58.0	1	.5670	.2520	.5995	.0325
59.0	1	.6000	.3743	.6459	.0459
60.0	2	.6670	.4945	.6895	.0225
61.0	1	.7000	.6127	.7300	.0300
62.0	2	.7670	.7289	.7670	.0000
63.0	2	.8330	.8434	.8005	.0325
64.0	1	.8670	.9560	.8305	.0365
66.0	1	.9000	1.1760	.8802	.0198
69.0	1	.9330	1.4939	.9324	.0005
70.0	1	.9670	1.5958	.9448	.0222
71.0	1	1.0000	1.6982	.9553	.0447

TABLE B.1.5 LINK 5
FITTING LOG-NORMAL DISTRIBUTION TO
OBSERVED LINK TRAVEL TIMES

PARAMETERS OF LOG-NORMAL MODEL

MU = 3.9012 ; SIGMA = .2031

TIME (SEC.)	FREQ.	RELATIVE CUM.FREQ. S(X)	Z	NORMAL CDF F(X)	(K-S) D
36.0	1	.0330	-1.5643	.0589	.0259
38.0	2	.1000	-1.2981	.0971	.0029
39.0	1	.1330	-1.1703	.1210	.0120
41.0	3	.2330	-.9240	.1777	.0553
42.0	1	.2670	-.8154	.2103	.0567
44.0	1	.3000	-.5763	.2822	.0178
45.0	2	.3670	-.4657	.3207	.0463
46.0	1	.4000	-.3575	.3604	.0396
47.0	1	.4330	-.2516	.4007	.0323
48.0	1	.4670	-.1479	.4412	.0258
49.0	1	.5000	-.0464	.4815	.0185
50.0	2	.5670	.0531	.5212	.0458
51.0	3	.6670	.1506	.5598	.1072
52.0	1	.7000	.2462	.5972	.1028
53.0	1	.7330	.3400	.6331	.0999
55.0	2	.8000	.5223	.6993	.1007
57.0	2	.8670	.6982	.7575	.1095
59.0	1	.9000	.8682	.8073	.0927
72.0	1	.9330	1.8484	.9677	.0347
73.0	1	.9670	1.9153	.9723	.0053
91.0	1	1.0000	3.0015	.9987	.0013

TABLE B.1.6 LINK 6
FITTING LOG-NORMAL DISTRIBUTION TO
OBSERVED LINK TRAVEL TIMES

PARAMETERS OF LOG-NORMAL MODEL

MU = 3.7912 ; SIGMA = .1523

TIME (SEC.)	FREQ.	RELATIVE CUM.FREQ. S(X)	Z	NORMAL CDF F(X)	(K-S) D
34.0	1	.0330	-1.7392	.0410	.0080
35.0	1	.0670	-1.5488	.0607	.0063
37.0	2	.1330	-1.1838	.1182	.0148
38.0	2	.2000	-1.0087	.1566	.0434
39.0	1	.2330	-.8381	.2010	.0320
40.0	2	.3000	-.6718	.2508	.0492
41.0	4	.4330	-.5096	.3051	.1279
42.0	1	.4670	-.3514	.3626	.1044
43.0	2	.5330	-.1968	.4220	.1110
44.0	1	.5670	-.0459	.4817	.0853
45.0	1	.6000	.1017	.5405	.0595
46.0	1	.6330	.2461	.5972	.0358
47.0	1	.6670	.3873	.6507	.0163
49.0	2	.7330	.6610	.7457	.0127
50.0	1	.7670	.7937	.7863	.0193
51.0	2	.8330	.9237	.8222	.0108
53.0	1	.8670	1.1764	.8803	.0133
54.0	1	.9000	1.2991	.9031	.0031
55.0	1	.9330	1.4197	.9221	.0109
60.0	1	.9670	1.9911	.9768	.0098
61.0	1	1.0000	2.0997	.9821	.0179

TABLE B.1.7 LINK 7
FITTING LOG-NORMAL DISTRIBUTION TO
OBSERVED LINK TRAVEL TIMES

PARAMETERS OF LOG-NORMAL MODEL

MU = 4.0481 ; SIGMA = .1510

TIME (SEC.)	FREQ.	RELATIVE CUM. FREQ. S(X)	Z	NORMAL CDF F(X)	(K-S) D
46.0	2	.0670	-1.4531	.0731	.0061
48.0	2	.1330	-1.1713	.1207	.0123
49.0	2	.2000	-1.0348	.1504	.0495
50.0	1	.2330	-.9010	.1838	.0492
51.0	2	.3000	-.7699	.2207	.0793
53.0	1	.3330	-.5152	.3032	.0298
54.0	3	.4330	-.3915	.3477	.0853
55.0	1	.4670	-.2700	.3936	.0734
56.0	3	.5670	-.1507	.4401	.1269
57.0	1	.6000	-.0335	.4866	.1134
58.0	1	.6330	.0816	.5325	.1005
61.0	1	.6670	.4155	.6611	.0059
62.0	2	.7330	.5232	.6996	.0334
63.0	2	.8000	.6291	.7354	.0646
65.0	1	.8330	.8360	.7984	.0346
66.0	1	.8670	.9371	.8256	.0414
71.0	1	.9000	1.4205	.9223	.0223
76.0	1	.9330	1.8711	.9593	.0363
79.0	1	.9670	2.1274	.9833	.0163
80.0	1	1.0000	2.2167	.9865	.0135

TABLE B.1.8 LINK 8
FITTING LOG-NORMAL DISTRIBUTION TO
OBSERVED LINK TRAVEL TIMES

PARAMETERS OF LOG-NORMAL MODEL

MU = 4.1164 ; SIGMA = .1325

TIME (SEC.)	FREQ.	RELATIVE CUM. FREQ. S(X)	Z	NORMAL CDF F(X)	(K-S) D
49.0	1	.0333	-1.6951	.0450	.0121
50.0	1	.0570	-1.5426	.0615	.0055
51.0	1	.1001	-1.3932	.0818	.0182
52.0	1	.1330	-1.2466	.1063	.0267
53.0	1	.1671	-1.1128	.1351	.0319
54.0	1	.2000	-.9617	.1681	.0319
57.0	2	.2671	-.5536	.2899	.0229
58.0	2	.3330	-.4223	.3364	.0034
59.0	2	.4001	-.2933	.3847	.0153
60.0	2	.4670	-.1654	.4339	.0331
61.0	2	.5330	-.0416	.4834	.0495
62.0	2	.6000	.0811	.5323	.0677
63.0	2	.6670	.2019	.5800	.0870
64.0	1	.7000	.3208	.6258	.0742
65.0	3	.8000	.4378	.6692	.1308
66.0	1	.8330	.5530	.7099	.1231
67.0	1	.8670	.6666	.7475	.1195
71.0	1	.9001	1.1043	.8653	.0347
74.0	1	.9330	1.4167	.9217	.0113
78.0	1	.9671	1.8140	.9652	.0018
93.0	1	1.0000	3.1417	.9992	.0008

TABLE B.1.9 LINK 9
FITTING LOG-NORMAL DISTRIBUTION TO
OBSERVED LINK TRAVEL TIMES

PARAMETERS OF LOG-NORMAL MODEL

MU = 3.5811 ; SIGMA = .1729

TIME (SEC.)	FREQ.	RELATIVE CUM.FREQ. S(X)	Z	NORMAL CDF F(X)	(K-S) D
27.0	1	.0330	-1.6503	.0494	.0164
28.0	2	.1000	-1.4399	.0749	.0251
29.0	2	.1670	-1.2369	.1081	.0589
30.0	2	.2330	-1.0408	.1490	.0840
31.0	1	.2670	-.8511	.1973	.0697
32.0	1	.3000	-.6675	.2522	.0478
33.0	1	.3330	-.4895	.3122	.0208
34.0	2	.4000	-.3158	.3757	.0243
35.0	1	.4330	-.1491	.4407	.0077
36.0	3	.5330	.0139	.5055	.0275
37.0	2	.6000	.1723	.5584	.0416
38.0	2	.6670	.3266	.6280	.0390
39.0	3	.7670	.4759	.6833	.0837
40.0	1	.8000	.6233	.7335	.0665
41.0	1	.8330	.7662	.7782	.0548
42.0	1	.8670	.9056	.8174	.0496
46.0	2	.9330	1.4318	.9239	.0091
48.0	1	.9670	1.6780	.9533	.0137
56.0	1	1.0000	2.5697	.9949	.0051

TABLE B.1.10 LINK 10
FITTING LOG-NORMAL DISTRIBUTION TO
OBSERVED LINK TRAVEL TIMES

PARAMETERS OF LOG-NORMAL MODEL

MU = 4.0148 ; SIGMA = .1256

TIME (SEC.)	FREQ.	RELATIVE CUM.FREQ. S(X)	Z	NORMAL CDF F(X)	(K-S) D
45.0	2	.0670	-1.5782	.0573	.0097
46.0	1	.1000	-1.4032	.0803	.0197
47.0	1	.1330	-1.2319	.1090	.0240
48.0	2	.2000	-1.0642	.1436	.0564
49.0	1	.2330	-.9000	.1841	.0489
50.0	2	.3000	-.7391	.2299	.0701
51.0	2	.3670	-.5814	.2805	.0865
52.0	2	.4330	-.4267	.3348	.0982
53.0	1	.4670	-.2750	.3916	.0754
54.0	1	.5000	-.1262	.4498	.0502
55.0	1	.5330	.0200	.5080	.0250
56.0	3	.6330	.1635	.5649	.0681
57.0	1	.6670	.3144	.6196	.0474
59.0	1	.7000	.5791	.7187	.0187
60.0	2	.7670	.7129	.7621	.0049
61.0	1	.8000	.8446	.8008	.0008
63.0	1	.8330	1.1015	.8647	.0317
65.0	1	.8670	1.3504	.9116	.0446
66.0	1	.9000	1.4720	.9295	.0295
67.0	1	.9330	1.5917	.9443	.0113
68.0	1	.9670	1.7097	.9563	.0107
69.0	1	1.0000	1.8260	.9661	.0339

TABLE B.1.11 LINK 11
FITTING LOG-NORMAL DISTRIBUTION TO
OBSERVED LINK TRAVEL TIMES

PARAMETERS OF LOG-NORMAL MODEL

MU = 4.0222 ; SIGMA = .1423

TIME (SEC.)	FREQ.	RELATIVE CUM. FREQ. S(X)	Z	NORMAL CDF F(X)	(K-S) D
45.0	1	.0330	-1.5141	.0650	.0320
46.0	2	.1000	-1.3597	.0870	-.0130
47.0	1	.1330	-1.2186	.1134	-.0196
48.0	2	.2000	-1.0607	.1444	-.0556
49.0	1	.2330	-.9159	.1799	-.0531
50.0	2	.3000	-.7739	.2195	-.0805
51.0	2	.3670	-.6348	.2628	-.1042
52.0	1	.4000	-.4984	.3091	-.0909
53.0	1	.4330	-.3646	.3577	-.0753
54.0	2	.5000	-.2332	.4078	-.0922
55.0	2	.5670	-.1043	.4585	-.1085
56.0	1	.6000	.0223	.5089	-.0911
59.0	1	.6330	.3889	.6513	.0183
60.0	2	.7000	.5070	.6939	-.0061
61.0	1	.7330	.6231	.7334	.0004
63.0	1	.7670	.8498	.8023	.0353
64.0	1	.8000	.9604	.8316	.0316
65.0	1	.8330	1.0693	.8575	.0245
66.0	1	.8670	1.1756	.8803	.0133
67.0	1	.9000	1.2822	.9001	.0001
69.0	1	.9330	1.4889	.9317	-.0013
73.0	1	.9670	1.8848	.9703	.0033
75.0	1	1.0000	2.0747	.9810	.0190

TABLE B.1.12 LINK 12
FITTING LOG-NORMAL DISTRIBUTION TO
OBSERVED LINK TRAVEL TIMES

PARAMETERS OF LOG-NORMAL MODEL

MU = 4.1915 ; SIGMA = .3043

TIME (SEC.)	FREQ.	RELATIVE CUM.FREQ. S(X)	Z	NORMAL CDF F(X)	(K-S) D
45.0	1	.0330	-1.2651	.1029	.0699
46.0	1	.0670	-1.1929	.1165	.0495
48.0	1	.1000	-1.0531	.1462	.0462
49.0	2	.1670	-.9853	.1622	.0048
50.0	1	.2000	-.9189	.1791	.0209
51.0	1	.2330	-.8538	.1966	.0364
56.0	2	.3000	-.5465	.2924	.0076
57.0	1	.3330	-.4883	.3127	.0203
58.0	1	.3670	-.4311	.3332	.0338
59.0	1	.4000	-.3750	.3538	.0462
60.0	1	.4330	-.3197	.3746	.0584
61.0	1	.4670	-.2654	.3953	.0717
63.0	2	.5330	-.1594	.4367	.0963
64.0	2	.6000	-.1077	.4571	.1429
68.0	2	.6670	.0916	.5365	.1305
70.0	1	.7000	.1858	.5741	.1259
71.0	1	.7330	.2334	.5923	.1407
72.0	1	.7670	.2794	.6100	.1570
75.0	1	.8000	.4135	.6604	.1396
79.0	1	.8330	.5843	.7205	.1125
80.0	1	.8670	.6256	.7342	.1328
93.0	1	.9000	1.1204	.8687	.0313
113.0	1	.9330	1.7605	.9608	.0278
120.0	1	.9670	1.9580	.9749	.0079
187.0	1	1.0000	3.4158	.9997	.0003

TABLE B.1.13 LINK 13
FITTING LOG-NORMAL DISTRIBUTION TO
OBSERVED LINK TRAVEL TIMES

PARAMETERS OF LOG-NORMAL MODEL

MU = 4.3741 ; SIGMA = .2299

TIME (SEC.)	FREQ.	RELATIVE CUM.FREQ. S(X)	Z	NORMAL CDF F(X)	(K-S) D
55.0	1	.0331	-1.5952	.0553	.0223
56.0	1	.0670	-1.5158	.0647	.0023
60.0	1	.1000	-1.2167	.1119	.0119
61.0	1	.1330	-1.1448	.1261	.0069
62.0	1	.1570	-1.0741	.1414	.0156
63.0	1	.2000	-1.0045	.1576	.0424
66.0	1	.2330	-.8121	.2112	.0218
68.0	1	.2670	-.6723	.2507	.0163
69.0	1	.3000	-.6188	.2713	.0287
70.0	2	.3670	-.5462	.2925	.0745
71.0	1	.4000	-.4845	.3140	.0860
73.0	1	.4330	-.3637	.3581	.0749
76.0	1	.4670	-.1885	.4253	.0417
79.0	1	.5000	-.0201	.4920	.0080
80.0	1	.5330	.0346	.5138	.0192
81.0	1	.5670	.0887	.5353	.0317
82.0	1	.6000	.1420	.5565	.0435
83.0	1	.6330	.1948	.5772	.0558
84.0	1	.6670	.2469	.5975	.0695
85.0	1	.7000	.2983	.6173	.0827
86.0	1	.7330	.3492	.6365	.0965
87.0	1	.7570	.3995	.6552	.1118
94.0	1	.8000	.7361	.7692	.0308
95.0	1	.8330	.7821	.7829	.0501
103.0	1	.8670	1.1338	.8716	.0046
104.0	1	.9000	1.1759	.8802	.0198
114.0	1	.9330	1.5752	.9424	.0094
125.0	1	.9571	1.9759	.9759	.0089
146.0	1	1.0000	2.6514	.9960	.0040

TABLE B.1.14 LINK 14
FITTING LOG-NORMAL DISTRIBUTION TO
OBSERVED LINK TRAVEL TIMES

PARAMETERS OF LOG-NORMAL MODEL

MU = 3.3640 ; SIGMA = .2025

TIME (SEC.)	FREQ.	RELATIVE CUM. FREQ. S(X)	Z	NORMAL CDF F(X)	(K-S) D
20.0	1	.0330	-1.8184	.0345	.0015
21.0	1	.0570	-1.5774	.0573	.0003
22.0	2	.1330	-1.3477	.0889	.0441
23.0	2	.2000	-1.1282	.1296	.0704
24.0	2	.2670	-.9181	.1793	.0877
25.0	1	.3000	-.7165	.2368	.0632
26.0	3	.4000	-.5228	.3005	.0995
27.0	1	.4330	-.3365	.3683	.0647
28.0	1	.4670	-.1559	.4377	.0293
29.0	1	.5000	.0164	.5065	.0065
30.0	1	.5330	.1838	.5729	.0399
31.0	4	.6670	.3457	.6352	.0318
32.0	1	.7000	.5125	.6923	.0077
33.0	1	.7330	.6544	.7436	.0106
35.0	2	.8000	.9450	.8277	.0277
36.0	1	.8330	1.0841	.8608	.0278
37.0	2	.9000	1.2194	.8886	.0114
39.0	1	.9330	1.4793	.9305	.0025
40.0	1	.9670	1.6043	.9457	.0213
41.0	1	1.0000	1.7262	.9578	.0422

TABLE B.1.15 LINK 15
FITTING LOG-NORMAL DISTRIBUTION TO
OBSERVED LINK TRAVEL TIMES

PARAMETERS OF LOG-NORMAL MODEL

MU = 3.1645 ; SIGMA = .2210

TIME (SEC.)	FREQ.	RELATIVE CUM.FREQ. S(X)	Z	NORMAL CDF F(X)	(K-S) D
15.0	1	.0331	-2.0657	.0194	.0136
17.0	2	.1000	-1.4993	.0669	.0331
18.0	1	.1331	-1.2416	.1074	.0256
19.0	1	.1671	-.9959	.1596	.0074
20.0	3	.2671	-.7638	.2225	.0445
21.0	2	.3330	-.5430	.2936	.0394
22.0	3	.4330	-.3325	.3698	.0632
23.0	3	.5330	-.1313	.4478	.0852
24.0	3	.5330	.0513	.5244	.1085
25.0	1	.5670	.2460	.5972	.0698
26.0	1	.7000	.4235	.6640	.0360
27.0	1	.7330	.5943	.7239	.0091
28.0	1	.7671	.7589	.7760	.0090
29.0	1	.8000	.9177	.8206	.0206
30.0	1	.8331	1.0711	.8579	.0249
31.0	1	.8670	1.2195	.8887	.0217
32.0	1	.9001	1.3632	.9136	.0136
33.0	1	.9330	1.5025	.9335	.0005
35.0	1	.9671	1.7688	.9615	.0055
37.0	1	1.0000	2.0202	.9783	.0217

TABLE B.1.16 LINK 16
FITTING LOG-NORMAL DISTRIBUTION TO
OBSERVED LINK TRAVEL TIMES

PARAMETERS OF LOG-NORMAL MODEL

MU = 4.1940 ; SIGMA = .2095

TIME (SEC.)	FREQ.	RELATIVE CUM. FREQ. S(X)	Z	NORMAL CDF F(X)	(K-S) D
41.0	1	.0330	-2.2933	.0109	.0221
48.0	1	.0670	-1.5408	.0617	.0053
49.0	1	.1000	-1.4424	.0746	.0254
50.0	1	.1330	-1.3460	.0892	.0438
52.0	1	.1670	-1.1587	.1233	.0437
53.0	1	.2000	-1.0678	.1428	.0572
56.0	1	.2330	-.8150	.2104	.0226
58.0	2	.3000	-.6375	.2619	.0381
59.0	1	.3330	-.5559	.2892	.0438
63.0	2	.4000	-.2427	.4041	.0041
64.0	1	.4330	-.1675	.4335	.0005
66.0	2	.5000	-.0207	.4918	.0082
69.0	2	.5670	.1915	.5759	.0089
70.0	1	.6000	.2602	.6027	.0027
71.0	1	.6330	.3279	.6285	.0045
72.0	1	.6670	.3947	.6535	.0135
73.0	1	.7000	.4605	.6774	.0226
74.0	1	.7330	.5255	.7004	.0326
75.0	1	.7670	.5896	.7223	.0447
79.0	1	.8000	.8376	.7989	.0011
84.0	1	.8330	1.1305	.8709	.0379
85.0	2	.9000	1.1870	.8824	.0176
90.0	1	.9330	1.4599	.9278	.0052
94.0	1	.9670	1.6675	.9523	.0147
96.0	1	1.0000	1.7680	.9615	.0385

TABLE B.1.17 LINK 17
FITTING LOG-NORMAL DISTRIBUTION TO
OBSERVED LINK TRAVEL TIMES

PARAMETERS OF LOG-NORMAL MODEL

MU = 4.0699 ; SIGMA = .2439

TIME (SEC.)	FREQ.	RELATIVE CUM. FREQ. S(X)	Z	NORMAL CDF F(X)	(K-S) D
36.0	1	.0330	-1.9943	.0231	.0099
40.0	1	.0670	-1.5623	.0591	.0079
44.0	1	.1000	-1.1715	.1207	.0207
45.0	1	.1330	-1.0793	.1402	.0072
48.0	1	.1670	-.8147	.2076	.0406
49.0	1	.2000	-.7302	.2326	.0326
50.0	2	.2570	-.6473	.2587	.0083
51.0	1	.3000	-.5651	.2857	.0143
52.0	1	.3330	-.4855	.3133	.0197
53.0	1	.3570	-.4084	.3415	.0255
54.0	1	.4000	-.3318	.3700	.0300
55.0	1	.4330	-.2565	.3988	.0342
56.0	1	.4570	-.1826	.4275	.0395
57.0	1	.5000	-.1101	.4562	.0438
58.0	1	.5330	-.0387	.4845	.0485
59.0	2	.6000	.0313	.5125	.0875
60.0	1	.6330	.1003	.5399	.0931
61.0	1	.6670	.1680	.5667	.1003
62.0	1	.7000	.2347	.5928	.1072
63.0	1	.7330	.3003	.6180	.1150
65.0	1	.7570	.4285	.6658	.1012
67.0	1	.8000	.5527	.7098	.0902
68.0	1	.8330	.6135	.7302	.1028
72.0	1	.8670	.8479	.8017	.0653
82.0	1	.9000	1.3811	.9164	.0164
84.0	1	.9330	1.4799	.9306	.0024
95.0	1	.9570	1.9845	.9764	.0094
118.0	1	1.0000	2.8735	.9980	.0020

TABLE B.1.18 LINK 18
FITTING LOG-NORMAL DISTRIBUTION TO
OBSERVED LINK TRAVEL TIMES

PARAMETERS OF LOG-NORMAL MODEL

MU = 3.4273 ; SIGMA = .2390

TIME (SEC.)	FREQ.	RELATIVE CUM. FREQ. S(X)	Z	NORMAL CDF F(X)	(K-S) D
20.0	1	.0331	-1.8159	.0355	.0025
21.0	1	.0670	-1.6017	.0546	.0124
22.0	1	.1000	-1.4071	.0797	.0203
23.0	1	.1330	-1.2211	.1110	.0220
24.0	2	.2000	-1.0438	.1485	.0515
25.0	1	.2330	-.8722	.1915	.0415
26.0	2	.3000	-.7081	.2394	.0606
27.0	1	.3330	-.5502	.2911	.0419
28.0	1	.3671	-.3980	.3453	.0217
29.0	3	.4670	-.2512	.4008	.0662
30.0	1	.5000	-.1194	.4565	.0435
31.0	2	.5670	.0278	.5111	.0559
32.0	2	.6330	.1607	.5538	.0692
33.0	1	.6670	.2894	.6139	.0531
34.0	1	.7000	.4143	.6607	.0393
35.0	1	.7330	.5356	.7039	.0291
37.0	1	.7670	.7681	.7788	.0118
39.0	1	.8000	.9884	.8385	.0385
40.0	1	.8330	1.0943	.8631	.0301
41.0	1	.8670	1.1977	.8845	.0175
42.0	1	.9000	1.2985	.9029	.0029
44.0	2	.9671	1.4931	.9323	.0347
53.0	1	1.0000	2.2718	.9885	.0115

TABLE B.1.19 LINK 13
FITTING LOG-NORMAL DISTRIBUTION TO
OBSERVED LINK TRAVEL TIMES

PARAMETERS OF LOG-NORMAL MODEL

MU = 4.0365 ; SIGMA = .2031

TIME (SEC.)	FREQ.	RELATIVE CUM. FREQ. S(X)	Z	NORMAL CDF F(X)	(K-S) D
33.0	1	.0330	-2.6600	.0039	.0291
37.0	1	.0670	-2.0966	.0186	.0484
42.0	1	.1000	-1.4723	.0705	.0295
46.0	1	.1330	-1.0243	.1528	.0198
48.0	1	.1670	-.8147	.2076	.0406
50.0	1	.2000	-.6137	.2697	.0697
51.0	5	.3670	-.5162	.3029	.0641
52.0	1	.4000	-.4205	.3371	.0629
54.0	1	.4330	-.2347	.4072	.0258
55.0	1	.4670	-.1443	.4426	.0244
57.0	1	.5000	-.0316	.5126	.0126
58.0	1	.5330	.1173	.5467	.0137
60.0	1	.5670	.2842	.6119	.0449
61.0	1	.6000	.3656	.6427	.0427
62.0	2	.6670	.4457	.6721	.0051
63.0	1	.7000	.5245	.7000	.0000
64.0	1	.7330	.6021	.7264	.0066
65.0	1	.7670	.6784	.7512	.0158
67.0	1	.8000	.8277	.7961	.0039
69.0	1	.8330	.9725	.8346	.0016
70.0	1	.8670	1.0434	.8516	.0154
74.0	2	.9330	1.3171	.9061	.0269
77.0	1	.9670	1.5128	.9348	.0322
78.0	1	1.0000	1.5763	.9425	.0575

TABLE B.1.20 LINK 20
FITTING LOG-NORMAL DISTRIBUTION TO
OBSERVED LINK TRAVEL TIMES

PARAMETERS OF LOG-NORMAL MODEL

MU = 4.0211 ; SIGMA = .2495

TIME (SEC.)	FREQ.	RELATIVE CUM.FREQ. S(X)	Z	NORMAL CDF F(X)	(K-S) D
36.0	1	.0330	-1.7538	.0397	.0067
37.0	1	.0570	-1.6440	.0501	.0169
40.0	1	.1000	-1.3315	.0915	.0085
41.0	1	.1330	-1.2325	.1089	.0241
42.0	2	.2000	-1.1360	.1280	.0720
44.0	1	.2330	-.9495	.1712	.0618
46.0	1	.2570	-.7714	.2202	.0468
48.0	1	.3000	-.6008	.2740	.0260
49.0	1	.3330	-.5182	.3022	.0308
50.0	1	.3670	-.4372	.3310	.0360
51.0	1	.4000	-.3578	.3602	.0398
52.0	1	.4330	-.2800	.3897	.0433
54.0	1	.4570	-.1288	.4488	.0182
55.0	1	.5000	-.0552	.4780	.0220
57.0	1	.5330	.0879	.5350	.0020
59.0	1	.5570	.2261	.5895	.0225
61.0	1	.6000	.3597	.6405	.0405
62.0	1	.5330	.4249	.6645	.0315
63.0	1	.6570	.4890	.6876	.0206
65.0	3	.7670	.6143	.7305	.0365
66.0	1	.8000	.6755	.7503	.0497
68.0	1	.8330	.7951	.7867	.0463
69.0	1	.8570	.8535	.8033	.0637
71.0	1	.9000	.9681	.8335	.0665
75.0	1	.9330	1.1878	.8825	.0505
91.0	1	.9670	1.9628	.9752	.0082
103.0	1	1.0000	2.4592	.9930	.0070

TABLE B.1.21 LINK 21
FITTING LOG-NORMAL DISTRIBUTION TO
OBSERVED LINK TRAVEL TIMES

PARAMETERS OF LOG-NORMAL MODEL

MU = 3.9030 ; SIGMA = .1561

TIME (SEC.)	FREQ.	RELATIVE CUM.FREQ. S(X)	Z	NORMAL CDF F(X)	(K-S) D
32.0	1	.0330	-2.8227	.0025	.0305
36.0	1	.0670	-2.0462	.0204	.0465
41.0	1	.1000	-1.2131	.1125	.0125
42.0	1	.1330	-1.0588	.1448	.0118
43.0	1	.1670	-.9081	.1819	.0149
44.0	2	.2330	-.7608	.2234	.0095
45.0	2	.3000	-.6158	.2687	.0313
46.0	1	.3330	-.4760	.3170	.0160
47.0	1	.3670	-.3383	.3676	.0006
48.0	2	.4330	-.2034	.4194	.0136
49.0	1	.4670	-.0713	.4716	.0045
50.0	1	.5000	.0581	.5232	.0232
51.0	2	.5670	.1849	.5734	.0064
52.0	1	.6000	.3093	.6215	.0215
53.0	2	.6670	.4313	.6669	.0001
54.0	1	.7000	.5511	.7092	.0092
55.0	1	.7330	.6686	.7481	.0151
57.0	1	.7670	.8974	.8152	.0482
58.0	1	.8000	1.0088	.8435	.0435
59.0	3	.9000	1.1183	.8683	.0317
60.0	1	.9330	1.2260	.8899	.0431
61.0	1	.9670	1.3319	.9085	.0585
62.0	1	1.0000	1.4360	.9245	.0755

TABLE B.1.22 LINK 22
FITTING LOG-NORMAL DISTRIBUTION TO
OBSERVED LINK TRAVEL TIMES

PARAMETERS OF LOG-NORMAL MODEL

MU = 3.6581 ; SIGMA = .1484

TIME (SEC.)	FREQ.	RELATIVE CUM. FREQ. S(X)	Z	NORMAL CDF F(X)	(K-S) D
30.0	1	.0330	-1.7312	.0417	.0087
32.0	2	.1000	-1.2964	.0974	.0026
33.0	3	.2000	-1.0890	.1381	.0619
34.0	2	.2670	-.8879	.1873	.0797
35.0	1	.3000	-.6926	.2443	.0557
36.0	1	.3330	-.5028	.3076	.0254
37.0	3	.4330	-.3182	.3752	.0578
38.0	2	.5000	-.1385	.4449	.0551
39.0	2	.5670	.0366	.5146	.0524
40.0	3	.6670	.2071	.5820	.0850
41.0	2	.7330	.3735	.6456	.0874
42.0	1	.7670	.5359	.7040	.0630
44.0	1	.8000	.8493	.8021	.0021
45.0	1	.8330	1.0007	.8415	.0085
46.0	2	.9000	1.1488	.8747	.0253
47.0	1	.9330	1.2937	.9021	.0309
48.0	1	.9671	1.4356	.9244	.0426
60.0	1	1.0000	2.9391	.9384	.0616

TABLE B.1.23 LINK 23
FITTING LOG-NORMAL DISTRIBUTION TO
OBSERVED LINK TRAVEL TIMES

PARAMETERS OF LOG-NORMAL MODEL

MU = 4.6839 ; SIGMA = .0902

TIME (SEC.)	FREQ.	RELATIVE CUM.FREQ. S(X)	Z	NORMAL CDF F(X)	(K-S) D
93.0	1	.0330	-1.6771	.0468	.0138
94.0	1	.0670	-1.5584	.0596	.0074
95.0	1	.1000	-1.4411	.0748	.0252
97.0	1	.1330	-1.2102	.1131	.0199
99.0	2	.2000	-.9839	.1626	.0374
101.0	1	.2330	-.7622	.2230	.0100
102.0	2	.3000	-.6530	.2569	.0431
103.0	2	.3670	-.5448	.2929	.0741
104.0	2	.4330	-.4377	.3308	.1022
105.0	1	.4670	-.3316	.3701	.0969
106.0	1	.5000	-.2255	.4104	.0896
107.0	1	.5330	-.1224	.4513	.0817
108.0	1	.5670	-.0193	.4923	.0747
109.0	1	.6000	.0829	.5330	.0670
112.0	1	.6330	.3839	.6495	.0165
113.0	2	.7000	.4824	.6852	.0148
114.0	1	.7330	.5801	.7191	.0139
115.0	1	.7670	.6769	.7508	.0162
116.0	1	.8001	.7729	.7802	.0198
119.0	1	.8330	1.0559	.8545	.0215
121.0	1	.8670	1.2407	.8926	.0256
123.0	1	.9000	1.4224	.9226	.0226
124.0	1	.9330	1.5122	.9348	.0018
126.0	1	.9670	1.6896	.9544	.0125
132.0	1	1.0000	2.2053	.9863	.0137

TABLE B.1.24 LINK 24
FITTING LOG-NORMAL DISTRIBUTION TO
OBSERVED LINK TRAVEL TIMES

PARAMETERS OF LOG-NORMAL MODEL

MU = 4.0139 ; SIGMA = .1495

TIME (SEC.)	FREQ.	RELATIVE CUM.FREQ. S(X)	Z	NORMAL CDF F(X)	(K-S) D
40.0	2	.0670	-2.1745	.0148	.0522
44.0	1	.1000	-1.5368	.0622	.0378
46.0	1	.1330	-1.2393	.1076	.0254
48.0	2	.2000	-.9546	.1699	.0301
49.0	1	.2330	-.8166	.2071	.0259
50.0	2	.3000	-.6814	.2478	.0522
51.0	1	.3330	-.5489	.2915	.0415
52.0	1	.3670	-.4190	.3376	.0294
53.0	1	.4000	-.2916	.3853	.0147
55.0	1	.4330	-.0437	.4826	.0496
56.0	2	.5000	.0768	.5306	.0306
57.0	1	.5330	.1953	.5774	.0444
58.0	2	.6000	.3116	.6223	.0223
59.0	1	.6330	.4260	.6649	.0319
60.0	1	.6671	.5385	.7049	.0379
61.0	1	.7000	.6491	.7418	.0418
62.0	1	.7330	.7579	.7757	.0427
63.0	1	.7670	.8649	.8065	.0395
64.0	2	.8330	.9703	.8340	.0010
65.0	1	.8670	1.0741	.8586	.0084
66.0	2	.9330	1.1762	.8802	.0528
67.0	1	.9670	1.2768	.8992	.0678
71.0	1	1.0000	1.6648	.9520	.0480

TABLE B.16.25 LINK 25
FITTING LOG-NORMAL DISTRIBUTION TO
OBSERVED LINK TRAVEL TIMES

PARAMETERS OF LOG-NORMAL MODEL

MU = 3.4935 ; SIGMA = .1263

TIME (SEC.)	FREQ.	RELATIVE CUM.FREQ. S(X)	Z	NORMAL CDF F(X)	(K-S) D
26.0	1	.0330	-1.8641	.0312	.0018
27.0	1	.0670	-1.5653	.0588	.0082
28.0	3	.1670	-1.2773	.1007	.0663
29.0	2	.2330	-.9995	.1588	.0742
30.0	1	.2670	-.7311	.2324	.0345
31.0	3	.3670	-.4714	.3187	.0483
32.0	4	.5000	-.2201	.4129	.0871
33.0	2	.5670	.0236	.5094	.0576
34.0	3	.6670	.2599	.6025	.0645
35.0	2	.7330	.4894	.6877	.0453
36.0	2	.8000	.7125	.7619	.0381
38.0	1	.8330	1.1406	.8730	.0400
39.0	2	.9000	1.3462	.9109	.0109
40.0	1	.9330	1.5467	.9390	.0060
41.0	1	.9670	1.7422	.9593	.0077
42.0	1	1.0000	1.9330	.9734	.0266

TABLE B.1.25 LINK 26
FITTING LOG-NORMAL DISTRIBUTION TO
OBSERVED LINK TRAVEL TIMES

PARAMETERS OF LOG-NORMAL MODEL

MU = 4.1092 ; SIGMA = .1428

TIME (SEC.)	FREQ.	RELATIVE CUM. FREQ. S(X)	Z	NORMAL CDF F(X)	(K-S) D
46.0	1	.0330	-1.9643	.0247	.0083
50.0	1	.0670	-1.3804	.0837	.0167
51.0	1	.1000	-1.2417	.1072	.0072
52.0	1	.1330	-1.1058	.1344	.0014
53.0	1	.1670	-.9724	.1654	.0016
54.0	2	.2330	-.8415	.2000	.0330
55.0	1	.2670	-.7130	.2379	.0291
56.0	2	.3330	-.5868	.2787	.0543
57.0	2	.4000	-.4629	.3217	.0783
58.0	1	.4330	-.3411	.3665	.0665
59.0	1	.4670	-.2214	.4124	.0546
60.0	2	.5330	-.1037	.4587	.0743
61.0	2	.5000	.0121	.5048	.0952
62.0	1	.5330	.1259	.5501	.0829
63.0	1	.5670	.2380	.5940	.0730
64.0	1	.7000	.3482	.6362	.0638
65.0	1	.7330	.4558	.6761	.0569
68.0	1	.7670	.7728	.7802	.0132
69.0	1	.8000	.8750	.8092	.0092
71.0	2	.8670	1.0751	.8588	.0082
72.0	1	.9000	1.1737	.8796	.0204
73.0	1	.9330	1.2696	.8979	.0351
81.0	1	.9670	1.9978	.9771	.0101
87.0	1	1.0000	2.4982	.9938	.0062

TABLE B.1.27 LINK 27
FITTING LOG-NORMAL DISTRIBUTION TO
OBSERVED LINK TRAVEL TIMES

PARAMETERS OF LOG-NORMAL MODEL

MU = 4.0162 ; SIGMA = .1757

TIME (SEC.)	FREQ.	RELATIVE CUM. FREQ. S(X)	Z	NORMAL CDF F(X)	(K-S) D
40.0	1	.0333	-1.8628	.0312	.0018
42.0	1	.0670	-1.5851	.0565	.0105
44.0	1	.1000	-1.3283	.0934	.0065
45.0	1	.1330	-1.1924	.1166	.0164
47.0	1	.1670	-.9449	.1724	.0054
48.0	1	.2000	-.8250	.2047	.0047
49.0	2	.2670	-.7177	.2396	.0274
50.0	2	.3330	-.5927	.2767	.0563
51.0	1	.3670	-.4800	.3156	.0514
52.0	1	.4000	-.3695	.3559	.0441
53.0	2	.4670	-.2610	.3970	.0700
54.0	2	.5330	-.1546	.4386	.0944
55.0	1	.5670	-.0502	.4800	.0870
56.0	1	.6000	.0524	.5209	.0791
57.0	2	.6670	.1531	.5608	.1062
58.0	1	.7000	.2521	.5995	.1005
61.0	1	.7330	.5391	.7051	.0279
62.0	1	.7670	.6317	.7362	.0308
65.0	1	.8001	.9006	.8161	.0161
68.0	1	.8330	1.1575	.8765	.0435
69.0	1	.8670	1.2406	.8926	.0256
70.0	1	.9000	1.3225	.9070	.0070
74.0	1	.9330	1.6388	.9494	.0164
78.0	1	.9670	1.9384	.9737	.0067
80.0	1	1.0000	2.0825	.9814	.0186

TABLE B.1.28 LINK 28
FITTING LOG-NORMAL DISTRIBUTION TO
OBSERVED LINK TRAVEL TIMES

PARAMETERS OF LOG-NORMAL MODEL

MU = 4.4459 ; SIGMA = .1904

TIME (SEC.)	FREQ.	RELATIVE CUM. FREQ. S(X)	Z	NORMAL CDF F(X)	(K-S) D
61.0	1	.0330	-1.7597	.0392	.0062
64.0	1	.0670	-1.5075	.0658	.0012
67.0	1	.1000	-1.2669	.1026	.0026
69.0	1	.1330	-1.1124	.1330	.0000
71.0	2	.2000	-.9624	.1679	.0321
73.0	1	.2330	-.8165	.2071	.0259
74.0	1	.2670	-.7450	.2281	.0389
75.0	1	.3000	-.6745	.2500	.0500
76.0	1	.3330	-.6049	.2726	.0604
77.0	2	.4000	-.5363	.2959	.1041
78.0	1	.4330	-.4685	.3197	.1133
80.0	2	.5000	-.3355	.3686	.1314
83.0	1	.5330	-.1422	.4435	.0895
87.0	1	.5670	.1051	.5418	.0252
88.0	1	.6000	.1651	.5656	.0344
89.0	1	.6330	.2244	.5888	.0442
91.0	1	.6670	.3412	.6335	.0335
94.0	1	.7000	.5115	.6955	.0045
98.0	1	.7330	.7314	.7674	.0344
99.0	1	.7670	.7837	.7834	.0164
101.0	1	.8000	.8888	.8129	.0129
103.0	1	.8330	.9918	.8393	.0063
105.0	1	.8670	1.0928	.8628	.0042
111.0	1	.9000	1.3846	.9169	.0169
117.0	1	.9330	1.6611	.9517	.0187
120.0	1	.9670	1.7941	.9636	.0034
127.0	1	1.0000	2.0919	.9818	.0182

TABLE B.1.29 LINK 29
FITTING LOG-NORMAL DISTRIBUTION TO
OBSERVED LINK TRAVEL TIMES

PARAMETERS OF LOG-NORMAL MODEL

MU = 3.9327 ; SIGMA = .2045

TIME (SEC.)	FREQ.	RELATIVE CUM. FREQ. S(X)	Z	NORMAL CDF F(X)	(K-S) D
35.0	1	.0330	-1.8453	.0325	.0005
37.0	1	.0670	-1.5735	.0578	.0092
39.0	1	.1000	-1.3151	.0941	.0059
40.0	1	.1330	-1.1923	.1166	.0164
41.0	1	.1670	-1.0715	.1420	.0250
42.0	1	.2000	-.9537	.1701	.0299
43.0	1	.2330	-.8386	.2008	.0322
44.0	1	.2670	-.7262	.2339	.0331
45.0	3	.3670	-.6153	.2689	.0981
46.0	1	.4000	-.5088	.3054	.0946
48.0	1	.4330	-.3997	.3818	.0512
49.0	2	.5000	-.1999	.4208	.0792
51.0	2	.5670	-.0042	.4983	.0687
53.0	1	.6000	.1839	.5729	.0271
55.0	1	.6330	.3651	.6425	.0095
57.0	3	.7330	.5397	.7053	.0277
58.0	1	.7670	.6247	.7339	.0331
59.0	1	.8000	.7083	.7606	.0394
64.0	1	.8330	1.1061	.8657	.0327
66.0	1	.8670	1.2566	.8956	.0286
69.0	1	.9000	1.4740	.9298	.0298
71.0	1	.9330	1.6137	.9467	.0137
74.0	2	1.0000	1.8161	.9653	.0347

TABLE B.1.30 LINK 30
FITTING LOG-NORMAL DISTRIBUTION TO
OBSERVED LINK TRAVEL TIMES

PARAMETERS OF LOG-NORMAL MODEL

MU = 3.6068 ; SIGMA = .1993

TIME (SEC.)	FREQ.	RELATIVE CUM.FREQ. S(X)	Z	NORMAL CDF F(X)	(K-S) D
27.0	2	.0670	-1.5597	.0594	.0076
28.0	1	.1000	-1.3772	.0842	.0158
29.0	1	.1330	-1.2012	.1148	.0182
30.0	2	.2000	-1.0312	.1512	.0488
31.0	2	.2670	-.8667	.1931	.0739
32.0	2	.3330	-.7074	.2397	.0933
33.0	1	.3670	-.5530	.2901	.0769
34.0	1	.4000	-.4033	.3434	.0565
35.0	2	.4670	-.2579	.3982	.0688
36.0	1	.5000	-.1166	.4536	.0464
37.0	3	.6000	.0209	.5083	.0917
39.0	1	.6330	.2849	.6122	.0208
40.0	1	.6670	.4119	.6598	.0072
41.0	1	.7000	.5358	.7040	.0040
42.0	1	.7330	.6567	.7443	.0113
43.0	1	.7670	.7747	.7808	.0138
44.0	1	.8000	.8901	.8133	.0133
45.0	1	.8330	1.0028	.8420	.0090
46.0	1	.8670	1.1130	.8672	.0002
48.0	1	.9000	1.3265	.9077	.0077
50.0	1	.9330	1.5313	.9372	.0042
51.0	1	.9670	1.6306	.9485	.0185
58.0	1	1.0000	2.2758	.9886	.0114

TABLE B.1.31 LINK 31
FITTING LOG-NORMAL DISTRIBUTION TO
OBSERVED LINK TRAVEL TIMES

PARAMETERS OF LOG-NORMAL MODEL

MU = 3.8642 ; SIGMA = .2325

TIME (SEC.)	FREQ.	RELATIVE CUM.FREQ. S(X)	Z	NORMAL CDF F(X)	(K-S) D
33.0	2	.0670	-1.5815	.0569	.0101
34.0	1	.1000	-1.4531	.0731	.0269
36.0	1	.1330	-1.2173	.1137	.0193
37.0	1	.1670	-1.0895	.1380	.0290
38.0	1	.2000	-.9748	.1648	.0352
39.0	1	.2330	-.8631	.1941	.0389
41.0	2	.3000	-.6480	.2585	.0415
42.0	2	.3670	-.5443	.2931	.0739
43.0	1	.4000	-.4431	.3288	.0712
44.0	2	.4670	-.3442	.3653	.1017
46.0	1	.5000	-.1531	.4392	.0608
47.0	1	.5330	-.0606	.4759	.0571
48.0	1	.5670	.0300	.5120	.0550
49.0	1	.6000	.1187	.5472	.0528
50.0	1	.6330	.2055	.5814	.0515
51.0	1	.6670	.2907	.6144	.0525
54.0	1	.7000	.5365	.7042	.0042
55.0	1	.7330	.6155	.7309	.0021
56.0	1	.7670	.6930	.7558	.0112
60.0	2	.8330	.9897	.8388	.0058
62.0	1	.8670	1.1307	.8709	.0039
67.0	1	.9000	1.4643	.9284	.0284
69.0	1	.9330	1.5908	.9442	.0112
70.0	1	.9670	1.6527	.9508	.0162
79.0	1	1.0000	2.1729	.9851	.0149

TABLE B.1.32 LINK 32
FITTING LOG-NORMAL DISTRIBUTION TO
OBSERVED LINK TRAVEL TIMES

PARAMETERS OF LOG-NORMAL MODEL

MU = 4.0551 ; SIGMA = .2050

TIME (SEC.)	FREQ.	RELATIVE CUM.FREQ. S(X)	Z	NORMAL CDF F(X)	(K-S) D
41.0	1	.0330	-1.6664	.0478	.0148
42.0	1	.0670	-1.5489	.0607	.0063
43.0	1	.1000	-1.4340	.0758	.0242
45.0	1	.1330	-1.2122	.1127	.0203
47.0	1	.1670	-1.0061	.1586	.0084
48.0	1	.2000	-.8973	.1848	.0152
49.0	2	.2670	-.7967	.2128	.0542
50.0	1	.3000	-.6982	.2425	.0575
51.0	1	.3330	-.6015	.2737	.0593
52.0	1	.3670	-.5058	.3062	.0608
53.0	1	.4000	-.4138	.3395	.0605
54.0	2	.4670	-.3226	.3735	.0935
55.0	1	.5000	-.2331	.4078	.0922
56.0	1	.5330	-.1452	.4423	.0907
57.0	1	.5670	-.0588	.4765	.0905
58.0	1	.6000	.0250	.5104	.0896
59.0	1	.6330	.1094	.5436	.0894
61.0	1	.6670	.2721	.6072	.0598
66.0	1	.7000	.6565	.7442	.0442
68.0	1	.7330	.8021	.7888	.0558
69.0	1	.7670	.8734	.8088	.0418
70.0	1	.8000	.9436	.8273	.0273
73.0	1	.8330	1.1483	.8746	.0415
75.0	1	.8670	1.2802	.8998	.0328
77.0	1	.9000	1.4086	.9205	.0205
78.0	1	.9330	1.4716	.9294	.0036
82.0	1	.9670	1.7156	.9569	.0101
86.0	1	1.0000	1.9479	.9743	.0257

TABLE B.1.33 LINK 33
FITTING LOG-NORMAL DISTRIBUTION TO
OBSERVED LINK TRAVEL TIMES

PARAMETERS OF LOG-NORMAL MODEL

MU = 3.8937 ; SIGMA = .2016

TIME (SEC.)	FREQ.	RELATIVE CUM.FREQ. S(X)	Z	NORMAL CDF F(X)	(K-S) D
35.0	1	.0330	-1.6782	.0467	.0137
36.0	1	.0670	-1.5384	.0620	.0050
37.0	1	.1000	-1.4026	.0804	.0196
38.0	1	.1330	-1.2703	.1020	.0310
40.0	5	.3000	-1.0159	.1548	.1452
43.0	1	.3330	-.6572	.2555	.0775
44.0	2	.4000	-.5431	.2935	.1065
48.0	2	.4670	-.1116	.4556	.0114
50.0	1	.5000	.0909	.5362	.0362
51.0	2	.5670	.1891	.5750	.0080
52.0	1	.6000	.2854	.6123	.0123
53.0	1	.6330	.3799	.6480	.0150
54.0	1	.6670	.4726	.6818	.0148
55.0	1	.7000	.5636	.7135	.0135
57.0	2	.7670	.7428	.7706	.0035
59.0	1	.8000	.9118	.8191	.0191
60.0	1	.8330	.9952	.8402	.0072
61.0	1	.8670	1.0772	.8593	.0077
63.0	1	.9000	1.2372	.8920	.0080
67.0	1	.9330	1.5425	.9385	.0055
70.0	2	1.0000	1.7597	.9608	.0392

TABLE B.1.34 LINK 34
FITTING LOG-NORMAL DISTRIBUTION TO
OBSERVED LINK TRAVEL TIMES

PARAMETERS OF LOG-NORMAL MODEL

MU = 4.0864 ; SIGMA = .2895

TIME (SEC.)	FREQ.	RELATIVE CUM.FREQ. S(X)	Z	NORMAL CDF F(X)	(K-S) D
35.0	1	.0330	-1.8342	.0333	.0003
36.0	1	.0670	-1.7359	.0412	.0258
39.0	1	.1000	-1.4604	.0721	.0279
41.0	1	.1330	-1.2877	.0989	.0341
42.0	1	.1670	-1.2044	.1142	.0528
44.0	1	.2000	-1.0437	.1483	.0517
45.0	1	.2330	-.9661	.1671	.0660
47.0	1	.2670	-.8159	.2073	.0597
48.0	1	.3000	-.7432	.2287	.0713
49.0	1	.3330	-.6720	.2508	.0822
51.0	1	.3670	-.5338	.2967	.0703
56.0	1	.4000	-.2107	.4165	.0165
59.0	2	.4670	-.0315	.4878	.0208
61.0	1	.5000	.0847	.5337	.0337
63.0	1	.5330	.1951	.5777	.0447
68.0	3	.6330	.4599	.6772	.0442
70.0	1	.6670	.5611	.7123	.0453
71.0	1	.7000	.6090	.7287	.0287
72.0	1	.7330	.6573	.7445	.0115
73.0	1	.7670	.7050	.7596	.0074
74.0	2	.8330	.7520	.7740	.0590
77.0	1	.8670	.8892	.8131	.0539
80.0	1	.9000	1.0212	.8464	.0536
86.0	2	.9670	1.2710	.8981	.0689
120.0	1	1.0000	2.4217	.9923	.0077

TABLE B.2
FREQUENCY DISTRIBUTION OF DATA ON LINK TRAVEL TIMES
STADIUM-STATION (1.4[-KM)

INTERVAL (SEC.)	FREQUENCY	RELATIVE FREQUENCY	RELATIVE CUMULATIVE FREQUENCY
90.0 ≤ 95.0	2	.0670	.0670
95.0 ≤ 100.0	4	.1330	.2000
1[[.[≤ 1[5.0	7	.2330	.4330
1[5.[≤ 11[.[	5	.167[	.6000
11[.[≤ 115.[	4	.133[	.733[
115.0 ≤ 12[.[	3	.1[[[	.833[
12[.[≤ 125.[	3	.1[[[	.933[
125.0 ≤ 130.0	1	.0330	.9670
130.[≤ 135.[	1	.033[	1.0000

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C     ******************************************************************
C     THIS PROGRAM EVALUATES THE MEAN AND VARIANCE OF LINK TRAVEL
C     TIMES USING THE MAXIMUM LIKELIHOOD ESTIMATES OF THE LOG-NORMAL
C     MODEL AND THE REGRESSION MODELS DEVELOPED FOR THE PARAMETERS OF
C     LOG-NORMAL DISTRIBUTION MODEL.

C     NOTATION USED IN THIS PROGRAM:
C               AMEW   LOG-NORMAL MODEL PARAMETER MU
C               ZIG    LOG-NORMAL MODEL PARAMETER SIGMA
C               CUM    CUMULATIVE DISTRIBUTION
C               IFREQ  FREQUENCY
C               ALEN   LINK LENGTH (KM.)
C               ANRBT  NUMBER OF ROUNDABOUTS ALONG A LINK
C               PMEW1  PREDICTED VALUE OF PARAMETER MU:MODEL(5.3B)
C               PMEW2  PREDICTED VALUE OF PARAMETER MU:MODEL(5.4B)
C               PMEW3  PREDICTED VALUE OF PARAMETER MU:MODEL(5.5B)
C               PZIG5  PREDICTED VALUE OF PARAMETER SIGMA:MODEL(5.6)
C               PZIG1  PREDICTED VALUE OF SIGMA:MODEL(5.3B) AND
C                      EQUATION(A.2.2B)
C               PZIG2  PREDICTED VALUE OF SIGMA:MODEL(5.4B) AND
C                      EQUATION(A.2.2B)
C               PZIG3  PREDICTED VALUE OF SIGMA:MODEL(5.5B) AND
C                      EQUATION(A.2.2B)
C               PM2    PREDICTED MEAN: MODEL(5.4B),EQUATION(A.2.2B)
C               PM3    PREDICTED MEAN: MODEL(5.5B),EQUATION(A.2.2B)
C               PM25   PREDICTED MEAN: MODELS(5.4B) AND (5.6)
C               PM35   PREDICTED MEAN: MODELS(5.5B) AND (5.6)
C               PVAR2  PREDICTED VARIANCE:MODEL(5.4B),EQUATION(A.2.2B)
C               PVAR3  PREDICTED VARIANCE:MODEL(5.5B),EQUATION(A.2.2B)
C               PVAR25 PREDICTED VARIANCE:MODELS(5.4B) AND (5.6)
C               PVAR35 PREDICTED VARIANCE:MODELS(5.5B) AND (5.6)
C               OM     OBSERVED MEAN
C               OVAR   OBSERVED VARIANCE
C     ******************************************************************
      PROGRAM MIT(LOGDATA,OUTF,PDATA,PPDATA,TAPE5=LOGDATA,TAPE6=OUTF
     1,TAPE7=PDATA,TAPE8=PPDATA)
      DIMENSION AMEW(29),ZIG(29),PMEW1(29),PZIG1(29),ALEN(29),ANRBT(29
     1,TIME(870),IFREQ(870),ANLOG(870),PZIG2(29),PMEW2(29),PZIG3(29)
     2, PZIG5(29),OM(29),OVAR(29),OSD(29),PM1(29)
     3,PVAR1(29),PSD1(29),PMEW3(29),EM(29),EVAR(29)
     4,PM2(29),PVAR2(29),PM3(29),PVAR3(29)
     5,PM25(29),PVAR25(29),PM35(29),PVAR35(29)
      K=1
   90 READ(5,100,END=999) TIME(K),IFREQ(K)
      K=K+1
      GO TO 90
  999 DO 500 I=1,K
  500 ANLOG(I)=ALOG(TIME(I))
  100 FORMAT(29X,F3.0,8X,I1/)
      J1=1
      J2=1
      J3=1
      DO 600 JJ=1,29
      READ(7,95)AMEW(JJ),ZIG(JJ),ALEN(JJ),ANRBT(JJ)
      PMEW1(JJ)=3.3656+1.0139*ALEN(JJ)+0.1842*ANRBT(JJ)
      PMEW2(JJ)=4.3638+0.6498*ALOG(ALEN(JJ))+0.1987*ANRBT(JJ)
      PMEW3(JJ)=ALOG(20.5480+59.4695*ALEN(JJ)+10.2664*ANRBT(JJ))
      PZIG5(JJ)=SQRT(1/(71.0694*ALEN(JJ)))
      SUM1=0.0
      SUM2=0.0
      SUM3=0.0
      KOUNT1=0
      KOUNT2=0
```

```
      KOUNT3=0
601   SUM1=SUM1+((ANLOG(J1)-PMEW1(JJ))**2)*IFREQ(J1)
      KOUNT1=KOUNT1+IFREQ(J1)
      J1=J1+1
      IF(KOUNT1.LT.30) GO TO 601
602   SUM2=SUM2+((ANLOG(J2)-PMEW2(JJ))**2)*IFREQ(J2)
      KOUNT2=KOUNT2+IFREQ(J2)
      J2=J2+1
      IF(KOUNT2.LT.30) GO TO 602
603   SUM3=SUM3+((ANLOG(J3)-PMEW3(JJ))**2)*IFREQ(J3)
      KOUNT3=KOUNT3+IFREQ(J3)
      J3=J3+1
      IF(KOUNT3.LT.30) GO TO 603
      PZIG1(JJ)=SQRT(SUM1/FLOAT(KOUNT1))
      PZIG2(JJ)=SQRT(SUM2/FLOAT(KOUNT2))
      PZIG3(JJ)=SQRT(SUM3/FLOAT(KOUNT3))
600   CONTINUE
 95   FORMAT(2F7.5,F4.2,F1.0)
      WRITE(6,80)
 80   FORMAT(1H1,/////24X,*LOG-NORMAL MODEL PARAMETERS FOR LINKS *,
     1*STUDIED*//36X,*MAXIMUM*,12X,*PREDICTED*/36X,*LIKELIHOOD*,9X,
     2*VALUES*/24X,* LINK*,7X,*ESTIMATES*/24X,* NUMBER *,1X,*MU *,6X,
     3*SIGMA*,5X,*MU *,6X,*SIGMA*,2X,*SIGMA(M)*/)
      DO 700 KK=1,29
      WRITE(6,60)KK,AMEW(KK),ZIG(KK),PMEW1(KK),PZIG1(KK),PZIG5(KK)
700   CONTINUE
      WRITE(6,80)
      DO 702 K=1,29
      WRITE(6,60)K,AMEW(K),ZIG(K),PMEW2(K),PZIG2(K),PZIG5(K)
702   CONTINUE
      WRITE(6,80)
      DO 703 K=1,29
      WRITE(6,60)K,AMEW(K),ZIG(K),PMEW3(K),PZIG3(K),PZIG5(K)
703   CONTINUE
 60   FORMAT(27X,I2,4X,F6.4,2X,F5.4,5X,F6.4,2X,F6.4,2X,F6.4)
      DO 300 LL=1,29
      EM(LL)=EXP(AMEW(LL)+0.5*(ZIG(LL)**2))
      EVAR(LL)=(EM(LL)**2)*(EXP(ZIG(LL)**2)-1)
      PM2(LL)=EXP(PMEW2(LL)+0.5*(PZIG2(LL)**2))
      PVAR2(LL)=(PM2(LL)**2)*(EXP(PZIG2(LL)**2)-1)
      PM3(LL)=EXP(PMEW3(LL)+0.5*(PZIG3(LL)**2))
      PVAR3(LL)=(PM3(LL)**2)*(EXP(PZIG3(LL)**2)-1)
      PM25(LL)=EXP(PMEW2(LL)+0.5*(PZIG5(LL)**2))
      PVAR25(LL)=(PM25(LL)**2)*(EXP(PZIG5(LL)**2)-1)
      PM35(LL)=EXP(PMEW3(LL)+0.5*(PZIG5(LL)**2))
      PVAR35(LL)=(PM35(LL)**2)*(EXP(PZIG5(LL)**2)-1)
300   CONTINUE
      READ(8,50)(OM(I),OVAR(I),OSD(I),I=1,29)
 50   FORMAT(F6.2,F7.3,F6.3)
 40   FORMAT(1H1,/////26X,*OBSERVED ESTIMATED AND PREDICTED LINK *,
     1*TRAVEL TIMES*//31X,*OBSERVED*,12X,*ESTIMATED*,11X,*PREDICTED*
     2,//20X,*LINK*,3(4X,*MEAN*,4X,*VARIANCE*)/20X,*NUMBER*,2X,
     3*(SEC.)*,2X,*(SEC.SEC)*,2(3X,*(SEC.)*,2X,*(SEC.SEC)*)/)
      WRITE(6,40)
      DO 202 J=1,29
      WRITE(6,30)J,OM(J),OVAR(J),EM(J),EVAR(J),PM2(J),PVAR2(J)
202   CONTINUE
      WRITE(6,40)
      DO 203 J=1,29
      WRITE(6,30)J,OM(J),OVAR(J),EM(J),EVAR(J),PM3(J),PVAR3(J)
203   CONTINUE
      WRITE(6,40)
      DO 206 J=1,29
      WRITE(6,30)J,OM(J),OVAR(J),EM(J),EVAR(J),PM25(J),PVAR25(J)
```

```
206   CONTINUE
      WRITE(6,40)
      DO 207 J=1,29
      WRITE(6,30)J,OM(J),OVAR(J),EM(J),EVAR(J),PM35(J),PVAR35(J)
207   CONTINUE
 30   FORMAT(22X,I2,4X,F6.2,2X,F5.2,6X,F6.2,2X,F6.2,6X,F6.2
     1,2X,F7.2)
      STOP
      END
```

TABLE 8.3.1
LOG-NORMAL MODEL PARAMETERS FOR LINKS STUDIED

LINK NUMBER	MAXIMUM LIKELIHOOD ESTIMATES MU	SIGMA	PREDICTED VALUES MU'	SIGMA"	SIGMA(M)""
1	4.1539	.2111	3.8726	.3517	.1678
2	3.6914	.1709	3.7712	.1886	.1875
3	3.8[illegible]5	.239[illegible]	3.8726	.2496	.1678
4	4.0252	.1398	3.9739	.1489	.1532
5	3.9[illegible]12	.2031	3.9739	.2157	.1532
6	3.7912	.1523	3.9739	.2379	.1532
7	4.[illegible]481	.151[illegible]	3.9739	.1683	.1532
8	4.1164	.1325	4.1260	.1328	.1370
9	3.5811	.1729	3.7712	.2569	.1875
10	4.0048	.1256	4.1581	.1982	.1532
11	4.[illegible]0222	.1423	4.[illegible]753	.1519	.1418
12	4.1917	.3043	4.0567	.3329	.1678
13	4.3741	.2299	4.2[illegible]88	.2831	.1472
14	3.3640	.2025	3.6698	.3668	.2165
15	3.1645	.221[illegible]	3.5684	.4604	.2653
16	4.194[illegible]	.2095	4.0061	.2814	.1769
17	4.0699	.2439	3.7712	.3856	.1875
18	3.4273	.239[illegible]	3.6698	.3404	.2165
19	4.[illegible]366	.2031	3.9554	.2187	.1875
20	4.[illegible]211	.2495	3.9554	.258[illegible]	.1875
21	3.9030	.1561	3.7712	.2043	.1875
22	3.6581	.1484	3.854[illegible]	.2457	.2165
23	4.6839	.0902	4.9693	.2993	.1003
24	4.[illegible]139	.1495	3.8726	.2[illegible]57	.1678
25	3.4935	.1263	3.7205	.2597	.2005
26	4.1091	.1428	4.[illegible]753	.1468	.1418
27	4.0162	.1757	3.7712	.3015	.1875
28	4.4459	.1904	4.126[illegible]	.3723	.1371
29	3.9327	.2045	3.6698	.3331	.2165

' DERIVED FROM MODEL(5.38)

" DERIVED FROM EQUATION(A.2.28)

"" DERIVED FROM MODEL(5.6)

TABLE B.3.2
LOGENORMAL MODEL PARAMETERS FOR LINKS STUDIED

LINK NUMBER	MAXIMUM LIKELIHOOD ESTIMATES MU	SIGMA	PREDICTED VALUES MU'	SIGMA"	SIGMA(M)""
1	4.1539	.2111	3.9134	.3200	.1678
2	3.6914	.1709	3.7684	.1875	.1875
3	3.8005	.2390	3.9134	.2643	.1678
4	4.0252	.1398	4.0319	.1400	.1532
5	3.9012	.2031	4.0319	.2415	.1532
6	3.7912	.1523	4.0319	.2848	.1532
7	4.0481	.1510	4.0319	.1519	.1532
8	4.1164	.1325	4.1769	.1456	.1378
9	3.5811	.1729	3.7684	.2549	.1875
10	4.0048	.1256	4.2306	.2583	.1532
11	4.0222	.1423	4.1320	.1798	.1418
12	4.1917	.3043	4.1121	.3145	.1678
13	4.3741	.2299	4.2826	.2474	.1472
14	3.3640	.2025	3.5815	.2972	.2165
15	3.1645	.2210	3.3180	.2690	.2653
16	4.1940	.2095	4.0436	.2579	.1763
17	4.0699	.2439	3.7684	.3878	.1875
18	3.4273	.2390	3.5815	.2844	.2165
19	4.0366	.2031	3.9671	.2146	.1875
20	4.0211	.2495	3.9671	.2553	.1875
21	3.9030	.1561	3.7684	.2361	.1875
22	3.6581	.1484	3.7802	.1921	.2165
23	4.6839	.0902	4.7811	.1327	.1083
24	4.0139	.1495	3.9134	.1801	.1678
25	3.4935	.1263	3.6816	.2265	.2005
26	4.1091	.1428	4.1320	.1446	.1418
27	4.0162	.1757	3.7684	.3037	.1875
28	4.4459	.1914	4.1759	.3295	.1373
29	3.9327	.2045	3.5815	.4064	.2165

' DERIVED FROM MODEL(5.4B)

" DERIVED FROM EQUATION(A.2.2B)

"" DERIVED FROM MODEL(5.6)

TABLE B.3.3
LOG-NORMAL MODEL PARAMETERS FOR LINKS STUDIED

LINK NUMBER	MAXIMUM LIKELIHOOD ESTIMATES MU	SIGMA	PREDICTED VALUES MU '	SIGMA"	SIGMA(M)""
1	4.1539	.2111	3.9177	.3168	.1678
2	3.6914	.1709	3.7918	.1982	.1875
3	3.8005	.2390	3.9177	.2662	.1678
4	4.0252	.1398	4.0294	.1399	.1532
5	3.9012	.2031	4.0294	.2402	.1532
6	3.7912	.1523	4.0294	.2828	.1532
7	4.0481	.1510	4.0294	.1522	.1532
8	4.1164	.1325	4.1767	.1456	.1378
9	3.5811	.1729	3.7918	.2725	.1875
10	4.0048	.1256	4.1971	.2297	.1532
11	4.0222	.1423	4.1300	.1786	.1418
12	4.1917	.3043	4.1035	.3168	.1678
13	4.3741	.2299	4.2409	.2657	.1472
14	3.3640	.2025	3.6478	.3486	.2165
15	3.1645	.2210	3.4795	.3847	.2653
16	4.1940	.2095	4.0531	.2525	.1769
17	4.0699	.2439	3.7918	.3699	.1875
18	3.4273	.2390	3.6478	.3251	.2165
19	4.0366	.2031	4.0001	.2063	.1875
20	4.0211	.2495	4.0001	.2504	.1875
21	3.9030	.1561	3.7918	.1915	.1875
22	3.6581	.1484	3.8848	.2709	.2165
23	4.6839	.0902	4.7368	.1046	.1003
24	4.0139	.1495	3.9177	.1777	.1678
25	3.4935	.1263	3.7224	.2614	.2005
26	4.1091	.1428	4.1300	.1443	.1418
27	4.0162	.1757	3.7918	.2850	.1875
28	4.4459	.1914	4.1767	.3297	.1378
29	3.9327	.2045	3.6478	.3507	.2165

' DERIVED FROM MODEL(5.58)

" DERIVED FROM EQUATION(A.2.28)

"" DERIVED FROM MODEL(5.6)

TABLE B.4.1
OBSERVED ESTIMATED AND PREDICTED LINK TRAVEL TIMES

LINK NUMBER	OBSERVED MEAN (SEC.)	OBSERVED VARIANCE (SEC.SEC)	ESTIMATED" MEAN (SEC.)	ESTIMATED" VARIANCE (SEC.SEC)	PREDICTED"" MEAN (SEC.)	PREDICTED"" VARIANCE (SEC.SEC)
1	65.13	206.81	65.11	193.31	52.78	299.48
2	40.70	52.98	40.69	49.08	44.08	69.49
3	46.10	154.02	46.12	124.51	51.85	194.56
4	56.53	62.12	56.54	63.13	56.92	64.13
5	50.57	136.46	50.43	107.37	58.03	202.23
6	44.83	50.83	44.82	47.13	58.70	291.14
7	57.97	87.21	57.95	77.48	57.02	75.91
8	61.90	79.47	61.88	67.79	65.85	92.96
9	36.47	44.46	36.45	40.31	44.74	134.33
10	55.31	51.18	55.32	48.59	71.09	348.70
11	56.40	70.04	56.39	65.09	63.32	131.73
12	69.83	813.11	59.27	465.52	64.17	428.24
13	81.60	431.35	81.49	360.43	74.68	352.10
14	29.50	36.52	29.50	36.44	37.55	130.16
15	24.27	30.82	24.26	29.46	28.62	61.50
16	67.73	201.10	67.75	215.97	58.95	239.01
17	60.43	280.94	60.32	222.96	46.69	353.79
18	31.70	63.05	31.69	59.02	37.41	117.88
19	57.77	127.91	57.81	140.69	54.05	137.78
20	57.57	234.32	57.53	212.59	54.58	200.63
21	50.13	57.09	51.16	62.06	44.24	84.93
22	39.23	39.29	39.22	34.26	44.64	74.94
23	108.63	102.24	106.63	96.40	120.23	256.90
24	55.97	67.41	55.98	70.80	50.89	85.36
25	33.17	18.49	33.17	17.69	40.74	87.44
26	61.53	86.10	61.52	77.98	62.95	83.79
27	56.37	108.17	56.35	99.54	45.35	198.80
28	86.87	301.43	86.84	278.35	68.80	543.15
29	52.13	122.05	52.12	116.01	39.02	273.45

" USING MAXIMUM LIKELIHOOD ESTIMATES OF LOG-NORMAL MODEL AND EQUATIONS(3.12),&(3.13)

"" USING MODEL(5.4B) AND EQUATIONS(A.2.2B),(3.12),&(3.13)

TABLE B.4.2
OBSERVED ESTIMATED AND PREDICTED LINK TRAVEL TIMES

LINK NUMBER	OBSERVED MEAN (SEC.)	OBSERVED VARIANCE (SEC.SEC)	ESTIMATED" MEAN (SEC.)	ESTIMATED" VARIANCE (SEC.SEC)	PREDICTED"" MEAN (SEC.)	PREDICTED"" VARIANCE (SEC.SEC)
1	65.13	206.81	65.11	193.31	52.87	295.14
2	45.70	52.98	40.69	49.08	45.22	81.93
3	46.10	154.02	46.02	124.51	52.10	199.28
4	56.53	62.12	56.54	63.13	56.78	63.73
5	50.57	136.46	50.49	107.37	57.88	198.91
6	44.83	50.83	44.82	47.13	58.52	285.10
7	57.97	87.21	57.95	77.48	56.88	75.83
8	51.90	79.47	51.88	67.79	65.84	92.83
9	36.47	44.46	36.45	40.31	46.01	163.22
10	55.30	51.18	55.32	48.59	68.27	252.49
11	56.40	70.04	56.39	65.09	63.18	129.29
12	69.83	803.11	59.27	465.52	63.67	428.01
13	81.60	431.35	81.49	360.43	71.97	378.79
14	29.50	36.60	29.50	36.44	40.79	215.08
15	24.27	30.82	24.25	29.46	34.93	194.59
16	67.73	201.10	67.75	205.97	59.44	232.50
17	60.43	280.94	60.32	222.96	47.47	330.46
18	31.70	63.05	31.69	59.02	40.47	182.64
19	57.77	127.91	57.81	140.69	55.78	135.29
20	57.57	234.32	57.53	212.59	56.34	205.41
21	50.13	57.09	50.16	62.06	45.15	76.29
22	39.23	39.29	39.22	34.26	50.47	193.99
23	108.53	102.24	108.63	95.40	114.71	144.72
24	55.97	67.41	55.98	70.80	51.08	83.76
25	33.17	18.49	33.17	17.69	42.80	129.52
26	61.53	86.10	61.52	77.98	62.83	83.07
27	56.37	108.17	56.35	99.54	46.17	180.35
28	86.87	301.43	86.84	278.35	68.79	543.50
29	52.13	122.05	52.12	116.01	40.82	218.13

" USING MAXIMUM LIKELIHOOD ESTIMATES OF LOG-NORMAL MODEL AND EQUATIONS(3.12),&(3.13)

"" USING MODEL(5.5B) AND EQUATIONS(A.2.28),(3.12),&(3.13)

TABLE B.4.3
OBSERVED ESTIMATED AND PREDICTED LINK TRAVEL TIMES

LINK NUMBER	OBSERVED MEAN (SEC.)	OBSERVED VARIANCE (SEC.SEC)	ESTIMATED" MEAN (SEC.)	ESTIMATED" VARIANCE (SEC.SEC)	PREDICTED"" MEAN (SEC.)	PREDICTED"" VARIANCE (SEC.SEC)
1	65.13	206.81	65.11	193.31	50.78	73.62
2	40.70	52.98	40.69	49.08	44.08	69.59
3	46.10	154.02	46.02	124.51	50.78	73.62
4	56.53	62.12	56.54	63.13	57.03	77.20
5	50.57	136.46	50.49	107.37	57.03	77.20
6	44.83	50.83	44.82	47.13	57.03	77.20
7	57.97	87.21	57.95	77.48	57.03	77.20
8	51.90	79.47	51.88	67.79	65.78	81.96
9	36.47	44.46	36.45	40.31	44.08	69.59
10	55.30	51.18	55.30	48.59	69.57	114.87
11	56.40	70.04	56.33	65.09	62.93	80.45
12	69.83	803.11	69.27	465.52	61.94	109.54
13	81.60	431.35	81.49	360.43	73.22	117.34
14	29.50	36.60	29.50	36.44	36.78	64.98
15	24.27	30.82	24.26	29.46	28.59	59.51
16	67.73	201.10	67.76	205.97	57.93	106.63
17	60.43	280.94	60.32	222.96	44.08	69.59
18	31.70	63.05	31.69	59.02	36.78	64.98
19	57.77	127.91	57.81	140.69	53.77	103.54
20	57.57	234.32	57.53	212.59	53.77	103.54
21	50.13	57.09	50.15	62.06	44.08	69.59
22	39.23	39.29	39.22	34.26	44.85	96.68
23	118.63	102.24	108.63	96.40	119.84	145.12
24	55.97	67.41	55.98	70.80	50.78	73.62
25	33.17	18.49	33.17	17.69	48.52	67.36
26	61.53	86.10	61.52	77.98	62.93	80.45
27	56.37	108.17	56.35	99.54	44.08	69.59
28	86.87	301.43	86.84	278.35	65.78	81.96
29	52.13	122.05	52.12	116.61	36.78	64.98

" USING MAXIMUM LIKELIHOOD ESTIMATES OF LOG-NORMAL MODEL AND EQUATIONS(3.12),&(3.13)

"" USING MODELS(5.48),&(5.6) AND EQUATIONS(3.12),&(3.13)

TABLE B.4.4
OBSERVED ESTIMATED AND PREDICTED LINK TRAVEL TIMES

	OBSERVED		ESTIMATED"		PREDICTED""	
LINK NUMBER	MEAN (SEC.)	VARIANCE (SEC.SEC)	MEAN (SEC.)	VARIANCE (SEC.SEC)	MEAN (SEC.)	VARIANCE (SEC.SEC)
1	65.13	206.81	65.11	193.31	51.00	74.25
2	40.71	52.98	40.69	49.08	45.12	72.92
3	46.10	154.02	46.02	124.51	51.00	74.25
4	56.53	62.12	56.54	63.13	56.89	76.83
5	50.57	136.46	50.49	107.37	56.89	76.83
6	44.83	50.83	44.82	47.13	56.89	76.83
7	57.97	87.21	57.95	77.48	56.89	76.83
8	61.90	79.47	61.88	67.79	65.75	81.93
9	36.47	44.46	36.45	40.31	45.12	72.92
10	55.30	51.18	55.30	48.59	67.28	107.45
11	56.40	70.04	56.39	65.09	62.80	80.12
12	69.83	803.11	59.27	465.52	61.41	107.66
13	81.60	431.35	81.49	360.43	70.23	107.96
14	29.50	36.60	29.50	36.44	39.30	74.19
15	24.27	30.82	24.25	29.46	33.60	82.33
16	67.73	201.10	67.75	205.97	58.48	108.67
17	60.43	280.94	60.32	222.96	45.12	72.92
18	31.70	63.05	31.69	59.02	39.30	74.19
19	57.77	127.91	57.81	140.69	55.57	110.60
20	57.57	234.32	57.53	212.59	55.57	110.60
21	50.13	57.09	50.16	62.06	45.12	72.92
22	39.23	39.29	39.22	34.26	49.81	119.18
23	108.63	102.24	108.63	96.40	114.65	132.81
24	55.97	67.41	55.98	70.80	51.00	74.25
25	33.17	18.49	33.17	17.69	42.20	73.08
26	61.53	86.10	61.52	77.98	62.80	80.12
27	56.37	108.17	56.35	99.54	45.12	72.92
28	86.87	301.43	86.84	278.35	65.75	81.93
29	52.13	122.05	52.12	116.01	39.30	74.19

" USING MAXIMUM LIKELIHOOD ESTIMATES OF LOG-NORMAL MODEL AND EQUATIONS(3.12),&(3.13)

"" USING MODELS(5.5B),&(5.6) AND EQUATIONS(3.12),&(3.13)

TABLE B.5

LINKS ON ROUTE ONE

LINK NUMBER	LINK NAME
1.	KAWO/ AREWA
2.	AREWA/ST. PETERS
3.	ST.PETERS/NEW BARRACK
4.	NEW BARRACK/JAMA'ATUL ISLAM
5.	JAMA'ATUL ISLAM/NIGERIAN DEFENCE ACADEMY
6.	NIGERIAN DEFENCE ACADEMY/UNGWAN KANAWA
7.	UNGWAN KANAWA/UNGWAN SARIKI
8.	UNGWAN SARIKI/UNGWAN SHANU
9.	UNGWAN SHANU/SCOA
10.	SCOA/ABAKPA
11.	ABAKPA/UAC
12.	UAC/KINGSWAY
13.	KINGSWAY/10 STOREYS
14.	10 STOREYS/LENNARDS
15.	LENNARDS/BATA
16.	BATA/LEVENTIS
17.	LEVENTIS/KANO ROAD
18.	KANO ROAD/KATSINA ROAD
19.	KATSINA ROAD/MAGAJIN GARI
20.	MAGAJIN GARI/KAZAURE ROAD
21.	KAZAURE ROAD/SARDAUNA CRESCENT
22.	SARDAUNA CRESCENT/STADIUM

23. STADIUM/STATION

24. STATION/AGIP

25. AGIP/IGL

26. IGL/AFTER CORNER

27. AFTER CORNER/BREWERY

28. BREWERY/UNITED TEXTILE

29. UNITED TEXTILE/BIBLE SOCIETY

30. BIBLE SOCIETY/GENERAL PUMP

31. GENERAL PUMP/AREWA

32. AREWA/AIR FORCE

33. AIR FORCE/KAKURI

34. KAKURI/AFTER RAIL.

TABLE B.6

LINK ATTRIBUTES

LINK NUMBER	LINK LENGTH (KM)	NUMBER OF ROUNDABOUTS	NUMBER OF INTERSECTIONS
1	.50	0	0
2	.40	0	0
3	.50	0	0
4	.60	0	0
5	.60	0	1.
6	.60	0	1.
7	.60	0	1.
8	.75	0	1.
9	.40	0	1.
10	.60	1.	1.
11	.70	0	1.
12	.50	1.	2.
13	.85	1.	1.
14	.30	0	0
15	.20	0	0
16	.45	1.	2.
17	.40	0	2.
18	.30	0	2.
19	.40	1.	2.
20	.40	1.	2.
21	.40	0	1.
22	.30	1.	1.
23	1.40	1.	1.
24	.50	0	0
25	.35	0	0
26	.70	0	1.
27	.40	0	0
28	.75	0	0
29	.30	0	1

APPENDIX C

C.1 ANALYSIS OF STOP DWELL TIMES

DEAD TIMES

Null Hypothesis H_o, Alternative Hypothesis H_1

H_o: Empirical data and Exponential (or Gamma) distribution are not statistically different.

H_1: Empirical data and Exponential (or Gamma) distribution are statistically different.

K - S Statistic D and Decision rule for $\alpha = 0.05$

D_n^α $D_{60}^{.05} = 0.176$ From Statistical Tables ($\alpha = 0.05$, n=60)

$D_{106}^{.05} = 0.132$ From Statistical Tables ($\alpha = 0.05$, n=106)

$D_n = \max|S(X) - F(X)|$

REJECT H_o if $D_n > D_n^\alpha$; otherwise, DO NOT REJECT H_o

Exponential Model

$$\hat{\lambda} = 1/\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right) \qquad \text{(C.1.1)}$$

Cumulative Distribution Function (CDF):

$$F(X) = 1 - e^{-\lambda x} \qquad \text{(C.1.2)}$$

For example, the Exponential model parameter for Pre-boarding dead times is:

$\hat{\lambda} = 1/10.477 = 0.09545/sec$

Gamma Model

The maximum likelihood estimates of the parameters of Gamma model are thus

$$\hat{K}/\hat{\lambda} = \bar{x}$$

$$\ln(\bar{x}/G) = \ln\hat{K} - \psi(\hat{K}) = g(\hat{K})$$

where $\bar{x} = \sum_{i=1}^{n} x_i/n$, $G = \prod_{i=1}^{n} x_i^{1/n}$

For Pre-boarding dead times,

$\bar{x} = 10.477$, $G = 7.5031$

$\hat{K}/\hat{\lambda} = 10.477 = \bar{x}$

$\ln(\bar{x}/G) = 0.3339 = \ln\hat{K} - \psi(\hat{K})$

By inverse interpolation from tables of Gamma and Diagamma functions:

$\hat{K} = 1.7929$; $\hat{\lambda} = \hat{K}/\bar{x} = 0.1711$/sec

From Incomplete Gamma Function Tables:

$$F_G(x;K,\lambda) = F_{\chi^2}(2\lambda x, \nu = 2K)$$

$$F_G(x; 1.7929, 0.1711) = F_{\chi^2}(0.3422x, 3.5858)$$

Thus, using Incomplete Gamma Function Tables, the Gamma CDF is evaluated.

C.2 CONFIDENCE INTERVAL ON PARAMETER $\hat{\lambda}$ OF EXPONENTIAL MODEL

The $(100 - \alpha)\%$ Confidence interval on the scale parameter $\hat{\lambda}$ of Exponential model is given by:

TABLE C.1.1
DATA COLLECTED ON BUS DWELL TIMES DURING BOARDING EVENTS

NUMBER OF BOARDING PASSENGERS	BOARDING TIME (SEC.)	DEAD TIME(SEC.) PRE-BOARDING	POST-BOARDING	TOTAL	STOP DWELL TIME(SEC.)
2	5.9	13.0	2.8	15.8	21.7
2	5.1	13.0	53.8	66.8	71.9
1	1.7	21.0	4.0	25.0	26.7
1	3.9	4.0	10.0	14.0	17.9
3	37.0	7.0	20.0	27.0	64.0
2	7.9	6.0	9.0	15.0	22.9
1	4.0	9.0	1.0	10.0	14.0
1	3.9	7.0	4.0	11.0	14.9
2	7.0	14.0	9.0	23.0	30.0
3	6.5	1.0	38.4	39.4	45.9
3	7.6	36.0	44.1	80.1	87.7
4	16.9	18.0	17.0	35.0	51.9
3	9.3	5.6	10.0	15.6	24.9
3	11.9	17.0	8.0	25.0	36.9
3	11.4	14.0	6.0	20.0	31.4
21	82.7	7.0	35.0	42.0	124.7
2	4.9	8.0	17.0	25.0	29.9
2	6.7	3.0	21.0	24.0	30.7
2	10.9	8.0	11.0	18.0	28.9
1	4.0	9.0	7.0	16.0	20.0
2	6.1	4.0	6.0	10.0	16.1
3	9.2	17.0	5.0	22.0	31.2
2	7.1	36.0	10.0	46.0	43.1
2	4.7	1.0	7.0	8.0	12.7
1	3.5	9.0	2.0	11.0	14.5
1	3.9	4.0	5.0	9.0	12.9
3	10.2	5.0	26.0	31.0	41.2
1	4.9	2.0	1.0	3.0	7.9

2	5.[illegible]	42.0	5.0	47.0	52.0	184
1	4.1	14.0	4.0	18.0	22.1	
1	3.9	7.[illegible]	[illegible].0	8.0	11.9	
10	39.9	16.[illegible]	26.[illegible]	42.[illegible]	81.9	
2	6.[illegible]4	2.[illegible]	5.[illegible]	7.[illegible]	13.4	
3	8.9	3.0	2.0	5.0	13.9	
5	20.6	4.[illegible]	6.[illegible]	10.[illegible]	30.6	
2	5.7	5.0	4.[illegible]	9.0	14.7	

TABLE C.1.2
DATA COLLECTED ON BUS DWELL TIMES DURING ALIGHTING EVENTS

NUMBER OF ALIGHTING PASSENGERS	ALIGHTING TIME (SEC.)	DEAD TIME(SEC.) PRE-ALIGHTING	POST-ALIGHTING	TOTAL	STOP DWELL TIME(SEC.)
1	1.4	4.0	3.0	7.0	8.4
3	6.9	17.0	2.0	19.0	25.9
1	2.1	14.0	2.0	16.0	18.1
2	10.9	4.0	9.0	13.0	23.9
4	16.9	14.0	9.0	23.0	39.9
27	55.8	1.0	3.0	4.0	59.8
10	33.6	4.0	9.0	3.0	46.6
12	42.4	15.0	39.0	54.0	96.4
5	11.4	4.0	13.0	17.0	28.4
3	4.5	3.0	6.0	9.0	13.5
7	39.7	8.0	20.0	28.0	67.7
3	12.4	9.0	4.0	13.0	25.4
1	1.5	11.0	2.0	13.0	14.5
1	2.3	5.0	2.0	7.0	9.3
1	2.5	13.0	2.0	15.0	17.5
3	5.4	3.0	1.0	4.0	9.4
1	1.7	10.0	3.0	13.0	14.7
2	4.4	8.0	3.0	11.0	15.4
6	21.5	13.0	11.0	24.0	45.5
2	4.6	10.0	5.0	15.0	19.6
2	7.4	8.0	1.0	9.0	16.4
4	6.0	5.0	13.0	18.0	24.0
8	30.7	34.0	10.0	44.0	74.7
1	2.4	12.0	10.0	22.0	24.4

19	68.7	3.0	32.0	35.0	103.7
3	6.4	7.0	5.0	12.0	18.4
2	4.4	2.0	5.0	7.0	11.4
3	10.6	4.0	28.0	32.0	42.6
2	6.2	6.0	6.0	12.0	18.2
3	8.6	7.0	12.0	19.0	27.6
6	38.4	15.0	29.0	44.0	82.4
3	7.7	2.0	2.0	4.0	11.7
1	4.4	5.0	13.0	18.0	22.4
1	3.8	3.0	30.0	33.0	36.8
5	13.5	1.0	2.0	3.0	16.5
1	2.9	1.0	4.0	5.0	7.9
6	14.4	4.0	2.0	6.0	20.4
3	7.3	4.0	8.0	12.0	19.3
2	3.7	6.0	10.0	16.0	19.7
1	1.3	4.0	6.0	10.0	11.3
2	2.3	3.0	5.0	8.0	10.3
1	1.8	9.0	5.0	14.0	15.8
1	3.3	3.0	5.0	8.0	11.3
2	7.9	8.0	7.0	15.0	22.9
1	2.3	16.0	6.0	22.0	24.3
2	4.8	5.0	4.0	9.0	13.8
1	3.0	4.0	5.0	9.0	12.0
2	2.9	3.0	4.0	7.0	9.9
1	2.6	1.0	3.0	4.0	6.6
4	8.8	19.0	10.0	29.0	37.8
2	5.0	4.0	6.0	10.0	15.0
1	3.6	1.0	1.0	2.0	5.6
1	2.4	7.0	10.0	17.0	19.4
1	2.0	6.0	3.0	9.0	11.0
2	4.0	7.0	6.0	13.0	17.0
1	1.9	4.0	3.0	7.0	8.9
2	3.0	6.0	11.0	17.0	20.0
2	6.4	7.0	2.0	9.0	15.4
2	7.1	8.0	1.0	9.0	16.1
1	3.0	10.0	6.0	16.0	19.0
1	1.4	6.0	4.0	10.0	11.4
2	4.8	1.0	2.0	3.0	7.8
2	6.5	2.0	10.0	12.0	18.5
1	1.4	15.0	13.0	28.0	29.4
2	5.8	3.0	9.0	12.0	17.8
1	3.0	7.0	6.0	13.0	16.0
1	2.7	15.0	4.0	19.0	21.7
2	4.5	1.0	7.0	8.0	12.5
3	8.3	1.0	7.0	8.0	16.3
1	1.5	9.0	4.0	13.0	14.5
3	7.0	3.0	5.0	8.0	15.0
2	6.7	2.0	5.0	7.0	13.0
1	2.8	26.0	3.0	29.0	31.8
2	6.0	14.0	37.0	51.0	57.0
2	6.2	3.0	1.0	4.0	10.2
5	8.4	3.0	7.0	10.0	18.4
4	11.0	8.0	4.0	12.0	23.0
4	15.3	6.0	15.0	21.0	36.3

$\frac{\chi^2_{\alpha/2}}{2nT_1} < \hat{\lambda} < \frac{\chi^2_{1\alpha/2}}{2nT_1}$ with degree of freedom (d.f.)$\nu = 2n$

χ^2_α = lower percentile of Chi-square distribution

$\chi^2_{1-\alpha/2}$ = Upper percentile of Chi-square distribution

n = number of observations

$T_1 = \bar{X}$ = Test Statistic (Mean)

At 95% confidence level (i.e. $\alpha = 0.05$):

For $n = 60$, $\chi^2_{.025,120} = 91.567$ d.f. $= \nu = 2n = 120$

$\chi^2_{.975,120} = 152.21$

For $n = 106$ d.f. $= \nu = 2n = 212$

$\chi^2_{.025,212} = 173.56$

$\chi^2_{.975,212} = 254.22$

Boarding Process

(a) Pre-boarding:

$$\frac{91.567}{2(60)(10.477)} < \hat{\lambda} < \frac{152.21}{2(60)(10.477)}$$

$$0.0728 < \hat{\lambda} < 0.1211$$

(b) Post-boarding:

$$\frac{91.567}{2(60)(11.617)} < \hat{\lambda} < \frac{152.21}{2(60)(11.617}$$

$$0.0657 < \hat{\lambda} < 0.1092$$

Alighting Process

(a) Pre-alighting:

$$\frac{173.56}{2(106)(6.8585)} < \hat{\lambda} < \frac{254.22}{2(106)(6.8585)}$$

$$0.1194 < \hat{\lambda} < 0.1748$$

(b) Post-alighting

$$\frac{173.56}{2(106)(7.6038)} < \hat{\lambda} < \frac{254.22}{2(106)(7.6038)}$$

$$0.1077 < \hat{\lambda} < 0.1577$$

C.3 WEIGHTED AVERAGES OF $\hat{\lambda}$ OF EXPONENTIAL DEAD TIME MODELS

Boarding Process

Pre-boarding $\hat{\lambda}$ = 0.0954/sec n = 60 observations

Post-boarding $\hat{\lambda}$ = 0.08608/sec n = 60 observations

Weighted Average of $\hat{\lambda}_{wb} = \frac{0.09545\ (60) + 0.08608(60)}{60 + 60}$

= 0.090765/sec

Alighting Process

Pre-alighting: $\hat{\lambda}$ = 0.1458/sec n = 106 Observations

Post-alighting: $\hat{\lambda}$ = 0.1315/sec n = 106 observations

Weighted Average of $\hat{\lambda}_{wa} = \frac{0.1458(106)+0.1315(106)}{106 + 106}$

= 0.13865/sec

The weighted average values of $\hat{\lambda}$ fall within the respective 95% confidence Intervals.

Boarding and Alighting Process

$$\text{Overall weighted average of } \hat{\lambda}_w = \frac{0.09545(60) + 0.08608(160) + 0.1315(106)}{60 + 60 + 106 + 106}$$

$$= 0.12134/\text{sec}$$

This overall weighted average of $\hat{\lambda}$ falls within the 95% confidence intervals for only the pre- and post-boarding processes.

C.4 PROBABILITY THAT ONE EXPONENTIAL RANDOM VARIABLE IS SMALLER THAN ANOTHER

Suppose X_1 and X_2 are independent Exponential random variables with respective means $1/\lambda_1$ and $1/\lambda_2$. By conditioning on Y, the probability that event X is less than Y is (Ross, 1972)

$$P\{X_1 < X_2\} = \frac{\lambda_1}{\lambda_1 + \lambda_2} \qquad \text{(C.4.1)}$$

If λ_1 is approximately equal to λ_2 (i.e., $\lambda_1 \simeq \lambda_2 = \lambda$) in which case X_1 and X_2 can be assumed to identically distributed, then from (C.4.1)

$$P\{X_1 < X_2\} = \lambda/(\lambda + \lambda) \qquad \text{(C.4.2)}$$

$$= \lambda/2\lambda$$

$$= 0.50$$

That is, if two Exponential random variables are almost identical, the probability that one is smaller than the other is approximately 50%

(1) Boarding events

Pre-boarding λ = 0.09545/sec, Post-boarding λ = 0.08608/sec

Probability {Pre-boarding < Post-boarding} $= \frac{0.09545}{0.09545+0.08608} = 0.5258$

(2) Alighting events

Pre-alighting λ = 0.1458/sec, Post-alighting λ = 0.1315/sec

Probability {Pre-alighting< Post-alighting} =

$$= \frac{0.1458}{0.1458 + 0.1315} = 0.5258$$

C.5 COMPUTATION OF MEAN AND STANDARD DEVIATION OF DEAD TIMES

Pre-boarding Dead Time

Exponential model

$E(X) = 1/\lambda = 10.477$sec.

$Var(X) = 1/\lambda^2 = 109.768 sec^2$.

$SD = \sqrt{Var(X)} = 10.477$sec.

Observed Measurements

Sample mean $\bar{x}$

$$\bar{x} = \sum_{i=1}^{n} f_i x_i / \sum_{i=1}^{n} f_i$$

Sample variance S^2 (Grouped data)

$$S^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n} f_i x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} f_i x_i\right)^2\right)$$

where f_i = frequency, x_i = measurement.

Pre-boarding Dead Times

$\bar{x}$ = 10.667sec.

S^2 = 88.107sec^2.

S = 9.387

Height of Histogram = $\frac{\text{Relative Frequency}}{\text{Width of Histogram (Interval)}}$

TABLE C.2.1
FREQUENCY DISTRIBUTION OF DATA ON PRE-BOARDING DEAD TIMES

INTERVAL (SEC.)	FREQUENCY	RELATIVE FREQUENCY	HEIGHT OF HISTOGRAM
0.0 ≤ 5.0	18	.300	.060
5.0 ≤ 10.0	20	.333	.067
10.0 ≤ 15.0	8	.133	.027
15.0 ≤ 20.0	6	.100	.020
20.0 ≤ 25.0	4	.067	.013
25.0 ≤ 30.0	0	0	0
30.0 ≤ 35.0	1	.017	.003
35.0 ≤ 40.0	2	.033	.007
40.0 ≤ 45.0	1	.017	.003

TABLE C.2.2
FREQUENCY DISTRIBUTION OF DATA ON POST-BOARDING DEAD TIMES

INTERVAL (SEC.)	FREQUENCY	RELATIVE FREQUENCY	HEIGHT OF HISTOGRAM
0.0 ≤ 5.0	20	.333	.067
5.0 ≤ 10.0	19	.317	.063
10.0 ≤ 15.0	5	.083	.017
15.0 ≤ 20.0	4	.067	.013
20.0 ≤ 25.0	4	.067	.013
25.0 ≤ 30.0	2	.033	.007
30.0 ≤ 35.0	0	0	0
35.0 ≤ 40.0	3	.050	.010
40.0 ≤ 45.0	1	.017	.003
45.0 ≤ 50.0	1	.017	.003
50.0 ≤ 55.0	1	.017	.003

TABLE C.2.3
FREQUENCY DISTRIBUTION OF DATA ON PRE-ALIGHTING DEAD TIMES

INTERVAL (SEC.)	FREQUENCY	RELATIVE FREQUENCY	HEIGHT OF HISTOGRAM
0.0 - 5.0	44	.415	.083
5.0 - 10.0	37	.349	.070
10.0 - 15.0	15	.142	.028
15.0 - 20.0	8	.075	.015
20.0 - 25.0	0	0	0
25.0 - 30.0	1	.009	.002
30.0 - 35.0	1	.009	.002

TABLE C.2.4
FREQUENCY DISTRIBUTION OF DATA ON POST-ALIGHTING DEAD TIMES

INTERVAL (SEC.)	FREQUENCY	RELATIVE FREQUENCY	HEIGHT OF HISTOGRAM
0.0 - 5.0	44	.415	.083
5.0 - 10.0	37	.349	.070
10.0 - 15.0	15	.142	.028
15.0 - 20.0	2	.019	.004
20.0 - 25.0	2	.019	.004
25.0 - 30.0	3	.028	.006
30.0 - 35.0	2	.019	.004
35.0 - 40.0	2	.019	.004

TABLE C.3.1
GOODNESS-OF-FIT OF EXPONENTIAL MODEL
PRE-BOARDING DEAD TIMES

PARAMETER λ = .09545/SEC.

TIME (SEC.)	FREQ.	CUM. FREQ.	REL. CUM. FREQ. S(X)	EXPN. CDF F(X)	K-S D
1.0	2	2	.0333	.0918	.0577
2.0	3	5	.0833	.1738	.0905
3.0	4	9	.1500	.2490	.0990
4.0	9	18	.3000	.3174	.0174
5.0	3	21	.3500	.3795	.0295
5.6	1	22	.3667	.4141	.0474
6.0	4	25	.4333	.4360	.0027
7.0	7	33	.5500	.4873	.0627
8.0	2	35	.5833	.5340	.0493
9.0	3	38	.6333	.5764	.0569
10.0	1	39	.6500	.6150	.0350
11.0	2	41	.6833	.6500	.0333
13.0	2	43	.7167	.7109	.0058
14.0	3	46	.7667	.7372	.0295
16.0	1	47	.7833	.7829	.0004
17.0	3	50	.8333	.8026	.0307
18.0	2	52	.8667	.8206	.0461
20.0	1	53	.8833	.8518	.0315
21.0	1	54	.9000	.8653	.0347
22.0	1	55	.9167	.8775	.0392
24.0	1	56	.9333	.8988	.0345
32.0	1	57	.9500	.9528	.0028
36.0	2	59	.9833	.9678	.0155
42.0	1	60	1.0000	.9818	.0182

TABLE C.3.2
GOODNESS-OF-FIT OF EXPONENTIAL MODEL
POST-BOARDING DEAD TIMES

PARAMETER λ =.08608/SEC.

TIME (SEC.)	FREQ.	CUM. FREQ.	REL. CUM. FREQ. S(X)	EXPN. CDF F(X)	K-S D
1.0	3	3	.0500	.0825	.0325
1.4	1	4	.0667	.1135	.0468
2.0	4	8	.1333	.1582	.0249
2.8	1	9	.1500	.2142	.0642
3.0	4	13	.2167	.2276	.0109
3.5	1	14	.2333	.2601	.0268
4.0	6	20	.3333	.2913	.0420
5.0	6	26	.4333	.3498	.0835
6.0	4	30	.5000	.4034	.0966
7.0	3	33	.5500	.4526	.0974
8.0	2	35	.5833	.4978	.0855
9.0	4	39	.6500	.5392	.1108
10.0	3	42	.7000	.5772	.1228
11.0	2	44	.7333	.6121	.1262
15.0	1	45	.7500	.7251	.0249
17.0	3	48	.8000	.7586	.0314
20.0	1	49	.8167	.8212	.0045
21.0	1	50	.8333	.8360	.0027
24.0	2	52	.8667	.8733	.0066
26.0	2	54	.9000	.8933	.0067
35.0	1	55	.9167	.9509	.0342
37.0	1	56	.9333	.9586	.0253
38.4	1	57	.9500	.9633	.0133
44.1	1	58	.9667	.9775	.0108
48.0	1	59	.9833	.9839	.0006
53.8	1	60	1.0000	.9903	.0097

TABLE C.3.3
GOODNESS-OF-FIT OF GAMMA MODEL
PRE-BOARDING DEAD TIMES

PARAMETERS K=1.7929/SEC. λ =.1711/SEC.

TIME (SEC.)	FREQ.	CUM. FREQ.	REL.CUM. FREQ. S(X)	EXPN. CDF F(X)	K-S D
1.0	2	2	.0333	.0400	.0067
2.0	3	5	.0833	.1268	.0435
3.0	4	9	.1500	.1988	.0488
4.0	9	18	.3000	.2945	.0055
5.0	3	21	.3500	.3631	.0131
5.6	1	22	.3667	.4066	.0399
6.0	4	25	.4333	.4276	.0057
7.0	7	33	.5500	.5064	.0436
8.0	2	35	.5833	.5765	.0068
9.0	3	38	.6333	.6084	.0249
10.0	1	39	.6500	.6660	.0160
11.0	2	41	.6833	.7161	.0328
13.0	2	43	.7167	.7786	.0619
14.0	3	46	.7667	.8130	.0463
16.0	1	47	.7833	.8553	.0720
17.0	3	50	.8333	.8782	.0449
18.0	2	52	.8667	.8977	.0310
20.0	1	53	.8833	.9214	.0381
21.0	1	54	.9000	.9342	.0342
22.0	1	55	.9167	.9398	.0231
24.0	1	56	.9333	.9579	.0246
32.0	1	57	.9500	.9814	.0314
36.0	2	59	.9833	.9936	.0103
42.0	1	60	1.0000	.9975	.0025

TABLE C.3.4
GOODNESS-OF-FIT OF GAMMA MODEL
POST-BOARDING DEAD TIMES

PARAMETERS K=1.2976/SEC. λ =.1117/SEC.

TIME (SEC.)	FREQ.	CUM. FREQ.	REL. CUM. FREQ. S(X)	GAMMA CDF F(X)	K-S D
1.0	3	3	.0500	.0952	.0452
1.4	1	4	.0667	.1393	.0726
2.0	4	8	.1333	.1813	.0480
2.8	1	9	.1500	.2592	.1092
3.0	4	13	.2167	.2953	.0786
3.5	1	14	.2333	.3297	.0964
4.0	6	20	.3333	.3624	.0291
5.0	5	25	.4333	.4231	.0102
6.0	4	30	.5000	.4780	.0220
7.0	3	33	.5500	.5507	.0007
8.0	2	35	.5833	.5934	.0101
9.0	4	39	.6500	.6321	.0179
10.0	3	42	.7000	.6671	.0329
11.0	2	44	.7333	.6988	.0345
15.0	1	45	.7500	.8173	.0673
17.0	3	48	.8000	.8514	.0514
20.0	1	49	.8167	.8892	.0725
21.0	1	50	.8333	.8997	.0664
24.0	2	52	.8667	.9328	.0661
26.0	2	54	.9000	.9450	.0450
35.0	1	55	.9167	.9798	.0631
37.0	1	56	.9333	.9834	.0501
38.4	1	57	.9500	.9864	.0364
44.1	1	58	.9667	.9926	.0259
48.0	1	59	.9833	.9948	.0115
53.8	1	60	1.0000	.9975	.0025

TABLE C.3.5
GOODNESS-OF-FIT OF EXPONENTIAL MODEL
PRE-ALIGHTING DEAD TIMES

PARAMETER λ = .1458/SEC.

TIME (SEC.)	FREQ.	CUM. FREQ.	REL. CUM. FREQ. S(X)	EXPN. CDF F(X)	K-S D
1.0	11	11	.1038	.1357	.0319
2.0	8	19	.1792	.2529	.0737
3.0	13	32	.3019	.3543	.0524
4.0	12	44	.4151	.4419	.0268
5.0	8	52	.4906	.5176	.0270
6.0	9	61	.5755	.5831	.0076
7.0	8	69	.6509	.6396	.0113
8.0	7	75	.7170	.6885	.0285
9.0	5	81	.7642	.7308	.0334
10.0	6	87	.8208	.7673	.0529
11.0	1	88	.8302	.7989	.0313
12.0	3	91	.8585	.8262	.0323
13.0	2	93	.8774	.8498	.0276
14.0	3	96	.9057	.8701	.0356
15.0	5	101	.9528	.8878	.0650
16.0	1	102	.9623	.9030	.0593
17.0	1	103	.9717	.9161	.0556
19.0	1	104	.9811	.9374	.0437
26.0	1	105	.9906	.9774	.0132
34.0	1	106	1.0000	.9930	.0070

TABLE C.3.6
GOODNESS-OF-FIT OF EXPONENTIAL MODEL
POST-ALIGHTING DEAD TIMES

PARAMETER λ = .1315/SEC.

TIME (SEC.)	FREQ.	CUM. FREQ.	REL. CUM. FREQ. S(X)	EXPN. CDF F(X)	K-S D
1.0	9	9	.0849	.1232	.0383
2.0	12	21	.1981	.2313	.0332
3.0	13	34	.3208	.3260	.0052
4.0	10	44	.4151	.4091	.0060
5.0	10	54	.5094	.4819	.0275
6.0	9	63	.5943	.5457	.0486
7.0	8	71	.6698	.6017	.0681
8.0	4	75	.7075	.6508	.0567
9.0	6	81	.7642	.6938	.0704
10.0	6	87	.8208	.7316	.0892
11.0	2	89	.8396	.7646	.0750
12.0	1	90	.8491	.7936	.0555
13.0	5	95	.8962	.8191	.0771
14.0	1	96	.9057	.8414	.0643
15.0	1	97	.9151	.8609	.0542
18.0	1	98	.9245	.9036	.0182
20.0	1	99	.9340	.9279	.0061
27.0	1	100	.9434	.9713	.0279
28.0	1	101	.9528	.9748	.0220
29.0	1	102	.9623	.9779	.0156
30.0	1	103	.9717	.9807	.0090
32.0	1	104	.9811	.9851	.0040
37.0	1	105	.9906	.9923	.0017
39.0	1	106	1.0000	.9941	.0059

TABLE C.3.7
GOODNESS-OF-FIT OF GAMMA MODEL
PRE-ALIGHTING DEAD TIMES

PARAMETERS K=1.8922/SEC. λ =.2769/SEC.

TIME (SEC.)	FREQ.	CUM. FREQ.	REL.CUM. FREQ. S(X)	GAMMA CDF F(X)	K-S D
1.0	11	11	.1038	.0321	.0717
2.0	8	19	.1792	.1070	.0722
3.0	13	32	.3019	.2023	.0996
4.0	12	44	.4151	.3037	.1114
5.0	8	52	.4905	.4027	.0879
6.0	9	61	.5755	.4945	.0810
7.0	8	69	.6509	.5769	.0740
8.0	7	76	.7170	.6490	.0680
9.0	5	81	.7642	.7110	.0532
10.0	6	87	.8208	.7635	.0573
11.0	1	88	.8302	.8075	.0227
12.0	3	91	.8585	.8441	.0144
13.0	2	93	.8774	.8743	.0031
14.0	3	96	.9057	.8989	.0068
15.0	5	101	.9528	.9181	.0347
16.0	1	102	.9623	.9353	.0270
17.0	1	103	.9717	.9485	.0232
19.0	1	104	.9811	.9675	.0136
26.0	1	105	.9905	.9938	.0032
34.0	1	106	1.0000	.9991	.0009

TABLE C.3.8
GOODNESS OF FIT OF GAMMA MODEL
POST ALIGHTING DEAD TIMES

PARAMETERS K=1.6515/SEC. λ =.2172/SEC.

TIME (SEC.)	FREQ.	CUM. FREQ.	REL. CUM. FREQ. S(X)	GAMMA CDF F(X)	K-S D
1.0	9	9	.0849	.0671	.0178
2.0	12	21	.1981	.1617	.0310
3.0	13	34	.3208	.2716	.0492
4.0	10	44	.4151	.3713	.0438
5.0	10	54	.5094	.4624	.0470
6.0	9	63	.5943	.5436	.0507
7.0	8	71	.6698	.6144	.0554
8.0	4	75	.7075	.6758	.0317
9.0	6	81	.7642	.7284	.0358
10.0	6	87	.8208	.7732	.0476
11.0	2	89	.8396	.8112	.0284
12.0	1	90	.8491	.8431	.0060
13.0	5	95	.8962	.8698	.0264
14.0	1	96	.9057	.8922	.0135
15.0	1	97	.9151	.9109	.0042
18.0	1	98	.9245	.9501	.0256
20.0	1	99	.9340	.9662	.0322
27.0	1	100	.9434	.9929	.0495
28.0	1	101	.9528	.9931	.0403
29.0	1	102	.9623	.9956	.0323
30.0	1	103	.9717	.9954	.0237
32.0	1	104	.9811	.9969	.0158
37.0	1	105	.9906	.9989	.0083
39.0	1	106	1.0000	.9993	.0007

TABLE C.3.9
GOODNESS-OF-FIT OF EXPONENTIAL MODEL
PRE-BOARDING DEAD TIMES

WEIGHTED PARAMETER $\hat{\lambda}_{wb}$=.09765/SEC.
OVERALL WEIGHTED PARAMETER $\hat{\lambda}_{w}$.12134/SEC.

TIME (SEC.)	FREQ.	CUM. FREQ.	REL. CUM. FREQ. S(X)	EXPN. CDF F(X)	K-S D	EXPN'. CDF F'(X)	K-S D'
1.0	2	2	.0333	.0868	.0535	.1143	.0810
2.0	3	5	.0833	.1661	.0827	.2155	.1322
3.0	4	9	.1500	.2384	.0884	.0351	.1551
4.0	9	18	.3000	.3845	.0845	.3845	.0845
5.0	3	21	.3500	.3648	.0148	.4549	.1049
5.6	1	22	.3667	.3985	.0318	.4931	.1264
6.0	4	26	.4333	.4199	.0134	.5171	.0838
7.0	7	33	.5500	.4703	.0797	.5723	.0223
8.0	2	35	.5833	.5162	.0671	.6212	.0379
9.0	3	38	.6333	.5582	.0751	.6645	.0312
10.0	1	39	.6500	.5965	.0535	.7028	.0528
11.0	2	41	.6833	.6513	.0518	.7368	.0535
13.0	2	43	.7167	.6927	.0240	.7935	.0758
14.0	3	46	.7667	.7137	.0473	.8171	.0504
16.0	1	47	.7833	.7660	.0173	.8565	.0732
17.0	3	50	.8333	.7863	.0470	.8729	.0396
18.0	2	52	.8667	.8048	.0619	.8874	.0207
20.0	1	53	.8833	.8372	.0461	.9117	.0284
21.0	1	54	.9000	.8513	.0487	.9218	.0218
22.0	1	55	.9167	.8642	.0525	.9307	.0140
24.0	1	56	.9333	.8868	.0465	.9456	.0123
32.0	1	57	.9500	.9452	.0048	.9794	.0294
36.0	2	59	.9833	.9619	.0214	.9873	.0040
42.0	1	60	1.0000	.9779	.0221	.9939	.0061

TABLE C.3.10
GOODNESS-OF-FIT OF EXPONENTIAL MODEL
POST-BOARDING DEAD TIMES

WEIGHTED PARAMETER $\hat{\lambda}_{wb}$=.090765/SEC.
OVERALL WEIGHTED PARAMETER $\hat{\lambda}_{w}$ =.12134/SEC.

TIME (SEC.)	FREQ.	CUM. FREQ.	REL.CUM. FREQ. S(X)	EXPN. CDF F(X)	K-S D	EXPN'. CDF F'(X)	K-S D'
1.0	3	3	.0500	.0868	.0368	.1143	.0643
1.4	1	4	.0667	.1193	.0526	.1562	.0895
2.0	4	8	.1333	.1660	.0327	.2155	.0822
2.8	1	9	.1500	.2244	.0744	.2881	.1381
3.0	4	13	.2167	.2384	.0217	.3051	.0884
3.5	1	14	.2333	.2722	.0389	.3460	.1127
4.0	6	20	.3333	.3045	.0288	.3845	.0512
5.6	6	26	.4333	.3648	.0685	.4549	.0216
6.0	4	30	.5000	.4199	.0801	.5171	.0171
7.0	3	33	.5500	.4703	.0797	.5723	.0223
8.0	2	35	.5833	.5162	.0671	.6212	.0379
9.0	4	39	.6500	.5582	.0918	.6645	.0145
10.0	3	42	.7000	.5965	.1035	.7028	.0028
11.0	2	44	.7333	.6315	.1018	.7368	.0035
15.0	1	45	.7500	.7437	.0063	.8380	.0880
17.0	3	48	.8000	.7863	.0137	.8729	.0729
20.0	1	49	.8167	.8372	.0205	.9117	.0950
21.0	1	50	.8333	.8513	.0180	.9218	.0885
24.0	2	52	.8667	.8868	.0201	.9456	.0789
26.0	2	54	.9000	.9156	.0056	.9574	.0574
35.0	1	55	.9167	.9583	.0416	.9857	.0690
37.0	1	56	.9333	.9552	.0319	.9888	.0555
38.4	1	57	.9500	.9694	.0194	.9905	.0405
44.1	1	58	.9667	.9817	.0150	.9953	.0286
48.0	1	59	.9833	.9872	.0039	.9970	.0137
53.8	1	60	1.0000	.9924	.0076	.9985	.0015

TABLE C.3.11
GOODNESS-OF-FIT OF EXPONENTIAL MODEL
PRE-ALIGHTING DEAD TIMES

WEIGHTED PARAMETER $\hat{\lambda}_{wa}$ = .13865/SEC.
OVERALL WEIGHTED PARAETER $\hat{\lambda}_{w}$ .12134/SEC.

TIME (SEC.)	FREQ.	CUM. FREQ.	REL. CUM. FREQ. S(X)	EXPN. CDF F(X)	K-S D	EXPN'. CDF F'(X)	K-S D'
1.0	11	11	.1038	.1295	.0257	.1143	.0105
2.0	8	19	.1792	.2422	.0630	.2155	.0363
3.0	13	32	.3019	.3403	.0384	.3051	.0032
4.0	12	44	.4151	.5648	.1497	.5171	.1020
5.0	8	52	.4906	.5001	.0095	.4549	.0357
6.0	9	61	.5755	.5648	.0107	.5171	.0584
7.0	8	69	.6509	.6211	.0298	.5723	.0786
8.0	7	76	.7170	.6702	.0468	.6212	.0958
9.0	5	81	.7642	.7129	.0513	.6645	.0997
10.0	6	87	.8208	.7501	.0707	.7028	.1180
11.0	1	88	.8302	.7824	.0478	.7368	.0934
12.0	3	91	.8585	.8106	.0479	.7669	.0916
13.0	2	93	.8774	.8531	.0423	.7935	.0839
14.0	3	96	.9057	.8565	.0492	.8171	.0886
15.0	5	101	.9528	.8750	.0778	.8380	.1148
16.0	1	102	.9623	.8912	.0711	.8565	.1058
17.0	1	103	.9717	.9053	.0664	.8729	.0988
19.0	1	104	.9811	.9282	.0529	.9003	.0808
26.0	1	105	.9905	.9728	.0178	.9574	.0332
34.0	1	106	1.0000	.9910	.0090	.9838	.0162

TABLE C.3.12
GOODNESS-OF-FIT OF EXPONENTIAL MODEL
POST-ALIGHTING DEAD TIMES

WEIGHTED PARAMETER $\hat{\lambda}_{wa}$ =.13855/SEC.
OVERALL WEIGHTED PARAMETER $\hat{\lambda}_{w}$ =.12134/SEC.

TIME (SEC.)	FREQ.	CUM. FREQ.	REL.CUM. FREQ. S(X)	EXPN. CDF F(X)	K-S D	EXPN'. CDF F'(X)	K-S D'
1.0	9	9	.1849	.1295	.0446	.1143	.0294
2.0	12	21	.1981	.2422	.0441	.2155	.0174
3.0	13	34	.3208	.3403	.0195	.3051	.0157
4.0	10	44	.4151	.4257	.0106	.3845	.0306
5.0	10	54	.5094	.5001	.0093	.4549	.0545
6.0	9	63	.5943	.5648	.0295	.5171	.0772
7.0	8	71	.6698	.6212	.0487	.5723	.0975
8.0	4	75	.7075	.6702	.0373	.6212	.0863
9.0	6	81	.7642	.7129	.0513	.6645	.0997
10.0	6	87	.8208	.7511	.0707	.7028	.1180
11.0	2	89	.8396	.7824	.0572	.7368	.1028
12.0	1	90	.8491	.8116	.0385	.7669	.0822
13.0	5	95	.8962	.8351	.0611	.7935	.1027
14.0	1	96	.9057	.8565	.0492	.8171	.0886
15.0	1	97	.9151	.8750	.0401	.8380	.0771
18.0	1	98	.9245	.9176	.0069	.8874	.0371
20.0	1	99	.9340	.9375	.0035	.9117	.0223
27.0	1	100	.9434	.9763	.0329	.9622	.0188
28.0	1	101	.9528	.9794	.0266	.9665	.0137
29.0	1	102	.9623	.9812	.0198	.9704	.0081
30.0	1	103	.9717	.9844	.0127	.9738	.0021
32.0	1	104	.9811	.9882	.0071	.9794	.0017
37.0	1	105	.9906	.9941	.0035	.9888	.0018
39.0	1	106	1.0000	.9955	.0045	.9912	.0088

TABLE C.4.1

OBSERVED AND ESTIMATED MEAN AND STANDARD DEVIATION OF DEAD TIME FOR BOARDING PROCESS

DISTRIBUTION	PRE-BOARDING DEAD TIME		POST-BOARDING DEAD TIME	
	MEAN (Sec)	STANDARD DEVIATION (Sec)	MEAN (Sec)	STANDARD DEVIATION (Sec)
OBSERVED	10.667	9.387	12.083	12.361
EXPONENTIAL	10.477	10.477	11.617	11.617
GAMMA	10.477	7.826	11.617	10.198

TABLE C.4.2

OBSERVED AND ESTIMATED MEAN AND STANDARD DEVIATION OF DEAD TIME FOR ALIGHTING PROCESSES.

DISTRIBUTION	PRE-ALIGHTING DEAD TIME		POST-ALIGHTING DEAD TIME	
	MEAN (Sec)	STANDARD DEVIATION (Sec)	MEAN (Sec)	STANDARD DEVIATION (Sec)
OBSERVED	7.311	5.603	8.066	7.632
EXPONENTIAL	6.859	6.859	7.604	7.604
GAMMA	6.859	4.977	7.604	5.916

TABLE C.5

95% CONFIDENCE INTERVAL ON PARAMETER OF EXPONENTIAL DEAD TIME MODELS:

		LOWER	UPPER
BOARDING PROCESS	PRE-BOARDING	0.0728	0.1211
	POST-BOARDING	0.0657	0.1092
ALIGHTING PROCESS	PRE-BOARDING	0.1194	0.1748
	POST-BOARDING	0.1077	0.1577