

STOCHASTIC ASSESSMENT OF NIGERIAN TIMBERS FOR BRIDGE DECKS IN
ACCORDANCE TO AASHTO LFRD

BY

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M.SC/ENG/02386/2010-2011

M. Sc. THESIS SUBMITTED TO THE SCHOOL OF POSTGRADUATE STUDIES
AHMADU BELLO UNIVERSITY, ZARIA

IN PARTIAL FULFILLMENT FOR THE AWARD OF MASTER OF SCIENCE IN CIVIL
ENGINEERING

DEPARTMENT OF CIVIL ENGINEERING,
FACULTY OF ENGINEERING
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DECEMBER, 2014

DECLARATION

I hereby declare that this work entitled '**Stochastic Assessment of Nigerian Timbers for Bridge Decks in Accordance to AASHTO LFRD**' is my own research work and has been composed by me. It has not been presented in any previous work for the award of a degree.

All sources of information utilized in this thesis that are not part of my conception are specifically acknowledged by means of reference.

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CERTIFICATION

This project thesis entitled ‘Stochastic Assessment of Nigerian Timbers for Bridge Decks in Accordance to AASHTO LFRD’ by Peter Abayomi Owoeye, has been read and approved as meeting the partial requirement of the Department of Civil Engineering of Ahmadu Bello University, zaria, for the Degree of Masters of Science (M.Sc) in Civil Engineering.

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ACKNOWLEDGEMENT

My deepest gratitude goes to the Almighty GOD, for HIS guidance and protection throughout my studies in this school and also for HIS grace on me to complete this MSc program successfully.

To Dr. O. S. ABEJIDE, and Dr. A. T. OLOWOSULU my supervisors, I wish to express my sincere thanks for patiently and encouragingly guiding me towards a successful completion of this project. Their assistance, fatherly advice and timely corrections proved to be very effective and would not be forgotten. To my departmental lecturers and the entire staff of Civil Engineering department, Ahmadu Bello University, zaria, for all their kind support and encouragement.

My deep felt appreciation goes to my parents Mr. and Deaconess Owoeye for their encouragement and support towards my studies.

ABSTRACT

This work provides results for stochastic safety evaluation of Nigerian timbers for bridge decks in accordance to American Association of State Transportation and Highway Officials (AASHTO) Load Resistance Factored Design (LRFD) design specifications. A timber bridge is modeled in accordance to AASHTO LRFD, 2010, to represent real life experiment in order to depict the structural behavior of planks when used as a bridge deck. This model was then subjected to some degree of entropy using Advanced Second Moment Reliability Assessment (ASMRA) method, which was subsequently analysed using JAVA library with the help of Flanagan polynomial. Experimental data collected from literature was compared with the current Nigerian Code of Practice, NCP 2, 1973 for timber, and were used for the assessment. It was observed that, strength classes, timber thicknesses and stringer spacing are the major factors among others influencing the structural behavior of Nigerian timber proposed as bridge decks. Therefore, the major classes of Nigerian timber recommended for bridge decks are timber within the strength classes N_1 to N_4 with dimensions ranging from 100 x 250mm to 150 x 300mm on stringers spaced not greater than 300mm, depending on the strength class adopted; with timber belonging to the higher strength classes taking the lower dimension. An exception to this, are timber which belong to the strength class N_1 , where the stringers can be spaced at 450mm using timber with dimension not lower than 100mm thick and width not less than 250mm. The recommended strength classes with associated material properties can be a source of sustainable bridge deck material over a reasonable period of time as indicated by the probability of failure as a result of damage due to load accumulation. In view of this, timber which is a locally available material can be used as substitute for the expensive concrete and steel which are the most commonly used materials.

TABLE OF CONTENTS

DECLARATION	ii
CERTIFICATION	iii
ACKNOWLEDGEMENT	iv
ABSTRACT	v
TABLE OF CONTENTS	vi
LIST OF FIGURES	ix
LIST OF TABLES	x
NOTATIONS	xi
CHAPTER 1	1
INTRODUCTION	1
1.1 GENERAL OVER VIEW	1
1.2 JUSTIFICATION OF STUDY	3
1.3 AIM AND OBJECTIVES	5
1.4 SCOPE AND LIMITATION	5
CHAPTER 2	7
2.0 LITERATURE REVIEW	7
2.1 INTRODUCTION	7
2.2 CLEAR WOOD SPECIMENS AND THEIR PROPERTIES	8
2.2.1 Moisture Content	9
2.2.2 Shrinkage and Swelling	9
2.2.3 Density	10
2.2.4 Strength and Stiffness Properties	10
2.2.5 Moisture Content and Mechanical Properties	11
2.3 STRUCTURAL TIMBER	12
2.4 GROWTH IRREGULARITIES IN TIMBER STRUCTURAL COMPONENTS	13
2.4.1 Knots	13
2.4.2 Cross Grain	13
2.4.3 Distortion	14
2.4.4 Wane	14
2.4.5 Permanent Compressive Yield	14

2.5	NIGERIAN TIMBER	15
2.6	BRIDGES IN RETROSPECTIVE	15
2.7	BRIEF HISTORY OF TIMBER BRIDGES	17
2.8	DECLINE OF TIMBER BRIDGE BUILDING	19
2.9	REVIVAL OF TIMBER BRIDGE BUILDING	20
2.10	TIMBER BRIDGES	21
CHAPTER 3		28
3.0	RESEARCH METHODOLOGY	28
3.1	INTRODUCTION	28
3.2	ADVANCE SECOND MOMENT RELIABILITY ASSESSMENT METHOD (ASMRAM)	30
3.3	PLANK DECK DESIGN MODEL	39
3.3.1	LRFD AASHTO SPECIFICATIONS (2010)	40
3.3.2	TIRE CONTACT AREA	44
3.3.3	RELIABILITY ANALYSIS	46
3.3.4	DAMAGE ACCUMULATION MODEL	48
CHAPTER 4		51
4.0	DATA SOURCE, ANALYSIS AND RESULTS	51
4.1	INTRODUCTION	51
4.2	DATA SOURCE	52
4.3	ANALYSIS AND RESULTS	54
4.3.1	General reliability assessment	54
4.3.2	Reliability assessment of Nigerian timbers in relation to stringer spacing under varying loads	56
4.3.3	Reliability assessment of Nigerian timbers in relation to stringer spacing under constant loads	57
4.3.4	Reliability assessment of Nigerian timbers in relation to plank thickness at 0.3m stringer spacing	58
4.3.5	Reliability assessment of Nigerian timbers in relation to plank thickness at constant stringer spacing of 0.45m	59
4.3.6	Reliability assessment of Nigerian timbers in relation to plank width at constant stringer spacing	60
4.3.7	Damage accumulation reliability analysis results	61

CHAPTER 5	62
5.0 DISCUSSION, CONCLUSIONS AND RECOMMENDATIONS	62
5.1 DISCUSSION	62
5.2 CONCLUSIONS	66
5.3 RECOMMENDATIONS	67
REFERENCE	67
APPENDIX	71
Appendix I: Stochastic assessment data and reliability index	71
Appendix II: Java safety assessment programs	78

LIST OF FIGURES

Figure 3.1 - Performance function	33
Figure 3.2- Typical Transverse Plank Deck	39
Figure 3.3- Typical Longitudinal Plank Deck	40
Figure 3.4 - Tire Contact Area	46
Figure 4.1 Reliability index – Stringer spacing under varying loading	57
Figure 4.2 - Reliability index – Stringer spacing under constant loading.	58
Figure 4.3 - Reliability index – Plank thickness at 0.3m stringer spacing.	59
Figure 4.4- Reliability index – Plank thickness at 0.45m stringer spacing.	60
Figure 4.5 - Reliability index – Plank width (m)	61

LIST OF TABLES

Table 2.1 - Strength Properties and Density of Some Structural Materials	11
Table 2.2 - Approximate Change (%) of Clear Wood Properties	12
Table 4.1 – Reference Dry Grade (80%) Stress	52
Table 4.2 – NCP 1973, Basic Bending Stress Parallel to Grain (Grade 80)	53
Table 4.3 - Other Design Data	54
Table 4.4 – Reliability Assessment of Nigeria Timber (EKKI, APA, IROKO and ABURA).	55
Table 4.5 – Damage Accumulation Reliability Assessment Results of Nigerian Timber	61
Table A-1 - Reliability Assessment of Nigerian Timbers for Bridge Decks In Relation to Stringer Spacing Under Varying Load	71
Table A-2 - Reliability Assessment of Nigerian Timber for Bridge Deck at Constant Stringer Spacing Under Constant Loading	72
Table A-3 - Reliability Assessment of Nigerian Timbers for Bridge Deck at 0.3m Stringer Spacing for Various Plank Thicknesses	74
Table A-4 - Reliability Assessment of Nigerian Timbers for Bridge Deck at 0.45m Stringer Spacing for Various Plank Thicknesses	75
Table A-5 - Reliability Assessment of Nigerian Timbers for Bridge Deck at Fixed Stringer Spacing for Various Plank Widths	77

NOTATIONS

$Z(X)$ = structural performance function

$R(X)$ = material resistances function

$L(X)$ = load effects function

X_i = basic random variables

P_f = failure probability

μ_{xi} = mean of random variable

μ_z = mean performance function

σ = standerd deviation

β = reliability index

r = reliability index on graph

α = directional cosine on an hyperplane

γ_{X_i} = safety factor for variable X_i

ϑ_{X_i} = coefficient of variation for variable X_i

DC = dead load moment of structural and nonstructural components

DW = dead load moment of wearing surfaces and utilities

IM = vehicular dynamic load moment

LL = vehicular live load moment

λ_i = load modifier

γ_i = Load factor

Q_i = Force effect

ϕ = Resistance factor

$R_r =$ Factored resistance

$R_n =$ Nominal resistance

$C_l =$ Beam stability factor

$f_b =$ adjusted flexure

$S =$ sectional modulus

$s =$ stringer center to center spacing

$b_s =$ stringers width

$f_{bo} =$ reference design value

$C_{kf} =$ format conversion factor

$C_m =$ wet service factor

$C_f =$ size factor for visually graded

$C_{fu} =$ flat use factor

$C_i =$ incising factor

$C_d =$ deck factor

$C_\lambda =$ time effect factor

$\phi =$ resistance factor for flexure member

$\gamma =$ load factor

$IM =$ dynamic load allowance percent

$P =$ design wheel load

$\rho_p =$ unit weight of plank deck

$\rho_s =$ unit weight of surfacing material

$w_p =$ width of plank deck

$w_l = \text{lane load}$

$t_p = \text{depth of plank deck}$

$t_s = \text{depth of surfacing}$

$b_t = \text{width of tire contact area}$

CHAPTER 1

INTRODUCTION

1.1 GENERAL OVER VIEW

The need for local content in construction of engineering infrastructure is now a serious engineering challenge in Nigeria. This is because vast quantities of local raw materials, which must be processed and used for cost effective construction abound. Construction activities based on these locally available raw materials are major steps towards industrialisation and economic independence for developing countries (Aguwa and Sadiku, 2011).

Timber is one of the natural occurring raw materials which abound in Nigeria and it had been put to use as a building material for construction since prehistoric times. It is available in large quantities in the forested parts of the country. The extent of its usage by professionals in the building industry is determined not only by their understanding of the material, but also by their perception of the material (Adedeji and Ogunsote, 2004).

The major use of timber in Nigeria and most part of the third world has been limited to domestic use as an alternative source of heat energy or household kitchen appliance. In this respect, timber which is supposed to be used to sustain the development of the economy is being burnt as coal. With critical analysis of our environment and careful exploration of the structural properties of timber, one can adequately establish and design an environmentally friendly structure which is cost effective. This is why Afolayan (1999), describes timber as a low density, cellular, polymeric composite which does not fall into any one class of materials; rather it tends

to overlap a number of classes. Because of its high strength performance and low cost, timber is found to be the world most successful fiber composite, (Afolayan, 1999).

Timber, like other building materials, has inherent advantages that make it especially attractive in specific applications (Aguwa and Sadiku, 2011: Afolayan and Adeyeye, 1998). Structural timber is the timber used in framing and load-bearing structures, where strength is the major factor in its selection and use. Trees are the only sources of timber, and those that carry naked seeds are called softwoods, while those with seeds inside a fruit are termed hardwoods(Aguwa and Sadiku, 2011).

With the current dispensation and increasing global development, the public demands a safe and efficient transportation system. With increasing traffic demand, it is clear that engineers and political leaders must find ways to upgrade bridges and do so with fewer resources at hand.

The use of timber (which is a locally available material) as component for expensive civil engineering bridge project (for example in bridge decks), will be a cost effective bridge type for rural bridge rehabilitation and sustainability. The benefits of timber bridge are primarily derived from the use of native timber species which are renewable, reusable, replaceable recyclable, environmentally friendly, sustainable and maintainable.

The main advantages of timber bridges relative to other bridge materials as pointed out by DelDOT(2005) which is a construction company in Delaware, include Ease of construction, Ease of maintenance, Pleasing appearance, Renewable resource, it's use in Construction is not weather dependent, it is Lightweight among others.

The use of this renewable lightweight natural resource as a bridge deck, will not only be a new strategy for development in the third world but also as a sustainable development which will help to overcome the exclusion of timber technology in modern times, thereby strengthening its inclusion through research and practical application.

In view of the above, this work focuses on the possibility of using Nigerian timber for bridge decks. As a result, timber decks was proposed and evaluated to assess the possibility and use of timber bridge decks for the construction and or rehabilitation of road bridges both in rural and urban settings of Nigeria (and other parts of the country) in order to open up new roads while at the same time rehabilitating old and or abandoned bridges.

1.2 JUSTIFICATION OF STUDY

Due to the current dispensation and increased challenges in global development in all sectors of the economy, construction companies inclusive, there exist these motives to build bridges faster that will last longer, for less money and with aesthetics appeal which has led to the quest for the perfect bridge material.

Reinforced concrete and steel which has edged out timber as a construction component for bridges was reported by Bell (2007) as not been an everlasting material they were assumed to be. This is because many countries have experienced serious problems with concrete bridges built in the 1960's and 70's. This was backed with the assertions stated in Vermont (2008) local road fact sheet. It was clearly stated in this local fact sheet that, properly treated, timber is stable and durable under the most severe weather and site conditions, which is one of its attractive performance features for bridges as it is completely resistant to the deicing salts, decay and insect

attacks. It had also been noticed, that deicing salts have caused significant and surprisingly rapid deterioration of both steel and concrete bridges and components. It was reported too that, when a larger structural timber is exposed to fire, there is some delay as it chars and eventually flames. As burning continues, the charred layer has an insulative effect, and the burning slows to an average rate of about 1/40 inch (0.6mm) per minute (or 1 ½ (38mm) per hour), for average structural timber species. These slow rates of fire penetration means that timber structural members subjected to fire maintain a high percentage of their original strength for considerable periods of time. In contrast, structural steel becomes plastic when exposed to a heat of 1,000°F (340°C) and it yields almost immediately.

Treated timber bridge decks are built in days not weeks, because materials are low energy, certified, reusable and renewable, where components are shop manufactured under controlled conditions to maintain quality (David,2011). As a result the use of timber will reduce detour times and construction inspection expense, because controlled shop conditions will improve quality and thus, reduce the need for site inspection. Considering the availability of timber as a renewable, sustainable natural resource which can be formed locally even with simple hand tools, remote parts of the country can easily be linked and reached through the construction of short span timber bridges at a reasonable low cost.

Currently, the use of timber as a bridge deck or bridge component has not been a common practice in Africa, though with few physical examples associated with scarce historical documentation. The last two decades have seen a growing interest in timber bridges in many European countries (Bell, 2007). However, in 2005, DelDOT Construction Company Delaware,

reported that its construction of timber bridges has decreased in proportion to its use of other materials over the last 50 years.

1.3 AIM AND OBJECTIVES

The aim of this research is to assess the possibility of using various Nigerian timber species as sustainable bridge decks.

In this regard, the objectives of the research include,

- i. To develop a suitable stochastic model for the analysis of Nigerian timber species when used as bridge decks, taking appropriate variables into consideration.
- ii. Identify the suitability of Nigerian timber for use as material for bridge decks.
- iii. To estimate safety indices and probability of failure of using Nigerian timber for bridge decks
- iv. To predict and or evaluate the long-term performance of Nigerian timber when use as a bridge deck material.

1.4 SCOPE AND LIMITATION

The scope of this research is to use a stochastic method of analysis to model a real world experiment consisting of limit states which occur randomly. In relation to timber, the major parameters which determine performance of wood structures are modulus of rupture (MOR), modulus of elasticity (MOE), and loads. These parameters are subject to considerable variation and therefore, they are treated as random variables. (Nowak and Saraf, 1996). Abejide (2007), and Martenson (1992), had both iterated that “the effect of load duration and moisture influence on the strength and formation of timber members in a structure affect the load carrying

capacity of structural timber”. A bridge deck is always subjected to impact and dynamic load of varying magnitude at random intervals of time and seasonal changes. These parameters will be subjected to a stochastic model in order to evaluate the structural behavior of Nigerian woods when used as a bridge deck.

This work is limited to the structural behavior of plank decks when used as bridge decks. Girder or stringer designs are not included, so also are effects of preservatives and perish ability of the timbers are not included in this work. The stochastic data used for this work are based on real tests from existing literatures.

The design considerations and properties of timber will be based on NCP 2 (1973), and LRFD AASHTO (2010).

CHAPTER 2

2.0 LITERATURE REVIEW

2.1 INTRODUCTION

The source of any structural timber is the wood which is obtained from a tree. A tree can be seen as a structure that faces an ongoing optimisation process over millenniums(Kohler, 2007).Its function is to expose leafs to sunlight and due to the competitive situation with other plants, leafs have to be lifted up far above the ground. Every part of the tree has different mechanical and biological functions. The uptake of water and minerals in the roots, the transport of these nutritional agents, the sap, through the stem and the branches and the photosynthesis in the leaf exposed to the sunlight describes briefly the biological function of the different parts. Mechanically, the leafs have to resist the direct exposure to wind and rain, whereas the branches and the stem have to support the crown and transfer the corresponding load effects to the roots where the tree is anchored into the ground.

However, trees have developed the ability to react continuously on every specific boundary condition; that is, during growth, trees are able to optimize their shape and mechanical properties. Regarding this aspect, we can adequately distinguish between two classes of growth; the primary and secondary growth. The so called secondary growth in the thickness direction of the tree components is of high importance, because, the thickness of branches and stems determines the strength and stiffness (or the supporting qualities) of the tree(Kohler, 2007). Secondary growth takes place just below the outer surface of the stem or branch in the cambium, where phloem and finally bark are produced outwards, and wood is grown inwards (Niemz, 2004). The inwardly grown wood is an annual tree ring consisting mostly of more porous early wood, and

denser and stronger late wood. Aiming at an even stress distribution in the supporting part of the tree the mechanical performance is continuously reassessed and more and stronger material is accumulated by secondary growth on the locations where the stresses are the highest (Kohler, 2007). Simultaneously, the primary growth is aspiring to a tree design which is balanced between light exposure and affordable moment forces in the supporting parts (Mattheck, 1998).

However wood can be seen as a hard fibrous tissue found in many trees. It has been used for hundreds of years for both fuel and as a construction material. On every scale it can be recognized that tree as a source of wood is a material with highly optimized properties as long as the original purpose is aimed at the mechanical support and the transportation of sap in a tree (Kohler, 2007). However, the material properties become sub-optimal when wood material is reused for human purposes, e.g. as timber for building structures (Kohler, 2007).

2.2 CLEAR WOOD SPECIMENS AND THEIR PROPERTIES

The characteristics of wood are usually obtained from tests of small pieces of wood termed “clear” or “straight grained” (Kohler, 2007), because they are assumed not to contain characteristics that lowers the technical quality or commercial value of the wood. These test pieces have anatomical characteristics such as growth rings that occur in consistent patterns within each piece. Clear wood specimens are usually considered “homogeneous” in wood mechanics (Niemz, 2004). The properties of a clear wood specimen include the following, but not limited to; moisture content, shrinkage and swelling, density, strength and stiffness properties, moisture content and mechanical properties. These are explained below in the next subsections.

2.2.1 Moisture Content

The moisture content is defined as the ratio of the mass of removable water to the oven dry mass of the wood. The oven dry mass is obtained by oven drying the specimen. The moisture content may be expressed as a fraction or in percentage terms. In so-called green wood, that is, wood of a freshly cut tree, the moisture content could be higher than 100% (Kohler, 2007). At a moisture content of around 26%, the so-called fiber saturation point, the wood cell walls are still saturated, whereas there is no free water that is moisture has been removed from the cell cavity. Below the fiber saturation point the cell wall is losing water which has a major influence on the mechanical properties of the wood. Above the fiber saturation point the mechanical properties are nearly independent of moisture content (Hoffmeyer, 1995).

2.2.2 Shrinkage and Swelling

Wood is dimensionally stable when the moisture content is greater than the fiber saturation point (Kohler, 2007). Wood changes dimension as it gains or loses moisture below that point. It shrinks when losing moisture from the cell walls and swells when gaining moisture in the cell walls. Shrinkage and swelling are also termed movements. The magnitude of these movements is dependent on the direction to the wood cells and is mainly governed by the substantial layer of the wood cell wall (Kohler, 2007). Wood prisms are deformed due to shrinkage in different manner, depending on the orientation of the annual rings in the prism; these deformations are in general referred to as distortions (Hoffmeyer, 1995).

2.2.3 Density

The wood density is the most important physical characteristic of wood (Kohler, 2007). The density ρ_{den} is defined as, the ratio of mass to that of its corresponding volume:

$$\rho_{den} = \frac{m_g}{v_{vol}} \quad (2.1)$$

Where m_g is the mass in kg and v_{vol} is the volume in m^3 . According to Nigerian standard code of practice for the use of timber for construction, the density $\rho_{den,18}$ at 18% moisture content are most frequently used. While the density of the cell wall material is relatively constant among wood species, which is about 1500 kg/m^3 (Kohler, 2007; Kollmann et al., 1968), the density of wood depends mainly on the ratio between cell cavity and cell wall. The density ranges from 200 kg/m^3 to 650 kg/m^3 for coniferous wood and from 300 kg/m^3 to 1100 kg/m^3 for deciduous wood (Kohler, 2007).

2.2.4 Strength and Stiffness Properties

The strength and stiffness properties of wood depend on which direction the fibers are stressed. For small clear wood specimens it is in general distinguished between properties parallel and perpendicular to the grain. The Table 2.1 below gives data on some structural materials including timber.

Due to the fact that wood in a tree is a highly optimized structural material, it is no surprise that its performance is impressive if it is stressed as in nature, for example, clear wood exhibits high strength and stiffness in tension parallel to the grain; the ratio between strength and density is even higher than for steel (see Table 2.1). On the other hand, if clear wood is

loaded perpendicular to the grain the load carrying capacity is very low, (Thelandersson and Larsen, 2004).

Table 2.1- Strength Properties and Density of Some Structural Materials

Material	Density <i>kg/m³</i>	Strength <i>MPa</i>	Strength/Density <i>10⁻³MPa.m³/kg</i>
Structural steel	7800	400-1000	50-30
Aluminum	2700	100-300	40-110
Concrete, compression	2300	30-120	13-50
Clear coniferous wood, tension parallel to the grain	400-600	40-200	100-300
Clear coniferous wood, compression parallel to the grain	400-600	30-90	70-150
Clear coniferous wood, tension perpendicular to the grain	400-600	2-8	5-10

(Thelandersson And Larsen,2004).

2.2.5 Moisture Content and Mechanical Properties

The mechanical properties of wood depend on the moisture content. An increase in moisture leads to lower values for strength and stiffness properties. Water, when penetrating the cell wall, weakens the coherence of the cell wall. Moisture variations above the fiber saturation point of around 26% have no effect on the mechanical properties, since such variations are related to free water in the cell cavity. Not only the moisture content is important for the strength properties of wood specimen; fast changes cause moisture gradients in wood which induce stresses perpendicular to the grain direction. Wood is a hygroscopic material that takes water from the surrounding air, and the moisture content of wood tends to attain equilibrium with the air humidity.

The effect of moisture on the mechanical properties of wood is given in Table 2.2. For moisture contents between 8% and 20% a linear relationship between moisture content and strength can be assumed (Hoffmeyer, 1995).

Table 2.2 - Approximate Change (%) of Clear Wood Properties

Material property	Change (%)
Compression strength parallel to the grain	5
Compression strength perpendicular to the grain	5
Bending strength parallel to the grain	4
Tension strength perpendicular to the grain	2.5
Tension strength parallel to the grain	2
Modulus of elasticity parallel to the grain	1.5

The above table is for a one percentage change of moisture content. (Hoffmeyer, 1995).

2.3 STRUCTURAL TIMBER

Timber is as different from wood as concrete is different from cement (Madsen *et. al*, 1992). Structural timber is the timber used in framing and load-bearing structures, where strength is the major factor in its selection and use (Aguwa and Sadiku, 2011). Structural timber components are sawn from logs. Apart from some exceptions they have a prismatic shape with a rectangular cross section. In contrast to small clear wood specimen timber components have structural dimensions. The maximum possible dimension of the timber components is determined by the size of the trees in the forest, for example, one hundred years ago timber components with a length of 20m and a rectangular cross section of about 150 x 450 mm were commonly available (Kohler, 2007). Nowadays, the forestry strategy has changed and in most countries

timber with a cross section over 75 x 225 mm and a length more than 5 m attracts a cost premium due to scarcity, Steer, e.t.c., (Hoffmeyer, 1995). Timber components are applied for load carrying functions in structures; therefore there are several provisions in the production line to obtain appropriate structural elements, for example, it is aimed for, that the longitudinal axis of the timber structural component coincides with the grain direction of the wood cells (Kohler, 2007). Due to the dimensions of the timber components, this is mostly not strictly possible. The so called growth irregularities as knots and grain deviations are affecting the uniform and parallel grain direction. It should be remembered that these 'growth irregularities' are just part of the excellent property of a tree to react on stress peaks and special load conditions constantly during its growth (Kohler, 2007). However, for a sawn piece of timber, these growth irregularities are suboptimal.

2.4 GROWTH IRREGULARITIES IN TIMBER STRUCTURAL COMPONENTS

2.4.1 Knots

A portion of a branch enclosed in the wood by the natural growth of the tree. Being an optimal stress reducing connection of the stem with the branches in a tree, knots in sawn structural timber are by far the most important defects affecting the mechanical properties (Kohler, 2007).

2.4.2 Cross Grain

The phenomenon, when the mean grain direction within a large section is not coinciding with the longitudinal axis of the timber component is referred to as cross grain. This is quite common, but only recognized at a certain magnitude. The reason for cross grain might be technical, that

is, due to sawing configurations, or due to growth particularities, for example, helix-growth, or a cone shaped stem of the tree.

2.4.3 Distortion

This is generally due to moisture movement as a result of shrinkage and swelling. As for clear wood prisms, distortion due to inhomogeneous shrinkage and swelling properties is also present for timber structural elements.

2.4.4 Wane

Timber structural elements, in general, have a rectangular shaped cross section. Due to sawing practice it is possible that the edge of the timber log becomes visible after cutting the rectangular shaped elements from the conical shaped log. This is in general referred to as wane (Kohler, 2007).

2.4.5 Permanent Compressive Yield

Permanent compressive yield is a local phenomenon due to an instant compression in the stem of the living tree, for example, during a storm the compression strength can be reached on a spot where local buckling of the fibers occurs. This happens without further consequences for the tree. In a sawn timber component these spots can be very disadvantageous due to very low strength characteristics (Kohler, 2007). Permanent compressive yield is very hard to detect by visual quality control (Kohler, 2007; Arnold and Steiger, 2005).

2.5 NIGERIAN TIMBER

Nigerian timber can be grouped in terms of properties and characteristics. The strength group is one of seven groups designated N_1, N_2, \dots, N_7 into which it has been found convenient to divide Nigerian timbers. The strongest of the group is N_1 while the weakest is N_7 (NCP2; 1973). One basic disadvantage of timber in Nigeria is that they are usually exported to other parts of the world and basically Europe (Abejide, 2007). Also they could be adapted and grown elsewhere in the tropics. Each species is classified according to the ability of the heartwood to resist attack by fungi and insect. Five durability classes are used as follows: very durable, durable, moderately durable, non-durable, and perishable. The ease with which they can be impregnated with preservative is of importance when they are used under conditions which favor decay or attack by insects or marine borers. Five classes indicating the resistance of the heartwood to impregnation are as follows: permeable, moderately resistant, resistant, very resistant, and extremely resistant (Abejide, 2007).

2.6 BRIDGES IN RETROSPECTIVE

Road bridges certainly played an important role in the transport systems of old cultures, but amazingly little is known about them. Exceptions are Caesar's bridge over the Rhine or Trojan's bridge over the Danube (Bell, 2007; Meierhofer, 1996).

Considering the entire history of men, Meierhofer(1996) reiterated that timber has been undoubtedly the most frequently used material for bridge building; even the Romans, who developed a highly sophisticated building technology with natural stones, used a lot of timber not only for bridges but also for other major structures like the Coliseum in Rome. After the collapse

of the Roman Empire their amazing road system broke apart due to the lack of maintenance and many of the building skills of the Romans were lost. More simple methods were applied later on, techniques which had been used at all times by craftsmen for less important crossings. These techniques can be characterized by short span, low load bearing capacity, cheap material, little workmanship and low durability.

Very little is known about the art of bridge building between the Roman times and the Middle Ages (approx. 1200 A.D.) as noticed by Meierhofer(1996). By then the road systems had gained importance again and included safe and durable bridges. From that period, there are considerable evidence of timber bridges, mainly of covered ones. The roof obviously was recognized as being a most efficient means of protecting the bearing structure from the adverse influence of the weather. These bridges mostly had an abutment of stone to separate the timber structure of the bridge from the moist soil. Those builders knew perfectly well that - besides hostile men - rot is the most dangerous enemy of timber bridges - and can be prevented by keeping the wood reasonably dry.

The evidences of those timber bridges are not restricted to pictures and written documents, for the durability achieved with the technology of that time allowed many of these structures to stay in service until now. Meierhofer(1996) sighted one of the famous examples as the Chapel-bridge in Lucerne, which lasted until 1993 when most of it was destroyed by a fire. By the way, this has been the fate of most of the historical timber bridges: They were not destroyed by the ravages of time, but were burnt down in the many quarrels and wars of European history.

One of Switzerland famous timber bridge builder as pointed out by Meierhofer(1996) are the Hans Ulrich Grubenmann family, which represents certain climax of craft skilled timber bridges up to the 19th century. This family built the famous bridge over the Rhine at Schaffhausen in Switzerland in 1758, which had a length of 120 m (400ft), though longer bridges were built by other builders. Then new techniques were introduced, triggered by an increased availability of iron at a considerably reduced price due to new manufacturing methods.

2.7 BRIEF HISTORY OF TIMBER BRIDGES

The first bridges constructed by mankind were probably woodstructures (Howard, 1997). Before the advent of first cast iron bridge (the Iron Bridge near Coalbrookdale on the river Severn, opened in 1781), then steel bridge (one of the first major steel bridges, the King Albert Bridge over the Elbe, was opened in 1893), and finally reinforced concrete, stone and timber were the only available materials for bridge building (Bell, 2007). While there are many fine examples of very old stone bridges, this is, for obvious reasons, not the case for timber bridges - albeit some covered timber bridges have survived for a remarkably long time, the most famous example being the Chapel-bridge in Lucerne, built in the period between 1300 and 1333, which is still standing (restored after a fire in 1993) most of the timber bridges of the past have disappeared (Meierhofer, 1996; Bell, 2007). Caesar's bridge over the Rhine, built around 50 B.C., is quoted as one of the first major timber bridges in Europe. Andrea Palladio (1508-80), an Italian architect, is often mentioned in connection with timber bridges of the past, both for his introduction of the timber truss bridge design, and for some famous bridges, like the Ponte degli Alpini built in 1567 at Bassano (Bell, 2007).

While in the United States, the building of the railroads in the latter part of the 1800's produced many, quite large timber bridges, which include the Cascade Bridge, built by Thomson Brown as early as 1845- was a truss-arch-truss bridge with a span of 90 meters-, which was quoted by a visiting Swiss engineer to be one of the finest timber structures in the US(Bell, 2007).

However, there was a decline of timber bridge building which was triggered by the development of competing materials: steel and concrete. With steel and reinforced concrete on the scene, road and railway bridges made of timber more or less disappeared in the 20th century- except for very short spans which exist in very large numbers both in North America and Australia; but most of them are fairly insignificant structures and hardly considered to be bridges by the traveling public - not until around 1990(Bell, 2007).

Meierhofer(1996) believes that, building bridges is a matter - among other things - of topography, transportation needs, economic potential, cultural heritage, availability of suitable materials and experienced craftsmen, fashions, prestige, preferences and prejudices. Bridges often have a symbolic character and reflect the spirit of the times in which they were built. Thus as a result of optimistic demand for sustainable resources, the last two decades have seen a growing interest in timber bridges in many European countries. This is why Howard (1997), was quick to point out that, there has been a dramatic increase in the number of new timber bridges constructed in the latter portion of the 20th century. There are several reasons for this, as noted by Howard (1997), that the competitive challenges posed by modern engineering encourage the development and use of timber for construction; while Bell (2007) stated his reasons to include; the growing interest in environmental questions and sustainability has definitely paved the road

for more use of structural timber, but also new and innovative use of timber, such as the stress laminated timber deck and better connections, have played an important role; the fact that reinforced concrete did not turn out to be an everlasting material - many countries have experienced serious problems with concrete bridges built in the 1960's and 70's - and this is another factor; however for the last, but not least, the enthusiasm of individuals should also be acknowledged.

2.8 DECLINE OF TIMBER BRIDGE BUILDING

The most crucial challenge for a timber bridge is perhaps the question of longevity which brings in protection and durability as major concerns. The decline in use of timber bridges was promoted by decrease in the price of iron and later steel, associated with quick technology development; iron and steel became increasingly popular for bridge building and the use of timber declined rapidly in the second half of the 19th century (Meierhofer 1996). However, Bell (2004) also makes his assertion by concluding that, with steel and reinforced concrete on the scene, road and railway bridges made of timber more or less disappeared in the 20th century, except for very short spans.

Steel and reinforced concrete represented modern times and technology. The seemingly unlimited new possibilities obtained with the use of both steel and concrete have edged timber completely out of the minds of bridge engineers. Besides the image of being outdated - and not offering any superlatives (strongest, biggest, longest) -, Meierhofer (1996) believes that, timber as a bridge material was disregarded for several other reasons, which include: the traffic loads had multiplied, urging the use of high strength materials, scientific research and development resulted in a high level technology of steel- and concrete production, large and powerful steel and

concrete industries developed, and absorbed and implemented new knowledge. Tough competition kept (and still keeps) the price of steel and concrete low and professional training in steel and concrete technology and design is thorough and well recognized. In the same period, Meierhofer(1996) reiterated that wood and timber engineering remained a research topic of low significance. Production - except that of the wood derived panel products - remained in the hands of numerous local saw-millers and craftsmen, lacking a substantial capital base and innovative drive. Professional training at the crafts-men's level was (and is) satisfactory but traditional –this is evident in the local craft men moulding various house hold and kitchen appliances.

Meierhofer(1996) noted that, for at least half a century the art of timber bridge building remained in a state of hibernation, fifty years during which an enormous increase of all building activities took place in the industrialized part of the world - a period strictly dominated by concrete and steel.

2.9 REVIVAL OF TIMBER BRIDGE BUILDING

Timber bridges are becoming more prevalent as new bridges are constructed for short to medium spans (Howard, 1997). A gradual revival of Timber Bridge building as noticed in European and American countries by Meierhofer(1996) started about 1980. Some 20-25 years ago timber became an interesting material also for longer bridge spans, both in Europe and North America, and one now finds a fair number of medium size timber bridges, even on major roads(Bell, 2007). It was based mainly on earlier developments in timber engineering of glued laminated timber. Numerous innovative projects resulted in some spectacular structures, particularly roof structures for sport stadiums. All these structures have one advantage bridges don't have: The carrying structure is normally well protected from the weather and presents few

complications regarding durability. Even though adequate durability is one of the major tasks in timber bridge construction, there were and certainly still are more dominating obstacles to a more frequent use of timber (Meierhofer, 1996). As stated by Meierhofer(1996) considering Europe, it is well known that, not only in Africa, but in most countries the limited knowledge and experience of most design engineers and officials in highway departments leads to a generally negative attitude towards the use of timber for bridges.

The renaissance of the timber bridge demonstrates that technical as well as mental obstacles can be overcome according to Meierhofer(1996).

2.10 TIMBER BRIDGES

So why consider timber? Many people choose timber for various reasons, chief among them; cost, strength, durability, environmental friendliness and aesthetics e.t.c. Gilbert (2006) noted that, in some cases, aesthetic concerns can make timber bridges a preferred alternative over other types. The motive to build bridges faster, to last longer, for less money and with aesthetic appeal has led to the quest for the perfect bridge material. High strength steel, form liners for concrete, new coatings for reinforcing bars, plastic composites all enter the discussion. One material with a proven track record is often overlooked – treated timber (David, 2011). Developed from years of monitored installations, innovations in materials and advances in design and construction, treated timber provides all the attributes of a quality bridge material (David, 2011). With recent escalations in the cost of steel and concrete, timber bridges are benefiting from increased scrutiny as a design alternative in European and American countries. In addition to the abundant supply of raw material, timber bridges are often quicker to build. Since timber components are smaller in size and lighter in weight than their steel and concrete counterparts, smaller equipment is required

to install and service them. Alternative designs for timber bridges, though numerous, benefit from many standardized design and construction techniques, also helping to keep installation costs low (Gilbert, 2006).

Meierhofer (1996) further outlines a considerable number of arguments to encourage the use of timber for bridge construction today. This includes technical and other requirements such as;

I. Technical arguments

- a) The availability of new types of wood-based products with engineered properties.
- b) The fun of rediscovering a “new-old” fascinating material with many interesting properties.
- c) the better knowledge of the properties of wood and the increasing ability to take advantage of specific characteristics of wood in an appropriate manner, new possibilities of modifying wood and combining it with other materials to fulfill special tasks within the timber structures.
- d) New ways and means of manufacturing, quality control, transporting and erecting wood structures.

II. Other arguments

- a) The rediscovery of timber as an environmentally friendly, renewable resource with virtually unlimited supplies.
- b) A fashion-like preference for architectural features expressing the characteristics of timber structures.
- c) The potential of wood and timber structures to promote a certain design quality which cannot be achieved with other materials and last but not least.

- d) Some loss of image of the materials competing with wood, particularly the increasing awareness that concrete is by far not the everlasting material it was expected to be and the high cost of steel production, compelled with depleting resources..

Critical analysis of the circumstances leading to the current increase in the use of wood as construction material for bridges in Central Europe, Meierhofer(1996) noticed that three dominating factors are responsible for it, two of which are of a technical nature:

The first factor is the development and use of pressure treated glulam for which a base was laid in extensive tests in the seventies. This has allowed the use of glulam elements in the more hazardous circumstances normally occurring in bridge constructions.

The second factor has been the development of the stress laminated bridge deck, a development which started also in the seventies in Ontario. A later development was the glued laminated bridge deck. Compared to former techniques, such bridge decks are characterized by a certain monolithic solidity, allowing an efficient plate action of the deck, that is a better load distribution and providing also a solid base for the moisture barrier and the wearing surface (pavement). In view of the everyday problems of bridge maintenance and repair, this factor should not be underestimated. Today, laminated bridge decks are important for new timber bridges, as well as for retrofitting existing ones.

The third and last factor has been a slowly changing attitude of building owners, politicians and especially the responsible officials in highway departments, which lead to a more positive attitude towards the use of timber.

For a long time, the knock against timber was that it had a short service life. However, by its very nature, timber is an extremely durable material. Properly designed and detailed, treated timber bridges have demonstrated durability with installations in service for more than fifty years (David, 2011). Timber has a remarkable ability to handle short-term overloads without adverse effects and has the added advantage of holding up to freezing and thawing cycles while resisting deterioration from deicing agents, this is in combination with the advent of new wood preservation techniques, which makes timber bridges last much longer (Gilbert, 2006).

Another benefit of timber construction is that it is generally less disruptive to the environment. When compared to the production of steel and reinforced concrete, the energy required to produce and transport treated wood is far less (David, 2011). Unlike alternative designs that utilize long spans and large piers, timber bridges utilize short spans and pile bents that do not require large excavations, cofferdams, dewatering or other procedures disruptive to sensitive environments (Gilbert, 2006). In many instances, a timber bridge can be built entirely from the deck down, with minimal disturbance to ground-dwelling plants and animals (David, 2011).

Arguably, according to Howard (1997), timber bridges do have some disadvantages compared to other construction materials. Convincingly, Howard (1997), was able to point out that wood is an organic material and is susceptible to damage by fungi, insects, marine borers, fire and wear. The fungal damage will occur only when the moisture content of the wood approaches or is greater than the fiber saturation point of around twenty percent to eighteen percent. Chemical preservation of wood is used to prevent or retard damage by fungi, insects, marine borers and fire. A protecting surface, such as asphalt, is applied to timber bridge decks to

prevent wearing. Attention must be given to avoid excessmoisture buildup between the timber elements and the wearing surface. Also,deflections of the timber deck must be limited to prevent premature cracking of the asphalt surface; this is a visible problem with common timber bridges.Howard(1997) iterated that, these cracks are probablycaused by the deflection of one timber deck panel with respect to an adjacent timber deck panel, a phenomenon he termed interpanel differential deck deflection.

Undoubtedly the improved durability image of outdoor timber structures - among them also bridges - is based to a considerable extent on chemical treatments for preservation. However, considering timber bridges, durability is more and more a dominating design factor. In the last twenty years, it was observed by Meierhofer(1996) that environmental concerns have clearly spoken increasingly against the use of toxic chemicals for wood preservation, which has recently prompted some less experienced designers to omit chemical treatments completely, without taking into account the far reaching consequences: Some design configurations just don't work without the help of chemical treatment, and completely different designs may be necessary. But it should be clear that improper approaches to any design may result in substantial damage within a short time. Thus, improper definition of the preservative to use may quickly and thoroughly destroy the 'durability image' of timber bridges. Over time,treated timber has experienced changes in preservatives and treating practices. General usepreservatives such as copper naphthenate are replacing restricted use preservatives likecreosote (David, 2011). The requirement to minimize or to avoid completely chemical preservation clearly reduces the multiplicity of design possibilities and enhances the importance of constructive measures, i.e.

structural detailing. Nevertheless one hope that the innovative drive in timber bridge design observed in the last few years will not lose its momentum(Meierhofer, 1996).

Looking at other perspectives of Central European timber bridge design - beyond durability considerations - it seems that engineers have acquired a new freedom of combining the various materials available to them: timber, steel, concrete, wood derived materials, and last but not least, high performance fibers(Meierhofer, 1996), which are increasingly the subject of rehabilitation and design and full scale implementation also in the building domain.

Composite structures is certainly one of the distinct trends considering economy, environmental friendliness and ease of manipulation in the present dispensation, based on newly developed technologies for joining various materials by gluing or more conventionally with mechanical fasteners. Concrete bridge decks on glulam girders are in many cases a viable alternative to a wooden deck on a glulam timber girders as the monolithic deck can provide an efficient weather protection for the underlying timber structure, but one of the important details still in need of improvement is the efficient and economic shear transfer between the deck and girders(Meierhofer, 1996).

Treated timber bridge decks are built in days not weeks, because materials are low energy, certified, reusable and renewable, where components are shop manufactured under controlled conditions to maintain quality (David, 2011). As a result, the use of timber will reduce detour times and construction inspection expense, because controlled shop conditions will improve quality and thus, reduce the need for site inspection. This is why David (2011) iterated that, accelerated bridge construction has long been an attribute of treated timber bridges in which the design details have been refined for over forty years. However, according to Meierhofer(1996),

substantial efforts are necessary to maintain and consolidate the current rather favorable general attitude towards timber bridge building. For such an objective, it is highly desirable that many engineers with enthusiasm and solid knowledge of timber construction create and try out new attractive designs. On the other hand it may be less spectacular but still of higher importance regarding the marketing potential that the respective industries get interested in developing, producing and marketing technically and economically convincing utility bridges. While the quest continues for the perfect bridge material, treated timber already provides many of the advantages desired of new materials and construction techniques (David, 2011).

It is in this bid that this work propose and evaluate the possible use of timber bridges and decks for the construction and or rehabilitation of road bridges both in rural and urban settings of Nigeria (and other parts of the country) in order to open up new roads while at the same time rehabilitate old and or abandoned bridges.

The advantages of this include linking all remote areas for economic development and providing prompt government accessibility to such areas.

Because of the variability in the types of wood and timber structures, the design parameters for bridges and/or rehabilitation, it is of necessity that a stochastic procedure that will optimize each parameter in order to achieve or obtain an optimum design for the required timber in each strength class of timber needs to be employed.

The method for the stochastic process used in this work is presented in chapter three.

CHAPTER 3

3.0 RESEARCH METHODOLOGY

3.1 INTRODUCTION

In this work, the concept of Advance Second Moment Reliability Assessment (ASMRA) method is used to assess the reliability (or the possible existence) of using different suitable species of Nigerian timber for bridge decks. There is a need for a fully probability approach to reliability assessment of structural timber decks, because many factors, such as climate, material composition of structural timber, occurrence of natural disasters and impacts of man-made technology control the existence of human beings and the quality of the surrounding environment of the bridge deck. Many of these quantities cannot be represented adequately by deterministic values or relationships, thus, the variability in these quantities needs to be included in models of both nature and the built environment (Anagnos and Marek, 1996).

As it was observed by Nowak and Saraf (1996), that truck parameters which is one of the major variables influencing the performance function of timber bridge decks are strongly site-specific, so also, are modulus of rupture (MOR), modulus of elasticity (MOE), the variation of moisture content and durability of load combined effect in time (which are both nonlinear dependency functions) lead to the reduction of timber strength over time (Lokja and Marek, 2006).

To define the variability of these random variables as they affect timber decks, the various model aspects of stochastic process listed below will be examined, which includes;

- i. Dependencies between variables in the sequence. This defines the combined effect of those factors which will undermine the durability of timber bridge decks.
- ii. Various kinds of long-term averages. This is examined in terms of probability of failure over time which will help to determine the sustainability of the bridge decks.
- iii. The frequency at which “boundary events” will occur. The boundary events are the limit states which are defined by the performance functions of the bridge decks, while the frequency of occurrence are examined either in service or at collapse (that is, serviceability and ultimate limit state respectively).

The above can be achieved through numerous stochastic processes which define a particular Probability Distribution Function (PDF). The processes include the following, but not limited to;

1. Arrival processes such as the Bernoulli and Poisson processes which model the frequency of “arrivals” or “successes” in some time-dependent model. In general, we consider models in which interarrival times are independent random variables.
2. Markov processes, which are sequences where the next value depends on the past only through the current value.
3. Martingales are processes where the expectation of the next value is exactly the same as the current value, e.t.c.

With adequate PDF to represent the possible variability of each individual random variables, the combined effect of these variables on the performance function of a bridge can be used to study the reliability or possibility of using Nigerian timbers for bridge decks. The reliability which can be seen as the detection of rare physical events such as failures, which

usually occur with low probability, play key roles in the probabilistic reliability assessment of engineering structures (Praks and Brozovsky, 2010).

3.2 ADVANCE SECOND MOMENT RELIABILITY ASSESSMENT METHOD (ASM RAM)

This method is also referred to as mean value first-order second moment method, and it is based on the first order Taylor series approximation of the performance function linearized at the mean values of the random variables. It uses only second-moment statistics (mean and variance) of the random variables. Originally, Cornell (1969) used the simple two variable approaches. On the basic assumption that the resulting probability of performance function $Z(X)$, is a normal distribution, by some relevant virtue of the central limit theorem, Cornell (1969) defined the reliability index as the ratio of the expected value of $Z(X)$, over its standard deviation.

Reliability concept uses a function, $Z(X)$, which determines the performance of the structural system. Thus it is also known as the performance function of the system. Therefore the performance function of the structural system is always defined as

$$Z(X) = R(X) - L(X) \tag{3.1}$$

Where;

R(X) and L(X) are material resistances and load effects which depend on the random variable X

At the extreme point of failure (or the limit state), $R(X) = L(X)$ and $Z(X) = 0$

Thus, the point of interest can be defined as $Z(X) = 0$.

When $Z(X) < 0$, the element is in the failure state, and when $Z(X) > 0$, it is in the survival state.

If the joint probability density function for the basic random variables X_i 's is $f_{x_1, x_2, \dots, x_n}(x_1, x_2, \dots, x_n)$, then the possibility of failure known as the failure probability P_f of the element can be given by the integral

$$P_f = \int \dots \int f_{x_1, x_2, \dots, x_n}(x_1, x_2, \dots, x_n) d_{x_1, x_2} \dots d_{x_n} \quad (3.2)$$

Where the integration is performed over the region in which $Z(X) < 0$.

In general, the joint probability density function is unknown, and the integral is a formidable task (Assakkaf, 2004).

For practical purposes, alternate method of evaluating P_f is necessary. The possibility of survival (existence) of the structure known as the RELIABILITY is assessed as one minus the failure probability.

Instead of integrating equation (3.2), directly, the performance function $Z(X)$ in Equation(3.1) can be expanded using Taylor series about the mean value of X s and then truncated at the linear term. Therefore, according to (Assakkaf, 2004), the first order approximation for the mean and variance of the performance function are as follows:

$$\mu_z \approx Z(\mu_{x_1, x_2, \dots, x_n}) \quad (3.3)$$

$$\sigma_z^2 \approx \sum_{i=1}^n \sum_{j=1}^n \left(\frac{\partial Z}{\partial x_i} \right) \left(\frac{\partial Z}{\partial x_j} \right) Cov(X_i, X_j) \quad (3.4a)$$

Where,

$\mu = \text{mean of random variable}$

$\mu_Z = \text{mean of } Z$

$\sigma_Z^2 = \text{variance of } Z$

$\text{Cov}(X_i, X_j) = \text{Covariance of } X_1 \text{ and } X_2$

$\frac{\partial Z}{\partial X_i} = \text{partial derivative evaluated at the mean of random variable}$

For uncorrelated random variables, the variance can be expressed as

$$\sigma_Z^2 \approx \sum_{i=1}^n \sigma_{x_i}^2 \left(\frac{\partial Z}{\partial X_i} \right)^2 \quad (3.4b)$$

The reliability index β can be computed from:

$$\beta = \frac{\mu_Z}{\sigma_Z} \quad (3.5)$$

$$P_f = 1 - \Phi(\beta) \quad (3.6)$$

The equation above holds only when Z is linear and normally distributed (Assakkaf, 2004)

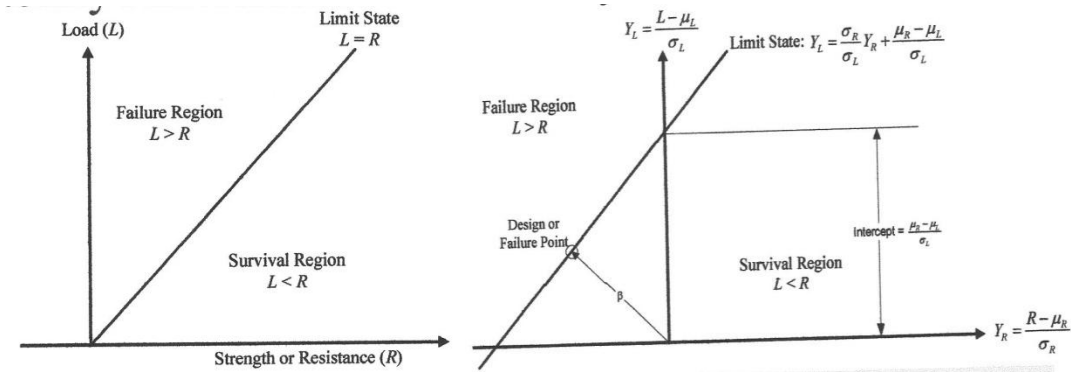
For nonlinear performance functions, the Taylor series expansion of $Z(X)$ is linearized at some point on the failure surface referred to as the design point or checking point or the most probable failure point rather than at the mean.

Assuming the performance function for material resistance and load effect is given by $Z(X_i) = R(X_i) - L(X_i)$, where X_i are uncorrelated random variables; the following transformation to reduce or normalise coordinates can be used:

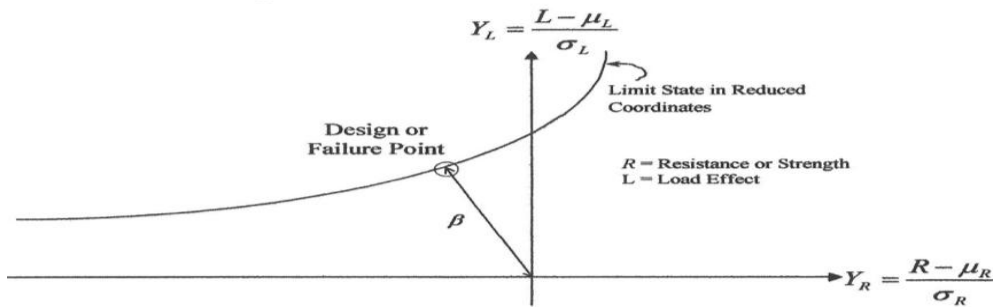
$$Y_R = \frac{R(X_i) - \mu_{X_R}}{\sigma_{X_R}} \text{ And } Y_L = \frac{L(X_i) - \mu_{X_L}}{\sigma_{X_L}} \quad (3.7a)$$

Where $\mu_{Y_i} = 0$ and $\sigma_{Y_i} = 1$

At the failure surface $R(X_i) - L(X_i) = 0$ or $R(X_i) = L(X_i)$



(a) (b)



(c)

Figure 3.1 - Performance function

- (a) Performance function for a linear, two normally distributed random variable case. (b) Performance function for a linear, two random variable case in normalized coordinates. (c) Performance function for a nonlinear, two random variable case in a normalized coordinates.

This implies that

$$Y_L = \frac{\sigma_{X_R}}{\sigma_{X_L}} Y_R + \frac{\mu_{X_R} - \mu_{X_L}}{\sigma_{X_L}} \text{ or } Y_L \sigma_{X_L} - Y_R \sigma_{X_R} - (\mu_{X_R} - \mu_{X_L}) = 0 \quad (3.7b)$$

Equation (3.7b) is equation of a straight line.

Considering figure 3.1(b) and dividing Equation(3.7b) through by $\sqrt{\sigma_{X_R}^2 + \sigma_{X_L}^2}$, we have

$$\frac{Y_L \sigma_{X_L}}{\sqrt{\sigma_{X_R}^2 + \sigma_{X_L}^2}} - \frac{Y_R \sigma_{X_R}}{\sqrt{\sigma_{X_R}^2 + \sigma_{X_L}^2}} - \frac{(\mu_{X_R} - \mu_{X_L})}{\sqrt{\sigma_{X_R}^2 + \sigma_{X_L}^2}} = 0$$

This implies that $Y_L \alpha_{X_L} - Y_R \alpha_{X_R} - P = 0$

$$\text{Where; } \alpha_{X_L} = \frac{\sigma_{X_L}}{\sqrt{\sigma_{X_R}^2 + \sigma_{X_L}^2}}, \alpha_{X_R} = \frac{\sigma_{X_R}}{\sqrt{\sigma_{X_R}^2 + \sigma_{X_L}^2}}, \text{ and } P = \frac{(\mu_{X_R} - \mu_{X_L})}{\sqrt{\sigma_{X_R}^2 + \sigma_{X_L}^2}},$$

α_{X_i} = directional cosine of the perpendicular distance from the origin and

P = perpendicular distance from the origin

The perpendicular distance of the linearly normalized performance function from the origin which is also the shortest distance from the origin, is a measure of the possible design point (failure point) given by $Z(X_i^*) = (X_1^*, X_2^*, \dots, X_n^*)$ from the origin (mean). This indicates that P is a measure of the system reliability, thus;

$$P = \beta = \frac{\mu_{X_R} - \mu_{X_L}}{\sqrt{\sigma_{X_R}^2 + \sigma_{X_L}^2}} \quad (3.8)$$

The concept of perpendicular and shortest distance applies to linear performance function; while for nonlinear performance function only the shortest distance dictate the possible failure point, as shown in figure 3.1(c).

Equation (3.8) can only be used for a two variant performance function in a linear plane. But for a multi-variant performance function on a hyperplane, β is always given as in Equation(3.5), that is;

$\beta = \frac{\mu_Z}{\sigma_Z}$, But from (3.4b), $\sigma_Z^2 \approx \sum_{i=1}^n \sigma_{x_i}^2 \left(\frac{\partial Z}{\partial X_i}\right)^2$, thus

$$\beta = \frac{\mu_Z}{\left\{ \sum_{i=1}^n \sigma_{x_i}^2 \left(\frac{\partial Z}{\partial X_i}\right)^2 \right\}^{1/2}} \text{ or } \mu_Z = \beta \left\{ \sum_{i=1}^n \sigma_{x_i}^2 \left(\frac{\partial Z}{\partial X_i}\right)^2 \right\}^{1/2}$$

$$\mu_Z = \beta \left\{ \sum_{i=1}^n \sigma_{x_i}^2 \left(\frac{\partial Z}{\partial X_i}\right)^2 \right\}^{1/2} \times \frac{\sum_{i=1}^n \sigma_{x_i}^2 \left(\frac{\partial Z}{\partial X_i}\right)^2}{\sum_{i=1}^n \sigma_{x_i}^2 \left(\frac{\partial Z}{\partial X_i}\right)^2}$$

$$\mu_Z = \beta \times \frac{\sum_{i=1}^n \sigma_{x_i} \left(\frac{\partial Z}{\partial X_i}\right) \times \sum_{i=1}^n \sigma_{x_i} \left(\frac{\partial Z}{\partial X_i}\right)}{\left\{ \sum_{i=1}^n \sigma_{x_i}^2 \left(\frac{\partial Z}{\partial X_i}\right)^2 \right\}^{1/2}}$$

$$\text{Thus, } \mu_Z = \beta \alpha_Z \sum_{i=1}^n \sigma_{x_i} \left(\frac{\partial Z}{\partial X_i}\right) \quad (3.9)$$

$$\text{Where } \alpha_Z = \frac{\sum_{i=1}^n \sigma_{x_i} \left(\frac{\partial Z}{\partial X_i}\right)}{\left\{ \sum_{i=1}^n \sigma_{x_i}^2 \left(\frac{\partial Z}{\partial X_i}\right)^2 \right\}^{1/2}}, \quad (3.10)$$

$\alpha_Z = \text{directional cosine on an hyperplane}$

On the failure surface, at the possible failure point

$$Z(X_1^*, X_2^*, \dots, X_n^*) = 0 \quad (3.11)$$

But also from Eq. (3.9)

$$\mu_Z - \beta \alpha_Z \sum_{i=1}^n \sigma_{X_i} \left(\frac{\partial Z}{\partial X_i} \right) = 0$$

Therefore,

$$Z(X_1^*, X_2^*, \dots, X_n^*) = \mu_Z - \beta \alpha_Z \sum_{i=1}^n \sigma_{X_i} \left(\frac{\partial Z}{\partial X_i} \right)$$

Thus, the corresponding failure points (design point) X_i^* , can be written as

$$X_i^* = \mu_{X_i} - \beta \alpha_{X_i} \sigma_{X_i} \quad (3.12)$$

Also;

$$\frac{X_i^*}{\mu_{X_i}} = \gamma_{X_i} = 1 - \beta \alpha_{X_i} \vartheta_{X_i} \quad (3.13)$$

Where;

γ_{X_i} = safety factor of variable X_i

$\vartheta_{X_i} = \frac{\mu_{X_i}}{\sigma_{X_i}}$ = coefficient of variation of variable X_i

The partial derivatives are evaluated at the design point, X_i^* .

The reliability index β and the design point $(X_1^*, X_2^*, \dots, X_n^*)$ can be determined by solving the nonlinear Equation (3.10), (3.11) and (3.12) iteratively for β .

Also, partial safety factors, γ for individual variable can be calculated from Equation (3.13).

Generally, partial safety factors take on values larger than 1 for loads, and less than 1 for strengths.

Equation (3.6) can be used to compute the probability of failure, P_f corresponding to the deterministic design point $Z(X_1^*, X_2^*, \dots, X_n^*)$.

The directional cosines are considered as a measure of the importance of the corresponding random variable in determining the reliability index β .

However, the above formulation is limited to normally distributed random variables.

For random variable X which is not normally distributed must be transformed to an equivalent normally distributed random variable.

If the parameters of the equivalent normal distribution are

$$\mu_{X_i}^N \text{ and } \sigma_{X_i}^N$$

Assakkaf (2004), states that these parameters can be estimated by imposing two conditions; viz;

$$\Phi\left(\frac{X_i^* - \mu_{X_i}^N}{\sigma_{X_i}^N}\right) = F_i(X_i^*) \quad (3.14a)$$

And

$$\phi\left(\frac{X_i^* - \mu_{X_i}^N}{\sigma_{X_i}^N}\right) = f_i(X_i^*) \quad (3.14b)$$

Where

$F_i =$ non normal cumulative distribution function

$f_i =$ non normal probability density function

$\Phi =$ cumulative distribution function of the standard normal variate

$\phi =$ probability density function of the standard normal variate

The standard deviation and mean of equivalent normal distributions are given by

$$\sigma_{x_i}^N = \frac{\phi(\Phi^{-1}[F_i(X_i^*)])}{f_i(X_i^*)} \quad (3.15a)$$

$$\mu_{X_i}^N = X_i^* - \Phi^{-1}[F_i(X_i^*)]\sigma_{X_i}^N \quad (3.15b)$$

Once $\sigma_{x_i}^N$ and $\mu_{X_i}^N$, are determined for each random variable, β can be solved following the same procedure for normal distribution.

ASMRAM follow the same principles as First Order Reliability Method(FORM), however, as a logical extension of FORM the failure surface is expanded to the second order in the designpoint. The result of ASMRAM analysis may be given as the FORM β multiplied with a correction factor evaluated on the basis of the second order partial derivatives of the failure surface in the design point. Obviously the ASMRAM analysis becomes exact for failure surfaces, which may be given as second order polynomial of the basic random variables.

In general the result of ASMRAM analysis can be shown to be asymptotically exact for any shape of the failure surface as β approaches infinity (Madsen et al. (1986)).

3.3 PLANK DECK DESIGN MODEL

The major part of the load model is axle load, and in particular wheel load. For plank decks, live load consideration is focused on axle weights and wheel loads rather than whole vehicles (Nowak and Saraf, 1996).

A typical plank or timber deck consists of planks placed on stringers as shown in Figure 3.2 and 3.3 according to Nowak and Saraf (1996). There are two categories of plank decks depending on the direction of planks versus the direction of traffic: transverse deck and longitudinal deck. For a typical transverse plank deck the span of the deck is perpendicular to the direction of traffic.

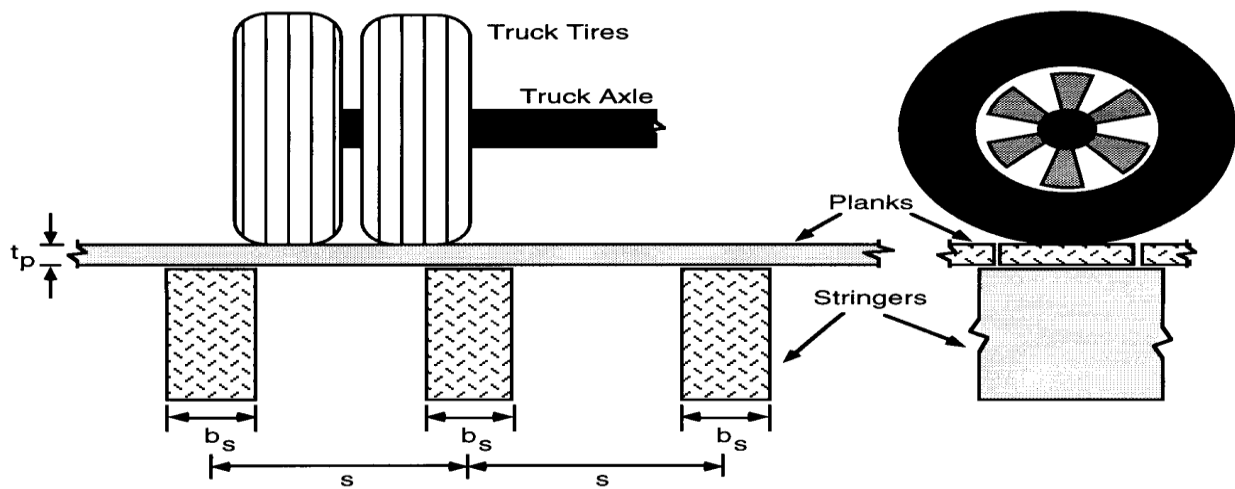


Figure 3.2- Typical Transverse Plank Deck

A longitudinal plank deck, as shown in Figure 3.3, is placed parallel to the direction of traffic. It is assumed that stringers have an adequate load carrying capacity and that they provide a sufficient support for planks.

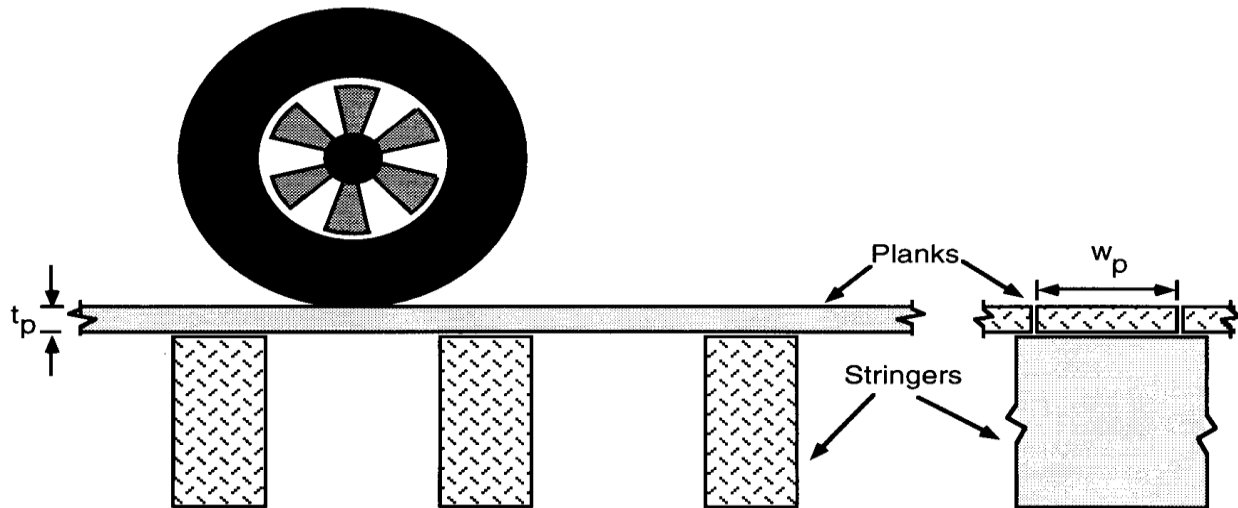


Figure 3.3- Typical Longitudinal Plank Deck

3.3.1 LRFD AASHTO SPECIFICATIONS (2010)

The LRFD AASHTO specification (2010) for design of highway bridges presents the following notations:

DC = dead load moment of structural components and nonstructural attachments

DW = dead load moment of wearing surfaces and utilities

IM = vehicular dynamic load moment

LL = vehicular live load moment

$\lambda_i = \text{load modifier}$

$\gamma_i = \text{Load factor}$

$Q_i = \text{Force effect}$

$\phi = \text{Resistance factor}$

$R_r = \text{Factored resistance}$

$R_n = \text{Nominal resistance}$

$C_l = \text{Beam stability factor}$

$f_b = \text{adjusted flexure}$

$S = \text{sectional modulus}$

$f_{bo} = \text{reference design value}$

The basis of LRFD methodology is given in Article 1.3.2-1 of ASSHTO LRFD (2010) as

$$\sum \lambda_i \gamma_i Q_i \leq \phi R_n = R_r \quad (3.16)$$

For plank decks, the dead load moment shall include the weight of all components of the structure, ASSHTO LRFD (2010) table 3.4.1-1, and 3.4.1-2;

$$\text{Therefore, } \sum \lambda_i \gamma_i Q_i = 1.25DC + 1.5DW + 1.75(LL + IM) \quad (3.17)$$

For strength level I (Basic load combination relating to the normal vehicular use of the bridge without wind)

For flexural member therefore,

$$1.25DC + 1.5DW + 1.75(LL + IM) \leq \phi R_n \quad (3.18)$$

$$\text{But, } R_n = f_b S C_l \quad (3.19)$$

(ASSHTO LRFD, 2010 Equation 8.6.2-1)

Where the depth of a flexural component does not exceed its width, or where lateral movement of the compression zone is prevented by continuous support and where points of bearing have lateral support to prevent rotation, the stability factor, $C_l = 1.0$ as in Article 8.6.2 of ASSHTO LRFD(2010)

Also, for the strip wood component under flexure, ASSHTO LRFD, 2010, Equation 8.4.4.1-1

$$f_b = f_{bo} C_{kf} C_m C_f C_{fu} C_i C_d C_\lambda \quad (3.20)$$

So that, $R_n = f_{bo} C_{kf} C_m C_f C_{fu} C_i C_d C_\lambda S$, but

$$C_{kf} = \text{format conversion factor} = \frac{2.5}{\phi} \quad (\text{LRFD, 2010, article 8.4.4.2})$$

$$C_m = \text{wet service factor} = 1.0 \quad (\text{LRFD, 2010, article 8.4.4.3})$$

$$C_f = \text{size factor for visually graded} = 1.0 \quad (\text{LRFD, 2010, article 4.4.4.4})$$

$$C_{fu} = \text{flat use factor} = 1.0 \quad (\text{LRFD, 2010, table 8.4.4.6 - 1})$$

$$C_i = \text{incising factor} = 0.8 \quad (\text{LRFD, 2010, table 8.4.4.7 - 1})$$

$$C_d = \text{deck factor} = \text{varied} \quad (\text{LRFD, 2010, Article 8.4.4.8, table 8.4.4.8 - 2})$$

$C_\lambda = \text{time effect factor} = 0.8$ (LRFD, 2010 table 8.4.4.9 – 1)

$\phi = \text{resistance factor for flexure member} = 0.85$ (LRFD, 2010, Art 8.5.2.2)

Thus, the design formula for a plank deck is given as

$$1.25DC + 1.5DW + 1.75(LL + IM) \leq 1.6f_{bo}SC_d \quad (3.21)$$

According to LRFD (2010) Article 3.2.3.6, dynamic load allowance need not be applied to wood components. Clause C3.6.2.3 of LRFD (2010), states that, wood structures are known to experience reduced dynamic wheel load effects due to internal friction between the components and the damping characteristics of wood. Additionally, wood is stronger for short duration loads, as compared to longer duration loads. This increase in strength is greater than the increase in force effects resulting from the dynamic load allowance.

Therefore,

$$1.25DC + 1.5DW + 1.75LL \leq 1.6f_{bo}SC_d \text{ Or}$$

$$1.6f_{bo}SC_d - 1.25DC - 1.5DW - 1.75LL \geq 0 \quad (3.22)$$

The plank deck is modeled as a simply supported beam on elastic supports. It is assumed that the elastic support parameters vary depending on stiffness of individual stringers.

With reference to AASHTO LFRD, 2010, article 4.6.2.1.3, for equivalent strip design, the wood plank decks shall be designed for the wheel load of the design truck distributed over the tire contact area. For transverse planks, that is, planks perpendicular to traffic direction:

- i. If $w_p \geq 10.0 \text{ in. (250mm)}$, the full plank width shall be assumed to carry the wheel load.

- ii. If $w_p < 10.0 \text{ in. (250mm)}$, the portion of the wheel load carried by a plank shall be determined as the ratio of w_p and 10.0 in. (250mm).

For longitudinal planks:

- i. If $w_p \geq 20.0 \text{ in. (500mm)}$, the full plank width shall be assumed to carry the wheel load.
- ii. If $w_p < 20.0 \text{ in. (500mm)}$, the portion of the wheel load carried by a plank shall be determined as the ratio of w_p and 20.0 in. (500mm).

Where: w_p = plank width

Only the wheel load is specified for plank decks. Addition of lane load will cause a negligible increase in force effects; however, it may be added for uniformity of the Code (LRFD 2010, clause C4.6.2.1.3).

3.3.2 TIRE CONTACT AREA

The tire contact area of a wheel consisting of one or two tires shall be assumed to be a single rectangle, whose width is 20.0 in. (500mm), and whose length is 10.0 in. (250mm). The tire pressure shall be assumed to be uniformly distributed over the contact area. The tire pressure shall be assumed to be distributed as follows:

- i. On continuous surfaces, uniformly over the specified contact area, and
- ii. On interrupted surfaces, uniformly over the actual contact area within the footprint with the pressure increased in the ratio of the specified to actual contact areas. (LRFD 2010 article 3.6.1.2.5)

According to clause C3.6.1.2.5 of LRFD (2010), 'The area load applies only to the design truck and tandem. For other design vehicles, the tire contact area should be determined by the engineer. As a guideline for other truck loads, the tire area in square inches may be calculated from the following dimensions:

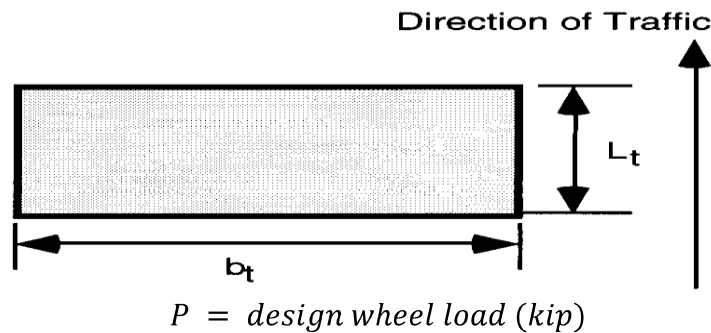
$$\text{Tire width} = P/0.8 \quad (3.23)$$

$$\text{Tire length} = 6.4\gamma(1 + IM/100) \quad (3.24)$$

Where:

$\gamma = \text{load factor}$

$IM = \text{dynamic load allowance percent}$



The tire contact area is defined as a rectangle as shown in Figure 3.4.

Figure3.4 - Tire Contact Area

3.3.3 RELIABILITY ANALYSIS

Where;

$\rho_p = \text{unit weight of plank deck}$

$\rho_s = \text{unit weight of surfacing material}$

$w_p = \text{width of plank deck}$

$w_l = \text{lane load}$

$t_p = \text{depth of plank deck}$

$s = \text{stringer spacing}$

$t_s = \text{depth of surfacing}$

$b_t = \text{width of tire contact area}$

$P = \text{wheel load}$

The load moment are calculated for a simply supported beam, with each span equal to the center-to-center distance between the stringers. For plank decks, the dead load moment shall include the weight of all components of the structure, and the live load is specified as the moment due to HL-93 which shall consist of the design truck coincident with the design lane load. From Equation (3.22)

$$1.6f_{bo}SC_d - 1.25DC - 1.5DW - 1.75LL \geq 0$$

But where,

$$S = \frac{w_p t_p^2}{6}, DC = \rho_p \frac{w_p t_p s^2}{8}, DW = \rho_s \frac{w_t t_s s^2}{8}, \text{ and } LL = \left\{ \frac{P}{8} (2s - b_t) + \frac{w_l s^2}{8} \right\}, \text{ Equation (3.22)}$$

become

$$1.6f_{bo} \frac{w_p t_p^2}{6} C_d - 1.25\rho_p \frac{w_p t_p s^2}{8} - 1.5\rho_s \frac{w_t t_s s^2}{8} - 1.75 \left\{ \frac{P}{8} (2s - b_t) + \frac{w_l s^2}{8} \right\} \geq 0$$

Therefore,

$$f(x_i) = 0.2667f_{bo}w_p t_p^2 C_d - 0.15625\rho_p w_p t_p s^2 - 0.1875\rho_s w_t t_s s^2 - 0.21875\{P(2s - b_t) + w_l s^2\} \quad (3.25)$$

With reference to AASHTO LRFD (2010), this is the reliability equation for plank decks under flexure, where the depth of the flexural component does not exceed its width, or where lateral movement of the compression zone is prevented and where points of bearing have lateral support to prevent rotation. As a result of this, the structural performance of the plank deck is determined by loads and resistance. Both load and resistance are random variables. Therefore, reliability analysis can be used to evaluate the current design provisions. The reliability index, β , is considered as the measure of safety.

3.3.4 DAMAGE ACCUMULATION MODEL

The damage accumulation model is used to mathematically describe the long term strength reduction as a function of stress level and duration of loading. In this work, the concept of Gerhards (1979) model is adopted. Gerhards (1979) uses the concept that the rate of damage accumulation is exponentially dependent on stress level. The damage accumulation model presented by Gerhards (1979) is written as:

$$\frac{\partial \alpha}{\partial t} = \exp \left(-A + B \frac{\sigma(t)}{f_0} \right) \quad (3.26)$$

Where;

$$\frac{\partial \alpha}{\partial t} = \text{rate of damage acculation}$$

$$\sigma(t) = \text{applied load history}$$

$$f_0 = \text{static strenght (as a function time, grain sizes, moisture content e. t. c.)}$$

A and B are constants determined from experiment

The characteristic of the damage model is that α , is defined as the degree of damage, i.e. $\alpha = 0$ stands for no damage and $\alpha = 1$ stands for total damage or failure.

The solution of the differential Equation (3.26) is given by:

$$\frac{\sigma(t)}{f_0} = \frac{A}{B} - \frac{\ln 10}{B} \log t = a - b \log_{10} t_f \quad (3.27)$$

Where

$t_f = \text{time to failure}$

$$a = \frac{A}{B} \text{ and } b = \frac{\ln 10}{B}$$

Wood L. W (1951) published the investigation for the relationship between stress level and logarithmic of time for timber materials. The almost linear relationship between stress level and logarithmic of time commonly known as Madison curve is given by;

$$SL = 90.4 - 6.3 \log_{10} t_f \quad (3.28)$$

Where;

$SL = \text{actual stress level over predicted short time strength in \%}$

$t_f = \text{time to failure in hours}$

Comparing equation (3.27) and (3.28), the solution of equation (3.26) given by equation (3.27) is similar to Woods (1951) prediction in equation (3.28), where;

$$a = 90.4, b = 6.3 \text{ and } SL = \frac{\sigma(t)}{f_0}$$

Hoffmeyer (2007), tested hundreds of timber species with different construction sizes and various moisture contents with following results;

$$SL = 95.0 - 6.36 \log_{10} t_f \quad (3.29)$$

For timber species with constant moisture content of 11%

$$SL = 84.5 - 5.11 \log_{10} t_f \quad (3.40)$$

For timber species with constant moisture content of 20%

$$SL = 88.5 - 7.85 \log_{10} t_f \quad (3.41)$$

For timber species with varying moisture contents between 11 - 20%.

For timber decks, the working condition was assumed to be within the range of 11 - 20% moisture content. Therefore, the damage accumulation model can be given as;

$$\frac{\sigma(t)}{f_0} = 88.5 - 7.85 \log_{10} t_f \text{ Or}$$

$$88.5 - 7.85 \log_{10} t_f - \frac{\sigma(t)}{f_0} \geq 0 \quad (3.42)$$

For tire contact area of 0.5 x 0.25mm, the applied tire stress, $\sigma(t) = \frac{p}{0.25 \times 0.5}$. also, $f_0 = C_{kf} C_i C_\lambda f_b$, where, $C_{kf} = \text{format conversion factor} = \frac{2.5}{\phi}$, $C_i = \text{incising factor} = 0.8$ and $C_\lambda = \text{time effect factor} = 0.6$

Therefore, equation (3.42) become

$$88.5 - 7.85 \log_{10} t_f - 4901.960784 \frac{p(t)}{f_b} \geq 0 \quad (3.43)$$

Thus, the safety equation which define the time to failure as a result of load accumulation is given as,

$$f(x_1, x_2, \dots, x_n) = 88.5 - 7.85 \log_{10} t_f - 4901.960784 \frac{p(t)}{f_b} \quad (3.44)$$

Equation (3.44) was used to assess the reliability of the system under the specified stress level at a predictable time of failure (50 years) to check the long term viability and economic performance of using timber for bridge decks.

CHAPTER 4

4.0 DATA SOURCE, ANALYSIS AND RESULTS

4.1 INTRODUCTION

For plank decks, the dead load moment shall include the weight of all components of the structure, and the live load is specified as the moment due to HL-93 which shall consist of the design truck coincident with the design lane load.

For HL-93, the load includes the HS-20 44 (where H stands for highway, S for semi-trailer, 20 TON (40 kips, 176 kN) weight tractor (first two axles) and was proposed in 1994) truck load and lane load of 640 lb/ft (9.3 kN/m) which is uniformly distributed over 10 ft. (3.00m) transversely. The major part of the load model is axle load, and in particular wheel load. For plank decks, live load consideration is focused on axle weights and wheel loads rather than

whole vehicles. The maximum axle weight is 32 kips (140.8 kN) with a wheel load of 16 kips (70.4 kN).

Nowak and Saraf (1996) iterated that, this wheel load (P) is strongly site specific, and as such they are treated as random variables. Also treated as random variables is reference design strength (f_{bo}), plank width (w_t), plank thickness (t_t), surfacing thickness (t_s), stringers spacing (S), unit weight of plank (ρ_t), and unit weight of surfacing (ρ_s). While the depth factor (C_d), lane load ($w_l = 9.3$ kN/m) and tire width ($b_t = 500$ mm) are all constant. Therefore Equation (3.25) become

$$0.2667f_{bo}w_t t_t^2 C_d - 0.15625\rho_t w_t t_t S^2 - 0.1875\rho_s w_t t_s S^2 - 0.21875\{P(2S - 0.5) + 9.3S^2\} \geq 0 \quad (4.1)$$

4.2 DATA SOURCE

Table 4.1–ReferenceDry Grade (80%)Stress

			FLEXURAL STRENGTH, f_{bo} (grade 80) kN/m^2		UNIT WEIGHT, $\rho_t (kN/m^3)$		STRENGTH GROUP	DURABILITY
DISTRIBUTION TYPE			LOGNORMAL		NORMAL			
DATA SOURCE	BOTANICAL NAME	STANDARD NAME	μ	σ	μ	σ		
Aguwa J. I. et al, 2011	Lophira alata	Ekki	29960	3295.6	11.33	0.6798	N_1	Very durable

Aguwa J. I. 2012	Afzelia bipindensis	Apa	23940	3112.2	7.98	0.8778	N_2	Very durable
Tropix 7 report, 2012	Chlorophora exceisa	Iroko	18507	2525.5	6.40	0.6000	N_3	Very durable
Tropix 7 report, 2012	Mitragyna ciliata	Abura	14478	2375.9	6.00	0.500	N_4	Non-durable
Babatola O. et al 2011	Eucalyptus camaldulensis	River red gum	26484	3602.0	9.776	0.2476	Not available	Not available

Where μ equal mean of the variable data and σ the standard deviation of the variables

For Table 4.1, the data are exact extract from the referred literature. The dry basic stresses are calculated from failure static stress in bending as given by Ozelton and Baird, (1981) as

$$f_{bo} = \frac{f_m - 2.33\sigma}{2.25} \quad (4.2)$$

Where; f_{bo} = basic bending stress parallel to the grain, f_m = mean value of the failure stresses and σ = standard deviation of the failure stresses

While, coefficient of variation for dry basic stress is taken as 80% of that of the failure stress in bending.

Table 4.2–NCP 1973, Basic Bending Stress Parallel to Grain (Grade 80)

TIMBER NAME	EKKI	APA	IROKO	ABURA
STRENGTH GROUP	N_1	N_2	N_3	N_4
DRY GRADE STRESS, BENDING PARALLEL TO GRAIN (GRADE 80) kN/m^2	28000	22400	18000	14000
UNIT WEIGHT kN/m^3	11.36	8.64	6.88	5.76

Source: NCP 1973, table 2 and table 10

Table 4.3 - Other Design Data

SOURCE	DATA	VALUE			DISTRIBUTION TYPE
		μ	COV (%)	σ	
Nowak A. S. and Saraf V.1996	Width of timber	250-300mm	12	30.00	LOGNORMAL
Nowak A. S. and Saraf V.1996	Timber thickness	100mm	7	7.00	LOGNORMAL
Nowak A. S. and Saraf V.1996	Stringer spacing	300-600mm	23.33	70.00	LOGNORMAL
AAHSTO LRFD 2010	Unit weight of surfacing material	22.426 kN/m^3	24.53	5.5	NORMAL
AAHSTO LRFD 2010	Surfacing thickness	70mm	14.98	10.00	NORMAL
AAHSTO LRFD 2010	Wheel load	70kN	14	9.8	LOGNORMAL

4.3 ANALYSIS AND RESULTS

4.3.1 General reliability assessment

The general reliability assessment of using four Nigerian timbers, namely, EKKI, APA, IROKO and ABURA for bridge deck was assessed in relation to Equation (4.1) by ASMRA method using JAVA library with the help of Flanagan polynomial. The results of the general stochastic assessments are displayed on Table 4.4 to indicate possible design point (failure point) and corresponding safety indices.

Table 4.4–Reliability Assessment of Nigeria Timber (EKKI, APA, IROKO and ABURA).

β	f_{bo}	w_t	t_t	ρ_t	S	ρ_s	t_s	P
EKKI								
4.51619	29960	0.25	0.1	11.33	0.3	22.426	0.07	70
3.97876	22759.1651	0.17860	0.08070998	11.3301	0.41087	22.4329	0.07000	70.9440
3.86841	24825.3080	0.20184	0.08539701	11.3302	0.49374	22.4399	0.07001	70.9734
3.85186	25186.4961	0.20446	0.08669217	11.3303	0.51219	22.4459	0.07002	70.7435
3.84950	25244.1376	0.20505	0.08685331	11.3304	0.51517	22.4518	0.07002	70.7003
3.84916	25252.7246	0.20511	0.08688318	11.3305	0.51561	22.4576	0.07003	70.6935
3.84911	25253.9364	0.20512	0.08688663	11.3306	0.51568	22.4634	0.07004	70.6925
3.84910	25254.1332	0.20512	0.08688730	11.3307	0.51569	22.4692	0.07004	70.6923
3.84910	25254.1584	0.20512	0.08688737	11.3308	0.51569	22.4750	0.07005	70.6923
3.8491	25254.1636	0.20512	0.08688739	11.331	0.51569	22.4808	0.07005	70.6923
APA								
3.66764	23940	0.25	0.1	7.98	0.3	22.426	0.07	70
3.25391	17872.6451	0.19597	0.08546123	7.98019	0.40362	22.4325	0.07000	70.8454
3.19515	19832.8101	0.21171	0.08873686	7.98038	0.46550	22.4387	0.07001	70.7947
3.18812	19991.2493	0.21367	0.08959614	7.98055	0.47604	22.4443	0.07001	70.6285
3.18732	20023.2617	0.21394	0.08967215	7.98072	0.47739	22.4497	0.07002	70.6033
3.18723	20025.2190	0.21397	0.08968478	7.98088	0.47755	22.4552	0.07003	70.6001
3.18721	20025.7264	0.21397	0.08968579	7.98104	0.47756	22.4606	0.07003	70.5997
3.18721	20025.7450	0.21397	0.08968598	7.98121	0.47757	22.4660	0.07004	70.5996
3.18721	20025.7541	0.21397	0.08968599	7.98137	0.47757	22.4715	0.07004	70.5996
3.18721	20025.7547	0.21397	0.08968600	7.98154	0.47757	22.4769	0.07005	70.5996
IROKO								
2.86339	18507	0.25	0.1	6.4	0.3	22.426	0.07	70
2.55362	14714.6272	0.21032	0.08938767	6.40008	0.39672	22.4321	0.07000	70.7512
2.52834	15916.8588	0.22131	0.09182965	6.40016	0.43814	22.4376	0.07001	70.6091
2.52620	15980.2464	0.22243	0.09228624	6.40023	0.44306	22.4426	0.07001	70.5018

2.52602	15991.0386	0.22253	0.09231518	6.40030	0.44352	22.4476	0.07002	70.4896
2.52600	15991.3786	0.22254	0.09231887	6.40037	0.44356	22.4525	0.07002	70.4885
2.52600	15991.4820	0.22254	0.09231907	6.40044	0.44356	22.4575	0.07003	70.4884
2.52600	15991.4826	0.22254	0.09231911	6.40051	0.44356	22.4625	0.07003	70.4884
2.52600	15991.4842	0.22254	0.09231911	6.40058	0.44356	22.4674	0.07004	70.4884
2.526	15991.4846	0.22254	0.09231911	6.40065	0.44356	22.4724	0.07004	70.4884
ABURA								
2.110136	14478	0.25	0.1	6	0.3	22.426	0.07	70
1.93072	11638.6104	0.22363	0.09303018	6.00005	0.37884	22.4310	0.07000	70.5069
1.92325	12499.2393	0.22942	0.09433318	6.00009	0.40383	22.4356	0.07001	70.3871
1.92270	12507.0797	0.23004	0.09456108	6.00013	0.40566	22.4399	0.070015	70.3268
1.92267	12511.9668	0.23006	0.09456719	6.00017	0.40580	22.4442	0.07001	70.3224
1.92267	12511.7904	0.23006	0.09456829	6.00022	0.40580	22.4484	0.07002	70.3220
1.92267	12511.8316	0.23006	0.09456829	6.00026	0.40580	22.4527	0.07002	70.3220
1.92267	12511.8288	0.23006	0.09456830	6.00030	0.40580	22.4570	0.07003	70.3220
1.92266	12511.8297	0.23006	0.09456830	6.00034	0.40580	22.4613	0.07003	70.3220
1.92266	12511.8300	0.23006	0.09456830	6.00039	0.40580	22.4655	0.07004	70.3220

It was observed from Table 4.4, that the major factors which influence the performance function of planks for bridge decks are, the grade stress, plank thickness and stringer spacing among other factors. Thus, the effect of these factors on reliability index was examined in section 4.3.2 and 4.3.6.

4.3.2 Reliability assessment of Nigerian timbers in relation to stringer spacing under varying loads

The results of these reliability assessments are displayed on Table A-1 of the appendix. This indicates the reliability assessment of investigated wood types for bridge deck in relation to stringer spacing under varying load.

However, Figures 4.1 indicates clearly in a pictorial form, the reliability indices in relation to the stringer spacing for each wood class evaluated for use as bridge decks. Also, the possible

best fit curve (trend lines) and related equations defining the relationship between the reliability indices(r) and stringer spacing (s) for each class of wood evaluated was indicated.

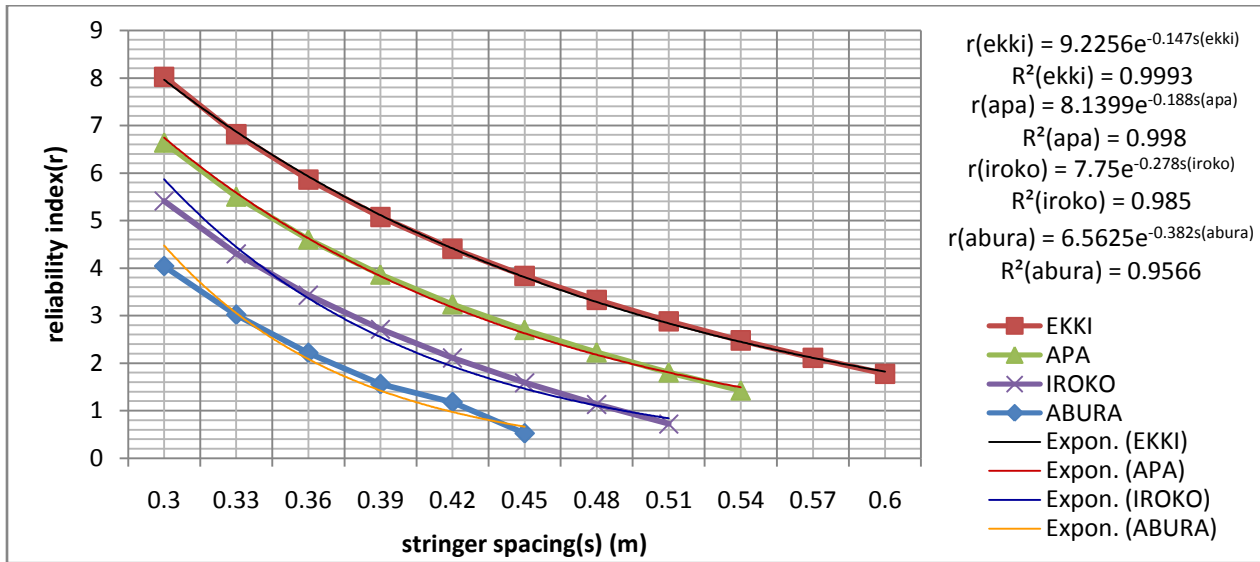


Figure 4.1 Reliability index – Stringer spacing under varying loading

4.3.3 Reliability assessment of Nigerian timbers in relation to stringer spacing under constant loads

The results of these reliability assessments are displayed on Table A-2 of the appendix. This indicates the reliability assessment of investigated wood types for bridge deck in relation to stringer spacing under constant load.

However, Figures 4.2 indicates clearly in a pictorial form, the indicated safety or reliability indices in relation to the stringer spacing for each wood class evaluated for use as bridge decks.

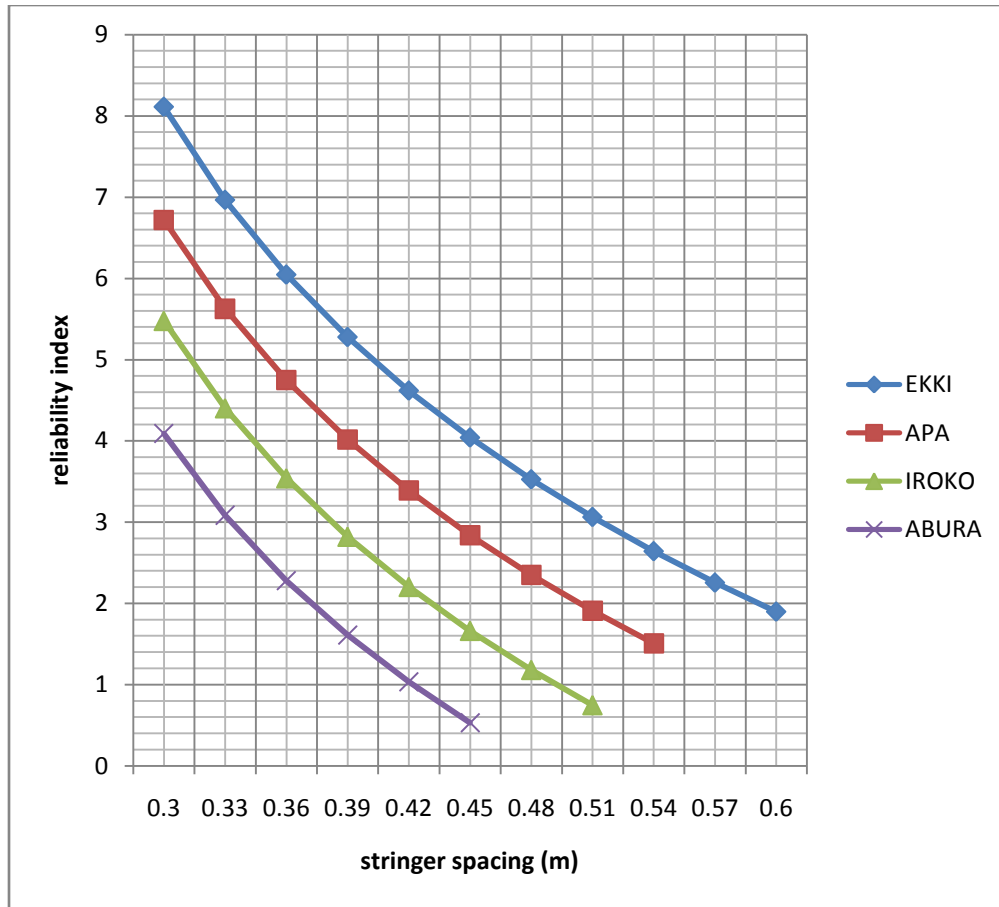


Figure 4.2 - Reliability index – Stringer spacing under constant loading.

4.3.4 Reliability assessment of Nigerian timbers in relation to plank thickness at 0.3m stringer spacing

The results of these reliability assessments are displayed on Table A-3 of the appendix. This indicates the reliability assessment of investigated wood types for bridge deck in relation to plank thickness at constant stringer spacing of 0.3m.

However, Figures 4.3 indicates clearly in a pictorial form, the indicated safety or reliability indices in relation to plank thickness for each wood class evaluated for use as bridge decks.

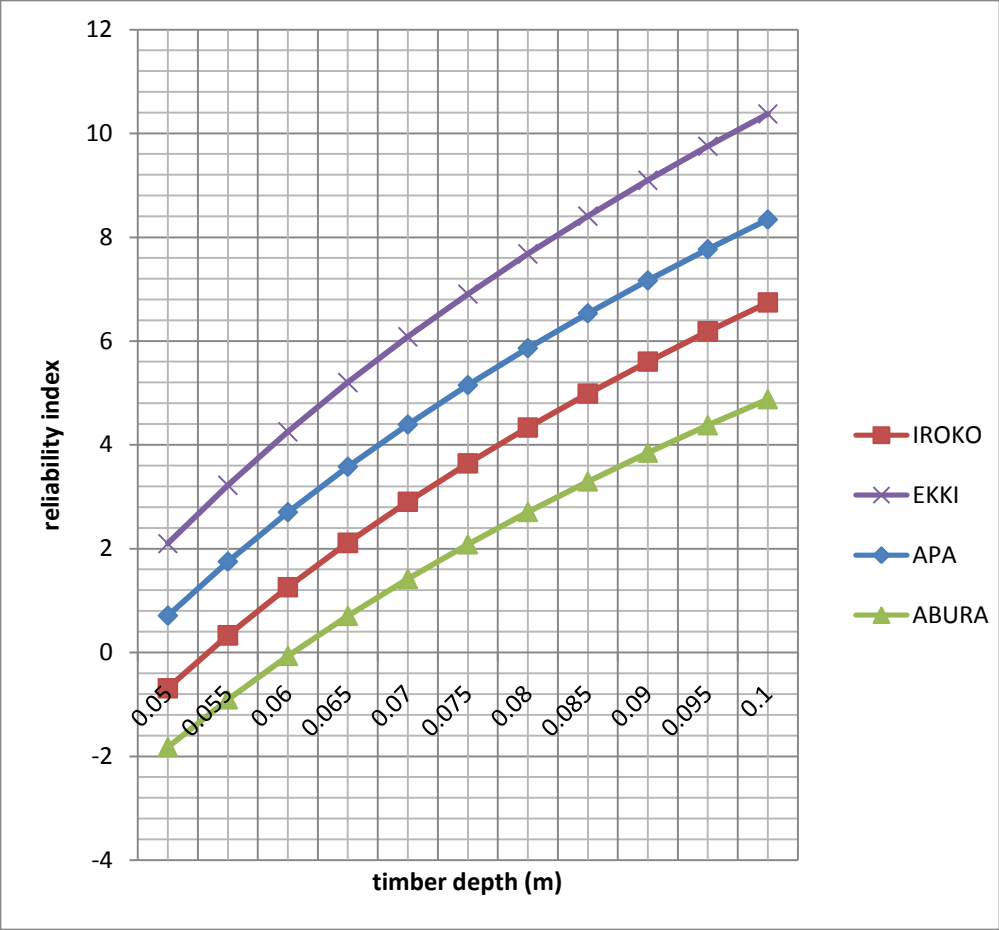


Figure 4.3 - Reliability index – Plank thickness at 0.3m stringer spacing.

4.3.5 Reliability assessment of Nigerian timbers in relation to plank thickness at constant stringer spacing of 0.45m

The results of these reliability assessments are displayed on Table A-4 of the appendix. This indicates the reliability assessment of investigated wood types for bridge deck in relation to plank thickness at constant stringer spacing of 0.45m.

However, Figures 4.4 indicates clearly in a pictorial form, the indicated safety or reliability indices in relation to plank thickness for each wood class evaluated for use as bridge decks.

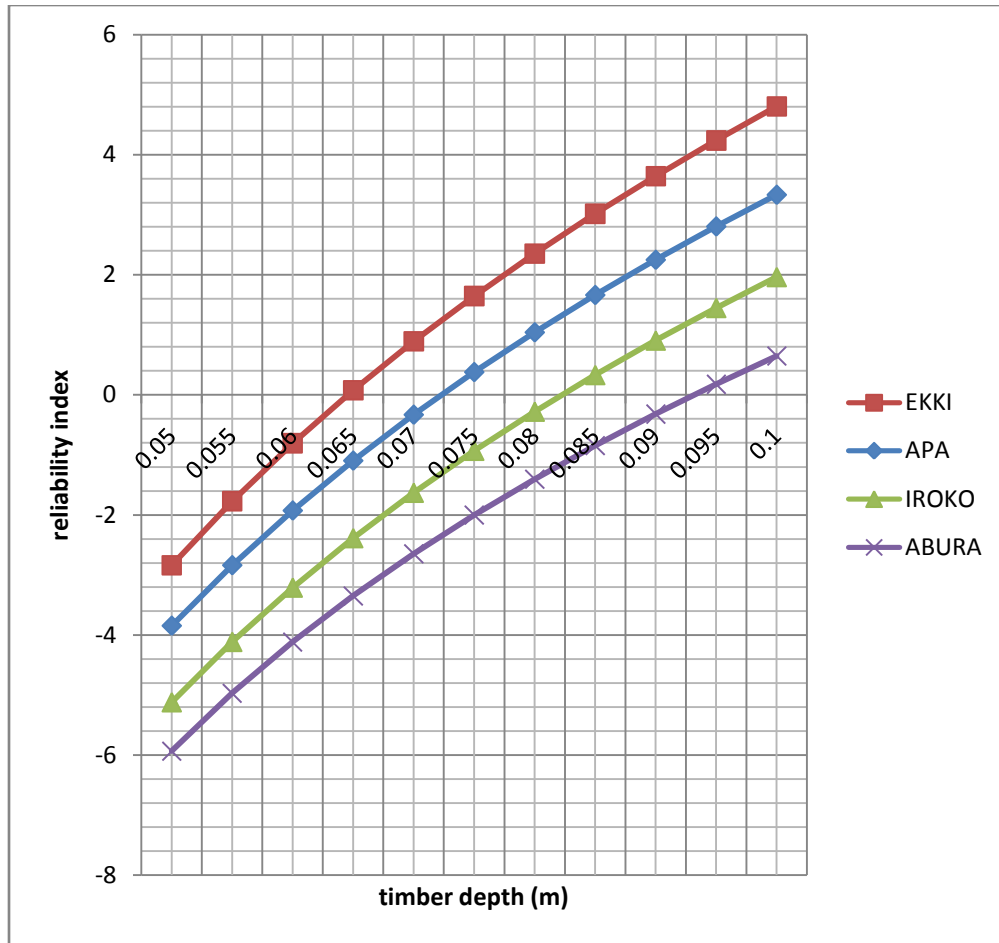


Figure 4.4- Reliability index – Plank thickness at 0.45m stringer spacing.

4.3.6 Reliability assessment of Nigerian timbers in relation to plank width at constant stringer spacing

The results of these reliability assessments are displayed on Table A-5 of the appendix. This indicates the reliability assessment of investigated wood types for bridge deck in relation to plank width at constant stringer spacing of 0.45m and 0.3m.

However, Figures 4.5 indicates clearly in a pictorial form, the indicated safety or reliability indices in relation to plank width for each wood class evaluated for use as bridge decks.

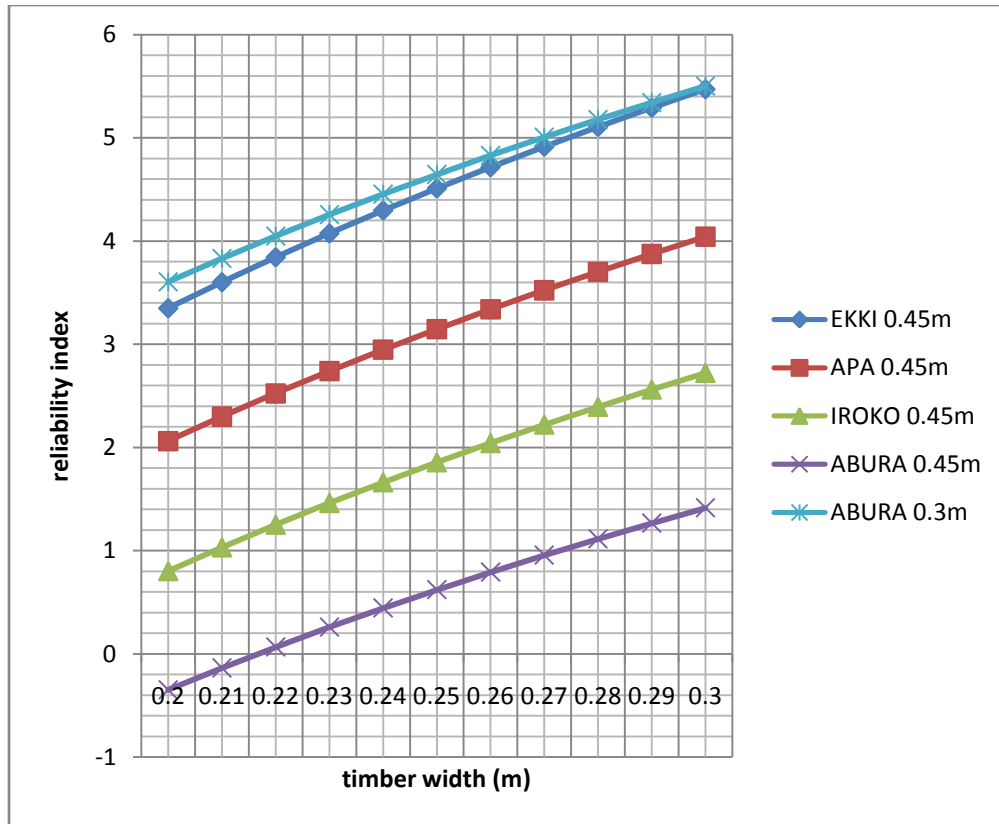


Figure 4.5- Reliability index – Plank width (m)

4.3.7 Damage accumulation reliability analysis results

Results of damage safety assessment for the effect of load accumulation over time with reference to bridge load for the four assessed timber proposed for bridge decks are giving in Table 4.5.

Table 4.5– Damage Accumulation Reliability Assessment Results of Nigerian Timber

t_f	$\sigma(t)$	f_0	β	p_f
EKKI				
438000	95	23940	6.812120214	3.08433E-09
556617.3	118.0818	12877.62701	7.885119	2.89E-12

854777.4	128.7875	14500.72	7.885175	2.89E-12
964323.2	129.0599	14665.11	7.890448	2.78E-12
978154.3	129.0197	14676.49	7.89147	2.76E-12
APA				
438000	95	23940	4.182466392	1.44182E-05
491442	109.4275	11825.95669	5.564637	1.31E-08
666261.9	114.9092	12698.86	5.561052	1.34E-08
706784.6	114.9654	12760.99	5.562589	1.33E-08
709507.5	114.9578	12763.81	5.562738	1.33E-08
IROKO				
438000	95	18507	3.019065248	0.00126778
460607	104.8846	11281.94424	3.747934	8.91E-05
572768.8	107.359	11732.81	3.74454	9.04E-05
589840.6	107.3488	11757.04	3.745052	9.02E-05
590489.2	107.3469	11757.78	3.745078	9.02E-05
ABURA				
438000	95	14478	1.625037246	0.052077311
437605.9	99.26417	10638.01509	1.893234	0.029163
486483.7	99.92608	10791.19	1.892287	0.029226
489753.4	99.92157	10795.94	1.892337	0.029223
489796.9	99.92146	10796	1.892337	0.029223

CHAPTER 5

5.0 DISCUSSION, CONCLUSIONS AND RECOMMENDATIONS

5.1 DISCUSSION

Table 4.4 shows the general stochastic reliability assessment of four Nigerian timbers representing different strength classes in progressive order; EKKI for strength class N_1 , APA for class N_2 , IROKO for class N_3 and ABURA for class N_4 . According to Melchers (1987), the target reliability (β) for timber members ranges from 2.0 to 3.0 with strong mean of 2.5, a target

reliability index of 2.5 was set for this reliability assessment forecast using ASMRA method under the same and specified design condition of loading and geometrical properties in accordance to AASHTO LRFD (2010) specification.

It was observed from the stochastic assessment, that the major factor influencing the performance function of bridge deck in accordance to AASTHO LRFD, 2010 specification are strength classes, timber thickness and stringer spacing among other factors. The reliability indices was found to strongly depend on strength classes: with EKKI (class N_1) having the highest reliability index of approximately 3.8491 at an appreciable possible point of failure with a timber depth of 0.087m, width of 0.205m and stringer spacing of 0.516m under an ultimate wheel load of 70.692kN. This is followed by APA (class N_2) with reliability index of 3.187, timber thickness of 0.090m, width of 0.214m, stringer spacing of 0.478m and ultimate load wheel load 70.599kN at the possible point of failure, then IROKO (N_3) with reliability index of 2.526, timber thickness 0.092m, width of 0.222m, stringer spacing of 0.444m and ultimate load of 70.488kN at the possible point of failure and lastly ABURA (N_4) with reliability index of 1.923, timber thickness of 0.095m, width of 0.230m stringer spacing of 0.406m under an ultimate load of 70.322kN at possible point of failure. Nowak et al (1996) states that stringers are spaced at, 0.3-0.6m center to center, mostly 0.3-0.45m, while the planks are typically 0.1x0.25m or 0.1x0.3m. Thus, in relation to this, it will be observed that Nigerian timbers with strength class N_4 or lower will not be suitable for use as bridge decks, but there exist the possibility of using strength class N_4 , grade 80% for bridge decks at stringer spacing lower than 0.4m with an appreciable increased timber thickness above 0.23m.

From clear examination of Figure 4.1, it will be observed that Nigerian timbers of strength class N_1 and N_2 , grade 80% (that is EKKI and APA) can comfortably be used for bridge decks with stringer spacing ranging from 0.3 - 0.45m; with EKKI having a reliability index of 8.019 (probability of failure, $p_f = 1 - \Phi(\beta) \approx 0$) at stringer spacing of 0.3m, timber thickness of 0.068m with an appreciable width of 0.150m, and reliability index of 3.833 ($p_f \approx 6.33346E - 05$) at stringer spacing of 0.45m, timber thickness of 0.084m and width of 0.195m, while for APA, the reliability index is 6.642 ($p_f \approx 1.54651E - 11$) at stringer spacing of 0.3m, timber thickness of 0.073m, width of 0.165m and reliability index of 2.703 ($p_f \approx 0.003436885$) at stringer spacing of 0.45m, timber thickness of 0.089m, width of 0.211m. Also, for strength class N_3 (IROKO), the reliability index is 5.419 ($p_f \approx 3.175224E - 08$) for timber with 0.078m thickness, width of 0.178m at a stringer spacing of 0.3m, but at any depth greater than 0.4m, the reliability index fall below the target index of 2.5. Lastly, ABURA has a reliability index of 4.040 ($p_f \approx 2.67552E - 05$) for timber with 0.084 thickness, width of 0.195m at a stringer spacing of 0.3m but at any depth greater than 0.34m, the reliability index fall below the target reliability.

Also, the possible best fit curve (trend lines) and related equations defining the relationship between the reliability indices(r) and stringer spacing (s) for each class of wood was established. The best fit curves are define by an exponential relationship, which are given as $r(ekki) = 9.2256e^{-0.147s(ekki)}$ with a strong r-squared (that is the coefficient of determination, which is a measure of the goodness of fit of the trend line to the data, where a value of 1 is a perfect fit) value of $R^2(ekki) = 0.9993$ for Ekki timber, $r(apa) = 8.1399e^{-0.188s(apa)}$ with r-squared value of $R^2(apa) = 0.998$ for Apa timber, $r(iroko) = 7.75e^{-0.278s(iroko)}$ with r-squared value

of $R^2(\text{iroko}) = 0.985$ for Iroko timber, $r(\text{abura}) = 6.5625e^{-0.382s(\text{abura})}$ with r-squared value of $R^2(\text{abura}) = 0.9566$ for Abura timber.

In order to assess the possibility of failure at specific stringer spacing, the timber thickness was varied for constant stringer spacing of 0.3m and 0.45m respectively. It was observed from Figure 4.3, that the strength classes observed can all be used for bridge decking at stringer spacing of 0.3m. While from Figure 4.4, it was observed that only Nigerian timber with strength class N_1 (EKKI) and N_2 (APA) can be used for bridge decking at stringer spacing of 0.45m.

From Figure 4.5, it was observed that, although the timber width may be reduced for ABURA at stringer spacing of 0.3m to a minimum of 0.2m, the structure will not meet the required safety index at a stringer spacing of 0.45m; not even with a timber thickness of 0.1m and a width of 0.3m.

Lastly, Table 4.5 shows the safety indices and approximate probability of failure over a predictable design period (50 years). This probability of failure is based on effect of load accumulation over time. Thus, it was observed that the possibility of failure over time still depends on the strength classes; with timber within the high strength classes having low probability of failure over time compared to timber with low strength classes: that is, strength class N_1 (EKKI timber) with probability of failure, $p_f = 1 - \Phi(\beta) \approx 0$ over a period of 978154 hours (100 years), strength class N_3 (APA timber) with probability of failure, $p_f \approx 1.33E - 08$ over a period of 709508 hours (80 years), strength class N_3 (IROKO timber) with probability of failure, $p_f \approx 9.02E - 05$ over a period of 590489 hours (65 years) and strength class N_4

(ABURA timber) with probability of failure, $p_f \approx 0.029223$ over a period of 489797 hours (55 years).

5.2 CONCLUSIONS

The aim of this work is to assess the possibility of using various Nigerian timber species as sustainable bridge deck material and the major objective is to check the long time viability of using Nigerian timber for bridge decks. Therefore, some suitable species were identified as possible source of sustainable and renewable planks for bridge decks. These include timbers within the range of strength class N1 – N4 out of the listed seven strength classes obtainable in the country. However, other classes may be used if they are upgraded to the requirement of classes N1 to N4 by available engineering treatment for timber.

The recommended strength classes with associated material properties can be a source of sustainable bridge deck material over a reasonable period of time as indicated by the probability of failure as a result of damage due to load accumulation.

The limited amount of information available for the unknown timbers found at some geographical part of the country such as *Eucalyptus camaldulensis* found in the northern part of the country indicates that there is a need for additional research to identify more timbers which can be found within the required strength classes. Such research should involve load damage accumulation model, so that the effect of load accumulation over time can be incorporated into the design model of planks for bridge decks. This will enhance the suitability of Nigerian timber as innovative, sustainable and cost-effective material for bridge decks, therefore promoting interest in the use of wood as a competitive bridge construction material by adding value to the

use of local resources, and establishing a means of overcoming exclusion while also strengthening the inclusion of the use of timber in bridge construction industries.

5.3 RECOMMENDATIONS

The following recommendations should be considered in the construction of timber bridge decks using Nigerian timbers.

1. The most suitable strength classes for bridge decks are timber with strength class N_1 and any other timber with similar strength characteristics.
2. Timbers with strength class lower than that of class N_4 should not be used for bridge decking.
3. When using Nigerian timber for bridge decks, the stringers spacing should not be greater than 300mm with timber thickness not less than 100mm, except for timber with strength class N_1 , where the spacing can be increased to 450mm with a timber thickness not less than 100mm
4. The width of the planks for the timber decking should not be less than 250mm.

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APPENDIX

Appendix I: Stochastic assessment data and reliability index

Table A-1- Reliability Assessment of NigerianTimbers for Bridge Decks In Relation to Stringer Spacing Under Varying Load

β	f_{bo}	w_t	t_t	ρ_t	S	ρ_s	t_s	P
Reliability assessment of Nigerian EKKI timber for bridge deck in relation to stringer spacing under varying load								
8.019364	19103.39	0.150701	0.067921864	11.33321	0.3	22.60254	0.070187	74.60792
6.816664	20428.04	0.162118	0.072139579	11.33311	0.33	22.59681	0.070181	76.26103
5.860383	21553.96	0.171939	0.075669132	11.33302	0.36	22.59225	0.070176	77.22571
5.072928	22531.27	0.180563	0.07869012	11.33294	0.39	22.58791	0.070171	77.70228
4.407249	23393.16	0.188254	0.081320554	11.33286	0.42	22.58325	0.070167	77.82418
3.832846	24162.81	0.195192	0.083642138	11.33276	0.45	22.57795	0.070161	77.68476
3.328942	24857.1	0.201511	0.08571383	11.33265	0.48	22.57181	0.070154	77.35142

2.880821	25488.71	0.207313	0.087579594	11.33252	0.51	22.56469	0.070147	76.8739
2.477732	26067.45	0.212673	0.089273076	11.33237	0.54	22.55647	0.070138	76.28943
2.11162	26601.02	0.217656	0.090820588	11.3322	0.57	22.5471	0.070128	75.62624
1.77632	27095.63	0.22231	0.092243069	11.33201	0.6	22.53649	0.070117	74.90588
Reliability assessment of Nigerian APA timber for bridge deck in relation to stringer spacing under varying load								
6.642073	15032.71	0.164874	0.073138351	7.985037	0.3	22.5924	0.070176	74.24605
5.505718	16215.09	0.17679	0.077378002	7.984751	0.33	22.58295	0.070166	75.58204
4.605594	17235.03	0.186935	0.080874698	7.984497	0.36	22.57457	0.070157	76.2097
3.866124	18132.65	0.195753	0.083829038	7.984245	0.39	22.56623	0.070149	76.35307
3.241687	18934.59	0.203541	0.08637182	7.983976	0.42	22.55736	0.070139	76.15873
2.702899	19659.58	0.210504	0.088592882	7.983681	0.45	22.54762	0.070129	75.72665
2.22988	20321.36	0.216795	0.090556488	7.983353	0.48	22.53678	0.070117	75.1269
1.808656	20930.31	0.222527	0.092310109	7.982987	0.51	22.52469	0.070105	74.40958
1.429087	21494.51	0.227787	0.093889728	7.98258	0.54	22.51123	0.07009	73.61099
Reliability assessment of Nigerian IROKO timber for bridge deck in relation to stringer spacing under varying load								
5.408641	12289.95	0.17804	0.077814356	6.402309	0.3	22.58895	0.070173	74.099
4.301295	13302.83	0.190681	0.082139472	6.402087	0.33	22.57331	0.070156	75.10136
3.42832	14178.56	0.201337	0.085659096	6.40188	0.36	22.55871	0.070141	75.32334
2.713195	14950.39	0.210513	0.088596095	6.401671	0.39	22.54395	0.070125	75.04041
2.109951	15640.93	0.218544	0.091095882	6.40145	0.42	22.52834	0.070108	74.42829
1.589244	16266.25	0.225669	0.093257879	6.401211	0.45	22.51148	0.070091	73.60237
1.13143	16838.22	0.23206	0.09515265	6.400951	0.48	22.49311	0.070071	72.63993
0.722847	17365.79	0.237847	0.096831816	6.400666	0.51	22.47303	0.07005	71.59354
Reliability assessment of Nigerian ABURA timber for bridge deck in relation to stringer spacing under varying load								
4.039741	9703.84	0.19489	0.083543785	6.001368	0.3	22.56506	0.070147	73.31906
3.018839	10642.96	0.207583	0.087668173	6.001159	0.33	22.54385	0.070125	73.80468
2.216754	11475.09	0.218043	0.090942353	6.00096	0.36	22.52355	0.070103	73.55568
1.559922	12225.73	0.226862	0.093615314	6.000756	0.39	22.50287	0.070081	72.86707
1.180517	12689.82	0.23203	0.09514429	6.000615	0.41	22.4885	0.070066	72.25481
0.524383	13548.06	0.241031	0.097741516	6.000243	0.45	22.45068	0.070026	70.81523

Table A-2 - Reliability Assessment of Nigerian Timber for Bridge Deck at Constant Stringer Spacing Under Constant Loading

β	f_{bo}	w_t	t_t	ρ_t	S	ρ_s	t_s	P
Reliability assessment of Nigerian EKKI timber for bridge deck in relation to stringer spacing under constant load								
8.109905	18990.65	0.149736	0.067559743	11.33327	0.3	22.6057	0.07019	70
6.964293	20224.29	0.160354	0.071495411	11.33321	0.33	22.60266	0.070187	70
6.044879	21275.12	0.169497	0.074799967	11.33318	0.36	22.60058	0.070185	70
5.276915	22196.67	0.177601	0.077660504	11.33314	0.39	22.5984	0.070183	70
4.617496	23021.36	0.184929	0.080189937	11.33308	0.42	22.59554	0.07018	70
4.039702	23770.41	0.191649	0.082461925	11.33301	0.45	22.59167	0.070175	70
3.525528	24458.53	0.197879	0.084527352	11.33292	0.48	22.58655	0.07017	70
3.062335	25096.37	0.203706	0.086422949	11.3328	0.51	22.58004	0.070163	70
2.64091	25691.92	0.209193	0.088176162	11.33266	0.54	22.57202	0.070155	70
2.254332	26251.34	0.214389	0.089808075	11.33248	0.57	22.56239	0.070144	70
1.897266	26779.47	0.219334	0.09133527	11.33228	0.6	22.5511	0.070132	70
Reliability assessment of Nigerian APA timber for bridge deck in relation to stringer spacing under constant load								
6.718757	14939.99	0.163933	0.072797567	7.985129	0.3	22.59545	0.070179	70
5.625805	16049.63	0.175132	0.076796715	7.984916	0.33	22.58843	0.070172	70
4.749118	17012.44	0.184733	0.080124366	7.984725	0.36	22.5821	0.070165	70
4.017157	17871.57	0.193201	0.082981653	7.984521	0.39	22.57535	0.070158	70
3.388872	18652.98	0.200818	0.085489286	7.984285	0.42	22.56757	0.07015	70
2.838512	19373.65	0.207768	0.087725742	7.984008	0.45	22.55842	0.07014	70
2.348863	20045.4	0.214181	0.089745107	7.983682	0.48	22.54766	0.070129	70
1.907843	20676.82	0.220149	0.091586343	7.983303	0.51	22.53513	0.070116	70
1.506648	21274.35	0.225741	0.093278514	7.982867	0.54	22.5207	0.0701	70
Reliability assessment of Nigerian IROKO timber for bridge deck in relation to stringer spacing under constant load								
5.477696	12212.99	0.177067	0.077474474	6.402356	0.3	22.5923	0.070176	70
4.402416	13171.35	0.189059	0.081593588	6.402168	0.33	22.579	0.070162	70
3.540168	14010.94	0.199319	0.085001008	6.401984	0.36	22.56606	0.070148	70
2.82042	14766.68	0.208349	0.087910876	6.401788	0.39	22.5522	0.070134	70
2.202707	15459.69	0.216454	0.090451352	6.401569	0.42	22.53674	0.070117	70
1.661655	16103.86	0.223833	0.092705631	6.401322	0.45	22.51928	0.070099	70
1.180301	16708.86	0.230626	0.094730986	6.401042	0.48	22.49955	0.070078	70
0.746743	17281.73	0.236932	0.096568644	6.400728	0.51	22.47738	0.070054	70
Reliability assessment of Nigerian ABURA timber for bridge deck in relation to stringer spacing under constant load								
4.088184	9645.941	0.194077	0.083273689	6.001396	0.3	22.56787	0.07015	70
3.082831	10548.73	0.206353	0.087275521	6.001203	0.33	22.54823	0.070129	70

2.278311	11364.14	0.216691	0.090525308	6.00101	0.36	22.52861	0.070109	70
1.607764	12119.19	0.225646	0.093251325	6.000804	0.39	22.5077	0.070087	70
1.03288	12830.37	0.233554	0.095589665	6.000579	0.42	22.48483	0.070062	70
0.52966	13508.61	0.240633	0.097628384	6.000331	0.45	22.4596	0.070036	70

Table A-3 - Reliability Assessment of Nigerian Timbers for Bridge Deck at 0.3m Stringer Spacing for Various Plank Thicknesses

β	f_{bo}	w_t	t_t	ρ_t	S	ρ_s	t_s	P
Reliability Assessment of Nigerian EKKI Timbers for Bridge Deck at 0.3m Stringer Spacing for Various Plank Thicknesses								
2.100356	25566.39	0.208034	0.05	11.3317	0.3	22.61305	0.070198	74.76794
3.224567	23508.05	0.189298	0.055	11.33258	0.3	22.68401	0.070273	77.29948
4.252803	21746.28	0.173646	0.06	11.33337	0.3	22.73501	0.070327	79.34167
5.20126	20222.17	0.160356	0.065	11.33409	0.3	22.7722	0.070367	80.9797
6.082227	18891.87	0.148918	0.07	11.33476	0.3	22.79958	0.070396	82.28534
6.90526	17722.82	0.138953	0.075	11.33537	0.3	22.8198	0.070417	83.31826
7.67797	16691.23	0.130165	0.08	11.33595	0.3	22.83469	0.070433	84.12763
8.406546	15780.56	0.12231	0.085	11.33649	0.3	22.8455	0.070444	84.75354
9.096099	14980.39	0.115177	0.09	11.33699	0.3	22.85315	0.070452	85.22806
9.750989	14364.78	0.107964	0.095	11.33568	0.3	22.75468	0.070348	85.55816
10.37334	13694.17	0.102328	0.1	11.33792	0.3	22.86132	0.070461	85.81296
Reliability Assessment of Nigerian APA Timbers for Bridge Deck at 0.3m Stringer Spacing for Various Plank Thicknesses								
0.708562	22248.71	0.234739	0.05	7.981006	0.3	22.49245	0.07007	71.09396
1.750322	20228.83	0.215923	0.055	7.982452	0.3	22.57327	0.070156	73.49607
2.701912	18547.09	0.199799	0.06	7.983744	0.3	22.63215	0.070218	75.45988
3.57861	17122.95	0.18584	0.065	7.984915	0.3	22.67582	0.070265	77.06096
4.392018	15900.24	0.173646	0.07	7.985988	0.3	22.70863	0.070299	78.36254
5.151149	14838.25	0.16291	0.075	7.986981	0.3	22.7335	0.070326	79.4171
5.863155	13906.39	0.153397	0.08	7.987905	0.3	22.75244	0.070346	80.26793
6.533819	13080.77	0.144924	0.085	7.98877	0.3	22.76687	0.070361	80.9506
7.167904	12341.98	0.137358	0.09	7.989584	0.3	22.77781	0.070373	81.49427
7.769398	11673.57	0.1306	0.095	7.990352	0.3	22.78601	0.070381	81.92267
8.341668	11061.01	0.124587	0.1	7.991079	0.3	22.79202	0.070388	82.25492
Reliability Assessment of Nigerian IROKO Timbers for Bridge Deck at 0.3m Stringer Spacing for Various Plank Thicknesses								
-0.68698	19625.69	0.261413	0.05	6.399483	0.3	22.35308	0.069923	67.55682
0.330747	17750.59	0.241999	0.055	6.400245	0.3	22.45745	0.070033	70.14148

1.258873	16211.98	0.22506	0.06	6.400922	0.3	22.53443	0.070115	72.30741
2.112876	14922.98	0.210198	0.065	6.401531	0.3	22.5923	0.070176	74.11841
2.904449	13825.33	0.197082	0.07	6.402087	0.3	22.63647	0.070223	75.62977
3.642642	12878.2	0.185434	0.075	6.402599	0.3	22.6706	0.070259	76.88876
4.334608	12051.91	0.175034	0.08	6.403074	0.3	22.69719	0.070287	77.93541
4.986104	11324.12	0.165699	0.085	6.403517	0.3	22.71804	0.070309	78.80344
5.601839	10677.52	0.157287	0.09	6.403933	0.3	22.73443	0.070327	79.52111
6.185733	10098.22	0.149683	0.095	6.404325	0.3	22.74731	0.07034	80.11207
6.741106	9574.604	0.142803	0.1	6.404695	0.3	22.75738	0.070351	80.59601
Reliability Assessment of Nigerian ABURA Timbers for Bridge Deck at 0.3m Stringer Spacing for Various Plank Thicknesses								
-1.82476	18180.32	0.278383	0.05	5.999023	0.3	22.22737	0.06979	64.89813
-0.90315	16062.36	0.263501	0.055	5.99952	0.3	22.33731	0.069906	67.20383
-0.06536	14406.7	0.249337	0.06	5.999966	0.3	22.42016	0.069994	69.17607
0.704507	13071.92	0.236127	0.065	6.000369	0.3	22.48364	0.070061	70.85604
1.417829	11969.93	0.223926	0.07	6.000737	0.3	22.533	0.070113	72.28356
2.083137	11042.82	0.212708	0.075	6.001076	0.3	22.57183	0.070154	73.49474
2.707024	10250.87	0.202409	0.08	6.001391	0.3	22.60268	0.070187	74.52126
3.29472	9565.787	0.192953	0.085	6.001684	0.3	22.62736	0.070213	75.39044
3.850466	8966.768	0.184262	0.09	6.001959	0.3	22.64721	0.070234	76.12564
4.377761	8438.073	0.176266	0.095	6.002217	0.3	22.6632	0.070251	76.74664
4.879547	7967.409	0.168902	0.1	6.00246	0.3	22.67608	0.070265	77.27016

Table A-4- Reliability Assessment of Nigerian Timbers for Bridge Deck at 0.45m Stringer Spacing for Various Plank Thicknesses

β	f_{bo}	w_t	t_t	ρ_t	S	ρ_s	t_s	P
Reliability Assessment of Nigerian EKKI Timbers for Bridge Deck at 0.45m Stringer Spacing for Various Plank Thicknesses								
-2.84038	35635.52	0.309797	0.05	11.32664	0.45	22.05616	0.069608	57.05533
-1.77314	33382.24	0.285106	0.055	11.32794	0.45	22.21973	0.069782	61.60669
-0.80769	31396.8	0.264433	0.06	11.32908	0.45	22.34124	0.06991	65.80217
0.074657	29633.16	0.246767	0.065	11.33008	0.45	22.43313	0.070008	69.64849
0.888081	28054.52	0.23143	0.07	11.33099	0.45	22.50373	0.070082	73.15797
1.643565	26631.74	0.217944	0.075	11.33181	0.45	22.55877	0.070141	76.3466
2.349743	25341.59	0.205963	0.08	11.33256	0.45	22.60223	0.070187	79.2326
3.013505	24165.44	0.195227	0.085	11.33326	0.45	22.63695	0.070223	81.83544
3.640409	23088.18	0.185538	0.09	11.33391	0.45	22.66495	0.070253	84.17506
4.234995	22097.4	0.176739	0.095	11.33452	0.45	22.68773	0.070277	86.2713

4.801001	21182.86	0.168707	0.1	11.3351	0.45	22.70639	0.070297	88.14352
Reliability Assessment of Nigerian APA Timbers for Bridge Deck at 0.45m Stringer Spacing for Various Plank Thicknesses								
-3.84856	33027.87	0.322782	0.05	7.97194	0.45	21.89341	0.069436	53.45614
-2.84059	30286.37	0.302627	0.055	7.974142	0.45	22.07409	0.069627	57.55594
-1.9299	28016.24	0.284702	0.06	7.976079	0.45	22.21009	0.069771	61.33353
-1.09868	26092.18	0.268703	0.065	7.977799	0.45	22.31411	0.069882	64.79785
-0.33341	24432.35	0.254348	0.07	7.97934	0.45	22.39487	0.069967	67.9623
0.376356	22980.46	0.241398	0.075	7.980736	0.45	22.45842	0.070034	70.84302
1.038835	21696.18	0.229655	0.08	7.982011	0.45	22.50904	0.070088	73.45757
1.660595	20549.66	0.218954	0.085	7.983185	0.45	22.54981	0.070131	75.82417
2.246946	19518.24	0.20916	0.09	7.984276	0.45	22.58297	0.070166	77.96107
2.802229	18584.29	0.20016	0.095	7.985296	0.45	22.61018	0.070195	79.88613
3.330032	17733.86	0.191861	0.1	7.986256	0.45	22.63267	0.070219	81.61654
Reliability Assessment of Nigerian IROKO Timbers for Bridge Deck at 0.45m Stringer Spacing for Various Plank Thicknesses								
-5.12247	29823.09	0.341012	0.05	6.394357	0.45	21.62935	0.069156	48.78023
-4.11695	27007.01	0.323564	0.055	6.39551	0.45	21.84984	0.06939	52.63082
-3.21209	24760.19	0.307146	0.06	6.396539	0.45	22.01879	0.069569	56.23464
-2.38869	22907.94	0.291944	0.065	6.397457	0.45	22.14987	0.069708	59.58783
-1.63261	21343.61	0.277955	0.07	6.398283	0.45	22.25283	0.069817	62.69395
-0.93304	19997.8	0.265098	0.075	6.39903	0.45	22.33467	0.069903	65.56094
-0.28155	18823.04	0.253272	0.08	6.39971	0.45	22.40043	0.069973	68.19929
0.328606	17785.51	0.242373	0.085	6.400335	0.45	22.45381	0.070029	70.62101
0.902867	16860.3	0.232306	0.09	6.400707	0.45	22.48148	0.070059	72.8389
1.445667	16028.63	0.222982	0.095	6.40145	0.45	22.5337	0.070114	74.86591
1.960706	15275.9	0.214327	0.1	6.401952	0.45	22.56381	0.070146	76.71513
Reliability Assessment of Nigerian ABURA Timbers for Bridge Deck at 0.45m Stringer Spacing for Various Plank Thicknesses								
-5.93326	31685.39	0.318012	0.05	5.995615	0.45	21.53463	0.069056	47.90894
-4.97073	27483.41	0.312667	0.055	5.996276	0.45	21.73788	0.069271	50.96806
-4.1173	24341.9	0.305456	0.06	5.9969	0.45	21.90081	0.069444	53.91384
-3.34885	21903.72	0.297172	0.065	5.997478	0.45	22.03158	0.069582	56.71164
-2.64835	19951.24	0.288369	0.07	5.998011	0.45	22.13714	0.069694	59.34323
-2.00347	18347.05	0.279409	0.075	5.998502	0.45	22.22294	0.069785	61.80146
-1.405	17001.13	0.270517	0.08	5.998955	0.45	22.29321	0.069859	64.08617
-0.84594	15852.33	0.261832	0.085	5.999374	0.45	22.35118	0.069921	66.20155
-0.32076	14857.76	0.253432	0.09	5.999764	0.45	22.39936	0.069972	68.15438
0.174944	13986.47	0.245357	0.095	6.000128	0.45	22.43968	0.070014	69.9529

0.644744	13215.49	0.237629	0.1	6.000469	0.45	22.47367	0.07005	71.60611
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Table A-5 - Reliability Assessment of Nigerian Timbers for Bridge Deck at Fixed Stringer Spacing for Various Plank Widths

β	f_{bo}	w_t	t_r	ρ_t	S	ρ_s	t_s	P
Reliability Assessment of Nigerian EKKI Timbers for Bridge Deck at 0.45m Stringer Spacing for Various Plank Widths								
3.34951	24136.95	0.2	0.083564585	11.33279	0.45	22.57968	0.070163	80.40002
3.601806	23751.14	0.21	0.082403796	11.33305	0.45	22.59388	0.070178	80.99567
3.843017	23387.51	0.22	0.081303766	11.33331	0.45	22.60784	0.070193	81.53113
4.074138	23043.9	0.23	0.080259001	11.33356	0.45	22.62157	0.070207	82.01303
4.296029	22718.44	0.24	0.079264711	11.3338	0.45	22.63508	0.070221	82.44717
4.509444	22409.5	0.25	0.078316697	11.33404	0.45	22.64839	0.070236	82.83859
4.715042	22115.66	0.26	0.077411254	11.33428	0.45	22.6615	0.070249	83.19171
4.913409	21835.67	0.27	0.076545101	11.33452	0.45	22.67443	0.070263	83.5104
5.105061	21568.42	0.28	0.075715312	11.33475	0.45	22.68718	0.070277	83.79809
5.290462	21312.93	0.29	0.074919275	11.33498	0.45	22.69976	0.07029	84.05779
5.470027	21068.33	0.3	0.074154639	11.3352	0.45	22.71218	0.070303	84.29218
Reliability Assessment of Nigerian APA Timbers for Bridge Deck at 0.45m Stringer Spacing for Various Plank Widths								
2.062819	20130.84	0.2	0.089997267	7.983158	0.45	22.53035	0.070111	76.22642
2.298831	19757.92	0.21	0.088888475	7.983577	0.45	22.54416	0.070125	76.87751
2.524279	19408.84	0.22	0.08783308	7.983988	0.45	22.55775	0.07014	77.47029
2.740126	19081.05	0.23	0.08682659	7.984392	0.45	22.57111	0.070154	78.01086
2.947208	18772.37	0.24	0.085865048	7.98479	0.45	22.58426	0.070168	78.5045
3.146253	18480.96	0.25	0.084944955	7.985183	0.45	22.59722	0.070181	78.95587
3.337898	18205.18	0.26	0.084063202	7.985569	0.45	22.60999	0.070195	79.36906
3.522708	17943.63	0.27	0.083217011	7.98595	0.45	22.62258	0.070208	79.74769
3.701181	17695.1	0.28	0.082403892	7.986326	0.45	22.63501	0.070221	80.09496
3.873763	17458.48	0.29	0.081621604	7.986698	0.45	22.64727	0.070234	80.41373
4.040851	17232.83	0.3	0.080868122	7.987064	0.45	22.65937	0.070247	80.70652
Reliability Assessment of Nigerian IROKO Timbers for Bridge Deck at 0.45m Stringer Spacing for Various Plank Widths								
0.801969	17108.27	0.2	0.096019257	6.401433	0.45	22.47334	0.07005	72.18297
1.032138	16773.43	0.21	0.094940682	6.401874	0.45	22.48793	0.070066	72.94977
1.251727	16461.03	0.22	0.093910514	6.402309	0.45	22.50228	0.070081	73.65697
1.461722	16168.58	0.23	0.092924923	6.402736	0.45	22.51641	0.070096	74.31022
1.662977	15893.94	0.24	0.091980509	6.403158	0.45	22.53032	0.07011	74.91453

1.856237	15635.3	0.25	0.091074241	6.403573	0.45	22.54403	0.070125	75.47429
2.042151	15391.09	0.26	0.090203409	6.403982	0.45	22.55755	0.070139	75.99345
2.221292	15159.98	0.27	0.089365575	6.404386	0.45	22.57089	0.070153	76.4755
2.394169	14940.78	0.28	0.088558541	6.404784	0.45	22.58406	0.070167	76.92357
2.561233	14732.47	0.29	0.087780322	6.405178	0.45	22.59706	0.070181	77.34045
2.722885	14534.14	0.3	0.087029111	6.405567	0.45	22.6099	0.070195	77.72869
Reliability Assessment of Nigerian ABURA Timbers for Bridge Deck at 0.45m Stringer Spacing for Various Plank Widths								
-0.34705	14873.34	0.2	0.101220846	5.999782	0.45	22.40386	0.069977	68.11052
-0.13508	14511.45	0.21	0.100330311	5.999914	0.45	22.41722	0.069991	68.85718
0.066875	14177.44	0.22	0.099469656	6.000043	0.45	22.43042	0.070005	69.55224
0.259785	13867.86	0.23	0.098637411	6.000172	0.45	22.44345	0.070018	70.2002
0.444472	13579.82	0.24	0.097832154	6.000298	0.45	22.45632	0.070032	70.8051
0.621649	13310.9	0.25	0.097052523	6.000423	0.45	22.46904	0.070046	71.37051
0.791944	13059.05	0.26	0.096297227	6.000547	0.45	22.48162	0.070059	71.89966
0.955904	12822.5	0.27	0.095565042	6.00067	0.45	22.49405	0.070072	72.39543
1.114016	12599.75	0.28	0.094854817	6.00079	0.45	22.50635	0.070085	72.86044
1.26671	12389.49	0.29	0.094165469	6.00091	0.45	22.51851	0.070098	73.29702
1.414371	12190.56	0.3	0.093495981	6.001029	0.45	22.53054	0.070111	73.70731
Reliability Assessment of Nigerian ABURA Timbers for Bridge Deck at 0.3m Stringer Spacing for Various Plank Widths								
3.604195	9614.064	0.2	0.083123867	6.001412	0.3	22.56953	0.070152	74.62467
3.830642	9397.193	0.21	0.082091078	6.001525	0.3	22.58097	0.070164	74.80623
4.04701	9196.266	0.22	0.08110985	6.001635	0.3	22.59219	0.070176	74.96518
4.25418	9009.397	0.23	0.080175886	6.001743	0.3	22.6032	0.070188	75.10452
4.45292	8834.998	0.24	0.079285385	6.00185	0.3	22.61401	0.070199	75.22678
4.643905	8671.719	0.25	0.078434975	6.001954	0.3	22.62463	0.07021	75.3341
4.827729	8518.403	0.26	0.077621649	6.002057	0.3	22.63506	0.070221	75.4283
5.004922	8374.054	0.27	0.076842722	6.002158	0.3	22.64532	0.070232	75.51094
5.175956	8237.806	0.28	0.076095785	6.002257	0.3	22.65541	0.070243	75.58338
5.341252	8108.906	0.29	0.075378672	6.002355	0.3	22.66533	0.070253	75.64677
5.501191	7986.691	0.3	0.07468943	6.002451	0.3	22.6751	0.070264	212

Appendix II: Java safety assessment programs

```
import flanagan.complex.Complex;
import flanagan.math.Polynomial;
import java.io.IOException;
```

```

import java.util.Arrays;
import java.util.HashMap;
import java.util.StringTokenizer;
import javax.swing.JTextArea;
import javax.swing.table.DefaultTableModel;

public class Main extends javax.swing.JFrame { private Thread t;
    private javax.swing.table.DefaultTableModel tableModel = new
javax.swing.table.DefaultTableModel(new Object[][]{},new
String[]{"Beta", "X1", "X2", "X3", "X4", "X5", "X6", "X7", "X8"})
{Class[] types = new Class[]{java.lang.String.class,
java.lang.String.class,mjava.lang.String.class,
java.lang.String.class, java.lang.String.class,
java.lang.String.class, java.lang.String.class,
java.lang.String.class, java.lang.String.class}; boolean[] canEdit =
new boolean[]{false, false, false, false, false, false, false, false,
false};

        @Override
        public Class getColumnClass(int columnIndex) {return
types[columnIndex];}

        @Override
        public boolean isCellEditable(int rowIndex, int columnIndex) {
return canEdit[columnIndex]; } };

/**
 * Creates new form Main
 */
public Main() { initComponents(); }

/**
 * This method is called from within the constructor to initialize
the form.
 * WARNING: Do NOT modify this code. The content of this method is
always
 * regenerated by the Form Editor.
 */
@SuppressWarnings("unchecked")
// <editor-fold defaultstate="collapsed" desc="Generated
Code">//GEN-BEGIN: initComponents
private void initComponents() {

    jTabledPanel = new javax.swing.JTabledPane();
    jScrollPane1 = new javax.swing.JScrollPane();
    jPanel7 = new javax.swing.JPanel();
    jPanel11 = new javax.swing.JPanel();
    jPanel12 = new javax.swing.JPanel();
    jPanel6 = new javax.swing.JPanel();
    jLabel1 = new javax.swing.JLabel();

```

```

jLabel2 = new javax.swing.JLabel();
jLabel3 = new javax.swing.JLabel();
jLabel4 = new javax.swing.JLabel();
jLabel5 = new javax.swing.JLabel();
jLabel6 = new javax.swing.JLabel();
jLabel7 = new javax.swing.JLabel();
var1 = new javax.swing.JFormattedTextField();
var2 = new javax.swing.JFormattedTextField();
var4 = new javax.swing.JFormattedTextField();
var3 = new javax.swing.JFormattedTextField();
var5 = new javax.swing.JFormattedTextField();
var6 = new javax.swing.JFormattedTextField();
var7 = new javax.swing.JFormattedTextField();
jLabel17 = new javax.swing.JLabel();
var8 = new javax.swing.JFormattedTextField();
jPanel9 = new javax.swing.JPanel();
jButton1 = new javax.swing.JButton();
jLabel18 = new javax.swing.JLabel();
iteration = new javax.swing.JSpinner();
progress = new javax.swing.JProgressBar();
jPanel3 = new javax.swing.JPanel();
jPanel4 = new javax.swing.JPanel();
jLabel8 = new javax.swing.JLabel();
jLabel9 = new javax.swing.JLabel();
jLabel10 = new javax.swing.JLabel();
jLabel11 = new javax.swing.JLabel();
x1_dist = new javax.swing.JComboBox();
x2_dist = new javax.swing.JComboBox();
x3_dist = new javax.swing.JComboBox();
x4_dist = new javax.swing.JComboBox();
x1 = new javax.swing.JFormattedTextField();
x2 = new javax.swing.JFormattedTextField();
x3 = new javax.swing.JFormattedTextField();
x4 = new javax.swing.JFormattedTextField();
jPanel5 = new javax.swing.JPanel();
jLabel12 = new javax.swing.JLabel();
jLabel13 = new javax.swing.JLabel();
jLabel14 = new javax.swing.JLabel();
x7_dist = new javax.swing.JComboBox();
x6_dist = new javax.swing.JComboBox();
x5_dist = new javax.swing.JComboBox();
jLabel15 = new javax.swing.JLabel();
cD = new javax.swing.JFormattedTextField();
x5 = new javax.swing.JFormattedTextField();
x6 = new javax.swing.JFormattedTextField();
x7 = new javax.swing.JFormattedTextField();
jLabel16 = new javax.swing.JLabel();
x8 = new javax.swing.JFormattedTextField();
x8_dist = new javax.swing.JComboBox();
jPanel11 = new javax.swing.JPanel();

```

```

jScrollPane2 = new javax.swing.JScrollPane();
jTextArea1 = new javax.swing.JTextArea();
jSeparator1 = new javax.swing.JSeparator();
jPanel8 = new javax.swing.JPanel();
jScrollPane3 = new javax.swing.JScrollPane();
jTable1 = new javax.swing.JTable();
jMenuBar1 = new javax.swing.JMenuBar();
jMenu1 = new javax.swing.JMenu();
 jMenuItem1 = new javax.swing.JMenuItem();

setDefaultCloseOperation(javax.swing.WindowConstants.EXIT_ON_CLOSE);
setBackground(new java.awt.Color(255, 255, 255));
getContentPane().setLayout(new java.awt.CardLayout());

jTabbedPane1.setBackground(new java.awt.Color(255, 255, 255));
jTabbedPane1.setTabPlacement(javax.swing.JTabbedPane.BOTTOM);

jScrollPane1.setBackground(new java.awt.Color(255, 255, 255));

jPanel7.setBackground(new java.awt.Color(255, 255, 255));

jPanel11.setBackground(new java.awt.Color(255, 255, 255));

jPanel11.setBorder(javax.swing.BorderFactory.createEtchedBorder());
jPanel11.setMinimumSize(new java.awt.Dimension(1146, 616));
jPanel11.setPreferredSize(new java.awt.Dimension(1146, 616));
jPanel11.setLayout(new
org.netbeans.lib.awtextra.AbsoluteLayout());

jPanel2.setBackground(new java.awt.Color(255, 255, 255));

jPanel2.setBorder(javax.swing.BorderFactory.createTitledBorder("Variance"));

jPanel6.setBackground(new java.awt.Color(255, 255, 255));

jPanel6.setBorder(javax.swing.BorderFactory.createBevelBorder(javax.swing.border.BevelBorder.RAISED));
jPanel6.setToolTipText("Enter value for variance of Xi");

jLabel1.setText("Var1");

jLabel2.setText("Var2");

jLabel3.setText("Var3");

jLabel4.setText("Var4");

jLabel5.setText("Var5");

```

```

jLabel6.setText("Var6");

jLabel7.setText("Var7");

var1.setFormatterFactory(new
javax.swing.text.DefaultFormatterFactory(new
javax.swing.text.NumberFormatter(new
java.text.DecimalFormat("#0.000000000000000000"))));

var2.setFormatterFactory(new
javax.swing.text.DefaultFormatterFactory(new
javax.swing.text.NumberFormatter(new
java.text.DecimalFormat("#0.000000000000000000"))));

var4.setFormatterFactory(new
javax.swing.text.DefaultFormatterFactory(new
javax.swing.text.NumberFormatter(new
java.text.DecimalFormat("#0.000000000000000000"))));

var3.setFormatterFactory(new
javax.swing.text.DefaultFormatterFactory(new
javax.swing.text.NumberFormatter(new
java.text.DecimalFormat("#0.000000000000000000"))));
var3.addActionListener(new java.awt.event.ActionListener() {
public void actionPerformed(java.awt.event.ActionEvent
evt) {var3ActionPerformed(evt);});

var5.setFormatterFactory(new
javax.swing.text.DefaultFormatterFactory(new
javax.swing.text.NumberFormatter(new
java.text.DecimalFormat("#0.000000000000000000"))));

var6.setFormatterFactory(new
javax.swing.text.DefaultFormatterFactory(new
javax.swing.text.NumberFormatter(new
java.text.DecimalFormat("#0.000000000000000000"))));

var7.setFormatterFactory(new
javax.swing.text.DefaultFormatterFactory(new
javax.swing.text.NumberFormatter(new
java.text.DecimalFormat("#0.000000000000000000"))));

jLabel17.setText("Var8");

var8.setFormatterFactory(new
javax.swing.text.DefaultFormatterFactory(new
javax.swing.text.NumberFormatter(new
java.text.DecimalFormat("#0.000000000000000000"))));

```

```

        javax.swing.GroupLayout jPanel6Layout = new
javax.swing.GroupLayout (jPanel6);
        jPanel6.setLayout (jPanel6Layout);
        jPanel6Layout.setHorizontalGroup(

jPanel6Layout.createParallelGroup(javax.swing.GroupLayout.Alignment.LE
ADING)
            .addGroup (jPanel6Layout.createSequentialGroup())
                .addGap(2, 2, 2)
                .addComponent (jLabel1)
                .addGap(18, 18, 18)
                .addComponent (var1,
javax.swing.GroupLayout.PREFERRED_SIZE, 48,
javax.swing.GroupLayout.PREFERRED_SIZE)
                .addGap(18, 18, 18)
                .addComponent (jLabel2,
javax.swing.GroupLayout.PREFERRED_SIZE, 30,
javax.swing.GroupLayout.PREFERRED_SIZE)
                .addGap(2, 2, 2)
                .addComponent (var2,
javax.swing.GroupLayout.PREFERRED_SIZE, 48,
javax.swing.GroupLayout.PREFERRED_SIZE)
                .addGap(18, 18, 18)
                .addComponent (jLabel3,
javax.swing.GroupLayout.PREFERRED_SIZE, 28,
javax.swing.GroupLayout.PREFERRED_SIZE)

            .addPreferredGap (javax.swing.LayoutStyle.ComponentPlacement.RELATED)
                .addComponent (var3,
javax.swing.GroupLayout.PREFERRED_SIZE, 48,
javax.swing.GroupLayout.PREFERRED_SIZE)
                .addGap(18, 18, 18)
                .addComponent (jLabel4)

            .addPreferredGap (javax.swing.LayoutStyle.ComponentPlacement.UNRELATED)
                .addComponent (var4,
javax.swing.GroupLayout.PREFERRED_SIZE, 48,
javax.swing.GroupLayout.PREFERRED_SIZE)
                .addGap(18, 18, 18)
                .addComponent (jLabel5)

            .addPreferredGap (javax.swing.LayoutStyle.ComponentPlacement.UNRELATED)
                .addComponent (var5,
javax.swing.GroupLayout.PREFERRED_SIZE, 48,
javax.swing.GroupLayout.PREFERRED_SIZE)
                .addGap(18, 18, 18)
                .addComponent (jLabel6)

            .addPreferredGap (javax.swing.LayoutStyle.ComponentPlacement.UNRELATED)

```

```

        .addComponent (var6,
javax.swing.GroupLayout.PREFERRED_SIZE, 48,
javax.swing.GroupLayout.PREFERRED_SIZE)
        .addGap (18, 18, 18)
        .addComponent (jLabel7)

.addPreferredGap (javax.swing.LayoutStyle.ComponentPlacement.UNRELATED)
        .addComponent (var7,
javax.swing.GroupLayout.PREFERRED_SIZE, 48,
javax.swing.GroupLayout.PREFERRED_SIZE)
        .addGap (18, 18, 18)
        .addComponent (jLabel17)

.addPreferredGap (javax.swing.LayoutStyle.ComponentPlacement.UNRELATED)
        .addComponent (var8,
javax.swing.GroupLayout.PREFERRED_SIZE, 48,
javax.swing.GroupLayout.PREFERRED_SIZE)
        .addContainerGap (77, Short.MAX_VALUE)) );
jPanel6Layout.setVerticalGroup (jPanel6Layout.createParallelGroup (javax
.swing.GroupLayout.Alignment.LEADING)
        .addGroup (jPanel6Layout.createSequentialGroup ()
        .addContainerGap ()

.addGroup (jPanel6Layout.createParallelGroup (javax.swing.GroupLayout.Al
ignment.CENTER)
        .addComponent (jLabel1)
        .addComponent (var1,
javax.swing.GroupLayout.PREFERRED_SIZE,
javax.swing.GroupLayout.DEFAULT_SIZE,
javax.swing.GroupLayout.PREFERRED_SIZE)
        .addComponent (jLabel2)
        .addComponent (var2,
javax.swing.GroupLayout.PREFERRED_SIZE,
javax.swing.GroupLayout.DEFAULT_SIZE,
javax.swing.GroupLayout.PREFERRED_SIZE)
        .addComponent (jLabel3)
        .addComponent (var3,
javax.swing.GroupLayout.PREFERRED_SIZE,
javax.swing.GroupLayout.DEFAULT_SIZE,
javax.swing.GroupLayout.PREFERRED_SIZE)
        .addComponent (jLabel4)
        .addComponent (var4,
javax.swing.GroupLayout.PREFERRED_SIZE,
javax.swing.GroupLayout.DEFAULT_SIZE,
javax.swing.GroupLayout.PREFERRED_SIZE)
        .addComponent (var5,
javax.swing.GroupLayout.PREFERRED_SIZE,
javax.swing.GroupLayout.DEFAULT_SIZE,
javax.swing.GroupLayout.PREFERRED_SIZE)
        .addComponent (jLabel5)

```

```

        .addComponent(jLabel6)
        .addComponent(var6,
javax.swing.GroupLayout.PREFERRED_SIZE,
javax.swing.GroupLayout.DEFAULT_SIZE,
javax.swing.GroupLayout.PREFERRED_SIZE)
        .addComponent(jLabel7)
        .addComponent(var7,
javax.swing.GroupLayout.PREFERRED_SIZE,
javax.swing.GroupLayout.DEFAULT_SIZE,
javax.swing.GroupLayout.PREFERRED_SIZE)
        .addComponent(jLabel17)
        .addComponent(var8,
javax.swing.GroupLayout.PREFERRED_SIZE,
javax.swing.GroupLayout.DEFAULT_SIZE,
javax.swing.GroupLayout.PREFERRED_SIZE))
        .addContainerGap(37, Short.MAX_VALUE))
    );

    jPanel9.setBackground(new java.awt.Color(255, 255, 255));
    jPanel9.setBorder(new
javax.swing.border.SoftBevelBorder(javax.swing.border.BevelBorder.RAISE
ED));

    jPanel9.setMinimumSize(new java.awt.Dimension(100, 73));
    jPanel9.setPreferredSize(new java.awt.Dimension(100, 73));

    jButton1.setFont(new java.awt.Font("Tahoma", 1, 11)); //
NOI18N
    jButton1.setText("SOLVE");
    jButton1.setToolTipText("Click to compute beta value for
number of iteration");
    jButton1.addActionListener(new java.awt.event.ActionListener()
{ public void actionPerformed(java.awt.event.ActionEvent evt) {
        jButton1ActionPerformed(evt); }});

    jLabel18.setText("Iteration(s):");

    iteration.setModel(new javax.swing.SpinnerNumberModel(5, 0,
1000, 1));
    iteration.setToolTipText("Select number of iteration(s)");

    javax.swing.GroupLayout jPanel9Layout = new
javax.swing.GroupLayout(jPanel9);
    jPanel9.setLayout(jPanel9Layout);
    jPanel9Layout.setHorizontalGroup(

jPanel9Layout.createParallelGroup(javax.swing.GroupLayout.Alignment.LEADING)
        .addGroup(jPanel9Layout.createSequentialGroup()
            .addContainerGap()

```

```

.addGroup(jPanel9Layout.createParallelGroup(javax.swing.GroupLayout.Alignment.LEADING)
        .addGroup(jPanel9Layout.createSequentialGroup()
            .addComponent(iteration,
                javax.swing.GroupLayout.PREFERRED_SIZE, 59,
                javax.swing.GroupLayout.PREFERRED_SIZE)
            .addContainerGap(javax.swing.GroupLayout.DEFAULT_SIZE,
                Short.MAX_VALUE)
            .addGroup(jPanel9Layout.createSequentialGroup()

                .addGroup(jPanel9Layout.createParallelGroup(javax.swing.GroupLayout.Alignment.LEADING)
                    .addComponent(jLabel18)
                    .addComponent(jButton1,
                        javax.swing.GroupLayout.DEFAULT_SIZE, 74, Short.MAX_VALUE))
                .addContainerGap(javax.swing.GroupLayout.DEFAULT_SIZE,
                    Short.MAX_VALUE)))
        );
jPanel9Layout.setVerticalGroup(

jPanel9Layout.createParallelGroup(javax.swing.GroupLayout.Alignment.LEADING)
        .addGroup(jPanel9Layout.createSequentialGroup()
            .addGap(20, 20, 20)
            .addComponent(jButton1)

            .addPreferredGap(javax.swing.LayoutStyle.ComponentPlacement.RELATED,
                javax.swing.GroupLayout.DEFAULT_SIZE, Short.MAX_VALUE)
            .addComponent(jLabel18)

            .addPreferredGap(javax.swing.LayoutStyle.ComponentPlacement.RELATED)
            .addComponent(iteration,
                javax.swing.GroupLayout.PREFERRED_SIZE,
                javax.swing.GroupLayout.DEFAULT_SIZE,
                javax.swing.GroupLayout.PREFERRED_SIZE)
        );

        progress.setStringPainted(true);

        javax.swing.GroupLayout jPanel2Layout = new
javax.swing.GroupLayout(jPanel2);
        jPanel2.setLayout(jPanel2Layout);
        jPanel2Layout.setHorizontalGroup(

jPanel2Layout.createParallelGroup(javax.swing.GroupLayout.Alignment.LEADING)
        .addGroup(jPanel2Layout.createSequentialGroup()

```

```

        .addGap(78, 78, 78)

    .addGroup(jPanel2Layout.createParallelGroup(javax.swing.GroupLayout.Alignment.LEADING, false)
        .addComponent(progress,
            javax.swing.GroupLayout.DEFAULT_SIZE,
            javax.swing.GroupLayout.DEFAULT_SIZE, Short.MAX_VALUE)
        .addComponent(jPanel6,
            javax.swing.GroupLayout.DEFAULT_SIZE,
            javax.swing.GroupLayout.DEFAULT_SIZE, Short.MAX_VALUE))
        .addGap(5, 5, 5)
        .addComponent(jPanel9,
            javax.swing.GroupLayout.PREFERRED_SIZE,
            javax.swing.GroupLayout.DEFAULT_SIZE,
            javax.swing.GroupLayout.PREFERRED_SIZE)
        .addContainerGap())
    );
    jPanel2Layout.setVerticalGroup(

jPanel2Layout.createParallelGroup(javax.swing.GroupLayout.Alignment.LEADING)
        .addGroup(jPanel2Layout.createSequentialGroup()
            .addGap(5, 5, 5)

    .addGroup(jPanel2Layout.createParallelGroup(javax.swing.GroupLayout.Alignment.TRAILING, false)
        .addComponent(jPanel9,
            javax.swing.GroupLayout.DEFAULT_SIZE, 108, Short.MAX_VALUE)
        .addGroup(jPanel2Layout.createSequentialGroup()
            .addComponent(jPanel6,
                javax.swing.GroupLayout.PREFERRED_SIZE,
                javax.swing.GroupLayout.DEFAULT_SIZE,
                javax.swing.GroupLayout.PREFERRED_SIZE)
            .addGap(19, 19, 19)
            .addComponent(progress,
                javax.swing.GroupLayout.DEFAULT_SIZE,
                javax.swing.GroupLayout.DEFAULT_SIZE, Short.MAX_VALUE)))
        .addContainerGap(24, Short.MAX_VALUE))
    );

    jPanel1.add(jPanel2, new
org.netbeans.lib.awtextra.AbsoluteConstraints(10, 230, 1130, 160));

    jPanel3.setBackground(new java.awt.Color(255, 255, 255));

jPanel3.setBorder(javax.swing.BorderFactory.createTitledBorder("Information for Variable x"));
jPanel3.setToolTipText("Input initial value of Xi");

jPanel4.setBackground(new java.awt.Color(255, 255, 255));

```

```

jPanel4.setBorder(javax.swing.BorderFactory.createBevelBorder(javax.swing.border.BevelBorder.RAISED));
    jPanel4.setPreferredSize(new java.awt.Dimension(220, 172));

    jLabel8.setText("X1");

    jLabel9.setText("X2");

    jLabel10.setText("X3");

    jLabel11.setText("X4");

    x1_dist.setModel(new javax.swing.DefaultComboBoxModel(new String[] { "Normal", "Log Normal" }));

    x2_dist.setModel(new javax.swing.DefaultComboBoxModel(new String[] { "Normal", "Log Normal" }));

    x3_dist.setModel(new javax.swing.DefaultComboBoxModel(new String[] { "Normal", "Log Normal" }));

    x4_dist.setModel(new javax.swing.DefaultComboBoxModel(new String[] { "Normal", "Log Normal" }));

    x1.setFormatterFactory(new javax.swing.text.DefaultFormatterFactory(new javax.swing.text.NumberFormatter(new java.text.DecimalFormat("#0.0000000000000000"))));
    x1.addActionListener(new java.awt.event.ActionListener() {
        public void actionPerformed(java.awt.event.ActionEvent evt) {
            x1ActionPerformed(evt);
        }
    });

    x2.setFormatterFactory(new javax.swing.text.DefaultFormatterFactory(new javax.swing.text.NumberFormatter(new java.text.DecimalFormat("#0.0000000000000000"))));
    x2.addActionListener(new java.awt.event.ActionListener() {
        public void actionPerformed(java.awt.event.ActionEvent evt) {
            x2ActionPerformed(evt);
        }
    });

    x3.setFormatterFactory(new javax.swing.text.DefaultFormatterFactory(new

```

```

javax.swing.text.NumberFormatter(new
java.text.DecimalFormat("#0.000000000000000000"));
    x3.addActionListener(new java.awt.event.ActionListener() {
        public void actionPerformed(java.awt.event.ActionEvent
evt) {
            x3ActionPerformed(evt);
        }
    });

    x4.setFormatterFactory(new
javax.swing.text.DefaultFormatterFactory(new
javax.swing.text.NumberFormatter(new
java.text.DecimalFormat("#0.000000000000000000"));
    x4.addActionListener(new java.awt.event.ActionListener() {
        public void actionPerformed(java.awt.event.ActionEvent
evt) {
            x4ActionPerformed(evt);
        }
    });

    javax.swing.GroupLayout jPanel4Layout = new
javax.swing.GroupLayout(jPanel4);
    jPanel4.setLayout(jPanel4Layout);
    jPanel4Layout.setHorizontalGroup(

jPanel4Layout.createParallelGroup(javax.swing.GroupLayout.Alignment.LE
ADING)
        .addGroup(jPanel4Layout.createSequentialGroup()
            .addGroup(jPanel4Layout.createParallelGroup(javax.swing.Grou
pLayout.Alignment.LEADING)
                .addComponent(jLabel8)
                .addComponent(jLabel9)
                .addComponent(jLabel10)
                .addComponent(jLabel11))
            .addContainerGap())
        .addPreferredGap(javax.swing.LayoutStyle.ComponentPlacement.REL
ATED)
        .addGroup(jPanel4Layout.createParallelGroup(javax.swing.Grou
pLayout.Alignment.LEADING)
            .addComponent(x1,
javax.swing.GroupLayout.PREFERRED_SIZE, 52,
javax.swing.GroupLayout.PREFERRED_SIZE)
            .addComponent(x2,
javax.swing.GroupLayout.PREFERRED_SIZE, 52,
javax.swing.GroupLayout.PREFERRED_SIZE)
            .addComponent(x3,
javax.swing.GroupLayout.PREFERRED_SIZE, 52,
javax.swing.GroupLayout.PREFERRED_SIZE)

```

```

        .addComponent(x4,
javax.swing.GroupLayout.PREFERRED_SIZE, 52,
javax.swing.GroupLayout.PREFERRED_SIZE))
        .addGap(18, 18, 18)

.addGroup(jPanel4Layout.createParallelGroup(javax.swing.GroupLayout.Ali
ignment.LEADING)
        .addComponent(x1_dist,
javax.swing.GroupLayout.PREFERRED_SIZE, 94,
javax.swing.GroupLayout.PREFERRED_SIZE)
        .addComponent(x2_dist,
javax.swing.GroupLayout.PREFERRED_SIZE, 94,
javax.swing.GroupLayout.PREFERRED_SIZE)
        .addComponent(x3_dist,
javax.swing.GroupLayout.PREFERRED_SIZE, 94,
javax.swing.GroupLayout.PREFERRED_SIZE)
        .addComponent(x4_dist,
javax.swing.GroupLayout.PREFERRED_SIZE, 94,
javax.swing.GroupLayout.PREFERRED_SIZE)
        .addContainerGap(18, Short.MAX_VALUE))
    );
    jPanel4Layout.setVerticalGroup(

jPanel4Layout.createParallelGroup(javax.swing.GroupLayout.Alignment.LE
ADING)
        .addGroup(jPanel4Layout.createSequentialGroup()
        .addGap(20, 20, 20)

.addGroup(jPanel4Layout.createParallelGroup(javax.swing.GroupLayout.Ali
ignment.LEADING)
        .addComponent(jLabel8,
javax.swing.GroupLayout.Alignment.TRAILING)
        .addComponent(x1,
javax.swing.GroupLayout.Alignment.TRAILING,
javax.swing.GroupLayout.PREFERRED_SIZE,
javax.swing.GroupLayout.DEFAULT_SIZE,
javax.swing.GroupLayout.PREFERRED_SIZE)
        .addComponent(x1_dist,
javax.swing.GroupLayout.Alignment.TRAILING,
javax.swing.GroupLayout.PREFERRED_SIZE,
javax.swing.GroupLayout.DEFAULT_SIZE,
javax.swing.GroupLayout.PREFERRED_SIZE))

.addPreferredGap(javax.swing.LayoutStyle.ComponentPlacement.RELATED)

.addGroup(jPanel4Layout.createParallelGroup(javax.swing.GroupLayout.Ali
ignment.LEADING)
        .addComponent(jLabel9,
javax.swing.GroupLayout.Alignment.TRAILING)

```

```

        .addComponent(x2,
javax.swing.GroupLayout.Alignment.TRAILING,
javax.swing.GroupLayout.PREFERRED_SIZE,
javax.swing.GroupLayout.DEFAULT_SIZE,
javax.swing.GroupLayout.PREFERRED_SIZE)
        .addComponent(x2_dist,
javax.swing.GroupLayout.Alignment.TRAILING,
javax.swing.GroupLayout.PREFERRED_SIZE,
javax.swing.GroupLayout.DEFAULT_SIZE,
javax.swing.GroupLayout.PREFERRED_SIZE) )

.addPreferredGap(javax.swing.LayoutStyle.ComponentPlacement.RELATED)

.addGroup(jPanel4Layout.createParallelGroup(javax.swing.GroupLayout.Al
ignment.LEADING)
        .addComponent(jLabel10,
javax.swing.GroupLayout.Alignment.TRAILING)
        .addComponent(x3,
javax.swing.GroupLayout.Alignment.TRAILING,
javax.swing.GroupLayout.PREFERRED_SIZE,
javax.swing.GroupLayout.DEFAULT_SIZE,
javax.swing.GroupLayout.PREFERRED_SIZE)
        .addComponent(x3_dist,
javax.swing.GroupLayout.Alignment.TRAILING,
javax.swing.GroupLayout.PREFERRED_SIZE,
javax.swing.GroupLayout.DEFAULT_SIZE,
javax.swing.GroupLayout.PREFERRED_SIZE) )

.addPreferredGap(javax.swing.LayoutStyle.ComponentPlacement.RELATED)

.addGroup(jPanel4Layout.createParallelGroup(javax.swing.GroupLayout.Al
ignment.LEADING)
        .addComponent(x4_dist,
javax.swing.GroupLayout.Alignment.TRAILING,
javax.swing.GroupLayout.PREFERRED_SIZE,
javax.swing.GroupLayout.DEFAULT_SIZE,
javax.swing.GroupLayout.PREFERRED_SIZE)
        .addComponent(x4,
javax.swing.GroupLayout.Alignment.TRAILING,
javax.swing.GroupLayout.PREFERRED_SIZE,
javax.swing.GroupLayout.DEFAULT_SIZE,
javax.swing.GroupLayout.PREFERRED_SIZE)
        .addComponent(jLabel11,
javax.swing.GroupLayout.Alignment.TRAILING) )
        .addContainerGap(50, Short.MAX_VALUE)
);

jPanel5.setBackground(new java.awt.Color(255, 255, 255));

```

```

jPanel5.setBorder(javax.swing.BorderFactory.createBevelBorder(javax.swing.border.BevelBorder.RAISED));
    jPanel5.setPreferredSize(new java.awt.Dimension(220, 172));

    jLabel12.setText("X5");

    jLabel13.setText("X6");

    jLabel14.setText("X7");

    x7_dist.setModel(new javax.swing.DefaultComboBoxModel(new String[] { "Normal", "Log Normal" }));

    x6_dist.setModel(new javax.swing.DefaultComboBoxModel(new String[] { "Normal", "Log Normal" }));

    x5_dist.setModel(new javax.swing.DefaultComboBoxModel(new String[] { "Normal", "Log Normal" }));

    jLabel15.setText("Cd");

    cD.setFormatterFactory(new
javax.swing.text.DefaultFormatterFactory(new
javax.swing.text.NumberFormatter(new
java.text.DecimalFormat("#0.000000000000000000"))));
    cD.addActionListener(new java.awt.event.ActionListener() {
        public void actionPerformed(java.awt.event.ActionEvent
evt) {
            cDActionPerformed(evt);
        }
    });

    x5.setFormatterFactory(new
javax.swing.text.DefaultFormatterFactory(new
javax.swing.text.NumberFormatter(new
java.text.DecimalFormat("#0.000000000000000000"))));
    x5.addActionListener(new java.awt.event.ActionListener() {
        public void actionPerformed(java.awt.event.ActionEvent
evt) {
            x5ActionPerformed(evt);
        }
    });

    x6.setFormatterFactory(new
javax.swing.text.DefaultFormatterFactory(new
javax.swing.text.NumberFormatter(new
java.text.DecimalFormat("#0.000000000000000000"))));
    x6.addActionListener(new java.awt.event.ActionListener() {

```



```

        .addComponent(jLabel14)
        .addComponent(jLabel13)
        .addComponent(jLabel12))

.addPreferredGap(javax.swing.LayoutStyle.ComponentPlacement.RELATED)

.addGroup(jPanel5Layout.createParallelGroup(javax.swing.GroupLayout.Alignment.LEADING)
        .addComponent(cD,
javax.swing.GroupLayout.PREFERRED_SIZE, 52,
javax.swing.GroupLayout.PREFERRED_SIZE)

.addGroup(jPanel5Layout.createSequentialGroup())

.addGroup(jPanel5Layout.createParallelGroup(javax.swing.GroupLayout.Alignment.LEADING)
        .addComponent(x6,
javax.swing.GroupLayout.PREFERRED_SIZE, 52,
javax.swing.GroupLayout.PREFERRED_SIZE)
        .addComponent(x7,
javax.swing.GroupLayout.PREFERRED_SIZE, 52,
javax.swing.GroupLayout.PREFERRED_SIZE)
        .addComponent(x5,
javax.swing.GroupLayout.PREFERRED_SIZE, 52,
javax.swing.GroupLayout.PREFERRED_SIZE))

.addPreferredGap(javax.swing.LayoutStyle.ComponentPlacement.UNRELATED)

.addGroup(jPanel5Layout.createParallelGroup(javax.swing.GroupLayout.Alignment.LEADING)
        .addComponent(x5_dist,
javax.swing.GroupLayout.PREFERRED_SIZE, 94,
javax.swing.GroupLayout.PREFERRED_SIZE)
        .addComponent(x6_dist,
javax.swing.GroupLayout.PREFERRED_SIZE, 94,
javax.swing.GroupLayout.PREFERRED_SIZE)
        .addComponent(x7_dist,
javax.swing.GroupLayout.PREFERRED_SIZE, 94,
javax.swing.GroupLayout.PREFERRED_SIZE))))
        .addGroup(jPanel5Layout.createSequentialGroup())
        .addComponent(jLabel16)

.addPreferredGap(javax.swing.LayoutStyle.ComponentPlacement.RELATED)
        .addComponent(x8,
javax.swing.GroupLayout.PREFERRED_SIZE, 52,
javax.swing.GroupLayout.PREFERRED_SIZE)

.addPreferredGap(javax.swing.LayoutStyle.ComponentPlacement.UNRELATED)

```

```

        .addComponent(x8_dist,
javax.swing.GroupLayout.PREFERRED_SIZE, 94,
javax.swing.GroupLayout.PREFERRED_SIZE))
        .addContainerGap(34, Short.MAX_VALUE))
    );
    jPanel5Layout.setVerticalGroup(

jPanel5Layout.createParallelGroup(javax.swing.GroupLayout.Alignment.LEADING)
        .addGroup(jPanel5Layout.createSequentialGroup()
            .addContainerGap()

.addGroup(jPanel5Layout.createParallelGroup(javax.swing.GroupLayout.Alignment.LEADING)
            .addComponent(x5,
javax.swing.GroupLayout.Alignment.TRAILING,
javax.swing.GroupLayout.PREFERRED_SIZE,
javax.swing.GroupLayout.DEFAULT_SIZE,
javax.swing.GroupLayout.PREFERRED_SIZE)
                .addComponent(jLabel12,
javax.swing.GroupLayout.Alignment.TRAILING)
                    .addComponent(x5_dist,
javax.swing.GroupLayout.Alignment.TRAILING,
javax.swing.GroupLayout.PREFERRED_SIZE,
javax.swing.GroupLayout.DEFAULT_SIZE,
javax.swing.GroupLayout.PREFERRED_SIZE))

.addPreferredGap(javax.swing.LayoutStyle.ComponentPlacement.RELATED)

.addGroup(jPanel5Layout.createParallelGroup(javax.swing.GroupLayout.Alignment.LEADING)
            .addComponent(jLabel13,
javax.swing.GroupLayout.Alignment.TRAILING)
                .addComponent(x6,
javax.swing.GroupLayout.Alignment.TRAILING,
javax.swing.GroupLayout.PREFERRED_SIZE,
javax.swing.GroupLayout.DEFAULT_SIZE,
javax.swing.GroupLayout.PREFERRED_SIZE)
                    .addComponent(x6_dist,
javax.swing.GroupLayout.Alignment.TRAILING,
javax.swing.GroupLayout.PREFERRED_SIZE,
javax.swing.GroupLayout.DEFAULT_SIZE,
javax.swing.GroupLayout.PREFERRED_SIZE))

.addPreferredGap(javax.swing.LayoutStyle.ComponentPlacement.RELATED)

.addGroup(jPanel5Layout.createParallelGroup(javax.swing.GroupLayout.Alignment.LEADING)
            .addComponent(jLabel14,
javax.swing.GroupLayout.Alignment.TRAILING)

```

```

        .addComponent(x7,
javax.swing.GroupLayout.Alignment.TRAILING,
javax.swing.GroupLayout.PREFERRED_SIZE,
javax.swing.GroupLayout.DEFAULT_SIZE,
javax.swing.GroupLayout.PREFERRED_SIZE)
        .addComponent(x7_dist,
javax.swing.GroupLayout.Alignment.TRAILING,
javax.swing.GroupLayout.PREFERRED_SIZE,
javax.swing.GroupLayout.DEFAULT_SIZE,
javax.swing.GroupLayout.PREFERRED_SIZE)
        .addGap(3, 3, 3)

.addGroup(jPanel5Layout.createParallelGroup(javax.swing.GroupLayout.Ali
ignment.LEADING)
        .addComponent(jLabel16,
javax.swing.GroupLayout.Alignment.TRAILING)

.addGroup(javax.swing.GroupLayout.Alignment.TRAILING,
jPanel5Layout.createParallelGroup(javax.swing.GroupLayout.Alignment.BA
SELINE)
        .addComponent(x8,
javax.swing.GroupLayout.PREFERRED_SIZE,
javax.swing.GroupLayout.DEFAULT_SIZE,
javax.swing.GroupLayout.PREFERRED_SIZE)
        .addComponent(x8_dist,
javax.swing.GroupLayout.PREFERRED_SIZE,
javax.swing.GroupLayout.DEFAULT_SIZE,
javax.swing.GroupLayout.PREFERRED_SIZE)))

.addPreferredGap(javax.swing.LayoutStyle.ComponentPlacement.RELATED)

.addGroup(jPanel5Layout.createParallelGroup(javax.swing.GroupLayout.Ali
ignment.LEADING)
        .addComponent(jLabel15,
javax.swing.GroupLayout.Alignment.TRAILING)
        .addComponent(cD,
javax.swing.GroupLayout.Alignment.TRAILING,
javax.swing.GroupLayout.PREFERRED_SIZE,
javax.swing.GroupLayout.DEFAULT_SIZE,
javax.swing.GroupLayout.PREFERRED_SIZE)
        .addContainerGap(36, Short.MAX_VALUE))
    );

jPanel11.setBorder(javax.swing.BorderFactory.createTitledBorder(javax.
swing.BorderFactory.createBevelBorder(javax.swing.border.BevelBorder.R
AISED), "Output"));
jPanel11.setPreferredSize(new java.awt.Dimension(400, 170));

jTextArea1.setEditable(false);

```

```

        jTextArea1.setColumns(20);
        jTextArea1.setRows(5);
        jScrollPane2.setViewportView(jTextArea1);

        javax.swing.GroupLayout jPanel11Layout = new
javax.swing.GroupLayout(jPanel11);
        jPanel11.setLayout(jPanel11Layout);
        jPanel11Layout.setHorizontalGroup(

jPanel11Layout.createParallelGroup(javax.swing.GroupLayout.Alignment.L
EADING)
            .addComponent(jScrollPane2,
javax.swing.GroupLayout.DEFAULT_SIZE, 388, Short.MAX_VALUE)
            );
        jPanel11Layout.setVerticalGroup(

jPanel11Layout.createParallelGroup(javax.swing.GroupLayout.Alignment.L
EADING)
            .addComponent(jScrollPane2,
javax.swing.GroupLayout.DEFAULT_SIZE, 148, Short.MAX_VALUE)
            );

        javax.swing.GroupLayout jPanel3Layout = new
javax.swing.GroupLayout(jPanel3);
        jPanel3.setLayout(jPanel3Layout);
        jPanel3Layout.setHorizontalGroup(

jPanel3Layout.createParallelGroup(javax.swing.GroupLayout.Alignment.LE
ADING)
            .addGroup(jPanel3Layout.createSequentialGroup())
                .addGap(134, 134, 134)
                .addComponent(jPanel4,
javax.swing.GroupLayout.PREFERRED_SIZE,
javax.swing.GroupLayout.DEFAULT_SIZE,
javax.swing.GroupLayout.PREFERRED_SIZE)
                .addGap(5, 5, 5)
                .addComponent(jPanel5,
javax.swing.GroupLayout.PREFERRED_SIZE,
javax.swing.GroupLayout.DEFAULT_SIZE,
javax.swing.GroupLayout.PREFERRED_SIZE)
                .addGap(5, 5, 5)
                .addComponent(jPanel11,
javax.swing.GroupLayout.PREFERRED_SIZE,
javax.swing.GroupLayout.DEFAULT_SIZE,
javax.swing.GroupLayout.PREFERRED_SIZE)
            );
        jPanel3Layout.setVerticalGroup(

jPanel3Layout.createParallelGroup(javax.swing.GroupLayout.Alignment.LE
ADING)

```

```

        .addGroup(jPanel3Layout.createSequentialGroup()
            .addGap(5, 5, 5)

        .addGroup(jPanel3Layout.createParallelGroup(javax.swing.GroupLayout.Alignment.LEADING)
            .addComponent(jPanel4,
                javax.swing.GroupLayout.PREFERRED_SIZE,
                javax.swing.GroupLayout.DEFAULT_SIZE,
                javax.swing.GroupLayout.PREFERRED_SIZE)
            .addComponent(jPanel5,
                javax.swing.GroupLayout.PREFERRED_SIZE,
                javax.swing.GroupLayout.DEFAULT_SIZE,
                javax.swing.GroupLayout.PREFERRED_SIZE)
            .addComponent(jPanel11,
                javax.swing.GroupLayout.PREFERRED_SIZE,
                javax.swing.GroupLayout.DEFAULT_SIZE,
                javax.swing.GroupLayout.PREFERRED_SIZE)))
        );

        jPanel11.add(jPanel3, new
            org.netbeans.lib.awtextra.AbsoluteConstraints(10, 11, 1130, 220));
        jPanel11.add(jSeparator1, new
            org.netbeans.lib.awtextra.AbsoluteConstraints(0, 400, 1140, 10));

        jPanel8.setBackground(new java.awt.Color(255, 255, 255));

jPanel8.setBorder(javax.swing.BorderFactory.createTitledBorder("Result
"));
        jPanel8.setLayout(new java.awt.CardLayout());

        jTable1.setModel(tableModel);
        jTable1.setSelectionBackground(new java.awt.Color(0, 51,
255));
        jScrollPane3.setViewportView(jTable1);

        jPanel8.add(jScrollPane3, "card2");

        jPanel11.add(jPanel8, new
            org.netbeans.lib.awtextra.AbsoluteConstraints(10, 410, 1130, 200));

        jPanel7.add(jPanel11);

        jScrollPane1.setViewportView(jPanel7);

        jTabbedPane1.addTab("Variable Settings", jScrollPane1);

        getContentPane().add(jTabbedPane1, "card2");

        jMenuBar1.setBackground(new java.awt.Color(255, 255, 255));

```

```

jMenu1.setText("File");

jMenuItem1.setAccelerator(javax.swing.KeyStroke.getKeyStroke(java.awt.
event.KeyEvent.VK_Q, java.awt.event.InputEvent.CTRL_MASK));
    jMenuItem1.setText("Exit");
    jMenuItem1.addActionListener(new
java.awt.event.ActionListener() {
        public void actionPerformed(java.awt.event.ActionEvent
evt) {
            jMenuItem1ActionPerformed(evt);
        }
    });
jMenu1.add(jMenuItem1);

jMenuBar1.add(jMenu1);

setJMenuBar(jMenuBar1);

pack();
} // </editor-fold> // GEN-END: initComponents

private void jMenuItem1ActionPerformed(java.awt.event.ActionEvent
evt) { // GEN-FIRST:event_jMenuItem1ActionPerformed
    // TODO add your handling code here:
    System.exit(0);
} // GEN-LAST:event_jMenuItem1ActionPerformed

private void jButton1ActionPerformed(java.awt.event.ActionEvent
evt) { // GEN-FIRST:event_jButton1ActionPerformed
    // TODO add your handling code here:
    if (t != null) {
        if (t.isAlive()) {
            System.out.println("System Busy");
            t.interrupt();
        }
    }
    int size = tableModel.getRowCount();
    for (int i = size - 1; i >= 0; i--) {
        tableModel.removeRow(i);
    }
    jTextArea1.setText("");
    final int times = (int) iteration.getValue();

    t = new Thread(new Runnable() {
        @Override
        public void run() {
//step 1 - 3
            double[] xi = new double[8];

```

```

double[] varXi = new double[8];
double[] varXiy = new double[8];
double[] miuXiy = new double[8];
double[] dzdxi = new double[8];
double[] dzdxie = new double[8];
double[] varXie = new double[8];
double[] miuXie = new double[8];
double[] alfaXi = new double[8];
Double[] tb_row1 = new Double[9];
String[] xi_dist = new String[8];
double varZ, beta1, miuZ, cd, sumVar, sumDzxie;
//value of x
xi[0] = Double.parseDouble(x1.getText());
xi[1] = Double.parseDouble(x2.getText());
xi[2] = Double.parseDouble(x3.getText());
xi[3] = Double.parseDouble(x4.getText());
xi[4] = Double.parseDouble(x5.getText());
xi[5] = Double.parseDouble(x6.getText());
xi[6] = Double.parseDouble(x7.getText());
xi[7] = Double.parseDouble(x8.getText());
cd = Double.parseDouble(cd.getText());

//variance of x
varXi[0] = Double.parseDouble(var1.getText());
varXi[1] = Double.parseDouble(var2.getText());
varXi[2] = Double.parseDouble(var3.getText());
varXi[3] = Double.parseDouble(var4.getText());
varXi[4] = Double.parseDouble(var5.getText());
varXi[5] = Double.parseDouble(var6.getText());
varXi[6] = Double.parseDouble(var7.getText());
varXi[7] = Double.parseDouble(var8.getText());

xi_dist[0] = x1_dist.getSelectedItem().toString();
xi_dist[1] = x2_dist.getSelectedItem().toString();
xi_dist[2] = x3_dist.getSelectedItem().toString();
xi_dist[3] = x4_dist.getSelectedItem().toString();
xi_dist[4] = x5_dist.getSelectedItem().toString();
xi_dist[5] = x6_dist.getSelectedItem().toString();
xi_dist[6] = x7_dist.getSelectedItem().toString();
xi_dist[7] = x8_dist.getSelectedItem().toString();

//end of step 3
//step 4
for (int i = 0; i < varXiy.length; i++) {
    varXiy[i] = (xi_dist[i].equals("Log Normal") ?
Math.pow(Math.log(1 + Math.pow(varXi[i] / xi[i], 2)), 0.5) :
varXi[i]);
}
for (int i = 0; i < miuXiy.length; i++) {

```

```

        miuXiy[i] = (xi_dist[i].equals("Log Normal") ?
Math.log(xi[i]) - 0.5 * Math.pow(varXiy[i], 2) : xi[i]);
    }

//step 5
        miuZ =
cd
        0.2667 * xi[0] * xi[1] * Math.pow(xi[2], 2) *
Math.pow(xi[4], 2)
        - 0.15625 * xi[1] * xi[2] * xi[3] *
* xi[6]
        - 0.1875 * xi[1] * Math.pow(xi[4], 2) * xi[5]
        - 0.21875 * (xi[7] * (2 * xi[4] - 0.5) + 9.3 *
Math.pow(xi[4], 2));
//step 6
        dzdxi[0] = 0.2667 * xi[1] * Math.pow(xi[2], 2) * cd;

        dzdxi[1] = 0.2667 * xi[0] * Math.pow(xi[2], 2) * cd -
0.15626 * xi[2] * xi[3] * Math.pow(xi[4], 2) - 0.1875 *
Math.pow(xi[4], 2) * xi[5] * xi[6];

        dzdxi[2] = 0.5334 * xi[0] * xi[1] * xi[2] * cd -
0.15625 * xi[1] * xi[3] * Math.pow(xi[4], 2);

        dzdxi[3] = -0.15625 * xi[1] * xi[2] * Math.pow(xi[4],
2);

        dzdxi[4] = -0.3125 * xi[1] * xi[2] * xi[3] * xi[4] -
0.375 * xi[1] * xi[4] * xi[5] * xi[6] - 0.21875 * (2 * xi[7] + 18.6 *
xi[4]);

        dzdxi[5] = -0.1875 * xi[1] * Math.pow(xi[4], 2) *
xi[6];

        dzdxi[6] = -0.1875 * xi[1] * Math.pow(xi[4], 2) *
xi[5];

        dzdxi[7] = -0.21875 * (2 * xi[4] - 0.5);

//step 7
        sumVar = 0;
        for (int i = 0; i < dzdxi.length; i++) {
            sumVar += Math.pow(varXi[i], 2) *
Math.pow(dzdxi[i], 2);
        }

        varZ = Math.pow(sumVar, 0.5);
        jTextArea1.setText("miuZ = " + miuZ);
        jTextArea1.setText(jTextArea1.getText() + "\nvarZ = "
+ varZ);

```

```

//step 8
        betal = miuZ / varZ;
//step 9

        JTextArea1.setText(JTextArea1.getText() + "\nInitial
value for beta = " + betal);
//        addToOutput(JTextArea1, betal+"", "Initial value
for beta");

        for (int k = 0; k < times; k++) {
            if (k > 0) {
                for (int i = 0; i < xi.length; i++) {
                    xi[i] = miuXie[i] - (betal * alfaXi[i] *
varXie[i]);
//                    System.out.println("xi[" + i + "] =" +
xi[i]);
                }
            }
//
            for (int i = 0; i < xi.length; i++) {
                JTextArea1.setText(JTextArea1.getText() +
"\nx" + (i + 1) + " = " + xi[i]);
            }
            for (int i = 0; i < varXi.length; i++) {
                JTextArea1.setText(JTextArea1.getText() +
"\nvarXi" + (i + 1) + " = " + varXi[i]);
            }
            for (int i = 0; i < xi_dist.length; i++) {
                JTextArea1.setText(JTextArea1.getText() +
"\nx" + (i + 1) + "_distribution = " + xi_dist[i]);
            }
            for (int i = 0; i < dzdxi.length; i++) {
                JTextArea1.setText(JTextArea1.getText() +
"\ndzdx" + (i + 1) + " = " + dzdxi[i]);
            }

//

//step 10
            for (int i = 0; i < varXie.length; i++) {
                varXie[i] = (xi_dist[i].equals("Log Normal") ?
varXiy[i] * xi[i] : varXi[i]);
                JTextArea1.setText(JTextArea1.getText() +
"\nvarX" + (i + 1) + "e = " + varXie[i]);
            }

            for (int i = 0; i < miuXie.length; i++) {

```

```

        miuXie[i] = (xi_dist[i].equals("Log Normal") ?
xi[i] - varXie[i] / varXiy[i] * (Math.log(xi[i]) - miuXiy[i]) :
xi[i]);
        JTextArea1.setText(jTextArea1.getText() +
"\nmiuX" + (i + 1) + "e = " + miuXie[i]);
    }

//step 11
        dzdxie[0] = 0.2667 * miuXie[1] *
Math.pow(miuXie[2], 2) * cd;

        dzdxie[1] = 0.2667 * miuXie[0] *
Math.pow(miuXie[2], 2) * cd - 0.15626 * miuXie[2] * miuXie[3] *
Math.pow(miuXie[4], 2) - 0.1875 * Math.pow(miuXie[4], 2) * miuXie[5] *
miuXie[6];

        dzdxie[2] = 0.5334 * miuXie[0] * miuXie[1] *
miuXie[2] * cd - 0.15625 * miuXie[1] * miuXie[3] * Math.pow(miuXie[4],
2);

        dzdxie[3] = -0.15625 * miuXie[1] * miuXie[2] *
Math.pow(miuXie[4], 2);

        dzdxie[4] = -0.3125 * miuXie[1] * miuXie[2] *
miuXie[3] * Math.pow(miuXie[4], 1) - 0.375 * miuXie[1] * miuXie[6] *
Math.pow(miuXie[4], 1) * miuXie[5] - 0.21875 * (2 * miuXie[7] + 18.6 *
miuXie[4]);

        dzdxie[5] = -0.1875 * miuXie[1] *
Math.pow(miuXie[4], 2) * miuXie[6];

        dzdxie[6] = -0.1875 * miuXie[1] *
Math.pow(miuXie[4], 2) * miuXie[5];

        dzdxie[7] = -0.21875 * (2 * miuXie[4] - 0.5);

        sumDzxie = 0;
        for (int i = 0; i < dzdxie.length; i++) {
            sumDzxie += Math.pow(varXie[i], 2) *
Math.pow(dzdxie[i], 2);
            JTextArea1.setText(jTextArea1.getText() +
"\ndzdx" + (i + 1) + "e = " + dzdxie[i]);
        }

//step 12
        for (int i = 0; i < alfaXi.length; i++) {
            alfaXi[i] = varXie[i] * dzdxie[i] /
Math.pow(sumDzxie, 0.5);
            JTextArea1.setText(jTextArea1.getText() +
"\nalfaX" + (i + 1) + " = " + alfaXi[i]);

```

```

}

Poly c0 = new Poly(0, 0);
Poly c1 = new Poly(0.2667, 0);
Poly c2 = new Poly(0.15625, 0);
Poly c3 = new Poly(0.1875, 0);
Poly c4 = new Poly(0.21875, 0);
Poly c5 = new Poly(2, 0);
Poly c6 = new Poly(0.5, 0);
Poly c7 = new Poly(9.3, 0);
Poly variableBetaPower1 = new Poly(1, 1);

Poly px1e = new Poly(miuXie[0], 0);
Poly px2e = new Poly(miuXie[1], 0);
Poly px3e = new Poly(miuXie[2], 0);
Poly px4e = new Poly(miuXie[3], 0);
Poly px5e = new Poly(miuXie[4], 0);
Poly px6e = new Poly(miuXie[5], 0);
Poly px7e = new Poly(miuXie[6], 0);
Poly px8e = new Poly(miuXie[7], 0);
Poly pCd = new Poly(cd, 0);

Poly pVarX1e = new Poly(varXie[0], 0);
Poly pVarX2e = new Poly(varXie[1], 0);
Poly pVarX3e = new Poly(varXie[2], 0);
Poly pVarX4e = new Poly(varXie[3], 0);
Poly pVarX5e = new Poly(varXie[4], 0);
Poly pVarX6e = new Poly(varXie[5], 0);
Poly pVarX7e = new Poly(varXie[6], 0);
Poly pVarX8e = new Poly(varXie[7], 0);

Poly pAlfaX1 = new Poly(alfaXi[0], 0);
Poly pAlfaX2 = new Poly(alfaXi[1], 0);
Poly pAlfaX3 = new Poly(alfaXi[2], 0);
Poly pAlfaX4 = new Poly(alfaXi[3], 0);
Poly pAlfaX5 = new Poly(alfaXi[4], 0);
Poly pAlfaX6 = new Poly(alfaXi[5], 0);
Poly pAlfaX7 = new Poly(alfaXi[6], 0);
Poly pAlfaX8 = new Poly(alfaXi[7], 0);

//step 13
Poly res =
c1.times(px1e.minus(pAlfaX1.times(pVarX1e).times(variableBetaPower1)))
.times(px2e.minus(pAlfaX2.times(pVarX2e).times(variableBetaPower1))).times
imes(px3e.minus(pAlfaX3.times(pVarX3e).times(variableBetaPower1))).times
es(px3e.minus(pAlfaX3.times(pVarX3e).times(variableBetaPower1))).times
(pCd)

.minus(c2.times(px2e.minus(pAlfaX2.times(pVarX2e).times(variableBetaPower1))).times
(px3e.minus(pAlfaX3.times(pVarX3e).times(variableBetaPower1))).times

```

```

r1))) .times (px4e.minus (pAlfaX4.times (pVarX4e) .times (variableBetaPower1
))) .times (px5e.minus (pAlfaX5.times (pVarX5e) .times (variableBetaPower1))
) .times (px5e.minus (pAlfaX5.times (pVarX5e) .times (variableBetaPower1)))

.minus (c3.times (px2e.minus (pAlfaX2.times (pVarX2e) .times (variableBetaPo
wer1))) .times (px5e.minus (pAlfaX5.times (pVarX5e) .times (variableBetaPowe
r1))) .times (px5e.minus (pAlfaX5.times (pVarX5e) .times (variableBetaPower1
))) .times (px6e.minus (pAlfaX6.times (pVarX6e) .times (variableBetaPower1))
) .times (px7e.minus (pAlfaX7.times (pVarX7e) .times (variableBetaPower1)))

.minus (c4.times ((px8e.minus (pAlfaX8.times (pVarX8e) .times (variableBetaP
ower1))) .times (c5.times (px5e.minus (pAlfaX5.times (pVarX5e) .times (variab
leBetaPower1))) .minus (c6))) .plus (c7.times (px5e.minus (pAlfaX5.times (pVa
rX5e) .times (variableBetaPower1)))));

```

```

        JTextArea1.setText (JTextArea1.getText () + "\n
Polynomial = " + res);

        StringTokenizer st = new
StringTokenizer (res.toString ());
        String content;
        HashMap<Object, Object> map = new HashMap<> ();
        int x = 0;
        while (st.hasMoreTokens ()) {
            content = st.nextToken ();
            if (content.contains ("x")) {
                String val = (content.substring (0,
content.indexOf ('x')));
                map.put (++x, val);
            } else {
                map.put (++x, content);
            }
        }
        double [] arr = new double [6];
        int p = 0;
        for (int i = x; i > 0; i--) {
            if (i - 1 > 0) {
                if (map.get (i - 1).toString ().equals ("-"))
{
                    if (!map.get (i).toString ().equals ("-"))
&& !map.get (i).toString ().equals ("+")) {
                        arr [p++] =
Double.parseDouble ((String) map.get (i)) * -1;
                    }
                } else {
                    if (map.get (i -
1).toString ().equals ("+")) {
                        arr [p++] =
Double.parseDouble ((String) map.get (i));

```

```

        }
    }
    } else {
        arr[p++] = Double.parseDouble((String)
map.get(i));
    }
}

Polynomial pol = new Polynomial(arr);
Complex[] roots = pol.roots();//cp.roots();
Double[] rts = new Double[6];
rts[0] = beta1;

    for (int i = 1; i < rts.length; i++) {
//        System.out.println(roots[i]);
        StringTokenizer tokenizer = new
StringTokenizer(roots[i - 1].toString());
        String token = tokenizer.nextToken();
//        System.out.println(token);
        rts[i] = Double.parseDouble(token);
    }

//
System.out.println("=====");
    Arrays.sort(rts);
    JTextArea1.setText(JTextArea1.getText() + "\nRoots
with beta" + Arrays.deepToString(rts));
//        addToOutput(jTextArea1,
Arrays.deepToString(rts), "Roots with beta");

        beta1 = getClosest(rts, beta1);

        JTextArea1.setText(JTextArea1.getText() + "\n Beta
value " + beta1);

        JTextArea1.setText(JTextArea1.getText() + "\n
=====");
//        addToOutput(jTextArea1, beta1+"", "Beta value");
//        addToOutput(jTextArea1, "",
"=====");

        tb_row1[0] = beta1;
        for (int i = 1; i < tb_row1.length; i++) {
            tb_row1[i] = xi[i - 1];
        }
//        addToTable(tableModel, tb_row1);
        tableModel.addRow(tb_row1);

        double per = (double) k / times * 100;
        progress.setValue((int) per);
        if (k == times - 1) {

```

```

        progress.setValue(100);
    }
}
});
t.start();

} //GEN-LAST:event_jButton1ActionPerformed

private void var3ActionPerformed(java.awt.event.ActionEvent evt)
{ //GEN-FIRST:event_var3ActionPerformed
    // TODO add your handling code here:
} //GEN-LAST:event_var3ActionPerformed

private void x8ActionPerformed(java.awt.event.ActionEvent evt)
{ //GEN-FIRST:event_x8ActionPerformed
    // TODO add your handling code here:
} //GEN-LAST:event_x8ActionPerformed

private void x7ActionPerformed(java.awt.event.ActionEvent evt)
{ //GEN-FIRST:event_x7ActionPerformed
    // TODO add your handling code here:
} //GEN-LAST:event_x7ActionPerformed

private void x6ActionPerformed(java.awt.event.ActionEvent evt)
{ //GEN-FIRST:event_x6ActionPerformed
    // TODO add your handling code here:
} //GEN-LAST:event_x6ActionPerformed

private void x5ActionPerformed(java.awt.event.ActionEvent evt)
{ //GEN-FIRST:event_x5ActionPerformed
    // TODO add your handling code here:
} //GEN-LAST:event_x5ActionPerformed

private void cDActionPerformed(java.awt.event.ActionEvent evt)
{ //GEN-FIRST:event_cDActionPerformed
    // TODO add your handling code here:
} //GEN-LAST:event_cDActionPerformed

private void x4ActionPerformed(java.awt.event.ActionEvent evt)
{ //GEN-FIRST:event_x4ActionPerformed
    // TODO add your handling code here:
} //GEN-LAST:event_x4ActionPerformed

private void x3ActionPerformed(java.awt.event.ActionEvent evt)
{ //GEN-FIRST:event_x3ActionPerformed
    // TODO add your handling code here:
} //GEN-LAST:event_x3ActionPerformed

```

```

    private void x2ActionPerformed(java.awt.event.ActionEvent evt)
{
//GEN-FIRST:event_x2ActionPerformed
    // TODO add your handling code here:
}
//GEN-LAST:event_x2ActionPerformed

    private void x1ActionPerformed(java.awt.event.ActionEvent evt)
{
//GEN-FIRST:event_x1ActionPerformed
    // TODO add your handling code here:
}
//GEN-LAST:event_x1ActionPerformed

/**
 * @param args the command line arguments
 */
public static void main(String args[]) {
    /*
     * Set the Nimbus look and feel
     */
    //<editor-fold defaultstate="collapsed" desc=" Look and feel
setting code (optional) ">
    /*
     * If Nimbus (introduced in Java SE 6) is not available, stay
with the
     * default look and feel. For details see
     *
http://download.oracle.com/javase/tutorial/uiswing/lookandfeel/plaf.html
     */
    try {
        for (javax.swing.UIManager.LookAndFeelInfo info :
javax.swing.UIManager.getInstalledLookAndFeels()) {
            if ("Nimbus".equals(info.getName())) {
                javax.swing.UIManager.setLookAndFeel(info.getClassName());
                break;
            }
        }
    } catch (ClassNotFoundException ex) {
        java.util.logging.Logger.getLogger(Main.class.getName()).log(java.util
.logging.Level.SEVERE, null, ex);
    } catch (InstantiationException ex) {
        java.util.logging.Logger.getLogger(Main.class.getName()).log(java.util
.logging.Level.SEVERE, null, ex);
    } catch (IllegalAccessException ex) {
        java.util.logging.Logger.getLogger(Main.class.getName()).log(java.util
.logging.Level.SEVERE, null, ex);
    } catch (javax.swing.UnsupportedLookAndFeelException ex) {

```

```

java.util.logging.Logger.getLogger(Main.class.getName()).log(java.util
.logging.Level.SEVERE, null, ex);
    }
    //</editor-fold>

    /*
    * Create and display the form
    */
    java.awt.EventQueue.invokeLater(new Runnable() {
        public void run() {
            new Main().setVisible(true);
        }
    });
}

private void addToOutput(final JTextArea ouput, final String
value, final String naration) {
    Thread tt = new Thread(new Runnable() {
        @Override
        public void run() {
            if (ouput.getText().trim().equals("")) {
                ouput.setText(naration + "\n" + value);
            } else {
                ouput.setText(ouput.getText() + "\n" + naration +
"\n" + value);
            }
        }
    });
    tt.start();
}

private void addToTable(final DefaultTableModel tableModel, final
Double[] row) {
    Thread tt = new Thread(new Runnable() {
        @Override
        public void run() {
            tableModel.addRow(row);
        }
    });
    tt.start();
}

private double getClosest(Double[] roots, double res) {
    int index = Arrays.binarySearch(roots, res);
    double[] x = new double[2];
    if (index == 0) {
        return roots[index + 1];
    }
}

```

```

    } else if (index == roots.length - 1) {
        return roots[index - 1];
    } else {
        x[0] = res - roots[index - 1];
        x[1] = res - roots[index + 1];

        return Math.abs(x[0]) < Math.abs(x[1]) ? roots[index - 1]
: roots[index + 1];
    }
}
// Variables declaration - do not modify//GEN-BEGIN:variables
private javax.swing.JFormattedTextField cD;
private javax.swing.JSpinner iteration;
private javax.swing.JButton jButton1;
private javax.swing.JLabel jLabel1;
private javax.swing.JLabel jLabel10;
private javax.swing.JLabel jLabel11;
private javax.swing.JLabel jLabel12;
private javax.swing.JLabel jLabel13;
private javax.swing.JLabel jLabel14;
private javax.swing.JLabel jLabel15;
private javax.swing.JLabel jLabel16;
private javax.swing.JLabel jLabel17;
private javax.swing.JLabel jLabel18;
private javax.swing.JLabel jLabel2;
private javax.swing.JLabel jLabel3;
private javax.swing.JLabel jLabel4;
private javax.swing.JLabel jLabel5;
private javax.swing.JLabel jLabel6;
private javax.swing.JLabel jLabel7;
private javax.swing.JLabel jLabel8;
private javax.swing.JLabel jLabel9;
private javax.swing.JMenu jMenuItem1;
private javax.swing.JMenuBar jMenuItemBar1;
private javax.swing.JMenuItem jMenuItem1;
private javax.swing.JPanel jPanel1;
private javax.swing.JPanel jPanel11;
private javax.swing.JPanel jPanel2;
private javax.swing.JPanel jPanel3;
private javax.swing.JPanel jPanel4;
private javax.swing.JPanel jPanel5;
private javax.swing.JPanel jPanel6;
private javax.swing.JPanel jPanel7;
private javax.swing.JPanel jPanel8;
private javax.swing.JPanel jPanel9;
private javax.swing.JScrollPane jScrollPane1;
private javax.swing.JScrollPane jScrollPane2;
private javax.swing.JScrollPane jScrollPane3;
private javax.swing.JSeparator jSeparator1;
private javax.swing.JTabbedPane jTabbedPane1;

```

```

private javax.swing.JTable jTable1;
private javax.swing.JTextArea jTextArea1;
private javax.swing.JProgressBar progress;
private javax.swing.JFormattedTextField var1;
private javax.swing.JFormattedTextField var2;
private javax.swing.JFormattedTextField var3;
private javax.swing.JFormattedTextField var4;
private javax.swing.JFormattedTextField var5;
private javax.swing.JFormattedTextField var6;
private javax.swing.JFormattedTextField var7;
private javax.swing.JFormattedTextField var8;
private javax.swing.JFormattedTextField x1;
private javax.swing.JComboBox x1_dist;
private javax.swing.JFormattedTextField x2;
private javax.swing.JComboBox x2_dist;
private javax.swing.JFormattedTextField x3;
private javax.swing.JComboBox x3_dist;
private javax.swing.JFormattedTextField x4;
private javax.swing.JComboBox x4_dist;
private javax.swing.JFormattedTextField x5;
private javax.swing.JComboBox x5_dist;
private javax.swing.JFormattedTextField x6;
private javax.swing.JComboBox x6_dist;
private javax.swing.JFormattedTextField x7;
private javax.swing.JComboBox x7_dist;
private javax.swing.JFormattedTextField x8;
private javax.swing.JComboBox x8_dist;
// End of variables declaration//GEN-END:variables
}

/*=====

*Poly.java

*/

package yomi;

/*****
****
* Compilation: javac Polynomial.java
* Execution: java Polynomial
*
* Polynomials with integer coefficients.
*
* % java Polynomial
* zero(x) = 0
* p(x) = 4x^3 + 3x^2 + 2x + 1

```

```

* q(x) =          3x^2 + 5
* p(x) + q(x) = 4x^3 + 6x^2 + 2x + 6
* p(x) * q(x) = 12x^5 + 9x^4 + 26x^3 + 18x^2 + 10x + 5
* p(q(x))      = 108x^6 + 567x^4 + 996x^2 + 586
* 0 - p(x)     = -4x^3 - 3x^2 - 2x - 1
* p(3)         = 142
* p'(x)        = 12x^2 + 6x + 2
* p''(x)       = 24x + 6
*

```

```

*****
***/

```

```

public class Poly {private double[] coef; // coefficients
    private int deg; // degree of polynomial (0 for the zero
polynomial) // a * x^b public Poly(double a, int b) { coef = new
double[b+1];coef[b] = a;deg = degree();}

```

```

// return the degree of this polynomial (0 for the zero
polynomial)

```

```

public int degree() {
    int d = 0;
    for (int i = 0; i < coef.length; i++)
        if (coef[i] != 0) d = i;
    return d; }

```

```

// return c = a + b

```

```

public Poly plus(Poly b) {
    Poly a = this;
    Poly c = new Poly(0, Math.max(a.deg, b.deg));
    for (int i = 0; i <= a.deg; i++) c.coef[i] += a.coef[i];
    for (int i = 0; i <= b.deg; i++) c.coef[i] += b.coef[i];
    c.deg = c.degree();
    return c; }

```

```

// return (a - b)

```

```

public Poly minus(Poly b) {
    Poly a = this;
    Poly c = new Poly(0, Math.max(a.deg, b.deg));
    for (int i = 0; i <= a.deg; i++) c.coef[i] += a.coef[i];
    for (int i = 0; i <= b.deg; i++) c.coef[i] -= b.coef[i];
    c.deg = c.degree();
    return c; }

```

```

// return (a * b)

```

```

public Poly times(Poly b) {
    Poly a = this;
    Poly c = new Poly(0, a.deg + b.deg);
    for (int i = 0; i <= a.deg; i++)
        for (int j = 0; j <= b.deg; j++)

```

```

        c.coef[i+j] += (a.coef[i] * b.coef[j]);
    c.deg = c.degree();
    return c; }

// return a(b(x)) - compute using Horner's method
public Poly compose(Poly b) {
    Poly a = this;
    Poly c = new Poly(0, 0);
    for (int i = a.deg; i >= 0; i--) {
        Poly term = new Poly(a.coef[i], 0);
    c = term.plus(b.times(c));    }
    return c;}

// do a and b represent the same polynomial?
public boolean eq(Poly b) {
    Poly a = this;
    if (a.deg != b.deg) return false;
    for (int i = a.deg; i >= 0; i--)
        if (a.coef[i] != b.coef[i]) return false;
    return true; }

// use Horner's method to compute and return the polynomial
evaluated at x
public double evaluate(int x) {
    double p = 0;
    for (int i = deg; i >= 0; i--)
        p = coef[i] + (x * p);
    return p; }

// differentiate this polynomial and return it
public Poly differentiate() {
    if (deg == 0) return new Poly(0, 0);
    Poly deriv = new Poly(0, deg - 1);
    deriv.deg = deg - 1;
    for (int i = 0; i < deg; i++)
        deriv.coef[i] = (i + 1) * coef[i + 1];
    return deriv;}

// convert to string representation
public String toString() {
    if (deg == 0) return "" + coef[0];
    if (deg == 1) return coef[1] + "x + " + coef[0];
    String s = coef[deg] + "x^" + deg;
    for (int i = deg-1; i >= 0; i--) {
        if (coef[i] == 0) continue;
        else if (coef[i] > 0) s = s + " + " + (coef[i]);
        else if (coef[i] < 0) s = s + " - " + (-coef[i]);
        if (i == 1) s = s + "x";
    }
}

```

```
    else if (i > 1) s = s + "x^" + i; }  
return s;}}
```