A STUDY ON THE NIGERIAN NAIRA PER US-DOLLAR EXCHANGE RATE USING ARFIMA-GARCH AND ARFIMA-FIGARCH MODELS

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DECLARATION

I hereby declare that this dissertation entitled "A Study on the Nigerian Naira per US-Dollar Exchange Rate using ARFIMA-GARCH and ARFIMA-FIGARCH Models" was carried out by me in the Department of Statistics, Ahmadu Bello University, Zaria, under the supervision of Prof. H. G. Dikko and Dr. U. A. Danbaba. The information derived from the literature was duly acknowledged in the text and a list of references provided. No part of this dissertation was previously presented for another degree or diploma at this or any other institution.

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AHMAD, MaimunaAliyu Date

CERTIFICATION

This dissertation entitled "A STUDY ON THE NIGERIAN NAIRA PER US-DOLLAR EXCHANGE RATE USING ARFIMA-GARCH AND ARFIMA-FIGARCH MODELS" by AHMAD MaimunaAliyu meets the regulations governing the award of the degree of Master of Science in Statitstics of the Ahmadu Bello University, Zaria and is approved for its contribution to knowledge and literary presentation.

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ABSTRACT

Autoregressive Fractionally Integrated Moving Average (ARFIMA) model is widely used in the study of long memory processes but it is not suitable for series exhibiting high periods of volatility. Exchange rate series are characterized by periods of stability followed by periods of instability in volatility which can be modeled by Autoregressive Conditional Heteroskedastic (ARCH) model. A parsimonious generalization of the ARCH model is Generalized ARCH (GARCH), but still, neither ARCH nor GARCH can handle the presence of long memory in volatility. This research investigated the presence of long memory both in mean and volatility of the Nigerian Naira per US-Dollar exchange rate series using the hybrid models of ARFIMA, GARCH and Fractionally Integrated GARCH (FIGARCH) origins. Long memory tests were carried out on fractionally differenced and volatility series. The result of GPH estimator indicated the existence of significant Long Memory in the exchange rate data. Classical ARFIMA model was fitted to the data but the results showed the presence of serial autocorrelation and ARCH effects, signifying the limitations of fitting the ARFIMA model. Hybrid ARFIMA models with conditional variance following GARCH and FIGARCH processes were then respectively fitted to the exchange rate series with much improvement in model fitting. Autocorrelation of residuals and ARCH effects were insignificant showing the adequacy of the fitted hybrid models. At the end of the research, the forecasting performance measures of the fitted ARFIMA-GARCH and ARFIMA-FIGARCH models were determined in terms of RMSE. ARFIMA-GARCH demonstrated a better performance.

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CHAPTER ONE

INTRODUCTION

1.1 Background of the Study

Time series analysis is the scientific way to forecast future values of a given variable over time. Models of various kind are used to achieve this objective. In time series modeling and forecasting, the models describepattern of datasets recorded over time and are used to provide reliable forecasts. In the literature, the Autoregressive Integrated Moving Average (ARIMA) models are used to describe or study nonstationary series especially when the series length is sometimes short (Adebiyi*et al.*,2002;Alwadi, 2015; andOmekara*et al.*,2016). Thus, the ARIMA models could be seen as short memory models in real life application. However, when the series is large, trendy and deterministic, the autocorrelation function (ACF) exhibits a slow decay contrary to the fast decay that is common with Short Memory models. In that case, the series will be said to have long term or long range dependence and Long Memory (LM) models like the Autoregressive Fractionally Integrated Moving Average (ARFIMA) model will be appropriate, specifically if the series pass the Long Memory test (see Barunik and Hlinkova, 2016 and Jibrin*et al.,* 2015). Any financial series, economic index or time series that display slow decaying autocorrelation function is referred to as a LM process (Chiawa*et al.*, 2010). Another definition of LM by Box *et al.* (2008) attributed LM process toany series with autocorrelations decaying at a hyperbolic rate.

LM models are statistical models that describe strong dependence across time series data. The LM process as described above is attributed to a decreasing tendency at a hyperbolic rate in the ACF of return and volatility. In essence, LM models exhibit slow decay and slow mean reversion in the returns, which is an evidence of long term dependence in the return series. For example, when a positive autocorrelation exists between price movement and stock market price, we say that there exists a long-term dependence in the price movement which assumes a predictable structure. Thus, historical or previous prices in the past may be used to estimate price in future (Baillie *et al*., 2007). Bollerslev and Mikkelsen (1996) proposed the Fractionally Integrated Exponential Generalized Autoregressive Conditional Heteroscedasticity (FIEGARCH) model as simply the Fractionally Integrated Generalized Autoregressive Conditional Heteroscedasticity (FIGARCH) model using the Exponential Generalized Autoregressive Conditional Heteroscedasticity (EGARCH) approach. They applied the FIEGARCH to model the LM in volatility of S&P500 returns in United States stock market. Some other researchers including Resende and Teixeira (2002) also investigated the existence of LM in Brazilian stock market using Autoregressive Fractionally Integrated Moving Average (ARFIMA) model, which shows the efficiency of the Market. Kumar (2014) shows the evidence of the existence of LM and conditional heteroscedasticity in returns of Indian Stock Market by using ARFIMA-GARCH model, while Caporale and Gil-Alana (2014) proved the existence of the same LM in the daily S&P500 index for the period between 1828 and 1991.Macheshchandra (2012) also investigated the presence of LM in the Indian stock market and studied its pattern using ARFIMA-FIGARCH model. The results show the absence of LM in the mean return series, but the presence of LM in the volatility of the same returns.

For any Time-Series process, discrete in nature, with ACF $\rho(h)$ at lag h, the process has LM if the quantity $\lim_{n\to\infty}\sum_{h=n}^{n} l\rho(h)$ is infinite. The spectral density $f(\omega)$ is therefore made to be unbounded at low frequencies. Brockwell and Davis (1991) also defined LM to be a weakly stationary process whose ACF decay hyperbolically (i.e. $\rho(h)(h)ch^{2d-1}$) as $h \to \infty$, where $c \neq 0$ and $d < 0.5$. On the other hand, a weakly stationary process is said to have a short memory if its ACF is geometrically bounded (i.e. $\rho(h) \leq cr^h$) for some $c > 0, 0 < r < 1$.

Furthermore, according to Laurent (2004), Karanasos and Kim (2006), Ding (2011), the heteroscedasticity, volatility and hyperbolic decaying ACF of the observed volatility are modeled usingAutoregressive Conditional Heteroscedasticity (ARCH), GARCH and FIGARCH models respectively.

1.2 Statement of the Problem

The presence of Long Memory in the mean exchange rate and volatility implies the existence of a strong relationship between many observations widely separated in time (Shittu and Yaya, 2009). Granger and Joyeux (1980) and Hosking (1981) proposed the Autoregressive fractionally integrated moving average (ARFIMA) model and revealed that fractionally integrated series could produce the features of long range dependence. Wiri and Tuaneh (2022) have applied ARFIMA for modeling and forecasting of monthly exchange rate series of Naira to United States Dollar. However,the ARFIMA model is only concentrated and centered on the assumptions of stationarity, homoscedasticity of error variance and linearity. Incorporating all these assumptions is quite impossible with series exhibiting high periods of instability also called volatility, which is a common feature of exchange rate series. Exchange rate series show periods of stability followed by the periods of instability in volatility, which can be tackled using Autoregressive conditional heteroscedastic (ARCH) model. However, the ARCH is characterized with having large number of parameters to give satisfactory forecast which leads to the emergence of comparatively more parsimonious model called the Generalized ARCH (GARCH) model of Bollerslev (1986). However, the main drawback of GARCH model is that it cannot handle the long memory present in the volatility. For modeling and forecasting of exchange rate series with long memory in volatility, Fractionally Integrated GARCH (FIGARCH) model of Baillie (1996) is generally used. Balancing the trade-off between long memory and volatility in exchange rate or any time series data requires an integrated approach to ensure good conceptual frame work and accuracy of forecast values.

In this work, we adopted a hybrid approach of modeling (Baillie *et al*., 1996 and Baillie *et al*., 2002). The first architecture is based on ARFIMA method, which is used to estimate the mean equation. The second one implements GARCH or FIGARCH modelsto estimate the volatilityusingsingle or double long memory parameter(s) respectively. Thisdemonstrated the flexibility of the resulting hybrid models in simultaneously exploring the long range dependence in mean and volatility.

1.3 Aim and Objectives

Theresearch aimed at investigating a study on the Nigerian Naira per US-Dollar exchange rate (NGNUSDER) using ARFIMA-GARCH and ARFIMA-FIGARCH models. The aim was achieved through the following objectives:

i. Testing and evaluatingpresence of long memory and determiningwhether there is a warning for keeping investments in Naira as compared to Dollar in the long run.

ii. Fittingthe hybrid ARFIMA-GARCH and ARFIMA-FIGARCH models in order to examine the possibility of dual long memory in exchange rate.

iii. Generatingthe one-step-ahead volatility forecast of the fitted models.

iv. Comparingthe fitted hybrid models based on forecast performance measures.

1.4 Significance of the Study

This study considers a hybrid time series model for modeling the Long Memory effects in mean and volatility of the NGNUSDER using the ARFIMA-GARCH and ARFIMA-FIGARCH models, providing a useful way of explaining the connection between the conditional mean and conditional variance of a process characterized with long memory. The significance of the hybrid models is to investigate the interaction between conditional mean and volatility, and account for the long memory impact on the returns and volatility of the exchange rate series.This we hope will add to the literature on the study of the currencyexchange rate in the case where the impact of long memory behavior is of interest.

CHAPTER TWO

LITERATURE REVIEW

Fractional integration and modeling LM phenomenon have become an area where researchers devoted huge time over the decades. The reviews by Baillie *et al.* (1996) and Guegan (2005) provided adequate explanation in the studies of LM. The studies of Granger and Joyeux (1980), Granger (1980) and Hosking (1981) proposed the ARFIMA model by extending the differencing filter of the ARIMA model of Box and Jenkins (1976) to handle the fractional order. Since then, LM series has been modeled by using the fractional order and LM models. The LM test, fractional differencing and modeling are applied in many fields such as real exchange rate, crude oil prices, stock index, meteorological data and health index. Studies including Aloy*et al.* (2011), Delavari*et al.* (2013), Aloui and Ben Hamida (2014), Kruse (2015) and Barunik and Hlinkova (2016) are few of many researchers that carried out LM modeling.

According to Baillie *et al.* (1996), a combination of ARFIMA and GARCH models can be used to describe both the LM and variability in financial and economic time series. In the literature, several studies including Danielson (2011), Kumar (2014) and Ambach and Schmid (2015) concurrently study the LM and heteroscedasticity by combining the mean and variance models so that the variance specification provides substantial improvements in terms of fit and diagnostics test. For example, Zhou and He (2009) recommended the hybrid of Autoregressive Moving Average (ARMA) and Asymmetric Power ARCH (APARCH) models in forecasting S&P 500 stock index, and assumed the errors to be skew-t distributed. Duppati*et al.* (2016) studied the persistence in Asian stock markets using high-frequency data and ARFIMA-APARCH models. Their finding confirmed the presence of LM in the volatility, based on five minutes intra-day returns.

Babatunde and Akinwade (2011) examined the consistence and persistence, as well as volatility of naira per dollar exchange rate for the period between 1986 and 2008 using ARCH and GARCH models. They also used purchasing power parity to analyze the long run consistence of the exchange rate, and Augmented Dickey Fuller (ADF) was used to examine the time series data for stationarity. According to their findings, they used the models to investigate the degree of volatility on first differencing, as well as standard and coefficient of deviations. The result shows the presence of over shooting shocks and the analysis further revealed that the nominal and real exchange rates were not consistent with the traditional long run purchasing power parity model.

Adnan and Erdost (2007) provided additional information that the Turkish Stock Exchange has presence of dual long memory property and suggested the application of ARFIMA-FIGARCH model for modelling the return and volatility alike. Their finding shows strong evidence of LM in both the return and volatility. Long memory in volatility is an evidence showing the importance of risk as a determinant of the behavior of daily data, while the long memory in return indicates that the data follow an expected behavior that is not consistent with the efficient market hypothesis.

Serpil and Mesut (2014) examine the weak-form market efficiency of the Karachi Stock Exchange in Pakistan Stock Market for the period between the year 2010 to 2013 using ARFIMA-FIGARCH models. According to their findings, ARFIMA model does not support LM behavior of the stock market return, while the FIGARCH model supports LM behavior in volatility. The predictable structure of volatility in this Market reflects on its weak-form nature.

Ishida and Watanabe (2009) used the ARFIMA model to study LM in the Nikkei 225 future realizes volatility (RV) data. The results show that the residuals of model indicates evidence of heteroscedasticity, therefore, ARFIMA-GARCH was found to capture persistence and volatility concurrently.

Olowe (2009) studied the NGN per USD exchange rate for the period January 1970 to December 2007 using several variants of GARCH model. According to their findings, GARCH-family models reveal persistence in volatility. This behavior was consistently found in the fixed exchange rate and manage float rate regimes.

Macheshchandra (2012) investigate that the Indian stock market has long memory based on ARFIMA-FIGARCH models. The ARFIMA model indicates the absence of LM in the mean return series, while the FIGARCH model indicates the presence of LM in volatility of the stock returns.

Tasiu*et al.* (2017) studied some of the stylized facts of volatility that needs to be incorporated in a model, they used daily data on NGN per USD exchange rate to illustrate these stylized facts and investigated the capability of GARCH-family models to apprehend these features. The volatility of the exchange rate was found to be quiet persistent, they fitted two models: GARCH (1,1) and EGARCH (1,1), they found the EGARCH (1,1) to be the best model.

Caporale and Gil-Alana (2010) studied the Nigerian naira, South Africa rand, Moroccan dirham and Ghana cedi to United States (US) dollar exchange rate series respectively. They proved that all the series are dominated by long-range dependence behavior and therefore described by using the ARFIMA LM models. However, they did not look at the Market shocks impact on the four exchange rates return and volatility.

The understanding derived from these related literatures were that ARFIMA model account only for Long Memory in mean but fails to capture volatility. Consequently, it fails to account for the Long Memory in volatility as well. Theresults also signified the limitations of fitting ARFIMA model to exchange rate series and signaled the importance of testing the existence of Long Memory in volatility.This study therefore investigated the Long Memory features of the exchange rate not only in mean but also in volatility so as to examine their simultaneous accumulation consequences.

CHAPTER THREE

METHODOLOGY

3.1 Preamble

This section explains the techniques that would be adopted in this study. In previous chapters, LM process in terms of ACF with positive autocorrelations that decay monotonically & hyperbolically to zero were defined. The modeling techniques, ARFIMA-FIGARCH (with the ability to account for LM in theconditional mean, assuming the volatility is a fractionally integrated process) and ARFIMA-GARCH (that account for LM in the conditional mean assuming the volatility follows GARCH process) would be adopted to estimate the hybrid of mean and volatility ofNGNUSDER.

3.2 Definition of the Hybrid and Related Models

3.2.1 The ARFIMA Model

The general ARIMA model was described by Whittle (1951) and it was popularized in the book of Box and Jenkins (1976). The ARIMA model is a hybridization of Autoregressive (AR) and Moving Average (MA) models for an Integrated (I) time series data.

A time series is "stationary" if all of its statistical properties—mean, variance, autocorrelations, etc.—are constant in time. Thus, it has no trend and no heteroscedasticity. A series which can be stationarized is called an "integrated" (I) series. The most general class of forecasting models for time series that can be stationarized by transformations such as differencing, logging and/or deflating constitute the ARIMA models.

Most of the well-known class of stationary and invertible ARIMA time series processes have autocorrelations that decay at an exponential rate. A process $\{y_t\}, t = 1,...,T$ may be generated by an ARIMA process if $\Phi(B) y_t = \Theta(B)\varepsilon_t$, where ε_t is a white noise with variance σ_ε^2 and B denoting the backward shift operator. On the other hand, the process y_t which has lag k autocorrelations(ρ_k)is a Long Memory processif $\lim_{n\to\infty}\left(\sum_{k=-n}^{n}|\rho_k|\right)$ is non-finite.

For any stationary Long Memory process, the ACF decays at an hyperbolic rate. Granger and Joyeux (1980) introduced the filter $(1-B)^d=\varepsilon_t$ for $0 < d < 0.5$, where $d$is the fractional difference (Long Memory) parameter and ε_t is distributed as white noise process. The ARFIMA (p, d, q) model is defined in terms of backward shift operator as

$$\Phi(B)(1-B)^d\ y_t=\Theta(B)\varepsilon_t \qquad (3.1)$$

where $\Phi(B)=1-\phi_1B-\phi_2B^2-...-\phi_pB^P$ and $\Theta(B)=1-\theta_1B-\theta_2B^2-...-\theta_qB^q$. The roots of $\Phi(B)$ and $\Theta(B)$ must lie outside the unit-root circle. The filter $(1-B)^d$ is responsible for fractionally differencing the process $y_t, y_{t-1}, y_{t-2},..., y_{t-n}$ that exhibits a slow decaying ACF.

3.2.2 The GARCH Model

The process $\{\varepsilon_t\}$ is ARCH(q) if its conditional distribution given the available information (Ω_{t-1}) is $\varepsilon_t\,|\,\Omega_{t-1}$ which follows $N(0,h_t)$such that $\varepsilon_t=h_t^{1/2}\xi_t$, where ξ_t is a white noise process. In Generalized ARCH (GARCH) model, the conditional variance is a linear function of its own lags and past squared residuals. The GARCH model was defined by Bollerslev (1986) as

$$h_t=\alpha_0+\sum_{i=1}^{p}\alpha_ih_{t-i}+\sum_{j=1}^{q}\beta_j\varepsilon_{t-j}^2\ , \qquad (3.2)$$

where

h_t is the volatility with respect to time t, $\alpha_i,\ \beta_j \geq 0$; $\left(\alpha_i + \beta_j\right) < 1\ \forall i = 1,\ldots,p$ and $j = 1,\ldots,q$.

ε_{t-j}^2 = squared innovation at i[th] lag time.

h_{t-i} = the volatility with respect to time $t-i$.

3.2.3 FIGARCH Model

Engle and Bollerslev (1986) proposed a particular class of GARCH model referred to as integrated GARCH (IGARCH) model whose unconditional variance does not exist given that $\alpha_i + \beta_j = 1$ in Eq.(3.2) above. The IGARCH model implies infinite persistence of the conditional variance to a shock in squared returns. An integrated GARCH (IGARCH) process can be written as

$$\Phi\left(B\right)\left(1-B\right)\varepsilon_t^2 = \omega + \left[1-\beta\left(B\right)\right]v_t \qquad (3.3)$$

The original work that introduces the concept of Long Memory in volatility model is (Baillie *et al*., 1996) wherein the FIGARCH (p, d, q) model was proposedby replacing the first difference operator $\left(1-B\right)$ with the fractional differencing operator $\left(1-B\right)^d$, where d is a fraction $0 < d < 1$. The model in terms of backward shift operator is

$$\Phi\left(B\right)\left(1-B\right)^d\ \varepsilon_t^2 = \omega + \left[1-\beta\left(B\right)\right]v_t \qquad (3.4)$$

where $v_t = \varepsilon_t^2 - h_t$.

The v_t is a process that can be interpreted as innovations of the volatility with zero mean serially uncorrelated. To warrant covariance-stationarity, the roots of $\Theta\left(B\right)$ and $\left\{1-\beta(B)\right\}$ are controlled to lie outside the unit-root circle. The FIGARCH model offers better flexibility for modeling volatility. The FIGARCH model which is defined in Eq. (3.4) reduces to an IGARCH model when$= 1$. The FIGARCH (p, d, q) model enforces an ARFIMA structure on ε_t^2 , leading to the hybrid ARFIMA-FIGARCH model.

According to Baillie *et al*. (1996), the impact of a shock on volatility in the FIGARCH (p,d,q) frame work decreases at a hyperbolic rate when $0 \leq d < 1$. Thus, the long-term dynamics of the volatility would be accounted for by the fractional integration parameter d, while the short-term dynamics is modeled via the classical GARCH parameters.

3.2.4 ARFIMA-GARCH Model

The general ARFIMA(p, d, q)-GARCH(P, Q) process defined by Baillie *et al*. (1996) has the assumption that the time-dependent heteroskedasticity h_t follows the GARCH(P, Q) model defined in equation (3.2) above. The ARFIMA(p, d, q)-GARCH(P, Q)model is given by

$$\Phi(B)(1-B)^d\, y_t = \Theta(B)\varepsilon_t\,, \quad h_t = \alpha_0 + \sum_{i=1}^{p}\alpha_i h_{t-i} + \sum_{j=1}^{q}\beta_j \varepsilon_{t-j}^2\,, \qquad (3.5)$$

where y_t is a covariance-stationary process, $\varepsilon_t \mid \Omega_{t-1}$ ~ $D(0, h_t)$, $\Phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^P$, $\Theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q$ and all the roots of $\Phi(B),\ \Theta(B)$ must lie outside the unit-root circle.The error terms are assumed to follow a conditional density $D,$ which is either Normal or Student t depending on the degree of kurtosis in the data.

3.2.5 ARFIMA-FIGARCH Model

The ARFIMA(p, d, q)-FIGARCH(P, D, Q) process is defined in the same way and manner the ARFIMA(p, d, q)-GARCH(P, Q) process was defined. The only difference is that the conditional time-dependent variance of the process(h_t) is specified by the FIGARCH model defined in equation (3.4).The ARFIMA(p, d, q)-FIGARCH(P, D, Q) model is given by

$$\Phi(B)(1-B)^d (y_t - \mu) = \Theta(B)\varepsilon_t, \quad \Phi(B)(1-B)^d \varepsilon_t^2 = \omega + [1-\beta(B)]v_t, \qquad (3.6)$$

where $v_t = \varepsilon_t^2 - h_t$.

The autoregressive process for h_t is initialized by its unconditional mean, which is replaced by the corresponding sample mean in the estimation process.

3.3 Data for the Study

We used the daily exchange rate of NGNUSDER series for the period 20th October 2008 to April 6th, 2021 which consist of 3252 observations (5 days per week) and obtained from the CBN websitewww.cbn.gov.ng. The return on exchange rate is defined as:

$$r_t = \log\left(\frac{e_t}{e_{t-1}}\right), (3.7)$$

where e_t represents the exchange rate at time t and e_{t-1} represents exchange rate at time $t-1$.

3.4 Long Memory Estimator

This research used the Geweke and Porter-Hudak (GPH) methodto estimate the fractional (i.e. long memory) parameter *d*. The GPH estimator is based on the regression equation using the periodogram function as an estimate of the spectral density, see (Geweke and Porter-Hudak, 1983) for detailed derivation. In order to test for the presence of Long Memory, Hurst statistic is used. For finite variance process, fractional difference parameter d has a closed relation with the Hurst parameter H given in equation (3.8).

$$d = H - 1/2 \tag{3.8}$$

The incidence of Long Memory is said to exist if the H statistic lies in the interval 0.5 < H < 1. The closer the value is to 1, the greater the incidence of Long Memory.

3.5Return and Actual Daily Volatility

Persistence in various type of returns attracts the attention of numerous studies both theoretically and empirically. The return series defined in equation (3.7) can be re-expressed as a logarithmic percentage given by

$$r_t = 100 * \log\left(\frac{e_t}{e_{t-1}}\right) \tag{3.9}$$

where r_t is the returns for each exchange rate at time t. However, Sadorsky (2006) and Kang and Yoon (2013) defined the Actual Daily Volatility (ADV) also called the variance to be the daily squared returns $\left(r_t^2\right)$ given by

$$h_t = \sigma_t^2 = r_t^2 \tag{3.10}$$

The definition in equation (3.10) is used for volatility throughout this work.

3.6Jarque-Bera Test

Jarque and Bera (1987) have developed a test for normality based on third moment (skewness) and fourth moment (kurtosis) of a distribution. The test checks the pair of hypothesis $H_0 : E\left(\varepsilon_t^2\right)^3 = 0$ and $E\left(\varepsilon_t^2\right)^4 = 3$ (the distribution is symmetry and therefore normal) vs $H_i : E(\varepsilon_t^2)^3 \neq 0$ and $E\left(\varepsilon_t^2\right)^4 = 3$ (the distribution is asymmetric and therefore non-normal). The test statistic is given as

$$JB = \frac{T}{6}\left[T^{-1}\sum_{t=1}^{T}\left(\hat{\varepsilon}_t^2\right)^T + \frac{T}{24}\left(T^{-1}\sum_{t=1}^{T}\left(\hat{\varepsilon}_t^2\right)^4 - 3\right)^T\right] \tag{3.11}$$

which has an asymptotic χ^2 distribution with 2 degrees of freedom under the null hypothesis. The Jarque-Bera statistic can be used to test for the normality assumption in residuals of fitted models.

3.7 Portmanteau Test.

The Portmanteau test investigates the presence of serial autocorrelation in residuals of any fitted model. Supposing that the autocorrelation between the error terms ε_t and ε_{t-k} is $\rho_k = corr\left(\varepsilon_t, \varepsilon_{t-k}\right)$, then, H_0 states that all lagged correlations are zero and at least one lag correlation is not zero and is designed as $H_0 : \rho_1 = \rho_2 = ... = \rho_k = 0$. The test statistic, a modified Q statistic, originally developed by Box and Pierce in (1970) is given by

$$Q_1 = m\left(m+2\right)\sum_{j=1}^{k}\left(m-j\right)^{-1}\hat{\rho}_{\hat{\varepsilon}}^2(j) \tag{3.12}$$

The Q_1 statistic approximately follows a χ^2distribution with $K-p-q$ degrees of freedom depending only on the number of parameters in the model.

3.8 Mean Square Error (MSE)

The MSE is defined as the measure of the accuracy of forecasts. The MSE of any proposed forecasting model ought to be very small. As mentioned by Saigal and Mehrotra (2012), the accuracy of forecasts from a model is associated to the MSE. The deviations in forecasting errors explain the relationship between the historical observations and the fitted values. The following equation is used in estimating the MSE.

$$MSE = \frac{1}{n}\sum_{t-1}^{n}\varepsilon_t^2 \qquad (3.13)$$

whereε_t is the random error obtained by using the relation $\varepsilon_t = y_t - \hat{y}_t$, y_t is the output, $\hat{y}_t$ is the forecast output and n is the sample size.

The root of MSE gives another measure calledRMSE whichis defined as the estimate of deviation of errors in forecasting. It is calculated by obtaining the square root of the difference among predicted and historical observations that are both squared and averaged over the sample. A smaller RMSE indicates a better model estimate. The RMSE expressed below in (3.13) is used to obtain the RMSE in this study.

$$RMSE = \sqrt{MSE} = \sqrt{\frac{1}{n}\sum_{t=1}^{n}\varepsilon_t^2} \qquad (3.14)$$

3.9The Akaike Information Criterion (AIC)

The Akaike Information Criterion (AIC) is an estimator of prediction error and thereby relative quality of statistical models for a given set of data. Given a collection of models for the data, AIC estimates the quality of each model, relative to each of the other models.

Thus, AIC provides a means for model selection.The AIC will be used to select the best model

$$AIC = 2k - 2\log(L) \tag{3.15}$$

where k is the number of parameters in the fitted model, and L is the log-likelihoodfunction.

3.10Augmented Dickey-Fuller Test

In statistics, an augmented Dickey–Fuller test (ADF) tests the null hypothesis that a unit root is present in a time series sample. The alternative hypothesis is different depending on which version of the test is used, but is usually stationarity or trend-stationarity. It is an augmented version of the Dickey–Fuller test for a larger and more complicated set of time series models.

The ADF statistic, used in the test, is a negative number. The more negative it is, the stronger the rejection of the hypothesis that there is a unit root at some level of confidence.

CHAPTER FOUR

RESULTS AND DISCUSSION OF FINDINGS

4.1 Preamble

This chapter deals with results and discussion of findings of the study. An exploratory model selection procedure is adopted using Akaike Information Criterion (AIC) while fixing AR and MA terms at the maximum of 3 for parsimony. This procedure has been adopted by many researchers including Ali*et al*. (2017) in the selection of Long Memory models. In the study, we used a daily data ranging from 20th October, 2008 to April 6th, 2021 of Nigerian Naira per US-Dollar exchange rate, consisting of a total of 3252 observations.

4.2Data Summary and Descriptive Statistics

Table 4.1 provides a summary statistics of the exchange rate series. The standard deviation is 81.54 and the coefficient of variation is 36.18 indicating the presence of period of instability or volatility in the original data. The series ispositively skewed and platykurtic, indicating light tails as compared to normal distribution. The excess kurtosis, which is the difference between a given distribution's kurtosis and the kurtosis of a normal distribution is found to be -1.25, indicating small risk a trader of exchange rate should expect in the Nigerian exchange rate market. In other words, the platykurtic distribution here shows that the exchange rate series will not be very extreme, which is great for investors who do not want to take a lot of risk.

Table 4.1: Descriptive Statistics for the Daily NGNUSDER

Variable	Min.	Max.	Mean	SD	Skewness	Ex. Kurtosis
NGNUSDE	117.69	403.97	225.4	81.54	0.58	-1.25

4.3 Test for Stationarity

The time series plot of the NGNUSDER is shown in Fig. 4.1. The graph exhibits deterministic trend behavior and evidence of the presence of unit root. It is important to transform the series by applying first differencing or fractional differencing depending on the type of model (ARIMA or ARFIMA) to be estimated.

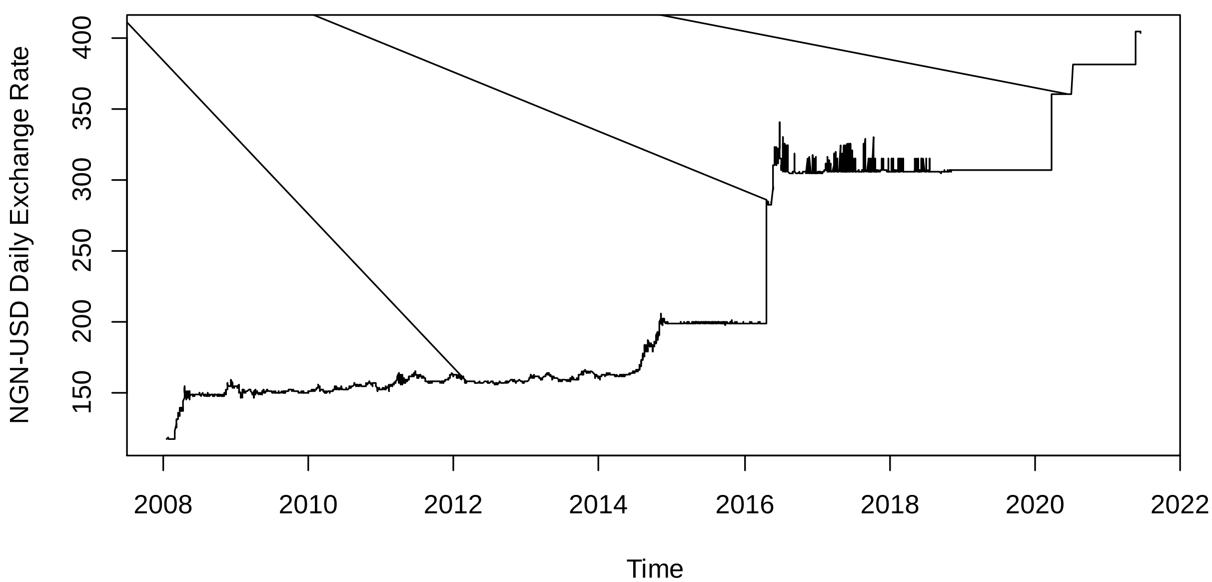


Figure 4.1: Plot of Daily Time Series of NGNUSDER.

After independently taking first difference and fractional difference, the Augmented Dickey-Fuller (ADF) test is carried out as follows

$H_0 : |\phi| = 1$ (datacontains unit root)

$H_1 : |\phi| < 1$ (datadoes not contain unitroot)

The ADF test is applied to see the presence of non-seasonal unit root in the return series.The results shown in Table 4.2 are found significant at 1% level of significance (since all the p-values are less than 0.01) indicating stationarity of the differenced series.

Table 4.2: Results of Stationarity Test

First difference		Fractional difference	
ADF (with constant)	-8.7800 (0.000)	ADF (with constant)	-4.3338 (0.0004)
ADF (with constant and trend)	-8.7807 (0.000)	ADF (with constant and trend)	-4.6841 (0.0007)

Figure 4.2 shows the plot of the first differenced series. It is evident from figure 4.2 that the Nigerian Exchange rate market displayed periods of relatively low volatilities until mid of 2016 through 2019 when the volatility became high with wide spread breadth. The volatility stabilized for some time, and then periods of high volatility were again seen in 2020 and 2021. Thus, there is evidence of volatility clustering in the series which may be caused by Nigerian government policy on Naira devaluation or otherwise.

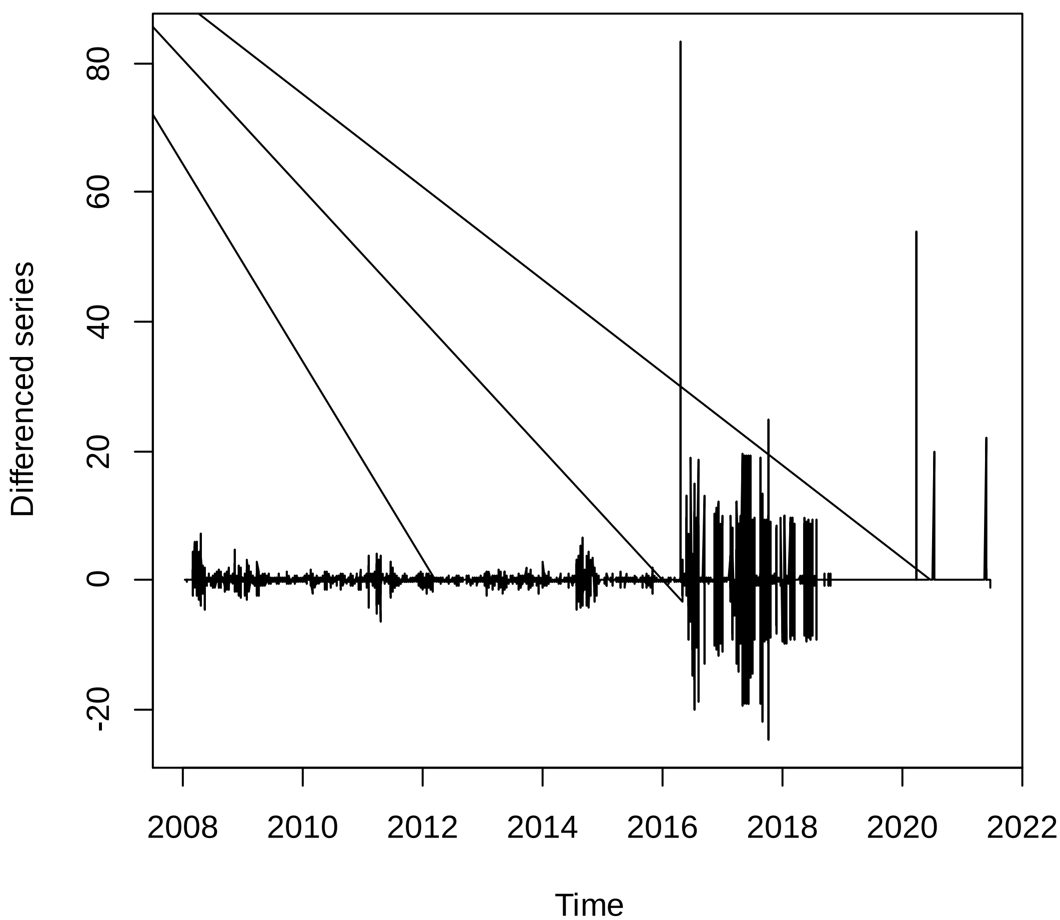


Figure 4.2: The First Differenced Series Showing Volatilities with Extensive Amplitudes from the year 2016

4.4 Autocorrelation Function of the Differenced Series

The ACF of the level exchange rate series shown in figure 4.3indicates a slow decay in ACF which is an evidence of Long Memory process.

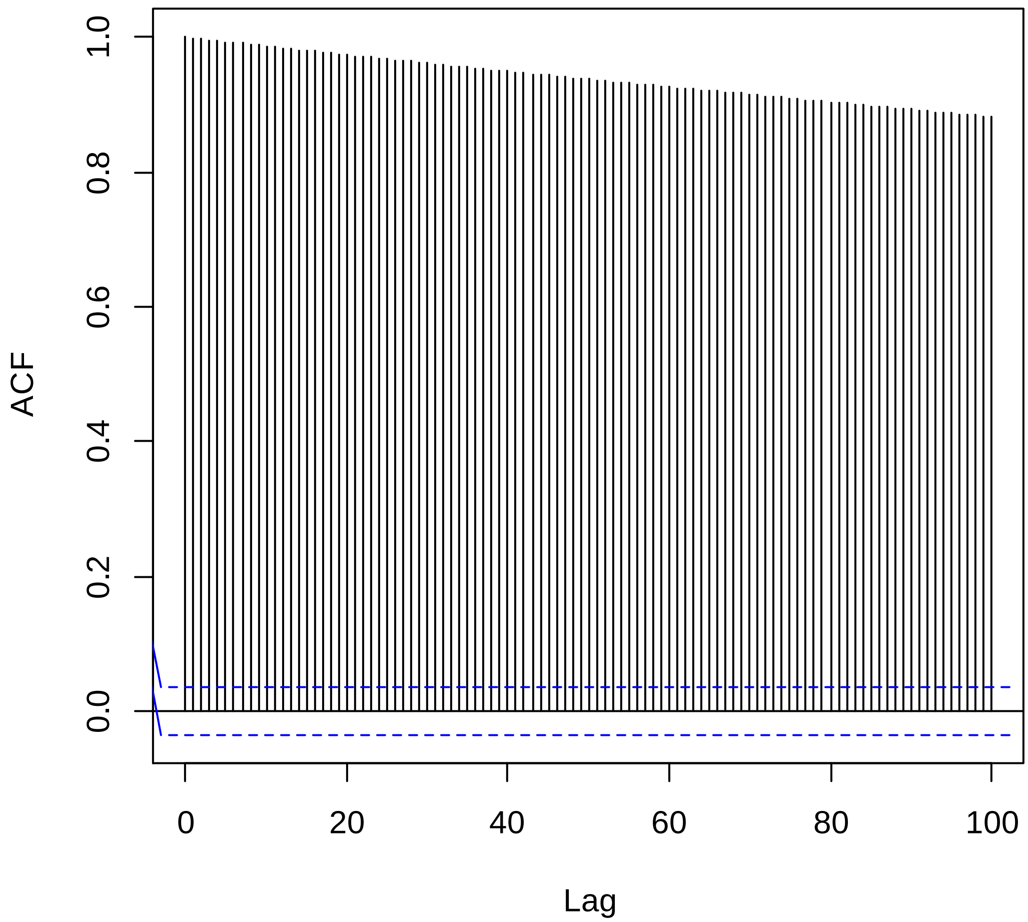
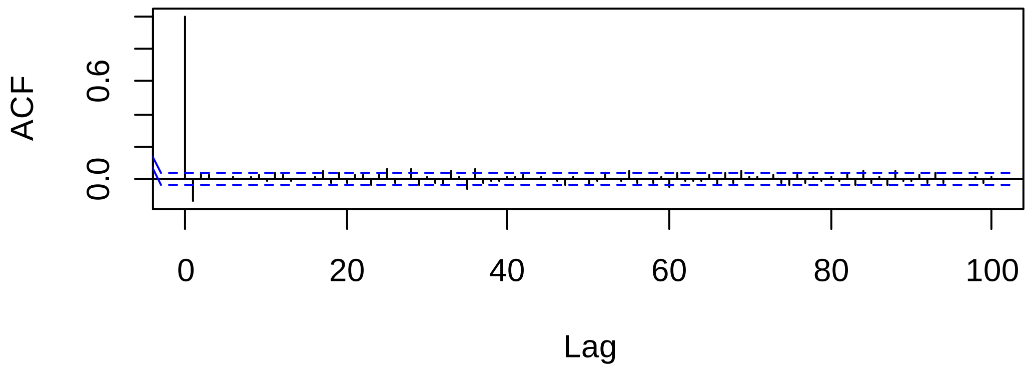


Figure 4.3: ACF for the Exchange Rate Series

Similarly, time series data that show this type of behavior can produce fractional difference value in the interval $0 < d < 1$. After taking the fractional difference, the series display a clearer tendency of the volatilities to be on the high side (see figure 4.2), and the slow decay in the autocorrelations in figure 4.3 disappeared as shown in figure 4.4, indicating the strength of fractional difference in tackling Long Memory in the series.



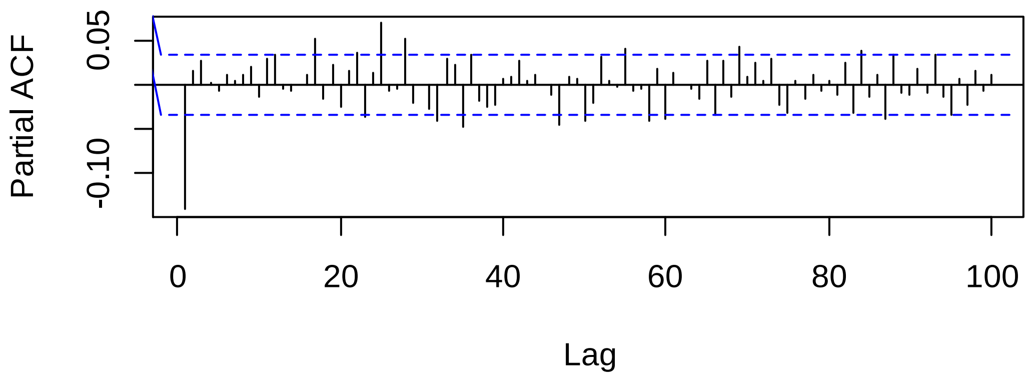


Figure 4.4: ACF and PACF plots of the Differenced Series

The ACF and PACF of the differenced and volatility series are studied to investigate the distributional characteristic of the exchange rate series. The ACF and PACF plots are shown in Figures 4.4 and 4.5.

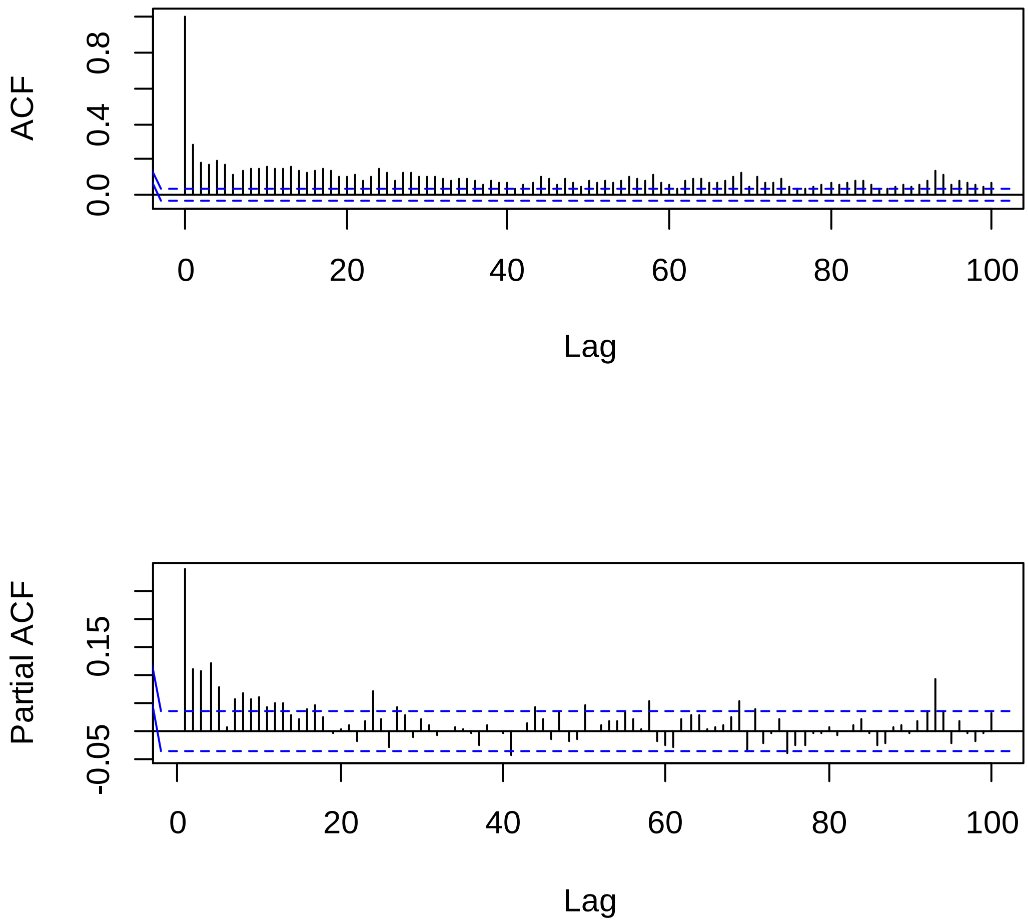


Figure 4.5: ACF and PACF plots of Daily Volatility of NGN per USD Exchange Rate

Since the autocorrelation and partial autocorrelation functions are significant at distant lags, there is a clear indication of Long Memory in the volatility series. The dotted lines in the ACF and PACF plots represent the 95% critical values of the test statistic.

4.5 Testing for Long Memory

After visualizing the ACF and PACF of the differenced and volatility series, Long Memory parameters to be used for the hybrid models (namely ARFIMA-GARCH and ARFIMA-FIGARCH)are estimated using GPH estimator. The results for testing the presence of Long Memory using Hurst statistic are provided in Table 4.3.

Table 4.3: Results of Long Memory Test

	Differenced series	Volatility
d	0.012	0.048
SE	0.087	0.065
H	0.512	0.548

Judging from Table 4.3,all the Hurst values for testing the incidence of Long Memory lie in the interval $0.5 < H < 1$, further indicating the presence of Long Memory in the exchange rate series. Accordingly, ARFIMA model is fitted to the exchange rate series.

4.6 Fitting of ARFIMA Model

4.6.1 ARFIMA Model Identification

The identified ARFIMA models for the NGNUSDER series and their AIC values are displayed in Table 4.4. The best ARFIMA model is selected based on minimum AIC values. The AIC of the identified ARFIMA models are displayed in Table 4.4. The values are compared where ARFIMA(0, d, 1) and ARFIMA(2, d, 3) appear to be the models with the minimum AIC. This suggests that the two ARFIMA models can be considered for studying the daily NGNUSDER.

Table 4.4: AIC Values for ARFIMA(*p*,*d*,*q*) Models

S/N	ARFIMA(p,d,q)	AIC
1	ARFIMA(1,0.047,0)	16521.43
2	**ARFIMA(0,0.064,1)**	**16435.32**
3	ARFIMA(1,0.064,1)	16437.32
4	ARFIMA(2,0.052,0)	16441.73
5	ARFIMA(0,0.054,2)	16436.93
6	ARFIMA(2,0.059,1)	16438.48
7	ARFIMA(1,0.050,2)	16439.44
8	ARFIMA(2,0.061,2)	16441.38
9	ARFIMA(3,0.043,0)	16439.68
10	ARFIMA(0,0.053,3)	16439.33
11	ARFIMA(3,0.060,1)	16439.33
12	ARFIMA(1,0.066,3)	16438.30
13	ARFIMA(3,0.064,2)	16441.13
14	**ARFIMA(2,0.077,3)**	**16429.48**
15	ARFIMA(3,0.019,3)	16441.85

Next is the parameter estimation and diagnostics of these models and are presented in the subsection 4.6.2.

4.6.2 ARFIMA Model Estimation and Diagnostic Analysis

The results of the parameter estimation and diagnostic test of the best two ARFIMA models are given in Table 4.5.All parameter estimates in the ARFIMA models are significant at 1% level. However, the results of Portmanteau test show the presence of serial autocorrelation due to the rejection of the white noise null hypothesis of the residuals. It therefore means the ARFIMA model had failed to capture all the patterns in the exchange rate series.

Table 4.5: Estimated Results of the ARFIMA Models

ARFIMA (0, 0.064, 1)			ARFIMA (2, 0.077, 3)		
Coefficient	Estimate	SE	Coefficient	Estimate	SE
d-ARFIMA	0.064**	0.012	d-ARFIMA	0.077**	0.00008
MA(1)	0.290**	0.021	AR(1)	1.919**	0.02055
			AR(2)	-0.935**	0.01777
			MA(1)	2.224**	0.01486
			MA(2)	-1.501**	0.00297
			MA(3)	2.663**	0.02001
ln(L)	-8215		ln(L)	-8208	
Portmanteau test	25.834	0.000	Portmanteau test	26.707	0.000
ARCH-LM test	46.83	0.000	ARCH-LM test	84.80	0.000

**Represents significance at 1% level of significance level.

ARCH statistic from the ARCH-LM test is also highly significant, indicating the existence of ARCH effects in the standardized residuals or the presence ofheteroskedasticity in the residuals. These results signify the limitations of fitting ARFIMA model in the exchange rate series and signal the importance of testing the existence of Long Memory in volatility.Therefore, the persistence in the volatility can well be captured by fitting a FIGARCH model to the return series.

4.7 Fitting of ARFIMA-GARCH and ARFIMA-FIGARCH Models

The plot in figure 4.2 shows evidence of volatility clustering indicating that the exchange rate series is volatile. The observed volatility shows that GARCH is a candidate model. Again, the significant correlation for many lags in figure 4.3 indicates evidence of long memory in the series. Therefore, both the long memory and volatility observed in the original series indicates that the hybrid ARFIMA-GARCH model is a better candidate to simultaneously study the Long Memory and volatility in the series. Long Memory could either exist in mean or in both the mean and variance. While ARFIMA-GARCH model tackles the Long Memory existing in mean, ARFIMA-FIGARCH is another hybrid model that can be used to study Long Memory existing in both the mean and variance of the series.

The results in table 4.5 indicate that the ARFIMA model alone is not strong enough to eliminate the variability and noise signals in the exchange rate series. Some of the factors that may cause the serial correlation and heterogeneity effect of the ARFIMA model residuals are volatility clustering, magnitude of volatility observed in the fractional differenced series and evidence of Long Memory. In view of these observed characteristics, analyses based on hybrid models, the ARFIMA-GARCH and ARFIMA-FIGARCH are carried out.

4.7.1 ARFIMA-GARCH and ARFIMA-FIGARCH Model Identification

The criteria for fitting the ARFIMA-GARCH and ARFIMA-FIGARCH models to the exchange rate series are reported in Tables 4.6 and 4.7 respectively. The models are selected using the AIC while fixing AR and MA at the maximum of 3. The models having minimum AIC were ARFIMA(2, 0.012, 1)-GARCH(1,1) and ARFIMA(3, 0.012, 3)-

FIGARCH(1,1) and can therefore be regarded to be the best in tackling the Long Memory process in the series.

Table 4.6: AIC Values for ARFIMA(*p*,*d*,*q*)-GARCH(1,1) Models

S/N	ARFIMA(*p*,*d*,*q*)-GARCH(1,1)	AIC
1	ARFIMA(1, 0.012,0)-GARCH(1,1)	2.2522
2	ARFIMA(0, 0.012,1)-GARCH(1,1)	2.2413
3	ARFIMA(1, 0.012,1)-GARCH(1,1)	2.2436
4	ARFIMA(2, 0.012,0)-GARCH(1,1)	2.6124
5	ARFIMA(0, 0.012,2)-GARCH(1,1)	NA
6	**ARFIMA(2,0.012,1)-GARCH(1,1)**	**1.9906**
7	ARFIMA(1, 0.012,2)-GARCH(1,1)	2.1716
8	ARFIMA(2, 0.012,2)-GARCH(1,1)	NA
9	ARFIMA(3, 0.012,0)-GARCH(1,1)	2.1445
10	ARFIMA(0, 0.012,3)-GARCH(1,1)	2.1848
11	ARFIMA(3, 0.012,1)-GARCH(1,1)	NA
12	ARFIMA(1, 0.012,3)-GARCH(1,1)	2.0580
13	ARFIMA(3, 0.012,2)-GARCH(1,1)	NA
14	ARFIMA(2, 0.012,3)-GARCH(1,1)	2.5635
15	ARFIMA(3, 0.012,3)-GARCH(1,1)	2.0238

Table 4.7: AIC Values for ARFIMA(*p*,*d*,*q*)-FIGARCH(1,D,1) Models

S/N	ARFIMA(*p*,*d*,*q*)-FIGARCH(1,D,1)	AIC
1	ARFIMA(1, 0.012,0)-FIGARCH(1, 0.048,1)	2.3599
2	ARFIMA(0, 0.012,1)-FIGARCH(1, 0.048,1)	2.3600
3	ARFIMA(1, 0.012, 1)-FIGARCH(1, 0.048,1)	2.3607
4	ARFIMA(2, 0.012,0)-FIGARCH(1, 0.048,1)	2.3249
5	ARFIMA(0, 0.012,2)-FIGARCH(1, 0.048,1)	2.3263
6	ARFIMA(2, 0.012,1)-FIGARCH(1, 0.048,1)	2.3249
7	ARFIMA(1, 0.012,2)-FIGARCH(1, 0.048,1)	2.3268
8	ARFIMA(2, 0.012,2)-FIGARCH(1, 0.048,1)	2.2535
9	ARFIMA(3, 0.012,0)-FIGARCH(1, 0.048,1)	2.3251
10	ARFIMA(0, 0.012,3)-FIGARCH(1, 0.048,1)	2.3261
11	ARFIMA(3, 0.012,1)-FIGARCH(1, 0.048,1)	2.3013
12	ARFIMA(1, 0.012,3)-FIGARCH(1, 0.048,1)	2.3267

13	ARFIMA(3, 0.012,2)-FIGARCH(1, 0.048,1)	2.2998
14	ARFIMA(2, 0.012,3)-FIGARCH(1, 0.048,1)	2.2411
15	**ARFIMA(3,0.012,3)-FIGARCH(1,0.048,1)**	**2.2229**

4.7.2 ARFIMA-GARCH and ARFIMA-FIGARCH Model Estimation and Diagnostics

As shown in Table 4.8, the parameter d (i.e. d-Arfima) is significant at 5% level of significance, revealing the presence of Long Memory in the mean exchange rate. For the volatility component, the Long Memory parameter (d-Figarch) is also significant at 5% significance level, indicating the long-range dependence phenomenon for volatilities.

Table 4.8: Estimation Results of the ARFIMA-GARCH and ARFIMA-FIGARCH Models

ARFIMA(2, 0.012,1)-GARCH(1,1)			ARFIMA(3, 0.012,3)-FIGARCH(1, 0.048,1)		
Coefficient	Estimate	SE	Coefficient	Estimate	p-values
Mean equation			Mean Equation		
d-ARFIMA	0.012*	0.08700	d-ARFIMA	0.012*	0.08700
AR(1)	0.9816**	0.00054	AR(1)	1.5453**	0.00032
AR(2)	0.0085**	0.00006	AR(2)	-0.1871**	0.00013
MA(1)	-0.9318**	0.00378	AR(3)	-0.3594**	0.00014
			MA(1)	-1.6077**	0.00001
			MA(2)	0.2578**	0.00002
			MA(3)	0.3528**	0.00002
Variance equation			Variance equation		
ARCH (alpha1)	0.1048**	0.00215	d-FIGARCH	0.048*	0.06500
			ARCH (alpha1)	0.1931**	0.00450
GARCH (beta1)	0.8928**	0.00037	GARCH (beta1)	0.9030**	0.00103
			OMEGA	1.0000**	0.00002
ln(L)	-3223.199		ln(L)	-4800.936	
Portmanteau test	0.000321	0.9857	Portmanteau test	0.001679	0.9673

ARCH-LM test	0.000308	0.9860	ARCH-LM test	0.000910	0.9759

**Represents significance at 1% level of significance level and * represents significance at 5% level of significance level.

Since all these Long Memory parameters lie between 0 and 0.5, there is significant presence of stationary Long Memory both in mean and volatility.The existence of Long Memory in both mean and volatility supports the application of the hybrid models with which the future mean and volatility values of the exchange rate series are predictable.

The parameters of the hybrid ARFIMA-GARCH and ARFIMA-FIGARCH models in Table 4.8were also found significant and come with smaller standard errors relative to parameters of the ARFIMA models in Table 4.5, indicating the adequacy of the hybrid models. Also, the log-likehood values in Table 4.8 are larger compared to the log-likehood values of ARFIMA models in Table 4.5 indicating the goodness-of-fit of the hybrid models to the Nigeria Naira to USD exchange rate. These results show evidence of improvement in model fitting as a result of introducing the GARCH and FIGARCH models to the ARFIMA model.

For residual diagnosis, the fitted hybrid models are verified by plotting the autocorrelations at various lags of the residuals obtained from the fitted ARFIMA(2, 0.012, 1)-GARCH(1, 1) model as well as ARFIMA(3, 0.012, 3)-FIGARCH(1, 0.048, 1) model. The ACF plots of residuals for the two models are given in the figures 4.6 and 4.7 respectively. Since all the autocorrelations are insignificant, it has been proved that the selected ARFIMA-GARCH and ARFIMA-FIGARCH modelswere appropriate models for capturing dual long memory and volatility present in the exchange rate data.

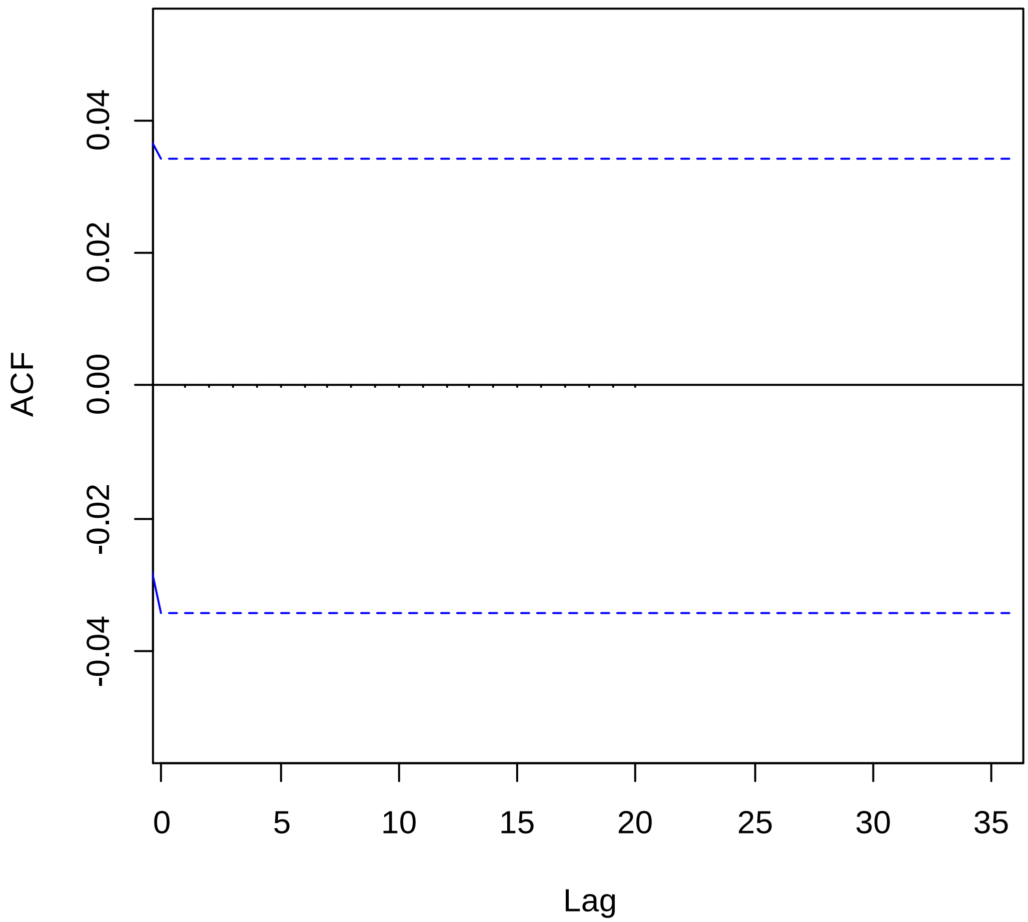


Figure 4.6: ACF of Standardized Residuals of ARFIMA(2, 0.012,1)-GARCH(1,1) Model

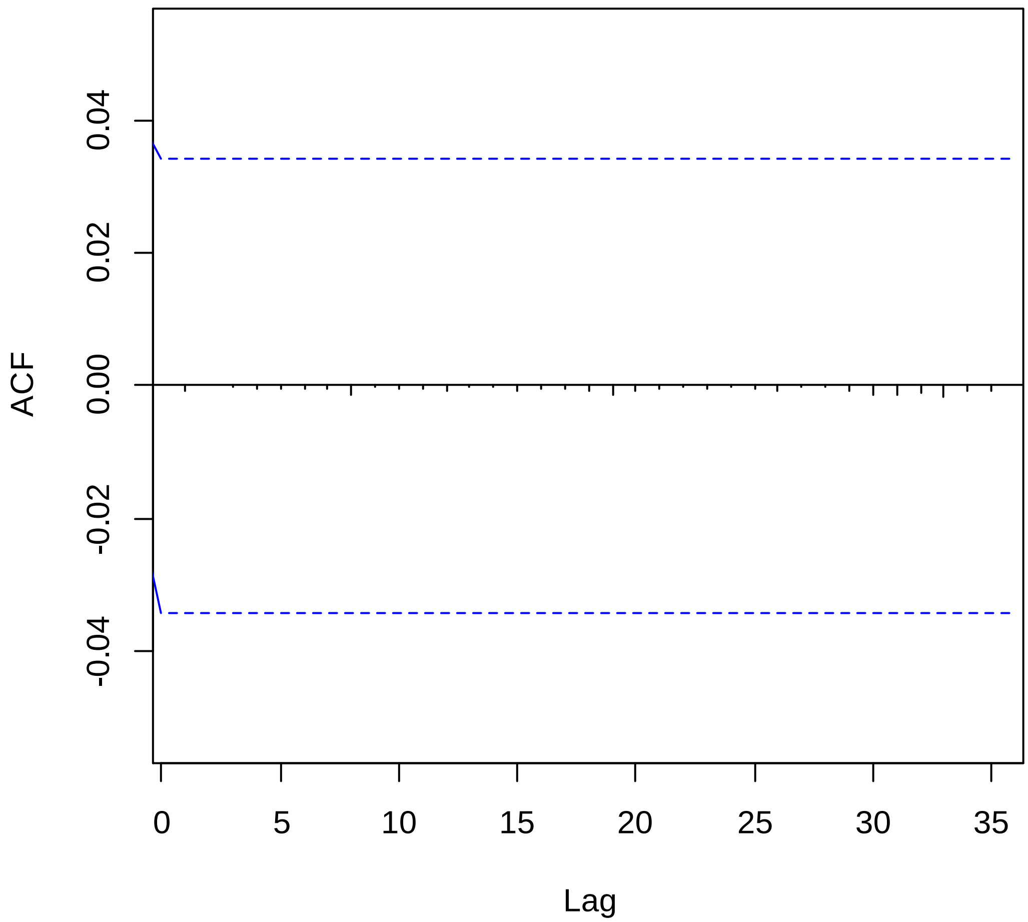


Figure 4.7: ACF of Standardized Residuals of ARFIMA(3, 0.012,3)-FIGARCH(1, 0.048, 1) Model

4.7.3 The Forecast of the Hybrid Models

One-step ahead forecast results of Nigerian Naira to US-Dollar exchange rate using the hybrid ARFIMA-GARCH and ARFIMA-FIGARCH models are shown in Table 4.9.

Table 4.9: Forecast for Exchange Rate using the Fitted ARFIMA-GARCH and ARFIMA-FIGARCH Models

No of Days	ARFIMA-GARCH		ARFIMA-FIGARCH	
	NGNUSDER	VOLATILITY	NGNUSDER	VOLATILITY
7th April, 2021	402.8	1.579	403.4	1.692
8th April, 2021	402.8	1.867	403.7	1.915
9th April, 2021	402.8	2.009	404	2.043
10th April, 2021	402.8	2.101	404.3	2.131
11th April, 2021	402.8	2.166	404.7	2.195
12th April, 2021	402.8	2.215	405.0	2.245
13th April, 2021	402.8	2.252	405.4	2.284
14th April, 2021	402.8	2.283	405.7	2.317
15th April, 2021	402.8	2.307	406.1	2.343
16th April, 2021	402.8	2.325	406.4	2.364

The accuracy of the fitted models is measured in terms of RMSE using the formulae defined in equation (3.14). The accuracy measures are shown in Table 4.10. Compared to the other candidate hybrid models, the ARFIMA (1,0.012,0)-GARCH(1,1) model produces a better forecast with minimum RMSE.

Table 4.10: Forecast Accuracy Measures for Hybrid Models

Models	RMSE
ARFIMA(2, 0.012, 1)-GARCH(1, 1)	0.8672
ARFIMA(3, 0.012, 3)-FIGARCH(1, 0.048, 1)	0.8881

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 SUMMARY

In the present study the relevance of capturing long-range dependence pattern in modeling and forecasting of daily exchange rate of Nigerian Naira to United States Dollar is investigated. Significant results of GPH test indicate the presence of Long Memory in mean as well as in volatility. After fitting ARFIMA model to the fractionally differenced series, the residuals are obtained from the fitted mean model and tested for possible presence of serial autocorrelation and ARCH effect. It is found that both the serial autocorrelation and ARCH effect are significant. Accordingly, ARFIMA-GARCH and ARFIMA-FIGARCH modelsare fitted and the best modelsare selected on the basis of minimum AIC value. The lesser values of RMSE ensure good predictability of the fitted models. Finally, one-ahead forecasts are calculated using the best fitted models and it has been observed that the forecasts are very close to the actual values, indicating better fitness and sufficiency of the models.

5.2 CONCLUSION

Compared to the well-known and classicalfractionally integrated order model(generally called ARFIMA model) which can only capture long range dependence in mean, the hybrid ARFIMA-GARCH or ARFIMA-FIGARCH models give a better fit and result when dealing with the time series data which possess the Long Memory property. The flexibility of the hybrid models is in their ability to simultaneously explore the long range dependence existing both in mean and volatility of the series. Therefore, a comprehensive study and evaluation of the hybrid models using exchange rate datais presented in thiswork.The results of fitting ARFIMA model to the series show the presence of serial autocorrelation and the existence of ARCHeffects (heteroscedasticity) in the standardized residuals, signifying the limitations of fitting ARFIMA model to the exchange rate series and signaling the importance of testing the existence of Long Memory in volatility. The Long Memory parameter for the volatility was estimated using GPH method and found significant at 5% level of significance. The fitted hybrid models were found adequate and sufficient for the exchange rate data.

In conclusion, the study showed a significant interaction between the conditional mean and volatility of the exchange rate series, and that the hybrid ARFIMA-GARCH model has accounted for presence of Long Memory in the conditional mean and presence of volatility in the exchange rate series. Although, both hybrid models considered in this study showed significant improvement over the classical ARFIMA model, the ARFIMA-GARCH specification was also found flexible enough to capture the dual Long Memory observed in the exchange rate series.

5.3 RECOMMENDATIONS

The results show that even though some series may appear to be stationary through integer or non-fractional differencing and supported by ADF test, they could still exhibit the characteristics of Long Memory process as indicated in the result of this study. It is therefore recommended that efficient data exploratory exercise be taken very crucial before carrying out time series data analysis of any kind. This would reveal all the hidden characteristics of the series which could assist in the choice of the appropriate model that would yield optimal forecast values.

This study has neglected to test for the impact of structural breaks evident in figure 4.2. Some researchers opined that LongMemory results could be biased as a result of overlookingstructural breaks. Caporale and Gil-Alana (2008) discussed a test procedure for possible presence of structural breaks in Long Memory models. Caporale and Skare (2014) adopted the procedure and found that even when accounting for breaks in that way, the Long Memory results were essentially the same.To empirically confirm such claim for the models developed in this study is left for further research.

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Appendix

```
library(forecast)

library(tseries)

library(archlmtest)

library(rugarch)

library(fBasics)
```

```
library(fracdiff)

ngntousddexc1<-read.csv(file.choose(),header=T)

basicStats(ngntousddexc1)

ngntousddexcdated=ts(ngntousddexc1,start=c(2008,10,20),frequency=240)

ts.plot(ngntousddexcdated,ylab="NGN-USD Daily Exchange Rate",xlab=c("Time"))

par(mfrow=c(1,2))

acf(as.vector(ngntousddexcdated),50,main=paste(" "),xlab=c("(a) Plot of the ACF for Daily NGN to
USD Exchange Rate."))

pacf(as.vector(ngntousddexcdated),50,main=paste(" "),xlab=c("(b) Plot of the PACF for NGN to
USD Exchange Rate."))


###Fitting ARIMA Models


logtransformed=log(ngntousddexcdated)

ts.plot(logtransformed,ylab="Log of NGN-USD Exchange Rate",xlab=c("Time"))

nonseasonaldifferencengntousddexc=diff(log(ngntousddexcdated))

ts.plot(nonseasonaldifferencengntousddexc,ylab="Nonseasonally Differenced NGN-USD Exchange
Rate",xlab=c("Time"))

acf(as.vector(nonseasonaldifferencengntousddexc),50, main=paste(" "),xlab=c("(a)ACF for
Nonseasonal Difference NGN-USD Exchange Rate."))

pacf(as.vector(nonseasonaldifferencengntousddexc),50, main=paste(" "),xlab=c("(b)PACF for
Nonseasonal Difference NGN-USD Exchange Rate."))

arima(ngntousddexcdated,order=c(2,1,7),method='ML')

arima(ngntousddexcdated,order=c(0,1,7),method='ML')

arima(ngntousddexcdated,order=c(2,1,6),method='ML')


###Fitting ARFIMA Models


d=fdGPH(ngntousddexcdated)$d

fractionaldifferencengntousddexc=diffseries(ngntousddexcdated,d=fdGPH(ngntousddexcdated)$d
)
```

```
ts.plot(fractionaldifferencengntousddexc,ylab="Fractionally Differenced NGN-USD Exchange Rate",xlab=c("Time"))

acf(as.vector(fractionaldifferencengntousddexc),50, main=paste(" "),xlab=c("(a)ACF for Fractional Difference NGN-USD Exchange Rate."))

pacf(as.vector(fractionaldifferencengntousddexc),50, main=paste(" "),xlab=c("(b)PACF for Fractional Difference NGN-USD Exchange Rate."))


#Long Memory in Volatility

volatility=abs(diff(log(ngntousddexcdated)))

d=fdGPH(volatility)$d


#ARFIMA Model Identificationand Estimation

arfimangntousddexc10=fracdiff(ngntousddexcdated,nar=1,nma=0)

arfimangntousddexc01=fracdiff(ngntousddexcdated,nar=0,nma=1)

arfimangntousddexc11=fracdiff(ngntousddexcdated,nar=1,nma=1)

arfimangntousddexc20=fracdiff(ngntousddexcdated,nar=2,nma=0)

arfimangntousddexc02=fracdiff(ngntousddexcdated,nar=0,nma=2)

arfimangntousddexc21=fracdiff(ngntousddexcdated,nar=2,nma=1)

arfimangntousddexc12=fracdiff(ngntousddexcdated,nar=1,nma=2)

arfimangntousddexc22=fracdiff(ngntousddexcdated,nar=2,nma=2)

arfimangntousddexc30=fracdiff(ngntousddexcdated,nar=3,nma=0)

arfimangntousddexc03=fracdiff(ngntousddexcdated,nar=0,nma=3)

arfimangntousddexc31=fracdiff(ngntousddexcdated,nar=3,nma=1)

arfimangntousddexc13=fracdiff(ngntousddexcdated,nar=1,nma=3)

arfimangntousddexc32=fracdiff(ngntousddexcdated,nar=3,nma=2)

arfimangntousddexc23=fracdiff(ngntousddexcdated,nar=2,nma=3)


summary(arfimangntousddexc10)

summary(arfimangntousddexc01)
```

```
summary(arfimangntousddexc11)
summary(arfimangntousddexc20)
summary(arfimangntousddexc02)
summary(arfimangntousddexc22)
summary(arfimangntousddexc21)
summary(arfimangntousddexc12)
summary(arfimangntousddexc30)
summary(arfimangntousddexc03)
summary(arfimangntousddexc31)
summary(arfimangntousddexc13)
summary(arfimangntousddexc32)
summary(arfimangntousddexc23)


#ARFIMA Model Diagnostic Analysis
forecastsarfimangntousddexc02=forecast::forecast(arfimangntousddexc02)
forecastsarfimangntousddexc03=forecast::forecast(arfimangntousddexc03)
residualsarfimangntousddexc02=residuals(forecastsarfimangntousddexc02)
residualsarfimangntousddexc03=residuals(forecastsarfimangntousddexc03)
Box.test(residualsarfimangntousddexc02,lag=2,type="Ljung-Box")
Box.test(residualsarfimangntousddexc03,lag=2,type="Ljung-Box")
archlmtest(residualsarfimangntousddexc02,lag=5)
archlmtest(residualsarfimangntousddexc03,lag=5)


### Fitting ARFIMA-GARCH Models


ARFIMA(0,d,2)-GARCH(1,1)
garch0211.spec = ugarchspec(mean.model = list(armaOrder = c(0,0)),
```

```
variance.model = list(garchOrder = c(1,1),

            model = "sGARCH"), distribution.model = "norm")

garch.fit0211 = ugarchfit(garch0211.spec, data = residualsarfimangntousddexc02,na.rm = TRUE,
mean=FALSE, fit.control=list(scale=TRUE))

residualsgarch.fit0211=residuals(garch.fit0211)

Box.test(residualsgarch.fit0211,lag=1,type="Ljung-Box")

archlmtest(residualsgarch.fit0211,lag=1)


ARFIMA(0,d,3)-GARCH(1,1)

garch0311.spec = ugarchspec(mean.model = list(armaOrder = c(0,0)),

variance.model = list(garchOrder = c(1,1),

            model = "sGARCH"), distribution.model = "norm")

garch.fit0311 = ugarchfit(garch0311.spec, data = residualsarfimangntousddexc03, mean=FALSE,
fit.control=list(scale=TRUE))

residualsgarch.fit0311=residuals(garch.fit0311)

Box.test(residualsgarch.fit0311,lag=1,type="Ljung-Box")

archlmtest(residualsgarch.fit0311,lag=1)



### Fitting ARFIMA-FIGARCH Models


ARFIMA(0,d,2)-FIGARCH(1,d,1)

figarch0211.spec = ugarchspec(mean.model = list(armaOrder = c(0,0)),

variance.model = list(garchOrder = c(1,1),

            model = "fiGARCH"), distribution.model = "norm")

figarch.fit0211 = ugarchfit(figarch0211.spec, data = residualsarfimangntousddexc02, mean= FALSE,
fit.control=list(scale=TRUE))


residualsfigarch.fit0211 =residuals(figarch.fit0211)

Box.test(residualsfigarch.fit0211,lag=1,type="Ljung-Box")
```

```
archlmtest(residualsfigarch.fit0211,lag=1)


ARFIMA(0,d,3)-FIGARCH(1,d,1)

figarch0311.spec = ugarchspec(mean.model = list(armaOrder = c(0,0)),

variance.model = list(garchOrder = c(1,1),

            model = "fiGARCH"), distribution.model = "norm")

figarch.fit0311 = ugarchfit(figarch0311.spec, data = residualsarfimangntousddexc03, mean= FALSE,
fit.control=list(scale=TRUE))


residualsfigarch.fit0311 =residuals(figarch.fit0311)

Box.test(residualsfigarch.fit0311,lag=1,type="Ljung-Box")

archlmtest(residualsfigarch.fit0311,lag=1)




library(arfima)

y = log(ngntousddexcdated)-mean(log(ngntousddexcdated))

acf2(y) ## ACF and PACF of the data

## Estimate d:

exratefd = arfima(y)

summary(exratefd)

d = summary(exratefd)$coef[[1]][1] #d = 0.4990953

d #prints the value of d to the screen

se.d = summary(exratefd)$coef[[1]][1,2] #se = 6.320601e-07

se.d #prints the standard error of d to the screen


##Residuals

resids = resid(exratefd)[[1]]
```

```
#resid(exratefd) is a list and [[1]] accesses the residuals as a vector
plot.ts(resids)
acf(resids)
plot.ts(y)
```