

**INFLUENCE OF EFFECTIVE LENGTHS ON RELIABILITY OF HOT-ROLLED STEEL
COLUMNS**

BY

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(M.Sc/ENG/58678/2005-06)

DEPARTMENT OF CIVIL ENGINEERING
FACULTY OF ENGINEERING
AHMADU BELLO UNIVERSITY, ZARIA
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IN PARTIAL FULFILLMENT OF THE REQUIREMENT FOR THE AWARD OF MASTER OF
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DEPARTMENT OF CIVIL ENGINEERING
FACULTY OF ENGINEERING
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DECLARATION

I hereby declare that the work in this thesis titled: “Influence of Effective Lengths on Reliability of Hot-Rolled Steel Columns was carried out by me in the Department of Civil Engineering, under the Supervision of Dr.O.S. Abejide and Dr. I. Abubakar. The information derived from the literature has been duly acknowledged in the text and list of references provided. No. part of this work has been presented for another degree at any institution.

SALIHU ABUBAKAR ONEMISI

CERTIFICATION

This project thesis “Influence of Effective Length on Reliability of Hot-Rolled Steel Columns” meets the regulations government the award of the degree of Masters of Science (Civil Engineering) of the Ahmadu Bello University, and is approved for its contribution to knowledge and literary presentation.

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DEDICATION

This work is dedicated to my parents, lovely wife, children, and well wishers whose support and words of encouragement led to the completion of the project.

ACKNOWLEDGEMENTS

It may be almost be impossible for a work of this type to be completed without the support of some people, in eight physical, moral, intellectual and financial ways.

Gratitude goes to my supervisor Dr. O.S. Abejide for all the guidance and support he gave me in the course of this research.

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H.S. I thank all my lecturers and members of staff of the Department of Civil Engineering, Ahmadu Bello University, Zaria for their support.

ABSTRACT

This work was carried out in order to check the adequacy or otherwise of the current prediction of effective length factor K on hot-rolled steel columns subjected to the axial load and see to their suitability in application to designing steel columns or compressive member in general. Firstly the compressive strength of steel columns for Square Hollow Sections (SHS), Rectangular Hollow Section (RHS) and Circular Hollow Sections (CHS) were re-calculated using Euler's formula for values of effective length factor K ranging from 0.2 to 2.0 and for story height of 3m, 6m, 9m, 12, and 15m. Secondly, two samples each of 3mm ϕ diameter and 4mm ϕ diameter rod were prepared the lengths of which ranged between 24cm to 36cm. These two samples of steel which were obtained from market were also confirmed with the vernier calipers.

The buckling loads of each sample were determined using the apparatus shown fig.4.19 in the CCECC Nigeria Limited Laboratory in Damaturu, Yobe State. From the relationship between the compressive strength and the slenderness ratios with varying values of K , the limit slenderness ratio was found to be 15 at optimum effective length factor of 2.2 for SHS at 15.0m. Also tests indicated that the minimum effective length factor K at buckling is 2.0 for pin-ended column which is 100% increase or twice the current value being used in the design. However, the buckling loads were found to be decreasing with increasing lengths for a given columns size. It is therefore suggested that, the minimum effective length factor of K should not be less than 2.0 when using the Euler's formula for designing compression member. In addition to this, the allowable slenderness ratio of 180 should not be exclusively be applied for checking the stability of steel columns, because at 165 value of slenderness ratio, the theoretical value of K was 2.2 as shown above are to 1.0 provided pin ended columns.

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GENERAL INFORMATION AND SYMBOLS

K	=	Effective length factor
E	=	Modulus of elasticity (constant) N/mm ²)
r _{min}	=	Minimum radius of gyration (mm)
I	=	Moment of inertia (mm ⁴)
P _r	=	Required axial compressive strength (N)
P _c	=	Compressive strength (N)
P _{r(calc)}	=	Calculated axial compressive
λ	=	Slenderness Ratio
A _g	=	Gross Area (mm ²)
L	=	Story Heights (mm)
π	=	3.142 (constant)
CHS	=	Circular Hollow Section
SHS	=	Square Hollow Section
RHS	=	Rectangular Hollow Section
UC	=	Universal Column

$$r = \sqrt{I/A}$$

CHAPTER ONE:

INTRODUCTION

1.1 Background/ General

In response to increasingly stringent architectural and cost requirement in modern buildings, the use of slender building elements, has become more common in steel sections, thus making it necessary to provide a comprehensive evaluation of the behaviour of these columns in compression (Khalil et.al, 2001).

The aim of structural design is first to provide, with due regards to economy, a structure capable of fulfilling it's intended functions and sustaining the design loads for it's intending life time (BS 5950, 1990). The requirement that a structure performs effectively without failure during its design life is fundamental. Failure is implied in the sense of exceeding a certain limit state corresponding, for example, to a measure of un-serviceability or instability (Abejide, 2007; Ellingwood, 1978; Melchers 1974). When making calculations to determine the sizes of structural members to resist tension, compression, shear, bending, or other structural loads, the engineer refers to standard specifications, code of practices and by –laws (Morgan, 1979). These permissible working stresses are based on knowledge of the ultimate, or failing strengths of the materials, obtained by testing them in tension, compression, shear, and or on research into the natures, properties and behaviour of such materials.

The current prescription of effective length (K) factors, is widely employed in engineering practice even when the assumptions on which it is based are violated in the alignment chart. The AISC (1989) for the Allowable Stress Design (ASD) and AISC (1999) for Load and Resistance Factor Design (LRFD) specifications, and the EUROCODE 3 (1995), includes alignment charts to determine the K factor to be applied in the calculation of the buckling for a particular compressed member in a frame (Raul, 2003). Effective length factors (K -factor) of columns must be estimated to evaluate the stability of columns in frames with rigid and semi-rigid connections, as opined by (Degertekin and Hayalioglu, 2004).

Hot-rolled steel columns are very efficient structurally in carrying compression loads and moments and are widely used in the construction of framed structures in office and industrial buildings (Kodur, 1998) shows that hollow sections are more efficient than solid sections, provided that the wall is not made so thin to induce local crumpling.

1.2 Problem Statement and Justification of Study

All members resisting axial compression load would buckle if the applied axial force/stress were greater than allowable for given sizes and lengths. Stability checks are the calculations to check that the resistance to buckling is greater than the applied forces, when investigating the stability of a column (NSC, 2001). Although, engineers are accustomed to checking the buckling resistance of columns using effective lengths (NSC, 1981), the effect or influence of the effective length coefficient for the chosen hot-rolled section are based on the theoretical value of K given by Euler. Hence there is need to adequately determine practically the required effective length factors for design of steel columns.

1.3 Aim and Objectives

In this project, the aim is to determine the effect of the effective length factors (K) on the compressive strength of hot-rolled steel columns. The objective will herein include the following :

1. Evaluate the effective lengths for a storey height of 3, 6, 9, 12, 15, with k factor ranging from 0.2 and 2.0 using chosen sections of hot-rolled steel columns (CHS and RHS) for grade 275, 325 and 355 as obtained in BS 5950 (2000).
2. Evaluate the slenderness ratios corresponding to the effectiveness length.
3. Estimating the compressive strength for the chosen column size as the slenderness ratio increases.
4. Determine the optimum effective length factor k corresponding to the limit slenderness ratio for each steel section.
5. Experimental determination of the k factor using sample column models.
6. Correlation between the present theoretical value of $k = 1$ for pin ended column and the experimental result.

1.4 Scope and Limitations

The scope of this project covers the determination of values of slenderness ratios for the values of K-factor ranging from 0.2 to 2.0 and the corresponding values of theoretical compressive strengths for Rectangular Hollow Section (RHS), Square Hollow Section (SHS) and Circular Hollow Sections (CHS). In addition to this, experiments were performed to determine practically the value of K corresponding to the buckling loads for column models. The samples (models) were made of 10 specimens of 30mmØ and 40mmØ solid circular section and of length ranging from 24cm – 36cm. However, the experiments did not cover fixed ends conditions.

CHAPTER TWO:

LITERATURE REVIEW

2.1 General

Structural steel products are manufactured to conform to BS 4360 (1990); Steel is composed of about 98% iron with the main alloying element; carbon, silicon and manganese. Struts, stanchions and columns are the terms applied most commonly to compression members. Strut is a general term, but columns and stanchions are reserved for the main vertical members carrying loads from the roof, floors and the walls in buildings (Currie and Sharpe, 1990; Bhatt and Nelson, 1990). As an overview, a short, stocky stanchion fails by squashing or crushing of the material; and for long slender stanchion failures occur by overall buckling. Other forms of failure are torsion buckling and local buckling. Davidson and Owens (2005) hinted that, the axial load-carrying capacity for simple compression members largely depends on its slenderness, material strength, cross sectional shape and method of fabrications.

It is customary to consider that a column may buckle in either one of two ways: (a) by plate buckling of its component webs and flanges in shorter half-waves (local or plate buckling) (b) by deflection of the entire column in a half-wave of length equal to the effective column length (overall buckling). For a given column, buckling is supposed to occur at the lower of the two critical stresses, local or overall (Yang et al, 2002). In reality, however, there is an interaction between these two modes of buckling, so that the actual buckling stress will be smaller than either of the buckling stresses. Imperfections play a significant role in interaction buckling. Column strength is characterized by the maximum axial force that can be supported without excessive lateral deformations (Yang et al, 2002). In view of the fact that post-buckling strength of a flat plate is available for structural members to carry additional loads,

cold-formed steel sections are normally designed on the basis of the post-buckling strength of the plate elements rather than based on the local buckling stress.

Attempt have been made in this directions, for example, the resistance factor for structural steel was set at 0.90 with the inception of limit states design in Canadian Standard Association (CSA) Standard S16.1 (CSA 1974), as described in the landmark study by Kennedy and Gad (1980). Steel structures designed using a resistance factor of 0.90 has demonstrated satisfactory performance, indicating that a more conservative value is not required (Schmidt and Bartlett, 2002).

Conventionally, design specifications provide data upon which a material is supposed to be used for design of a structure, which are often provided in design code literatures preferred by the professional or the expert. The Code specifications of compromise between theoretical considerations and practical requirements, and therefore are not a complete remedy for all design problems. Ronald (1998) observed that the value of loads and allowable stresses are generally based on past experience and test data, and has to be modified periodically to agree with the latest finding. It is on this background that this research proposal tends to investigate or evaluate the effective length factors (K) of hot-rolled steel columns to BS 5950 (2000) specifications.

2.2 Elastic Buckling of slender compression members

The first qualitative remarks on column strength and stability were given by Erone Alexandria (75 BC) and similar descriptions of buckling columns by Leonardo Da Vinci (1452 – 1519).

The theory of column buckling was first originated by Euler (1744 – 1759) (Euler 1759).

Euler consider an ideal column with the following attributes :

1. Material is isotropic and homogenous and is assumed to be perfectly elastic.
2. The column is initially straight and the load acts along the Centroidal at axis (i.e No eccentricity of loads)
3. Column has no imperfections
4. Columns ends are linged

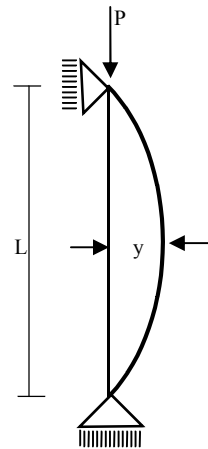


Figure 2.1

At any location x, the bending moment M on the member bent slightly about the principal axis is

$$M = PY \text{ ----- 2.1}$$

and since

$$d^2y/dx^2 = - \left(\frac{M}{EI} \right) \text{----- 2.2}$$

or

$$EI \frac{d^2 y}{dx^2} = -M \quad \text{-----} 2.3$$

I is the moment of the inertia and E is the young's Modulus.

$$\text{Then, } EI \frac{d^2 y}{dx^2} + M = 0$$

$$\Rightarrow \frac{EI d^2 y}{dx^2} + PY = 0$$

$$\frac{d^2 y}{dx^2} + \frac{PY}{EI} = 0$$

$$\frac{d^2 y}{dx^2} + \left[\frac{P}{EI} \right] Y = 0 \quad \text{-----} 2.4$$

$$\text{let } k^2 = \frac{P}{EI}$$

$$\frac{d^2 y}{dx^2} + k^2 y = 0$$

The solution of this 2nd order differential equation becomes

$$y = A_1 \sin kx + A_2 \cos kx \quad \text{-----} 2.5$$

Where A_1 and A_2 are unknown coefficient

$$(a) \ y = 0 \text{ at } x = 0, \quad 0 = 0 + A_2, \quad A_2 = 0$$

$$(b) \ y = 0 \text{ at } x = L, \quad 0 = A_1 \sin kx + 0$$

$$A_1 \sin kx = 0 \text{ ----- 2.6}$$

or

$$A_1 \sin kx = 0, \quad A_1 = 0$$

Then

$$\sin kL = 0 \quad kL = n\pi \text{ to satisfy the equation.}$$

$$\text{Hence } k = \frac{n\pi}{L} \quad \text{from above}$$

$$\text{But } k^2 = \frac{P}{EI}; \quad P = k^2 EI \text{ ----- 2.7}$$

$$\text{Substitute for } k = \frac{n\pi}{L} \text{ from 2.7}$$

$$P = \frac{n^2 \pi^2 EI}{L^2} \quad n = (1, 2, 3, \dots)$$

$$\text{Hence } P = \frac{\pi^2 EI}{L^2}, \frac{4\pi^2 EI}{L^2}, \frac{9\pi^2 EI}{L^2}$$

All the terms of the right hand side, $\frac{\pi^2 EI}{L^2}, \frac{4\pi^2 EI}{L^2}, \frac{9\pi^2 EI}{L^2}$ are possible values of Euler critical bulking load.

But fundamental buckling load with a single curvature deflection ($y = A_1 \sin Kx = A_1 \sin \pi x$) will occur when $n = 1$. Thus, the Euler critical load for a column with both ends hinged is

$$P_{cr} = \frac{n^2 \pi^2 EI}{L^2} \text{ ----- 2.8}$$

Here it is noted that no mention made about the effective length factor throughout the derivation of this formula.

CHAPTER THREE:

METHODOLOGY

3.1 General

Elastic buckling analysis reveals at least three buckling modes; local, distortional, and Euler (flexural, torsional, or flexural-torsional) that must be considered in design (AISI, 2004). The concept that will be employed is based on the general principle of compression member as obtained in the clause 4.7 of BS 5950 (2000). The concept is that steel columns or compression members fail either by crushing or buckling about the axis perpendicular to the applied loads. The design should basically involve checking that the applied compressive force is less than the maximum theoretical compressive strength of the section in question for a given height. This means that

$$\alpha P_r \leq 0.15 P_{cr}$$

P_r = required axial compressive strength

(LRFD or ASD)

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} \quad [\text{in the plane of bending (Euler)}] \quad \dots\dots\dots (3.1)$$

$$\text{Hence } \frac{P_r}{P_{cr}} \leq 0.15 \quad \alpha = 1.0(\text{LRFD}) \text{ or } 1.6(\text{ASD}) \quad \text{Take } \alpha = 1.0$$

$$P_r \leq \frac{0.15 \pi^2 EI}{(KL)^2} \quad \dots\dots\dots (3.2)$$

- (i) The required axial compressive strengths P_r shall be calculated for values of K ranging from 0.2 to 2.0.

- (ii) The corresponding slenderness ratios $= \frac{KL}{r_{\min}}$ shall also be determined for each length of column considered.
- (iii) The calculated slenderness ratios shall be compared to the limit or allowable slenderness ratios in the code.
- (iv) The optimum effective length factors (K) corresponding to the limit slenderness ratio for each steel section shall be deduced from the graphs of compressive strengths versus slenderness ratios at each height for a chosen section.
- (v) Necessary recommendations or suggestions shall be made based on the results obtained on the optimum, as compared to code specifications as per the effective length factors to be used in hot-rolled steel columns.

3.2 Steel sections considered

The steel sections considered in this work are as shown below.

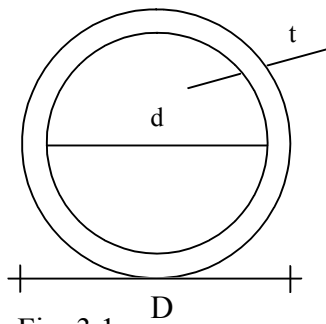


Fig. 3.1

CHS
 $d = D - 2t$

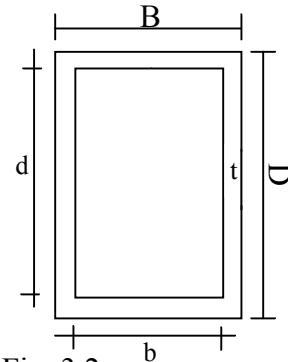


Fig. 3.2

RHS
 $b = B - 2t$
 $d = D - 2t$

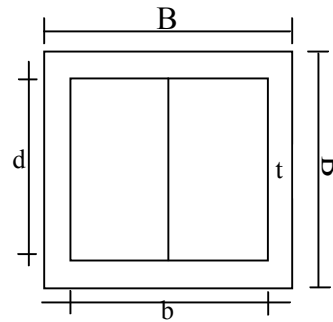
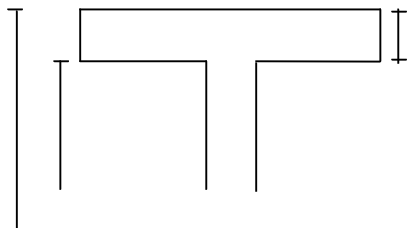
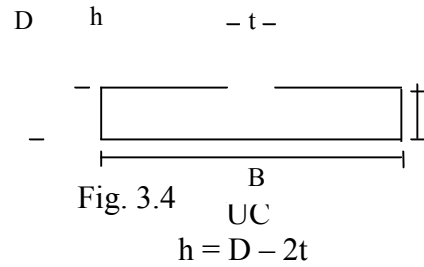


Fig. 3.3

SHS
 $b = B - 2t$
 $d = B - 2t$





CHS, RHS, SHS, and UC as defined in page ii.

3.2.1 *Effective lengths*

There are two other factors which affect the buckling tendency of stanchions

- i) Actual length (L), which is taken as the distance between the points of effective resistance forming into members.
- ii) End condition.

These combine to produce what is termed “*the effective length*”. In design, end conditions are accounted for through the use of effective length factors which when multiplied by the effective length (L) gives the effective length (L_{eff}) for the buckling. The fixity of the joints and the rigidity of adjacent members may be taken into account for the purpose of calculating effective lengths of compression members. Thus, guidance for the choice of factors is given in Table 24 of BS 5950 (2000), in addition to the defined nominal effective lengths for compression members as obtained also in Table 22 of BS 5950 (2000).

3.2.2 *Radius of gyration*

This is a geometrical property depending upon the size and shape of the section; and is one of the measures of effectiveness in resisting buckling (Currie and Sharpe, 1990). It is used in column design and is expressed by the formula:

$$r = \sqrt{I/A} \dots\dots\dots(3.3)$$

Where I is the second moment of area or moment of inertia

A stands for the area of the section

3.2.3 *Slenderness*

This is the absolute measurement of the column tendency to buckle and is given by:

$$\lambda = \frac{L_{eff}}{r} \dots\dots\dots(3.4)$$

In practice it is necessary to consider the slenderness about both $X-X$ and $Y-Y$ Axis (Currie and Sharpe, 1990). At high slenderness, struts become very flexible and deflect more easily, and for this reason upper limits are proposed. BS5950 (2000) limits the maximum slenderness permissible in struts to the following.

- 180: for members resisting loads other than winds
- 250: for members resisting weight and wind loads
- 350: for members acting as tie but subject to load reversal under wind action

However, some of the typical values given for structural design are high (Ben, 1997), when compared with those values given by elastic stability theory. Hence one can easily see that these values are conservative and higher for those cases in which reliance is placed on externally provided rotational fixity, which is in recognition of the practical difficulties on externally provided rotational fixity conditions.

3.3 Critical load

The critical load will be evaluated using the “Euler formula” which is derived for an ideal or perfect column. That is, it is assumed that the column is long, slender, straight, homogeneous, and elastic and is subjected to concentric axial compressive loads. It is also assumed that the two ends of the columns are hinged, as shown in Figure 2.1.

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} \dots\dots\dots (3.5)$$

where KL is the distance between the points of zero moment, or inflection points. The length KL is known as the effective length of the column. The dimensionless coefficient K is called the *effective length factor*. Dividing the critical load P_{cr} by the cross-sectional area of the column A , we can find the critical stress f_{cr} :

$$f_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{(KL)^2 A} = \frac{\pi^2 E}{(KL/r)^2} \dots\dots\dots (3.6)$$

3.4 Compression resistance

The compression resistance P_c should be taken as:

$$P_c = A_g \rho_c \dots\dots\dots(3.7)$$

Where A_g is the cross sectional area and ρ_c is the compressive strength obtained from Tables 27(a)-(d) of BS 5950 (2000).

3.5 Classification of Sections

To prevent local buckling, limiting width to thickness ratio for flanges, and web in compression are given in Table 7 of BS 5950 (2000). Classification depends not only on the dimension of the section but also on the dimension of the stress across the section and position of the neutral axis. For the analysis in this study Rectangular Hollow Section (RHS) and Circular Hollow Section (CHS) will be adopted as shown in the Figure 1. Table 12 of BS 5950 (2000) gives the limiting width to thickness ratio for CHS, whereas for RHS, the dimension b and d should be taken as defined in BS 5950 to conform to the specifications and CEN (1995).

Since the permissible stresses for compression members increases with decreasing l/r ratio, the section should be proportioned to have a higher radius of gyration. A circular pipe has best cross section in this regards as it's radius of gyration is higher for a given cross section and is equal in all directions (Negi, 1995). However, in actual practice it is not preferred due to difficulties in joining with other members, but this should not limit educative findings on them.

3.6 Practical determination of buckling load and k-factors for selected column samples

After the theoretical determination of the slenderness ratios of different column height for a range of 0.2 to 2.0 effective length factors, it was observed that the required compressive strength P_r still varies inversely with the slenderness ratio λ .

However, attempts were made here to study the behaviour of some column. Samples models in relation to the axially applied load.

3.7 Apparatus Used

1. Digital scale
2. Hydraulic jack
3. Fabricated rectangular frame (0.3m x 0.6m)
4. Tape
5. Plumbulb
6. Column models

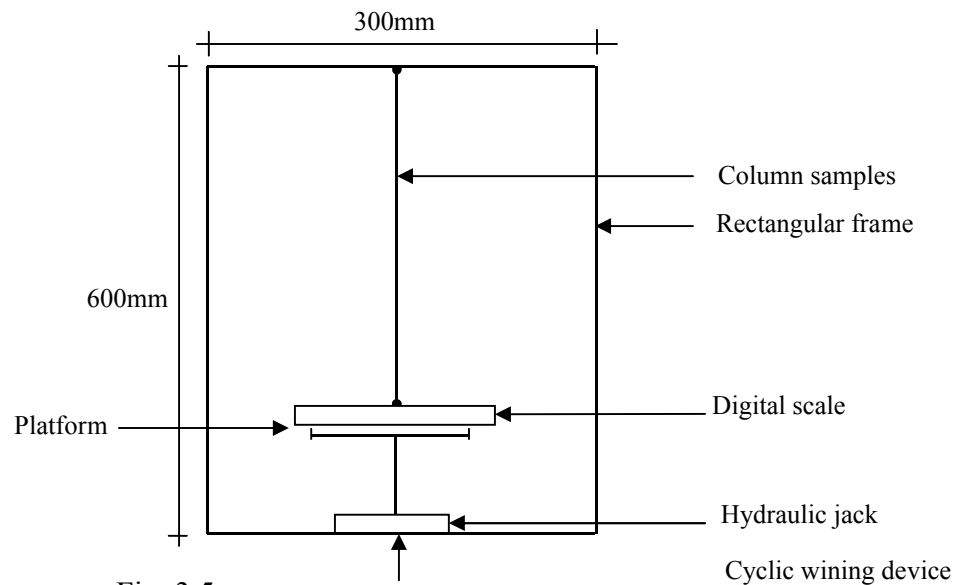


Fig. 3.5

Practical determination of buckling load for column samples

3.8 Experimental procedure

The practical determination of the buckling loads for selected samples of 3mmØ and 4mmØ rod was carried out using the following method: see Fig 3.5

1. Various lengths of chosen column sample were cut using pocket tape and hacksaw.
2. The average sizes of the column sample were pre-determined using the vernier calliper.
3. The apparatus was then arranged as showing in Fig. 3.5, with the digital scale being placed on the platform which was mounted on hydraulic jack.
4. Each sample was carefully placed between the top board of the rectangular frame and the digital scale after its self weight is read on the scale.
5. By means of the winding handle/device, the load was carefully and gradually applied to the column sample.
6. A plumbulb is carefully suspended from the top of the frame to measure the deviation 7mm.
7. The corresponding buckling load for 7mm deviation were recorded.
8. Ten (10) samples were tested for the 3mmØ and 4mmØ respectively.
9. The average result were given in Table 4.10 and 4.11.

10. The graph of buckling loads (kg) against the lengths (cm) were also plotted.

CHAPTER FOUR:

DESIGN AND ANALYSIS

4.1 General

To provide a point of comparison between various heights chosen from 3 m to 15m, selecting different section within the scope of this study, certain performance parameters were computed for all the height and section under consideration.

The effective length factors k , were varied from 0.2 to 2.0 at an interval of 0.2 for all the sections under considerations with respect to the storey height in questions. The slenderness ratio and axial load carrying capacities were computed as obtained in the proceeding section of the write up.

4.2 Sections Analysis

The sections chosen for the analysis are Rectangular Hollow Section (RHS), Circular Hollow Section (CHS) and Square Hollow Section (SHS) respectively.

4.2.1 *Square Hollow Section*

A typical SHS of $400 \times 400 \times 16mm$ sections is selected for the analysis as shown in figure 4.1 below.

Equations used, recalled from chapter 3

$$\lambda = KL$$

$$P_r = \frac{0.15\pi^2 EI}{(KL)^2}$$

$$f_{cr} = \frac{\pi^2 EI}{\left(\frac{KL}{r}\right)^2}$$

$$P_c = \frac{\pi^2 EI}{LE^2}$$

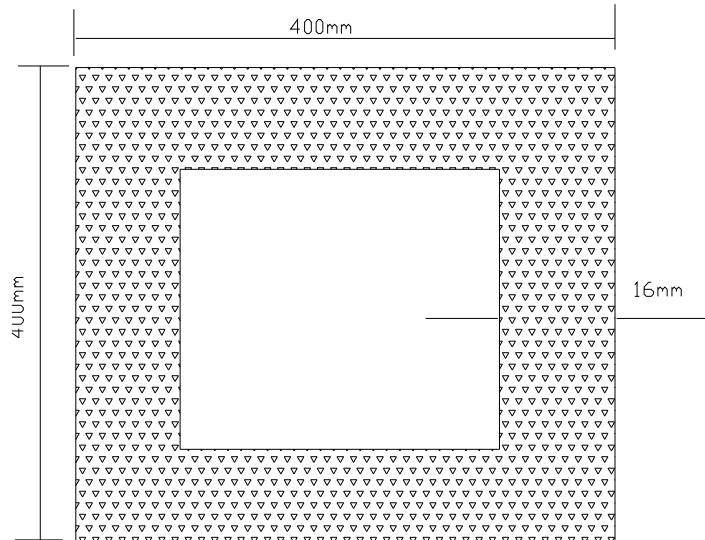


Figure 4.1 Typical Square Hollow section (SHS)

Section properties

Cross sectional Area (A) = $295cm^2$

Moment of inertia, $I_{xx} = I_{yy} = 59910cm^4$

Section modulus ($Z_{xx} = Z_{yy} = 2995m^3$)

Radius of gyration ($r_{xx} = r_{yy} = 15.7$)

4.2.1.1 Analysis for SHS

The Table 4.1A to 4.5A and Figures 4.2 to 4.6 presented herewith represent the complete analysis as regards to SHS for the respective height within the scope of the study area. K- Factor was evaluated at increment of 0.2 in range of 0.2 – 2.0 using equations 3.1 and 3.2.

Table 4.1.A: Data for SHS @3m height (L=3000mm)

K	KL (mm)	$(KL)^2$ (mm ²)	λ (KL/r)	Pr (N)	Fr (N/mm ²)
0.2	600	360000	3.821656	5.05E+08	138391.7
0.4	120	1440000	7.643312	1.26E+08	34597.92
0.6	180	3240000	11.46497	56060727	15376.85
0.8	2400	5760000	15.28662	31534159	8649.48
1	3000	9000000	19.10828	20181862	5535.667
1.2	3600	12960000	22.92994	14015182	3844.213
1.4	4200	17640000	26.75159	10296868	2824.32
1.6	4800	23040000	30.57325	7883540	2162.37

1.8	5400	29160000	34.3949	6228970	1708.539
2	6000	36000000	38.21656	5045465	1383.917

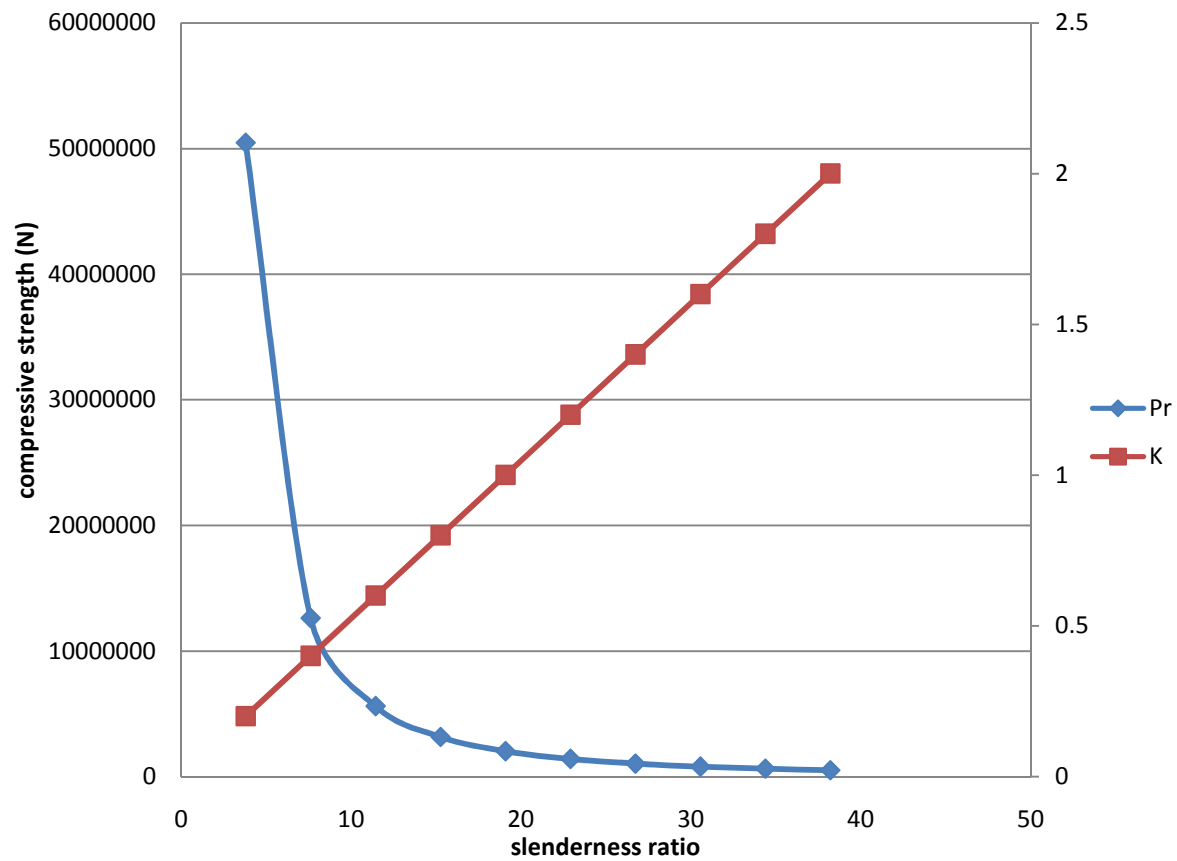


Figure 4.2: Slenderness ratio variation with respect to the compressive strength for SHS @3m

Table 4.2.A: Data for SHS @6m height (L=6000mm)

K	KL (mm)	$(KL)^2$ (mm ²)	λ (KL/r)	Pr (N)	fr (N/mm ²)
0.2	1200	1440000	7.643312	1.26E+08	34597.92
0.4	2400	5760000	15.28662	31534159	8649.48
0.6	3600	12960000	22.92994	14015182	3844.213
0.8	4800	23040000	30.57325	7883540	2162.37
1	6000	36000000	38.21656	5045465	1383.917
1.2	7200	51840000	45.85987	3503795	961.0533
1.4	8400	70560000	53.50318	2574217	706.08
1.6	9600	92160000	61.1465	1970885	540.5925
1.8	10800	116640000	68.78981	1557242	427.1348
2	12000	144000000	76.43312	1261366	345.9792

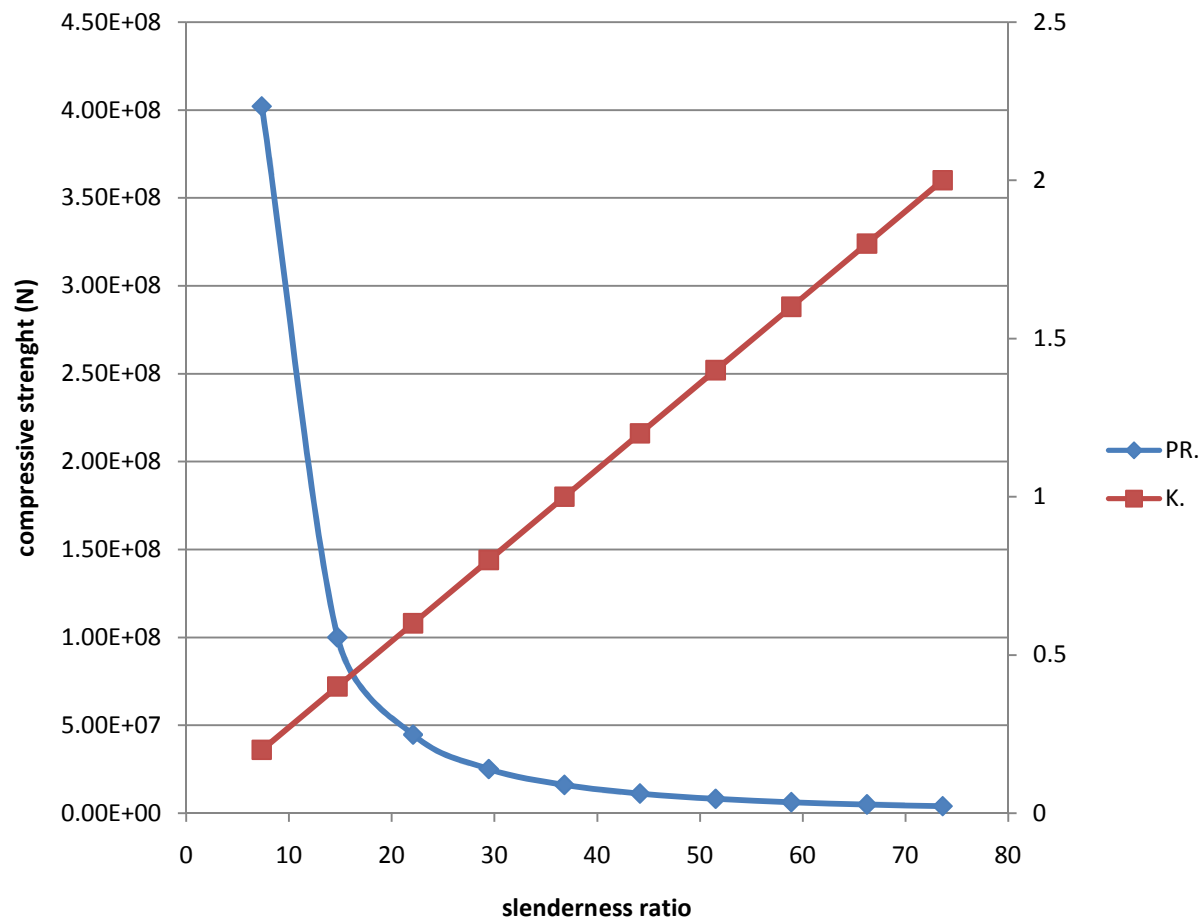


Figure 4.3: Slenderness ratio variation with respect to the compressive strength for SHS @6m

Table 4.3.A: Data for SHS @9m height (L=9000mm)

K	KL (mm)	$(KL)^2$ (mm ²)	λ (KL/r)	Pr (N)	fr (N/mm ²)
0.2	1800	3240000	11.46497	56060727	15376.85
0.4	3600	12960000	22.92994	14015182	3844.213
0.6	5400	29160000	34.3949	6228970	1708.539
0.8	7200	51840000	45.85987	3503795	961.0533
1	9000	81000000	57.32484	2242429	615.0741
1.2	10800	116640000	68.78981	1557242	427.1348
1.4	12600	158760000	80.25478	1144096	313.8133
1.6	14400	207360000	91.71975	875948.9	240.2633
1.8	16200	262440000	103.1847	692107.7	189.8377
2	18000	324000000	114.6497	560607.3	153.7685

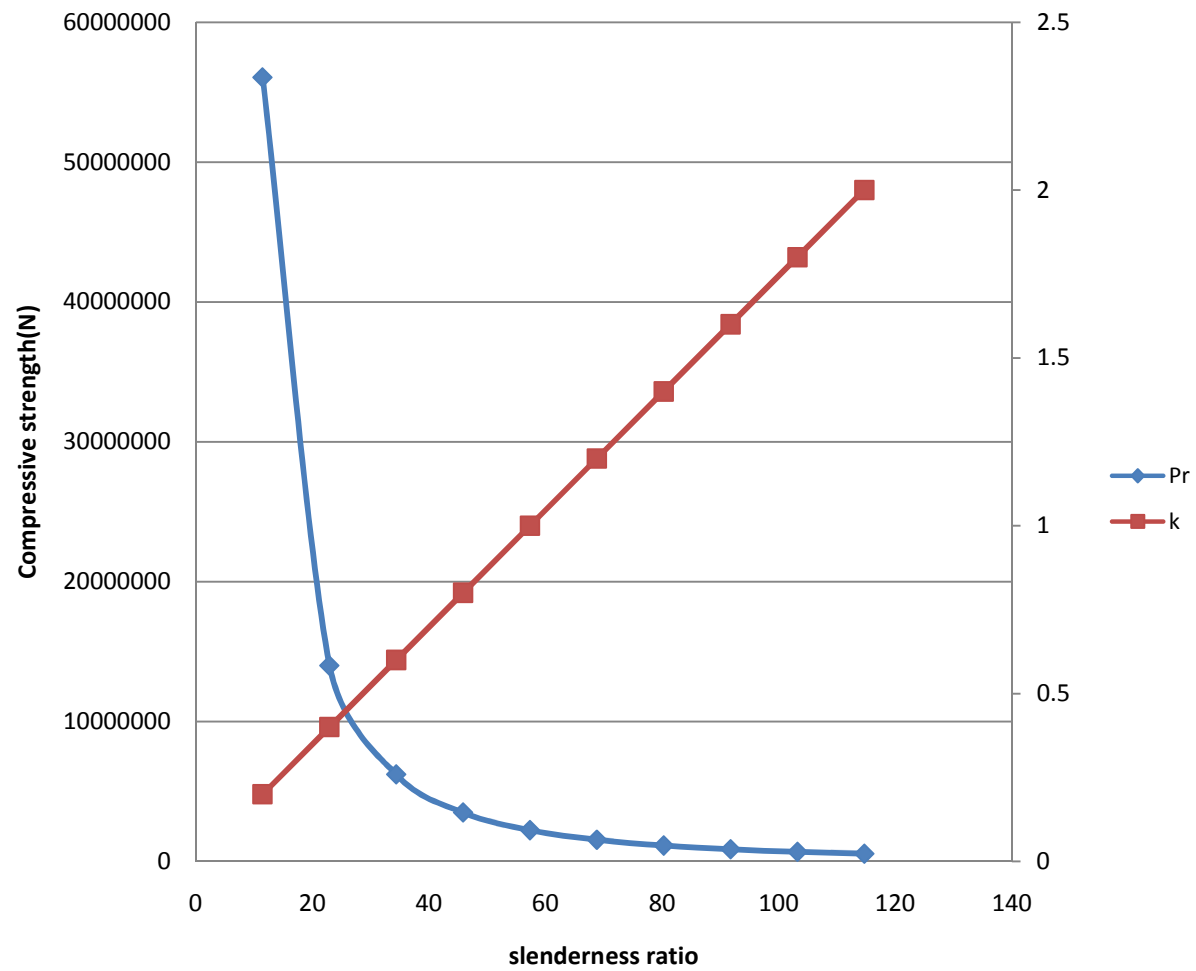


Figure 4.4: Slenderness ratio variation with respect to the compressive strength for SHS @9m

Table 4.4.A: Data for SHS @12m height (L=12000mm)

K	KL (mm)	$(KL)^2$ (mm ²)	λ (KL/r)	Pr (N)	Fr (N/mm ²)
0.2	2400	5760000	15.28662	31534159	8649.48
0.4	4800	23040000	30.57325	7883540	2162.37
0.6	7200	51840000	45.85987	3503795	961.0533
0.8	9600	92160000	61.1465	1970885	540.5925
1	12000	144000000	76.43312	1261366	345.9792
1.2	14400	207360000	91.71975	875948.9	240.2633
1.4	16800	282240000	107.0064	643554.3	176.52
1.6	19200	368640000	122.293	492721.2	135.1481
1.8	21600	466560000	137.5796	389310.6	106.7837
2	24000	576000000	152.8662	315341.6	86.4948

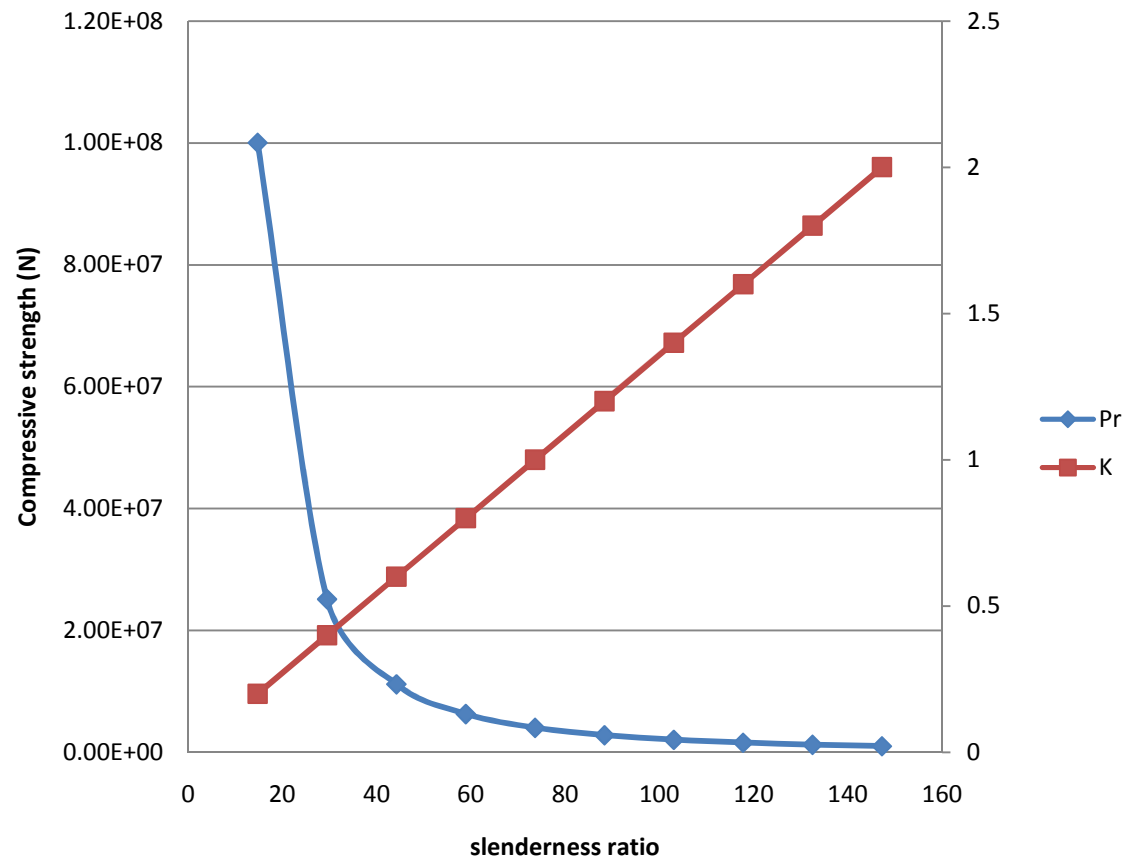


Figure 4.5: Slenderness ratio variation with respect to the compressive strength for SHS @12m

Table 4.5.A: Data for SHS @15m height (L=15000mm)

K	KL (mm)	$(KL)^2$ (mm ²)	λ (KL/r)	Pr (N)	fr (N/mm ²)
0.2	3000	9000000	19.10828	20181862	5535.667
0.4	6000	36000000	38.21656	5045465	1383.917
0.6	9000	81000000	57.32484	2242429	615.0741
0.8	12000	144000000	76.43312	1261366	345.9792
1	15000	225000000	95.5414	807274.5	221.4267
1.2	18000	324000000	114.6497	560607.3	153.7685
1.4	21000	441000000	133.758	411874.7	112.9728
1.6	24000	576000000	152.8662	315341.6	86.4948
1.8	27000	729000000	171.9745	249158.8	68.34157
2	30000	900000000	191.0828	201818.6	55.35667

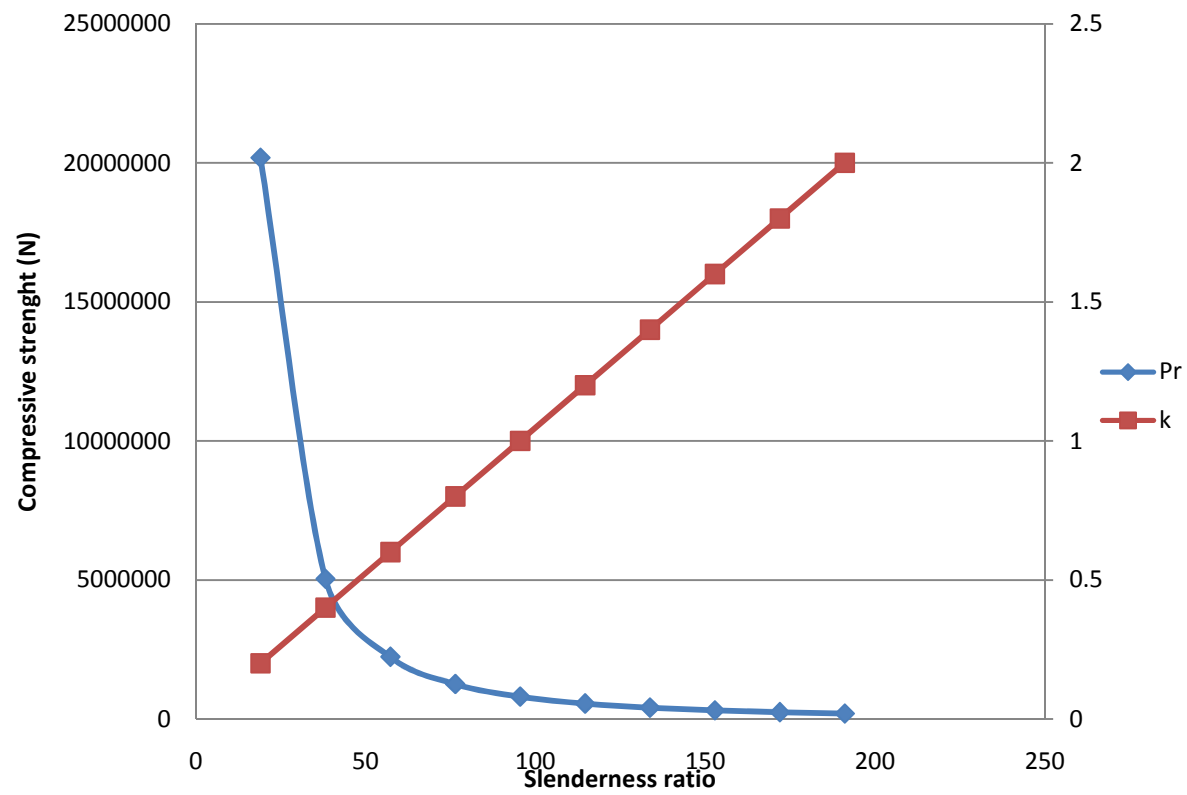


Figure 4.6: Slenderness ratio variation with respect to the compressive strength for SHS @15m

4.2.2 Analysis for CHS

A typical CHS of $508 \times 508 \times 50\text{mm}$ sections is selected for the analysis as shown in figure 4.7 below.

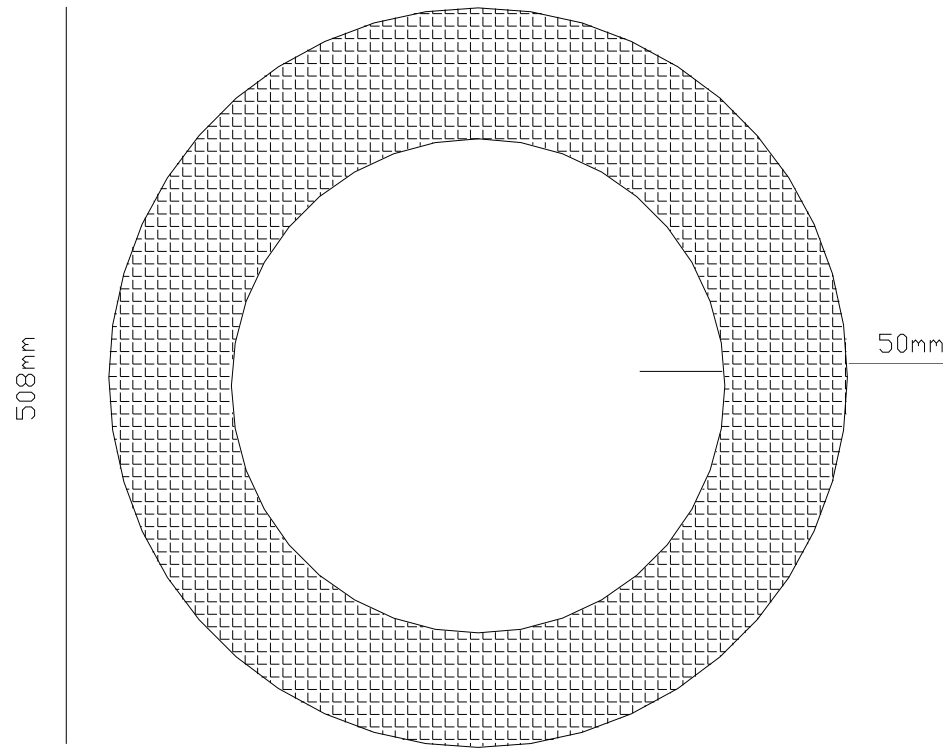


Figure 4.7 Typical Circular Hollow Sections (CHS)

Section properties

Cross sectional Area (A) = 719cm^2

Moment of inertia, $I_{xx} = I_{yy} = 190900\text{cm}^4$

Section modulus ($Z_{xx} = Z_{yy} = 7515m^3$)

Radius of gyration ($r_{xx} = r_{yy} = 16.3$)

4.2.2.1 Analysis for CHS

The figures Table 4.6B to 4.10B and 4.8 to 4.12 presented herewith represent the complete analysis as regards to CHS for the respective height within the scope of the study area.

K- Factor was evaluated at increment of 0.2 in the range of 0.2 – 2.0

using equation 3.1 and 3.2.

Table 4.6.B: Data for CHS @3m height (L=3000mm)

K	KL (mm)	$(KL)^2$ (mm ²)	λ (KL/r)	Pr (N)	fr (N/mm ²)
0.2	600	360000	3.680982	1.61E+09	149177.6
0.4	1200	1440000	7.361963	4.02E+08	37294.39
0.6	1800	3240000	11.04294	1.79E+08	16575.28
0.8	2400	5760000	14.72393	1E+08	9323.597
1	3000	9000000	18.40491	64311028	5967.102
1.2	3600	12960000	22.08589	44660436	4143.821
1.4	4200	17640000	25.76687	32811749	3044.44

1.6	4800	23040000	29.44785	25121495	2330.899
1.8	5400	29160000	33.12883	19849083	1841.698
2	6000	36000000	36.80982	16077757	1491.776

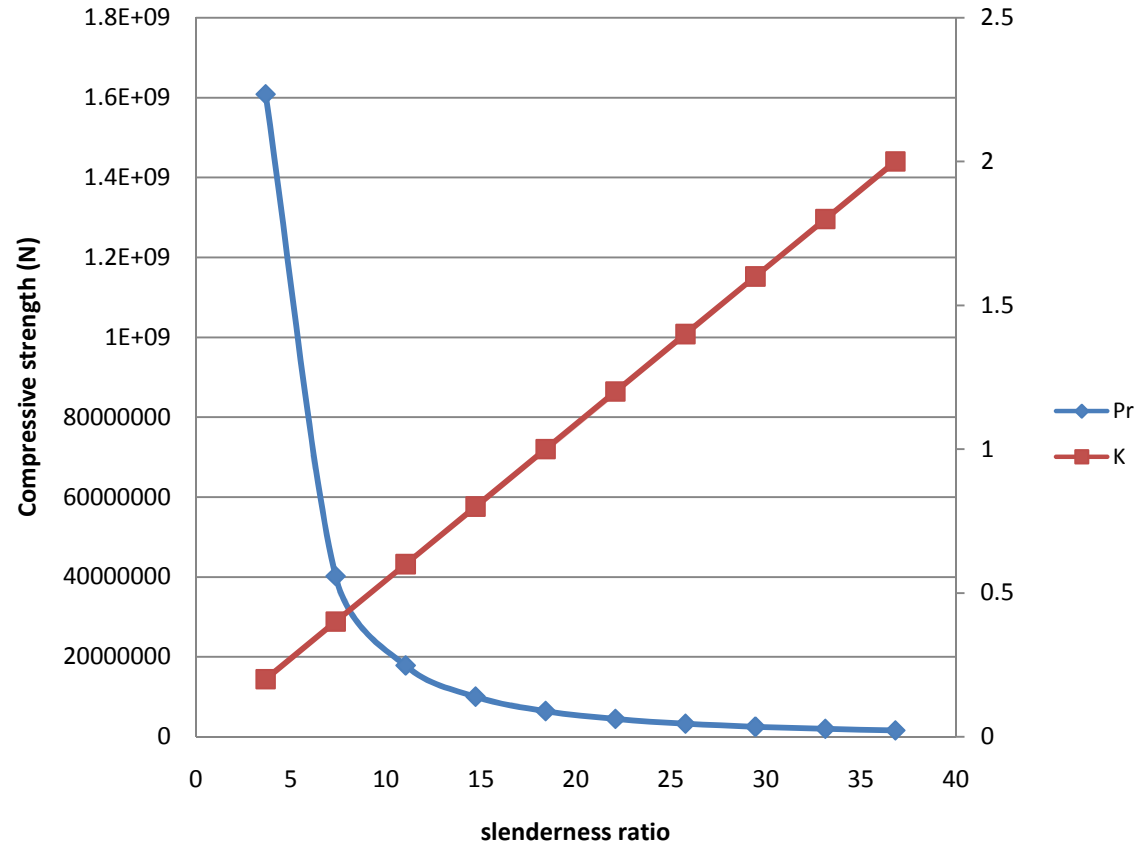


Figure 4.8: Slenderness ratio variation with respect to the compressive strength for CHS @3m

Table 4.7.B: Data for CHS @6m height (L=6000mm)

K	KL (mm)	$(KL)^2$ (mm ²)	λ (KL/r)	Pr (N)	Fr (N/mm ²)
0.2	1200	1440000	7.361963	4.02E+08	37294.39
0.4	2400	5760000	14.72393	1.00E+08	9323.597
0.6	3600	12960000	22.08589	44660436	4143.821
0.8	4800	23040000	29.44785	25121495	2330.899
1	6000	36000000	36.80982	16077757	1491.776
1.2	7200	51840000	44.17178	11165109	1035.955
1.4	8400	70560000	51.53374	8202937	761.11
1.6	9600	92160000	58.89571	6280374	582.7248
1.8	10800	116640000	66.25767	4962271	460.4246
2	12000	144000000	73.61963	4019439	372.9439

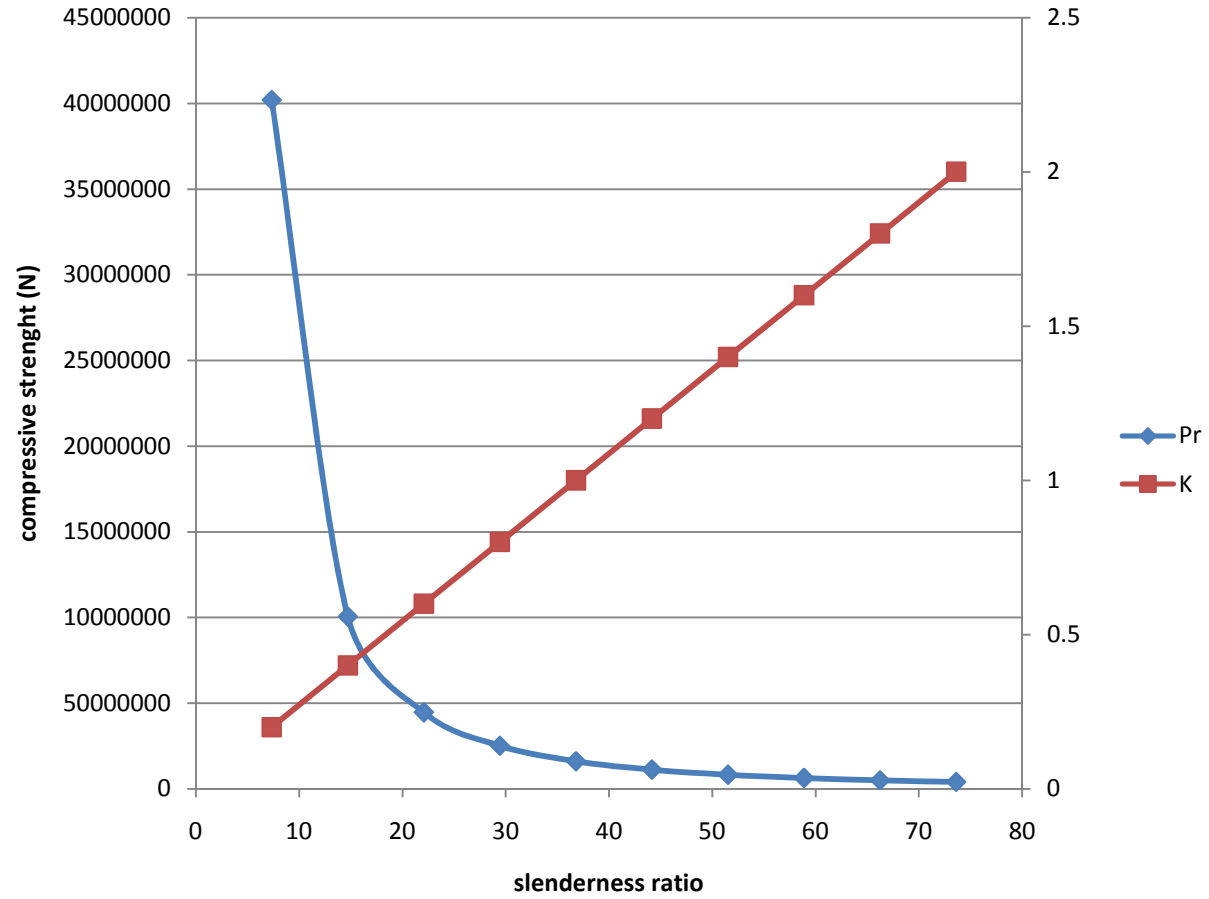


Figure 4.9: Slenderness ratio variation with respect to the compressive strength for CHS @6m

Table 4.8.B: Data for CHS @9m height (L=9000mm)

K	KL (mm)	$(KL)^2$ (mm ²)	λ (KL/r)	Pr (N)	Fr (N/mm ²)
0.2	1800	324000	11.04294	1.79E+08	16575.28
0.4	3600	1296000	22.08589	44660436	4143.821
0.6	5400	2916000	33.12883	19849083	1841.698
0.8	7200	5184000	44.17178	11165109	1035.955
1	9000	8100000	55.21472	7145670	663.0114
1.2	10800	11664000	66.25767	4962271	460.4246
1.4	12600	15876000	77.30061	3645750	338.2711
1.6	14400	20736000	88.34356	2791277	258.9888
1.8	16200	26244000	99.3865	2205454	204.6331
2	18000	32400000	110.4294	1786417	165.7528

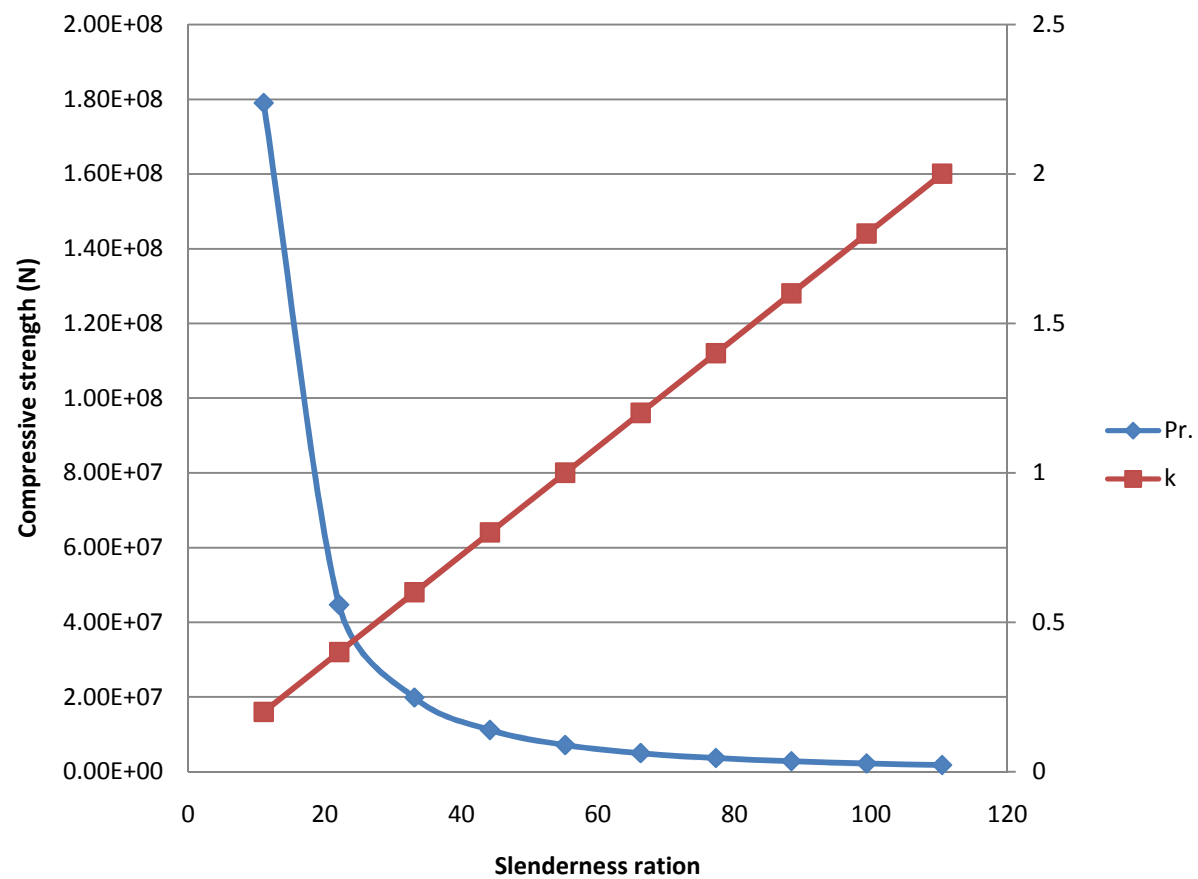


Figure 4.10: Slenderness ratio variation with respect to the compressive strength for CHS @9m

Table 4.9.B: Data for CHS @12m height (L=12000mm)

K	KL (mm)	$(KL)^2$ (mm ²)	λ (KL/r)	Pr (N)	Fr (N/mm ²)
0.2	2400	5760000	14.72393	1.00E+08	9323.597
0.4	4800	23040000	29.44785	25121495	2330.899
0.6	7200	51840000	44.17178	11165109	1035.955
0.8	9600	92160000	58.89571	6280374	582.7248
1	12000	144000000	73.61963	4019439	372.9439
1.2	14400	207360000	88.34356	2791277	258.9888
1.4	16800	282240000	103.0675	2050734	190.2775
1.6	19200	368640000	117.7914	1570093	145.6812
1.8	21600	466560000	132.5153	1240568	115.1061
2	24000	576000000	147.2393	1004860	93.23597

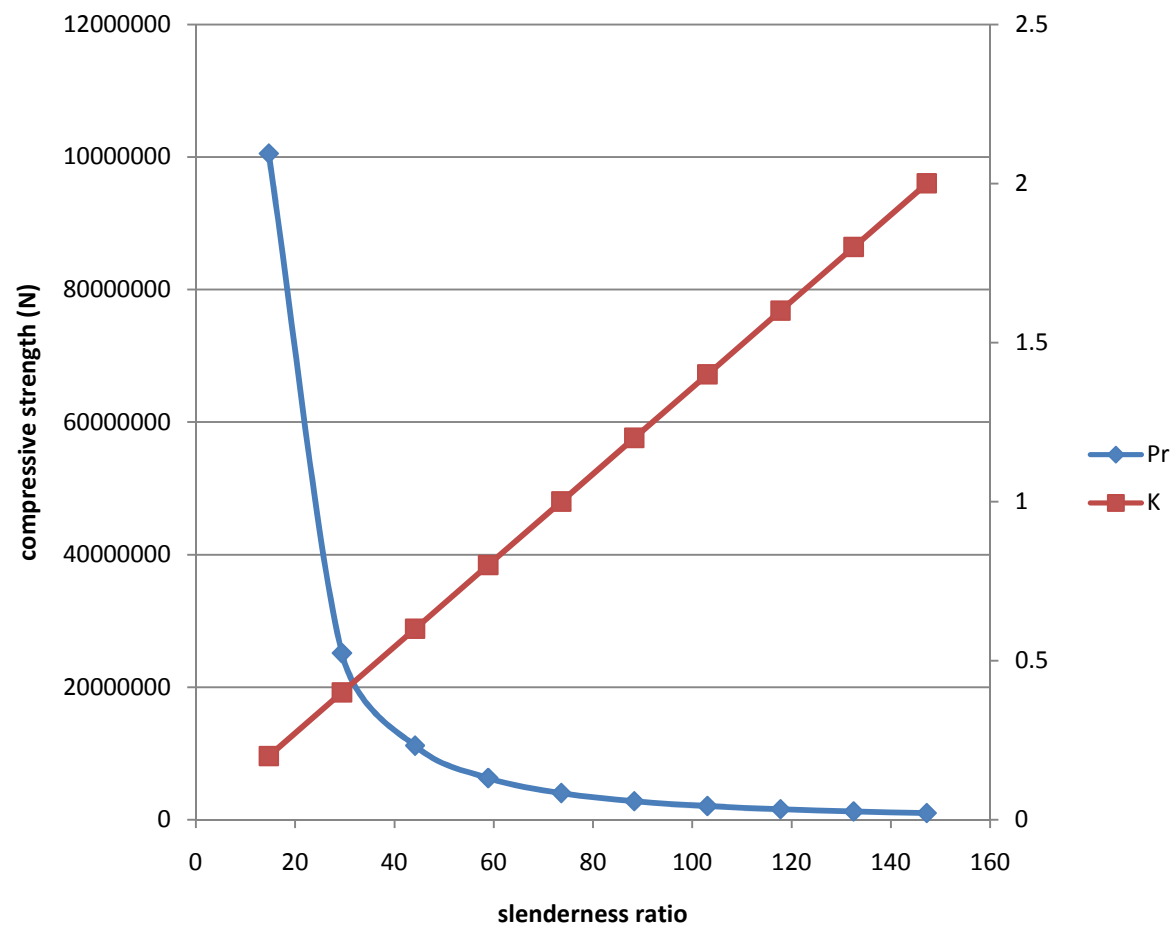


Figure 4.11: Slenderness ratio variation with respect to the compressive strength for CHS @12m

Table 4.10.B: Data for CHS @15m height (L=15000mm)

K	KL (mm)	$(KL)^2$ (mm ²)	λ (KL/r)	Pr (N)	Fr (N/mm ²)
0.2	3000	9000000	18.40491	64311028	5967.102
0.4	6000	36000000	36.80982	16077757	1491.776
0.6	9000	81000000	55.21472	7145670	663.0114
0.8	12000	144000000	73.61963	4019439	372.9439
1	15000	225000000	92.02454	2572441	238.6841
1.2	18000	324000000	110.4294	1786417	165.7528
1.4	21000	441000000	128.8344	1312470	121.7776
1.6	24000	576000000	147.2393	1004860	93.23597
1.8	27000	729000000	165.6442	793963.3	73.66793
2	30000	900000000	184.0491	643110.3	59.67102

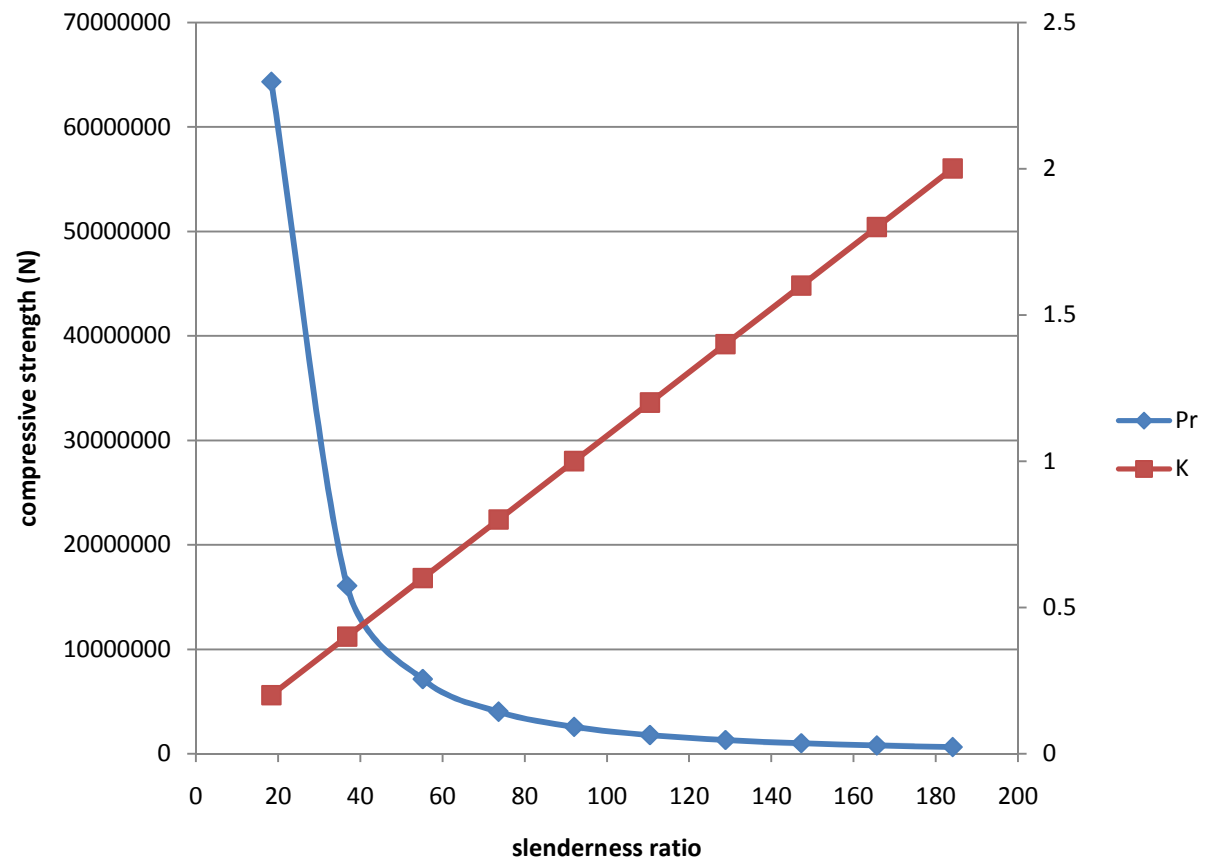


Figure 4.12: Slenderness ratio variation with respect to the compressive strength for CHS @15m

4.2.3 Analysis for RHS

A typical RHS of $500 \times 300 \times 16\text{mm}$ sections is selected for the analysis as shown in figure 4.13 below.

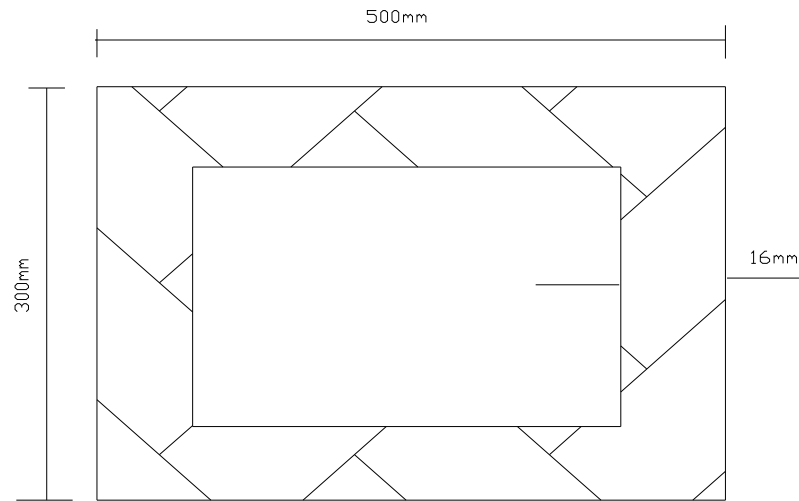


Figure 4.13 typical Rectangular Hollow Section (RHS)

Section properties

Cross sectional Area (A) = 245cm^2

Moment of inertia, $I_{xx} = 82670\text{cm}^4$, $I_{yy} = 37080\text{cm}^4$

Section modulus $Z_{xx} = 3307m^3$, $Z_{yy} = 2472m^3$

Radius of gyration $r_{xx} = 18.4$, $r_{yy} = 12.3$

4.2.2.1 Analysis for RHS

The Figures Table 4.11C to 4.15C and 4.14 to 4.18 presented herewith represent the complete analysis as regards to CHS for the respective height within the scope of the study area.

K- Factor was evaluated at increment of 0.2 in the range of 0.2 – 2.0 using equations 3.1 and 3.2.

Table 4.11.C: Data forRHS @3m height (L=3000mm)

K	KL (mm)	$(KL)^2$ (mm ²)	$\lambda_x(KL/r_x)$	$\lambda_y(KL/r_y)$	Pr (N)	Fr (N/mm ²)
0.2	600	360000	3.26087	4.878049	6.96E+08	619840.2
0.4	1200	1440000	6.521739	9.756098	1.74E+08	309920.1
0.6	1800	3240000	9.782609	14.63415	77358376	206613.4
0.8	2400	5760000	13.04348	19.5122	43514086	154960
1	3000	9000000	16.30435	24.39024	27849015	123968
1.2	3600	12960000	19.56522	29.26829	19339594	103306.7
1.4	4200	17640000	22.82609	34.14634	14208681	88548.6

1.6	4800	23040000	26.08696	39.02439	10878522	77480.02
1.8	5400	29160000	29.34783	43.90244	8595375	68871.13
2	6000	36000000	32.6087	48.78049	6962254	61984.02

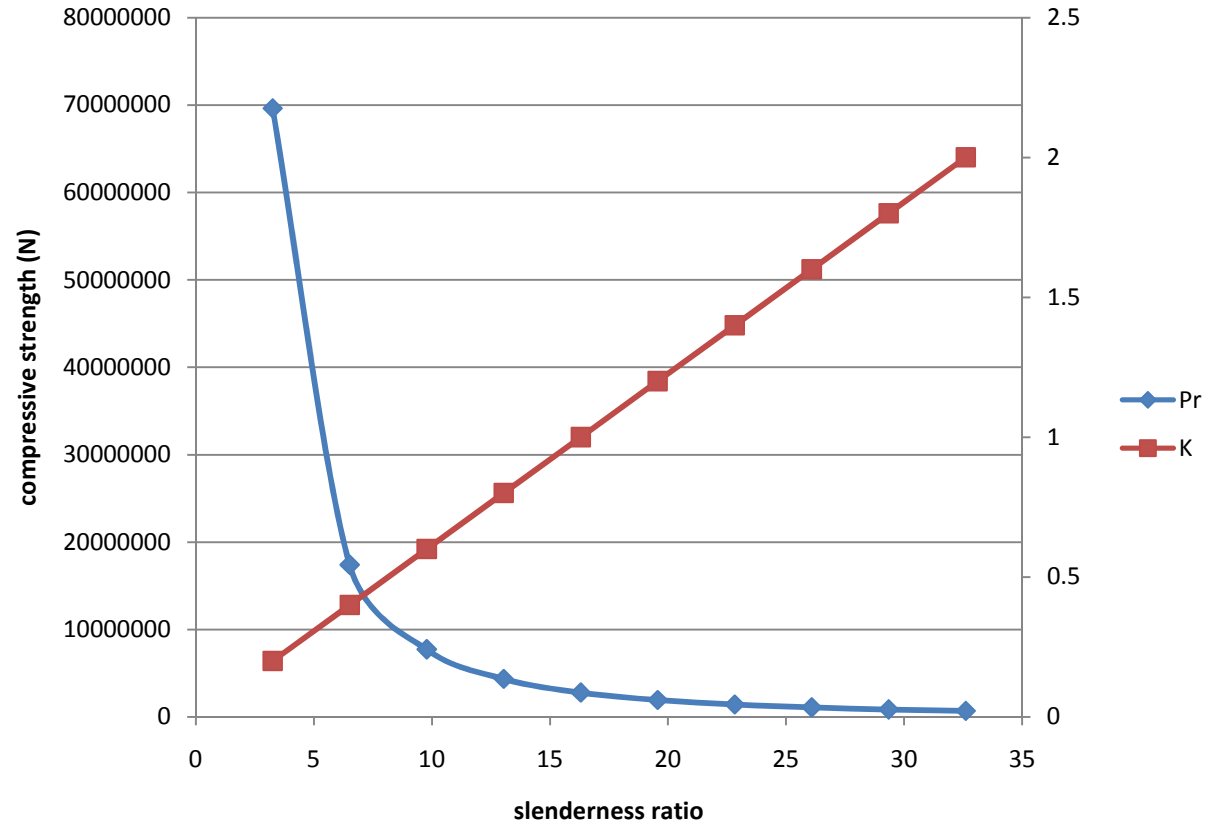


Figure 4.14: Slenderness ratio variation with respect to the compressive strength for RHS @3m

Table 4.12.C: Data for RHS @6m height (L=6000mm)

K	KL (mm)	$(KL)^2$ (mm ²)	$\lambda_x(KL/r_x)$	$\lambda_y(KL/r_y)$	Pr (N)	Fr (N/mm ²)
0.2	1200	1440000	6.521739	9.756098	1.74E+08	309920.1
0.4	2400	5760000	13.04348	19.5122	43514086	154960
0.6	3600	12960000	19.56522	29.26829	19339594	103306.7
0.8	4800	23040000	26.08696	39.02439	10878522	77480.02
1	6000	36000000	32.6087	48.78049	6962254	61984.02
1.2	7200	51840000	39.13043	58.53659	4834898	51653.35
1.4	8400	70560000	45.65217	68.29268	3552170	44274.3
1.6	9600	92160000	52.17391	78.04878	2719630	38740.01
1.8	10800	116640000	58.69565	87.80488	2148844	34435.57
2	12000	144000000	65.21739	97.56098	1740563	30992.01

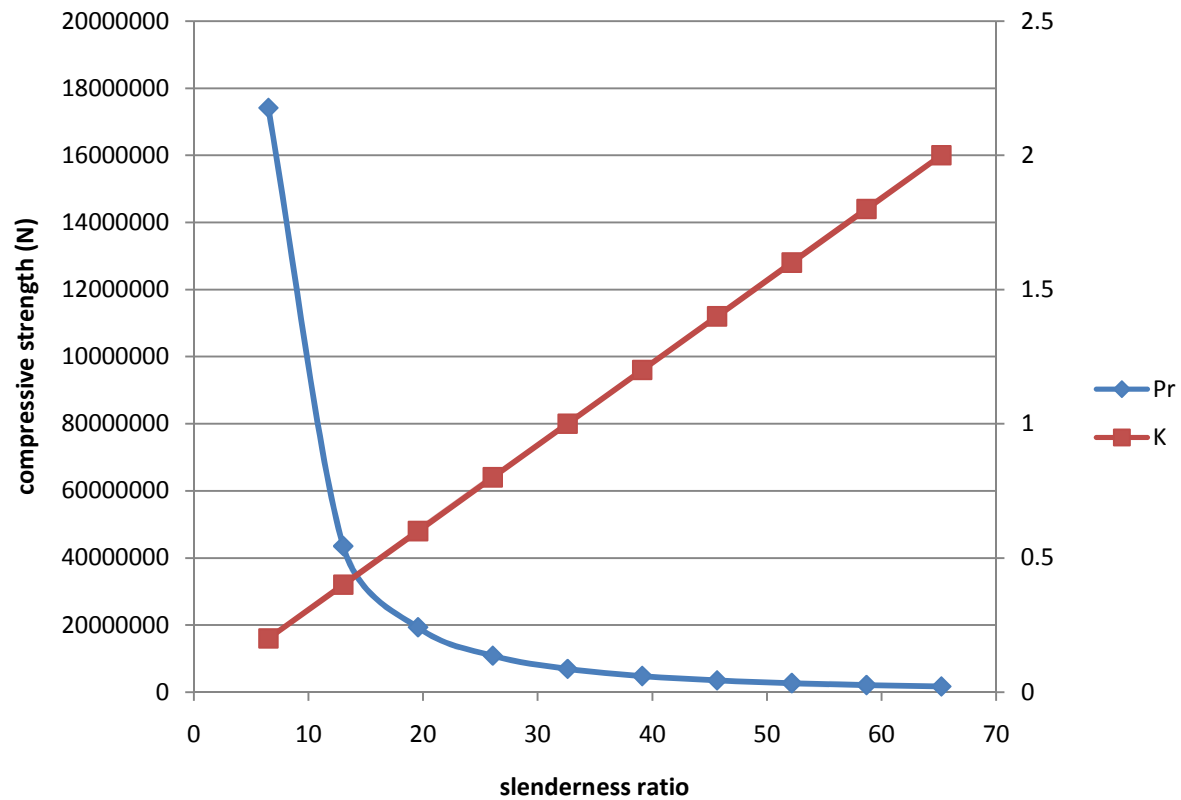


Figure 4.15: Slenderness ratio variation with respect to the compressive strength for RHS @6m

Table 4.13.C: Data for RHS @9m height (L=9000mm)

K	KL (mm)	$(KL)^2$ (mm ²)	$\lambda_x(KL/r_x)$	$\lambda_y(KL/r_y)$	Pr (N)	Fr (N/mm ²)
0.2	1800	324000	9.782609	14.63415	77358376	206613.4
0.4	3600	1296000	19.56522	29.26829	19339594	103306.7
0.6	5400	2916000	29.34783	43.90244	8595375	68871.13
0.8	7200	5184000	39.13043	58.53659	4834898	51653.35
1	9000	8100000	48.91304	73.17073	3094335	41322.68
1.2	10800	11664000	58.69565	87.80488	2148844	34435.57
1.4	12600	15876000	68.47826	102.439	1578742	29516.2
1.6	14400	20736000	78.26087	117.0732	1208725	25826.67
1.8	16200	26244000	88.04348	131.7073	955041.7	22957.04
2	18000	32400000	97.82609	146.3415	773583.8	20661.34

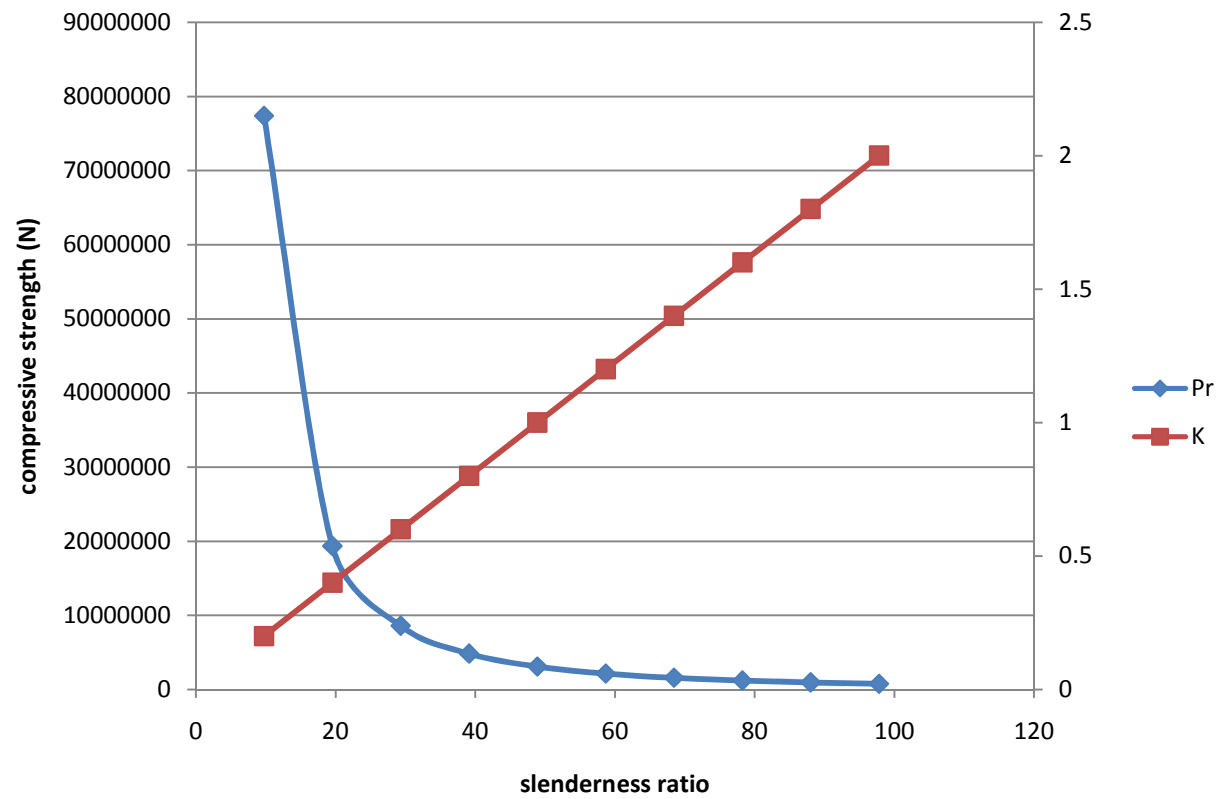


Figure 4.16: Slenderness ratio variation with respect to the compressive strength for RHS @9m

Table 4.14.C: Data for RHS @12m height (L=12000mm)

K	KL (mm)	$(KL)^2$ (mm ²)	$\lambda_x(KL/r_x)$	$\lambda_y(KL/r_y)$	Pr (N)	Fr (N/mm ²)
0.2	2400	5760000	13.04348	19.5122	43514086	154960
0.4	4800	23040000	26.08696	39.02439	10878522	77480.02
0.6	7200	51840000	39.13043	58.53659	4834898	51653.35
0.8	9600	92160000	52.17391	78.04878	2719630	38740.01
1	12000	144000000	65.21739	97.56098	1740563	30992.01
1.2	14400	207360000	78.26087	117.0732	1208725	25826.67
1.4	16800	282240000	91.30435	136.5854	888042.6	22137.15
1.6	19200	368640000	104.3478	156.0976	679907.6	19370.01
1.8	21600	466560000	117.3913	175.6098	537210.9	17217.78
2	24000	576000000	130.4348	195.122	435140.9	15496

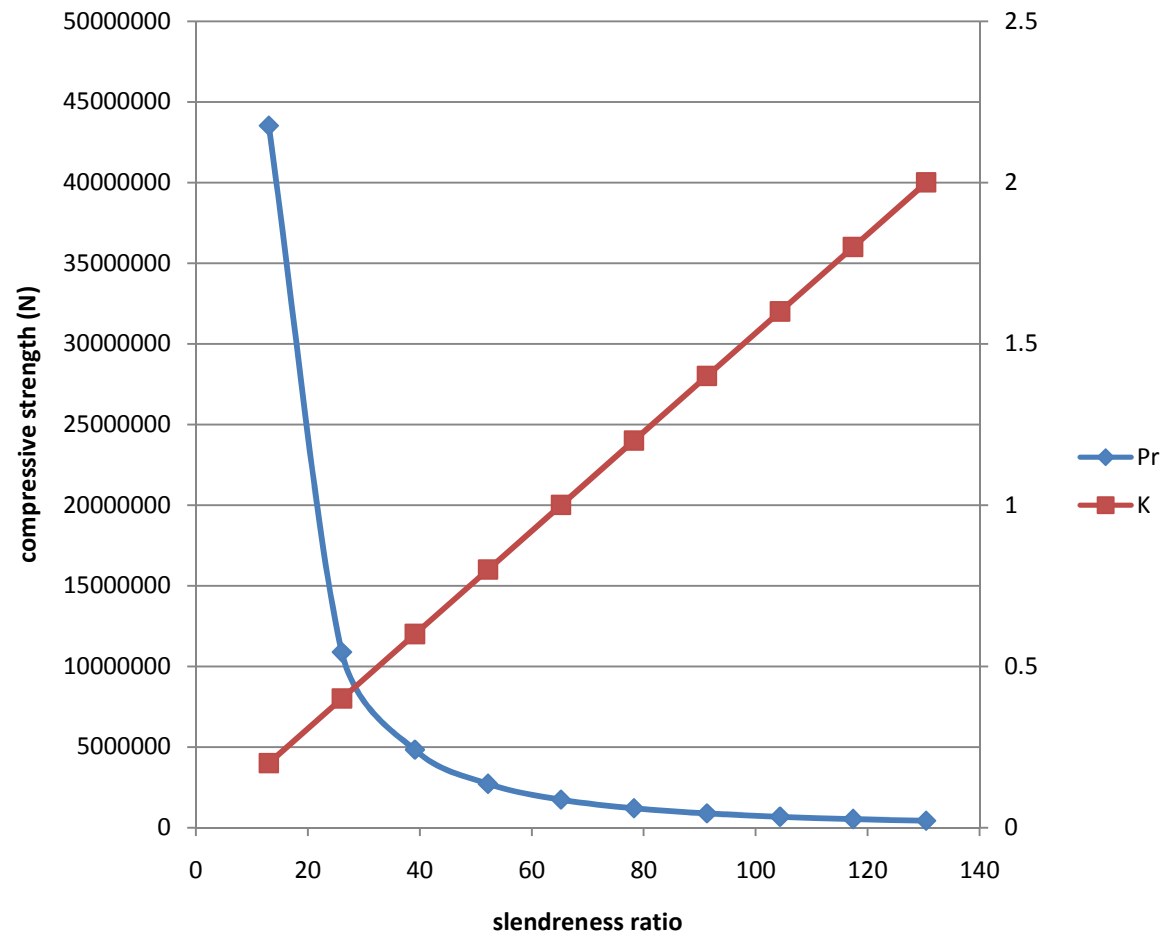


Figure 4.17: Slenderness ratio variation with respect to the compressive strength for RHS @12m

Table 4.15.C: Data for RHS @15m height (L=15000mm)

K	KL (mm)	$(KL)^2$ (mm ²)	$\lambda_x(KL/r_x)$	$\lambda_y(KL/r_y)$	Pr (N)	Fr (N/mm ²)
0.2	3000	9000000	16.30435	24.39024	27849015	123968
0.4	6000	36000000	32.6087	48.78049	6962254	61984.02
0.6	9000	81000000	48.91304	73.17073	3094335	41322.68
0.8	12000	144000000	65.21739	97.56098	1740563	30992.01
1	15000	225000000	81.52174	121.9512	1113961	24793.61
1.2	18000	324000000	97.82609	146.3415	773583.8	20661.34
1.4	21000	441000000	114.1304	170.7317	568347.3	17709.72
1.6	24000	576000000	130.4348	195.122	435140.9	15496
1.8	27000	729000000	146.7391	219.5122	343815	13774.23
2	30000	900000000	163.0435	243.9024	278490.2	12396.8

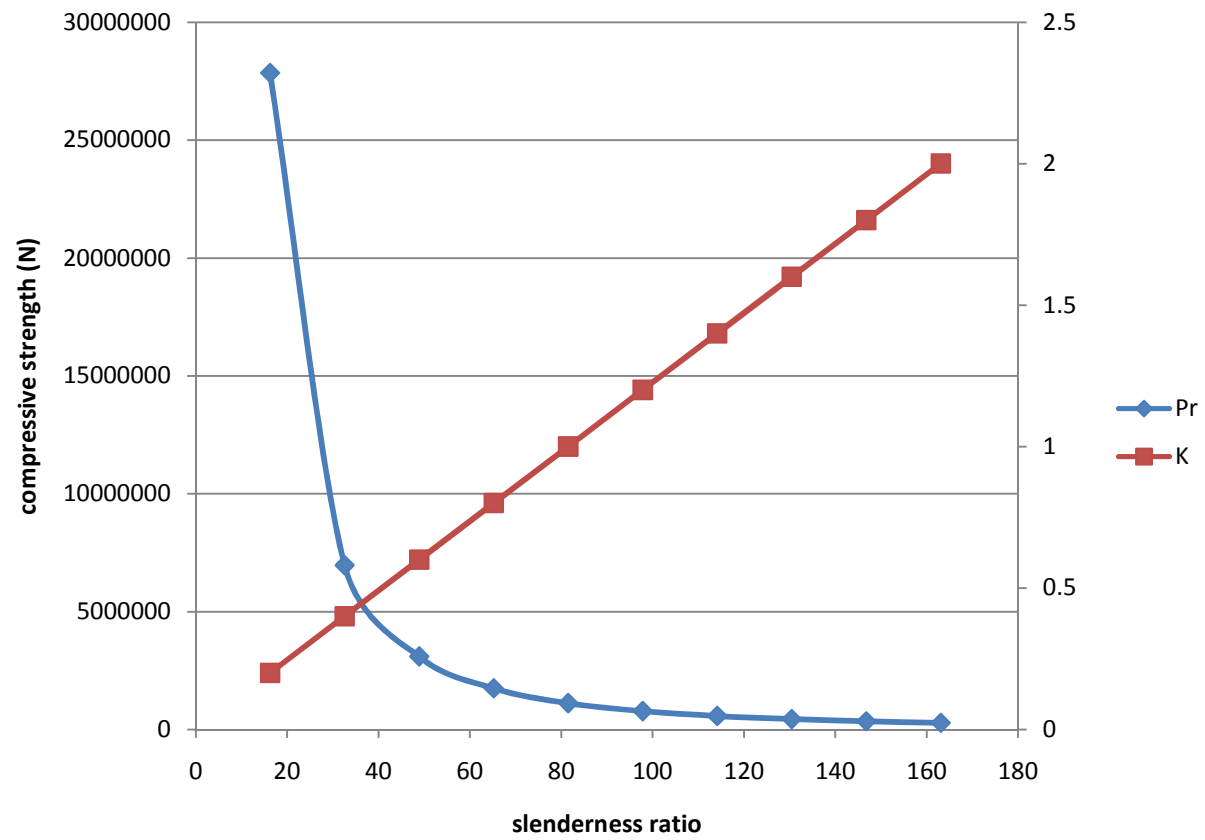


Figure 4.18: Slenderness ratio variation with respect to the compressive strength for RHS @15m

Table 4.16D: Buckling loads for 3mmØ rod

SIZE Ø (Diam) (a)	LENGTH (cm) (b)	SELF Wt (kg) (c)	SCALE READING (kg) on buckling (d)	BUCKLING LOAD (kg) (e) (d) – (c)	K-VALUE EXPERIMENTAL
3mm	36cm	0.017kg	2.28kg	2.26kg	1.96
3mm	34.5cm	0.0165kg	2.41kg	2.39kg	1.98
3mm	33cm	0.016kg	2.53kg	2.51kg	2.09
3mm	31.5cm	0.0155kg	2.94kg	2.92kg	2.10
3mm	30cm	0.015kg	3.34kg	3.33kg	1.94
3mm	28.5cm	0.0145kg	3.49kg	3.48kg	2.03
3mm	27cm	0.014kg	3.64kg	3.63kg	1.96
3mm	25.5cm	0.0135kg	4.23kg	4.22kg	2.12
3mm	24cm	0.013kg	4.81kg	4.80kg	2.02
3mm	22.5m	0.0125kg	5.39kg	5.38kg	1.86

Average K = 2.01

Table 4.17D: Buckling loads for 4mmØ rod

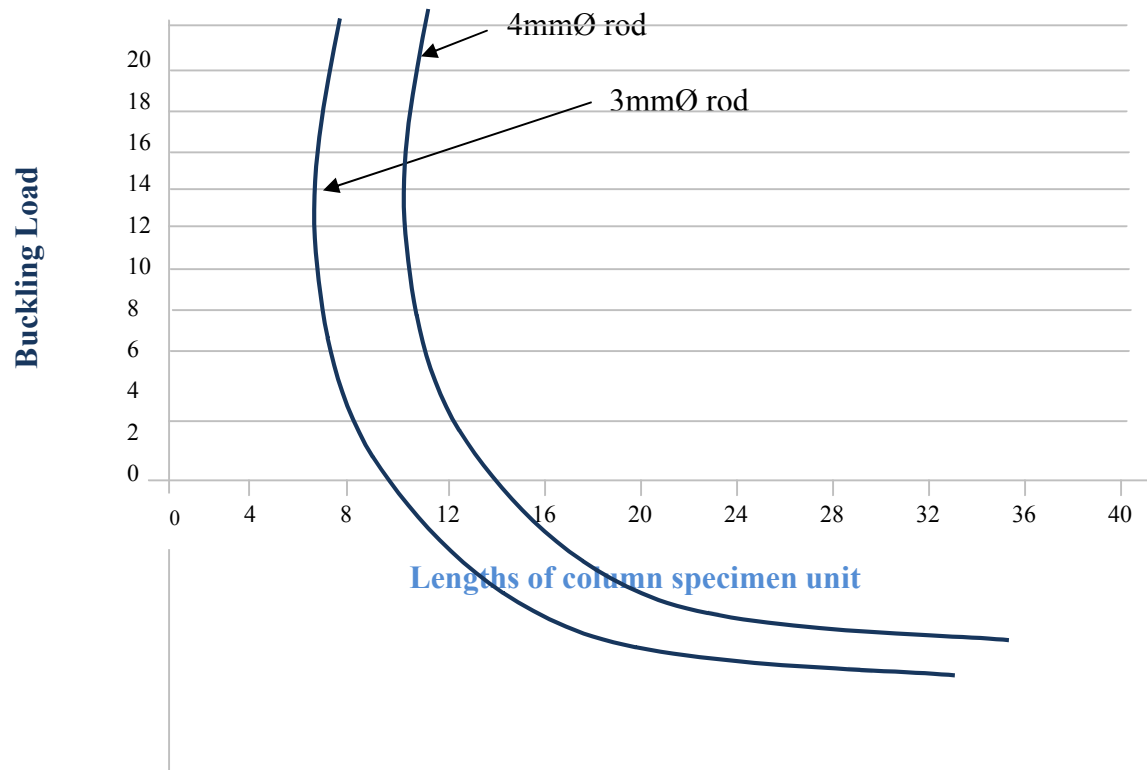
SIZE Ø (Diam) (a)	LENGTH (cm) (b)	SELF Wt (kg) (c)	SCALE READING (kg) (d)	BUCKLING LOAD (kg) (e) (d) – (c)	K-VALUE EXPERIMENTAL
4mm	36cm	0.023kg	6.65kg	6.65kg	2.03
4mm	34.5cm	0.022kg	7.44kg	7.42kg	2.01
4mm	33cm	0.021kg	8.23kg	8.21kg	1.99
4mm	31.5cm	0.020kg	9.78kg	9.76kg	1.98
4mm	30cm	0.019kg	11.32kg	11.31kg	1.86
4mm	28.5cm	0.018kg	11.87kg	11.85kg	2.15
4mm	27cm	0.017kg	12.23kg	12.23kg	1.99
4mm	25.5cm	0.016kg	13.28kg	13.26kg	2.02
4mm	24cm	0.016kg	14.32kg	14.32kg	2.07
4mm	22.5cm	0.014kg	15.37kg	15.36kg	2.06

Average K = 2.02

A factor of 0.7 was used for buckling loads to cater for experimental errors. $K = \sqrt{\frac{\pi^2 EI}{P_c L^2}}$

Fig. 4.19

This shown in the graph of buckling load versus length of column



4.3 Analysis of Result

The results are obtained as shown from the Tables 4.1A to 4.5A, Figures 4.2 to 4.6; Table 4.6B to 4.10B, Figures 4.8 to 4.12, and Table 4.11C to 4.15C, Figure 4.14 to 4.18 for SHS, CHS and RHS respectively. These graphs show a plot of axial carrying capacities against the slenderness ratios of the respective sections under different parametric condition of height and K-factors. The result shows that, from Figure 4.2 to 4.6 for SHS ($400 \times 400 \times 16\text{mm}$), Figure 4.8 to 4.12 for CHS ($508 \times 508 \times 50\text{mm}$) and Figure 4.14 to 4.18 for RHS ($500 \times 300 \times 16\text{mm}$), the load carrying capacities of the section under considerations decreases as the storey height increases, which also result in increasing in slenderness ratio as the height increases. Further more it is important to note the code specification of 180 as the maximum required for member resisting load other wind have not been violated for the section under considerations. The limit slenderness ratio for RHS at 15.0m was found to be 165 and the corresponding effective length factor was 2.2 Fig. 4.18. The results for the recalculation of axial compressive strength using Euler's formula and that of experiments performed on column samples are summarized below:

1. The limit slenderness ratio λ for Rectangular Hollow Section RHS at 15.0m was found to be 165. Table 4.15 and Figure 4.18 the corresponding optimum effective length factor k was 2.2.
2. Throughout the derivation process of critical buckling load of a given column by Euler, column, no mention was made about the effective length factor.
3. The general solution of the critical buckling load as derived by Euler is

$$P_{cr} = \frac{n^2 \pi^2 EI}{L^2} \quad \text{for } \left[n = 1, 2, 3, \dots \right]$$

so $\frac{n^2 EI}{L^2}$, $\frac{4\pi^2 EI}{L^2}$ } are all solution for pin ended column.

4. Experimental result shown in Table 4.16D & 4.17D and Figure 4.19 agree with the mathematical relationship between the axial buckling load and the lengths earlier derived by Euler i.e $P \propto \frac{1}{L}$
5. The effective length factor k determined from experimental result using Euler's formula on the averages is 2.0.

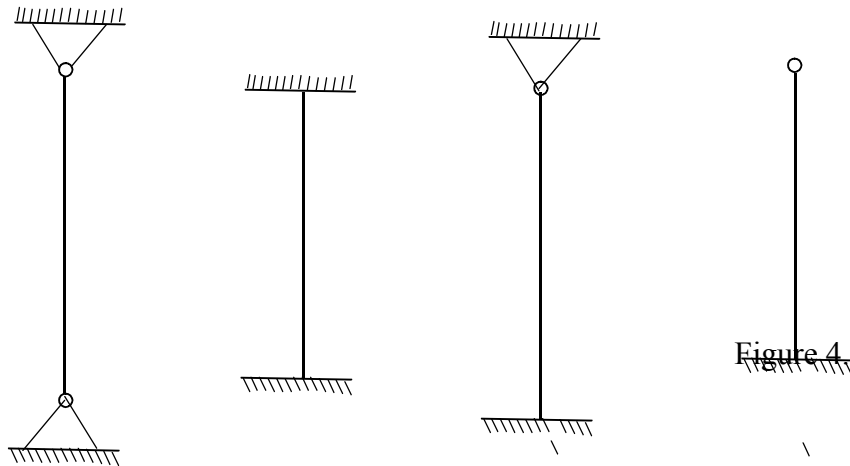


Figure 4.20

K (RESEARCH)	2.0	1.63	1.63	5.0
K (THEORETICAL)	1.0	0.5	0.7	2.0
K – DESIGN	0.8	0.65	0.8	2.0

CHAPTER FIVE:

CONCLUSION AND RECOMMENDATION

1. The findings indicate that the load carrying capacity decreases as the height increases. Furthermore it is important to note that the code specification of 180 as the maximum slenderness ratio required for member resisting load other than wind have not been violated for the section under consideration.
2. For the given sections and the actual height, the axial load carrying capacities decrease as the slenderness ratio increase due to increasing value of effective length k-factor.
3. The code specification of 180 as the maximum slenderness ratio required for member resisting load other than wind have not been violated for the section under considerations.

RECOMMENDATION

1. There is need to review the effective length factors (k) used as strength reducing factor.
2. Further research into the influence of effective length factor with respect to various grades as obtained in BS5950 would be of great interest, so as to find out whether the strength variation would further spelt another findings into possible reduction in the value of k.
3. The recommended K factors for various and condition from pin ended column to one end fixed and the other free is as shown in Table 4.18.

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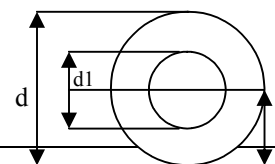
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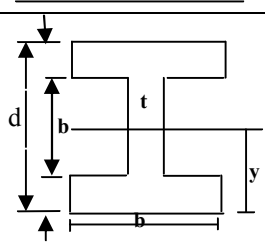
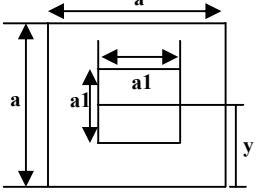
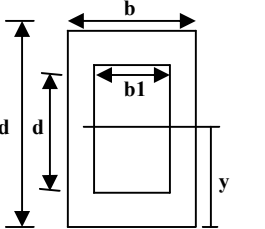
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MOMENT OF INERTIA AND SECTION PROPERTIES

Appendix i

SECTION	AREA A	DISTANCE FROM AXIS TO EXTREME FIBRES Y AND Y ¹	MOMENT OF INERTIA I	SECTION MODULUS Z = I/s	RADIUS OF GYRATION $R = \sqrt{\frac{I}{A}}$
	$\frac{n(d^2 - d_1^2)}{4}$ $= 0.785(d^2 - d_1^2)$	$Y = \frac{d}{2}$	$\frac{n(d^4 - d_1^4)}{4}$ $= 0.49(d^4 - d_1^4)$	$\frac{n(d^4 - d_1^4)}{4d}$ $= \frac{0.098(d^4 - d_1^4)}{d}$	$\sqrt{\frac{d^2 - d_1^2}{4}}$

	$b \cdot d - h(b - t)$	$Y = \frac{d}{2}$	$\frac{b \cdot d^3 - h(b - t)}{12}$	$\frac{b \cdot d^3 - h^3(b - t)}{6d}$	$\sqrt{\frac{b \cdot d^3 - h^3(b - t)}{12(b \cdot d - h(b - t))}}$
	$a^2 - a_1^2$	$Y = \frac{a}{2}$	$\frac{a^4 - a_1^4}{12}$	$\frac{a^4 - a_1^4}{6a}$	$\sqrt{\frac{a^2 - a_1^2}{12}}$
	$b \cdot d - b_1 d_1$	$Y = \frac{d}{2}$	$\frac{b \cdot d^3 - b_1 d_1^3}{12}$	$\frac{b \cdot d^3 - b_1 d_1^3}{6d}$	$\sqrt{\frac{b \cdot d^3 - b_1 d_1^3}{12(b \cdot d - b_1 d_1)}}$



DETERMINATION OF BUCKLING LOAD FOR COLUMN SPECIMENS



DETERMINATION OF BUCKLING LOAD FOR COLUMN SPECIMENS

Appendix iii



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DETERMINATION OF BUCKLING LOAD FOR COLUMN SPECIMENS



DETERMINATION OF BUCKLING LOAD FOR COLUMN SPECIMENS