ANALYTICAL STUDY ON THERMO-SOLUTAL NATURAL CONVECTION FLOW IN AN INCLINED CHANNEL FILLED WITH POROUS MATERIAL

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A DISSERTATION SUBMITTED TO THE SCHOOL OF POSTGRADUATE STUDIES AHMADU BELLO UNIVERSITY, ZARIA, IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE AWARD OF MASTER OF SCIENCE DEGREE IN

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DEPARTMENT OF MATHEMATICS,

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Declaration

I declare that the work in this dissertation titled “Analytical Study on Thermo-Solutal Natural Convection Flow in an Inclined Channel Filled with Porous Material” has been carried out by me in the Department of Mathematics under the supervision of Prof. A.O. Ajibade and Dr. H.M Jibril. The information derived from the literature has been duly acknowledged in the text and a list of references provided. No part of this dissertation was previously presented for another degree or diploma at this or any other Institution.

Sanusi ABDULLAHI _____ _____

(Name of Student) Signature Date

Certification

This dissertation titled “Analytical Study on Thermo-Solutal Natural Convection Flow in an Inclined Channel Filled with Porous Material” by Sanusi ABDULLAHI meets the regulations governing the award of the degree of Master of Science in Mathematics of the Ahmadu Bello University, and is approved for its contribution to knowledge and literary presentation.

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Dedication

This dissertation is dedicated to Almighty Allah, through whom all things were made possible. Also to my beloved parents Abdullahi and Amina, my lovely aunty Mrs. Fatima Mohammed whose ministries made me to discover myself, hence this dissertation.

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Abstract

This dissertation presents an analytical study on thermo-solutal natural convection flow in an inclined channel filled with porous material. Two problems are considered in this work. In both problems, the flow is assumed to be hydrodynamically and thermally fully developed and the mathematical equations governing the flow were formulated. The first problem focused on Soret effect on steady natural convection of double-diffusive flow of heat generating fluid. While the second problem examines the transpiration and Soret effects on thermo-solutal convection flow of heat generating fluid in an inclined porous channel filled with porous material. Using appropriate transformations, the differential equations governing the flow were transformed and solved analytically to obtain an expression for velocity, concentration, temperature, skin friction, Nusselt number, Sherwood number as well as volumetric flow rate. The effects of various governing parameters on fluid velocity, concentration, temperature, skin friction, Nusselt number, local Sherwood number and volumetric flow rate are shown in graphs and tables and analyzed in detail. During the course of this investigation, it is found that Soret effect, angle of inclination and viscosity of the porous material are very effective in controlling the hydrodynamics within the channel.Furthermore, there is a rise in the skin friction on both plates due to increase in heat generation parameter δ, permeability parameter Da and angle of inclination a while a fall is observed in skin friction on both plates due to increase in Prandtl number Pr, Soret number So and Schmidt number Sc.

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Nomenclatures and Greek Letters

a : angle of inclination

C_0 : ambient concentration

C : concentration

C_w : concentration at the wall

C_p : specific heat constant pressure [$m^2 s^{-2} K^{-1}$]

D_m : mass diffusion parameter [$m^2 s^{-1}$]

Da : Darcy number

Pr : Prandtl number

g: acceleration due to gravity [ms^{-2}]

h : characteristic length [m]

k : thermal conductivity [W/mK]

K : permeability of porous medium [m^2]

N : sustentation parameter

q : heat flux

Q : non-dimensional mass flux

Q_0 : coefficient of heat generation [$kgm^{-1}s^{-3}K^{-1}$]

S^* : dimensional Soret effect

Sc : Schmidt number

So : Soret number

T : fluid temperature [K]

T_0 : ambient temperature [K]

T_w : temperature at the wall [K]

u : velocity of the fluid along the x' -direction in non-dimensional form

u' : velocity of the fluid along the x' -direction [ms^{-1}]

$V'o$: suction velocity[ms^{-1}]

Vo : suction/injection parameter

Nu_0, Nu_1 : Nusselt number at $y = 0$ and $y = 1$ respectively

y : non-dimensional coordinate in y' -direction

Greek Letters

α : thermal diffusivity [$m^2 s^{-1}$]

β : thermal expansion coefficient[K^{-1}]

β_0 : coefficient of solutal expansion

δ : heat generating parameter

ϕ : dimensionless concentration

γ : viscosity ratio

θ : dimensionless temperature

ϑ : kinematic viscosity of the fluid [$m^2 s^{-1}$]

ϑ_{eff} : effective kinematic viscosity of the porous medium [$m^2 s^{-1}$]

ρ : density of the fluid[kgm^{-3}]

τ_0 : dimensionless skin friction at the plate $y = 0$

τ_1 : dimensionless skin friction at the plate $y = 1$

Dimensionless Quantities.

Dimensionless quantities are widely used in many fields, such as fluid mechanics, physics, engineering and economics to determine the relationship between parameters so that the results can be generalized and hence use to predict the performance for different system. This dissertation has used many dimensionless quantities. We give short notes on some of the dimensionless quantities used.

1.Prandtl Number (Pr): is a dimensionless number named after the German physicist Ludwig Prandtl, defined as the ratio of momentum diffusivity to thermal diffusivity. Mathematically, Prandtl number is expressed as $\Pr = \frac{\vartheta}{\alpha}$, where α and ϑ are defined in the nomenclature.

2. Schmidt Number (Sc): is a dimensionless number defined as the ratio of momentum diffusivity (viscosity) and mass diffusivity, and is used to characterize fluid flows in which there are simultaneous momentum and mass diffusion convection processes. It was named after the German engineer Ernst Heinrich Wilhelm Schmidt (1892-1975). Mathematically, it is given as $Sc = \frac{\vartheta}{D_m}$.

3. Darcy Number(Da): in fluid dynamics through porous media, the Darcy number represents the relative effect of the permeability of the medium versus its cross-sectional area-commonly the diameter squared. The number is named after Henry Darcy and is found from nondimensionalizing the differential form of Darcy's law. It is expressed as $Da = \frac{K}{h^2}$.

CHAPTER ONE

GENERAL INTRODUCTION

1.1 Background of the Study

Natural convection flow is caused by density differences induced by the temperature gradients, chemical composition gradients and material compositions. It is a type of heat transport in which the fluid motion is not generated by an external source (pump, fan, suction devices). The driving force is buoyancy.

Fluid flows generated by combined temperature and concentration gradients are referred to as thermo-solutal convection. Considerable attention has been paid to natural convection for over ten decades as it plays important roles in different applications, such as the early stages of melting and in the transient heating of insulating air gap by heat input at the start-up of furnaces. Also, time dependent laminar natural convection is likely to find wider use as it could provide the flow mechanism in some types of solar heating and ventilating passive systems. In modern electronic equipment, the vertical circuit boards include heat generating elements, and this situation can be modeled by parallel heated plates with upward flow in the intervening space.

Heat generation/absorption plays significant role in engineering problems such as plasma studies, nuclear reactors, geothermal energy extraction and the boundary layer control in the field of aerodynamics.

The diffusion of mass due to temperature gradient is called Soret or thermal diffusion effect. Soret effect arises when the mass flux contains a term that depends on the temperature gradient. The major focus of our study is the effect on natural steady convection flow with injection of some diffusing species. Convection in binary fluids is considerably more complicated than that

in pure fluids. Both temperature and concentration gradients contribute to the initiation of convection and each may have stabilizing or destabilizing effects. Even when a concentration gradient is not externally imposed (thermosolutal problem) it can be created by applied thermal gradient via the Soret effect.

1.2 Statement of the Problem

Several studies have been conducted on fluid flow involving heat and mass transfer and many of the works have neglected thermal diffusion effects. However, when the temperature of fluid increases its concentration is not affected.Therefore, this study investigates the thermal diffusion (Soret effect) on steady natural convection double diffusive flow of heat generating/absorbing fluid in an inclined channel filled with porous material. This dissertation will further consider the opposing flow between heat and mass transfer with suction/injection.

1.3 Aim and Objectives of the Study

This work is aimed at studying the hydrodynamics, thermodynamics and solutal behaviour of fluid in a channel formed by two inclined parallel plates. This will be achieved through the following objectives which are to:

i. investigate the influence of suction/injection in the control of fluid flow and the thermodynamics in an inclined channel.
ii. investigate the effects of Soret parameter on the hydrodynamics in inclined channel.
iii. determine the influence of porous material on skin friction, mass flux, Nusselt number and Sherwood number within the inclined channel.

1.4 Research Methodology

The methodology adopted in the realization of the set of objectives is characterized as follows. Firstly, we review existing literatures and extend them to include different physical geometries. To obtain the analytical solutions, we use the method of undetermined coefficients and direct integration. We then obtain the numerical values of the analytical solutions and the computer package, MATLABR2015a is used to plot the graphs. The last part has to do with physical interpretation of the graphs so as to discuss the influence of the governing parameters on the flow and draw conclusions

1.5 Organization of the Dissertation

This dissertation is grouped into five chapters with references and appendices. Chapter one contains the general introduction of the dissertation. Chapter two is the literature review while chapter three contains the problem formulations and solution. The results and discussion are contained in chapter four. Chapter five is the summary, conclusion and recommendation, while references and appendices follow.

1.6 Significance of the Study

This study reveals the role of Soret on steady natural convection of double-diffusive flow of heat generating fluid. It would provide further details on the behavior of fluid in an inclined channel with heat generation as well as suction and injection. This research work has many applications in geophysics, oil reservoir, ground water system and boundary layer control in the field of aerodynamics. Further, it is hoped that the results obtained will not only provide useful information for engineering applications but also serve as an improvement on the previous studies.

1.7 Scope of the Study

The formulation, analysis and results obtained in this dissertation are theoretical. No experimental work has been carried out on this dissertation. The governing ordinary differential equations were employed and conclusions were drawn based on the behavior of the graphs and tables.

1.8 Basic Definitions

i. **Buoyancy:**is the force that causes objects to float.

ii. **Boussinesq approximation:** is the assumption that the fluid is considered under variations of temperature and density.

iii. **Double diffusive convection**: fluid flow generated by combined temperature and concentration gradients is referred to as double-diffusive convection or thermol-solutal convection.

iv. **Dimensionless quantity:** is a quantity without an associated physical dimension.

v. **Free or Natural convection**: Free convection arises in the fluid when temperature changes cause density variation leading to buoyancy forces acting on the fluid elements.

vi. **Heat generation/Absorption**: refers to the energy transferred as a result of a difference in temperature.

vii. **Heat flux**: is defined as the amount of heat transferred per unit area per unit time from or to a surface.

viii. **Injection**: is an infusion method of putting fluid in to the body.

ix. **Inclined channel**: is a channel inclined at an angle to the horizontal.

x. **Mass flux**: is the rate of mass flow per unit area.

xi. **Nusselt number**: is the rate of heat transfer.

xii. **Porous medium or porous material**: is a material containing pores (voids).

xiii. **Suction**: is the flow of a fluid in to partial vacuum or region of low pressure.

xiv. **Skin friction**: is the shear stress that occurs between the fluid and the solid surface of boundaries.

xv. **Sherwood number:** is a dimensionless number used in mass transfer operation.

xvi. **Soret or thermal diffusion effect:** the diffusion of mass due to temperature gradient is called Soret or thermal diffusioneffect.

xvii. **Isothermal/Isoflux thermal boundary condition:** is one that takes place at constant temperature.

1.9 Basic Equations

These four equations all together fully described the fundamental characteristic of fluid flow in an inclined channel.

i. Continuity equation

the continuity equation in Cartesian coordinates is given as:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \vec{v}\right) = 0 \quad (1.1)$$

For viscous incompressible fluid flow, the continuity equation (1.1) reduces to:

$$\nabla \cdot \vec{v} = 0 \quad (1.2)$$

ii. Momentum equation

The general expression for conservation of momentum (The Navier-Stokes equation) is:

$$\frac{\partial u}{\partial t} + u \cdot \nabla u = \frac{1}{\rho}\nabla p + \vartheta \nabla^2 u + \frac{1}{\rho} F \qquad (1.3)$$

Where ϑ is the kinematic viscosity, ρ is fluid density and F is the sum of any body forces. In the present problem, a steady flow is considered so that time derivative of the velocity, $\frac{\partial u}{\partial t} = 0$. In addition, the flow is hydrodynamically fully developed so that the advection terms are ignored and the momentum equation (1.3) reduces to:

$$\vartheta \nabla^2 u + \frac{1}{\rho} F = 0 \quad (1.4)$$

Where the pressure gradient is ignored in a natural convection problem. Here, the density is assumed to be partially fixed and partially dependent on the fluid temperature as well as concentration so that the equation of state is given as:

$$\rho = \rho_0 - \beta \rho_0 (T - T_0) - \beta_0 \rho_0 (C - C_0) \quad (1.5)$$

Where ρ_0 is the fluid density at temperature T_0 and concentration C_0.
From (1.5), it is observed that

$$\rho - \rho_0 \approx -\rho_0 [\beta (T - T_0) - \beta_0 (C - C_0)] \quad (1.6)$$

In natural convection problem, the body force term $F = (\rho - \rho_0) g$ is the gravitational body force, acting in the opposite direction of the buoyancy induced flow so that

$$\vartheta \nabla^2 u - \frac{(\rho - \rho_0) g}{\rho} = 0 \quad (1.7)$$

So that:

$$\vartheta \nabla^2 u + \frac{\rho_0 [\beta (T - T_0) + \beta_0 (C - C_0)] g}{\rho} = 0 \quad (1.8)$$

Invoking Boussinesq approximation in which the change in density is negligible so that $\rho \approx \rho_0$, then the momentum equation becomes:

$$\vartheta\nabla^2 u + g\beta(T - T_0) + g\beta_0(C - C_0) \quad (1.9)$$

iii. **Energy equation**

The energy equation that describes the conservation of energy in the system is given as:

$$\rho Cp\frac{DT}{Dt} = k\nabla^2 T + \zeta(1.10)$$

Where Cp is the specific heat constant pressure, k is the thermal conductivity and ζ is the dissipation function. Here, the dissipation function is ignored since the effect of pressure on the density is neglected. Also, the flow is steady and hydrodynamically fully developed, the energy equation (1.10) reduces to:

$$\frac{k}{\rho Cp}\nabla^2 T = 0\,(1.11)$$

Equation (1.11) can be written as:

$$\alpha\nabla^2 T = 0 \text{ where } \alpha = \frac{k}{\rho Cp} \quad (1.12)$$

iv. **Concentration equation**

The general equation of concentration is:

$$\frac{\partial c}{\partial t} = \nabla\cdot(D\cdot\nabla c) - \nabla\cdot(\vec{v}c) + R \quad (1.13)$$

Where c is the variable of interest (species concentration for mass transfer, temperature for heat transfer), D is the diffusivity (also called diffusion coefficient), such as mass diffusivity for particle motion or thermal diffusivity for heat transport. $\vec{v}$ is the velocity field that the quantity is moving with. It is also a function of time and space. R describes " sources" or " sinks" of the quantity c. Here, the diffusion coefficient is constant, there are no sources or sinks, and the velocity field describes an incompressible flow (i.e., it has zero divergence). Then the formula simplifies to:

$$\frac{\partial c}{\partial t} = D\nabla^2 c - \vec{v}\cdot\nabla c \quad (1.14)$$

Since the flow is steady and hydrodynamically fully developed, $\frac{\partial c}{\partial t} = 0$, so equation

(1.14) reduces to:

$$D\nabla^2 c = 0 \quad (1.15)$$

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

This chapter explores some of the related literature of previous research and finding. Due to the applications of fluid flow in geophysics, oil reservoir, ground water system and boundary layer control in the field of aerodynamics, it has continued to gain extensive attention over the past years. In order to have inevitable guide to a study of this nature, this chapter therefore explains some of the related literature of some previous researchers. This review is discussed in four parts.

2.2 Natural Convection Flow in Channel

Natural convection is of great importance in many industrial applications. Applications of natural convection in engineering can be found in the solar collectors, furnaces, building heating and cooling system, heat exchangers and so on. Buoyancy-induced flow and heat transfer in enclosures have received considerable attention by many researchers both experimentally and numerically. For example, Acharya and Goldstein (1985) studied numerically the natural convection in an externally heated vertical or inclined enclosure possessing internal energy sources and concluded that the average heat flux at the cold wall increases with an increase in the external Rayleigh number. Theirinvestigation also reveals that the local heat flux ratio along a surface attains its maximum value in the vicinity or the region where the heated (cooled) fluid from the opposite walls or from the interior encounters the surface. Sundaravelu and Kadaswamy (2003) investigated the natural convection heat transfer in pure water, around its density

maximum, contained in an inclined enclosure with internal heat generation and they reported on the existence of multiple fluid cells inside the enclosure due to the density maximum of water occurring in the operating temperature region. The size of the cell depends strongly on the inclination angle. They also found the average heat transfer rate to be a maximum for the inclination angle of 150^0. Natural convection heat transfers in an inclined channel containing centrally placed solid square heat generating body is studied by Kandaswamy and Sivajankaran in (2006) and they showed that the heat transfer rate increases when the source strength and the tilting angle of the channel takes lower negative values. Ouriemi et al. (2005) presented an analytical and numerical study of natural convection of a binary fluid confined in a tall enclosure, slightly inclined about the gravity field where they showed that for a given Raleigh number, both "natural" flow circulating anticlockwise and "anti-natural" flow circulating anticlockwise are possible provided the angle of inclination is small. Mamou et al. (1998) have studied the onset of double-diffusive convection in an inclined porous channel. In their study, they have dealt with a particular situation where buoyancy forces induced by thermal and solutal effects were opposing and of equal intensity. The aim of their study was to investigate the critical stability of their system in terms of the inclination angle, aspect ratio and Lewis number. An investigation of natural convection inside a triangular enclosure with linear heating/cooling on the inclined surfaces is investigated by Basak et al. (2007). Mamou et al. (1995) have investigated double diffusion convection in an inclined channel filled with porous medium.

2.3 Heat Generation

Heat generation/absorption plays significant role in various physical phenomena such as fluid undergoing exothermic or endothermic chemical reactions. Natural convection flow of heat generating/absorbing fluid was studied by Vajravelu (1979). Vajravelu concluded that the heat

generating/absorbing play an important role in delaying the velocity and the temperature to reach the steady state condition. Jha (2001) discusses and analyzed the flow behavior of transient free convective flow of viscous incompressible fluid in a vertical channel due to asymmetric heating of the channel walls in the presence of temperature dependent heat sink. He observed that as heat sink parameter (S) increases both the skin friction (τ) and Nusselt number (Nu) decreases. Chandran et al. (2005) examined natural convection near a vertical plate with ramped temperature and obtained two different solutions, one valid for the fluid of Prandtl number different from the unity and the other for which the Prandtl number is unity. In their study, they obtained analytical solutions under the assumption that the velocity and temperature conditions on the wall are continuous and well defined and they concluded that the solutions for non-dimensional velocity and temperature variables depend upon the Prandtl number of the fluid, and the expression of the fluid velocity is not uniformly valid for all values of Pr. In their work, heat source or sink is absent. However, when the temperature differences are appreciably large, the volumetric heat generation or absorption term may exert strong influence on the heat transfer and as a consequence, on the fluid flow as well. Alam et al. (2006) studied the problem of free convection heat and mass transfer flow past an inclined semi-infinite heated surface of an electrically conducting and steady viscous incompressible fluid in the presence of magnetic field and heat generation.

2.4 Soret Effect on Double-diffusive Free Convection Flow

The diffusion of mass due to temperature gradients is called Soret or thermo-diffusion effect and this effect might become significant when large density differences exist in the flow regime. For example, Soret effect can be significant when species are introduced at a surface in fluid domain, with a density lower than the surrounding fluid. The Soret parameter has been utilized for isotope

separation and in a mixture between gases with very light molecular weight (H_2, H_e) and of medium molecular weight $\left(N_{2,} air\right)$. Boutana et al. (2004) investigated the Soret effect and double-diffusive medium filled with a binary fluid both analytically and numerically. They reported that the range of buoyancy ratio for the existence of multiple solutions depends on the type of convection induced by the solute gradients i.e. on the constant a. In particular, it was demonstrated that for opposing flows near the buoyancy ratio greater than -1, multiple solutions are possible for a range of R_t which depends on the other governing parameters. Nithyadevi and Yang (2009) considered the effect of double diffusive natural convection in a partially heated enclosure with Soret and Dufour coefficient numerically. Nithyadevi and Yang concluded that the fluid particle moves with less velocity and high mass transfer rate in the presence of Soret coefficient when the partially heated active vertical left side wall has lower concentration while the opposite side is observed when the partially heated active vertical left side wall has higher concentration. Basu and Layek (2013) gave a theoretical analysis to investigate the onset of double-diffusive convection at the marginal state in presence of cross-diffusive terms (Soret and Dufour effects) in a horizontal fluid layer and concluded that the Soret parameter destabilizes the oscillatory convection while the Dufour does the opposite. Chen (2010) studied the Soret and Dufour effects on the boundary layer flow due to free convection heat and mass transfer over a vertical cylinder in a porous medium saturated with Newtonian fluids with constant wall temperature and concentration. Their results showed that an increase in the Soret number leads to a decrease in the local Sherwood number and an increase in the local Nusselt number. In a related work, Chen (2012), in his study of free convection boundary layer flow over an arbitrarily inclined heated plate in a porous medium with Soret and Dufour effects pointed out that an increase in the Dufour parameter tends to decrease the local heat transfer rate and

increase in the Soret parameter tends to decrease the local mass transfer rate. Mahdy (2010) presented a non-similar boundary layer analysis to study the flow, heat and mass transfer characteristics of non-Darcian mixed convection of a non-Newtonian fluid from a vertical isothermal plate imbedded in a homogeneous porous medium with the effect of Soret and Dufour and in the presence of either surface injection or suction. They reported that as the Soret parameter increases and the Dufour parameter decreases, both velocity and concentration decreases, whereas, the temperature distribution increases. In their study of double diffusive convection in a horizontal sparsely produced porous layer in presence of Soret effect Gaikwad and Kamble (2012) revealed that the increase of Soret effect is to destabilize the system. Chen et al. (2010) investigated numerically the double-diffusive natural convection with opposing temperature and concentration gradients. Prasad et al. (2011) considered thermo-diffusion and diffusion-thermo effects on MHD free convection flow past a vertical porous plate embedded in a non-Darcian porous medium.

2.5 Suction/Injection

The significance of suction/injection on the boundary layer control in the field of aerodynamics and space science is well recognized. In general, suction tends to increase the skin friction and heat transfer coefficients whereas injection act in the opposite manner. The effect of lateral mass flux (fluid suction/injection) on the free convection boundary layers induced by a heated vertical plate embedded in a saturated porous medium with an exponential decaying heat generation is studied by Postelnicu et al (2000), Ali (2007)and Achemlal et al (2012). El-Arabawy (2009) examined the effects of suction/injection and chemical reaction on mass transfer over a stretching surface. Vajravelu and Hadjnicolaou (1993) studied the heat transfer characteristic in the laminar boundary layer of a viscous fluid over a linearly stretching surface with variable wall

temperature subject to suction or blowing. They considered the effects of viscous dissipation and internal heat generation. Al-Sanea (2005) investigated the steady laminar flow and heat transfer characteristics of a continuously moving vertical sheet of extruded material and found that the skin-friction and heat transfers coefficient increases with suction and the reverse is the case with injection. Shojaefard et al. (2005) investigated flow control on a subsonic airfoil by suction and injection and concluded that suction significantly increases the lift coefficient while injection decrease the surface skin friction, which transitively resulted in a considerable reduction in energy consumed during flights of subsonic aircraft. However, to our best findings, no studies have been carried out analytically on thermo-solutal natural convection flow in an inclined channel filled with porous material. To this end, this prompts this dissertation.

CHAPTER THREE

PROBLEM FORMULATIONS AND SOLUTION

3.1 Introduction

This chapter is concerned with the derivation of the velocity, concentration and temperature equations of the system. The non-dimensional equations are derived from the dimensional equations using appropriate non-dimensional parameters.

3.2 Natural convection double-diffusive flow of heat generating fluid in an inclined channel filled with porous material

The main objective of this section is to explore the Soret effect on steady natural convection double-diffusive flow of heat generating fluid in an inclined channel filled with porous material.

3.2.1Mathematical formulation

We consider the steady natural convection flow of heat generating fluid between two inclined parallel plates. The x' axis is taken vertically upward along the plates and the y' axis is normal to it. The distance between the plates is h . The channel is presented in an inclined position making an angle a with the positive y' axis as shown in the figure 3.1. The plate $y' = h$ is heated to a temperature T_w with concentration C_W while the other plate $y' = 0$ is kept at the temperature of the ambient temperature T_0 and ambient concentration C_0 . The fluid is assumed to be steady, thermally and hydrodynamically fully developed.

$C = C_o$ $C = C_o$ g

$T = T_0$ $T = T_w$

$u' = 0$ $u' = 0$

a

$y' = 0$ $y' = h$

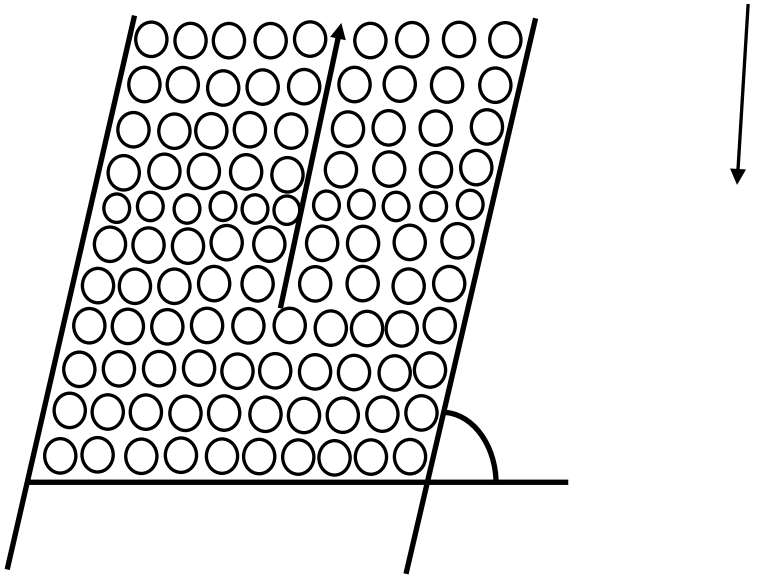


Figure 3.1: Schematic diagram of the problem

The governing equations of the system are given, assuming the usual Boussinesq approximation as:

$$\vartheta_{eff} \frac{d^2u'}{dy'^2} - \frac{\vartheta u'}{K} + g\sin(a)\left[\beta(T - T_0) + \beta_0(C - C_0)\right] = 0$$

(3.1)

$$D_m \frac{d^2C}{dy'^2} + S^* \frac{d^2T}{dy'^2} = 0$$

(3.2)

$$\alpha \frac{d^2T}{dy'^2} + \frac{Q_0}{\rho Cp}(T - T_0) = 0 \qquad (3.3)$$

Where the momentum equation (3.1) follows from the basic equation (1.9) in which the acceleration due to gravity is resolved in the direction of flow to have $g\sin(a)$, where a is the angle of inclination of the channel with horizontal. Also, the channel is filled with porous material, therefore, additional term $\frac{\vartheta u'}{K}$ was introduced in equation (3.1) to capture the porosity of the medium. The terms $S^*\frac{d^2T}{dy'^2}$ and $Q_0(T-T_0)$ were introduced in equation (3.2) and (3.3) to capture the Soret effect and heat generation respectively.

With the boundary conditions:

$$y'=0: T=T_0, u'=0, C=C_0 \quad (3.4)$$

$$y'=h: T=T_w, u'=0, C=C_w \quad (3.5)$$

3.3 Transpiration and Soret effects on thermo-solutal convection flow of heat generating fluid in an inclined channel filled with porous material

This present study is an extension of the first study by including the effect of suction and injection.

3.3.1 Mathematical formulation

The present study considers a steady natural convective flow of heat generating fluid between two inclined parallel porous plates with constant suction having suction velocity V_0'. The x' axis is taken vertically upward along the walls and the y' axis is normal to it. The distance between the porous walls is h and the channel is filled with porous materials. The channel is presented in an inclined position making an angle a with the positive y' axis as shown in the figure 3.2. The flow is induced by the combined effects of buoyancy force due to temperature gradient as well as

concentration gradient caused by the isothermal/isoflux thermal boundary condition and asymmetric but constant concentration on the boundary. In addition, mass flux is governed by temperature leading to diffusion-thermo (Soret) effects. Also, the flow is assumed to be hydrodynamically and thermally fully developed so that its equations are dependent on y' only .

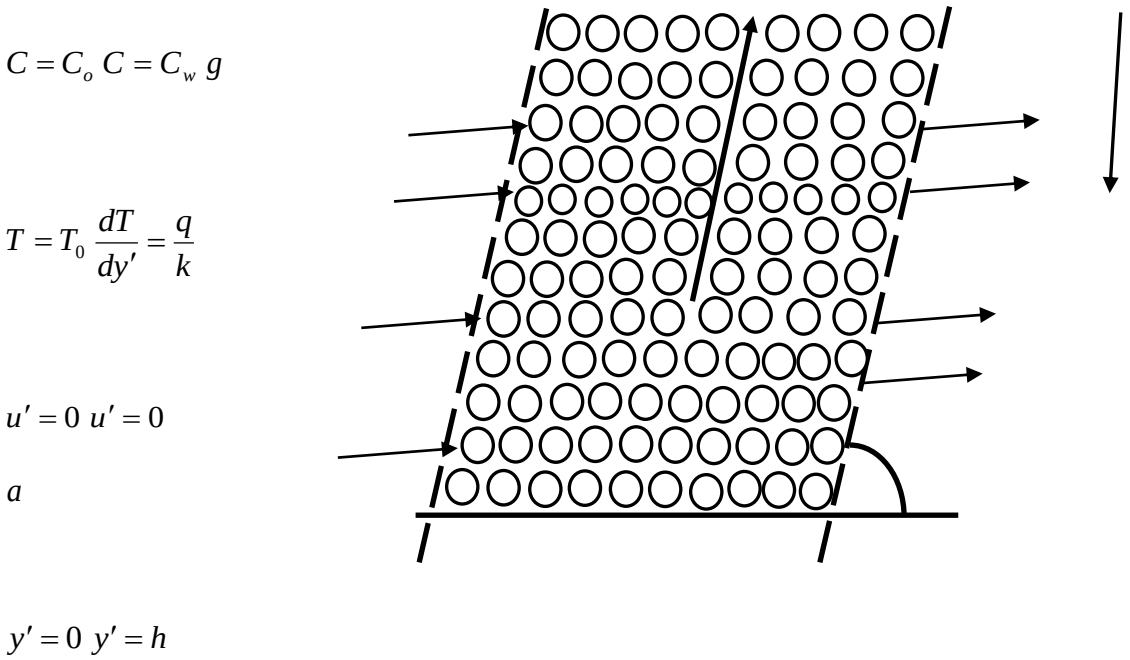


Figure 3.2: Schematic diagram of the problem

The governing equations for the flow of heat generating fluid through an inclined channel filled with porous material in the presence of suction/injection and Soret effects are presented in a no-slip regime as:

$$\vartheta_{eff}\frac{d^2u'}{dy'^2} - V'o\frac{du'}{dy'} - \vartheta\frac{u'}{K} + g\sin(a)\left[\beta(T - T_o) + \beta_o(C - C_o)\right] = 0 \quad (3.6)$$

$$D_m\frac{d^2C}{dy'^2} - Vo'\frac{dC}{dy'} + S^*\frac{d^2T}{dy'^2} = 0 \quad (3.7)$$

$$\alpha \frac{d^2T}{dy'^2} - V'o\frac{dT}{dy'} + \frac{Q_o}{\rho Cp}(T - T_o) = 0 \quad (3.8)$$

Along with the boundary conditions:

$$y' = 0 : u' = 0, T = T_o, C = C_o$$

(3.9)

$$y' = h : u' = 0, \frac{dT}{dy'} = \frac{q}{k}, C = C_w \quad (3.10)$$

3.4 Dimensionless Analysis

In the equations (3.6), (3.7), (3.8), (3.9) and (3.10), u' is the velocity of the fluid along x'-direction, γ is the viscosity ratio, α is the thermal diffusivity, β is the thermal expansion coefficient, β_0 is the coefficient of solutal expansion, S^* is the dimensional Soret effect, N is the sustentation parameter, D_m is the mass diffusion parameter, a is the angle of inclination, K is the permeability of porous medium, C is the fluid concentration, T is the fluid temperature, C_p is the specific heat constant pressure, ρ is the density of the fluid, ϑ is the kinematic viscosity of the fluid, ϑ_{eff} is the effective kinematic viscosity of the porous medium and Q_0 is the coefficient of heat generation.

We introduce non-dimensional quantities:

$$y=\frac{y'}{h},\gamma=\frac{\vartheta_{eff}}{\vartheta},\theta=\frac{T-T_o}{\Delta q},\phi=\frac{C-C_o}{C_w-C_0},u=\frac{\vartheta u'}{g\beta h^2\left(T_w-T_o\right)},N=\frac{\beta_o\left(C_w-C_o\right)}{\beta\left(T_w-T_o\right)},$$

$$So=\frac{S^*\left(T_w-T_o\right)}{D_m\left(C_w-C_o\right)},\delta=\frac{Q_o h^2}{k},Da=\frac{K}{h^2},\alpha=\frac{k}{\rho Cp},Vo=\frac{Vo'h}{\vartheta},\Pr=\frac{\vartheta}{\alpha},Sc=\frac{\vartheta}{Dm}' \quad (3.11)$$

$$\Delta q=\frac{qh}{k}$$

3.5 Governing Equations in Dimensionless Form

By substituting the dimensionless parameters in equation (3.11) into (3.1), (3.2), (3.3), (3.4), (3.5), (3.6), (3.7), (3.8), (3.9) and (3.10), the governing equations for both problems in dimensionless form are as follow:

3.5.1 Governing equations in dimensionless form of problem 3.2

$$\gamma\frac{d^2u}{dy^2}-\frac{u}{Da}+\left(\theta+N\phi\right)\sin\left(a\right)=0 \quad (3.12)$$

$$\frac{d^2\phi}{dy^2}+So\frac{d^2\theta}{dy^2}=0 \quad (3.13)$$

$$\frac{d^2\theta}{dy^2}+\delta\theta=0 \quad (3.14)$$

Subjectto the following dimensionless boundary conditions:

$$y=0:\theta=0,u=0,\phi=0$$

(3.15) $y=1:\theta=1,u=0,\phi=1$ (3.16)

Now, equations (3.12), (3.13) and (3.14) together with their appropriate boundary conditions (3.15) and (3.16), have the following analytical solutions for velocity, concentration and temperature profiles respectively as:

$$u(y)=\frac{1}{\exp\left(\frac{1}{\sqrt{\gamma Da}}\right)-\exp\left(\frac{-1}{\sqrt{\gamma Da}}\right)}\left(\frac{\sin a(1-NSo)}{\left(\gamma\delta+\frac{1}{Da}\right)}+(N+NSo)Da\sin a\right)\left(\exp\left(\frac{-y}{\sqrt{\gamma Da}}\right)-\exp\left(\frac{y}{\sqrt{\gamma Da}}\right)\right)$$

$$+\frac{\sin a(1-NSo)\sin\left(y\sqrt{\delta}\right)}{\left(\gamma\delta+\frac{1}{Da}\right)\sin\left(\sqrt{\delta}\right)}+y(N+NSo)Da\sin(a)$$

$$\phi(y)=(1+So)y-\frac{So\sin\left(y\sqrt{\delta}\right)}{\sin\left(\sqrt{\delta}\right)} \quad (3.18)$$

$$\theta(y)=\frac{\sin\left(y\sqrt{\delta}\right)}{\sin\left(\sqrt{\delta}\right)}$$

(3.19)

3.5.2 Skin-friction of problem 3.2

The skin-friction (τ) on both walls in non-dimensional form is obtained by differentiating equation (4.1) and are given by:

$$\tau_0=\left(\frac{du}{dy}\right)_{y=0}=\frac{(1-NSo)\sqrt{\delta}\sin(a)}{\sin\left(\sqrt{\delta}\right)\left(\gamma\delta+\frac{1}{Da}\right)}+(N+NSo)Da\sin(a)-$$

$$\frac{2}{\left(\exp\left(\frac{1}{\sqrt{\gamma Da}}\right)-\exp\left(\frac{-1}{\sqrt{\gamma Da}}\right)\right)\sqrt{\gamma Da}}\left(\frac{(1-NSo)\sin(a)}{\left(\gamma\delta+\frac{1}{Da}\right)}+(N+NSo)Da\sin(a)\right) \quad (3.20)$$

$$\tau_1 = -\left(\frac{du}{dy}\right)_{y=1} = \frac{\exp\left(\frac{1}{\sqrt{\gamma Da}}\right) + \exp\left(\frac{-1}{\sqrt{\gamma Da}}\right)}{\left(\exp\left(\frac{1}{\sqrt{\gamma Da}}\right) - \exp\left(\frac{-1}{\sqrt{\gamma Da}}\right)\right)\sqrt{\gamma Da}}$$

$$\left(\frac{(1-NSo)\sin(a)}{\left(\gamma\delta + \frac{1}{Da}\right)} + (N+NSo)Da\sin(a)\right) - \frac{\left(\sqrt{\delta}\right)(1-NSo)\cot\left(\sqrt{\delta}\right)\sin(a)}{\left(\gamma\delta + \frac{1}{Da}\right)} -$$

$$(N+NSo)Da\sin(a)$$

(3.21)

3.5.3 Volumetric flow rate of problem 3.2

Another interesting phenomenon in this study is the volumetric flow rate. The volumetric flow rate within the channel is obtained by evaluating the integral $Q = \int_0^1 u dy$ and is given by:

$$Q = \frac{\sqrt{\gamma Da}}{\left(\exp\left(\frac{1}{\sqrt{\gamma Da}}\right) - \exp\left(\frac{-1}{\sqrt{\gamma Da}}\right)\right)}\left(\frac{(1-NSo)\sin(a)}{\left(\gamma\delta + \frac{1}{Da}\right)} + (N+NSo)Da\sin(a)\right)$$

$$\left(2 - \exp\left(\frac{1}{\sqrt{\gamma Da}}\right) - \exp\left(\frac{-1}{\sqrt{\gamma Da}}\right)\right) + \frac{(1-NSo)\left(1-\cos\left(\sqrt{\delta}\right)\right)\sin(a)}{\left(\sqrt{\delta}\right)\left(\gamma\delta + \frac{1}{Da}\right)\sin\left(\sqrt{\delta}\right)} +$$

$$\frac{(N+NSo)Da\sin(a)}{2}$$

(3.22)

3.5.4Nusselt number of problem 3.2

It is also interesting to study the rate of heat transfer on boundary surfaces which is expressed as Nusselt number and is obtained as:

$$Nu_0 = \left(\frac{d\theta}{dy}\right)_{y=0} = \left(\sqrt{\delta}\right)\cos ec\left(\sqrt{\delta}\right) \qquad (3.23)$$

$$Nu_1 = \left(\frac{d\theta}{dy}\right)_{y=1} = \left(\sqrt{\delta}\right)\cot\left(\sqrt{\delta}\right) \qquad (3.24)$$

3.5.5Sherwood number of problem 3.2

We now study the Sherwood number sh whichis given in non-dimensional formby:

$$sh_0 = \left(\frac{d\phi}{dy}\right)_{y=0} = (1+So) - \left(\sqrt{\delta}\right)\cos ec\left(\sqrt{\delta}\right)So \qquad (3.25)$$

$$sh_1 = \left(\frac{d\phi}{dy}\right)_{y=1} = (1+So) - \left(\sqrt{\delta}\right)\cot\left(\sqrt{\delta}\right)So \qquad (3.26)$$

3.5.6Governing equations in dimensionless form of problem 3.3

$$\gamma\frac{d^2u}{dy^2} - Vo\frac{du}{dy} - \frac{u}{Da} + (\theta+\phi)\sin(a) = 0 \quad (3.27)$$

$$\frac{d^2\phi}{dy^2} - VoSc\frac{d\phi}{dy} + So\frac{d^2\theta}{dy^2} = 0 \quad (3.28)$$

$$\frac{d^2\theta}{dy^2} - Vo\,\mathrm{Pr}\frac{d\theta}{dy} + \delta\theta = 0 \quad (3.29)$$

Subject to the followingdimensionless boundary conditions:

$$y = 0 : u = 0, \theta = 0, \phi = 0 \quad (3.30)$$

$$y = 0, : u = 0, \frac{d\theta}{dy} = 1, \phi = 1$$

(3.31)

Similarly, equations (3.27), (3.28) and (3.29) subject to their appropriate boundary conditions (3.30) and (3.31), have the analytical solutions for velocity, concentration and temperature profiles as:

$$u(y) = E\exp(m_7 y) + F\exp(m_8 y) + G\exp(m_1 y) + H\exp(m_2 y) + I\exp(VoScy) + J \quad (3.32)$$

$$\phi(y) = C + D\exp(VoScy) + \alpha\exp(m_1 y) + \beta\exp(m_2 y)$$

(3.33)

$$\theta(y) = A\exp(m_1 y) + B\exp(m_2 y)$$

(3.34)

Where the constants A, B, C, D, E, F, G, H, I, J, α, β, m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8, m_9, m_{10}, m_{11} and m_{12} are defined in Appendix.

3.5.7Skin-friction of problem 3.3

Similarly, the skin-friction on both plates in non-dimensional form are given by:

$$\tau_0 = \frac{du}{dy}\Big|_{y=0} = m_7 E + m_8 F + m_1 G + m_2 H + VoScI \quad (3.35)$$

$$\tau_1 = -\frac{du}{dy}\Big|_{y=1} = -\begin{pmatrix} m_7 E\exp(m_7) + m_8 F\exp(m_8) + m_1 G\exp(m_1) + m_2 H\exp(m_2) + \\ VoScI\exp(VoSc) \end{pmatrix} \quad (3.36)$$

3.5.8Volumetric flow rate of problem 3.3

In similar manner, the volumetric flow rate (mass flux) in non-dimensional form is given by:

$$Q = \int_0^1 u dy = \frac{E}{m_7}\left(\exp(m_7) - 1\right) + \frac{F}{m_8}\left(\exp(m_8) - 1\right) + \frac{G}{m_1}\left(\exp(m_1) - 1\right) + \frac{H}{m_2}\left(\exp(m_2) - 1\right) + \frac{I}{VoSc}\left(\exp(VoSc) - 1\right) + J$$

(3.37)

3.5.9Nusselt number of problem 3.3

Similarly, the dimensionless rate of heat transfer, presented as the Nusselt number is given by:

$$Nu_0 = \left(\frac{d\theta}{du}\right)_{y=0} = m_1 A + m_2 B \qquad (3.38)$$

$$Nu_1 = \left(\frac{d\theta}{du}\right)_{y=1} = m_1 A \exp(m_1) + m_2 B \exp(m_2) \qquad (3.39)$$

3.5.10 Sherwood number of problem 3.3

It is also important to study the Sherwood number which is given in non-dimensional form by:

$$Sh_0 = \left(\frac{d\phi}{dy}\right)_{y=0} = VoScD + \alpha m_1 + \beta m_2 \qquad (3.39)$$

$$Sh_0 = \left(\frac{d\phi}{dy}\right)_{y=0} = VoScD\exp(VoSc) + \alpha m_1 \exp(m_1) + \beta m_2 \exp(m_2) \qquad (3.40)$$

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 Introduction

This chapter presents the physical interpretations of the solutions obtained in chapter three. The results are shown in figures and tables and are analyzed in detail. From these graphs and tables, we deduced the physical interpretations.

4.2Natural convection double-diffusive flow of heat generating fluid in an inclined channel filled with porous material

The basic physical parameters controlling the present situation are heat generation parameter (δ), Soret parameter (So), porous medium permeability (Da), viscosity ratio (γ),the channel inclination angle (a) and sustentation parameter (N).For the purpose of this discussion, the values of the heat generation parameter (δ) has been taken between 0.5 and 2.0; Soret parameter (So), is taken between 0.015 and 1.0; porous medium permeability (Da) is taken between 0.001 and 0.5; the viscosity ratio (γ) is taken between 1.0 and 4.0; the channel inclination angle is taken between 0 and $\frac{\pi}{2}$. If $a = 0$, the channel is horizontal and natural convection effect is not considered while $a = \frac{\pi}{2}$ denotes natural convection in vertical channeland finally, the sustentation parameter (N) is taken between 1.0 and 3.0. The parameter affecting the temperature distribution is the heat generation parameter (δ) only and the effect of this parameter on the temperature profile is shown in the graph. Figure 4.1 shows the effect of heat generation parameter on the temperature distribution. It is found that increase in heat generation parameter leads to an increase in temperature within the channel. Figures 4.2 and 4.3 respectively show the response of concentration with variations in heat generation parameter and Soret number. It is seen that concentration profile decreases with increase in heat generation parameter. This result is expected since an increase in δ causes temperature increase which transitively lead to decrease in the concentration in the channel. Similarly, for a fixed value of heat generation parameter, growing the Soret effect amplifies the effect of temperature so that increase in fluid temperature will decrease the concentration within the channel. Figures 4.4-4.9 show the variation of velocity with heat generation parameter, Soret number, Darcy number, viscosity ratio, angle of inclination

and sustentation parameter. Figure 4.4 depicts the variation of the velocity with the heat generation parameter. It is observed that the velocity decreases with increase in heat generation parameter. This is due to the fact that growing heat generation parameter (δ) decreases fluid concentration which in turns weakens the buoyancy force due to mass transfer, hence a decrease in velocity results. The velocity distribution is shown in figure 4.5 for various values of So. The figure shows that the increase in the Soret number causes a decrease in the velocity profile. This is the consequence of decrease in fluid concentration which acts to decrease the buoyancy effects due to mass transfer so that velocity decrease is recorded. Figure 4.6 presents the variation of the velocity with the Darcy number. The figure shows that the velocity profile increases with an increase in the value of Darcy number. Physically speaking, growing the permeability of the porous medium acts to reduce the impedance to the flow presented by the porous material. Also the velocity profile decreases with increase in the value of the viscosity ratio parameter as shown in the figure 4.7. An increase in viscosity ratio is caused by the increase of the effective viscosity of the porous medium which tends to amplify the porous medium resistance to flow resulting in a decrease in fluid velocity. In the figures 4.8 and 4.9 the velocity profile shows a similar behavior of increasing with increase in the value of the angle of inclination $(a)$and with the increase in the value of the sustentation parameter (N) respectively. This is physically true since buoyancy effect is highest in a vertical channel while it diminishes as the angle of inclination approaches zero. On the other hand, an increase in the sustentation parameter amplifies the convection currents due to mass transfer effect so that velocity of the fluid increases as N increases.

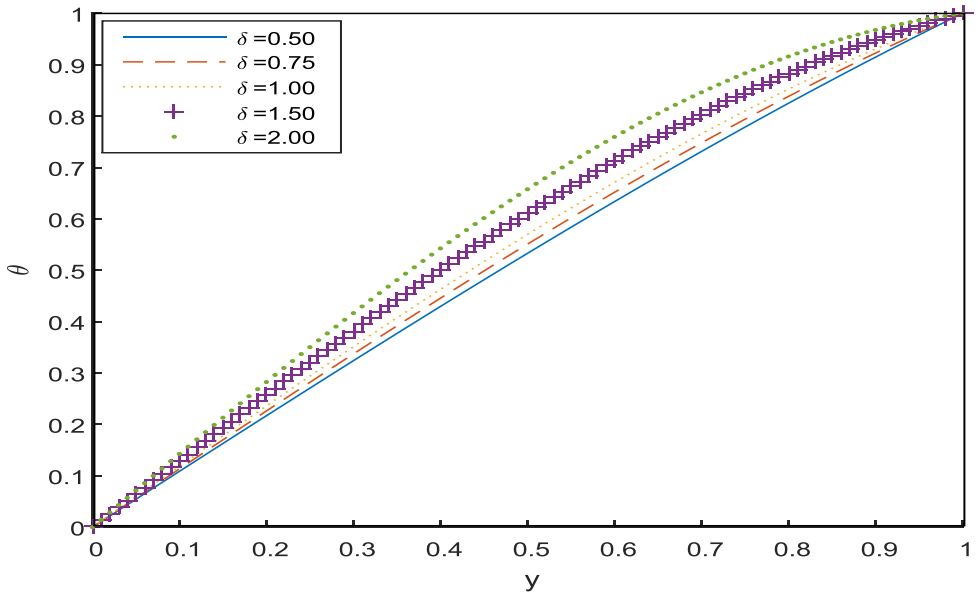


Figure 4.1: Variation of temperature with heat generation parameter δ

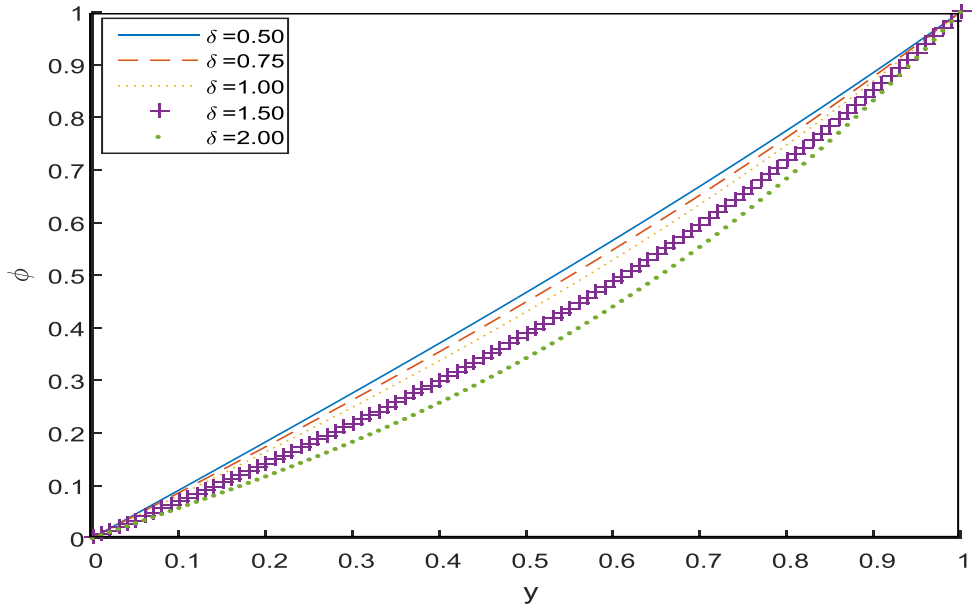


Figure 4.2: Variation of concentration with heat generation parameter δ at $So = 1.0$

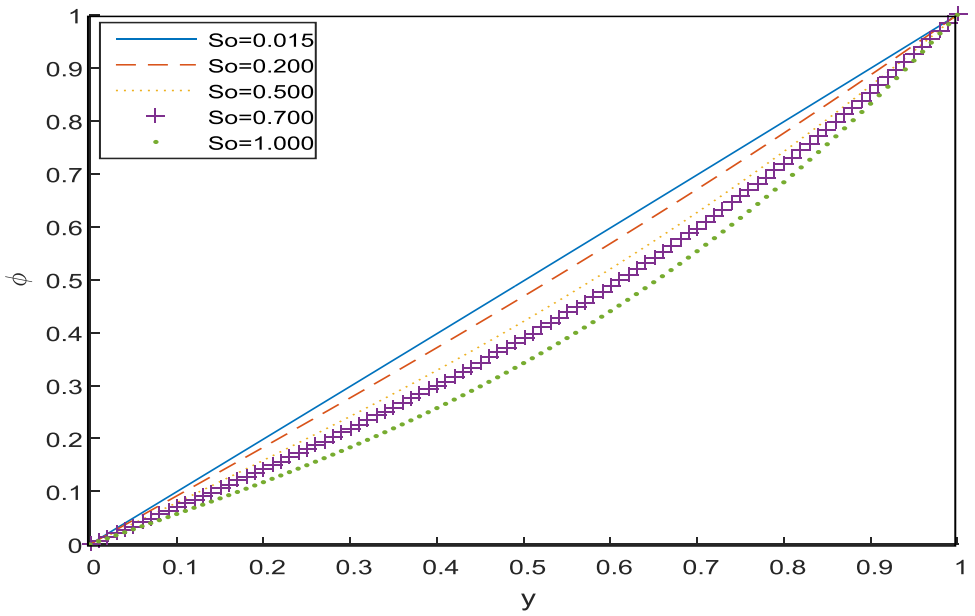


Figure 4.3: Variation of concentration with Soret number So **at** $\delta = 2.0$

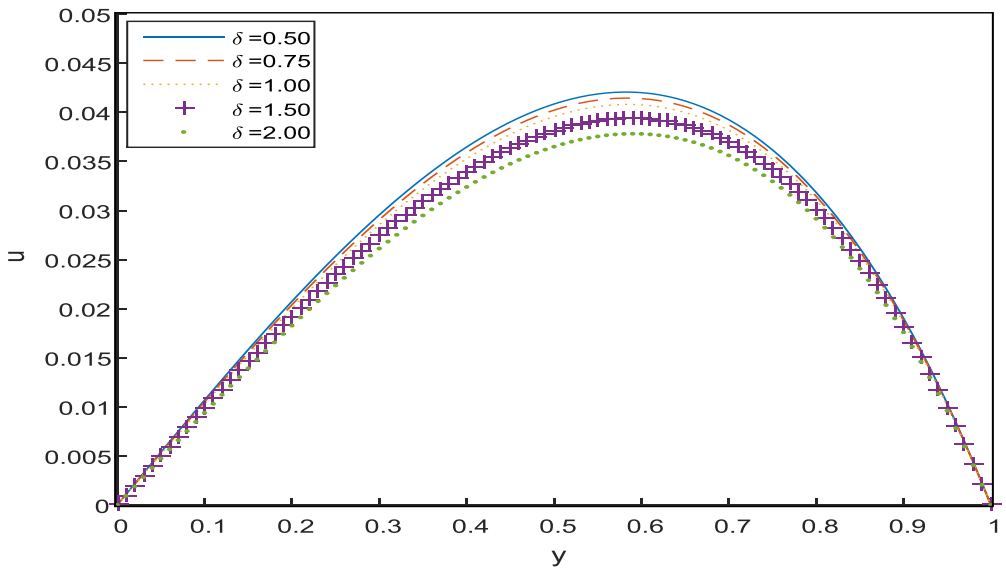


Figure 4.4: Variation of velocity with heat generation parameter δ **at**

$So = 1.0, Da = 0.5, \gamma = 4.0, N = 3.0$ **and** $a = \frac{\pi}{4}$

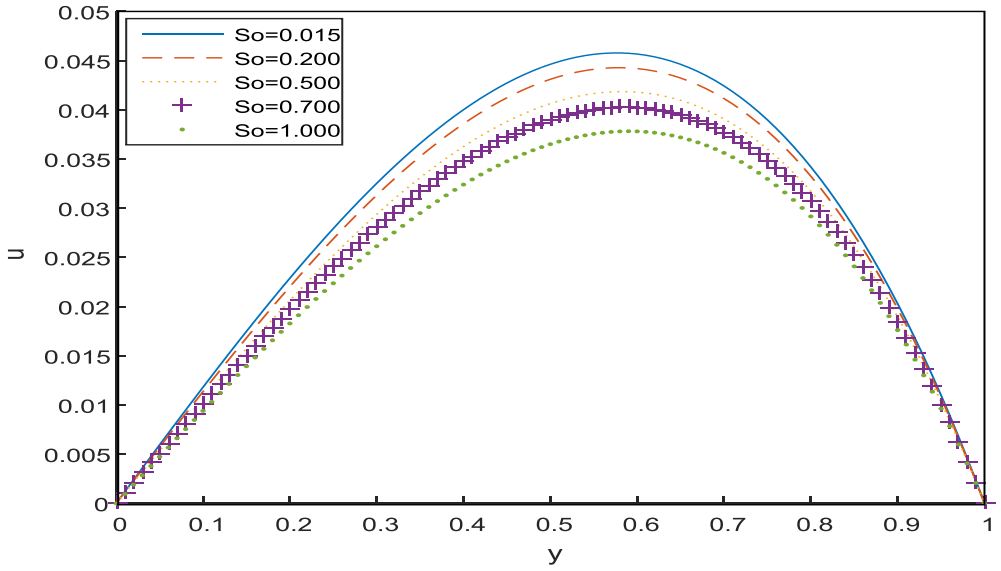


Figure 4.5: Variation of velocity with Soret number So **at**

$\delta = 2.0, Da = 0.5, \gamma = 4.0, N = 3.0$ **and** $a = \frac{\pi}{4}$

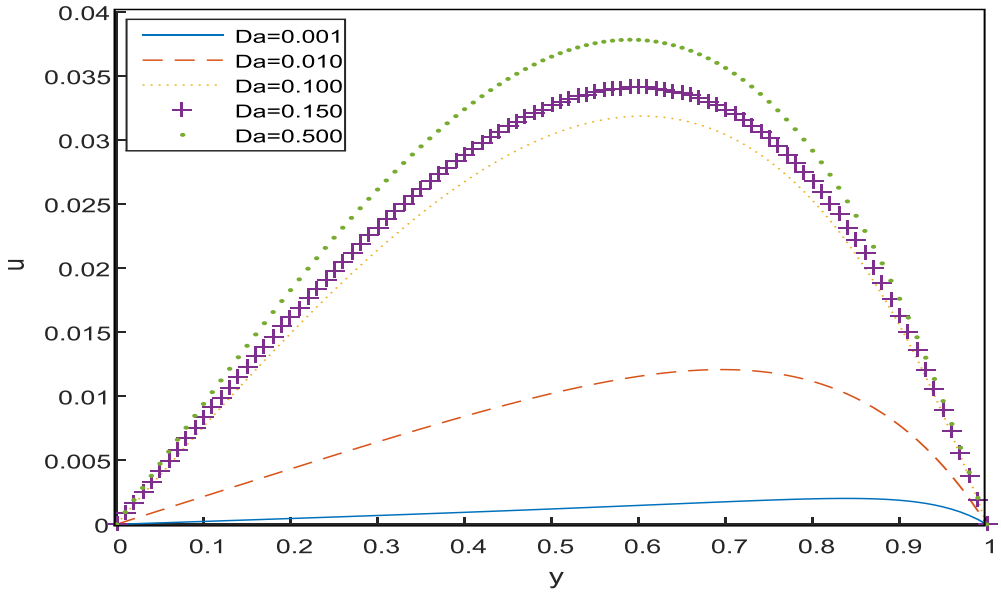


Figure 4.6: Variation of velocity with Darcy number Da **at**

$\delta = 2.0, So = 1.0, \gamma = 4.0, N = 3.0$ **and** $a = \frac{\pi}{4}$

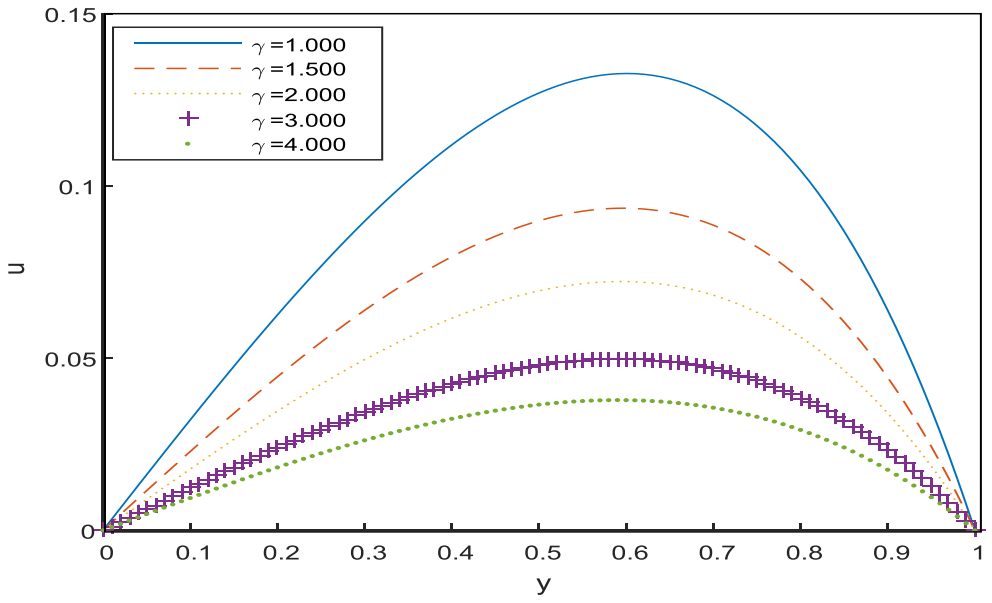


Figure 4.7: Variation of velocity with viscosity ratio γ at $\delta = 2.0, So = 1.0, Da = 0.5, N = 3.0$ and $a = \frac{\pi}{4}$

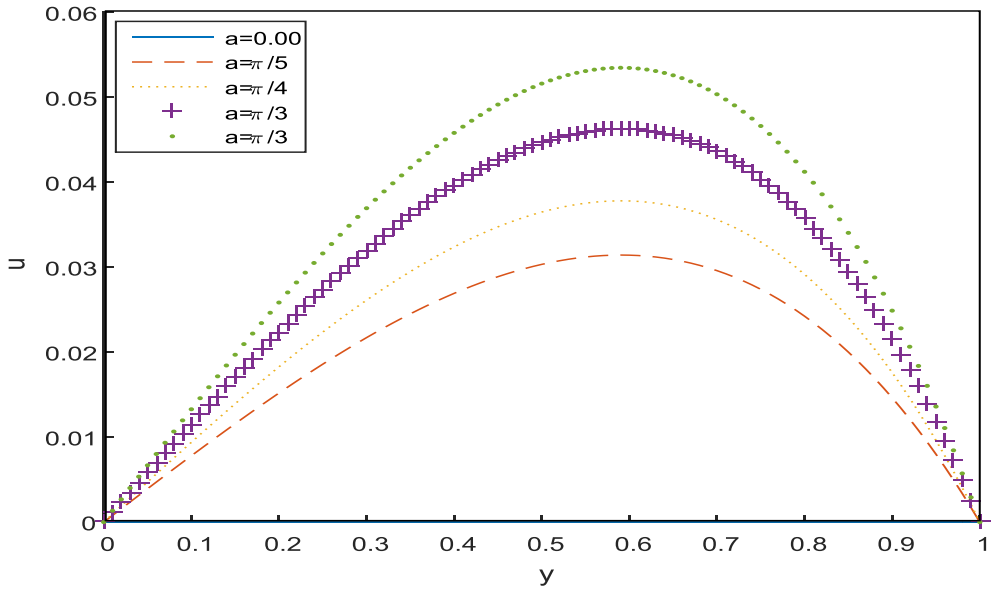


Figure 4.8: Variation of velocity with angle of inclination a at $\delta = 2.0, So = 1.0, Da = 0.5, \gamma = 4.0$ and $N = 3.0$

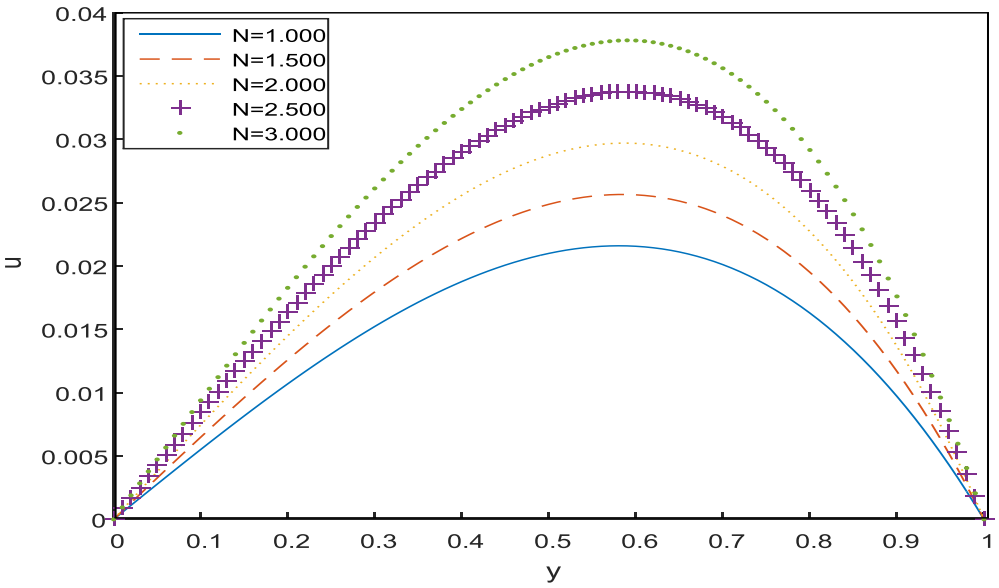


Figure 4.9: Variation of velocity with sustentation parameter N **at** $\delta = 2.0, So = 1.0, Da = 0.5, \gamma = 4.0$ **and** $a = \frac{\pi}{4}$

In order to show the effects of the basic physical parameters on skin friction, mass flux, Nusselt number and local Sherwood number, tables are depicted. Table 4.1 shows the effect of heat generation parameter (δ), Soret number (So), Darcy number (Da) and viscosity ratio (γ) on the skin friction on the surfaces of the two plates. This table clearly shows that the skin friction on both plates decreases with increase in the value of heat generation parameter and Soret number respectively. This is premised on the physical fact that fluid velocity decreases with growing δ. With the decrease in velocity caused by growing heat source, the shear stress decreases with it.The table also reveals that as the value of Darcy number increases, the skin friction increases on both plates while with increase in the value of viscosity ratio causes the skin friction to decrease on both boundary plates. This is true since growing the viscosity ratio causes a decrease in fluid velocity thereby decreasing the shear stress on the surfaces of the boundary plates. The effects of angle of inclination (a) and sustentation parameter (N) on the skin friction are shown

in table 4.2. It is observed that as the value of a increases, the skin friction increases on both plates. Further, with increase in the value of N, the skin friction increases on both boundary plates as well. This is because fluid velocity increases within the channel with an increase in the sustentation parameter. Also, the skin friction on the surface of vertical channel is highest because of velocity which is also highest when $a = \frac{\pi}{2}$.

Table 4.1: Effect of δ, So, Da and γ on skin friction ($N = 3.0$ and $a = \frac{\pi}{4}$)

	So	Da	$\gamma = 1$		$\gamma = 2$		$\gamma = 4$	
			τ_0	τ_1	τ_0	τ_1	τ_0	τ_1
$\delta = 0.5$	0.015	0.001	0.0029	0.0867	0.0029	0.0605	0.0029	0.0420
		0.100	0.2103	0.6189	0.1483	0.3669	0.0921	0.2055
		0.500	0.3864	0.8437	0.2140	0.4463	0.1130	0.2301
	0.500	0.001	0.0028	0.0866	0.0028	0.0604	0.0028	0.0418
		0.100	0.2052	0.6126	0.1449	0.3628	0.0900	0.2031
		0.500	0.3777	0.8336	0.2092	0.4408	0.1105	0.2272
	1.00	0.001	0.0027	0.0864	0.0027	0.0602	0.0027	0.0417
		0.100	0.2000	0.6062	0.1414	0.3586	0.0878	0.2006
		0.500	0.3687	0.8232	0.2043	0.4352	0.1079	0.2243
		0.001	0.0031	0.0871	0.0031	0.0609	0.0031	0.0423

$\delta = 2.0$	0.015	0.100	0.2230	0.6340	0.1569	0.3768	0.0973	0.2114
		0.500	0.4081	0.8683	0.2259	0.4597	0.1192	0.2371
	0.500	0.001	0.0027	0.0864	0.0027	0.0602	0.0027	0.0417
		0.100	0.1986	0.6047	0.1404	0.3576	0.0872	0.2000
		0.500	0.3663	0.8207	0.2030	0.4338	0.1072	0.2236
	1.00	0.001	0.0022	0.0856	0.0022	0.0594	0.0022	0.0410
		0.100	0.1734	0.5745	0.1233	0.3378	0.0769	0.1882
		0.500	0.3232	0.7717	0.1794	0.4072	0.0948	0.2096

Table 4.2: Effect of a and N on skin friction ($\delta = 2.0$, $So = 1.0$, $Da = 0.5$, and $\gamma = 4$)

a	$N = 1$		$N = 2$		$N = 3$	
	τ_0	τ_1	τ_0	τ_1	τ_0	τ_1
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$\frac{\pi}{5}$	0.0463	0.0948	0.0625	0.1346	0.0788	0.1743
$\frac{\pi}{4}$	0.0557	0.1141	0.0752	0.1619	0.0948	0.2096

The effects of heat generation parameter (δ), Soret number (So), Darcy number (Da) and viscosity ratio (γ) on the volumetric flow rate are shown in the table 4.3. It is found that increase in the value of heat generation parameter and Soret number respectively lead to decrease in the

volumetric flow rate. This is a direct consequence of velocity decrease which characterized the growth of these two parameters. It is also seen from this table that as the value of Darcy number increases, the volumetric flow rate increases while with increase in the value of viscosity ratio, the volumetric flow rate is found to decrease. A similar trend is observed for angle of inclination and sustentation parameter in which mass flux is highest with vertical orientation of the channel while it decreases as the inclination tilts towards a horizontal presentation as shown in table 4.4. The table further shows the mass flux grows with increase in sustentation parameter. This is due to the strengthening of convection due to mass transfer and subsequent velocity increase caused by increasing N .

Table 4.3: Effect of δ , So , Da and γ on mass flux ($N = 3.0$ and $a = \frac{\pi}{4}$)

	So	*Da*	$\gamma = 1$	$\gamma = 2$	$\gamma = 4$
$\delta = 0.5$	0.015	0.001	0.0013	0.0013	0.0012
		0.100	0.0600	0.0398	0.0239
		0.500	0.0995	0.0542	0.0284
	0.500	0.001	0.0013	0.0013	0.0012
		0.100	0.0589	0.0391	0.0234
		0.500	0.0976	0.0532	0.0279
	1.00	0.001	0.0013	0.0013	0.0012
		0.100	0.0577	0.0383	0.0230
		0.500	0.0957	0.0522	0.0273

$\delta = 2.0$	0.015	0.001 0.100 0.500	0.0014 0.0628 0.1041	0.0014 0.0417 0.0568	0.0013 0.0250 0.0297
	0.500	0.001 0.100 0.500	0.0013 0.0574 0.0952	0.0013 0.0381 0.0519	0.0012 0.0228 0.0272
	1.00	0.001 0.100 0.500	0.0012 0.0519 0.0860	0.0011 0.0345 0.0469	0.0011 0.0206 0.0245

Table 4.4: Effect of a and N on mass flux ($\delta = 2.0,\ So = 1.0,\ Da = 0.5,$ and $\gamma = 4$)

a	$N = 1$	$N = 2$	$N = 3$
0	0.0000	0.0000	0.0000
$\frac{\pi}{5}$	0.0117	0.0160	0.0204
$\frac{\pi}{4}$	0.0140	0.0193	0.0245

The numerical values of Nu_0 and Nu_1 are listed in Table 4.5 for different values of heat generation parameter.

TABLE 4.5: NUMERICAL VALUES OF Nu_0 AND Nu_1

δ	Nu_0	Nu_1
0.50	1.0885	0.8275
0.75	1.1369	0.7365
1.00	1.1884	0.6421
1.50	1.3019	0.4416
2.00	1.4317	0.2233

We observe from table 4.5 that an increase in heat generation parameter δ leads to an increase in the Nusselt number Nu_0 whereas under similar conditions, the value of the Nusselt number Nu_1 decreases. This is attributed to temperature increase with growing heat generation which in turn increases the temperature difference between the fluid and the cold plate $(y = 0)$. In the same situation, the temperature difference is being decreased between the warming fluid and the heated boundary plate. We have evaluated sh_0 and sh_1 numerically and these are listed in Table 4.6.

TABLE 4.6: NUMERICAL VALUES OF sh_0 AND sh_1

$So = 1.0$ $\delta = 2.0$					
δ	sh_0	sh_1	So	sh_0	sh_1
0.50	0.9115	1.1725	0.015	0.9935	1.0117
0.75	0.8631	1.2635	0.200	0.9137	1.1553
1.00	0.8116	1.3579	0.500	0.7841	1.3884
1.50	0.6981	1.5584	0.700	0.6978	1.5437
2.00	0.5683	1.7767	1.000	0.5683	1.7767

We observe from table 4.6that the Sherwood number decreases on the plate at $y = 0$ with increasing the value of heat generation parameter but rises on the plate at $y = 1$. This is physically true since fluid concentration decreases with increase in heat generation. This transitively decreases the gradient of concentration on the surface of the cold plate $(y = 0)$while it increases the concentration difference near the heated plate. Further with increase in the value of Soret number, the Sherwood number decreases on the plate at $y = 0$ and increases on the plate at $y = 1$. This is due to the fact that fluid concentration decreases with growing Soret effect.

4.3 Transpiration and Soret effects on thermo-solutal convection flow of heat generating fluid in an inclined channel filled with porous material

It is seen from equations (3.22)-(3.40) that the basic parameters controlling the present study are heat generation parameter(δ), suction/injection parameter(Vo), Prandtl number(Pr), Soret parameter(So), Schmidt number(Sc), porous medium permeability(Da), viscosity ratio(γ), the channel inclination angle(a) and sustentation parameter(N).In the present study we have chosen the values of the heat generation parameter(δ)between 0.25 and 1.00; the suction/injection parameter(Vo) between -1.00 and 1.00; Prandtl number(Pr) between 0.01 and 2.0; Soret parameter (So)between 0.100 and 2.00; Schmidt number$(Sc)$is taken between 0.01 and 2.00;Porous medium permeability(Da) is taken between 0.001 and 0.10; Viscosity ratio(γ) between 1.00 and 2.00; Sustentation parameter$(N)$is taken between -2.00 and 2.00 and the channel inclination angle$(a)$is taken between 0 and $\frac{\pi}{2}$. If $a = 0,$the channel is horizontal and the natural convection effect is not considered while $a = \frac{\pi}{2}$ denotes natural convection in a vertical

channel. The temperature profiles for different values of heat generation parameter (δ) are described in figure 4.10. It is seen that an increase in δ leads to an increase in fluid temperature within the channel. This is evident from the fact that the heat generation parameter is temperature dependent. Therefore, an increase in δ acts to amplify the supplied heat flux from the porous channel plates. The temperature profiles for different values of suction/injection parameter (Vo) are described in figure 4.11. We observed from this figure that temperature rises when injection of fluid takes place through the heated plate while it decreases when suction takes place through the heated plate $(y=1)$. This result is due to the fact that an increase in injection through the heated plate is to inject warm fluid in to the system and taking out cold fluid through the cold plate $(y=0)$. On the other hand, for $Vo>0$, warm fluid is taken away from the system and replaced with cold fluid by injection through the cold plate. This causes the temperature to rise for $Vo<0$ and fall for $Vo>0$. For different values of Prandtl number (Pr), the temperature profiles are plotted in figure 4.12. Prandtl number defines the ratio of momentum diffusivity to thermal diffusivity. It is observed from figure 4.12 that the temperature decreases with an increase in Pr. This implies that an increase in Prandtl number leads to a fall in the thermal boundary layer flow. This is because fluid with large Pr have low thermal diffusivity which causes low heat generation resulting in reduced thermal boundary layer. The effect of heat generation parameter (δ) on concentration profiles is shown in figure 4.13. From this figure we see that as δ increases the concentration profiles decrease. Physically, this is true since an increase in δ increases the temperature which increases the diffusion of the concentration due to Soret effect resulting in decrease in the concentration in the channel. Figure 4.14 shows the concentration profiles with suction parameter (Vo), it is seen that concentration profile is found to decrease with increase in suction through the cold porous plate $(Vo<0)$. However, in the case

of injection through the heated porous plate $(V0 > 0)$, fluid concentration increases. Figure 4.15 illustrates the effect of Prandtl number (Pr) on concentration profiles for different values. We see that the concentration increases as parameter Pr increases. Prandtl number is a thermophysical property whose increase acts to decrease fluid temperature thereby decreasing Soret effect resulting in increase in the concentration. Figure 4.16 represents the concentration profiles for different values of Soret number (So). From this figure we observe that the concentration profiles decrease with an increase in Soret number. Figure 4.17 shows the effect of Schmidt number (Sc) on concentration profiles. It is found that an increase in Sc leads to a decrease in the concentration distribution. This is because increasing the values of Sc is equivalent to decreasing the chemical molecular diffusivity which acts to decrease the solutal boundary layer. Figure 4.18-4.26 show the velocity profiles with the parameters occurring in the governing equations. Figure 4.18 shows the velocity profiles across the channel for different values of heat generation parameter (δ). The results show that the effect of increasing values of δ results in increasing the velocity. This is physically true since increase in heat generation parameter contributes an increase in temperature as well as an increase in the fluid concentration. This tends to act in favour of the thermal buoyancy as well as buoyancy due to fluid concentration. The resultant effect is the strengthening of the convection current within the channel and hence velocity increase is recorded. The effect of suction parameter (Vo) on the velocity profiles is shown in figure 4.19. From this figure we observe that velocity profiles increase whenever injection of fluid into the system takes place through the heated plate $(Vo < 0)$ On the other hand, for $Vo > 0$, which denotes suction through the heated plate, it is observed that fluid velocity decreases with growing Vo. This clearly explains the direct relationship between fluid temperature which increases with growing injection of warm fluid through the heated plate and velocity that

increases under the same physical condition. Figure 4.20 shows the velocity profiles for different values of Prandtl number $\Pr$. An increase in $\Pr$ leads to decrease the velocity profiles. This happens because the increase in Prandtl number is due to increase in viscosity of the fluid which makes the fluid thick so that diffusivities of heat and diffusing chemical specie are decreased and hence causes a decrease in the velocity of the fluid. Figure 4.21 shows the effect of Soret number (So) on the velocity distribution. The Soret number defines the effect of the temperature gradients inducing significant mass diffusion effects. It can be seen that the velocity profiles decreases with an increase in So. For different values of Schmidt number (Sc) ,the velocity profiles are plotted in figure 4.22. It is obvious that the effect of increasing values of Sc results in decreasing velocity distribution in the channel. This is the consequence of decrease in mass diffusivity of the fluid which decreases the boundary layer of the diffusing specie and hence buoyancy force due to mass transfer decreases resulting in overall decrease in fluid velocity. Figure 4.23 depicts the effect of Darcy number (Da) on the velocity profiles. It is observed from this figure that as Da increases, the velocity profiles increase. The physical consequence of increase in the permeability parameter Da rests on the fact that increasing Da signifies an increase in the diameters of the pores of the medium through which fluid flow occurs. This means that resistance to flow offered by the porous material are gradually decreasing so that fluid velocity increases with growing Da. Figure 4.24 shows the effect of viscosity ratio (γ) on the velocity profiles. It is observed from this figure that an increase in the values of γ leads to a fall in the values of velocity. This is the response of fluid flow to increase in the effective viscosity of the porous medium. It is to be noted that ϑ_{eff} constitutes a certain drag to the fluid motion as it moves through the pores of the porous medium. This drag effect increases with growing γ so that fluid velocity decreases as γ increases. The effects of sustentation parameter

(N) are shown in figure 4.25. We observe from this figure that the velocity decreases when N is negative and increases when N is positive. It should be noted that for $N > 0$, the buoyancy effects due to heat transfer and that of mass transfer are supporting each other, therefore as $N > 0$ increases, the convection current due to mass transfer is strengthened and velocity increase is achieved. On the other hand, for negative value of N, convection current due to mass transfer acts in opposition to that of heat transfer and this acts to retard the fluid motion. Figure 4.26 shows the velocity profiles for different values of angle of inclination (a). We observe that the velocity profiles increase significantly with an increase in a. It should be noted that when the angle of inclination $(a = 0)$, the channel is in horizontal presentation and the fluid remains at rest. However, when $a = \frac{\pi}{2}$, the channel is vertical and the buoyancy influence is maximum so that fluid velocity become maximum.

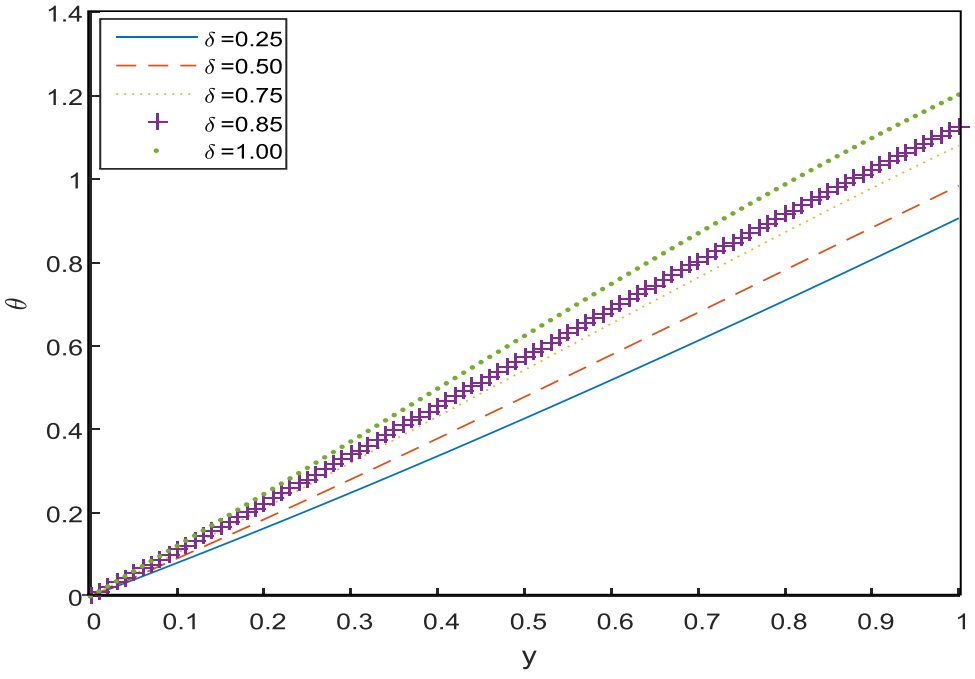


Figure 4.10: Temperature profiles for different values of δ at $Vo = 0.5$ and $\Pr = 0.71$

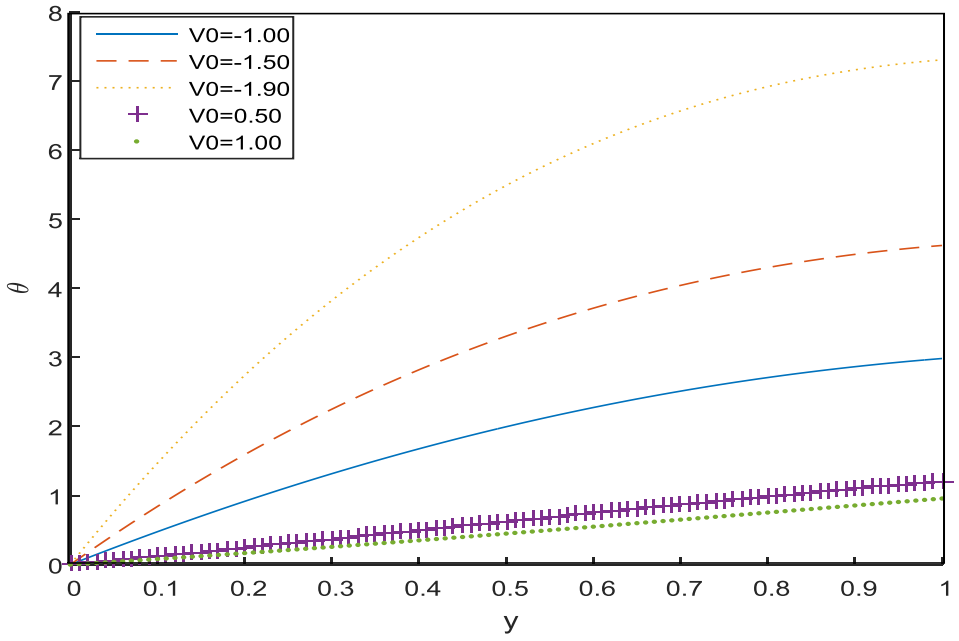


Figure 4.11: Temperature profiles for different values of V_0 at $\delta = 1.0$ and $\Pr = 0.71$

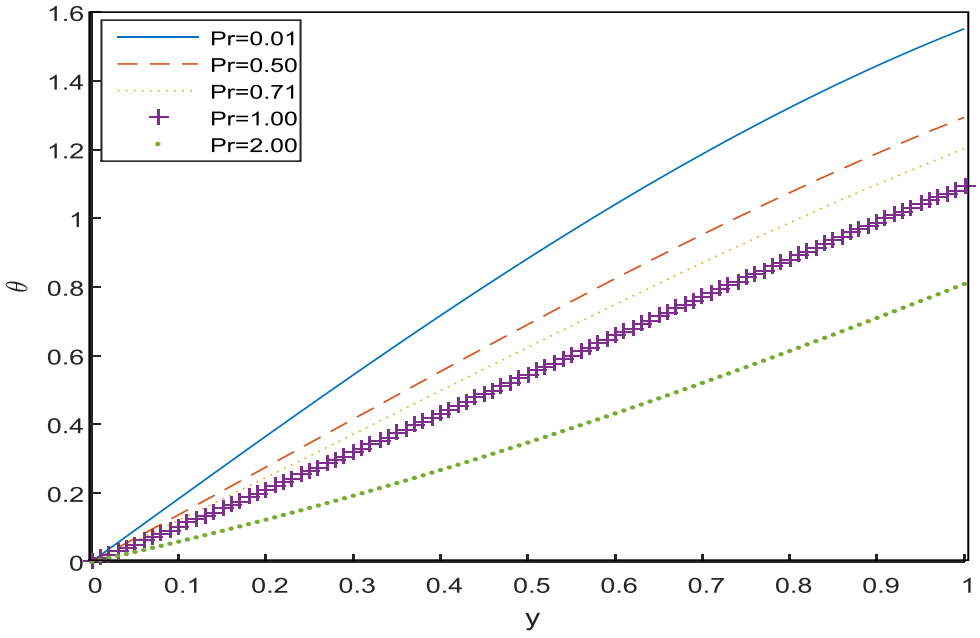


Figure 4.12 Temperature profiles for different values of $\Pr$ at $\delta = 1.0$ and $Vo = 0.5$

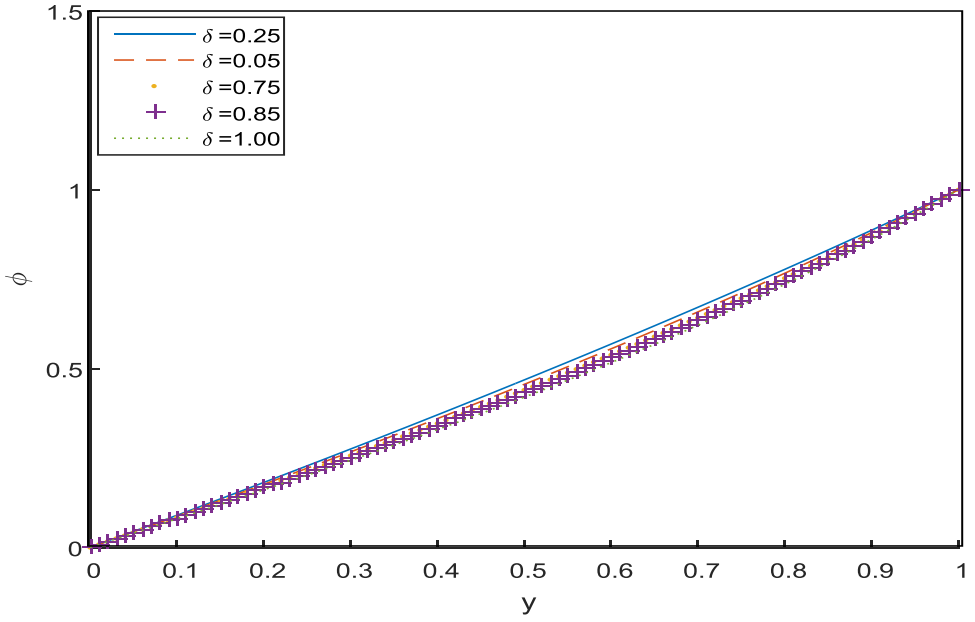


Figure 4.13: Concentration profiles for different values of δ at $Vo = 0.5, \Pr = 0.71, So = 1.0$ and $Sc = 0.94$

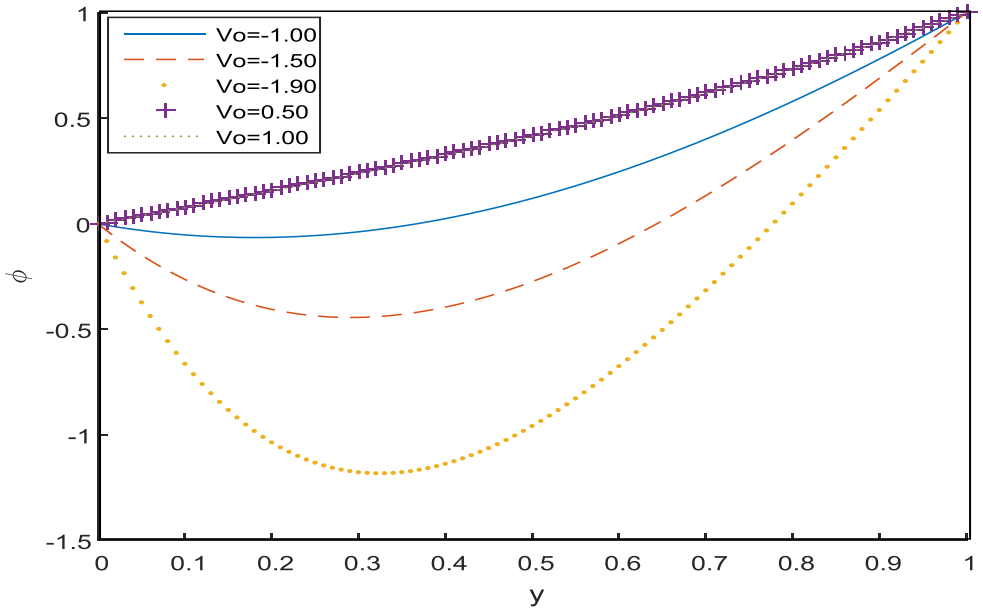


Figure 4.14: Concentration profiles for different values of Vo at $\delta = 1.0, \Pr = 0.71, So = 1.0$ and $Sc = 0.94$

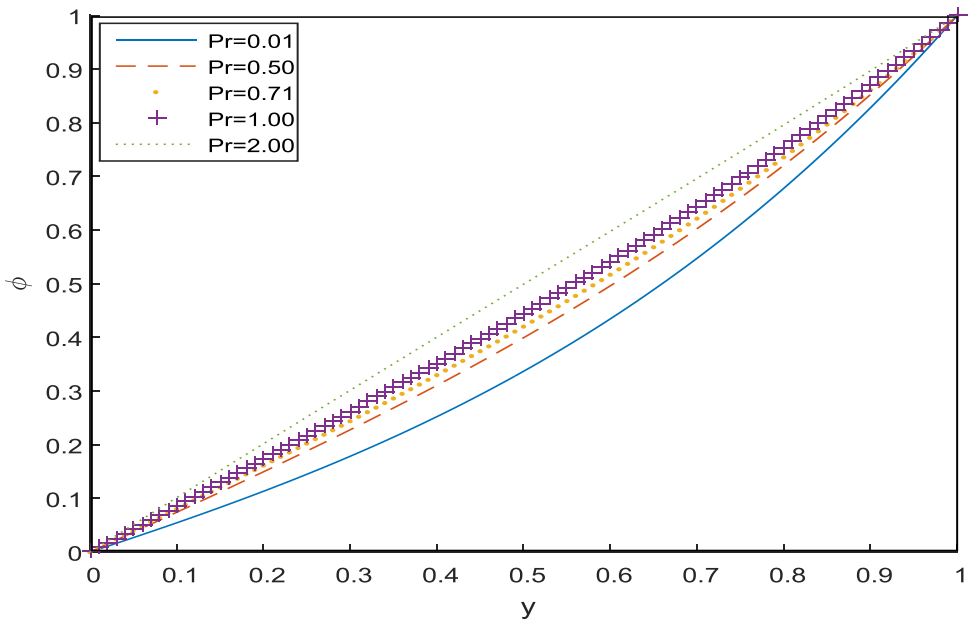


Figure 4.15: Concentration profiles for different values of $\Pr$ **at** $\delta = 1.0, Vo = 0.5, So = 1.0$ **and** $Sc = 0.94$

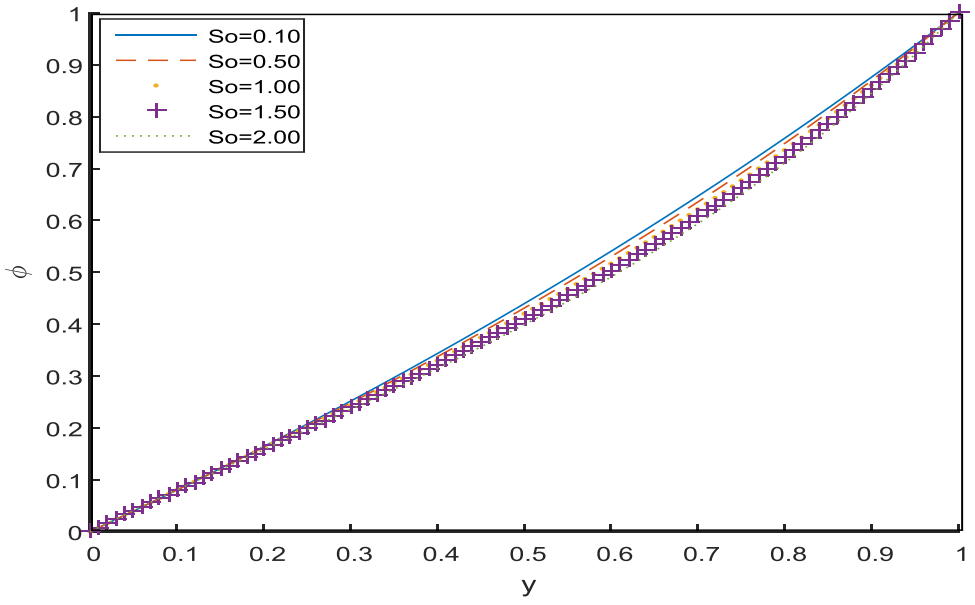


Figure 4.16: Concentration profiles for different values of So **at** $\delta = 1.0, Vo = 0.5 \Pr = 0.71$ **and** $Sc = 0.94$

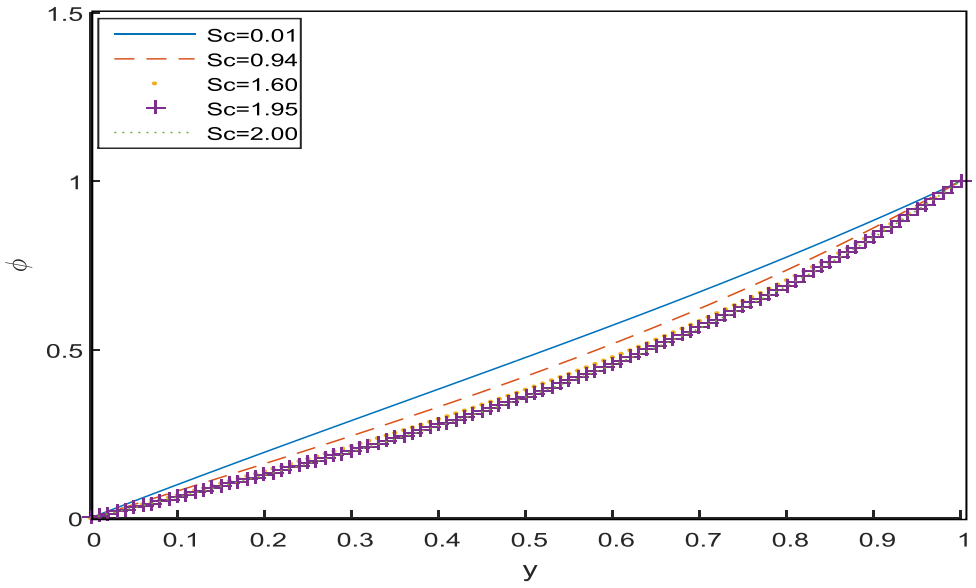


Figure 4.17: Concentration profiles for different values of Sc **at** $\delta = 1.0, Vo = 0.5 \Pr = 0.71$ **and** $So = 1.0$

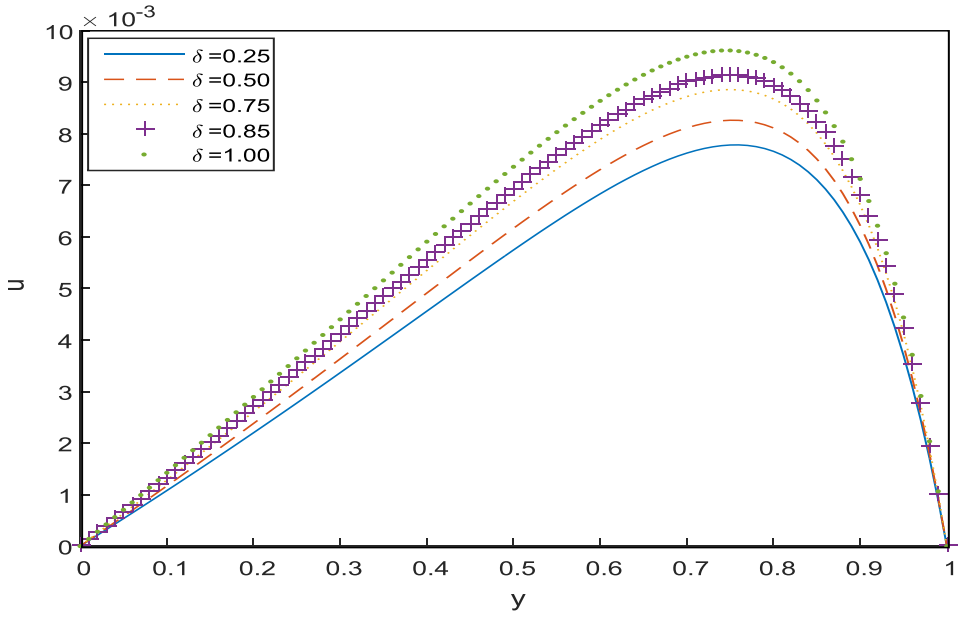


Figure 4.18: Velocity profiles for different values of δ **at**

$Vo = 0.5, \Pr = 0.71, So = 1.0, Sc = 0.94, Da = 0.01, \gamma = 1.5, N = 1.0$ **and** $a = \frac{\pi}{4}$

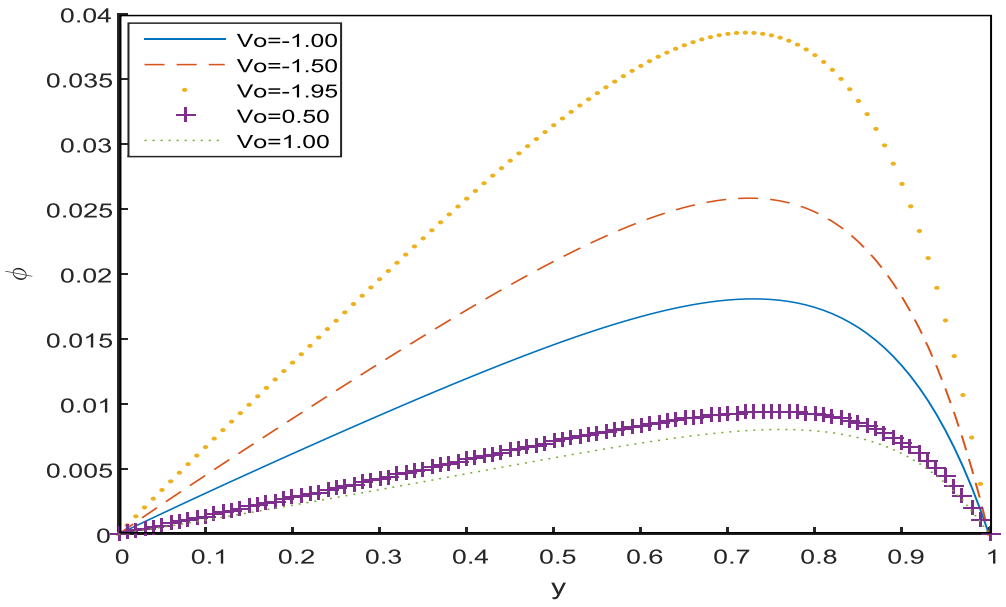


Figure 4.19: Velocity profiles for different values of Vo at $\delta = 1.0, \Pr = 0.71, So = 1.0, Sc = 0.94, Da = 0.01, \gamma = 1.5, N = 1.0$ and $a = \frac{\pi}{4}$

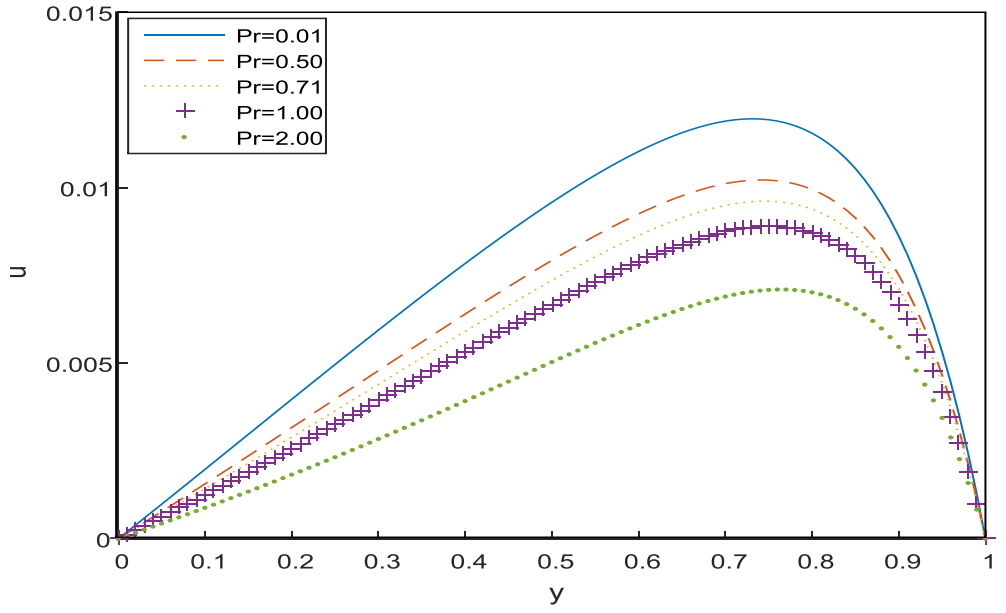


Figure 4.20: Velocity profiles for different values of $\Pr$ at $\delta = 1.0, Vo = 0.5, So = 1.0, Sc = 0.94, Da = 0.01, \gamma = 1.5, N = 1.0$ and $a = \frac{\pi}{4}$

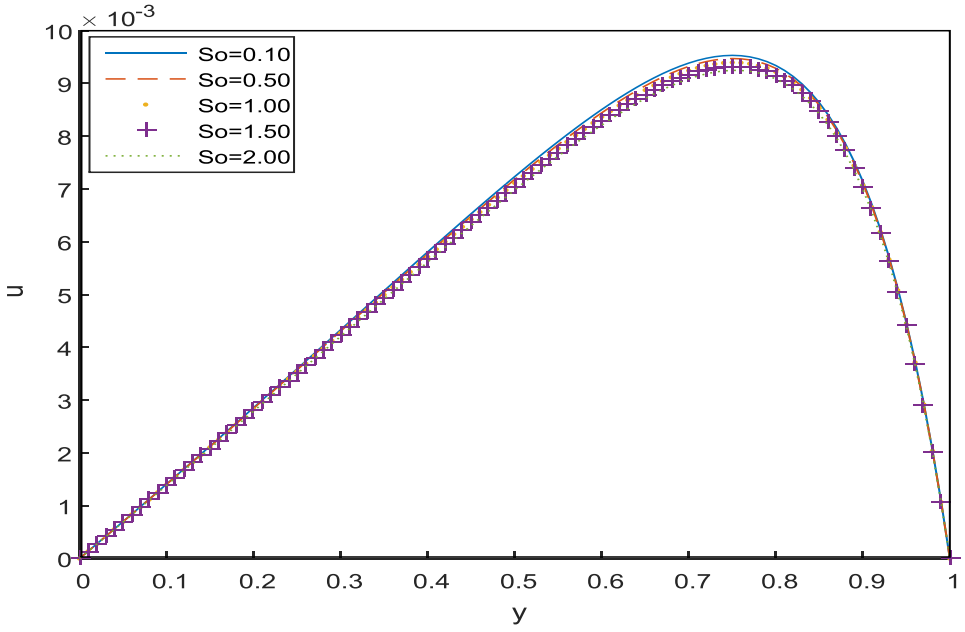


Figure 4.21: Velocity profiles fordifferent values of So **at** $\delta = 1.0, Vo = 0.5, \Pr = 0.71 Sc = 0.94, Da = 0.01, \gamma = 1.5, N = 1.0$ **and** $a = \frac{\pi}{4}$

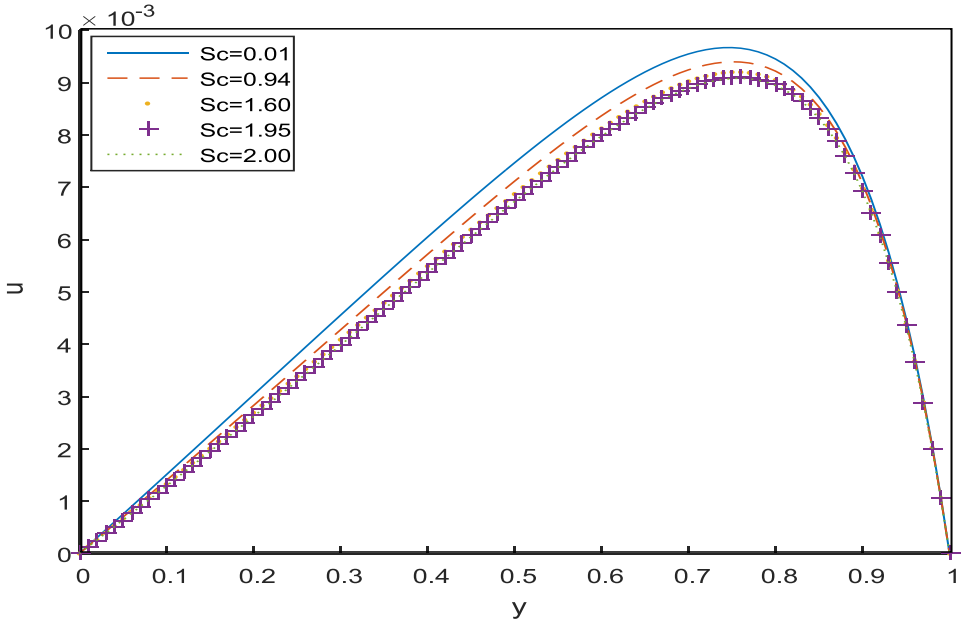


Figure 4.22: Velocity profiles for different values of Sc **at** $\delta = 1.0, Vo = 0.5, \Pr = 0.71, So = 1.0, Da = 0.01, \gamma = 1.5, N = 1.0$ **and** $a = \frac{\pi}{4}$

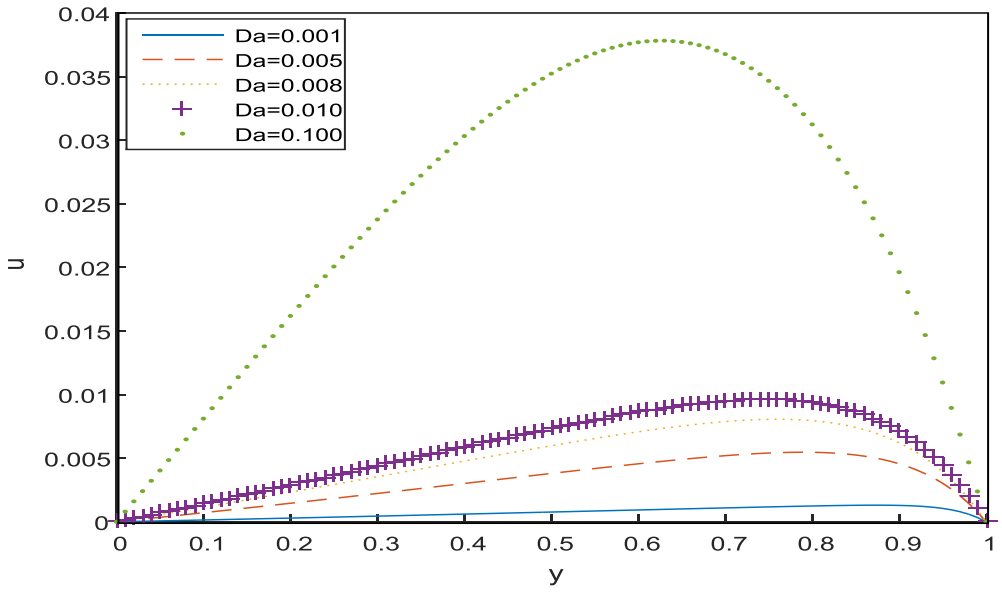


Figure 4.23: Velocity profiles for different values of Da **at** $\delta = 1.0, Vo = 0.5, \Pr = 0.71, So = 1.0, Sc = 0.94, \gamma = 1.5, N = 1.0$ **and** $a = \frac{\pi}{4}$

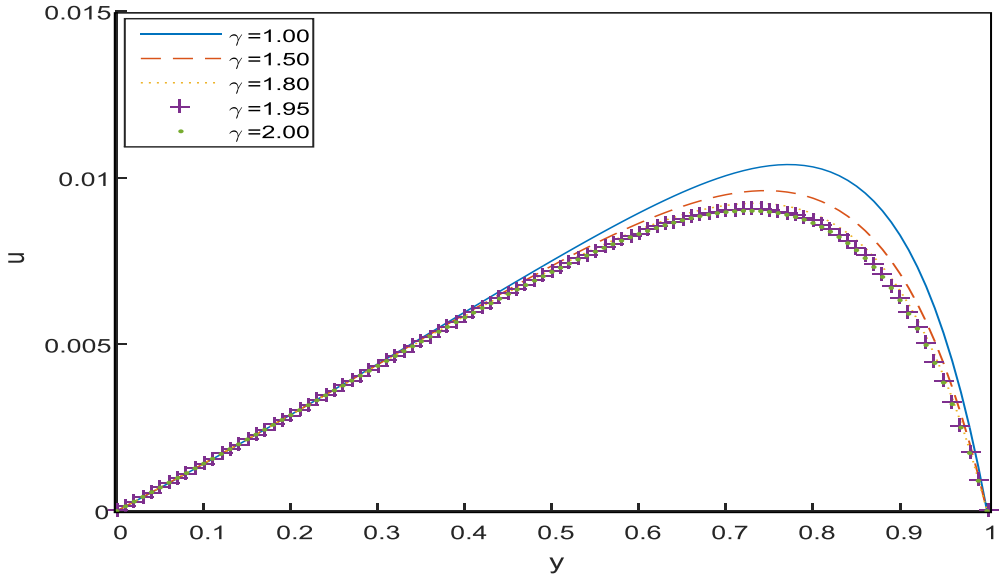


Figure 4.24: Velocity profiles for different values of γ **at** $\delta = 1.0, Vo = 0.5, \Pr = 0.71, So = 1.0, Sc = 0.94, Da = 0.01, N = 1.0$ **and** $a = \frac{\pi}{4}$

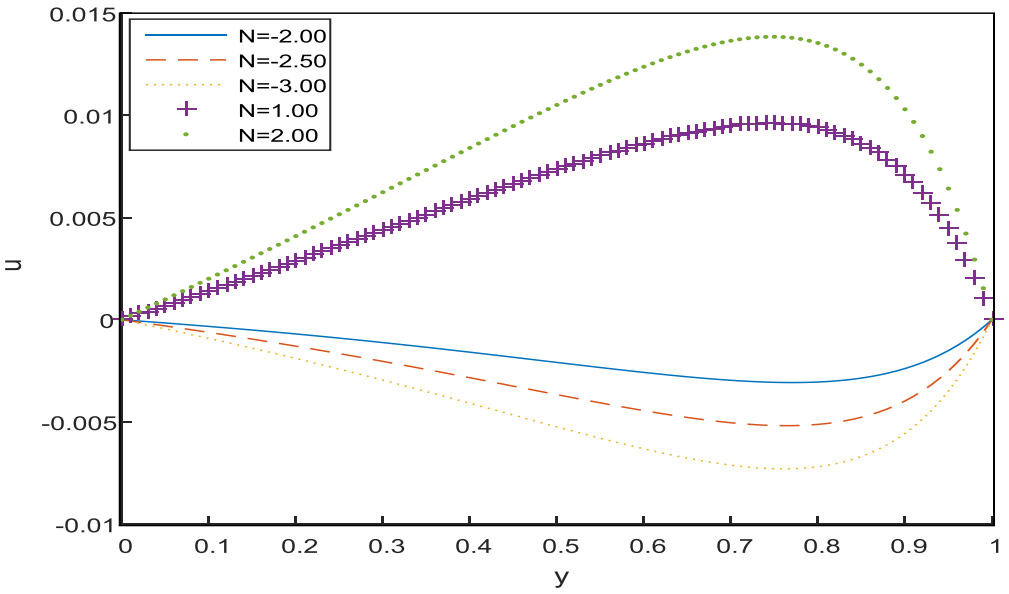


Figure 4.25: Velocity profiles for different values of N at $\delta = 1.0, Vo = 0.5, \Pr = 0.71, So = 1.0, Sc = 0.94, Da = 0.01, \gamma = 1.5$ and $a = \frac{\pi}{4}$

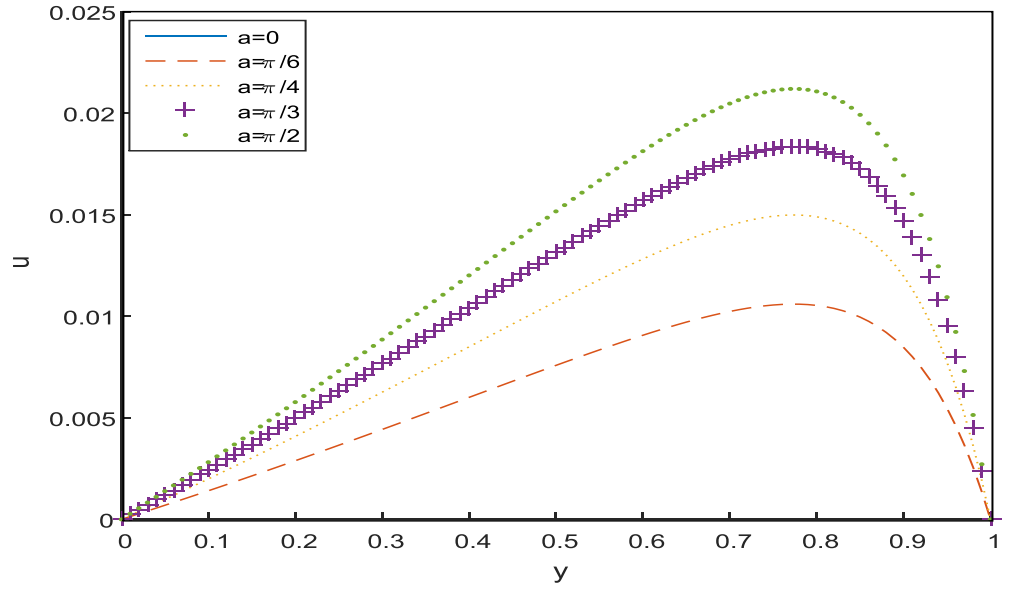


Figure 4.26: Velocity profiles for different values of a at $\delta = 1.0, Vo = 0.5, \Pr = 0.71, So = 1.0, Sc = 0.94, Da = 0.01, \gamma = 1.5$ and $N = 1.0$

Numerical simulations were done on the skin-friction using the expressions in equations (3.35) and (3.36) and results of the simulation was presented in tables 4.7-4.9.

Table 4.7 depicts the effects of heat generation parameter, suction parameter, Prandtl number and Soret number on the skin-friction on the two plates. It is observed from this table that as δ increases, the skin-friction increases on both channel plates. This is due to the fact that fluid velocity increases with growing heat generation parameter. Subsequent to this, velocity gradient and hence skin-friction increases on both boundary plates as δ increases. The table further reveals that skin-friction increases on both plates as injection increases through the heated plate $(Vo < 0)$ while skin-friction decreases on the surface of both plates as suction is increased through the heated plate. The physical explanation for such a behavior, is that in case of suction, the heated fluid is pushed towards the wall where the buoyancy forces can act to retard the fluid due to high influence of viscosity. This effect acts to decrease the wall shear stress and local skin-friction. In addition, the table shows the effects of Prandtl number and Soret number on the skin-friction. This table clearly shows that with increase in the value of $\Pr$, the skin-friction decreases on both plates. This is the consequence of temperature decrease which causes decrease in fluid velocity and transitively, a decrease in the shear stress on the boundary surface of the channel. Further, an increase in the value of So causes the skin-friction to decrease on both plates. The effects of Schmidt number, Darcy number, viscosity ratio and sustentation parameter on the skin-friction are shown in table 4.8. It is seen from this table that as Sc increases, the skin-friction decreases on both plates while it increases on both plates with increase in the value of Da. We also observe from this table that as γ increases, the skin-friction increases on the plate at $y = 0$ and decreases on the plate at $y = 1$. However, with increase in the value of N, the skin-friction increases on both plates. This is expected since an increase in the viscosity ratio

parameter γ brings about a decrease in the fluid velocity. The influence of γ as reveals in figure 4.24 is clear at fluid sections adjacent to the heated plate. However, the influence is negligible at fluid sections near the boundary surfaces. This explains the negligible change observed in the skin-friction on the surface of both plates. Table 4.9 shows the effect of inclination angle on the skin-friction. We observe that as a increases, the skin-friction increases on both the plates. That is, the skin-friction is maximum when the channel orientation is vertical. This is because convection currents are maximum and fluid velocity highest in vertical orientation of the channel.

Table4.7: Effects of $\delta, V_0, \Pr$ **and** So **on skin friction** $\left(Sc = 0.94, Da = 0.01, \gamma = 1.5, N = 1.0, a = \frac{\pi}{4} \right)$

	V_0	$\Pr = 0.71$ and $So = 1.0$		$\Pr = 0.71$ and $So = 2.0$		$\Pr = 1.0$ and $So = 2.0$	
		τ_0	τ_1	τ_0	τ_1	τ_0	τ_1
$\delta = 0.75$	-1.00	0.0277	0.1660	0.0193	0.1600	0.0185	0.1921
	-1.50	0.0370	0.2094	0.0200	0.2001	0.0154	0.2944
	0.50	0.0129	0.1059	0.0131	0.1057	0.0128	0.1015
	1.00	0.0100	0.0958	0.0109	0.0967	0.0103	0.0908
$\delta = 1.00$	-1.00	0.0324	0.1980	0.0201	0.1890	0.0191	0.2448
	-1.50	0.0470	0.2761	0.0202	0.2611	0.0110	0.5038

	0.50	0.0138	0.1122	0.0137	0.1113	0.0133	0.1063
	1.00	0.0106	0.0999	0.0113	0.1004	0.0106	0.0934

Table 4.8: Effects of Sc, Da, γ and N on skin friction $\left(\delta = 1.0, V_0 = 0.5, \Pr = 0.71, So = 1.0, a = \frac{\pi}{4}\right)$

	Da	$Sc = 0.94$ and $N = 1.0$		$Sc = 0.94$ and $N = 2.0$		$Sc = 2.0$ and $N = 2.0$	
		τ_0	τ_1	τ_0	τ_1	τ_0	τ_1
$\gamma = 1.0$	0.001	0.0014	0.0482	0.0020	0.0700	0.0017	0.0694
	0.005	0.0064	0.1032	0.0096	0.1491	0.0084	0.1472
	0.010	0.0136	0.1418	0.0191	0.2045	0.0168	0.2009
	0.100	0.0962	0.3473	0.1351	0.4967	0.1227	0.4792
	0.001	0.0014	0.0333	0.0020	0.0482	0.0017	0.0478
	0.005	0.0070	0.0701	0.0098	0.1011	0.0086	0.0994

$\gamma = 2.0$	0.010	0.0138	0.0947	0.0193	0.1362	0.0172	0.1331
	0.100	0.0717	0.2011	0.1008	0.2873	0.0921	0.2762

Table 4.9: Effects of inclination angle on skin-friction

$\delta = 1.0, Vo = 0.5, \Pr = 0.71, So = 1.0, Sc = 0.94, Da = 0.01, \gamma = 1.5, N = 3.5.$

a	τ_0	τ_1
0	0.0000	0.0000
$\frac{\pi}{6}$	0.0097	0.0793
$\frac{\pi}{4}$	0.0138	0.1122
$\frac{\pi}{3}$	0.0169	0.1374
$\frac{\pi}{2}$	0.0195	0.1587

The expression for the volumetric flow rate equation (3.37) is presented numerically for different values of the governing parameters and its values are presented in tables 4.10 and 4.11.

The effects of heat generation parameter, suction parameter, Prandtl number, Soret number and Schmidt number on volumetric flow rate is presented in table 4.10. We observe that as the value of δ increases, the volumetric flow rate increases. However, as injection of warm fluid increases through the heated plate, the volumetric flow rate increases while it decreases as suction grows through the heated plate. From the table,it is found that an increase in the value of $\Pr$ leads to decrease in the volumetric flow rate while an increase in the value of So causes the volumetric flow rate to decreases. The table further shows that as the value of Sc increases, the volumetric flow rate decreases. The effects of viscosity ratio, sustentation parameter, Darcy number and channel inclination on volumetric flow rate are shown in table 4.11. This table shows that the volumetric flow rate decreases with increase in the value of γ and increases with increase in the value of N . This actually shows the resultant effect of increase in the effective viscosity of the porous medium which was found to institute a drag effect and subsequently decrease the fluid velocity within the inclined channel. The table further reveals that the volumetric flow rate increases with increase in the permeability parameter Da . However, it is found that as angle of inclination(a) increases, the volumetric flow rate is found to increase. This is due to the physical fact that growing the porosity of the porous medium causes an increase in the velocity of flowing fluid.

Table4.10: Effects of $\delta, V_0, \Pr, So$ and Sc on volumetric flow rate $\left(Da = 0.01, \gamma = 1.5, N = 1.0, a = \frac{\pi}{4} \right)$

	V_0	$\Pr = 0.71, So = 1.0$ and $Sc = 0.94$	$\Pr = 0.71, So = 1.0$ and $Sc = 2.0$	$\Pr = 0.71, So = 2.0$ and $Sc = 2.0$	$\Pr = 1.00, So = 2.0$ and $Sc = 2.0$

$\delta = 0.75$	-1.00	0.0095	0.0100	0.0087	0.0099
	-1.50	0.0123	0.0132	0.0111	0.0150
	0.50	0.0053	0.0050	0.0050	0.0049
	1.00	0.0045	0.0040	0.0042	0.0039
$\delta = 1.00$	-1.00	0.0112	0.0118	0.0099	0.0120
	-1.50	0.0160	0.0171	0.0137	0.0239
	0.50	0.0056	0.0054	0.0053	0.0051
	1.00	0.0047	0.0042	0.0044	0.0041

Table 4.11:Effects of γ , N , Da, and $a = \frac{\pi}{4}$ on volumetric flow rate

$(\delta = 1.0, V_0 = 0.5, \Pr = 0.71, So = 1.0, Sc = 0.94)$

	Da	$N = 1.00$ and $a = \frac{\pi}{6}$	$N = 1.00$ and $a = \frac{\pi}{4}$	$N = 2.00$ and $a = \frac{\pi}{4}$
$\gamma = 1.0$	0.001	4.9647e-04	7.0212e-04	9.9542e-04
	0.005	0.0023	0.0032	0.0045
	0.010	0.0042	0.0060	0.0084
	0.100	0.0218	0.0308	0.0436

$\gamma = 2.0$	0.001	4.8248e-04	6.8233e-04	9.6709e-04
	0.005	0.0021	0.0030	0.0042
	0.010	0.0038	0.0053	0.0076
	0.100	0.0146	0.0206	0.0291

Table 4.12 shows the effects of heat generation parameter, suction parameter and Prandtl number on Nusselt number. It is observed that increasing δ increases the Nusselt number on the cold plate (Nu_0). This is physically true since growing heat generation parameter acts to increase temperature difference between the warm fluid and the plate $y = 0$. The influence of suction and injection on Nusselt number on the cold plate shows that heat transfer increases on the cold plate whenever injection takes place through the heated plate. This is due to the fact that growing $(Vo < 0)$ injection increases the fluid temperature which increase the temperature gradient on the cold plate and decreases whenever suction takes place. Also, growing Pr has been observed to exhibit a decreasing effect on the heat transfer on the cold surface. This is due to the decreasing thermal diffusivity and low temperature which characterizes fluid with large Prandtl number.

Table 4.12: Effects of δ, V_0 and Pr on Nusselt number

	V_0	Pr = 0.01		Pr = 0.71		Pr = 1.00	
		Nu_0	Nu_1	Nu_0	Nu_1	Nu_0	Nu_1
	-1.00	1.5619	1.0000	3.7074	1.0000	5.4796	1.0000

$\delta = 0.75$	-1.50	1.5711	1.0000	6.0046	1.0000	11.7010	1.0000
	0.50	1.5345	1.0000	1.0227	1.0000	0.8674	1.0000
	1.00	1.5255	1.0000	0.6853	1.0000	0.4973	1.0000
$\delta = 1.00$	-1.00	1.8747	1.0000	4.9448	1.0000	7.9244	1.0000
	-1.50	1.8867	1.0000	8.8936	1.0000	22.8584	1.0000
	0.50	1.8390	1.0000	1.1896	1.0000	0.9991	1.0000
	1.00	1.8273	1.0000	0.7796	1.0000	0.5576	1.0000

The effects of heat generation parameter, suction parameter, Prandtl number, Soret numberand Schmidt number on Sherwood number are shown in table 4.13. We observe from this table that as δ increases, the value of Sh_0 decreases while the value of Sh_1 increases. The table also reveals that both Sh_0 and Sh_1 increases when $Vo < 0$ while Sh_0 decreases and Sh_1 increases when $Vo > 0$. We also observe from this table that as $\Pr$ increases, the value of the Sherwood number Sh_0 increases and the value of the Sherwood number Sh_1 decreases while with increase in the value of So, both the Sherwood number Sh_0 and Sh_1 increases. It is also seen from this table that as Sc increases, the Sherwood number Sh_0 decreases and Sh_1 increases.

Table 4.13: Effects of $\delta, V_0, \Pr, So$ and Sc on Sherwood number

	So	$\Pr = 0.71, V_0 = 0.5$ and $Sc = 0.94$	$\Pr = 0.71, V_0 = 0.5$ and $Sc = 2.00$	$\Pr = 0.71, V_0 = 1.00$ and $Sc = 2.00$	$\Pr = 1.00, V_0 = 1.0$ and $Sc = 2.00$

		Sh_0 Sh_1		Sh_0 Sh_1		Sh_0 Sh_1		Sh_0 Sh_1	
$\delta = 0.75$	0.10	0.7891	1.2614	0.5878	1.5900	0.3271	2.2956	0.3306	2.2804
	0.50	0.8123	1.2937	0.6109	1.6223	0.3834	2.2260	0.4010	2.1497
	1.50	0.8703	1.3744	0.6688	1.7029	0.5241	2.0521	0.5769	1.8229
	2.00	0.8993	1.4148	0.6978	1.7432	0.5944	1.9651	0.6649	1.6595
$\delta = 1.00$	0.10	0.7853	1.2743	0.5846	1.6036	0.3267	2.3047	0.3308	2.2866
	0.50	0.7931	1.3579	0.5950	1.6899	0.3813	2.2711	0.4020	2.1808
	1.50	0.8125	1.5671	0.6210	1.9056	0.5179	2.1873	0.5800	1.9164
	2.00	0.8222	1.6717	0.6340	2.0135	0.5862	2.1453	0.6689	1.7842

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 Summary

Two problems have been considered in this dissertation. The first problem investigates the Soret effect on steady natural convection double-diffusive flow of heat generating fluid in an inclined channel filled with porous material. The second problem is an extension of the first problem by including the effects of suction and injection. The mathematical equations governing the flow are derived and analytical solutions are obtained by the method of undetermined coefficients for

velocity, concentration and temperature profiles. The skin-friction, mass flux, Nusselt number as well as local Sherwood number are evaluated in each case. The effects of the basic parameters controlling both problems are shown in graphs and tables and analyzed in detail.

5.2 Conclusion

This dissertation has examined the analytical study on thermo-solutal natural convection in an inclined channel filled with porous material. The first problem has considered the steady natural convection double-diffusive flow of heat generating fluid in an inclined channel by taking in to account the effect of Soret number. From this investigation, it is concluded that increase in the heat generation parameter (δ) leads to an increase in the temperature within the channel. Also, with increase in heat generation parameter (δ) and the Soret number (So), the concentration profile decreases. This shows that the concentration decrease is required for optimal performance of a system; it is achievable by increasing the heat generating capacity of the working fluid. The velocity profile increases with increase in the Darcy number (Da), the angle of inclination (a), and sustentation parameter (N), while it decreases with increase in the heat generation parameter (δ), the Soret number (So), and viscosity ratio (γ). In addition, the skin-friction increases on the surfaces of the two plates with increase in the heat generation parameter (δ), Soret number (So) ,Darcy number (Da), the angle of inclination (a) and sustentation parameter (N) while it decreases with increase in the viscosity ratio. Further, the volumetric flow rate increases with increase in the Darcy number (Da), angle of inclination (a) and sustentation parameter (N) but decreases with increase in the heat generation parameter (δ), the Soret number (So), and viscosity ratio (γ). It is also observed that the Sherwood number decreases on the plate $y=0$ with increasing the value of the heat generation parameter (δ) but rises on the plate at $y=1$.

Further, with increase in the value of the Soret number, the Sherwood number decreases on the plate at $y = 0$ and increases on the plate at $y = 1$. The problem has been extended to include the effects of suction and injection and it is found that velocity profiles increased due to increase in heat generation parameter δ, Soret number So, permeability parameter Da and angle of inclination a while it decreased due to increase in Prandtl number $\Pr$, Schmidt number Sc and viscosity ratio γ. Concentration profiles increased due to increase in Prandtl number $\Pr$ while it decreased due to increase in heat generation parameter δ, Soret number So and Schmidt number Sc. An increase in temperature profile is a function of an increase in heat generation parameter δ while it decreased due to Prandtl number $\Pr$.There is a rise in the skin-friction coefficient on both plates due to increase in heat generation parameter δ, permeability parameter Da and angle of inclination a while a fall is observed in skin-friction on both plates due to increase in Prandtl number $\Pr$, Soret number So and Schmidt number Sc. The mass flux increased with increase in heat generation parameter δ, permeability parameter Da and angle of inclination a while it decreased with increase in Prandtl number $\Pr$, Soret number So, Schmidt number Sc and viscosity ratio γ. The Nusselt number increased due to increase in heat generation parameter δ while it decreased due to increase in Prandtl number $\Pr$ and the Sherwood number increased due to increase in Soret number So.

5.3 Recommendations

This dissertation discussed in detail the effect of Soret on steady natural convection of double-diffusive flow of heat generating fluid. Exact solution of velocity, concentration, temperature, rate of heat transfer, skin-friction, volumetric flow rate as well as local Sherwood number are obtained. I recommend this work to scientist in geophysics and aerodynamics.

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APPENDICES

APPENDIX I

CONSTANTS USED IN PROBLEM 3.3

$$m_1 = \frac{1}{2}Vo\,\mathrm{Pr} + \frac{1}{2}\sqrt{(Vo\,\mathrm{Pr})^2 - 4\delta}\,, \qquad m_2 = \frac{1}{2}Vo\,\mathrm{Pr} - \frac{1}{2}\sqrt{(Vo\,\mathrm{Pr})^2 - 4\delta}$$

$$C_2 = \frac{1}{m_2\exp(m_2) - m_1\exp(m_1)}\,, \qquad m_3 = VoSc,\ m_4 = \frac{C_2 m_1 m_3 - C_2 m_1 m_2}{(m_3 - m_1)(m_3 - m_2)}$$

$$m_5 = \frac{C_2 m_1 m_2 - C_2 m_2 m_3}{(m_3 - m_1)(m_3 - m_2)}, \qquad m_6 = \frac{m_3 - m_1 - m_2 + \dfrac{m_1 m_2}{m_3}}{(m_3 - m_1)(m_3 - m_1)},$$

$$m_7 = \frac{1}{\exp(m_3) - 1}\left[1 - m_4(\exp(m_1) - 1) - m_5(\exp(m_2) - 1)\right], \quad m_8 = C_2 \sin a - N m_4 \sin a$$

$$m_9 = C_2 \sin a + N m_5 \sin a, \quad m_{10} = N m_7 \sin a, \quad m_{11} = N m_4 \sin a + N m_5 \sin a + N m_7 \sin a$$

$$m_{12} = \frac{VoDa + \sqrt{(VoDa)^2 + 4\gamma Da}}{2\gamma Da}, \quad m_{13} = \frac{VoDa - \sqrt{(VoDa)^2 + 4\gamma Da}}{2\gamma Da}$$

$$m_{14} = \frac{m_8}{\gamma m_1^{\,2} - Vom_1 - \dfrac{1}{Da}}, \qquad m_{15} = \frac{m_9}{\gamma m_2^{\,2} - Vom_2 - \dfrac{1}{Da}}, \qquad m_{16} = \frac{m_{10}}{\gamma m_3^{\,2} - Vom_3 - \dfrac{1}{Da}}$$

$$m_{17} = m_{11} Da - m_{14} + m_{15} + m_{16},$$

$$m_{18} = \frac{1}{\exp(m_{12}) - \exp(m_{13})}\left(m_{14}\exp(m_1) - m_{15}\exp(m_2) - m_{16}\exp(m_3) + m_{17}\exp(m_{12}) - m_{11}Da\right)$$

APPENDIX II

MATLAB PROGRAM FOR PROBLEM 3.2

```
%SORET EFFECT ON STEADY NATURAL CONVECTION DOUBLE DIFFUSIVE FLOW OF HEAT
 GENERATING FLUID IN AN INCLINED CHANNEL FILLED WITH POROUS MATERIAL
%Velocity profiles for different values of delta
clc;
y=0:0.01:1;
delta=2.0;
gamma=4.0;
Da=0.5;
N=3.0;
So=0.5;
```

```
a=pi./4;
m1=sin(sqrt(delta));
m2=gamma.*delta+1./Da;
m3=(1-N.*So).*sin(a);
m4=(N+N.*So).*Da.*sin(a);
m5=exp(1./sqrt(gamma.*delta))-exp(-1./sqrt(gamma.*delta));
m6=exp(-y./sqrt(gamma.*delta))-exp(y./sqrt(gamma.*delta));
u=m6/m5.*((m3./m2)+m4)+(m3.*sin(y.*sqrt(delta)))./(m2.*m1)+m4.*y;
hold on;
plot(y,u,'k');
hold off


%SORET EFFECT ON STEADY NATURAL CONVECTION DOUBLE DIFFUSIVE FLOW OF HEAT
 GENERATING FLUID IN AN INCLINED CHANNEL FILLED WITH POROUS MATERIAL
%Concentration profiles for different values of Pr
clc;
y=0:0.01:1;
delta=2.0;
gamma=4.0;
Da=0.5;
N=3.0;
So=0.5;
a=pi./4;
m1=sin(sqrt(delta));
m2=gamma.*delta+1./Da;
m3=(1-N.*So).*sin(a);
m4=(N+N.*So).*Da.*sin(a);
m5=exp(1./sqrt(gamma.*delta))-exp(-1./sqrt(gamma.*delta));
m6=exp(-y./sqrt(gamma.*delta))-exp(y./sqrt(gamma.*delta));
Cs=(1+So).*y-So.*sin(y.*sqrt(delta))./m1;
hold on;
plot(y,Cs,'k');
hold off


%SORET EFFECT ON STEADY NATURAL CONVECTION DOUBLE DIFFUSIVE FLOW OF HEAT
 GENERATING FLUID IN AN INCLINED CHANNEL FILLED WITH POROUS MATERIAL
%Temperature profiles for different values of So
clc;
y=0:0.01:1;
delta=2.0;
gamma=4.0;
Da=0.5;
N=3.0;
So=0.5;
a=pi./4;
m1=sin(sqrt(delta));
m2=gamma.*delta+1./Da;
m3=(1-N.*So).*sin(a);
m4=(N+N.*So).*Da.*sin(a);
m5=exp(1./sqrt(gamma.*delta))-exp(-1./sqrt(gamma.*delta));
m6=exp(-y./sqrt(gamma.*delta))-exp(y./sqrt(gamma.*delta));
T=sin(y.*sqrt(delta))./m1;
hold on;
plot(y,T,'k');
```

```
hold off

%SORET EFFECT ON STEADY NATURAL CONVECTION DOUBLE DIFFUSIVE FLOW OF HEAT
 GENERATING FLUID IN AN INCLINED CHANNEL FILLED WITH POROUS MATERIAL
%skin friction for different values of So at y=0
y=0;
delta=0.5;
Da=0.001;
gamma=1.0;
So=0.015;
N=3.0;
a=pi./4;
m1=sqrt(gamma.*Da);
m2=(1-N.*So).*sin(a);
m3=(N+N.*So).*Da.*sin(a);
m4=gamma.*delta+(1./Da);
m5=exp(1./m1)-exp(-1./m1);
m6=exp(-y./m1)-exp(y./m1);
m7=sin(sqrt(delta));
m8=cos(y.*sqrt(delta));
m9=m2./m4;
sf=(1./m5).*(m9+m3).*((-1./m1).*exp(-y./m1)-
(1./m1).*exp(y./m1))+((m2.*sqrt(delta).*cos(y.*sqrt(delta))./(m4.*m7))+m3

%SORET EFFECT ON STEADY NATURAL CONVECTION DOUBLE DIFFUSIVE FLOW OF HEAT
 GENERATING FLUID IN AN INCLINED CHANNEL FILLED WITH POROUS MATERIAL
%skin friction for different values of gamma at y=1
y=1;
delta=0.5;
Da=0.001;
gamma=1.0;
So=0.5;
N=3.0;
a=pi./4;
m1=sqrt(gamma.*Da);
m2=(1-N.*So).*sin(a);
m3=(N+N.*So).*Da.*sin(a);
m4=gamma.*delta+(1./Da);
m5=exp(1./m1)-exp(-1./m1);
m6=exp(-y./m1)-exp(y./m1);
m7=sin(sqrt(delta));
m8=cos(y.*sqrt(delta));
m9=m2./m4;
sf=(1./m5).*(m9+m3).*((-1./m1).*exp(-y./m1)-
(1./m1).*exp(y./m1))+((m2.*sqrt(delta).*cos(y.*sqrt(delta))./(m4.*m7))+m3

%SORET EFFECT ON STEADY NATURAL CONVECTION DOUBLE DIFFUSIVE FLOW OF HEAT
 GENERATING FLUID IN AN INCLINED CHANNEL FILLED WITH POROUS MATERIAL
%The volumetric flow rate for different values of N
delta=0.5;
Da=0.001;
gamma=1.0;
So=0.015;
N=3.0;
a=pi./4;
m1=sqrt(gamma.*Da);
```

```
m2=(1-N.*So).*sin(a);
m3=(N+N.*So).*Da.*sin(a);
m4=gamma.*delta+(1./Da);
m5=exp(1./m1)-exp(-1./m1);
m6=exp(-y./m1)-exp(y./m1);
m7=sin(sqrt(delta));
m8=cos(y.*sqrt(delta));
m9=m2./m4;
Q=(m1./m5).*((m2./m4)+m3).*(2-exp(1./m1)-exp(-1./m1))+(m2.*(1-
cos(sqrt(delta)))).(sqrt(delta).*m4.*sin(sqrt(delta)))+(m3./2)

%SORET EFFECT ON STEADY NATURAL CONVECTION DOUBLE DIFFUSIVE FLOW OF HEAT
 GENERATING FLUID IN AN INCLINED CHANNEL FILLED WITH POROUS MATERIAL
%Nusselt number on the plate at y=0 for different values of delta
clc;
y=0
delta=0.5;
Nu=sqrt(delta).*csc(sqrt(delta))

%SORET EFFECT ON STEADY NATURAL CONVECTION DOUBLE DIFFUSIVE FLOW OF HEAT
 GENERATING FLUID IN AN INCLINED CHANNEL FILLED WITH POROUS MATERIAL
%Nusselt number on the plate at y=1 for different values of delta
y=1
delta=0.5
Nu=sqrt(delta).*cot(sqrt(delta))

%SORET EFFECT ON STEADY NATURAL CONVECTION DOUBLE DIFFUSIVE FLOW OF HEAT
 GENERATING FLUID IN AN INCLINED CHANNEL FILLED WITH POROUS MATERIAL
%Sherwood number on the plate at y=0 for different values of delta
So=1.0;
delta=0.5;
sho=(1+So)-sqrt(delta).*csc(sqrt(delta)).*So

%SORET EFFECT ON STEADY NATURAL CONVECTION DOUBLE DIFFUSIVE FLOW OF HEAT
 GENERATING FLUID IN AN INCLINED CHANNEL FILLED WITH POROUS MATERIAL
%Sherwood number on the plate at y=1 for different values of So
So=1.0;
delta=2.0;
sh1=(1+So)-sqrt(delta).*cot(sqrt(delta)).*So
```