DIURNAL, SEASONAL AND TOPOGRAPHICAL EFFECTS ON VERTICAL REFRACTION

BY

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As thesis submitted to the Post Graduate School, Ahmadu Bello University in partial fulfillment of the requirements for the degree of Master of Science in Land Surveying.

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DECLARATION

I hereby declare that this thesis has been composed by myself and that it is a record of my own research work. It has not been accepted in any previous application for a higher degree. All quotations are distinguished by quotation marks and the sources of informations are specifically acknowledged by means of references.

O. O. ODETOLA

CERTIFICATION

This thesis entitled "Diurnal, Seasonal and *Topographical Effects on Vertical Refraction*" by Odetola O. O. meets the regulation governing the award of the degree of Master of Science of Ahmadu Bello University, and is approved for its contribution to knowledge and literary presentation.

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Member, Supervisory Committee Date

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Head of Department Date

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Dean, Postgraduate School Date

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ABSTRACT

In this investigation, the effect of season, time of the day and topography has been examined. Two lines, one on water of length 1098.650 meters and the other 1287.05 meters on land were selected for the study. Coefficient of refractions on these two lines for different time of the day - 0800 hours to 1600 hours and for different seasons - dry and wet seasons were computed. Analysis of data revealled that variations of refraction on the two lines was less in wet seasons than in dry season. Also refraction was found to be minimum and more steady between the hours of 1200 - 1600. This confirmed the statement found in most text books and survey literatures. The values of coefficient of refraction obtained from this study are higher than the traditional values. This may be due to the fact that the lines are relatively short.

The importance of meteorology in high order work is much emphasised.

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Abreviation and Symbols

H	=	Difference in Height between two points
S	=	Length of line of sight
Ø	=	Observed Vertical angles
θ	=	Computed plane angle
r	=	Refraction
R	=	Radius of the earth
K	=	Coefficient of Refraction
t	=	Temperature
T	=	Absolute Temperature
∽	=	Refraction Angle
i_A	=	Height of Instrument
g_B	=	Height of Target
σ	=	Standard Error/Radius of Currature
B	=	Angle Substended at the center of the Earth by line of sight
P	=	Pressure
g	=	Acceleration due to gravity
M.S.L	=	Mean Sea Level

CHAPTER ONE

INTRODUCTION

1.0 OBJECTIVE OF THE STUDY

(i) To determind the effects of topography, seasons of the year and time of the day on the vertical refraction of light rays using the vertical angles measured on two lines of different configurations in two different seasons. The crucial factors in this study is the value of coefficient of vertical refraction. This value will be computed for each observation.

(ii) To compare the values obtained in (i) above with those obtained on similar lines and to test statistically if the mean value computed for each configuration for different months show any significant difference.

(iii) To analyse the results obtained in (i) above graphically with a view to recommending the best time to observe vertical angles for height determination in Zaria.

1.1 The need for the research into vertical refraction.

Surveyors rightly distrust refraction. They are careful to elliminate it from high order work by equalising level sights and observing simulteneously

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reciprocal vertical angles. But is its power to distrust the humbler task of photo control sometimes overlooked?(Rogers 1977)

A common problem in medium scale mapping is to fix height of points far away from control. The accuracy of heighting is determined by the contour interval and thus by the scale of mapping. Modern photogrammetry demands much better heights.

A knowledge of the accurate heights of points on the surface of the earth is required among other reasons:

(i) To enable survey measurements to be reduced to the horizontal sea level.

(ii) To enable drainage works to be surveyed so that water, petroleun and other liquid may flow in the desired direction.

(iii) To enable relief to be depicted on topographical and air maps.

(iv) To provide information for scientists concerning the shapes and structures of the earth.

(v) To enable the surveyors to calculate whether two points are intervisible on the ground.

(vi) To enable roads and railways to be cons-

tructed in such a manner that steep hills are avoided.

1.2 TRIGONOMETRICAL HEIGHTING

Trigonometrical heighting is one of the methods extensively used to determine the relative heights of survey points. A common method of determining (ΔH) is to observe the vertical angle to a survey beacon and compute the difference in height between the theodolite and the top of the beacon by the relationship.

$$\Delta H = \text{Stan } \emptyset \quad \ldots\ldots\ldots\ldots \quad 1.1$$

where S is the horizontal distance between the stations or from the relationship.

$$\Delta H = S \sin \emptyset \quad \ldots\ldots\ldots\ldots \quad 1.2$$

Where S is the slanting distance as measured by a tellurometer. In figure I below H_A and H_B are the heights of the stations A and B above the horizontal datum line DE which is considered straight for the time being. H_A is the height of the trunion axis of a theodolites at A referred to some reference mark on station A. h_B is the height of signal at B.

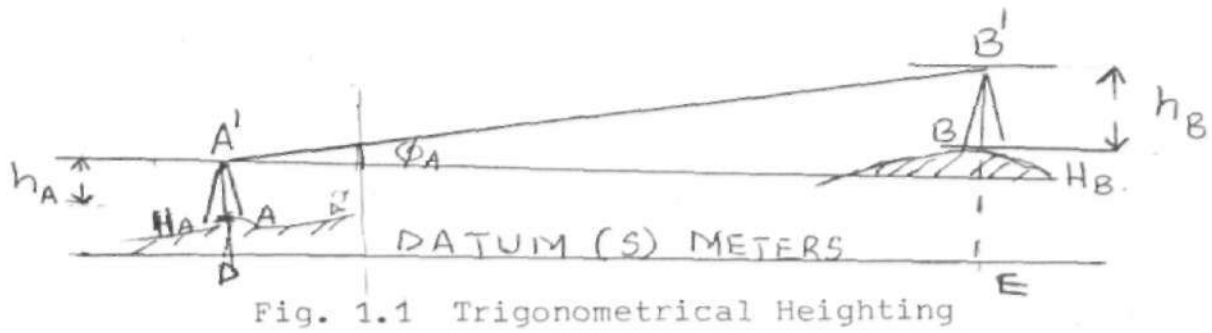


Fig. 1.1 Trigonometrical Heighting

The vertical angle observed at A is ϕ_A and the horizontal distance DE is smaller. From the figure it can be seen that

$$H_A + h_A + \text{Stan } \phi_A = h_B + H_B \text{ .....}$$

Putting $H_B - H_A = (\triangle H)$

$$\triangle H = \text{Stan } \phi_A + h_A - h_B \text{ ..... } 1.3$$

Accurate determination of this relative difference in height between two stations is not an easy matter as the measurement of the vertical angle ϕ_A is very much prone to vertical atmospheric refraction error. To minimise this error simultaneous reciprocal vertical angle observations can be made at both end of the line. This is not always possible and it can only at best minimise the errors. It cannot remove the errors completely. It is therefore necessary to study the behaviour of refraction in different localities in order to know the favourable time the observed vertical angle will be least affected by refraction

and to be able to apply the correct refraction corrections. Hence the need for this study.

1.3 CURVATURE AND REFRACTION

Lines of sights such as A' B' in figure 1 above are not strictly straight lines but curved lines. This is as a result of two phenonena, one geometrical and the other physical -
The geometrical one is the curvature of the earth and the physical one is the refraction which is the main concern of this study.

1.3.1 Curvature

The reference datum line DE in figure 1 is not straight as we supposed but is curved as in figure 2 below

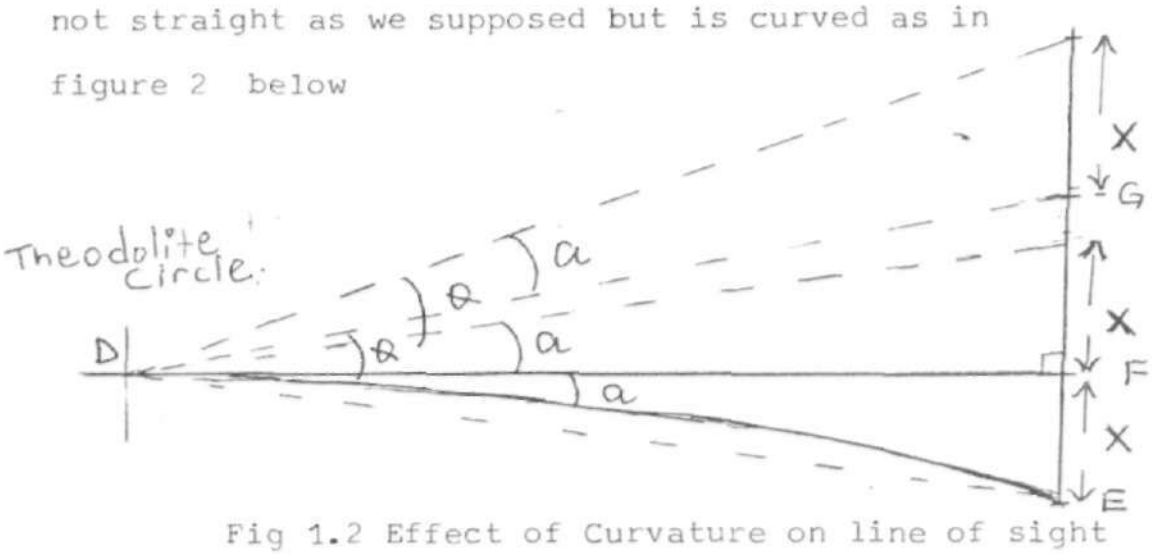


Fig 1.2 Effect of Curvature on line of sight

The linear amount of the earth's curvature FE = X is given by

$$X = \frac{S^2}{2R} \qquad \ldots\ldots\ldots\ldots 1.4$$

Allan et al (1968).

Where R is the radius of the earth and S is the length of the line. If the datum line DE is considered as equivalent straight datum line DF, the problem is reduced to that of figure 1.1. If we consider E raised to F, the angle Ø has to be considered as increased by the curvature angle which is given by

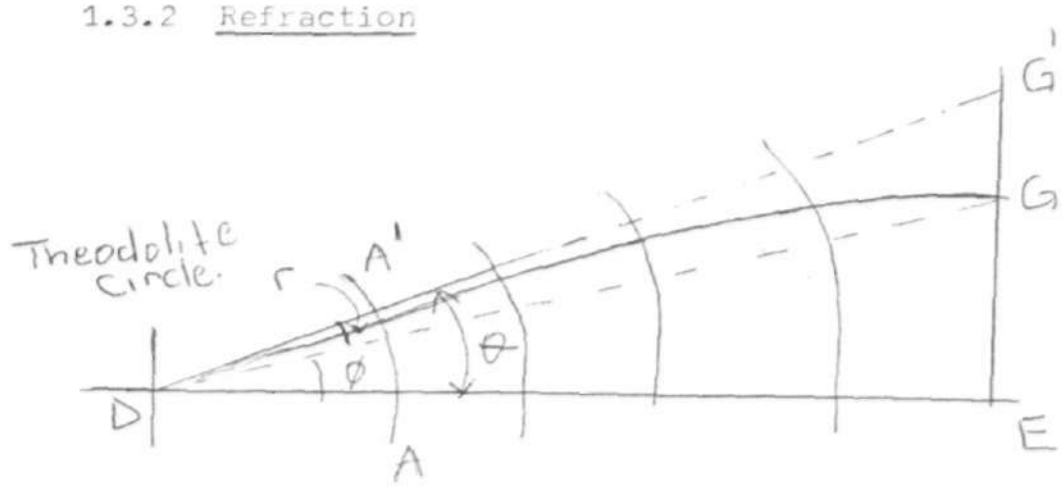
$$a^{11} = \frac{206265S}{2R} \qquad \ldots\ldots\ldots 1.5$$

$$\text{or } a^{11} = PS \text{ egn } 1.6 \qquad \ldots\ldots\ldots 1.6$$

$$\text{where } P = \frac{206265}{2R} = \text{Constant}$$

Hence the observed angle Ø referred to a curved datum DE, can be considered with respect to the straight datum DE if increased by a^{11}. The angle will always be considered positive though Ø may be of either sign. If Ø is negative and less than a numerically, the sign of the effective observed angle Ø + a will be changed.

1.3.2 Refraction



Effect of refraction on rays of light

Fig 1.3

In the figure 3 above, the light path from D to G is shown a curved line instead of a straight line. This is in practice due to the refraction effect of atmosphere. If the density of the atmosphere is uniform, then there would be no refractive effect on rays of light laying entirely within it, but since the density decreases with height, rays of light pass through continuously changing density and therefore curved upwards into layers of lower density. In figure 3 above, D and G are two points on the earth's surfacre at different elevation. In the absence of the atmospheric refraction, the ray of light will be straight line The concentric arcs from AA' outwards represent layers of decreasing atmospheric density, which because the density varies continuously should be

regarded as infinitesimally close together.
Because of this inifinitesimally close layers with continuously changing refractive index line of sight follows the curve path DG. At D object G would actually appear at G^1. Where DG^1 is the tangent to curve parth at D. The vertical angle to B measured by theodolite is therefore too great by the angular amount DGG^1 called the angle of refraction. This angle of refraction is the difference between the observed angle and the plane angle which we could have observed in the absence of the effect of curvature and refraction.

Refraction could be either positive or negative. Positive refraction is downward curvature of a ray. i.e. the same direction as earth curvature. It is caused because, except in air very near the ground the density of the atmoshere decreases with height. Negative refraction occurs when the density increases with height. This happens under the conditions of insolation when the ground becomes very hot (Geiger 1959). The atmosphere is then heated from the ground and a turbulent system developes in which the lower layer of air are less dense then the air above. Coulman and Hell (1967). This condition may prevail for attitudes up to 100 ft (about 34.0 meters) above the surface. The effect decreases with height until

it is no longer evident and at higher altitudes, the density starts decreasing with height in the normal manner. This is the case where the measured angle is less than the plane angle.

It is therefore necessary to subtract the refraction angle r from the observed angle in order to obtain the equivalent angle free from refraction. Allan et al (1968) gave the refractive angle by the formula.

$$r = \frac{KS}{R} \qquad \ldots\ldots\ldots\ldots 1.7$$

$$r \quad \text{seconds} = \frac{KS206265}{R}$$

$$r \quad \text{seconds} = 2\,KPS \quad \ldots\ldots 1.8$$

where P has the same constant as earlier mentioned and K is the coefficient of refraction which is defined in details in the next section.

1.3.3 Combined Curvature and Refraction

To reduce observed angle Ø to its equivalent plane angle Ø which we could have observed in the absence of the refractive effect, we have the following relationship:

corrected plane angle = observed angle + curvature - refraction

i.e $\theta = \emptyset + PS - 2KPS$

$$\theta = \emptyset + PS\,(1-2K) \qquad \ldots\ldots\ldots\ldots 1.9$$

1.4 COEFFICIENT OF VERTICAL REFRACTION

1.4.1 Definition

Ramsay (1971) defines coefficient of vertical refraction as that factor K which when multiplised by the angle subtended at the centre of the earth by the line of sight provides the same amount of circular variation due to the effect of refraction.

Abdulrauf (1982) defines the coefficient of vertical refraction as the ratio between the refraction angle to the angle which the two stationsof observation subtend at the centre of the earth.

1.4.2 Variations in the Values of K

The values of K varies with seasons, time of the day and topography of the ray. For a ray crossing a deep valley or lying well above the surrounding ground level the value of refration is likely to be larger than the normal whereas if the ray is grazing or lies close to the ground level the coefficient will be lower than normal (Pamphlet No. 1, 1956). The value of K also varies inversely as the square of the abslute temperature (Pamphlet No. 1, 1986).

$$\therefore \quad K \propto \frac{1}{T^2} \qquad \ldots\ldots\ldots 1.10$$

The value of K is greater for night observation and for rays over water. In Great Britian it varies from about 0.065 to 0.081 on lines over water. Ramsay (1971).

1.4.3 Formulae for computing the K - values

The values of K can be computed in various ways from different formulae. One of such formulae was given in Bomford (1977) as:

$$\alpha = KB$$

$$K = \frac{\alpha}{B} \qquad \text{.....} \quad 1.11$$

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where α is the refraction angle and B is the angle subtended at the centre of the earth by the line of sight. It can be shown that B is approximately equal to S/R radians where R is the radius of the earth.

$$\therefore \alpha = \frac{KS}{R} \qquad \text{.....} \quad 1.12$$

The problem with this formula is that the refraction must be known in order to compute for K

(ii) From equation 1.9 K can be expressed as

$$K = \tfrac{1}{2}\left(1 - \frac{\theta - \emptyset}{PS}\right) \qquad \text{.....} \quad 1.13$$

Where θ = Corrected plane angle using the fomula

$$\emptyset = Sin^{-1}(h/s)$$

Here h is the height difference between the two stations and S is the length of the line.

$\emptyset$ = observed angle

R = Radius of the earth

P = The same constant given in equation 6.

The formula is very practicable since $\emptyset$ and S are the quantities measured directly from the field and O and P can be easily computed. This formula is used in this work.

(iii) The part of light through a medium in which the refractive index n varied is determined by the law:

$\int nds$ along the path is minimum.- Bomford (1973)

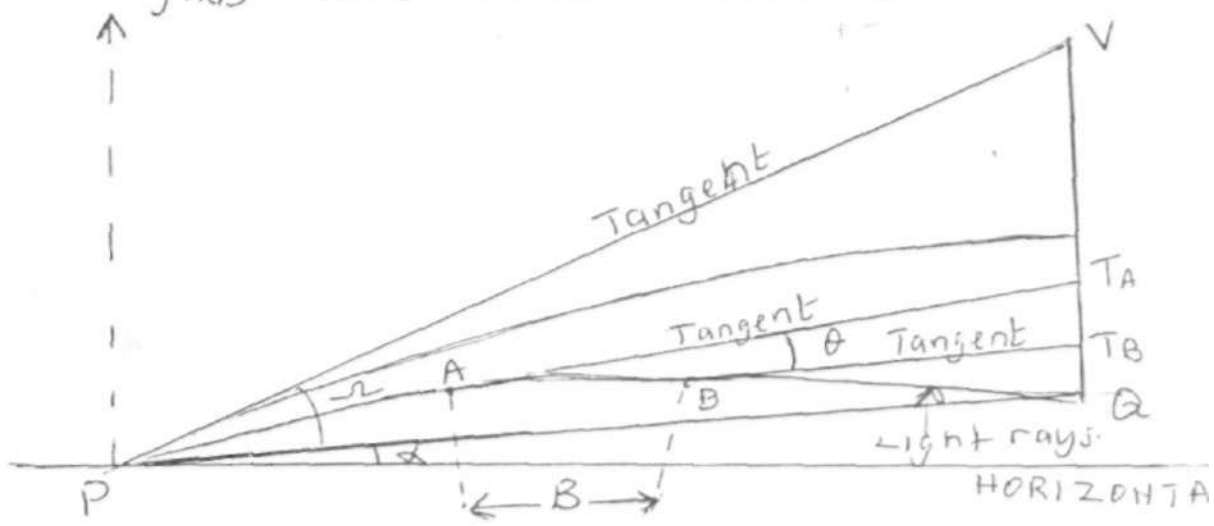


Fig 4 Curfacture of light rays.

In figure 1.4 above arc AB of length L_1 is a short element of the path of a ray of light PQ. If the radius of curvature is σ whish is for the moment taken constant and is large compared with L_1, so that the angle B between the tangents at both end of (AB) is L_1/σ . Let the inclination of PQ and of (AB) to the surface of equal n be α . So the gradient of n across (AB) is dn/dh cosα, α is small angle. The solutions of equation 1.14 is given by

$$\frac{1}{\sigma} = 16.3 \frac{P}{T^2} (0.0342 + \frac{dT}{dh}) \text{ CosBsec/m} \quad \ldots\ldots 1.15$$

where P is in millibars in ^{o}K and h in metres. CosB can be ordinarily taken aa 1 and SO

$$\frac{1}{\sigma} = 16.3 \frac{P}{T^2} (0.0342 + \frac{dT}{dh}) \text{ sec/m} \quad \ldots\ldots 1.16$$

If we consider the whole length PQ along which σ varies, the angle of refraction at P is given by

$$= (\frac{V}{P}) \text{ Cosec } 1'' = \frac{1}{L} \int_{\alpha}^{L} \frac{L-l}{\sigma} dl \text{ secs} \quad \ldots\ldots\ldots 1.17$$

where $\frac{1}{\sigma}$ is expressed in sec/m. It has been shown that = KB = K S/R.

If PQ is sufficiently horizontal for P and T to be treated as constant as when h_1 and h_2 are the same within 100 or 200 metres, and if dT/dh is regarded as

constant equation 1.17 becomes

$$\rho = \frac{L}{2\sigma} = \frac{RB}{2\sigma} \qquad \ldots\ldots\ldots\ldots 1.18$$

$$\therefore \quad K = \frac{R}{2\sigma} \sin 1'' = 256 \frac{P}{T^2}\left(0.0342 + \frac{dT}{dh}\right) \ldots 1.19$$

Treating T, P and dT/dh as constant, equation 1.19 agrees with equation 1.10 which is the statement earlier made that the coefficient of refraction varies inversely with the square of the absolute temperature.

For inclined lines, i.e. h_1 and h_2 are so different that P and T cannot be treated as constant, P/T^2 and consequently $1/\sigma$ (since $\frac{dT}{dh}$ is still being treated as constant) may be considered to vary linearly with height and a good value of K will be obtained by substituting in equation 1.19 the estimated values of P and T at one third of the distance along the line i.e. at a height $\frac{1}{3}(2h_1 + h_2)$ for observation at P and $\frac{1}{3}(h_1 + 2h_2)$ for observation at Q. Bomford (1977).

In practice a table is made Graaff-Hunter(1941) P.8 giving K for the normal values of P and T at different deights, with the differences resulting from departure the normal of 1" (30mb) in pressure

and 10^{o} in temperature to enable the tabular values to be corrected by actual observation. Then K is computed for the appropriate height as above. This will give satisfactory results provided lines are from one steep-sided hill to another, not grazing the ground in between, and provided the observations are made at both P and Q at the time of minimum refraction, 1200 hours to 1600 hours.

Equations (1.17) and (1.19) are the fundamental formulae if P and T and dT/dh are known at every point along the line, or is determinate. Actually P is generally known to 1 per cent or better. T can be measured at P and the average value of T along the path in ^{o}K can probably be estimated within 1 per cent too. The doubtful item is dT/dh. The adiabatic gradient for dry air which might be expected to be the actual gradient in afternoon is -0.010 or ^{o}C per 100m. but apart from abnormal values near the ground -0.0055 is more usually observed Bomford (1977).

CHAPTER TWO

LITERATURE REVIEW

2.0 PREAMBLE

A lot of studies have been made by many researchers into the nature and behaviours of the vertical refractions. It is therefore necessary to look into some details some of these researches.

2.1 VARIATIONS IN TEMPERATURE GRADIENT

Measurements in the atmosphere leads us to expect T and P (Temperature and Pressure) and to be generally sufficiently constant over long distance and over the range of heigh traversed by survey lines. But the same cannot be said of the gradient dT/dh. Infact the diurnal change in refraction is mostly due to change in dT/dh, the usual source of which is the varying temperature of the ground and the consequent periodic upward transfer of heat.

dT/dh varries over a wide range primarily with height above the ground surface and time of the day. Some figures for Durbar given by P. V. Angus Leppan (1961) should illustrate the variation. In the layers 1525-4570 above M.S.L. the gradient is - 0.0032^{o}F/ft (-0.0096^{o}F/m), and the diurnal and seasonal divergence from this mean value is small. With decreasing height

the diurnal variation about this value increases and the rate of change increases so that very close to the ground the gradient can be hundred of times as large. At 1.5m above the ground surface mean value for the month of February 1957 gave a minimum gradient (mid-day) of $-0.35^{\circ}F/ft$ ($-1.06^{\circ}F/m$) and a maximum (before dawn) of $+0.0.15^{\circ}F/ft$ ($0.46^{\circ}F/m$).

These gradients would obviously yield widely diverging values for the coefficient of refraction. This explains why it is necessary to confine observation for trigonometric heights to a limited period during the day and to avoid the "grazing" rays which passes close to the ground surface.

2.1.1 Observed Values of The Temperature Gradient

The following are some observed values of the gradient near the ground in $^{\circ}C$ per 100m. Negative indicates the normal decrease with height.

(i) A Dehra Dun, India, Bomford (1977) observed in March 11 and 16 hours that at height of 4, 9, 15, 20, and 26 metre above the ground the temperature gradients were -14, -3, -1.6, -1.3 and -2.3 respectively. This is a normal state in Sunny weather,

apart from the small increase at 26m.

(ii) At Ganges Bridge, Bomford (1977) observed in March between 10 and 1500 hours that at height of 2½, 5 and 8 metre above water temperature gradients, are +20, 0, -2 respectively. At 1½m above the water vertical angles between points of known heights gave an average of +13" and a maximum of +36". This shows inversion over water, which in this case persisted only to a small height because the extent of water was small.

(iii) At Punjab Plan walker (1870) in winter deduced a temperature gradient of about -7 from refraction at height of 6 to 12 meters above the ground at midday,.but that higher negative value had occured an hour earlier when marks were below the horizon. At sun-rises and at night typically +13^{o}C to +23^{o}C was recorded. This illustrates inversion at night. It should be noted that a temperature inversion of +13^{o}C per 100m gives a coefficient of refraction of 0.5 and the curvature of the ray equals that of the earth.

It is apparent from (i) and (ii) that a temperature gradient measured at a station is little guide to the gradient 10m higher up, or generally to the gradient a few 100m along the ray of uniform height above the ground quantitatively but these are not conditions in which useful numerical results can be got.

To solve the problem of accurate determination or dT/dh_1 Bomford, (1962) proposed that progress can be made on the following lines:

(a) on the inclined line, by flying along the line of sight and measuring the air temperature. If the slope is sufficient, useful values of dT/dh may be possibly got.

(b) by using an aircraft flying along the line to measure dT/dh by e.g. a pair of thermocouples as used on ground.

2.1.2 Factors Causing the Variations in Temperature Gradient

Angus-Lappan (1961) discusses the problem of the variation of temperature gradient. He said that the gradient depends fundamentally upon the amound of heating of the ground surface and the rate at which the heating is transferred upwards. These in turns depends upon a number of factors

interrelated in a complex fashion.

A. Temperature of the Surface

(1) Insolation varies regularly with the seasons of the year and irregularly because of cloudiness, suspended matter and vapour in the atmosphere.

(2) Topography and aspect affect the amount of shadding from the sun and the angle between the sun's rays and the surface being healed.

(3) Nature of the Surface: colour, heat properties of the materials and underneath the surface vegetation covering or lack there of, all affect the amount of heat received directly, absorbed and radiated and the temperature attained by the surface.

B. Turbulence The amount of heat convected upwards will depend upon the nature of the turbulent eddies, their size, frequency etc. Factors affecting this are:

(1) The surface temperature

(2) Wind velocities and gustiness

(3) The aerodynamic "smoothness" or "roughness" of the surface, on a small scale (e.g. grass or sand) and on a large scale (e.g.

scattered single trees, forest, buildings).

(4) Topography which funnels wind over certain areas and shelters other.

C. Movement of air Masses

(1) In moving masses of air the wind causes the gradient to be determined by the above factors at the point in question, but some distance away.

(2) There is mixing of bodies of air during the time of passage over this distance. These variations appear very complex and it is unnecessary to account for every single factors, but merely to find an approximation that the gradient dT/dh is a constant. Angus-Leppan developed a temperature function which has taken the account of most of these factors. Details of this is given in appendix I.

2.1.3 Temperature Gradient in Nigeria

Morley (1937) gave the average lapse-rate of temperature of the ground level with H near noon, in Nigeria to be about 6.2°C (43.2°F) per 1000 meters, although the numerical values of this lapse rate evidently varies with locality

and nature of vegetation. In Nigeria and in Tropics generally it is generally recognised by the meterorogists that abnormal lapse-rate of temperature do frequently occur in the lower air. Morley (1937) gave the following figures for Mombassa being unit of 100m and t in degree centrigrade.

Table 2.1 Temperature Gradient and Rate of Change of Temperature Gradient (Mombassa)

h	dt/dh.	d^2t/dh^2
0	-2.39	+1.03
1	-1.55	0.65
2	-1.03	0.40
3	-0.72	0.23
4	-0.55	0.10
5	-0.50	0

2.2 COMPUTED VALUES AND VARIATIONS OF COEFFICIENT OF REFRACTION

Some researchers have developed formulae for the computation of co-efficient of refractions and formulae to show refraction co-efficient varies with height of the ray above the ground.

2.2.1 EVALUATION OF COEFFICIENT OF REFRACTION

Morley (1937) developed an equation which defines the temperature at a point in terms of the height of the point above ground level for Mombassa. The equation is:

$$t_h = 21.8 + 1.09 \text{ Sinh } 0.45(5-h) \quad \ldots\ldots 2.1$$

He used this equation to compute the lapse rate for Mombassa and he computed the coefficient of refraction using the formula -

$$K = 0.16847 \, Rf_1 \, PT^2 + 0.39 \, Rf_1 f_2 P^2 S^2 T^{-5} \quad \ldots\ldots 2.2$$

where

P = atmospheric pressure in inches

R = radius of the earth in meters

S = Length of the ray in miles

T = absolute temperature (Fahrenheit)

f_1 = $\sqrt{(1.869-100\ dT/dh)(1.869-200\ dT/dh) - 10^4 T\ d^2T/d^2h}\ /\ 1.042$.

h = height measured in feet.

The values of T, dT/dh^1, d^2T/dh^2 used in the above equation are those deduced from Mombassa aerological soundings. Table below shows the values he computed for Mombassa.

Table 2.2 Values of K Computed for Mombassa

Height above ground in hundred of meters	Coefficient of Refraction (K)
0	0.0263
0-1	0.0482
1-2	0.0614
2-3	0.0694

The values for Nigeria obtained from Trigonometrical observations are given below:

Table 2.3 K Value for Nigeria

Height above ground in hundred of meters	Coefficient of refraction (K)
0	0.02 approx.
0-1	0.0512
1-2	0.0584
2-3	0.0622

The similarity between the sets of K for Nigeria and Mombassa is remarkable, especially in view of the fact that they were obtained by two entirely different methods.

2.2.2 Variations of K-Values with Height

The coefficient of refraction (K) depends on the inclination of the ray which is constantly changing and also in free atmosphere the coefficient decreases with height.

Assuming the lapse-rate to be constant throughout the range of eleveation of two points different in height by H, Grzaff Hunter gave the relation between pressure and temperature for convective equilibrium (when true minimum coefficient of refraction occurs) to be $- (P_H/P_o) (R_x/g) = 1 - x^H/to$ 2.3

where P_o and t_o are the pressure and temperature at lower point, P_H the pressure at higher point, X the constant lapse rate, R the gas constant and g the gravitational attraction. Then the relation between the coefficient of refraction K_H and K_o is

$$\frac{H_h}{K_o} = \frac{P_h\ t^2_o}{P_o\ t^2 h} = 1 - \frac{H}{t_o} (g/R - 2x) \quad \ldots\ 2.4$$

Strictly speaking g/R varies slightly with height but we have assumed it is constant and equals to 1/30 in c.g.s. units. This is greater than 2x under condition of convective equilibrium in free air, and thus under these conditions refraction decreases with height. However, it is usually assumed that K decreases with increase in height above the Mean-Sea-

Level surface. From the analysis of the result obtained by varying h but holding the height above Mean-Sea-Level fixed; A. J. Morley came to the conclusion that the variations in K could only be due to abnormal value of dt/dh and d^2t/dh^2, "t" denoting the temperature of air and h the height above the ground.

Table 2.4 below shows the values computed for different height above the mean-sea-level and Table 2.5 shows the values computed for different heights above the ground.

Table 2.4 : Values for K for Different Attitudes

Height of ray above M.S.L.	Computed Coefficient (K)	No. of Rays
180÷360m	0.0576	30
360-540m	0.0577	32
540-720m	0.0577	52
720-900m	0.0568	17

Here there is no obvious decrease in K with increase in altitude. A detailed comparison was therefore made between K and the mean heights of the rays above ground level. For every individual case there is a striking relationship between this mean height and the value of K. The results are as follows:

Table 2.5 Value of K for different Heights above the ground level

Mean Ht. of Ray above ground	Computed K	No of Rays
0 - 100 meters	0.0512	15
100 - 200 "	0.0567	14
200 - 300 "	0.0624	15

The above variations in K cannot be due to pressure changes; they must be due to variations in temperature due to change in height above the ground level.

2.2.3 Variations of K values along the Ray

Morley (1941) discusses the variation of k-values along the line of observation. He defined the coefficient of refraction (K_m) along the line AB in the direction A to B to be "the fixed value of K which would give the same perpendicular displacement from A to B as that actually produced by the true value of K along the line". He then derived an expression for K_m as

$$k_m = k_o + K/3 - BZ^2/6 - S^2 \frac{(5A+4BZ)}{60R} \quad \ldots\ldots 2.5$$

If the effect of curvature is ignored and if K varies linearly with h (that is B is zero) then the coefficient of refraction from A to B is equal to the coefficient of refraction at a plus one third of the difference between the coefficient of refraction at both ends. The survey of India use this result for the computation of height difference by single ray method. The rule is applicable for rays high above the ground otherwise the effect of coefficient B is unlikely to be negligible and in addition $S^2/2R$ will not be small in comparison with h.

If Z = K = 0 is substituted in equation 2.5 it is seen that the diffierence between the coefficient of refraction along a ray whose terminal are at the same level, and the coefficient at one end, is equal to $As^2/I2R$ and this expression vanishes when S=0. In practice its numerical value is unlikely to exceed 0.002, for A tends to decrease as S increases, a result which follows from the fact that h increases with S.

Morley (1941) also emphasised another very important consideration to be keptin mind. If the height H above the plain level varies, the values of K will be affected, or to be more precise, any departure of the atmospheric pressure P, and temperature

t, from certain fixed datum values will produce corresponding changes in K. The actual values of P and T ought always to be determined in conjunction with vertical observations taken along a primary chain, but if they are not, it will be necessary to estimate their values from (H-h) and the standard lapse rate of temperature and pressure and then correct K accordingly. In Nigeria the lapse rates are about 1 inch and 3.4^{o}F per 1000 feet respectively. Naturally the method of obtaining P and T from the standard lapse rates is not satisfactory as that of direct measurement; the general nature of the ground surface, or its distance from the coast, may appreciably affect the magnitude of the air temperature which will, in turn produce a deviation in K from its "normal" value. Thus, the values of K in the arid, sandy regions of Northern Nigeria may be as much as 15 precent smaller than those obtained at a similar level in areas close to the sea. For this reason is tis better for the observer to measure the temperature and pressure directly, rather than to rely on values derived from the standard temperature and pressure gradient.

2.2.4 Diurnal Variation of Refraction

Angus-Leppan (1961) carried out some investigation to measure simulteneously the angle of refraction and atmospheric conditions on two lines. One was 2835m long and 3.7m. above the ground. The other was 1097m long and 2.1m above ground. The site was stanfort hill aerodrome Durban. The surface texture was well cropped grass about 1½ inch thick over sandy soil. Observations were designed to have a probable error of less than one second. Wild T_4 instrument was used. Reciprocal angles were measured at interval from sun rise to sun set and tergets at lower and higher levels were sighted. The conclussion drawn out from the experiments are:

(a) The diurnal variation in the angle of refraction was spectacular (see figures 2.2 and 2.2 page 32) being more than 120 seconds of are, equivilent of a variation in the coefficient of refraction of 3.0 (the accepted value for South Africa is 0.13).

(b) There was an increase in refraction on lines to lower targets and a marked decreased to the higher target.

2.2.5 Seasonal Variation of Refraction

The damping coefficient as deduced by Angus-Leppan (see the appendix I) fell into two groups with one group nearly 2.0 and the others about 3.5. The difference was ascribed to the fact the 2.0 values were in dry years, and the others in humid years. Humidity thus affect the demping coefficient b, being greater in wet season than dry season. The radiation from the ground is rapidly damped down by the water vapour present in the atmosphere. There is less variation in temperature. Under this condition we will expect less diurnal variation in refraction in wet season than in season.

2.2.6 Variation over Land and Water

McIntosh et al (1973) discussed the diurnal variation of temperature over the surfaces of land and water. According to them the depth of soil reached by the daily cycle of solar heating and also the temperature rise within the top layer depends on the thermal conductiviey of the soil and on its heat capacity. If for example, the conductivity and heat capacity are both low, the heat does not penetrate far and there is a large temperature rise within the surface layer (top few centimeter). In such a case the land's reservoir action

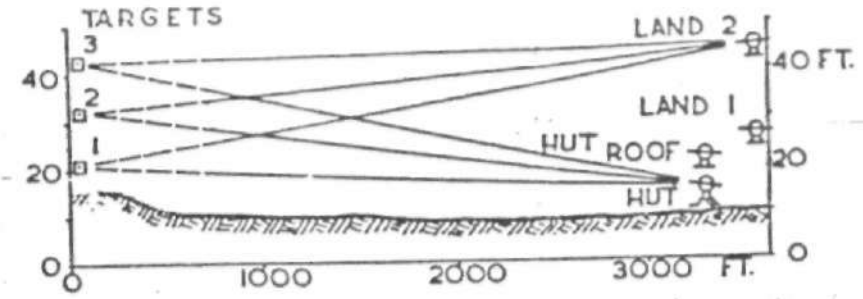
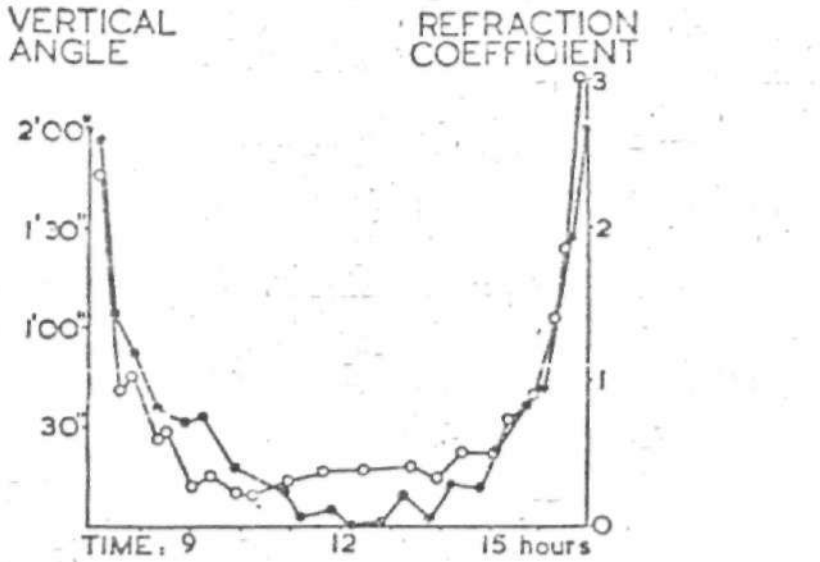
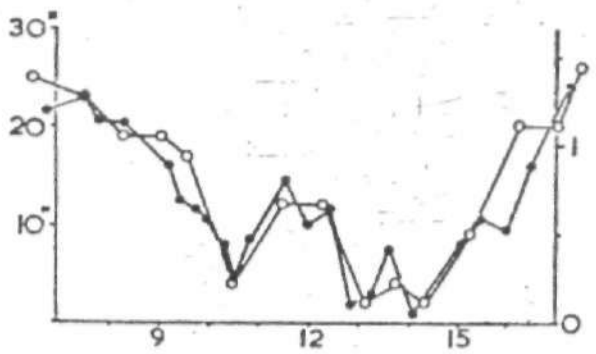


FIGURE 2 - 1 SIGHT LINES



A. LENGTH OF LINE 9300 FT. (2834.64m)



B. LENGTH OF LINE 3600 FT. (1097.28m)

FIGURE: 2 - 2

"Culled out from Survey Review Vol. 16 NO. 120".

is almost absent and temperature falls quickly at night to a low level if atmospheric conditions favours a net radiative loss from the surface.

The air content of a soil is also important since air is a poor conductor and has a low heat capacity. Thus the surface layer of a dry soil is warmer during the day and colder during the night than that of a wet soil of the same type.

Ground which is covered by vegetation is protected from direct solar rays and therefore has a low daily range of temperature.

In the light of the above discussion it is very much likely that the values of K vary very low over dry land covered by vegetation than over exposed dry land.

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They also discussed the various factor which restrict the temperature rise of water surface exposed to sun shine. First, a substantial fraction of energy is used in evaporating water from the surfaces, second, much of the radiation penetrates a long way before it is completely absorbed; third, water has a high specific heat; fourth, and most important, movement of the surface of water, by wind mixes the absorbed heat through a deep layer, and also surfaced water which is cooled

by radiation is replaced by warmer water from below. All these effects combined to make the diurnal temperature variations of water surface (and of the air in contact with it) very small indeed. The variation are much more smaller than those over land. The coefficient of refraction on lines over water is therefore expected to vary less than on lines over dry land, even though the coefficient may be higher on lines over water than on lines over the land.

2.2.7 Effect of Topography on Refraction

Topograph affects the amount of shading from the sun and the angle between the sun rays and the surface being heated. Some topographies funnel, wind over certain area and shelter other areas. All these affect the temperature gradient and consequently the refraction patterns of the area. Uneven or undulating topography will have effect on the refraction while flat, level sites with even surface condition will have little or no effect on the refraction pattern of the area.

2.3 INVESTIGATION IN KADUNA STATE

Some people have done some researches into the variation of refraction in Kaduna State. Inubiaraye (1983) made some observation in Kajuru. Details

of his results are tabulated below. The first three lines are from hill to hill which are on the average 800 meter above the M.S.L. The fourth line runs from the top of hill K_{15} via a narrow and grazing valley down to a traverse point on the ground.

Table 2.6: Computed Values of K for Kajuru

Line	Length	Mean	Variations
1	8.40Km	0.032	0.017-0.057
2	6.3 Km	0.055	0.045-0.075
3	5.0 Km	0.050	0.028-0.071
4	2.1 Km	-0.024	0.007-0.10

Iyaji 1982 carried out some investigation into the variation of vertical refraction in Zaria. He made observation on four lines. The first line is 1098.650m. long over water. This line was also used by the author of this thesis. The line is about 1.50m above water. The second line is 1.669m long with 1.100m of its length over water and the rest over dry land. The third line is 648.0m long over land and it is 1.60m above the ground. The fourth line is 4.42Km long and has an estimated

height of 700.00 above the mean-sea-level and 40.0 meter above the ground. Details of the results are tabulated below.

Table 2.7: Values of K for Zaria

Line	K - Values		
	August	October	December
1	0.10±0.01	0.04±0.06	0.09±0.03
2	0.16±0.03	0.15±0.02	0.14±0.03
3	0.08±0.07	0.33±0.07	0.28±0.07
4	0.13±0.02	0.13±0.02	0.09±0.01

CHAPTER THREE

3.0 FIELD OPERATIONS

3.1 LOCATION OF LINES

Two lines of different configurations were located at two different places in Zaria for the investigation.

3.1.1 Line 1 (D_1-D_2

This line (designated line 1) is located at the university dam-site (Kubani dam). The line extends from one end of the dam to the other. 96% of the length of the line is over water. The line is over one kilometer long. The exact length is 1.098.629m. It lies at an estimated height of about 2120 feet (642.2m) above mean-sea-level. This is an estimate from an available contour map of Zaria. In February when water level was low the line was about 1.70 meter above the water level. In September when the water level had risen, the line was about 1.50 meter above the water level. The line is almost 1/500. This line was chosen in order to investigate the behaviour of refraction along the line that passes over water and make comparison with the behaviour of refraction on line of similar length that passes over dry land and to

make reccommendation for the best time to observe vertical engles on such lines for the purpose of height determination .

The diagram of the location of the line is given in figure 3.1 below.

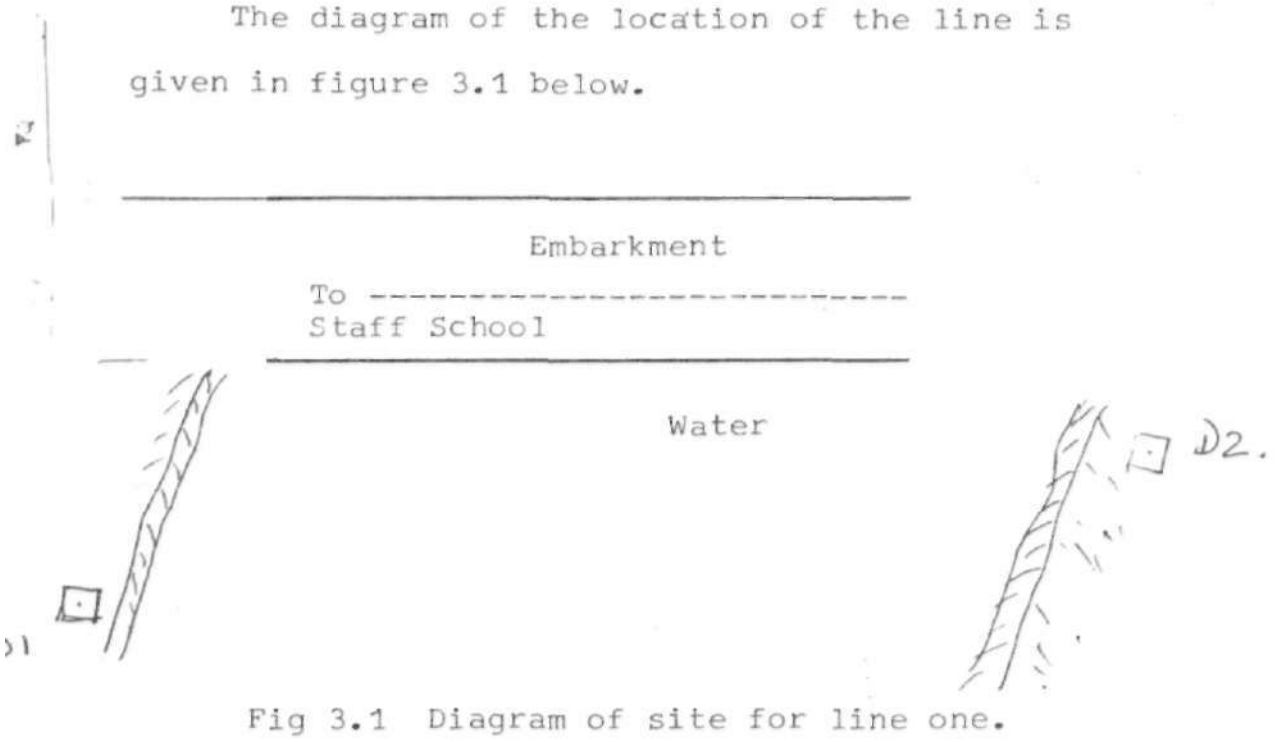


Fig 3.1 Diagram of site for line one.

3.1.2 Line Two ($P - I_4$)

Line two is also over one kilometer long. It is 1287.0m long. About 700 meter of the line passes over a deep gulley (about 9.0 meter deep). The line is defined by a pillar (designated P_1) established near the A.B.U. Press and a minesfield beacons I_4 which is at the other side of the gulley. The minefield beacon is about 3.1 meters high above the ground level and very close to the gulley. It is approximately 20.0 meters from the edge of the

gulley. The line runs over a scrub and vegetation cover. The elevation of the Minesfield beacon (I_4) is about 2120 feet above the mean-sea-level. (about 646.0m). The elevation of the pillar P_1 that was established near the A.B.U. Press is about 2145 feet above mean sea level. (about 654.0m). The line is slightly inclined with a slope of about 1/100: About half of the line is about 3.0 meters above the ground and the other half is 12.0 meters well above the ground. The line was selected so that the effect of vertical refraction on lines passing over scrubs and vegetation cover can be investigated and therefore make appropriate recommendation for the best time to observe vertical angles for the purpose of height determination.

The diagram of the locaiion of the line is given in figure 3.2 below.

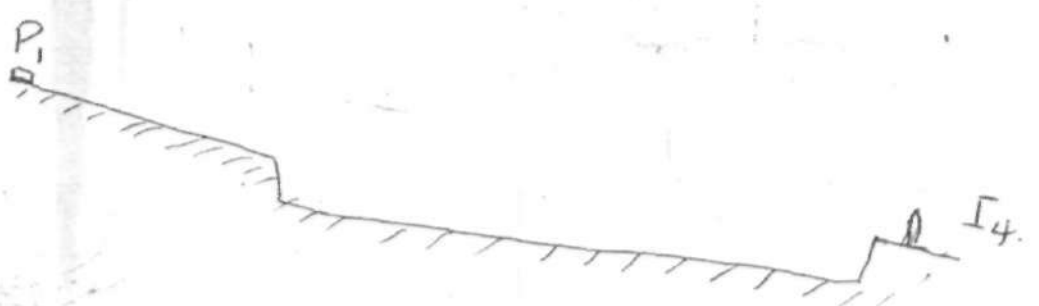


Fig. 3.2 Diagram for site for line two.

3.2 EQUIPMENT USED

The following equipments were used for the field work in the project.

(a) D.K.M.2A. Theodolite (No. AF.29071, was used for the measurement of vertical angles to a least count of ± 0.1 seconds of arcs.

(b) Wild T_2 traverse forget. This was set up on T_2 tripod and vertical angles were observed to them to a least count of ± 0.1 second of arc.

(c) Tape. Linear tape (30.0 meter) was used to measure the heights of targe and theodolite above the pillars on which they were set up to the nearest centimeter.

(d) Zeiss Eldi 1. This instrument with its reflector was used to measure the lengths of the line of sight.

(e) Altimeter and Thermometer. Hand altimeter and ordinary thermometer were used to measure the pressure and temperature respectively.

(f) Automatic Level and Levelling Stare. Direct levelling was carried out using automatic level to determine the difference in height between the terminals of line (D_1-D_2).

3.2.1 Adjustment and Standardisation of the Instrument

The D.K.M.2A was tested for vertical collimation error. The difference between face left and face right of the vertical circle readings was two seconds (2 seconds of arc). This is within the allowable difference and as such no adjustment was carried ot out on the theodolite.

3.2.2 Caliloration of Eldi 1

The Eldi 1 was calibrated using the 30.0 meter base line in the Department of Civil Engineering. However, a base line of 100.0 meter is the ideal one but since this is not available the 30.0 meter base line was therefore used. The standard error of the instrument is given by:

$$\sigma_2 = \left[\frac{1}{n-1} \sum_{i=1}^{n} ei \right]^{1/2}$$

Blackshut et al (1945)

where ei = error in each length combination,

n = number of stations (here n=4)

Four stations were created as A, B, C, & D.

The lines AB, AC, AD, BC, BD, CD were measured (see table below).

AD = 30.0M

Line	Observed Distances	$D=H^2/25$	Adjusted Length	Errors	Zero error (Z_o)
AC	16.897	16.897	16.899		
AP	8.964	8.964	8.966	AB-(AC+CD)=0.002	0.001
PD	21.030	21.032	21.033	AD-(AB+BD)=0.001	
PC	7.932	7.932	7.934	AD-(AB+BC+CD)= 0.001	
CD	13.096	13.096	13.099		

3.3 OBSERVATION PROCEDURE

3.3.1 Vertical Angle Measurement

Observations were made from 8.00a.m. to 4.00p.m. at hourly interval on each day of observations. For line 1 the instrument was set upon D_1 and the target on D_2. D_1 is lower than D_2 and as such angle of elevation was always observed on this line. For line 2 there was already a target on the mines-field beacon so that no target was needed here. The established pillar P_1 was higher in elevation than the top of the target on the Minesfield beacon and as such angle of depression was always observed on this line. Ten sets of vertical angles on each face were observed to the targets at an hourly interval on each day of observations. This makes a total of

twenty readings for each hour. Readings always commenced five minutes to the hour and five minutes after each hour. The average time lapse for the whole set of twenty readings was ten minutes. The heights of both the theodolite and the targets above the pillars on which they were set were also measured with linear tape and recorded in the field book.

3.3.2 Temperature and Pressure

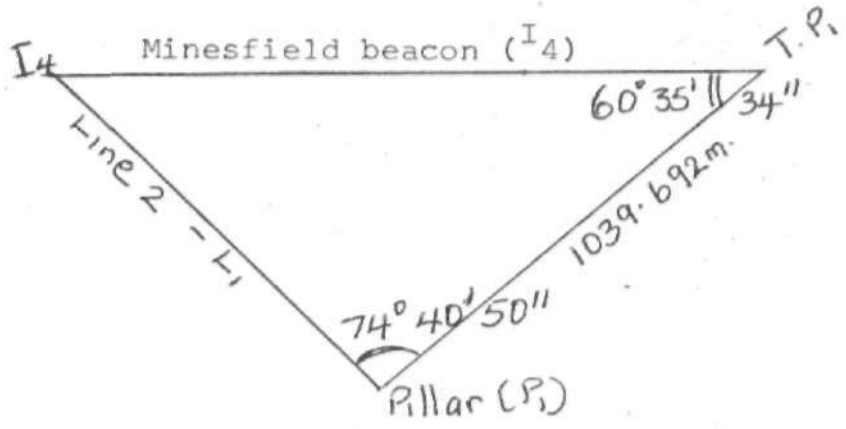
Temperature and Pressure readings were taken and recorded just before and immediately after each set of twenty vertical angle observations. The mean of the two readings is then taken to represent the temperature and pressure for that short period.

3.3.3 Time Interval Between the Observation on the Two Lines

In order to have good basis for comparison observations for vertical angles, temperature and pressure as mentioned above are made on the two lines one after the other in two consective days. This is to ensure that the weather conditions have not changed so much for the two lines. Two days in the months were spent on each line.

3.3.4 Linear Measurements

Direct measurement of line 1 was made using the Eldi 1. The values obtained for the mean of six readings taken on the field was 1098.627 $\pm$ 0.01m. Direct measurement of line 2 was impossible as the reflector could not be levelled on the Minesfield beacon. An indirect measurement of the line was then made by setting out a triangle on the field as shown below.



Line P_1 to $T.P_1$ (Temporary peg 1) was measured with Eldi 1. The mean value of 1039.692 $\pm$ 0.002 was obtained from the six measured values.

Using sine rule

$$\text{Line 2} = \frac{1039.639 \sin 60^{\circ}35'34''}{\sin 44^{\circ}43'36''}$$

$$= 1287.05 \text{ metres.}$$

In the above exercise only angles at P_1 and $T.P_1$ could be measured on the field. For the reason mentioned above the angle at the Minesfield

beacon could not be measured. This angle was then deduced.

Error term in L_1

Let the measured side be x

$60^{\circ}\ 35'\ 34'' = A.$

$44^{\circ}\ 43'\ 36'' = B.$

$\therefore L_1 \quad \frac{X \sin A}{\sin B}$

differentiating L_1 partially

$$dL_1 = \frac{\sin A}{\sin B} dx + \frac{X \cos A\, dA}{\sin B} - \frac{X \sin A \cos B\, dB}{\sin^2 B}$$

A Now assume $dx = 0.01m$ $dA = dB = 1''$

$$dL_1 = \frac{\sin 60^{\circ}\ 35'\ 34''}{\sin 44^{\circ}\ 43'\ 36''}\ 0.01 +$$

$$1039.692\ \frac{\cos 60^{\circ}\ 35'\ 36''}{\sin 44^{\circ}\ 43'\ 36''}\ \frac{1}{206265}$$

$$- 1287.05\ \frac{\sin 60^{\circ}\ 35'\ 54''\ \cos 44^{\circ}\ 43'\ 36''}{\sin^2 44^{\circ}\ 43'\ 36''} \times \frac{1}{206265}$$

$= 0.01 + 0.004 - 0.006$

$= + 0.008m.$

3.3.5 Measurement of Difference in Height

(i) Levelling In order to determine the difference in height between D_1 and D_2 in the two months of observations, direct levelling was carried out from PM18 at the university staff school to connect D_1 and D_2 and back to PM18 thereby forming a loop. This exercise offers an additional opportunity of determining the elevations of D_1 and D_2 absolutely, The two pillars are strategically placed at the dam site.

(ii) Reciprocal Vertical Angle Observation

For line 2 it was not possible to level from P_1 to the Minesfield beacon because of the topography. Also for reasons mentioned earlier on, it was impossible to set up on the Minesfield beacon. Temporary pegs were then created at points close to the Mines-field beacon and clearly visible from the pillar (P_1) to observe reciprocal vertical angle to determine the difference in height between the pillar (P_1) and the temporary pegs. Levelling was then run between the temporary pegs and the Minesfield beacon to determine the difference in height betwenn

the temporary pegs and the Minesfield beacon. The difference in heigh between the Minesfield beacon and the pillar (P_1) was then deduced.

CHAPTER FOUR

4.1 COMPUTATION AND DATA ANALYSIS

The difference in height between D_1 and D_2 the terminal of line 1 was obtained from the computed reduced level of D_1 and D_2. The difference in heighs for the months of March and September are 0.408 and 0.410 meters respectively. See the field book.

The fifference in heights between the Pillar (P_1) and the Minesfield beacon, the terminal of line 2 was computed using the reciprocal vertical angles which were simulteneously observed at P_1 and the temporary peg together with the levelling between the temporary peg and the Minesfield beacon.

The formula Allan et al (1968);

$$\Delta H = S\tan \phi_A + i_A - g_B \quad \ldots\ldots\ 4.1$$

was used to compute the difference in height between the Pillar (P_1) and the temporary pegs (TP). In this formula

AB = length of line

θ_A = angle observed at end point A

i_A = height of instrument at end point A

and g_B = height of target at the other end point P.

The mean of the observed six values of $\emptyset_A$ was used in the formula 4.1. The mean of the two values computed from either end was used as the true difference in height petween P_1 and the temporary peg.

To obtain the standard error of the determined difference in heights, formula 4.2 is used to compute the error in θ (the corrected plane angle computed for line P_1 to the temporary peg (TP)). This error was then used in equation 4.6 to compute the error in difference in height determined.

ΔH = Stan θ 4.2

where S = the length of line.

ΔH = difference in height between the two points defining the line.

θ = Corrected plane angle. Differentiate 4.2 with respect to θ

$d(DH) = S \sec^2 \emptyset \, d\emptyset$ 4.3

Differentiate 4.2 with respect to S.

$d(DH) =$ ten θ ds 4.4

squaring 4.3 and 4.4 and adding

$$d(\Delta H)^2 = \tfrac{1}{2}\left[S \sec^4\theta \left(\frac{(\theta)}{206265}\right)^2 + \tan^2\theta \, (ds)^2 \right]$$

..... 4.5

Taking the square root of 4.5

$$d(\Delta H) = \tfrac{1}{2}\left[S^2 \sec^4\left(\frac{\theta}{206265}\right)^2 + \tan^2\theta\,(ds)\right]^{\frac{1}{2}} \quad \ldots\ldots\ldots 4.6$$

Assuming ds = 0.01 and substituting the values of O computed for March and September, the standard error in the difference in height are as tabulated below:

Table 4.1: Standard Error in (DH)

Line	March	September
1	0.002	0.0031
2	0.001	0.004

4.2 COMPUTATION OF THE COEFFICIENT OF REFRACTION (K)

The formula Allan et all (1968)

$$K = \tfrac{1}{2}\left[1 - \frac{(\theta - \phi)}{PS}\right] \quad \ldots\ldots\ldots 4.7$$

where K = Coefficient of refraction

θ = plan angle computed using the formula 4.8. (The configuration for the computation of θ is illustrated in the

appendix II

S = Length of the line

Ø = Observed angle

P = Cosec $\frac{1}{2R}$ = Constant

R = radius of the earth,

was used to compute the values of K. A computer programme was written for the computation of K using equation 4.7

Separate values of coefficient of refraction were computed for each of the ten observations made for each hour on the two lines. See appendix III for the program and computation.

Mean values of the ten values of K for the ten observations were computed for each hour and then standard errors of the mean for the particular hour were also computed (see Appendix). Thrice the standard errors of K values were obtained using the Casio FX 3600P hand calculator.

Some values of K among the set of K values for any particular hour that varied more than three times the standard error from the mean value were rejected and the new means and their standard errors were again computed for each hour and are tabulated in Tables 4.2 and 4.3.

Table 4.2A: Values of K Computed on Line 1 March 1986

Time	1st Half March	2nd Half March	Mean March
08.00	0.225 ± 0.025	0.245 ± 0.033	0.235 ± 0.014
09.00	0.205 ± 0.011	0.188 ± 0.026	0.197 ± 0.012
10.00	0.195 ± 0.020	0.177 ± 0.033	0.191 ± 0.012
11.00	0.164 ± 0.026	0.164 ± 0.020	0.164 ± 0.000
12.00	0.116 ± 0.021	0.127 ± 0.019	0.143 ± 0.025
13.00	0.115 ± 0.020	0.109 ± 0.031	0.112 ± 0.004
14.00	0.106 ± 0.020	0.108 ± 0.021	0.107 ± 0.001
15.00	0.102 ± 0.025	0.101 ± 0.021	0.102 ± 0.001
16.00	0.133 ± 0.021	0.172 ± 0.029	0.153 ± 0.028
MEAN	0.151 ± 0.047	0.155 ± 0.048	0.156 ± 0.046

Table 4.2B: Values of K Computed on Line 1 for the Month of Semptember 1986

Tine	1st Half of September	2nd Half of September	Mean for September
0800	0.191 ± 0.025	0.164 ± 0.027	0.178 ± 0.
0900	0.200 ± 0.015	0.145 ± 0.023	0.173 ± 0.039
1000	0.164 ± 0.018	0.128 ± 0.023	0.144 ± 0.029
1100	0.110 ± 0.023	0.128 ± 0.033	0.119 ± 0.013
1200	0.098 ± 0.033	0.113 ± 0.021	0.106 ± 0.011
1300	0.103 ± 0.027	0.114 ± 0.026	0.109 ± 0.008
1400	0.083 ± 0.020	0.090 ± 0.019	0.087 ± 0.005
1500	0.084 ± 0.016	0.106 ± 0.026	0.095 ± 0.016
1600	0.104 ± 0.025	0.113 ± 0.021	0.109 ± 0.006
Mean	0.126 ± 0.046	0.122 ± 0.022	0.123 ± 0.034

Table 4.3A : Values of K Computed for Line 2 for the Month of March 1986

Time	1st Half March	2nd Half March	Mean March
0800	0.208 ± 0.022	0.277 ± 0.022	0.243 ± 0.049
0900	0.193 ± 0.020	0.265 ± 0.016	0.229 ± 0.051
1000	0.145 ± 0.027	0.227 ± 0.021	0.186 ± 0.058
1100	0.133 ± 0.026	0.177 ± 0.022	0.155 ± 0.031
1200	0.124 ± 0.015	0.174 ± 0.017	0.149 ± 0.035
1300	0.114 ± 0.020	0.138 ± 0.018	0.126 ± 0.017
1400	0.111 ± 0.023	0.130 ± 0.015	0.125 ± 0.013
1500	0.112 ± 0.014	0.125 ± 0.017	0.119 ± 0.009
1600	0.121 ± 0.019	0.161 ± 0.016	0.141 ± 0.028
Mean	0.140 ± 0.036	0.186 ± 0.057	0.163 ± 0.046.

Table 4.3B : Value of K Computed for Line 2 for the Month September 1986

Time	1st Half September	2nd Half September	Mean September
08.00	0.162 ± 0.020	0.150 ± 0.016	0.156 ± 0.008.
09.00	0.141 ± 0.023	0.152 ± 0.015	0.147 ± 0.008.
10.00	0.127 ± 0.019	0.128 ± 0.016	0.128 ± 0.001.
11.00	0.110 ± 0.024	0.117 ± 0.019	0.114 ± 0.004
12.00	0.111 ± 0.016	0.116 ± 0.019	0.114 ± 0.004
13.00	0.088 ± 0.016	0.130 ± 0.014	0.109 ± 0.030
14.00	0.088 ± 0.013	0.113 ± 0.017	0.101 ± 0.018
15.00	0.083 ± 0.016	0.109 ± 0.012	0.096 ± 0.018
16.00	0.110 ± 0.020	0.107 ± 0.013	0.109 ± 0.002.
Mean	0.113 ± 0.026	0.125 ± 0.017	0.119 ± 0.020

4.3 INFERENCES FROM DATA

Some of the inferences that can be drawn from the data are as follows:

In the month of March, the diurnal variation of the values of K for line 1 is 0.102 - 0.235 corresponding to 3.3 secs/km - 7.6 secs/km. This variation is significant. The standard error varies from 0.001 - 0.028. The small values of the standard errors is an indication of the reliability of the observation. It shows that the computed values of K have little dispersion about the true values.

In the month of September the range of the values of K is 0.087 - 0.178 corresponding to 3.2 secs/km - 5.2 secs/km. This range is smaller that that of March. The standard errors vary from 0.005 - 0.039 which also gives an indication of better reliability and also an indication that the computed values of K are closer to the true values. The mean 0.123 obtained for the month compares with the mean 0.10 obtained for the same line in the month of August 1984 by Iyaji.

From the mean values of K for March and September it can be inferred that refraction is lower in September (rainy season) than in March (dry season). Also the range of variation of refraction in rainy season is lower than the range in dry season. This is in agreement with the fact mentioned earlier on that there is less diurnal variation in refraction in wet season than in dry season. Also the standard error of the mean in March for line 1 is $\pm$0.046 and the standard error of the mean for the same line in September is $\pm$0.034. The lower standard error in September is a indication that observations are more reliable in September than in March. In March, observation could be adversely affected by foggy weather. A more accurate bis-section of the target can be achieved on a clear day in wet season than on foggy day in dry season. It then means that vertical angles for the purpose of height determination is better observed on a clear day during wet season than in dry season when it is generally foggy.

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In the month of March the diurnal variations of K - values on line 2 is 0.119 - 0.243 corresponding to 3.85 secs/km - 7.86 secs/km. The standard errors vary from 0.009 - 0.058. The small range of standard

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error is an indication of reliability of the observation and small deviation of the K values from the true values.

Also the range of 0.096 - 0.156 of the computed values for K corresponding to 3.16 secs/km - 6.50 secs/km for line 2 in the month of September is smaller than the range for March. The range of standard errors 0.001 - 0.030 is also smaller. These are also indications of reliability of observation and closeness of the computed values for K to the true values.

The mean K values for March is 0.156 ± 0.046 and the mean for September is 0.123 ± 0.034. Refraction is higher in March than in September as mentioned above. Also the higher standard error in March suggests that observation is more reliable in September than in March (Dry Season).

In the month of March, refraction on lines 1 and 2 varies as follows:

for line 1, the diurnal variation is 3.3 secs/km - 7.6 secs/km, and for line 2 the variation is 3.85 secs/km - 7.86 secs/km. They seem to vary the same way with slightly higher refraction on line 2 than on line 1. This is expected as line 2 is more inclined than

line 1.

Refractions increases with the angle of inclination Brown (1970).

In the month of September refraction on the two lines varies as follows:

For line 1, the diurnal variation is 3.2 secs/km - 5.2 secs/km and for line 2, the range is 3.16 secs/km - 6.50 secs/km. The variation for line 2 is more significant than the variation for line 1 during this month. Here again refraction is very much higher in line 2 than on line 1 for reason mentioned above.

4.3.0 Measure of Reliability of Observation

Attempt is made to estimate the measure of reliability of the observation by estimating the possible observational errors that can be introduced into the work. This work is prone to some errors which are instrumental,natural and physical. Example of instrumental errors is the Collimation errors. In order to elliminate this error observations are made on both face left and right. Examples of errors due to physical conditions is shimmer. This is beyond the control of the observers. Errors arising from the natural conditions are those due to the personal equation of the observer. This includes

the following:

(i) Inaccurate bisection of the target

(ii) Centering of the bubble and

(iii) Reading the micrometer.

The error (a) above depends on the length of the line and on the size of the target. The target drawn below is not drawn to scale:

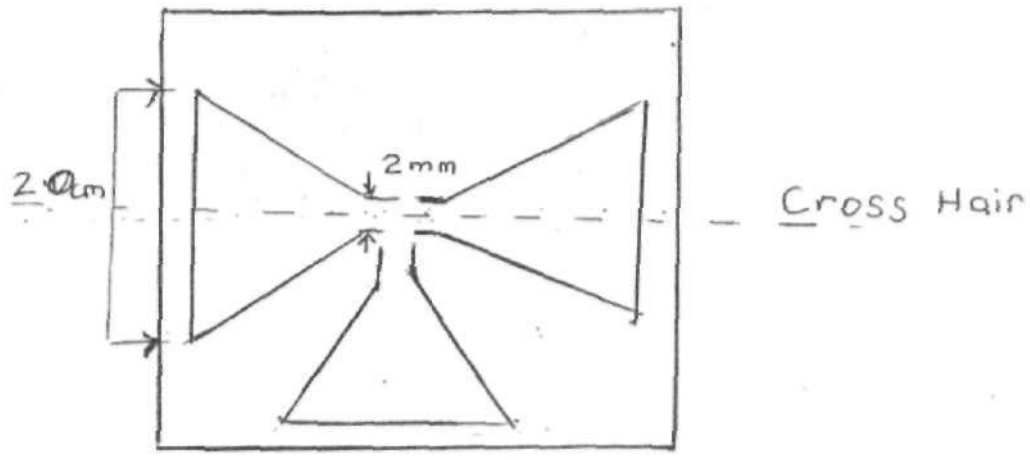


Fig. 4.1 Target T_2

The error is proportional to the length of the line. Ideally the target is well bisected as shown above. At a distance of over one kilometer the ideal bisection is difficult to achieve. In This work it was ensured that the white triangles were well focussed. In bisecting the two horizontal adjascent triangles at a distance of over one kilometer max. error of $\pm$ 0.5 cm can be assumed.

The angle subtended at the centre of the theodolite by an intercept of Xcm on the target is given by

$$\emptyset = \frac{X}{S} \qquad \ldots\ldots\ldots\ldots 4.7A$$

where $\emptyset$ is the angle subtended at the centre.

X = intercept on the target

S = length of the line.

Differentiating 4.1 with respect to X

$$d\emptyset = dx/_{S}.$$

$$d\emptyset = \frac{0.005}{1098.627}$$

$$= 0.99 \text{ sec.}$$

To estimate the errors arising from centring of the bubble and reading the micrometer, a test observation was carried out to observe the behaviour of the theodolite. The theodolite was set up and levelled and fixed in a direction. Micrometer readings were taken ten times each time relevelling the theodolite and re-establishing symmetry in the panel. This was done in three different directions and the readings are tabulated below with their mean and standard errors.

Table 4.4 : Micrometer Readings of Test Observation

In Seconds of arc		
Set I	Set II	Set III
38"	19	45
39	18	46
38	19	45
40	19	45
39	18	46
39	18	46
39	19	45
39	17	45
40	19	46
39	19	45
38.9 ± 0.74	18.5 ± 0.71	45.4 ± 0.52

The mean of the three standard errors is 0.66.

The combined estimated errors in observation is:

$$\sigma = (0.99^2 + 0.66^2)^{\frac{1}{2}}$$

$$= 1.19 \text{ sec}$$

$$= 1.2 \text{ secs}$$

$$= 1.08 \text{ sec/km}$$

= 1.08 x 1.96 = 2.12 secs at
95% confidence interval.

4.3.1 Computed error Terms in Parameter Used to Determine K - values

The formula $\theta = \sin^{-1}(\frac{h}{s})$ 4.8

where

θ = Computed plane angle

h = difference in height between two end points of the line

s = length of the line, was differentiated partially with respect to h and s to obtain the error term in the angle θ computed using the measured slope distance and the computed difference in height.

Differentiating equation 4.8.

$$\cos\theta\, d\theta = 1/S\, dh - \frac{h}{s^2} ds \quad ..... \quad 4.9$$

Squaring and ignoring the cross product terms and using sigma notation for the error terms.

$$\cos^2\theta\, d\phi^2 = \frac{1}{S}\, \sigma_h^2 + \frac{h^2}{S^4}\, \sigma_s^2$$

$$\sigma_{\phi}^2 = \frac{\sigma_h^2}{S^2\cos^2\phi} + \frac{h^2}{S^4\cos\phi}\,\sigma_S^2 \quad \ldots\ldots\ldots \quad 4.10$$

Now $\cos^2 + \sin^2 \phi = 1.$

$\therefore \cos^2 \theta = 1-\sin^2 \theta$

but $\sin \theta = h/_S$ from 4.8

$$\therefore \cos^2 = 1-h^2/S^2 = \frac{S^2 - h^2}{S^2} \quad \ldots\ldots\ldots \quad 4.11$$

putting equation 4.11 into 4.10

$$\sigma_{\phi}^2 = \frac{1}{\frac{S^2(S^2-h^2)}{S^2}}\,\sigma_h^2 + \frac{h^2}{\frac{S^4(S^2-h^2)}{S^2}}\,\sigma_S^2$$

= Simplifying

$$\sigma_{\phi}^2 = \frac{1}{S^2 - h^2}\,\sigma_h^2 + \frac{h^2\,\sigma_s^2}{S^2(S^2-h^2)}$$

$$= \frac{\sigma_h^2}{S^2 - h^2} + \frac{h^2\,\sigma_S^2}{S^4-S^2h^2}$$

$$\sigma_\theta = \left[\frac{\sigma_h^2}{(S^2-h^2)} + \frac{h^2\,\sigma_s^2}{S^2(S^2-h^2)}\right]^{\frac{1}{2}} \quad \ldots\ldots\ldots \quad 4.12$$

Assuming $\sigma_h = \sigma_s = 0.01$m. Linean tape was used to measure the heights of target and the trunion axis of the theodolite above the pillar. Eldi 1 was used to measure the distance. Substituting the values of S and h the standard errors of the computed plane angles are tabulated in the Table 4.5 below. Casio FX 3600P was used to compute the error term in θ.

Table 4.5 : Computed Error Term in θ in seconds of arc

Line	1st Half March	2nd Half March	1st Half Sept.	2nd Half Sept.
1	± 1.86	± 1.86	± 1.26	± 1.86
2	± 1.65	± 1.65	± 1.65	± 1.64

4.3.2 Statistical Analysis

The varianaces of the observed vertical angles and the difference between the plan angle and the mean values of the observed vertical angles ($\theta - \overline{\emptyset}$) that were used to compute the values of K were computed and tabulated in the table below:

Table 4.6 : Variances of ($\theta - \bar{\theta}$) Angles in Second of arc.

Line	1st Half March	2nd Half March	1st Half Sept.	2nd Half Sept.
1	3.16	2.85	2.81	1.23
2	2.34	5.80	1.34	0.84

Table 4.7 : Mean Seconds of arc. of Observed Angles

Line	1st Half March	2nd Half March	1st Half Sept.	2nd Half Sept.
1	17.32	8.26	21.42	58.68
2	37.58	26.89	17.58	48.14

Table 4.8 : $\theta - \bar{\theta}$ in Seconds

Line	1st Half March	2nd Half March	1st Half Sept.	2nd Half Sept.
1	12.61	12.28	13.20	13.42
2	15.10	13.21	16.11	15.69

The standard deviation of the sample distribution of the means were computed using the formula (Norclliffe 1977).

$$\sigma = \sqrt{\frac{S_1^2}{N_1} + \frac{S_2^2}{N_2}} \qquad \ldots\ldots\ldots\ldots \quad 4.13$$

were S_1^2 & S_2^2 are variances of the samples,

σ = standard deviation. The students t values were computed using the formula (Nordiff 1977).

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sigma}$$

where t = student's t values. $\bar{x}_1$ and $\bar{x}_2$ are the mean values of the sample (θ - ϕ)

σ = standard deviation computed from 4.13.

The values are tabulated below in table 4.9.

Table 4.9 : Cumputed Student's t Values

Line	Between (sample groups)	Student's t
1	1st Half March - 2n Half March	
	1st Half Sept. - 1st Half Sept	0.79.
	1st Half March - 1st Half Sept.	2.29.
	1st Half March - 2nd Half Sept.	3.67.
2	1st Half March - 2nd Half March	6.28.
	1st Half Sept. - 2nd Half Sept.	1.06.
	1st Half March - 1st Half Sept.	4.99
	1st Half March - 2nd Half Sept.	3.14.
1	1st Half March - 1st Half March	10.07.
and	2nd Half March - 2nd Half March	2.90
2	1st Half March - 1st Half Sept.	15.65.
	1st Half March - 2nd Half Sept.	14.61.
	2nd Half Sept. - 2nd Half Sept.	14.96.

A two tailed test using the student's t distribution at 95% confidence Interval was used to test whether the mean values of coefficient of

refraction for any particular line for different period or the coefficient of refraction for the two lines for different or the same periods show any significant difference. The values of t computed in table 4.9 are compared with the values of t with $N_1 + N_2 - 2$ degree of freedom in statistical table. N_1 and N_2 are the sample sizes. A comparison of t computed in table 4.9 and the critical value of t at 95% confidence interval shows the following results:

1. The critical value at 95% confidence interval is 1.960. It then means that all the t - values greater than this shows significant difference.
2. The t-values for the two halves of March and for the two halves of September are less than the critical value. The values of K for these periods do not show any significant difference. This is what is expected.
3. The t-values for the two halves of September for line 2 is also less than the critical value. This is also expected. However, the t-value for the two halves of March is above the critical value. The difference

for those two period is significant. This may be due to the fact that towards the end of the month rainy season had set in. There had been a gradual change of weather.

4. All the t-values for the two lines for different periods or the same period are greater than the student's values. The differences in the value of for these two lines for the different periods or the same period are significant at 95% confidence interval.

4.4 GRAPHICAL ANALYSIS

The following graphs were ploted:

(i) Values of the coefficient of refraction against time of the day.

(ii) Values of the coefficient of refraction against $1/_T{}^2$.

where T is the absolute temperature.

The values of $1/_T{}^2$ computed for different hours are tabulated below. For ease of reference the values of K are incoporated into the table.

Table 4-9. Computed Values of $^1/T^2$ Line 1 March

Time	Computed K-Values	T	$^1/T^2$
08	0.235 ± 0.014	306	107x 10^{-7}
09	0.197 ± 0.012	307	106 "
10	0.191 ± 0.012	307.5	106 "
11	0.164 ± 0.000	308	105 "
12	0.143 ± 0.025	309	104 "
13	0.112 ± 0.004	311	103 "
14	0.107 ± 0.001	311	103 "
15	0.102 ± 00001	310	104 "
16	0.153 ± 0:028	309.5	104 "

Table 4-10. Computed Values of $^1/T^2$ Line 1 September

Time	Computed K-Values	T	$^1/T^2$
08	0.174 ± 0.019	309	105 x 10^{-7}
09	0.173 ± 0.039	310	104 x 10^{-7}
10	0.144 ± 0.029	311	103 x 10^{-7}
11	0.119 ± 0.013	311.5	103 x 10^{-7}
12	0.106 ± 0.011	314	101 x 10^{-7}
13	0.109 ± 0.009	314	101 x 10^{-7}
14	0.087 ± 0.005	315	100 x 10^{-7}
15	0.095 ± 0.016	314	101 x 10^{-7}
16	0.109 ± 0.006	314	101 x 10^{-7}

Table 4-11. Computed Value of $^{1}/T^{2}$ Line 2 March

Time	Computed Values of K.	T	$^{1}/T^{2}$
08	0.243 ± 0.049	304	108×10^{-7}
09	0.229 ± 0.051	304.5	108×10^{-7}
10	0.186 ± 0.058	305.5	107×10^{-7}
11	0.155 ± 0.031	307.0	106×10^{-7}
12	0.149 ± 0.035	307.0	106×10^{-7}
13	0.126 ± 0.017	308.0	105×10^{-7}
14	0.121 ± 0.013	308.0	105×10^{-7}
15	0.119 ± 0.009	306.0	107×10^{-7}
16	0.141 ± 0.028	306.0	107×10^{-7}

Table 4-12. Computed Values of $^{1}/T^{2}$ Line 2 September

Time	Computed K-Values	T	$^{1}/T^{2}$
08	0.156 ± 0.008	305	107×10^{-7}
09	0.147 ± 0.008	305	107×10^{-7}
10	0.128 ± 0.001	306	106×10^{-7}
11	0.114 ± 00004	306	106×10^{-7}
12	0.114 ± 0.004	307	106×10^{-7}
13	0.109 ± 0.030	309	105×10^{-7}
14	0.101 ± 0.018	309	105×10^{-7}
15	0.096 ± 0.018	307	106×10^{-7}
16	0.109 ± 0.002	305	107×10^{-7}

The trends exhibited by the graphs are discussed below:

Line 1

Line 1 with 96% of its length over water can be assumed to be passing through a medium of almost uniform density especially at the period that the atmosphere has been heated that is is experiencing low temperature gradient change and hence low refraction effect. Small refraction variation is therefore expected.

In the month of March, the diurnal variation is larger than in September. The reason is due to the damping effect discussed in 2.2.5.

The general trend exhibited by March and September graphs is that diurnal variation of refraction is smaller between 12.00 hours and 15.00 hours with the least refraction between 13.00 hours and 14.00 hours. This confirms the statement in Allan at al (1968) that refraction is minimum and varies least between 12.00 hours and 16.00 hours. The graphs shows the same trends in diurnal variation as that depicted in figure 2.2 by Angus-Leppan (1961). Between 8.00 and 11.00 the K-values vary from 0.23 - 0.16 in March and from 0.18 - 0.12 in September. These are much greater variation. This is so

because during this period temperature gradient is changing rapidly.

The graphs of K-values against $1/T^2$ for line 1 for March and September shows that the k-value is approximately proportinal to the inverse of the square of absolute temperature. This confirms the statement in Pamphlet No. 1, (1956) that the value of K varies inversely as the square of the absolute temperature. This statement was put in the methematical form in equation 1.10. From the examination of the two graphs for March and September the values of K that obey the inverse proportional law are those values for the period 1100 hours to 14.00 hours. It then probably means that the telationship is only true during the time steady and minimum temperature gradient change.

The graphs of K-values against $1/T^2$ for this line for the months of March and September show one major difference. The deviations of the points from the best line of fit are more for March line than for September line. This may be due to one or both of these reasons.

(i) Temperature lapse rate for March is more eratic than for September.

(ii) Observations are not as reliable in March

as in September due to bad weather.

Line 2

About 700.00 meters of this line passes over a gully of about 8.0 m deep. The entire line passes over scrub vegetation. The graphs of the values of K against time show much wider variations in March than in September as for line 1. However, the smaller diurnal variations in refraction for line 2 during the two months was between 10.00 and 15.00 hours as against 12.00 and 16.00 hours for line 1. The longer period for the much smaller variations of refraction could be due to the effect of surfaces over which the two lines pass. That line 1 experience smaller variation over a longer period is in order with the statement of McIntosh and Thom (1973) that diurnal temperature variation over water surfaces (and of the air in contact with it) are very much smaller than those over the land.

The graphs of K against $1/T^2$ for the two months for the line also shows the same pattens as for line 1 above. The plotted points show wider spread about the best line of fit than those of line 1.

GRAPH OF K-VALUES VS TIME.

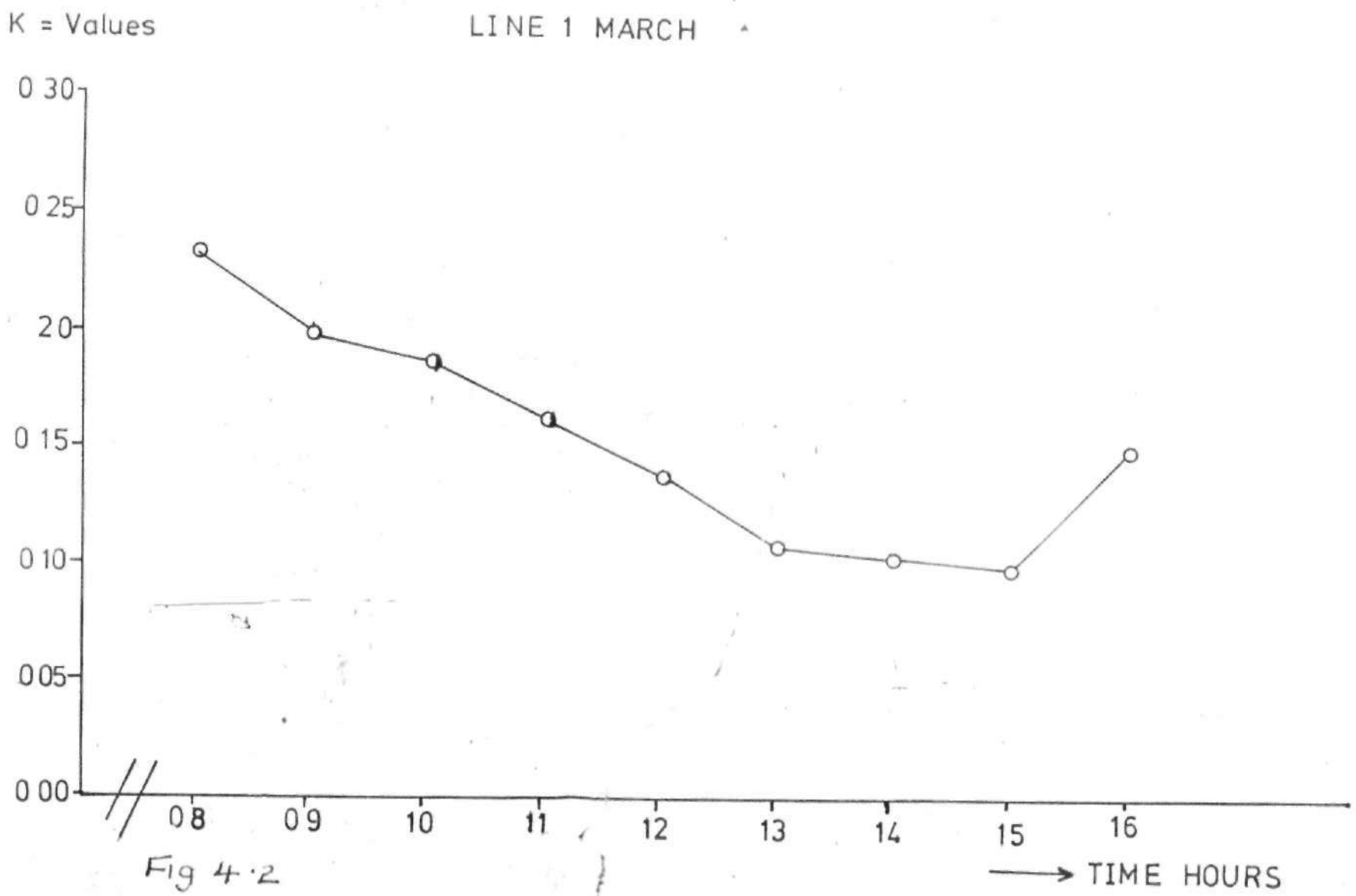


Fig 4·2

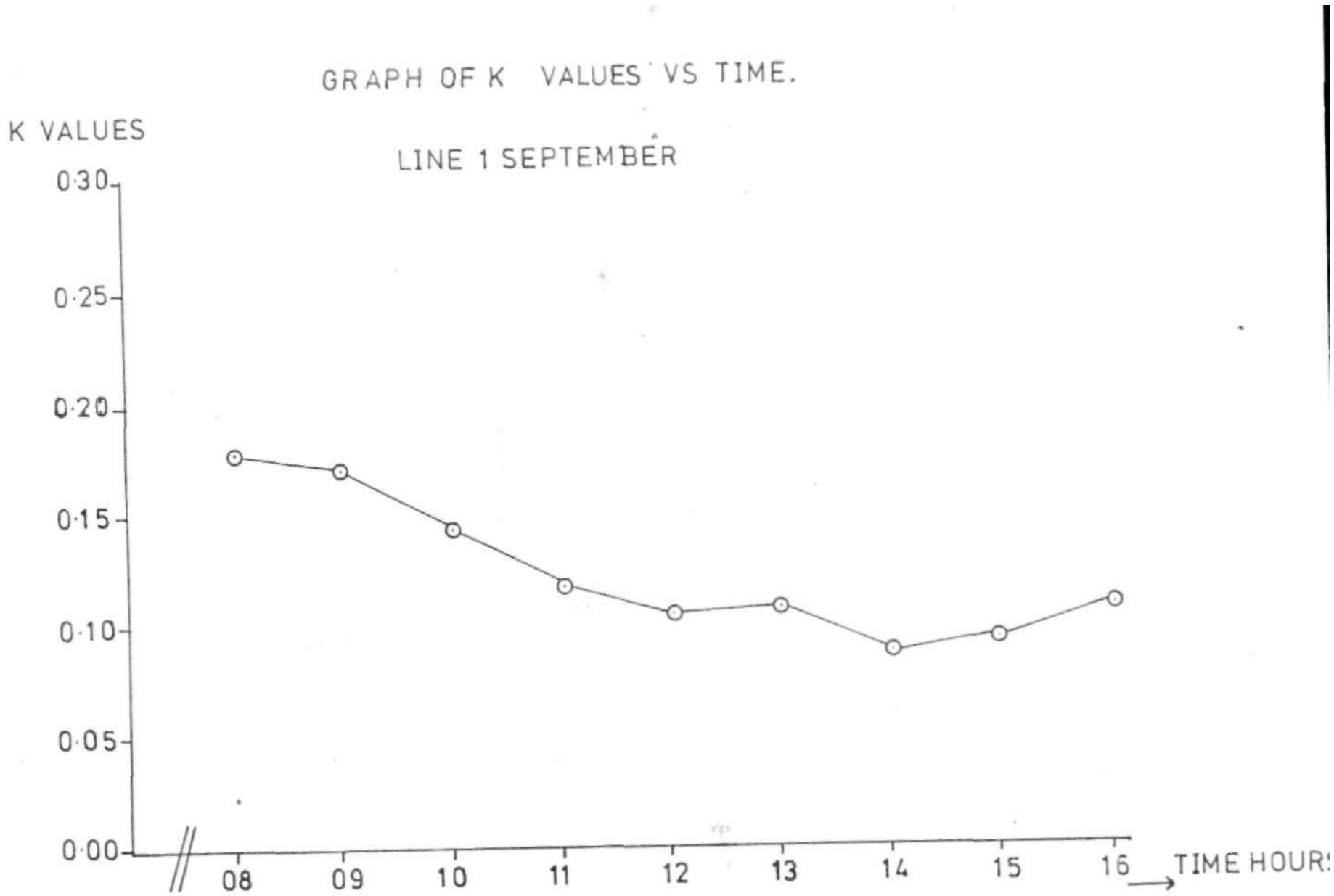

GRAPH OF K VALUES VS TIME.
LINE 1 SEPTEMBER
K VALUES
0·30
0·25
0·20
0·15
0·10
0·05
0·00
08
09
10
11
12
13
14
15
16
TIME HOURS

Fig 4·3

GRAPH OF K VALUES VS TIME

LINE 2 MARCH

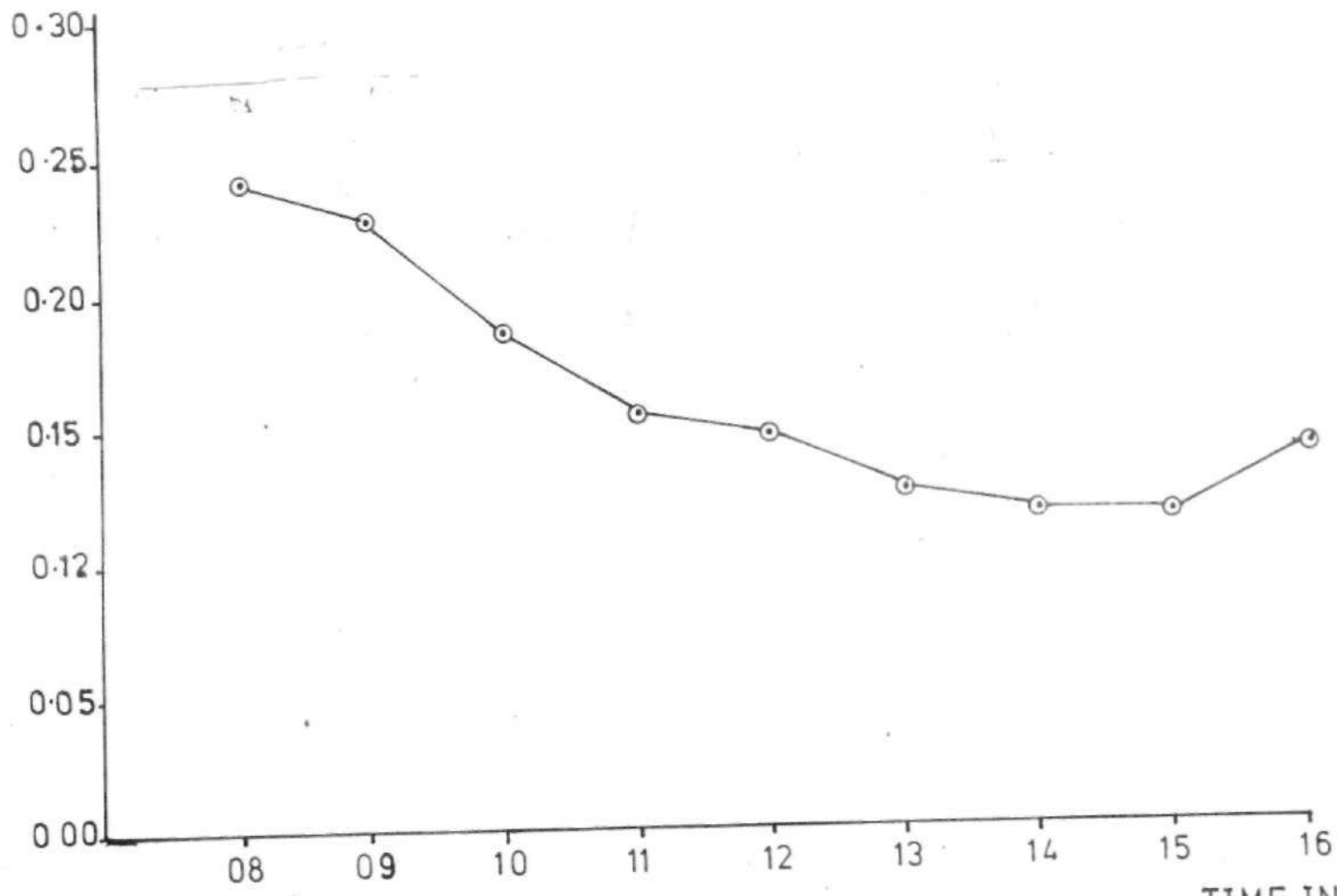


Fig 4·4

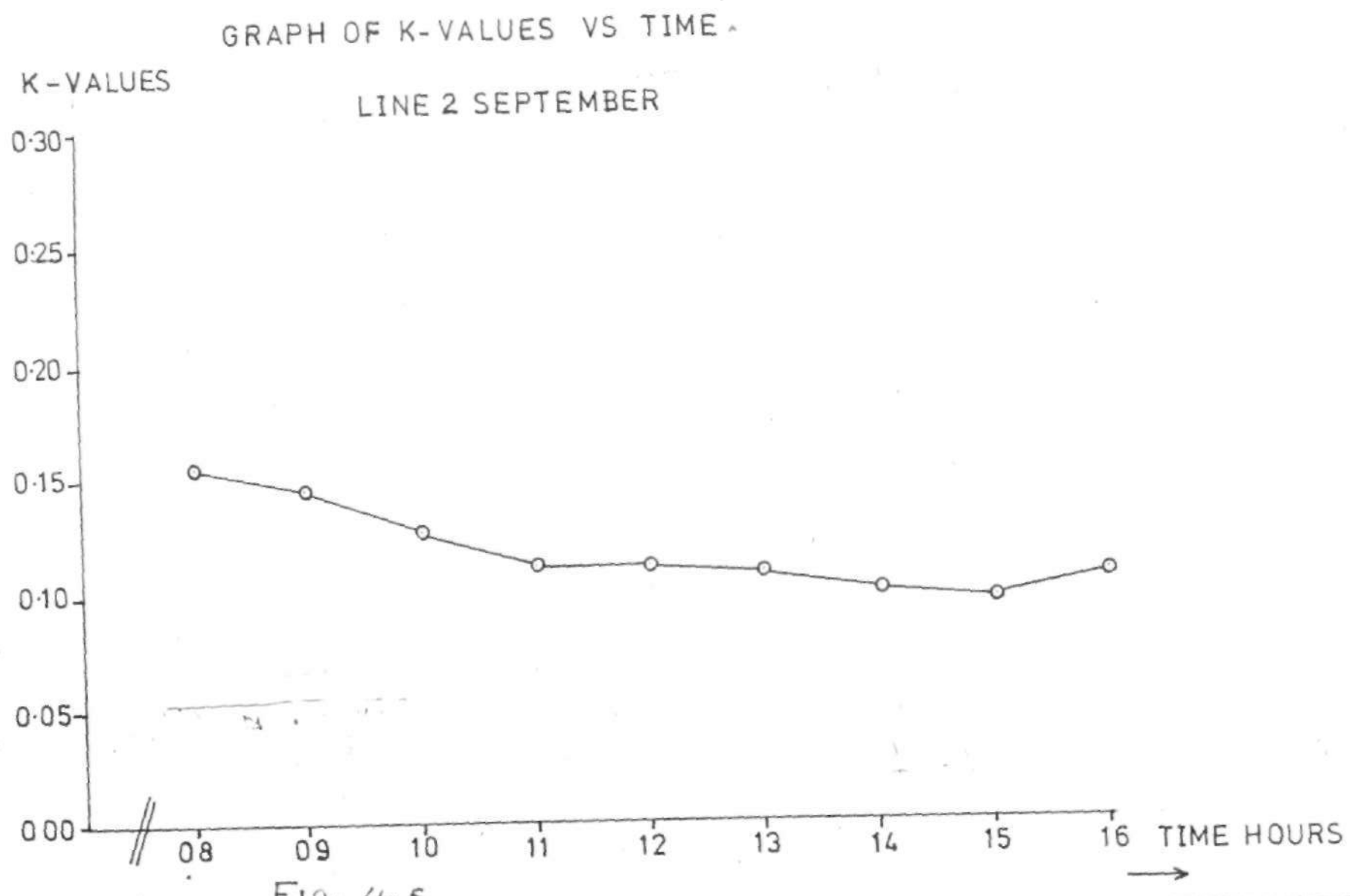

GRAPH OF K-VALUES VS TIME
K-VALUES
LINE 2 SEPTEMBER
0·30
0·25
0·20
0·15
0·10
0·05
0·00
08
09
10
11
12
13
14
15
16
TIME HOURS

Fig. 4.5

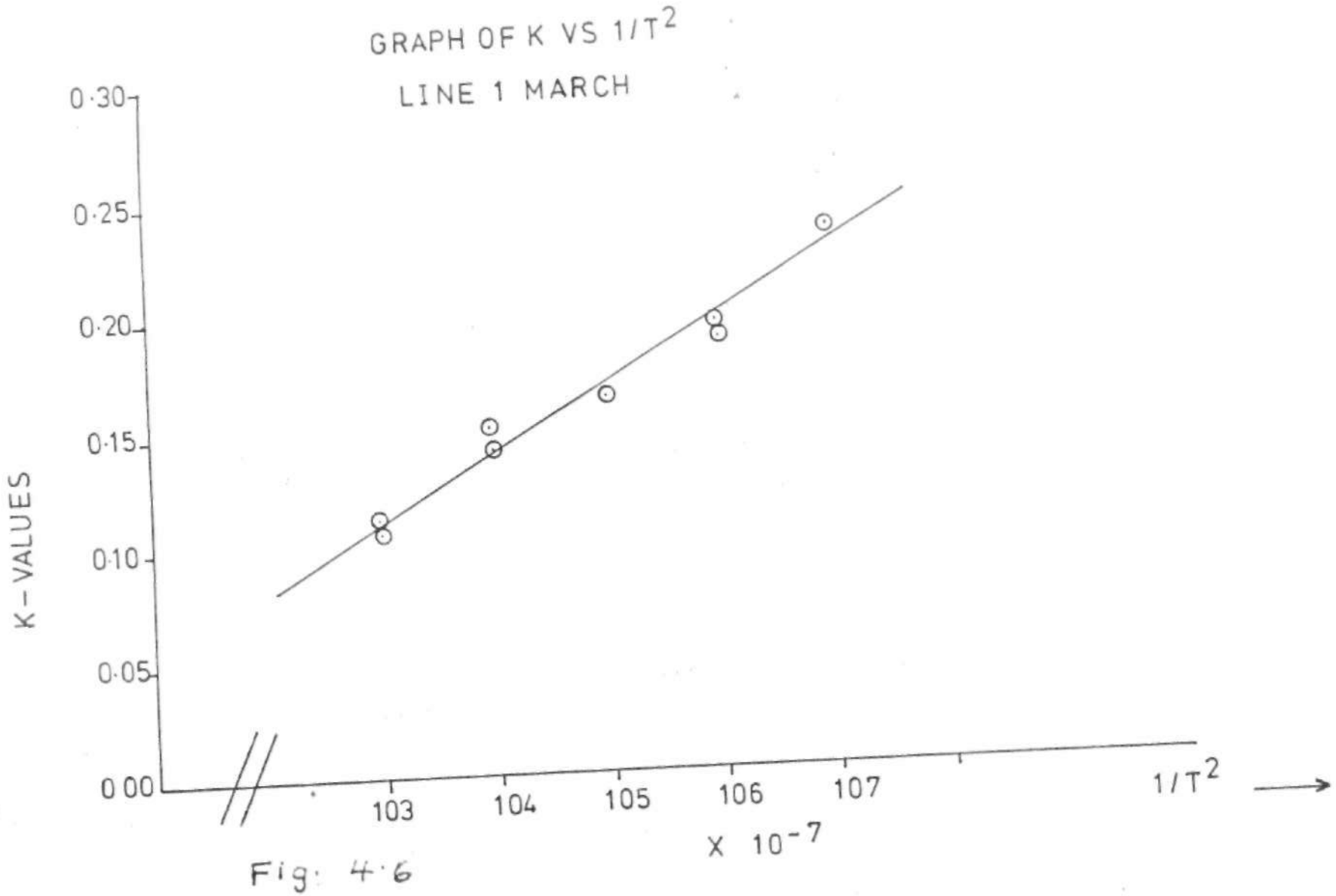

GRAPH OF K VS 1/T2
LINE 1 MARCH
0·30
0·25
0·20
0·15
0·10
0·05
0 00
K-VALUES
103
104
105
106
107
1/T2
X 10-7

Fig: 4·6

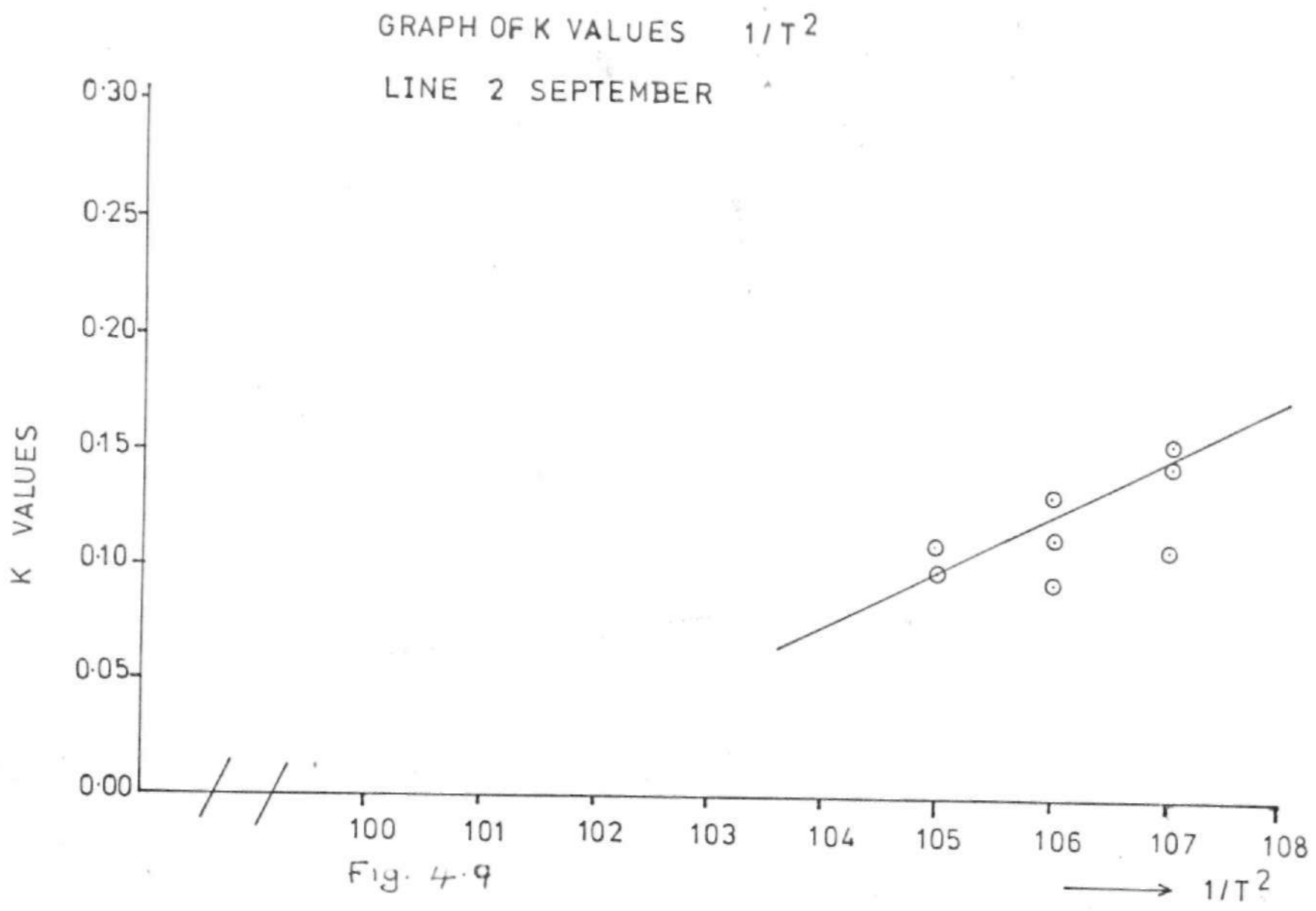

GRAPH OF K VALUES 1/T2
LINE 2 SEPTEMBER
K VALUES
0·30
0·25
0·20
0·15
0·10
0·05
0·00
100
101
102
103
104
105
106
107
108
1/T2

Fig. 4·9

CHAPTER FIVE

CONCLUSION AND RECOMMENDATION

The conclusions drawn on the results of this investigation are discussed below:

1. The diurnal variation of refraction on line 1 for March is 0.102(3.3 Secs/km to 0.235 (7.6 secs/km). This is significant. The September refraction varies from 0.087 (3.2 secs/km) to 0.178 (5.2 secs/km). This is also significant. The diurnal variation in March is more significant than the variation in September. The minimum refraction for the months are the same. On the basis of this vertical angles for the purpose of height determination can be observed in any of the two months, from 12.00 hours to 15.00 hours. However, on the basis of diurnal variations, September which is the month with less variation will be a better choice for vertical angle observations on lines passing entirely over water in Zaria.

2. For line 2, the variation during the day is between 0.119 (3.85 secs/km) to 0.243 (7.8 secs/km) for the month of March. The variation for September is 0.096 (3.16 secs/km) to 0.156 (6.50 secs/km). These variation are also

significant. The minimum for the two months for this line are the same. The maximum for March is higher than the maximum for September. Also on the basis of minimum refraction the two months scores are the same. But just as for line 1 the variation in March is more than the variation in September. On the account of this smaller range of variations September will be a better time to observe vertical angles for height determination in Zaria on lines passing over scrub vegetations.

From the result of 1 and 2 above the effect of season is very obvious. In wet season refraction tends to very less than in dry season. Wet season may then be a better month for vertical angle observation.

The effect of topography (surface types) is not very obvious in this work. This may be due to the fact that the sites of the two lines are adjacent and close to each other. The wind blowing from the water will have effect on line above the adjourning land. The atmospheric conditions of the medium through which the line 2 passes may not be too much different from that of line 1. It is therefore recommended that the locations for these two types of lines for future work should be very far from each other.

The values of K got from this research compares very well with the vaues obtained by earlier researchers in Kaduna State. The values got by Innbiaraye (1983) range from 0.007 to 0.10 for a line of 2.1 km. Iyaji 1984 got 0.10 $\pm$ 0.01 for the same line 1 in August 1984.

The values of K obtained are higher than the traditional values often found in text book and some values found in literature review. The reason for this difference may be due to the fact that the lines are very short comparatively.

This work confirms the findings of many authors that refraction varies according to seasons and time of the day with the minimum and steady refraction between 12.00 and 16.00 hours.

Finally, the investigation indicates the importance of meteorology in relation to observing. In the opinion of the author of this thesis every observer engaged on first order work ought to be something of a meteorologist. The time may not be far distant when air sounding may be considered an integral part of geodetic work-especially in the tropics where external conditions play such an important part in the standard of accuracy obtained. After all there are quite as likely to be serious

anomalies in the curvature of a ray of light as in the curvature of the earth, and surely this is a matter of some moment when triangulation is used for geodetic purposes.

In the absence of any attempt of correct for atmospheric errors, it is questionable, whether the term "Primary triangulation" infers the same angular degree of precision in different countries. What for example, is the use of defining such observations employed - such as the number of zeros to be employed when the external errors may be so much greater, as possibly they are in certain climates?

REFERENCES

1. Allan, A. L., Hollway, J. R., Maynes J. H. B, (1968) Practical Field Surveying and Computation Heimemanni London pp. 689.

2. Angus - Leppan (1961) A Study of Refraction in the Lower Atmosphere in Survey-Review Vol. 16 No. 120.

3. Annales du Burear Central Metroloque Pour (1894)

4. Asplund Lars (1957-1959). Reports on #Geodentic Operations in Sweeden Published in I.A.G. Helsinki 1960 at ed.

5. Black J. J. (1945) Urban Surveying and Mapping Springer Verlag New York Inc.

6. Bomford G. (1928) Three Sources of Errors in Precise Levelling in Survey of India Professional Paper Vol. 22.

7. Bomford G. (1977) Geodesy Oxford University Press pp. 731.

8. Brown M. S. (1970) The Variation of Refraction with Elevation Angle in Arid Climate in Survey Review Vol. 20 No. 158 pp 366-376.

9. Coulman C. E. (1967) Optical Effect of Thermal Structure in the Lower Atmosphere in Applied Optics Vol. 6 No. 3.

10. De-Graaff - Hunter J. E. (1941) Topographic Survey Tables in Survey of India Anxilliary Table 7th Ed. Part III.

11. Gerger R. (1950) The Climate near the Ground Harvard University Press.

12. Iyaji P. (1985) Investigation of the Variation of Geofficient of Vertical Refraction. Unpublished Thesis, Ahmadu Bello University, Zaria, Nigeria.

13. Levallois J. J. De Massin G (1953) D' Antume Etude Sur la Refraction Geodesique et la nivellement barometrique Parts.

14. McIntosh D. H. & Thom A. S. (1973) Essentials of Meteorology Wykeham Publications (London) pp. 393.

15. Morley A. J. (1937) On Evaluation of the Coefficient of Terrestial Refraction in Empire Survey Review Vol. 4 Nos. 23,25,40.

16. Morley A. J. (1941) On Evaluation of Coefficient of Terrestial Refraction in Empire

Survey Review Vol. 6 No. 40

17. Norcliffe G. S. (1977) Inferential Statistics for Geographers. Nutchinson London pp. 272.

18. Pamphlet No. 1 (1956) Instruction for Primary Triangulation Survey Department

19. Paferson W. S. B. (1955) Atmospheres Refraction Above the Inland Ice in North Greenland Bull Geod 38

20. Ramsay J. P. Wilson (1971) Land Surveying M. E. Hand Book.

21. Rogers H. H. (1977) On Vertical Refraction on Long and Short Lines In Survey Review Vol. 24 No. 183.

22. Sutton O. G. (1949) Atmosphere Turbulence Methuen Monograph London.

23. Walker J. J. (1879) On Operation of the Great Trigonometrical Survey (ii) Reduction of the Principal Triangulation in Survey of India.

APPENDIX I

DAMPING COEFFICIENT b

The diurnal range of temperature R(Z) at height Z is given by

$$R(Z) = R(Z_o)\, e^{-b(z-z_o)}$$

where b is the damping Goefficient from 1

$$b = {}^{1}\log_{10} \frac{(R_{(Z_o)}/R_{(Z)}}{0.343(Z-Z_o)}$$

using the Trappes data Anggus-Leppan (1961)=determined b for various years and for various layers. The results are independent of height but fall into five groups, one with values nearly 2.0 others of about 3.5. The difference was ascribed to the fact that the 2.0 values were in dry years, the others in humid years. Humidity thus affects b and Angot 1894 has shown that humidity is proportional to temperature. Anggus-Leppan also obtained a mean of six values of b for each height as which temperature observations were taken. The mean values was tested by substitution back. The average error in the amplitude calculated in this way was about $0.5^{o}C$.

Table I shows the computed values of b for various places and months.

The seasonal variation of b and the variation from place to place is unfortunately irregular, having no simple correlations with latitude temperature, nearness to the sea, attitude of the ground surface, humidity etc. The amount of variation from year to year had to be assessed from those stations where two figures were available for the same month. Only four cases are available.

	Santa Monica	Norfolk	Pretons	Cape Town
First Year	0.0048	0.0028	0.0032	0.0032
Second Year	0.0007	0.0028	0.0033	0.0037

In the last three cases there is no significant difference but in the first case there is a very large difference.

TABLE I

DAMPING COEFFICIENTS

For 2 in meters

A. Summer Months January or July

Station	Year	b
Norfolk va	1956	0.0088
Norfolk	1957	0.0028
Amarillo	1957	0.0022
Santa Monica	1956	0.0046
Santa Monica	1957	0.0007
Montgomery	1957	0.0039
Fort Worth	1957	0.0009
Pretoria	1957	0.0023

B Whole Year

Month	Cape Town	Pretoria
January	0.0035	0.0023
February	0.0025	0.0025
March	0.0023	0.0035
April	0.0025	0.0030
May	0.0039	0.0041
June	0.0053	0.0041
July	0.0037	0.0032
August	0.0041	0.0029
September	0.0025	0.0032
October	0.0021	0.0032
November	0.0032	0.0028

Appendix II

A typical Configuration for computation of θ.

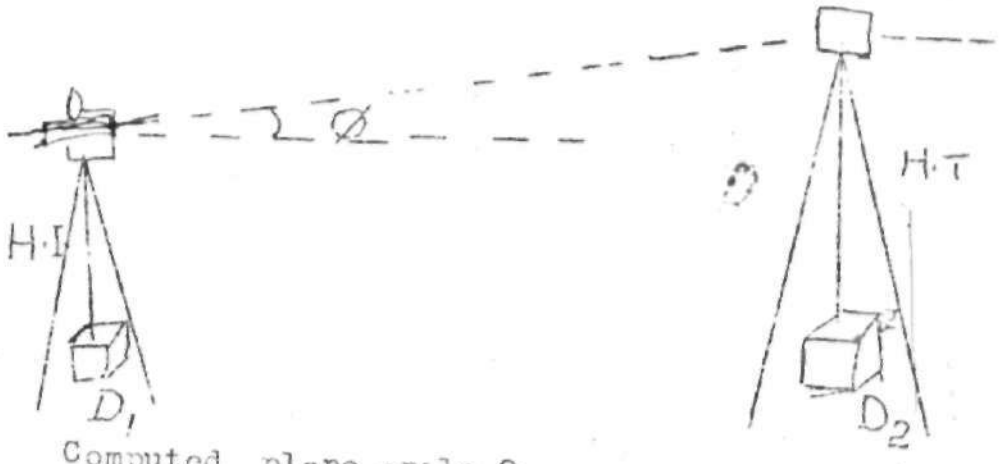


Fig. 4.10

Computed plane angle θ

ΔH [(elevation of D_2 + H.T)-(elevation of D_1 + H.I.)]

$\theta = \text{Sin}^{-1} \frac{\Delta H}{}$

APPENDIX III

```
]LIST

10 V$ = "KVALUE-ODETOLA"
20  HOME : HTAB 15: VTAB 15: PRINT
     V$: VTAB 17
30  DIM K(10)
40 D$ =  CHR$ (4)
50  INPUT "PAPER OUTPUT? ";O$
60  IF O$ = "Y" THEN 80
70  IF O$ <  > "N" THEN 50
80  DEF  FN A(X) =  INT (X * (10 ^
     F0) + 0.5) / (10 ^ F0)
90 F0 = 3
100 M0 = 60
110  FOR L = 1 TO 2
120 X$ = "LINE" +  STR$ (L) + "VE
    RTODE"
130  READ F0
140  FOR S = 1 TO 4
150  READ PH1
160 PH1 = M0 + PH1
170  IF O$ = "Y" THEN  PRINT D$;"
    PR#1"
180  PRINT : PRINT "LINE ";L;"  S
    ET ";S
190  FOR H = 1 TO 9
200  PRINT "HOUR "H + 7
210 REC = H + 9 * (S - 1)
220  PRINT D$;" OPEN";X$;",L150"
230  PRINT D$;"READ";X$;",R";REC
240 M = 0
250  FOR A = 1 TO 10
260 'INPUT W
270 W =  ABS (W) * 3600
280 W = (PH1 - W)
290 K(A) = (1 - W / F0) / 2
300 M = M + K(A)
310  NEXT A
320  PRINT D$;"CLOSE ";X$
330 M = M / 10
340  FOR A = 1 TO 10
350  PRINT  FN A(K(A));"  ";: NEXT
    A
360  PRINT : PRINT  FN A(M)
370  NEXT H
380  NEXT S
390 M0 = 900
400  NEXT L
410  DATA   17,75,29,83,20,54,34,
    6 ,          2,12.1,20.57,52
    ,6,40.1,33.65,          65.
    82
420  PRINT : PRINT D$;"PR#0"
430  END
```

LINE 1 SET 1
HOUR 8
.277 .22 .206 .235 .235 .206 .249 .206 .192 .22
Mean .225
HOUR 9
.22 .206 .192 .206 .22 .192 .192 .206 .22 .192
.205
HOUR 10
.206 .22 .192 .178 .164 .178 .206 810.229 .192 .22
81.199 0.195
HOUR 11
.15 .164 .136 .108 .122 .136 .122 .094 .094 .136
.126
HOUR 12
.122 .136 .094 .136 .136 .108 .08 .136 .108 .108
.116
HOUR 13
.108 .08 .136 .108 .136 .122 .136 .094 .108 .122
.115
HOUR 14
.08 .122 .108 .08 .08 .108 .136 .136 .108 .108
.106
HOUR 15
.108 .136 .108 .08 .066 .108 .108 .066 .108 .136
.102
HOUR 16
.136 .136 .108 .164 .136 .136 .108 .164 .136 .108
.133

LINE 1 SET 2
HOUR 8
.26 .231 .288 .288 .245 .231 .302 .203 .231 .26
.245
HOUR 9
.217 .203 .189 .217 .147 .175 .147 .203 .175 .203
.188
HOUR 10
.203 .119 .175 .231 .175 .203 .189 .147 .147 .175
.177
HOUR 11
.175 .175 .147 .147 .203 .147 .175 .147 .175 .147
.164
HOUR 12
.147 .147 .119 .119 .147 .119 .119 .147 .0909999999 .11
.127
HOUR 13
.077 .147 .119 .133 .147 .063 .119 .0909999999 .0909999
.108
HOUR 14
.119 .105 .105 .147 .133 .0909999999 .0909999999 .0909995
.108
HOUR 15

.0909999999 .077 .119 .133 .0909999999 .0909999999 .119 .11
.077
.101
HOUR 16
.147 .119 .175 .217 .189 .175 .147 .175 .203 .175
.172

LINE 1 SET 3
HOUR 8
.201 .145 .201 .201 .229 .173 .201 .201 .159 .201
.191
HOUR 9
.201 .201 .159 .245 .229 .187 .201 .173 .201 .201
.2
HOUR 10
.173 .145 .159 .173 .201 .159 .173 .145 .173 .145
.164
HOUR 11
.145 .117 .0879999999 .074 .102 .145 .102 .102 .102 .117
.11
HOUR 12
.102 -.024 .145 .102 .117 .0879999999 .117 .0879999999 .10

.0929999799
HOUR 13
.074 .06 .117- .0879999999 .0879999999 .117 .117 .145 .117
.101
HOUR 14
.0879999999 .0879999999 .117 .06 .0879999999 .06 .06 .08799
9999 .0879999999
.083
HOUR 15
.0879999999 .06 .0879999999 .102 .06 .0879999997 .0879999999
.074 .102
.084
HOUR 16
.117 .145 .0879999999 .117 .06 .0879999999 .117 .0879999999
.117
.102

LINE 1 SET 4
HOUR 8
.159 .145 .187 .159 .173 .131 .131 .216 .187 .145
.164
HOUR 9
.131 .145 .159 .173 .173 .145 .103 .159 .117 .145
.145
HOUR 10
.131 .145 .159 .0889999999 .117 .103 .103 .131 .145 .103
.123
HOUR 11
.159 .117 .145 .145 .202 .117 .704 .202 .117 .103
.8859999999
HOUR 12
.131 .103 .145 .103 .173 .103 .131 .103 .131 .103
.113
HOUR 13

.103 .131 .159 .103 .145 .103 .075 .131 .103 .0889999997
.114
HOUR 14
.075 .075 .103 .131 .103 .075 .075 .0889999999 .0889999997

.0899999999
HOUR 15
.075 .103 .0889999999 .145 .075 .145 .117 .0889999999 .1
.106
HOUR 16
.117 .103 .117 .075 .103 .117 .145 .0889999999 .131 .13
.113

LINE 2 SET 1
HOUR 8
.198 .222 .198 .174 .222 .21 .174 .21 .222 .246
.208
HOUR 9
.198 .186 .222 .186 .21 .15 .21 .198 .186 .186
.193
HOUR 10
.186 .174 .126 .138 .102 .126 .126 .15 .174 .15
.145
HOUR 11
.15 .138 .174 .102 .126 .162 .102 .126 .15 .102
.133
HOUR 12
.126 .102 .126 .15 .126 .114 .126 .102 .126 .138
.124
HOUR 13
.15 .126 .126 .102 .102 .0779999999 .102 .126 .126 .102
.114
HOUR 14
.102 .126 .102 .138 .15 .114 .0899999999 .102 .0779999997
.111
HOUR 15
.102 .138 .102 .0899999999 .102 .114 .126 .102 .126 .126
.113
HOUR 16
.126 .114 .15 .15 .126 .102 .114 .0899999999 .114 .126
.121

LINE 2 SET 2
HOUR 8
.234 .306 .258 .282 .282 .306 .282 .258 .282 .282
.277
HOUR 9
.282 .27 .234 .258 .27 .282 .282 .258 .27 .246
.265
HOUR 10
.222 .258 .234 .21 .222 .234 .198 .258 .234 .198
.227
HOUR 11
.198 .102 .21 .186 .162 .162 .138 .174 .198 .174
.177

HOUR 12
.174 .174 .162 .174 .186 .15 .186 .21 .162 .162
.174
HOUR 13
.162 .126 .162 .138 .114 .138 .138 38.243 .114 .15
38.947 0.138.
HOUR 14
.138 .126 .126 .138 .114 .126 .162 .138 .114 .114
.13
HOUR 15
.126 .138 .11 .138 .15 .138 .126 .114 .0899999999 .114
.125
HOUR 16
.186 .162 .15 .138 .162 .138 .162 .186 .162 .162
.161

LINE 2 SET 3
HOUR 8
.172 .148 .172 .172 .124 .148 .172 .172 .148 .196
.162
HOUR 9
.148 .124 .124 .148 .196 .124 .148 .124 .124 .148
.141
HOUR 10
.136 .0879999999 .136 .112 .112 .136 .136 .148 .148 .124
.128
HOUR 11
.1 .0879999999 .148 .112 .112 .136 .124 .112 .064 .1
.11
HOUR 12
.1 .1 .124 .1 .112 .124 .1 .118 .1 .1
.111
HOUR 13
.1 .076 .076 .0879999999 .076 .124 .076 .076 .1 .0879999999
.0879999999
HOUR 14
.076 .076 .1 .1 .076 .1 .076 .076 .1 .1
.0879999999
HOUR 15
.1 4E-03 .124 .1 .076 .076 .076 .1 .076 .1
.083
HOUR 16
.1 .124 .124 .1 .1 .076 .124 .1 .148 .1
.11

LINE 2 SET 4
HOUR 8
.169 .169 .148 .148 .133 .169 .145 .121 .145 .157
.15
HOUR 9
.181 .145 .133 .169 .157 .145 .133 .133 .169 .145
.152
HOUR 10
.121 .121 .1 .121 .133 .145 .121 .145 .121 .0899999999
.128

HOUR 11
.0969999999 .121 .121 .109 .0969999999 .145 .145 .121 .09

.117
HOUR 12
.121 .0969999999 .145 .145 .0969999999 .121 .0969999999 .1
..121
.115
HOUR 13
.121 .145 .121 .121 .109 .145 .145 .121 .121 .145
.129
HOUR 14
.0969999999 .0969999999 .121 .145 .121 .109 .121 .09699999
999999
.113
HOUR 15
.0969999999 .0969999999 .121 .0969999999 .121 .0969999999 .
1 .121 .121
.109
HOUR 16
.121 .0969999999 .0969999999 .121 .0969999999 .121 .096999
9 .121 .0969999999
.107