"STRESS ANALYSIS OF A REINFORCED CONCRETE ARCHED STRUCTURE FOR AIRCRAFT HANGARS".

A THESIS PRESENTED TO THE DEPARTMENT OF BUILDING, FACULTY OF ENVIRONMENTAL DESIGN, AHMADU BELLO UNIVERSITY, ZARIA IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE AWARD OF THE DEGREE OF MASTER OF SCIENCE CONSTRUCTION TECHNOLOGY.

BY

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SUPERVISORS DR. I. FERENS.

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ABOUT THE AUTHOR

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This thesis is dedicated to John Kamang Kawai and Nechen Kutubai Kamang, my father and mother respectively for their love and patience.

ABBREVIATIONS

de = thickness of arch at the Crown

ds = lenght of arch

dx = lenght of a straight element

E = Young's Modulus of elasticity

F = Cross-sectional area

J = Moment of inertia

Jc = moment of inertia at the Crown

Jc = the Section at the support

Jx = moment of inertia of a Section Situated at a distance x from the coordinate origin.

Li = one half of the arch span

$\bar{M}i$ = Bending stresses due to a load unity at Section i

$\bar{N}i$ = Normal stresses due to load unity at section i

$\bar{Q}i$ = Shear stresses due to load unity at section i

Mp = Bending stresses due to actual state

Np = Normal stresses due to actual state

Qp = Shear stresses due to actual state

Θi = angle of the tangent of an arbitrary section i

x = obscissa of the metral line referred to a Coordinate origin coinciding with the centroid of the crown section

ip = relative deflections and rolations of cut ends due to applied load.

ii = relative horizontal, vertical or rotational displacement due to unit loads on cut ends.

GLOSSARY

Actual state = referred to the case when a structure is acted by the combination of actions effectively applied.

Imaginary state = referred to the case when a load unity is applied to a structure.

Redundant structure = Redundant here does not mean Superfluous or useless but Simply refers to a structure requiring additional equations namely deformation equations for the computation of the stresses set up.

Influence lines = A line showing the variation of the parameters of reactions and deflections due to moving load or loads on a Structure.

ABSTRACT

This study has briefly outlined the development of theories for the stress analysis of arches from the eighteenth century to the present day. The method of forces has been presented as an exact means of analyzing the stresses set up in redundant structures (eg. fixed end arches). The elastic centre, use of symmetry, symmetrical and unsymmetrical loading and load transformation are explained with examples to illustrate how their use simplifies considerably the calculations based on the method of forces.

The effects of temperature changes, shrinkage and settlement of supports which can be disastrous to redundant structures when not properly taken care of, have been explained.

Sometimes, arches neutral line and law of cross-sectional variation cannot be expressed by analytical equations fit for practical purposes, it thus become necessary to resoft to approximate methods, two of which ~~have been~~ described and examplified ~~in Chapter four.~~

The use of influence lines as explained in ~~Chapter three~~, and one of the approximate methods described in ~~Chapter four~~, have been utilized for the full stress analysis of the arched beam of the City aircraft hangar of San Diego having a clear span of 102m and a clear height of 27m at mid span.

Finally some constructional aspects of arches such as centering, formwork, reinforcement, foundation selection, thermal insulation and waterproofing are discussed.

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CHAPTER ONE

INTRODUCTION

1.1 PURPOSE OF STUDY

The existence of arch structures dates back centuries ago because of the advantages they have among which is the possibility of using them to cover very large spaces. Ever since their inception numerous changes have taken place both in terms of the materials of construction and their application. Since arches are often used in present day structures, the necessity of structural engineers getting themselves equipped fully with the stress analysis, design and methods of erection of these type of structures requires no overemphasis and argument.

Thus this study is aimed at learning mainly from literature, the stress analysis of fixed-end arches under simple and complicated loading and effects of some physical phenomena like temperature changes, shrinkage and settlement of supports. Also the development of the theories for the stress analysis and design of arches and some practical problems associated with the design and erection of arches will be learned. A full stress analysis for an existing arched structure for an aircraft hanger of 102 metres clear span and 27 metres clear he right at mid-span will be presented.

1.2 SCOPE AND LIMITATION OF STUDY

The presentation of the development of theories for the stress analysis and design of arches from before the eighteenth century to the present day has been based mainly on literature.

The method of forces as an exact method in the stress analysis of redundant structures, and most suitable for arches is the main theme of chapter three. The use of the elastic centre method, influence lines, use of symmetry, grouping of unknown symmetrical and antisymmetrical loading, and load transformation method which allows to simplify very much the calculations based on the method of forces is presented in this chapter. Each of the methods have been illustrated with working example. For the simplicity of these calculations the advantage of using the elastic centre method, influence lines, load transformation and other possibilities of simplification are illustrated with working examples on mostly frames, though some examples are for arches. Effects like temperature changes, shrinkage and support settlement in strees analysis are limited to reinforced concrete structures only.

Chapter three deals with two approximate methods of stress analysis of fixed-end redundant arches. Both methods are presented with working examples. These two approximate methods can only be applied for reinforced concrete arches.

A complete stress analysis of a beam from a practical existing bigger arched structure, for an aircraft hanger, with a clear span of 102 metres and a clear height of 27 metress at

mid span is presented in chapter five to demonstrate the practical application of the methods presented in chapter three and four.

Lastly, the presentation of some practical considerations is the design and erection of reinforced concrete arches like formwork concreting and thermal insulation.

1.3 DEFINITION OF ARCHES AND TYPES

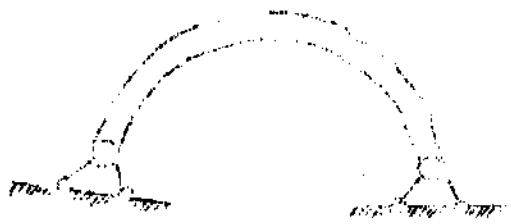
Arches are thrust developing structures whose general form is that of a curve. Depending on the number of hinges an arch may be either a three-hinged arch, a two-hinged arch a one-hinged arch or a fixed end arch i.e. a hingeless arch. Also an arch may be symmetrical or unsymmetrical, right or skew, or a single arch or one of a series of arches mutually dependent upon each other. Arch construction may comprise an arch slab (or vault) or a series of parallel arch ribs.

Three-hinged arch. Is an arch with a hinge at each springing and with a hinge at the crown, and statically determinate. The thrusts on the abutments and therefore the bending moments and shearing forces on the arch itself, are not affected by a small movement of one abutment relative to the other. For any load in any position the thrust on the abutments can be determined from the statical equations of equilibrium.



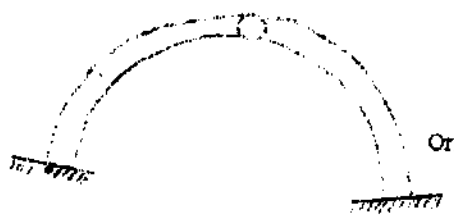
Three-hinged arch.

Two-hinged Arch. The hinges are placed at the abutments and thus only thrusts are transmitted to the abutments, with no bending moments at the springings.



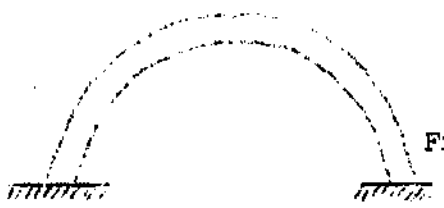
Two-hinged arch.

One-hinged arch. Has only one hinge placed at the crown.



One-hinged arch.

Hingless fixed-end arch. They have no hinges and are fixed at their two ends. The fixed ends exert bending moments on the abutment in addition to the vertical and horizontal thrust.



Fixed-end arch.

All arches except the three-hinged ones are statically indeterminate.

In all calculations in solid masonry or reinforced concrete arches it is customary to consider strips of unit separated in imagination from the rest by two parallel planes. The whole structure becomes thus replaced by a series of parallel arches, the defections of which remain strickly identical as long as each one of them carries the same load. In this way the stress analysis

remain exactly the same for all the arched structures of the same type irrespective of their depths.

1.4 THE CHOICE OF ARCH NUETRAL LINE

Great care should be taken when designing arches, so as to reach as close coincidence as possible between the outline of the axis (nuetral line or centre line) of the arch and the pressure line (or equilibrium polygon). Such coincidence provides an arch with maximum economy. Complete coincidence can only be achieved in the case of three-hinged arches. It is impossible to obtain full coincidence of the arch axis with the pressure line in the case of statically indeterminate arches, for bending moments in such arches are absolutely unavoidable. Therefore the most economical design of an arch will be the one providing minimum fibre stresses in the arch.

The preassure line for statically indeterminate arches can be obtained only if the redundant reactions are already determinated. However, these reactions depend on the deformations of the arch and for this reason it becomes extremely difficult to find the most economical configuration of an arch of this type even when dead loads are involved. The problem can be solved by a series of approximations. This may be done by selecting first some arbitrary curve (usually a parabola) for the arch nuetral line which is then corrected on the grounds of comparison with the pressure line obtained for that particular arch. Alternatively, the centre line of the arch may, in the first approximation, follow the pressure line of a three hinged arch of the same span

and rise. Upon correction of the arch nuetral line a new pressure polygon is constructed for the corrected arch. The operation is repeated as many times as is necessary to obtain a satisfactory coincidence of the two lines.

For arches carrying moving loads the choice of the nuetral line becomes even more complicated. In actual practice this choice is most frequently based on the simple comparison of several arches differing both in outline and cross sectional dimensions.

The configuration of masonry arches must be selected with the view of maintaining the pressure line for all possible load combinations as close as possible to the nuetral core boundaries, masonry being incapable of resisting tensile stresses of any appreciable magnitude.

1.5 ARCHES WITH VARIABLE CROSS-SECTIONAL DIMENSIONS

The coefficients to the unknowns and free terms of the simultaneous equations used for purposes of stress analysis of redundant arches depend on their cross-sectional dimensions and the moments of inertia of the structure. Neither of these two remain constant throughout the whole lenght of the arch. Thus, for instance, in fixed-end arches the height of the section and consequently its moment of inertia increase frequently from the crown to the abutments because the bending moments are as a rule much smaller at mid span than the immediate neighboorhood of the supports. On the other hand, the thickness of two-hinged arches

decreases usually from the crown to the abutments following the bending moment diagram.

Direct computation of the coefficients to the unknowns and of the free terms of the simultaneous equations requires the integration of expressions containing the values of F and J and therefore it becomes necessary to express mathematically the variation of these quantities along the arch. The following equation has been found very useful in practical design.

$$Jx = \frac{Jc}{1-(1-n)\frac{x}{4}} \quad Cos\theta$$

$$n = \frac{Jc}{Cos\theta}$$

Modifying the value "n" we modify at the same time the law governing the variation of cross-sectional dimension along the arch. Frequently, "n" is taken equal to unity in which case the expression for Jx becomes

$$Jx = \frac{Jc}{Cos\theta x}$$

For a rectangular arch of constant width b, Jx may be replaced by $\frac{dx^3 b}{12}$ and Jc by $\frac{dc^3 b}{12}$

When n = 1

$$\frac{dx^3 b}{12} = \frac{dc^3 b}{12 Cos\theta x}$$

where from $d^3 = \frac{d_c^3}{Cos\theta x}$

Fc and Fx become in this case Fx = bdx and Fc = bdc

It follows that $F^3 = \frac{F_c^3}{Cos\Theta x}$

where from $Fx = \frac{Fc}{\sqrt[3]{Cos\Theta x}}$

For simplicity this expression is very frequently replaced by

$$Fx = \frac{Fc}{Cos\Theta x}$$

It has been proved that this simplification entails an error in the bending moment and thrust values which does not exceed 1 persent. When the rise of an arch is less than $\frac{1}{3}$ of its span (flat arches) the value of $Cos\Theta x$ for all the cross-sections will remain very close to unity, thus permitting the adoptation of a constant thickness of the arch throughout and therefore $Fx = Fc = constant$.

In the design of flat arches the lenght of the elementary segment ds is also usually replaced by the lenght of its horizontal projection dx.

CHAPTER TWO

DEVELOPMENT OF THEORIES FOR THE STRESS ANALYSIS AND DESIGN OF ARCHES

From the earliest times when people started to build, it was found necessary to have information regarding the stronght of structural materials so that rules for determining safe dimensions of members could be drawn up though this information was lacking, some of the structures erected many centuries back are still in use to the present day. This chapter will present briefly the developments that have taken place in the theories used for the stress analysis and design of arches from before the eighteenth century to the present day.

2.1 ARCHES BEFORE THE EIGHTEENTH CENTURY

There is no known theory that was used before the eighteenth century for the stress analysis of arches and consequently the design. And of all the arch structures erected within this period, the design was based mainly on empirical rules for determining the safe dimensions. Though, the stress analysis that enable accurate design of arches was lacking, it is a known fact that some of the structures still exist to the present day.

The Romans who were reckoned great builders during this period often used arches in their buildings. The arches in the famous Pont du Gard, a bridge in Southern France is still in service up to the present day.

A comparison of the proportions of the Roman arches with those of the present times indicates that nowadays much lighter structures are built, this is because the Romans had not the advantages provided by stress analysis. They did not know how to select the proper shape and usually took semi circular arches of comparatively small spans.

The materials of construction used were bricks made up of clay and stone which was extensively used by the Roman. The clay bricks were prepared by adding water and treading-in chopped straw or dung to prevent warping or cracking. The bricks were then shaped usually two at a time, in a rectangular wooden mould with no top or bottom, and were afterwards left to dry, being turned over from time to time. With a large addition of water the same clay mixture was used both for mortar and plastering. These bricks were then used in erecting the corbelled false-arch, with which small doomed burial chambers were built up of limestone rubble and the true-arch - surmounting gateways through walls. Brick vaults were made out of arches, in which each successive semi-circle of bricks was tilted slightly backwards so that no temporary wooden framework was needed for centering.

The Romans were fortunate in having available a volcanic earth called bozzolana, which when mixed with lime formed a cement that resisted both water and fire. Blended with brick or stone, it made concrete as hard as stone or stone itself. Such concrete was used between brick arches or ribs served in

particular to reduce the weight of vaulted structures.

Arches of wedged-shaped stones, requiring little mortaring were extended to form a vault merely by the provision of additional material of timber framework during construction. The result was not only dark but dangerous because of the outward thrust upon the walls on which the vault rested. The Romans therefore constructed cross-vaults to buttress the walls.

Most of the knowledge that the Greeks and Romans accumulated in the art of building in general was lost during the middle ages. The advent of the Renaissance revived the economic life of the Europeans and it became necessary for the builders to relearn a new the art of building.

2.2 ARCHES IN THE EIGHTEEN CENTURY

Within this period the materials of erection were the same with those of before the eighteenth century. It was in this period that some French engineers made attempts to evolve a theory for the stress analysis of arches.

A member of the french academy named Lahire (1640-1718) was the first to apply statics to the solution of arch problems. Lahire employed the use of funicular polygon for the first time in the stress analysis of arches.

He considered a semi-circular arch and assumed that the surfaces of contact of the wedges forming it, are perfectly smooth, so that only normal pressures can be transmitted through them.

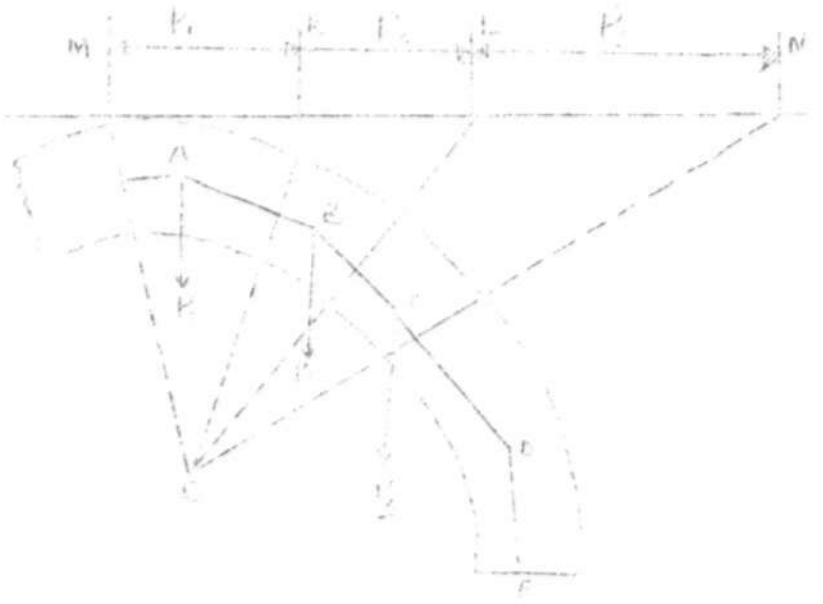


Figure 2.1

With that assumption Lahire investigated the following problem: what weights P_1, P_2, P_3... must the wedges forming the arch have inorder to ensure stability of the structure. The problem was solved geometrically as shown in figure 2.1.

Drawing a horizontal tangentMN to the outer surface and extending the radii which subdivide the arch into wedges to the intersection points M,K,L,N. Lahire found that the requirement of stability is met if the weights P_1, P_2... are in the same ratio as the lenghts Mk, Kl, LN. He also found out that the pressures acting between the wedges of the arch are represented in such a case by the lenghts CK, CL and CN.

Using todays interpretation it will be stated that figure ABCDE is the funicuar polygon constructed for the system of vertical forces P_1, P_2, P_3 and that the figure MKLNC represents the polygon of forces rotated by 90 degrees about the pole C.

This method of stress analysis and design by Lahire was first put to practical use and suggested that the angle in figure 2.2 should be taken equal 45^{o}. Lahire's method was also, later used by Perronet and Chezy in a preparation of a table for use in calculations of the thickness of the arch.

Further progress was done by Coulomb in the theory of arches. It was known that the typical failures of arches are shown in figure 2.3.

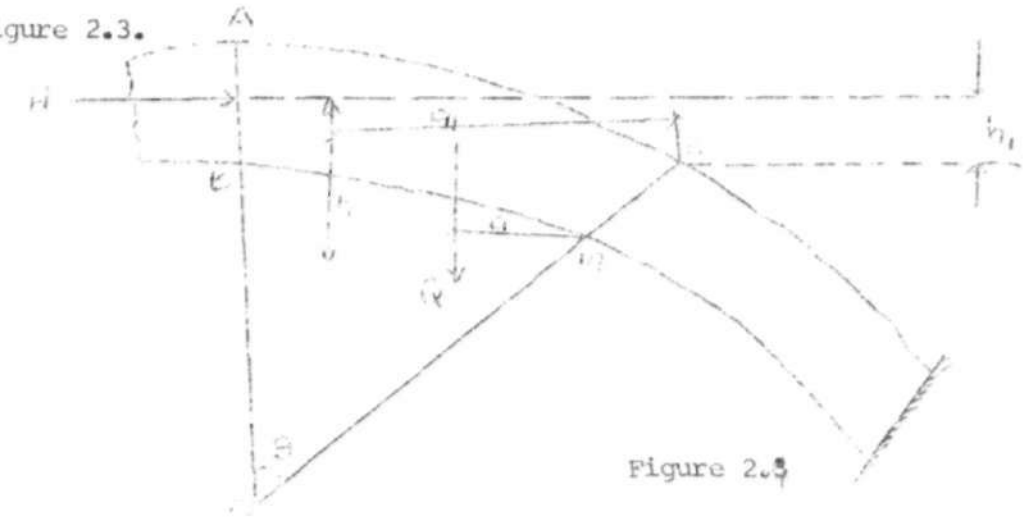


Figure 2.3

From figure 2.3 in investigating arch stability it is not sufficient only to consider relative sliding of the wedges, but also the possibility of relative rotation must be checked.

Coulomb, in 1773 took all this into consideration.



Figure 2.4

In figure 2.4 ABDE represents one-half of a symmetrical arch which is symmetrically loaded. Denoting the horizontal thrust at the cross-section AB by H and the weight of the portion ABmn of the arch by Q, Coulomb found out that the normal and tangential components of the resultant force acting on the plane mn are HCosΘ + QSin and QCosΘ - HSin respectively, Θ is the angle between this plane and the vertical AC. The smallest value of H required to prevent the portion ABmn of the arch from sliding down along the plane mn is given by the equation, QCosΘ - HSinΘ = U (HCosΘ + QSinΘ) + rA. V = coefficient of friction and rA the total resistence of the arch to shearing along the plane mn. From this equation H was found to be $H = \frac{QCos\Theta - VQSin\Theta + rA}{Sin\Theta + VCos\Theta}$

2.3 ARCHES IN THE PERIOD 1800-1833

During the period 1800-1833, designers of arches followed Coulomb's theory and assumed that if an arch fails it does so as figure 2.5 and breaks into four pieces. The main difficulty in the stress analysis laid in finding the location of the cross-section of fracture Bc of figure 2.5.

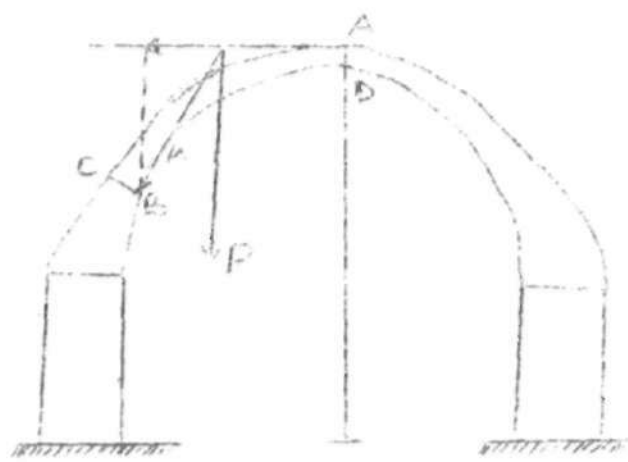


Figure 2.5

The theory supposes that the horizontal thrust H, applied at the top most point A of the cross-section AD, together with the weight P of the portion ABCD of the arch and the load on that portion gives a resultant R passing through the point D. The cross-section BC is located such that H has its maximum value. The problem of finding the location of this location of the cross-section of fracture was usually solved by a trial and error method.

For some assumed position of the section, the weight P and the point of its application can be found and the correspinding value of the thrust H can be calculated from the equation of statics. To find the maximum value of H with sufficient accuracy, this calculation must be repeated several times, and since this work demanded the expenditures of much times. To simplify the work, tables were prepared giving the weight and the position of the centre of gravity of the portion BCAD of the arch for any location BC in certain types of arches. When the cross-section of fracture and the corresponding value H max have been found, the dimension EF of the lower portion EFCB of the structure must be fixed such that the moment of the weight of the half arch ADEF with respect to the axis F will withstand the action of H max applied at A with a factor of safety.

Some interesting extensions of this theory were made by M.G. Lame and E. Clapeyron who at that time were in the service of the Russian Government. Both of them were Professors at the Institute of Engineers of Ways of Communication in St. Petersburg.

They were asked to investigate the stability of cylindrical arches of the dome of the St. Isaac Cathedral which at that time was being constructed. For a circular arch of constant cross-section, they found the position of the cross-section of fracture analytically and gave a formula for Hmax. They also showed that for symmetrical arches of any shape, the calculation of the position of the cross-section of fracture can be greatly simplified, if instead of radial cross-section vertical ones are contemplated. They proved that if this were done the required cross-section would be found from the condition that if the tangent, to the intrados at B passes through the intersection point of the forces P and H. A very simple graphic method of locating the cross-section BC follows from this face.

In 1826 Navier added important discussions on stresses in arches. Coulomb had already mentioned that the forces H and P must at some distance from points A and B so that there will be sufficient areas for the distribution of stresses. Navier assumed that normal stresses along the cross-sections AD and DC are distributed according to a linear law and that they vanish at points D and C where according to Coulomb's theory. Splitting commences when failure occurs. From this it follows that at point A and B, the stresses are twice as great as those which would be obtained on the assumption of a uniform distribution of stresses (over the cross-section AD and CB). It also follows that the resultant forces H and R must be applied at distances of one-third of the depth of the cross-sections from points A and B.

Taking these new lines of action of the forces H and R. Navier finds somewhat larger values for Hmax than those arrived at hitherto. He applies those new values in computing the dimension EF of the structure. The new positions of the forces H and R won general acceptance and were adopted in future work on the stress analysis of arches. It is worth mentioning that the solution of the problem of eccentric compression of bars obtained by Thomas Young (1607) was not noticed by Navier who arrived at his conclusion regarding the significance of the third points quite independently.

Navier was the first to solve several problems of the deformation of curved bars, it did not occur to him that these solutions could be applied in calculating the magnitude of the thrust H in stone arches.

The view that arches should be treated as curved elastic bar was expressed, perhaps for the first time by Poncelet. Though it took a considerable lenght of time to effect the practical design of arches.

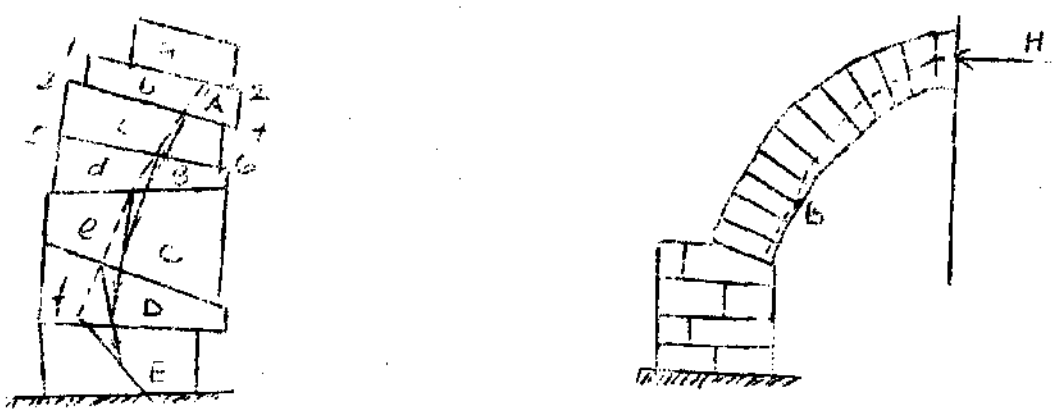
2.4 ARCHES IN THE PERIOD 1833-1867

Within this period though Bresse had given a complete solution for an elastic arch with built-in ends, engineers continued to consider stone arches as a systems of absolutely rigid stone blocks in the design of arches. In the year 1830 F.J. Gestner introduced the notion of line of pressure and a line of resistance into the analysis of arch design.

He started his analysis on suspension bridges, treated the catenary and gave tables for the curve construction. He then indicated that the inverted curve was the proper one for an arch of constant cross-section. And under the action of its own weight such an arch will work under compression only. He investigated the question of how the intensity of load should be distributed on circular and elliptic arches, in such a way that circles or ellipses are pressure lines. In practice he showed what the load distribution differs from that obtained theoritically for the ideal case. This meant that in conjunction with compression there was bending. He showed that the problem is statically indeterminate and that it was possible to construct an infinite number of pressures lines which satisfy the conditions of equilibrium and pass through various points of the keystone and of the abutments. To each of these curves, there corresponds a value of the horizontal thrust H. To make the problem definite, Gerstner made some arbitrary assumption regarding the position of the true pressure curve.

An English Scientist Henry Moseley brought some general idea regarding the position of the true pressure line.

In his paper titled "On the theory of the Arch" written in 1839 and published in the first volume of "The theory Practice and Architecture of Bridges" edited by John Wale in 1843, it was shown for the first time that the pressure line and the resistance line are two different curves. The sketch in the next page illustrates this.



The Arrow a represents the force acting on the cross-section 1-2 at point a. Adding the weight of the stone 1234 to this force, the force B acting on the cross-section 3-4 at point b is obtained. Continuing in this way the forces C, D, E... applied at points c, d, c... are obtained.

The polygon formed by the intersections of forces A, B, C, D, E... gives us in the limit, the line of pressure and the polygon a, b, c, d, e... similarly gives the line of resistance.

For the construction of the pressure line for a symmetrical arch Moseley began with an arbitrary chosen point C and applies a thrust H of such magnitude that the pressure line becomes tangent to the intrados of the arch at a certain point B. By changing the position of C an infinite number of pressure lines were obtained. To select the true position curve from all those possible Moseley used his principle of least resistance and states that the required one is that curve corresponding to the minimum value of H. By moving the point C upward we shall dimmish H, and so Moseley finally found that the true pressure lines passes through the points A and B. This conclusion is in agreement with the Basic assumption of Coulomb's theory which he said was unknown to him at the time he carried his researches.

German engineers were interested in Moseley's work and his book "The Mechanical Principles of Engineering and Architectural" was translated into German by Scheffler (1844) who later improved Morseley's theory by introducing a consideration of the strenght of the material of the arch. Scheffler noted that if the pressure line passes through points A and B the stresses most be infinitely large at those points. In a bid to eliminate difficulties, Scheffler proposed that the pressure line be moved into the arch and considers that line for which the maximum stress at A and B reaches the value of the ultimate strenght of the material as the true pressure line.

Further progress in arch analysis was made by Poncelet who introduced graphic methods in his investigation. He used Coulomb's theory as the basis for his analysis and showed that the position of the cross-section of rupture could be determined graphically.

It was proved in his investigation, the theories that the point on the intrados of an arch, corresponding to the cross-section of rupture is that at which the tangent to the intrados passes through the point of intersection of the horizontal thrust, acting at the keystone, and the gravity force of the portion of the arch between the arch stone and the cross-section of rupture.

K. Culmann made an improvement in the graphical analysis of arches. Assuming that the material of an arch cannot withstand tensile stresses, he deduced that the extremes position of

the pressure line is that for which it passes through the upper third point at the keystone and the lower third point at the cross-section of rupture. Using those two points he constructed the funicular polygon for the gravity forces of the stone and of the external load and determines the forces acting on (and the corresponding stresses) each cross-section of the arch in this way.

With the work of K. Culmann, that period in the development of the theory of arches during which no regard was given to elastic deformation was brought to a conclusion.

In 1840 Yvon Villarcean made important contributions to the analysis of arches. In his work on arches he was mainly interested in the selection of the most favourable shape of the center line of an arch. He saw clearly that the problem is statically indeterminate and that its complete solution requires consideration of the elastic deformations. But concluded that there was insufficient knowledge of the elastic properties of arch materials and therefore decides to treat the problem by considering the arch stones as being absolutely rigid. He believed that the forms found in this way would be sufficiently good when the elastic deformation were taken into consideration. This lead Villarceau into considering cases in which the pressure curve coincides with the center line of the arch. Villarceau went further and considers not only the external pressure, which he assumes acts normally to the extrados

(or external surface), but also the weight of the arch. Under this assumptions, the differential equation of the pressure line becomes involved and its integration requires the use of elliptic functions. To render the solution useful in practice, he calculated numberous tables which enabled practical engineers to obtain the required center line and the necessary thickness of the arch in each particular case. With these tables, Villarceau applied his theory to many existing bridges and showed that often the thickness of the arch could be reduced without increasing stresses by using his shape of the center line. He discussed the effect of the elasticity of the materials which was neglected in his theory and considers the increase which must be given to the height of the arch in order to get the correct shape of the center line after defection. Villarceau's idea of taking the funicular curve constructed for the assumed external loads for the center line is widely used. And by this means the preliminary shape of an arch and its thickness are usually established and the selected dimensions are later checked by means of the elastic theory of arches.

Although Bresse drew up the theory of curved beam and discussed as examples a two-hunged arch and an arch with built in ends. But in his time engineers did not consider that an elastic theory could be applied to the design of stone arches and continued to threat them as if composed of absolutely rigid bodies. Very gradually however after considerable experimental work by Winkler and de PeroOil and especially after extensive tests

made by a special committee of the society of Austrian Engineers and Architects, engineers came to accept the theory of elastic curved bars as giving the proper dimensional of stone arches with accuracy. Winkler and Mohr were chiefly responsible for the introduction of this theory into practice.

Winkler treated two-hinged arches and hingless arches and applied the notion of influence lines for the stress analysis of arches. Winkler also used the principle of least energy to investigate the positions of pressure lines of arches. Mohr also contributed to the stress analysis of arches using graphical methods.

2.5 ARCHES FROM 1867 TO THE PRESENT DAY

The introduction of Reinforced concrete into structural engineering practice has made arches have a wide application especially in bridge construction. The methods of stress analysis are still an object of research. In hingless arches, there is three statically indeterminate quantities and analysis.

He showed that if the reactive forces acting at an arch abutment are represented by a force applied at its elastic center and by a couple, the three unknowns (two components of the force and the couple) may be determined, each from one equation with one unknown. For determining the position of the elastic center and for calculating the reactions, he used a graphic method in which fictitions forces, similar to those used in calculating the deflection of beams, are applied. Various methods of arch analysis have been presented by E. Morsch, J. Melan and by A. Strassner. Numerous tables for thrust and bending

moments in arches of various proportions are given in their books.

It is advantageous to make the centre line of an arch coincide with the funicular of permanent loads. If this condition is fulfilled in a three-hinged arch, the resultant force on each cross section is tangential to the centre line, producing only uniformly distributed compressive stresses. In a hingless arch, internal forces depend on the deformation of the structure and bending stresses cannot entirely be eliminated. If the centre line of such an arch coincides with the funicular curve for dead load initially, there will be a certain contraction of the axis of the arch due to compression of the material, shrinkage of the concrete, and lowering of temperature. The thrust of the arch calculated as for a three-hinged structure, will be somewhat diminished. As a result of this the thrust line of a hingless arch is displaced upward at the crown and downwards at the abutment from the assumed centre line (coinciding with the funicular curve for the dead load). This displacement becomes particularly great in thick and flat arches where we may expect the appearance of undesirable tensile stresses at the abutments. Various methods have been developed to reduce these displacements in large arch structures. Sometimes temporary hinges are put at the crown and abutments, these permit free rotation during decentering. In this way displacement of the thrust line, which occur as a result of the compression of the centre line by the dead load, are eliminated. By filling up the gabs at the temporary hinges with concrete after decentering

we obtain a hingeless arch, the center line of which coincides with the funicular curve for dead load. To allow for the thrust line displacements due to shrinkage and lowering of temperature, the temporary hinges must be placed with proper eccentricities such that when the gabs have been filled after decentering, bending moments are produced at the crown and the abutments, the signs of which are opposite to those expected later from shrinkage and temperature drop. E. Freyssinet suggest putting jacks in the temporary gab at the crown between the two halub of the arch. By using them, the most favorable position of the thrust line at the crown can be ascertained before filling up the gab with concretes. In the case of an arch with a tie beam, F. Dischinger proposed the use of tie cables, by stretching these, the bending moments resulting from dead load action and shrinkage of concrete can be greatly reduced.

Designers of arches usually assumed that the displacements which result from elastic deformations are small and consider the initial underdeformed center line of the arch in their analysis. In large-span arches, the effects of deflections on the magnitude of the redundant forces may become considerable. If this is so, calculations made on the basis of the initial undeformed centre line can be taken only as a first approximation using them, better accuracy can be obtained by taking into account the deflections indicated by the first approximation.

CHAPTER THREE

STRESS ANALYSIS OF FIXED END ARCHES BY THE METHOD OF FORCES

Fixed end arches are statically indeterminate or redundant structures, hence the computation of the stresses set up in them requires the use of additional equations namely deformation equations. In the method of forces these equations are obtained through the transformation of the given structure to the n^{th} degree into a statically determinate one.

The method of forces is one of the exact methods that can be used in the analysis of the stresses induced in these redundant structures. The elastic centre method based on the method of forces simplifies considerably the computation of these stresses set up. Influence lines quickens also the pace of work involved in the stress computation of redundant structures, hence will be presented in this chapter and examplified.

The use of symmetry, symmetrical, anti-symmetrical loading, grouping of unknowns and load transformation are also other methods that simplify the calculations of the method of forces and consequently they are explained in this chapter with some examples. The effect of physical phenomena like temperature changes, shrinkage and settlement of supports will be presented.

3.1 CANONICAL EQUATIONS

Canonical equations are a standard form of a system of simultaneous linear expressions for unknowns which are the reactive

forces developed by the eliminated constraints. The number of these equations normally equals the degree of redundancy of any given structure. It is important to note that both the number of terms in each of the separate equations and the total number of these equations depend solely on the degree of redundancy of the structure and are in no way influenced by any other of its peculiarities.

To obtain these canonical equations the redundant structure has to be transformed into a simple statically determinate one. The elimination of constraints introduces no alterations neither in the distribution of stresses nor in the strains and deflections if in place of the removed constraints, forces equivalent to the reactions developed by these constraints are introduced.

The displacements of the conjugate simple statically determinate structure is zero because no displacement of the given redundant structure is possible. This indicates that the reactions developed by the redundant constraints will be such as necessary to render nil the deformations of the statically determinate structure along the direction of these reactions.

The equations translating the above statement into a mathematical symbol is as follows: $\Delta i = \Delta i1 + \Delta i2 + \ldots + \Delta 1,n-1 + \Delta in + \Delta ip = 0$ .... (3.1).

In the expression the letter i following Δ shows the direction of the displacement (the latter coinciding with the eliminated constraints) and the second one the action causing this displacement. Thus Δik indicates a displacement along the

direction i caused by the reaction of the constraint k and Δip will indicate the displacement along the direction of constraint i caused by applied loads.

Let the magnitude of the reaction developed by the constraint k be indicated by Xk. Let also the displacement caused by unit action be designated by δik. Therefore Δik can be replaced by $X_k \delta ik$ and consequently expression (3.1) becomes $\Delta i =$ $X1\delta ik + X2\delta i2 + ... + X_{n-1}\delta i,n-1 + X_n\delta in + \Delta ip = 0$-(3.2).

By this way the equivalence of the original structure and of the simple statically determinate one will be mathematically interpreted by a system of n linear equations

$$X1\delta 11 + X2\delta 12 + ... + X_n\delta 1_n + \Delta 1p = 0$$
$$X1\delta 21 + X2\delta 22 + ... + X_n\delta 2_n + \Delta 2p = 0$$
$$X1\delta 31 + X2\delta 32 + X3\delta 33 + ... + X_n\delta 3_n + \Delta 3p = 0$$

main diagonal

secondary diagonals

Equations (3.3) constitute the additional expressions based on the deformations of the system which permits complete determination of all support reactions and all stresses induced by the given system of loads in the original redundant structure. All the above equations expresses the idea that the displacement of the simple structure along the direction of the eliminated

constraints are all equal to zero. These are the equations called Canonical equations. The coefficients to the unknowns of equations (3.3) represent the deflection of the simple structure obtained by the elimination of the redundant members. These deflections being due to unit loads and moments acting in the direction of the eliminated constraints.

The coefficients δik entering the above equations will represent the deflection along the direction k. The unit displacement δii situated in the main diagonal of the canonical equations are characterised by two identical indices will be called principal deflections whereas δik standing in the secondary diagonals of the aforesaid equations will be termed secondary deflections. Due to Maxwells theories of reciprocal works, secondary deflections situated symmetrically about the main diagonal will always equal themselves $\delta ik = \delta ki$. By this the volume of work necessary to determine the coefficients to the unknowns is considerably reduced.

The diagrams of bending moments induced in the conjugate simple structure by each of the unit actions ($X_1 = 1$) will be traced separately each of those graphs bearing the number of the eliminated constraint, the same applying to the actual loading (the M_p graph). The unit deflections δik will be obtained through the multiplication of the unit graph Mi by the graph $\bar{M}k$ whereas the deflection due to the applied loads Δip by the graph of the actual bending moment Mp.

The main or principal deflections will be always positive whilst the secondary ones as well as those due applied loads might be positive or negatives.

When all the coefficients to the unknowns entering the simultaneous equations (unit displacements) as well as the deflections due to applied loads are known, one may proceed with the solution of said equations. The roots of these equations will furnish the values of the unknown stresses acting in the redundant members. This will permit the construction of the bending moments diagrams induced by X_1, X_2.... Xi etc. in all the necessary members of the structure.

Several simple structures may be used for the computation of the same redundant structure, these different structures being obtained by the elimination of different members regarded as redundant. Its important to choose the one leading to the greatest possible simplification of the computation through the reduction of a maximum number of secondary deflections. Also an endeavour to obtain bending moment graphs of the simplest possible configuration for the members of the simple structure.

The portal frame appearing below will be used to clear the above statement. The portal frame is similar to fixed end archesi.e the sense that both are redundant to the third degree and only different in terms of its members is the members in the portal frame are straight while those of a fixed end arch are curved.

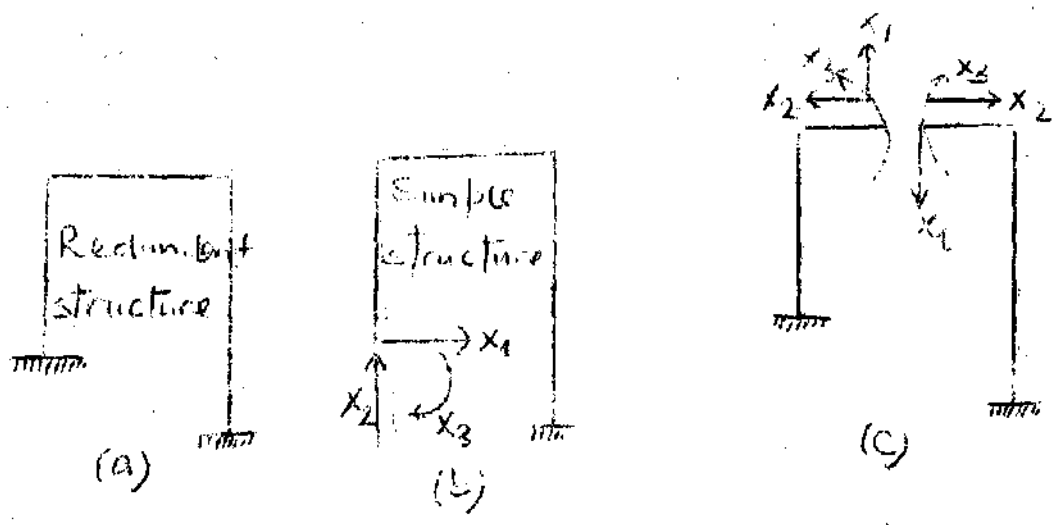


Figure 3.1

To begin with, eliminate the three constraints which prevent both the horizontal and the vertical movements and the angular rotation of the extremity of the left hand column. The simple structure obtained in that case appears in figure 3.1b. The unknowns X_1, X_2 and X_3 will represent the reactions developed by the eliminated constraints and the simultaneous equations will express the idea that the deflections and rotation along the directions of the eliminated constraints remain nil.

Another way still could be used to render the redundant structure statically determinate by cutting in two the top bar as indicated in figure 3.1c. This is equivalent to the elimination of three constraints preventing mutual displacement of the two faces of the cross-bar situated to the right and to the left of the cut. Hence each of the unknowns X_1, X_2 and X_3 will represent in this case a group of two opposite forces or couples acting over the two cross-sections just mentioned. The canonical equations will remain in the same form regardless of the way in which the simple statically determinate structure

has been obtained. In the case of the simple structure of figure 3.1b the δ_{12} represents the horizontal motion of the lower end of the left column caused by the vertical unit load $X_2 = 1$.

3.2 METHODS OF DISPLACEMENT COMPUTATION

Two different states of one and the same system are considered. The first is acted upon by any number of loads and moments whose values are chosen at will (figure 3.2a) and in the second by one single load unity p.2 (figure 3.2b) compute the work A_{21} produced by the load unity P2 along displacement A_{21} due to all the actions of state 1.

$$A_{21} = P2 \quad _{21} = 1 \quad _{21} = \quad _{21} \text{ ..... } (3.2)$$

This work is expressed in terms of internal formulas

$$A_{21} = \int \bar{M}_2 \frac{Midx}{EJ} + \int \frac{1}{N_2} \frac{N1 \ x}{EF} + \int \frac{1}{Q_2} \frac{Q1 \ x}{GF} \quad (3.3)$$

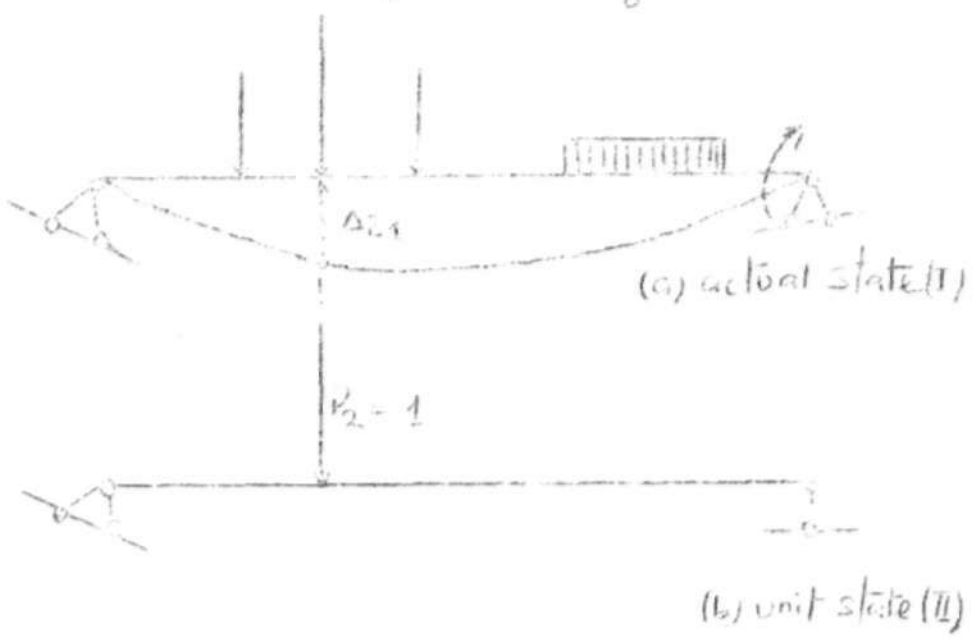


Figure 3.2

The displacement caused by any combination of loads may by expressed in terms of the stresses developed by the said combination and those due to a load unity. The line of action of this load unity must coincide with the direction of the displacement under consideration.

When a linear displacement is required line a deflection at any point of the system, the load unity must be a concentrated non-dimensional load acting at this particular point. If on the other hand, it is required to find the angular rotation of a certain cross-section, the unit action must be a non-dimensional concentrated moment applied to this section.

The state when a single load unity is applied to the system is referred to as the imaginary or unit state, while the case when the same system is acted upon by the combination of actions effectively applied will be refered to as the real or actual state.

In certain cases it is more convinient to use alphabetical indices in expression 3.4

instead of numerical ones say m, and n or P and d k. The expression then becomes;

$$\Delta mn = \sum\int \frac{1}{\bar{M}m} \frac{Mndx}{EJ} + \sum\int \frac{1}{\bar{N}m} \frac{Nndx}{EJ} + \sum\int \frac{1}{\bar{Q}m} \frac{Qndx}{GF} \quad \ldots (3.4)$$

when the cross-sections of all the members remain constant the expression (3.4) can be written as

$$\Delta mn = \sum \frac{1}{EJ} \int \bar{M}mMndx + \sum \frac{1}{EF} \int \bar{M}mNndx + \sum \frac{1}{GF} \int \bar{Q}mQndx \quad (3.5).$$

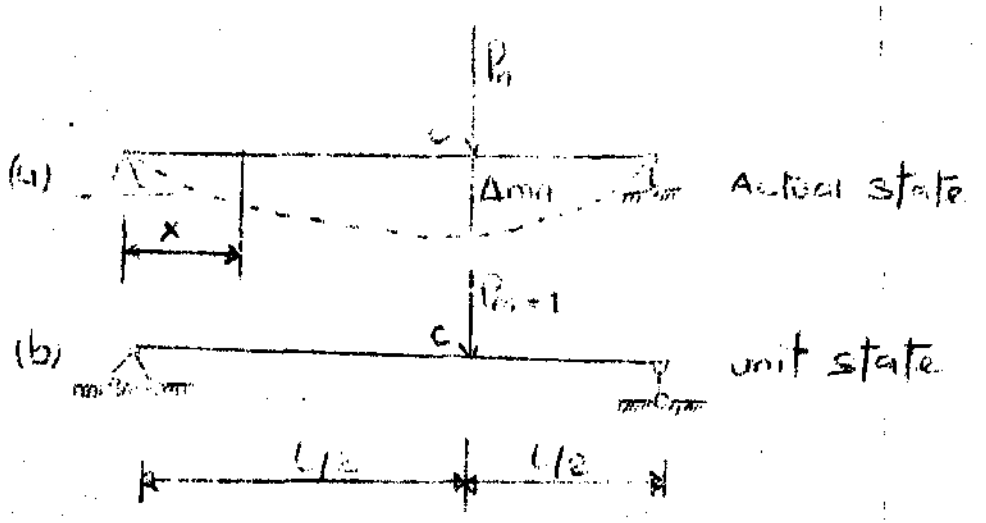
The above three equations are referred to as the general displacements equations or Mohrs equation. For the computation of displacement with the help of these expressions the following sequence will be adopted.

1. Determine the stresses Mn, Nn and Qn due to the applied loads for an arbitrary cross-section in terms of its asscissa x.
2. Apply a unity action at the cross-section whose deflection or angular rotation is required, a concentrated load corresponding to deflection or any other translation and a moment due to an angular rotation.
3. Compute the stresses $\bar{M}m$, $\bar{N}$, and $\bar{Q}m$ due to unit actions for the same cross-section situated a distance x from the origin of co-ordinates.
4. Introduce the values of the stresses Mn, Nn, and Qn as well as those of $\bar{M}m$, $\bar{N}m$ and $\bar{Q}m$ in one of the three expressions and integrate along all the elements of the entire structure. When the displacement Δmn thus

obtained is positive, its direction coincides with that adopted for the unit action and when it is negative, it is opposite to the one adopted for the unit action.

In majority of cases only one term of Mohrs formula has to be retained. Thus the structure works mainly in bending (this being generally the case for rectilinear beams portal frames, building frames and flat arches); it will suffice to use only the term containing bending moments. Similarly when the members of the structure work mainly in direct tension or compression as in the case of all hinged systems, both terms in the bending moment and shears may be neglected without any appreciable reduction in the accuracy of the results obtained.

If the element under consideration is a curved bar whose radius of curvature is at least ten times as great as the depth of its cross-section, Mohrs formula for rectilinear bars may be used provided the lenght of the straight element dx is replaced by the lenght of the arch ds.



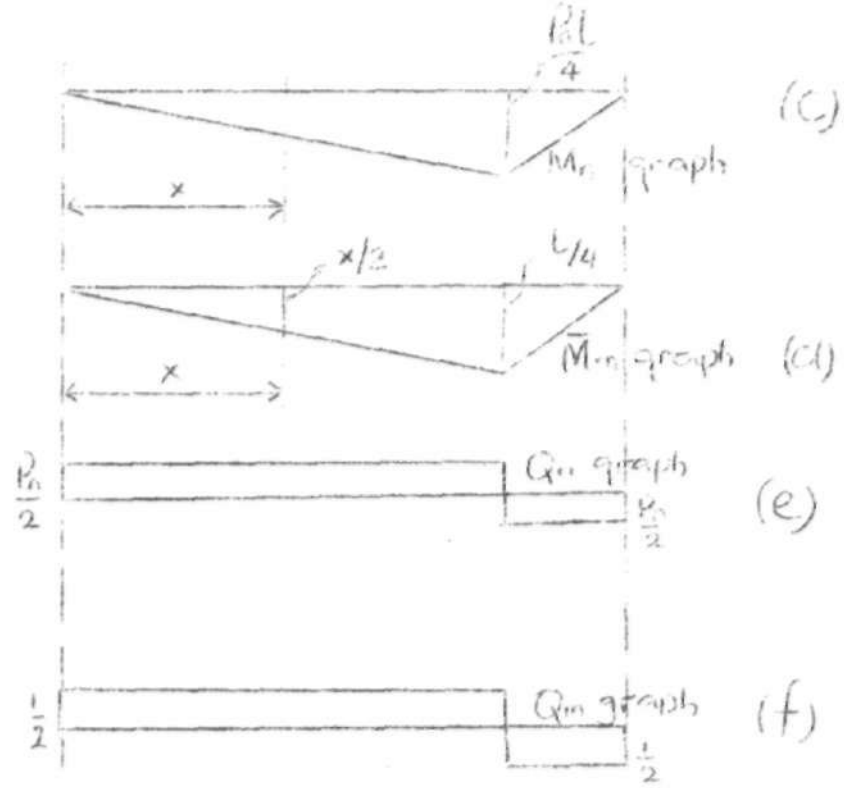


Figure 3.3

Example

Determine the deflection of an end supported beam of constant cross-section acted upon by a concentrated load Pn at midspan.

The maginary state of this beam will correspond to the application of a concentrated load unity in the direction of the deflection required. The normal stresses will remain constantly nil and therefore Mohrs formula will become

$$/\ mn = \sum \frac{1}{EJ} \int_0^l MmMndx + \sum \frac{1}{GF} \int_0^l QmQndx = \Delta^M_{mn} + \Delta^Q_{mn}$$

Where $\Delta^{M}_{mn} = \sum \frac{1}{EF} \int_0^1 \bar{M}_m M_n dx$ is the deflection in pure bending while $\sum \frac{n}{GF} \int_0^1 \bar{Q}_m Q_n dx$ is the part of the total deflection caused solely by shearing forces.

For all the cross-sections of the beam to the left of point C the bending moments Mn and $\bar{M}m$ and the shearing forces Qn and Qm are given by

$$\left.\begin{array}{ll} \bar{M}n = \frac{Pnx}{2} & \bar{M}m = \frac{1}{2}x \\ Qn = \frac{Pn}{2} & \bar{Q}m = \frac{1}{2} \end{array}\right\} \text{These values are given in the corresponding graphs}$$

Introducing those values into the deflections:

$$\Delta^{M}_{mn} = \frac{2}{EJ} \int_0^{1/2} \frac{x\ Pnxdx}{2\ 2} = \frac{Pnl^3}{48EJ}$$

$$\Delta^{Q}_{mn} = \frac{2}{GF} \int_0^{1/2} \frac{1}{2}\ \frac{Pndx}{2} = \frac{Pn\,l}{4GF}$$

3.3 DISPLACEMENT COMPUTATION TECHNIQUES

In some cases displacement computations can be simplified by the introduction of a special technique for the calculation of the integrals belonging to the type $\int_0^l \bar{M}_m M_n dx$, $\int_0^l \bar{N}_m N_n dx$ and $\int_0^l \bar{Q}_m Q_n dx$.

The technique is called graph multiplication method because it is based on the fact that the expression preceeded by the sign of the integral contains the product of two ordinates to the $\bar{M}m$ and $\bar{M}n$ curves. This technique applies on the condition that at least one of the curves reduces to a straight line. The other diagram may be bounded by any curve or broken line.

The ordinate to any straight line may be always expressed by

$\bar{M}m = x\tan\theta$

Introducing this value into the integral under consideration we obtain

$$\int_0^l \bar{M}mMndx = \tan\theta \int_0^l xMndx = \tan\theta \int_0^l xd\Omega_n,$$

where $Mndx = d\Omega_n$ represents the differential of the area bounded by the Mn curve (figure 3.4).

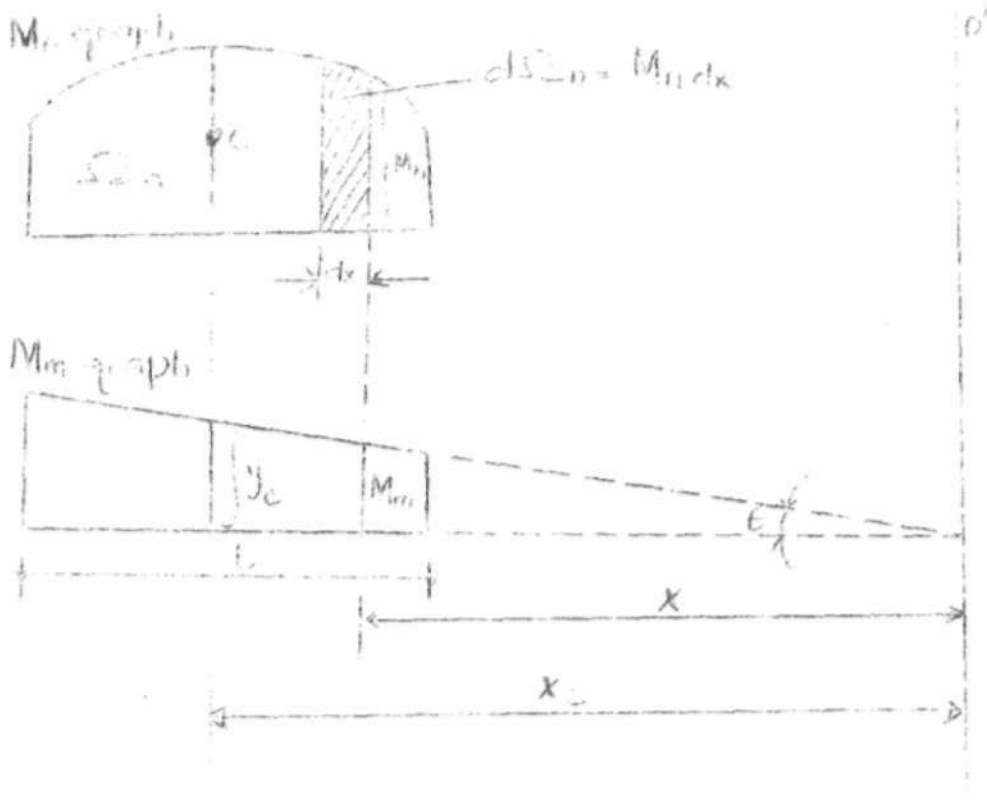


Figure 3.4

The expression $\int_0^l xd\Omega_n$ represents the statistical moment of the graph area about the 0-0 axis (figure 3.4). This statical moment may be expressed by $\int_0^l xd\Omega_n = \Omega_n xc$

xc represents the obscissa of the graph centroid. From this it follows that $\int_0^l \bar{M}mMndx = xc\tan\theta\,\Omega_n$ but since $xc\tan\theta = yc$ we obtain finally $\int_0^l \bar{M}mMndx = \Omega_n yc$.

Hence the product of the graph multiplication of two graphs one of which is at least bounded by a straight line equals the area bounded by the graph of arbitrary outline multiplied by the ordinate yc to the first graph measured along the vertical through the centroid of the second one. The product is reckoned positive when the graph of arbitrary outline and the ordinate to the rectilinear graph are both of the same sign. This method is called vereschagin's method. In the final expression $\int \bar{M}_m M_n dx = \omega y_c$ the left part differs from Mohrs expression by the lack of $\frac{1}{EJ}$. Hence the result of the graph multiplication carried out by vereschagins method must be divided later by EJ.

The ordinate yc must be measured on the graph bounded by a straight line. If both graphs are bounded by straight lines the ordinate yc could be measured on any one of the two.

3.4 THE ELASTIC CENTRE METHOD

The stress analysis of redundant structures require the simultaneous solution of several equations with several unknowns. The higher the degree of indeterminacy the greater the number of these equations, the harder their solution and the lower the accuracy of results. Attempts are therefore made frequently to form the above equations in such a way that each of them should contain only one unknown, or, alternatively that the system of these equations should fall into separate groups each containing a reduced number of unknowns not entering the next group.

To investigate the possibility of simplifying the analysis of a closed contour of arbitrary configuration such as shown, for instance in figure 3.5.

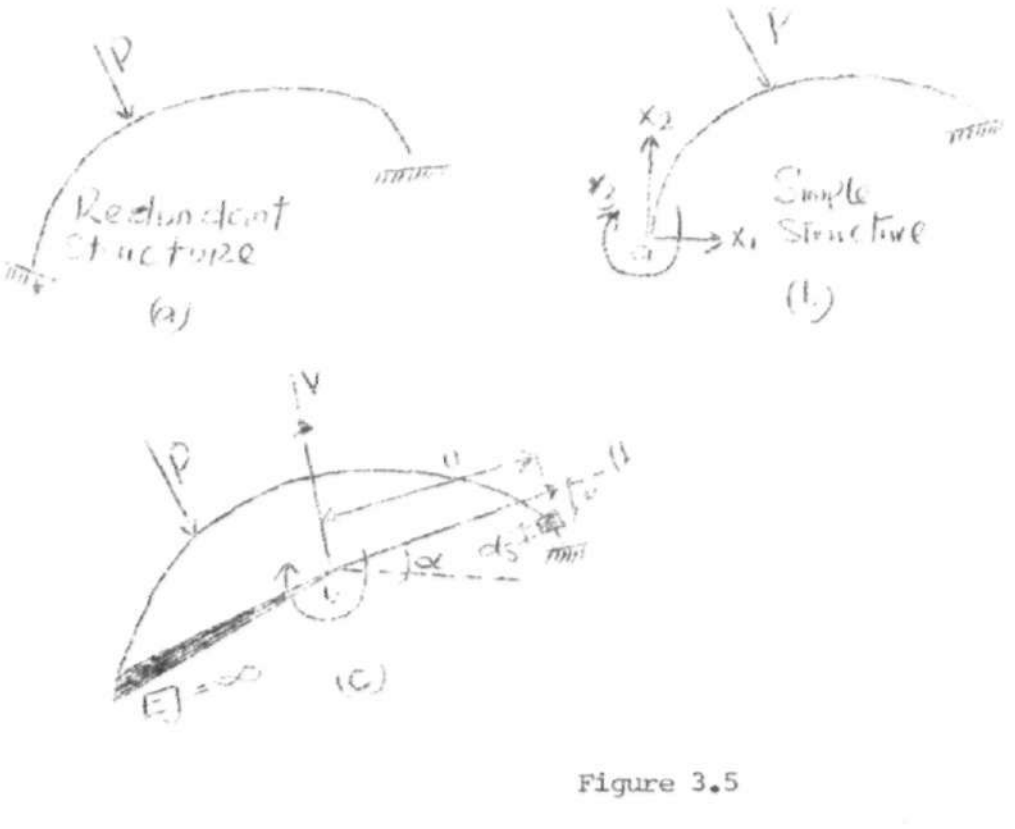


Figure 3.5

Having eliminated the constraints at the left hand abutment and having replaced them with three unknown reactions X_1, X_2 and X_3 as indicated in figure 3.5b, three standard equations should be normally formed with three unknowns.

In order to avoid the simultaneous solution of these equations let us fix to the free end of the simple structure an infinitely sliff bracket as both the lenght and direction of which remain as unknown.

At the free end b of this bracket apply at right angles two forces Z_1 and Z_2 (the direction of these forces coinciding with that of a new set of coordinates axes u and v) and moment Z_3.

If the magnitude of this actions were such in a way that they would unmobilize completely the end b of the bracket, preventing both its rotation and translation, the left extremity of the simple structure (point a) would also become fixed and thus from the view point of their deflections the given redundant structure and the simple structure of figure 3.5c would be in the same conditions. In other words the structure figure 3.5a and figure 3.5b acted upon both by load P the forces Z_1 and Z_2 and the moment Z_3 would be equivalent.

The system of equations expressing that the end b of the bracket is held fast is as follows.

$$\left.\begin{aligned} Z_1 \delta 11 + Z_2 \delta 12 + Z_3 \delta 13 + \Delta 1p = 0 \\ Z_1 \delta 21 + Z_2 \delta 22 + Z_3 \delta 23 + \Delta 2p = 0 \\ Z_1 \delta 31 + Z_2 \delta 32 + Z_3 \delta 33 + \Delta 3p = 0 \end{aligned}\right\} \quad 3.4$$

The coefficients δik entering these equations represent the displacement of the free end of bracket as induced by load unities following the direction of the unknowns Z_1, Z_2 and Z_3. The magnitude of these displacements depends of course, both on the size of the bracket and on the direction of the axes u and v. The parameters are chosen in such a way as to render hill all the secondary displacements of point b. In this case each of the three equations above will contain only one unknown, these equations reducing to

$$\left.\begin{aligned} Z_1 = \frac{-\Delta 1p}{\delta 11} \\ Z_2 = \frac{-\Delta 2p}{\delta 22} \\ Z_3 = \frac{-\Delta 3p}{\delta 33} \end{aligned}\right\} \quad 3.5$$

Below are the mathematical expression for the conditions governing the choice of the aforesaid parameters. Denoting by u and v the coordinates of an element ds of the given structure it can be written,

$$\delta_{12} = \int_0^l \frac{\bar{M}_1 \bar{M}_2 ds}{EJ} ds = \int_0^l \frac{v.u}{EJ} = 0$$

$$\delta_{13} = \int_0^l \frac{\bar{M}_1 \bar{M}_3}{EJ} ds = \int_0^l \frac{v.1 ds}{EJ} = 0$$

$$\delta_{23} = \int_0^l \frac{M_2 M_3}{EJ} ds = \int_0^l \frac{u.1 ds}{EJ} = 0$$

Dividing each of these equations by E we obtain

$$u\frac{ds}{J} = 0, \qquad v\frac{ds}{J} = 0 \text{ and } \quad uv\frac{ds}{J} = 0$$

For these expressed conditions its easy to find a geometrical interpretation. For this purpose subdivide the whole structure into elements ds and apply at the centre of gravity of each of these elements an imaginary load $\frac{ds}{J}$. In that case the first two of the above integrals will mean that the statistical moments of these imaginary loads about the coordinate axes u and v having for origin point b are nil. This will happen only when point b coincides with the centre of gravity of loads $\frac{ds}{J}$. The third of the integrals means that the polar moment of these loads about point b is equally nil, this becoming possible only when the coordinate axes coincide with the principal axes of inertia of the system.

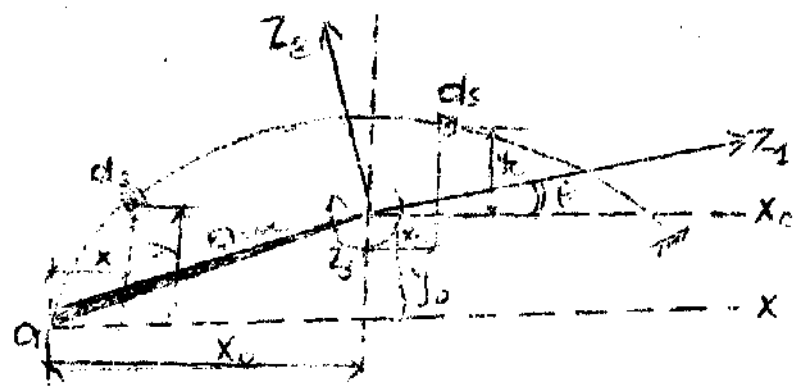


Figure 3.6

It follows that all secondary displacements of a structure redundant to the third degree and forming a close contour will reduce to zero provided the new origin of coordinate axis is brought in coincidence with the centre of gravity of the imaginary loads $\frac{ds}{J}$ and the direction of this axes with that of the principal axes of inertia of these same loads. If the structure is symmetrical, the principal axes of inertia will coincide with the axes of symmetry.

The origin of the new coordinate axes may be determined using formulas provided by theoretical mechanics, thus using the notation of figure 3.6 the centre of gravity of the loads $\frac{ds}{J}$ will be given by

$$x_o = \frac{\int x \frac{ds}{J}}{\int \frac{ds}{J}} \qquad y_o = \frac{\int y \frac{ds}{J}}{\int \frac{ds}{J}}$$

In each case the numerator represents the sum of the statical moments of the imaginary loads $\frac{ds}{J}$ about some axis of coordinates

(the summation being carried over the whole lenght of the contour) and the denominator equals the total of the some loads.

The angle formed by the principal axes of inertia with the x-axis is given by the following formula developed in the treatises on the strenght of materials

$$\tan 2\varphi = \frac{-2Jxcyc}{Jxc-Jyc}$$

$$Jxcyc = \int Xcyc\frac{ds}{J}$$

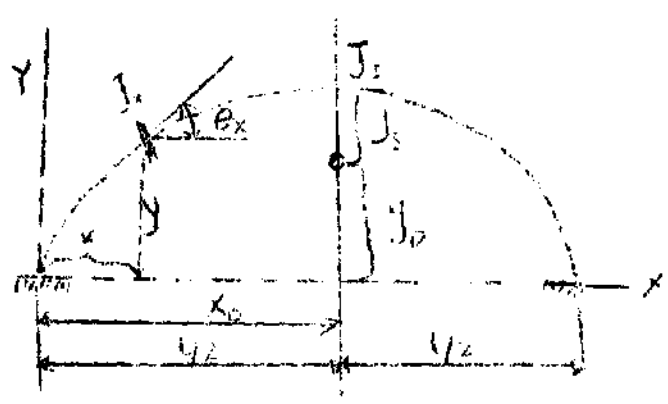
$$J_{xc} = \int y_c^2 \frac{ds}{J}$$

$$J_{yc} = \int x_c^2 \frac{ds}{J}$$

The imaginary loads $\frac{ds}{J}$ are some times called elastic loads or masses and the centre of gravity of these elastic masses - the elastic centre of gravity or simply the elastic centre.

For unsymmetrical systems the amount of work required to determine the position of the new coordinate axes may become so tedious that it outweigh the advantages gained from the simplification of equations. The elastic centre method yields tangible results for symmetrical structures.

Example



Determine the position of the elastic centre for the fixed end arch of the figure above. The neutral line of this arch follows a conic parabola by $y = \frac{4f(1-x)x}{12}$ and its cross-sectional moments of inertia vary inversely to $\cos\theta_x$

$$J_x = \frac{J_s}{\cos\theta_x}$$

Solution: Start with determining the position of the elastic centre with reference to the coordinate axes xoy. Owing to the symmetry of the structure both as regards its dimensions and the stiffness of its cross-sections, one of the coordinate axes is known off hand, it is vertical in direction and passes through the crown. Therefore, the abscissa of the elastic centre xo equals y_2

$$Dls = \frac{dx}{\cos\theta x} \text{ and } J_x = \frac{J_s}{\cos\theta}$$

$$Y_o \quad \frac{\frac{4f}{12}(1-x)x\,\frac{ds}{Js}}{\frac{ds}{Js}} = \frac{2}{3}f$$

3.5 USE OF INFLUENCE LINES

The use of influence lines simplifies very much the calculation of stresses induced in a structure and most useful when the loading applied to the structure is a moving type. It fasten the checking of stresses of any given section of the structure. The use of influence lines is best illustrated by an example.

Required the influence lines for the redundant reactions X_1, X_2 and X_3 as well as for the stresses Mk, Nk and Qk acting at section K of the arch presented below (figure 3.7) the obcissa of this section is taken equal to 6m.

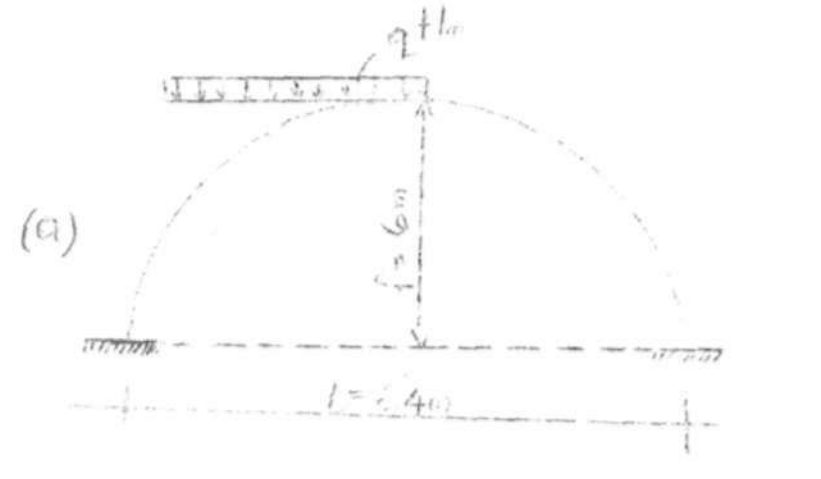


Figure 3.7

Solution.

The coordinates of the inscribed polygon as well as the Y_s are assumed to have been determined and has the values written on figure 3.7b.

$$Y_s = 2.0625m$$

The simultaneous equations permitting the determination of the redundant reactions due to a moving load unity P become

$$X_1 \delta 11 + \delta 1p = 0$$

$$X_2 \delta 22 + \delta 2p = 0$$

$$X_3 \delta 33 + \delta 3p = 0$$

from which $X_1 = \frac{-\delta 1p}{\delta 11}$

$$X_2 = \frac{-\delta 2p}{\delta 22}$$

and $X_3 \quad \frac{-\delta 3p}{\delta 33}$

or $X_1 = \frac{\delta p1}{\delta 11}$

$X_2 = \frac{\delta p2}{\delta 22}$

and $X_3 = \frac{\delta p3}{\delta 33}$

The graphs $\delta p1$, $\delta p2$ and $\delta p3$ for various positions of the load unity P constitute the deflection graphs of the simple structure due to the application of the redundant reactions $X_1 = 1$, $X_2 = 1$ and $X_3 = 1$ respectively. Divided by $-\delta 11$, $-\delta 22$ and $-\delta 33$ the ordinates to this graphs will represent the ordinates to the influence lines for X_1, X_2 and X_3.

The deflection graphs for this purpose would be obtained using the elastic load methods. To achieve this purpose the M_1, M_2 and M_3 graphs due to unit loads X_1, X_2 and X_3 will have to be constructed and elastic loads will have to be calculated using the expressions set out below.

$$W_o = \frac{a}{6EJc}(2M_o + M_1)$$

$$W_4 = \frac{a}{6EJc}(M3 + 2M_4)$$

$$W_n = \frac{a}{6EJc}(M_{n-1} + 4M_n + M_{n+1})$$

Below also are the unit diagrams due to X_1, X_2 and X_3.

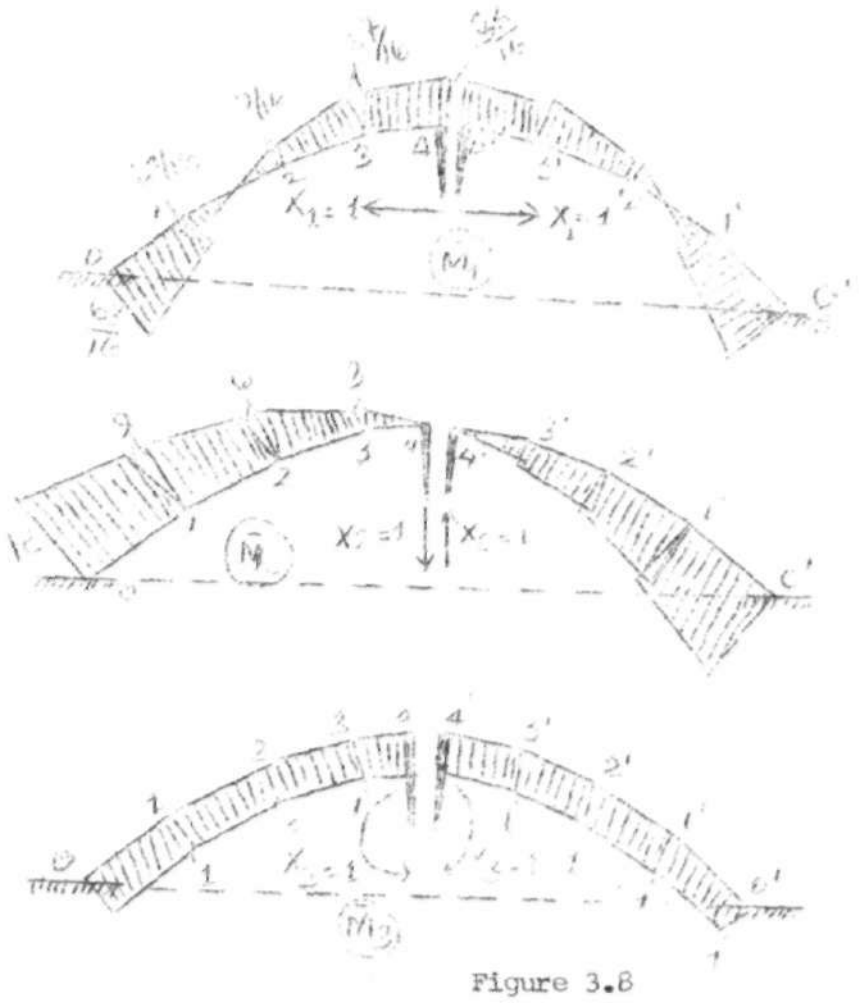


Figure 3.8

The elastic load W_2 (at point 2) when the simple structure is loaded by X_1=1 is given by

$$W_2 = \frac{a}{6EJc}(M1 + 4M_2 + M_3) = \frac{3}{6EJc}\left(\frac{21}{16} - 4x\frac{9}{16} - \frac{27}{16}\right) = \frac{-21}{16EJc}$$

W_4 due to the application of forces $X_2 = 1$ to the structure will amount to

$$W_4 = \frac{a}{6EJc}(M_3 + 2M_4) = \frac{3}{6EJc}(-3 + 2x0) = \frac{-3}{2EJc}$$

The values of elastic loads is as follows for all points of application.

TABLE 3.1

Load point	Unit forces		
	X_1	X_2	X_3
0	$\frac{73.5}{16EJc}$	$\frac{-33}{2EJc}$	3/2EJc
1	$\frac{69}{16EJc}$	$\frac{-54}{2EJc}$	6/2EJc
2	$\frac{-21}{16EJc}$	$\frac{-36}{2EJc}$	6/2EJc
3	$\frac{-75}{16EJc}$	$\frac{-18}{2EJc}$	6/2EJc
4	$\frac{-46}{16EJc}$	$\frac{-3}{2EJc}$	$\frac{3}{2EJc}$

In order to obtain the deflection diagram $\delta p1$ apply to the left hand part of the imaginary structure the elastic load corresponding to the unit action X_1. These elastic loads must be parallel to the displacement in question and must be directed towards the more extended fibres of the simple structure (figure 3.9a).

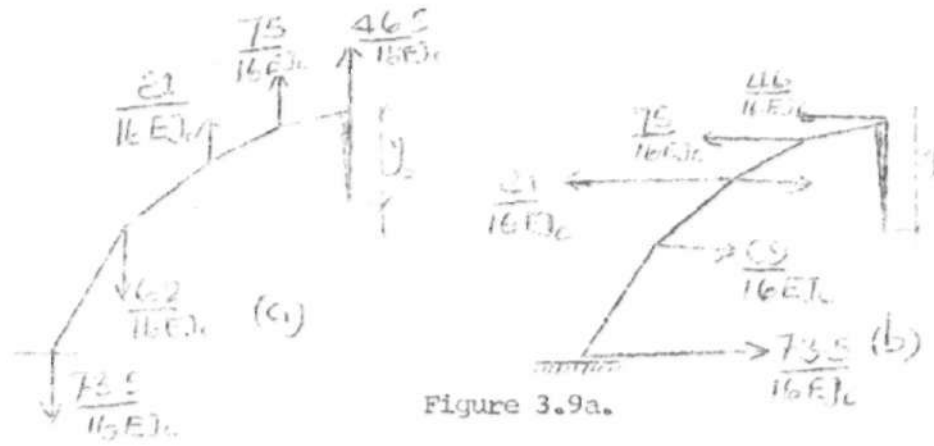


Figure 3.9a.

Since the rigid bracket fixed to the simple structure may not be rotated by the unit load X_1, the algebraic sum of all the elastic loads applied to the system figure 3.9(a) must equal zero

$$W = \frac{7350}{16EJc} + \frac{69}{16EJc} \quad \frac{-21}{16EJc} \quad \frac{-75}{16EJc} \quad \frac{-46.50}{16EJc} = 0$$

This relation is used as a check on the accuracy of the computed elastic loads.

The vertical displacement $\delta p1$ due to the unit action X_1 will equal the ordinates to the diagram of bending moments induced in the imaginary structure by the elastic loads.

$$\delta p1 \text{ at point } 1 = \frac{73.5x3}{16EJc} = \frac{2205}{16.EJc}$$

$$\delta p1 \text{ at point } 2 = \frac{73.5x6 + 69x3}{16EJc} = \frac{648}{16EJc}$$

$$\delta p1 \text{ at point } 3 = \frac{73.5x9 + 69x6 - 21x3}{16EJc} = \frac{1012.50}{16EJc}$$

$$\delta p1 \text{ at point } 4 = \frac{73.5x12 + 69x9 - 21x6-75x3}{16.EJ} = \frac{1,152}{16EJc}$$

The displacement $\delta 11$ is obtained rotating the imaginary loads through 90° until they become horizontal as in figure 3.9c calculate the moment of these loads about the elastic centre and double its value, for the displacement $\delta 11$ represents the total change in the distance between the lower ends of the bracket along the direction of the horizontal unit loads X_1.

$$\delta 11 = 2 \left[\frac{73.5}{16EJc} \left(\frac{48}{8} - \frac{16.5}{8}\right) + \frac{69}{16EJc} \left(\frac{27}{8} - \frac{16.5}{8}\right) + \frac{21}{16EJc} \left(\frac{16.5}{8} - \frac{12}{8}\right) + \frac{75}{16EJc} \left(\frac{16.5}{8} - \frac{3}{8}\right) + \frac{16.5}{16EJc} x \frac{16.5}{8} \right] = \frac{1228.50}{16EJc}$$

Dividing all the ordinates to the $\delta p1$ graph ($-\delta 11$) we obtain the ordinates to the influence line for X_1. This influence line

shows the variation of X_1 when the unit load P travels along the arch. See figure 3.10.

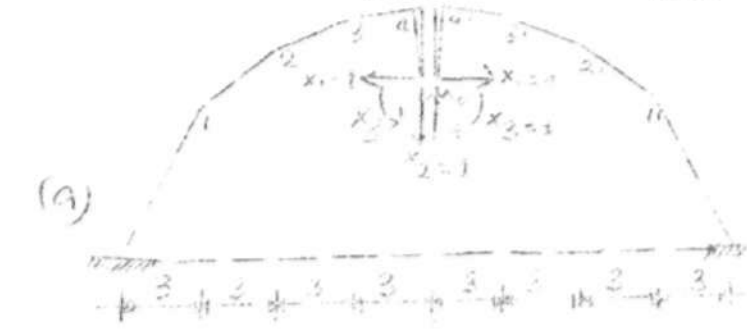
Following the same procedure the displacement δ_{p2} is determined:

$$\delta p2 \text{ at } 1 = \frac{33x3}{2EJc} = \frac{99}{2EJc}$$

$$\delta p2 \text{ at } 2 = \frac{33x6 + 54x3}{2EJc} = \frac{360}{2EJc}$$

$$\delta p2 \text{ at } 3 = \frac{33x9 + 54x6 + 36x3}{2EJc} = \frac{729}{2EJc}$$

$$\delta p2 \text{ at } 4 = \frac{33x12 + 54x9 + 36x6 + 10x3}{2EJc} = \frac{1152}{2EJc}$$



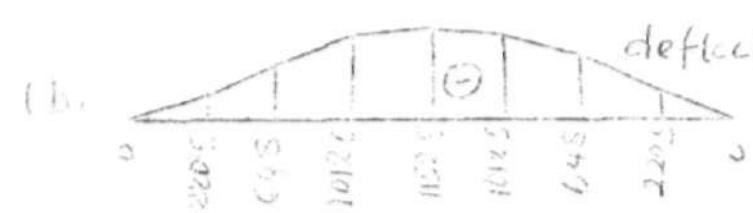


Figure 3.10

The ordinates are set up as in figure 3.10. The displacement δ_{22} representing the mutual translation of the free ends of the brackets along the direction of X_2. This displacement must be the double of δ_{p2} at point 4

$$\delta_{22} = 2\,\delta_{p2} \text{ at point } 4 = \frac{2x1,152}{2EJc} = \frac{1,152}{EJc}$$

The influence lines for X_2 are then determined by dividing δ_{p2} by $-\delta_{22}$.

Similarly δ_{p3} at $1 = \frac{3x3}{2EJc} = \frac{9}{2EJc}$

$$\delta_{p3} \text{ at } 2 = \frac{3x6 + 6x3}{2EJ} = \frac{36}{2EJc}$$

$$\delta_{p3} \text{ at } 3 = \frac{3x9 + 6x6 + 6x3}{2EJc} = \frac{81}{2EJc}$$

$$\delta_{p3} \text{ at } 4 = \frac{3x12 + 6x9 + 6x6 + 6x3}{2Jc} = \frac{144}{2EJc}$$

The displacement graph of δ_{p3} is figure 3.10(d). The displacement δ_{33} representing the mutual angular rotation of the brackets (faces 4 and 4^1 of the semi arches) its value will be equal to the sum of the elastic loads).

$$\delta_{33} = 2\left(\frac{3x2}{2EJ} + \frac{6x3}{2EJc}\right) = \frac{24}{EJc}$$

The influence line for X_3 is obtained by dividing the ordinates to the δ_{p3} by $(-\delta_{33})$.

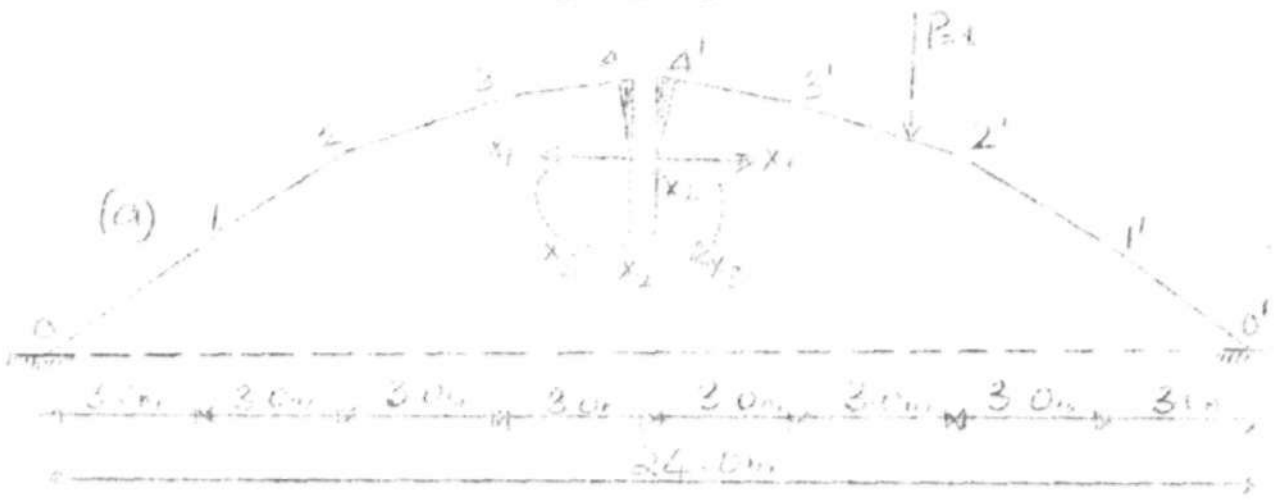
Once the influence lines for all the redundant reactions X_1, X_2 and X_3 have been found, the construction of the influence lines for bending moments, shearing and normal stresses at any cross section of the arch can be proceeded.

The influence lines for any section of the arch can now be plotted with the determination of the redundant reactions.

Lets constructs the three influence lines for section K situated 6m to the right from the left-hand abutment section 2 of figure 3.7b.

The angle θ_2 between the neutral line of the arch and the horizontal equals for this section $26^{o}34^{1}$ and in $\theta_2 = 0.447$ and $\cos\theta_2 = 0.894$. The lever arm of the force X_1 with reference to section K equals

$$y_s - y_2 = \frac{33}{16} - \frac{3}{2} = \frac{9}{16}\text{m}$$





Influence line for X_1

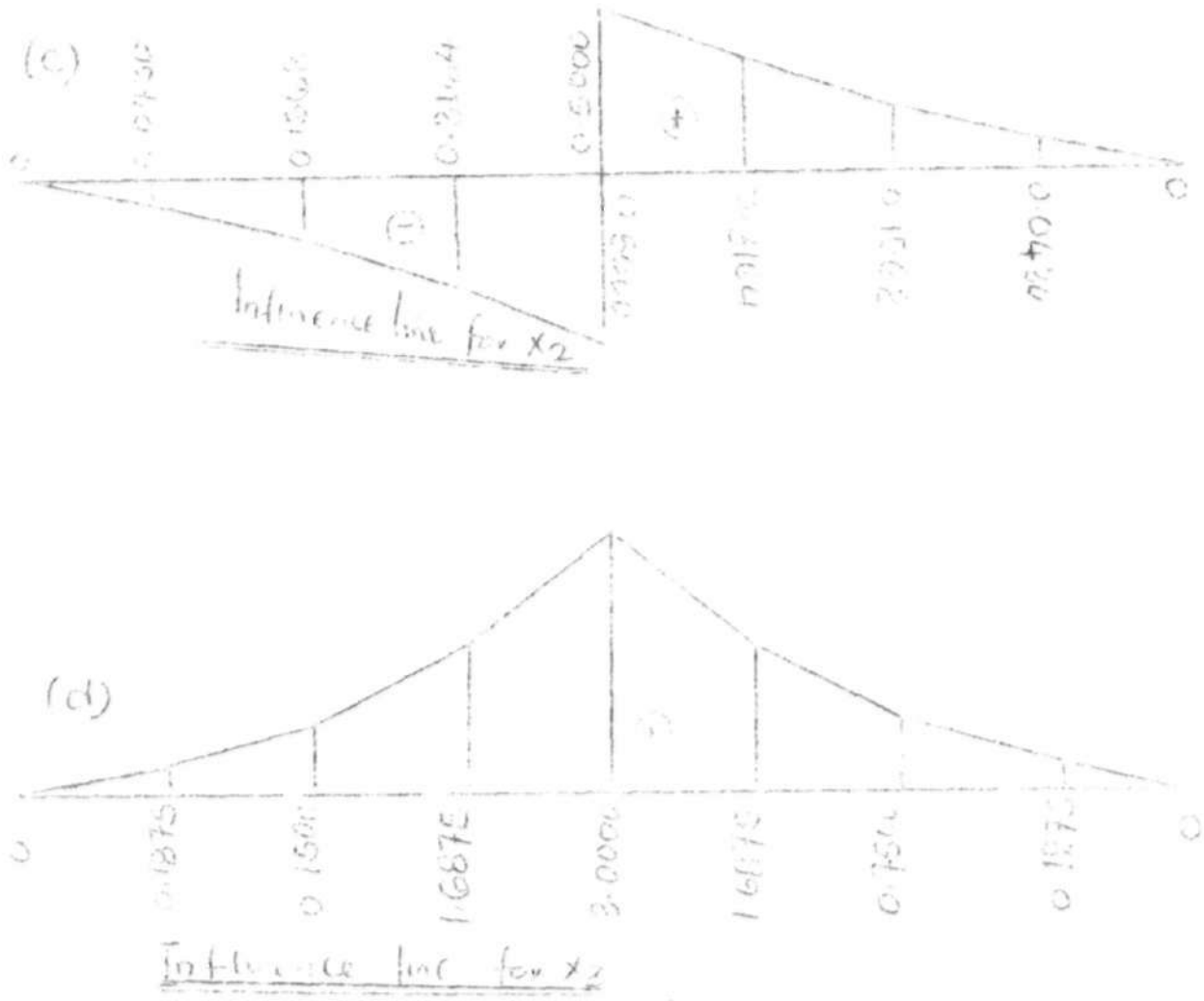


Figure 3.12

and lever arm of force X_2 with reference to the same section equals 6metres. The ordinates to the influence lines Mk, Qn and Nk are calculated using the following expressions.

(a) When unit load P is between the left-hand abutment and section K or when it has shifted to the right-hand semiarch

$$Mk = -X\frac{9}{116} - X_2 6 + X_3$$

$$Qk = X_1 Sin\Theta_2 + X_2 Cos\Theta_2 = -X_1 0.447 + X_2 0.894$$

$$Nk = X_1 Cos\Theta_2 + X_2 Sin\Theta_2 = X_1 0.894 + X_2 0.447$$

(b) when the unit load P is between section K and the crown $Mk = -X_1 \frac{9}{16} - X_2 6 + X_3 - 1r$

where r is the lever arm of the unit load P about section K and

$$Qk = X_1 0.447 + X_2 0.894 + 1x0.894$$

$$Nk = -X_1 0.894 + X_2 0.447 + 1x0.447$$

The ordinates to the Mk, Qk and Nk are tabulated below

TABLE 3.1

Ordinates to the Mk Influence line

Load point	X_1	$-X_1 \frac{9}{116}$	X_2	$-X_2 6$	X_3	$-r$	Ordinates to the Mk influence lines, metres
0	0	0	0	0	0	-	-
1	0.1795	-0.101	-0.0430	0.258	0.187	-	0.344
2	0.5275	-0.297	-0.1562	0.937	0.750	0	1.390
3	0.8242	-0.464	-0.3164	1.898	1.687	-3	0.124
4	0.9377	-0.528	-0.5000	3.000	3.000	-6	-0.528
4^1	0.9377	-0.528	-0.5000	-3.000	3.000	-	-0.528
3^1	0.8242	-0.464	0.3164	-1.898	1.687	-	-0.675
2^1	0.5275	-0.297	0.1562	-0.937	0.750	-	-0.404
1^1	0.1795	-0.101	0.0430	-0.258	0.187	-	-0.172
0	0	0	0	0	0	0	0

TABLE 3.2

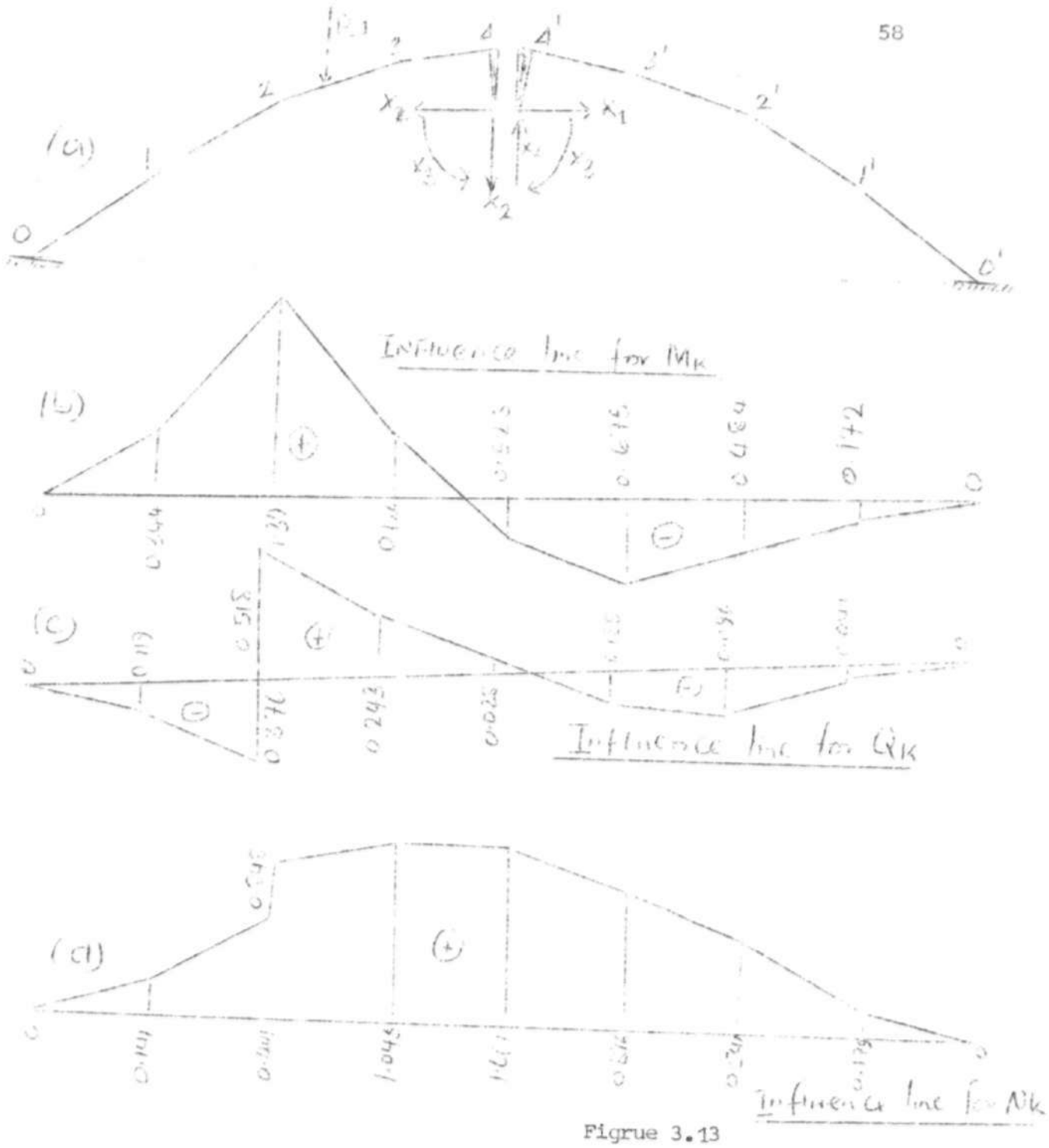
Ordinates to the Qk Influence line

Load point	X_1	$-x_1 0.447$	X_2	$X_2 0.894$	0.894	Ordinates to the Qk influence line
0	0	0	0	0	0	0
1	0.1795	-0.080	-0.0430	-0.039	-	-0.119
Immediately to the left of section 2	0.3275	-0.236	-0.1562	-0.140	-	0.376
Immediately to right of section 2	0.5275	-0.236	-0.1562	-0.140	0.894	0.518
3	0.8242	-0.368	-0.3164	-0.283	0.894	0.243
4	0.9371	-0.419	-0.5000	-0.447	0.894	0.028
4^1	0.9377	-0.419	0.5000	0.447	-	0.028
3^1	0.8242	-0.368	0.3164	0.283	-	-0.085
2^1	0.5275	-0.236	0.1562	0.140	-	-0.096
1^1	0.1795	-0.080	0.0430	0.039	-	-0.041
0	0	0	0	0	0	0

TABLE 3.3

Ordinates to the Mk Influence line

Load point	X_1	X_1 0.894	X_2	X_2 0.447	0.447	Ordinates to the Mk influence line
0	0	0	0	0	0	0
1	0.1795	0.160	-0.0430	-0.019	-	0.141
Immediately to the left of section 2	0.5275	0.471	-0.1562	-0.070	-	0.401
Immediately to the right of section 2	0.5275	0.471	-0.1562	-0.070	0.447	0.848
3	0.8242	0.737	-0.3164	-0.141	0.447	1.043
4	0.9377	0.838	-0.5000	-0.223	0.447	1.061
4^1	0.9377	0.838	0.5000	0.223	-	1.061
3^1	0.8242	0.737	0.3164	0.141	-	0.875
2^1	0.5275	0.471	0.1562	0.070	-	0.541
1^1	0.1795	0.160	0.0430	0.019	-	0.179
0	0	0	0	0	0	0



Figrue 3.13

Using the entries in last column of tables 3.1, 3.2 and 3.3 the influence lines appearing in figure 3.13 have been constructed.

Now using these influence lines the magnitudes of Mk, Qk and Nk induced by uniformly distributed loads acting over the whole of the semiarch, the intensity being equal to 2 tons per metre. For this purpose the uniformly distributed loads are replaced by concentrated ones acting at points 0, 1, 2, 3 and 4 of the polygonal arch. These concentrated loads will amount to $P_o = P_4 = \frac{a}{2} = \frac{2x3}{2} = 3$ tons

$$P_1 = P_2 = P_3 = a = 2x3 = 6 \text{ tons}$$

The desired values of Mk, Qk, and Nk will be obtained multiplying the magnitudes of these concentrated loads by the corresponding ordinates to the influence lines which are then summed up as below:

Mk = 3x0 + 6x0.344 + 6x1.390 + 6x0.124 − 3x0.528 = 9.564 ton-metres.

Qk = $3x0 - 6x0.119 + 6x\frac{-0.376+0.518}{2} + 6x0.243 + 3x0.028 =$ 1.254 tons.

Nk = $3x0 + 6x0.114 + 6x\frac{0.401 + 0.848}{2} + 6x1.043 + 3x1.061 =$ 14.034 tons.

3.6 USE OF SYMMETRY

In analyzing the stresses arising in structures with a large number of redundant constraints, a large number of simultaneous equations equal to the structures degree of redundancy each of these equations containing the same number of unknowns are involved.

For example figure 3.14.

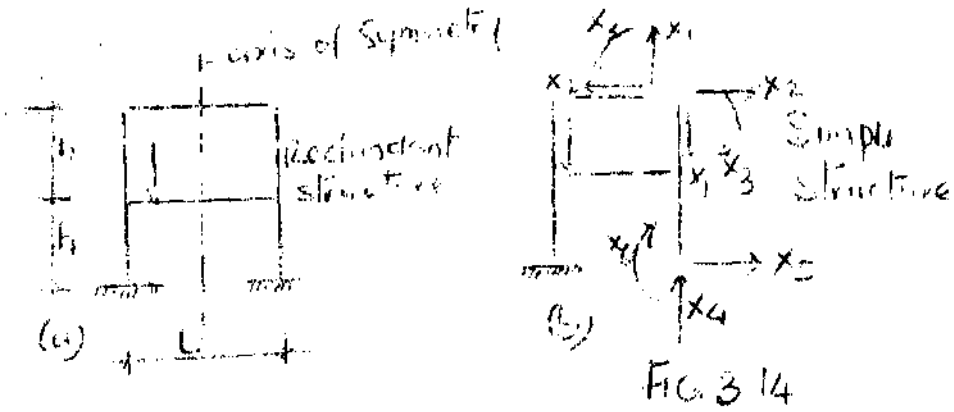


FIG 3.14

In figure 3.14(a) the structure consist of two closed contours and consequently is redundant to the sixth degree. If all the computations pertaining to this frame were carried out, adopting as conjugate statically determinate structure the one given in figure 3.14(b) it would be necessary to form and to solve a system of six simultaneous equations with six unknowns each.

$$X_1\delta 11 + X_2\delta 12 + X_3\delta 13 + X_4\delta 14 + X_5\delta 15 + X_6\delta 16 + \Delta 1p = 0$$
$$X_1\delta 21 + X_2\delta 22 + X_3\delta 23 + X_4\delta 24 + X_5\delta 25 + X_6\delta 26 + \Delta 2p = 0$$
$$X_1\delta 31 + X_2\delta 32 + X_3\delta 33 + X_4\delta 34 + X_5\delta 35 + X_6\delta 36 + \Delta 3p = 0$$
$$X_1\delta 41 + X_2\delta 42 + X_3\delta 43 + X_4\delta 44 + X_5\delta 45 + X_6\delta 46 + \Delta 4p = 0$$
$$X_1\delta 51 + X_2\delta 52 + X_3\delta 53 + X_4\delta 54 + X_5\delta 55 + X_6\delta 56 + \Delta 5p = 0$$
$$X_1\delta 61 + X_2\delta 62 + X_3\delta 63 + X_4\delta 64 + X_5\delta 65 + X_6\delta 66 + \Delta 6p = 0$$

The solution of these system of equation would be extremely labourious and require a lot of time. The work can be simplified very considerably due to symmetry. In a symmetrical structure not

only the arrangement of its members but also their cross-sectional rigidity are symmetrical about a certain axis. The simplification is based on the possibility of finding a conjugate statically determinate structure for which the Mi diagram for each redundant reaction Xi = 1 will be either symmetrical or antisymmetrical.

Thus for the frame under consideration figure 3.14(a) a simple structure of figure 3.15 could be adopted.

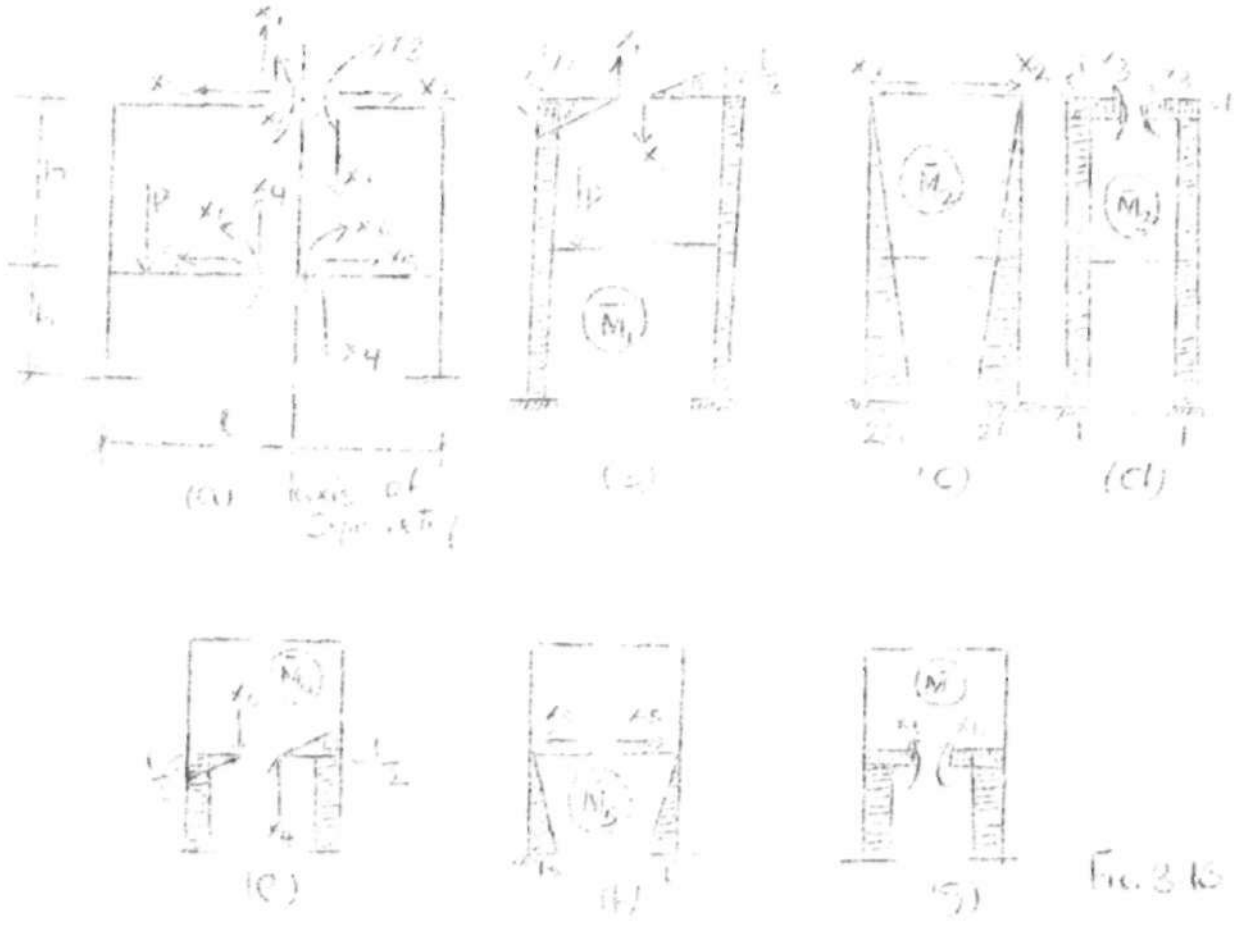


Fig. 3.15

In this case $\bar{M}_2$, $\bar{M}_3$, $\bar{M}_5$ and $\bar{M}_6$ diagrams due to symmetrical unit forces X_2, X_3, X_5 and X_6 would themselves be symmetrical (figure 3.15 c, d and g) while the $\bar{M}_1$ and M_4 diagrams induced by antisymmetrical unit forces X_1 and X_4 would also be antisymmetrical unit forces X_1 and X_4 would also be antisymmetrical figure 3.15 b and e.

The product of a symmetrical graph by antisymmetrical one is always nil. Thus for instance if the $\bar{M}_1$ graph were multiplied by the $\bar{M}_2$ graph the product pertaining to the left-hand half frame would equal.

$= + 2h \frac{2h}{2} \frac{1}{2} = +h^2 1$ while that pertaining to the right hand frame would amount to $-2h \frac{2h}{2} \frac{1}{2} = -h^2 1$. As a result the displacement δ_{12} equal to the algebraic sum of these two amounts to nil. For the same reason all the other secondary displacements whose values are obtained multiplying graphs by antisymmetrical graphs will also reduce to zero.

For the frame under consideration such will be the case for displacements δ_{12}, δ_{13}, δ_{15}, δ_{16}, δ_{21}, δ_{24}, δ_{31}, δ_{34}, δ_{43} δ_{43}, δ_{46}, δ_{51}, δ_{54} δ_{61} and δ_{64}. If follows that the system of simultaneous equations would become.

$$X_1\delta_{11} + X_4\delta_{14} + \Delta_{1p} = 0$$
$$X_2\delta_{22} + X_3\delta_{23} + X_5\delta_{35} + X_6\delta_{26} + \Delta_{2p} = 0$$
$$X_2\delta_{32} + X_3\delta_{33} + X_5\delta_{35} + X_6\delta_{36} + \Delta_{3p} = 0$$
$$X_1\delta_{41} + X_4\delta_{44} + \Delta_{4p} = 0$$
$$X_2\delta_{52} + X_3\delta_{53} + X_5\delta_{55} + X_6\delta_{56} + \Delta_{5p} = 0$$
$$X_2\delta_{62} + X_3\delta_{63} + X_5\delta_{65} + X_6\delta_{66} + \Delta_{6p} = 0$$

and would consequently fall into two independent systems.

$$X_1\delta_{11} + X_4\delta_{14} + \Delta_{4p} = 0$$
$$X_1\delta_{41} + X_4\delta_{44} + \Delta_{4p} = 0$$
$$X_2\delta_{22} + X_3\delta_{23} + X_5\delta_{25} + X_6\delta_{26} + \Delta_{2p} = 0$$
$$X_2\delta_{32} + X_3\delta_{33} + X_5\delta_{35} + X_6\delta_{36} + \Delta_{3p} = 0$$

$$X_2\delta 52 + X_3\delta 53 + X_5\delta 55 + X_6\delta 56 + \Delta 5p = 0$$

$$X_2\delta 62 + X_3\delta 63 + X_5\delta 65 + X_6\delta 66 + \Delta 6p = 0$$

The first of these systems contains two equations with two antisymmetrical unknowns and the second four equations with four symmetrical unknowns.

Thus the choice of an adequate simple structure of symmetrical pattern has resulted in the replacement of a system of six simultaneous equations containing six unknowns each by two independent systems, the first comprising two equations with two unknowns and the second four equations with four unknowns. This simplifies enormously the computations enhancing at the same time very considerably the precision of the results obtained. Additional reduction of computation work has been obtained due to the fact that all displacements can be calculated for one-half of the conjugate simple structure only. The total displacement will be the double of that for the half frame.

If the symmetrical frame contains a central column, the displacement due to the redundant antisymmetrical reactions can be computed multiplying at first the diagrams relative to one half of the frame (without the central column), where after the product obtained should be doubled and increased by the product of graph multiplication pertaining the central column.

3.7 GROUPING OF UNKNOWNS

If a structure consists of several spans it becomes impossible to transfer the points of application of all the redundant reactions to the axis of symmetry. But symmetrical and antisymmetrical bending moment diagrams may still be obtained if the unknowns representing a single force or couple are replaced by unknowns representing whole groups of forces.

The structure appearing below is six times redundant, if the conjugate structure appearing beside it is adopted six simultaneous equations with six nonsymmetrical unknowns X_1, X_2 X_3, X_4, X_5 and X_6 will have to be solved. And in these equations none of the coefficients would normally equal zero.

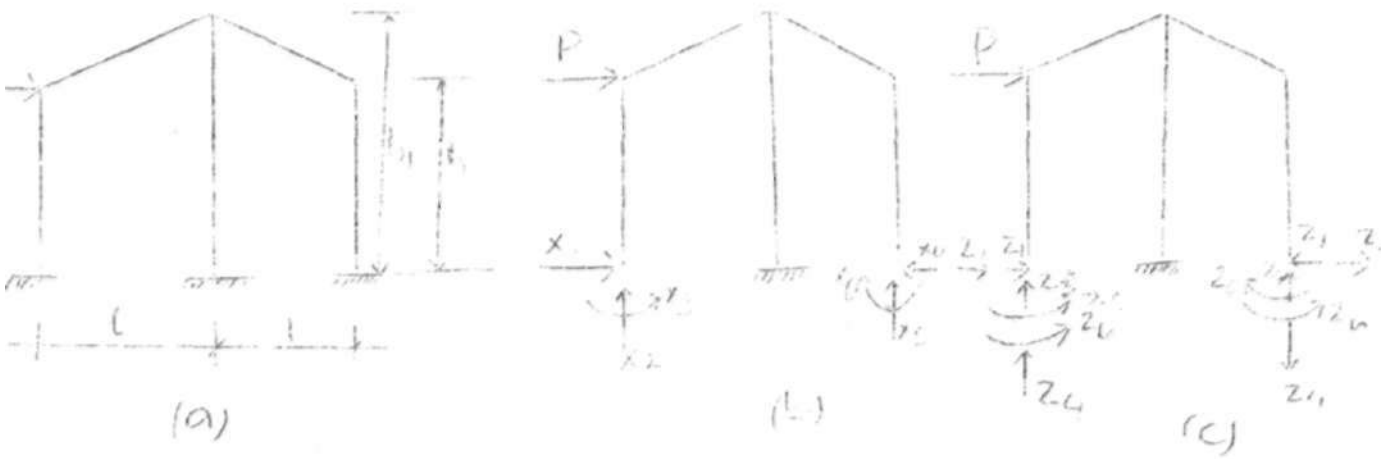


Figure 3.16

However if the groups of unknown forces Z_1, Z_2, Z_3, Z_4, Z_5 and Z_6 shown in figure 3.16(c) are adopted as the unknowns, a very large number of secondary displacements in the simultaneous equations would reduce to zero, for these displacements (coefficients) would result from the multiplication of symmetrical graphs by antisymmetrical ones.

The unknown Z_1 represents two horizontal forces X_1 and X_4 equal in value and opposite in sign.

Z_2 = two horizontal forces equal both in value and in sign

Z_3 = two vertical forces of equal amount, both directed upwards.

Z_4 = two vertical forces equal in amount and opposite in direction.

Z_5 = two couples equal in amount and opposite in direction, and

Z_6 = two couples of the same magnitude and acting in the same direction.

The bending moment diagram due to the above groups of unit forces are given in figure 3.17. It is readily observed that the $\bar{M}_1$ $\bar{M}_2$ and $\bar{M}_5$ diagrams are symmetrical while the $\bar{M}_2$, $\bar{M}_4$ and $\bar{M}_6$ diagrams are antisymmetrical.

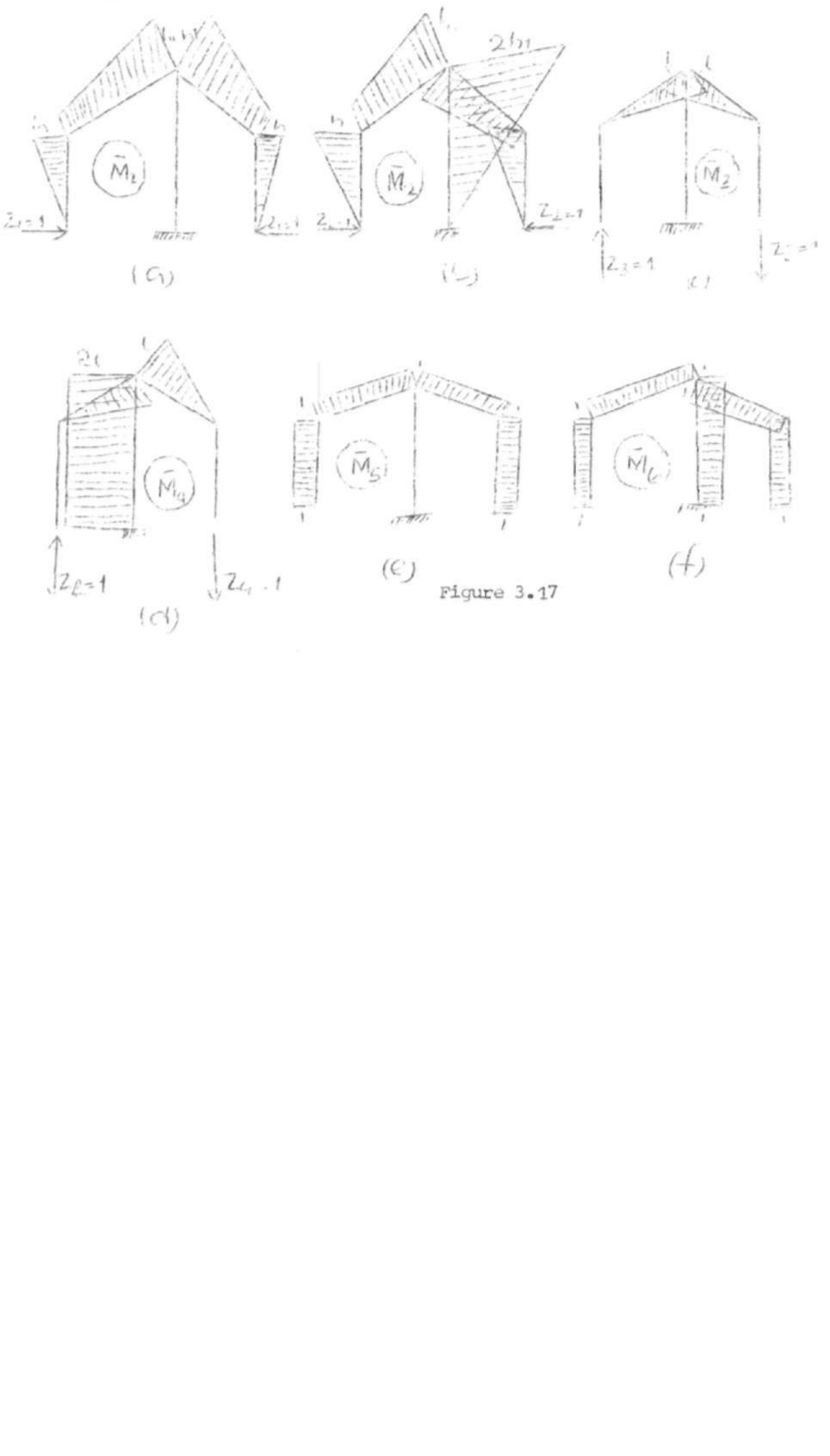


Figure 3.17

Comparing the structures in figure 3.17(a) and (b) the following relationships exist between the unknowns X and Z.

$$X_1 = Z_1 + Z_2;\ X_4 = Z_1 - Z_2$$
$$X_2 = Z_3 + Z_4;\ X_5 = Z_3 - Z_4$$
$$X_3 = Z_5 + 56;\ X_6 = Z_5 - Z_6$$

This can be rewritten as:

$$\frac{Z_1 = X_1 + X_4}{2};\ \frac{Z_4 = X_2 - X_5}{2}$$

$$\frac{Z_2 = X_1 - X_4}{2};\ \frac{Z_5 = X_3 + X_6}{2}$$

$$\frac{Z_3 = X_2 + X_5}{2};\ \frac{Z_6 = X_3 - X_6}{2}$$

Grouping the unknowns as indicated above permits the replacement of a single system formed by six simultaneous equations by two independent ones. The first containing symmetrical ones and the second antisymmetrical ones.

(a) The first system:

$$Z_1 (\delta 11) + Z_3 (\delta 13) + Z_5 (\delta 15) + (\Delta 1p) = 0$$
$$Z_1 (\delta 31) + Z_3 (\delta 33) + Z_5 (\delta 35) + (\Delta 3p) = 0$$
$$Z_1 (\delta 51) + Z_3 (\delta 53) + Z_5 (\delta 55) + (\Delta 5p) = 0$$

(b) the second system:

$$Z_2 (\delta 22) + Z_4 (\delta 24) + Z_6 (\delta 26) + (\Delta 2p) = 0$$
$$Z_2 (\delta 42) + Z_4 (\delta 44) + Z_6 (\delta 46) + (\Delta 4p) = 0$$
$$Z_2 (\delta 62) + Z_4 (\delta 64) + Z_6 (\delta 66) + (\Delta 6p) = 0$$

In the above expressions the coefficients (δik) and the free terms (Δip) represent the displacements induced by and along the aforesaid groups of unknown forces.

Thus the system of six equations with six unknowns has been reduced to two independent systems of three equations with three unknowns.

The displacements are computed as usual, multiplying diagrams pertaining to one-half of the structure and doubling the results obtained. If the frame contains a central column, graphs pertaining to one half-frame (excluding the central column) will be first multipled one by the other and the result will obtained will be added to the product of graphs for the central column.

3.8 SYMMETRICAL AND UNSYMMETRICAL LOADING

If a system of load acting on a structure is either symmetrical or antisymmetrical all the computations are further simplified, because in this case it becomes possible to find a conjugate simple structure for which the bending moment diagrams due both to the unit actions and to the actual loading become either symmetrical or antisymmetrical. As a result a number of free terms of the simultaneous equations together with some coefficients to the unknowns will reduce to zero.

The sixth degree redundant frame (figure 3.18) will illustrate the above description. This frame is acted by a system of symmetrical loads.

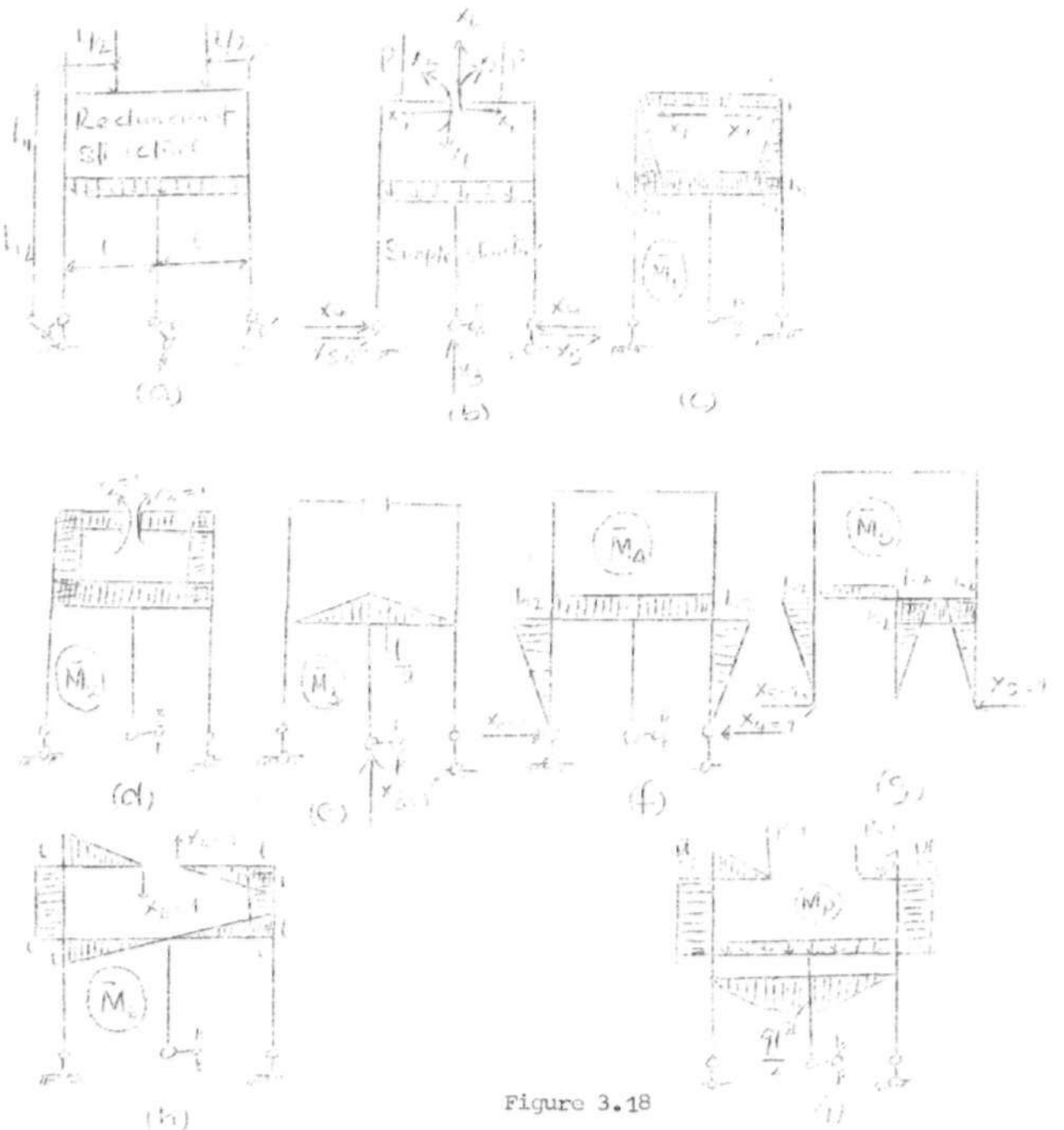


Figure 3.18

A symmetrical statically determinate structure with symmetrical and unsymmetrical unknowns may be obtained cutting in two the upper cross-bar and eliminating three constraints at the supports as indicated in figure 3.18(b).

The redundant reactions which cannot be transferred to the axis of symmetry such as the horizontal components of the left and of the right-hand column reactions will be replaced by two group of forces X_4 and X_5.

The bending moment diagrams for the conjugate structure due both to the unit actions following the direction of the unknows are shown in figure 3.18 c, d, e, f, g, h, and i. All the unknowns being either symmetrical (X_1, X_2, X_3 and X_4) or antisymmetrical (X_5 and X_6), the simultaneous equations form two independent systems given hereunder:

$$\left.\begin{aligned} X_1\delta 11 + X_2\delta 12 + X_3\delta 13 + X_4\delta 14 + \Delta 1p = 0 \\ X_1\delta 21 + X_2\delta 22 + X_3\delta 23 + X_4\delta 24 + \Delta 2p = 0 \\ X_1\delta 31 + X_2\delta 32 + X_3\delta 33 + X_4\delta 34 + \Delta 3p = 0 \\ X_1\delta 41 + X_2\delta 42 + X_3\delta 43 + X_4\delta 44 + \Delta 4p = 0 \end{aligned}\right\} \text{Symmetrical}$$

$$\left.\begin{aligned} X_5\delta 55 + X_6\delta 56 + \Delta 5p = 0 \\ X_6\delta 66 + X_6\delta 66 + \Delta 6p = 0 \end{aligned}\right\} \text{antisymmetrical.}$$

In the last two systems the displacements $\Delta 5p$ and $\Delta 6p$ are both nil, their value being obtained multiplying antisymmetrical graphs $\bar{M}_5$ and $\bar{M}_6$ by the symmetrical graph M_p due to the applied loads (figure 3.18i). Consequently, those two equations become

$$X_5\delta 55 + X_6\delta 56 = 0$$
$$X_6\delta 65 + X_6\delta 66 = 0$$

This indicates that both antisymmetrical unknowns X_5 and X_6 are also nil, were the frame of figure 3.18(a) acted upon by a system of antisymmetrical loads it would be the symmetrical unknowns that

would become nil.

From the above the following rules can be formulated:

1. When a symmetrical structure is acted upon by a symmetrical system of loads only those of the unknowns which represent the symmetrical redundant reactions remain different from zero, while all the unknowns representing antisymmetrical reactions become nil.
2. When a symmetrical structure is acted by antisymmetrical system of loads, only those of the unknowns which represent the antisymmetrical redundant reactions remain different from zero, while all the unknowns representing symmetrical unknowns become nil.

3.9 LOAD TRANSFORMATION METHOD

The two rules formulated above are applicable to any symmetrical structure regardless of the actual distribution of loading, for the reason that any system of loads can be easily replaced by an equivalent combination of two separate systems, one of which is symmetrical and the other antisymmetrical.

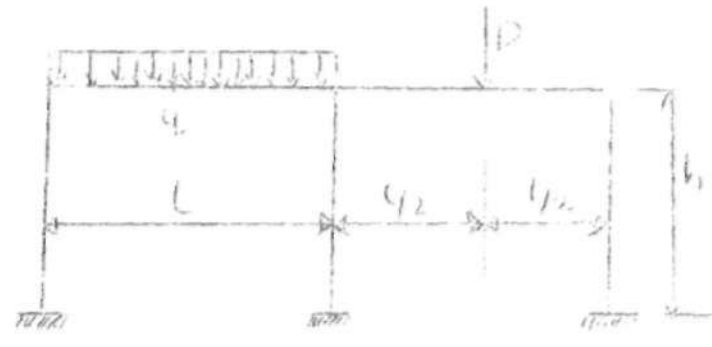


Figure 3.19

The symmetrical structure of figure 3.19 is acted by a uniformly distributed load of qkn per unit lenght and by one concentrated load P, both are nonsymmetrical. These two loads may be replaced by two groups of components appearing in figure 3.20a and b. The first (a) being symmetrical and the second (b) being antisymmetrical.

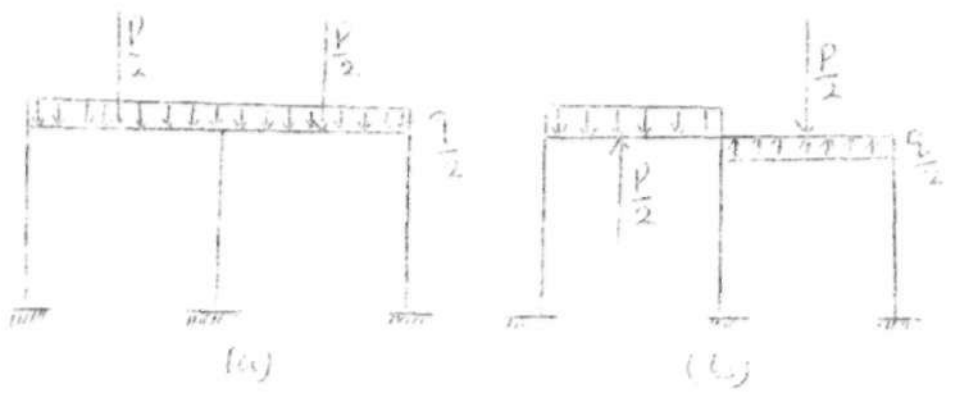


Figure 3.20

When a symmetrical structure is acted upon by symmetrical loads alone the symmetrical unknowns remain different from zero. Consequently, for the simple statically determinate structure of figure 3.19(a) acted upon by the loads appearing in figure 3.20 only the symmetrical unknowns X_1, X_2 and X_3 must be calculated. The corresponding equations become:

$$X_1\delta 11 + X_2\delta 12 + X_3\delta 13 + \Delta 1p = 0$$
$$X_1\delta 21 + X_2\delta 22 + X_3\delta 23 + \Delta 2p = 0$$
$$X_1\delta 31 + X_2\delta 32 + X_3\delta 33 + \Delta 3p = 0$$

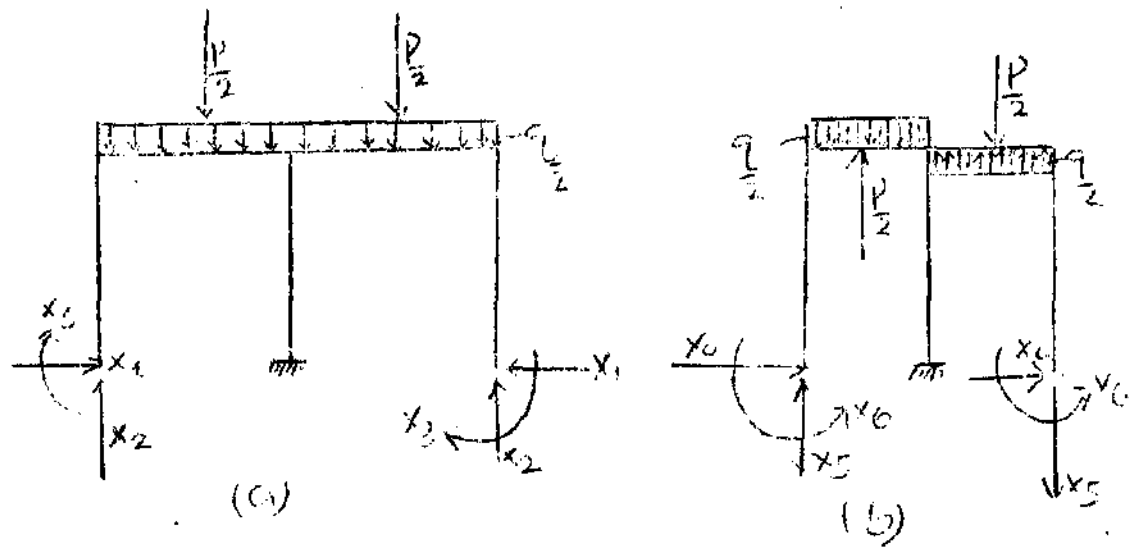


Figure 3.21

For the same reason the same reason the system of loads in figure 3.20b applied to the simple structure of figure 3.21b will provide three antisymmetrical unknowns X_4, X_5 and X_6 consequently, these equations become:

$$X_4\delta 44 + X_5\delta 45 + X_6\delta 46 + \Delta 4p = 0$$
$$X_4\delta 54 + X_5\delta 55 + X_6\delta 56 + \Delta 5p = 0$$
$$X_4\delta 64 + X_5\delta 65 + X_6\delta 66 + \Delta 6p = 0$$

In certain cases the replacement of loads by their symmetrical and antisymmetrical components may complicate the computations instead of simplifying them and as a result the application of this procedure cannot be recommended unconditionally.

3.10 ACCURACY CONTROL OF ALL THE TERMS ENTERING THE SIMULTANEOUS EQUATIONS

The coefficients to the unknown and the free terms of the simultaneous equations represent displacements induced in the simple statically determinate structure by unit forces acting along the direction of the unknown reactions as well as those due to the actual loads. These displacements are usually obtained through

the multiplication of the corresponding bending moment diagrams. Errors which occur while carrying out these operations influence further calculations and therefore render erroneous the values of the redundant reactions finally obtained.

Errors in the displacement values can usually be detected using the procedure described hereunder.

Suppose that for the analysis of a structure redundant to the nth degree reactions or groups of reactions X_1, X_2... X_i X_n, have been adopted as unknowns. Apply to the simple statically determinate structure all the unit forces corresponding to these reactions and construct the diagram of the resulting bending moment. This diagram is given the name summary unit bending moment diagram designating its ordinates by $\bar{M}_s$. In any cross-section the ordinate $\bar{M}_s$ will thus equal the algebraic sum of all the ordinates $\bar{M}_1$, $\bar{M}_2$... $\bar{M}_i$..., $\bar{M}_n$.

Multiply successively the M_s graph area by the unit bonding moment diagrams $\bar{M}_1$, $\bar{M}_2$ etc. each of the products this obtained will be equal to the algebraic sum of all the coefficients to the unknowns of the corresponding equations because

$$\sum_{i=1}^{i=n} \delta_{1i} = \delta_{11} + \delta_{12} + \delta_{13} + \dots + \delta_{in} = \sum \bar{M}_1 \bar{M}_1 \frac{ds}{EJ} +$$

$$\sum \bar{M}_1 \bar{M}_2 \frac{ds}{EJ} + \dots + \sum \bar{M}_1 \bar{M}_n \frac{ds}{EJ} = \sum \bar{M}_1 (\bar{M}_1 + \bar{M}_2 + \dots + \bar{M}_n) \frac{ds}{EJ} =$$

$$\sum \bar{M}_1 \bar{M}_2 \frac{ds}{EJ} = \delta_{is}$$

For the same reason we may write

$$\delta_{21} + \delta_{22} + \delta_{23} + \dots + \delta_{2n} = \delta_{2s}$$

It follows that the algebraic sum of all the coefficients to the unknowns contained in equation i must be equal to δ_{is} where

$$\delta_{is} = \int \bar{M}_i M_s \frac{ds}{EJ}$$

Hence the values of all the unit displacements forming the coefficients to the unknowns of the first of the simultaneous equations can be checked comparing their sum with the value of δ_{is}

$$\sum_{i=i}^{i=n} \delta_{1i} = \delta_{1s}$$

The algebraic sums of the coefficients to the unknowns of all the other simultaneous equations may be checked in exactly the same way.

The above procedure permits the verification of the coefficients to the unknowns of each equation separately.

$\sum$ denotes the sum of all the values $\delta_{1s}, \delta_{2s} \ldots, \delta_{ns}$

i.e. $\sum = \delta_{1s} + \delta_{2s} + \delta_{3s} + \ldots + \delta_{ns}$

But since $\delta_{1s} = \int \bar{M}_1 \bar{M}_s \frac{ds}{EJ}$

$$\delta_{2s} = \int \bar{M}_2 \bar{M}_s \frac{ds}{EJ}$$

$$\delta_{ns} = \int \bar{M}_n \bar{M}_s \frac{ds}{EJ}$$

Its found out that

$$\sum \delta_{ss} = \int \bar{M}_s \bar{M}_1 \frac{ds}{EJ} + \int \bar{M}_s M_2 \frac{ds}{EJ} + \ldots + \int \bar{M}_s \bar{M}_n \frac{ds}{EJ} =$$

$$\int \bar{M}_s (\bar{M}_1 + \bar{M}_2 + \ldots + \bar{M}_n) \frac{ds}{EJ} = \int \bar{M}_s^2 \frac{2ds}{EJ} = \delta_{ss}$$

Consequently $\delta_{ss} = \int \bar{M}_s^2 \frac{s}{EJ}$

and $\sum \delta = \delta_{ss}$

The last expression permits a simultaneous check on the accuracy of all the coefficients to the unknowns contained in all the equations of the given system. The check is carried out as follows:

(a) Find the algebraic sum of all the coefficients to the unknowns (unit displacements) contained in all the equations of the given system.

$$\sum \delta = (\delta_{11} + \delta_{22} + \delta_{33} + \dots + \delta_{nn}) + 2(\delta_{12} + \delta_{13} + \dots + \delta_{in} + \delta_{23} + \delta_{24} + \dots + \delta_{2n} + \delta_{34} + \dots + \delta_{n-1,n}).$$

(b) Using the method of diagram multiplication find the values of $\delta_{ss} = \int \frac{\bar{M}_s^2 ds}{EJ}$

(c) Compare the two values obtained as described above.

In a number of cases the following inequality may help in finding $\frac{\text{erroneous}}{}$ coefficients to the unknowns.

$$\delta_{ii} \times \delta_{kk} < \delta_{ik}^2$$

The control of the free terms of the canonical equations will be carried out in a similar way.

(a) First compute the value of $\Delta_{sp} = \int \frac{\bar{M}_s M_p ds}{EJ}$ where M_p represents the bending moments induced in the conjugate simple structure by the actual loads.

(b) Check whether $\sum \Delta = \Delta_{1p} + \Delta_{2p} + \dots + \Delta_{np} = \Delta_{sp}$.

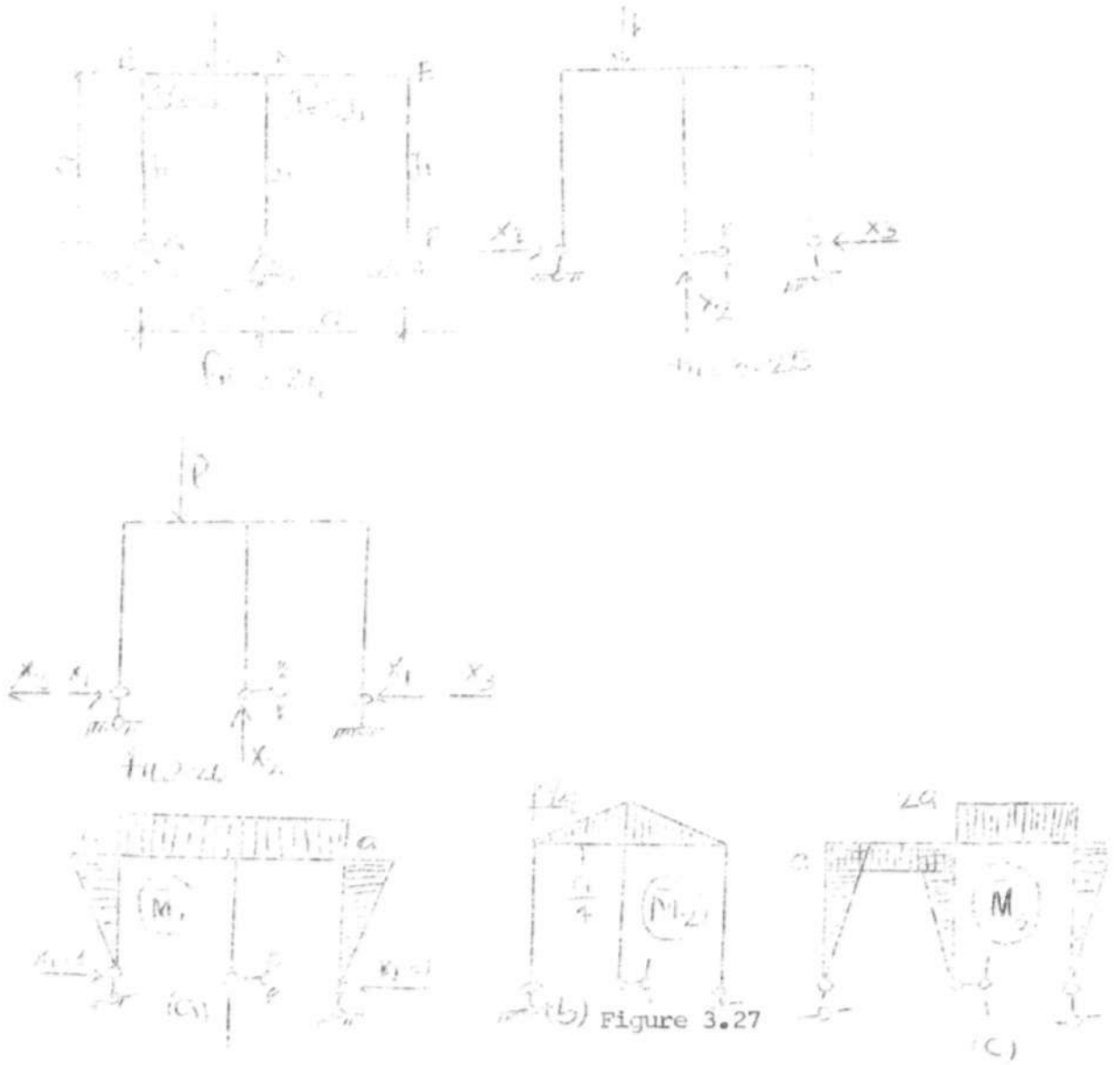
In general it is quite sufficient to verify simultaneously the coefficients to the unknowns of the whole system of equations. If this check reveals an error, it is recommended to verify these

coefficients equation by equation as described at the beginning of the present article.

3.11 EXAMPLES

The examples below will be used to illustrate the various methods meant for the simplification of the calculations of the method of forces as described in this chapter.

Example 1: Construct and check the M, Q and N diagrams for the double span symmetrical frame shown in figure 3.24.



(b) Figure 3.27

(c)

Solution

The frame under consideration is three times statically indeterminate figure 3.25 shows one of the simple structures which could be adopted for the solution of the problem. This will involve the simultaneous solution of three equations with three unknowns. The problem will be considerably simplified if the unknowns are grouped as indicated as in figure 3.26.

In this case the canonical equations become

$$X_1 \delta 11 + X_2 \delta 12 + \Delta 1p = 0$$

$$X_1 \delta 12 + X_2 \delta 22 + \Delta 2p = 0$$

$$X_3 \delta 33 + \Delta 3p = 0$$

The unit bending moment diagrams $\bar{M}_1$, $\bar{M}_2$ and $\bar{M}_3$ corresponding to the case under consideration are given in figure 3.27a, b and c, while that due to the applied load is presented in figure 3.28



Figure 3.28

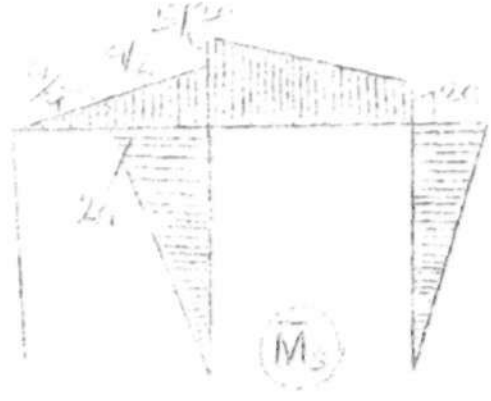


Figure 3.29

Multiplying the appropriate graphs one by the other we shall obtain the values both of the coefficients to the unknowns and of the free terms of the above equations.

$$\delta 11 = 2(a\cdot\frac{a}{2}\cdot\frac{2}{3}a\,\frac{1}{EJ_1} + a\cdot a\cdot a\frac{1}{2EJ_1}) = \frac{5a^3}{3EJ1}$$

$$\delta 12 = \frac{a}{2}\cdot\frac{2a}{2}\cdot\frac{a}{2EJ_1} = \frac{9^3}{4EJ_1}$$

$$\delta 22 = 2\cdot\frac{a}{2}\cdot\frac{a}{2}\cdot\frac{2}{3}\cdot\frac{a}{2}\cdot\frac{1}{EJ_1} = \frac{a^3}{12EJ_1}$$

$$\delta 33 = 2(a\frac{a}{2}\cdot\frac{2}{3}a\,\frac{1}{EJ_1} + a\cdot a\cdot a\,\frac{1}{2EJ1}) + 2a\frac{a}{2}\cdot\frac{2}{3}\cdot\frac{2a}{EJ_1} = \frac{3a^3}{EJ_1}$$

$$\Delta 1p = \frac{-3Pa}{8}\cdot\frac{2a}{2}\cdot\frac{a}{2EJ_1} = \frac{-3P_a^3}{16EJ_1}$$

$$\Delta 2p = \frac{-1}{2EJ_1}(\frac{3Pa}{8}\cdot\frac{a}{2}\cdot\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{a}{4} + \frac{a}{26}(2\cdot\frac{3}{8}P\,a\frac{a}{4} + 2\frac{1}{4}\cdot P\,a\frac{a}{2} + \frac{3P}{8}a\,\frac{a}{2} + \frac{1P}{4}a\frac{a}{4}) + \frac{1P}{4}a\frac{a}{2}\cdot\frac{2}{3}\cdot\frac{a}{2}) = \frac{-11P_a^3}{192EJ_1}$$

$$\Delta 3p = (\frac{3P}{8}a\,\frac{a}{2}\cdot\frac{1}{2} + \frac{\frac{3p}{8}a + \frac{1P}{4}a}{2}\cdot\frac{a}{2} - \frac{1P}{4}a\,\frac{a}{2})\,\frac{a}{2EJ_1} = \frac{P_a^3}{16EJ_1}$$

The displacements thus computed are checked using the "summary" unit bending moment diagram $\bar{M}_s$ due to the simultaneous application of all unit forces along the unknowns (figure 3.29). The value of δss is obtained raising to the second power the $\bar{M}_s$ diagram.

$$\delta ss = \int \bar{M}\frac{2ds}{sEJ} = (\frac{a}{2}\cdot\frac{a}{2}\cdot\frac{2}{3}\cdot\frac{a}{2} + \frac{a}{6}(2\cdot\frac{5}{2}a\cdot\frac{5a}{2}a + 2\cdot 2a\cdot 2a + 2\cdot\frac{5}{2}a\cdot 2a)\,\frac{1}{2EJ_1} + 2\cdot 2a\cdot\frac{a}{2}\cdot\frac{2}{3}\cdot\frac{2a}{EJ_1} + \frac{21a^3}{4EJ_1}$$

checking that the condition $= \delta ss$

$$= \frac{21a^3}{4EJ1} = \frac{5a^3}{3EJ_1} + \frac{a^3}{12EJ_1} + \frac{3a^3}{EJ1} + 2\cdot\frac{a^3}{4EJ1} \text{ or } \frac{21a^3}{4EJ1} = \frac{21a^3}{EJ1}$$

It follows that all the unit displacements are correct.

Determine also the value of Δsp by multiplying the $\bar{M}s$ diagram (figure 3.29) by the M_p one (figure 3.28)

$$\Delta sp = \sum \int \frac{\bar{M}_s M_p ds}{EJ} = \frac{-1}{2EJ_1}\left(\frac{3P}{8}a\cdot\frac{a}{2}\cdot\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{a}{4} + \frac{a}{2}\cdot 6\left(2\cdot\frac{3P}{8}a\cdot\frac{a}{4} + 2\cdot\frac{1P}{4}a\cdot\frac{a}{2} + \frac{3P}{8}a\cdot\frac{a}{2} + \frac{1P}{4}a\cdot\frac{a}{4}\right) + \frac{1P}{4}a\cdot\frac{a}{2}\left(\frac{2}{3}\cdot\frac{5}{2}a + \frac{1}{3}\cdot 2a\right)\right) = \frac{-35P_a^3}{192EJ_1}$$

check also that the condition $\Delta sp = \sum \Delta$ is also satisfied.

$$\frac{-35P_a^3}{192EJ_1} = \frac{-3P_a^3}{16EJ_1} - \frac{11P_a^3}{192EJ_1} + \frac{P_a^3}{16EJ_1} \text{ or } \frac{-35P_a^3}{192EJ_1} = \frac{-35P_a^3}{192EJ_1}$$

This also implies that the displacements due to the applied loads are also correct. Introduce the values of the coefficients to the unknowns and free terms into the system of canonical equations deduced for this problem.

Multiplying the first of these equations by $\frac{192EJ_1}{a^3}$ and the last one by $\frac{16EJ_1}{a^3}$ we obtain

$$320X_1 + 48X_2 - 36P = 0$$

$$48X_1 + 16X_2 - 11P = 0$$

$$48X_3 + P = 0$$

The solution of these equations leads to

$$X_1 = \frac{3P}{176};\ X_2 = \frac{7P}{11};\ X_3 = \frac{-P}{48}$$

The construction of the resulting bending moment diagram for the given redundant structure will be carried out in the following sequence. First, determine the bending moments induced in the simple structure by the redundant reactions X_1, X_2 and X_3 obtained

above. For this purpose multiply all the ordinates to the unit bending moment graphs $\bar{M}_1$, $\bar{M}_2$ and $\bar{M}_3$ by the respective magnitudes of these reactions. The three diagrams thus obtained are represented in figure 3.30, 3.31 and 3.32.

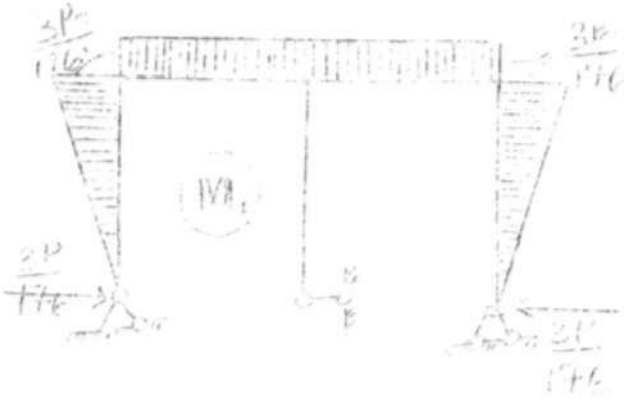


Fig 3.30

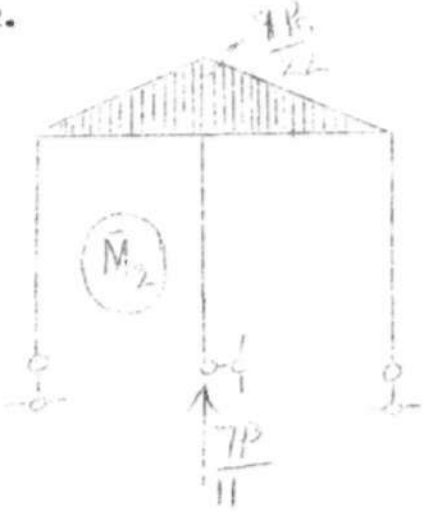


Fig 3.31

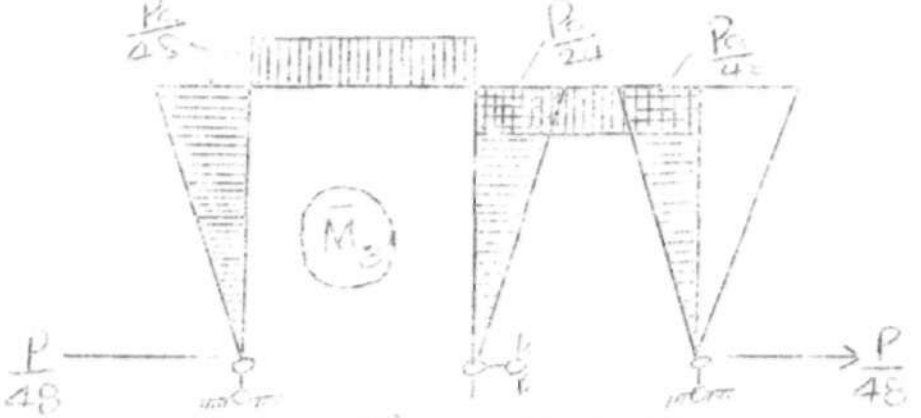


Fig 3.32

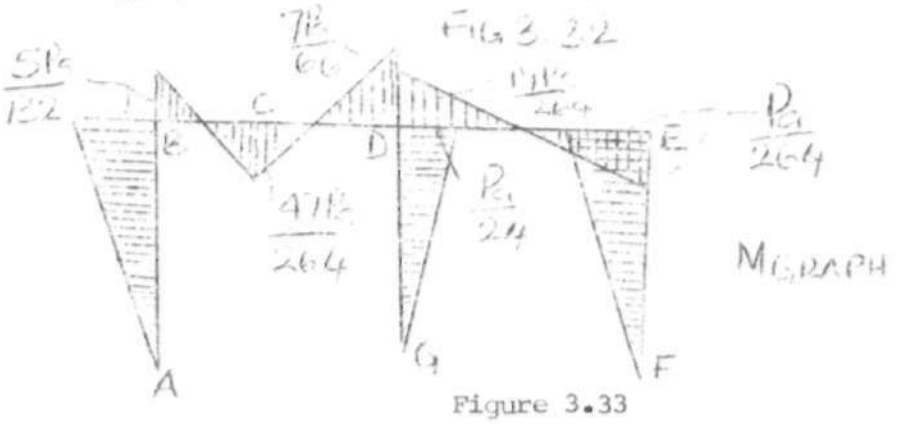


Figure 3.33

The ordinates to the resulting bending moment diagram can now be obtained summing up the ordinates to these diagrams with those to the M_p diagram given previously in figure 3.28.

$$MCB = MCD = \frac{-3P_a}{176} \frac{-7P_a}{44} \frac{-P_a}{48} + \frac{3P_a}{8}$$

$$MBA = MBC = \frac{-3P_a}{176} + 0 \frac{-P_a}{48} + 0 = \frac{-5P_a}{132}$$

$$MDC = \frac{-3P_a}{176} \frac{-7P_a}{22} \frac{-P_a}{48} + \frac{P_a}{4} = \frac{-7P_a}{66}$$

$$MDE = \frac{-3P_a}{176} \frac{-7P_a}{22} + \frac{P_a}{48} + \frac{P_a}{4} = \frac{-17P_a}{264}$$

$$MDG = 0 + 0 + \frac{P_a}{24} + 0 = \frac{P_a}{24}$$

$$MED = MEF = \frac{-3P_a}{176} + 0 + \frac{P_a}{48} + 0 = \frac{P_a}{264}$$

In all the above expressions MBA denotes the bending moment at cross-section B of member BA, MBC denotes the bending moment at cross-section B of member BC and so forth. The diagram of the resulting bending moments are plotted as in figure 3.33.

After the bending moment determination, that of the shearing forces are proceeded as shown below.

$$QAB = QBA = \frac{MBA-MAB}{a} = \frac{\frac{-5P_a}{132} - 0}{a} = \frac{-5P}{132}$$

$$QBC = QCB = (\frac{47Pa}{264} + \frac{5P_a}{132})\frac{1}{a/2} = \frac{57P}{132}$$

$$QCD = QDC = (\frac{7Pa}{66} + \frac{47P_a}{264})\frac{1}{a/2} = \frac{-75P}{132}$$

$$QDE = QED = (\frac{Pa}{264} + \frac{17P_a}{264})\frac{1}{a} = \frac{3P}{44}$$

$$QEF = QFE = \frac{-Pa}{264}\cdot\frac{1}{a} = \frac{-P}{264}$$

$$QDG = QGD = \frac{Pa}{24}\cdot\frac{1}{a} = \frac{P}{24}$$

These values are plotted as in figure 3.34.

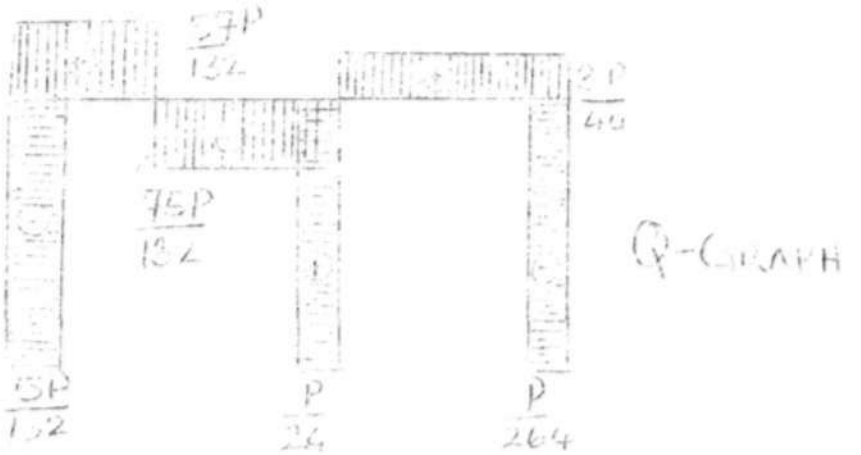


Figure 3.34

The values of the normal forces will be derived from the equilibrium of joints B, D and E isolated in succession (figure 3.35a, b and c). In those computations unknown normal stresses will be reckoned positive i.e. causing the extension of the corresponding member.

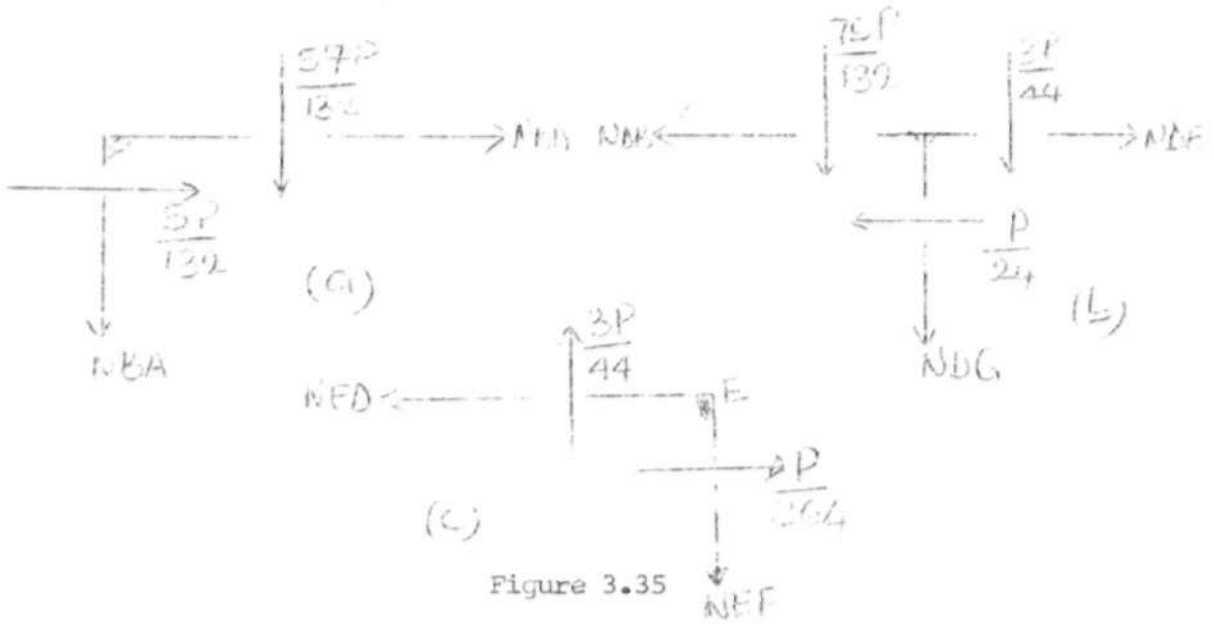


Figure 3.35

Joint B (figure 3.35a).

$$= -NBA - \frac{57P}{132} = 0$$

where from NBA = $\frac{-57P}{132}$ (compression).

$= \frac{5P}{132} + NBD = 0$ therefore NBD = $\frac{-5P}{132}$ (compression)

Joint D (figure 3.35b)

$$Y = -NDG - \frac{75P}{132} - \frac{3P}{44} = 0$$

where from NDG = $\frac{-7P}{11}$ (compression)

$= -NDB + NDE - \frac{P}{24} = 0$ but since NDB = NBD we finally have

NDE = $\frac{P}{24} \frac{-5P}{132} = \frac{P}{264}$ (extension).

Joint E (figure 3.35c).

$Y = \frac{3P}{44} - NEF = 0$ wherefrom NEF = $\frac{3P}{44}$

(extension).

$X = -NED + \frac{P}{264} = 0$ therefore NED = $\frac{P}{264}$

(extension).

The diagram of normal stresses is given in figure 3.36.

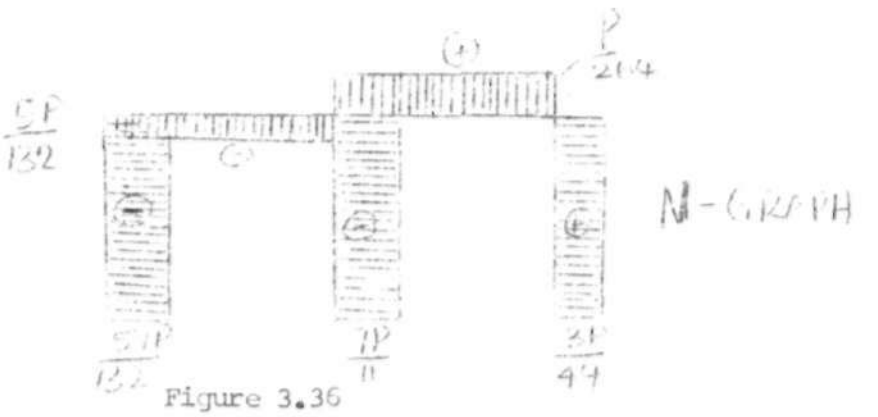


Figure 3.36

Example 2. Required the complete stress analysis for the two storey frame for a factory building loaded unsymmetrically

along the top crossbeam. The different rigiditio of all the frame members are indicated in figure 3.37.



Figure 3.37(a)

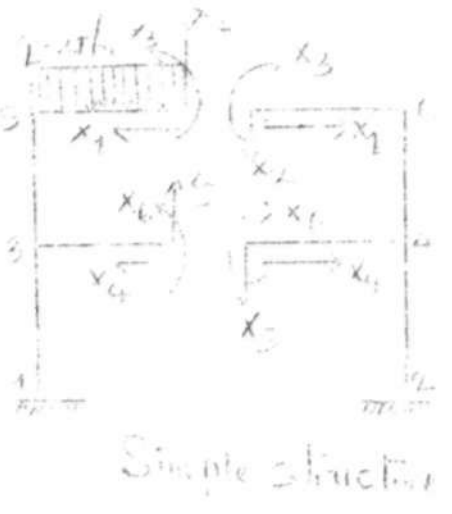
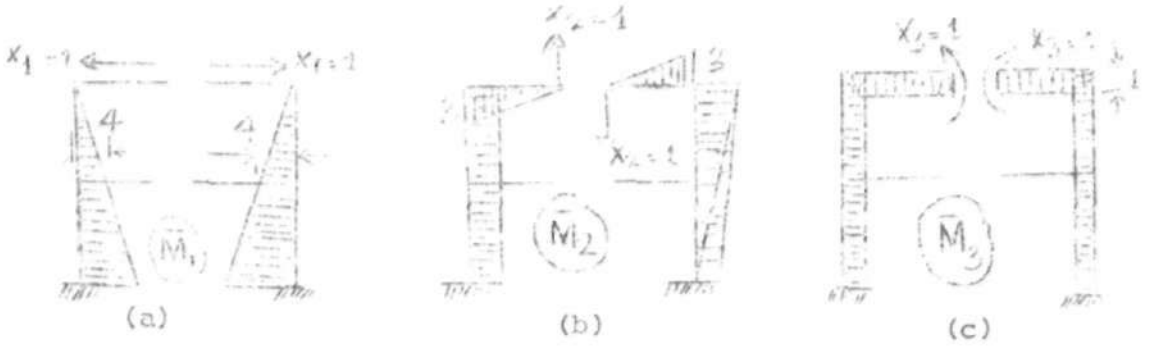


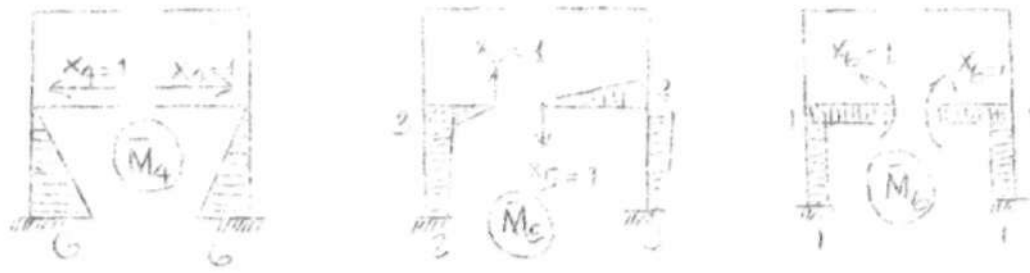
Figure 3.37(b)

Solution:

The conjugate simple structure will be obtained sectioning both cross bars at mid span figure 3.37(b). The bending moment diagrams due to unknown unit forces are given in figure 3.38. The diagram of the bending moments induced in the same simple structure by the actual loading is given in figure 3.39. Owing to the proper choice of the simple structure all the following secondary displacements reduce to zero, δ_{12}, δ_{15}, δ_{23}, δ_{24}, δ_{26}, δ_{35}, δ_{45} and δ_{56}.



(a) (b) (c)



(d) (e) (f)

Figure 3.38

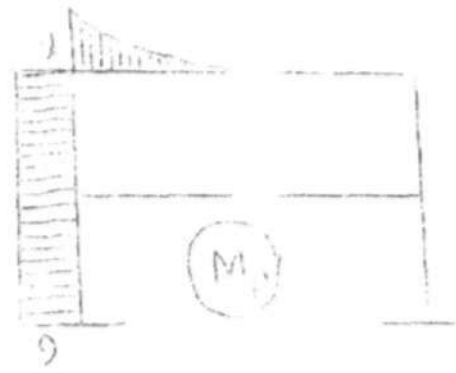


Figure 3.39

In this case the system of simultaneous equations will fall into two separate groups, the first containing four unknowns out of the six and the second two unknowns only.

(a) The first group:

$$X_1\delta 11 + X_3\delta 13 + X_4\delta 14 + X_6\delta 16 = -\Delta 1p$$
$$X_1\delta 31 + X_3\delta 33 + X_4\delta 34 + X_6\delta 36 = -\Delta 3p$$
$$X_1\delta 41 + X_3\delta 43 + X_4\delta 44 + X_6\delta 46 = -\Delta 4p$$
$$X_1\delta 61 + X_3\delta 63 + X_4\delta 64 + X_6\delta 66 = -\Delta 6p$$

(b) the second group

$$X_2\delta 22 + X_5\delta 25 = -\Delta 2p$$
$$X_2\delta 52 + X_5\delta 55 = -\Delta 5p$$

All the displacements will be calculated assuming that EJ = 1 ton sq m.

$\delta 11 = \frac{4x4}{2} \times \frac{2}{3} \times 4x\frac{2}{1} + \frac{6}{6}(2x4^2+2x10^2+2x4x10)\frac{2}{2} = 354.70$

$\delta 13 = \frac{4x4}{2} \times 1 \times \frac{2}{1} + 6 \times \frac{4+10}{2} \times 1 \times \frac{2}{2} = 58$

$\delta 14 = \frac{6x6}{2}(4+\frac{2}{3}x6)\frac{2}{2} = 144$

$\delta 16 = 6 \times 1 \times 1 \times \frac{1}{2} + 4 \times 1 \times 1 \times + 6 \times 1 \times 1 \times 1 = 17$

$\delta 34 = \frac{6x6}{2} \times \frac{1}{2} \times 2 = 18$

$\delta 36 = 1 \times 6 \times \frac{1}{2} \times 2 = 6$

$\delta 44 = \frac{6x6}{2} \times \frac{2}{3} \times \frac{6x2}{2} = 72$

$\delta 46 = \frac{6x6}{2} \times \frac{1}{2} \times 2 = 18$

$\delta 66 = 6 \times 1 \times 1 \times 1 \times \frac{1}{4} + 6 \times 1 \times 1 \times \frac{1}{2} \times 2 = 75$

$\delta 22 = \frac{3x3}{2} \times \frac{2}{3} \times 3 \times \frac{1}{2} \times 2 + 4 \times 3 \times 3 \times 2 + 6 \times 3 \times 3 \times 1 = 135$

$\delta 25 = 3 \times 6 \times 3 \times \frac{1}{2} \times 2 = 54$

$\delta 55 = \frac{3x3}{2} \times \frac{2}{3} \times 3 \times \frac{1}{4} \times 2 + 6 \times 3 \times 3 \times \frac{1}{2} \times 2 = 58.50$

$\Delta 1p = \frac{4x4x9}{2x1} - \frac{6(10+4)x9}{2x2} = 261$

$\Delta 2p = \frac{-9x3}{3x2} \times \frac{3}{4} \times 3 - 4x9x3 - \frac{6x9x3}{2} = 199.12$

$\Delta 3p = \frac{-3x9x1}{3x2} - \frac{4x9x1}{1} - \frac{6x9x3}{2} = -67.50$

$\Delta 4p = \frac{-6x6x9}{2x2} = -81$

$\Delta 5p = \frac{-6x9x3}{2} = -81$

$\Delta 6p = \frac{-6x9x1}{2} = -27$

The values of these displacements are checked to make sure to the expressions

$$\Delta ss = \sum \bar{M}_s \frac{2ds}{EJ} \text{ and } \Delta sp = \sum \bar{M}_s \bar{M}_p \frac{ds}{EJ}$$

i.e. $\Delta ss = \Sigma$ and $\Delta sp = \Sigma A$. Both of these conditions being satisfied, we may be sure that no error has been committed in calculating the unit displacements. Substituting the values of these displacements in the two groups of simultaneous equations, we obtain

$$354.7X_1 + 58X_3 + 144X_4 + 42X_6 = +261$$

$$58X_1 + 17X_3 + 18X_4 + 6X_6 = 67.5$$

$$144X_1 + 18X_3 + 72X_4 + 18X_6 = 81$$

$$42X_1 + 6X_3 + 18X_4 + 75X_6 = 27$$

$$135X_2 + 54X_5 = 199.12$$

$$54X_2 + 58.5X_5 = 81$$

The solution of these equations leads to the following redundant reactions

$X_1 = 0.607$ tons $\qquad X_4 = -0.692$ ton

$X_2 = 1.46$ tons $\qquad X_5 = +0.037$ ton

$X_3 = 2.753$ ton-metres $\qquad X_6 = -0.340$ ton-metre.

Applying all the forces to the simple statically determinate structure and calculating the bending moments induced both by the reactions just obtained and the actual loads we find.

M56 = +1.46x3 + 2.753-9 = 1.87 ton-metres

M65 = -1.46x3 + 2.753 = -1.63 ton-metres

M34 = 10.037x3 - 0.340 = -0.23 ton-metre.

M43 = −0.037x3 − 0.340 = −0.45 ton-metre

M35 = +0.607x4 + 1.460x3 + 2.753−9 = +0.56 ton-metre

M46 = +0.607x4 − 1.460x3 + 2.753 = +0.80 ton-metre

M31 = +0.607x4 + 1.460x3 + 2.753 + 0.037x3 − 0.340−9 = +0.33 ton-metre

M42 = +0.607x4 − 1.460x3 + 2.753 − 0.037x3 − 0.340 = +0.36 ton-metre

M13 = +0.607x10 + 1.460x3 + 2.753 − 0.692x6 + 0.037x3 − 0.340−9 =

0.18 ton-metre

M24 = +0.607x10 − 1.460x3 + 2.753 − 0.692x6 − 0.037x3 − 0.340 =

−0.16 ton-metre

The final bending moments are entered into figure 3.40.

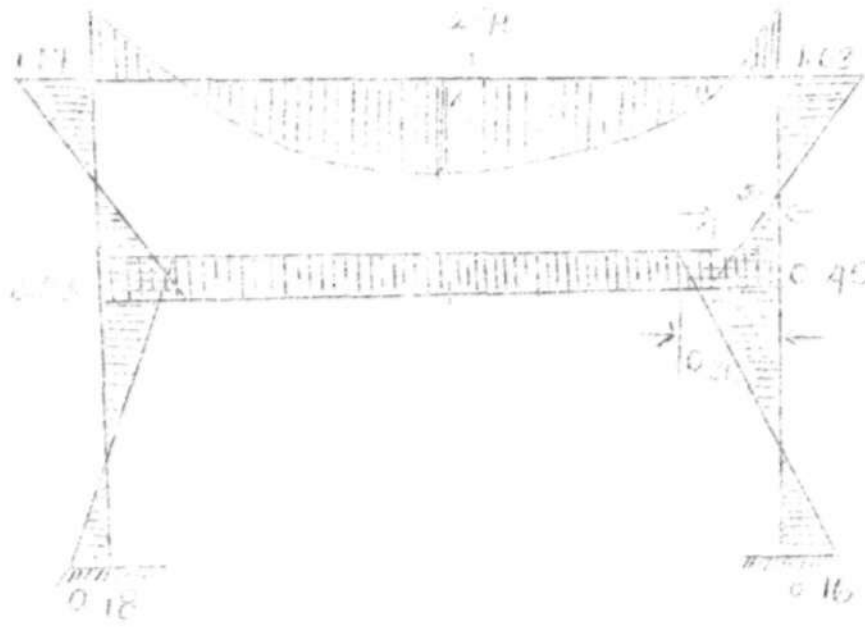
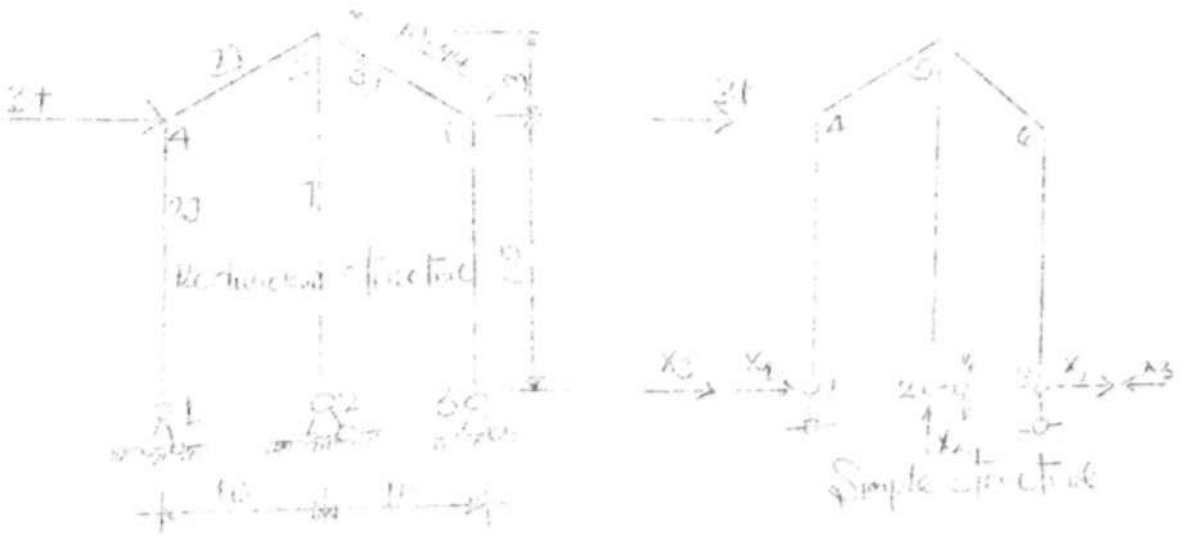


Figure 3.40

Example 3

Required the complete stress analysis of a double-span symmetrical frame loaded with one horizontal force figure 3.41.



(a) (b)

Figure 3.41

Solution:

The frame under consideration is redundant to the third degree. The statically determinate structure adopted is represented in figure 3.41(b). The bending moment diagrams induced by unit reactions will be constructed for the following groups of unknowns:

X_1 consisting of two horizontal antisymmetrical forces.

X_2 consisting of two vertical reaction at the central support.

X_3 consisting of two two symmetrical horizontal forces.

The corresponding unit bending moment diagrams are shown in figure 3.42. The force acting on the frame is resolved into two groups of components, one symmetrical and the other antisymmetrical as indicated in figure 3.43(a) and (b). The bending moment diagrams due to these components are also given

in the same figure.

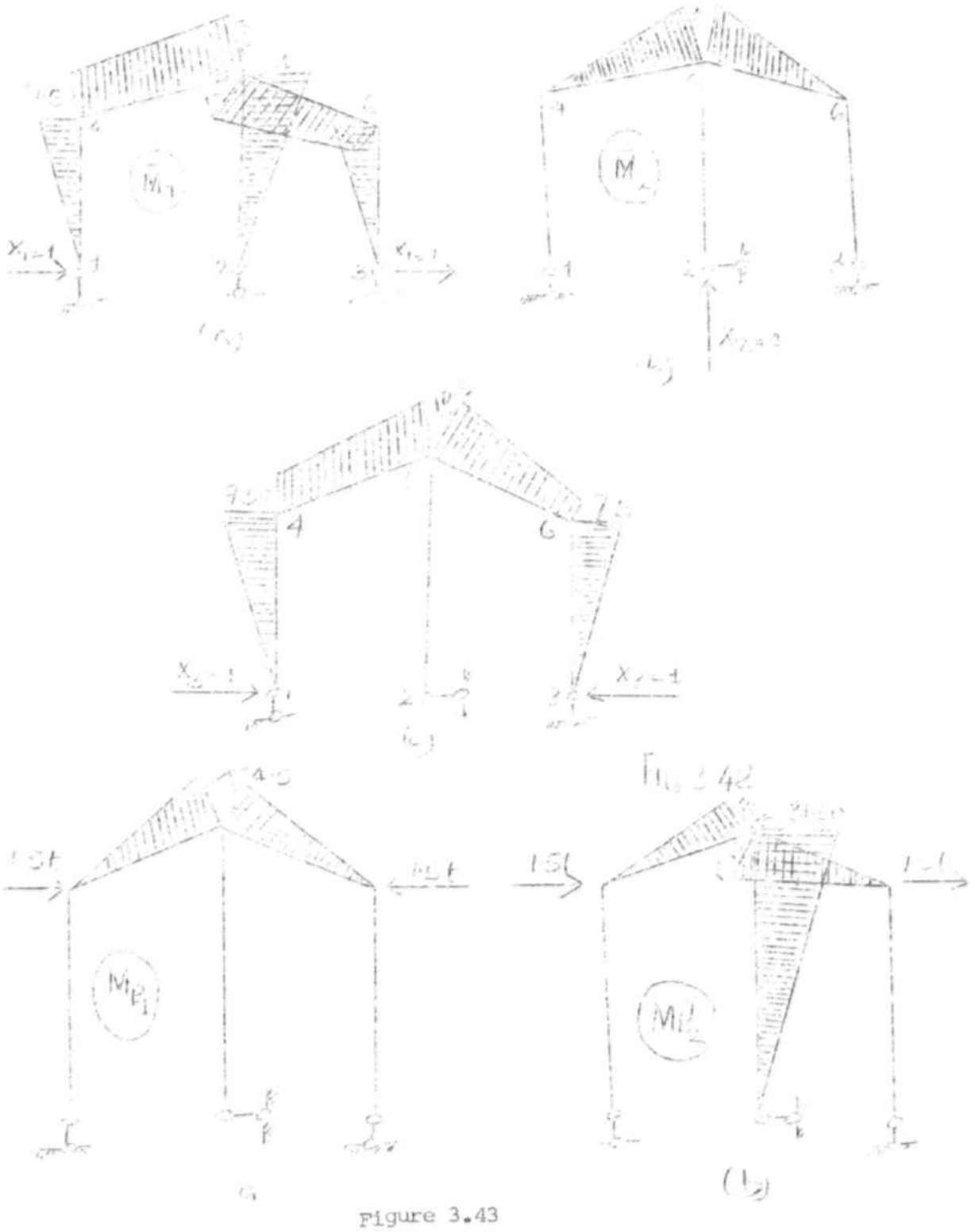


Figure 3.43

The displacements induced by the actual loads will be obtained multiplying the bending moment graph due to antisymmetrical unit forces X_1 figure 3.42(a) by the graph of figure 3.43(b) due to the antisymmetrical component of the actual load. The same operation is repeated with the graphs due to the symmetrical unknowns $X_2 = 1$ and $X_3 = 1$ on one hand, and the graph due to the symmetrical components of force P given in figure 3.43(a) on the other. The secondary displacements 12 and 13 will reduce to zero. Consequently, the three simultaneous equations become

$$X_1 \delta 11 + \Delta 1p = 0$$

$$X_2 \delta 22 + X_3 \delta 23 + \Delta 2p = 0$$

$$X_2 \delta 32 + X_3 \delta 33 + \Delta 3p = 0$$

The displacements are calculated assuming that EJ = 1 ton sqm

$$\delta 11 = 2 \times \frac{10.44 \times 7.5x9}{3} + 2 \times \frac{10.44x3x9.5}{2x3} + \frac{2x7.5x7.5x5}{2x2} +$$

$$\frac{10.5x21x14}{2x1} = +2253.1$$

$$\delta 22 = 2 \times \frac{10.44x5x\frac{2x5}{3}}{2x3} = +58$$

$$\delta 33 = 2 \times \frac{10.44x7.5x9}{3} + 2 \times \frac{10.44x3x9.5}{2x3} + 2 \times \frac{7.5x7.5x5}{2x2} = +709.60$$

$$\delta 23 = 2 \times \frac{10.44x5x9.5}{2x3} = +165.3$$

$$\Delta 1p = 2 \times \frac{10.44x15.75x9.5}{2x3} + \frac{10.50x31.50x14}{2x1} = +2835.90$$

$$\Delta 2p = 2 \times \frac{10.44x5x3}{2x3} = +52.20$$

$$\Delta 3p = 2 \times \frac{10.44x4.50x9.50}{2x3} = +148.77$$

Substituting these values into the above system of equations we obtain

$$2253.1X_1 = -2835.90$$

$$58X_2 + 165.3X_3 = -52.2$$

$$165.3X_2 + 709.6X_3 = -148.77$$

The solution of these equations yields $X_1 = 1.259$; $X_2 = -0.900 = 0.9$; $X_3 = 0$.

The bending moments at different cross-sections of the structure will be

M45 = +1.259 x 7.50 = +9.44 ton-metres

M54 = +1.259 x 10.5 + 0.9x5 - 4.5-15.75 = -2.53 ton-metres

M56 = -1.259 x 10.5 + 0.9x5 - 4.50 + 15.75 = +2.53 ton-metres

M65 = 1.259 x 7.5 = -9.44 ton-metres

M52 = -1.259 x 21 + 31.50 = 5.06 ton-metres

The corresponding bending moment graph is represented in figure 3.44.

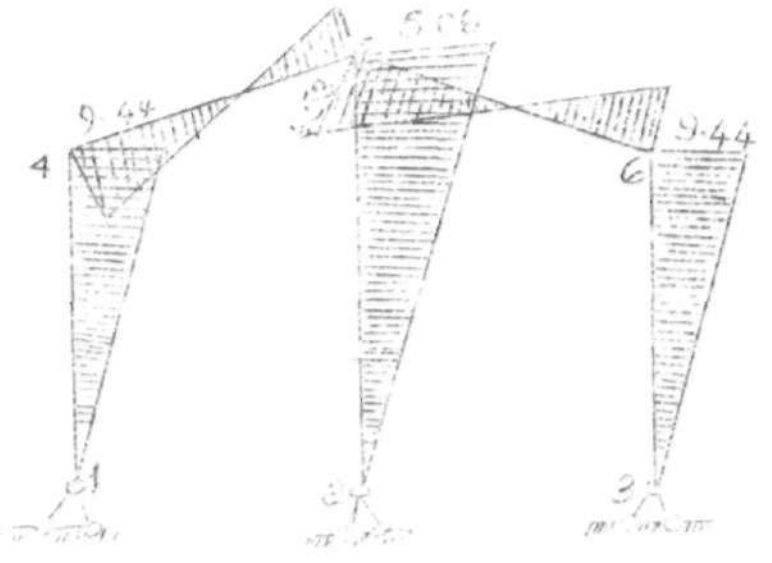


Figure 3.44

3.12 EFFECTS OF TEMPERATURE CHANGES, SHRINKAGE AND SETTLEMENT OF SUPPORTS ON FIXED-END REINFORCED CONCRETE ARCHES

Temperature changes, shrinkage and settlement of supports leads to the appearance of stresses in fixed arches.

Temperature Changes

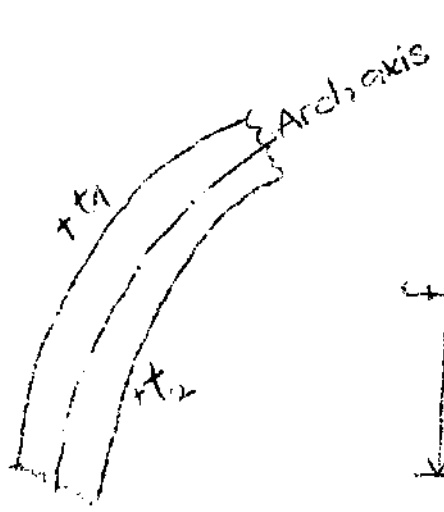


Figure 3.22

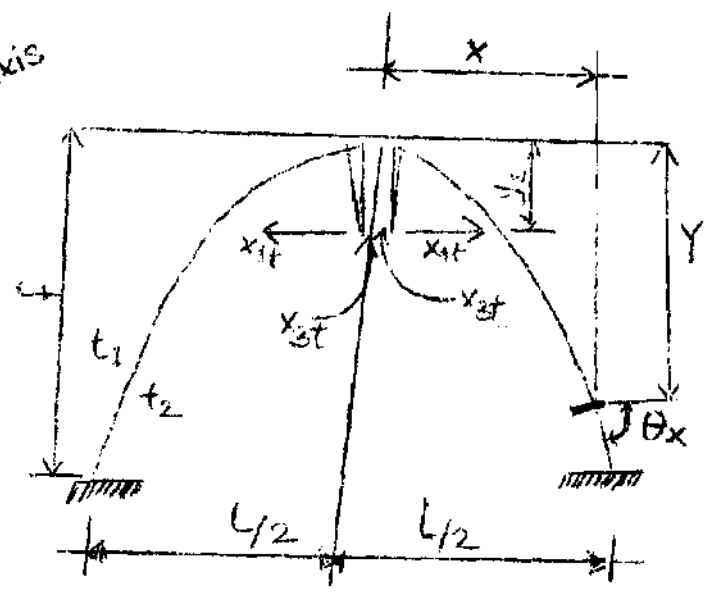


Figure 3.23

Expressions will be established to enable the prediction of these stresses. For this purpose assume that the temperature at the extrados has been increased by t_1° while at the intrados by t_2°. It is admitted that within the thickness of the arch the temperature varies linearly and therefore the increase in temperature at the neutral line will equal $\frac{t_1 + t_2}{2}$. The difference in the temperature $(t_1 - t_2)$ is denoted by t, and half the sum of these temperatures by t. The simultaneous equations expressing that the displacements along the redundant constraints transferred to

the elastic centre of the arch are nil figure 3.23 become in this case

$$X_1 t \delta 11 + \Delta_1 t = 0$$

$$X_2 t \delta 22 + \Delta_2 t = 0$$

$$X_3 t \delta 33 + \Delta_3 t = 0$$

Since all the displacements of the arch caused by temperature change are symmetrical, the displacement Δt_2 must be equal zero and consequently $X_2 t$ is equally nil. The deflections due to a temperature change are given by

$$\Delta_1 t = \alpha (t_1 - t_2) \int \bar{M}_1 \frac{ds}{h} + \alpha \frac{t_1 + t_2}{2} \int \bar{N}_1 ds$$

$$\Delta_3 t = \alpha (t_1 - t_2) \int \bar{M}_3 \frac{ds}{h}$$

or $\Delta_1 t = -\alpha \Delta t \int (Y - y_s) \frac{ds}{h}) \frac{ds}{h} - \alpha t \int \text{Cos}\theta x ds$

$$\Delta_3 t = \alpha \Delta t \int \frac{ds}{h}$$

In this expressions h represents the thickness of the arch and α is the coefficient of thermal expension. The value $\delta 11$ (with due regard to the influence of normal stresses) is given by

$$\delta 11 \int (y - y_s)^2 \frac{ds}{EJ} + \int \text{Cos}^2 \theta x \frac{ds}{EF}$$

while the value of $\delta 33$ is provided by the expression $\delta 33 = \int \frac{ds}{EJ}$.

Introducing the values of $\Delta_1 t$, $\Delta_3 t$, $\delta 11$ and $\delta 33$ in the simultaneous equations of the redundant constraints we obtain

$$\frac{\alpha \Delta t \int (y - y_s) \frac{ds}{h} + \alpha t \int \text{Cos}\theta x ds}{\int (y - y_s)^2 \frac{ds}{EJ} + \int \text{Cos}^2 \theta x \frac{ds}{EF}} = \frac{-\Delta 1t}{\delta 11} = X_1 t$$

$$X_3t = \frac{-\Delta_3{}^t}{\delta_{33}} = \frac{\alpha\Delta t \int \frac{ds}{h}}{\int \frac{ds}{EJ}}$$

The fixed end moment due to a temperature change can be computed using the expression

$$MAt = X_3t + X_1t\ (f-ys)$$

For a parabolic arch whose neutral line follows a curve given by $Y = \frac{4f}{12}x^2$ and whose cross-sectional moments of inertia and thickness vary is accordance with the expression

$$J = \frac{Jc}{Cos\theta}j \qquad h = \frac{hc}{Cos\theta}$$

Jc and hc represents the moment of inertia and the thickness at the crown respectively

X_1t and X_3t are given in this case by

$$X_1t = \frac{\alpha t l}{(1+u)\frac{4}{45}f^2\frac{1}{EJc}} = \frac{45\alpha tEJc}{4(1+u)f2} \quad \ldots\ (1)$$

$$X_3t = \frac{1}{hc}\alpha\Delta tEJc$$

where $u = \frac{\int Cos^2\theta x \frac{ds}{F}}{\int (y-y_s)^2 \frac{ds}{J}}$

The first expression indicates that the thrust due to a temperature change increases with the rigidity of the arch and with a redunction of its rise. While a reduction in the cross-sectional dimensions and the use of the materials with lower modulus of elasticity will reduce the stresses caused by temperature changes.

Shrinkage

Stresses set up in a reinforced concrete arch by the shrinkage of concrete can be calculated in the same way as those due to a temperature change. If α is the coefficient of thermal expansion of concrete, t would be the strain per unit lenght caused by a change in temperature equal to $t^{o}c$.

The shrinkage of concrete leads to a shortening of all linear dimensions by approximately 0.025 percent. If the coefficient of thermal expansion of concrete is equal to 0.00001, the shrinkage may be regarded as equivalent to a drop in temperature of about $25^{o}c$. But in actual design this is usually reduced to 10 or $15^{o}c$ for in reality the arches are cast section by section and therefore only a certain fraction of the total shrinkage must be taken into consideration. Thus the stresses set by shrinkage in a fixed end arch may be computed in exactly the same way as the stresses due to a drop in temperature from 10 to $15^{o}c$.

Settlement of Supports

Statically indeterminate arches may become severely stressed not only due to the application of external loads or due to temperature changes or shrinkage but also in the event when one or more of their supports suffers a linear translation, an angular rotation or both. Using the figure below, if supports B moves and attends a new position B'. The horizontal and vertical displacements of the support will be taken equal to 'a' and 'b' respectively and its angular rotation to θ.

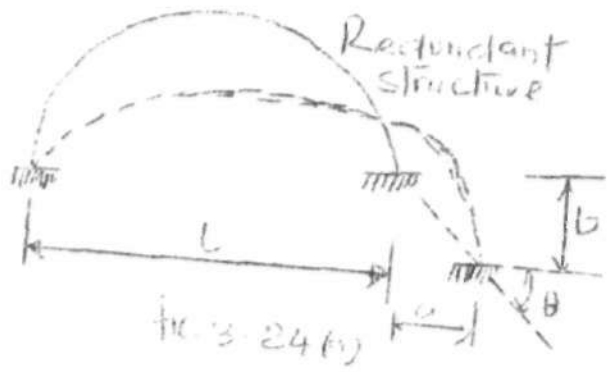
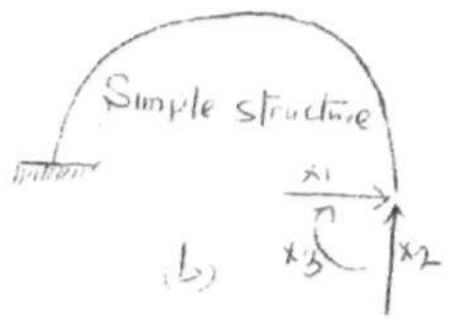
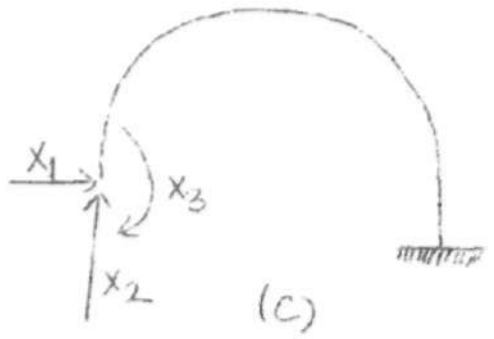


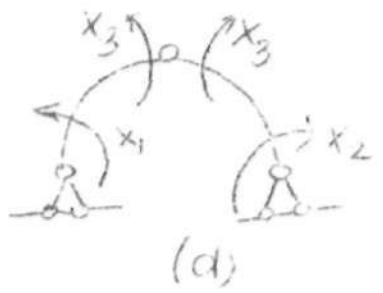
fig. 3.24 (a)



(b)



(c)



(d)

The influence this movements has on the structure will examined using figure 3.24b, c and d. In the first case the direction of the redundant. The canonical equations will then have the following shape.

$$X_1\delta_{11} + X_2\delta_{12} + X_3\delta_{13} + a = 0$$

$$X_1\delta_{21} + X_2\delta_{22} + X_3\delta_{23} + -b + 1\theta = 0$$

$$X_1\delta_{31} + X_2\delta_{32} + X_3\delta_{33} + +\theta = 0$$

for the simple structure of figure 3.24d this same equations would become

$$X_1\delta_{11} + X_2\delta_{12} + X_3\delta_{13} + \Delta_{1\Delta} = 0$$
$$X_1\delta_{21} + X_2\delta_{22} + X_3\delta_{23} + \Delta_{2\Delta} = 0$$
$$X_1\delta_{31} + X_2\delta_{32} + X_3\delta_{33} + \Delta_{3\Delta} = 0$$

$\Delta_{1\Delta}$, $\Delta_{2\Delta}$ and $\Delta_{3\Delta}$ are the displacements of the conjugate simple structure along the directions of X_1, X_2 and X_3 due to the horizontal and vertical morements of the right hand support.

The displacements are readily computed using the expression

$$X_i\Delta_{i\Delta} + \sum k\Delta = 0$$

the computed values of $\Delta_{1\Delta}$, $\Delta_{2\Delta}$ and $\Delta_{3\Delta}$ are as below

$\Delta_{2\Delta} = \frac{-a}{2f} + \frac{b}{l} = 0$ $\Delta_{1\Delta} = \frac{-a}{2f} - \frac{b}{l}$ and $\Delta_{3\Delta} = \frac{a}{h}$

Introducing these values into the simultaneous equations we obtain.

Constraints coincide exactly with those of the support displacements the unknown reaction X_1 follows the direction of the horizontal displacement, the reaction X_2 that of the vertical one (though being opposite in sign) while the moment X_3 acts along the direction of the rotation suffered by the cross-section at the support. The magnitudes of these reactions must be such as to render the displacements of the simple statically determinate structure exactly equal to these stipulated in the problem. The canonical equations expressing this idea will be in the following form.

$$X_1\delta_{11} + X_2\delta_{12} + X_3\delta_{13} = a$$
$$X_1\delta_{21} + X_2\delta_{22} + X_3\delta_{23} = b$$
$$X_1\delta_{31} + X_2\delta_{32} + X_3\delta_{33} = 0$$

The negative value of the last term of the second equation is due to the fact that reaction X_2 is directed upwards while the support moves downwards.

If the simple structure of figure 3.24c were adopted it would become necessary to regard the displacements of support b as a system of external loads. This would be reflected by the introduction into the canonical equations of free terms corresponding to the said loads, these terms being designated as usual by $\Delta_{1\Delta}$, $\Delta_{2\Delta}$ and $\Delta_{3\Delta}$ and these terms will have the following values

$$\Delta_{1\Delta} = a; \quad \Delta_{2\Delta} = -b + 1\theta; \quad \Delta_{3\Delta} = \theta$$

$$X_1\delta 11 + X_2\delta 12 + X_3\delta 13 - (\frac{a}{2h} + \frac{b}{l}) = 0$$

$$X_1\delta 21 + X_2\delta 22 + X_3\delta 23 - (\frac{a}{2h} - \frac{b}{l}) = 0$$

$$X_1\delta 31 + X_2\delta 32 + X_3\delta 33 + \frac{a}{h} = 0$$

From the foregoing chapter it can be seen that the method of forces is a very convinient method that can be used as an exact means for the computation of the stresses induced on redundant structures. The elastic centre method as has been presented is most convinient for arched structures only. Methods like the use of symmetry, grouping of unknowns, symmetrical and unsymmetrical loading and load transformation have been examplified using highly redundant frames which are similar to arches in many aspects.

CHAPTER FOUR

APPROXIMATE METHODS OF STRESS ANALYSIS OF REDUNDANT ARCHES

Resort is often taken to approximate methods of stress analysis of redundant arches because the nuetral lines of these arches and the law governing their cross-sectional variation cannot be expressed by analytical equations which befits practical purposes. Consequently the analysis of the deflections of the conjugate simple structure is rendered impossible, because this analysis is based on integral calculus.

Two approximate methods of stress analysis of redundant arches will be described and illustrated with examples as a means to explaining concisely their applications. Thereafter one of these methods will be employed in the stress analysis of an aircraft hangar in chapter five.

4.1 FIRST APPROXIMATE METHOD WITH EXAMPLE

The arch neutral line is replaced by a polygon of from 8 to 20 sides. The cross-sectional areas remain constant along each of these sides. Their dimensions are equal to those of the given arch as measured over the centre of that particular side. All the loads applied to the arch are replaced by concentrated loads acting at the apices of the polygon. The magnitude of these loads is taken equal to the combined support reaction of two contiguous simple beams carrying the same loads and having the

same lenght as the corresponding portion of the arch. A simple conjugate structure is adopted with or without the transfer of the redundant constraints to the elastic centre. The deflections and rotations can be calculated using vereshchagin's method of graph multiplication. The stresses obtained by this produce are practically equal to those induced in the curved arch.

EXAMPLE BASED ON THE ABOVE METHOD

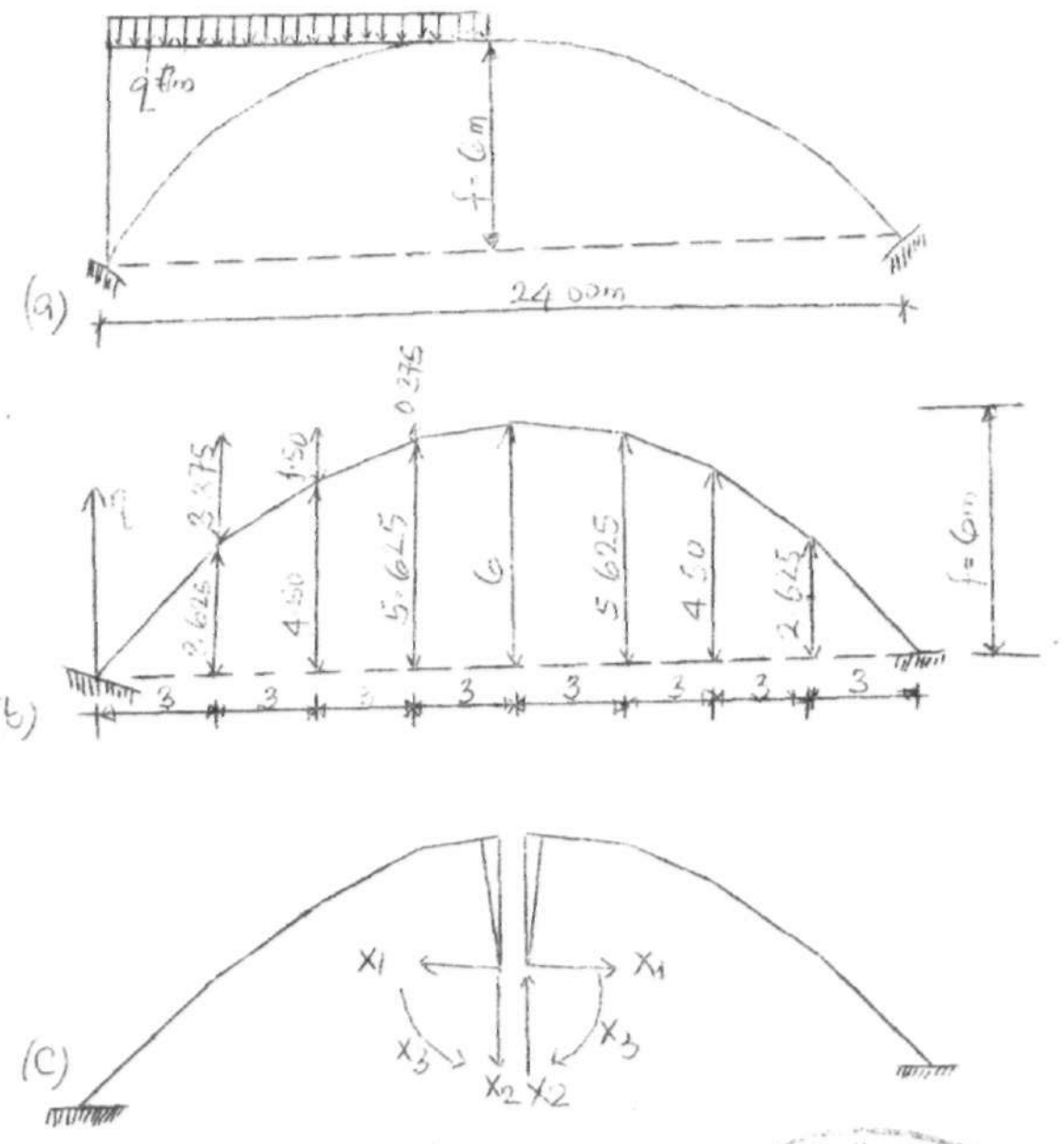


Figure 4.1

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Figure 4.1 is a structure (arch) with a 24m span and 6m rise. The nuetral line of this arch follows a conic parabola and the cross-sectional moments of inertia vary in accordance with the expression $Jx = \frac{Jc}{Cos\theta x}$. The left semiarch carries a uniformly distributed load of 2 tons per metre run. The bending moments (M), the shear (Q) and the normal (N) stresses are required.

To compute the stresses the arch span has been divided into 8 equal parts thus adopting n = 8 and a = 3 metres. Inscribed into the given arch a polygonal one as indicated in figure 4.16. The equation of conic parabola in the coordinate system $\eta 0 \xi$ whose origin coincides with the centroid of the section at the left hand support becomes:

$$= \frac{4f}{2}(1 - \frac{\xi}{2})\xi = \frac{4x6}{24^2}(24-\xi)\xi = \frac{24-\xi}{24}\xi$$

The values of η at the boundaries of all the different segments are indicated in the same figure. A conjugate simple structure is obtained by assuming that the arch has been cut at the crown as indicated in figure 4.1c with the transfer of the redundant constraints to the elastic centre.

The ordinate ys of this elastic centre in the coordinate system xy is computed by the expression

$$ys = \frac{\sum \int y\frac{ds}{J}}{\sum \int \frac{ds}{J}}$$

Since Ji is assumed constant for each side of the polygonal arch the expression for ys becomes

$$ys = \frac{\sum_{i=1}^{i=n} \frac{yi-1 + yi}{2} \cdot \frac{Si}{Ji}}{\sum_{i=1}^{i=n} \frac{Si}{Ji}}$$

$$i=1 \quad Ji$$

where Si = lenght of the segment

Ji = moment of inertia of the same section

$\frac{yi-1 + yi}{2}$ and the ordinate to the centre of this side.

In the case under consideration where $Ji = \frac{Jc}{Cos\Theta i}$ where

Θi is the inclination of side i of the polygon to the horizontal

and therefore $\frac{Si}{Ji} = \frac{SiCos\Theta i}{Jc} = \frac{a}{Jc}$

hence $ys = \frac{\frac{a}{2Jc} \sum_{i=1}^{i=n} (yi- + yi)}{\frac{a}{Jc} n} = \frac{\sum_{i=1}^{i=n} (yi-1 + yi)}{2n}$

or with due regard to the symmetry of the arch

$$ys = \frac{2 \sum_{i=1}^{i=n/2} (yi-1 + yi)}{2n} = \frac{1}{n} \sum_{i=1}^{i=n} (yi-1 + yi)$$

wherefrom $ys = \frac{1}{8} (6 + \frac{27}{8} \times 2 + 15 \times 2 + 0.375 \times 2 + 0) =$

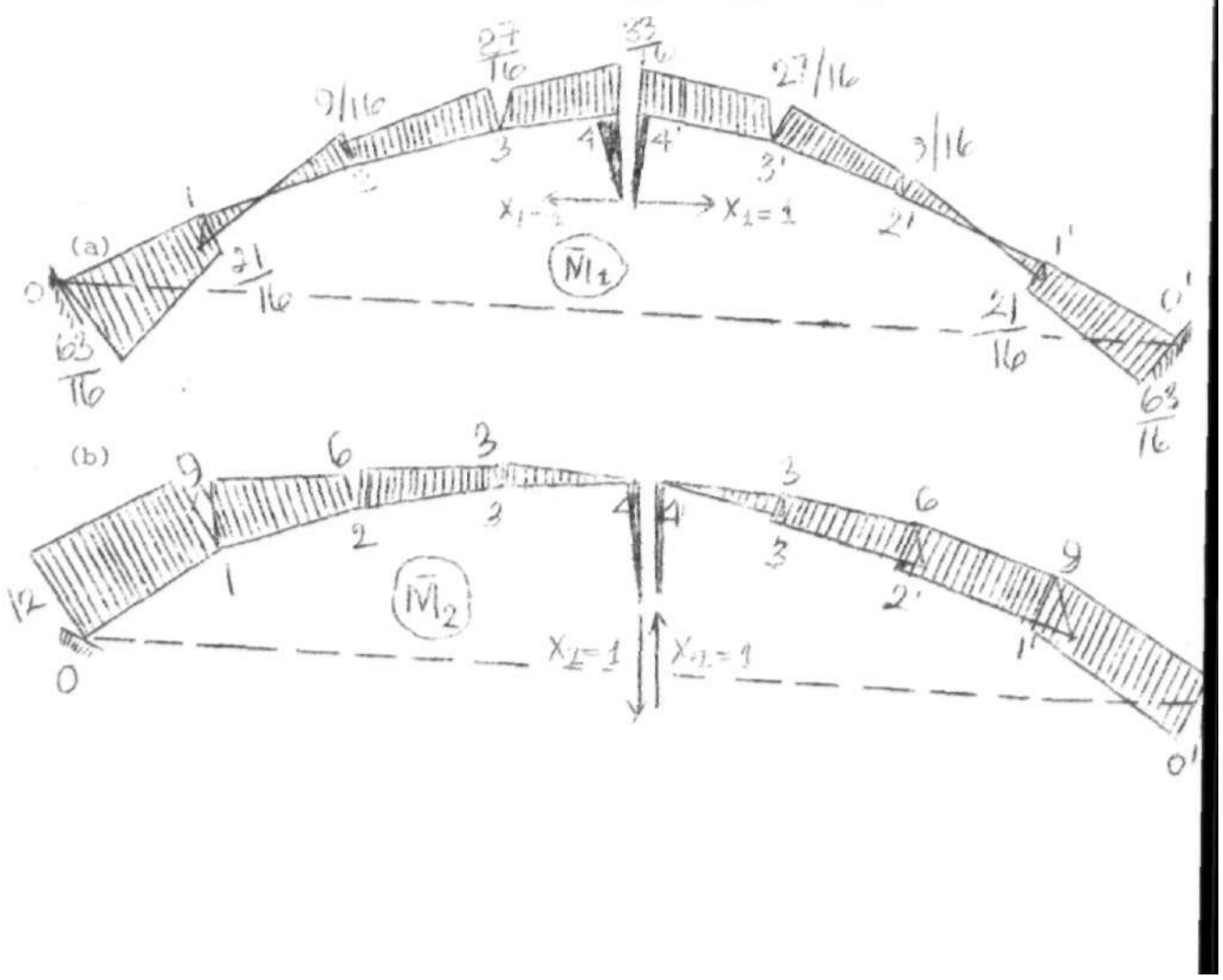
2.0625 metres

The exact value of ys for a parabolic arch whose moment of inertia vary in accordance with the $Jx = \frac{Jc}{Cos\Theta x}$ and equal to $\frac{1}{3}$f

The figure 4.2 (a-c) represent the bending moments induced in the simple structure by unit actions applied at the elastic centre. Figure 4.3 represent those induced by the actual loads concentrated at the apices of the left half of the polygonal arch. At both ends of this arch (points 0 and 4) these loads are $\frac{qa}{2}$ = 3 tons and at the intermediate points (1, 2, 3) these loads amount to qa = 6 tons. The reaction of the unknown constraints will be given by:

$$X_1 = \frac{-\Delta 1q}{\delta 11};\quad X_2 = \frac{-\Delta 2q}{\delta 22} \text{ and } X_3 = \frac{-\Delta 3q}{\delta 33}$$

Displacements δ and Δ will be obtained by multiplying the graphs of figure 4.2(a-c) by those of figure 4.3 using vereshchagin's method. The ratio between the lenght of each side of the polygon and the moment of inertia of the corresponding cross-section remains constant and equal to $\frac{Si}{Ji} = \frac{SiCos\Theta i}{Ji} = \frac{a}{Jc}$.



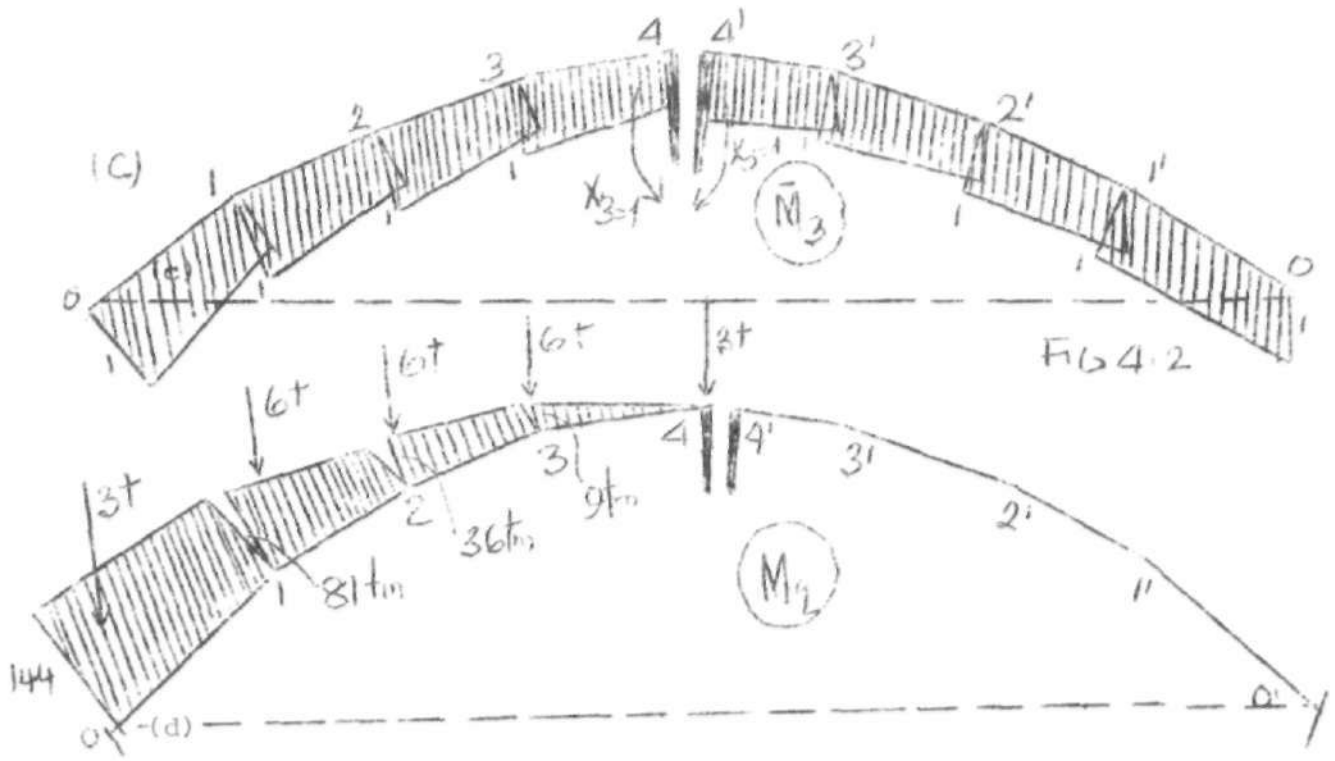


Fig 4.2

Figure 4.3

The unit displacements will be obtained by using formulas peculiar to trapezoidal stress diagrams.

Therefore by raising the $\bar{M}_1$ graph to the second power we obtain,

$$\delta 11 = \frac{2a}{6EJc}\left(2\cdot\frac{63}{16}\cdot\frac{63}{16} + 2\cdot\frac{21}{16}\cdot\frac{21}{16} + 2\cdot\frac{63}{16}\cdot\frac{21}{16}\right) + \left(2\cdot\frac{21}{16}\cdot\frac{21}{16} + 2\cdot\frac{9}{16}\cdot\frac{9}{16} - \frac{21}{16}\cdot\frac{9}{16}\right) + \left(2\cdot\frac{9}{16}\cdot\frac{9}{16} + 2\cdot\frac{27}{16}\cdot\frac{27}{16} + 2\cdot\frac{9}{16}\cdot\frac{27}{16}\right) + \left(2\cdot\frac{27}{16}\cdot\frac{27}{16} + 2\cdot\frac{33}{16}\cdot\frac{33}{16} + 2\cdot\frac{27}{16}\cdot\frac{33}{16}\right)) = \frac{2,457}{32EJc}$$

The values of $\delta 22$ and $\delta 33$ will be obtained in the same way i.e. raising to the second power the $\bar{M}_2$ graphs i.e. figure 4.2 (b and c):

$$\delta 22 = \frac{2a}{6EJc}\,(2.12.12 + 2.9.9 + 2.12.9) + (2.9.9 + 2.6.6 + 2.9.6) + (2.6.6 + 2.3.3 + 2.6.3) + (2.3.3) = \frac{1,152}{EJc}$$

$$\delta 33 = \frac{a}{EJc} \times 8 = \frac{24}{EJc}$$

The displacements $\Delta 1q$, $\Delta 2q$ and $\Delta 3q$ due to the actual loading will be given by the product of the $\bar{M}_1$, $\bar{M}_2$ and $\bar{M}_3$ graphs by the M_q graph (figure 4.3.3).

$$\Delta 1q = \frac{a}{6EJc}\,(-2.\frac{63}{16}\,144 - 2.\frac{21}{16}.81 - \frac{63}{16}.18 - \frac{21}{16}.144) + (-2.\frac{21}{18}.81 + 2.\frac{9}{16}.36 - \frac{21}{16}.36 + \frac{9}{16}.81) + (2.\frac{9}{16}.36 + 2.\frac{27}{16}.9 + \frac{9}{16}.9 + \frac{27}{16}.36) + (2.\frac{27}{16}.9 + \frac{33}{16}.9) = \frac{-7,371}{8EJc}$$

$$\Delta 2q = \frac{a}{6EJc}\,(2.12.144 + 2.9.81 + 12.81 + 9.144) + (2.9.81 + 2.6.36 + 9.36 + 6.81) + (2.6.36 + 2.3.9 + 6.9 + 3.36) + (2.3.9) = \frac{5,292}{EJc}$$

$$\Delta 3q = \frac{a}{EJc}\,(\frac{144}{2} + 81 + 36 + 9) = \frac{-594}{EJc}$$

From these values of displacements are used to compute the redundant constraints.

$$X_1 = -\frac{1q}{11} = \frac{7,371 \times 32}{8 \times 2,457} = 12 \text{ tons}$$

$$X_2 = -\frac{2q}{22} = \frac{5,292}{1,152} = -4.594 \text{ tons}$$

$$X_3 = -\frac{3q}{33} = \frac{594}{24} = 24.75 \text{ tons-metres}$$

The bending moment diagram (M) the shear force diagram (Q) and the normal forces diagram (N) can now be obtained by applying

to the elastic centre of the conjugate structure two forces X_1 = 12 tons, X_2 = -4.594 tons and a couple X_3 = 24.75 ton-metres together with the uniform loads distributed over the left semiarch.

The following formulas may used provided, the ordinates pass through apices of the polygonal arch:

(a) for the left semiarch (figure 4.4)

$$M = X_1 (y-ys) + X_2 x + X_3 - q\frac{x^2}{2}$$

$$Q = X_1 \sin\theta + X_2 \cos\theta - qx\cos\theta$$

$$N = X_1 \cos\theta + X_2 \sin\theta - qx\sin\theta$$

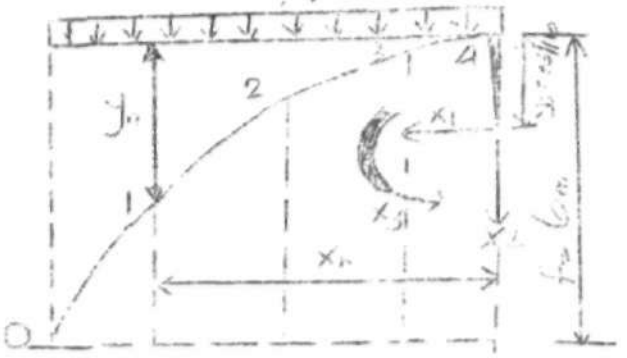


Figure 4.4a

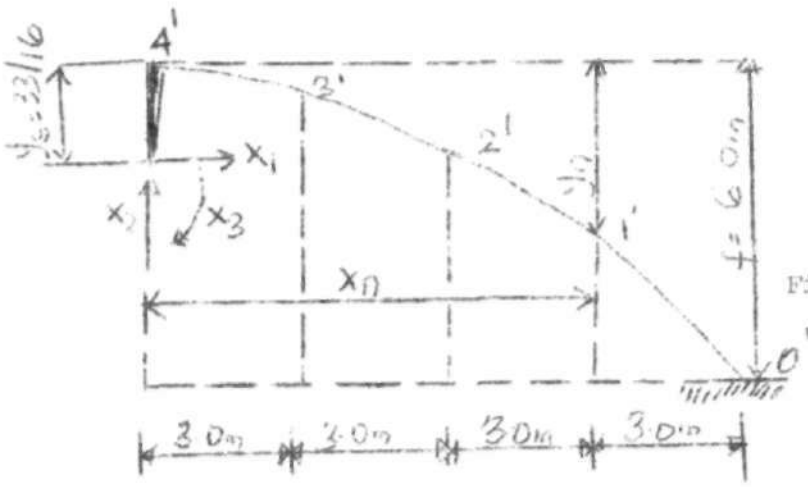


Figure 4.4b

(b) For the right semi arch (figure 4.4b).

$$M = X_1 (y-ys) + X_2 x + X_3$$

$$Q = - X_1 \sin\theta + X_2 \cos\theta$$

$$N = X_1 \cos\theta + X_2 \sin\theta$$

In the expressions above x and y are the coordinates of the nuetral line of the arch, x becoming negative to the left of the axis of symmetry (see figure 4.4b), while θ is the angle between the tangent to this nuetral line and the horizontal. The values of this angle are positive for the left half of the arch and negative for the right one.

Normal stresses are reckoned positive when they cause compression of the arch. The stresses induced in the left semiarch is different from the right semiarch because of the distributed load on the left semiarch. The values of the angle θ will be deduced from the nuetral line equation

$$= \frac{4f}{2}(1-\xi)\xi$$

wherefrom $\tan\theta = \frac{dn}{d\xi} = \frac{4f}{2}(1-2\xi) = \frac{4x6}{24^2}(24-2\xi) = 1-\frac{\xi}{12}$

The values of x, y and θ corresponding to different cross-sections of the arch

TABLE 4.1

Section No.	EM	tan θ	θ	Sinθ	Cosθ	xm	ym
0	0	1	45°	0.707	0.707	-12	6
1	3	0.75	$36^{\circ}52'$	0.600	0.800	-9	3.375
2	6	0.50	$26^{\circ}34'$	0.447	0.894	-6	1.500
3	9	0.25	$14^{\circ}02$	0.2425	0.970	-3	0.375
4,4'	12	0	0°	0	1	0	0
3'	15	-0.25	$-14^{\circ}02'$	-0.2425	0.970	3	0.375
2'	18	-0.50	$-26^{\circ}34'$	-0.447	0.894	6	1.50
1'	21	-0.75	$-36^{\circ}52'$	-0.600	0.800	9	3.375
0'	24	-1	45°	-0.707	0.707	12	6

TABLE 4.2

Ordinates to the M Diagram

Section No.	X_1	y-ys	$X_1(y-ys)$	x_2	x	X_2x	X_3	$-\frac{qx^2}{2}-x^2$ For left semiarch	Ordinates to the M diagram for metres
0		3.9575	47.25		-12.00	55.13		-144.00	-16.87
1		1.3125	15.75		-9.00	41.35		-81.00	0.85
2		-0.5625	-6.75		-6.00	27.56		-36.00	9.56
3		-1.6875	-20.25	-4.594	-3.00	13.78	24.75	-9.00	9.28
4.4'	12.00	2.0625	-24.75		0	0		0	0
3'		-1.6875	-20.25		3.00	-13.78		-	-9.28
2'		-0.5625	-6.75		6.00	-27.56		-	-9.56
1'		1.3125	15.75		9.00	-41.35		-	-0.85
0'		3.9375	47.25		12.00	-55.13		-	16.87

TABLE 4.3

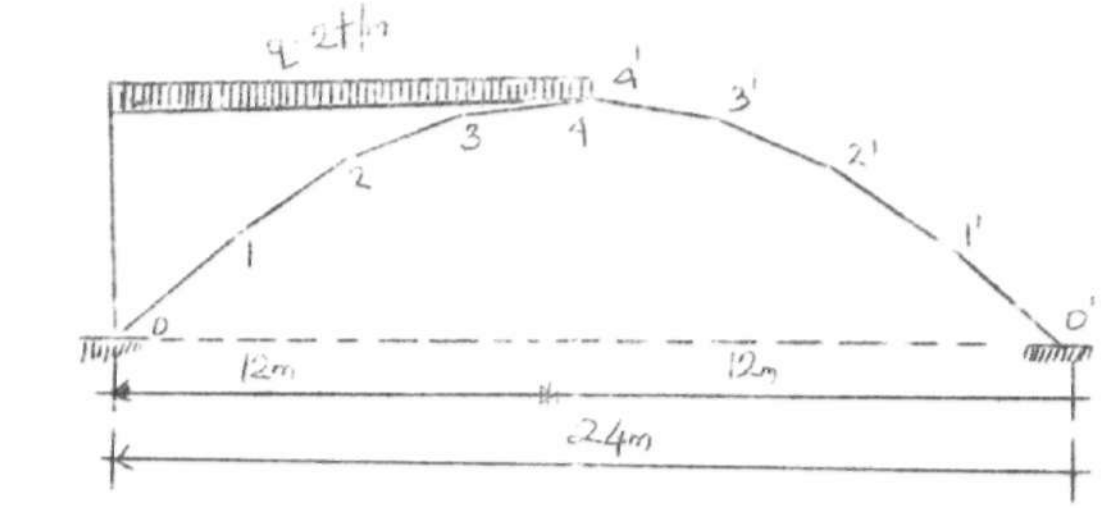
Ordinates to the Q Diagram

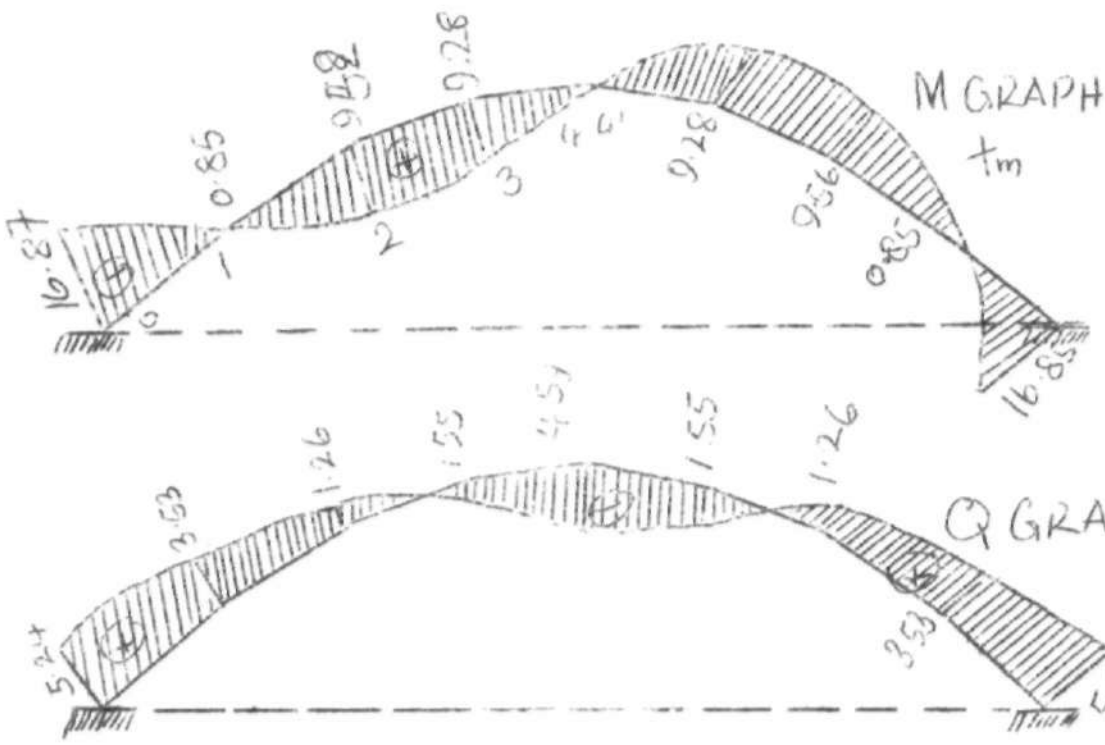
Section No.	X_1	Sinθ	$-X_1$ Sinθ	X_2	Cosθ	X_2Cosθ	qx	-qxCosθ For the left semi arch	Ordinates to the Q diagram in tons
0	12.000	0.707	-8.484	-4.594	0.707	-3.248	-24.00	16.968	5.24
1		0.600	-7.200		0.800	-3.675	-18.00	14.400	3.53
2		0.447	-5.364		0.894	-4.109	-12.00	10.728	1.26
3		0.2425	-2.910		0.910	-4.457	-6.00	5.820	-1.55
4,4'		0	0		1.000	-4.594	0	0	-4.59
3		-0.2425	2.910		0.970	-4.457	-	-	1.55
2		-0.447	5.364		0.894	-4.109	-	-	1.26
1		-0.600	7.200		0.800	-3.675	-	-	3.53
0		0.707	8.484		0.707	-3.248	-	-	5.24

TABLE 4.4

Ordinates to the N Diagram

Section No.	X_1	Cosθ	X_1 Cosθ	X_2	Sinθ	X_2Sinθ	qx	-qxSinθ For the left semi arch	Ordinates to the N diagram tons
0	12.00	0.707	8.484	-4.594	0.707	-3.248	-24.000	16.960	22.21
1		0.800	9.600		0.600	-2.756	-18.000	10.800	17.64
2		0.894	10.728		0.447	-2.054	-12.000	5.364	14.04
3		0.970	11.640		0.2425	-1.114	-6.000	1.455	11.98
4,4'		1.000	12.000		0	0	0	0	12.00
3		0.970	11.640		-0.2425	1.114	-	-	12.76
2		0.894	10.728		-0.447	2.054	-	-	12.79
1		0.800	9.600		-0.600	2.756	-	-	12.36
0		0.707	8.464		-0.707	3.248	-	-	11.73


2.2t/m
4'
3'
2'
1'
0'
3
4
2
1
0
12m
12m
24m


M GRAPH
tm
10.87
0.85
9.80
9.28
3
4
2
1
0
9.28
9.56
0.85
16.85
Q GRAPH, t
5.24
3.53
1.26
1.55
4.54
1.55
1.26
3.53
5.24
N-GRAPH, t
17.64
14.04
11.95
12
12.76
12.79
12.36
11.73
27.21

4.2 SECOND APPROXIMATE METHOD

In this method the arch is subdivided into a number of segments generally consisting of 8 and 20. An appropriate Simple structure deemed fit for the purposes is chosen and the Construction of stress diagram due to the actual load and unit loads is proceeded. After this has been done, the calculations of the coefficients and free terms of the simultaneous equations are carried out assuming that within the limits of each segment the expression under the integral sign vary linearly. Consequently, the corresponding integral will be equal to the lenght of the segment multiplied by half the sum of the values of the expression under the integral sign calculated for the sections limiting this particular section. Thus for instance the displacement δ_{12} will be equal

$$\delta_{12} = \int_0^s \frac{M1\ M2}{EJ} ds \qquad \sum_{i-1}^{i=n} \frac{Si}{2} \left(\frac{M1,i-1\ M2,i-1}{EJi-1} \frac{M1,iM2,i-1}{EJi} \right)$$

$$\sum_{i=o}^{i=n} \frac{M1,iM2,i}{EJi} \left(\frac{Si}{2} + \frac{Si+1}{2} \right)$$

where Si = lenght of the segment i limited by sections (i-1) and i

M1, i and M2,i = bending moments induced at section i by the unknown X1 and X2

Ji = moment of mertia of the same section

n = number of segments into which the arch has been divided

The above expression can also be written as

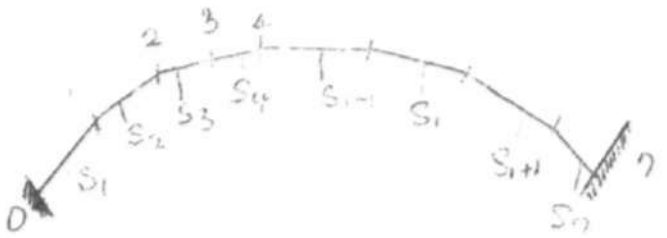
$$\delta_{12} = \sum_{i=o}^{i=n} M1,i\ M2,\ i\ \frac{Si}{EJi}$$

Si is half the sum of the segment lengths contigous to section i

$$\bar{S}i = \frac{Si + Si + 1}{2}$$

Adherance to the following will enable the determination of the displacements δ or Δ.

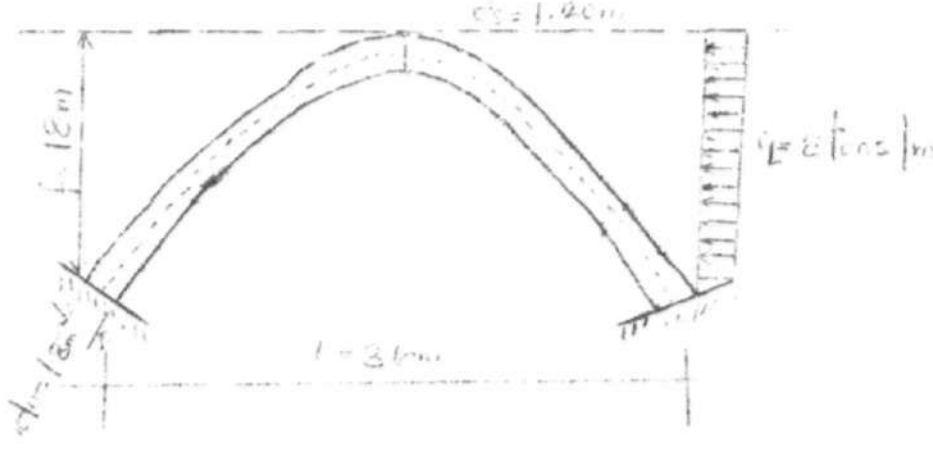
1. Compute the values of the expressions under the integral sign corresponding to each section situated at the boundaries of the segments forming the arch.
2. Multiply each of the values obtained in this way by half the sum of the contiquous segment lenghts.
3. Calculate the sum of all the values obtained as explained above.



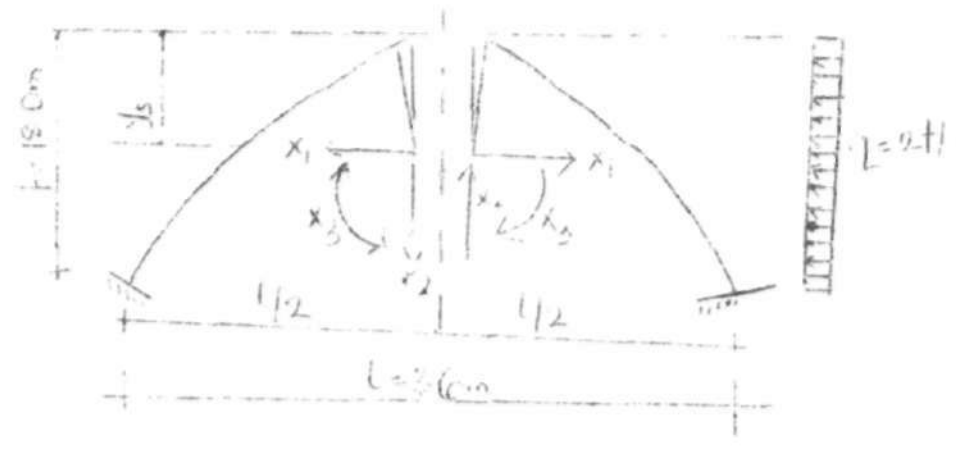
4.2 (b) Example: Below is a parabolic arch carrying a horizontal load of q=2 tons per sq metre. The span of the arch i=36m, its rise f=18m. The thichness at the crown is 1.20m and at the adutments it is 1.80m thick. The M, Q and N i.e. the Moments the slear and the Normal stresses respectively are required.

At intermediate sections the thickness of the arch is given by $dx = dc + \frac{da-dc}{0.51} \left| x \right|$.

x = the horizontal distance of the cross-section under consideration to the crown. The width of the arch (in the direction normal to the plane of drawing) will be assumed equal to 1.0m.



The Confugate Simple structure with all redundant reactions transferred to the elastic centre appears below.



The Corresponding Canomical equations become

$$X_1 \delta_{11} + \Delta 1q = o$$

$$X_2 \delta_{22} + \Delta 2q = o$$

$$X_3 \delta 33 + \Delta 3q = o$$

therefore $X_1 = - \frac{\Delta 1_q}{\delta 11}$

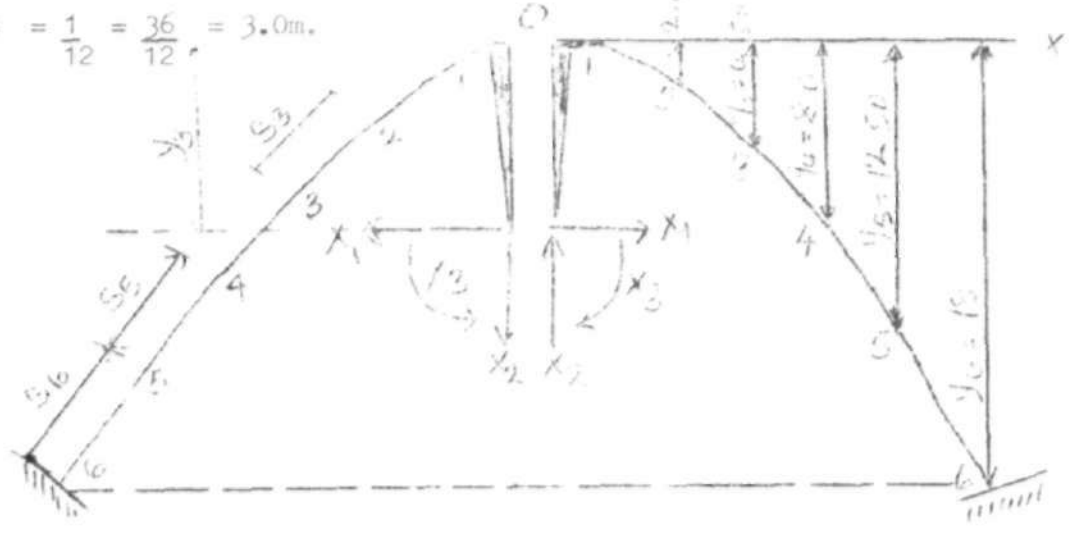
$$X_2 = - \frac{\Delta 2q}{\delta 22}$$

$$X_3 = - \frac{\Delta 3q}{\delta 33}$$

To determine the displacements $\delta 11, \delta 22, \delta 33, \Delta 1q, \Delta 2q, \Delta 3q$

Subdivided the arch into twelve divisions having equal horizontal projections as below.

$$a = \frac{1}{12} = \frac{36}{12} = 3.0m.$$



The ordinates to the metral line of the arch presented above, are computed by using the above expression.

$$Y = \frac{4f}{12} x^2 = \frac{4x18}{36x36} x^2 = \frac{x^2}{18} =$$

The angle θ between the tangent to the unctral line of the arch and the horizontal is computed using the expression:

$$\tan \quad = \frac{dy}{dx} = \frac{2x}{18} = \frac{x}{9} =$$

The data necessary for further calculation is given in the tasle next page. For the right semiarch tan θ is positive and for the left semiarch they are negative. The mean values of segment lenghts contiguous to section subdividing the arch will be calculated using the approximate relation $\delta i = \frac{a}{\cos}$ with the exception of sections 0 and 6 for which $\delta o = \frac{a}{2\cos} = 1.50m$ and

$$\delta_6 = \frac{a}{2\cos} = \frac{3}{2x0.4472} = 3.35m$$

The arch thickness at different cross-sections has been calculated using the relationship specified above

$$d = 1.20 + \frac{1.80 - 1.20}{18} x! = 1.20 + \frac{1x1}{30}$$

The moments of inertia are given by $= \frac{d^3}{12}$

The Coordinates of the elastic Centre with reference to the adopted axes will be $X_s=0$ and

$$Ys = \frac{\sum Y\frac{ds}{J}}{\sum \frac{ds}{J}} = \frac{\sum Y\frac{S}{J}}{\sum \frac{S}{J}} = \frac{496.70}{91.90} = 5.40m.$$

The values 496.70 and 91.90 i.e. the numerator and the denominator in the above expression have been taken from table 4.5 in page 117.

Table 4.5

Section no	X,m		Y,m	tan right semi arch	Cos θ	Sin θ		S m	d, m	J, m^4	$\frac{S}{J}$	$Y\frac{S}{J}$
	Left, semi arch	Right semi arch				Left, semi arch	Right, semi arch					
0	0	0	0	0	1	0	0	1.50	1.20	0.144	10.40	0
1	-3	3	0.5	0.3333	0.9487	-0.3162	0.3162	3.16	1.30	0.183	17.30	8.70
2	-6	6	2.0	0.6667	0.8321	-0.5547	0.5547	3.61	1.40	0.229	15.80	31.60
3	-9	9	4.50	1.0000	0.7071	-0.7071	0.7071	4.24	1.50	0.281	15.10	67.90
4	-12	12	8.0	1.3333	0.6000	-0.8000	0.8000	5.00	1.60	0.341	14.60	116.8
5	-15	15	12.50	1.6667	0.5145	-0.8575	0.8575	5.83	1.70	0.409	11.80	147.50
6	-18	18	18	2.0000	0.4472	-0.8944	0.8944	3.35	1.80	0.486	6.90	124.20
									TOTALS		91.90	496.70

Computation of unit displacements

Displacement δ_{11} will be obtained using the relation

$$E\delta_{11} = 2\left(\sum M_1^2 \frac{\bar{S}}{J} + \sum \bar{N}_1^2 \frac{\bar{S}}{F}\right)$$

The above expression takes care of the normal stresses

M1 = 1(Y-Ys) i.e. X1 (Y-Ys)

N1 = 1(cos θ) i.e. X1

F = 1d F = Cross-sectional area.

$$E\delta_{11} = 2\left[\sum (Y-Ys)^2 \frac{\bar{S}}{J} + \sum \cos^2 \theta \frac{\bar{S}}{F}\right]$$

The term in bracket is multiplied by 2 because the summation is carried along half the arch only. All calculations relating to $E\delta_{11}$ have been entered into taste 4.6

Using the data obtained from this taste

$$E\delta_{11} = 2(2702.4 + 9.02) = 5422.80$$

The displacement δ_{22} will be computed neglecting the influence of the normal stresses, consequently $E\delta_{22} = 2\sum \bar{M}_2^2 \frac{\bar{S}}{J}$

Since $\bar{M}_2 = 1_x$, this expression reduces to $E\delta_{22} = 2\sum x^2 \frac{\bar{S}}{J}$.

The corresponding calculation is entered into taste 4.6

Using the total shown at the foot of the last column we obtain.

$$E\delta_{22} = 2x8,941 = 17,882$$

Displacement δ_{33} will be obtained in exactly the same way

$$E\delta_{33} = 2\sum \bar{M}_3^2 \frac{\bar{S}}{J}$$

where $M_3 = 1$, leading to $E\delta_{33} = 2\sum \frac{\bar{S}}{J}$

Using the values of tasle 4.7 we obtain $E\delta_{33} = 2x91.9 =$ 183.8.

Table 4.6

Section no.	Y,m	Y-Ys	$(Y-Ys)^2$	$\frac{S}{J}$	$(Y-Ys)^2\frac{S}{J}$	Cosφ	$Cos^2\varphi$	$\bar{S}$,m	F,m^2	$\frac{\bar{S}}{F}$	$Cos^2\varphi\ \frac{\bar{S}}{F}$
0	0	-5.40	29.16	10.40	303.3	1	1	1.50	1.20	1.25	1.25
1	0.5	-4.90	24.01	17.30	415.4	0.949	0.901	3.16	1.30	2.43	2.18
2	2	-3.40	11.56	15.8	182.6	0.832	0.692	3.61	1.40	2.58	1.79
3	4.5	0.90	0.81	15.10	12.2	0.707	0.500	4.24	1.50	2.83	1.41
4	8	2.60	6.76	14.60	98.7	0.600	0.360	5.00	1.60	3.12	1.12
5	12.50	7.10	50.41	11.80	594.8	0.514	0.264	5.83	1.70	3.43	0.90
6	18	12.60	158.76	6.90	1095.4	0.447	0.200	3.35	1.80	1.86	0.37
				TOTAL 2702.40		-	-	-	-	-	9.02

TABLE 4.7

Section no	X,m	X^2	$\frac{S}{J}$	$X^2 \frac{S}{J}$
0	0	0	10.4	0
1	3	9	17.3	156
2	6	36	15.8	569
3	9	81	15.1	1223
4	12	144	14.6	2102
5	15	225	11.8	2655
6	18	324	6.9	2236
		Total	91.90	8941

Displacements due to applied load

The unit displacements E , and will be obtained by using the following expressions.

$$E\Delta_{1q} = \sum \bar{M}_1 M_q \frac{S}{J}$$

$$E\Delta_{2q} = \sum \bar{M}_2 M_q \frac{S}{J}$$

$$E\Delta_{3q} = \sum \bar{M}_3 M_q \frac{S}{J}$$

$\bar{M}_1 = Y - Ys$ $\bar{M}_2 = X$, and $\bar{M}_3 = -\frac{Y^2}{2} = \frac{2Y^2}{2} = -Y^2$

Consequently

$$E\Delta_{1q} = -\sum (Y-Ys)\, Y^2 \frac{S}{J}$$

$$E\Delta_{2q} = -\sum X\, Y^2 \frac{S}{J} \qquad E\,3q = -xy^2 \frac{S}{J}$$

In all three of these expressions the Summation will be carried over the right Semi arch only. The bending moments remaining constrantly nil.

The corresponding calculations are entered into table 4.8

Section No	X	Y	$\frac{S}{J}$	Y-Ys	Y^2	$Y^2 \frac{S}{J}$	(Y-Ys)x	$xY^2 \frac{S}{J}$
0	0	0	10.4	-5.40	0	0	0	0
1	3	0.50	17.3	-4.90	0.25	4.30	-21	13
2	6	2.00	15.8	-3.40	4.00	63.20	-215	379
3	9	4.50	15.10	-0.90	20.25	305.8	-275	2752
4	12	8	14.60	2.60	64	934.4	2429	11213
5	15	12.50	11.80	7.10	156.25	1843.8	15091	27657
6	18	18	6.90	12.60	324	2235.60	28169	40241
					TOTAL	5387.10	43178	82255

Using the results of this calculation we obtain

$\sum \Delta_{1q} = -\ 43,178,\ \sum \Delta_{2q} = -\ 82,255;\ \sum \Delta_{3q} = -\ 5,387.10$

Introducting these values into the expressions for deriving the redundant constraints, the magnitude of the unknown redundant reactions

$$X_1 = -\frac{\Delta_{1q}}{\delta_{11}} = -\frac{43,178}{5422.8} = 7.96 \text{ tons}$$

$$X_2 = -\frac{\Delta_{2q}}{\delta_{22}} = -\frac{82,255}{17.882} = 4.60 \text{ tons}$$

$$X_3 = -\frac{\Delta_{3q}}{\delta_{33}} = -\frac{5387.10}{183.80} = 29.31 \text{ ton-metres}$$

These forces are applied at the elastic centre of the statically determinate conjugate structure. Together with the uniform loads applied to the arch, they constitute the complete system of loads permitting the computation of all the stresses and reactions.

The ordinates to the M, A and N graphs will be obtained using the following equations

a) For the left Semi-arch

$$M = X_1 (Y-Ys) + X_2 x + X_3$$

$$Q = X_1 \sin\theta + X_2 \sin\theta$$

b) For the right Semi arch

$$M = X_1 (Y-Ys) + X_2 x + X_3 - \frac{qy^2}{2}$$

$$Q = X_1 \sin\theta + X_2 \cos\theta - qy \sin\theta^2$$

$$N = X_1 \cos\theta - X_2 \sin\theta - qy \cos\theta$$

The expressions for the left semi arch differ from those for the right one solely because of the term accounting for the uniformly distributed load q

All Calculations relative to the ordinates M, N and Q Diagrams are carried out in tables 4.9 4.10 and 4.11

TABLE 4.9 ORDINATES TO THE M DIAGRAM

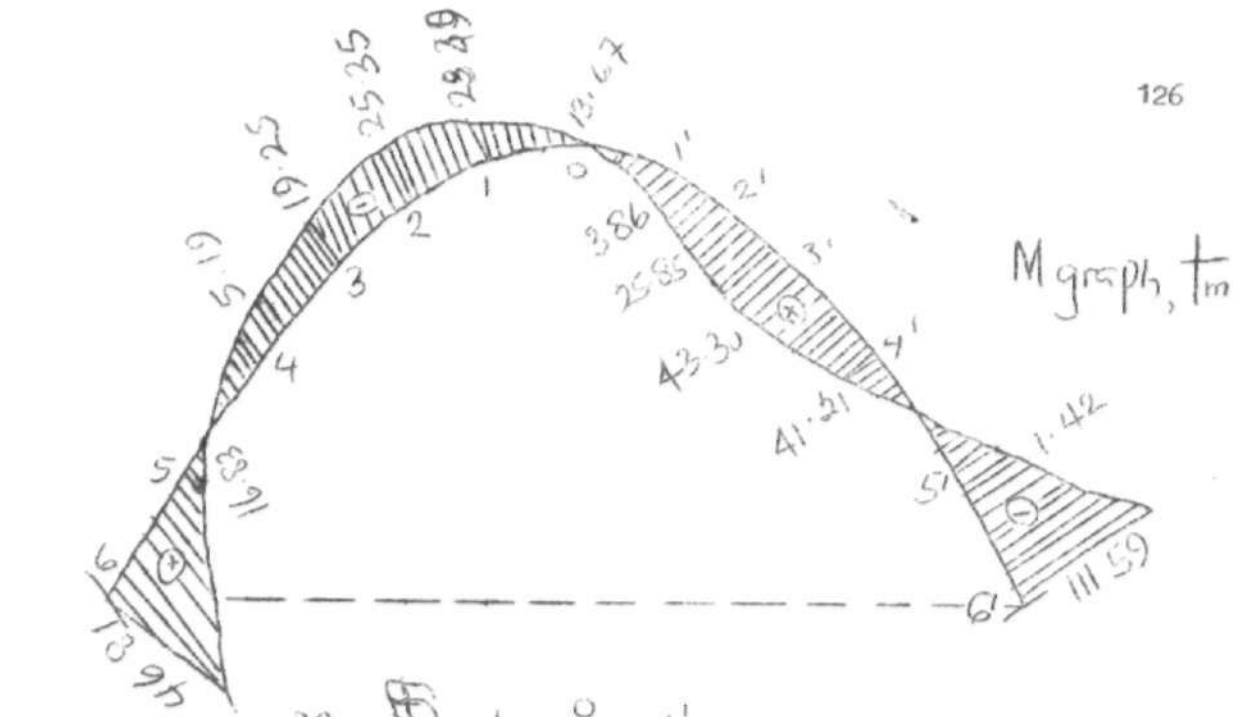
	Section no.	X_1	Y-Ys	$X_1(Y-Ys)$	X_2	X	X_2x	X_3	$-\frac{qy^2}{2}=y^2$ For the right semi arch	Ordinates to M diagram ton-metres
Left semi arch	0	7.96	-5.40	-42.96	4.60	0	0	29.31	-	-13.67
	1		-4.00	-39.00		-3	-13.80		-	-23.49
	2		-3.40	-27.06		-6	-27.60		-	-25.25
	3		-0.90	-7.16		-9	-41.40		-	-19.25
	4		7.60	20.70		-12	-55.20		-	-5.19
	5		7.10	56.52		-15	-69.00		-	16.83
	6		12.60	100.30		-18	-82.80		-	46.81
Right semi arch	0		-5.40	-42.96		0	0		0	-13.67
	1		-4.90	-39.00		3	13.80		-0.25	3.86
	2		-3.40	-27.06		6	27.60		-4	25.85
	3		-0.90	-7.16		9	41.40		-20.25	43.30
	4		2.60	20.70		12	55.20		-64	41.21
	5		7.10	56.52		15	69.00		-156.25	-1.42
	6		12.60	100.30		16	82.80		-324.00	-111.59

TABLE 4.10 ORDINATES TO THE Q, DIAGRAM

Section no.		X_1	Sinθ	X_1Sinθ	X_2	X_2 Cosθ	-qys = - 2y	-qysin For the right semi arch	Ordinates to the Q diagram tons
Left semi arch	0	7.96	0	0	4.60	1	-	-	4.60
	1		-0.316	-2.52		0.949	-	-	1.85
	2		-0.555	-4.42		0.832	-	-	-0.59
	3		-0.707	-5.63		0.707	-	-	-2.38
	4		-0.800	-6.37		0.600	-	-	-3.61
	5		-0.858	-6.83		0.514	-	-	-4.47
	6		-0.894	-7.11		0.447	-	-	-5.05
Right semi arch	0		0	0		1	0	0	4.60
	1		0.316	2.52		0.949	-1	-0.32	6.57
	2		0.555	4.42		0.832	-4	-2.22	6.03
	3		0.707	5.63		0.707	-9	-6.36	2.52
	4		0.800	6.37		0.600	-16	-12.80	-3.67
	5		0.858	6.83		0.514	-25	-21.45	-12.26
	6		0.894	7.11		0.447	-36	-32.18	-23.01

TABLE 4.11 ORDINATES TO THE N DIAGRAM

	Section number	X_1	Cos θ	X_1 Cosθ	$-X_2$	Sin θ	$-X_2$ Sinθ	$-qy = -2y$	-qybs For right semi arch	Crdinates to the N diagram, tons
Left semi arch	0		1	7.96		0	0	-	-	7.96
	1		0.949	7.55		-0.316	1.45	-	-	9.00
	2		0.832	6.63		-0.555	2.55	-	-	9.18
	3		0.707	5.63		-0707	3.25	-	-	8.88
	4		0.600	4.78		-0.800	3.68	-	-	8.46
	5		0.514	4.08		-0.858	3.95	-	-	8.04
	6	7.96	0.447	3.56	-4.60	-0.894	4.11	-	-	8.04
Right semi arch	0		1	7.96		0	0	0	0	7.96
	1		0.949	7.55		0.316	-1.45	-1	-0.95	5.15
	2		0.832	6.63		0.555	-2.55	-4	-3.33	0.75
	3		0.707	5.63		0.707	-3.25	-9	-6.36	-3.98
	4		0.600	4.78		0.800	-3.68	-16	-9.60	-8.60
	5		0.514	4.08		0.858	-3.95	-25	-12.85	-12.71
	6		0.447	3.56		0.894	-4.11	-36	-16.09	-16.64


M graph, tm
13.67
25.85
43.30
41.21
111.59
Q graph, t
4.85
4.60
6.51
6.03
2.52
3.67
12.26
23.01
N graph, t
7.96
3.96
8.60
12.71
16.64

In the two approximate methods of the stress analysis of reinforced concreted fixed end arches described in this chapter, the work i.e. the calculations have been considerably reduced by the various approximations. Though these approximations affect the accuracy of the final results but the errors are so slight and pause no dangerto the structure if the results are used for the design.

CHAPTER FIVE: STRESS ANALYSIS OF AN ARCHED BEAM OF A REINFORCED CONCRETE HANGAR.

The first approximate method described and illustrated with the example in Chapter four will be used for the stress analysis of the arched beam of the hangar structure. Effects due to wind on the structure will best be calculated using the second approximate method of chapter four. Influence lines will also be used for the stress analysis of this structure. The arched beam refered to is one of the structural members of the Rapid City hangar erected between 1947 and 1948 in San Diego U.S.A.

5.1. DATA

The rapid City Hangar is supported on piles, with tie rods connecting the arch abutments. The arch ribs vary from 1.50m to 2.10m in depth and are about 60cm wide spaced 7.50metres on centres. The slab is shaped like an inverted catenary 17.50cm at the spring line and 12.50cm at the crown.

The hangar comprises reinforced concrete abutments supporting the arch ribs and extending up to the spring line. In order to simplify the construction of forms and facilitate their movement as the slab is cast section by section, the ribs project above the roof. This results in a smooth unobstructed intrados. Contraction pints are placed at 15metres intervals. Those that fall at third points extend through the abutment to the floor of the hangar the intermediate joints are carried only to the tops of the abutments. The structure has a clear span of 102meters and a clear height of 27metres at mid-span.

5.2. ASSUMPTIONS

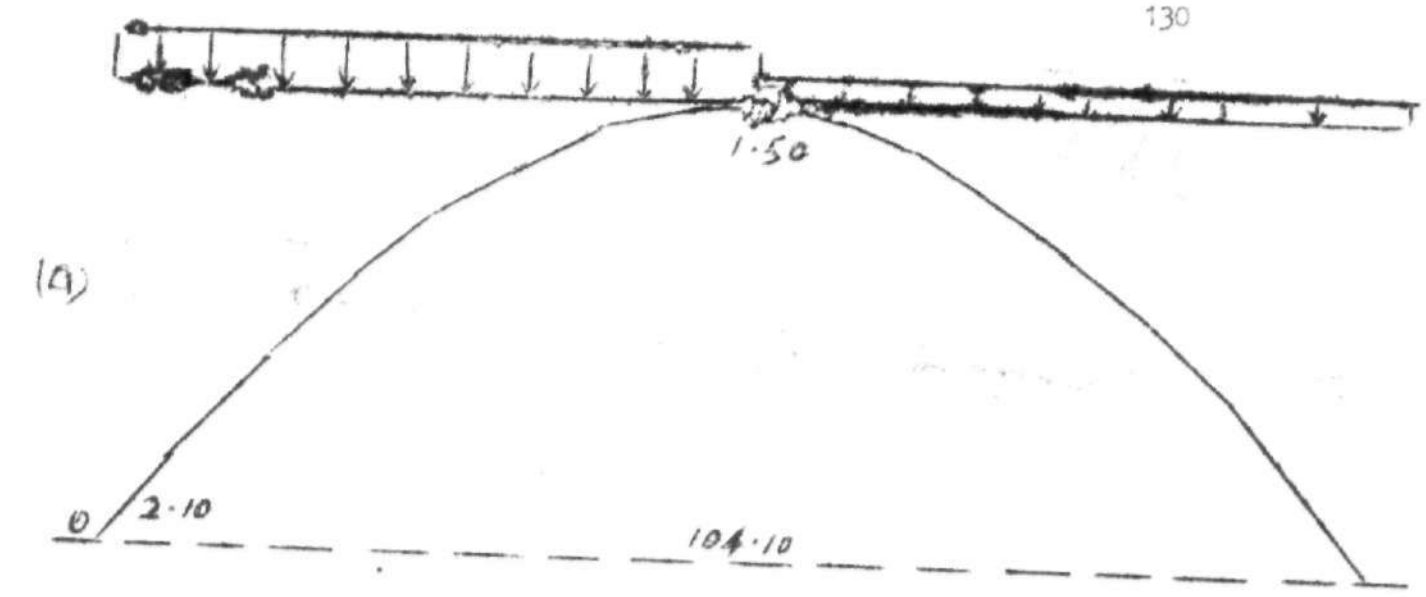
For the purpose of the stress analysis, the arched beam has been divided into 16 segments. The appropriate simple conjugate structure for this redundant structure is assumed to have been cut at the crown. The cross-sectional dimensions are assumed to remain constant along each of these sides, their dimensions being equal to those of the given arch as measured over the centre of that particular side. All the loads applied to the arch are replaced by concentrated loads acting at the apices of the polyson. All the deflections and rotations will be calculated using Vereshchagin's method of graph multiplication.

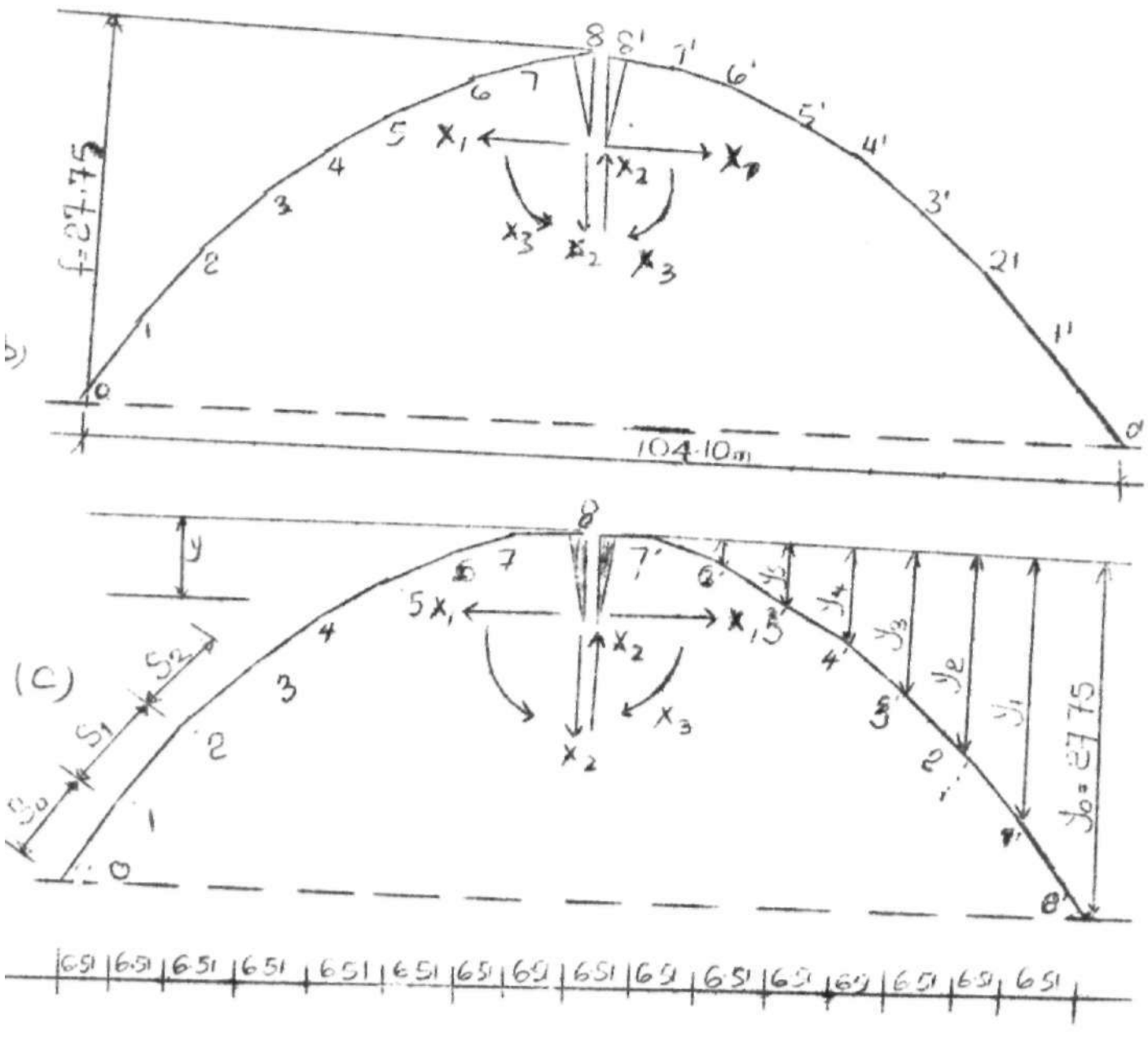
The structure is assumed situated in Zaria for the purpose of determining the wind pressure. The situation of the structure is also considered exposed with a design wind velocity of 144km/h.

The rise of the arch is greater than one-fifth of its span, consequently the calculation of the deflections and angular rotations may be carried out neglecting the influence of shears and normal stresses. But shear and normal stresses are assumed to be present for the purposes of familiarization with their computation.

The most critical loading for this structure is taken to be, when one half of the beam is loaded with the imposed load and dead load while the other is only loaded with the dead loads.

The imposed load used is $0.75KN/m^2$.


(A)
1·50
0
2·10
104·10


f=27.75
8
8'
7'
6'
5'
4'
3'
21
1'
6
7
5
4
3
2
1
0
X1
X2
X3
104·10m
(C)
y
S2
S1
S0
y1
y2
y3
y4
y5
y6
y0=27.75
6.51

5.3. STRESS ANALYSIS BY THE FIRST APPROXIMATE METHOD

The nuetral line of this arch follows a conic parabola and the cross-sectional moments of inertiasvary in accordance with the expression $Jx = \frac{Jc}{Cos\theta}$ The M, C and N calculations will be done assuming that the left semi-arch and the right one are uniformly loaded with the dead load.

The arched beam has been divided into 16 equal parts thus adopting n = 16 and a = 6.51 metres. The equation of a conic parabola in the co-ordinate system $\eta O \xi$ whose origin coincides with the centroid of the section at the left hand abutment becomes

$$= \frac{4f}{L2} \quad (L - \xi)\xi = \frac{4x27.75}{27.75x27.75}(27.75-\xi)\xi = \frac{4(27.75-\xi)\xi}{27.75}$$

The ordinate Ys of this elastic centre in the coordinate system xy

$$Ys = \frac{\sum\int Y\frac{\delta s}{J}}{\sum\int \frac{\delta s}{J}}$$

Since Ji is constant for each side of the polygonal arch the expression becomes

$$Ys = \frac{\sum_{i=1}^{i=n} \frac{Yi-1+Yi}{2} \times \frac{Si}{J}}{\sum_{i=1}^{i=n} \frac{Si}{Ji}}$$

Si represents the lenght of side i and $\frac{Yi-1+Yi}{2}$ is the ordinate to the centre of this side.

In the case under consideration we have $Ji = \frac{Jc}{Cos\theta i}$ where θi is the inclination of side i of the polygon to the horizontal

and therefore $\frac{Si}{Ji} = \frac{SiCos\theta i}{Jc} = \frac{a}{Jc}$

hence

$$Ys = \frac{\frac{a}{2Jc}\sum_{i=1}^{i=n}(Yi-1+Yi)}{\frac{a}{Jc}\,n} = \frac{\sum_{i=1}^{i=n}(Yi-1+Yi)}{2n}$$

with one regard to the symmetry of the arch

$$Ys = \frac{\sum_{i=1}^{i=n/2}(Yi-1+Yi)}{2n} = 1/n\sum_{i=1}^{i=n}(Yi-1+Yi)$$

tan θ is obtained from the expression

$$\frac{dn}{d\xi} = \frac{4f}{l^2}(l - 2\xi)$$

η is also obtained from the expression

$$\eta = \frac{4f}{l^2}(l - \quad) \quad \text{or} \quad Y = \frac{4fx^2}{l^2}$$

The values of x, y and θ corresponding to different cross sections of the arch appear in table 5.1

Table 5.1

Section No.	ξ	tan θ	θ	Sin θ	Cos θ	Xm	Ym
0	0	1.07	45°	0.7071	0.7071	-52.08	27.75
1	6.51	0.9398	$43^{\circ}13^{1}$	0.6847	0.7288	-45.57	21.27
2	13.02	0.8096	39^{0}	0.6293	0.7771	-39.06	15.62
3	19.53	0.6794	34°	0.5592	0.8290	-32.55	10.85

4	26.04	0.5492	$28^{\circ}46^{1}$	0.4813	0.8766	-26.04	6.94
5	32.55	0.4190	$22^{\circ}40^{1}$	0.3854	0.9227	-19.53	3.91
6	39.06	0.2888	$16^{\circ}5^{1}$	0.2770	0.9607	-13.02	1.74
7	45.57	0.1586	$9^{\circ}1^{1}$	0.1567	0.9877	-6.51	0.43
$8,8^{1}$	52.08	0.0284	0°	0	1.000	0	0
7^{1}	58.59	0.1586	$9^{\circ}\ 1^{1}$	0.1567	0.9877	6.51	0.43
6^{1}	65.08	0.2888	$16^{\circ}51$	0.2770	0.9607	13.02	1.74
5^{1}	71.51	0.4990	$22^{\circ}40^{1}$	0.3854	0.9227	19.53	3.91
4^{1}	78.1	0.5492	$28^{\circ}40^{1}$	0.4813	0.8766	26.04	6.94
3^{1}	84.61	0.6794	34°	0.5592	0.8290	32.55	10.85
2^{1}	91.12	0.8096	39°	0.6293	0.7771	39.06	15.62
1^{1}	97.63	0.9398	$43^{\circ}13^{1}$	0.6847	0.7288	45.57	21.27
0	104.10	1.07	45°	0.7071	0.7071	52.08	27.75

From table 5.1

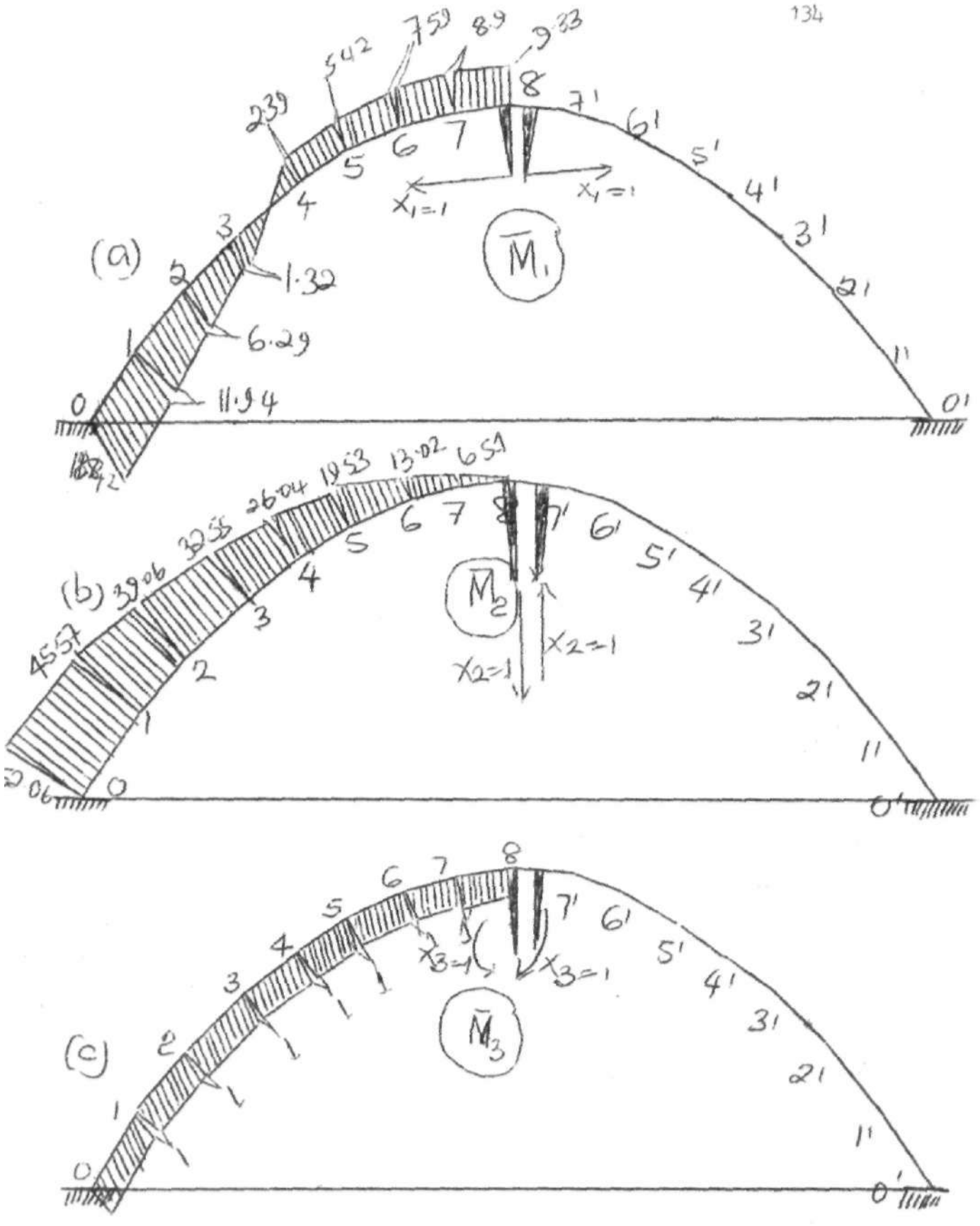
$$Ys = \frac{1}{8}\ (27.75 + 21.27x2 + 15.62x2 + 10.85x2 + 6.94x2 + 3.91 \times 2 + 1.74x2 + 0.43x2 + 0) =$$

$$= \frac{149.27}{16} = 9.33m$$

The exact value of Ys for a parabolic arch whose moments vary in accordance with the $Jx = \frac{Jc}{Cos\theta_x}$ 6 equal $\frac{1}{8}$f which in this case is greater by 8cm. This difference can be further reduced by increasing the sides of the polygonal arch.

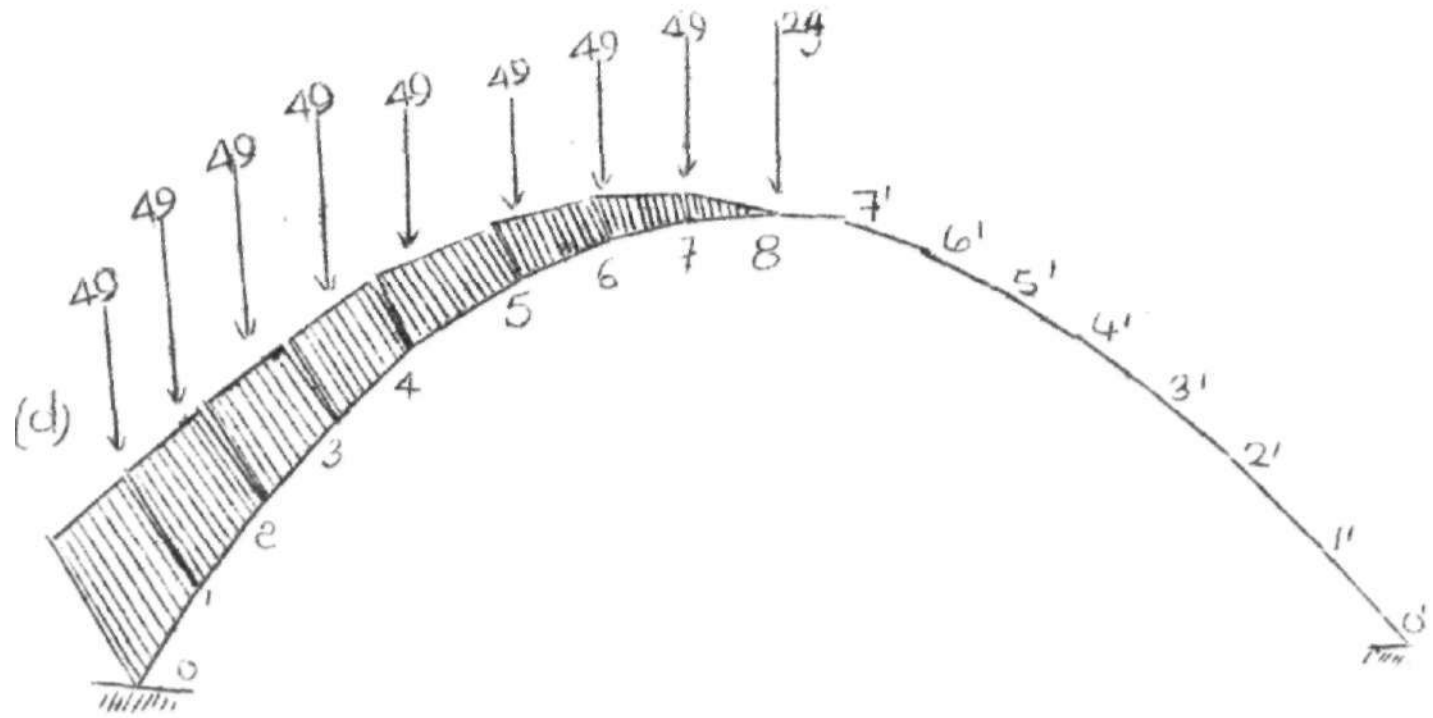
Fig. 5.2a, b and c represents the bending moments induced in the simple structure by unit actions applied at the elastic centre. Fig 5.3 represent those induced by the actual loads concentrated at the apices of the arch. At both ends of the arch (points 0 and 8) these loads are $\frac{a}{2}$

and those of the intermediate joints ie 1,2, 3, 4, 5, 6 and 7 = qa.


(a)
0
1
2
3
4
5
6
7
8
0'
1'
2'
3'
4'
5'
6'
7'
1.32
2.39
5.42
7.59
8.9
9.33
6.29
11.94
$\bar{M}_1$
$X_1=1$
$X_1=1$
(b)
$\bar{M}_2$
$X_2=1$
$X_2=1$
(c)
$\bar{M}_3$
$X_3=1$
$X_3=1$

The unknown reactions are given by

$$X1 = -\frac{\Delta 1q}{\delta 11}; \quad X2 = -\frac{\Delta 2q}{\delta 22}; \quad X3 = -\frac{\Delta 3q}{\delta 33}$$



DEAD LOAD CALCULATION

The arch tickness at different cross sections will be calculated using the expression $di = dc + \frac{da-dc}{0.51} \times$

di = thickness at the considered section

dc = " " " crown

da = " " " abutment

x = distance of given section from considered section.

l = span of arch structure

Using the above expression the following results have been obtained.

$d8 = 1.50m, \quad d_7 = 1.58m, \quad d_6 = 1.65$

$d5 = 1.73m, \quad d_4 = 1.80m, \quad d_3 = 1.88m$

$d2 = 1.95m, \quad d_1 = 2.03m, \quad d_0 = 2.10m$

From which the weight of the beam at the various cross sections are obtained

Crown Section	8	=	1.50m x 0.60 x 2.40KN/m^3	= 2.16KN/m
-11-	7	=	1.58 x 0.60 x 2.4 "	= 2.28KN/m
-11-	6	=	1.65 x 0.60 x 2.4 "	= 2.38
-11-	5	=	1.73 x 0.60 x 2.4 "	= 2.49
-11-	4	=	1.80 x 0.60 x 2.4 "	= 2.59
-11-	3	=	1.88 x 0.60 x 2.4 "	= 2.71
-11-	2	=	1.95 x 0.60 x 2.4 "	= 2.81
-11-	1	=	2.03 x 0.60 x 2.4 ".	= 2.92
-11-	0	=	2.10 x 0.60 x 2.4 "	= 3.02

For the purpose of simplicity the average load will be used

= 2.60KN/m.

Self weight of rib = 2.60KN/m

Thermal insulation = fireboards = 0.058KN/mwt due to slab = 2.70KN/m

Plaster = 0.02 x 2.20 = 0.044KN/m

Water proofing asphalt = 0.02x22.6x75 = 0.005KN/m

Total Dead Load = 5.41KN/m

Live Load is taken to be 0.75KN/m.

Design dead load = 5.41KN/m x 1.4 = 7.57 KN/m.

The loads acting at the apices are computed as

Section 0 and 8 = $\frac{qa}{2} = \frac{7.57x6.51}{2}$ = 24.64KN/m

The intermediate sections are:

Qa = 7.57 x 6.5/m = 49.28KN

Moments due to applied loads.

Section 8 = 24.64x0 = 0

" 7 = 24.64x6.51 = 160KNm

" 6 = 24.64x13.02 + 49.28 x 6.51 = 828KNm

" 5 = 24.64 x19.53 + 49.28x13.02 + 49.28 x 6.51 = 1143 KNm.

" 4 = 24.64 x26.04 + 49.28x19.53 + 49.28 x 13.02 + 49.28x6.51 =

= 2567KNm.

3 = 32.55 x 24.64 + 49.28 x 26.04 + 49.28 x 19.53 +

49.28 x 13.02 + 49.28 x 6.51 = 4010 8KNm

2 = 39.06 x 24.64 + 49.28 x 32.55 + 49.28 x 26.04 +

49.28 x 19.53 + 49.28 x 13.02 + 49.28 x 6.51 =

= 5775 KNm

Section 1 = 24.64 x 45.57 + 49.28 x 39.06 + 49.28 x 32.55

+ 49.28 x 26.04 + 49.28 x 19.53 + 49.28 x 13.02 +

+ 49.28 x 6.51 = 7860 KNm

Section 0 (abutment)= 24.64 x 52.08 + 49.28 x 45.57 +

+ 49.28 x 39.06 + 49.28 x 32.55 + 49.28 x 26.04

+ 49.28 x 19.53 + 49.28 x 13.02 + 49.28 x 6.5a =

= 10266 KNm.

The unit displacements will be determined using formulas peculiar to the trapezoidal stress diagrams. Thus rising to the second power. The M_1 graph. But first the ordinates of the centroids of the various cross-sections will have to be determined by substracting the y-coordinate of the elastic centres from Y.

Section 0 = Y-Ys = 27.75 - 9.33 = 18.43

1 = -11- = 21.27 - 9.33 = 11.94

2 = -11- = 15.62 - 9.33 = 6.29

3 = -11- = 10.85 - 9.33 = 1.52

4 = -11- = 6.94 - 9.33 = 2.39

5 = -11- = 3.91 - 9.33 = -5.42

6 = -11- = 1.74 - 9.33 = -7.59

7 = -11- = 0.43 - 9.33 = -8.90

8 = -11- = 0 - 9.33 = -9.33

$$\delta_{11} = \frac{2a}{6EJc}\Big[(2x18.43x18.43 + 11.94x11.94x2 + 2.18.43 \times 11.94)+$$
$$+(2x11.94x11.94 + 6.29x6.29x2 + 11.94x6.29 \times 2)+$$
$$+(2x6.29 \times 6.29 + 1.52 \times 1.52.2 + 1.52 \times 6.29 \times 2)+$$
$$+(2x1.52 \times 1.52 + 2x2.39 \times 2.39 + 2x1.52 \times 2.39)+$$
$$+(2x2.39 \times 2.39 + 2x5.42 \times 5.42 + 2.x\ 5.42 \times 2.39)+$$
$$+(2x5.42 \times 5.42 + 2x7.59 \times 7.59 + 5.42 \times 7.59 \times 2\ )+$$
$$+(2x7.59 \times 7.59 + 2x8.90 \times 8.90 \times 8.90 + 2x7.59 \times 8.90)+$$
$$+(2x8.90 \times 8.90 + 2x9.33 \times 9.33 + 2 \times 8.90 \times 9.33)\Big]=$$

$$= \frac{3795 \times 2 \times 6.51}{6EJc}$$

$$= \frac{8235.15}{EJc}$$

The value of $\delta 22$ and $\delta 33$ are obtained in a similar way by raising to the second power the M_2 and M_3 graphs (fig 5. 1b and c)

$$\delta_{22} = \frac{2a}{6EJ}\ (52.08 \times 52.08 \times 2 + 45.57 \times 45.57 \times 2 + 52.08 \times 45.57 \times 2\ )$$
$$+(45.57 \times 45.57 \times 2 + 39.06 \times 39.06 \times 2 + 45.57 \times 39.06 \times 2\ )+$$
$$+(39.06 \times 39.06 \times 2 + 32.55 \times 32.55 \times 32.55 \times 2 + 39.06c32.55\ x2)+$$
$$+(32.55 \times 32.55x2 + 26.04x26.04x2 + 32.55 \times 26.04 \times 2\ )+$$
$$+(26.04x26.04x2 + 19.53x19.53x2 + 26.04x19.53 \times 2)+$$
$$+(19.53x19.53x2 + 13.02 \times 13.02 \times 2 + 13.02x2 + 19.53 \times 13.02\ x2)+$$
$$+(13.02x13.02x2 + 6.51 \times 6.51 \times 2 + 13.02 \times 6.51x2)+$$

$$+(6.51 \times 6.51 \times 2) = \frac{43397 \times 2x6.51}{6EJc}$$

$$= \frac{94171}{EJc}$$

$$\delta 33 = \frac{a}{EJc} \times 16 = \frac{104.10}{EJc.}$$

The displacements $\Delta 1q$, $\Delta 2q$ and $\Delta 3q$, due to the actual loading will be given by the product of the M_1, M_2 and M_3 graphs by the Mq graph fig 5.3

$$\Delta 1q = \frac{a}{6EJc}\Big[(-2x10266 \times 18.43 - 2x7860x11.94 -$$
$$10266 \times 11.94 - 7860 \times 18.43)+$$
$$+(-2 \times 7860 \times 11.94 - 6.29 \times 5775 \times 2 - 7860 \times 6.29 -$$
$$- 5775 \times 11.94) + (-2x 5775 \times 6.29 - 1.52 \times 4010 +$$
$$+ 2.39 \times 2567 \times 2 - 1.52 \times 2567 + 2.39 \times 4010)+$$
$$+ (2x 2.39 \times 4010 + 2x5.42 \times 1143 + 2.39 \times 1143 +$$
$$+ 4010 \times 5.42) + (2x5.42 \times 1143 + 2x7.59 \times 828 +$$
$$+ 5.42 \times 828 + 7.59 \times 1143) + (2x7.59 \times 828 +$$
$$+ 2x8.90 \times 160 + 8.90 \times 828 + 7.59 \times 160) +$$
$$(8.90 \times 160 + 9.33 \times 160)\Big] = - \frac{1280652}{EJc}$$

$$\Delta 2q = \frac{a}{6EJc}\Big[(2x10266 \times 52.08 + 2 \times 7860 \times 45.57 + 10266 \times$$
$$\times 45.57 + 7860 \times 52.08) + (45.57 \times 7860x2 + 5775 \times$$
$$\times 39.06x2 + 7860 \times 39.06 + 5775 \times 45.57) + (39.06 \times$$
$$\times 5775 \times 2 + 32.55 \times 4010 \times 2 + 32.55 \times 5775 + 39.06 \times$$
$$\times 4010) + (32.55 \times 4010 \times 2 + 26.04 \times 2567 \times 2 + 32.55 \times$$
$$\times 2567 + 26.04 \times 4010) + (26.04 \times 2567 \times 2 + 19.53 \times$$
$$\times 1143 \times 2 + 26.04 \times 1143 + 19.53 \times 2567)\Big]+$$

$$+ (19.53 \times 1143 \times 2 + 13.02 \times 828 \times 2 + 19.53 \times 823 + 13.02 \times$$

$$\times 1143) + (13.02 \times 828 \times 2 + 6.51 \times 160 \times 2 + 13.02 \times$$

$$\times 160 + 6.51 \times 828) + (6.51 \times 160 \times 2)] =$$

$$= \frac{6400729 \times 6.51}{6EJc}$$

$$= \frac{6944790.5}{EJc}$$

$$\Delta 3q = \frac{a}{EJc}\left[\left(\frac{10266}{2} + 7860 + 5775 + 4010 + 2567 + 1143 + 828 + 160\right)\right] = \frac{178869}{EJc}$$

Introducing these values into the displacement equations are obtain

$$X_1 = -\frac{\Delta 1q}{\delta 11} = -\frac{1280652}{8235} = 156KN$$

$$X_2 = -\frac{\Delta 2q}{22} = -\frac{6944791}{94171} = -74KN$$

$$X_3 = -\frac{\Delta 3q}{22} = \frac{178869}{104.10} = 1718KNm$$

The M, Q and N diagrams will now be obtained by applying to the elastic centre of the conjugate structure two forces $X_1 = 156KN$ $X_2 = -74KN$ and a couple $X_3 = 1718$ KN-m.

a) For the left arch and right since they are symmetrical

$$M = X_1 (Y-Ys) + X_2 x + X_3 - \frac{qx^2}{2}$$

$$Q = X_1 \text{ Sin } \theta + X_2 \text{ Cos } \theta - qx \text{ Cos } \theta$$

ORDINATES TO THE M-DIAGRAM Table 5.2

Section No.	X_1	$\underline{Y}_s$-Y	$X_1(Y_s-Y)$	X_2	X	X_2x	X_3	$-qx^2/2$	Ordinates to the M diagram
0		-18.43	-2875		-52.08	3854		-10266	-7569
1		-11.94	-1863		-45.57	3372		-7860	-4633
2		- 6.29	-981		-39.06	2890		-5775	-2148
3		- 1.52	-237		-32.55	2409		-4010	-120
4	156	2.39	373	-74	-26.04	1927	1718	-2567	1451
5		5.42	846		-19.53	1445		-1444	2565
6		7.59	1184		-13.02	963		-642	3223
7		8.90	1388		-6.51	482		-160	3428
8		9.33	1455		0	0		0	3173

ORDINATES TO THE Q - DIAGRAM TABLE 5.3

Section No.	X_1	Sin θ	$-X_1$ Sin θ	X_2	Cos θ	X_2Cosθ	qx	-qxCosθ	Ordinate to the Qk-Dia.
0		-0.7071	-110		0.7071	-52	-394	279	117
1		-0.6847	-107		0.7288	-54	-345	-251	90
2		-0.6293	- 98		0.7771	-56	-296	228	74
3		-0.5592	- 87		0.8290	-61	-246	204	56
4	156	-0.4813	- 75	-74	0.8766	-65	-197	173	33
5		-0.3854	- 60		0.9227	-68	-148	137	9
6		-0.2770	- 43		0.9607	-71	- 99	.95	-19
7		-0.1567	- 24		0.9877	-73	- 49	48	-49
8		0	0		1.0000	-74	0	0	-74

ORDINATES TO THE N-DIAGRAM TABLE 5.4

Section No.	X_1	Cosθ	X_1 Cosθ	X_2	Sinθ	X_2Sinθ	qx	-qxSinθ	Ordinate to the N-diagram
0		0.7071	110		0.7071	.52	-394	279	337
1		0.7288	114		0.6847	-51	-345	236	299
2		0.7771	121		0.6293	-47	-296	186	260
3		0.8290	129		0.5592	-41	-246	138	226
4		0.8766	137		0.4813	-36	-197	95	196
5	156	0.9227	144	-74	0.3854	-29	-148	57	172
6		0.607	150		0.2770	-20	-99	27	157
7		0.9877	154		0.1567	-12	-49	8	150
8		1.0000	156		0	0	0	0	156

The Bending moments, shear force and normal force induced in the structure due to the live loads will be calculated assuming that the left semi-arch is loaded with a live load of 0.75KN/mx1.6 while the right semi-arch is not loaded.

The live load = 0.75 x 1.60 = 1.20KN/m

δ_{11}, δ_{22} and δ_{33} will remain the same with those of the dead load because they are due to the unit actions applied at the elastic centre.

The displacements Δ_{1q}, Δ_{2q} and Δ_{3q}, due to the live loading will be given by the product of the M1, M2 and M3 graphs by the Mq graph.

The loads at the apices of section 0 and 8 = $\frac{qa}{2} = \frac{1.20 \times 6.51}{2} = 3.906 =$

= 4KN/m

and those of section 1, 2, 3, 4, 5, 6 and 7 are = qa = 1.20 x 6.51 =

7.8 = 8KN/m

Moments due to applied live load

section 8 = 4 x 0 = 0

" 7 = 4 x 6.51 = 26KNm

" 6 = 4 x 13.02 + x 6.51 = 104 KNm

" 5 = 4 x 19.53 + 8 x 13.02 + 8 x 6.51 = 234KNm

" 4 = 4 x 26.04 + 8x19.53 + 8x13.02 + 8x6.51 = 417KNm

" 3 = 4 x 32.55 + 8x26.04 + 8x19.53 + 8x13.02 +
+ 8 x 6.51 = 651KNm

" 2 = 4 x 39.06 + 8x32.55 + 8x26.04 + 8x19.53 +
+ 8 x 13.02 + 8x6.51 = 937KNm

" 1 = 4 x 45.57 + 8x39.06 + 8x32.55 +
+ 8 x 26.04 + 8 x 19.53 + 8x 13.02 +
+ 8 x 6.51 = 1276 KNm

" 0 = 4 x 52.08 + 8x45.57 + 8 x 39.06 + 8 x 32.55 +
+ 8 x 26.04 + 8 x 19.53 + 8 x 13.02 + 8x 6.51 =
= 1667KNm

$$\Delta 1q = \frac{a}{6EJc}\Big[(-2 \times 1667 \times 18.43 - 2 \times 1276 \times 11.94 - 1667 \times 11.94 - 1276 \times 18.43) + (-2x1276 \times 11.94 - 2 \times 937 \times 6.27 - 1276 \times 6.27 - 937 \times 11.94) + (-2 \times 937 \times 6.27 - 2 \times 651 \times 1.52 - 937 \times 1.52 - 651 \times 6.27) + (2x\ 651 \times 1.52 + 2 \times 417 \times 2.39 - 6.51 \times 5.42 + 417 \times 1.52) + (2 \times 417 \times 2.39 + 2 \times 234 \times 5.42 + 417 \times 5.42 + 234 \times 2.39) + (2 \times 234 \times 5.42 + 2 \times 104 \times 7.59 + 234 \times 7.59 + 104 \times 5.42) + (2x\ 104 \times 7.59 + 2 \times 26 \times 8.90 + 104 \times 8.\ 8.90 + 26 \times 7.59)(26 \times 8.9 + 26 \times 9.33)\Big] =$$

$$= \frac{6.51}{6EJc} \times 194964 = \frac{211536}{EJc}$$

$$\Delta 2q = \frac{a}{6EJc}\Big[(2 \times 1667 \times 52.08 + 2 \times 1276 \times 45.57 + 1667 \times 45.57 + 1276 \times 52.08) + (2 \times 1276 \times 45.57 + 2 \times 937 \times 39.06 + 1276 \times 39.06 + 937 \times 45.57) + (2 \times 937 \times 39.06 + 2 \times 651 \times 32.55 + 937 \times 32.55 + 651 \times 39.06) + (2 \times 651 \times 32.55 + 417 \times 26.04 \times 2 + 651 \times 26.04 + 417 \times 32.55) + (2 \times 417 \times 26.04 + 2 \times 234 \times 19.53 + 417 \times 19.53 + 234 \times 26.04) + (2 \times 234 \times 19.53 + 2 \times 104 \times 13.02 + 234 \times 13.02 + 104 \times 19.53) + (2 \times 104 \times 13.02 + 2 \times 26 \times 6.51 + 104 \times 6.51 + 26 \times 13.02) + (2 \times 26 \times 6.51)\Big] = \frac{1135927}{EJc}$$

$$\Delta 3q = \frac{a}{EJc}\Big[(1667 + 1276 + 937 + 651 + 417 + 234 + 104 + 26)\Big] = \frac{5312 \times 6.51}{EJc}$$

$$= \frac{34581}{EJc}$$

$$X_1 \text{ due to live load} = -\frac{\Delta 1q}{\delta 11} = -\frac{211536}{8235} = 26KN$$

$$X_2 \text{ due to live load} = -\frac{\Delta 2q}{\delta 22} = \frac{113592}{94171} = -1.2KN$$

$$X_3 \text{ due to live load} = -\frac{\Delta 3q}{\delta 33} = \frac{34581}{104.10} = 332KNm.$$

The M, Q and N diagrams will now be obtained by applying to the elastic of the conjugate structure two forces X_1 = 26KN, X_2 - 1.2KN and a couple X_3 = 332KNm.

a) For the left semi-arch

$$M = X_1 (y - ys) + X_2 x + X_3 - qs^2$$

$$Q = -X_1 \text{ Sin } \theta + X_2 \text{ Cos}\theta - qx\text{Cos } \theta$$

$$N = X_1 \text{ Cos}\theta + X_2 \text{ Sin } \theta - qx\text{Sin}\theta$$

b) For the right Semi-arch.

$M = X_1 (y - ys) + X_2 x + X_3$

$Q = -X_1 \text{ Sin } \theta + X_2 \text{ Cos } \theta$

$N = X_1 \text{ Cos}\theta + X_2 \text{ Sin } \theta$

ORDINATES TO M DIAGRAM TABLE 5.5

Section No.	X_1	$y_s - y$	$X_1(y_s - y)$	X_2	X	$X_2 x$	X_3	$-\frac{qx^2}{2}$	Ordinates to the M-diagram in KNm
0		-18.43	-479		-52.08	62		-1627	-1712
1		-11.94	-310		-45.57	55		-1246	- 1177
2		- 6.29	-162		-39.06	47		-915	-698
3		- 1.52	-40		-32.55	39		-636	-305
4		2.39	62		26.04	31		-407	-106
5		5.42	141		-19.53	23		-229	267
6		7.59	197		-13.02	16		-102	443
7		8.90	231		- 6.51	8		- 25	546
8,8¹	26	9.33	243	-1.20	0	0	332	0	575
7¹		8.90	231		6.51	-8		-	555
6¹		7.59	197		13.02	-16		-	513
5¹		5.42	141		19.53	-23		-	450
4¹		2.39	62		26.04	-31		-	363
3¹		-1.52	-40		32.55	-39		-	253
2¹		-6.29	-162		39.06	-47		-	123
1¹		-11.94	-310		45.57	-55		-	-33
0¹		-18.43	-479		52.08	-62		-	-209

left semi arch / right semi arch

ORDINATES TO THE Q - DIAGRAM Table 5.6

Section No.	X_1	Sin θ	$-X_1$ Sinθ	X_2	Cosθ	X_2Cosθ	qx	-qxCosθ	Ordinates to the Q Diagram
0		0.7071	-18		0.7071	-0.80	-62	44	25.20
1		0.6847	-18		0.7288	-0.90	-55	40	21.10
2		0.6293	-16		0.7771	-0.90	-47	37	20.10
3		0.5592	-15		0.8290	-1.00	-39	32	16.00
4		0.4813	-13		0.8766	-1.05	-31	27	12.95
5		0.3854	-10		0.9227	-1.12	-23	21	9.88
6	26	0.2770	-7	-1.20	0.9607	-1.15	-16	15	6.85
7		0.1567	-4		0.9877	-1.19	- 8	8	2.81
$8,8^1$		0	0		1.00	-1.20	0	0	-1.20
7^1		-0.1567	4		0.9877	1.19	-		5.19
6^1		-0.2770	7		0.9607	1.15	-		8.15
5^1		-0.3854	10		0.9227	1.12	-		11.12
4^1		-0.4813	13		0.8766	1.05	-		14.05
3^1		-0.5592	15		0.8290	1.00	-		16.00
$2^1 8$		-0.6293	16		0.7771	0.90	-		16.90
1^1		-0.6847	18		0.7288	0.90	-		18.90
0^1		-0.7071	18		0.7071	0.80	-		18.80

right semi arch

left semi arch

ORDINATES TO THE N-DIAGRAM TABLE 5.7

	Section No.	X_1	Cos θ	X_1 Cos θ	X_2	Sin θ	X_2Sinθ	qx	-qxSinθ	Ordinates to N diagram in FN
right semi arch	0		0.7071	18		0.7071	-0.85	-62	44.00	61
	1		0.7288	19		0.6847	-0.82	-55	38.00	56
	2		0.7771	20		0.6293	-0.76	-47	30.00	49
	3		0.8290	22		0.5592	-0.67	-39	22.00	43
	4		0.8766	23		0.4818	-0.58	-31	15.00	37
	5		0.9227	24		0.3854	-0.46	-23	9.00	33
	6		0.9607	25		0.2770	-0.33	-16	4.00	29
	7		0.9877	26	-1.20	0.1567	-0.19	- 8	1.25	27
	$8,8^1$	26	1.0000	26		0	0	0	0	26.00
left semi arch	7^1		0.9877	26		-0.1567	0.19	-	-	26.19
	6^1		0.9607	25		-0.2770	0.33	-	-	25.33
	5^1		0.9227	24		-0.3854	0.46	-	-	24.46
	4^1		0.8766	23		-0.4818	0.58	-	-	23.58
	3^1		0.8290	22		-0.5592	0.67	-	-	22.67
	2^1		0.7771	20		-0.6293	0.76	-	-	20.76
	1^1		0.7288	19		-0.6847	0.82	-	-	19.82
	0^1		0.7071	18		-0.7071	0.85	-	-	18.85

WIND FORCE CALCULATIONS

The first method of approximate stress analysis of fixed end arches, described will be used for the calculation of the wind forces. The Design wind velocity is 144Km per hour (40mls). For a wind velocity of this magnitude the wind pressures corresponding to this wind velocity for various heights are given below. (NCP)

0	-	3.0m	=	$70N/m^2$
3m	-	6m	=	$80N/m^2$
6m	-	9m	=	$95N/m^2$
9m	-	12m	=	$110N/m^2$
12m	-	15m	=	$125N/m^2$
15m	-	18m	=	$140N/m^2$
18m	-	21m	=	$155N/m^2$
21m	-	24m	=	$158N/m^2$
24m	-	27m	=	$170N/m^2$

Wind ward load			lee ward load
70 x 7.5 x 0.67	=	352N/m	173N/m
80 x - " -	=	396N/m	198N/m
95 - " -	=	477N/m	235N/m
110 - " -	=	553N/m	272N/m
125 - " -	=	628N/m	309N/m
140 - " -	=	704N/m	347N/m
155 - " -	=	779N/m	384N/m
158 - " -	=	791N/m	390N/m
170 - " -	=	854N/m	421N/m

The wind load is taken to be uniformly distributed on the whole structure's height, with every two successive segments having the same value of the UDL due to the wind.

For segment 1 and 2 the UDL due to the wind is (352 + 396 + 477 + 553) 0.25 = 445N/m

Segment 3 and = (553 + 628 + 704) ÷3 = 630N/m

Segment 5 and 6 = (704 + 779 + 791 + 854) 0.25 = 782N/m

Segment 7 and 8 = 854N/m

For the leeward side :-

Segment 1 and 2 = (173 + 198 + 235 + 272) 0.25
= 220N/m

Segment 3 and 4 = (272 + 309 + 347) x $\frac{1}{3}$ = 309N/m

Segment 5 and 6 = (347 + 384 + 390 + 421) 0.25 = 386N/m

Segment 7 and 8 = 421N/m.

The unit displacements have been calculated and are

$$\delta_{11} = \frac{8235.15}{EJc},$$

$$\delta_{22} = \frac{94171}{EJc}$$

and $$\delta_{33} = \frac{104.10}{EJc}$$

The displacements $\Delta 1q$, $\Delta 2q$ and $\Delta 3q$ due to the actual loading will be given by the product of the $\bar{M}1$, $\bar{M}2$ and $\bar{M}3$ graphs by the Mq graph (fig 5.3)

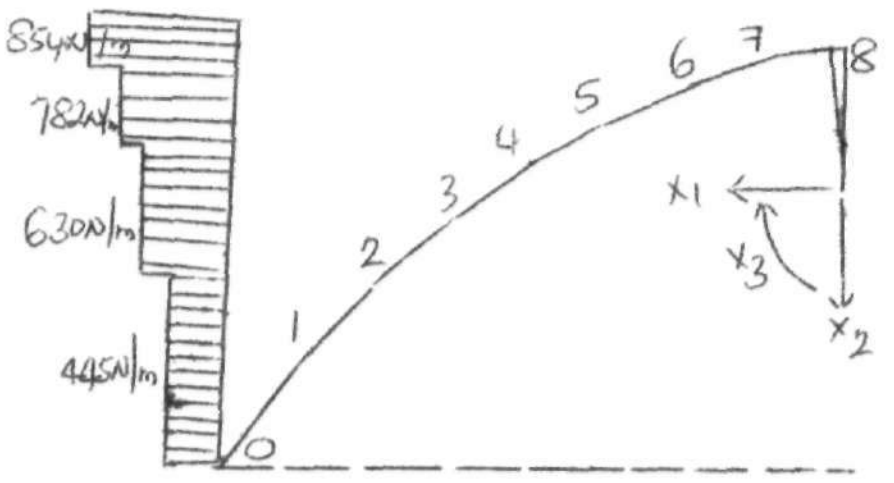


FIG 5.3

Lenghts of Sections = 1 = 9.20m

2 = 8.93m

3 = 8.38m

4 = 7.85m

5 = 7.43m

6 = 7.06m

7 = 6.78m

8 = 6.59m

loads at the points:-

Point 0 = $445 \times \frac{9.20}{2}$ - 2.05KN

Point 1 = $445 \times \frac{9.20 + 8.93}{2} = 4.03KN$

Point 2 = $445 \times \frac{8.93 + 8.38}{2} = 3.85KN$

Point 3 = $630 \times \frac{8.38 + 7.85}{2} = 5.11$KN

Point 4 = $630 \times \frac{7.85 + 7.43}{2} = 4.81$KN

Point 5 = $782 \times \frac{7.43 + 7.06}{2} = 5.67$KN

Point 6 = $782 \times \frac{7.06 + 6.78}{2} = 5.41$KN

Point 7 = $854 \times \frac{6.78 + 6.59}{2} = 5.71$KN

Point 8 = $854 \times \frac{6.59}{2} = 2.81$KN

Moments due to the wind loads :-

Section 8 = 2.81 x 0 = 0

Section 7 = 2.81 x 0.43 = 1,20KNm

Section 6 = 2.81 x 1.74 + 5.71 x 1.31 = 7.48KNm = 7KNm

Section 5 = 2.81 x 3.91 + 5.71 x 3.48 + 5.41 x 2.17 = 43KNm

Section 4 = 2.81 x 6.94 + 5.71 x 6.51 + 5.41 x 4.77 +
+ 5.67 x 3.03 = 142KNm

Section 3 = 2.81 x 10.85 + 5.71 x 10.42 + 5.41 x 8.68 +
+ 5.67 x 6.94 + 4.81 x 3.91 = 195KNm

Section 2 = 2.81 x 15.62 + 5.71 x 15.19 + 5.41 x 13.45 +
+ 5.67 x 11.71 + 4.81 x 8.68 + 5.11 x 4.77 =
= 336KNm

Section 1 = 2.81 x 21.27 + 5.71 x 20.84 + 5.41 x 19.10 + 5.67 x
x 17.36 + 4.81 x 14.33 + 5.11 x 10.42 + 3.85 x
x 5.65 = 524KNm

Section 0 = 2.81 x 27.27 + 5.71 x 27.32 + 5.41 x 25.58 +
+ 5.67 x 23.84 + 4.81 x 20.81 + 5.11 x 16.90 +
+ 3.85 x 12.13 + 4.03 x 6.48 = 767KNm.

$\Delta_{1q} = \frac{a}{6EJc}$ [(-2x767 x 18.43 - 2 x 524 x 11.9 - 767 x 11.94 -
- 524 x 18.43)+ (-2x336 x 6.29 - 2 x 195 x 1.52 -
- 336 x 1.52 - 195 x 6.29) + (-2 x 195 x 152 +
+ 2 x 142 x 2.39 - 195 x 2.39 + 142 x 1.52) +
+ (2 x 142 x 2.39 + 2 x 43 x 5.42 + 142 x 5.42 +
+ 43 x 2.39) + (2x 43 x 5.42 + 2 x 7 x 7.59 +
+ 43 x 7.59 + 7 x 5.42) + (2 x 7 x 7.59 + 1.20 x
x 8.90 x 2 + 7 x 8.90 + 1.20 x 7.59) + (1.20 x
x 8.90 + 9.33 x 1.20)]

$= \frac{-94864}{EJc}$

$\Delta_{2q} = \frac{6.51}{6EJc}$ (2 x 767 x 52.08 + 2 x 524 x 45.57 + 767 x 45.57 +
+ 524 x 52.08) + (2x524 x 45.57 + 2 x 336 x 39.06 +
+ 524 x 39.06 + 336 x 45.57) + (2 x 336 x 39.06 +
+ 2 x 195 x 32.55 + 336 x 32.55 + 195 x 39.06)
+ (2 x 195 x 32.55 + 2 x 142 x 26.04 + 195 x
x 26.04 + 142 x 32.55) + (2 x 142 x 26.04 + 2 x 43 x
x 19.53 + 142 x 19.53 + 142 x 19.53 + 43 x 26.04) +
+ (2 x 43 x 19.53 + 2 x 7 x 13.02 + 43 x 13.02 +
+ 7 x 19.53) + (2 x 7 x 13.02 + 2 x 1.20 x 6.51 +
+ 7 x 6.51 + 1.20 + 1.20 x 13.02) + (2x 1.20
x 6.51) =

$= \frac{436997}{EJc}$

$$\Delta 3q = \frac{6,51}{EJc} \left(767 + 524 + 336 + 195 + 142 + 43 + 7 + 1.20 \right)$$

$$= \frac{13119}{EJc}.$$

Introducing these values into the displacement equations we obtain:-

X_1 for the wind ward side $= -\frac{\Delta 1q}{11} = \frac{.94864}{8235}$

$= 11.52 = 12KN.$

X_1 for the leeward side will be approximate 6KN.

X2 for the windward side $= -\frac{436997}{94171} =$

$= -4.64 = -5KN$

X2 for the leeward side = -2.5KN

X3 for the windward side $= \frac{13119}{104.10} = 126KNm$

X3 for the leeward side = 63KNm.

The M, Q and N diagrams will now be constructed or obtained by applying to the elastic centre of the conjugate structure two forces 12KN, -5KN and couple 126KNm for the wind ward side and 6KN -2.5KN and 63KNm for the leeward side.

a) For the left semi-arch and right semi-arch

$$M = X1 (y_s - y) + X_2 x + X_3 - \frac{qy^2}{2}$$

$$Q = X_1 \text{ Sin } \theta + X_2 \text{ Cos } \theta - qy \cos?$$

$$N = X_1 \cos \theta + X_2 \text{ Sin } \theta - qy \sin 0$$

All calculations relative to the ordinates to the M, Q and N diagrams are entered into tables 5.8, 5.9 and 5.10 respectively.

ORDINATES TO THE M-DIAGRAM Table 5.8

Section No.	X_1	y_s-y	$X_1(y_s-y)$	X_2	x	X_2x	X_3	$-\frac{qy^2}{2}$	Ordinates to the M-diagram in KNm
0		-18.43	-221		-52.08	260		-171	-6
1		-11.94	-143		-45.57	228		-101	110
2		- 6.29	- 75		-39.06	195		- 77	169
3		- 1.52	- 18		-32.55	163		- 37	234
4		2.39	29		-26.04	130		- 15	270
5		5.42	65		19.53	98		- 6	283
6		7.59	79		13.02	65		- 1	269
7		8,90	107		-6.51	33		-0.08	266
8		9.33	112		0	0		0	238
8^1		9.33	56		0	0		0	119
7^1		8.90	54		6.51	-17		-0.04	100
6^1		7,59	40		13.02	-33		-0.5	70
5^1		5.42	33		19.53	-49		-3	44
4^1		2.39	15		26.04	-65		-8	5
3^1		1.52	-9		32.55	-81		-19	-49
2^1		-6.29	-38		39.06	-98		-39	-112
1^1		-11.94	-72		45.57	-114		-52	-175
0^1		-18.43	-111		52.08	-130		-86	-378

right semi arch: X_1 = 12; X_2 = 5; X_3 = 126

left semi arch: X_1 = 6; X_2 = -2.5; X_3 = 63

ORDINATES TO THE Q-DIAGRAM Table 5.9

Section No.	X_1	Sinθ	$-X_1$Sinθ	X_2	Cosθ	X_2 Cos θ	-qy	-qy Cos θ	Ordinates to the Q diagram
0	12	0.7071	-8.50	-5	0.7071	-3.54	-12	-8.50	-20.54
1		0.6847	-8.20		0.7288	-3.64	- 9	-6.56	-18.40
2		0.6293	-7.60		0.7771	-3.89	- 7	-5.44	-16.93
3		0.5592	-6.70		0.8290	-4.14	- 7	-5.80	-16.65
4		0.4813	-5.80		0.8766	-4.38	- 4	-3.51	-13.69
5		0.3854	-4.60		0.9227	-4.61	- 3	-2.77	-11.98
6		0.2770	-3.30		0.9607	-4.80	- 1	-0.96	- 9,06
7		0.1567	-1.90		0.9877	-4.94	- 0	-	- 6.84
8		0	0		1.0000	-5.00	θ	-	- 5.00
8^1	6	0	0	-2.5	1.0000	-2.50	-	-	- 2.50
7^1		0.1567	0.85		0.9877	-2.47	-	-	- 1.62
6^1		0.2770	1.65		0.9607	-2.40	-0.50	-0.48	- 1.23
5^1		0.3854	2.30		0.9227	-2.31	-1.50	-1.89	- 1.90
4^1		0.4813	2.90		0.8766	-2.19	-2.0	-1.86	- 1.15
3^1		0.5592	3.35		0.8290	-2.10	-3.50	-2.40	- 1.15
2^1		0.6293	3.80		0.7771	-1995	-3.50	-2.72	- 0.87
1^1		0.6847	4.10		0.7288	-1.32	-4.50	-3.28	- 0.50
0^1		0.7071	4.25		0.7071	-1.77	-6.00	-4.25	- 1.77

Left semi arch / right semi arch

ORDINATES TO THE N- DIAGRAM Table 5.10

Section No.	X_1	Cosθ	X_2Cosθ	X_2	Sin θ	X_2Sinθ	-qy	-qy Sinθ	Ordinates to the Q diagram
0		0.7071	8.50		0.7071	-3.54	-12	8.49	13.45
1		0.7288	8.70		0.6847	-3.42	- 9	6.16	11.44
2		0.7771	9.30		0.6293	-3.15	-7	4.41	10.56
3		0.8290	9.90		0.5592	-2.80	- 7	3.91	11.01
4		0.8766	10.50		0.4813	-2.41	- 4	1.93	10.02
5		0.9227	11.10		0.3854	-1.93	- 3	1.16	10.33
6		0.9607	11.50		0.2770	-1.39	- 1	0.28	10.39
7		0.9877	11.90		0.1567	-0.78	0.23	0.04	11.16
8		1.0000	12.00		0	0	0	0	12.00
8^1		1.0000	6.00		0	0	0	0	6.00
7^1		0.9877	5.95		-0.1567	0.99	-	-0.02	6.32
6^1		0.9607	5.75		-0.2770	0.66	-0.50	-0.14	6.27
5^1		0.9227	5.55		-0.3854	0.97	-1.50	-0.58	5.94
4^1		0.8766	5.25		-0.4813	1.21	-2.00	-0.97	5.49
3^1		0.8290	4.95		-0.5592	1.40	-3.50	-1.96	4.20
2^1		0.7771	4.65		-0.6293	1.58	-3.50	-2.21	4.02
1^1		0.7288	4.35		-0.6847	1.71	-4.50	-3.08	2.98
0^1		0.7071	4.25		-0.7071	1.77	-1.77	-4.25	1.77

left semi arch: X_1 = 12, X_2 = -5

right semi arch: X_1 = 6, X_2 = -2.50

The bending moments, shears and Normal forces due to the combined effects of dead load, live load and wind loads are given tables 5.11, 5.12 and 5.13 respectively.

TOTAL ORDINATES TO THE BENDING MOMENTS DIAGRAM Table 5.11

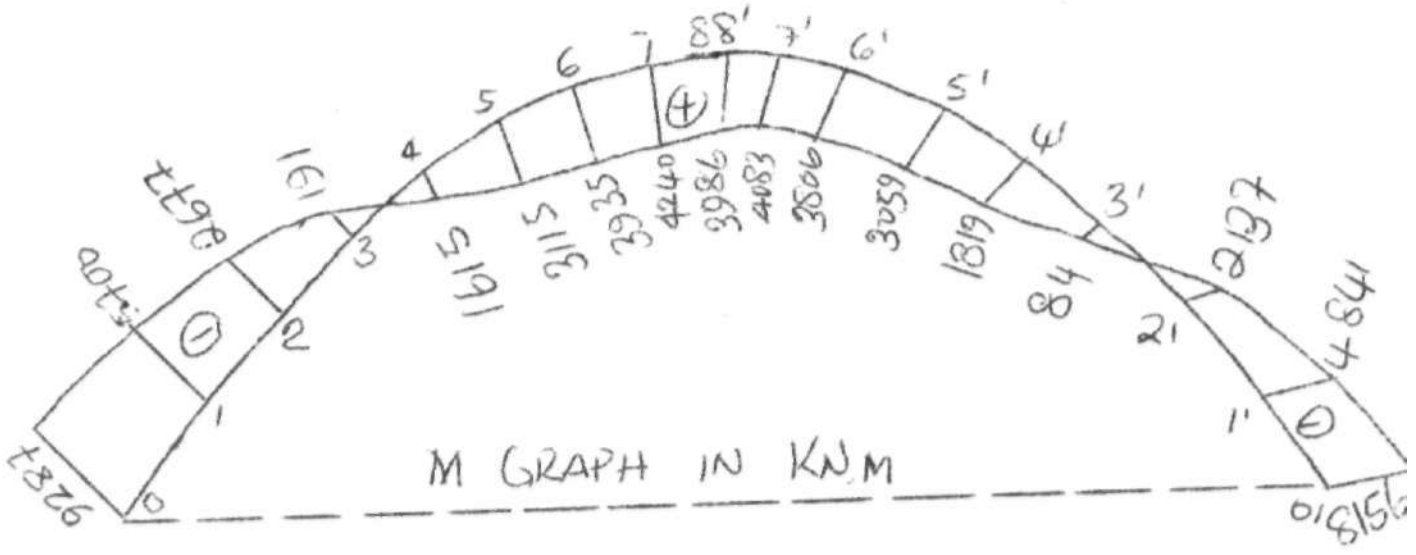
Section No.		Moments due to dead load	Moments due to live load	Moments due to wind load	Total Moment IN KNM
right semi arch	0	-7569	-1712	- -6	-9287
	1	-4633	-1177	110	-5700
	2	-2148	-698	169	-2677
	3	-120	-305	234	-191
	4	1451	-106	270	1615
	5	2565	267	283	3115
	6	3223	443	269	3935
	7	3428	546	266	4240
	8	3173	575	238	3986
left semi arch	0	-7569	-209	-378	-8156
	1	-4633	- 33	-175	-4841
	2	-2148	123	-112	-2137
	3	-120	253	- 49	84
	4	1451	363	5	1819
	5	2565	450	44	3059
	6	3223	513	70	3806
	7	3428	555	100	4083
	8	3173	575	119	3867

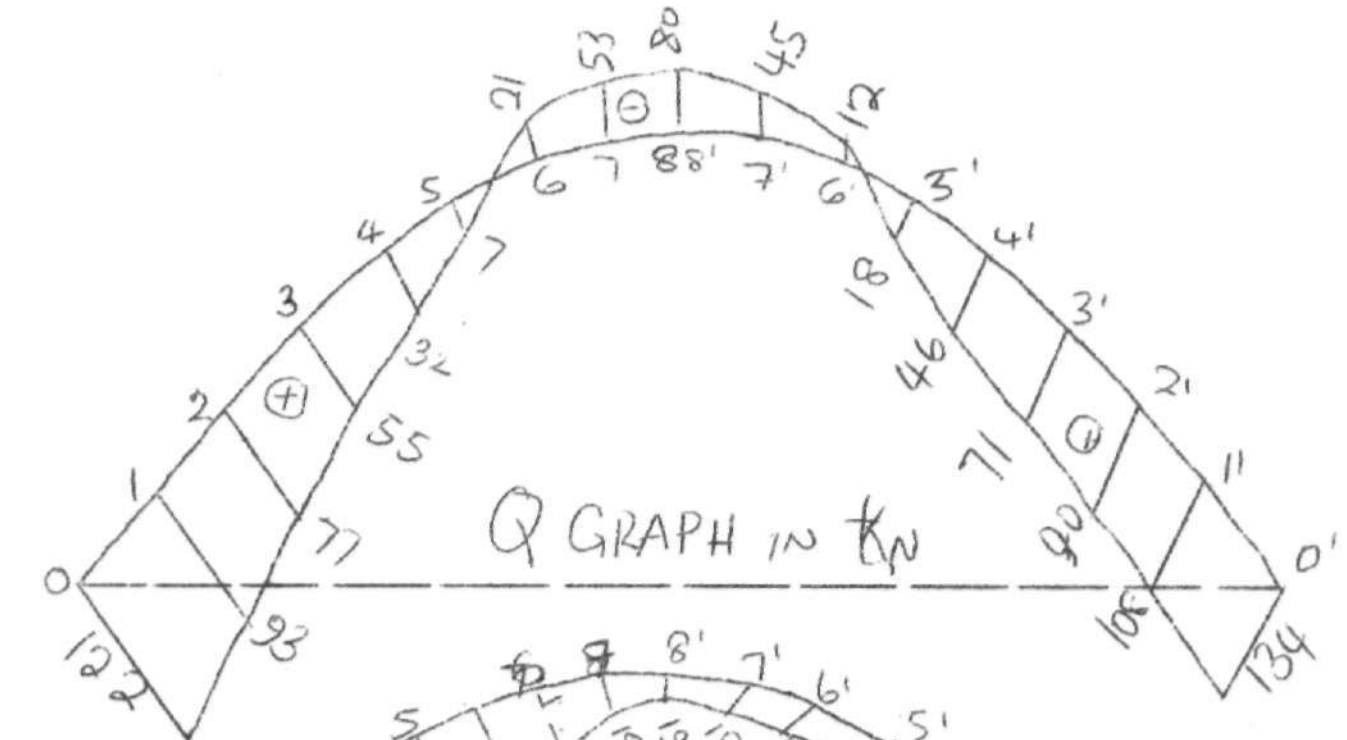
TOTAL ORDINATES TO THE SHEAR DIAGRAM TABLE 5.12

	Section No.	Shear due to the dead load	Shear due to the live load	Shear due to the wind load	Total Shear in KN
left semi arch	0	117	25.20	-20.54	122
	1	90	21.10	-18.40	93
	2	74	20.10	-16.93	77
	3	56	16.00	-16.65	55
	4	33	12.95	-13.69	32
	5	9	9.88	-11.98	7
	6	-19	6.85	-9.06	21
	7	-49	2.81	-6.84	-53
	8	-74	-1.20	-5.00	-80
right semi arch	8'	-74	-1.20	-2.50	-78
	7'	-49	5.19	-1.62	-45
	6'	-19	8.15	-1.23	-12
	5'	9	11.12	-1.90	18
	4'	33	14.05	-1.15	46
	3'	56	16.00	-1.15	71
	2'	74	16.90	-0.87	90
	1'	90	1890	-0.50	108
	0'	117	18.80	-1.77	134

TOTAL ORDINATES TO THE N-DIAGRAM Table 5.13

	Section No.	Normal force due to dead load	Normal force due to live load	Normal force due to wind load	Total Normal force
left semi arch	0	337	61 1	13.45	411
	1	290	56	11.44	366
	2	260	49	10.56	320
	3	226	43	11.01	280
	4	196	37	10.02	243
	5	172	33	10.33	215
	6	157	29	10.39	196
	7	150	27	11.16	188
	8	156	26	12.00	194
right semi arch	8^1	156	26	6.00	188
	7^1	150	26.19	6.32	182
	6^1	157	25.33	6.27	188
	5^1	172	24.46	5.94	202
	4^1	196	23.67	5.49	225
	3^1	226	22.67	4.20	253
	2^1	260	20.76	4.02	285
	1^1	299	19.82	2.98	322
	0^1	337	18.85	1.77	358


M GRAPH IN KN.M


Q GRAPH IN KN

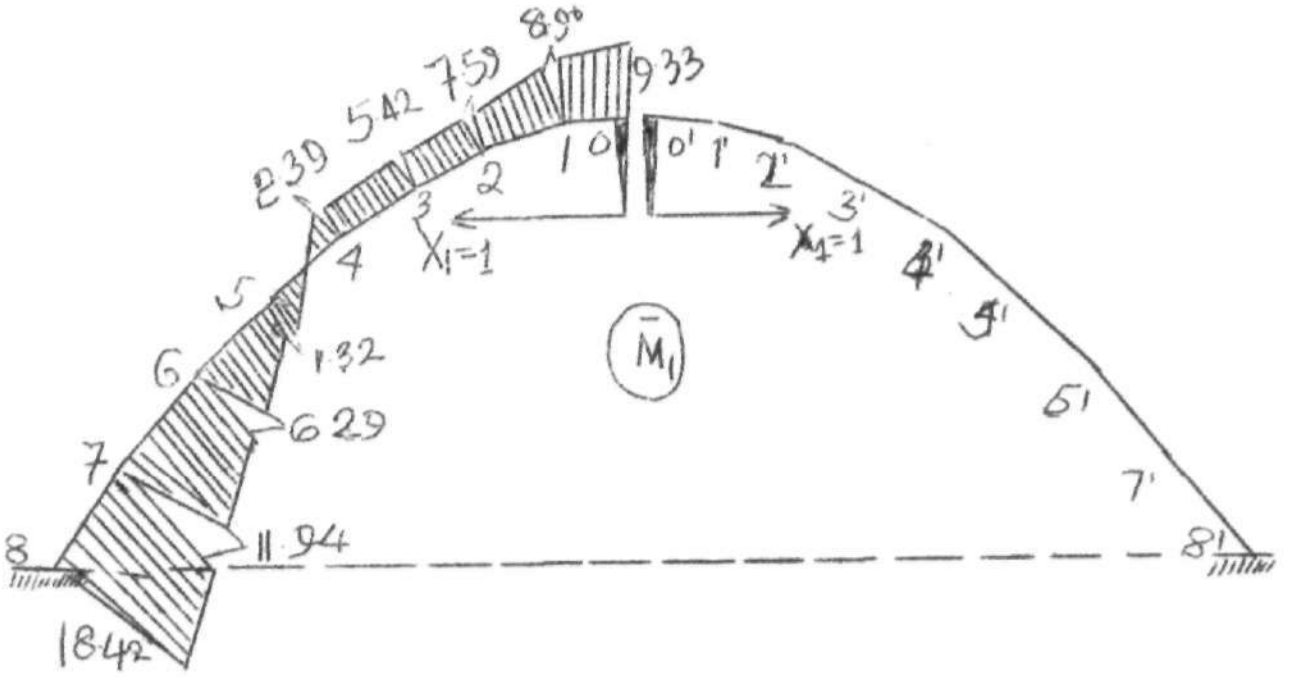

N- GRAPH IN KN

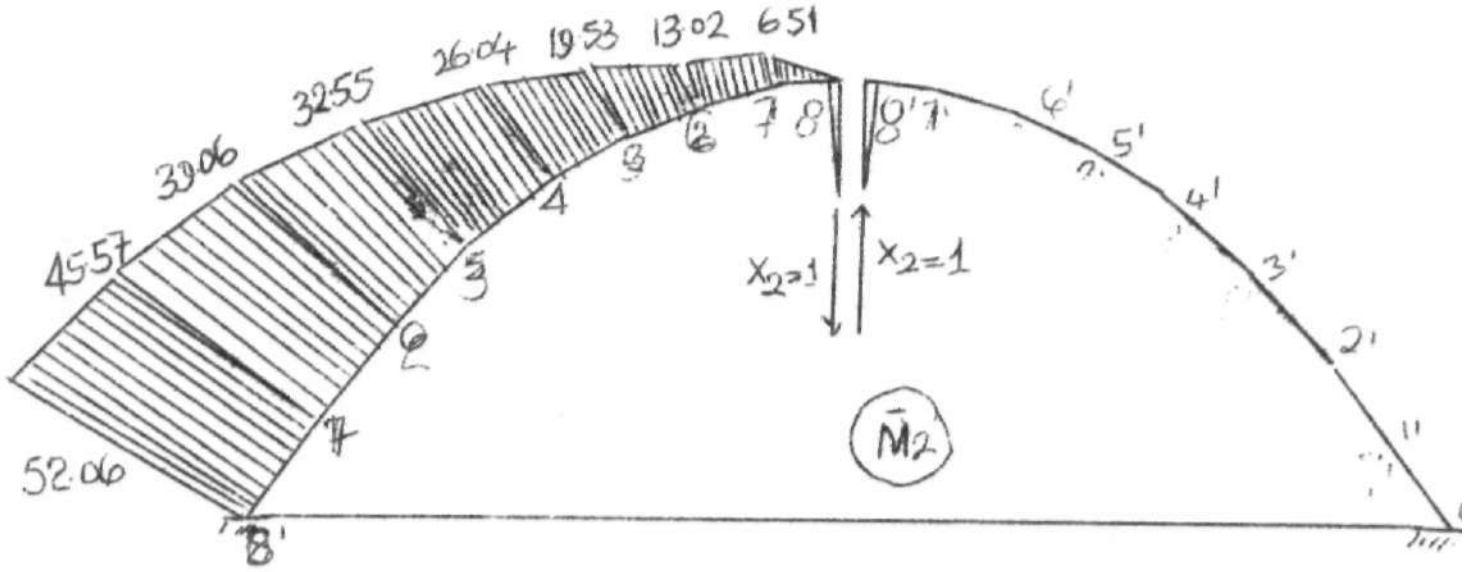
5.4. STRESS ANALYSIS OF ARCHED BEAM BY INFLUENCE LINES

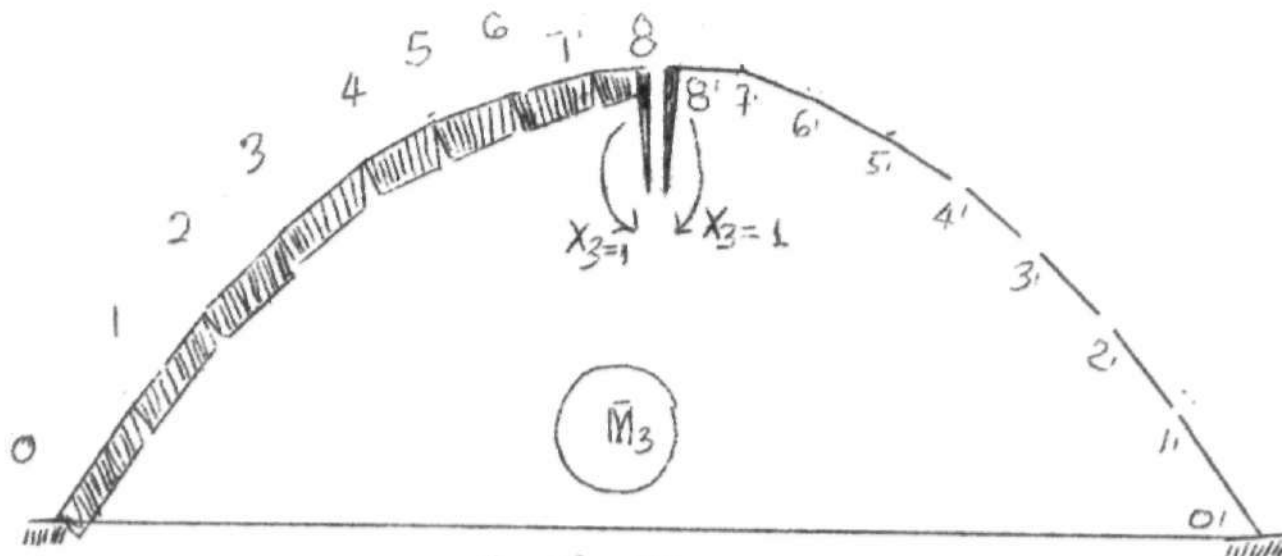
Under this section the influence lines for the redundant reactions X_1, X_2 and X_3 as well as those for the stresses Mk, Nk and Qk acting at section K of the arch represented in Fig. 5.1 will be calculated and drawn. The Section K is taken at 13.02m from the left abutment.

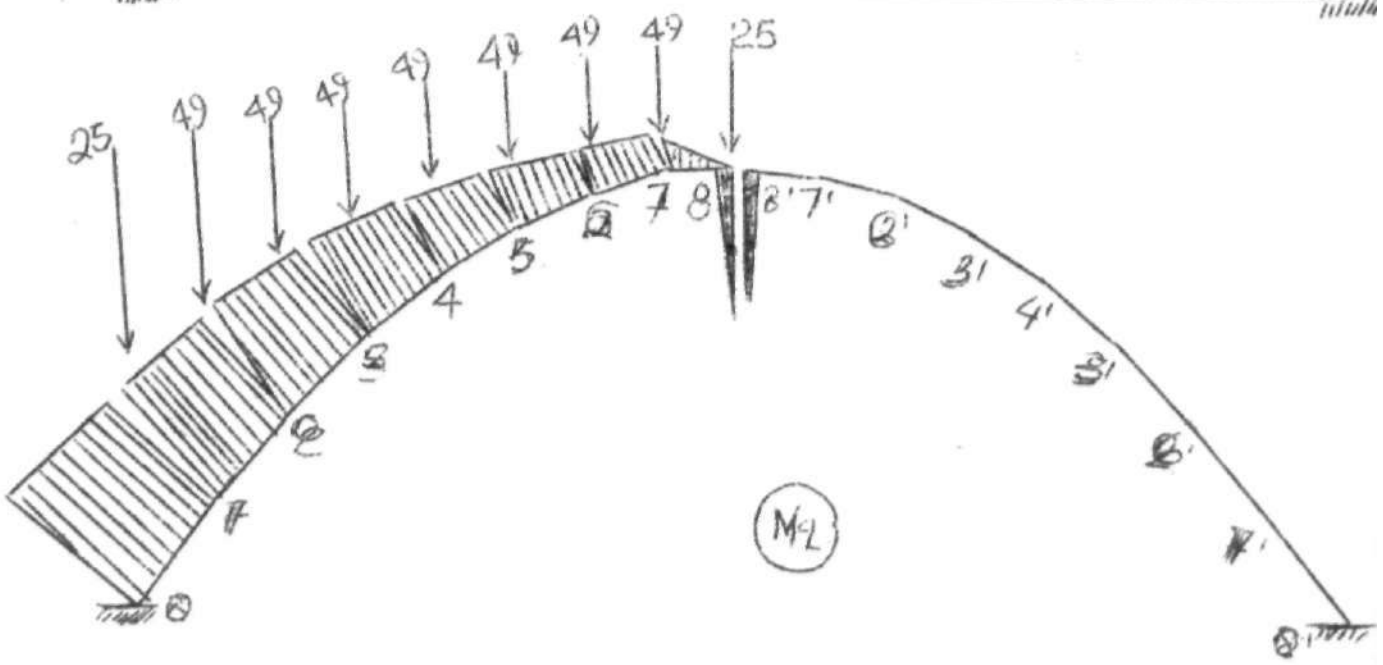
The coordinates of the inscribed polygen apices as well as the ordinate ys of the elastic centre of the arch have been calculated under Section 5.3 in Figs. 5.1b and c.

The bending moments induced in the structure due to the unit actions X_1, X_2 and X_3 applied along the redundant constraints are given in the figures below:




52.06
45.57
39.06
32.55
26.04
19.53
13.02
6.51
$X_2=1$
$X_2=1$
$\bar{M}_2$


$X_3=1$
$X_3=1$
$\bar{M}_3$


25
49
49
49
49
49
49
49
25
M_q

The Simplified expressions for the elastic loads will be used.

$$Wn = \frac{Sn}{6EJn}(Mn-1 + 2Mn) + \frac{Sn+1}{6EJn+1}(2Mn+Mn+1) - En\tan\beta n + En+1\tan\beta n$$

The longitudinal strains of the arch are neglected because $\frac{Si}{Ji}$ ratios remain Constant and equal to $\frac{a}{Jc}$ the above expression thus becomes

$$Wn = \frac{a}{6EJc}(2Mo + M1)$$

at the Crown and the abutments Sn = 0 and Sn + 1 = 0 and therefore the elastic loads will equal

$$Wo = \frac{a}{6EJc}(2Mo + M1)$$

$$W_8 = \frac{a}{6EJc}(M_7 + 2M_8)$$

The Simultaneous equations permitting the determination of the redundant reactions due to a load unity P become

$$X_1\delta 11 + \Delta 1p = 0$$

$$X_2\delta 22 + \Delta 2p = 0$$

$$X_3\delta 33 + \Delta 3p = 0$$

wherefrom

$$X_1 = -\frac{\Delta 1p}{\delta 11} \text{ or } -\frac{\delta p1}{\delta 11}$$

$$X_2 = -\frac{\delta 2p}{\delta 22} \text{ or } -\frac{\delta p2}{\delta 22}$$

$$\text{and } X_3 = -\frac{\delta 3p}{\delta 33} \text{ or } -\frac{\delta 3p}{\delta 33}$$

The graphs of $\delta p1$, $\delta p2$ and $\delta p3$ for various positions of load unity P constitute the defection graphs of the simple structure due to the application of the redundant reactions $X_1 = 1$, $X_2 = 1$ and $X_3 = 1$ respectively. Divided by $-\delta 11$, $-\delta 22$, and $-\delta 33$ the ordinates to these graphs will represent the ordinates to the influence line for X_1, X_2 and X_3.

These deflection graphs will be obtained by the use of the elastic load method.

Values of the elastic loads

Using the expressions

for the Crown Wo $= \frac{a}{6EJc}(2Mo + M1)$

Abutment $= W_8 = \frac{a}{6EJc}(M_7 + 2M_8)$

for intermediate spans $= Wn = \frac{a}{6EJc}(Mn-1 + 4Mn + Mn+1)$.

Values of Elastic loads (multiplied by EJc) Table 14

Load Point	Unit Forces		
	X_1	X_2	X_3
8	-30	-7.06	3.26
7	-57	-42.38	6.51
6	-49	-84.76	6.51
5	-34	-127.14	6.51
4	-15	-69.52	6.51
3	11	-211.90	6.51
2	42	-252	6.51
1	79	-296.66	6.51
0	53	-162.46	3.26

The algebraic Sum of all the elastic load applied to system must equal Zero $(-30) + (-57) + (-49) + (-34) + (-15) + 11 + 42 + 79 + 53 = 0$

This relation is utilized as a check on the accuracy of the computed elastic loads.

The vertical displacements δ_{p1} of the simple structure due to unit action X_1 will equal the ordinates to the diagram of bending moments induced in the imaginary structure by the elastic loads

$$\delta_{p1}(7) = 53 \times 6.51 = \frac{345}{EJc}$$

$$\delta_{p1}(6) = 53 \times 13.02 + 79 \times 6.51 = \frac{1204}{EJc}$$

$$\delta_{p1}(5) = 53 \times 19.53 + 79 \times 13.02 + 42 \times 6.51 = \frac{2337}{EJc}$$

$$\delta_{p1}(4) = 53 \times 26.04 + 79 \times 19.53 + 42 \times 13.02 + 11 \times 6.51 = \frac{3541}{EJc}$$

$$\delta_{p1}(3) = 53 \times 32.55 + 79 \times 26.04 + 42 \times 19.53 + 11 \times 13.02 - 15 \times 6.51 = \frac{4648}{EJc}$$

$$\delta_{p1}(2) = 53 \times 39.06 + 79 \times 32.55 + 42 \times 26.04 + 11 \times 19.53 - 15 \times 13.02 - 34 \times 6.51 = \frac{5534}{EJc}$$

$$\delta_{p1}(1) = 53 \times 45.57 + 79 \times 39.06 + 42 \times 32.55 + 11 \times 26.04 - 15 \times 19.53 - 34 \times 13.02 - 48 \times 6.51 = \frac{6393}{EJc}$$

$$\delta_{p1}(o) = 53 \times 52.08 + 79 \times 45.57 + 42 \times 39.06 + 11 \times 32.55 - 15 \times 26.04 - 34 \times 19.53 - 48 \times 13.02 - 57 \times 6.51 = \frac{6666}{EJc}.$$

The displacement δ_{11} is obtained by rotating the imaginary loads through an angle of 90° until they become horizontal. Calculate the moment of these loads about the elastic centre and double its result.

$$\delta_{11} = 2\left[(30 \times 9.33) + (57 \times 8.90) + (49 \times 7.59) + (34 \times 5.42) + (15 \times 2.39) + (11 \times 1.52) + (42 \times 6.29) + (79 \times 11.94) + (53 \times 18.42)\right] = 3580 \times 2 = \frac{7160}{EJc}$$

The ordinates to the δ_{p1} are divided by $(-\delta_{11})$ to obtain the ordinates to the influence line shows the variation of X_1 when the unit load P travels along the arch.

ORDINATES TO THE INFLUENCE LINE FOR X_1

Section No

8	=	$\frac{6666}{7160}$	=	0.93
7	=	$\frac{6393}{7160}$	=	0.89
6	=	$\frac{5534}{7160}$	=	0.77
5	=	$\frac{4648}{7160}$	=	0.65
4	=	$\frac{3541}{7160}$	=	0.49
3	=	$\frac{2337}{7160}$	=	0.33
2	=	$\frac{1204}{7160}$	=	0.17
8	=	$\frac{345}{7160}$	=	0.05
0	=	0		

The same procedure is adopted as above to obtain the displacement $\delta p2$. The elastic loads will be applied once more to the imaginary structure and the corresponding bending moment will be determined. For this purpose the extended fibres of the simple structure acted upon by the force X2 will be situated at the extrados and therefore the elastic loads are directed upwards.

$$\delta_{p_2}(7) = \frac{162.46 \times 6.51}{EJc} = \frac{1058}{EJc}$$

$$\delta_{p_2}(6) = \frac{162.46 \times 13.02 + 296.66 \times 6.51}{EJc} = \frac{5978}{EJc}$$

$$\delta_{p_2}(5) = \frac{162.46 \times 19.53 + 296.66 \times 13.02 + 252.28 \times 6.51}{EJc} = \frac{8678}{EJc}$$

$$\delta_{p_2}(4) = \frac{162.46 \times 26.04 + 296.66 \times 19.53 + 252.28 \times 13.02 \times 211.9 \times 6.51}{EJc}$$

$$= \frac{14688}{EJc}$$

$$\delta_{p_2}(3) = \frac{162.46 \times 32.55 + 296.6 \times 26.04 + 252.28 \times 19.53 + 211.90 \times}{EJc}$$

$$\frac{13.02 + 169.52 \times 6.51}{EJc} = \frac{21803}{EJc}$$

$$\delta_{p_2}(2) = \frac{162.46 \times 39.06 + 296.66 \times 32.55 + 252.28 \times 26.04 + 211.90 \times}{EJc}$$

$$\frac{19.53 + 169.52 \times 13.02 + 127.14 \times 6.51}{EJc} = \frac{29745}{EJc}$$

$$\delta_{p_2}(1) = \frac{162.46 \times 45.57 + 296.66 \times 39.06 + 252.28 \times 32.55 + 211.90 \times}{EJc}$$

$$\frac{26.04 + 169.52 \times 19.53 + 127.14 \times 13.02 + 84.76 \times 6.51}{EJc}$$

$$= \frac{38238}{EJc}$$

$$\delta p_2(o) = \frac{162.46 \times 52.08 + 296.66 \times 45.57 + 252.28 \times 39.06 + 211.90 \times}{EJc}$$

$$\frac{32.55 + 169.52 \times 26.04 + 127.14\ 127.14 \times 19.53 + 84.76 \times 13.02 +}{EJc}$$

$$\frac{42.38 \times 6.51}{} = \frac{47008}{EJc}$$

The displacement $\delta 22$ representing the mutual translation of the free ends of the brackets along the direction of x_2, this displacement must be the double of $\delta p_2{}^{(o)}$

$$\delta 22 = 2\delta p_{22}(o) = 2 \times 47008$$

$$= 94016$$

Ordinates of the influence lines for the X_2.

(8)	=	$\frac{47008}{94016}$	= 0.50	3	=	$\frac{8678}{94016}$	= 0.09
7	=	$\frac{38238}{94016}$	= 0.41	2	=	$\frac{5978}{94016}$	= 0.06
6	=	$\frac{29745}{94016}$	= 0.32	1	=	$\frac{1058}{94016}$	= 0.01
5	=	$\frac{21803}{94016}$	= 0.23				
4	=	$\frac{14688}{94016}$	= 0.16				

The EJc is eliminated in both the numerator and denominator

In order to find the displacement δp_3 the elastic loads must be applied to the imaginary structure, these loads being directed as usual towards the extended fibres that means downward.

$$\delta p_3^7 = \frac{3.26 \times 6.51}{EJc} = \frac{21}{EJc}$$

$$\delta p_3^6 = \frac{3.26 \times 13.02 + 6.51 \times 6.51}{EJc} = \frac{85}{EJc}$$

$$\delta p_3^5 = \frac{3.26 \times 19.53 + 6.51 \times 13.02 + 6.51 \times 6.51}{EJc} = \frac{191}{EJc}$$

$$\delta p_3^4 = \frac{3.26 \times 26.04 + 6.51 \times 19.53 + 6.51 \times 13.02 + 6.51 \times 6.51}{EJc}$$

$$= \frac{339}{EJc}$$

$$\delta p_3^3 = \frac{3.26 \times 32.55 + 6.51 \times 26.04 + 6.51 \times 19.53 + 6.51 \times 13.02 +}{EJc}$$

$$\frac{+6.51 \times 6.51}{EJc} = \frac{530}{EJc}$$

$$\delta p_3^2 = \frac{3.26 \times 29.06 + 6.51 \times 32.55 + 6.51 \times 26.04 + 6.51 \times 19.53 +}{EJc}$$

$$\frac{+ 6.51 \times 13.02 + 6.51 \times 6.51}{EJc} = \frac{763}{EJc}$$

$$\delta p_3^1 = \frac{3.26 \times 45.57 + 651 \times 39.06 + 6.51 \times 32.55 + 6.51 \times 26.04 +}{EJc}$$

$$\frac{6.51 \times 19.53 + 6.51 \times 13.02 + 6.51 \times 6.51}{EJc} = \frac{1039}{EJc}$$

$$\delta p^{(o)} = \frac{3.26 \times 52.08 + 6.51 \times 45.57 + 6.51 \times 39.06 + 6.51 \times 32.55 +}{EJc}$$

$$\frac{6.51 \times 26.04 + 6.51 \times 19.53 + 6.51 \times 13.02 +}{EJc}$$

$$\frac{+ 6.51 \times 6.51}{EJc} = \frac{1356}{EJc}$$

Ordinates to the influence line for X3 are obtained by dividing δp_3 by $(\delta 33)$

The displacement $\delta 33$ representing the angular rotation of brackets (faces O and O' of the semi circles), itsvalue will be equal to the sum of the elastic loads

$$\delta 33 = 2\ \frac{(3.26 \times 2 + 6.51 \times 7)}{EJc} = \frac{104}{EJc}$$

The Ordinates:

$8 = \frac{1356}{104} = 13$ $\qquad$ $0 = \frac{21}{104} = 0.2$

$7 = \frac{1039}{104} = 10$

$6 = \frac{763}{104} = 7$

$5 = \frac{530}{104} = 5$

$4 = \frac{339}{104} = 3$

$3 = \frac{191}{104} = 2$

$2 = \frac{85}{104} = 1$

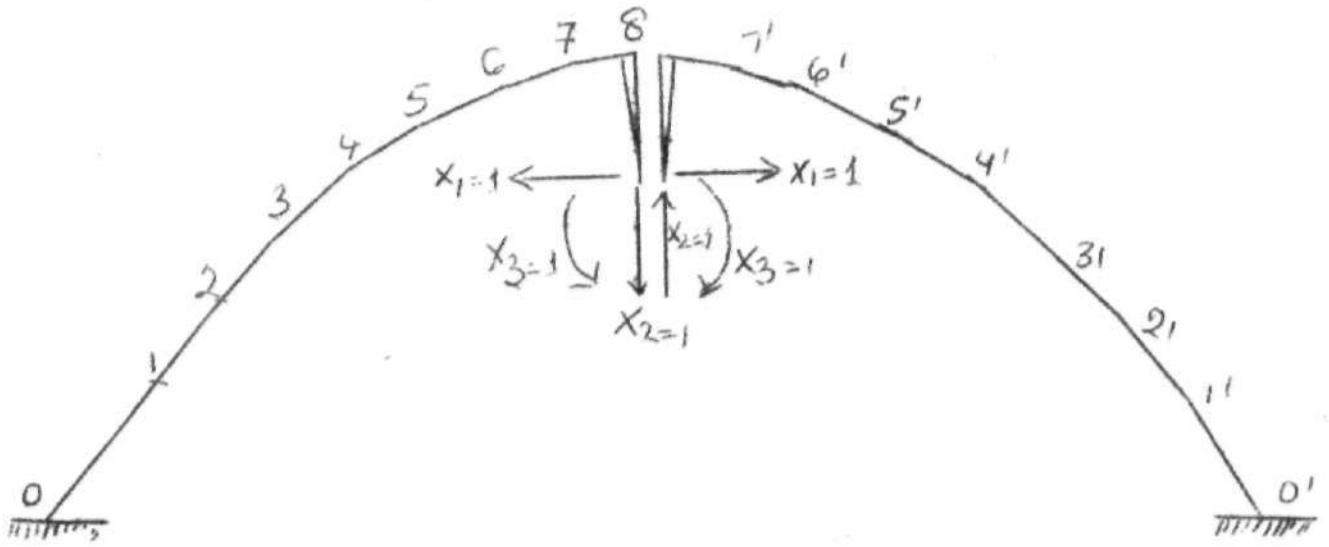
The construction of the influence line for Section K situated at 13.02 metres from the left abutment. The angle $\theta 2$ between the tangent to the nuetral line of the arch, and the horizontal equals for this section 39°. Sin 0.2 = 0.6293 and Cos θ_2 = 0.7771. The lever arm of the force X1 with reference to section K equals.

ys - yg = 9.33 - 15.62 = 6.29m. and the lever arm of force X_2 with reference to the same section equals 39.06m to the right from the left hand abutment. The ordinates to the influence line will be calculated using the following expressions

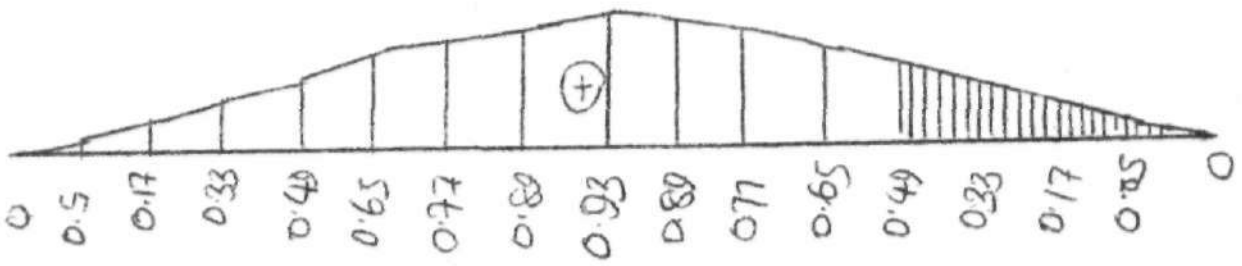
a) When the unit load is between the left-hand abutment and section K or when it is shifted to the right-hand semi-arch.

$$Mk = - X_1 \times 6.29 - X_2 \times 13.02 + X_3$$

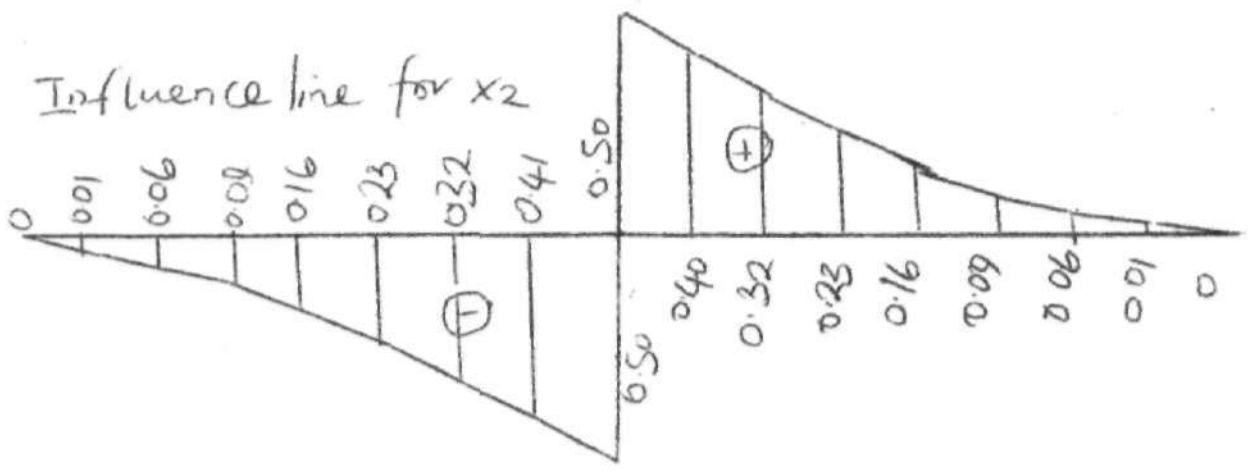
$$Qk = - X_1 \text{ Sin } 0.2 + X_2 \text{ Cos}_{12} = - X_1\ 0.6293 + X_2\ 0.7771$$



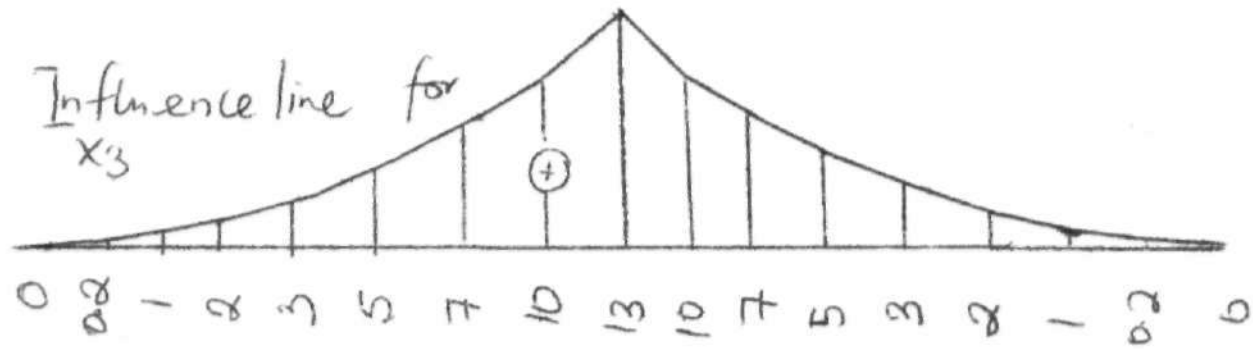
Influence line for X_1



Influence line for X_2



Influence line for X_3



(b) when the unit load P is between section K and the crown

$$Mk = - X_1 \times 6.29 - X_2 \times 39.06 + X_3 - 1r$$

r is the lever arm of the unit load P about section K and

$$Qk = - X_1\ 0.6293 + X_2\ 0.7771 + 0.7771$$

$$Nk = - X_1\ 0.7771 + X_2\ 0.6293 + 0.629$$

ORDINATES TO THE Mk INFLUENCE LINE TABLE 15

load Point	X_1	$-X_1 6.29$	X_2	$-X_2 39.06$	X_3	$-r$	Ordinates to the Mk influence line in m
8	0.93	-5.85	-0.50	19.53	13	-39.06	-12.38
7	0.89	-5.60	-0.41	16.01	10	-32.55	-12.14
6	0.77	-4.84	-0.32	12.50	7	-26.04	-11.38
5	0.65	-4.09	-0.23	8.80	5	-19.53	- 9.72
4	0.49	-3.08	-0.16	6.25	3	-13.02	- 6.85
3	0.33	-2.08	-0.09	3.52	2	- 6.51	- 3.07
2	0.17	-1.07	-0.06	2.34	1	0	+ 2.27
1	0.05	-0.31	-0.01	0.39	0.2	-	+0.28
0	0	0	0	0	0	-	0
0'	0	0	0	0	0	-	0
1'	0.05	-0.31	0.01	-0.39	0.2	-	-0.50
2'	0.17	-1.07	0.06	-2.34	1	-	-2.41
3'	0.33	-2.08	0.09	-3.52	2	-	-3.60
4'	0.49	-3.08	0.16	-6.25	3	-	-6.33
5'	0.65	-4.09	0.23	-8.89	5	-	-7.98
6'	0.77	-4.84	0.32	-12.89	7	-	-10.73
7'	0.89	-5.60	0.41	-16.01	10	-	-11.61
8'	0.93	-5.85	0.50	-19.53	13	-	-12.38

ORDINATES TO THE Qk INFLUENCE LINE TABLE

Load Point	X_1	$-X_1$ 0.6293	X_2	X_2 0.7771	0.7771	Ordinates to the Qk influence line
8	0.93	-0.59	-0.50	-0.39	0.7771	-0.20
7	0.89	-0.56	-0.41	-0.32	0.7771	-0.10
6	0.77	-0.48	-0.32	-0.25	0.7771	+0.05
5	0.65	-0.41	-0.23	-0.18	0.7771	0.19
4	0.49	-0.31	-0.16	-0.12	0.7771	0.35
3	0.33	-0.21	-0.09	-0.07	0.7771	0.50
Immediately to the Right of Section						
2	0.17	-0.11	-0.06	-0.05	0.7771	0.62
Immediately to the left of section						
2	0.17	-0.11	-0.06	-0.05	0	-0.16
1	0.05	-0.03	-0.01	-0.01	-	-0.04
0	0	0	0	0	0	0
1'	0.05	-0.03	0.01	0.01	-	-0.02
2'	0.17	-0.11	0.06	0.05	-	-0.06
3'	0.33	-0.21	0.09	0.07	-	-0.14
4'	0.49	-0.31	0.16	0.12	-	-0.19
5'	0.65	-0.41	0.23	0.18	-	-0.23
6'	0.77	-0.48	0.32	0.25	-	-0.23
7'	0.89	-0.56	0.41	0.32	-	-0.24
8'	0.93	-0.59	0.50	0.39	-	-0.20

ORDINATES TO THE Nk INFLUENCE LINE TABLE 16

Load Point	X_1	$X_1$0.7771	X_2	X_{20}.6293	0.6293	Ordinates to the Nk influence line
8	0.92	0.72	0.50	-0.31	0.6293	1.04
7	0.89	0.69	-0.41	-0.26	0.6293	1.06
6	0.77	0.60	-0.32	-0.20	0.6293	1.03
5	0.65	0.51	-0.23	-0.14	0.6293	1.00
4	0.49	0.38	-0.16	-0.10	0.6293	0.91
3	0.33	0.26	-0.09	-0.06	0.6293	0.83
Immediately to the left of Section 2	0.17	0.13	-0.06	-0.04	0.6293	0.72
Immediately to the left of Section 2	0.17	0.13	-0.06	-0.04	-	0.09
1	0.05	0.04	-0.01	-0.01	-	0.03
0,0'	0	0	0	0	-	0
1'	0.05	0.04	0.01	0.01	-	0.03
2'	0.17	0.13	0.06	0.04	-	0.17
3'	0.33	0.26	0.09	0.06	-	0.32
4'	0.49	0.38	0.16	0.10	-	0.48
5'	0.65	0.51	0.23	0.14	-	0.65
6'	0.77	0.60	0.32	0.20	-	0.65
7'	0.89	0.69	0.41	0.26	-	0.95
8'	0.93	0.72	0.50	0.31	-	1.03

Using these influence lines the magnitudes of Mk, Qk and Nk induced by the uniformly distributed loads acting over the whole arch.

For this purpose the uniformly distributed load are replaced by concentrated onces acting at Points 0, 1, 2, 3, 4, 5, 6, 7 and 8. These concentrated loads will amount to

$$Pa = P_8 = 9\frac{9}{2} = 8.77 \times \frac{6.51}{2} = 29kn$$

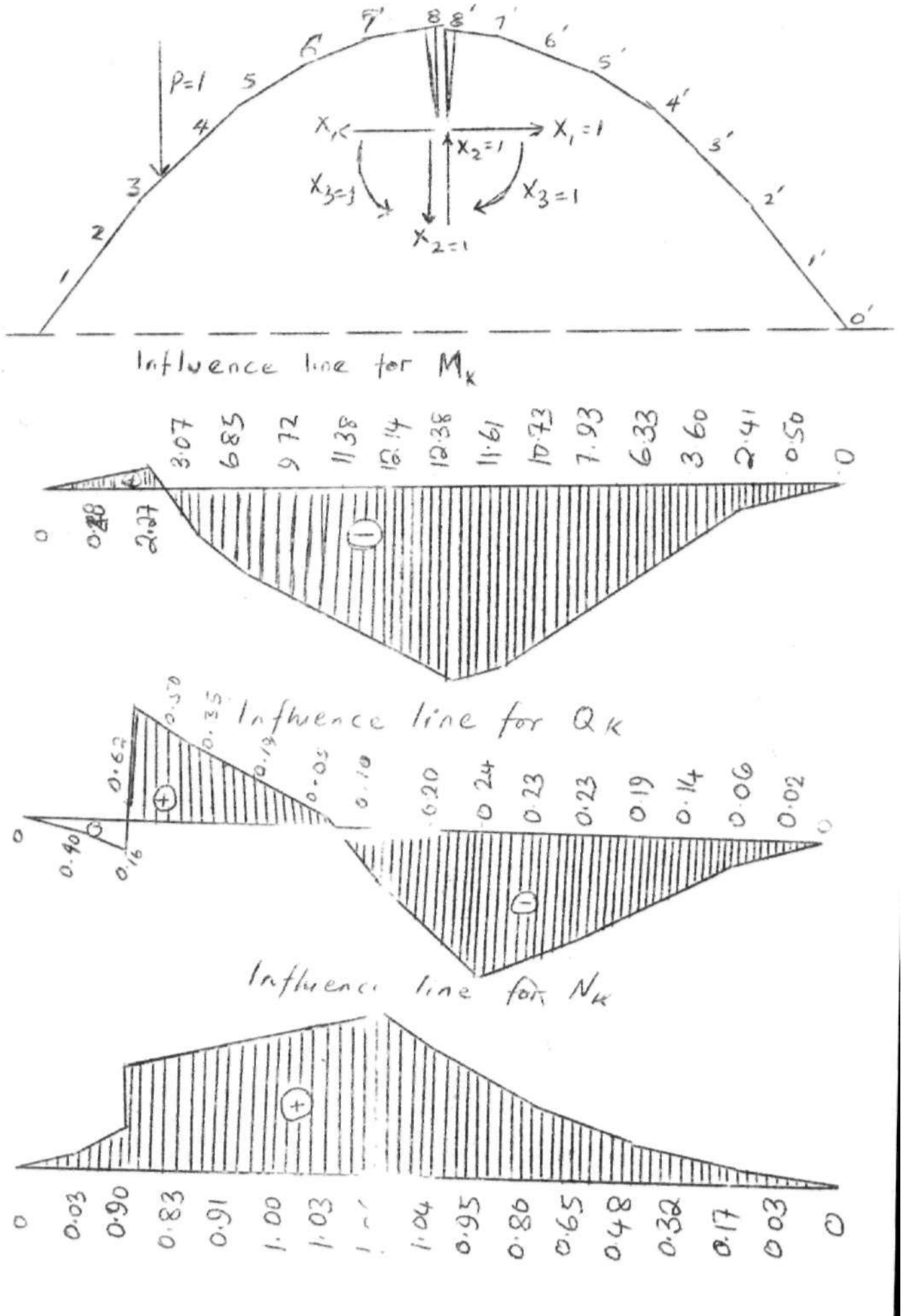
$$P_1 = P_2 = P_3 = P_4 = P_5 = P_6 = P_7 = 8.77 \times 6.51 = 57KN$$

The desried values of Mk, Qk and Nk will be obtained multiplying the magnitudes of these concentrated loads by the corresponding ordinates to the influence lines which are then Summed up as indicated below

$$Mk = (0.28 \times 57) + (2.27 \times 57) - (3.07 \times 57) - (6.85 \times 57) - (9.72 \times 57) - (11.38 \times 57) - (12.14 \times 57) - (12.38 \times 29) = 2819.14 \text{ Knm.}$$

$$Qk = - (0.04 \times 57) - (0.16 \times 57) + (0.62 \times 57) + (0.50 \times 57) + (0.3 \times 37) + (0.19 \times 57) + (0.05 \times 57) - (0.10 \times 57) - (0.20 \times 29) = 74.57Kn$$

$$Nk = (0.03 \times 57) + (0.09 \times 57) + (0.72 \times 57) + (0.83 \times 57) + (0.91 \times 57) + (1.00 \times 57) + (1.03 \times 57) + (1.06 \times 57) + (1.04 \times 29) = 353.35Kn.$$


P=1
X1=1
X2=1
X3=1
Influence line for Mk
Influence line for Qk
Influence line for Nk

CHAPTER SIX: SOME PRACTICAL CONSIDERATION IN THE DESIGN AND ERECTION OF REINFORCED CONCRETE ARCHES.

A considerable portion of cost of an arch is due to the false work i.e. centering and formwork, etc. Consequently to ensure rigidity and maximum economy great care should be taken in their design. Also to minimise stresses set up in the final structure the specified method of concreting in arches should be strickly followed. In this chapter the centering of arches, formwork for arches, reinforcement in arches, concreting arches, thermal insulation and waterproofing are discussed.

6.1 ARCH CENTERING

The principles underlying the construction of centering for arch construction are

i) absolute rigidity

ii) ease of removal

iii) economy

Because a considerable portion of cost of an arch is due to the false work, rigidity and economy are better assured if the centering and formwork is properly designed.

The main supporting members are called soleplates and they should be so designed that they can efficiently carry the estimated weight of the false work, the weight of the wet concrete, and any incidental loads that may be present during construction. Arrangements must be made to enable the centering to be eased off easily before it is dismantled. For this purpose wedges or Jacks may be used.

All the post and struts should be well braced, and unless calculations are made to cover cases of slender columns it is

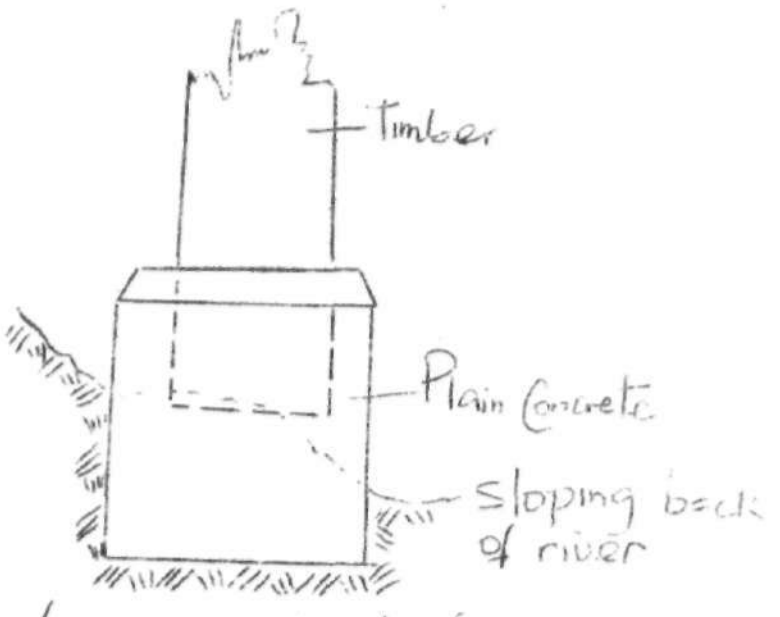
advisable, if the struts are to carry their full load, to arrange the bracing so that no piece has an unbraced lenght of more than 15 times its width. Thus with 150 mm x 150 mm post braces should be 2.10 m a apart in both directions. With 230 mm x 75mm post the braces in the 230mm direction must be placed at 3 meters centres or less while those in the 75mm direction must not be more than 1.05m apart.

It is also important to take into account the fact that the strenght of timber which has become wet due to exposure such as may be experienced with arch centering is very much lower than when the wood is dry.

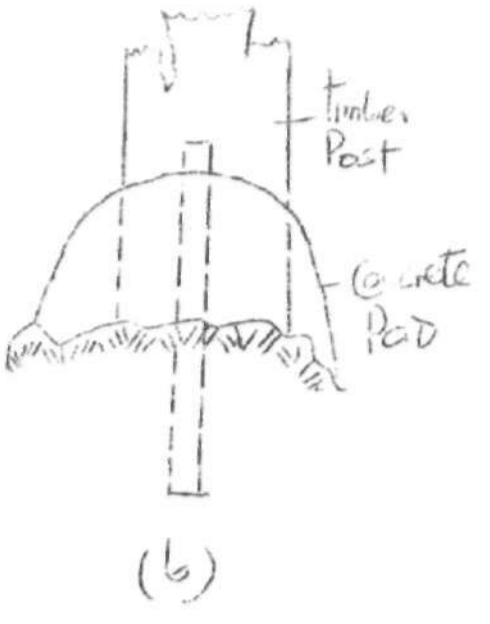
The effect of wind should also be allowed for in the design of the main centering especially in high narrow bridges and hangars in exposed positions. Centering composed of steel scaffolding or steel trusses and girders is frequently used and may be economical when wide waterways or other clearances have to be spanned.

The principal supporting members of arch centering follow one of three principal arrangements, but numerous variations of these methods occur in practice depending on site conditions and requirements. The arrangements of fig. 6.1 illustrate the principal arrangements.

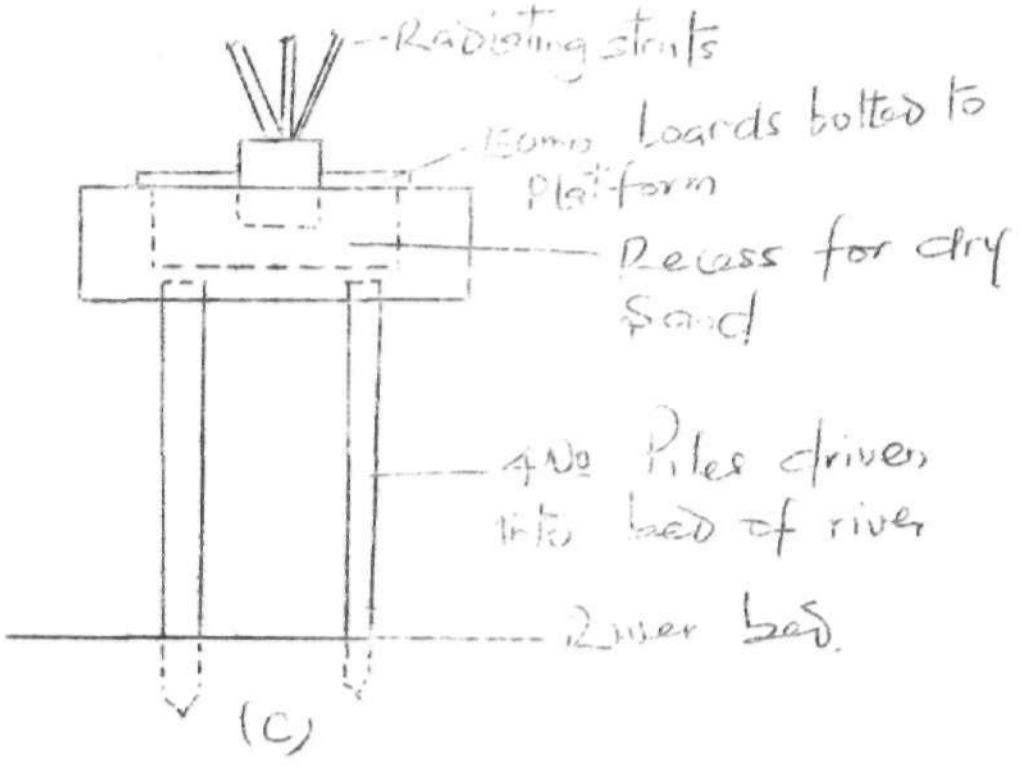
Type (a). its suitability is when little clearance is required and where the ground beneath is capable of providing a satisfactory foundation for the post of a continuous trestle. This is usually in timber, but tubular steel scaffolding may be used for this purpose. Suitable timber sizes for this design are 150 mm x 150 mm or 200 mm x 200 mm. Pine posts of 3.60 m lenghts, the post and horizontal braces being arranged to form panels about 3 metres square. Each panel is doubly crossed braced with 150mm x 50mm, 200mm x 75mm or



(a) Excavation to firm ground (level of excavation to be deep enough to obviate risk of lateral buckling)



(b)



(c)

FIG. 6.1 FOUNDATION FOR SUPPORT OF ARCH CENTERING

230mm x 75mm braces. The transverse and longitudina. braces being either 150mm square or 230mm x 75mm. All connections are bolted where practicable. Economy of material and time have been claimed for the use of nailed joints throughout.

When the arch spans over a wide waterway type c is more suitable, type b is used when large spans extend over deep valleys.

Type (b) consist of a support at each end of the span with one or more intermediate piers supporting longitudinal members which in turn carry the braced posts supporting the formwork. In this type the minimum number of principal supports is founded on the bottom of the valley or in the river bed, and struts are spread out fanwise to produce a series of close supports for the arch soffit to produce a series of close - supports for the arch soffit, the pictures on the bottom of the second photostat copy shows this type of centering.

Foundation of the posts or struts for centering should be given careful consideration. For type (a) mentioned above, when the bottom of the valley is firm ground,the post can be anchored into a block of concrete. Excavation is taken down to firm ground and levelled off. When the post bears on a rocky surface a mild steel dowel-rod driven into the rock and penetrating the foot of the post as in Fig. 6.1b can be used to secure the foot of the post. Sometimes the construction of a pier for centering is most effected by driving three, four, or more timber piles into the river bed and bracing the piles above low water level as in Fig. 6.1c.

6.2 FORMWORK

The main stagings are templates or profiles carrying in the case of arch slabs, the sheeting to the soffit or in the case of independent

arch ribs, the boxing for the ribs. The usual arrangements is to form the templates from longitudinal boards laid on edge as shown in Fig. 6.2a

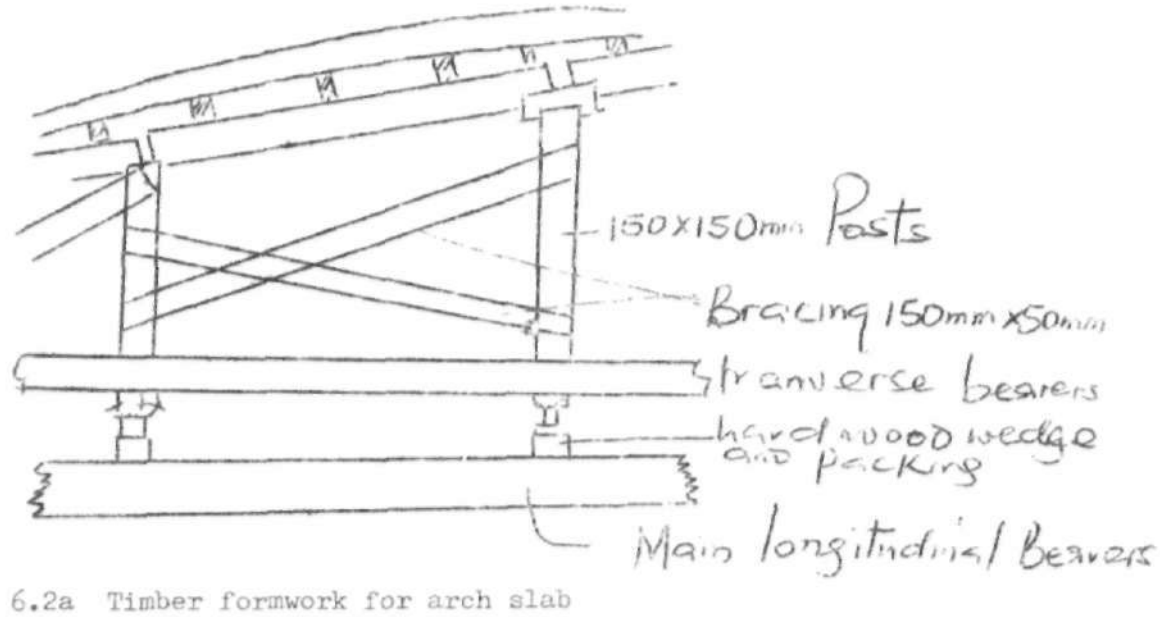


Fig. 6.2a Timber formwork for arch slab

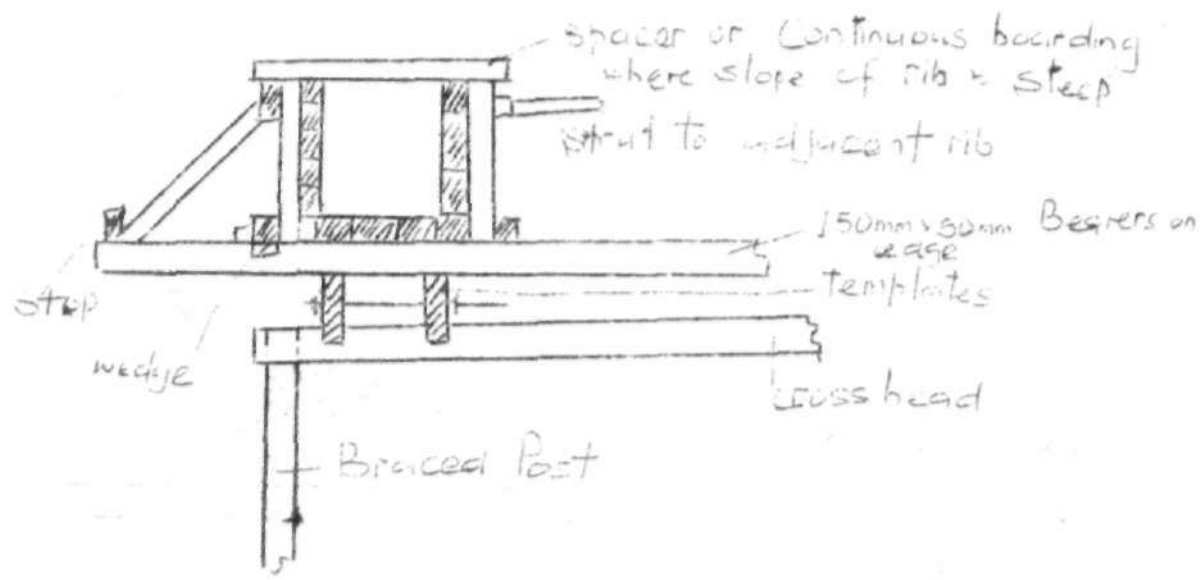


Fig. 6.2b Boxing for arch rib

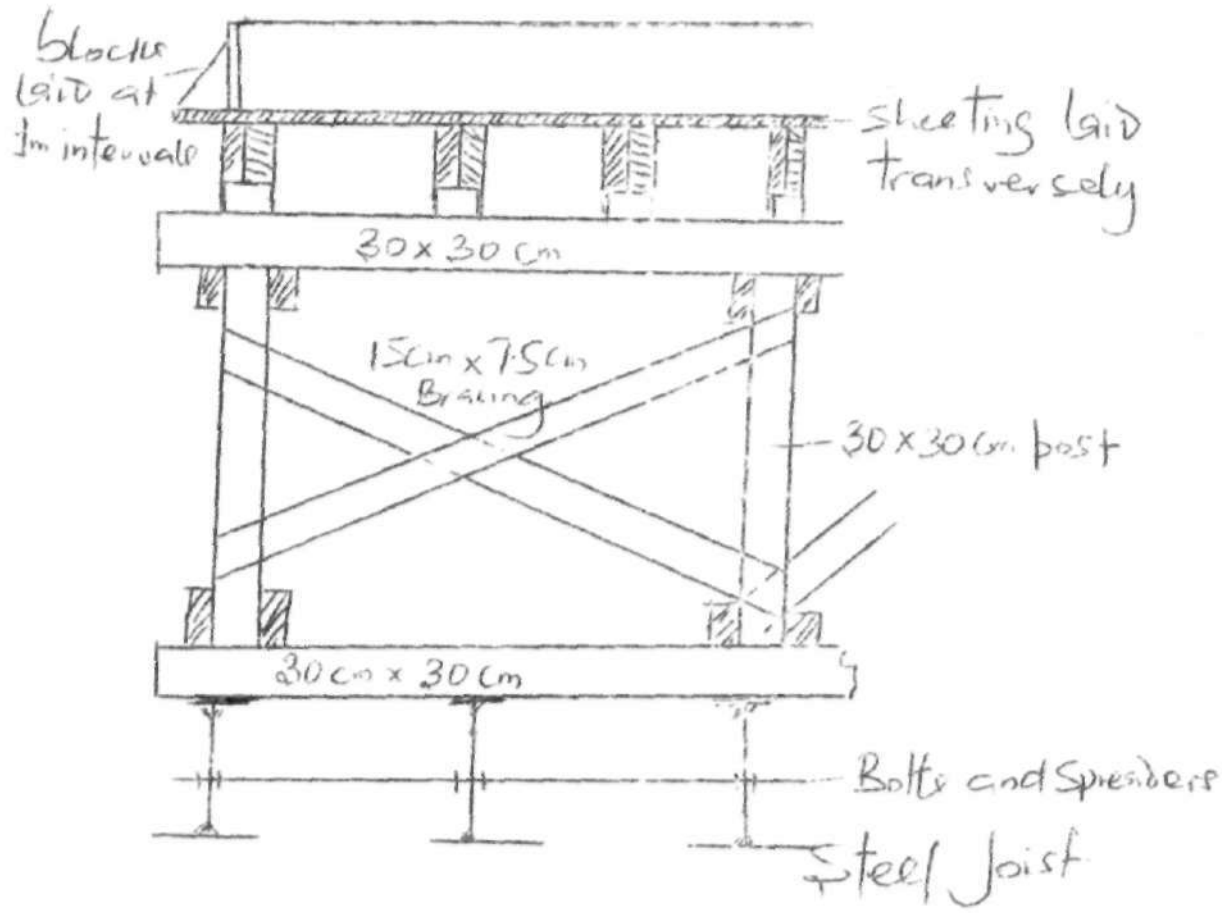


Fig. 6.2c Arch slab formwork supported on steel beams.

The upper edge is cut to the required profile making due allowance for the combined thickness of the transverse bearers and sheeting. If the template joists are placed sufficiently close together, the sheeting can be laid directly on templates but if the templates are placed too far apart for the thickness of the boarding, the sheeting would be carried on bearers spanning between the templates. In the construction of some large spans, steel girders with the upper booms accurately curred to the arch profile have been used in place of built-up timber templates Fig. 6.2b shows an interesting adaptation of steel beams as templates as can be seen in Fig. 276(b) on the photostat copy.

Various ways of arranging the formwork are given in Fig. 6.2.

Vertical posts, wedged off the main longitudinal bearers, are braced diagonally and horizontally. Suitable sizes being given in the diagram. Cross-heads bolted across the heads of the posts support the shaped ends of the templates. The templates boards could be in short lenghts between the cross-heads. Where the adjacent templates overlap they are bolted together.

Sheeting, although necessarily - substantial does not usually exceed 50 mm in thickness as it must have sufficient spring to conform to the soffit profile when laid longitudinal.
When a good surface to the suffit is particularly required, the sheeting can be covered with plywood or other smooth lining in which case wrought boarding is necessary and adjacent boards need not be bolted together.

An alternative detail for an arch slab is shown in the cross-section of the formwork illustrated in Fig. 6.2b. Here the principal longitudinal bearers are steel beams connected by bolts and spreaders and supporting a timber centering similar to that of Fig. 6.1a, except the posts are supported on heavy transverse carried on the steel beams. The wedges in this case are directly under the templates, the latter consisting of pairs bolted together and extending the full span of the structure under construction.

Since the templates are closely spaced, the sheeting spans transversely, an overhang at the outer edge of the arch providing a support for blocks maintaining the facing block in position. When the structure is of arch-rib design with ribs closely spaced, a completely sheeted soffit as for an arch is sometimes provided and

upon this surface the boxing for the sides of the ribs is erected. Bearers placed on edge carry the curved bottom boards and the side forms are kept in position by inclined struts or by props from adjacent boxes. Occasional spacers between the tops of corresponding side board mountain the shape of the box. Where the slope of the rib is steep, as near the springings, short lenghts of transverse boarding are nailed across the top of the side boards as concreting proceeds.

6.3 REINFORCEMENT IN ARCHES

Reinforcement meant for fixed end arches should not be excéssive, a little more concrete of higher strenght is preferable to closely packed bars for longitudinal thrusts. The bars must be properly bonded and tied. A reasonable amount of transverse reinforcement perhaps 0.25 per cent of the corresponding of concrete. Narrow deep ribs do not need such heavy transverse steel, reasonable stirrups being adequate. Solid barrel arches should not be too wide perhaps 15m to 18m otherwise a longitudinal contraction joint may be desirable to prevent shrinkage and temperature cracking of the arch when on strong relatively non-deforming foundation especially rock.

6.4 CONCRETING IN ARCHES

The specified method of concreting arches should be strickly followed, since it is devised to minimise stresses due to shrinkage in the final structure and to prevent distortion of the centering during construction.

Concreting should proceed symmetrically about the centre of the arch, commencing either at the crown or at the springings. In large arch slabs it is best to commence concreting on the longitudinal centre line and work outwards towards the edges: in arch ribs if possible the central rib should be concreted first and the remainder in outwards successions: that is with seven ribs, No. 4 would be first, No. 3 and 5 would follow at the same time then Nos. 2 and 6 and finally Nos. 1 and 7. Alternatively work can be started at the outer ribs and continued inwards, reversing the foregoing order. When there are only two ribs supported on a common centering, both should be concreted at the same time.

Each rib should be divided into an even number of parts constituting a days placing. Two adjacent parts should be concreted consecutively. Thus the concrete in a number of isolated parts will be placed first the intervening gabs being filled as late as possible afterwards. This allows each part to undergo its initial shrinkage before it is bonded to the completed parts.

The illustrations in Fig. 6.3 shows an arch rib in parts with 1.20m spaces left to be filled in after the bulk of the shrinkage has taken place. Fig. 6.3 shows typical examples of the procedure. In some cases as Fig. 6.3e the parts are more or less equal in length or in volume, while in others the parts in the first series to be placed are longer than the final closure or shrinkage keys, the later being 0.6m to 1.50m long. These structures comprise of spans from 6m to 60m. To ensure a better key for the parts concrete later, the faces of the first parts are grooved.

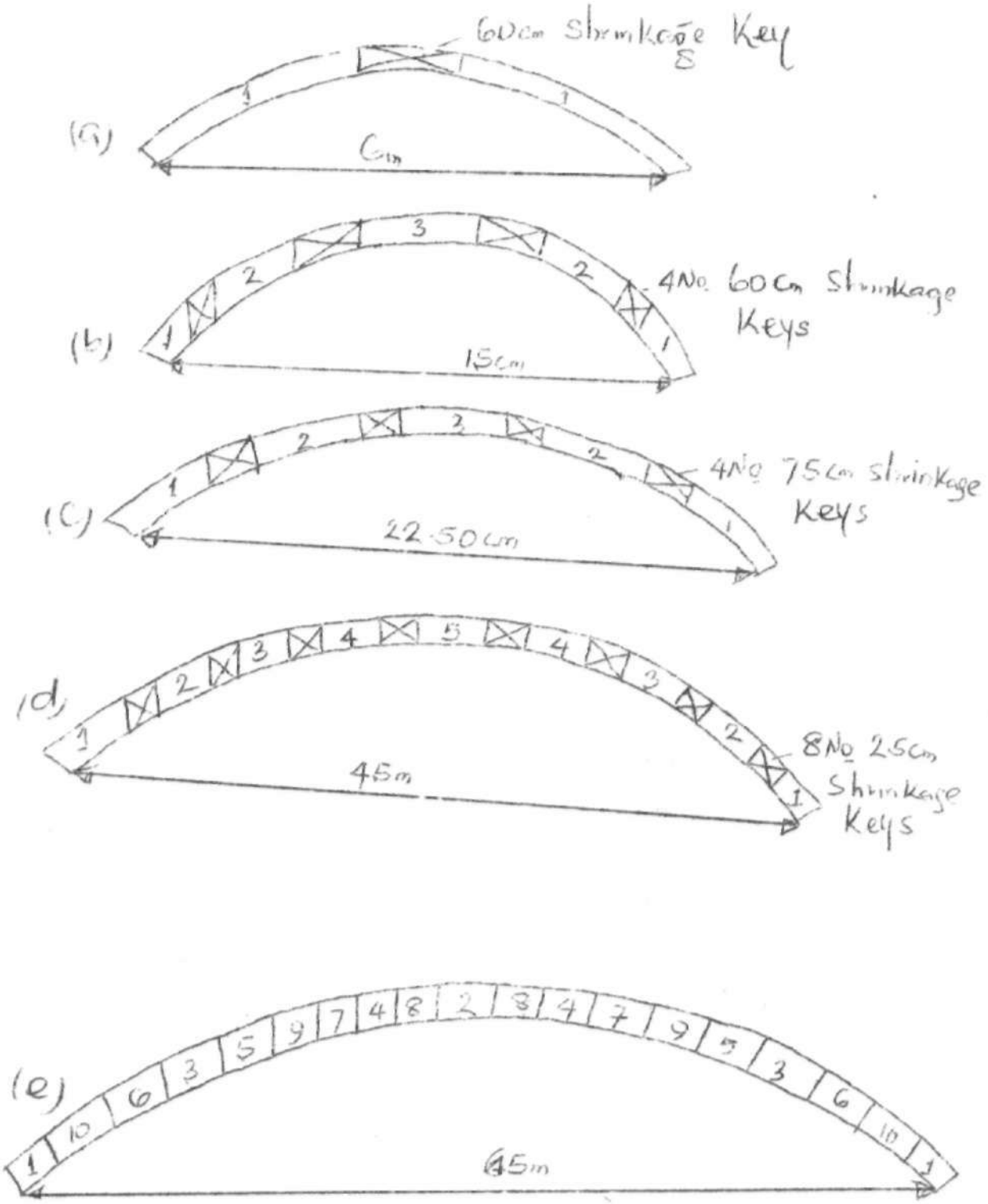


Fig. 6.3 Concreting arches

As a precaution against movement of the piers during construction a symmetrical sequence of construction in adjacent spans or by shoring against unbalanced forces is adopted. If the arches are designed without hinge at the springings small movements of the piers either translational or rotational, produce appreciable concrete stresses for which the arches may not have been designed. These stresses may occur at a time when the concrete is not fully matured and unable to resist overstrain. It is possible to relieve piers of unbalanced, thrusts from arches by retaining the centering in position until adjacent spans are complete. The stresses in an arch due to its weight and some other factors are sometimes reduced by introducing in the arch during construction stresses of the opposite sense. For this purpose one or more jacks inserted in a gab (afterwards filled in) at the crown are operated simultaneously. The placing and operation of th: Jacks must be ca:efully controlled by the engineer.

6.5 FOUNDATION TYPES FOR ARCH STRUCTURES

This is very important especially for fixed end arches or structures. The sketches below are some of the arrangements that have been used. Sketch (a) is a Simple footing on rock, the thrust is transmitted directly to the rock, but the depth of footing should be sufficient to develop the dowels which transfer the stresses out of the longitudinal reinforcement to the enlarged section of concrete. End moments may be with stood by shifting of the reaction Ra so that Ma = hae where e is some eccentricity. This causes a tendency to increase the edge pressure so that bars "a" may be useful in tieing the footing together. Another way to resist Ma may be by

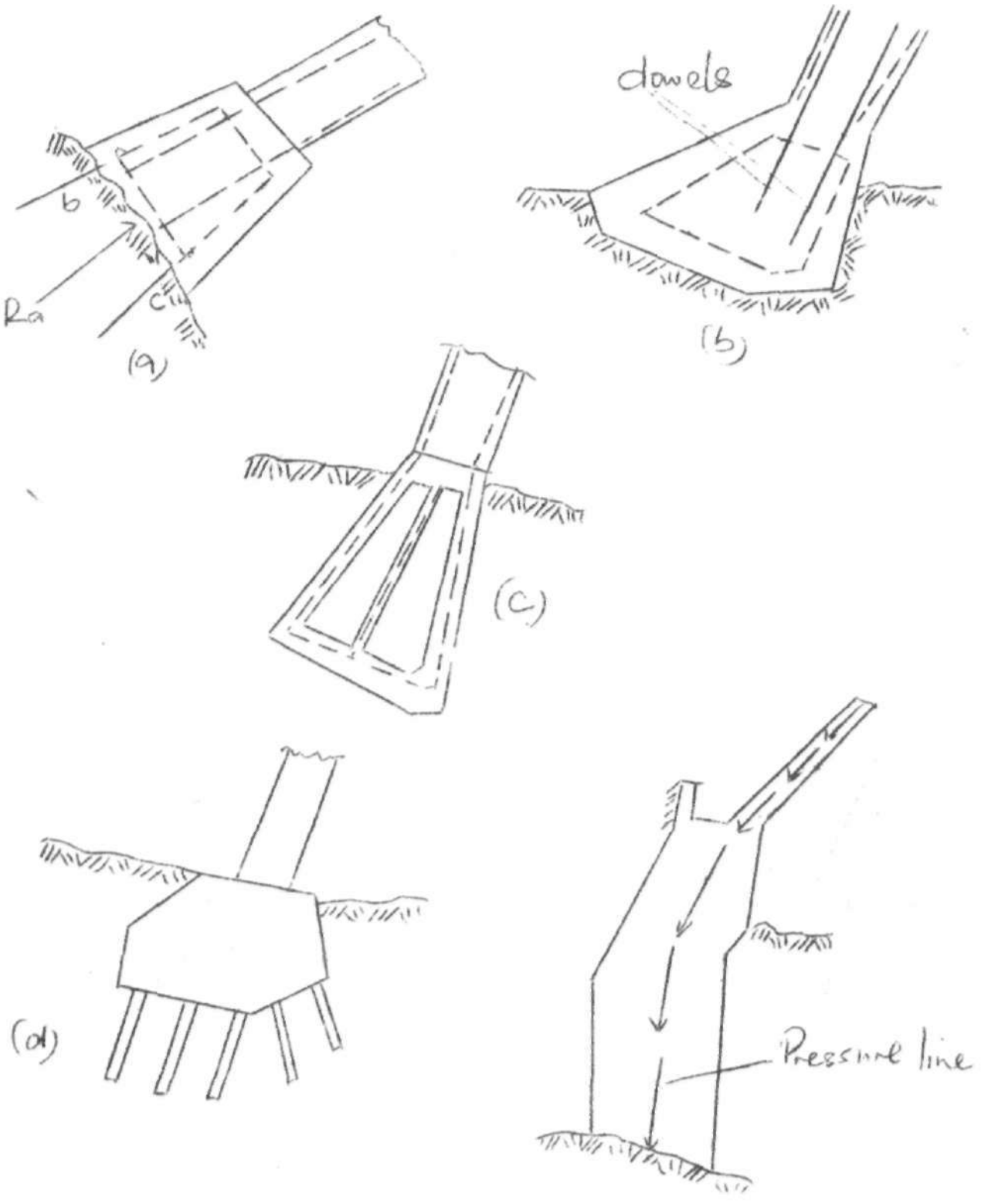


Fig. 6.4

means of rock anchors shown by bars "b" and "c" which are grouted into holes drilled in the rocks. However, these anchors are no better than the rock to which they are attached, and the later may be seriously weakened against pulling out because of cracks or seams. Sketch (b) shows a simple footing placed on gravel or coarse sand. Part CD is needed to resist the vertical reaction during erection as well as to help react against the thrust and bending moments. Sketch (c) is a hollow box like footing which was used because the required bearing area was so large and the good soil so deep that this was considered preferable to a solid concrete footing. Sketch (d) shows the general features of an arch foundation which was built on piles. In such cases the piles must be stiff and durable, and they should go to a strong end beaving.

Sketch (e) shows the left foundations used under a bridge. The rock was 6 metres below grade. A massive block of concrete was used so as to "pull" the pressure line downward to the bearing area.

6.6 THERMAL INSULATION AND WATER PROOFING

Heat insulation is a necessity in arch structures because it reduces considerably the temperature variations that can be present and consequently reduces cracks that may result. The materials that can be used for insulating purposes can be briefly classifed into two groups. The first comprises those with low density and comparatively low conductivity. They offer resistance to the passage of heat by conduction. The second group has a more limited but still valuable application and comprises metallic surfaces used in boundaries of air spaces within the construction of walls, ceilings or roofs. They act as reflectors of the radiant heat waves.

The first group is divided into three classes identified as rigid insulation, flexible types and insulating fills. The second group has no classifications but various metals are used to accomplished the purpose.

RIGID INSULATION: These are generally in the shape of boards 12.5mm to 25mm thick. One of the varieties is composed of wood, Cane and other vegetable fibres pressed into sheets. Sufficiently rigid to be nailed upnn joists or studding. Many of these insulating boards may be used under built up waterproofing or as a base for plaster on ceilings or exterior walls, and some are attractively finished one face for use as exposed interior surfaces. Others are coated with bitumen or enclosed in a waterproof paper to guard against the passage of waste vapour. Some insulation boards of this type are furnished with one face surfaced with aluminium foil in order to combine the functions of low heat conduction with the reflection of radiant heat. This same groups includes units such as blocks and short boards or slabs formed of inorganic materials. One of the better varieties in this class is foainglass, supplied in blocks of 30cm x 46cm and 30cm x 92cm, varying in thickness from 2.5cm to 10cm. These two materials compete with slabs of comparable size made of pressed granulated cork. They are mainly used for roof insulation and are laid in coal tar pitch or asphalt, between the roof deck and the built-up waterproofing.

FLEXIBLE INSULATION: These types of insulation are not sufficiently rigid to be used directly beneath built-up water-proofing, but have their application as fillers between wood studs in exterior wall construction and between the joists of flat roofs and rafters of

pitched roofs. Although very widely used in connection with wood-frame structures, they are also appropriate for application between furing strips on the inside of mansory walls and between floor or roof joists in steel framed assemblies if provision is made for their attachment.

One type is composed of various organic materials such as wood fibre and cotton, treated in a manner to render them very light and fire resistant. Another variety is derived from inorganic materials and identified as mineral, rock or glass wool. Mineral wool is in the form of fire interlaced mineral fibres.

When installing either the rigid or the flexible type of insulation, particular attention should be given to avoiding any cracks or openings between the sheets, batts, or blankets through which heat transfer can occur, including filling around floor bridging. Care should be taken to place the insulation in a manner to protect water pipes.

Insulating fills: Many insulating materials are furnished in a granulated form. Often the size of small pellets. Several of the newer varieties particularly vermiculite and pertite are mixed with portland cement to form an insulating fill over concrete roof decks. Pumice is also employed for the same purpose.

FRELECTIVE INSULATION

Several types of metals are used for this purpose. They are generally in form of foil mounted on paper, or very thin all-metal sheets. The better types maintain their effectiveness under service conditions. If installed as continous surfaces, they also provide an excellent vapour seal. Aluminium foil mounted upon a strong paper backing is a popular design.

CONCRETE WATERPROOFING:-

Concrete structures like arches without adequate waterproof protection cannot be claimed to have any satisfactory waterproofed qualities when exposed to rigorous water conditions. There are three methods of waterproofing generally in use at the present times.

1. Integral waterproofing
2. Internal Coatings) Surface waterproofing.
3. External Coatings)

Integral waterproofers:- These consist of materials added to the cement or concrete. The addition to the cement is usually made at the works, and the waterproofer in this case is in the form of a waterproofed cement. When the waterproofed is added to the concrete during mixing at the site. In this case, much depends on the skill of the persons at the mixer, and the deligent observance of strickly correct proportions of waterproofer to concrete. It is assumed that the admixture fills all the voids through which the water may pass.

Integral waterproofers may contain calcium chloride, soaps, hydrated lime etc.

Internal surface Coatings:- They are usually employed to stop leaks. Mortars with impervious mixtures in them, paints, with waterglass or other kinds of chemicals, ferric compounds etc are the common types.

External Surface Coatings:- These are usually bituminous or asphalt coatings with or without strenghtening membranous layers inside the layers of the surfacing. It is essential that the concrete should be sufficiently dry. Most external water-proofers are usually iron compounds, cement coatings, transparent coatings, paints and vanishes, but the most commonly used are asphalt emulsions

or butiminous solutions.

One of the most widely used water proofer is mastic asphalt. It is a physical mixture of mineral aggregate and bitumen. Asphalt when hardening on cooling does not chemically change the original nature of the material. There is no risk of a loss of waterproofing ability from this cause. The plasticity and ductile properties of asphalt are sufficient for it to conform to any normal movements of the structure. Asphalt has a very little heat insulation value but this can be considerably improved by placing a good insulating medium underneath it. As asphalt has the property of clinging securely to vertical or steeply sloped surfaces, it can be employed on sloping and vertical sections of roofs.

CONCLUSION

Though, there are many methods available for the stress analysis of redundant structure, the method of forces has been used as an exact method because of its suitability.

The use of the elastic centre, and the application of influence lines which enables fast checking of internal forces or deflections at any given section of the redundant arch structure, have been learned and presented.

The use of Symmetry, Symmetrical and antisymmetrical loading, grouping of unknowns and load transformation have been used as methods for Simplifying the calculations based on the method of forces. Physical phenomena, like temperature changes, shrinkage and settlement of support have effects on redundant structures and therefore a careful consideration of their effects should be carefully examined as done in this thesis.

The two approximate methods which are useful for the stress analysis of reinforced concrete arches are used when the nuetral line and the law of cross-sectional variation cannot be expressed by analytical equations fit for practical use. With the use of one of the approximate methods a full stress analysis of the arched beam of the aircraft hanger has been presented.

Some vital consideration in erection of reinforced concrete arches has also been learned and presented.

Arches have undergone changes since their inception, both in the materials of construction, the theories used for the stress analysis and design. In the event of any changes the future, structural engineers might be faced with challenges in the development of new

theories or new methods in the stress analysis, design and erection of arches. It is in the light of this that the importance, design and erection of arches used in buildings and other engineering structure can be clearly seen.

This study has helped the author immensely with the knowledge of the stress analysis, design and methods of erection of fixed end arches and has consequently equipped him with adequate knowledge to meet future challenges in this field.

BIBLIOGRAPHY

BENJAMIN J.R: Statically indeterminate structures.
McGraw-Hill Book Company, Inc. 1959.

THEODORE CRANE: Architectural Construction - The Choice of Structural Design. John Wiley and Sons Inc. and Chapman and Hall Limited. Second edition 1956.

A. DARKOV AND V. KUZNETSON: Structural Mechanics.
MIR Publishers Moscow 1969.

T.K. DERRY and TREVOR I. WILLIAMS: A short History of Technology from the earliest Times to A.D. 1900. Oxford University Press 1970.

CLARENCE WHITING DUNHAM - Advanced reinforced concrete.
McGraw-Hill Book Company 1964.

LESLIE ERNEST HUNTER: Concrete waterproofing
Sir Isaac Pitman and Sons, Ltd 1954.

VALERIAN LEONTOVICH: Frames and Arches. McGraw-Hill Book Comapny, Inc. 1959.

R.L. PEURIFOY: Formwork for Concrete Structures
McGraw-Hill Book Company 1964.

CHARLES E. REYNOLDS and JAMES C. STEEDAM: Reinforced Concrete Designers Handbook. A viewpoint Publication, eighth edition 1976.

CHAS E. REYNOLDS: Concrete construction Concrete Publications limited London. Third edition 1967.

Late FREDERICK WINSLOW TAYLOR, LATE SANFORD E. THOMPSON, LATE EDWARD SMULSKI and HENRY C. ROBINSON: Plain and reinforced concrete. John wiley and Sons. INC., fourth edition 1955.

STEPHEN PROKOP TIMOSHENKO: History of Strenght of materials.
McGraw-Hill Book Company INC 1953.

CHU-KIA WANG: Statically indeterminate structure
McGraw-Hill KogaKusha GD 1953.