RELIABILITY ANALYSIS OF OPTIMALLY DESIGNED REINFORCED CONCRETE FRAMES IN ACCORDANCE WITH EUROCODE 2 (2004)

BY

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A Thesis submitted to the School of Postgraduate Studies, Ahmadu Bello University, Zaria
in partial fulfillment of the requirements for the Award of
Doctor of Philosophy Degree in Civil Engineering.
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January, 2016

Declaration

I hereby declare that the work in this thesis entitled “Reliability Analysis of Optimally Designed Reinforced Concrete Frames in Accordance with Eurocode 2 (2004)” was carried out by me in the Department of Civil Engineering, under the supervision of:Prof. Idris Abubakar, Prof. Stephen PinderEjeh and Dr.Amana Ocholi. The information derived from the literature has been duly acknowledged in the text and a list of references provided. No part of this work has been presented for the award of Ph.D at any institution.

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Certification

Thethesis entitled "RELIABILITY ANALYSIS OF OPTIMALLY DESIGNED REINFORCED CONCRETE FRAMES IN ACCORDANCE WITH EUROCODE 2 (2004)" meets the regulations governing the award of the degree of Doctor of Philosophy of the Ahmadu Bello University, and is approved for its contribution to knowledge and literary presentation.

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Dedication

To my dear wife, Mrs Rita Rauta Titus and our childrenNicholas Nenfot, Mary Mwarotdang, Chris Gerdima and Patience Nenrit.

Acknowledgements

First and foremost, praise and thanks be to God my creator, the giver of wisdom who strengthened me despite the ill health during the course of the study.

I would like to express my sincere gratitude and appreciation to my supervisor, Prof. Idris Abubakarwhose drive for excellence spurred me relentlessly throughout the course of this research work to do a good job. I am also grateful to other members of the supervisory committee: Prof. S.P.Ejeh and Dr. A. Ocholi for their meticulous and timely assessment anytime I submitted the work to them thereby meeting the required standard. I also thank Dr. JibrinKaura who helped me with useful literature materials and also taught me the application of system reliability and code calibration software. I am also grateful to Engr. E. E. Okoro and late Engr. C. U. Chikezie who were former fellow Post Graduate students for their assistance and concern.

I really appreciate my dear wife, Mrs Rita Rauta especially for her understandingwhich enabled me to concentrate and commit resources to this work. My children, Nicholas Nenfot, Mary Mwarotdang, Chris Gerdima,Patience Nenrit, Mr& Mrs Solomon Daniel Adegun and SarayaBuazaTurdimma, I thank them all. My mum, Mrs Esther Ketkukah and my siblings, Sawul, Luka and Danjuma I thank them for their support, prayers and understanding.

I thank Prof. Sulaiman E. Bogoro for his moral and financial support, His Highness the District Head of Tapshin, NdeDauda Saleh Godit and my brother the Village Head of Kong Tapshin, NdeJurbe T. M. Dangana for their good wishes and prayers.

I am indebted to Rev. LawiPokti, Rev John Yilkpuba and RevYakubuKapalafor their prayers for me for the success of the study.

I extend my gratitude to Master Warrant Officer Samuel GofupAdu,Arc. P. J. Bot,Mr and Mrs Samson Nghalmi, Engr. Simon DatauGodenaan and all my friends not mentioned here for their encouragement and prayers.

I also thank my Dean of Engineering, University of Jos, Engr. Prof Stephen Mallo and the Coordinator of the Department of Civil Engineering Engr. Dr, Rose Daffi for their encouragement.

Abstract

This work presents reliability analysis of optimally designed reinforced concrete frames in accordance with Eurocode 2 (2004). A computer program in MATLAB (2004) was developed to analyze the frames based on direct stiffness method. The mathematical models of the design optimization problem were formulated and computer programs developed to obtain economical cross sectional dimensions and the corresponding amount of reinforcing steel using optimization process. MATLAB optimization tool box,$fmincon$was employed to perform the optimization. Reliability analysis based on the First-Order Reliability Method of analysis as in FORM5 (Gollwitzer, *et al.*, 1988) was employed to determine reliability levels of optimized frame elements. The results showedthat optimum design resulted in cost savings of 5.1% in beams and 3.2% in columns.The reliability indices of the critical beam in flexural failure mode were3.96 and 3.43 for deterministic and optimum designs respectively. The results of column axial failure mode gave reliability indices of 5.83 and 5.04 for deterministic and optimum designs respectively.The results of system reliability analysis gave system reliability index value of 3.33 as against the minimum component reliability index of 3.43 for the critical member. The optimally designed reinforced concrete frames resulted in cost savings but the component and system reliability indices were both lower than the reliability indices of the deterministic design.

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Notations

A_{bf} Surface area of formwork of the beam

A_{cf} Surface area of formwork of the column

A_t Cross sectional area of bars used for ties

A_{gb} Gross sectional area of beam (Concrete and reinforcement)

A_{st} Bottom reinforcement in the span of beams

A_s^- Top reinforcement in support of beams

A_{sc} Area of longitudinal reinforcement in column

A_{sw} Cross sectional area of shear reinforcement in the form of links

A_v Cross sectional area of bars used for stirrups

b_b Width of beam

b_c Width of column

C_bCost of a critical beam in the frame

C_cCost of concrete per unit volume

C_{column} Cost of a critical column in the frame

C_fCost of form work unit surface area

C_sCost of steel, ties and stirrups per unit weight

d_b Effective depth of beams

d_c Effective depth of columns

d' Depthof compression reinforcement

d_{lb} Diameter of longitudinal reinforcing bar in beams

d_{lc} Diameter of longitudinal reinforcing bar in columns

d_s Diameter of beams used for stirrups

d_t Diameter of bars used for ties

f_{ck} Characteristic compressive cylinder strength of concrete at 28 days

f_{yk} Characteristic steel yield strength

h_b Depth of beam

h_c Depth of column

L_n Clear Span of beam

L_b Length of beam between column centre lines

L_v Length of one stirrups

l Length over which stirrups are distributed

L_{bars} Length of longitudinal reinforcing steel bars

$L_{inclined}$Length of inclined portion of steel bars in column

L_u Unsupported length (clear height) of column

L_c Length of Column between centrelines of beams

M_u Factored bending moment at the most critical location in beams

N_{Ed} Design axial compression force

P_u Factored axial load on the most critical column at a given eccentricity

S_v Spacing of stirrups (links)

S_t Vertical spacing of ties.

V_{bs} Volume of longitudinal steal in the beam

V_{cs} Volume of longitudinal reinforcing/ steel in a column

V_v Volume of stirrups in beam

V_t Volume of laterals ties in a column

V_n Nominal shear strength of the cross section=$V_c + V_s$

V_c Nominal shear strength provided by concrete =$0.24 b_b d_b f_{ck}$

V_s Nominal shear strength provided by stirrups =$0.87 f_{tk} A_{sv}$

V_u Factorized shear force in the most critical beam

γ_sUnit weight of steel

n_s Number of stirrups

n_t Number of ties in one column

ρ Tension reinforcement ratio

ρ^0 Reference reinforcement ratio

ρ' Compression reinforcement ratio

φ Strength reduction factor

CHAPTER ONE

INTRODUCTION

1.2 Preamble

Reinforced concrete (RC) is one of the most widely used modern building materials. The wide use of RC in construction stems from the wide availability of reinforcing steel as well as concrete ingredients (cement, sand, gravel and water). It has the advantage that any shape can be formed (MacGregor and Wight, 2009).

RC Frames consist of horizontal beam elements and vertical column elements connected by rigid joints. These structures are cast monolithically, that is, the beams and columns are cast in a single operation in order to act in unison. Design applications range from low – rise buildings that are few stories to high-rise buildings with many floors.They provide resistance to both gravity and lateral loads through bending in the beams andcolumns(Nawy, 1996).

The design of structural components in a building frame is based on bending moment, shearforce and other load effects at each critical section in beams and columns. The critical sections for bending in columns are located at the top and bottomarears near the joints. For beams, the critical sections for positive and negative moments are assumed to be located at mid-spans and near the joints, respectively.

The trend in structural design has been towards improving the final design to the maximum degree possible without impairing the functional purposes the structure is supposed to serve

(Camp, *et al.*, 2003). This has led to the emergence of the concept of structural optimization. An optimum structure is that which satisfies the performance requirements related to safety and serviceability while economy (minimum cost or material consumption) is maximized (Moharrami, 1993). The objective of reinforced concrete optimization is usually to find the concrete cross-sectional dimensions and the corresponding amounts of reinforcing steel. The objective function is minimized under a set of conditions called constraints. The most important of these are the design constraints such as the Eurocode 2 (2004) requirements. These include design variables such as the area of steel and the cross sectional dimension of the members. The design constraints on dimensions, strength capacities and areas of reinforcement were based on the specifications of Eurocode 2 (2004).

Reliability – based structural design is necessary if uncertainties exist in loads, material and geometric properties and /or mathematical models (Agrawal and Bhattacharya, 2010). Structural reliability is the probability of not attaining the acceptable limits for safety and serviceability requirements of the structure before failure occurs.Structural analysis deals with the relation between the loads a structure must carry and its ability to carry the loads. It is usually measured by the reliability index, β which is a relative measure of the reliability or confidence in the ability of a structure to perform its function in a satisfactory manner.

The partial safety factors in Eurocodes are based on a linear analysis of structures and the satisfaction of ultimateand serviceability limit states using a semi-probabilisticapproach (Castro,*et al.*, 2005).By means of structural reliability methods the design equations, characteristic values, partial safety factors and load combination factors may be chosen such that the level of reliability of all structures designed according to the design codes is homogeneous and independent of the choice of material and the prevailing loading, operational and environmental conditions. The partial factors are calculated so that the reliability of structures is at the predetermined target level (Nowak and Szerszen, 2000).

A computer program was developed to perform the optimization process. The formulationwas programmed using MATLAB's intrinsic optimization toolbox function, fmincon (MATLAB, 2004).

1.2 Statement of Research Problem

Codes of practice for example Eurocode 2 (2004) and ACI 318 (2011) are used for the design of RC members. The codes of practice use the concept of the semi-probabilistic method. In this method, the random variables such as the strength of the materials and the loads are treated as deterministic values after obtaining their design values from characteristic values. The transformation of a characteristic value of a random variable into a design value is performed by multiplying or dividing it by a partial factor related to this variable. The resulting deterministic solution is usually associated with a high chance of failure of the member being designed, due to the influence of controllable uncertainties

(e.g. dimensions and material properties), and due to model uncertainties and errors associated with semi-probabilistic design. Reliability design methodologies address these problems (Melchers, 1999; Oberkampf, *et al.*, 2000).

Probability is incorporated in the design of these codes as represented by factors of safety, which results in overdesign of structural elementsas stresses needed to balance or offset the applied loads are very much in excess of what they should be. The effect of overdesign is increase in the cost of the members.Optimum design addresses the problem of overdesign by designing for optimum materials thereby reducing costs.

In the load factor design, when the partial safety factors are high, the reliability of the structure will be also high. Therefore, the reliability of the load factor design is not adequate(Almeida, *et al.*, 2008).Furthermore, deterministic design depends on the experience of the design engineer. For example, if the flexural strength of a beam is inadequate, the decision about whether to increase the section depth or the amount of reinforcement is the choice of the designer. In optimum design, however, this decision is made by the optimization algorithm that guides the modification. The reliability analyses therefore results in efficient and economical use of materials.

1.3 Justification of the Study

The cost of a reinforced concrete structure is influenced by the cost of formwork, concrete and reinforcing bars. Minimizing consumption of these materials therefore leads to cost saving.

Structural optimization is employed to achieve cost saving.A design is optimum if it is proportioned in size and reinforcement such that its cost is minimized while satisfying all design requirements (Fadaee and Grierson, 1996)). The optimization procedure involves minimization of an objective function, which takes into account the structure cost due to concrete, steel reinforcement and form work subject to performance requirements.

Optimum design also has the advantage that it leads to automation, that is, the design variables are chosen by the optimization process and not by the engineer.

Deterministic design is semi-probabilistic. In this method, the random variables such as the strength of the materials and the loads are treated as deterministic values after obtaining their design values from characteristic values. There is no direct link between uncertainties in design parameters and reliability. The resulting deterministic solution is usually associated with a high chance of failure of the member being designed (Babu and Basha, 2008)).

Uncertainty arising from randomness in structural materials and applied loads as well as from errors in behavioural models is inevitable and most be properly accounted to assuresafety and reliability. Reliability – based structural design accounts for such uncertainties (Agrawal and Bhattacharya, 2010).

1.4Aim and Objectives

1.4.1 Aim

The aim of this study is to carry out reliability analysis of optimized reinforced concrete frames in accordance with the provision of Eurocode 2 (2004).

1.4.2 Objectives

The objectives are to:

i. Develop a computer program to analyze plane frames based on stiffness method using MATLAB (2004) and design the frame elements according to Eurocode 2 (2004) design method

ii. Formulate the design optimization problem, develop the mathematical model and a computer program to obtain economical cross sectional dimensions and the corresponding amount of reinforcing steel using optimization process

iii. Develop optimum design charts for design of reinforced concrete elements

iv. Perform reliability analysis and establish the safety levels for the optimum design.

1.5 Scope and Limitations

1.5.1 Scope

A computer program in MATLAB was developed to analyze a three bay one storey reinforced concrete plane frame using the direct stiffness method. Optimization functions were formulated and modeled. MATLAB optimization tool box, $fmincon$was employed to perform the optimization.Reliability analysis was also carried out based on the First Order Reliability Analysis Method as in FORM5 (Gollwitzer,*et al*., 1988).

1.5.2 Limitations

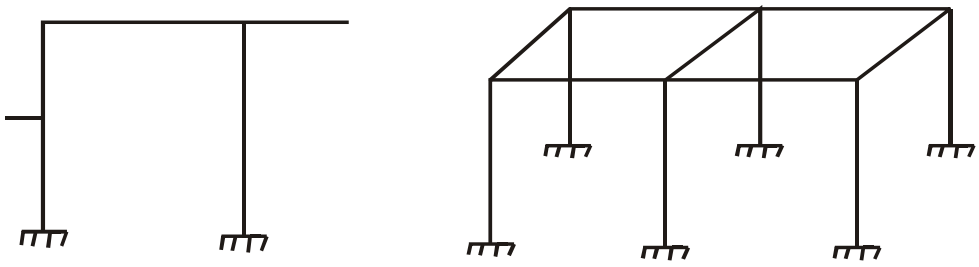
The reliability based optimum design was done only on one storey reinforced concrete plane frames.The statistical data (expected values, coefficient of variation and distribution models) used in the reliability analysis were obtained from literature.

CHAPTER TWO

LITERATURE REVIEW

2.1 Analysis of Frames

It is customary to divide structures into statically determinate and statically indeterminate. A statically determinate system means a system for which all the reactions of supports can be determined by equations of equilibrium and the internal forces also can be found by method of sections.Consider the structures shown in Figure 2.1.



(a) Plane frame (b) Space frame

Figure 2.1 Statically Indeterminate Structures

For plane structures (acting in the x-y plane) subject to forces, three equations of static equilibrium can be written. These equations are;

$$\Sigma F_x = 0\ , \Sigma F_y = 0\ , \Sigma M_z = 0 \qquad (2.1)$$

For space structures subjected to forces, six equations of static equilibrium can be written. The structure will be in equilibrium if the components of the resultant in the three orthogonal directions x, y, and z are equal to zero;

$$\Sigma F_x = 0\ , \Sigma F_y = 0\ , \Sigma F_z = 0 \quad (2.2a)$$

$$\Sigma M_x = 0\ , \Sigma M_y = 0\ , \Sigma M_z = 0 \ (2.2b)$$

A statically indeterminatesystem means that the reactions and internal forces cannot be analyzed by the application of the equations of static alone. The indeterminacy of the structure may be either external, internal, or both. A space structure is externally indeterminate if the number of the reaction components is more than six. The corresponding number in a plane structure is three. The plane structure shown in Figure 2.2 is externally indeterminate. The numbers of the reaction components are nine hence the degree of indeterminacy is six.

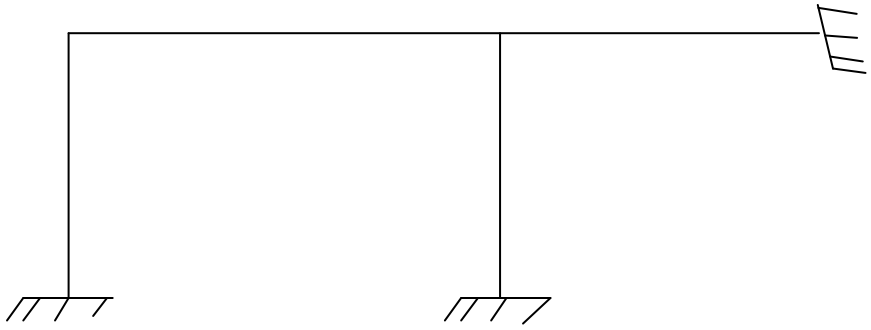


Figure 2.2: External Statically indeterminate System

Some structures are built with intermediate hinges; each hinge provides an additional equation of static equilibrium and allows the determination of additional reaction component. For instance, the frame given in Figure 2.3 with three intermediate hinges is a statically determinate structure with six reaction components.

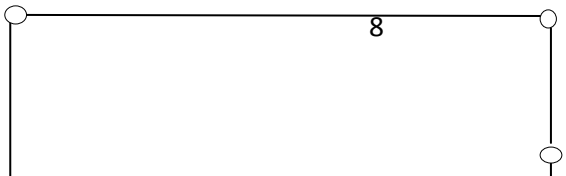


Figure 2.3: Frame built with intermediate hinges

A closed plane frame given in the Figure 2.4 which is externally statically determinate but internally indeterminate gives three degrees of indeterminacy. For closed plane frames, each closed part gives three degrees of indeterminacy.The number of indeterminacy decreases by introducing intermediate hinges. Each intermediate hinge removes the bending moment at each respective point.

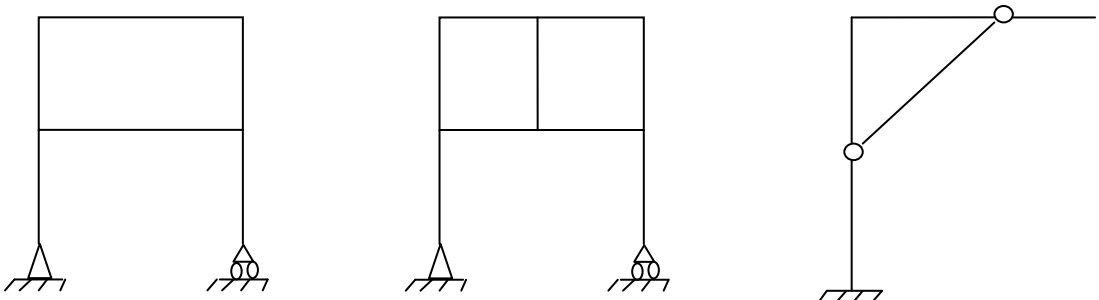


Figure 2.4: Internal Statically indeterminate System

The continuity of displacements in a given structure must be satisfied in a correct structural analysis. This is sometimes referred to as conditions of geometry. In a given structural frame joint, the displacements and rotations at the connection joint must be the same or consistent with each other. There are two types of indeterminacy static or kinematic indeterminacy depending on whether actions or displacements are of interest. When actions are the unknowns, then static indeterminacy is of paramount interest. A criterion in

determining static indeterminacy for two dimensional frames is given by the expression in equation (2.3).

$$D_s = (3\mathrm{b} + \mathrm{r}) - 3j \qquad (2.3)$$

Where D_s is Degree of static indeterminacy, b is number of members, r is the number of reactions and j is the number of joints.

For three dimensional frames the expression in equation (2.4).

$$D_s = (6\mathrm{b} + \mathrm{r}) - 6j \qquad (2.4)$$

When the displacements at the joints are unknown quantities, kinematic indeterminacy is important. A structure is said to be kinematically indeterminate if the displacement components of its joints cannot be determined by compatibility equation alone. In order to evaluate displacement components, it is necessary to consider the equations of static equilibrium which include number of unknown joint displacements which results to the degree of kinematic indeterminacy.

Kinematic Indeterminacy is the number of unknown displacement quantities or the number of Degree of Freedom that a structure has.When a structure is subjected to loads each joint may undergo translations andor rotations. At supports some displacements will be known, others will not. The number of unknown joint displacements corresponds to the kinematic indeterminacy of structure. The criterion in determining static indeterminacy for two dimensional frames isas follows

For pin jointed structure: Every joint has two displacement components and no rotation.

The expression is as in equation (2.5).

$$D_k = 2j - e \qquad (2.5)$$

Where D_k is the degree of kinematic indeterminacy, e is the number of equations of compatibility which is the number of reaction components plus constraints due to inextensibility.

For rigid jointed structure:Every joint has three displacement components,two displacements and one rotation. The expression is in equation (2.6).

$$D_k = 3j - e \qquad (2.6)$$

There are two distinct methods of analysis for statically indeterminate structures. These methods are the force or flexibility method and the displacement or stiffness method. The latter is employed to analyze the frame in this study therefore it is discussed in most detailed.

2.1.1 Force or Flexibility method.

In the force method of analysis, primary unknowns are reaction forces. Static indeterminacy is of paramount interest. Compatibility equations are written for displacement and rotations (which are calculated by force displacement equations). The degree of static indeterminacy of the structure is determined and the redundant identified. Release the indeterminate constraints, and the resulting deformation discontinuity calculated. Then redundant actions are then replaced to restore the continuity and the resulting compatibility equations solved for the redundant force actions. If the entire redundant forceis removed, the resulting structure known as released structure is statically determinate. This released structure is also known as basic determinate structure. From the

principle of super-imposition, the net displacement at any point in statically indeterminate structure results to displacements in the basic structure due to the applied loads and the redundant.

2.1.2 Displacement or Stiffness Method

In this method of analysis, the primary unknowns are the displacements.The structure is considered to be kinematically indeterminate to the nth degree where n is the total number of independent displacements. Additional restrains are added to fix all the degrees of freedom and the values of these restraints calculated. The restraints are removed to allow deformations and restore equilibrium. The resulting equilibrium equations are solved for the displacements and subsequently the force actions are determined satisfying the compatibility conditions and force displacement relations. The displacement-based method is amenable to computer programming and hence the method is being widely used in the modern day structural analysis. There are three methods for the stiffness based analysis of a structure: slope deflection, moment distribution and direct stiffness methods. The slope deflection and moment distribution methods lend themselves to hand calculations and direct stiffness method to computer based analysis.

a) Slope Deflection: Which results in a system of n linear equations with n unknowns, where n is the degree of kinematic indeterminacy (total number of independent displacements or rotation) consists of series of simultaneous equations, each expressing the relation between the moments acting at the ends of the members written in terms of slope & deflection. The solution of slope deflection equations along with equilibrium equations

gives the values of unknown rotations of the joints. Knowing these rotations, the end moments are calculated using slope deflection equations.

b) Moment Distribution: This is an iterative method to solve for the n displacements and corresponding internal forces in flexural structures. This method was first introduced by Prof. Hardy Cross and is widely used for the analysis of intermediate structures. In this method, the structural system is reduced to its kinematically determinate form; this is accomplished by assuming all the joints to be fully restrained,the fixed end moments are calculated for thestructure, the joints are allowed to deflect or rotate one after the other by releasing them successively. Furthermore, the unbalanced moment at the joint is shared by the members connected at the joint when it is released.

c) Direct Stiffness method: This is by far the most powerful method of structural analysis. The basic stiffness equation expresses equilibrium of each of the nodes in terms of nodal stiffness influence coefficients and unknown nodal displacements. First each of the degrees of freedom of the structure is given a number. The displacements corresponding to each of the degrees of freedom are the unknowns to be determined. The second step is to compute the stiffness influence coefficients. When degree of freedom, $j$is given a unit displacement, the forces corresponding to each of the degrees of freedomK_{ij} are found. The equilibrium equations are then formulated as in equation (2.7).

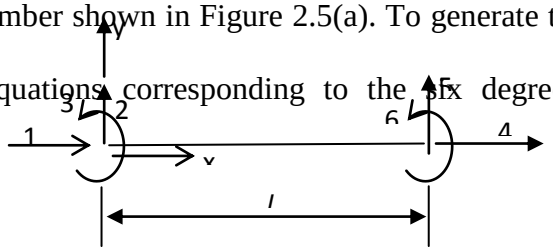
$$K_{11}D_1 + K_{12}D_2 + \ldots + K_{1n}D_n = Q_1$$

$$K_{21}D_1 + K_{22}D_2 + \ldots + K_{2n}D_n = Q_2 \qquad (2.7)$$

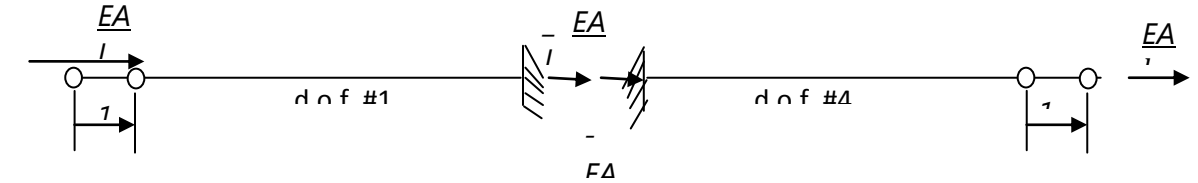
$$K_{n1}D_1 + K_{n2}D_2 + \ ... + K_{nn}D_n = Q_n$$

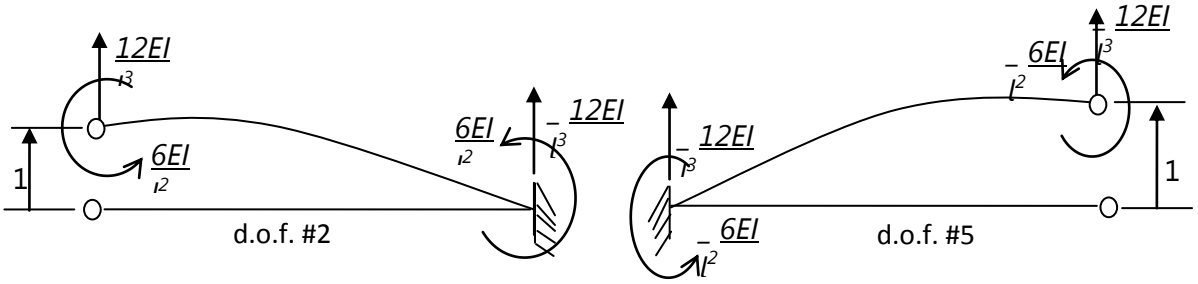
Where the terms D_1 to D_n represent the n unknown displacements, and the terms Q_1 to Q_n are the nodal loads.

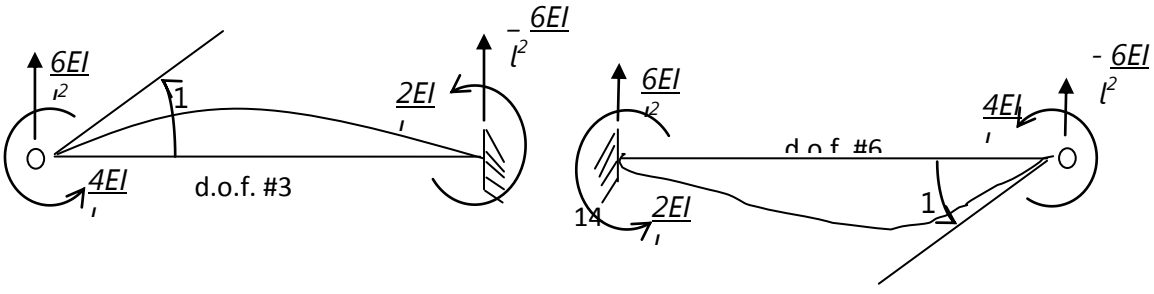
To determine the nodal stiffness influence coefficients, consider a prismatic member of length$l$, area A, moment of inertia I about the z axis and modulus of elasticity E as shown for a simple member shown in Figure 2.5(a). To generate the member stiffness matrix, six displacement equations corresponding to the six degrees of freedom are defined in Figure2.5(b).



(a)







(b)

Figure 2.5: Member stiffness Influence Coefficients (White et al., 1976)

Every two-dimensional framed structure with prismatic members is made up of elements of this type, so that once the member stiffness matrix has been computed, it may be used repeatedly for each of the members in the structure. The member stiffness matrix is computed by imposing a unit displacement corresponding degree of freedom, j. Figure 2.5b indicates each of the unit displacement in turn, with the corresponding forces and couples required to impose the displacement.

The member stiffness may be written directly from the information displayed in Figure 2.5(b)as shown in equation (2.8), (White, et al., 1976):

$$R_T = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \quad (2.8)$$

It is assumed here that deformations due to shear forces are negligible compared to those due to bending. Consequently, the stiffness matrix in equation (2.8) does not include the effect of shear deformations.

Computing the applied nodal loads, consider the one storey frame shown in Figure 2.6. To accomplish the conversion to nodal loads, the superposition principle for the related two cases of loadings as shown in Figure 2.6 (b) and (c). This is because the structure is linear and the loads in cases I and II add to give the loads of Figure 2.6(a). Case I is constructed by imposing on the structure the fixed end reactions from the uniform load as they act on the rest of the structure instead of the uniform load itself. Case II is constructed by imposing the uniform load and in addition, imposing the fixed end reactions as they act on the loaded member to prevent joint rotation and translation. Case I is analyzed by the direct stiffness method and case II are simply those of fixed beam.

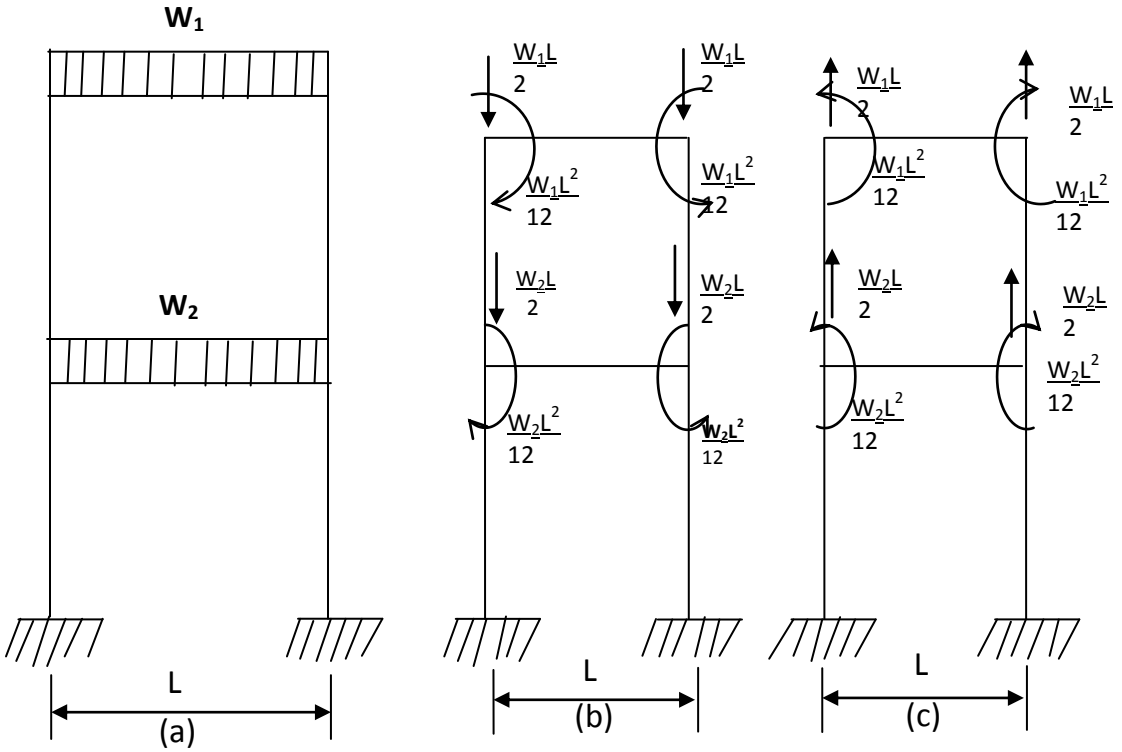


Figure 2.6 Transformation to nodal loads (White et al, 1976)

The member stiffness array expressed in the member coordinate system cannot be worked into the structure nodal stiffness matrix until it has been transformed to the structure coordinate system; however, two force components leading to degrees of freedom 1 and 2 of the member is considered. Member end forces are related to member end displacementsd' through the member stiffness matrix shown in equation (2.9),(White, et al., 1976):

$$\begin{bmatrix} q_1' \\ q_2' \end{bmatrix} = \begin{bmatrix} k_{11}' & k_{12}' \\ k_{21}' & k_{22}' \end{bmatrix} \begin{bmatrix} d_1' \\ d_2' \end{bmatrix} \qquad (2.9)$$

Whereq', d' and k' are member end forces , member end displacements and member stiffness matrix respectively in the local structure coordinate system

Consider the vector V_1 and V_2 in the global coordinate system of Figure 2.7 (called the global coordinate system) in equation (2.7).

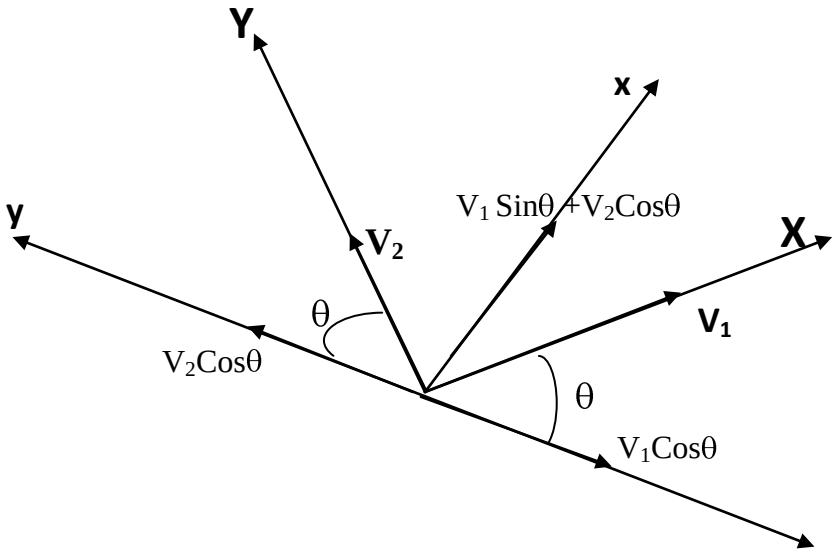


Figure 2.7: Coordinate transformation

Their components in the x, y coordinate system (local coordinate) also indicated in the figure.

The transformation from the global to the local system is simply by resolution of forces from Figure 3.7 as shown in equation (2.10).

$$V_1' = V_1 cos\theta + V_2 sin\theta \quad (2.10)$$

$$V_2' = -V_1 sin\theta + V_2 cos\theta \quad (2.11)$$

In matrix terms, it is shown in equation (2.12).

$$\begin{bmatrix} V_1' \\ V_2' \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad (2.12)$$

Where the V terms with primeare the transformations to local coordinate system.

Since both forces and displacements are vector quantities, they may be transformed in the same manner as the vector, V. The member end forces in the member coordinate system are written in equation (2.13).:

$$\begin{bmatrix} q_1' \\ q_2' \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \quad (2.13)$$

Where q is member force in the global structure coordinate system at ends 1 and 2. Similarly, the member end displacements in the member coordinate system may be written as shown in in equation (2.14), (White, et al., 1976):

$$\begin{bmatrix} d_1' \\ d_2' \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \quad (2.14)$$

Where d is member endforcein the global structure coordinate system at the ends 1 and 2. Substituting equations (2.13) and (2.14) into (2.9), we obtain the member force displacement relationship in equation (2.15),(White, et al., 1976):

$$\begin{bmatrix} \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} k'_{11} & k'_{12} \\ k'_{21} & k'_{22} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix} \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \qquad (2.15)$$

In matrix shorthand form, we have

$$[r][q] = [k'][r][d] \qquad (2.16)$$

Where$[r]$ is the rotation matrix, $[k']$ the member stiffness matrix in member coordinates, $[q]$the member end forces in the structure coordinate system and $[d]$ the end displacements in the structure coordinate system.

Pre-multiplying by$[r]^{-1}$, we have

$$[r]^{-1}[r][q] = [r]^{-1}[k'][r][d] \qquad (2.17)$$

$[r]^{-1}[r]$ is unity, therefore;

$$[q] = [r]^{-1}[k'][r][d] \qquad (2.18)$$

Since $[q]$ and $[d]$ are the member end forces and displscements in the structures coordinate system, we interpret $[r]^{-1}[k'][r]$ as the member stiffness matrix in the structure coordinate system, $[k]$.

The transformation array may be expanded into $6x6$ matrix as(White, et al., 1976):

$$R_T = \begin{bmatrix} cos\theta sin\theta & 0 & 0 & 0 & 0 & 0 \\ -sin\theta cos\theta & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & cos\theta sin\theta & 0 & 0 \\ 0 & 0 & 0 & -sin\theta & cos\theta & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (2.19)$$

Applying the $6x6$ matrix we have

$$[k] = [R]^{-1}[k'][R] \qquad (2.20)$$

The inverse form of $[R]$ is equal to its transpose, therefore we have

$$[k] = [R]^{T}[k'][R] \qquad (2.21)$$

To obtain the displacements[D], $[K]$has to be expanded to include degrees of freedom that are actually restrained. Equation (2.21) may be partitioned as shown in Equation (2.22).

$$\begin{bmatrix} K_{ff} & K_{fr} \\ K_{rf} & K_{rr} \end{bmatrix} \begin{bmatrix} D \\ Dr \end{bmatrix} = \begin{bmatrix} Q \\ Qr \end{bmatrix} \qquad (2.22)$$

From equation (2.22), we have

$$[\mathrm{K_{ff}}][\mathrm{D}] + [\mathrm{K_{fr}}][\mathrm{D_r}] = [\mathrm{Q}] \qquad (2.23)$$

Equation (2.23) is solved for displacement,

$$[\mathrm{D}] = [\mathrm{K_{ff}}]^{-1}([\mathrm{Q}] - [\mathrm{K_{fr}}][\mathrm{D_r}]) \qquad (2.24)$$

The member forces in a plane frame member can be obtained from the following expression (Laursen, 1988):

$$M_i = E\left[\frac{6I}{L^2}\left(-sd_i^1 + cd_i^2\right) + \frac{4I}{L}d_i^3 - \frac{6I}{L^2}\left(-sd_i^1 + cd_i^2\right) + \frac{2I}{L}d_i^3\right] \qquad (2.25)$$

$$M_j = E\left[\frac{6I}{L^2}\left(-sd_i^1 + cd_i^2\right) + \frac{2I}{L}d_i^3 - \frac{6I}{L^2}\left(-sd_i^1 + cd_i^2\right) + \frac{4I}{L}d_i^3\right] \qquad (2.26)$$

$$P = P_j = \frac{EA}{L}\left(-cd_i^1 - sd_i^2 + cd_j^1 + sd_j^2\right) \qquad (2.27)$$

Where s=sinθ, c= cosθ, $d_i^1, d_i^2 and\ d_i^3$;$d_j^1, d_j^2 and\ d_j^3$ are global components of displacement at the member ends in the X,Y and Z directions respectively. The subscripts i and j are the first and last ends of a member respectively. P is the axial force and M is the member-end moment.

2.2 Structural Optimization

Structural optimization is an exercise by which an optimum design is derived from an initial design through some iterative process. The definition of the optimum structure is that which satisfies the performance requirements provided in design codes while economy (minimum cost or material consumption) is maximized (Arora,1976). Generally, any optimization problem consists of an objective function and some constraints formulated in terms of the design variables.

Parameters chosen to describe the design of a system are called design variables. Once these variables are assigned numerical values, a design of the system is known. If the specified values do not satisfy all the constraints of the problem, the design is not feasible. If the constraints are satisfied, we have a feasible (workable) design.

There can be many feasible design for a system and some are better than others. To make such a claim we must have a criterion to compare various designs. Such a criterion is called an objective function for the optimum design problem, and it is represented by$f$or$f(x)$, to emphasize its dependence on the design variable vector x. The objective function will always be minimized in this text. A function that is to be minimized is called the cost function. Several objective functions are used-minimized cost,maximize profit,

minimize weight, etc. We always want to minimize the cost of manufacturing goods or maximize return on an investment.

In structural design optimization, the minimum weight or minimum cost is usually taken as the objective. The objective function to be minimized expresses the goal or performance criterion in terms of the decision variables. This is expressed in terms of concrete volume, steel weight, form work surface area as well as their unit costs. The mathematical form of the objective function for the design of reinforced concrete frames is given by the following expressions[Camp, *et al.*, 2003]:

Minimize $F = f(p_m, p_j, p_s)$

$$F = \sum_{element}\left(C_c lbh + C_s l A_s + 2C_f l(b+h)\right) \quad (2.28)$$

Subject to $c_1 \leq 0,\ c_2 \leq 0\ ,\ \dots\ , c_n \leq 0$

Where F: Objective function representing cost of reinforced concrete element in terms of cost of concrete, reinforcing steel and formwork.

p_m, p_j, p_s: Material properties, connection characteristics and structural characteristic respectively

b,h : Width and depth of the cross section respectively.

l, A_s : Length of member and area of reinforcing bars respectively

C_c: Cost of concrete per unit volume

C_s: Cost of steel, ties and stirrups per unit weight

C_f: Cost of form work unit surface area

The constraints concern limitations imposed on structural behaviour bythe design performance requirements of the governing code of practice. There is a rather broad class of constraints, some of which are limitations on stress or strength capacity, deflection, buckling, natural frequency and dynamic instability. These constraints, are generally formulated as inequality constraints and. in their basic form, are often implicit functions of the design variables. To facilitate computer solution of the optimization problem, the constraints must first be transformed into explicit functions of the design variables.

The main ideas in structural optimization can be traced to the time of Galileo who proposed the uniform strength criterion for optimum form, and determined in 1638 the optimal form of a bent beam of uniform strength (Wasiutynski and Brandt, 1963).

By 1960 it was known that a broad class of structural design optimization problems could be stated as mathematical programming problems. In his pioneering work, Schmit (1960) proposed a rather general approach to structural optimization which has served as a conceptual foundation for the development of many modern structural optimization methods. The idea of coupling finite element structural analysis and nonlinear mathematical programming to create automated optimum design capabilities for a broad class of structural systems were introduced. It was shown that the minimum weight optimum design of elastic statically indeterminate structures could be posed as a nonlinear mathematical programming problem accounting for multiple loading and various types of

constraints, including sizing conditions. Furthermore, he found that because of the non-linearity of the constraints the optimum does not necessarily lie at a vertex in design space. This discovery brought attention to the fact that the SFM method, which was then the prevailing approach to aircraft structural component optimization, does not necessarily guarantee an optimal solution. The application of nonlinear mathematical programming techniques in structural optimization became widespread during the period 1960-1970. A good history of the progress in this period is reported by Schmitt(1971), which contains a review of the development of nonlinear mathematical programming, including sequential linear programming, the penalty function method, the method of usable-feasible directions, and the gradient projection method.

By 1970, attention became focused on the efficiency of algorithms. This was particularly important because the currently available structural optimization techniques required long computer run times to solve design problems for even modest size structures. Three different ways were proposed to overcome this difficulty. These three approaches are design oriented structural analysis, the approximation concept approach and the optimality criteria method:

2.2.1 Design oriented structural analysis approach

Attention is given to the idea that structural analysis should generate with minimum effort an estimate of the critical and potentially critical response quantities adequate to guide the design modification process. Developments in the design oriented structural analysis approach (DOSA) fall into two main categories:

a) Methods for obtaining rates of change of response quantities with respect to design variables Fox(1965)

b) Re-analysts techniques for constructing new analysis. This technique results in less effort than a complete analysis (Arora, 1976).

2.2.2The approximation concept approach

There arethree major obstacles to the development of efficient computer-automated structural optimization techniques using mathematical programming algorithms. The first obstacle is the large number of design variables, the second is the large number of inequality constraints and the third is the fact that the inequality constraints are generally only implicit functions of the design variables. The challenge was how to overcome these obstacle over the years.

Much work was done to overcome these difficulties, but it was not until the mid-1970's that Schmitt and Farshi (1974) and later Schmitt and Miura (1976) developed the approximation concept approach (ACA) technique base on mathematical programming principles. The main idea was to replace the original optimization problem with a sequence of relatively small, explicit but approximate problems that preserve the essential features of the original design problem. The basic features of ACA include reduction of the number of independent design variables by means of a design variable linking technique, temporary deletion of inactive constraints at each design stage, and expressing implicit constraints explicitly in terms of design variables using first-order Taylor series expansions. In this way, it was found that the number of structural re-analyses, design variables and constraints could be reduced significantly.

2.2.3OptimalityCriteria methods

The methods focuses on the satisfaction of prescribed conditions for the structure without explicit concern for an objective function. In other words, it is accepted that if the optimality conditions are satisfied, then an objective function relating to weight or cost of the structure is satisfied indirectly and the corresponding design is optimum (Agarwal, 2004).

Design modifications in these methods are usually based on a recursive resizing formula derived from the optimality criterion. Among the optimality criteria (OC) methods, the Fully Stressed Design (FSD) method has a long history (since the time of Galileo) and for some time was thought to provide the true optimum for the elastic design of structures (Fadaee and Grierson, 1996). The ease of implementation and fast convergence of this method makes it a viable alternative to formal optimization algorithms and is widely used along with the Simultaneous Failure Mode (SFM) method until 1960, when Schmittquestioned the optimality of the FSD and SFM results. However, the need for efficient algorithms attention was then focused on finding other optimal criteria methods that have rigorous mathematical basis.Razani (1965) investigated the possible relationship between fully stressed design and minimum weight design and suggested a procedure to modify the FSD solution to the real optimum design.

Further advances in modern structural optimization were obtained by Sander and Fleury (1978) proposed the mixture of optimality criteria and mathematical programming

methods in a dual formulation of the design problem. In this way,they overcame the difficulty that the optimality criteria method experiences in identifying critical constraints a priori. Later, Schmit and Fleury (1980) combined approximation concepts and dual optimization methods to achieve further efficiency for the dual method.

The use of the OC method has become widespread and has been applied, with a variety of modifications, to various fields of structural optimization, including building structures. The main reason for the popularity of this method is its ease of implementation and fast convergence, when the method is applicable and when it works.

2.2.4Works on Optimization of Reinforced Concrete Structures

Hassanain(1992) employed second-order method to analyse and design RC frames. The frames were formulated as a non-linear programming programme according to the ACI code provisions. Concrete dimensions and steel areas for columns and beams were the design variables. The column widths were assigned the same design variables as well as each of the column depths, beam widths and beam depths. The objective function was the sum of all the costs for each column and beam.

Modern optimization algorithms and software were utilized to develop an optimization system that was capable of analysing and designing economical RC frames of moderate heights. It was proven that the minimum total cost criterion was the most economical design for RC frames. While analysis and design using moment distribution was found to generate lower-cost RC frames than no moment distribution, the reduction in cost did not justify the extra computational effort to reach an optimal solution. For the case of the study

there was 3.5% reduction in cost while CPU times to reach an optimum solution increased by 5% for no moment distribution.

Balling and Yao (1997) carried out a study on the optimization of three dimensional (3D) reinforced concrete frames. The validity of the assumption that optimum concrete –section dimensions are insensitive to the number, diameter, and longitudinal distribution (topology) of reinforced bars is examined. The optimum results from a method that simultaneously optimizes concrete section dimensions and the number, diameter and topology of the reinforcing bars is compared to a traditional method that represents the number diameter and topology of the reinforced bars through the single design variable, A_s the total number of steel. The results gave maximum difference in optimum cost for one story as 4.2% and average difference in optimum width, bar depth, $h$as 14.7%. They represented a new method, referred to as the simplified method as the most efficient method for theoptimization of reinforced concrete frames. The method applied to 3D one-two and four- story frames subjected to load combinations involving dead, live, snow and seismic loads.

The reinforcing steel bar number and the number of bars or topology of the reinforcement were used by Camp, *et al.* (2003) as design variables with the width and the thickness of the sections, for the design of reinforced concrete frames using the genetic algorithm. The limitations and specifications of the ACI code are formulated as a series of constraints to the discrete cost optimization problem and applied as penalties on fitness function (structural cost) of the genetic algorithm. It was associated with GA design might be

viewed as insignificant in the total cost of the structure the systematic and automatic verification of the ACI code limitations and restrictions can provide an increased level of confidence in the integrity of the design.

Guerra and Kiousis(2006) presented a novel optimization approach for the design of reinforced concrete (RC) structures. Optimal sizing and reinforcing for beam and column members in multi-bay and multistory RC structures incorporates optimal stiffness correlation among structural members and results in cost savings over typical state-of-the-practice design solutions. The design procedures for RC structures adapted began by assuming initial stiffness for the structural skeleton elements. This was necessary to calculate the internal forces of a statically indeterminate structure. The final member dimensions were then designed to resist the internal forces that were the result of the assumed stiffness distribution.

The formulation, including the ACI-318(1996) member sizing and the cost evaluation, was programmed in MATLAB (2004) and was solved to obtain the minimum cost design using the sequential quadratic programming (SQP)algorithm implemented in MATLAB's intrinsic optimization function fmincon, which is designed to solve problems of the form:

To find a minimum of a constrained nonlinear multivariable function, $f(X)$,

Subject to

$$g(X) = 0 \qquad (2.29)$$

$$h(X) \leq 0 \qquad (2.30)$$

$$lb \leq X \leq ub \qquad (2.31)$$

Where X are the decision variables, $g(X)$ and $h(X)$ are constraint functions, $f(X)$ is a nonlinear objective function that returns a scalar (cost), and lb and ub are the lower and upper bounds on the decision variables. All variables in the optimization model must be continuous.

The SQP method approximates the problem as a quadratic function with linear constraints within each iteration; in order to determine the search direction and distance to travel (Edgar and Himmelblau, 1998). Two dimensional reinforced concrete frames with i members of length, L_i were considered. Each member had a rectangular cross section with width $b_i$and depth, h_i reinforced with compressive and tensile steel reinforcing bars, $A_{s1.i}$, and $A_{s2,i}$ respectively. The set ofb_i, h_i, $A_{s1,i}$, and $A_{s2,i}$constitute the decision variables.

They found out that optimal designs result in cost savings of a little less 1% for the 8-meter span to 17% for the 24- meter span. It is very interesting to note that for certain span lengths the typical design costs were relatively close to the optimal design costs. A beam length of eight meters for a portal frame with four meter long columns appears to be the most efficient structure for the typical practice assumptions as it results in a cost closest to the optimal cost. Substantial costs savings over typical design was demonstrated for multi-bay structures. After reaching the lowest typical design cost of approximately $9700 for the six-bay structure, the costs increased linearly with each additional bay. The linear increase indicates that minimum dimensions have been reached for the typical design assumptions. Thus, each additional bay increased the total cost by the cost of one column

and one beam. The two-through six-story structures showed cost savings of 11.6%, 3.0%, 9.4%, 6.9%, and 6.2%, respectively.

Galeb and Atiyah (2011) carried out optimum design of reinforced concrete (two-way ribbed) waffle slabs using the built-in genetic algorithm tool box of MATLAB. A computer program was written using MATLAB to perform the structural Analysis and design of waffle slabs by the direct design method. The design variables were taken as the effective depth of the slab, ribs width, the spacing between ribs, top slab thickness, the area of flexural reinforcement at the moment critical sections, the band beams with and the area of steel reinforcement of the beams. The optimum value of effective depth was found to be between (280-450) mm within the practical range of (300-600) mm for span length up to 15m (Cement and Concrete Association of Australia, 2003). The effective depth to span length was found to be between (1/33 – 1/18) also in conformity with the provision of the Australian code. It was found that the increase in the ratio of concrete cost relative to the steel cost caused a decrease in the rib spacing and the cross-sectional area of the rib.

Yousif and Najem (2014)examined the optimum cost design of reinforced concrete columns with various loading conditions using the Generic Algorithms (GAs). Many design constraints were used to cover all the reliable design results, such as limiting the cross sectional dimensions, limiting the reinforcement ratio and even the behaviour of the optimally designed sections. The optimum results were compared with other published works and the reduction in design cost of the biaxially loaded columns of about 26% was achieved using the GA design (1-3%) was achieved for the uniaxially designed in the

axially loaded columns. It was proved that GA is capable for designing optimum columns sections.

Most of the works presented above were based on ACI 318-83(1998) Building Code requirements for Reinforced Concrete. Booz, *et al.*(1984) applied the German design regulations to optimize the concrete elements while Kanagasundaram and Karihaloo (1990) performed their studies based on Australian design regulations. The study by Almeida, et al (2008) was carried out according to the NBR6118:2004 and was treated deterministically in both the deterministic design optimization (DDO) and reliability-based design optimization RBDO).

The work presented in this study is carried out in accordance with Eurocode 2 (2004). The frame was analyzed using direct stiffness method while the optimization procedure was implemented using MATLAB's intrinsic optimization function *fmincon* and Generic Algorithm in MATLAB environment. The use of MATLAB's intrinsic toolbox optimization has been employed by other authors such asas in Guerra and Kiousis (2006), Babu and Basha (2008) and Galeb and Atiyah (2011).

2.3 Reliability Analysis

Structural reliability is the probability of not attaining the acceptable limits for safety and serviceability requirements of the structure before failure occurs(Pander, 2008). Structural analysis deals with the relation between the loads a structure must carry and its ability to carry the loads. It is usually measured by the Reliability Index, β (Baecher and Christian,

2003)which is a relative measure of the reliability or confidence in the ability of a structure to perform its function in a satisfactory manner.

If the loading to which an engineering system is exposed is *Q* and the available resistance is *R*. The values of both *R* and *Q* are uncertain, so the result of their interaction is also uncertain. These variables have mean or expected values, variances and covariance. The notations are as follow:

μ_R, μ_Q: mean values of *R* and *Q*, respectively.

E[*R*], *E*[*Q*] : expected values of *R* and *Q*, respectively = μR, μQ.

σ_R, σ_Q: standard deviations of *R* and *Q*, respectively.

Ω: coefficient of variation = σ/μ.

σ_R^2, σ_Q^2 : variances of *R* and *Q*, respectively, also denoted *Var* [*Q*], *Var* [*R*]

ρ_{RQ}: correlation coefficient between *R* and *Q*.

Cov[*R, Q*]: covariance of *R* and *Q* = $\rho_{RQ}\sigma_R\sigma_Q$

The margin of safety, M, is the difference between the resistance and the load:

$$M = R - Q \tag{2.32}$$

From the elementary definitions of mean and variance, it follows that regardless of the probability distributions of R and Q the mean value of M is (Baecher and Christian, 2003)

$$\mu_M = \mu_R - \mu_Q \tag{2.33}$$

and the variance of M is

$$\sigma_M^2 = \sigma_R^2 + \sigma_Q^2 - 2\rho_{RQ}\sigma_R\sigma_Q \tag{2.34}$$

A reliability index, β , is defined as

$$\beta = \frac{\mu_M}{\sigma_M} = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2 - 2\rho_{RQ}\sigma_R\sigma_Q}} \quad (2.35)$$

which expresses the distance of the mean margin of safety from its critical value (M = 0) in units of standard deviation. If the load and resistance are uncorrelated, the correlation coefficient is zero, and

$$\beta = \frac{\mu_M}{\sigma_M} = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \quad (2.36)$$

Figure 2.8 shows plots of typical probability distributions of reaction(R) and load(Q). Figure 2.9 (a) shows the resulting probability distribution of *M*. The probability of failure must be the probability that *M* is less than 0.0, which is the shaded area in Figure 2.9(a).

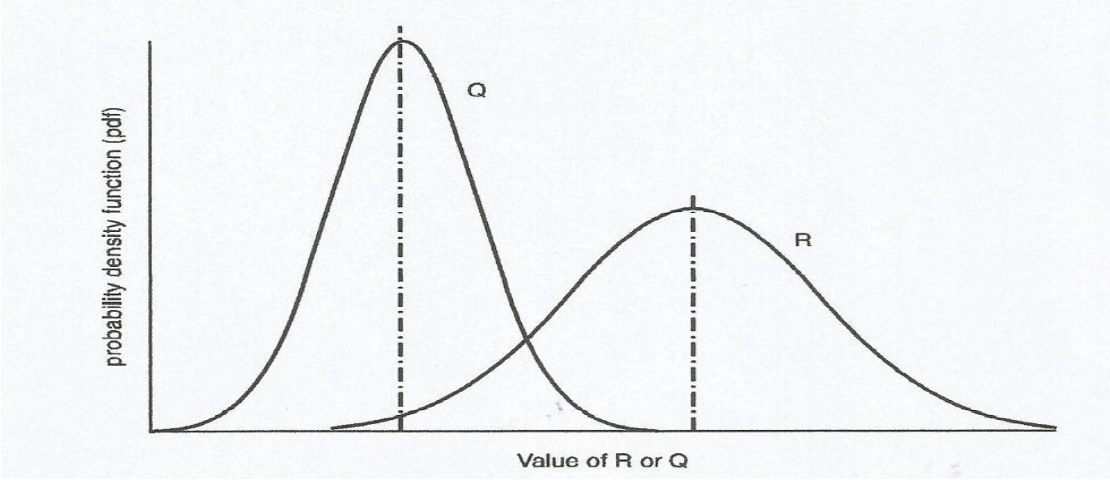


Figure 2.8:Probability densities for typical resistance (*R*) and load (*Q*) (Baecher and Christian, 2003)

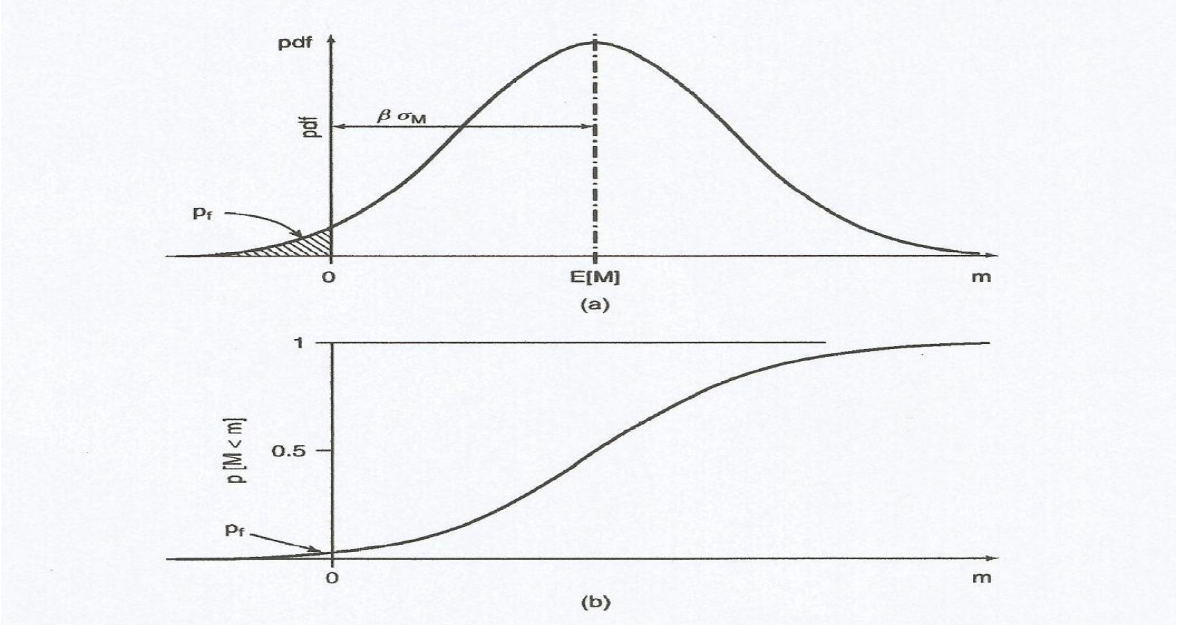


Figure 2.9: (a) Probability density and (b) cumulative probability for margin (M) (Baecher and Christian, 2003).

Note that the area in Figure 2.9(a) under the curve and to the left of the axis is the probability of failure identified in (b).Figure 2.9(b) shows the cumulative distribution corresponding to the distribution function of Figure 2.9(a). The probability of failure is the intercept of the cumulative distribution function with the vertical axis at $M = 0$. In the special case that R and Q are normally distributed, M is normally distributed as well. Thus, the reliability index, β, which normalizes M with respect to its standard deviation, is a standard Normal variate, usually designated Z.

We want the probability of failure, P_f, which is the integral between $-\infty$ and values of the parameter Zlocated below the mean value, that is, negative values of$Z$. Due to the symmetry of the normal distribution, the probability of failure is:

$P_f = 1 - \Phi(\beta) = \Phi(-\beta)$(2.37)

The reliability analyses are performed by the First Order Reliability Method (FORM). The method FORM, although an approximate method, leads to an accurate evaluation of the probability of failure in most reliability problems (Almeida, *et al.*, 2008).

The reliability of structural component is evaluated with respect to one or more limit states. The limit state equation is referred to as the performance or state function and expressed as

$g(X) = g(X_1, X_2, \dots, X_n) = R - S$(2.38)

Where X_i for $i = 1,2, \dots, n$ represent the basic design variables. S is the load and R is the resistance on the structural system or element.

2.3.1 Target Reliability

In terms of a reliability based approach the structural risk acceptance criteria correspond to a required minimum reliability defined as target reliability. The requirements to the safety of the structure are consequently expressed in terms of the accepted minimum reliability index or the accepted maximum failure probability.

The levels of reliability that apply to a particular structure may be specified by the classification of the structure as a whole or by the classification of its components (Eurocode 2, 2004).For the purpose of reliability differentiation, consequences classes (CC) may be established by considering the consequences of failure or malfunction of the structure as given in Table 2.1.

Table 2.1: Definition of consequences classes (Eurocode 0, 2002)

Consequences Class	Description	Examples of buildings and civil engineering works
CC3	**High** consequence for loss of human life,*or* economic, social or environmental consequences **very great**	Grandstands, public buildings whereconsequences of failure are high (e.g. a concert hall)
CC2	**Medium** consequence for loss of human life, economic, social or environmental consequences **considerable**	Residential and office buildings, public buildings where consequences of failure are medium (e.g. an office building)
CC1	**Low** consequence for loss of human life, *and* economic, social or environmental consequences **small or negligible**	Agricultural buildings where people do not normally enter (e.g. storage buildings), greenhouses

Three reliability classes RC1, RC2 and RC3 may be associated with the three consequences classes CC1, CC2 and CC3. Tables 2.2gives recommended minimum values of reliability index associated with reliability classeswhile Table 2.3 gives the target reliability index,β for Class RC2 structural members

Table 2.2: Recommended minimum values for reliability index β(ultimate limit states) (Eurocode 0, 2002)

	Minimum values for β	
	1 year reference period	50 years reference period
RC3	5.2	4.3
RC2	4.7	3.8

RC1	4.2	3.3

Table 2.3 Target reliability index β for Class RC2 structural members atultimate limit states (Eurocode 0, 2002)

	Target reliability index	
	1 year reference period	*50* yearsreference period
Ultimate	4.7	3.8
Fatigue	-	1.5 to 3.8
Serviceability (irreversible)	2.9	1.5

2.3.2Component Reliability Analysis

Component reliability is the reliability of one single structural member which has one dominating failure mode. For each of the failure elements, the element reliability index (β) is estimated using the first-order reliability method as in FORM5 (Gollwitzer *et al*., 1988).

2.3.3 System Reliability Analysis

Probabilistic structural design is primarily concerned with component behavior. System behavior is, however, of concern because it is usually the most serious consequence of structural failure. Therefore the livelihood of system failure following an initial component failure should be assessed. In particular, it is necessary to determine the system characteristics in relation to acceptable tolerance (Cizmar, *et al.*,2009).

System reliability (β_{sys}) is the reliability of a structural system composed of a number of components or the reliability of a single component which has several failure modes of nearly equal importance.

Any mechanical system may be assigned to one of the following three categories: series system, parallel systems or combination of series and parallel system (also referred to as a hybrid system). In series systems failure of any element leads to failure of the system. Parallel systems are those systems in which combined failure of each and every element the system results in failure of the systems.

For a series system, the probability of a union of failure domain can be computed from (Gollwitzer and Rackwitz, 1988):

$$P(F_S) = 1 - P\left(\bigcup_{k=1}^{m} V_k\right) = 1 - P\left(\bigcup_{k=1}^{m} \bar{V}_k\right) = 1 - P\left(\bigcap_{k=1}^{m} \{\underline{\alpha}_k \underline{\cup} + \beta_k > 0\}\right)$$

$$= 1 - P\left(\bigcap_{k=1}^{m} \{Z_k > -\beta_k > 0\}\right) = 1 - P\left(\bigcap_{k=1}^{m} \{Z_k > -\beta_k > 0\}\right)$$

$$= 1 - \Phi_m\left(\underline{\beta}; \underline{R}\right) \leq \sum_{k=1}^{m} \Phi(-\beta_k) \leq 1 \qquad (2.39)$$

If a series system is considered for n failure elements, then the probability of failure of the series system is defined as the intersection of the individual failure events (Cizmar, *et al.*, 2009):

$$P_f = P[\cup_{k=1}^{m}\{M_i \leq 0\}] = P[\cup_{k=1}^{m}\{g_i(X) \leq 0\}] \quad (2.40)$$

The FORM approximation of a parallel system can be written:

$$P_f = P\left[\left[\cup_{k=1}^{m}\{\beta_i^J - \alpha_i^T.\cup \leq 0\}\right]\right] = \Phi_{n,A}(-\beta^J, \rho)(2.41)$$

Where $\Phi_{n,A}$ is the multivariate n-dimensional normal distribution function and ρ is the correlation coefficient matrix where the correlation coefficients are obtained from the alpha vectors, α_i and α_j.

According to Cizmar, *et al.* (2009), the probability of failure of the series system is assessed using upper and lower bounds. The FORM approximation is

$P_f \geq max[P(F_{i=1}^n)](2.42)$

$P_f \leq 1 - \prod_{i=1}^{n}\left(1 - P(F_i)\right)(2.43)$

For a parallel system, the probability of an intersection of failure domain can be computed from (Gollwitzer and Rackwitz,1988):

$$P(F_P) = P\left(\bigcap_{k=1}^{m} V_k\right) \approx P\left(\bigcap_{k=1}^{m}\{\underline{\alpha}_k \underline{\cup} + \beta_k > 0\}\right)$$

$$= P\left(\bigcap_{k=1}^{m}\{Z_k < -\beta_k\}\right)$$

$= -\Phi_m\left(\underline{\beta}; \underline{R}\right) \leq min_{k=1}^{m}\{\Phi(-\beta_k)\}(2.44)$

Where $\underline{\beta} = (\beta_1, \dots, \beta_m)^T$ and $\underline{R} = \left\{\rho_{ij} = corr\left[Z_i, Z_j\right] = \underline{\alpha_i}^T \underline{\alpha_j} : i, j = 1,2, \dots, m\right\}$

If a parallel system is considered for n failure elements, then the probability of failure of the parallel system is defined as the intersection of the individual failure events (Cizmar, *et al.*, 2009):

$$P_f = P[\cap_{i=1}^{n}\{M_i \leq 0\}] = P[\cap_{i=1}^{n}\{g_i(X) \leq 0\}](2.45)$$

The FORM approximation of a parallel system can be written:

$$P_f = P\left[\cap_{i=1}^{n}\left\{\beta_i^J - \alpha_i^T.\cup \leq 0\right\}\right] = \Phi_{n,A}(-\beta^J, \rho) \quad (2.46)$$

Where $\Phi_{n,A}$ is the multivariate n-dimensional normal distribution function and ρ is the correlation coefficient matrix where the correlation coefficients are obtained from the alpha vectors, α_i and α_j.

According to Cizmar, *et al.* (2009), the probability of failure of the parallel system is assessed using upper and lower bounds. The FORM approximation is

$$P_f \geq min[P(F_{i=1}^{n})] \quad (2.47)$$

$$P_f \leq 1 - \prod_{i=1}^{n}\left(1 - P(F_i)\right) \quad (2.48)$$

Where a lower and upper bounds correspond respectively to fully correlated and uncorrelated safety margins. An estimate of the failure probability is obtained as the arithmetic mean of the upper and lower probability bounds.

2.3.4Works on Reliability Analysis of Reinforced Concrete Structures

Lu, *et al.* (1994) evaluated the time invariant reliability of reinforced concrete beams designed under the provision of the ACI Building Code. First Order Reliability Method (FORM) and Second Order Reliability Method(SORM) were developed to approximately evaluate the probability of failure, given by the expression:

$$\int_{(g(x)\leq 0)} f_x(x)dx \quad (2.47)$$

Where the random vector X contains all the uncertain basic variables such as material properties, loads, member dimensions, and model uncertainty parameters; $f_x(x)$ represents the joint probability density of the basic random variables; and $g(x)$is the limit-state function corresponding to the failure mode considered and defined such that the failure event corresponds to $g \leq 0$. FORM reliability method was developed to approximately evaluate the probability of failure. FORM analysis has the most important feature of readily providing the sensitivities of the reliability index β with respect to both probability distribution and limit-state function parameters. This sensitivity is very useful in identifying the relative importance of the basic variables in terms of the failure probability and can be used to suggest design alternatives to improve the reliability of the system.

The limit state functions were formulated for bending, shear and torsion failure modes. The general purpose structural reliability analysis program CALREL(Lu, *et al.*, 1990) was employed to compute component and system reliability indices for various loading conditions in accordance to ACI Building Code requirements.

The results showed a drop of the reliability index at the lower end of the shear reliability index versus live-to-dead load ratio. This was explained by the fact that in this range, shear resistance is increasingly contributed by concrete which has a wider strength distribution than steel.

The largest sensitivity of the reliability index corresponds to the distribution parameter $\sigma_L$for all four failure modes. The result indicate the extreme importance of an accurate

estimation of the extreme live load variability prior to design, since overestimation of this variability results into a large loss of reliability. The bending and shear reliability indices were mostly sensitive to the mean and standard deviation of the three variables *L*, *B*, and material strength f_y for bending and $\sqrt{f_c'}$ for shear, whereas the torsion and shear–torsion reliability indices are mostly influence by L,$\sqrt{f_c'}$ and f_y.

The large sensitivity of the reliability indices with respect to f_y suggests that poor manufacturing of steel inducing a larger variability of its yield strength results into a significant loss of reliability. The importance large variability of its yield strength results into a significant loss of reliability. The importance of the concrete strength parameter emphasizes the significance of quality control in producing this material. The normalized sensitivity results also indicate that the sensitivity of β with respect to the standard deviation of a basic variable tends to be larger than the sensitivity with respect to the mean of this variable.

For the probability distribution used and the values of the steel ratio adopted in the design, the probabilities of light-and over-reinforcement are negligible compared to the probability of flexure failure under moderate reinforcement and was neglected. The reliability index was found to be relatively insensitive to the total lateral reinforcement ratio and to the torsion- to- shear reinforcement ratio. The reliability index for shear-to- torsion interaction was mostly influenced by the live load, the material strength and the model uncertainty factor.

Based on the results obtained it was found out that for all the failure modes(flexure, shear and torsion), the reliability indices remain confined in a relatively narrow band and varies slowly over a wide realistic range of design parameter values. The geometric parameters do not influence significantly the reliability index. However, the most influential among them are the longitudinal tension reinforcement are$A$, and the effective beam depth d for flexure, the shear stirrup area per unit length A_v/s and d for shear, and the tensional stirrup area per unit length A_v/s and the center-to-center stirrup dimensions x_1 and y_1 for torsion and shear-torsion. All component reliabilities computed vary from 3.2 to 4.2. This indicates that, for the particular type of structure studied, ACI Code 318-89 fulfils its objective of approaching a uniform reliability for many design situations.

Finally, the reliability of the beam as a system was examined. The beam was designed for the simultaneous effects of bending moment, shear force, and torque. The influence of the live-to-dead load ratio and material strengths was considered here. The reliability indices β of thecomponential failure modes (flexure and combined shear-torsion) and of the system defined as a series system of flexure and combined shear-torsion, displayed against the live-to-dead load ratio for two sets of material strengths showed results influenced mostly by live load, material strength and model uncertainty factor. The results showed that at lower live-to-dead load, the system effect is more pronounced than the component effect for high strength than for low strength materials.

SriVidya and Ranganathan (1994) developed a methodology for reliability based optimum design (RBOD) of reinforced concrete frames which considers the failure of structure both

at the component and system levels simultaneously, under two different live load and wind load combination. Optimum values of the moment capacities of critical sections are obtained based on the minimization of the total structural cost of the frame using he cost functions for member sections as a function of their moment capacities. The observed that reliability based optimal design carried out for allowable system reliability, β_s of 3.72 and component reliability, β_c of 3.0 has yielded a cost reduction of about 13.4%. It is also observed that the depth of the bean-section increased for high requirement of system reliability.

Zhao and Ono (1998) carried out a study on the reliability of ductile frames using Response Surface approach on portal frame. Ductile frame structures are considered elasto-plastic. The failure of a ductile frame is defined as the formation of a kinematically admissible mechanism due to the formation of plastic hinges at a certain number of sections.

The response surface approach is a collection of statistical analysis method that examines the relationship between experimental response and the variations in the values of the input variables. A practical approach was developed in which a failure mode independent performance function was defined using load factor obtained by limit analysis and a response surface approach was used to approximate the performance function. With the performance function of explicit second degree polynomial, the failure probability was obtained usingFirst and Second Order Reliability Methods (FORM/SORM). The results showed that the response surface approach could be used to obtain the most likely failure mode of frame structures and that the FORM reliability index corresponding to the

response surface was a good approximation of the FORM reliability index corresponding to the most likely collapse mode.

Royset,*et al.* (2006) develop gradient-based optimization algorithms for estimating the solution of three classes of optimization problems in the case of continuous design variables. Their approach was based on a sequence of approximating design problems, which was constructed and then solved by a semi-infinite optimization algorithm. The construction consisted of two steps: First, the failure probability terms in the objective function were replaced by auxiliary variables resulting in a simplified objective function. The auxiliary variables were determined automatically by the optimization algorithm. Second, the failure probability constraints were replaced by a parameterized first-order approximation. After repeatedly solving the approximating problem, an approximate solution of the original design problem ws found, which satisfied the failure probability constraints at a precision level corresponding to the selected reliability method. The approach was illustrated by a series of examples involving optimal design and maintenance planning of a reinforced concrete bridge girder. A significant advantage of the algorithms was the flexibility in the selection of the method for computing failure probabilities.

Almeida, *et al.* (2008) compared the Deterministic Design Optimization (DDO) with the Reliability-Based Design Optimization (RBDO) of reinforced concrete plane frames. For the structural analysis performed in the optimization process the plane frame was modeled using finite elements considering both geometric and material nonlinearities. The random variables of the RBDO problem were the concrete compressive strength, the steel

compressive and tensile strength, as well as some applied loads. The design variables were the areas of the cross sections of both the longitudinal steel bars and stirrups as well as the depths of the cross sections of the beams and columns in the frame.

The design of the reinforcement bars of the reinforced concrete cross sections were carried out according to the concrete NBR6118 (2004) and is treated deterministically in both the DDO and the RBDO problems. The reliability analyses were performed by the First Order Reliability Method, FORM (Melchers, 2002). The random variables of the problem were the steel and concrete compressive strength, the steel tension strength and the external loads.

The results showed that the active constraints in the RBDO problem were: the probabilistic constraint related to the displacement, the deterministic constraint related to minimum shear-transverse reinforcement in the elements and the deterministic constraint related to the combined action of the axial force and the bending moment in nodes. The optimum design obtained via DDO satisfied, as expected, the deterministic constraints, but, on verifying the reliability of the DDO project, it was noticed that the reliability index related to the maximum displacement was less than the recommended value ($\beta_{\mathrm{ELS}} = 1.18 < \beta_{\mathrm{TARGET}=1.50}$) while the reliability index related to the critical load factor is excessively big ($\beta_{\mathrm{ELU}} = 10.81 > \beta_{TARGET}$). As expected, the RBDO design satisfied both the deterministic and probabilistic constraints resulting in an economical and reliable design.

Babu and Basha (2008) conducted reliability-based design optimization of reinforced concrete cantilever retaining wall. They assessed the effect of uncertainties in design parameters on the probability of failure of cantilever retaining walls using a reliability based design optimization of retaining walls (RBDORET) program written in MATLAB (2004). Using the statistics of input parameters and code developed for FORM, reliability indices with respect to all failure modes were computed, followed by a verification of the target reliability index. The termination tolerance for the convergence of the reliability index was taken as10^{-3}. The execution of eachiteration took 12–18seconds and FORM scheme took 3–8 iterations for the convergence of the reliability index corresponding to each failure mode depending upon the nonlinearity of the constraints.

It was shown that curves representing variation in the reliability index with respect to sliding, bearing, eccentricity, toe shear, toe moment, heel shear, heel moment, stem shear, and stem moment failure modes for COV_{φ}=5 and 10%, respectively, converge at the targeted value of the reliability index (β_t=3 – 3.2). A comparative study showed that optimized sections have less areas of cross section compared to those obtained from specifications on dimensioning of retaining walls available in literature. A comparative study of savings in terms of areas of cross section, showed a substantial amount of reduction in the cross-sectional area for COV_{φ}=5%. For COV_{φ}=10%, the reduction is in the range of 2.5–25% with respect to conventional design specifications indicated in the literature.

CHAPTER THREE

MATERIALS AND METHODS

3.1Analysis of the Frame

The basic objective of frame analysis is to determine the bending moment, shear, thrust, and theaxial loads at each critical section in columns and beams.In this study, the direct stiffness method was used to analyze a frame.The direct stiffness method lends itself to computer based analysis. The basic structural frame is shown in Figure 3.1. The members are numbered I to VIII referring to members 1 to 8 respectively. L_1 and L_2 are spans of the beams and H_1 and H_2 are heights of the columns. The frame consists of beams with width breadth and depth of 225mm, 225mm and 450mm respectively. The first floor and columns have breadth, width and depth dimensions of 225mm, 225mm and 450mm respectively.

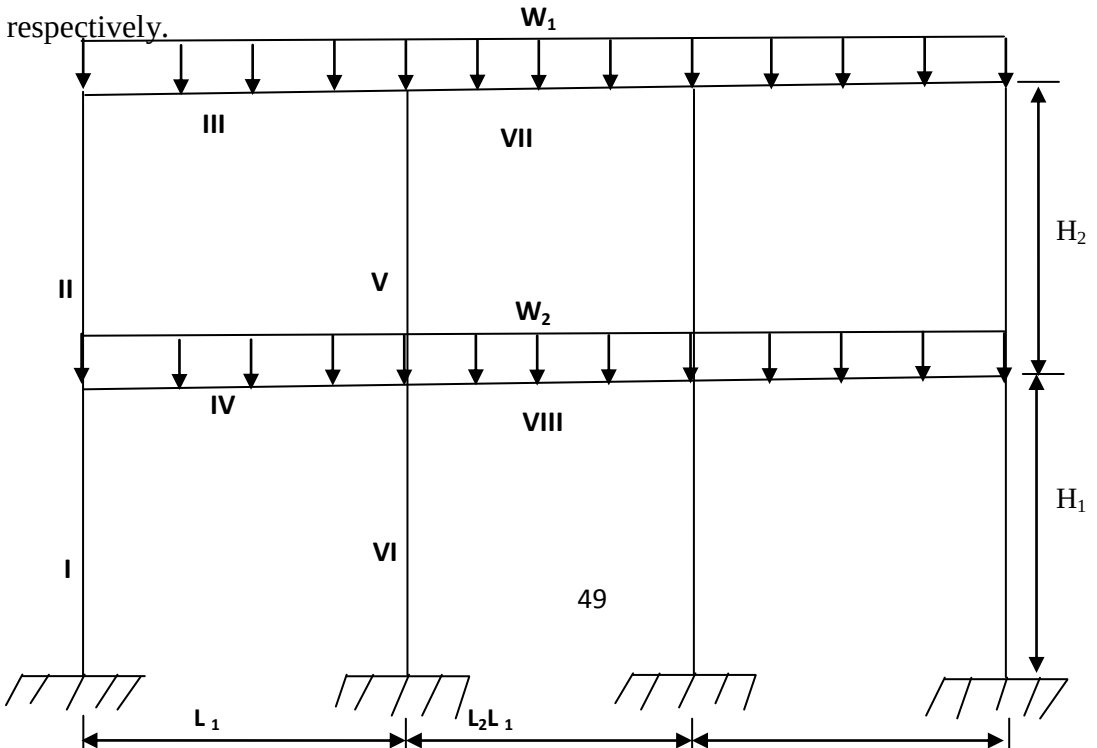


Figure 3.1: The basic frame

The computational analysis of the frame by direct stiffness method is presented in Appendix A. A computer program was developed in MATLAB (2004) to analyze the frame. The computer program is as shown in Appendix B.

3.2 Formulation of Optimum Design Functions

The design problem is formulated in an optimization format which is a mathematical representation of the objective function and performance constraints for the design. The objective function and constraints are explicitly formulated in terms of design variables and constraints.

3.2.1 Objective function

The objective function to be minimized expresses the goal or performance criterion in terms of the decision variables. It is expressed in terms of concrete volume, steel weight, form work surface area as well as their unit costs. The mathematical form of the objective function for the design of reinforced concrete frames is given by the following expressions [Camp, *et al.*, 2003] is:

$$F = \sum_{element}\left(C_c lbh + C_s l A_s + 2C_f l(b+h)\right) \quad (3.1)$$

Subject to $c_1 \leq 0,\ c_2 \leq 0\ ,\ \dots\ , c_n \leq 0$

Where F: Objective function representing cost of reinforced concrete element in terms of cost of concrete, reinforcing steel and formwork.

p_m, p_j, p_s are material properties, connection characteristics and structural characteristic respectively, b and $h$are width and height of the section respectively,l and $A_s$are length of member and area of reinforcing bars respectively, $C_c$is cost of concrete per unit volume, $C_s$is cost of steel, ties and stirrups per unit weight and $C_f$is cost of formwork per unit surface area.

The objective functions minimized in terms of the decision variables are the cost of the frame elements determined as follows:

$$C_{beam} = C_c(V_{bc} - V_{bs} - V_v) + C_s\gamma_s(V_{bs} + V_v) + C_f A_{bf} \quad (3.2)$$

and

$$C_{Column} = C_c(V_{cc} - V_{cs} - V_t) + C_s\gamma_s(V_{cs} + V_t) + C_f A_{cf} \quad (3.3)$$

Where C_{beam} is the cost of a critical beam in the frame, C_{column} is the cost of a critical column in the frame,$C_f$is the cost of form work; V_{bc} andV_{cc} are volumes of concrete in the beam and column respectively; V_{bs} andV_{cs} arevolume of longitudinal steal in the beam and column respectively; V_v andV_t arevolume of stirrups in beam and column respectively; A_{bf} andA_{cf} arethe surface area of formwork for the beam and column respectively and γ_s is the unit weight of steel.

Expressions for volumes of concrete, longitudinal steel, stirrups and surface area of formwork are obtained from the following Figure 3.2:

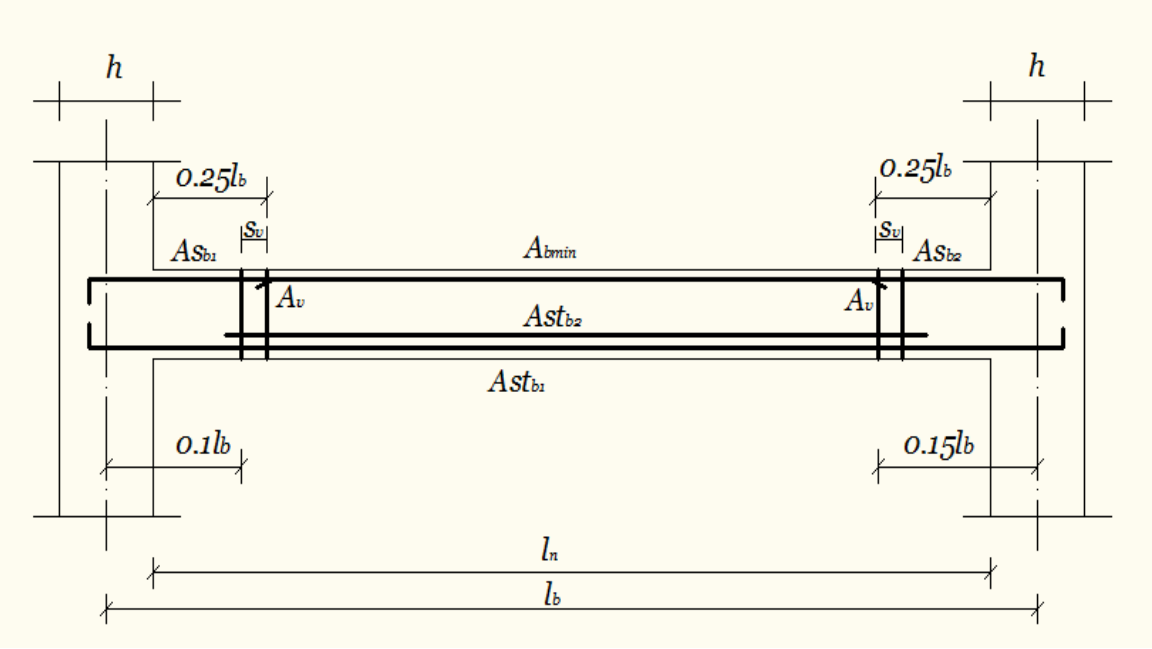


Figure: 3.2 Beam details for singly reinforced section

The lengths and curtailing of reinforcement are provided from EC 2 (Mosley, et al., 2007) as follows:

1. Top Reinforcement terminate $0.25L_b$ from face of column
2. Half of bottom reinforcement terminates $0.10L_b$ and $0.15L_b$ from centre of end and inner columns respectively as shown in the Figure 3.2.

If L_n is clear span of beam, L_b span of beam centre to centre,S_v spacing of links, $A_v$is cross sectional area of bars used for stirrups, A_{sb1}^{-} and A_{sb2}^{-} are top reinforcements at left and right supports respectively; $A_{sb1}$and A_{sb2} are bottom reinforcements curtailed and full respectively and A_{smin} hanger reinforcement, then:

The total volume of a beam can be expressed as:

$$V_{bg} = A_{gb}L_b = b_w(d_b + d_b')L_b \quad (3.4)$$

Where A_{gb} is the gross cross sectional area of beam,b_w, d_b and $d'_b$are width of beam, effective depth and distance from top fibre to compression reinforcement of the beam respectively. $L_b$is the length of beam between column centre lines.

The volume of concrete in a beam can be expressed as

$$V_{bc} = V_{bg} - V_{bs} - V_v \quad (3.5)$$

The volume of longitudinal steel in a beam can be computed as follows:

$$V_{bs} = A_{sb1}(L_b - 0.10L_b - 0.15L_b) + A_{sb2}L_b + A_{smin}(L_n - 0.25L_b - 0.25L_b) + A^-_{sb1}\left(0.25L_b + \frac{h_c}{2}\right) + A^-_{sb2}\left(0.25L_b + \frac{h_c}{2}\right) \quad (3.6)$$

Where $A^-_{sb1}$and $A^-_{sb2}$are top steel areas at left and right ends of the span respectively,

Half of bottom reinforcement curtailed and half across full length. Therefore:

$$A_{sb1} = A_{sb2} = \frac{A_{sb}}{2} \quad (3.7)$$

Thewidth of the column is the sum of effective depth plus the cover. That is:

$$h_c = d_c + d'_c \quad (3.8)$$

Where h_c, d_c and d'_c are column width, effective depth and cover respectively.

The length of beam between column centres is clear span of beam plus width of column. Therefore:

$$L_b = L_n + h_c = L_n + d_c + d'_c \quad (3.9)$$

Making equation L_n the subject of equation (3.9) we obtain:

$$L_n = L_b - (d_c + d'_c) = L_b - d_c - d'_c \quad (3.10)$$

Equation (3.10) is therefore:

$$V_{bs} = (0.75A_{stb1} + A_{stb2})L_b + A_{smin}(0.84L_b - d_c - d'_c) + (A^-_{sb1} + A^-_{sb2})\left(0.25L_b + \frac{dc+d'c}{2}\right) \quad (3.11)$$

Considering when there is compression reinforcement in the beam then, $A_{smin}=A_{scb}$.

Consider a stirrups in beam shown in Figure 3.3:

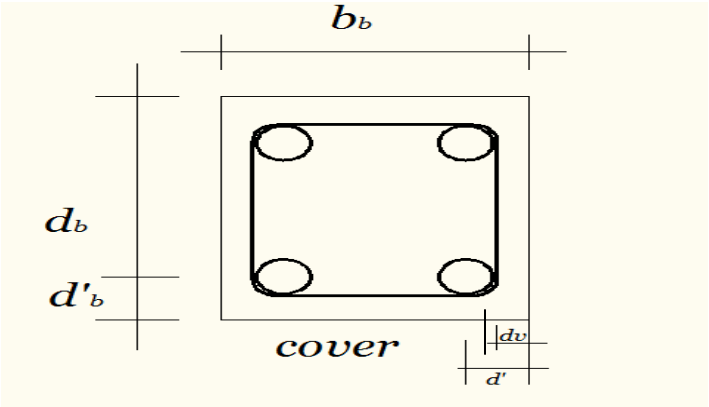


Figure: 3.3 Stirrup details

The volume of stirrups in beam,V_v is computed as follows:

$$V_v = A_v L_v n_s \quad (3.12)$$

Where A_v,L_v are cross sectional area and length of bars used for stirrups;$n_s$is the number of stirrups. $L_v$is determined from Figure 3.3 as follows:

If d_{lb}= diameter of longitudinal bar,

d_v= diameter of stirrups

l_b= length over which stirrups are distributed and

s_v= spacing of stirrups

$$L_v = 2(b_b + d_b + d'_b) - 4(2d'_b) + 4\left(\frac{2d_{lb}}{2}\right) + 4\,(2d_v) \qquad (3.13)$$

Simplifying equation (3.13), we obtain:

$$L_v = 2(b_b + d_b + d'_b) - 8\left(d'_b - \frac{d_{lb}}{2} - d_v\right) \qquad (3.14)$$

Therefore equation (3.12) becomes:

$$V_v = A_v\left[(2(b_b + d_b + d'_b) - 8\left(d'_b - \frac{d_{lb}}{2} - d_v\right)\right]\left(\frac{L_b}{s_v} + 1\right) \qquad (3.15)$$

The surface area of framework for the beam can be computed from

$$A_{bf} = b_b L_n + 2(d_b + d'_b)L_b = b_b[L_b - (d_c + d'_c)] + 2(d_b + d'_b)L_b \qquad (3.16)$$

The objective function of the columnis derived from Figure 3.4. Let A_{gc} be gross cross sectional area of column, $L_u$be unsupported length (clear height) of column and L_c the length of Column between centre lines of beams.

The volume of concrete in a column can be expressed as

$$V_{c\,c} = A_{gc}L_u = b_c(d_c + d'_c)\left(L_c - \frac{2h_b}{2}\right)$$

$$= b_c(d_c + d'_c)(L_c - d_b - d'_b) \qquad (3.17)$$

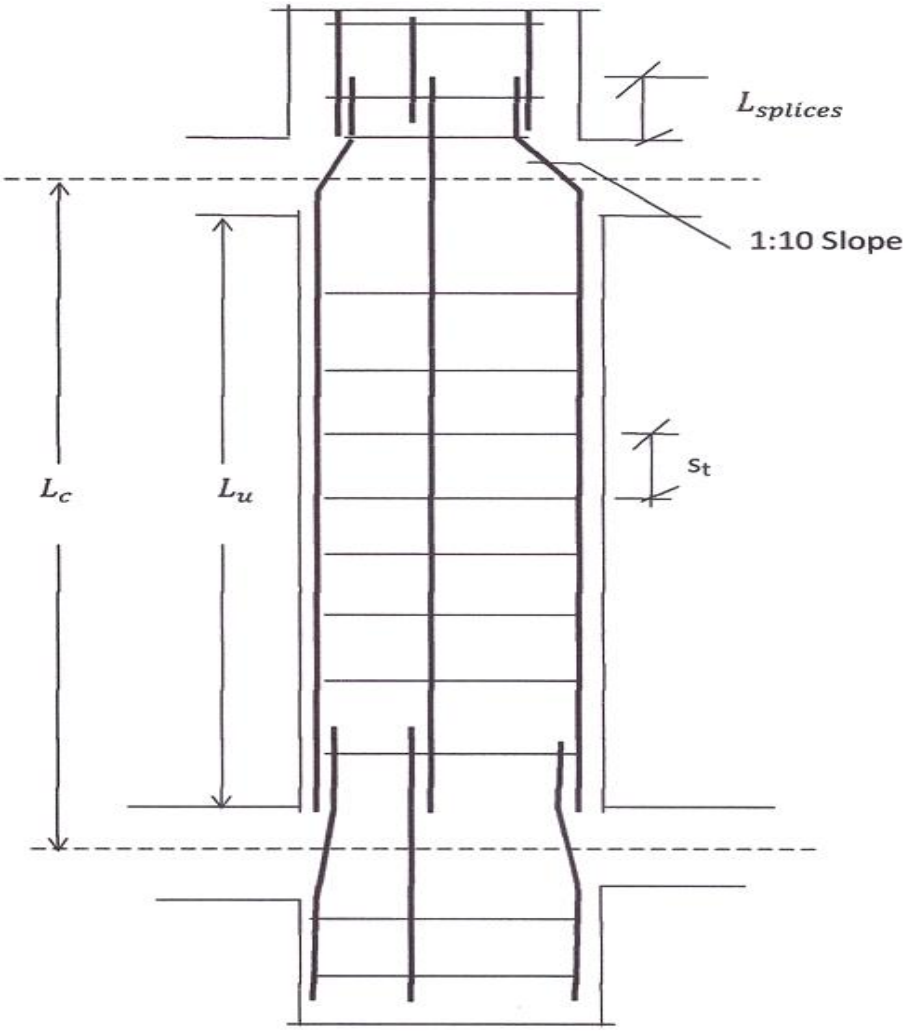


Figure 3.4: Column details

The Volume of longitudinal reinforcing steel in a column

$$V_{sc} = A_{sc}L_{bars} \quad (3.18)$$

Where $A_{sc}$is the area of longitudinal reinforcing steel and $L_{bars}$is the length of longitudinal reinforcing steel bars

$$L_{bars} = L_u + L_{inclined} + L_{splices} \quad (3.19)$$

Where $L_{inclined}$and $L_{splices}$are lengths of inclined and spliced portions of steel bars determined as follows:

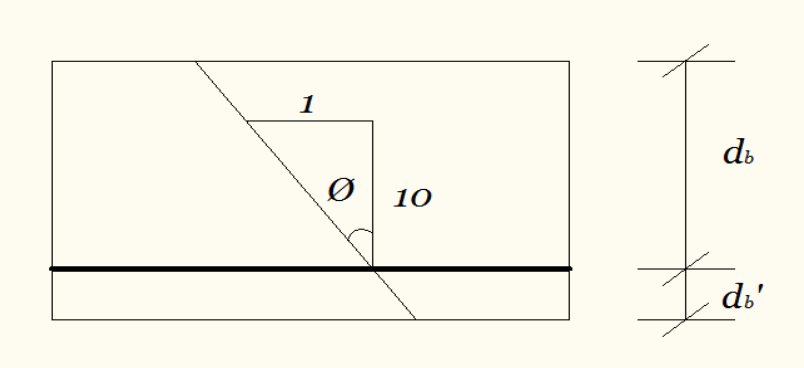


Figure 3.5: Length of bent up bars in columns

The slope of the inclined portion of longitudinal steel bars is 1:10 (Mosley et al, 2007).

From Figure 4.4. Hypotenuse = $\sqrt{1^2 + 10^2} = \sqrt{101}$

For ratio 10, $L_{inclined} = \sqrt{101}$

Therefore, for length (d_b +d_b'),

$$L_{inclined} = \frac{\sqrt{101}}{10}(d_b + d_b') = 1.005\ (d_b + d_b') \quad (3.20)$$

Let the clear height of column be L_u, then:

$$L_u = L_c - d_{b1} - d_{b2} \quad (3.21)$$

Where d_{b1} and d_{b2} are effective depths of beams in the upper and lower stories

$L_{splice} = K_A$ x d_{lc}

Where$K_A$is a coefficient and$d_{lc}$is diameter of longitudinal reinforcing bars.

For f_{ck} = 25N/mm², K_A = 40 (Mosley et al, 2007).

Therefore, $L_{spice} = 40d_{lc}$

And from equation (3.18),

$$V_{cs} = A_{sc}[(\mathrm{L_c} - \mathrm{d_b} - \mathrm{d'_b}) + 1.005(d_b + \mathrm{d'_b}) + 40d_{lc}] \qquad (3.22)$$

The volume of laterals ties in a column is computed as in the case of beams with little modification:

$$V_t = A_t L_{tie} n_t \qquad (3.23)$$

Where$A_t$is the cross area of bars used for ties, $L_{tie}$is the length of the tie and $n_t$is the number of ties in one column.

$$L_{tie} = 2(\mathrm{b_c} + d_c + d'_c) - 8(d'_c - \frac{d_{lc}}{2} - d_t) \text{ and } n_t = \frac{L_u}{S_v} + 1,$$

Therefore

$$\mathrm{V_t} = \mathrm{A_t}[2(\mathrm{b_c} + d_c + d'_c) - 8(d'_c - \frac{d_{lc}}{2} - d_t)](\frac{L_u}{S_v} + 1) \qquad (3.24)$$

Simplifying results to:

$$\mathrm{V_t} = \mathrm{A_t}[2(\mathrm{b_c} + d_c + d'_c) - 8(d'_c - \frac{d_{lc}}{2} - d_t](\frac{\mathrm{L_c} - \mathrm{d_b} - \mathrm{d'_b}}{s_t} + 1) \qquad (3.25)$$

Whered_t is the diameter of bars used for ties and S_t is the vertical spacing of ties.

The surface area of framework for the column can be computed from

$$\mathrm{A_{cf}} = 2(b_c + d_c + \mathrm{d'_c})L_u = 2(b_c + d_c + \mathrm{d'_c})(\mathrm{L_c} - \mathrm{d_b} - \mathrm{d'_b}) \qquad (3.26)$$

3.2.2 Design Constraints

a) Beam constraints

i) Geometric constraint

The engineer must check that the beam sizes are adequate to carry the loadings or alternatively, decide on sizes that are adequate. Consideration of maximum material

economy often leads to proportions with effective depth d in the range of 1.5 to 2.0 times the width of beam, b_b (Hassanain, 1992).

Therefore, $1.5b_b \leq d_b \leq 2.0$ b. Hence,

$$d_b - 2.0\ b_b \leq 0 \qquad (3.27)$$

ii) **Flexural constrain**

For a singly reinforced beam section shown in Figure 3.6.

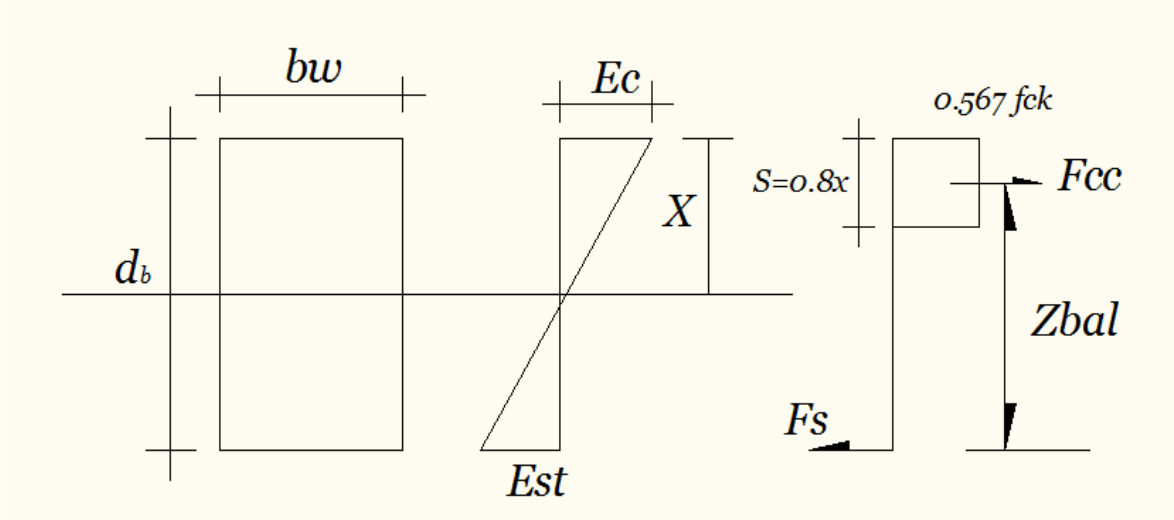


Figure 3.6: Rectangular Stress – Block for Singly Reinforced Section (Mosley, et al., 2007)

The ultimate design stress of concrete and reinforcing steel are $0.567f_{ck}$ and $0.87f_{yk}$respectively (Eurocode 2, 2004). Bending of the section will induce a resultant compressive force F_{cc} in the concrete and resultant tensile force F_{st} in reinforcing steel.

Force= stress x area of action.

Therefore, from Figure 3.6,

$$F_{cc} = 0.567f_{ck}bs \qquad (3.28)$$

And

$$F_{st} = 0.87 f_{yk} A_s \quad (3.29)$$

But $F_{cc} = F_{st}$

Therefore, $0.567 f_{ck} bs = 0.87 f_{yk} A_s$

Hence,

$$s = \frac{0.87 f_{yk} A_s}{0.567 f_{ck} b}$$

Taking moment about centre of F_{cc} gives $M = F_{st} z$

Therefore,

$$M \leq M_u = F_{st} z = 0.87 f_{yk} A_s (d - \frac{s}{2}) = 0.87 f_{yk} A_s \left(d - \frac{0.87 f_{yk} A_s}{1.134 f_{ck} b}\right) \quad (3.30)$$

Consider a rectangular section with compression reinforcement at the ultimate limit state as shown in Figure 3.7.

The compression reinforcement is

$$A'_s = \frac{M - 0.167 f_{ck} bd^2}{0.87 f_{yk} (d - d)} = \frac{M - 5.01 bd^2}{435 (d - d)} \quad (3.31)$$

The tension reinforcement is

$$A_s = \frac{K_{bal} f_{ck} bd^2}{0.87 f_{yk} z} + \frac{M - 5.01 bd^2}{435 (d - d)} \quad (3.32)$$

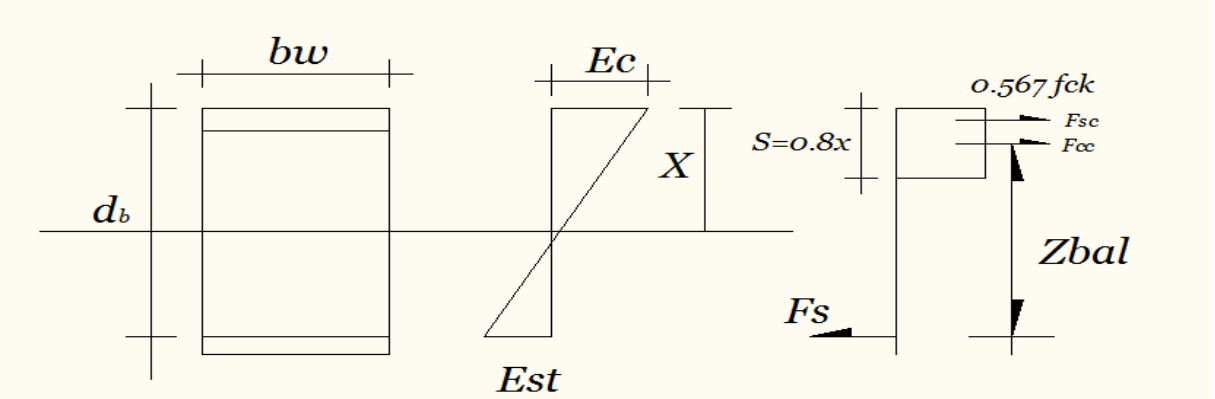


Figure 3.7: Rectangular Stress – Blockor doubly Reinforced Section (Mosley, et al. 2007)

iii) Shear strength requirement

The maximum shear capacity of a beam is:

$$`V_u \leq \frac{1}{2} V f_{cd}(0.9\text{bd}) \quad (3.33)$$

Where V is efficiency factor $= 0.7 - \frac{f_{yk}}{200} = 0.55$(Mosley, et al. 2007)

For $f_{ck} = 25\, N/mm^2$, $f_{cd} = f_{ck}/1.5 = 16.667\ \text{N}/mm^2$

Substituting f_{cd}in equation (3.33), we obtain:

$$\text{V} \leq V_u = 4.125\text{bd} \quad (3.34)$$

iv) **Minimum Reinforcing steel Area constraint,**

If the cracking moment in a beam is exceeded, for example by accidental overload, the beam will fail suddenly by rupture of the steel. To prevent such brittle failure, a lower limit is established for steel ratio.

For concrete class, $C_{25/30}$

$$\rho_{min} = \frac{A_{s,min}}{b_b d_b} \geq 0.26 \frac{f_{ctm}}{f_{yk}}$$

Therefore

$$\frac{0.26 f_{ctm}}{f_{yk}} - \frac{A_{s,min}}{b_b d_b} \leq 0 \quad (3.35)$$

Where f_{yk} = 500N/mm^2 and

f_{ctm} = Mean value of the axial tensile strength of concrete = 2.6N/mm (EC 2, 2004).

b) Column constraints

i) Geometric constraint

In order to ensure that the width of the column will not exceed its depth (which is assumed to be in the direction of bonding), the column dimensions are constrained as follows:

$$b_c \leq h_c \qquad (3.36)$$

But $b_c = d_c + d'$, Therefore,

$$b_c \leq d_c + d'$$

Therefore,

$$b_c - (d_c + d') \leq 0 \qquad (3.37)$$

Where

b_c is the width of column, h_c is the depth of column,d_c is the effective depth and d' is the depth of compression reinforcement.

ii) Strength constraint

The Ultimate Load Capacity of a section, Eurocode 2 (2004), clause 4.3.5.6.3 is:

$$N_{ud} = 0.567 f_{ck} A_c + 0.87 A_{st} f_{yk} \qquad (3.38)$$

The design load should be less or equal to the Ultimate Load Capacity

Therefore,

$$N_{Ed} \leq N_{ud} = 0.567 f_{ck} b_c (d_c + d') + 0.87 A_{sc} f_{yk} \qquad (3.39)$$

The maximum axial load from clause 4.3.5.6.3 is

$$N_{bal} = 0.4 f_{cd} A_c = 0.4 \frac{f_{ck}}{1.5} A_c = 0.267 f_{ck} A_c$$

$$N_{Ed} \leq N_{bal} = 0.267 f_{ck} A_c \qquad (3.40)$$

A rectangular reinforced concrete column section is similar to that of a doubly reinforced beam shown Figure 3.7.

Taking moments about centre of tensile steel,

$$M = F_{cc}\left(d - \frac{s}{2}\right) + F_{sc}\left(d - d'\right) \quad (3.41)$$

The concrete section with depth of neutral axis x=45d is referred to as balanced section (Mosley, *et al.*, 2007). Therefore from Figure 3.7, the depth of stress block is:

$s = 0.8(0.45d) = 0.36d$

The amount of reinforcement in column section are equal about the neutral axis, therefore

$A'_s = \frac{A_{sc}}{2}$

Hence, M_{Ed}=5.103bd$\left(d - \frac{0.36d}{2}\right)$ + 0.87$f_{yk}\frac{A_{sc}}{2}$ (d-50)

And simplifying further we obtain

$$M_{Ed} = 4.184bd^2 - 10875A_{sc} + 217.5A_{sc}d \quad (3.42)$$

iii) Minimum Reinforcing steel area constraint

The minimum reinforcing steel area is provided by the expression:

$$A_{s,min} \geq \frac{0.01N_{Ed}}{0.87f_{yk}} \geq 0.002A_c \quad (3.43)$$

Where N_{Ed} = the design axial compression force

$0.002A_c \leq -A_{smin}$ or

$$0.002b(d + d') - A_{smin} \leq 0 \quad (3.44)$$

3.3 Optimum Design Input Parameters

An input data is prepared describing the material properties, the geometric layout as well as the loading.

a) Initial input parameters

If L_b is span of beam in metre, Q_k the floor live load in kN/m^2, f_{ck} , f_{ctm} , f_{yk} , γ_c, γ_s , γ_f, b_b, b_c, h_b , h_c , d_b', d_c', d_b , d_c ,d_{lb} ,d_{lc} , and d_v are terms as defined in the list of notations, then from deterministic design we have initial input values as follows:

L_b=5400mm

Q_k=3.0kN/m^2

f_{ck}=25N/mm^2

f_{ctm} =2.6 N/mm^2

f_{yk} = 500 N/mm^2

γ_c=25 kN/m^3

γ_s = 77kN/m^3

γ_f=5 kN/m^2

b_b =b_c =225mm

h_b =300mm

h_c =225mm

d_b'=d_c'=50mm

d_b =300mm

d_c =175mm

d_{lb} =20mm

d_{lc} = 16mm

d_v= d_t= 8mm

b) Initial Input Parameters of material cost

The cost of materials were adopted from Newpro Quants Consultants (2008). The market survey rates for concrete work, reinforcement and formwork are as follows:

1. Cost of plain concrete (1:2:4-19mm aggregate) in super structure = ₦24,000/m^2

   But 1000mm = 1m or 1,000,000mm^2=1m^2

   Therefore, cost of plain concrete (1:2:4-19mm aggregate) in super structure per m^3 = ₦24,000/(1,000,000mm^2)=₦24,000(1.0E-9) per mm^3
2. Cost of reinforcement = ₦420.00 per kilogram(kg)

   1kg weight is approximately 10Newton(N)

   Therefore, Cost of reinforcement =₦420.00/10N= N42.00/N

   But 1000N=1kilo Newton(kN)

   Therefore Cost of reinforcement = ₦42.00/(1/1000)kN= ₦42,000/kN.
3. Cost of Formwork= ₦950 per m^2= ₦950 (1.0E-6)per mm^2

3.4 Formulation of Explicit Design Functions

Let$x_1 = b_b$,$x_2 = d_b$,$x_3 = A_{sb}$,$x_4 = A_{sbc}$, $x_5 = A_{sb1}^-$ and$x_6 = A_{sb2}^-$ then the design functions for the beams are obtained from objective functions as follows:

From equations (3.4),

V_{gb}=270000*x_1+5400*x_1x_2 (3.45)

From equations (3.11),

V_{bs} =725*x_3+4311*x_4+1462.5*x_5+1462.5*x_6 (3. 46)

From equations (3.15),

$V_v = 6384*x_1+6384*x_2 -549024$ (3.47)

From equations (3.16),

$A_{bf}=5175*x_1+10800*x_2+540000$ (3.48)

Therefore, the objective function for beam design from equations (3.2) is

$f =4000*1.0E-9(270000x_1+5400x_1x_2-4725x_3-4311x_4-1462.5x_5-1462.5x_6$

$-6384*x_1-6384*x_2+549024) +42*77*1.0E-9*(4725x_3+4311x_4+$

$1462.5x_5+1462.5x_6+6384*x_1+6384*x_2-549024) +$

$950*1.0E-6(5175x_1+10800x_2+540000)$(3.49)

The constraints for the design of beams in terms of the design variables are as follows:

From equation (3.27),

$x_1 -2x_2 \leq 0$ (3.50)

From equation (3.30),

$M_j - 435x_n(x_2-15.344x_n/x_1) \leq 0$ (3.51)

Where M_j is the applied moment (M) and the subscript j refers to the point of application: M_{mid} for span, M_{ls} for left support and M_{rs} for right support. X_n is the tension reinforcement (A_{st}), the subscript n refers to the section where tension reinforcement is required; 3 for span,5 for left support and 6 right support.

From equation (3.32), substituting the variables and rearranging, we obtain

$X_n - 0.012x_1x_2-(M_j- 4.175x_1x_2^2)/(435x_2- 21750)=0$ (3.52)

Where X_n refers to area of reinforcement and the subscript n refers to the section where tension reinforcement is required: 3 for span, 5 for left support and 6 right support. The subscript j refers to the applied moment: Mmid for span, Mls for left support and Mrs forright support. The terms $x_1$and x_2 are width and effective depth of section as earlier defined.

Half of span reinforcement is continuous to the supports which take care of compression reinforcement at the supports.

If $x_4 = A_{bsc} = A_{smin}$, then:

From equation (3.34),

$V - 4.125x_1x_2 \leq 0$ (3.53)

From equation (3.35),

$0.00135x_1x_2 - 102 \leq 0$ (3.54)

Let$x_1 = b_c$,$x_2 = d_c$ and$x_3 = A_{sc}$, then the design functions for the columns are as follows:

From equation (3.17),

$V_{cc} = 140000x_1 + 2800x_1x_2$(3.55)

From equation (3.22),

$V_{cs} = 4151.75x_3$(3.56)

From equation (3.25),

$V_t = 3472x_1 + 3472x_2 - 236088.38$ (3.57)

From equation (3.26),

$A_{cf} = 5600x_1 + 5600x_2 + 280000$ (3.58)

Therefore, the objective function for column design from equations (3.3) is:

$f = 24000 * 1.0E\text{-}9(140000x_1 + 2800x_1x_2 - 4151.75x_3 - 3472x_1 - 3472x_2 + 236088.38)$

+42000*77*1.0E-9(4151.75x_3+3472x_1+3472x_2-236088.38)

+950*1.0E-6(5600x_1+5600x_2+280000) (3.59)

The constraints for the design of columns in terms of the design variables are as follows:

From equation (3.37),$x_1 - x_2 - 50 \leq 0$ (3.60)

From equation (3. 39),N_{Ed} – 708.75x_1 -14.175x_1x_2 – 435x_3(3.61)

From equation (3. 40),N_{Ed} – 333.75x_1 – 6.675x_1x_2 (3.62)

From equation (3. 41), M_{Ed} – 4.175$x_1{x_2}^2$-217.5x_2x_3+10875$x_3 \leq 0$(3.63)

From equation (3. 42),0.1x_1+0.002x_1x_2-452 $\leq$ 0 (3.64)

A MATLAB program for the optimization is shown in Appendix B. The steps for matlab optimization for the beam for instance are as follows:

1. Create M-File for beam objective function , function f=bfun(x)

   Where f=beam objective function
2. Create M-File for constraint function, function [c,ceq]=confun(x)

   Where c= nonlinear inequality constraints and

   Ceq=nonlinear equality constraints
3. Invoke constrained optimization routine, $fmincon$ that minimizes the objective function.

3.5 Reliability Analysis

The reliability analysis is based on the First Order Reliability Analysis Method as in FORM5 (Gollwitzer, *et al.*, 1988). The limit state equation is referred to as the performance or state function and is expressed as

$$g(X) = g(X_1, X_2, \dots, X_n) = R - S \qquad (3.65)$$

Where X_i for $i = 1,2, \dots, n$ represent the basic design variables. S is the load and R is the resistance on the structural element or system.

3.5.1 Beam Failure Modes and Limit States

3.5.1.1 Beam flexure Failure Modes

The ultimate capacity for a singly reinforced beam Eurocode 2 (2004) is

$$M_u = 0.167 f_{ck} b d^2 \qquad (3.66)$$

$$M_{Ed} = 0.87 f_{yk} A_{s1} Z \qquad (3.67)$$

Where the terms:

M_u, f_{ck} and f_{yk} are ultimate moment capacity, characteristic concrete strength and steel yield strength respectively; b and d are width of section and effective depth respectively while M_{Ed}, A_{s1} and z are design bending moment, area of tension reinforcement and lever arm respectively.

$$Z = d\left\{0.5 + \sqrt{0.25 - \frac{k}{1.134}}\right\} \qquad (3.68)$$

Where

$$k = \frac{M_{Ed}}{b d^2 f_{ck}} \qquad (3.69)$$

Having obtained support moments by the stiffness method the shears and the maximum span moments can be obtained from Figure 3.9 as follows (Mosley,*et al.,* 2007):

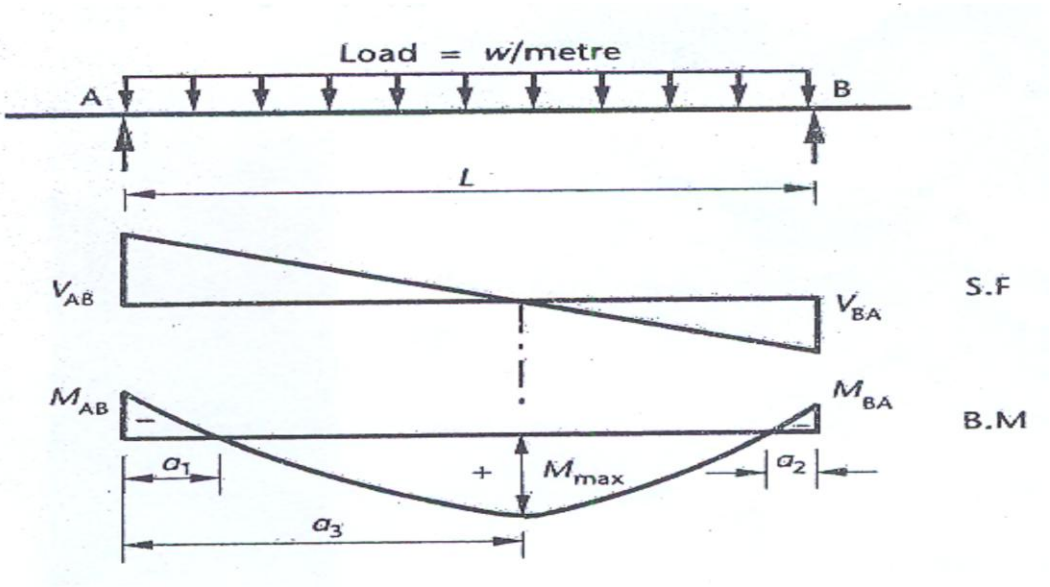


Figure 3.8: Shears and moments in a continuous beam

Taking moments about support B, we have:

$$V_{AB}L - \frac{wL^2}{2} + M_{AB} - M_{BA} = 0 \quad (3.70)$$

Therefore

$$V_{AB} = \frac{wL}{2} - \frac{(M_{AB} - M_{BA})}{L} \quad (3.71)$$

and

$$V_{BA} = wL - V_{AB} \quad (3.72)$$

$$M_{max} = \frac{{V_{AB}}^2}{2w} + M_{AB} \quad (3.73)$$

Where w is uniformly distributed load, M_{AB} is the moment at left support, M_{BA} the moment at right support, $V_{AB}$is the shear from left support, $V_{BA}$is the shear from right support and $M_{max}$is the maximum span moment.

From results of frame analysis typically shown in Appendix B1,

$M_{BA} > M_{AB}$, therefore,

$$M_{smax} = M_{BA} \quad (3.74)$$

Where M_{smax} is the maximum support moment.

Taking category C_2 for floor areas where people may congregate (e.g. lecture hall) as case of study, then from Table 6.2 of Eurocode 1 (2002), imposed load from slab is 3.0kN/m^2.

For frame spacing of 3.6m, the uniformly distributed load,w on the beam is:

$w = 3.6(1.35G_k + 1.5Q_k) = 3.6Q_k(1.35 \propto +1.5)$,

Where, $\propto = \frac{G_k}{Q_k}$

$w = 10.8(1.35 \propto +1.5)$ (3.75)

Substituting equations (3.67) and (3.68) into equation (3.65), we obtain for singly reinforced section,

The limit state equation is:

$$g(x) = 0.167f_{ck}bd^2 - 0.87f_{yk}A_{s1}d\left\{0.5 + \sqrt{0.25 - \frac{k}{1.134}}\right\} \quad (3.76)$$

For doubly reinforced beam, the moment capacity is:

$$M_d = 0.167f_{ck}bd^2 + 0.87f_{yk}\,A_{sc} \quad (3.77)$$

Where A_{sc} is the area of longitudinal reinforcement.

Therefore the limit state function for doubly reinforced section in flexure is given as:

$$g(x) = 0.167f_{ck}bd^2 + 0.87f_{yk}A_{sc} - 0.87f_{yk}A_{s1}d\left\{0.5 + \sqrt{0.25 - \frac{k}{1.134}}\right\} \quad (3.78)$$

3.5.1.2Beam Shear Failure Mode

The ultimate shear capacity in the beam is according to Eurocode 2 (2004) is

$$V_u = 0.3\left(0.7 - \frac{f_{ck}}{200}\right) f_{ck} bd \quad (3.79)$$

Where is ultimate shear capacity.

From Equation (3.52) and substituting $Q_k(1.35 \propto +1.5)$for uniformly distributed load, we obtain:

$$V_{AB} = \frac{Q_k(1.35\propto+1.5)\,\mathrm{L}}{2} - \frac{(M_{AB}-M_{BA})}{L} \quad (3.80)$$

Where $\propto$ is load ratio (dead load/live load).

Therefore, the limit state function for shear failure mode is:

$$g(x) = 0.3\left(0.7 - \frac{f_{ck}}{200}\right) f_{ck} bd - \left[\frac{Q_k(1.35\propto+1.5)\mathrm{L}}{2} - \frac{(M_{AB}-M_{BA})}{L}\right] \quad (3.81)$$

3.5.1.3 Beam Deflection Control

The limiting span/depth ratio may be estimated using Equation (3.82a) and (3.82b) obtained from Eurocode 2 (2004), clause 7.4.2. These equations are multiplied by correction factors to allow for the amount of reinforcement, length of member and characteristic strength.

$$\frac{l}{d} = K\left[11 + 1.5\sqrt{f_{ck}}\frac{\rho_0}{\rho} + 3.2\sqrt{f_{ck}}\left(\frac{\rho_0}{\rho} - 1\right)^{3/2}\right] \text{if } \rho \le \rho_0 \quad (3.82a)$$

$$\frac{l}{d} = K\left[11 + 1.5\sqrt{f_{ck}}\frac{\rho_0}{\rho-\rho'} + \frac{1}{12}\sqrt{f_{ck}}\sqrt{\frac{\rho'}{\rho_0}}\right] \quad \text{if} \rho > \rho_0 (3.82b)$$

Where:

$\frac{l}{d}$is the limit span/depth ratio

K is the correction factor to take into account for the different structural systems

ρ_0 is the reference reinforcement ratio$= 10^{-3}\sqrt{f_{ck}}$

f_{ck} is the characteristic concrete strength

$\rho$is the required tension reinforcement ratio at mid-span to resist the moment due to the design loads (at support for cantilevers) =A_{s1}/bd

A_{s1}, b and d are tension reinforcement, width and effective depth respectively

$\rho'$is the required compression reinforcement ratio at mid-span to resist the moment due to design loads

The correction factor,K in Equations (3.63a) and (3.63b) is calculated as provided in clause 7.4.2, Eurocode 2 (2004) as follows:

i) The expressions have been derived on the assumption that $f_{yk}$is$500Mpa$. If other stress strengths other than $500Mpa$ are used, then the values obtained should be multiplied by $500/f_{yk}$.

ii) For beams and slabs, other than flat slabs, with spans exceeding 7m, which support partitions liable to be damaged by excessive deflections, the values of limiting $\frac{l}{d}$should be multiplied by $\frac{7}{l_{eff}}$

Where l_{eff} is the effective span of the beam

iii) If $A_{s,prov} > A_{s,req}$ at the ultimate limit state, the values of limiting $\frac{l}{d}$should be multiplied $\frac{A_{s,prov}}{A_{s,req}}$ (upper limit=1.5)

Where $A_{s,prov}$ and $A_{s,req}$ are provided and required reinforcements respectively.

The actual span depth ratio is $\frac{l}{d}$. Therefore, the limit states are:

$$g(x) = K\left[11 + 1.5\sqrt{f_{ck}}\frac{\rho_0}{\rho} + 3.2\sqrt{f_{ck}}\left(\frac{\rho_0}{\rho} - 1\right)^{3/2}\right] - \frac{l}{d} \text{ if } \rho_0 \geq \rho \quad (3.83a)$$

and

$$g(x) = K\left[11 + 1.5\sqrt{f_{ck}}\frac{\rho_0}{\rho - \rho'} + \frac{1}{12}\sqrt{f_{ck}}\sqrt{\frac{\rho'}{\rho_0}}\right] - \frac{l}{d} \text{ if} \rho_0 < \rho \quad (3.83b)$$

3.5.2 Column Failure Modes and Limit States

3.5.2.1 Flexure Failure Modes

To obtain column moments, let us consider a substitute frame of Figure 1 shown in Figure 6. If the cross sections of members are width, b and depth d, then the member stiffness are as follows:

$$\frac{K_{AB}}{2} = \frac{1}{2}\left(\frac{bh^3}{12L_{AB}}\right) = \frac{1}{2}\left(\frac{bh^3}{12L_1}\right) \quad (3.84)$$

$$\frac{K_{BC}}{2} = \frac{1}{2}\left(\frac{bh^3}{12L_{BC}}\right) = \frac{1}{2}\left(\frac{bh^3}{12L_2}\right) \quad (3.85)$$

and

$$K_{COL} == \frac{bh^3}{12L_{COL}} = \frac{bh^3}{12L} \quad (3.86)$$

Where K_{AB}, K_{BC} and K_{COL} are stiffnesses of beam AB, Beam BC and the column respectively.

Distribution factor of the column is

$$DF_{COL} = \frac{K_{COL}}{\sum K} \quad (3.87)$$

Where the terms K_{COL} and $\sum K$ are column stiffness and sum of member stiffness at node B respectively.

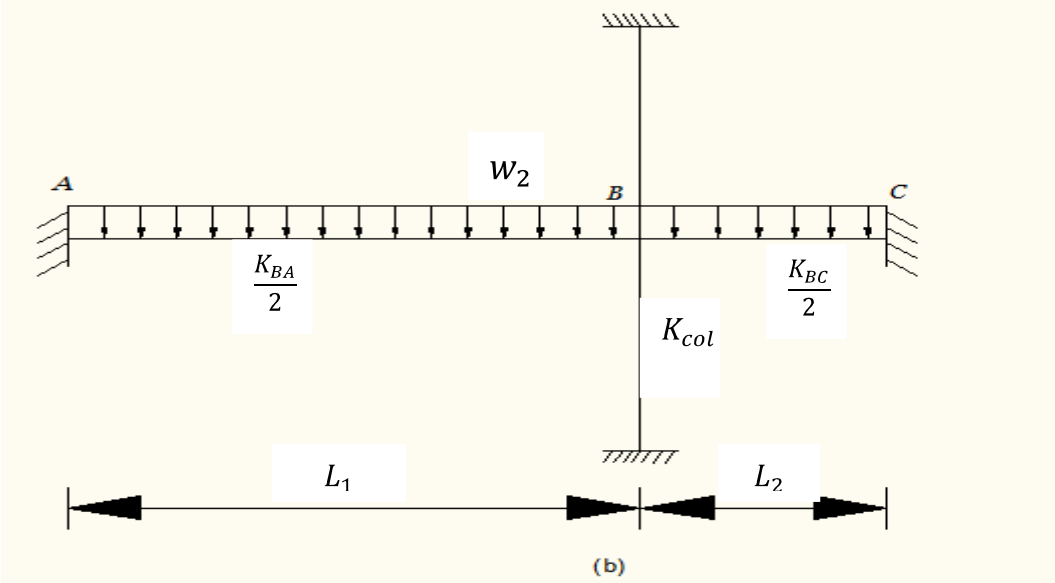


Figure: 3.9: Substitute frame

If K_{AB} and K_{BA} are stiffnesses of members AB and BA, $L_1$and L_2 lengths of the members respectively as shown in Figure 3.9, then:

$$\sum K = \frac{K_{AB}}{2} + \frac{K_{BC}}{2} + K_{COL} \quad (3.88)$$

Fixed end moments(FEM) at node B are

$$\mathrm{FEM_{BA}} = \frac{wL_1^2}{12} \quad (3.89)$$

and

$$\mathrm{FEM_{BC}} = \frac{\mathrm{wL_2^2}}{12} \quad (3.90)$$

Where w_2 and w_1 are ultimate uniformly distributed load at first floor and roof level beams respectively as shown in Figure 3.1.

The column moment is

$$M_{Ed} = DF_{COL}(FEM_{BA} - FEM_{BC}) = DF_{COL}\left(\frac{w_2}{12}\right)(L_1^2 - L_2^2) \quad (3.91)$$

The cross sectional areas of beam is 225mm times 450mm and that of the column is 225mmx225mm.

Therefore, the moment of inertia of the beam(I_b) is

$I_b = \frac{bh^3}{12} = 0.225 \times 0.45^3/12 = 0.0017m^3$

For $L_1$and L_2 of 5.4m and 1.8m respectively, the stiffness of the beam at joint B is

K_{bjb}=0.5(I_b/5.4+I_b/1.8) = $6.3281x10^{-04}m^2$

Moment of inertia of the column (I_c) is

$$I_c = \frac{bh^3}{12} = 0.225 \times 0.225^3/12 = 2.1357x10^{-04}m^2$$

For $H_1$and H_2 both 3.15m, the stiffness of the column at joint B is

$$K_{col} = I_c/3.15 = 6.7801x10^{-04}m^2$$

The sum of stiffnesses at Joint B = $\sum K$=$K_{bjb} + K_c = 7.0061x10^{-04}m^2$

DF_{col}= $K_c/\sum K$=0.0968

Therefore, the design bending moment is

$$M_{Ed} = 0.0968\left(\frac{w_2}{12}\right)(L_1^2 - L_2^2) \quad (3.92)$$

The moment of resistance derived from sectional properties is

$$M = 0.567bhf_{ck}(0.5d + 25) + 0.87A_{sc}f_{yk}(d - 50) \quad (3.93)$$

Where h,d, A_{sc} are height of section, effective depth and area of longitudinal reinforcement respectively.

Limit state equation for a column failure due to flexure from Equations (92) and (93) is

$$g(x) = 0.567bhf_{ck}(0.5d + 25) + 0.87A_{sc}f_{yk}(d - 50) - \left[0.0968\left(\frac{w_2}{12}\right)(L_1^2 - L_2^2)\right] \quad (3.94)$$

3.5.2.2 Axial Failure Modes

The ultimate axial capacity N_{ud} of an axially loaded column according to Eurocode 2 (2004) is

$$N_{ud} = 0.567A_cf_{ck} + 0.87A_{sc}f_{yk} \quad (3.95)$$

And the design axial force is

$$N_{Ed} = \frac{1}{2}(W_1L_1 + W_1L_2) + \frac{1}{2}(W_2L_1 + W_2L_2) \quad (3.96)$$

Where W_2 and W_1 are ultimate uniformly distributed load at first floor and roof level beams respectively as shown in Figure 3.1.

Limit state equation for a column failure due to axial load from Equations (3.95) and (3.96) is

$$g(x) = 0.567A_cf_{ck} + 0.87A_{sc}f_{yk} - \left[\frac{1}{2}(W_1L_1 + W_1L_2) + \frac{1}{2}(W_2L_1 + W_2L_2)\right] \quad (3.97)$$

The long term effect, g_k and the variable action, q_k at roof level are $1.5kN/m^2$ and $0.75kN/m^2$ respectively (Oyenuga, 2005). Self-weight of roof level beam with cross section 225mm by 225mm is:

$$25x0.225^2 = 1.266\,kN/m$$

Maximum ultimate load at roof floor for frame spacing of 3.6m is

$$w_1 = 1.35(1.5x3.6 + 1.266) + 1.5x0.75x3.6 = 13.049kN/m \quad (3.98)$$

Maximum ultimate load at first floor for panel spacing of 3.6m is

$$w_2 = q_k(1.35\alpha + 1.5)L = 3.6q_k(1.35\alpha + 1.5) \quad (3.99)$$

3.5.2.3 Combined flexure and Axial Failure Modes

The Limit state equation for a column due to combined axial force plus flexure is therefore given by the following expression:

$$g(x) = 0.567A_c f_{ck} + 0.87A_{sc}f_{yk} + 0.567bhf_{ck}(0.5d + 25) + 0.87A_{sc}f_{yk}(d - 50) - \begin{bmatrix} \frac{1}{2}(W_1L_1 + W_1L_2) + \frac{1}{2}(W_2L_1 + W_2L_2) + \\ 0.0968\left(\frac{w_2}{12}\right)(L_1^2 - L_2^2) \end{bmatrix} \quad (3.100)$$

3.6 System Reliability Analysis of the Frame

The frame is rigid hence the failure of an element does not lead to failure of the system. The system is therefore parallel. Consider a system given a unions and intersections of individual eventsE_k. The primary task is to rearrange these combinations such that the event of interest can be given as:

$$E_k = \cap\cup\left(E_{ij}\right) \quad (3.101)$$

And is minimal in the sense that no combined set contains another as a genuine subset. The formation of such a set can always be achieved by application of the distributive (expansion) rules of algebra, that is, by

$$E_i \cap \left(E_j \cup E_k\right) = \left(E_i \cap E_j\right) \cup (E_i \cap E_k) \quad (3.102)$$

$$E_i \cup \left(E_j \cap E_k\right) = \left(E_i \cup E_j\right) \cap (E_i \cup E_k) \quad (3.103)$$

And the absorption rules, if

$E_i \; C \; E_j$ and $E_k \; C \; E_j$ (3.104)

Then

$(E_i \cup E_k)\ C\ E_j$ and $(E_i \cap E_k)\ C\ E_j$ (3.105)

Let $\underline{X} = (X_1, \dots, X_n)^T$ be a vector of basic uncertainty variables describing all random parameters in a given system. Furthermore, let the system failure event $F = \{\underline{X} \in V\}$ be given as a minimum combination of unions of intersections of componential failure events$F_k == \{\underline{X} \in V_k\}$, k=1… q where V_k is described by a componential state function $h_k(\underline{X})$ such that $h_k(\underline{X}) > 0$ denote componential safe states and $h_k(\underline{X}) = 0$ denotes limit state function. The system failure event can be described as

$$F = \{\underline{X} \in V\} = \{\underline{X} \in \cup_i^m \cap_{j\in l_i}^n V_{ij}\} = \cup_i^m \cap_{j\in l_i}^n \{h_{ij}(\underline{X}) \le 0\} \quad (3.106)$$

The probability of an intersection of failure domain can be computed from (Gollwitzer and Rackwitz, 1988):

$$P(F_P) = P\left(\bigcap_{k=1}^{m} V_k\right) \approx P\left(\bigcap_{k=1}^{m} \{\underline{\alpha}_k \underline{\cup} + \beta_k > 0\}\right)$$

$$= P\left(\bigcap_{k=1}^{m} \{Z_k < -\beta_k\}\right)$$

$= -\Phi_m\left(\underline{\beta}; \underline{R}\right) \le min_{k=1}^{m}\{\Phi(-\beta_k)\}$(3.107)

Where $\underline{\beta} = (\beta_1, \dots, \beta_m)^T$ and $\underline{R} = \left\{\rho_{ij} = corr[Z_i, Z_j] = \underline{\alpha_i}^T \underline{\alpha_j} : i, j = 1,2, \dots, m\right\}$

The fault tree of the frame is shown in Figure 3.10.

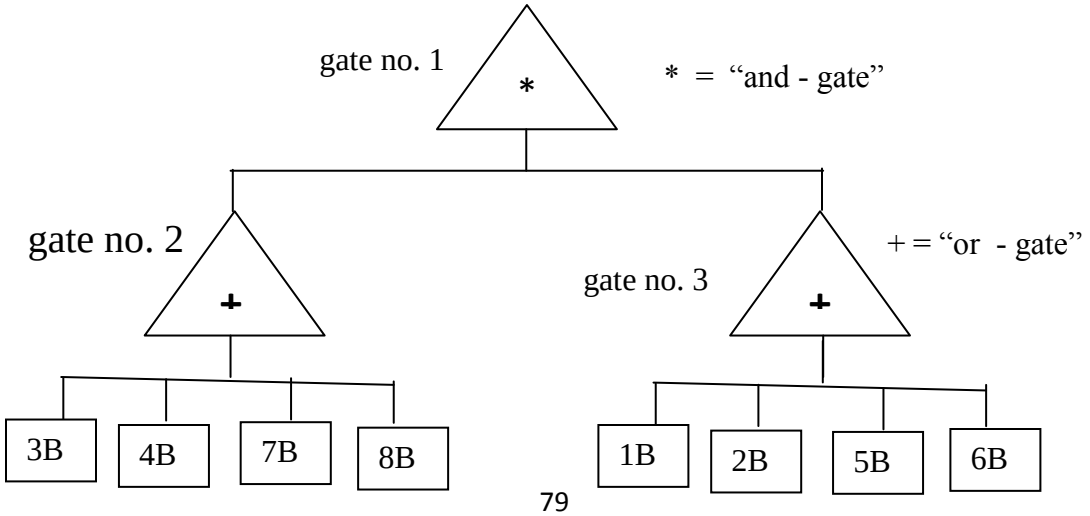


Figure 3.10: Fault- tree of the frame

3.10: Fault-tree of the frame

Gate 2 is flexural failure mode in beams and gate 3 is the axial failure mode in columns.

We obtain the following expression:

$(E_3 \cup E_4 \cup E_7 \cup E_8) \cap (E_1 \cup E_2 \cup E_5 \cup E_6) = \cap\cup \left(E_{ij}\right)$(3.102)

According to Cizmar, *et al.* (2009), the probability of failure of the parallel system is assessed using upper and lower bounds. The FORM approximation is

$P_f \geq min[P(F_{i=1}^n)]$ (3.103)

$P_f \leq 1 - \prod_{i=1}^n \left(1 - P(F_i)\right)$ (3.104)

Where a lower and upper bounds correspond respectively to fully correlated and un-correlated safety margins. An estimate of the failure probability is obtained as the arithmetic mean of the upper and lower probability bounds. The MATLAB program is shown in Appendix E (5).

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 Frame Analysis

The beam spans for the frame shown in Figure 3.1 were varied from 5400mm to 9400mm with 1000mm increment in order to determine critical spans of the beam. Floor live loads were also varied from 3kN/m^2 to 7kN/m^2 with 1.0 kN/m^2 increment. The results of the frame analysis obtained from MATLAB program of Appendix B are presented in Table 4.1

Table 4.1: Results of Analysis for beams

Span	Parameter	Live Loads (kN/m^2)				
		3.0	4.0	5.0	6.0	7.0
5.4	Midpan Moment(kNm)	60.811	64.013	72.085	80.161	88.238
	L/Support Moment(kNm) -	41.127	48.221	53.151	56.082	63.013
	R/Support Moment(kNm) -	120.450	146.673	166.304	185.935	205.566
	Shear Force(kN)	88.100	98.520	110.379	122.237	134.094
6.4	Midpan Moment(kNm)	90.175	102.030	113.885	125.740	137.592
	L/Support Moment(kNm) -	84.504	95.634	106.764	117.893	129.023
	R/Support Moment(kNm) -	157.307	181.591	205.875	230.158	254.442
	Shear Force(kN)	107.290	122.515	137.740	152.964	168.189
	Midpan Moment(kNm)	124.532	141.051	157.570	174.089	190.608
	L/Support Moment(kNm) -	118.928	134.738	150.547	166.357	182.167

7.4	R/Support Moment(kNm) -	208.308	240.364	272.420	304.476	336.532
	Shear Force(kN)	125.129	142.913	160.698	178.482	196.267
8.4	Midpan Moment(kNm)	164.911	186.957	209.003	231.049	253.095
	L/Support Moment(kNm) -	159.366	180.718	202.070	223.422	244.774
	R/Support Moment(kNm) -	265.640	306.404	347.168	387.931	428.695
	Shear Force(kN)	143.097	163.466	183.835	204.204	224.573
9.4	Midpan Moment(kNm)	211.344	239.789	268.234	296.678	325.123
	L/Support Moment(kNm) -	205.850	233.615	261.380	289.145	316.910
	R/Support Moment(kNm) -	329.271	379.671	430.070	480.469	530.868
	Shear Force(kN)	161.160	184.132	207.104	230.076	253.049

4.2 Deterministic and Optimum Designs of Beams

The deterministic design of the critical beam was performed in accordance with EC 2 specification. The calculation sheetis presented in Appendix C1. The optimum design was done as enumerated in chapter three. The results of the design of the beam using the two methods at different live loadsare shown in Table 4.2.

Table 4.2: Comparison of Design Variables of the Design Methods (beam span of 5.4m).

Design Method	Variable	Live Loads (kN/mm^2)				
		3.0	4.0	5.0	6.0	7.0
	$X_1=b_b$(mm)	225	225	225	225	225

Deterministic Design	$X_2=d_b$(mm)	400	400	400	400	400
	$X_3=A_{sbt}$(mm^2)	539	624	712	790	864
	$X_4=A_{sbc}$(mm^2)	102	102	102	102	102
	$X_5=\bar{A_{b1}}$(mm^2)	482	556	632	712	798
	$X_6=\bar{A_{b2}}$(mm^2)	802	1197	1357	1518	1678
	f=Cost	₦40776	₦43169	₦45612	₦47926	₦50,203
Optimum Design	$X_1=b_b$(mm)	200	200	200	200	200
	$X_2=d_b$(mm)	400	400	400	400	400
	$X_3=A_{sbt}$(mm^2)	550	640	735	759	833
	$X_4=A_{sbc}$(mm^2)	102	102	102	102	173
	$X_5=\bar{A_{b1}}$(mm^2)	491	568	649	736	753
	$X_6=\bar{A_{b2}}$(mm^2)	1065	1226	1386	1487	1675
	f=Cost	₦38,666	₦41,139	₦43,716	₦44,934	₦47,702
Reduction in cost of beam (%)		5.17	4.70	4.16	6.24	4.98

There was decrease in cross sections which resulted in increase in areas of reinforcement as seen in Table 4.2. The decrease in cost was basically due to decrease in cross section. The optimum design resulted in lower cost than the deterministic design. The results indicate that cost savings of an average of 5.10 in the beam was achieved. The decrease in cost was due to decrease in cross section as a result of optimization. Figure 4.1 is the graph showing optimized cost of beam for various floor live loads. The graph shows linear increase in cost as floor live load increased. It is seen that the optimum design gave lower costs than the deterministic design.

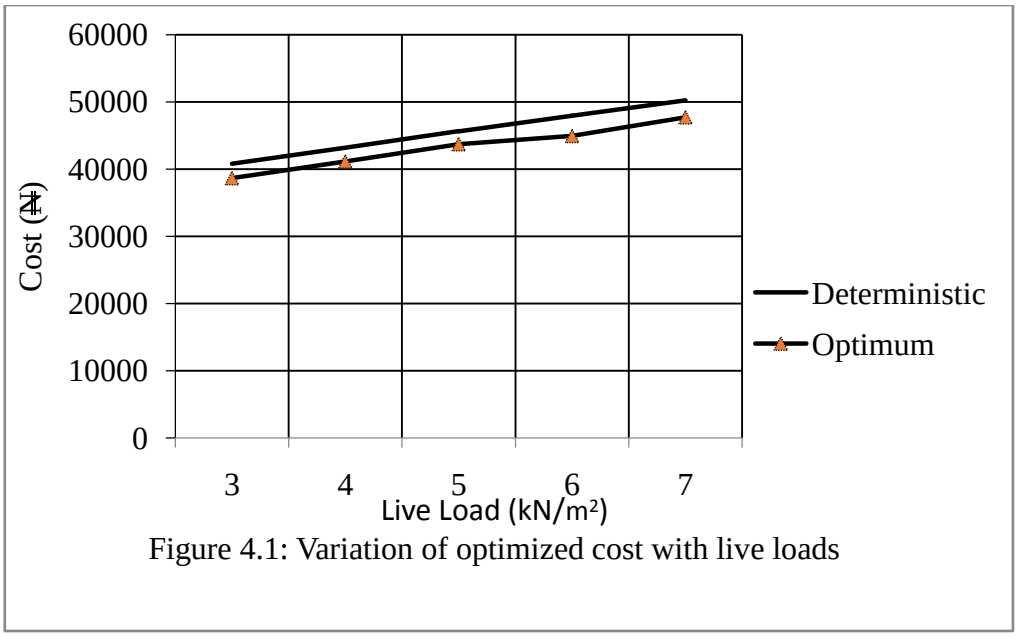


Figure 4.1: Variation of optimized cost with live loads

The various optimum design variables are compared with the deterministic values as presented in Figures 4.2(a) to 4.2(e). The iterative procedure consists of size population of individual finite strings enclosing possible solutions in a given space. The population can be viewed as points in the search space of all solutions to the optimization problem. Each individual in the population has a fitness value defined by a fitness function. At the end of iteration, the best fitness values are obtained as shown in the figure. The term, q_k refers to Live loads.

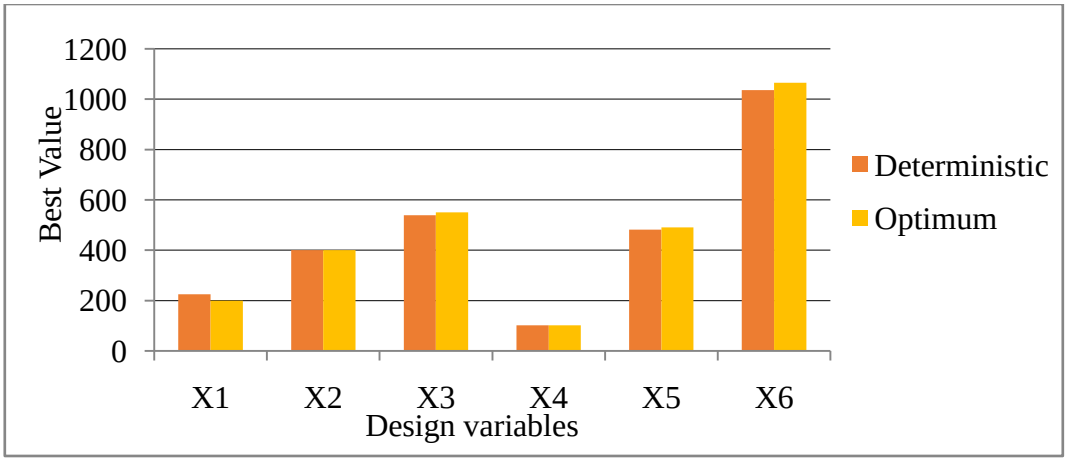


Figure 4.2a: Variation of best values of design variables forq_k of 3kN/m^2

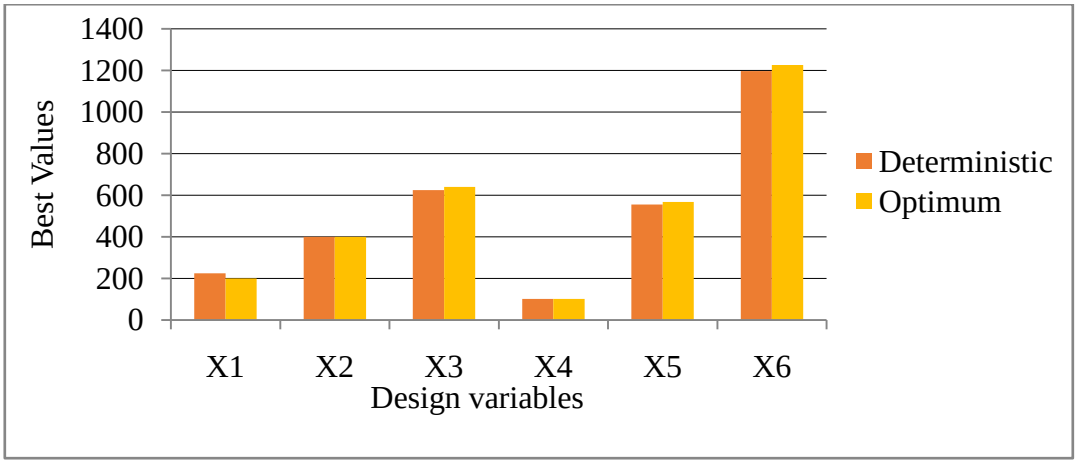


Figure 4.2a: Variation of best values of design variables for q_k of 4kN/m^2

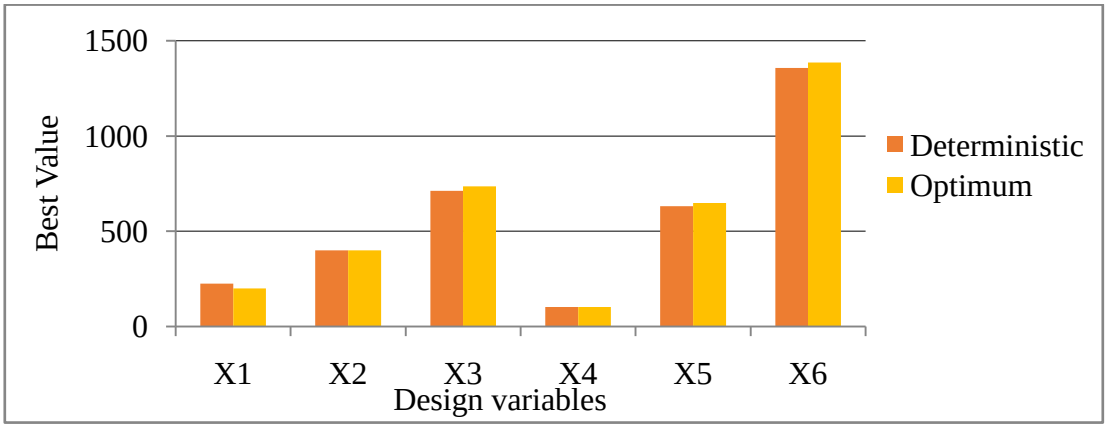


Figure 4.2c: Variation of best values of design variables for q_k of 5kN/m^2

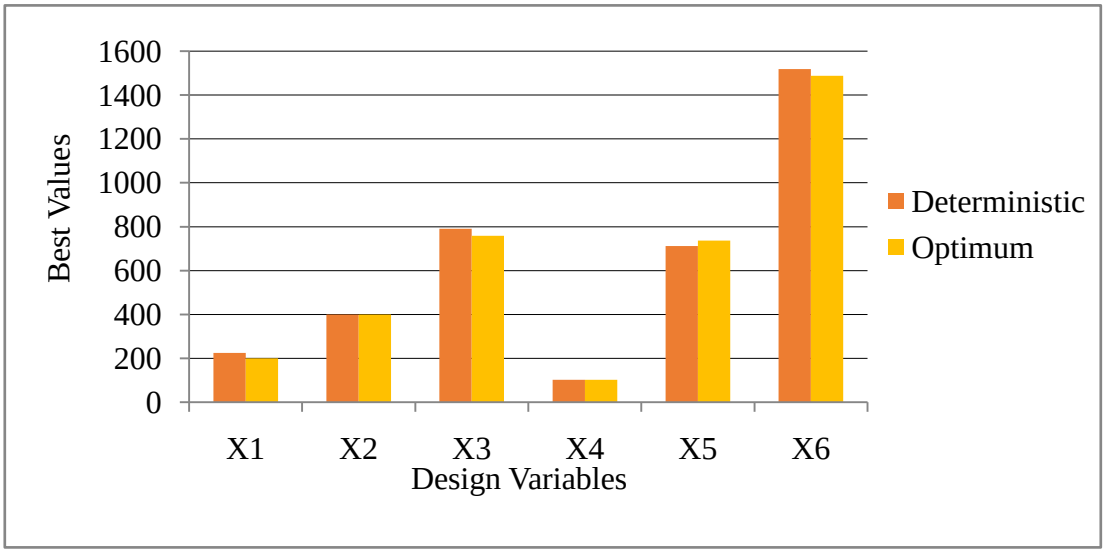


Figure 4.2d: Variation of best values of design variables for q_k of 6kN/m^2

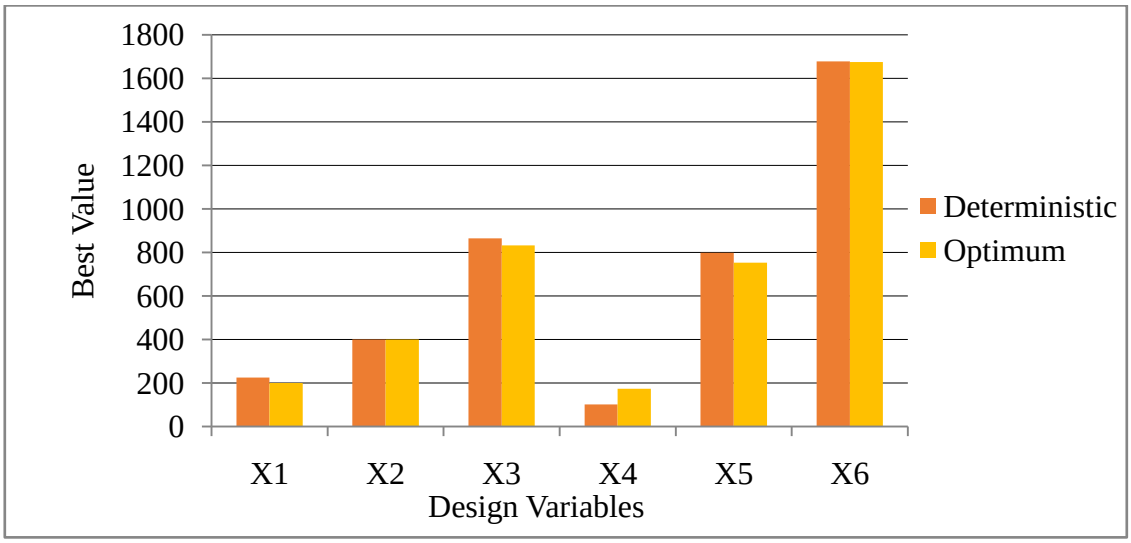


Figure 4.2e: Variation of best values of design variables for q_k of 7kN/m^2

4.3 Formulation of Expressions for Optimum Design of Beams

The optimum design results of the critical beam at various spans of the beam are presented in Table 4.3.

Table 4.3 Beam results for optimum design at various spans of beam

Span (m)	Variable	Live Loads (kN/mm^2)				
		3.0	4.0	5.0	6.0	7.0
5.4	$X_1=b_b$(mm)	200	200	200	200	200
	$X_2=d_b$(mm)	400	400	400	400	400
	$X_3=A_{sbt}(mm^2)$	550	640	735	759	833
	$X_4=A_{sbc}(mm^2)$	102	102	102	102	173
	$X_5 = A_{b1}^{-}\ (mm^2)$	491	568	649	736	753

	$X_5 = A_{b2}^{-}(mm^2)$	1065	1226	1386	1487	1675
	f=Cost	₦38,666	₦41,139	₦43,716	₦44,934	₦47,702
6.4	X_1=b_b(mm)	200	200	210	209	200
	X_2=d_b(mm)	400	400	400	400	400
	X_3=$A_{sbt}(mm^2)$	798	907	1015	1124	1234
	X_4=$A_{sbc}(mm^2)$	138	247	320	442	574
	$X_5 = A_{b1}^{-}\ (mm^2)$	819	848	949	1052	1155
	$X_5 = A_{b2}^{-}\ (mm^2)$	1416	1642	1860	2084	2309
	f=Cost	₦45,945	₦49,805	₦54,348	₦58,403	₦62,354
7.4	X_1=b_b(mm)	200	200	200	200	200
	X_2=d_b(mm)	400	400	400	400	400
	X_3=$A_{sbt}(mm^2)$	1114	1266	1418	1570	1722
	X_4=$A_{sbc}(mm^2)$	454	606	758	910	1062
	$X_5 = A_{b1}^{-}\ (mm^2)$	1063	1208	1353	1499	1644
	$X_5 = A_{b2}^{-}\ (mm^2)$	1884	2179	2474	2769	3064
	f=Cost	₦56,997	₦62,769	₦68,541	₦74,313	₦80,085
8.4	X_1=b_b(mm)	200	200	200	200	200
	X_2=d_b(mm)	400	400	400	400	400
	X_3=$A_{sbt}(mm^2)$	1485	1688	1891	2094	2296
	X_4=$A_{sbc}(mm^2)$	825	1028	1231	1434	1636
	$X_5 = A_{b1}^{-}\ (mm^2)$	1434	1631	1827	2023	2220
	$X_5 = A_{b2}^{-}\ (mm^2)$	2417	2787	3161	3536	3911
	f=Cost	₦70,275	₦77,903	₦85,530	₦93,157	₦100,780
9.4	X_1=b_b(mm)	200	200	200	200	205
	X_2=d_b(mm)	400	400	400	400	400
	X_3=$A_{sbt}(mm^2)$	1912	2174	2436	2697	2958
	X_4=$A_{sbc}(mm^2)$	1252	1514	1776	2037	2283

	$X_5 = A_{b1}^{-}\ (mm^2)$	1862	2117	2373	2628	2882
	$X_5 = A_{b2}^{-}\ (mm^2)$	2997	3460	3924	4387	4850
	f=Cost	₦85,445	₦95,201	₦104,960	₦114,710	₦124,620

4.3.1 Expression for Optimum Cost of Beams

The Cost of beam are sorted out from Tables 4.3 as shown graphically in in Figure 4.3.

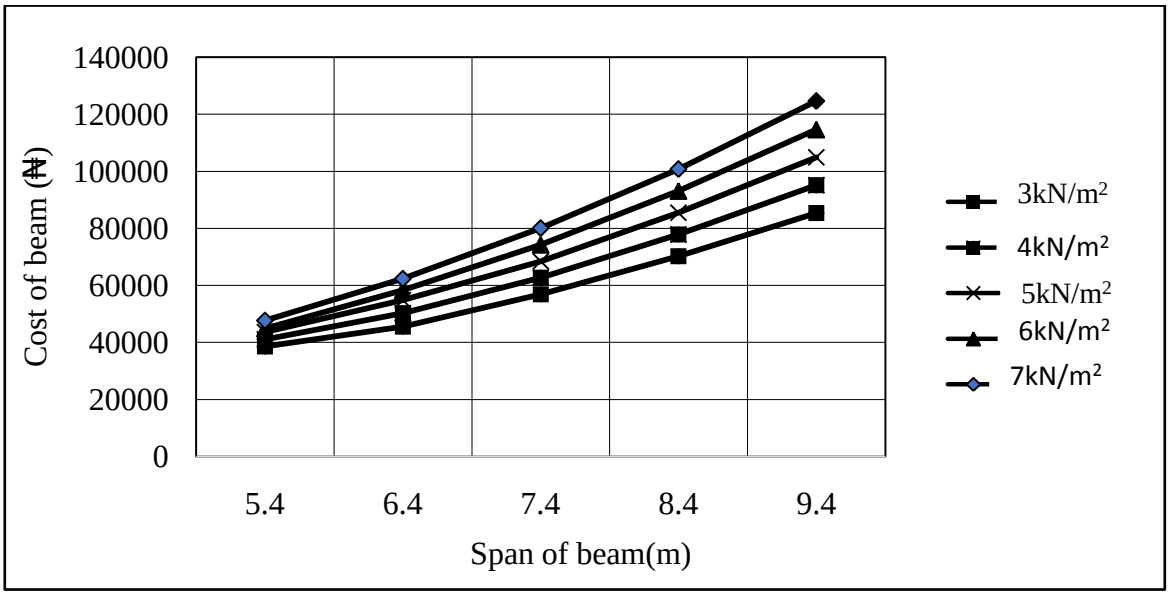


Figure 4.3: Variation of cost with the change of beam span

Expressions for computation of cost of reinforced concrete beam elements are derived as follows:

The variation in Figure 4.3 can be expressed in the form of a linear equation (Hussanian, 1992)

$$C_b = C_1 L_b + C_2 \quad (4.66)$$

Where $C_b$is the cost of beam, C_1 and $C_2$are constants. The constantsC_1 and C_2can be determined by linear fitting each line of Figure 4.3.

For live load of 3KN/m^2, C_1 and C_2 are 11789 and -27772 respectively. Therefore,

C_b= 11789L_b -27772

For live load of 4kN/m^2, C_1 and C_2 are 13622 and -35441 respectively. Therefore,

C_b=13622 L_b -35441

For live load of 5kN/m^2, C_1 and C_2 are 15367 and -42297 respectively. Therefore,

C_b=15367 L_b -42297

For live load of 6kN/m^2, C_1 and C_2 are 17431 and -51883 respectively. Therefore,

C_b=17431L_b -51883

For live load of 7kN/m^2, C_1 and C_2 are 19226 and -59166 respectively. Therefore,C_b=19226L_b-59166

Each pair of C_1 and $C_2$is plotted with respect to floor live loads as shown in Figure 4.4 and 4.5.

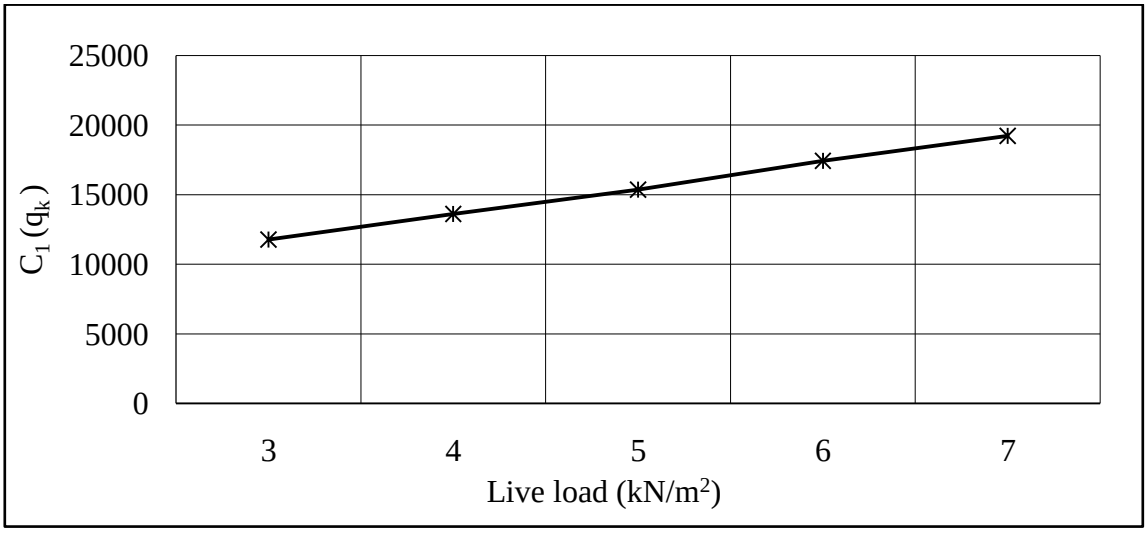


Figure 4.4: Constants C_1 for cost of beam expression

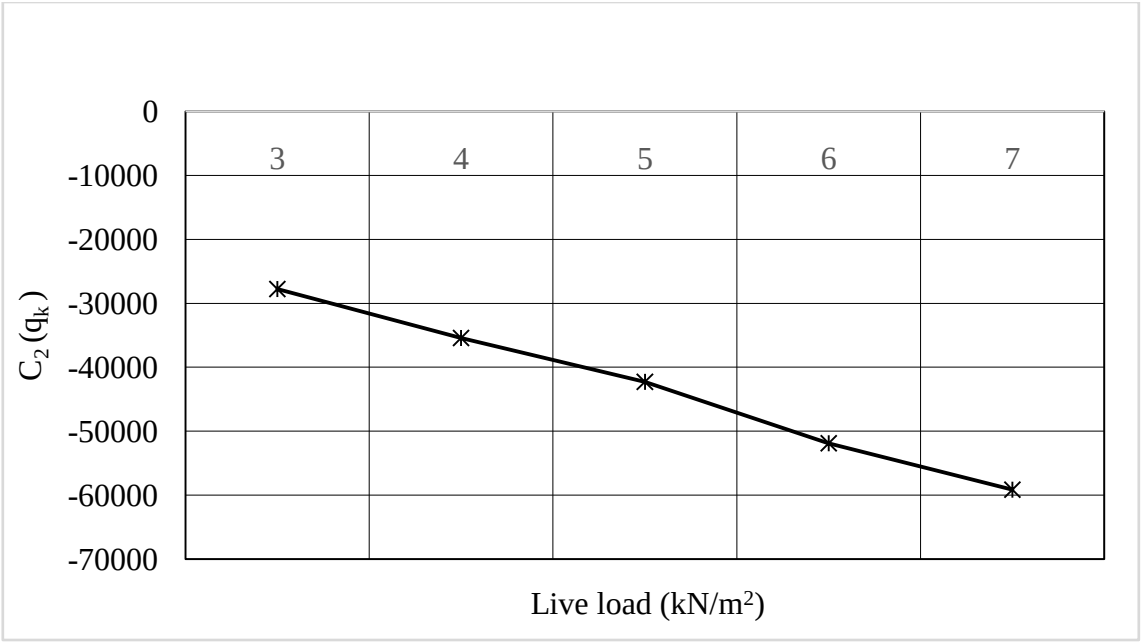


Figure 4.5: Constants C_2 for cost of beam expression

Linearly fitting Figure 4.4 gives the coefficients 1868 and 6146for C_1. Linearly fitting Figure 4.5 gives the coefficients -7923 and -3697 forC_2.

Therefore,

C1=1868q_k+6146 and C_2=-7923q_k-3697 (4.67)

Where $q_k$is live load in kN/m^2

The cost function is therefore

$C_b = C_1 L_b + C_2$**=** **(**1868q_k+6146**)**L_b -7923q_k-3697 (4.68)

4.3.2Expression for BeamSpan to Effective Depth Ratios

Expression for beam span to effective depth ratio. Deflection is controlled by placing a limit on the ratio of span-effective depth of the beam.

The optimum span - effective depth ratios are shown in Figure 4.6.

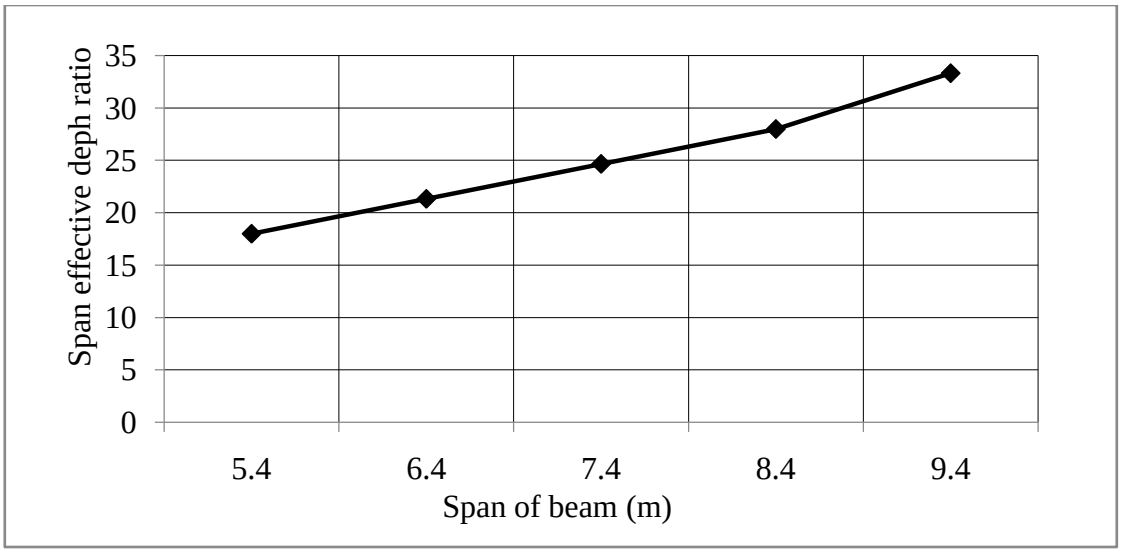


Figure 4.6 Variation of span - effective depth ratio with span

As in the case of cost function, the variation in the curve can be expressed in the form of a linear equation, that is:

$$\frac{L_b}{d_b} = C_1 L_b + C_2$$

Where $C_1$and $C_2$are constants. Linearly fitting Figure 4.6 gives the Values of $C_1$and C_2as 3.733 and -2.558 respectively.

The optimum span effective depth is therefore

$L_b/d_b = C_1 L_b + C_2 = 3.733 L_b$ -2.558

4.3.3Expression for Beam Reinforcement Ratios.

Expression for beam reinforcement ratios were formulated. Limits to reinforcement quantities ensure that a structure has satisfactory durability and serviceability performance under normal circumstances.

The reinforcement ratios for the beam are shown in Figure 4.7.

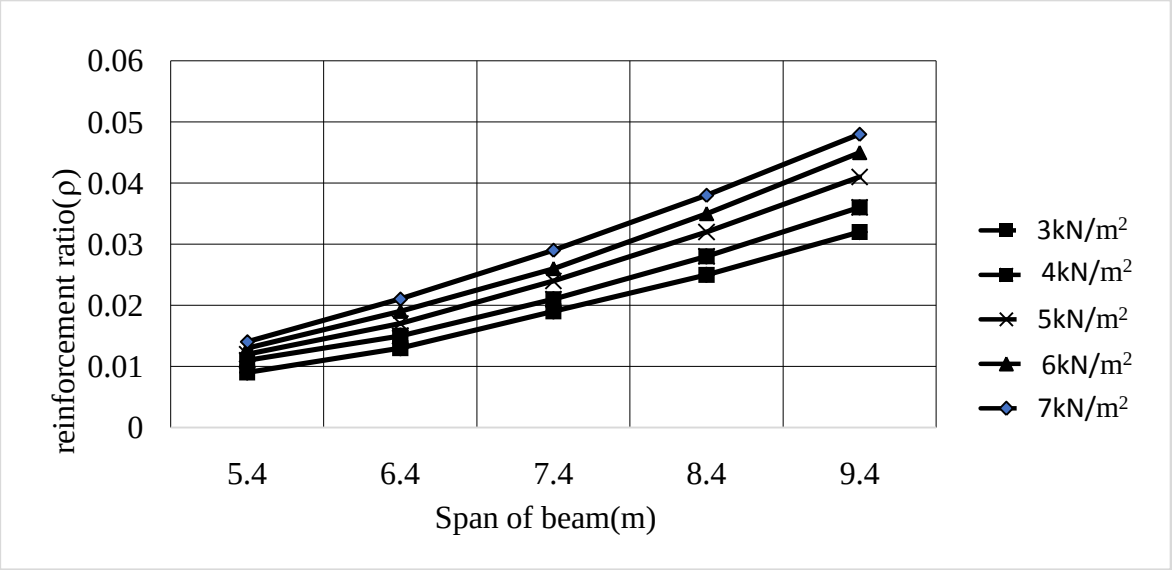


Figure 4.7: Variation of reinforcement ratios with the change of span

The optimal beam reinforcement ratio, ρ is also expressed in the form of linear equations as:

$\rho = C_1 L_b + C_2$

Where C_1 and C_2 are coefficients determined by linearly fitting each curve in Figure 4.7.

For live load of 3kN/m^2, C_1 and C_2 are 0.0058 and -0.0233 respectively. Therefore,

C_b= 0.0058Lb - 0.0233

For live load of 4kN/m^2, C_1 and C_2 are 0.0063 and -0.0244 respectively. Therefore,

C_b=0.0063Lb -0.0244

For live load of 5kN/m^2, C_1 and C_2 are 0.0073 and -0.0288 respectively. Therefore,

C_b=0.0073Lb -0.0288

For live load of 6kN/m^2, C_1 and C_2 are 0.0080 and -0.0316 respectively. Therefore,

C_b=0.0080Lb -0.0316

For live load of 7kN/m^2, C_1 and C_2 are 0.0085 and -0.0329 respectively. Therefore,

C_b=0.0085Lb -0.0329

Values of C_1 and C_2 thus obtained are plotted as shown in Figure 4.8 and 4.9.

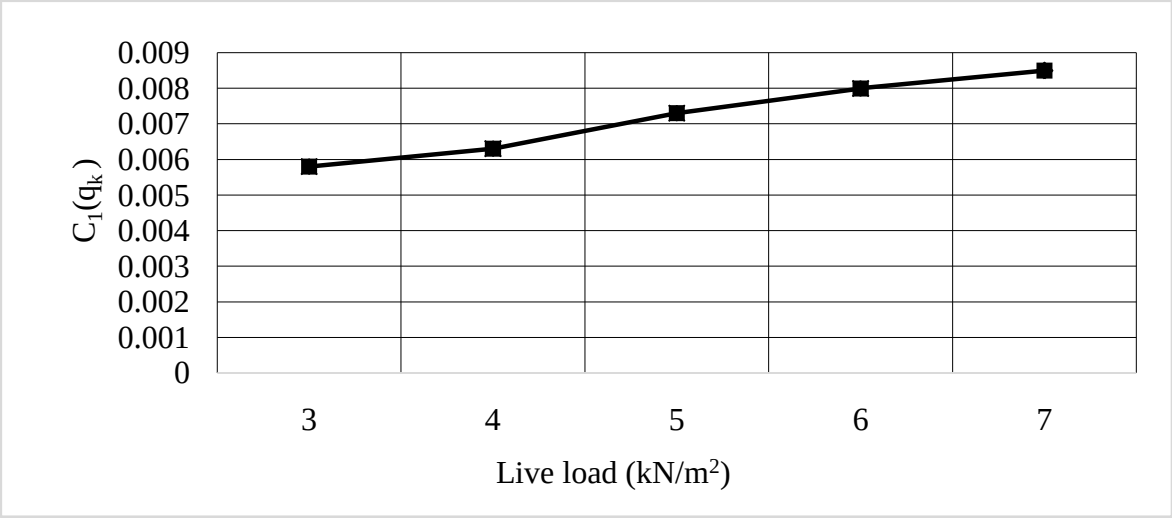


Figure 4.8: Constants C_1 for reinforcement ratio expression

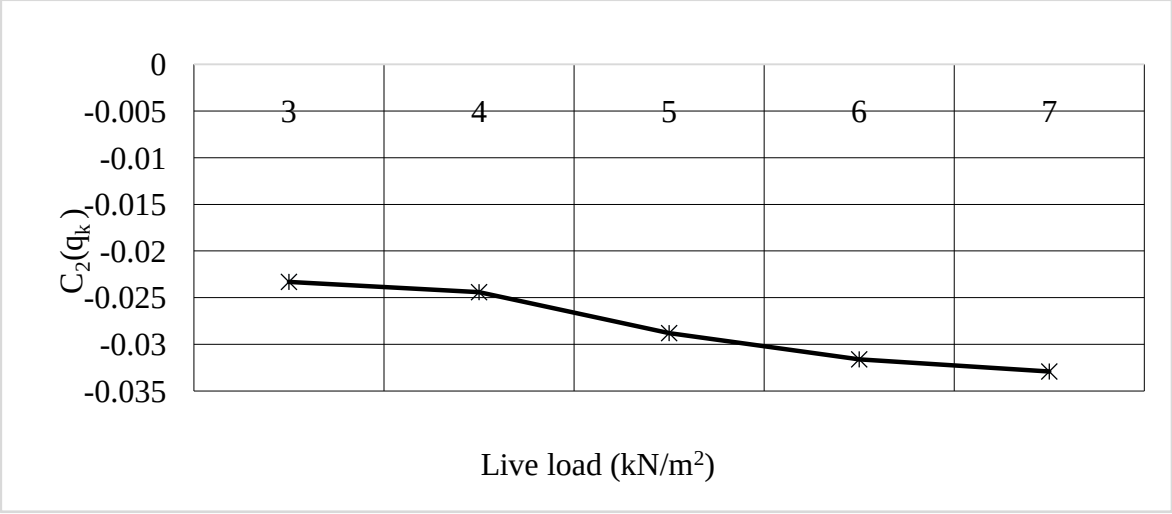


Figure 4.9: Constants C_2 for reinforcement ratio expression

Linearly fitting the curve C_1 gives the coefficients 0.0007 and 0.0036. Similarly linearly fitting the C_2 gives the coefficients0.0105 and -0.0676. Therefore:

C_1= 0.0007q_k - 0.0036 and C_2=0.0105q_k - 0.0676

The reinforcement ratio is therefore

$\rho = C_1 L_b + C_2$ =(0.0007q_k - 0.0036)L_b + 0.0105q_k - 0.0676

4.3.4Beam Design charts

To plot a beam deign chart for optimum design of reinforced concrete section, M/bd^2, 100A_{st}/bd and 100A'_{st}/bd were computed for the various span of beams and the results are presented in Table 4.4.

Table 4.4: Design chart Data

Span of Beam	Axis	Live load (kN/m^2)				
		3.0	4.0	5.0	6.0	7.0
5.4m	M/bd^2 (kN/m^2)	3.428	3.880	4.327	4.773	5.220
	100A_s/bd^2 (m^{-1})	0.917	1.067	1.225	1.265	1.389
	100A'_s/bd^2 (m^{-1})	0	0	0	0	0.289
6.4m	M/bd^2 (kN/m^2)	5.010	5.668	6.327	6.986	7.644
	100A_s/bd^2 (m^{-1})	1.330	1.512	1.692	1.874	2.057
	100A'_s/bd^2 (m^{-1})	00.230	0.412	0.534	0.775	0.957
7.4m	M/bd^2 (kN/m^2)	6.918	7.836	8.754	9.672	10.589
	100A_s/bd^2 (m^{-1})	1.857	2.110	2.364	2.617	2.870
	100A'_s/bd^2 (m^{-1})	0.757	1.010	1.264	1.517	1.770
8.4m	M/bd^2 (kN/m^2)	9.162	10.387	11.611	12.836	14.061
	100A_s/bd^2 (m^{-1})	2.475	2.814	3.152	3.490	3.827
	100A'_s/bd^2 (m^{-1})	1.375	1.714	2.052	2.390	2.727
9.4m	M/bd^2 (kN/m^2)	11.741	13.322	14.902	16.482	17.622
	100A_s/bd^2 (m^{-1})	3.187	3.624	4.060	4.495	4.810
	100A'_s/bd^2 (m^{-1})	2.087	2.524	2.960	3.395	3.713

The values of 100A_s/bd^2, 100A'_s/bd^2 and M/bd^2 are employed to plot the optimum beam design chartsshown in Figures 4.10 and 4.11.

Figure 4.10: Typical optimum design chart for tension

Figure 4.11: Typical optimum beam design for compression reinforcement

4.4 Deterministic and Optimum Design of Critical Column

The floor live loads were varied from 3kN/m^2 to 7kN/m^2 with 1.0 kN/m^2 increment. The result of the frame analysis obtained from MATLAB program of Appendix B is presented in Table 4.5

Table 4.5 Column Results of Frame Analysis

Span of beam	Parameter	Live Loads (kN/m^2)				
		3.0	4.0	5.0	6.0	7.0
5.4	Bending Moment(kNm)	24.522	28.031	31.520	35.019	38.518
	Axial Force (kN)	299.547	333.241	366.937	400.631	434.326
	Shear Force(kN)	4.290	7.294	8.373	9.477	10.581

The results of Table 4.5is for critical column. The calculation sheetfor the deterministic design is presented in Appendix C2. The optimum design was done as enumerated in chapter three. The results are shown in Table 4.6.

Table 4.6:The results of the deterministic and optimum designs of the column

Type of Design	Variable	Live load (kN/m^2)				
		3.0	4.0	5.0	6.0	7.0
Deterministic EC2 Design	$X_1=b_c$ (mm)	225	225	225	225	225
	$X_2=d_c$ (mm)	175	175	175	175	175
	$X_3=A_{sc}$ (mm^2)	542	542	542	542	609
	f=Cost	₦15118	₦15118	₦15118	₦15118	₦16378
Optimum Design	$X_1=b_c$ (mm)	200	200	200	200	200
	$X_2=d_c$ (mm)	175	175	177	189	200
	$X_3=A_{sc}$ (mm^2)	542	542	542	542	609
	f=Cost	₦14730	₦14730	₦14802	₦15143	₦15568
Reduction in cost (%)		2.57	2.57	2.08	3.66	4.95

The results showed that optimal design resulted in cost savings of an average of 3.2% for the column. The decrease in cost was due to decrease in cross section as a result of optimization.

4.5 Reliability Analysis

4.5.1 Beam Reliability Analysis

Geometric variables were considered normally distributed, characteristic strength of concrete and steel yield strength as lognormal while live loads are gamma distributions (Holický and Vrouwenvelder, 1996). The Coefficients of variations for width, effective depth of section and areas of reinforcement were each taken as 0.05. Coefficient of variations for characteristic strength of concrete and steel yield strengths were both taken as 0.15, coefficient of variations for dead and live loads were both taken as 0.3 and coefficients of variation for moments and shear forces taken as 0.24. ((Royset, et al., 2006, Almeida, et al., 2008). Taking the results of deterministic design shown Table 4.1 as mean values, statistical data for reliability analysis are presented in Tables 4.7 and 4.8 for deterministic and optimum designs respectively.

Table 4.7: Statistical Data for Deterministic Design of Beams (Load Ratio α=1.5)

Parameter	Mean	COV	Standard Deviation	Probability Distribution
f_{ck}(mm)	25	0.15	3.75	Lognormal
$f_{yk}(N/mm^2)$	500	0.15	75.00	Lognormal
b_b(mm)	225	0.05	11.25	Normal
d_b(mm)	400	0.05	20.00	Normal
As1(mm^2)	373	0.05	18.65	Normal
$Q_k(kN/m^2)$	10.800	0.3	3.24	Gamma
M_B(Nmm)	41127000	0.24	9870000	Lognormal
M_E(Nmm)	120450000	0.24	28908000	Lognormal

Table 4.8: Statistical Data for Optimum Design of Beams for Load Ratio α=1.5

Parameter	Mean	COV	Standard Deviation	Probability Distribution
f_{ck} (mm)	25	0.15	3.75	Lognormal
f_{yk} (N/mm^2)	500	0.15	75.00	Lognormal
b_b (mm)	200	0.05	10.0	Normal
d_b (mm)	400	0.05	20.0	Normal
As1 (mm^2)	377	0.05	18.85	Normal
Q_k (kN/m^2)	10.800	0.3	3.24	Gamma
M_B(Nmm)	41127000	0.24	9870000	Lognormal
M_E(Nmm)	120450000	0.24	28908000	Lognormal

The component reliability indices for flexure failure, shear failure and deflection control in beams are shown inFigures 4.12 to 4.14. The results of the beam analysis showed that the flexure failure mode governs design. For this limit state, the Reliability indices for the deterministic design decreased from 6.694 to 3.961 by changing the load ratio from 0.3 to 1.5. For the optimum design, the reliability indices decreased from 6.172 to 3.427. The reliability index of 3.961 for deterministic design is adequate being greater than the recommended value of 3.8 in Euro code 0 (2004). The corresponding value of 3.427 for optimum design is inadequate at ultimate limit state.

The shear and deflection reliability indices for both deterministic and optimum designs are adequate being greater than the recommended value of 3.7 at ultimate limit state. Flexure failure mode therefore governs design.

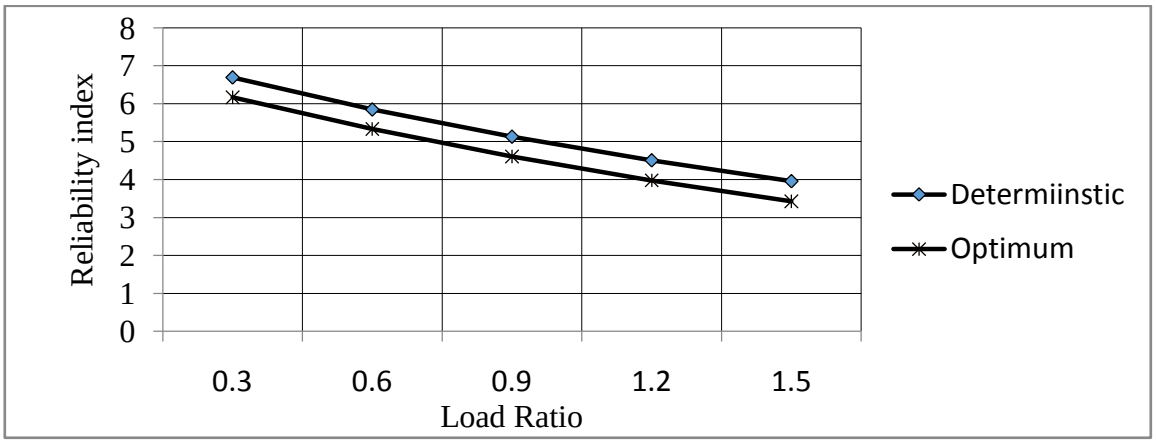


Figure 4.12: Beam flexure failure reliability index versusload ratio

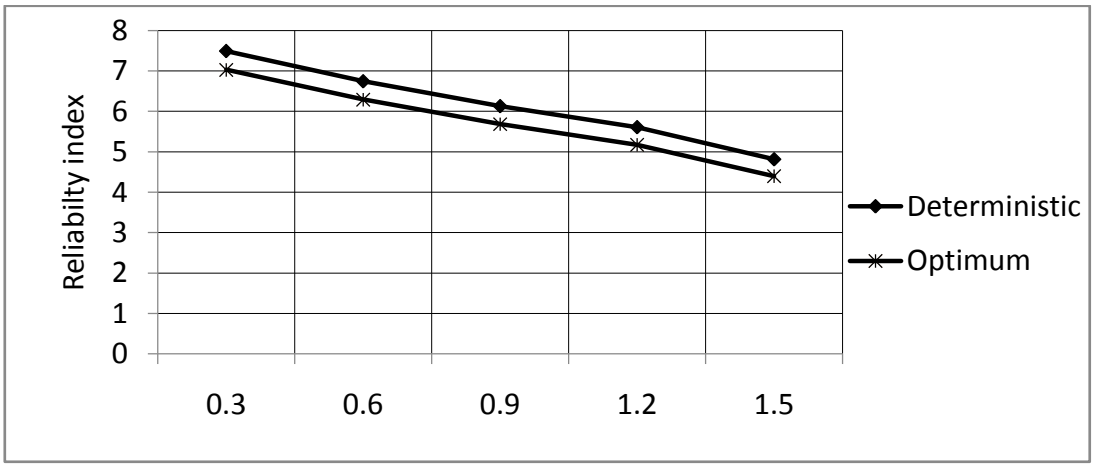


Figure 4.13: Beam shear failure reliability index versus load ratio

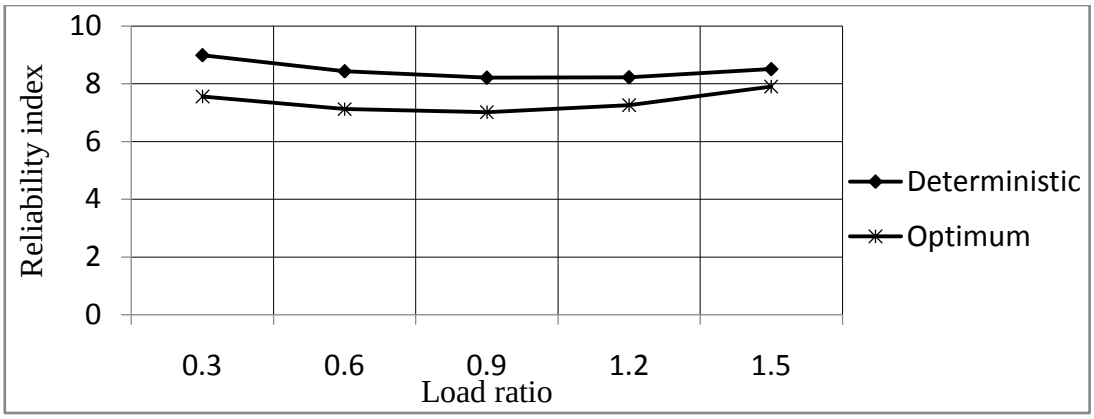


Figure 4.14: Span -effective depth reliability index versus load ratio

4.5.2 Column Reliability Analysis

The results of frame analysis from MATLAB program of Appendix B for the critical column are shown in Table 4.9. The results of the deterministic and optimum designs are shown in Table 4.10.

Table 4.9: Results of frame analysis for critical column

Parameter	Live Loads (kN/m^2)				
	3.0	4.0	5.0	6.0	7.0
Bending Moment(kNm)	24.522	28.031	31.520	35.019	38.518
Axial Force (kN)	299.547	333.241	366.937	400.631	434.326
Shear Force(kN)	4.290	7.294	8.373	9.477	10.581

Table 4.10: Results of deterministic and optimum design ofcritical column

Type of Design	Variable	Load Ratio				
		0.1	0.2	0.3	0.4	0.5
Deterministic Design	$X_1=b_b$	225	225	225	225	225
	$X_2=d_b$	175	175	175	175	175

	$X_3=A_{sc}$	557	987	1316	1873	2340
Optimum Design	$X_1=b_b$	200	212	225	225	225
	$X_2=d_b$	150	164	175	175	175
	$X_3=A_{sc}$	452	452	599	933	1247

Taking the results of the deterministic design in Table 4.10 as mean values, statistical data for reliability analysis of the column are presented in Tables 4.11 and 4.12.

Table 4.11: Statistical Data for Deterministic Design of Columns: $\alpha = 1.5$

Parameter	Mean	COV	Standard Deviation	Probability Distribution
$f_{ck}(N/mm^2)$	25	0.15	3.75	Lognormal
$f_{yk}(N/mm^2)$	500	0.15	75.00	Lognormal
b_c(mm)	225	0.05	11.25	Normal
d_c(mm)	175	0.05	8.75	Normal
$A_{sc}mm^2$	557	0.05	27.85	Normal
$Q_k(kN/m^2)$	25	0.3	7.5	Gamma

Table 4.12: Statistical Data for Optimum Design of Columns: $\alpha = 1.5$

Parameter	Mean	COV	Standard Deviation	Probability Distribution
$f_{ck}(N/mm^2)$	25	0.15	3.75	Lognormal
$f_{yk}(N/mm^2)$	500	0.15	75.00	Lognormal
b_c(mm)	200	0.05	10.0	Normal
d_c(mm)	150	0.05	7.5	Normal
$A_{sc}mm^2$	452	0.05	22.6	Normal
$Q_k(kN/m^2)$	15	0.3	4.5	Gamma

The results of the reliability analysis are presented in Figures 4.15 to 4.17. It can be seen from the results that axial load failure mode governs design. For this mode of failure, the reliability indices are not very sensitive to load ratio increase.

The reliability indices for the deterministic design are at an average of 3.088 for load ratio ranging from 0.3 to 1.5. The corresponding average reliability index for optimum design is 1.914. This is because the limit state functions for the limit states in terms of resistance R and load effect S are functions of longitudinal reinforcement, A_{sc} and uniformly distributed load, W_2 respectively which simultaneously increases as load ratios increase.

The flexural and axial plus flexural reliability indices for both deterministic and optimum designs are adequate being greater than the recommended value of 3.8 in Annex C, (Euro code 0, 2004) at ultimate limit state.

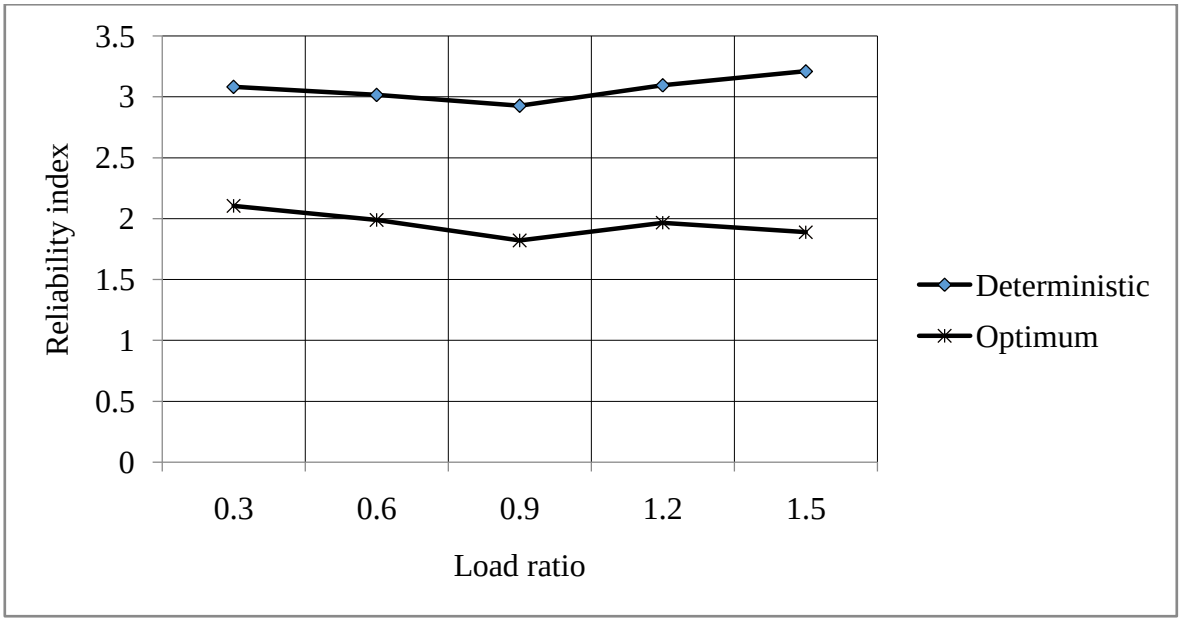


Figure 4.15: Column axial load failure reliability index versusload ratio

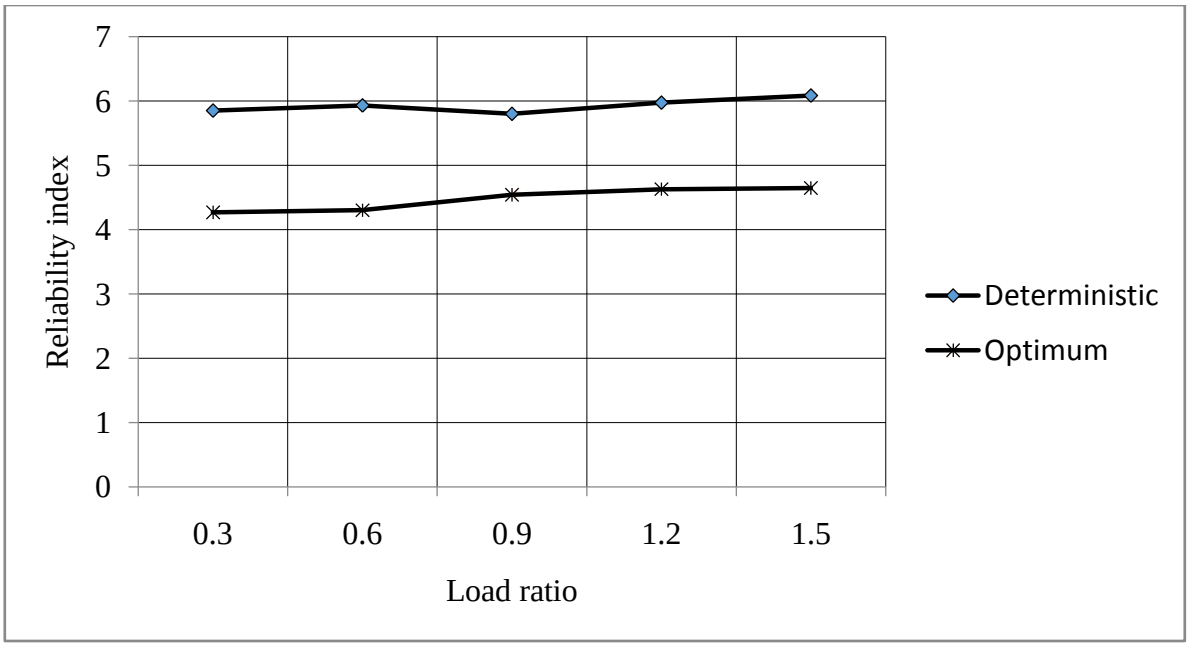


Figure 4.16: Column flexural failure reliability index versus load ratio

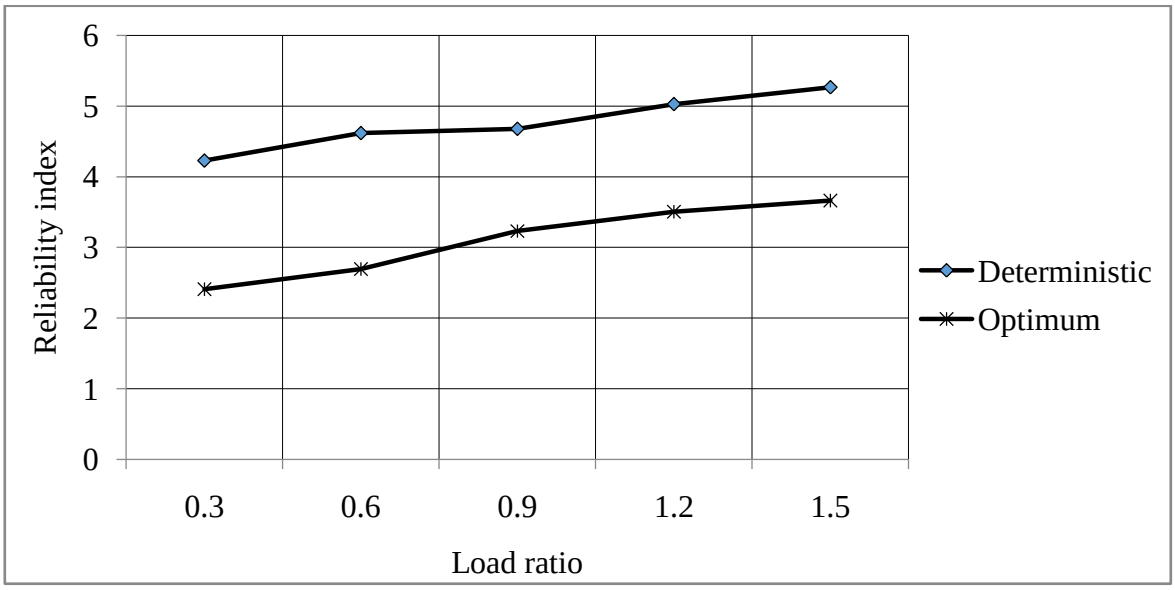


Figure 4.17: Column Axial plus flexural failure reliability index versus dead -to-live load ratio

4.5.3 System Reliability Analysis

The component reliability computations were carried out with load ratios varying from 0.3 to 1.5. The results obtained were subsequently used to determine overall system safety index "β_{Sys}" for the various load ratios using system reliability MATLAB program presented in Appendix E5. The results are shown in Table 4.13

The results of component reliability analysis of the reinforced concrete frames gave the lowest value of 3.43 at load ratio of 1.5 in member four (4). The corresponding systems reliability index is 3.33.

Table 4.13 Components and system reliability index values

Member	Load Ratios				
	0.3	0.6	0.9	1.2	1.5
1	5.035	5.035	5.035	5.035	5.035
2	5.035	5.035	5.035	5.035	5.035
3	4.882	4.882	4.882	4.882	4.882
4	6.081	5.330	4.605	3.967	3.427
5	5.035	5.035	5.035	5.035	5.035
6	5.035	5.035	5.035	5.035	5.035
7	8.835	8.835	8.835	8.835	8.835
8	7.035	6.168	5.418	4.793	3.443
9	5.035	5.035	5.035	5.035	5.035
10	5.035	5.035	5.035	5.035	5.035

11	6.081	5.330	4.605	3.967	3.427
12	4.882	4.882	4.882	4.882	4.882
13	5.035	5.035	5.035	5.035	5.035
14	5.035	5.035	5.035	5.035	5.035
System	4.91	4.90	4.60	3.97	3.33

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

An optimization problem was solved by formulating design variables for a structural plane frame, the objective function that was minimized and the design constraints imposed on the frame. The objective functions were the cost of the frame elements in terms of design variables. The Eurocode 2(2004) requirements for safety and serviceability as well as other performance requirements constituted the constraints.

MATLAB optimization tool box ($fmincon$) was employed. The optimization toolbox is a collection of functions that extend the capability of the MATLAB numeric computing environment. The toolbox consists of functions that perform minimization on general nonlinear functions.

The beam spans were varied from 5.4m to 9.4m with 1.0m increment. Service live loads were varied from 3kN/m^2 to 7kN/m^2 with 1.0 kN/m^2 increment. The totals of 25 optimal cases were considered.The results showed that there was decrease in cross sections of optimized members which resulted in increase in areas of reinforcement. The optimum designresulted in lower cost than the deterministic design based on the Eurocode 2 design. The results indicate that cost savings of an average of 5.1% in the beam and 3.2% in columns were achieved. The decrease in cost were due to decrease in cross section as a result of optimization.

Optimum beam design chart were derived for tension and compression reinforcement in beams. Expression for beam span - effective depth ratio and beam reinforcement ratios were formulated. Deflection is controlled by placing a limit on the ratios of span-effective

depth of the beam. The expression obtained to calculate maximum optimum span-effective depth ratio is:

L_b/d_b= 3.733L_b -2.558

The term L_b/d_b refers span-effective depth ratio and L_b is the span of the beam in metre

Limits to reinforcement quantities ensure that a structure has satisfactory durability and serviceability performance under normal circumstances. The expression obtained to calculate minimum reinforcement ratio in the beam is:

ρ= (0.0007q_k - 0.0036)L_b + 0.0105q_k - 0.0676

The term ρ refers reinforcement ratio, L_b span of the beam in metre andq_k is the live load in kN/m^2.

The reliability levels of the optimized components were performed to ascertain the reliability of the optimum design. The reliability analysis was performed using First order Reliability Method (FORM). The results of reliability analysis showed that flexure failure mode governs the design of beams. Reliability index for the deterministic design of the critical beam was 3.96 at design load which was adequate being greater than the recommended value of 3.8 in Annex C, (Eurocode 0, 2002). The corresponding value for optimum design was 3.43 which is not adequate being lower than the recommended value of 3.8 at ultimate limit state. Reliability index for the deterministic design of the critical column was 5.83 while the corresponding value for optimum design was 5.03. In all cases the reliability index values for optimum design were lower than that of deterministic design. It is established as expected that optimization of reinforced concrete elements gave reliability indexes lower than deterministic values.

The results of overall system reliability analysis gave reliability index valuelower than the component reliability index valuefor the critical member. The component reliability index of the critical beam was 3.43. The corresponding systems reliability was 3.33.

5.2 Recommendations

The following recommendation is made:

Design charts based on optimum design concept will be valuable for low cost design of reinforced concrete structures. Furthermore, expressions for computations of span - effective depth and reinforcement ratios obtained can be used for preliminary optimum design of reinforced concrete structures.

References

ACI - 318 (2011). *Building Code Requirements for Structural Concrete. ACI Standard 318-11,* American Concrete Institute, Detroit, Michigan.

Agrawal, G.and Bhattacharya, B. (2010).Partial safety factor design of rectangular partially prestressed concretebeams in ultimate flexural limit state. *Journal of Structural Engineering*,37(4):257–267

Agarwal, H. (2004). Reliability based design optimization: formulations and Methodologies. Ph.D Thesis in Aerospace and Mechanical Engineering. University of Notre Dame. Indiana.

Almeida, A. F., Vaz, L. E. and Velasco, Marta S. L. (2008). Deterministic and reliability-based design optimization of reinforced concrete plane frames. *International Conference on Engineering Optimization*, Rio de Janeiro, Brazil, 01 - 05 June 2008.

Arora J.S. (1976). Survey of Structural Reanalysis Techniques, ASCE Proceedings, Journal of Structural Division, 102(4), 783-802.

Babu , G. L. S.and Basha, B. M. (2008).Optimum design of cantilever retaining walls using target reliability approach . *International Journal of Geomechanics*,8(4):240-252.

Balling, R. J. and Yao, X. (1997). Optimization of Reinforced Concrete Frames. *Journal of Structural Engineering*,123(2): 193-202.

Baecher, G. B. and Christian, J. T.(2003) *Reliability and Statistics in Geotechnical Engineering.* John Wiley & Sons, London. PP. 301-470

Booz, W., Legewie, G. and Thierauf, G. (1984).Optimization of Reinforced Concrete Structures According to German Design Regulations, 'Proceeding of the International Conference on Computer Aided Analysis and Design of concrete structures' Yugoslavia, pp. 761-773.

Camp, C. V., Pezeshk, S. and Hansson, H. (2003). Flexural design of reinforced concrete frames using a genetic algorithm.*Journal of structural Engineering,* 129(1):105-110.

Castro, P.M.R.P., Delgado,R.M. and César de Sá , J.M. A.(2005). A partial factors methodology for structural safety assessment in non-linear analysis.*Computers and Concrete,*2 (1)*:*31-53

Cizmar, D., Kirkgaard, P.H. and Sorensen, J.D. (2009). *Reliability and Robustness Evaluation of Timber Structures*. Short Term Scientific Mission, COST E55 Action. University of Aalborg, Aalborg.

Ditleven, O. and Madsen, H.O.(2005).Structural *Reliability Methods.*John Wiley & Sons, Chichester.

Edgar, T.F., and Himmelblau, D.M. (1998).*Optimization of Chemical Processes*, 2nd edition. New York, McGraw Hill.

Eurocode 0 (2002).*EN 1990: Basis of structural design.*CEN, Brussels,

Eurocode 1 (2002). *EN 1991-1-1: Actions on structures, part 1-1: Densities, self-weight and imposed loads for buildings.* CEN, Brussels,

Eurocode 2 (2004).*EN 1992-1-1: Design of Reinforced Concrete structures, part 1-1, General Rules and Rules foe Buildings.* CEN, Brussels,

Faber, M.H. and Sorensen, J.D. (2003). Reliability-based Code Calibration. Joint Committee on Structural Safety. Technical University Denmark.

Fadaee, M. J., and Grierson, D. E. (1996). Design Optimization of 3D Reinforced Concrete Structures.*Structural Optimization,* (12). 127–134.

Fox, R. L. (1965). Constraint Surface Normals for Structural Synthesis Techniques. *AIAA Journal 3,* 1516-1517.

Galeb, A. C. and Atiyah, Z. F. (2011). Optimum Design of reinforced Concrete Waffle Slabs. International *Journal of Civil and structural Engineering*. (4): 862-880.

Gayton, N., Mohamed, A., Sorensen, J.D., Pendola, M. and Lemaire, M. (2004). Calibration Methods for Reliability-based Design Codes. *Structural Safety,26.* 91–121

Gollwitzer S, Abdo, T. and Rackwitz, R. (1988).*First Order Reliability Method: FORM Software.* User Manual.Munich,

Gollwitzer S. and Rackwitz, R. (1988). *Systems Reliability and CUT Algorithm Manual, RCP GMBH.* Munich, West Germany.

Guerra, A. and Kiousis, P. D.(2006). Design optimization of reinforced concrete structures'. *Computers and concrete, 3(5).* 313-33.

Hassanian , M. A. (1992). Optimal Design of Reinforced Concrete Frames. M.Sc Thesis, King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia.

Holický, M. and Vrouwenvelder, T. (1996). Reliability analysis of a reinforced concrete column designed according to Eurocodes. *IABSE reports,* (74).

Kanagasundaram, S. and Karihaloo, B.,L. (1990). Minimum Cost Design of Reinforced Concrete structures. Structural Optilmization, .2(3), 173-184

Laursen, H. I.(1988). *Structural Analysis*, Third Edition, McGraw-Hill Inc, New York. Pp 354-408.

Lu, P. L., Lin, H.Z. and Kiureghian, A. D. (1990). CALREL, User Manual, Research Report.No. UCB/SEMM-89/18. Structural Engineering, Mechanic and Materials,Department of Civil Engineering, University of California, Berkeley.

Lu, R., Luo, Y. and Conte, J.P. (1994). Reliability Evaluation of Reinforced Concrete Beam.

Structural Safety 4. 277-298.

MacGregor, J.G.and Wight, J.K.(2009). *Reinforced Concrete Mechanics and Design*. 5th Edition. Pearrson International Edition, London. PP. 17-28.

MATLAB (2004) .The MathWorks Inc., 24 Prime Park Way, Natick, USA.

Melchers, R.E. (1999). *StructuralReliability Analysis and Prediction. Second Edition,* John Wiley & Sons, Chichester, UK.

Mohammed, J.K. (2014). Reliability-based Analysis and Calibration of Eurocode 5 Design Criteria of Solid Timber Portal Frame.Ph.D Thesis, Department of Civil Engineering, Ahmadu Bello University, Zaria, Nigeria.

Moharrami, H. (1993). Design optimization of Reinforced concrete Building Frames. Ph.D Thesis, University of Waterloo, Canada. Pp 3-10

Mosley, B., Bungey, J. and Hulse, R. (2007). *Reinforced Concrete Design*, 6th edition, Book Power, London. Pp258-259.

Nawy, E. G. (1996)*Reinforced concrete-A fundamental approach*, 3rd Ed., Prentice–Hall, Upper Saddle River, N. J.

NBR6118 (2004). Design of structural concrete. *Brazilian Concrete Standard,* Rio de Janeiro, Brazil.

Newpro Quants Consultants (2008).*New comprehensive Construction Cost Book*, 3rd edition, Baraka press and publishers Ltd, Kaduna. Pp443-664.

Nowak, A.S. and Szerszen, M.M.(2000). Structural Reliablilty as Applied to Highway, Prog. *Struct. Eng. Mater 2:218 -224*.

Oberkampf, W. L., DeLand, S. M., Rutherford, B. M., Diegert, K. V. and Alvin, K. F.(2000). Estimation of total uncertainty in modeling and simulation, *Technical Report SAND 2000-0824*. Sandia National Laboratories.

Oyenuga, V. O. (2005), *Simplified Reinforced Concrete Design: A consultant, computer-based approach, 2nd Edition*. Vassions concept Engineering series, Lagos.

Razani, R. (1965). The Behaviour of Fully Stressed Design of Structures and its Relationship to Minimum Weight Design. AIAA Journal, 3, 2262-2268.

Royset, J. O., Kiureghian, A. D. and Polak E. (2006).Optimal Design with probabilistic objective and constraints. *Journal of Engineering Mechanics, 132(1)*, 107-118.

Sander. G. and Fleury .C. (1978). A Mixed Method in Structural Optimization. *International Journal of Numerical Methods in Engineering, 13.* 385-404.

Schmit, L.A. (1960). Structural Design by Systematic Synthesis, Proceedings of Second National Conference on Electronic Computation, ASCE. pp. 105-122.

Schmit L.A (1971). Structural Synthesis 1959-1969: A Decade in Progress, Recent Advances in Matrix Methods of Structural Analysis and Design, University of Alabama Press, Huntsville. Alabama,.

Schmit, L.A. and Farshi, B. (1974). Some Approximation Concepts for Structural Synthesis.

AIAA Journal, 12(5). 1252-1260. .

Schmit, L.A. and Miura, H. (1976). A New Structural Analysis/Synthesis Capability Access-1.*AIAA Journal, 14(5)*. 661-671.

Schmit, L.A. and Fleury, C. (1980). Structural Synthesis by Combining Approximation Concepts and Dual Methods.*AIAA Journal, 18(10)*. 1252-1260.

SriVidya , A. and Ranganathan, R. (1994). Reliability based optimal design of reinforced concrete frames. *Computers and Structures, 57(7)*, 651-661.

Wasiutynski, Z. and Brandt, A. (1963). The Present State of Knowledge in the Field of Optimum Structural Design, *Applied Mechanics Reviews, 16*, 341-350.

White, R.N., Gergely, P. and Sexsmith, R. G. (1976). *Structural Engineering*, Combined Edition, John Wiley and sons, New York. Pp 531-565

Yousif, S. T. and Najem, R. M. (2014). Optimum Cost Design of Reinforced Concrete Coulumns using Generic Algorithm , *Al-Rafidain Engineering, 22(1)*, 123-141

Zhao, Y. and Ono, T. (1998).System Reliability Evaluation of Ductile Frame Structures Journal *of Structural Division, ASCE, Structural Division. 1245(6).*

Appendix A: Computation of Frame Analysis Matrices

The computational analysis of the frame by the direct stiffness method is hereby presented. The frame shown in Figure A1.The members are numbered I to VII. It is spaced 3.6m and consists of beam sections 225mm x 450mm at first floor and 225mm x 225mm at roof level while the column sections are 225mm x 225mm. Span L_1 and L_2 are 5.4m and 1.8m respectively. The heights of columns,$H_1$and $H_2$are 3.15m each.

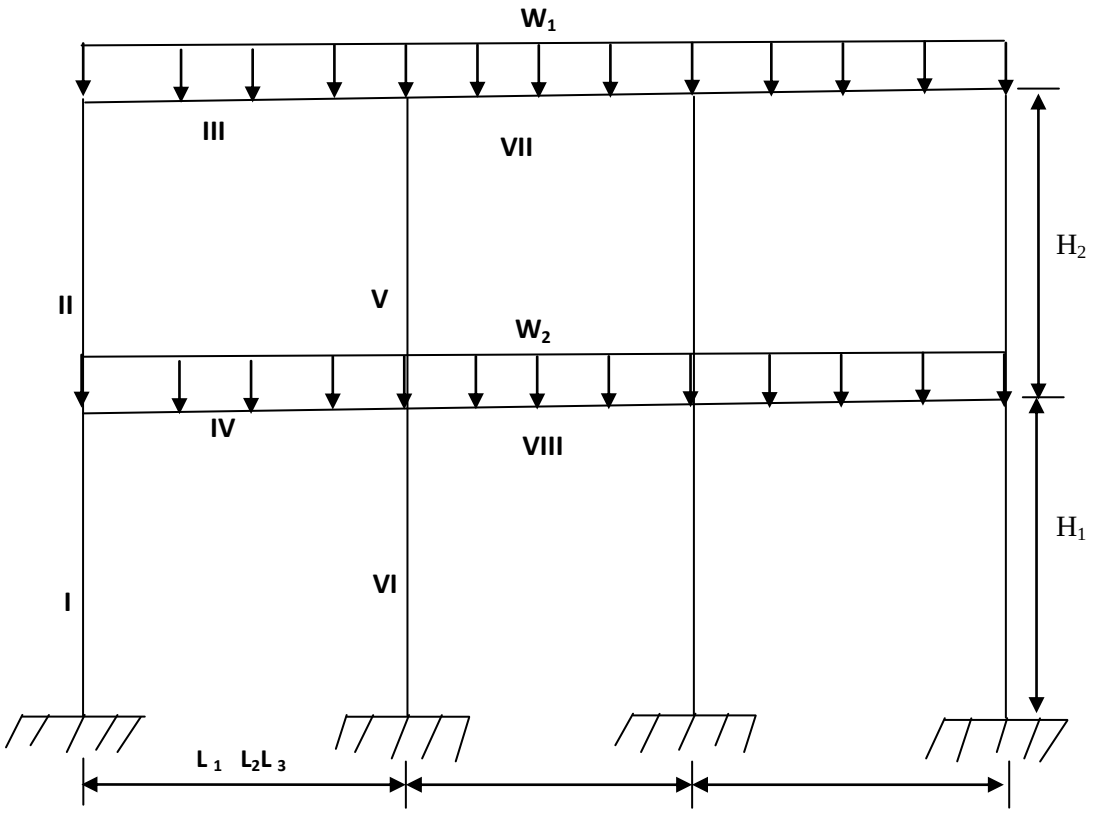


Figure A1: Basic frame

According to Eurocode 2 EN 1992-1-1(2002)

Dead load from roof=2.5KN/m^2

Design Load = $1.35G_k+1.5Q_k$

Design load from roof = 1.35 x 2.5 x3.6 +1.5 x0.75x3.6 = 16.20KN/m

=From EC 1, Table 4.1, Weight of reinforced concrete = 25 KN/m^3

Let W_1 be the ultimate weight of second storey beam of dimension 225mmx225mm

Self weight of beam=1.35 x 25 x 0.225^2 = 1.71KN/m

∴ W_1 = 16.20+ 1.71 =17.91KN/m

Let W_2 be the loadings on roof level beam of dimension 225mm x450mm

Self weight = 25 x 0.225x0.450 = 2.531KN/m

Let the spacing of the beams be 3.6m from centre to centre,

Dead load from first floor slab = 25 x 0.15 x 3.6 = 13.50KN/m

Total dead load =2.531+13.50= 16.031 KN/m

Again from Eurocode 1 , Table 6.1, Imposed load from slab (1^{st} floor) = 3.0KN/m^2 = 3.0 x 3.6 = 10.8 KN/m

∴ W_2= 1.35 x 16.031 + 1.5 x 10.80 = 37.842KN/m

For columns, the cross sections are 225mm x 225mm.

Therefore, $A = 0.225x0.225 = 5.06x10^{-2}m^2$

$$I = 0.225x0.225^3/12 = 2.14x10^{-4}m^4$$

Using class C25/30 concrete, from EC2, Part1, Table 6.1,

E_c = 31Gpa = 31 x 10^9 pa = 31 x 10^9 N/m^2= 31 x 10^6 KN/m^2

EA = 31 x 10^6 x 5.06 x 10^{-2} = 156.9 x 10^4 KN=1569 x 10^3 KN

EI = 31 x 10^6 x 2.14 x 10^{-4} = 66.34 x 10^2 KNm^2 =6.634 x 10^3 KN m^2

H = 3.15m

$$K_{Mi} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \quad \text{(A1)}$$

Where K_{Mi} is stiffness matrix for member, mi.

The members are numbered as show in Figure 3.1. Therefore, the stiffness matrices for columns, SMC (MI, MII, MV and MVI) are:

$$K_{MC} = 10^3 \begin{bmatrix} 498.10 & 0 & 0 & -498.10 & 0 & 0 \\ 0 & 2.55 & 4.01 & 0 & -2.55 & 4.01 \\ 0 & 4.01 & 8.42 & 0 & -4.01 & 4.21 \\ -498.10 & 0 & 0 & 498.10 & 0 & 0 \\ 0 & -2.55 & -4.01 & 0 & 2.55 & -4.01 \\ 0 & 4.01 & 4.21 & 0 & -2.55 & 8.42 \end{bmatrix} \quad \text{(A2)}$$

The roof beams are taken to have the same dimensions as the columns

The cross section is 225mm x 225mm.

Therefore, $A = 0.225x0.225 = 5.06x10^{-2}m^2$

$$I = 0.225x0.225^3/12 = 2.14x10^{-4}m^4$$

Again, using class C25/30 concrete, from EC2, Part1, Table 6.1,

E_c = 31Gpa = 31 x 10^9 pa = 31 x 10^9 N/m^2= 31 x 10^6 KN/m^2

EA = 31 x 10^6 x 5.06 x 10^{-2} = 156.9375 x 10^4 KN=1569.375 x 10^3 KN

EI = 31 x 10^6 x 2.14 x 10^{-4} = 66.34 x 10^2 KNm^2 =6.634 x 10^3 KN m^2

For beam length, L=5.4m

Therefore, the stiffness matrix for the roof level beam taken as member III substituting EA , EA and L IN Equation (3.1) is:

$$K_{MIII} = 10^3 \begin{bmatrix} 290.56 & 0.00 & 0.00 & -290.56 & 0.00 & 0.00 \\ 0.00 & 0.51 & 1.37 & 0.00 & -0.51 & 1.37 \\ 0.00 & 1.37 & 4.91 & 0.00 & -1.37 & 2.46 \\ -290.56 & 0 & 0.00 & 290.56 & 0.00 & 0.00 \\ 0.00 & -0.51 & -1.37 & 0.00 & 0.51 & -1.37 \\ 0.00 & 1.37 & 2.46 & 0.00 & -0.51 & 4.91 \end{bmatrix} \quad \text{(A3)}$$

The First floor Beams have cross sectional dimensions of 225mm x 450mm.

Therefore, $A = 0.225x0.450 = 10.125x10^{-2}m^2$

$$I = 0.225x0.450^3/12 = 17.086x10^{-4}m^4$$

Again, using class C25/30 concrete, from EC2, Part1, Table 6.1,

E_c = 31Gpa = 31 x 10^9 pa = 31 x 10^9 N/m^2= 31 x 10^6 KN/m^2

EA = 31 x 10^6 x 10.125x 10^{-2} = 3138.75 x 10^3 KN

EA = 31 x 10^6 x 17.086x 10^{-4} = 52.966 x 10^3 KN m^2

Let the first floor beam be taken as member IV. For, L=5.4m

Therefore, the stiffness matrix for member IV substituting EA , EA and L IN Equation (3.1) gives:

$$K_{MBIV} = 10^3 \begin{bmatrix} 581.250 & 0.00 & 0.00 & -581.250 & 0.00 & 0.00 \\ 0.00 & 4.04 & 10.90 & 0.00 & -4.04 & 10.90 \\ 0.00 & 10.90 & 39.23 & 0.00 & -10.90 & 19.62 \\ -581.250 & 0.00 & 0.00 & 581.250 & 0.00 & 0.00 \\ 0.00 & -4.04 & -10.90 & 0.00 & 4.04 & -10.90 \\ 0.00 & 10.90 & 9.23 & 0.00 & -4.04 & 39.23 \end{bmatrix} \quad \text{(A4)}$$

From equation (2.19), the rotation transformation matrix is given as,

$$R_T = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & \cos\theta & \sin\theta & 0 \\ 0 & 0 & 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{(A5)}$$

For Element I, $\gamma = 90^0$

Therefore,

$$R_{TI} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{(A6)}$$

The member stiffness matrix for structure coordinate for member I, SMSI from equation (2.21) is:

$S_{MSI}=R_{TI'}{}^{T}*S_{MC}*R_{TI}$

$$S_{MSI} = 10^3 \begin{bmatrix} 2.55 & 0.00 & -4.01 & -2.55 & 0.00 & -4.01 \\ 0.00 & 498.1 & 0.00 & 0.00 & -498.1 & 0.00 \\ -4.01 & 0.00 & 8.42 & 4.01 & 0.00 & 4.21 \\ -2.55 & 0.00 & 4.01 & 2.55 & 0.00 & 4.01 \\ 0.00 & -498.1 & 0.00 & 0.00 & 498.1 & 0.00 \\ -4.01 & 0.00 & 4.21 & 2.55 & 0.00 & 8.42 \end{bmatrix}$$

(A7)

For member II, $\gamma = 90^0$

RTII =RTI

$S_{MSII}=R_{TII'}{}^{T}*S_{MC}*R_{TII}=S_{MSI}$

$$\therefore\ S_{MSII} = 10^3 \begin{bmatrix} 290.63 & 0 & 0.00 & -290.63 & 0.00 & 0.00 \\ 0.00 & .51 & 1.37 & 0.00 & -0.51 & 1.37 \\ 0.00 & 1.37 & 4.91 & 0.00 & -1.37 & 2.47 \\ -290.63 & 0.00 & 0.00 & 290.63 & 0.00 & 0.00 \\ 0.00 & -0.51 & 1.37 & 0.00 & 0.51 & -1.37 \\ 0.00 & 1.37 & 2.46 & 0.00 & -0.51 & 4.91 \end{bmatrix} \quad \text{(A8)}$$

For Element III, $\gamma = 0^0$, Equation (3.5) gives

$$R_{TiiI} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{(A9)}$$

$S_{MSIII}=R_{TIII}{}^{T}*S_{MBIII}*R_{TIII}$

$$S_{MSIII} = 10^3 \begin{bmatrix} 290.56 & 0.00 & 0.00 & -290.56 & 0.00 & 0.00 \\ 0.00 & 0.51 & 1.37 & 0.00 & -0.51 & 1.37 \\ 0.00 & 1.37 & 4.91 & 0.00 & -1.37 & 2.46 \\ -290.56 & 0.00 & 0.00 & 290.56 & 0.00 & 0.00 \\ 0.00 & -0.51 & -1.37 & 0.00 & 0.51 & -1.37 \\ 0.00 & 1.37 & 2.46 & 0.00 & -0.51 & 4.91 \end{bmatrix} \quad \text{(A10)}$$

For Element IV, $\gamma = 0^0$

$$R_{TIV} = R_{TIII}$$

$S_{MSIV}=R_{TIV}{}^{T}*S_{MBIV}*R_{TIV}$

$$S_{MBIV} = 10^3 \begin{bmatrix} 581.250 & 0.00 & 0.00 & -581.250 & 0.00 & 0.00 \\ 0.00 & 4.04 & 10.90 & 0.00 & -4.04 & 10.90 \\ 0.00 & 10.90 & 39.23 & 0.00 & -10.90 & 19.62 \\ -581.250 & 0.00 & 0.00 & 581.250 & 0.00 & 0.00 \\ 0.00 & -4.04 & -10.90 & 0.00 & 4.04 & -10.90 \\ 0.00 & 10.90 & 19.62 & 0.00 & -4.040 & 39..23 \end{bmatrix} \quad \text{(A11)}$$

For Element V,

$$\gamma = 270^0$$

$$R_{TV} = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \text{(A12)}$$

$S_{MSV}=R_{TV}{}^{T}*S_{MC}*R_{TV}$

$$S_{MSV} = 10^3 \begin{bmatrix} 2.55 & 0.00 & 4.01 & -2.55 & 0.00 & 4.01 \\ 0.00 & 498.1 & 0.00 & 0.00 & -498.1 & 0.00 \\ 4.01 & 0.00 & 8.42 & -4.01 & 0.00 & 4.21 \\ -2.55 & 0.00 & -4.01 & 2.55 & 0.00 & -4.01 \\ 0.00 & -498.1 & 0.00 & 0.00 & 498.1 & 0.00 \\ 4.01 & 0.00 & 4.21 & -2.55 & 0.00 & 8.42 \end{bmatrix} \qquad \text{(A13)}$$

For Element VI, $\gamma = 270^0$

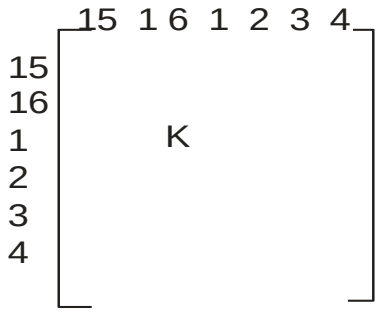
$R_{TVI} = R_{TV}$, therefore $S_{MSVI} = S_{MSV}$

$$\therefore\ S_{MSVI} = 10^3 \begin{bmatrix} 2.55 & 0.00 & 4.01 & -2.55 & 0.00 & 4.01 \\ 0.00 & 498.1 & 0.00 & 0.00 & -498.1 & 0.00 \\ 4.01 & 0.00 & 8.42 & -4.01 & 0.00 & 4.21 \\ -2.55 & 0.00 & -4.01 & 2.55 & 0.00 & -4.01 \\ 0.00 & -498.1 & 0.00 & 0.00 & 498.1 & 0.00 \\ 4.01 & 0.00 & 4.21 & -2.55 & 0.00 & 8.42 \end{bmatrix} \qquad \text{(A14)}$$

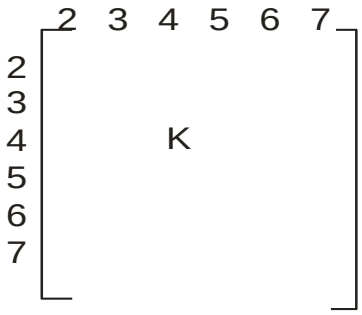
The values of S_{MSI} to S_{MSVI} of Equations (A7) to (A14) are placed in the matrices of Table A1 corresponding to Figure 3.2 respectively. For instance the value of S_{MSI} is place in element 1. This information is required to place the elements into the K-Matrix as shown in Equation (15). For element 1, element 1,1 is placed into position 15,15, element 1,2 is placed into position 15,16, and so on for each of the 36 elements of the member stiffness array. After 36 elements from member 1 have been written into the structure stiffness matrix, member 2 is treated. When writing into an already filled slot, the new contribution

is added to the existing value. A computer program is developed to perform the computation incorporated in the analysis program shown in Appendix B.

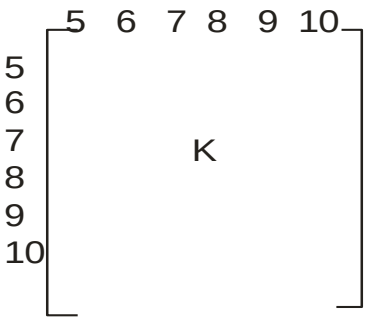
Table A1: Stiffness matrices and the corresponding structure degrees of freedom



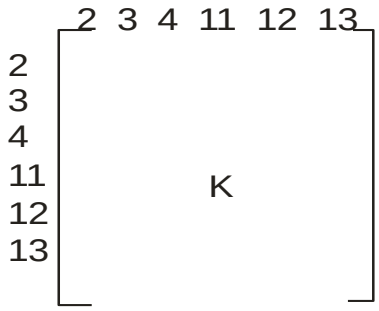
Element 1



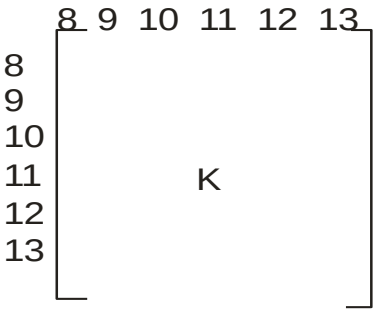
Element 2



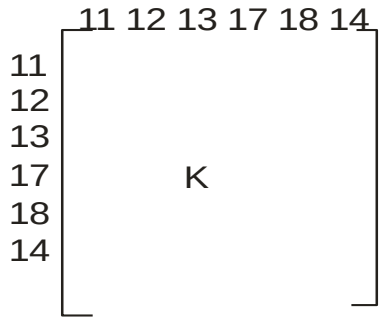
Element 3



Element 4



Element 5



Element 6

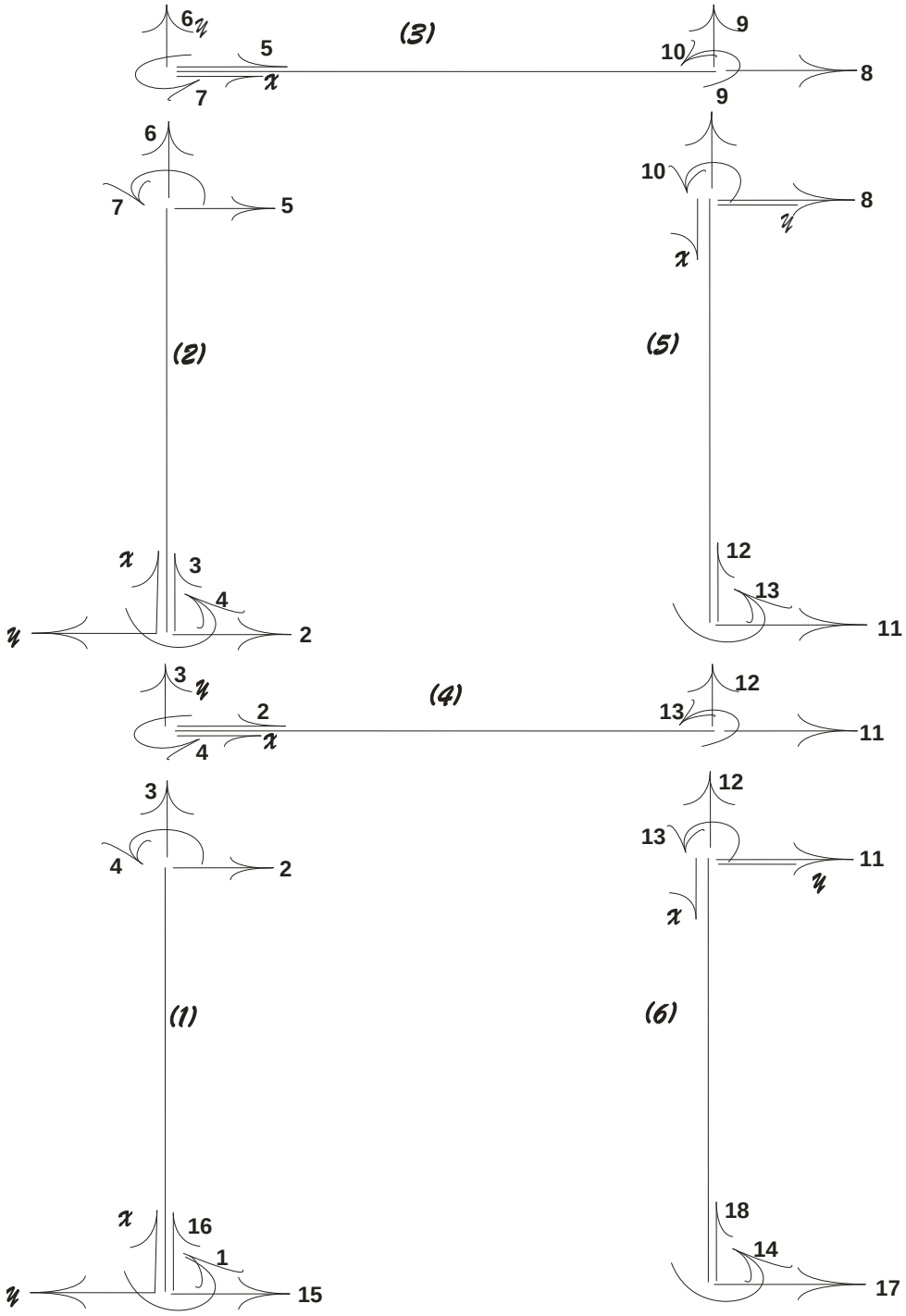


Figure 3.2: Member Stiffness Coefficients

Print Equation (A15)

Print Equation (A16)

Print Equation (A17)

Print Equation (A18)

Print Equation (A19)

To obtain the displacements$[D]$, $[K]$has been expanded to include degrees of freedom that are actually restrained. Equation (3.1) may be partitioned as shown in Equation (3.20).

$$\begin{bmatrix} K_{ff} & K_{fr} \\ K_{rf} & K_{rr} \end{bmatrix} \begin{bmatrix} D \\ Dr \end{bmatrix} = \begin{bmatrix} Q \\ Qr \end{bmatrix} \qquad \text{(A20)}$$

From equation (4.20), we have

$$[K_{ff}][D] + [K_{fr}][D_r] = [Q] \qquad \text{(A21)}$$

Equation (4.21) is solved for displacement,

$$[D] = [K_{ff}]^{-1}([Q] - [K_{fr}][D_r]) \qquad \text{(A22)}$$

To obtain nodal loads as indicated in Figures 2.6,

$$Q_1 = 0 \qquad \text{(A23)}$$

$$Q_2 = 0 \qquad \text{(A24)}$$

$$Q_3 = -\text{w2} * \text{L1}/2 = -100.1241 \qquad \text{(A25)}$$

$Q_4 = -\text{w2} * \text{L1\^2}/12$ = -90.1117 (A26)

Q_5=0 (A27)

$Q_6 = -\text{w1} * \text{L1}/2$ = 48.3570 (A28)

$Q_7 = -\text{w1} * \text{L1\^2}/12$= -43.5213 (A29)

Q_8=0 (A30)

$Q_9 = \text{w1} * (\text{L1} + \text{L2})/2$= 64.4760 (A31)

$Q_{10} = \text{w1} * (\text{L1\^2} - \text{L2\^2})/12$ = 38.6856 (A32)

Q_{11}=0 (A33)

$Q_{12} = \text{w2} * (\text{L1} + \text{L2})/2$= 133.4988 (A34)

$Q_{13} = \text{w2} * (\text{L1\^2} - \text{L2\^2})/12$ =80.0993 (A35)

Q_{14}=0 (A36)

From Figure A3, the vector of forces at restrained degrees of freedom, Q15, Q16, Q17 and Q18 are unknown.

Let A_{RC} be the vector of forces at restrained degrees of freedom,

$$A_{RC} = \begin{bmatrix} Q_{15} \\ Q_{16} \\ Q_{17} \\ Q_{18} \end{bmatrix} \quad \text{(A37)}$$

If A_{FC}, is the vector of forces at unrestrained degrees of freedom, then from the results from the MATLAB from are as follows:

$$\therefore A_{FC} = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \\ Q_9 \\ Q_{10} \\ Q_{11} \\ Q_{12} \\ Q_{13} \\ Q_{14} \end{bmatrix} = \begin{bmatrix} 0.000 \\ 0.000 \\ -102.1734 \\ -91.9561 \\ 0.000 \\ -48.3570 \\ -43.5213 \\ 0.000 \\ 64.4760 \\ 38.6856 \\ 0.000 \\ 136.2312 \\ 81.7387 \\ 0.000 \end{bmatrix} \quad \text{(A38)}$$

The vector of displacement at unrestrained degrees of freedom D_F is:

$$D_F = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \\ D_8 \\ D_9 \\ D_{10} \\ D_{11} \\ D_{12} \\ D_{13} \\ D_{14} \end{bmatrix} \quad \text{(A39)}$$

And the vector of displacement at restrained degrees of freedom is D_R. This is usually zero.

$$\therefore D_R = \begin{bmatrix} D_{15} \\ D_{16} \\ D_{17} \\ D_{18} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{(A40)}$$

From equations 33 - 34

$A_{FC} = S_{FF}D_F + S_{FR}D_R$ (A41) $A_{RC} = S_{RF}D_F + S_{RR}D_R$

(A42)

and

$D_F = S_{FF}^{-1}(A_{FC} - S_{FR}D_R)$ (A43)

The term D_F is obtained from the Mat lab program (Appendix B) as follows:

$$D_F = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \\ D_8 \\ D_9 \\ D_{10} \\ D_{11} \\ D_{12} \\ D_{13} \\ D_{14} \end{bmatrix} = \begin{bmatrix} 0.0011 \\ -0.0001 \\ -0.0003 \\ -0.0021 \\ 0.0002 \\ -0.0004 \\ -0.0031 \\ 0.0001 \\ 0.0005 \\ 0.0028 \\ 0.0000 \\ 0.0004 \\ 0.0021 \\ -0.0011 \end{bmatrix} \quad \text{(A44)}$$

Therefore, $A_{RC} = S_{RF}D_F + S_{RR}D_R$ (A45)

Again, the termA_{RC} is obtained from the Mat lab program (Appendix C),

$$A_{RC} = \begin{bmatrix} Q_{15} \\ Q_{16} \\ Q_{17} \\ Q_{18} \end{bmatrix} = \begin{bmatrix} 4.2903 \\ 147.1974 \\ -4.2093 \\ -197.3742 \end{bmatrix} \quad \text{(A46)}$$

$$D = \begin{bmatrix} D_F \\ D_R \end{bmatrix} = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \\ D_8 \\ D_9 \\ D_{10} \\ D_{11} \\ D_{12} \\ D_{13} \\ D_{14} \\ D_{15} \\ D_{16} \\ D_{17} \\ D_{18} \end{bmatrix} = \begin{bmatrix} 0.0011 \\ -0.0001 \\ -0.0003 \\ -0.0021 \\ 0.0002 \\ -0.0004 \\ -0.0031 \\ 0.0001 \\ 0.0005 \\ 0.0028 \\ 0.0000 \\ 0.0004 \\ 0.0021 \\ -0.0011 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{(A47)}$$

$$D_I = \begin{bmatrix} D_{15} \\ D_{16} \\ D_1 \\ D_2 \\ D_3 \\ D_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0.0011 \\ -0.0001 \\ -0.0003 \\ -0.0021 \end{bmatrix} \quad \text{(A48)}$$

$$D_{II} = \begin{bmatrix} D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \end{bmatrix} = \begin{bmatrix} -0.0001 \\ -0.0003 \\ -0.0021 \\ 0.0002 \\ -0.0004 \\ -0.0031 \end{bmatrix} \quad \text{(A49)}$$

$$D_{III} = \begin{bmatrix} D_5 \\ D_6 \\ D_7 \\ D_8 \\ D_9 \\ D_{10} \end{bmatrix} = \begin{bmatrix} 0.0002 \\ -0.0004 \\ -0.0031 \\ 0.0001 \\ 0.0005 \\ 0.0028 \end{bmatrix} \quad \text{(A50)}$$

$$D_{IV} = \begin{bmatrix} D_2 \\ D_3 \\ D_4 \\ D_{11} \\ D_{12} \\ D13 \end{bmatrix} = \begin{bmatrix} -0.0001 \\ -0.0003 \\ -0.0021 \\ 0.0000 \\ 0.0004 \\ 0.0021 \end{bmatrix} \quad \text{(A51)}$$

$$D_V = \begin{bmatrix} D_8 \\ D_9 \\ D_{10} \\ D_{11} \\ D_{12} \\ D_{13} \end{bmatrix} = \begin{bmatrix} 0.0001 \\ 0.0005 \\ 0.0028 \\ 0.0000 \\ 0.0004 \\ 0.0021 \end{bmatrix} \quad \text{(A52)}$$

$$D_{VI} = \begin{bmatrix} D_{11} \\ D_{12} \\ D_{13} \\ D_{17} \\ D_{18} \\ D_{14} \end{bmatrix} = \begin{bmatrix} 0.0000 \\ 0.0004 \\ 0.0021 \\ 0 \\ 0 \\ -0.0011 \end{bmatrix} \quad \text{(A53)}$$

$$d = \begin{bmatrix} 0.000 & 0.00 & -0.011 & -0.0001 & -0.0003 & -0.021 \\ -0.0001 & -0.0003 & -0.0021 & 0.0002 & -0.0004 & -0.0031 \\ 0.0002 & -0.0004 & -0.0031 & 0.0001 & 0.0005 & 0.0028 \\ -0.0001 & -0.0003 & -0.021 & 0.0000 & 0.0004 & 0.0021 \\ 0.0001 & 0.0005 & 0.0028 & 0.0000 & 0.004 & 0.0021 \\ 0.0000 & 0.0004 & 0.0021 & 0.0000 & 0.0000 & -0.0011 \end{bmatrix} \quad \text{(A54)}$$

Appendix B: Frame Analysis Program

```
%%  THIS PROGRAM ANALYZES THE FRAME
clc
clear
tic;
% Data representation = [Column1, Column2, Beam3, Beam4, Column5, Column6]
EA = [1569375, 1569375, 1569375, 2442800, 1569375, 1569375] % EA
EI = [6634,6634,6634,24921,6634,6634] % EI
y = [90,90, 0, 0,270,270]            % degree of freedom
L = [3.15, 3.15, 9.4, 9.4, 3.15, 3.15]   % Hight of column = 3.15, length of beam = 5.4
w1 = 17.91;          % KN/m
w2 = 58.683          % KN/m
L1 = 9.4;            % m
L2 = 1.8;            % m
%     a construction
for i = 1:6
 a = zeros(6,6);          % initializing
a(1,1) = EA(1,i)/ L(1,i);
a(1,4) = - a(1,1);
a(2,2) = 12*EI(1,i)/L(1,i)^3;
a(2,3) = 6*EI(1,i)/L(1,i)^2;
a(2,5) = - a(2,2);
a(2,6) = a(2,3);
```

```
a(3,2) = a(2,3);

a(3,3) = 4*EI(1,i)/L(1,i);

a(3,5) = - a(3,2);

a(3,6) = 2*EI(1,i)/L(1,i);

a(4,1) = - a(1,1);

a(4,4) = a(1,1);

a(5,2) = - a(2,2);

a(5,3) = a(3,5);

a(5,5) = a(2,2);

a(5,6) = - a(2,3);

a(6,2) = a(2,6);

a(6,3) = a(3,6);

a(6,5) = - a(2,2);

a(6,6) = a(3,3);

 a

 R = zeros(6,6);

R(1,1) = cosd(y(1,i));

R(1,2) = sind(y(1,i));

R(2,1) = - sind(y(1,i));

R(2,2) = cosd(y(1,i));

R(3,3) = 1;

R(4,4) = cosd(y(1,i));

R(4,5) = sind(y(1,i));
```

```
R(5,4) = - sind(y(1,i));

R(5,5) = cosd(y(1,i));

R(6,6) = 1;

 R

 S = R' * a * R ;

Sm (:,:,i) = S ;

end

S1 = Sm(:,:,1)

S2 = Sm(:,:,2)

S3 = Sm(:,:,3)

S4 = Sm(:,:,4)

S5 = Sm(:,:,5)

S6 = Sm(:,:,6)

 %-----------------

k1 = zeros(18,18);

k2 = zeros(18,18);

k3 = zeros(18,18);

k4 = zeros(18,18);

k5 = zeros(18,18);

k6 = zeros(18,18);

 k1(1:4,1:4) = S1(3:6,3:6);     % COMPONENT OF S1 IN K

k1(1:4,15:16) = S1(3:6,1:2);     % COMPONENT OF S1 IN K

k1(15:16,1:4) = S1(1:2,3:6);     % COMPONENT OF S1 IN K
```

```
k1(15:16,15:16) = S1(1:2,1:2);     % COMPONENT OF S1 IN K

k2(2:7,2:7) = S2;              % COMPONENT OF S2 IN K

k3(5:10,5:10) = S3;           % COMPONENT OF S3 IN K

k4(2:4,2:4) = S4(1:3,1:3);     % % COMPONENT OF S4 IN K

k4(2:4,11:13) = S4(1:3,4:6);     % % COMPONENT OF S4 IN K

k4(11:13,2:4) = S4(4:6,1:3); % % COMPONENT OF S4 IN K

k4(11:13,11:13) = S4(4:6,4:6); % % COMPONENT OF S4 IN K

k5(8:13,8:13) = S5(1:6,1:6);  % COMPONENT OF S5 IN K

k6(11:13,11:13) = S6(1:3,1:3); % COMPONENT OF S6 IN K

k6(14,11:13) = S6(6,1:3);;  % COMPONENT OF S6 IN K

k6(17:18,11:13) = S6(4:5,1:3);  % COMPONENT OF S6 IN K

k6(17:18,14) = S6(4:5,6);    % COMPONENT OF S6 IN K

k6(11:13,14) = S6(1:3,6);    % COMPONENT OF S6 IN K

k6(14,14) = S6(6,6);     % COMPONENT OF S6 IN K

k6(11:13,17:18) = S6(1:3,4:5);  % COMPONENT OF S6 IN K

k6(14,17:18) = S6(6,4:5);   % COMPONENT OF S6 IN K

k6(17:18,17:18) = S6(4:5,4:5);  % COMPONENT OF S6 IN K

 k = k1 + k2 + k3 + k4 + k5 + k6;   % SUM OF ALL COMPONENTS

%-------------------------------------------------------

k

SFF = k(1:14,1:14)

SFR = k(1:14,15:18)

SRF = k(15:18,1:14)
```

```
SRR = k(15:18,15:18)
 q1 = 0;
q2 = 0;
q3 =-w2*L1/2;
q4 =-w2*L1^2/12;
q5 = 0;
q6 =-w1*L1/2;
q7 =-w1*L1^2/12;
q8 = 0;
q9 =w1*(L1+L2)/2;
q10 =w1*(L1^2-L2^2)/12;
q11 = 0;
q12 =w2*(L1+L2)/2;
q13 =w2*(L1^2-L2^2)/12;
q14 = 0;
 %-------------------------------------------------
 AFC = [q1,q2,q3,q4,q5,q6,q7,q8,q9,q10,q11,q12,q13,q14]'
DR = [0,0,0,0]'
DF = inv(SFF)*(AFC-SFR*DR)
ARC =(SRF)*DF+SRR*DR
D = [DF(:,:);DR(:,:)]
%-----------------------------------------------
 d1 = [D(15),D(16),D(1),D(2),D(3),D(4)]'
```

```
d2 = [D(2),D(3),D(4),D(5),D(6),D(7)]'

d3 = [D(5),D(6),D(7),D(8),D(9),D(10)]'

d4 = [D(2),D(3),D(4),D(11),D(12),D(13)]'

d5 = [D(8),D(9),D(10),D(11),D(12),D(13)]'

d6 = [D(11),D(12),D(13),D(17),D(18),D(14)]'

 d(1,:) = d1

d(2,:) = d2

d(3,:) = d3

d(4,:) = d4

d(5,:) = d5

d(6,:) = d6

 %-------------------------------------------------------

 for i = 1:6

    s = sind (y(1,i));

    c = cosd (y(1,i));

    dx = d(i,:);

    MBiA(1,i)=EI(1,i)*(6/L(1,i)^2    *(-s*dx(1,1)+c*dx(1,2))+4/L(1,i)*dx(1,3)-6/L(1,i)^2*(-
s*dx(1,4)+c*dx(1,5))+2/L(1,i)*dx(1,6));% Analyzed moment at beginning of members 1-6

    MEiA(1,i)=-EI(1,i)*(6/L(1,i)^2   *(-s*dx(1,1)+c*dx(1,2))+2/L(1,i)*dx(1,3)-6/L(1,i)^2*(-
s*dx(1,4)+c*dx(1,5))+4/L(1,i)*dx(1,6)); % Analyzed moment at end of members 1-6

end


for i = 1:6
```

```
    s = sind (y(1,i));
    c = cosd (y(1,i));
    dx = d(i,:);
    PA(1,i) = EA(1,i)/L(1,i)*(-c*dx(1,1) - s*dx(1,2)+ c*dx(1,4)+ s*dx(1,5)); % Axial forces due to frame analysis in members 1-6
    end
PA
MBiA % Moment at left end of members 1-6
MEiA % Mome nt at right end of members 1-6
MBiF=[0,0,w1*L1^2/12,w2*L1^2/12,0,0] % Fixed end moment at left end of members 1-6 due to applied force
MEiF=[0,0,-w1/12*L1^2,-w2/12*L1^2,0,0] % Fixed end moment at right end of members 1-6 due to applied force
PF=[-w2*L1/2,-w1*L1/2,0,0, w1*L1/2,w2*L1/2]% Member axial forces
P=PA+PF
MB=MBiA +MBiF % Total moment at left end of members 1-6
ME=MEiA +MEiF % Total moment at right end of members 1-6
MM=
[0,0,((w1*L(1,3)^2)/12+0.5*(ME(1,3)+MB(1,3))),((w2*L(1,4)^2)/12+0.5*(ME(1,4)+MB(1,4))),0,0] % midspan moments
SF1=(MB(1,1)-ME(1,1))/L(1,1)
SF2=(MB(1,2)-ME(1,2))/L(1,2)
SF3=w1*L(1,3)/2-((MB(1,3)-ME(1,3))/L(1,3))
```

```
SF4=w2*L(1,4)/2-((MB(1,4)-ME(1,4))/L(1,4))

SF5=(MB(1,5)-ME(1,5))/L(1,5)

SF6=(MB(1,6)-ME(1,6))/L(1,6)

MM= [0,0,(SF3)^2/(2*w1)+MB(1,3),(SF4)^2/(2*w2)+MB(1,4),0,0] % midspan moments

BEAMI=0.225*0.45^3/12

KBJC=0.5*(BEAMI/5.4+BEAMI/1.8)

COLUMNI=0.225^4/12

KCOL=COLUMNI/3.15

KSUM=KBJC+KCOL

DFC=KCOL/KSUM

MCOL6=DFC*q13 % Moment in member 6

toc
```

Appendix C: Design Information

Intended Use:	Office
Design Code:	EC 2 (2004)
Design Stresses:	Concrete ,f_{ck}=25KN/ $m^2$and Steel, f_{yk}=500KN/m^2
Fire Resistance:	One hour for all elements
Cross- sections	Roof level beams -225mm x 225mm First floor beams -225mm x 450mm Columns -225mm x 225mm
Exposure: Condition:	Mild for all elements Cover: beams and colums-50mm
General loading Conditions	Floor Live load -3.0 KN/m^2 Roof (load Dead)-1.5K N/m^2 Roof load(Live load)=.75KN/m^2
Design Data	K_u=0.167 $K = M/f_{ck}bd^2$, $K \leq K_u$ $A_s = \frac{M}{0.87f_{yk}l_a d}$;; otherwise $l_a = 0.5 + \sqrt{(0.25 - k/0.25)}$ $A'_s = \frac{(K - K_u)f_{ck}bd^2}{0.87f_{yk}(d - d')}$ $A_s = \frac{K_u f_{ck} bd^2}{0.87f_{yk}Z_u} + A'_s$; $Z_u = d\left(0.5 + \sqrt{(0.25 - k/0.25)}\right)$

C1: Deterministic Design of Critical Beam

Member Ref.	Calculations	Output
Member IV	**For Span of 5.4m and Live load of 3KN/m^2** Left support moment=56.047KNm K=$\frac{M}{bd^2f_{ck}}=\frac{56.047x10^6}{225x300^2x25}$=0.1107<0.167 Compression steel is not required z=300$\left[0.5+\sqrt{0.25-0.1107/1.134}\right]$=269mm $A_s=\frac{56.047\ x\ 10^6}{0.87\ x\ 500x269}$=482mm^2 Midspan Moment =61.701KNm K=$\frac{M}{bd^2f_{ck}}=\frac{61.701x10^6}{225x300^2x25}$=0.1219<0.167 Compression steel is not required z=300$\left[0.5+\sqrt{0.25-0.1219/1.134}\right]$=263mm $A_s=\frac{69.390\ x\ 10^6}{0.87\ x\ 500x259.73}$=539mm^2 Right support moment=112.687KNm K=$\frac{M}{bd^2f_{ck}}=\frac{112.6870x10^6}{225x300^2x25}$=0.2226>0.167	b=225mm h=450mm d=400mm A_s=482mm^2 A_s=539mm^2 A'_s =539mm^2 A_s=539mm^2

(Beam)	Compression steel is required $A'_s = \frac{(K-k_{bal})bd^2f_{ck}}{0.87f_{yk}(d-d')} = \frac{(0.2226-0.167)x30x225x300^2}{0.87x500x(300-50)}$=259mm^2 $A_s = \frac{K_{bal}f_{ck}bd^2}{0.87f_{yk}z} + A'_s$ $A_s = \frac{0.167x\ 25\ x\ 225\ x\ 300^2}{0.87\ x\ 500x(300-50)}$+772 = 1030mm^2 **Check for Longitudinal Reinforcement:** $\frac{A_{Sb,min}}{b_t d} \geq 0.26\frac{f_{ctm}}{f_{yk}}$ $A_{Sb,min} \geq 0.26\frac{b_t d f_{ctm}}{f_{yk}}$ $A_{Sb,min} \geq \frac{0.26x2.9x225x300}{500} = 102\text{mm}^2$ Shear Force= 89.635KN Shear reinforcement is computed from $$\frac{A_{sw}}{S} = \frac{V}{0.78df_{yk}\,cot\theta}$$ For most cases of beams with predominantly uniformly distributed loads, The angle θ is taken as 22^o $\therefore$ cotθ = 2.5	A'_s =259mm^2 A_s=1030mm^2 A_s=102mm^2 S=300mm

	Assume link spacing, S = 300mm Shear reinforcement = $\frac{89.635\ x\ 10^3 x300}{0.78\ x\ \ 300\ x\ 500\ x\ 2.5}$ = 82mm^2/m	V_S=89.635KN A_{sv} =82mm^2/m

C2: Deterministic Design of Critical Column

Member Ref.	**Calculations**	**Output**
MemberVI Column	The loads are resisted by a frame action rigidly connected columns and beams. Therefore the column is considered as braced. b=h=225mm Take d'= 50mm as for beam, then D=h- d'=225-50=175mm $I_{column} = \frac{bh^3}{12} = \frac{225^4}{12} = 213.574\ x\ 10^6\ mm^4$ $I_{beam} = \frac{bh^3}{12} = \frac{225x350^3}{12} = 803.906\ x\ 10^6\ mm^4$	b=225mm h=225mm d'= 50mm d=175mm

	A_{column}= 225 x 225 = 50625mm^2 A_{beam}= 225 x 350 = 50625mm^2 Stiffnesses $(I/L)_{column} = \frac{213.574X10^6}{3150}$ =0.068 $x10^6$mm^3 $(I/L)_{beamAB} = \frac{803.906X10^6}{5400}$ =0.149 $x10^6$mm^3 $(I/L)_{beamBC} = \frac{803.906X10^6}{1800}$ =0.447 $x10^6$mm^3 For braced columns $$l_0 = 0.5L\sqrt{\left(1+\frac{k_1}{0.45+k_1}\right)\left(1+\frac{k_2}{0.45+k_2}\right)}$$ Minimum value of 0.1 is recommended for k_1 and k_2 L_o = effective height of the column At the top of column $$K_1 = \frac{Column\ Stiffness}{\sum beam\ stifness} = \frac{(I/L)Column}{\sum 2(I/L)beam}$$ =0.068/(0.149+0.4477)=0.114 At the bottom of column K_2 =0.1(minimum)	L_o =1.89m

	∴ L_o = 1.890 Radius of gyration, $i = \sqrt{\frac{I_{column}}{A_{column}}}$ =65.03mm Slenderness ratio, $\boldsymbol{\lambda}$ =$\frac{Lo}{i}$=1890/65.03=29.06 Minimum limiting value of $\boldsymbol{\lambda}$; $\boldsymbol{\lambda}_{min}=26.2/\sqrt{N_{Ed}/A_c f_{cd}}$ $=26.2/\sqrt{295.188/225^2 x\ 25/1.5}$ =44.295 $\boldsymbol{\lambda}>\boldsymbol{\lambda}_{min}$ The slenderness ratio is greater than the minimum required. Hence the column is short. Second order moment effects are not taken into account in the design of the column. Axial Force = 295.188KN Moment = 19.422KNm Shear Force= 6.166KN $\frac{N_{Ed}}{bhf_{ck}}=\frac{295.177x10^3}{225^2x30}$ =0.23	Axial Force = 295.177KN Moment = 19.422KNm Shear Force=6.166KN Longitudinal Reinforcment

	$\frac{M_{Ed}}{bh^2 f_{ck}}$=$\frac{80.0828x10^6}{225^3 x30}$=0.07 From column design chart, $\frac{A_s f_{yk}}{bh^2 f_{ck}}$=0.02 Therefore, $A_s = \frac{0.02x225^2x25}{500}$ = 51mm^2 Provide $A_{sc,min}$, 4y12=452 mm^2 Shear reinforcement is obtained from the expression: $$\frac{A_{sw}}{S} = \frac{V}{0.78df_{yk}\,cot\theta}$$ The angle θ is taken as 22^o , ∴ cotθ = 2.5 Maximum spacing ≤ 20 x size of compression bar(20x25= 500mm) or 400mm ∴ maximum spacing < 400mm Provide S = 300mm $$A_{sw} = \frac{300\ x\ 6.166\ x\ 10^3}{0.78\ x\ 175\ x\ 500\ x\ 2.5} = 06mm^2$$ Provide $A_{sw,min}$,Y8 @ 300mm (Area = 168mm^2) C**heck for longitudinal Reinforcement**	=452mm^2 Shear Reinforcment =168mm^2

	$A_{s,min} \geq \frac{0.1N_{Ed}}{0.87f_{yk}}$ $A_{sc,min} = \frac{0.10\ x\ 295.188x\ 10^3}{0.87\ x\ 500} = 67.85mm^2$ $< 0.002A_c$ $\therefore A_{s,min} = 67.85mm^2 < 101mm^2$	

AppendixD: MATLAB Optimization Program

% Program to optimize cost function

function f=bfunL1(x)

f=24000*1.0E-9*(270000*x(1)+5400*x(1)*x(2)-4725*x(3)-2875*x(4)-1462.5*x(5)-

1462.5*x(6)-6384*x(1)-6384*x(2)+549024)+42000*77*1.0E-

```
9*(4725*x(3)+2875*x(4)+1462.5*x(5)+1462.5*x(6)+6384*x(1)+6384*x(2)-
549023)+950*1.0E-6*(5175*x(1)-x(1)*x(2)+10800*x(2)+540000);
function[c,ceq]=constbfunsd(x)
% nonlinear inequality constraints
c=[x(2)-2*x(1);127732-4.125*x(1)*x(2);77951000-435*x(5)*(x(2)-15.344*x(5)/x(1))];
% nonlinear equality constraints
ceq=[x(3)-0.011*x(1)*x(2)-((85922000-4.175*x(1)*x(2)^2)/(435*x(2)-21750));85922000-
4.175*x(1)*x(2)^2-435*x(4)*(x(2)-50);x(6)-0.011*x(1)*x(2)-((165062000-
4.175*x(1)*x(2)^2)/(435*x(2)-21750))];
x0=[0 0 0 0 0 0];  % make a starting guess at the solution
lb=[200 275 91 91 91 91];  % set lower bounds
ub=[225 300 3150 3150 3150 3150],  % Set upper bounds
options=optimset('Display','Iter','Algorithm','Active-set')
[x,fval]=fmincon(@bfunL,x0,[],[],[],[],lb,ub,@constbfunsd,options
```

Appendix E: Reliability Analysis Programs

E1: PROGRAM FRAMEB

```
C     FLEXURAL FAILURE SAFETY ANALYSIS FOR RC SS REINF BEAM
C     FOR DETERMINISTIC AND OPTIMUM DESIGNS AT SPSN
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      EXTERNAL GKNICK
      DIMENSION X(8),EX(8),SX(8),VP(10,8),COV(8,8),ZES(3),UU(8),
     +          EIVEC(8,8),IV(2,8)
```

```
      CHARACTER*10 PRT
      COMMON/CKNICK/S,ALPHA
      DATA EX/25.0D0,500.0D0,200.0D0,400.0D0,284.0D0,10.8D0,33.139D6,
     +     91.010D6/,SX/3.75D0,75.0D0,10.0D0,20.0D0,14.2D0,3.24D0,
     +     7.953D6,21.842D6/,N/8/,NC/8/,NE/8/,IRHO/0/
      S=5.4D3
      ALPHA=0.9D0
C     RESULTS WRITTEN IN FILE NUMBER 7
      NAUS=7
      ICRT=0
      OPEN(7,FILE='FRAMEbf1.DAT',STATUS='OLD',ERR=10)
      GOTO 20
 10   OPEN(7,FILE='FRAMEbf1.DAT',STATUS='NEW')
C
C     PRESETTING VARIABLES VP,COV AND IV IS DONE USING YINIT
 20   CALL YINIT(N,IV,VP,IRHO,COV,NC)
      IV(1,1)=3
      IV(1,2)=3
      IV(1,6)=5
      IV(1,7)=3
      IV(1,8)=3
      DO 100 I=1,N
C
 100  X(I)=EX(I)
      V1=1.D0
      BETA=1.D0
C
      WRITE (NAUS,5000)
 5000 FORMAT (////,5X,70('*'),/,30X,'FORM5',/,5X,70('*'))
C
      CALL YKOPF(NAUS,N,IV,EX,SX,VP,IRHO)
C     PRINT ALSO TO SCREEN
      WRITE(ICRT,*)'START OF FORM5'
      WRITE(ICRT,*)'STOCHASTIC MODEL'
      CALL YKOPF(ICRT,N,IV,EX,SX,VP,IRHO)
C     THE MATRIX COV IS PRINTED USING 'YMAUS"
      PRT='COV'
      CALL YMAUS(NAUS,NC,N,COV,PRT)
C     THE MAIN SUBROUTINE IN FORM IS CALLED
      CALL FORM5(N,IV,EX,SX,VP,GKNICK,IRHO,COV,NC,EIVEC,NE,V1,
     +           NAUS,BETA,X,UU,ZES,IER)
C     THE COORDINATE OF THE BETA POINT ARE PRINTED WITH THE
C      TITLE VECTOR UU
      PRT=' UU'
      CALL YFAUS(NAUS,N,UU,PRT)
C     THE VECTOR ZES IS PRINTED WITH THE TITLE VECTOR ZES
```

```
      PRT=' ZES '
      CALL YFAUS(NAUS,3,ZES,PRT)
      WRITE(ICRT,*)'END OF FORM5:IER=',IER
      WRITE(ICRT,*)'RESULTS SEE FILE FRAMEbf1.DAT'
      STOP
      END

      SUBROUTINE GKNICK (N,X,FX,IER)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION X(N)
      COMMON/CKNICK/S,ALPHA
C
      F1=X(1)*X(3)*X(4)**2
      A1=0.167D0*F1
      W2=X(6)*(1.35*ALPHA+1.5D0)
      F2=X(8)-X(7)
      F3=W2*S/2-F2/S
      F4=F3**2/(2.0*W2)-X(7)
      FK=F4/F1
      F5=0.25D0-FK/1.134
      A2=0.87*X(2)*X(4)*X(5)*(0.5D0+F5**0.5D0)
C     CHECK ERROR, CALCULATE FX
      IF(F1.GT.0.AND.F2.NE.0)THEN
      FX=A1-A2
      IER=0
      ELSE
      FX=1.0D+20
      END IF
      RETURN
      END
```

E2: PROGRAMFRAMEB

```
C     SHEAR SAFETY ANALYSIS FOR RC REINF BEAM
C     FOR BOTH DETERMINISTIC AND OPTIMUM DESIGNS
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      EXTERNAL GKNICK
      DIMENSION X(7),EX(7),SX(7),VP(10,7),COV(7,7),ZES(3),UU(7),
     +          EIVEC(7,7),IV(2,7)
      CHARACTER*10 PRT
      COMMON/CKNICK/S,ALPHA
```

```
      DATA EX/25.0D0,500.0D0,200.0D0,400.0D0,10.8D0,41.127D6,
     +     120.45D6/,SX/3.75D0,75.0D0,10.0D0,20.0D0,3.24D0,
     +     9.87D6,28.908D6/,N/7/,NC/7/,NE/7/,IRHO/0/
      S=5.4D3
      ALPHA=1.5D0
C     RESULTS WRITTEN IN FILE NUMBER 7
      NAUS=7
      ICRT=0
      OPEN(7,FILE='FRAMEbs.DAT',STATUS='OLD',ERR=10)
      GOTO 20
 10   OPEN(7,FILE='FRAMEbs.DAT',STATUS='NEW')
C
C     PRESETTING VARIABLES VP,COV AND IV IS DONE USING YINIT
 20   CALL YINIT(N,IV,VP,IRHO,COV,NC)
      IV(1,1)=3
      IV(1,2)=3
      IV(1,5)=7
      IV(1,6)=3
      IV(1,7)=3
      DO 100 I=1,N
C
 100  X(I)=EX(I)
      V1=1.D0
      BETA=1.D0
C
      WRITE (NAUS,5000)
 5000 FORMAT (////,5X,70('*'),/,30X,'FORM5',/,5X,70('*'))
C
      CALL YKOPF(NAUS,N,IV,EX,SX,VP,IRHO)
C     PRINT ALSO TO SCREEN
      WRITE(ICRT,*)'START OF FORM5'
      WRITE(ICRT,*)'STOCHASTIC MODEL'
      CALL YKOPF(ICRT,N,IV,EX,SX,VP,IRHO)
C     THE MATRIX COV IS PRINTED USING 'YMAUS"
      PRT='COV'
      CALL YMAUS(NAUS,NC,N,COV,PRT)
C     THE MAIN SUBROUTINE IN FORM IS CALLED
      CALL FORM5(N,IV,EX,SX,VP,GKNICK,IRHO,COV,NC,EIVEC,NE,V1,
     +           NAUS,BETA,X,UU,ZES,IER)
C     THE COORDINATE OF THE BETA POINT ARE PRINTED WITH THE
C       TITLE VECTOR UU
      PRT=' UU'
      CALL YFAUS(NAUS,N,UU,PRT)
C     THE VECTOR ZES IS PRINTED WITH THE TITLE VECTOR ZES
      PRT=' ZES '
      CALL YFAUS(NAUS,3,ZES,PRT)
```

```
      WRITE(ICRT,*)'END OF FORM5:IER=',IER
      WRITE(ICRT,*)'RESULTS SEE FILE FRAMEbs.DAT'
      STOP
      END

      SUBROUTINE GKNICK (N,X,FX,IER)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION X(N)
      COMMON/CKNICK/S,ALPHA
C
      F1=0.7-X(1)/200.
      A1=0.3*F1*X(1)*X(3)*X(4)
      W2=X(5)*(1.35*ALPHA+1.5)
      F2=X(7)-X(6)
      A2=W2*S/2.-F2/S
C     CHECK ERROR, CALCULATE FX
      IF(A1.NE.0.AND.A2.NE.0)THEN
      FX=A1-A2
      IER=0
      ELSE
      FX=1.0D+20
      END IF
      RETURN
      END
```

E3: PROGRAMFRAMEB

```
C
PROGRAM KNICK
C     DEFLECTION CONTROL FOR RC BEAMS RHO LESS THAN RHO-0
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      EXTERNAL GKNICK
      DIMENSION X(5),EX(5),SX(5),VP(10,5),COV(5,5),ZES(3),UU(5),
     +          EIVEC(5,5),IV(2,5)
      CHARACTER*10 PRT
      COMMON/CKNICK/F
      DATA EX/25.0D0,225.0D0,400.0D0,379.0D0,5400.0D0/,
     +     SX/3.75D0,11.25D0,20.0D0,18.95D0,324.0D0/,
     +     N/5/,NC/5/,NE/5/,IRHO/0/
      F=1.5D0
C     RESULTS WRITTEN IN FILE NUMBER 7
      NAUS=7
      ICRT=0
      OPEN(7,FILE='FRAMEDC1.DAT',STATUS='OLD',ERR=10)
      GOTO 20
 10   OPEN(7,FILE='FRAMEDC1.DAT',STATUS='NEW')
C
```

```
C     PRESETTING VARIABLES VP,COV AND IV IS DONE USING YINIT
 20   CALL YINIT(N,IV,VP,IRHO,COV,NC)
      IV(1,1)=3
      DO 100 I=1,N
C
 100  X(I)=EX(I)
      V1=1.D0
      BETA=1.D0
C
      WRITE (NAUS,5000)
 5000 FORMAT (////,5X,70('*'),/,30X,'FORM5',/,5X,70('*'))
C
      CALL YKOPF(NAUS,N,IV,EX,SX,VP,IRHO)
C     PRINT ALSO TO SCREEN
      WRITE(ICRT,*)'START OF FORM5'
      WRITE(ICRT,*)'STOCHASTIC MODEL'
      CALL YKOPF(ICRT,N,IV,EX,SX,VP,IRHO)
C     THE MATRIX COV IS PRINTED USING 'YMAUS"
      PRT='COV'
      CALL YMAUS(NAUS,NC,N,COV,PRT)
C     THE MAIN SUBROUTINE IN FORM IS CALLED
      CALL FORM5(N,IV,EX,SX,VP,GKNICK,IRHO,COV,NC,EIVEC,NE,V1,
     +           NAUS,BETA,X,UU,ZES,IER)
C     THE COORDINATE OF THE BETA POINT ARE PRINTED WITH THE
C       TITLE VECTOR UU
      PRT=' UU'
      CALL YFAUS(NAUS,N,UU,PRT)
C     THE VECTOR ZES IS PRINTED WITH THE TITLE VECTOR ZES
      PRT=' ZES '
      CALL YFAUS(NAUS,3,ZES,PRT)
      WRITE(ICRT,*)'END OF FORM5:IER=',IER
      WRITE(ICRT,*)'RESULTS SEE FILE FRAMEDC1.DAT'
      STOP
      END

      SUBROUTINE GKNICK (N,X,FX,IER)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION X(N)
      COMMON/CKNICK/F
C
      F1=X(1)**0.5D0
      F2=F1*X(2)*X(3)*1.0D-3
      F3=11+1.5D0*F1*(F2/X(4)+3.2D0*F1*(F2/X(4)-1.0D0))
      A1=F*F3
      A2=X(5)/X(3)
C     CHECK ERROR, CALCULATE FX
```

```
      IF(F1.GT.0.AND.F2.GT.0)THEN
      FX=A1-A2
      IER=0
      ELSE
      FX=1.0D+20
      END IF
      RETURN
      END
```

E4: PROGRAM FRAMEC

```
C     PROBABILITY SAFETY ANALYSIS FOR OPTIMUM DESIGN RC COLUMN
C     IN AXIAL FAILURE MODE(c6a)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      EXTERNAL GKNICK
      DIMENSION X(6),EX(6),SX(6),VP(10,6),COV(6,6),ZES(3),UU(6),
     +          EIVEC(6,6),IV(2,6)
      CHARACTER*10 PRT
      COMMON/CKNICK/S1,S2,ALPHA
      DATA EX/25.0D0,500.0D0,200.0D0,150.0D0,452.0D0,10.8D0/,
     +     SX/3.75D0,75.0D0,10.0D0,7.5D0,22.6D0,3.24D0/,
     +     N/6/,NC/6/,NE/6/,IRHO/0/
      S1=5.4D+3
      S2=1.8D+3
      ALPHA=1.5D0
C     RESULTS WRITTEN IN FILE NUMBER 7
      NAUS=7
      ICRT=0
      OPEN(7,FILE='framec6a.DAT',STATUS='OLD',ERR=10)
      GOTO 20
 10   OPEN(7,FILE='framec6a.DAT',STATUS='NEW')
C
C     PRESETTING VARIABLES VP,COV AND IV IS DONE USING YINIT
 20   CALL YINIT(N,IV,VP,IRHO,COV,NC)
      IV(1,1)=3
      IV(1,2)=3
      IV(1,6)=7
      DO 100 I=1,N
C
 100  X(I)=EX(I)
      V1=1.D0
      BETA=1.D0
C
      WRITE (NAUS,5000)
 5000 FORMAT (////,5X,70('*'),/,30X,'FORM5',/,5X,70('*'))
C
      CALL YKOPF(NAUS,N,IV,EX,SX,VP,IRHO)
```

```
C     PRINT ALSO TO SCREEN
      WRITE(ICRT,*)'START OF FORM5'
      WRITE(ICRT,*)'STOCHASTIC MODEL'
      CALL YKOPF(ICRT,N,IV,EX,SX,VP,IRHO)
C     THE MATRIX COV IS PRINTED USING 'YMAUS"
      PRT='COV'
      CALL YMAUS(NAUS,NC,N,COV,PRT)
C     THE MAIN SUBROUTINE IN FORM IS CALLED
      CALL FORM5(N,IV,EX,SX,VP,GKNICK,IRHO,COV,NC,EIVEC,NE,V1,
     +           NAUS,BETA,X,UU,ZES,IER)
C     THE COORDINATE OF THE BETA POINT ARE PRINTED WITH THE
C       TITLE VECTOR UU
      PRT=' UU'
      CALL YFAUS(NAUS,N,UU,PRT)
C     THE VECTOR ZES IS PRINTED WITH THE TITLE VECTOR ZES
      PRT=' ZES '
      CALL YFAUS(NAUS,3,ZES,PRT)
      WRITE(ICRT,*)'END OF FORM5:IER=',IER
      WRITE(ICRT,*)'RESULTS SEE FILE framec6a.DAT'
      STOP
      END

      SUBROUTINE GKNICK (N,X,FX,IER)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION X(N)
      COMMON/CKNICK/S1,S2,ALPHA
C
      F1=0.567D0*X(1)*X(3)*(X(4)+50)
      F2=0.87D0*X(2)*X(5)
      A1=F1+F2
      W1=13.049D0
      F3=0.5D0*W1*(S1+S2)
      W2=X(6)*(1.35D0*ALPHA+1.5D0)
      F4=0.5D0*W2*(S1+S2)
      A2=F3+F4
C     GX=A1-A2
C     CHECK ERROR, CALCULATE FX
      IF(F1.GT.0.AND.F2.NE.0)THEN
      FX=A1-A2
      IER=0
      ELSE
      FX=1.0D+20
      END IF
      RETURN
      END
```

APPENDIX E5: SYSTEM RELIABILITY PROGRAM

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%THIS IS A MATLAB PROGRAM FOR SYSTEM RELIABILITY
%           ANALYSIS OF A REINFORCED CONCRETE PLANE FRAME
%  CODE DEVELOPED BY: KETKUKAH, T.S.
%  PhD/ENG/00564/2006-07
%  MAJOR SUPERVISOR
%           DR. IDRIS ABUBAKAR
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
disp('   ')
pf1 = input('   input the probability of failure for failure memeber 1');
disp('   ')
disp('   ')
pf2 = input('   input the probability of failure for failure memeber 2');
disp('   ')
disp('   ')
pf3 = input('   input the probability of failure for failure memeber 3');
disp('   ')
disp('   ')
pf4 = input('   input the probability of failure for failure memeber 4');
disp('   ')
disp('   ')
pf5 = input('   input the probability of failure for failure memeber 5');
disp('   ')
disp('   ')
pf6 = input('   input the probability of failure for failure memeber 6');
disp('   ')
disp('   ')
pf7 = input('   input the probability of failure for failure memeber 7');
disp('   ')
disp('   ')
pf8 = input('   input the probability of failure for failure memeber 8');
disp('   ')
disp('   ')
pf9 = input('   input the probability of failure for failure memeber 9');
disp('   ')
disp('   ')
pf10 = input('   input the probability of failure for failure memeber 10');
disp('   ')
disp('   ')
pf11 = input('   input the probability of failure for failure memeber 11');
disp('   ')
disp('   ')
```

```
pf12 = input('   input the probability of failure for failure memeber 12');
disp('   ')
disp('   ')
pf13 = input('   input the probability of failure for failure memeber 13');
disp('   ')
disp('   ')
pf14 = input('   input the probability of failure for failure memeber 14');
disp('   ')
%
pf_set = [pf1,pf2,pf3,pf4,pf5,pf6,pf7,pf8,pf9,pf10,pf11,pf12,pf13,pf14];
%
p_upper_bound = min(pf_set);
%
%
p_lower_bound = 1.0 - ((1.0-pf1) * (1.0-pf2) * (1.0-pf3) * (1.0-pf4)...
       * (1.0-pf5)* (1.0-pf6) * (1.0-pf7)* (1.0-pf8) * (1.0-pf9)...
       * (1.0-pf10)* (1.0-pf11) * (1.0-pf12)* (1.0-pf13) * (1.0-pf14));
%
%
Psys = (p_upper_bound + p_lower_bound)/2.0;
disp('   ')
beta_sys = system_safety_index(Psys);
disp('   ')
disp('  THIS IS THE SYSTEM PROBABILITY OF FAILURE')
disp('   ')
disp(Psys)
disp('    ')
disp('   THIS IS THE STSTEM RELIABILITY INDEX')
disp('   ')
disp(beta_sys)
```

APPENDIX E6: SYSTEM RELIABILITY OUTPUT

input the probability of failure for failure memeber 10.206e-8

input the probability of failure for failure memeber 20.206e-8

input the probability of failure for failure memeber 30.525e-6

input the probability of failure for failure memeber 40.305e-3

input the probability of failure for failure memeber 50.206e-8

input the probability of failure for failure memeber 60.206e-8

input the probability of failure for failure memeber 70.509e-18

input the probability of failure for failure memeber 80.289e-3

input the probability of failure for failure memeber 90.206e-8

input the probability of failure for failure memeber 100.206e-8

input the probability of failure for failure memeber 110.305e-3

input the probability of failure for failure memeber 120.525e-6

input the probability of failure for failure memeber 130.206e-8

input the probability of failure for failure memeber 140.206e-8

THIS IS THE SYSTEM PROBABILITY OF FAILURE 4.4990e-004

THIS IS THE STSTEM RELIABILITY INDEX 3.3300

F: DETAIL OF RESULTS

TableF1(4.4): Values of Optimum Cost of beam

Span of beam (m)	Live Loads (KN/mm^2)				
	3.0	4.0	5.0	6.0	7.0
5.4	₦38,666	₦41,139	₦43,716	₦44,934	₦47,702

6.4	₦45,945	₦49,805	₦54,348	₦58,403	₦62,354
7.4	₦56,997	₦62,769	₦68,541	₦74,313	₦80,085
8.4	₦70,275	₦77,903	₦85,530	₦93,157	₦100,780
9.4	₦85,445	₦95,201	₦104,960	₦114,710	₦124,620

Table F2: Curve fitting coefficients of the cost of beam

C1	C2	Live Loads (kN/m^2)
11789	-27772	3.0
13622	-35441	4.0
15367	-42297	5.0
17431	-51883	6.0
19226	-59166	7.0

Table F3: Values of Span Effective Depth Ratio

Span of beam (m)	Live Loads (KN/mm^2)				
	3.0	4.0	5.0	6.0	7.0
5.4	18	18	18	18	18
6.4	21.33	21.33	21.33	21.33	21.33
7.4	24.67	24.67	24.67	24.67	24.67
8.4	28	28	28	28	28
9.4	33.33	33.33	33.33	33.33	33.33

Table F4: Values of reinforcement ratios

Span of eam (m)	Live Loads (KN/m^2)				
	3	4	5	6	7
5.4	0.009	0.011	0.012	0.013	0.014
6.4	0.013	0.015	0.017	0.019	0.021
7.4	0.019	0.021	0.024	0.026	0.029
8.4	0.025	0.028	0.032	0.035	0.038
9.4	0.032	0.036	0.041	0.045	0.048

Table F5: Curve fitting coefficients of reinforcement ratio

C_1	C_2	Q_k
0.0058	-0.0233	3.0
0.0063	-0.0244	4.0
0.0073	-0.0288	5.0
0.0080	-0.0316	6.0
0.0085	-0.0329	7.0

Table F6: Values of $100A_s/bd^2$ re-arranged in ascending

S/Number	M/bd^2	$100A_{st}/bd$	$100A'_{st}/bd$
1	3.428	1.003	0
2	3.880	1.180	0
3	4.327	1.367	0
4	4.773	1.523	0
5	5.010	1.570	0
6	5.220	1.678	0.278
7	5.668	1.003	0.303
8	6.327	1.180	0.385
9	6.918	1.367	0.590
10	6.986	1.523	0.618
11	7.644	1.570	0.938
12	7.836	2.017	1.158
13	8.754	2.338	1.523
14	9.162	2.558	1.807
15	9.672	2.923	1.888
16	10.387	2.017	1.158
17	10.589	2.338	1.523
18	11.611	3.207	2.253
19	11.741	3.288	2.263
20	12.836	3.653	2.618
21	13.322	3.663	2.253
22	14.061	4.018	2.263
23	14.902	4.122	2.618
24	16.482	4.578	3.207
25	17.623	4.578	3.288

Table F7: Reliability Analysis for the beam span flexure mode of failure

Type of Design	Variable	Load ratios				
		0.3	0.6	0.9	1.2	1.5
Deterministic Design	β	6.694	5.849	5.133	4.511	3.961
	P_f	0.109E-10	0.248E-8	0.143E-6	0.324E-5	0.374E-4
Optimum Design	β	6.172	5.333	4.607	3.980	3.427
	P_f	0.339E-9	0.485E-7	0.204E-5	0.345E-4	0.305E-3

Table F8: Reliability Analysis for the beam in shear mode of failure

Type of Design	Variable	Load ratios				
		0.3	0.6	0.9	1.2	1.5
Deterministic Design	β	7.495	6.746	6.130	5.610	4.835
	P_f	0.335E-13	0.765E-11	0.441E-09	0.101E-07	0.67E-06
Optimum Design	β	7.030	6.291	5.685	5.173	4.397
	P_f	0.104E-11	0.159E-09	0.658E-08	0.115E-06	0.55E-05

Figure 4.9: Typical optimum beam design for compression reinforcement

Table F9: Reliability Analysis for deflection control in the beam

Type of Design	Variable	Span of beam (m)				
		0.3	0.6	0.9	1.2	1.5
Deterministic Design	β	8.985	8.430	8.009	8.223	8.503
	P_f	0.131E-08	0.135E-16	0.113E-15	0.999E-16	0.94E-16
Optimum Design	β	7.553	7.116	7.009	7.255	7.897
	P_f	0.214E-13	0.561E-12	0.120E-11	0.203E-12	0.14E-14

Table F10: Reliability Analysis for the Column in Axial Failure

Type of Design	Variable	Load Ratio				
		0.3	0.6	0.9	1.2	1.5
Deterministic Design	β	3.083	3.017	2.928	3.096	3.21
	P_f	0.103E-2	0.002E-3	0.170E-2	0.982E-3	0.663E-3
Optimum Design	β	2.106	1.989	1.821	1.967	1.889
	P_f	0.176E-1	0.360E-1	0.343E-1	0.246E-1	0.295E-1

Table F11:Reliability Analysis for the Column in Flexure Failure

Type of Design	Variable	Load Ratio				
		0.3	0.6	0.9	1.2	1.5
Deterministic Design	β	5.853	5.933	5.803	5.976	6.086
	P_f	0.243E-8	0.149E-8	0.327E-8	0.115E-8	0.580E-9
Optimum Design	β	4.267	4.301	4.540	4.628	4.647
	P_f	0.629E-5	0.851E-5	0.281E-5	0.185E-5	0.169E-5

Table F12: Reliability Analysis for the Column in Combined Axial and Flexure Failure

Type of Design	Variable	Load Ratio				
		0.3	0.6	0.9	1.2	1.5
Deterministic Design	β	4.231	4.620	4.680	5.030	5.268
	P_f	0.116E-4	0.192E-5	0.144E-5	0.246E-6	0.690E-7
Optimum Design	β	2.409	2.695	3.231	3.505	3.663
	P_f	0.799E-2	0.352E-2	0.616E-3	0.228E-3	0.125E-3