APPLICATION OF FIBRE REINFORCED CONCRETE IN THE DESIGN AND CONSTRUCTION OF TENSION CONTROLLED STRUCTURES

BY

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CERTIFICATION

This thesis entitled "Application of Fibre-Reinforced-Concrete in the Design and Construction of Tension Controlled Structures" by IBRAHIM, Yakubu, meet with the regulations governing the award of the Master of Science (Civil Engineering Construction Materials) of the Ahmadu Bello University, Zaria and is approved for its contribution to knowledge and literary presentation.

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I hereby declare that the content of this thesis has been prepared by me and that it is a record of my own work. It has not been accepted in any previous application for a higher degree. And all sources of information are specifically acknowledged by means of references.

IBRAHIM, Yakubu

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DEDICATION

This work is dedicated to my daughters, Khadijat and Hauwa'u for the painful loss of paternal attachment during my long absence.

ACKNOWLEDGEMENT

My sincere and heartfelt gratitude goes to the team of Mrs. Nytko, Prof. Abatan and Mr. Letcha for their candid and sincere guidance during the period of this study. Their devotion and unreserved support is appreciated.

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Above all, I am grateful to Allah, the Most High, the Most Merciful, the Most Forgiving, the Most Rewarding. Allah, it is by your grace this work has been carried through. Allah, from thee we cometh and to thee we shall return. Allah, we seek your guidance and reward. Amen.

ABSTRACT

Linear-elastic theory of fracture mechanics was reviewed and a proposed method for designing structural elements subjected principally to tension presented, based on the strain-energy-release-rate for the fibrous concrete mixes. Eighteen experimental models were provided and of which seventeen were tested and reported including the load ranges and associated circumferential strains upto failure of the experimental hollow cylindrical models. Test results and theoretical values are found to be in good agreement within the scope of the investigation, It is concluded that inclusion of steel fibres in the concrete mixes does not necessarily affect the cracking and crack propagation. It is also concluded that for a specific fibre diameter, using a higher aspect ratio for the given fibre favours failure of specimen by fibre fracture rather than pull-out. Decrease in the strain energy release rate is obtained and the economics of fibre concrete mixes was appraised without recourse to financial consideration.

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CHAPTER 1

1.1 INTRODUCTION

It has long been observed that concrete is rather poor when subjected to tension; cracks are known to develop within the concrete mass even before any appreciable load is applied to it. Despite this observation, it is astonishing that no serious attempt has been made, until recently, to find an answer to the question: "How does concrete fail"? It is generally believed that in axial compression concrete fails by shear as in the case of metals and soils; this misconception is probably due to the conical shape of the fracture. It is even more astonishing when it is considered that at the time most engineers were applying the various shear theories to concrete, scientists on the other hand were already investigating the brictle fracture of such materials as metals.

The modern technique of fracture mechanics began with A.A. Griffith (1920) and has continued to be applied successfully to various materials including ceramics, polymers and metals. And now it has been recognised that under certain set of conditions even ordinary plastic material can have a brittle fracture, yet until recently, it has not entered the consciousness of concrete engineers and researchers. Two reasons may be responsible for this.

In contrast to many other materials, concrete is heterogenous, hence any attempt to apply the Griffith model will immediately be encountered with the difficulty of choosing which value to replace the surface tension term. Secondly and since concrete members are usually designed to fail in compression, the Griffith analysis and those that follow are not strictly applicable to concrete failures, being confined to failures in tension and shear.

Despite these shortcomings, M.F. Kaplan (10) has calculated the critical strain-energy-release-rate (Gc) of three concrete mixes by the notch-bend method, applied them in predicting beam strengths, and early in 1962 Glocklich (4) attempted to adopt the Griffith approach to concrete by considering the propagation of cracks in a compression field and inhomogeneities. Interestingly, and while this was going on Romualdi and Baston (16), utilising the Griffith concept along with (4) developed the crack arrest concept.

By considering these facts, it seems plausible and reasonable to assume that, having answered the question of which term is to replace the surface-energy in the balance of energies, a rational design method can be developed for concrete structures that are essentially subjected to tensile stresses. This view is further reinforced when it is considered that, since the development of the crack arrest concept (5), significant improvement in the composite strength of concrete has been achieved by using fibres as reinforcement. Thus, this study is geared towards the use of concrete containing fibrous reinforcement as crack arrestors. Consequently, the question of the use of fibre-reinforced-concretes in the design and construction of tension controlled structures will be addressed, utilising the concepts developed in the mechanics of fracture and crack arrest in concrete, coupled with the developments in the behaviour at the interface between the concrete and the reinforcement.

1.2 Literature Review:

In his paper, A.A. Griffith (6) has shown, using the concept of energy balance, that the condition for unstable growth of cracks in ideally brittle bodies, without taking into account the stress-state at the crack tip, three energy components are at work viz: (i) the surface energy of the crack,

(ii) the energy of the applied forces and (iii) the elastic energy of the deformed body. It is worthwhile to observe that of these three components the second favours crack development, the first acts in opposite of the second and the third can act in direction depending on the condition of loading. Without considering heat losses, it is known that with glass, which was Grifith material, the strain energy is transformed into surface energy; the same transformation applies to other ceremaic materials and polymers. With metals, Orowan (15) and Irwin (9) have shown that the surface energy is only a negligible quantity within the total energy absorption, with most of the energy expended in producing plastic strains in the highly stressed zone(s).

It is easily seen that the above cannot wholly hold in the case of concrete because of its heterogenous nature. Recognising this discrepancy, Kaplan (10) calculated the critical strain-energy-release-rate for three different mixes of concrete by the notch-bend method and used these in predicting the strength of beams. Glocklich (4) attempted to apply the Griffith concept while considering the propagation of cracks in a compression field and by considering inhomogeneities, and later in the the same year he was able to show the differences in behaviour of concrete when subjected to tension and compression fields (5) hence giving an insight into why concrete strength in tension is orders of magnitude lower than in compression. In the same article he proposed that the cement-gel is similar to glass thus anticipated that it should fracture similarly. In contrast, however, the fracture is not limited to one crack but covers the entire highly stressed zone in the form of multiple microcracks. Hence, in the balance of energies on the expense side is mainly surface-energy, but the area of a newly formed surface may be ten times larger than the effective fracture area.

Glocklich (5) particularly proposed that:

(i) The strength determining property in concrete for all types of loading is the critical strain-energy-release-rate;

(ii) In concrete the strain-energy is transformed almost entirely into surface-energy with the new surfaces much larger in area than the effective fracture surface;

(iii) The increase of the microcrack zone and the heterogeneity of the material contribute to the relatively high value of the critical strain-energy-release-rate in concrete;

(iv) In tension, the driving force, i.e. the rate of strain-energy release increases with crack length;

(v) In tension, the firswt crack to grow is the fatal one, but preceding its fast growth is a stage of slow growth that is responsible for the slight curvature of the stress-strain curve.

An investigation conducted with a microscope (8) revealed the following facts regarding the fracture of concrete in compression:-

(i) Bond strength is less than matrix strength which, in turn, is less than the aggregate strength;

(ii) Some bond seperation occur before any load application;

(iii) Cracks develop almost entirely in the matrix-aggregate interfacial bond to approximately seventy percent of the breaking load;

(iv) Above this load level, cracks develop in the matrix and bridge the existing bond cracks;

(v) Cracks rarely penetrate into aggregates; and

(vi) The matrix cracks select the shortest distance between bond cracks to be bridged, with preferred bridging occurring between the larger aggregates so that a continuous crack would grow through a minimum distance within the matrix.

In the year 1962, Slayter proposed the "Two-phased Material" Concept (19) and showed that the overall performance of a composite is dictated by the individual performances of each phase. Therefore, Romualdi and Mandel (17), combining the concepts in (19), (16,18) developed the crack arrest concept, showing paralled in fibre-glass. They concluded that the application of linear-elastic fracture mechanics to reinforced concrete indicates that the low tensile strength of concrete is not inherent to the material, pointing out that this can be avoided by suitable reinforcement arrangement i.e. at approximately small spacings incipient flaws are prevented from enlarging and propagating through the tension zone, thereby resulting in a truelly two-phased material exhibiting strength properties not restricted by the characteristics of each seperate matrerial. It is important to note here that their validation was based on flexural tests as distinct from direct tension test.

Walkus et al (21) in their study of fibrous concrete subjected to uniaxial tension concluded that, apart from diameter and volume fraction of fibre, the load carrying capacity of fibre-reinforced concrete specimen tested after 28 days is reached when crack occurs and at failure the strength of the fibre is not reached because of loss of bond between fibre and matrix, with the location of crack depending on orientation and number of fibres. They also concluded that the tensile strength of fibre reinforced concrete is up to fifty percent increased below a certain micro-reinforcement content while modulus of elasticity decreases when volume fraction of fibre increases.

The behaviour of fibrous composites after reaching the ultimate yield strain is governed bythe relative interaction of the fibre and the matrix components.

In the case of concrete, matrix strain is less than the ultimate fibre strain so that matrix cracking occurs first, followed by progressive debonding and pull-out of randomly dispersed fibres. Attempts (1,2,3, 4,11,12) have been made to improve the efficiency of fibre reinforcement by increasing the shear strength of the fibre-matrix interface through some chemical and/or mechanical treatment. However, when such treated fibres were used in concrete, the observed improvement in the composite action was much less significant than expected (14). Naaman et al (13) have concluded that the efficiency of fibres orientation cannot be predicted from elastic continuum consideration because the fibre contribution in such brittle matrices is significant only after matrix cracking and because the pull-out mechanism of inclined fibres is substantially different from that of aligned fibres. Further, for a large number of fibres the fibre contribution depends significantly on the capacity of the matrix to withstand the forces induced by the fibres bridging the cracked surface, observing that spalling and disruption of the mortar matrix leads to a substantial reduction in the pull-out resistance. Previous theoretical efficiency ratios have been suggested for randomly dispersed fibres (1,2,3,7,11,12)

1.3 Aims and Objectives:

This study is intended to evaluate the influence of locally available steel fibres reinforcement on the response of concrete under the action of tensile stresses upto failure. The high point of the study is to develop a rational design approach for tension control members and structures such as silos, pressure-vessels etc. However, the study considers only fibre content as variable.

With this in mind it is intended to look at the probable economics of the procedure using the plain concrete data as base information without the necessity of looking at the fiscal aspect. It is hoped that this study would lead to a reasonable utilization of fibrous concrete to achieve a saving in weight and consequently cost hence a prudent use of materials and other scarce resources.

CHAPTER 2

2.0 GENERAL THEORETICAL CONSIDERATION OF FRACTURE

2.1 Definition of Fracture:

Fracture can be, generally, defined as the disintegration of a body into pieces under the action of some applied stress(es). Since the phenomenon is very diversed in its effects, a more rigid and accurate definition seems rather illusive, as even the most generalised definition does not cover cases when disintegration of bodies caused, not only by mechanical, but by some other actions like corrosion etc. In this context therefore, it seems that the concept of fracture should necessarily be specified by indicating the particular operative conditions.

I t is relevant, and most fruitful, to consider fracture as the process which develops in time and passes successively through the preliminary, critical and supercritical stages. This approach makes it possible to understand completely the mechanism of fracture and of course the effects of various factors, both internal and external, on the fracture resistance of a material. A crack can pass either through the bodies of grains or along their boundaries; various stages of the process can be detected viz: initial, developed or full fracture. The surrounding medium can simultaneously influence the process etc. With single action of a monotonically increasing load, two principal types of fracture are possible, either brittle or ductile (plastic), the terms ductile and plastic here being synonymous. As this study is based on concrete the first type of fracture is mainly of concern herein.

2.2 Brittle Fracture In Concretes

Concrete by its very nature and composition is not homogenous. It is therefore inadmissible, to assume it isotropic. It is known that the components of the concrete differ significantly in their mechanical properties. Therefore, it is significant to distinguish between the two modes of fracture in concrete tensile and compressive viz-a-viz the differences of concrete as against other materials.

2.2.1 Difference Between Concrete And Glass

In approaching the problem of concrete fracture, the first difficulty is to establish the form of energy into which the strain-energy is transformed in the fracture process. Neglecting heat losses, it is known that with glass (Griffith subject of investigation) the strain-energy is transformed into surface-energy; the same transformation applies to other ceramic materials and polymers. In metals, it has been shown (9,15) that the surface-energy is only a negligible quantity within the total energy absorption, with most of the energy expended in producing plastic strains in the highly stressed regions. The cement-gel is similar to glass thus is anticipated to behave similalry. Basically therefore concrete is not different from glass and in fracture, the strain energy is transformed into surface (tension) energy. However, in contrast to glass, the fracture is not limited to one crack but covers the entire highly stressed zone in the form of multitude of microcracks. Hence only surface energy applies but the area of the newly formed surface is several folds larger than the effective fracture area.

The balance of energies for an ideal elastic and homogenous material (Griffith case) is shown in Figure 2.1 this is an example of elliptical flaw of semi-major axis, c, perpendicular to a tension field σ_t.

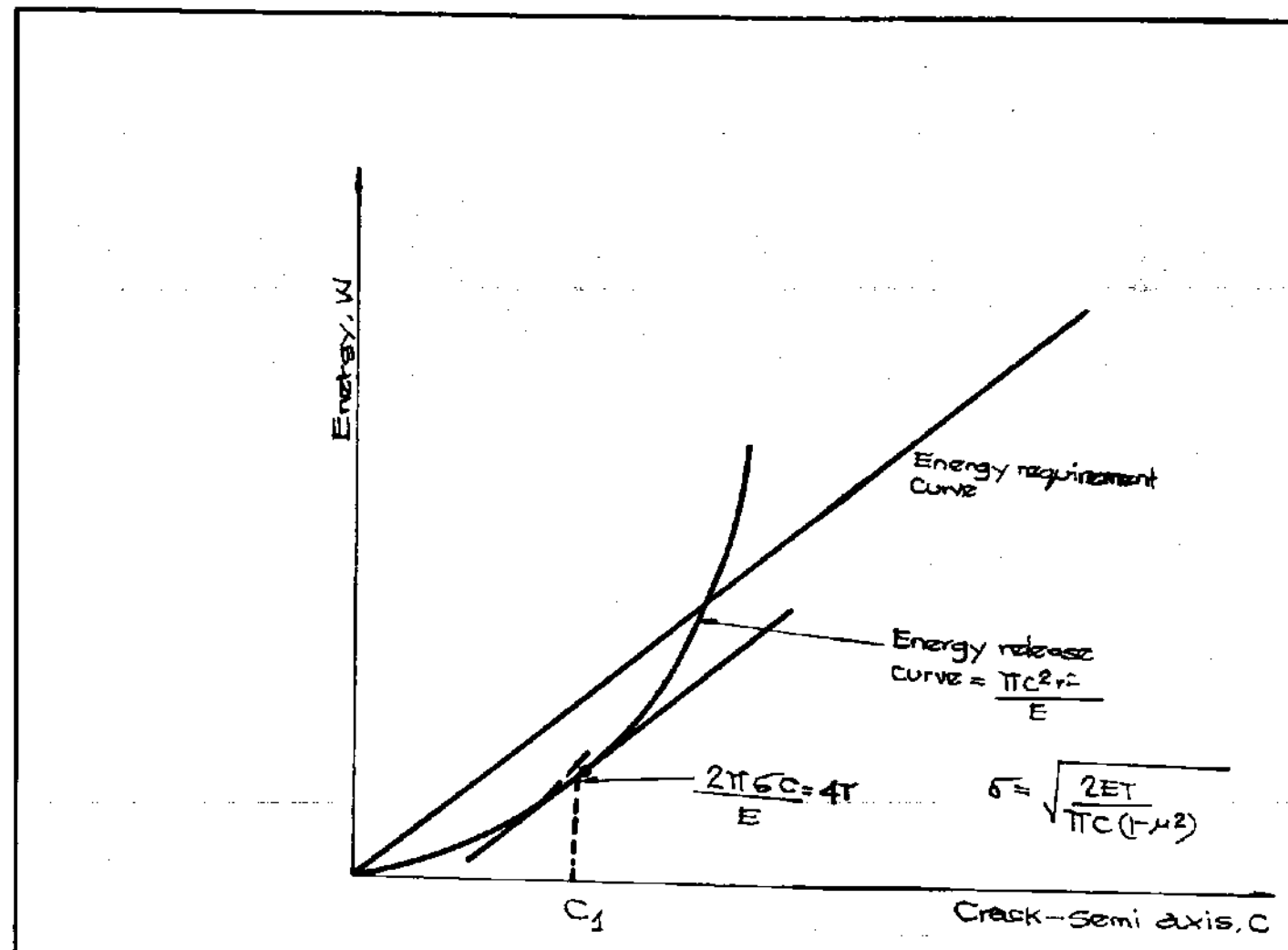


Fig. 2.1: The Giffith Case

The energy requirement curve is a straight line with slope equal to 4T in which T is the surface tension. The energy-release curve is a second power parabolar. On-set of instability occurs when the slopes of this two curves become equal. With concrete, a difference is certain.

It can be shown that, as the crack grows, the area of the highly stressed zone grows with it: and because the cracking process is not limited to the main crack, but includes the entire zone, it is clear that the rate of energy absorption increases with the crack size. Hence the energy requirement curve for concrete will not be a straight line but a curve with an ascending slope as illustrated in Figure 2.2

An initial crack of length C_o will begin to grow under a stress . This growth will stop at C_1 as a result of increase in energy demand. For the crack to continue in its growth therefore, there has to be an increase in the stress to σ_1 to enable growth to C_2 ; this continues until the crack grows to length C_n. This is a continuous process rather than stepped as shown in figure 2.2. At a point "A", the slope of the energy requirement curve ceases to increase such that further increase of stress beyond σ_n is required and the crack grows spontaneously. The slope ceases to increase because the highly stressed zone has now reached its maximum size which is determined by the geometry of the specimen among other things.

The strength determining property is therefore complicated. In an ideal material, surface-tension (T) and Young Modulus (E) alone determine the strength or "toughness", as the resistance to seperation fracture is called.

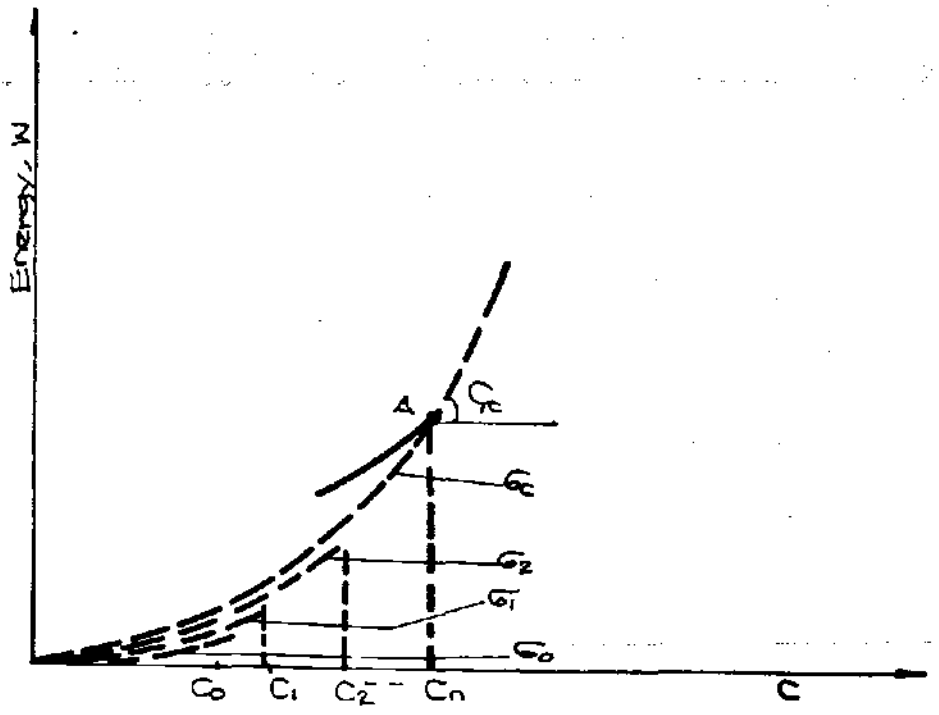


Fig. 2.2: Forced Growth of a Crack

In a material such as concrete, the equivalent to T would be the rate opf energy absorption, the value of which is not a constant and is only siginificant at the onset of instability or fracture. At this stage it is then referred to as G_c, the critical strain-energy-release-rate. This is therefore the maximum slope attained by the energy requirement curve. Like in metals, this is definitely a characteristic material constant for all types of loading.

The second problem to contend with is the heterogeneity of concrete. Assuming that the strain-energy is transformed mainly to surface-energy, the question of which surface-energy arises; that is:- is it transformed into that of the hardened cement gel, that of the aggregates or perhaps that of the aggregate-cement interface! Figure 2.3 shows a theoretical case of a crack propagating in a bi-phase material. The initial crack size, C_o, starts growing under a stress σ_o . This will be a forced growth when the requirement curve is concave upwards so that the stress will be increased slightly. At C_I, the stress must be increased drastically to σ_i , since the energy demand will increase suddenly as the crack enters a tougher phase. However, the extent to which this theoretical condition is operative in the case of concrete will depend on the type of aggregates used; for less strong aggregates, cracks will penetrate rather than detour round them. Cracks first tend to develop where the demand for energy is least and then continue into more resistant media. Because the divisions between the various media are distinct in concrete, it is possible for a jump to occur in the energy requirement as shown in figure 2.3 to be actually present.

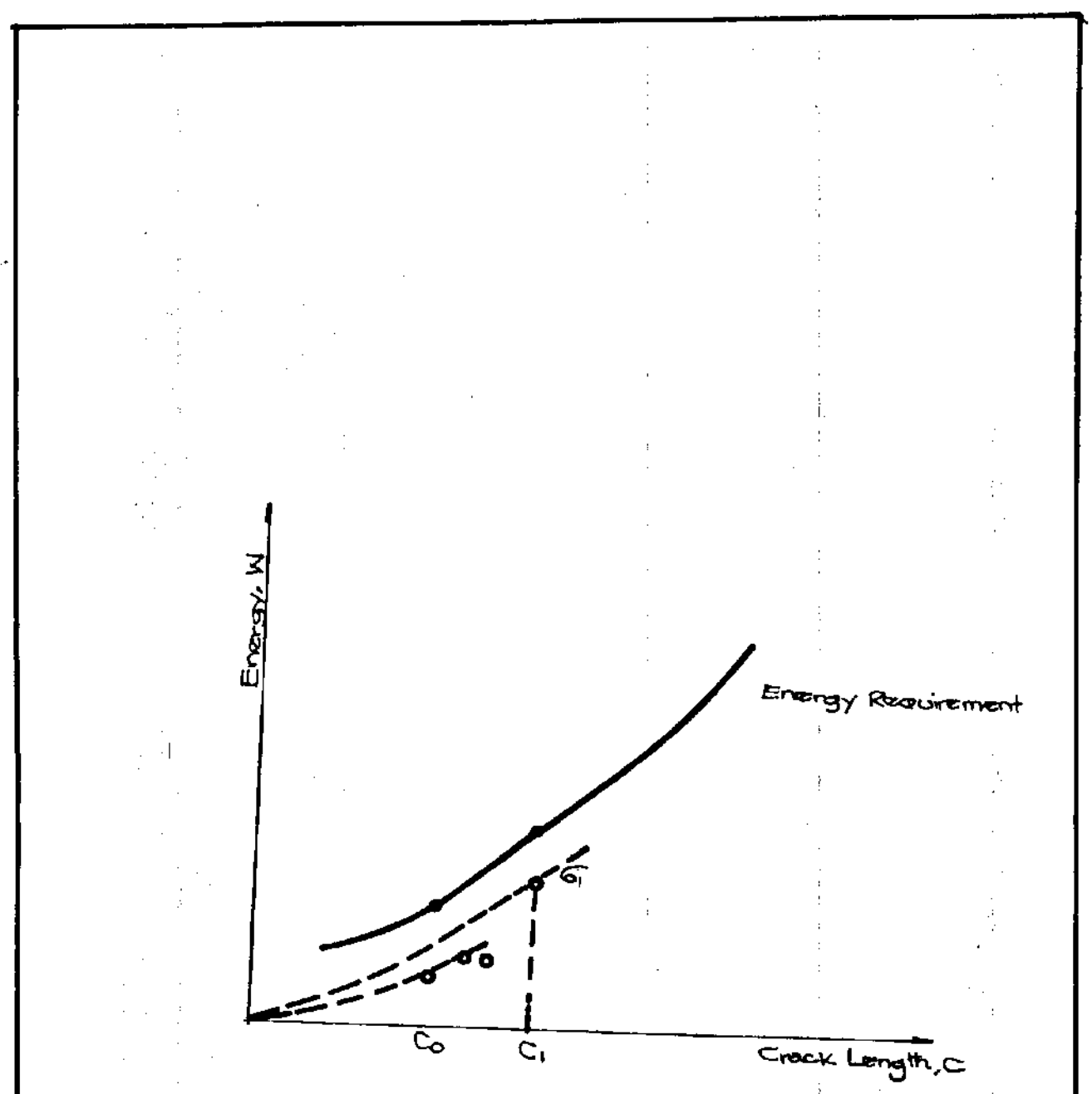


Fig. 2.3: Crack Propagating in a Bi-phase Material

Two examples are significant to illustrate this point. When a crack develops around an aggregate, the energy demand is quite low, but when it has to force its way into the matrix, the demand is suddenly increased. Secondly, a crack propagating in a matrix and encountering a zone of higher strength; if the crack penetrates this zone, the demand will increase, but if it detours round it the demand will also increase because the actual area of the newly formed surface is larger than the effective area. These two examples illustrate the premature occurrence of a running crack due to inertia forces.

The two phenomena outlined above, i.e. the increase in the microcracking zone near the crack tip due to increase of the crack size and the various crack arrestors contributed to the very high strength attained in concrete. It can then be conceived that in the absence of these two phenomena, the first crack to start growing will be the fatal one since the growth of a running crack with linearly increasing resistance is a process of unstable equilibrium as shown in figure 2.1. Therefore it could be assumed that these two phenomena lead to the gradual increase of slope of the energy requirement. The final slope of the curve or the G_c value is therefore the sum of all individual slopes caused by the various factors that influence it such as type and size of aggregates, water-cement ratio etc. Nevertheless G_c can be regarded as a material constant for concrete.

2.2.2 Tensile Fracture of Concrete

In a homogenous tension field, the most severe pre-existing crack will be the one normal to the stress with the largest radius (the bond cracks that form before load application often approximate the ideal elliptic crack as assumed by Griffith). Considering the lack of homogeniety of concrete however, the most severe crack will then be the one in which $EG_c/c\sigma_t^2$ have a minimum value.

Considering a mass of concrete subjected to a uniform tension stress σ_t in which:

$$\sigma_t^2 = [2ET/\ c(1 - \mu^2)]^{\frac{1}{2}} \quad \ldots \quad (2.1)$$

the behaviour of the concrete will be linear-elastic and the stress-strain relationship will be a straight line. This case represents plain strain situation (see figure 2.4). From linear-elastic fracture mechanics, let U represent the reduction in the stored elastic energy of the stressed system because of the crack, V the energy of the crack surface and c refers to the radius of the internal crack. Then the criterion for no crack extension is

$$\frac{\partial}{\partial c}(U - V) = 0 \quad \ldots \quad \ldots \quad (2.2)$$

Like in other brittle materials, the work required to extend a crack is related to the surface free energy of the newly created crack surfaces; considering this and neglecting the plastic enclave the energy of a crack surface in a two dimensional plate element on a unit thickness bases is:

$$V = 4TC \quad \ldots \quad \ldots \quad (2.3)$$

where T is the surface tension.

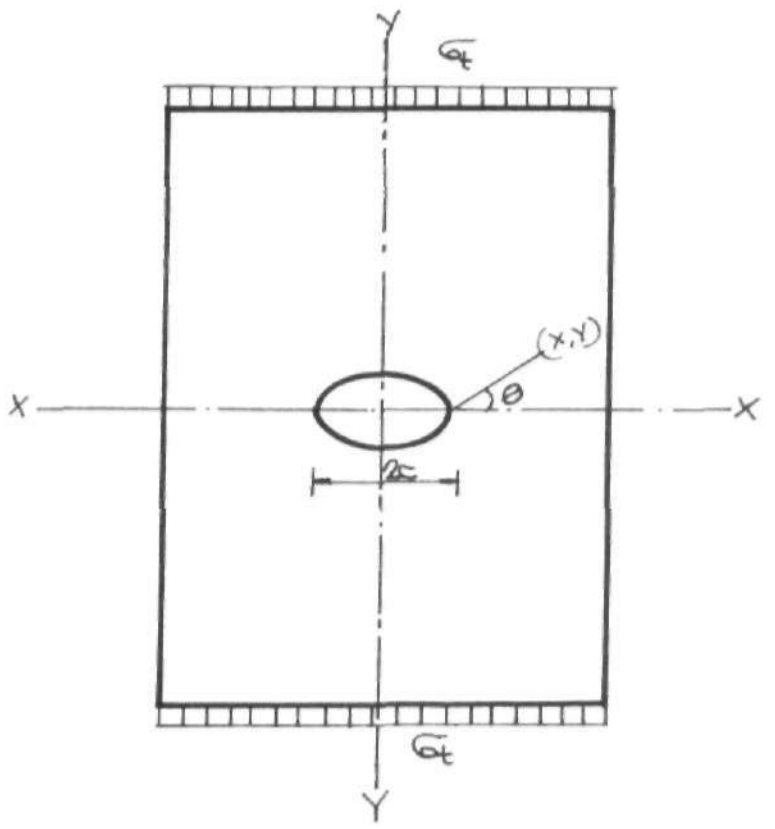


Fig. 2.4: Propagation of an elliptic crack in a Uniform Tension field

To estimate the reduction in strength of concrete (a brittle material) in the presence of a crack from equation (2.2) it is only necessary to have an analytic expression for U and a knowledge of T's magnitude, Therefore for the plain strain situation of a crack of length 2c in a thin stressed plate that is subjected to a tensile stress σ_t as shown in figure 2.4 the reduction is stored elastic energy is

$$U = \sigma_t^2 C^2/E \quad \ldots \quad \ldots \quad (2.4)$$

where E is the modulus of elasticity. The condition for crack initiation for the situation of figure 2.4 is

$$\frac{\pi \sigma_t^2 C}{E} = 2T \quad \ldots \quad \ldots \quad (2.5)$$

Equations (2.4) and (2.5) represent the strain-energy-release and its rate respectively. At the stress given by equation (2.1), the stress-strain curve deviates from linearity. Due to reduction in thickness as a result of crack formation or propagation the stress increases faster than the applied load. The release of strain-energy and its rate increase even faster. At a certain stage the strain-energy release equals the energy requirement, thus:

$$G_c = (2\pi \sigma^2 C)/E \quad \ldots \quad \ldots \quad (2.6)$$

Sheddon (21) has shown that the presence of a disk shaped crack of radius C in the interior of a solid and perpendicular to the applied tensile stress lowers the potential energy of the medium by

$$U = \left[8 (1 - \mu^2) \sigma_t^2 C^2\right] /3E \quad \ldots \quad \ldots \quad (2.7)$$

But the crack extension force tendency is

$$G = \frac{\partial u}{\partial A} = \frac{\partial U}{dc} \cdot \frac{dc}{\partial A} \quad \ldots \quad \ldots \quad (2.8a)$$

where A is the area of the newly created crack surface associated with crack extension. Also, for a circular crack of radius C

$$\frac{dc}{dA} = \frac{1}{2\pi c} \quad \ldots \quad \ldots \quad (2.8b)$$

and with $\partial U/\partial A$ obtained from equation (2.7) then

$$G = \frac{4\,(1-\mu^2)\,\sigma_\varepsilon^{\,2} C}{E} \quad \ldots \quad \ldots \quad (2.9)$$

Equation (2.9) gives the strain-energy release rate in tension. At the point of crack initiation or propagation, G attains its critical value of G_c and is the driving force for a running crack . It is this driving force which therefore has to be controlled or reversed, thus leading to the proposal of the crack arrest concept. In essence, this is the importance of fibrous reinforcement in fibre-reinforced concretes; the control of cracking.

Steel-fibre-reinforced-concretes (SFRC) have been used in structural units like beams, beam-column connections, heavy duty slabs and plates, the beneficial interactions, between the matrix and the reinforcing fibres being harnessed to achieve particular objectives. To the authors best knowledge, no report about usage of SFRC in tension control structural units have been published.

In this study, the following are recognised:

(i) that the concrete by itself is hardly homogenous in its properties (ii) that the composite acts in recognition of the contributions of the matrix, fibrous reinforcement and the matrix-fibre interface with the contribution of the matrix and fibres according to their available proportions, and (iii) that the contribution of the matrix-fibre interphase is through interfacial shear.

2.3 Strengthof SFRC

The behaviour of SFRC is governed by the relative interaction between the concrete and the fibrous reinforcement dispersed within the concrete mass. In a composite material consisting of a matrix reinforced with uniformly distributed and undirectionally continuous fibres, the cracking stress of the composite in direct tension is given by the law of admixtures as

$$\sigma_{cr} = \sigma_{mu} \cdot V_m + \sigma'_{f} . V_f \quad \ldots \quad \ldots \quad (2.10)$$

in which

σ_{mu} = cracking stress of matrix

σ'_f = stress in fibres at matrix cracking;

v_m = volume fraction of matrix and

v_f = volume fraction of fibres.

When the matrix cracks, it may be assumed that the entire stress is carried by the fibres alone. Therefore, the ultimate strength of the composite is given by the fracture strength of the fibres, fu ;

thus

$$\sigma_{cu} = \sigma'_{fu} . v_f \quad \ldots \quad \ldots \quad (2.11)$$

Assuming perfect bond between fibre and matrix

σ_{fu} is given by:

$$\sigma_{fu} = 2\tau \frac{(l_f)}{df}. \quad v_f$$

hence equation (2.11) becomes;

$$\sigma_{cu} = 2\tau \frac{(l_f)}{d_f}. \; v_f \qquad \ldots \qquad \ldots \qquad (2.12)$$

But, and for short length of fibres in particular, the following observations can be made;

(i) that the fibres will be inefficiently oriented with respect to the principal tensile stress;

(ii) for a finite length of fibre, the stress distribution along the length of the fibre will not be uniform throughout with the stress being highest at the ends;

(iii) for different fibre shapes bond efficiency between matrix and fibre will be different due to the difference in their surface deformation.

With these observations in mind it is obvious that to be meaningful equation (2.12) has to be modified to account for these, thus:

$$\sigma_{cu} = n_o.n_i.n_b \cdot 2\tau \; (l_{f/d_f}). \; v_f \qquad \ldots \qquad \ldots \quad (2.13)$$

in which

n_o = Orientation factor to account for fibre misorientation;

n_i = Length correction factor given by equation (2.14)

n_b = Bond efficiency factor.

The length correction factor is a function of the critical length of the fibre and is related to the bond stress as shown in equation (2.12). It has been shown by Swammy et al (1981) that the interfacial bond stress is influenced by the composition of the matrix and its strength properties and the correction factor must therefore account for the dimension of the fibre as well as the matrix. Cox (2) proposed the following function to evaluate this factor:

$$n_1 = 1 - \frac{\tanh (\alpha . l_f / 2)}{(\alpha . l_f / 2)} \quad \ldots \quad \ldots \quad (2.14)$$

where

$$\alpha = \left[2\pi G_m / (E_f . A_f . \ln (s/r_f)) \right]^{\frac{1}{2}} \quad \ldots \quad (2.15)$$

and

G_m = matrix shear modulus;

A_f = fibre-cross-sectional area;

r_f = radius of fibre and

S = mean fibre centroidal spacing

$$= 25 \left(d_f/(v_f . l_f)\right)^{\frac{1}{2}} \quad \ldots \quad \ldots \quad (2.16)$$

2.4 Method of Analysis

In order to use the materials of the SFRC to the best advantage, it is reasonable to carry out the analysis beyond the elastic range. This approach allows the optimisation of materials usuage with regards to their strength determining parameters of the type of structure/ section considered.

This allows the determination of the ultimate strength condition, the procedure adopted being as follows:

(i) Evaluate fibre spacing from equation (2.16);

(ii) Evaluate α and hence n_1 from equations (2.15) and (2.14) respectively;

(iii) Select values of n_o and n_b appropriate to the orientation and condition of fibre;

(iv) Evaluate τ from modulus of rapture or any relevant test like pull-out test otherwise from established values;

(v) Evaluate σ_{cu} from equation (2.13)

(vi) Evaluate forces in tension from

$$F_t = \sigma_{cu} \cdot b \cdot t. \quad \ldots \quad \ldots \quad (2.17)$$

CHAPTER 3

EXPERIMENTAL PROGRAM

3.1 Experimental Program:

To investigate the proposed approach, an experimental program was set-up using steel fibre-reinforced-concrete (SFRC) hollow cylinder models. The models have an average internal to external diameter ($d/_D$) ratio of 410:450 with an average wall thickness of 20mm. However, some of the models were found to have varied thicknesses along the vertical axis and this is attributable to the movement of the formwork during the vibro-compaction process.

In addition to the test models, three solid cylinders (150 x 300mm^3) and three cubes (150x150x150mm^3) were also prepared for each mix, cured and tested not earlier than 28 days after casting. Both the test models and the cubes and solid cylinders were kept immersed in water for the first ten days and subsequently allowed to stand in the open atmosphere of the curing room until tested. The fibre content for the mixes used were 0.0%, 0.25%, 0.5%, 1.0%, 1.5% and 2.0% by weight, and the water: cement ratio was modified with the addition of fibre to improve the workability of the mixes. The weight proportion for these mixes are shown in table 3.2.

3.2 Materials:

The materials used for the S.F.R.C. mixes were crushed granite, river sand, ordinary portland cement (Ashaka Brand) and steel fibres in the form of yarns. The length to diameter ratio of the fibres is given as mean aspect ratio in paragraph 3.2.4.

3.2.1 Coarse Aggregate:

The coarse aggregate used was crushed granite. For the models, a practical maximum size of coarse aggregate was 10mm. However, to allow for a more uniform dispersion of the material, all sizes passing 6.3mm sieve but retained on sieve size 4.76mm was used for the coarse aggregate.

3.2.2 Fines:

The fine aggregate used was river sand obtained within the Zaria locality. All sizes passing sieve size 4.76mm but not smaller than 0.002mm were used. Traces of lateritic aggregates were observed in the fines.

3.2.3 Cement

The cement used was ordinary portland cement produced at the Ashaka Cement Works.

3.2.4 Fibres:

The fibres used were steel fibres (yarns) obtained from burnt tyres. These were then cut into approximately equal lengths, soaked in a solvent (DPK) for three days and then subsequently charged into a drum mixer with aggregates which provided the abrassion for the cleaning of the surface of the fibres. The washed fibres were later allowed to dry in the sun. Due to previous usage and heat treatment during extraction it is expected that the mechanical properties of the fibres would have been modified.

Thirty five pieces of the cut lengths were then picked at random and their lengths and diameters measured. The mean length and diameter of the sample were found to be 90mm and 0.655mm with standard deviation of 3mm and 0.054mm respectively. This combination of mean length and mean diameter resulted in a mean aspect ratio for the fibres of 137.4.

Subsidiary test results gave the density of the fibre as 6580 kg/m^3 and tensile strength as shown in table 3.1

Table 3.1 - Tensile Test Result For Fibres

Test No.	Fibre Ø (mm)	Failure Load(KN)	(N/mm^2) Failure Stress	Remarks
1.	0.655	0.12	356.13	
2	-do-	0.16	474.84	
3	-do-	0.16	474.84	Average failure stress = 416.835N/mm^2
4	-do-	0.13	385.81	
5	-do-	0.125	370.97	
6	-do-	0.15	445.16	Standard deviation = 50.402N/mm^2
7	-do-	0.13	385.81	
8	-do-	0.125	370.97	
9	-do-	0.125	370.97	
10	-do-	0.15	445.16	
11	-do-	0.17	504.52	

3.3 Mix Proportions:

The mix proportion cement: fines: coarse aggregates was kept constant throughout the mixes. However, these quantities were reduced to their kilogram equivalent and thus all fibre contents were calculated on weight percentage basis. The water: cement ratio was varied only in as much as it improved the workability of the mixes as the fibre content increased. The material quantities for the mixes are as shown in table 3.2

Table 3.2 - Mix Proportion By Weight (kg)

Mix	Cement	Fines	Coarse	Fibre	Water	Remarks
A/0.00/0.6	25.8	60.1	40.06	0.00	15.48	Control
A/0.25/0.6	25.8	60.1	40.06	0.318	15.48	
A/0.50/0.6	25.8	60.1	40.06	0.6298	15.48	
A/1.0/0.65	25.8	60.1	40.06	1.2596	16.77	
A/1.5/0.7	25.8	60.1	40.06	1.8894	18.06	
A/2.0/0.7	25.8	60.1	40.06	2.5192	18.06	

3.4 Mix And Specimen Preparation:

Batching for the mixes was done by weight as shown in table 3.2 The cement, fine and coarse aggregates were first determined by volume and subsequently reduced to their kilogram equivalent. The obtained quantities were then reduced in proportion to provide sufficient amounts of concrete to produce three test models, three 150mmx300mm solid cylinders and three 150mm x150mm x150mm cubes per batch.

The total weight of fine, coarse aggregates and cement per batch amounted to 125.96kg. Based on this combined weight the fibre content for the various mixes, by weight percentage, were calculated and are also shown in table 3.2. Weight fractions of 0.25%, 0.5%, 1.0%, 1.5% and 2.0% were used. The water-cement ratio. used was 0.6 but as the fibre content was increased the mix became stiffer hence more water was added to make the mix more workable. The range of water-cement ratio used was therefore 0.6 to 0.7 with only one mix using 0.65

The hollow cylinder models were produced using a double skin form work with a 20mm space between the two skins. The casting was done in two layers. In some cases the spacing between the inner and outer skin of the forms varied and this is attributed to shifting of the formwork thus causing a variation in the thickness of the models. Details of the formwork is shown in plate 3.1.

3.4.1 Mix Preparation

The concrete was mixed in a rotary pan mixer. The cement, fine and coarse aggregate were charged into the mixer without regard to order. The mixer was then allowed to run so as to provide preliminary dispersion of the cement, fine and coarse aggregates. After a reasonably good distribution was achieved, the fibres were then charged into the mixer while the mixer was kept runing. The fibres were manually loaded by hand, with shredding as the charging was continued. This procedure helped in reducing the possibility of "balling" and fibre congestion to the minimum. Mixing water was then added and the mixing continued.

The mixer was then stopped and the mix visually checked for consistency. Where the consistency was seen to be poor, the mix was then collected in the middle of the mixing pan, using hand trowel, and then the mixer operated again. This was repeated until a satisfactory mix was obtained.

3.4.2 Specimen Preparation:

The test models were hollow cylinders with an internal and external diameters of 410mm and 450mm respectively. However, some of the cylinders after casting were found to have variable thicknesses generally ranging from about 13mm to 27mm. This variation in thickness was attributed to the shifting of the skin of the formwork due to vibro compaction. Casting of the models was completed in the layers per model with each layer vibrated with a Kango kammer before the next layer was placed. The small size of the coarse aggregate prompted the use of the Kango hammer since the use of the high frequency vibration table could have resulted in floatation of the coarse aggregates. The details of the formwork construction is as shown on Plate 3.1 as well as the cast models before stripping the formwork.

3.5 Test Set-up and Instrumentation:

To obtain an approximately uniform pressure distribution through the test model height it was necessary to equalise the load from the two ends of the cylinder models. This was achieved by allowing free vertical displacement of the two discs placed at the top and bottom of the models. Because the inside surface of the models was sufficiently smooth, friction between the surfaces and the aggregayes was small and therefore could be neglected.

3.5.1 Test-Set-Up

The test models were placed with their longitudinal axis vertically positioned and centred. Two disc shaped plugs were provided whose diameters were averagely 10mm less than the internal diameter of the test models. The space between the bottom plug and the inner face of the model was prevented from being occupied by the gravel fill using a packing of paper. This arrangement allowed the free movement of the plugs both at the top and bottom when the load was applied vertically giving an approximately uniform pressure throughout the depth of the infill. The vertical load was appliued at an incremental step of 5kN until failure and circumferential strain readings were taken at mid height of the specimen for each load incremental. Details of the test set-up is shown in plate 3.2.

3.5.2 Instrumentation;

For each specimen four Demec point bridges were placed on a bridge length of 101.6mm (4") and using a 101,6mm (4") Demec Mechanical Gauge the strains at each stage of loading were taken. These readings were also used as crack width indicators. Plate 4.1. shows the Demec points disposition.

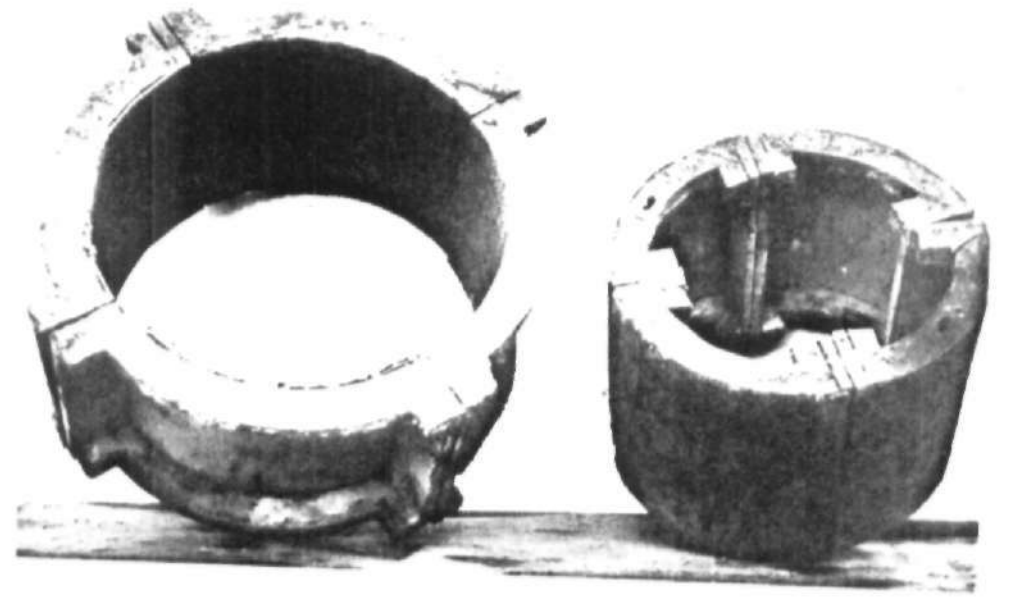


PLATE 3.1: Formwork : Left-Outer skin

Right-Inner Skin.



PLATE 3.2: Loading Arrangement.

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CHAPTER 4

4.0 TEST RESULTS AND DISCUSSIONS:

A large amount of data was obtained from the test results and analysed; only those relevant to this study and representative of the behaviour of the test specimens are presented herein.

4.1 Test Results

4.1.1 Deformation Characteristics:

Typical load extension curves for the specimens are shown in figure 4.1. These curves were derived directly using the applied vertical load as the horizontal pressure is directly related to it. Table 4.1 shows the readings obtained at each load stage. All strain readins were taken approximately mid height of the specimens using a Demec Mechanical Strain Gauge with a gauge factor of 2.64 $x10^{-5}$ mm/mm. The reduced strain values are also shown on table 4.1. These strain measurements formed a reasonable basis for estimating the crack width and consequently reflect the crack radii.

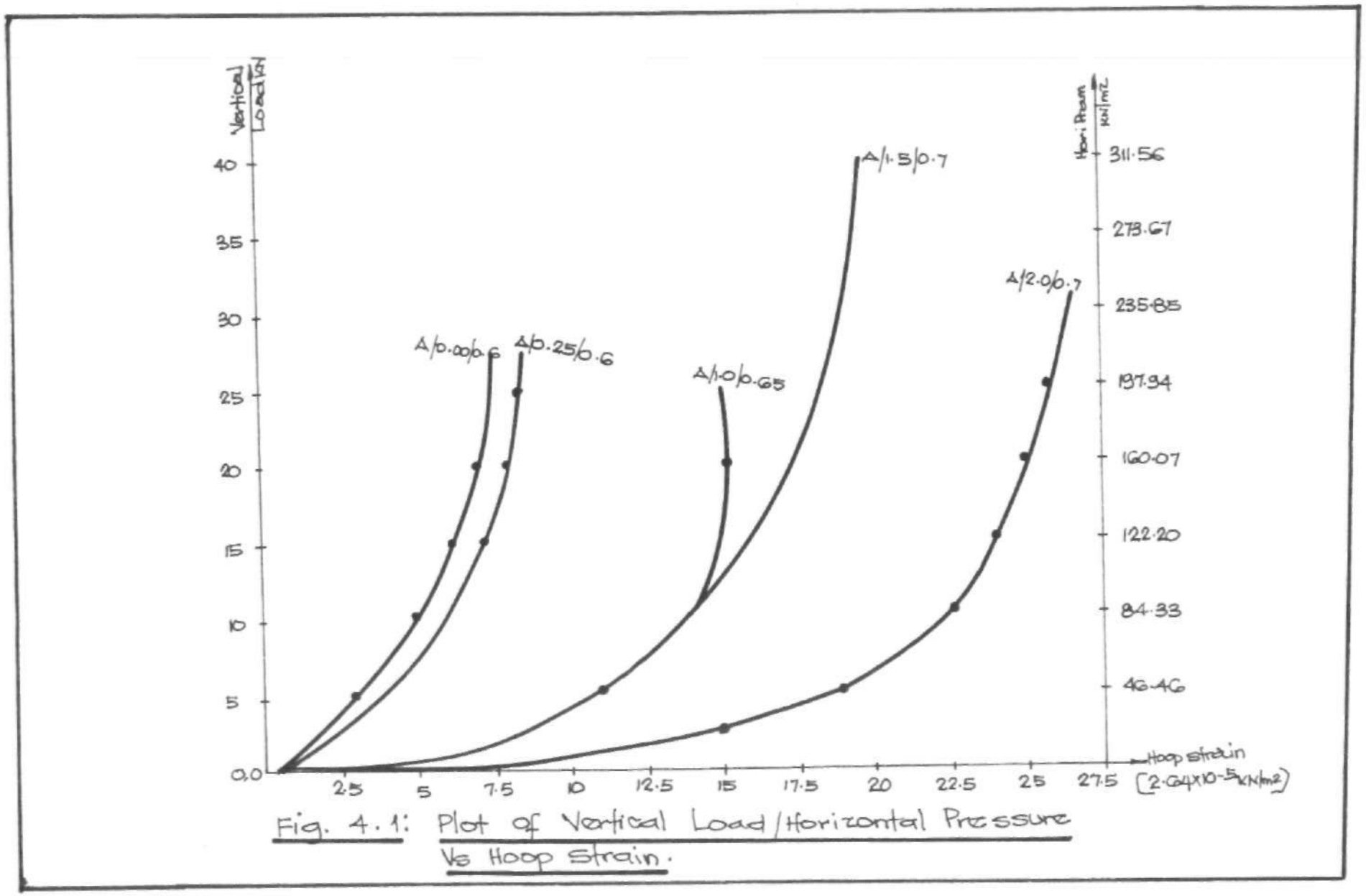


Fig. 4.1: Plot of Vertical Load/Horizontal Pressure Vs Hoop Strain.

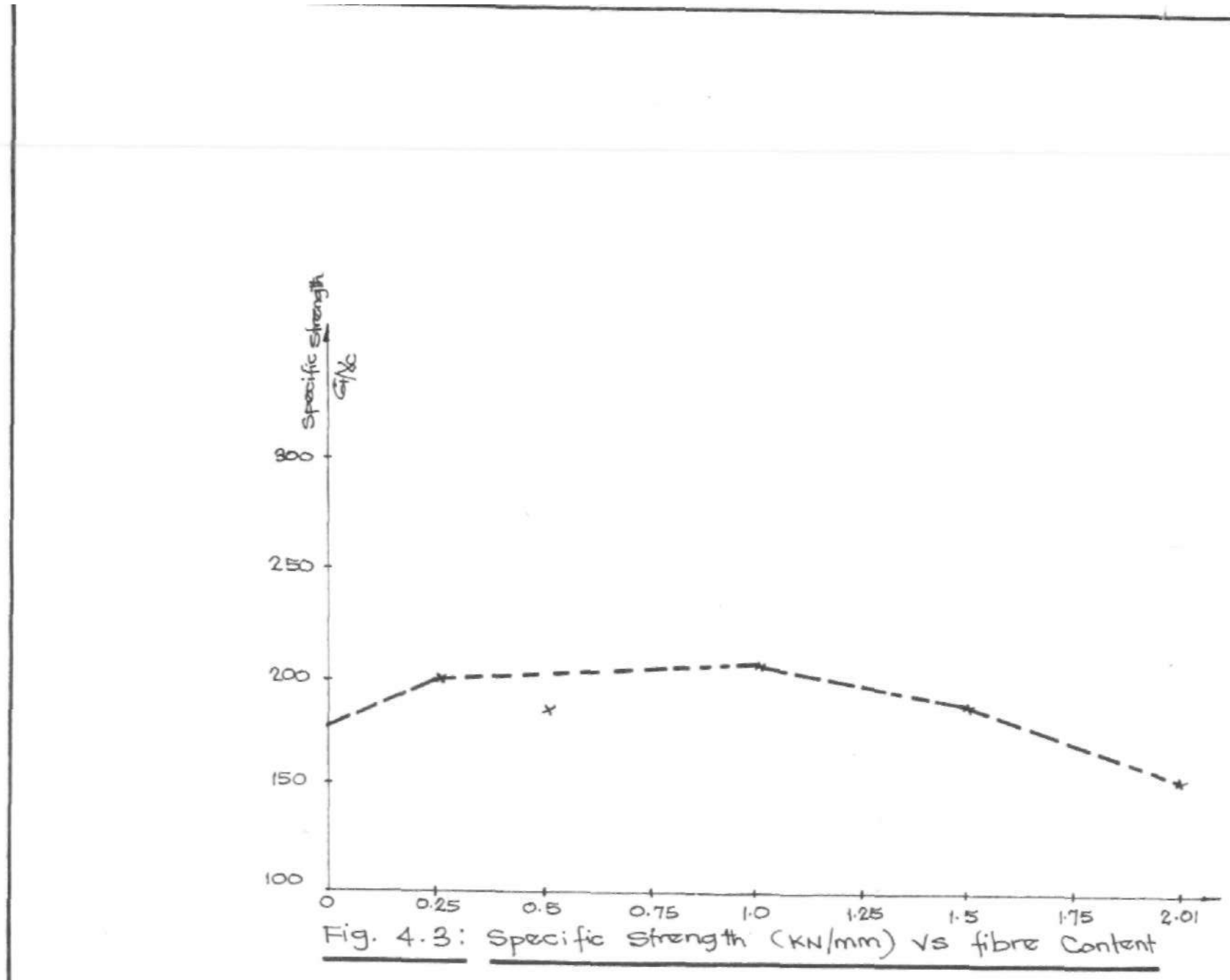


Fig. 4.3: Specific Strength (KN/mm) vs fibre Content

Table 4.1 - Experimental Readings

Specimen Identification		A/0.00/0.6	A/0.25/0.6	A/0.5/0.6	A/1.0/0.65	A/1.5/0.7	A/2.0/0.7
Vertical Load kN	Hor.Press. kN/m^2	Average Strain ($x2.64x10^{-5}$)mm/mm					
0	8,586	0.00	0.00	0.00	0.00	0.00	0.00
5	46.475	3.5	4.0	0.00	11.00	13.00	19.00
10	84.329	5.0	6.0	2.4	14.0	14.00	22.5
15	122.200	6.0	7.5	4.0	15.0	16.0	24.0
20	160.072	7.0	8.0	4.0	15.0	17.5	25.0
25	197.944	8.0	8.5	7.0	15.0	18.5	26.0
30	235.815	-	11.0	-	-	19	-
35	273.686	-	-	-	-	19.5	-
40	311.558	-	-	-	-	20.25	-
45	349.429	-	-	-	-	-	-

Looking both at table 4.1 and figure 4.1 for the experimentally obtained values, the following facts are observable;

(a) That at each load/stress level there is an increase in obtained strain as the fibre content of the mix in the specimen increases;

(b) That as the load/stress level increases, there is also an increase in obtained strain; and

(c) Traversing the table 4.1 diagonally also reveals an increase in strain level as both stress and fibre content of the mix increase.

(d) From figure 4.1, it is seen that inclusion of steel fibres in the fibrous concrete puts additional ductility to the material. This effect is however seen to be more pronounced at higher fibre contents.

The curves for fibre content of 0.5% and 1.0% (by weight) are of doubtful circumstance and are not necessarily considered in this case.

4.1.2 Mode of Failure

All the S.F.R.C. specimens failed in a gradual manner, with the rate of failure becoming more gradual as the fibre content of the mix increased. This is against the mode of failure of the control specimen with zero fibre content in which case a violent and catastrophic failure was observed. The planes of failure were all observed to be parallel to the longitudinal axis and in the case of the fibrous concrete some failure planes were still held together by fibres. It is noteworthy that complete failure of the specimens containing fibres was found to be by fibre fracture rather than pull-out.

4.1.3 Strength Characteristics:

Table 4.2 shows that the strength improvement of the SFRC mixes, at least within the range of fibre content considered, is only marginal for the concrete matrix. In essence, first cracking occurs at the maximum strain attainable by the matrix. After cracking, the fibres take over and the tensile strength of the mix in the specimen is governed by the interfacial shear at the matrix fibre interface Because of the nature and type of fibre used, additional factors come into play. For instance, since the fibres used were in the form of "yarns" vacant spaces could exist between the individual fibre strands, hence when such are used as microreinforcement veins of mortar penetrate through the spaces between these strands thus providing additional 'locking' effect which could lead to additional strength gain, however it is also recognised that the reverse may also be possible especially when this locking breaks down.

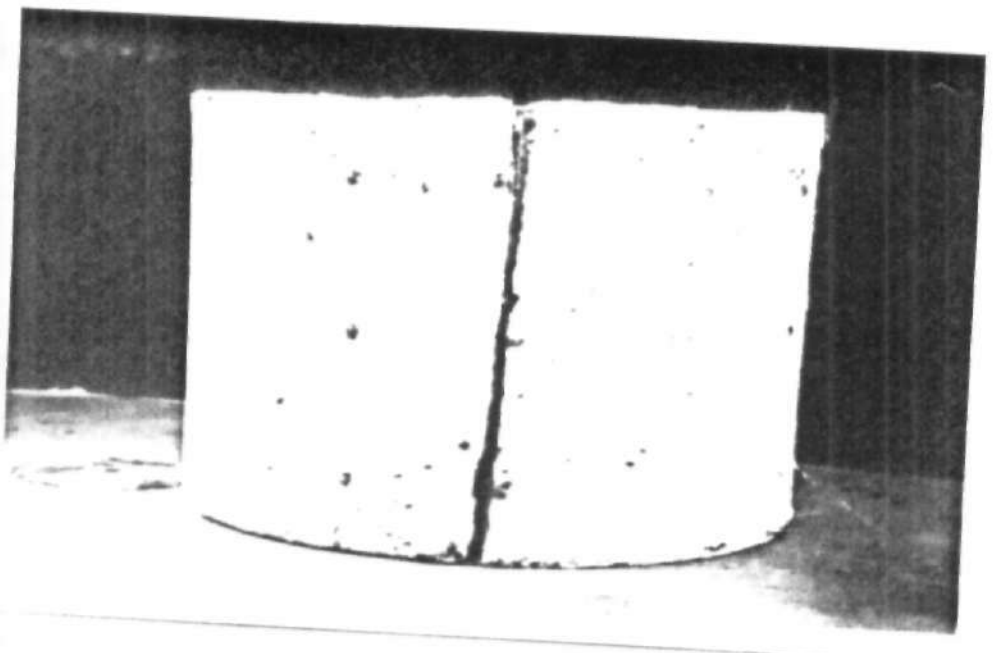


PLATE 4.1: Typical Failure Crack for
Fibrous Concrete Cylinder

Table 4.2 - Comparison of Theoretical and Experimental Values

	A/0.00/0.6	A/0.25/0.6	A/0.5/0/6	A/1.00/0.65	A/1.5/0.7	A/2.0/0.7
1. Vertical failure Pressure (N/mm^2)	30	35.6	31	41	30	26
2. Hor. Failure Pressure (N/mm^2)	0.206	0.279	0.243	0.319	0.236	0.206
3. Failure Stress (N/mm^2)	3.841	4.357	4.213	4.468	4.186	3.359
4. Thickness of failure plane (mm)	13	14	12	15	12	13
5. Theoretical Strength (N/mm^2)	-	4.568	4.813	5.4	6.241	6.732
6. Ratio (3)/(5)	-	0.954	0.875	0.827	0.671	0.499
7. Specific Strength Strength : Density	175	198	187	205	186	152

Secondly as the individual strands are normally twisted to form the yarns, the true length of the fibre is much greater than the measured length which in effect means that a much greater aspect ratio than determined is operational and consequently increases the bond force required to pull out the fibre. Table 4.2 shows both the predicted and the experimentally determined tensile strengths of the mixes.

4.2 Discussion of Test Results:

Table 4.2 shows the ultimate tensile strengths as predicted by the proposed method as well as the experimentally determined strengths for the different micro reinforcement contents. The same table shows the gain in strength of the SFRC with increase in fibre content while figure 4.1 shows the gain in ductility.

From these facts, the following observations could be made:

(a) That essentially, the cracking strength of the matrix per se is not in fact improved; however, after cracking propagation of the crack is retarded due to the bridging action of fibres thus reducing the rate at which the crack propagates;

(b) That before cracking, both the matrix and the micro reinforcement are simultaneously active with the micro reinforcement becoming more active as the stress level increases until the matrix cracks. At this point the overall behaviour along the crack length is governed by the interfacial shear strength along the length of the microreinforcement.

Hence the length of the fibre as well as the interfacial shear determine the mode of failure. If the length of the microreinforcement is greater than the critical length then failure of the whole specimen is governed by fracturing strength of the fibre while if the length is much less than the critical length then failure by pull-out can be expected. The fibres used in this experiment have an average length of 90mm with an aspect ratio of about 137. This value of aspect ratio lies within the recommended limits of 50-150. In all cases for the fibrous concrete, failure was seen to be by fibre fracture;

(c) That the behaviour of the fibrous concrete is governed by the mechanical properties of both the matrix and the micro reinforcement before cracking, but after cracking the behaviour of the specimen is principally governed by the properties of the microreinforcement.

(d) From figure 4.2 it is seen that for a constant stress level the sustainable strain increases as the fibre content increases but as the stress level increases the rate of straining increases even faster. This in physical terms implies that as the fibre content increases, more of the matrix is substituted for. As this proceeds a stage is reached when the micro reinforcement predominates with the original matrix now behaving similarly to a binder as well as a filler which has a parallel in glass-reinforced-plastics.

It can be observed that as the fibre fraction is increased the attainable strain also increases with it for a constant load level up to a maximum at which point the plot becomes practically parallel to the strain axis. It is also noted that as the load level increases the rate of climb of the plot falls. Physically the second observation implies that as the load level increases there is a gradual breakdown of composite action. This breakdown is in any of two ways viz: either there is a breakdown in the interfacial bond or there is yielding of the microreinforcement.

(e) From the analysis of the plot shown on figure 4.3 it is clear that the best specific strength is obtained within the range of 0.925 and 1.05 percent fibre content. As the plot is concave upward it is obvious that this calls for a maximisation function to obtain the best approximation to the correct fibre content and this can be achieved through linear or quadratic programming with maximisation of constraints.

CHAPTER 5

5.0 CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions:

A large amount of experimental result was obtained and analysed in Chapter 4. From the experimental as well as the theoretical values so found the following conclusions were drawn.

(a) That the proposed method in chapter 2 was capable of predicting the tensile strength of the SFRC at least within the limit of the fibre content tested;

(b) That the fibres were effective in that they retard the progress of a running crack thus providing a better and controlled energy absorption;

(c) That because of (b), an increase in ductility was achieved in the composite and this seemed to improve as the fibre content increased;

(d) That even though the actual tensile strength of the matrix was not substantially increased by itself, improved bonding strength between the matrix and the fibres seemed to favour failure by fibre fracture rather than by pull-out.

(e) That due to the effect of the second part of (d) above the tensile strength of the specimen is reached only when the strength of the fibre bridging the crack was reached;

(f) That the gradual loss of experimentally obtained strength was probably related to the thinness of the section. This was because for the thin section used in the experiment, as the fibre fraction increased the tendency for the fibres to cluster together also increased, with the result that the average spacing between the individual fibres decreased also. This subsequently became a source of poor fibre distribution within the cross-section. This, coupled with an increase in the water-cement ratio, resulted in honeycombing of the fibrous concrete which formed a potential source of weakness. This phenomenon was seen to have occurred within the model A/2.00/0.7 which showed a particularly very poor experimental strength as seen in table 4.2 as compared to the theoretically obtained value,

(g) From table 4.2 and figure 4.3 it was concluded that the optimum fibre content in tension lies within 0.925-1.05 percent as against in compression which is generally about 6.4 (2.0% by volume) percent.

5.2 Recommendations:

To fully appreciate and effectively utilize the advantages deriveable from the use of SFRC, particularly with regards to its proposed use in structures in tension, certain properties of the material have to be ascertained. Further, the influence of the different variables that come into play in the tensile strength determination of this material must be fully investigated and their extent of influence established. Therefore, the following areas are thus recommended for further study:

(a) Creep: As creep is a time as well as stress dependent property of materials generally used in engineering works, it is important to understamd and appreciate the creep behaviour of SFRC, particularly moreso when it is considered against the background of its use relative to particular environmental factors like temperature related stresses,etc.

(b) Fatique Characteristics: As the condition of use may be cyclic in nature, it is important to establish the fatique resistance of SFRC. This information will form a reasonable basis for assessment of the material when used where the loading is pulsating as in surge tanks and pressure vessels.

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