A STUDY ON NON LINEAR CONVECTION IN INTERNAL AND EXTERNAL FLOWS WITH EITHER SORET OR DUFOUR EFFECT

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A THESIS SUBMITTED TO THE SCHOOL OF POSTGRADUATE STUDIES, AHMADU BELLO UNIVERSITY, ZARIA, IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE AWARD OF DOCTOR OF PHILOSOPHY DEGREE IN MATHEMATICS

DEPARTMENT OF MATHEMATICS,

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MARCH, 2023

DECLARATION

I declare that work in this thesis titled“**A Study on NonlinearConvection in Internal and External Flows with Either Soret or Dufour Effect”**has been performed by me in the Department of Mathematics.The information extracted from the literature has been duly acknowledged in the text and a list of references provided. No part of this thesis was previously presented for another degree or diploma at this or any other Institution.

_____ _____

Muhammad Nasiru Sarki Date

CERTIFICATION

This thesis thesis titled "**A Study on NonlinearConvection in Internal and External Flows with Either Soret or Dufour Effect"** by Muhammad Nasiru Sarki with Registration number P15SCMT9023 meets the regulationsgoverning the award of degree of Doctor of Philosophy in Mathematics of Ahmadu Bello University, and is approved for its contribution to knowledge and literary presentation.

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DEDICATION

Dedicated to all my family members and friends

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NOMENCLATURE

T = Dimensionless temperature of the fluid

T' = Dimensional fluid temperature

T'_w = Temperature at the porous plate $(y = 0)$

T'_0 = Temperature at the porous plate $(y = h)$

T_∞ = Ambient value of T

C = Dimensionless fluid concentration

C' =Dimensional fluid concentration

C'_w = Concentration at the porous plate $(y = 0)$

C'_0 = Concentration at the porous plate $(y = h)$

C_∞ = Ambient value of C

D = Coefficient of mass diffusion

A^* = Nonlinear density variation with temperature (NDT)

B^* = Nonlinear density variation with concentration (NDC)

D_f = Dufour number (Diffusion thermo effect)

$S = D_t$ = Soret effect (Thermal diffusion effect)

N = Sustentation parameter (buoyancy ratio parameter)

D^* = Dimensional coefficient of the diffusion thermo effect

g = Acceleration due to gravity

K = Dimensionless chemical reaction parameter

k_e = Chemical reaction parameter

k = Thermal conductivity of the fluid

Gc = Mass Grashof number

Gr = Thermal Grashof number

Pr = Prandtl number

Sc = Schmidt number

U = Dimensionless velocity

u_w = Velocity of the vertical porous plate at $(y=0)$

x = Spatial co-ordinate along the vertical porous plate

y = Spatial co-ordinate normal to the vertical porous plate

Y =Dimensionless co-ordinate perpendicular to the plate

V_0 = Suction/injection velocity

τ_0 = Dimensionless skin friction at $(Y=0)$

τ_H = Dimensionless skin friction at $(Y=H)$

Nu_0 = Dimensionless Nusselt number at $(Y=0)$

Nu_H = Dimensionless Nusselt number at $(Y=H)$

Sh_0 = Dimensionless Sherwood number at $(Y=0)$

Sh_H = Dimensionless Sherwood number at $(Y=H)$

Greek Symbols

β_0 , β_1 = Thermal expansion coefficients

β_0^* , β_1^* = Solutal expansionCoefficients

υ = Kinematic viscosity

α = Thermal diffusivity

ABSTRACT

In this study, fully developed steady nonlinear free convection heat and mass transfer flow near a vertical moving porous plate with chemical reaction, in the presence of Dufour and Soret effectshave been analytically investigated, by taking into consideration the nonlinear convection parameter. The dimensionless momentum, temperature and concentration equations for the present physical situation arepresented under relevant boundary conditions and solved exactly in terms of exponential functions. The effect of various flow parameters entering into the problem such as nonlinear density variation with temperature parameter, nonlinear density variation with concentration parameter, Soret effect, Dufoureffect, Prandtl number, Schmidt number, chemical reaction parameter and sustentation parameter are discussed with the aid of line graphs, contours and tables. The analysis of the result shows that the velocity and skin-friction have higher values in case of nonlinear variation of density with temperature and concentration in comparison to linear variation of density with temperature and concentration. It has beenobserved that the velocity and skin-friction increase withincrease in Soret and Dufour parameters.

CHAPTER ONE

INTRODUCTION

1.1 Background of the Study

The movement of heat and mass transfer has been a subject of studies due to the increasing demand associated with the transport processes existing in nature and industrial applications. The study of such processes helpsin the field of transportation, microelectronic and manufacturing industries. It is alsoof great importance in various fields of Science, such as Chemical Engineering and Medical Sciences. There are many practical and experimental applications available in literature,which can be modelled, remodelled,or approximated as transport through a vertical channel under different conditions. Large numbers of studies were carriedouton various kinds of fluid at different situations in heat transfer processes. However, heat and mass transferplay an important role in several manufacturing industries and engineering processes, such as the design of fins, steel rolling, nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, spacecraft design, satellites and polymer.Das *et al.* (1994) and Subhas*et al.*(2010).Therefore,a clear picture of the behaviour and interaction between the energy and mass fluxes is necessary to control these processes.

Comprehensive research works were carriedout connected to different kinds of fluid under differentconditions and various types of channels.We find several industrial applications of magnetic field such as polymer technology and metallurgy Ambethkar and Singh (2011). In recent years, the problem of two-dimensional free convection flow along a vertical plate with different boundary conditions has attracted the attention of many researchers. This is connected with a wide range of naturally occurring phenomena and practical applications. The combined heat and mass transfer flow in linear free convection in a vertical channel has a wide range of applications in the field

of Science and Engineering. Astudyon the steady free-convective flow along a semi-infinite vertical plate by integral method was analysed by Das *et al.* (1994). Subsequently, a study on the Soret effect (thermal diffusion) and Dufour effect (diffusion thermo)in free convection and mass transfer flow for Stokes problem of an infinite vertical plate, taking into account mainly the Soret effectwas then conducted by Jha and Singh,(1990). The problem was connected with the effect of free convection currents caused by temperature difference only.

The flow is also affected by the concentration gradient. For example, in atmospheric flows there exists a difference in the water concentration and hence the flow is affected by such concentration difference. The effect of mass transfer on transient free convective flow along a semi-infinite vertical isothermal plate, were studied with the help of explicit finite difference method by Palani and Srikanth, (2009). They found that the mass of fluidmay also be diffused at a constant rate from the surface.

In many transport processes existing in nature and in industrial applications, heat and mass transfer is often due to buoyancy effects caused by density variations with temperature and density variations with concentration. The study and investigation of such processes are useful for improving a large number of chemical technologies, such as polymer production, enhanced oil recovery, underground energy transport, manufacturing of ceramic and food processing. Therefore, a clear understanding ofthe nature and behaviour of interaction between heat flux and mass fux is necessary to ascertain these processes. A phenomenon of thermal diffusion in liquids was presented by Soret 1879, (Platten, 2006),whoobserved that mass transfercan be generatedby temperature gradient. A reciprocal effect, in which heat transfer is caused by the

gradient of species concentration, was developed by Dufour in 1872, (De Groot and Mazur, 2011).The thermal diffusion (Soret effect) is a relative motion of the components of a gaseous mixture or solution. It has been proved that an energy flux can be generated not only by temperature gradient but also by composition gradients as well. When heat transfer is caused by concentration gradient, it is called diffusion-thermo (Dufour effect) and mass transfer being created by temperature gradients in such case is called thermo-diffusion (Soret effect). However, the thermal diffusion and diffusion thermo effects have been found to be very important, as the thermal diffusion effect is utilized for isotope separation and gaseous mixture with very light molecular weight (H_2, He)and for medium molecular weight (N_2, air).Hence, the diffusion thermo effect constitutes a substantial effect such that it cannot be ignored. Moreover, the thermo-diffusion and diffusion-thermo effects are much neglected in heat and mass transfer processes, since the order of magnitude is smaller than the effect described by Fourier's and Fick's laws. However, such effects become significant when the species having lower density than the surrounding liquids are introduced at surface of fluid medium Moorthy andSenthilvadivu,(2012).

The necessary part of nonlinear convection or nonlinearBoussinesq approximation is that, there are flows in which the temperature dependent density and concentration dependent density relation vary linearly with temperature and concentration. In such cases, it is linear density variations with temperature and concentration. Suppose that the temperature and spacies concentration of fluid and the surface are moderate and very high, the density varies nonlinearly with temperature and concentration. For example, the heat released by the viscous dissipation is one among the reasons for the density temperature and concentration relationship to become nonlinear.

Whenever there is chemical reaction between two species, heat is assumed to be generated. In most chemical reaction processes,the reaction rate depends on the concentration of species itself. A reaction is said to be first order if the rate of reaction is directly proportional to concentration Gebhart and Pera, (1971). Chemical reactions can be modelled as either homogeneous or heterogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. A homogeneous chemical reaction is one that takes place in a solution, where the species generatedare the same as internal source of heat generation. On the other hand, a heterogeneous reaction takes place in an interface,but in the present study we consider first order homomgenous chemical reaction. The study of heat and mass transfer with chemical reaction is of great practical importance to Engineers and Scientists because of its universal occurrence in many branches of Science and Engineering. Chamkha (2010) studied the effect of mass transfer on the flow past impulsively started infinite vertical plate with melting and radiation in vertical surface. Therefore, heat and mass fluxes applications are widely used in industries and science related fields.

1.2 Statement of the Problem

The influence of Soret orDufour effects on nonlinear natural convection heat and mass transfer flow in vertical channel is of great importance in the field of Engineering, Science and Technology.Different approaches to the problemwhich were connected and associated withSoret effect, Dufour effect, linear and nonlinear have been discussed byTayeba *et al.* (2019)and Rajakumar *et al.*(2020).Jha and Ajibade (2011) gave analytical solutions for natural convection and mass transfer flow in vertical channel formed by two vertical parallel plates, considering the Dufour effect. It was found

thatthe steady state of the problem was independent of the Dufour effect,andthe Dufour effect contributed towards enhancing the temperature of the working fluid, which resulted in a significant reduction in time taken to reach the steady state. Muthucumaraswamy (2002) reported analytical solutions for natural convection and mass transfer flow near a moving infinite vertical porous plate in the presence of chemical reaction for linear density variation with temperature LDT and concentrationLDC. Also Partha (2010) studied nonlinear convection in a non-Darcy porous medium due to nonlinear density variation in temperature and concentration. The purpose of this study is to generalize the model used in Muthucumaraswamy (2002) to includenonlinear density variation with temperature and concentration, Dufour effect and Soret effect. The influence of density variation with temperature and concentration are restricted to the body-force term only, in accordance with the nonlinear Boussinesq approximation approach.

1.3 Aim and Objectives of the Study

The aim of this research is to investigate the enhancement of heat and mass transfer flow rate on nonlinear natural convection flow in vertical channel withSoret, Dufour effect and chemical reaction. This aim will be achieved through the following specific objectives,

i. determination of the nonlinear natural convection and mass transfer flow near a vertical moving porous plate with chemical reaction and Soret effect.
ii. investigation of the combined effect of chemical reaction and Soret effect on nonlinear free convection mass transfer Couette flow in a vertical channel.
iii. examinationof the chemical reaction and Dufour effects on nonlinear free convection heat and mass transfer flow near a vertical moving porous plate.

iv. analysis on the influence of Dufour and chemical reaction effects on nonlinear natural convection heat and mass transfer Couette flow in a vertical channel.

v. exploring of the impact ofSoret effect on nonlinear natural convection mass transfer flow near an infinite vertical porous plate with chemical reaction.

vi. discussion on the effect of nonlinear natural convection mass transfer flow in a vertical channel with chemical reaction and thermal diffusion effect.

vii. inspection of the effects of Dufour effect on nonlinear natural convection heat and mass transfer flow near an infinite vertical porous plate with chemical reaction.

viii. discovery of the influence of chemical reaction and Dufour effect on nonlinear natural convection heat and mass transfer flow in a vertical channel.

1.4 Methodology of the Study

The methodology adopted in this investigation includes:

i. Reviewing and modifying the related literature to include stated objectives.

ii. Solving the ordinary differential equations by the method of undetermined coefficients

iii. Obtaining contours, line graphs and tables to verify the effects of various governing parameters.

iv. Discussion and interpretation of the effects of each of the governing parameters on flow formation.

v. Justifying the accuracy of results by comparing with published works in literature.

1.5 Motivation

Since Dufour orSoret effects contribute towards enhancing convection process, thusheat and mass transfer flow tend to be faster in the presence of the nonlinear convection parameter. The investigation of such process helps in the fields of heat transfer, microelectronic and manufacturing industries. The Soret and Dufour effects have been found to be very important, as Soret effect is utilized for isotope separation, in gaseous mixture and in composition mixture.The Dufour effect constitutes a substantialeffect such that it cannot be ignored. Different kinds of base fluids such as water, ethylene glycol and engine oil have very low thermal conductivity. The process of heat and mass transfer in these base fluids can be enhanced by improving the thermal properties, such as density, thermal conductivity, specific heat and viscosity. All these motivated us to carry out this study. It has been shown in this study that the effects of Soret, Dufour and chemical reaction parameters on nonlinear natural convection heat and mass transfer flow past near an infinite vertical porous plates are very significant.

1.6 Scope and Limitation

The scope of this work encompasses laminar flow, with vertical single and channel porous plates only. Exact solutionsfor the velocity, concentration and temperature have been obtained,theresults have been used to computeand draw contours, line graphs, and tables. No experimental work has been carried out in this study. The limitation of this work is non-consideration of the combined effects of Soret and Dufour parameters on nonlinear natural convection heat and mass transfer process.

1.7 Significance of the Study

This study reveals theanalytical solutionfor the steady fully developed nonlinear natural convection, heat and mass transfer flow near a vertical moving or stationary porous plate in the presence of chemical reaction, Soret effect or Dufour effect. Theresults obtained in this researchwouldbridge the gap between theoretical works in the literature and practices in chemical engineeringand industrial applications.The present work will be extremely usefulfor validation of laboratory experiments.Moreover, it is hoped that the results obtained in this work will be of great importance to engineers, more especially in cooling of electronic instruments and devices, and also serve as an improvement on previous studies.

1.8 Basic Definitions

1.8.1 Boussinessq approximation: Boussinessq approximationis the assumption that the fluid flow is considered under little variation of temperature and density.Khan *et al.,* (2017)

1.8.2 Channel: channels are passages through which a fluid passes while moving from one point to another in specified boundaries. There can be different orientations of the plates such as vertical, horizontal among others. Jha and Ajibade (2011)

1.9.1 Fluid: is a substance which is capable of flowing or is a substance which deforms continuously when subjected to external shearing force Rajput (1998).

1.9.2 Fluid Flow: is the motion of a gas or liquid. Zhang *et al.* (2010).

1.9.3 Steady Flow: A flow is said to be steady if the velocity and other flow parameters such as pressure and acceleration are independent of time but may depend on position.

1.9.4 Couette flow: Couette flow is the laminar flow of a viscous fluidin the space between two parallel plates, one of which is moving relative to the other. Attia (2006).

1.9.5 Laminar flow: Laminar flow occurs when fluid flows at low velocity, in parallel layers with no disruption between the layers.

1.9.6 Fully developed flows: Implies that the velocity profile does not change in the fluid flow direction. When the boundary layers expand to fill the entire pipe, the developing flow transforms to fully developed flow.

1.10 Heat and mass transfer

1.10.1 Heat: is defined in physics as the transfer of thermal energy across a well defined boundary around a thermodynamic system Potelnicu (2004).

1.10.2 Heat Transfer: Is transfer of thermal energy from one physics system to another. Heat transfer is classified into three modes namely; Heat conduction, convection and Radiation. Heat conduction is the transfer of energy between objects that are in physical contact. And convection is the transfer of energy between objects and its environment due to fluid motion.While radiation is the transfer of energy from a body by means of emission Lie hard and John (2008).

1.10.3 Mass Transfer: is the transfer of mass as a result of the species concentration difference in a system/ mixture. So long as there is concentration differences mass transfer will occur Rajput (1998).

1.10.4 Convection: This refers to fluid motion caused by an external force such as gravity. Alternatively, refers to as heat transfer in a gas or liquid by the circulation of currents from one region to another.

1.10.5 Natural Convection: Natural convection is a mechanism in which fluid motion is generated only by density difference in fluid caused by temperature gradient.

It is a type of heat transport in which the fluid motion is not generated by an external source (pump, fan, suction devices and so on). Gebhard and Pera (1971)

1.10.6 Porous Medium: A porous medium or porous material is a solid (often called frame or matrix) permeated by an interconnected network of pores (voids) filled with a fluid (gas or liquid).

1.11 Dimensionless Parameters: is a quantity without an associated physical dimension

1.11.1 Chemical Reaction: Is a process by which atoms or group of atoms are redistributed, resulting in a change in the molecular composition of substances. Muthucumaraswamy (2002)

1.11.2 Schmidt Number (Sc): is the ratio of shear component for diffusivity to viscosity density for mass transfer D. It physically relates the thickness of the hydrodynamic layer and mass transfer boundary layer $Sc = \frac{\nu}{D} = \frac{\mu}{\rho D}$

Muthucumaraswamy (2002)

1.11.3 Thermal diffusion (D_t): It is the mass flux generated by temperature gradient, leading to the thermal diffusion effect (Soret effect). Platten (2006) it is defined as $\left(D_t = \frac{D^* \left(T'_w - T'_\infty \right)}{\left(C'_w - C'_\infty \right)} \frac{V_o}{\nu} \right)$

1.11.4 Diffusion-thermo (D_f): Diffusion-thermo is the energy flux generated not only by temperature gradient but also by concentration gradient leading to the Dufour effect. De Groot and Mazur (2011)

It is defined as $\left(D_f = \frac{D^* \left(C_w' - C'_0 \right)}{\left(T'_w - T'_0 \right)} \frac{V_o}{\alpha} \right)$

1.11.5 Nusselt Number (Nu): Nusselt numberis the rate of heat transfer between the fluid and the surfaces of the channel. It is the ratio of heat transfer by convection to the heat transfer by conduction

1.11.6 Skin-friction (C_f): Skin-friction is defined as the friction (drag) between a fluid and the surface of a solid moving through it or between a moving fluid and its enclosing surface.

1.11.7 Prandtl number (Pr): This is a dimensionless number approximating the ratio of kinematics viscosity (ν) to thermal diffusivity (α).

$$\Pr = \frac{\mu C_p}{k} = \frac{\rho \nu C_p}{k} = \frac{\nu}{k \diagup pC_p} = \frac{\nu}{\alpha}$$

Prandtl number provides a measure of the relative effectiveness of the

momentum and energy transport by diffusion.

1.11.8 Sherwood Number (Sh): Sherwood numberis the rate of mass transfer between the fluid and the surfaces of the plates. It is the ratio of mass transfer by convection to mass transfer by diffusion

1.11.9 Grashof number (Gr): The Grashof number (Gr) is a dimensionless number in fluid dynamics and heat transfer which approximates the ratio of the buoyancy toviscous force acting on a fluid. It frequently arises in the study of situations involving natural convection. It's believed to be named after Franz Grashof. It is defined as $Gr = \frac{g\beta\upsilon(T_i - T_0)}{V_o^3}$

1.11.10 Modified grashof number (Gc): The modified grashof number (Gc) is a dimensionless number in fluid dynamics and mass transfer problems. In the case

of mass transfer, natural convection is by concentration gradients rather than temperature gradients. Muthucumaraswamy (2002) It is defined as

$$Gc = \frac{g\beta^{*}\upsilon\left(C_i - C_0\right)}{V_o^{3}}$$

1.11.11Buoyancy ratio (N): Buoyancy is property of an object that enables it to float on the surface of a liquid, or ascend through and remain freely suspended in a fuid such as the atnmosphere, quatitatively, it may be expreesd as the ratio of the specific weight of the fluid to the specific weight of the fluid to the specific weight of the object; or, in another manner by the weight of the fluid displaced minus the weight of object.It is defined as

$$\left(N = \frac{\beta_o^{*}\left(C_w' - C_0'\right)}{\beta_o\left(T_w' - T_0'\right)} \right)$$

1.11.12Non-linear density variation: Non-linear convection is the assumption that the fluid flow is considered under moderate, very high temperature and concentration variations with density Tayeba *et al.* (2019)

$$\left(B^{*} = \frac{\beta_1^{*}}{\beta_0^{*}}\left(C_w' - C_\infty'\right) \right)$$

$$\left(A^{*} = \frac{\beta_1}{\beta_0}\left(T_w' - T_\infty'\right) \right)$$

1.12 Outline of the Thesis

The thesis is divided into five chapters as follows:

Chapter 1 outlines what the thesis is all about, that is, it provides the background of the study and clearly establishes the gap in the literature that study fills. It also includesthe statement of the problem, aim and objective of the study, methodology, motivation, significant of the study,scope and limitation among others.

Chapter 2 summerizes the relevant literature which is associated with the research work, and bringing out where the gaps in the literature are, andhow our research helps to fill in these gaps.

Chapter 3 presents the methods we used in solving the governing mathematical model equations. In the chapter also, the governing equations describing the physical situation have been modified and solved.

In chapter 4 we analyse the findngs of our study. Contours, line graphs and tables have been used in the presentation of results. The findings of the study are discussed in details, which show the contributions of our study to the existing literature.

Chapter 5 summarizes the thesis in a concise form.It also gives conclusion that gives asummary of all answers to the research questions. It alsohighlights the implications of all the findings in the study and presents recommendations for future research

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CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

In this chapter,relevant literaturewhich is directly related to the research work has been considered.The history, philosophy, procedures and phenomena that are connectedor associated with the present work are also considered. Furthermore, a review is made for nonlinear natural convection, thermal diffusion, diffusion thermo and chemical reaction effects.

2.2 Nonlinear Natural Convection

The study of nonlinear natural convection heat and mass transfer is gaining attention due to its interesting applications in the field of Engineering and Sciences. Cheng and Minkowcyz (1997) discussed the natural convection flow formation near a vertical plate embedded in a porous media assuming linear density variation with temperature. The effect of lateral mass flux on natural convection boundary layers in saturated porous media under linear Boussinesq approximation has been analyzed by (Cheng, 1977). In an interesting article, Bejan and Khair (1985) discussed the natural convection and mass transfer flow in a porous media assuming linear density variation with temperature and concentration.They found that the heat and mass transfertook place near a thin layer. However, if the temperature and concentration differences are appreciably large, the densities vary nonlinearly with temperature and concentration, leading to nonlinear density variation with temperature and concentration. The density variations caused by temperature and concentration differences, have many important applications such as, cooling of electronic instruments and devices, electrochemical processes, and in solar energy collectors.Some comprehensive studies related withthe current investigation are

found in (Angirasa *et al.*, 1977; Lai and Kulacki, 1991; Latour *et al.*, 1976; Yen, 1974; Nelson and wood(1989), Borght (1980);Lee and Yan, (1994); Magyari and Keller, (1998); Murthy, (2000); Singh and Poul (2006);Mustafa *et al.*, (2014);Kameswaran *et al.*,(2014); Khan *et al.*,(2017).Heat and mass transfer in the boundary layers on an exponentially stretching continuous surface have been investigated by Magyari *et al.*,(2001).The combined effects of thermal diffusion and diffusion thermo in non-Darcy porosity have been investigated by Partha *et al.*,(2006). Partha (2010) considered nonlinear convection due to heat and mass transfer.The effects of nonlinear temperature difference, nonlinear concentration difference, the thermal diffusion and diffusion thermo on heat and mass transfer has been found to be more significant in Darcy porosity compared with those in non-Darcy porosity.The unsteady free convection flow of nanofluid over a vertical plate due to time exponentialtemperature has been reported by Abdul Rauf and Yasir,(2018).

The effects of the Brownian motion and the thermophoresis had been considered in the model. The effects of various physical parameters involved in the problem, such as Lewis number, Prandtl number, buoyancy ratio parameter, thermophoresis parameter and Brownian motion parameter on the concentration profile, temperature and velocity profiles were presented graphically and in tables. It was found that decrease in values of Brownian diffusion coefficient, and, for every increase in values of Lewis number, the temperature and concentration of particles decrease. Ram *et al.* (2018)analyzed the steady state solution of the boundary layer problems of two dimensional flow, the governing equations were non-dimensionalized into non-similar form, in order to have an easy way to solve the model equations numerically by the successive linearization method (SLM). It was found that, the velocity, heat and mass transfer rates decrease,

and, there is positive enhancement on the skin friction, thermal and solutal boundary layer thicknesses with increase of inclination angle. Tayeba *et al.* (2019),made efforts to study the effect of nonlinear density and temperature variations (NDT) on the magnetohydrodynamic (MHD) free convective flow of coupled fluids with nanoparticles in a vertical porous channel taking into account the problem with Darcy-Forchheimer flow. The nonlinear partial differential equations governing the flow formation weretransformed into ordinary differential equationsand solved numerically using the finate differences method and accurate model of LOBATTO III. They found that velocity decreases at a great order of magnitude as couple stress parameter increases. This is due to the fact that rotation field of the velocity was produced by the forced fluid. It was foundout that the fluid temperature and nanoparticles concentration were enhanced with increase in couple stress parameter.

Hayat *et al.* (2018), reported investigation on heat transfer analysis for heat generation/absorption and nonlinear thermal radiation under the influence of nonlinear mixed convection flow due to Riga plate with double stratification. They employed the Homotopy perturbation techniqueto transform the partial differential equations into ordinary differential equations and solved. However, it was indicated that enhancement in the velocity and temperature profiles decreased the strength of Lorentz force. Also, the reverse characteristic of stratification was noticed for temperature and concentration distributions.

Shah *et al.* (2017)analyzed free convection flow of a fractional viscous fluid over an infinite vertical plate with exponential heating using a fractional derivative with non-singular kernel. The influence of the dimensionless velocity, temperature profile and

Nusselt number were restricted to body force term only in accordance with Boussinesq approximation approach. It was analysed that the obtained results can generate exact solution for any motion with technical relevance of this type. Moreover, fluid velocity was presented as a sum of its mechanical and thermal components. A semi analytical solution based on the Stehfest's formula for the inverse Laplace transform was also obtained. Finally, the influence of fractional parameter on fluid motion as well as the contributions of mechanical and thermal components of velocity were graphically presented and discussed.

2.3 Thermal Diffusion and Diffusion Thermo Effects

The earliest studies on the Soret and Dufoureffects in fluid flow for transports of heat and mass transfer were developed from the kinetic theory of gases by Chapman and Cowling,(1952), Gebhart and Pera, (1971).

The swiss Scientist Charles Soret discovered in 1879, (Soret, 1879; Platten, 2006) that a salt solution contained in a tube with the two ends at different temperatures did not remain uniform in composition. The salt is more concentrated near the cold end than the hot end of the tube. He concluded that a flux of salt was generated by a temperature gradient resulting in steady state conditions, in a concentration gradient. A reciprocal to that effect was also developed by Dufour in 1873 (De Groot and Mazur, 2011)that an energy flux can be generated not only by temperature gradients but by composition gradients as well.

The investigation on the influence of thermal diffusion effect on transient free convective heat and mass transfer flow was found in the study of Jha and Singh (1990).They examined the free convection and mass transfer flow for Stokes problem of

an infinite vertical plate taking into account the Soret effects. Some research scholars devoted much time and energy to study critically the behaviour and influence of Soret and Dufour effects on heat and mass transfer processes. Notable ones among them are the works of Dursunkaya and Worek (1992),Rastogi and Poulikakos (1995),Jha (1992) and Jha (1990).

Adrian (2004) analysed the effect of MHD on heat transfer in the presence of Soret and Dufour effects. Related studies on the influence of diffusion-thermo effect on transient free convective heat and mass transfer flow in a channel bounded by two infinite vertical parallel plates was conducted by Jha and Ajibade (2011). They discovered that the steady state of the problem was independent of the Dufour effect. The Dufour effect contributed towards enhancing the temperature of the working fluid, which resulted in a significant reduction in time taken to reach the steady state. The double diffusive convection in a two-component couple stress liquid layer with Soret effect was investigated by Malashetty *et al*. (2006) using both linear and non-linear stability analysis. The linear theory is based on normal mode technique and the non-linear analysis is based on a minimal representation of double Fourier series. Their investigation led to the conclusion that the effects of couple stress are quite large, and positive Soret number enhances the stabilitywhile the negative Soret number induces instability.

Many researchworks have been committed to study the effects of diffusion thermo and thermal diffusion on the flow formation heat and mass transfer, especially those by Postelnicu (2004), Hayat *et al.* (2017), Palani and Srikanth (2009), Beg *et al.* (2009), Jha and Ajibade (2010), Wang and Tan (2011), and Hayat *et al.* (2016). Alam and

Rahman (2006) examined the effects of Dufour and Soret on mixed convection flow past a vertical porous flat plate with variable suction. The governing nonlinear partial differential equations have been transformed in to a system of ordinary differential equations by a similarity transformation, and solved numerically by applying Naachtseim-Swigert shooting iteration technique together with sixth order Runge-kutta integration scheme. In view of the importance of above mentioned effects, some works related to the present investigation are found in the literature of Attia (2006) and Partha *et al.* (2006). Vempati and Laxmi(2010) studied the Soret and Dufour effects on unsteady MHD flow past an infinite vertical porous plate with thermal radiation. The expressions for the dimensionless velocity, temperature and concentration have been solved numerically using finite difference element method. The result shows that velocity increases with the increase of the thermal Grashof number and solutal Grashof number in the presence of cooling and heating of the plate. The concentration decreases with an increase in the Schmidt number. Also, an increase in the Soret number leads to increase in the concentration.

Alam *et al.* (2007)studied the diffusion thermo and thermal diffusion effects on unsteady free convection and mass transfer flow along an accelerated vertical porous plate embedded in a porous medium.Their analysis showed that the Dufour and Soret effects appreciably influence the flow field. Hence, they concluded that for fluids with medium molecular weight, the Dufour and Soret effects should not be neglected. Abreu *et al.* (2006) examined the laminar boundary layer flows in the presence of Dufour and Soret effects force and free convection heat and mass transfer flow over flat plate.Alloui *et al.* (2011) reported an investigation of the fully developed natural convection heat and mass transfer of a micropolar fluid in a vertical channel. Asymmetric temperature and

concentration boundary conditions are applied to the walls of the channel. The cases of double diffusion and Soret-induced convection were both considered. Pandya and Shukla (2014)investigated the combined effects of thermal diffusion, diffusion thermo and radiation on time dependent MHD flow past a moving porous plate by considering the effect of viscous dissipation. They concluded that velocity increases rapidly with increase in Soret effect parameter, and also noticed skin-friction to increase with increase in Dufour effect parameter. The exact solution of the problem was found, using Laplace transform techniques for MHD transient natural convection and mass transfer flow.This was for incompressible and electrically conducting fluids past an infinite vertical plate taking into consideration the Soret effect and thermal radiation and was reported byAhmed and Kalita(2012). It was noticed that in the studythatthe thermal radiation effect delayed the fluid flow where by the Soret effect speed up the flow. The viscous forces due to plate were increasedwith the Soret and magnetic field effects, but the thermal radiation effect decreased the skin friction. In addition, the rate of heat transfer at the plate accelerated with the thermal radiation effect. It was also noticed that the presence of radiation parameter and Soret effect resulted in the mass transfer enhancement between the fluid and plate with time independence.

Recently, related relationship on time dependent two dimensional flows past a continuously moving plate in the absence of viscous dissipation effect has been studied by Mukesh and Shukla (2015). Pandya and Yadav, (2015) conducted a similar investigation in the absence of radiation effect and viscous dissipation by taking into account, the thermal diffusion and diffusion thermo effects. They concluded that skin-friction and rate of change of heat increase with increase in thermal diffusion effect, and so also Sherwood number is found to decrease. In another development, a recent article

written byHayat *et al*. (2018) analysedthe effect of nonlinear free and forced convection flow by taking into consideration the thermophoresis and Brownian movement.

Sulochana *et al.* (2017) discussed the effect of thermophoresis and Brownian moment on 2D MHD nanofluid flow over an elongated sheet. Tanveer *et al.* (2017),studied the modified Darcy's law utilization in peristalsis of sisko fluid. They observed decreasing response of magnetic field and increasing impact of Darcy number on fluid velocity and temperature. The consequences of the effect of frictional heating on mixed convection flow of chemically reacting radioactive Casson nanofluid over an inclined porous plate was analyzed by Sulochana *et al.* (2017). Hayat *et al.* (2016) investigated the influences of hall current and chemical reaction in mixed convective peristaltic flow of Prandtl fluid. Furthermore, combined influence of radiation absorption and Hall current effects on MHD double-diffusive free convective flow past a stretching sheet has been studied by Sreedevi *el al.* (2018). Shahid *et al.* (2018)analyzed the chemical reaction for Carreau-Yasuda nanofluid flow past a nonlinear stretching foil considering Joule heating.

Khan *et al.* (2017) explored the mixed convection flow of Eyring-Powell nanofluid over a cone and plate with chemical reactive species. It was observed that concentration profile reduces for higher values of Brownian motion and Lewis number. Sulochana *et al.* (2017) discussed the effect of frictional heating on mixed convection flow of chemically reacting radiative Casson nanofluid over an inclined porous plate. In another article, Samrat *et al.*(2018)examined the impact of thermal radiation and chemical reaction on unsteady 2D flow of magnetic-nanofluids over an elongated plate embedded with ferrous nanoparticles. Sulochana and Ashwinkumar (2017) analyzed the

Carreau model for liquid thin film flow of dissipative magnetic-nanofluids over a stretching sheet.However, in another development hydromagnetic-oscillatory flow of a nanofluid in presence of Hall effects and thermal radiation past vertical plate in a rotating porous medium was studied by Sreedevi *et al.* (2017).

In another work, Sreedevi *et al.* (2018), analyzed combined Soret and Dufour effects on MHD flow with heat and mass transfer past a permeable stretching sheet in the presence of thermal radiation. Transient free convection flow in a vertical channel was studied by Gholiniaa *et al.* (2018), Mousazadeha *et al.* (2018), Marneni*et al.* (2015), Gholiniaa *et al.* (2019), Derakhshana *et al.* (2019), Zangooeea *et al.* (2019), Gholiniaa *et al.* (2019), Ghadikolaei *et al.* (2018), Asma and Sharidan(2015), Rahimi *et al.* (2017) and Hsiao *et al.* (2014). Some works connected to the present investigations are found in the literature, for instance Choudhury and Das (2000), Biswas *et al.* (2018), Chamkha *et al.* (2010), Chamkha and Al-Humoud (2007), Tai and Char (2010), Hatami and Ganji (2014), Biswas *et al.* (2019), Ahmmed and Biswas (2019), John *et al.* (2017), Mittal and Patel (2020), Platten (2006), Kataria and Patel (2018).

Rajakumar *et al.*(2020)studied the Soret effect in the presence of MHD on steady natural convection flow ohmic heating and viscous dissipative in a vertical porous plate.It was reported that increase in the value of Schmidt number, result to decrease in velocity profile. It was found that the velocity rises due to increase of Ion-slip parameter.The influences of Dufour/Soret and thermal radiation effects with non-Darcy and magneto hydrodynamic forces on natural convection non-Newtonian fluids over a vertical plate porous medium were conducted by Chuo (2018). The study shows that increase in the non-Darcy parameter or the magnetic field parameter resulted to

reduction in the local Nusselt number and the local Sherwood number. In addition, increase in the thermal radiation parameter enhanced the local Nusselt number and the local Sherwood number respectively.

2.4 Chemical Reaction Effect

The rate of diffusion can be tremendously altered by chemical reaction. In most cases associated withchemical reaction, the reaction rate depends on the concentration of species itself. A reaction is said to be first order if the rate of reaction is directly proportional to concentration itself. Chemical reactions can be classified as either homogeneous or heterogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. A homogeneous reaction is one that takes place in a solution. The species generation in a homogeneous reaction is the same as internal source of heat generation. On the other hand, a heterogeneous reaction takes place in a restricted area or within the boundary of a phase (or at an interface). It can therefore be treated as a boundary condition similar to the constant heat flux condition in heat transfer. Studies in many chemical engineering processes indicated that there is a chemical reaction between a foreign mass and the fluid. These processes take place in numerous industrial applications such as manufacturing of ceramics, food processing and polymer production.

The combined effects of chemical reaction and mass transfer play an important role in design of chemical processing equipment, food processing, cooling towers, tubular reactors, oxidation of solid materials, and synthesis of ceramic materials, among others in that direction. There are many studies on combined effects of chemical reaction and mass transfer. Soundalgekar (1977)investigatedthe free convection effects on the Stokes

problem for an infinite vertical plate due to temperature difference using convection current. Later, Soundalgekar (1979) extended this problem by incorporating theeffects of mass transfer and free convection currents on the flow past an impulsively started vertical plate. It is found out that the result of decrease in shear stress and mass transfer conductance is due to injecting foreign gases into the fluid flow. Das *et al.* (1994) presented an exact solution to the flow due to impulsive motion of an infinite vertical plate in the presence of chemical reaction, species concentration and constant heat flux at the plate. They observed that by increasing the Schmidt number and chemical reaction parameter, velocity decreases. The analysis of combined fully-developed natural convection heat and mass transfer on the flow formation can be found in the literature (Eckert and Drake,(1972), Soundalgekar and Akolkar (1983); Hossain and Begum (1985); Hajji and Worek (1988)).

Muthucumaraswamy and Ganesan (2001) studied the first-order chemical reaction on flow past an impulsively started vertical plate with uniform heat and mass flux. The dimensionless governing equations are solved by implicit finite difference scheme of Crank-Nicolson type. The velocity was found to be decreasing with an increase in the value of chemical reaction parameter, in the presence of first order chemical reaction.
Muthucumaraswamy (2002) investigated the effects of chemical reaction on a moving isothermal vertical surface with suction. They established that velocity and concentration increase during the generative reaction and decrease in the destructive reaction. There are some investigations related to free convection flow of an electrically conducting fluid with chemical reaction (Chamkha (2003) and Subba *et al.* (2007)). Theystudied the numerical aspect of MHD flow of uniformly stretching vertical permeable surface in the presence of heat generation/absorption and a chemical

reaction. Ibrahim *et al.*(2008),have studied the effect of chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi- infinite vertical permeable moving plate with heat source and suction. They found that the velocity profiles increase due to decrease in chemical reaction parameter, the Schmidt number, heat absorption coefficient, magnetic field parameter and permeability parameter while they increase due to increases in absorption radiation parameter.Ahmad and Kalita (2012)considered Laplace technique on magnetohydrodynamic (MHD) radiating and chemically reacting fluid over an infinite vertical surface.

The transient MHD natural convection flow of a viscous incompressible electrically conducting fluid confined between vertical walls heated/cooled asymmetrically has been studied by (Sarkar *et al.* 2012). They considered two different cases (i) when one of the walls is stationary and (ii) when one of the walls starts to move impulsively. It was observed that magnetic field has a retarding influence whereas buoyancy force has an accelerating influence on the fluid velocity for both stationary wall and likewise for impulsive motion of one of the walls.

CHAPTER THREE

PROBLEM FORMULATION AND MATHEMATICAL ANALYSIS

In reaching the stated objectives of the study in section 1.3, effort has been made to computethe following results:

3.1 Introduction

In this section, fully developed steady natural convection and mass transfer flow near a vertical moving porous plate with chemical reaction and Soret effect (thermal diffusion effect)was analytically investigated by taking into consideration the nonlinear variation of density with temperature and concentration. The dimensionless momentum, temperature and concentration equations for the present physical situation are presented under relevant boundary conditions and solved exactly in terms of exponential functions. The effect of various flow parameters entering into the problem as well as skin-friction are discussed with the aid of line graphs and contours.

3.2 Nonlinear natural convection and mass transfer flow near a vertical moving porous plate with chemical reaction and Soret effect

Consider the nonlinear natural convection and mass transfer flow near a vertical moving porous plate with chemical reaction and Soret effect. The nonlinear density variation with temperature NDT and concentration NDCare assumed to be large and the convection region is thick. The flow is assumed to be in the $x-$ direction, which is taken to be vertically upward along the channel wall, and the $y-$ axis is taken to be normal to the porous plate. It is assumed that the amount of fluid particles removed from the plate is so limited that only fluid particles near the boundary of the plate are sucked away. Initially, the fluid is at rest and is at the constant, ambient temperature, T_∞ and concentration, C_∞. The temperature and concentration at the porous plate increase or

decrease to T_w and C_w respectively. It is also assumed that a chemically reactive species areemitted from the moving porous plate in form of fluid motion, and diffuses into the fluid where it goes into first order chemical reaction.The flow configuration is shown in Figure 3.1.Following Muthucumaraswamy (2002) and considering nonlinear convection due to heat and mass transfer(Partha, 2010).

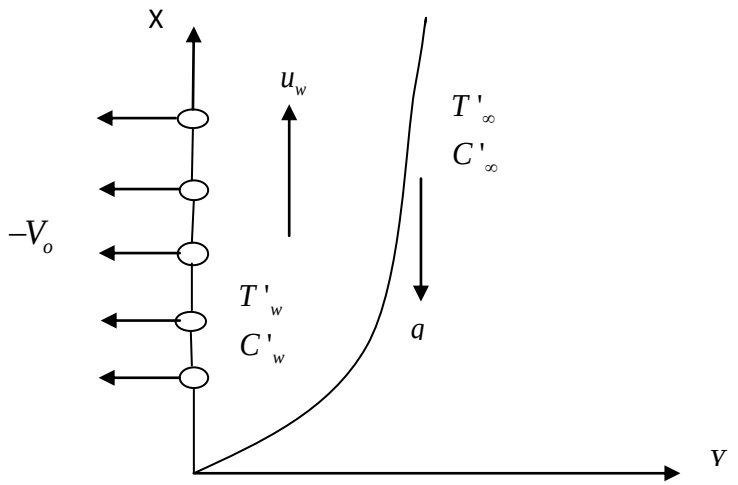


Figure 3.1: Flow configuration

The equations for the present physical situation under nonlinear Boussinesq's approximations are:

$$-V_o \frac{du}{dy} = g\left[\beta_o\left(T'-T'_\infty\right)+\beta_1\left(T'-T'_\infty\right)^2+\beta_o^*\left(C'-C'_\infty\right)+\beta_1^*\left(C'-C'_\infty\right)^2\right]+\upsilon\frac{d^2u}{dy^2},$$

(3.1)

$$-V_o \frac{dT'}{dy} = \alpha \frac{d^2T'}{dy^2} \quad (3.2)$$

$$-V_o \frac{dC'}{dy} = D\frac{d^2C'}{dy^2} - k_e\left(C'-C'_\infty\right)+D^*\frac{d^2T'}{dy^2}, \quad (3.3)$$

The equations (3.1), (3.2) and (3.3) are governing equations used in Muthucumaraswamy (2002); when $\beta_1 = \beta_1^* = 0$ and $D^* = 0$

3.2.1Dimensionless analysis

In dimensional analysis, a dimensionless quantity is a quantity without an associated physical dimension.Dimensionless quantities are widely used in Mathematics, Physics, Engineering, Economics, and in everyday life (such as in counting). This parameter helps in describing and investigating natures and characteristics of fluid. The following dimensionless parameters are used:

$$\left.\begin{aligned} &Y=\frac{yV_o}{\upsilon},\quad \Pr=\frac{\upsilon}{\alpha},\quad Sc=\frac{\upsilon}{D},\quad U=\frac{u}{u_w}\\ &C=\frac{C'-C'_\infty}{C'_w-C'_\infty},\qquad T=\frac{T'-T'_\infty}{T'_w-T'_\infty},\quad A^*=\frac{\beta_1}{\beta_0}\left(T'_w-T'_\infty\right),\\ &B^*=\frac{\beta_1^*}{\beta_0^*}\left(C'_w-C'_\infty\right),Gr=\frac{g\beta_0\upsilon\left(T'_w-T'_\infty\right)}{V_o^{\,2}u_w},K=\frac{k_e\upsilon}{V_o^{\,2}},\\ &S=\frac{D^*\left(T'_w-T'_\infty\right)}{\left(C'_w-C'_\infty\right)}\frac{1}{\upsilon},\\ &\qquad Gc=\frac{g\beta_0^{\,*}\upsilon\left(C'_w-C'_\infty\right)}{V_o^{\,2}u_w}, \end{aligned}\right\} \tag{3.4}$$

The boundary conditions in dimensional form are

$$\left.\begin{aligned} &u=u_w \qquad T'=T'_w \qquad C'=C'_w \qquad at\ \ y=0\\ &u\to 0 \qquad T'\to T'_\infty \qquad C'\to C'_\infty \qquad as\ \ y\to\infty \end{aligned}\right\} \tag{3.5}$$

Equations (3.1) to (3.5) are reduced to the following non-dimensional form:

$$\frac{-V_o^{\,2}}{\upsilon}\frac{dUu_w}{dY}=\frac{\upsilon}{\upsilon^2}\frac{d^2UV_o^{\,2}u_w}{dY^2}+g\left[\begin{aligned}&\beta_o\left(T'_w-T'_\infty\right)T+\beta_1\left(T'_w-T'_\infty\right)T^2+\\&\beta_o^{\,*}\left(C'_w-C'_\infty\right)C+\beta_1^{\,*}\left(C'_w-C'_\infty\right)C^2\end{aligned}\right]$$

$$\frac{-V_o^{\,2}}{\upsilon V_o^{\,2}u_w}\frac{dUu_w\upsilon}{dY}=\frac{\upsilon}{\upsilon^2}\frac{d^2UV_o^{\,2}u_w}{dY^2}+\frac{g\upsilon}{V_o^{\,2}u_w}\left[\begin{matrix}\beta_o\left(T'_w-T'_\infty\right)T+\beta_1\left(T'_w-T'_\infty\right)T^2+\\ \beta_o^{\,*}\left(C'_w-C'_\infty\right)C+\beta_1^{\,*}\left(C'_w-C'_\infty\right)C^2\end{matrix}\right]$$

$$\frac{d^2U}{dY^2}+\frac{dU}{dY}+\frac{g\beta_o\upsilon\left(T'_w-T'_\infty\right)}{V_o^{\,2}u_w}\left[T+\frac{\beta_1}{\beta_o}\left(T'_w-T'_\infty\right)T^2\right]$$
$$+\frac{g\beta_o^{\,*}\upsilon\left(C'_w-C'_\infty\right)}{V_o^{\,2}u_w}\left[C+\frac{\beta_1^{\,*}}{\beta_o^{\,*}}\left(C'_w-C'_\infty\right)C^2\right]=0$$

Where; $A^*=\frac{\beta_1}{\beta_0}\left(T'_w-T'_\infty\right), B^*=\frac{\beta_1^*}{\beta_0^*}\left(C'_w-C'_\infty\right), Gr=\frac{g\beta_0\upsilon\left(T'_w-T'_\infty\right)}{V_o^{\,2}u_w},$

$$Gc=\frac{g\beta_0^{\,*}\upsilon\left(C'_w-C'_\infty\right)}{V_o^{\,2}u_w}.$$

Therefore, we obtain

$$\frac{d^2U}{dY^2}+\frac{dU}{dY}+Gr\left[T+A^*T^2\right]+Gc\left[C+B^*C^2\right]=0, \quad (3.6)$$

From the dimensional energy equation (3.2) we obtain

$$\frac{-V_o^{\,2}\left(T_w-T_\infty\right)}{\upsilon}\frac{dT}{dY}=\frac{\alpha V_o^{\,2}}{\upsilon^2}\frac{d^2T\left(T_w-T_\infty\right)}{dY^2}$$

$$-\frac{dT}{dY}=\frac{\alpha}{\upsilon}\frac{d^2T}{dY^2}$$

Where, $\Pr=\frac{\upsilon}{\alpha}$

$$\frac{1}{\Pr}\frac{d^2T}{dY^2}+\frac{dT}{dY}=0$$

$$\frac{d^2T}{dY^2}+\Pr\frac{dT}{dY}=0, \qquad (3.7)$$

From the dimensional temperature equation (3.3) we get

$$\frac{-V_o^{\,2}\left(C_w - C_\infty\right)}{\upsilon}\frac{dC}{dY} = \frac{D\left(C_w - C_\infty\right)}{\upsilon^2}\frac{V_o^{\,2}d^2C}{dY^2} - ke\left(C'_w - C'_\infty\right)C + D^*\left(T'_w - T'_\infty\right)\frac{d^2T}{dY^2}\frac{V_o^{\,2}}{\upsilon^2}$$

$$-\frac{dC}{dY} = \frac{1}{Sc}\frac{d^2C}{dY^2} - ke\frac{\upsilon}{V_o^{\,2}}C + D^*\frac{\left(T'_w - T'_\infty\right)}{\left(C'_w - C'_\infty\right)}\frac{1}{\upsilon}\frac{d^2T}{dY^2}$$

Where $S = D^*\dfrac{\left(T'_w - T'_\infty\right)}{\left(C'_w - C'_\infty\right)}\dfrac{1}{\upsilon}$, $K = ke\dfrac{\upsilon}{V_o^{\,2}}$, $Sc = \dfrac{\upsilon}{D}$

Hence, we obtain:

$$\frac{d^2C}{dY^2} + Sc\frac{dC}{dY} - KScC + S\frac{d^2T}{dY^2} = 0, \quad (3.8)$$

It is important to mention here that A^* represents nonlinear density variation with temperature parameter (NDT), B^* is nonlinear density variation with concentration parameter (NDC), C is dimensionless species concentration of the fluid, S is Soret effect**,** Gc is modified Grashof number (due to mass transfer)**,** Gr is Grashof number (due to heat transfer)and K is dimensionless chemical reaction parameter, Prandtl number $\Pr$, Schmidt number Sc.

The corresponding boundary conditions in non-dimensional form are

$$\left.\begin{array}{llll} U = 1 & T = 1 & C = 1 & at\ \ Y = 0 \\ U \to 0 & T \to 0 & C \to 0 & as\ \ Y \to \infty \end{array}\right\} \quad (3.9)$$

3.2.2 Solution to the problem

The solutions forordinary differential equations (3.6), (3.7) and(3.8) with boundary conditions (3.9) are obtained by applying the method of undetermined coefficients.

The solution to ordinary differential equations (3.7) is given as

Let $m = \frac{dT}{dY}$

$$m^2 + \Pr m = 0$$

$$m\left(m + \Pr\right) = 0$$

$$T\left(Y\right) = A_1 + A_2 \exp\left(-\Pr Y\right) \quad (3.10)$$

To find the constants in equation (3.10), that is

A_1 and A_2 we apply the boundary conditions as $\begin{matrix} T = 1 \; at \; Y = 0 \\ T = 0 \; as \; Y \to \infty \end{matrix}$

$$1 = A_1 + A_2$$

$$0 = A_1 + 0$$

$$A_1 = 0$$
$$A_2 = 1$$

Therefore the general solution of equation (3.23) is given as

$$T\left(Y\right) = \exp\left(-\Pr Y\right). \quad (3.11)$$

The solution to ordinary differential equation (3.8) isobtained as

$$\frac{d^2C}{dY^2} + Sc\frac{dC}{dY} - KSc\,C = -S\frac{d^2T}{dY^2}$$

The solution to homogenous part of (3.8) is given as

$$C\left(Y\right) = A_3 \exp\left(\delta_1 Y\right) + A_4 \exp\left(-\delta_2 Y\right) - \frac{S\Pr^2 \exp\left(-\Pr Y\right)}{\left[\Pr^2 - \Pr Sc - KSc\right]} \quad (3.12)$$

Where,

$$\delta_1 = \frac{1}{2}\left[-Sc + \sqrt{Sc^2 + 4KSc}\right], \qquad -\delta_2 = \frac{1}{2}\left[Sc + \sqrt{Sc^2 + 4KSc}\right] \quad (3.13)$$

Therefore,

$$C(h) = A_3 \exp(\delta_1 Y) + A_4 \exp(-\delta_2 Y) \quad (3.14)$$

The particular solution to (3.8) is obtained as:

$$C(p) = -\frac{S\,\mathrm{Pr}^2 \exp(-\mathrm{Pr}\,Y)}{\left[\mathrm{Pr}^2 - \mathrm{Pr}\,Sc - KSc\right]} \quad (3.15)$$

$$C(Y) = C(h) + C(p)$$

To find the constant A_3 and A_4 we apply the boundary conditions:

$C = 1$ *at* $Y = 0$

$C = 0$ *as* $Y \to \infty$

$$1 = A_3 + A_4 - \frac{S\,\mathrm{Pr}^2}{\left[\mathrm{Pr}^2 - \mathrm{Pr}\,Sc - KSc\right]}$$

$$0 = A_3 \exp(\delta_1 \infty) + A_4 \exp(-\infty \delta_2) - \frac{S\,\mathrm{Pr}^2 \exp(-\infty\,\mathrm{Pr})}{\left[\mathrm{Pr}^2 - \mathrm{Pr}\,Sc - KSc\right]}$$

In order to satisfy $C = 0$ *as* $Y \to \infty$

We obtain $A_3 = 0$ and $A_4 = 1 + \dfrac{S\,\mathrm{Pr}^2}{\left[\mathrm{Pr}^2 - \mathrm{Pr}\,Sc - KSc\right]}$

The general solution to ordinary differential equation (3.8)is obtained as:

$$C(Y) = \exp(-\delta_2 Y) + \frac{S\,\mathrm{Pr}^2}{a_{00}}\left[\exp(-\delta_2 Y) - \exp(-\mathrm{Pr}\,Y)\right]. \quad (3.16)$$

The solution to ordinary differential equation (3.6) is given as

$$\frac{d^2U}{dY^2} + \frac{dU}{dY} + Gr\left[T + A^* T^2\right] + Gc\left[C + B^* C^2\right] = 0$$

$$\frac{d^2U}{dY^2}+\frac{dU}{dY}=-Gr\left[T+A^*T^2\right]-Gc\left[C+B^*C^2\right] \quad (3.17)$$

But from equation (3.11) we have

$$T=\exp\left(-\Pr Y\right)$$

$$T^2=\exp\left(-2\Pr Y\right) \quad (3.18)$$

From equation (3.16) we have

$$C=\exp\left(-\delta_2 Y\right)+\frac{S\,\Pr^2}{a_{00}}\left[\exp\left(-\delta_2 Y\right)-\exp\left(-\Pr Y\right)\right]$$

Let

$$-\delta=-\ \delta_2=\frac{1}{2}\left[Sc+\sqrt{Sc^2+4KSc}\right]$$

Therefore, we obtain

$$C^2=\left[\exp\left(-\delta Y\right)+a_{01}\exp\left(-\delta Y\right)-a_{01}\exp\left(-\Pr Y\right)\right]$$
$$\left[\exp\left(-\delta Y\right)+a_{01}\exp\left(-\delta Y\right)-a_{01}\exp\left(-\Pr Y\right)\right]$$

$$C^2=a_{02}\exp\left(-2\delta Y\right)+a_{03}\exp\left(-\left(\Pr+\delta\right)\right)+a_{01}{}^2\exp\left(-2\Pr Y\right) \quad (3.19)$$

Substituting equations (3.11), (3.16), (3.18) and (3.19) we obtained

$$\frac{d^2U}{dY^2}+\frac{dU}{dY}=-Gr\left[T+A^*T^2\right]-Gc\left[C+B^*C^2\right]$$

$$\frac{d^2U}{dY^2}+\frac{dU}{dY}=-Gr\left[\exp\left(-\Pr Y\right)+A^*\exp\left(-2\Pr Y\right)\right]$$
$$-Gc\left[\begin{array}{l}\exp\left(-\delta Y\right)+a_{01}\exp\left(-\delta Y\right)-a_{01}\exp\left(-\Pr Y\right)\\ +B^*\left[a_{02}\exp\left(-2\delta Y\right)+a_{03}\exp\left(-\left(\Pr+\delta\right)\right)+a_{01}{}^2\exp\left(-2\Pr Y\right)\right]\end{array}\right]$$

(3.20)

Taking the integral of equation (3.20) we obtained

$$\frac{dU}{dY}+U=-Gr\frac{\exp(-\Pr Y)}{-\Pr}-GrA^{*}\frac{\exp(-2\Pr Y)}{-2\Pr}$$
$$-Gc\frac{\exp(-\delta Y)}{-\delta}-Gc\frac{a_{01}\exp(-\delta Y)}{-\delta}+Gc\frac{a_{01}\exp(-\Pr Y)}{-\Pr}$$
$$-GcB^{*}\frac{a_{02}\exp(-2\delta Y)}{-2\delta}-GcB^{*}\frac{a_{03}\exp(-(\Pr+\delta))}{-(\Pr+\delta)}$$
$$-GcB^{*}\frac{a_{01}{}^{2}\exp(-2\Pr Y)}{-2\Pr}$$

(3.21)

Soving equation (3.21),the general solution of momentum equation is obtained as

$$U(Y)=\exp(-Y)+\frac{GrA^{*}}{2\Pr(2\Pr-1)}\left[\exp(-Y)-\exp(-2\Pr Y)\right]$$
$$+\frac{GcB^{*}a_{02}}{2\delta(2\delta-1)}\left[\exp(-Y)-\exp(-2\delta Y)\right]$$
$$+\frac{GcB^{*}a_{03}}{(\Pr+\delta)\left[(\Pr+\delta)-1\right]}\left[\left(\exp(-(\Pr+\delta)Y)-\exp(-Y)\right)\right]$$
$$+\frac{GcB^{*}a_{01}{}^{2}}{2\Pr(2\Pr-1)}\left[\exp(-Y)-\exp(-2\Pr Y)\right]$$
$$+\frac{Gr}{\Pr(\Pr-1)}\left[\left(\exp(-Y)-\exp(-\Pr Y)\right)\right]+\frac{Gc}{\delta(\delta-1)}\left[\left(\exp(-Y)-\exp(-\delta Y)\right)\right]$$
$$+\frac{Gca_{01}}{\delta(\delta-1)}\left[\left(\exp(-Y)-\exp(-\delta Y)\right)\right]+\frac{Gca_{01}}{\Pr(\Pr-1)}\left[\left(\exp(-\Pr Y)-\exp(-Y)\right)\right].$$

(3.22)

Where, Sh is theSherwood numbercalled mass transfer, whichrepresents the ratio of the convective mass transfer to the rate of diffusive mass transport.It is also a transport material between a boundary surface and a moving fluid. The Sherwood number at boundary surface $(Y=0)$ is defined as

$$Sh=\left(\frac{dC}{dY}\right)_{Y=0}=\frac{S\Pr^{2}}{a_{o}}\left[\Pr-\delta\right]-\delta$$

Therefore, fluid flowing against a surface would experience viscous forces called skin friction. It is the fluid force against the direction of motion of an object. Therefore, the dimensionless skin friction at the surface wall $(Y=0)$ is defined as

$$\tau=\left(\frac{dU}{dY}\right)_{Y=0}=\frac{GrA^{*}}{2\Pr(2\Pr-1)}[2\Pr-1]+\frac{GcB^{*}a_{02}}{2\delta(2\delta-1)}[2\delta-1]$$
$$+\frac{GcB^{*}a_{03}}{(\Pr+\delta)[(\Pr+\delta)-1]}[(1-(\Pr+\delta))]+\frac{GcB^{*}a_{01}{}^{2}}{2\Pr(2\Pr-1)}[(2\Pr-1)]$$
$$+\frac{Gr}{\Pr(\Pr-1)}[(\Pr-1)]+\frac{Gc}{\delta(\delta-1)}[\delta-1]+\frac{Gca_{01}}{\delta(\delta-1)}[\delta-1]$$
$$+\frac{Gca_{1}}{\Pr(\Pr-1)}[1-\Pr]-1.$$

(3.23)

3.3Combined effect of chemical reaction and Soret effect on nonlinear natural convection and mass transfer couette flow

In the present investigation we consider the nonlinear natural convection mass transfer Couette flow in a vertical channel with chemical reaction and Soret effect was considered. The nonlinear density variation with temperature(NDT) and nonlinear density variation with concentration(NDC) are set to be large. The fluid is assumed to be removed through one plate and simultaneously injected through the other plate with a constant velocity, V_0. Both the fluid and porous plates are subjected to a temperature T_0 and concentration C_0. The temperature and concentration of the porous plate situated at $y=0$ suddenly raises to T_w and C_w while the other porous plate at a distance h is maintained at T_0 and C_0 respectively. The flow is assumed to be in the $x-$direction, which is taken to be vertically along the channel wall, and the $y-$axis is taken to be normal to it.It is also assumed that a chemically reactive species isemitted from the

moving porous plate in form of fluid motion, and diffuses into the fluid where it moves into first order chemical reaction.The flow configuration is shown in Figure 3.2.

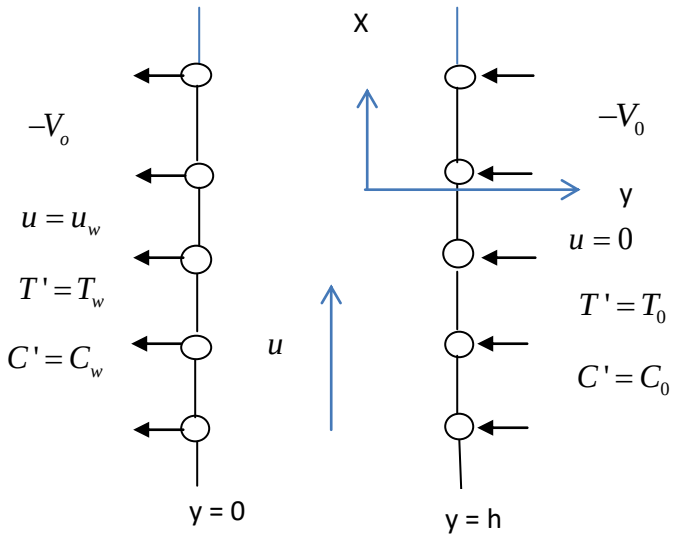


Figure 3.2 Flow configuration

Following the analysis in section 3.1the equations for the present physical situation under nonlinear Boussinesq's approximations are given in equations (3.1), (3.2) and (3.3) respectively.

The dimensionless quantities requiredin equations (3.1), (3.2) and (3.3) are:

$$\left.\begin{aligned} &H = \frac{hV_o}{\upsilon}, \quad \Pr = \frac{\upsilon}{\alpha}, \quad Sc = \frac{\upsilon}{D}, \quad U = \frac{u}{u_w} \\ &C = \frac{C'-C_0}{C'_w-C_0}, \quad T = \frac{T'-T_0}{T_w-T_0}, \quad A^* = \frac{\beta_1}{\beta_0}(T_w-T_0), \\ &B^* = \frac{\beta_1^*}{\beta_0^*}(C_w-C_0), Gr = \frac{g\beta_0\upsilon(T_w-T_0)}{V_o^{\,2}u_w}, K = \frac{k_e\upsilon}{V_o^{\,2}}, \\ &S = \frac{D^*(T_w-T_0)}{(C_w-C_0)}\frac{1}{\upsilon}, \\ &Gc = \frac{g\beta_0^{\,*}\upsilon(C_w-C_0)}{V_o^{\,2}u_w}, \end{aligned}\right\} \quad (3.24)$$

The boundary conditions in dimensional form are

$$\left.\begin{array}{llll} u = u_w & T' = T_w & C' = C_w & at\ \ y = 0 \\ u = 0 & T' = T_0 & C' = C_0 & as\ \ y = h \end{array}\right\} \quad (3.25)$$

Equations (3.1), (3.2) and (3.3) together with (3.24) and (3.25) are reduced to the following non-dimensional form in equations (3.6), (3.7) and (3.8)

The boundary conditions in dimensionless form is

$$\left.\begin{array}{llll} U = 1 & T = 1 & C = 1 & at\ \ Y = 0 \\ U = 0 & T = 0 & C = 0 & as\ \ Y = H \end{array}\right\} \quad (3.26)$$

3.3.1 Solution to the problem

The solution for equations (3.6), (3.7) and (3.8) with boundary conditions (3.26) are obtained by applying the method of undetermined coefficients.

$$T(Y) = \frac{\exp(-\Pr Y) + a_1}{a_0}. \qquad (3.27)$$

where

$$\delta_1 = \frac{1}{2}\left[-Sc + \sqrt{Sc^2 + 4KSc}\right], \qquad -\delta_2 = \frac{1}{2}\left[Sc + \sqrt{Sc^2 + 4KSc}\right]$$

Therefore, we have

$$C(Y) = a_8\left[\exp(\delta_1 Y)\right] + a_7\left[\exp(-\delta_2 Y)\right] - a_3\left[\exp(-\Pr Y)\right]. \quad (3.28)$$

But from equation (3.42) and (3.43) we have

$$T^2 = \left[\frac{\left(\exp(-\Pr Y) + a_1\right)}{a_0}\right]\left[\frac{\left(\exp(-\Pr Y) + a_1\right)}{a_0}\right]$$

$$T^2 = \frac{a_1^{\,2} + 2a_1\exp(-\Pr Y) + \exp(-2\Pr Y)}{a_0^{\,2}} \quad (3.29)$$

$$C^2 = \left[a_8 \left[\exp\left(\delta_1 Y\right) \right] + a_7 \left[\exp\left(-\delta_2 Y\right) \right] - a_3 \left[\exp\left(-\Pr Y\right) \right] \right]$$
$$\left[a_8 \left[\exp\left(\delta_1 Y\right) \right] + a_7 \left[\exp\left(-\delta_2 Y\right) \right] - a_3 \left[\exp\left(-\Pr Y\right) \right] \right]$$

$$C^2 = a_8{}^2 \left(2\delta_1 Y\right) + a_9 - a_{10} \exp\left(\left(\delta_1 - \Pr\right) Y\right) + a_7{}^2 \exp\left(-2\delta_2 Y\right) - a_{11} e \exp\left(-\left(\Pr + \delta_2\right) Y\right) + a_3{}^2 \exp\left(-2\Pr Y\right) \quad (3.30)$$

Subtituting equations (3.27),(3.28), (3.29) and (3.30) into equation (3.17) we have

$$\frac{d^2U}{dY^2} + \frac{dU}{dY} = -Gr\left[T + A^* T^2\right] - Gc\left[C + B^* C^2\right]$$

$$\frac{d^2U}{dY^2} + \frac{dU}{dY} = -Gr\left[\frac{\left(\exp\left(-\Pr Y\right) - a_1\right)}{a_0}\right] - GrA^*\left[\frac{a_1{}^2 + 2a_1 \exp\left(-\Pr Y\right) + \exp\left(-2\Pr Y\right)}{a_0{}^2}\right]$$
$$- Gc\left[a_8\left[\exp\left(\delta_1 Y\right)\right] + a_7\left[\exp\left(-\delta_2 Y\right)\right] - a_6\left[\exp\left(-\Pr Y\right)\right]\right]$$
$$- GcB^*\left[\begin{array}{c} a_8{}^2\left(2\delta_1 Y\right) + a_9 - a_{10}\exp\left(\left(\delta_1 - \Pr\right)Y\right) + a_7{}^2\exp\left(-2\delta_2 Y\right) \\ - a_{11} e \exp\left(-\left(\Pr + \delta_2\right)Y\right) + a_3{}^2 \exp\left(-2\Pr Y\right) \end{array}\right]$$

(3.31)

Taking the integral of both side of equation (3.31) obtained

$$\frac{dU}{dY} + U = -Gr\left[\frac{\left(\exp\left(-\Pr Y\right) + a_1\right)}{-a_0 \Pr}\right] - \frac{GrA^* a_1{}^2}{a_0{}^2} Y - \frac{GrA^* 2a_1 \exp\left(-\Pr Y\right)}{-a_0{}^2 \Pr} - \frac{GrA^* \exp\left(-2\Pr Y\right)}{-2\Pr a_0{}^2}$$
$$- Gc\left[\frac{a_8\left[\exp\left(\delta_1 Y\right)\right]}{\delta} + \frac{a_7\left[\exp\left(-\delta_2 Y\right)\right]}{-\delta} - \frac{a_6\left[\exp\left(-\Pr Y\right)\right]}{-\Pr}\right]$$
$$- GcB^* a_8{}^2 Y - \frac{GcB^* 2a_8 a_7 \exp\left(-\delta Y\right)}{-\delta} + \frac{GcB^* 2a_6 a_8 \exp\left(-\Pr Y\right)}{-\Pr} - \frac{a_7{}^2 \exp\left(-2\delta Y\right)}{-2\delta}$$
$$- GcB^*\left[\begin{array}{c} \frac{a_8{}^2\left(\delta_1 Y\right)}{\delta} + a_9 Y - \frac{a_{10}\exp\left(\left(\delta_1 - \Pr\right)Y\right)}{\left(\delta_1 - \Pr\right)} + \frac{a_7{}^2\exp\left(-2\delta_2 Y\right)}{-2\delta_2} \\ - \frac{a_{11} e \exp\left(-\left(\Pr + \delta_2\right)Y\right)}{-\left(\Pr + \delta_2\right)} + \frac{a_3{}^2 \exp\left(-2\Pr Y\right)}{-2\Pr} \end{array}\right] + C_1$$

(3.32)

From equation (3.32)

$$Ue^{Y} = -\frac{GrA^{*}a_1^{\,2}}{a_0^{\,2}}(Y-1)e^{Y} - Gr\frac{\exp(1-\Pr)Y}{-a_0\Pr(1-\Pr)} - \frac{Gra_1}{a_0}(Y-1)e^{Y} - \frac{GrA^{*}2a_1\exp(1-\Pr)Y}{a_0^{\,2}\Pr(\Pr-1)}$$

$$-\frac{GrA^{*}\exp(1-2\Pr)Y}{2\Pr(2\Pr-1)a_0^{\,2}}$$

$$-Gc\left[\frac{a_8\left[\exp(1+\delta_1)Y\right]}{\delta(1+\delta_1)} + \frac{a_7\left[\exp(1-\delta_2)Y\right]}{\delta(\delta-1)} - \frac{a_6\left[\exp(1-\Pr)Y\right]}{\Pr(\Pr-1)}\right]$$

$$-GcB^{*}\left[\begin{array}{l} \dfrac{a_8^{\,2}(1+\delta_1)Y}{\delta(1+\delta_1)} + a_9(Y-1)e^{Y} - \dfrac{a_{10}\exp\left(1+(\delta_1-\Pr)\right)Y}{(\delta_1-\Pr)\left[(1+(\delta_1-\Pr))\right]} \\ \qquad\qquad + \dfrac{a_7^{\,2}\exp(1-2\delta_2)Y}{2\delta_2(2\delta_2-1)} \\ -\dfrac{a_{11}e\exp\left(1-(\Pr+\delta_2)\right)Y}{(\Pr+\delta_2)\left[(\Pr+\delta_2)-1\right]} + \dfrac{a_3^{\,2}\exp(1-2\Pr)Y}{2\Pr(2\Pr-1)} \end{array}\right] + C_1e^{Y} + C_2$$

(3.33)

Solving equation (3.33) we obtain

$$U = -\frac{GrA^{*}a_1^{\,2}(Y-1)}{a_0^{\,2}} - \frac{GrA^{*}2a_1\exp(-\Pr Y)}{a_0^{\,2}\Pr(\Pr-1)} - \frac{GrA^{*}\exp(-2\Pr Y)}{2\Pr(2\Pr-1)a_0^{\,2}} - Gr\frac{(\exp(-\Pr Y))}{a_0\Pr(\Pr-1)} - Gr\frac{a_1}{a_0}(Y-1) - Gc\frac{a_8[\exp(\delta_1)Y]}{\delta(1+\delta_1)} - Gc\frac{a_7[\exp(-\delta_2)Y]}{\delta(\delta-1)} + Gc\frac{a_6[\exp(-\Pr)Y]}{\Pr(\Pr-1)} - GcB^{*}\frac{a_8^{\,2}(\delta_1)Y}{\delta(1+\delta_1)} - GcB^{*}a_9(Y-1) + GcB^{*}\frac{a_{10}\exp((\delta_1-\Pr))Y}{(\delta_1-\Pr)[(1+(\delta_1-\Pr))]} - GcB^{*}\frac{a_7^{\,2}\exp(-2\delta_2)Y}{2\delta_2(2\delta_2-1)} + GcB^{*}\frac{a_{11}e\,\mathrm{xp}((\Pr+\delta_2))Y}{(\Pr+\delta_2)[(\Pr+\delta_2)-1]} - GcB^{*}\frac{a_3^{\,2}\exp(-2\Pr)Y}{2\Pr(2\Pr-1)} + C_1 + C_2e^{-Y} \quad (3.34)$$

From equation (3.34) we obtain

$$U = -a_{26}(Y-1) + a_{27}\exp(-\Pr Y) - a_{28}\exp(-2\Pr Y) - a_{15}\exp(2\delta_1 Y) - a_{18}\exp(-2\delta_2 Y) - a_{23}\exp(\delta_1 Y) - a_{24}\exp(-\delta_2 Y) + a_{17}\exp((\delta_1-\Pr)Y) + a_{19}\exp(-(\Pr+\delta_1)Y) + C_1 + C_2e^{-Y}$$

$$U(Y) = -a_{26}(Y-1) + a_{27}\exp(-\Pr Y) - a_{28}\exp(-2\Pr Y) - a_{15}\exp(2\delta_1 Y) - a_{18}\exp(-2\delta_2 Y) - a_{23}\exp(\delta_1 Y) - a_{24}\exp(-\delta_2 Y) + a_{17}\exp((\delta_1 - \Pr)Y) + a_{17}\exp(-(\Pr + \delta_1)Y) + C_1 + C_2 e^{-Y}$$

(3.35)

To find the constants C_1 and C_2 in equation (3.35) we apply the boundary conditions (3.26) as

$U = 1 \quad at \quad Y = 0$

$U = 0 \quad at \quad Y = H$

Using equations (3.35) and (3.26), then

$$1 = a_{26} + a_{27} - a_{28} - a_{15} - a_{18} - a_{23} - a_{24} + a_{17} + a_{17} + C_1 + C_2$$

$$0 = -a_{26}(H-1) + a_{27}\exp(-\Pr H) - a_{28}\exp(-2\Pr H) - a_{15}\exp(2\delta_1 H) - a_{18}\exp(-2\delta_2 H) - a_{23}\exp(\delta_1 H) - a_{24}\exp(-\delta_2 H) + a_{17}\exp((\delta_1 - \Pr)H) + a_{17}\exp(-(\Pr + \delta_1)H) + C_1 + C_2 e^{-H}$$

We obtain

$$U(Y)=\frac{(1-\exp(-H))-(1-\exp(-Y))}{(1-\exp(-H))}-a_{26}\left\{\frac{(1-\exp(-H))-H(1-\exp(-Y))}{(1-\exp(-H))}+(Y-1)\right\}$$
$$+a_{27}\left\{\frac{(1-\exp(-H))-(1-\exp(-\mathrm{Pr}\,H))(1-\exp(-Y))}{(1-\exp(-H))}-\exp(-\mathrm{Pr}\,Y)\right\}$$
$$-a_{28}\left\{\frac{(1-\exp(-H))-(1-\exp(-2\,\mathrm{Pr}\,H))(1-\exp(-Y))}{(1-\exp(-H))}-\exp(-2\,\mathrm{Pr}\,Y)\right\}$$
$$-a_{15}\left\{\frac{(1-\exp(-H))-(1-\exp(2\delta_1 H))(1-\exp(-Y))}{(1-\exp(-H))}-\exp(2\delta_1 Y)\right\}$$
$$-a_{18}\left\{\frac{(1-\exp(-H))-(1-\exp(-2\delta_2 H))(1-\exp(-Y))}{(1-\exp(-H))}-\exp(-2\delta_2 Y)\right\}$$
$$-a_{23}\left\{\frac{(1-\exp(-H))-(1-\exp(\delta_1 H))(1-\exp(-Y))}{(1-\exp(-H))}-\exp((\delta_1)Y)\right\}$$

$$-a_{24}\left\{\frac{(1-\exp(-H))-(1-\exp(-\delta_2 H))(1-\exp(-Y))}{(1-\exp(-H))}-\exp((-\delta_2)Y)\right\}$$
$$+a_{17}\left\{\frac{(1-\exp(-H))-(1-\exp(\delta_1-\mathrm{Pr})H)(1-\exp(-Y))}{(1-\exp(-H))}-\exp((\delta_1-\mathrm{Pr})Y)\right\}$$
$$+a_{19}\left\{\frac{(1-\exp(-H))-(1-\exp(-(\mathrm{Pr}+\delta_2))H)(1-\exp(-Y))}{(1-\exp(-H))}-\exp(-(\mathrm{Pr}+\delta_2)Y)\right\}$$

(3.36)

To have physical insight for the transport phenomenabetween a boundary surface and the moving fluid, the mass transferis calculated. The Sherwood number at boundary surface $(Y=0)$ and $(Y=H)$ is obtained as

$$Sh_0=\left(\frac{dC}{dY}\right)_{Y=0}=a_3\,\mathrm{Pr}+a_8\delta_1-a_7\delta_2$$

(3.37)

$$Sh_H=\left(\frac{dC}{dY}\right)_{Y=H}=a_3\,\mathrm{Pr}\exp(-\mathrm{Pr}\,H)+a_8\delta_1\exp(\delta_1 H)-a_7\delta_2\exp(-\delta_2 H)$$

To find the fluid force against the direction of motion of an object, the dimensionless skin friction at the surface wall $(Y=0)$ and $(Y=H)$ is obtained as

$$\tau_0=\left(\frac{dU}{dY}\right)_{Y=0}=\frac{(-1)}{(1-\exp(-H))}-a_{26}\left\{\frac{-H}{(1-\exp(-H))}+1\right\}+a_{27}\left\{\Pr-\frac{(1-\exp(-\Pr H))}{(1-\exp(-H))}\right\}$$
$$-a_{28}\left\{2\Pr-\frac{(1-\exp(-2\Pr H))}{(1-\exp(-H))}\right\}-a_{15}\left\{-\frac{(1-\exp(2\delta_1 H))}{(1-\exp(-H))}-2\delta_1\right\}$$
$$-a_{18}\left\{2\delta_2-\frac{(1-\exp(-2\delta_2 H))}{(1-\exp(-H))}\right\}$$
$$-a_{23}\left\{-\frac{(1-\exp(\delta_1 H))}{(1-\exp(-H))}-\delta_1\right\}-a_{24}\left\{\delta_2-\frac{(1-\exp(-\delta_2 H))}{(1-\exp(-H))}\right\}$$
$$+a_{17}\left\{-\frac{(1-\exp(\delta_1-\Pr)H)}{(1-\exp(-H))}-(\delta_1-\Pr)\right\}+a_{19}\left\{(\Pr+\delta_2)+\frac{(1-\exp(-(\Pr+\delta_2))H)}{(1-\exp(-H))}\right\}$$

(3.38)

$$\tau_H = \left(\frac{dU}{dY}\right)_{Y=H} = \frac{\left(-\exp(-H)\right)}{\left(1-\exp(-H)\right)} - a_{26}\left\{\frac{-H\exp(-H)}{\left(1-\exp(-H)\right)}+1\right\}$$
$$+ a_{27}\left\{\Pr\exp(-\Pr H) - \frac{\left(1-\exp(-\Pr H)\right)}{\left(1-\exp(-H)\right)}\right\}$$
$$- a_{28}\left\{2\Pr\exp(-2\Pr H) - \frac{\left(1-\exp(-2\Pr H)\right)}{\left(1-\exp(-H)\right)}\right\}$$
$$- a_{15}\left\{-\frac{\left(1-\exp(2\delta_1 H)\right)}{\left(1-\exp(-H)\right)} - 2\delta_1\exp(2\delta_1 H)\right\}$$
$$- a_{18}\left\{2\delta_2\exp(-2\delta_2 H) - \frac{\left(1-\exp(-2\delta_2 H)\right)}{\left(1-\exp(-H)\right)}\right\}$$
$$- a_{23}\left\{-\frac{\left(1-\exp(\delta_1 H)\right)}{\left(1-\exp(-H)\right)} - \delta_1\exp(\delta_1 H)\right\}$$
$$- a_{24}\left\{\delta_2\exp(-\delta_2 H) - \frac{\left(1-\exp(-\delta_2 H)\right)}{\left(1-\exp(-H)\right)}\right\}$$
$$+ a_{17}\left\{-\frac{\left(1-\exp(\delta_1-\Pr)H\right)}{\left(1-\exp(-H)\right)} - (\delta_1-\Pr)\exp(\delta_1-\Pr)H\right\}$$
$$+ a_{19}\left\{(\Pr+\delta_2)\exp(-(\Pr+\delta_2)) + \frac{\left(1-\exp(-(\Pr+\delta_2))H\right)}{\left(1-\exp(-H)\right)}\right\}$$

(3.39)

3.4 Combined effect of chemical reaction and Dufour effect on non-linear free convection heat and mass transfer flow near a vertical moving porous plate.

Consider the steady fully developed natural convection and mass transfer flow near an infinite vertical moving porous plate in the presence of Dufour effect and first order chemical reaction. The temperature and concentration difference between the porous plate and the fluid far away from the plate are assumed to be large. The $x-$axis is taken along the vertical moving porous plate and the $y-$axis is normal to it. The flow configuration is shown in Figure 3.1. The porous plate and fluids are maintained at

uniform constant temperature T_w' and at a uniform constant species concentration, C_w' of a foreign fluid, which are higher than the corresponding values of temperature, T_∞' and concentration, C_∞' far away from the moving porous plate. It is also assumed that a chemically reactive species isemitted from the moving porous plate in form of fluid motion, and diffuses into the fluid where it moves into first order chemical reaction.The influence of density variation with temperature and concentration are restricted to the body-force term only, in accordance with the nonlinear Boussinesq approximation approach discussed by Partha (2010).

Muthucumaraswamy (2002) reported analytical solutions for natural convection and mass transfer flow near a moving infinite vertical porous plate in the presence of first order chemical reaction for linear temperature density variation and linear concentration density variation. Following Muthucumaraswamy (2002) and Partha (2010), and incorporating of Dufour effect, the mathematical model responsible for the present physical situation under nonlinear Boussinesq approximations is given in equations (3.1), (3.40) and (3.41)

$$-V_o \frac{dT'}{dy} = \alpha \frac{d^2T'}{dy^2} + D^* \frac{d^2C}{dy^2} \quad (3.40)$$

$$-V_o \frac{dC'}{dy} = D \frac{d^2C'}{dy^2} - k_e \left(C' - C'_\infty\right) \quad (3.41)$$

The boundary conditions in dimensional form are

$$\left.\begin{array}{llll} u = u_w & T' = T'_w & C' = C'_w & at \;\; y = 0 \\ u \to 0 & T' \to T'_\infty & C' \to C'_\infty & as \;\; y \to \infty \end{array}\right\} \quad (3.42)$$

Introducing the dimensionless quantities in the above equations defined as

$$\left.\begin{aligned} &Y=\frac{yV_o}{\upsilon},\quad \Pr=\frac{\upsilon}{\alpha},\quad Sc=\frac{\upsilon}{D},\quad U=\frac{u}{u_w}\\ &C=\frac{C'-C'_\infty}{C'_w-C'_\infty},\quad T=\frac{T'-T'_\infty}{T'_w-T'_\infty},\quad K=\frac{k_e\upsilon}{V_o^{\,2}}\\ &D_f=\frac{D^*\left(C'_w-C'_\infty\right)}{\left(T'_w-T'_\infty\right)}\frac{V_o}{\alpha},\ Gr=\frac{g\beta_0\upsilon\left(T'_w-T'_\infty\right)}{V_o^{\,2}u_w}\\ &Gc=\frac{g\beta_0^{\,*}\upsilon\left(C'_w-C'_\infty\right)}{V_o^{\,2}u_w},\ A^*=\frac{\beta_1}{\beta_0}\left(T'_w-T'_\infty\right),\\ &B^*=\frac{\beta_1^*}{\beta_0^*}\left(C'_w-C'_\infty\right), \end{aligned}\right\} \quad (3.43)$$

Equations (3.1), (3.40) – (3.43) are reduced to the following non-dimensional form:

Equation(3.6)

$$\frac{d^2T}{dY^2}+\Pr\frac{dT}{dY}+D_f\frac{d^2C}{dY^2}=0 \quad (3.44)$$

$$\frac{d^2C}{dY^2}+Sc\frac{dC}{dY}-KScC=0 \quad (3.45)$$

The boundary conditions in dimensionless form are

$$\left.\begin{aligned} &U=1 \quad T=1 \quad C=1 \quad at\ \ Y=0\\ &U\to 0 \quad T\to 0 \quad C\to 0 \quad as\ \ Y\to\infty \end{aligned}\right\} \quad (3.46)$$

wherefor $A^*=0,\ B^*=0$ the above mathematical expression represents linear natural convection and mass transfer flow (linear density variation with temperature and concentration), and for, $A^*=1,\ B^*=1$ the expression represents non-linear natural convection and mass transfer flow (nonlinear density variation with temperature and concentration).

3.4.1 Solution to the problem

The solutions of equations (3.6), (3.44) and (3.45) subject to their relevant boundary conditions (3.46) are obtained as:

$$C(Y) = \exp(-\delta Y) \qquad (3.47)$$

$$T(Y) = \exp(-\Pr Y) + \frac{D_f \delta^2}{(\delta^2 - \Pr \delta)}\left[\exp(-\Pr Y) - \exp(-\delta Y)\right] \quad (3.48)$$

$$\begin{aligned} U(Y) = {} & \exp(-Y) + \frac{GrA^* b_{01}}{2\Pr(2\Pr - 1)}\left[\exp(-Y) - \exp(-2\Pr Y)\right] \\ & + \frac{GrA^* b_{02}}{(\Pr + \delta)\left[(\Pr + \delta) - 1\right]}\left[\exp(-Y) - \exp\left(-(\Pr + \delta)Y\right)\right] \\ & + \frac{GrA^* b_{00}{}^2}{2\delta(2\delta - 1)}\left[\left(\exp(-Y) - \exp(-2\delta Y)\right)\right] \\ & + \frac{GcB^*}{2\delta(2\delta - 1)}\left[\exp(-Y) - \exp(-2\delta Y)\right] \\ & + \frac{Gr}{\Pr(\Pr - 1)}\left[\left(\exp(-Y) - \exp(-\Pr Y)\right)\right] \\ & + \frac{Grb_{00}}{\Pr(\Pr - 1)}\left[\left(\exp(-Y) - \exp(-\Pr Y)\right)\right] \\ & + \frac{Grb_{00}}{\delta(\delta - 1)}\left[\left(\exp(-\delta Y) - \exp(-Y)\right)\right] \\ & + \frac{Gc}{\delta(\delta - 1)}\left[\left(\exp(-Y) - \exp(-\delta Y)\right)\right] \end{aligned}$$

(3.49)

The dimensionless rate of heat transfer(Nusselt number) is given by:

$$Nu = \left(\frac{dT}{dY}\right)_{Y=0} = \frac{D_f \delta^2}{(\delta^2 - \Pr \delta)}\left[\delta - \Pr\right] - \Pr \quad (3.50)$$

The dimensionless rate of mass transfer(Sherwood number) is given by:

$Sh = -\delta$ (3.51)

The dimensionless skin friction at the moving porous plate is given by:

$$\tau = \left(\frac{dU}{dY}\right)_{Y=0} = \frac{GrA^{*}b_{01}}{2\Pr(2\Pr-1)}\left[(2\Pr-1)\right] + \frac{GrA^{*}b_{02}}{(\Pr+\delta)\left[(\Pr+\delta)-1\right]}\left[\left((\Pr+\delta)-1\right)\right]$$
$$+ \frac{GrA^{*}b_{00}{}^{2}}{2\delta(2\delta-1)}\left[(2\delta-1)\right] + \frac{GcB^{*}}{2\delta(2\delta-1)}\left[(2\delta-1)\right] + \frac{Gr}{\Pr(\Pr-1)}\left[(\Pr-1)\right]$$
$$+ \frac{Grb_{00}}{\Pr(\Pr-1)}\left[(\Pr-1)\right] + \frac{Grb_{00}}{\delta(\delta-1)}\left[(1-\delta)\right] + \frac{Gc}{\delta(\delta-1)}\left[(\delta-1)\right] - 1$$

(3.52)

3.5Combined effects of Dufour and chemical reaction on nonlinear natural convection heat and mass transfer couette flow in a vertical channel

The problem under investigation is a steady fully developed nonlinear free convection and mass transfer Couette flow formed by two vertical porous plates with chemical reaction and diffusion-thermo effects. The temperature difference and concentration difference between the porous plates and fluid are noticed to be large. Therefore, the convection area is thick. The fluid is assumed to be removed through one plate and simultaneously injected through the other plate with a constant velocity V_0. Both the fluid and porous plates are assumed to be at temperature T_0' and concentration C_0'. The temperature and concentration of the porous plate situated at $y = 0$ suddenly raises to T_w' and C_w' while the other porous plate at a distance is maintained at T_0' and C_0' respectively. The flow was set on to be in the $x-$ direction and put on to be vertically upward along the channel wall and the $y-$ axis is taken to be normal to the porous plate. It is also assumed that a chemically reactive species isemitted from the moving porous plate in form of fluid motion, and diffuses into the fluid where it moves into first order chemical reaction.

The flow configuration is shown in Figure 3.2. The influence of variation of density with temperature and concentration are restricted to the body force term only, in accordance with the nonlinear Boussinesq's approximation approach. To the governing temperature equation, the temperature is driven by concentration, leading to the diffusion-thermo effect. Mathematical model responsible for heat and mass transfer effects on a continuously moving isothermal vertical porous plate in the presence of chemical reaction was discussed in Muthucumaraswamy (2002).

Following Muthucumaraswamy (2002)and considering nonlinear convection due to heat and mass transfer Partha (2010), the governing equations for the present physical situation under nonlinear Boussinesq's approximations are given in equations (3.1), (3.40) and (3.41) respectively

Non-dimensional quantities introduced are given as

$$\left.\begin{aligned} &Y=\frac{hV_o}{\upsilon}=H, \quad \Pr=\frac{\upsilon}{\alpha}, \quad Sc=\frac{\upsilon}{D}, \quad U=\frac{u}{u_w} \\ &C=\frac{C'-C'_0}{C'_w-C'_0}, \quad T=\frac{T'-T'_0}{T'_w-T'_0}, \; A^*=\frac{\beta_1}{\beta_0}\left(T'_w-T'_0\right), \\ &B^*=\frac{\beta_1^*}{\beta_0^*}\left(C'_w-C'_0\right), \quad K=\frac{k_e\upsilon}{V_o^{\,2}} \\ &D_f=\frac{D^*\left(C_w'-C'_0\right)}{\left(T'_w-T'_0\right)}\frac{V_o}{\alpha}, \; Gr=\frac{g\beta\upsilon\left(T'_w-T'_0\right)}{V_o^{\,2}u_w} \\ &Gc=\frac{g\beta^*\upsilon\left(C'_w-C'_0\right)}{V_o^{\,2}u_w} \end{aligned}\right\} \quad (3.53)$$

The boundary conditions are given as

$$\left.\begin{aligned} &u=u_w \quad T'=T'_w \quad C'=C'_w \quad at \;\; y=0 \\ &u=0 \quad T'=T'_w \quad C'=C'_w \quad as \;\; y=h \end{aligned}\right\} \quad (3.54)$$

wherefor $A^* = 0,\ B^* = 0$ the above mathematical expression represents linear natural convection and mass transfer flow (linear density variation with temperature and concentration), and for, $A^* = 1,\ B^* = 1$, the expression represents non-linear natural convection and mass transfer flow (nonlinear density variation with temperature and concentration).

Equations (3.1), (3.40), (3.41) together with (3.53) and (3.54) are reduced to the following non-dimensional equations (3.6), (3.44) and (3.45)

The boundary conditions in non-dimensional form are

$$\left.\begin{array}{llll} U = 1 & T = 1 & C = 1 & at \quad Y = 0 \\ U = 0 & T = 0 & C = 0 & as \quad Y = H \end{array}\right\} \quad (3.55)$$

3.5.1 Solution to the problem

The solutions to equation (3.6), (3.44) and (3.45) is obtained by the method of undetermined coefficients

$$C(Y) = B_1 \exp(\delta_1 Y) + B_2 \exp(-\delta_2 Y)$$

$$\delta_1 = \frac{1}{2}\left[-Sc + \sqrt{Sc^2 + 4KSc}\right] \quad and \quad -\delta_2 = \frac{1}{2}\left[Sc + \sqrt{Sc^2 + 4KSc}\right]$$

To find the contants B_1 and B_2

we apply the boundary conditions as $\left.\begin{array}{lll} C = 1 & at & Y = 0 \\ C = 0 & as & Y = H \end{array}\right\}$

$$1 = B_1 + B_2$$

$$0 = B_1 \exp(\delta H) + B_2 \exp(-\delta H)$$

$$B_1 = \frac{(b_2 - 1)}{(b_2 - b_1)} = b_4$$

$$B_2 = \frac{(1 - b_1)}{(b_2 - b_1)} = b_3$$

$$C(Y) = b_4 \exp(\delta_1 Y) + b_3 \exp(-\delta_2 Y) \quad (3.56)$$

From equation (3.44) we have

$$\frac{d^2T}{dY^2} + \Pr \frac{dT}{dY} = -D_f \frac{d^2C}{dY^2}$$

The homogeneous part of equation (3.44) is obtained as:

$$\frac{d^2T}{dY^2} + \Pr \frac{dT}{dY} = 0$$

$$T(h) = B_3 + -B_4 \exp(-\Pr Y)$$

The particular solution of equation (3.44) is obtained as

$$T(p_1) = \frac{-D_f \delta^2 b_4}{(\delta^2 + \Pr \delta)}$$

$$T(p_2) = \frac{-D_f \delta^2 b_3}{(\delta^2 - \Pr \delta)}$$

$$T(Y) = T(h) + T(p_1) + T(p_2)$$

$$T(Y) = B_3 + B_4 \exp(-\Pr Y) - \frac{D_f \delta^2 b_4 \exp(\delta_1 Y)}{(\delta^2 + \Pr \delta)} - \frac{D_f \delta^2 b_3 \exp(-\delta_2 Y)}{(\delta^2 - \Pr \delta)} \quad (3.57)$$

B_3 and B_4 are constants to be determined.

The general solution of equation (3.67) is given as

$$T(Y) = b_{11} + b_{10} \exp(-\Pr Y) - b_5 \exp(\delta_1 Y) - b_6 \exp(-\delta_2 Y). \quad (3.58)$$

From equation (3.6) we obtain

$$\frac{d^2U}{dY^2} + \frac{dU}{dY} = -Gr\left[T + A^* T^2\right] - Gc\left[C + B^* C^2\right]$$

$$C^2 = b_{12} + b_4^{\,2}\exp\left(2\delta_1 Y\right) + b_3^{\,2}\exp\left(-2\delta_2 Y\right) \quad (3.59)$$

$$\begin{aligned} T^2 = {} & b_5^{\,2}\exp\left(2\delta_1 Y\right) + b_6^{\,2}\exp\left(-2\delta_1 Y\right) + b_{10}^{\,2}\exp\left(-2\,\mathrm{Pr}\,Y\right) + b_{13}\exp\left(-\mathrm{Pr}\,Y\right) \\ & - b_{14}\exp\left(-\delta Y\right) - b_{16}\exp\left(\delta - \mathrm{Pr}\right)Y - b_{17}\exp\left(-\left(\delta + \mathrm{Pr}\right)Y + b_{18}\right) \end{aligned} \quad (3.60)$$

Substituting equations (3.56), (3.58), (3.59) and (3.60) into equation (3.6) we have

$$\frac{d^2U}{dY^2} + \frac{dU}{dY} = -Gr\left[b_{11} + b_{10}\exp\left(-\mathrm{Pr}\,Y\right) - b_5\exp\left(\delta_1 Y\right) - b_6\exp\left(-\delta_2 Y\right)\right]$$

$$-GrA^*\left[\begin{array}{l} b_5^{\,2}\exp\left(2\delta_1 Y\right) + b_6^{\,2}\exp\left(-2\delta_1 Y\right) + b_{10}^{\,2}\exp\left(-2\,\mathrm{Pr}\,Y\right) + b_{13}\exp\left(-\mathrm{Pr}\,Y\right) \\ \\ \quad - b_{14}\exp\left(-\delta Y\right) - b_{16}\exp\left(\delta - \mathrm{Pr}\right)Y - b_{17}\exp\left(-\left(\delta + \mathrm{Pr}\right)Y\right) + b_{18} \end{array}\right]$$

$$-Gc\left[b_4\exp\left(\delta_1 Y\right) + b_3\exp\left(-\delta_2 Y\right)\right]$$

$$-GcB^*\left[b_{12} + b_4^{\,2}\exp\left(2\delta_1 Y\right) + b_3^{\,2}\exp\left(-2\delta_2 Y\right)\right]$$

(3.61)Taking the integral of both side of equation (3.61) we obtain

$$\frac{dU}{dY}+U=-Gr\left[b_{11}Y+\frac{b_{10}\exp\left(-\Pr Y\right)}{-\Pr}-\frac{b_5\exp\left(\delta_1 Y\right)}{\delta_1}-\frac{b_6\exp\left(-\delta_2 Y\right)}{-\delta_2}\right]$$

$$-GrA^*\left[\begin{array}{c}\frac{{b_5}^2\exp\left(2\delta_1 Y\right)}{2\delta_1}+\frac{{b_6}^2\exp\left(-2\delta_1 Y\right)}{-2\delta_1}+\frac{{b_{10}}^2\exp\left(-2\Pr Y\right)}{-2\Pr}+\frac{b_{13}\exp\left(-\Pr Y\right)}{-\Pr}\\ -\frac{b_{14}\exp\left(-\delta Y\right)}{-\delta}-\frac{b_{16}\exp\left(\delta_1-\Pr\right)Y}{\left(\delta_1-\Pr\right)}-\frac{b_{17}\exp\left(-\left(\delta_2+\Pr\right)Y\right)}{-\left(\delta_2+\Pr\right)}+b_{18}Y\end{array}\right]$$

$$-Gc\left[\frac{b_4\exp\left(\delta_1 Y\right)}{\delta_1}+\frac{b_3\exp\left(-\delta_2 Y\right)}{-\delta_2}\right]$$

$$-GcB^*\left[b_{12}Y+\frac{{b_4}^2\exp\left(2\delta_1 Y\right)}{2\delta_1}+\frac{{b_3}^2\exp\left(-2\delta_2 Y\right)}{-2\delta_2}\right]+C_1$$

$$Ue^Y=-Gr\left[\begin{array}{c}b_{11}\left(Y-1\right)e^Y+\frac{b_{10}\exp\left(1-\Pr Y\right)}{-\Pr\left(1-\Pr\right)}-\frac{b_5\exp\left(1+\delta_1 Y\right)}{\delta_1\left(1+\delta_1\right)}\\ -\frac{b_6\exp\left(1-\delta_2 Y\right)}{-\delta_2\left(1-\delta_2\right)}\end{array}\right]$$

$$-GrA^*\frac{{b_5}^2\exp\left(1+2\delta_1 Y\right)}{2\delta_1\left(1+2\delta_1\right)}-GrA^*\frac{{b_6}^2\exp\left(1-2\delta_1 Y\right)}{-2\delta_1\left(1-2\delta_1\right)}$$

$$-GrA^*\frac{{b_{10}}^2\exp\left(1-2\Pr Y\right)}{-2\Pr\left(1-2\Pr\right)}-GrA^*\frac{b_{13}\exp\left(1-\Pr Y\right)}{-\Pr\left(1-\Pr\right)}$$

$$+GrA^*\frac{b_{14}\exp\left(1-\delta_2 Y\right)}{-\delta\left(1-\delta_2\right)}+GrA^*\frac{b_{16}\exp\left(1+\left(\delta_1-\Pr\right)\right)Y}{\left(\delta-\Pr\right)\left(1+\left(\delta_1-\Pr\right)\right)}$$

$$+GrA^*\frac{b_{17}\exp\left(1-\left(\delta_2+\Pr\right)Y\right)}{-\left(\delta_2+\Pr\right)\left(1-\left(\delta_2+\Pr\right)\right)}-GrA^*b_{18}\left(Y-1\right)e^Y$$

$$-Gc\frac{b_4\exp\left(1+\delta_1 Y\right)}{\delta_1\left(1+\delta_1\right)}-Gc\frac{b_3\exp\left(1-\delta_2 Y\right)}{-\delta_2\left(1-\delta_2\right)}$$

$$-GcB^*b_{12}\left(Y-1\right)e^Y-GcB^*\frac{{b_4}^2\exp\left(1+2\delta_1 Y\right)}{2\delta_1\left(1+2\delta_1\right)}$$

$$-GcB^*\frac{{b_3}^2\exp\left(1-2\delta_2 Y\right)}{-2\delta_2\left(1-2\delta_2\right)}+C_1e^Y+C_2 \qquad (3.62)$$

$$Ue^{Y} = -Grb_{11}\left(Y-1\right)e^{Y} - Gr\frac{b_{10}\exp\left(1-\Pr Y\right)}{-\Pr\left(1-\Pr\right)} + Gr\frac{b_{5}\exp\left(1+\delta_1 Y\right)}{\delta_1\left(1+\delta_1\right)}$$

$$+ Gr\frac{b_{6}\exp\left(1-\delta_2 Y\right)}{-\delta_2\left(1-\delta_2\right)}$$

$$- GrA^{*}\frac{b_{5}^{\,2}\exp\left(1+2\delta_1 Y\right)}{2\delta_1\left(1+2\delta_1\right)} - GrA^{*}\frac{b_{6}^{\,2}\exp\left(1-2\delta_1 Y\right)}{-2\delta_1\left(1-2\delta_1\right)}$$

$$- GrA^{*}\frac{b_{10}^{\,2}\exp\left(1-2\Pr Y\right)}{-2\Pr\left(1-2\Pr\right)} - GrA^{*}\frac{b_{13}\exp\left(1-\Pr Y\right)}{-\Pr\left(1-\Pr\right)}$$

$$+ GrA^{*}\frac{b_{14}\exp\left(1-\delta_2 Y\right)}{-\delta_2\left(1-\delta_2\right)} + GrA^{*}\frac{b_{16}\exp\left(1+\left(\delta_1-\Pr\right)\right)Y}{\left(\delta-\Pr\right)\left(1+\left(\delta_1-\Pr\right)\right)}$$

$$+ GrA^{*}\frac{b_{17}\exp\left(1-\left(\delta_2+\Pr\right)Y\right)}{-\left(\delta_2+\Pr\right)\left(1-\left(\delta_2+\Pr\right)\right)} - GrA^{*}b_{18}\left(Y-1\right)e^{Y}$$

$$- Gc\frac{b_{4}\exp\left(1+\delta_1 Y\right)}{\delta_1\left(1+\delta_1\right)} - Gc\frac{b_{3}\exp\left(1-\delta_2 Y\right)}{-\delta_2\left(1-\delta_2\right)}$$

$$- GcB^{*}b_{12}\left(Y-1\right)e^{Y} - GcB^{*}\frac{b_{4}^{\,2}\exp\left(1+2\delta_1 Y\right)}{2\delta_1\left(1+2\delta_1\right)}$$

$$- GcB^{*}\frac{b_{3}^{\,2}\exp\left(1-2\delta_2 Y\right)}{-2\delta_2\left(1-2\delta_2\right)} + C_1 e^{Y} + C_2$$

From equation (3.62) we obtain

$$U = -Grb_{11}(Y-1) - Gr\frac{b_{10}\exp(\Pr Y)}{\Pr(\Pr-1)} + Gr\frac{b_5\exp(\delta_1 Y)}{\delta_1(1+\delta_1)} + Gr\frac{b_6\exp(-\delta_2 Y)}{\delta_2(\delta_2-1)}$$
$$- GrA^*\frac{{b_5}^2\exp(2\delta_1 Y)}{2\delta_1(1+2\delta_1)} - GrA^*\frac{{b_6}^2\exp(-2\delta_1 Y)}{2\delta_1(2\delta_1-1)}$$
$$- GrA^*\frac{{b_{10}}^2\exp(-2\Pr Y)}{2\Pr(2\Pr-1)} - GrA^*\frac{b_{13}\exp(-\Pr Y)}{\Pr(\Pr-1)}$$
$$+ GrA^*\frac{b_{14}\exp(-\delta_2 Y)}{\delta_2(\delta_2-1)} + GrA^*\frac{b_{16}\exp((\delta_1-\Pr))Y}{(\delta-\Pr)(1+(\delta-\Pr))}$$
$$+ GrA^*\frac{b_{17}\exp(-(\delta_2+\Pr)Y)}{(\delta_2+\Pr)[(\delta_2+\Pr)-1]} + b_{18}(Y-1)$$
$$- Gc\frac{b_4\exp(1+\delta_1 Y)}{\delta_1(1+\delta_1)} - Gc\frac{b_3\exp(1-\delta_2 Y)}{\delta_2(\delta_2-1)}$$
$$- GcB^* b_{12}(Y-1) - GcB^*\frac{{b_4}^2\exp(1+2\delta_1 Y)}{2\delta_1(1+2\delta_1)}$$
$$- GcB^*\frac{{b_3}^2\exp(-2\delta_2 Y)}{2\delta_2(2\delta_2-1)} + C_1 + C_2 e^{-Y}$$

$$U(Y) = -b_{42}(Y-1) - b_{37}\exp(2\delta_2 Y) - b_{38}\exp(-2\delta_2 Y)$$
$$- b_{21}\exp(-2\Pr Y)$$
$$- b_{39}\exp(-\Pr Y) - b_{40}\exp(\delta_1 Y) - b_{41}\exp(-\delta_2 Y)$$

$$+ b_{25}\exp(\delta_1-\Pr)Y + b_{26}\exp(-(\delta_2+\Pr)Y) + C_1 + C_2 e^{-Y}$$

(3.63)

To find the constants C_1 and C_2 we apply the boundary conditions $\begin{matrix} U=1 & at & Y=0 \\ U=0 & as & Y=H \end{matrix}$

Using equations (3.68) and (3.63) we obtained

$$1 = b_{42} - b_{37} - b_{38} - b_{21} - b_{39} - b_{40} - b_{41} + b_{25} + b_{26} + C_1 + C_2$$

$$\begin{aligned} 0 = &-b_{42}\left(H-1\right) - b_{37}\exp\left(2\delta_2 H\right) - b_{38}\exp\left(-2\delta_2 H\right) \\ &- b_{21}\exp\left(-2\,\mathrm{Pr}\,H\right) \\ &- b_{39}\exp\left(-\mathrm{Pr}\,H\right) - b_{40}\exp\left(\delta_1 H\right) - b_{41}\exp\left(-\delta_2 H\right) \\ &+ b_{25}\exp\left(\left(\delta_1 - \mathrm{Pr}\right)H\right) + b_{26}\exp\left(-\left(\delta_2 + \mathrm{Pr}\right)H\right) + C_1 + C_2 e^{-H} \end{aligned}$$

The general solution for momentum equation is obtained as:

$$\begin{aligned} U\left(Y\right) = &\frac{\left(1-\exp\left(-H\right)\right)-\left(1-\exp\left(-Y\right)\right)}{\left(1-\exp\left(-H\right)\right)} - b_{42}\left\{\frac{\left(1-\exp\left(-H\right)\right)-H\left(1-\exp\left(-Y\right)\right)}{\left(1-\exp\left(-H\right)\right)} + \left(Y-1\right)\right\} \\ &- b_{37}\left\{\frac{\left(1-\exp\left(-H\right)\right)-\left(1-\exp\left(2\delta_1 H\right)\right)\left(1-\exp\left(-Y\right)\right)}{\left(1-\exp\left(-H\right)\right)} - \exp\left(2\delta_1 Y\right)\right\} \\ &- b_{38}\left\{\frac{\left(1-\exp\left(-H\right)\right)-\left(1-\exp\left(-2\delta_2 H\right)\right)\left(1-\exp\left(-Y\right)\right)}{\left(1-\exp\left(-H\right)\right)} - \exp\left(-2\delta_2 Y\right)\right\} \\ &- b_{21}\left\{\frac{\left(1-\exp\left(-H\right)\right)-\left(1-\exp\left(-2\,\mathrm{Pr}\,H\right)\right)\left(1-\exp\left(-Y\right)\right)}{\left(1-\exp\left(-H\right)\right)} - \exp\left(-2\,\mathrm{Pr}\,Y\right)\right\} \\ &- b_{39}\left\{\frac{\left(1-\exp\left(-H\right)\right)-\left(1-\exp\left(-\left(\mathrm{Pr}\right)H\right)\right)\left(1-\exp\left(-Y\right)\right)}{\left(1-\exp\left(-H\right)\right)} - \exp\left(-\left(\mathrm{Pr}\right)Y\right)\right\} \\ &- b_{40}\left\{\frac{\left(1-\exp\left(-H\right)\right)-\left(1-\exp\left(\delta_1 H\right)\right)\left(1-\exp\left(-Y\right)\right)}{\left(1-\exp\left(-H\right)\right)} - \exp\left(\delta_1 Y\right)\right\} \\ &- b_{41}\left\{\frac{\left(1-\exp\left(-H\right)\right)-\left(1-\exp\left(-\delta_2 H\right)\right)\left(1-\exp\left(-Y\right)\right)}{\left(1-\exp\left(-H\right)\right)} - \exp\left(-\delta_2 Y\right)\right\} \\ &+ b_{25}\left\{\frac{\left(1-\exp\left(-H\right)\right)-\left(1-\exp\left(\left(\delta_1 - \mathrm{Pr}\right)H\right)\right)\left(1-\exp\left(-Y\right)\right)}{\left(1-\exp\left(-H\right)\right)} - \exp\left(\left(\delta_1 - \mathrm{Pr}\right)Y\right)\right\} \\ &+ b_{26}\left\{\frac{\left(1-\exp\left(-H\right)\right)-\left(1-\exp\left(-\left(\mathrm{Pr}+\delta_2\right)H\right)\right)\left(1-\exp\left(-Y\right)\right)}{\left(1-\exp\left(-H\right)\right)} - \exp\left(-\left(\mathrm{Pr}+\delta_2\right)Y\right)\right\} \end{aligned}$$

(3.64)

To find the fluid force against the direction of motion of an object, the dimensionless skin friction at the surface wall $(Y=0)$ and $(Y=H)$ is given by:

$$\tau_0 = \left(\frac{dU}{dY}\right)_{Y=0} = \frac{(-1)}{(1-\exp(-H))} - b_{42}\left\{\frac{-H}{(1-\exp(-H))} + (1)\right\}$$
$$- b_{37}\left\{-\frac{(1-\exp(2\delta_1 H))}{(1-\exp(-H))} - 2\delta\right\}$$
$$- b_{38}\left\{2\delta_2 - \frac{(1-\exp(-2\delta_2 H))}{(1-\exp(-H))}\right\} - b_{21}\left\{2\,\mathrm{Pr} - \frac{(1-\exp(-2\,\mathrm{Pr}\,H))}{(1-\exp(-H))}\right\}$$
$$- b_{39}\left\{(\mathrm{Pr}) - \frac{(1-\exp(-(\mathrm{Pr})H))}{(1-\exp(-H))}\right\} - b_{40}\left\{-\frac{(1-\exp(\delta_1 H))}{(1-\exp(-H))} - \delta_1\right\}$$

$$- b_{41}\left\{\delta_2 - \frac{(1-\exp(-\delta_2 H))}{(1-\exp(-H))}\right\} + b_{25}\left\{-\frac{(1-\exp((\delta_1 - \mathrm{Pr})H))}{(1-\exp(-H))} - (\delta_1 - \mathrm{Pr})\right\}$$
$$+ b_{26}\left\{(\mathrm{Pr}+\delta_2) - \frac{(1-\exp(-(\mathrm{Pr}+\delta_2)H))}{(1-\exp(-H))}\right\}$$

(3.65)

$$\tau_H = \left(\frac{dU}{dY}\right)_{Y=H} = \frac{\left(-\exp(-H)\right)}{\left(1-\exp(-H)\right)} - b_{42}\left\{\frac{-H\exp(-H)}{\left(1-\exp(-H)\right)} + (1)\right\}$$
$$- b_{37}\left\{-\frac{\left(1-\exp(2\delta_1 H)\right)}{\left(1-\exp(-H)\right)} - 2\delta\exp(2\delta_1 H)\right\}$$
$$- b_{38}\left\{2\delta\exp(-2\delta_2 H)_2 - \frac{\left(1-\exp(-2\delta_2 H)\right)}{\left(1-\exp(-H)\right)}\right\}$$
$$- b_{21}\left\{2\,\mathrm{Pr}\exp(-2\,\mathrm{Pr}\,H) - \frac{\left(1-\exp(-2\,\mathrm{Pr}\,H)\right)}{\left(1-\exp(-H)\right)}\right\}$$
$$- b_{39}\left\{\mathrm{Pr}\exp\left(-(\mathrm{Pr})H\right) - \frac{\left(1-\exp\left(-(\mathrm{Pr})H\right)\right)}{\left(1-\exp(-H)\right)}\right\}$$
$$- b_{40}\left\{-\frac{\left(1-\exp(\delta_1 H)\right)}{\left(1-\exp(-H)\right)} - \delta_1\exp(\delta_1 H)\right\}$$
$$- b_{41}\left\{\delta_2\exp(-\delta_2 H) - \frac{\left(1-\exp(-\delta_2 H)\right)}{\left(1-\exp(-H)\right)}\right\}$$
$$+ b_{25}\left\{-\frac{\left(1-\exp\left((\delta_1-\mathrm{Pr})H\right)\right)}{\left(1-\exp(-H)\right)} - (\delta_1-\mathrm{Pr})\exp\left((\delta_1-\mathrm{Pr})H\right)\right\}$$
$$+ b_{26}\left\{(\mathrm{Pr}+\delta_2)\exp\left(-(\mathrm{Pr}+\delta_2)H\right) - \frac{\left(1-\exp\left(-(\mathrm{Pr}+\delta_2)H\right)\right)}{\left(1-\exp(-H)\right)}\right\}$$

(3.66)

The heat transfer rate produced is given in equation (3.58) below:

$$Nu_0 = \left(\frac{dT}{dY}\right)_{Y=0} = b_6\delta_2 - \delta_1 b_5 - \mathrm{Pr}\,b_{10}$$

$$Nu_H = \left(\frac{dT}{dY}\right)_{Y=H} = \delta_2 b_6\exp(-\delta_2 H) - b_5\delta_1\exp(\delta_1 H) - \mathrm{Pr}\,b_{10}\exp(-\mathrm{Pr}\,H)$$

(3.67)

3.6Thermal diffusion effect on nonlinear natural convection mass transfer flow near an infinite vertical porous plate with chemical reaction

The study considers the nonlinear natural convection and mass transfer flow near vertical porous plate with chemical reaction and Soret effect. The flow is assumed to be in the $x-$ direction, which is taken to be vertically upward along the channel wall, and the $y-$ axis is taken to be normal to the porous plate. It is assumed that the amount of fluid particles removed from the plate is so limited that only fluid particles near the boundary of the plate are sucked away. Initially, the fluid is at rest and is at the uniform constant temperature T_w and concentration C_w which are greater than the corresponding values of temperature, T_∞ and concentration, C_∞. It is also assumed that chemically reactive species is emitted from the porous plate in form of fluid motion and itthen diffuses into the fluid where it goes into first order chemical reaction.Theflowconfiguration is shown in Figure3.3. Muthucumaraswamy (2002) reported analytical solutions for natural convection and mass transfer flow near a moving infinite vertical porous plate in the presence of chemical reaction for linear density variation of temperature (LDT) and concentration (LDC). Recently, Partha (2010) studied the non-linear convection in a non-Darcy porous medium due to nonlinear density variation in temperature and concentration. The purpose of this section is to generalize the model used by Muthucumaraswamy (2002) to include nonlinear density variation with temperature and concentration and thermal diffusion effect. In the governing concentration equation, the concentration is governed by temperature, leading to the thermal diffusion effect. Following Muthucumaraswamy (2002) and considering nonlinear convection due to heat and mass transfer Partha (2010).

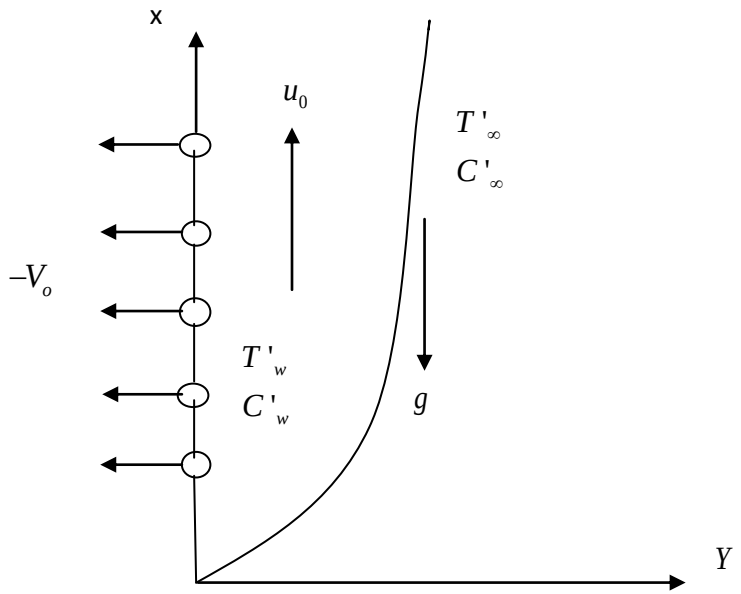


Figure 3.3 Flow configuration

The equations for the present physical situation under nonlinear Boussinesq's approximations are: equations (3.1), (3.2), (3.3)

Thedimensionless quantitiesintroduced in the governing equations are defined as

$$\left.\begin{aligned} &Y=\frac{yV_o}{\upsilon},\quad \Pr=\frac{\upsilon}{\alpha},\quad Sc=\frac{\upsilon}{D},\quad U=\frac{u}{u_0},\\ &u_0=\frac{g\beta_0\left(T'_w-T'_\infty\right)}{V_o^{\,2}},\ C=\frac{C'-C'_\infty}{C'_w-C'_\infty},\ T=\frac{T'-T'_\infty}{T'_w-T'_\infty},\\ &A^*=\frac{\beta_1}{\beta_0}\left(T'_w-T'_\infty\right),\ B^*=\frac{\beta_1^*}{\beta_0^*}\left(C'_w-C'_\infty\right),\\ &K=\frac{k_e\upsilon}{V_o^{\,2}},\ D_t=\frac{D^*\left(T'_w-T'_\infty\right)}{D\left(C'_w-C'_\infty\right)},\ N=\frac{\beta_o^{\,*}\left(C_w'-C'_\infty\right)}{\beta_o\left(T'_w-T'_\infty\right)} \end{aligned}\right\} \quad (3.68)$$

The boundary conditions in dimensional form are:

$$\left.\begin{aligned} &u=0 \qquad T'=T'_w \qquad C'=C'_w \qquad at\ \ y=0\\ &u\to 0 \qquad T'\to T'_\infty \qquad C'\to C'_\infty \qquad as\ \ y\to\infty \end{aligned}\right\} \quad (3.69)$$

Equations (3.1), (3.2), (3.3), (3.68) and (3.69) are reduced to the following non-dimensional form:

$$\frac{d^2U}{dY^2}+\frac{dU}{dY}+\left[T+A^*T^2\right]-N\left[C+B^*C^2\right]=0, \quad (3.70)$$

$$\frac{d^2T}{dY^2}+\Pr\frac{dT}{dY}=0, \quad (3.71)$$

$$\frac{d^2C}{dY^2}+Sc\frac{dC}{dY}-KScC+D_t\frac{d^2T}{dY^2}=0, \quad (3.72)$$

where, A^* represents non-linear density variation with temperature parameter (NDT), B^* is non-linear density variation with concentration parameter (NDC), C is dimensionless species concentration of the fluid, D_t is Soret effect parameter**,** N is Sustentation parameterand K is dimensionless chemical reaction parameter.

The corresponding boundary conditions in non-dimensional form are

$$\left.\begin{array}{llll} U=0 & T=1 & C=1 & at\ \ Y=0 \\ U\to 0 & T\to 0 & C\to 0 & as\ \ Y\to\infty \end{array}\right\} \quad (3.73)$$

3.6.1 Solution to the problem

The solutions for ordinary differential equation (3.70) - (3.72) with boundary conditions (3.73) is obtained as

$$T(Y)=\exp(-\Pr Y). \quad (3.74)$$

$$C(Y)=\exp(-\delta Y)+\frac{D_t\Pr^2}{d_o}\left[\exp(-\delta Y)-\exp(-\Pr Y)\right]. \quad (3.75)$$

$$U(Y)=\frac{A^{*}}{2\Pr(2\Pr-1)}\left[\exp(-Y)-\exp(-2\Pr Y)\right]$$
$$+\frac{NB^{*}d_2}{2\delta(2\delta-1)}\left[\exp(-Y)-\exp(-2\delta Y)\right]$$
$$+\frac{NB^{*}d_3}{(\Pr+\delta)\left[(\Pr+\delta)-1\right]}\left[\left(\exp\left(-(\Pr+\delta)Y\right)-\exp(-Y)\right)\right]$$
$$+\frac{NB^{*}d_1^{\,2}}{2\Pr(2\Pr-1)}\left[\exp(-Y)-\exp(-2\Pr Y)\right]$$
$$+\frac{1}{\Pr(\Pr-1)}\left[\left(\exp(-Y)-\exp(-\Pr Y)\right)\right]+\frac{N}{\delta(\delta-1)}\left[\left(\exp(-Y)-\exp(-\delta Y)\right)\right]$$
$$+\frac{Nd_1}{\delta(\delta-1)}\left[\left(\exp(-Y)-\exp(-\delta Y)\right)\right]+\frac{Nd_1}{\Pr(\Pr-1)}\left[\left(\exp(-\Pr Y)-\exp(-Y)\right)\right].$$

(3.76)

Sherwood number (Sh) also called mass transfer, represents the ratio of the convective mass transfer to the rate of diffusive mass transport. It is also a transport material between a boundary surface and a moving fluid. The Sherwood number at boundary surface when $(Y=0)$ is obtained as

$$Sh=\left(\frac{dC}{dY}\right)_{Y=0}=\frac{D_t\,\Pr^2}{d_o}\left[\Pr-\delta\right]-\delta \quad (3.78)$$

Therefore, fluid flowing against the boundary surface would experience viscous forces called skin friction. It is the fluid forces against the direction of wall surface, and is obtained by

$$\tau=\left(\frac{dU}{dY}\right)_{Y=0}=\frac{A^{*}}{2\Pr}+NB^{*}\left(\frac{d_2}{2\delta}+\frac{d_3}{(\Pr+\delta)}+\frac{d_1^{\,2}}{2\Pr}\right)+\frac{N}{\delta}(1+d_1)+\frac{1}{\Pr}\left[(1+Nd_1)\right] \quad (3.79)$$

3.7Nonlinear natural convection mass transfer flow in vertical channel with chemical reaction and thermal diffusion effect

The study considers thenonlinear natural convection and mass transfer flow near a vertical porous plate with chemical reaction and Soret effect under nonlinear Boussinesq's approximations approach are studied.Flow is assumed to be in the $x-$ direction, which is taken to be vertically upward along the channel wall, and the $y-$ axis is taken to be normal to the porous plate. The fluid is assumed to be removed through one plate and simultaneously injected through the other plate with a constant velocity V_0. It is assumed that the amount of fluid particles removed from the plate is so limited that only fluid particles near the boundary of the plate are sucked away. Initially, the fluid is at rest and is at the uniform constant temperature T_w and concentration C_w which are greater than the corresponding values of temperature T_∞ and concentration C_∞

.

It is also assumed that a chemically reactive species is emitted from the porous plate in form of fluid motion which then diffuses into the fluid where it undergoes into first order chemical reaction. The flow configuration is shown in Figure3.4. Following Muthucumaraswamy (2002) and considering nonlinear convection due to heat and mass transferPartha (2010),

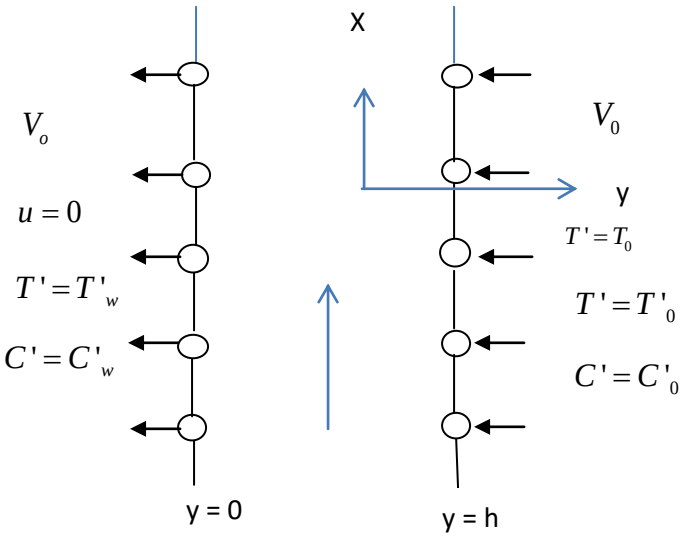


Figure 3.4 Flow configuration

The equations for the present physical situation under nonlinear Boussinesq's approximations are given in equations (3.1), (3.80), (3.81)

$$-V_o \frac{dT'}{dy} = \alpha \frac{d^2T'}{dy^2} \qquad (3.80)$$

$$-V_o \frac{dC'}{dy} = D\frac{d^2C'}{dy^2} - k_e\left(C' - C_0\right) + D^* \frac{d^2T'}{dy^2}, \quad (3.81)$$

The dimensionless quantities introduced in the above equations are defined as

$$\left.\begin{aligned} &Y = \frac{yV_o}{\upsilon},\ H = \frac{hV_o}{\upsilon},\ \mathrm{Pr} = \frac{\upsilon}{\alpha},\ Sc = \frac{\upsilon}{D},\ U = \frac{u}{u_0}, \\ &u_0 = \frac{g\beta_0\left(T'_w - T'_\infty\right)}{V_o^{\,2}},\ C = \frac{C' - C'_0}{C'_w - C'_0},\ T = \frac{T' - T'_0}{T'_w - T'_0}, \\ &A^* = \frac{\beta_1}{\beta_0}\left(T'_w - T'_0\right),\ B^* = \frac{\beta_1^*}{\beta_0^*}\left(C'_w - C'_0\right),\ K = \frac{k_e\upsilon}{V_o^{\,2}}, \\ &D_t = \frac{D^*\left(T'_w - T'_0\right)}{D\left(C'_w - C'_0\right)},\quad N = \frac{\beta_o^*\left(C_w' - C'_0\right)}{\beta_o\left(T'_w - T'_0\right)} \end{aligned}\right\} \quad (3.82)$$

The boundary conditions in dimensional form are:

$$\left.\begin{array}{llll} u=0 & T'=T_w & C'=C_w & at\ \ y=0 \\ u=0 & T'=T_0 & C'=C_0 & as\ \ y=h \end{array}\right\} (3.83)$$

Equations (3.1) and (3.80- 3.83) are reduced to (3.70), (3.71), (3.72) in non-dimensional form.

Where, A^* represents non-linear density variation with temperature parameter (NDT), B^* is the non-linear density variation with concentration parameter (NDC), C is dimensionless species concentration of the fluid, D_t is Soret effect**,** N is Sustentation parameterand K is dimensionless chemical reaction parameter.

The corresponding boundary conditions in non-dimensional form are:

$$\left.\begin{array}{llll} U=0 & T=1 & C=1 & at\ \ Y=0 \\ U=0 & T=0 & C=0 & as\ \ Y=H \end{array}\right\} (3.84)$$

3.7.1 Solution to the problem

The solutions for ordinary differential equation(3.70), (3.71) and (3.72) with boundary conditions (3.84) is obtained as

$$T(Y)=\frac{\exp(-\Pr Y)+h_1}{h_0}. \qquad (3.85)$$

$$C(Y)=h_5\left[\exp(\delta_1 Y)\right]+h_4\left[\exp(-\delta_2 Y)\right]-h_3\left[\exp(-\Pr Y)\right]. (3.86)$$

Where,

$$\delta_1=\frac{1}{2}\left[-Sc+\sqrt{Sc^2+4KSc}\right], \qquad -\delta_2=\frac{1}{2}\left[Sc+\sqrt{Sc^2+4KSc}\right] (3.87)$$

The solution to equation (3.70) would be given as:

$$
\begin{aligned}
U(Y) = -h_{26}&\left\{\frac{\left(1-\exp(-H)\right)-H\left(1-\exp(-Y)\right)}{\left(1-\exp(-H)\right)}+(Y-1)\right\}\\
&+h_{27}\left\{\frac{\left(1-\exp(-H)\right)-\left(1-\exp(-\mathrm{Pr}\,H)\right)\left(1-\exp(-Y)\right)}{\left(1-\exp(-H)\right)}-\exp(-\mathrm{Pr}\,Y)\right\}\\
&-h_{28}\left\{\frac{\left(1-\exp(-H)\right)-\left(1-\exp(-2\,\mathrm{Pr}\,H)\right)\left(1-\exp(-Y)\right)}{\left(1-\exp(-H)\right)}-\exp(-2\,\mathrm{Pr}\,Y)\right\}\\
&-h_{15}\left\{\frac{\left(1-\exp(-H)\right)-\left(1-\exp(2\delta_1 H)\right)\left(1-\exp(-Y)\right)}{\left(1-\exp(-H)\right)}-\exp(2\delta_1 Y)\right\}\\
&-h_{18}\left\{\frac{\left(1-\exp(-H)\right)-\left(1-\exp(-2\delta_2 H)\right)\left(1-\exp(-Y)\right)}{\left(1-\exp(-H)\right)}-\exp(-2\delta_2 Y)\right\}\\
&-h_{23}\left\{\frac{\left(1-\exp(-H)\right)-\left(1-\exp(\delta_1 H)\right)\left(1-\exp(-Y)\right)}{\left(1-\exp(-H)\right)}-\exp\left((\delta_1)Y\right)\right\}\\
&-h_{24}\left\{\frac{\left(1-\exp(-H)\right)-\left(1-\exp(-\delta_2 H)\right)\left(1-\exp(-Y)\right)}{\left(1-\exp(-H)\right)}-\exp\left((-\delta_2)Y\right)\right\}\\
&+h_{17}\left\{\frac{\left(1-\exp(-H)\right)-\left(1-\exp(\delta_1-\mathrm{Pr})H\right)\left(1-\exp(-Y)\right)}{\left(1-\exp(-H)\right)}-\exp\left((\delta_1-\mathrm{Pr})Y\right)\right\}\\
&+h_{19}\left\{\frac{\left(1-\exp(-H)\right)-\left(1-\exp(-(\mathrm{Pr}+\delta_2))H\right)\left(1-\exp(-Y)\right)}{\left(1-\exp(-H)\right)}-\exp\left(-(\mathrm{Pr}+\delta_2)Y\right)\right\}
\end{aligned}
$$

(3.88)

Therefore, fluid flowing against the boundary surface would experience viscous forcescalled skin friction. It is the fluid forces against the direction of wall surface, is obtained as

$$\tau_0 = \left(\frac{dU}{dY}\right)_{Y=0} = -h_{26}\left\{\frac{-H}{\left(1-\exp\left(-H\right)\right)}+1\right\}+h_{27}\left\{\Pr-\frac{\left(1-\exp\left(-\Pr H\right)\right)}{\left(1-\exp\left(-H\right)\right)}\right\}$$
$$-h_{28}\left\{2\Pr-\frac{\left(1-\exp\left(-2\Pr H\right)\right)}{\left(1-\exp\left(-H\right)\right)}\right\}-h_{15}\left\{-\frac{\left(1-\exp\left(2\delta_1 H\right)\right)}{\left(1-\exp\left(-H\right)\right)}-2\delta_1\right\}$$
$$-h_{18}\left\{2\delta_2-\frac{\left(1-\exp\left(-2\delta_2 H\right)\right)}{\left(1-\exp\left(-H\right)\right)}\right\}$$
$$-h_{23}\left\{-\frac{\left(1-\exp\left(\delta_1 H\right)\right)}{\left(1-\exp\left(-H\right)\right)}-\delta_1\right\}-h_{24}\left\{\delta_2-\frac{\left(1-\exp\left(-\delta_2 H\right)\right)}{\left(1-\exp\left(-H\right)\right)}\right\}$$
$$+h_{17}\left\{-\frac{\left(1-\exp\left(\delta_1-\Pr\right)H\right)}{\left(1-\exp\left(-H\right)\right)}-\left(\delta_1-\Pr\right)\right\}+h_{19}\left\{\left(\Pr+\delta_2\right)+\frac{\left(1-\exp(-\left(\Pr+\delta_2\right))H\right)}{\left(1-\exp\left(-H\right)\right)}\right\}$$

(3.89)

$$\tau_H = \left(\frac{dU}{dY}\right)_{Y=H} = -h_{26}\left\{\frac{-H\exp\left(-H\right)}{\left(1-\exp\left(-H\right)\right)}+1\right\}+h_{27}\left\{\Pr\exp\left(-\Pr H\right)-\frac{\left(1-\exp\left(-\Pr H\right)\right)}{\left(1-\exp\left(-H\right)\right)}\right\}$$
$$-h_{28}\left\{2\Pr\exp\left(-2\Pr H\right)-\frac{\left(1-\exp\left(-2\Pr H\right)\right)}{\left(1-\exp\left(-H\right)\right)}\right\}$$
$$-h_{15}\left\{-\frac{\left(1-\exp\left(2\delta_1 H\right)\right)}{\left(1-\exp\left(-H\right)\right)}-2\delta_1\exp\left(2\delta_1 H\right)\right\}$$
$$-h_{18}\left\{2\delta_2\exp\left(-2\delta_2 H\right)-\frac{\left(1-\exp\left(-2\delta_2 H\right)\right)}{\left(1-\exp\left(-H\right)\right)}\right\}$$
$$-h_{23}\left\{-\frac{\left(1-\exp\left(\delta_1 H\right)\right)}{\left(1-\exp\left(-H\right)\right)}-\delta_1\exp\left(\delta_1 H\right)\right\}$$
$$-h_{24}\left\{\delta_2\exp\left(-\delta_2 H\right)-\frac{\left(1-\exp\left(-\delta_2 H\right)\right)}{\left(1-\exp\left(-H\right)\right)}\right\}$$
$$+h_{17}\left\{-\frac{\left(1-\exp\left(\delta_1-\Pr\right)H\right)}{\left(1-\exp\left(-H\right)\right)}-\left(\delta_1-\Pr\right)\exp\left(\delta_1-\Pr\right)H\right\}$$
$$+h_{19}\left\{\left(\Pr+\delta_2\right)\exp\left(-\left(\Pr+\delta_2\right)\right)+\frac{\left(1-\exp(-\left(\Pr+\delta_2\right))H\right)}{\left(1-\exp\left(-H\right)\right)}\right\}$$

(3.90)

Sherwood number (Sh) also called mass transfer, represents the ratio of the convective mass transfer to the rate of diffusive mass transport. It is also the transport material between a boundary surface and a moving fluid. The Sherwood number at boundary surface is obtained as

$$Sh_0 = \left(\frac{dC}{dY}\right)_{Y=0} = h_3 \Pr + h_8\delta_1 - h_7\delta_2 \tag{3.91}$$

$$Sh_H = \left(\frac{dC}{dY}\right)_{Y=H} = h_3 \Pr \exp(-\Pr H) + h_8\delta_1 \exp(\delta_1 H) - h_7\delta_2 \exp(-\delta_2 H) \tag{3.92}$$

3.8Dufour effect on nonlinear natural convection heat and mass transfer flow near an infinite vertical porous plate with chemical reaction

The study considers the nonlinear natural convection and mass transfer flow near a vertical porous plate with chemical reaction and Dufour effect. The flow is assumed to be in the $x-$direction, which is taken to be vertically upward along the channel wall, and the $y-$axis is taken to be normal to the porous plate. It is assumed that the amount of fluid particles removed from the plate is so limited that only fluid particles near the boundary of the plate are sucked away. Initially, the fluid is at rest and is at the uniform constant temperature T_w and concentration C_w which are greater than the corresponding values of temperature T_∞ and concentration C_∞. It is also assumed that chemically reactive species is emitted from the porous plate in form of fluid motion, which diffuses into the fluid where it udergoes into first order chemical reaction. The flow configuration is shown in Figure3.3.

Muthucumaraswamy (2002) reported analytical solutions for natural convection and mass transfer flow near a moving infinite vertical porous plate in the presence of chemical reaction for linear density variation of temperature LDT and concentration LDC. Recently, Partha (2010) studied the non-linear convection in a non-Darcy porous medium due to nonlinear density variation in temperature and concentration.

Thisproblemgeneralized the model used in Muthucumaraswamy (2002) to include nonlinear density variation with temperature and concentration and Dufour effect. In the governing temperature equation, the temperature is governed by concentration, leading to the Dufour effect. Following Muthucumaraswamy (2002) and considering nonlinear convection due to heat and mass transferPartha (2010), the equations for the present physical situation under nonlinear Boussinesq's approximations is obtained in dimensionless equation (3.1)

$$-V_o \frac{dT'}{dy} = \alpha \frac{d^2T'}{dy^2} + D^* \frac{d^2C'}{dy^2} \quad (3.93)$$

$$-V_o \frac{dC'}{dy} = D \frac{d^2C'}{dy^2} - k_e \left(C' - C'_\infty\right) \quad (3.94)$$

The dimensionless quantities introduced in the above equations are defined as

$$\left.\begin{aligned} &Y = \frac{yV_o}{\upsilon}, \quad \Pr = \frac{\upsilon}{\alpha}, \quad Sc = \frac{\upsilon}{D}, \quad U = \frac{u}{u_0}, \\ &u_0 = \frac{g\beta_0\left(T'_w - T'_\infty\right)}{V_o^{\,2}}, C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \\ &T = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \quad K = \frac{k_e\upsilon}{V_o^{\,2}} \\ &D_f = \frac{D^*\left(C'_w - C'_\infty\right)}{\left(T'_w - T'_\infty\right)\alpha}, \quad A^* = \frac{\beta_1}{\beta_0}\left(T'_w - T'_\infty\right), \\ &B^* = \frac{\beta_1^*}{\beta_0^*}\left(C'_w - C'_\infty\right), \quad N = \frac{\beta_o^*\left(C_w' - C'_\infty\right)}{\beta_o\left(T'_w - T'_\infty\right)} \end{aligned}\right\} \quad (3.95)$$

The boundary conditions are

$$\left.\begin{array}{llll} u=0 & T'=T'_w & C'=C'_w & at\ \ y=0 \\ u\to 0 & T'\to T'_w & C'\to C'_w & as\ \ y\to\infty \end{array}\right\} (3.96)$$

Equations (3.92) to (3.95) are reduced to the following non-dimensional form:

$$\frac{d^2U}{dY^2}+\frac{dU}{dY}+\left[T+A^*T^2\right]-N\left[C+B^*C^2\right]=0 \quad (3.97)$$

$$\frac{d^2T}{dY^2}+\Pr\frac{dT}{dY}+D_f\frac{d^2C}{dY^2}=0 \quad (3.98)$$

$$\frac{d^2C}{dY^2}+Sc\frac{dC}{dY}-KScC=0 \quad (3.99)$$

where, A^* represents non-linear density variation with temperature parameter (NDT), B^* is non-linear density variation with concentration parameter (NDC), C is dimensionless species concentration of the fluid, D_f is Dufour effect parameter**,** N isSustentation parameterand K is dimensionless chemical reaction parameter.

The corresponding boundary conditions in non-dimensional form are

$$\left.\begin{array}{llll} U=0 & T=1 & C=1 & at\ \ Y=0 \\ U\to 0 & T\to 0 & C\to 0 & as\ \ Y\to\infty \end{array}\right\} (3.100)$$

3.8.1 Solution to the problem

The solutions for ordinary differential equation (3.97), (3.98) and (3.99) subject to the relevant boundary conditions (3.100) fornatural convection nonlinear density variation with temperature and concentration are summerized as follows

$$C(Y)=\exp(-\delta Y) \quad (3.101)$$

$$T(Y)=\exp(-\Pr Y)+\frac{D_f\delta^2}{\left(\delta^2-\Pr\delta\right)}\left[\exp(-\Pr Y)-\exp(-\delta Y)\right] \quad (3.102)$$

$$U(Y)=\frac{A^{*}e_{01}}{2\,\mathrm{Pr}(2\,\mathrm{Pr}-1)}\left[\exp(-Y)-\exp(-2\,\mathrm{Pr}\,Y)\right]$$
$$+\frac{A^{*}e_{02}}{(\mathrm{Pr}+\delta)\left[(\mathrm{Pr}+\delta)-1\right]}\left[\exp(-Y)-\exp\left(-(\mathrm{Pr}+\delta)Y\right)\right]$$
$$+\frac{A^{*}e_{00}{}^{2}}{2\delta(2\delta-1)}\left[\left(\exp(-Y)-\exp(-2\delta Y)\right)\right]+\frac{NB^{*}}{2\delta(2\delta-1)}\left[\exp(-Y)-\exp(-2\delta Y)\right]$$
$$+\frac{1}{\mathrm{Pr}(\mathrm{Pr}-1)}\left[\left(\exp(-Y)-\exp(-\mathrm{Pr}\,Y)\right)\right]+\frac{e_{00}}{\mathrm{Pr}(\mathrm{Pr}-1)}\left[\left(\exp(-Y)-\exp(-\mathrm{Pr}\,Y)\right)\right]$$
$$+\frac{e_{00}}{\delta(\delta-1)}\left[\left(\exp(-\delta Y)-\exp(-Y)\right)\right]+\frac{N}{\delta(\delta-1)}\left[\left(\exp(-Y)-\exp(-\delta Y)\right)\right]$$

(3.103)

Nusselt number represents the ratio of heat transfer by fluid when heat change through fluids by convection. The dimensionless rate of heat transfer (Nusselt number) is obtained as

$$Nu=\left(\frac{dT}{dY}\right)_{Y=0}=\frac{D_f\,\delta^{2}}{\left(\delta^{2}-\mathrm{Pr}\,\delta\right)}\left[\delta-\mathrm{Pr}\right]-\mathrm{Pr}\quad(3.104)$$

Fluid flowing against the boundary surface would experience viscous forcescalled skin friction. It is the fluid forces against the direction of wall surface, and is obtained as

$$\tau=\left(\frac{dU}{dY}\right)_{Y=0}=\frac{A^{*}e_{01}}{2\,\mathrm{Pr}(2\,\mathrm{Pr}-1)}\left[(2\,\mathrm{Pr}-1)\right]+\frac{A^{*}e_{02}}{(\mathrm{Pr}+\delta)\left[(\mathrm{Pr}+\delta)-1\right]}\left[\left((\mathrm{Pr}+\delta)-1\right)\right]$$
$$+\frac{A^{*}e_{00}{}^{2}}{2\delta(2\delta-1)}\left[(2\delta-1)\right]+\frac{NB^{*}}{2\delta(2\delta-1)}\left[(2\delta-1)\right]+\frac{1}{\mathrm{Pr}(\mathrm{Pr}-1)}\left[(\mathrm{Pr}-1)\right]$$
$$+\frac{e_{00}}{\mathrm{Pr}(\mathrm{Pr}-1)}\left[(\mathrm{Pr}-1)\right]+\frac{e_{00}}{\delta(\delta-1)}\left[(1-\delta)\right]+\frac{N}{\delta(\delta-1)}\left[(\delta-1)\right]$$

(3.105)

3.9The influence of chemical reaction and Dufour effect on nonlinear natural convection heat and mass transfer flow in vertical channel

The nonlinear free convection heat and mass transfer flow near a vertical porous plate with chemical reaction and Dufour effect are presented, under the nonlinear Boussinesq's approximations approach.The flow is assumed to be in the $x-$direction, which is taken to be vertically upward along the channel wall, and the $y-$axis is taken to be normal to the porous plate. The fluid is assumed to be removed through one plate and simultaneously injected through the other plate with a constant velocity V_0. It is assumed that the amount of fluid particles removed from the plate is so limited that only fluid particles near the boundary of the plate are sucked away. Initially, the fluid is at rest and is at the uniform constant temperature T_w and concentration C_w which are greater than the corresponding values of temperature T_∞ and concentration C_∞. It is also assumed that a chemically reactive species is emitted from the porous plate in form of fluid motion, and diffuses into the fluid as simple isothermal homogenous chemical reaction.

The flow configuration is shown in Figure3.4. In the governing temperature equation, the temperature is governed by concentration, leading to the Dufour effect, following Muthucumaraswamy (2002) and considering nonlinear convection due to heat and mass transferPartha (2010), the equations for the present physical situation under nonlinear Boussinesq's approximations are give in equation (3.1), (3.93) and (3.94)

The dimensionless quantities introduced in the above equations are defined as

$$\left.\begin{aligned}
&Y=\frac{yV_o}{\upsilon},\ H=\frac{hV_o}{\upsilon} \quad \Pr=\frac{\upsilon}{\alpha}, \quad Sc=\frac{\upsilon}{D}, \quad U=\frac{u}{u_0},\\
&u_0=\frac{g\beta_0\left(T'_w-T'_\infty\right)}{V_o^{\,2}}, C=\frac{C'-C'_0}{C'_w-C'_0}, \quad T=\frac{T'-T'_0}{T'_w-T'_0},\\
&A^*=\frac{\beta_1}{\beta_0}\left(T'_w-T'_0\right),\ B^*=\frac{\beta_1^*}{\beta_0^*}\left(C'_w-C'_0\right), \quad K=\frac{k_e\upsilon}{V_o^{\,2}}\\
&D_f=\frac{D^*\left(C_w'-C'_0\right)}{\left(T'_w-T'_0\right)}\frac{}{\alpha}, \quad N=\frac{\beta_o^{\,*}\left(C_w'-C'_0\right)}{\beta_o\left(T'_w-T'_0\right)}
\end{aligned}\right\} \quad (3.106)$$

The boundary conditions in dimensional form are:

$$\left.\begin{aligned}
&u=0 \qquad T'=T'_w \qquad C'=C'_w \qquad at\ \ y=0\\
&u=0 \qquad T'=T'_w \qquad C'=C'_w \qquad as\ \ y=h
\end{aligned}\right\} \quad (3.107)$$

Equations (3.92), (3.93), (3.94), (3.106) and (3.107) are reduced to the following non-dimensional form:

$$\frac{d^2U}{dY^2}+\frac{dU}{dY}=-\left[T+A^*T^2\right]-N\left[C+B^*C^2\right]$$

$$\frac{d^2T}{dY^2}+\Pr\frac{dT}{dY}+D_f\frac{d^2C}{dY^2}=0$$

$$\frac{d^2C}{dY^2}+Sc\frac{dC}{dY}-KScC=0$$

where, A^* represents non-linear density variation with temperature parameter (NDT), B^* is non-linear density variation with concentration parameter (NDC), C is dimensionless species concentration of the fluid, D_f is Dufour effect**,** N issustentation parameterand K is dimensionless chemical reaction parameter.

The corresponding boundary conditions in non-dimensional form are

$$\left.\begin{aligned}
&U=0 \qquad T=1 \qquad C=1 \qquad at \quad Y=0\\
&U=0 \qquad T=0 \qquad C=0 \qquad as \quad Y=H
\end{aligned}\right\} (3.108)$$

3.9.1 Solution to the problem

The solutions for ordinary differential equation (3.97)-(3.99) with boundary conditions (3.108) are obtained as

$$T(Y) = c_{11} + c_{10}\exp(-\Pr Y) - c_5\exp(\delta_1 Y) - c_6\exp(-\delta_2 Y) \quad (3.109)$$

$$C(Y) = c_4\exp(\delta_1 Y) + c_3\exp(-\delta_2 Y) \quad (3.110)$$

$$\begin{aligned}
U(Y) = {} & -c_{42}\left\{\frac{\left(1-\exp(-H)\right)-H\left(1-\exp(-Y)\right)}{\left(1-\exp(-H)\right)}+(Y-1)\right\} \\
& -c_{37}\left\{\frac{\left(1-\exp(-H)\right)-\left(1-\exp(2\delta_1 H)\right)\left(1-\exp(-Y)\right)}{\left(1-\exp(-H)\right)}-\exp(2\delta_1 Y)\right\} \\
& -c_{38}\left\{\frac{\left(1-\exp(-H)\right)-\left(1-\exp(-2\delta_2 H)\right)\left(1-\exp(-Y)\right)}{\left(1-\exp(-H)\right)}-\exp(-2\delta_2 Y)\right\} \\
& -c_{21}\left\{\frac{\left(1-\exp(-H)\right)-\left(1-\exp(-2\Pr H)\right)\left(1-\exp(-Y)\right)}{\left(1-\exp(-H)\right)}-\exp(-2\Pr Y)\right\} \\
& -c_{39}\left\{\frac{\left(1-\exp(-H)\right)-\left(1-\exp(-(\Pr)H)\right)\left(1-\exp(-Y)\right)}{\left(1-\exp(-H)\right)}-\exp(-(\Pr)Y)\right\} \\
& -c_{40}\left\{\frac{\left(1-\exp(-H)\right)-\left(1-\exp(\delta_1 H)\right)\left(1-\exp(-Y)\right)}{\left(1-\exp(-H)\right)}-\exp(\delta_1 Y)\right\} \\
& -c_{41}\left\{\frac{\left(1-\exp(-H)\right)-\left(1-\exp(-\delta_2 H)\right)\left(1-\exp(-Y)\right)}{\left(1-\exp(-H)\right)}-\exp(-\delta_2 Y)\right\} \\
& +c_{25}\left\{\frac{\left(1-\exp(-H)\right)-\left(1-\exp((\delta_1-\Pr)H)\right)\left(1-\exp(-Y)\right)}{\left(1-\exp(-H)\right)}-\exp((\delta_1-\Pr)Y)\right\} \\
& +c_{26}\left\{\frac{\left(1-\exp(-H)\right)-\left(1-\exp(-(\Pr+\delta_2)H)\right)\left(1-\exp(-Y)\right)}{\left(1-\exp(-H)\right)}-\exp(-(\Pr+\delta_2)Y)\right\}
\end{aligned}$$

(3.111)

Therefore, fluid flowing against the boundary surface would experience viscous forcescalled skin friction. It is the fluid forces against the direction of wall surface, and is obtained as

$$\tau_0 = \left(\frac{dU}{dY}\right)_{Y=0} = -c_{42}\left\{\frac{-H}{\left(1-\exp\left(-H\right)\right)}+\left(1\right)\right\} - c_{37}\left\{-\frac{\left(1-\exp\left(2\delta_1 H\right)\right)}{\left(1-\exp\left(-H\right)\right)}-2\delta\right\}$$
$$- c_{38}\left\{2\delta_2 - \frac{\left(1-\exp\left(-2\delta_2 H\right)\right)}{\left(1-\exp\left(-H\right)\right)}\right\} - c_{21}\left\{2\,\mathrm{Pr} - \frac{\left(1-\exp\left(-2\,\mathrm{Pr}\,H\right)\right)}{\left(1-\exp\left(-H\right)\right)}\right\}$$
$$- c_{39}\left\{\left(\mathrm{Pr}\right) - \frac{\left(1-\exp\left(-\left(\mathrm{Pr}\right)H\right)\right)}{\left(1-\exp\left(-H\right)\right)}\right\} - c_{40}\left\{-\frac{\left(1-\exp\left(\delta_1 H\right)\right)}{\left(1-\exp\left(-H\right)\right)}-\delta_1\right\}$$

$$- c_{41}\left\{\delta_2 - \frac{\left(1-\exp\left(-\delta_2 H\right)\right)}{\left(1-\exp\left(-H\right)\right)}\right\} + c_{25}\left\{-\frac{\left(1-\exp\left(\left(\delta_1-\mathrm{Pr}\right)H\right)\right)}{\left(1-\exp\left(-H\right)\right)} - \left(\delta_1-\mathrm{Pr}\right)\right\}$$
$$+ c_{26}\left\{\left(\mathrm{Pr}+\delta_2\right) - \frac{\left(1-\exp\left(-\left(\mathrm{Pr}+\delta_2\right)H\right)\right)}{\left(1-\exp\left(-H\right)\right)}\right\}$$

(3.112)

$$\tau_H = \left(\frac{dU}{dY}\right)_{Y=H} = -c_{42}\left\{\frac{-H\exp(-H)}{(1-\exp(-H))}+(1)\right\} - c_{37}\left\{-\frac{(1-\exp(2\delta_1 H))}{(1-\exp(-H))} - 2\delta\exp(2\delta_1 H)\right\}$$

$$- c_{38}\left\{2\delta\exp(-2\delta_2 H)_2 - \frac{(1-\exp(-2\delta_2 H))}{(1-\exp(-H))}\right\}$$

$$- c_{21}\left\{2\,\mathrm{Pr}\exp(-2\,\mathrm{Pr}\,H) - \frac{(1-\exp(-2\,\mathrm{Pr}\,H))}{(1-\exp(-H))}\right\}$$

$$- c_{39}\left\{\mathrm{Pr}\exp(-(\mathrm{Pr})H) - \frac{(1-\exp(-(\mathrm{Pr})H))}{(1-\exp(-H))}\right\}$$

$$- c_{40}\left\{-\frac{(1-\exp(\delta_1 H))}{(1-\exp(-H))} - \delta_1\exp(\delta_1 H)\right\}$$

$$- c_{41}\left\{\delta_2\exp(-\delta_2 H) - \frac{(1-\exp(-\delta_2 H))}{(1-\exp(-H))}\right\}$$

$$+ c_{25}\left\{-\frac{(1-\exp((\delta_1-\mathrm{Pr})H))}{(1-\exp(-H))} - (\delta_1-\mathrm{Pr})\exp((\delta_1-\mathrm{Pr})H)\right\}$$

$$+ c_{26}\left\{(\mathrm{Pr}+\delta_2)\exp(-(\mathrm{Pr}+\delta_2)H) - \frac{(1-\exp(-(\mathrm{Pr}+\delta_2)H))}{(1-\exp(-H))}\right\}$$

(3.113)

The heat transfer rate produced is given as the ratio of heat transfer by convection and heat transfer by conduction

$$Nu_0 = \left(\frac{dT}{dY}\right)_{Y=0} = c_6\delta_2 - \delta_1 c_5 - \mathrm{Pr}\,c_{10}$$

$$Nu_H = \left(\frac{dT}{dY}\right)_{Y=H} = \delta_2 c_6\exp(-\delta_2 H) - c_5\delta_1\exp(\delta_1 H) - \mathrm{Pr}\,c_{10}\exp(-\mathrm{Pr}\,H)$$

(3.114)

CHAPTER FOUR

RESULTS AND DISCUSSIONS

4.1Nonlinear natural convection and mass transfer flow near a vertical moving porous plate with chemical reaction and Soret effect

In order to have a physical insight to the problem, a comprehensive analytical study was conducted and the results are reported in terms of lines graphs, contours and tables. Numerical calculations have been carriedout for different values of the nonlinear density variation with temperature (NDT) and concentration (NDC), Soret parameter (S), chemical reaction parameter (K), Schmidt number (Sc) and Prandtl number $(\Pr)$. A comparative analysis has been made to fully understand the influence of (NDT) and (NDC) with linear density variation with temperature (LDT) and linear density variation with concentration (LDC) as presented in Table1 and Table2. Since air and water are two very common fluids, the choice of Prandtl numbers are 0.71 and 7.0 respectively. The diffusing chemical species of common interest that have Schmidt number ranging from 0.1 to 1.0, Muthucumaraswamy (2002).The selected values of Soret parameter S and nonlinear density variation with temperatureand concentration for the present analysis are chosen arbitrarily in such a way that their product is constant, provided that the temperature remains constant.

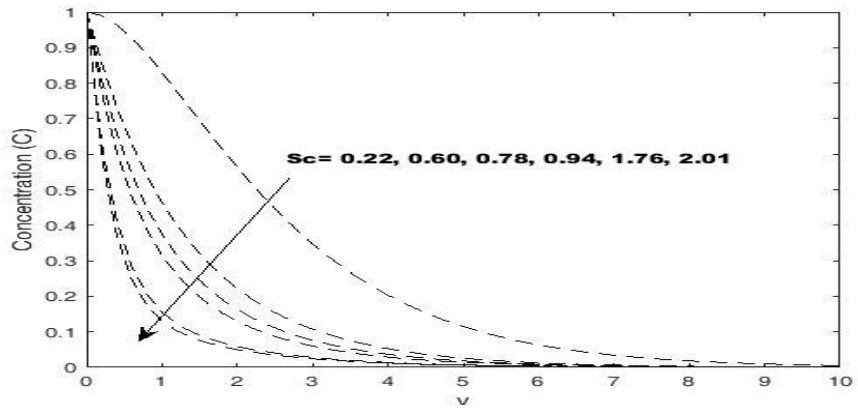


Figure 4.1 Concentration profile for different values of Sc and the fixed values $K = 2.0,\ S = 2.0,\ \Pr = 0.71$

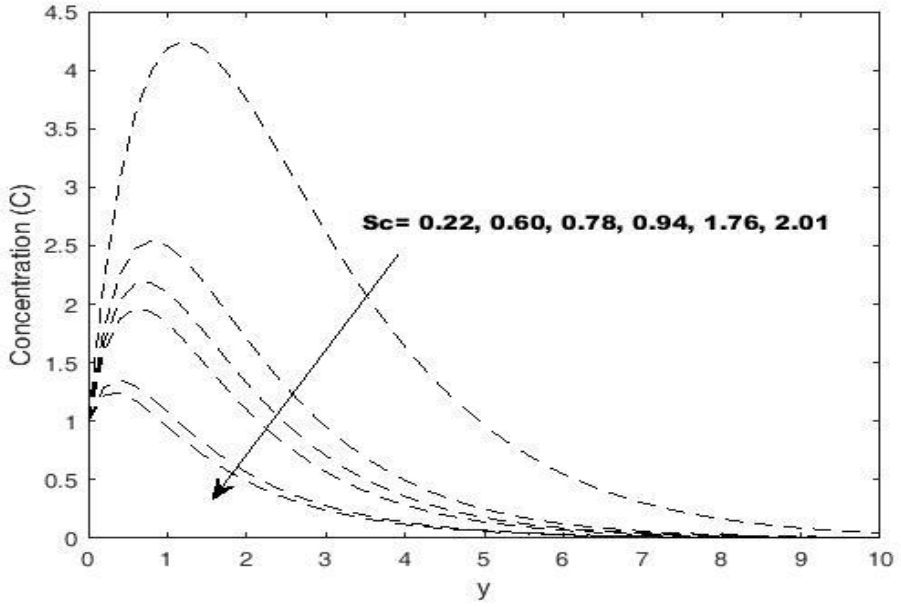


Figure 4.2 Concentration profile for different values of Sc and fixed values $K = 2.0,\ S = 10.0,\ \Pr = 0.71$

Figure 4.1shows the concentration distribution for different values of Schmidt number (Sc) for small value of Soret parameter $(S = 2)$. It is observed that the concentration ofthe fluid is inversely proportional to the Schmidt number. This show that increase in ratio of momentum diffusivity to mass diffusion decreases the concentration profile. Figure 4.2 shows the variation of concentration with Schmidt number for large value of Soret parameter $(S = 10)$. It isobserved that concentration boundary layer thickness increases with increase in Soret parameter.A comparison of Figure 4.1 withFigure4.2 indicates that the maximum value of concentration is at the porous plate for small value of Soret parameter $(S = 2)$which is not true for large value of Soret parameter $(S = 10)$.

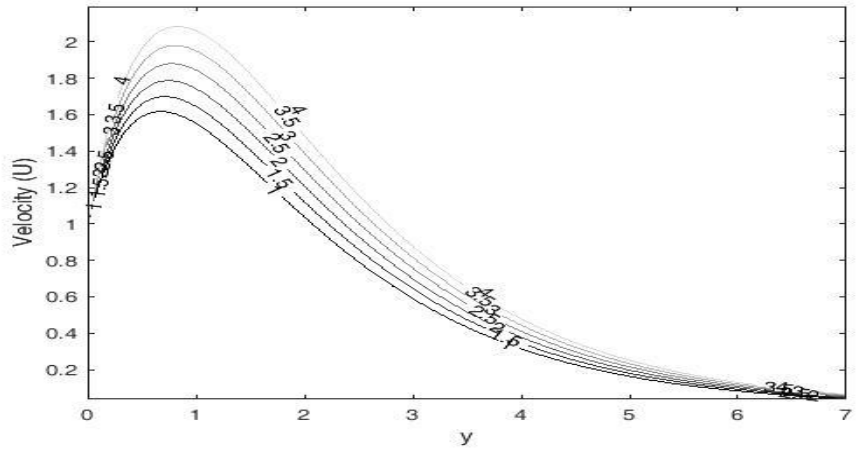


Figure 4.3 Velocity profile of nonlinear density variation with temperature NDT and concentration NDC for different values of S and fixed values $K = 2.0,\ Sc = 0.78,\ \Pr = 0.71,\ Gr = Gc = 1.0,\ A^* = B^* = 1$

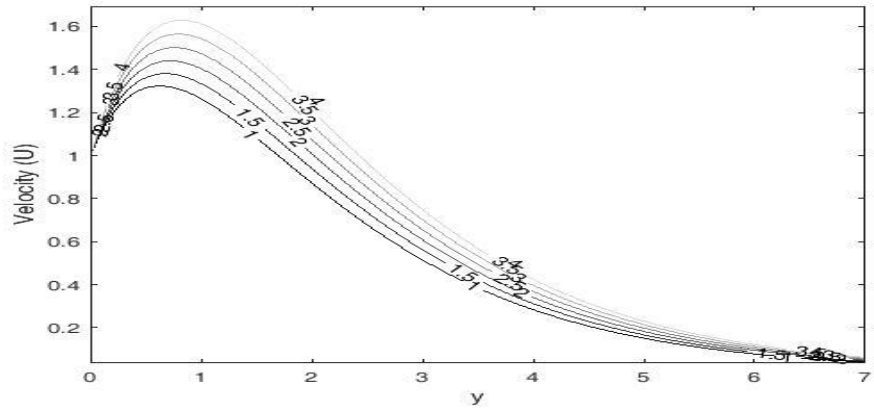


Figure 4.4 Velocity profile of linear density variation with temperature LDT and concentration LDC for different values of S and fixed values $K = 2.0,\ Sc = 0.78,\ \Pr = 0.71,\ Gr = Gc = 1.0,\ A^* = B^* = 0$

The Figures 4.3 and 4.4characterize the influence of S on velocity distribution for non-linear density variation with temperature and concentration, linear density variation with temperature and concentration respectively. It is observed that the momentum boundary layer thickness increases by incorporating nonlinear variation of density with temperature and concentration. Furthermore, fluid velocity increases with increase in

Soret parameter. This is due to the fact that as Soret parameter increases, convection current is stronger.

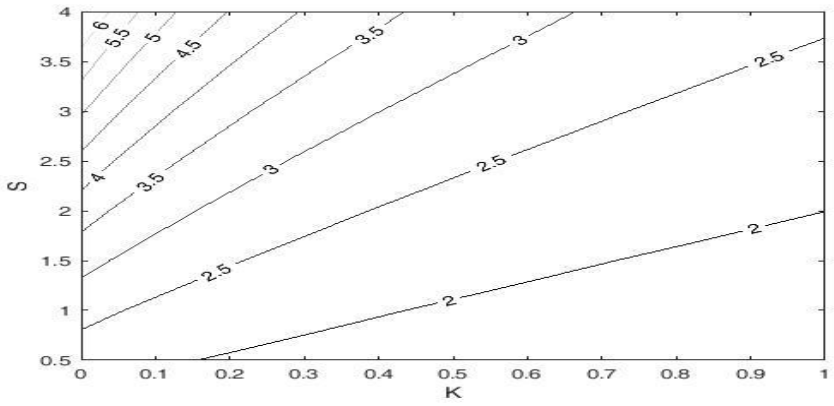


Figure 4.5 Contours of velocity profiles of nonlinear density variation with temperature NDT and concentration NDC for different values of S and K and fixed values $Y = 0.5,\ Sc = 0.78,\ \Pr = 0.71,\ Gr = Gc = 1.0,\ A^* = B^* = 1$

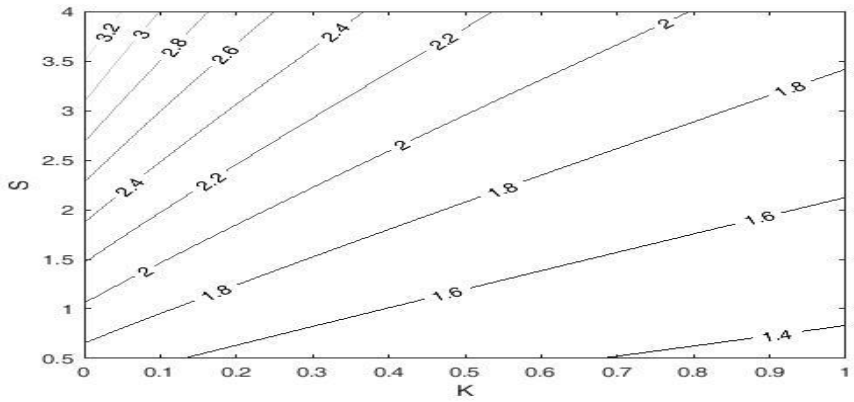


Figure 4.6 Contours of velocity profiles of linear density variation with temperature LDT and concentration LDC for different values of S and K and fixed values $Y = 0.5,\ Sc = 0.78,\ \Pr = 0.71,\ Gr = Gc = 1.0,\ A^* = B^* = 0$

Figures4.5 and 4.6 depict the contours of velocity distribution for nonlinear density variation with temperature (NDT), concentration (NDC)and linear density variation with temperature(LDT), concentration(LDC) for different values of S and K. It is observed that velocity increases with increase in S and decreases with increase in K

.This is due to the fact that convection current is stronger for higher values of Soret parameter while it is weaker for higher values of chemical reaction parameter. Mass diffusion equation shows that when $K = 0$ indicated no reaction, when $K < 0$ represented generative reaction and $K > 0$ denotes destructive reactionMuthucumaraswamy (2002), but in the present study, $K > 0$ has been considered.

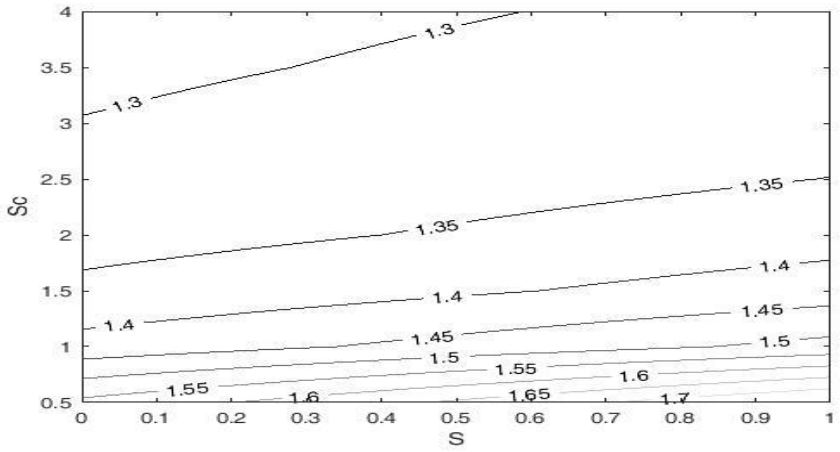


Figure 4.7 Contours of velocity profile of nonlinear density variation with temperature NDT and concentration NDC for different values of Sc and S and fixed values $Y = 0.5,\ \Pr = 0.71,\ K = 2.0,\ Gr = Gc = 1.0,\ A^* = B^* = 1$

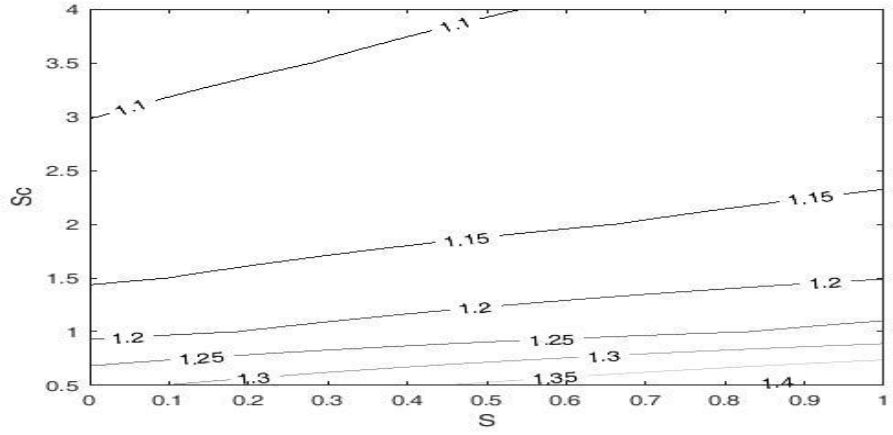


Figure 4.8 Contours of velocity profile of linear density variation with temperature LDT and concentration LDC for different values of Sc and S and fixed values $Y = 0.5,\ \Pr = 0.71,\ K = 2.0,\ Gr = Gc = 1.0,\ A^* = B^* = 0$

Figures 4.7 and 4.8reveal the variation of velocity profiles on nonlinear density variation with temperature (NDT), concentration (NDC),linear density variation with temperature(LDT), concentration(LDC) for different values of Sc and S. It is interesting to note that increasingSchmidt number decreases the velocity. This is physically true because as the ratio of momentum diffusivity to mass diffusion increase the velocity profile decreases. On the other hand, increasing thermal diffusion parameter S increases velocity profile. This is due to the fact that as Soret parameter increases, intensity of convection current increases.

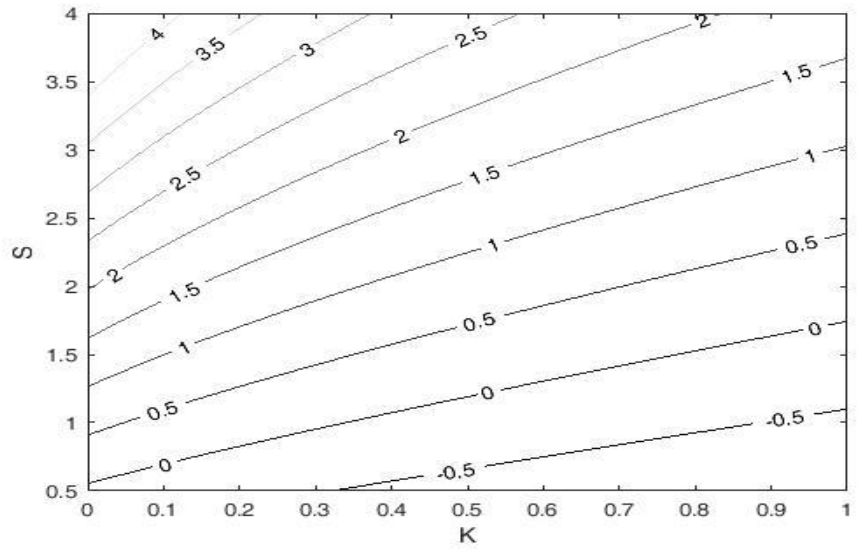


Figure 4.9 Contours showing the influence of S and K on the Sherwood number for fixed values of $Sc = 0.78,\ \Pr = 0.71$

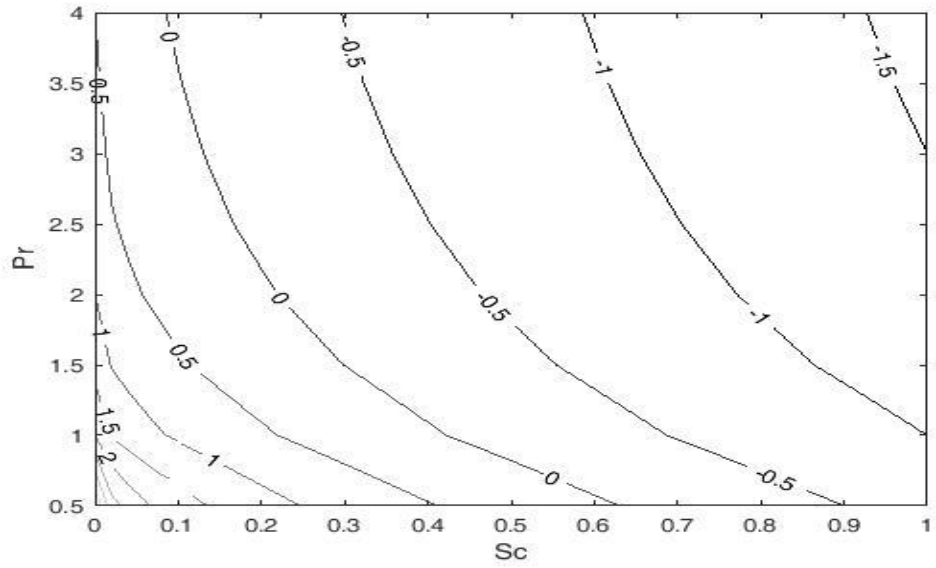


Figure 4.10 Contours showing the influence of Pr and Sc on the Sherwood number for fixed values of $S = 2.0,\ K = 2.0$

Figure 4.9 depicts the variation of Sherwood number for different values of S and K. Also, variation of Sherwood number for different values of Pr and Sc are shown in Figure 4.10. It is observed from these figures that, the ratio of mass transfer by convection and mass transfer by diffusion decreases as K, Pr and Sc increases, while the presence of S increasesmass transfer rate.

Table 4.1.1 Numerical comparison of the values of skin-friction for different values S in the presence and absence of NDT and NDC

Case (I - IV)	$S\ \tau = \left(\frac{dU}{dY}\right)_{Y=0}$
$A^* = 1,\ B^* = 1$ 0.00	1.9958
0.50	2.1661
1.50	2.5315
$A^* = 0,\ B^* = 0$ 0.00	0.9972
0.50 1.1256	
1.50	1.3823
$A^* = 1,\ B^* = 0$ 0.001.7014	
0.501.8298	
1.50 2.0865	
$A^* = 0,\ B^* = 1$ 0.00	1.2916
0.50	1.4619
1.50	1.8273

Table 4.1.2 Numerical comparison of the values of velocity obtained in the present work with those of Muthucumaraswamy (2002)

Y	Previous work U(Y) (2002)	Present work U(Y) $A^* = B^* = S = 0$	Present work U(Y) $A^* = B^* = S = 1.0$
0.1	1.0856	1.0856	1.2048
0.2	1.1462	1.1462	1.3573
0.3	1.1859	1.1859	1.4672
0.4	1.2085	1.2085	1.5422
0.5	1.2169	1.2169	1.5886
0.6	1.2138	1.2138	1.6118
0.7	1.2013	1.2013	1.6164
0.8	1.1813	1.1813	1.6058
0.9	1.1554	1.1554	1.5832
1.0	1.1248	1.1248	1.5512

Table 4.1.1 depicts a numerical comparison of skin-friction at the moving porous plate for different values of Soret parameter in the presence or absence of nonlinear density variation with temperature (NDT) and concentration (NDC). It can be observed from the table that the numerical values of skin-friction are higher in presence of NDT and NDC in relative to LDT and LDC respectively. Also, by increasing Soret parameter, frictional force on the boundary increases.

Table 4.1.2 depicts a numerical comparison of velocity obtained in the present work when $A^* = B^* = S = 0.0$ and compared with those of Muthucumaraswamy (2002) $A^* = B^* = S = 1.0$. It is observed from the table that the numerical values of fluid velocity in the absence of NDT and NDC as well as Soret effect reported in the present work are in excellent agreement with the numerical values of dimensionless velocity reported in Muthucumaraswamy (2002). Furthermore, the presence of nonlinear variation with temperature and concentration as well as Soret effect increases the velocity profile.

4.2Combined effect of chemical reaction and Soret effect on nonlinear natural convection and mass transfer couette flow

In this section, the interactive effects of some physical parameters such as, the nonlinear densityvariation with temperature parameter (A^*) and nonlinear density variation with concentration (B^*), Soret parameter (S), chemical reaction (K), Schmidt number (Sc) and Prandtl number (Pr) are investigated.Theinfluence of NDT and NDC (nonlinear density variation with temperature and concentration) on skin friction, rate of mass

transfer, and velocity are presented in Table 4.2.1, Table 4.2.2 and Table 4.2.3 respectively. The fluids considered in this investigation are air $(\mathrm{Pr}=0.71)$ and water $(\mathrm{Pr}=7.0)$. The chemical species of common interest also have Schmidt numbers in air ranging from 0.1 to 1.0 Muthucumaraswamy (2002). The selected values of Soret parameter S and nonlinear density variation with temperatureand concentration for the present analysis are chosen arbitrarilyin such a way that their product is constant, provided that the temperature remains constant.

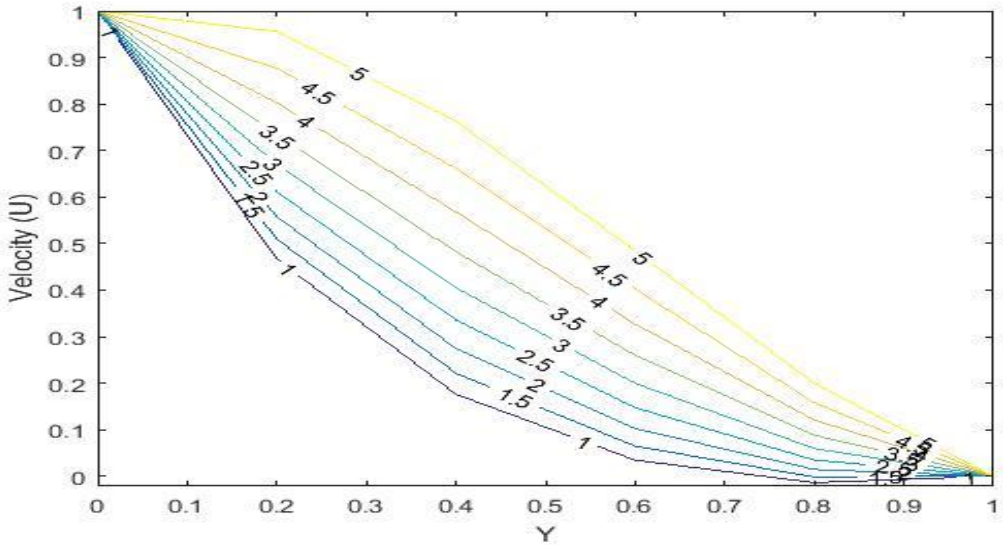


Figure 4.11 Velocity profile of nonlinear density variation with temperature (NDT) and concentration (NDC) for different values of S and the fixed values $H=1.0,\ K=2.0,\ Sc=0.78,\ \mathrm{Pr}=0.71,\ Gr=Gc=1.0,\ A^{*}=B^{*}=1$

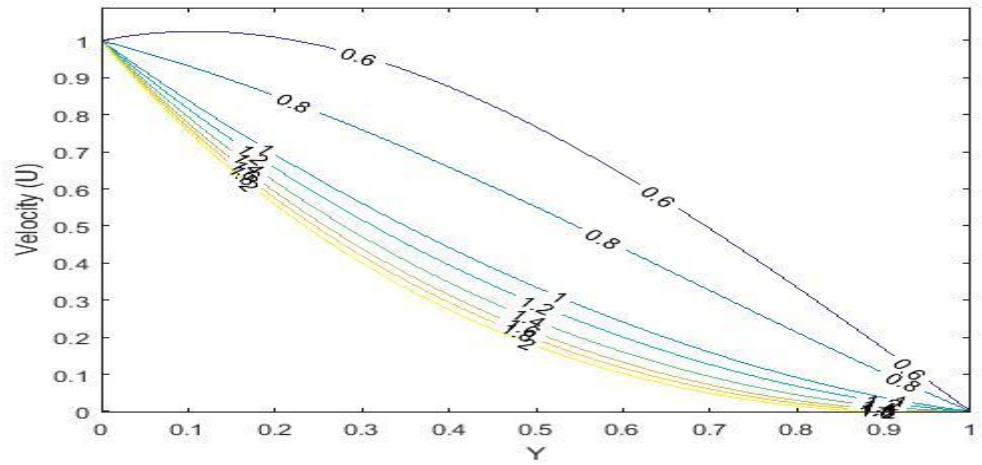


Figure 4.12Velocity profile of nonlinear density variation with temperature (NDT) and concentration (NDC) for different values of K and the fixed values $H = 1.0,\ S = 2.0,\ Sc = 0.78,\ \Pr = 0.71,\ Gr = Gc = 1.0,\ A^* = B^* = 1$

Figure 4.11shows the velocity distribution for different values of thermal diffusion parameter(S). Velocity profile increases with increase in the thermal diffusion parameter, which shows that buoyancy forces increase,compared to what is obtainable in Muthucumaraswamy(2002) investigation in the presence of Soret effect, nonlinear density variation with temperature (NDT), concentration (NDC). It is observed that as the thermal diffusion parameter increases, the convection currents also become stronger. Figure 4.12 depicts the velocity profile of nonlinear density variation with temperature (NDT), concentration (NDC) for different values of chemical reaction parameter. It is observed thatfor the destructive reaction, velocity decreases with an increase in chemical reaction parameter, which shows that chemical reactive species emitted from the porous plate diffuse into the fluids and resulted to decrease in velocity profile. It is also noticed that velocity increases near a moving porous plate and sufficiently decreases far away from the moving porous plate with an increase in chemical reaction parameter.

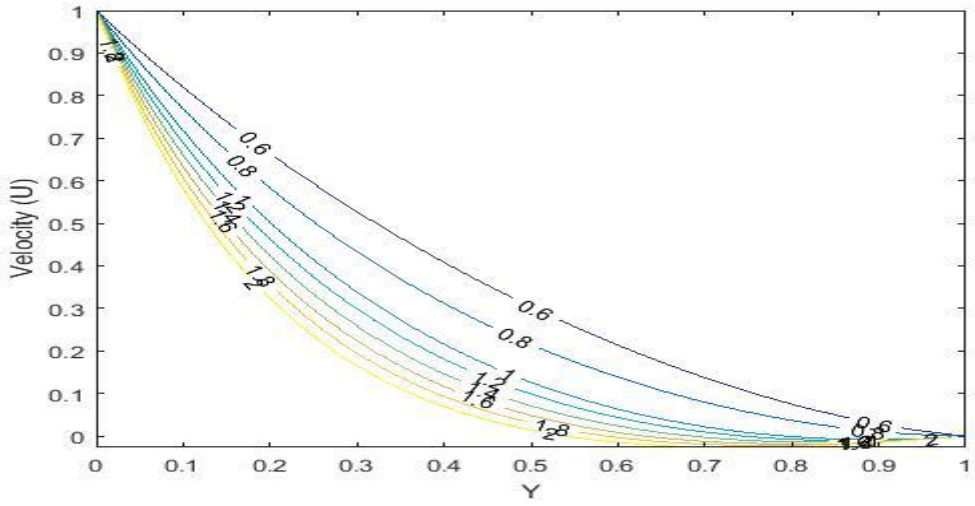


Figure 4.13 Velocity profile of nonlinear density variation with temperature (NDT) and concentration (NDC) for different values of Sc and the fixed values $H = 1.0,\ S = 2.0,\ K = 2,\ \Pr = 0.71,\ Gr = Gc = 1.0,\ A^* = B^* = 1$

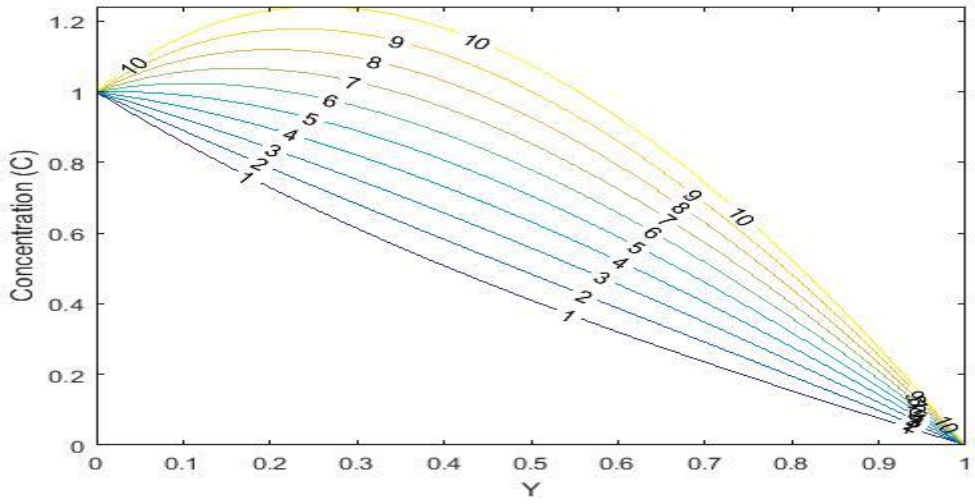


Figure 4.14 Concentration profile for different values of (S) and the values of $H = 1.0,\ K = 2.0,\ Sc = 0.78,\ \Pr = 0.71$

Figure 4.13 also shows that velocity profile decreases with increase in Schmidt number Sc. This is physically true due to the fact that the Schmidt number is the ratio of momentum diffusitivity to mass diffusition convection process. It has been observed that the velocity profile decreases with increase in Schmidt number sufficiently far away

from a moving porous plate. Figure 4.14 portrays the concentration profile for different valuesof Soret parameter(S). It is noticed that concentration profile is slightly enhanced with increase in Soret parameter. However, this occurs due to the physical fact that, as the thermal diffusion parameter increases, the mass diffusivity of the fluid increases which results to simultaneous increase in the concentration boundary layer.

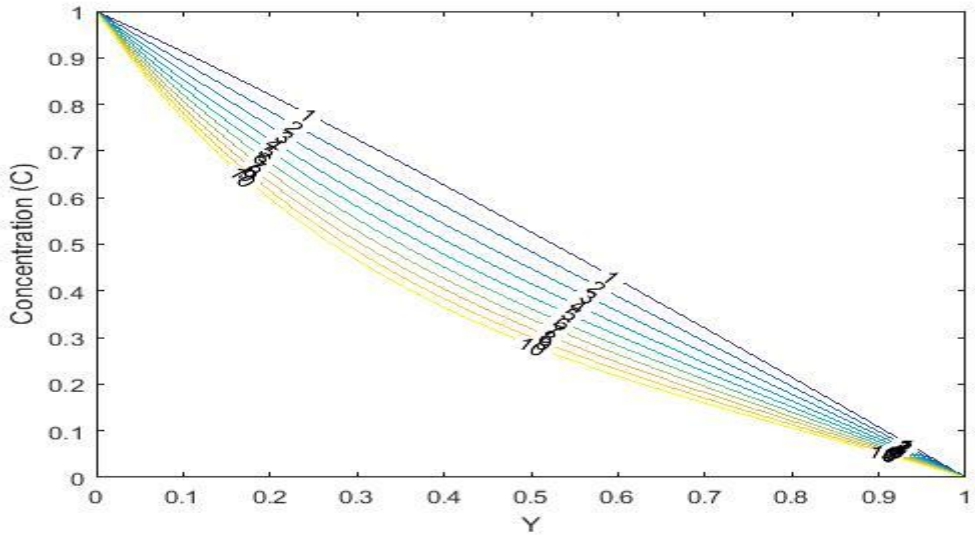


Figure 4.15 Concentration profile for different values of(K) and the values $H = 1.0,\ S = 2.0,\ Sc = 0.78,\ \Pr = 0.71$

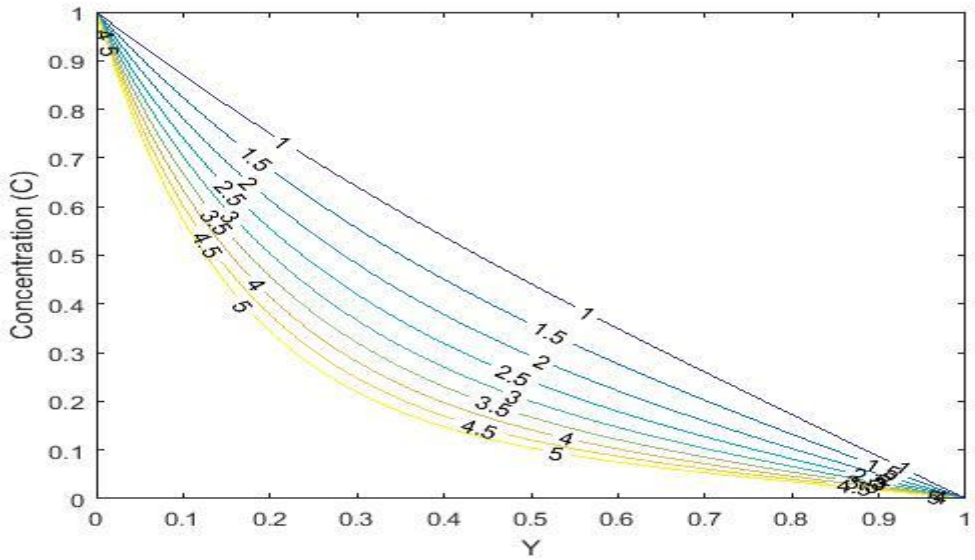


Figure 4.16 Concentration profile for different values of(Sc) and the values $H = 1.0,\ S = 2.0,\ K = 2.0,\ \Pr = 0.71$

Figure 4.15 shows that, concentration profile decrease in the presence of destructive reaction. It has been found that, increase in chemical reaction parameter decreases the mass diffusion rate convection process. This phenomenon agrees with Muthucumaraswamy(2002) in the absence of nonlinear temperature density variation, nonlinear concentration density variation and Soret parameter. Figure 4.16 shows the concentration profile for different values of Sc . It is observed that concentration profile decreases due to increase in the value of Schmidt number. Therefore, decrease inratio of momentum diffusivity to mass diffusion rateresulted to increase in the concentration profile.

Table 4.2.1 Comparison of skin friction (τ) at (Y=0) and (Y=H) for fixed values $Sc = 0.78,\ \Pr = 0.71,\ Gr = Gc = 1.0,\ H = 10$

S	$A^* = B^* = 0\ K = 0$		$A^* = B^* = 0\ K = 1$		$A^* = B^* = 1\ K = 0$		$A^* = B^* = 1\ K = 1$	
	τ_0	τ_H	τ_0	τ_H	τ_0	τ_H	τ_0	τ_H
0	1.6751	0.0030	1.1360	0.0019	3.0186	0.0032	2.2079	0.0020
1	2.9077	0.0092	1.5371	0.0030	5.1529	0.0098	2.7870	0.0031
2	4.1404	0.0155	1.9382	0.0040	7.8815	0.0170	3.4428	0.0042

Table 4.2.2 Rate of mass transfer (Sh) at (Y=0) and (Y=H) for fixed values of $\Pr = 0.71,\ Gr = Gc = 1.0,\ H = 1.0$

Sc	$S=0\ \ K=0$		$S=0\ \ K=2$		$S=1\ \ K=0$		$S=1\ \ K=2$	
	Sh_0	Sh_H	Sh_0	Sh_H	Sh_0	Sh_H	Sh_0	Sh_H
0.6	1.3298	0.7298	1.8841	0.4483	1.1105	0.8714	1.7727	0.5027
0.78	1.4402	0.6602	2.0788	0.3803	1.2157	0.7967	1.9702	0.4291
1.0	1.5820	0.5820	2.3130	0.3130	1.3513	0.7123	2.2066	0.3564

Table 4.2.3 Shows the velocity (U) for different values of channel width (H) where $Sc = 0.78,\ \Pr = 0.71,\ Gr = Gc = 1.0, Y = 0.5$

Ref.(2002)Present work												
K	$H=5$	$H=10$	$H=20$	$A^*=B^*=0$ $S=0$		$A^*=B^*=1$ $S=1$		$A^*=B^*=0$ $S=0$		$A^*=B^*=1$ $S=1$		$A^*=B^*=0$ $S=0$		$A^*=B^*=1$ $S=1$	
				$A^*=B^*=0$ $S=0$		$A^*=B^*=1$ $S=1$									
0	1.4772	2.0653	1.4518	1.7491	1.4779	2.0578	1.4772	2.0653							
1	1.2712	1.7266	1.2766	1.7068	1.2768	1.7249	1.2712	1.7266							
2	1.2169	1.7277	1.2259	1.7024	1.2185	1.7285	1.2169	1.7277							
3	1.1883	1.7424	1.1983	1.7020	1.1900	1.7433	1.1883	1.7424							
4	1.1699	1.7565	1.1801	1.7084	1.1718	1.7573	1.1699	1.7565							

The influence of nonlinear density variation with temperature (NDT), concentration (NDC), Soret parameter, chemical reaction parameter, skin-friction (shear stress), rate of mass transfer on the surface of the channel porous plates are presented in Tables 4.2.1 and 4.2.2 respectively. It is observed that in Table 4.2.1,the drag caused by thefriction of fluid against the surface of the channel plates increases with increase in thermal diffusion parameter at $\tau_0\ (Y=0)$ and $\tau_H\ (Y=H)$ respectively. However, the numerical values of the skin friction at $\tau_0\ (Y=0)$ are higher at all levels in comparison to $\tau_H\ (Y=H)$.

The effects of chemical reaction and nonlinear convection parameter arealso considered in Table 4.2.1. It is noted that the skin friction is singnificantlyenhanced in the presence of nonlinear density variation parameter, in comparison to chemical reaction effect. The influence of Schmidt number, linear and nonlinear density variation, chemical reaction parameter, Soret effect pesentedin Table 4.2.2, it is noticed that, the rate of mass transfer is enhanced withincrease in Schmidt number at the plate Sh_0 and decreases at the plate

Sh_H . This result also reports that, mass diffusion convection increases in thepresence of chemical reaction parameter at the hot plate Sh_0 and decreases in the absence of chemical reaction parameter at the plate Sh_H .It hasalso confirmed that themass diffusion decreases in the presence of Soret parameter at the plate Sh_0 and increases at the plate Sh_H . However,in Table 4.2.3 comparison of present results is made with the previous workin the literature, which justifiesthe accuracy of the analytical method employed in this study. It is observed that when the channel gap (H) is larg, the result for the velocity of present situation coincides with that of Muthucumaraswamy(2002).

4.3Combined effect of chemical reaction and Dufour effect on non-linear free convection heat and mass transfer flow near a vertical moving porous plate

Exact solutions have been derived for the mathematical model responsible for non-linear natural convection flow due to heat and mass transfer near an infinite vertical moving porous plate in the presence of first order chemical reaction and Dufour effect. To have a clear picture of the problem, we computed tables for the Skin fiction, Nusselt number and Sherwood number. A comparative analysis for skin-friction has been made for the influence of non-linear density variation with temperature (NDT), concentration (NDC) and linear density variation with temperature (LDT), concentration (LDC) with diffusion-thermo effect as presented in Table 4.3.1. The selected values of Prandtl number are 0.71 and 7.0 which represent air and water respectively. The specific kind of chemical species have the Schmidt numbers 0.1-1.0Muthucumaraswamy(2002).

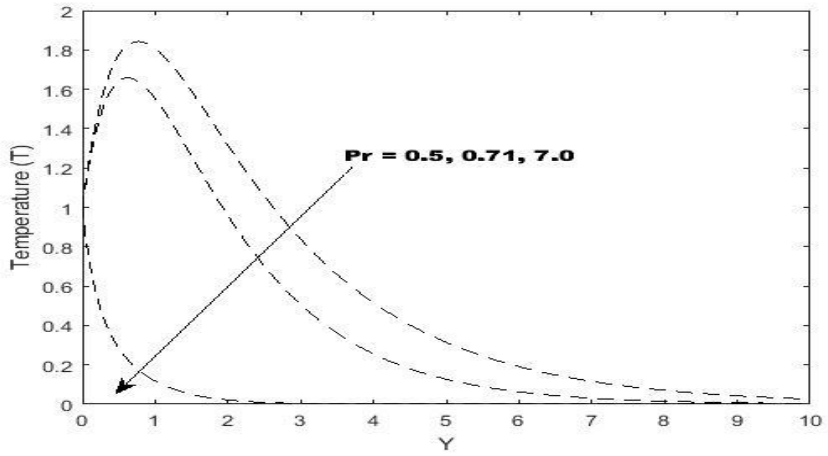


Figure 4.17Temperature profile for different values of Pr for fixed values of $K = 2.0,\ D_f = 2.0,\ Sc = 0.78$

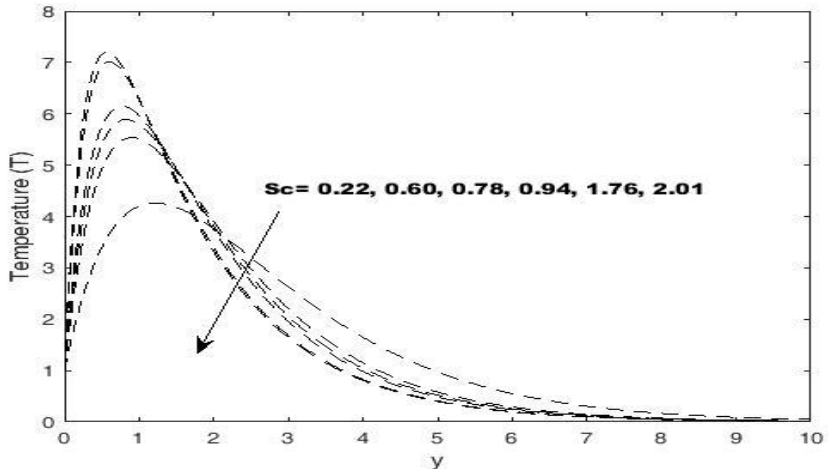


Figure4.18Temperature profile for different values Sc for fixed values of $K = 2.0,\ D_f = 10.0,\ \Pr = 0.71$

It is observed that increase in Prandtl number decreases the temperature as shown in Figure4.17. This is due to the fact that Prandtl number is the ratio of momentum transport tothermal diffusion. Therefore,decrease in Prandtl number increases the thermal boundary layer thickness, and causes the significant increase of heat diffusion process within the system. Figure 4.18indicates that increase in Schmidt number

increases the temperature near the porous plate and decreases the temperature far away from the moving porous plate.

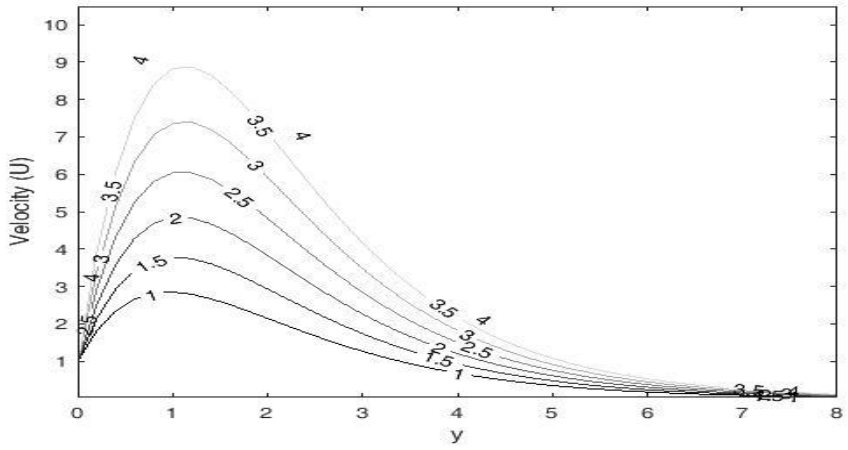


Figure 4.19 Velocity profile of non-lineardensity variation with temperature and non-linear density variation with concentration for different values of D_f and fixed values of

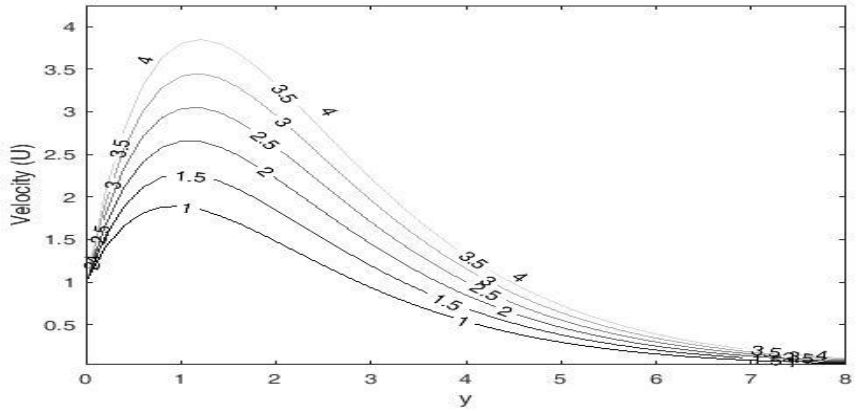


Figure 4.20 Velocity profile of lineardensity variation with temperature and linear density variation with concentration for different values of D_f and fixed values of

$$K = 2.0,\ Sc = 0.78,\ \Pr = 0.71,\ Gr = Gc = 1.0,\ A^* = B^* = 0$$

Figures 4.19 and 4.20demonstrate the influence of Dufour effect on the velocity profile for non-linear density variation with temperature,concentration, linear density variation

with temperature and concentration respectively.It is observed that velocity increases with increase in Dufour number. This is due to the fact that as the Dufour number increases, the convection current would be stronger. It is observed from these figures that the numerical values of velocity are higher in the presence of non-linear density variation with temperature and concentration in relative to the linear density variation with temperature and concentration.

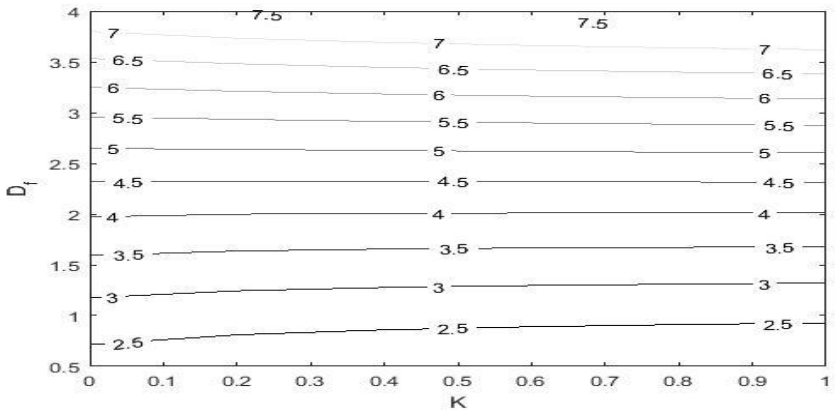


Figure 4.21 Contours of velocity profile for non-lineardensity variation with temperature and non-linear density variation with concentration for different values of D_f and K for fixed values of $Y = 0.5,\ Sc = 0.78,\ \Pr = 0.71,\ Gr = Gc = 1.0,\ A^* = B^* = 1$

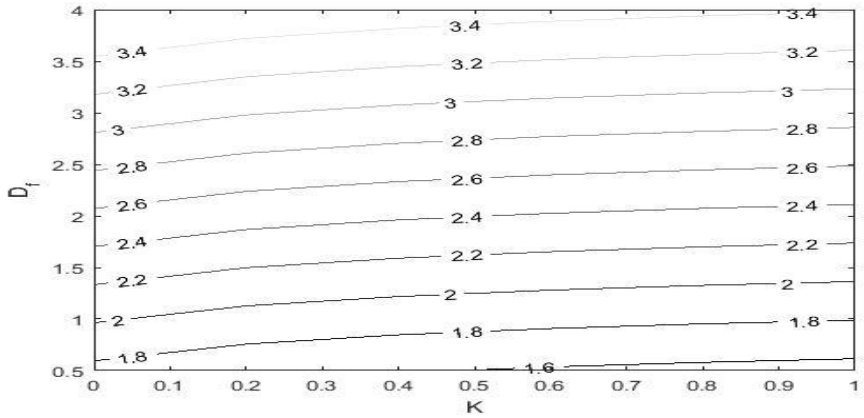


Figure 4.22 Contours of velocity profile forlineardensity variation with temperature and linear density variation with concentration for different values of D_f and K for fixed values of $Y = 0.5,\ Sc = 0.78,\ \Pr = 0.71,\ Gr = Gc = 1.0,\ A^* = B^* = 0$

Figures 4.21 and 4.22 depict the contours of velocity variation for non-linear density variation with temperature, concentration, linear temperature density variation with temperature and concentration respectively for different values of D_f and K. It is observed from these Figures that the velocity increases with increase in Dufour number and decreases with increase in chemical reaction parameter. It is also noticed that in the presence of non-linear density variation with temperature and concentration, velocity is higher in comparison to linear density variation with temperature and concentration.

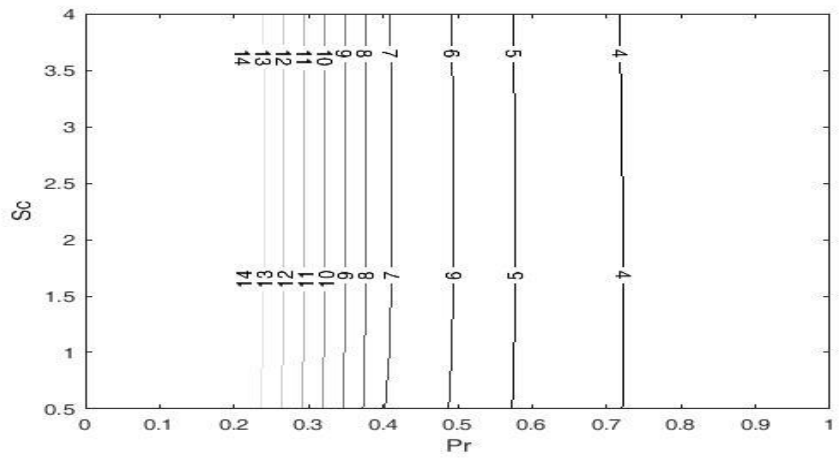


Figure 4.23 Contours of velocity profile of non-lineardensity variation with temperature and non-linear density variation with concentration for different values of Sc and Pr for fixed values of $Y = 0.5,\ D_f = 2.0,\ K = 2.0,\ Gr = Gc = 1.0,\ A^* = B^* = 1$

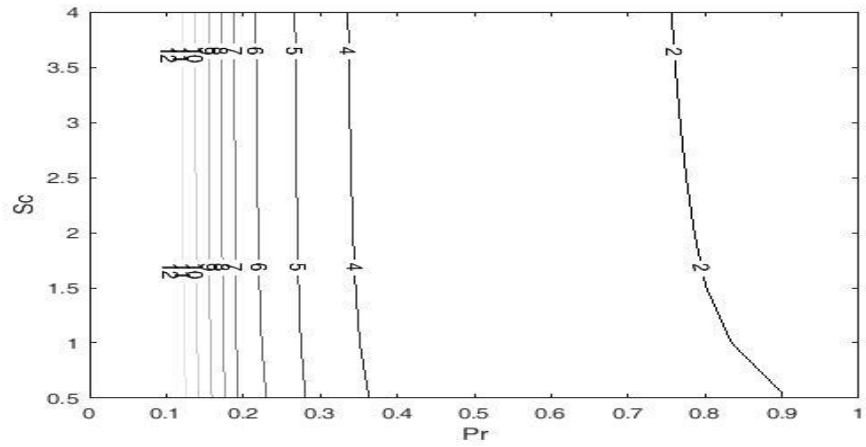


Figure 4.24 Contours of velocity profile oflineardensity variation with temperature and linear density variation with concentration for different values of Sc and Pr for fixed values of $Y = 0.5,\ D_f = 2.0,\ K = 2.0,\ Gr = Gc = 1.0,\ A^* = B^* = 0$

Figures 4.23 and 4.24 show the contours of velocity variation due to nonlinear density variation with temperature, concentration, linear density variation with temperature and concentration for different values of Sc and $\Pr$. It is noticed that increase in the values of Sc and $\Pr$ decrease the velocity profile.This is due to the fact that as $\Pr$ increases, convection current due to heat transfer decreases, similarly, as Sc increases, convection current due to mass transfer decreases.

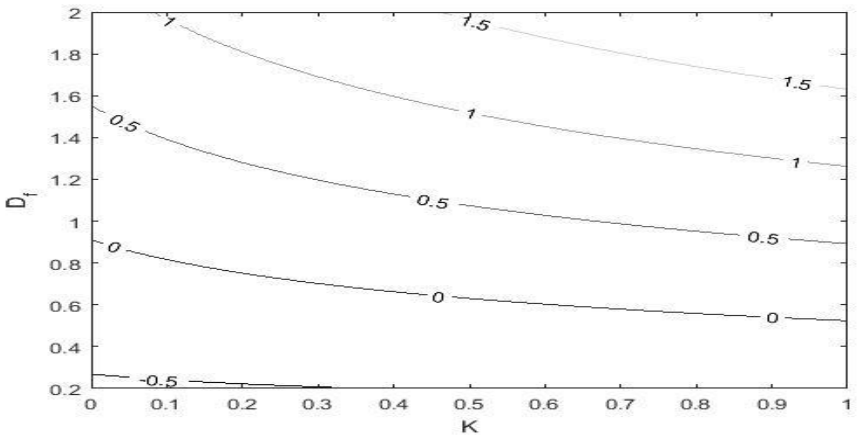


Figure 4.25 Contours showing the effect of D_f and K on the Nusselt number for fixed values of $Y = 0.5,\ Sc = 0.78,\ \Pr = 0.71$

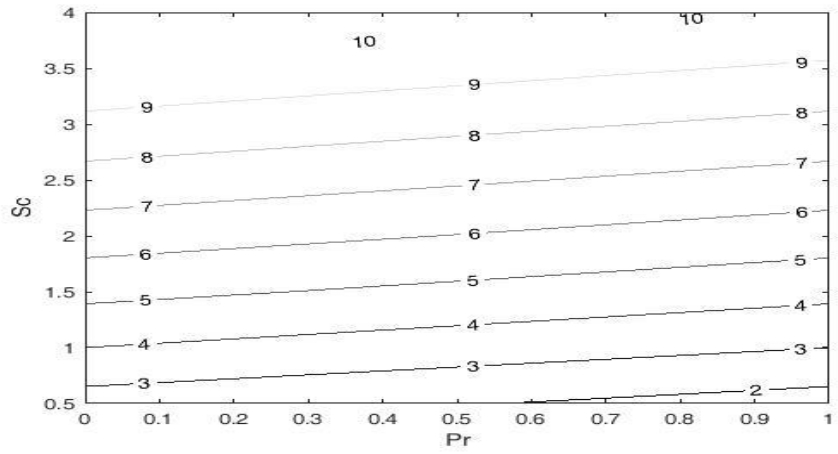


Figure 4.26 Contours showing the effect of Pr and Sc on the Nusselt number for fixed values of $Y = 0.5,\ D_f = 2.0,\ K = 2.0$

Figures 4.25 and 4.26 represent the contours showing the influence of the parameters D_f, K, Sc and Pr on the ratio of heat transfer by convection to heat transfer by conduction. It is observed that the heat transfer diffusion increases with increase in D_f, K and Sc while it decreases with increase in Prandtl number.

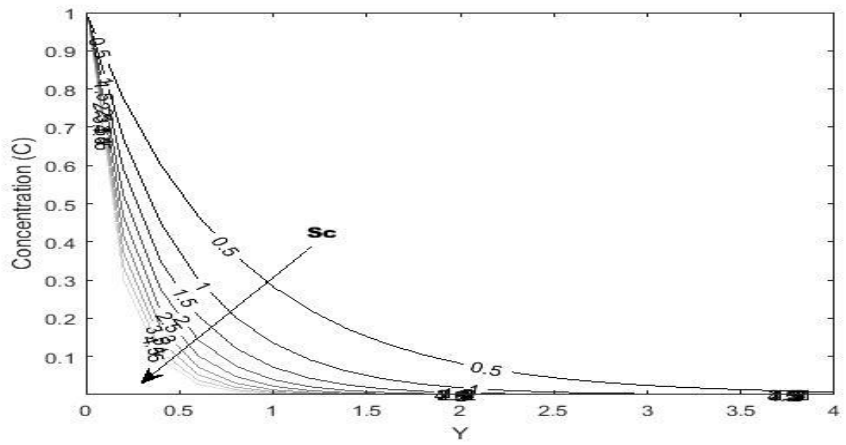


Figure 4.27 Concentration profile for different values of Sc for fixed value of $K = 2.0$

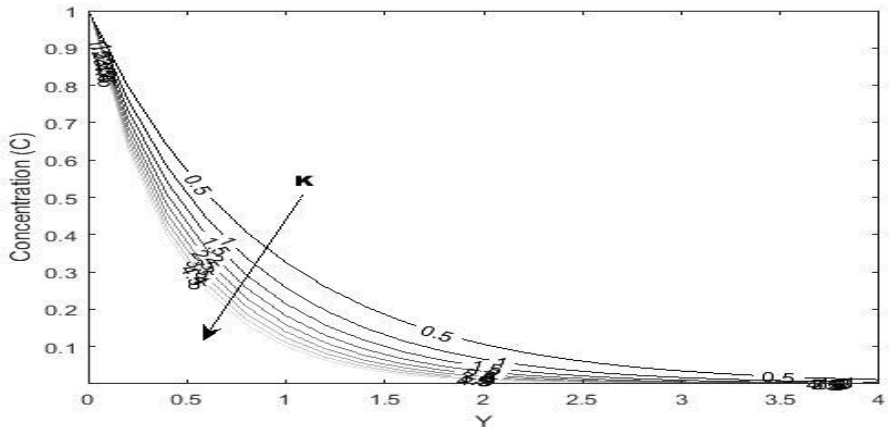


Figure 4.28 Concentration profile for different values of K for fixed value of $Sc = 0.78$

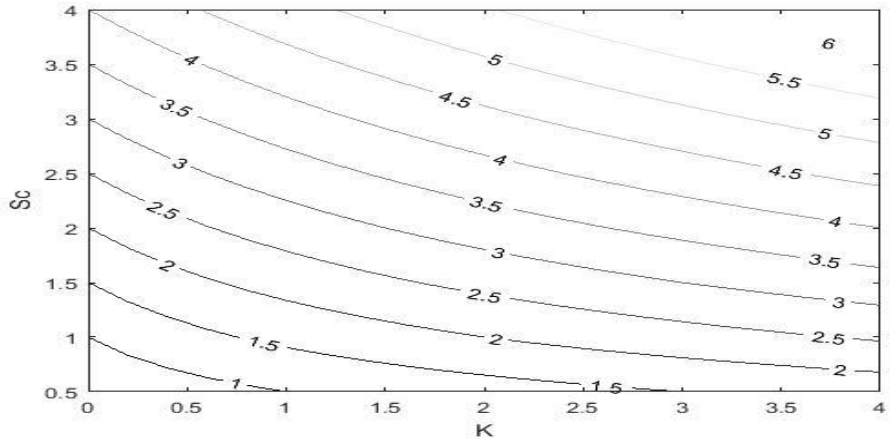


Figure 4.29 Contours showing the effect of Sc and K on the Sherwood number.

The effects of Schmidt number and chemical reaction parameter on the concentration profile are shown in Figures 4.27 and 4.28 respectively. The concentration profile isfound to decrease with increase in K and Sc . Figure 4.29 shows that the ratio of mass transfer by convection and mass transfer by diffusion increases with an increasein chemical reaction parameter andSchmidt number.

Table 4.3.1 Variation of skin friction for fixed values of $Sc = 0.78,\ \Pr = 0.71,\ K = 2.0,\ Gr = Gc = 1.0$

Case (I - IV)	$(D_f)\ \tau = \left(\frac{dU}{dY}\right)_{Y=0}$.		
$A^* = 1,\ B^* = 1$	0.00	1.9958	
	0.50	3.3208	
	1.50	6.7158	
$A^* = 0,\ B^* = 0$	0.00	0.9972	
	0.50	1.7014	
	1.50	3.1099	
$A^* = 1,\ B^* = 0$	0.00	1.7014	
	0.50	3.0264	
	1.50	6.4214	
$A^* = 0,\ B^* = 1$	0.00	1.2916	
	0.50	1.9958	
	1.50	3.4043	

A comparison of numerical values of skin-friction at the porous plate for different values of D_f ,in case of linear density variation with temperature and concentration ($A^* = B^* = 0$) and non-linear density variation with temperature and concentration ($A^* = B^* = 1$)is presented in Table 4.3.1. This Table reveals that the numerical values of skin-friction are higher in the presence of non-linear density variation with temperature and concentration relative to linear density variationwith temperature and concentration.

Table 4.3.2 Comparison for the values of velocity in the present work with those of Muthucumaraswamy (2002) for fixed values of $Sc = 0.78,\ \Pr = 0.71,\ K = 2.0,\ Gr = Gc = 1.0,\ (A^* = B^* = D_f = 0)$

Y	Previous work	Present work U(Y)

U(Y) (2002)		$A^* = B^* = D_f = 0$
0.1	1.0856	1.0856
0.2	1.1462	1.1462
0.3	1.1859	1.1859
0.4 1.2085		1.2085
0.5	1.2169	1.2169
0.6	1.2138	1.2138
0.71.2013		1.2013
0.8	1.1813	1.1813
0.9	1.1554	1.1554
1.0	1.1248	1.1248

Table 4.3.2 presents numerical results of non-dimensional velocity obtained in this work compared with those of Muthucumaraswamy (2002)in the absence of Dufour effect and in the presence of linear density variation with temperature and concentration ($A^* = B^* = 0$).Details from the table show that in the absence of diffusion thermo effect ($D_f = 0$) and linear density variation with temperature and concentration ($A^* = B^* = 0$), numerical values of present work are in excellent agreement with those of Muthucumaraswamy (2002), hence validating the correctness of the solution obtained herein.

4.4 Combined effects of Dufour and chemical reaction on nonlinear natural convection heat and mass transfer couette flow in a vertical channel

The ordinary differential equations (3.6), (3.44) and (3.45) along with the boundary condition (3.47) are solved exactly in terms of exponential function by the method of undetermined coefficients. The solutions obtained are functions of the governing non-dimensional parameters, such as nonlinear variation with temperature $\left(A^*\right)$, nonlinear

variation with concentration $\left(B^{*}\right)$, Dufour number $\left(D_f\right)$, chemical reaction parameter $\left(K\right)$, Schmidt number $\left(Sc\right)$ and Prandtl number $\left(\mathrm{Pr}\right)$.

The influenceof these parameters on the velocity, temperature, skin friction and Nussselt number are extensively discussed.To have a clear picture of the problem, a numerical investigation is conducted and the results are reported in terms of lines graphs, contours and tables. The air and water are two mutual fluids, used in this study with values Prandtl number 0.71 and 7.0 respectively. The selection of Schmidt number considered ranges from 0.1 to 1.0Muthucumaraswamy (2002).

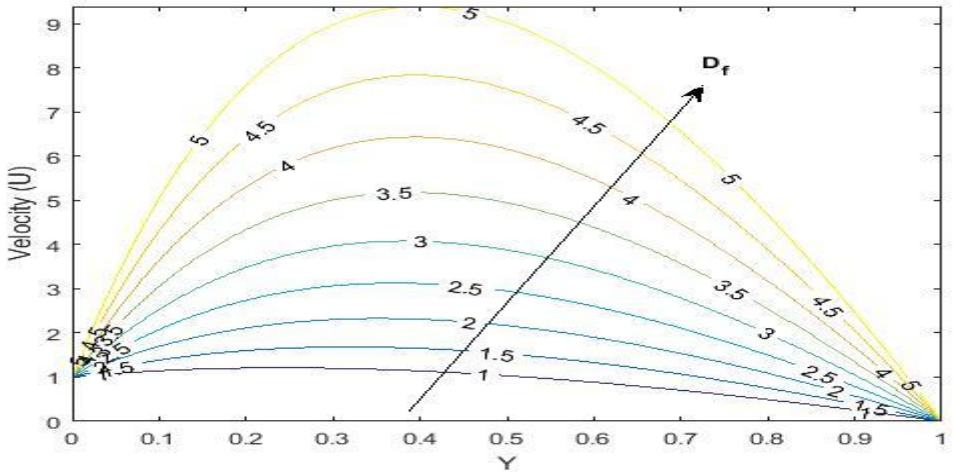


Figure 4.30 Velocity profile for nonlinear density variation with temperature (NDT) and concentration (NDC) for different values of D_f and the fixed value $H=1.0,\ K=2.0,\ Sc=0.78,\ \mathrm{Pr}=0.71,\ Gr=Gc=1.0,\ A^{*}=B^{*}=1$

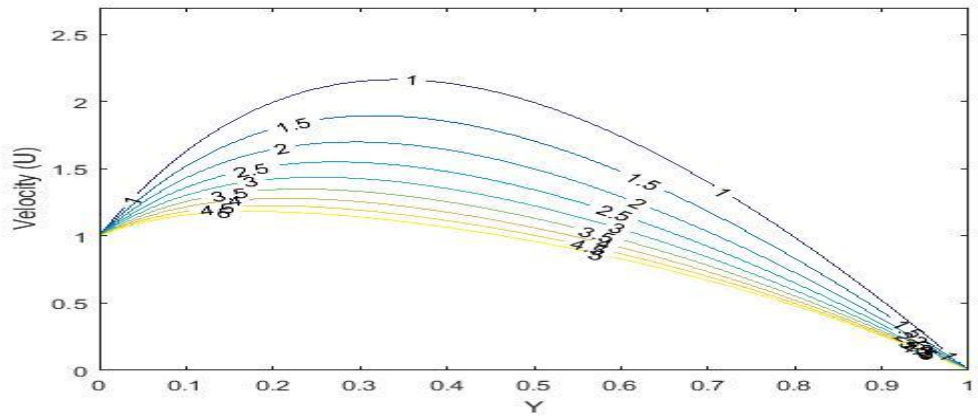


Figure 4.31 Velocity profile for non-linear density variation with temperature (NDT) and concentration (NDC) for different values of Sc and the fixed values $H = 1.0,\ K = 2.0,\ D_f = 2.0,\ \Pr = 0.71,\ Gr = Gc = 1.0,\ A^* = B^* = 0$

Figure4.30shows the velocity variation for different values of diffusion thermo effect.It is observed that velocity increases with increase in Dufour number. However, this significant effect is due to nonlinear density variation with temperature and nonlinear density variation with concentration. The heat enhancement is found to be higher in nonlinear density variation with temperature and concentration when comparedto linear density variation with temperature (LDT) and concentration (LDC). This attributed to the fact that, for an increasein Dufour effect, the convection due heat transport increase.Figure 4.31 depicts the physical influence of Schmidt numberon velocity profile in the presence of nonlinear variation with temperature and concentration. It is found that increase in Schmidt number decreases velocity profile.

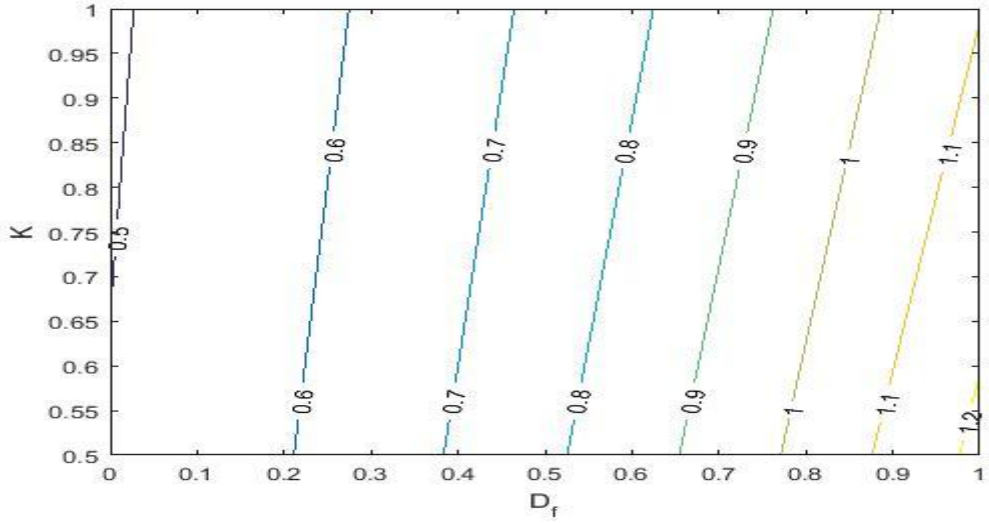


Figure 4.32 Contours of velocity profile for nonlinear density variation with temperature (NDT) and concentration (NDC) of different values of K, D_f and the fixed values $Y = 0.5,\ Sc = 0.78,\ \Pr = 0.71,\ Gr = Gc = 1.0,\ A^* = B^* = 1$

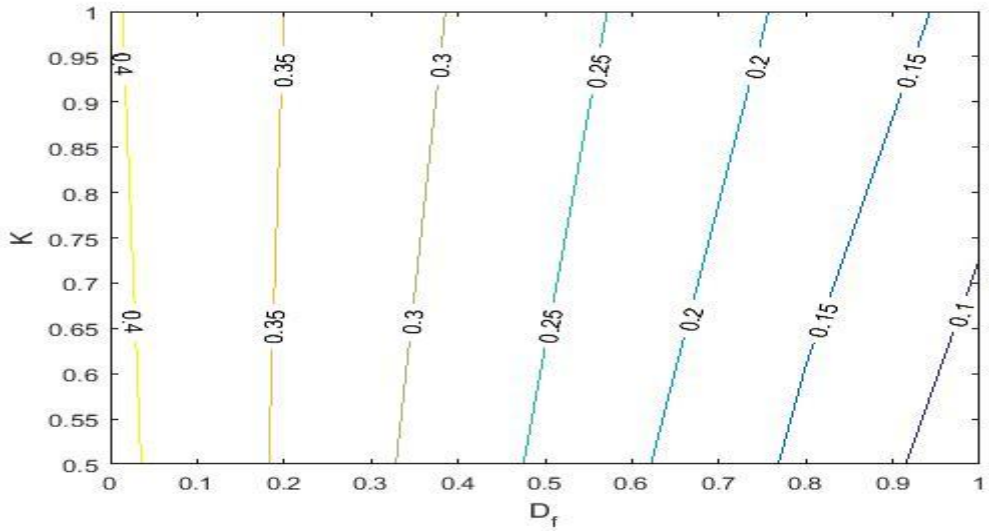


Figure 4.33 Contours of velocity profile for linear density variation with temperature (LDT) and concentration (LDC) of different values of K, D_f and the fixed values $Y = 0.5,\ Sc = 0.78,\ \Pr = 0.71,\ Gr = Gc = 1.0,\ A^* = B^* = 0$

The contours of velocity profile for nonlinear variation with temperature, nonlinear density variation with temperature, concentration,linear density variation with temperature and concentration for different values of K (chemical reaction parameter)

and D_f (Dufour number) are presented in Figures 4.32and 4.33 respectively. It was obvious from these figures that increase in positive value of chemical reaction parameter decreases the velocity profile for the destructive reaction, while increase in Dufour number increases the buoyancy forces, which resulted to increase in the velocity profile. In addition, the presence of nonlinear variation with temperature and nonlinear variation with concentration enhances the velocity.

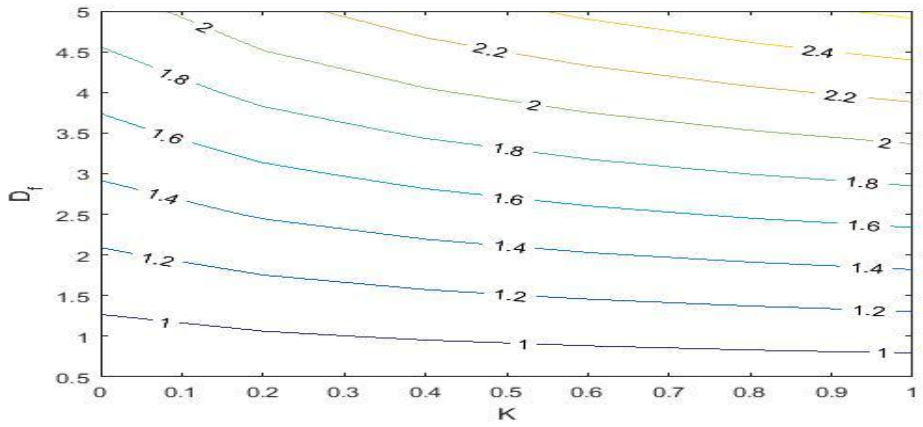


Figure 4.34 Contours of Temperature profile showing the influence of K, D_f and the fixed values $Sc = 0.78,\ \mathrm{Pr} = 0.71,\ \ Y = 0.5$

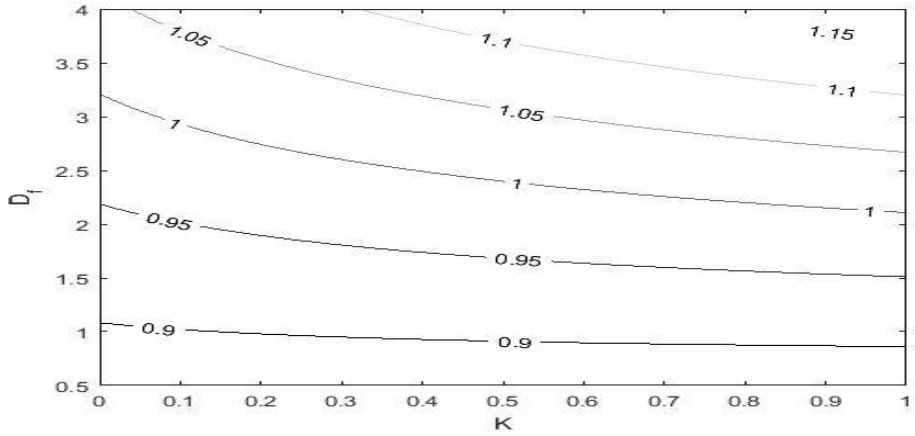


Figure 4.35 Contours of skin-friction showing the influence of K, D_f and the fixed values $Sc = 0.78,\ \mathrm{Pr} = 0.71,\ Gr = Gc = 1.0,\ A^* = B^* = 1,\ Y = 0.5$

Figures 4.34 and 4.35 depict the contours showing the variation of K and D_f on temperature profile and skin-friction. It is noticed that the temperature profile and drag forces caused by friction of fluids against the surface of the platesdecrease with increase in chemical reaction parameter, but increases with increase in Dufour number.

Table 4.4.1 skin friction (τ) at $Y=0$ and $Y=H$ for different values of D_f where $Sc=0.78,\ \Pr=0.71,\ Gr=Gc=1.0,\ H=5.0$

D_f	$A^*=B^*=0$ $K=0$		$A^*=B^*=0$ $K=1$		$A^*=B^*=1$ $K=0$		$A^*=B^*=1$ $K=1$	
	τ_0	τ_H	τ_0	τ_H	τ_0	τ_H	τ_0	τ_H
0	1.6751	0.0030	1.1360	0.0019	3.0183	0.0032	2.2079	0.0020
1	3.0304	0.0099	2.5235	0.0055	5.4620	0.0106	4.9743	0.0060
2	4.3857	0.0167	3.9111	0.0091	8.6241	0.0186	8.6581	0.0103

Table 4.4.2 Nusselt number (Nu) at (Y=0) and (Y=H) for different values of D_f where $Sc=0.78,\ Gr=Gc=1,\ H=1.0$

Pr	$D_f=0$ $K=0$		$D_f=0$ $K=1$		$D_f=1$ $K=0$		$D_f=1$ $K=1$	
	Nu_0	Nu_H	Nu_0	Nu_H	Nu_0	Nu_H	Nu_0	Nu_H
0.70	1.3905	0.6905	1.3905	0.6905	0.9060	0.9860	0.4892	1.1447
0.71	1.3967	0.6867	1.3967	0.6867	0.9116	0.9816	0.4943	1.1398
7.0	7.0064	0.0064	7.0064	0.0064	6.3084	0.0884	5.7625	0.1179

Table 4.4.3 Shows the velocity U for different values of channel width H where $Sc=0.78,\ \Pr=0.71,\ Gr=Gc=1.0, Y=0.5$

Ref. [2002]			Present work					
$H=5$			$H=10$			$H=20$		
K	$A^*=B^*=0$ $D_f=0$	$A^*=B^*=1$ $D_f=1$	$A^*=B^*=0$ $D_f=0$	$A^*=B^*=1$ $D_f=1$	$A^*=B^*=0$ $D_f=0$	$A^*=B^*=1$ $D_f=1$	$A^*=B^*=0$ $D_f=0$	$A^*=B^*=1$ $D_f=1$
0	1.4772	2.7948	1.34682.2869	1.4711	2.7603	1.4772	2.7947	
1	1.2712	2.5819	1.1887	2.2454	1.2672	2.5662	1.2712	2.5818
2	1.2169	2.5324	1.1380	2.2302	1.2129	2.5187	1.2169	2.5324
3	1.1883	2.5073	1.1104	2.2211	1.1843	2.4945	1.1883	2.5073
4	1.1699	2.4914	1.0923	2.2148	1.1659	2.4790	1.1699	2.4914

The analysis for the effects of linear variation of density, nonlinear variation of density, Dufour number, chemical reaction effect, skin-friction and Nusselt number at the surface of the channels are presented in Tables 4.4.1 and 4.4.2. It is observed from Table 4.4.1 that the skin-frictions at $\tau_0\ (Y=0)$ and $\tau_H\ (Y=H)$ increases with increase in Dufour number. Also the skin-friction is higher in the presence of nonlinear variation with temperature and nonlinear variation with concentration relative to linear variation with temperature and linear variation with concentration.

The influences of Prandtl number, chemical reaction effect, and Dufour number are demonstrated in Table 4.4.2. This table reveals that, heat diffusion rate increases with an increase in Prandtl number at $Y=0\ (Nu_0)$and decreases at $Y=H\ (Nu_H)$. The Table also shows that heat diffusion rate increases at (Nu_0)due to presence of Dufour effect, and decreases in the presence of chemical reaction effect. Table 4.4.3 presents numerical values of the present study which is the generalization ofMuthucumaraswamy

(2002). It has been concluded that as the channel distance (H) increases, the result of velocity for the present investigation coincides with that of Muthucumaraswamy (2002) in the absence of Dufour number for linear variation with temperature and concentration.

4.5 Thermal diffusion effect on nonlinear natural convection mass transfer flow near an infinite vertical porous plate with chemical reaction

In order to have a clear view of the present physical situation, comprehensive study was conducted, and the results are reported in terms of line graphs, contours and tables. Numerical values of the dimensionless velocity have been analysed. The results are used to determine the influence of different values of the nonlinear density variation with temperature (NDT), concentration (NDC), Soret parameter (D_t), chemical reaction parameter (K), Schmidt number (Sc) and Prandtl number (Pr) on the flow formation. A comparative analysis has been made for the effects of NDT and NDC in comparison to linear density variation with temperature and linear density variation with concentration,as presented in the Table 4.5.The fluids considered in this investigation are air $(\mathrm{Pr}=0.71)$ and water $(\mathrm{Pr}=7.0)$. The diffusing chemical species of common interest have Schmidt number between 0.1 – 1.0 Muthucumaraswamy (2002).To have a clear picture of the problem, a numerical investigation is conducted and the results are reported in terms of lines graphs, contours and tables.

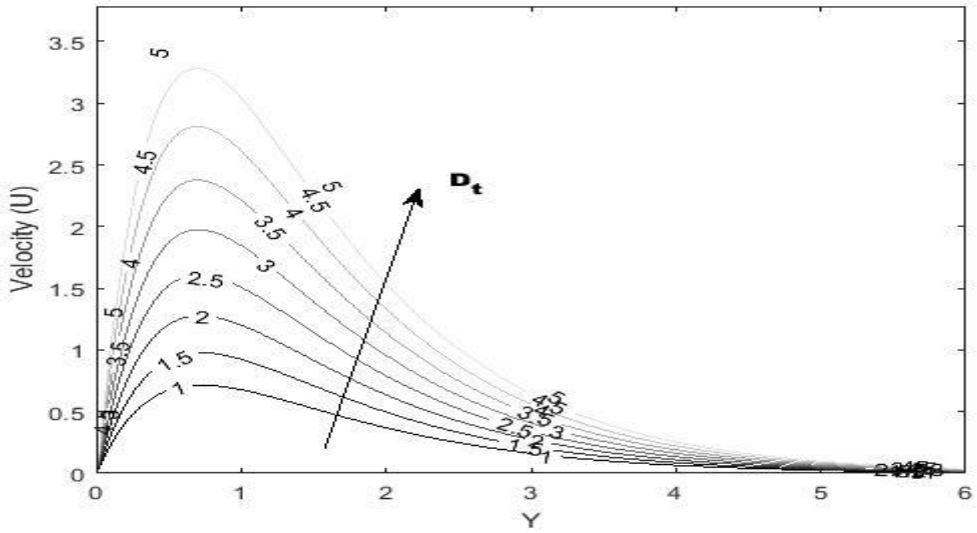


Figure 4.36 Velocity profile of non-linear density variation with temperature NDT and concentration NDC for different values of D_t and fixed values of $K = 2.0,\ Sc = 0.78,\ \Pr = 0.71,\ N = 1.0,\ A^* = B^* = 1$

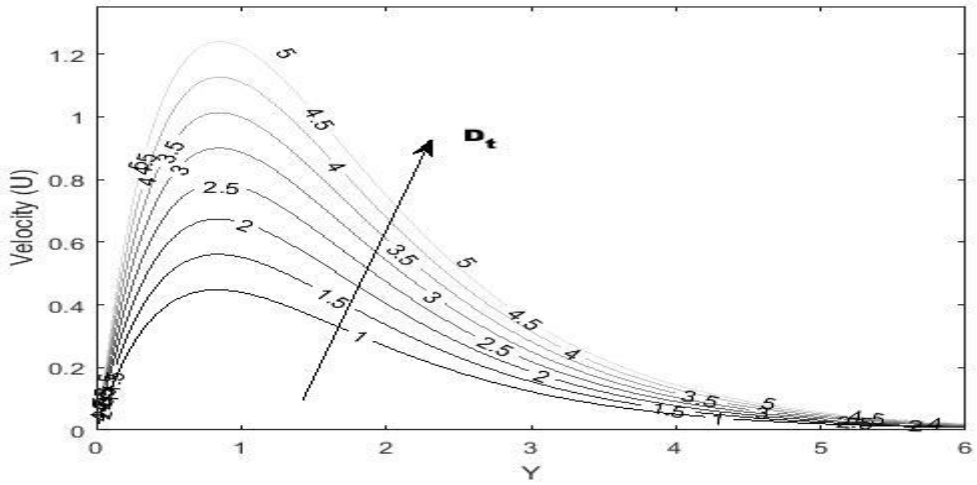


Figure 4.37 Velocity profile of linear density variation with temperature LDT and concentration LDC for different values of D_t and the fixed values $K = 2.0,\ Sc = 0.78,\ \Pr = 0.71,\ N = 1.0,\ A^* = B^* = 0$

Figures 4.36 and 4.37 show the influence of Soret parameter in the presence andabsence of nonlinear density variation with temperature and concentration on velocity profile respectively. It is noticed that the velocity profile increases with an increase in Soret parameter. While the significant effect of Soret parameter is physically observed, in the

presence of nonlinear density variation with temperature and concentration in comparison to linear density variation with temperature and concentration as clearly presented in the Figures 4.36 and 4.37,on velocity profiles. The enhancement of fluid flow rate is due toSoret effect,which results to the strong convection..

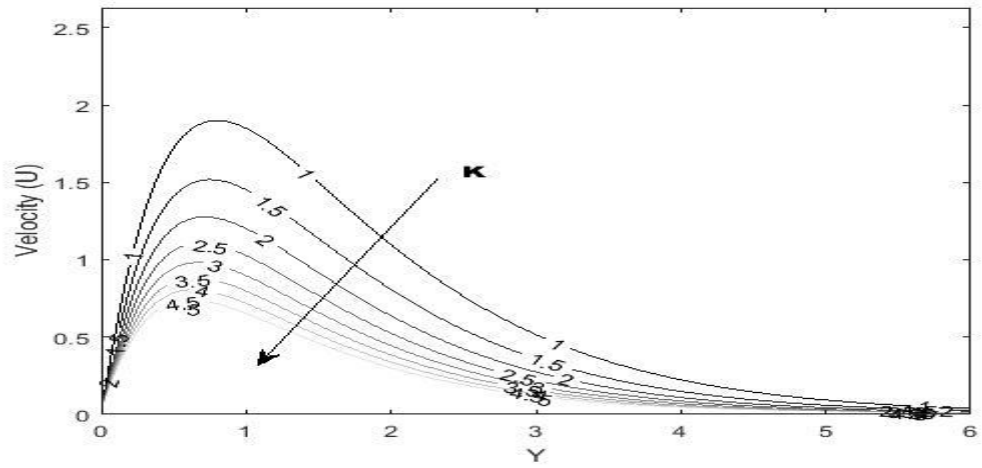


Figure 4.38 Velocity profile of non-linear density variation with temperature NDT and concentration NDC for different values of K and the fixed values $Sc = 0.78,\ \Pr = 0.71,\ N = 1.0,\ A^* = B^* = 1$

The effect of chemical reaction parameter is demonstrated in Figure 4.38. The velocity profile is noticed to reduce with an increament inthe chemical reaction parameter. It is found that the increase in the chemical reactive species emitted from the porous plate led to decrease in velocity profile for the destructive reaction.

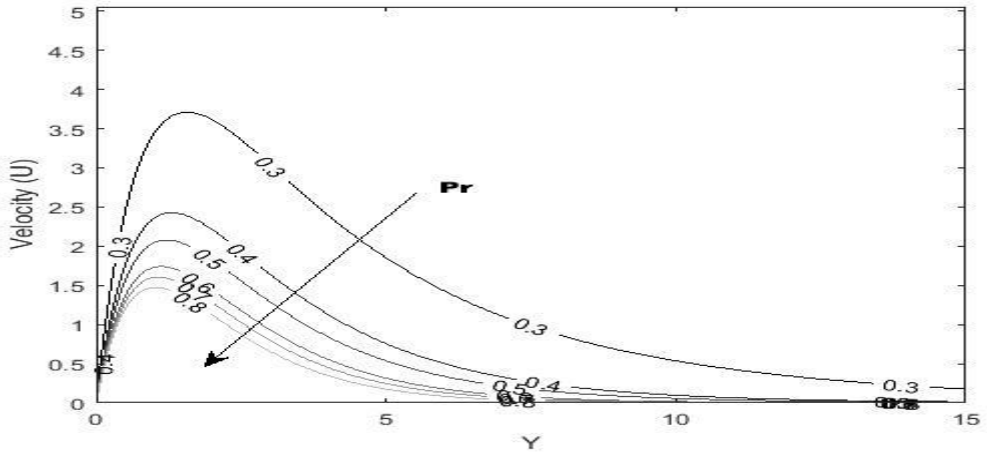


Figure 4.39 Velocity profile of non-linear density variation with temperature NDT and concentration NDC for different values of $\Pr$ and the fixed values $Sc = 0.78,\ N = 1.0,\ A^* = B^* = 1$

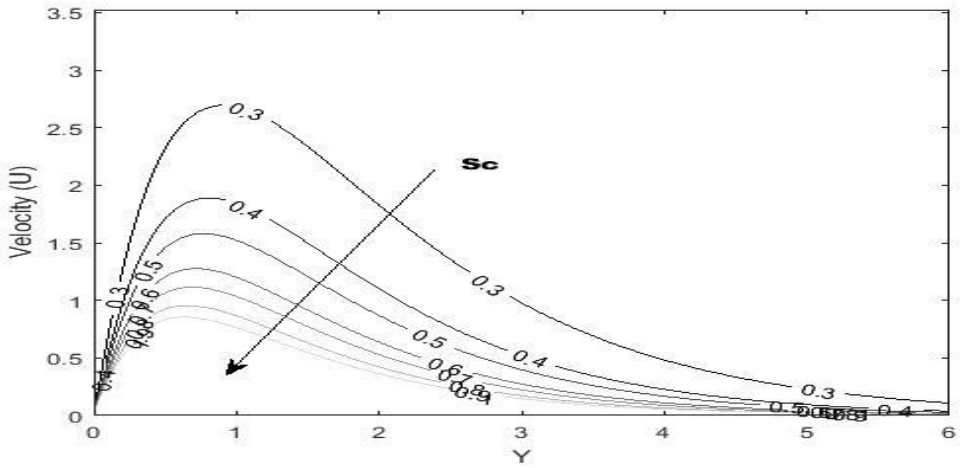


Figure 4.40 Velocity profile of non-linear density variation with temperature (NDT) and concentration (NDC) for different values of K and the fixed values $\Pr = 0.71,\ N = 1.0,\ A^* = B^* = 1$

Figures 4.39 and 4.40indicate the influence and behaviors of Prandtl number and Schmidt number on the velocity profile. It is noticed that the velocity decreases with increase in the Prandtl number and Schmidt number. This shows that the Prandtl number and Schmidt number parameters affect the velocity profile.

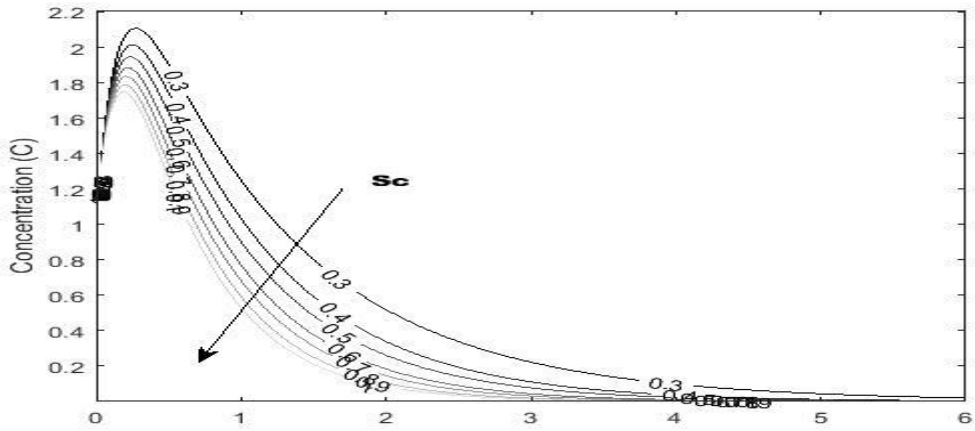


Figure 4.41 Concentration profile for different values (Sc) and the fixed values $K = 2.0,\ D_t = 2.0,\ \Pr = 0.71$

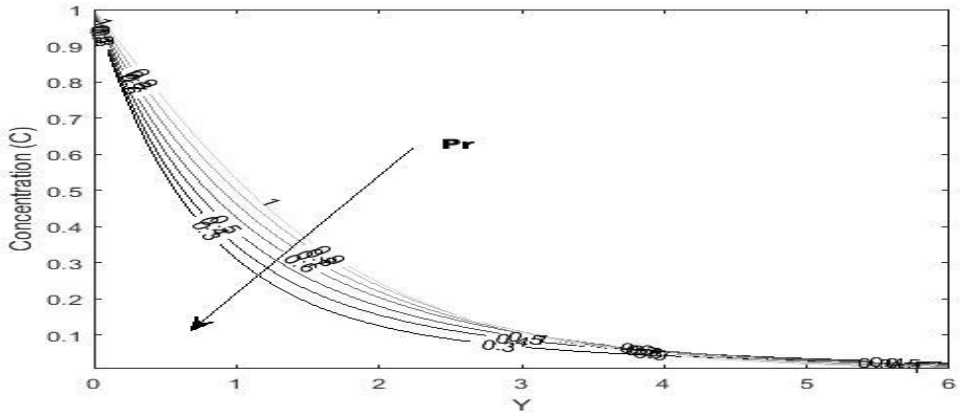


Figure 4.42 Concentration profile for different values $(\Pr)$ and the fixed values $K = 2.0,\ D_t = 2.0,\ Sc = 0.78$

The effect of Schmidt number on the concentration profile has been analyzed from Figure 4.41. It is shown that increase in the ratio of momentum diffusitivity and mass diffusion rate decreases the concentration profile, which indicates that decrease in Schmidt number inreases the mass diffusion convection process in the concentration profile. However, it is also noticed that the diffusion rate is affected by Schmidt number.In Figure 4.42, the influence of Prandtl number on the concentration profile isverified. The significanteffect of Prandtl number observedin Figure 4.42is due to the mass flux generated by temperature difference resulting in a steady state condition in concentration gradient.

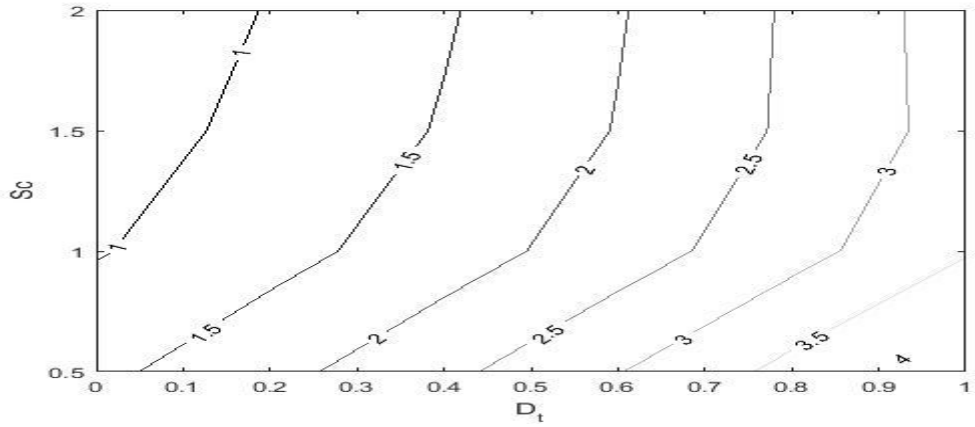


Figure 4.43 Contours showing the influence of D_t and Sc on the skin friction and the fixed values $\Pr = 0.71,\ N = 1.0,\ A^* = B^* = 1$

In Figure 4.43 it isnoted that the skin friction increases with increase in Soret parameter and decreases with increase in Schmidt number. It is also noticed from this figure that the skin friction appreciates with an increase in Soret effect.

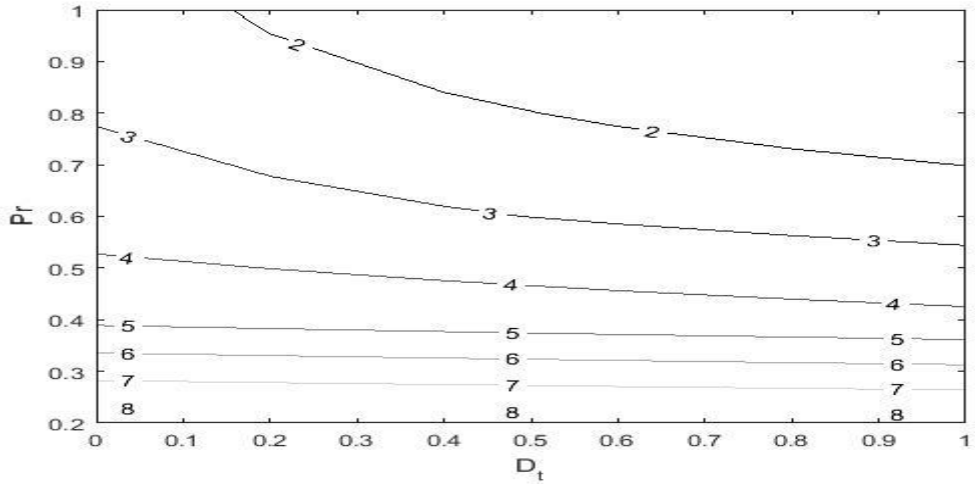


Figure 4.44 Contours showing the influence of D_t and $\Pr$ on the skin friction and the fixed values $Sc = 0.78,\ N = 1.0,\ A^* = B^* = 1$

Figure 4.44 represents contour plot showing the influences of Soret number and Prandtl number on the skin friction. It is noted that increase in Soret parameter enhances the skin friction,while increase in Prandtl number decreases the skin-friction.

Table 4.5Shows the velocity U(Y) for different values of
$Sc = 0.78,\ \Pr = 0.71,\ N = 1.0$

Y	Muthucumaraswamy (2002)	Present work $A^* = B^* = D_t = 0$	$A^* = B^* = D_t = 1$
0	0.0000	0.0000	0.0000
1	0.39340.39340.9430		
2	0.39080.39080.8025		
3	0.27730.27730.5134		
4	0.17060.17060.2944		
5	0.09720.09720.1597		
6	0.05290.05290.0839		
7	0.02790.02790.0432		
8	0.01440.01440.0219		
9	0.00730.00730.0110		
10	0.00370.00370.0055		

In order to validate the accuracy of present study,Table 4.5 presents numerical values for the dimensionless velocity obtained in this work when compared with those of Muthucumaraswamy (2002) in the absence of Soret effect, linear density variation with temperature and concentration that means $A^* = B^* = 0$. Details from the Table suggested that in the absence of Soret effect, nonlinear density variation with temperature and concentration $A^* = B^* = 0$, in the present work,gives excellent agreement with the work of Muthucumaraswamy (2002).

4.6Nonlinear natural convection mass transfer flow in vertical channel with chemical reaction and thermal diffusion effect

This studyreveals the analytical solution of the dimensionless velocity, concentration and temperature for nonlinear free convection mass transfer flow in vertical channel with Soret and chemical reaction effects.The influence of different physical parameters appearing in the problem such as nonlinear density variation with temperature (NDT), and concentration (NDC), Soret parameter(D_t), sustentation parameter(N), chemical reaction parameter(K), Schmidt number$(Sc)$and Prandtl number(Pr)on the flow formation are discussed. Since air and water are two very common fluids, the choice of Prandtl number are 0.71 and 7.0 which correspond to air and water respectively. The diffusing chemical species of common interest have Schmidt numbers in the range of 0.1 to 1.0 Muthucumaraswamy (2002). Table 4.6 validates the accuracy of the current workwhen compared to the work of Muthucumaraswamy (2002).

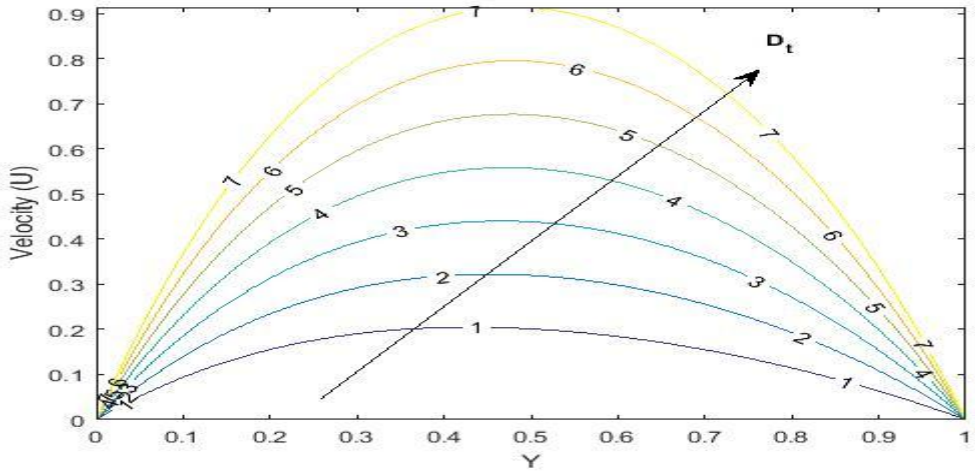


Figure 4.45 Velocity profile of nonlinear density variation with temperature NDT and concentration NDC for different values of D_t and the fixed values $H=1.0,\ K=2.0,\ N=1.0,\ Sc=0.78,\ \mathrm{Pr}=0.71,\ A^*=B^*=1$

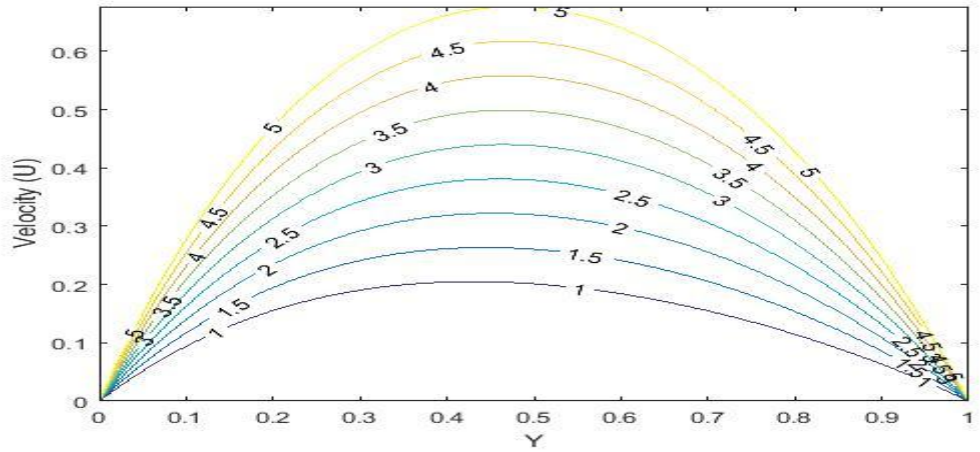


Figure 4.46 Velocity profile of nonlinear density variation with temperature NDT and concentration NDC for different values of N and the fixed values $H = 1.0,\ D_t = 2.0,\ K = 2.0,\ Sc = 0.78,\ \Pr = 0.71,\ A^* = B^* = 1$

Figure 4.45 portrays the velocity profile of nonlinear density variation with temperature (NDT) and concentration (NDC) for different values of Soret parameter. It is noted that increase in Soret parameter increases the velocity profile significantly. Figure 4.46 shows that the contribution of buoyancy ratio parameter on the velocity profile in the presence of nonlinear density variation with temperature and concentration increases the buoyancy forces which result to increase in convection process.

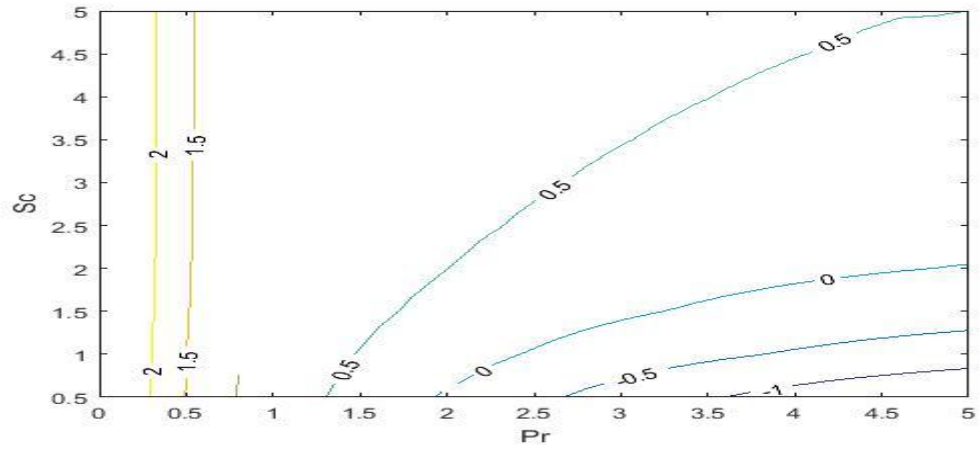


Figure 4.47 Contour of velocity profile of nonlinear density variation with temperature NDT and concentration NDC for different values of $\Pr$, Sc and the fixed values $K = 2.0,\ D_t = 2,\ N = 1.0,\ A^* = B^* = 1,\ Y = 0.5$

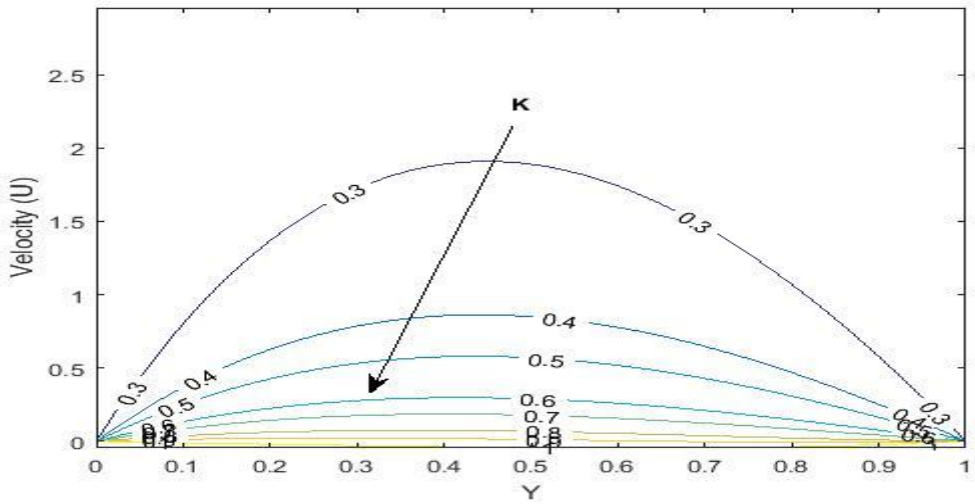


Figure 4.48 Velocity profile of nonlinear density variation with temperature NDT and concentration NDC for different values of K and the fixed values $H = 1.0,\ D_t = 2,\ Sc = 0.78,\ \Pr = 0.71,\ N = 1.0$

Figure 4.47 depicts the physical impact of Schmidt numberand Prandtl number on velocity profile in the presence of nonlinear density variation with temperature and concentration. It is noticed that the velocity profile reduces with an increase in Schmidt numberand Prandtl number. It isnoticed that for Prandtl number less than one $(\Pr < 1)$ the thermal diffusion rate is significant over the viscous diffusion rate, and for the $(\Pr > 1)$.This means that viscous diffusion rate dominated the flow. It is also clearly observed from Figure 4.48 that chemical reaction parameter has strong effect in the velocity. Therefore, a significant change in chemical reaction parameter reduces the velocity profile.

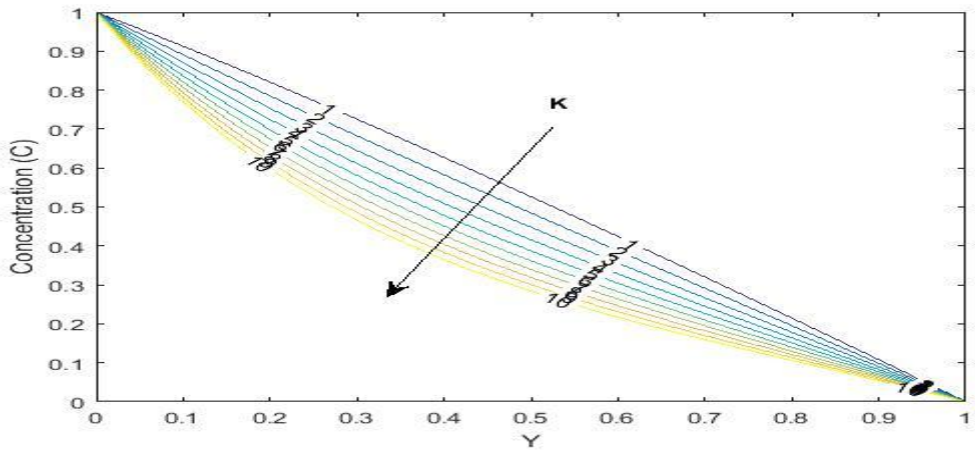


Figure 4.49 Concentration profile for different values K and the fixed values $H = 1.0,\ Sc = 0.78,\ D_t = 2.0,\ \Pr = 0.71$

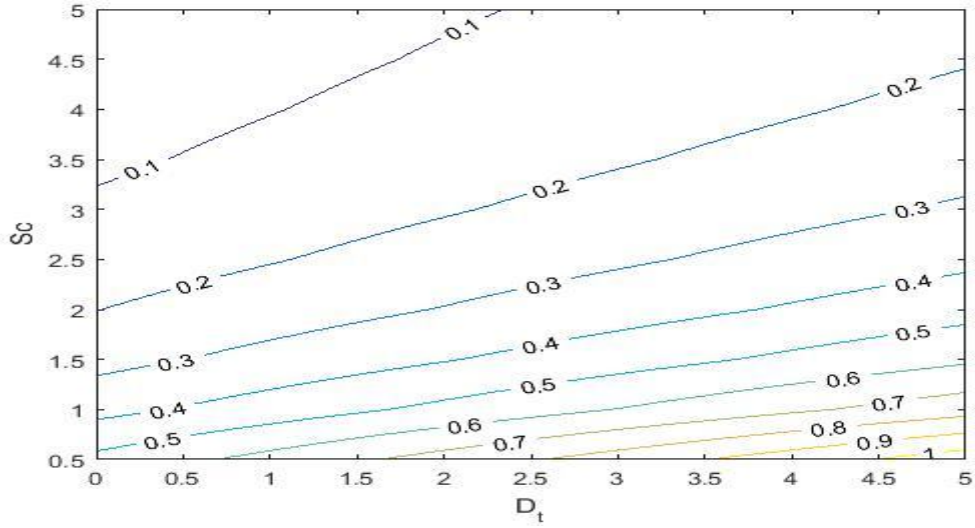


Figure 4.50Contours of concentration profile for different values D_t, Sc and the fixed value of $\Pr = 0.71,\ Y = 0.5,\ H = 1.0$

Figure 4.49 depicted the effect of chemical reaction parameter on concentration profile. It is observedthat decrease in chemical reaction effect increases the concentration profile.Figure 4.50 shows the contours of concentration profile for Soret parameter and Schmidt number respectively. The figure shows that dimensionless concentration profile increases with increament in the Soret parameter, and decreases with increament in chemical reaction and Schmidt number**.** This implies that

dimensionless concentration profile is significantly influenced by chemical reaction and Schmidt number.

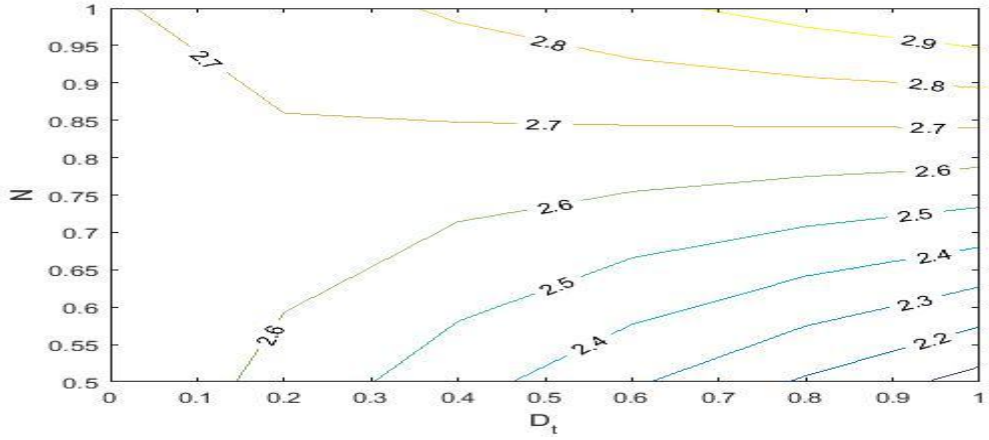


Figure 4.51 Contours showing the influence of D_t and N on the skin friction and the fixed values $Sc = 0.78,\ \Pr = 0.71,\ A^* = B^* = 1$ at $(Y = H)$

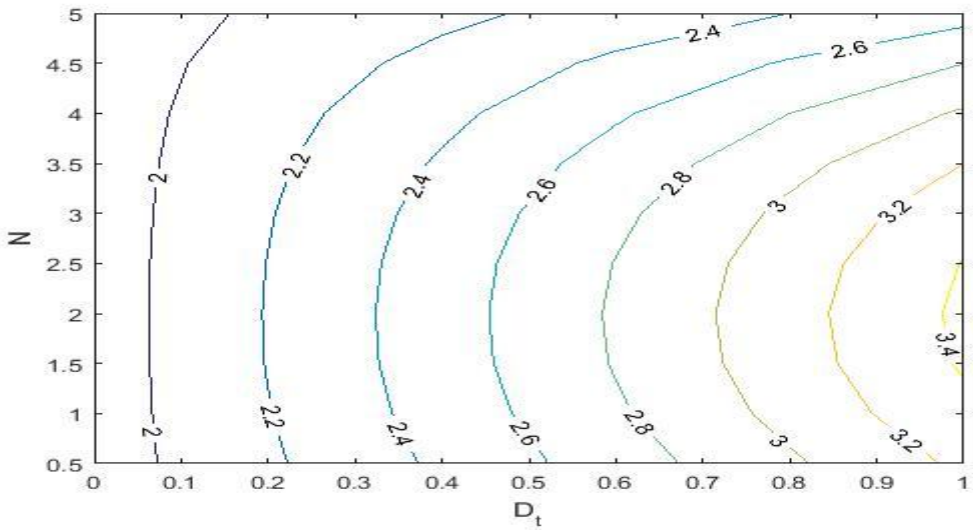


Figure 4.52 Contours showing the influence of D_t and N on the skin friction and the fixed values $Sc = 0.78,\ \Pr = 0.71,\ A^* = B^* = 1$ at $(Y = 0)$

Figure 4.51 shows the contours for the influences of Soret effect and buoyancy ratio parameter on the skin friction at $(Y = H)$. It is noticed that the drag force caused by friction of fluid against the surface of channel plates increases with increment in Soret

parameter and buoyancy ration parameter at $(Y = H)$. It is also found that the drag forcecaused by friction of the fluid against the surface of channel plates improves with increment in Soret effect and buoyancy ratio parameter at $(Y = 0)$ as shown in Figure 4.52.

Table 4.6 Show the velocity U(Y) for different values of $Sc = 0.78,\ \Pr = 0.71,\ N = 1.0,\ Y = 0.5$

K	Ref (2002)	Present work $H = 5$		$H = 10$		$H = 20$	
	$A^* = B^* = 0$ $D_t = 0$	$A^* = B^* = 1$ $D_t = 1$	$A^* = B^* = 0$ $D_t = 0$	$A^* = B^* = 1$ $D_t = 1$	$A^* = B^* = 0$ $D_t = 0$	$A^* = B^* = 1$ $D_t = 1$	$A^* = B^* = 0$ $D_t = 0$	$A^* = B^* = 1$ $D_t = 1$
0	0.8706	1.4588	0.8479	1.1452	0.8714	1.4513	0.8706	1.4588
1	0.6646	1.1200	0.6737	1.0838	0.6663	1.1204	0.6646	1.1200
2	0.6104	1.1212	0.6220	1.0986	0.6121	1.1220	0.6104	1.1212
3	0.5818	1.1358	0.5942	1.1181	0.5835	1.1368	0.5818	1.1358
4	0.5634	1.1604	0.5762	1.1346	0.5651	1.1508	0.5634	1.1604

Table 4.6 is computed to validate the present study which is the generalization of the work of Muthucumaraswamy(2002). It has been noticed that when the channel gap (H) is increased,the result of velocity for the present investigation is in confirmity with that of Muthucumaraswamy (2002).

4.7Dufour effect on nonlinear natural convection heat and mass transfer flow near an infinite vertical porous plate with chemical reaction

The ordinary differential equations(3.110) –(3.112) under relevant boundary conditions (3.113) weresolved by the method of undetermined coefficients. Numerical values for the non-dimensional velocity, concentration and temperature was thenstudied. The effect of each physical parameter appearing in the problem, such as; nonlinear density variation with temperature (NDT) and concentration (NDC), Dufour parameter$\left(D_f\right)$, chemical reaction parameter$\left(K\right)$, Schmidt number$\left(Sc\right)$, Prandtl number$\left(\Pr\right)$and Sustentation parameter$\left(N\right)$were examined on the flow formation. Since air and water are two very common fluids, the choices of Prandtl numbersare0.71 and 7.0 which correspond to air and water respectively.

The diffusing chemical species of common interest have Schmidt numbers in the range 0.1 to 10.To have a clear picture of the problem, a numerical investigation is conducted and the results are reported in terms of lines graphs, contours and tables.

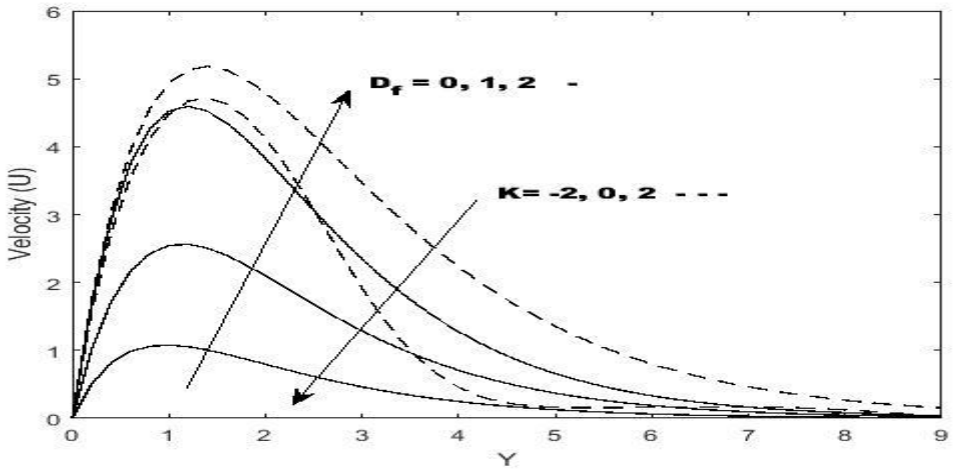


Figure 4.53 Velocity profile of nonlinear density variation with temperature NDT and concentration NDC for different values of D_f, K and fixed values of $Sc = 0.78,\ \Pr = 0.71,\ N = 1.0,\ A^* = B^* = 1$

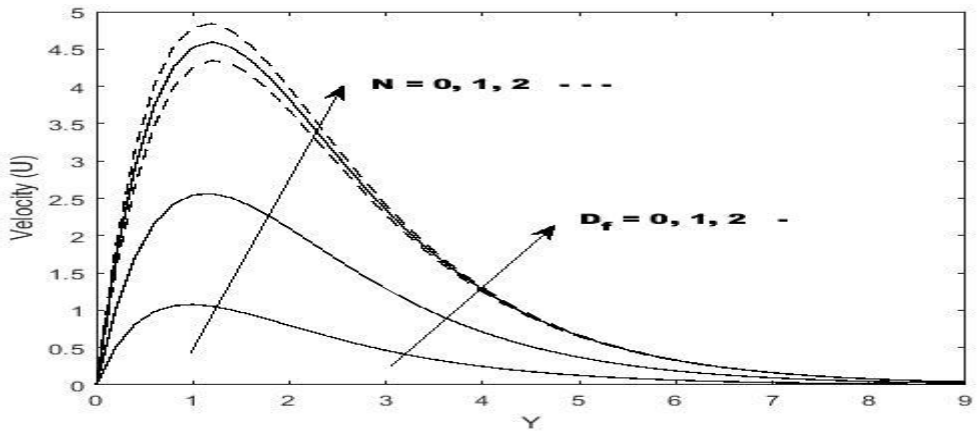


Figure 4.54 Velocity profile of nonlinear density variation with temperature NDT and concentration NDC for different values of N, D_f and fixed values of $K = 2.0,\ Sc = 0.78,\ \Pr = 0.71,\ A^* = B^* = 1$

Figure 4.53 depicts the velocity profile of nonlinear density variation with temperature (NDT) and concentration (NDC) for different values of Dufour effect and chemical reaction parameter. It is noticed that the velocity profile increases with an increase in Dufour number, and decreases with increase in chemical reaction parameter. This implies thatchemical reaction parameter hassignificant effect on velocity profile. The velocity profile increases with increasingDufour parameter. It is also observed that the convection current is stronger for increasein Dufour number as shown in Figure 4.53. Figure 4.54 showsthe influence of Dufour number and sustentation parameter on the velocity profile. It is found that the velocity profile increases with an increase in Dufour number and sustentation parameter. But the contribution of buoyancy ratio parameter (N) on the flow rate is higher than the Dufour effect, as being reflected in Figure 4.54.

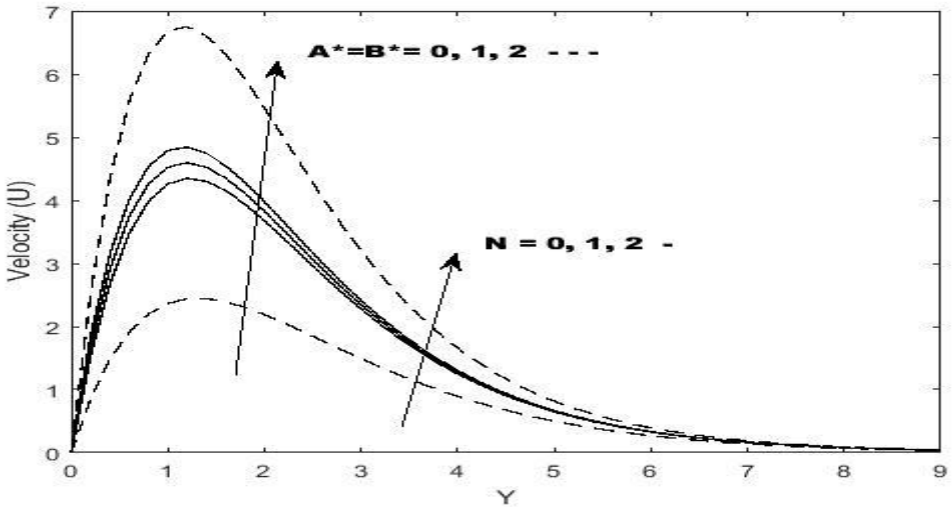


Figure 4.55 Velocity profile of nonlinear density variation with temperature NDT and concentration NDC for different values of N, A^*, B^* and fixed values of $K = 2.0,\ Sc = 0.78,\ \Pr = 0.71,\ N = 1.0,\ D_f = 1$

The behaviors of sustentation parameter and nonlinear density variation with temperature (NDT) and concentration (NDC) on the velocity profile are demonstrated in Figure 4.55. It is noticed thatthe velocity profile increase with an increase in buoyancy ratio parameter and nonlinear variation of density with temperature and concentration parameters. However, the flow formation is higher in nonlinear variation of density with temperature and concentration in comparison to buoyancy ratio parameter (N).

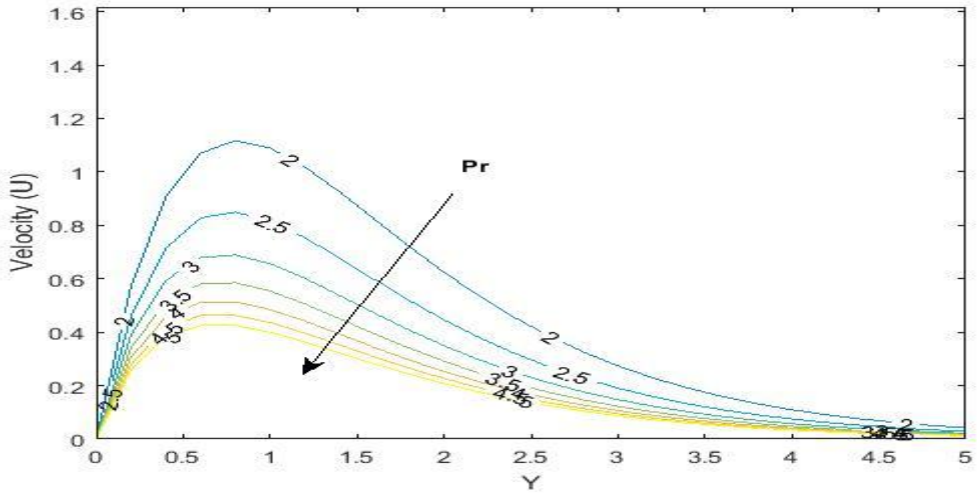


Figure 4.56 Velocity profile of nonlinear density variation with temperature NDT and concentration NDC for different values of Pr and fixed values of $K = 2.0,\ D_f = 1,\ N = 1.0,\ A^* = B^* = 1$

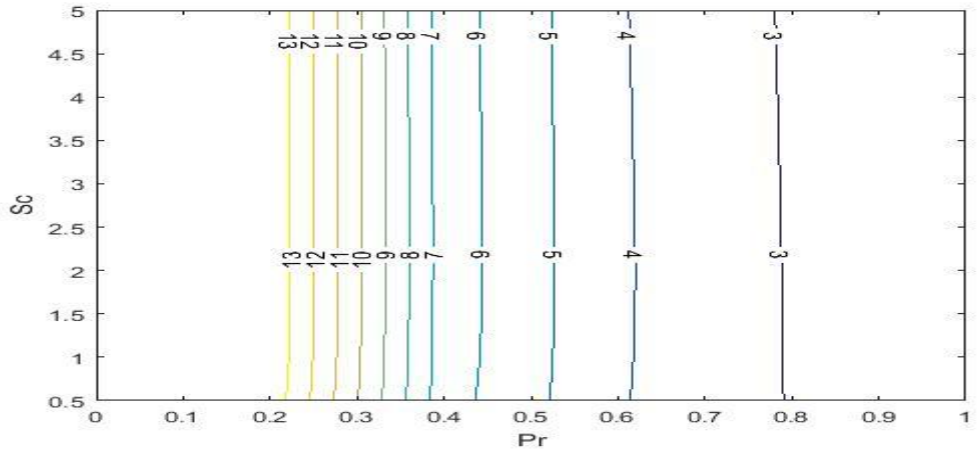


Figure 4.57 Velocity profile of nonlinear density variation with temperature NDT and concentration NDC for different values of Sc, Pr and fixed values of $K = 2.0,\ D_f = 2,\ N = 1.0,\ A^* = B^* = 1$

Figure 4.56 shows the velocity profile for nonlinear density variation with temperature and concentration for different values of Prandtl number(Pr). The velocity profile increases with decrease in Prandtl number. Since the Prandtl number value is smaller in

air compared to water, heat tends to diffuse faster in air. Figure 4.57 shows that increase in Prandtl number and Schmidt number decrease the velocity profile.

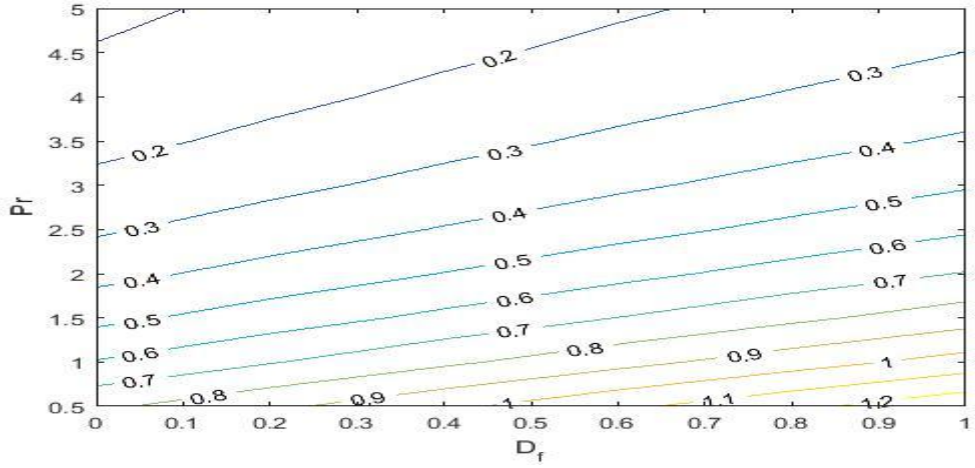


Figure 4.58 Temperature profile for different values of D_f and Pr for fixed values of $K = 2.0,\ Sc = 0.78$

Figure 4.58 shows contours of temperature profile for the variation of Dufour effect and Prandtl number. It is noticed that increase in Dufour parameter results to an increase in temperature, however higher values of Prandtl number decrease the temperature.

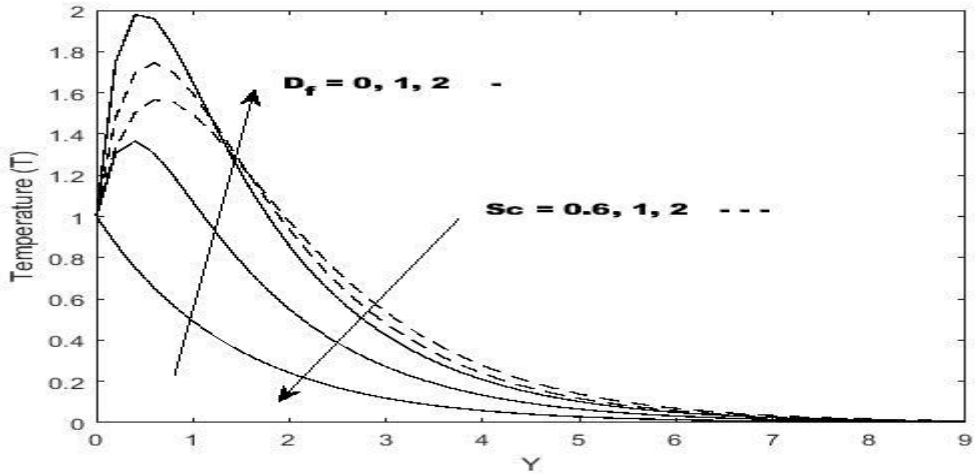


Figure 4.59 Temperature profile for different values of D_f and Sc for fixed values of $K = 2.0,\ \Pr = 0.71$

In Figure 4.59, increase in Schmidt number decreases the temperature far away from the porous plate, and it also leads to increase in temperature near the porous plate.Thisphenomenon occurs in the presence of Dufour effect. It is also found that increase in Dufour number increases the temperature.

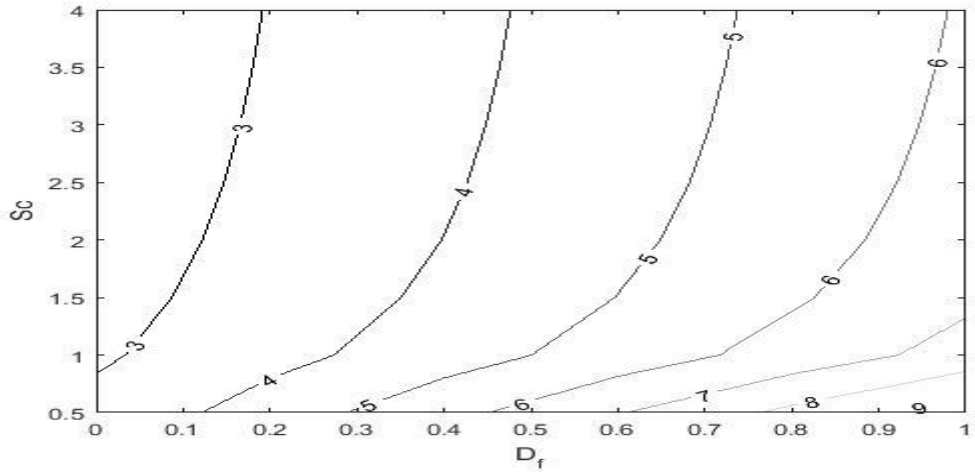


Figure 4.60 Contours showing the influence of D_f and Sc on the skin friction at (Y=0) for fixed values of $K = 2.0,\ N = 1,\ A^* = B^* = 1,\ \Pr = 0.71$

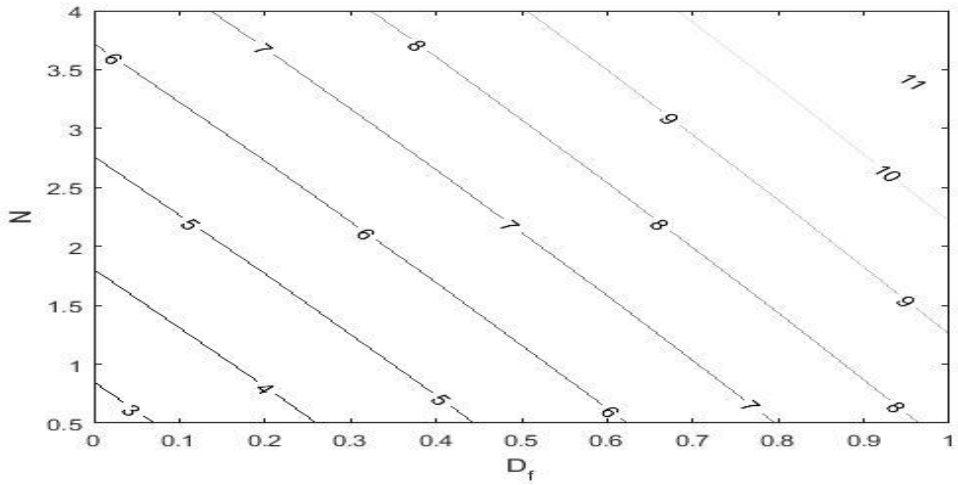


Figure 4.61 Contours showing the influence of N and D_f on the skin friction at (Y=0) for fixed values of $K = 2.0,\ Sc = 0.78,\ A^* = B^* = 1,\ \Pr = 0.71$

Figures 4.60 and 4.61 show the influence of Dufour effect, Schmidt number, and sustentation parameteronskin friction at (Y = 0). It is found that there is an increament

in the skin friction with an increase in sustentation parameter or Dufour number. Increase in Schmidt number or chemical reaction parameter led to decrease in skin friction.

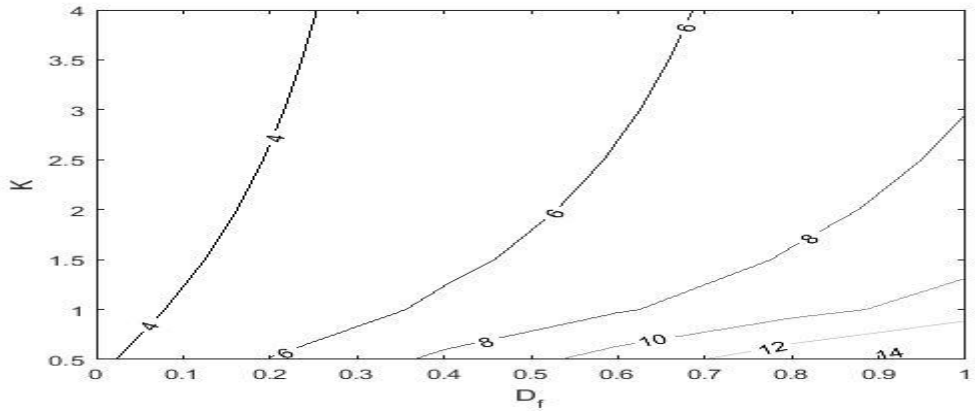


Figure 4.62 Contours showing the influence of K and D_f on the skin friction at (Y=0) for fixed values of $N = 1.0,\ Sc = 0.78,\ A^* = B^* = 1,\ \Pr = 0.71$

Figure 4.62 shows the contours on the influence of Dufour effect, chemical reaction effect on the skin friction at (Y = 0). It is found that skin friction is enhanced with increase in Dufour number, and decreases with increase in chemical reaction parameter.

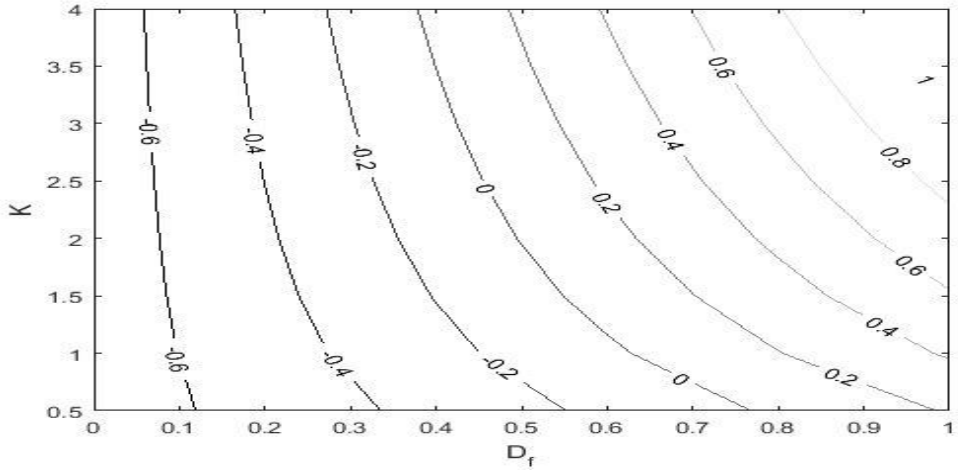


Figure 4.63 Contours showing the influence of D_f and K on the heat transfer rate for fixed values of $Y = 0.5,\ Sc = 0.78,\ \Pr = 0.71$

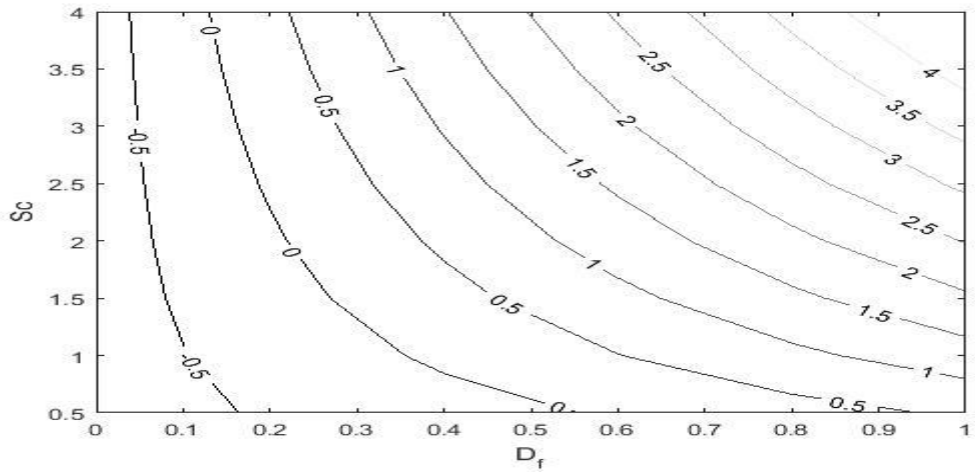


Figure 4.64Contours showing the influence of D_f and Sc on the heat transfer rate for fixed values of $Y = 0.5,\ Sc = 0.78,\ K = 2.0$

Figures 4.63and 4.64 depicted the contours of the behaviors of Sc and K on the heat diffusion rate. It is noticed that the heat diffusion rate increases with increase inDufour number, and decreases with increase of Schmidt number and chemical reaction effect.

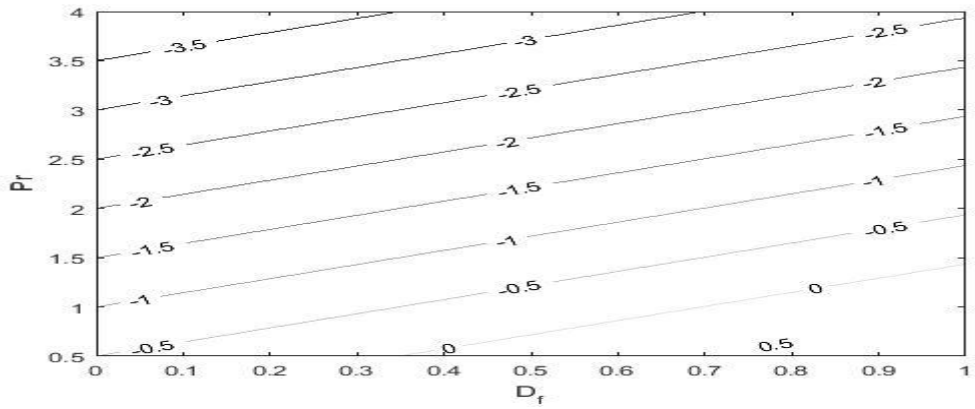


Figure 4.65Contours showing the influence of D_f and $\Pr$ on the heat transfer rate for fixed values of $Y = 0.5,\ Sc = 0.78,\ K = 2.0$

The impact of Dufour parameter and Prandtl number on heat transfer is demonstrated in Figure 4.65. It isobserved that heat transfer rate increases with increase in Dufour effect and decreases with increase in Prandtl number.

Table 4.7.1 Variation of skin friction τat (Y=0) for different values of D_f where $Sc = 0.78,\ \Pr = 0.71,\ K = 2.0$

D_f	$N = A^* = B^* = 1$	$A^* = B^* = 1$	$N = 0$	$A^* = B^* = 0$	$N = 1$	$N = A^* = B^* = 0$
0.0	3.1574	2.1127	2.1049		1.4358	
1.0	8.7351	7.6904	6.2690	5.5726		
2.0	15.2553	14.2106	10.4331	9.7367		

Table 4.7.2 Show the velocity U(Y) for different values of $Sc = 0.78,\ \Pr = 0.71,\ N = 1.0$

Y	Ref (2002)	Present work $A^* = B^* = D_f = 0$	$A^* = B^* = D_f = 1$
0	0.0000	0.0000	0.0000
1	0.8088	0.8088	2.5455
2	0.6425	0.6425	2.0972
3	0.3934	0.3934	1.2868
4	0.2190	0.2190	0.7080
5	0.1163	0.1163	0.3706
6	0.0602	0.0602	0.1891
7	0.0307	0.0307	0.0952
8	0.0155	0.0155	0.0475
9	0.0077	0.0077	0.0236
10	0.0039	0.0039	0.0117

Table 4.7.1shows the variation of skin friction (τ) at $Y = 0$ for different values of D_f in the presence or absence of sustentation parameter, nonlinear density variation with temperature and concentration. It isfound that the numerical values of skin friction are

larger in the presence of sustentation parameter, nonlinear density variation with temperature and concentration in comparison to linear density variation with temperature and concentration. The skin friction isalso higher in the presence of Dufour effect. Table 4.7.2 shows the numerical comparison of the velocity obtained in the present work, with those of Muthucumaraswamy (2002) in the absence ofDufour effect and in the presence of linear density variation with temperature and concentration $A^* = B^* = 0$. Details from the table suggested that in the absence of diffusion thermo effect $D_f = 0$ and linear density variation with temperature and concentration $\left(A^* = B^* = 0\right)$, there is excellent agreement with the results of Muthucumaraswamy (2002).

4.8The influence of chemical reaction and Dufour effect on nonlinear natural convection heat and mass transfer flow in vertical channel

This section examined the nonlinear free convection and mass transfer flow in a vertical channel with chemical reaction in the presence of Dufour effect. The results obtained are reported in terms of lines graphs, contours and Tables. The effect of nonlinear density variation with temperature (NDT) and concentration (NDC), Dufour parameter, chemical reaction parameter, Schmidt numberand Prandtl numberon the dimensionless velocity, concentration and temperature profiles have been studied. Since air and water are two very common fluids, the choice of Prandtl numbers are 0.71 and 7.0 which corresponds to air and water respectively. The diffusing chemical species of common interest have Schmidt numbers in the range of 0.1 to 1.0 Muthucumaraswamy (2002).

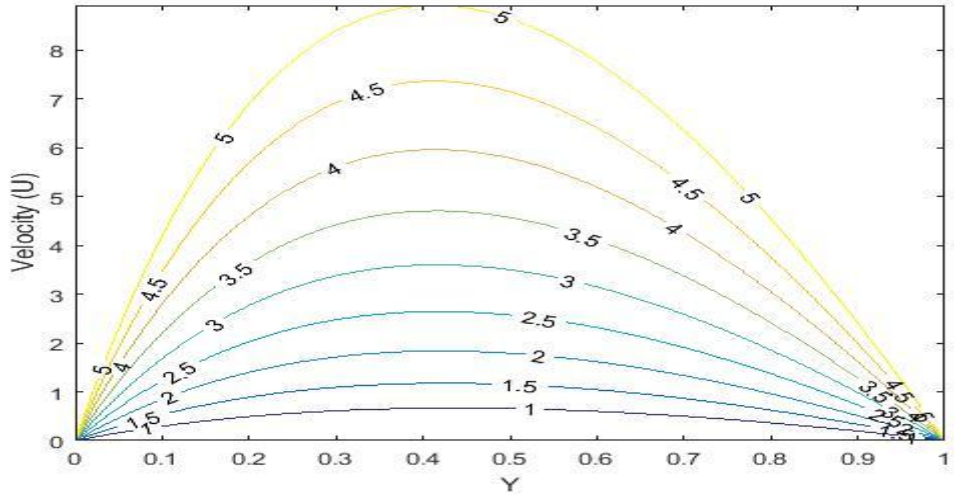


Figure 4.66 Velocity profile of nonlinear density variation with temperature NDT and concentration NDC for different values of D_f and the fixed values $H = 1.0,\ N = 1.0,\ K = 2.0,\ Sc = 0.78,\ \Pr = 0.71,\ A^* = B^* = 1$

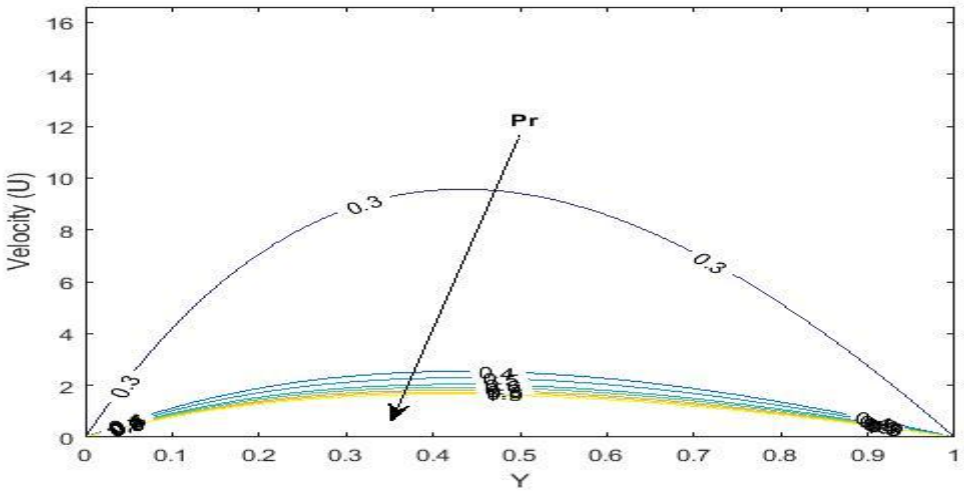


Figure 4.67 Velocity profile of nonlinear density variation with temperature (NDT) and concentration (NDC) for different values of $(\Pr)$ and the fixed values $H = 1.0,\ N = 1.0,\ K = 2.0,\ Sc = 0.78,\ D_f = 2.0,\ A^* = B^* = 1$

Figure 4.66shows the velocity profile of nonlinear density variation with temperature (NDT) andconcentration (NDC) for different values of Dufour number. It is observed that the velocity profile increases with increase in Dufour number. The flow rate is found to be more significant in the presence ofnonlinear density variation with

temperature and concentration in comparedtolinear density variation with temperature and concentration. Figure 4.67 displays the effect of Prandtl numberon the dimensionless velocity, where it is clearly seen that the velocity profile decreases with increase in the ratio of momentum diffusion to heat diffusion convection process.Also, it is observed thatwhen the selected value of Prandtl number is small,heat quickly diffuses.

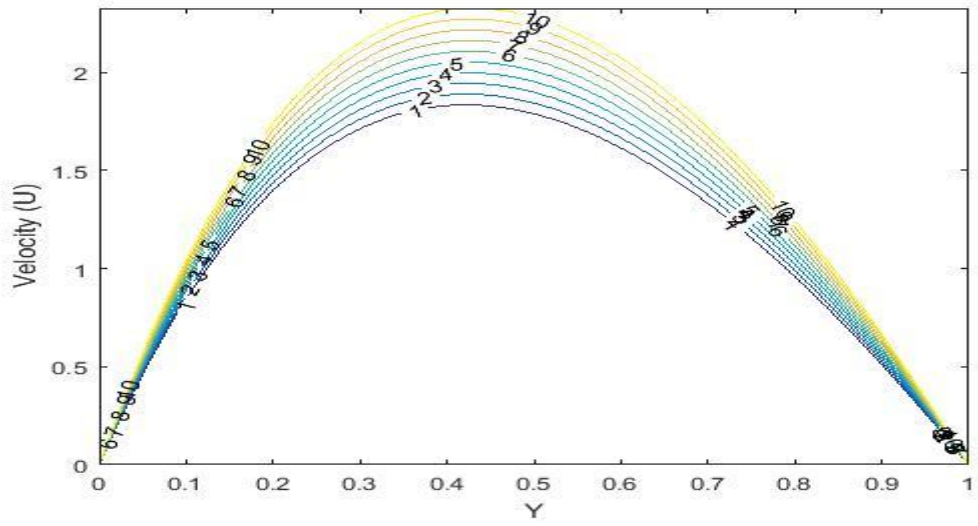


Figure 4.68 Velocity profile of nonlinear density variation with temperature NDT and concentration NDC for different values of N and the fixed values $H = 1.0,\ D_f = 2.0,\ K = 2.0,\ Sc = 0.78,\ \Pr = 0.71,\ A^* = B^* = 1$

Figure 4.68depicts the velocity profile of nonlinear density variation with temperature (NDT) and concentration (NDC) andbuoyancy ratio parameter. It isobvious that the velocity profile increases with increase in buoyancy ratio parameter. This implies that convection due to heat and mass transfer is supporting each other with increase in Sustentation parameter.

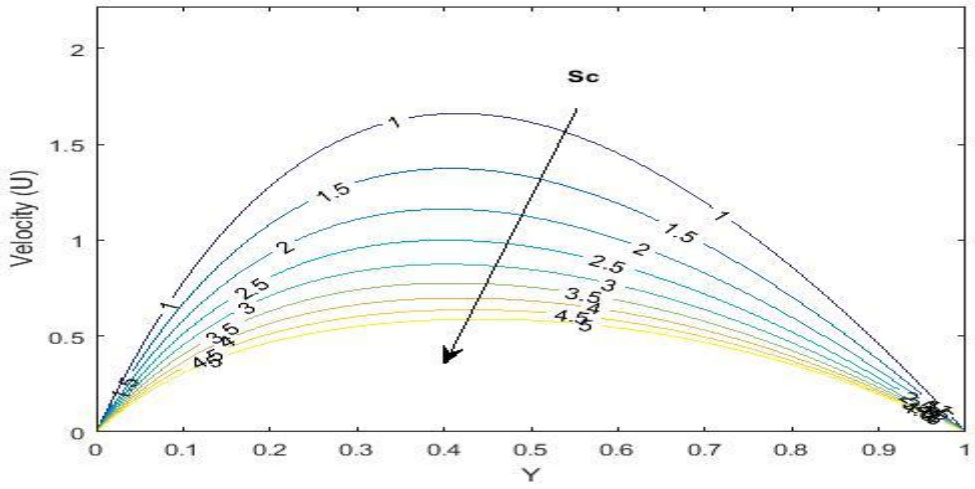


Figure 4.69 Velocity profile of nonlinear density variation with temperature NDT and concentration NDC for different values of (Sc) and the fixed values $H = 1.0,\ \Pr = 0.71,\ K = 2.0,\ N = 1.0,\ D_f = 2.0,\ A^* = B^* = 1$

The effects of Schmidt numberon the velocity profile is demonstrated in Figure 4.69. It is noticed that the fluid velocity decreases with an increasein the ratio of kinematics viscosity to mass diffusion convection process.

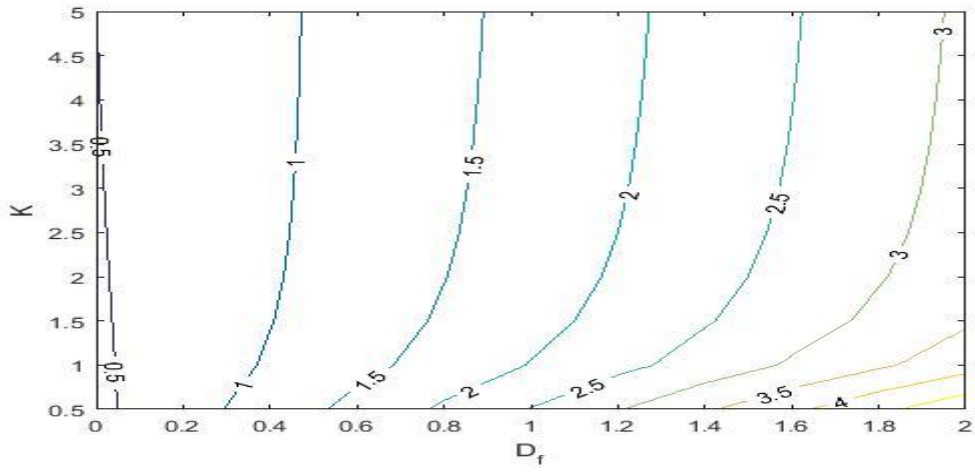


Figure 4.70Contours of velocity profile for nonlinear density variation with temperature (NDT) and concentration (NDC) for different values of K and D_f the fixed values $\Pr = 0.71,\ Sc = 0.78,\ N = 1.0,\ A^* = B^* = 1,\ Y = 0.5$

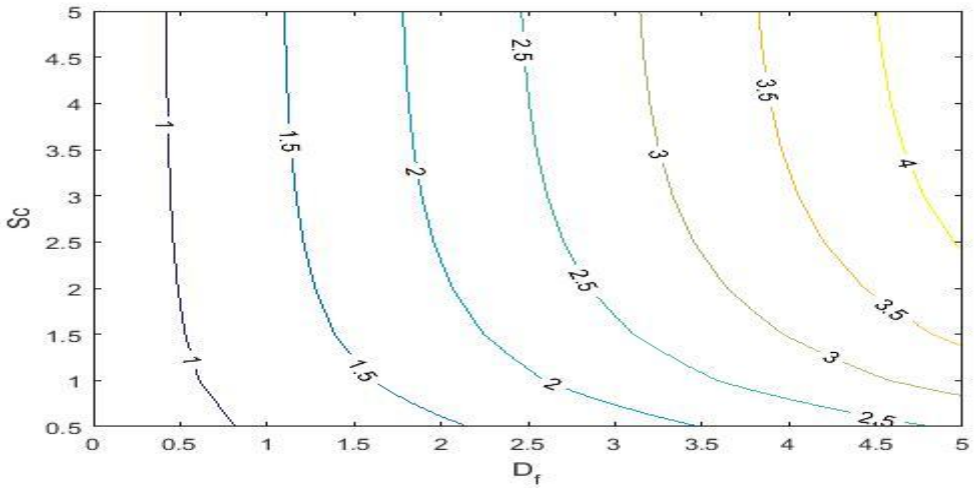


Figure 4.71 Contours of velocity profile for nonlinear density variation with temperature (NDT) and concentration (NDC) for different values of Sc and D_f the fixed values $K = 2.0,\ \Pr = 0.71,\ \ N = 1.0,\ \ A^* = B^* = 1,\ Y = 0.5$

Figures4.70and 4.71show the contours of velocity profile for nonlinear density variation with temperature (NDT) and concentration (NDC) for different values of chemical reaction parameter, Schmidt number and Dufour effect. It is observed that the velocity profile increases with increase in Dufour effect, and decreases with increase in positive value of chemical reaction parameter for the destructive reaction.

It is also found that increase in the ratio of momentum diffusion and mass diffusion convection process results to decrease in velocity profile. Theseresults reveal that due to the presence of nonlinear density variation with temperature and concentration parameter the velocity profile accelerates rapidly, and decelerates withan increase in chemical reactive species emitted from the porous plate.

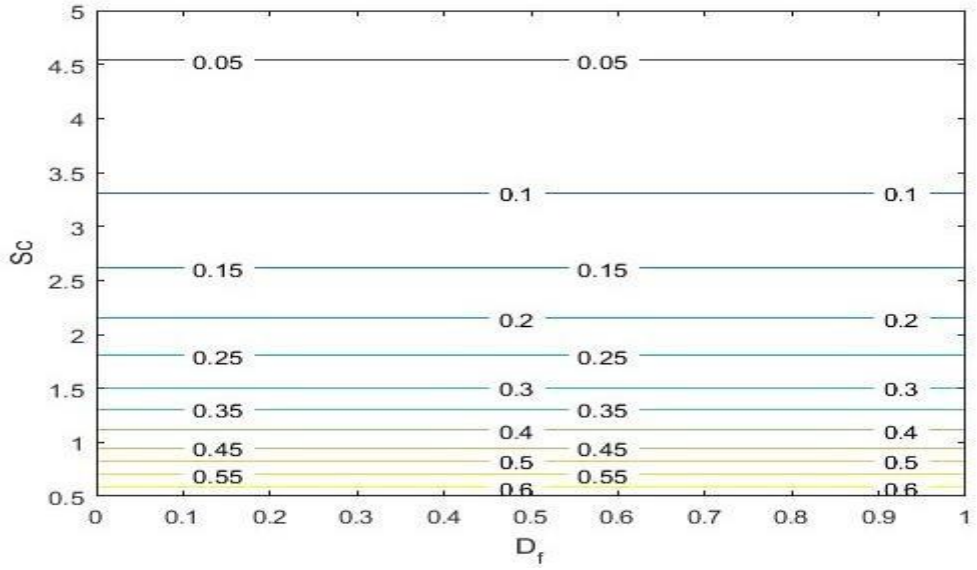


Figure 4.72 Contours of concentration profile for different values D_f and Sc and fixed value of $\Pr = 0.71,\ Y = 0.5,\ H = 1.0$

Figure 4.72 depicts the contours of concentration profile for different values ofDufour number and Schmidt number respectively. It isfound that the concentrationprofile decreasesrapidly with an increasein Schmidt number near the porous plate.The result also revealed that the non- dimensional composition equation is independent of Dufour effect.

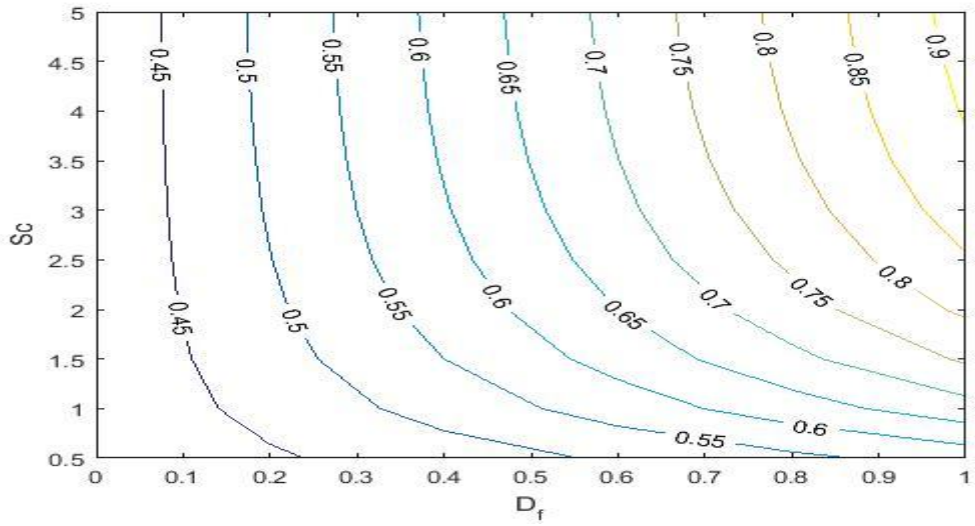


Figure 4.73 Contours of temperature profile for different values (D_f) and (Sc) the fixed value of $H = 1.0,\ \Pr = 0.71,\ Y = 0.5$

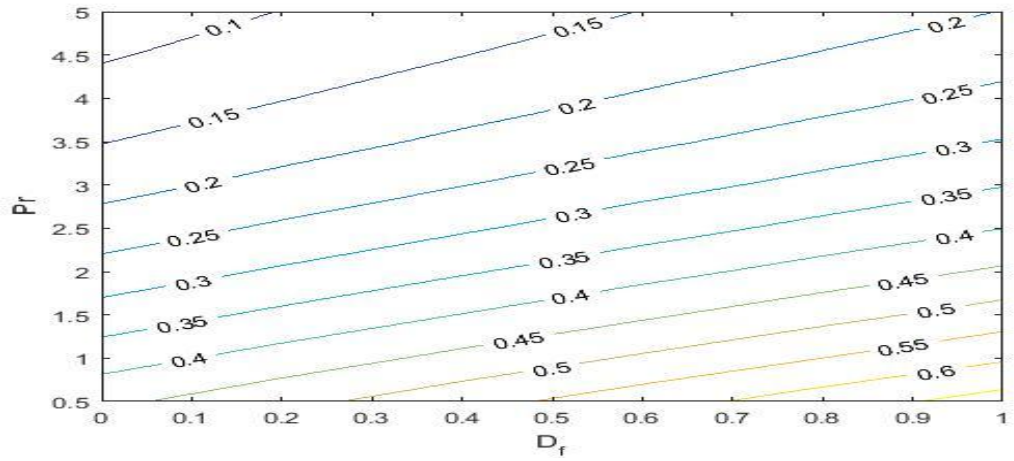


Figure 4.74Contours of temperature profile for different values (D_f) and$(\Pr)$ the fixed value of $H = 1.0,\ Y = 0.5,\ Sc = 0.78$

The temperature profile for different values ofDufour effect is studied in Figures 4.73 and 4.74. It is noticed that the temperature profile increases with increasein Dufour effect. It is also observed that temperature profile reduces with increase in the ratio of momentum diffusion to heat diffusion convection, and ratio of kinematics viscosity to mass diffusion convection process respectively. Therefore, for small values of Prandtl number heat diffusion convection process dominates the system, and for small values ofSchmidt numbermass diffusion convection process dominates the flow.

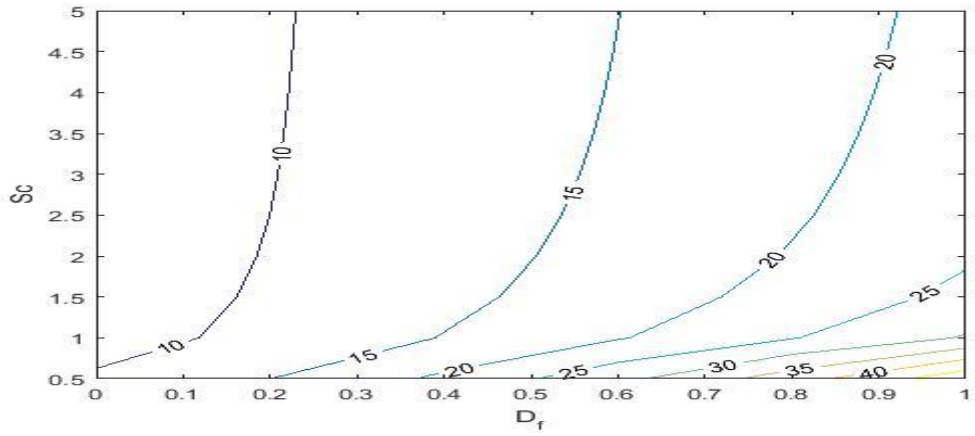


Figure 4.75 Contours of skin-friction for different values D_f, Sc and the fixed value of $K = 2.0,\ \mathrm{Pr} = 0.71,\ \ N = 1.0,\ \ A^* = B^* = 1,\ Y = 0.5 \,\mathrm{at}\,(Y = H)$.

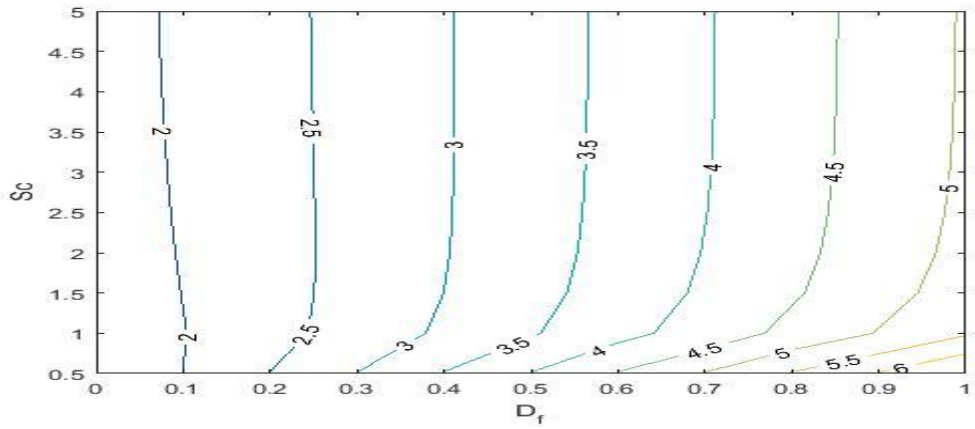


Figure 4.76Contours of skin-friction for different values D_f, Sc and the fixed value of $K = 2.0,\ \mathrm{Pr} = 0.71,\ \ N = 1.0,\ \ A^* = B^* = 1,\ Y = 0.5 \,\mathrm{at}\,(Y = 0)$

The contours showing the variations of Dufour and Schmidt number parameters on the skin friction are demonstrated in Figure 4.75. The result from these contours revealed that the skin friction appreciates at $(Y = H)$ due to higher values of Dufour and depreciates with increase inSchmidt number parameter. Figure 4.76 displays the contours showing the variations of Dufour and Schmidt number parameters on the skin friction. It is noted that the skin friction increaseat porous plate $(Y = 0)$ due to increase inDufour effect, and decreases with an increase in Schmidt number parameter.

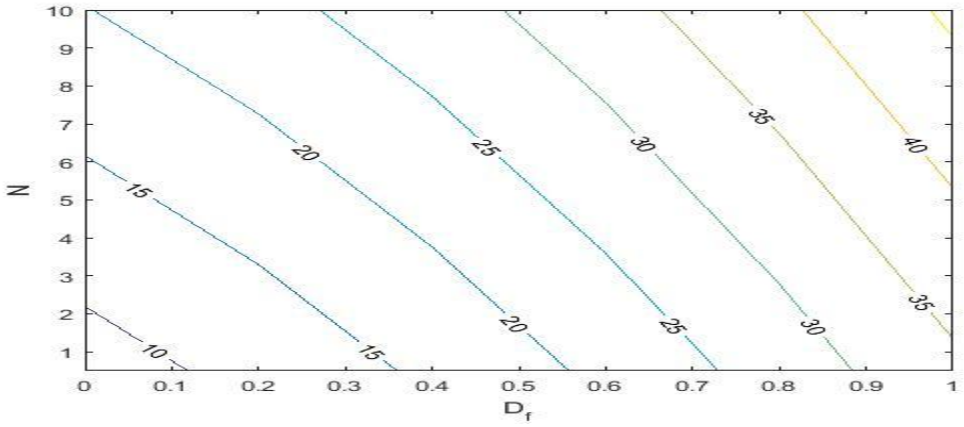


Figure 4.77 Contours showing the influence of D_f and N on the skin friction and the fixed values $Sc = 0.78,\ \Pr = 0.71,\ A^* = B^* = 1$ at $(Y = H)$.

The contours showing the variations of Dufour and buoyancy ratio parameter on the skin friction is analysed in Figure 4.77. The result from the contours reveal that the skin friction increase at the porous plate $(Y = H)$ in the presence of Dufour and buoyancy ratio parameters.

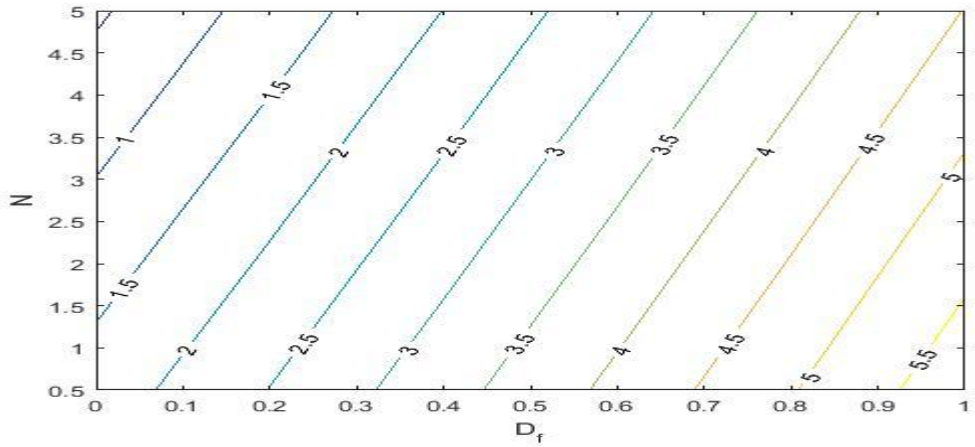


Figure 4.78 Contours showing the influence of D_f and N on the skin friction and the fixed values $Sc = 0.78,\ \Pr = 0.71,\ A^* = B^* = 1$ at $(Y = 0)$

Figure 4.78 displays the contours showing the variations of Dufour and buoyancy ratio parameter on the skin friction. It is observed that the skin friction increaseat the

porousplate $(Y=0)$ in the presence of Dufour and decreases in the presence of buoyancy ratio parameter

Table 4.8 Show the velocity U(Y) for different values of $Sc=0.78,\ \Pr=0.71,\ N=1.0,\ Y=0.5$

	Muthucumaraswamy(2002)			Present work			
K	$H=5$		$H=10$		$H=20$		
	$A^*=B^*=0$ $D_f=0$	$A^*=B^*=1$ $D_f=1$	$A^*=B^*=0$ $D_f=0$	$A^*=B^*=1$ $D_f=1$	$A^*=B^*=0$ $D_f=0$	$A^*=B^*=1$ $D_f=1$	$A^*=B^*=0$ $D_f=0$	$A^*=B^*=1$ $D_f=1$
0	0.8706	2.1881	0.7351	0.5390	0.8645	2.1037	0.8706	2.1881
1	0 .6646	1.9753	0.5846	1.5660	0.6606	1.9575	0.6646	1.9753
2	0.6104	1.9259	0.5341	1.5854	0.6064	1.9110	0.6104	1.9259
3	0.5818	1.9008	0.5065	1.5886	0.5778	1.8871	0.5818	1.9008
4	0.5634	1.8849	0.4886	1.5887	0.5594	1.8719	0.5634	1.8849

Table 4.8 gives numerical validation of the present study with those of Muthucumaraswamy (2002). It is noticed that when the channel gap (H) is larger, the results for velocity of present section is in linewith those of Muthucumaraswamy(2002).

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 Summary

In this work, the physicalbehaviours of Soret or Dufour effects on nonlinear natural convection heat and mass transfer flow in vertical porous channel with chemical

reaction, under different conditions have beenstudied. The different conditions considered in this work comprise;stationary single vertical porous plate, moving single vertical porous plate, Couette flow in vertical channel porous plates, and fixed porous platevertical channel. The dimensionless velocity, temperature and concentration equations for the present physical situation have been considered under relevant boundary conditions, andsolved by the method of undetermined coefficients.

The solutions of dimensionless velocity, temperature, concentration equations, and the results obtained have been presented in graphs, contours and tables.The result of velocity for the present study is in conformity with that of Muthucumaraswamy(2002) in the absence of Soret or Dufourand nonlinear convection parameter.

The effect of the various physical parameters, which include nonlinear density variation with temperature NDT andnonlinear density variation concentration NDC parameter, chemical reaction parameter, Soret parameter, Dufour parameter,chemical reaction parameter, Schmidt number, Prandtl numberand buoyancy ratio parameter have been discussed with the aid of line graphs, contours and table.

5.2 Conclusion

This study investigatesthe influence of Dufour effect or Soret effect on steady fully developed nonlinear natural convection heat and mass transfer flow past nearan infinite vertical porous plate in the presence of first order chemical reaction. Thisresearch work is ageneralization of the mathematical models ofMuthucumaraswamy(2002).

Analytical solutions have been obtained for steady fully developed nonlinear natural convection heat and mass transfer flow past near a vertical moving or stationary porous plate with chemical reaction, Soret effect and Dufour effect. The expressions for dimensionless velocity, concentration, temperature, Sherwood number, Nusselt number and skin friction have been obtained by solving the present mathematical models under relevant boundary conditions.

The effects of the nonlinear density variation with temperature (NDT) and concentration (NDC) parameter, chemical reaction parameter,Soret parameter and Dufour parameter have been discussed with the aid of line graphs, contours and tables. Based on the graphical representations and results obtained in the tables, the following conclusions are drawn.

i. Velocity and skin-friction are higher in case of nonlinear variation of density with temperature and concentration in comparison to linear variation of density with temperature and concentration.
ii. Increase inSchmidt number or chemical reaction parameter,decreases mass diffusivity.
iii. Increase in Soret parameter leads to increase in the velocity, skin-friction and mass diffusion.
iv. Velocity, skin-friction and heat transfer diffusion increase with increase in Dufour parameter
v. The presence of nonlinear convection parameter, Soreteffect and Dufour effect increaseconvection current.

vi. The drag force caused by friction of fluid against the surface of channel porous plates increases with increase in Dufour, Soret, and buoyancy ratio parameters at $(Y = 0)$ and decreases at $(Y = H)$

vii. Dufour parameter contributes positively towards enhancing the temperature of the moving fluid.

viii. Temperature and concentration profiles decrease with increase in Schmidt number and Prandtl number

ix. Flow rate is higher forthe moving porous plate relative to stationary porous plate, in the presence of nonlinear convection, Soret and Dufour parameters.

x. It was found that when the channel distance is very large, the result converges to single porous plate, for both moving and stationry porous plates.

5.3 Recommendations

This investigation evaluatestheinfluence of Dufour or Soret effects on steady fully developed nonlinear natural convection heat and mass transfer flow near an infinite vertical porous plate in the presence of first order homogeneous chemical reaction.

The influences of various flow parameters contained in the problemsare presented with the aid of contours, line graphs and tables.

1. It is interesting to reconstruct the mathematical model that couples Soret and Dufour effects on unsteady nonlinear convection flow near vertical porous channel.
2. Combine effects of Soret and Dufour effects on transient nonlinear natural convection flow near vertical porous surface.
3. Combine effects of Soret and Dufour effects on transient nonlinear natural convection flow near vertical porous surface in the presence of MHD (Magnetohydrodynamic) and Magnetic field
4. It is however recommended that more experimental works should be carried out.

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APPENDICES

Appendix 1: The constant used in section 3.1 and 3.2

$$a_{00} = \left(\mathrm{Pr}^2 - \mathrm{Pr}\,Sc - KSc\right)$$

$$a_{01} = \frac{S\,\mathrm{Pr}^2}{a_{00}}$$

$$a_{02} = \left(1 + 2a_1 + a_1^{\,2}\right)$$

$$a_{03} = 2\left(a_{01} + a_{01}^{\,2}\right)$$

$$a_0 = \left(1 - \exp\left(-\mathrm{Pr}\right)\right)$$

$$a_2 = \left(\mathrm{Pr}^2 - \mathrm{Pr}\, Sc - KSc\right)$$

$$a_1 = \left(a_0 - 1\right)$$

$$a_3 = \frac{S\,\mathrm{Pr}^2}{a_0 a_2}$$

$$a_4 = \left(1 - \exp\left(\delta_1 H\right)\right)$$

$$a_5 = \left(1 - \exp\left(-\delta_2 H\right)\right),$$

$$a_6 = \left(1 - \exp\left(-\mathrm{Pr}\, H\right)\right)$$

$$a_7 = \frac{\left(1 - a_4 - a_3 a_4 + a_3 a_6\right)}{\left(a_5 - a_4\right)}$$

$$a_8 = \left(1 - a_7 + a_3\right),$$

$$a_9 = 2a_7 a_8,$$

$$a_{10} = 2a_8 a_3,$$

$$a_{11} = 2a_7 a_3$$

$$a_{12} = \frac{GrA^* {a_1}^2}{{a_0}^2},$$

$$a_{13} = \frac{GrA^* 2a_1}{{a_0}^2 \,\mathrm{Pr}\left(\mathrm{Pr}-1\right)},$$

$$a_{14} = \frac{GrA^*}{{a_0}^2 2\,\mathrm{Pr}\left(2\,\mathrm{Pr}-1\right)},$$

$$a_{15} = \frac{GcB^* {a_8}^2}{2\delta_1\left(1+2\delta_1\right)}$$

$$a_{16} = GcB^* a_9,$$

$$a_{17} = \frac{GcB^* 2a_{10}}{\left(\delta_1-\mathrm{Pr}\right)\left[1+\left(\delta_1-\mathrm{Pr}\right)\right]},$$

$$a_{18} = \frac{GcB^* {a_7}^2}{2\delta_2\left(2\delta_2-1\right)},$$

$$a_{19} = \frac{GcB^* a_{11}}{\left(\mathrm{Pr}+\delta_2\right)\left[\left(\mathrm{Pr}+\delta_2\right)-1\right]},$$

$$a_{20} = \frac{GcB^* {a_3}^2}{2\,\mathrm{Pr}\left(2\,\mathrm{Pr}-1\right)},$$

$$a_{21} = \frac{Gr}{a_0\,\mathrm{Pr}\left(\mathrm{Pr}-1\right)},$$

$$a_{22} = \frac{Gra_1}{a_0},$$

$$a_{23} = \frac{Gca_8}{\delta_1\left(1+\delta_1\right)},$$

$$a_{24} = \frac{Gca_7}{\delta_2\left(\delta_2-1\right)},$$

$$a_{25} = \frac{Gca_3}{\mathrm{Pr}\left(\mathrm{Pr}-1\right)}$$

$$a_{26} = \left(a_{12} + a_{16} + a_{22} \right),$$
$$a_{27} = \left(a_{13} + a_{25} - a_{21} \right),$$
$$a_{28} = \left(a_{14} + a_{20} \right),$$

Appendix 2: The constant used in section 3.3 and 3.4

$$b_{00} = \frac{D_f \delta^2}{\left(\delta^2 - \Pr \delta \right)},$$

$$b_{01} = \left(1 + 2b_{00} + {b_{00}}^2 \right),$$

$$b_{02} = \left(-2b_{00} - 2{b_{00}}^2 \right),$$

$$b_1 = \left(1 + \exp\left(\delta_1 H \right) \right)$$
$$b_2 = \left(1 + \exp\left(-\delta_2 H \right) \right)$$
$$b_3 = \frac{\left(1 - b_1 \right)}{\left(b_2 - b_1 \right)},$$
$$b_4 = \frac{\left(b_2 - 1 \right)}{\left(b_2 - b_1 \right)},$$
$$b_5 = \frac{D_f {\delta_1}^2 b_4}{{\delta_1}^2 + \Pr \delta},$$
$$b_6 = \frac{D_f {\delta_2}^2 b_3}{{\delta_2}^2 - \Pr \delta},$$

$$b_7 = \left(1 - \exp\left(-\Pr H \right) \right)$$
$$b_8 = \left(1 - \exp\left(\delta_1 H \right) \right)$$
$$b_9 = \left(1 - \exp\left(-\delta_2 H \right) \right)$$

$$b_{10} = \frac{\left(1 + b_5 b_8 + b_6 b_9\right)}{b_7}$$

$$b_{11} = GrA^* {b_5}^2,$$
$$b_{12} = \frac{GrA^* b_8}{\mathrm{Pr}\left(\mathrm{Pr}-1\right)},$$
$$b_{13} = \frac{GrA^* b_9}{\delta\left(\delta-1\right)}$$
$$b_{14} = \frac{GrA^* {b_4}^2}{2\,\mathrm{Pr}\left(2\,\mathrm{Pr}-1\right)},$$

$$b_{11} = \left(1 + b_5 + b_6 - b_{10}\right)$$
$$b_{12} = 2b_4 b_3,\ \ b_{13} = 2b_{11} b_{10}$$
$$b_{14} = 2b_{11} b_5,\ \ b_{15} = 2b_{11} b_6$$
$$b_{16} = 2b_5 b_{10},\ \ b_{17} = 2b_6 b_{10}$$
$$b_{18} = {b_{11}}^2 + 2b_5 b_6$$

$$b_{18} = \frac{GrA^* {b_5}^2}{2\delta_1\left(2\delta_1+1\right)},$$
$$b_{19} = \frac{GrA^* {b_6}^2}{2\delta_2\left(2\delta_2-1\right)},$$
$$b_{21} = \frac{GrA^* {b_{10}}^2}{2\,\mathrm{Pr}\left(2\,\mathrm{Pr}-1\right)},$$
$$b_{22} = \frac{GrA^* b_{13}}{\mathrm{Pr}\left(\mathrm{Pr}-1\right)},$$

$$b_{23} = \frac{GrA^* b_{14}}{\delta_1(\delta_1+1)},$$

$$b_{24} = \frac{GrA^* b_{15}}{\delta_2(\delta_2-1)},$$

$$b_{25} = \frac{GrA^*}{(\delta_1-\mathrm{Pr})\left[1+(\delta_1-\mathrm{Pr})\right]},$$

$$b_{26} = \frac{GrA^* b_{17}}{(\mathrm{Pr}+\delta_2)\left[(\mathrm{Pr}+\delta_2)-1\right]},$$

$$b_{27} = GrA^* b_{18}(Y-1),$$

$$b_{28} = GcB^* b_{12}(Y-1),$$

$$b_{29} = \frac{GcB^* b_4{}^2}{2\delta_1(2\delta_1+1)},$$

$$b_{30} = \frac{GcB^* b_3{}^2}{2\delta_2(2\delta_2-1)},$$

$$b_{31} = Grb_{11}(Y-1),$$

$$b_{32} = \frac{Grb_{10}}{\mathrm{Pr}(\mathrm{Pr}-1)},$$

$$b_{33} = \frac{Grb_5}{\delta_1(\delta_1+1)},$$

$$b_{34} = \frac{Grb_6}{\delta_2(\delta_2-1)},$$

$$b_{35} = \frac{Gcb_4}{\delta_1(\delta_1+1)},$$

$$b_{36} = \frac{Gcb_5}{\delta_1(\delta_1+1)},$$

$$b_{37} = (b_{19} + b_{29}),$$
$$b_{38} = (b_{20} + b_{30})$$
$$b_{39} = (b_{22} + b_{32}),$$
$$b_{40} = (b_{23} + b_{33} - b_{35})$$
$$b_{41} = (b_{24} + b_{34} - b_{36})$$
$$b_{42} = (b_{27} + b_{28} - b_{31})$$

Appendix 3: The constant used in section 3.5 and 3.6

$$d_{00} = \left(\Pr^2 - \Pr Sc - KSc\right),\ d_{01} = \frac{D_t \Pr^2}{d_{00}},$$
$$d_{02} = \left(1 + 2d_{01} + {d_{01}}^2\right), \quad d_{03} = 2\left(d_{01} + {d_{01}}^2\right)$$

$$h_0 = \left(1 - \exp\left(-\Pr\right)\right)$$

$$h_1 = \left(h_0 - 1\right)$$

$$h_2 = \left(\Pr^2 - \Pr Sc - KSc\right)$$

$$h_3 = \frac{S \Pr^2}{h_0 h_2}$$

$$h_4 = \left(1 - \exp\left(\delta_1 H\right)\right)$$

$$h_5 = \left(1 - \exp\left(-\delta_2 H\right)\right),$$
$$h_6 = \left(1 - \exp\left(-\Pr H\right)\right)$$
$$h_7 = \frac{\left(1 - h_4 - h_3 h_4 + h_3 h_6\right)}{\left(h_5 - h_4\right)}$$
$$h_8 = \left(1 - h_7 + h_3\right),$$

$$h_9 = 2h_7 h_8, \quad h_{10} = 2h_8 h_3, \quad h_{11} = 2h_7 h_3$$

$$h_{12} = \frac{GrA^{*} h_1^{\,2}}{h_0^{\,2}},$$

$$h_{13} = \frac{GrA^{*} 2h_1}{h_o^{\,2}\,\mathrm{Pr}\left(\mathrm{Pr}-1\right)},$$

$$h_{14} = \frac{GrA^{*}}{h_0^{\,2} 2\,\mathrm{Pr}\left(2\,\mathrm{Pr}-1\right)},$$

$$h_{15} = \frac{GcB^{*} h_8^{\,2}}{2\delta_1\left(1+2\delta_1\right)}$$

$$h_{16} = GcB^{*} h_9,$$

$$h_{17} = \frac{GcB^{*} 2h_{10}}{\left(\delta_1-\mathrm{Pr}\right)\left[1+\left(\delta_1-\mathrm{Pr}\right)\right]},$$

$$h_{18} = \frac{GcB^{*} h_7^{\,2}}{2\delta_2\left(2\delta_2-1\right)},$$

$$h_{19} = \frac{GcB^{*} h_{11}}{\left(\mathrm{Pr}+\delta_2\right)\left[\left(\mathrm{Pr}+\delta_2\right)-1\right]},$$

$$h_{20} = \frac{GcB^{*} h_3^{\,2}}{2\,\mathrm{Pr}\left(2\,\mathrm{Pr}-1\right)},$$

$$h_{21} = \frac{Gr}{h_0\,\mathrm{Pr}\left(\mathrm{Pr}-1\right)},$$

$$h_{22} = \frac{Grh_1}{h_0},$$

$$h_{23} = \frac{Gch_8}{\delta_1\left(1+\delta_1\right)},$$

$$h_{24} = \frac{Gch_7}{\delta_2\left(\delta_2-1\right)},$$

$$h_{25} = \frac{Gch_3}{\mathrm{Pr}\left(\mathrm{Pr}-1\right)}$$

$$h_{26} = \left(h_{12} + h_{16} - h_{22} \right),$$
$$h_{27} = \left(h_{13} + h_{25} + h_{21} \right),$$
$$h_{28} = \left(h_{14} + h_{20} \right),$$

Appendix 4: The constant used in section 3.7 and 3.8

$$e_{00} = \frac{D_f \delta^2}{\left(\delta^2 - \Pr \delta \right)},$$

$$e_{01} = \left(1 + 2b_{00} + {b_{00}}^2 \right),$$

$$e_{02} = \left(-2b_{00} - 2{b_{00}}^2 \right),$$

$$c_1 = \left(1 + \exp\left(\delta_1 H \right) \right)$$
$$c_2 = \left(1 + \exp\left(-\delta_2 H \right) \right)$$
$$c_3 = \frac{\left(1 - c_1 \right)}{\left(c_2 - c_1 \right)},$$
$$c_4 = \frac{\left(c_2 - 1 \right)}{\left(c_2 - c_1 \right)},$$
$$c_5 = \frac{D_f {\delta_1}^2 c_4}{{\delta_1}^2 + \Pr \delta},$$
$$c_6 = \frac{D_f {\delta_2}^2 c_3}{{\delta_2}^2 - \Pr \delta},$$

$$c_7 = \left(1 - \exp\left(-\Pr H\right)\right)$$
$$c_8 = \left(1 - \exp\left(\delta_1 H\right)\right)$$
$$c_9 = \left(1 - \exp\left(-\delta_2 H\right)\right)$$

$$c_{10} = \frac{\left(1 + c_5 c_8 + c_6 c_9\right)}{c_7}$$

$$c_{11} = GrA^* {c_5}^2,$$
$$c_{12} = \frac{GrA^* c_8}{\Pr\left(\Pr - 1\right)},$$
$$c_{13} = \frac{GrA^* c_9}{\delta\left(\delta - 1\right)}$$
$$c_{14} = \frac{GrA^* {c_4}^2}{2\Pr\left(2\Pr - 1\right)},$$

$$c_{11} = \left(1 + c_5 + c_6 - c_{10}\right)$$
$$c_{12} = 2c_4 c_3, \;\; c_{13} = 2c_{11} c_{10}$$
$$c_{14} = 2c_{11} c_5, \;\; c_{15} = 2c_{11} c_6$$
$$c_{16} = 2c_5 c_{10}, \;\; c_{17} = 2c_6 c_{10}$$
$$c_{18} = {c_{11}}^2 + 2c_5 c_6$$

$$c_{18} = \frac{GrA^* {c_5}^2}{2\delta_1\left(2\delta_1 + 1\right)},$$
$$c_{19} = \frac{GrA^* {c_6}^2}{2\delta_2\left(2\delta_2 - 1\right)},$$
$$c_{21} = \frac{GrA^* {c_{10}}^2}{2\Pr\left(2\Pr - 1\right)},$$
$$c_{22} = \frac{GrA^* c_{13}}{\Pr\left(\Pr - 1\right)},$$

$$c_{23} = \frac{GrA^* c_{14}}{\delta_1(\delta_1+1)},$$

$$c_{24} = \frac{GrA^* c_{15}}{\delta_2(\delta_2-1)},$$

$$c_{25} = \frac{GrA^*}{(\delta_1-\mathrm{Pr})\left[1+(\delta_1-\mathrm{Pr})\right]},$$

$$c_{26} = \frac{GrA^* b_{17}}{(\mathrm{Pr}+\delta_2)\left[(\mathrm{Pr}+\delta_2)-1\right]},$$

$$c_{27} = GrA^* c_{18}(Y-1),$$

$$c_{28} = GcB^* c_{12}(Y-1),$$

$$c_{29} = \frac{GcB^* c_4{}^2}{2\delta_1(2\delta_1+1)},$$

$$c_{30} = \frac{GcB^* c_3{}^2}{2\delta_2(2\delta_2-1)},$$

$$c_{31} = Grc_{11}(Y-1),$$

$$c_{32} = \frac{Grc_{10}}{\mathrm{Pr}(\mathrm{Pr}-1)},$$

$$c_{33} = \frac{Grc_5}{\delta_1(\delta_1+1)},$$

$$c_{34} = \frac{Grc_6}{\delta_2(\delta_2-1)},$$

$$c_{35} = \frac{Gcc_4}{\delta_1(\delta_1+1)},$$

$$c_{36} = \frac{Gcc_5}{\delta_1(\delta_1+1)},$$

$$c_{37} = (c_{19} + c_{29}),$$

$$c_{38} = (c_{20} + c_{30})$$

$$c_{39} = (c_{22} + c_{32}),$$

$$c_{40} = (c_{23} + c_{33} - c_{35})$$

$$c_{41} = (c_{24} + c_{34} - c_{36})$$

$$c_{42} = (c_{27} + c_{28} - c_{31})$$

Appendix 5: Matlab Codes for the Analytical Solutions

```
clc

clear


%[k,Df]=meshgrid(0.0:0.2:1.0,0.5:0.5:5.0);

%[Df,sc]=meshgrid(0.0:0.2:1.0,0.5:0.5:1.0);

%[y,Df]=meshgrid(0.0:0.2:1,0.5:0.5:5.0);

%[y,Df]=meshgrid(0.0:0.2:5,0.5:0.5:5.0);

     [y,sc]=meshgrid(0.0:0.02:1,0.5:0.5:5.0);

%[pr,sc]=meshgrid(0.0:0.2:1.0,0.5:0.5:5.0);

%[pr,sc]=meshgrid(0.0:0.1:1.0,0.5:0.5:4.0);

%w=(k-k+1).*(s-s+1);

%w=(Df-Df+1).*(sc-sc+1);

%w=(y-y+1).*(Df-Df+1);

%w=(y-y+1).*(sc-sc+1);

%w=(y-y+1).*(pr-pr+1);

      w=(y-y+1).*(sc-sc+1);

%w=(k-k+1).*(Df-Df+1);

%w=(pr-pr+1).*(sc-sc+1);

%y=0.5;

       H=1;

%sc=0.78;

pr=0.71;

Df=2.0;

       k=2;

       Gr=1.0;

Gc=1.0;

       A=1.0;

       B=1.0;
```

```
x1=sqrt((sc.*sc)+(4.*k.*sc));
     d1=0.5.*(-sc+x1);
     d2=0.5.*(sc+x1);
%r1=(Df.*d.*d);
     a1=(1+(exp(d1.*H)));
     a2=(1-(exp(-d2.*H)));
     a3=(1-a1)./(a2-a1);
     a4=(a2-1)./(a2-a1);
con=(a4.*exp(d1.*y))+(a3.*exp(-d2.*y));
     a5=(Df.*(d1.*d1).*a4)./((d1.*d1)+(pr.*d1));
     a6=(Df.*(d2.*d2).*a3)./((d2.*d2)-(pr.*d2));
     a7=(1-(exp(-pr.*H)));
     a8=(1-(exp(d2.*H)));
     a9=(1-(exp(-d2.*H)));
     a10=(1+(a5.*a8)+(a6.*a9))./a7;
     a11=(1+a5+a6-a10);
     r2=(a10.*exp(-pr.*y));
     r3=(a5.*exp(d1.*y));
     r4=(a6.*exp(-d2.*y));
     Temp=a11+r2-r3-r4;
     a12=(2.*a4.*a3);
     a13=(2.*a11.*a10);
     a14=(2.*a11.*a5);
     a15=(2.*a11.*a6);
     a16=(2.*a5.*a10);
     a17=(2.*a6.*a10);
     a18=(2.*a5.*a6);
     r5=(Gr.*A);
     r6=(Gc.*B);
     r7=(2.*d1).*(1+(2.*d1));
     a19=(r5.*(a5.*a5))./r7;
```

```
r8=(2.*d2).*((2.*d2)-1);

a20=(r5.*(a6.*a6))./r8;

r9=(2.*pr).*((2.*pr)-1);

a21=(r5.*(a10.*a10))./r9;

r10=(pr.*(pr-1));

a22=(r5.*a13)./r10;

r11=(d1.*(1+d1));

a23=(r5.*a14)./r11;

r12=(d2.*(d2-1));

a24=(r5.*a15)./r12;

r13=(d1-pr).*(1+(d1-pr));

a25=(r5.*a16)./r13;

r14=(d2+pr).*((d2+pr)-1);

a26=(r5.*a17)./r14;

a27=(r5.*a18);

a28=(r6.*a12);

a29=(r6.*(a4.*a4))./r7;

a30=(r6.*(a3.*a3))./r8;

a31=(Gr.*a11);

a32=(Gr.*a10)./r10;

a33=(Gr.*a5)./r11;

a34=(Gr.*a6)./r12;

a35=(Gc.*a4)./r11;

a36=(Gc.*a3)./r12;

a37=(a19+a29);

a38=(a20+a30);

a39=(a22+a32);

a40=(a23+a33-a35);

a41=(a24+a34-a36);

a42=(a27+a28-a31);

k1=exp(-d2.*y);
```

```
     k2=exp(-d2.*H);
     k3=exp(-pr.*y);
     k4=exp(-pr.*H);
     k5=exp(-2.*pr.*y);
     k6=exp(-2.*pr.*H);
     k7=exp(-2.*d2.*y);
     k8=exp(-2.*d2.*H);
     k9=exp(-(pr+d2).*y);
     k10=exp(-(pr+d2).*H);
     k11=exp(-y);
     k12=exp(-H);
     k13=exp(2.*d1.*y);
     k14=exp(2.*d1.*H);
     k15=exp((d1-pr).*y);
     k16=exp((d1-pr).*H);
     k17=exp(d1.*y);
     k18=exp(d1.*H);
     r15=(((1-k12))-H.*(1-k11))./(1-k12);
x2=-a42.*(r15+(y-1));
     r17=(((1-k12))-(1-k14).*(1-k11))./(1-k12);
x3=-a37.*(r17-k13);
     r18=(((1-k12))-(1-k8).*(1-k11))./(1-k12);
x4=-a38.*(r18-k7);
     r19=(((1-k12))-(1-k6).*(1-k11))./(1-k12);
x5=-a21.*(r19-k5);
     r20=(((1-k12))-(1-k4).*(1-k11))./(1-k12);
x6=-a39.*(r20-k3);
     r21=(((1-k12))-(1-k18).*(1-k11))./(1-k12);
x7=-a40.*(r21-k17);
     r22=(((1-k12))-(1-k2).*(1-k11))./(1-k12);
x8=-a41.*(r22-k1);
```

```
     r23=(((1-k12))-(1-k16).*(1-k11))./(1-k12);
x9=a25.*(r23-k15);
     r24=(((1-k12))-(1-k10).*(1-k11))./(1-k12);
x10=a26.*(r24-k9);
x11=(((1-k12))-(1-k11))./(1-k12);
     u=x11+x2+x3+x4+x5+x6+x7+x8+x9+x10;
%[C,h]=contour(k,Df,u);
[C,h]=contour(y,u,sc);
set(h,'showText','on','textStep',get(h,'levelStep')*1);
%colormap gray

clc
clear
    %Soret Effect
     [y,sc]=meshgrid(0.0:0.02:1,0.5:0.5:5.0);
      w=(y-y+1).*(sc-sc+1);
%y=0.5;
       H=1;
%sc=0.78;
pr=0.71;
       s=2.0;
       k=2;
       Gr=1.0;
Gc=1.0;
       A=1.0;
       B=1.0;
x1=sqrt((sc.*sc)+(4.*k.*sc));
     d1=0.5.*(-sc+x1);
     d2=0.5.*(sc+x1);
ao=(1-(exp(-pr.*H)));
     a1=(ao-1);
```

```
      T=(exp(-pr.*y)+a1)./ao;

      a2=((pr.*pr)-(pr.*sc)-(k.*sc));

      a3=(s.*(pr.*pr))./(ao.*a2);

      a4=(1-(exp(d1.*H)));

      a5=(1-(exp(-d2.*H)));

      a6=(1-(exp(-pr.*H)));

      a7=(1-a4-(a3.*a4)+(a3.*a6))./(a5-a4);

      a8=(1-a7+a3);

con=(a8.*exp(d1.*y))+(a7.*exp(-d2.*y))-(a3.*exp(-pr.*y));

      a9=(2.*a7.*a8);

      a10=(2.*a8.*a3);

      a11=(2.*a7.*a3);

      r1=(Gr.*A);

      r2=(Gc.*B);

      a12=(r1.*(a1.*a1))./(ao.*ao);

      a13=(r1.*2.*a1)./((ao.*ao).*(pr.*(pr-1)));

      a14=(r1)./((ao.*ao).*(2.*pr.*(2.*pr-1)));

      a15=(r2.*(a8.*a8))./(2.*d1).*(1+(2.*d1));

      a16=(r2.*a9);

      a17=(r2.*a10)./(d1-pr).*(1+(d1-pr));

      a18=(r2.*(a7.*a7))./(2.*d2).*((2.*d2)-1);

      a19=(r2.*a11)./(d2+pr).*((d2+pr)-1);

      a20=(r2.*(a3.*a3))./(2.*pr).*((2.*pr)-1);

      a21=Gr./(ao.*(pr.*(pr-1)));

      a22=(Gr.*a1)./(ao);

      a23=(Gc.*a8)./(d1.*(1+d1));

      a24=(Gc.*a7)./(d2.*(d2-1));

      a25=(Gc.*a3)./(pr.*(pr-1));

      a26=(a12+a16+a22);

      a27=(-a13+a25-a21);

      a28=(a14+a20);
```

```
     k1=exp(-d2.*y);
     k2=exp(-d2.*H);
     k3=exp(-pr.*y);
     k4=exp(-pr.*H);
     k5=exp(-2.*pr.*y);
     k6=exp(-2.*pr.*H);
     k7=exp(-2.*d2.*y);
     k8=exp(-2.*d2.*H);
     k9=exp(-(pr+d2).*y);
     k10=exp(-(pr+d2).*H);
     k11=exp(-y);
     k12=exp(-H);
     k13=exp(2.*d1.*y);
     k14=exp(2.*d1.*H);
     k15=exp((d1-pr).*y);
     k16=exp((d1-pr).*H);
     k17=exp(d1.*y);
     k18=exp(d1.*H);
     r15=(((1-k12))-H.*(1-k11))./(1-k12);
x2=-a26.*(r15+(y-1));
     r17=(((1-k12))-(1-k4).*(1-k11))./(1-k12);
x3=+a27.*(r17-k3);
     r18=(((1-k12))-(1-k6).*(1-k11))./(1-k12);
x4=-a28.*(r18-k5);
     r19=(((1-k12))-(1-k14).*(1-k11))./(1-k12);
x5=-a15.*(r19-k13);
     r20=(((1-k12))-(1-k8).*(1-k11))./(1-k12);
x6=-a18.*(r20-k7);
     r21=(((1-k12))-(1-k18).*(1-k11))./(1-k12);
x7=-a23.*(r21-k17);
     r22=(((1-k12))-(1-k2).*(1-k11))./(1-k12);
```

```
x8=-a24.*(r22-k1);
     r23=(((1-k12))-(1-k16).*(1-k11))./(1-k12);
x9=a17.*(r23-k15);
     r24=(((1-k12))-(1-k10).*(1-k11))./(1-k12);
x10=a19.*(r24-k9);
x11=(((1-k12))-(1-k11))./(1-k12);
     u=x11+x2+x3+x4+x5+x6+x7+x8+x9+x10;
%[C,h]=contour(s,sc,u);
      [C,h]=contour(y,u,sc);
set(h,'showText','on','textStep',get(h,'levelStep')*1);
%colormap gray
```

Appendix 5: List of publications

1.Non-linear Natural Convection and Mass Transfer Flow near a Vertical Moving Porous Plate with Chemical Reaction and Soret Effect, *Multidiscipline modeling in materials and structures*,Vol. 15 No 5, pp. 846 – 858,https://doi.org/10.1108/mmms-04-2018-0063

2.Chemical Reaction and Dufour Effects on Nonlinear Free Convection Heat and Mass Transfer Flow near a Vertical Moving Porous Plate,https://doi.0rg/10.1002/htj.21649 (doi:10.1002/htj.21649) *Heat Transfer – Asian Research*, 49(B14)