A STUDY ON SOME FLOW BEHAVIOUR OF CONDUCTING FLUIDS

BY
JIBRIL, HARUNA MOHAMMED
B.SC (A.B.U, 1987), M.SC (A.B.U, 1992)
PHD/SCIEN/34370/2002-2003

A DISSERTATION SUBMITTED TO THE
SCHOOL OF POSTGRADUATE STUDIES,
AHMADU BELLO UNIVRSITY,
ZARIA, NIGERIA

IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE AWARD OF

DOCTOR OF PHILOSOPHY DEGREE IN MATHEMATICS
DEPARTMENT OF MATHEMATICS,
FACULTY OF SCIENCE
AHMADU BELLO UNIVERSITY, ZARIA, NIGERIA

Supervisory Committee
PROFESSOR B.K.JHA: Chairman
PROFESSOR D.SINGH: Member
PROFESSOR G.U.GARBA: Member

MAY, 2012

DECLARATION

I declare that the work in this dissertation titled, "A study on some flow behavior of conducting fluids" was carried out by me in the department of Mathematics, Ahmadu Bello University, Zaria, Nigeria, under the supervision of Professor B.K.Jha, Prof. D.Singh and Prof. G.U.Garba. The information obtained from the literature has been duly acknowledged in the text and a list of references provided. No part of this work has been presented for another degree or diploma at any institution.

Haruna Mohammed Jibril May, 2012.

CERTIFICATION

This dissertation titled, "A study on some flow behaviour of conducting fluids" by JIBRIL, Haruna Mohammed meets the regulations governing the award of the degree of Doctor of Philosophy of Ahmadu Bello University, and is approved for its contribution to knowledge and literary presentation.

_____

Chairman, Supervisory Committee
(Prof. B.K.Jha)

_____

Member, Supervisory Committee
(Prof.D.Singh)

_____

Member, Supervisory Committee
(Prof. G.U.Garba)

_____

External Examiner
(Prof M.Y.Bello)

_____

Head of Department
(Dr. B. Sani)

_____

Dean, School of Postgraduate Studies
(Prof. A.A.Joshua)

ACKNOWLEDGEMENTS

Praise is to the Almighty God for all the favours bestowed on me and the rare opportunities as well as the potential to pursue this research work to its logical conclusion. May Allah continue to Guide us to the right path.

My sincere appreciation goes to my Late Father of blessed memory and to my aged Mother for their guidance, prayers and moral support. May the Almighty God grant you the best in you final abode. Ameen.

My profound gratitude goes to my Major supervisor, Prof. B. K. Jha, for his untiring efforts, consistent advice and support. The excellent tutelage provided by you will continue to guide me in my current and future Endeavors. Once more I sincerely say thank you for all the efforts geared towards me for the successful completion of this work.

My appreciation also goes to my minor- supervisors, Prof. D. Singh for his fatherly advice and invaluable contribution towards this project, and Prof. G. U Garba for his encouragement.

My special thanks go to the Head of Department, Dr Babangida Sani for his continuous support and encouragement right from my undergraduate days to date, and to all my Colleagues in the department for the wonderful, brotherly and sisterly advice I enjoyed from you. My gratitude also goes to the Postgraduate Coordinator and his deputy in the persons of Dr M.A Ibrahim and Dr Abubakar yahaya respectively and the seminar coordinator, Dr Abdul Mohammed for their advice and support. I also wish to thank my office mate in the persons of Mal. A.M. Abdullahi Chafe for keeping me company and for his encouragement.

I also wish to thank all those that contributed to the success of this exercise in one way or the other too numerous to mention especially, Mr Chibuike Nnamani and Mal. Abdullahi Mohammed for their help in the preparation of this manuscript and to all research scholars in our team of Computational Mathematics for their moral and professional support.

I also appreciate the contribution of my valuable friends for their encouragement.

Finally my deep appreciation goes to my immediate family especially my wife for her understanding in enduring my long absence, especially in the night in the process of making this dream come true. May God bless you All.

DEDICATION

This work is dedicated to my parents for their support throughout my educational pursuits. To my teachers right form nursery school for their tutelage. May God bless you all.

ABSTRACT

The unsteady flow of a viscous, incompressible and electrically conducting fluid subject to uniform transverse magnetic field under different physical situations was examined. The motion of this fluid near a horizontal flat plate having ramped motion is considered with or without transpiration. Similarly, the unsteady hydromagnetic Couette flow between two parallel plates due to ramped motion of one of the plates was also investigated, when the plates are porous and non-porous. A unified closed form analytical solution have been obtained for velocity field and the skin-friction corresponding to the cases of magnetic field fixed relative to the fluid or the moving boundary. Under the assumption of negligible induced magnetic field, the governing momentum equation has been solved exactly by using Laplace-transform technique. The flow formation near a ramped motion of the plate has also been compared with the flow formation near a plate having constant motion(impulsive motion).It is observed that the velocity of fluid in the case of ramped motion of the plate is always less than that of the flow induced by impulsive motion of the plate. Next, we considered the combined effects of viscous and ohmic dissipation on magnetohydrodynamics (MHD) Couette flow and heat transfer in a parallel plate channel formed by two horizontal porous plates. An analytical solution for time dependent velocity and time independent temperature field are derived by taking into account the effect of viscous and ohmic dissipation. The effect of various flow parameters on the velocity field, temperature distribution, skin friction coefficient and Nusselt number at the walls are discussed with the aid of graphs. Finally the time dependent two phase MHD flow in a parallel plate channel partially filled with conducting fluid and remaining with non-conducting fluid was studied, both the phases were incompressible and the flow is assumed to be time dependent. A closed

form expressions for velocity in both phases were obtained using Green's function approach for cases corresponding to closed and open circuit.

SYMBOLS/NOTATIONS

h = distance between the porous plates

B_0 = magnetic field strength

E_z =Electric field

u' = dimensional velocity of the fluid

u = dimensionless velocity of the fluid

t' = dimensional time

t = dimensionless time

V_0 = suction/injection velocity

U_0 = constant velocity of the porous plate at $y' = 1$

y' =dimensional transverse co-ordinate

y = dimensionless transverse co-ordinate

M =Hartmann number

Pr =Prandtl number

R=dimensionless electric load parameter

Ec =Eckert number

Re =Reynolds number

Nu_0 =Nusselt number at $y = 0$

Nu_1 = Nusselt number at $y = 1$

T' = dimensional temperature

C_p = specific heat capacity

d = width of the non-conducting fluid

h' = total width of channel

H = dimensionless total width of the channel $\left(\frac{h}{d}\right)$

u_1 = velocity field of conducting fluid in phase I

u_2 = velocity field of conducting fluid in phase II

U_1 = dimensionless velocity field of conducting fluid in phase I $\left(\frac{u_1 d}{\nu_1}\right)$

U_2 =dimensionless velocity field of conducting fluid in phase II $\left(\frac{u_2 d}{\nu_1}\right)$

y =dimensional transverse co-ordinate

Y = dimensionless transverse co-ordinate $\left(\frac{y}{d}\right)$

$\frac{\partial p}{\partial x}$ =dimensional axial pressure gradient

P =dimensionless axial pressure gradient $\left(\frac{\partial p}{\partial x}\left(\frac{d^3}{\rho_1 \nu_1^2}\right)\right)$

M =Hartman number $\left(B_0 d\sqrt{\frac{\sigma}{\mu_1}}\right)$

$f_1^*(\tau)$= velocity of lower bounding wall

$f_2^*(\tau)$= velocity of upper bounding wall

$\overline{F_1}(y)$ = initial velocity in non-conducting fluid

$\overline{F_2}(y)$ = initial velocity in conducting fluid

$f_1(t)$=dimensionless velocity of lower bounding wall $\left(\frac{f_1^*(\tau)d}{\nu_1}\right)$

$f_2(t)$=dimensionless velocity of upper bounding wall $\left(\frac{f_2^*(\tau)d}{\nu_1}\right)$

$F_1(Y)$=dimensionless initial velocity in non-conducting fluid $\left(\frac{\overline{F_1}(y)d}{\nu_1}\right)$

$F_2(Y)$=dimensionless initial velocity in conducting fluid $\left(\frac{\overline{F_2}(y)d}{\nu_1}\right)$

t =dimensionless time $\left(\frac{\tau \nu_1}{d^2}\right)$

Greek Symbols

τ_0 = dimensionless skin friction at $y = 0$

τ_1 = dimensionless skin friction at $y = 1$

μ =dynamic viscosity of the fluid

ν = kinematic viscosity

θ =dimensionless temperature

ρ =density of fluid

σ =electrical-conductivity of conducting fluid

α = thermal conductivity of the fluid

τ = time

ρ_1 =density of conducting fluid phase I

ρ_2 =density of conducting fluid phase II

σ_1 =electrical-conductivity of conducting fluid phase I

σ_2 =electrical-conductivity of conducting fluid phase II

μ_1 =dynamic viscosity of conducting fluid phase I

μ_2 =dynamic viscosity of conducting fluid phase II

γ =ratio of dynamic viscosities $\left(\frac{\mu_2}{\mu_1}\right)$

α =ratio of densities of fluids $\left(\frac{\rho_1}{\rho_2}\right)$

δ =ratio of electrical conductivities of fluids $\left(\frac{\sigma_2}{\sigma_1}\right)$

Subscripts

1= conducting fluid phase I

2= conducting fluid phase II

DIMENSIONLESS QUANTITIES

These are quantities used often in the study of fluids in motion to understand the inter relationship between parameters of the flow in order to allow for generalization of results. The dimensionless quantities in this work are:

1. Hartmann number

This is the most important non-dimensional number in hydromagnetics, first introduced by Hartmann. It is the ratio of electromagnetic force to the viscous force. The Hydromagnetic effects are significant if the Hartmann number is significant. It is defined by:

$$M = B_0 L\sqrt{\frac{\sigma}{\mu}}$$

Where B_0 is the magnetic field, L is the characteristic length scale, σ is the electrical conductivity and μ is the viscosity

2. Reynolds number

The Reynolds numberRe, named after Osborne Reynolds (1842–1912), is a dimensionless number that gives a measure of the ratio of inertial forces to viscous forces and consequently quantifies the relative importance of these two types of forces for given flow conditions.

It is also used to characterize different flow regimes, such as laminar or turbulent flow. Laminar flow occurs at low Reynolds numbers, where viscous forces are dominant, and is characterized by smooth, constant fluid motion, while turbulent flow occurs at high Reynolds numbers and is dominated by inertial forces, which tend to produce chaoticeddies, vortices and other flow instabilities. It is defined as: $\mathrm{Re} = \frac{VL}{\nu}$

Where V is the mean fluid velocity, L is a characteristic linear dimension and ν is the kinematic viscosity

3. Prandtl number

The Prandtl number Pr, named after the German physicist Ludwig Prandtl, is the ratio of momentum diffusivity (kinematic viscosity) to thermal diffusivity. In heat transfer problems, the Prandtl number controls the relative thickness of the momentum and thermal boundary layers. When *Pr* is small, it means that the heat diffuses very quickly compared to the velocity (momentum).

The thermal boundary layer thickness is similarly the distance from the body at which the temperature is 99% of the temperature found from an inviscid solution. The ratio of the two thicknesses is governed by the Prandtl number. If the Prandtl number is 1, the two boundary layers are the same thickness. If the Prandtl number is greater than 1, the thermal boundary layer is thinner than the velocity boundary layer. If the Prandtl number is less than 1, which is the case for air at standard conditions, the thermal boundary layer is thicker than the velocity boundary layer. The Prandtl number is defined as:

$$\mathrm{Pr} = \frac{\nu}{\alpha} = \frac{\mu C_p}{k}$$

Where ν is the kinematic viscosity, α the thermal diffusivity, *k the* thermal conductivity and c_p the specific heat capacity.

4. Magnetic Reynolds number

The Magnetic Reynolds number is a dimensionless quantity that occurs in magnetohydrodynamics. It gives an estimate of the effects of magnetic advection to magnetic diffusion. If the magnetic Reynolds number is small, it means the magnetic field is not changed appreciably by the flow and so the induced magnetic field is zero. The Magntic Reynolds number is typically defined by: $R_m = \frac{UL}{\eta}$

Where U is a typical velocity scale of the flow, L the typical length scale of the flow and η the magnetic diffusivity.

5. Nusselt number

Given two parallel plates at different temperatures, the Nusselt number gives the ratio of actual heat transferred between the plates by a moving fluid to the heat transfer that would occur by conduction. It is defined by

$$Nu = \frac{Hd}{k(T_2 - T_1)}$$

where H heat transfer per unit area per unit time, d is the distance scale, $T_2 - T_1$ is the temperature differential, k is the thermal conductivity

6. Eckert number

The Eckert number, named after Ernst R. G. Eckert, is a dimensionless number used in fluid dynamics. It expresses the relationship between a flow's kinetic energy and enthalpy, and is used to characterize dissipation.

It is defined as $E_c = \frac{V^2}{C_p \Delta T}$

Where V is a characteristic velocity of the flow, c_p the constant-pressure specific heat of the flow and ΔT is a characteristic temperature difference of the flow.

Table of Contents

Declaration ... ii
Certification ... iii
Acknowledgements ... iv
Abstract ... vii
Dimensionless quantities ... xii
1. Hartmann number ... xii
2. Reynolds number ... xii
3. Prandtl number ... xiii
4. Magnetic Reynolds number ... xiii
5. Nusselt number ... xiv
6. Eckert number ... xiv
List of Figures ... **Error! Bookmark not defined.**
List of Appendices ... **Error! Bookmark not defined.**
CHAPTER ONE ... 1
1.0 INTRODUCTION ... **1**
1.1 The Magnetohydrodynamic Basic Equations ... **2**
1.2 Overview of Basic Concepts ... **9**
1.3 Mathematical Methods ... **15**
1.4 Objectives of the Study ... **21**
1.5 Research Methodology ... **22**
1.6 Organization of the Dissertation ... **23**
CHAPTER TWO ... 25
LITERATURE REVIEW ... **25**
2.0 Introduction ... **25**
2.1 Literature review on MHD flow due to the ramped motion of a horizontal flat plate ... **25**
2.2 Literature review on unsteady MHD Couette flow between two parallel plates due to the ramped motion of one of the boundaries ... **28**
2.3 Literature review on the combined effects of viscous, Joulean and ohmic dissipation on MHD Couette flow with transpiration ... **31**
2.4 Literature review on transient Two- phase flow in magneto hydrodynamics: A Green's function approach ... **36**
CHAPTER THREE ... 38
MHD FLOW DUE TO THE RAMPED MOTION OF AN INFINITE HORIZONTAL PLATE ... **38**
3.0 Introduction. ... **38**

3.1 Time Dependent MHD flow due to the ramped motion of an infinite horizontal flat plate under the influence of transverse magnetic field. 38
3.1.0 Introduction 38
3.1.1 Mathematical Analysis 38
3.1.2 Solution of the problem 41
3.1.3 Skin friction 42
3.1.4 Special Case 43
3.1.5 Results and Discussion 43
3.1.6 Summary and Conclusion 49
3.2 Effects of Transpiration on the MHD flow near a Porous Plate having ramped motion 50
3.2.0 Introduction 50
3.2.1 Governing Equations and Solutions 50
3.2.2 Solution of the Problem 53
3.2.3 Skin friction 54
3.2.4 Special Case 55
3.2.5 Results and Discussions 56
3.2.6 Summary and Conclusion 64
CHAPTER FOUR 65
UNSTEADY MHD COUETTE FLOW BETWEEN TWO PARALLEL PLATES DUE TO RAMPED MOTION OF ONE OF THE BOUNDARIES 65
4.0 Introduction 65
4.1 Time dependent MHD Couette flow due to ramped motion of one of the boundaries 65
4.1.0 Introduction 65
4.1.1 Formulation of the Problem 65
4.1.2 Mathematical solution 68
4.1.3 Special case: Impulsive motion of one of the boundaries 71
4.1.4 Skin friction 72
4.1.5 Steady State Solution 73
4.1.6. Results and Discussion 73
4.1.7 Summary and Conclusion 85
4.2 Unsteady Hydromagnetic Couette flow due to the ramped motion of one of the porous plates 86
4.2.0 Introduction 86
4.2.1 Governing Equations and Solutions 86
4.2.2 Solution of the problem 89
4.2.3 Skin friction 90

4.2.4 Special case: Impulsive motion of one of the porous plates 92
4.2.6 Steady State Solution 94
4.2.7 Results and Discussions 95
4.2.8 Summary and Conclusion 105
CHAPTER FIVE 106
COMBINED EFFECTS OF VISCOUS, JOULEAN AND OHMIC DISSIPATION ON MHD COUETTE FLOW WITH TRANSPIRATION 106
5.0 Introduction 106
5.1 Combined Effects of Viscous and Joulean Dissipation on MHD Couette flow with transpiration. 106
5.1.0 Introduction 106
5.1.1 Formulation of the problem 106
5.1.2 Analytical Solutions. 109
5. 1.3 Steady state temperature 111
5.1.4 Nusselt Number 113
5.1.5 Results and Discussion 114
5.1.6 Summary and Conclusion 135
5.2 Effects of viscous dissipation and ohmic heating on MHD Couette flow in a porous channel under the influence of electric field. 136
5.2.0 Introduction 136
5.2.1. Formulation of the problem 136
5.2.2. Analytical Solutions. 139
5.2.3 Skin friction 141
5.2.4 Steady state temperature 142
5.2.5 Nusselt Number 143
5.2.6 Discussion of Results 144
5.2.8 Summary and Conclusion 179
CHAPTER SIX 181
TWO PHASE FLOW IN MAGNETOHYDRODYNAMICS: A GREENS FUNCTION APPROACH 181
6.0 Intoduction 181
6.1 Time dependent two phase flow in magnetohydrodynamics: A Greens function approach 181
6.1.0 Introduction 181
6.1.1 Governing equations and solutions 181
6.1.2 Particular Case:Uniform pressure gradient 186
6.1.3. Steady- State Solution 187
6.1.4 Concluding remarks 188

6.2 Unsteady MHD two- phase flow of conducting fluids in a horizontal channel..... 188
6.2.0 Introduction 188
6.2.1. Governing equations and solutions 189
6.2.2. Particular Case 194
6.2.3. Steady state solution 195
6.2.4 Concluding remarks 196
CHAPTER SEVEN 197
SUMMARY AND CONCLUDING REMARKS 197
7.1 Summary 197
7.2 Conclusion 198
7.3 Directions for future research 199
REFERENCES 200
APPENDIX 209

LIST OF FIGURES

Figure 3.1.1 Physical configuration of the flow……...39

Fig.3.1.2 Velocity (u) versus distance (y), for different time (t),

(M =0.5 and K =0.0 & K =1.0)…….…….....44

Fig.3.1.3 Velocity (u) versus distance (y) for different time (t),

(M =0.5 and K =0.0)…….45

Fig.3.1.4 Velocity (u) versus distance (y) for different time (t),

(M =0.5 and $K = 1.0$) …… 45

Fig.3.1.5 Velocity (u) versus distance (y) for different (t),(M =0.5 and K =0.0)…..46

Fig. 3.1.6 velocity (u) versus distance (y) for different (t),

($K = 0.0$ and M =0.5)…….....47

Fig.3.1.7 Velocity (u) versus distance (y) for different values of M and t ,

(K =0.0 & (K =1.0)……...……48

Fig.3.1.8 Skin friction (τ) versus time (t) for different values of M ,

($K = 0.0$ and $K = 1.0$.)……...……...…..49

Fig.3.2.1 Schematic view of the flow……..……..…...51

Fig.3.2.2 Velocity (u) versus distance (y) for different values of time (t)…..……...57

Fig.3.2.3 Velocity (u) versus distance (y) for different values of time (t)……....57

Fig.3.2.4 Velocity (u) versus distance (y) for different values of time (t)……..…..58

Fig.3.2.5 Velocity (u) versus distance (y) for different values of time (t)……..…..58

Fig.3.2.6 Velocity (u) versus distance (y) for different values of M, K and S..……60

Fig.3.2.7 Velocity (u) versus distance (y) for different values of S and t ……...…61

Fig.3.2.8 Velocity (u) versus distance (y) for different values of S and t ……...…..61

Fig.3.2.9 Skin friction (τ) versus time (t) for different values of S and K ……...….63

Fig.3.2.10 Skin friction (τ) versus time (t) for different values S, K and M ……..63

Fig.4.1.1 Physical configuration of flow……..…....66

Fig.4.1.2 Velocity (u) versus distance (y) for different time,
($M = 0.5$ and $K = 0.0$)……..…..73
Fig.4.1.3 Velocity (u) versus distance (y) for different time,
($M = 0.5$ and $K = 1.0$)…….73
Fig.4.1.4 Velocity (u) versus distance (y) for different time,
($M = 0.5$ and $K = 0.0$ or $K = 1.0$)…….……...76
Fig. 4.1.5. Velocity (u) versus distance (y) for different time,
($M = 0.5$ and $K = 0.0$))……..…….76
Fig.4.1.6 Velocity (u) versus distance (y) for different time,
($M = 0.5$ and $K = 1.0$)……..78
Fig.4.1.7 Velocity (u) versus distance (y) for different M,
(t =0.2 & $K = 0.0$ and $K = 1.0$)…….79
Fig.4.1.8 Velocity (u) versus distance (y) for different time,
($M = 0.5$ and $K = 0.0$ or $K = 1.0$)……..…..80
Fig.4.1.9 Velocity (u) versus distance (y) for different M and t ,
($K = 0.0$ or $K = 1.0$)…….81
Fig.4.1.10 Skin-friction versus time t for different M ,
($K = 0.0$ or $K = 1.0$ & $y = 0$)…….82
Fig.4.1.11 Skin-friction (τ_0) versus time t for different M ,
($K = 0.0$ or $K = 1.0$ & $y = 0$)…….83
Fig.4.1.12 Skin-friction versus time t for different M ,
($K = 0.0$ or $K = 1.0$ & $y = 1$)…….84
Fig.4.1.13 Skin-friction versus time t for different M ,
($K = 0.0$ or $K = 1.0$ & $y = 1$)……..85
Fig.4.2.1 Flow configuration.…….…….....86
Fig.4.2.2 Velocity for different values of S ($M = 0.5$ and $K = 0.0$ or $K = 1.0$) …….96
Fig.4.2.3 Variation of velocity for different values of t ,
($S = 5.0$, $M = 0.5$ and $K = 0.0$ or $K = 1.0$)……..…….97
Fig.4.2.4 Velocity for different time ($S = -5.0$, $M = 0.5$ and $K = 0.0$ or $K = 1.0$)….97
Fig.4.2.5 Velocity for different S and t ($M = 0.5$ and $K = 0.0$ or $K = 1.0$)……..…...98

Fig.4.2.6 Velocity for different S and t ($M = 0.5$ and $K = 0.0$)……99

Fig.4.2.7 Velocity for different S and t ($M = 0.5$ and $K = 1.0$)……..100

Fig.4.2.8 Skin-friction for different S ($M = 0.5$, $K = 0.0$ or $K = 1.0$ at $y = 0$)……101

Fig.4.2.9 Skin-friction for different S ($M = 0.5$, $K = 0.0$ or $K = 1.0$ at $y = 1.0$) 102

Fig.4.2.10 Skin-friction for different S ($M = 0.5$, $K = 0.0$ or $K = 1.0$ at $y = 0$)……104

Fig.4.2.11 Skin-friction for different S ($M = 0.5$ $K = 0.0$ or $K = 1.0$ at $y = 1.0$)……104

Fig.5.1.1 Physical configuration of flow……..107

Fig.5.1.2a Transient velocity (u_t) versus distance (y) for different t

(M =2.0, Re = 5.0) ……115

Fig.5.1.2b Variation of transient velocity u_t versus distance (y) for different t,

(M =2.0, Re =-5.0)……116

Fig.5.1.3a Velocity u versus distance y for different t (M =2.0, Re =-5.0).....117

Fig.5.1.3b Transient velocity (u_t) versus distance y for different Re

(t =0.05 and M = 2.0) 118

Fig.5.1.4a Transient velocity (u_t) versus distance y for different M,

(t =0.05 and Re =5.0)……..…….119

Fig.5.1.4b Transient velocity u_t versus distance y for different M,

(t =0.05, Re =-5.0)…….....120

Fig.5.1.5a Steady state temperature (θ) versus distance (y) for different Re,

(M =2.0, Pr =0.71, Ec =0.0)……..121

Fig.5.1.5b Steady state temperature (θ) versus distance (y) for different Re,

(M =2.0, Pr =7.0, Ec =0.0)……..121

Fig.5.1.6a Steady state temperature (θ) versus distance (y) for different Re,

(M =2.0, Pr =7.0 (water) and Ec =2.0)……122

Fig. 5.1.6b Steady state temperature (θ) versus distance (y) for different Re,

(M =2.0, Pr =0.7(air), Ec =2.0)……123

Fig.5.1.7a Steady state temperature (θ) versus distance (y) for different Pr,

(M =2.0, Re=2.0, Ec=0.0)……......124

Fig.5.1.7b Steady state temperature (θ) versus distance (y) for different Pr ,

(M =2.0, Re=-2.0, Ec=0.0)……....124

Fig.5.1.8aSteady state temperature (θ) versus distance (y) for different Pr ,

(M =2.0, Re=-2.0, Ec=2.0)……125

Fig.5.1.8b Steady state temperature (θ) versus distance (y) for different Pr ,

(M =2.0 Re = 2.0 , Ec=2.0)……....126

Fig.5.1.9a Skin friction (τ_0) versus (Re) for different M at y =0(t =0.05)…....127

Fig.5.1.9b Skin friction (τ_1) versus (Re)different M at y =1 (t=0.05)……...127

Fig.5.1.10a Skin friction (τ_0) versus (M) for different Re at y =0 (t=0.05)….....128

Fig.5.1.10b Skin friction (τ_1) versus(M) for different Re t =0.05, at y =1……...129

Fig.5.1.11a Nusselt number(Nu_0) versus (Re) for different Pr at $y = 0$

(M =2.0 , Ec=2.0)……130

Fig.5.1.11b Nusselt number(Nu_1)versus (Re) for different Pr at y=1

(M =2.0, Ec=2.0)……130

Fig.5.1.12a Nusselt number(Nu_0) versus Re for different M at $y = 0$

(Pr=0.7, Ec=0.2)……131

Fig.5.1.12b Nusselt number(Nu_1) versus Re for different M at $y = 1$

(Pr=0.7, Ec=2.0)……132

Fig.5.1.13a Steady state temperature (θ) versus distance (y) for different Ec

(M =2.0, Pr=0.71, Re=-5.0)……133

Fig.5.1.13b Steady state temperature (θ) versus distance (y) for different Ec

(M =2.0, Pr=0.71, Re=5.0)……...133

Fig.5.1.14a Steady state temperature (θ) versus distance (y) for different Ec ,

(M =2.0, Pr=7.0, Re=-1.0)……134

Fig.5.1.14b Steady state temperature (θ) versus distance (y) for different Ec ,

(M =2.0, Pr=7.0 Re=1.0)……135

Fig.5.2.1 Physical configuration of flow…….……..……..137

Fig.5.2.2a Velocity (u) versus distance (y) for different t,
(Re=-5.0 M =2.0 and R=1.0)……146
Fig.5.2.2b Velocity (u) versus distance (y) for different t ,
(Re= -5.0 M =2.0 and R=-1.0)……..146
Fig.5.2.3a Transient velocity u_t versus distance y for different M ,
(t =0.05, Re =-5.0, R=1.0)…….....147
Fig.5.2.3b Transient velocity (u_t) versus distance (y) for different M ,
(t =0.05 and Re =-5.0, R=-1.0)…….……147
Fig.5.2.4a Transient velocity (u_t) versus distance (y) for different M
(t =0.05 Re =5.0, R=1.0)…….…….148
Fig.5.2.4b Transient velocity (u_t) versus distance (y) for different M
(t =0.05 and Re =5.0, R=-1.0)……..……149
Fig.5.2.5a Velocity (u) versus distance (y) for different M
(Re= -5.0, t=0.05, R=1.0)…….150
Fig.5.2.5b Velocity (u) versus distance (y) for different M
(Re= -5.0, t=0.05 and R=-1.0)……..150
Fig.5.2.6a Velocity (u) versus distance (y) for different M (Re= 5.0, t=0.05,
R=1.0)……...151
Fig.5.2.6b Velocity (u) versus distance (y) for different M
Re= 5.0, t=0.05, R= -1.0)…….152
Fig.5.2.7a Transient velocity (u_t) versus distance (y) for different Re
(t =0.05 and M =2.0, R=1.0)…….....153
Fig.5.2.7b Transient velocity (u_t) versus distance (y) for different Re
(t =0.05, M =2.0, R=-1.0)……153
Fig.5.2.8a Velocity (u) versus distance (y) for different Re
(t =0.05 and M =2.0 R=1.0)……154
Fig.5.2.8b Velocity (u) versus distance y for different Re
(t =0.05, M =2.0 R=-1.0)……154

Fig.5.2.9a Skin friction (τ_0) versus (Re) for different M at y =0
(t =0.05, R=1.0)……....155
Fig.5.2.9b Skin friction (τ_0) versus (Re) for different M at y =0
(t =0.05,R=-1.0)……156
Fig.5.2.10a Skin friction (τ_1) versus Re for different M at y =1
(t =0.05, R=1.0)……..157
Fig.5.2.10b Skin friction (τ_1) versus (Re) for different M at y =1
(t =0.05, R =-1.0)……..157
Fig.5.2.11a Steady state Temperature (θ) versus distance (y) for different M
(Ec=2.0, Pr =0.7, Re=-5.0, R=1.0)……...158
Fig.5.2.11b Steady state Temperature (θ) versus distance (y) for different M
(Ec=2.0, Pr =0.7 and Re=-5.0, R=-1.0)……159
Fig.5.2.12a Steady state Temperature (θ) versus distance (y) for different M
(Ec=2.0, Pr =7.0, Re=-5.0 , R=1.0)……..160
Fig.5.2.12b Steady state temperature (θ) versus distance (y) for different M
(Ec=2.0, Pr =7.0 Re=-5.0, R=-1.0)……...160
Fig.5.2.13a Steady state Temperature (θ) versus distance (y) for different M
(Ec=2.0, Pr =0.7, Re=5.0, R=1.0)……..161
Fig.5.2.13b Steady state Temperature (θ) versus distance (y) for different M
(Ec=2.0, Pr =0.7, Re=5.0, R=-1.0)…….162
Fig.5.2.14a Steady state temperature (θ) versus distance (y) for different M
(Ec=2.0, Pr =7.0 , Re=5.0, R=1.0)…….162
Fig.5.2.14b Steady state temperature (θ) versus distance (y) for different M
(Ec=2.0, Pr =7.0 , Re=5.0 , R=-1.0)…….....163
Fig.5.2.15a Steady state temperature (θ) versus distance (y) for different Re
(Ec=2.0, Pr =0.7, M=2.0,R=1.0)……164
Fig.5.2.15b Steady state temperature (θ) versus distance (y) for different Re
(Ec=2.0, Pr =0.7, M=2.0, R=-1.0)…….....164
Fig.5.2.16a Steady state temperature (θ) versus distance (y) for different Re
(Ec=2.0, Pr =7.0, M=2.0, R=1.0)……..165
Fig.5.2.16b Steady state temperature (θ) versus distance (y) for different Re
(Ec=2.0, Pr =7.0, M=2.0, R=-1.0)…….166

Fig.5.2.17a Steady state temperature (θ) versus distance (y) for different Pr
(Re=-5.0, Ec=2.0, M=2.0, R=1.0)……167
Fig.5.2.17b Steay state temperature (θ) versus distance (y) for different Pr
(Re=-5.0, Ec=2.0, M=2. R=-1.0)……..167
Fig.5.2.18a Steady state temperature (θ) versus distance y for different Pr
(Re=1.0, Ec=2.0, M=2.0 , R=1.0)…….168
Fig.5.2.18b Steady state temperature (θ) versus distance y for different Pr,
(Re =1.0, Ec=2.0, M=2.0 , R=-1.0)……....169
Fig.5.2.19a Steady state temperature (θ) versus distance (y) for different Ec
(Re=-5.0, Pr=0.7, M=2.0 , R=1.0)……170
Fig.5.2.19b Steady state temperature (θ) versus distance (y) for different Ec
(Re=-5.0, Pr=0.7, M=2.0 , R=-1.0)……170
Fig.5.2.20a Steady state temperature (θ) versus distance (y) for different Ec
(Re=5.0, Pr=0.7, M=2.0 , R=1.0)…….171
Fig.5.2.20b Steady state temperature versus distance y for different Ec
(Re=5.0, Pr=0.7, M=2.0 , R=-1.0)…….172
Fig.5.2.21a Steady state temperature (θ) versus distance y for different Ec
(Re=5.0, Pr=7.0, M=2.0 , R=1.0)…….173
Fig.5.2.21bSteady state temperature (θ) versus distance (y) for different Ec
(Re=5.0, Pr=7.0, M=2.0 , R=-1.0)…….....173
Fig.5.2.22a Steady state temperature (θ) versus distance (y) for different Ec
(Re=-5.0, Pr=7.0, M=2.0 , R=1.0)……..174
Fig.5.2.22b Steady state temperature (θ) versus distance (y) for different Ec
(Re=-1.0, Pr=7.0, M=2.0 , R=-1.0)…….....174
Fig.5.2.23a Nusselt number(Nu_0)versus (M)for different Ec at $y = 0$
(Re=-5.0, Pr=0.7, R=1.0)…….....175
Fig.5.2.23b Nusselt number ($Nu_{0)}$ versus M for different Ec
(Re=-5.0, Pr=0.7, R=-1.0 at $y = 0$) …….....175
Fig.5.2.24a Nusselt number (Nu_0)versus (M)for different Ec at $y = 0$
(Re=5.0, Pr=0.7 , R=1.0)……..176
Fig.5.2.24b Nusselt number ($Nu_{0)}$ versus M for different Ec at $y = 0$

(Re=5.0, Pr=0.7, R=-1.0)……176

Fig.5.2.25a Nusselt number(Nu_1)versus M for different Ec at $y = 1$

(Re=-5.0, Pr=0.7,R=1.0)……177

Fig.5.2.25b Nusselt number(Nu_1) versus M for different Ec at $y = 1$

(Re=-5.0, Pr=0.7 R=-1.0)……177

Fig.5.2.26a Nusselt number(Nu_1) versus M for different Ec at $y = 1$

(Re=5.0, Pr =0.7 R=1.0)……....178

Fig.5.2.26b Nusselt number (Nu_1) versus M for different Ec at $y = 1$

(Re=5.0, Pr =0.7 and R=-1.0)……178

Fig.6.1.1 Physical configuration of the flow……..182

Fig.6.2 .1 Physical model…….....189

LIST OF APPENDICES

Appendix I The functional, used to define velocity and skin-friction in Chapters three and Four…….209

Appendix II Constants used to define velocity, skin friction, temperature and Nusselt number in Chapter five, problem 5.1…….210

Appendix III Constants used to define temperature, velocity, Nusselt number and skin friction in Chapter five, problem 5.2……211

Appendix VI FORTRAN and MATLAB programme used to generate the numerical values for the analytical solutions ……212

CHAPTER ONE

PRELIMINARIES AND BASIC EQUATIONS

1.0 Introduction

This chapter deals with the basic definitions and derivations of the governing equations which will provide the background for the succeeding chapters.

The study of the motion of electrically conducting fluids such as plasma and liquid metals under the influence of external magnetic field, known as Magnetohydrodynamics (MHD), is receiving considerable attention of many researchers. This is due to its numerous practical applications in science and Engineering such as in the design of nuclear reactors, MHD power generators, MHD pumps and accelerators, aerodynamic heating, electrostatic precipitation, polymer technology, petroleum industry, purification of crude oil and fluid droplets and sprays, Astrophysical and Geophysical studies, etc

Magnetohydrodynamics (MHD) is the physical/mathematical framework that concerns the dynamics of magnetic fields in conducting fluids. The word magnetohydrodynamics is comprised of the words magneto- meaning magnetic, hydro- meaning water (or liquid) and dynamics referring to the movement of an object by forces.

The motion of an electrically conducting fluid, like mercury, under the influence of magnetic field in general gives rise to induced electric current on which mechanical forces are exerted by the magnetic field. On the other hand the induced electric currents also produces induced magnetic and thus the original magnetic field are also changed. Thus there is a two-way interaction between the flow field and the magnetic

field, the magnetic field exerts force on the fluid by producing induced currents and the induced currents changed the original fluid. The study of Hydromagnetics goes back to Michael Faraday who predicted induced currents in the ocean due to the Earth's magnetic field. Hartmann (1937) was the first to discuss both experimentally and theoretically the hydromagnetic flow between two parallel plates. But real boost was given by Alfven in 1942, when he established the transverse waves in electrically conducting fluids and explain many astrophysical phenomena with it. Since then the literature on MHD has increased many folds.

The study of flow formation in the presence of mass leakage has been a subject of many theoretical and expiremental studies owing to its varied applications in industry as well as in nature. For example, permeable porous plates are used in filtration processes in industries and also for a heated body to keep its temperature constant and to make the heat insulation processes more effective. Furthermore, the problem of boundary layer growth on a flat plate has important application in the field of aerodynamics as the application of suction is widely used in order to prevent separation and to delay transition to turbulence by means of which we get much larger maximum lift and reduced drag. On the other hand injection can be used at the wall to cool the body in a high temperature drag.

1.1 THE MAGNETOHYDRODYNAMIC BASIC EQUATIONS

The set of equations which describe the unsteady MHD flow of an incompressible, viscous, electrically conducting fluid are the mass conservation equation (continuity equation), Momentum equation, the Energy equation, the Maxwell's equations and the Ohm's law respectively:

1.1.1 Equation of Continuity

The continuity equation also called the mass conservation equation is derived based on the laws of conservation of mass which presupposes that matter can neither be created nor destroyed but conserved.

The equation can be derived by considering an arbitrary closed surface S enclosing a volume V of the fluid. Suppose that the fluid is uniform, i.e. contains the same particles uniformly. Let ρ be the density of the fluid in an elementary volume and $\vec{q}$ be the velocity of the fluid at any point P. The equation is derived under the assumption that i) the fluid is continuous ii) mass is conserved.

The vol. of the fluid which passes through the surface is equal to the total mass of fluid coming out of the surface of the volume. Thus

$$\frac{d}{dt}\int_V \rho dV = -\int \rho \vec{q} n ds \qquad (1.1.1.1)$$

where n is the unit normal vector to the surface element ds. According to the Gauss divergence theorem the integral on the right can be replaced by the volume integral $\int \nabla.(\rho \vec{q}) dV$

Thus since the control volume is fixed in space, the ordinary derivative can be brought inside of the integral and changed into partial derivative, and allowing both sides of the equation to be consolidated within the integral gives ,

$$\int_V (\frac{\partial \rho}{\partial t} + \nabla.(\rho \vec{q}) dV) = 0 . \qquad (1.1.1.2)$$

This reduces to $\frac{\partial \rho}{\partial t} + \nabla.(\rho \vec{q}) = 0$, (1.1.1.3)

the so-called Euler's continuity equation.

If the fluid is incompressible, i.e. the density is constant, then $\frac{\partial \rho}{\partial t} = 0$ and the equation becomes

$$\nabla.\vec{q} = 0 . \quad (1.1.1.4)$$

1.1.2 Euler's Momentum equation

The Euler's Momentum equation(Equation of motion) is based on the assumption that the fluid is incompressible and non-viscous and is derived from the Newton's second law of motion which states that the rate change of linear momentum is equal to the total forces acting on the fluid. In this case the forces acting on the fluid are two types. Body(external) forces such as forces due to gravity, magnetic forces etc and Surface forces including both normal forces due to pressure and tangential forces due to viscous shear which are absent in non-viscous fluids.

Considering an arbitrary closed surface S in the region (of volume V) occupied by an incompressible and non-viscous fluid of density ρ moving with velocity $\bar{q}$. If $\overline{F}$ and P are respectively the body and normal forces (due to pressure), then the Newton's second law of motion gives

$$\frac{d}{dt}\int_V \rho \vec{q} dV = \int_V \vec{F} \rho dV - \int pn\, ds, \quad (1.1.2.1)$$

where the integral on the left represents the rate of change of linear momentum, the first on the right, the total external forces acting on the body of the fluid and the total

normal forces due to pressure respectively. Furthermore the second integral on the right can be replaced by the volume integral $-\int_V \nabla p \, dV$

,

Hence equation 1.1.2.1 becomes

$$\frac{d}{dt}(\rho\vec{q}) = \vec{F}\rho - \nabla p \ . \qquad (1.1.2.2)$$

Again since the fluid is incompressible (ρ is constant) and the derivative $\frac{d}{dt}(\rho\vec{q}) = \rho(\frac{\partial\vec{q}}{\partial t} + (\vec{q}\cdot\nabla)\vec{q})$ the Euler's equation becomes

$$\rho(\frac{\partial\vec{q}}{\partial t} + (\vec{q}\cdot\nabla)\vec{q}) = \vec{F}\rho - \nabla p \ . \qquad (1.1.2.3)$$

1.1.3 Navier-Stokes Equations

In fluid dynamics the Navier–Stokes equations, named after Claude-Louis Navier and George Gabriel Stokes, are a set of equations describing the motion of viscous fluids. These equations arise from applying Newton's second law to fluid motion, together with the assumption that the fluid stress is the sum of a diffusingviscous term (proportional to the gradient of velocity), plus a pressure term.

The equations are useful because they describe the physics of many things of academic and economic interest. They may be used to model the weather, ocean currents, water flow in a pipe, air flow around a wing , and motion of stars inside a galaxy. The Navier–Stokes equations in their full and simplified forms help with the design of aircraft and cars, the study of blood flow, the design of power stations, the

analysis of pollution, and many other things. Coupled with Maxwell's equations they can be used to model and study magneto hydrodynamics.

The Navier-stokes (Momentum) equation for Non-Newtonian fluids in vector form are expressed as

$$\rho \frac{\partial \bar{q}}{\partial t} + \rho(\vec{q}\nabla).\vec{q} = \rho\vec{F} - \nabla P + \mu \nabla^2 \vec{q} \qquad (1.1.3.1)$$

where $\vec{F} = (\bar{J} \times \bar{B})$, for electrically conducting fluids in addition to other body forces acting on the fluid.

If the fluid is non-viscous or in viscid and $\mu = 0$, the above equation reduces to the Euler's momentum equation

$$\rho \frac{\partial \vec{q}}{\partial t} + \rho(\vec{q}\nabla).\vec{q} = \rho\vec{F} - \nabla P \qquad (1.1.3.2)$$

1.1.4 Maxwell's Equation

The Maxwell's Equations describe the behavior of electric field, $\vec{E}$ and Magnetic field $\vec{B}$ .The detail explanation of these equations can be obtained from Sherman and Sutton (1965). However the simplified versions of the Maxwell's equations are as follows:

Ampere's circuital law $\nabla \times \vec{B} = \mu_0 \bar{J}$ (1.1.4.1)

Faraday's law $-\frac{\partial \vec{B}}{\partial t} = \nabla \times \bar{E}$ (1.1.4.2)

Solenoid Law $\nabla \bullet \bar{B} = 0$ (1.1.4.3)

Ohm's law $$\bar{J} = \sigma(E + \vec{q} \times B) \qquad (1.1.4.4)$$

Lorentz force $$\vec{F} = \bar{J} \times \bar{B} \qquad (1.1.4.5)$$

It should be noted that the total magnetic field $\vec{B}$ as $\vec{B} = \mu_0 \vec{H}$. In this work the polarization effects are negligible ($\bar{E} = 0$) and the magnetic Reynolds number is very small i.e induced magnetic field is also negligible.

1.1.5 Energy Equation

The Equation of Energy (Temperature) is derived based on the assumption that the energy of the fluid is conserved during its motion. The energy equation is represented by

$$\rho C_\rho \left(\frac{\partial T}{\partial t} + (\bar{V}\nabla) \cdot T \right) = \mu(\nabla \bar{V})^2 + k\nabla^2 T + Q_T \qquad (1.1.5.1)$$

If the flow is one-dimensional the Energy equation reduces to

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{\nu}{C_p} \left(\frac{\partial u}{\partial y} \right)^2 \qquad (1.1.5.2)$$

1.1.6 Cartesian coordinates

Many times in discussing any particular flow we may find that one particular coordinate system more useful than the other. Therefore for future reference, the above equations are given in Cartesian form. The Cartesian equations seem to follow directly from the vector equations above.

In Cartesian coordinates, the mass conservation equation (1.1.1.3) becomes

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0 \qquad (1.1.6.1)$$

When the flow is at steady-state, ρ does not change with respect to time. The continuity equation is reduced to:

$$\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0 \qquad (1.1.6.2)$$

When the flow is incompressible, ρ is constant and does not change with respect to space. The continuity equation is reduced to:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0\,. \qquad (1.1.6.3)$$

The Momentum (Navier-Stokes) equations for Newtonian fluids (1.1.3.1) reduce to:

$$\text{In the } x\text{-direction } \rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=\rho F_x-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right) \qquad (1.1.6.4)$$

$$\text{In the } y\text{-direction } \rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=\rho F_y-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right) \qquad (1.1.6.5)$$

$$\text{In the } z\text{-direction } \rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=\rho F_z-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right) \qquad (1.1.6.6)$$

Note that gravity has been accounted for as a body force, and the values of F_x, F_y, F_z will depend on the orientation of gravity with respect to the chosen set of coordinates.

Again the one- dimensional energy (1.1.5.1) equation reduces to:

$$\frac{\partial T}{\partial t} = \frac{k}{\mu C_p}\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{C_p}(\frac{\partial u}{\partial y})^2 \qquad (1.1.6.7)$$

In this case $\overline{H}, \overline{J}, \overline{E}, \overline{B}, \overline{V}, T, k, C_p, \sigma, \rho, \beta, \mu$ are the magnetic field, current density, electric field, magnetic flux, the velocity, the temperature, thermal conductivity, specific heat capacity, the electrical conductivity, the density of the fluid and the viscosity of the fluid respectively.

1.2 OVERVIEW OF BASIC CONCEPTS

In this section the definition of some of the concepts used in the thesis is given.

1.2.1 MHD Couette flow

The Couette flow refers to the laminar flow of a viscous fluid in a parallel wall channel, one wall is stationary and the other is moving. The flow is driven by virtue of viscous drag force acting on the fluid and the applied pressure gradient parallel to the plates. This type of flow is named in honor of Maurice Marie Alfred Couette, a Professor of Physics at the French university of Angers in the late 19th century. The MHD Couette flow refers to the Couette flow of a conducting fluid in the presence of a magnetic field applied normal to the two parallel walls.

1.2.2 Lorentz Force

The Farady's law says that if a current $\vec{J}$ is passing through a conductor under a magnetic flux $\vec{B}$, then the conductor experiences a force perpendicular to both of them and is proportional to their magnitude. Thus we can write the force F acting on the conductor as $F = \vec{J} \times \vec{B}$ .This force is called the Lorentz force and is a body force acting on the fluid and it has significant influence on the velocity of the flow.

1.2.3 Boundary Layer

A boundary layer is the layer of fluid in the immediate vicinity of a bounding surface where the effects of viscosity are significant. In the Earth's atmosphere, the planetary boundary layer is the air layer near the ground affected by diurnal heat, moisture or momentum transfer to or from the surface. On an aircraftwing the boundary layer is the part of the flow close to the wing, where viscousforces distort the surrounding non-viscous flow.

The aerodynamic boundary layer was first defined by Ludwig Prandtl. This definition simplifies the equations of fluid flow by dividing the flow field into two areas: one inside the boundary layer, dominated by viscosity and creating the majority of drag experienced by the boundary body; and one outside the boundary layer, where viscosity can be neglected without significant effects on the solution. This allows a closed-form solution for the flow in both areas, a significant simplification of the full Navier–Stokes equations. The majority of the heat transfer to and from a body also takes place within the boundary layer, again allowing the equations to be simplified in the flow field outside the boundary layer.

1.2.4 Compressible and Incompressible fluids

Compressible fluids are Fluids in which changes in pressure, temperature and/or other physical properties will result in changes in density. However, in many situations the changes in pressure and temperature are sufficiently small that the changes in density are negligible. In this case the flow can be modeled as an incompressible flow.

Mathematically, incompressibility is expressed by saying that the density ρ of a fluid parcel does not change as it moves in the flow field, i.e.,

$$\frac{D\rho}{Dt} = 0$$

Where $\frac{D}{Dt}$ is the substantial derivative, which is the sum of local and convective derivatives.

1.2.5 Viscous and non-viscous (Inviscid) fluids

Viscosity is a measure of the resistance of a fluid which is being deformed by either shear stress or tensile stress. It describes a fluid's internal resistance to flow and may be thought of as a measure of fluid friction. All real fluids (except super fluids) have some resistance to stress and therefore are viscous, but a fluid which has no resistance to shear stress is known as an ideal fluid or inviscid fluid. The Reynolds number can be used to evaluate whether viscous or in viscid equations are appropriate to the problem.

Stokes flow is flow at very low Reynolds numbers, such that inertial forces can be neglected compared to viscous forces.

On the contrary, high Reynolds numbers indicate that the inertial forces are more significant than the viscous (friction) forces. Therefore, we may assume the flow to be an inviscid flow, an approximation in which we neglect viscosity at all, compared to inertial terms.

1.2.6 Suction/Injection

Suction and injection is the process of removing or introducing fluid in to the body. It serves as a very active but simple flow control system as they significantly affect the flow field. They affect particularly the shape of the velocity profile near the plate and change the boundary layer. The influence of suction on rate of heat transfer and skin friction is opposite that of injection. The equation of motion and energy equation considering the effect of constant suction/injection on the horizontal component of velocity are respectively $\frac{\partial u}{\partial t} = v_0 \frac{\partial u}{\partial y} + \nu \frac{\partial^2 u}{\partial y^2}$ (1.2.6.1)

$$\frac{\partial T}{\partial t} = v_0 \frac{\partial T}{\partial y} + \frac{k}{\rho c_p} \frac{\partial^2 u}{\partial y^2}, \quad (1.2.6.2)$$

In this case v_0 - is the suction/injection velocity.

1.2.7 Viscous Dissipation:

Viscous dissipation also called sheared heating is the transformation of kinetic energy into internal energy (heating up the fluid) due to viscosity. It is the work done by the velocity against the viscous stresses. In other words it is an irreversible process where Mechanical energy is turned into thermal energy. This energy transformed into heat will raise the temperature of the fluid. It can be important in flows of polymers and

oils. Gephardt shows that viscous dissipation heat cannot be neglected for fluids with high Reynolds number or low gravitational field.

1.2.8 Ohmic (Joule) Dissipation.

Ohmic dissipation (also known as Joule heating) is the Loss of electrical energy when current flows through a resistance due to the conversion of electricity in to heat. It was first studied by James Prescott Joule in 1841. Joule heating is caused by interactions between the moving particles that form the current (usually, but not always, electrons) and the atomicionsthat make up the body of the conductor. Charged particles in an electric circuit are accelerated by an electric field but give up some of their kinetic energy each time they collide with an ion. The increase in the kinetic or vibration energy of the ions manifests itself as heat and a rise in the temperature of the conductor. Hence energy is transferred from the electrical power supply to the conductor and any materials with which it is in thermal contact. Joule heating is referred to as ohmic heating or resistive heating because of its relationship to Ohm's Law.

1.2.9 Skin friction

Skin friction is a component of drag, the force resisting the motion of a solid body through a fluid. It arises from the friction of the fluid against the skin of the object that is moving through it. Skin friction arises from the interaction between the fluid and the skin of the body, and is directly related to the area of the surface of the body that is in contact with the fluid. Skin friction follows the drag equation and rises with the square of the velocity. There are two ways to decrease skin friction: the first is to shape the moving body so that smooth flow is possible, like an airfoil.

1.2.10 Two-phase flow

The term two-phase flow applies to the motion of a mixture of different fluids having different phases, such as air and water, or oil and natural gas occurs in a system containing gas and liquid with a meniscus separating the two phases. Two-phase flow is a particular example of multiphase flow. Nuclear reactors use water to remove heat from the reactor core using two-phase flow. Two-phase flow is also applied in climate systems such as clouds, and in groundwater flow, in which the movement of water and air through the soil is studied.

1.2.11 Newtonian and Non-Newtonian Fluids.

A Newtonian fluid (named for Isaac Newton) is a fluid whose stress versus rate of strain curve is linear and passes through the origin. The constant of proportionality is known as the viscosity.

The shear stress exerted by the Newtonian fluid is described by the equation.

$\tau = \mu \frac{du}{dy}$, where τ the shear stress is exerted by the fluid, μ is the fluid viscosity and $\frac{du}{dy}$ is the velocity gradient perpendicular to the direction of shear. Thus the viscosity, for a Newtonian fluid, depends only on temperature and pressure (and also the chemical composition of the fluid if the fluid is not a pure substance), not on the inertial forces acting upon it. Example of Newtonian fluids includes water. Non-Newtonian fluids, on the other hand, do not obey this relation. They are fluids whose flow behavior cannot be described by a single constant of viscosity. These types of fluids include polymer solutions, molten polymers, many solid suspensions and most highly viscous fluids.

1.3 Mathematical Methods

1.3.0 Introduction

In this section the outline of the Mathematical methods use in solving the resulting equations in this work is given. These methods include: The Laplace transformation method, Fourier series method, etc.

1.3.1 Laplace Transform

The Laplace transform, named after the French Mathematician, Pierre Simon Marquis de Laplace (1749-1827), is an easy and effective means of solving many mathematical problems arising from science and engineering. This integral transform reduces a differential equation into an algebraic equation which is easier to solve.

1.3.1.1 Definition

Let $f(t)$ be defined and single valued in the interval $0 \le t < \infty$ .Then the Laplace transform of $f(t)$ with respect to t, denoted by $L\{f(t)\} = \hat{F}(s)$, where $\hat{F}(s) = \int_0^\infty f(t)e^{-st}dt = \quad Lt \;\; M \to \infty \quad \int_0^M f(t)e^{-st}dt$, $s > 0$ s is a parameter which may be real or complex. The Laplace transform exist if the above integral converges, otherwise it does not exist.

1.3.1.2 Exponential order

Suppose $f(t)$ is piecewise continuous function, then the function $f(t)$ is said to be of exponential order if there exist constants α , $M > 0$, and $T > 0$ such that $|f(t)| \le Me^{\alpha t}, \forall t > T$

1.3.1.3 Existence of Laplace transform

Let $f(t)$ be a function piecewise continuous on the interval $[0,\infty)$ and of exponential order. Then, the Laplace transform $L\{f(t)\} = \int_0^\infty f(t)e^{-st}dt$ exist for $s > \alpha$, that is

$$\{s > \alpha\} \subset Domain(L(f)) \qquad (1.3.1.3)$$

Proof:

By the additive interval property of definite integrals

$$L\{f(t)\} = \int_0^\infty f(t)e^{-st}dt = \int_0^T f(t)e^{-st}dt + \int_T^\infty f(t)e^{-st}dt$$

The first integral on the right exists because it can be written as a sum of integrals over intervals on which $f(t)e^{-st}$ is continuous. Now $f(t)$ is of exponential order , so there exist constant α $M > 0$, and $T > 0$, so that $|f(t)| \le Me^{\alpha t}, \forall t > T$,the second integral on the right can be written as

$$\int_T^\infty f(t)e^{-st}dt \le \int_T^\infty \left|f(t)e^{-st}\right|dt \le M\int_T^\infty e^{-st}e^{\alpha t}dt = M\int_T^\infty e^{-(s-\alpha)t}dt = M\frac{e^{-(s-\alpha)T}}{s-\alpha},$$

for $s > \alpha$. Since the integral $M\int_T^\infty e^{-(s-\alpha)t}dt$ converges, the integral $\int_T^\infty \left|f(t)e^{-st}\right|dt$, converges by the comparison test for improper integrals. This in turn implies that the second integral exists for $s > \alpha$. The existence of the first and second integral implies that $L\{f(t)\} = \int_0^\infty f(t)e^{-st}dt$ exists for $s > \alpha$.

1.3.1.4 Linearity of the Laplace transforms

If $f(t)$ and $g(t)$ are any two functions whose Laplace transform exist, then

$$L\{c_1 f(t) + c_2 g(t)\} = c_1 L\{f(t)\} + c_2\{Lg(t)\} \qquad (1.3.1.4)$$

where c_1 *and* c_2 are constants.

Proof: $L\{c_1 f(t) + c_2 g(t)\} = \int_0^\infty \{c_1 f(t) + c_2 g(t)\} e^{-st} dt$

$=$

$$c_1 \int_0^\infty f(t) e^{-st} dt + c_2 \int_0^\infty g(t) e^{-st} dt = c_1 L\{f(t)\} + c_2 \{Lg(t)\}$$

1.3.1.5 Inverse Laplace transform

The inverse Laplace transform is a function whose transform is $\hat{F}(s)$. Thus if $L\{f(t)\} = \hat{F}(s)$, then $f(t) = L^{-1}\{\hat{F}(s)\}$ (1.3.1.5)

1.3.1.6 Linearity of the Inverse Laplace transforms

If $L^{-1}\{c_1 \hat{F}(s) + c_2 \hat{G}(s)\} = c_1 L^{-1}\{\hat{F}(s)\} + c_2 L^{-1}\{\hat{G}(s)\}$ and $\hat{G}(s)$ is $f(t)$ and $g(t)$ is any two functions whose Laplace transform exist, then $L\{c_1 f(t) + c_2 g(t)\} = c_1 L\{f(t)\} + c_2 \{Lg(t)\}$ where c_1 *and* c_2 are constants.

1.3.1.7 Transform of a derivative.

If $f(t), f'(t), f''(t), \ldots\ldots\ldots f^{(n-1)}(t)$ are continuous on $0 \le t < \infty$ and are of exponential order and if $f^{(n)}(t)$ is piecewise continuous on the interval $0 \le t < \infty$ then $L\{f^{(n)}(t)\} = s^n \hat{F}(s) - s^{n-1} f(0) - s^{n-2} f(0) \ldots\ldots - f^{n-1}(0)$ where $L\{f(t)\} = \hat{F}(s)$

(1.3.1.7).

13.2 Fourier Series,

1.3.2.0 Introduction

Fourier series is named in honour of the French Mathematician Joseph Fourier (1768–1830), who made important contributions to the study of trigonometric series. This series decomposes periodic functions or periodic signals into the sum of a (possibly infinite) set of simple oscillating functions, namely Sines and Cosines (or complex exponentials). Fourier introduced the series for the purpose of solving the heat equation in a metal plate.

1.3.2.1 Definition

A function $f(x)$ is said to be periodic with a period T if for all x, $f(x+T)=f(x),\ T$ is a positive constant. The least positive value of T is called the least period of $f(x)$.

1.3.2.2 Definition

Let $f(x)$ be defined in the interval $[-L, L]$ and outside the interval by $f(x+2L)=f(x)$, ie. Assume that $f(x)$ has period 2L. Then the Fourier series expansion of $f(x)$ is given by $f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos\frac{n\pi x}{L}+b_n\sin\frac{n\pi x}{L})$, where the Fourier coefficients a_0, a_n and b_n are given by $a_0=\frac{1}{L}\int_{-L}^{L}f(x)dx$,

$$a_n=\frac{1}{L}\int_{-L}^{L}f(x)\cos\frac{n\pi x}{L}dx,\quad b_n=\frac{1}{L}\int_{-L}^{L}f(x)\sin\frac{n\pi x}{L}dx$$

(1.3.2.2)

1.3.2.3 Half Rang Fourier series

If $f(x)$ is defined in the half interval $[0, L]$, then the half range Fourier Cosine and series representation for $f(x)$ are respectively given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos\frac{n\pi x}{L}), \text{ where } \quad a_0 = \frac{2}{L}\int_0^L f(x)dx, \ \& a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx$$

$$and \quad f(x) = \sum_{n=1}^{\infty}(b_n \sin\frac{n\pi x}{L}), \text{ in this case }, \ b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx \qquad (1.3.2.3)$$

1.3.3 Green's functions

Green's function (GF) is a basic solution to a linear differential equation, a building block that can be used to construct many useful solutions. For heat conduction, the GF is proportional to the temperature caused by a concentrated energy source. The exact form of the GF depends on the differential equation, the body shape, and the type of boundary conditions present. Green's functions are named in honour of English mathematician and physicist George Green (1793-1841).

1.3.3.1 Definition

Let $Lu(x)$ be a given linear differential operator in n variables $x = (x_1, x_2, ...x_n)$ then the Green's function of $Lu(x)$ denoted by $G(x,t)$ is defined by $L_x G(x,t) = \delta(x-t)$, where $\delta(x-t)$ is the dirac delta function.

The Function $G(x,t)$, which is given by $G(x,t) = \begin{cases} G_1(x), when\ x < t \\ G_2(x), when\ x > t \end{cases}$ has the following properties

i) $G_1 \, and \, G_2$ Satisfy the question $L(G) = 0$ in their defined intervals i.e. $L(G_1) = 0$ $when \; x < t$ and

$L(G_2) = 0 \; when \; x > t$

ii) $G_1 \, and \, G_2$ Satisfy the boundary conditions at the end points $x = a \, and \; x = b$ respectively.

iii) $G(x,t)$ is continuous at $x = t$ i.e $G_1(t) = G_2(t)$

vi) The derivative of G is continuous at every point within the range of x except at $x = t$ i.e $G_2^{'}(t) - G_1^{'}(t)$

1.3.3.2 Definition.

If $G(x,t)$ exists as above, then the differential equation $L(\phi(x)) = f(x)$, for a known function $f(x)$, has the solution of the form $\phi(x) = \int G(x,t) f(x) dt$. (1.3.3.2)

1.4 Objectives of the Study

The aim of this work is to investigate and analyse some flow behavior of viscous, incompressible and electrically conducting fluids in a single and multi-phase rectangular channels(with or without porosity). The flow is assumed to be influenced by a transverse Magnetic and electric field. The major objectives of the study are to investigate:

1. Time dependent MHD flow due to the ramped motion of an infinite horizontal plate under the influence of an externally applied magnetic field.

2. Effects of Transpiration (suction/injection) on the MHD Flow near a Porous plate having ramped Motion.

3. Time dependent MHD Couette flow due to ramped motion of one of the plates.

4. Unsteady hydromagnetic Couette flow due to ramped motion of one of the porous plates.

5. The effects of transverse magnetic field on the transient velocity and viscous and Joulean dissipation on the steady state temperature for the Couette flow of an incompressible, viscous and electrically conducting fluid between two infinite porous plates.

6. The effects of viscous and Ohmic dissipation on the transient velocity and steady state temperature for the Couette flow of incompressible, viscous and electrically conducting fluid between two infinite porous plates in the presence of magnetic and applied electric field.

7. The time-dependent (unsteady) two- phase Magnetohydrodynamic (MHD)flow of two immiscible and incompressible fluids in a horizontal parallel plate channel:
    a. One phase is occupied partly by electrically conducting fluid and the other by electrically non-conducting fluid corresponding to zero electric load parameter($E = 0$) (short circuit case)
    b. Both phases are occupied partly by electrically conducting fluids corresponding to non-zero electric load parameter ($E \neq 0$) open circuit case.

1.5 RESEARCH METHODOLOGY

The Methodology adopted in order to realize the above set objectives is as follows. In the first phase the existing literature related to the research work is extensively reviewed to include different physical and boundary conditions. In the second phase each of the physical cases (1) to (7) above is modeled into partial differential equations using the Navier- Stokes equations, the Maxwell equations, the continuity equation and the energy equations (temperature)as the case may be. The dimensional momentum and energy equations together with the boundary conditions are reduced to non-dimensional form by using suitable parameters. The resultant non-dimensional equations were solved analytically using the relevant mathematical methods such as Laplace transformation techniques, Decoupling method, Fourier series method as well as the Green's Functions approach. The operator/undetermined coefficient methods were used to determine the particular solutions. In phase three the FORTRAN programming language and the MATLAB computer package were used to obtain the numerical values from the analytical/ exact solutions. The numerical values so

obtained are graphically displayed using Microsoft EXCEL as well as MATLAB. The graphs obtained are used to interpret and discuss the flow behavior and the influence of the different flow conditions/parameters.

1.6 Organization of the Dissertation

This thesis is divided into seven chapters. Chapter one is an introductory chapter which deals with the basic definitions of terms and concepts frequently used in this work as well as derivations of equations of motion which will provide the background for succeeding chapters. The chapter also contains the methodology employed to achieve the set objectives of the work.

In Chapter two the literature review of the work is given.

In Chapter three the time dependent MHD flow of an incompressible, viscous fluid due to the ramped motion of an infinite horizontal flat plate under the influence of an externally applied transverse magnetic field was examined in the absence or presence of mass suction and injection at the plate.

Chapter four deals with the time dependent MHD Couette flow of an incompressible, viscous and electrically conducting fluid in between two parallel infinite horizontal plates. The motion is induce by the ramped motion of one of the boundaries with or without suction/injection.

The objective of Chapter five is to discuss the combined effects of viscous and Joulean dissipation on MHD Couette flow of an electrically conducting fluid in between two infinite horizontal porous plates with transpiration under the influence of transverse magnetic field in the absence or presence of electric field.

The work of Chapter six is concerned with the mathematical modeling of the time dependent two phase magneto-hydrodynamic (MHD) flow in a parallel plate channel

filled with two immiscible fluids for the cases when one of the fluids is conducting and the other is not or both fluids are electrically conducting. The resulting partial differential equations are solved using the Greens function approach.

Finally Chapter seven gives a summary and concluding remarks and also suggest some future directions on the research work.

CHAPTER TWO

LITERATURE REVIEW

2.0 Introduction

In this chapter an extensive review is carried out, describing the earlier works done on the MHD flow of conducting fluids under the influence of magnetic and electric field in the various areas considered in this work.

2.1 Literature review on MHD flow due to the ramped motion of a horizontal flat plate

The study of the effects of magnetic field on the laminar flow of an incompressible electrically conducting viscous fluid is an important problem that is related to many practical applications in science and Engineering. These applications occur in design of nuclear reactors, MHD power generators, MHD pumps, accelerators, aerodynamic heating, electrostatic precipitation, polymer technology, and petroleum industry, purification of crude oil and fluid droplets and sprays, Astrophysical and Geophysical studies. Furthermore, the use of electrically conducting fluids under the in influence of magnetic fields has gained interest in various industrial applications such as the semiconductor industry and the purification of molten metals from nonmetallic inclusions, as discussed by Seddeek and Salem (2006)

In addition the boundary layer flows of fluids past flat plates have been discussed extensively in the literature due to largely the relative simplicity of their analyses coupled with their utility as idealized models for studies of more complicated fluid body interactions. When the fluid flow takes place near regular boundaries such as the horizontal plate considered here, it can be shown that the governing equations reduce to linear partial differential equations and can be analyzed exactly, in many cases. The

solutions of the resulting equations are still dependent on the type of initial and boundary conditions to be imposed. The flow of an incompressible viscous fluid past an infinite horizontal plate, impulsively started form rest into motion, in its own plane, with a constant velocity, was studied first by Stokes (1851).It is also known as Rayleigh's problem in literature. This problem has also been discussed by several other Authors, with or without the influence of magnetic field in different physical situations.Ong and Nichola s(1959), Hasimoto (1957), White (1991),Turbatu et al., (1998) and Deka et al (1999). Erdogan (2000) studied the unsteady motion of a viscous fluid caused by the sinusoidal oscillation of a plane wall. Of recent Chandran et al (2005) and Dass et al (2006) discussed the mass transfer effects of unsteady flow past a flat plate. However all the above analyses considered the hydrodynamic flow of an incompressible fluid bounded by an infinite plate in the absence of magnetic field. The Rayleigh problem in the presence of a magnetic field was first investigated by Rossow (1958) and (1960). This work by Rossow was carried out under the assumption that the electrically conducting, viscous and incompressible fluid has constant properties throughout the flow field on the infinite flat plate and is subject to a uniform and constant magnetic field in the direction normal to the plate. He also assumed that the induced magnetic field is negligible and no external electric field or pressure gradient is present. Chuang (1971) discussed the effects of transpiration on the MHD flow near an oscillating plate. Kumar et al. (2002) studied the MHD flow and heat transfer on a moving vertical surface/plate with suction. He observed that the velocity decreases in the presence of magnetic field as compared to its absence.Al-odat et al (2004) discussed the Effect of Magnetic field on entropy generation due to laminar forced convection past a horizontal flat plate. Deka (2008) analyzed the effects of Hall currents on MHD flow past an accelerated horizontal flat plate.Of

recent Jat and Chaudhary (2009) studied the MHD boundary layer flow past a porous substrate with Beavers-Joseph boundary condition under the influence of transverse magnetic field. His analysis was however done on the impulsive motion of the plate. Kumaran, et al (2010) studied the transition effect of boundary layer flow past a stretching sheet due to a suddenly imposed magnetic field.

The study of flow formation in the presence of mass leakage is receiving considerable attention of many researchers because of its varied applications in industry as well as in nature. This is because permeable porous plates are used in filtration processes in industries and also for a heated body to keep its temperature constant and to make the heat insulation processes more effective. Furthermore, the problem of boundary layer growth on a flat plate has important application in the field of aerodynamics as the application of suction is widely used in order to prevent separation and to delay transition to turbulence by means of which we get much larger maximum lift and reduced drag. On the other hand injection can be used at the wall to cool the body in a high temperature drag.

Xenos, et al (2001) investigated the effect of adverse pressure gradient and mass transfer on the MHD compressible laminar boundary-layer flow over a permeable plate. He observed that the presence of magnetic field always increases frictional drag on the wall either in case of suction or injection. This analysis is however time independent. In a related work, Attia (2003) investigated the hydro magnetic stagnation point flow with heat transfer over a permeable surface. In recent past, Fang (2004a) has studied the hydro magnetic flow over an oscillating flat plate with arbitrary periodic oscillation. Recently, Hayat et al (2008) presented an analytical solution for unsteady magneto hydrodynamic flow of a non-Newtonian fluid in a rotating porous medium. As the suction or injection of fluid through the body surface

is known to be an efficient mechanism to alter the structure of the boundary layer along a body. As a consequence, the drag of the body and the heat exchange rate between the body and the surrounding fluid are modified.

Furthermore, the boundary-layer flow may be made to separate or attach to the body surface. Abdel-Azez (2004) analysed the effects of thermal conductivity and temperature dependent viscosity on combined heat and mass transfer in MHD three dimensional flow over a stretching surface with ohmic heating. This analysis was however time independent. Tsai (2009) discussed the effects of Viscosity and thermal conductivity on heat transfer for hydromagnetic flow over a continuous moving porous plate with ohmic heating.

2.2 Literature review on unsteady MHD Couette flow between two parallel plates due to the ramped motion of one of the boundaries

The flow of conducting fluid between two infinite parallel, stationary, and insulated plates under the influence of a transverse uniform magnetic field is of special technical significance because of its frequent occurrence in many industrial applications such as MHD power generators and Pumps, cooling of nuclear reactors, geothermal systems, waste disposal, petroleum and polymer technology, heat exchangers and others. One of the earliest studies in this field was carried out by Hartmann and Lazarus (1937). They analyzed the influence of the effects of a uniform magnetic field on the flow of a viscous incompressible electrically fluid existing through parallel stationary plates that are insulated. Since then this problem was extended in numerous ways.

The simple geometry in this analysis designated by a parallel wall channel renders many a problem in fluid dynamics to the solution of a linear differential equation. The

equation is an ordinary differential equation (ODE) if the flow is assumed steady and a partial differential equation (PDE) if the flow is unsteady. Even with the introduction of several complicating factors such as a suction velocity at the walls, an applied magnetic field, a time dependent pressure gradient, an arbitrary wall velocity, and an arbitrary initial velocity, the equation still remains a linear PDE. The solutions of the resulting equations are still dependent on the type of initial and boundary conditions to be imposed. One of these important and fundamental fluid flow situations is the Couette flow which can occur, for example, between two parallel flat plates, one of which is at rest, and the other is moving in its own plane with a constant or time dependent velocity. It is an important classical example of exact solutions for time dependent Navier-Stokes equation and reveals how the velocity profile varies with time, approaching a linear distribution asymptotically, and how the boundary layer spreads throughout the flow field.

The steady and unsteady Couette-type flow in hydrodynamics and magneto-hydrodynamics, subjected to wall suction have also been discussed by several authors due to their applications in transpiration cooling and gaseous diffusion processes.

These include Sutton and Sherman (1965), Longwell (1966), Cramer and Pai (1973), White (1991), Davidson (1987), Panton (1996) and Schlichting (2000). The Couette flow of a viscous incompressible and electrically conducting fluid between two infinite parallel plates in the presence of transverse magnetic field when one of the plate's starts impulsively from rest was also studied by Baral(1962) and Katagiri (1962). Katagiri presented his analysis by taking the magnetic lines of force fixed relative to the fluid. Singh and Kumar (1983) analyzed the problem discussed by Katagiri, by assuming magnetic lines of force fixed relative to the moving plate having impulsive or accelerated motion.

Attia (1996) studied the steady MHD flow between two parallel plates with heat transfer. The effects of Hall current on the temperature and velocity fields of an unsteady Hartmann flow was also investigated by Attia (2005), he also considered the effects of ion slip on transient Hartmann flow with heat transfer. Shalybkon (2006) discussed the stability of the hydrodynamic and hydromagnetic Couette flow. In a related development a detailed study of the effects of rotation and the magnetic field on an unsteady MHD Couette flow was carried out by Chandran et al (1993), Singh et al (1994) and Guria et al (2006).The work by Guria et al was further studied by Jha and Apere (2010) by taking into account the additional effects of Hall and ion slip currents. They observed that the skin friction on the moving plate decreases with increase in Hall and ion slip current, when the magnetic field is fixed relative to the fluid in contrast to the case when the magnetic lines of force are fixed relative to the moving plate. Recently, Jha and Jibril (2010) investigated the hydro magnetic flow due to ramp motion of the boundary. They observed that the velocity of fluid in the case of ramp motion of the plate is always less than that of the flow induced by impulsive motion of the plate.

The steady and unsteady Couette–type flow in hydrodynamics and magneto hydrodynamics , subjected to wall suction/injection have also been discussed by several authors due to their numerous applications in industry in areas such as in transpiration cooling and gaseous diffusion processes, dialysis of blood in artificial kidneys, etc. A theoretical analysis of an unsteady MHD flow through and bounded by two parallel porous plates the lower plate being stationary and the other moving in its own plane was also presented by Agrawal (1965) and Hassanien and Mansour (1990).

A flow formation due to Couette motion between porous walls has also been investigated by Fang (2004b), where he discussed the incompressible Couette flow with porous walls. The effects of pressure gradient on the flow between two parallel plates subject to suction and injection was also considered by Fang (2004c). Attia (2005) studied the effect of suction and injection on the unsteady MHD Couette flow between two parallel plates with variable properties, taking the viscosity as temperature dependent. Singh (2004) analysed the influence of moving magnetic field on time dependent three dimensional MHD Couette flow of an incompressible, viscous fluid bounded by two parallel porous plates in which the magnetic field is fixed with respect to the upper moving porous plate. He observed among others that the velocity profile decreases with increase in injection/suction parameter and increases with increase in Hartmann number. .

In all the above discussed works the flow formation was considered to be due to impulsive/ accelerated motion of the plate. In this chapter the unsteady hydro magnetic Couette flow between two parallel plates due to ramped motion of one of the porous plates is considered with or without suction/injection. It is assumed that both plates are uniformly permeable and that fluid enters and leaves the porous plates at the same constant rate.

2.3 Literature review on the combined effects of viscous, Joulean and ohmic dissipation on MHD Couette flow with transpiration

The study of MHD flow and heat transfer of a viscous, incompressible fluid between parallel plates is of considerable interest due to its frequent occurrence in industrial, technological and geothermal applications. These applications include nuclear engineering in the cooling of Nuclear reactors, manufacture of MHD pumps, petroleum industry and in aerodynamic heating. The pioneer work on the study of the

flow of an incompressible, viscous and electrically conducting fluid in between two parallel plates in the presence of transverse magnetic field was done by Hartmann in 1937. Since then there has been a lot of discussion on this problem in different physical situations. The flow of electrically conducting fluid between two parallel plates in which one of the plates is moving and the other stationary, is known as the MHD Couette flow. Many researchers have considered Couette problem in the presence or absence of magnetic field under different physical situations. These include Schlichting and Gersten(2000),Agrawal(1965),Singh and Kumar(1983), Katagiri (1962) and Hashimoto(1957).

Das (2009) discussed the effect of suction and injection on MHD Couette flow and heat transfer through a porous medium. He concluded among others that the effect of increasing suction/injection parameter and the Prandtl number is to reduce the temperature of the flow field at all points.

Furthermore Chamkha (1997) investigated the effects of Hartmann number on the velocity, temperature, skin friction and the Nusselt number on two dimensional MHD-free convection in a vertical plate embedded in a thermally stratified porous medium with Hall effects. The analyses of both Das and Chamkha are however time independent. The natural convection in an unsteady Couette flow of an incompressible fluid for impulsive start of one of the plates was studied by Singh (1988). His work is extended for electrically conducting fluids, when the magnetic lines of force are fixed either relative to the fluid or plate, by Jha (2001). In his investigation Jha discovered among others that the velocity decreases by increasing the Hartmann number when the magnetic lines of force are fixed relative to the fluid but the reverse is the case when the magnetic lines of force are fixed relative to the plate.

Attia (2005) studied the effect of suction and injection on the unsteady MHD Couette flow between two parallel plates with variable properties, taking the viscosity as temperature dependent. Jha and Apere (2010a) studied the effect of Hall and ion-slip currents on unsteady MHD Couette flows in a rotating system. In recent past Fang (2004b) discussed and analysed the effect of mass transfer on the transient velocity and steady state temperature for an incompressible viscous Couette flow with porous boundaries, where the upper boundary is impulsively set in motion. He observed that large Eckert number leads to large fluid temperature at the upper wall under mass injection. In another article Fang (2004c) studied the pressure driven flow when the two plates are subjected to equal mass injection and suction at the porous plates respectively. These analyses by Fang where however carried out in the absence of Magnetic or electric field.

The study of MHD flows with viscous and ohmic dissipation has received the attention of many researchers because of its applications in many industrial, technological and geothermal processes such as in high temperature plasmas, cooling of nuclear reactors, liquid metals fluids, MHD accelerators and power generation systems. Setayashi (1990) studied the effect of temperature dependent transport properties on MHD flow and heat transfer in a parallel –plate channel under the influence of applied electric field. Umavathi (1996) did his work on combined effect of viscous and ohmic dissipation on MHD flow in a vertical channel in the presence of applied electrical and magnetic field under steady state conditions. He observed among others that the effect of increasing the Hartmann number is to accelerate the flow in the positive direction for negative values of the electric load parameter and in the negative direction for positive values of the electric load parameter. His analysis was however time independent. El-Amin (2003) discussed the combined effects of

viscous dissipation and ohmic (Joule) dissipation on MHD forced convection over a non-isothermal horizontal cylinder embedded in a fluid saturated porous medium. The combined heat and Mass transfer in MHD free convection from a vertical surface with Ohmic heating and viscous dissipation was also discussed by Chen (2004).

Pantokratoras (2007) investigated the fully developed flow of electrically conducting fluid between two parallel plates under the action of a transverse magnetic field. Sharma and Singh (2009) discussed the effects of ohmic heating and viscous dissipation on steady MHD flow of viscous, incompressible and electrically conducting fluids near a stagnation point on an isothermal stretching sheet. This analysis is however time independent. Devi and Ganga (2009) investigated the effects of Joulean (ohmic) and viscous dissipation on MHD nonlinear flow and heat transfer past a porous surface with prescribed heat flux. He observed that the flow becomes faster with increase in Eckert number and an increase in Prandtl number reduces the temperature distribution and heat transfer rate. His analysis was however carried out in the absence of electric field.

Adhikary and Misra (2011) investigated the effects of porosity of the channel wall, the presence of an external magnetic field and radiative heat transfer on unsteady flow of an electrically conducting fluid through a (horizontal) channel.

Sibande and Makinde (2010) studied the MHD steady flow and heat transfer characteristics of an incompressible viscous and electrically conducting fluid past a rotating disk in a porous medium with ohmic heating, Hall current and viscous dissipation. They discovered among others that both the magnetic field and Eckert number tends to enhance the heat transfer rate. In a related article Makinde and Onyejekwe (2011) studied the MHD generalized Couette flow and heat transfer with

variable viscosity and electrical conductivity. This analysis is however done in the absence of suction/injection.

Barletta and Celli Abel et al (2008) considered and investigated the effects of viscous and ohmic (Joule) dissipation on MHD laminar flow over a porous nonlinear vertical stretching sheet with partial slip. This analysis was however carried out in the absence of electrical field. Pal and Mondal (2010) investigated the hydromagnetic non-Darcy flow and heat transfer over stretching sheet in the presence of thermal and ohmic dissipation. He concluded that the effect of electric load parameter is to increase the velocity while the temperature distribution decreases.

Farzandeh-Gord, et al (2009) studied the effects of Eckert number, Reynolds number, electric load parameter , Prandtl number, Hartmann number on Magneto hydrodynamics flow and heat transfer over a stretching sheet. He observed that increasing the values of the local Reynolds number and the electric load parameter decreased the skin friction and in the presence of magnetic field, the effect of the electric field is to decrease the temperature near the stretching boundary. This analysis is however time independent. Of recent Abel et al(2011) analysed the effects of buoyancy, viscous and Joulean dissipation on MHD flow and heat transfer over a non-linear vertical stretching porous sheet with partial slip.

Recently Zivojin *et al* (2010) investigated the MHD flow and heat transfer of two immiscible fluids between horizontal moving plates in the presence of applied electric field. He observed that for a fixed Hartmann number, positive values of the electric load factor will decrease the pressure while negative values will increase it.

2.4 Literature review on transient Two- phase flow in magneto hydrodynamics: A Green's function approach

The time dependent two phase flow is of great importance because of their applications in several industrial and physical fields. The flow and heat transfer aspects of immiscible fluids is of special importance in the petroleum extraction and transport. For example the reservoir (rock) of an oil field always contains several immiscible fluids in its pores. Part of the pore volume is occupied by water and the rest by crude oil or natural gas or both. Due to its practical applications, in last thirty years two-phase flow has been widely researched and reported in literature under several idealized assumptions. These include Charles and Lillelcht (1965), Benwich (1964), Packhamand Shail (1971) and Shail (1973). Shail (1973) considered the two-phase flow of conducting and non-conducting incompressible fluids in a horizontal channel in the presence of transverse magnetic field. It was shown theoretically that significant increase in the flow rate of conducting fluid can be obtained for suitable choice of ratio of viscosities and the depth of the two phases. Lohrasbi and Sahai (1988) studied the thermal characteristics of Shail (1973), by accommodating viscous and Ohmic-dissipation in the energy equation. The two- phase flow was further examined by Malashetty and Leela (1991) and Malashetty and Leela (1992) by considering both phases as electrically conducting and solved the momentum and energy equations analytically for short circuit and open circuit cases respectively. Furthermore Umavathi *et al (2005)* investigated the unsteady two-fluid flow and heat transfer in a horizontal channel. In another article Umavathi et al (2006) investigated the unsteady Hartmann flow of two immiscible fluids through a horizontal channel with time dependent oscillatory wall transpiration velocity. Chamkha (2000)

discussed the flow of two-immiscible and electrically conducting fluids in porous and non-porous channels. He observed that increase in Hartmann number reduced flow velocities. Recently Zivojin et al (2010) studied the MHD flow and heat transfer of two immiscible electrically conducting fluids between moving plates in the presence of an applied electric field. In all these above discussed works momentum and energy equations are assumed to be independent of time.

CHAPTER THREE

MHD FLOW DUE TO THE RAMPED MOTION OF AN INFINITE HORIZONTAL PLATE

3.0 Introduction.

The aim of this chapter is to discuss the time dependent MHD flow of an incompressible, viscous and conducting fluid due to the ramped motion of an infinite horizontal flat plate. The flow is assumed to be under the influence of an externally applied transverse magnetic field. In section one the flow was considered in the absence of suction/injection on the plate. While in the second case the effect of suction/injection was taken into consideration.

3.1 Time Dependent MHD flow due to the ramped motion of an infinite horizontal flat plate under the influence of transverse magnetic field.

3.1.0 Introduction

In this section the time dependent MHD flow of an incompressible, viscous and electrically conducting fluid due to the ramped motion of an infinite horizontal flat plate under the influence of an externally appliedtransverse magnetic field has been investigated.

3.1.1 Mathematical Analysis

In this work the flow of a viscous, incompressible and electrically fluid due to ramped motion of an infinite horizontal plate is considered. At time $t' \leq 0$ the fluid, the plate and the magnetic lines of force are assumed to be at rest. At time $t' > 0$ the plate begins to move in its own plane with a velocity $U_0 f(t')$, where U_0 *is* constant. A uniform magnetic field of strength B_0 is acting perpendicular to the plate as shown in figure 3.1.1. The magnetic Reynolds number of the flow is assumed to be small

enough so that the induced magnetic field can be neglected. The flow is assumed to be in the x' direction along the plate and the y' axis is normal to the plate. Since the plate is infinite in extent all the physical variables are taken to be of $y' \& t'$

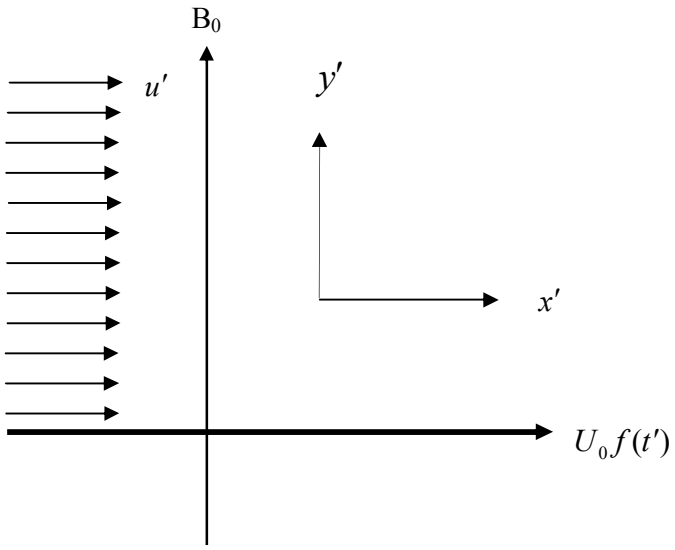


Figure 3.1.1 Physical configuration of the flow

Under the above physical conditions the flow is governed by the following Equations

$$\frac{\partial u'}{\partial t} = \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^{\,2}}{\rho} u' \qquad (3.1.1)$$

Equation (3.1.1) is valid when the magnetic lines of force are fixed relative to the fluid.

If the magnetic field is fixed relative to the moving plate, equation (3.1.1) can be replaced by

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^{\,2}}{\rho} \left[u' - U_0 f(t')\right] \qquad (3.1.2)$$

Equation (3.1.1) and (3.1.2) can be combined together to obtain

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^{\,2}}{\rho}\left[u' - KU_0 f(t')\right] \qquad (3.1.3)$$

Where

$$K = \begin{cases} 0 \text{ if magnetic field is fixed relative to the fluid} \\ 1 \text{ if magnetic field is fixed relative to the plate} \end{cases}$$

The initial and boundary conditions relevant to the problem are

for $t' \le 0;\ \ u' = 0$ for all $y' \ge 0$

$$t' > 0;\ \begin{cases} u' = U_0 f(t') \ \text{ at } y' = 0 \\ u' \to 0 \quad \text{as } y' \to \infty \end{cases}$$

(3.1.4)

Introducing the non-dimensionless quantities,

$$y = \frac{y'U_0}{\nu}, t = \frac{t'U_0^2}{\nu}, u = \frac{u'}{U_0}, M^2 = \frac{\sigma \mu_e^2 B_0^2 \nu}{\rho U_0^2}, \qquad (3.1.5)$$

Where all the quantities are defined in nomenclature, the momentum equation becomes

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - M^2\left[u - Kf(t)\right] \qquad (3.1.6)$$

And the initial and boundary conditions above in the dimensionless form for the physical system are:

for $t \le 0;\ \ u = 0$ for all $y \ge 0$

$$t > 0;\ \begin{cases} u = f(t) \ \text{ at } y = 0 \\ u \to 0 \quad \text{as } y \to \infty \end{cases} \qquad (3.1.7)$$

In the present work $f(t)$ is specified as the ramp function:

$$f(t)=\frac{t}{t_0} \quad \text{if } 0\le t\le t_0$$
$$=1 \quad \text{if } t\ge t_0 \tag{3.1.8}$$

Eq. (3.1.8) can be written as:

$$f(t)=H(t)\left(\frac{t}{t_0}\right)-\left(\frac{1.}{t_0}\right)(t-t_0)H(t-t_0) \tag{3.1.9}$$

Where $H(t)$ is the Heaviside unit step function defined by $H(t)=\begin{Bmatrix}0,\ t<0\\1,\ t\ge 0\end{Bmatrix}$ (3.1.10)

3.1.2 Solution of the problem

The solution of the partial differential Eq.(3.1.6) subject to conditions (3.1.7) is attempted by the operational method of Laplace transformation. Let

$$L[u(y,t)]=\bar{u}(y,s)=\int_0^\infty u(y,t)Exp(-st)dt \tag{3.1.11}$$

be the Laplace transform of $u(y,t)$, then Eqs. (3.1.6), (3.1.7) and (3.1.9) reduce to:

$$\frac{d^2\bar{u}}{dy^2}-(M^2+s)\bar{u}=-KM^2\bar{f}(s) \tag{3.1.12}$$

$t\le 0;\ \bar{u}=0$ for all $y\ge 0$

$$t>0;\ \begin{cases}\bar{u}=f(s) & \text{at } y=0\\ \bar{u}\to 0 & \text{as } y\to\infty\end{cases} \tag{3.1.13}$$

$$\bar{f}(s)=\frac{[1.-Exp(-st_0)]}{s^2t_0} \tag{3.1.14}$$

The solution of Eq. (3.1.12), under the conditions (3.1.13) and (3.1.14), is:

$$\bar{u}=\frac{1}{t_0s^2}[1.-Exp(-st_0)]\left[1.-\frac{KM^2}{(M^2+s)}\right]Exp\left(-y\sqrt{M^2+s}\right)$$
$$+\frac{KM^2}{(M^2+s)t_0s^2}[1.-Exp(-st_0)] \tag{3.1.15}$$

The exact solution for velocity field can be obtained from Eq. (3.1.15), by taking the inverse transforms as inAbramowitz and Stegun(1965) and Carslaw and Jaeger (1963).After detailed simplifications and shifting on the t-axis, the solution can be expressed as:

$$u(y,t)=\frac{[1-K]}{t_0}\left[F_1(y;M;t;0.0)-F_1(y;M;t:t_0)H(t-t_0)\right]$$

$$+\frac{K}{M^2}\frac{1}{t_0}\left[F_2(y;M;t;0.0)-Exp\left(-M^2t\right)F_2(y;0.0;t:0.0)\right]$$

$$+\frac{K}{M^2}\frac{1}{t_0}\left[Exp\left(-M^2(t-t_0)\right)F_2(y;0.0;t;t_0)-F_2(y;M,t,t_0)\right]H(t-t_0)$$

$$+\frac{K}{t_0}\left[\frac{Exp\left(-M^2t\right)}{M^2}-\frac{1}{M^2}+t\right]$$

$$+\frac{K}{t_0}H(t-t_0)\left[\frac{1}{M^2}-\frac{Exp\left(-M^2(t-t_0)\right)}{M^2}-(t-t_0)\right] \qquad (3.1.16)$$

3.1.3 Skin friction

Using the expression (3.1.16), the shear-stress at the moving plate is:

$$\tau=-\left.\frac{\partial u}{\partial y}\right|_{y=0}=\frac{[1-K]}{t_0}\left[F_4(M;t;0.0)-F_4(M;t:t_0)H(t-t_0)\right]$$

$$+\frac{K}{M^2}\frac{1}{t_0}\left[F_3(M;t;0.0)-Exp\left(-M^2t\right)F_3(0.0;t:0.0)\right]$$

$$+\frac{K}{M^2}\frac{1}{t_0}\left[Exp\left(-M^2(t-t_0)\right)F_3(0.0;t;t_0)-F_3(M,t,t_0)\right]H(t-t_0) \qquad (3.1.17)$$

The equation (3.1.16) and (3.1.17) represents the unified expressions for velocity and skin-friction at the plate of conducting fluid due to ramped motion of the horizontal plate in presence of transverse magnetic field fixed with respect to fluid or moving plate.

3.1.4 Special Case

For $t_0 = 0$, the equation (3.1.16) is

$$u(y,t) = [1. - K]F_2(y; M; t; 0.0) + K + Exp(-M^2 t)\left[Erfc\left(\frac{y}{2\sqrt{t}}\right) - 1. \right], \quad (3.1.18)$$

and the equation (3.1.17) is

$$\tau = -\left.\frac{\partial u}{\partial y}\right|_{y=0} = [1. - K]F_3(M; t; 0.0) - \frac{K}{\sqrt{\pi t}} Exp(-M^2 t). \quad (3.1.19)$$

The equation (3.1.18) and (3.1.19) represent the unified expression for velocity field and skin-friction at the plate of conducting fluid due to impulsive motion of the horizontal plate, in the presence of transverse magnetic field either fixed with respect to fluid or moving plate. Furthermore for $K = 0$, the solution represents the Rayleigh problem in magneto-hydrodynamics.Also for $M^2 = 0$, the solution boils down to Rayleigh problem in hydro-dynamics.

3.1.5 Results and Discussion

In this section, some representative results are presented to illustrate the effects of magnetic field on velocity and skin-friction when $t_0 = 2.0$. A FORTRAN program was written to compute the numerical values of the boundary layer velocity and the skin friction, using equations (3.1.16) and (3.1.17) respectively. The velocity profiles and skin frictions in Figs. 3.1.2 to 3.1.8 are the MS Excel plots of these computed values.

Fig.3.1.2 shows the variation of velocity (u) versus distance (y), for different time (t). From this figure it is clear that velocity increases with time whether the magnetic lines of force are fixed with respect to the fluid or plate. Also it reflects that the

velocity field is greater when magnetic lines of force are fixed with respect to the moving plate in comparison to magnetic lines of force fixed with respect to the fluid.

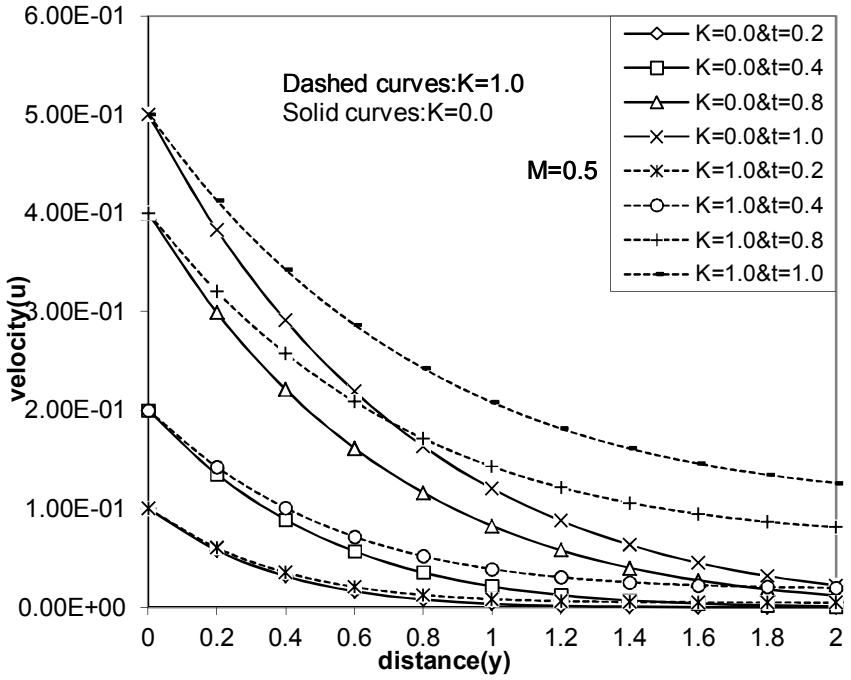


Fig.3.1.2 velocity (u) versus distance (y), for different time (t). M =0.5 and K =1.0

Fig.3.1.3. shows the variation of velocity profiles for ramped motion of the plate when magnetic lines of force are fixed with respect to the fluid for different values of time. As seen from Fig.3.1.3 an increase in time results in a proportionate increase in the velocity, and this velocity steadily decreases to zero in layers of variable depths.

Fig.3.1.4 reflects the variation of velocity profiles for ramped motion of the plate when magnetic lines of force are fixed with respect to moving plate for various values of time. A comparison of Fig.3.1.3 and 3.1.4 reveals that the approach of the boundary layer fluid velocity profile to its free stream value (zero or finite) depends on the mode of application of magnetic field.

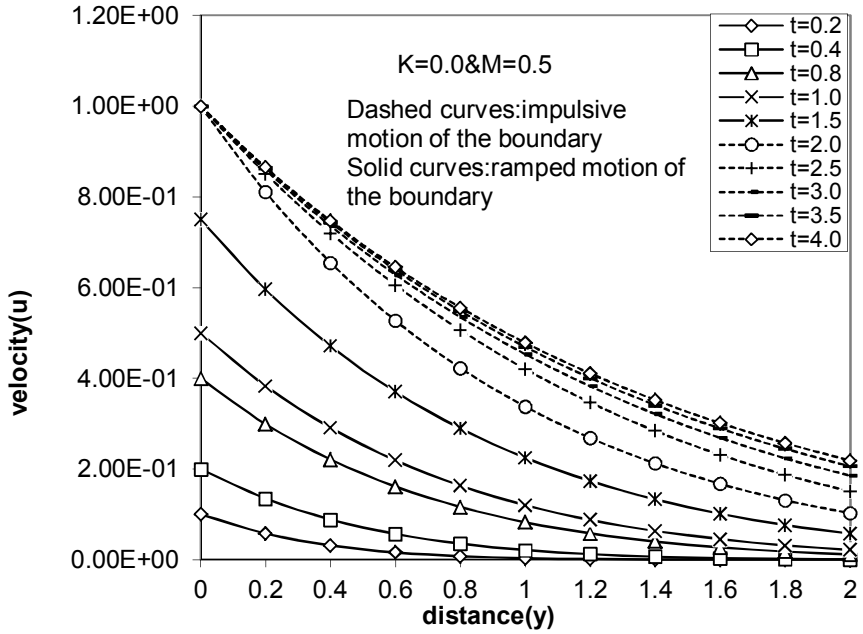


Fig.3.1.3Velocity (u) versus distance (y) for different time (t), M =0.5 and K =1.0

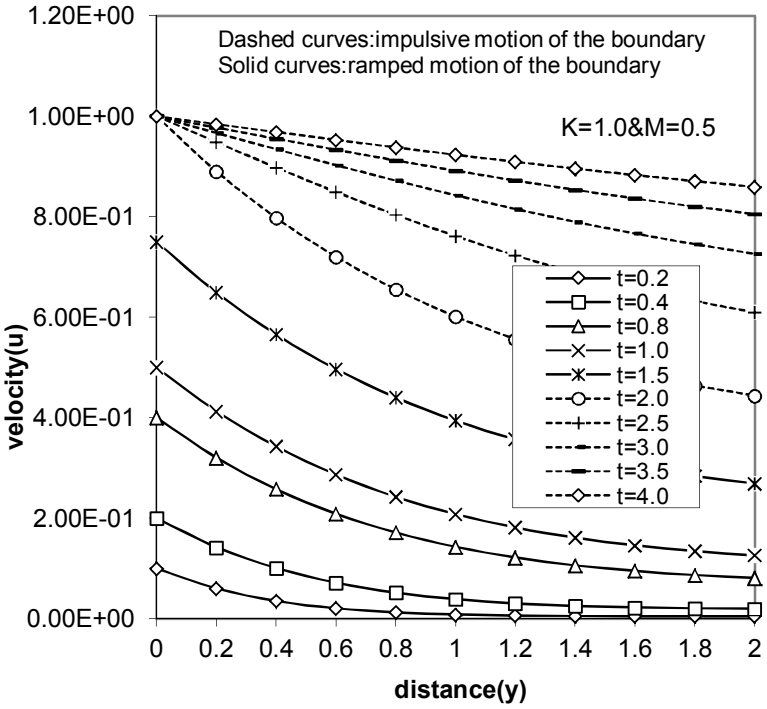


Fig.3.1.4 velocity (u) versus distance (y) for different time (t) (M =0.5and K = 1.0)

In order to compare the influence of ramped motion versus constant motion of the boundary on the fluid velocity, velocity profiles are plotted in Fig.3.1.5. and Fig.3.1.6.when the magnetic lines are fixed with respect to fluid and moving plate respectively. These figures disclose that velocity field in ramped motion of the boundary is always less than that of the flow induced by the impulsive (constant) motion of the boundary. This should be expected since in the case of ramped motion, the velocity of the plate takes place more gradually than in the impulsive motion.

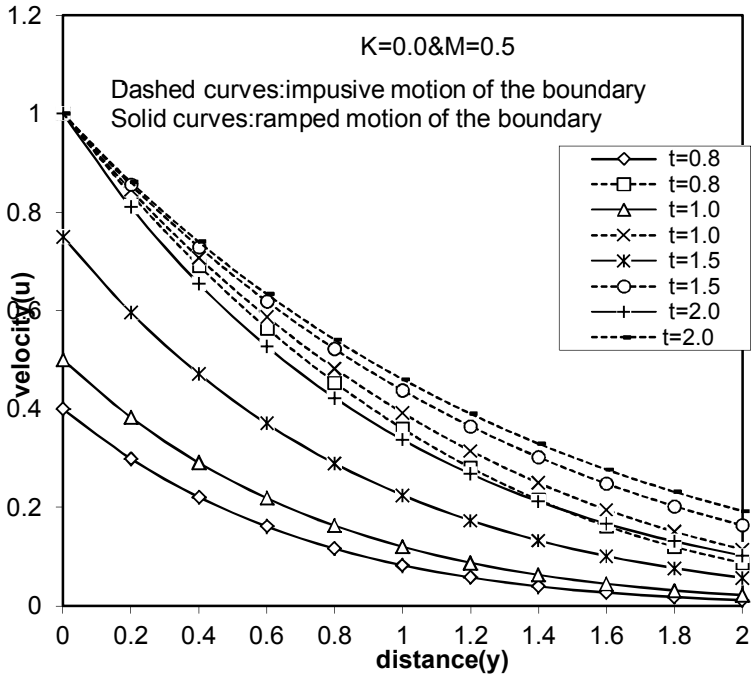


Fig. 3.1.5Velocity (u) versus distance (y) for different (t) (M =0.5 and K =0.0)

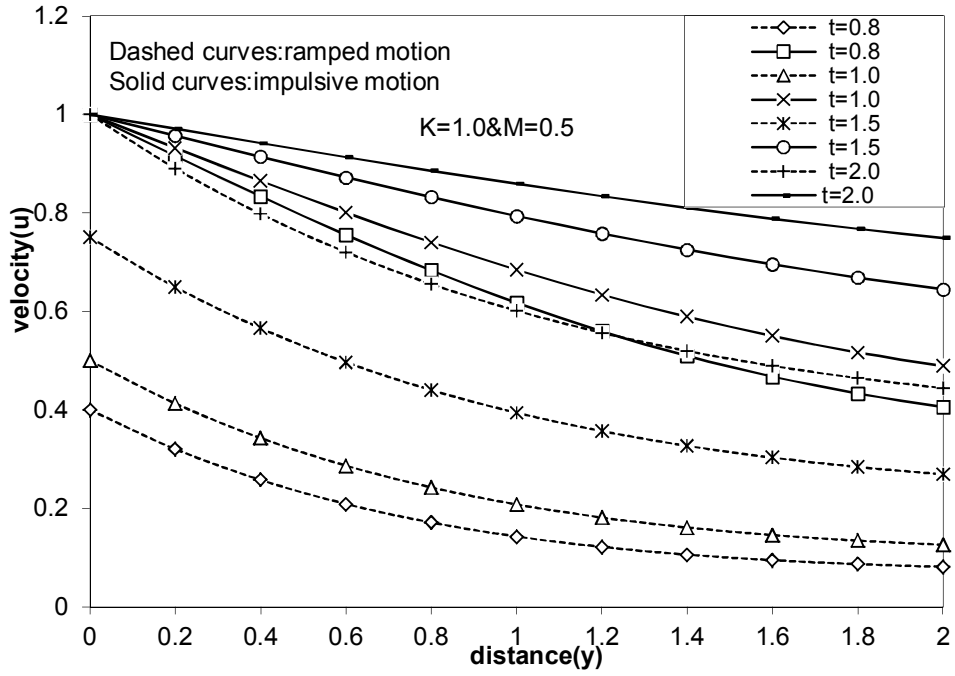


Fig. 3.1.6 Velocity (u) versus distance (y) for different (t) ($K = 0.0$ and M =0.5.

In order to give a quantitative estimate of the dependence of the velocity profile, on the mode of application of magnetic field, the velocity profiles are shown in Fig.3.1.7. It is observed that velocity increases with increase of magnetic field when the magnetic lines of force are fixed with respect to moving boundary while the results are just reverse when the magnetic lines are fixed with respect to the fluid.

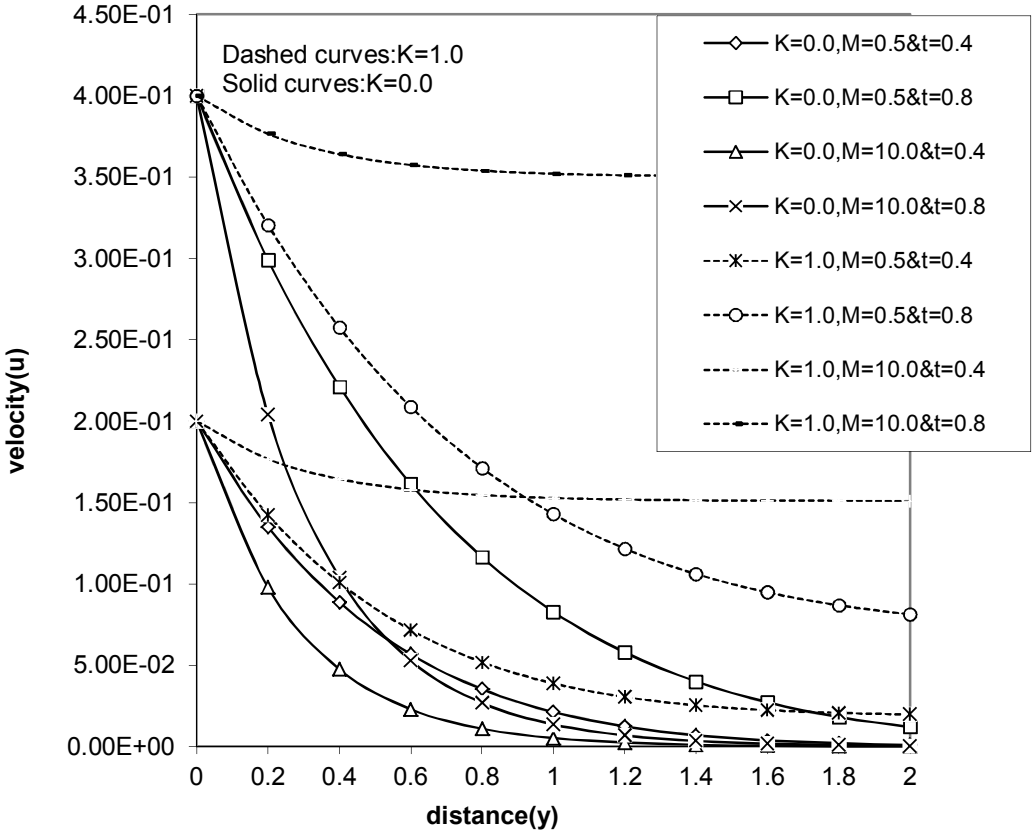


Fig.3.1.7 Velocity(u) versus distance (y) for different values of M and t.

Fig.3.1.8 explains the variation of skin-friction with time for different values of magnetic parameter (M) when magnetic lines are fixed with fluid or moving boundary. This figure reflects that skin-friction increases as the magnetic field increases for constant and ramped motion of the boundary when magnetic lines of force are fixed with respect to fluid which is just reverse to the case when magnetic lines are fixed with moving boundary. Furthermore, skin-friction is less for ramped motion of the boundary in comparison to constant motion of the boundary for small

values of time and as time increases its value increases and attains its maximum value at $t = t_0$ and then decreases and coincide the skin-friction for constant motion of the plate.

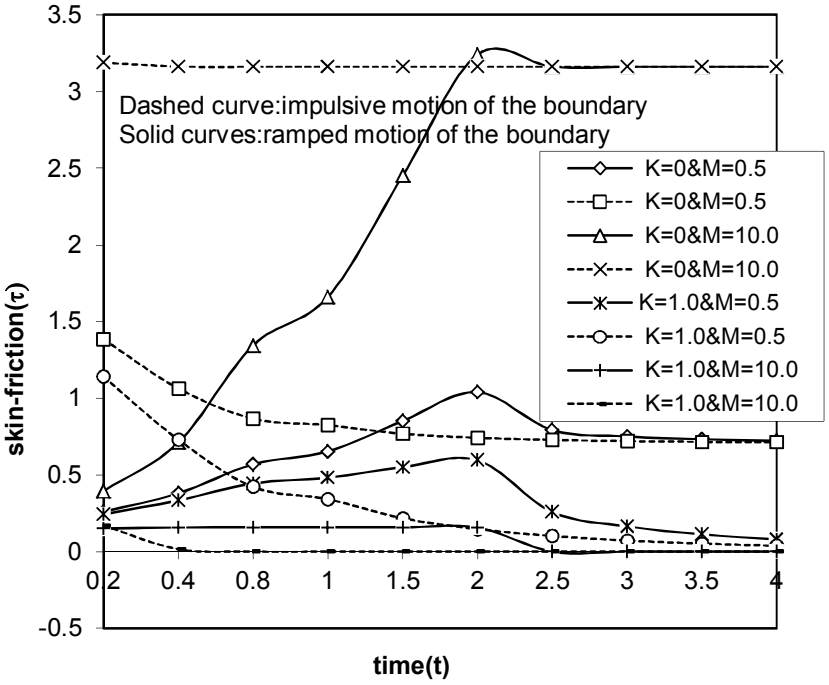


Fig.3.1.8 Skin friction (τ) versus time (t) for different values of M ($K = 0.0$ and $K = 1.0$.)

3.1.6 Summary and Conclusion

In this section the analytical solution of an unsteadyflow of a viscous, incompressible and electrically conducting fluid subject to uniform transverse magnetic field near a horizontal plate having ramped motion has been considered. A unified closed form analytical solution have been obtained for velocity field and the skin-friction corresponding to the cases of magnetic field fixed relative to the fluid or the moving

boundary. Under the assumption of negligible induced magnetic field, the governing momentum equation has been solved exactly by using Laplace-transform technique. The flow formation near a ramped motion of the plate has also been compared with the flow formation near a plate having constant motion(impulsive motion).It is observed that velocity of fluid in the case of ramped motion of the plate is always less than that of the flow induced by impulsive motion of the plate. It is observed that velocity increases with increase of magnetic field when the magnetic lines of force are fixed with respect to moving boundary while the results are just reverse when the magnetic lines are fixed with respect to fluid. It has also been observed that the skin friction increases as the magnetic field increases for constant and ramped motion of the boundary when magnetic lines of force are fixed with respect to fluid which is just reverse to the case when magnetic lines are fixed with moving boundary.

3.2 Effects of Transpiration on the MHD flow near a Porous Plate having ramped motion.

3.2.0 Introduction

The objective of this section is to investigate and analyse the effects of viscosity, magnetic field and mass leakage on the unsteady flow of an incompressible and electrically conducting fluid in the presence of transverse magnetic field.The flow is generated due to the ramped motion of the horizontal porous plate.

3.2.1 Governing Equations and Solutions

Let us consider the motion of a viscous, incompressible and electrically conducting fluid due to the ramped motion of an infinite horizontal porous plate. At time $t' \leq 0$, the fluid, the porous plate and the magnetic lines of force are assumed to be at rest. At time $t' > 0$, the porous plate begins to move in its plane with a velocity $U_0 f(t')$, where U_0 is a constant and $f(t')$ is the dimensional ramped function. A uniform magnetic

field of strength B_0 is acting perpendicular to the porous plate. The magnetic Reynolds's number of the flow is assumed to be small enough so that the induced magnetic field can be neglected. The flow is assumed to be in the x' direction along the porous plate and the y' axis is normal to the porous plate as shown in figure 3.2.1.

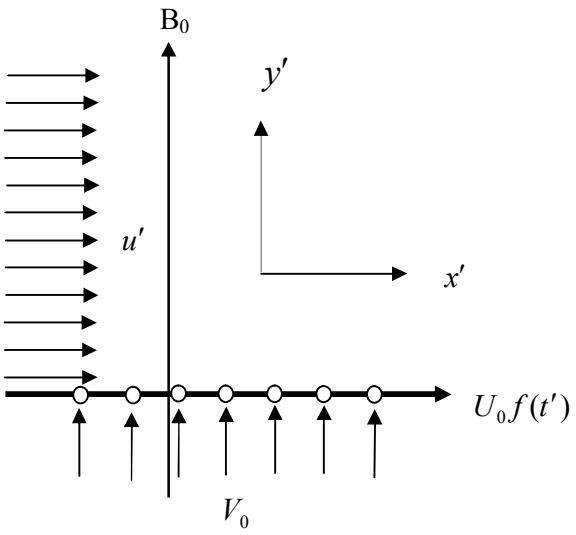


Fig.3.2.1 Schematic view of the flow

The Momentum equation for this investigation based on the above physical conditions is:

$$\frac{\partial u'}{\partial t'} + V_0 \frac{\partial u'}{\partial y'} = \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^{\,2}}{\rho} u' \qquad (3.2.1)$$

Equation (3.2.1) is valid when the magnetic lines of force are fixed relative to the fluid. If the magnetic field is also having ramp motion with the same velocity as the porous plate the relative motion must be accounted for. In this case equation (3.2.1) is replaced by

$$\frac{\partial u'}{\partial t'} + V_0 \frac{\partial u'}{\partial y'} = \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^{\,2}}{\rho} \left[u' - U_0 f(t')\right] \qquad (3.2.2)$$

This is valid when the magnetic lines of force are fixed relative to the moving porous plate.

Equation (3.2.1) and (3.2.2) can be combined together to obtain

$$\frac{\partial u'}{\partial t'} + V_0 \frac{\partial u'}{\partial y'} = \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^{\,2}}{\rho} \left[u' - K U_0 f(t')\right] \qquad (3.2.3)$$

Where $u', t', V_0,$ are the dimensional velocity of the fluid in the x'-direction, dimensional time variable and the suction/injection parameter respectively. Similarly ρ, σ, ν, B_0 are respectively, the density of the fluid, the electrical conductivity of the plate, the kinematic viscosity of the fluid and the applied transverse magnetic field, and

$$K = \begin{cases} 0 \text{ if magnetic field is fixed relative to the fluid} \\ 1 \text{ if magnetic field is fixed relative to the plate} \end{cases}$$

The initial and boundary conditions are:

For $t' \le 0;\ \ u' = 0$ for all $y' \ge 0$

$$t' > 0; \begin{cases} u' = U_0 f(t') \ \text{ at } y' = 0 \\ u' \to 0 \quad \text{as } y' \to \infty \end{cases} \qquad (3.2.4)$$

Introducing the non-dimensionless quantities,

$$y = \frac{y' U_0}{\nu}, t = \frac{t' U_0^2}{\nu}, u = \frac{u'}{U_0}, M^2 = \frac{\sigma B_0^2 \nu}{\rho U_0^2}, \ S = \frac{V_0}{U_0} \qquad (3.2.5)$$

Under these conditions the equation of motion for magnetic lines of force fixed with fluid or porous plate in dimensionless form is

$$\frac{\partial u}{\partial t} + S \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - M^2 \left[u - K f(t)\right] \qquad (3.2.6)$$

The initial and boundary conditions in the dimensionless form for the physical system are:

for $t \le 0;\ \ u = 0$ for all $y \ge 0$

$$t>0;\ \begin{cases} u = f(t) & \text{at } y = 0 \\ u \to 0 & \text{as } y \to \infty \end{cases} \qquad (3.2.7)$$

In the above, u, S and M are the dimensionless velocity, suction/injection and magnetic parameter respectively.

In the present work $f(t)$ is specified as the ramp function:

$$\begin{aligned} f(t) &= \frac{t}{t_0} \quad \text{if } 0 \le t \le t_0 \\ &= 1 \quad \text{if } t \ge t_0 \end{aligned} \qquad (3.2.8)$$

Eq.(3.2.8) can be written as:

$$f(t) = H(t)\left(\frac{t}{t_0}\right) - \left(\frac{1.}{t_0}\right)(t - t_0)H(t - t_0) \qquad (3.2.9)$$

Where $H(t) = 0$ for $t \le 0$, and $H(t) = 1$ for $t>0$.

3.2.2 Solution of the Problem

The solution of the partial differential Equation (3.2.6) subject to conditions (3.2.7) is attempted by the operational method of Laplace transformation. Let

$$L[u(y,t)] = \bar{u}(y,s) = \int_0^\infty u(y,t)Exp(-st)dt \qquad (3.2.10)$$

be the Laplace transform of $u(y,t)$ and $s>0$ is the Laplace parameter, then Equations.(3.2.6), (3.2.7) and (3.2.9) reduce to:

$$\frac{d^2\bar{u}}{dy^2} - S\frac{d\bar{u}}{dy} - (M^2 + s)\bar{u} = -KM^2\bar{f}(s) \qquad (3.2.11)$$

$t \le 0;\ \bar{u} = 0$ for all $y \ge 0$

$$t>0;\ \begin{cases}\bar{u}=f(s) & \text{at } y=0\\ \bar{u}\to 0 & \text{as } y\to\infty\end{cases} \tag{3.2.12}$$

$$\bar{f}(s)=\frac{[1.-Exp(-st_0)]}{s^2 t_0} \tag{3.2.13}$$

The solution of Equation (3.2.11), under the conditions (3.2.12) and (3.2.13), is:

$$\bar{u}=\frac{1}{t_0 s^2}[1.-Exp(-st_0)]\left[1.-\frac{KM^2}{(M^2+s)}\right]Exp\left(-y\sqrt{C^2+s}\right)Exp\left(\frac{Sy}{2}\right)$$

$$+\frac{KM^2}{(M^2+s)t_0 s^2}[1.-Exp(-st_0)] \tag{3.2.14}$$

where

$$C^2=\frac{S^2}{4.}+M^2$$

The exact solution for velocity field can be obtained from Eq. (3.2.14), by taking the inverse transforms as in Abramowitz and Stegun (1965) and Carslaw and Jaeger(1963).After detailed simplifications and shifting on the t-axis, the solution can be expressed as:

$$u(y,t)=\frac{[1-K]}{t_0}Exp\left(\frac{Sy}{2.}\right)[F_1(y;C;t;0.0)-F_1(y;C;t:t_0)H(t-t_0)]$$
$$+\frac{K}{M^2 t_0}Exp\left(\frac{Sy}{2.}\right)\left[F_2(y;C;t;0.0)-Exp(-M^2 t)F_2\left(y;\frac{S}{2};t:0.0\right)\right]$$

$$+\frac{K}{M^2 t_0}Exp\left(\frac{Sy}{2.}\right)\left[Exp(-M^2(t-t_0))F_2\left(y;\frac{S}{2.};t;t_0\right)-F_2(y;C;t;t_0)\right]H(t-t_0)$$

$$+\frac{K}{t_0}\left[\frac{Exp(-M^2 t)}{M^2}-\frac{1}{M^2}+t\right]$$

$$+\frac{K}{t_0}H(t-t_0)\left[\frac{1}{M^2}-\frac{Exp(-M^2(t-t_0))}{M^2}-(t-t_0)\right] \tag{3.2.15}$$

3.2.3 Skin friction

Using the expression (3.2.15), the shear-stress at the moving porous plate is:

$$\tau = -\left.\frac{\partial u}{\partial y}\right|_{y=0}$$

$$= \frac{[1-K]}{t_0}\left[F_4(C;t;0.0) - \frac{S}{2.}F_1(0.0;C;t;0.0) - \left(F_4(C;t;t_0) - \frac{S}{2.}F_1(0.0;C;t;t_0)\right)H(t-t_0)\right]$$

$$+ \frac{K}{M^2}\frac{1}{t_0}\left[F_3(C;t;0.0) - \frac{S}{2}F_2(0.0;C;t;0.0) - Exp(-M^2 t)\left(F_3\left(\frac{S}{2.};t;0.0\right) - \frac{S}{2.}F_2\left(0.0;\frac{S}{2.};t;0.0\right)\right)\right]$$

$$+ \frac{K}{M^2}\frac{1}{t_0}\left[Exp(-M^2(t-t_0))\left(F_3\left(\frac{S}{2.};t;t_0\right) - \frac{S}{2.}F_2\left(0.0;\frac{S}{2.};t;t_0\right)\right)\right.$$

$$\left. - \left(F_3(C,t,t_0) - \frac{S}{2.}F_2(0.0;C;t;t_0)\right)\right]H(t-t_0) \qquad (3.2.16)$$

The equation (3.2.15) and (3.2.16) represents the unified expressions for velocity and skin-friction at the porous plate of conducting fluid due to ramped motion of the horizontal porous plate in presence of transverse magnetic field fixed with respect to fluid or moving porous plate.

3.2.4 Special Case

For $t_0 = 0$, the equation (3.2.15) is

$$u(y,t) = Exp\left(\frac{S}{2}y\right)[1.-K]F_2(y;C;t;0.0) + K\left[1.-Exp(-M^2 t)\right]$$
$$+ Exp\left(\frac{S}{2}y\right)Exp(-M^2 t)F_2\left(y;\frac{S}{2.};t;0.0\right) \qquad (3.2.17)$$

and the equation (3.2.16) is

$$\tau = -\left.\frac{\partial u}{\partial y}\right|_{y=0}$$

$$= [1.-K]\left[F_3(C;t;0.0) - \frac{S}{2.}F_2(0.0,C;t;0.0)\right]$$

$$+ KExp\left(-M^2 t\right)\left[F_3\left(\frac{S}{2.};t;0.0\right)-\frac{S}{2.}F_2\left(0.0,\frac{S}{2.};t;0.0\right)\right]. \qquad (3.2.18)$$

The equation (3.2.17) and (3.2.18) represent the unified expression for velocity field and skin-friction of the conducting fluid due to impulsive motion of the horizontal porous plate, in the presence of transverse magnetic field either fixed with respect to fluid or moving porous plate. Furthermore, for $K = 0.0$, the solution represents the Rayleigh problem in magneto-hydrodynamics with suction/injection also for $M^2 = 0$, solution boils down to Rayleigh problem in hydro-dynamics with suction/injection.

3.2.5 Results and Discussions

In this section, some representative results are presented to illustrate the effects of the controlling parameters when $t_0 = 2.0$. A FORTRAN program was written to compute the boundary layer velocity and the skin friction, using equations (3.2.15) and (3.2.16) respectively. The velocity profiles and skin frictions in Figs. 3.2.2 to 3.2.10 are the MS Excel plots of these computed values.

Figure 3.2.2 shows the temporal and spatial variations of the boundary layer velocity for ramped motion of the porous plate subjected to injection when the magnetic field is fixed relative to the fluid while Fig.3.2.3 shows the corresponding results for the magnetic field fixed with respect to the moving porous plate. The time dependent flow formation is such that the velocity decreases from its initial value at the boundary to its free-stream stationary value through layers of different depths and the boundary layer thicknesses depend on the initial conditions as well as the mode of application of magnetic field. As far as the temporal variations are concerned, the velocity increases gradually in the boundary layer.

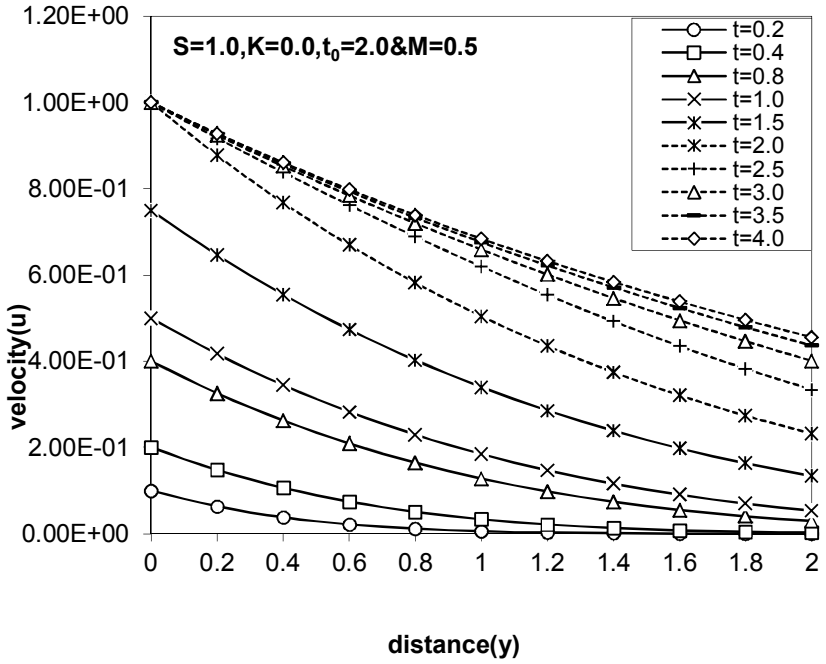


Fig.3.2.2Velocity (u) versus distance (y) for different values of time (t)

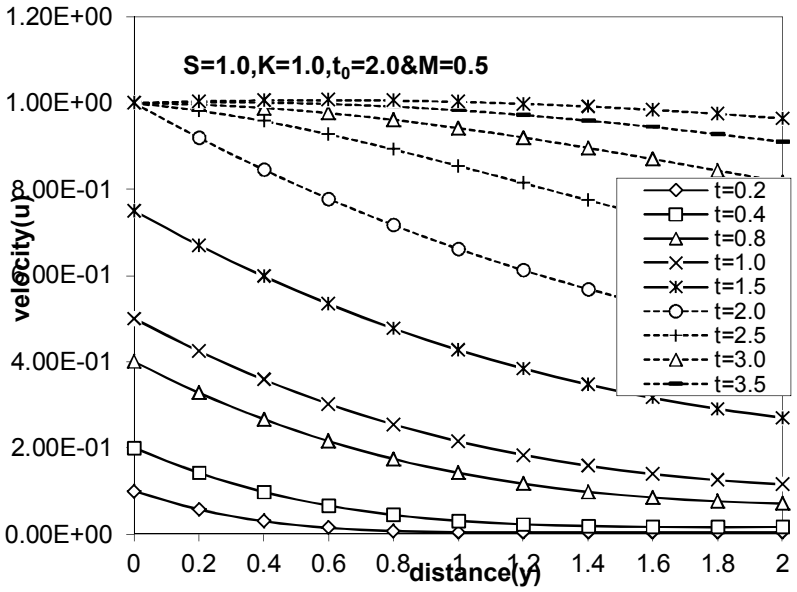


Fig.3.2.3 Velocity (u) versus distance (y) for different values of time (t).

Fig.3.2.4 and 3.2.5 illustrates the typical velocity profiles in the presence of suction at the porous plate when the magnetic field is fixed with respect to fluid and moving porous plate respectively.

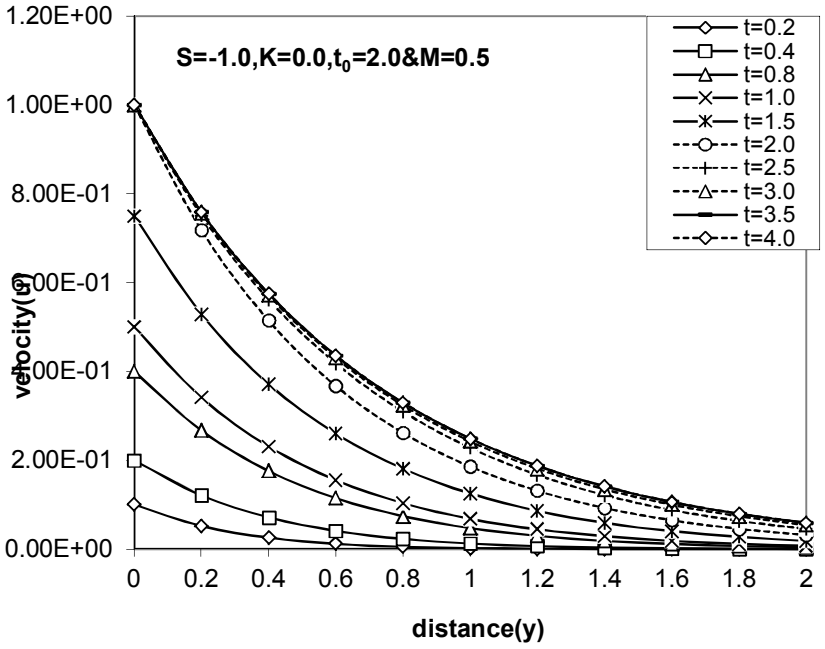


Fig.3.2.4Velocity (u) versus distance (y) for different values of time (t)

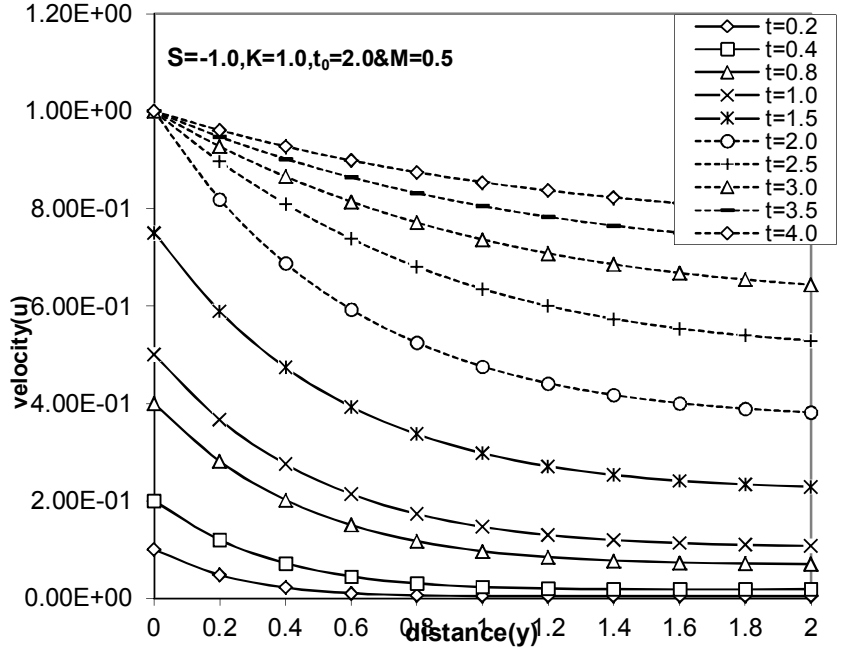


Fig.3.2.5 Velocity (u) versus distance (y) for different values of time (t)

The effect of the magnetic field and suction/injection on the flow formation is shown in fig.3.2.6. A comparative study reflects that the effect of injection is to increase the velocity while the suction decreases it. The physical interpretation of velocity increase

due to injection is borne out of the fact that injection increase thickens the momentum boundary layer which eventually increases the fluid velocity while suction causes a thinning of the boundary layer which leads to a decrease in fluid velocity.Furthermore the impact of magnetic field is to increase the velocity when the magnetic field is fixed with respect to moving porous plate while the result is just reverse when it is fixed with respect to fluid. This is physically true because when magnetic lines of force move with the same velocity of the porous plate ($K = 1.0$), the electromagnetic force tends to accelerate the fluid motion. On the other hand, when the magnetic lines of force are fixed with fluid, the resulting Lorentz force is a retarding force. This retarding effect is manifested by the reduction of the velocity profile, as can be seen from equation (3.2.6). For $K = 0.0$, equation (3.2.6) contains $-M^2u$ which reduces the velocity as magnetic field increases. On the other hand, for $K = 1.0$, equation (3.2.6) contains $-M^2u + M^2 f(t)$, which supports the velocity as magnetic field increases.

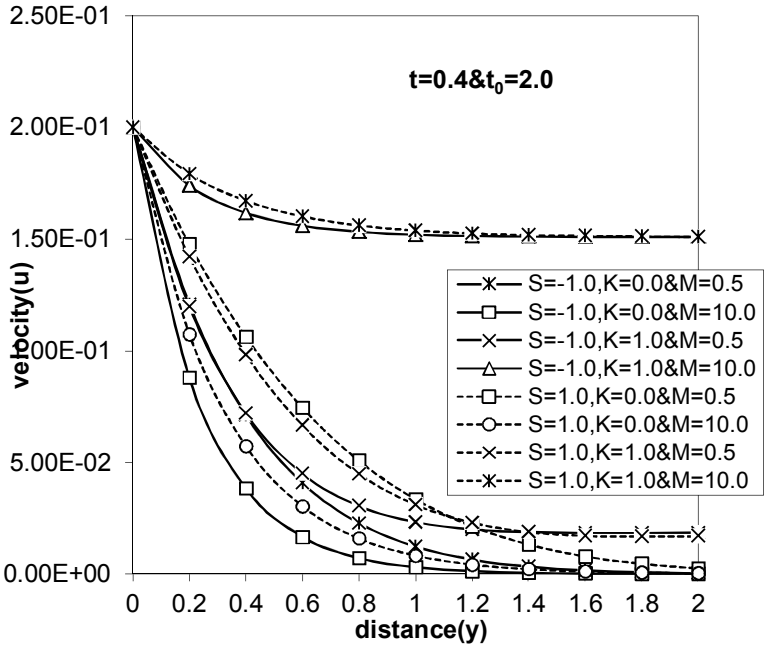


Fig.3.2.6 Velocity (u) versus distance (y) for different values of M, K and S.

In order to compare the influence of ramped motion of the porous plate versus constant motion of the porous plate on the fluid motion, velocity profiles are depicted in fig.3.2.7 and fig.3.2.8 for magnetic field fixed with respect to fluid and porous plate respectively. From these figures it is observed that the velocity in the case of ramped motion of the porous plate is always less than that of the flow induced by constant motion of the porous plate in the resence of suction/injection. This should be physically true since in case of ramped motion of the porous plate, the motion takes place more gradually than in the constant motion of the porous plate.

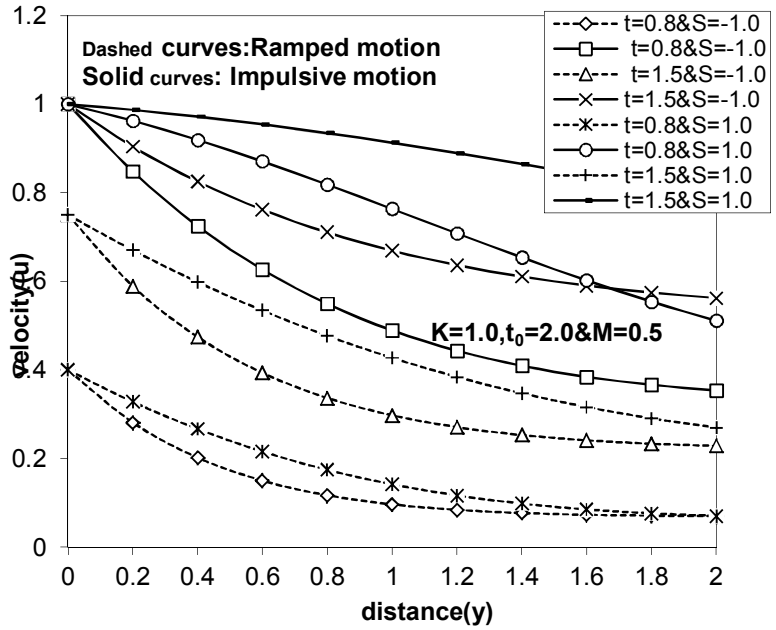


Fig.3.2.7 Velocity (u) versus distance (y) for different values of S and t

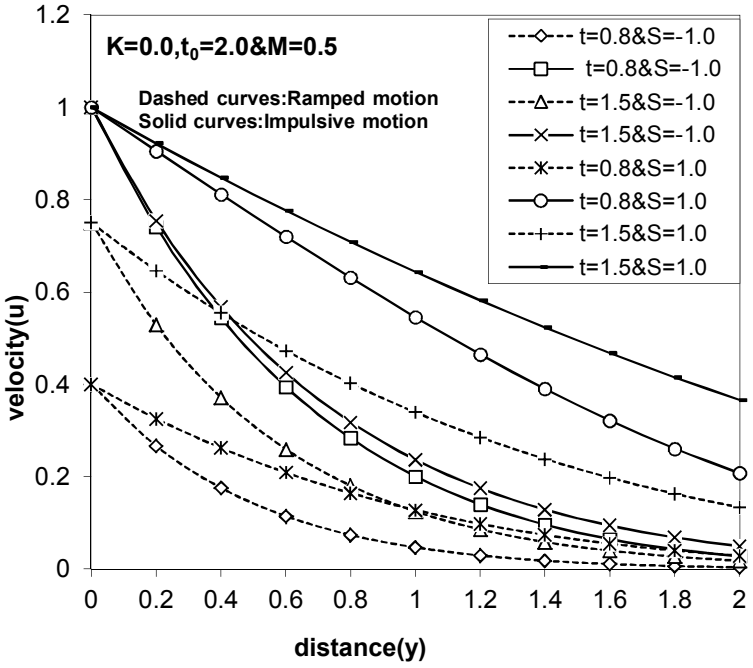


Fig.3.2.8 Velocity (u) versus distance (y) for different values of S and t

The variation of skin frictions are shown in figure 2.2.9 and 2.2.10. Fig 2.2.9 reflects that skin friction increases for $0<t<t_0$, and decreases for $t>t_0$, for all values of S and K. On the other hand, skin friction is greater in case of suction than injection. This cause is explained by the fact that momentum boundary layer becomes thinner with increasing suction and hence, the velocity gradient becomes higher near the moving porous plate leading to an increase in skin friction. Figure 2.2.9 also indicates that skin friction is higher, when the magnetic field is fixed with respect to the fluid in comparison to magnetic field fixed with respect to the moving porous plate. This is due to the fact that when magnetic lines of force are fixed with moving porous plate, velocity gradient is smaller (see figure 2.2.6) consequently skin friction is lower.

In order to see the effect of porous plate on skin friction, variations of skin friction with time for different values of S, K and M are shown in fig.2.2.10, for impulsive and ramped motion of the porous plate. A clear indication from it is that skin friction increases with time and attains its maximum at $t = t_0$ and then decreases, for ramped motion of the porous plate. Moreover, skin friction decreases for constant motion of the porous plate as time increases. Physically speaking, as time increases fluid velocity increases, thereby decreasing the velocity gradient on the moving porous plate and consequently a decrease in skin friction in the result. It is interesting to note that skin-friction due to ramped motion and constant motion coincides as time increases. This is anexpected result because the effect of ramped motion of the porous plate on fluid motion tends to that of constant motion of the porous plate when $t \geq t_0$.

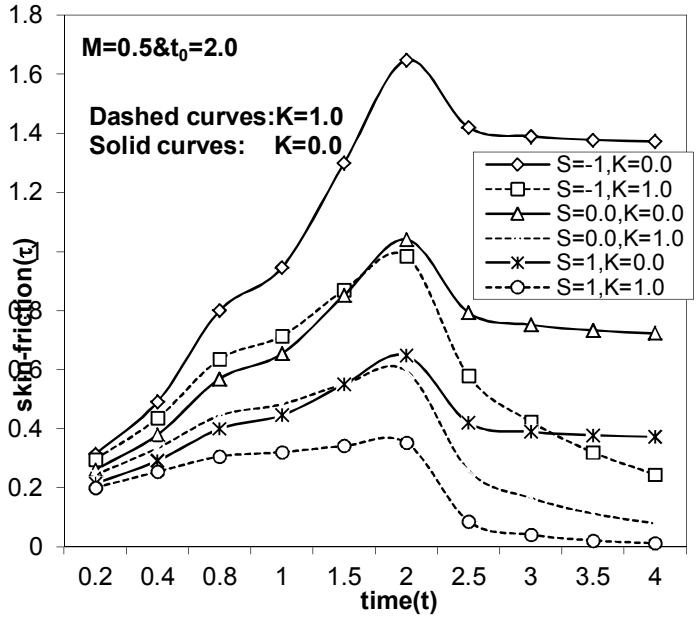


Fig.3.2.9 Skin friction (τ) versus time (t) for different values of S and K

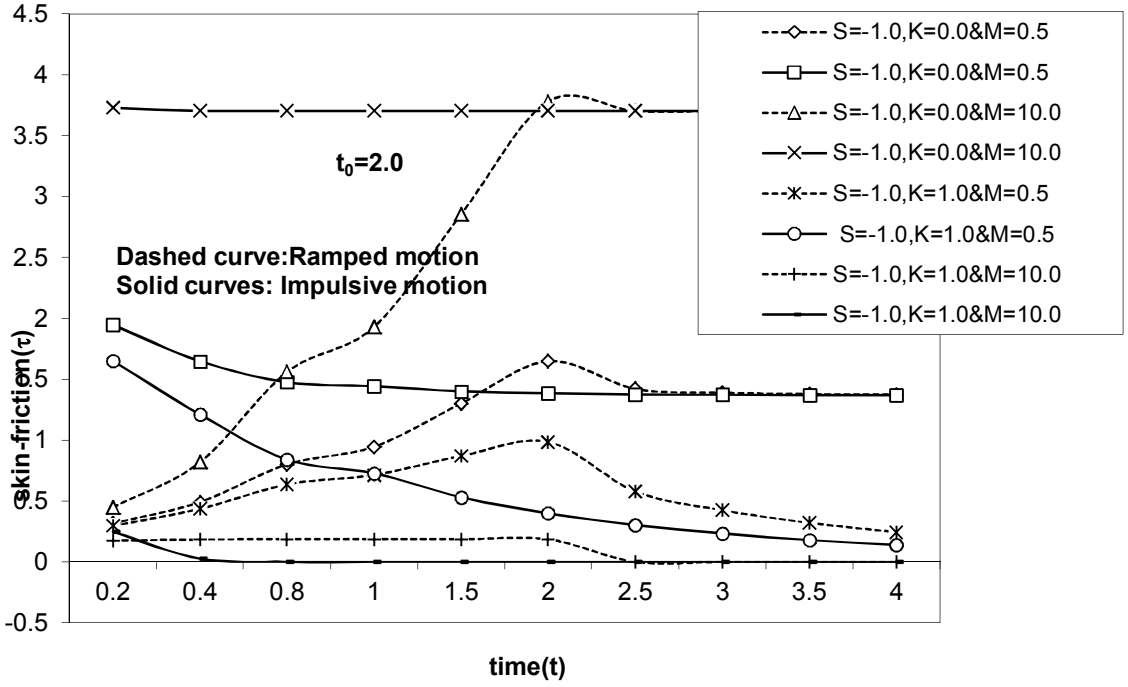


Fig.3.2.10 Skin friction (τ) versus time (t) for different values S, K *and* M

3.2.6 Summary and Conclusion

The present article analyzed the effects of magnetic field and mass leakage on the unsteady flow formation of an incompressible, viscous and electrically conducting fluid in the presence of transverse magnetic field. The flow is generated due to the ramped motion of the horizontal porous plate. It is observed that the impact of magnetic field is to increase the velocity when the magnetic field is fixed with respect to the moving porous plate. The result is just the reverse when it is fixed with respect to the fluid. It is also seen that the velocity in the case of ramped motion of the porous plate is always less than that of the flow induced by the constant motion of the porous plate.

CHAPTER FOUR

UNSTEADY MHD COUETTE FLOW BETWEEN TWO PARALLEL PLATES DUE TO RAMPED MOTION OF ONE OF THE BOUNDARIES

4.0 Introduction

This chapter is concerned with the unsteady hydromagnetic Couette flow between two parallel plates due to ramped motion of the lower plate. Two cases were considered.The first part discussed the case one when the channel is non-permeable and case two when the plates are porous.

4.1 Time dependent MHD Couette flow due to ramped motion of one of the boundaries

4.1.0 Introduction

In this section the unsteady hydromagnetic Couette flow between two parallel plates due to ramped motion of the lower plate is considered. It is assumed that both plates are uniformly permeable and that fluid enters and leaves the plates at the same constant rate.

4.1.1 Formulation of the Problem

The time dependent hydromagnetic flow of a viscous, incompressible and electrically conducting fluid, bounded by two infinite parallel plates, distant d apart, due to the ramped motion of one of the plates ($y' = 0$) is considered , as shown in Fig 4.1.1.

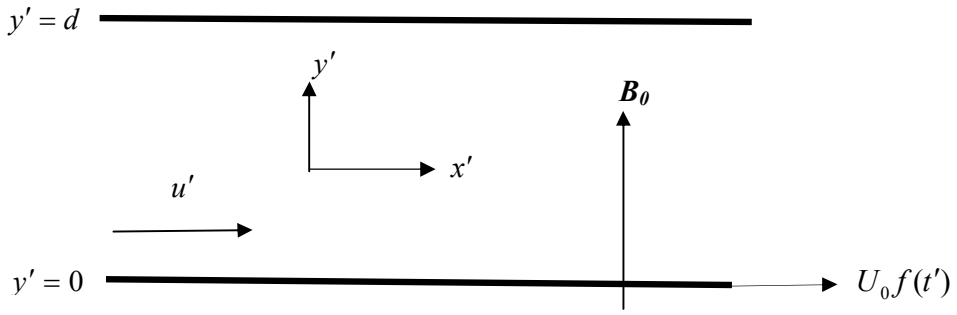


Fig.4.1.1 Physical configuration of flow

The x'-axis is chosen along the lower plate in the horizontal direction and the y'-axis normal to it. The fluid flows between the two plates in the x'-direction in the presence of a uniform magnetic field acting perpendicular to the main flow direction. It is assumed that the plates are electrically non-conducting and that the applied uniform magnetic field B_0 is acting perpendicular to the flow direction. Since the plates are infinite in extent all the variable quantities are taken to be functions of y' and t'only. By assuming a small magnetic Reynolds number the induced magnetic field is neglected in comparison to the applied magnetic field. Therefore the uniform magnetic B is a constant B= $(0, B_0, 0)$ and is considered as the total magnetic field acting on the fluid. Also since no applied or polarization voltage is imposed on the flow field, the electric field vector $E = 0$. This corresponds to the case when no energy is added or extracted from the fluid by the electric field.

Initially, $(t' \le 0)$, both the plates as well as the fluid are assumed to be at rest. When $t' > 0$, the lower plate starts moving in its own plane with a velocity $U_0 f(t')$, where U_0 *is* constant and the upper plate remain stationary.

Under the above physical conditions the flow is governed by the following Equations:

$$\frac{\partial u'}{\partial t} = \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma {B_0}^2}{\rho} u' \qquad (4.1.1)$$

Equation (4.1.1) is valid when the magnetic lines of force are fixed relative to the fluid.

If the magnetic field is fixed relative to the moving plate , equation(4.1.1) can be replaced by

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma {B_0}^2}{\rho} \left[u' - U_0 f(t')\right] \qquad (4.1.2)$$

Equation (4.1.1) and (4.1.2) can be combined together to obtain

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma {B_0}^2}{\rho} \left[u' - K U_0 f(t')\right] \qquad 43.1.3)$$

Where

$$K = \begin{cases} 0 \text{ if magnetic field is fixed relative to the fluid} \\ 1 \text{ if magnetic field is fixed relative to the plate} \end{cases}$$

The initial and boundary conditions relevant to the problem are

$$t' \le 0: \quad u' = 0, \ for \quad 0 \le y' \le d$$

$$t'>0; \begin{cases} u' = U_0 f(t') & \text{at } y' = 0 \\ u' = 0 & \text{at } y' = d \end{cases} \qquad (4.1.4)$$

Introducing the non-dimensionless quantities,

$$y = \frac{y' U_0}{\nu}, t = \frac{t' U_0^2}{\nu}, u = \frac{u'}{U_0}, M^2 = \frac{\sigma B_0^2 \nu}{\rho U_0^2}, h = \frac{d U_0}{\nu}, \qquad (4.1.5)$$

Where all the quantities are defined in nomenclature, the momentum equation becomes

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - M^2 \left[u - K f(t)\right] \qquad (4.1.6)$$

The initial and boundary conditions in equation (4.1.4) in the dimensionless form become

For $t \le 0;\ \ u = 0$ for all $y \ge 0$

$$t>0;\ \begin{cases} u = f(t) & \text{at } y = 0 \\ u = 0 & \text{at } y = h \end{cases} \qquad (4.1.7)$$

In the present work $f(t)$ is specified as the ramp function:

$$f(t) = \frac{t}{t_0} \quad \text{if } 0 \le t \le t_0$$
$$= 1 \quad \text{if } t \ge t_0 \qquad (4.1.8)$$

Eq.(4.1.8) can be written as:

$$f(t) = H(t)\left(\frac{t}{t_0}\right) - \left(\frac{1.}{t_0}\right)(t - t_0)H(t - t_0) \qquad (4.1.9)$$

Where $H(t)$ is the Heaviside unit step function defined by

$$H(t) = \begin{Bmatrix} 0, & t < 0 \\ 1, & t \ge 0 \end{Bmatrix} \qquad (4.1.10)$$

4.1.2 Mathematical solution

The solution of the partial differential Eq.(4.1.6) subject to conditions (4.1.7) is attempted by the operational method of Laplace transformation. Let

$$L[u(y,t)] = \bar{u}(y,s) = \int_0^\infty u(y,t)\exp(-st)dt \qquad (4.1.11)$$

be the Laplace transform of $u(y,t)$, then Eqns. (4.1.5), (4.1.6) and (4.1.9) reduce to:

$$\frac{d^2\bar{u}}{dy^2} - (M^2 + s)\bar{u} = -KM^2\bar{f}(s) \qquad (4.1.12)$$

$t \le 0;\ \bar{u} = 0$ for all $y \ge 0$

$$t>0;\ \begin{cases} \bar{u} = f(s) & \text{at } y = 0 \\ \bar{u} = 0 & \text{at } y = h \end{cases} \qquad (4.1.13)$$

$$\bar{f}(s)=\frac{[1.-Exp(-st_0)]}{s^2 t_0} \tag{4.1.14}$$

The solution of Eq. (4.1.12), under the conditions (4.1.13) and (4.1.14), is:

$$\bar{u}=\frac{1}{t_0 s^2}[1.-Exp(-st_0)]\sum_{m=0}^{\infty}\left[\left(1.-\frac{KM^2}{(M^2+s)}\right)\left(Exp\left(-a\sqrt{M^2+s}\right)-Exp\left(-b\sqrt{M^2+s}\right)\right)\right.$$

$$\left.+\left(\frac{KM^2}{(M^2+s)}\right)\left(Exp\left(-c\sqrt{M^2+s}\right)-Exp\left(-d\sqrt{M^2+s}\right)\right)\right]$$

$$+\frac{KM^2}{(M^2+s)t_0 s^2}[1.-Exp(-st_0)] \tag{4.1.15}$$

where

$$a=2mh+y, b=2mh+2h-y, c=2mh+h+y, d=2mh+h-y. \tag{4.1.16}$$

In equation (4.1.15), summation term is due to expansion of term $\left[1.-Exp\left(-2h\sqrt{M^2+s}\right)\right]^{-1}$ by Binomial theorem.

The exact solution for velocity field can be obtained from Eq. (4.1.15), by taking the inverse transforms Abramowitz and Stegun(1965) and Carslaw and Jaeger(1963). After detailed simplifications and shifting on the t-axis, the solution can be expressed as:

$$u(y,t)=\frac{1}{t_0}\sum_{m=0}^{\infty}[F_1(a;M;t;0.0)-F_1(b;M;t:0.0)-H(t-t_0)(F_1(a;M;t;t_0)-F_1(b;M;t:t_0))$$

$$-\frac{K}{t_0}(F_1(a;M;t;0.0)-F_1(b;M;t;0.0)-F_1(c;M;t;0.0)+F_1(d;M;t;0.0))$$

$$+\frac{K}{t_0}H(t-t_0)(F_1(a;M;t;t_0)-F_1(b;M;t;t_0)-F_1(c;M;t;t_0)+F_1(d;M;t;t_0))$$

$$+\frac{K}{t_0 M^2}(F_2(a;M;t;0.0)-F_2(b;M;t;0.0)-F_2(c;M;t;0.0)+F_2(d;M;t;0.0))$$

$$-\frac{K}{t_0}H(t-t_0)\left(F_2(a;M;t;t_0)-F_2(b;M;t;t_0)-F_2(c;M;t;t_0)+F_2(d;M;t;t_0)\right)$$

$$-\frac{K}{t_0M^2}Exp\left(-M^2t\right)\left(Erfc\left(\frac{a}{2\sqrt{t}}\right)-Erfc\left(\frac{b}{2\sqrt{t}}\right)-Erfc\left(\frac{c}{2\sqrt{t}}\right)+Erfc\left(\frac{d}{2\sqrt{t}}\right)\right)$$

$$+\frac{K}{t_0M^2}H(t-t_0)Exp\left(-M^2(t-t_0)\right)\left(\begin{array}{l}Erfc\left(\frac{a}{2\sqrt{t-t_0}}\right)-Erfc\left(\frac{b}{2\sqrt{t-t_0}}\right)\\-Erfc\left(\frac{c}{2\sqrt{t-t_0}}\right)+Erfc\left(\frac{d}{2\sqrt{t-t_0}}\right)\end{array}\right)$$

$$-\frac{K}{t_0}\left[\frac{1}{M^2}-\frac{Exp\left(-M^2t\right)}{M^2}-t\right]+\frac{K}{t_0}H(t-t_0)\left[\frac{1}{M^2}-\frac{Exp\left(-M^2(t-t_0)\right)}{M^2}-(t-t_0)\right]$$

(4.1.17)

Using the expression (4.1.17), the frictional force acting at the lower and upper plates are:

$$\tau_0=-\left.\frac{\partial u}{\partial y}\right|_{y=0}$$

$$=\frac{1}{t_0}\sum_{m=0}^{\infty}\left[F_3(b_1;M;t;0.0)-F_3(a_1;M;t:0.0)+H(t-t_0)\left(F_3(a_1;M;t;0.0)+F_3(b_1;M;t:t_0)\right)\right.$$

$$+\frac{K}{t_0}\left(F_3(a_1;M;t;0.0)+F_3(b_1;M;t:0.0)-2F_3(c_1;M;t;0.0)\right)$$

$$-\frac{K}{t_0}H(t-t_0)\left(F_3(a_1;M;t;t_0)+F_3(b_1;M;t:t_0)-2F_3(c_1;M;t;t_0)\right)$$

$$-\frac{K}{t_0M^2}\left(F_4(a_1;M;t;0.0)+F_4(b_1;M;t:0.0)-2F_4(c_1;M;t;0.0)\right)$$

$$+\frac{K}{t_0M^2}H(t-t_0)\left(F_4(a_1;M;t;t_0)+F_4(b_1;M;t:t_0)-2F_4(c_1;M;t;t_0)\right)$$

$$+\frac{K}{t_0M^2}Exp\left(-M^2t\right)\left(F_4(a_1;0.0;t;0.0)+F_4(b_1;0.0;t:0.0)-2F_4(c_1;0.0;t;0.0)\right)$$

$$-\frac{K}{t_0 M^2}Exp\left(-M^2(t-t_0)\right)H(t-t_0)\begin{pmatrix}F_4(a_1;0.0;t;t_0)\\+F_4(b_1;0.0;t:t_0)-2F_4(c_1;0.0;t;t_0)\end{pmatrix}\Bigg] \quad (4.1.18)$$

$$\tau_h = \left.\frac{\partial u}{\partial y}\right|_{y=h}$$

$$=\frac{1}{t_0}\sum_{m=0}^{\infty}\left[2F_3(c_1;M;t;0.0)+H(t-t_0)\left(F_3(c_1;M;t;0.0)\right)\right.$$

$$-\frac{K}{t_0}\left(2F_3(c_1;M;t;0.0)-F_3(b_1;M;t:0.0)-F_3(a_1;M;t;0.0)\right)$$

$$+\frac{K}{t_0 M^2}\left(2F_4(c_1;M;t;0.0)-F_4(b_1;M;t:0.0)-F_4(a_1;M;t;0.0)\right)$$

$$-\frac{K}{t_0 M^2}H(t-t_0)\left(F_4(c_1;M;t;t_0)-F_4(b_1;M;t:t_0)-F_4(a_1;M;t;t_0)\right)$$

$$-\frac{K}{t_0 M^2}Exp\left(-M^2 t\right)\left(2F_4(c_1;0.0;t;0.0)-F_4(b_1;0.0;t:0.0)-F_4(a_1;0.0;t;0.0)\right)$$

$$+\frac{K}{t_0 M^2}Exp\left(-M^2(t-t_0)\right)H(t-t_0)\begin{pmatrix}2F_4(c_1;0.0;t;t_0)\\-F_4(b_1;0.0;t:t_0)-F_4(c_1;0.0;t;t_0)\end{pmatrix}\Bigg] \quad (4.1.19)$$

Where

$a_1 = 2mh, b_1 = 2mh+2h, c_1 = 2mh+h.$

4.1.3 Special case: Impulsive motion of one of the boundaries

For $t_0 = 0$, the expression (4.1.17) reduces to

$$u(y,t)=\sum_{m=0}^{\infty}\left[(1.-K)\left[F_2(a;M;t;0.0)-F_2(b;M;t;0;0)\right]\right.$$

$$+KExp\left(-M^2 t\right)\left[Erfc\left(\frac{a}{2\sqrt{t}}\right)-Erfc\left(\frac{a}{2\sqrt{t}}\right)\right]$$

$$+K\left[F_2(c;M;t;0.0)-F_2(d;M;t;0;0)\right]$$

$$-KExp\left(-M^2 t\right)\left[Erfc\left(\frac{c}{2\sqrt{t}}\right)-Erfc\left(\frac{d}{2\sqrt{t}}\right)\right]$$

$$+K\left[1-Exp\left(-M^2t\right)\right]\Big] \qquad (4.1.20)$$

4.1.4 Skin friction

The skin friction at the lower and upper plates is obtained from equation (4.1.20) respectively as follows:

$$\tau_0 = -\left.\frac{\partial u}{\partial y}\right|_{y=0}$$

$$=\sum_{m=0}^{\infty}[(K-1)\left[F_4\left(a_1;M;t;0.0\right)+F_4\left(b_1;M;t:0,0\right)\right]$$

$$+\frac{KExp\left(-M^2t\right)}{\sqrt{\pi t}}\left[EXP\left(\frac{-a_1^{\,2}}{4t}\right)+EXP\left(\frac{-b_1^{\,2}}{4t}\right)\right]$$

$$-2K\left[F_4\left(c_1;M;t;0.0\right)\right]$$

$$-\frac{2KExp\left(-M^2t\right)}{\sqrt{\pi t}}\left[EXP\left(\frac{-c_1^{\,2}}{4t}\right)\right]] \qquad (4.1.21)$$

$$\tau_h = \left.\frac{\partial u}{\partial y}\right|_{y=h}$$

$$=\sum_{m=0}^{\infty}[2(K-1)F_4\left(c_1;M;t;0.0\right)+\frac{2K\,Exp\left(-M^2t\right)}{\sqrt{\pi t}}\left[EXP\left(\frac{-c_1^{\,2}}{4t}\right)\right]$$

$$-K[F_4\left(b_1;M;t;0.0\right)+F_4\left(a_1;M;t:0,0\right)]$$

$$-\frac{2KExp\left(-M^2t\right)}{\sqrt{\pi t}}\left[EXP\left(\frac{-c_1^{\,2}}{4t}\right)\right]]$$

$$+\frac{KExp\left(-M^2t\right)}{\sqrt{\pi t}}\left[EXP\left(\frac{-a_1^{\,2}}{4t}\right)+EXP\left(\frac{-b_1^{\,2}}{4t}\right)\right]] \qquad (4.1.22)$$

4.1.5 Steady State Solution

As time increases, the response of the velocity with respect to time becomes insignificant. This implies that the flow velocity has attained its steady state value at that time. Thus, expression for velocity distribution of the problem at this state can be obtained by setting $\frac{\partial u}{\partial t} = 0$ in equation (4.1.6).This reduce the equation to

$$\frac{d^2u}{dy^2} - M^2u = -KM^2 \tag{4.1.23}$$

Equation (4.1.23) is solved subject to the boundary conditions:

$$\begin{aligned} u &= 1 \quad at \quad y = 0 \\ u &= 0 \quad at \quad y = h \end{aligned} \tag{4.1.24}$$

The solution of Equation (4.1.23) subject to the boundary conditions (4.1.24) is expressed as:

$$u(y) = \frac{(1-K)\sinh(M(h-y))}{\sinh(Mh)} + K[1 - \frac{\sinh(My)}{\sinh(hM)}] \tag{4.1.25}$$

Using expression (4.1.25), the frictional forces at the lower and upper plate is:

$$\tau_0 = -\left.\frac{du}{dy}\right|_{y=0} = \frac{KM + M(1-K)\cosh(hM)}{\sinh(Mh)} \tag{4.1.26}$$

$$\tau_h = -\left.\frac{du}{dy}\right|_{y=h} = \frac{M(1-K) + MK\cosh(hM)}{\sinh(Mh)} \tag{4.1.27}$$

4.1.6. Results and Discussion

In order to study the effects of magnetic field on velocity and skin-friction, due to ramped motion of one of the boundaries, numerical values of the velocity and skin fiction where computed taking the values of $t_0 = 2.0$ and h=1.0, for various values of time (t) and Hartmann number (M).The velocity profiles due to ramp and impulsive motion of the horizontal plate are plotted as shown in Fig.4.1.2 to Fig.4.1.13

. Fig.4.1.2 shows the variation of velocity profile due to the ramp motion of the plate for different values of time when the magnetic lines of force are fixed with respect to the fluid for fixed value of magnetic field. Fig.4.1.3 shows the variation of velocity field due to the ramp motion of the plate for different values of time when the magnetic lines of force are fixed with respect to the moving plate for fixed value of magnetic field. From these figures we conclude that as time increases velocity increases whether the magnetic field is fixed with respect to the fluid or moving plate.

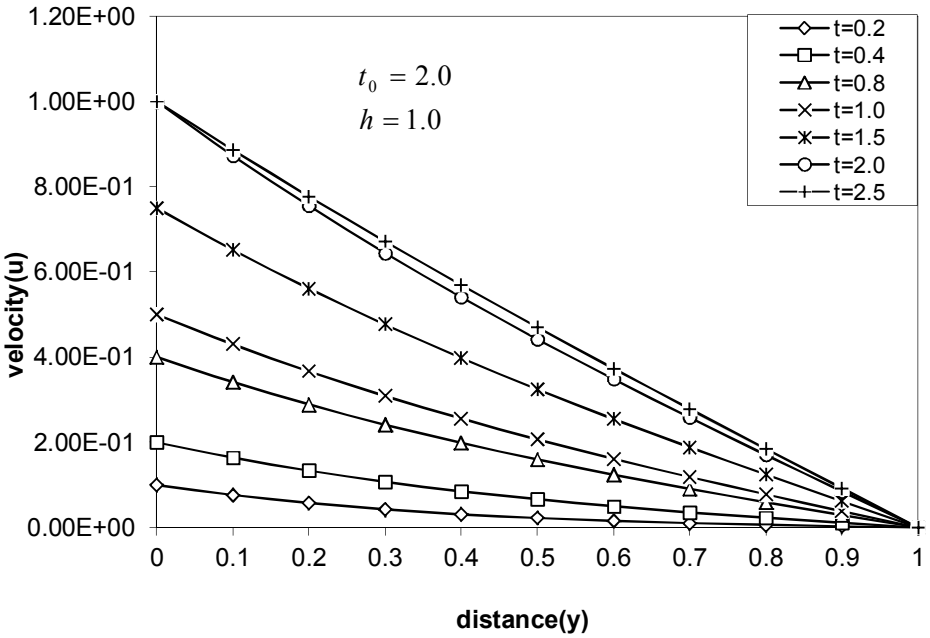


Fig.4.1.2Velocity (u)versus distance (y)for different time ($M = 0.5$ and $K = 0.0$)

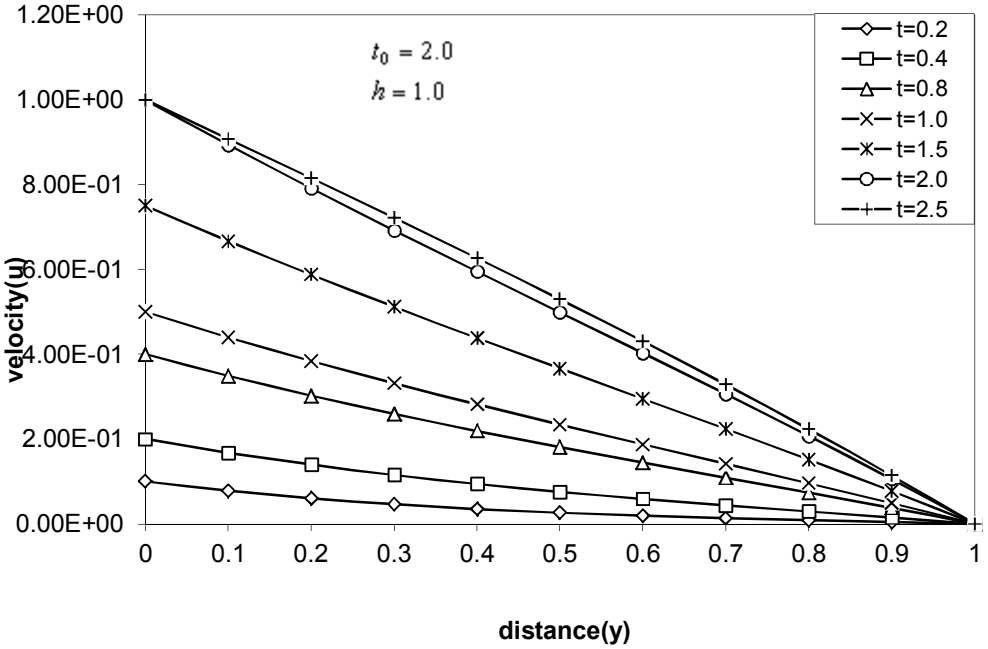


Fig.4.1.3Velocity (u)versus distance (y) for different time ($M = 0.5$ and $K = 1.0$)

Fig.4.1.4 reflects the velocity variation with time due to ramp motion of the boundary when magnetic lines of force are fixed with fluid or moving plate for fixed values of the magnetic

field. This figure indicates that velocity is higher when the magnetic lines of force are fixed relative to the moving plate in comparison to when the magnetic lines of force are fixed relative to the fluid.

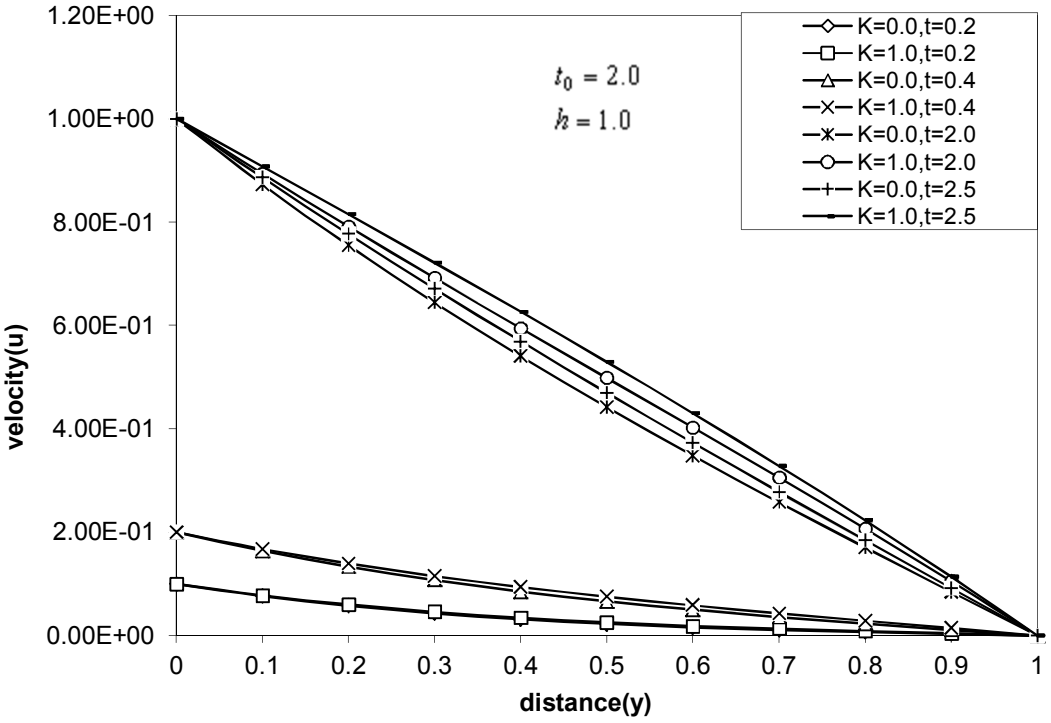


Fig.4.1.4 Velocity (u) versus distance (y) for different time ($M = 0.5$ and $K = 0.0$ or $K = 1.0$)

In order to see the impact of ramp motion of the boundary on flow formation we compared the velocity profile due to ramp as well as impulsive motion of the boundary in fig.4.1.5 for fixed value of M when the magnetic lines of force are fixed with respect to the fluid. Fig.4.1.6 shows the variation of velocity due to ramp as well

as impulsive motion of the boundary for different time for fixed value of M when the magnetic lines of force are fixed with respect to the moving plate.

From these figures we concluded that the velocity field is higher in case of impulsive motion of the plate in comparison to the ramp motion of the plate whether the magnetic field is fixed with respect to the fluid or moving plate. These should be expected since in the ramp motion of the plate, the velocity of the fluid takes place more gradually than the impulsive motion of the plate.

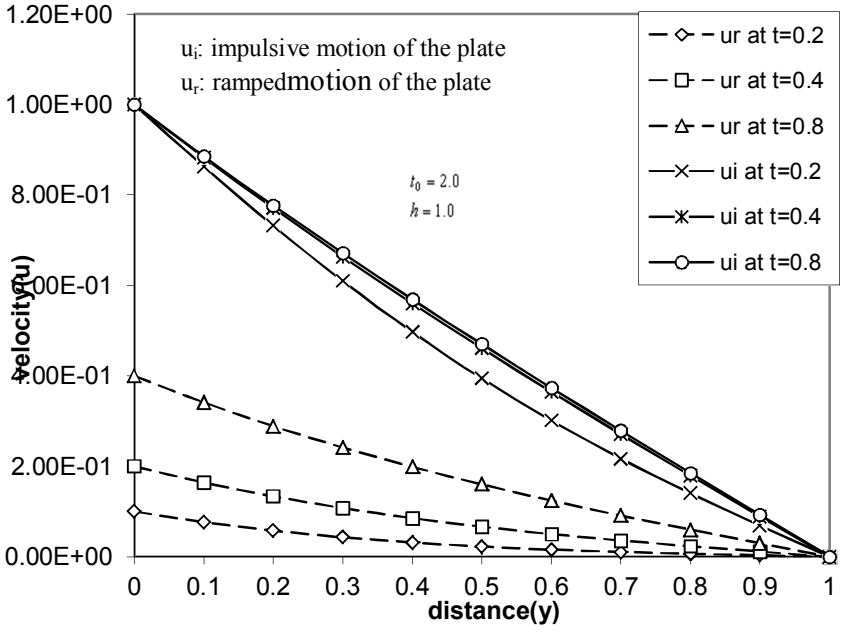


Fig 4.1.5. Velocity (u) versus distance (y) for different time ($M = 0.5$ and $K = 0.0$))

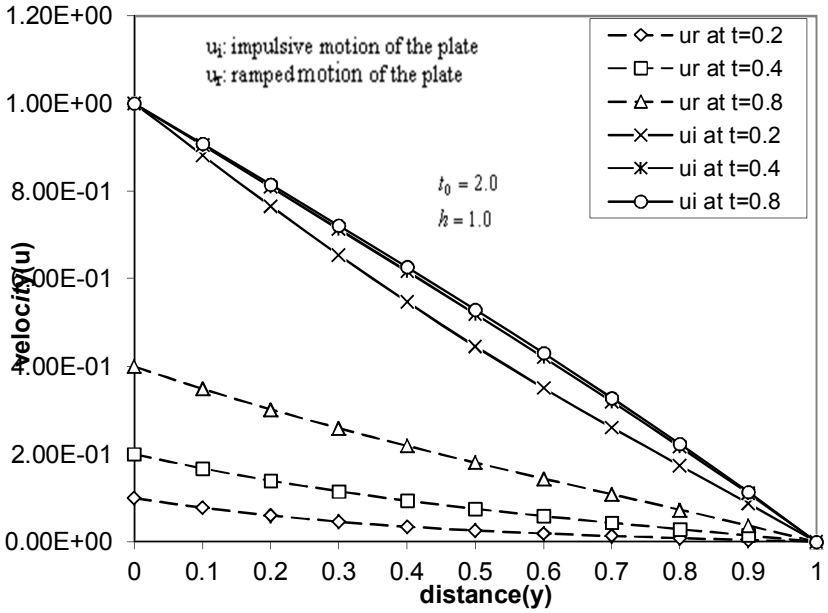


Fig.4.1.6 Velocity (u) versus distance (y) for different time ($M = 0.5$ and $K = 1.0$)

The effect of the Hartmann number, on the velocity profile, due to the ramp and impulsive motion of the boundary for fixed value of time, when the magnetic field is fixed with respect to the fluid or moving plate, is depicted in fig.4.1.7. This figure reflects that fluid velocity is always higher in case of impulsive motion of the plate in comparison to the ramp motion of the plate whether the magnetic field is fixed with respect to the fluid or moving plate. Furthermore we also conclude that as magnetic field increases velocity increases for impulsive as well as ramp motion of the plate when magnetic lines of force are fixed relative to the moving plate, while the result is just the contrast when magnetic lines of force are fixed with respect to the fluid.

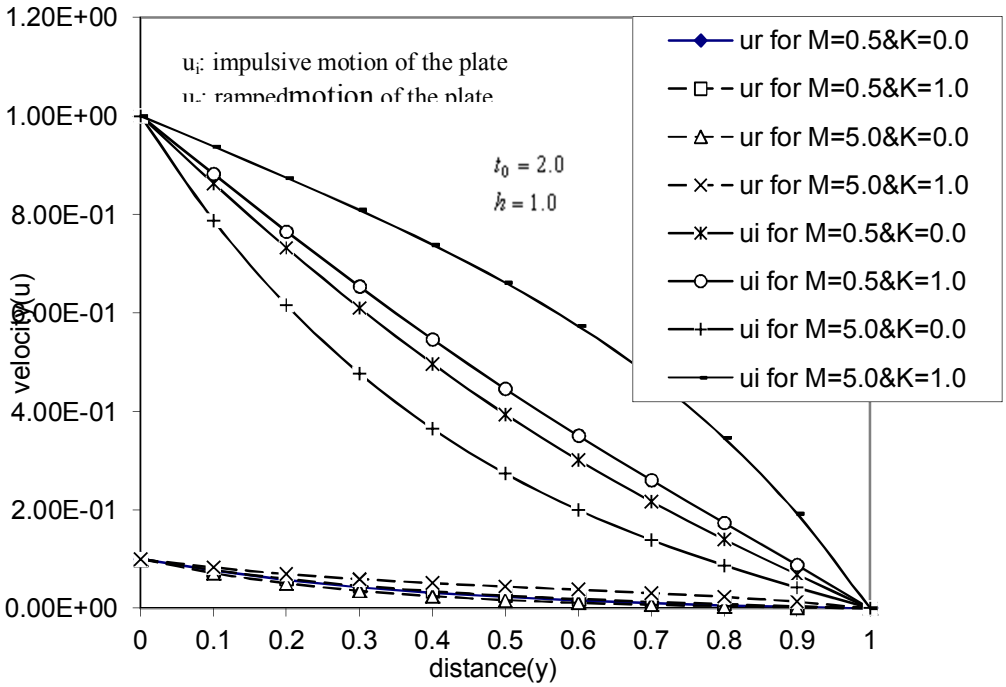


Fig.4.1.7 Velocity (u) versus distance (y) for different M (t =0.2 & $K = 0.0$ and $K = 1.0$)

Fig.4.1.8 shows the flow formation due to ramp motion of the plate for different values of time when the magnetic lines of force are fixed with fluid or moving plate. From this figure it is clear that velocity field is always higher when magnetic field is fixed with moving plate in comparison with to velocity field, when magnetic field is fixed with fluid for any fixed value of time.

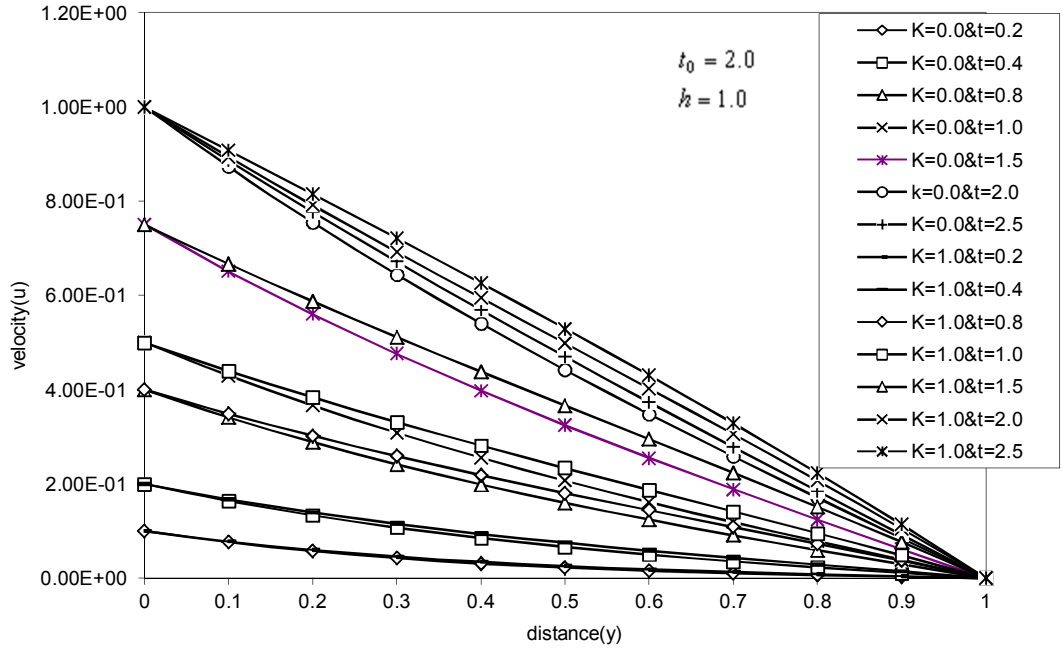


Fig.4.1.8 Velocity (u)versus distance (y) for different time ($M = 0.5$ and $K = 0.0$ or $K = 1.0$)

Fig.4.1.9 shows the velocity variation due to ramp motion of the plate for various values of Hartmann number and time when magnetic lines of force are fixed with respect to the moving plate or fluid. This figure shows that increase of magnetic field suppress the velocity field when $K = 0.0$, while increase of magnetic field enhance the velocity field when $K = 1.0$. Also from these figures we conclude that velocity increases with time for both $K = 0.0$ and $K = 1.0$

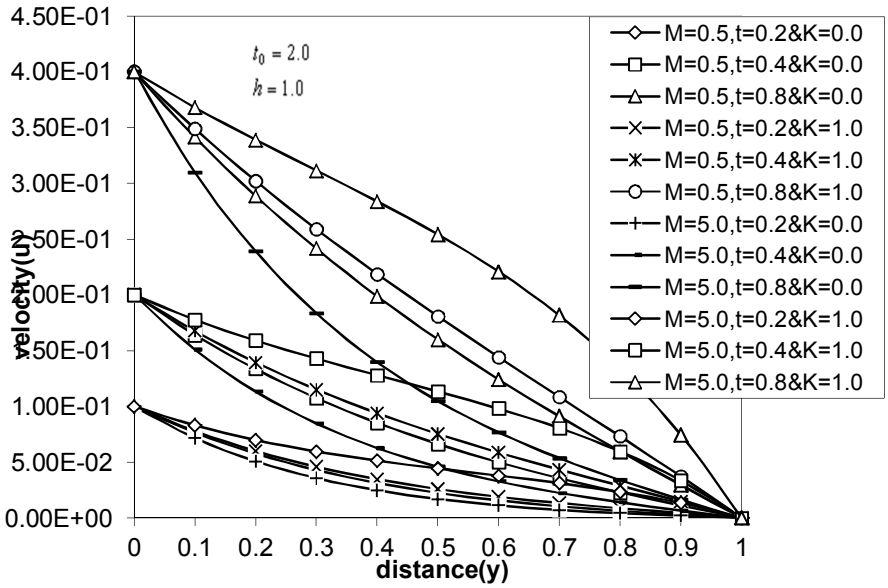


Fig.4.1.9 Velocity (u)versus distance (y) for different M and t ($K = 0.0$ or $K = 1.0$)

Fig.4.1.10 reflects the skin friction variation with time at the moving plate for different values of Hartmann number when magnetic lines of force are fixed with respect to fluid. This figure shows that as time increases skin friction increases and finally reaches its maximum value and then decreases and reaches its steady state value. This figure also reveals that as M increases skin friction increases when magnetic lines of force are fixed relative to the fluid which is physically true because when magnetic lines of force are fixed with fluid, increase in magnetic field decreases the velocity consequently skin friction decreases, while the result is just contrast when magnetic lines of force are fixed relative to the moving plate.

Fig.4.1.11 compares the skin friction versus time at the moving plate when the plate is having ramp or impulsive motion. From this figure it is observed that, due to the ramp motion of the plate, the skin friction increases with time, attains its maximum value,

then decreases and reaches its steady state value. This is the case whether $K = 0.0$ or $K = 1.0$. However for the impulsive motion of the plate, the skin friction decreases with time and finally reaches its steady state value. Furthermore it is also clear that skin friction due to ramp motion as well as impulsive motion of the boundary coincide at large value of time. Hence we conclude that in case of steady state the skin friction is almost same whether the plate is having impulsive or ramp motion.

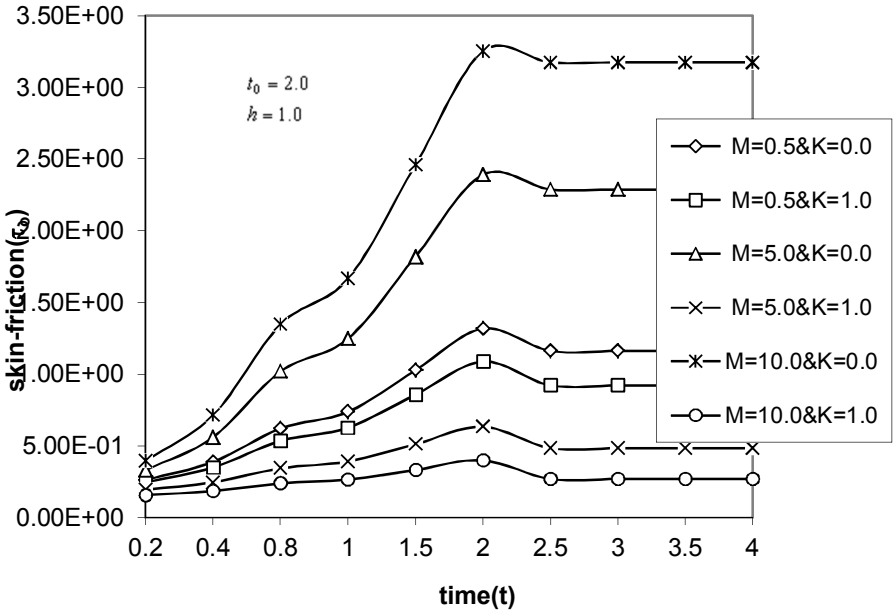


Fig.4.1.10 Skin-friction versus time t for different M ($K = 0.0$ or $K = 1.0$ & $y = 0$)

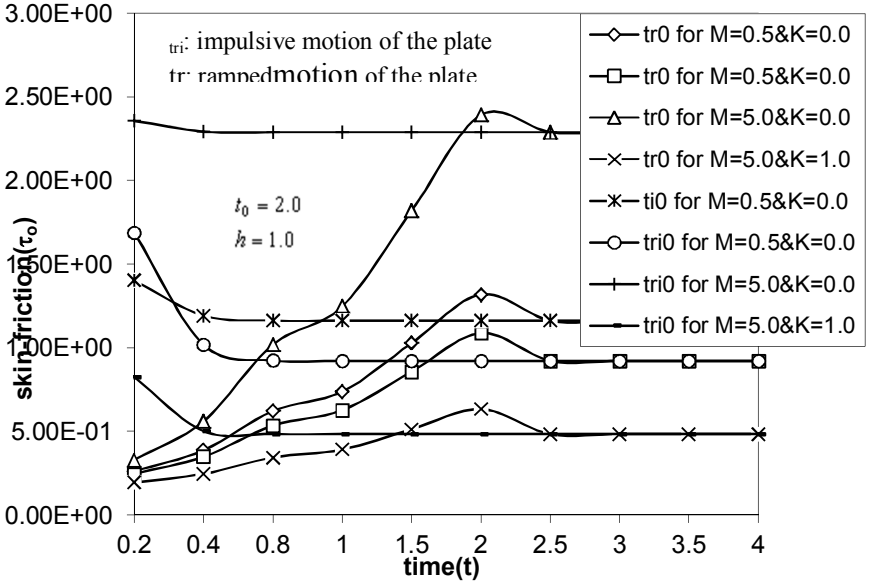


Fig.4.1.11 Skin-friction (τ_0) versus time t for different M ($K = 0.0$ or $K = 1.0$ & $y = 0$)

The variation of skin friction at the upper plate with time for different values of magnetic field M is shown in fig.4.1.12, when magnetic lines of force are fixed relative to fluid or moving plate. This figure reveals that as magnetic field increases skin friction decreases for certain range of time ($(t = t_0)$ and further increase in time, skin friction increases with increase of magnetic field, when magnetic lines of force are fixed relative to moving plate while the result is just the reverse when magnetic lines of force are fixed relative to fluid.In order to give a qualitative estimate of the dependence of the skin friction at the upper plate on the mode of application of the magnetic field on movement of the lower plate, the variation of the skin friction for different values of magnetic field M, when $K = 0.0$ or $K = 1.0$, due to the ramp and impulsive motion of the plate is shown in fig.4.1.13. It is observed that the skin friction increases with increase of time and reaches its steady state value quickly

when the lower plate is moving impulsively for both $K = 0.0$ and $K = 1.0$ .From this figure it is interesting to note that skin friction at the upper plate, due to ramp motion of the lower plate, decreases with increase in time, for certain range of time, and further increase in time, the skin friction exceeds the skin friction due to impulsive motion and then decreases and finally coincides with the skin friction due to the impulsive motion.

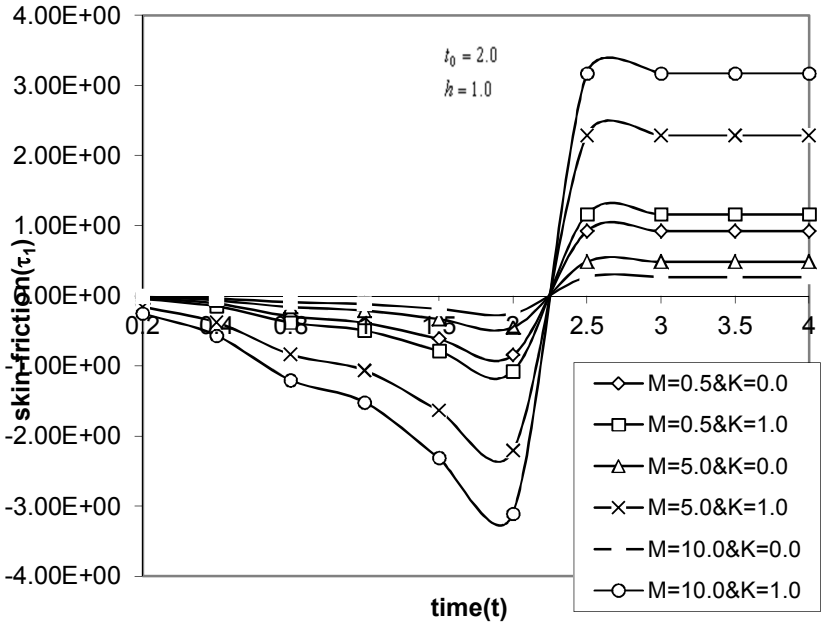


Fig.4.1.12 Skin-friction versus time t for different M ($K = 0.0$ or $K = 1.0$ & $y = 1$)

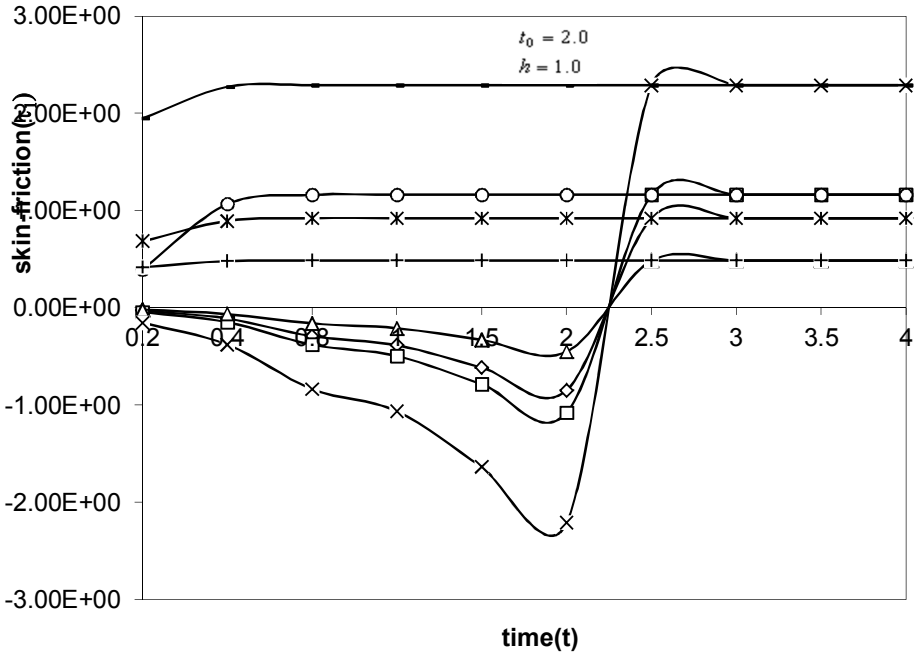


Fig.4.1.13 Skin-friction versus time t for different M ($K = 0.0$ or $K = 1.0$ & $y = 1$)

4.1.7 Summary and Conclusion

In this study the effects of transverse magnetic field on the Couette flow of an electrically conducting fluid between two infinite parallel plates have been discussed when one of the plates has been set in to ramped motion. Under the assumption of negligible induced magnetic field and applied magnetic field being fixed relative to the fluid or moving plate, the governing momentum equation has been solved exactly. The expressions for velocity field and skin-friction have been presented. The transient features of velocity and skin-friction have been discussed for different values of Hartmann number. It is seen that velocity field and skin-friction increases with increase of Hartmann number when the magnetic field is fixed with respect to moving plate while the result is just reverse when it is fixed with respect to fluid. It has also been observed that the velocity field is higher in case of impulsive motion of the plate

in comparison to the ramp motion of the plate whether the magnetic field is fixed with respect to the fluid or moving plate

4.2 Unsteady Hydromagnetic Couette flow due to the ramped motion of one of the porous plates

4.2.0 Introduction

In this section the effects of suction and injection on theunsteady MHD flow Couette flow between two parallel plates due to ramped motion of one of the porous plates is considered.

4.2.1 Governing Equations and Solutions

The physical configuration of the flow is shown in fig.4.2.1 in which we considered the motion of a viscous, incompressible and electrically conducting fluid between two infinite parallel porous plates due to the ramped motion of the lower moving plate.

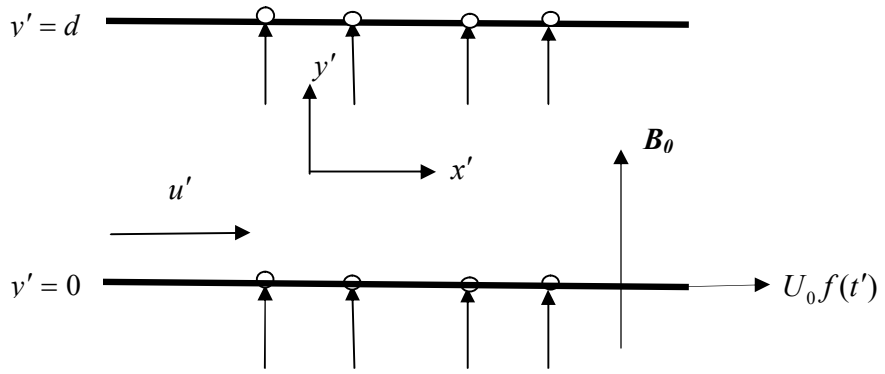


Fig.4.2.1 Flow configuration

The x'-axis is chosen along the lower plate in the horizontal direction and the y'-axis normal to it. The fluid flows between the two porous plates in the x'-direction in the presence of a uniform magnetic field acting normal to the flow direction. At time t' ≤ 0, the fluid, the porous plates and the magnetic lines of force are assumed to be at

rest. At time $t'>0$ the porous plates begins to move in its own plane with a velocity proportional to $U_0 f(t')$ .A uniform magnetic field of strength B_0 is acting perpendicular to the porous plate. The magnetic Reynolds number of the flow is assumed to be small enough so that the induced magnetic field can be neglected. The flow is assumed to be in the x' direction along the porous plate and the y' axis is normal to the porous plate. Since the porous plates are infinite in extent all physical variables are functions of y' and t' only.

The Momentum equation for this investigation based on the above physical conditions is:

$$\frac{\partial u'}{\partial t'} + V_0 \frac{\partial u'}{\partial y'} = \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^{\,2}}{\rho} u' \qquad (4.2.1)$$

Equation (4.2.1) is valid when the magnetic lines of force are fixed relative to the fluid. If the magnetic field is also having ramp motion with the same velocity as the porous plate the relative motion must be accounted for. In this case equation (4.2.1) is replaced by the equation]

$$\frac{\partial u'}{\partial t'} + V_0 \frac{\partial u'}{\partial y'} = \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^{\,2}}{\rho} \left[u' - U_0 f(t')\right] \qquad (4.2.2)$$

Which is valid when the magnetic lines of force are fixed relative to the moving porous plate.

Equation (4.2.1) and (4.2.2) can be combined together to obtain

$$\frac{\partial u'}{\partial t'} + V_0 \frac{\partial u'}{\partial y'} = \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^{\,2}}{\rho} \left[u' - K U_0 f(t')\right] \qquad (4.2.3)$$

Where

$$K = \begin{cases} 0 \text{ if magnetic field is fixed relative to the fluid} \\ 1 \text{ if magnetic field is fixed relative to the plate} \end{cases}$$

The initial and boundary conditions are:

$$t' \le 0: \quad u' = 0, \ for \quad 0 \le y' \le d$$

$$t' > 0; \begin{cases} u' = U_0 f(t') & \text{at } y' = 0 \\ u' = 0 & \text{at } y' = d \end{cases} \tag{4.2.4}$$

Introducing the non-dimensionless quantities,

$$y = \frac{y' U_0}{\nu}, t = \frac{t' U_0^2}{\nu}, u = \frac{u'}{U_0}, M^2 = \frac{\sigma B_0^2 \nu}{\rho U_0^2}, h = \frac{d U_0}{\nu}, \ S = \frac{V_0}{U_0} \tag{4.2.5}$$

where all the physical quantities are defined in nomenclature, the momentum equation (4.2.3) becomes

$$\frac{\partial u}{\partial t} + S \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - M^2 [u - K f(t)] \tag{4.2.6}$$

The initial and boundary conditions (4.2.4) in the dimensionless form for the physical system considered in the present study are:

For $t \le 0; \ u = 0$ for all $y \ge 0$

$$t > 0; \begin{cases} u = f(t) & \text{at } y = 0 \\ u = 0 & \text{at } y = h \end{cases} \tag{4.2.7}$$

In the present work $f(t)$ is specified as the ramp function:

$$f(t) = \begin{cases} \dfrac{t}{t_0} & \text{if } 0 \le t \le t_0 \\ = 1 & \text{if } t > t_0 \end{cases} \tag{4.2.8}$$

Eq. (4.2.8) can be written as:

$$f(t) = H(t)\left(\frac{t}{t_0}\right) - \left(\frac{1.}{t_0}\right)(t - t_0) H(t - t_0) \tag{4.2.9}$$

4.2.2 Solution of the problem

The solution of the partial differential Eq. (4.2.6) subject to conditions (4.2.7) is attempted by the operational method of Laplace transformation. If

$$L[u(y,t)] = \bar{u}(y,s) = \int_0^\infty u(y,t)\exp(-st)dt \tag{4.2.10}$$

is the Laplace transform of $u(y,t)$ then Eqs. (4.2..6), (4.2.7) and (4.2.9) reduce to:

$$\frac{d^2\bar{u}}{dy^2} - S\frac{d\bar{u}}{dy} - (M^2 + s)\bar{u} = -KM^2\bar{f}(s) \tag{4.2.11}$$

$t \le 0;\ \bar{u} = 0$ for all $y \ge 0$

$$t>0;\ \begin{cases} \bar{u} = f(s) & \text{at } y = 0 \\ \bar{u} = 0 & \text{at } y = h \end{cases} \tag{4.2.12}$$

$$\bar{f}(s) = \frac{[1. - Exp(-st_0)]}{s^2 t_0} \tag{4.2.13}$$

The solution of Eq. (4.2.11), under the conditions (4.2.12) and (4.2..13), is:

$$\bar{u} = \frac{Exp\left(\frac{Sy}{2}\right)}{t_0 s^2}[1. - Exp(-st_0)]\sum_{m=0}^{\infty}\left[\left(1. - \frac{KM^2}{(M^2+s)}\right)\left(Exp\left(-a_1\sqrt{C^2+s}\right) - Exp\left(-a_2\sqrt{C^2+s}\right)\right)\right.$$

$$\left. + \left(\frac{KM^2}{(M^2+s)}\right)Exp\left(\frac{Sh}{2}\right)\left(Exp\left(-a_3\sqrt{C^2+s}\right) - Exp\left(-a_4\sqrt{C^2+s}\right)\right)\right]$$

$$+ \frac{KM^2}{(M^2+s)t_0 s^2}[1. - Exp(-st_0)] \tag{4.2.14}$$

Where

$$a_1 = 2mh + y, a_2 = 2mh + 2h - y, a_3 = 2mh + h + y, a_4 = 2mh + h - y, C^2 = \left(\frac{S^2}{4} + M^2\right).$$

In equation (4.2.14), summation term is due to expansion of term $\left[1. - Exp\left(-2h\sqrt{C^2+s}\right)\right]^{-1}$ by Binomial theorem.

The exact solution for velocity field can be obtained from Eq. (4.2.14), by taking the inverse transforms Abramowitz and Stegun(1965),Carslaw and Jaeger (1963). After detailed simplifications and shifting on the t-axis, the solution can be expressed as:

$$u(y,t)=\frac{Exp\left(\frac{Sy}{2}\right)}{t_0}\sum_{m=0}^{\infty}\left[F_1(a_1;C;t;0.0)-F_1(a_2;C;t:0.0)-H(t-t_0)\left(F_1(a_1;C;t;t_0)-F_1(a_2;C;t:t_0)\right)\right.$$

$$-K\left(F_1(a_1;C;t;0.0)-F_1(a_2;C;t;0.0)-Exp\left(-\frac{Sh}{2}\right)\left(F_1(a_3;C;t;0.0)-F_1(a_4;C;t;0.0)\right)\right)$$

$$+KH(t-t_0)\left(F_1(a_1;C;t;t_0)-F_1(a_2;C;t;t_0)-Exp\left(-\frac{Sh}{2}\right)\left(F_1(a_3;C;t;t_0)-F_1(a_4;C;t;t_0)\right)\right)$$

$$+\frac{K}{M^2}\left(F_2(a_1;C;t;0.0)-F_2(a_2;C;t;0.0)-Exp\left(-\frac{Sh}{2}\right)\left(F_2(a_3;C;t;0.0)-F_2(a_4;C;t;0.0)\right)\right)$$

$$-\frac{K}{M^2}H(t-t_0)\left(F_2(a_1;C;t;t_0)-F_2(a_2;C;t;t_0)-Exp\left(-\frac{Sh}{2}\right)\left(F_2(a_3;C;t;t_0)-F_2(a_4;C;t;t_0)\right)\right)$$

$$-\frac{K}{M^2}Exp\left(-M^2t\right)\left(\begin{array}{l}F_2\left(a_1;\frac{S}{2};t;0.0\right)-F_2\left(a_2;\frac{S}{2};t;0.0\right)-\\ Exp\left(-\frac{Sh}{2}\right)\left(F_2\left(a_3;\frac{S}{2};t;0.0\right)-F_2\left(a_4;\frac{S}{2};t;0.0\right)\right)\end{array}\right)$$

$$+\frac{K}{M^2}Exp\left(-M^2(t-t_0)\right)H(t-t_0)\left(\begin{array}{l}F_2\left(a_1;\frac{S}{2};t;t_0\right)-F_2\left(a_2;\frac{S}{2};t;t_0\right)-\\ Exp\left(-\frac{Sh}{2}\right)\left(F_2\left(a_3;\frac{S}{2};t;t_0\right)-F_2\left(a_4;\frac{S}{2};t;t_0\right)\right)\end{array}\right)\Bigg]$$

$$-\frac{K}{t_0}\left[\frac{1}{M^2}-\frac{Exp\left(-M^2t\right)}{M^2}-t\right]+\frac{K}{t_0}H(t-t_0)\left[\frac{1}{M^2}-\frac{Exp\left(-M^2(t-t_0)\right)}{M^2}-(t-t_0)\right]$$

(4.2.15)

4.2.3 Skin friction

Using the expression (4.2.15), the frictional force acting at the lower and upper plates is:

$$\tau_0=\left.\frac{\partial u}{\partial y}\right|_{y=0}$$

$$= \frac{1}{t_0}\sum_{m=0}^{\infty}\left[\frac{S}{2}F_1(b_1;C;t;0.0)+F_3(b_1;C;t:0.0)-\left(\frac{S}{2}F_1(b_2;C;t;0.0)-F_3(b_2;C;t:0.0)\right)\right.$$

$$-H(t-t_0)\left(\frac{S}{2}F_1(b_1;C;t;0.0)+F_3(b_1;C;t:0.0)-\left(\frac{S}{2}F_1(b_2;C;t;t_0)-F_3(b_2;C;t:t_0)\right)\right)$$

$$-K\left(\frac{S}{2}F_1(b_1;C;t;0.0)+F_3(b_1;C;t:0.0)-\left(\frac{S}{2}F_1(b_2;C;t;0.0)-F_3(b_2;C;t:0.0)\right)\right.$$

$$\left.-2Exp\left(-\frac{Sh}{2}\right)F_3(b_3;C;t:0.0)\right)$$

$$+KH(t-t_0)\left(\frac{S}{2}F_1(b_1;C;t;t_0)+F_3(b_1;C;t:t_0)-\left(\frac{S}{2}F_1(b_2;C;t;t_0)-F_3(b_2;C;t:t_0)\right)\right.$$

$$\left.-2Exp\left(-\frac{Sh}{2}\right)F_3(b_3;C;t:t_0)\right)$$

$$+\frac{K}{M^2}\left(\frac{S}{2}F_2(b_1;C;t;0.0)+F_4(b_1;C;t:0.0)-\left(\frac{S}{2}F_2(b_2;C;t;0.0)-F_4(b_2;C;t:0.0)\right)\right.$$

$$\left.-2Exp\left(-\frac{Sh}{2}\right)F_4(b_3;C;t:0.0)\right)$$

$$-\frac{K}{M^2}H(t-t_0)\left(\frac{S}{2}F_2(b_1;C;t;t_0)+F_4(b_1;C;t:t_0)-\left(\frac{S}{2}F_2(b_2;C;t;t_0)-F_4(b_2;C;t:t_0)\right)\right.$$

$$\left.-2Exp\left(-\frac{Sh}{2}\right)F_4(b_3;C;t:t_0)\right)$$

$$-\frac{KExp\left(-M^2t\right)}{M^2}\left(\frac{S}{2}F_2\left(b_1;\frac{S}{2};t;0.0\right)+F_4\left(b_1;\frac{S}{2};t:0.0\right)-\left(\frac{S}{2}F_2\left(b_2;\frac{S}{2};t;0.0\right)-F_4\left(b_2;\frac{S}{2};t:0.0\right)\right)\right.$$

$$\left.-2Exp\left(-\frac{Sh}{2}\right)F_4\left(b_3;\frac{S}{2};t:0.0\right)\right)$$

$$+\frac{KExp\left(-M^2(t-t_0)\right)}{M^2}H(t-t_0)\left(\frac{S}{2}F_2\left(b_1;\frac{S}{2};t;t_0\right)+F_4\left(b_1;\frac{S}{2};t:t_0\right)-\left(\frac{S}{2}F_2\left(b_2;\frac{S}{2};t;t_0\right)-F_4\left(b_2;\frac{S}{2};t:t_0\right)\right)\right.$$

$$\left.\left.-2Exp\left(-\frac{Sh}{2}\right)F_4\left(b_3;\frac{S}{2};t:t_0\right)\right)\right] \qquad (4.2.16)$$

$$\tau_h = \left.\frac{\partial u}{\partial y}\right|_{y=h}$$

$$= \frac{1}{t_0}\sum_{m=0}^{\infty}\Big[2\big(F_3(c_1;C;t:0.0) - H(t-t_0)F_3(c_1;C;t:0.0)\big)$$

$$-K\left(\begin{array}{l} 2F_3(c_1;C;t:0.0) - Exp\left(-\frac{Sh}{2}\right)\left\{\frac{S}{2}F_1(c_3;C;t;0.0)\right. \\ \left. + F_3(c_3;C;t:0.0) - \left(\frac{S}{2}F_1(c_4;C;t;0.0) - F_3(c_4;C;t:0.0)\right)\right\}\end{array}\right)$$

$$+KH(t-t_0)\left(\begin{array}{l} 2F_3(c_1;C;t:t_0) - Exp\left(-\frac{Sh}{2}\right)\left\{\frac{S}{2}F_1(c_3;C;t;t_0)\right. \\ \left. + F_3(c_3;C;t:t_0) - \left(\frac{S}{2}F_1(c_4;C;t;t_0) - F_3(c_4;C;t:t_0)\right)\right\}\end{array}\right)$$

$$+\frac{K}{M^2}\left(\begin{array}{l} 2F_4(c_1;C;t:0.0) - Exp\left(-\frac{Sh}{2}\right)\left\{\frac{S}{2}F_2(c_3;C;t;0.0)\right. \\ \left. + F_4(c_3;C;t:0.0) - \left(\frac{S}{2}F_2(c_4;C;t;0.0) - F_4(c_4;C;t:0.0)\right)\right\}\end{array}\right)$$

$$-\frac{K}{M^2}H(t-t_0)\left(\begin{array}{l} 2F_4(c_1;C;t:t_0) - Exp\left(-\frac{Sh}{2}\right)\left\{\frac{S}{2}F_2(c_3;C;t;t_0)\right. \\ \left. + F_4(c_3;C;t:t_0) - \left(\frac{S}{2}F_2(c_4;C;t;t_0) - F_4(c_4;C;t:t_0)\right)\right\}\end{array}\right)$$

$$-\frac{KExp\left(-M^2t\right)}{M^2}\left(\begin{array}{l} 2F_4\left(c_1;\frac{S}{2};t:0.0\right) - Exp\left(-\frac{Sh}{2}\right)\left\{\frac{S}{2}F_2\left(c_3;\frac{S}{2};t;0.0\right)\right. \\ \left. + F_4\left(c_3;\frac{S}{2};t:0.0\right) - \left(\frac{S}{2}F_2\left(c_4;\frac{S}{2};t;0.0\right) - F_4\left(c_4;\frac{S}{2};t:0.0\right)\right)\right\}\end{array}\right)$$

$$+\frac{KExp\left(-M^2(t-t_0)\right)}{M^2}H(t-t_0)\left(\begin{array}{l} 2F_4\left(c_1;\frac{S}{2};t:t_0\right) - Exp\left(-\frac{Sh}{2}\right)\left\{\frac{S}{2}F_2\left(c_3;\frac{S}{2};t;t_0\right)\right. \\ \left. + F_4\left(c_3;\frac{S}{2};t:t_0\right) - \left(\frac{S}{2}F_2\left(c_4;\frac{S}{2};t;t_0\right) - F_4\left(c_4;\frac{S}{2};t:t_0\right)\right)\right\}\end{array}\right)\Bigg] \quad (4.2.17)$$

Where

$$b_1 = 2mh, \qquad b_2 = 2mh + 2h, \qquad b_3 = 2mh + h,$$

$$c_1 = 2mh + h,\ c_3 = 2mh + 2h, \qquad c_4 = 2mh. \quad (4.2.18)$$

4.2.4 Special case: Impulsive motion of one of the porous plates

For $t_0 = 0$, the expression (4.2.15) reduces to

$$u(y,t) = Exp(\frac{Sy}{2})\sum_{m=0}^{\infty}[[(1.-K)[F_2(a_1;C;t;0.0) - F_2(a_2;C;t;0;0)]$$

$$+ KExp(-M^2t)\left[F_2\left(a_1;\frac{S}{2};t;0.0\right) - F_2\left(a_2;\frac{S}{2};t;0.0\right)\right]$$

$$+ KExp\left(-\frac{Sh}{2}\right)[F_2(a_3;c;t;0.0) - F_2(a_4;c;t;0.0)]$$

$$+ K\exp\left(-\frac{Sh}{2}\right)Exp(-M^2t)\left[F_2\left(a_{3_1};\frac{S}{2};t;0.0\right) - F_2\left(a_4;\frac{S}{2};t;0.0\right)\right]]$$

$$+\left[KExp(-M^2t)\right] \qquad (4.2.19)$$

And the skin friction at the lower and upper plates is respectively:

$$\tau_{0i} = -\left.\frac{\partial u}{\partial y}\right|_{y=0}$$

$$= \sum_{m=0}^{\infty}[\frac{S}{2}\left[(1.-K)\left[F_2\left(b_{1_1};C;t;0.0\right) - F_2(b_2;C;t;0;0)\right]\right.$$

$$+ KExp(-M^2t)\left[F_2\left(b_1;\frac{S}{2};t;0.0\right) - F_2\left(b_2;\frac{S}{2};t;0.0\right)\right]]$$

$$+\left[(1.-K)\left[F_{4_2}(b_1;C;t;0.0) + F_4(b_2;C;t;0;0)\right]\right.$$

$$+ KExp(-M^2t)\left[F_4\left(b_1;\frac{S}{2};t;0.0\right) + F_4\left(b_2;\frac{S}{2};t;0.0\right)\right]$$

$$+ 2KExp\left(-\frac{Sh}{2}\right)[F_4(b_3;c;t;0.0)]$$

$$- 2K\exp\left(-\frac{Sh}{2}\right)Exp(-M^2t)\left[F_4\left(b_{3_1};\frac{S}{2};t;0.0\right)\right]] \qquad . \qquad (4.2.20)$$

$$\tau_{hi} = -\left.\frac{\partial u}{\partial y}\right|_{y=h}$$

$$= \sum_{m=0}^{\infty} [\frac{S}{2} Exp(\frac{Sh}{2}\left[\left(KExp(\frac{-Sh}{2}) \right)[F_2(C_3;C;t;0.0) - F_2(C_4;C;t;0;0)]\right.$$

$$\left. - KExp(-M^2 t)\left[F_2\left(C_3;\frac{S}{2};t;0.0 \right) - F_2\left(C_4;\frac{S}{2};t;0.0 \right)\right]\right]$$

$$+ 2(1-K)Exp(\frac{Sh}{2})\left[F_4\ (c_1;c;t;0.0)\right] \quad + 2KExp(-M^2 t)\left[F_4\left(C_1;\frac{S}{2};t;0.0 \right)\right]$$

$$+ K[F_4(c_3;c;t;0.0) + F_4(c_4;c;t;0.0)]$$

$$- KExp(-M^2 t)\left[F_4\left(c_3;\frac{S}{2};t;0.0 \right) + F_4\left(C_4;\frac{S}{2};t;0.0 \right)\right]] \qquad (4.2.21)$$

4.2.6 Steady State Solution

As time increases, the response of the velocity with respect to time becomes insignificant. This implies that the flow velocity has attained its steady state value at that time. In this case the expression for velocity distribution of the problem at this state can be obtained by setting $\frac{\partial u}{\partial t} = 0$ in equation (4.2.6).This reduces the equation to:

$$\frac{d^2u}{dy^2} - S\frac{du}{dy} - M^2 u = -KM^2 \qquad (4.2.22)$$

Equation (4.2.27) is solved under the boundary conditions:

$$\begin{aligned} u &= 1 \quad at \quad y = 0 \\ u &= 0 \quad at \quad y = h \end{aligned} \qquad (4.2.23)$$

The solution of Equation (4.2.22) subject to the boundary conditions (4.2.23) is expressed as:

$$u = \frac{\exp(\frac{Sy}{2})(1-K)\sinh(C(h-y))}{\sinh(Ch)} + K[1 - \frac{\exp(-\frac{Sh}{2})\exp(\frac{Sy}{2})\sinh(Cy)}{\sinh(Ch)}] \qquad (4.2.24)$$

Where C=$\sqrt{\frac{\lambda^2}{4} + M^2}$ (4.2.25)

Using expression (4.2.24), the frictional forces at the lower and upper plate is :

$$\tau_0 = \left.\frac{du}{dy}\right|_{y=0}$$

$$= \frac{(1-K)}{\sinh(Ch)}\left[\frac{S}{2}\sinh(Ch) - C\cosh(CH)\right] - \frac{KC\exp(-\frac{Sh}{2})}{\sinh(Ch)} \qquad (4.2.26)$$

$$\tau_h = \left.\frac{du}{dy}\right|_{y=h}$$

$$= \frac{C}{\sinh(Ch)}[\ (K-1)\exp(\frac{Sh}{2}) - K\cosh(Ch)] - \frac{S}{2}K \qquad (4.2.27)$$

4.2.7 Results and Discussions

In order to illustrate the effects of the controlling parameters, such as the Hartmann number, the suction/injection parameter, the velocity and skin friction where computed taking the value of $t_0 = 2$ and $h = 1.0$. The results are illustrated in figures 4.2.2 to 4.2.11.

Figure 4.2.2 shows the variation of the velocity due the ramp motion of the moving porous plate for different values of the suction/injection parameter S when the magnetic lines of force are fixed relative to the moving porous plate or fluid for fixed value of the magnetic field M. It is seen from this figure that the velocity increases in the presence of injection parameter and decreases with suction at the porous moving plate, whether the magnetic lines of force are fixed with respect to the fluid or moving porous plate. It also shows that velocity is higher when the magnetic lines of force are fixed relative to the moving porous plate than when fixed relative to the fluid in the presence of suction or injection. Furthermore it reflects that the effect of injection increases the velocity while suction decreases it.

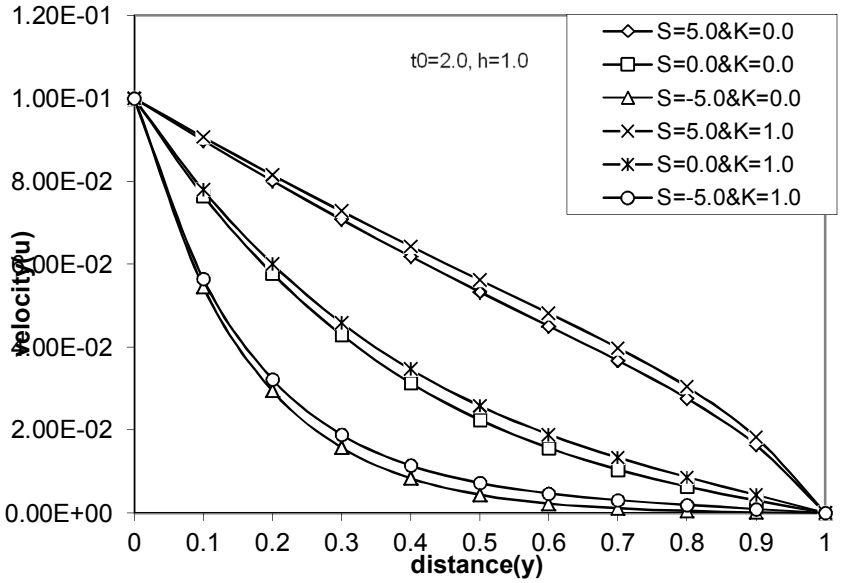


Fig.4.2.2 Velocity for different values of S ($M = 0.5$ and $K = 0.0$ or $K = 1.0$)

The variation of velocity due to the ramp motion of the porous moving plate for different values of time for fixed magnetic field M with injection and suction when the magnetic lines of force are fixed relative to the fluid or moving porous plate is depicted in fig.4.2.3 and 4.2.4 respectively.

From these figures it is evident that velocity increases with increase in time for a fixed value of suction/injection parameter. Furthermore these figures also reveal that the velocity is higher when the magnetic lines of force are fixed with respect to the moving porous plate than when fixed relative to the fluid.

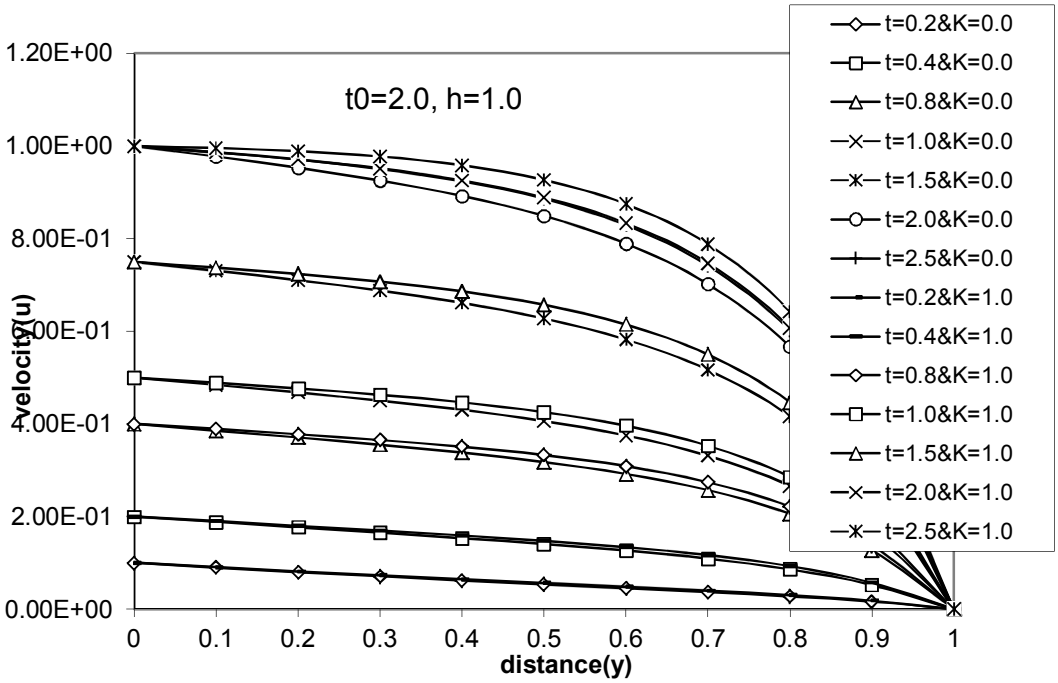


Fig.4.2.3 Variation of velocity for different values of t ($S = 5.0$, $M = 0.5$ and $K = 0.0$ or $K = 1.0$)

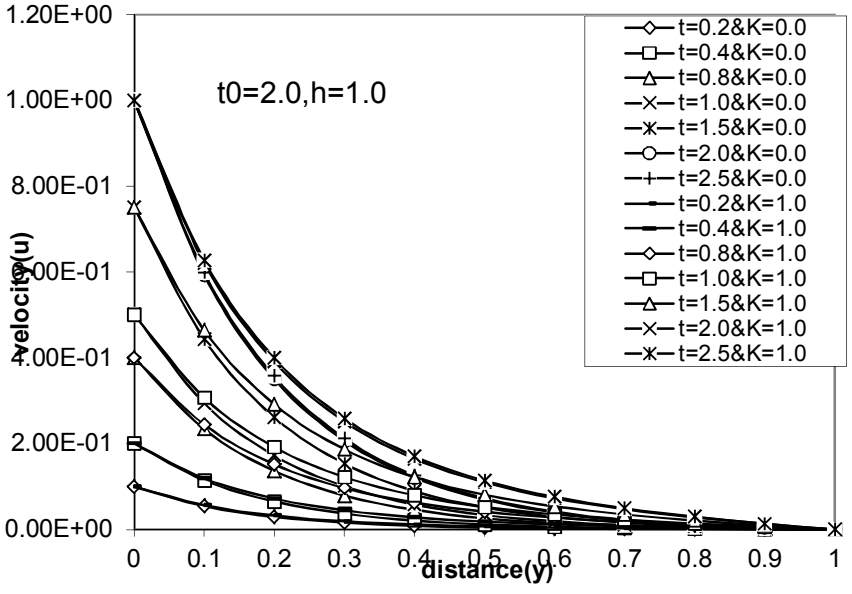


Fig.4.2.4 Velocity for different time ($S = -5.0$, $M = 0.5$ and $K = 0.0$ or $K = 1.0$)

Fig.4.2.5 presents the development of velocity due to the ramp motion of the moving porous plate for different time t and S, when the magnetic field lines are fixed relative to the fluid or moving porous plate for fixed value of the Hartmann number M .This velocity profile shows under mass suction or injection velocity increases with increase in time, whether the magnetic lines of force are fixed relative to the fluid ($K = 0$) or moving porous plate ($K = 1$). Furthermore it is seen that the velocity is higher when the magnetic lines of force are fixed relative to the moving porous plate than when fixed relative to the fluid whether there is injection or suction at the moving porous plate.

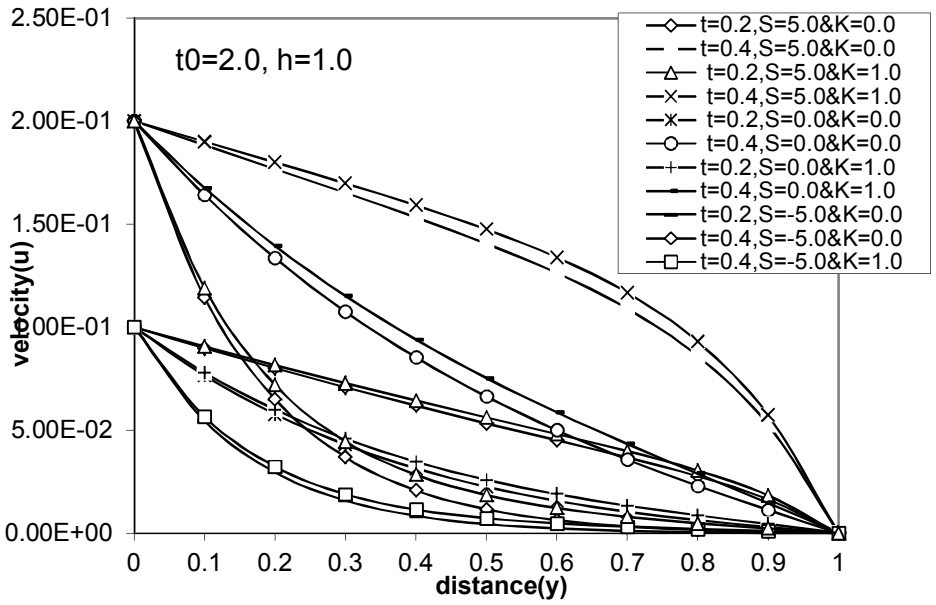


Fig.4.2.5 Velocity for different S and t ($M = 0.5$ and $K = 0.0$ or $K = 1.0$)

Fig.4.2.6 and 4.2.7 illustrate velocity variation due to the ramp and impulsive motion of the boundary for different values of time, and injection/suction parameter, for fixed M for $K = 0.0$ or $K = 1.0$. From these figures it is observed that when the magnetic lines of force are fixed relative to the fluid ($K = 0.0$) or relative to the moving porous plate ($K = 1.0$) velocity increases with increase in time, in the presence of mass injection or suction, due to the ramp or impulsive motion of the porous plate. These figures also show that injection increases the velocity while suction reduces it. Furthermore the velocity due to the impulsive motion of the porous plate is higher than the velocity due to the ramp motion of the porous plate when the magnetic lines of force are fixed with respect to the moving porous plate or fluid.

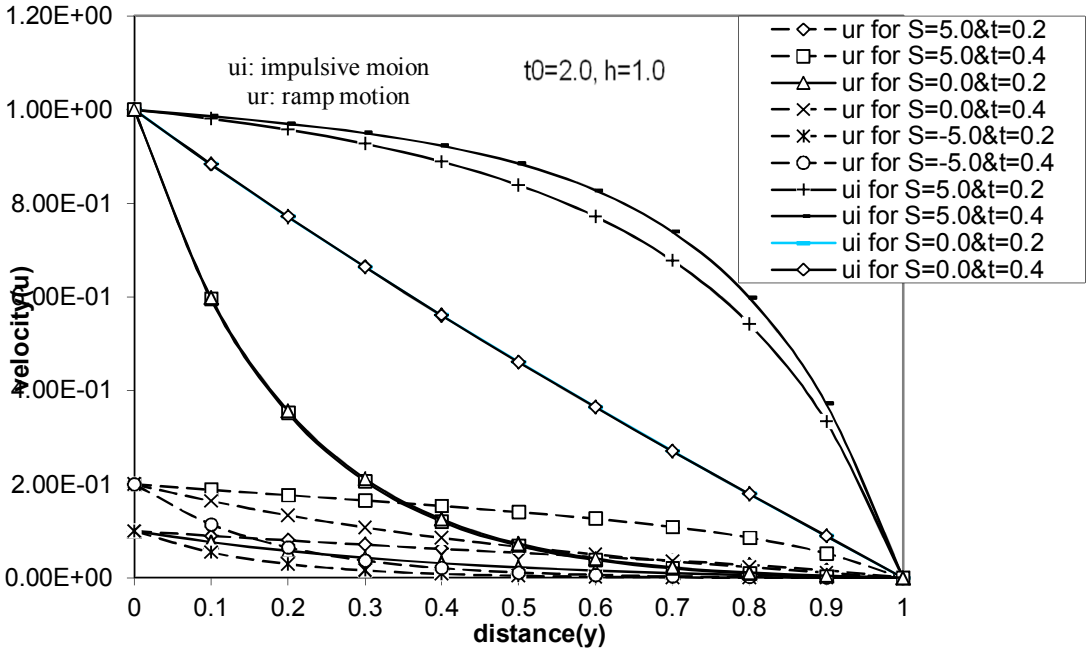


Fig.4.2.6 Velocity for different S and t ($M = 0.5$ and $K = 0.0$)

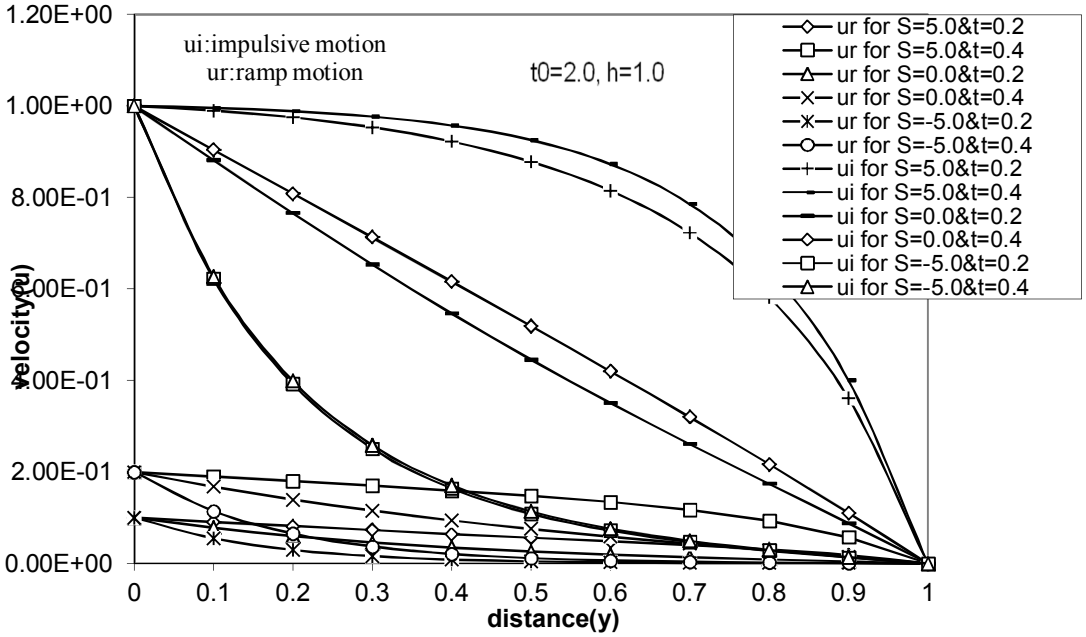


Fig.4.2.7 Velocity for different S and t ($M = 0.5$ and $K = 1.0$)

The variation of the skin friction at the lower and upper porous plate for different values of the injection/suction parameter for fixed value of the magnetic field M, when $K = 0.0$ 0r $K = 1.0$ is shown in figures 4.2.8 and 4.2.9.

Fig.4.2.8 shows that at the lower porous plate, in the presence of suction/injection, the skin friction decreases with increase in time for certain range of time $(t \cong t_0)$ and attain its minimum value. Further increase in time, skin friction increases with increase in time and reaches its steady state value. This figure also reveals that the magnitude of skin friction at the lower moving porous plate is smaller when the magnetic lines of force are fixed relative to the moving porous plate than when fixed relative to the fluid. The steady state skin friction is higher when the magnetic lines of force are fixed relative to the fluid than when fixed relative to the moving porous plate .From figure 4.2.8 it is also evident that the magnitude of skin friction is higher in case of suction at the moving porous plate in comparison to injection at the moving

porous plate whether magnetic field is fixed with fluid or moving porous plate while the result is just the reverse at the stationary porous plate (see fig.4.2.9).

Fig.4.2.9 reveals that at the upper porous plate, in the presence of injection/suction the skin friction decreases with increase in time for certain range of time $(t \cong t_0)$ and attain its minimum value. Further increase in time, skin friction increases with increase in time and reaches its steady state value. This figure also reveals that the magnitude of skin friction at the upper plate is higher when the magnetic lines of force are fixed relative to the fluid than when fixed relative to the moving porous plate.

The steady state skin friction is however higher when the magnetic lines of force are fixed relative to the moving porous plate than when fixed relative to the fluid.

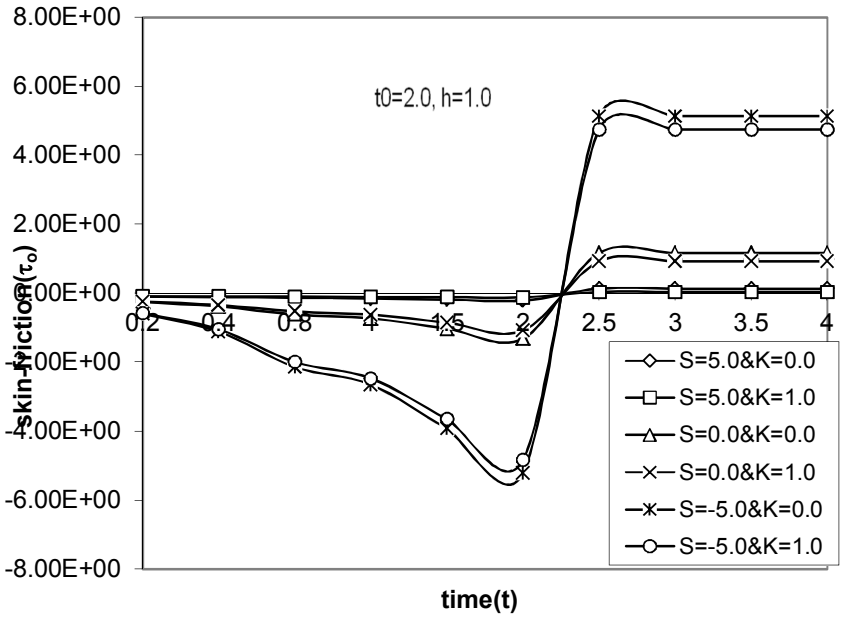


Fig.4.2.8 Skin-friction for different S ($M = 0.5$, $K = 0.0$ or $K = 1.0$ at $y = 0$)

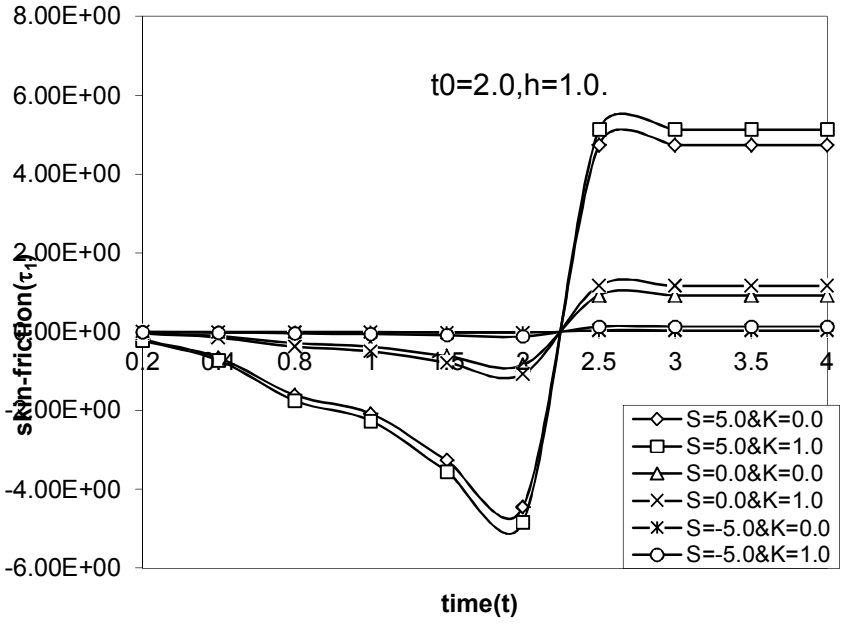


Fig.4.2.9 Skin-friction for different S ($M = 0.5$, $K = 0.0$ or $K = 1.0$ at $y = 1.0$)

Fig.4.2.10 and 4.2.11 give a comparative view of the skin friction at the lower moving and upper (stationary) porous plates, for different values of S, when the plate is having ramp or impulsive motion ($K = 0.0$ and $K = 1.0$).

From fig.4.2.10 we concluded that the skin friction due to ramp motion of the lower boundary decreases with increase in time and attain its minimum value and then increases and reaches its steady state value, whether $K = 0.0$ or $K = 1.0$. Furthermore it is also clear that skin friction due to the ramp motion of the boundary coincide with impulsive motion of the boundary at large value of time. Hence we concluded that in case of steady state the skin friction is almost same whether the plate is having impulsive or ramp motion.

Fig.4.2.11 shows the variation of the skin friction versus time at the upper (stationary) plate, for different values of S, when the plate is having ramp or impulsive motion ($K = 0.0$ and $K = 1.0$). It is observed that the skin-friction decreases with increase of time under certain range of time ($t \cong t_0$) and attain its minimum value and further increase in time, it increases and reaches its maximum value and then decreases and reaches its steady state value the presence of mass injection the skin friction decreases with increase in time and attain its min. value and then increases and reaches its steady state value when $K = 0.0$ or $K = 1.0$. It is also clear that the skin friction due to ramp motion as well as impulsive motion of the boundary coincide at large value of time. Hence we concluded that in case of steady state the skin friction is almost same whether the plate is having impulsive or ramp motion.

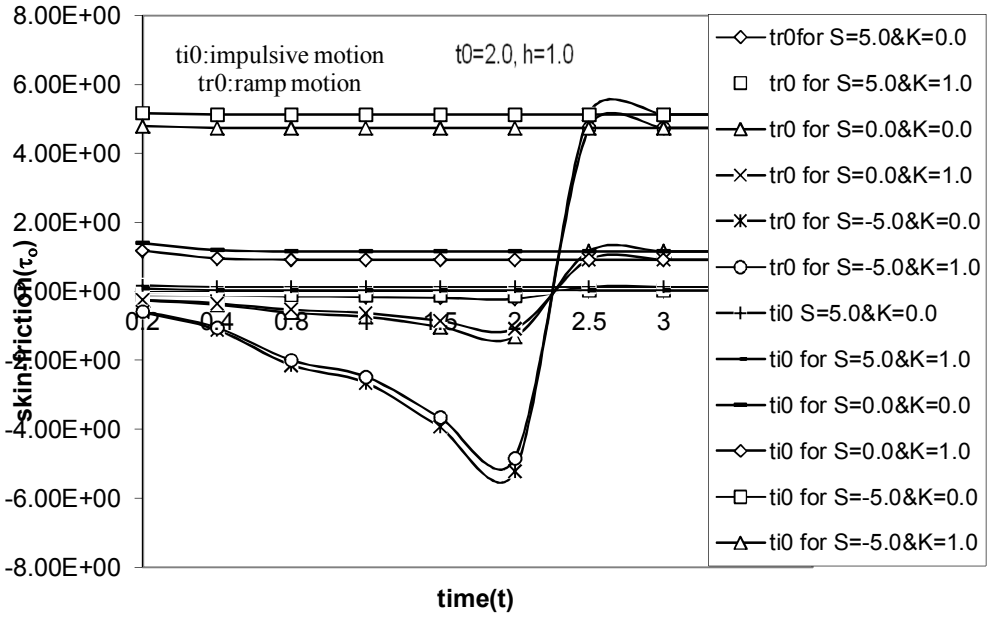


Fig.4.2.10 Skin-friction for different S ($M = 0.5$, $K = 0.0$ or $K = 1.0$ at $y = 0$)

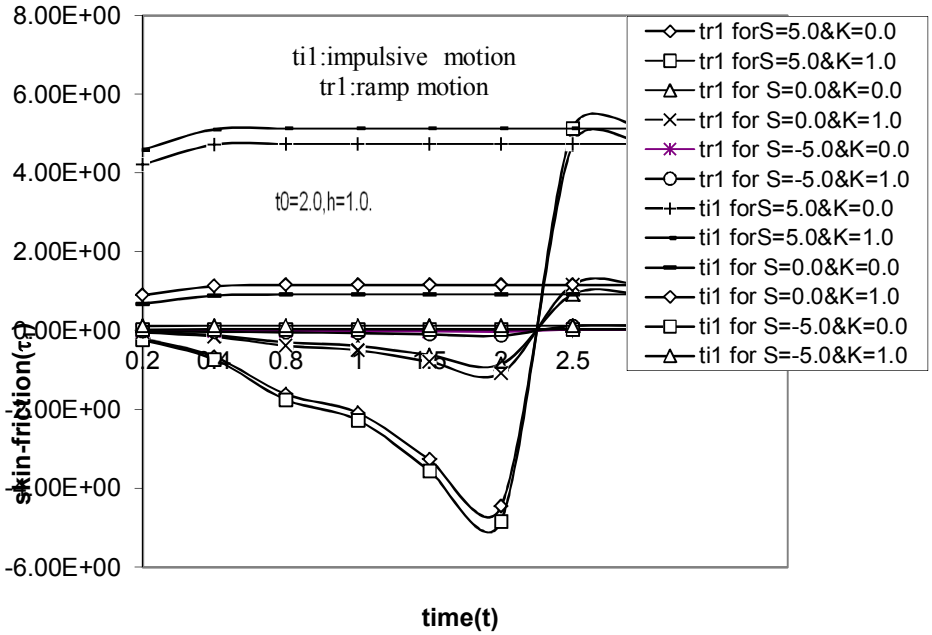


Fig.4.2.11 Skin-friction for different S ($M = 0.5$, $K = 0.0$ or $K = 1.0$ at $y = 1.0$)

4.2.8 Summary and Conclusion

The effects of viscosity, magnetic field and mass leakage on the unsteady flow of an incompressible and electrically conducting fluid have been analyzed when the flow is generated due to ramped motion of the horizontal porous plate. A unified closed form analytical solution have been obtained for fluid velocity and skin-friction corresponding to the cases of magnetic field fixed relative to the fluid or moving porous plate. The variations of velocity and skin-friction have been discussed for different value of magnetic and suction/injection parameters. The flow formation near a ramped motion of porous plate has also been compared with the flow formation near constant motion of porous plate. It has been observed that the effect of injection increases the velocity while suction decreases it. It is seen from this figure that the velocity increases in the presence of injection parameter and decreases with suction at the porous moving plate, whether the magnetic lines of force are fixed with respect to the fluid or moving porous plate. It also shows that velocity is higher when the magnetic lines of force are fixed relative to the moving porous plate than when fixed relative to the fluid in the presence of suction or injection. Furthermore it reflects that the effect of injection increases the velocity while suction decreases it.

CHAPTER FIVE

COMBINED EFFECTS OF VISCOUS, JOULEAN AND OHMIC DISSIPATION ON MHD COUETTE FLOW WITH TRANSPIRATION

5.0 Introduction

The aim of this chapter is to discuss the effects of transverse magnetic field on the transient velocity and viscous and Joulean dissipation on the steady state temperature for the Couette flow of incompressible, viscous and electrically conducting fluid between two infinite porous plates. In the first section the flow was investigated under the influence of transverse magnetic field while in the second section the additional effect of ohmic dissipation (electric field) was also considered.

5.1 Combined Effects of Viscous and Joulean Dissipation on MHD Couette flow with transpiration.

5.1.0 Introduction

In this section the influence of transverse magnetic field on the transient velocity and viscous and Joulean dissipation on the steady state temperature for the Couette flow of incompressible, viscous and electrically conducting fluid between two infinite porous plates is investigated .It is the extension of Fang article (Fang2004b) for MHD case.

5.1.1 Formulation of the problem

The physical configuration considered in this paper is shown in fig.5.1.1.It consists of a time dependent, viscous, incompressible Couette flow of an electrically conducting fluid trapped between two infinite horizontal parallel porous plates separated by a distance h. The lower porous plate is fixed and subjected to a mass injection velocity, V_0 with temperature T_1 and there is equal mass suction (with velocity, V_0) at the

upper wall which is maintained at constant temperature T_2. A uniform magnetic field B_0 is applied perpendicular to the porous plates. It is assumed that there is no pressure gradient and no external electric field acting on the fluid. The flow is assumed to be time dependent, unidirectional and fully developed. Since the porous plates are infinite in extent all physical quantities are functions of y' and t' only.

Both plates are assumed to be stationary at $t' \le 0$, and there is no fluid motion. At $t' > 0$, the upper plate is started impulsively with a constant velocity U_0, the viscous conducting fluid flow between the porous plate is due the impulsive motion of one of the porous plate and retarded by magnetic field acting perpendicular to the flow direction.

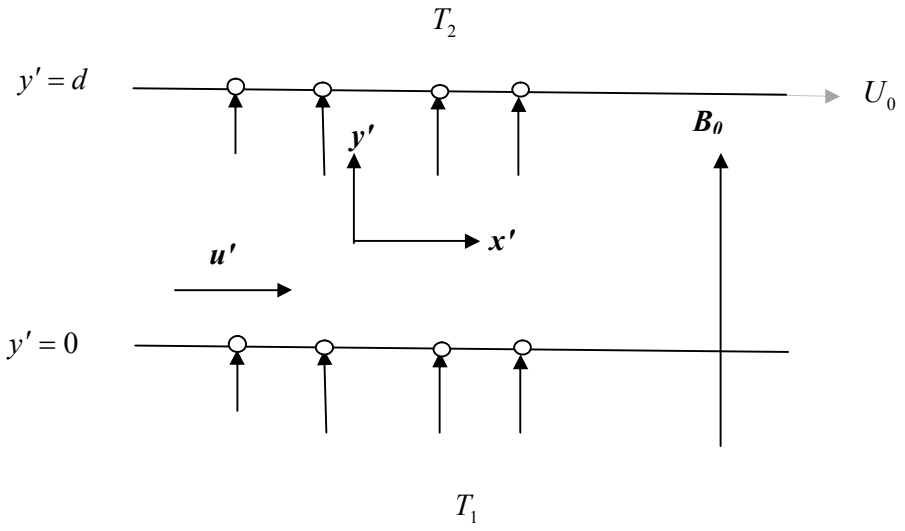


Fig.5.1.1 Physical configuration of flow

Neglecting the induced magnetic field (which is allowed at small Reynolds numbers) the continuity, momentum and energy equations in dimensional form are respectively:

$$\frac{\partial u'}{\partial x'} = 0 \qquad (5.1.1)$$

$$\frac{\partial u'}{\partial t'} + V_0 \frac{\partial u'}{\partial y'} = \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma \beta_0^{\,2}}{\rho} u' \tag{5.1.2}$$

$$\frac{\partial T}{\partial t'} + V_0 \frac{\partial T}{\partial y'} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y'^2} + \frac{\sigma}{\rho c_p} B_0^{\,2} u'^2 + \frac{\mu}{\rho c_p} \left(\frac{\partial u'}{\partial y'} \right)^2 \tag{5.1.3}$$

The initial and boundary conditions are

$$t' \le 0 : u' = 0, T' = T_1 \quad for\, 0 \le y' \le h$$

$$t' > 0 \begin{cases} u' = 0 : T' = T_1 & \text{at } y' = 0 \\ u' = U_0 ; T' = T_2 & \text{at } y' = h \end{cases} \tag{5.1.4}$$

It is convenient to make the governing equations (5.1.2) and (5.1.3) and initial and boundary conditions (5.1.4) dimensionless by using the non-dimensional quantities

$$y = \frac{y'}{h}, t = \frac{t' U_0^2}{\nu}, u = \frac{u'}{U_0}, M^2 = \frac{\sigma B_0^2 \nu}{\rho U_0^2}, \theta = \frac{(T' - T_1)}{T_2 - T_1}, \text{Re} = \frac{V_0 h}{\nu}, \; E_c = \frac{U_0^{\,2}}{(T_2 - T_1) C_P} \tag{5.1.5}$$

Again using expressions (5.1.5) in equations (5.1.2) and (5.1.3) the dimensionless momentum and energy equations are:

$$\frac{\partial u}{\partial t} + \text{Re} \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - M^2 u \tag{5.1.6}$$

$$\frac{\partial \theta}{\partial t} + \text{Pr}\,\text{Re} \frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2} + \text{Pr}\, Ec\, M^2 u^2 + \text{Pr}\, Ec \left(\frac{\partial u}{\partial y} \right)^2 \tag{5.1.7}$$

The initial and boundary conditions (5.1.4) reduce to:

$$t \le 0 : u = 0, \theta = 0 \quad for \;\; 0 \le y \le 1$$

$$t > 0 \begin{cases} u = 0 : \theta = 0 & \text{at } y = 0 \\ u' = 1; \;\; \theta = 1 & \text{at } y = 1 \end{cases} \tag{5.1.8}$$

Here Pr is the Prandtl number which is inversely proportional to the thermal conductivity; Re is the dimensionless Reynolds number which signifies suction and

injection velocity. Positive values of Re denote injection at the lower porous plate with a corresponding injection on the upper porous plate y=1while negative values of Re denote suction at the lower porous plate y=0 and injection at y=1, M, the Hartmann number and Ec is the Eckert number which is the

measure of viscous dissipation in the system. While θ is the dimensionless temperature of the fluid.

5.1.2 Analytical Solutions.

The analytical solution of equation (5.1.6) can be obtained by decomposing the solution in to two parts, say a transient part and a steady-state part, $u(y,t) = u_s(y) + u_t(y,t)$ as in (Fang2004b) where u_t and u_s represents the transient and steady state solution respectively. In this case we have two separate problems:

$$\mathrm{Re}\frac{du_s}{dy} = \frac{d^2u_s}{dy^2} - M^2u_s \tag{5.1.9}$$

With boundary and initial conditions

$$\begin{aligned} u_s &= 0 \ at \ y = 0 \\ u_s &= 1 \ at \ y = 1 \end{aligned} \tag{5.1.10}$$

The solution of Equation (5.1.9) with BCs (5.1.10) can be obtained as

$$u_s(y) = \frac{Exp(\frac{\mathrm{Re}}{2}(y-1))\sinh(dy)}{\sinh(d)} \tag{5.1.11}$$

Where $d = \sqrt{M^2 + \frac{\mathrm{Re}^2}{4}}$ (5.1.12)

The transient part of the problem therefore becomes

$$\frac{\partial u_t}{\partial t} + \mathrm{Re}\frac{\partial u_t}{\partial y} = \frac{\partial^2 u_t}{\partial y^2} - M^2u_t \tag{5.1.13}$$

With the boundary and initial conditions

$$u_t(0,t)=0\ ;\ u_t(1,t)=0\ \ and\ \ u_t(y,0)=-u_s(y)$$

$$=-\frac{Exp(\mathrm{Re}/2\,(y-1))\sinh(dy)}{\sinh(d)} \tag{5.1.14}$$

Also the solution of Equation (5.1.13) can be obtained using the transformation

$$u_t(y,t)=\bar{u}(y,t)Exp(\frac{\mathrm{Re}\,y}{2}-d^2t) \tag{5.1.15}$$

Using Eq.(5.1.15) in Eq.(5.1.13) reduces it to

$$\frac{\partial\bar{u}}{\partial t}=\frac{\partial^2\bar{u}}{\partial y^2} \tag{5.1.16}$$

With the BCs $\bar{u}(0,t)=0$; $\bar{u}(1,t)=0$ and $\bar{u}(y,0)=-\overline{u_s}Exp(-\frac{\mathrm{Re}}{2}y)$ (5.1.17)

The solution of Eq.(5.1.16) can be derived using the Fourier series method Carslaw and Jaeger(1963)as

$$u_t(y,t)=2\pi\sum_{n=0}^{\infty}\ \frac{Exp(\frac{\mathrm{Re}(y-1)}{2}-d^2t))}{d^2+n^2\pi^2}n(-1)^n\,Exp(-n^2\pi^2t)\sin n\pi y \tag{5.1.18}$$

Therefore the overall transient solution is

$$u(y,t)=\frac{Exp(\frac{\mathrm{Re}}{2}(y-1))\sinh(dy)}{\sinh(d)}$$

$$+2\pi^2\sum_{n=0}^{\infty}\ \frac{Exp(\frac{\mathrm{Re}(y-1)}{2}-d^2t))}{d^2+n^2\pi^2}n(-1)^n\,Exp(-n^2\pi^2t)\sin n\pi\,y \tag{5.1.19}$$

When $M=0$, the solution reduces to the solution in [Fang 2004b]

The skin frictions at the lower and upper moving porous plates are respectively

$$\tau_0 = \left.\frac{\partial u}{\partial y}\right|_{y=0}$$

$$= \frac{dExp(-\frac{\mathrm{Re}}{2})}{\sinh(d)} + 2\pi^2 \sum_{n=0}^{\infty} \frac{Exp(-(\frac{\mathrm{Re}}{2} + d^2 t))}{d^2 + n^2\pi^2} n^2 (-1)^n Exp(-n^2\pi^2 t) \qquad (5.1.20)$$

$$\tau_1 = \left.\frac{\partial u}{\partial y}\right|_{y=1}$$

$$= \frac{\mathrm{Re}}{2} + \frac{d\cos(d)}{\sinh(d)} + 2\pi^2 \; Exp(-d^2 t) \sum_{n=0}^{\infty} \frac{n^2 Exp(-n^2\pi^2 t)}{d^2 + n^2\pi^2} \qquad (5.1.21)$$

5. 1.3 Steady state temperature

It is difficult to derive the exact solution of the transient Energy Equation in (5.1.7) with viscous and Joulean dissipation using the transient velocity profile in (5.1.19). Therefore, since the flow will develop after a certain period it will be practically expedient to solve the steady state temperature equation.

The dimensionless steady state energy equation by taking $\frac{\partial \theta}{\partial t} = 0$, becomes

$$\mathrm{Pr}\,\mathrm{Re}\frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2} + \mathrm{Pr}\, Ec\, M^2 [u_s]^2 + \mathrm{Pr}\, Ec(\frac{du_s}{dy})^2 \qquad (5.1.22)$$

Substituting equation (5.1.11) into equation (5.1.22) gives

$$\Pr \mathrm{Re}\frac{d\theta}{dy} = \frac{d^2\theta}{dy^2} + \Pr Ec M^2 \left[\frac{Exp(\frac{\mathrm{Re}}{2}(y-1))\sinh(dy)}{\sinh(d)} \right]^2$$

$$+ \Pr Ec(R \left[\frac{\frac{\mathrm{Re}}{2}Exp(\frac{\mathrm{Re}}{2}(y-1))\sinh(dy)}{\sinh(d)} + \frac{dExp(\frac{\mathrm{Re}}{2}(y-1))\cosh(dy)}{\sinh(d)} \right]^2 \quad (5.1.23)$$

Simplifying (5.1.23) yields

$$\frac{d^2\theta}{dy^2} - \Pr \mathrm{Re}\frac{d\theta}{dy} =$$
$$-\frac{Ec\Pr}{4Exp(\mathrm{Re})\sinh^2(d)}[d(2d+\mathrm{Re})\exp(2d+\mathrm{Re})y + d(2d-\mathrm{Re})\exp(-(2d-\mathrm{Re})y)]$$

(5.1.24)

With the boundary conditions

$$\theta(0) = 0 \quad and \ \theta(1) = 1 \quad (5.1.25)$$

The general solution of Equation (5.1.24) can be obtained as

$$\theta(y) = A_1 + A_2 Exp(\Pr \mathrm{Re}\, y) - \frac{Ec\Pr}{c}\{c_1 Exp(2d+\mathrm{Re})y + c_2 Exp(-(2d-\mathrm{Re})y)\} \quad (5.1.26)$$

Therefore combining equation (5.1.26) with BCs (5.1.25), gives

$$A_1 = \frac{c_1\{Exp(\Pr \mathrm{Re}) - Exp(2d+\mathrm{Re})\} + c_2\{Exp(\Pr \mathrm{Re}) - Exp - (2d-\mathrm{Re})\} - 1}{Exp(\Pr \mathrm{Re}) - 1}$$

$$A_2 = \frac{c_1\{Exp(2d+\mathrm{Re}) - 1\} + c_2\{Exp(-(2d-\mathrm{Re})) - 1\} - 1}{Exp(\Pr \mathrm{Re}) - 1}$$

Therefore the steady –state temperature distribution for the flow is

$$\theta(y) = \frac{Exp(\Pr \mathrm{Re}\, y) - 1}{Exp(\Pr \mathrm{Re}) - 1} + \frac{Ec\Pr}{c(Exp(\Pr \mathrm{Re}) - 1)}[c_1\{Exp(\Pr \mathrm{Re}) - Exp(2d+\mathrm{Re})$$

$$+ Exp(2d + \mathrm{Re}(1 + \mathrm{Pr}\, y)) - Exp((2d + \mathrm{Re})y + \mathrm{Pr}\,\mathrm{Re}) + Exp(2d + \mathrm{Re})y - Exp(\mathrm{Pr}\,\mathrm{Re})y)\}$$

$$+ c_2\{Exp(\mathrm{Pr}\,\mathrm{Re}) - Exp(-(2d - \mathrm{Re})) + Exp(-(2d - \mathrm{Re}) + \mathrm{Pr}\,\mathrm{Re}\, y)$$

$$- Exp(-(2d - \mathrm{Re})y + \mathrm{Pr}\,\mathrm{Re}) + Exp(-(2d - \mathrm{Re})y) - Exp(\mathrm{Pr}\,\mathrm{Re}\, y)] \qquad (5.1.27)$$

$$c = \frac{1}{4Exp(\mathrm{Re})\sinh^2(d)}\ ;\ c_1 = \frac{d}{2d + \mathrm{Re}(1 - \mathrm{Pr})}\ ;\ c_2 = \frac{d}{2d - \mathrm{Re}(1 - \mathrm{Pr})} \qquad (5.1.28)$$

It can be seen from (5.1.27) that the temperature profile is linearly dependent on the Eckert number.

5.1.4 Nusselt Number

The Nusselt number, which is the rate of heat transfer on the porous plates, can be obtained from

$$Nu = \left[\frac{d\theta}{dy}\right]_{y=0,1}$$

Therefore the Nusselt number for the bottom plate is

$$Nu_0 = \left[\frac{d\theta}{dy}\right]_{y=0}$$

$$= \frac{\mathrm{Pr}\,\mathrm{Re}}{Exp(\mathrm{Pr}\,\mathrm{Re}) - 1}\left[1 + \frac{Ec\,\mathrm{Pr}}{c}\{c_1(Exp(2d + \mathrm{Re}) - 1) + c_2\{Exp(-(2d - \mathrm{Re})) - 1)\}\right]$$

$$- \frac{Ec\,\mathrm{Pr}}{c}[c_1(2d + \mathrm{Re}) - c_2(2d - \mathrm{Re})] \qquad (5.1.29)$$

and for the upper porous plate is

$$Nu_1 = \left[\frac{d\theta}{dy}\right]_{y=1}$$

$$= \frac{\Pr \operatorname{Re} \exp(\Pr \operatorname{Re})}{\exp(\Pr \operatorname{Re}) - 1}\left[1 + \frac{Ec \Pr}{c}\{c_1(\exp(2d + \operatorname{Re}) - 1) + c_2\{\exp(-(2d - \operatorname{Re})) - 1)\right]$$

$$- \frac{Ec \Pr}{c}\left[c_1(2d + \operatorname{Re}) \exp(2d + \operatorname{Re}) - c_2(2d - \operatorname{Re}) \exp(-(2d - \operatorname{Re}))\right] \tag{5.1.30}$$

5.1.5 Results and Discussion

The role of suction and injection and other parameters on time dependent velocity profiles and steady state temperature on the MHD Couette flow of a viscous incompressible fluid in a horizontal channel formed by two infinite porous plates that is subjected to asymmetric thermal boundary conditions is presented. The dimensionless closed form expression for transient velocity profiles and steady state temperature are derived exactly for the MHD Couette flow.

The non –dimensional parameters that govern the flow are the Prandtl number ($\Pr$) which is inversely proportional to the thermal diffusivity of the fluid. The suction/injection parameter, Re , Hartmann number, M and the Nusselt number Nu . In this segment computational results are presented for different values of the suction/injection parameter, Re , Hartmann number, M , Prandtl number , $\Pr$,Nusselt number Nu and Eckert number, Ec .

For the purpose of discussion some numerical calculations are carried out for dimensional temperature and velocity. In order to be realistic the values of Pr are chosen between the non-dimensional values of 0.001 and 7.0 to accommodate fluids like mercury (0.008-0.041), water vapour (0.882-0.994), oxygen (0.729-0.757), air (0.703-0.784), water (5.18-8.91), etc. Similarly the values of S are chosen between the non-dimensional values of -5 to +5 to represent suction as well as injection. The results are shown in figures fig.5.1.2 to fig.5.1.16.

Figures 5.1.2a and 5.1.2b Show the variation of transient velocity (u_t) versus distance (y) for different values of time when Re=5.0 and Re=-5.0 respectively, for fixed values of t=0.05 and the Hartmann number, M =2.0. These figures show that velocity increases with increase in time in the presence of injection or suction at the lower porous plate respectively.

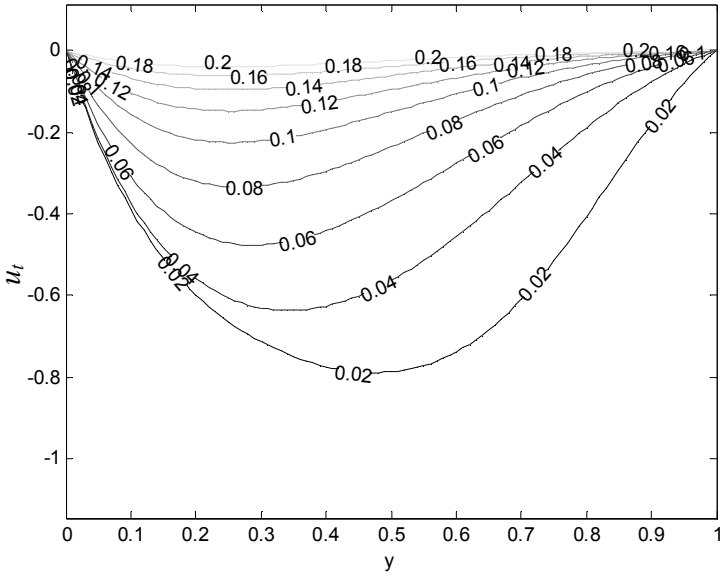


Fig.5.1.2a Ttransient velocity (u_t)versus distance(y) for different (M =2.0, Re=5.0)

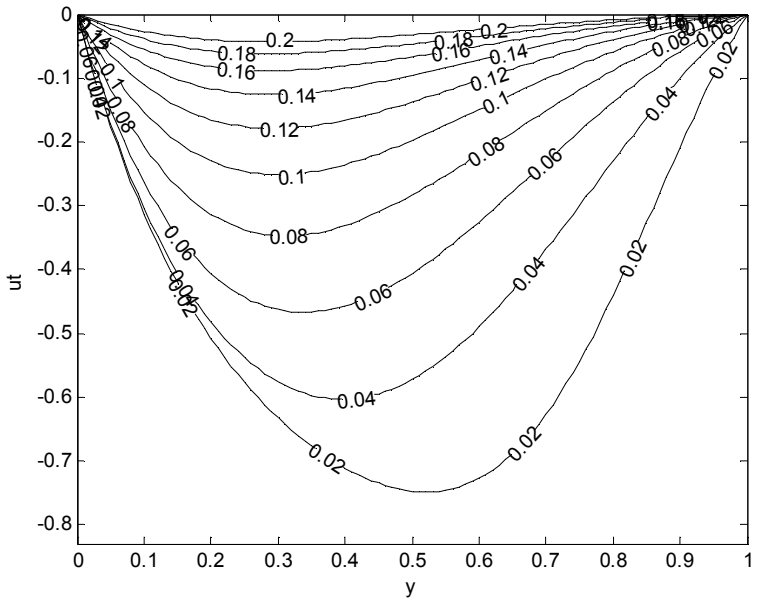


Fig.5.1.2b Variation of transient velocity (u_t) versus distance (y) for different t (M =2.0, Re=-5.0)

Fig. 5.1.3a shows the variation of velocity (u) versus distance (y) for different values of time for fixed values of $\mathrm{Re} = -5.0$ and the Hartmann number, M =2.0.This figure shows that the velocity (u) increases with increase in time and approach the steady state case for higher values of t.

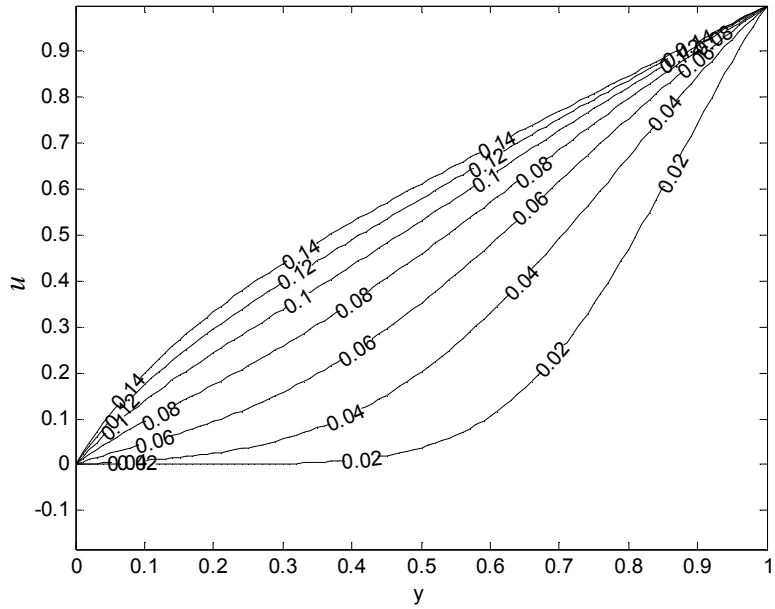


Fig.5.1.3a Velocity (u_t) versus distance (y) for different t (M =2.0, Re=-5.0)

Fig.5.1.3b shows the variation of transient velocity (u_t) versus distance (y) for different values of the injection/ suction parameter Re for fixed values of t=0.05 and the Hartmann number, M =2.0. This figure shows that velocity increases as injection at the lower porous plate increases while it decreases with mass suction at the lower porous plate. It is also observed that in the absence of suction/injection ($\mathrm{Re} = 0$), the velocity profile is parabolic and symmetrically distributed. It is interesting to note that when $\mathrm{Re} < 0$ (under mass suction) the transient velocity reaches a maximum value for a critical value of the Reynolds number. When the magnitude of the Reynolds number is greater than or less than this critical Reynolds number the transient velocity will become weaker. This critical Reynolds number depends on time. The critical Reynolds number for this problem is $\mathrm{Re} = -8.0$ when t=0.05 which is lower than the critical number ($\mathrm{Re} = -6.5$) obtained by Fang (2004b). This is due to the influence of magnetic field M in this investigation which slows the system.

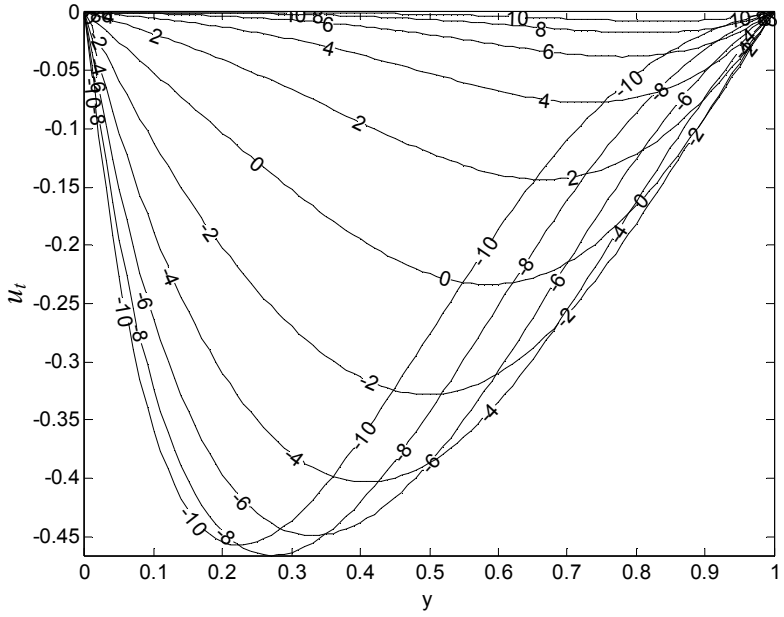


Fig.5.1.3b Transient velocity (u_t) versus distance (y) for different Re (t =0.05 and M =2.0)

Fig.5.1.4a and 5.1.4b depict the variation of transient velocity versus distance for different values of the Hartmann number (M) and fixed value of time (t) in the presence of mass suction or injection.

These figures show that under mass injection or suction on the lower porous plate, the magnitude of transient velocity decreases with increase in M. This is because the application of transverse magnetic field normal to the flow has a tendency to create a drag-like force called the Lorentz force. This force has a decreasing effect on the flow velocity. It is also observed that the magnitude of transient velocity is higher under mass suction in contrast to the case when there is mass injection at the lower porous plate.

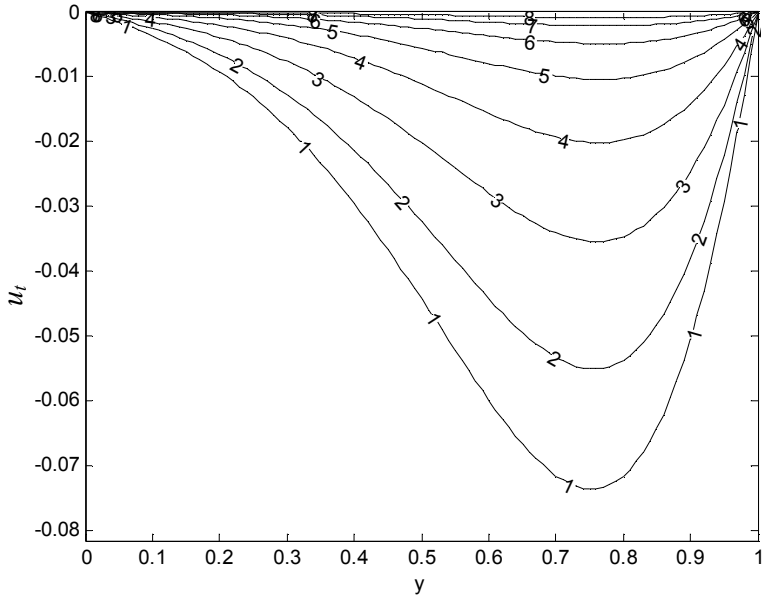


Fig.5.1.4a Transient velocity (u_t) versus distance (y) for different M (t=0.05 and Re =5.0)

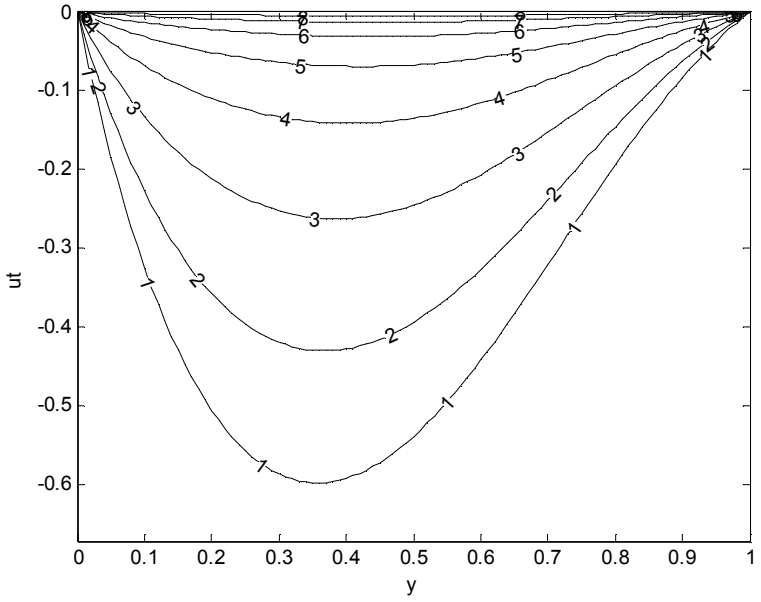


Fig.5.1.4b Transient velocity (u_t) versus distance (y) for different M (t=0.05, Re=-5.0)

Fig.5.1.5a and 5.1.5b repectively show the comparative effects of mass injection and suction at the lower porous plate on steady state temperature for fixed values of $M = 2.0$, $Ec = 0.0$, $\mathrm{Pr} = 0.7$ .and $\mathrm{Pr} = 7.0$.

These figures show that if the Eckert number is neglected ($Ec = 0.0$) for the same magnitude of mass transfer (Re=5.0 and Re=-5.0), the temperature profile is symmetrical and that the temperature of the fluid increases with increase in suction while it decreases with increase in injection at the lower porous plate. This is physically true since suction of fluid increases the fluid velocity thereby increasing the sheared heating within the porous plates. Consequently there is accumulation of heat within the channel leading to increase in temperature.

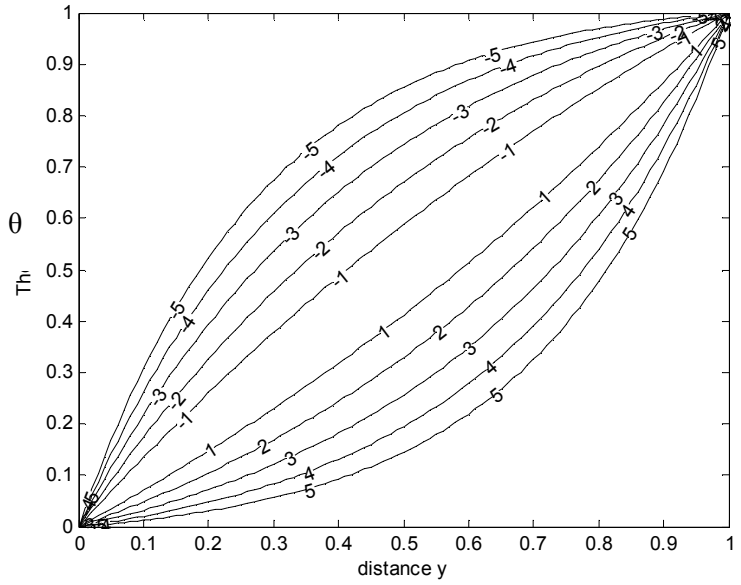


Fig.5.1.5a Steady state temperature (θ) versus distance (y) for different Re (M =2.0, Pr =0.71, Ec =0.0)

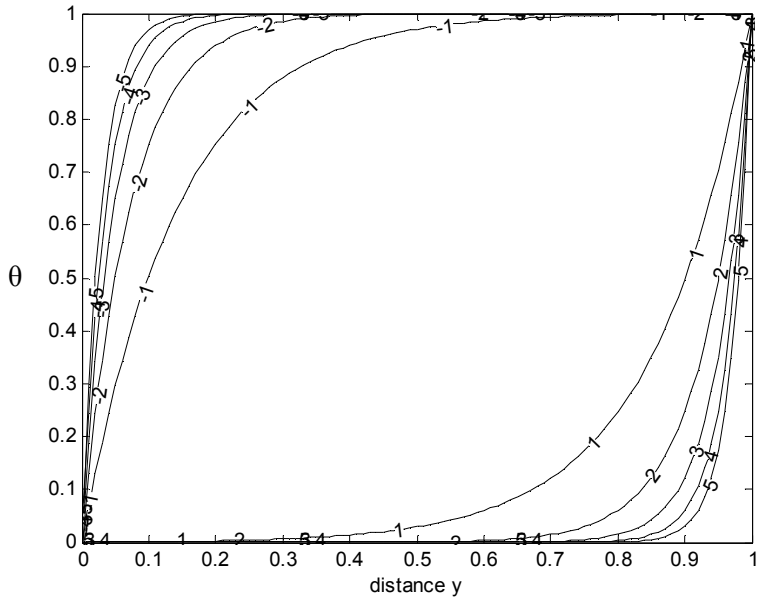


Fig 5.1.5b Steady state temperature (θ) versus distance (y) for different Re (M =2.0, Pr =7.0, Ec =0.0)

Figures 5.1.6a and 5.1.6b respectively show the effects of Eckert number on the steady state temperature for fixed values of $M = 2.0$, $Ec = 2.0$, $\mathrm{Pr} = 7.0$ .and $\mathrm{Pr} = 0.7$ These figures show that the steady state temperature decreases with increase in suction or injection at the lower porous plate.

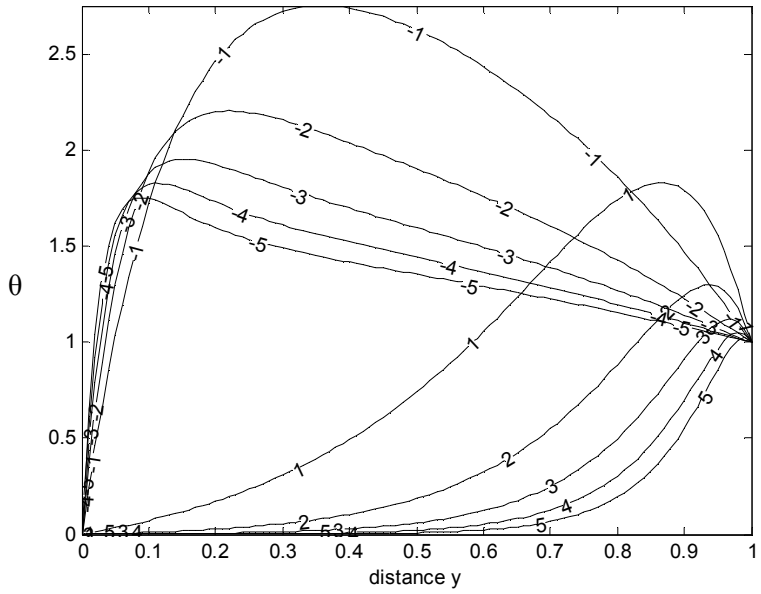


Fig. 5.1.6a Steady state temperature (θ) versus distance (y) for different Re (M =2.0, Pr =7.0 (water) and Ec =2.0)

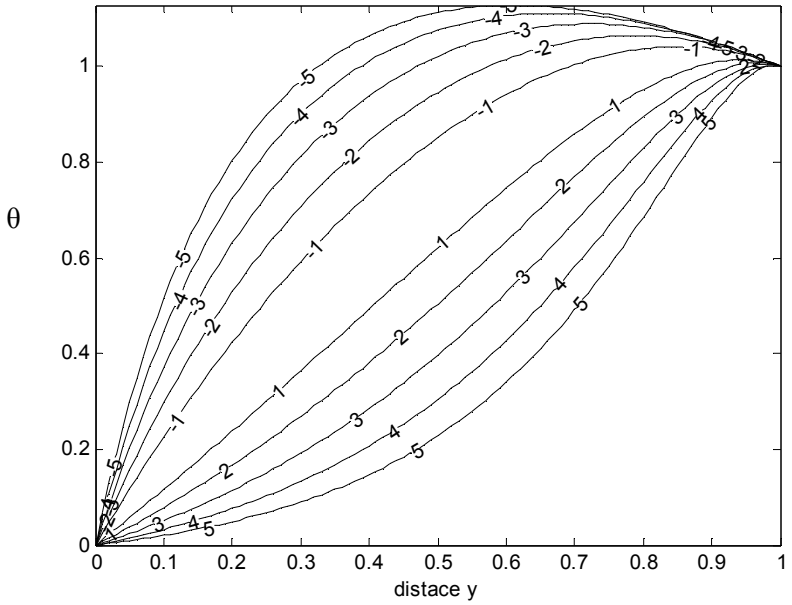


Fig.5.1.6bSteady state temperature (θ) versus distance (y) for different Re (M =2.0, Pr =0.7(air), Ec =2.0)

In order to compare the effect of Prandtl number on temperature field, the variation of the steady state temperature for different values of the Prandtl number and fixed values of the Hartmann number $M = 2.0$, $Ec = 0.0$, $\mathrm{Re} = 2.0$ and $\mathrm{Re} = -2.0$ i.e. under negligible viscous dissipation, in the presence of injection or suction at the lower porous plate is shown in figures 5.1.7a and 5.1.7b respectively.

Figure 5.1.7a reveals that when the viscous term is neglected, under mass injection at the lower porous plate the temperature decreases with increase in Pr. The reverse is however the case under mass suction when the temperature increases with increase in Pr as depicted in fig. 5.1.7b

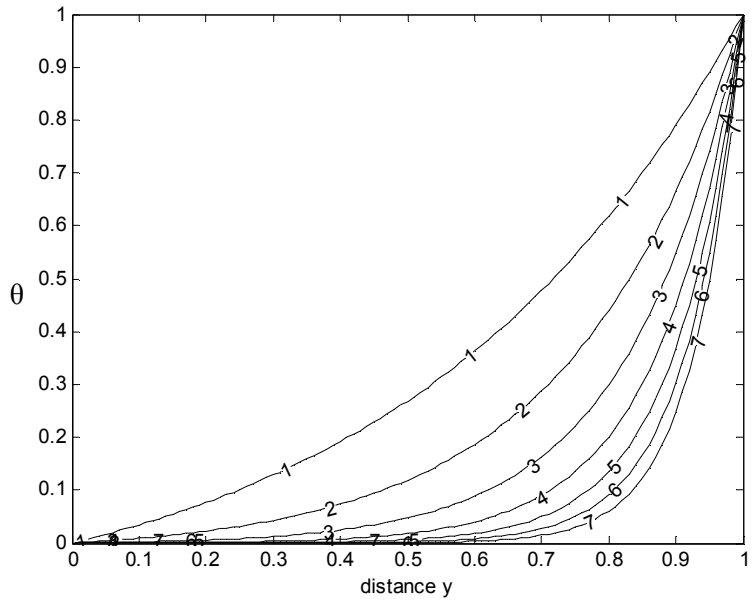


Fig.5.1.7a Steady state temperature (θ) versus distance (y) for different Pr (M =2.0, Re =2.0, Ec =0.0)

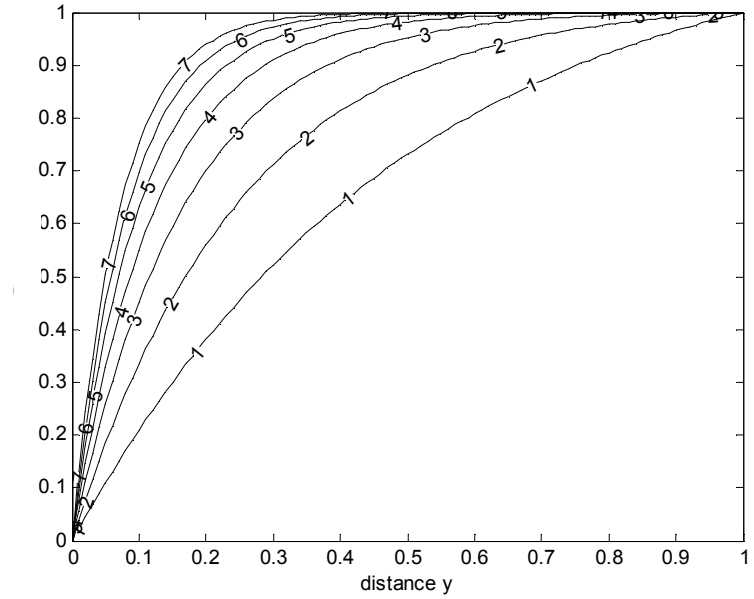


Fig.5.1.7bSteady state temperature (θ) versus distance (y) for different Pr (M =2.0, Re =-2.0, Ec =0.0)

Fig.5.1.8a and 5.1.8b show the influence of mass injection and suction at the lower porous plate on steady state temperature for different values of Pr when M =2.0, $Ec = 2.0$, $\mathrm{Re} = 2.0 \,\&\, \mathrm{Re} = -2.0$ Fig.5.1.8a shows that under mass suction at the lower porous plate the temperature increases with increase in Pr. The temperature reaches a maximum, decreases and then coincides, for different Pr, near the upper moving porous plate. The reverse is however the case under mass injection when the temperature starts to decrease with increase in Pr near the lower stationary plate, reaches maximum near the upper moving porous plate where the temperature increases with increase in Pr as depicted by figure 5.1.8b.

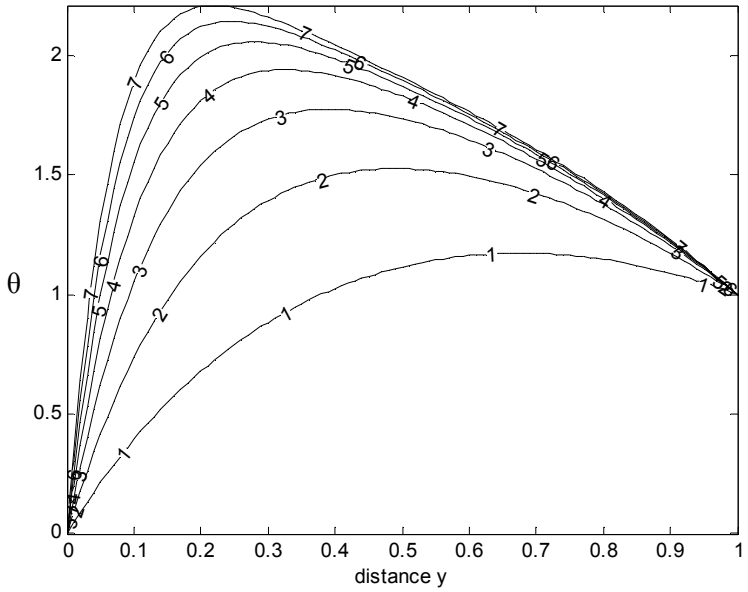


Fig.5.1.8aSteady state temperature (θ) versus distance (y) for diferent Pr (M =2.0, Re =-2.0, Ec =2.0)

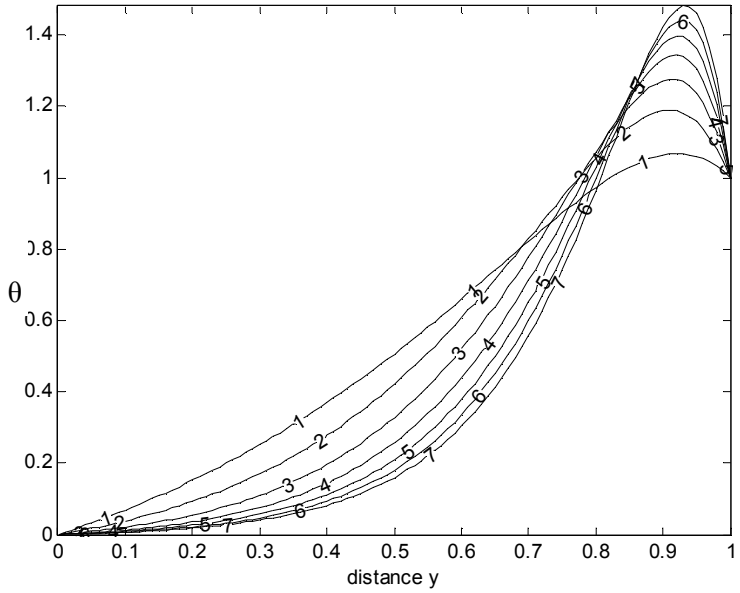


Fig.5.1.8b Steady state temperature (θ) versus distance (y) for different Pr (M =2.0 $\mathrm{Re} = 2.0$, Ec =2.0)

The effects of the Hartmann number M on the skin friction, at the lower and upper porous plates are shown in figures 5.1.9a and 5.1.9b respectively. Figure 5.1.9a reveals that as M increases the skin friction at the lower porous plate decreases. On the other hand Fig.5.1.9b shows that the skin friction at the upper moving porous plate increases as M increases. This is the case since Hartmann number increases velocity gradient increases near the moving porous plate consequently skin friction increases, while the result is just reverse at the stationary porous plate. Furthermore, the role of Hartmann number on skin friction is negligible in case of injection at the stationary plate. It is interesting to mention that skin friction is higher in case of injection at the moving plate in comparison to suction at the moving plate.

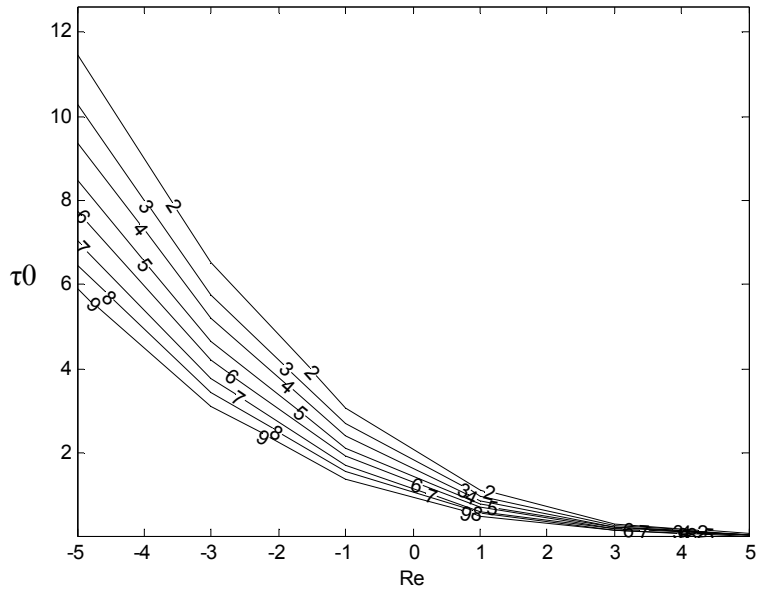


Fig.5.1.9a Skin friction (τ_0) versus (Re) for different M at y =0(t =0.05)

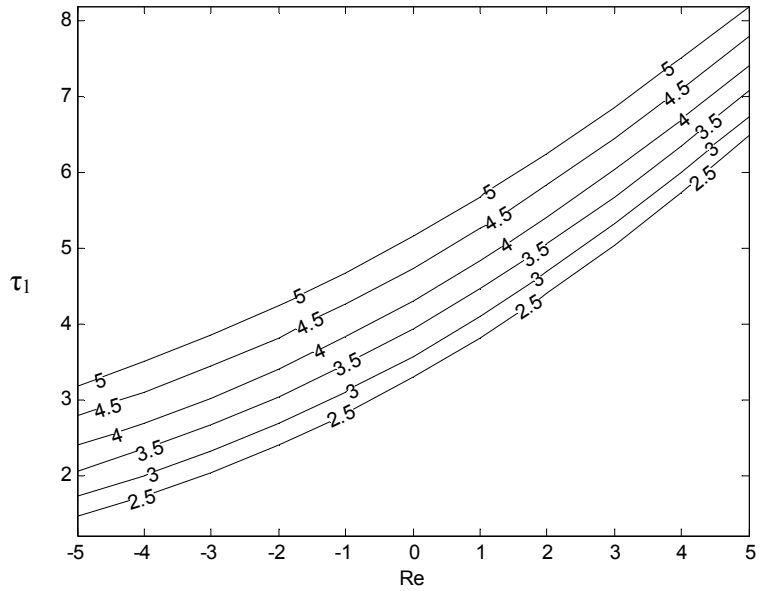


Fig.5.1.9b Skin friction (τ_1) versus (Re)different M at y =1 (t=0.05)

The effects of the suction/injection parameter, Re, on the skin friction at the lower and upper porous plates is displayed in figures 5.1.10a and 5.1.10b. These figures show that at the lower porous plate skin friction decreases as Re increases in contrast with the case at the upper moving porous plate where skin friction increases with increases in Re. Thus these figures reveal that injection at the lower porous plate suppresses skin friction while suction enhances it and the reverse is the case at the upper moving porous plate where injection enhances skin friction while suction suppresses it.

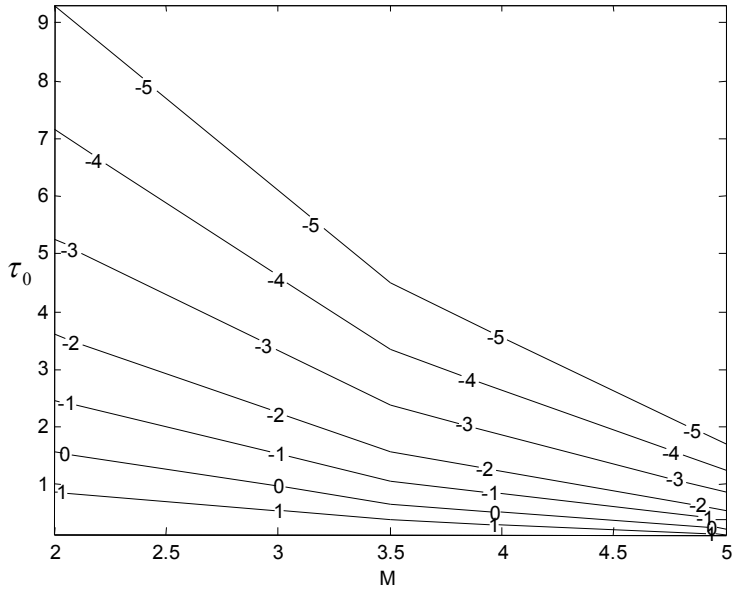


Fig.5.1.10a Skin friction (τ_0) versus (M) for different Re at y =0 (t=0.05)

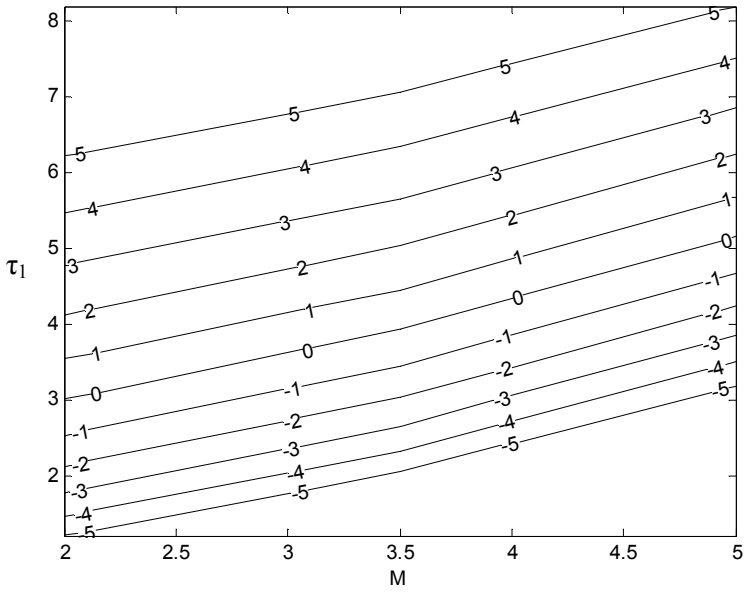


Fig.5.1.10b Skin friction (τ_1) versus (M) for different Re t =0.05, at y =1

The role of the Prandtl number Pr , as well as the suction/injection parameter, Re , on the Nusselt number is captured in figures 5.1.11a and 5.1.11b. These Figures reveal that the Nusselt number increases with increase in Pr both at the lower and upper moving porous plate. This is physically true since as Pr increase thermal conductivity decreases hence temperature gradient between fluid and porous plates is high. Thus rate of heat transfer increases with increase of Pr . Furthermore these figures show that the Nusselt number at the lower porous plate decreases with increase in the Reynolds number while the reverse is the case at the upper moving boundary where the Nusselt number increases with increase in Re .

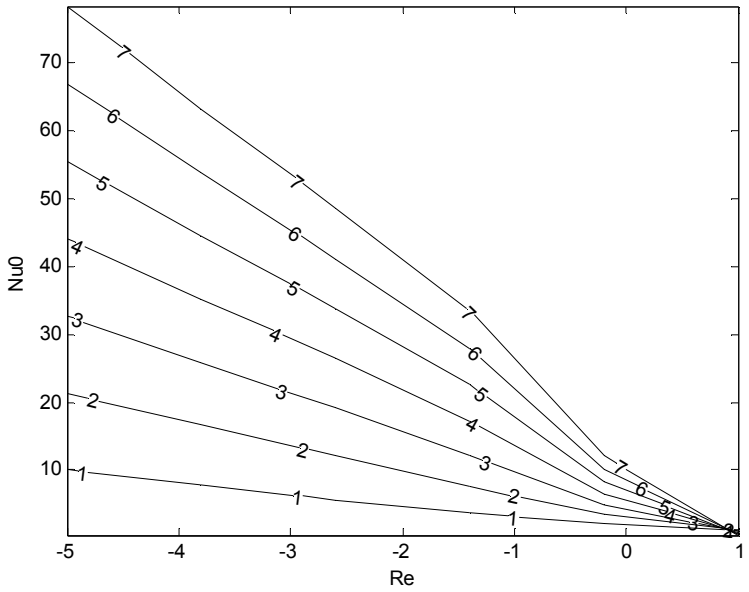


Fig.5.1.11a Nusselt number(Nu_0) versus (Re) for different Pr at $y = 0$ (M =2.0 , Ec =2.0)

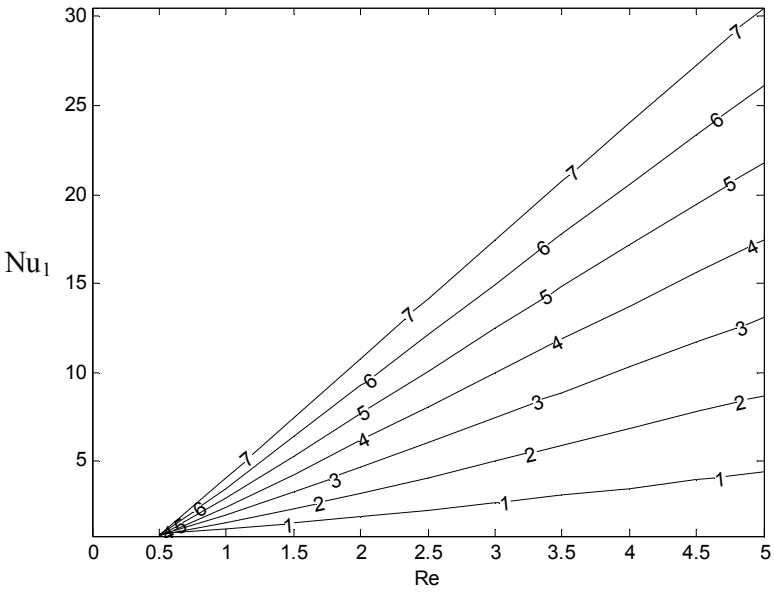


Fig.5.1.11b Nusselt number(Nu_1)versus (Re) for different Pr at y=1 (M =2.0, Ec =2.0)

Figures 5.1.12a and 5.1.12b show the variation of Nusselt number versus Re for different values of the Hartmann number M for fixed values of $\mathrm{Pr}=0.7$ and $Ec=0.2$, on the lower and upper porous plate. These figures show that the Nusselt number on the lower as well as the upper moving porous plate decreases with increase in the Hartmann number M . This is physically true since as M increases the dissipation is high so the temperature gradient is less between the fluid and porous plate leading to less heat transfer.

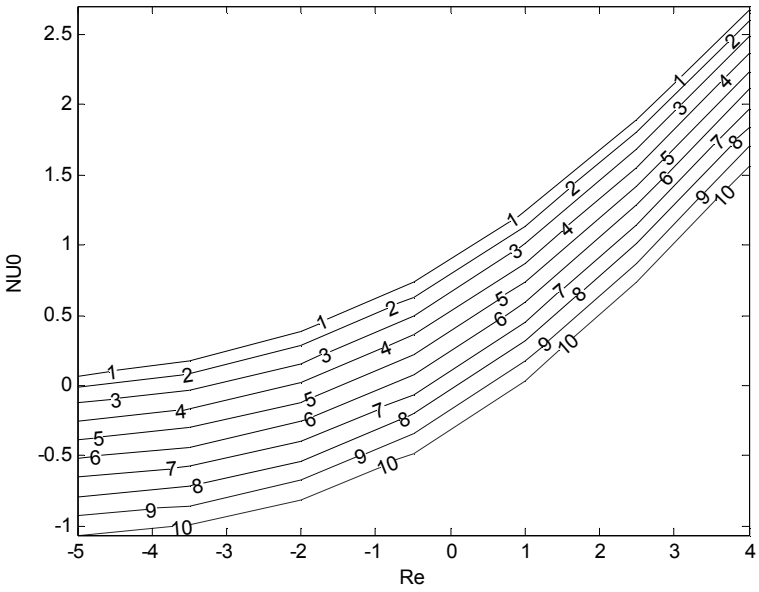


Fig.5.1.12a Nusselt number(Nu_0) versus (Re) for different M at $y=0$ ($\mathrm{Pr}=0.7, Ec=0.2$)

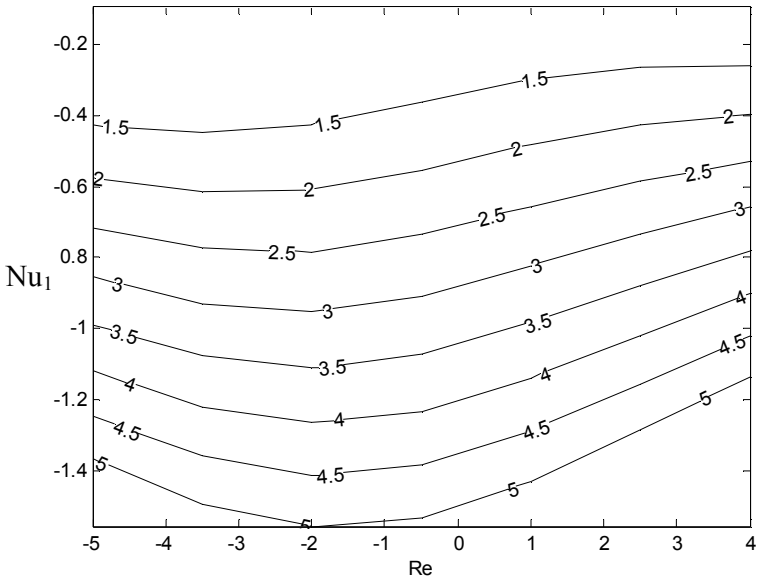


Fig.5.1.12b Nusselt number(Nu_1) versus (Re) for different M at $y = 1$ (Pr =0.7, Ec =2.0)

Figures 5.1.13a and 5.1.13b present the representative profile of the fluid temperature for different values of the Eckert number Ec and fixed values of M =2.0, Pr =0.7, Re =-5.0 & Re =5.0 respectively. These two figures reveal that in the presence of mass suction or injection at the lower porous plate the fluid temperature increases with increase in the Eckert number.

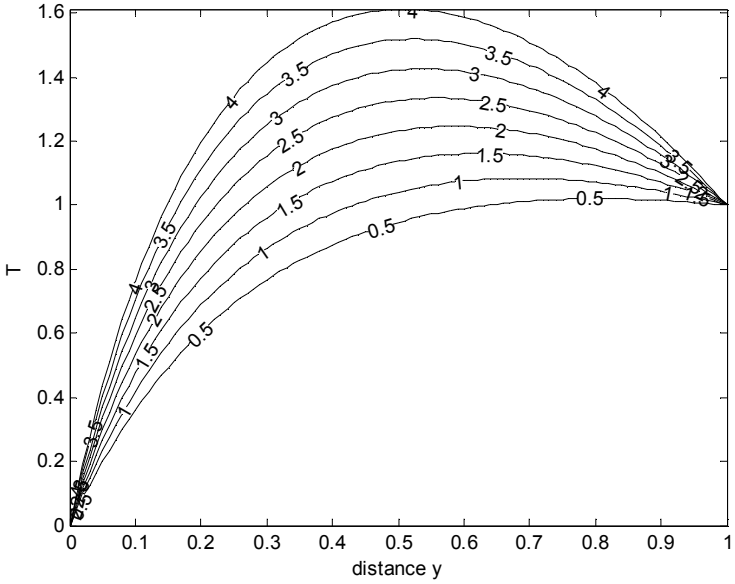


Fig.5.1.13a Steady state temperature (θ) versus distance (y) for diffirent Ec (M =2.0, $\Pr$ =0.71, Re =-5.0)

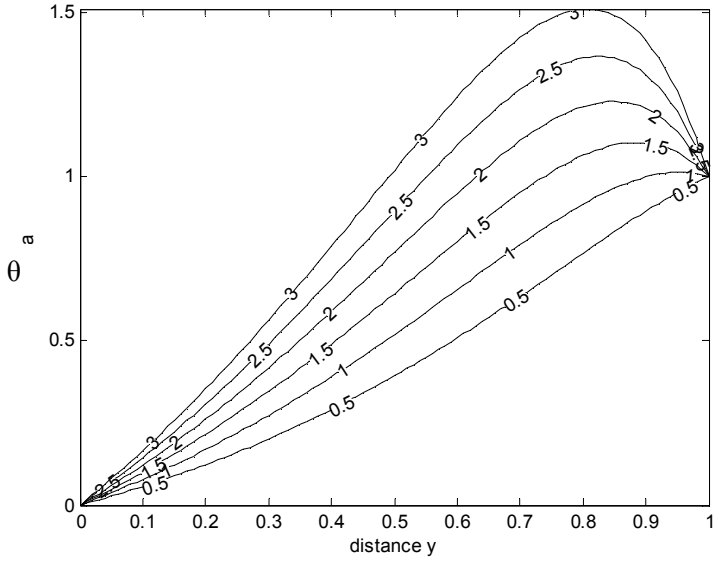


Fig.5.1.13b Steady state temperature (θ) versus distance (y) for different Ec (M =2.0, $\Pr$ =0.71, Re =5.0)

Figures 5.1.14a and 5.1.14b show the profile of the fluid temperature for different values of the Eckert number Ec and fixed values of M =2.0, Pr =7.0, Re =-1.0 & Re =1.0 respectively. Both Figures reveals that, for a higher value of Pr , in the presence of mass injection at the lower porous plate the fluid temperature increases with increase in Ec . Furthermore figure 5.1.14a shows that in the presence of suction, the temperature increases, reach the maximum near lower porous plate, and then decreases to the plate temperature, while figure 5.1.14b shows that in the presence of injection/suction the temperature increases, reaches the maximum near upper moving porous plate, and then decreases to the plate temperature.

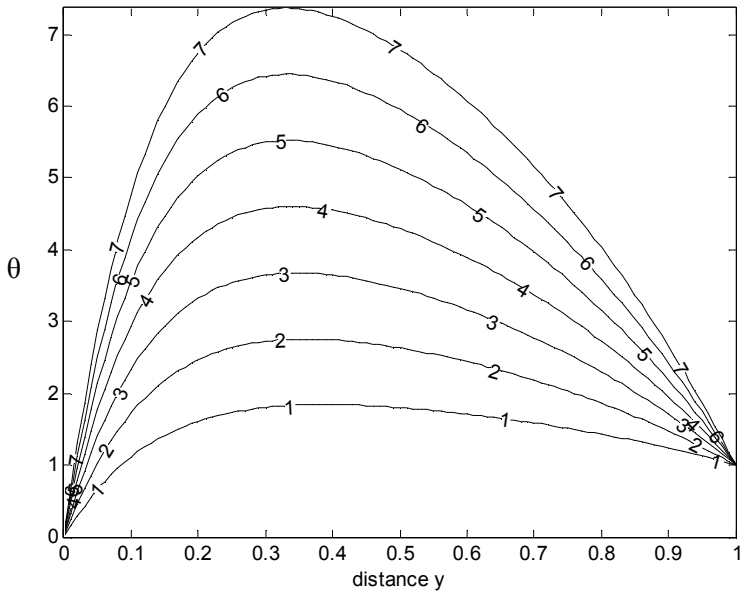


Fig.5.1.14a Steady state temperature (θ) versus distance (y) for different Ec , (M =2.0, Pr =7.0, Re =-1.0)

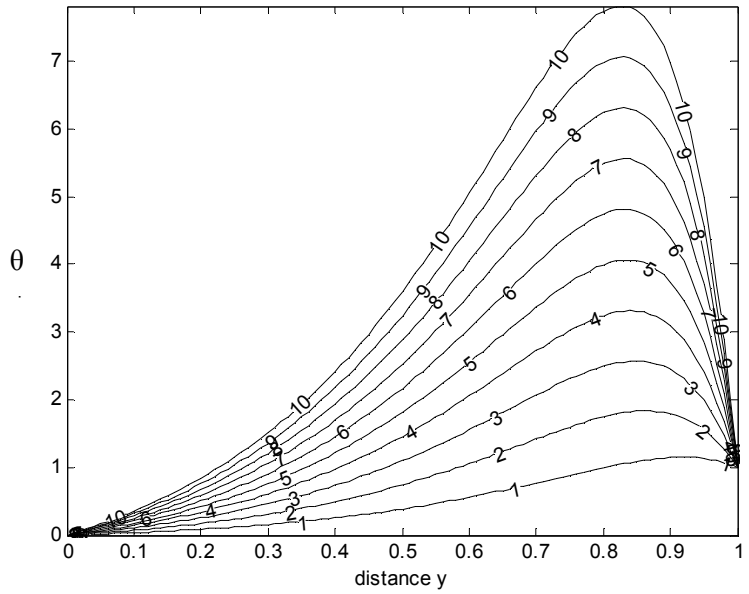


Fig.5.1.14b Steady state temperature (θ) versus distance (y) for different Ec (M =2.0, $\Pr$ =7.0 Re =1.0)

5.1.6 Summary and Conclusion

In this section, the effects of viscous and Joulean dissipation on unsteady MHD Couette flow of a viscous incompressible electrically conducting fluid between parallel porous plates when one of the porous plates is set into constant motion has been considered. The influence of the suction/injection parameter and the Hartmann number on transient velocity is analysed and discussed. Also the steady state temperature equation is solved exactly. It is fascinating to note that the Hartmann number suction/injection and viscous and Joulean dissipation greatly affect the fluid motion and heat transfer characteristics. It was observed that:

The Hartmann number M decreases the transient velocity of the flow in the presence of suction/injection due to the magnetic pull of the Lorentz force acting on the flow field.

The transient velocity of the flow increases with injection at the lower porous plate while it decreases with mass suction.

The steady state temperature increases with increase in suction while it decreases with increase in injection at the lower porous plate.

The skin friction coefficient at the lower porous plate decreases with increase in the Hartmann number M, while it increases with increase in M at the upper moving porous plate.

5.2 Effects of viscous dissipation and ohmic heating on MHD Couette flow in a porous channel under the influence of electric field.

5.2.0 Introduction

In this section the effects of viscous and Ohmic dissipation on the transient velocity and steady state temperature for the Couette flow of incompressible, viscous and electrically conducting fluid between two infinite porous plates in the presence of magnetic and applied electric field are investigated. This is a generalization of the Fang article (2004b)

5.2.1. Formulation of the problem

The physical configuration considered in this problem is shown in fig.5.2.1.It consists of a time dependent (unsteady), incompressible, viscous Couette flow of an electrically conducting fluid trapped between two infinite horizontal parallel porous plates separated by a distance h. The lower porous plate is fixed and subjected to a mass injection velocity, V_0 with temperature T_1 and there is equal mass suction (with velocity, V_0) at the upper wall which is maintained at constant temperature T_2. A uniform magnetic field, $\vec{B} = (0, B_0, 0)$ is applied across the channel perpendicular to the porous plates and a uniform electric field $\vec{E} = (0,0, E_z)$.

It is assumed that there is no pressure gradient and the flow is assumed to be time dependent, unidirectional and fully developed. Such applications of electric and magnetic fields stabilize the boundary layer flow. The magnetic field, $\vec{B} = (0, B_0, 0)$ and electric field, $\vec{E} = (0, 0, E_z)$, satisfy the MHD (Maxwell's) equations $\nabla \bullet \vec{B} = 0$ and $\nabla \bullet \vec{E} = 0$, respectively. When the magnetic field is not so strong, then the magnetic field and electric field obey Ohm's law $J = \sigma(\vec{E} + \vec{q} \times \vec{B})$, where J is the Ohmic (Joule) current, σ, is the electrical conductivity of the fluid and $\vec{q}$ is the velocity of the fluid. We assume that the magnetic Reynolds number of the fluid is so small so that induced magnetic field and Hall effect can be neglected. .

And the since the porous plates are infinite in extent all physical quantities are functions of y' and t' only.

Both plates are assumed to be stationary at $t' \le 0$, and there is mass transfer in the y' -direction. At $t' > 0$, the upper plate is started impulsively with a constant velocity U_0 , the viscous conducting fluid flow in the channel is driven by impulsive motion of one of the porous plate and retarded by magnetic forces.

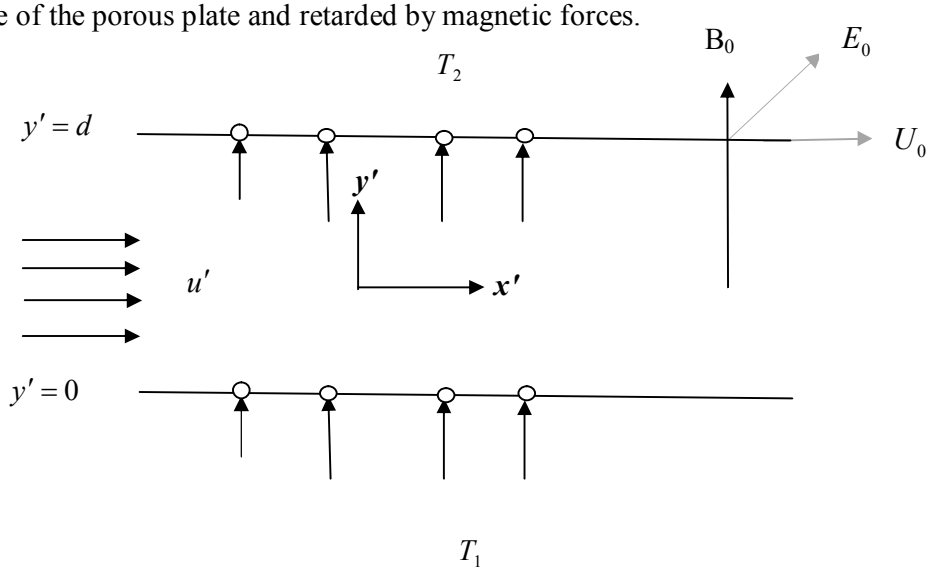


Fig. 5.2.1 Physical configuration of flow

Under the above stated physical situations, the governing boundary layer equations for continuity, momentum and energy equations in dimensional form are respectively:

$$\frac{\partial u'}{\partial x'} = 0 \tag{5.2.1}$$

$$\frac{\partial u'}{\partial t'} + V_0 \frac{\partial u'}{\partial y'} = \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma \beta_0{}^2 h^2}{\rho}(u' + \frac{E_z}{U_0}) \tag{5.2.2}$$

$$\frac{\partial T}{\partial t'} + V_0 \frac{\partial T}{\partial y'} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y'^2} + \frac{\sigma}{\rho c_p}\left[E_z + u' B_0\right]^2 + \frac{\mu}{\rho c_p}(\frac{\partial u'}{\partial y'})^2 \tag{5.2.3}$$

The initial and boundary conditions of the flow are

$$t' \le 0 : u' = 0, T' = T_1 \quad for\ 0 \le y' \le h$$

$$t' > 0 \begin{cases} u' = 0 : T = T_1 & \text{at } y' = 0 \\ u' = U_0 ; T = T_2 & \text{at } y' = h \end{cases} \tag{5.2.4}$$

It is convenient to make the governing equation (5.2.2), (5.2.3), and initial and boundary conditions (5.2.4) dimensionless by using the non-dimensionless quantities

$$y = \frac{y' U_0}{\nu}, t = \frac{t' U_0^2}{\nu}, u = \frac{u'}{U_0}, M^2 = \frac{\sigma B_0^2 \nu}{\rho U_0^2}, h = \frac{d U_0}{\nu}, \theta = \frac{(T - T_1)}{T_2 - T_1}$$

$$\mathrm{Re} = \frac{V_0 h}{\nu},\ E_c = \frac{U_0{}^2}{(T_2 - T_1) C_P}\ ,\ R = \frac{E_z}{U_0 B_0} \tag{5.2.5}$$

Using equation (5.2.5) in equations (5.2.2) and (5.2.3) the dimensionless momentum and energy equations are:

$$\frac{\partial u}{\partial t} + \mathrm{Re}\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - M^2 (R + u) \tag{5.2.6}$$

$$\frac{\partial \theta}{\partial t} + \mathrm{Pr}\,\mathrm{Re}\frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2} + \mathrm{Pr}\, Ec\, M^2 (R + u_s)^2 + \mathrm{Pr}\, Ec (\frac{\partial u_s}{\partial y})^2 \tag{5.2.7}$$

The initial and boundary conditions (5.2.4) reduce to:

$$t \le 0 : u = 0, \theta = 0 \quad for \; 0 \le y \le 1$$

$$t > 0 \begin{cases} u = 0 : \theta = 0 \quad \text{at } y = 0 \\ u = 1; \; \theta = 1 \text{ at } y = 1 \end{cases} \tag{5.2.8}$$

5.2.2. Analytical Solutions.

The analytical solution of equation (5.2.6) can be obtained by decomposing the solution in to two parts, say a transient part and a steady-state part, $u(y,t) = u_s(y) + u_t(y,t)$ as in Fang (2004b) where u_t and u_s represents the transient and steady state solution respectively. In this case we have two separate problems:

$$\mathrm{Re}\frac{du_s}{dy} = \frac{d^2 u_s}{dy^2} - M^2(R + u_s) \tag{5.2.9}$$

With boundary and initial conditions

$$\begin{aligned} u_s &= 0 \; at \; y = 0 \\ u_s &= 1 \; at \; y = 1 \end{aligned} \tag{5.2.10}$$

The solution of Eq. (5.2.9) with BCs (5.2.10) can be obtained as

$$u_s(y) = (R+1)\frac{Exp(\frac{\mathrm{Re}}{2}(y-1))\sinh(dy)}{\sinh(d)}$$

$$- R\left[\frac{\sinh(dy)}{\sinh(d)} Exp(-(d - \frac{\mathrm{Re}}{2}y)) - Exp(-(d - \frac{\mathrm{Re}}{2})y) + 1\right] \tag{5.2.11}$$

Where $d = \sqrt{M^2 + \frac{\mathrm{Re}^2}{4}}$ (5.2.12)

It should be noted that if the electric load parameter is equal to zero (R=0) *i.e* in the absence of electric field equation (5.2.11) will reduce to the solution Jha and Jibril (2011).

The transient part of the problem therefore becomes

$$\frac{\partial u_t}{\partial t} + \mathrm{Re}\frac{\partial u_t}{\partial y} = \frac{\partial^2 u_t}{\partial y^2} - M^2 u_t \tag{5.2.13}$$

With the boundary and initial conditions

$$u_t(0,t) = 0 \;;\; u_t(1,t) = 0 \;\; and \;\; u_t(y,0) = -u_s(y)$$

$$= -\{(R+1)\frac{Exp(\frac{\mathrm{Re}}{2}(y-1))\sinh(dy)}{\sinh(d)}$$

$$+ R\left[\frac{\sinh(dy)}{\sinh(d)}Exp(-(d-\frac{\mathrm{Re}}{2}y)) - Exp(-(d-\frac{\mathrm{Re}}{2})y) + 1\right]\} \tag{5.2.14}$$

Also the solution of Eq. (5.2.13) can be obtained using the transformation

$$u_t(y,t) = \bar{u}(y,t)Exp(\frac{\mathrm{Re}\,y}{2} - d^2 t) \tag{5.2.15}$$

Using Eq. (5.2.15) in Eq. (5.2.13) reduces it to

$$\frac{\partial \bar{u}}{\partial t} = \frac{\partial^2 \bar{u}}{\partial y^2} \tag{5.2.16}$$

With the BCs $\bar{u}(0,t) = 0$; $\bar{u}(1,t) = 0$ and $\bar{u}(y,0) = -u_s Exp(-\frac{\mathrm{Re}}{2}y)$ (5.2.17)

The solution of Eq. (5.2.16) can be derived using the Fourier series method as

$$u_t(y,t) = 2\pi\sum_{n=0}^{\infty}\left[\frac{((R+1)(-1)^n Exp(-(\frac{\mathrm{Re}}{2})) - R)}{d^2 + n^2\pi^2} + R\frac{(1-(-1)^n Exp(-(\frac{\mathrm{Re}}{2})))}{d^2 + n^2\pi^2 - M^2}\right] \tag{5.2.18}$$

$$n\,Exp(\frac{\mathrm{Re}\,y}{2} - (d^2 + n^2\pi^2)t)\sin n\pi\, y$$

Therefore the overall transient solution is

$$u(y,t) = (R+!)\frac{Exp(\mathrm{Re}/2(y-1))\sinh(dy)}{\sinh(d)} -$$

$$R\left[\frac{\sinh(dy)}{\sinh(d)}Exp(-(d-\frac{\mathrm{Re}}{2}y)) - Exp(-(d-\frac{\mathrm{Re}}{2})y)+1\right]$$

$$+2\pi\sum_{n=0}^{\infty}\left[\frac{\{(R+1)(-1)^n Exp(-(\frac{\mathrm{Re}}{2})) - R\}}{d^2+n^2\pi^2} + R\frac{\{1-(-1)^n Exp(-(\frac{\mathrm{Re}}{2}))\}}{d^2+n^2\pi^2-M^2}\right] \qquad (5.2.19)$$

$$n\,Exp(\frac{\mathrm{Re}\,y}{2} - (d^2+n^2\pi^2)t)\sin n\pi\, y$$

When $R \to 0$, the solution reduces to the solution in Jha and Jibril (2011)

5.2.3 Skin friction

The skin frictions at the lower and upper moving porous plates can be obtained from eq. (5.2.19) respectively as follows:

$$\tau_0 = \left.\frac{\partial u}{\partial y}\right|_{y=0}$$

$$= \frac{(R+1)dExp(-\frac{\mathrm{Re}}{2})}{\sinh(d)} - R\{\frac{dExp(-d)}{\sinh(d)} + d - \frac{\mathrm{Re}}{2}\}$$

$$+2\pi^2\sum_{n=0}^{\infty}\left[\frac{(\{R+1)(-1)^n Exp(-(\frac{\mathrm{Re}}{2})) - R\}}{d^2+n^2\pi^2} + \frac{\{1-(-1)^n Exp(-(\frac{\mathrm{Re}}{2}))\}}{d^2+n^2\pi^2-M^2}\right] n^2 Exp(-(d^2+n^2\pi^2 t)) \qquad (5.2.20)$$

$$\tau_1 = \left.\frac{\partial u}{\partial y}\right|_{y=1}$$

$$= (1+R)\{\frac{\mathrm{Re}}{2} + \frac{d\cosh(d)}{\sinh(d)}\} - R\{\frac{\cosh(d)}{\sinh(d)} + 1\}\, d(1 - Exp(-(d-\frac{\mathrm{Re}}{2})))$$

$$+2\pi^2\sum_{n=0}^{\infty}\left[\frac{\{(R+1)Exp(-(\frac{\mathrm{Re}}{2}))-(-1)^n R\}}{d^2+n^2\pi^2}+R\frac{\{(-1)^n-Exp(-(\frac{\mathrm{Re}}{2}))\}}{d^2+n^2\pi^2-M^2}\right]n^2Exp(\frac{\mathrm{Re}}{2}-(d^2+n^2\pi^2t)) \quad (5.2.21)$$

5.2.4 Steady state temperature

It is difficult to determine the exact solution of the transient energy equation in (5.2.7) with viscous and Joulean dissipation using the transient velocity profile in (5.2.19). Therefore, since the flow will develop after a certain period it will be practically expedient to solve the steady state temperature equation..

The dimensionless steady state energy equation becomes

$$\mathrm{Pr}\,\mathrm{Re}\frac{\partial\theta}{\partial y}=\frac{\partial^2\theta}{\partial y^2}+\mathrm{Pr}\,Ec\,M^2[u_s+R]^2+\mathrm{Pr}\,Ec(\frac{du_s}{dy})^2 \quad (5.2.22)$$

Substituting equation (5.2.11) into equation (5.2.22) after simplification yields

Simplifying (5.2.23) yields

$$\frac{d^2\theta}{dy^2}-\mathrm{Pr}\,\mathrm{Re}\frac{d\theta}{dy}=$$
$$-Ec\,\mathrm{Pr}[a_1{}^2d(2d+\mathrm{Re})\exp(2d+\mathrm{Re})y+\ a^2{}_2d(2d-\mathrm{Re})\exp(-(2d-\mathrm{Re})y) \quad (5.2.23)$$

With BCs as

$$\theta(0)=0\ \ and\ \ \theta(1)=1 \quad (5.2.24)$$

The general solution of Equation (5.2.23) can be obtained as

$$\theta(y)=B_1+B_2Exp(\mathrm{Pr}\,\mathrm{Re}\,y)-Ec\,\mathrm{Pr}\,d\{k_1Exp(2d+\mathrm{Re})y+k_2Exp(-(2d-\mathrm{Re})y)\} \quad (5.2.25)$$

Therefore combining equation (5.2.25) with BCs (5.2.24), gives

$$B_1 = \frac{k_1\{Exp(\Pr \mathrm{Re}) - Exp(2d + \mathrm{Re})\} + k_2\{Exp(\Pr \mathrm{Re}) - Exp - (2d - \mathrm{Re})\} - 1}{Exp(\Pr \mathrm{Re}) - 1}$$

$$B_2 = \frac{k_1\{Exp(2d + \mathrm{Re}) - 1\} + k_2\{Exp(-(2d - \mathrm{Re})) - 1\} - 1}{Exp(\Pr \mathrm{Re}) - 1}$$

Substituting the above values into equation (5.2.25), the steady state temperature can be expressed as

$$\theta(y) = \frac{Exp(\Pr \mathrm{Re}\, y) - 1}{Exp(\Pr \mathrm{Re}) - 1} + \frac{Ec \Pr}{(Exp(\Pr \mathrm{Re}) - 1)}[k_1\{Exp(\Pr \mathrm{Re}) - Exp(2d + \mathrm{Re})$$

$$+ Exp(2d + \mathrm{Re}(1 + \Pr y)) - Exp((2d + \mathrm{Re})y + \Pr \mathrm{Re}) + Exp(2d + \mathrm{Re})y - Exp(\Pr \mathrm{Re})y)\}$$

$$+ k_2\{Exp(\Pr \mathrm{Re}) - Exp(-(2d - \mathrm{Re})) + Exp(-(2d - \mathrm{Re}) + \Pr \mathrm{Re}\, y)$$

$$- Exp(-(2d - \mathrm{Re})y + \Pr \mathrm{Re}) + Exp(-(2d - \mathrm{Re})y) - Exp(\Pr \mathrm{Re}\, y)\}] \quad (5.2.26)$$

Where all the constants occurring in equation (5.2.26) are defined as

$$d = \sqrt{M^2 + \frac{\mathrm{Re}^2}{4}} \; ; d_1 = \frac{1}{2Exp(\mathrm{Re}/2)\sinh(d)} \, , \, d_2 = \frac{1}{2Exp(d)\sinh(d)} \; ;$$

$$a_1 = (R+1)d_1 - Rd_2 \; ; \; a_2 = (R+1)d_1 - Rd_2 - R$$

$$k_1 = \frac{{a_1}^2 d}{2d + \mathrm{Re}(1 - \Pr)} \; ; \; k_2 = \frac{{a_2}^2 d}{2d - \mathrm{Re}(1 - \Pr)} \qquad (4.2.27)$$

It can be seen from (5.2.26) that the steady state temperature profile is linearly dependent on the Eckert number.

5.2.5 Nusselt Number

The Nusselt number, which is the rate of heat transfer on the lower and moving upper porous plates, can be obtained from equation (5.2.26) as

$$Nu = \left| \frac{d\theta}{dy} \right|_{y=\alpha}$$

Therefore the Nusselt number for the lower plate is

$$Nu_0 = \left| \frac{d\theta}{dy} \right|_{y=0}$$

$$\frac{\Pr \mathrm{Re}}{Exp(\Pr \mathrm{Re}) - 1}\left[1 + Ec \Pr\{k_1 (Exp(2d + \mathrm{Re}) - 1) + k_2 \{Exp(-(2d - \mathrm{Re})) - 1)\}\right]$$

$$- Ec \Pr[k_1 (2d + \mathrm{Re}) - k_2 (2d - \mathrm{Re})] \tag{5.2.27}$$

And for the upper moving porous plate is

$$Nu_1 = \left| \frac{d\theta}{dy} \right|_{y=1}$$

$$= \frac{\Pr \mathrm{Re} \exp(\Pr \mathrm{Re})}{\exp(\Pr \mathrm{Re}) - 1}\left[1 + Ec \Pr\{k_1 (\exp(2d + \mathrm{Re}) - 1) + k_2 \{\exp(-(2d - \mathrm{Re})) - 1)\right]$$

$$- Ec \Pr\left[k_1 (2d + \mathrm{Re}) \exp(2d + \mathrm{Re}) - k_2 (2d - \mathrm{Re}) \exp(-(2d - \mathrm{Re}))\right] \tag{5.2.28}$$

5.2.6 Discussion of Results

In this section the results of the MHD Couette flow of an incompressible, viscous fluid in a porous channel formed by two infinite horizontal porous plates under the influence of transverse magnetic field and applied electric field with viscous and ohmic dissipation is considered. The dimensionless velocity (5.2.19) and temperature distributions (5.2.26) are obtained by solving equations (5.2.6) and (5.2.7) analytically using the separation of parameters method and Fourier series technique. The effects of the various flow parameters such as the Hartmann number M, Eckert number Ec, Reynolds number Re, Prandtl number Pr, and electric load parameter R, on the

dimensionless velocity and temperature distributions, the skin friction coefficient on the lower and upper porous plates and Nusselt number has been presented and discussed with the aid of graphs for different values of Pr =0.7 and Pr=7.0 corresponding to water and air respectively. Three cases for the dimensionless Electric load parameter R, are considered. In the first case R is set equal to zero(R=0) representing the short circuit case in which there is no electric field. The second case R= -1, is typical of situations, of which the MHD generator channel is an example, in which the electric field E_Z is opposed to $\vec{q} \times \vec{B}$ where $\underline{q}$ and $\underline{B}$ represents the velocity and magnetic field vectors respectively, Lohrasbi and Sahai (1989). When R=1, the channel operates in the open circuit condition. In this case the current flow is quite large and the flow in addition to viscous and Joule dissipation is also affected by Ohmic heating.

Fig.5.2.2a and 5.2.2b respectively, show the variation of transient velocity versus distance for different values of time for fixed values of M=2.0 and Re=-5.0, when R=1.0 and R=-1.0. These figures show that transient velocity increases with increase in time for all values of R. Also a comparative study of fig.5.2.2a & fig.5.2.2b reveal that velocity is higher in case of R=-1.0 in comparison to R=1.0.

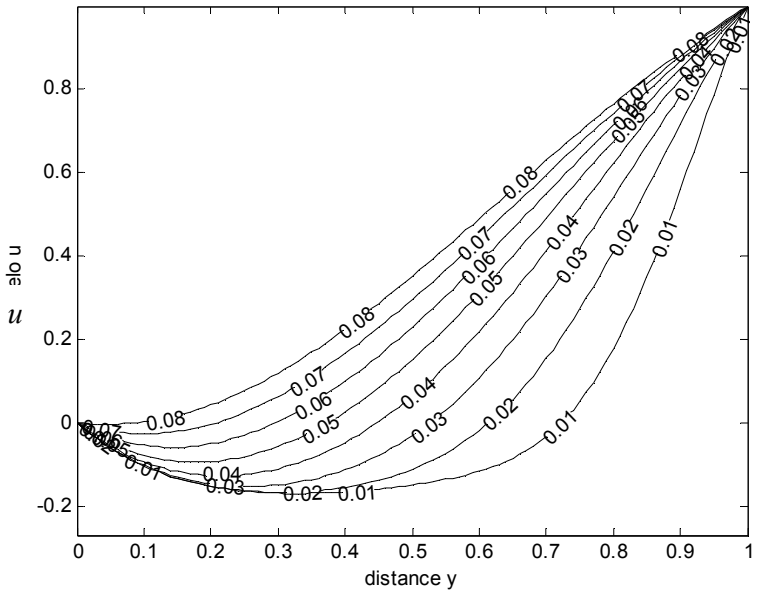


Fig.5.2.2a Velocity (u) versus distance (y) for different t, (Re=-5.0 M =2.0 and R=1.0)

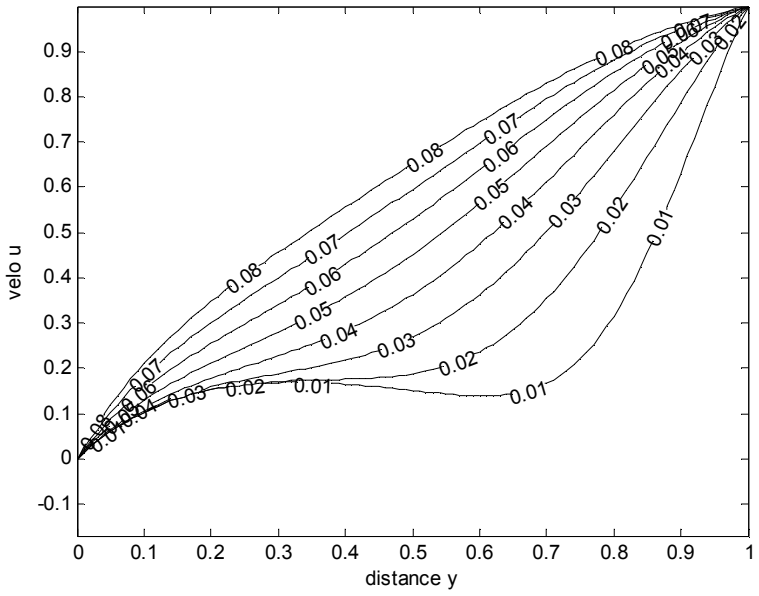


Fig.5.2.2b Velocity (u) versus distance (y) for different t (Re= -5.0 M =2.0 and R=-1.0)

Fig.5.2.3a and 5.2.3b show the variation of transient velocity (u_t) versus distance for different values of Hartmann number M, for fixed values of t=0.05, Re=-5.0, when the electric load parameter R= 1.0 and -1.0 respectively. These figures show that the magnitude of velocity decreases with increase in M, when R= 1.0 or R=-1.0.

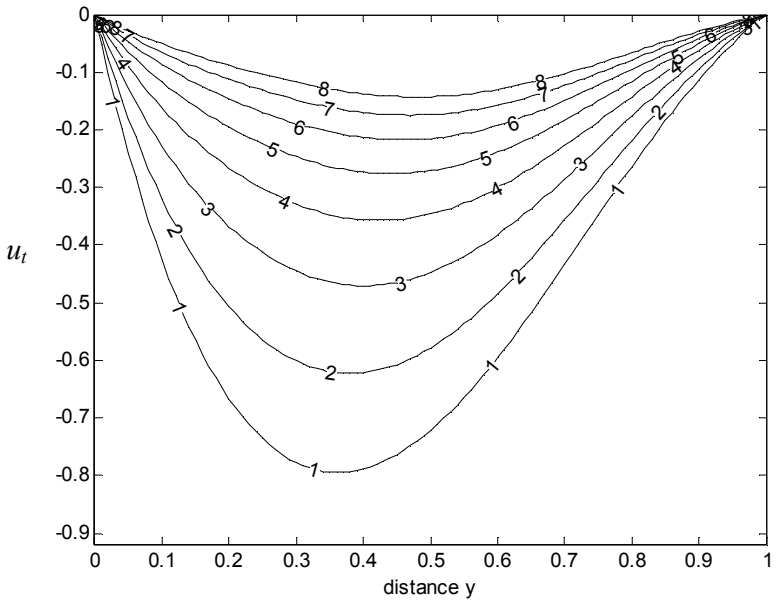


Fig.5.2.3a Transient velocity u_t versus distance y for different M (t=0.05, Re=-5.0, R=1.

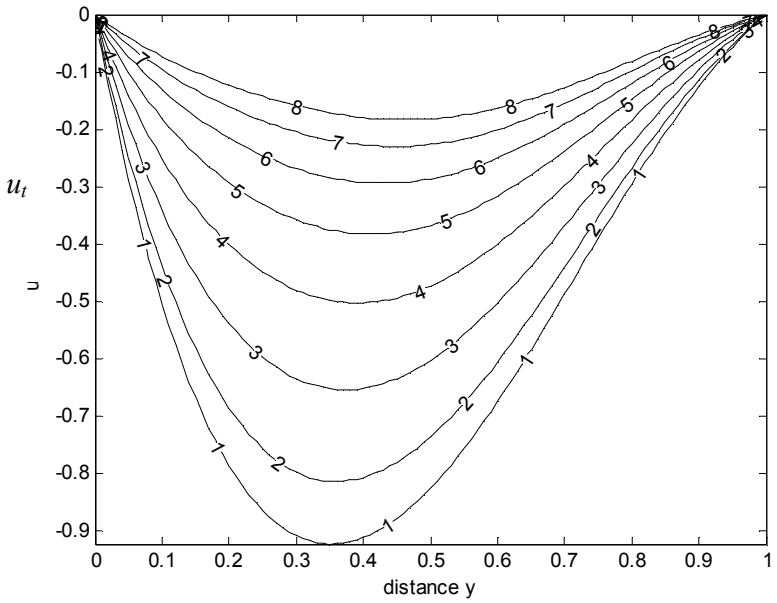


Fig.5.2.3b velocity (u_t) versus distance (y) for different M (t=0.05 and Re=-5.0, R=-1.0)

Fig.5.2.4a and 5.2.4b show the variation of transient velocity (u_t) versus distance for different values of Hartmann number M, for fixed values of t=0.05 and Re= 5.0, for electric load parameter R= 1.0 and -1.0 respectively. These figures shows that under mass injection, the magnitude of velocity increases with increase in M for both cases of R=1.0 or -1.0. However there is a "backflow" when R=-1.0 as shown in fig.5.2.4a.

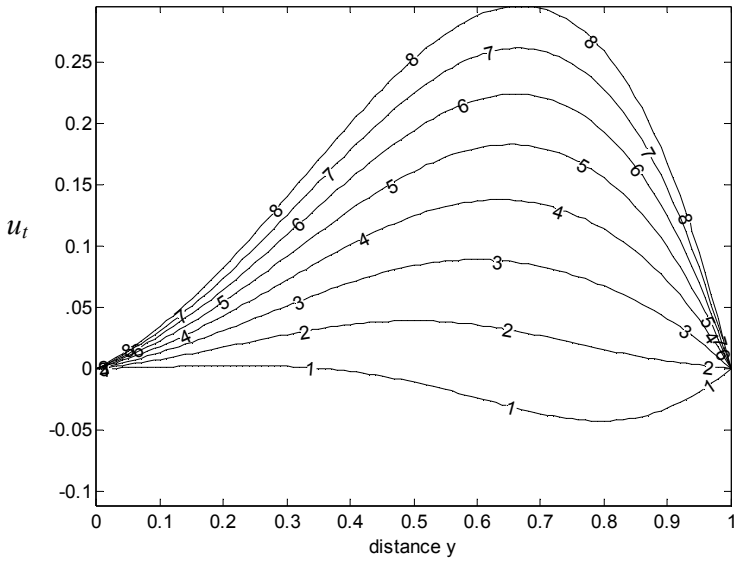


Fig.5.2.4a Transient velocity (u_t) versus distance (y) for different M (t=0.05 Re=5.0, R=1.0)

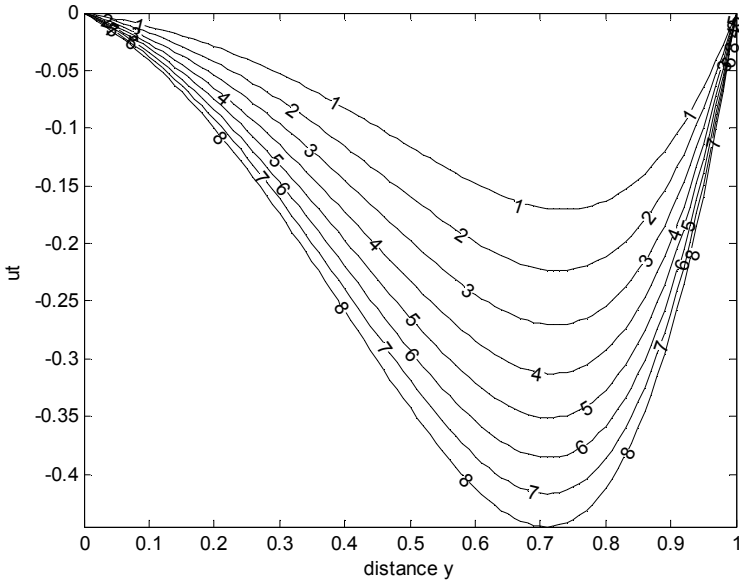


Fig.5.2.4b Transient velocity (u_t) versus distance (y) for different M (t=0.05 and Re =5.0, R=-1.0)

Fig.5.2.5-5.2.6 shows the effects of Hartmann number on the overall velocity (u).Fig.5.2.5a and 5.2.5b show the variation of velocity (u) versus distance y for different values of Hartmann number M, for fixed values of t=0.05, Re=-5.0, when the electric load parameter R= 1.0 and -1.0 respectively. Fig.5.2.5a shows that the velocity decreases with increase in M, when R= 1.0, while fig.5.2.5b shows that the reverse is the case when R=-1.0, that is the velocity increases with increase in M. This is due to the fact that the effect of the electromagnetic force when R=-1.0 accelerate the flow, while the flow is decelerated when R=1.

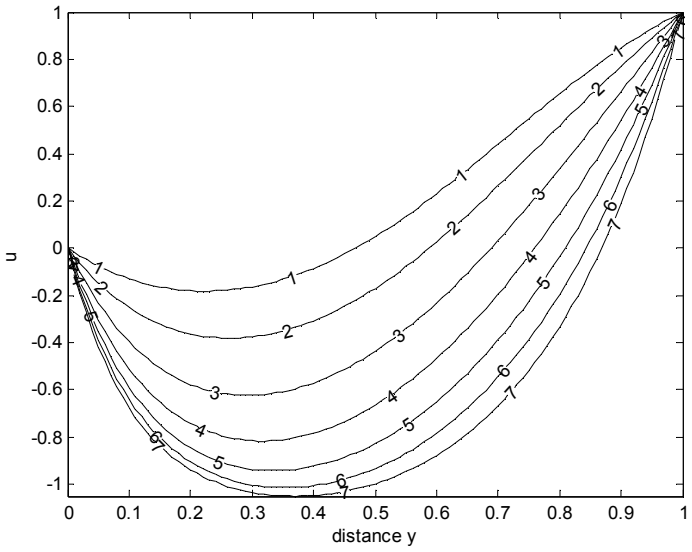


Fig.5.2.5a Velocity (u) versus distance (y) for different M (Re= -5.0, t=0.05,R=1.0)

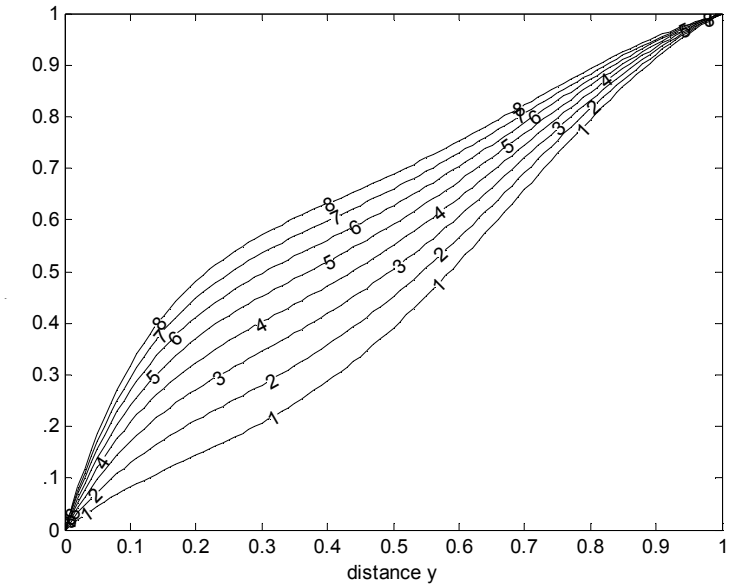


Fig.5.2.5b Velocity (u) versus distance (y) for different M (Re= -5.0, t=0.05 and R=-1.0)

Fig.5.2.6a and 5.2.6b show the variation of velocity versus distance for different values of Hartmann number M, for fixed values of t=0.05 and Re= 5.0, for electric load parameter R= 1.0 and -1.0 respectively. Fig.5.2.6a shows that under mass injection, for the open circuit case (R=1.0), the velocity decreases with increase in M. The reverse is however the case when R=-1.0 where the velocity increase with increase in Hartmann number M as depicted in fig. 5.2.6b. This is due to the fact that when R=-1.0 the resulting electromagnetic force accelerates the flow.

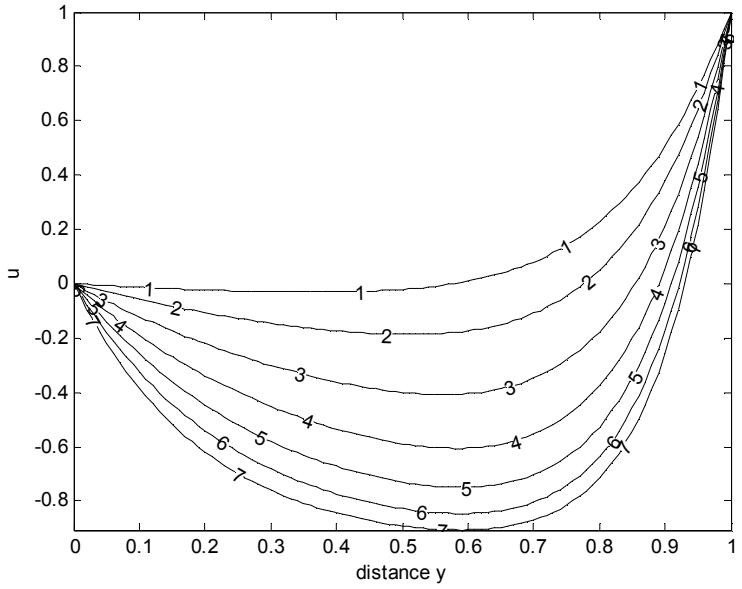


Fig.5.2.6a Velocity (u) versus distance (y) for different M (Re= 5.0, t=0.05, R=1.0)

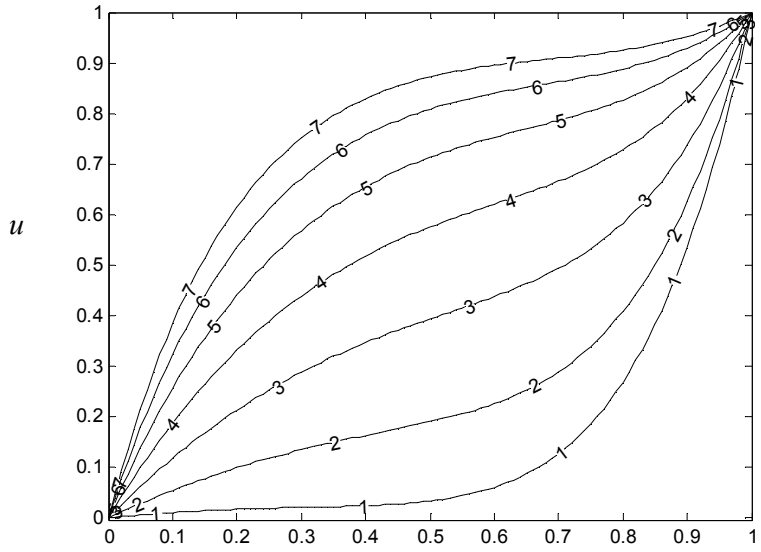


Fig.5.2.6b Velocity (u) versus distance (y) for different M Re= 5.0, t=0.05, R=-1.0)

Fig.5.2.7a and 5.2.7b shows the variation of the transient velocity (u_t) versus distance for different values of the suction/injection parameter Re, for fixed values of M=2.0 and t = 0.05 when R=1 and R= -1 respectively. These figures reveal that the velocity increases with increase in the suction/injection parameter whether R=1.0. or R=-1.0.

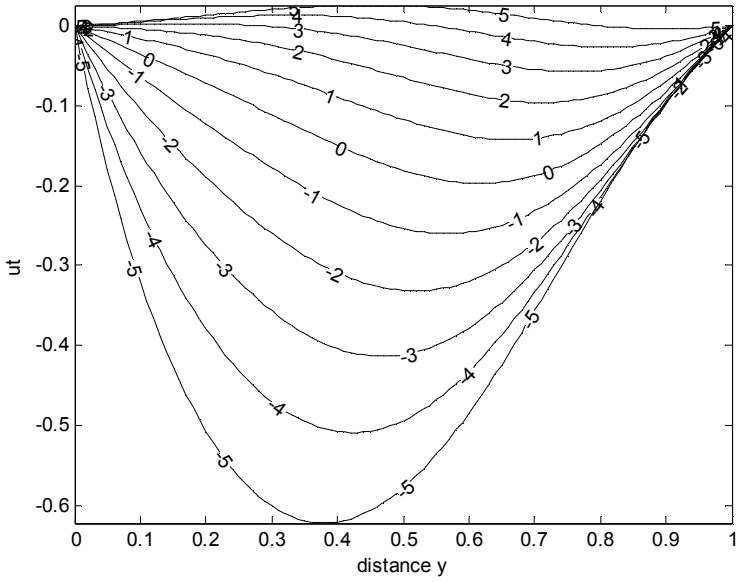


Fig.5.2.7a Transient velocity (u_t) versus distance (y) for different Re (t =0.05 M =2.0, R=1.0)

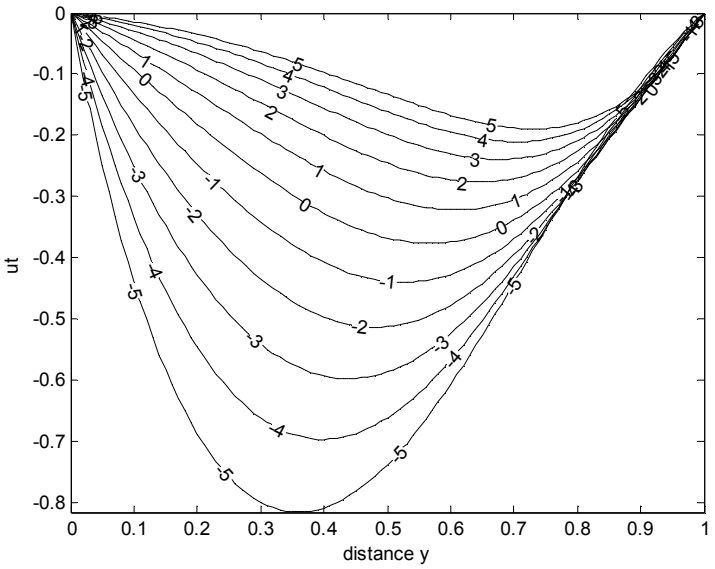


Fig.5.2.7b Transient velocity (u_t) versus distance(y) for different Re (t =0.05, M =2.0, R=-1.0)

Fig.5.2.8a and 5.2.8b shows the variation of velocity versus distance for different values of the suction/injection parameter Re, for fixed values of M=2.0 and t = 0.05

when R=1.0 and R= -1.0 respectively. These figures reveal that the velocity decreases with increase in the suction/injection parameter whether R=1.0. or R=-1.0.

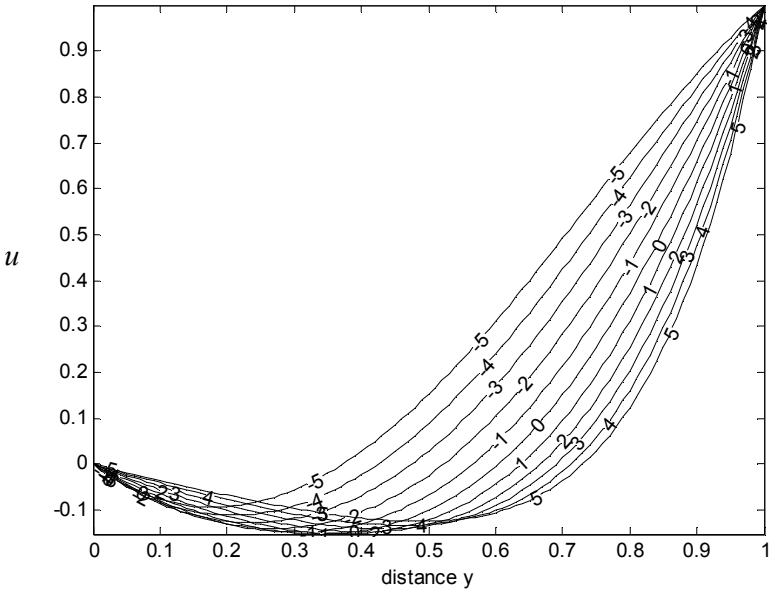


Fig.5.2.8a Velocity (u) versus distance (y) for different Re (t =0.05 and M =2.0 R=1.0)

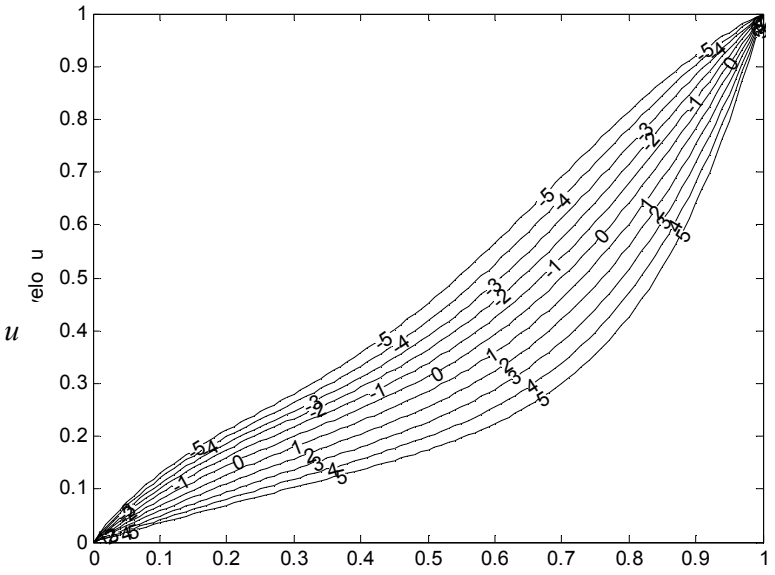


Fig.5.2.8b Velocity (u) versus distance (y) for different Re (t =0.05, M =2.0 R=-1.0)

Fig.5.2.9-5.2.10 illustrate the effect of M and Re on the skin friction coefficient at the lower porous wall(y=0) and upper moving porous wall(y=1) for fixed values of t=0.05 and R=1.0&R=-1.Fig.5.2.9a shows that when R=1.0 the skin friction coefficient on the lower porous wall decreases with increase in M and increases with increase in Re, while in case of R=-1.0, the skin friction coefficient increases with increase in M and decreases with increase in Re, as shown in fig. 5.2.9a and 5.2.9b.

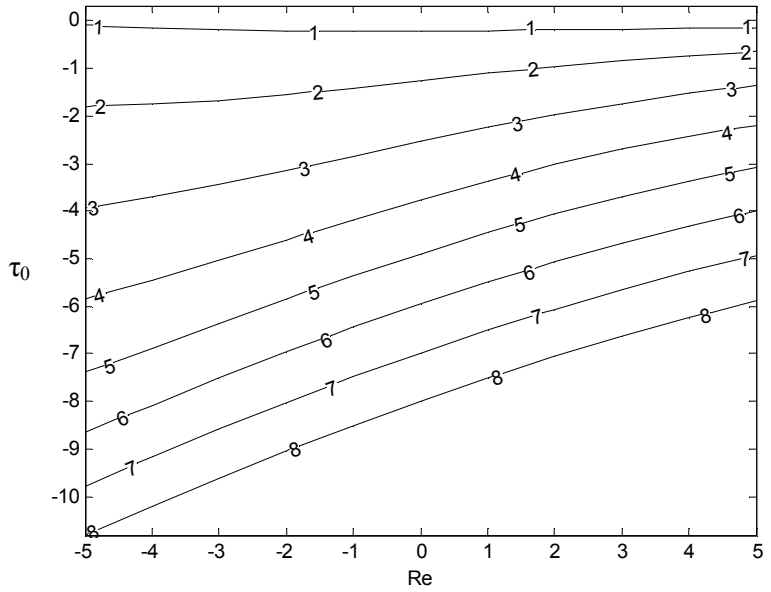


Fig.5.2.9a Skin friction (τ_0) versus (Re) for different M at y =0(t =0.05, R=1.0)

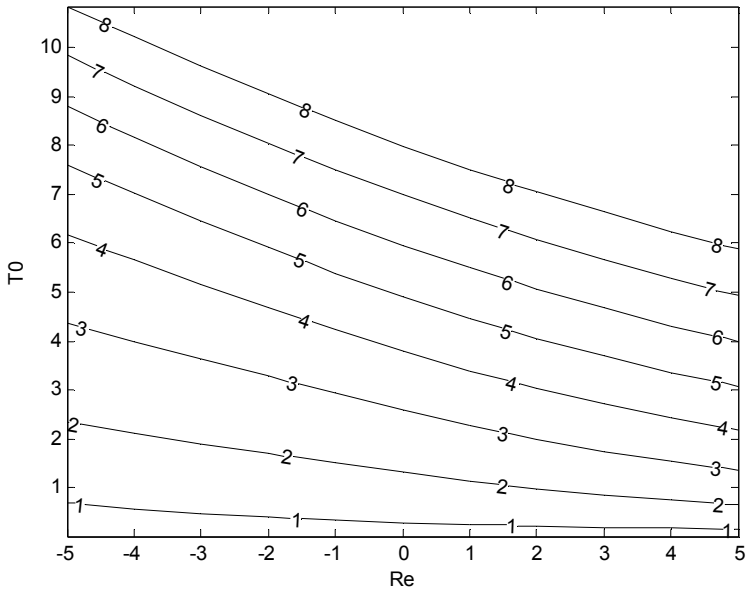


Fig.5.2.9b Skin friction (τ_0) versus (Re) for different M at y =0(t =0.05, R=-1.0)

Fig.5.2.10a shows the variation of skin friction at the upper moving porous plate versus Re for different values of M, for fixed values of t=0.05, R=1.0. This fig. indicates that the skin friction increases with increase in M and increase in Re. while for R=-1.0 the effect of M is dependent on Re. When Re>0(under mass injection) the skin friction decreases with increase in M and increases with increase in Re and for Re<0 (under mass suction) the reverse is the case where the skin friction increases with increase in M and decreases with increase in Re (as depicted in figure 5.2.10b.

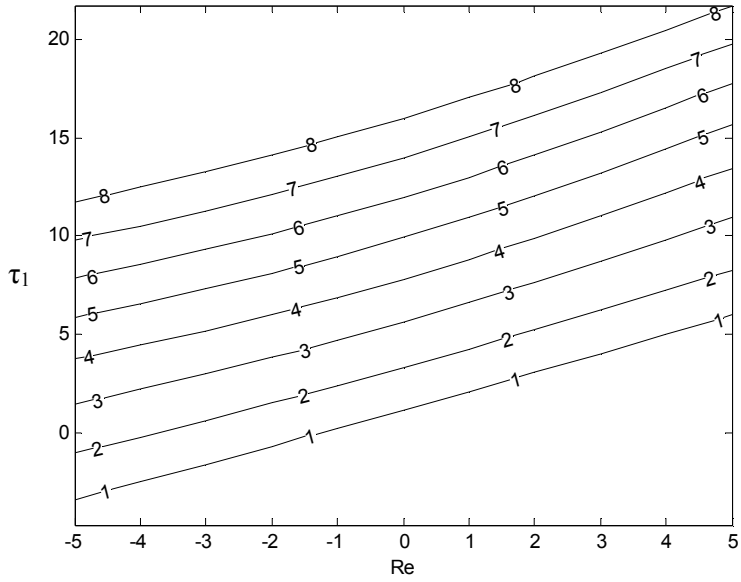


Fig.5.2.10a Skin friction (τ_1) versus (Re) for different M at y =1(t =0.05, R=1.0)

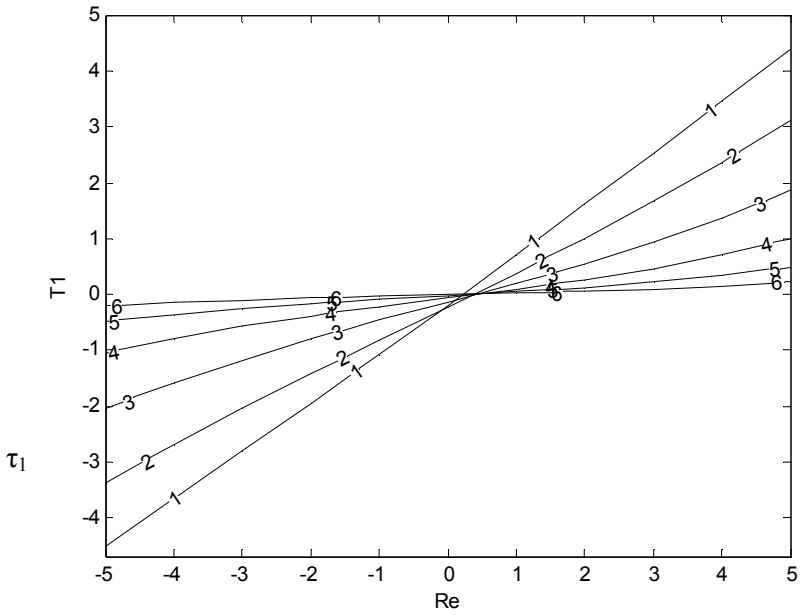


Fig.5.2.10b Skin friction ($\tau_{1)}$ versus (Re) for different M at y =1 (t =0.05, R =-1.0)

The effects of the electric load parameter R, suction/injection parameter Re and Hartmann number on the steady state temperature for fixed values of Ec=2.0 and Pr=0.7 and Pr=7.0 (corresponding to air and water) is displayed in fig..5.2.11- 5.2.12. Fig.5.2.11a and 5.2.11b respectively, show that when Pr=0.7, under mass suction at the lower porous wall, the steady state temperature increases with increase in M whether R=1.0 or -1.0 .This is because when R=1.0, the Lorentz force arising due to electric field reduces frictional resistance which helps in reducing temperature distribution in the boundary layer. On the other hand when R=-1.0 the current flow is so small that viscous dissipation dominates the temperature. Furthermore the temperature is higher when R=1.0 than when R=-1.0.

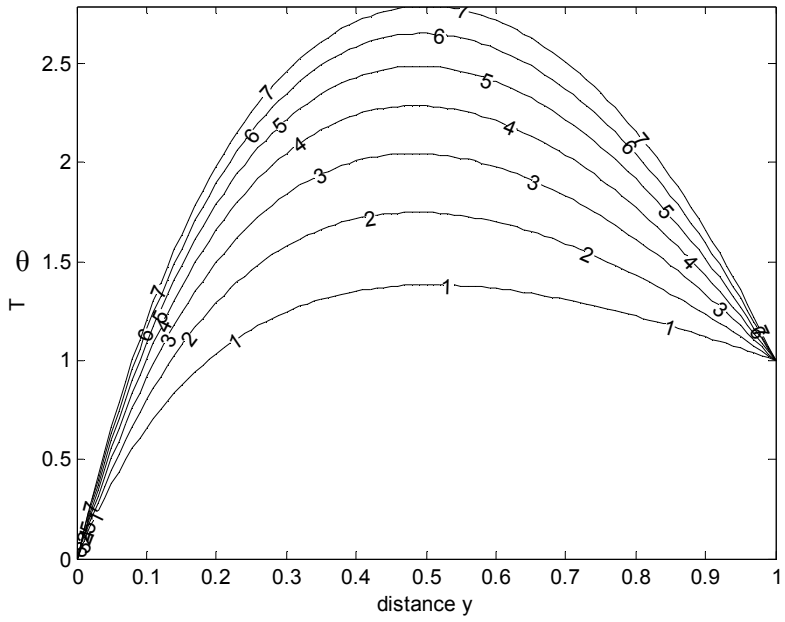


Fig.5.2.11a Steady state Temperature (θ) versus distance (y) for differentM (*Ec*=2.0, Pr =0.7, Re=-5.0, R=1.0)

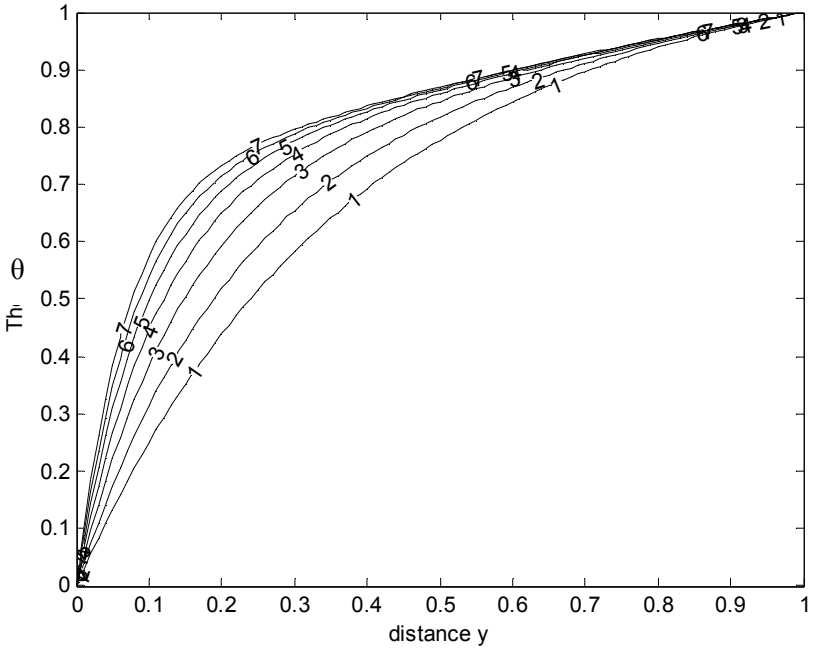


Fig.5.2.11b Steady state Temperature (θ) versus distance (y) for diffenet M (Ec=2.0, $\Pr$=0.7 and Re=-5.0, R=-1.0)

Fig.5.2.12a and 5.2.12b respectively, show the influence of Hartmann number M and Electric load parameter R, on the steady state temperature for fixed values of Ec=2.0, Re=-5.0, when Pr =7.0. Figure 5.2.12a indicates that when R=1.0, the temperature increases with increase in M. Fig.5.2.12b shows that when R=-1.0, the temperature increases with increase of M near the lower porous plate when $0 \le y \le 3$, after which it decreases with increase of M.

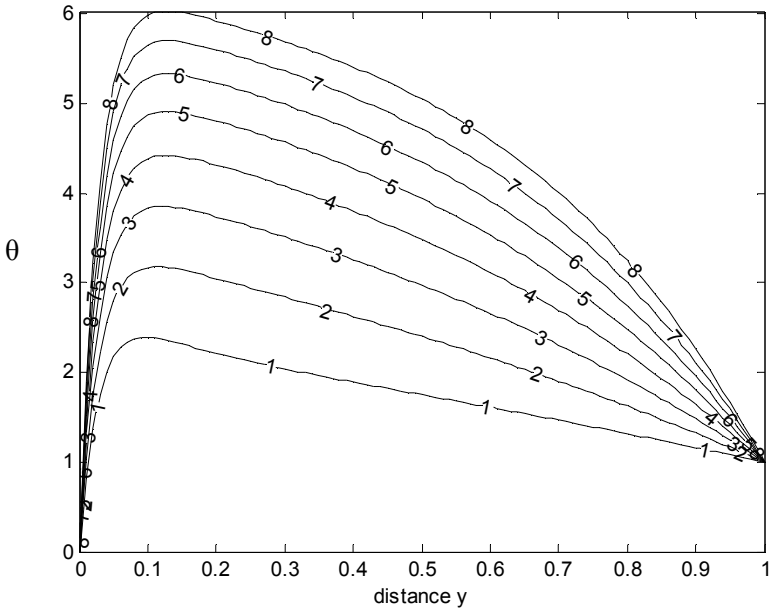


Fig.5.2.12a Steady state Temperature (θ) versus distance (y) for different M (Ec=2.0, $\Pr$=7.0, Re=-5.0 , R=1.0)

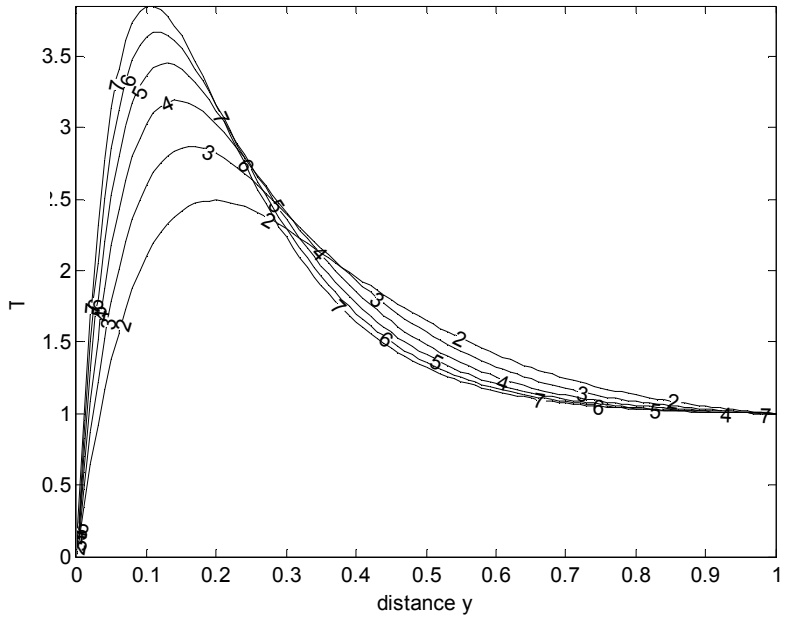


Fig.5.2.12b Steady state temperature (θ) versus distance (y) for differentM (Ec=2.0, $\Pr$=7.0 Re=-5.0, R=-1.0)

Fig.5.2.13-5.2.14 plot the temperature versus distance for different values of M, when R=1.0 and R=-1.0 for fixed values of Re=1.0 , Ec=2.0 and Pr=0.7 and Pr=7.0.

Fig.5.2.13a and 5.2.13b show that in the presence of mass injection the steady state temperature increases with increase in M when R=1.0 and R=-1.0 but the temperature is h ;her when R=-1.0. For fixed values of Pr=0.7.This is because when R= -1.0 the electromagnetic force tends to accelerate the flow leading to higher dissipation.

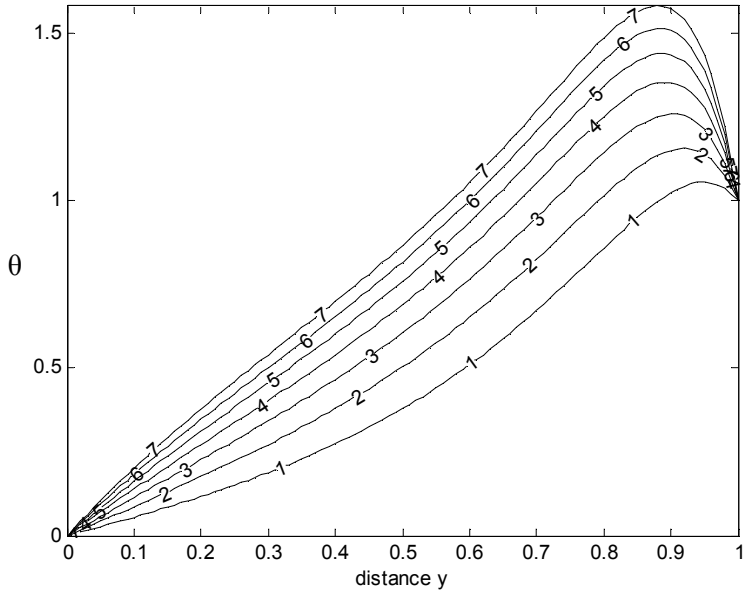


Fig.5.2.13a Steady state Temperature(θ) versus distance (y) for different M (Ec=2.0, $\Pr$=0.7, Re=5.0, R=1.0)

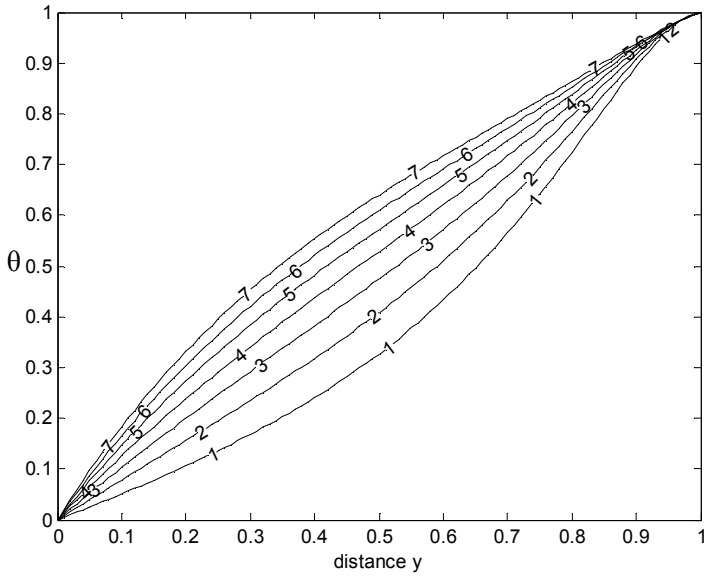


Fig.5.2.13b Steady state Temperature (θ) versus distance (y) for different M (Ec=2.0, $\Pr$=0.7, Re=5.0, R=-1.0)

Fig.5.2.14a and 5.2.14b show that under mass injection the temperature of the fluid increases with increase in M when R=1.0 and R=-1.0 for fixed values of Pr=7.0

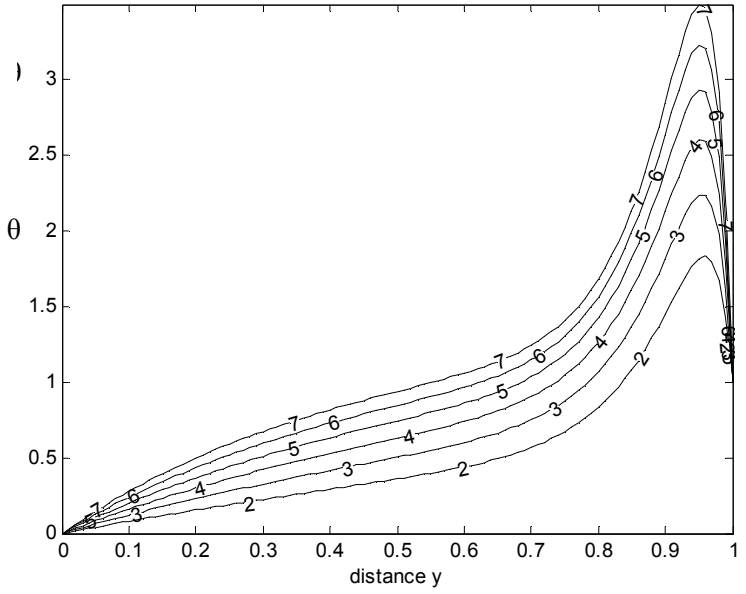


Fig.5.2.14a Steady state temperature (θ) versus distance (y) for different M (Ec=2.0, $\Pr$=7.0 , Re=5.0, R=1.0)

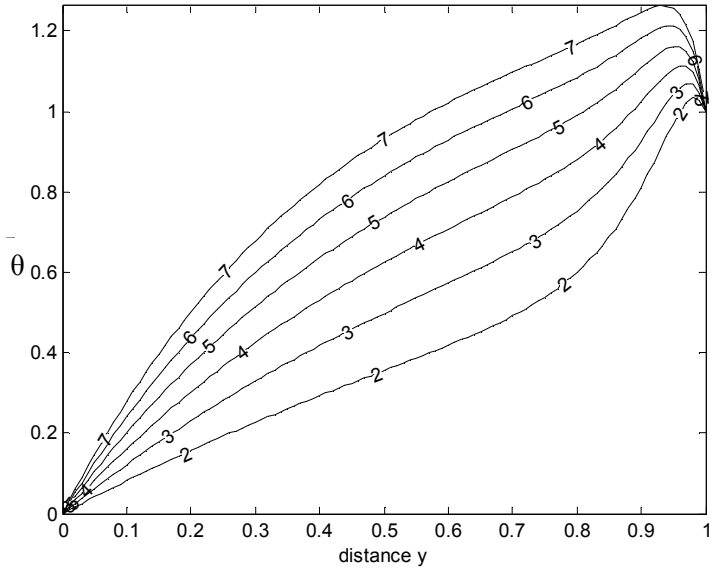


Fig.5.2.14b Steady state temperature(θ) versus distance (y) for different M (Ec=2.0, $\Pr$=7.0 , Re=5.0 , R=-1.0)

Fig.5.2.15a and 5.2.15b respectively, illustrates the effects of the suction/injection parameter on the temperature distribution for fixed values of Pr = 0.7, Ec=2.0 and M=2.0. Both figures show that when Pr=0.7, the temperature decreases with increase in the suction/injection parameter, Re, when R=1.0 and R=-1.0. The temperature is however higher when R=1.0 than when R=-1.0. These figures show that suction enhances the temperature while injection reduces it.

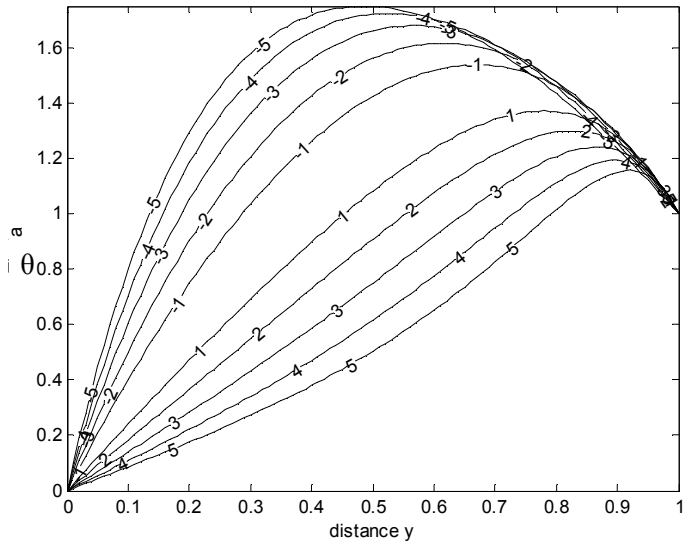


Fig.5.2.15a Steady state temperature (θ) versus distance (y) for different Re (Ec=2.0, Pr =0.7, M=2.0, R=1.0)

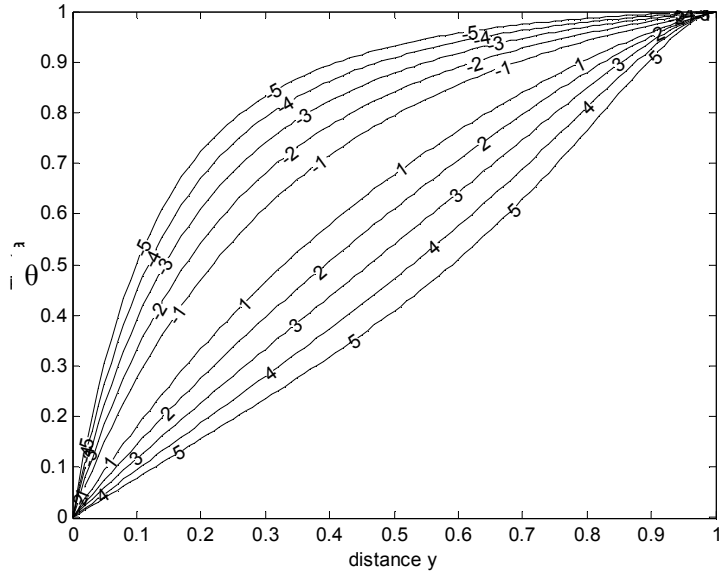


Fig.5.2.15b Steady state temperature (θ) versus distance (y) for different Re (Ec=2.0, Pr =0.7, M=2.0, R=-1.0)

Fig.5.2.16a and 5.2.16b illustrates the effects of the suction/injection parameter on the temperature distribution for higher value of Pr (Pr=7.0), Ec=2.0 and M=2.0. These figures show that the temperature decreases with increase in suction/injection for both cases of R=1.0 and R=-1.0.

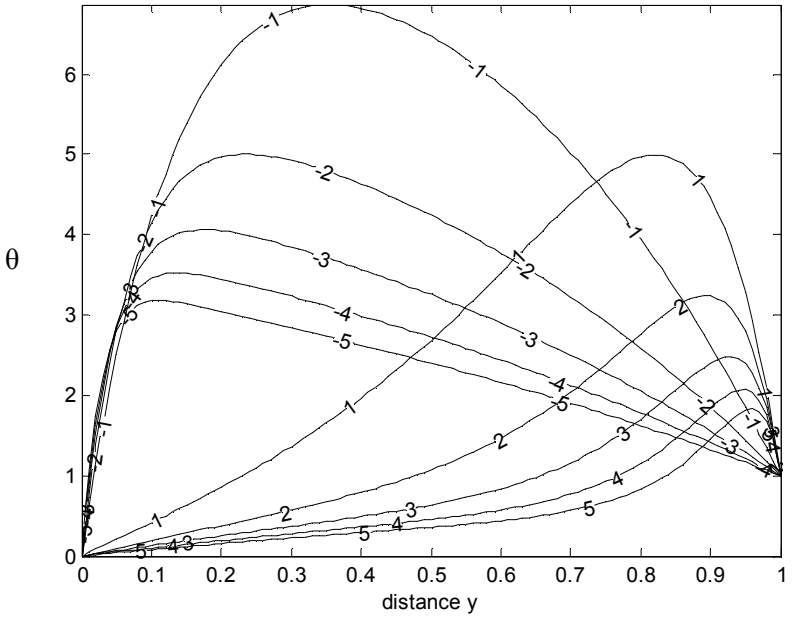


Fig.5.2.16a Steady state temperature (θ) versus distance (y) for different Re (*Ec* =2.0, Pr =7.0, M=2.0, R=1.0)

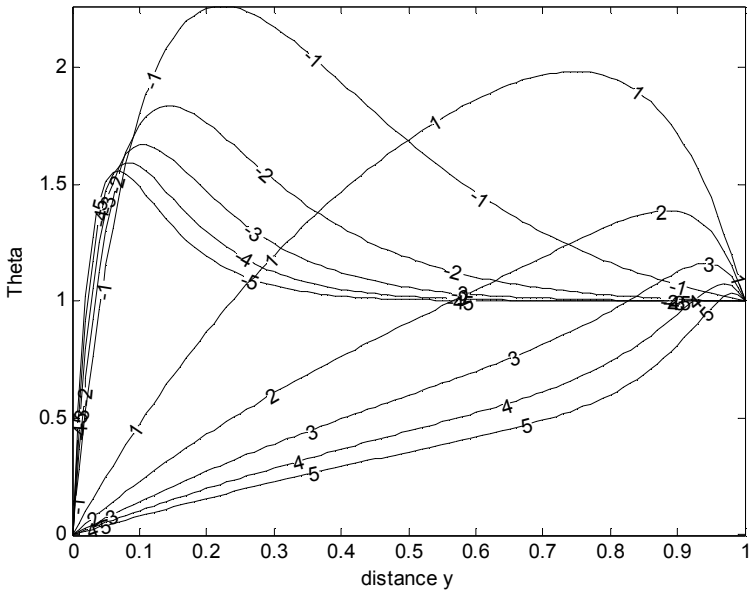


Fig.5.2.16b Steady state temperature (θ) versus distance (y) for different Re (Ec=2.0, Pr =7.0, M=2.0, R=-1.0)

Fig.5.2.17a and 5.2.17b respectively, represent the graph of temperature profile for different values of Prandtl number Pr, for fixed values of Ec and M, under mass suction. These figures reveal that the temperature for both cases whether R=1.0 and R= -1.0

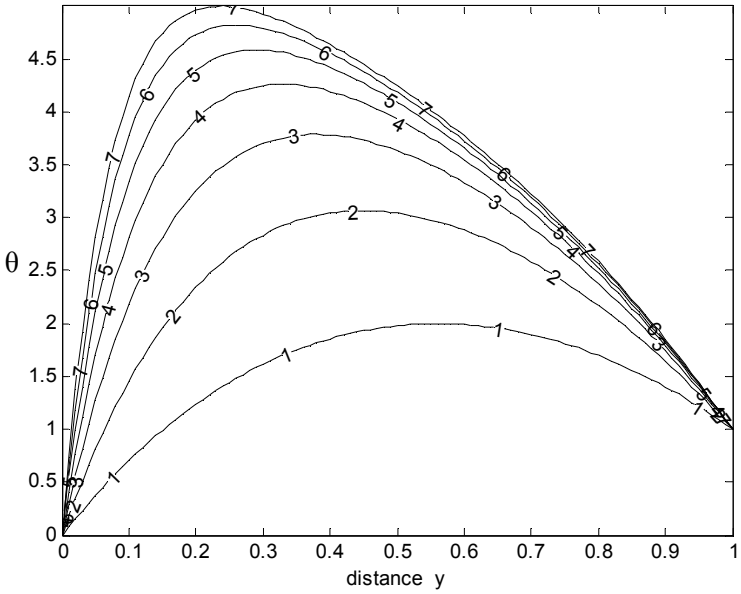


Fig.5.2.17a Steady state temperature (θ) versus distance (y) for different Pr (Re=-5.0, Ec=2.0, M=2.0, R=1.0)

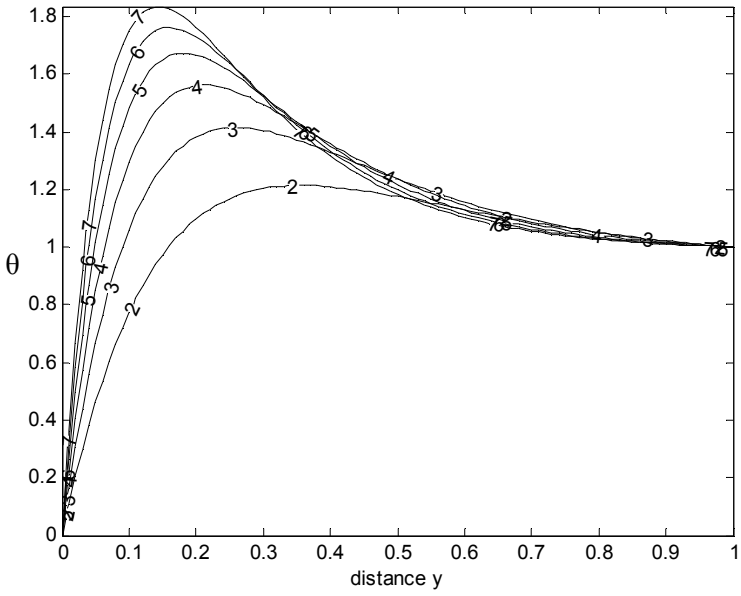


Fig.5.2.17b Steay state temperature (θ) versus distance (y) for different Pr (Re=-5.0, Ec=2.0, M=2. R=-1.0)

The effect of Prandtl number on the steady state temperature under mass injection for different values of the Electric load parameter is illustrated in ig.5.2.18a and 5.2.18b. Both figures indicate that the temperature increases with increase in Pr. The temperature is however higher when R=1.0 than R=-1.0.

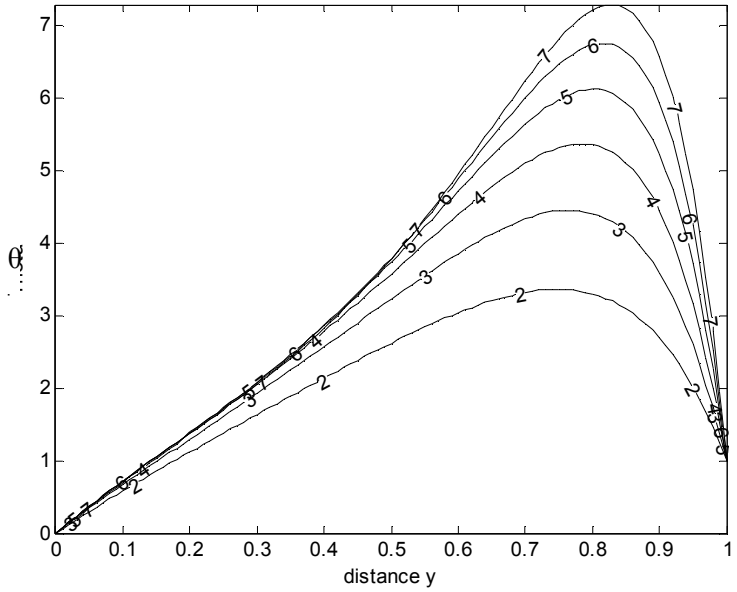


Fig.5.2.18a Steady state temperature (θ) versus distance (y) (for different Pr (Re=1.0, Ec=2.0, M=2.0, R=1.0)

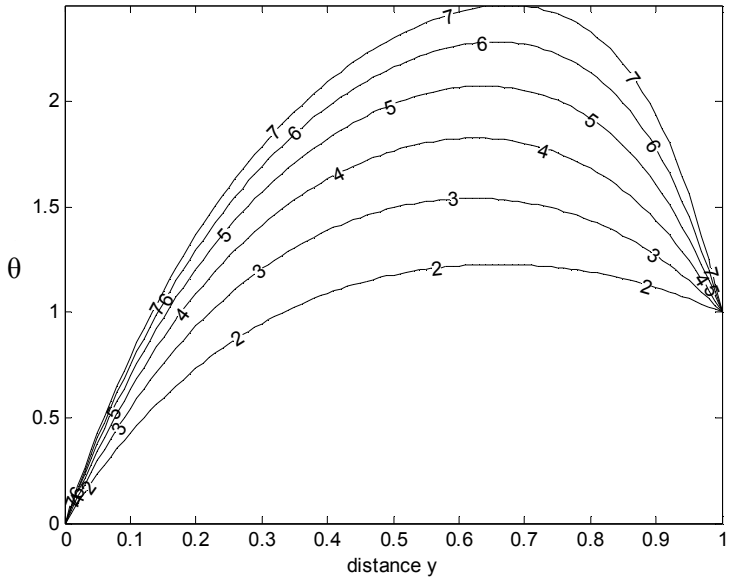


Fig.5.2.18b Steady state temperature (θ) versus distance (y) for different Pr (Re=1.0 , Ec=2.0, M=2.0 , R=-1.0)

The effect of Eckert number on the temperature variation, in the presence of mass suction, for fixed values of Pr=0.7, Re= -5.0, M=2.0 is depicted in fig.5.2.19a and 5.2.19b for R=1.0 and R= -1.0 respectively. These figures show that the temperature increases with increase in the Ec, when R=1.0 and R=-1.0 . The temperature is however higher when R=1.0 than R=-1.0.

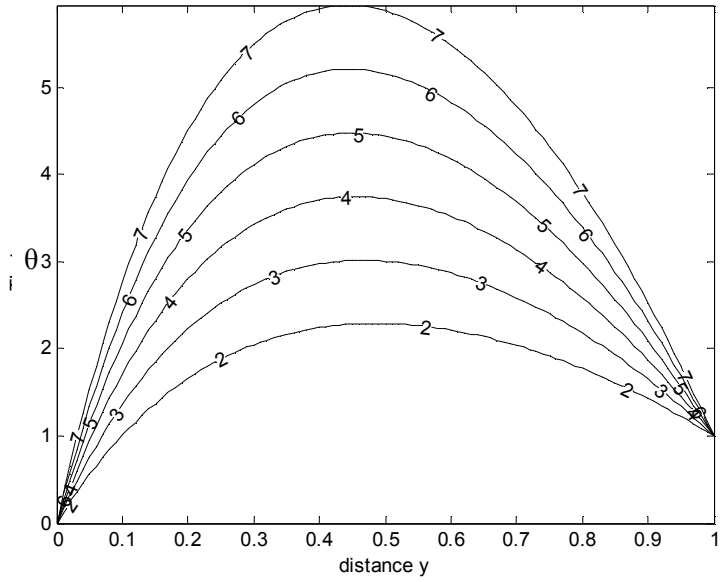


Fig.5.2.19a Steady state temperature (θ) versus distance (y) for different Ec (Re=-5.0, Pr=0.7, M=2.0 , R=1.0)

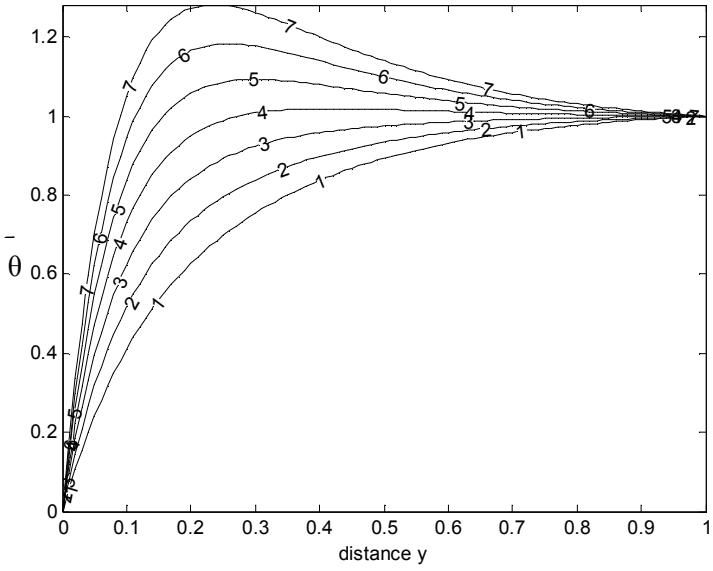


Fig.5.2.19b Steady state temperature (θ) versus distance (y) for different Ec (Re=-5.0, Pr=0.7, M=2.0 , R=-1.0)

The combined effects of injection and Eckert number on the temperature distribution for fixed values of Pr=0.7, M=2.0 is displayed in Fig.5.2.20a and 5.2.20b for R=1.0 and R= -1.0 respectively. These figures revealed that under mass injection at the lower moving porous wall, the temperature increases with increase in the Eckert number Ec when R=1.0 or R=-1.0, the temperature is however higher when R=1.0, than R=-1.0. This is due to the fact the flow is dominated by dissipation.

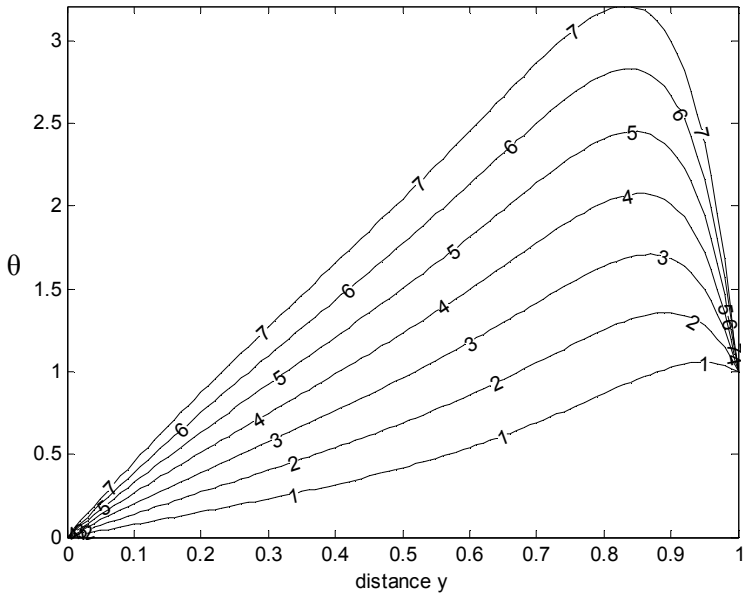


Fig.5.2.20a Steady state temperature (θ) versus distance (y) for different *Ec* (Re=5.0, Pr=0.7, M=2.0 , R=1.0)

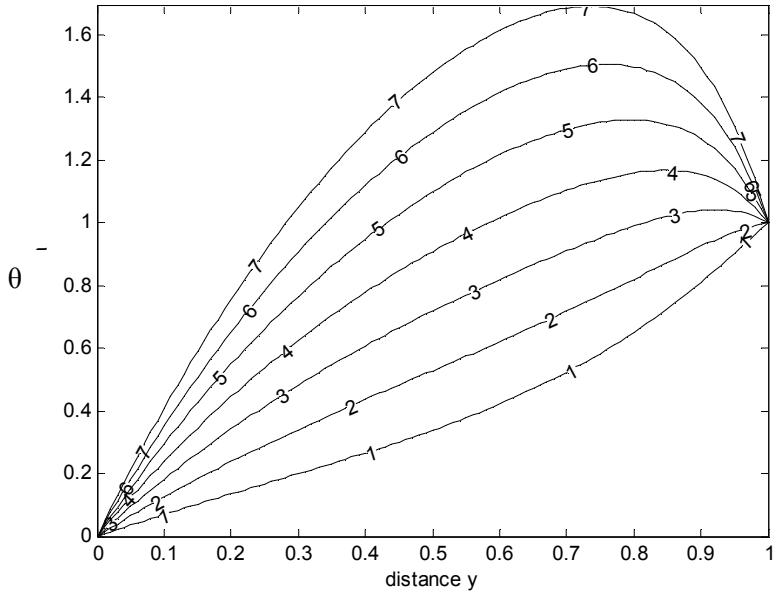


Fig.5.2.20b Steady state temperatureversus distance y for different Ec(Re=5.0, Pr=0.7, M=2.0 , R=-1.0)

Fig.5.2.21-5.2.22 show that in the presence of mass suction or injection, for fixed Pr, the temperature increases with increase in Ec, whether R=1.0 or R=-1.0. Furthermore figures 5.2.22a and 5.2.22b indicate that the temperature is higher near the upper moving plate in the presence of mass injection while the reverse is the case under mass suction where the temperature is higher near the lower plate as indicated in figures 5.2.21a and 5.2.21b.

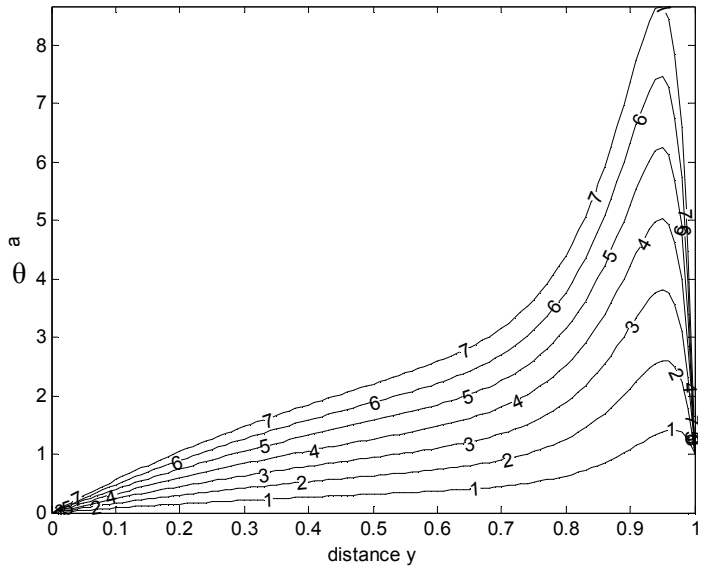


Fig.5.2.21a Steady state temperature (θ) versus distance y for different Ec (Re=5.0, Pr=7.0, M=2.0 , R=1.0)

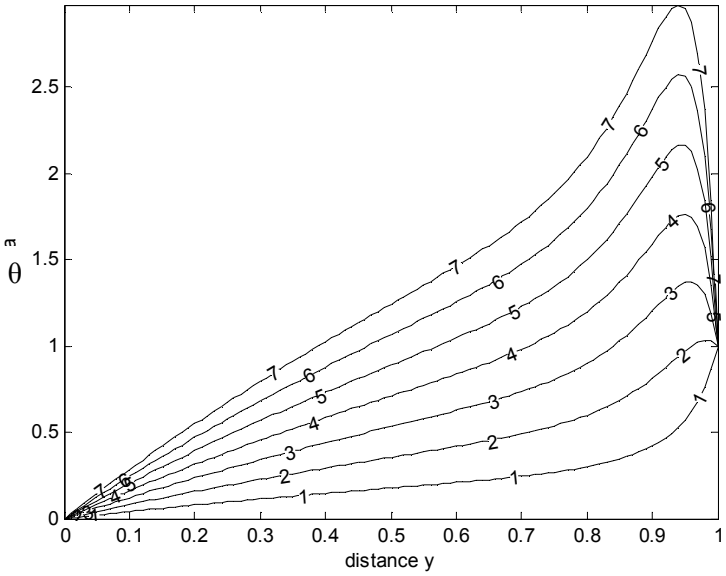


Fig.5.2.21bSteady state temperature (θ) versus distance (y) for different Ec (Re=5.0, Pr=7.0, M=2.0 , R=-1.0)

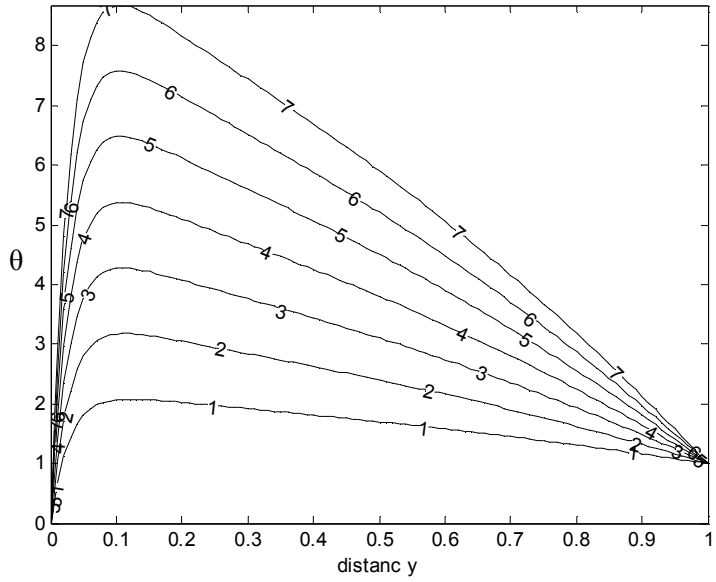


Fig.5.2.22a Steady state temperature (θ) versus distance (y) for different *Ec* (Re=-5.0, Pr=7.0, M=2.0, R=1.0)

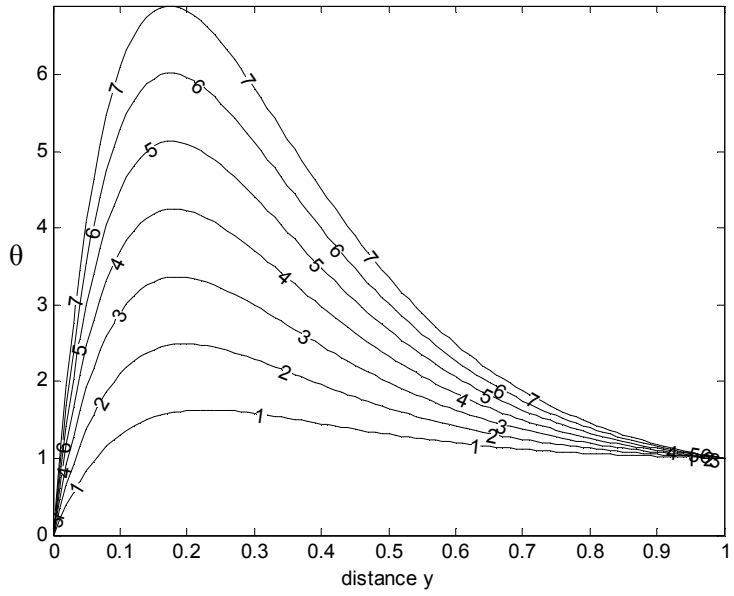


Fig.5.2.22b Steady state temperature (θ) versus distance (y) for different *Ec* (Re=-1.0, Pr=7.0, M=2.0, R=-1.0)

The effects of Eckert number, Ec, Hartmann number, *M* and the Electric load parameter, R, in the presence of mass injection/suction, on the Nusselt number at the

lower and upper moving porous plate, for fixed values of Re=-5.0, Re=5.0 and Pr=0.7, is displayed in Figs.5.2.23-5.2.24.These figures show that at the lower porous wall in the presence of mass suction or injection, the Nusselt number increases with increase in Ec and M , whether R=1.0 or R=-1.0.

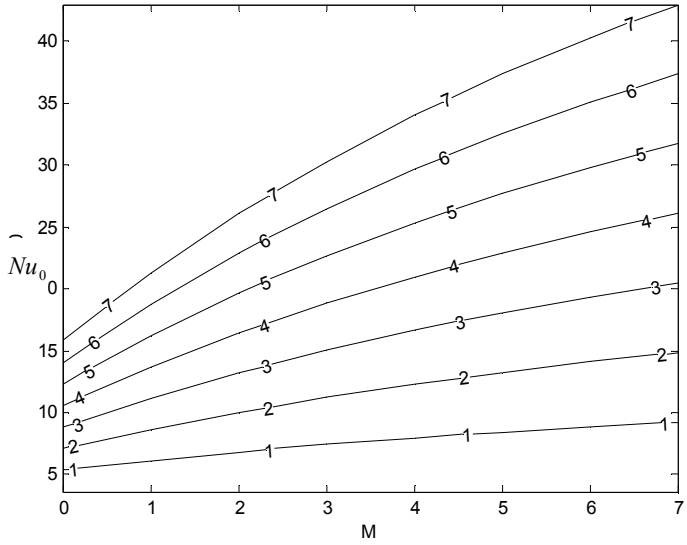


Fig.5.2.23a Nusselt number(Nu_0)versus (M)for different Ec at $y = 0$ (Re=-5.0, Pr=0.7, R=1.0)

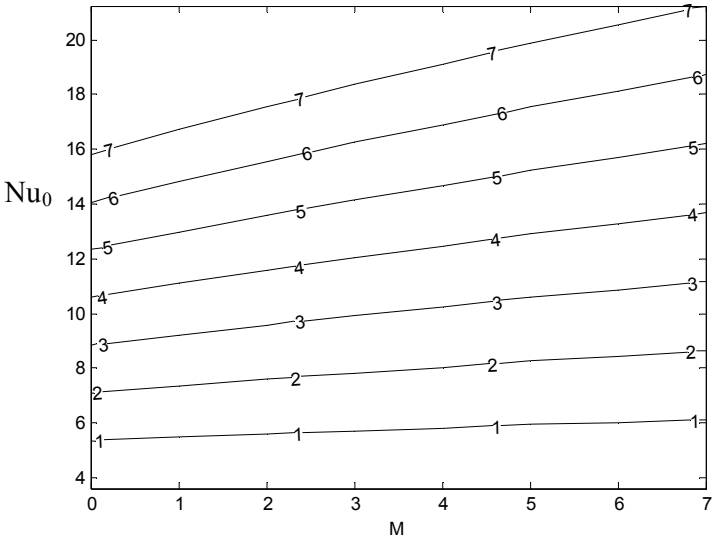


Fig.5.2.23b Nusselt number ($Nu_{0)}$ versus M for different Ec (Re=-5.0, Pr=0.7, R=-1.0 at $y = 0$)

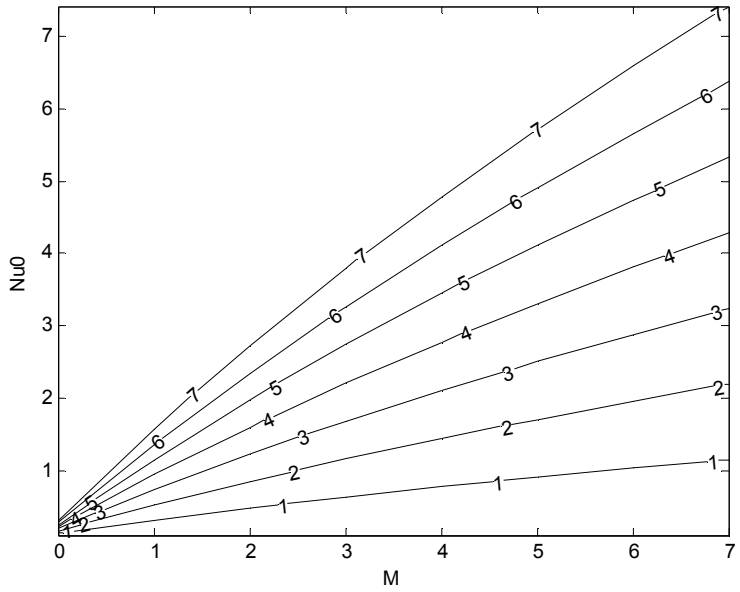


Fig.5.2.24a Nusselt number (Nu_0)versus (M)for different Ec at $y = 0$ (Re=5.0, Pr=0.7 , R=1.0)

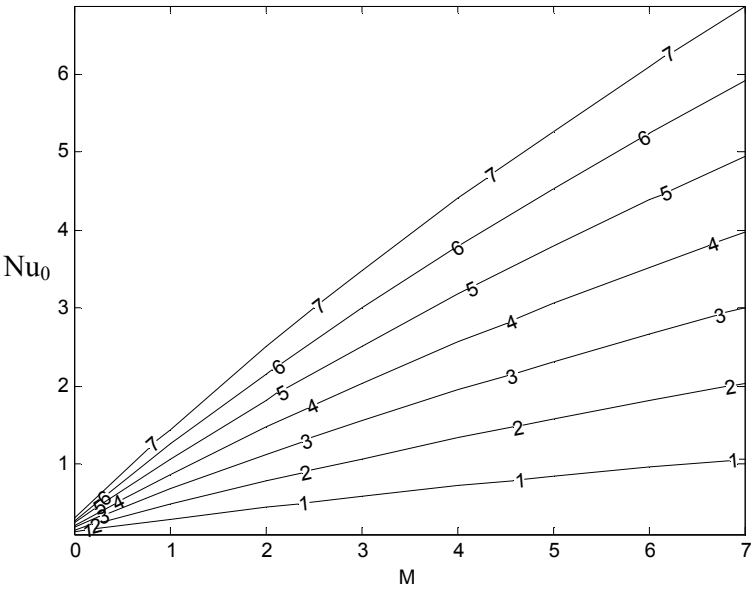


Fig.5.2.24b Nusselt number ($Nu_{0)}$ versus (M) for different Ec at $y = 0$ (Re=5.0, Pr=0.7, R=-1.0)

Figs.5.2.25-5.2.26. display the effects of Eckert number, Ec, Hartmann number, M and the Electric load parameter R in the presence of Mass injection/suction on the

Nusselt number at the upper moving porous wall, for fixed values of Re=-5.0, Re=5.0 and Pr=0.7.These figures show that at the upper moving porous wall in the presence of mass suction or injection, the Nusselt number decreases with increase in Ec, when R=1.0 or R=-1.0.

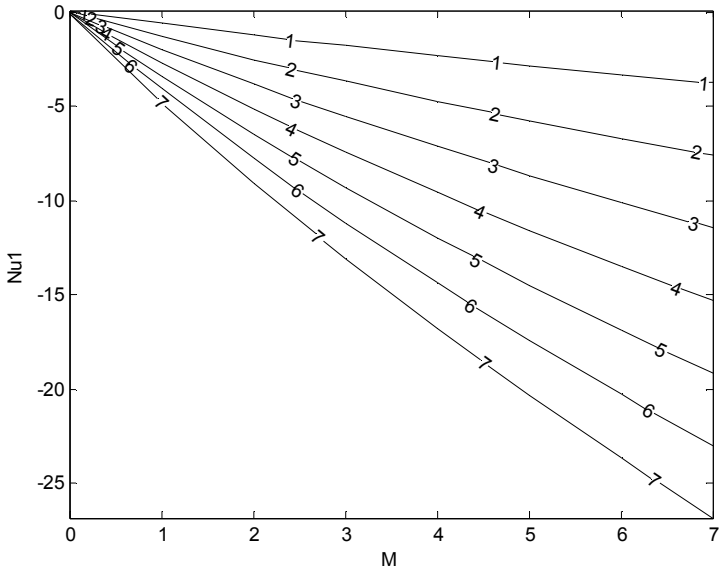


Fig.5.2.25a Nusselt number(Nu_1)versus (M) for different Ec at $y = 1$ (Re=-5.0, Pr=0.7,R=1.0)

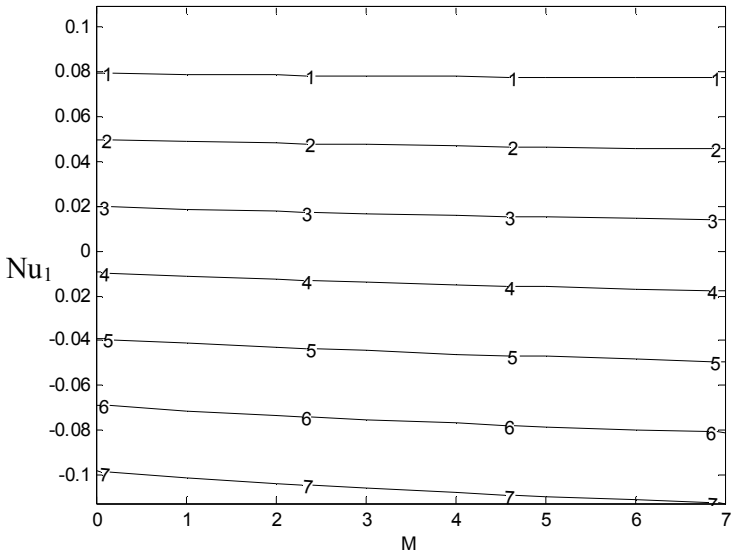


Fig.5.2.25b Nusselt number(Nu_1) versus (M) for different Ec at $y = 1$ (Re=-5.0, Pr=0.7 R=-1.0)

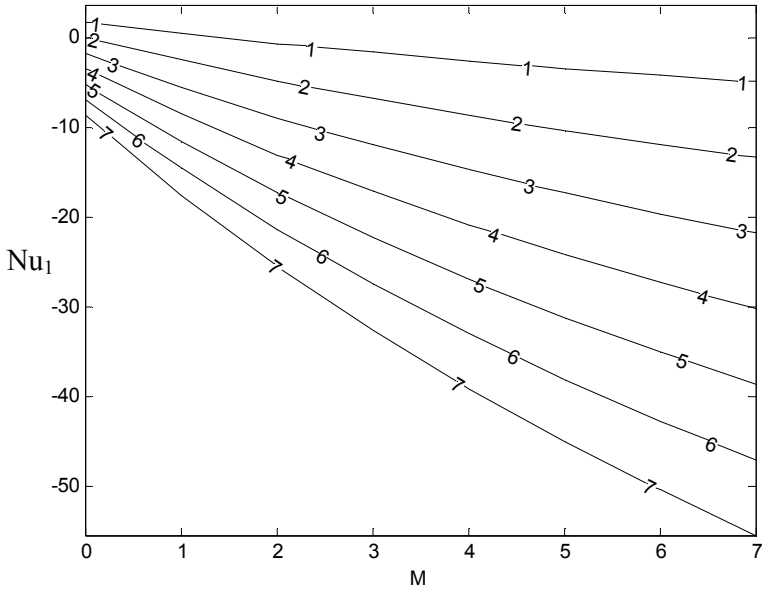


Fig.5.2.26aNusselt number(Nu_1) versus (M) for different Ec at $y = 1$ (Re=5.0, Pr =0.7 R=1.0)

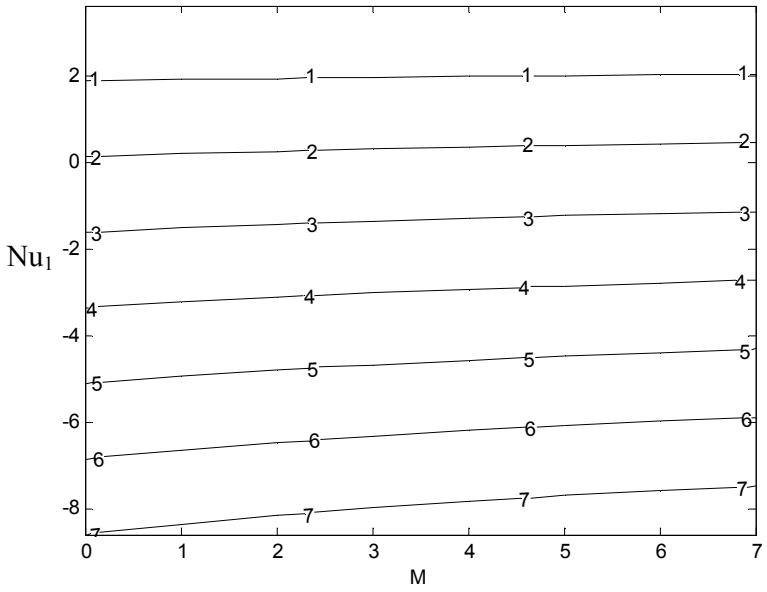


Fig.5.2.26b Nusselt number (Nu_1) versus (M) for different Ec at $y = 1$ (Re=5.0, Pr =0.7 and R=-1.0)

5.2.8 Summary and Conclusion

In this work the MHD Couette flow of an incompressible, viscous and electrically conducting fluid in a porous channel under the influence of transverse magnetic field and applied electric field taking into account the effects of viscous and Ohmic dissipation is considered. The resulting MHD boundary layer equations of momentum and energy transfer are solved analytically and closed form expressions obtained for time dependent velocity and steady state temperature under prescribed boundary conditions. Computation of results were carried out for different values of the Electric Load parameter R, suction/injection parameter, Re ,Hartmann number M, Prandtl number , Pr and Eckert number, Ec, in order to examine the effect of these parameters on the transient velocity, steady state temperature, the Nusselt number and the skin friction on the walls.

It was observed that:

- Under mass injection, for the open circuit case(R=1.0), the velocity decreases with increase in M. The reverse is however the case when R=-1.0, where the velocity increases with increase in the Hartmann number M.
- When R=1.0 the skin friction coefficient on the lower porous wall decreases with increase in M and increases with increase in Re, while the reverse is the case when R=-1.0, where the skin friction coefficient increases with increase in M and decrease with increase in Re.
- Under mass suction or injection, the temperature increases with increase in Pr for both R=1.0 and R= -1.0. The temperature is however higher when R=-1.0 than R=1.0.
- The Nusselt number at the lower and upper moving porous plate decreases with increase in the Prandtl number.

- Under mass injection at the lower moving plate, the temperature increases with increase in the Eckert number (Ec) when R=1.0 or R=-1.0. The temperature is however higher when R=1.0 than when R= -1.0.

CHAPTER SIX

TWO PHASE FLOW IN MAGNETOHYDRODYNAMICS: A GREENS FUNCTION APPROACH

6.0 Intoduction

The present chapter addresses the transient two phase flow of viscous, incompressible fluid in a horizontal parallel plate channel. Two cases were considered i) the channel having one phase occupied by electrically- conducting fluid and other by non-conducting fluid. ii) The two fluids in the channel are both electrically conducting. A closed form solution is obtained using Green's function approach.. This work is devoted to study the time dependent MHD two phase flows of viscous, incompressible and electrically conducting fluids in a horizontal parallel plate channel having both phases occupied by electrically conducting fluids of different nature. An exact solution is obtained using Green's function approach.

6.1 Time dependent two phase flow in magnetohydrodynamics: A Greens function approach

6.1.0 Introduction

This section presents a mathematical model to study the time dependent two phase magneto-hydrodynamic (MHD) flow in a parallel plate channel partially filled with conducting fluid and remaining with non-conducting fluid. Both the phases were incompressible and the flow is assumed to be time dependent.

6.1.1 Governing equations and solutions

The physical situation corresponds to that of the time dependent two phase MHD (magneto-hydrodynamic) flow in a horizontal parallel plate channel partially filled with conducting fluid. The physical model shown in Fig.6.1.1 consists of two infinite parallel plates extending x-and z-directions. The region $0 \le y \le d$ and $d \le y \le h$ are

occupied by two different fluids of density ρ_1 and ρ_2 which are electrically non-conducting and conducting respectively. The viscosities of both fluids are also assumed to be different. A constant magnetic field of uniform strength B_o is applied in the y-direction. The flow formation inside the channel is due to the application of constant external pressure gradient or (and) by the sudden movement of the boundaries.

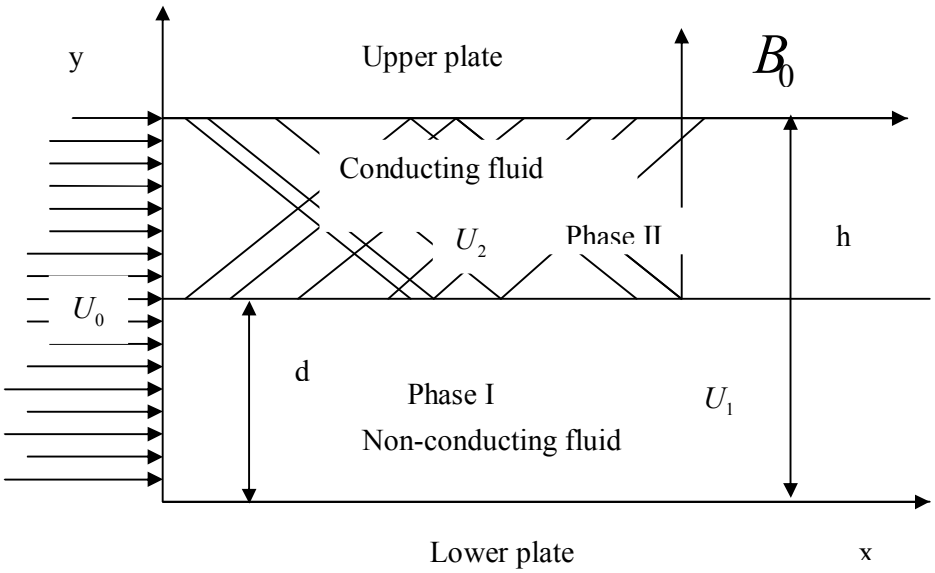


Fig.6.1.1 Physical configuration of the flow

With these assumptions, the dimensionless governing equations for both phases and corresponding initial, boundary and interface conditions are:

$$\frac{\partial U_1}{\partial t} = -P + \frac{\partial^2 U_1}{\partial Y^2}_1 \tag{6.1.1}$$

$$\frac{\partial U_2}{\partial t} = -P\alpha + \gamma\alpha \frac{\partial^2 U_2}{\partial Y^2} - \alpha M^2 U_2 \tag{6.1.2}$$

The Initial, boundary and interface conditions are:

$$U_1(0,Y)=F_1(Y),U_2(0,Y)=F_2(Y),\ \begin{matrix}U_1(t,0)=f_1(t),\\ U_2(t,H)=f_2(t),\end{matrix} \tag{6.1.3}$$

$$U_1(t,1)=U_2(t,1),\quad \frac{\partial U_1(t,1)}{\partial Y}=\gamma\frac{\partial U_2(t,1)}{\partial Y}, \tag{6.1.4}$$

The dimensional quantities introduced in equations (6.1.1-6.1.4) are given in list of symbols.

Equation (6.1.3) represents the general nature of initial and boundary conditions. A specific situation can be handled by proper selection of initial and boundary conditions.

It is more convenient to solve the problem with homogeneous boundary conditions. To attain, this let

$$U_i(Y,t)=V_i(Y,t)+\phi_i(Y)f_1(t)+\varepsilon_i(Y)f_2(t),i=1,2 \tag{6.1.5}$$

This relation transforms the governing equations (6.1.1-6.1.4) into the following set of equations

$$\frac{d^2\phi_1}{dY^2}_1=0 \tag{6.1.6}$$

$$\gamma\frac{d^2\phi_2}{dY^2}-M^2\phi_2=0 \tag{6.1.7}$$

$$\left.\begin{matrix}\phi_1(0)=1;\phi_2(H)=0,\\ \phi_1(1)=\phi_2(1),\frac{d\phi_1(1)}{dY}=\gamma\frac{d\phi_2(1)}{dY}\end{matrix}\right\}. \tag{6.1.8}$$

$$\frac{d^2\varepsilon_1}{dY^2}_1=0 \tag{6.1.9}$$

$$\gamma\frac{d^2\varepsilon_2}{dY^2}-M^2\varepsilon_2=0 \tag{6.1.10}$$

$$\left.\begin{matrix}\varepsilon_1(0)=0;\varepsilon_2(H)=1,\\ \varepsilon_1(1)=\varepsilon_2(1),\frac{d\varepsilon_1(1)}{dY}=\gamma\frac{d\varepsilon_2(1)}{dY}\end{matrix}\right\} \tag{6.1.11}$$

$$\frac{\partial V_1}{\partial t} = -P_1 + \frac{\partial^2 V_1}{\partial Y^2} \tag{6.1.12}$$

$$\frac{\partial V_2}{\partial t} = -P_2 + \gamma\alpha \frac{\partial^2 V_2}{\partial Y^2} - \alpha M^2 V_2 \tag{6.1.13}$$

$$V_1(t,0) = 0, V_2(t,H) = 0, V_1(t,1) = V_2(t,1)$$

$$\frac{dV_1(t,1)}{dY} = \mu_r \frac{dV_2(t,1)}{dY} \tag{6.1.14}$$

$$V_i(0,Y) = F_i(Y) - \phi_i(Y) f_1(0) - \varepsilon_i(Y) f_2(0) = F_i^*, i = 1,2 \tag{6.1.15}$$

$$P_1 = P + \phi_1 \frac{df_1}{dt} + \varepsilon_1 \frac{df_2}{dt} \tag{6.1.16}$$

$$P_2 = P\alpha + \phi_1 \frac{df_1}{dt} + \varepsilon_1 \frac{df_2}{dt} \tag{6.1.17}$$

The solutions of equations (6.1.6-6.1.8) are:

$$\phi_1(Y) = A_1 + A_2 Y, \tag{6.1.18}$$

$$\phi_2(Y) = A_3 Exp\left(\frac{MY}{\sqrt{\gamma}}\right) + A_4 Exp\left(-\frac{MY}{\sqrt{\gamma}}\right), \tag{6.1.19}$$

where $A_2 = 1$, and A_1, A_3 and A_4 are found from the solution of the following set of equations:

$$\begin{bmatrix} 0 & Exp\left(\frac{MH}{\sqrt{\gamma}}\right) & Exp\left(-\frac{MH}{\sqrt{\gamma}}\right) \\ -1 & Exp\left(\frac{M}{\sqrt{\gamma}}\right) & Exp\left(-\frac{M}{\sqrt{\gamma}}\right) \\ -1 & M\sqrt{\gamma}\,Exp\left(\frac{M}{\sqrt{\gamma}}\right) & -M\sqrt{\gamma}\,Exp\left(-\frac{M}{\sqrt{\gamma}}\right) \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Also, the solutions of equations (6.1.9-6.1.11) are:

$$\varepsilon_1(Y) = A_5 Y + A_6, \tag{6.1.20}$$

$$\varepsilon_2(Y) = A_7 Exp\left(\frac{MY}{\sqrt{\gamma}}\right) + A_8 Exp\left(-\frac{MY}{\sqrt{\gamma}}\right), \tag{6.1.21}$$

where $A_6 = 0$, and A_5, A_7 and A_8 are found from the solution of the following set of equations:

$$\begin{bmatrix} 0 & Exp\left(\frac{MH}{\sqrt{\gamma}}\right) & Exp\left(-\frac{MH}{\sqrt{\gamma}}\right) \\ -1 & Exp\left(\frac{M}{\sqrt{\gamma}}\right) & -Exp\left(-\frac{M}{\sqrt{\gamma}}\right) \\ -1 & M\sqrt{\gamma}Exp\left(\frac{M}{\sqrt{\gamma}}\right) & -M\sqrt{\gamma}Exp\left(-\frac{M}{\sqrt{\gamma}}\right) \end{bmatrix} \begin{bmatrix} A_5 \\ A_7 \\ A_8 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Using Green's function method, Equations (6.1.12-6.1.15) assume the following form:

$$\begin{aligned} V_1(t,Y) = & \int_0^1 G_{11}(Y,t/Y',t')_{t'=0} F_1^*(Y')dY' \\ & - \int_{t'=0}^{t} dt' \int_0^1 G_{11}(Y,t/Y',t')(P_1)dY' \\ & + \int_1^H G_{12}(Y,t/Y',t')_{t'=0} F_2^*(Y')dY' \\ & - \int_{t'=0}^{t} dt' \int_1^H G_{12}(Y,t/Y',t')(P_2)dY' \end{aligned}$$

$$\begin{aligned} V_2(t,Y) = & \int_0^1 G_{21}(Y,t/Y',t')_{t'=0} F_1^*(Y')dY' \\ & - \int_{t''=0'}^{t} dt' \int_0^1 G_{21}(Y,t/Y',t')(P_1)dY' \\ & + \int_1^H G_{22}(Y,t/Y',t')_{t'=0} F_2^*(Y')dY' \\ & - \int_{t''=0}^{t} dt' \int_1^H G_{22}(Y,t/Y',t')(P_2)dY' \end{aligned} \tag{6.1.23}$$

Where G_{ij} is the appropriate Green's function found from the homogeneous version of the governing equations (6.1.12-6.1.15) are:

$$G_{ij}\left(Y,t/Y',t'\right)=\sum_{n=1}^{\infty}\frac{Exp\left(-\beta_n^2\left(t-t'\right)\right)}{N_n}\eta_i\psi_{in}\left(Y\right)\psi_{jn}\left(Y'\right),\ .$$

Where i = 1,2 & j = 1,2, $\eta_1=1,\eta_2=\frac{1}{\alpha}.$ (6.1.24)

$$N_n=\int_0^1\psi_{1n}^2\left(Y'\right)dY'+\left(\frac{1.}{\alpha}\right)\int_1^H\psi_{2n}^2\left(Y'\right)dY' \tag{6.1.25}$$

$$\psi_{1n}\left(Y\right)=Sin\left(\beta_nY\right), \tag{6.1.26}$$

$$\psi_{2n}\left(Y\right)=A_{2n}Cos\left(\lambda_nY\right)+B_{2n}Sin\left(\lambda_nY\right), \tag{6.1.27}$$

Where $\lambda_n^2=\left(\frac{\beta_n^2}{\alpha\gamma}\right)-\left(\frac{M^2}{\gamma}\right).$

The values of constants A_{2n} and B_{2n} are found from the solution of the following set of equations:

$$\begin{bmatrix}Cos\left(\lambda_n\right) & Sin\left(\lambda_n\right)\\ -\gamma\lambda_nSin\left(\lambda_n\right) & \gamma\lambda_nCos\left(\lambda_n\right)\end{bmatrix}\begin{bmatrix}A_{2n}\\ B_{2n}\end{bmatrix}=\begin{bmatrix}Sin\left(\beta_n\right)\\ \beta_{2n}Cos\left(\beta_n\right)\end{bmatrix}.$$

The eigen-values λ_n and β_n are found as the roots of

$$A_{2n}Cos\left(H\lambda_n\right)+B_{2n}Sin\left(H\lambda_n\right)=0. \tag{6.1.28}$$

6.1.2 Particular Case:Uniform pressure gradient

The physical situation, in which flow inside the channel is solely caused by uniform pressure gradient, i.e., $F_1\left(Y\right)=F_2\left(Y\right)=f_1\left(t\right)=f_2\left(t\right)=0$, which yields:

$P_1=P,P_2=\alpha P$ and $U_i=V_i,i=1,2.$

Using these values in equations (6.1.22) and (6.1.23) the dimensional velocity in both phases are:

$$U_1\left(Y,t\right)=-\int_{t'=0}^{t}dt'\int_0^1G_{11}\left(Y,t/Y',t'\right)\psi_{1n}\left(Y\right)\psi_{1n}\left(Y'\right)PdY'$$

$$- \int_{t'=0}^{t} dt' \int_{1}^{H} G_{12}\left(Y,t/Y',t'\right)\psi_{1n}(Y)\psi_{2n}\left(Y'\right)PdY' \qquad (6.1.29)$$

$$U_2(Y,t) = -\int_{t'}^{t} dt' \int_{0}^{1} G_{21}\left(Y,t/Y',t'\right)\psi_{2n}(Y)\psi_{1n}\left(Y'\right)PdY'$$

$$- \int_{t'}^{t} dt' \int_{1}^{H} G_{22}\left(Y,t/Y',t'\right)\psi_{2n}(Y)\psi_{2n}\left(Y'\right)PdY' \qquad (6.1.30)$$

$$U_1(Y,t) = -P\sum_{n=1}^{\infty} \frac{\left[1.-Exp\left(-\beta_n^2 t\right)\right]}{\beta_n^3 N_n} Sin(\beta_n Y)\left[1.-Cos(\beta_n)\right]$$

$$- P\sum_{n=1}^{\infty} \frac{\left[1.-Exp\left(-\beta_n^2 t\right)\right]}{\beta_n^2 N_n \lambda_n} Sin(\beta_n Y)$$

$$\left[A_{2n}\left\{Sin(\lambda_n H)-Sin(\lambda_n)\right\}-B_{2n}\left\{Cos(\lambda_n H)-Cos(\lambda_n)\right\}\right] \qquad (6.1.31)$$

$$U_2(Y,t) = -P\sum_{n=1}^{\infty} \frac{\left[1.-Exp\left(-\beta_n^2 t\right)\right]\left[A_{2n}Cos(\beta_n Y)+B_{2n}Sin(\beta_n Y)\right]\left[1.-Cos(\beta_n)\right]}{\beta^3 N_n}$$

$$- P\sum_{n=1}^{\infty} \frac{\left[1.-Exp\left(-\beta_n^2 t\right)\right]\left[A_{2n}Cos(\lambda_n Y)+B_{2n}Sin(\lambda_n Y)\right]}{\beta^2 N_n \lambda_n}$$

$$\left[A_{2n}\left\{Sin(\lambda_n H)-Sin(\lambda_n)\right\}-B_{2n}\left\{Cos(\lambda_n H)-Cos(\lambda_n)\right\}\right] \qquad (6.1.32)$$

$$N_n = \int_0^1 \psi_{1n}^2\left(Y'\right)dY' + \left(\frac{1.}{\alpha}\right)\int_1^H \psi_{2n}^2\left(Y'\right)dY'$$

$$= \left[\frac{Y'}{2} - \frac{Sin\left(2C_n Y'\right)}{4}\right]_0^1 + \left(\frac{A_{2n}^2}{\alpha}\right)\left[\frac{Y'}{2} - \frac{Sin\left(2\lambda_n Y'\right)}{4\lambda_n}\right]_1^H$$

$$+ \left(\frac{B_{2n}^2}{\alpha}\right)\left[\frac{Y'}{2} - \frac{Sin\left(2\lambda_n Y'\right)}{4\lambda_n}\right]_1^H - \left(\frac{A_{2n}B_{2n}}{\alpha\lambda_n}\right)\left[Cos\left(2\lambda_n Y'\right)\right]_1^H$$

6.1.3. Steady- State Solution

$$\frac{\partial^2 U_1}{\partial Y^2} = P, \qquad (6.1.33)$$

$$\gamma \frac{\partial^2 U_2}{\partial Y^2} - M^2 U_2 = P, \tag{6.1.34}$$

$$U_1 = \frac{PY^2}{2} + C_1 Y + C_2, \tag{5.1.35}$$

$$U_2 = C_3 Cosh\left(\frac{MY}{\sqrt{\gamma}}\right) + C_4 Sinh\left(\frac{MY}{\sqrt{\gamma}}\right) - \frac{\left[P + \mathrm{Re}\, M^2\right]}{M^2}, \tag{6.1.36}$$

where $C_2 = 0$, and C_1, C_3 and C_4 are found from the solution of following set of equations:

$$\begin{bmatrix} 0 & Cosh\left(\frac{MH}{\sqrt{\gamma}}\right) & Sinh\left(\frac{MH}{\sqrt{\gamma}}\right) \\ -1 & Cosh\left(\frac{M}{\sqrt{\gamma}}\right) & Sinh\left(\frac{M.}{\sqrt{\gamma}}\right) \\ -1 & M\sqrt{\gamma} Sinh\left(\frac{M}{\sqrt{\gamma}}\right) & M\sqrt{\gamma} Cosh\left(\frac{M}{\sqrt{\gamma}}\right) \end{bmatrix} \begin{bmatrix} C_1 \\ C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} \frac{P}{M^2} \\ \frac{P}{2} + \frac{P}{M^2} \\ P \end{bmatrix}. \tag{6.1.37}$$

6.1.4 Concluding remarks

The novel feature of this work is to present an analytical solution of unsteady two phase flow where one phase is conducting and other phase is non-conducting using Green's function approach.

6.2 Unsteady MHD two- phase flow of conducting fluids in a horizontal channel

6.2.0 Introduction

This section is devoted to the study of unsteady MHD two phase flows of viscous, incompressible and electrically conducting fluids in a horizontal parallel plate channel having both phases occupied by electrically conducting fluids of different nature. An exact solution is obtained using Green's function approach.

6.2.1. Governing equations and solutions

The physical situation corresponds to that of the time dependent MHD two phase flows in a horizontal parallel plate channel filled with two different conducting fluids. The physical model is shown in fig.6.2.1. The direction of the flow is taken along the x'-axis coinciding with the lower horizontal plate while y'-axis is taken normal to the flow direction. The regions $0 \le y \le d$ and $d \le y \le h$ are occupied by two different conducting fluids having different densities, viscosities and electrical conductivities. An external magnetic field of uniform strength B_0, is applied parallel to y'-axis. The magnetic field is assumed to be small so that the induced magnetic field can be neglected compared to the applied magnetic field. The unsteadiness in the fluid motion is caused either by a sudden change in the imposed pressure gradient along x'-axis or (and) by a sudden change in the velocity of the channel boundaries.

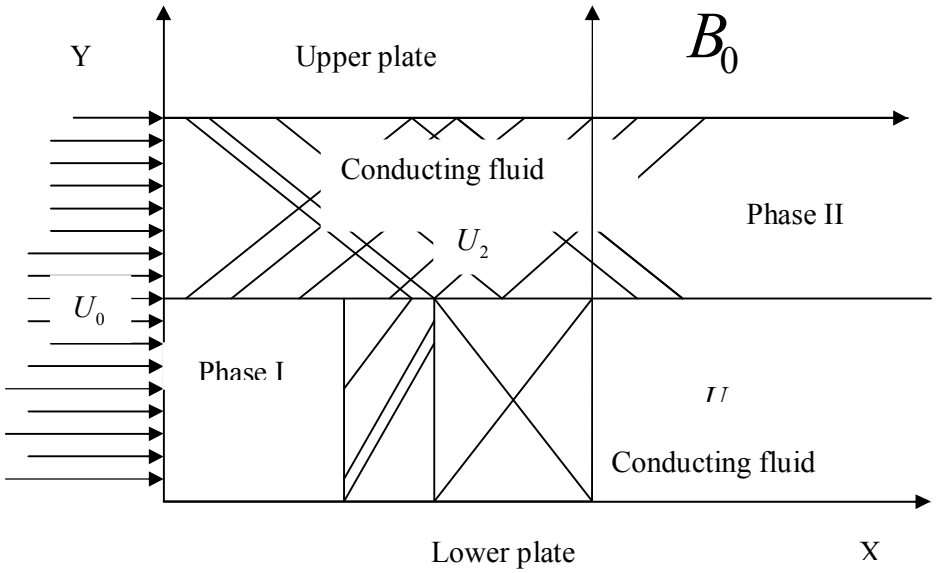


Fig.6.2 .1 Physical model

Under the assumptions discussed above the one dimensional time dependent flow for both phases in short-circuit case are governed by the following non-dimensional equations:

$$\frac{\partial U_1}{\partial t} = -P + \frac{\partial^2 U_1}{\partial Y^2} - M^2 U_1 \tag{6.2.1}$$

$$\frac{\partial U_2}{\partial t} = -\alpha P + \gamma\alpha \frac{\partial^2 U_2}{\partial Y^2} - \alpha\delta M^2 U_2 \tag{6.2.2}$$

The Initial, boundary and interface conditions are:

$$U_1(0,Y) = F_1(Y), U_2(0,Y) = F_2(Y),$$

$$\begin{aligned} U_1(t,0) &= f_1(t), \\ U_2(t,H) &= f_2(t), \end{aligned} \tag{6.2.3}$$

$$U_1(t,1) = U_2(t,1), \frac{\partial U_1(t,1)}{\partial Y} = \gamma \frac{\partial U_2(t,1)}{\partial Y}, \tag{6.2.4}$$

The dimensionless quantities introduced in equations (6.2.1-6.2.4) are given in list of symbols.

Equation (6.2.3) represents the general nature of initial and boundary conditions. A specific situation can be handled by proper selection of initial and boundary conditions.

It is more convenient to solve the problem with homogeneous boundary conditions. To attain this let

$$U_i(Y,t) = V_i(Y,t) + \phi_i(Y) f_1(t) + \varepsilon_i(Y) f_2(t), i = 1,2 \tag{6.2.5}$$

This relation transforms the governing equations (6.2.1-6.2.4) in to the following set of equations

$$\frac{d^2\phi_1}{dY^2} - M^2\phi_1 = 0 \tag{6.2.6}$$

$$\gamma \frac{d^2\phi_1}{dY^2} - \delta M^2 \phi_1 = 0 \tag{6.2.7}$$

$$\left.\begin{aligned}&\phi_1(0)=1;\phi_2(H)=0,\\&\phi_1(1)=\phi_2(1),\frac{d\phi_1(1)}{dY}=\gamma\frac{d\phi_2(1)}{dY}\end{aligned}\right\}. \tag{6.2.8}$$

$$\frac{d^2\varepsilon_1}{dY^2}-M^2\varepsilon_1=0 \tag{6.2.9}$$

$$\gamma\frac{d^2\varepsilon_2}{dY^2}-\delta M^2\varepsilon_2=0 \tag{6.2.10}$$

$$\left.\begin{aligned}&\varepsilon_1(0)=0;\varepsilon_2(H)=1,\\&\varepsilon_1(1)=\varepsilon_2(1),\frac{d\varepsilon_1(1)}{dY}=\gamma\frac{d\varepsilon_2(1)}{dY}\end{aligned}\right\}. \tag{6.2.11}$$

$$\frac{\partial V_1}{\partial t}=-P_1+\frac{\partial^2 V_1}{\partial Y^2}-M^2V_1 \tag{5.2.12}$$

$$\frac{\partial V_2}{\partial t}=-P_2+\gamma\alpha\frac{\partial^2 V_2}{\partial Y^2}-\alpha\delta M^2V_2 \tag{6.2.13}$$

$$V_1(t,0)=0,V_2(t,H)=0,V_1(t,1)=V_2(t,1)$$

$$\frac{dV_1(t,1)}{dY}=\mu_r\frac{dV_2(t,1)}{dY} \tag{6.2.14}$$

$$V_i(0,Y)=F_i(Y)-\phi_i(Y)f_1(0)-\varepsilon_i(Y)f_2(0)=F_i^*,i=1,2 \tag{6.2.15}$$

$$P_1=P+\phi_1\frac{df_1}{dt}+\varepsilon_1\frac{df_2}{dt} \tag{6.2.16}$$

$$P_2=P\alpha+\phi_2\frac{df_1}{dt}+\varepsilon_2\frac{df_2}{dt} \tag{6.2.17}$$

The solutions of equation (6.2.6-6.2.8) are

$$\phi_1(Y)=A_1Exp(MY)+A_2Exp(-MY) \tag{6.2.18}$$

$$\phi_2(Y)=A_3Exp\left(MY\sqrt{\frac{\delta}{\gamma}}\right)+A_4Exp\left(-MY\sqrt{\frac{\delta}{\gamma}}\right) \tag{6.2.19}$$

where $A_1, A_2, A_3,$ and A_4 are found from the solution of the following set of equations:

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ Exp(M) & Exp(-M) & -Exp\left(M\sqrt{\frac{\delta}{\gamma}}\right) & -Exp\left(M\sqrt{\frac{\delta}{\gamma}}\right) \\ Exp(M) & -Exp(-M) & -\sqrt{\gamma\delta}\,Exp\left(M\sqrt{\frac{\delta}{\gamma}}\right) & \sqrt{\gamma\delta}\,Exp\left(-M\sqrt{\frac{\delta}{\gamma}}\right) \\ 0 & 0 & Exp\left(MH\sqrt{\frac{\delta}{\gamma}}\right) & Exp\left(-MH\sqrt{\frac{\delta}{\gamma}}\right) \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (6.2.20)$$

Also, the solutions of equation (6.2.9-6.2.11) are

$$\varepsilon_1(Y) = A_5 Exp(MY) + A_6 Exp(-MY) \quad (6.2.21)$$

$$\varepsilon_2(Y) = A_7 Exp\left(MY\sqrt{\frac{\delta}{\gamma}}\right) + A_8 Exp\left(-MY\sqrt{\frac{\delta}{\gamma}}\right) \quad (6.2.22)$$

where $A_5, A_6, A_7,$ and A_8 are found from the solution of the following set of equations:

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ Exp(M) & Exp(-M) & -Exp\left(M\sqrt{\frac{\delta}{\gamma}}\right) & -Exp\left(M\sqrt{\frac{\delta}{\gamma}}\right) \\ Exp(M) & -Exp(-M) & -\sqrt{\gamma\delta}\,Exp\left(M\sqrt{\frac{\delta}{\gamma}}\right) & \sqrt{\gamma\delta}\,Exp\left(-M\sqrt{\frac{\delta}{\gamma}}\right) \\ 0 & 0 & Exp\left(MH\sqrt{\frac{\delta}{\gamma}}\right) & Exp\left(-MH\sqrt{\frac{\delta}{\gamma}}\right) \end{bmatrix} \begin{bmatrix} A_5 \\ A_6 \\ A_7 \\ A_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad (6.2.23)$$

Using Green's function approach, Equation (6.2.12-6.2.15) assumes the following form:

$$
\begin{aligned}
V_1(t,Y) &= \int_0^1 G_{11}(Y,t/Y',t')_{t'_{=0}} F_1^*(Y')dY' \\
&\quad - \int_{t'=0}^{t} dt' \int_0^1 G_{11}(Y,t/Y',t')(P_1)dY' \\
&\quad + \int_1^H G_{12}(Y,t/Y',t')_{t'_{=0}} F_2^*(Y')dY' \\
&\quad - \int_{t'=0}^{t} dt' \int_1^H G_{12}(Y,t/Y',t')(P_2)dY'
\end{aligned}
\qquad (6.2.24)
$$

$$
\begin{aligned}
V_2(t,Y) &= \int_0^1 G_{21}(Y,t/Y',t')_{t'_{=0}} F_1^*(Y')dY' \\
&\quad - \int_{t''=0'}^{t} dt' \int_0^1 G_{21}(Y,t/Y',t')(P_1)dY' \\
&\quad + \int_1^H G_{22}(Y,t/Y',t')_{t'_{=0}} F_2^*(Y')dY' \\
&\quad - \int_{t''=0}^{t} dt' \int_1^H G_{22}(Y,t/Y',t')(P_2)dY'
\end{aligned}
\qquad (6.2.25)
$$

Where G_{ij} is the appropriate Green's function found from the homogeneous version of the governing equations (6.2.12-6.2.15) are:

$$
G_{ij}(Y,t/Y',t') = \sum_{n=1}^{\infty} \frac{Exp\left(-\beta_n^2(t-t')\right)}{N_n} \eta_i \psi_{in}(Y)\psi_{jn}(Y'), \ .
$$

where $\mathrm{i} = 1,2$ & $\mathrm{j} = 1,2, \eta_1 = 1, \eta_2 = \frac{1}{\alpha}.$ (6.2.26)

$$
N_n = \int_0^1 \psi_{1n}^2(Y')dY' + \left(\frac{1.}{\alpha}\right)\int_1^H \psi_{2n}^2(Y')dY' \qquad (6.2.27)
$$

$$
\psi_{1n}(Y) = Sin(C_n Y), \text{ where } C_n^2 = \beta_n^2 - M^2 \qquad (6.2.28)
$$

$$
\psi_{2n} = A_{2n}Cos(\lambda_n Y) + B_{2n}Sin(\lambda_n Y), \text{ where } \lambda_n^2 = \left[\frac{\beta_n^2}{\alpha\gamma} - \frac{\delta M^2}{\gamma}\right] \qquad (6.2.29)
$$

The values of constant A_{2n} and B_{2n} are found from the solution of following set of equations

$$\begin{bmatrix} Cos(\lambda_n) & Sin(\lambda_n) \\ -\gamma\lambda_n Sin(\lambda_n) & \gamma\lambda_n Sin(\lambda_n) \end{bmatrix}\begin{bmatrix} A_{2n} \\ B_{2n} \end{bmatrix} = \begin{bmatrix} Sin(C_n) \\ C_n Cos(C_n) \end{bmatrix} \tag{6.2.30}$$

The eigen-values λ_n and β_n are found as the roots of

$$A_{2n}Cos(H\lambda_n) + B_{2n}Sin(H\lambda_n) = 0. \tag{6.2.31}$$

6.2.2. Particular Case

The physical situation, in which flow inside the channel is solely caused by uniform pressure gradient, i.e., $F_1(Y) = F_2(Y) = f_1(t) = f_2(t) = 0$, which yields:

$P_1 = P, P_2 = P\alpha$ and $U_i = V_i, i = 1,2.$

Using these values in equations (6.2.22) and (6.2.23) the dimensional velocity in both phases are:

$$U_1(Y,t) = -\int_{t'}^{t} dt' \int_0^1 G_{11}(Y,t/Y',t')\psi_{1n}(Y)\psi_{1n}(Y')PdY'$$

$$-\int_{t'}^{t} dt' \int_1^H G_{12}(Y,t/Y',t')\psi_{1n}(Y)\psi_{2n}(Y')P\alpha dY' \tag{6.2.32}$$

$$U_2(Y,t) = -\int_{t'}^{t} dt' \int_0^1 G_{21}(Y,t/Y',t')\psi_{2n}(Y)\psi_{1n}(Y')PdY'$$

$$-\int_{t'}^{t} dt' \int_1^H G_{22}(Y,t/Y',t')\psi_{2n}(Y)\psi_{2n}(Y')P\alpha dY' \tag{6.2.33}$$

$$U_1(Y,t) = -P\sum_{n=1}^{\infty} \frac{\left[1. - Exp(-\beta_n^2 t)\right]}{\beta_n^2 N_n C_n} Sin(C_n Y)\left[1. - Cos(C_n)\right]$$

$$-P\sum_{n=1}^{\infty} \frac{\left[1. - Exp(-\beta_n^2 t)\right]}{\beta_n^2 N_n \lambda_n} Sin(C_n Y)$$

$$\left[A_{2n}\{Sin(\lambda_n H) - Sin(\lambda_n)\} - B_{2n}\{Cos(\lambda_n H) - Cos(\lambda_n)\}\right] \tag{6.2.34}$$

$$U_2(Y,t) = -P\sum_{n=1}^{\infty} \frac{\left[1. - Exp(-\beta_n^2 t)\right]\left[A_{2n}Cos(C_n Y) + B_{2n}Sin(C_n Y)\right]\left[1. - Cos(C_n)\right]}{\beta_n^2 N_n C_n}$$

$$- P\sum_{n=1}^{\infty}\frac{\left[1.-Exp\left(-\beta_n^2 t\right)\right]\left[A_{2n}Cos\left(C_nY\right)+B_{2n}Sin\left(C_nY\right)\right]}{\beta_n^2 N_n\lambda_n}$$

$$\left[A_{2n}\left\{Sin\left(\lambda_n H\right)-Sin\left(\lambda_n\right)\right\}-B_{2n}\left\{Cos\left(\lambda_n H\right)-Cos\left(\lambda_n\right)\right\}\right] \qquad (6.2.35)$$

$$N_n=\int_0^1\psi_{1n}^2\left(Y'\right)dY'+\left(\frac{1.}{\alpha}\right)\int_1^H\psi_{2n}^2\left(Y'\right)dY'$$

$$=\left[\frac{Y'}{2}-\frac{Sin\left(2C_nY'\right)}{4}\right]_0^1+\left(\frac{A_{2n}^2}{\alpha}\right)\left[\frac{Y'}{2}-\frac{Sin\left(2\lambda_nY'\right)}{4\lambda_n}\right]_1^H$$

$$+\left(\frac{B_{2n}^2}{\alpha}\right)\left[\frac{Y'}{2}-\frac{Sin\left(2\lambda_nY'\right)}{4\lambda_n}\right]_1^H-\left(\frac{A_{2n}B_{2n}}{\alpha\lambda_n}\right)\left[Cos\left(2\lambda_nY'\right)\right]_1^H \qquad (6.2.36)$$

6.2.3. Steady state solution

$$\frac{\partial^2U_1}{\partial Y^2}-M^2U_1=P, \qquad (6.2.37)$$

$$\gamma\frac{\partial^2U_2}{\partial Y^2}-M^2U_2=P, \qquad (6.2.38)$$

$$U_1=C_1Sinh\left(MY\right)-\frac{P}{M^2}\left[Cosh\left(MY\right)-1.\right], \qquad (6.2.39)$$

$$U_2=C_3Cosh\left(MY\sqrt{\frac{\delta}{\gamma}}\right)+C_4Sinh\left(MY\sqrt{\frac{\delta}{\gamma}}\right)-\frac{P}{M^2\delta}, \qquad (6.2.40)$$

where C_1, C_3 and C_4 are found from the solution of following set of equations:

$$\begin{bmatrix} 0 & Cosh\left(MH\sqrt{\frac{\delta}{\gamma}}\right) & Sinh\left(MH\sqrt{\frac{\delta}{\gamma}}\right) \\ -Sinh\left(M\right) & Cosh\left(M\sqrt{\frac{\delta}{\gamma}}\right) & Sinh\left(M\sqrt{\frac{\delta}{\gamma}}\right) \\ -MCosh\left(M\right) & M\sqrt{\gamma\delta}Sinh\left(M\sqrt{\frac{\delta}{\gamma}}\right) & M\sqrt{\gamma\delta}Cosh\left(M\sqrt{\frac{\delta}{\gamma}}\right) \end{bmatrix}\begin{bmatrix} C_1 \\ C_3 \\ C_4 \end{bmatrix}=\begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} \quad 6.2.41)$$

Where $Z_1=\frac{P}{M^2\delta}$

$$Z_2 = \frac{P}{M^2}\left[Cosh(M) - 1.\right] + Z_1$$

$$Z_3 = \frac{P}{M} Sinh(M) \qquad (6.2.42)$$

6.2.4 Concluding remarks

In this work a closed form solution of time-dependent MHD two phase flows where both phases are electrically conducting and having different physical properties has been considered and analysed.

CHAPTER SEVEN

SUMMARY AND CONCLUDING REMARKS

7.1 Summary

This thesis has investigated the MHD Couette flow of incompressible, viscous fluid under the influence of transverse magnetic due to the ramped motion of the plate with or without transpiration. The MHD Couette flow of this type of fluid in between two infinite parallel plates due to the ramped motion of the lower boundary as well as the MHD two -phase flow of conducting fluids under different physical situations were considered. The different physical conditions considered include: ramped and impulsive motion of the (lower) boundary, magnetic lines of force fixed relative to the fluid or plate, suction and injection, thermal input, influence of applied magnetic and electric fields, viscous and ohmic dissipation. The work also considered the influence of moving plates and porous plates on the flow velocity. Finally the work also analysed the two –phase flow of conducting and non-conducting fluids.

Each of the cases above were modeled into partial differential equations and the resulting governing equations are analytically solved by the use of Laplace transforms, Fourier series and the Green's functions techniques, while the particular solutions are obtained using the method of undetermined coefficients. The numerical values of the analytical solutions of velocity, temperature, skin-friction and Nusselt number are obtained using MATLAB and FORTRAN programming. The graphs for the problems are plotted using Microsoft Excel and MATLAB. The graphs are interpreted in accordance with the physical situations.

7.2 Conclusion

The problems investigated in this dissertation are governed by different dimensionless parameters. Some of these parameters relate with the properties of the fluid such as the Reynolds number, Prandtl number etc. While others, such as the suction/injection parameter, Hartmann number, electric load parameter etc, are related with the physical situations. The conclusions drawn in this investigation were given in detail at the end of each section. However some of the general conclusions drawn in this work are summarized as follows:

1. The velocity field in the ramped motion of the boundary is always less than that of the flow induced by the impulsive (constant) motion of the boundary and velocity increases with increase of magnetic field when the magnetic lines of force are fixed with respect to the moving boundary while the results are just reverse when the magnetic lines are fixed with respect to the fluid.
2. The effect of injection is to increase the velocity while suction decreases it.
3. The skin friction is higher, when the magnetic field is fixed with respect to the fluid in comparison to magnetic field fixed with respect to the moving porous plate.
4. Injection at the lower moving porous plate suppresses skin friction while suction enhances it and the reverse is the case at the upper porous plate where injection enhances skin friction while suction suppresses it.
5. In case of heat and mass transfer, the temperature of the fluid increases with increase in suction while it decreases with increase in injection at the lower porous plate.
6. The Nusselt number on the lower as well as the upper moving porous plate decreases with increase in the Hartmann number M rate of heat transfer increases with increase of $\Pr$.

7. The skin friction coefficient at the lower porous plate decreases with increase in the Hartmann number M, while it increases with increase in M at the upper porous plate.

8. In case of heat and mass transfer under the influence of magnetic and electric field, under mass injection, for the open circuit case (R=1.0), the velocity decreases with increase in M. The reverse is however the case when R=-1.0 where the velocity increase with increase in Hartmann number M.

9. Under mass suction or injection, the temperature increases with increase in Pr for both R=1.0 and R= -1.0. The temperature is however higher when R=-1.0 than R=1.0.

10. Lastly the results of Fang (2004b), Jha and Jibril (2010) can be recovered as special cases of the different problems considered in this dissertation.

7.3 Directions for future research

In this work, the MHD Flow of an incompressible, viscous fluid due to the ramped motion of the plate was considered. The work can be extended by taking into consideration the effect of applied electric field on the ramped motion of the plate.

The effect of induced magnetic fields can also be taken into consideration for future research. Future research work can also be carried out by taking the fluid to be compressible.

REFERENCES

Abel, M.S, Kumar, K.A, Ravikumara, R. (2011). MHD Flow and heat transfer with effects of Buoyancy, viscous and Joules dissipation over a non-linear vertical stretching porous sheet with partial slip. *Engineering, 3, 285-291.*

Abdel-Aziz, M. (2007).Temperature dependent viscosity and thermal conductivity effects on combined heat and mass transfer in MHD three-dimensional flow over a stretching surface with ohmic heating. *Mechanica, 42, 375-386.*

Abramowitz, B.M., and Stegun, I.A. (1965). *Handbook of Mathematical Functions*, New York, Dover publications.

Adhikary, S.D. and Misra, J.C, (2011).Unsteady two dimensional hydromagnetic flow and heat transfer of a fluid. I*nternational Journal of Applied Mathematics and Mechanics, 7(4), 1-20.*

Agrawal, J.P. (1965). On Generalised Incompressible Couette Flow in Hydromagnetics.*Applied Science Research, Section B (9), 255-266.*

Al-odat, M.Q, Damseh, R.A, and Al-Nimr, M.A (2004). Effect of Magnetic field on

Entropy generation due to laminar forced convection past a horizontal flat plate. *Entropy,* 4(3), 293-303.

Attia, H.A and Kotb, N.A (1996) MHD flow between two parallel porous plates with heat transfer. *Acta Mechanica*, 117, 215-220.

Attia, H.A. (2005). The effects of Suction and injection on the unsteady flow between two Parallel plates with variable properties. *Tamkang Journal of science and engineering, 8*(1), 17-22.

Attia, H, A. (2008).Transient MHD flow between parallel porous plates with heat Transverse under exponential decaying pressure gradient and the ion-slip. *Kragujevac Jr. Sci., 30, 5-16.*

Baral, M.C.(1962).On hte unsteady flow of conducting liquid between two parallel plates. *Appl.Scient. Research* 35(A), 3, 422-427.

Barletta, A. and Celli, M. (2008).Mixed convection MHD flow in a vertical channel: Effects of Joule heating and viscous dissipation. *International Journal of heat and mass transfer* 51, 6110-6117.

Carslaw H.S. and Jaeger J.C. (1963). *Operational Method in applied mathematics*, 2nd Edition. Dover, New York.

Chamkha, A.J (1997). MHD free Convection from a vertical plate embedded in a Thermally stratified porous medium with Hall effects. *Applied Mathematical Modelling*, 21,603-609.

Chamkha, A. J. (2000). Flow of immiscible fluids in porous and Non-porous Channels. *Journal of fluids Engineering*, vol.122, 117-124.

Chandran P. Sacheti N.C. and Singh, A. K.(1993).Effects of Rotation on unsteady hydromagnetic Couette flow. *Astrophysics and space science*, 202, 1-10.

Chandran,P, Sacheti, N.C and Singh A.K.(2005).Natural Convection near a vertical plate with ramped wall temperature. *Heat and Mass transfer,* 41, 459-464.

Charles, M.E. and Lilleleht, L.U.(1965). *Canadian Jr. of Chem. Engineering* 43,110-119.

Chen, C-H.(2004). Combined heat and Mass transfer in MHD free convection from a

Vertical surface with Ohmic heating and viscous dissipation. *International Journal of Engineering,* 42, 699-713.

Chuang, H. (1971). Effects of transpiration on the MHD flow near an oscillating plate. *International Jr. of heat mass transfer, 15*, 562-563.

Cramer, K.R and Pai, S.I. (1973). *Magneto fluid dynamics for engineers and Applied Physicists,* McGraw-Hill, New York.

Das, S.S, (2009). Effect of suction and Injection on MHD three dimensional Couette flow and heat transfer through a porous medium.*Journal of Naval Architecture and Marine Engineering* 6: 41-50

DAS, S.S, Sahoo, S.K. and Dash, G.C. (2006).Numerical soln. of mass transfer effects of unsteady flow past an accelerated vertical porous plate with suction. *Bulletin of Malaysian Mathematical Science Society,* 2(29), 1, 33-42.

Deka, R.K.(2008).Hall effects on MHD flow past an accelerated plat. *Theoret. Appl. Mech., Belgrade,* 35(4), 333-346.

Deka, R.K., Gupta, A.S, Takbar, H.S, and Soundalkekar, V.M.(1999). Flow past an accelerated horizontal plate in a rotating fluid. *Acta Mechanica*, 138, 13-19.

Devi and Ganga (2010). Dissipation Effects on MHD Non-linear flow and heat transfer past a porous surface with prescribed heat flux. *Journal of Applied fluid Mechanics,* vol.3 (1), 1-6.

El-Amin, M.F.(2003). Combined effects of viscous dissipation and Joule heating on MHD forced convection over a non-isothermal horizontal cylinder embedded in a fluid saturated porous medium. *Journal of Magnetism and Magnetic Materials.* 26(3), 337-343.

Erdogan, M.E.(2000). A note on an unsteady flow of a viscous fluid due to an oscillating plane wall. *International Jr. of non-linear mechanics*, 35, 1-6.

Fang, T. (2004b). A note on the incompressible Couette flow with Porous walls. *Int. Comm.Heat Mass Transfer,* 31(1), 31- 41.

Fang, T. (2004c).Further note on the incompressible pressure-driven flow in a channel with porous walls. *Int. Comm. Heat Mass Transfer*, 31(4), 487-500.

Farzaneh-Gord, M., Joneidi, A.A and Haghighi, B.(2010). Investigating the effects of the important parameters on Magneto hydrodynamics flow and heat transfer over a stretching sheet. *Proceedings of the Institute of Mechanical Engineers, Part E: Journal of process of Mechanical Engineering*, 224, 1-9.

Guria M., Jana R.N. and Ghosh S.K.(2006). Unsteady Couette flow in a rotating system. *Int. Jr. of Non-linear Mechanics*, 41, 838-843.

Hartmann, L.(1937). Hg-Dynamics1. *Theory of laminar flow of an electrically Conductive Liquid in a homogeneous magnetic field*, K. Dan vidensk, Selsk. Math Fys. Medd. 15(1), 1-28.

Hashimoto, H. (1957).Boundary layer Growth on a flat plate with suction or injection. *Jr. of the physical society of Japan,* vol.12 (1), 68-72.

Hassanien, A. and Mansour, M.A.(1990). Unsteady magneto hydrodynamic flow through a porous medium between two parallel plates. *Astrophysics and space sciences*, 163, 241-246.

Hayat T., Fetecau, C.and Sajid M. (2008).On MHD transient flow of a Maxwell fluid

in a Porous medium and rotating frame..*Physics Letters A*, 372(10), 1639-1644.

Jat, R.N and Chaudhary, S. (2009). Magneto hydrodynamics boundary layer flow past a Porous substrate with Beavers –Joseph boundary condition. *Indian Journal of Pure and Applied Physics*, 47, 624-630.

Jha B. K.and Jibril H.M. (2010). Hydromagnetic flow due to ramped motion of the boundary. *Int.J.of Applied Mechanics and Engineering*, 15(14), 1101-1109.

Jha B. K. and Apere, C.A. (2010a). Unsteady MHD Couette flows in annuli: The Riemann-sum Approximation Approach. *J. Phys. Soc. Jpn*, 79(10), 124403-124405.

Jha, B. K. and Apere, C. A.(2010b).Combined Effect of Hall and ion slip Currents on unsteady MHD Couette flows in a rotating system. Jr. *Phys. Soc. Jpn*, 79(9), 104401-104409.

Katagiri M. (1962). Flow formation in Couette motion in Magneto-hydrodynamics. *Jr. of Physical Soc. of Japan*, 17(11) 393-401.

Kumar, B.R., Raghurahman, D.R.S and Muthucumaraswamy, R.(2002). Hydromagnetic flow and heat transfer on a continuously moving vertical surface. *Acta Mechanica*, 153, 249-253.

Kumaran, V., Kumar, A.V and Pop, I.(2010). Transition of MHD boundary layer flow past a stretching sheet. *Commun.Nonlinear Sci.Numer Simulation*, 15,300-311.

Lohrasbi, J. and Sahai,V.(1987). Magnetohydrodynamic heat transfer in two-phase Flow between parallel plates. *Appl. Sci. Research,* 45,53-66.

Lohrasbi, J. and Sahai, V.(1989). Magneto hydrodynamic heat transfer in two-phase flow between parallel plates. Appl.Sci.Research, 45, 53-78
Longwell P.A (1966). *Mechanics of Fluid Flow.* McGraw-Hill Company , New York.

Makinde, O.D and Onyejekwe, O.O.(2011). A numerical study of MHD generalized Couette flow and heat transfer with variable viscosity and electrical conductivity. *Journal of Magnetism and Magnetic Materials,* 323, 2757-2763.

Malashetty, M.S. and Lella, V.(1991). Magnetohydrodynamic heat transfer in two – fluid Flow.Proceedings *of AICHE and ASME27th National heat transfer Conference and Exposition on,* 28-31, July. 159-167.

Malashetty, M.S. and Lella, V.(1992). Magnetohydrodynamic heat transfer in two-Phase Flow.International *Journal of Engineering Science*, 30, 371-377.

Mccune J.E. and Sears W.R. (1959). On the concepts of moving electric and magnetic Fields in magneto-hydrodynamics. *Jr. Aero/Space Sci,*26, 674-675.

McLachlan N.W. (1947). *Complex Variable and Operational Calculus with Technical applications*. Camb. Univ. Press, London.

Ong, R.S.and Nicholls J.A.(1959). On the flow of hydromagnetic fluid near an oscillating flat Plate. *Journal of AeroSpace Science*, 26, 313-314.

Pal D. and Mondal, H.(2010). Hydromagnetic non-Darcy flow and heat transfer over a stretching sheet in the presence of thermal and ohmic dissipation. *Commun.Nonlinear Sci Numer simulate,* 15, 1197-1200.

Pantokratoras, A. (2007). Some new flows in a weakly conducting fluid with

exponentially decaying Lorentz force. *Mathematical Problems in Engineering*, 3, 1-14.

Panton R.L. (1996). *Incompressible Flow*, second edition, J. Wiley, New York.

Rossow, V. J. (1958).On flow of electrically conducting fluids over a flat in the presence of a transverse magnetic field. *NACA Report,* 1358.

Rossow, V.J. (1960). On Rayleigh's problem in Magnetohydrodynamics. *Physics of Fluids*, 3, 395-398.

Schlichting H. and Gerstein K. (2000). *Boundary Layer Theory*, 8th revised and enlarged edition (English), Springer- Verlag, New York.

Seddeek, M.A. and Salem, A.M. (2006).Further results on the variable viscosity with Magnetic field on the flow and heat transfer to a continuous moving flat plate. *Physics Letters A*, 353(4), 337-340.

Setayashi, A. and Sahai B.(1990). Heat transfer in developing Magneto hydrodynamic Poiseuille flow and variable transport properties. *Int. J. Heat and Mass transfer, 33(8),* 1711-1720.

Shail, R. (1973). On laminar two-phase flows in Magnetohydrodynamics. *Int. Jr. of Engineering Science,* 11, 1103-1108.

Shalybkov D.A. (2006).Hydrodynamic and hydromagnetic stability of the Couette flow. *Physiks-Uspekhi*, 52(9), 915-935.

Sharma, R. and Singh, G. (2009). Effects of ohmic heating and viscous dissipation on

steady MHD flow of viscous, incompressible and electrically conducting fluids near a stagnation point on an isothermal stretching sheet. *Thermal Science*, vol.13, 231-244

Singh, A.K. and Kumar, N. (1983). Unsteady Magneto-hydrodynamics Couette flow. *Wear*, 89,125.

Singh A.K., Sacheti N.C. and Chandran, P.(1994). Transient effects on magneto hydrodynamic Couette flow with rotation: Accelerated motion. *Int. Jr. Eng. Sciences,* 32,133-139.

Singh, A.K. (1988). Natural convection in unsteady coquette motion. *Def Sci J., No.1, January,* 35-41.

Singh, K.D.(2004). Influence of moving magnetic field on three dimensional Couette flow. *Zeitschrift Angewandte Mathematical Physics*, 55,894-902.

Stokes, G.G. (1851). On the effect of the internal friction of fluids on the motion of Pendulums. *Camb. Phil. Soc. Trans.*, IX, 8-106.

Sutto,n G.W. and Sherman, A.(1965). *Engineering Magneto-hydrodynamics*.1st Ed. McGraw-Hill. New York.

Turbatu, S. Buhler, K.R and Zierep, J.(1998). New solutions of the II. Stokes problem for an Oscillating flat plate. *Acta Mechanica, 1*29, 25-30.

Tsai, R., Huang, K.H. and Huang, J.S.(2009). The effects of Viscosity and thermal conductivity on heat transfer for hydromagnetic flow over a continuous moving porous plate with ohmic heating. *Applied Thermal Engineering*, 29, 1921-1926.

Umavathi, J.C, Abdul Mateen, Chamkha, A.J and Al-Mudhaf,A. A.(2006),Oscillatory Hartmann two-fluid flow and heat transfer in a horizontal Channel. *Int.J. of Applied Mechanics and Engineering*, 11(1), 155-178.

Umavathi, J.C, Chamkha, A.J, Mateen, A. and Al-Mudhaf, A.A.(2005). Unsteady Two-fluid flow and heat transfer in a horizontal Channel. *Heat mass transfer*, 42, 81-90.

Umavathi, J.C. (1996). A note on magnetoconvection in a vertical enclosure, *International Journal of Non-linear Mechanics,* 31(3), 371-376.

White, F.M. (1991).*Viscous Fluid Flow,* second edition, McGraw-Hill Company, New York.

Xenos, M., Kafoussias, N. and Karahalios, G.(2001). Magnetohydrodynamic Compressible laminar boundary layer adiabatic flow with adverse pressure gradient and continuous or localized transfer, *Canadian Journal of Physics, 79, 1247-1263*.

Zivojin, S.M. *et al*. (2010). MHD flow and heat transfer of two immiscible fluids between Moving Plates. *Transaction of the Canadian society for mechanical Engineering*, vol.34 (No.3-4), 351-372.

APPENDIX

The functional F_1, F_2, F_3 and F_4 used in chapter three and four to define the velocity and skin fiction are defined as:

$$F_1(X_1;X_2;X_3;X_4)=\frac{1.}{2.}\left[Exp(X_1.X_2)Erfc\left(\frac{X_1}{2\sqrt{X_3-X_4}}+X_2\sqrt{X_3-X_4}\right)\left[(X_3-X_4)+\frac{X_1}{2.X_2}\right]\right.$$

$$\left.+Exp(-X_1.X_2)Erfc\left(\frac{X_1}{2\sqrt{X_3-X_4}}-X_2\sqrt{X_3-X_4}\right)\left[(X_3-X_4)-\frac{X_1}{2.X_2}\right]\right]$$

$$F_2(X_1;X_2;X_3;X_4)=\frac{1.}{2.}\left[Exp(X_1.X_2)Erfc\left(\frac{X_1}{2\sqrt{X_3-X_4}}+X_2\sqrt{X_3-X_4}\right)\right.$$

$$\left.+Exp(-X_1.X_2)Erfc\left(\frac{X_1}{2\sqrt{X_3-X_4}}-X_2\sqrt{X_3-X_4}\right)\right]$$

$$F_3(X_1;X_2;X_3)=\left[X_1-X_1Erfc\left(X_1\sqrt{X_2-X_3}\right)+\frac{Exp\left(-X_1^2(X_2-X_3)\right)}{\sqrt{\pi(X_2-X_3)}}\right]$$

$$F_4(X_1;X_2;X_3)=\left[X_1(X_2-X_3)+\left(\frac{1}{2X_1}\right)+\sqrt{\frac{(X_2-X_3)}{\pi}}Exp\left(-X_1^2(X_2-X_3)\right)\right.$$

$$\left.-\left(X_1(X_2-X_3)+\left(\frac{1}{2X_1}\right)\right)+\sqrt{\frac{(X_2-X_3)}{\pi}}Exp\left(-X_1^2(X_2-X_3)\right)\right]$$

where $Erfc$ is complementary error function defined as:

$$Erfc(\eta)=\frac{2}{\sqrt{\pi}}\int_{\eta}^{\infty}Exp(-X^2)dX$$

APPENDIXII

Constants used to define temperature, velocity, Nusslt number and skin friction in 5.1

$$d = \sqrt{M^2 + \frac{\mathrm{Re}^2}{4}}$$

$$c = \frac{1}{4Exp(\mathrm{Re})\sinh^2(d)} \; ; \; c_1 = \frac{d}{2d + \mathrm{Re}(1 - \mathrm{Pr})} \; ;$$

$$c_2 = \frac{d}{2d - \mathrm{Re}(1 - \mathrm{Pr})}$$

$$A_1 = \frac{c_1\{Exp(\mathrm{Pr}\,\mathrm{Re}) - Exp(2d + \mathrm{Re})\} + c_2\{Exp(\mathrm{Pr}\,\mathrm{Re}) - Exp - (2d - \mathrm{Re})\} - 1}{Exp(\mathrm{Pr}\,\mathrm{Re}) - 1}$$

$$A_2 = \frac{c_1\{Exp(2d + \mathrm{Re}) - 1\} + c_2\{Exp(-(2d - \mathrm{Re})) - 1\} - 1}{Exp(\mathrm{Pr}\,\mathrm{Re}) - 1}$$

APPENDIXIII

Constants used to define temperature, velocity, Nusslt number and skin friction in 5.2

$$d = \sqrt{M^2 + \frac{\mathrm{Re}^2}{4}} \;; \qquad d_1 = \frac{1}{2Exp(\mathrm{Re}/2)\sinh(d)},$$

$$d_2 = \frac{1}{2Exp(d)\sinh(d)}$$

$$k_1 = \frac{a_1{}^2 d}{2d + \mathrm{Re}(1-\mathrm{Pr})} \;;\; k_2 = \frac{a_2{}^2 d}{2d - \mathrm{Re}(1-\mathrm{Pr})}$$

$$B_1 = \frac{k_1\{Exp(\mathrm{Pr}\,\mathrm{Re}) - Exp(2d + \mathrm{Re})\} + k_2\{Exp(\mathrm{Pr}\,\mathrm{Re}) - Exp - (2d - \mathrm{Re})\} - 1}{Exp(\mathrm{Pr}\,\mathrm{Re}) - 1}$$

$$B_2 = \frac{k_1\{Exp(2d + \mathrm{Re}) - 1\} + k_2\{Exp(-(2d - \mathrm{Re})) - 1\} - 1}{Exp(\mathrm{Pr}\,\mathrm{Re}) - 1}$$

$$a_1 = (R+1)d_1 - Rd_2 \;; \qquad a_2 = (R+1)d_1 - Rd_2 - R$$

APPENDIX VI

FORTRAN and MATLAB programme used to generate the numerical values for the analytical solutions

a. FORTRAN programme used to generate the numerical values for the velocity in problem 3.1

```
C       MHD COUETTE FLOW DUE TO RAMPED MOTION OF THE PLATE
C       Program to compute the velocity of the flow
          program abcd
         dimension ay(11),am(3),at(10),aa(2)
         open(6,file='rmhd1.out')
         t0=2.0
         h=1.0
         data am/0.5,5.0,10.0/
         data aa/0.0,1.0/
         data at/0.2,0.4,0.8,1.0,1.5,2.0,2.5,3.0,3.5,4.0/
         data
        ay/0.0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1.0/
         do 10 i=1,3
         do 20 j=1,2
         do 30 j1=1,10
         do 35 k=1,11
         v=am(i)
         a1=aa(j)
         t=at(j1)
         y=ay(k)
         v1=sqrt(v)
         sum=0.0
         n=0
 14      an=n
         a=2.*an*h+y
         b=2.*an*h+2.*h-y
         c=2.*an*h+h+y
         d=2.*an*h+h-y
         if (t .le.  t0) then
         x1=f1(a,v1,t,0.0)-f1(b,v1,t,0.0)
         x2=f1(a,v1,t,0.0)-f1(b,v1,t,0.0)
         x3=f1(d,v1,t,0.0)-f1(c,v1,t,0.0)
         x4=-a1*(x2+x3)
         x5=f2(a,v1,t,0.0)-f2(b,v1,t,0.0)
         x6=f2(d,v1,t,0.0)-f2(c,v1,t,0.0)
         x7=(a1/v)*(x5+x6)
         x71=2.*sqrt(t)
         x8=erfc(a/x71)-erfc(b/x71)
         x9=erfc(d/x71)-erfc(c/x71)
         x10=-(a1/v)*exp(-v*t)*(x8+x9)
         x11=(a1/t0)*(t+(exp(-v*t)/v)-(1./v))
```

```
      u=(x1+x4+x7+x10)
       sum=sum+u
      n=n+1
       if(abs(u) .gt. 0.000001) go to 14
      uu=(sum/t0)+x11
      else
      x1=f1(a,v1,t,0.0)-f1(b,v1,t,0.0)
      x2=f1(a,v1,t,0.0)-f1(b,v1,t,0.0)
      x3=f1(d,v1,t,0.0)-f1(c,v1,t,0.0)
      x4=-a1*(x2+x3)
      x5=f2(a,v1,t,0.0)-f2(b,v1,t,0.0)
      x6=f2(d,v1,t,0.0)-f2(c,v1,t,0.0)
      x7=(a1/v)*(x5+x6)
      x71=2.*sqrt(t)
      x8=erfc(a/x71)-erfc(b/x71)
      x9=erfc(d/x71)-erfc(c/x71)
      x10=-(a1/v)*exp(-v*t)*(x8+x9)
      x11=(a1/t0)*(t+(exp(-v*t)/v)-(1./v))
      x111=x1-(f1(a,v1,t,t0)-f1(b,v1,t,t0))
      x12=f1(a,v1,t,t0)-f1(b,v1,t,t0)
      x13=f1(d,v1,t,t0)-f1(c,v1,t,t0)
      x14=a1*(x12+x13)+x4
      x15=f2(a,v1,t,t0)-f2(b,v1,t,t0)
      x16=f2(d,v1,t,t0)-f2(c,v1,t,t0)
      x17=-(a1/v)*(x15+x16)+x7
      xx=2.*sqrt(t-t0)
      x18=erfc(a/xx)-erfc(b/xx)
      x19=erfc(d/xx)-erfc(c/xx)
      x20=(a1/v)*exp(-v*(t-t0))*(x18+x19)+x10
      x21=t-t0
      x22=-(a1/t0)*(x21+exp(-v*x21)*(1./v)-(1./v))
      u=(x111+x14+x17+x20)
       sum=sum+u
       n=n+1
       if(abs(u) .gt. 0.000001) go to 14
       uu=(sum/t0)+x11+x22
       endif
       x23=sinh(v1*(h-y))
       x24=sinh(h*v1)
       x25=sinh(v1*y)
       us=(1.-a1)*(x23/x24)+a1*(1.-(x25/x24))
       write(6,60)am(i),aa(j),at(j1),ay(k),uu,us
60     format(1x,4f6.2,2e14.5)
35    continue
30    continue
20    continue
10    continue
      stop
         end
          function f2(x1,x2,x3,x4)
          z1=exp(x1*x2)
```

```
            z2=erfc((x1/(2.*sqrt(x3-x4)))+x2*sqrt(x3-x4))
            z3=exp(-x1*x2)
            z4=erfc((x1/(2.*sqrt(x3-x4)))-x2*sqrt(x3-x4))
            f2=0.5*(z1*z2+z3*z4)
            return
          end
            function f1(x1,x2,x3,x4)
            z1=exp(x1*x2)*((x1*(0.5/x2))+(x3-x4))
            z2=erfc((x1/(2.*sqrt(x3-x4)))+x2*sqrt(x3-x4))
            z3=exp(-x1*x2)*((x3-x4)-(x1*(0.5/x2)))
            z4=erfc((x1/(2.*sqrt(x3-x4)))-x2*sqrt(x3-x4))
            f1=0.5*(z1*z2+z3*z4)
            return
           end
            function erfc(f)
          ac=abs(f)
          t1=1./(1.00+0.500*ac)
          erfc=t1*exp(-ac*ac-
        1.2655122300+t1*(1.0000236800+t1*(
        +0.3740919600+t1*(0.0967841800+t1*(-
        0.1862880600+t1*(
        +0.2788680700+t1*(-
        1.1352039800+t1*(1.4885158700+t1*(-
             0.822+1522300+t1*(0.170872700)))))))))
         if(f.lt.0.000) erfc=2.00-erfc
         return
         end

C      MHD COUETTE FLOW DUE TO RAMPED MOTION OF THE
        PLATE
C       Programme to calculate the skin friction of the
        plate
C       SKINFRICTION AT Y=0
        program abcd
        dimension am(3),at(10),aa(2)
        open(6,file='rmhd3.out')
        t0=2.0
        h=1.0
        data am/0.5,5.0,10.0/
        data aa/0.0,1.0/
        data at/0.2,0.4,0.8,1.0,1.5,2.0,2.5,3.0,3.5,4.0/
        do 10 i=1,3
        do 20 j=1,2
        do 30 j1=1,10
        v=am(i)
        vv=aa(j)
        t=at(j1)
        v1=sqrt(v)
        sum=0.0
       n=0
 14     an=n
```

```
      a1=2.*an*h
      b1=2.*an*h+2.*h
      c1=2.*an*h+h
      if (t .le.  t0) then
      x1=-(f3(a1,v1,t,0.0)+f3(b1,v1,t,0.0))
      x2=f3(a1,v1,t,0.0)+f3(b1,v1,t,0.0)-
      2.*f3(c1,v1,t,0.0)
      x3=f4(a1,v1,t,0.0)+f4(b1,v1,t,0.0)-
      2.*f4(c1,v1,t,0.0)
      x41=f4(a1,0.0,t,0.0)
      x42=f4(b1,0.0,t,0.0)-2.*f4(c1,0.0,t,0.0)
      x4=exp(-v*t)*(x41+x42)
      x5=x1+vv*x2-(vv/v)*(x3-x4)
       sum=sum+x5
      n=n+1
       if(abs(x5) .gt. 0.000001) go to 14
      tau=sum/t0
      else
      x1=-(f3(a1,v1,t,0.0)+f3(b1,v1,t,0.0))
      x2=f3(a1,v1,t,0.0)+f3(b1,v1,t,0.0)-
      2.*f3(c1,v1,t,0.0)
      x3=f4(a1,v1,t,0.0)+f4(b1,v1,t,0.0)-
      2.*f4(c1,v1,t,0.0)
      x41=f4(a1,0.0,t,0.0)
      x42=f4(b1,0.0,t,0.0)-2.*f4(c1,0.0,t,0.0)
      x4=exp(-v*t)*(x41+x42)
      x6=f3(a1,v1,t,t0)+f3(b1,v1,t,t0)
       x7=f3(a1,v1,t,t0)+f3(b1,v1,t,t0)-
       2.*f3(c1,v1,t,t0)
       x8=f4(a1,v1,t,t0)+f4(b1,v1,t,t0)-
       2.*f4(c1,v1,t,t0)
      x9=f4(a1,0.0,t,t0)+f4(b1,0.0,t,t0)-
      2.*f4(c1,0.0,t,t0)
      x10=exp(-v*(t-t0))*x9
      x11=x1+x6+vv*(x2-x7)-(vv/v)*(x3-x4-x8+x10)
       sum=sum+x11
       n=n+1
       if(abs(x11) .gt. 0.000001) go to 14
       tau=(sum/t0)
       endif
       x12=vv*v1+v1*(1-vv)*cosh(h*v1)
       ts=x12/(sinh(h*v1))
       write(6,60)am(i),aa(j),at(j1),tau,ts
60     format(1x,3f6.2,2e14.5)
30    continue
20    continue
10    continue
      stop
         end
          function f4(x1,x2,x3,x4)
          p1=4.*atan(1.)
```

```
      z1=x2*exp(x1*x2)
      z2=erfc((x1/(2.*sqrt(x3-x4)))+x2*sqrt(x3-x4))
      z3=-x2*exp(-x1*x2)
      z4=erfc((x1/(2.*sqrt(x3-x4)))-x2*sqrt(x3-x4))
      z5=-(x1*x1+4.*x2*x2*(x3-x4)**2)/(4.*(x3-x4))
      z6=-2./(sqrt(p1*(x3-x4)))
      z7=z6*exp(z5)
      f4=0.5*(z1*z2+z3*z4+z7)
      return
    end
      function f3(x1,x2,x3,x4)
      p1=4.*atan(1.)
      z1=x2*exp(x1*x2)*((x1*(0.5/x2))+(x3-x4))
      z2=erfc((x1/(2.*sqrt(x3-x4)))+x2*sqrt(x3-x4))
      z3=-x2*exp(-x1*x2)*((x3-x4)-(x1*(0.5/x2)))
      z4=erfc((x1/(2.*sqrt(x3-x4)))-x2*sqrt(x3-x4))
      z5=erfc((x1/(2.*sqrt(x3-x4)))+x2*sqrt(x3-x4))
      z6=erfc((x1/(2.*sqrt(x3-x4)))-x2*sqrt(x3-x4))
      z7=exp(x1*x2)/(2.*x2)
      z8=-exp(-x1*x2)/(2.*x2)
      z9=z5*z7+z6*z8
      z10=-(x1*x1+4.*x2*x2*(x3-x4)**2)/(4.*(x3-x4))
      z11=-2.*sqrt((x3-x4)/p1)
      z12=z11*exp(z10)
      f3=0.5*(z12+z9+z1*z2+z3*z4)
      return
      end
      function erfc(f)
      ac=abs(f)
     t1=1./(1.00+0.500*ac)
     erfc=t1*exp(-ac*ac-
     1.2655122300+t1*(1.0000236800+t1*(
     +0.3740919600+t1*(0.0967841800+t1*(-
     0.1862880600+t1*(
     +0.2788680700+t1*(-
     1.1352039800+t1*(1.4885158700+t1*(-     0.822
     +1522300+t1*(0.170872700))))))))))
      if(f.lt.0.000) erfc=2.00-erfc
     return
     end
```

b. FORTRAN Programme used to generate the numerical values for the velocity in problem 3.2

```
C      MHD  COUETTE  FLOW  DUE  TO  RAMPED  MOTION  OF  THE
       POROUS PLATE
C      Programme to velocity of the porous plate
       program abcd
       dimension ay(11),as(3),am(3),at(10),aa(2)
       open(6,file='rmhd2.out')
       t0=2.0
```

```
      h=1.0
      data am/0.5,5.0,10.0/
      data as/5.0,0.0,-5.0/
      data aa/0.0,1.0/
      data at/0.2,0.4,0.8,1.0,1.5,2.0,2.5,3.0,3.5,4.0/
      data
        ay/0.0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1.0/
      do 5  i1=1,3
      do 10 i=1,3
      do 20 j=1,2
      do 30 j1=1,10
      do 35 k=1,11
      v=am(i1)
      s=as(i)
      a1=aa(j)
      t=at(j1)
      y=ay(k)
      sum=0.0
     n=0
14    an=n
      a=2.*an*h+y
      b=2.*an*h+2.*h-y
      c=2.*an*h+h+y
      d=2.*an*h+h-y
      g=(s*s/4.)+v
      g1=sqrt(g)
      z1=exp(s*y*0.5)
      z2=exp(-s*h*0.5)
      if (t .le.  t0) then
      x1=f1(a,g1,t,0.0)-f1(b,g1,t,0.0)
      x2=f1(a,g1,t,0.0)-f1(b,g1,t,0.0)
      x3=f1(d,g1,t,0.0)-f1(c,g1,t,0.0)
      x4=-a1*(x2+x3*z2)
      x5=f2(a,g1,t,0.0)-f2(b,g1,t,0.0)
      x6=f2(d,g1,t,0.0)-f2(c,g1,t,0.0)
      x7=(a1/v)*(x5+x6*z2)
      x72=0.5*s
      x8=f2(a,x72,t,0.0)-f2(b,x72,t,0.0)
      x9=f2(d,x72,t,0.0)-f2(c,x72,t,0.0)
      x10=-(a1/v)*exp(-v*t)*(x8+x9*z2)
      x11=(a1/t0)*(t+(exp(-v*t)/v)-(1./v))
      u=(x1+x4+x7+x10)
       sum=sum+u
      n=n+1
       if(abs(u) .gt. 0.000001) go to 14
      uu=z1*(sum/t0)+x11
      else
      x1=f1(a,g1,t,0.0)-f1(b,g1,t,0.0)
      x2=f1(a,g1,t,0.0)-f1(b,g1,t,0.0)
      x3=f1(d,g1,t,0.0)-f1(c,g1,t,0.0)
      x4=-a1*(x2+x3*z2)
```

```
      x5=f2(a,g1,t,0.0)-f2(b,g1,t,0.0)
      x6=f2(d,g1,t,0.0)-f2(c,g1,t,0.0)
      x7=(a1/v)*(x5+x6*z2)
      x72=0.5*s
      x8=f2(a,x72,t,0.0)-f2(b,x72,t,0.0)
      x9=f2(d,x72,t,0.0)-f2(c,x72,t,0.0)
      x10=-(a1/v)*exp(-v*t)*(x8+x9*z2)
      x11=(a1/t0)*(t+(exp(-v*t)/v)-(1./v))
      x111=x1-(f1(a,g1,t,t0)-f1(b,g1,t,t0))
      x12=f1(a,g1,t,t0)-f1(b,g1,t,t0)
      x13=f1(d,g1,t,t0)-f1(c,g1,t,t0)
      x14=a1*(x12+x13*z2)+x4
      x15=f2(a,g1,t,t0)-f2(b,g1,t,t0)
      x16=f2(d,g1,t,t0)-f2(c,g1,t,t0)
      x17=-(a1/v)*(x15+x16*z2)+x7
      x18=f2(a,x72,t,t0)-f2(b,x72,t,t0)
      x19=f2(d,x72,t,t0)-f2(c,x72,t,t0)
      x20=(a1/v)*exp(-v*(t-t0))*(x18+x19*z2)+x10
      x21=t-t0
      x22=-(a1/t0)*(x21+exp(-v*x21)*(1./v)-(1./v))
      u=(x111+x14+x17+x20)
       sum=sum+u
       n=n+1
       if(abs(u) .gt. 0.000001) go to 14
       uu=z1*(sum/t0)+x11+x22
       endif
       x23=sinh(g1*(h-y))
       x24=sinh(h*g1)
       x25=sinh(g1*y)
       x26=z1*(1-a1)*(x23/x24)
       x27=1-((z1*z2*x25)/x24)
       us=x26+a1*x27
       write(6,60)am(i1),as(i),aa(j),at(j1),ay(k),uu,us
60     format(1x,5f6.2,2e14.5)
35     continue
30     continue
20     continue
10     continue
5      continue
      stop
        end
         function f2(x1,x2,x3,x4)
         z1=exp(x1*x2)
         z2=erfc((x1/(2.*sqrt(x3-x4)))+x2*sqrt(x3-x4))
         z3=exp(-x1*x2)
         z4=erfc((x1/(2.*sqrt(x3-x4)))-x2*sqrt(x3-x4))
         f2=0.5*(z1*z2+z3*z4)
         return
        end
         function f1(x1,x2,x3,x4)
         z1=exp(x1*x2)*((x1*(0.5/x2))+(x3-x4))
```

```
          z2=erfc((x1/(2.*sqrt(x3-x4)))+x2*sqrt(x3-x4))
          z3=exp(-x1*x2)*((x3-x4)-(x1*(0.5/x2)))
          z4=erfc((x1/(2.*sqrt(x3-x4)))-x2*sqrt(x3-x4))
          f1=0.5*(z1*z2+z3*z4)
          return
          end
          function erfc(f)
          ac=abs(f)
          t1=1./(1.00+0.500*ac)
         erfc=t1*exp(-ac*ac-
         1.2655122300+t1*(1.0000236800+t1*(
         +0.3740919600+t1*(0.0967841800+t1*(-
         0.1862880600+t1*(+0.2788680700+t1*(-
         1.1352039800+t1*(1.4885158700+t1*(-0.822
         +1522300+t1*(0.170872700)))))))))
         if(f.lt.0.000) erfc=2.00-erfc
         return
         end

C     MHD COUETTE FLOW DUE TO RAMPED MOTION OF THE POROUS
      PLATE
C     Programme to compute skin friction of the porous
      plate
C     SKINFRICTION AT Y=H
      program abcd
      dimension am(3),at(10),aa(2)
        open(6,file='rmhd4.out')
        t0=2.0
        h=1.0
        data am/0.5,5.0,10.0/
        data aa/0.0,1.0/
        data at/0.2,0.4,0.8,1.0,1.5,2.0,2.5,3.0,3.5,4.0/
        do 10 i=1,3
        do 20 j=1,2
        do 30 j1=1,10
        v=am(i)
        vv=aa(j)
        t=at(j1)
        v1=sqrt(v)
        sum=0.0
       n=0
 14     an=n
        a1=2.*an*h
        b1=2.*an*h+2.*h
        c1=2.*an*h+h
        if (t .le.  t0) then
        x1=2.*f3(c1,v1,t,0.0)
        x2=2.*f3(c1,v1,t,0.0)-f3(b1,v1,t,0.0)-
        f3(a1,v1,t,0.0)
        x3=2.*f4(c1,v1,t,0.0)-f4(b1,v1,t,0.0)-
        f4(a1,v1,t,0.0)
```

```
      x41=2.*f4(c1,0.0,t,0.0)-f4(b1,0.0,t,0.0)-
           f4(a1,0.0,t,0.0)
      x4=exp(-v*t)*x41
      x5=x1-vv*x2+(vv/v)*(x3-x4)
       sum=sum+x5
      n=n+1
       if(abs(x5) .gt. 0.000001) go to 14
      tau=sum/t0
      else
      x1=2.*f3(c1,v1,t,0.0)
      x2=2.*f3(c1,v1,t,0.0)-f3(b1,v1,t,0.0)-
      f3(a1,v1,t,0.0)
      x3=2.*f4(c1,v1,t,0.0)-f4(b1,v1,t,0.0)-
      f4(a1,v1,t,0.0)
      x41=2.*f4(c1,0.0,t,0.0)-f4(b1,0.0,t,0.0)-
           f4(a1,0.0,t,0.0)
      x4=exp(-v*t)*x41
      x6=-2.*f3(c1,v1,t,t0)
       x7=2.*f3(c1,v1,t,t0)-f3(b1,v1,t,t0)-
       f3(a1,v1,t,t0)
       x8=2.*f4(c1,v1,t,t0)-f4(b1,v1,t,t0)-
       f4(a1,v1,t,t0)
      x9=2.*f4(c1,0.0,t,t0)-f4(b1,0.0,t,t0)-
      f4(a1,0.0,t,t0)
      x10=exp(-v*(t-t0))*x9
      x11=x1+x6+vv*(x7-x2)+(vv/v)*(x3-x8+x10-x4)
       sum=sum+x11
       n=n+1
       if(abs(x11) .gt. 0.000001) go to 14
       tau=-(sum/t0)
       endif
       x12=(v1*(1-vv)+vv*v1*(cosh(v1*h)))
       ts=x12/(sinh(h*v1))
       write(6,60)am(i),aa(j),at(j1),tau,ts
60     format(1x,3f6.2,2e14.5)
30     continue
20     continue
10     continue
       stop
        end
         function f4(x1,x2,x3,x4)
         p1=4.*atan(1.)
         z1=x2*exp(x1*x2)
         z2=erfc((x1/(2.*sqrt(x3-x4)))+x2*sqrt(x3-x4))
         z3=-x2*exp(-x1*x2)
         z4=erfc((x1/(2.*sqrt(x3-x4)))-x2*sqrt(x3-x4))
         z5=-(x1*x1+4.*x2*x2*(x3-x4)**2)/(4.*(x3-x4))
         z6=-2./(sqrt(p1*(x3-x4)))
         z7=z6*exp(z5)
         f4=0.5*(z1*z2+z3*z4+z7)
         return
```

```
      end
       function f3(x1,x2,x3,x4)
       p1=4.*atan(1.)
       z1=x2*exp(x1*x2)*((x1*(0.5/x2))+(x3-x4))
       z2=erfc((x1/(2.*sqrt(x3-x4)))+x2*sqrt(x3-x4))
       z3=-x2*exp(-x1*x2)*((x3-x4)-(x1*(0.5/x2)))
       z4=erfc((x1/(2.*sqrt(x3-x4)))-x2*sqrt(x3-x4))
       z5=erfc((x1/(2.*sqrt(x3-x4)))+x2*sqrt(x3-x4))
       z6=erfc((x1/(2.*sqrt(x3-x4)))-x2*sqrt(x3-x4))
       z7=exp(x1*x2)/(2.*x2)
       z8=-exp(-x1*x2)/(2.*x2)
       z9=z5*z7+z6*z8
       z10=-(x1*x1+4.*x2*x2*(x3-x4)**2)/(4.*(x3-x4))
       z11=-2.*sqrt((x3-x4)/p1)
       z12=z11*exp(z10)
       f3=0.5*(z12+z9+z1*z2+z3*z4)
       return
      end
       function erfc(f)
      ac=abs(f)
      t1=1./(1.00+0.500*ac)
      erfc=t1*exp(-ac*ac-
      1.2655122300+t1*(1.0000236800+t1*(
      +0.3740919600+t1*(0.0967841800+t1*(-
      0.1862880600+t1*(+0.2788680700+t1*(-
       1.1352039800+t1*(1.4885158700+t1*(-0.822
      +1522300+t1*(0.170872700)))))))))
       if(f.lt.0.000) erfc=2.00-erfc
       return
       end
```

c. FORTRAN programme to generate the numerical values of the velocity in problem 4.1

```
C       MHD COUETTE FLOW BETWEEN TWO PARALLEL PLATES DUE
        TO RAMPED   MOTION OF ONE THE PLATE
C       Program to compute the ramped velocity of the
        fluid
C       MAGNETIC FIELD FIXED WITH FLUID OR PLATE
        program abcd
        dimension ay(11),at(10),am(2),aa(2)
        open(6,file='rramp41.out')
        t0=2.0
C        a=0.0
        data aa/0.0,1.0/
        data am/0.5,10.0/
        data at/0.2,0.4,0.8,1.0,1.5,2.0,2.5,3.0,3.5,4.0/
      dataay/0.0,0.2,0.4,0.6,0.8,1.0,1.2,1.4,1.6,1.8,2.0/
        do 20 i=1,2
        do 25 j=1,2
```

```
      do 30 j1=1,10
      do 35 k=1,11
      a=aa(i)
      v=am(j)
      t=at(j1)
      y=ay(k)
      z1=(1.-a)/t0
      z2=a/(t0*v)
      z3=(a/t0)*(t+(exp(-v*t)/v)-(1./v))
      v1=sqrt(v)
      if (t .le.  t0) then
      u1=z1*f1(y,v1,t,0.0)
      u2=z2*(f2(y,v1,t,0.0)-exp(-v*t)*f2(y,0.0,t,0.0))
      +z3
      u=u1+u2
      else
      u1=z1*(f1(y,v1,t,0.0)-f1(y,v1,t,t0))
      u2=z2*(f2(y,v1,t,0.0)-exp(-v*t)*f2(y,0.0,t,0.0))
      u3=z2*(exp(-v*(t-t0))*f2(y,0.0,t,t0)-
      f2(y,v1,t,t0))
      u4=(a/t0)*((1./v)-(t-t0)-(1./v)*exp(-v*(t-
      t0)))+z3
      u=u1+u2+u3+u4
      endif
      tt=t/t0
C     IMPULSIVE MOTION OF THE PLATE
      x=y/(2.*sqrt(t))
      ui=(1-a)*f2(y,v1,t,0.0)+a+a*exp(-v*t)*(erfc(x)1.)
      write(6,60)aa(i),am(j),at(j1),ay(k),u,tt,ui
 60   format(1x,4f8.3,3f10.5)
 35   continue
 30   continue
 25   continue
 20   continue
     stop
        end
         function f1(d1,d2,d3,d4)
         z1=exp(-d1*d2)*((d3-d4)-0.5*(d1/d2))
         z2=erfc(d1*0.5/sqrt(d3-d4)-d2*sqrt(d3-d4))
         z3=exp(d1*d2)*((d3-d4)+0.5*(d1/d2))
         z4=erfc(d1*0.5/sqrt(d3-d4)+d2*sqrt(d3-d4))
         f1=0.5*(z1*z2+z3*z4)
         return
      end
         function f2(d1,d2,d3,d4)
         z1=exp(-d1*d2)
         z2=erfc(d1*0.5/sqrt(d3-d4)-d2*sqrt(d3-d4))
         z3=exp(d1*d2)
         z4=erfc(d1*0.5/sqrt(d3-d4)+d2*sqrt(d3-d4))
         f2=0.5*(z1*z2+z3*z4)
         return
```

```
      end

      function erfc(f)
      ac=abs(f)
      t1=1./(1.00+0.500*ac)
      erfc=t1*exp(-ac*ac-
      1.2655122300+t1*(1.0000236800+t1*(
      +0.3740919600+t1*(0.0967841800+t1*(-
      0.1862880600+t1*(
      +0.2788680700+t1*(-
      1.1352039800+t1*(1.4885158700+t1*(-       0.822
     +1522300+t1*(0.170872700))))))))))
     if(f.lt.0.000) erfc=2.00-erfc
     return
     end

C    MHD COUETTE FLOW BETWEEN TWO PARALLEL PLATES DUE
     TO RAMPED MOTION OF ONE THE PLATE
C    Programme to compute the skin friction at the
     lower plate
C    SKIN FRICTION(Y=0.0)
     program abcd
      dimension as(3),am(3),at(10),aa(2)
      open(6,file='rmhd5.out')
      t0=2.0
      h=1.0
      data as/5.0,0.0,-5.0/
      data am/0.5,5.0,10.0/
      data aa/0.0,1.0/
      data at/0.2,0.4,0.8,1.0,1.5,2.0,2.5,3.0,3.5,4.0/
      do 5  i1=1,3
      do 10 i=1,3
      do 20 j=1,2
      do 30 j1=1,10
      s=as(i1)
      v=am(i)
      a1=aa(j)
      t=at(j1)
      sum=0.0
     n=0
 14   an=n
      b1=2.*an*h
      b2=2.*an*h+2.*h
      b3=2.*an*h+h
      g2=s*0.5
      g=(s*s/4.)+v
      g1=sqrt(g)
      z2=exp(-s*h*0.5)
      if (t .le.  t0) then
      x1=0.5*s*f1(b1,g1,t,0.0)+f3(b1,g1,t,0.0)
      x2=-0.5*s*f1(b2,g1,t,0.0)+f3(b2,g1,t,0.0)
```

```
x3=0.5*s*f1(b1,g1,t,0.0)+f3(b1,g1,t,0.0)
x4=-0.5*s*f1(b2,g1,t,0.0)+f3(b2,g1,t,0.0)
x5=0.5*s*f1(b3,g1,t,0.0)+f3(b3,g1,t,0.0)
x6=-0.5*s*f1(b3,g1,t,0.0)+f3(b3,g1,t,0.0)
x7=x1+x2-a1*(x3+x4-z2*(x5+x6))
x8=0.5*s*f2(b1,g1,t,0.0)+f4(b1,g1,t,0.0)
x9=-0.5*s*f2(b2,g1,t,0.0)+f4(b2,g1,t,0.0)
x10=0.5*s*f2(b3,g1,t,0.0)+f4(b3,g1,t,0.0)
x11=-0.5*s*f2(b3,g1,t,0.0)+f4(b3,g1,t,0.0)
x12=(a1/v)*(x8+x9-z2*(x10+x11))
g2=0.5*s
x13=0.5*s*f2(b1,g2,t,0.0)+f4(b1,g2,t,0.0)
x14=-0.5*s*f2(b2,g2,t,0.0)+f4(b2,g2,t,0.0)
x15=0.5*s*f2(b3,g2,t,0.0)+f4(b3,g2,t,0.0)
x16=-0.5*s*f2(b3,g2,t,0.0)+f4(b3,g2,t,0.0)
x17=-(a1/v)*exp(-v*t)*(x13+x14-z2*(x15+x16))
tou=(x7+x12+x17)
 sum=sum+tou
n=n+1
 if(abs(tou) .gt. 0.000001) go to 14
tou0=(sum/t0)
else
x1=0.5*s*f1(b1,g1,t,0.0)+f3(b1,g1,t,0.0)
x2=-0.5*s*f1(b2,g1,t,0.0)+f3(b2,g1,t,0.0)
x3=0.5*s*f1(b1,g1,t,0.0)+f3(b1,g1,t,0.0)
x4=-0.5*s*f1(b2,g1,t,0.0)+f3(b2,g1,t,0.0)
x5=0.5*s*f1(b3,g1,t,0.0)+f3(b3,g1,t,0.0)
x6=-0.5*s*f1(b3,g1,t,0.0)+f3(b3,g1,t,0.0)
x8=0.5*s*f2(b1,g1,t,0.0)+f4(b1,g1,t,0.0)
x9=-0.5*s*f2(b2,g1,t,0.0)+f4(b2,g1,t,0.0)
x10=0.5*s*f2(b3,g1,t,0.0)+f4(b3,g1,t,0.0)
x11=-0.5*s*f2(b3,g1,t,0.0)+f4(b3,g1,t,0.0)
g2=0.5*s
x13=0.5*s*f2(b1,g2,t,0.0)+f4(b1,g2,t,0.0)
x14=-0.5*s*f2(b2,g2,t,0.0)+f4(b2,g2,t,0.0)
x15=0.5*s*f2(b3,g2,t,0.0)+f4(b3,g2,t,0.0)
x16=-0.5*s*f2(b3,g2,t,0.0)+f4(b3,g2,t,0.0)
x181=0.5*s*f1(b1,g1,t,t0)+f3(b1,g1,t,t0)
x182=-0.5*s*f1(b2,g1,t,t0)+f3(b2,g1,t,t0)
zz1=x1+x2-(x181+x182)
x183=0.5*s*f1(b1,g1,t,t0)+f3(b1,g1,t,t0)
x184=-0.5*s*f1(b2,g1,t,t0)+f3(b2,g1,t,t0)
x185=0.5*s*f1(b3,g1,t,t0)+f3(b3,g1,t,t0)
x186=-0.5*s*f1(b3,g1,t,t0)+f3(b3,g1,t,t0)
zz2=-a1*(x3+x4-z2*(x5+x6))+a1*(x183+x184-
z2*(x185+x186))
x188=0.5*s*f2(b1,g1,t,t0)+f4(b1,g1,t,t0)
x189=-0.5*s*f2(b2,g1,t,t0)+f4(b2,g1,t,t0)
x1810=0.5*s*f2(b3,g1,t,t0)+f4(b3,g1,t,t0)
x1811=-0.5*s*f2(b3,g1,t,t0)+f4(b3,g1,t,t0)
```

```
      zz3=(a1/v)*(x8+x9-z2*(x10+x11)-(x188+x189-
           z2*(x1810+x1811)))
      x1813=0.5*s*f2(b1,g2,t,t0)+f4(b1,g2,t,t0)
      x1814=-0.5*s*f2(b2,g2,t,t0)+f4(b2,g2,t,t0)
      x1815=0.5*s*f2(b3,g2,t,t0)+f4(b3,g2,t,t0)
      x1816=-0.5*s*f2(b3,g2,t,t0)+f4(b3,g2,t,t0)
      x1817=(a1/v)*exp(-v*(t-t0))*(x1813+x1814-
           z2*(x1815+x1816))
      x1818=-(a1/v)*exp(-v*t)*(x13+x14-z2*(x15+x16))
      zz4=x1817+x1818
      tou=zz1+zz2+zz3+zz4
       sum=sum+tou
       n=n+1
       if(abs(tou) .gt. 0.000001) go to 14
       tou0=-(sum/t0)
       endif
       y1=g2*(1-a1)*sinh(g1*h)
       y2=-g1*(1-a1)*cosh(g1*h)
       y3=-a1*g1*z2
       write(*,*)y3
       ts=-(y1+y2+y3)/sinh(g1*h)
       write(6,60)as(i1),am(i),aa(j),at(j1),tou0,ts
60     format(1x,4f6.2,2e14.5)
30     continue
20     continue
10     continue
5      continue
       stop
        end
         function f2(x1,x2,x3,x4)
         z1=exp(x1*x2)
         z2=erfc((x1/(2.*sqrt(x3-x4)))+x2*sqrt(x3-x4))
         z3=exp(-x1*x2)
         z4=erfc((x1/(2.*sqrt(x3-x4)))-x2*sqrt(x3-x4))
         f2=0.5*(z1*z2+z3*z4)
         return
        end
         function f1(x1,x2,x3,x4)
         z1=exp(x1*x2)*((x1*(0.5/x2))+(x3-x4))
         z2=erfc((x1/(2.*sqrt(x3-x4)))+x2*sqrt(x3-x4))
         z3=exp(-x1*x2)*((x3-x4)-(x1*(0.5/x2)))
         z4=erfc((x1/(2.*sqrt(x3-x4)))-x2*sqrt(x3-x4))
         f1=0.5*(z1*z2+z3*z4)
         return
         end
         function f4(x1,x2,x3,x4)
         p1=4.*atan(1.)
         z1=x2*exp(x1*x2)
         z2=erfc((x1/(2.*sqrt(x3-x4)))+x2*sqrt(x3-x4))
         z3=-x2*exp(-x1*x2)
         z4=erfc((x1/(2.*sqrt(x3-x4)))-x2*sqrt(x3-x4))
```

```
          z5=-(x1*x1+4.*x2*x2*(x3-x4)**2)/(4.*(x3-x4))
          z6=-2./(sqrt(p1*(x3-x4)))
          z7=z6*exp(z5)
          f4=0.5*(z1*z2+z3*z4+z7)
          return
          end
          function f3(x1,x2,x3,x4)
          p1=4.*atan(1.)
          z1=x2*exp(x1*x2)*((x1*(0.5/x2))+(x3-x4))
          z2=erfc((x1/(2.*sqrt(x3-x4)))+x2*sqrt(x3-x4))
          z3=-x2*exp(-x1*x2)*((x3-x4)-(x1*(0.5/x2)))
          z4=erfc((x1/(2.*sqrt(x3-x4)))-x2*sqrt(x3-x4))
          z5=erfc((x1/(2.*sqrt(x3-x4)))+x2*sqrt(x3-x4))
          z6=erfc((x1/(2.*sqrt(x3-x4)))-x2*sqrt(x3-x4))
          z7=exp(x1*x2)/(2.*x2)
          z8=-exp(-x1*x2)/(2.*x2)
          z9=z5*z7+z6*z8
          z10=-(x1*x1+4.*x2*x2*(x3-x4)**2)/(4.*(x3-x4))
          z11=-2.*sqrt((x3-x4)/p1)
          z12=z11*exp(z10)
          f3=0.5*(z12+z9+z1*z2+z3*z4)
          return
         end

         function erfc(f)
         ac=abs(f)
         t1=1./(1.00+0.500*ac)
         erfc=t1*exp(-ac*ac-
         1.2655122300+t1*(1.0000236800+t1*(
         +0.3740919600+t1*(0.0967841800+t1*(-
         0.1862880600+t1*(
         +0.2788680700+t1*(-
         1.1352039800+t1*(1.4885158700+t1*(-
         0.822+1522300+t1*(0.170872700)))))))))
         if(f.lt.0.000) erfc=2.00-erfc
       return
       end

C      MHD COUETTE FLOW BETWEEN TWO PARALLEL PLATES DUE
       TO RAMPED    MOTION OF ONE THE PLATES
C      Programme to compute skin friction at the
       upper plate.
C       SKINFRICTION(Y=h)
        program abcd
        dimension as(3),am(3),at(10),aa(2)
        open(6,file='rmhd6.out')
        t0=2.0
        h=1.0
        data as/5.0,0.0,-5.0/
C        data as/2.0,0.0,-2.0/
        data am/0.5,5.0,10.0/
```

```
      data aa/0.0,1.0/
      data at/0.2,0.4,0.8,1.0,1.5,2.0,2.5,3.0,3.5,4.0/
      do 5  i1=1,3
      do 10 i=1,3
      do 20 j=1,2
      do 30 j1=1,10
      s=as(i1)
      v=am(i)
      a1=aa(j)
      t=at(j1)
      sum=0.0
      n=0
14    an=n
      b1=2.*an*h+h
      b3=2.*an*h+2.*h
      b4=2.*an*h
      g2=s*0.5
      g=(s*s/4.)+v
      g1=sqrt(g)
      z2=exp(-s*h*0.5)
      if (t .le.  t0) then
      x1=2.*f3(b1,g1,t,0.0)
      x2=2.*f3(b1,g1,t,0.0)
      x3=0.5*s*f1(b3,g1,t,0.0)+f3(b3,g1,t,0.0)
      x4=-0.5*s*f1(b4,g1,t,0.0)+f3(b4,g1,t,0.0)
      x5=x1-a1*(x2-z2*(x3+x4))
      x6=2.*f4(b1,g1,t,0.0)
      x7=0.5*s*f2(b3,g1,t,0.0)+f4(b3,g1,t,0.0)
      x8=-0.5*s*f2(b4,g1,t,0.0)+f4(b4,g1,t,0.0)
      x9=(a1/v)*(x6-z2*(x7+x8))
      g2=0.5*s
      x10=2.*f4(b1,g2,t,0.0)
      x11=0.5*s*f2(b3,g2,t,0.0)+f4(b3,g2,t,0.0)
      x12=-0.5*s*f2(b4,g2,t,0.0)+f4(b4,g2,t,0.0)
      x13=-(a1/v)*exp(-v*t)*(x10-z2*(x11+x12))
      tou=(x5+x9+x13)
       sum=sum+tou
       n=n+1
       if(abs(tou) .gt. 0.000001) go to 14
      tou0=exp(s*0.5*h)*(sum/t0)
      else
      x1=2.*f3(b1,g1,t,0.0)
      x2=2.*f3(b1,g1,t,0.0)
      x3=0.5*s*f1(b3,g1,t,0.0)+f3(b3,g1,t,0.0)
      x4=-0.5*s*f1(b4,g1,t,0.0)+f3(b4,g1,t,0.0)
      x5=x1-a1*(x2-z2*(x3+x4))
      x6=2.*f4(b1,g1,t,0.0)
      x7=0.5*s*f2(b3,g1,t,0.0)+f4(b3,g1,t,0.0)
      x8=-0.5*s*f2(b4,g1,t,0.0)+f4(b4,g1,t,0.0)
      x9=(a1/v)*(x6-z2*(x7+x8))
      g2=0.5*s
```

```
      x10=2.*f4(b1,g2,t,0.0)
      x11=0.5*s*f2(b3,g2,t,0.0)+f4(b3,g2,t,0.0)
      x12=-0.5*s*f2(b4,g2,t,0.0)+f4(b4,g2,t,0.0)
      x13=-(a1/v)*exp(-v*t)*(x10-z2*(x11+x12))
      x14=(x5+x9+x13)
      x15=2.*f3(b1,g1,t,t0)
      x16=2.*f3(b1,g1,t,t0)
      x17=0.5*s*f1(b3,g1,t,t0)+f3(b3,g1,t,t0)
      x18=-0.5*s*f1(b4,g1,t,t0)+f3(b4,g1,t,t0)
      x19=-x15+a1*(x16-z2*(x17+x18))
      x20=2.*f4(b1,g1,t,t0)
      x21=0.5*s*f2(b3,g1,t,t0)+f4(b3,g1,t,t0)
      x22=-0.5*s*f2(b4,g1,t,t0)+f4(b4,g1,t,t0)
      x23=-(a1/v)*(x20-z2*(x21+x22))
      g2=0.5*s
      x24=2.*f4(b1,g2,t,t0)
      x25=0.5*s*f2(b3,g2,t,t0)+f4(b3,g2,t,t0)
      x26=-0.5*s*f2(b4,g2,t,t0)+f4(b4,g2,t,t0)
      x27=(a1/v)*exp(-v*(t-t0))*(x24-z2*(x25+x26))
      tou=x14+x19+x23+x27
       sum=sum+tou
       n=n+1
       if(abs(tou) .gt. 0.000001) go to 14
       tou0=-exp(s*h*0.5)*(sum/t0)
       endif
       y1=-g1*exp(s*h*0.5)*(1-a1)
       y2=-a1*g2
       y3=-a1*g1*cosh(g1*h)
       ts=-((y1+y3)/sinh(g1*h))+y2
       write(6,60)as(i1),am(i),aa(j),at(j1),tou0,ts
60     format(1x,4f6.2,2e14.5)
30    continue
20    continue
10    continue
5     continue
      stop
         end
       function f2(x1,x2,x3,x4)
          z1=exp(x1*x2)
          z2=erfc((x1/(2.*sqrt(x3-x4)))+x2*sqrt(x3-x4))
          z3=exp(-x1*x2)
          z4=erfc((x1/(2.*sqrt(x3-x4)))-x2*sqrt(x3-x4))
          f2=0.5*(z1*z2+z3*z4)
          return
          end
          function f1(x1,x2,x3,x4)
          z1=exp(x1*x2)*((x1*(0.5/x2))+(x3-x4))
          z2=erfc((x1/(2.*sqrt(x3-x4)))+x2*sqrt(x3-x4))
          z3=exp(-x1*x2)*((x3-x4)-(x1*(0.5/x2)))
          z4=erfc((x1/(2.*sqrt(x3-x4)))-x2*sqrt(x3-x4))
          f1=0.5*(z1*z2+z3*z4)
```

```
          return
         end
          function f4(x1,x2,x3,x4)
          p1=4.*atan(1.)
          z1=x2*exp(x1*x2)
          z2=erfc((x1/(2.*sqrt(x3-x4)))+x2*sqrt(x3-x4))
          z3=-x2*exp(-x1*x2)
          z4=erfc((x1/(2.*sqrt(x3-x4)))-x2*sqrt(x3-x4))
          z5=-(x1*x1+4.*x2*x2*(x3-x4)**2)/(4.*(x3-x4))
          z6=-2./(sqrt(p1*(x3-x4)))
          z7=z6*exp(z5)
          f4=0.5*(z1*z2+z3*z4+z7)
          return
        end
          function f3(x1,x2,x3,x4)
          p1=4.*atan(1.)
          z1=x2*exp(x1*x2)*((x1*(0.5/x2))+(x3-x4))
          z2=erfc((x1/(2.*sqrt(x3-x4)))+x2*sqrt(x3-x4))
          z3=-x2*exp(-x1*x2)*((x3-x4)-(x1*(0.5/x2)))
          z4=erfc((x1/(2.*sqrt(x3-x4)))-x2*sqrt(x3-x4))
          z5=erfc((x1/(2.*sqrt(x3-x4)))+x2*sqrt(x3-x4))
          z6=erfc((x1/(2.*sqrt(x3-x4)))-x2*sqrt(x3-x4))
          z7=exp(x1*x2)/(2.*x2)
          z8=-exp(-x1*x2)/(2.*x2)
          z9=z5*z7+z6*z8
          z10=-(x1*x1+4.*x2*x2*(x3-x4)**2)/(4.*(x3-x4))
          z11=-2.*sqrt((x3-x4)/p1)
          z12=z11*exp(z10)
          f3=0.5*(z12+z9+z1*z2+z3*z4)
          return
        end

          function erfc(f)
           ac=abs(f)
         t1=1./(1.00+0.500*ac)
       erfc=t1*exp(-ac*ac-
       1.2655122300+t1*(1.0000236800+t1*(
       +0.3740919600+t1*(0.0967841800+t1*(-
      0.1862880600+t1*(
       +0.2788680700+t1*(-
       1.1352039800+t1*(1.4885158700+t1*(-
       0.822+1522300+t1*(0.170872700)))))))))
        if(f.lt.0.000) erfc=2.00-erfc
        return
        end
```

d. FORTRAN programme to generate the numerical values of the velocity in problem 4.2

```
C    MHD COUETTE FLOW BETWEEN TWO PARALLEL PLATES DUE TO
      RAMPEDMOTION OF ONE OF THE POROUS PLATE
C     Programme to compute velocity of the porous flow
C     MAGNETIC FIELD FIXED WITH FLUID OR PLATE
      program abcd
        dimension ay(11),at(10),am(2),aa(2),as(3)
        open(6,file='rramp42.out')
        t0=2.0
        data am/0.5,10.0/
        data as/-1.0,0.0,1.0/
        data aa/0.0,1.0/
         data at/0.2,0.4,0.8,1.0,1.5,2.0,2.5,3.0,3.5,4.0/
        data
        ay/0.0,0.2,0.4,0.6,0.8,1.0,1.2,1.4,1.6,1.8,2.0/
        do 20 i=1,3
        do 21 i1=1,2
        do 25 j=1,1
        do 30 j1=1,10
        do 35 k=1,11
        s=as(i)
        a=aa(i1)
        v=am(j)
        t=at(j1)
        y=ay(k)
        c=((s*s/4.)+v)
        z1=(1.-a)/t0
        z2=a/(t0*v)
        z3=(a/t0)*(t+(exp(-v*t)/v)-(1./v))
        v1=sqrt(c)
        z4=exp(s*0.5*y)
        if (t .le.  t0) then
        u1=z1*f1(y,v1,t,0.0)*z4
        u2=z2*z4*(f2(y,v1,t,0.0)-exp(-
        v*t)*f2(y,0.0,t,0.0))+z3
        u=u1+u2
        else
        u1=z1*z4*(f1(y,v1,t,0.0)-f1(y,v1,t,t0))
          u2=z2*z4*(f2(y,v1,t,0.0)-exp(-
          v*t)*f2(y,0.0,t,0.0))
        u3=z2*z4*(exp(-v*(t-t0))*f2(y,0.0,t,t0)-
        f2(y,v1,t,t0))
       u4=(a/t0)*((1./v)-(t-t0)-(1./v)*exp(-v*(t-t0)))+z3
        u=u1+u2+u3+u4
        endif
        tt=t/t0
C       IMPULSIVE MOTION OF POROUS PLATE
         xx=s*0.5
         xx1=(1.-a)*z4*f2(y,v1,t,0.0)+a*(1.-exp(-V*t))
         xx2=a*z4*exp(-v*t)*f2(y,xx,t,0.0)
         ui=xx1+xx2
```

```
      write(6,60)as(i),aa(i1),am(j),at(j1),ay(k),u,tt,ui
   60    format(1x,5f8.3,3f10.5)
   35    continue
   30    continue
   25    continue
   21    continue
   20    continue
        stop
           end
            function f1(d1,d2,d3,d4)
            z1=exp(-d1*d2)*((d3-d4)-0.5*(d1/d2))
            z2=erfc(d1*0.5/sqrt(d3-d4)-d2*sqrt(d3-d4))
            z3=exp(d1*d2)*((d3-d4)+0.5*(d1/d2))
            z4=erfc(d1*0.5/sqrt(d3-d4)+d2*sqrt(d3-d4))
            f1=0.5*(z1*z2+z3*z4)
            return
          end
            function f2(d1,d2,d3,d4)
            z1=exp(-d1*d2)
            z2=erfc(d1*0.5/sqrt(d3-d4)-d2*sqrt(d3-d4))
            z3=exp(d1*d2)
            z4=erfc(d1*0.5/sqrt(d3-d4)+d2*sqrt(d3-d4))
            f2=0.5*(z1*z2+z3*z4)
            return
          end

          function erfc(f)
          ac=abs(f)
          t1=1./(1.00+0.500*ac)
        erfc=t1*exp(-ac*ac-
        1.2655122300+t1*(1.0000236800+t1*(
        +0.3740919600+t1*(0.0967841800+t1*(-
        0.1862880600+t1*(
        +0.2788680700+t1*(-
        1.1352039800+t1*(1.4885158700+t1*(-0.822
        +1522300+t1*(0.170872700)))))))))
          if(f.lt.0.000) erfc=2.00-erfc
          return
          end

C     MHD COUETTE FLOW BETWEEN TWO PARALLEL PLATES DUE TO
      RAMPED MOTION OF ONE THE PLATE
C      Programme to compute the skin friction at the
      lowerporous plate
C         MAGNETIC FIELD FIXED WITH FLUID OR PLATE
C         SKIN-FRICTION Y=0.0
           program abcd
          dimension at(10),am(2),aa(2)
          open(6,file='rramp43.out')
          t0=2.0
```

```
      p=4.*atan(1)
      data aa/0.0,1.0/
      data am/0.5,10.0/
       data at/0.2,0.4,0.8,1.0,1.5,2.0,2.5,3.0,3.5,4.0/
      do 20 i=1,2
      do 25 j=1,2
      do 30 j1=1,10
      a=aa(i)
      v=am(j)
      t=at(j1)
      z1=(1.-a)/t0
      z2=a/(t0*v)
      v1=sqrt(v)
      if (t .le.  t0) then
      u1=f4(v1,t,0.0)
      u2=f3(v1,t,0.0)-exp(-v*t)*f3(0.0,t,0.0)
      tau=z1*u1+z2*u2
      else
      u1=f4(v1,t,0.0)-f4(v1,t,t0)
      u2=f3(v1,t,0.0)-exp(-v*t)*f3(0.0,t,0.0)
      u3=exp(-v*(t-t0))*f3(0.0,t,t0)-f3(v1,t,t0)
      tau=z1*u1+z2*(u2+u3)
      endif
C     IMPULSIVE MOTION
      x=SQRT(p*t)
      ti=(1-a)*f3(v1,t,0.0)+(a/x)*exp(-v*t)
       write(6,60)aa(i),am(j),at(j1),tau,ti
   60   format(1x,3f8.3,2f14.5)
   30   continue
   25   continue
   20   continue
       stop
         end
          function f3(d1,d2,d3)
          p1=4.*atan(1.)
          z1=d1-d1*erfc(d1*sqrt(d2-d3))
          z2=exp(-d1*d1*(d2-d3))
          z3=sqrt(p1*(d2-d3))
          z4=z2/z3
          f3=z1+z4
          return
       end
          function f4(d1,d2,d3)
          p1=4.*atan(1.)
          z1=d1*(d2-d3)+(0.5/d1)
          z2=sqrt((d2-d3)/p1)*exp(-d1*d1*(d2-d3))
          f4=z1+z2-z1*erfc(d1*sqrt(d2-d3))
          return
       end
       function erfc(f)
       ac=abs(f)
```

```
      t1=1./(1.00+0.500*ac)
           erfc=t1*exp(-ac*ac-
     1.2655122300+t1*(1.0000236800+t1*(
     +0.3740919600+t1*(0.0967841800+t1*(-
     0.1862880600+t1*(
     +0.2788680700+t1*(-
     1.1352039800+t1*(1.4885158700+t1*(-          0.822
     +1522300+t1*(0.170872700))))))))))
       if(f.lt.0.000) erfc=2.00-erfc
       return
       end
C     MHD COUETTE FLOW BETWEEN TWO PARALLEL PLATES DUE TO
      RAMPED MOTION   OF ONE THE PLATE
    C     Programme to compute the skin friction at the
          upper porous plate
C         MAGNETIC FIELD FIXED WITH FLUID OR PLATE
C         (SKIN-FRICTION Y=H)
          program abcd
          dimension at(10),am(2),aa(2),as(3)
        open(6,file='rramp44.out')
        t0=2.0
        data am/0.5,10.0/
        data as/-1.0,0.0,1.0/
        data aa/0.0,1.0/
         data at/0.2,0.4,0.8,1.0,1.5,2.0,2.5,3.0,3.5,4.0/
        do 20 i=1,3
        do 21 i1=1,2
        do 25 j=1,2
        do 30 j1=1,10
        s=as(i)
        a=aa(i1)
        v=am(j)
        t=at(j1)
        c=((s*s/4.)+v)
        z1=(1.-a)/t0
        z2=a/(t0*v)
        v1=sqrt(c)
        if (t .le.  t0) then
        u1=z1*(f4(v1,t,0.0)-0.5*s*f1(0.0,v1,t,0.0))
        u2=f3(v1,t,0.0)-0.5*s*f2(0.0,v1,t,0.0)
        z3=0.5*s
        u3=exp(-v*t)*(f3(z3,t,0.0)-z3*f2(0.0,z3,t,0.0))
        tt=z2*(u2-u3)+u1
        else
        u1=f4(v1,t,0.0)-0.5*s*f1(0.0,v1,t,0.0)
        u2=f3(v1,t,0.0)-0.5*s*f2(0.0,v1,t,0.0)
        z3=0.5*s
        u3=exp(-v*t)*(f3(z3,t,0.0)-z3*f2(0.0,z3,t,0.0))
        u4=f4(v1,t,t0)-0.5*s*f1(0.0,v1,t,t0)
        u5=f3(v1,t,t0)-0.5*s*f2(0.0,v1,t,t0)
```

```
          u6=exp(-v*(t-t0))*(f3(z3,t,t0)-
          z3*f2(0.0,z3,t,t0))
        tt=z2*(u2-u3+u6-u5)+z1*(u1-u4)
        endif

C       IMPULSIVE MOTION OF THE PLATE
         x=f3(v1,t,0.0)-z3*f2(0.0,v1,t,0.0)
         xx=f3(z3,t,0.0)-z3*f2(0.0,z3,t,0.0)
         ti=(1-a)*x+a*exp(-v*t)*xx
         write(6,60)as(i),aa(i1),am(j),at(j1),tt,ti
   60    format(1x,4f8.3,2f14.5)
   30    continue
   25    continue
   21    continue
   20    continue
        stop
          end
           function f3(d1,d2,d3)
           p1=4.*atan(1.)
           z1=d1-d1*erfc(d1*sqrt(d2-d3))
           z2=exp(-d1*d1*(d2-d3))
           z3=sqrt(p1*(d2-d3))
           z4=z2/z3
           f3=z1+z4
           return
          end
           function f4(d1,d2,d3)
           p1=4.*atan(1.)
           z1=d1*(d2-d3)+(0.5/d1)
           z2=sqrt((d2-d3)/p1)*exp(-d1*d1*(d2-d3))
           f4=z1+z2-z1*erfc(d1*sqrt(d2-d3))
           return
           end
           function f1(d1,d2,d3,d4)
           z1=exp(-d1*d2)*((d3-d4)-0.5*(d1/d2))
           z2=erfc(d1*0.5/sqrt(d3-d4)-d2*sqrt(d3-d4))
           z3=exp(d1*d2)*((d3-d4)+0.5*(d1/d2))
           z4=erfc(d1*0.5/sqrt(d3-d4)+d2*sqrt(d3-d4))
           f1=0.5*(z1*z2+z3*z4)
           return
           end
           function f2(d1,d2,d3,d4)
           z1=exp(-d1*d2)
           z2=erfc(d1*0.5/sqrt(d3-d4)-d2*sqrt(d3-d4))
           z3=exp(d1*d2)
           z4=erfc(d1*0.5/sqrt(d3-d4)+d2*sqrt(d3-d4))
           f2=0.5*(z1*z2+z3*z4)
           return
           end
           function erfc(f)
        ac=abs(f)
```

```
   t1=1./(1.00+0.500*ac)
erfc=t1*exp(-ac*ac-
1.2655122300+t1*(1.0000236800+t1*(
+0.3740919600+t1*(0.0967841800+t1*(-
0.1862880600+t1*(
+0.2788680700+t1*(-
1.1352039800+t1*(1.4885158700+t1*(-0.822
+1522300+t1*(0.170872700)))))))))
  if(f.lt.0.000) erfc=2.00-erfc
  return
  end
```

e. MATLAB Programme to generate the numerical values in problem 5.1

```
C    COMBINED EFFECTS OF VISCOUS AND JOULEAN DISSIPATION
     ON MHD COUETTE FLOW WITH TRANSPIRATION
C    MATLAB programme to compute the velocity of the flow
     [y,t]=mesh grid (0.0:0.1:1, 0.1:0.1:1.0);
     w=(y-y+1).*(t-t+1);
     re=5.0*w;
     m=2.*w;
     d =sqrt(((re.^2)/4)+m.^2);
     X3=exp(((re./2.).*(y-1))-(d.^2.).*t);
     Sum=0.0.*w;
     n=1;
     While n<60;
     d2= (d.^2)+((n^2.)*(pi^2.));
     x1=exp(-((n.^2)*(pi^2).*t));
     x2=sin(n*pi*y);

     x=(((-1.0).^n)*n.*x1.*x2.*x3)./d2;
      sum=sum+x;
      n=n+1;
        end
        u1=2.*pi.*(sum+x);
     u2=exp((re./2).*(y-1)).*(sinh(d.*y))./(sinh(d));
        u=u1+u2;
       [C,h]=contour(y,t,u)

seth,'showText','on','textStep',get(h,'levelStep')*1);
       colormap gray

C    COMBINED EFFECTS OF VISCOUS AND JOULEAN DISSIPATION
     ON MHD COUETTE FLOW WITH TRANSPIRATION
C    programme to compute skin friction at y=0
      [m,t]=meshgrid(10.0:1.0:12,0.1:0.1:1.0);
       w=(m-m+1).*(t-t+1);
       re=5.0*w;
       %m=10.0*w;
```

```
        d =sqrt(((re.^2.)/4)+(m));
        x2=exp(-((re./2.)+(d.^2.).*t));
        sum=0.0.*w;
        n=1;
         while n<60;
        d2=((d.^2)+((n.^2.).*(pi^2.)));
       x1=exp(-((n.^2)*(pi.^2).*t));
       x=((n.^2).*x1)./d2;
       sum=sum+x;
       n=n+1;
         end
         s1=2.*(pi^2).*x2.*(sum);
         s2=(exp(-(re./2)).*d)./(sinh(d));
         s=s1+s2
        [C,h]=contour(m,s,t);

     set(h,'showText','on','textStep',get(h,'levelStep')*
     1);colormap gray

C    COMBINED EFFECTS OF VISCOUS AND JOULEAN DISSIPATION
     ON MHD COUETTE FLOW WITH TRANSPIRATION
C    programme to compute skin friction at y=1
       m,t]=meshgrid(10.0:1.0:12,0.1:0.1:1.0);
       w=(m-m+1).*(t-t+1);
       re=5.0*w;
       %m=10.0*w;
       d =sqrt(((re.^2.)/4)+(m));
       x2=exp(-((d.^2.).*t));
       sum=0.0.*w;
       n=1;
        while n<60;
     d2=((d.^2)+((n.^2.).*(pi^2.)));
     x1=exp(-((n.^2)*(pi.^2).*t));
     x=((n.^2).*x1)./d2;
     sum=sum+x;
     n=n+1;
        end
     s1=2.*(pi^2).*x2.*(sum);
     s2=(d.*cosh(d))./(sinh(d));
      s=(re./2)+s2+s1
     [C,h]=contour(m,s,t);

set(h,'showText','on','textStep',get(h,'levelStep')*1);
     colormap gray

C    COMBINED EFFECTS OF VISCOUS AND JOULEAN DISSIPATION
     ON MHD COUETTE FLOW WITH TRANSPIRATION
C    MATLAB programme used to generate the numerical
     values for     the steady state temperature
       %MHD flow and heat transfer with suction I
       [y,pr]=mesh grid (0.0:0.001:1,1.0:2.0:6.0);
```

```
  w=(y-y+1).*(pr-pr+1);
  ec=2.0.*w;
  %pr=5.0*w
  re=5.0.*w;
  m=2.0.*w;
  d =sqrt((re.^2./4)+(m.^2));
  d1=4.*(exp(re)).*((sinh(d)).^2);
  d2=(2.*d+(1.-pr).*re);
  d3=(2.*d-(1.-pr).*re);
  c=exp(pr.*re)-1.;
  c1=(d./d2);
  c2=(d./d3);
  x2=exp((2.*d)+re);
  x1=exp(pr.*re);
  f1=exp(pr.*re.*y);
  x3=exp(-((2.*d)-re));
  f5=exp(-(2.*d-re).*y);
  f6=exp(-(2.*d-re)+(pr.*re.*y));
  f2=exp((2.*d+re)+pr.*re.*y);
  f3=exp((2.*d+re).*y+pr.*re);
  f4=exp((2.*d+re).*y);
  f7=exp(-(2.*d-re).*y+pr.*re);
  t1=x1+f2+f4;
  t2=x2+f3+f1;
  t3=x1+f6+f5;
  t4=x3+f1+f7;
  temp1=c1.*(t1-t2);
  temp2=c2.*(t3-t4);
  tem=(((f1-1)+(ec./d1).*pr.*(temp1+temp2))./c
  [C,h]=contour(y,tem,pr,4);

set(h,'showText','on','textStep',get(h,'levelStep')*
1);
  colormap gray

C    COMBINED EFFECTS OF VISCOUS AND JOULEAN DISSIPATION
     ON MHD COUETTE FLOW WITH TRANSPIRATION
C    Programme to compute the Nusselt number
  %Nusselt number at y=0
  [m,pr]=meshgrid(10.0:1.0:12,2.0:1.0:3.0);
  w=(m-m+1).*(pr-pr+1);
  ec=2.0.*w;
  %pr=5.0*w;
  re=5.0.*w;
  %m=10.0.*w;
  d =sqrt((re.^2.)/4+(m));
  d1=4.*(exp(re)).*((sinh(d)).^2);
  c1=d./(2.*d+re.*(1-pr));
  c2=d./(2.*d-re.*(1-pr));
  c=exp(pr.*re)-1.;
  x1=exp(2.*d+re);
```

```
x2=exp(-(2.*d-re));
n1=(pr.*re).*(exp(pr.*re))./c;
n2=(c1.*(x1-1))+(c2.*(x2-1));
n3=(c2.*(2.*d-re).*x2)-(c1.*(2.*d+re).*x1);
n4=n1.*(1.+((ec.*pr.*n2)./d1));
n=n4+((ec.*pr.*n3)./d1)
[C,h]=contour(m,n,pr,4);

   set(h,'showText','on','textStep',get(h,'levelSt
   ep')*1);
 colormap gray

%Nusselt number at y=1
[m,pr]=meshgrid(10.0:1.0:12.0,2.0:1.0:3.0);
w=(m-m+1).*(pr-pr+1);
ec=2.0.*w;
%pr=5.0*w;
re=5.0.*w;
%m=10.0.*w;
d =sqrt(((re.^2.)/4)+m);
d1=4.*(exp(re)).*((sinh(d)).^2);
d3=4.*(d.^2.)-(((1.-pr).^2.).*(re.^2.));
c=exp(pr.*re)-1.;
x1=4.*(d.^2).*(exp(re)).*(cosh(2.*d));
x2=(2.*d .*(exp(re)).*re.*(1-pr).*(sinh(2.*d)));
x3=8.*(d.^3).*exp(re).*sinh(2.*d);
x4=4.*(d.^2).*exp(re).*pr.*re.*cosh(2.*d);
x5=2.*d.*exp(re).*(re.^2).*(1-pr).*(sinh(2.*d));
n1=(pr.*re).*(exp(pr.*re))./c;
n2=x1-x2-(4.*(d.^2));
n3=(ec.*pr).*(x3+x4-x5)./(d1.*d3);
n4=n1.*(1.+(ec.*pr.*n2)./(d1.*d3));
n=n4-n3
[C,h]=contour(m,n,pr);

   set(h,'showText','on','textStep',get(h,'levelSt
   ep'  )*1);
colormap gray
```

f. Programme to generate the numerical values for the problem in 5.2

C. COMBINED EFFECTS OF VISCOUS AND OHMIC DISSIPATION ON MHD COUETTE FLOW WITH TRANSPIRATION

```
C    programme to compute numerical values of the
velocity(R)
%mhd flow and heat transfer with suction I
[y,t]=meshgrid(0.0:0.1:1,0.1:0.1:1.0);
w=(y-y+1).*(t-t+1);
re=5.0*w;
```

```
  m=2.0*w;
  R=0.0.*w;
  d =sqrt(((re.^2)/4)+m.^2);
  x3=exp(((re./2.).*(y-1))-(d.^2.).*t);
  sum=0.0.*w;
  n=1;
   while n<60;
 d2=d.^2.+((n^2.)*(pi^2.));
 x1=exp(-((n.^2)*(pi^2).*t));
  x2=sin(n*pi*y);

x=(((-1.0).^n).*n.*x1.*x2)./d2;
sum=sum+x;
n=n+1;
end
 u1=2.*pi.*x3.*(sum);
 x4=exp((re./2).*(y-1)).*(sinh(d.*y))./(sinh(d));
x5=(sinh(d.*y))./(sinh(d));
  x6=exp(-(d-(re./2.)).*y);
x7=((1-x5).*x6)-1;
u2=(1+R).*x4+R.*x7;
u=u1+u2
[C,h]=contour(y,u,t);
set(h,'showText','on','textStep',get(h,'levelStep')*
1);
  colormap gray

C    COMBINED EFFECTS OF VISCOUS, JOULEAN AND OHMIC
     DISSIPATION ON MHD COUETTE FLOW WITH TRANSPIRATION
C    MATLAB Programme to compute skin friction at y=0
 %mhd cuoette flow skinfriction1
  [m,t]=meshgrid(10.0:1.0:12,0.1:0.1:1.0);
  w=(m-m+1).*(t-t+1);
  re=5.0*w;
  R=1.0*w;
  %m=10.0*w;
  d =sqrt(((re.^2.)/4)+(m));
  x2=exp(-((d.^2.).*t));
  sum=0.0.*w;
  n=1;
   while n<60;
d2=((d.^2)+((n.^2.).*(pi^2.)));
x1=exp(-((n.^2)*(pi.^2).*t));
x=((n.^2).*x1)./d2;
sum=sum+x;
n=n+1;
end
s1=2.*(pi.^2).*x2.*(sum);
s2=((R+1).*d.*(cosh(d)))./(sinh(d))+(R+1).*(re./2);
s3=(R.*d.*(cosh(d))).*(exp(-(d-re./2)))./(sinh(d));
s=s1+s2-s3
```

```
        [C,h]=contour(m,s,t);

set(h,'showText','on','textStep',get(h,'levelStep')*1);
      colormap gray

C     COMBINED EFFECTS OF VISCOUS, JOULEAN AND OHMIC
      DISSIPATION ON MHD COUETTE FLOW WITH
      TRANSPIRATION
C     Programme to compute skin friction at y=1
      [m,t]=meshgrid(10.0:1.0:12,0.1:0.1:1.0);
      w=(m-m+1).*(t-t+1);
        re=5.0*w;
        R=2.0*w;
        %m=10.0*w;
        d =sqrt(((re.^2.)/4)+(m));
        x2=exp(-(d.^2.).*t);
        sum=0.0.*w;
        n=1;
         while n<60;
      d2=((d.^2)+((n.^2.).*(pi^2.)));
      x1=exp(-((n.^2)*(pi.^2).*t));
      x=((n.^2).*x1)./d2;
      sum=sum+x;
      n=n+1;
      end
      s1=2.*(pi^2).*x2.*(sum);
      s2=((R+1).*d.*(cosh(d))./(sinh(d))+(R+1).*(re./2.);
      s3=sinh(d)-((cosh(d)).^2)./(sinh(d));
      s4=s3.*(d.*R.*(exp(re./2)));
       s=s1+s2+s4
      [C,h]=contour(m,s,t);
      set(h,'showText','on','textStep',get(h,'levelStep')*
      1);
      colormap gray

C     COMBINED EFFECTS OF VISCOUS, JOULEAN AND OHMIC
      DISSIPATION ON MHD COUETTE FLOW WITH TRANSPIRATION
C     programme to compute steady state temperature(R)
       % MHD flow nd heat transfer with suction I
       [y,pr]=meshgrid(0.0:0.001:1,1.0:2.0:6.0);
        w=(y-y+1).*(pr-pr+1);
        ec=2.0.*w;
        %pr=5.0*w
        re=5.0.*w;
        m=2.0.*w;
        d =sqrt((re.^2./4)+(m.^2));
        d1=4.*(exp(re)).*((sinh(d)).^2);
        d2=(2.*d+(1.-pr).*re);
        d3=(2.*d-(1.-pr).*re);
        c=exp(pr.*re)-1.;
        c1=(d./d2);
```

```
        c2=(d./d3);
        x2=exp((2.*d)+re);
        x1=exp(pr.*re);
        f1=exp(pr.*re.*y);
        x3=exp(-((2.*d)-re));
        f5=exp(-(2.*d-re).*y);
        f6=exp(-(2.*d-re)+(pr.*re.*y));
        f2=exp((2.*d+re)+pr.*re.*y);
        f3=exp((2.*d+re).*y+pr.*re);
         f4=exp((2.*d+re).*y);
        f7=exp(-(2.*d-re).*y+pr.*re);
        t1=x1+f2+f4;
        t2=x2+f3+f1;
        t3=x1+f6+f5;
        t4=x3+f1+f7;
        temp1=c1.*(t1-t2);
        temp2=c2.*(t3-t4);
        tem=(((f1-1)+(ec./d1).*pr.*(temp1+temp2))./c
        [C,h]=contour(y,tem,pr);

     set(h,'showText','on','textStep',get(h,'levelStep')*
     1)
        colormap gray

C    COMBINED EFFECTS OF VISCOUS, JOULEAN AND OHMIC
     DISSIPATION ON MHD COUETTE FLOW WITH TRANSPIRATION

C    %mhd flow nd heat transfer with suctionI
C    %Programme to compute Nusselt number
C    %Nusselt number at y=0
        [m,pr]=meshgrid(10.0:1.0:12,2.0:1.0:3.0);
        w=(m-m+1).*(pr-pr+1);
        ec=2.0.*w;
        %pr=5.0*w;
        re=5.0.*w;
        %m=10.0.*w;
        d =sqrt((re.^2.)/4+(m));
        d1=4.*(exp(re)).*((sinh(d)).^2);
        c1=d./(2.*d+re.*(1-pr));
        c2=d./(2.*d-re.*(1-pr));
        c=exp(pr.*re)-1.;
        x1=exp(2.*d+re);
        x2=exp(-(2.*d-re));
        n1=(pr.*re).*(exp(pr.*re))./c;
        n2=(c1.*(x1-1))+(c2.*(x2-1));
        n3=(c2.*(2.*d-re).*x2)-(c1.*(2.*d+re).*x1);
        n4=n1.*(1.+((ec.*pr.*n2)./d1));
        n=n4+((ec.*pr.*n3)./d1)
        [C,h]=contour(m,n,pr,4);
  set(h,'showText','on','textStep',get(h,'levelStep')*1);
        colormap gray
```

```
C    COMBINED EFFECTS OF VISCOUS, JOULEAN AND OHMIC
     DISSIPATION ON MHD COUETTE FLOW WITH TRANSPIRATION
C    %MHD flow nd heat transfer with suction I
C    %Programme to compute Nusselt number
C    %Nusselt number at y=1
      [m,pr]=meshgrid(10.0:1.0:12.0,2.0:1.0:3.0);
       w=(m-m+1).*(pr-pr+1);
       ec=2.0.*w;
       %pr=5.0*w;
       re=5.0.*w;
       %m=10.0.*w;
       d =sqrt(((re.^2.)/4)+m);
       d1=4.*(exp(re)).*((sinh(d)).^2);
       d3=4.*(d.^2.)-(((1.-pr).^2.).*(re.^2.));
       c=exp(pr.*re)-1.;
       x1=4.*(d.^2).*(exp(re)).*(cosh(2.*d));
       x2=(2.*d .*(exp(re)).*re.*(1-pr).*(sinh(2.*d)));
       x3=8.*(d.^3).*exp(re).*sinh(2.*d);
       x4=4.*(d.^2).*exp(re).*pr.*re.*cosh(2.*d);
       x5=2.*d.*exp(re).*(re.^2).*(1-pr).*(sinh(2.*d));
       n1=(pr.*re).*(exp(pr.*re))./c;
       n2=x1-x2-(4.*(d.^2));
       n3=(ec.*pr).*(x3+x4-x5)./(d1.*d3);
       n4=n1.*(1.+(ec.*pr.*n2)./(d1.*d3));
       n=n4-n3
       [C,h]=contour(m,n,pr);

          set(h,'showText','on','textStep',get(h,'levelSt
          ep')*1)colormap gray
```