TRANSVERSE REINFORCEMENT IN SHORT CONCRETE COLUMNS

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CONTENTS

		PAGE
	ACKNOWLEDGEMENTS	6
CHAPTER 1:	ABSTRACTS	7
	1.10 Quotations	7
	1.20 Key Words	7
	1.30 Synopsis	7
	1.40 Introduction	8
CHAPTER 2:	OBJECTIVE AND LIMITATIONS	13
	2.10 Objective	13
	2.11 Significance	15
	2.20 Limitations	16
	2.21 Scope	16
CHAPTER 3:	SUMMARY OF PREVIOUS INVESTIGATIONS ...	19
	3.10 A Considere of France (1899-1902) ...	19
	3.20 Talbot (1907)	20
	3.30 Bach of Stuttgart (1905 and 1913) ...	21
	3.40 Other Investigations between 1907-1927	21
	3.50 Talbot (1927)	22
	3.60 J.W.H. King (1946-1949) ...	23
	3.70 S. Mackey (1954)	26
	3.80 T. Szulczynski of Gdansk (1961) ...	30
	3.90 Blume, J.A., Newmark, N.M., Corning, L.H.,(1961)	31
	3.100 Roy, H.E.H. and Sozen M.E. (1964) ...	31

	3.200 N.G. Bunni (1964)	32
	3.300 Fred. M. Hudson (1965)	33
	3.400 Norman F. Somes (1970)	35
	3.500 M.Sargin, S.K. Ghosh and V.K. Handa (1971)	36
	3.600 Sylwester Oleszkiewicz (1973) ...	38
	3.700 J. Oleszkiewicz (1973)	40
	3.800 Sharifullah Najib(1973)	43
CHAPTER 4:	DISCUSSION OF PREVIOUS RECOMMENDATIONS ...	45
CHAPTER 5:	MATERIALS AND CONCRETE MIXES	54
	5.10 Materials	54
	5.11 Aggregates	54
	5.12 Cement	55
	5.13 Reinforcement	55
	5.14 The Universal Testing Machine ...	55
	5.15 Denison Compression Testing Machine ...	57
	5.16 Strain Gauges	57
	5.20 Concrete Mix Design	61
	5.30 Method of casting	65
	5.40 Curing Conditions	68
CHAPTER 6:	TEST PROGRAMME	69
	6.10 Variables and Factors	69
	6.20 Arrangement of Specimens ...	71
	6.21 Identification Marks	71

	6.30	Design and Description of Test Specimens	71
	6.31	Reinforcement Details	72
	6.32	Series "A" Columns	75
	6.33	Series "B" and "C" Columns ...	77
	6.40	Test Procedure	77
CHAPTER 7:		PRESENTATION OF RESULTS	83
	7.10	Observations	83
	7.20	Modes of Failure	84
	7.30	Test Results	92
	7.31	Concrete Strength	92
	7.32	Steel Strength	92
	7.33	Column Deformations	93
	7.40	Ultimate Bearing Capacities ...	94
CHAPTER 8:		DISCUSSION OF RESULTS	110
	8.10	General	110
	8.20	Load-Strain Relations	115
	8.30	Discussion of the Theory of the Formula derived by Blume, Newmark and Corning - "B.N.C. Equation"	120
	8.40	Theoritical Ultimate loads Based on the Above Formula	123
	8.50	Discussion Based on the Theory Behind Oleszkiewicz's Formula	125
	8.51	Ultimate Loads	140

8.60 Theoritical Ultimate Loads Based on Oleszkiewicz's Formula 142

8.70 Theoritical Ultimate Loads Based on Present Code Requirements ... 145

8.80 Comparison of Calculated and observed ultimate Loads 145

CHAPTER 9: RECOMMENDATIONS 154

9.10 Recommendations 154

9.20 Economy 157

9.30 Typical Comparative Designs 160

9.40 General Effect of Method of Reinforcing on Total Quantity of Steel 164

CHAPTER 10: CONCLUSIONS 170

10.10 Findings 170

10.20 Suggestions for Further Research ... 173

REFERENCES 175

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CHAPTER 1

A B S T R A C T S

1.10 Quotations:

"Lateral ties can be effectively utilised to a much better advantage in the reinforcement of short, axially loaded, rectangular columns than the present Nigerian Standard Code of Practice for Reinforced Concrete Structures recommends".

"A more intensive transverse reinforcement and less of longitudinal bars will prove to be more economical in the long run".

1.20. Key Words:

Short columns, lateral ties, cube crushing and cylinder strengths, yield stresses, previous works, analytical equations, testing, deformations, failure modes, ductility, buckling phenomenon, Poissons ratio, ultimate loads.

1.30 Synopsis:

This thesis brings into focus the knowns and unknowns in the host of technical informations available on the effect of lateral reinforcement on the performance of reinforced concrete columns. A critical review of the Nigerian Standard Code of Practice and of previous researches and their recommendations on the subject was made and areas of disagreement discussed. As a basis for theoretical analysis, equations put forward by previous investigators to define the ultimate load of reinforced concrete columns with lateral ties were studied.

An experimental programme was designed to investigate the complete load-strain relationship of confined concrete under the influence of certain factors that could affect this relationship. A total of twenty-seven columns were tested to destruction; the failure loads, modes of failure and strains being noted. Variable parameters in the tests were: tie spacing, number of legs in one link of tie, column size and side/width ratio, percentage and number of longitudinal reinforcement. The results of this experimental work is compared with equations derived from analytical considerations. An equation was recommended for possible adoption by the Nigerian Standards Organisation in a revised Nigerian Standard Code of Practice N.C.P-1.

1.40. Introduction:

As a result of the inability of plain concrete to absorb tensile punishments caused by different types of loading or disturbances, it is usually reinforced with a predetermined quantity of longitudinal steel bars. Also, adequate transverse reinforcement in the form of lateral ties or closely spaced helix or spiral binders is incorporated to prevent premature column failure due to buckling of the individual longitudinal bars or splitting of concrete. These reinforcements increase the load bearing capacity of reinforced concrete members.

With respect to column design, the International Codes of Practice [41] specify a lower and upper limit within which the cross-sectional area of longitudinal reinforcements should fall for satisfactory column design and necessary consistence* of concrete.

For axially loaded short columns reinforced with longitudinal bars and lateral ties, the Nigerian Standard Code of Practice, N.C.P. 1: 1973 [32] specifies that the permissible axial load should not exceed that given by the following equation:

$$P_o = p_{cc} A_c + p_{sc} A_{sc} \quad \text{----------(1)}$$

Where:

P_o - is the permissible axial load on a short column.

p_{cc} - is the permissible stress for the concrete in direct compression.

A_c - is the cross-sectional area of the concrete excluding any finishing material and reinforcing steel.

p_{sc} - is the permissible compression stress for column bars.

A_{sc} - is the cross-sectional area of longitudinal steel.

In addition, for axially loaded short columns with helical reinforcements, the Code recommends the following equation for calculating the permissible load:

*Applies to the traditional conception of "workability" as proposed in [50]

$$P_o = p_{cc} A_k + p_{sc} A_{sc} + 27000 A_b^{**} \text{ ----------(2)}$$

Where: A_k - is the cross-sectional area of concrete in the core excluding the area of longitudinal reinforcements.

A_b - is the equivalent area of helical reinforcement(volume of helix per unit length of the column).

Also, the sum of the terms $p_{cc} A_k + 27000 A_b^{**}$ should not exceed $0.5 U_w A_c$ where U_w is the 28-day cube strength of the concrete.

Equation (2) shows that for columns with helical reinforcements, the axial load is shared by the three materials which constitute the column, namely: the concrete in the column core, the longitudinal steel, and the helical reinforcement.

It is staggering to note that equation (1) is completely innocent of the term defining the effect of lateral reinforcement. This omission is common in all codes of practice. For instance, the American Concrete Institute (A.C.I.) Code [2] gives the following equation for calculating the ultimate load of axially loaded columns:

$$P_u = 0.85 f'_c A'_c + f'_y A_s$$

**The author considers the term $27000 A_b$ an obvious error since the factor "27000" applies to P.S.I. units only whereas the code's specifications are in N/mm^2. $27000 A_b$ should read $189 A_b$ since 27000 p.s.i = $189 N/mm^2$ approx.

Where:

P_u - is the ultimate strength of column

f_c^1 - is the 28- day cylinder strength.

A_c^1 - is the cross-sectional area of concrete excluding any finishing material and reinforcing steel.

f_y^1 - is the yield point of longitudinal reinforcement.

A_s - is the total area of longitudinal reinforcement.

while the British Standard Code, C.P. 114 (1969) [7] gives

$$P_o = p_{cc} A_c + p_{sc} A_{sc}$$

All these equations do not include the effect of transverse reinforcements. The common omission is therefore deliberate but not necessarily proper as shown by the theoritical and experimental findings of this thesis and those of previous researchers.

Also, the National Code [32] specifies that a reinforced concrete column should have transverse reinforcement "so disposed" as to provide restraint against the buckling of each of the longitudinal reinforcements. The phrase, "so disposed" creates a loop-hole in the specification and suggests that more investigations are still necessary. Moreover, the stipulation seems to imply that the only function of transverse reinforcements is prevention of buckling of the longitudinal bars. The stipulation therefore ends with the recommendation that the pitch of transverse reinforcements

should not be more than the least of the three following distances:

1. the least lateral dimension of the column
2. twelve times the diameter of the smallest longitudinal reinforcement in the column.
3. 300mm.

Experimental results have shown that the present code equations for ultimate load calculation is fairly satisfactory if and only if tie spacing is not less than the code's requirements. For increased tie spacing beyond the present stipulations, the ability of the ties to prevent buckling of the longitudinal reinforcements is very much reduced. A heavy transverse reinforcement, however, definitely produces a significant change in the load-strain relation of short columns, increasing the load carrying capacity to an extent that negligence of its effect in the permissible or ultimate load equations becomes intolerably unrealistic.

It seems therefore, that since a more intensive lateral reinforcement increases the load carying capacity of a short reinforced concrete column, and improves its overall performance a review of the code stipulations on transverse reinforcements is necessary. An increased percentage of lateral reinforcement should be recommended and its effect introduced directly in the load equations.

CHAPTER 2

OBJECTIVE AND LIMITATIONS

2.10. Objective:

As discussed in the proceeding chapter, equations in the National Design Codes of Practice for calculating the capacity of axially loaded short reinforced concrete columns do not contain terms expressing the effect of lateral reinforcement. To compensate for this contingency, the codes recommend a maximum value for the s spacing of lateral ties which offers a very low percentage of transverse steel. For such low percentages of binders, the effect of ties tends towards negligible and the codes recommendations are therefore fool-proof.

However, a decrease in the percentage of binding steel must be compensated for by a corresponding increase in the percentage of longitudinal bars since in that case the load has to be carried by only the longitudinal bars and the concrete. This method does not ensure full utilization of properties of all materials and will not, therefore, lead to economy. Results of experiments carried out in different research centres as will be discussed in the next chapter have shown that intensive transverse reinforcements improve the performance of reinforced concrete columns [30, 44, etc.]

Initially, the objective of this research was to find out possible effects of variation in lateral tie spacing on the capacity

of axially loaded, reinforced concrete columns. It was also intended to find out if a modification of the present code stipulations is necessary so as to include any effects of tie spacing.

However, after the preliminary stage of the work which was a careful study of related litterature on the subject, it was found that so much has been done already. The topic in its different forms has been subject of graduate research in different Universities, Research centres and Technological Schools [14, 19, 22, 24, 26, 30 33, 45, 48 etc.] The findings, more often than not, are that modification of the present code equations for ultimate load is necessary so as to include the effect of transverse reinforcements. In addition different researchers have recommended different formulae and proposed different hypotheses. This investigation started as far back as the 18th century and up till today there is no final ruling in any country.

The author therefore sincerely felt that as a positive step forward in the search for a better design procedure, time has come for critical reviews of all the proposed formulae with a view to approving one of them for adoption in design codes of Practice. For this exercise to improve design procedures, objective and bitter pruning is inevitable. The author therefore re-directed his objective which in a broad view can be summarized as follows:

A critical review and investigation of some proposed formulae for calculating the capacity of axially loaded

short reinforced concrete columns with a view to recommending one of them for adoption by the Nigerian Standards Organisation in a revised National Code of Practice, N.C.P. 1.

A formula is to be selected after theoritical considerations and then subjected to experimental investigations. A selected formula may or may not need adjustments depending on economic factors, possibility of use in design of offices, and experimental results.

2.11. Significance:

The above objective is necessary so that the anomaly in the present Code equation for reinforced concrete column capacity will be rectified to reflect the contribution of the ties and ensure that the properties of all column constituents are fully tapped to the best economic advantage of building materials. In the light of the foregoing discussion, it looks unnecessary to continue with the investigation on whether lateral ties have any material effect on the load-strain diagram of short columns since the results of such investigations more often than not have been "YES".

A change in method of facing the problem is not only important now but urgent. Time has come when the recommendations made so far by previous investigators should be critically reviewed and areas of disagreement resolved by carefully controlled tests with a view to improving design and construction procedures. This thesis lays the foundation upon which further investigations will be based.

2.20. Limitations:

As will be seen in the next chapter, over twenty different investigations have so far been carried out to examine the influence of transverse reinforcements on the performance of short concrete columns and at least seven different formulae have already been proposed by different researchers. It will not be possible and, in fact, not adviseable in one programme to aim at experimental investigations of all the proposed formulae especially as some of the equations are less likely to apply to generalized conditions and many will be too cumbersome to use in practice.

The sizes of columns tested in this investigation were limited by the maximum capacity of the available testing machine and by other factors like the volumetric capacity of the available concrete mixer and size of the vibrating table. Efforts were made, however, to simulate prototype sizes as much as possible.

2.21. Scope.

A careful study of all the available litterature on the subject showed that most of the proposed formulae were extracted from particular experimental results under specific conditions. Unfortunately, all the tests to date were carried out only on square and relatively small specimens and without regard to the number of legs in one link of tie. Also, most of the equations are characterized by use of arbitrary constants. Typical examples are those of King (1946) which expressed with familiar symbols will be as follows: [24]

$$P_u = A_c\, uc + 0.883\, A_s\, f_y + \frac{1{\cdot}5\, f_{yt}}{s_t} + 0.3 A_s\, f_y \frac{A_{st}}{s_t} f_{yt}$$

and Bunni (1964): [34]

$$P_u = 0.715\, x_1 + 0.928\, x_2 + 0.013 \frac{x_1^{\frac{1}{2}} x_3^{\frac{1}{2}}}{x_4^2} (x_2 + 115) - \frac{7.66}{10^4} \frac{x_1 x_2^{\frac{1}{2}} x_3 x_5^{\frac{1}{2}}}{x_4^2}$$

and others discussed in the next chapter. It is not likely that equations extracted from experimental results alone will urgently yield unchallengeable formulae since no single experiments can consider all necessary parameters and all other conditions discussed later in chapter 4.

It appears that a more brilliant method of attack is to face the problem from a purely analytical point of view based on accepted assumptions as was the case for beams and then to confirm the derived equation with experiments. In this thesis the Nigerian code equation as well as the only three equations derived through theoritical considerations alone were used as a basis for analysis. The three other equations are those derived by Blume, Newmark and Corning in 1961 expressed as follows:

$$f_c = 0.85\, f_c^1 + 4{\cdot}1 \frac{A_{st}\, f_{yt}}{a\ S_t}$$

The equation will often be referred to as the "PNC Equation" in this thesis and was recommended for adoption in the American Code by N.F. Sones [48] in 1970. The next equation was derived by S. Cleszkiewicz in 1973 [33] and

expressed as follows for all stages of loading:

$$P = A_b R_c \left(2 - \frac{\varepsilon_1}{\varepsilon_{bu}}\right) \frac{\varepsilon_1}{\varepsilon_{bu}} + \left(A_s + \frac{nA_{st}}{S_t} a_t\right) E_s \varepsilon_1$$

From this load-strain equation the ultimate load was expressed thus to give the last equation used in this thesis:

$$P_u = A_b R_c + A_s f_y + \frac{nA_{st}}{V S_t} a_t f_{yt}$$

The last two equations were investigated by Sharifullah Najib at the Technological University of Cracow. Different researchers may find different formulae more inviting and interesting.

Three column sizes of 10cm x 10cm; 10 x 15cm and 10 x 20 cm were selected to give side-width ratios of 1:1.0; 1:1.5; and 1:2.0. These dimensions were dictated by the limitations discussed in 2.20 and to satisfy the requirements of the preliminary calculations that the ultimate loads will lie within the 100-ton maximum capacity of the available testing machine and at the same time be fairly large enough to simulate prototype column sizes provided it will be possible to mix and vibrate each group of nine columns at the same time using one mixer and the mould fitting properly on the vibrating table.

CHAPTER 3

SUMMARY OF PREVIOUS INVESTIGATIONS

3.10. A. Considere of French (1899-1902):

The study of the effect of transverse reinforcement on the capacity of reinforced concrete columns started in France in the last century. Between 1899 and 1902, A. Considere carried out a series of tests on helically bound columns and from his results arrived at the following conclusions:

1. Helical reinforcements increase the strength of plain concrete columns considerably. This was because the helical reinforcement acted as a lateral support to the concrete and so decreased its lateral and longitudinal deformation.
2. This type of reinforcement prevents lateral buckling of the longitudinal steel and therefore increases the ultimate strength of the column.

He summarized his findings as follows:

> "Hooped concrete has a high ductility, its crushing strength is very high and exceeds the sum of the resistance at the concrete used used for it, the resistance of longitudinal reinforcing rods stressed up to their elastic limit" [24]

Formula proposed for calculating the capacity of columns with helical reinforcement based on Considere's findings was of the form:

$$\text{LOAD} = CA_k u + A_s f_y + kA_{st} f_{yt}$$

where:

C = a constant which varies from 1.5 according to Considere to 0.65 according to Building Research Technical Paper No.23 of the D S IR.

A_k = The area of concrete inside the spiral binding, that is the core area.

u = The ultimate cube strength of the concrete.

A_s = The cross-sectional area of longitudinal steel.

f_y = The tensile yield stress of longitudinal steel.

k = a constant varying from 1.5 to 2.5 according to Universities of Lehigh and Illinois "Journal of ACI, Feb. 1933" taken as 2.0.

A_{st} = The volume of spiral per unit length of column usually called the equivalent area of spiral reinforcement.

f_{yt} = The tensile yield stress of the spiral steel.

In the above formula, the contribution of the spiral is taken to be a function of its equivalent area, independently of its actual size, the pitch of its turns, or the presence of other steel.

3.20 Talbot (1907)

In 1907, Talbot [30] tested 18 concrete columns which consisted of seven plain, five reinforced with only longitudinal bars and six reinforced with lateral reinforcement in addition to longitudinal bars.

In cross-section, the columns were 9" x 9" and 12" x 12" (23.0 x 23.0cm and 30.5 x 30.5cm) and the percentages of longitudinal reinforcement were 1.21% and 1.52% while the percentage of lateral reinforcement varied between 0.12% to 0.22%. The spacing of the ties were the same as the lateral dimension of the columns, that is 9in. and 12in. respectively. The concrete cube crushing strength varied between 1472 and 2172 lb/in^2 (98 kg/cm^2 and 178 kG/cm^2).

In his tests, buckling of the longitudinal bars occured between the ties and the conclusion he drew was that lateral reinforcement had no effect on the ultimate load of columns.

The basic weakness of his test was that the spacing of the lateral ties was too much and he even admitted that such very low percentage of ties would hardly be expected to affect the column ultimate capacity.

3.30. Bach of Stuttgart [1905 and 1913]

Professor Bach's work on circular and rectangular columns [30] confirmed the findings of Mr. A. Considere. His conclusion was as follows:

> "The decrease in the pitch, increases the failing stress of the reinforced concrete columns".

3.40 Between the year 1907-1927, several investigations were carried out in different parts of the world by Graf, Withey, Van Emperger, Van Thullie, Rudolf and others.

3.50. Talbot [1927]

In his Ph.D. Thesis tendered in 1927, Talbot again re-examined the effect of square binders on the ultimate strength of columns.

His programme included tests on 78 columns with lengths ranging from 3.0ft. to 7.0ft. (92.0 to 214.0 cm) and square cross-section of 4.5 in. x 4.5 in. (11.4 cm x 11.4 cm), while the core section was 2.75 in. x 2.75 in., that is 7.0 cm x 7.0 cm. Four $\frac{1}{4}$" (6.35 mm) diameter bars consisted the longitudinal reinforcements.

As a transverse reinforcement in the form of binders, he used wires of cross-sectional dimension varying between 0.04" and 0.188 in. (1.0 mm and 4.8 mm) with spacings of 0.5" and 5.5" (1.27 cm and 14 cm). The concrete crushing strength was kept constant as much as possible throughout the investigation.

His conclusions were as follows:

1. By increasing the size of lateral reinforcement from 0.08" to 0.125" (that is 2.0 mm to 3.18 mm) the load carrying capacity was increased by 8.2%.
2. By decreasing the spacing of the lateral reinforcement from 5.5" to 0.5" (14.0 cm to 1.27 cm) the ultimate load of the column was increased by 33%.

However, Talbot failed to record both strain measurements and concrete and steel strengths for each column tested. This constituted a weakness and the effects of the lateral reinforcement could not have been trully determined. His conclusions were mere comparisons of ultimate loads. Moreover the percentage of the longitudinal reinforcement was rather too low, i.e. 0.97% which limited his investigation to a very small range of the problem.

3.60. J.W.H. King[1946-1949]

In 1946, Jeffrey Willian Hitchen King [22, 23, 24, 14] submitted the result of his tests to destruction of nearly 200 small columns with varying reinforcement which the author considers, for practical purposes, the premier proof that the ordinary hoop form of binder gives any material contribution to the ultimate strength of the column, contrary to the normal formula which is of the form:

$$\text{LOAD} = A_c uc + A_s f_y$$

where

A_c is the gross concrete area

u is the ultimate cube or cylinder strength of concrete

C is a constant which varies from 0.65 to 0.85

A_s is the area of longitudinal steel

f_y is its tensile yield stress.

Mr. King's results showed that the normal binders, if placed at a sufficiently small pitch, can contribute considerably to the load carrying capacity.

From the results of his tests, Mr. King evolved a formula which fitted the observed values of ultimate load of the columns within 4 percent. The columns from which the formula was derived were a standard size, 11.7 in. long x 3.5 in. square, having longitudinal bars, four in number, arranged as a square of overall dimensions 2.5 in. x 2.5 in. and binders of the same internal dimension. Working with a concrete giving a cube strength on 4 in. cubes at 28 days of about 4,000 lbs. per sq.in., and a range of longitudinal steel from $\frac{1}{2}$ in. bars to zero, with binders of pitches from $\frac{3}{4}$ in. to $11\frac{1}{2}$ in. of 14 to 4 gauge bright drawn wire, the conclusions were:

1. If at close enough pitch, normal binders appear to have a material effect on the strength of reinforced columns, and that such effect is a function of their yield load, their pitch, and the strength of the longitudinal steel.
2. The manner of failure of the main steel is influenced by the amount of lateral steel, which if sufficiently important may cause the main bars to fail by direct compression rather than buckling, in which case recovery of the column after overload is present.

3. The effect of all factors within the range of the experiments obeyed the law

$$LOAD = A.w + Bx + C.y + D.z$$

where these have significance as follows:

w = Concrete area x cube strength = Ac.u

x = Total tensile yield load of all main steel = Y_1

y = Yield load of binders steel/pitch^2 = Y_b/p^2

z = Product of x and y = $Y_1 \ Y_b/p^2$

A = A factor which varied from 0.84 to 0.94, and averaged 0.865.

B = 0.883, C = 1.5, and D = 0.30.

4. The effect of the main steel can be apparently increased to well above its yield load if binders are sufficiently closely spaced. Owing to the practical difficulty of reinforcing too heavily, and to the fact that binders are naturally more expensive than the same weight of main bar, it would not appear to be a really practical proposition to make such columns for normal work. Where the property of recovery after overload is desirable, it may be useful to have such heavily bound columns, in which case it is felt that the binder effect should be allowed on the lines indicated. In columns as normally constructed today, it is insignificant.

5. A heavily bound column will still sustain considerable percentage of its crippling load, even immediately after the application of such a load, to such an extent as to cause

a strain of over 4%, whereas a lightly bound column is much more liable to fail completely with such treatment.

6. Provided the binders are adequately secured at all contacts with the main steel, it does not appear to be important whether they are welded, just tied, or are in the form of a continuous helix, square in plan, instead of being individual binders at each pitch.

7. The terms, sudden failure, and gradual failure, applied to brittle or semi-brittle specimens tested to destruction, have little significance, unless all factors are known, since it is much more likely to be a fuction of the testing machine than of the specimen. The resilence of the machine itself appears to play a most important part in all such tests.

3.70. S. Mackey [1954]

In 1954, Dr. S. Mackey [25, 26, 27] published the results of his series of investigations on 4 in. square columns reinforced longitudinally with four bright steel rods, $\frac{3}{4}$ in. diameter and binders of 8 gauge soft iron wire welded to the longitudinal rods at $1\frac{1}{2}$ in. centres. His expression for the yield or cracking load for a short practical column was as follows:

$$P_u = 0.68 A_c u + k A_s f_y \qquad \text{----------[4]}$$

where

P_u - yield load of column

A_c - cross-sectional area of concrete in column

u - cube crushing strength

A_s - cross-sectional area of compression steel

f_y - yield stress of compression steel

k - a constant, the value of which will depend upon the quality and arrangement of the lateral reinforcement and the strength of the cover concrete.

If the ultimate column load (W) is in excess of the yield load and is obtained after the outer cover has spalled off, its value may be estimated by substituting the core area A_k for the gross area A_c in the above equation and taking 0.77 times the concrete cube strength to allow for the lateral restraint developed by the binders. The factor k should be reduced slightly below the corresponding value used in determining the yield load to allow for the loss of support afforded by the cover concrete, but this reduction will be offset by the substitution of the ultimate stress f_w for the yield stress f_y in the above equation. The final expression for P is then given by

$$P = 0.77 A_k u + k_1 A_c f_w \quad \text{----------} \quad [4a]$$

The values of k and k_1 to be adopted for practical design purposes can only be obtained from a comprehensive series of column tests and until these have been carried out over the range of variation of lateral reinforcement permitted in the code of Practice (C.P. 114), Dr. Mackey suggested a tentative value of k = 0.9 for estimation of the yield load of axially-loaded short reinforced concrete tied columns.

In another series of investigations carried out by Mackey, [26] with varying spacings and sizes of lateral reinforcement and slenderness ratio, he arrived at the following general conclusions:

1. A marked increase in the strength of reinforced columns is obtained with decrease in the slenderness ratio within the "short column" range. This increase is not very significant as the $^{L}/_{D}$ value is reduced from 15 to 12 but thereafter any further reduction yields a noticeable improvement in column carrying capacity.

   Determination of the extent of the "short column" range from tests on plain columns leads to conservative design and if full value is to be derived from the additional strength found in reinforced specimens, a modification of the present ruling on this matter is desirable.

2. Tied concrete columns require a certain minimum quantity of lateral reinforcement to develop the full yield-strength of the longitudinal steel. The actual quantity required is a function of the load-bearing capacity of the concrete and steel, and of the ratio of core area to gross area. In all cases examined it appears to be well above the minimum requirements for lateral reinforcement laid down in the British Standard Code C.P. 114. Any increase in the quantity of binders above this minimum value directly increases the ratio of ultimate column-load/yield load. Such increases cannot be used directly

for design purposes, since in this instance, the yield load must be regarded as the critical load. They do, however, give added security against collapse and can be utilised to reduce the factor of safety based on yielding to a practical minimum.

It is doubtful whether large increases in the quantity of lateral reinforcement will be employed for economic reasons, since spirally reinforced columns having a circular core are inherently better than columns with square cores and individual binders. When high tensile steel and steel having no defined yield point is used as main reinforcement, any increase in the percentage of binder steel will contribute directly to the ultimate column load due to the greater column strain developed before failure occurs. In determining the reinforcement for such columns it should be borne in mind that the critical binder percentage necessary to develop the full column yield load, increases with decreasing main steel percentage, indicating that the major function of the lateral binding is to increase the carrying capacity of the core concrete by restricting its lateral expansion. In columns having eight longitudinal bars symmetrically placed about the axes, a marked increase in load carrying capacity will be obtained if alternate binders are placed parallel to and at 45 degrees to the column faces.

3.80 T. Szulczynski of Gdansk [1961]

In 1961, Mr. T. Szulczynski presented the results of his tests on concrete prisms under axial compression [30] His findings on the effect of rectilinear transverse reinforcement on the unit strength of concrete in the specimen was expressed as follows:

$$f_c = R_c + 1.8 f_2 \qquad \text{[6.1]}$$

and

$$f_2 = \frac{2A_{st} f_{yt}}{a.S_t} \cdot \frac{1 + \frac{h}{a}}{1 - \left(\frac{h}{a}\right)^2} = \frac{P f_{yt}}{\left(\frac{a}{h} + \frac{h}{a}\right)^2} \qquad \text{[6.2]}$$

where:

f_c = Unit strength in compression

R_c = Unit strength of prism without reinforcement

A_{st} = cross sectional area of transverse reinforcement

a = width of the enclosed section

h = depth of the enclosed section

s_t = longitudinal spacing of the transverse reinforcement

p'' = volumetric ratio of the transverse reinforcement.

Szulczynski also indicates that "the behaviour of concrete in compression zone of beams with compression steel indicates that the combined action of longitudinal reinforcement with the ties may improve the ductility considerably [30]

3.90 Blume, J.A. Newmark, N.M., Corning L.H. [1961]

The equation put forward by the above authors from their research results was on compressive strength of confined concrete and expressed as follows: [48]

$$f_c = 0.85\ f'_c + \frac{4.1\ A_{sb}\ f_y}{a\ S_t} \qquad \text{----------[1]}$$

where

f'_c = compressive strength of concrete based on 6 by 12 in. cylinders

f_y = yield strength of hoop

A_{sb} = area of cross-section of a leg of hoop reinforcement

a = longer outside dimension of rectangular hoop

S_t = centre-to-centre spacing of hoop

3.100. Roy, H.E.H. and Sozen, M.A. [1964]

In 1964, Roy and Sozen submitted a paper captioned "Ductility of concrete" for discussion at the International Symposium on Flexural Mechanics of Reinforced Concrete [44, 49, 51]
Their primary objective was to obtain data on the load-deformation characteristics of concrete as related to the rotation capacity of reinforced concrete connections, in other words, to find out the effect of rectangular transverse reinforcement on the strength and ductility of confined concrete. The programme consisted of tests to destruction on 60 axially loaded columns of 5 by 5 by 25-in. prisms.

The variables in the experimental programme were the spacing of the ties and the amount of longitudinal reinforcement, the volumetric ratio of the transverse reinforcement being kept constant at 0.02 and the target cylinder strength was 3500 p.s.i. The centre-to-centre spacing of the ties was varied from 2 to 8 in. in 2-in. increments.

The conclusion drawn from the investigation was that the load-carrying capacity of the concrete in the axially-loaded specimens was not improved by the use of rectilinear ties, but there was a considerable increase of its ductility.

3.200. **N.G. Bunni of England [1964]**

In August 1964 Bunni [30] submitted his Ph.D. thesis on "The Effect of Lateral Reinforcement on the Strength of Reinforced Columns" to the University of London, Queen Mary College.

From the results of his investigations, he recommended the following equation for calculating the ultimate load of axially loaded columns.

$$Z_o = 0.715x_1 + 0.928x_2 + 0.013\frac{x_1^{\frac{1}{2}}x_3^{\frac{1}{2}}}{x_4^2}(x_2 + 115.0) - \frac{7.66}{10^4}\frac{x_1 x_2^{\frac{1}{2}} x_3^{\frac{1}{2}} x_5}{x_4^3}$$

where:

Z_o = is the ultimate load in 1000 lbf

x_1 = is the concrete load in 1000 lbf calculated as the net area of concrete times its 4 in. cube strength

x_2 - is the load in 1000 lbf, calculated as the area of longitudinal steel times its yield stress.

x_3 - is the binder load in 1000 lbf, calculated as the area of one binder times its yield stress.

x_4 - is the spacing of the lateral reinforcement in inches.

x_5 - is the slenderness ratio.

The constants 0.013; 115 and $7.66/10^4$ had the following dimensions: in^2/lbf; lbf; and in^3/lbf respectively. This equation consists of four terms, the first represents the load taken by concrete, the second represents the load taken by longitudinal reinforcement, the third gives the effect of lateral reinforcement on the concrete and the longitudinals, and the fourth term indicates the effect of the slenderness ratio on the ultimate strength of a column. Dunni noted: "The last term shows clearly the factors upon which the effect of the slenderness ratio depends. The most important reinforcement since it raised to the power of three which indicates that this effect decreases rapidly with an increase in spacing".

3.300. Fred. M. Hudson [Auburn University, 1965]

In the early 1960's a programme of tests was carried out at the Auburn University, Auburn, Ala. under Professor F.M. Hudson [19]. The first programme which was initiated as a subject of graduate research consisted of two investigations by two graduate students as follows:

1. Francis, R.C., Jr. "An Investigation of lateral Tie Spacing in Short Reinforced Concrete Columns".
2. Petrelis, C. "An Investigation of Lateral Tie Spacing of small Reinforced Concrete Columns Eccentrically Loaded. M.Sc. Thesis, Anburn University, 1962.

The above two research works were experimental studies of 4 in. square columns in which 16 specimens were tested in the axially loaded series and 16 in the eccentrically loaded tests. In programme II a total of 32 columns of cross-sectional dimension 6 in. x 6 in. were tested, 16 loaded axially and 16 eccentrially. Tie spacing was the only variable in programme I but in programme II the additional variables of concrete strength and percentage of longitudinal reinforcement were introduced. Tie spacings were 4 in. (Nigerian Standard Code of Practice Maximum), 6 in. 8 in. and 31 in. for each series of 16 columns in 4 groups of four columns each. The findings were as follows:

1. The influence of lateral ties on the ultimate load is negligible.
2. The ties exerted a strong influence on the mode of failure of the axially loaded members.
3. Axially loaded columns (max. length was 4 ft.) with only end ties failed almost invariably by vertical splitting along or close to the centre line. The final failure occured at a load roughly 20 percent greater than the load at which the split appeared to be complete.

3.40C Norman F. Somes [July, 1970]

In his investigations published in July, 1970 in the Proceedings of the American Society of Civil Engineers, Dr. Somes [46] considered only the following two variables, keeping all other factors constant:

1. Spacing of hoops
2. Cross-sectional area of hoops

His selected test specimen was a prism of square section 4 in. by 4 in. and height 12 in. with a concrete cover of zero. Within the limits of his experiments, he drew the following conclusions from the results:

1. The above factors, namely the spacing of the hoops and the cross-sectional area of the hoop influence the short term strength and ductility of concrete when the hoop spacing does not exceed a certain value. This spacing was approximately 6 in. for the prismatic specimens reported in Dr. Somes' publication for specimens of section 4 in. square and which were reinforced with square hoops.
2. The increase in strength over that for plain concrete was shown to be proportional to the cross-sectional area of the hoop and inversely proportional to the hoop spacing.
3. His results confirm the form of the equation put forward by Blume, Newmark and Corning (see article 3.90) in which the direct compressive strength of concrete confined in this

way is given by:

$$f_c = 0.85\, f'_c + \frac{4.1\, f_y}{a}\,(A_{sb})\left(\frac{1}{s}\right)$$

in which the parameters in parentheses indicate the variables which were studied by Dr. Somes. Dr. Somes showed that within the range of his investigations and for the variables considered, the above equation predicts the strength of concrete reinforced by hoops with satisfactory accuracy.

3.500. M. Sargin, S.K. Ghosh and Dr. V.K. Handa [1971]

In June 1971, Sargin, Ghosh and Handa all of the University of Waterloo, Ontario, Canada, submitted a paper on "Effects of Lateral Reinforcement upon the strength and deformation properties of concrete" to the "Magazine of Concrete Research" [45]. Their paper discussed the effect of rectilinear lateral reinforcement as interpreted from experimental findings and theoritical considerations. In the analysis of their test results, laterally reinforced specimens were treated as composite members consisting of core and cover. The parameters of an analytical stress-strain relationship which was found to describe the mechanical properties of plain and laterally reinforced concrete adequately, were determined from a regression analysis of the test results. The experimental results were compared with the strength of confined concrete as predicted by (1) a formula based on a theoritical analysis and (2) an equation which unifies several previously proposed formula giving the strength of laterally reinforced concrete.

In their investigation, a total of 63 specimens were tested in four different sets. The specimens were 125 x 125 x 510 mm concrete prisms, none having any longitudinal reinforcement . The principal variables in their investigation were:

1. strength of concrete;
2. type of lateral reinforcement (ties made of plain or deformed bars or plain sheet steel envelopes);
3. bar diameter or sheet thickness;
4. tie spacing;
5. type of steel used as lateral reinforcement;
6. enclosed core size or cover thickness;
7. strain gradients and
8. casting position.

The conclusions arrived at from their study are summarized as follows:

1. A laterally reinforced concrete member should be treated as a composite member consisting of a confined core and unconfined core.
2. The amount of confinement provided by lateral reinforcement, and hence the effects of lateral reinforcement upon the force-deformation characteristics of bound concrete are dependent not only upon the volumetric ratio of laterals, but also upon the type of lateral reinforcement (square or rectangular ties, spirals, envelopes), etc. the spacing and grade of lateral reinforcement, and the quality of confined concrete.

Spacing is the most important parameter, because the choice of bar sizes, and of qualities of concrete and steel is rather limited in practice. The effect of transverse reinforcement decreases considerably with increasing spacing and becomes negligible for spacings larger than the dimensions of the core.

4. Lateral reinforcement improves the strength and ductility of confined concrete, it has a detrimental effect upon the cover. Depending upon the arrangement of lateral reinforcement and the relative areas of core and cover, the over-all load-carrying capacity of a laterally reinforced concrete may well be less than that of an identical plain concrete member. In all cases, however, the presence of lateral reinforcement appears to improve the over-all ductility of a

5. It follows from the above that the ductility and hence rotation capacity of the so-called hinging regions in reinforced concrete members can be improved to a large extent through the use of lateral reinforcement.

3.600. Sylwester Oleszkiewicz [1973]

In 1973, Sylwester Oleskziewicz [33] approached the problem analytically by considering the pressures and internal forces of an externally loaded short reinforced concrete, column. Basing his analysis on Madrit's second degree parabolic equation

for the stress-strain diagram of ordinary concrete, he derived the following expressions for the load-strain relation and ultimate load of short axially loaded reinforced concrete columns.

1. $$N_u = A_b R_c + A_s f_y + \frac{mA_{st}\, a_t}{V\ S_t} f_{yt}$$

For a stress analysis of columns the following equation may be used.

2. $$N = A_b R_c \left(2 - \frac{\varepsilon_1}{\varepsilon_{bu}}\right) \frac{\varepsilon_1}{\varepsilon_{bu}} + \left(A_s + \frac{mA_{st}\, a_t}{S_t}\right) E_s \varepsilon_L$$

Conclusively, he added:

"The transverse reinforcement of short lightweight concrete columns should be stipulated by an inferior limit of lateral percentage of binding steel" [33] The minimum percentage of lateral reinforcement is recommended by the formula

3. $$P_t = 0{,}23\, R_c^{2/3} f_{yt}^{-1}$$

and additional stipulations for binding steel are therefore considered unnecessary.

Notations:

N_u - denotes the ultimate capacity of the columns

A_b - denotes the cross-sectional area of the columns

R_c - denotes the characteristic strength of concrete

A_s - denotes the sectional area of longitudinal reinforcement

f_y - denotes the yield strength of longitudinal reinforcement

n - denotes the leg number of one link

A_{st} - denotes the area of one leg of the link

a_t - denotes the lateral dimension of the column core

V - denotes the Poisson's ratio

S_t - denotes the link pitch

f_{yt} - denotes the yielding strength of the transverse reinforcement

N - denotes the actual load

ε_1 - denotes the actual longitudinal unit deformation of the column

ε_{bu} - denotes the ultimate unit feformation of concrete reached in compression at the peak load

E_s - Modulus of elasticity of reinforcing steel.

The relation (2) is valid for a range of longitudinal deformation ε_1 from zero to the lower of the two following calues ε_{bu} or $\frac{f_y}{E_s}$

and the core Area A_c is assumed as equal to a_t^2

As the above formulae form a base for theoritical considerations and experimental investigation in this thesis, it will be dealt with lengthily during discussion of results. The derivations will also be presented.

3.700. J. Oleszkiewicz [1973]

In 1973 J. Oleszkiewicz [30] carried out tests on 34 short reinforced and plain lightweight concrete columns of size 15 cm by 15 cm and slenderness retio of 4. Longitudinal reinforcement

consisted of 4 ϕ 10 and 4 ϕ 14 deformed steel while binders of 4.5 mm in diameter at spacings of 20.0 cm - 12.5 cm and 5.0 cm were used. A summary of his conclusions are as follows:

1. The lateral reinforcement of columns increases their ultimate strain, making them more ductile, thus avoiding a rather undesirable quality that is brittleness.
2. A richer lateral reinforcement is needed for a proper ductility of keramzite concrete columns. As it has been tested, heavy binders increase the strength of columns, this fact may be taken into account when considering dependence of columns bearing capacity upon their stirrups. The bearing capacities calculated according to the formula given by Polish Code of Reinforced Concrete, differ significantly from the test results obtained by the author. The differences were as high as 16.5% on the behalf of the test results because the work of stirrups had not been sufficiently taken into consideration.

3.800. Sharifullah Majib [1973]

In 1973, Sharifullah Majib [30] submitted his doctoral thesis on "Analysis of the Influence of Transverse Reinforcement on the Short Concrete Column Capacity". In his work 76 columns of 15 cm by 15 cm cross-section with a slenderness ratio of 4 were tested. The percentage of transverse reinforcement was varied between 0,059% to 1.68%, and the percentage of longitudinal

reinforcement was varied from 0.0% to 2.75%.

His conclusion are as follows:

1. A heavy transverse reinforcement decreases the lateral deformation of the column under a given axial load and increases its ultimate longitudinal strain as well as the bearing capacity. Within the range of his investigations, his result confirmed S. Oleszkiewicz's' formulae,

$$N = A_b R_c \left(2 - \frac{\varepsilon_1}{\varepsilon_{bu}}\right) \frac{\varepsilon_1}{\varepsilon_{bu}} + A_s E_s \varepsilon_1 + \frac{mA_{st}\, a_t}{S_t} E_s \varepsilon_1$$

and $$N_u = A_b R_c + A_s f_y + \frac{m A_{st}\, a_t}{V.S_t} f_{yt}$$

where the terms are as given in 3.500.

2. An intensive transverse reinforcement prevents the concrete column against premature failure and sudden collapse. He established the inferior limit for transverse reinforcement of square short reforced concrete column by the relation:

$$P_{t\,min} = \frac{m.A_{st}}{a_t\ S_t} \quad \frac{R_b}{f_{yt}} = \frac{R_c^2}{2.f_{yt}}$$

3. When too intensive transverse reforcement is applied, the concrete may be destroyed before the ultimate capacity of the column is reached. To avoid this phenomenon he established the upper limit for transverse reinforcement of axially loaded short concrete columns by the expression:

$$P_{tmax} = \frac{m.A_{st}}{a_t\ S_t} \quad V\ R_c\ f_{yt}^{-1}$$

4. For the inferior limits of longitudinal reinforcements recommended in Codes of Practices and for the inferior limit of transverse reinforcement recommended above, he recommended the following formula for determination of an inferior coefficient for todal reinforcement:

$$k_{min} = \frac{N_u - A_s R_c}{A_b f_y} \qquad P_{lmin} + \frac{C}{V} \frac{f_{yt}}{f_y} P_{tmin}$$

5. The ultimate longitudinal strain of short R.C. columns may be calculated by:

$$\varepsilon_{bu} = \varepsilon_{lu} + 0.05 \left(\frac{m.A_{st}}{a_t S_t}\right)^{\frac{1}{2}}$$

while the ultimate lateral deformation may be calculated from

$$\varepsilon_{tu} = V\left[\varepsilon_{lu} + 0.05\left(\frac{m.A_{st}}{a_t S_t}\right)^{\frac{1}{2}}\right]$$

The above formulae may be used for determining the maximum yielding strengths of reinforcement as follows:

$$f_y < \alpha E_s \varepsilon_{bu}$$

and $f_{yt} < \alpha E_s \varepsilon_{tu}$

where is the level of loading by which the yield strength of reinforcement has reached. In his investigations reached the values of 0.85 to 0.90 in columns with

closely spaced binders while the value was much higher for columns with widely spaced binders.

6. For the established k_{min} that is for P_{lmin}; P_{tmin}; f_{ymax}; f_{ytmax} and R_c, Najib suggested the following design prodedures:

- To assume k values and find the required cross-sectional area of the column from the expression

$$A_b = \frac{N_u}{R_c + k_{fy}}$$

or reverse to assume A_b and find the value of k.

- For the given or assumed k value to determine the required percentage of transverse reinforcement corresponding to established P_{lmin}, from the expression:

$$P_t = \frac{V}{C} (k - P_{lmin})$$

- To determine the number and size of longitudinal bars as well as the diameter and spacing of binders. These should meet the general structural requirements of the codes of practice. Minimum size and spacing of binders should not be less than that recommended by the codes for spiral columns.

CHAPTER 4

DISCUSSION OF PREVIOUS RECOMMENDATIONS

Discussions in the last chapter show that as far as the author is aware, about twenty different investigations and perhaps many more have been carried out to determine possible effects of transverse reinforcements in concrete columns. The conclusions as presented by different investigators are often confusing and contradicting and generally fall under one of the following two categories:

1. Lateral reinforcement has a significant effect on reinforced concrete columns.
2. Lateral reinforcement has a negligible effect on reinforced concrete columns. These two will be referred to in this thesis as "Conclusion I" and "Conclusion 2" respectively.

These conclusions are not hypothetical but backed by experimental findings, and yet only one of them is probably fool-proof and may be applicable in design procedure. This means that a particular conclusion depends greatly on the test procedure, interpretation of results and method of manipulating the salient parameters. For instance, researchers who arrived at conclusion 2 were mainly those that used very large spacing for the lateral reinforcement [19, 30, 44] while those that used small spacings or relatively large percentages of binders arrived at conclusion 1. [22, 24, 26, 27, 30, 33, 45, 48 etc.]

It means then that the value of results obtained by any investigator

is arrogantly dictated by the test arrangement. Secondly, it is not yet very clear which parameters are inevitable in experiments since there are up to sixteen and perhaps many more that would possibly affect the results, namely: (1) Spacing of the ties; (2) cross-sectional area of the ties (3) strain gradient accross the section; (4) the manner and rate of loading; (5) stress-strain curve for the hoop reinforcement; (6) effectiveness of the tie in confining the concrete; (7) area of concrete enclosed by the tie (8) shape of the enclosed area (9) concrete cover to the enclosed area (10) presence, size and location of the main reinforcement; (11) compressive strength of the concrete; (12) maximum size of aggregate; (13) slenderness ratio; (14) casting and curing procedures; (15) number of legs in one link of tie; (16) side-width ratio of cross-sectional dimension. No single investigation can consider all the parameters listed above and this problem limits the range of experimental deductions. For a general ultimate load equation to emerge from interpretation of experimental results alone, a wide range of the problem must be explored and the effect of all parameters investigated For instance, the "short" and "long" column ranges must be investigated and different cross-sections of the square and rectangular types must be tested and so forth. To crown all, the investigation should finally end with tests on normal-size prototype columns as encountered in practice because all the tests to date have been carried out on relatively small sections without regard to the area of concrete enclosed by the hoops. This procedure of purely experimental approach

will be very laborious and in some cases impossible to achieve, like testing very long, full sized, heavily-reinforced chlumns; as machines with enough capacity to crush such columns may be difficult to come by.

It appears, therefore, that a more clever method of attack is first to approach the problem from a purely analytical point of view based on unchallengeable assumptions or established facts. For this latter procedure, the only possible source of error will be in the method of viewing the stresses within an element. But then the possibility of hitting the right target at first trial is still there, but a final verdict must be backed by results of carefully controlled experimental programmes using random or arbitrarily selected cross-sections and covering all necessary parameters.

This approach presumes that a complete structural analysis of the forces inside and outside a loaded reinforced concrete column from theoritical considerations alone may be possible. If this is so, then the resulting equation must be confirmed through a systematic programme of experiments.

The author is not alone in this school of thought. The first attempt to derive a compressive strength equation was made in 1961 by Blume, Newmark and Corning [48] in which they expressed the compressive strength of confined concrete as follows:

$$f_o = 0.85\, f'_c + \frac{4.1\, A_{sb}\, f_y}{a.s}$$

Where the terms have the definitions given in article 3.90. This equation is one of those selected for experimental investigation and will be referred to as "the B.N.C. equation " in this thesis. Its derivation is discussed during the interpretation of experimental results.

Dr. Somes [48] in 1970 recommended the form of the B.N.C. equation. Somes' investigations kept the concrete cover constant at zero. The variables in his tests were spacing and size of tie which are perhaps the most important parameters in this type of investigation. Accurate strain measurements were not made rather, Somes recorded the overall contraction on his 12 in. column. and this overall contraction divided by 12 in. (total length of column) was termed "Nominal Strain". Somes had enough facilities and his programme was carefully planned. One of his three groups of columns with small diameter binders of 0.27 cm^2 showed that the B.N.C. equation often over estimates slightly the increase in strength due to the hoops. Somes' finding, however, was that the B.N.C. equation leads to a good estimation of the increase in strength due to the hoops. Perhaps, a point that exposes Somes test to challenge is the "Nominal Strain" procedure. It would have been better to record the strain at convenient load intervals in order to understand fully, the load strain relationship.

On the other hand, the failure of Professor Hudson's programme of experiments [19] to show any difference in the ultimate load of columns due to variations in tie spacings lies in the design of

his test specimens. Tie spacings in Hudson's investigations were 4 in. (Nigerian Standard Code of Practice maximum), 6 in., 8 in., and 31 in. (end ties only). Definitely, as tie spacing becomes too large, the effect of the ties becomes increasingly negligible. Had Hudson decreased instead of increasing the tie spacings below the code maximum, he could have observed some changes in the load-strain curves.

Moreover, he failed to obtain the actual yield strength of the steel used in his experiments; thus, the calculated ultimate load values are only approximate, at best.

An important aspect of the problem which has hitherto received little attention is the possibility of imposing not only a lower limit but also an upper limit to the amount of lateral reinforcement required in any particular design. So far, as far as the author is aware, Najib's investigation is the first to deal with this aspect. The author is also interested in finding out whether this aspect of the problem is necessary. Najib's equation [30] will therefore, be further dealt with during the discussion of experimental results.

The investigations of Roy and Sozen (article 3.100) confirm that concrete is a "ductile" material in that it can undergo large compressive strains, but contrary to the behaviour of steel, increasing strain beyond maximum stress is accompanied by a decrease in stress. The investigations of Vitelmo, V. Bertero and C. Polippa [51] on 3 x 3 x 12-in. beams confirmed that while the presence of longitudinal

reinforcement alone does not seem to enhance the ductility of concrete, ties without longitudinal reinforcement considerably improve the deformation capacity of confined concrete. In addition, however, they noted:

> "use of longitudinal reinforcement without ties does not affect the strength of the concrete. Ties with and without longitudinal reinforcement did enhance the load carrying capacity, of the concrete in the prisms" [51]

This contradicts Roy and Sozens findings on the effect of ties on restrained concrete.

The latest published and perhaps one of the most reliable attempts, as far as the author is aware, at an analytical derivation of an ultimate load formula for short axially loaded reinforced concrete columns was by Professor S. Cleszkiewicz, [33]. Based on the accepted stress-strain relation for normal concrete, he derived the following load-strain and ultimate load equations through consideration of the internal forces of an axially loaded column.

1. $N_u = A_b R_c + A_s f_y + \frac{m A_{st} a_t}{V S_t} f_{yt}$

2. $N = A_b R_c \left(2 - \frac{\varepsilon_1}{\varepsilon_{bu}}\right) \frac{\varepsilon_1}{\varepsilon_{bu}} + \left(A_{st} + \frac{m A_{st} a_t}{S_t}\right) E_s \varepsilon_1$

where the terms have the meanings given in article 3.600.

The above formulae are effectively utilised as a basis for interpretation of experimental results and as their derivations are discussed alongside the experimental results, they will not be presented

here to limit duplicity. This thesis does not assume that formulae recommended by other researchers through the interpretation of their test results are wrong. Perhaps, future researchers may find another equation more interesting or perhaps develop new ones. There are however, certain loop-holes that should not be over looked while considering any proposed equation for generalization and possible adoption.

For instance, based on Considere's findings (see 3.10), the formula proposed for the capacity of helically bound columns was:

$$P_u = A_k uC + A_s f_y + A_{st} f_{yt}$$

where the notations are as given in article 3.10.

The report of Committee 105 of the American Concrete Institute, [24] who were the initiators of the Lehigh and Illinois work, suggests that all columns be designed on the basis of an ultimate load arrived at by taking C as 0.85, k as zero, and using A_c the gross concrete area instead of A_k as suggested above, though a minority report recommends that the spiral effect should be counted in as above, with k = 2, for all spiral steel in excess of that required to give a strength equal to the concrete shell outside the spiral.

In the above formulae, the contribution of the spiral is taken to be a function of its equivalent area, independently of its actual size, the pitch of its turns, or the presense of other steel. Such an effect can be theoritically viewed by considering the spiral to be transformed into a cylinderical shell having the same volume of

steel, and considering the concrete to behave as a sand pile. Considere, in utilizing this justification stated that the connection between the active pressure and the resulting pressure in a perpendicular direction was $\tan^2(\phi/2)$, whereas Rankine's value is $\tan^2(\pi/4 - \phi/2)$. However, he subsequently made use of a figure of 4.8 for its reciprocal, which he stated was an experimental value, and thus arrived at a figure of 2.4 for k. He later claimed that his experiments fixed this figure at 2.1, which figure was approximately confirmed by Leihgh and Illinois Universities.

However, these findings were later criticised by Mr. J.W.H. King in 1946 [24]. Mr. King believed that the main contribution of the spiral is in improving the resistance of the longitudinal steel rather than that of concrete. The fact that k was then found to vary from 1.5 to 2.5 would hardly appear to be a justification for a figure of 2, under these circumstances. Also Considere's conclusions appear to be based on the results of only 13 tests which would not appear to have been very reliable in these days.

Another formula which is less bulky than those of Szulczynski and Bunni is Dr. Mackey's formula which was extracted from experimental results and given as:

$$W_1 = 0.68 A_g u + k A_c y$$

or $$W_1 = 0.77 A_q u + k_1 A_c w$$

depending on the conditions outlined in article 3.70. It is doubtful whether this formula can stand the test of generalization as provision

is not made for possible action of binders alone. Rather, the influence of binders is integrated into the effect of the longitudinal steel with a factor k or k_1 which also depends on the strength of the cover concrete. The value of k suggested by Mackey tentatively is 0.9 (less than unity) which implies that the binder decreases the yield load of the columns. The conclusions of his later experiments, [26] however, contradicted this implication. He later observed:

> "Any increase in the quantity of binders above this minimum value directly increases the ratio of ultimate column load/yield load".

Perhaps, it may not be in the interest of knowledge to deal further with Mackey's formula.

CHAPTER 5

MATERIALS AND CONCRETE MIXES

5.10 Materials:

All materials used in this investigation were handled and stored in such a manner as to prevent damage, deterioration or contamination as much as possible before being utilised.

5.11 Aggregates:

Materials used as aggregates were kept free from adherring coatings, clay lumps or impurities that may impair the strength of the concrete. Compositions were batched by weight using an "Avery" weighing machine, type 3205 CCE of maximum capacity 250kg. in 1 kg. divisions.

Fine Aggregate: River sand was used as fine aggregate and was free from silt, clay and coagulated lumps. The sand was obtained from a local Zaria stream. The grading was such that the maximum size was that which passed through mesh No.7 of the British Standard (B.S.) Sieves fitted to B.S.410.

Coarse Aggregate: The coarse aggregate consisted of crushed rock kept almost free from fine particles occuring either in a free state or as a loose adherent. The grading was such that the size was kept within the limits of 9.53 mm ($\frac{3}{8}$") maximum and 4.76mm (3/16") minimum. The crushed rock was obtained from Kabala quarry at Kaduna. The stones were sieved to the required sizes before utilization.

5.12 Cement:

Ordinary Portland cement, the "Elephant" brand manufactured by the "West African Portland cement Company Ltd." Lagos at their Ewekoro Factory in Abeokuta, Western Nigeria and delivered in paper bags was used for the tests. The exact age of the cement was unknown but consignments were used in the order of delivery.

5.13 Reinforcement

Steel reinforcements were plain round mild stool bars. The place of manufacture was not known and it was not possible in the case of longitudinal bars to obtain bright pieces but all reinforcements were as much as possible kept free from loose rust, grease, adherring earth or any material that may impair the bond between concrete and the reinforcement. The size and percentages of both longitudinal and transverse reinforcements are as described in chapter 6 (article 6.31) and shown in fig.4.

5.14 The "Avery" Universal Testing Machine:

The columns were tested by loading to failure in a large, electrically operated testing machine shown in plate 1.(p156) .The maximum capacity of the machine is 100-tons and it was despatched to Ahmadu Bello University Zaria from England in 1967. The machine is fitted with electrical autographic recorder, load pacing gear, automatic load control and is of the type 7104 DCJ

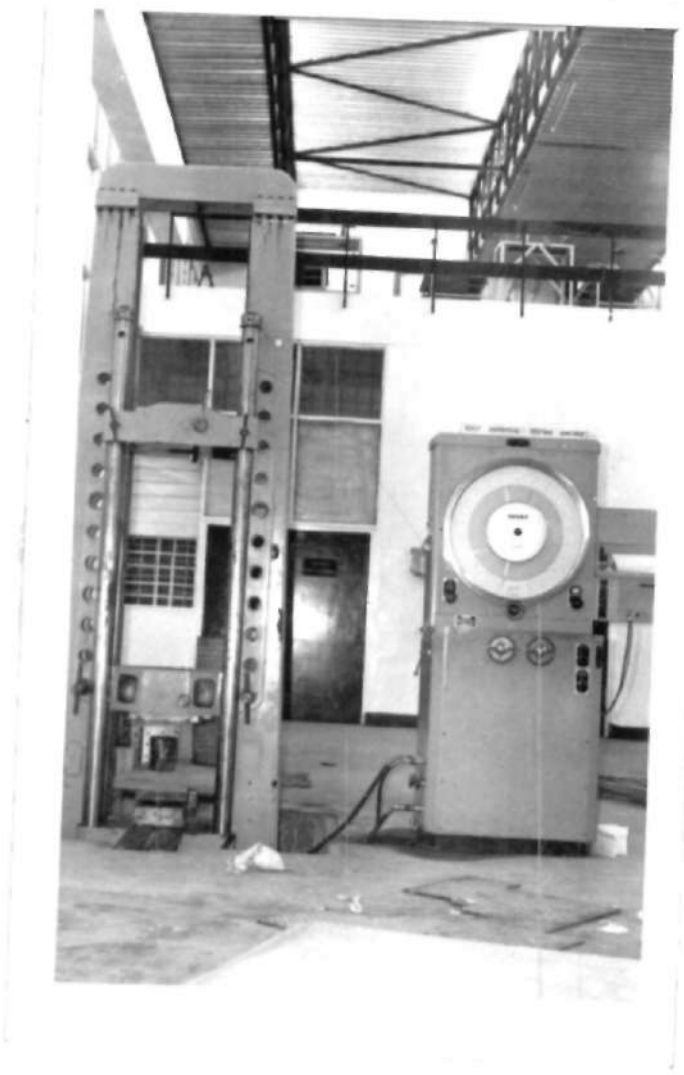


PLATE 1: THE "AVERY" SELF INDICATING
UNIVERSAL TESTING MACHINE

number E66200 made by Avery in Birmingham. The delivery number to Ahmadu Bello University is:

ADU 263/65-66/29/802501; AHM.ADU 6/21804/1

The machine has five load ranges of 5 tons in 0.01 ton divisions; 50 tons in 0.1 ton divisions and 100 tons in 0.2 ton divisions. The load range used throughout was 100 tons in 0.2ton divisions and the selected loading rate was 1 ton per minute but since the duration of the tests ranged from about 12 minutes to 30 minutes depending on the capacity of each column, it is clear that the loading rate control of the machine was faulty.

5.15 The Denison Compression Testing Machine

The 15.24 cm (6") cube crushing and 15.24 x 30.48 cm cylinder strengths of the concrete mixes were tested with a Denison Compression testing machine of the type shown in plate 2. (p.58) The maximum capacity of the machine is 250 tons and it was made by Saml. Denison and Sons Ltd. Hunslet Foundry Leeds 10, England. The model number is No. T.T.B/M.C. and the Machine No. is 29206. The specified maximum capacity in transverse is 10 tons.

5.16 Strain Gauges:

Longitudinal and lateral deformations were measured with TML temperature compensated strain gauges manufactured in Tokyo, Japan. The temperature compensations are accurate within approximately $\pm 1.8 \times 10^{-6}/^{\circ}C$. The types selected were the

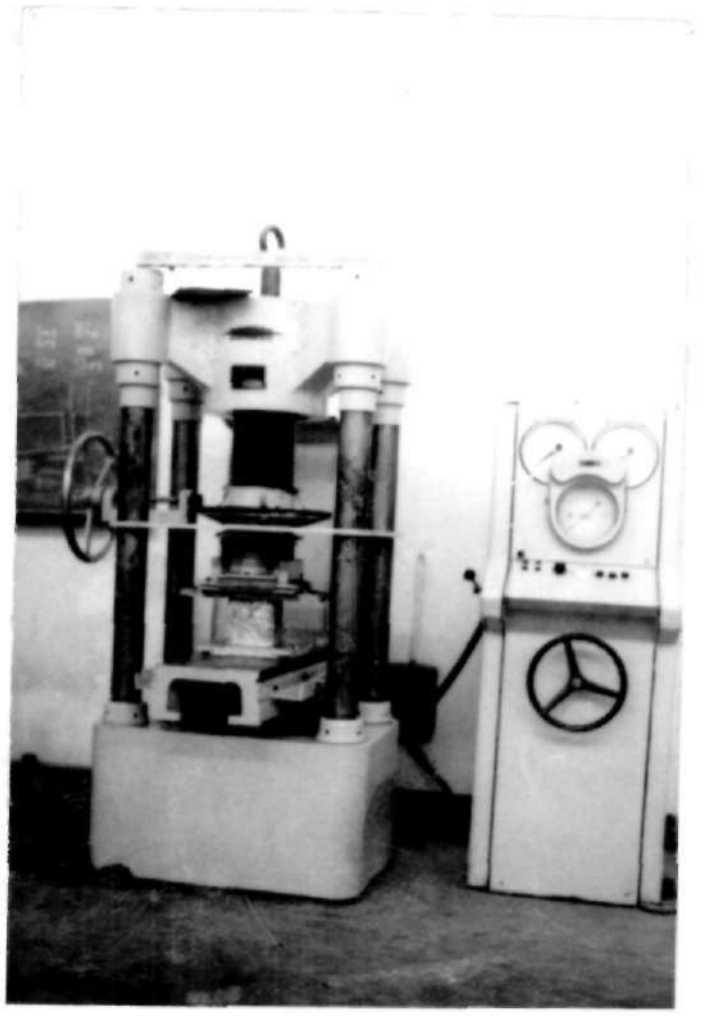


PLATE 2: THE DENISON COMPRESSION TESTING MACHINE

P-L types for measurement of transverse and longitudinal deformations and, in addition,, the mean column strains were measured over a 10.5 cm gauge length about mid-height of each column with dial gauges. These are mechanical gauges and only give approximate results. Their readings were therefore meant to serve as a check to those of the electrical resistance wire strain gauges.

The specifications of the TML gauges used are as follows:

1. TYPE PL-20-11

Gauge length	=	20 mm
Gauge width	=	3 mm
Base	=	31 x 7 mm
Gauge factor	=	2.09
Gauge resistance	=	120 $\pm$ 0.3 Ohms.

2. TYPE PL-5-11:

Gauge length	=	5 mm
Gauge width	=	3 mm
Base	=	15 x 16 mm
Gauge factor	=	2.05
Gauge resistance	=	120 $\pm$ 0.3 ohms

The suffix "11" added to the gauge types shows the coefficient of linear expansion of the test object (mild steel) and for which temperature compensation was made. All PL gauges are efficient within an operational temperature range of $-200^{\circ}C$ to $80^{\circ}C$.

Before fixing the gauges to the reinforcements, the bars were carefully sand-papered with emery cloth to get very smooth and bright surfaces and then thoroughly cleaned to remove all dirt. Two types of adhessives were used for the attachment. Initially, a two component epoxy resin/hardner mix called Araldite standard adhessive made by Ciba-Geigy Limited, Basle, Switzerland was used for A-4/2 and B-8/1 columns. This was very carefully utilised to ensure excellent humidity resistance. The rest of the gauges were fixed with P-2 adhessives as recommended by the TML gauge manufacturers. P-2 adhessives are products of Tokyo Sokki Kenkyujo Co., Ltd. Japan and is specified to have excellent humidity resistance.

For measurement of longitudinal deformations two PL type wire strain gauges of gauge length 20 mm were stuck each at mid height of longitudinal bars located at opposite corners of every column. Because of small spacings of binders of group 1 columns, PL-5 gauges were used for measuring the longitudinal deformations to ensure adequate contact with the main bars throughout the gauge length.

Strain gauges of gauge length 5 mm were used for measuring all transverse deformations of the columns. The gauges were attached to one binder at adjacent sides containing no hooks in series A and B while three gauges were stuck to one binder in series C. For group 1 and 2 columns with 4 cm and 8 cm tie spacing, strain gauges were similarly attached to two adjacent

binders about mid height of column while only one binder at mid height of each column was gauged in group 3 columns. Two dial gauges were used over a gauge length of 10.5 cm on two faces of each column. A total of 42 PL-20 gauges were used on all the longitudinal bars, 105 PL-5 gauges on all the lateral ties, 21 on the longitudinals and two dial gauges on each column.

Plates 3 and 4 show the arrangements of the reinforcements with the PL-strain gauges.

5.20 Concrete Mix Design

The concrete was made in the concrete laboratory of the Department of Civil Engineering, Ahmadu Bello University using an "Auto Menota" concrete mixer the spefication of which is given in the next article. A fairly low concrete strength was aimed at to ensure that the ultimate strength of the C-4 columns will be accommodated by the 100-ton Universal testing Machine. The water-cement ratio was therefore selected to give a target cube strength of 24 N/mm^2 (34 22 p.s.i). This strength was selected from a series of preliminary 28-day cube crushing strength tests on 15 cm cubes using trial mixes of varying water cement ratios.

The columns of each batch of concrete was $0.21m^3$ which is a little less than two-thirds of the maximum volumetric capacity

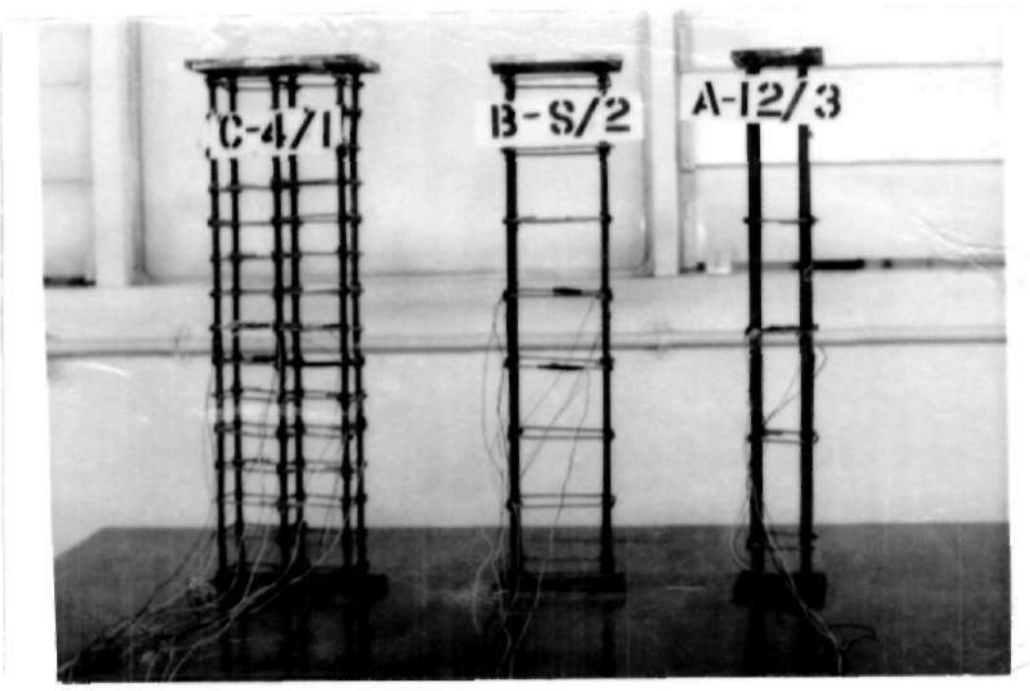


PLATE 3: ARRANGEMENT OF THE REINFORCEMENTS
WITH THE STRAIN GAUGES

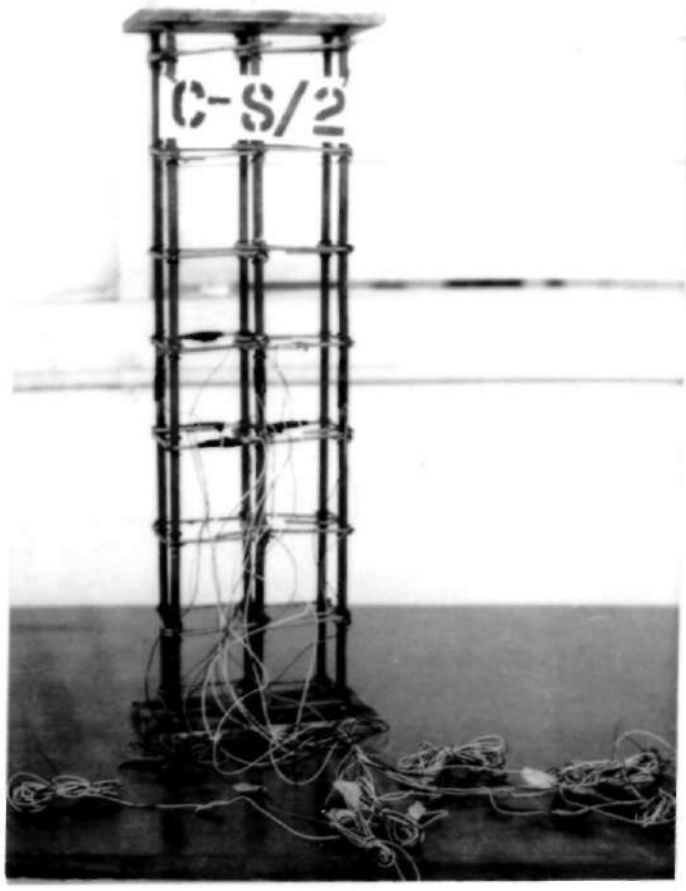


PLATE 4: REINFORCEMENT CAGE FOR C-8/2 COLUMNS SHOWING POSITION OF THE TML STRAIN GAUGES

of the available concrete mixer. It was intended that the mixer should not be filled beyond two-thirds of its total volume to ensure efficient performance during mixing operation The total volume of the mixer was 0.21 m^3 and two-thirds of of this volume is 0.14 m^3. The selected volume of 0.12m^3 for each batch is also a little greater than the total volume of all the columns of each group plus the volume of three 15 cm control cubes and three 15 cm x 30 cm control cylinders which amounted to 0.108m^3. The allowance for contigencies was therefore 11% - which is by no means extravagant - excluding total volume of the reinforcements which again provides increased security against under-estimation of batch volume. The estimated weight of 0.12m^3 of concrete was 288 kg. based on a concrete density of 2,400 KG/m^3.

The composition by weight of the ingredients of each batch was as follows:

Elephant Brand Portland cement	=	32.80 kg.
Fine Aggregates	=	81.80 kg.
Coarse Aggregates	=	147.20 kg.
Water	=	26.20 kg.

The mixing procedure was as follows:

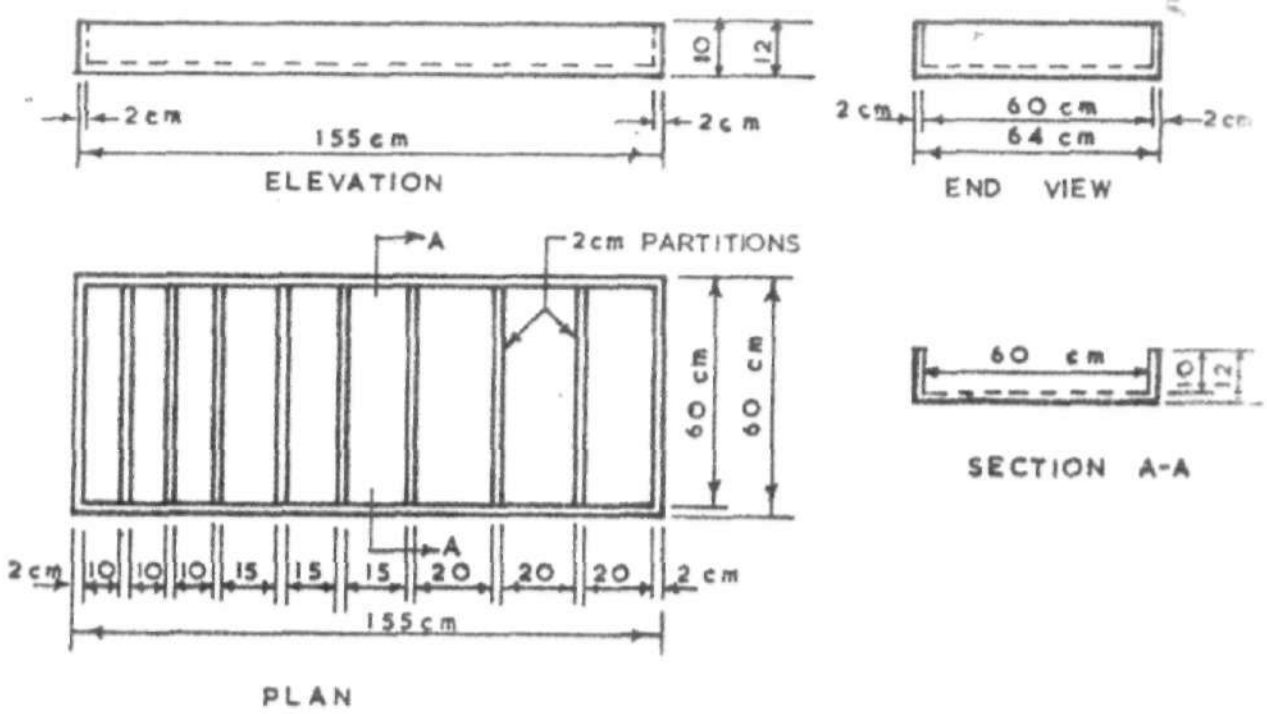
The mixing drum was thoroughly cleaned and placed on the platform of the 250 kg. weighing machine and the pointer adjusted to read zero. River sand was then poured into the drum till the pointer read 81.8 kg. Elephant Brand Portland Cement

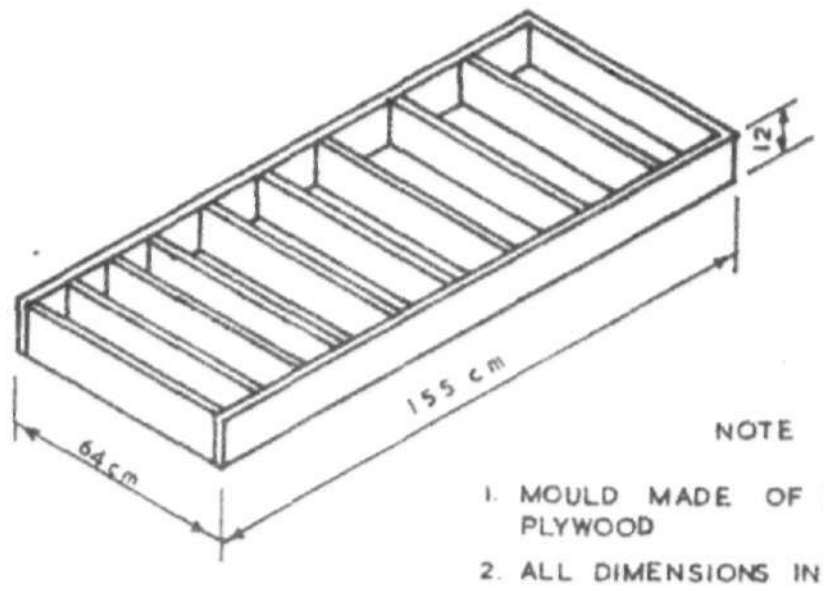
was next added till the pointer read 114.6 kg. Finally, the crushed rocks were poured into the contents of the drum till the pointer read 261.8 kg. The drum was then properly re-located on the mixing machine and running was started. 26.20 kg. of water was separately weighed out and introduced into the ingredients after two minutes of running. Mixing was continued for further three minutes so that the total mixing time was five minutes.

5.30. Method of Casting.

For simplification of formwork and casting, and to ensure better uniformity of strength, all the columns were cast and vibrated in a horrizontal position in a plywood mould(see fig.1 & plate 5) carefully designed to accommodate all nine columns of a group from one batch. 3 No. 15.24 cm control cubes and 3No. 15.24 x 30.48 cm control cylinders were also cast from the batch. The mould was made in such a way that it was easy to take appart. Mixing was facilitated by the laboratory concrete mixer which runs on trolly wheels. The trade mark of the mixer is "Auto Memota" 84 ADS series 5. The coil voltage varies from 380 - 440 with 4 - 8 amperes and 50 cycles. The mixer is a product of England and the total mixing time was five minutes.

Consolidation was by means of a vibrating table made by Mero Ltd. Engineers, Falkirk. The serial number of the vibrator is 36452 and the mould fitted exactly on its platform. Plate 5 shows the mould mounted on the vibrating table.





NOTE

1. MOULD MADE OF 2cm THICK PLYWOOD
2. ALL DIMENSIONS IN CENTIMETERS
3. SCALE 1 : 20

FIG. 1: DETAIL OF FORMWORK

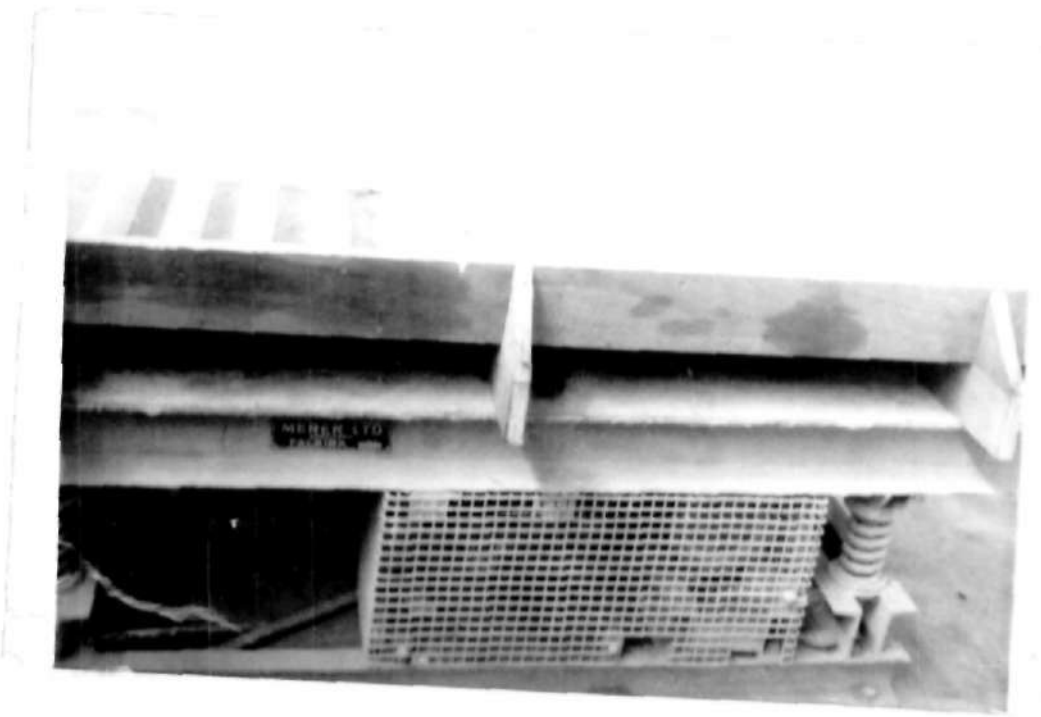


PLATE 5: THE MOULD MOUNTED ON THE VIBRATING TABLE

5.40. *Curing Conditions.*

The columns were removed from the moulds 24 hrs. after casting. They were then stored in water for fifteen days after which they were removed and kept in the laboratory to cure at room temperature until the time of testing. The laboratory temperature varied between 25.2 and 25.8 °C throughout the curing period. The temperature of the curing room was checked thrice daily by means of a mercury - in glass thermometer which was suspended in the neighbourhood. The control cubes were manufactured and cured simultaneously and in an identical manner with the columns. The age of the specimens at the time of testing ranged from 33 to 38 days.

CHAPTER 6

TEST PROGRAMME

6.10 Variables and Factors.

The investigation was aimed primarily at introducing the effect of lateral reinforcement in the equations for calculating the capacity of reinforced concrete columns with rectangular cross-sections. There are not less than sixteen different factors that may likely affect the results of the tests. These are:

1. Spacing of ties
2. Cross-sectional area of ties
3. Strain gradient across the section
4. The manner and rate of loading
5. Stress-strain curve for the lateral reinforcement
6. Effectiveness of the tie in confining the concrete
7. Area of concrete enclosed by the tie
8. Shape of the enclosed area
9. Concrete cover to the enclosed area
10. Presence, size and location of the main reinforcement
11. Compressive strength of concrete
12. Maximum size of aggregate
13. Slenderness ratio
14. Casting and curing procedures
15. Number of legs in one link of tie
16. Ratio of lateral dimensions of the column.

The last two factors have not yet received any attention from previous investigators as only square sections were being considered. It will not be possible to include all the factors above in any single investigation and so only the following have been selected for consideration in this programme:

1. Spacing of lateral ties
2. Number of legs in one link of tie
3. Percentage and number of longitudinal reinforcement
4. Ratio of lateral dimensions of the column.

The principal variable under investigation was the spacing of ties which varied from group to group within each series of the programme while the other factors varied from series to series.

The following were observed:

1. The longitudinal and lateral strains
2. Modes of failure
3. The ultimate loads.

These observations enabled the study of:

1. Load-strain diagrams of the columns
2. The effect of lateral ties on the ultimate loads
3. The ability of the ties to prevent the main bars from buckling.

6.20. Arrangement of Specimens:

The research was divided into three series, A, B and C according to size of column, and percentage and number of longitudinal bars. Each series is further subdivided into three groups arranged according to the spacing of lateral ties.

6.21. Column Marks.

For easy identification of the columns, each specimen is marked with three figures. For instance "A-4/1", "B-8/3", "C-12/2" etc. The first figure in the mark shows the series to which the column belongs and the second shows the spacing of ties in centimeters. The last figure after the stroke shows the numerical order of testing since three readings averaged to give each presented result.

6.30. Design and Description of Test Specimens:

All previous investigations on the effect of ties were based on square specimens as described in chapters 3 and 4. This investigation will be extended to cover rectangular cross-sections. A side-width ratio of 1:1 (i.e. square cross-sections) will be used in series A, a ratio of 1:1.5 will be used for columns of series B and 1:2.0 for series C. To achieve this, the least lateral dimension was kept constant at 10 cm while the adjacent side was 10 cm in series A, 15 cm in series B and 20 cm in series C. Loading was designed to be axial hence the strain gradient accross opposite sections was normally zero.

A constant slenderness ratio of 6 was used throughout the test which means that the columns tested fall under the "short-column" category with total lengths of 60 cm (23.6 ins.) each. (see fig.4).

6.31 Details of Reinforcements:

The size of the longitudinal reinforcement was kept constant at 7.73 mm (5/16") in diameter but the percentage and number was varied in series C. In series A and B, 4No. 7.73 mm (5/16") ϕ bars (A_s = 188 mm^2 or 0.307 in^2) were used while in series C, 6No. 7.73 mm ϕ bars were used to give an area of 282 mm^2 (0.460 in^2). The longitudinal steel ratio was therefore 1.88% for series A, 1.25% for series B and 1.41% for series C. A concrete cover of 15 mm was used for all bars.

The transverse reinforcements consisted of lateral ties made up of 3.1 mm ($\frac{1}{8}$") ϕ bars giving an area of 7.54 mm^2 per link of tie and tie spacings were 4 cm, 8 cm, and 12 cm, for group 1, 2 and 3 columns respectively. The maximum spacing recommended by the Nigerian Standard Code N.C.P. 1: 1973 [32] is 10 cm (4 in.). The ties were cold bent with an external closed loop dimension of 7 cm x 7 cm for columns of series A, 7 cm x 12 cm for series B and 7.0 x 8.5 cm for series C columns and carefully tied to the main bars at each contact point using a wire of cross-sectional dimension 1.2 mm ϕ The tie loops were formed by ends hooked at 180° to each other (see figs 2 and 3) before being tied to the main bars so as to check failure of the joints

and at the same time avoid the possibility of the yield strength of the ties being reduced if lap welded joints were used as was observed in an investigation elsewhere [27]

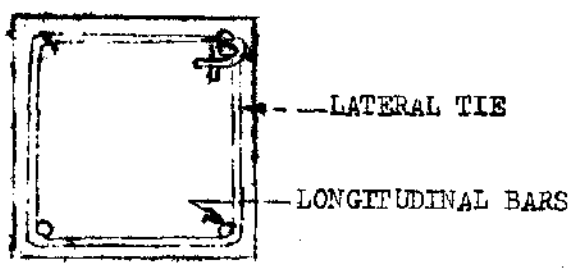


FIG 2: LOOP FORMATION

In an investigation on the ultimate strength of short axially loaded columns, Dr. S. Mackey [27] carried out tensile tests on specimens of steel used before and after undergoing welding processes. His result showed that the modulus of elasticity of steel is not affected by welding but the yield stress of the material is considerably reduced. Dr. Mackey's results are as follows:

Bar Diameter (in.)	No. of Tests	Yield Stress (lb/in^2)	Ultimate Stress (lb/in^2)	Modulus of Elasticity (lb/in^2)
$\frac{1}{4}$	2	74,500	81,800	29,200,000
$\frac{1}{4}$*	2	54,500		29,200,000

*Denotes bars subjected to welding as in columns

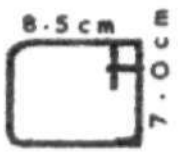
LOCATION	SIZE (mm)	TOTAL No	LENGTH (cm)	TOTAL LENGTH (m)	BENDING
ALL SERIES LONGITUDINAL BAR	7.73	126	58	73.1	58 cm — STRAIGHT
SERIES "A" TIES	3.1	84	34	30.22	
SERIES "B" TIES	3.1	84	46	38.7	
SERIES "C" TIES	3.1	168	40	67.2	

FIG. 3: BAR BENDING SCHEDULE

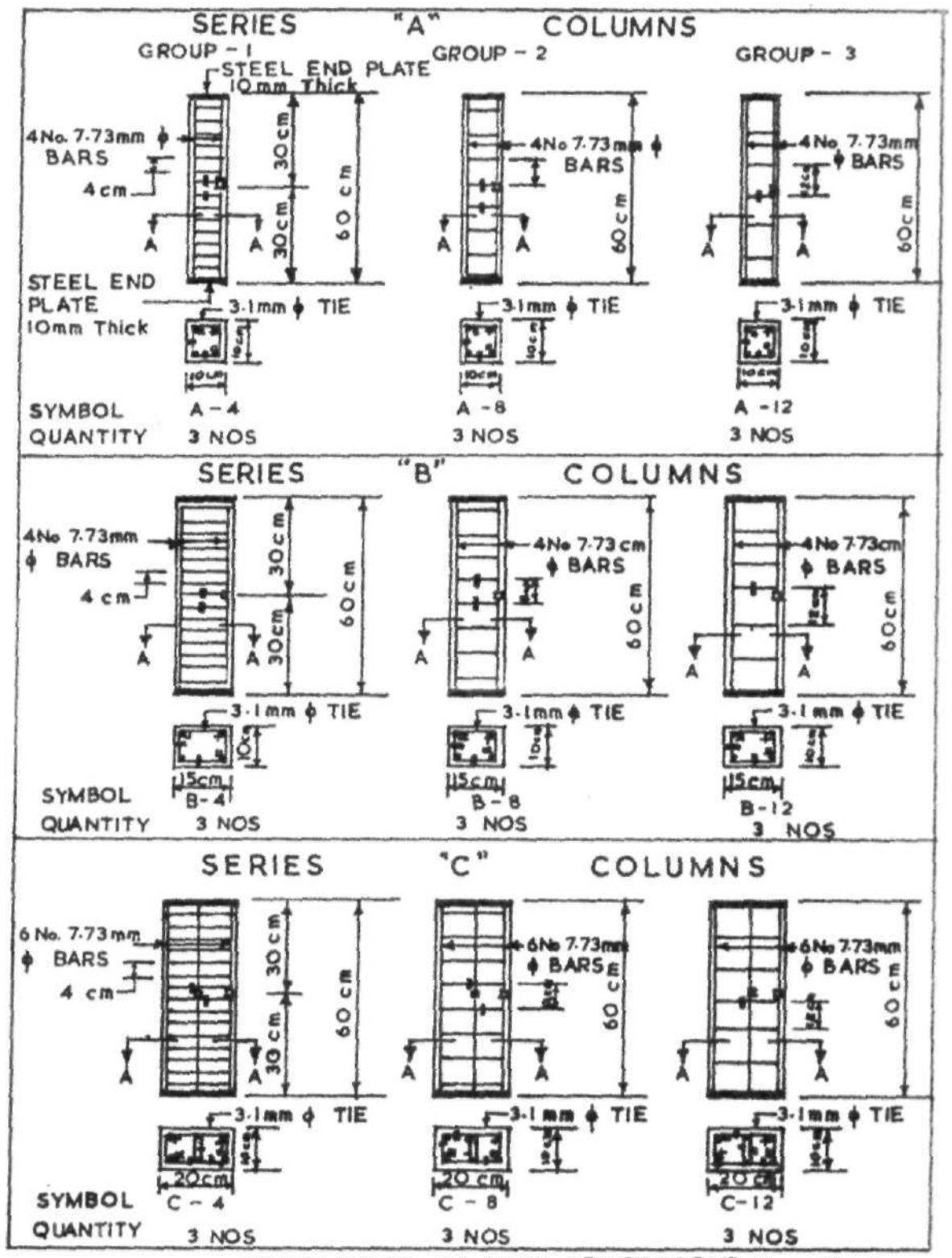
Also in an investigation similar to the authors work, many of S. Najib's [30] small diameter lateral ties out open from the lap welded joints before the ultimate load was reached.

The arrangement of the reinforcements in this investigation was such that the tie joints were staggered on all the four corners of the specimens. Schematic detail of the arrangement of the reinforcements is shown in fig.4.

For each diameter of steel used, six samples were tested to check their strengths. Results of the tensile tests on steel specimens used in this investigation are shown in table 2. Accurate centering of the reinforcing cages was effected by welding the main bars at right angles to their longitudinal axis to flat steel plates 10 mm.($\frac{3}{8}$") thick. The steel plates also served to ensure perfectly flat ends and thus uniform distribution of load over the full cross-sectional area of the column. At the same time, it served to prevent the longitudinal bars from damaging the concrete at the ends of the columns as would be expected to occur if ordinary concrete cover were provided at the ends of the column.

6.32. Series "A" Columns.

This series consisted of three groups of 3 No. 10 cm x 10 cm columns each with a side-width ratio of 1:1. Tie spacings were 4 cm for group 1, 8 cm for group 2 and 12 cm for group 3 columns.



NOTE: COVER TO MAIN REINFORCEMENT — 15 mm
SCALE: 1 : 20
□ = PL - 20 STRAIN GAUGES
▮ = PL 5 STRAIN GAUGES

FIG. 4: REINFORCEMENT DETAILS

Each group was cast from a different batch of concrete but all batches were designed to give a uniform target cube strength of 24 N/mm^2 (3422 p.s.i). Three control cubes and three control cylinders were also cast from each batch. The control cubes and cylinders enabled the determination of the actual strength of the concrete. Other salient particulars are as described in 6.30, 6.31 and shown in table 1. For each spacing of ties three columns were tested and the average of the three readings constituted one experimental result.

6.33 Series B and C Columns.

Series B and C were made as nearly identical as possible to series A and the casting and curing procedures were exactly the same. Subdivision into groups were as in series A and group distinction was dictated by the spacing of ties which were exactly the same as those of series A. That is 4 cm. for group 1 columns, 8 cm for group 2 and 12 cm for group 3.

Series "B" columns were 10 cm x 15 cm 1:1.5 cross-section with a side-width ratio of 1:2.0. Unlike series A and B, six bars were used as longitudinal reinforcements in series C and lateral ties consisted of four legs, 3.1 mm diameter bars instead of 2 legs as in A and B. Other details are as in table 1 and fig.4. Plate 6 shows the appearance of typical columns just before the tests.

6.40 Test Procedure.

All the columns were tested in a 100-ton Avery self-indicating universal testing machine and all the control cubes and

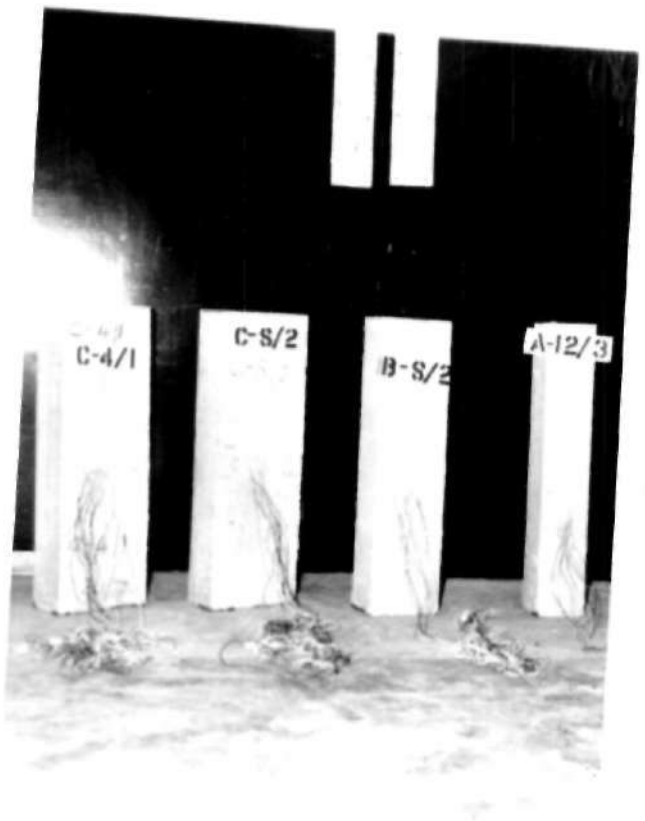


PLATE 6: APPEARANCE OF TYPICAL COLUMNS
JUST BEFORE THE TESTS

cylinders in a 250-ton Denison Compression testing machine. All tests were made when the concrete was at least 28 days old and since it took six days to test each group of columns, the actual age of the concrete at the time of testing was subject to slight variations. More detailed data on the exact age of concrete is given in tables 3-11. Prior to testing, both the straightness of the columns and variations in the widths were checked using a stretched wire. The resistances of all the electrical strain gauges were also checked.

For positioning the dial gauge, two demec points were fixed on each face of the columns; each at a distance of 10.5 cm from the other about mid-height of the specimens along the longitudinal axis. The separation between the grip holders of the machine was then lowered and fixed at a position where the clearance was just enough to accommodate the 60 cm long specimens. The intended load range of 100-tons in 0.2 ton divisions was selected by means of a selector knob.

After positioning the specimens in the machine such that the centre line of each test specimen was in line with the centres of the top and bottom plattens of the grip holders, the terminals of all the miniature stranded cables soldered to the electrical resistance strain gauges were connected to the TML static strain meter as shown in plate 7 and all readings were adjusted to obtain a satisfactory balance. Plate 8 shows

the appearance of a typical column just after the test. One strain gauge was used as a dummy throughout the test and coupled into the same bridge circuit with the active gauges in such a way that any dimensional change caused by temperature variation is compensated. Longitudinal and lateral deformations were measured on the main bars and binders inside the columns. As a check to strain measurements, longitudinal strains were also measured on two faces of each column in demec points mounted over a distance of 10.5 cm. about mid-height of each column with a dial gauge.

After carefully positioning the specimens on the machine they were preloaded to 10 tons and approximate equality of strains on opposite faces served as an indication of true concentric loading. The columns were then loaded to failure in 5-ton or 10-ton increments depending on the anticipated failure load. At the end of each load range, all strain readings were recorded.

In few instances, loading was continued after the ultimate load was reached resulting in large post-ultimate strains. Also, the crushed columns were often re-loaded to observe what percentage of their crushing load they could carry.

It was not possible to get a reliable measurement of strains near and at ultimate loads. However, a good estimation of the ultimate strains were made from the machine-recorded load-compression graph.

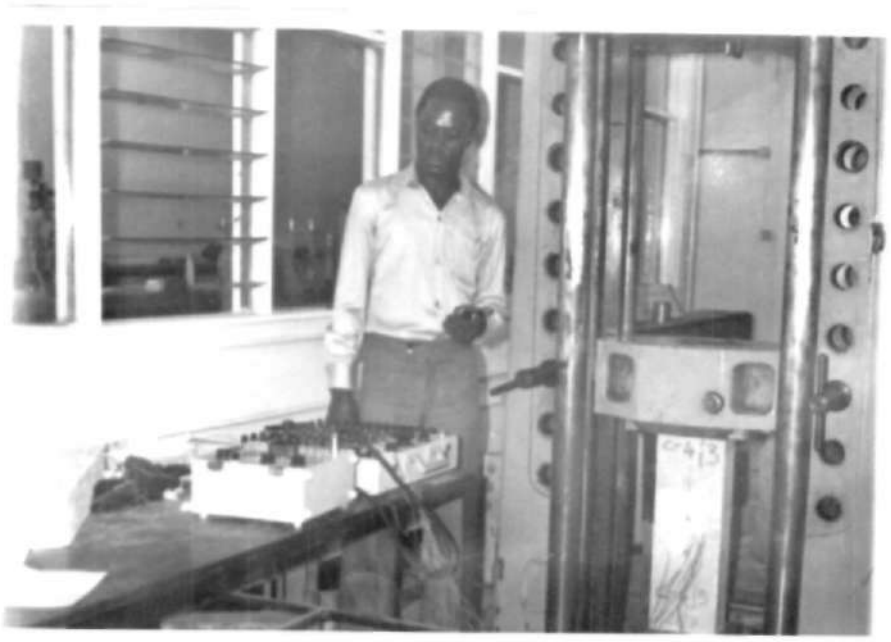


PLATE 7: ARRANGEMENT FOR THE TESTS

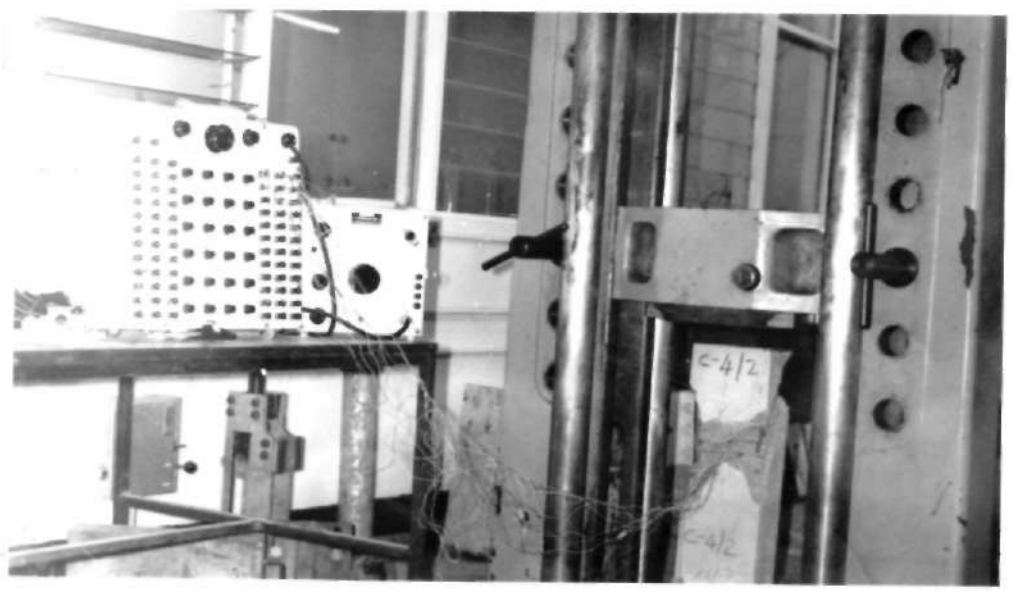


PLATE 8: APPEARANCE OF TYPICAL COLUMN AFTER THE TESTS

CHAPTER 7

PRESENTATION OF RESULTS

7.10 Observations:

The maximum temperature throughout the test was 25.6°C and the maximum variation was about 0.2°C. The temperature during the tests was checked by the aid of a mercury-in-glass thermometer, which was suspended in the neighbourhood. Such a low variation in temperature was possible because the tests were performed during a season when rapid changes in temperature were not expected. It is necessary that the temperature in the test room be as constant as possible so as to minimize the apparent strain produced by changes in temperature. The electrical strain gauges used, moreover, have temperature compensations that are accurate within approximately $\pm 1.8 \times 10^{-6}/°C$ which further reduced the fear of gauge sensitivity to temperature in addition to the compensations of the "dummy gauge" discussed in the last chapter.

In most cases, the appearance of cracks on the columns served as a warning that the ultimate load was being approached, the only exceptions being C-4/1; C-8/1; A-12/2 and C-12/2 which crushed suddenly without any warning of failure. In these instances of sudden failure the ultimate load usually exceeded those of corresponding specimens. In columns with 4 on tie spacing, cracks appeared at about 77% of the ultimate load while in columns with 8 on tie spacing, cracks were observed at about

80% of the ultimate load and in columns with 12 cm tie spacing, cracks appeared just before failure at about 96% of the ultimate load. These observations show that heavily bound columns give longer warning before collapse than columns with widely spaced lateral ties.

It is not clear why Najib's investigation [30] showed contradictory observations. In Najib's tests, cracks appeared at about 60% of the ultimate load in columns with widely spaced ties whereas in columns with heavy binders, cracks appeared at about 85% of the ultimate load.

In tests on 8" x 8" lightweight - aggregate (Lytag) concrete columns, Orangun [36] observed cracks in columns reinforced with $\frac{5}{8}$" Ø main bars and 5/16" Ø ties at $7\frac{1}{2}$" c/c at 76% of the ultimate load; in columns reinforced with $\frac{7}{8}$" Ø main bars and 5/16" Ø ties at 8" C/C, he observed cracks at 71% of the ultimate load while columns with $1\frac{1}{8}$" Ø main bars and 5/16" Ø ties at 8" C/C cracked at about 51% of the ultimate load. Orangun's tests showed that columns with higher percentage of longitudinal bars gave longer notice of failure. Orangun's results confirmed those of similar tests by Evans and Lawson [36] on gravel concrete columns, even though in the latter case cracks were recorded at 79% to 99% of the ultimate load.

7.20 Modes of Failure:

The general mode of failure observed was mainly diagonal failure though in some instances, compression failure occured.

The tested columns are shown in plates 9 to 14. Generally the failure of heavily tied columns was characterized by spalling of the outer concrete cover thus exposing the lateral ties and the core concrete as shown in plate 9. In all group 3 columns with wide spacing of ties and those of group 2 with .8 cm ties spacing, failures were characterized by buckling of the individual longitudinal bars as shown in plates 10 to 13. Spalling of the outer cover also occured in some columns of group 2 and 3 which may have been caused by the buckling phenomenon.

In few instances, straining was continued after the maximum load had been reached to study the behaviour of the reinforcements after much of the outer cover had spalled off. It was found that after a large post-yield strain, the ties still checked buckling of the longitudinal bars of group 1 columns with closely spaced ties until the specimen bent and a small buckling occured between the small spacing of the ties as shown in plate 14. In no case did failure of the ties at the joints occur as observed by Mackay [27]. This means that the method of hooking the ties through an angle of 180° and then adequately securing them to the main bars with wire ensures that the strength of the tie is not lost at the joint and is thus a recommendable process.

In Mackay's investigation with welded joints at the University of Leeds [27], buckling led to fracture of welded



PLATE 9: FAILURE OF GROUP 1 COLUMNS



PLATE 10: FAILURE OF GROUP 3 COLUMNS



PLATE 11: FAILURE OF A-12/3 COLUMN

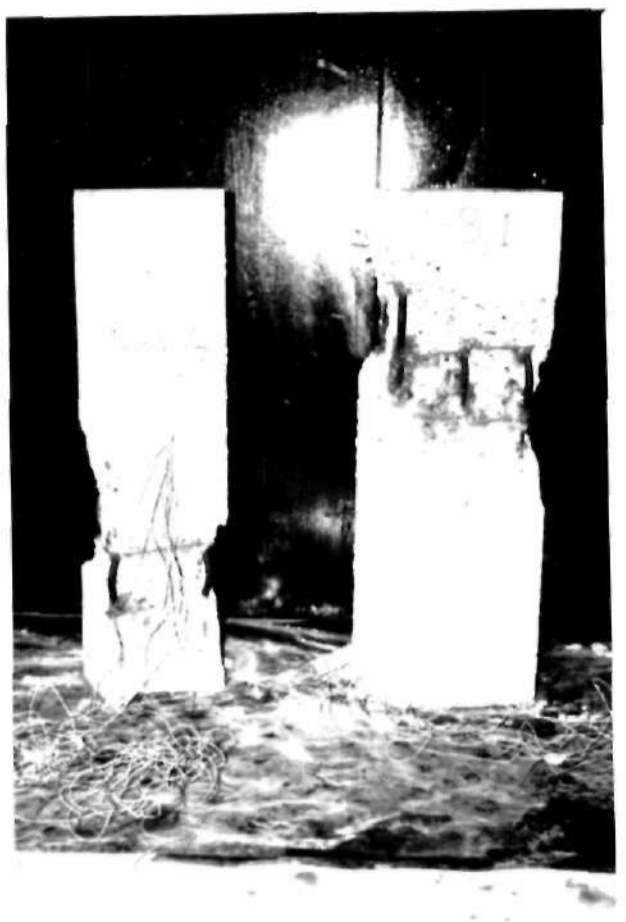


PLATE 12: FAILURE OF GROUP 2 COLUMNS

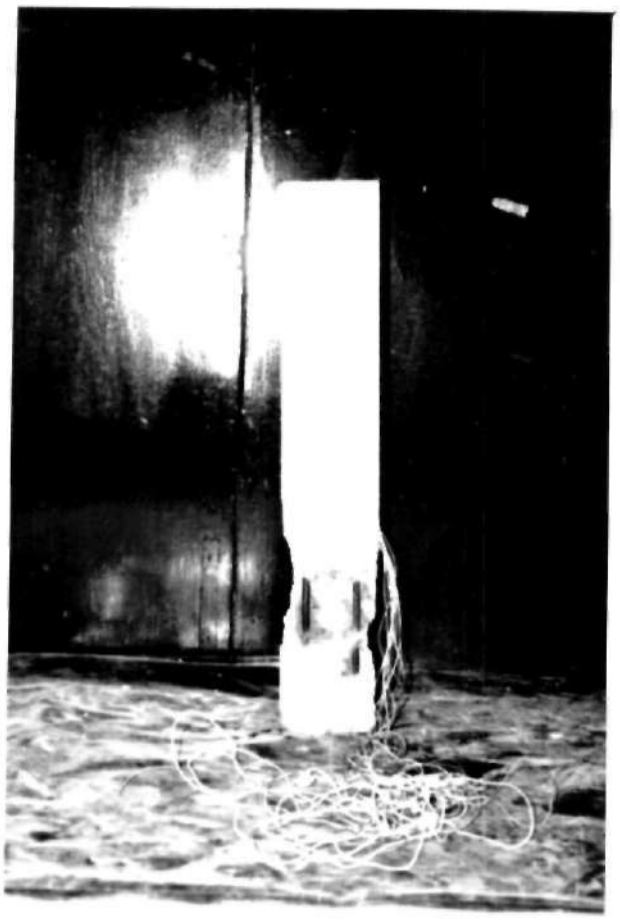


PLATE 13: FAILURE OF A-8/1 COLUMN



PLATE 14: OVERLOADED SPECIMEN (C-4/2)

adjacent joints in six instances. His binders consisted of 8 gauge soft iron wire with butt welded joints.

Also, in Najib's tests at the Technological University of Cracow [30] most small diameter stirrups - about 3.0mm diameter - got broken at the lap-welded joints leading to buckling of the main reinforcements in heavily bound columns.

It appears that in these circumstances, failure of the ties were caused by reduced strength of the ties at the welded portion and does not represent a general mode of failure.

7.30 Test Results.

7.31 Concrete Strength.

All columns of each group were cast at the same time and from the same batch of concrete. The mould was designed to accommodate all nine columns of each group in sets of three different sizes. For each batch of concrete, three 15.24 cm (6 in.) control cubes and three control cylinders 15.24 cm$\not\varnothing$ x 30.48 cm ($\varnothing$ 6" x 12") were cast. The control cubes and cylinders were tested after 28 days and the results are included in table 1, the mean cube crushing strength being 24.9 N/mm^2 and mean cylinder strength being 19 N/mm^2.

7.32 Steel Strength.

Six samples of each diameter of steel used were tested in tension and the results are presented in table 2. The actual strength of specimen 3 of the longitudinal steel could not be

obtained as a result of power failure during test. Similarly, five results were used from the tensile tests of the lateral ties as specimen 5 slipped from the grip of the machine during testing.

The average yield strengths are as follows:

Longitudinal Bars:

7.73 mm (5/16") ϕ mild steel bars $f_y = 283.6$ N/mm^2

Lateral Ties:

3.10 mm($\frac{1}{8}$") ϕ mild steel bars, $f_{yt} = 365.6$ N/mm^2

7.33 Column Deformations.

Strain measurements were made at each stage of loading and the mean of the results are presented in tables 3 to 11. Shrinkage stresses on steel reinforcements during drying have been neglected since these have no significant effect on the ultimate capacity of columns. Compressive steel stresses are therefore assumed to be zero before application of load and a perfect bond is assumed between concrete and steel but there could be very small relative displacements as was observed by P.V. Jensson in an investigation of bond between reinforcement and concrete [20]. However, the method of testing ensured that both concrete and steel were subjected to the same strain. Both longitudinal and lateral deformations were recorded. Since the columns were tested at the age of about one month, no allowance was made for any effect of shrinkage but there could be small variations in capacity due to shrinkage as

observed by Samuel P. Mauch and M.J. Holley in investigations on the effects of creep and shrinkage [29, 28]. Also creep effects during testing were neglected.

Tables 3 to 11 illustrate that deformations of columns of unequal size but with equal spacing of ties were not of the same magnitude but depends on the column cross-sectional dimensions. Larger sized columns have smaller pre-yield strains but greater ultimate strains. This shows that both size and shape of columns have marked effect on the magnitude of deformations.

Accurate strain measurements could not be made with the strain gauges up till the point of collapse as the load was released automatically by the machine as soon as the ultimate was reached resulting in reduced ultimate strains and moreover some of the gauges were no longer functioning efficiently near the ultimate load. However, ultimate deformations were estimated from the machine - recorded load-compression graph. From the readings, it was observed that for a particular column size and shape the ultimate deformation decreases with an increase in lateral tie.pitch. From these results, it can be deduced that the magnitude of deformations depends both on the size of columns and spacing of ties.

7.40 Ultimate Bearing capacities:

Results of ultimate load carried by the columns tested in this investigation are presented in table 12. The results are arranged according to series to enable easy comparison of the

ultimate loads of any particular column size with variation in the spacing of ties. From the table, it can be seen that for the 10 cm x 10 cm columns of series A, a decrease in lateral tie spacing from 12 cm. to 4 cm. resulted in a corresponding average increase in ultimate load of 3.52 tons (35.1 KN). For series B columns of 10 cm x 15 cm. lateral dimensions the increase is 2.8 tons (27.7 KN) while in the 10 cm x 20 cm. columns of series C, the increase is 7.7 tons (76.3 KN).

For every series of columns tested the only variable parameter is lateral tie spacing, other parameters remaining constant. The above increase in the ultimate loads is therefore due to the effect of tie spacing alone and will be discussed in detail in the next chapter.

The purpose of testing columns of series B and C is to find out whether column size, shape, and number of longitudinal bars affect the increase in ultimate loads due to decrease in lateral tie spacing. Table 12 shows that this increase applies to square as well as rectangular cross-sections irrespective of the side-width ratio, the number of longitudinal bars, and the number of legs in a link of tie.

The observed differences, however, show that the rise in ultimate loads for columns of series A and B are within the same neighbourhood while the rise for series C is just above twice these values. The main difference between columns of series C

and those of series A and B is the number of legs in one link of tie, other factors varying in all the three series. The number of legs in series C is double those of A and B. It appears then that the contribution of the ties is not a function of the longer side of the columns but among other things, depends on the least lateral dimension of the column - which is constant for all the three series - and the number of legs in one link of tie.

TABLE 1: SUMMARY OF MAIN TEST RESULTS

Col. Mark	No. Off	Main Bars Dia (mm)	Binders Dia (mm)	Pitch of Binders (mm)	Col. Size (cm)	Core Size (mm)	Ave. Cube Strength f_{cu} (N/mm^2)	Ave. Cylinder f'_c (N/mm^2)	Main Steel Ratio (%)	Binder Steel Ratio (%)	Av. Ultimate Load P_u (Tons)	Av. Ultimate strain ε_{bu} x 10^{-6}
A-4	3	4Ø 7.73	3.10	40	10x10	70x70	24.9	19.0	1.88	0.538	28.02	2080
A-8	3	"	"	80	"	"	"	"	"	0.269	26.77	1886
A-12	3	"	"	120	"	"	"	"	"	0.179	24.40	1886
D-4	3	"	"	40	10x15	70x120	"	"	1.25	0.538	38.40	2401
D-8	3	"	"	80	"	"	"	"	"	0.269	36.90	2133
D-12	3	"	"	120	"	"	"	"	"	0.179	35.60	1887
G-4	3	6 Ø 7.73	"	40	10x20	70x170	"	"	1.40	1.076	55.90	2691
G-8	3	"	"	80	"	"	"	"	"	0.538	52.17	2336
G-12	3	"	"	120	"	"	"	"	"	0.359	48.20	1993

TABLE 2: Tensile Test Results of Reinforcing Bars

Longitudinal Bars 8mm (5/16") dia; Length = 580						
No.	Diameter d_o (mm)	Area A_s (mm^2)	Yield load (N.)	Yield stress f_y (N/mm^2)	Ultimate Load (N.)	Length L (mm)
1	7.75	47.2	13680	280	18650	583
2	7.63	45.7	13680	299	18550	584
3*	7.59	45.3	-	-	-	584
4	7.73	47.0	13680	281	18460	585
5.	7.77	47.5	14120	280	18850	586
6	7.77	47.5	13680	278	18260	582
Average:	7.73	46.74	13,768	283.6	18,554	584

Lateral Ties 3.1 mm ($\frac{1}{8}$") dia; Length = 350 mm						
No.	d_o(mm)	A_{st} (mm^2)	Yield Load(N)	Yield stress f_{yt} (N/mm^2)	Ultimate Load (N)	Length L (mm)
1	3.10	7.54	2477.5	330	3770	355
2	3.10	7.54	2778	368	3870	349
3	3.10	7.54	3020	400	3970	356
4	3.10	7.54	2477.5	330	3670	355
5**	3.10	7.54	-	-	-	345
6	3.10	7.54	3020	400	3970	358
Average	3.10	7.54	2,754.6	365.6	3850	354.6

* Power failure interrupted test of specimen 3

** Specimen 5 slipped from its grip length during test

TABLE 3: SUMMARY OF AVERAGE STRAIN READINGS FOR GROUP 1 COLUMNS IDENTIFICATION MARK: A-4; 3 SPECIMENS

No.	Load (Tons.)	Longitudinal strain x 10^{-6}				Lateral Strain x 10^{-6}				$V=\frac{\varepsilon_t}{\varepsilon_l}$	$\frac{P}{P_u}$
		ε_{l-1}	ε_{l-2}	ε_{l-3}	ε_{l-mean}	ε_{t-1}	ε_{t-2}	ε_{t-3}	ε_{t-mean}		
1	5	252	178	235	222	66	25	25	39	0.178	0.179
2	10	520	398	460	459	118	50	58	75	0.163	0.357
3	15	827	700	763	763	177	115	103	132	0.164	0.536
4	20	1250	1190	1075	1172	226	135	179	180	0.154	0.714
5	25	1920	1725	1530	1725	331	328	325	328	0.190	0.893
6		2416	2103	1722	2080						

Notes:

Age of specimens = 37; 37; 37 days.

Spacing of binders = 40 mm.

Ultimate Loads:

P_{u-1} = 28.9 tons.

P_{u-2} = 26.2 tons

P_{u-3} = 28.95 tons.

TABLE 4: SUMMARY OF AVERAGE STRAIN READINGS FOR GROUP 1 COLUMNS
IDENTIFICATION MARK: D-4; 3 SPECIMENS

No.	Load	Longitudinal Strain x 10^{-6}				Lateral Strain x 10^{-6}				$V=\frac{\varepsilon_t}{\varepsilon_l}$	$\frac{P}{P_u}$
	(Tons)	ε_{l-1}	ε_{l-2}	ε_{l-3}	ε_{l-m}	ε_{t-1}	ε_{t-2}	ε_{t-3}	ε_{t-m}		
1	10	347	333	355	345	65	55	32	51	0.144	0.260
2	20	810	788	760	786	105	75	87	89	0.113	0.520
3	25	1087	1008	1001	1032	140	101	158	133	0.129	0.652
4	30	2200	1353	1383	1647	192	118	192	167	0.107	0.780
5		2595	2608	2240	2481						

Notes:

Age of specimens = 38; 37; 37 days

Spacing of ties = 40 mm.

Ultimate Loads:

P_{u-1} = 39.2 tons

P_{u-2} = 36.0 tons

P_{u-3} = 40.0 tons

TABLE 5: SUMMARY OF AVERAGE STRAIN READINGS FOR GROUP 1 COLUMNS
IDENTIFICATION MARK; C-4; 3 SPECIMENS

No.	Load	Longitudinal Strain x 10^{-6}				Lateral strain x 10^{-6}				$V=\frac{\varepsilon_t}{\varepsilon_l}$	$\frac{P}{P_u}$
	(Tons)	ε_{l-1}	ε_{l-2}	ε_{l-3}	ε_{l-m}	ε_{t-1}	ε_{t-2}	ε_{t-3}	ε_{t-m}		
1	10	276	233	169	229	27	38	76	47	0.205	0.179
2	20	598	543	389	510	70	49	144	88	0.145	0.358
3	30	1000	903	752	885	116	35	243	131	0.148	0.716
4	40	2045	1645	1553	1748	218	69	345	211	0.122	0.895
5	50	3275	5733	-	4505	-	187.5	-			
6		2710	2775	2589	2691						

Notes:

Age of Specimens = 35; 37; 37 days

Spacing of Binders = 40 mm.

Ultimate Loads:

P_{u-1} = 56.4 tons.

P_{u-2} = 54.2 tons.

P_{u-3} = 56.95 tons.

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TABLE 6: SUMMARY OF AVERAGE STRAIN READINGS FOR GROUP 2 COLUMNS
IDENTIFICATION MARK: A-8; NO. OF SPECIMENS = 3

No.	Load	Longitudinal Strain x 10^{-6}				Lateral Strain x 10^{-6}				$V=\frac{\varepsilon_t}{\varepsilon_l}$	$\frac{P}{P_u}$
	(Tons)	ε_{l-1}	ε_{l-2}	ε_{l-3}	ε_{l-m}	ε_{t-1}	ε_{t-2}	ε_{t-3}	ε_{t-m}		
1	5	203	225	252	227	51	47	28	42	0.185	0.187
2	10	475	480	502	486	80	105	70	85	0.175	0.374
3	15	813	817	827	819	116	152	105	124	0.152	0.561
4	20	1285	1078	1000	1121	167	196	195	186	0.166	0.748
5		1925	2030	1702	1886						

Notes:

Age of specimens = 36; 36; 36 days

Spacing of Ties = 80 mm.

Ultimate Loads:

P_{u-1} = 27.0 tons

P_{u-2} = 27.0 tons

P_{u-3} = 25.8 tons

TABLE 7: SUMMARY OF AVERAGE STRAIN RESULTS FOR GROUP 2 COLUMNS

IDENTIFICATION MARK: B-8; NO. OF SPECIMENS = 3

No.	Load	Longitudinal Strain $x10^{-6}$				Lateral Strain $x\ 10^{-6}$				$V=\frac{\varepsilon_2}{\varepsilon_1}$	$\frac{P}{P_u}$
	(Tons)	ε_{l-1}	ε_{l-2}	ε_{l-3}	ε_{l-m}	ε_{t-1}	ε_{t-2}	ε_{t-3}	ε_{t-m}		
1	10	330	380	315	342	68	36	35	46	0.135	0.271
2	15	582	625	520	578	96	59	59	77	0.133	0.407
3	20	769	932	733	811	137	95	93	108	0.133	0.542
4	25	1070	1355	1045	1200	-	174	135	160	0.133	0.678
5	30	1295	2615	1330	1747	421	236	199	285	0.163	0.813
6		1880	2030	2490	2133						

Notes:

Age of Specimens = 34; 36; 36 days

Spacing of Ties = 80 mm.

Ultimate Loads:

P_{u-1} = 38.0 tons.

P_{u-2} = 36.7 tons

P_{u-3} = 36.0 tons.

TABLE 8: SUMMARY OF AVERAGE STRAIN READINGS FOR GROUP 2 COLUMNS
IDENTIFICATION MARK: C-8; NO. OF SPECIMENS = 3

No.	Load	Longitudinal Strain x 10^{-6}				Lateral Strain x 10^{-6}				$V=\frac{\varepsilon_t}{\varepsilon_l}$	$\frac{P}{P_u}$
	(Tons)	ε_{l-1}	ε_{l-2}	ε_{l-3}	ε_{l-m}	ε_{t-1}	ε_{t-2}	ε_{t-3}	ε_{t-m}		
1	10	223	187	160	190	37	36	9	27	0.142	0.192
2	20	506	450	417	458	70	86	27	61	0.133	0.384
3	30	880	833	779	831	105	155	65	108	0.130	0.576
4	35	1075	1316	987	1126	120	213	99	144	0.128	0.672
5	40	1321	1976	1254	1517	145	285	129	186	0.123	0.768
6	45	1725	2696	1702	2041	182	376	275	278	0.136	0.864
7	50	2412	-	-		258					
8		2642	2280	2087	2336						

Notes:

Age of Specimens = 36; 36; 36 days

Spacing of Ties = 80 mm.

Ultimate Loads:

P_{u-2} = 54.5 tons

P_{u-2} = 52.0 tons

P_{u-3} = 50.0 tons

TABLE 9: SUMMARY OF AVERAGE STRAIN READINGS FOR GROUP 3 COLUMNS

IDENTIFICATION MARK: A-12; 3 SPECIMENS

No.	Load	Longitudinal Strain x 10^{-6}				Lateral Strain x 10^{-6}				$V=\frac{\varepsilon_t}{\varepsilon_1}$	$\frac{P}{P_u}$
	(Tons)	ε_{1-2}	ε_{1-2}	ε_{1-3}	ε_{1-m}	ε_{t-1}	ε_{t-2}	ε_{t-3}	ε_{t-m}		
1	5	390	195	213	266	45	35	20	33	0.155	0.205
2	10	500	425	453	459	75	73	40	66	0.144	0.410
3	15	730	731	732	731	145	109	70	108	0.148	0.615
4	20	1160	-	1201	1181	247	-	101	174	0.148	0.820
5		1940	-	1833	1886						

Notes:

Age of Specimens = 34; 34; 34 days

Spacing of Ties = 120 mm.

Ultimate Loads:

P_{u-1} = 26.4 tons

P_{u-2} = 20.0 tons

P_{u-3} = 26.9 tons

TABLE 10: SUMMARY OF AVERAGE STRAIN READINGS FOR GROUP 3 COLUMNS

IDENTIFICATION MARK: B-12; NO. OF SPECIMENS = 3

No.	Load (Tons)	Longitudinal Strain x 10^{-6}				Lateral Strain x 10^{-6}				$V=\frac{\varepsilon_t}{\varepsilon_1}$	$\frac{P}{P_u}$
		ε_{1-2}	ε_{1-2}	ε_{1-3}	ε_{1-m}	ε_{t-1}	ε_{t-2}	ε_{t-3}	ε_{t-m}		
1	10	320	337	318	325	105	95	20	73	0.224	0.281
2	15	525	505	525	518	170	137	38	115	0.203	0.423
3	20	777	740	753	757	222	182	66	157	0.207	0.563
4	25	1080	988	1010	1026	290	232	105	209	0.204	0.703
5	30	1450	1253	1350	1351	387	295	158	280	0.207	0.843
6		1880	2162	1618	1887						

Notes:

Age of Specimens = 34; 34; 34 days

Spacing of Ties = 120 mm.

Ultimate Loads:

P_{u-1} = 35.5 tons

P_{u-2} = 38.3 tons.

P_{u-3} = 33.0 tons.

TABLE 11: SUMMARY OF AVERAGE STRAIN READINGS FOR GROUP 3 COLUMNS

IDENTIFICATION MARK: C-12; NO. OF SPECIMENS = 3

No.	Load	Longitudinal Strain x 10^{-6}				Lateral Strain x 10^{-6}				$V=\frac{\varepsilon_l}{\varepsilon_l}$	$\frac{P}{P_u}$
	(Tons)	ε_{l-1}	ε_{l-2}	ε_{l-3}	ε_{l-m}	ε_{t-1}	ε_{t-2}	ε_{t-3}	ε_{t-m}		
1	10	443	276	264	320	36	30	32	33	0.103	0.208
2	20	778	574	605	652	81	101	54	79	0.121	0.416
3	30	1086	899	998	961	110	148	92	117	0.122	0.624
4	35	1333	799	1200	1111	116	168	109	131	0.118	0.728
5	40	1616	1345	1402	1454	124	193	126	148	0.102	0.832
6		1728	2076	2174	1993						

Notes:

Age of Specimens = 33; 34; 34 days

Spacing of Ties = 120 mm.

Ultimate Loads:

P_{u-1} = 41.9 tons

P_{u-2} = 54.22 tons

P_{u-3} = 48.60 tons

TABLE 12: ULTIMATE LOAD RESULTS

No.	Mark	Size of Specimen (mm)	Reinforcements		Ultimate Load P_u (tons)	P_n(mean) (Tons)
			Longitudinal Bars (No/mm)	3.1 mm Ø Lateral Tie - Spacing (mm)		
1	A-4/1	100 x 100	4 Ø 7.73	40	28.90	: .
2	A-4/2	"	"	"	26.20	28.02
3	A-4/3	"	"	"	28.95	
4	A-8/1	100 x 100	4 Ø 7.73	80	27.00	
5	A-8/2	"	"	"	27.00	26.77
6	A-8/3	"	"	"	25.80	
7	A-12/1	100 x 100	4 Ø 7.73	120	26.40	
8	A-12/2	"	"	"	20.00	24.40
9	A-12/3	"	"	"	26.90	
10	B-4/1	100 x 150	4 Ø 7.73	40	39.20	
11	B-4/2	"	"	"	36.00	38.40
12	B-4/3	"	"	"	40.00	
13	B-8/1	100 x 150	4 Ø 7.73	80	38.00	
14	B-8/2	"	"	"	36.70	36.90
15	B-8/3	"	"	"	36.00	
16	B-12/1	100 x 150	4 Ø 7.73	120	35.50	
17	B-12/2	"	"	"	38.30	35.60
18	B-12/3	"	"	"	33.00	
19	C-4/1	100 x 200	6 Ø 7.73	40	56.40	
20	C-4/2	"	"	"	54.20	55.90
21	C-4/3	"	"	"	56.95	

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TABLE 12 cotd.

No	Mark	Size of Specimen (mm)	Reinforcements		Ultimate Load P_u (Tons)	P_u(mean) (Tons)
			Longitudinal Bars (No/mm)	3.1 mm Ø Lateral Tie - Spacing (mm)		
22	C-8/1	100 x 200	6 Ø 7.73	80	54.50	
23	C-8/2	"	"	"	52.00	52.33
24	C-8/3	"	"	"	50.30	
25	C-12/1	100 x 200	6 Ø 7.73	120	41.90	
26	C-12/2	"	"	"	54.20	48.20
27	C-12/3	"	"	"	48.00	

CHAPTER 8

DISCUSSION OF RESULTS

8.10 General:

As mentioned in the introductory chapter, with respect to transverse reinforcement the Nigerian Standard Code of Practice N.C.P. -1 disregarded the effect of lateral ties in the recommended design equations for reinforced concrete columns. The code stipulates that the pitch of transverse reinforcement should be not more than the least of the following three distances:

(i) the least lateral dimension of the column;

(ii) twelve times the diameter of the smallest longitudinal reinforcement in the column;

(iii) 300 mm.

The diameter of the transverse reinforcements should be not less than one-quarter the diameter of the main rods, and in no case less than 5 mm.

In addition, the axial load p_o permissible on a short column reinforced with longitudinal bars and lateral ties should not exceed that given by the following equation:

$$P_o = p_{oc}A_c + p_{sc}A_{sc}$$

where p_{cc} is the permissible stress for the concrete in direct compression;

A_c is the cross-sectional area of concrete excluding any finishing material and reinforcing steel;

p_{sc} is the permissible compression stress for column bars;

A_{sc} is the cross-sectional area of the longitudinal steel

This omission of the effect of transverse reinforcements is common in all other standard Codes of Practice. For instance, the American Codes stipulate that the pitch should be:

(i) 16 diameters of longitudinal reinforcement.

(ii) 48 diameters of ties; where the diameter is equal or more than 6 mms.

(iii) the least dimention of the column.

The British Standard Code CP 114: Part 2: 1969 stipulates that the pitch should be not more than:

(i) the least lateral dimension of the column;

(ii) twelve times the diameter of the smallest longitudinal reinforcement in the column;

(iii) 300 mm.

The diameter of ties should be not less than one-quarter the diameter of the main rods and in no case less than 5 mm.

The Polish Code recommends that the spacing should be not more than:

(i) the least lateral dimension of the column and not more than 40 cms.

(ii) The dimension of a binder should not be less than 0.25 diameters of the longitudinal reinforcement and to the minimum of 4.5 mms.

The Russian Code stipulates that the pitch should be

(i) 15 diameters of longitudinal bars, with welded mats and cages.

(ii) the least dimension of the column but not more than 50 cms.

The size of binders should not be less than one-quarter of the longitudinal reinforcements or at least 5.00 mm.

Also the Russian Code adds:

"binders of great importance for the following reasons.

1. they ensure that the concrete will reach the cube crushing strength.
2. they prevent the longitudinal steel from buckling outwards".

Generally, the effect of transverse reinforcements on the bearing capacity of columns is not included in all recommended design equations. As discussed in chapters 3 and 4, over twenty different investigations have been carried out in different research centres to confirm or disprove this traditional design method of neglecting the effect of ties. The conclusions show that most researchers agree that lateral reinforcement has a significant effect on the capacity of reinforced concrete columns [22, 24, 26, 27, 30, 33, 43, 46 etc.]. However, each investigator arrived at a different formula for calculating column capacity.

Most of these formulae are direct deductions from particular experimental results and may or may not apply to general and prototype conditions.

The author considers it a better approach to face the problem from a purely analytical point of view and derive a formula from first principles based on accepted assumptions. Such a formula should then be subjected to series of experimental conditions to confirm or disprove its validity.

In this research, the author carried out axial loading tests on three different sizes of columns: 10 x 10 cm; 10 x 15 cm; and 10 x 20 cm. All tested columns were 60 cm. high and the main results summarized in table 1 show that for all sizes of columns, the smaller the spacing of ties, the greater the ultimate capacity. The increase in ultimate capacity is of the order discussed in 8.40. This result shows that the increase in ultimate capacity is valid not only for square cross-sections as observed by previous investigators but also for rectangular sections with more than two legs in one link of the tie.

Fig.5 shows the observed relations between the ultimate loads and tie spacings for the three series of columns. The lowest portion of the curves will correspond to the prism strength of unreinforced concrete columns. However, the limited number of test data made it difficult to say with some degree of certainty, what the lowest value will be as this has not been approached in fig.5. For instance, at tie spacings of 14 cm., fig 5 gives the ratios of the prism strengths of unreinforced columns to the cube

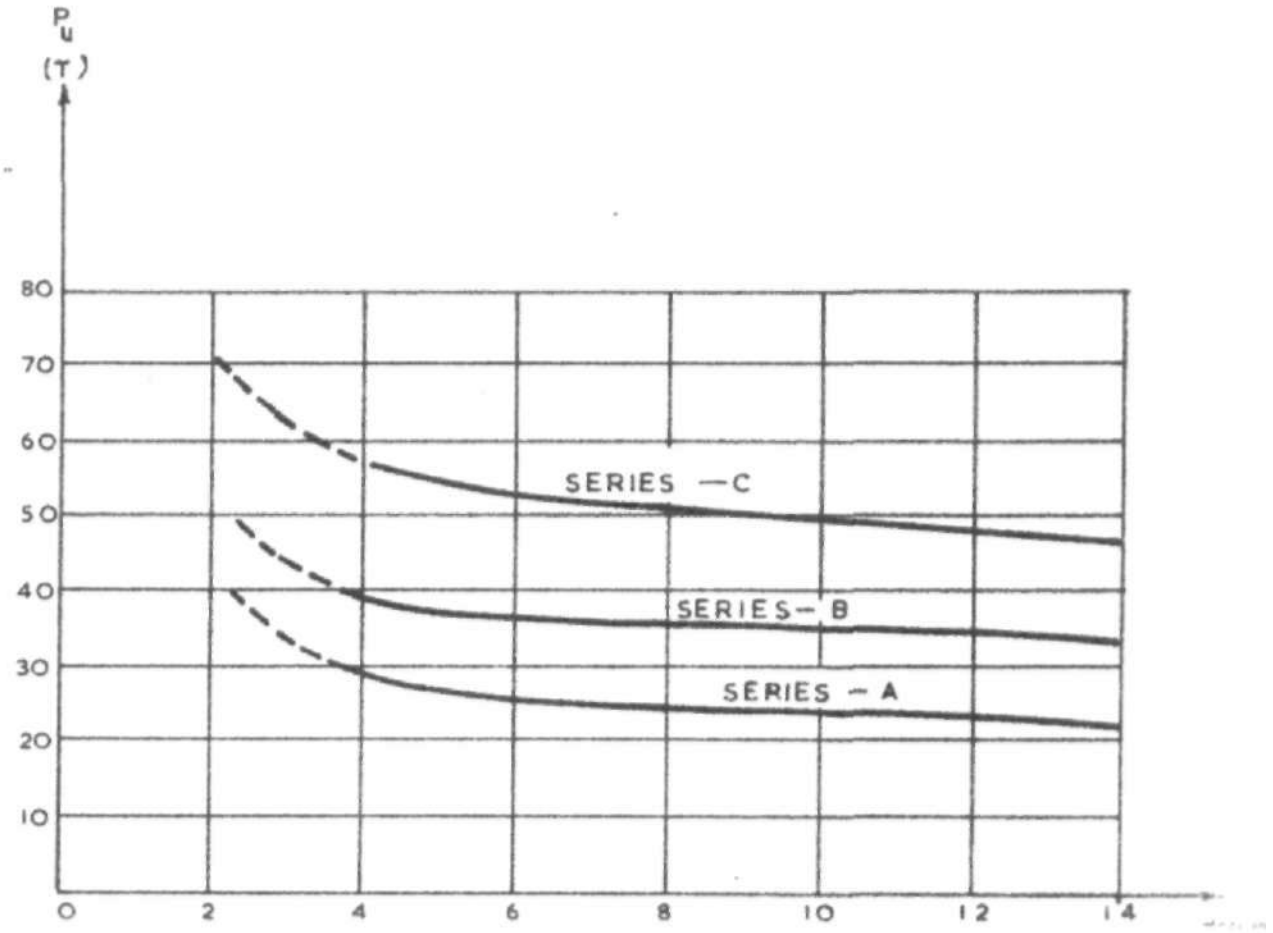


FIG. 5 : ULTIMATE LOAD — PITCH RELATIONS

crushing strengths as 0.865 for series A columns; 0.865 for series B and 0.885 for series C columns with an average of 0.87. The curves in fig 5, however, show that these ratios will be much lower for further increase in tie spacing. In tests on short lightweight concrete columns at the University of Leeds [36] Orangun obtained scattered values varying from 0.56 to 0.89 with average of 0.71 while tests on plain gravel concrete columns by Evans and Lawson [36] varied from 0.56 to 0.92 with an average of 0.68. The ratio of plain concrete column strength to cube crushing strength recommended in the code [32] is 0.67 which will be used in this work.

The curves in fig 5 show clearly that as tie spacing decreases, a considerable increase in the ultimate capacity of the columns is observed. It is also clear from the figure that for tie spacings from about 100% to 150% of the least lateral dimension of the column, the effect of pitch on ultimate load becomes less important irrespective of column size and shape. This perhaps accounts for Professor Hudson's [19] conclusion that "the influence of lateral ties on the ultimate load of short columns is negligible" since his tie spacing varied from 100% to 800% of the least dimension of the columns. In no case was tie spacing less than 100% of the least lateral dimension of columns in all of Hudson's tests.

8.20 Load-Strain Relations:

Figs 6, 7, and 8 show the load-strain diagrams of the tested columns plotted from the results given in tables 3 to 11.

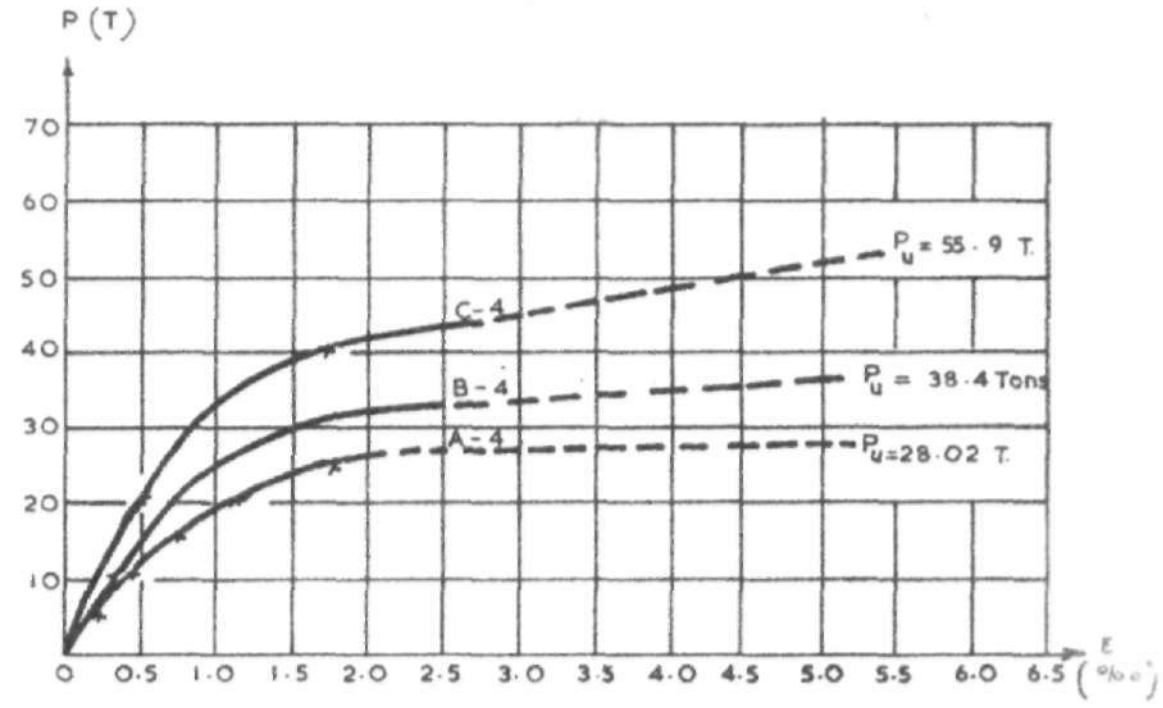


FIG. 6: LOAD – STRAIN RELATION FOR GROUP 1 COLUMNS

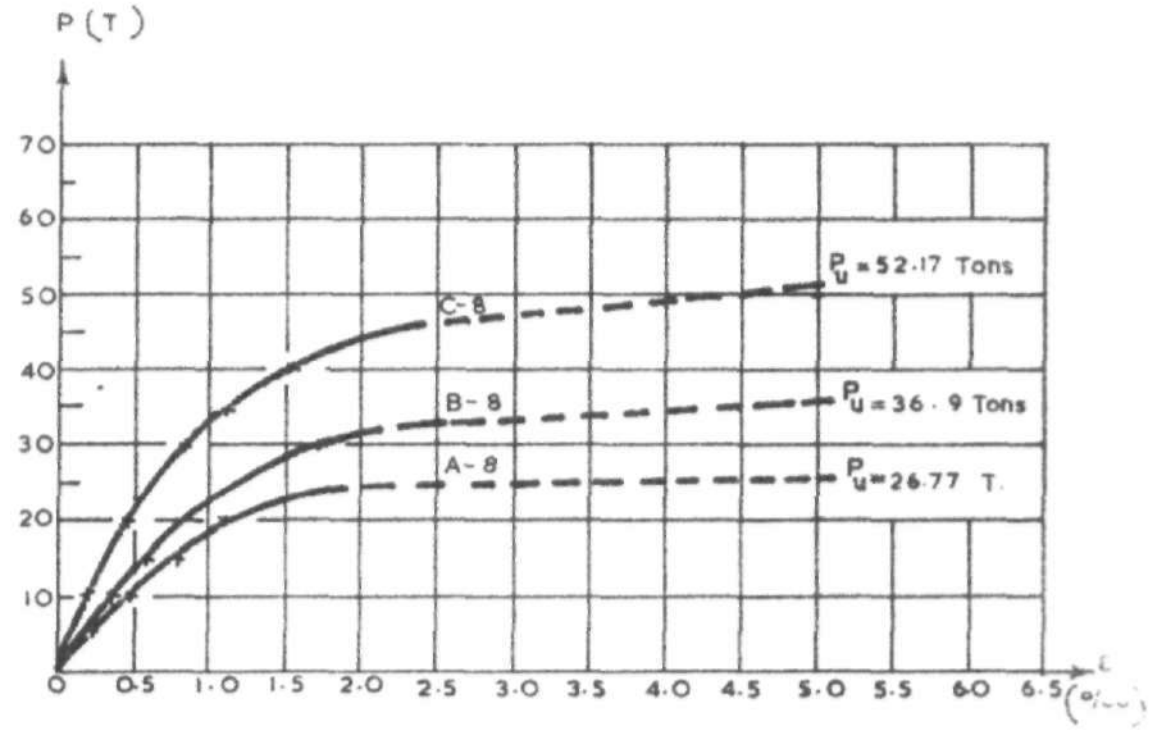


FIG. 7: LOAD – STRAIN RELATION FOR GROUP 2 COLUMNS

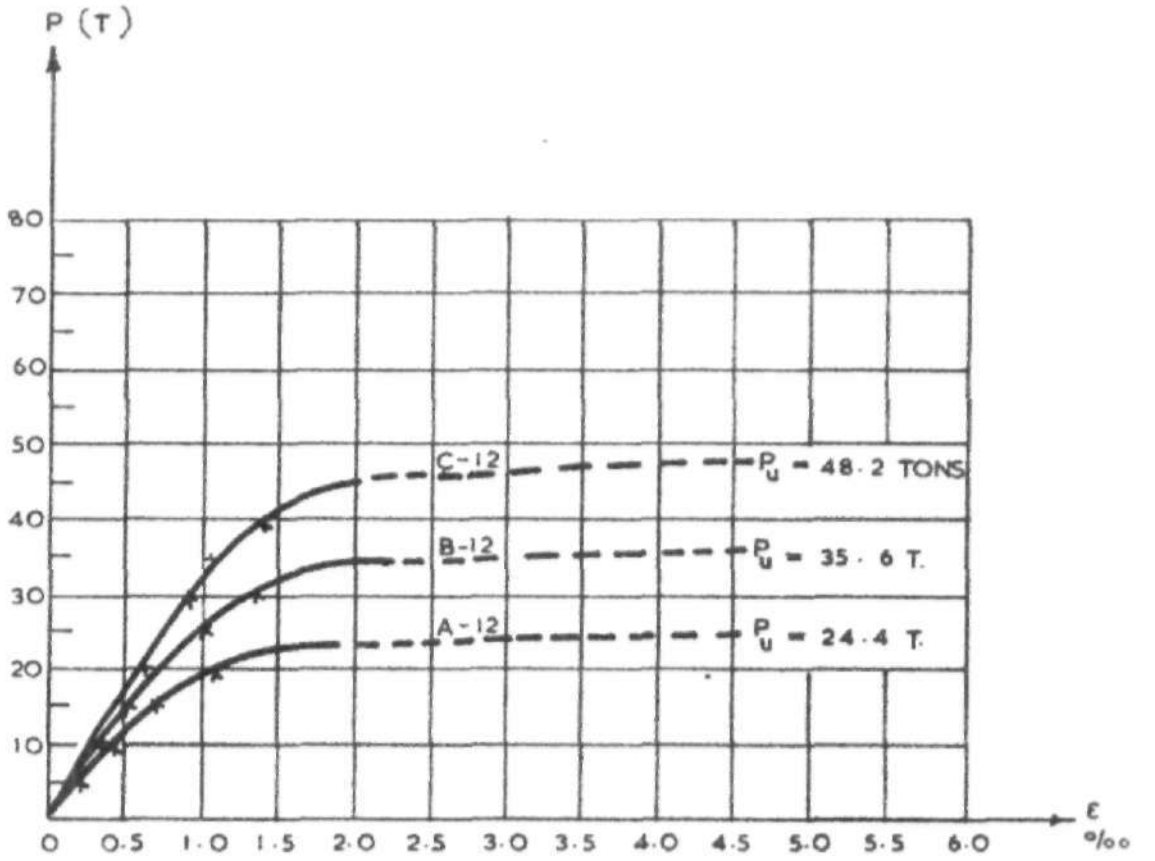


FIG. 8: LOAD – STRAIN RELATION GROUP 3 COLUMNS

The maximum strains at failure obtained from columns with 4 cm. tie spacing is higher than strains obtained from columns with 8 cm. tie spacing and much higher than those obtained from 12 cm. tie spacing. This is contrary to the observations of both Hudson [19] and Roy and Sozen [44] . Hudson's 4" x 4" and 6" x 6" axially loaded columns failed to reveal any marked effect of the presence or absence of ties on the deformations of the columns. He however admitted that the validity of his curves is open to challenge on the basis of the method employed in measuring the strains. Moreover, his tie spacings were so far apart that their effects became negligible.

The increase in ultimate strain with decrease in tie spacing observed in this investigation shows that the longitudinal steel is more effectively utilized when tie spacing is smaller resulting in higher ultimate loads, even though the results are fairly erratic. This observation of increase in strain with decrease in tie spacing is in line with the results of some previous works [30, 48 etc.] .

At the Technological University of Cracow, Najib [30] recorded ultimate strains of 5.3% to 6.2% for tie spacings of 5 cm., and about 4.6% for 12.5 cm. tie spacing, and 3.6% to 4.5% for 20 cm. tie spacing showing that the increase in ultimate strain has almost doubled for a decrease in tie spacing from 20 cm to 5 cm. with constant diameter of binders. Also his results showed that the amount of longitudinal reinforcement does not affect the deformation of columns.

For further theoritical analysis of the test results, two equations derived purely from theoritical considerations have been selected for investigation based on the test results of the author in this study. These equations as mentioned in previous chapter are:

(i) $f_c = 0.85\, f'_c + \frac{4.1\, A_{sb}\, f_y}{a\ s}$

derived by Blume, Newmark and Corning in 1961 [46] generally referred to as the "B.N.C. equation" in this thesis.

(ii) $N_u = A_b R_c + A_s f_y + \frac{m.\ A_{st}\ a_t}{V.\ S_t}\ f_{yt}$

derived by S. Cleszkiewicz in 1973 [33] .

It will be observed that the derivation of the "B N C Equation " was based on short columns containing no longitudinal reinforcements. In addition, no allowance was made for variations in the number of legs in one link of tie. The author has decided to modify the "B N C Equation" to include these missing salient parameters. The modified equation is then transformed into an equivalent ultimate load equation to ease comparison with observed ultimate values. These modifications are discussed in article 8.30.

8.30. Discussion of Theory of the Formula Derived by Blume, Newmark and Corning.

The development of the above equation contained in "Design" of Multi-story Reinforced concrete Buildings for Earthquake Motions" chapter 5, published by Portland Cement Association Chicago, 1961 as presented by N.F. Somes in [48] is discussed below together with the experimental results of this thesis.

In 1928, Richart, F.E. Brentzaeg, A., and Brown, R.L. published a paper entitioned "A Study of the Failure of Concrete Under Combined Compressive Stresses" in Bulletin No.185, University of Illinois Engineering Experimental Station Urbana, P.104. In this paper, Richart, et al. derived experimentally the following expression for the axial compressive strength of concrete cylinders subjected to lateral fluid pressure:

$$\bar{\sigma}_a = f'_c + 4.1\,\sigma_1 \qquad \text{----------(1)}$$

where:

$\bar{\sigma}_a$ = axial compressive strength;

f'_c = cylinder strength in the absence of lateral pressure;

$\bar{\sigma}_1$ = lateral confining pressure.

In this work, the lateral pressure applied to the concrete was active. Blume, Newmark and Corning applied this basic empirical result to the case of a passive lateral pressure applied on the concrete by the action of hoops. In doing this, they idealized as uniform the distribution of lateral pressure due to the hoops.

For rectangular hoops in rectangular sections, Blume, et al. proposed:

$$\sigma_1 = 0.5 \left(\frac{2 A_{sb} f_{sb}}{a. s}\right) \qquad \text{----------} \quad (2)$$

where:

a = the longer side of a rectangular concrete area enclosed by the hoop.

f_{sb} = the stress in the hoop

A_{sb} = area of cross-section of a leg of hoop reinforcement.

S = centre to centre spacing of hoop.

The term in parentheses is the lateral confining stress obtained on the basis of a free body diagram. The preceding 0.5 was introduced somewhat arbitrarily to take into account the reduced efficiency of rectangular hoops in confining a concrete core.

For the strength of the core of a rectangular column with closely spaced hoops, Blume, Newmark and Corning proposed

$$f_c = 0.85 f'_c + \frac{4.1 A_{sb} f_{sb}}{a. s.} \qquad \text{----------} (3)$$

and recommended that f_{sb} be taken to equal f_{yt}, the yield stress for the hoops.

Thus equation (3) becomes

$$f_c = 0.85 f'_c + \frac{4.1 A_{sb} f_{yt}}{a. s} \qquad \text{----------} (4)$$

The above equation does not include the effect of longitudinal reinforcement and refers only to laterally tied rectangular columns

without longitudinal bars. To enable a comparison of theoritical values using equation (4) and observed values in this investigation to be made, the author has introduced the term defining the effect of longitudinal reinforcement. In addition, the author feels it is not sufficient to limit the equation to only two legs of hoops as series C columns of this investigation have four legs of ties. The following modifications have therefore been made:

From equation (2), the lateral confining pressure σ_1 may be defined as

$$\sigma_1 = 0.5\left(\frac{m\, A_{sb}\, f_{sb}}{a.\ s}\right) \qquad \text{----------(5)}$$

where m denotes the number of legs in one link of hoop. Introducing this in equation (4), we have

$$f_c = 0.85\, f'_c + \frac{2.05\, m\, A_{sb}\, f_{yt}}{a.\ s} \qquad \text{----------(6)}$$

Applying these modifications, the author deduced that the ultimate load of an axially loaded short reinforced concrete column with closely spaced hoops may be calculated using the equation:

$$P_u = 0.85\, f'_c\, A_b + A_s\, f_y + \frac{2.05\, m\, A_k\, A_{sb}\, f_{yt}}{a.\ s} \qquad \text{-------(7)}$$

where:

P_u = ultimate strength of column

A_k = Cross-sectional area of column core

A_s = Area of compressive reinforcement

f_y = Yield stress of longitudinal reinforcement.

The third term in equation (7) represents the influence of hoops on the capacity of reinforced concrete columns. From the equation, it can be seen clearly that as the centre-to-centre spacing of hoops gets smaller, the capacity of reinforced concrete columns is improved. Table 13 shows the calculated ultimate loads using equation 7 as well as the actual ultimate loads from the test results.

8.40. Calculated Ultimate Loads Based on Modified Form of the "B N C Equation":

Applying the B N C equation in the form:

$$P_u = 0.85\, f'_c\, A_b + A_s f_y + \frac{2.05\, m\, A_k\, A_{sb}\, f_{yt}}{a\ s}$$

the calculated ultimate loads p_u(BNC) shown in table 13 below were obtained. The observed ultimate loads P_u are also shown in the table. From table 13, the error in using the B N C equation varies from 8.45% to 20.4%; all calculated values being lower bounds to the test data.

TABLE 13: EXPERIMENTAL AND THEORITICAL ULTIMATE LOADS USING MODIFIED FORM OF THE "B N C EQUATION".

No.	Column Mark	Ultimate Loads (Tons) Experimental p_u	Calculated P_u BNC	$\frac{P_u - P_u \text{ BNC}}{P_u \text{ BNC}} \times$
		SERIES A		
1	A-4	28.02	23.80	+ 17.70
2	A-8	26.77	22.80	+ 17.40
3	A-12	24.40	22.50	+ 8.45
		SERIES B		
4	B-4	38.40	31.90	+ 20.30
5	B-8	36.90	30.90	+ 19.40
6	B-12	35.60	30.60	+ 16.35
		SERIES C		
7	C-4	55.90	44.70	+ 25.00
8	C-8	52.17	43.20	+ 20.40
9	C-12	48.20	42.00	+ 14.75

8.50 Discussion Based on the Theory Behind Oleszkiewicz Formula

The stress-strain diagram for ordinary concrete due to external loads of short duration approximates to a second degree parabola,

$$\frac{\sigma}{R} = \frac{\varepsilon}{\varepsilon_u}\left(2 - \frac{\varepsilon}{\varepsilon_u}\right) \qquad \text{----------(8)}$$

where:

σ - the actual stress of concrete in N/mm^2

R - the compressive strength of concrete in N/mm^2

ε - the actual strain of concrete

ε_u - the concrete ultimate strain reached in compression at the peak load.

The above relation has been confirmed by experimental results in different Institution and is generally accepted as a fact.

From equation (8), the load-strain relation of short, longitudinally reinforced concrete columns may be approximately expressed by an equation as follows:

$$p = A_b R_c \left(2 - \frac{\varepsilon_1}{\varepsilon_{bu}}\right) \frac{\varepsilon_1}{\varepsilon_t} + A_s E_s \varepsilon_1 \qquad \text{----------(9)}$$

where:

p - denotes the actual load

A_b - denotes the total area of concrete

R_c - denotes the characteristic strength of concrete as defined by adequate ratios of cylinder or cube strength or measured on samples of plain columns or estimated from graph.

ε_1 - denotes the actual longitudinal unit deformation of the column

ε_{bu} - denotes the ultimate unit deformation of concrete reached in compression at the peak load.

A_s - denotes the area of longitudinal reinforcement

E_s - modulus of elasticity of reinforcing steel.

Equation (9) applies to plain columns or reinforced concrete columns with very low percentages of binders. The load-strain relation of columns with very low quantity of ties does not differ from those of plain columns as portrayed by Hudson's results. However, an increase of the amount of binders above a certain minimum percentage causes a significant change in the load-strain relation as can be observed from figs 6 to 8. Group 1 columns with 4 cm. tie spacing underwent larger deformations than group 2 columns with 8 cm. tie spacing and much larger deformations than columns of group 3 with 12 cm. tie spacing. This shows that a heavy transverse reinforcement increases the ultimate deformations. The increase in ultimate deformations is caused by the resistance of ties against the lateral deformation of the column core. This resistance is balanced by a lateral concrete pressure $_{bt}$ arising from additional axial stresses $_{b}$ in the concrete core.

Now, the strain in the tie is equivalent to the lateral deformation of the concrete. If these two are equated, then a relationship between the additional $_{b}$ stresses in the concrete

core and the transverse reinforcement in the column may be obtained. Usually, in plain columns it is assumed that the transverse deformation of concrete results from the uniformly distributed longitudinal stresses throughout the column cross-sectional area. If this is applied to reinforced concrete columns then the transverse deformation of concrete will be taken to result from the action of the axial $_b$ stresses uniformly distributed over the total cross-sectional area of the column and the resulting free body diagram will be as shown in fig 10.

Equal deformation of binders results from the additional $_b$ stresses causing lateral pressure $_{bt}$ in the concrete core. This may be expressed by

$$\varepsilon_t = V\frac{\sigma_b}{E_b} = V\varepsilon_1 = \frac{\sigma_{bt}\, a_t\, S_t}{m\, A_{st}\, E_s} \qquad \text{---------(10)}$$

The relation between the additional stresses $_b$ in the column core and the transverse reinforcement can be expressed as

$$\frac{\sigma_{bt}}{V} = \frac{m\, A_{st}}{a_t\, S_t}\, E_s\, \varepsilon_1 \qquad \text{----------(11)}$$

In expressions (10) and (11),

$\varepsilon_t = V\varepsilon_1$ - denotes transverse deformation of a column,

V - denotes the Poisson's ratio

ε_b - denotes the longitudinal stresses uniformly distributed over total area of the concrete.

E_b - denotes the modulus of concrete deformation

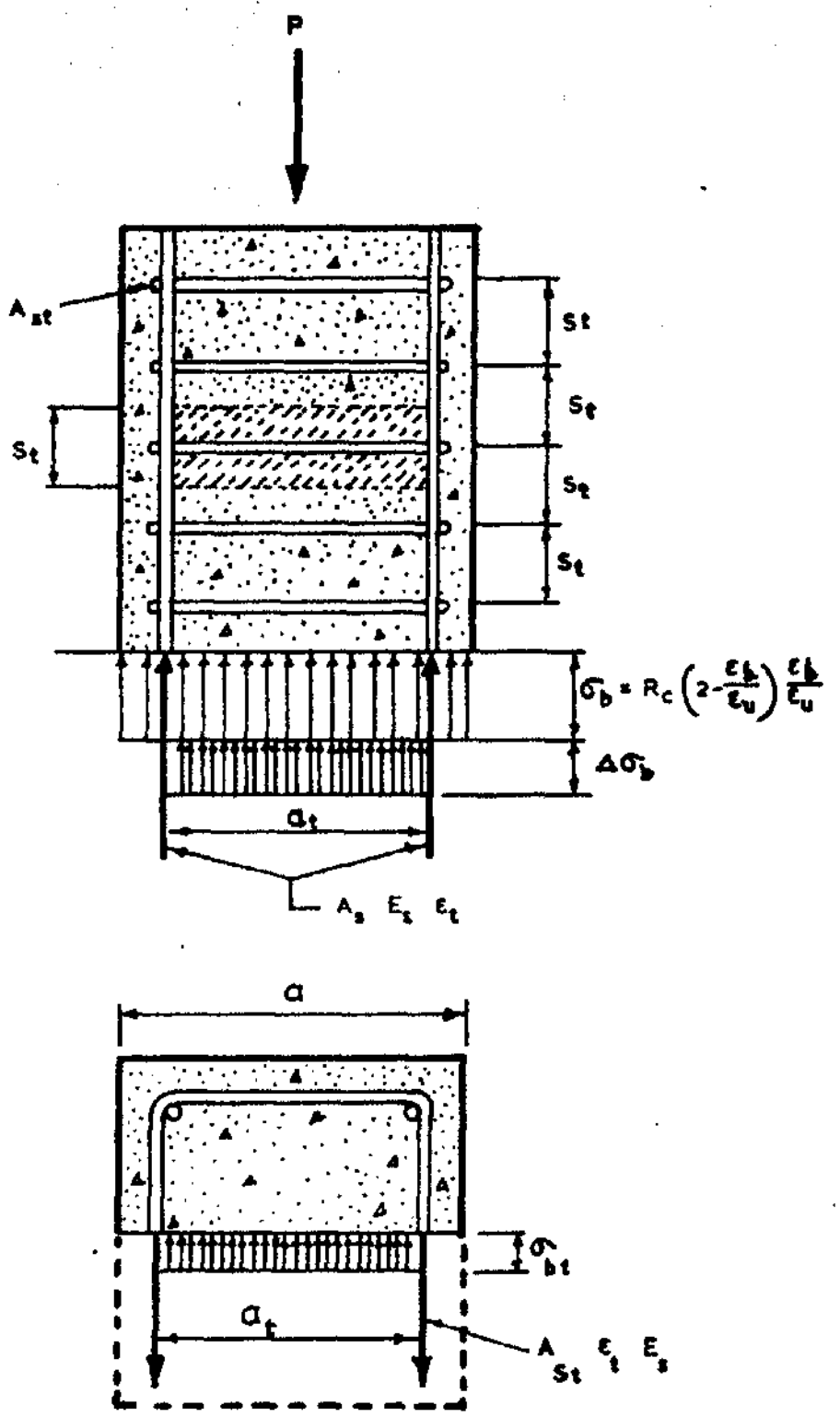


FIG. 10: ASSUMED STRESS DISTRIBUTION IN A REINFORCED CONCRETE COLUMN [33]

derived from theoritical considerations alone fits closely into the test results. The results and observations also show that at about the yield strength of the columns both the lateral and longitudinal deformations increase rapidly up to and after the

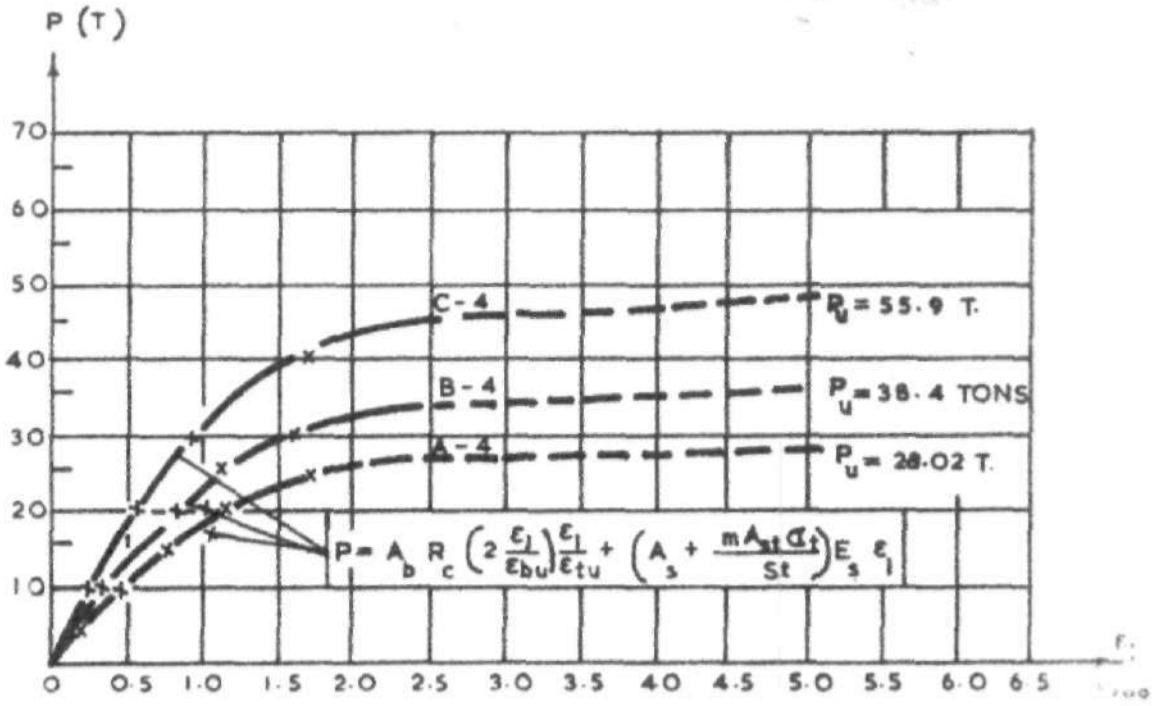


FIG. 11: EXPERIMENTAL AND THEORITICAL LOAD-STRAIN RELATION FOR GROUP 1 COLUMNS

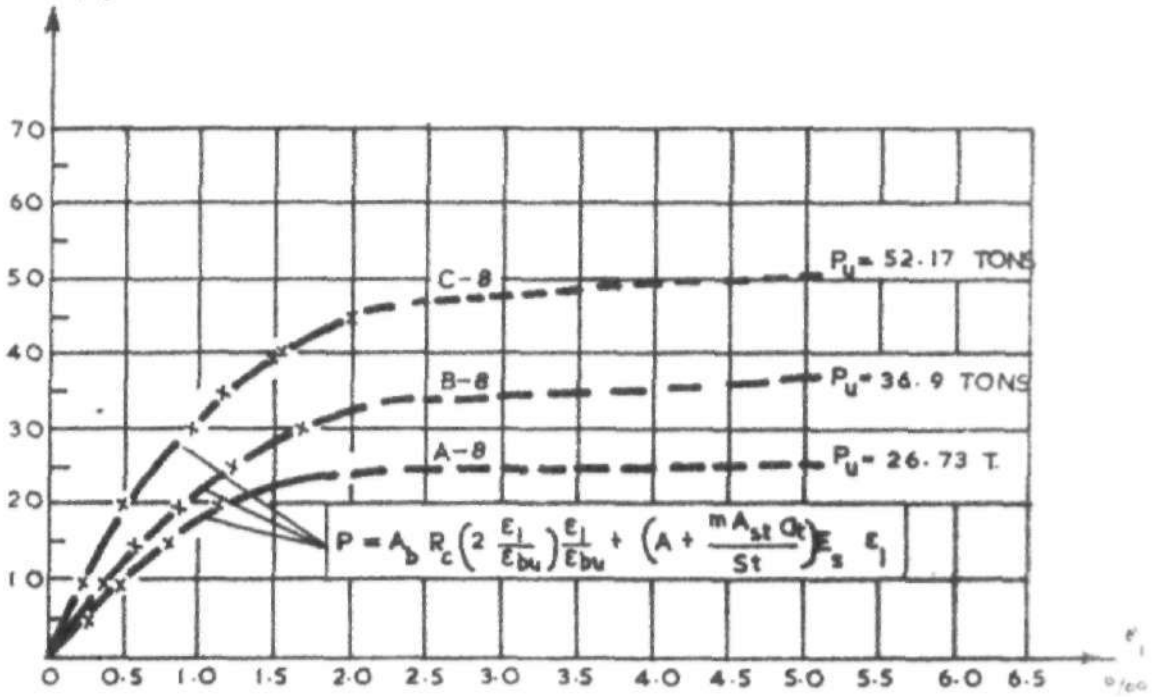


FIG. 12: EXPERIMENTAL AND THEORITICAL LOAD-STRAIN RELATION FOR GROUP 2 COLUMNS

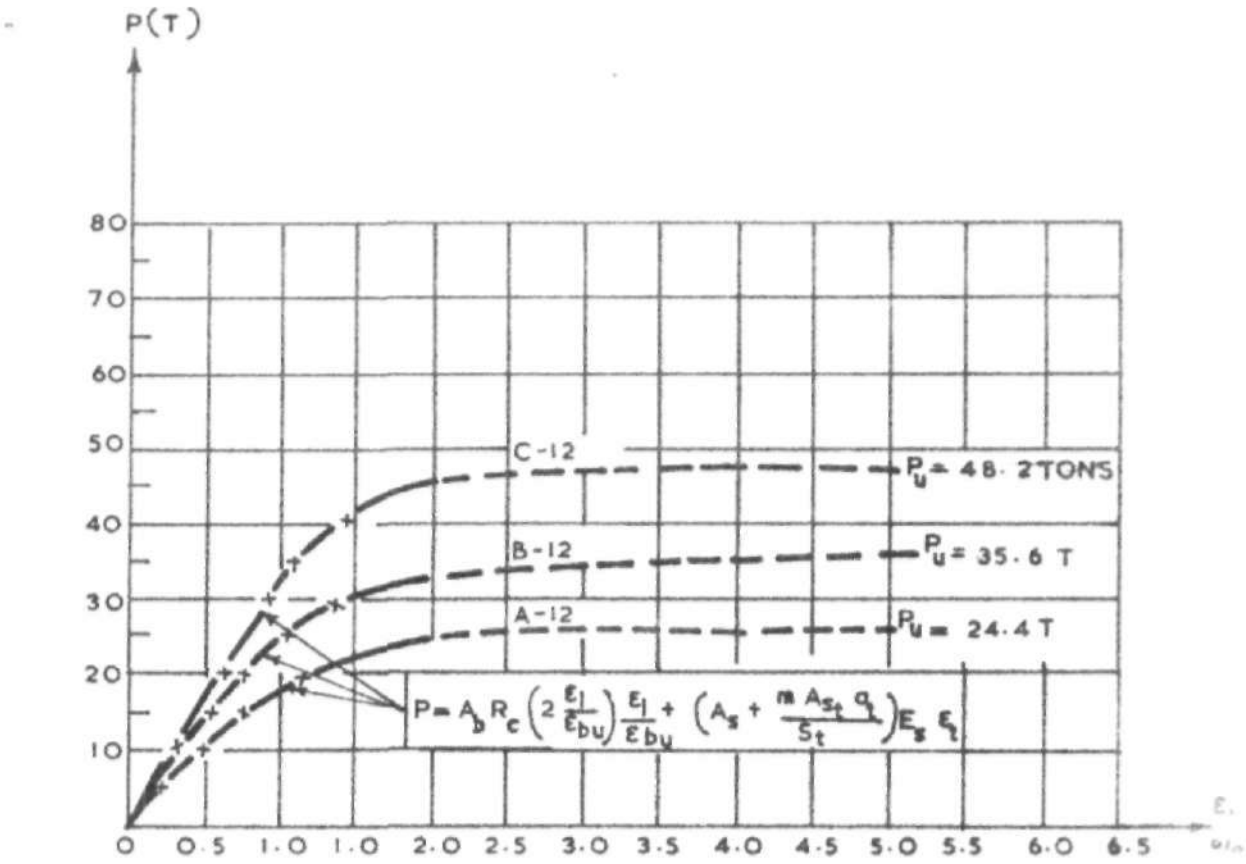


FIG. 13: EXPERIMENTAL AND THEORITICAL AND LOAD-STRAIN RELATION FOR GROUP 3 COLUMNS.

point of failure. These increases are represented by broken lines in the load-strain diagrams of figs 11, 12 and 13.

From results of strain measurements shown in tables 3 to 11, the rise in lateral deformation of columns of all sizes are more pronounced than the longitudinal deformation. Generally, the percentage increases in the transverse deformations are higher than those of the longitudinal deformation at each loading stage.

Figs 14, 15 and 16 show the increase in transverse deformation of the columns of each series at each stage of loading up to the failure load. The three curves in each of figs 14, 15 and 16 represent the three different column sizes of series A, B and C while each family of curves represents a different spacing of ties. The curves show that near the ultimate load the effect of ties become very significant. The deformation is larger for smaller spaced ties. Generally the deformation continues up to the crushing of the core concrete.

An illustration of the effectiveness of the lateral deformation in controlling the total deformation of the columns as the load increases is shown by the graph of poisson's ratio plotted against loading stages (figs 17 to 22).

In general, Poisson's ratio decreases with an increase in column size but increases with increased binder pitch. The observed values of Poisson's ratio show some scatter ranging from 0.133 in C-12 columns to 0.170 in A-4 series with a strange value

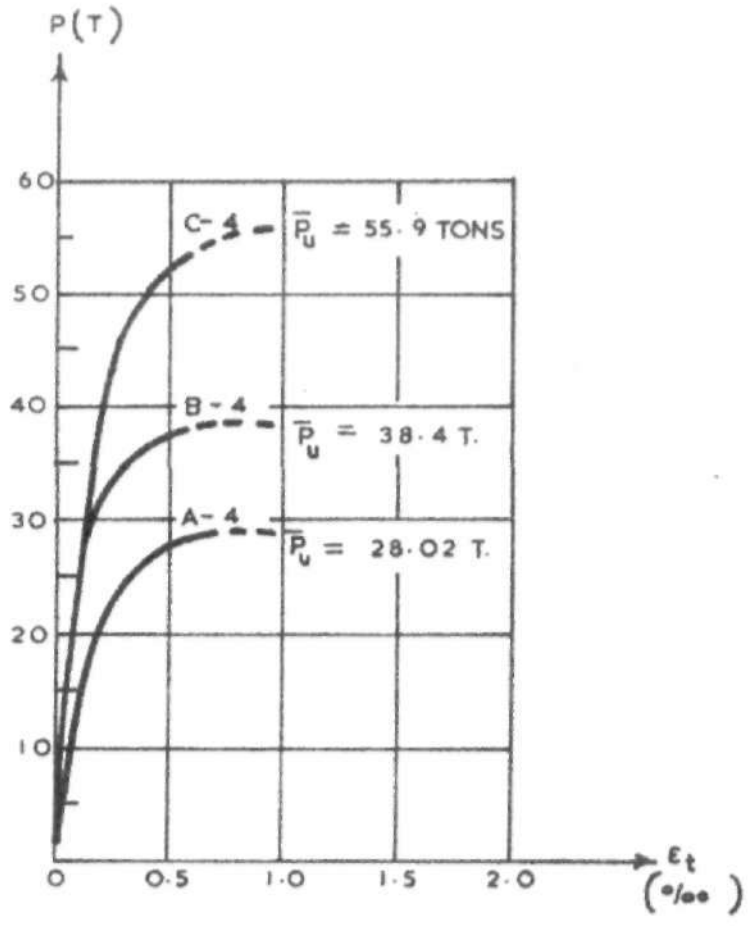


FIG 14: LOAD - TRANSVERSE STRAIN RELATION FOR GROUP I COLUMNS.

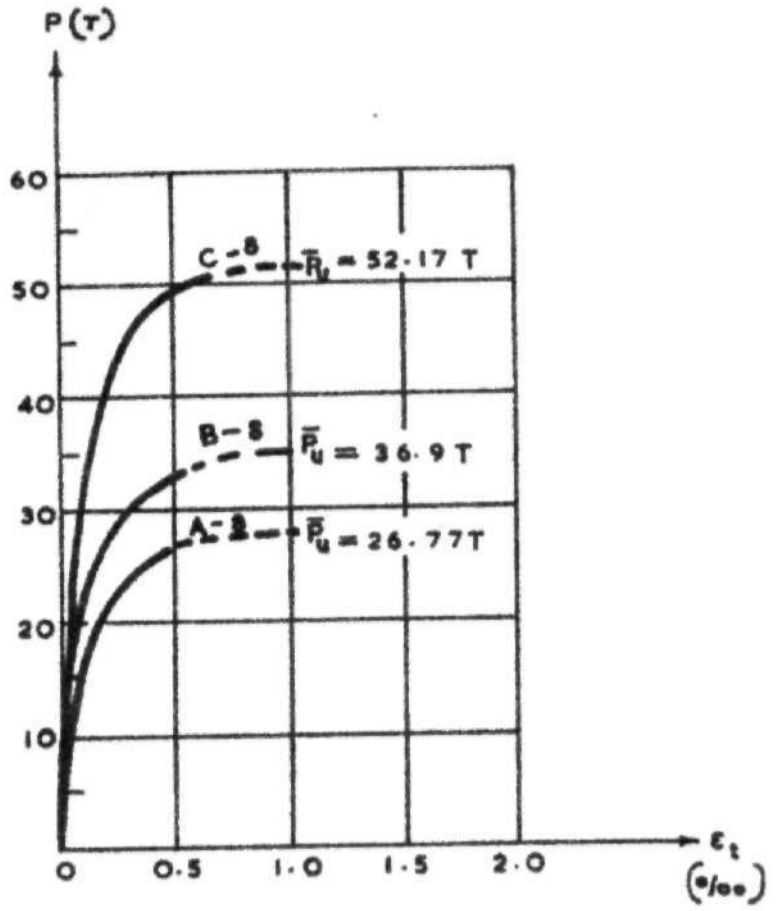


FIG. 15: LOAD – TRANSVERSE STRAIN RELATION FOR GROUP 2 COLUMNS.

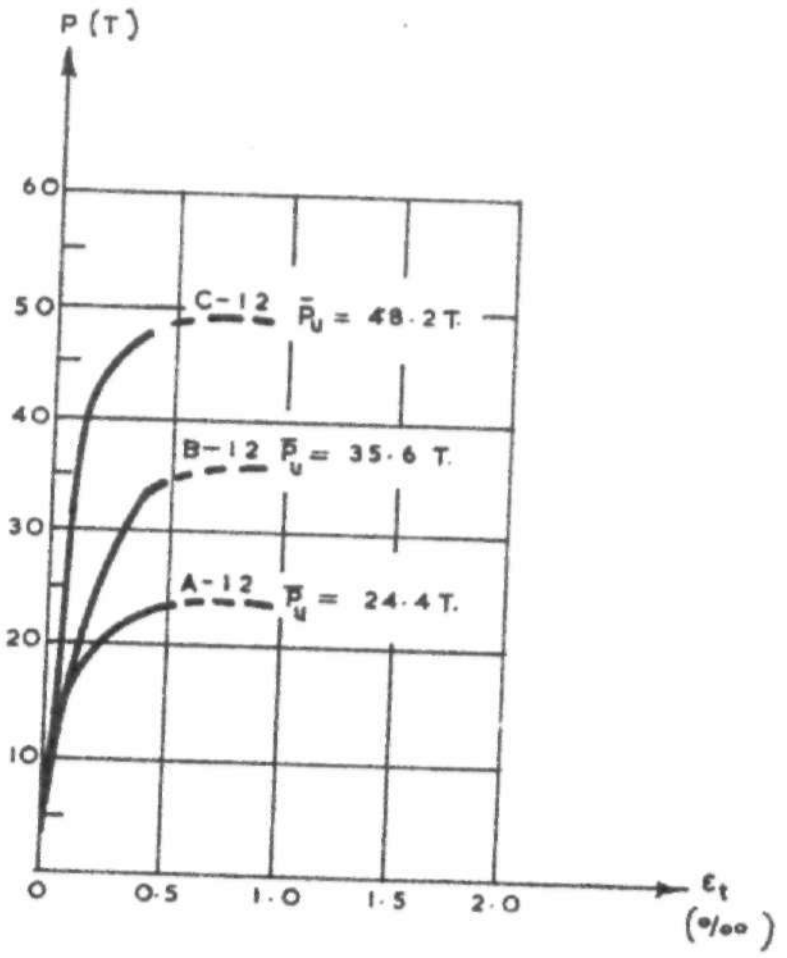


FIG. 16: LOAD – TRANSVERSE STRAIN RELATION FOR GROUP 3 COLUMNS

of 0.209 in B-12. For each column size and spacing of ties, however, Poisson's ratio is sensibly constant up to the point of collapse even though in some cases the last points are upper bound to the straight line. Figs 17 to 22 are representative diagrams.

These results are not in complete agreement with the Technological University of Cracow results [33] where Poisson's ratio was constant at 0.16 from zero up to about 60% of the ultimate load but increased rapidly to 0.4 (about 250% of its initial value) when the loading range varied from 0.6 to 1.00. The ultimate loads in that case varied from 56 to 91.7 tons as against 20 to 56.95 tons obtained in this study.

Figs 14 to 22 show that a heavy transverse reinforcement improves the ductility of reinforced concrete which agrees with the results of Roy, H.E.H. and Sozen, M.A. in their investigation on the "Ductility of Concrete" [44] . Their conclusion was:

"Although the load carrying capacity of the concrete in the axially-loaded specimens was not improved by the use of rectilinear ties, there was a considerable increase of its ductility".

It is surprising, anyway, that the tests carried out by Roy and Sozen showed no increase of strength of the concrete restrained by lateral reinforcement. Stockl [49] suggested that duration of the test may have affected the results even though Roy and Sozen did not state the duration of their tests.

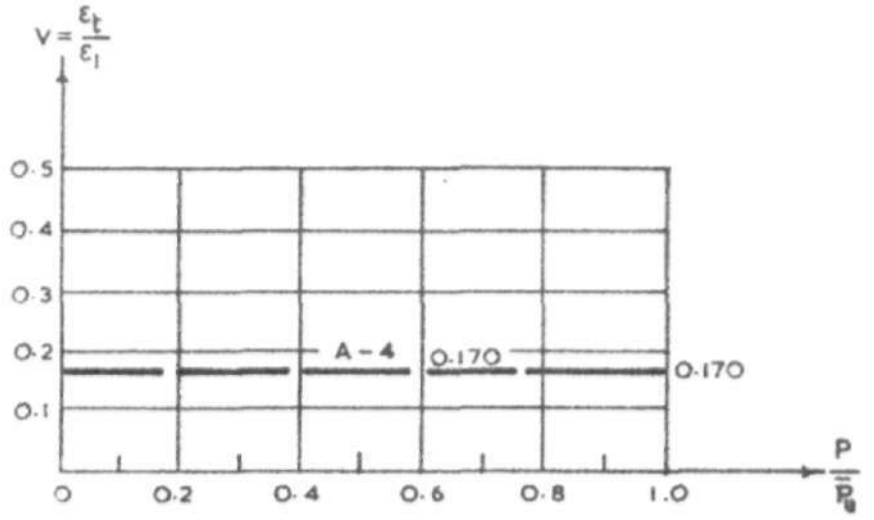


FIG. 17: POISSONS RATIO AGAINST LOADING STAGES FOR A – 4 COLUMNS

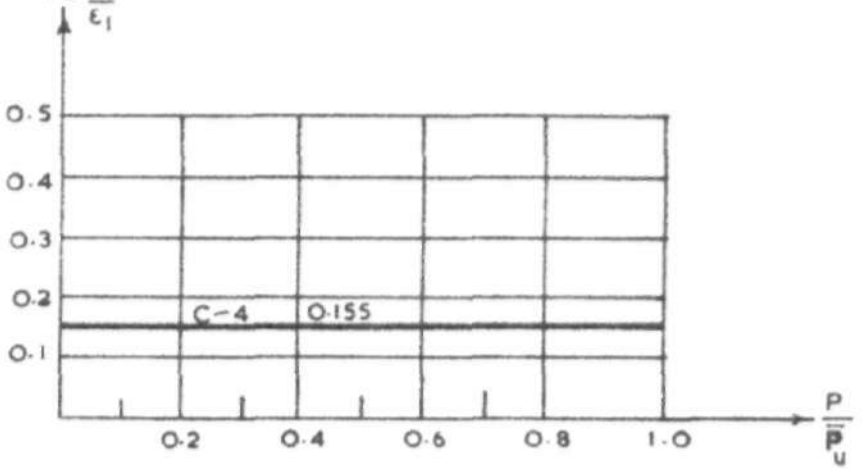


FIG. 18: POISSONS RATIO AGAINST LOADING STAGES FOR C – 4 COLUMNS

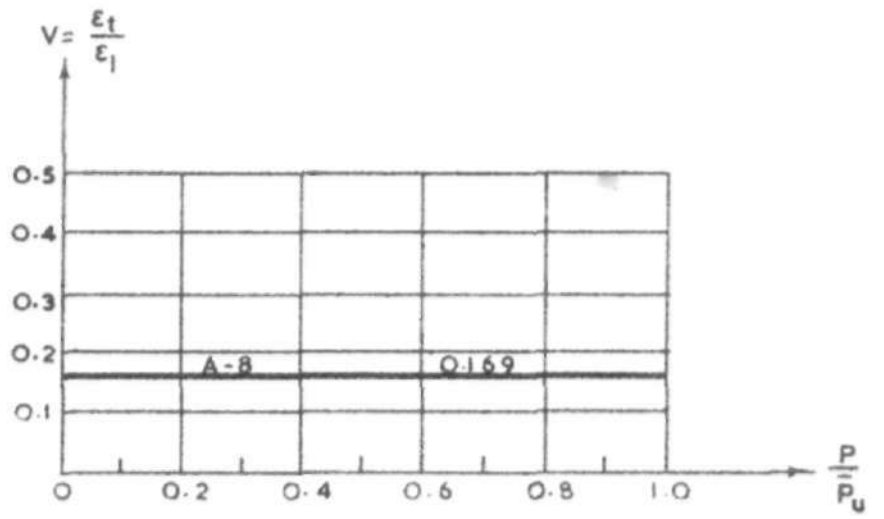


FIG. 19: POISSONS RATIO AGAINST LOADING STAGES FOR A - 8 COLUMNS

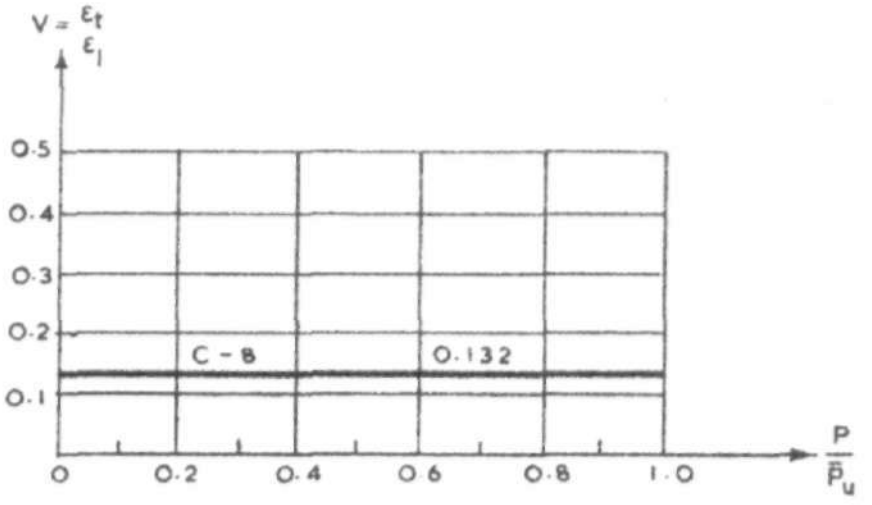


FIG 20: POISSONS RATIO AGAINST LOADING STAGES FOR C - 8 COLUMNS

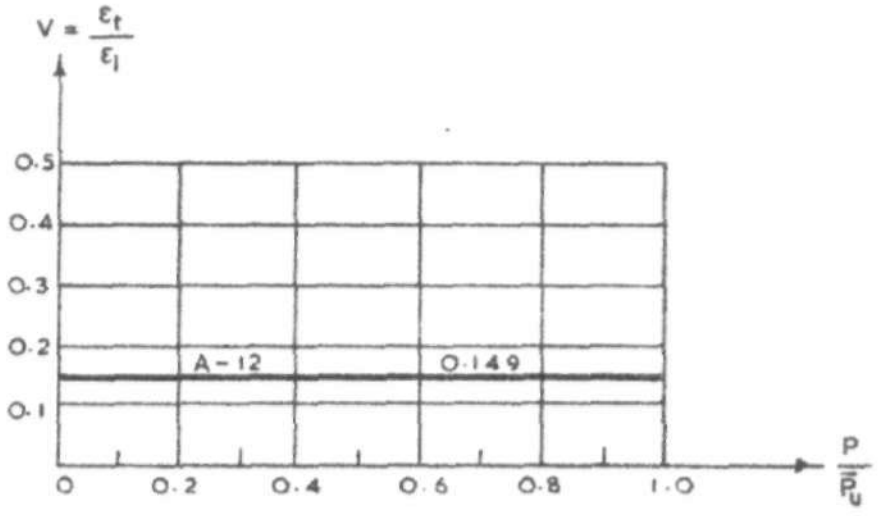


FIG 21: POISSONS RATIO AGAINST LOADING STAGES FOR A-12 COLUMNS

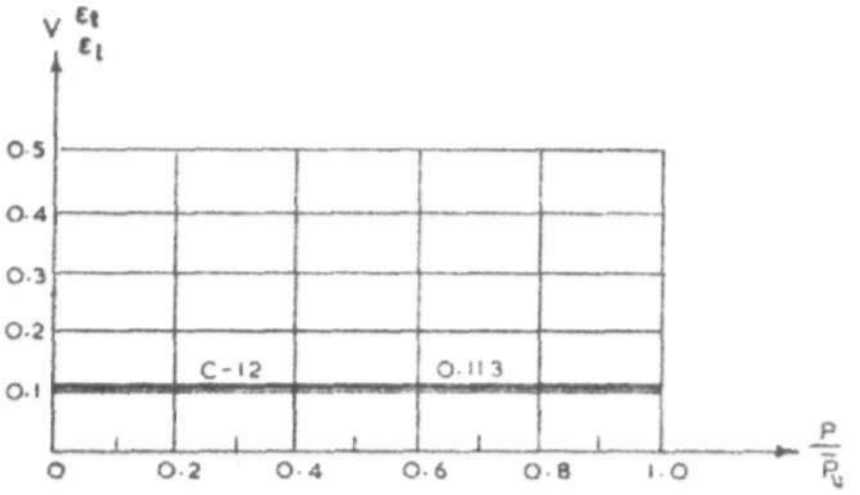


FIG. 22: POISSONS RATIO AGAINST LOADING STAGES FOR C-12 COLUMNS

8.51 Ultimate Load:

The modes of failure of columns in this investigation confirmed that for wide spacing of ties, the longitudinal bars buckled prematurely whereas in columns with closely spaced ties, buckling was effectively checked and failure occured after the yield strength of the main bars had been reached. This resulted in higher ultimate loads in heavily bound columns.

These observations confirmed that heavy transverse reinforcements improve the ductility of reinforced concrete columns significantly. The columns may fail after the yield stresses f_{yt} of the ties are passed. In this case the lateral pressure σ_{bt} in the column core reaches its ultimate value σ_{btu} where:

$$\sigma_{btu} = \frac{m A_{st} f_{yt}}{a_t \ S_t} = \text{Constant} \quad \text{----------(13)}$$

The additional core stress $\Delta\sigma_b$ in the concrete core becomes ultimate, and equals

$$\Delta \sigma_{bu} = \frac{\sigma_{btu}}{V} = \frac{m A_{st} f_{yt}}{V a_t \ S_t} \quad \text{----------(14)}$$

where:

f_y - denotes the yield strength of the longitudinal reinforcement

f_{yt} - denotes the yield strength of the transverse reinforcement.

Thus, assuming that at failure all the ingredients of the column reach their ultimate values then the column ultimate p_u may be

calculated from the expression:

$$P_u = A_b R_c + A_s f_y + \frac{m A_{st} a_t}{V S_t} \cdot f_{yt} \quad \text{----------(15)}$$

The above equation agrees that the ultimate load of reinforced concrete columns increases with a decrease in tie spacing.

As discussed earlier in chapter 7, spalling of the outer concrete cover often proceeded yielding of ties especially in columns with intensive transverse reinforcements. In this case, the column core area A_k should be used in calculations as the total working area of concrete at column failure and equation (15) becomes:

$$P_u = A_k R_c + A_s f_y + \frac{m A_{st} a_t}{V S_t} f_{yt} \quad \text{----------(16)}$$

According to the theory of [33] the concrete core area A_k is taken as $A_k = a_t^2$ as only square columns were considered. In this investigation consideration is not given to the theory of equation (16) because all columns tested have a uniform cover of 15 mm. and even though spalling of the outer cover occured in some tested columns as mentioned earlier, the nature of occurence was not uniform in any of the columns and computations based on these columns may give misleading results. More work on this aspect is recommended in which case the programme may be such that some groups of specimens have no cover at all as in Dr. Some's investigation [48] and those of Sargin, Ghosh and Handa [45] .

It will be noticed that Oleszkiewicz formula was derived with analysis based on square columns and previous investigations on the

formula were on square cross-sections. The author considered it necessary to include rectangular sections in this investigation as contained in series B and C. For a comparison of experimental and calculated results to be possible, it is necessary to re-define the term a_t contained in Oleszkiewicz formula (see equation 15). Instead of defining a_t as the lateral dimension of the column core, the author recommends that a_t be defined as the least lateral dimension of the column core; since, from the discussion in 7.40, it appears that the contribution of the ties is governed by the least lateral dimension of the column.

8.60. Theoritical Ultimate Loads Based on Oleszkiewicz's Formula:

Table 14 shows the calculated ultimate loads (p_{uo}) using equation (15) with a_t re-defined as suggested in 8.51 as well as the observed ultimate loads (p_u) of the tested columns. The table shows that a decrease in tie spacing results in a corresponding increase in the bearing capacity.

From the table, it can be seen that the errors in using Oleszkiewicz formula vary from 0.00% to 5.96% in series A columns while in series B columns with the strange values of Poisson's ratio, the errors vary from - 0.78% to 11.01%. In the 10 x 20 cm. columns of series C reinforced with six longitudinal bars and four legs in each link of tie, the errors vary from 0.13% to 5.83%.

In the test results obtained at the Technological University of Cracow using square specimens made of expanded clay (keramzit) light-weight aggregate [33] , the errors in using Cleszkiewicz formula ranged from - 1.01% to + 1.03% where negative errors were obtained from 20 cm. tie spacing. In the latter investigation, the actual concrete strengths were used as obtained from samples of plain columns.

TABLE 14: EXPERIMENTAL AND THEORITICAL ULTIMATE LOADS BASED ON CLESZKIEWICZ'S FORMULA

No.	Column Mark	Ultimate Loads (Tons)		$\frac{P_u - P_{uo}}{P_{uo}} \times 100\%$
		Experimental P_u	Calculated P_{uo}	
		SERIES A		
1	A-4	28.02	27.95	+ 0.25
2	A-8	26.77	25.24	+ 5.96
3	A-12	24.40	24.40	0.00
		SERIES B*		
1	B-4	38.40	38.70	- 0.78
2	B-8	36.90	34.40	+ 7.26
3	B-12	35.60	32.35	+11.01
		SERIES C		
1	C-4	55.90	54.50	+ 2.57
2	C-8	52.17	49.30	+ 5.83
3	C-12	48.20	47.60	+ 0.13

*strange values of Poisson's ratio

8.70. Theoritical Ultimate Loads Based on the Present Code Requirements (N.C.P.I: 1973)

From the recommendations of the Nigerian Standard Code of Practice, N.C.P. 1: 1973, the ultimate load of short, axially loaded, reinforced concrete column may be expressed as follows:

$$P_u = \frac{2}{3} uA_c + f_y A_{sc} \text{ ----------(17)}$$

where A_c is the cross-sectional area of the concrete.

Using the expression (17), the calculated ultimate loads (P_u NCP) shown in table 15 were obtained. The observed ultimate loads (p_u) are also shown in the table. From table 15, it can be seen that the errors in using the present code equation range from 15.1% to 32.14% in series A; 15.95% to 25.1% in series B and 15.3% to 33.7% in series C columns. Generally, the errors are worse in heavily bound columns and worst when four legs of ties are used with six longitudinal bars as in C-4.

8.80. Comparison of Calculated and Observed Ultimate Loads:

Tables 13 to 15 show the observed as well as calculated ultimate loads using equation 7, 15 and 17 respectively.

In table 16 the errors in using each of the above equations are tabulated. The table shows clearly that both the modified form of the B N C equation and the Cleszkiewicz equation give the ultimate capacity of short reinforced concrete columns with a better

TABLE 15: EXPERIMENTAL AND THEORITICAL ULTIMATE LOADS BASED ON THE PRESENT CODE REQUIREMENTS

No.	Column Mark	ULTIMATE Loads (Tons)		$\frac{P_u - P_{uNCP}}{P_{uNCP}} \times 100$ (%)
		Experimental P_u	Calculated $P_{u\ NCP}$	
		SERIES A		
1	A-4	28.02	21.20	+ 32.14
2	A-8	26.77	21.20	+ 26.30
3	A-12	24.40	21.20	+ 15.10
		SERIES B		
4	B-4	38.40	30.70	+ 25.10
5	B-8	36.90	30.70	+ 20.19
6	B-12	35.60	30.70	+ 15.95
		SERIES C		
7	C-4	55.90	41.80	+ 33.70
8	C-8	52.17	41.80	+ 24.30
9	C-12	48.20	41.80	+ 15.30

accuracy than the present code recommendations. The large errors in using the present code equation is as a result of the complete negligence of the effect of ties in the present code design equations.

From table 16, the error in using the formula proposed by Blume, Newmark and Corning (B.N.C. equation) vary from 8.45% to 7.7% for series A columns. For series B, the variation is 16.35% to 20.3% and for series C columns, the error is 14.75% to 25.00%. The error increases with increase in column cross-section just as Poisson's ratio decreases with increase in column size as discussed in 8.50.

Using the Oleszkiewicz's formula, the corresponding variations are 0.00% to 5.96% for series A; - 0.78% to 11.01% for series B and 0.13% to 5.83% for series C columns.

Applying the B.N.C. equation to the lightweight concrete columns tested at the Technological University of Cracow [34] the errors in using the equation are in the range of - 0.88% to + 2.77% while with Oleszkiewicz's formula, the errors ranged from - 0.01% to + 1.11% where all negative errors applied to the maximum tie spacing.

In general, the figures given by Oleszkiewez's equation yield better results than the B.N.C equation, even though the latter leads to a fair estimation of the increase in strength due to the ties when compared with the N.C.P. equation. Fig. 23 shows the

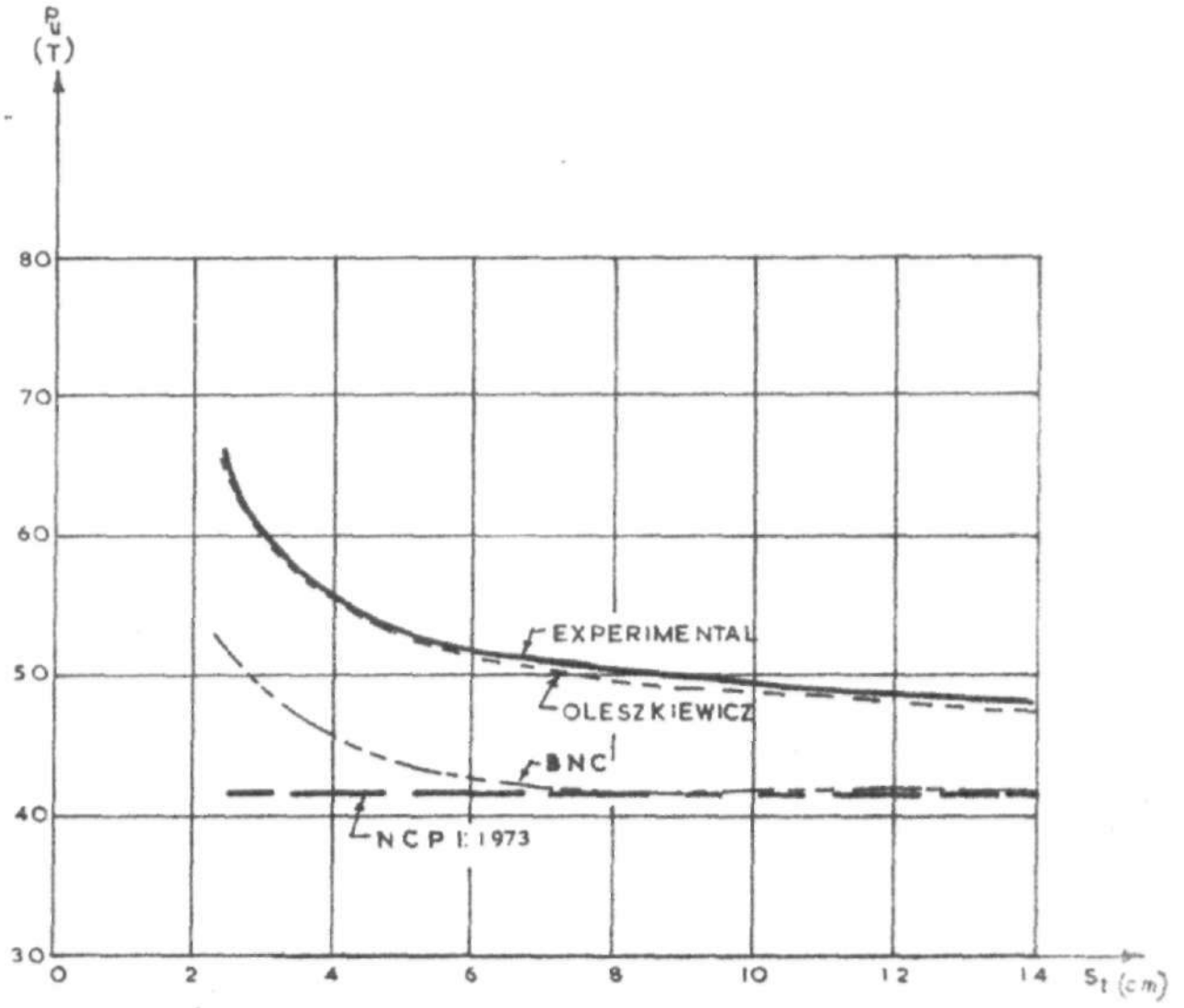


FIG. 23: EXPERIMENTAL AND THEORITICAL LOAD — PITCH RELATIONS FOR SERIES C COLUMNS.

experimental and theoritical ultimate load-pitch relations for series C columns.

TABLE 16: COMPARISON OF ERRORS IN CALCULATED ULTIMATE LOADS.

No.	Column Mark	Experimental P_u (Tons)	Percentage Errors in Calculated P_u (%)		
			Blume, Newmark and Corning (B.N.C. Equation) $\frac{P_u - P_{uBNC}}{P_{uBNC}} \times 100$	Cleszkiewicz $\frac{P_u - P_{uc}}{P_{uc}} \times 100$	N.C.P.1: 1973 $\frac{P_u - P_{uNCP}}{P_{uNCP}} \times 100$
1	A-4	28.02	+ 17.70	+ 0.25	+ 32.14
2	A-8	26.77	+ 17.40	+ 5.96	+ 26.30
3	A-12	24.40	+ 8.45	+ 0.00	+ 15.10
4	B-4	38.40	+ 20.30	- 0.78	+ 25.10
5	B-8	36.90	+ 19.40	+ 7.26	+ 20.19
6	B-12	35.60	+ 16.35	+11.01*	+ 15.95
7	C-4	55.90	+ 25.00	+ 2.57	+ 33.70
8	C-8	52.17	+ 20.40	+ 5.83	+ 24.30
9	C-12	48.20	+ 14.70	+ 0.13	+ 15.30

*Results affected by strange values of Poisson's ratio

It may be unsatisfactory in the long run to recommend at this juncture that the "B.N.C. Equation" be discarded without comparing the origins of the two equations to find out where they differ and confirm that for other ranges of investigations one will

continue to yield better results. Perhaps the B.N.C. equation may have other advantages like ease of application in practical design offices. In the next section, a comparison of the form of the above two equations is made to find out where they differ and which equation will be possible and/or better to apply in practice without pre-conditions like preliminary tests on models.

8.90 Comparison of the Form of B.N.C. and Oleszkiewicz Equation

In addition to the experimental results, it is necessary to compare the above two equations so as to find out where the difference in them lies and why one has advantage over the other and confirm that this advantage will apply to any other test conditions.

The extended form of the B.N.C. equation as recommended by the author is:

$$P_u = 0.85 f'_c A_b + A_s f_y + \frac{2.05 m A_k A_{sb}}{a_t \cdot S_t} f_{yt} \quad \text{------ (7)}$$

and the Oleszkiewicz's equation is

$$P_u = A_b R_c + A_s f_y + \frac{m A_{st} a_t}{V. S_t} f_{yt} \quad \text{----------(15)}$$

where a_t is re-defined by the author as the least lateral dimension of column core.

The first terms in each of equations (7) and (15) define the portion of the load carried by concrete. With this extension and re-definition of terms, it will be observed that the first terms

in each of equation (7) and (15) are identical. R_c in equation (15) which defines the concrete strength may be taken as 0.85 f'_c in terms of cylinder strength or $\frac{2}{3}$ u in terms of cube strength as is presently used in practice. In that case the first terms of each of the two equations are equal.

Also the second terms in each of the two equations define the portion of the load carried by the longitudinal reinforcement and are exactly the same in each of the two equations.

The only differing sections in the equations being the third term which defines the portion of the load carried by the transverse reinforcement. Both equations agree that contrary to code recommendations, transverse reinforcements carry a significant portion of the axial load. The B.N.C. equation has the amount of load carried by the ties as:

$$\frac{2.05\ m\ A_k\ A_{sb}}{a\ S_t}\ f_{yt} \qquad \text{----------(18)}$$

while Oleszkiewicz's equation has it as

$$\frac{m\ A_{st}\ a_t}{V\ S_t}\ f_{yt} \qquad \text{----------(19)}$$

Equation (18) can be further reduced to take the form of equation(19)

As a typical case, if square cross-sections are considered, A_k may be defined as a_t^2 and equation (18) therefore reduces to

$$2.05\ \frac{m.\ A_{sb}\ a_t}{S_t}\ f_{yt} \qquad \text{----------(20)}$$

Note that A_{sb} in the B.N.C. equation is the same as A_{st} in Cleszkiewicz's equation, that is, the area of one leg of tie.

Equations (19) and (20) show that the only difference between the B.N.C equation and Cleszkiewicz's equation is that the term $\frac{1}{V}$ in the latter is replaced by a factor of 2.05 in the former. This factor, 2.05 takes into account the reduced efficiency of rectangular hoops in confining a concrete core and is partly arbitrary while $\frac{1}{V}$, the inverse of Poisson's ratio is a measurable quantity.

The values of Poisson's ratio obtained in this investigation are within the range of 0.113 to 0.209 and $\frac{1}{V}$, therefore, varies from 8.85 to 4.78. These results show why the use of a constant figure of 2.05 recommended in the B.N.C. equation yields scattered results that under-estimated the contribution of the ties. It must be pointed out however that the value of Poisson's ratio depends on several factors and among other things, varies with the richness of the mix [37]. The University of Cracow investigation [33] showed a figure of 0.4 at a load near the ultimate capacity in which case $\frac{1}{V}$ becomes 2.5 which lies close to the B.N.C factor of 2.05.

If Poisson's ratio attains a value of 0.488, $\frac{1}{V}$ becomes 2.05 and both the B.N.C. and Cleszkiewicz's equations yield exactly the same results. However, for all values of Poisson's ratio below 0.488, the B.N.C. Equation under-estimates the ultimate load. On the other hand, if Poisson's

ratio exceeds a value of 0.488 in any particular condition, then the B.N.C. equation overestimates the ultimate load and the designer could be exposed to possible danger of failure. This places the use of the B.N.C. equation at a serious disadvantage.

However, it must be pointed out that even though Oleszkiewicz's formula gives a more flexible and thus better value for the ultimate loads, Poisson's ratio may remain an unknown quantity to a designer without actual laboratory tests on the designed concrete mix to determine the maximum ratio of its lateral strain to the strain in the dirrection of compression. Practical use of Oleszkiewicz's equation will therefore often be difficult.

It appears that in such circumstances, a better approach will be to replace $\frac{1}{\nu}$ by a factor as in the B.N.C. equation. The actual factor to be used for practical design purposes as said earlier should be an inverse of the maximum possible value of Poisson's ratio - and can only be obtained after a series of comprehensive tests on different concrete mixes. This means that charts in form of graphs or tables may be prepared for different concrete mixes from which the values of Poisson's ratio or its inverse can be obtained directly for any mix and other specified conditions. Work on this aspect is recommended.

It should be noted that in the B.N.C equation the term "a" is defined as the longer side of a rectangular column core. This is in line with the author's suggestion that "a_t" in Oleszkiewicz's

equation be defined as the least lateral dimension of the rectangular column core. This definition ensures necessary safety for columns.

Also, since it may not always be possible in practice to obtain the actual strength of the concrete from samples of plain columns, the author recommends that for practical purposes, the concrete strength should continue to be specified in terms of ratios of cube or cylinder strengths as obtains presently in the codes, that is two-thirds of the cube strength or 0.85 of the cylinder strength.

One other major advantage Olesskiewicz's equation has is that analysis of a column at all load ranges is possible using the load-strain expression of the equation. All other proposed equations are based only on the ultimate load of the column.

CHAPTER 9

R E C O M M E N D A T I O N S

9.10. Recommendations:

The findings of this research confirm that transverse reinforcements can be utilized to a much better advantage in short axially loaded concrete columns than the present National Code of Practice for Reinforced Concrete columns recommends. If effectively utilized, lateral ties can increase the ultimate capacity of short columns significantly by preventing premature failure due to buckling of the longitudinal bars. Also, ties can be effectively utilised to improve the ductility of short columns and ensure better utilization of the material properties of both concrete and steel used in the column.

The author therefore strongly recommends that the present code stipulations on the pitch of transverse reinforcements should be reviewed and a smaller spacing of ties stipulated to ensure that the above qualities are achieved. However, from practical considerations, it is reasonable that ties should not be placed too close to each other for economic reasons as discussed below and to avoid possible splitting of the cover concrete which not only reduces the strength but also is aesthetically unpleasant.

Perhaps, the only attempt so far to suggest both a lower and upper limit for tie spacing is the work done by Najib [30]. With respect to the establishment of a minimum limit, it was assumed that the

transverse reinforcement takes the splitting forces created in reinforced concrete columns during the process of axial loading in which case

$$m A_{st} f_{yt} > R_b a_t s_t \quad \text{----------(25)}$$

where:

R_b - denotes the splitting tensile strength of concrete. The splitting tensile strength of concrete is related to the compressive strength R_o by

$R_b = 0.23 R_c^{\frac{2}{3}}$ as proposed in [33] and the minimum percentage of transverse reinforcement, p_t therefore becomes

$$p_t = \frac{m A_{st}}{a_t s_t} > \frac{R_b}{f_{yt}} = 0.23 R_o^{\frac{2}{3}} f_{yt}^{-1} \quad \text{----------(26)}$$

Use of equation (26) is recommended for the calculation of the minimum percentage of ties.

A too intensive transverse reinforcement will not be economically advantageous. If tie spacing is reduced so much that p_t becomes too large, then the additional longitudinal stresses in the concrete core $\Delta\sigma_b$ will have reached their maximum limit and the core concrete ceases to be active. In this case the excess transverse reinforcement is simply wasted since it does not contribute any more to column efficiency as the concrete in the core will have reached its ultimate crushing strength resulting in column collapse. It is therefore recommended that transverse reinforcements should not be increased above the maximum lateral pressure σ_{btu} in the concrete core developed by additional

stresses $\Delta\sigma_b$. Since equation (26) ensures that this condition is satisfied, further increase in the percentage of ties beyond that given in equation (26) is not recommended.

The present code stipulations recommend a wide spacing for the pitch of ties which is not even sufficient to check buckling of the longitudinal bars effectively - as observed from group 2 columns with 8 cm tie spacing - and other advantages of ties above are not utilised. A more advantageous spacing of lateral ties can be easily derived from equation (26).

Since $p_t = \frac{m A_{st}}{a_t S_t}$ from equation (26),

then $S_t \leq \frac{m A_{st}}{p_t a_t}$ ----------(27)

Thus for design purposes, having designed or decided on the concrete mix and consequently the strength; and knowing the yielding strength of the material to be used as transverse reinforcement, p_t can be calculated from equation (26). Assuming the size of ties to conform to the present code stipulations, an effective spacing of the ties may be calculated using equation (27).

Equation (26) and previous discussions show that the minimum amount of transverse reinforcement depends on the following factors:

(i) Ratio of concrete strength to the yield stress of the ties.

(ii) Diameter and pitch of ties.

(iii) Lateral dimension of the column core and

(iv) Poisson's ratio.

The present code of practice takes into consideration the pitch and diameter of transverse reinfordements only.

Also, with respect to the code stipulations, as the column dimensions increase, the percentage of transverse reinforcements decrease. The theory of [33] illustrates this fact clearly with the graph shown in fig 24 where, for rectangular columns, "a" represents the least lateral dimension.

In fig 25 the amount of transverse reinforcement recommended in equation (26) is plotted against the cube crushing strength of concrete for the value of the yield stress of transverse reinforcement used in this investigation.

9.20 Economy:

It must be pointed out that the discussed recommendation for a more intensive transverse reinforcement implies a reduction in the percentage of longitudinal bars.

Any increase in the quantity of ties leads to a corresponding increase in the column ultimate capacity as shown by theoritical analysis and confirmed by experimental results. In other words the total amount of steel used in construction is not increased with reduced tie spacing since for a particular design load, if tie spacing is reduced, then the portion of the load carried by the ties is increased considerably. As the amount of the load carried by the concrete is always constant for each column size and concrete strength, the longitudinal reinforcements will have very little to contribute and its percentage may therefore be reduced considerably. A careful

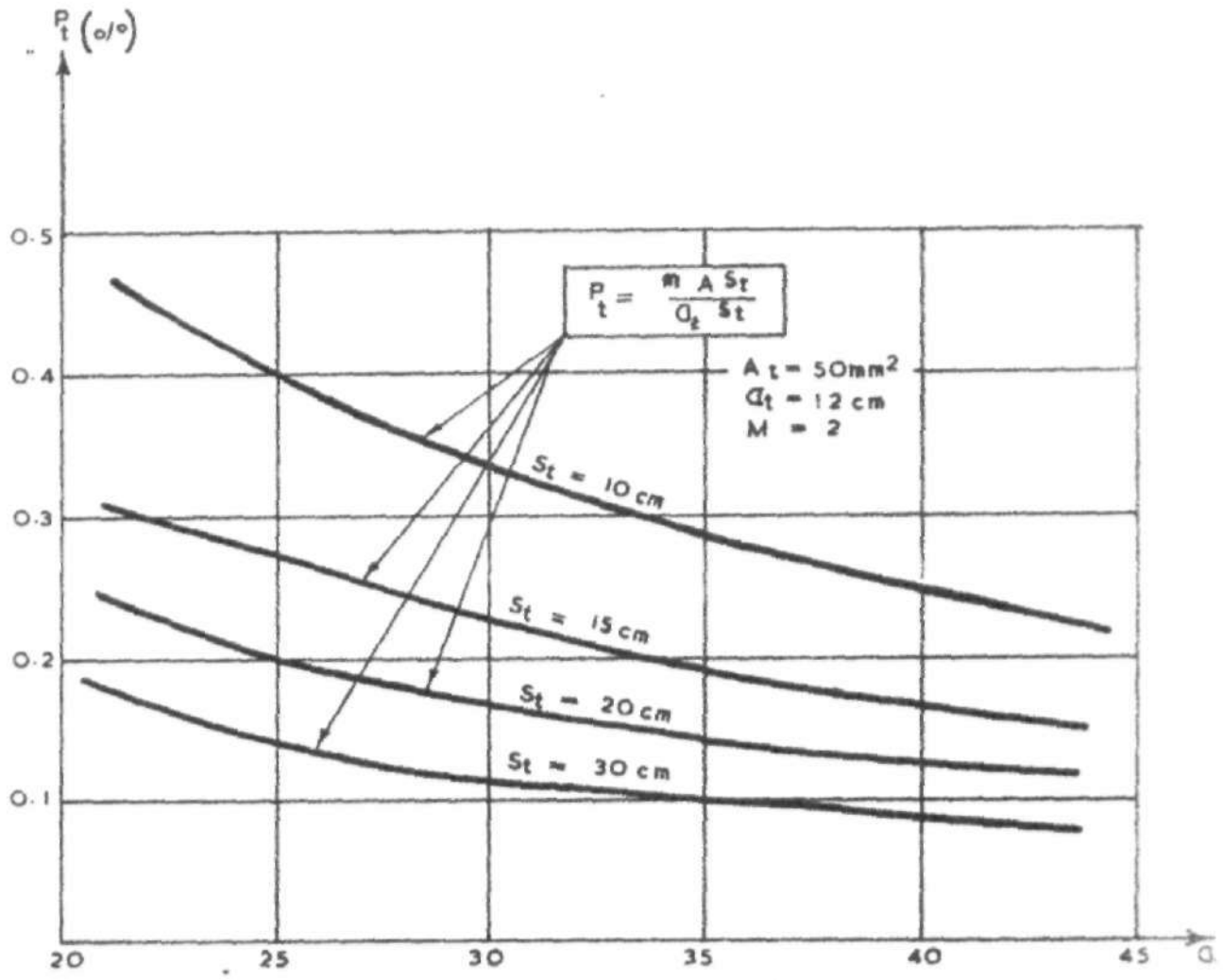


FIG. 24: PERCENTAGE OF LATERAL REINFORCEMENT AGAINST COLUMN DIMENSION AND BINDER PITCH

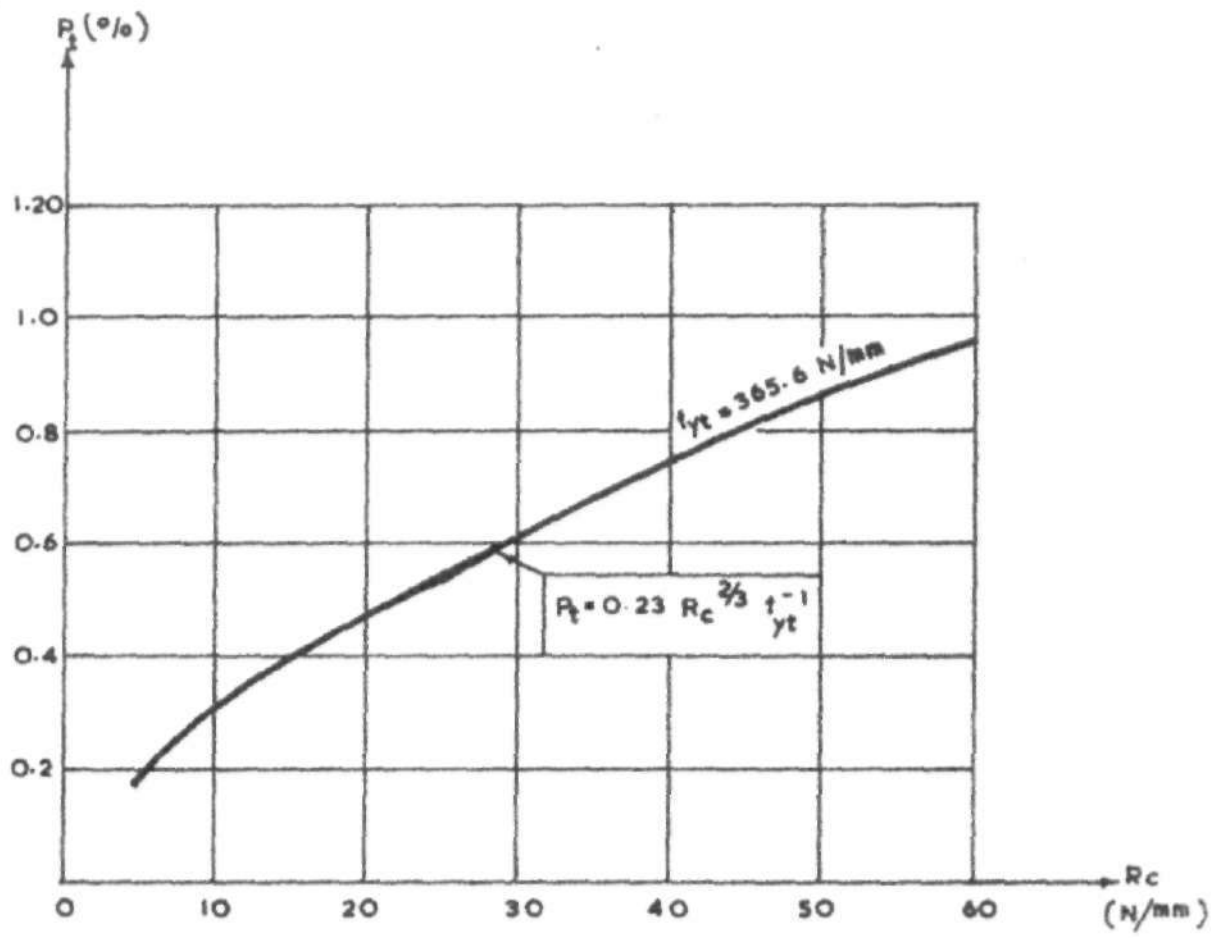


FIG. 25 PERCENTAGE OF TRANSVERSE REINFORCEMENT RECOMMENDED IN EQUATION 26

study of previous works [30, 45, 48, 51 etc] will show that the increase in column capacity with decrease in tie spacing obtains even in the absence of longitudinal bars.

It would have been necessary to investigate the possibility of reinforcing short columns only transversely but for the fact that the quantity of transverse reinforcements is limited by equation (26). Moreover, construction consideration requires the use of longitudinal bars to keep the ties in position.

Generally, however, the foregoing discussions and those in the early part of this chapter show that transverse reinforcements are far more active than the longitudinal ones. In addition to increasing the column capacity, lateral ties improve the ductility of columns, guarantees safety against buckling of the longitudinal bars and premature failure of columns, thus ensuring that all materials are effectively utilised.

The above discussions prove that a more intensive transverse reinforcement and less of longitudinal bars leads to economy.

There has been major advances in concrete technology since the end of the second world war and with modern facilities, previous practical problems of bar bending and fixing which would have overshadowed the recommendation for change in reinforcement procedure have been solve.

9.30 Typical Comparative Designs

A typical column will now be designed using both the present code requirements and the procedure recommended in this thesis.

The former recommends a high percentage of longitudinal bars and less of lateral reinforcements while the latter proposes the reverse.

Example

A 3.5 m column has to carry an axial load of 1955 KN. Assuming a 1:$2\frac{1}{2}$: $4\frac{1}{2}$ concrete mix of strength R_c = 24.9 N/mm^2 and steel strengths $f_y = f_{yt}$ = 350 N/mm^2, design the column:

(i) according to the code requirements

(ii) using the procedure recommended in this thesis.

Solutions:

(i) N.C.P. 1: 1973 requires that A_{sc} should lie between 0.8 and 8 percent of the gross cross-sectional area of the column. The Code also requires that longitudinal bars should be not less than 12 mm. in diameter.

Assuming A_{sc} = 4%, then $A_{sc} = \frac{4}{100} A_c$

$$P_u = A_c R_c + A_{sc} f_y \quad \text{----------(i)}$$

$$1955 \times 10^3 = 24.9 A_c + 350 \times \frac{4}{100} A_c$$

$$A_c = \frac{1955}{38.9} \times 10^3 = 49 \times 10^3 \text{ mm}^2$$

4 No. 12 mm ϕ bars will give A_{sc} = 0.231% which is much less than the minimum percentage of 0.8 and is therefore unsatisfactory.

Using square cross-section,

Length of side, $a = 4.9 \times 10^4$ = 220 mm or 22 cm.

concrete cover = 40 mm

$\therefore$ length of column core, a_t = 140 mm. or 14 cm.

Area of longitudinal steel $= \frac{4}{100} \times 49 \times 10^3$

$= 1960 \text{ mm}^2$

4 No. 25 mm Ø bars will give an area of 1960 mm^2

Weight of 25 mm Ø bars = 3.85 kg./m.

∴ Total weight of longitudinal bars

$= 4 \times 3.5 \times 3.85$

$= \underline{54} \text{ kg.}$

According to the Code,

diameter of lateral ties $= \frac{1}{4} \times 25 \text{ mm} = 6.25 \text{ mm.}$

Area $A_{st} = 30.6 \text{ mm}^2$

Pitch of ties = 220 mm or 22 cm.

No. of ties in 3.5 m length of column

$= \frac{3.5}{0.22} = 16$

Length of each tie $= [(140 \times 4) + 2(8 \times 6.25)]$

$= 0.66\text{m.}$

Total length of ties = 16×0.66 m

$= 10.6$ m

6.25 mm bar weights 0.231 kg/m

∴ Total weight of ties = 10.6×0.231 kg

$= 2.45$ kg.

Total weight of reinforcements using the present code design procedure

$= 54 + 2.45$ kg

$= \underline{56.45} \text{ kg.}$

(ii) Using the recommended design procedure of this thesis

with V = 0.13

$$P_u = A_c R_c + A_{sc} f_y + \frac{m A_{st}}{V \cdot S_t} f_{yt} \qquad \text{----------(11)}$$

Percentage of transverse reinforcements,

$$P_t = 0.23 R_c^{\frac{2}{3}} f_{yt}^{-1}$$

$$= \frac{0.23}{350} \times (\sqrt[3]{24.9})^2$$

$$= \frac{0.23}{350} \times \frac{8.5}{1} \times 100$$

$$= 0.56\%$$

$$S_t = \frac{m A_{st}}{P_t \, n_t}$$

$$= \frac{2 \times 30.6}{0.0056 \times 140}$$

$$= \underline{78 \text{ mm.}}$$

from equation (ii),

$$1955 \times 10^3 = 49 \times 10^3 \times 24.9 + A_{sc} \times 350 + \frac{2 \times 30.6 \times 140}{0.13 \times 78} \times 350$$

$$= 1220 \times 10^3 + 350 A_{sc} + 300000$$

from which $A_{sc} = 1240 \text{ mm}^2$

4 No. 20 mm ϕ bars will give $A_{sc} = 1260 \text{ mm}^2$

which is satisfactory

Total length of longitudinal bars = 4 x 3.5 m

Weight of 20 mm Ø bars = 2.47 kg/m

∴ Total weight of main bars = 2.47 x 4 x 3.5

= 34.6 kg.

No. of ties in 3.5 m length of column $= \frac{3.5}{0.078}$

Length of each tie = 0.66 m (obtained earlier)

Total length of tie required = 45 x 0.66 m

= 29.7 m.

6.25 mm bar weights 0.231 kg/m

∴ Total weight of ties = 29.7 x 0.231

= 6.85 kg.

Total weight of steel using the recommended design equations

= 34.6 + 6.85

= 41.45 kg.

Percentage saving in steel in using the recommended design procedure i.e. difference between (i) and (ii) methods of reinforcements

$$= \frac{56.45 - 41.45}{56.45} \times \frac{100}{1}$$

$$= 26.6\%$$

9.40. *General Effect of Method of Reinforcing on Total Quantity of Steel.*

In the typical example considered in 9.30, an intensive transverse reinforcement and less of longitudinals yielded a percentage saving in steel of 26.6%. In this section, a general

relationship between the longitudinal and lateral reinforcements will be sought for a most economical design based on the procedure recommended in this thesis.

Equation (15) may be expressed as follows:

$$\frac{P_u - A_b R_c}{A_b \; f_y} = \frac{A_s}{A_b} + \frac{1}{V} \frac{m\, A_{st}}{a_t \; S_t} \frac{A_k}{A_b} \frac{f_{yt}}{f_y} \qquad \text{----------(28a)}$$

where:

$\frac{A_s}{A_b} = p_l$ indicates the percentage of longitudinal reinforcements.

$\frac{m\, A_{st}}{a_t \; S_t} = p_t$ indicates the percentage of transverse reinforcements.

for $f_y = f_{yt}$ or $\frac{f_{yt}}{f_y} = 1$, (28a) may be expressed thus:

$$\frac{P_u - A_b \; R_c}{A_b \quad f_y} = p_l + \frac{C}{V} p_t \qquad \text{----------(28b)}$$

For any column of known cross-section and concrete and steel strength the term on the left hand side of equation (28b) is constant, say k, hence:

$$\frac{P_u - A_b \; R_c}{A_b \quad f_y} = k = p_l + \frac{C}{V} \; p_t \qquad \text{----------(28c)}$$

Equation(28c) gives the relationship between the percentages of longitudinal and transverse reinforcements. The equation shows clearly that as p_t increases, then a corresponding decrease in p_l occurs and verse versa. Fig 26 shows the recommended percentages of longitudinal and transverse reinforcements according to equation(28c) for some values of k and for Poisson's ratio V = 0.13 as used in the

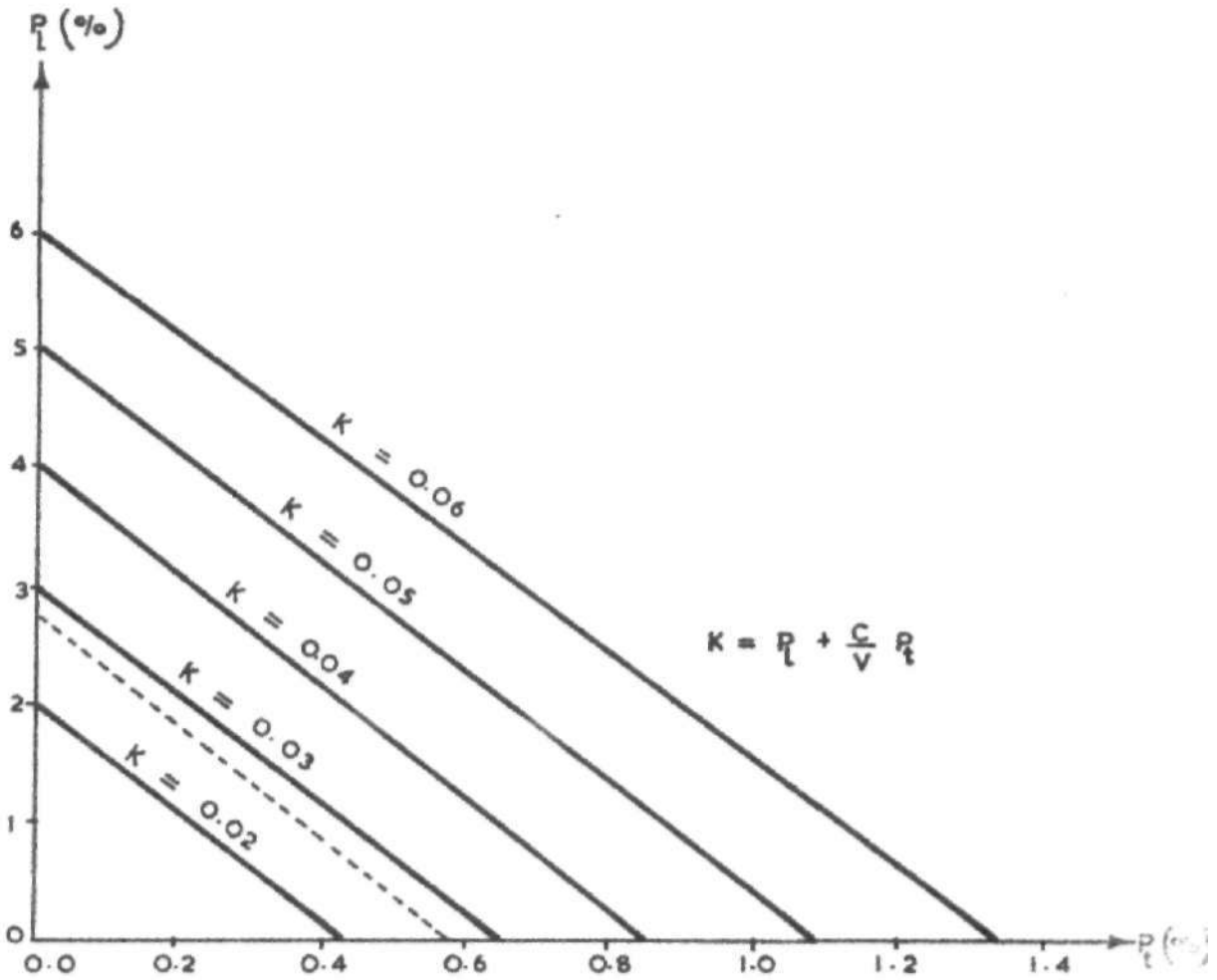


FIG. 26: LONGITUDINAL AND TRANSVERSE REINFORCEMENT RELATION

numerical example and C = 0.6. The values of C in the tested columns are 0.49 for series A; 0.56 for B and 0.60 for series C and the values of k are as follows: 0.04 for A-4 columns; 0.035 for A-8; 0.027 for A-12; 0.032 for B-4; 0.028 for B-8; 0.025 for B-12; 0.04 for C-4; 0.033 for C-8 and 0.026 for C-12 columns. In the last numerical example, the value of k was 0.045 and the relation between p_l and p_t is shown in broken lines in fig 26.

In the figure, for each value of k, the same capacity may be obtained at all points lying along the line representing that particular value of k. To determine the more economical reinforcing manner, it is necessary to know the total quantity of steel used for a required capacity.

In a unit length of column, the total volume of steel V_s may be calculated as:

$$V_s = (A_s + \frac{4\ A_{st}\ a_t}{S_t}).1 \qquad \text{----------(29a)}$$

$$= A_b\ (p_l + 2.\ \frac{2\ A_{st}}{S_t\ a_t}\ \frac{a_t^2}{a^2}).1 \qquad \text{----------(29b)}$$

For square sections,

$$a_t^2 = A_k \text{ and } a^2 = A_b$$

Therefore the volumetric percentage of reinforcements

$$p_v = \frac{V_s}{A_b} = p_l + 2\ c\ p_t \qquad \text{----------(30a)}$$

from (28c),

$$p_l = k - \frac{C}{V}\ p_t \qquad \text{----------(30b)}$$

and substituting this in (30a) gives

$$P_v = k + (2 - \frac{1}{\nu})c\, p_t \qquad \text{----------(30c)}$$

For very small percentages of lateral reinforcements ,

$p_t = 0$, $p_{vl} = p_l = k$ from (30c)

Similarly, for very small percentages of longitudinal reinforcements,

say $p_l = 0$, $p_{vt} = 2\, c\, p_t$ from (30a)

and from (30c) this means that $k - (\frac{1}{\nu})c\, p_t = 0$

or $c\, p_t = k\nu$

Hence for $p_l = 0$, $p_{vt} = 2\, c\, p_t = 2\, k\nu$

The maximum saving in steel when the percentage of longitudinal reinforcement tends to zero and consequently the percentage of transverse reinforcement takes its highest possible value is given by:

$$\Delta p_{v\ max} = \frac{p_{vl} - p_{vt}}{p_{vl}} = \frac{k - 2\,k\nu}{k} = 1 - 2\nu \qquad \text{------(31)}$$

where

Δp_v indicates the ratio of savings in steel

$p_{vl} = k$ indicates the volumetric percentage of reinforcement when $p_t = 0$.

$p_{vt} = 2k\nu$ indicates the volumetric percentage of reinforcement when $p_l = 0$.

For the numerical example considered in 9.30 where Poisson's ratio was 0.13, the maximum saving in steel if the column was reinforced only transversely will be given by

$$\triangle P_{v\ max} = 1 - 2\ V$$

$$= 1 - 2 \times 0.13 = 0.74 \text{ or } 74\%$$

This means that in the 22 x 22 cm column considered in the numerical example, if no longitudinal reinforcements were used and the column reinforced only transversely, a saving in steel of 74% would be achieved. However, practical considerations require that neither the longitudinal nor the transverse reinforcement should be zero. Hence the Nigerian Code stipulates a minimum percentage of longitudinal reinforcement to be 0.8% and in no case should the diameter be less than 12 mm.

The recommendation discussed in 9.10 is that the minimum percentage of transverse reinforcement be calculated from

$$p_t = 0.23\ R_c^{\frac{2}{3}}\ f_{yt}^{-1}$$

and the spacing, $s_t = \dfrac{m\ A_{st}}{p_t\ a_t}$

The only freedom left for the designer is to select the most practical percentage and spacings that satisfy the limitations above.

CHAPTER 10

C O N C L U S I O N S

10.10. Findings:

The basic conclusions drawn from the findings of this study are summarized as follows:

1. Lateral ties can be utilized to a much better advantage in short, axially loaded, rectangular columns than the present standard code of Practice for Reinforced Concrete structures recommends.
2. If intensively applied, ties will increase the ultimate capacity of short columns significantly. The strength of short reinforced concrete columns may therefore be considerably increased by reducing the pitch of transverse reinforcements. This increase is not significant for tie spacings between 100 to 150 percent and over of the least lateral dimension of the column but below this range any further reduction in the pitch of ties yields a marked increase in the ultimate load of the column.
3. The pitch of transverse reinforcements specified by the present code of Practice does not effectively check buckling of the longitudinal bars. An increase in the percentage of lateral ties above the quantity specified by the code of Practice enables the longitudinal bars to develop their full yield strength and thus premature column failure is prevented.

The amount of ties required to achieve this purpose depends on:

(i) Ratio of concrete strength to the yield stress of the ties.

(ii) Diameter and pitch of ties.

(iii) Least lateral dimension of the column core

(iv) Number of legs in one link of lateral reinforcement.

4. Transverse reinforcements can be effectively utilized to improve the ductility of short columns. A high percentage of transverse reinforcement results in large lateral deformation of the column and consequently an increased ductility of reinforced concrete columns.

5. The increase in the percentage of lateral ties leads to a decrease in the amount of longitudinal bars and since lateral reinforcements are far more active than the longitudinal ones, it appears that a more intensive transverse reinforcement and less of longitudinal bars will prove to be more economical in the long run.

6. Within the range of the tests, the equations derived by S. Oleszkiewicz leads to a most satisfactory estimation of the increase in strength due to lateral reinforcements with negligible deviations. The equation derived by Blume, newmark and Corning is satisfactory only when Poisson's ratio is in the neighbourhood of 0.488.

7. In view of the fact that the theory of S. Oleszkiewicz was based on a clear free body diagram and on already established fact, namely, the second degree parabolic stress-strain relation of concrete; and in view of the fact that the validity of the equations are confirmed by experimental test results; then taking practical applications into consideration, the author strongly recommends the adoption of Oleszkiewicz equation in the form given below for calculating the ultimate load of short, axially loaded rectangular columns to take into account the significant effect of transverse reinforcements:

$$P_u = \tfrac{2}{3}\, uA_c + A_{sc} f_y + \frac{m\, A_{st}}{V}\,\frac{a_t}{S_t}\, f_{yt}$$

Also, for analysis of a column at any stage of loading the load-strain relation of columns is well defined by Oleszkiewicz formula which may be used in the form:

$$p = \tfrac{2}{3}\, uA_c \left(2 - \frac{\varepsilon_1}{\varepsilon_{bu}}\right) \frac{\varepsilon_1}{\varepsilon_{bu}} + \left(A_{sc} + \frac{m\, S_{st}}{S_t}\,\frac{a_t}{}\right) E_s\, \varepsilon_1$$

The above two equations may also be expressed in terms of adequate ratios of cylinder strengths as follows:

$$P_u = 0.85\, f'_c\, A_c + A_{sc} f_y + \frac{m\, A_{st}}{V}\,\frac{a_t}{S_t}\, f_{yt}$$

and

$$p = 0.85\, f'_c\, A_c \left(2 - \frac{\varepsilon_1}{\varepsilon_{bu}}\right) \frac{\varepsilon_1}{\varepsilon_{bu}} + \left(A_{sc} + \frac{m\, A_{st}}{S_t}\,\frac{a_t}{}\right) E_s\, \varepsilon_1$$

8. The percentage of lateral reinforcement may be calculated using the equation:

$$p_t = 0.23 f_c^{\frac{2}{3}} f_{yt}^{-1}$$

as proposed in [33]

9. The effective but economical spacing for the ties may be calculated from:

$$S_t = \frac{m A_{st}}{p_t a_t}$$

10.20. Suggestions for Further Research

It should not be assumed that this research has provided a conclusive evidence that use of the formulae recommended herein is guaranteed for all ranges of construction beyond axially loaded, short column category. Before the formulae can be extended to cover other conditions, the following areas of research are suggested:

1. Effect of transverse reinforcements on the ultimate capacity of short reinforced concrete columns loaded eccentrically.
2. Effect of transverse reinforcement on long columns.
3. Transverse reinforcements may be more effective in columns which are part of a continuous structure.
4. Effects of creep and shrinkage on the capacity of short concrete columns with varied amount of transverse reinforcements are unknown for long time tests and may

be of great significance. As Mauch [29] found out; generally, reductions in carrying capacity due to creep are very significant in long columns. For very short columns, the only effect of creep is a very small reduction of the concrete strength.

5. Over 16 different parameters as listed in chapter 7 may likely affect the behaviour of tied columns. Only four of these which have been considered very essential have been studied in this research. It may be necessary to probe the influences of more.

6. Finally, to ease the application of Oleszkiewicz's equation, it will be necessary to carry out a series of comprehensive concrete tests so as to establish a chart in form of graphs or tables from which values of Poisson's ratio and/or its inverse for different concrete mixes and other specified salient conditions could be read off.

REFERENCES

1 A.C.I. COMMITTEE 105. "Reinforced Column Investigation" A.C.I. Journal, Proceedings V.29, No.5, Feb. 1933 pp.275-284.

2 A.C.I. COMMITTEE 318, "Building Code Requirements for Reinforced Concrete (A.C.I. 318-63), "American Concrete Institute, Detroit, 1963.

3 A.C.I. Publication S.P.-13: Symposium on Reinforced Concrete Columns. Copyright 1966.

4 Beedle, L.S., and Tall, L., "Basic Column Strength" Transactions, A.S.C.E., V.127, Part II, 1962, pp 138-179.

5 Bernard P.R., Discussion of "Ductility of Concrete" by H.E.H. Roy and M.E. Sozen, Proceedings of the International Symposium on Flexural Mechanics of Reinforced Concrete, Miami, Florida, Nov., 1964.

6. Bresler, B. and Gilbert, P.H. "Tie Requirements for Reinforced Concrete Columns", Journal of the A.C.I. Proceedings Vol.58, No.5. Nov.1961 pp.555-569.

7. British Standard Code of Practice, C P 114: Part 2: 1969. The Structural Use of Reinforced Concrete in Buildings, Published by the Council for Codes of Practice, British Standard Institution, B.S. House, 2 Park Street London W.I.

8. British Standard Code of Practice, B.S. 1881: 1952: Methods of Testing Concrete. B.S. Institution. 2 Park Street London, W.I.

9 Brooms, B., and Viest, I.M., "Design of Long Reinforced Concrete Columns", Proceedings A.S.C.E. V.84, No. ST 4, Paper 1694, July 1958 pp.1694-1 to 1694-28. Also Transactions, A.S.C.E. V.126, Part II, 1961, pp.367-400.

10 Chan, W.W.L., "The Ultimate Strength and Deformation of Plastic Hinges in Reinforced Concrete Frameworks", Magazine of Concrete Research Vol.7. No.21. Nov.1955 pp.121-132.

11 Chiwuzie, V.M., "A Central Library Building for Enugu" Dissertation for B.Sc. degree at the University of Nigeria, Nsukka. June, 1973.

12 Cowan, H.J., "Ultimate Strength Theory for Eccentrically Loaded Reinforced Concrete Columns". Magazine of Concrete Research, Cement and Concrete Association, London No.7: August 1951 pp.19-22.

13 Davies, J.D., "Structural Concrete", The Macmillan Company New York 1964.

14 Discussion on Effect of Lateral Reinforcement in Reinforced Concrete Columns/Discussion of Mr. J.W.H. King's Paper/The Structural Engineer, June, 1947.

15 Dunham, C.W., "Advanced Reinforced Concrete", MacGraw-Hill Civil Engineering Series, 1964.

16 Evans, R.H. and Wilby, C.B., "Concrete: Plain, Reinforced, Prestressed, Shell". London: Edward Arnold (Publishers) Ltd., 1963.

17 Grimmer, F.G., "Tests on Lightweight Concrete Columns". Structural Concrete, Journal of Reinforced Concrete Association Vol.2 No.12. Nov/Dec. 1965 p.503.

18 Hajnal-Koyni, K. "Tests on Concrete Columns Reinforced with square Twisted Steel and Mild Steel". Structural Engineer, Vol.26, July,1948.

19 Hudson, F.M. "Reinforced Concrete Columns: Effects of Lateral Tie Spacing on Ultimate Strength". A.C.I. Symposium on Reinforced Concrete Columns, Publication Sp-13, 1966 pp.235-244.

20 Johnsson, P.O. "Investigation of Bond Between Reinforcement and Concrete" Proceedings Swedish Cement and Concrete Research institute at the Royal Institute of Technology, Stockholm; 1957, No.29.

21 Keyser, C.A. "Material of Engineering, Properties, Fabrication, Uses and Testing. Prentice-Hall, Inc. Englewood Cliffs, N.J. 1965.

22 King, J.W.H. "Further Notes on Reinforced Concrete Columns" The Structural Engineer (London) Nov.1946, No.24.

23 King, J.W.H. "Some Investigations of the Effect of Core Size and Steel and Concrete Quality on Short Reinforced Concrete Column" Magazine of Concrete Research, June 1949 No.2.

24 King, J.W.H. "The Effect of Lateral Reinforcements in Reinforced concrete columns". The Structural Engineer (London) V.24, No.7. July 1946 p.355.

25 Mackey, S. "Investigation Into the Strength of Short Reinforced Concrete Columns Part I" Civil Engineering and Public Works Review, London Sept. 1954 Vol.49 No. 579. pp.945-948.

26 Mackey, S. "Investigations Into the Strength of Short Reinforced Concrete Columns (Part 2): Effect of Lateral Reinforcement". Civil Engineering and Public Works Review, London Oct. 1954. Vol.49 No.580. pp.1084-1085.

27 Mackey, S. "The Ultimate Strength of Short Axially Loaded Columns" Civil Engineering and Public Works Review, No. 1954, Vol;49. No.581 pp.1205-1207.

28 Mauch, S.P. and Holley M.J. "Creep Buckling of Reinforced Concrete Columns". Journal of the Structural Division, Proceedings of the A.S.C.E. Vol.89 No. ST 4. Aug. 1963. Part I pp.451-481.

29 Mauch, S.P. "Effect of Creep and Shrinkage on the capacity of Concrete Columns". Symposium on Reinforced Concrete Columns, A.C.I. Publication Sp-13, Detroit, 1966 pp.299-324.

30 Najib, S. "Analysis of the Influence of Transverse Reinforcement on the short concrete Column Capacity". Doctoral Thesis, Technological University of Cracow, 1973.

31 Neville, A.D., "Properties of Concrete". Sir Isaac Pitman & Sons Ltd., London, and John Willey & Sons Inc. New York, 1963, p.397.

32 Nigerian Standard Code of Practice, N.C.P. 1: 1973 Part I: "The Structural Use of Concrete in Buildings" Published by the Nigerian Standards Organisation. Federal Ministry of Industries, Lagos.

33 Oleszkiewicz, S., "Racjonalne Wymiarowanie Krepych Slupow Zelbetowych. "(Rational Design of Short Reinforced Concrete Columns). Journal of "Problemy Projektowe" 3/1973.

34 Oleszkiewicz, S. Rupert, J., and Najib, S., "The Ultimate Load of Short Lightweight Concrete Columns". Contribution to the Sympozjum CEB on Lightweight Concrete Vol.44. Technological University of Cracow Special Publication No.3. Cracow 1973 pp.97-109.

35 Oleszkiewicz, S., "The Analysis of Lightweight Concrete Deformation". Contribution to the Sympozjum CEB on Lightweight Concrete Vol.44 Technological University of Cracow Special Publication No.3. 1973 pp 60-69.

36 Orangun, C.O. "Axial Loading Tests on Short Reinforced Lightweight Aggregate (Lytag) Concrete Columns". The Structural Engineer, Vol.44, No.9, Sept. 1966 pp.291-294

37. Orchard, D.F. "Concrete Technology" 3rd ed., vol.1. Properties of Materials 1973 pp.154-258. Applied Science Publishers London.

38. Osha, A.R.A. "Concrete Mix Design in the Tropics". M.Sc. Thesis, Ahmadu Bello University, Zaria, December, 1970.

39. Pfrang, E.O. & Siess, C.P. "Behaviour of Restrained Reinforced Concrete Columns. "Proceedings, A.S.C.E. V.90 No. ST 5, Oct 1964 pp.113-136.

40. Pfrang, E.O. and Siess, S.P. "Predicting Structural Behaviour Analytically". Proceedings, A.S.C.E. V.90, No. ST 5 Oct.1964 pp 99-112.

41 Recommendations for an International Code of Practice for Reinforced Concrete. Published Jointly by the American Concrete Institute and

the cement and Concrete Association. 52 Grosvenor Gardens, London, S.W.1.

42. Reynolds, C.E. "Reinforced Concrete Designers" Handbook", 7th ed. (Revised). Cement and Concrete Association 52 Grosvenor Gardens, London, S.W.1 W/A9 1972.

43 Riad, L. "Eccentrically Loaded Reinforced Concrete Columns with Variable Cross-Section". Symposium on Reinforced Concrete Columns A.C.I. Publication SP-13, 1966 pp.245-262.

44 Roy, H.E.H. & Sozen, M.E., "Ductility of Concrete", Proceedings of the International Symposium on Flexural Mechanics, Miami, Florida No. 1964.

45. Sargin, M. Ghosh, S.K. and Handa, V.K. "Effects of Lateral Reinforcement Upon the Strength and Deformation Properties of Concrete". Magazine of Concrete Research Vol.23 No. 75-76 p.99. June-Sept. 1971.

46. Shanley, F.R. "Applied Column Theory". Transactions A.S.C.E., V.115, Paper 2410, 1950 pp.698-727.

47. Soliman, M.T.M. and Yu, C.W. "The Flexural Stress-strain Relationship of Concrete Confined by Rectangular Transverse Reinforcement" Magazine of Concrete Research Vol.19, No.61 December 1967 pp 223-238.

48 Somes, N.F. "Compression Tests on Hoop-Reinforced Concrete". Journal of the Structural Division, Proceedings of the American Society of Civil Engineers. Vol.96, No. ST 7 July 1970 pp 1495-1509.

49 Stockl, S. Discussion of "Ductility of Concrete" by H.E.H. Roy and M.E. Sozen, Proceedings of the International Symposium on Flexural Mechanics of Reinforced Concrete, Miami Florida, Nov. 1964.

50 Uzomaka, O.J. "Plastic Concrete" Journal of the Cement and Concrete Association London Vol.4 No.4, April 1970 pp 155-156.

51 Vitelmo, V., Bertero and Felipa, C. Discussion of "Ductility of Concrete" by H.E.H. Roy and M.E. Sozen. Proceedings of the International Symposium on Flexural Mechanics of Reinforced Concrete, Miami, Florida, Nov. 1964.