A STUDY ON NATURAL CONVECTION FLOW IN CYLINDRICALGEOMETRIESWITH TIME-PERIODIC HEAT INPUT

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A DISSERTATION SUBMITTED TO THE SCHOOL OF POSTGRADUATE STUDIES AHMADU BELLO UNIVERSITY, ZARIA, IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE AWARD OF

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Declaration

I declare that the work in this dissertation entitled "A Study on Natural Convection Flow in Cylindrical Geometries with Time-Periodic Heat Input" has been carried out by me in the Department of Mathematics. The information derived from the literature has been duly acknowledged in the text and a list of references provided. No part of this dissertation was previously presented for another degree or diploma at this or any other Institution.

Michael Oluwakayode ONI	__________________	__________________
(Name of Student)	Signature	Date

Certification

This dissertation entitled "A Study on Natural Convection Flow in Cylindrical Geometries with Time-Periodic Heat Input" by Michael Oluwakayode ONI meets the regulations governing the award of the degree of Master of Science in Mathematics of the Ahmadu Bello University, and is approved for its contribution to knowledge and literary presentation.

Prof. B.K. Jha	____________________	________________
Chairman, Supervisory Committee	Signature	Date
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Member, Supervisory Committee	Signature	Date
Prof. A.A. Tijjani	____________________	________________
Head of Department	Signature	Date
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External Examiner	Signature	Date
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Dean, School of Postgraduate Studies	Signature	Date

Dedication

This dissertation is dedicated to God Almighty, the Father of our Lord JESUS CHRIST, through whom all things were made possible. Also to my beloved mother Prophetess Mrs.E. Oni and the memory of my late father H.R.H. J.T. Oni (The Akinrin of Ekinrin-Adde II).

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Abstract

This dissertation investigatesthe effect of steady-periodic heating at the surfaces of the cylinders on natural convection flow in a tube and an annulus.Two problemsare considered in this work. In both problems, the fluid was assumed to be fully developed and the mathematical equations governing the flow were derived. The first problem presents the impact of time-periodic heating at the surface of a tube on fluid velocity, temperature, rate of heat transfer and skin friction.While the second problem examines the effect of periodic heating at the surfaces ofthe cylinders on natural convection flow in an annulus.Using suitable transformation, the partial differential governing equations were transformed to their corresponding ODE, representing the steady and periodic regimes.Closed-form expressions for velocity, temperature, skin-friction, mass flow rate and rate of heat transfer which was expressed as Nusselt number were obtained in terms of modified Bessel's function of first and second kinds for both cases.The solutions obtained were graphically represented and the effects of periodic heating (Strouhal number St), Prandtl number(Pr) and aspect ratio(λ) on fluid velocity, temperature, rate of heat transfer and skin-friction were investigated. Result indicates that the role of Strouhal number, Prandtl number and aspect ratio is to decrease fluid velocity, temperature, and skin-friction at the surfaces of the cylinders.Also, the rate of heat transfer between the surfaces of the cylinders and the fluid increases with increase in Strouhal number (St), Prandtl number (Pr) and aspect ratio (λ).Furthermore, for small Strouhal number (St),velocity, temperature, rate of heat transfer, mass flow rate and skin-friction of the time-periodic regime (periodic part) reduces to steady regime. At this point, the steady and periodic regimes make equal contributions on overall velocity. In addition, for large Strouhal number(St), the periodic regime contributes an adverse effect on the overall fluid temperature and velocity.

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Nomenclatures and Greek Letters

a	radius of inner cylinder
b	radius of outer cylinder
A	steady velocity profile
B	unsteady velocity profile
F	steady temperature profile
G	unsteady temperature profile
g	acceleration due to gravity
I_n	modified Bessel's function of first kind on order n, where $n = 0,1,2,3,\ldots$
K_n	modified Bessel's function of second kind on order n,where $n = 0,1,2,3,\ldots$
Pr	Prandtl number
r	dimensional coordinate
R	dimensionless coordinate
St	Strouhal number
t	time
T	temperature of the fluid
T_0	initial temperature
T_1	steady temperature on the surface of the tube
T_2	periodic temperature amplitude on the surface of the tube

u	dimensional axial velocity of fluid
v	dimensional radial velocity of fluid
$\vert\ \vert$	absolute value/modulus

Greek alphabets

α	thermal diffusivity
β	coefficient of thermal expansion
λ	aspect ratio $\left(b/a\right)$
ψ	phase of temperature
χ	phase of velocity
τ	skin friction
ν	kinematic viscosity
ω	frequency

Subscript

R	real part
i	imaginary part
1	outer surface of inner cylinder
λ	inner surface of inner cylinder

Dimensionless Quantities

Dimensionless quantities are used extensively in fluid Mechanics both in the development of theoretical models (to determine relationship between parameters) and in experimental work (to allow results to be generalized and hence use to predict performance for different system). This dissertation used quite a number of the dimensionless quantities in the course of the research. We give brief notes on the dimensionless quantities used.

1. **Prandtl Number ($\boldsymbol{Pr}$):** Prandtl number is a dimensionless number named after the German physicist Ludwig Prandtl. It is defined as the ratio of momentum diffusivity to thermal diffusivity.(Pr) is typically less than one for gases and greater than one for liquids. Mathematically, Prandtl number is given by the expression: $Pr = \frac{\nu}{\alpha}$; where α, and ν are defined in the nomenclature.

2. Strouhal Number$(St = \frac{a^2\omega}{\nu})$: The Strouhal number (St) is a dimensionless number describing oscillating flow mechanism. The parameter is named after Vincenc Strouhal, a Czech physicist (1878). It is the ratio of product of square of inner radius and dimensionless frequency to kinematic viscosity. The Strouhal number is directly proportional to the frequency of periodic heating.

3. Nusselt Number: Nusselt numberis the rate of heat transfer at the wall-fluid boundary. It is the ratio of convective to conductive heat transfer across the boundary.

Basic Equations

The set of equations that describes fluid flow are combination of the Navier-Stokes equations of fluid dynamics and energy equation of thermodynamics. These balance equations arises from applying Newton's second law to fluid motion, together with the assumption that the stress in the fluid is the sum of a diffusing viscous term and a pressure term. The Navier-Stokes equations were derived by Navier, Poisson, Saint Venant and Stokes between 1827 and 1848. These equationsarealways solved together with the continuity equation simultaneously either analytically or numerically. The Navier-Stokes equations represent the conservation of momentum while the continuity equation represents the conservation of mass.Finally, the energy equation represents the conservation of energy for laws of thermodynamics.

1. Continuity Equation

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho u) = 0 \qquad \text{(i)}$$

Ifρ is constant, then $\frac{\partial \rho}{\partial t} = 0$

$$(\nabla. u) = 0$$

In cylindrical coordinates, the continuity equation is given by

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r u) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v) + \frac{\partial}{\partial z}(\rho r w) = 0 \qquad \text{(ii)}$$

2. Navier-Stokes Equation

$The\ inertial\ forces = -pressure\ forces + viscous\ force$ +*external applied forces applied to the fluid*

$$\frac{\partial u}{\partial t} + u \cdot \nabla u = -\frac{1}{\rho}\nabla \bar{p} + \nu \nabla^2 u + \frac{1}{3}\nu \nabla(\nabla \cdot \mathrm{u}) + g \qquad \text{(iii)}$$

where the terms in (iii) signify;

$\frac{\partial u}{\partial t}$ variation

$u \cdot \nabla u$ convection

g external source

$\frac{1}{\rho}\nabla \bar{p}$ internal source

$\nu\nabla^2 u + \frac{1}{3}\nu\nabla(\nabla \cdot \mathrm{u})$ diffusion

Navier-Stokes equation in cylindrical coordinate:

The Navier-Stokes equation in cylindrical coordinate for constant density and viscosity is given as follows:

$$r: \quad \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \theta} + w\frac{\partial u}{\partial z} - \frac{v^2}{r}\right) = -\frac{\partial p}{\partial r} + \rho g_r + \mu\left(\nabla^2 u - \frac{u}{r^2} - \frac{2}{r^2}\frac{\partial v}{\partial \theta}\right)$$

$$\theta: \; \rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + \frac{v}{r}\frac{\partial v}{\partial \theta} + w\frac{\partial v}{\partial z} - \frac{uv}{r}\right) = -\frac{\partial p}{\partial \theta} + \rho g_\theta + \mu\left(\nabla^2 v - \frac{v}{r^2} + \frac{2}{r^2}\frac{\partial u}{\partial \theta}\right)$$

$$z: \; \rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + \frac{v}{r}\frac{\partial w}{\partial \theta} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \rho g_z + \mu(\nabla^2 w) \text{(iv)}$$

Where $\nabla^2 = \frac{1}{r}\frac{\partial}{\partial \mathrm{r}}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$

Letting velocity to be along one direction, $V = [0,0,w(r)]$, then neglecting pressure gradient, the Navier Stokes equation in cylindrical coordinate (iv) reduces to

$$\frac{\partial w}{\partial t} = \frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right) = 0 \qquad \text{(v)}$$

3. Conservation of Energy

The energy equation describes conservation of energy in the system. This equation is derived from laws of thermodynamics.

$$\frac{\partial T}{\partial t} + u \cdot \nabla T = \frac{\kappa}{\rho C_p} \nabla^2 T \qquad \text{(vi)}$$

CHAPTER ONE

GENERAL INTRODUCTION

1.1 Background of the Study

With advancement in science and technology, fluid dynamics has been of great importance over the decades due to its technological, industrial and engineering applications, which include calculating forces and moment of aircraft, determining the mass flow-rate of petroleum through pipelines, predicting weather pattern, modeling fission weapon detonation, traffic engineering, rocket engines, wind turbines, oil pipelines, air condition, turbine system in power generation and air condition.Ferzigerand Peric (1996), Nishigaki *et al.* (2013)

The investigation of laminar natural convection in vertical tubes or annuli has received a lot of attention because of its importance in many engineering applications. The main technical applications of these researches concern cooling systems for electronic devices, electrochemical processes, nuclear reactors, and in solar energy collectors.

This interest is due to the fact that researchers need a better understanding of the phenomena of heat and mass transfer in Engineering, Geophysical and Biological sciences.

1.2 Statement of the Problem

Wang (1998) investigated free convection between vertical plates with periodic heat input. In his work, he separated the solutions into steady and unsteady regime and stated the condition on which the periodic heat input is significant. But in real life situations, heat transfer is common through cylinders and annuli. It is therefore significant to study free convection flow in a vertical tube and annulus inspired by periodic heating at the surfaces of the cylinders.Hence, this dissertation endeavors to investigate the impact of periodic heating at the surfaces of the cylinders on natural convection flow formation in a tube and an annulus.

1.3 Aim and Objectives of the Study

The aim of this work is to study flow behavior of natural convection flow in a tube andan annulus subject to periodic heat input.

The objectives to attain the set aim are to:

i. derive the equations of motion governing the problems.
ii. investigate the effect of periodic heating at the surface of the tube on velocity, temperature, skin-friction and rate of heat transferin the tube.
iii. investigate the effect of periodic heating and aspect ratio on velocity, temperature, skin-friction and rate of heat transfer in the annulus.
iv. investigate fluid flow behaviour for different types of fluid dictated by the Prandtl number (Pr)
v. give the condition on which periodic heating on the surfaces of the cylinders is significant in both tube and annulus.

1.4 Research Methodology

The methodology adopted in the realization of the set of objectives is a five-phase approach. In phase one, we review existing literatures and extend them to include different physical geometries. Phase two is concerned with solving the mathematical model that described the study on some natural convectionflow in different geometries. To obtain the analytical solutions, we used the method of undetermined coefficient and direct integration method. In phase three, numerical values of the analytical solutions in phase two are obtained. Here, we used a computer package, MATLAB 12b to obtain the numerical values of the analytical solutions by writing a computer program for phase two. The numerical values obtained in phase three are presented graphically in phase four by the use of MATLAB 12b. The last phase has to do with physical interpretation of the graphs so as to discuss the influence of each of the governing parameters on the flow and draw conclusions.

1.5 Organization of the Dissertation

This dissertation is grouped into six chapters with references and appendices. Chapter one comprises basically the general introduction of the dissertation. Literature review in the research is basically contained in chapters two. Chapter three contains the mathematical analysis of the problems with the non-dimensionalisation. Chapters four and five contain the solutions of the problems and discussion of the results respectively. Chapter six gives the summary and concluding remarks of the study followed by references and appendices.

1.6 Significance of the Study

This study reveals the role of periodic heating at the surfaces of cylinders on natural convection flow in a vertical tube and annulus. The results in this research will be of great important to design engineers in improving electrical devices with thermostat and other appliances that require censors. Moreover, it is hoped that the results will not only provide useful information for industrial applications, but also serve as an improvement on the previous studies.

1.7 Scope of the Study

The formulation, analysis and results obtained in this work are theoretical. No experimental work has been carried on this work.The governing differential equations were employed and conclusions were drawn based on depicted lines and contour graphs.

1.8 Definition of Terms

i. **Annulus**: Annulus is the area bounded by two concentric cylinders

ii. **Boussinessq Approximation**: Boussinessq approximation is the assumption that the fluid flow is considered under little variations of temperature and density.

iii. **Compressible and Incompressible Fluids:** A compressible flow is a flow whose fluid density varies significantly within the flow field. While in an incompressible flow the density does not change.

iv. **Cylindrical Geometry:** A cylindrical geometry is any shape like a cylinder.

v. **Dimensionless Quantity:** Is a quantity without an associated physical dimension.

vi. **Free orNatural Convection**: Free or Natural convection is a mechanism in which fluid motion is generated only by density differences in fluid cause by temperature gradients.It is a type of heat transport in which the fluid motion is not generated by an external source (pump, fan, suction devices).

vii. **Laminar Flow:** Laminar flow occurs when fluid flows at low velocity, in parallel layers with no disruption between the layers.

viii. **Nusselt Number ($\boldsymbol{Nu}$)** : Nusselt numberis the rate of heat transfer between the fluid and the surfaces of the cylinders.

ix. **Periodic Function**:A function f is said to be periodic with period P (P being a nonzero constant) if we have$f(x + P) = f(x)$ for all x in the domain.

x. **Periodic Heating**: This is when the heating on the surfaces of the cylinders is sinusoidal or follows any periodic function.

xi. **Skin-friction**: Skin-friction is defined as the friction (drag) between a fluid and the surface of a solid moving through it or between a moving fluid and its enclosing surface.

xii. **Steady Flow**: A flow is said to be steady if the velocity and other flow parameters such as pressure and acceleration are independent of time but may depend on position.

xiii. **Unsteady Flow**: A flow is said to be unsteady if the velocity and other flow parameters such as pressure and acceleration are dependent on time.

xiv. **Volume Flow Rate:** Is the volume of fluid which passes through the geometry per unit time; usually represented by the symbol Q. The SI unit is m^3/s (cubic metres per second).

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

This chapter explores some of the related literature of previous research and findings. Due to the application of fluid flow in cooling systems for electronic devices and solar energy,natural convection has continued to gain extensive attention over the past years.

The engineer is interested in, transport of fluids from one location to another through pipes or open ducts, the pressure drops in the system and hence of the power required for pumping. Reynolds (1883) first showed that when a fluid is flowing through a tube over a surface, the pattern of the flow will vary with velocity, the physical properties of the fluid, and the geometry of the surface. In order to have inevitable guide to a study of this nature, this chapter therefore, explains some of the related literature of some previous researches.

2.2 Natural Convection Flow

Natural convection flow in cylindrical geometry has been studied for over fourdecades due to its applications in industries. Some related works on fully developed natural convection flow in vertical tubes and annulus include the work of Lighthill (1952), Aung et al. (1972), Miyatake and Fuzii (1972). Miyatake et al. (1973) gave their results for a steady free convective flow between vertical walls by applying different physical treatments for transport processes.El-Shaarawi and Sarhan(1981) investigatedthe fully developed free convection flow in a vertical annulus with isothermal and adiabatic boundaries. In the work of Jha et al. (2003), a closed form solution for transient free convective flow of a viscous and incompressible fluid in a vertical channel due to symmetric heating of channel walls was obtained. They developed correlation between steady state time and Prandtl number and concluded that nature of correlation is linear when Prandtl number is greater than one and cubic for Prandtl number less than one. In addition,Asmaa (2012) investigated the effect of free convection on the developing forced laminar upward and downward flow in a vertical annulus when the inner is heated uniformly and the motionless outer cylinder is kept adiabatically insulated. A survey on the related literature also indicate that Lee and Kuo (1998), Mahmud and Fraser (2002), Leong and Lai (2006) as well as Jha *et al.* (2011) have studied natural convection in annulus considering different physical situations.

2.3 Periodic Heat Input in Natural Convection Flow

In the course of modeling a real life situation of fluid flow, periodic heat input in a channel has captured the attention of many researchers due to its every day applications in electrical and electronic appliances such as oven, drying machines, thermostat component, automatic control and microchips. A number of researchers have studied the effect of periodic heat input on time dependent free convection flow in a channel. Sparrow and Gregg (1960) studied nearly Quasi-steady free convection heat transfer in gases. Menold and Yang (1962), Nanda and Sharma (1963) and Muhuri and Gupta (1979) investigated the effect of periodic heating on a single vertical plate with no edge on the boundary layer development. Chung and Anderson (1961) examined unsteady laminar free convection in a channel. Yang et al. (1974) studied laminar natural convection with oscillatory surface temperature using the finite difference approach. From miniaturization of electrical and electronic panels, Bar-Cohen and Rohsenow (1984) analyzed fully developed natural convection between two periodically heated parallel plates. Wang (1998) investigated free convection between vertical plates with periodic heat input. He presented the solutions of the temperature and velocity field of the fluid as function of Prandtl number, Strouhal number and indirectly Rayleigh number. He concluded that increasing Strouhal number decreases the unsteady temperature and velocity of the fluid and also for small Strouhal number, the effect of periodic heating corresponds exactly with the steady state results.

In recent past, Jha and Ajibade (2010) analyzed free convection flow between vertical porous plates with periodic heat input being an extension of Wang (1998) by including suction/injection and they concluded that inclusion of suction/injection distorted the symmetric nature of the flow considered by Wang (1998). Other research works on periodic heat input include Jha and Ajibade (2009) who investigated the effects of heat generating/absorbing fluid between vertical porous plates with periodic heat input. The same authors, (2012) studied the effect of viscous dissipation on natural convection flow between vertical parallel plates with time-periodic thermal boundary conditions and found that the influence of Strouhal number is suppressed on the flow when Eckert number assumed a relatively large positive value.

A coupled stress fluid modeling on free convection oscillatory hydromagnetic flow in inclined rotating channel was studied by Ahmed et al. (2014). To see the effect of periodic heat input on micro channel, Adesanya (2014) studied free convective flow of heat generating fluid through a porous vertical channel with velocity slip and temperature jump. He concluded that increase in slip and temperature jump parameters increases the flow velocity and fluid temperature respectively. Also, Adesanya et al. (2015) investigated the effect of atransversemagnetic field onthe flowofviscousin-compressible fluid flowing throughachannel subjected toperiodicheatingusingAdomian decomposition method. They found that bothskinfrictionandNusselt number decreasesatthewallwithincreasingvalueofmagnetic field parameter.

On periodic heat input in cylinders, Shiu and Wu(1996)discussed transient heat transfer in annular fins of various shapes with their bases subjected to a heat flux varying as a sinusoidal time function. Also, Cole and Crittenden (2009) investigated Steady-Periodic heating of cylinder presenting the solutions in term of Green's function for five different boundary conditions.

However, to our best findings, no studies have been carriedout analytically on natural convection flow in cylindrical geometries with time-periodic heat input. To this ends, this prompt this dissertation.

CHAPTER THREE

PROBLEM FORMULATIONS AND ANALYSIS

3.1 Introduction

This chapter is concerned with the derivation of the momentum and energy equations for different flow formation. The dimensional equations are transformed into the dimensionless equations using appropriate non-dimensional parameters.

3.2 Natural Convection Flow in a Vertical Tube with Periodic Heat Input

The objective of the present study is to investigate the impact of periodic heating on natural convection flow in a tube.

3.2.1 Mathematical formulation

Consider a natural convection flow of a viscous incompressible fluid in a vertical tube as shown in Figure3.1. Initially, the surface temperature of the tube and fluid is at T_0 and then assumed to be heated periodically to $T(r,t) = T_1 + T_2\cos(\omega t)$. The flow is inspired due to the temperature difference between the surface of the tube (with radius $r = a$) and fluid, this results to natural convection.

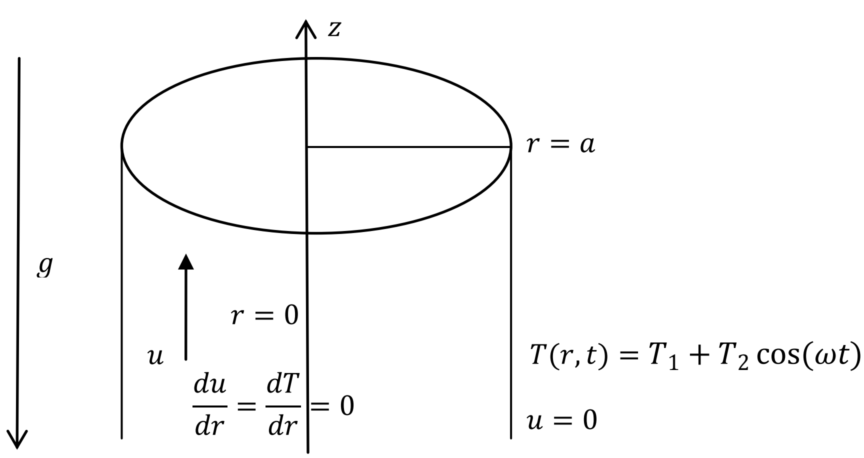

Figure 3.1 Schematic diagram of the problem

Every other parameters are assumed constant except otherwise stated and are presented in nomenclature.

The flow is assumed to be fully developed and the viscous dissipation term in the energy equation is also assumed to be negligible. The z and r-axes are respectively taken in the vertical upwards direction and radial direction.

Using Boussinesq's approximation, the governing continuity, momentum and energy equations respectively for the present physical situation in cylindrical geometry are:

$$\frac{d(rv)}{dr} = 0 \tag{3.1}$$

$$\frac{\partial u}{\partial t} = \nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)\right] + g\beta(T - T_0) \tag{3.2}$$

$$\frac{\partial T}{\partial t} = \alpha\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right] \tag{3.3}$$

Subject to the following relevant boundary conditions:

$$\begin{aligned} &\frac{\partial u}{\partial r} = 0 \qquad \frac{\partial T}{\partial r} = 0 \qquad && r = 0 \\ &u = 0 \qquad T = T_1 + T_2\cos(\omega t) \qquad && r = a \end{aligned} \tag{3.4}$$

3.3 Natural Convection Flow in a Vertical Annulus with Time-Periodic Boundary Conditions

The objective of the present study is to investigate the periodic flow formation inside vertical annulus due to periodic heating at the surfaces of the cylinders and to see the effect of annular gap and other governing parameters.

3.3.1 Mathematical formulation

An unsteady natural convection flow of viscous incompressible fluid in a vertical annulus with periodic heating of the surfaces of the cylinders as presented in Fig.3.2 is considered. The flow is assumed to be laminar and fully developed, such that fluid velocity u has non-vanishing component u_0 along the $z - axis$. The z$-axis$ is the axial coordinate which is parallel to the gravitational acceleration g but with opposite direction, while the $r - axis$ is the axis in radial direction. The radius of the inner and outer cylinders are a and b respectively. The surfaces of the cylinders are heated to a temperature $(T_1 + T_2 \cos(\omega t))$ greater than that of surrounding fluid having temperature (T_0). Due to this temperature difference, natural convection is setup in the vertical annular gap. The usual Boussinesq's approximation governing the mass conservation, momentum and energy equations respectively for the present physical situation in dimensional form are exactly the same with equations. (3.1 – 3.3) but with different boundary conditions:

$$\frac{d(rv)}{dr} = 0 \tag{3.5}$$

$$\frac{\partial u}{\partial t} = \nu \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)\right] + g\beta(T - T_0) \tag{3.6}$$

$$\frac{\partial T}{\partial t} = \alpha \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right] \tag{3.7}$$

Subject to the following relevant boundary conditions:

$$\begin{aligned} &u = 0 \qquad T = T_1 + T_2\cos(\omega t) \qquad r = a \\ &u = 0 \qquad T = T_1 + T_2\cos(\omega t) \qquad r = b \end{aligned} \tag{3.8}$$

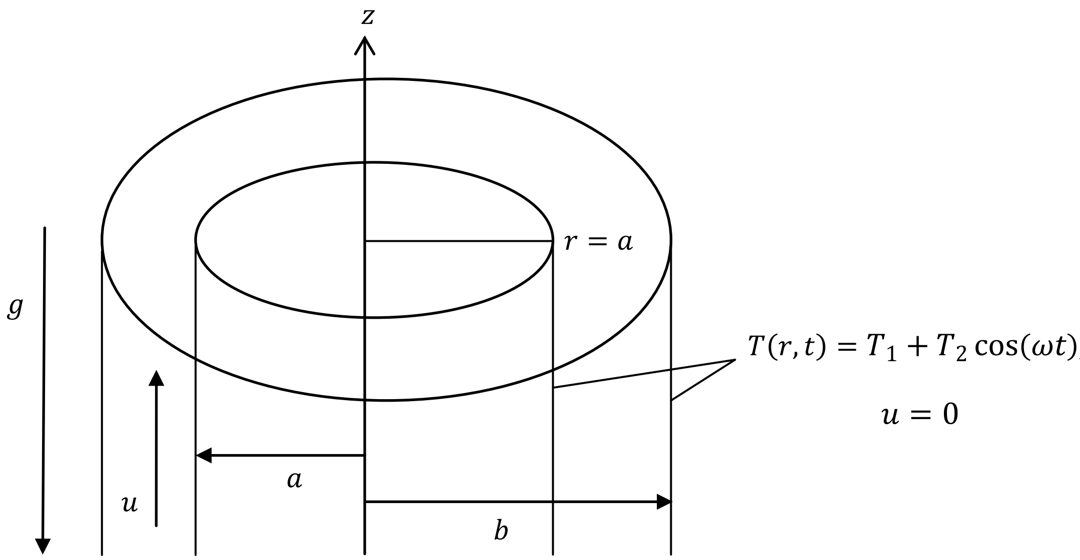

Figure 3.2:Physical model and coordinate system

3.4 Non- dimensionalization

In industries and research centres, the applications of fluid mechanics in designs make much use of empirical results from a lot of experiments. These results are presented in dimensional form. Dimensional analysis provides a strategy for choosing relevant data and how they should be presented. Due to the complexity of some engineering designs, results anddata obtained are often difficult to present in a readable form. Therefore, there is need to reduce the number of units in order to obtain a unified solution. So the governing equations of the above problems in dimensional form are non-dimensionalised to obtain their non-dimensional form.

3.4.1 Non-dimensionalization of problem 3.2

Using the following modelsintroduced by Wang (1998) to separate the velocity as well as temperature into steady and periodic parts:

$$u(r,t) = \frac{g\beta a^2}{\nu}\left[(T_1 - T_0)A(R) + T_2 B(R)e^{i\omega t}\right] \tag{3.9}$$

$$T(r,t) = T_0 + \left[(T_1 - T_0)F(R) + T_2 G(R)e^{i\omega t}\right] \tag{3.10}$$

$$R = \frac{r}{a}\ , \qquad Pr = \frac{\nu}{\alpha}\ , \qquad St = \frac{a^2\omega}{\nu} \tag{3.11}$$

where St is the Strouhal number which is directly proportional to the frequency of periodic heating and Pr is the Prandtl number which varies directly to kinematic viscosity and inversely proportional to thermal conductivity.

Substitutingequations(3.9) – (3.11) into (3.1) – (3.8), and equating the oscillatory and non-oscillatory parts, we have;

$$\left.\begin{aligned}
&\frac{d^2A(R)}{dR^2} + \frac{1}{R}\frac{dA(R)}{dR} = -F(R) \\
&\frac{d^2B(R)}{dR^2} + \frac{1}{R}\frac{dB(R)}{dR} - iStB(R) = -G(R) \\
&\frac{d^2F(R)}{dR^2} + \frac{1}{R}\frac{dF(R)}{dR} = 0 \\
&\frac{d^2G(R)}{dR^2} + \frac{1}{R}\frac{dG(R)}{dR} - iStPrG(R) = 0
\end{aligned}\right\} \tag{3.12}$$

subject to the following dimensionless boundary conditions

$$\left.\begin{aligned}
&\frac{dA}{dR} = \frac{dB}{dR} = \frac{dF}{dR} = \frac{dG}{dR} = 0 && \text{at} \quad R = 0 \\
&A(1) = B(1) = 0 && \text{at} \quad R = 1 \\
&F(1) = G(1) = 1 && \text{at} \quad R = 1
\end{aligned}\right\} \tag{3.13}$$

Equation (3.12) above gives the flow configuration in dimensionless form where $A(R)$, $F(R)$ represent the steady part and $B(R)$, $G(R)$represent the unsteady part of velocity and temperature profiles respectively.

3.4.2 Non-dimensionalization of problem 3.3

Using similar model and dimensionless parameters of equations (3.9-3.11) on problem 3.3, we have;

$$\left.\begin{aligned}
&\frac{d^2A(R)}{dR^2}+\frac{1}{R}\frac{dA(R)}{dR}=-F(R)\\
&\frac{d^2B(R)}{dR^2}+\frac{1}{R}\frac{dB(R)}{dR}-iStB(R)=-G(R)\\
&\frac{d^2F(R)}{dR^2}+\frac{1}{R}\frac{dF(R)}{dR}=0\\
&\frac{d^2G(R)}{dR^2}+\frac{1}{R}\frac{dG(R)}{dR}-iStPrG(R)=0
\end{aligned}\right\} \tag{3.15}$$

subject to the following dimensionless boundary conditions

$$\left.\begin{aligned}
&A(1)=B(1)=0\\
&F(1)=G(1)=1\\
&A(\lambda)=B(\lambda)=0\\
&F(\lambda)=G(\lambda)=1
\end{aligned}\right\} \tag{3.16}$$

where

$$\lambda=\frac{b}{a} \tag{3.17}$$

CHAPTER FOUR

SOLUTION OF THE PROBLEMS

4.1 Introduction

In this chapter, the non-dimensional equations obtained in chapter three are solved.It also presents solution of other quantities of interest such as volume flow rate, rate of heat transfer and skin-friction at surfaces of the cylinders.

4.2 Natural Convection Flow in a Vertical Tube with Periodic Heat Input

Equations (3.12) and boundary conditions (3.13), have the following analytical solutions for steady and unsteady temperature profiles respectively:

$$F(R) = 1 \tag{4.1}$$

$$G(R) = \frac{I_0(R\sqrt{iPrSt})}{I_0(\sqrt{iPrSt})} \tag{4.2}$$

substituting(4.1) and (4.2) into (3.12) with boundary conditions (3.13), the steady and the unsteady velocity profilesare given respectively as:

$$A(R) = \frac{1}{4}[1 - R^2] \tag{4.3}$$

$$B(R) = \frac{1}{iSt(Pr-1)}\left[\frac{I_0\left(R\sqrt{iSt}\right)}{I_0\left(\sqrt{iSt}\right)} - \frac{I_0\left(R\sqrt{iPrSt}\right)}{I_0\left(\sqrt{iPrSt}\right)}\right] \tag{4.4}$$

where I_0 is the modified Bessel's function of first kind of order zero.

4.2.1 Phase angle and amplitude of velocity and temperature for problem 3.2

Since the flow is periodic, in order to have a good insight on the problems, we compute the following: The phases ψ and χ which are the angles between the temperatures G_i and

G_R and velocities B_i and B_R respectively while the amplitudes $|G|$ and $|B|$ denote the absolute temperature and velocity at any given point in the cylinder.

The phase (ψ) of the temperature and the phase (χ) of the velocity are obtained by Jha and Ajibade (2010)

$$\psi = \tan^{-1}\left(\frac{G_i}{G_R}\right) \tag{4.5}$$

$$\chi = \tan^{-1}\left(\frac{B_i}{B_R}\right) \tag{4.6}$$

While the amplitudes of the unsteady temperature and unsteady velocity are given respectively by

$$|G| = \sqrt{\left({G_R}^2 + {G_i}^2\right)} \qquad \text{and} \qquad |B| = \sqrt{\left({B_R}^2 + {B_i}^2\right)} \tag{4.7}$$

The phase of the velocity χ is the angle between the velocities B_i and B_R while the amplitude $|B|$ denotes the absolute velocity at any given point in the cylinder.

4.2.2 Skin-friction of problem 3.2

The skin-friction (τ) at the cylinder wall in dimensionless form is obtained by differentiating the velocity as follows:

$$\tau = -\frac{dB_R}{dR}\bigg|_{R=1} = \frac{1}{\sqrt{iSt}(Pr-1)}\left[\frac{\sqrt{Pr}\, I_1\left(\sqrt{iPrSt}\right)}{I_0\left(\sqrt{iPrSt}\right)} - \frac{I_1\left(\sqrt{iSt}\right)}{I_0\left(\sqrt{iSt}\right)}\right] \tag{4.8}$$

4.2.3 Rate of heat transfer for problem3.2

Another quantity of interest is the rate of heat transfer (represented by Nusselt number in dimensionless form)at the cylinder wall. This quantity is obtained by differentiating the temperature as follows:

$$Nu = \frac{dG_R}{dR}\bigg|_{R=1} = \frac{\sqrt{iPrSt}\, I_1\left(\sqrt{iPrSt}\right)}{I_0\left(\sqrt{iPrSt}\right)} \tag{4.9}$$

4.2.4 Volume flow rate for problem 3.2

Also, the volume flow rate (amount of fluid passing through the tube per unit time) is given in dimensionless form by:

$$Q = \int_0^1 RB(R)dR = \frac{1}{(iSt)^{3/2}(Pr-1)}\left[\frac{I_1\left(\sqrt{iSt}\right)}{I_0\left(\sqrt{iSt}\right)} - \frac{I_1\left(\sqrt{iPrSt}\right)}{\sqrt{Pr}\, I_0\left(\sqrt{iPrSt}\right)}\right] \tag{4.10}$$

4.2.5 Limiting case of Prandtl number $(\mathbf{Pr})$ for velocity, skin-friction and volume flow rate of problem 3.2

It is good to note that equations (4.4), (4.8) and (4.10) become undefined when $Pr = 1$. In order to remove this singularity, the following expressions are obtained for velocity, skin-friction and volume flow rate respectively by taking limit $Pr \to 1$:

$$B(R) = \frac{I_0\left(R\sqrt{iSt}\right)I_1\left(\sqrt{iSt}\right) - RI_1\left(R\sqrt{iSt}\right)I_0\left(\sqrt{iSt}\right)}{2\sqrt{iSt}\, I_0\left(\sqrt{iSt}\right)} \tag{4.11}$$

$$\tau = \left[\frac{1}{2} - \frac{{I_1}^2\left(\sqrt{iSt}\right)}{2{I_0}^2\left(\sqrt{iSt}\right)}\right] \tag{4.12}$$

$$Q = \frac{i}{2St}\left[1 - \frac{{I_1}^2\left(\sqrt{iSt}\right)}{{I_0}^2\left(\sqrt{iSt}\right)} - \frac{2I_1\left(\sqrt{iSt}\right)}{\sqrt{iSt}\, I_0\left(\sqrt{iSt}\right)}\right] \tag{4.13}$$

4.3 Natural Convection Flow in a Vertical Annulus with Time-Periodic Boundary Conditions

The dimensionless equations (3.12) and (3.14) are seen to be the same but with different boundary conditions. Following the same method of solution as solutions 4.2, the temperature profiles as well as the velocity profiles for steady and unsteady part are given respectively as:

$$F(R) = 1 \tag{4.14}$$

$$G(R) = C_1 I_0\left(R\sqrt{iPrSt}\right) + C_2 K_0\left(R\sqrt{iPrSt}\right) \tag{4.15}$$

$$A(R) = \frac{1}{4}\left[1 - R^2 + \frac{(\lambda^2-1)}{\ln\lambda}\ln(R)\right] \tag{4.16}$$

$$B(R) = \frac{C_3 I_0\left(R\sqrt{iSt}\right) + C_4 K_0\left(R\sqrt{iSt}\right) - \left[C_1 I_0\left(R\sqrt{iPrSt}\right) + C_2 K_0\left(R\sqrt{iPrSt}\right)\right]}{iSt(Pr-1)} \tag{4.17}$$

where C_1, C_2, C_3, C_4 are constants defined in the appendix I.

The phase (ψ) of temperature and phase $(\chi)$of velocity are obtained by the expression (4.5) – (4.6) while the amplitude of the unsteady temperature and unsteady velocity are given respectively by (4.7).

4.3.1 Skin-friction of problem 3.3

In similar manner, the skin-frictions at the surfaces of cylinders are obtained by differentiating the unsteady velocity:

$$\tau_1 = \left.\frac{dB_R}{dR}\right|_{R=1} = \frac{1}{a_1(Pr-1)}\left[C_3 I_1(a_1) - C_4 K_1(a_1) - \sqrt{Pr}\left(C_1 I_1(m_1) - C_2 K_1(m_2)\right)\right] \tag{4.18}$$

$$\tau_\lambda = -\left.\frac{dB_R}{dR}\right|_{R=\lambda} = \frac{\left[C_4 K_1(a_{10}) + \sqrt{Pr}\left(C_1 I_1(\lambda m_1) + C_2 K_1(\lambda m_2)\right) - C_3 I_1(a_{10})\right]}{a_1(Pr-1)} \tag{4.19}$$

4.3.2 Rate of heat transfer for problem 3.3

The rate of heat transfer between the surfaces of the cylinders and the fluid is obtained by differentiating the unsteady temperature as follows:

$$Nu_1 = -\left.\frac{dG_R}{dR}\right|_{R=1} = \sqrt{iPrSt}\left(C_2 K_1\left(\sqrt{iPrSt}\right) - C_1 I_1\left(\sqrt{iPrSt}\right)\right) \tag{4.20}$$

$$Nu_\lambda = \frac{dG_R}{dR}\Big|_{R=\lambda} = \sqrt{iPrSt}\left(C_1 I_1\left(\lambda\sqrt{iPrSt}\right) - C_2 K_1\left(\lambda\sqrt{iPrSt}\right)\right) \tag{4.21}$$

4.3.3 Volume flow rate for problem 3.3

Also, the volume flow rate of the fluid in the annulus is given by:

$$Q = \int_1^\lambda RB(R)dR = \frac{1}{(iSt)^{3/2}(Pr-1)}[m_4 - m_5 - m_6] \tag{4.22}$$

where m_4, m_5, m_6 are constants defined in appendix I.

4.3.4 Limiting case of Prandtl number (Pr) for velocity and skin-friction of problem 3.3

In order to remove the discontinuity in equations (4.17) – (4.22), we obtained the following expressions for unsteady velocity and skin-friction at the surfaces of the cylinders respectively by taking limit $Pr \to 1$:

$$B(R) = \frac{iR}{2}\sqrt{\frac{i}{St}}\left[a_{13}K_0(a_1R) + a_{14}I_0(a_1R) + \frac{iSt\ R\left(a_4K_1(a_1R) - a_3I_1(a_1R)\right)}{a_5}\right] \tag{4.23}$$

$$\tau_1 = \frac{i\sqrt{i}}{\sqrt{St}}[a_{15} + a_{16}] \tag{4.24}$$

$$\tau_\lambda = -\frac{i\sqrt{i}}{\sqrt{St}}[a_{17} + a_{18}] \tag{4.25}$$

where $a_1, a_2, a_3, \ldots$ are constants defined in appendixI.

CHAPTER FIVE

RESULTS AND DISCUSSION

5.1 Introduction

This chapter gives the discussion and physical interpretation of the solutions obtained from chapter four. The results are presented in form of line graphs, contour graphs and tables.From these graphs, physical interpretations are deduced.

5.2 Natural Convection Flow in a Vertical Tube with Periodic Heat Input

It is seen from equations (4.1) – (4.13) that the basic parameters controlling the present physical situation are Strouhal number (St) and the Prandtl number (Pr).

Following Abramowitz &Stegun(1972), for small argument (x), we have:

$$I_0(x) \cong 1 \tag{5.1}$$

$$K_0(x) \cong -\ln\left(\frac{x}{2}\right) \tag{5.2}$$

For small Strouhal number, substituting (5.1) and (5.2) into (4.2) and (4.4), we have

$$G(R) \cong 1 \tag{5.3}$$

$$B(R) \cong 0 \tag{5.4}$$

These results in equations (5.3) and (5.4) correspond to the conclusion of Wang (1998); that for small Strouhal number, the periodic regime (unsteady part) corresponds to steady state and hence the simple steady state solution of equations (4.1) and (4.3) can be used for velocity and temperature respectively.

Also, following Abramowitz &Stegun(1972), for large argument (x), we have:

$$I_0(x) \cong \frac{e^x}{\sqrt{2\pi x}} \tag{5.5}$$

$$K_0(x) \cong \frac{\sqrt{\pi}e^x}{\sqrt{2x}} \tag{5.6}$$

Using (5.5) and (5.6), for large Strouhal number, we obtained the following expression for unsteady temperaturee and velocity fields respectively:

$$G(R) \cong \frac{e^{(R-1)\sqrt{iPrSt}}}{R^{1/2}} \tag{5.7}$$

$$B(R) \cong \frac{1}{iSt(Pr-1)R^{1/2}}\left(e^{(R-1)\sqrt{iSt}} - e^{(R-1)\sqrt{iPrSt}}\right) \tag{5.8}$$

To justify the result in equations (5.3) and (5.4), table 1 presents a comparison between steady and unsteady temperature and velocity for different value of St and this gives an excellent comparison for small Strouhal number (St).

Table 5.1:Comparison of velocity and temperature for steady and unsteady regimes for different values of Strouhal number (St) at $Pr = 0.71$

	$St = 2.0$				$St = 0.1$				$St = 0.01$			
R	$A(R)$	$B(R)$	$F(R)$	$G(R)$	$A(R)$	$B(R)$	$F(R)$	$G(R)$	$A(R)$	$B(R)$	$F(R)$	$G(R)$
0.0	0.2500	0.1868	1.0	0.9110	0.2500	0.2498	1.0	0.9998	0.2500	0.2500	1.0	1.0
0.2	0.2400	0.1804	1.0	0.9156	0.2400	0.2398	1.0	0.9998	0.2400	0.2400	1.0	1.0
0.4	0.2100	0.1604	1.0	0.9291	0.2100	0.2099	1.0	0.9998	0.2100	0.2100	1.0	1.0
0.6	0.1600	0.1254	1.0	0.9498	0.1600	0.1599	1.0	0.9999	0.1600	0.1600	1.0	1.0
0.8	0.0900	0.0729	1.0	0.9747	0.0900	0.0900	1.0	0.9999	0.0900	0.0900	1.0	1.0
1.0	0	0.0000	1.0	1.0000	0	0.0000	1.0	1.0000	0	0	1.0	1.0

From table 1 above, it can be seen that for small Strouhal number (which means for large time), the steady state velocity and temperature correspond to their unsteady states respectively.

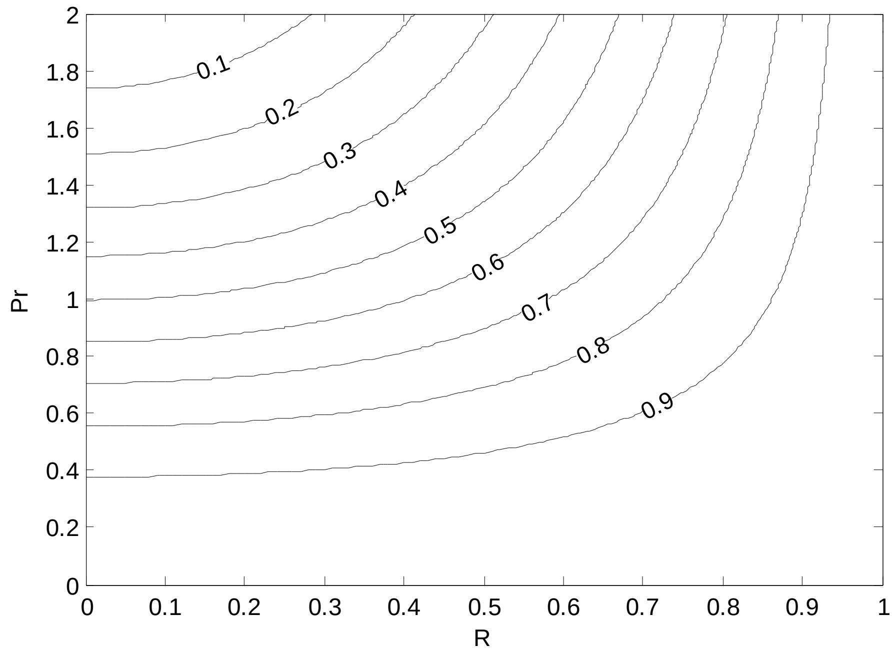

Figure 5.1:Temperature distribution(G_R) for different values of Prat $St = 4.0$

In order to reveal the influence of pertinent parameters, line graphs and contour graphs of unsteady temperature and velocity profiles are depicted. Figure 5.1 reveals the effect of Prandtl number (Pr) on temperature distribution in the tube at fixed value of Strouhal number (St). It is found that as Pr increases, the temperature of the fluid decreases. This result is due to the fact that Prandtl number is inversely proportional to thermal conductivity which in turn reduces temperature of the fluid. As expected, the same result is also noticed for the velocity profile in Figure 5.2.This is because the momentum equation is dependent on the energy equation.Also, for Prandtl number ($Pr = 1$), the periodic regime velocity is seen to be discontinuous; this explains the singularity point discussed in equation (4.4). In addition, for $(Pr > 1)$, the periodic regime velocity is seen to have an adverse effect on the overall velocity. Figures 5.3 and 5.4 on the other hand depict the effects of Strouhal number (St) on temperature and velocity distributions respectively. In both figures, it is seen that the temperature as well as the velocity are decreasing function of Strouhal number (St). This happens because as Strouhal number increases, frequency increases and this leads to decrease in intensity of heat in the tube. In addition, for large Strouhal number (St),the periodic regimes for temperature and velocity contributes a negative impact on the overall fluid temperature and velocity. Furthermore, from Figures 5.1-5.5, fluid temperature is higher at the region closer to the heated surface while the reverse is observed for velocity profiles.

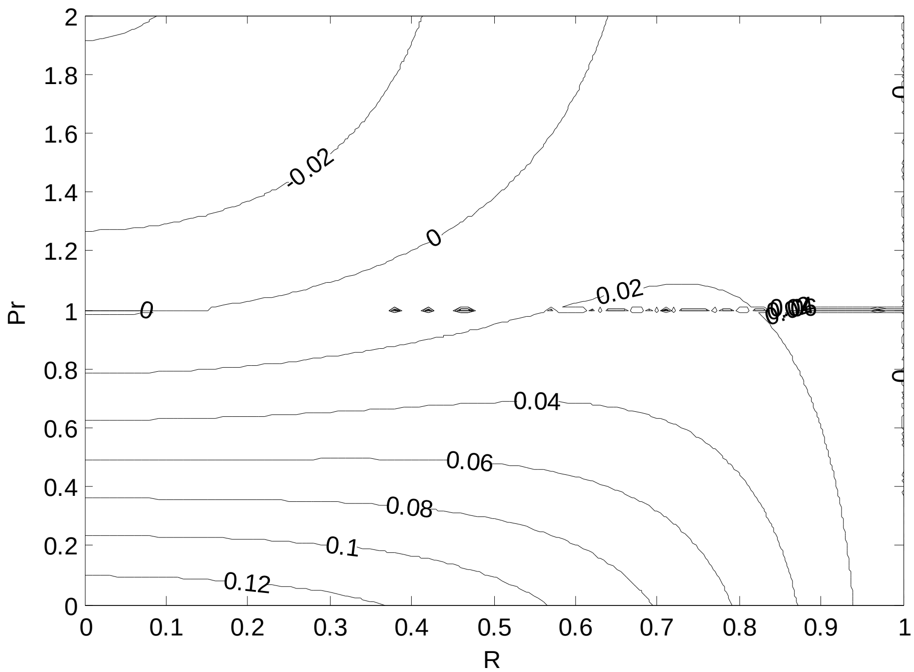

Figure 5.2:Velocity profiles (B_R) for different values of Pr at $St = 4.0$

Figure5.5 presents the effect of Strouhal number (St) on the phase of the velocity. It is observed that the fluid motion is characterized by a frequent phase change for $5 \leq St \leq 6$,this change is part of what results in negative contribution of the periodic velocity in figure 5.4.

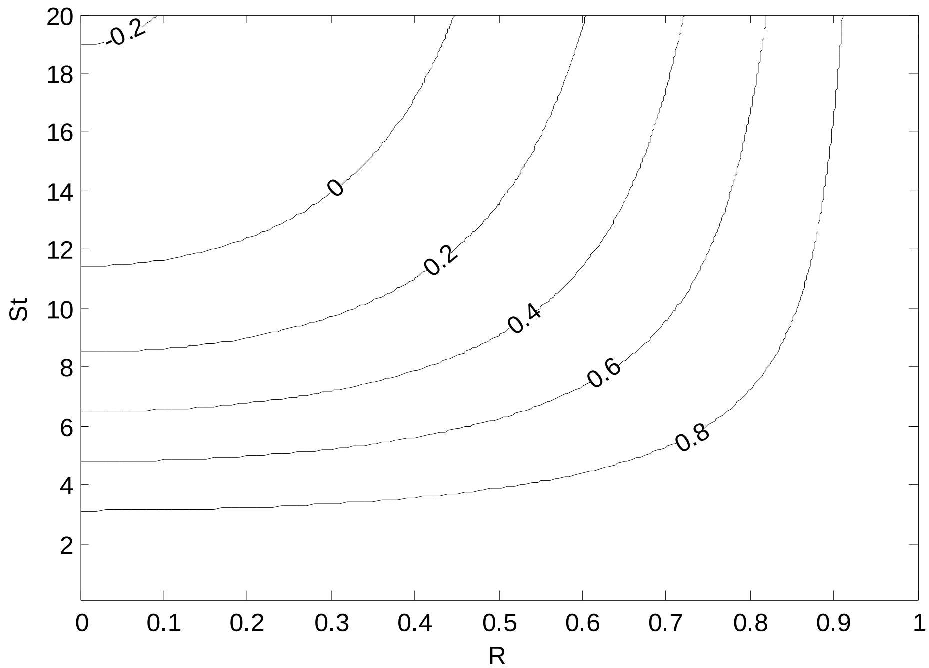

Figure 5.3:Temperature distribution (G_R) for different values of St at $Pr = 0.71$

The effect of Strouhal number(St) on out-of-phase velocity at constant Pr is depicted in Figure 5.6. It is revealed that the periodic velocity profile tends to zero for large Strouhal number. This means that the overall velocity is accounted for by steady regime only and at this point, the effect of periodic heating is insignificant.

Figure 5.7 gives the effect of Pr and St on skin-friction at the surface of the tube. It is observed that the effect of Pr and St is to decrease the skin-friction on the wall. Figure 5.8 on the other hand depicts rate of heat transfer between the fluid and the surface of the tube.

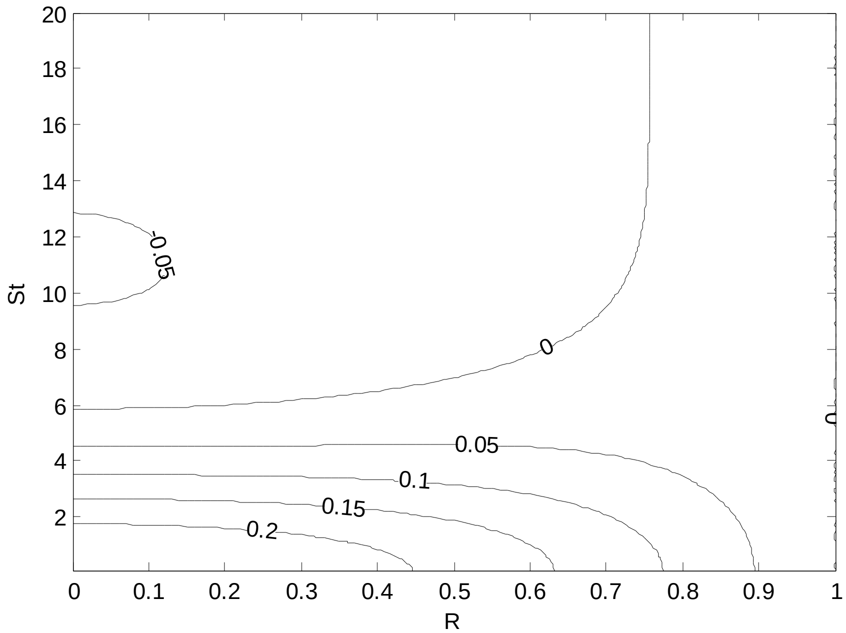

Figure 5.4:Velocity profiles (B_R) for different values of St at $Pr = 0.71$

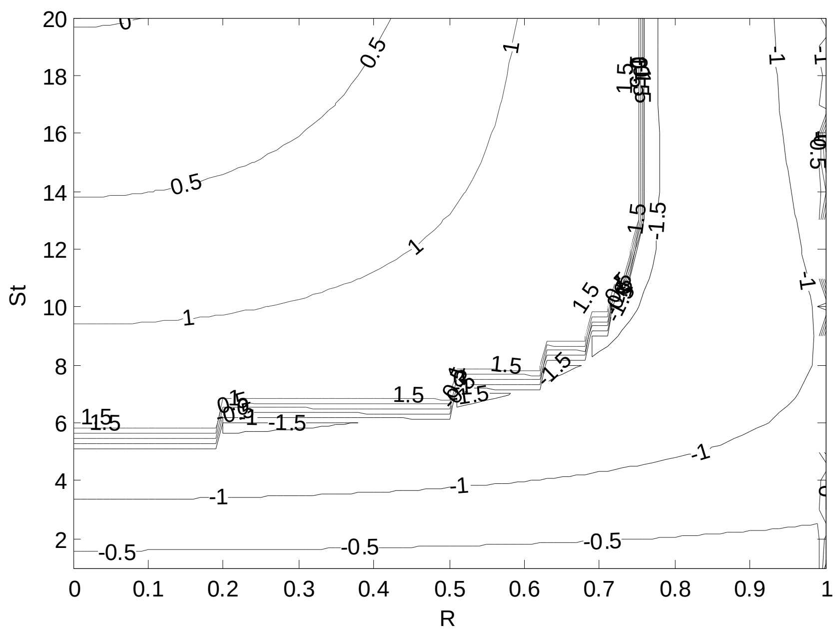

Figure 5.5:Phase of velocity (χ) for different values of St at $Pr = 0.71$

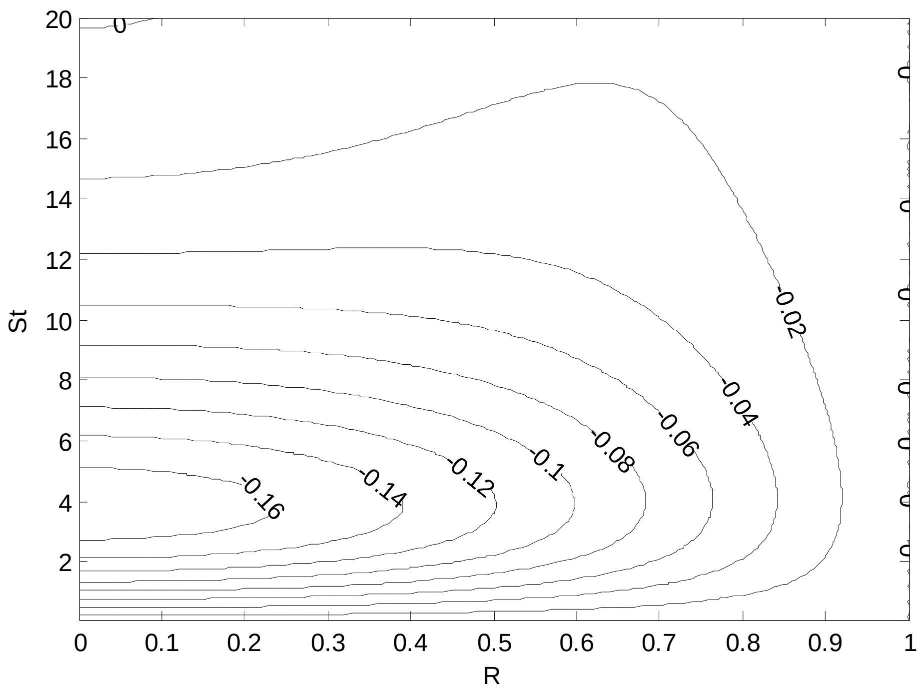

Figure 5.6: Out-of-phase velocity (B_i) for different values of St at $Pr = 0.71$

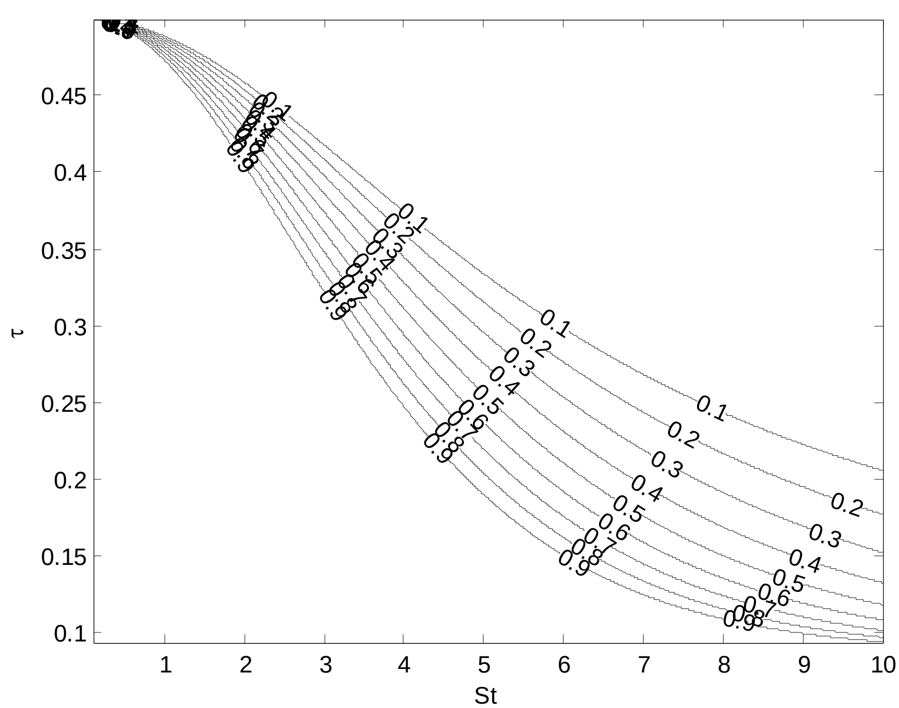

Figure 5.7:Skin-friction at the surface of the tube for different values of St and Pr

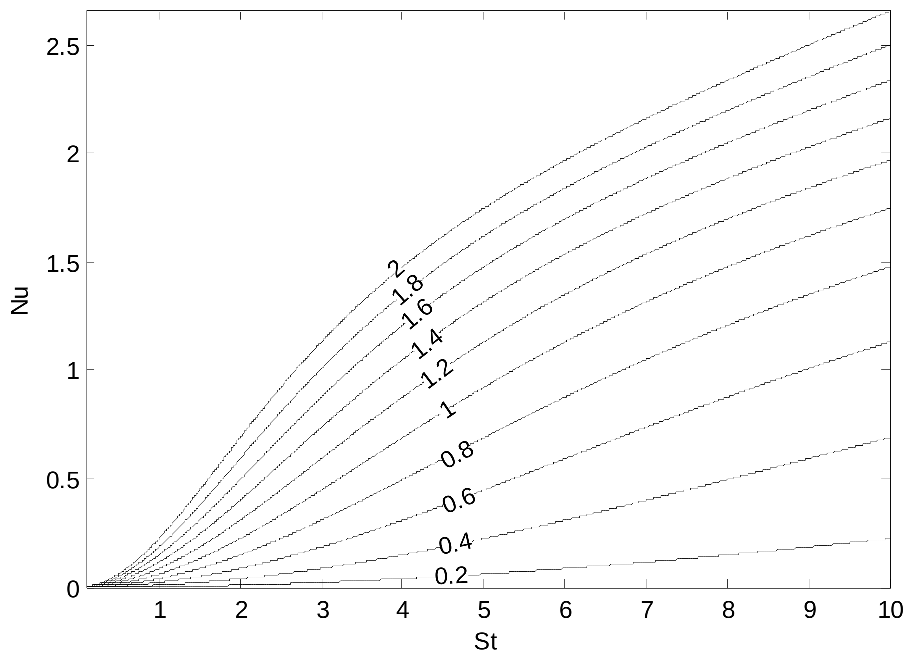

Figure 5.8:Rate of heat transfer for different values of St and Pr

It is found that the Nusselt number (Nu) is an increasing function of Pr and St. This is true because for higher values of Pr, higher temperature gradient exists near the wall thereby causing higher rate of heat transfer. In addition, Strouhal number (St) enhances rate of heat transfer because increasing frequency of heating results to increase in temperature gradient and therefore increases rate of heat transfer.

Figure 5.9 depicts the effect of frequency of oscillation on mass flow rate for different Prandtl number. The volume flow rate is seen to be a decreasing function of Strouhal number and Prandtl number. This is true because increasing Pr and St increases the density of the fluid thereby reducing the amount of fluid flowing through the tube.

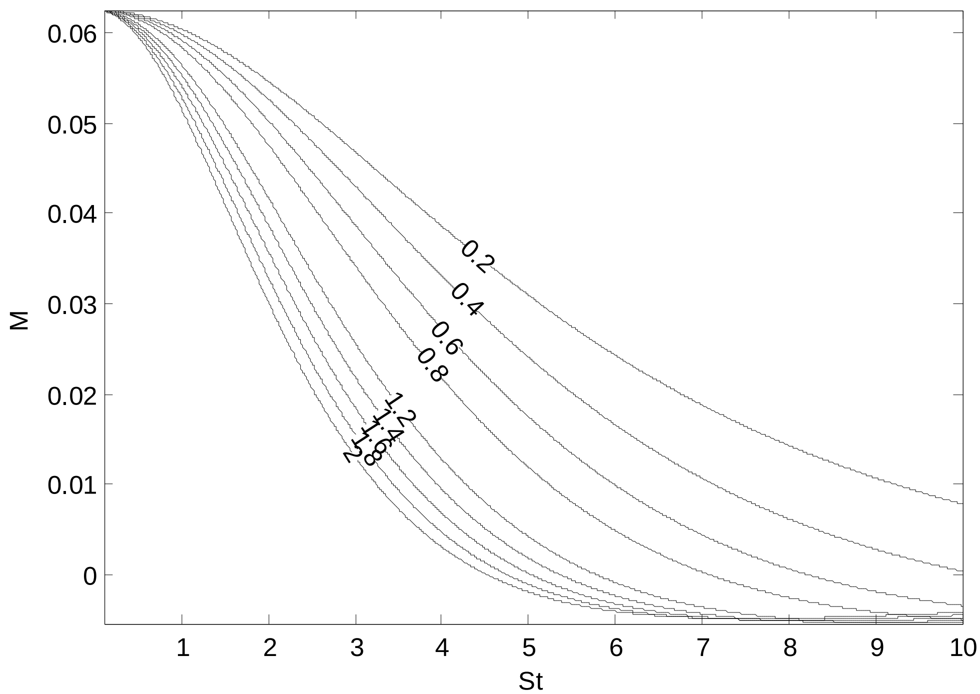

Figure 5.9:Volume flow rate for different values of St and Pr

5.3 Natural Convection Flow in a Vertical Annulus with Time-Periodic Boundary Conditions

From equations (4.14) – (4.25), the temperature distribution, velocity profile, skin-friction on both surfaces of the cylinders, volume flow rate and rate of heat transfer represented by Nusselt number is seen to be controlled by Prandtl number (Pr), Strouhal number (St) and aspect ratio (λ). We seek to know what happen for large and small frequency of heating, represented by Strouhal number (i.e. small time and large time respectively).

For small Strouhal number and relatively small Prandtl number and aspect ratio, using equations (5.1) and (5.2) on equations (4.15) and (4.17),after rigorous calculations and simplifications reduce to

$G(R) \cong 1$ (5.9)

$B(R) \cong 0$ (5.10)

In similar way, for large frequency of oscillation i.e. large Strouhal number, equation (4.15) and (4.17) after rigorous calculations and simplifications reduce to:

$G(R) \cong 0$ (5.11)

$B(R) \cong 0$ (5.12)

Physically, for small Strouhal number, the frequency of oscillation is very low and as such tends to the steady state results i.e. without periodic heat input (since frequency is inversely proportional to period of oscillation).

To have a better understanding of the results, line and contour graphs are plotted to give the effect of governing parameters.

Figure 5.10 presents temperature distribution for different Prandtl number (Pr) at constant Strouhal number (St) and aspect ratio (λ). As expected, it is observed that increasing Prandtl number decreases temperature distributions in the annulus. This result is physically true because increase in (Pr) decreases thermal conductivity which in turn decreases fluid temperature. Similar trend is noticed in figure 5.11 where temperature decreases with increase in Strouhal number (St). The symmetric nature of the depicted graphs are a result of constant periodic heat input at both surfaces of the cylinders.

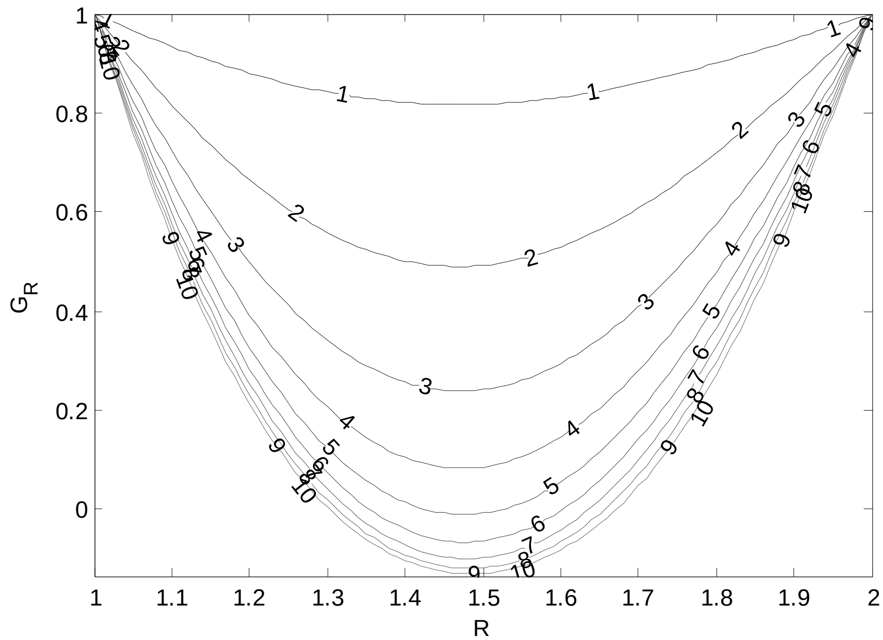

Figure 5.10: Temperature distribution (G_R) for different values of Pr at$St = 4.0$, $\lambda = 2.0$

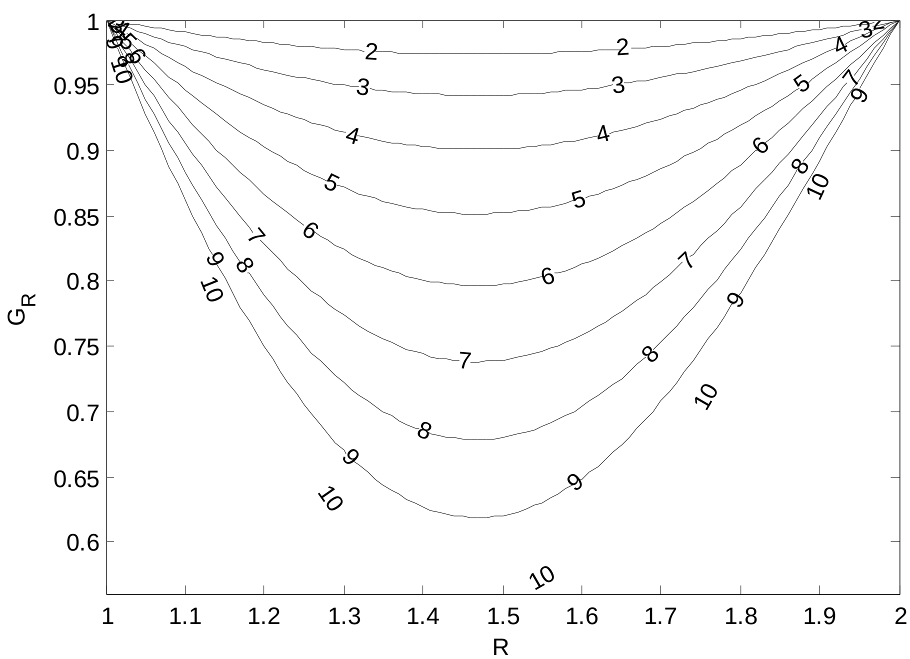

Figure 5.11: Temperature distribution (G_R) for different values of St at $Pr = 0.71$, $\lambda = 2.0$

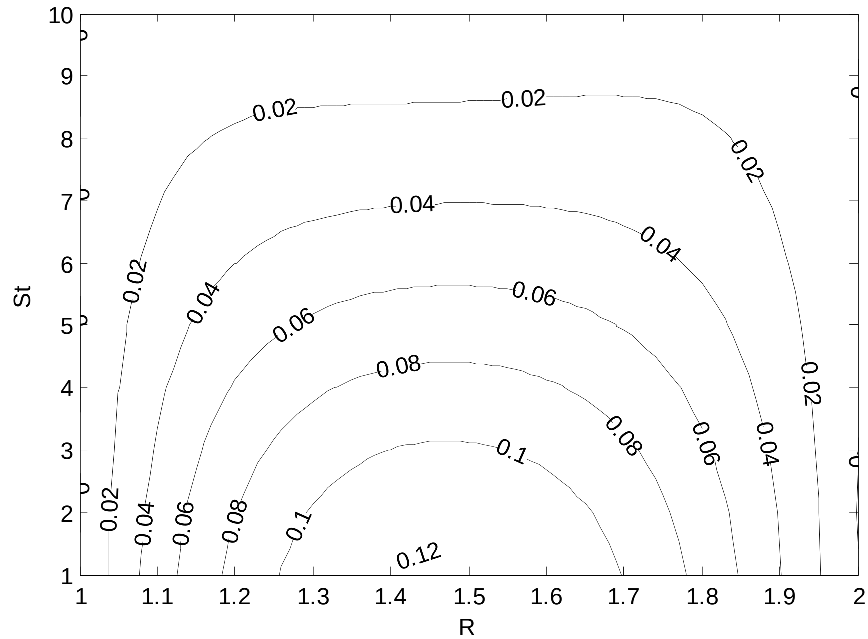

Figure 5.12: Velocity profiles (B_R) for different values of St at $Pr = 0.71$, $\lambda = 2.0$

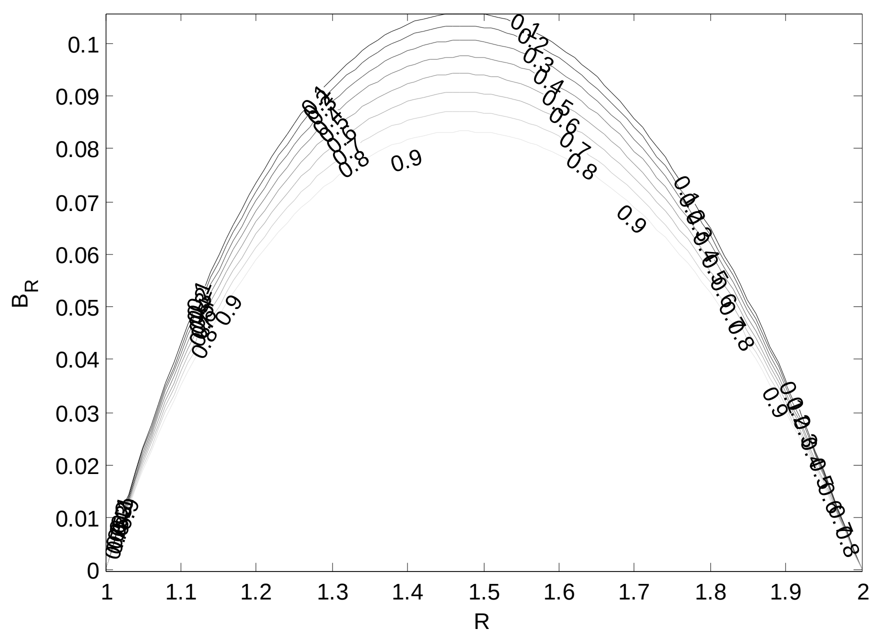

Figure 5.13: Velocity profiles (B_R) for different values of Pr at $St = 4.0$, $\lambda = 2.0$

Figure 5.12 depicts the effect of Strouhal number (St) on velocity of fluid flow in the annulus. It is evident from the contour graph that velocity decreases with increase in Strouhal number. This is physically true since $St$is inversely proportional to kinematic viscosity and hence increases fluid density, thereby retarding fluid motion.

Figure 5.13 gives velocity variation for different (Pr) at fixed value of Strouhal number (St)and aspect ratio(λ). It is observed from the figure that velocity also decreases with increase in Prandtl number.

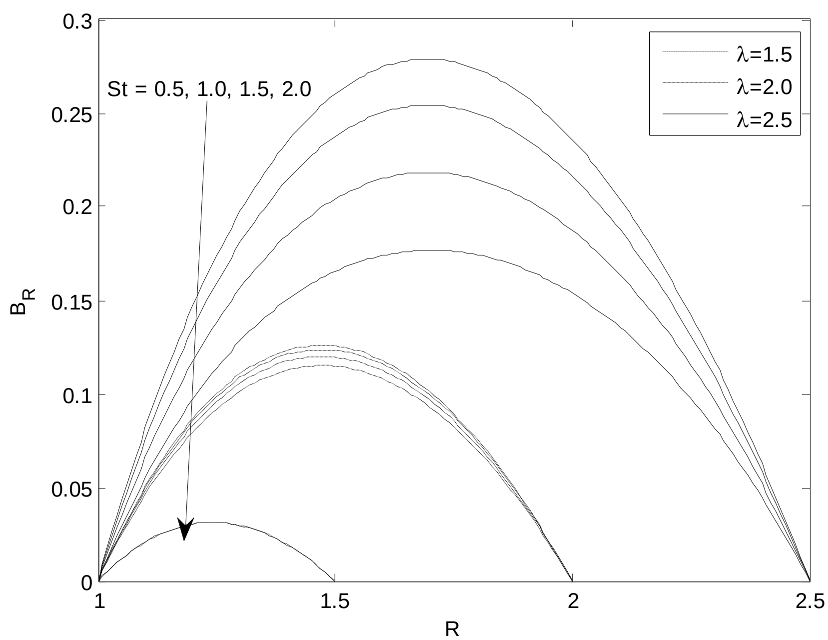

Figure 5.14a: Velocity profiles (B_R) for different values of λ and small St at $Pr = 0.71$.

Figures 5.14a and 5.14b show combined effect of aspect ratio and Strouhal number on fluid velocity for small and large Strouhal number respectively. It is found that the effect of aspect ratio and Strouhal number is to decrease fluid velocity for high frequency of heating.

It is interesting to note that for small Strouhal number, aspect ratio is an increasing function of velocity. This result is expected because for small Strouhal number, the intensity of heat is close to that of steady state and hence for large annular gap, fluid velocity is enhanced.

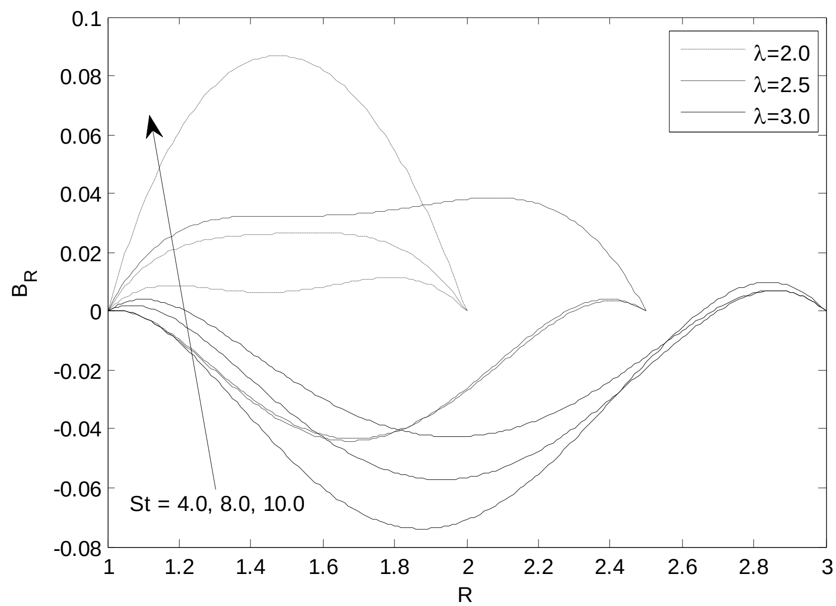

Figure 5.15b: Velocity profiles (B_R) for different values of$\lambda$and large Stat $Pr = 0.71$

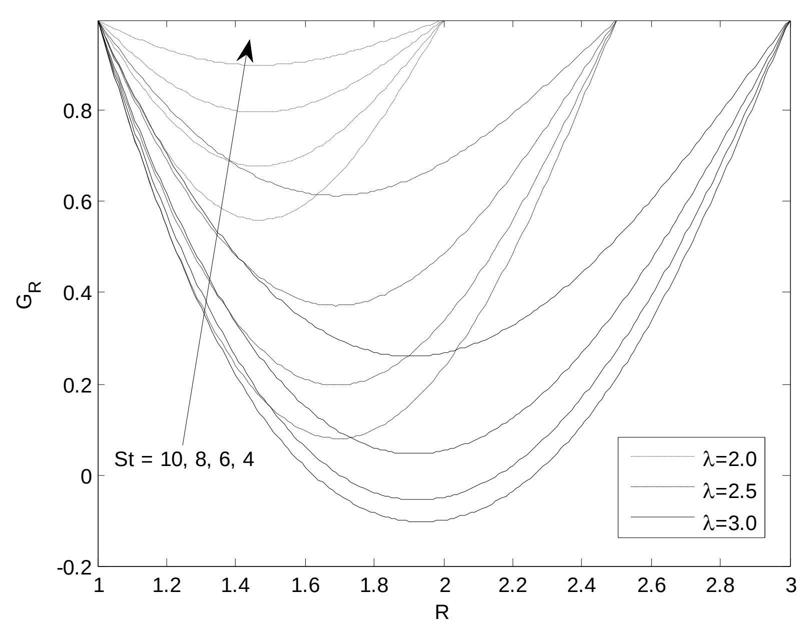

Figure 5.16: Temperature distributions (G_R) for different values of λ at $Pr = 0.71$, $St = 4.0$

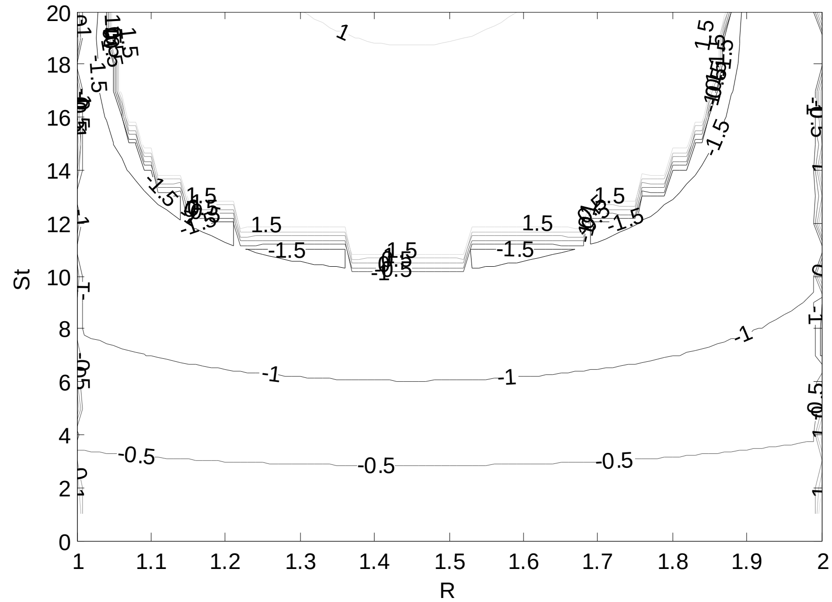

Figure 5.17: Phase of velocity (χ) for different values St at $Pr = 0.71,\ \lambda = 2.0$

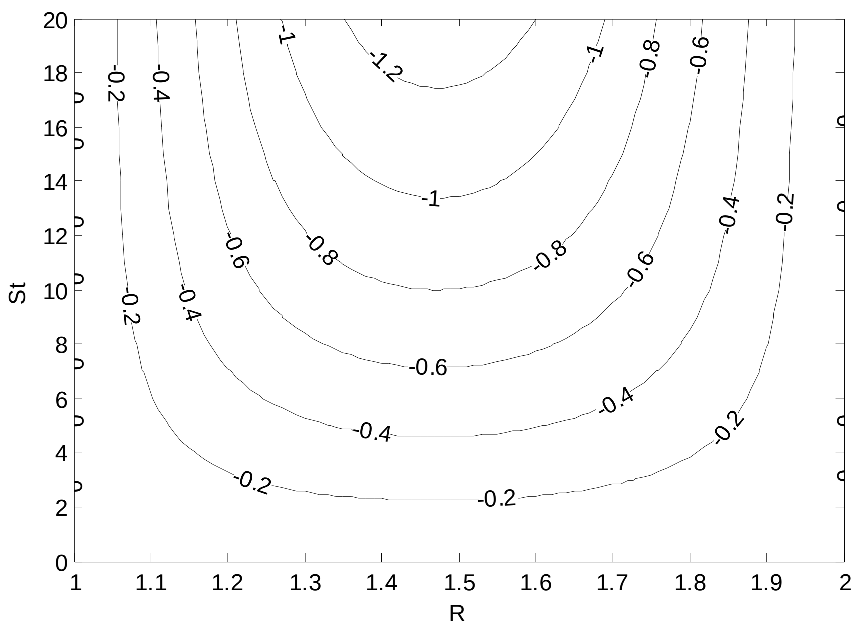

Figure 5.18: Phase of temperature (ψ) for different values St at $Pr = 0.71,\ \lambda = 2.0$

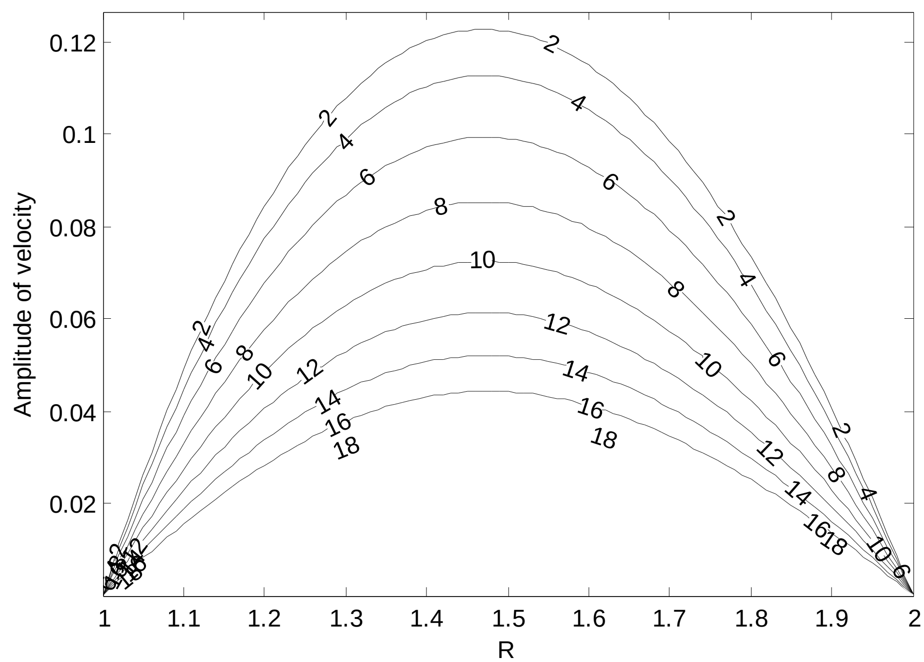

Figure 5.19: Amplitude of velocity for different values St at $Pr = 0.71,\ \lambda = 2.0$

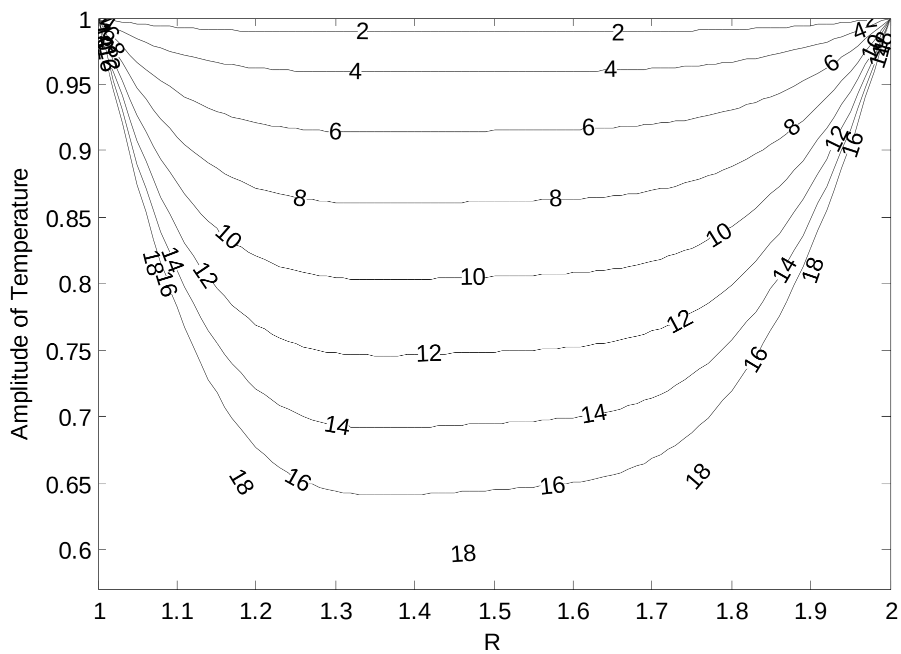

Figure 5.20: Amplitude of temperature for different values St at $Pr = 0.71,\ \lambda = 2.0$

Figure 5.15 on the other hand depicts combined effect of periodic heating and aspect ratio (λ) on steady-periodic fluid temperature at fixed value of Pr. The fluid velocity is observed to decrease with increase in aspect ratio. This result is expected since increasing aspect ratio widens the gap of the annulus, thereby leading to decrease in fluid temperature.

Figure 5.16 depicts the effect of Strouhal number (St) on the phase of velocity. It is seen that fluid motion is characterized by a frequent phase change for large Strouhal number (St). This change is caused by increase in frequency of heating. It is good to note that the velocities(B_R and B_i) are parallel to each other at this point of inflexion. On the other hand, phase of temperature is noticed to decrease with increase in Strouhal number (St) in figure 5.17.Unlike phase of velocity, the temperatures (G_R and G_i) are parallel to each other for small Strouhal number.

Figures 5.18 and 5.19 depict the effect of Strouhal number (St) on amplitude of velocity and temperature distributions respectively. In both cases, the effect of Strouhal number (St) is to decrease the amplitudes. These figures serve as a solution check for figures 5.12 and 5.13; it is seen to give an excellent comparison. In addition, the amplitudes contribute positively to the overall velocity and temperature respectively, because the amplitude computes the modulus of temperature and velocity.

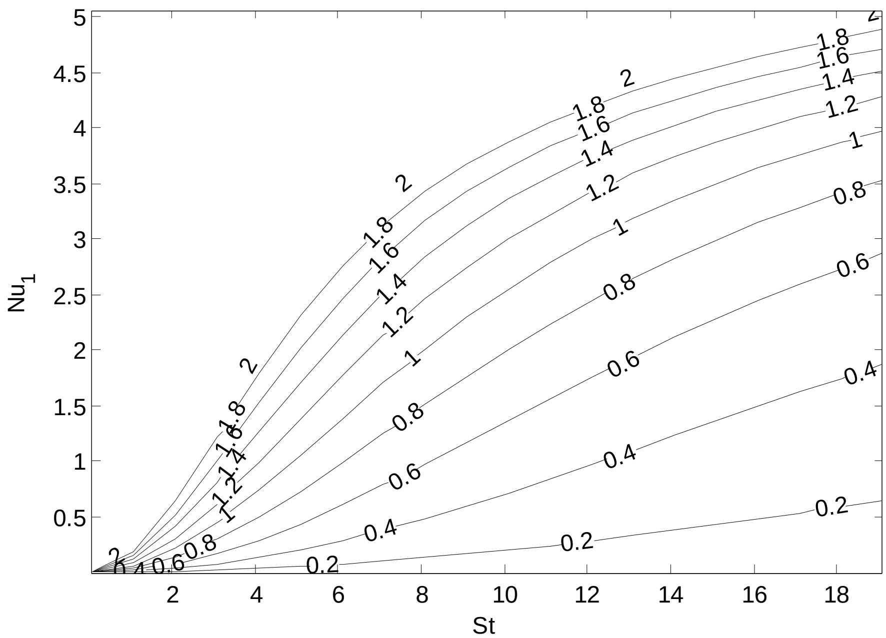

Figure 5.21: Nusselt number at the inner cylinder varying St and Pr at $\lambda = 2.0$

Figures 5.20 and 5.21 give the combined effects of Strouhal number (St) and Prandtl number (Pr) on rate of heat transfer between the fluid and the outer surface of the surfaces of the cylinders. It is seen that increase in these parameters increases the rate of heat transfer at both surfaces. Also, the rate of heat transfer at the outer surface of inner cylinder is seen to be higher compared to inner surface of outer cylinder.

In similar manner, the effect of aspect ratio and Strouhal number on rate of heat transfer at the outer surface of inner cylinder and inner surface of outer the cylinder is shown in figures 5.22 and 5.23 respectively. The impact of aspect ratio is seen to increase the rate of heat transfer at the both surfaces of the cylinders. This is true because increase in annular gap and Strouhal number increases the kinematic viscosity of the fluid thereby transferring heat faster.

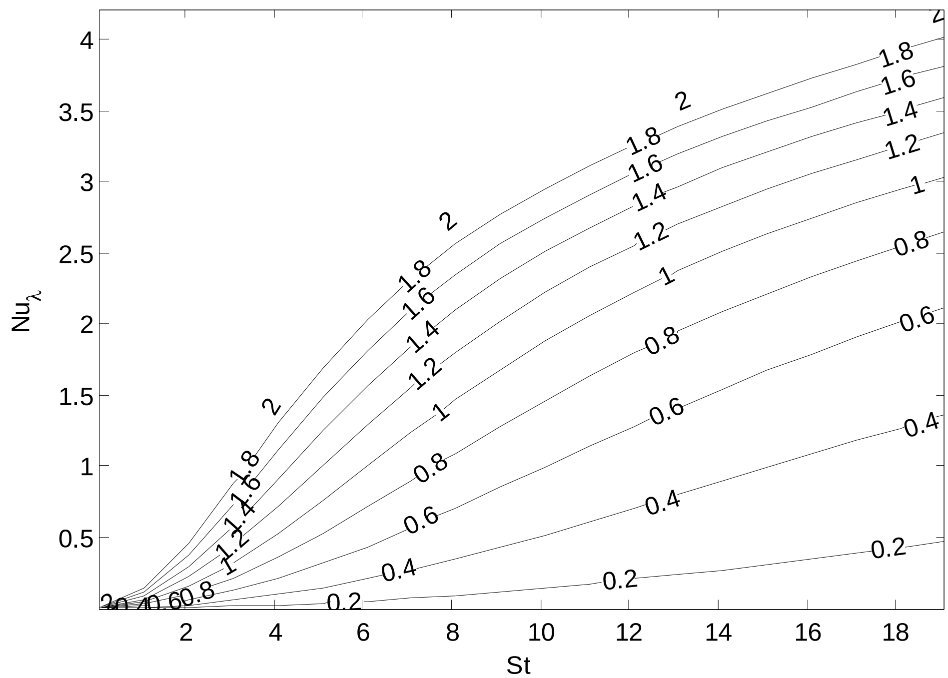

Figure 5.22: Nusselt number at the outer cylinder varying St and Pr at $\lambda = 2.0$

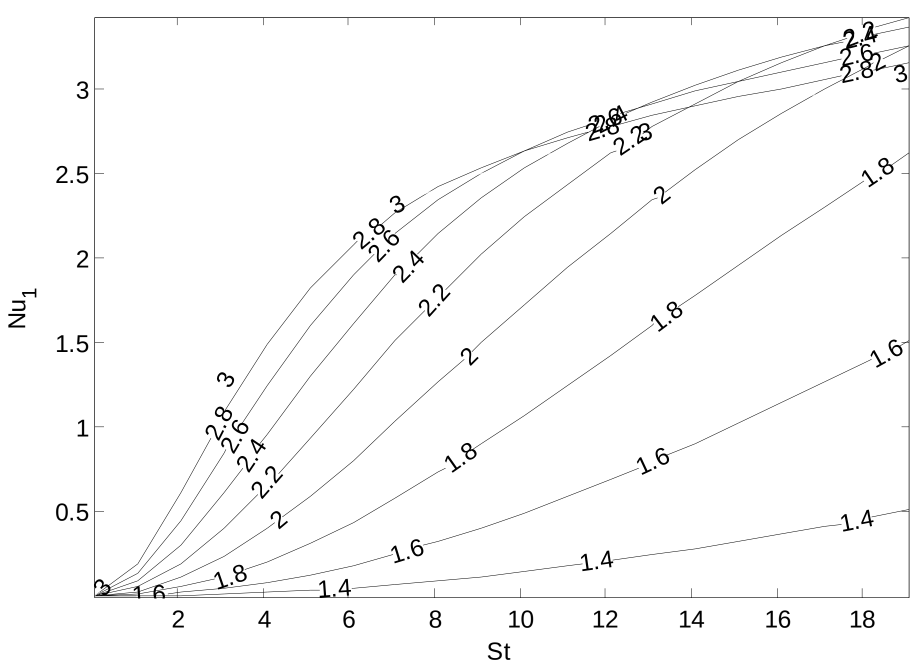

Figure 5.23: Nusselt number at the inner cylinder varying St and λ at $Pr = 0.71$

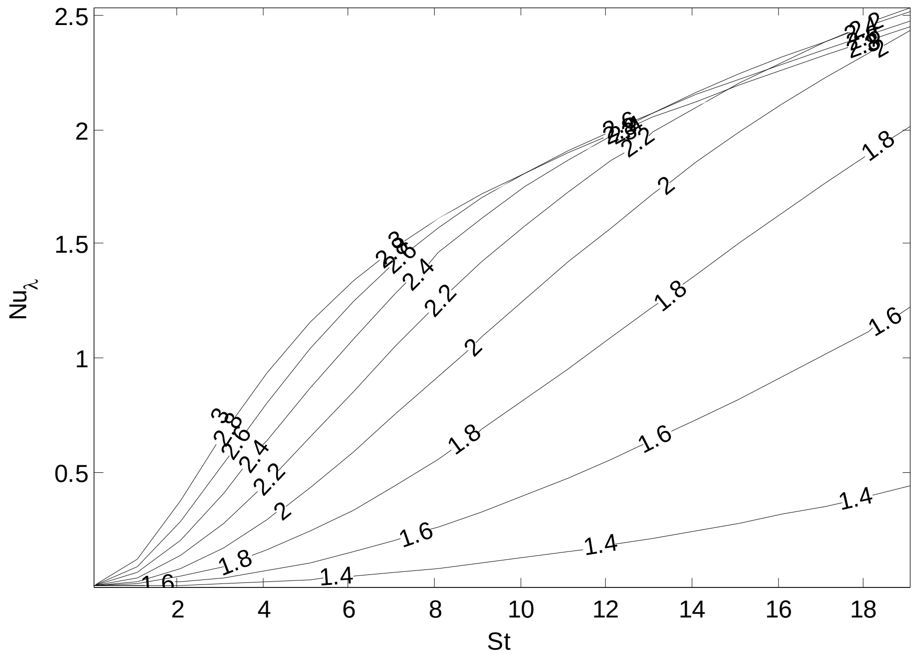

Figure 5.24: Nusselt number at the outer cylinder varying St and λ at $Pr = 0.71$

Variation of skin-friction with Strouhal number (St) and Prandtl number (Pr) at the outer surface of the inner cylinder (τ_1) and inner surface of outer cylinder (τ_λ) is depicted in figures 5.24 and 5.25 respectively. The effect of these parameters is to decrease the skin-friction at both surfaces of the cylinders. This result follows this pattern because for large Pr and St, fluid velocity decreases which in turn reduces the force at which the fluid strikes the surfaces of the cylinders.

Figures 5.26 and 5.27 plot variation of skin-friction with Strouhal number (St) and aspect ratio (λ) at fixed value of Pr. From these graphs, the effect of aspect ratio is to increase skin-friction at the surfaces of the cylinders. As expected, the skin-friction is higher on the outer surface of inner cylinder than inner surface of outer cylinder.

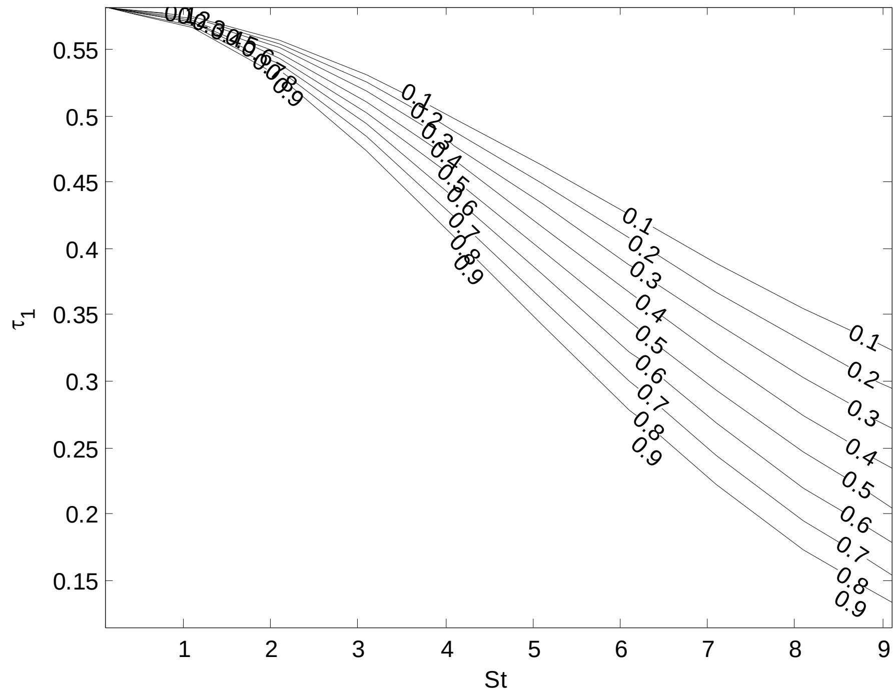

Figure 5.25: Skin-friction at the inner cylinder varying St and Pr at $\lambda = 2.0$

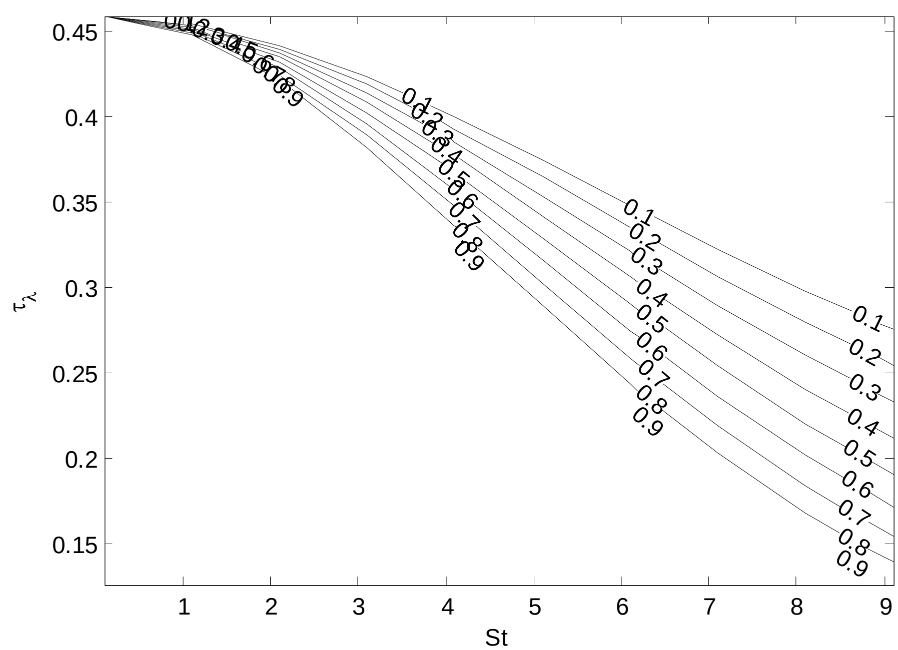

Figure 5.26: Skin-friction at the outer cylinder varying St and Pr at $\lambda = 2.0$

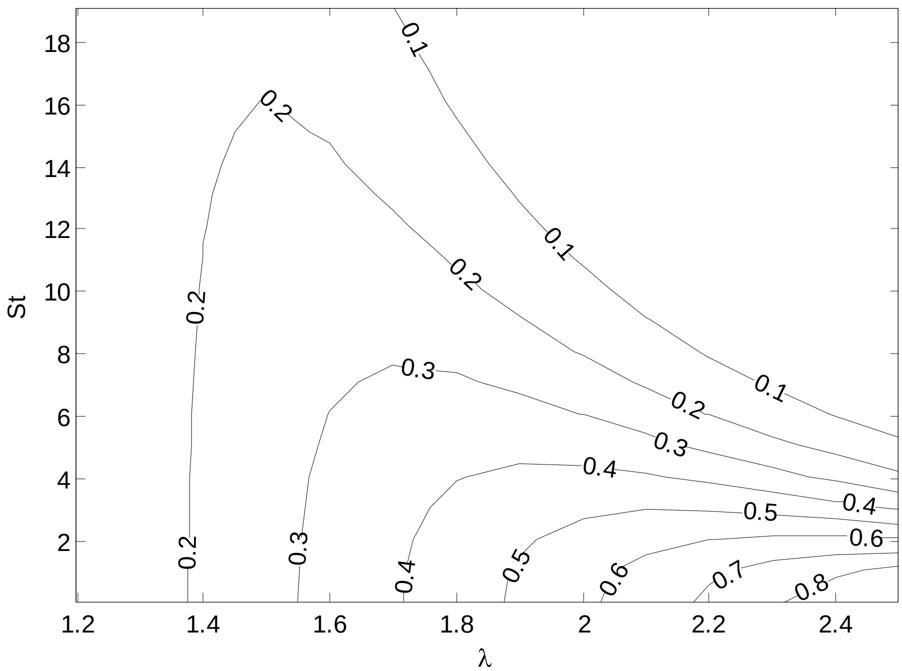

Figure 5.276: Skin-friction at the inner cylinder varying St and λ at $Pr = 0.71$

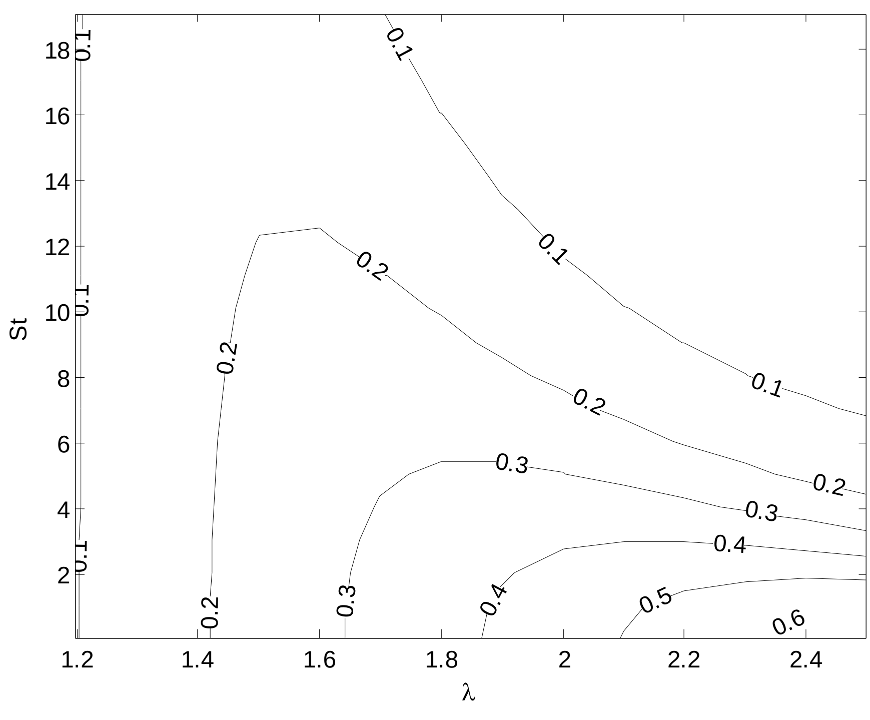

Figure 5.287: Skin-friction at the outer cylinder varying St and λ at $Pr = 0.71$

CHAPTER SIX

SUMMARY, CONCLUSION AND RECOMMENDATIONS

6.1 Summary

In this dissertation, two problems have been discussed; the first and second problemsanalyzed the natural convection flow inspired by time-periodic heating of the surfaces in a vertical tube and an annulus respectively. The mathematical equations governing the flow are derived and analytical solutions in terms of modified Bessel's functionof first and second kind are obtained.Closed-form expressions for velocity, temperature, skin-friction, volume flow rate and rate of heat transfer which is expressed as Nusselt number are obtained by solving the mathematical model after separating into steady and periodic regime. The effects of pertinent parameters such as Strouhal number (St), Prandtl number (Pr) and aspect ratio (λ) are shown with the use of contour and line graphs.

6.2 Conclusion

This dissertation studied flow formation of natural convection in tube and annulus subject to time-periodic heat input. The effects of Strouhal number (parameter governing the periodic heating), Prandtl number (Pr), and aspect ratio are discussed with the aid of contour and line graphs. Results show that velocity and temperature of the fluid are decreasing function of Strouhal number (St), Prandtl number (Pr) and aspect ratio (λ). Also, the rate of heat transfer between the surfaces of the cylinders and fluid increases with in Strouhal number(St), Prandtl number(Pr) and aspect ratio (λ). In addition, the skin-friction at the surfaces of the cylinders and volume flow rate decreases with increase in Strouhal number (St) and Prandtl number (Pr). Furthermore, for small Strouhal number

(St) ;velocity, temperature, rate of heat transfer, volume flow rate and skin-friction of the periodic regime (unsteady state) match with the steady regime (steady state). This means that for small Strouhal number, the steady state solution can be used, otherwise, the complicated unsteady state should be used. In addition, for small Strouhal number, the temperature (steady and periodic regimes) of the fluid is independent of the geometry used (annulus, tube or channel), otherwise the solutions obtained here are significant and different from that of Wang (1998).Finally, the rate of heat transfer and skin-friction are seen to be higher in annulus than in tube because the effect of aspect ratio is to increase the rate of heat transfer as well as skin-friction at the surfaces of the cylinder. From the above conclusions and based on graphs depicted, all the stated aim and objectives have been achieved.

6.3 Recommendations

This dissertation discussed in detail the effect of time-periodic heating at the surfaces of tube and annulus on natural convection flow. Exact solution of temperature, velocity, rate of heat transfer, skin-friction and volume flow rate of steady-periodic regimes are obtained. I recommend this work to engineers in design of cooling devices in tube and annulus. Also the inclusion of time-periodic heat input enhances the rate of heat transfer. Therefore, design engineers should introduce periodic heating in order to achieve an enhanced velocity, temperature, rate of heat transfer and can also be used to reduce skin-friction and volume flow rate.

6.4 Contribution to Knowledge

This dissertation obtained an analytical solution for natural convection fluid flow in a tube and annulus subject to time-periodic heat input.

1. We establishedthat for small Strouhal number (frequency of heating), velocity profile, temperature distribution, rate of heat transfer, volume flow rate and skin-friction of the periodic regime match with the steady regime (steady state). Otherwise, the complicated periodic regime solution obtained in this dissertation is significant and different from the steady state solution.
2. We obtained that temperature as well as velocity profile decreases with increase in Strouhal number and Prandtl number for both tube and annulus.
3. We found that the temperature distribution (steady and periodic regimes) of the fluid is independent of the geometry used (annulus, tube or parallel plates channel) for small Strouhal number.

6.5 Suggestions for Further Research

In this work, natural convection flow in a tube and an annulus is considered. The work can be extended by taking into account the dufour and Soret effects on unsteady natural/mixed convection flows in an annulus formed by two concentric cylinders. In addition, effect of viscous dissipation term which we neglected in both models is still an open problem. Moreover, combined effects of radiation and periodic heating of the surfaces of a cylinder with MHD can be studied in future. Finally, the fully developed flow is considered in this dissertation, study of developing flow for this dissertation is still an open problem.

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APPENDICES

APPENDIX I

CONSTANTS USED IN PROBLEM3.3

$$i = \sqrt{-1}$$

$m_1 = \sqrt{iPrSt}$, $a_1 = \sqrt{iSt}$

$C_1 = \frac{K_0(\lambda m_1) - K_0(m_1)}{I_0(m_1)K_0(\lambda m_1) - I_0(\lambda m_1)K_0(m_1)}$ $C_2 = \frac{I_0(\lambda m_1) - I_0(m_1)}{I_0(\lambda m_1)K_0(m_1) - I_0(m_1)K_0(\lambda m_1)}$

$P_1 = C_1 I_0(m_1) + C_2 K_0(m_1)$, $P_2 = C_1 I_0(\lambda m_1) + C_2 K_0(\lambda m_1)$

$C_3 = \frac{P_1[K_0(\lambda a_1)] - P_2[K_0(a_1)]}{I_0(a_1)K_0(\lambda a_1) - I_0(\lambda a_1)K_0(a_1)}$ $C_4 = \frac{P_1[I_0(\lambda a_1)] - P_2[I_0(a_1)]}{K_0(a_1)I_0(\lambda a_1) - K_0(\lambda a_1)I_0(a_1)}$

$m_2 = \left(C_1 I_0(m_1) - C_2 K_0(m_1)\right)$ $a_2 = \frac{iSt\left(a_7 I_0(a_{10}) + a_8 K_0(a_{10}) - \lambda a_{11} I_1(a_{10}) - \lambda a_{12} K_1(a_{10})\right)}{a_6}$

$m_3 = \left(C_1 I_0(\lambda m_1) - C_2 K_0(\lambda m_1)\right)$, $m_4 = C_3\left(\lambda I_1(a_{10}) - I_1(a_1)\right)$ $m_5 =$

$C_4\left(\lambda K_1(a_{10}) - K_1(a_1)\right)$, $m_6 = \frac{\left(C_1\left(\lambda I_1(\lambda m_1) - I_1(m_1)\right) - C_2\left(\lambda K_1(\lambda m_2) - K_1(m_2)\right)\right)}{\sqrt{Pr}}$

$a_3 = K_0(a_{10}) - a_{11}$ $a_4 = I_0(a_{10}) - a_{12}$

$a_5 = 2a_9\sqrt{iSt}$ $a_6 = 2\sqrt{iSt}$ $a_7 =$

$K_1\left(\sqrt{iSt}\right)$ $a_8 = I_1\left(\sqrt{iSt}\right)$

$a_9 = a_{11} I_0(a_{10}) - a_{12} K_0(a_{10})$ $a_{10} = \lambda\sqrt{iSt}$ $a_{11} =$

$K_0\left(\sqrt{iSt}\right)$ $a_{12} = I_0\left(\sqrt{iSt}\right)$ $a_{13} =$

$\frac{iSt\left(a_8 - \lambda I_1(a_{10})\right)}{a_6 a_9} - \frac{a_4 a_2}{a_9{}^2}$

$$a_{14} = \frac{iSt\left(a_7 - \lambda K_1(a_{10})\right)}{a_6 a_9} - \frac{a_3 a_2}{a_9{}^2}$$

$$a_{15} = a_3\left(\frac{a_6 a_8 - 2iSta_{12}}{2a_6 a_9} - \frac{a_8 a_2}{a_9{}^2}\right) + a_3\left(\frac{a_6 a_7 - 2iSta_{11}}{2a_6 a_9} - \frac{a_7 a_2}{a_9{}^2}\right)$$

$$a_{16} = \frac{iSt}{a_6 a_9}\left[\left(a_8 - \lambda I_1(a_{10})\right)a_7 - \left(a_7 - \lambda K_1(a_{10})\right)a_8\right]$$

$$a_{17} = K_1(a_{10})\left(\frac{iSt\left(a_8 - \lambda I_1(a_{10})\right)}{a_6 a_9} - \frac{a_4 a_2}{a_9{}^2}\right) + I_1(a_{10})\left(\frac{iSt\left(a_7 - \lambda K_1(a_{10})\right)}{a_6 a_9} + \frac{a_3 a_2}{a_9{}^2}\right)$$

$$a_{18} = \frac{a_4}{2a_6 a_9}\left[a_6 K_1(a_{10}) + 2\lambda iStK_0(a_{10})\right]$$

$I_0(x) = \frac{1}{\pi}\int_0^{\pi} \cosh(x\cos(\theta))\, d\theta \qquad I_1(x) = \frac{1}{\pi}\int_0^{\pi} \cos\exp(x\cos(\theta))\, d\theta$

$K_0(x) = \int_0^{\infty} \exp(-x\cos h(\theta))\, d\theta \qquad K_1(x) = \int_0^{\infty} \cosh(\exp(-x\cos h(\theta)))\, d\theta x > 0$

APPENDIX II

MATLAB PROGRAM FOR PROBLEM 3.2

```
%NATURAL CONVECTION IN A VERTICAL TUBE INSPIRED BY PERIODIC HEAT INPUT
%Temperature and Velocity varying Pr
[Pr,R]=meshgrid(0.0:0.1:0.9,0.0:0.01:1.0);
St=4.0;
x1=i.*Pr.*St;
l=sqrt(x1);
x11=besseli(0,l);
c1=1./x11;
G=c1.*besseli(0,l.*R);
Gr=real(G);
Gi=imag(G);
F=1;
x2=i.*St;
l2=sqrt(x2);
x3=(besseli(0,l)).*c1;
x4=besseli(0,l2);
x5=(l.^2)-x2;
x6=x4.*x5;
c5=x3./x6;
B=c5.*besseli(0,l2.*R)-((besseli(0,l.*R).*c1)./x5);
Br=real(B);
Bi=imag(B);
A=(1./4).*(1-(R.^2));
%AG is absolute G, AB absolute B, X is angle btw Bi and Br, P is angle
btw
%Gi and Gr
X=atan(Bi./Br);
P=atan(Gi./Gr);
AB=sqrt((Br).^2+(Bi).^2);
AG=sqrt((Gr).^2+(Gi).^2);
Nu=l.*besseli(0,l).*besseli(1,l);
Tau=l2.*c5.*besseli(1,l2)-(c1.*l.*besseli(1,l)./(l.^2-x2));
[C,h]=contour(R,Br,Pr);
   set(h,'showText','on','textStep',get(h,'levelStep'));
   colormap gray
```

```
   xlabel('R'), ylabel('T')
%NATURAL CONVECTION IN A VERTICAL TUBE INSPIRED BY PERIODIC HEAT INPUT
%Temperature and Velocity varying St
[St,R]=meshgrid(1.0:1.0:20.0,0.0:0.01:1.0);
Pr=0.7;
x1=i.*Pr.*St;
l=sqrt(x1);
x11=besseli(0,l);
c1=1./x11;
G=c1.*besseli(0,l.*R);
Gr=real(G);
Gi=imag(G);
F=1;
x2=i.*St;
l2=sqrt(x2);
x3=(besseli(0,l)).*c1;
x4=besseli(0,l2);
x5=(l.^2)-x2;
x6=x4.*x5;
c5=x3./x6;
B=c5.*besseli(0,l2.*R)-((besseli(0,l.*R).*c1)./x5);
Br=real(B);
Bi=imag(B);
A=(1./4).*(1-(R.^2));
%AG is absolute G, AB absolute B, X is angle btw Bi and Br, P is angle
btw
%Gi and Gr
X=atan(Bi./Br);
P=atan(Gi./Gr);
AB=sqrt((Br).^2+(Bi).^2);
AG=sqrt((Gr).^2+(Gi).^2);
Nu=l.*besseli(0,l).*besseli(1,l);
Tau=l2.*c5.*besseli(1,l2)-(c1.*l.*besseli(1,l)./(l.^2-x2));
[C,h]=contour(R,Br,Pr);
   set(h,'showText','on','textStep',get(h,'levelStep'));
   colormap gray
%plot(R,Gr,'k');
   xlabel('R'), ylabel('T')
```

```
%NATURAL CONVECTION IN A VERTICAL TUBE INSPIRED BY PERIODIC HEAT INPUT
%Skin-friction varying St and Pr
[Pr,St]=meshgrid(0.0:0.01:0.99,0.1:0.01:10.0);
x1=i.*Pr.*St;
l=sqrt(x1);
x11=besseli(0,l);
c1=1./x11;
x2=i.*St;
l2=sqrt(x2);
x3=(besseli(0,l)).*c1;
x4=besseli(0,l2);
x5=(l.^2)-x2;
x6=x4.*x5;
c5=x3./x6;
Nu1=l.*besseli(1,l)./besseli(0,l);
Nu=real(Nu1);
Tau1=(l2.*c5.*besseli(1,l2)-(c1.*l.*besseli(1,l)./(l.^2-x2)));
Tau=real(Tau1)
[C,h]=contour(St,Tau,Pr);
   set(h,'showText','on','textStep',get(h,'levelStep'));
   colormap gray
   xlabel('St'), ylabel('Tau')
```

```
%NATURAL CONVECTION IN A VERTICAL TUBE INSPIRED BY PERIODIC HEAT INPUT
%Nusselt number varying St and Pr
[Pr,St]=meshgrid(0.0:0.01:0.99,0.1:0.01:10.0);
x1=i.*Pr.*St;
l=sqrt(x1);
x11=besseli(0,l);
c1=1./x11;
x2=i.*St;
l2=sqrt(x2);
x3=(besseli(0,l)).*c1;
x4=besseli(0,l2);
x5=(l.^2)-x2;
x6=x4.*x5;
```

```
c5=x3./x6;
Nu1=l.*besseli(1,l)./besseli(0,l);
Nu=real(Nu1);
[C,h]=contour(St,Nu,Pr);
   set(h,'showText','on','textStep',get(h,'levelStep'));
   colormap gray
   xlabel('St'), ylabel('Nu')


%NATURAL CONVECTION IN A VERTICAL TUBE INSPIRED BY PERIODIC HEAT INPUT
%Mass flow rate varying St and Pr
[Pr,St]=meshgrid(0.0:0.01:0.9,0.1:0.01:10.0);
x1=i.*Pr.*St;
l=sqrt(x1);
x11=besseli(0,l);
x2=i.*St;
l2=sqrt(x2);
x3=besseli(0,l);
x4=besseli(0,l2);
x5=besseli(1,l);
x6=besseli(1,l2);
x7=x2.*l2.*(Pr-1);
x8=1./x7;
x9=x6./x4;
x10=x5./(sqrt(Pr).*x3);
M1=x8.*(x9-x10);
M=real(M1)
[C,h]=contour(St,M,Pr);
   set(h,'showText','on','textStep',get(h,'levelStep'));
   colormap gray
   xlabel('St'), ylabel('T')
```

```
%NATURAL CONVECTION IN A VERTICAL TUBE INSPIRED BY PERIODIC HEAT INPUT
%phase of temperature varying Pr and St
[Pr,R]=meshgrid(0.0:0.1:0.9,0.0:0.01:1.0);
St=1.0;
x1=i.*Pr.*St;
l=sqrt(x1);
x11=besseli(0,l);
c1=1./x11;
G=c1.*besseli(0,l.*R);
Gr=real(G);
Gi=imag(G);
F=1;
x2=i.*St;
l2=sqrt(x2);
x3=(besseli(0,l)).*c1;
x4=besseli(0,l2);
x5=(l.^2)-x2;
x6=x4.*x5;
c5=x3./x6;
B=c5.*besseli(0,l2.*R)-((besseli(0,l.*R).*c1)./x5);
Br=real(B);
Bi=imag(B);
A=(1./4).*(1-(R.^2));
%AG is absolute G, AB absolute B, X is angle btw Bi and Br, P is angle
btw
%Gi and Gr
X=atan(Bi./Br);
P=atan(Gi./Gr);
AB=sqrt((Br).^2+(Bi).^2);
AG=sqrt((Gr).^2+(Gi).^2);
Nu=l.*besseli(0,l).*besseli(1,l);
Tau=l2.*c5.*besseli(1,l2)-(c1.*l.*besseli(1,l)./(l.^2-x2));
[C,h]=contour(R,P,Pr);
   set(h,'showText','on','textStep',get(h,'levelStep'));
   colormap gray
      xlabel('R'), ylabel('T')
```

```
      St=3.0;
x1=i.*Pr.*St;
l=sqrt(x1);
x11=besseli(0,l);
c1=1./x11;
G=c1.*besseli(0,l.*R);
Gr=real(G);
Gi=imag(G);
F=1;
x2=i.*St;
l2=sqrt(x2);
x3=(besseli(0,l)).*c1;
x4=besseli(0,l2);
x5=(l.^2)-x2;
x6=x4.*x5;
c5=x3./x6;
B=c5.*besseli(0,l2.*R)-((besseli(0,l.*R).*c1)./x5);
Br=real(B);
Bi=imag(B);
A=(1./4).*(1-(R.^2));
%AG is absolute G, AB absolute B, X is angle btw Bi and Br, P is angle
btw
%Gi and Gr
X=atan(Bi./Br);
P=atan(Gi./Gr);
AB=sqrt((Br).^2+(Bi).^2);
AG=sqrt((Gr).^2+(Gi).^2);
Nu=l.*besseli(0,l).*besseli(1,l);
Tau=l2.*c5.*besseli(1,l2)-(c1.*l.*besseli(1,l)./(l.^2-x2));
[C,h]=contour(R,P,Pr);
   set(h,'showText','on','textStep',get(h,'levelStep'));
   colormap gray
      xlabel('R'), ylabel('T')

      St=6.0;
x1=i.*Pr.*St;
l=sqrt(x1);
x11=besseli(0,l);
```

```
c1=1./x11;
G=c1.*besseli(0,l.*R);
Gr=real(G);
Gi=imag(G);
F=1;
x2=i.*St;
l2=sqrt(x2);
x3=(besseli(0,l)).*c1;
x4=besseli(0,l2);
x5=(l.^2)-x2;
x6=x4.*x5;
c5=x3./x6;
B=c5.*besseli(0,l2.*R)-((besseli(0,l.*R).*c1)./x5);
Br=real(B);
Bi=imag(B);
A=(1./4).*(1-(R.^2));
%AG is absolute G, AB absolute B, X is angle btw Bi and Br, P is angle
btw
%Gi and Gr
X=atan(Bi./Br);
P=atan(Gi./Gr);
AB=sqrt((Br).^2+(Bi).^2);
AG=sqrt((Gr).^2+(Gi).^2);
Nu=l.*besseli(0,l).*besseli(1,l);
Tau=l2.*c5.*besseli(1,l2)-(c1.*l.*besseli(1,l)./(l.^2-x2));
[C,h]=contour(R,P,Pr);
   set(h,'showText','on','textStep',get(h,'levelStep'));
   colormap gray
      xlabel('R'), ylabel('T')


      St=10.0;
x1=i.*Pr.*St;
l=sqrt(x1);
x11=besseli(0,l);
c1=1./x11;
G=c1.*besseli(0,l.*R);
Gr=real(G);
Gi=imag(G);
```

```
F=1;
x2=i.*St;
l2=sqrt(x2);
x3=(besseli(0,l)).*c1;
x4=besseli(0,l2);
x5=(l.^2)-x2;
x6=x4.*x5;
c5=x3./x6;
B=c5.*besseli(0,l2.*R)-((besseli(0,l.*R).*c1)./x5);
Br=real(B);
Bi=imag(B);
A=(1./4).*(1-(R.^2));
%AG is absolute G, AB absolute B, X is angle btw Bi and Br, P is angle
btw
%Gi and Gr
X=atan(Bi./Br);
P=atan(Gi./Gr);
AB=sqrt((Br).^2+(Bi).^2);
AG=sqrt((Gr).^2+(Gi).^2);
Nu=l.*besseli(0,l).*besseli(1,l);
Tau=l2.*c5.*besseli(1,l2)-(c1.*l.*besseli(1,l)./(l.^2-x2));
[C,h]=contour(R,P,Pr);
   set(h,'showText','on','textStep',get(h,'levelStep'));
   colormap gray
      xlabel('R'), ylabel('T')


%NATURAL CONVECTION IN A VERTICAL TUBE INSPIRED BY PERIODIC HEAT INPUT
%phase of velocity varying Pr and St
[Pr,R]=meshgrid(0.0:0.1:0.9,0.0:0.01:1.0);
St=1.0;
x1=i.*Pr.*St;
l=sqrt(x1);
x11=besseli(0,l);
c1=1./x11;
G=c1.*besseli(0,l.*R);
Gr=real(G);
Gi=imag(G);
```

```
F=1;
x2=i.*St;
l2=sqrt(x2);
x3=(besseli(0,l)).*c1;
x4=besseli(0,l2);
x5=(l.^2)-x2;
x6=x4.*x5;
c5=x3./x6;
B=c5.*besseli(0,l2.*R)-((besseli(0,l.*R).*c1)./x5);
Br=real(B);
Bi=imag(B);
A=(1./4).*(1-(R.^2));
%AG is absolute G, AB absolute B, X is angle btw Bi and Br, P is angle
btw
%Gi and Gr
X=atan(Bi./Br);
P=atan(Gi./Gr);
AB=sqrt((Br).^2+(Bi).^2);
AG=sqrt((Gr).^2+(Gi).^2);
Nu=l.*besseli(0,l).*besseli(1,l);
Tau=l2.*c5.*besseli(1,l2)-(c1.*l.*besseli(1,l)./(l.^2-x2));
[C,h]=contour(R,X,Pr);
   set(h,'showText','on','textStep',get(h,'levelStep'));
   colormap gray
      xlabel('R'), ylabel('T')



      St=3.0;
x1=i.*Pr.*St;
l=sqrt(x1);
x11=besseli(0,l);
c1=1./x11;
G=c1.*besseli(0,l.*R);
Gr=real(G);
Gi=imag(G);
F=1;
x2=i.*St;
l2=sqrt(x2);
```

```
x3=(besseli(0,l)).*c1;
x4=besseli(0,l2);
x5=(l.^2)-x2;
x6=x4.*x5;
c5=x3./x6;
B=c5.*besseli(0,l2.*R)-((besseli(0,l.*R).*c1)./x5);
Br=real(B);
Bi=imag(B);
A=(1./4).*(1-(R.^2));
%AG is absolute G, AB absolute B, X is angle btw Bi and Br, P is angle
btw
%Gi and Gr
X=atan(Bi./Br);
P=atan(Gi./Gr);
AB=sqrt((Br).^2+(Bi).^2);
AG=sqrt((Gr).^2+(Gi).^2);
Nu=l.*besseli(0,l).*besseli(1,l);
Tau=l2.*c5.*besseli(1,l2)-(c1.*l.*besseli(1,l)./(l.^2-x2));
[C,h]=contour(R,X,Pr);
   set(h,'showText','on','textStep',get(h,'levelStep'));
   colormap gray
      xlabel('R'), ylabel('T')

      St=6.0;
x1=i.*Pr.*St;
l=sqrt(x1);
x11=besseli(0,l);
c1=1./x11;
G=c1.*besseli(0,l.*R);
Gr=real(G);
Gi=imag(G);
F=1;
x2=i.*St;
l2=sqrt(x2);
x3=(besseli(0,l)).*c1;
x4=besseli(0,l2);
x5=(l.^2)-x2;
x6=x4.*x5;
```

```
c5=x3./x6;
B=c5.*besseli(0,l2.*R)-((besseli(0,l.*R).*c1)./x5);
Br=real(B);
Bi=imag(B);
A=(1./4).*(1-(R.^2));
%AG is absolute G, AB absolute B, X is angle btw Bi and Br, P is angle
btw
%Gi and Gr
X=atan(Bi./Br);
P=atan(Gi./Gr);
AB=sqrt((Br).^2+(Bi).^2);
AG=sqrt((Gr).^2+(Gi).^2);
Nu=l.*besseli(0,l).*besseli(1,l);
Tau=l2.*c5.*besseli(1,l2)-(c1.*l.*besseli(1,l)./(l.^2-x2));
[C,h]=contour(R,X,Pr);
   set(h,'showText','on','textStep',get(h,'levelStep'));
   colormap gray
      xlabel('R'), ylabel('T')

      St=10.0;
x1=i.*Pr.*St;
l=sqrt(x1);
x11=besseli(0,l);
c1=1./x11;
G=c1.*besseli(0,l.*R);
Gr=real(G);
Gi=imag(G);
F=1;
x2=i.*St;
l2=sqrt(x2);
x3=(besseli(0,l)).*c1;
x4=besseli(0,l2);
x5=(l.^2)-x2;
x6=x4.*x5;
c5=x3./x6;
B=c5.*besseli(0,l2.*R)-((besseli(0,l.*R).*c1)./x5);
Br=real(B);
Bi=imag(B);
```

```
A=(1./4).*(1-(R.^2));
%AG is absolute G, AB absolute B, X is angle btw Bi and Br, P is angle
btw
%Gi and Gr
X=atan(Bi./Br);
P=atan(Gi./Gr);
AB=sqrt((Br).^2+(Bi).^2);
AG=sqrt((Gr).^2+(Gi).^2);
Nu=l.*besseli(0,l).*besseli(1,l);
Tau=l2.*c5.*besseli(1,l2)-(c1.*l.*besseli(1,l)./(l.^2-x2));
[C,h]=contour(R,X,Pr);
   set(h,'showText','on','textStep',get(h,'levelStep'));
   colormap gray
      xlabel('R'), ylabel('T')

%NATURAL CONVECTION IN A VERTICAL TUBE INSPIRED BY PERIODIC HEAT INPUT
%Absolute temperature varying Pr and St
[Pr,R]=meshgrid(0.0:0.1:0.9,0.0:0.01:1.0);
St=2.0;
x1=i.*Pr.*St;
l=sqrt(x1);
x11=besseli(0,l);
c1=1./x11;
G=c1.*besseli(0,l.*R);
Gr=real(G);
Gi=imag(G);
F=1;
x2=i.*St;
l2=sqrt(x2);
x3=(besseli(0,l)).*c1;
x4=besseli(0,l2);
x5=(l.^2)-x2;
x6=x4.*x5;
c5=x3./x6;
B=c5.*besseli(0,l2.*R)-((besseli(0,l.*R).*c1)./x5);
Br=real(B);
Bi=imag(B);
A=(1./4).*(1-(R.^2));
```

```
%AG is absolute G, AB absolute B, X is angle btw Bi and Br, P is angle
btw
%Gi and Gr
X=atan(Bi./Br);
P=atan(Gi./Gr);
AB=sqrt((Br).^2+(Bi).^2);
AG=sqrt((Gr).^2+(Gi).^2);
Nu=l.*besseli(0,l).*besseli(1,l);
Tau=l2.*c5.*besseli(1,l2)-(c1.*l.*besseli(1,l)./(l.^2-x2));
[C,h]=contour(R,AG,Pr);
   set(h,'showText','on','textStep',get(h,'levelStep'));
   colormap gray
      xlabel('R'), ylabel('T')



      St=4.0;
x1=i.*Pr.*St;
l=sqrt(x1);
x11=besseli(0,l);
c1=1./x11;
G=c1.*besseli(0,l.*R);
Gr=real(G);
Gi=imag(G);
F=1;
x2=i.*St;
l2=sqrt(x2);
x3=(besseli(0,l)).*c1;
x4=besseli(0,l2);
x5=(l.^2)-x2;
x6=x4.*x5;
c5=x3./x6;
B=c5.*besseli(0,l2.*R)-((besseli(0,l.*R).*c1)./x5);
Br=real(B);
Bi=imag(B);
A=(1./4).*(1-(R.^2));
%AG is absolute G, AB absolute B, X is angle btw Bi and Br, P is angle
btw
%Gi and Gr
```

```
X=atan(Bi./Br);
P=atan(Gi./Gr);
AB=sqrt((Br).^2+(Bi).^2);
AG=sqrt((Gr).^2+(Gi).^2);
Nu=l.*besseli(0,l).*besseli(1,l);
Tau=l2.*c5.*besseli(1,l2)-(c1.*l.*besseli(1,l)./(l.^2-x2));
[C,h]=contour(R,AG,Pr);
   set(h,'showText','on','textStep',get(h,'levelStep'));
   colormap gray
      xlabel('R'), ylabel('T')

      St=7.0;
x1=i.*Pr.*St;
l=sqrt(x1);
x11=besseli(0,l);
c1=1./x11;
G=c1.*besseli(0,l.*R);
Gr=real(G);
Gi=imag(G);
F=1;
x2=i.*St;
l2=sqrt(x2);
x3=(besseli(0,l)).*c1;
x4=besseli(0,l2);
x5=(l.^2)-x2;
x6=x4.*x5;
c5=x3./x6;
B=c5.*besseli(0,l2.*R)-((besseli(0,l.*R).*c1)./x5);
Br=real(B);
Bi=imag(B);
A=(1./4).*(1-(R.^2));
%AG is absolute G, AB absolute B, X is angle btw Bi and Br, P is angle
btw
%Gi and Gr
X=atan(Bi./Br);
P=atan(Gi./Gr);
AB=sqrt((Br).^2+(Bi).^2);
AG=sqrt((Gr).^2+(Gi).^2);
```

```
Nu=l.*besseli(0,l).*besseli(1,l);
Tau=l2.*c5.*besseli(1,l2)-(c1.*l.*besseli(1,l)./(l.^2-x2));
[C,h]=contour(R,AG,Pr);
   set(h,'showText','on','textStep',get(h,'levelStep'));
   colormap gray
      xlabel('R'), ylabel('T')


      St=10.0;
x1=i.*Pr.*St;
l=sqrt(x1);
x11=besseli(0,l);
c1=1./x11;
G=c1.*besseli(0,l.*R);
Gr=real(G);
Gi=imag(G);
F=1;
x2=i.*St;
l2=sqrt(x2);
x3=(besseli(0,l)).*c1;
x4=besseli(0,l2);
x5=(l.^2)-x2;
x6=x4.*x5;
c5=x3./x6;
B=c5.*besseli(0,l2.*R)-((besseli(0,l.*R).*c1)./x5);
Br=real(B);
Bi=imag(B);
A=(1./4).*(1-(R.^2));
X=atan(Bi./Br);
P=atan(Gi./Gr);
AB=sqrt((Br).^2+(Bi).^2);
AG=sqrt((Gr).^2+(Gi).^2);
Nu=l.*besseli(0,l).*besseli(1,l);
Tau=l2.*c5.*besseli(1,l2)-(c1.*l.*besseli(1,l)./(l.^2-x2));
[C,h]=contour(R,AG,Pr);
   set(h,'showText','on','textStep',get(h,'levelStep'));
   colormap gray
      xlabel('R'), ylabel('T')
```

APPENDIX III

MATLAB PROGRAM FOR PROBLEM 3.3

```
%%NATURAL CONVECTION IN A VERTICAL ANNULUS WITH TIME PERIODIC BOUNDARY
CONDITTIONS
%Temperature and Velocity Profiles varying St
[St,R]=meshgrid(0.1:1.0:20.0,1.0:0.01:2.0);
%g means lamda;
g=2.0;
Pr=0.71;
%Temperature
x1=i.*Pr.*St;
l=sqrt(x1);
x11=besseli(0,l);
x12=besseli(0,l.*g);
x13=besselk(0,l);
x14=besselk(0,l.*g);
x15=besseli(0,l.*g).*besselk(0,l);
x16=besseli(0,l).*besselk(0,l.*g);
x17=besseli(0,l.*g)-besseli(0,l);
x18=besselk(0,l.*g)-besselk(0,l);
c1=x18./(x16-x15);
c2=x17./(x15-x16);
G=c1.*besseli(0,l.*R)+c2.*besselk(0,l.*R);
Gr=real(G);
Gi=imag(G);
F=1;
%velocity
x2=i.*St;
l2=sqrt(x2);
 x19=(l.^2)-x2;
 k1=(c1.*besseli(0,l)+c2.*besselk(0,l))./x19;
 k2=(c1.*besseli(0,l.*g)+c2.*besselk(0,l.*g))./x19;
x20=besseli(0,l2.*g).*k1-besseli(0,l2).*k2;
x21=besseli(0,l2.*g).*besselk(0,l2)-besseli(0,l2).*besselk(0,l2.*g);
c6=x20./x21;
x22=besselk(0,l2.*g).*k1-besselk(0,l2).*k2;
x23=besselk(0,l2.*g).*besseli(0,l2)-besselk(0,l2).*besseli(0,l2.*g);
```

```
c5=x22./x23;
x24=G./x19;
B=c5.*besseli(0,l2.*R)+c6.*besselk(0,l2.*R)-x24;
Br=real(B)
Bi=imag(B);

c7=((1./(4.*log(g))).*((g.^2)-1));
c8=5./4;
A=-((R.^2)./4)+c7.*log(R)+c8;

%AG is absolute G, AB absolute B, X is angle btw Bi and Br, P is angle
btw
%Gi and Gr
X=atan(Bi./Br);
P=atan(Gi./Gr);
AB=sqrt((Br).^2+(Bi).^2);
AG=sqrt((Gr).^2+(Gi).^2);

[C,h]=contour(R,Br,St);
   set(h,'showText','on','textStep',get(h,'levelStep'));
   colormap gray
%plot(R,Gr,'k');
   xlabel('R'), ylabel('T')
```

```
%NATURAL CONVECTION IN A VERTICAL ANNULUS WITH TIME PERIODIC BOUNDARY CONDITTIONS
%Temperature and Velocity Profiles varying Pr
[Pr,R]=meshgrid(0.0:0.1:0.9,1.0:0.1:2.0);
%g means lamda;
g=2.0;
St=4.0;
%Temperature
x1=i.*Pr.*St;
l=sqrt(x1);
x11=besseli(0,l);
x12=besseli(0,l.*g);
x13=besselk(0,l);
x14=besselk(0,l.*g);
x15=besseli(0,l.*g).*besselk(0,l);
x16=besseli(0,l).*besselk(0,l.*g);
x17=besseli(0,l.*g)-besseli(0,l);
x18=besselk(0,l.*g)-besselk(0,l);
c1=x18./(x16-x15);
c2=x17./(x15-x16);
G=c1.*besseli(0,l.*R)+c2.*besselk(0,l.*R);
Gr=real(G);
Gi=imag(G);
 F=1;
%velocity
x2=i.*St;
l2=sqrt(x2);
 x19=(l.^2)-x2;
 k1=(c1.*besseli(0,l)+c2.*besselk(0,l))./x19;
 k2=(c1.*besseli(0,l.*g)+c2.*besselk(0,l.*g))./x19;
x20=besseli(0,l2.*g).*k1-besseli(0,l2).*k2;
x21=besseli(0,l2.*g).*besselk(0,l2)-besseli(0,l2).*besselk(0,l2.*g);
c6=x20./x21;
x22=besselk(0,l2.*g).*k1-besselk(0,l2).*k2;
x23=besselk(0,l2.*g).*besseli(0,l2)-besselk(0,l2).*besseli(0,l2.*g);
c5=x22./x23;
```

```
x24=G./x19;
B=c5.*besseli(0,l2.*R)+c6.*besselk(0,l2.*R)-x24;
Br=real(B)
Bi=imag(B);
c7=((1./(4.*log(g))).*((g.^2)-1));
c8=5./4;
A=-((R.^2)./4)+c7.*log(R)+c8;
%AG is absolute G, AB absolute B, X is angle btw Bi and Br, P is angle
btw
%Gi and Gr
X=atan(Bi./Br);
P=atan(Gi./Gr);
AB=sqrt((Br).^2+(Bi).^2);
AG=sqrt((Gr).^2+(Gi).^2);
[C,h]=contour(R,Br,St);
   set(h,'showText','on','textStep',get(h,'levelStep'));
   colormap gray
%plot(R,Gr,'k');
   xlabel('R'), ylabel('T')

%NATURAL CONVECTION IN A VERTICAL ANNULUS WITH TIME PERIODIC BOUNDARY
CONDITTIONS
%Phase of temperature varying Pr
[St,R]=meshgrid(0.1:1.0:20.0,1.0:0.01:2.0);
Pr=0.71;
%g mean lamda;
g=2.0;
%Temperature
x1=i.*Pr.*St;
l=sqrt(x1);
x11=besseli(0,l);
x12=besseli(0,l.*g);
x13=besselk(0,l);
x14=besselk(0,l.*g);
x15=besseli(0,l.*g).*besselk(0,l);
x16=besseli(0,l).*besselk(0,l.*g);
x17=besseli(0,l.*g)-besseli(0,l);
x18=besselk(0,l.*g)-besselk(0,l);
```

```
c1=x18./(x16-x15);
c2=x17./(x15-x16);
G=c1.*besseli(0,l.*R)+c2.*besselk(0,l.*R);
Gr=real(G);
Gi=imag(G);

F=1;

%velocity
x2=i.*St;
l2=sqrt(x2);
 x19=(l.^2)-x2;
 k1=(c1.*besseli(0,l)+c2.*besselk(0,l))./x19;
 k2=(c1.*besseli(0,l.*g)+c2.*besselk(0,l.*g))./x19;
x20=besseli(0,l2.*g).*k1-besseli(0,l2).*k2;
x21=besseli(0,l2.*g).*besselk(0,l2)-besseli(0,l2).*besselk(0,l2.*g);
c6=x20./x21;
x22=besselk(0,l2.*g).*k1-besselk(0,l2).*k2;
x23=besselk(0,l2.*g).*besseli(0,l2)-besselk(0,l2).*besseli(0,l2.*g);
c5=x22./x23;
x24=G./x19;
B=c5.*besseli(0,l2.*R)+c6.*besselk(0,l2.*R)-x24;
Br=real(B)
Bi=imag(B);

c7=((1./(4.*log(g))).*((g.^2)-1));
c8=5./4;
A=-((R.^2)./4)+c7.*log(R)+c8;

%AG is absolute G, AB absolute B, X is angle btw Bi and Br, P is angle
btw
%Gi and Gr
X=atan(Bi./Br);
P=atan(Gi./Gr);
AB=sqrt((Br).^2+(Bi).^2);
AG=sqrt((Gr).^2+(Gi).^2);
```

```
[C,h]=contour(R,P,St);
   set(h,'showText','on','textStep',get(h,'levelStep'));
   colormap gray
   xlabel('R'), ylabel('T')

   Pr=7.0;
%g mean lamda;
g=2.0;
%Temperature
x1=i.*Pr.*St;
l=sqrt(x1);
x11=besseli(0,l);
x12=besseli(0,l.*g);
x13=besselk(0,l);
x14=besselk(0,l.*g);
x15=besseli(0,l.*g).*besselk(0,l);
x16=besseli(0,l).*besselk(0,l.*g);
x17=besseli(0,l.*g)-besseli(0,l);
x18=besselk(0,l.*g)-besselk(0,l);
c1=x18./(x16-x15);
c2=x17./(x15-x16);
G=c1.*besseli(0,l.*R)+c2.*besselk(0,l.*R);
Gr=real(G);
Gi=imag(G);

F=1;

%velocity
x2=i.*St;
l2=sqrt(x2);
 x19=(l.^2)-x2;
 k1=(c1.*besseli(0,l)+c2.*besselk(0,l))./x19;
 k2=(c1.*besseli(0,l.*g)+c2.*besselk(0,l.*g))./x19;
x20=besseli(0,l2.*g).*k1-besseli(0,l2).*k2;
x21=besseli(0,l2.*g).*besselk(0,l2)-besseli(0,l2).*besselk(0,l2.*g);
c6=x20./x21;
x22=besselk(0,l2.*g).*k1-besselk(0,l2).*k2;
```

```
x23=besselk(0,l2.*g).*besseli(0,l2)-besselk(0,l2).*besseli(0,l2.*g);
c5=x22./x23;
x24=G./x19;
B=c5.*besseli(0,l2.*R)+c6.*besselk(0,l2.*R)-x24;
Br=real(B)
Bi=imag(B);

c7=((1./(4.*log(g))).*((g.^2)-1));
c8=5./4;
A=-((R.^2)./4)+c7.*log(R)+c8;

%AG is absolute G, AB absolute B, X is angle btw Bi and Br, P is angle
btw
%Gi and Gr
X=atan(Bi./Br);
P=atan(Gi./Gr);
AB=sqrt((Br).^2+(Bi).^2);
AG=sqrt((Gr).^2+(Gi).^2);

[C,h]=contour(R,P,St);
   set(h,'showText','on','textStep',get(h,'levelStep'));
   colormap gray
   xlabel('R'), ylabel('T')

%NATURAL CONVECTION IN A VERTICAL ANNULUS WITH TIME PERIODIC BOUNDARY
CONDITTIONS
%Phase of Velocity varying Pr
[St,R]=meshgrid(0.1:1.0:20.0,1.0:0.01:2.0);
Pr=0.71;
%g mean lamda;
g=2.0;
%Temperature
x1=i.*Pr.*St;
l=sqrt(x1);
x11=besseli(0,l);
x12=besseli(0,l.*g);
x13=besselk(0,l);
```

```
x14=besselk(0,l.*g);
x15=besseli(0,l.*g).*besselk(0,l);
x16=besseli(0,l).*besselk(0,l.*g);
x17=besseli(0,l.*g)-besseli(0,l);
x18=besselk(0,l.*g)-besselk(0,l);
c1=x18./(x16-x15);
c2=x17./(x15-x16);
G=c1.*besseli(0,l.*R)+c2.*besselk(0,l.*R);
Gr=real(G);
Gi=imag(G);

F=1;

%velocity
x2=i.*St;
l2=sqrt(x2);
 x19=(l.^2)-x2;
 k1=(c1.*besseli(0,l)+c2.*besselk(0,l))./x19;
 k2=(c1.*besseli(0,l.*g)+c2.*besselk(0,l.*g))./x19;
x20=besseli(0,l2.*g).*k1-besseli(0,l2).*k2;
x21=besseli(0,l2.*g).*besselk(0,l2)-besseli(0,l2).*besselk(0,l2.*g);
c6=x20./x21;
x22=besselk(0,l2.*g).*k1-besselk(0,l2).*k2;
x23=besselk(0,l2.*g).*besseli(0,l2)-besselk(0,l2).*besseli(0,l2.*g);
c5=x22./x23;
x24=G./x19;
B=c5.*besseli(0,l2.*R)+c6.*besselk(0,l2.*R)-x24;
Br=real(B)
Bi=imag(B);

c7=((1./(4.*log(g))).*((g.^2)-1));
c8=5./4;
A=-((R.^2)./4)+c7.*log(R)+c8;

%AG is absolute G, AB absolute B, X is angle btw Bi and Br, P is angle
btw
%Gi and Gr
```

```
X=atan(Bi./Br);
P=atan(Gi./Gr);
AB=sqrt((Br).^2+(Bi).^2);
AG=sqrt((Gr).^2+(Gi).^2);

[C,h]=contour(R,X,St);
   set(h,'showText','on','textStep',get(h,'levelStep'));
   colormap gray
   xlabel('R'), ylabel('T')

   Pr=7.0;
%g mean lamda;
g=2.0;
%Temperature
x1=i.*Pr.*St;
l=sqrt(x1);
x11=besseli(0,l);
x12=besseli(0,l.*g);
x13=besselk(0,l);
x14=besselk(0,l.*g);
x15=besseli(0,l.*g).*besselk(0,l);
x16=besseli(0,l).*besselk(0,l.*g);
x17=besseli(0,l.*g)-besseli(0,l);
x18=besselk(0,l.*g)-besselk(0,l);
c1=x18./(x16-x15);
c2=x17./(x15-x16);
G=c1.*besseli(0,l.*R)+c2.*besselk(0,l.*R);
Gr=real(G);
Gi=imag(G);

F=1;

%velocity
x2=i.*St;
l2=sqrt(x2);
 x19=(l.^2)-x2;
 k1=(c1.*besseli(0,l)+c2.*besselk(0,l))./x19;
```

```
 k2=(c1.*besseli(0,l.*g)+c2.*besselk(0,l.*g))./x19;
x20=besseli(0,l2.*g).*k1-besseli(0,l2).*k2;
x21=besseli(0,l2.*g).*besselk(0,l2)-besseli(0,l2).*besselk(0,l2.*g);
c6=x20./x21;
x22=besselk(0,l2.*g).*k1-besselk(0,l2).*k2;
x23=besselk(0,l2.*g).*besseli(0,l2)-besselk(0,l2).*besseli(0,l2.*g);
c5=x22./x23;
x24=G./x19;
B=c5.*besseli(0,l2.*R)+c6.*besselk(0,l2.*R)-x24;
Br=real(B)
Bi=imag(B);

c7=((1./(4.*log(g))).*((g.^2)-1));
c8=5./4;
A=-((R.^2)./4)+c7.*log(R)+c8;

%AG is absolute G, AB absolute B, X is angle btw Bi and Br, P is angle
btw
%Gi and Gr
X=atan(Bi./Br);
P=atan(Gi./Gr);
AB=sqrt((Br).^2+(Bi).^2);
AG=sqrt((Gr).^2+(Gi).^2);

[C,h]=contour(R,X,St);
   set(h,'showText','on','textStep',get(h,'levelStep'));
   colormap gray
   xlabel('R'), ylabel('T')

%NATURAL CONVECTION IN A VERTICAL ANNULUS WITH TIME PERIODIC BOUNDARY
CONDITTIONS
%Absolute temperature varying Pr
[St,R]=meshgrid(0.1:1.0:20.0,1.0:0.01:2.0);
Pr=0.71;
%g mean lamda;
g=2.0;
%Temperature
```

```
x1=i.*Pr.*St;
l=sqrt(x1);
x11=besseli(0,l);
x12=besseli(0,l.*g);
x13=besselk(0,l);
x14=besselk(0,l.*g);
x15=besseli(0,l.*g).*besselk(0,l);
x16=besseli(0,l).*besselk(0,l.*g);
x17=besseli(0,l.*g)-besseli(0,l);
x18=besselk(0,l.*g)-besselk(0,l);
c1=x18./(x16-x15);
c2=x17./(x15-x16);
G=c1.*besseli(0,l.*R)+c2.*besselk(0,l.*R);
Gr=real(G);
Gi=imag(G);

F=1;

%velocity
x2=i.*St;
l2=sqrt(x2);
 x19=(l.^2)-x2;
 k1=(c1.*besseli(0,l)+c2.*besselk(0,l))./x19;
 k2=(c1.*besseli(0,l.*g)+c2.*besselk(0,l.*g))./x19;
x20=besseli(0,l2.*g).*k1-besseli(0,l2).*k2;
x21=besseli(0,l2.*g).*besselk(0,l2)-besseli(0,l2).*besselk(0,l2.*g);
c6=x20./x21;
x22=besselk(0,l2.*g).*k1-besselk(0,l2).*k2;
x23=besselk(0,l2.*g).*besseli(0,l2)-besselk(0,l2).*besseli(0,l2.*g);
c5=x22./x23;
x24=G./x19;
B=c5.*besseli(0,l2.*R)+c6.*besselk(0,l2.*R)-x24;
Br=real(B)
Bi=imag(B);

c7=((1./(4.*log(g))).*((g.^2)-1));
c8=5./4;
```

```
A=-((R.^2)./4)+c7.*log(R)+c8;

%AG is absolute G, AB absolute B, X is angle btw Bi and Br, P is angle
btw
%Gi and Gr
X=atan(Bi./Br);
P=atan(Gi./Gr);
AB=sqrt((Br).^2+(Bi).^2);
AG=sqrt((Gr).^2+(Gi).^2);

[C,h]=contour(R,AG,St);
   set(h,'showText','on','textStep',get(h,'levelStep'));
   colormap gray
   xlabel('R'), ylabel('T')

   Pr=7.0;
%g mean lamda;
g=2.0;
%Temperature
x1=i.*Pr.*St;
l=sqrt(x1);
x11=besseli(0,l);
x12=besseli(0,l.*g);
x13=besselk(0,l);
x14=besselk(0,l.*g);
x15=besseli(0,l.*g).*besselk(0,l);
x16=besseli(0,l).*besselk(0,l.*g);
x17=besseli(0,l.*g)-besseli(0,l);
x18=besselk(0,l.*g)-besselk(0,l);
c1=x18./(x16-x15);
c2=x17./(x15-x16);
G=c1.*besseli(0,l.*R)+c2.*besselk(0,l.*R);
Gr=real(G);
Gi=imag(G);

F=1;
```

```
%velocity
x2=i.*St;
l2=sqrt(x2);
 x19=(l.^2)-x2;
 k1=(c1.*besseli(0,l)+c2.*besselk(0,l))./x19;
 k2=(c1.*besseli(0,l.*g)+c2.*besselk(0,l.*g))./x19;
x20=besseli(0,l2.*g).*k1-besseli(0,l2).*k2;
x21=besseli(0,l2.*g).*besselk(0,l2)-besseli(0,l2).*besselk(0,l2.*g);
c6=x20./x21;
x22=besselk(0,l2.*g).*k1-besselk(0,l2).*k2;
x23=besselk(0,l2.*g).*besseli(0,l2)-besselk(0,l2).*besseli(0,l2.*g);
c5=x22./x23;
x24=G./x19;
B=c5.*besseli(0,l2.*R)+c6.*besselk(0,l2.*R)-x24;
Br=real(B)
Bi=imag(B);

c7=((1./(4.*log(g))).*((g.^2)-1));
c8=5./4;
A=-((R.^2)./4)+c7.*log(R)+c8;

%AG is absolute G, AB absolute B, X is angle btw Bi and Br, P is angle
btw
%Gi and Gr
X=atan(Bi./Br);
P=atan(Gi./Gr);
AB=sqrt((Br).^2+(Bi).^2);
AG=sqrt((Gr).^2+(Gi).^2);

[C,h]=contour(R,AG,St);
   set(h,'showText','on','textStep',get(h,'levelStep'));
   colormap gray
      xlabel('R'), ylabel('T')


%NATURAL CONVECTION IN A VERTICAL ANNULUS WITH TIME PERIODIC BOUNDARY
CONDITTIONS
```

```
%Absolute velocity varying Pr
[St,R]=meshgrid(0.1:1.0:20.0,1.0:0.01:2.0);
Pr=0.71;
%g mean lamda;
g=2.0;
%Temperature
x1=i.*Pr.*St;
l=sqrt(x1);
x11=besseli(0,l);
x12=besseli(0,l.*g);
x13=besselk(0,l);
x14=besselk(0,l.*g);
x15=besseli(0,l.*g).*besselk(0,l);
x16=besseli(0,l).*besselk(0,l.*g);
x17=besseli(0,l.*g)-besseli(0,l);
x18=besselk(0,l.*g)-besselk(0,l);
c1=x18./(x16-x15);
c2=x17./(x15-x16);
G=c1.*besseli(0,l.*R)+c2.*besselk(0,l.*R);
Gr=real(G);
Gi=imag(G);

F=1;

%velocity
x2=i.*St;
l2=sqrt(x2);
 x19=(l.^2)-x2;
 k1=(c1.*besseli(0,l)+c2.*besselk(0,l))./x19;
 k2=(c1.*besseli(0,l.*g)+c2.*besselk(0,l.*g))./x19;
x20=besseli(0,l2.*g).*k1-besseli(0,l2).*k2;
x21=besseli(0,l2.*g).*besselk(0,l2)-besseli(0,l2).*besselk(0,l2.*g);
c6=x20./x21;
x22=besselk(0,l2.*g).*k1-besselk(0,l2).*k2;
x23=besselk(0,l2.*g).*besseli(0,l2)-besselk(0,l2).*besseli(0,l2.*g);
c5=x22./x23;
x24=G./x19;
B=c5.*besseli(0,l2.*R)+c6.*besselk(0,l2.*R)-x24;
```

```
Br=real(B)
Bi=imag(B);

c7=((1./(4.*log(g))).*((g.^2)-1));
c8=5./4;
A=-((R.^2)./4)+c7.*log(R)+c8;

%AG is absolute G, AB absolute B, X is angle btw Bi and Br, P is angle
btw
%Gi and Gr
X=atan(Bi./Br);
P=atan(Gi./Gr);
AB=sqrt((Br).^2+(Bi).^2);
AG=sqrt((Gr).^2+(Gi).^2);

[C,h]=contour(R,AB,St);
   set(h,'showText','on','textStep',get(h,'levelStep'));
   colormap gray
   xlabel('R'), ylabel('T')

   Pr=7.0;
%g mean lamda;
g=2.0;
%Temperature
x1=i.*Pr.*St;
l=sqrt(x1);
x11=besseli(0,l);
x12=besseli(0,l.*g);
x13=besselk(0,l);
x14=besselk(0,l.*g);
x15=besseli(0,l.*g).*besselk(0,l);
x16=besseli(0,l).*besselk(0,l.*g);
x17=besseli(0,l.*g)-besseli(0,l);
x18=besselk(0,l.*g)-besselk(0,l);
c1=x18./(x16-x15);
c2=x17./(x15-x16);
G=c1.*besseli(0,l.*R)+c2.*besselk(0,l.*R);
```

```
Gr=real(G);
Gi=imag(G);

F=1;

%velocity
x2=i.*St;
l2=sqrt(x2);
 x19=(l.^2)-x2;
 k1=(c1.*besseli(0,l)+c2.*besselk(0,l))./x19;
 k2=(c1.*besseli(0,l.*g)+c2.*besselk(0,l.*g))./x19;
x20=besseli(0,l2.*g).*k1-besseli(0,l2).*k2;
x21=besseli(0,l2.*g).*besselk(0,l2)-besseli(0,l2).*besselk(0,l2.*g);
c6=x20./x21;
x22=besselk(0,l2.*g).*k1-besselk(0,l2).*k2;
x23=besselk(0,l2.*g).*besseli(0,l2)-besselk(0,l2).*besseli(0,l2.*g);
c5=x22./x23;
x24=G./x19;
B=c5.*besseli(0,l2.*R)+c6.*besselk(0,l2.*R)-x24;
Br=real(B)
Bi=imag(B);

c7=((1./(4.*log(g))).*((g.^2)-1));
c8=5./4;
A=-((R.^2)./4)+c7.*log(R)+c8;

%AG is absolute G, AB absolute B, X is angle btw Bi and Br, P is angle
btw
%Gi and Gr
X=atan(Bi./Br);
P=atan(Gi./Gr);
AB=sqrt((Br).^2+(Bi).^2);
AG=sqrt((Gr).^2+(Gi).^2);

[C,h]=contour(R,AB,St);
   set(h,'showText','on','textStep',get(h,'levelStep'));
   colormap gray
```

```
      xlabel('R'), ylabel('T')

%NATURAL CONVECTION IN A VERTICAL ANNULUS WITH TIME PERIODIC BOUNDARY
CONDITTIONS
%Nusselt number at the surfaces of the cylinders varying Pr and St
[Pr,St]=meshgrid(0.0:0.1:0.9,0.1:1.0:20.0);
x1=i.*Pr.*St;
l=sqrt(x1);
g=2.0;
x15=besseli(0,l.*g).*besselk(0,l);
x16=besseli(0,l).*besselk(0,l.*g);
x17=besseli(0,l.*g)-besseli(0,l);
x18=besselk(0,l.*g)-besselk(0,l);
c1=x18./(x16-x15);
c2=x17./(x15-x16);
% inner cylinder
Nu=l.*(c1.*besseli(1,l)-c2.*besselk(1,l));
Nu1=real(Nu);
%for outer cylinder
Nuu=l.*(c1.*besseli(1,l.*g)-c2.*besselk(1,l.*g));
Nu2=real(Nuu);
[C,h]=contour(St,Nu2,g);
   set(h,'showText','on','textStep',get(h,'levelStep'));
%NATURAL CONVECTION IN A VERTICAL ANNULUS WITH TIME PERIODIC BOUNDARY
CONDITTIONS
%Nusselt number at the surfaces of the cylinders varying g and St
[g,St]=meshgrid(1.2:0.1:3.0,0.1:1.0:20.0);
Pr=0.71;
x1=i.*Pr.*St;
l=sqrt(x1);
x15=besseli(0,l.*g).*besselk(0,l);
x16=besseli(0,l).*besselk(0,l.*g);
x17=besseli(0,l.*g)-besseli(0,l);
x18=besselk(0,l.*g)-besselk(0,l);
c1=x18./(x16-x15);
c2=x17./(x15-x16);
% inner cylinder
Nu=l.*(c1.*besseli(1,l)-c2.*besselk(1,l));
```

```
Nu1=real(Nu);
%for outer cylinder
Nuu=l.*(c1.*besseli(1,l.*g)-c2.*besselk(1,l.*g));
Nu2=real(Nuu);
[C,h]=contour(St,Nu2,g);
   set(h,'showText','on','textStep',get(h,'levelStep'));
   colormap gray
   xlabel('St'), ylabel('Nu2')

%NATURAL CONVECTION IN A VERTICAL ANNULUS WITH TIME PERIODIC BOUNDARY
CONDITTIONS
%Skin-friction at the surfaces of the cylinders varying Pr and St
[Pr,St]=meshgrid(0.0:0.1:0.9,0.1:1.0:10.0);
x1=i.*Pr.*St;
x2=i.*St;
l2=sqrt(x2);
l=sqrt(x1);
g=2.0;
x15=besseli(0,l.*g).*besselk(0,l);
x16=besseli(0,l).*besselk(0,l.*g);
x17=besseli(0,l.*g)-besseli(0,l);
x18=besselk(0,l.*g)-besselk(0,l);
c1=x18./(x16-x15);
c2=x17./(x15-x16);
x19=(l.^2)-x2;
 k1=(c1.*besseli(0,l)+c2.*besselk(0,l))./x19;
 k2=(c1.*besseli(0,l.*g)+c2.*besselk(0,l.*g))./x19;
x20=besseli(0,l2.*g).*k1-besseli(0,l2).*k2;
x21=besseli(0,l2.*g).*besselk(0,l2)-besseli(0,l2).*besselk(0,l2.*g);
c6=x20./x21;
x22=besselk(0,l2.*g).*k1-besselk(0,l2).*k2;
x23=besselk(0,l2.*g).*besseli(0,l2)-besselk(0,l2).*besseli(0,l2.*g);
c5=x22./x23;

% inner cylinder
x31=c5.*besseli(1,l2)-c6.*besselk(1,l2);
x32=(c1.*besseli(1,l)-c2.*besselk(1,l))./x19;
tau=l2.*(x31-x32);
```

```
tau1=real(tau);

%for outer cylinder
x41=c5.*besseli(1,g.*l2)-c6.*besselk(1,g.*l2);
x42=(c1.*besseli(1,g.*l)-c2.*besselk(1,g.*l))./x19;
tauu=l2.*(x41-x42);
tau2=real(tauu);
[C,h]=contour(St,tau2,g);
   set(h,'showText','on','textStep',get(h,'levelStep'));
   colormap gray
   xlabel('St'), ylabel('Tau1')
```

```
%NATURAL CONVECTION IN A VERTICAL ANNULUS WITH TIME PERIODIC BOUNDARY
CONDITTIONS
%Skin-friction at the surfaces of the cylinders varying g and St
[g,St]=meshgrid(1.2:0.1:3.0,0.1:1.0:20.0);
x1=i.*Pr.*St;
x2=i.*St;
l2=sqrt(x2);
l=sqrt(x1);
Pr=0.71;
x15=besseli(0,l.*g).*besselk(0,l);
x16=besseli(0,l).*besselk(0,l.*g);
x17=besseli(0,l.*g)-besseli(0,l);
x18=besselk(0,l.*g)-besselk(0,l);
c1=x18./(x16-x15);
c2=x17./(x15-x16);
x19=(l.^2)-x2;
 k1=(c1.*besseli(0,l)+c2.*besselk(0,l))./x19;
 k2=(c1.*besseli(0,l.*g)+c2.*besselk(0,l.*g))./x19;
x20=besseli(0,l2.*g).*k1-besseli(0,l2).*k2;
x21=besseli(0,l2.*g).*besselk(0,l2)-besseli(0,l2).*besselk(0,l2.*g);
c6=x20./x21;
x22=besselk(0,l2.*g).*k1-besselk(0,l2).*k2;
x23=besselk(0,l2.*g).*besseli(0,l2)-besselk(0,l2).*besseli(0,l2.*g);
c5=x22./x23;
% inner cylinder
x31=c5.*besseli(1,l2)-c6.*besselk(1,l2);
x32=(c1.*besseli(1,l)-c2.*besselk(1,l))./x19;
tau=l2.*(x31-x32);
tau1=real(tau);
%for outer cylinder
x41=c5.*besseli(1,g.*l2)-c6.*besselk(1,g.*l2);
x42=(c1.*besseli(1,g.*l)-c2.*besselk(1,g.*l))./x19;
tauu=l2.*(x41-x42);
tau2=real(tauu);
[C,h]=contour(St,tau2,g);
   set(h,'showText','on','textStep',get(h,'levelStep'));
   colormap gray
   xlabel('St'), ylabel('Tau2')
```

```
%mass flow rate of Natural convection flow in a vertical annulus with
%time periodic boundary conditions varying Pr
St=0.1:0.1:10.0;
Pr=0.71;
x1=i.*Pr.*St;
x2=i.*St;
l2=sqrt(x2);
l=sqrt(x1);
g=2.0;
x15=besseli(0,l.*g).*besselk(0,l);
x16=besseli(0,l).*besselk(0,l.*g);
x17=besseli(0,l.*g)-besseli(0,l);
x18=besselk(0,l.*g)-besselk(0,l);
c1=x18./(x16-x15);
c2=x17./(x15-x16);
x19=(l.^2)-x2;
 k1=(c1.*besseli(0,l)+c2.*besselk(0,l))./x19;
 k2=(c1.*besseli(0,l.*g)+c2.*besselk(0,l.*g))./x19;
x20=besseli(0,l2.*g).*k1-besseli(0,l2).*k2;
x21=besseli(0,l2.*g).*besselk(0,l2)-besseli(0,l2).*besselk(0,l2.*g);
c4=x20./x21;
x22=besselk(0,l2.*g).*k1-besselk(0,l2).*k2;
x23=besselk(0,l2.*g).*besseli(0,l2)-besselk(0,l2).*besseli(0,l2.*g);
c3=x22./x23;
x24=g.*l;
x25=g.*l2;
 x26=c3.*(g.*besseli(1,x25)-besseli(1,l2));
 x27=c4.*(g.*besselk(1,x25)-besselk(1,l2));
x28=c1.*(g.*besseli(1,x24)-besseli(1,l));
x29=c2.*(g.*besselk(1,x24)-besselk(1,l));
 x30=(x28-x29)./sqrt(Pr);
M1=(x26-x27-x30)./(x19.*l2);
M=real(M1)
xlabel('St'), ylabel('T')
   plot(St,M,'k')
   hold on
```

```
%mass flow rate of Natural convection flow in a vertical annulus with
%time periodic boundary conditions varying aspect ratio
St=0.1:0.1:10.0;
g=2.0;
Pr=0.71;
x1=i.*Pr.*St;
x2=i.*St;
l2=sqrt(x2);
l=sqrt(x1);
x15=besseli(0,l.*g).*besselk(0,l);
x16=besseli(0,l).*besselk(0,l.*g);
x17=besseli(0,l.*g)-besseli(0,l);
x18=besselk(0,l.*g)-besselk(0,l);
c1=x18./(x16-x15);
c2=x17./(x15-x16);
x19=(l.^2)-x2;
 k1=(c1.*besseli(0,l)+c2.*besselk(0,l))./x19;
 k2=(c1.*besseli(0,l.*g)+c2.*besselk(0,l.*g))./x19;
x20=besseli(0,l2.*g).*k1-besseli(0,l2).*k2;
x21=besseli(0,l2.*g).*besselk(0,l2)-besseli(0,l2).*besselk(0,l2.*g);
c4=x20./x21;
x22=besselk(0,l2.*g).*k1-besselk(0,l2).*k2;
x23=besselk(0,l2.*g).*besseli(0,l2)-besselk(0,l2).*besseli(0,l2.*g);
c3=x22./x23;
x24=g.*l;
x25=g.*l2;
 x26=c3.*(g.*besseli(1,x25)-besseli(1,l2));
 x27=c4.*(g.*besselk(1,x25)-besselk(1,l2));
x28=c1.*(g.*besseli(1,x24)-besseli(1,l));
x29=c2.*(g.*besselk(1,x24)-besselk(1,l));
 x30=(x28-x29)./sqrt(Pr);
M1=(x26-x27-x30)./(x19.*l2);
M=real(M1)
   plot(St,M,'k')


   g=2.5;
Pr=0.71;
x1=i.*Pr.*St;
```

```
x2=i.*St;
l2=sqrt(x2);
l=sqrt(x1);
x15=besseli(0,l.*g).*besselk(0,l);
x16=besseli(0,l).*besselk(0,l.*g);
x17=besseli(0,l.*g)-besseli(0,l);
x18=besselk(0,l.*g)-besselk(0,l);
c1=x18./(x16-x15);
c2=x17./(x15-x16);
x19=(l.^2)-x2;
 k1=(c1.*besseli(0,l)+c2.*besselk(0,l))./x19;
 k2=(c1.*besseli(0,l.*g)+c2.*besselk(0,l.*g))./x19;
x20=besseli(0,l2.*g).*k1-besseli(0,l2).*k2;
x21=besseli(0,l2.*g).*besselk(0,l2)-besseli(0,l2).*besselk(0,l2.*g);
c4=x20./x21;
x22=besselk(0,l2.*g).*k1-besselk(0,l2).*k2;
x23=besselk(0,l2.*g).*besseli(0,l2)-besselk(0,l2).*besseli(0,l2.*g);
c3=x22./x23;
x24=g.*l;
x25=g.*l2;
 x26=c3.*(g.*besseli(1,x25)-besseli(1,l2));
 x27=c4.*(g.*besselk(1,x25)-besselk(1,l2));
x28=c1.*(g.*besseli(1,x24)-besseli(1,l));
x29=c2.*(g.*besselk(1,x24)-besselk(1,l));
 x30=(x28-x29)./sqrt(Pr);
M1=(x26-x27-x30)./(x19.*l2);
M=real(M1)
   plot(St,M,'k')


   g=3.0;
Pr=0.71;
x1=i.*Pr.*St;
x2=i.*St;
l2=sqrt(x2);
l=sqrt(x1);
x15=besseli(0,l.*g).*besselk(0,l);
x16=besseli(0,l).*besselk(0,l.*g);
x17=besseli(0,l.*g)-besseli(0,l);
```

```
x18=besselk(0,l.*g)-besselk(0,l);
c1=x18./(x16-x15);
c2=x17./(x15-x16);
x19=(l.^2)-x2;
 k1=(c1.*besseli(0,l)+c2.*besselk(0,l))./x19;
 k2=(c1.*besseli(0,l.*g)+c2.*besselk(0,l.*g))./x19;
x20=besseli(0,l2.*g).*k1-besseli(0,l2).*k2;
x21=besseli(0,l2.*g).*besselk(0,l2)-besseli(0,l2).*besselk(0,l2.*g);
c4=x20./x21;
x22=besselk(0,l2.*g).*k1-besselk(0,l2).*k2;
x23=besselk(0,l2.*g).*besseli(0,l2)-besselk(0,l2).*besseli(0,l2.*g);
c3=x22./x23;
x24=g.*l;
x25=g.*l2;
 x26=c3.*(g.*besseli(1,x25)-besseli(1,l2));
 x27=c4.*(g.*besselk(1,x25)-besselk(1,l2));
x28=c1.*(g.*besseli(1,x24)-besseli(1,l));
x29=c2.*(g.*besselk(1,x24)-besselk(1,l));
 x30=(x28-x29)./sqrt(Pr);
M1=(x26-x27-x30)./(x19.*l2);
M=real(M1)
   plot(St,M,'k')
```