COMPOSITE RISK MODELS OF OGEE TYPE SPILLWAY

BY

YUSSUFF MUMMUNNI IMAN

A Thesis Submitted to the Postgraduate School,
Ahmadu Bello University In Partial Fulfillment of
the Requirements for the Award of the Degree of Master
of Science in Water Resources and Environmental
Engineering

Department of Water Resources and Environmental Engineering,
Ahmadu Bello University,
Zaria, Nigeria.

JANUARY, 1990

DECLARATION

I hereby declare that this thesis has been executed by me and that it is a record of my own research work. It has not been presented in any previous publication for a higher degree.

M. I. YUSSUFF

Date : 11/1/1990

This declaration is confirmed by

Dr. M. Sowinski.

Date : 11/1/1990

CERTIFICATION.

This thesis entitled "Composite Risk Models of Ogee Type Spillway" by Yussuf, Mummunni Iman meets the regulations governing the award of the degree of Master of Science of Ahmadu Bello University, and is approved for its contribution to knowledge and literary presentation.

Chairman, Supervisory Committee
DR M. SOWINSKI — Date: 11/1/1990

Member, Supervisory Committee,
PROF. S. MUSTAPHA — Date: 12/1/90

Head of Department.
DR L.I. ODIGIE — Date: 15/1/90

Dean, Postgraduate School
PROF. D.I. SAROR — Date: 10-9-90

DEDICATION

To my dear parents.

Acknowledgements

Praise be to Allah, the magnificent, the merciful who has been my Guidance in all my affairs. This work would not have been possible without His usual infinite mercy.

My profound gratitude goes to my supervisor, Dr. M. Sowinski who supervised this work with an eagle's eye. Also, the contribution of Dr. S. Mustapha in this respect is highly acknowledged. Their immense contribution to the success of this work is unimagined.

The university authority of Abubakar Tafawa Balewa University, Bauchi is acknowledged for financing the thesis. Dr. C. Kanu, Programme Co-ordinator, Civil Engineering Programme, A.T.B.U., Bauchi is acknowledged for his co-operation throughout my course of study.

My best regards go to all members of my family and friends whose inspiration I received throughout the course.

ABSTRACT

In the existing design methods, consideration is given only on the stochastic process of flood flows imposed on the structure to account for hydrologic risk and the capacity of the structure is treated as deterministic. This thesis presents composite risk models which integrated hydrologic and hydraulic risks in the design of spillway structure. Both static and dynamic risk models were developed and applied to Ogee Type Spillway. While the static model concentrates on the occurrence of single (worse) flood magnitude, the dynamic model takes into account the repeated nature of the random flood magnitude imposed on the structure. Results show that risk evaluated by the conventional method which only considers hydrologic risk underestimates the risk of failure and this becomes significant when return period and safety factors are large. Risk, safety factor, service life, and return period relationships were established and these can be incorporated into optimisation model for determining minimum cost spillway systems.

TABLE OF CONTENTS

		PAGE
TITLE	. . . . .	i
DECLARATION	. . . . .	ii
CERTIFICATION	. . . . .	iii
DEDICATION	. . . . .	iv
ACKNOWLEDGEMENTS	. . . . .	v
ABSTRACT	. . . . .	vi
TABLE OF CONTENTS	. . . . .	vii
LIST OF TABLES	. . . . .	xii
LIST OF FIGURES	. . . . .	xiv
NOTATIONS	. . . . .	xvi
1. INTRODUCTION	. . . . .	1
1.1 Background of the Study	. . . . .	1
1.2 Research Objectives	. . . . .	2
1.3 Research Problem: Formulation	. . . . .	3
1.4 Scope of the Thesis	. . . . .	4
2. LITERATURE REVIEW	. . . . .	6
2.1 Hydrologic Risk	. . . . .	6
2.2 Hydraulic, Structural and Economic Risks	. . . . .	11
2.3 Composite Risk Model	. . . . .	13

3.	THEORY OF STATIC AND DYNAMIC MODELS	19
3.1	Definition of Risk and Reliability	19
3.2	Concept of Static Model	20
3.3	Concept of Dymamic Model	22
3.4	Estimation of Probability Distribution Parameters By First Order Analysis	28
4.	A CASE OF AN OGEE TYPE SPILLWAY	32
4.1	Hydrologic Model	32
4.1	Flood Frequency Analysis	32
4.2	Type of Probability Distribution for Maximum flood flows	33
4.2.1	Log - Pearson Type III Distribution	33
4.2.2	Lognormal Distribution	34
4.2.3	Extreme Type I (Gumbel) Distribution	35
4.3	Hydrologic Model Verification	36
4.3.1	Kolmogorov - Smirnov Test	37
4.3.2	Confidence Band	38
4.4	Hydraulics of Ogee Type Spillway	39
4.4.1	Spillway Discharge Coefficient	41
	Depth of Approach	41
	Effect of Heads differing from Design Head	42

	Effect of Upstream Face Shape	43
	Effect of Downstream Apron Interference and Submergence	44
4.4.2	Contraction Coefficients	45
	Pier Contraction Coefficients	46
	Abutment Contraction Coefficients	46
4.5	Hydraulic Model (for Computation of Probability Distribution Parameters)	47
5.	APPLICATION OF THE RISK THEORY TO DADIN - KOWA DAM	49
5.1	General Features of the Dam	49
5.1.1	Purpose	51
5.1.2	Spillway	51
5.2	Hydrological Model for Gongola River at Dadin - Kowa	52
5.2.1	Flood Frequency Analysis	52
5.2.2	Hydrological Model Verification	57
	Kolmogorov - Smirnov Test	60
	Confidence Band	63
5.3	Analysis of Hydraulic Unvertaintion and Estimation of Selected Probability Distribution Parameters	66
5.3.1	Hydraulic Model Equation	66
	Coefficient of Discharge Variable	66

	Regression Analysis for Discharge Coefficient	77
	Contraction Coefficient Variable	79
5.3.2	First - Order Analysis for the Computation of Coefficient of Variation of the Hydraulic Equation	82
	Determination of Hydraulic Model Error	86
	Computation of Coefficient of Variation	89
5.3.3	Estimation of the Selected Probability Distribution Parameters	93
5.4	Development of Static Model	94
5.4.1	Composite Risk Computations for Static Model	95
5.5	Development of Dynamic Model	97
5.5.1	Composite Risk Computations for Dynamic Model.	98
5.6	Development of Hydrologic Risk Model	104
5.6.1	Computation of Hydrologic Risk	105
5.7	Comparison of Hydrologic Risk and Composite Risk	106
6.	DISCUSSION	108
6.1	Selection of Hydrologic Probability Distribution....	108
6.2	Determination of Spillway Discharge Equation Variables	109

6.3	Static Composite Risk Model	110
6.4	Dynamic Composite Risk Model	111
6.5	Comparison of Hydrologic and Composite Risks	112
7.	CONCLUSIONS AND SUGGESTIONS	116
7.1	Conclusions	116
7.2	Suggestions	117
REFERENCES	. . . . .	119

LIST OF TABLES

		PAGE
TABLE 5.1 :	Result of the Flood Frequency Analysis for River Gongola at Dadin - Kowa	54
TABLE 5.2 :	Result of the Kolmogorov - Test for the Three Selected Distributions	61
TABLE 5.3 :	Table of Flood Values for the Construction of Confidence Band	64
TABLE 5.4 :	Values of Discharge Coefficients from Various Sources	72
TABLE 5.5 :	Discharge Coefficients from the fitted Equations	75
TABLE 5.6 :	Boundary Values for the Coefficients of Discharge	77
TABLE 5.7 :	Boundary and most Probable values of Pier Contraction Coefficient	80
TABLE 5.8 :	The Regressed Pier Contraction Coefficient	81
TABLE 5.9 :	Relative Error in Discharge Measurement	91
TABLE 5.10 :	Coefficient of Variation and the Expected Mean Capacity of the Spillway	92
TABLE 5.11 :	Parameters of Lognormal Distribution for the Spillway Capacity	94

TABLE 5.12 : Risk - Safety Factor Relationship for the Static Model (Composite Risk) 95

TABLE 5.13 : Risk - Safety factor Relationship for the Dynamic Model 100

TABLE 5.14 : Results of Hydrologic Risk with Various Head Ratio 106

TABLE 5.15 Comparison of Hydrologic Risk with Composite Risk 107

LIST OF FIGURES

			PAGE
FIGURE	3.1 :	Load Resistance Inference Model	20
FIGURE	4.1 :	Longitudinal Profile of Ogee Type Spillway . . .	41
FIGURE	4.2 :	Spillway Longitudinal Profile Showing Tail Water Depth . . .	42
FIGURE	5.1 :	Map of North Eastern Nigeria Showing Dadin - Kowa Dam	50
FIGURE	5.2 :	Dadin - Kowa Spillway General Arrangement and Plan View	53
FIGURE	5.3a,b,c:	Flood Frequency Curves for Annual Flood Values of R. Gongola at Dadin - Kowa Plotted in Gumbel, Lognormal, and Log-Pearson III Papers Respectively	57
FIGURE	5.4	Head Discharge Relation for Spillway Shape of Negligible depth of Approach	67
FIGURE	5.5 :	Coefficients of Discharge for straight Standard Dam Crest with No Velocity of Approach	67
FIGURE	5.6 :	Discharge Coefficients for Vertical Faced Ideal Crest	69
FIGURE	5.7 :	Coefficients of Discharge other than Design Head	70

FIGURE 5.8 : Dadin - Kowa Spillway Rating curve 71

FIGURE 5.9 : Coefficients of Discharge for Straight Dam Crest with no Velocity of Approach . . . 74

FIGURE 5.10 : Boundary Value curves for Coefficient of Discharge 78

FIGURE 5.11 : Coefficients of Fiers for Different Shapes 83

FIGURE 5.12 : Triangular Distribution 89

FIGURE 5.13 Risk - Safety Factor Relationship for Static Model 96

FIGURE 5.14a,b and c : Risk - Safety Factor Relationship for Dynamic Model 101

NOTATION.

The following symbols are used in this project.

a, b, c, = minimum, maximum and probable value respectively in the triangular distribution;

C, C_D = Coefficient of discharge and design discharge coefficient.

C_s, C_v = Coefficient of skewness and variation

$E(\bar{Q}_c)$ = Expected value of $\bar{Q}_c$

F( ) = Mathematic model function of variables x_1 to x_i ;

$\bar{F}$( ) = Expected values of F( );

f_L( ) = probability density function of loading;

F_L( ) = Cumulative distribution function of loading;

$f_{Q_c}(Q_c)$ = Probability density function of spillway;

$F_{Q_d}(Q_c)$ = Cumulative probability function of flood;

$f_{Q_d}(Q_d)$ = Probability density function of flood;

f_r( ) = Probability density function of resistance;

H , H_D = Total head and design head resp;

K_a = Abutment contraction coefficient,

K_p = Pier contraction coefficient;

L = loading ;

L_n = Effective crest length between gate bays ;

L_o = Nominal value of loading ;

L_t = Total clear length of crest between abutments and piers ;

N = No of piers ;

P( ) = Probability density function;

Q_c, $\bar{Q}_c$ = Capacity of the spillway and its mean value;

Q_d = Flood magnitude ;

r, $\bar{r}$ = Resistance of the structure and its mean value.

$\bar{R}$ = Risk of the spillway.

R = Reliability of the Spillway ;

R_n = Reliability after n-occurence of loading

R(t) = Reliability at time t for dynamic model

$\bar{R}(t)$ = Risk at time t for dynamic model.

S = Sample standard deviation ;

SF = Safety Factor ;

T_r = Return period

$Var(Q_c)$ = Variance of random variable Q_c;

α = Mean rate of occurence of event .

α_1, α_2 = the location and Scale paramaters of Gumbel distribution ;

α , β, γ = the scale, shape and location parameters of log - pearson type III distribution;

μ = population mean ;

μ_n, σ_n = Mean and standard deviation of the transformed variable ln X ;

σ = Population standard deviation ;

λ_m, $\bar{\lambda}_m$ = hydraulic model error and its mean value ;

Ω_{Q_c} = Coefficient of variation of Q_c ;

$\Omega_{(x_i)}$ = Coefficient of variation of, X_i ;

Ω_{λ_m} = Model coefficient of variation ;

CHAPTER ONE

INTRODUCTION

KASHIM IBRAHIM LIBRARY
AHMADU BELLO UNIVERSITY
ZARIA, NIGERIA.

1.1 Background of the Study

Flood occurrence results in large monetary damages, loss of life, and the disruption of community services. Major dams have failed as a result of flood magnitude exceeding the design flood event. The American Water Works Association (1966) reported that out of 293 dam failures in the US and other countries since 1799, about 20% of the failures were due to faulty spillway design. In a more recent study of over 300 dam disasters, Cicioni et al, (1973), it was found that 35% of the failures were due to inadequate spillway design.

The major concern of hydraulic structures designers is to design structures that will be safe. Nonfailure does not necessary mean an efficient design. Frequently structures are over designed and hence very safe, but also very expensive. An optimal design of hydraulic structures is much desirable.

Through flood frequency analysis in the design of structures within flood plain, adequate sizing of spillway could be achieved.

The simplest procedure in the frequency analysis estimation of spillway design floods is to select a high return period as a form of safety factor and use a theoretical probability distribution to estimate the corresponding event magnitude. This technique has been the practice of considering the hydrologic risk in spillway design. But the risk involved in spillway systems is not only hydrologic, but also hydraulic, economic and structural risks.

The theoretical concepts of risk analyses have been available for engineering practice for some time, however the application of the theory for the consideration of hydrologic and hydraulic risks in the design of hydraulic structures is in its infancy. It has been applied to flood conveyance hydraulic structures such as storm sewers, culverts, and flood levees. This thesis is to apply the risk theory to integrate both hydrologic and hydraulic uncertainties to facilitate the optimal design of spillway structure.

1.2 Research Objectives

The conventional return period method of design of spillway structure only considers the randomness of flood events and treats

the capacity of the structure as deterministic. This accounts for only hydrologic risk.

The intent of this thesis is :

(i) the application of risk theory to integrate the hydrologic and hydraulic uncertainties, in the design of spillway structure.

(ii) the selection of either Static or Dynamic Risk Model to integrate the two uncertainties into the design of spillway structure.

(iii) the establishment of risk - safety relationships, expected service life of the structure, and return periods which will facilitate further studies on risk based optimal design of spillway.

1.3 Research Problems Formulation

In order to meet the research objectives stated above, the following research problems were identified :

(1) Selection of hydrologic model for annual maximum flow so that more events can be estimated from the fitted distribution.

(2) Determination of the characteristic parameters of the independent random variables of the spillway discharge equation. The independent variables are the model error, coefficient of discharge and the crest length of the spillway.

(3) Performance of first - order analysis to estimate the mean and coefficient of variation of the dependent hydraulic variable of the hydraulic equation. The variable is the discharge capacity.

(4) Development of static and Dynamic Models for risk computations.

(5) Computations of risks, safety factors, expected service life of project and return periods relationship.

(6) Comparison of Static and Dynamic Models.

(7) Comparison of hydrologic risk and composite risk.

1.4 Scope of the Thesis

The risk theory is applied to the overflow (Ogee) spillway of Dadin - Kowa multipurpose dam. The dam is located 5 km north of the Dadin - Kowa Village near Combe on River Gongola in Bauchi State.

Three distributions were selected among various distributions used in hydrology to test their fits to River Gongola flood events. They are Log - Pearson Type III, Lognormal, Extremal Type 1 (Gumbel) distributions. Only one of these distributions which satisfie the model verification test is used to develop the hydrologic model. The theory of the three distributions and model verification are presented in chapter four.

The composite risk considered in this study consists of only hydrologic and hydraulic risks. Two models are considered in this study to describe the composite risk. They are static and Dynamic Models. While the Static Model concentrates on the occurrence of single (worse) loading, the dynamic model takes into account the repeated nature of the random loading imposed on the structure. In the development of dynamic model, the risk theory is restricted to random - independent loading and random-fixed resistance of the spillway capacity. The theory of the two models is presented in chapter three. The computation and the comparison of hydrologic risk and composite risk are presented in Chapter Five. The results, conclusions and suggestions are discussed in chapters six and seven respectively.

CHAPTER TWO

LITERATURE REVIEW.

Risk is an element that exists in all aspectsof engineering designs and planning, including water resources engineering. Risk in water resources engineering design can be attributed mainly to the inherently random behaviour of hydrologic processes such as flood, the lack of perfect knowledge about hydrologic process involved, and the lack of complete control of design and operation of hydraulic structures. This lack of knowledge is generally referred to as uncertainty.

2.1 Hydrologic Risk.

Hydraulic structures are designed with reference to some natural events, such as flood flows which are imposed on the structure during its expected service life. The natural events are stochastic in nature due to their randomness in occurrence. The probability of the values of the random phenomenon being greater than the design flood is referred to as the Hydrologic Risk. This risk has long been recognised in water resources engineering practice. A safety factor in the form of high return period has

been the conventional method to account for the hydrologic risk. Gumbel (1955) mentioned the simple risk involved in a hydraulic design; the calculated simple risk is a function of design period and service life of the structure.

In the risk evaluation there are two commonly used hydrologic risk methods. For the first model developed using the binomial law, the flood conveyance risk of a hydraulic structure over an expected service life is expressed as

$$R_1(n, T) = 1 - \left[1 - \frac{1}{T}\right]^n \qquad \text{— 2.1}$$

in which T is the return period and n is the expected service life of the structure. Eq. 2.1 was used by Yen (1970) to develop a series of curves relating risk to the return period and the expected service life for hydraulic structures. It is emphasized in the paper that there is an additional risk due to limited years of record, stochastic nature of the hydrologic system, and techniques utilized in the mathematical processing of data.

The second hydrologic risk method utilizes the poisson distribution for risk evaluation and this is expressed as :

$$R_2(t, T) = 1 - \exp\left(\frac{-t}{T}\right) \qquad \text{— 2.2}$$

in which t is the time period of interest which could be the expected service life of the hydraulic structure, and T is the design return period.

On the basis of eq. 2.2, Hall and Howell (1963) presented a risk evaluation procedure. For a fixed service life (t or n), the flow conveyance risk calculated by eq. 2.2 will asymptotically converge to that calculated by eq. 2.1 as the return period T gets larger.

A frequently used approximation of risk resulting from eq. 2.1 is

$$R_1 \simeq \frac{n}{T} \qquad — \ 2.3$$

in which n, and T are as defined before Gumbel (1955) termed this as the 'Design Quotient.'

The probabilities referred to above are all probabilities of occurrence of an event of a certain magnitude. Also of interest is the average probability of occurence of all events above that certain magnitude. The average probability of n' events which is exceeded is given as

$$R = \frac{1}{n'}\left[\frac{1}{n+1} + \frac{2}{n+1} + \frac{3}{n+1} + \ldots + \frac{n'}{n+1}\right] \qquad — \ 2.4$$

in which n is the number of years of the expected project life of n years.

Bemspm (1967) has shown that this expression can be reduced to

$$R = \frac{n + T}{2T(n+1)} \qquad — \quad 2.5$$

in which, and T are as defined before. As n approaches infinity, the average probability of occurrence becomes

$$R = \frac{1}{2T} \qquad — \quad 2.6$$

Thus, in general, the average probability of occurrence of all events above the T-year event is approximated by the probability of the 2T-year event.

When the probability of a hydrologic variable is described by a probability distribution, it is usually expressed as a function of statistical parameters that determine the distribution. However, probabilities may neither be expressed in terms of the statistical parameter of the variables nor of its initial probability distribution. Such probabilities may be described

as non-parametric or distribution free. Chow and Takse (1977) developed a model using the non-parametric approach which is also used to estimate the hydrologic risk. The probability that the extreme event observed in N past years would not be equalled or exceeded in a future M events is expressed as

$$P = \frac{N}{N + M} \qquad — \quad 2.7$$

It is noteworthy to mention here that the earlier expressions are also non-parametric.

If it is required to estimate the event magnitude corresponding to the design return period computed from, for example, eq. 2.1, then a probability distribution must be assumed. To show the wide variation possible in theresults of this assumption of a distribution, a design chart which shows the relationship between design quotient and the reduced variable is presented by Gumbel, (1955) for various distributions. The standard variable is given as

$$Z = \frac{X - \mu}{\sigma} \qquad — \quad 2.8$$

in which X is the flood magnitude, μ is the population mean

and σ is the population standard deviation. The design quotient is given before in eq. 2.3

Hydrologic risk evaluation in water resources structures has also been considered through Bayesian framework. This is known as Baye's hydrologic risk. Baye's methodology provides a framework for combining regional information with at - site historical records. The method allows the consideration of the uncertainties that constitute the risk of failure of hydraulic structure. Some works have been cited in literature which evaluated the hydrologic risk using Bayesian approach. Wood (1975a) provided a procedure for analysing and accounting for hydrologic risk of competing flood frequency models to determine the optimal size of local flood protection by using prior information from a regression model, historical data, and flood cost and damage functions.

All the above cited works have been the conventional return period method of considering hydrologic risk in water resources engineering designs.

KASHIM IBRAHIM LIBRARY
AHMADU BELLO UNIVERSITY
ZARIA, NIGERIA.

2.2 Hydraulic, Structural, and Economic Risks

Tung and Mays (1980a) pointed out that in addition to hydrologic risk, there are other risks associated with water

resources systems. These are hydraulic, structural, and economic risks.

(a) Hydraulic risk: Factors which are responsible for hydraulic risk are :

(i) the error in the use of hydraulic equation to describe the hydraulic model. This error results from the complicated and unsteady nature of flow through or over the hydraulic structure.

(ii) the errors in construction (misalinement of a hydraulic structure), material variability causing variations in the size and distribution of the surface roughness etc.

(b) Structural risk : Structural risk refers to the failure from structural weaknesses. Such failures can be caused by many factors such as (i) water saturation and loss of soil stability, (ii) erosion or hydraulic soil failures, (iii) wave action and (iv) hydraulic overloading.

(c) Economic risk : This risk can be as a result of uncertainties due to construction costs, damage costs, operation and maintenance costs, and inflation, etc.

2.2. Composite Risk Model

The concept of composite risk which is the integration of hydrologic and hydraulic risks or any other risk combination such as hydrologic and economic and/or structural risks is in its infancy in water resources systems designs. This concept is being considered in the design of hydraulic structures because of the belief of the underestimation of risk of failure when only hydrologic risk is considered. Such underestimated risk can consequently undervalue the economic losses if failure occurs. Additionally, it is also possible that from the composite risk model, risk - safety factor expected service life of the structure and design return period interrelationships can be determined which facilitate works in optimisation model design of hydraulic structures.

Few works have been reported in literatures which integrated various risks in the design of flood conveyance structures. There are two composite risk models which have been used in hydraulic structures design. They are static and Dynamic Models. Static Models are models which consider a single (worse) hydrologic event throughout the expected service life of the structure while dynamic models consider the repeated application of the hydrologic event over the expected service life of the structure.

Tang and Yen (1972) proposed a static risk model where hydrologic and hydraulic risks associated with the hydraulic design of storm sewers can be systematically analysed, combined, and incorporated in the evaluation of the composite risk - knowledge of this risk is essential in estimating the expected damages for properties and inconvinience to the public due to flooding over the expected service period of the sewer system. Later, Tang, et al (1975) incorporated this risk procedure into a dynamic programing approach for the optimal risk - based design of storm sewer systems. In the computation of risk values, lognormal distribution are assumed for both hydrologic event and Storm Sewers. From the findings in that work, it was reported that expected damage cost is large when a composite risk is considered. No comparison is made however between the expected damage cost for composite risk and when only hydrologic risk is considered.

Risk theory has also been applied to the design of drainage culverts. Young, et al (1974) developed an optimal model for highway drainage culverts that considered hydrologic and economic risks. This model was used to compute economic risks which are converted to yearly flood risks by using appropriate probability for each flood. In this work hydraulic risk is neglected but Hays (1979)

improved the work by incorporating hydraulic risk into the optimal risk design procedure of culvert systems.

Similar work on risk computation for culvert design was done by Tung and Mays (1980). An extension of the work was done by considering dynamic (Time - Dependent) model which accounts for the repeatability of hydrologic event (flood magnitude). In this work, extreme value Type 1 distribution was assumed for the hydrologic event, and lognormal distribution for the culvert capacity. A series of curves for risk - safety factor, return period and expected service life were obtained for the drainage culverts.

The composite risk concept has also been applied to another flood conveyance structure, flood levee systems. Tung and Mays (1981a) used the equations developed by Mays (1979) and Tung and Mays (1980) for culvert systems, to design flood levee systems. Two models were considered in that work, static and dynamic models. The two models systematically analysed the hydrologic and hydraulic risks involved in flood levee design. The components of the hydrologic risk which are model and parameter uncertainties were highlighted. Statistical goodness - of fit techniques such as Chi-square and Kolmogorov- Smirnov were suggested to account for the hydrological model uncertainty, and the provision of confidence

levels of selected probability distributions to obtain the best - fit distribution.

Another approach suggested in the same paper to compute a composite risk for both static and dynamic models is through Bayesian framework. In this case, it was suggested that a composite probability distribution of flood magnitude be used instead of only one best - fit distribution.

The work of Tung and Mays (1981b) went a step further to incorporate hydrologic and hydraulic risks into optimal design of flood levee systems to obtain minimum cost levee design. The risk-safety factor, expected service life, and return period relationships developed in the earlier work of Tung and Mays (1981a) were used in the optimisation model. In the paper the use of different design philosophies were examined; these include designs without considering any risk, designs connsidering only the hydrologic risk, and designs considering a composite risk which combines both hydrologic and hydraulic risks. From the findings reported in this paper, the annual expected damage cost computed by considering the composite risk was larger than that of considering no risk, or only hydrologic risk. This shows that economic consequences

of flood hazards can grossly be underestimated if no risk or hydrologic risk only is considered.

Tung and Mays (1985) developed dynamic risk model for flood levee structure incorporating both hydrologic and hydraulic risks using regional flood data. Dynamic model for composite risk was compared with the conventional return period methods based on binomial and poisson distributions eqs. 2.1 and 2.2. The comparison shows that considering only the hydrologic risk underestimates the risk of overtopping a levee. It is generally observed in that work that hydraulic risk can not be ignored in risk evaluation when its contribution is moderate or large as related to the capacity of the levee. Furthermore, a high price, in terms of a large value of safety factor, is needed to improve the risk level when the hydraulic risk is large than the case when the hydraulic risk is small for improving the total risk to the same degree.

The cited literatures on composite risk highlighted two important things :

(i) the application of composite risk theory to hydraulic structures which enables the examination of interrelationships between risk, safety factor, expected service life of the structure, and return period. This interrelationship is used in optimisation to obtain minimum cost hydraulic structures design.

(ii) the underestimation of risk when only hydrologic risk is considered and the consequent wrong evaluation of the economic losses of flood hazards.

Based on these two important points, the application of the composite risk theory to the design of spillway structure will be considered in this thesis, to establish the interrelationships between the risk, safety factor, service life, and return period to facilitate further work on the optimisation of spillway structure. It will also address the underestimation of the risk when only hydrologic risk is considered.

CHAPTER THREE

THEORY OF STATIC AND DYNAMIC MODELS.

3.1 Definition of Risk and Reliability.

The risk of a hydraulic component, subsystem, or system is defined as the probability that the loading l exceeds the resistance r, i.e., the probability of failure. Reliability is defined as the probability of the resistance exceeding the loading, i.e. the probability of survival.

The mathematical representation of the risk $\bar{R}$, can be expressed as

$$\bar{R} = P_r(L > r) = P_r(L - r > 0) \quad \ldots \quad — \quad 3.1$$

whose $P_r(\ )$ refers to probability. The relationship between risk and reliability, R is

$$R = 1 - \bar{R} \quad — \quad 3.2$$

The resistance of a hydraulic structure is essentially the capacity of the structure and the loading is the magnitude of flow over the structure e.g. from a hydrologic event. The magnitude of flood is the maximum annual flood which is imposed

on the structure. This is denoted as Q_d. The capacity of the structure is the discharge capacity of a hydraulic structure. This is denoted as Q_c.

Re-writing eq. (3.1) in terms of Q_c and Q_d ;

$$\bar{R} = P_r(Q_d > Q_c) = P_r(Q_d - Q_c > 0) \qquad — \quad 3.3$$

3.2 Concept of Static Model.

The loading applied to many hydraulic structures is a random variable and the repeated number of times such loading occurs is unknown and random. If the distribution of the loading and resistance is known, the resulting model is referred to as a Load - Resistance Inference Model. Fig. 3.1

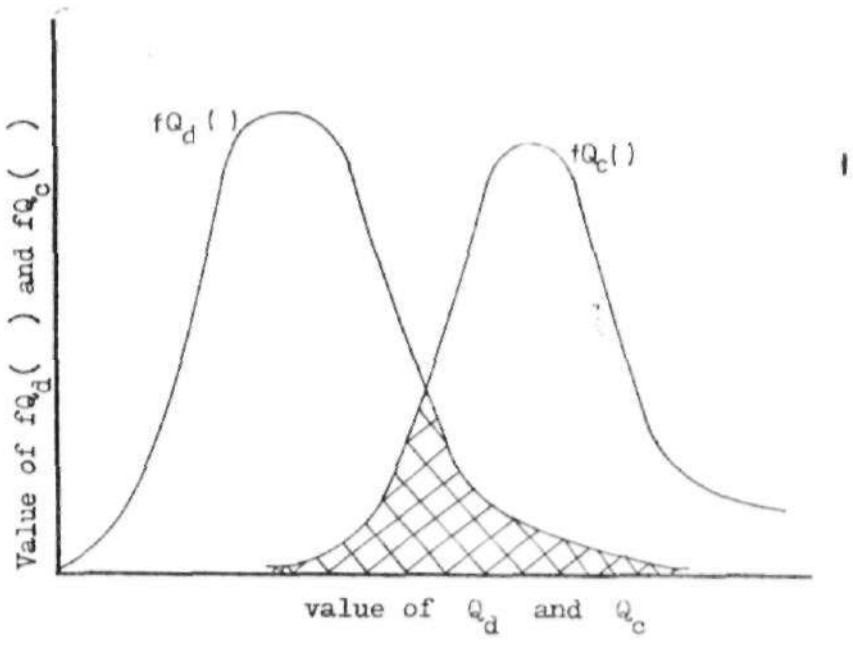

Fig 3.1: Load - Resistance Inference Model.

When a single loading is applied or when the worse single stress is applied, the resulting reliability model is known as 'Static Reliability Model' The reliability of a hydraulic structure can be expressed (Kapur and Lamberson, 1977) as :

$$R = \int_0^{\infty} f_{Q_c}(Q_c) \left[\int_0^{Q_c} f_{Q_d}(Q_d) dQ_d \right] dQ_c \qquad \text{--- } 3.4$$

or

$$R = \int_0^{\infty} f_{Q_d}(Q_d) \left[\int_{Q_d}^{\infty} f_{Q_c}(Q_c) dQ_c \right] dQ_d \qquad \text{--- } 3.5$$

and the risk is expressed as :

$$\bar{R} = \int_0^{\infty} f_{Q_d}(Q_d) \left[\int_0^{Q_d} f_{Q_c}(Q_c) dQ_c \right] dQ_d \qquad \text{--- } 3.6$$

or

$$\bar{R} = \int_0^{\infty} f_{Q_c}(Q_c) \left[\int_{Q_c}^{\infty} f_{Q_d}(Q_d) dQ_d \right] dQ_c \qquad \text{--- } 3.7$$

where $f_{Q_c}(\ )$ and $f_{Q_d}(\ )$ represent the probability density functions of resistance and loading respectively.

3.3 Concept of Dynamic Model

In the development of the reliability of static model considered previously, a single worse loading is imposed on the hydraulic structure. Dynamic models take into account the repeated nature of the random loading imposed on the structure; and also the change of the distribution of resistance with time. Since there is uncertainty about the loading and resistance random variables with respect to time and loading cycles, consideration for time - dependent risk model analysis is justified.

Repeated loadings are characterised by the time each load is applied and the behaviour of time intervals between the application of loads. The occurences may be classified as:

(i) the loads are applied at known times t_1, t_2,..., t_n. In this case, the changes may follow a cyclic pattern such as seasonal changes. The cycle times are known exactly before hand. The load occurrences may be constant or may vary from cycle to cycle. This is referred to as deterministic loading cycle.

(ii) the loads are applied at random times. Thus in this case, the cycle times are random and independent rather than known (deterministic).

The uncertainty about the loading and resistance variables may be classified in the following three categories, Kapur and Lamberson (1977).' (i) deterministic; (ii) random - fixed; and (iii) random - independent. For deterministic category, the variable assumes values that are exactly known a priori. This may suggest that the design process is fully controlled which is not the case for hydraulic structures subjected to hydrologic events. The two terms 'random - fixed' and 'random-independent' are used to describe the two uncertainties in the loading and the resistance random variables at any instant of time; and also about the behaviour of the random variables with respect to time. Random refers to the behaviour of the variable with respect to time and the word 'fixed' means that the behaviour of the random variable varies with time in a known manner. For random - independent category, the variable is not only random but unlike the random - fixed case, the successive values assumed by the variable are statistically independent. Observations of one loading or resistance value gives no information about the subsequent values. In otherwords, it means that the random behaviour of structural resistance is independent of the loading

condition.

The dynamic reliability model consists of such components: the design return period or the magnitude of design event; the service life of the hydraulic structure, a safety factor; and the probability density functions of the resistance as well as the loading.

In dynamic reliability model computations, the objective is to determine the reliability after n cycles or occurrences of loading R_n i.e., the probability of not having a failure on any one of the n cycles or loadings. Because the loadings on many hydraulic structures are basically independent hydrologic events, the random - independent case for loading is considered while the resistance is considered as random - fixed.

The model for deterministic cycles will be developed which will lead into the model for random cycle times.

For deterministically known cycle times the reliability at time t, R(t) is determined from the reliability after n cycles or occurrences of loading, R_n as :

$$R(t) = R_n \; ; \quad t_n \leq t \leq t_{n+1} \qquad \text{— } 3.8$$

where t_n = the instant time at which the nth cycle occurs

and $R_n = R^n$ — 3.9

R_n is given as :

$$R_n = P\left|(Q_{d_1} < Q_c) \cap (Q_{d_2} < Q_c) \cap \cdots \cap (Q_{d_n} < Q_c)\right|$$

$$R_n = P\left|\max(Q_{d_1}, \ldots, Q_{d_n})\ Q_c\right| \quad \text{— 3.10}$$

If $Q_{dmax} = \max(Q_{d_1}, Q_{d_2}, \ldots, Q_{d_n})$, the distribution

$F_n(Q_d)$ of Q_{dmax} from eq. 3.9 is :

$$F_n(Q_d) = \left[F_{Q_d}(Q_d)\right]^n \quad \text{— 3.11}$$

where $F_{Q_d}(Q_d)$ = cumulative distribution of the loadings.

Eq. 3.11 can be solved by integrating the probability distribution function, pdf, of the loadings. For deterministic cycles, the dynamic model is expressed, Kapur and Lamberson (1977) as :

$$R_n = \int_0^\infty f_{\omega_c}(\omega_c)\left[\int_0^{Q_c} f_{\omega_d}(\omega_d)\,d\omega_d\right]^n d\omega_c \quad \text{— 3.12}$$

or

$$\bar{R}_n = \int_0^\infty f_{\omega_c}(\omega_c)\left[\int_0^\infty f_{Q_d}(\omega_d)\,dQ_d\right]^n d\omega_c \quad \text{— 3.13}$$

Using cummulative distribution function of $\int_0^{Q_c} fQ_d(Q_d)dQ_d$ as $FQ_d(Q_c)$;

eq. 3.12 becomes

$$R_n = \int_0^{\infty} f_{Q_c}(Q_c)\left[FQ_d(Q_c)\right]^n dQ_c \qquad — \quad 3.14$$

This reliability determined for n cycles is related to random intervals of time i.e., if the cycle occurs at random times, then

$$R(t) = \sum_{n=0}^{\infty} \Pi_n(t)\, R_n \qquad — \quad 3.15$$

where $\Pi_n(t)$ is the probability of n loadings occuring in the time interval (0, t) and R_n is defined as before. Clearly, the case of deterministic cycle timen is a special case of eq. 3.15.

A poisson distribution is used to describe the probability of the number of events occuring in a given time interval. This distribution is given as

$$\Pi_n(t) = \frac{e^{-\alpha t}(\alpha t)^n}{n\,!} \qquad — \quad 3.16$$

in which α is the parameter equal to the mean occurrence per unit time.

Other distributions could be applicable but they lead to more complicated results. Tung (1985) used a binomial distribution to describe the same process.

If the annual data are used $\alpha = \frac{1}{T_r}$;

where T_r is the return period.

The mean and variance are both at where t can be considered as the expected service life of the hydraulic structure.

For random - fixed resitance and random - independent loading, substituting, eq. 3.14 and 3.16 in 3.15,

$$R(t) = \sum_{n=0}^{\infty} \frac{e^{-\alpha t}(\alpha t)}{n!} \int_0^{\infty} f_{Q_c}(Q_c) \left[F_{Q_d}(Q_c)\right]^n dQ_c \qquad -- 3.17$$

re-arranging 3.17 ;

$$R(t) = \int_0^{\infty} f_{Q_c}(Q_c) \sum_{n=0}^{\infty} e^{-\alpha t} \frac{\left[\alpha t \int_0^{Q_c} f_{Q_d}(Q_d) dQ_d\right]^n dQ_c}{n!} \qquad -- 3.18$$

where $F_{Q_d}(Q_c) = \int_0^{Q_c} f_{Q_d}(Q_d) dQ_d$.

Simplifying eq. 3.18 ;

$$R(t) = \int_0^{\infty} f_{Q_c}(q_c) \exp\left(-\alpha t \left[1 - F_{Q_d}(q_c)\right]\right) dq_c \quad — \quad 3.19$$

3.4 Estimation of Probability Distribution Parameters By First-Order Second Moment Analysis.

The capacity of a hydraulic structure is a function of several components that are considered random variables. By performing first-order second moment analysis the estimates of the mean values and coefficients of variation of the hydraulic structure can be determined.

In general function W, of random variables w_1, w_2,..., w_n whose properties are known can be expressed as :

$$W = f(w_1, w_2, \ldots, w_n) \quad — \quad 3.20$$

The vectors of the expected values and of the standard deviations of w_1, w_2, ..., w_n are denoted respectively as :

$$\mu = (\mu_1, \mu_2, \ldots, \mu_n)$$

$$\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_n)$$

Then by the Taylor's series expansion of eq. 3.20,

$$W = f(\mu_1, \ldots, \mu_n) + \sum_{i=1}^{n} \frac{\partial f(W)}{W_i} \Bigg|_{W=\mu} (w_i - \mu_i)$$

$$+ \frac{1}{2!} \sum_{j=1}^{n} \sum_{i=0}^{n} \frac{\partial^2 f(W)}{\partial W_i \, \partial W_j} \Bigg|_{W=\mu} (W_i - \mu_i)(W_j - \mu_j) + R \qquad — \quad 3.21a$$

where R = remainder.

Taking the expectation of eq. 3.21 ;

$$E(W) = f(\mu_1, \mu_2, \ldots, \mu_n) + \sum_{i=1}^{n} \frac{\partial f(W)}{W_i} \Bigg|_{W=\mu} E(W_i - \mu_i)$$

$$+ \frac{1}{2!} \sum_{j=1}^{n} \sum_{i=1}^{n} \frac{\partial^2 f(W)}{\partial W_i \cdot \partial W_j} \Bigg|_{W=\mu} E(W_i - \mu_i)(W_j - \mu_j)$$

$$+ E(R) \ . \qquad — \quad 3.21b$$

If $w_1, w_2, \ldots, w_n$ are independent random values,

eq. 3.21 becomes :

$$E(W) = f(\mu_1, \ldots, \mu_n) + \frac{1}{2!} \sum_{i=1}^{n} \frac{\partial^2 f(W)}{\partial W_i^2} \Bigg|_{W=\mu} V(W_i) + E(R)$$

— 3.22

where $V(W_i)$ is the variance of the independent variables.

Approximating by dropping the remainder term,

Approximating by dropping the remainder term,

$$E(W) \simeq f(\mu_1, \ldots, \mu_n) + \frac{1}{2!} \sum_{i=1}^{n} \left. \frac{\partial^2 f(W)}{\partial W_i^2} \right|_{W=\mu} \cdot V(W_i) \qquad \text{— 3.23}$$

Ignoring the second term in eq. 3.23

$$E(W) \approx f(\mu_1 \ldots, \mu_n) \qquad \text{— 3.24}$$

but $\mu = \mu_1, \ldots, \mu_n$.

$\therefore$ Eq. 3.24 is re-written as :

$$E(W) \simeq f(\mu) \qquad \text{— 3.25}$$

Now considering only the first two terms in eq. 3.21 and taking the variance, we have :

$$V(W) = V\left(f(W) \Big|_{W=\mu} + V \sum_{i=1}^{n} \left. \frac{\partial f(W)}{\partial W_i} \right|_{W=\mu} \cdot (W_i - \mu_i) \right)$$

$$V(W) = \sum_{i=1}^{n} \left[\left. \frac{\partial f(W)}{\partial W_i} \right|_{W=\mu} \right]^2 \cdot V(W_i) \qquad \text{— 3.26}$$

Assuming that all the w_i values are statistically

independent, eq. 3.26 can be simplified further to :

$$\sigma_W^2 = \sum_{i=1}^{n} \left[\frac{\partial f(W)}{\partial W_i} \Big|_{W=\mu} \right]^2 \sigma_{W_i}^2 \qquad — \quad 3.27$$

using $\Omega_W^2 = \dfrac{V(W)}{[E(w)]^2}$

where Ω_W = coefficient of variation.

$$\therefore \quad v(w) = [E(W)]^2 \, \Omega_W^2 = \sigma_w^2 \qquad — \quad 3.28$$

Sub eq. 3.28 in 3.27 ,

$$\Omega^2_W = \frac{1}{[E(W)]^2} \left(\sum_{i=1}^{n} \left[\frac{\partial f(W)}{\partial W_i} \Big|_{W=\mu} \right]^2 \mu_i^2 \; \Omega^2_{W_i} \right).$$

— 3.29 .

CHAPTER FOUR

A CASE OF OGEE SPILLWAY

4.1 Hydrological Model.

4.1.1 Flood Frequency Analysis.

The primary objectives of flood frequency analysis are to determine the return periods and then to estimate the magnitudes of events for design return periods beyond the recorded range. The intermediary between these two objectives is the theoretical probability distribution. The fitted distribution is then used to estimate event magnitudes corresponding to return periods greater than or less than those of the recorded events.

Maximum floods are accorded a ranking m, starting with m = 1 for the highest value, m = 2 for the next highest value and so on in descending order. The recurrence interval T_r can be calculated by Weibull formula :

$$T_r = \frac{n + 1}{m} \qquad — \quad 4.1$$

where m = event of ranking

n = number of events.

The probability of exceedence P% of an N-year event of return period T_r is expressed as :

$$P = \frac{100}{T_r}\% \qquad \text{— } 4.2$$

To establish the flood - frequency curve, the ranked flood magnitudes are plotted against the plotting positions (exceedence probability or return period) in probability distribution papers. Probability distribution papers provide properly scaled ordinates such that the cummulative distribution function of the probability law plots as straight lines.

4.2 Types of Probability Distribution for Maximum Flood Flows

There are many theoretical distributions used in hydrology to fit maximum flood events. Some of which are Log - Pearson Type III distribution, Lognormal distribution, and extreme value Type 1 distribution (Gumbel distribution).

KASHIM IBRAHIM LIBRARY
AHMADU BELLO UNIVERSITY
ZARIA, NIGERIA.

4.2.1 Log-Pearson Type III Distribution

This distribution involves the transformation of random variable x as a Pearson Type III from natural units to logarithmic units. The probability distribution function of the Log-Pearson

Type IIIdistribution is given as :

$$P(X) = \frac{1}{x\Gamma(\beta)}\left[\left[\frac{\ln x - \gamma}{\alpha}\right]^{\beta-1} \exp\left(\frac{\ln x - \gamma}{\alpha}\right)\right] \quad -- \quad 4.3$$

where: $\Gamma(\beta)$ is gamma function

where α , β , and γ are the scale, shape, location parameter resp.

The parameters of the distribution can be expressed, for instance, as :

$$\beta = \frac{1}{n\alpha} \sum_{i=1}^{n} (\ln X_i - \alpha).$$

$$\alpha = \frac{1}{n\Psi(\beta)} \sum_{i=1}^{n} (\ln X_i - \gamma)$$

where $\Psi( )$ is the psi function.

$$n = \alpha (\beta - 1) \sum_{i=1}^{n} \left[\frac{1}{(\ln X_i - \gamma)} \right].$$

4.2.2 Lognormal Distribution.

This distribution involves the transformation of random variable from natural units to logarithmic units. The transformed variable X which is normally distributed is ln X.

The probability density function pdf is given as :

$$P(X) = \frac{1}{X\sqrt{2\Pi}\,\sigma_n} \exp\left[-\frac{1}{2}\left(\frac{\ln X - \mu_n}{\sigma_n}\right)^2\right] \quad — \quad 4.4$$

where μ_n = Mean of the transformed random variable ln X

σ_n = Standard deviation of the transformed random variable ln X.

The justification of this distribution is obvious in model fitting since some hydrologic variables exhibit a marked right skewness, partly due to the influence of natural phenomena values greater than zero, or some other limits, are being unconstrained, theoretically in the upper range. In such cases, frequencies will not follow the normal distribution but logarithmic. The parameter, of the distribution can be expressed as

$$\mu = \exp\left[\mu_n + \frac{\sigma_n^2}{2}\right]$$

$$\sigma = \exp\left[2\mu_n + \sigma_n^2\right]\left[\exp(\sigma_n^2) - 1\right] .$$

4.2.3 Extreme Type 1 (Gumbel) Distribution.

The exponential probability function of largest values fits asymptotically the distributions of annual maximum flood

events.

The probability density function of this distribution is expressed as :

$$P(X) = \frac{1}{\alpha_2} \exp\left[- \left(\frac{X - \alpha_1}{\alpha_2}\right) - \exp\left(-\left(\frac{X - \alpha_1}{\alpha_2}\right)\right)\right] \quad \text{— 4.5}$$

where α_1 is a measure of central tendency, and α_2 is a concentration parameter.

The parameters of he distribution can be expressed as :

$\alpha_1 = \bar{X} - \gamma\alpha_2$ and $\alpha_2 = (\frac{\sqrt{6}}{\pi})\sigma$; where $\bar{X}$ is the sample mean, and γ is the Euler's constant, and σ is the sample standard deviation.

4.3 Hydrologic Model Verification

To test the validity of any particular model selected or chosen, quantitative rules that can lead to acceptance or rejection of the hypothesised model are applied. Goodness - of fit and construction of confidence band are essential tools for the validity test. Kolmogorov - Smirnov (K-S) test, and Chi-square test are some of the Goodness - of fit tests. Only the theory of Kolmogorov - Smirnov test is presented and applied in this study. The K - S test does not require

lumping of data, and comparing discrete categories, but rather compares all the data in an unaltered form, Benjamin and Cornell (1970).

4.3.1 Kolmogorov - Smirnov Test.

This test concentrates on the deviations between the hypothesised cummulative distribution function $F(Q_i)$ and the observed cummulative histogram ;

$$F^*(Q_i) = \frac{i}{n}$$

where Q_i = the ith largest observed value in the random sample of size n

i = the largest observed value

n = sample size.

To perform the K - S test, the statistic used is :

$$D_2 = \max_{i=1}^{n} F^*(Q_i) - F(Q_i) \qquad \text{— 4.7}$$

The hypothesis at a prescribed significance level α is stated as :

H_o : Q_i has a specified distribution

H_1 : the distribution of Q_i is other than that specified.

The form of operating rule is :

Accept the null hypothesis H_o if $D_2 \leq C$ where C = critical value.

If the largest of the observed K - S test statistic is greater than the critical value at a chosen level of significance, the null hypothesis is rejected.

4.3.2 Confidence Band

Confidence band is constructed for the fitted distribution around the flood - frequency curves to determine the reliability of points representing the annual maximum discharge X_{max} for a recurrence interval T_r. The plotted positions obtained from the line through the set of points in a sample, serves as the best estimate of the true position. But there is a possibility of sampling error, and the true position might be above or below i.e., there is a chance that an error has been made in estimating the discharge of a flood with a recurrence interval of T_r.

It is usually assumed that such sampling errors are normally distributed about the mean flood frequency curve. With this assumption, an interval $\pm$ X_{max} can be drawn within there is a probability of say P% that the true position of X_{max} lies. If this is computed for selected points and curves

are drawn, bands are defined within which there is a 1% certainty that the true discharge for a chosen probability of exceedence lies.

Approximate control curves which can be placed on frequency curves was proposed by Beard (1962). The method involved placing lines above and below the fitted curve to form a reliability band. He published coefficients for various levels of confidence which should be multiplied by the standard deviation and then added or subtracted from the discharges of the flood frequency curves for various percentage of exceedence at a given level of significance.

4.4 Hydraulics of Ogee Type Spillway.

An overflow spillway is a section of the dam designed to release excess water over the crest of the spillway. The overflow (Ogee) spillway discharge capacity is expressed by the equation

$$Q_c = C\, L_n\, H^{3/2} \quad m^3/sec \qquad — \ 4.8$$

where Q_c = spillway discharge capacity ;

C = coefficient of discharge ($m^{-1/2} s^{-1}$);

L_n = effective crest length between gate bays and it is expressed as :

$$L_n = L_t - 2(Nk_p + k_a)H \quad \text{—} \quad 4.9$$

where :

L_t = total clear length of crest between abutments and piers,

N = number of piers,

K_p = pier contraction coefficient ;

K_a = abutment contraction coefficient,

H = total head on the crest, Fig. 4.1

H is expressed as $H_D + H_v$ (m) — 4.10

where H_D = design head on the crest (m)

H_v = velocity head (m) in the approach channel and it is expressed as :

$$H_v = \frac{V^2}{2g} \quad \text{—} \quad 4.11$$

where v = velocity of water in the approach channel.

g = acceleration due to gravity.

h = height of spillway.

The ideal longitudinal profile of an overflow spillway follow along the same curve as the underside of the free- falling water nappe (Fig. 4.1) to minimise the pressure on the spillway surface. However caution must be taken to avoid negative pressure

on the surface which can result to damages.

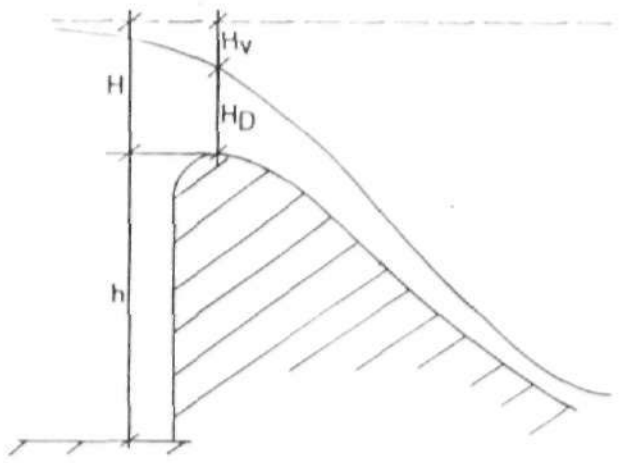

Fig. 4.1 Longitudinal Profile of the Spillway.

$$Q_c = C(L_t - 2(Nk_p + k_a)H)H^{3/2} \quad — \quad 4.12$$

Since the hydraulic conditions for flow over a spillway are complicated and highly unsteady, a model error λ_m is incorporated into the discharge formula and each variable of the formula can be treated as statistically independent random variable. Thus Eq. 4.12 is now re-written as :

$$Q_c = \lambda_m C(L_t - 2(Nk_p + k_a)H)H^{3/2} \quad — \quad 4.13$$

4.4.1 Spillway Discharge Coefficient

The discharge coefficient C is influenced by a number of factors. (i) velocity of approach (depth of approach); (ii) relation of the actual crest shape to the ideal nappe shape, (iii) the upstream faceslope, and (iv) the downstream apron interference and submergence, Chow (1959).

Depth of Approach.

For a high sharp - crested weir placed in a channel,

the velocity of approach is small and the underside of the nappe' flowing over the weir attains maximum vertical contraction. As the approach depth is decreased, the velocity of approach increases and the vertical contraction diminishes. For sharp - crested weirs whose heights are not less than about one - fifth the heads producing flows over them, C remains fairly constant although the contraction diminishes. For weir heights less than about one - fifth of the head, the contraction of the flow becomes increasingly suppressed and the crest coefficient decreases.

If the sharp crested weir coefficients are related to the head measured from the point of maximum contraction instead of the head above the crest, C application to profile of under nappe' for various approach velocities can be established Chow (1959), and USBR (1974). The velocity of approach is zero or $H_v = 0$ if the height of spillway is greater than $1.33\ H_D$, Chow (1959)

Effect of Heads differing from Design Head

When the crest has been shaped for a head larger or smaller than the one under consideration, coefficient of

discharge will differ from the design coefficient of discharge, C_D. Design coefficient of discharge is the coefficient of discharge at the design head, H_D. A widened shape will result in positive pressures along the crest contact surface, thereby reducing the discharge, with a narrow crest shape, negative pressures along the contact surface will occur, resulting in an increased discharge.

The variation of $\frac{C}{C_D}$ as related to head ratio, $H_r = \frac{H}{H_D}$ are obtainable from Chow (1959), Creager (1964), and USBR (1974).

Effect of Upstream Face Shape.

For small ratios of the approach depth to head on the crest, sloping the upstream face of the overflow results in an increase in discharge coefficient. For large ratios the effect is a decrease in discharge coefficient. Values of C for the equation applicable to spillways having sloping ustreams with or without velocity of approach are obtainable from the references quoted above.

Effect of Downstream Apron Interference and Submergence.

When the water level below an overflow weir is high enough to affect the discharge, the weir is said to be submerged. The vertical distance from the crest of the overflow to the downstream apron and the depth of flow in the downstream channel, as it relates to the head pool level, are factors which alter the coefficients of discharge. Fig. 4.2 shows the apron and tail water depth, d

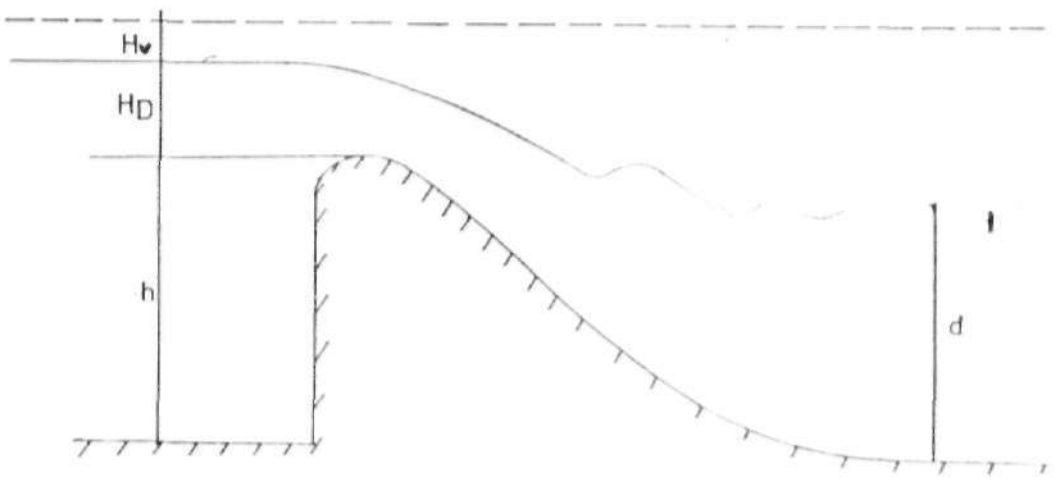

Fig. 4.2 Spillway Longitudinal profile showing tail water depth .

The USBR (1974) has classified the flow in submerged round crested weir into five distinct types according to the flow condition prevalent in the downstream apron: (i) flow will continue at supercritical stage; (ii) a partial or incomplete hydraulic jump will occur immediately downstream from the crest;

(iii) a true hydraulic jump will occur (iv) a drowned jump will occur in which the high velocity jet will follow the face of over flow and then continue in an erratic and fluctuating path for a considerable distance under and through the slower water; and (v) no jump will occur the jet will break away from the face of the overflow and ride along the surface for a short distance and then erratically intermingle with the slow moving water underneath. The relationship of the floor position and downstream submergence which provides these distinctive flows is obtainable from USBR (1974).

Where the downstream flow is at supercritical stage or where the hydraulic jump occurs, the decrease in coefficient of discharge due partially to the back - pressure effect of the downstream apron and is independent of any submergence effect due to tail - water.

Submergence of spillway will reduce the coefficient of the discharge of the corresponding unsubmerged flow. The percentage of reduction is published in references quoted earlier on.

4.4.2 Contraction Coefficients

In gated spillways, piers are needed to form the sides

of the gates. The effect of the piers is to contract the flow, hence, to alter the effective crest length L_n of the spillway.

Pier Contraction Coefficients.

Contraction coefficients of piers differ depending on the shapes of the piers, position of the pier nose, approach depth of flow, and the operation of the adjacent gates. The approximate values given by Creager (1964) ranges from 0.1 for thick, blunt noses to 0.04 for thin or pointed noses and 0.035 for round noses. These values apply to piers having a thickness equal to about one-third the head on the crest when all gates are opened. When one gate is open and the adjacent gates are closed, these values become roughly 2.5 times larger.

The US Army Engineers Waterways Experiment Station has conducted tests on different forms of pier nose for spillways. The effect of nose - shape other than round-nose shape on contraction coefficient is shown. This shows the variation of k_p with $\frac{H}{H_D}$, (Chow, 1957)

Abutment Contraction Coefficient

Coefficients of contraction for abutments are published

in USBR, (1974) for various types of abutment :

k_a = 0.20 for square abutments with head wall at 90°

k_a = 0.10 for rounded abutments with $H_D \leq r \leq 0.50\ H_D$; where r is the radius of abutment.

k_a = 0 for rounded abutment $r > 0.5\ H_D$ and headwall is placed not more than 45° to the direction of flow.

4.5 Hydraulic Model (For Computation of Probability Distribution Parameters).

The probability distribution of flood conveyance structure is assumed to be lognormal, Tung (1980).

The probability distribution function(pdf)of lognormal distribution as given before is :

$$P(X) = \frac{1}{X\sqrt{2\Pi}\sigma_n} \exp\left[-\frac{1}{2}\left(\frac{\ln X - \mu_n}{\sigma_n}\right)^2\right].$$

Using the spillway capacity variable Q_c in place of X in the above equation.

$$fQ_c(Q_c) = \frac{1}{\sqrt{2\Pi}Q_c\,\sigma_n} \cdot \exp\left[-\frac{1}{2}\left(\frac{\ln Q_c - \mu_n}{\sigma_n}\right)^2\right] \quad \text{— 4.14}$$

The parameters of the distribution are :

μ_n = means of the transformed random variable ln Q_c

σ_n = standard deviations of the transformed random variable ln Q_c.

By the method of moments, μ_n and σ_n can be expressed for instance Benjamin and Cornell (1977) as :

$$\mu_n = \ln \bar{Q}_c - \frac{1}{2}\sigma_n^2 \qquad \text{— 4.15}$$

$$\sigma_n = \left[\ln(\Omega_{Q_c}^2 + 1)\right]^{\frac{1}{2}} \qquad \text{— 4.16}$$

where $\bar{Q}_c$ = mean capacity of the spillway

Ω_{Qc} = coefficient of variation of the spillway capacity.

CHAPTER FIVE

5.0 Application of the Risk Theory to Dadin - Kowa Dam

In this chapter, the general features of Dadin - Kowa dam are highlighted, and the application of reliability theory to the computation of composite risk for static and dynamic models are considered. Hydrologic risk is computed and compared with composite risk.

5.1 General Features of the Dam

The Dadin - Kowa dam is located 5 km north of Dadin - Kowa, village near Gombe, on River Gongola in Bauchi State, Fig. 5.1. The reservoir extends from the dam site north - wards for a distance of about 65 km, approximately to where the existing Borno extension railway line to Maiduguri crosses the Gongola River in the vicinity of Ashaka.

The main dam has a crest length of 520m with a base 230m wide and a dam top width of 8m.

The catchment area of the Gongola river is approximately 56,000 km^2, 58.5% of which lies upstream of the dam.

The surface area of the reservoir is 300 km^2 with a

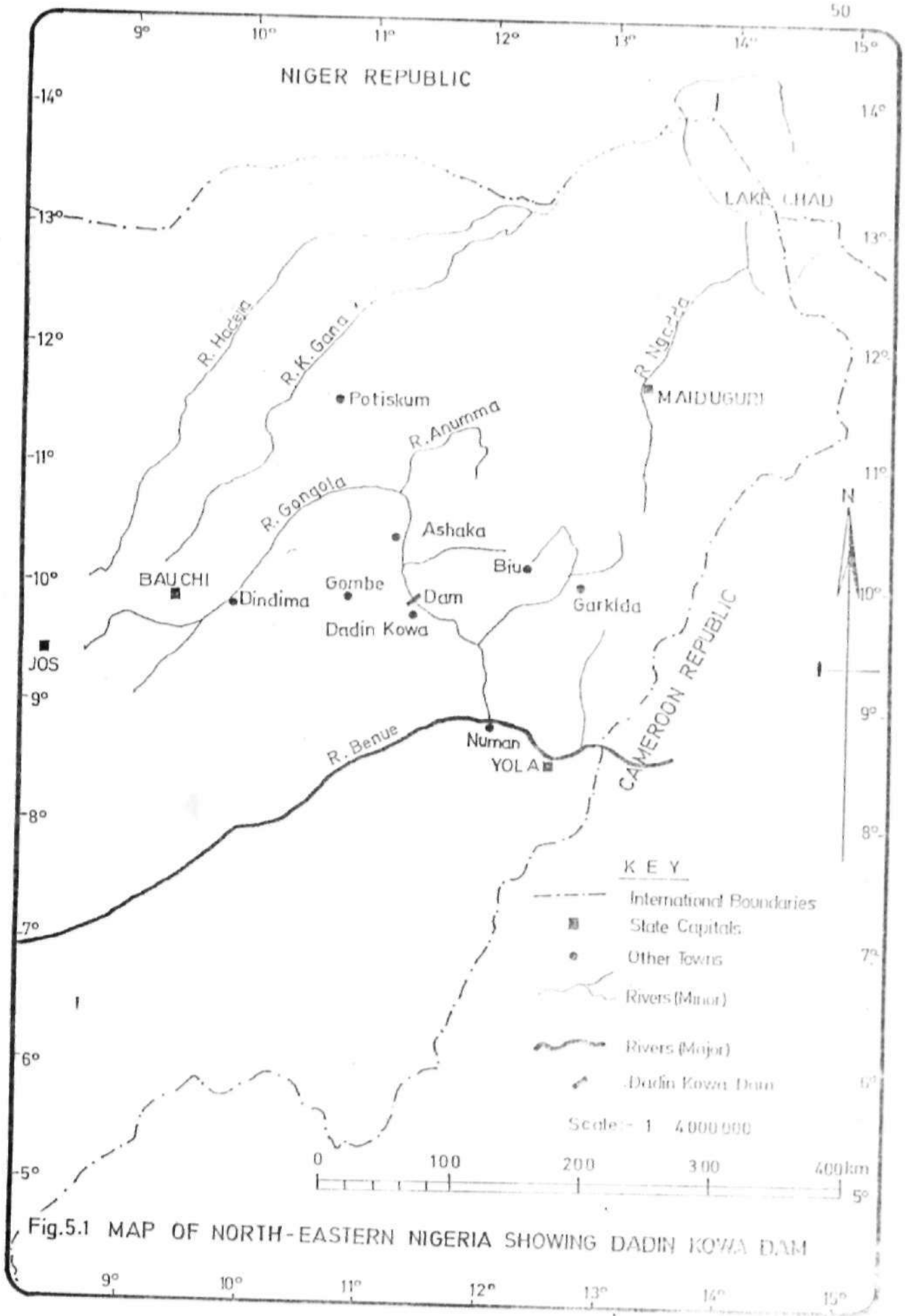

Fig.5.1 MAP OF NORTH-EASTERN NIGERIA SHOWING DADIN KOWA DAM

live storage of 1.77 billion m^3 of water. The reservoir under normal operating condition will fluctuate between low supply elevation 239.0m and full supply elevation 247.0m with occasional flood at elevation 249.0m.

5.1.1 Purpose.

The project as conceived is capable of providing an almost total control of Gongola River at Dadin - Kowa for benefit of :

(a) irrigated agriculture of about 44,000 hectres of fertile land downstream of Dadin - Kowa and upstream of the Kiri reservoir.

(b) hydro - electric power generation. It has two turbines with capacity 17 MW each.

(c) facilities for fishing and fisheries development,

(d) flood control and flow regulation.

(e) domestic and industrial water supply to Gombe urban area as well as rural areas around the dam, and

(f) recreation and other socio-economic benefits.

5.1.2 Spillway

The spillway crest section includes three radial gates 6m x 8 m high each seperated by two 2 m wide piers.

Fig. 5.2 shows the longitudinal profile of the spillway. The maximum discharge of the spillway is 1100 m^3/s at reservoir maximum flood level elevation (EL 249 asl) responding to the head. The design head of the spillway is 10 m and its total width is 22.0 m. The height of the spillway is 15.0 m.

5.2 Hydrological Model for Gongola River at Dadin- Kowa.

5.2.1 Flood Frequency Analysis

Annual maximum flow data for River Gongola at Dadin-Kowa for the period of 1956 - 1987 will be used for the hydrologic model. The source of the data is the record kept by the Hydrologic Division of Inland Water ways, Federal Ministry of Transport, Lokoja.

The annual maximum floods are ranked starting with m = 1 for the highest value and so on in descending order. The probability of exceedence which is used as the plotting position is then determined using

$$P(X) = \frac{m}{n+1}$$

where P = probability of exceedence

m = order of the rank

n = sample size of the ungrouped data.

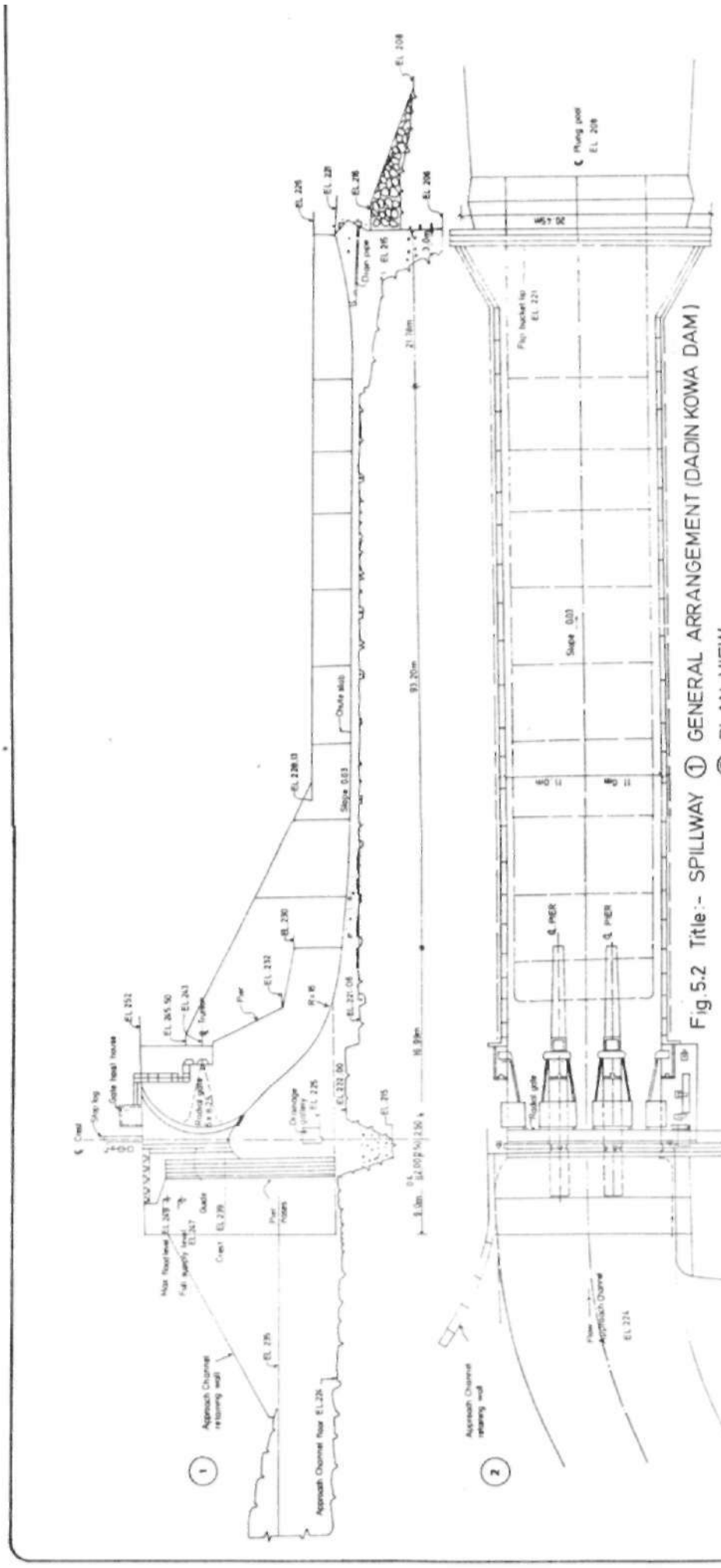

Fig.5.2 Title:- SPILLWAY

For the plotting position of eq. 5.1, the return period is

$$T_r = \frac{n + 1}{m} \qquad — 5.3$$

The result of the computations is shown in Table 5.1

Table 5.1: Result of the Flood Frequency Analysis for River Gongola at Jadin - Kowa. 5.1

Year of occurrence	Q_{d_i} m^3/sec	Order of Rank m	Return period T_r	Prob. of exceedence P_r
1981	950	1	32.00	0.03
1983	910	2	16.00	0.06
1984	760	3	10.60	0.94
1964	723	4	8.00	0.13
1985	700	5	6.40	0.16
1975	663	6	5.30	0.19
1974	620	7	4.60	0.22
1956	588	8	4.00	0.25
1958	579	9	3.60	0.28
1962	578	10	3.20	0.31
1961	555	11	2.90	0.34
1980	545	12	2.70	0.38
1982	540	13	2.46	0.41
1967	517	14	2.29	0.44
1970	495	15	2.13	0.47
1965	485	16	2.00	0.50

Table 5.1 (Contd.)

Year of occurrence	Q_{d_I}	Order of Rank m	Return period T_r	Prob. of exceedence
1971	482	17	1.88	0.53
1973	476	18	1.78	0.56
1969	457	19	1.68	0.59
1976	437	20	1.60	0.63
1957	436	21	1.52	0.66
1959	362	22	1.45	0.69
1979	360	23	1.39	0.72
1962	334	24	1.33	0.75
1960	328	25	1.28	0.78
1963	323	26	1.23	0.81
1983	320	27	1.19	0.84
1972	310	28	1.14	0.88
1966	299	29	1.10	0.91
1984	180	30	1.07	0.94
1987	175	31	1.03	0.97

KASHIM IBRAHIM LIBRARY
AHMADU BELLO UNIVERSITY
ZARIA, NIGERIA.

The mean discharge and the standard deviations of the flood are then computed which will be used to estimate the parameters of the fitted distribution.

The mean discharge

$$\bar{Q}_d = \frac{1}{n} \sum_{i=1}^{n} Q_{d_i} .$$

where n = number of years record

$$\bar{Q}_d = \frac{15492}{31}$$

$$\bar{Q}_d = 499.74 \ m^3/sec.$$

The sample standard deviation

$$S = \left[\frac{1}{n-1} (\Sigma Q_{d_i} - \bar{Q}_d)^2 \right]^{1/2}$$

$$S = 187.14 .$$

Using the values of Table 5.1, the ranked flood magnitudes are plotted on probability distribution papers using the plotting positions. Three distributions are selected to fit the flood magnitudes. Fig. 5.3a, 3 b, and 3 c show the frequency curves for the three distributions. The curves are fitted by visual straight line method. (Yevjevich, 1978).

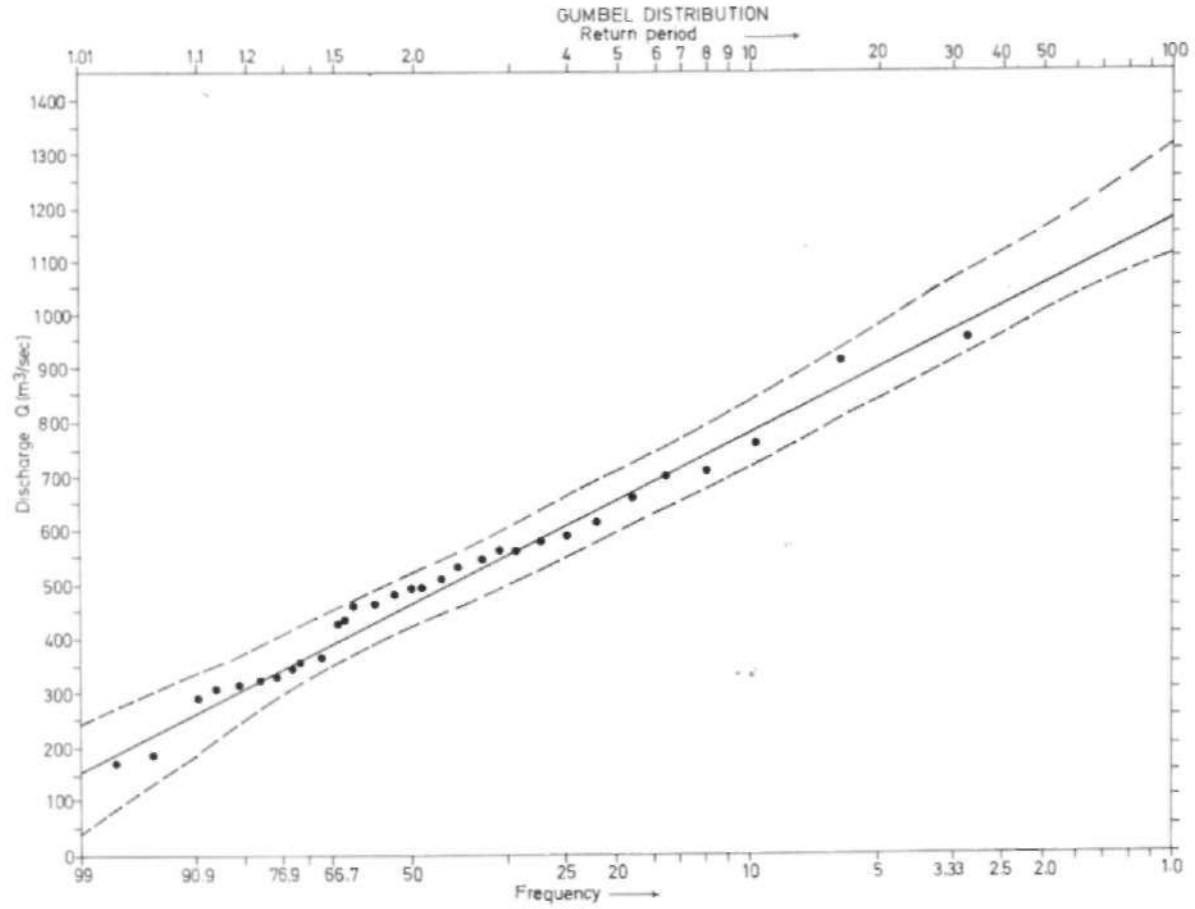

Fig. 5.3a Flood frequency curve for annual flood values of R. Gongola at Dadin-Kowa, Bauchi State. Dashed lines show zone within which there is 90% chance that the true value for that occurence interval lies.

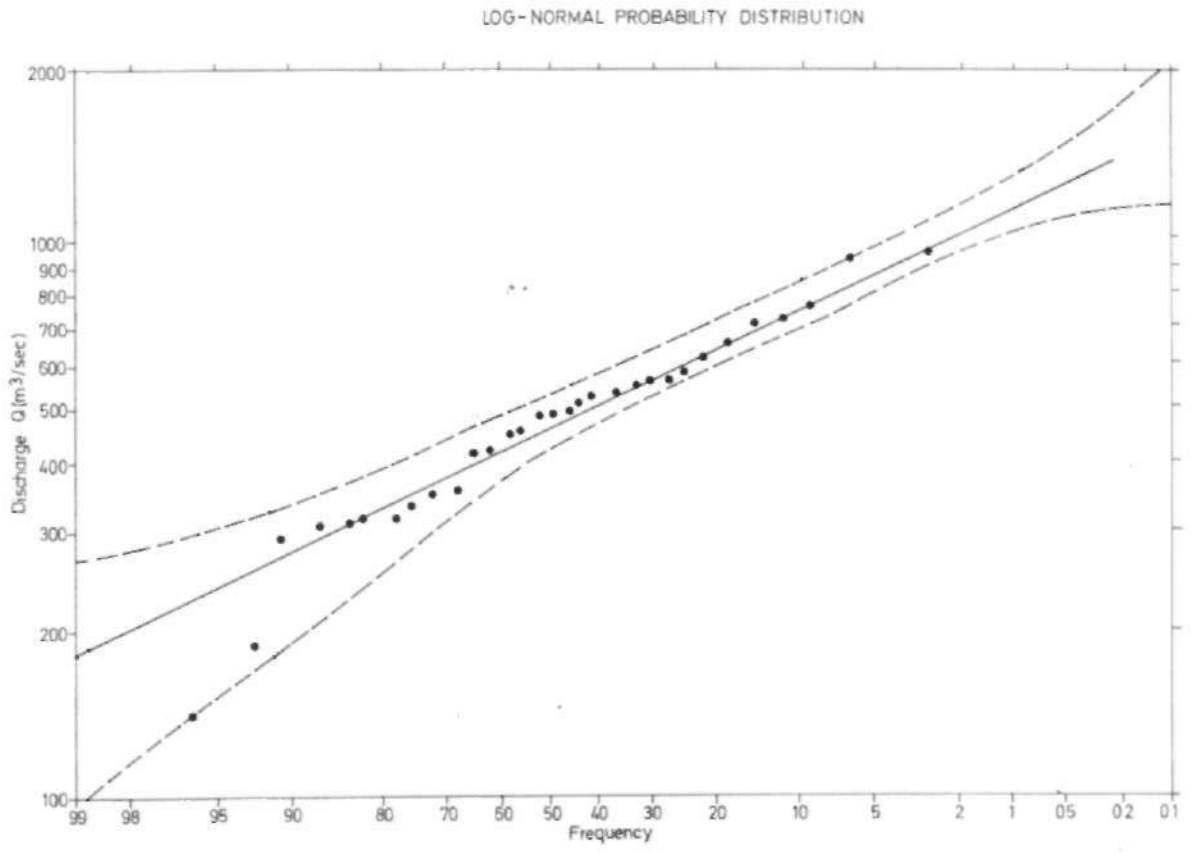

Fig. 5.3b Flood frequency curve for annual flood values of River Gongola at Dadin-Kowa, Bauchi State. Dashed lines show zone within which there is 90% chance that the true value for that occurrence interval lies

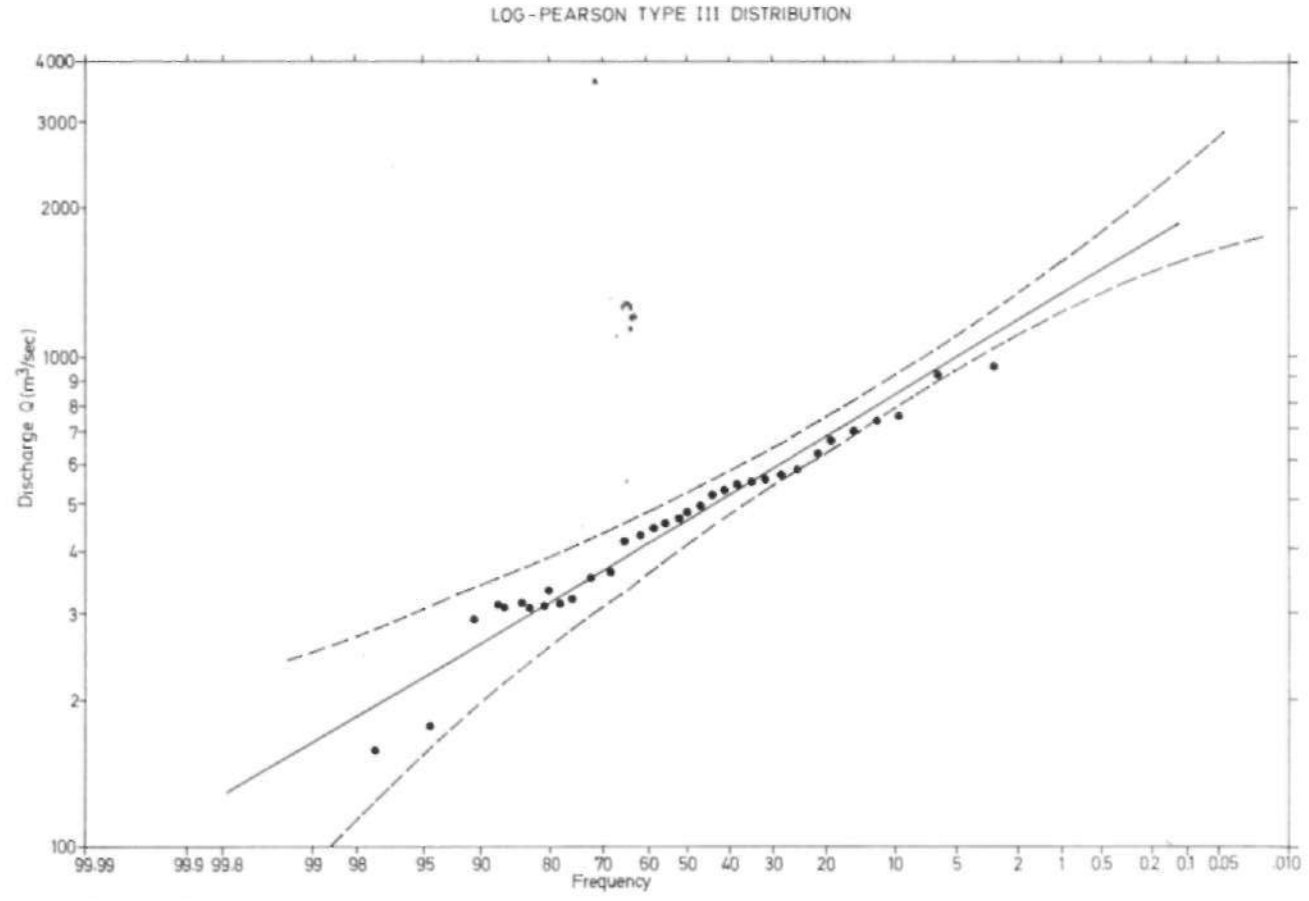

Fig. 5.3c Flood frequency curve for annual flood values of River Gongola at Dadin-Kowa, Bauchi State. Dashed lines show zone within which there is 90% chance that the true value for that occurrence interval lies.

5.2.2 Hydrologic Model Verification.

Among the three fitted curves, it is required to select the one that best fits the flood magnitudes. Kolmogorov - Smirnov Test is performed for the models verification

1 Kolmogorov - Smirnov Test.

The test statistic is given as in eq. 4.7

$$D_2 = \max_{i=1}^{n} \left[\left| F^*(Q_{d_i}) - F(Q_{d_i}) \right| \right].$$

$F^*(Q_i)$ is the observed cummulative freq. which is $\frac{1}{T_r}$; and $F(Q_{d_i})$ is the theoretical cummulative frequency which is obtained from the fitted curves.

The hypothesis test is :

H_o : Q_{d_i} has a specific distribution.

H_1 : the distribution of Qd_i is not the one specified.

Accept the null hypothesis H_o if D_2 is less than critical value C, otherwise reject it.

The values of $\left| F^*(Q_{d_1} - F(Q_i) \right|$ for the three distributions are shown in Table 5.2.

Table 5.2: Result of the Kolmogornov-Smirnov Test for the Three Selected Distributions.

Observed cumu. Freq. $F^*(q_{d_i})=\frac{1}{T_r}$	Theoretical cummu. Freq. ($F(q_{d_i})$			$\lvert F^*(q_{d_i}) - F(Q_{d_i}) \rvert$		
	Gumbel	Lognorm.	Log-Pear	Gumbel	Lognorm.	Log-Pears
0.030	0.038	0.031	0.06	0.008	0.01	0.03
0.060	0.041	0.044	0.075	0.018	0.016	0.015
0.094	0.110	0.94	0.15	0.016	0.00	0.056
0.13	0.140	0.13	0.18	0.01	0.00	0.05
0.16	0.150	0.151	0.20	0.01	0.01	0.04
0.19	0.180	0.19	0.22	0.01	0.00	0.03
0.22	0.25	0.22	0.27	0.03	0.00	0.05
0.25	0.29	0.26	0.30	0.04	0.01	0.05
0.28	0.281	0.25	0.32	0.001	0.01	0.04
0.31	0.311	0.31	0.34	0.001	0.00	0.03
0.34	0.345	0.34	0.36	0.005	0.00	0.02
0.38	0.378	0.36	0.38	0.002	0.02	0.00
0.41	0.390	0.40	0.41	0.01	0.01	0.00
0.44	0.42	0.42	0.43	0.02	0.02	0.01
0.47	0.44	0.45	0.46	0.03	0.02	0.01
0.50	0.48	0.460	0.51	0.02	0.04	0.01
0.53	0.49	0.50	0.53	0.04	0.03	0.00
0.56	0.53	0.52	0.56	0.03	0.04	0.00
0.59	0.56	0.54	0.57	0.03	0.05	0.02
0.63	0.55	0.59	0.59	0.050	0.04	0.04
0.66	0.64	0.66	0.63	0.02	0.05	0.03
0.69	0.70	0.72	0.70	0.01	0.03	0.01
0.72	0.72	0.73	0.72	0.00	0.01	0.00
0.75	0.76	0.78	0.79	0.01	0.03	0.04
0.78	0.79	0.80	0.81	0.01	0.02	0.02

Table 5.2 (Contd.)

Observed cumu.Freq $F^*(q_{d_i}) = \frac{1}{T_r}$	Theoretical cummu. Freq. $(F(q_{d_i})$			$\lvert F^*(q_{d_i}) - F(q_{d_i}) \rvert$		
	Gumbel	Lognorm.	Log-Pear	Gumbel	Lognorm.	Log-Pearson
0.81	0.81	0.82	0.82	0.00	0.01	0.01
0.84	0.82	0.83	0.83	0.02	0.01	0.01
0.88	0.84	0.84	0.84	0.04	0.04	0.04
0.91	0.87	0.88	0.855	0.04	0.03	0.055
0.94	0.97	0.987	0.986	0.03	0.053	0.046
0.97	0.98	0.99	0.992	0.01	0.02	0.02

The critical values C is obtained from K - S table (Yevjevich, 1978) at a level of significance $\alpha = 0.10$ as 0.219.

The maximum deviations D_2 for the three distributions are shown below :

Distribution	Max. Dev. D_2	Critical Value
Gumbel	0.050	0.219
Lognormal	0.053	0.219
Log-Pearson	0.056	0.219

Since the maximum deviation for each of the model is less than the critical value, the null hypothesis for each model is accepted which means that the three distributions fit the annual maximum flood of River- Gongola at Dadin - Kowa.

Confidence Band

Confidence band is constructed by multiplying the sample standard deviation and the coefficient of standard deviation for upper and lower bands at significance level $\alpha = 0.10$, Beard (1962). He published the coefficients of standard deviation as a function of percentage probability and the number of year records.

Sample Standard Deviation S = 187.14

Table 5.3 Table of flood values for the construction of confidence band.

	Percentage Probability P%				
	1.0	10	50	90.9	99
Coefficient of S.D. for Upper Band	0.70	0.47	0.29	0.33	0.45
Coefficient of S.D.for lower Band	-0.45	-0.34	-0.30	-0.48	-0.71

	Deviations of Discharge (m^3/sec)				
Upper Band	131	88	51	62	84
Lower Band	-84	-62	- 51	- 88	-131

$Q(m^3/sec)$

	P%	Gumbel	Lognormal	Log-Pearson III
Upper Band	1.0	1306	1306	1301
	10	868	838	938
	50	521	526	511
	90.9	332	332	337
	99	244	269	254
Lower Band	1.0	1086	1086	1186
	10	688	688	788
	50	424	424	409
	90.9	182	182	187
	99	30	54	39

The dashed lines in Fig. 5.3a, 5.3b, and 5.3c show the confidence band constructed for the three distributions.

For the Gumbel distribution, all the flood magnitudes fall within the band. For the lognormal distribution, all the flood values fall within the confidence band except one which lies on the lower band. And finally, for log-Pearson type III distributions, two points fall out of the band while three lie on the lower band.

Gumbel distribution is finally selected since all the flood values fall within the band. It is considered the best fit relative to the lognormal and log-Pearson distributions.

The probability density function of the flood magnitude is expressed as in eq. 4.5.

$$f(Q_d) = \frac{1}{\alpha_2} \exp\left[-\left(\frac{Q_d - \alpha_1}{\alpha_2}\right) - \exp\left(-\frac{Q_d - \alpha_1}{\alpha_2}\right)\right]$$

The parameters of the distribution α_1 and α_2 are given as :

$$\alpha_1 = \bar{Q}_d - \gamma\,\alpha_2 \; ; \quad \alpha_2 = \left(\frac{\sqrt{6}}{\pi}\right)\sigma_{Q_d}$$

$\bar{Q}_d$ = 499.74 ; σ_{Q_d} = 187.14 ; γ = Euler's constant

= 0.5772.

Thus $\alpha_2 = \frac{\sqrt{6}}{\pi} \cdot 187.14 = 145.9$.

$\alpha_1 = 499.74 - 0.5772\ (145.9) = 415.52.$

5.3 Analysis of Hydraulic Uncertainties and Estimation of Selected Probability Distribution Parameters

5.3.1 Hydraulic Model Equations.

The hydraulic model equation is given as in e.g. 4.13

$$q_c = \lambda_m\, c(l_t - 2(Nk_p + k_a)H)H^{3/2} .$$

Coefficient of Discharge Variable .

To compute coefficient of discharge, Chow (1959) provided a curve, Fig. 5.4 $\frac{c}{C_D}$ V_s $\frac{H}{H_D}$ for standard vertical face upstream, where :

C = Coefficient of discharge for head H other than the design head H_D.

C_D = design coefficient of discharge.

If the height of spillway h is greater than 1.33 H_D Chow (1959), the effect of approach velocity on the coefficient of discharge is negligible. The height of spillway h for Badin - Kowa Dam is 15.0 m. The design head = 10.0 m.

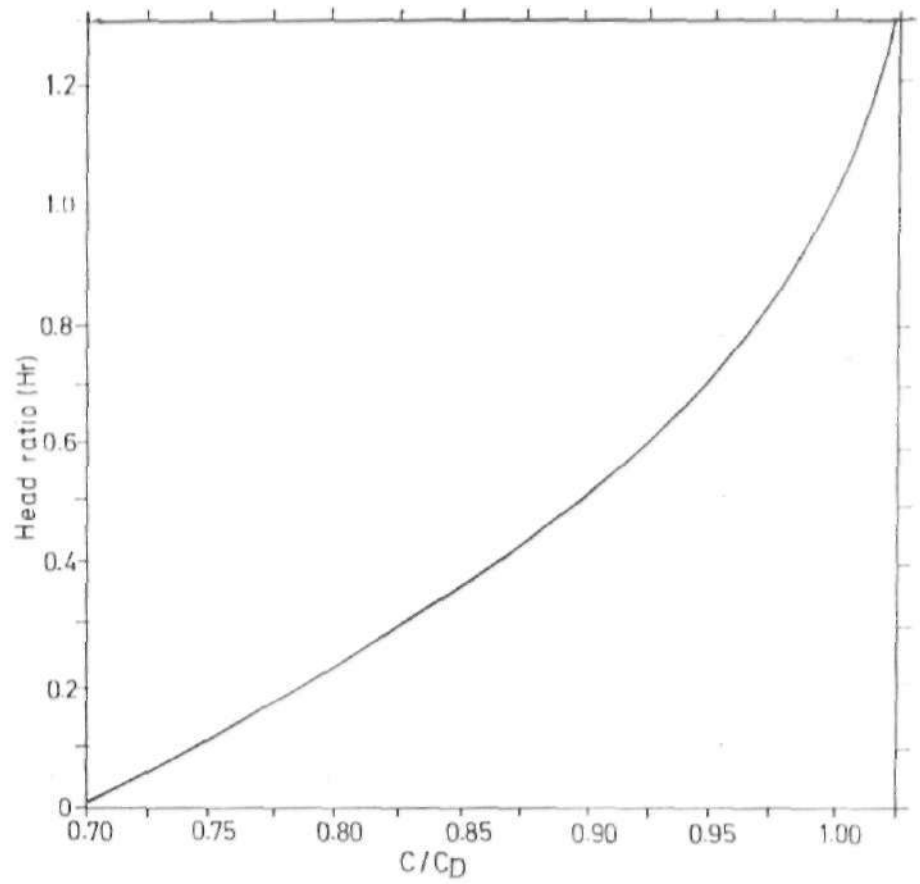

Fig. 5.4 Head-discharge Relation for Standard Spillway Shape for Negligible Depth of Approach. (Chow, 1959)

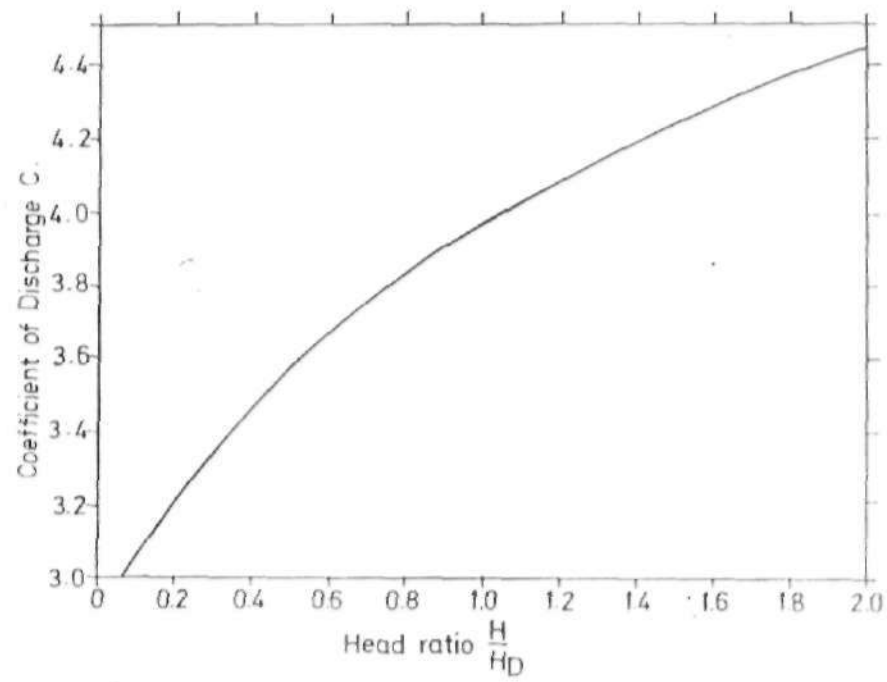

Fig 5.5 Coefficients of Discharge for Straight Standard dam crests with no Velocity of Approach. (Creager, 1964)

Since $h \geq 1.33\,(10) = 13.3$ m, the effect of upstream depth of approach on discharge coefficient is negligible.

For this condition, C_D is given by Chow (1959) as 2.21, C can then be computed from $C = C_D(Hr)$ — 5.1

where C_D is as defined before; H_r = head ratio H/H_D.

Similar curve for the computation of coefficient of discharge for head other than the design head is given by Creager (1964). Fig. 5.5, for negligible depth of approach and for a standard vertical upstream face.

The value of C_D given by Creager (1964) is 2.20

C is then computed from eq. 5.1

The USBR (1974) provided a similar curve for the computation of C, Fig. 5.7, for negligible depth of approach and standard vertical upstream face. The C_D obtained from the curve Fig. 5.6, is 2.16. C is again computed from eq. 5.1.

From the spillway rating curve, Fig. 5.8 obtained from Dadin - Kowa, C_D is 2.20.

The result of the computation of coefficient of discharge C obtained from the above sources are shown in Table 5.4

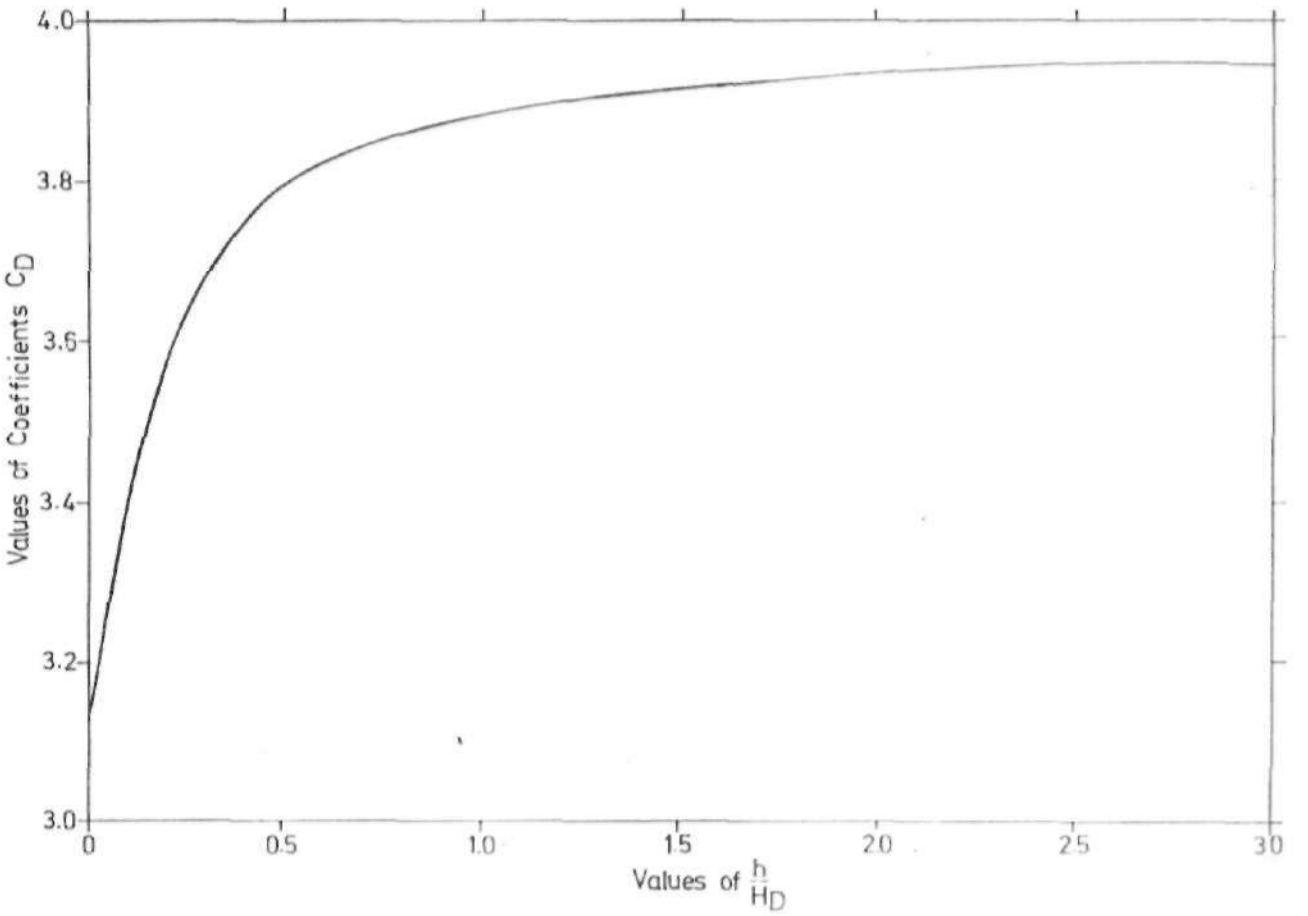

Fig. 5.6 Discharge Coefficients for Vertical faced Ideal crest (USBR, 1974)

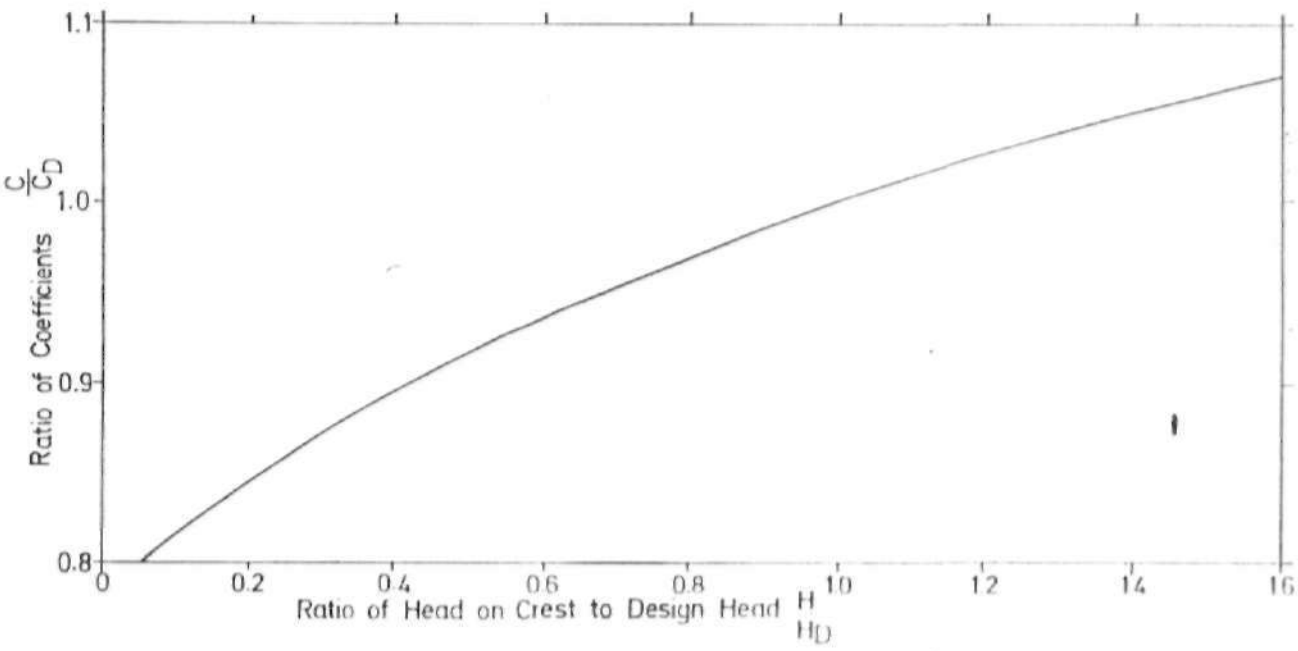

Fig 5.7 Coefficients of Discharge other than Design Head (USBR, 1974)

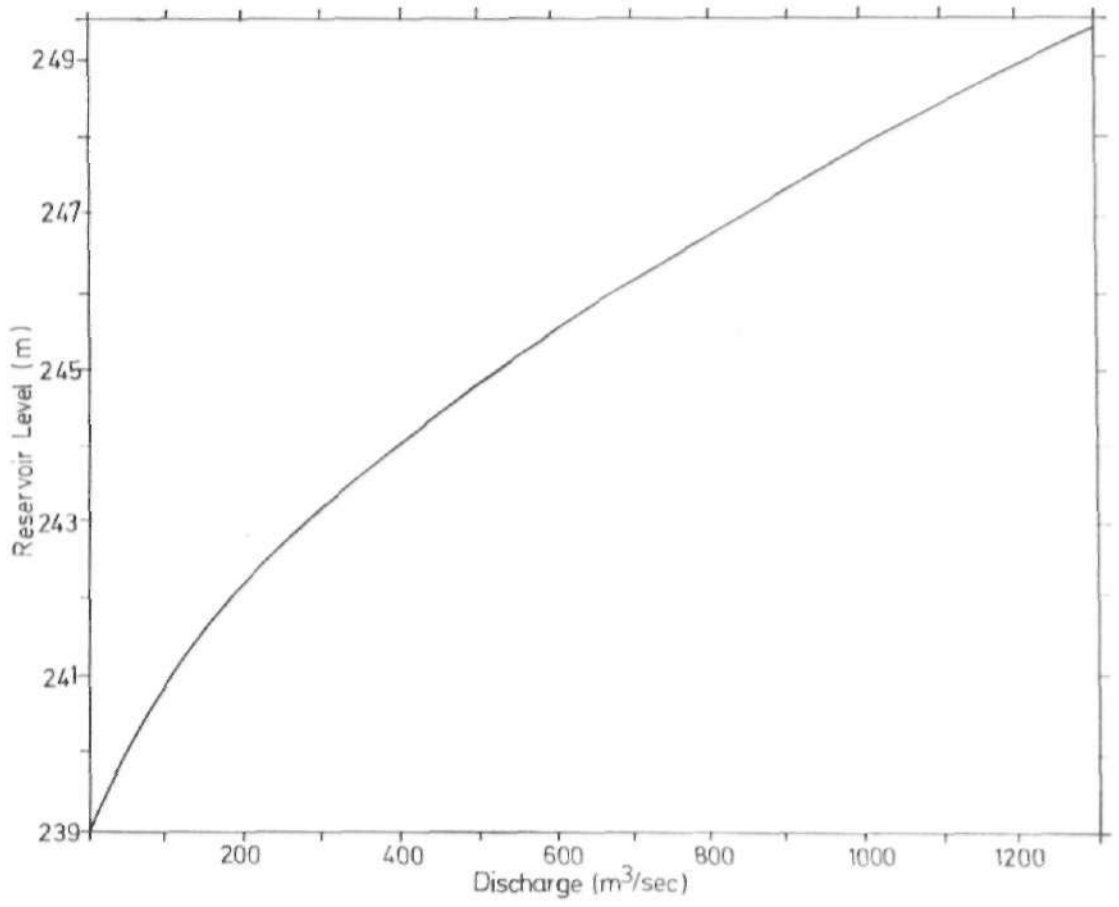

Fig. 5.8 Dadin-kowa Spillway Rating Curve

Table 5.4: Values of Discharge Coefficients from Various Sources.

Head ratio $H_r = \frac{H}{H_D}$	Total Head H(m)	Coefficient of Discharge C (S.I Units).			
		Creager	Chow	USBR	D. Kowa
0.3	3.0	1.85	1.85	1.90	1.95
0.4	4.0	1.92	1.93	1.95	2.02
0.5	5.0	2.00	2.00	1.99	2.04
0.6	6.0	2.03	2.06	2.03	2.09
0.7	7.0	2.08	2.11	2.08	2.14
0.8	8.0	2.12	2.15	2.10	2.16
0.9	9.0	2.16	2.19	2.14	2.19
1.0	10.0	2.20	2.21	2.16	2.20
1.9	11.0	2.23	2.24	2.20	2.24
1.2	12.0	2.26	2.27	2.21	2.26

From the above table, it can be observed that values of C differ for different sources under the same operating flow conditions i.e. (negligible approach depth) and standard vertical upstream face of the spillway. This confirms the uncertainty in the use of the discharge coefficient variable in determining the resistance of the structure.

The influence of downstream submergence and apron interference on the coefficient of discharge is neglected since the tail water depth is not high enough to cause submergence. Fig. 5.9 shows the coefficients of discharge for the various sources.

Regression Analysis for Discharge Coefficients

It is difficult to quantify the tolerance limit of the coefficients of discharge. To compute the tolerance limits, combining all the sources, a regression analysis is done to establish the relationship between the head ratio H_r and C.

The form of the proposed equation for the computation of coefficient of discharge C is :

$$C = A + B \ln H_r. \qquad — 5.2$$

where C = Coefficient of discharge.

A, B are regression constants.

To compute the regression constants, A, B, the following expressions are used :

$$A = \frac{\Sigma(C) - B\,\Sigma(\ln Hr)}{n} \qquad — 5.3$$

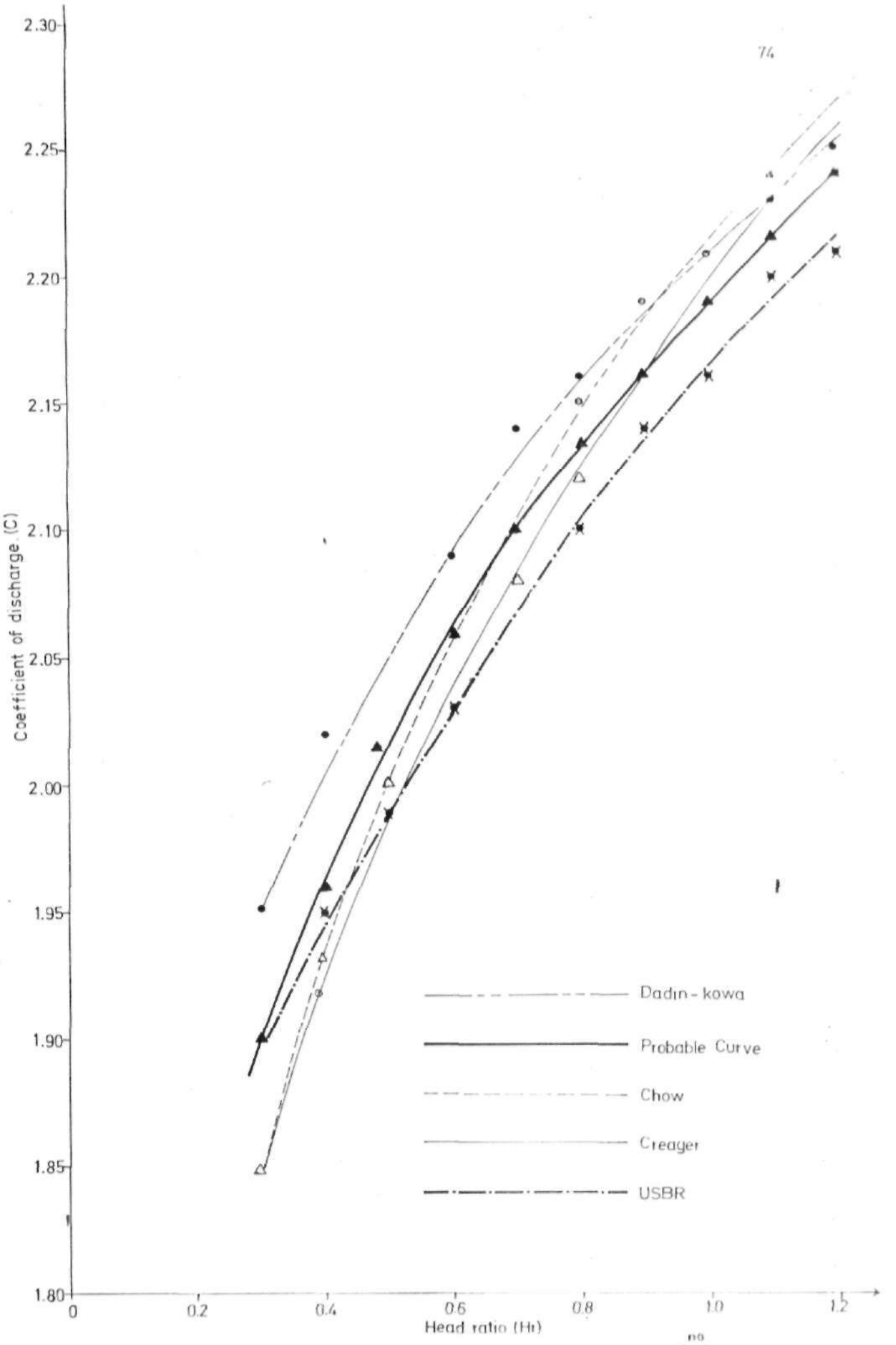

Fig. 5.9 Coefficients of discharge for straight standard dam crests with velocity of approach.

and

$$B = \frac{n\Sigma(\ln H_r)(C) - (\ln H_r)\Sigma(C)}{n\Sigma(\ln H_r)^2 - (\Sigma \ln H_r)^2} \quad \text{— 5.4}$$

Using data from Table 5.3, the following regression equations are obtained

$$\left.\begin{aligned} &\text{Creager:} \quad C = 2.195 + 0.297 \ln H_r \\ &\text{Chow :} \quad C = 2.215 + 0.305 \ln H_r \\ &\text{USBR :} \quad C = 2.163 + 0.232 \ln H_r \\ &\text{D. Kowa:} \quad C = 2.210 + 0.219 \ln H_r \end{aligned}\right\} \text{ - 5.5}$$

The fitted equations yield the following results of coefficient of discharge shown in Table 5.5

Table 5.5: Discharge Coefficients from the fitted Equations.

Head ratio H_r	Discharge Coefficients, C			
	Creager	Chow	USBR	Dadin Kowa
0.3	1.84	1.85	1.90	1.95
0.4	1.92	1.94	1.95	2.00
0.5	1.99	2.00	1.99	2.06
0.6	2.04	2.06	2.03	2.10
0.7	2.09	2.11	2.08	2.13
0.8	2.13	2.15	2.10	2.16
0.9	2.16	2.18	2.14	2.19
1.0	2.20	2.22	2.16	2.22
1.1	2.22	2.24	2.20	2.23
1.2	2.25	2.27	2.24	2.25

$$C_u = A_u + B_u \ln H_r$$

$$C_c = A_c + B_c \ln H_r$$

$$C_L = A_L + B_L \ln H_r \qquad \text{— 5.6}$$

where C_u = Upper boundary value of coefficient of discharge.

C_c = Probable value of coefficient of discharge.

C_L = Lower boundary value of coefficient of discharge.

A_u, A_c, and A_L are the upper, probable, and lower boundary values of the regression constant A.

B_u, B_c, and B_L are the upper, probable, and lower boundary values of the regression constant B.

To obtain upper and lower boundary values C_u and C_L, the outer and lower continuous curves of Fig. 5.2 are adopted respectively. The most probable curve for coefficient of discharge - head ratio curve is taken as a continuous curve C_c, between the upper and lower boundary curves.

The resulting C_u, C_c, and C_L curves are shown in Fig. 5.10.

Table 5.6 shows the boundary values of coefficients of discharge C_u, C_c and C_L.

Table 5.6: Boundary Values for the Coefficients of Discharge.

H_r	C_L	C_c	C_u
0.3	1.85	1.90	1.95
0.4	1.92	1.96	2.00
0.5	1.99	2.02	2.05
0.6	2.03	2.06	2.09
0.7	2.08	2.10	2.13
0.8	2.11	2.14	2.16
0.9	2.14	2.17	2.19
1.0	2.17	2.19	2.22
1.1	2.19	2.22	2.24
1.2	2.21	2.24	3.26

Regression analysis is performed to fit equations to the three boundary value curves of fig. 5.10. The regressed equations are shown below :

$$\left.\begin{aligned} C_L &= 2.167 + 0.263 \ln H_r \ ; \\ C_c &= 2.193 + 0.250 \ln H_r \ ; \\ C_u &= 2.215 + 0.230 \ln H_r \end{aligned}\right\} \quad \text{— 5.7}$$

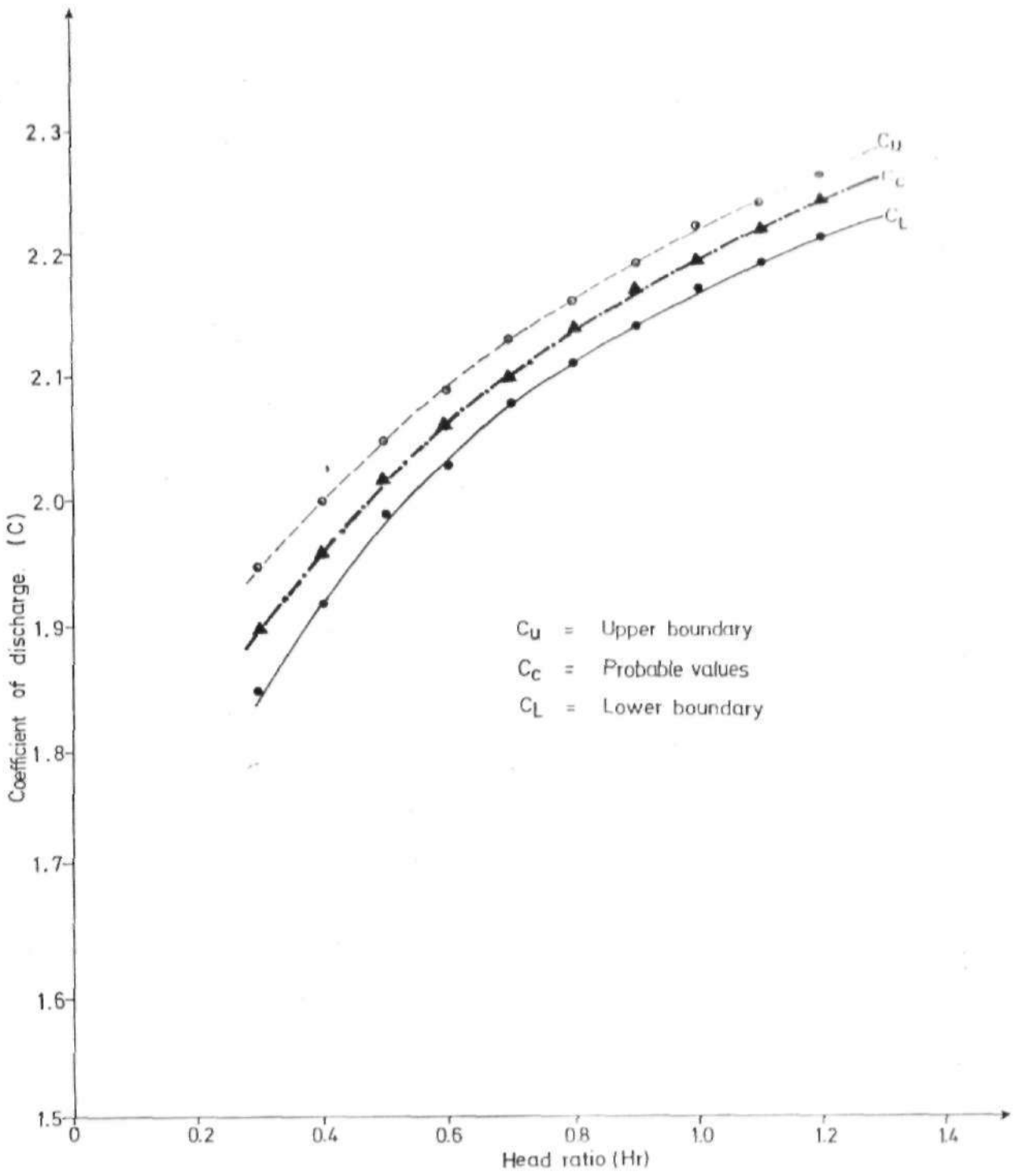

Fig. 5.10 Boundary values for coefficients of discharge.

From equation 5.6 and 5.7, A lies between 2.167 and 2.215 and B lies between 0.230 and 0.263. The upper boundary value for regression constant $A_u = 2.215$, the probable value $A_c = 2.193$, and $A_L = 2.167$. Similarly, $B_u = 0.263$; $B_c = 0.25$; and $B_L = 0.23$.

Contraction Coefficient Variable

The effective crest length L_n is expressed as in eq. 4.9

i.e., $L_n = L_t - 2(NK_p + k_a)H$

Abutment contruction coefficient $k_a = 0$ since the angle of the headwall for the right abutment 43°, is less than 45° and the radius of the left abutment r = 5.2 m is greater than $0.5H_D = 0.5(10) = 5.0$ m, USBR (1974).

The number of piers N = 2

Eq. 4.13 is now written as :

$$Q_c = \lambda_m C(L_t - 4k_pH)H^{\frac{3}{2}} \quad \text{—} \quad 5.8$$

Contraction coefficient k_p vary with nose shapes and head ratio.Chow (1959) shows these variations in a chart for different pier nose shapes.

The pier nose shape for Dadin - Kowa spillway lies between those of type 2 and type 3 given by Chow (1959). The Dadin- Kowa Pier shape is thus approximated to lie between round-nose type (2) and oval-pointed nose type (3) piers.

The values of k_p for type 2 are designated as lower boundary values k_L; and k_p for type 3 as the upper boundary values. The most probable values of k_p are designated as k_c.

Table 5.7 shows the variations of the boundary values with the head ratio.

Table 5.7: Boundary and most Probable values of Pier Contraction

H_r	K_L	K_c	K_u
0.3	0.064	0.070	0.078
0.4	0.049	0.053	0.058
0.5	0.031	0.037	0.044
0.6	0.024	0.030	0.035
0.7	0.018	0.023	0.028
0.8	0.010	0.017	0.024
0.9	0.004	0.01	0.018
1.0	-0.001	0.003	0.011
1.1	-0.008	-0.004	0.003
1.2	-0.013	-0.008	-0.002

Equations are fitted to these values using regression analysis and the following results are obtained.

$$\left.\begin{aligned} k_L &= -0.0025 - 0.054 \ln H_r \\ k_c &= 0.0047 - 0.054 \ln H_r \\ k_u &= 0.0098 - 0.054 \ln H_r \end{aligned}\right\} \quad \text{— 5.9}$$

The regressed equations yield the following result,

Table 5.8.

Table 5.8 . The regressed Pier contraction coefficients.

H_r	K_L	K_c	K_u
0.3	0.063	0.068	0.075
0.4	0.046	0.053	0.059
0.5	0.035	0.041	0.047
0.6	0.025	0.032	0.037
0.7	0.017	0.024	0.029
0.8	0.010	0.017	0.022
0.9	0.003	0.010	0.016
1.0	-0.003	0.005	0.010
1.1	-0.008	-0.0003	0.005
1.2	-0.012	-0.005	-0.0005

Fig. 5.11 shows the contraction coefficients for piers of various shapes.

In general, eq. 5.9 for boundary values of K_u, K_c and K_L can be re-written as :

$$K_p = K_A - 0.054 \ln H_r . \qquad -- \quad 5.10$$

where K_A lies between - 0.0025 and 0.0098. The most probable value of K_A = 0.0047.

5.3.2 First Order Analysis for the Computation of Coefficients of Variation of the Hydraulic Equation

The hydraulic equation whose variables are to be analysed can now be expressed after the substitution of eq. 5.2, and 5.10 in eq. 5.8 as :

$$Q_c = \lambda_m(A + B \ln H_r)(l_t - 4H(K_A - 0.054)\ln H_r)H^{3/2} \qquad -- \quad 5.11$$

First - order analysis of eq. 5.11 is performed to determine the mean and coefficient of variation of the components uncentainty. The component variables are A, B, L_t, K_A and λ_m.

From eq. 3.25, the expected values of eq. 5.11 is expressed as :

$$\bar{Q}_c = \bar{\lambda}_m(\bar{A} + \bar{B} \ln H_r)(\bar{L}_t - 4H(\bar{K}_A - 0.054 \ln H_r)H^{3/2} \qquad -- \quad 5.12$$

he- arranging eq. 5.12

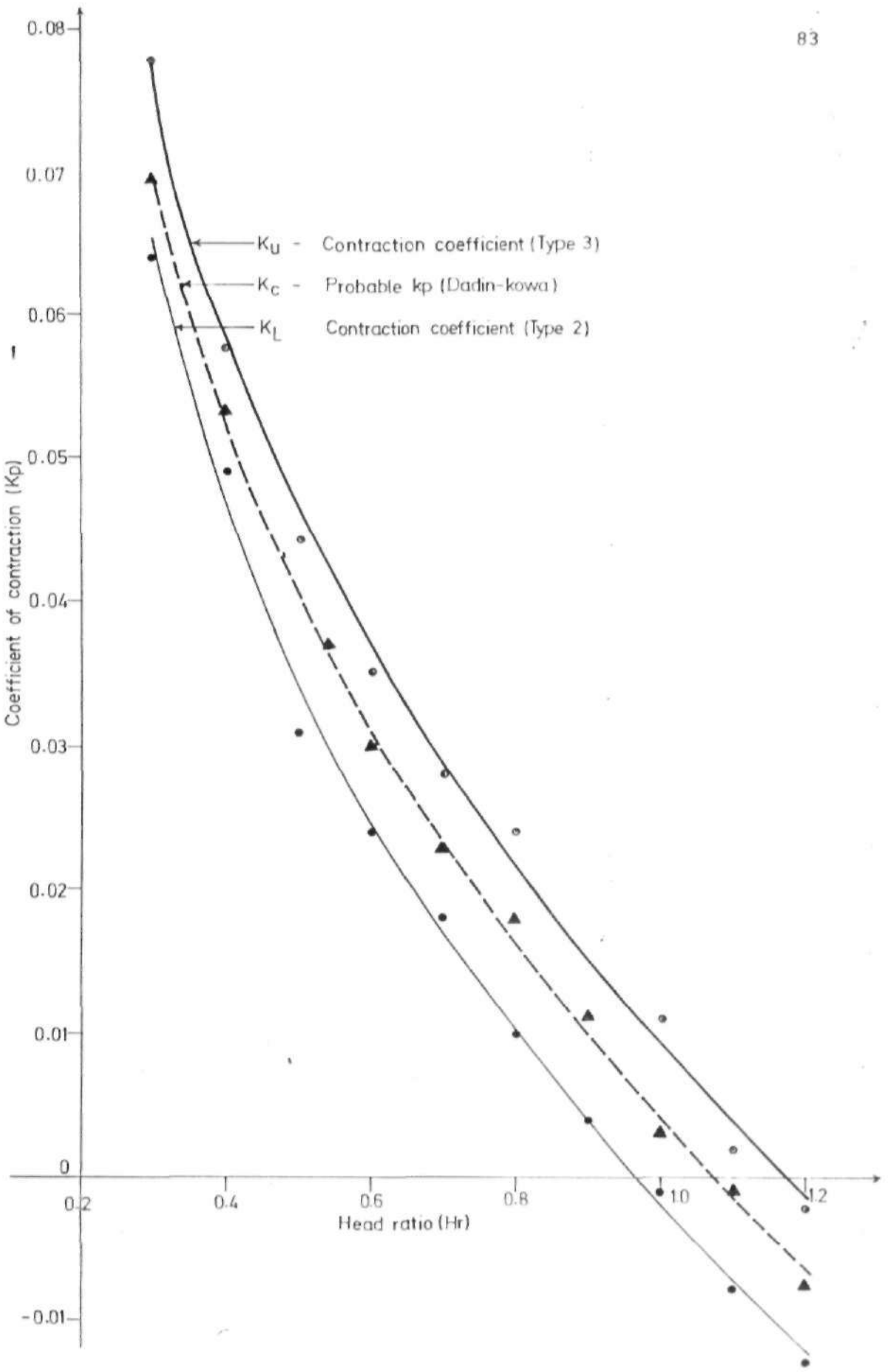

Fig. 5.11 Coefficients of piers for different shapes.

$$\bar{Q}_c = \bar{\lambda}_m \left[\bar{L}_t(\bar{A} + \bar{B} \ln H_r) - 4H(\bar{A} + \bar{B} \ln H_r)(\bar{K}_A - 0.054 \ln H_r) \right] H^{3/2}$$

$$\bar{Q}_c = \bar{\lambda}_m \left[\bar{L}_t(\bar{A} + \bar{B} \ln H_r) - 4H(\bar{A} + \bar{B} \ln H_r)\bar{K}_A + 4H(\bar{A}+\bar{B} \ln H_r) \right.$$

$$\left. 0.054 \ln H_r \right] H^{3/2} \qquad \text{— 5.13}$$

From equation 3.29, the coefficient of variation is given as :

$$\Omega^2_{Q_t} = \Omega^2_{\lambda_m} + \frac{1}{(E(W))^2} \sum \left(\frac{\partial w}{\partial w_i} \right)^2 \Bigg|_{w=\mu} \mu_i^2 \Omega^2_{w_i}$$

Taking the partial derivative of equation 5.13

$$\frac{\bar{Q}_c}{\bar{A}} \Bigg|_{\bar{A}} = \left[\bar{L}_t - 4H(\bar{K}_A - 0.054 \ln H_r) \right] H^{3/2} \qquad \text{— 5.14a.}$$

$$\frac{\bar{Q}_c}{\bar{B}} \Bigg|_{\bar{B}} = \left[\bar{L}_t - 4H(\bar{K}_A - 0.054 \ln H_r) \right] \ln H_r \, H^{3/2} \qquad \text{— 5.14b}$$

$$\frac{\bar{Q}_c}{\bar{K}_A} \Bigg|_{\bar{K}_A} = - 4H(\bar{A} + \bar{B} \ln H_r) H^{3/2} \qquad \text{— 5.14c.}$$

$$\frac{\bar{Q}_c}{\bar{L}_t} \Bigg|_{\bar{L}_t} = (\bar{A} + \bar{B} \ln H_r) H^{3/2} \qquad \text{— 5.14c}$$

Now, rewriting eq. 3.29, to compute α as :

$$\Omega^2_{Q_c} = \Omega^2 \lambda_m + \alpha^2_{w_i} \mu_i^2 \Omega^2_{w_i} \qquad \text{— 5.15}$$

where $\alpha^2_{W_i} = \frac{1}{(E(W))^2} \sum \left(\frac{\partial W}{\partial W_i} \right)^2_{W=\mu}$;

$$\alpha^2_{\bar{A}} = \frac{\left[\bar{L}_t - 4H(\bar{K}_A - 0.054 \ln H_r)H^{3/2}\right]^2}{\left[H^{3/2}\bar{L}_t(\bar{A} + \bar{B} \ln H_r) - 4H(\bar{A}+\bar{B}\ln H_r)\bar{K}_A + 4H\left[(\bar{A}+\bar{B}\ln H_r)0.054 \ln H_r\right]\right]^2}$$

let $\beta = \bar{A} + \bar{B} \ln H_r$ — 5.16

Thus $$\alpha^2_{\bar{A}} = \frac{\left[\bar{L}_t - 4H(\bar{K}_A - 0.054 \ln H_r)\right]^2}{\beta^2\left[\bar{L}_t - 4H(\bar{K}_A - 0.054 \ln H_r)\right]^2}$$

$$\alpha^2_{\bar{A}} = \frac{1}{\beta^2}$$ — 5.17a.

$$\alpha^2_{\bar{B}} = \frac{\left[\bar{L}_t - 4H(\bar{K}_A - 0.054 \ln H_r) \ln H_r H^{3/2}\right]^2}{\beta^2\left[\left[\bar{L}_t - 4H(\bar{K}_A - 0.054 \ln H_r)\right]H^{3/2}\right]^2}$$

$$\alpha^2_{\bar{B}} = \frac{(\ln H_r)^2}{\beta^2}$$ — 5.17b

$$\alpha^2_{\bar{K}_A} = \frac{\left[-4H(\bar{A} + \bar{B} \ln H_r)H^{3/2}\right]^2}{\beta^2\left[(\bar{L}_t - 4H(\bar{K}_A - 0.054 \ln H_r)^{3/2})\right]^2}$$

let $\gamma = \bar{L}_t - 4H(\bar{K}_A - 0.05 \ln H_r)$

$$\alpha^2_{\bar{K}_A} = \frac{16H^2\beta^2}{\beta^2\gamma^2} .$$

$$\alpha^2_{\bar{K}_A} = \frac{16H^2}{\gamma^2}$$ — 5.17c.

$$\alpha^2_{\bar{L}_t} = \frac{\left[(\bar{A} + \bar{B}\ \ln H_r)H^{3/2}\right]^2}{\left[(\bar{A} + \bar{B}\ \ln H_r)(\bar{L}_t - 4H(\bar{K}_A - 0.054\ \ln H_r))^{3/2}\right]^2}$$

$$= \frac{\beta^2}{\beta^2 \gamma^2} = \frac{1}{\gamma^2} \quad . \qquad \text{— 5.17d.}$$

Expressing eq. 5.15 in full i.e containing all the variables :

$$\Omega^2_{Q_c} = \Omega^2_{\lambda_m} + \alpha^2_{\bar{A}} \cdot \bar{A}^2 \cdot \Omega^2_A + \alpha^2_{\bar{B}} \cdot \bar{B}^2 \Omega^2_B + \alpha^2_{\bar{K}_A} \cdot \bar{K}^2_A \cdot \Omega^2_{K_A}$$

$$+ \alpha^2_{\bar{L}_t} \cdot \bar{L}^2_t \cdot \Omega^2_{L_t} \quad . \qquad \text{— 5.18}$$

Substituting eqs. 5.17a, b, c, d in 5.18 :

$$\Omega^2_{Q_c} = \Omega^2_{\lambda_m} + \frac{\bar{A}^2 \cdot \Omega^2_A}{\beta^2} + \frac{\bar{B}^2 \cdot \Omega^2_B (\ln H_r)^2}{\beta^2} + \frac{16\, H^2 \cdot \bar{K}^2_A \cdot \Omega^2_{K_A}}{\gamma^2}$$

$$+ \frac{\bar{L}^2_t \Omega^2_{L_t}}{\gamma^2}$$

Taking the square - root :

$$\Omega_{Q_c} = \left[\Omega^2_{\lambda_m} + \frac{\bar{A}^2 \Omega^2_A}{\beta^2} + \frac{\bar{B}^2 (\ln H_r)^2 \Omega^2_B}{\beta^2} + \frac{16\, H^2 \bar{K}^2_A \Omega^2_{K_A}}{\gamma^2} + \frac{\bar{L}^2_t \Omega^2_{L_t}}{\gamma^2} \right]^{1/2}$$

— 5.19.

Determination of Hydraulic Model Error λ_m.

The errors that are introduced in the discharge formula may be expressed as an absolute quantity or as a relative quantity (Rao, 1975). The absolute error may be considered while testing the inherent feature of a weir as a measuring device. However, the consideration of the relative error will be more important than the absolute error in most of the engineering problems wherein the relative magnitude is significant.

The relative error in discharge is obtained by differentiating the discharge - head relation with respect to the depth of flow, Rao (1975).

The percentage error

$$e = \frac{dQ}{Q} \Big/ \frac{dH}{H} \qquad \text{— 5.20}$$

The hydraulic equation is given as in eq. 5.12

$$Q_c = (\bar{A} + \bar{B} \ln H_r)(\bar{L}_t - 4H(\bar{K}_A - 0.054 \ln H_r))H^{\frac{3}{2}}$$

let $f = \bar{A} + \bar{B} \ln H_r$.

$g = \bar{K}_A - 0.054 \ln H_r$.

Then $Q_c = f(L_t - 4Hg)H^{\frac{3}{2}}$

$$\frac{dQ_c}{dH} = 1.5f(\bar{L}_t - 4Hg)H^{\frac{1}{2}} - 4fgH^{\frac{3}{2}} \qquad \text{— 5.21}$$

$$= 1.5f\bar{L}_t H^{1/2} - 6fg H^{3/2} - 4fgH^{3/2}$$

$$\frac{dQ_c}{dH} = 1.5f\bar{L}_t H^{1/2} - 10fgH^{3/2} \qquad \text{— 5.22}$$

Dividing eq. 5.21 by Q_c :

$$\frac{dQ_c \Big| dH}{Q_c} = \frac{1.5\, f\bar{L}_t H^{1/2} - 10fgH^{3/2}}{f(\bar{L}_t - 4Hg)H^{3/2}} \qquad \text{— 5.23}$$

Multiplying eq. 5.23 by H ;

$$\frac{dQ_c}{Q_c} \Big/ \frac{dH}{H} = e = \frac{(1.5\, f\bar{L}_t H^{1/2} - 10fgH^{1/2})H}{f(\bar{L}_t - 4Hg)H^{3/2}}$$

$$= \frac{1.5f\bar{L}_t H^{3/2} - 10fgH^{3/2}}{f(\bar{L}_t - 4Hg)H^{3/2}} .$$

$$= \frac{1.5\, fH^{3/2}(\bar{L}_t - 10Hg)}{fH^{3/2}(\bar{L}_t - 4Hg)}$$

$$e = \frac{1.5(\bar{L}_t - 10Hg)}{\bar{L}_t - 4Hg.} \%$$

Substituting in the expressions of f and g in the above equation :

$$e = \frac{1.5\, \bar{L}_t - 10H(\bar{K}_A - 0.05h \ln H_r)}{\bar{L}_t - 4H(\bar{K}_A - 0.05h \ln H_r)} \% \qquad \text{— 5.24}$$

Computation of Coefficients of Variation.

Triangular distribution is assumed to fit all the variables, Mays and Tung (1980a).

The mean $\bar{X}$ and coefficient of variation Ω_{w_i} are estimated using the following relationship, Tung and Mays (1980a)

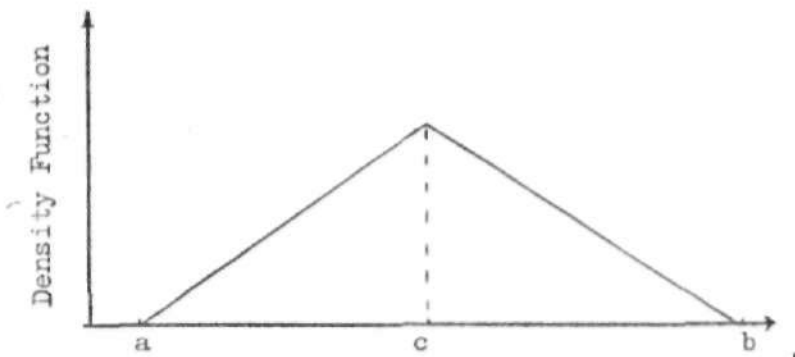

Fig. 5.12 Triangular Distribution.

If a, b, and c are the minimum, maximum, and the probable values of the variable. then

$$\bar{X} = \frac{1}{3}(a + b + c). \qquad — 5.25$$

$$\Omega_{w_i} = \left[\frac{1}{2} - \frac{ab + bc + ac}{6\bar{X}^2}\right]^{1/2} \qquad — 5.26.$$

Coefficient of variation of A :

From eq. 5.7 a = 2.167, b = 2.215, and c = 2.193

Thus $\bar{A}$ from (5.25) and (5.26)

$$\bar{A} = \frac{1}{3}(2.167 + 2.193 + 2.215)$$

$$\bar{A} = 2.192 .$$

and Ω_A

$$\Omega_A = \left[\frac{1}{2} - \frac{2.167(2.215) + 2.215(2.193) + 2.167(2.193)}{6(2.192)^2}\right]^{\frac{1}{2}}$$

$$\Omega_A = 0.013$$

(ii) Coefficient of variation of B :

From eq. 5.7 ; a = 0.230, b = 0.263, c = 0.25 .

Thus $\bar{B}$ from eq. 5.25 = 0.248

and Ω_B from eq. 5.26 = 0.046 .

(iii) Coefficient of variation of K_A :

From eq. 5.9 a = - 0.0025, b = 0.0098, c = 0.0047

Thus $\bar{K}_A$ from eq. 5.25 = 0.004

and Ω_{K_A} from eq. 5.26 = 0.630 .

(iv) Coefficient of Variation of L_t :

Since the exact measurement of length may not be possible even with most accurate instrument, a tolerance limit of linear measurement is assumed to be $\pm$ 0.05 m .

Thus a = 17.95; b = 18.05 ; and c = 18.0 m

From eq. 5.25 , $\bar{L}_t$ = 18.0 m

and η_t from eq. 5.26 = 0.001 .

(v) Coefficient of Variation of λ_m :

Using eq. 5.24 the tolerance limit of λ_m is calculated for various head ratio H_r. The result is shown in Table 5.9.

Table 5.9: Relative Error in the Discharge measurements.

H_r	H(m)	Error, e
0.3	3.0	0.01451
0.4	4.0	0.01456
0.5	5.0	0.01452
0.6	6.0	0.01456
0.7	7.0	0.01462
0.8	8.0	0.01471
0.9	9.0	0.01480
1.0	10.0	0.01491
1.1	11.0	0.01503
1.2	12.0	0.01504

From the above table, the spillway discharge measurement relative to the head is in error between 0.0145 and 0.015. If the average model error λ_m is 1.0, then λ_m varies between 1 - 0.0145 and 1 + 0.015 ;

Thus a = 0.985, b = 1.015; and c = 1.0.

λ_m = 1.0

Ω_{λ_m} from eq. 5.26 = 0.006 .

Substituting the mean values of the random variables in eq. 5.12 to compute the expected value $\bar{Q}_c$, the following equation is obtained :

$$\bar{Q}_c = (2.192 + 0.248 \ln H_r)(18.0 - 4H(0.004 - 0.054 \ln H_r))H^{3/2} \quad \text{— 5.27.}$$

The computed values of $\bar{Q}_c$ for various head ratio is shown in Table 5.10.

Substituting the coefficients of variation for all the variables, the coefficient of variation of the spillway capacity is calculated from eq. 5.19. The result is shown in Table 5.10.

Table 5.10: Coefficient of Variation and the Expected Mean Capacity of the Spillway.

Head ratio H_r	Total Head H(m)	Mean Cap. $\bar{Q}_c$ (m^3/sec)	Coeff. of Var. Ω_{Q_c}
0.3	3.0	168.95	0.0232
0.4	4.0	269.48	0.0220
0.5	5.0	387.82	0.0214
0.6	6.0	523.36	0.0210
0.7	7.0	675.87	0.0207
0.8	8.0	845.42	0.0206
0.9	9.0	1032.22	0.0207
1.0	10.0	1236.62	0.0210
1.1	11.0	1459.07	0.0207
1.2	12.0	1700.08	0.0208

5.3.3 Estimation of the Selected Probability Distribution Parameters

The probability distribution of the random variable, spillway capacity, is assumed to be lognormal. The mean and coefficient of variation of the spillway capacity have been computed and tabulated in Table5.10. Knowing these statistical parameters of the spillway capacity, the log-normal probability density function is expressed as in eq. 4.14 :

$$f(Q_c) = \frac{1}{\sqrt{2\pi}\, Q_c \sigma_n} \exp\left[-\frac{1}{2}\left(\frac{\ln Q_c - \mu_n}{\sigma_n}\right)^2\right].$$

μ_n = Means of the transformed random variable ln Q_c.

σ_n = Standard deviations of the transformed random variables.

The parameter μ_n is computed using the equation from eq. 4.15 $\mu_n = \ln \bar{Q}_c - \frac{1}{2}\sigma_n^2$. The coefficient of variation of the spillway capacity is used to compute the other parameter σ_n, using the relation as in eq. 4.16

$$\sigma_n = \left[\ln(\Omega_{Q_c}^2 + 1)\right]^{1/2}$$

Using eqs. 4.15 and 4.16, μ_n and σ_n are computed and the result is shown in Table 5.11

Table 5.11: Parameters of Lognormal Distribution for the Spillway Capacity

H_r	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
μ_n	5.12	5.59	5.75	6.25	6.51	6.73	6.93	7.11	7.30	7.43
σ_n	0.0232	0.022	0.0214	0.021	0.0207	0.0206	0.0207	0.021	0.0207	0.0208

5.4 Development of Static Model

The reliability of the spillway structure can be expressed from eq. 3.4 :

$$R = \int_0^\infty f_{Q_c}(Q_c)\left[F_{Q_d}(Q_c)\right]dQ_c \qquad — 5.28$$

where $F_{Q_d}(Q_c)$ = cummulative probability function for flood.

$$F_{Q_d}(Q_c) = \int_0^{Q_c} f_{Q_d}(Q_d)dQ_d \ ;$$

and $F_{Q_d}(Q_c)$ for the flood magnitude fitted to Gumbel distribution is

$$F_{Q_d}(Q_c) = \exp\left\{-\exp\left[-\frac{Q_c - \alpha_1}{\alpha_2}\right]\right\} \qquad — 5.29$$

The reliability for the structure is expressed below when eq. 4.14 and 5.29 are substituted in 5.28

$$R = \int_0^\infty \frac{1}{\sqrt{2\pi}\, Q_c\, \sigma_n} \exp\left[-\frac{1}{2}\left[\frac{\ln Q_c - \mu_n}{\sigma_n}\right]^2\right] \exp\left\{-\exp\left[-\frac{Q_c - \alpha_1}{\alpha_2}\right]\right\} dQ_c$$

— 5.30

Risk $\bar{R}$ is computed using $\bar{R} = 1 - R$.

5.4.1 Risk Computations for Static Model

Substituting $\alpha_1 = 415.52$; and $\alpha_2 = 145.0$ in eq. 5.30

$$\int_0^{\infty} \frac{1}{\sqrt{2\pi}\, Q_c\, \sigma_n} \exp\left[-\frac{1}{2}\left(\frac{\ln Q_c - \mu_n}{\sigma_n}\right)^2\right] \exp\left\{-\exp\left\{-\frac{Q_c - 415.52}{145.9}\right\}\right\} dQ_c \quad (5.31)$$

Eq. 5.31 is solved in conjunction with Table 5.11 by numerical integration using rectangular rule. The approximate solution is obtained using a developed computer program in Basic language. The computer model is AMSTRAD, PCW 9513, 7.

The computer result of the numerical integration is shown in Table 5.12. Fig. 5.13 shows the Risk-Safety Factor graph for Static Model.

H_r	μ_n	σ_n	$\bar{R}$	Safety Factor						
				T_r=2	T_r=5	T_r=10	T_r=20	T_r=30	T_r=50	T_r=100
0.3	5.12	0.0232	0.996	0.37	0.26	0.22	0.19	0.17	0.16	0.14
0.4	5.59	0.0220	0.94	0.59	0.42	0.35	0.30	0.28	0.26	0.23
0.5	5.95	0.0214	0.71	0.84	0.60	0.50	0.43	0.40	0.34	0.33
0.6	6.25	0.0210	0.39	1.14	0.81	0.68	0.58	0.54	0.50	0.45
0.7	5.51	0.0207	0.16	1.47	1.04	0.87	0.75	0.70	0.65	0.58
0.8	6.73	0.0206	0.054	1.84	1.30	1.09	0.94	0.87	0.87	0.72
0.9	6.93	0.0207	0.016	2.25	1.59	1.33	1.15	1.06	0.98	0.88
1.0	7.11	0.0210	0.004	2.69	1.90	1.60	1.38	1.28	1.18	1.05
1.1	7.30	0.0207	0.0007	3.17	2.23	1.88	1.62	1.50	1.39	1.24
1.2	7.43	0.0208	0.0002	3.70	2.62	2.19	1.89	1.75	1.62	1.45

Table 5.12 : Risk - Safety Factor relationship for the Static Model

(Composite Risk)

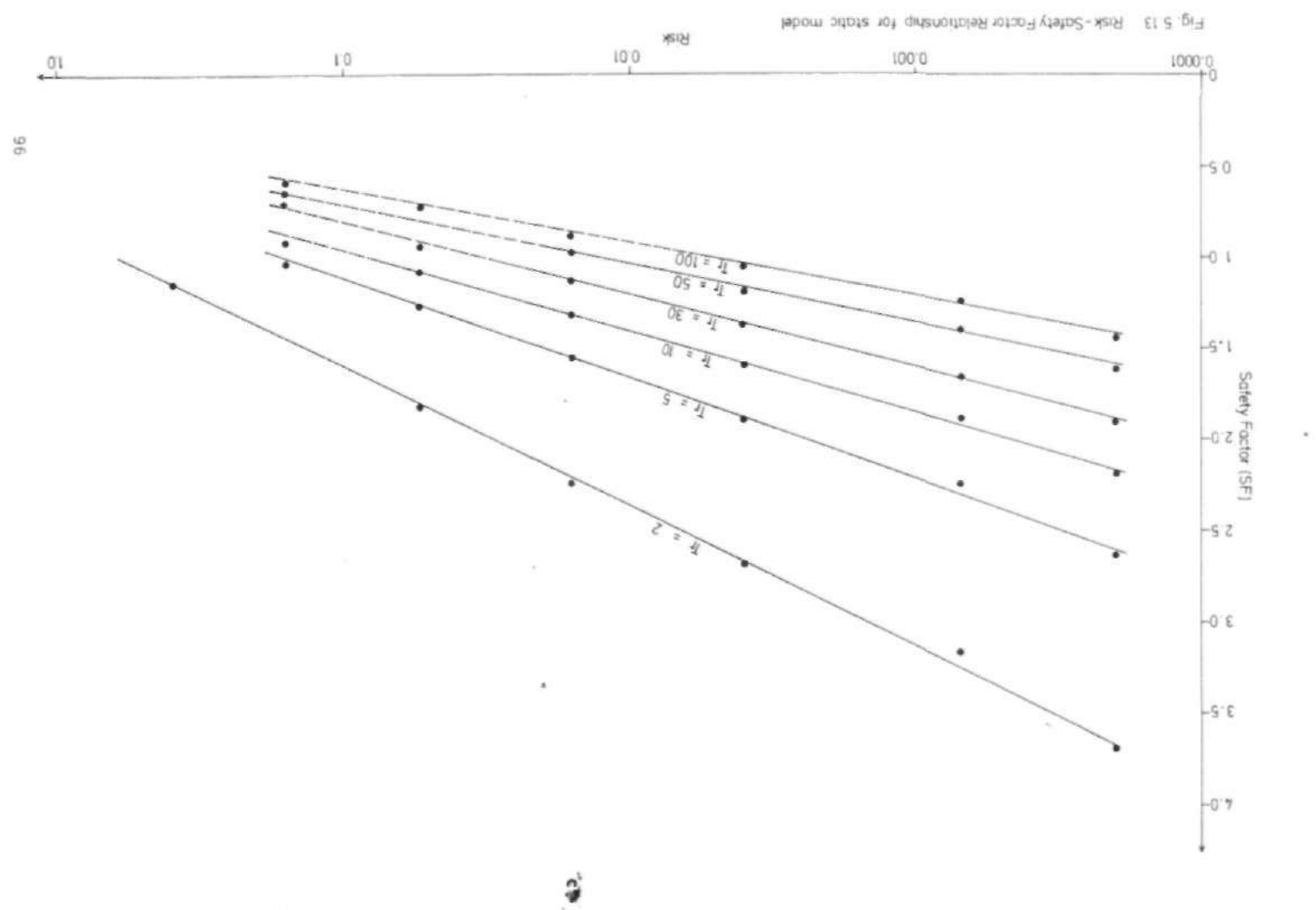

Fig 5.13 Risk - Safety Factor Relationship for static model

The safety factor in Table5.12 is computed using the relation given by Tung (1980a) as $SF = \frac{\bar{Q}_c}{Q_d(T_r)}$ — 5.32

5.5 Development of Dynamic Model.

The dynamic reliability model for extremal Type 1 (random-independent loading and lognormal (random - fixed) resistance is expressed as in eq. 3.19

$$R(t) = \int_0^\infty f_{Q_c}(Q_c) \exp\left[-\alpha t(1 - F_{Q_d}(Q_c))\right] dQ_c$$

The cummulative distribution function of the loading is expressed as in eq. 5.29

$$F_{Q_d}(Q_c) = \exp\left[-\exp\left(-\frac{Q_c - \alpha_1}{\alpha_2}\right)\right].$$

Substituting eq. 4.14 and 5.29 in 3.19 :

$$R(t) = \int_0^\infty \frac{1}{\sqrt{2\pi}Q_c\sigma_n} \exp\left[-\frac{1}{2}\left(\frac{\ln Q_c - \mu_n}{\sigma_n}\right)^2\right] \exp\left[-\alpha t\left(1-\exp\left\{-\exp\left[-\frac{Q_c - \alpha_1}{\alpha_2}\right]\right\}\right)\right] dQ_c \quad — \quad 5.33$$

Simplifying

$$R(t) = \frac{1}{\sqrt{2\pi}\sigma_n} \int_0^\infty \frac{e^{-m}}{Q_c} dQ_c \quad — \quad 5.34$$

where $m = \frac{1}{2}\left(\frac{\ln Q_c - \mu_n}{\sigma_n}\right)^2 + \alpha t \left[1 - \exp\left\{-\exp\left(\frac{-Q_c - \alpha_1}{\alpha_2}\right)\right\}\right]$ — 5.35

In the derivation of this equation, it was detected that the one provided by Tung (1980) is not correct. One exponential function is missing.

Though the error may be typographical, it is noteworthy to point it out especially for researchers/designers who will use the formula directly without the theoretical background.

The risk of thestructure is computed using eq. 3.2 as $\bar{R} = 1 - R$.

5.5.1 Composite Risk Computations for Dynamic Model.

By substituting $\alpha_1 = 415.52$ and $\alpha_2 = 145.9$ in eq. 5.35, m is now expressed as :

$$m = \frac{1}{2}\left(\frac{\ln q_c - \mu_n}{\sigma_n}\right)^2 + \alpha t\left[1 - \exp\left\{-\exp\left(\frac{-q_c - 415.52}{145.9}\right)\right\}\right]$$

The parameters μ_n and σ_n of the lognormal distribution for the resitance have previously been determined, Table 5.11.

The selected service years of the spillway :

t : 5, 10, 20, 30, 50 and 100 years, and the return periods T_r = 10, 25 and 100 yrs. The flood magnitude $q_d(T_r)$ is obtained from Fig. 5.3a.

$q_d(10) = 775$; $q_{d(25)} = 925$; and $q_{d(100)} = 1175$ m^3/sec.

Eq. 5.34 is solved by numerical integration. The approximate solution is obtained using a developed computer program mentioned earlier.

The result is shown in Table 5.13.

The safety factor SF in Table 5.13 is computed using eq. 5.32

$$SF = \frac{\bar{Q}_c}{Q_{d(T_r)}},$$

$\bar{Q}_c$ is obtained from Table 5.10

The risk - safety factor graph for different return periods and the service life of the spillway is shown in Fig. 5.14a, 5.14b, and 5.14c.

Table 5.13

Table 5.13: Risk - Safety factor Relationship for the Dynamic Model.

Mean μ_n	S. Dev σ_n	Safety Factor SF.	Return Period T_r = 100 yrs Service Life t (yrs)					
			5	10	20	30	50	100
			$X10^{-6}$	$X10^{-6}$	$X10^{-6}$	$X10^{-6}$	$X10^{-6}$	$X10^{-6}$
5.12	0.0232	0.14	910	1800	3600	5500	9100	18000
5.59	0.022	0.23	470	930	1900	2800	4600	9200
5.95	0.0214	0.33	210	410	830	1300	2100	4200
6.25	0.0210	0.45	84	170	330	500	830	1700
6.51	0.0207	0.58	29	58	130	1800	2900	580
6.73	0.0206	0.72	6.9	16	36	540	91	190
6.93	0.0207	0.88	3.9	3.4	8.0	140	24	50
7.11	0.0210	1.05	*	*	1.6	2.9	5.5	1.0
7.30	0.0207	1.24	*	*	*	*	1.1	*
7.43	0.0208	1.45	*	*	*	*	*	*

*The risk value is zero. This is due to low accuracy of the computer model used.

Table 5.13 (Contd.)

Return Period T_r = 10 yrs.								
Mean μ_n	S. Dev. σ_n	Safety Factor SF	Service Life t(yrs).					
			5	10	20	30	50	100
5.12	0.0232	0.22	0.0091	0.018	0.036	0.053	0.087	0.17
5.59	0.022	0.35	0.0046	0.0092	0.018	0.027	0.045	0.088
5.95	0.0211	0.50	0.0021	0.0042	0.0083	0.012	0.021	0.041
6.25	0.0210	0.68	0.00083	0.0017	0.0033	0.0049	0.0083	0.017
6.51	0.0207	0.87	0.00029	0.00058	0.0012	0.0018	0.0029	0.0058
6.73	0.0206	1.07	0.000091	0.00019	0.00037	0.00056	0.00094	0.0019
6.93	0.0207	1.33	0.000028	0.000054	0.00011	0.00016	0.00027	0.00053
7.11	0.0210	1.60	0.000055	0.000012	0.000025	0.000039	0.000016	0.000013
7.30	0.0207	1.88	0.0000026	0.0000038	0.0000061	0.0000085	0.0000013	0.000025
7.45	0.0208	2.19	*	*	*	*	*	*

Return Period T_r = 25 yrs.								
Mean (μ_n)	S. Dev (σ_n)	Safety Factor SF	Service life t (yrs)					
			5	10	20	30	50	100
5.12	0.032	0.18	0.0036	0.0073	0.015	0.022	0.036	0.07
5.59	0.022	0.29	0.0018	0.0037	0.0074	0.011	0.018	0.036
5.95	0.0214	0.42	0.00083	0.0017	0.0033	0.005	0.0083	0.017
6.25	0.0210	0.57	0.00033	0.00067	0.0013	0.002	0.0033	0.00066
6.51	0.0207	0.73	0.00012	0.00023	0.00046	0.0007	0.0012	0.0023
6.73	0.0206	0.91	0.000035	0.00073	0.00014	0.00022	0.00037	0.0075
6.93	0.0207	1.12	0.000012	0.000026	0.000044	0.000072	0.00011	0.00021
7.11	0.0210	1.34	0.0000042	0.0000091	0.000013	0.00002	0.000029	0.00052
7.30	0.0007	1.58	0.0000015	0.0000025	0.0000035	0.000053	0.0000079	0.000011
7.43	0.0209	1.84	*	*	*	*	*	*

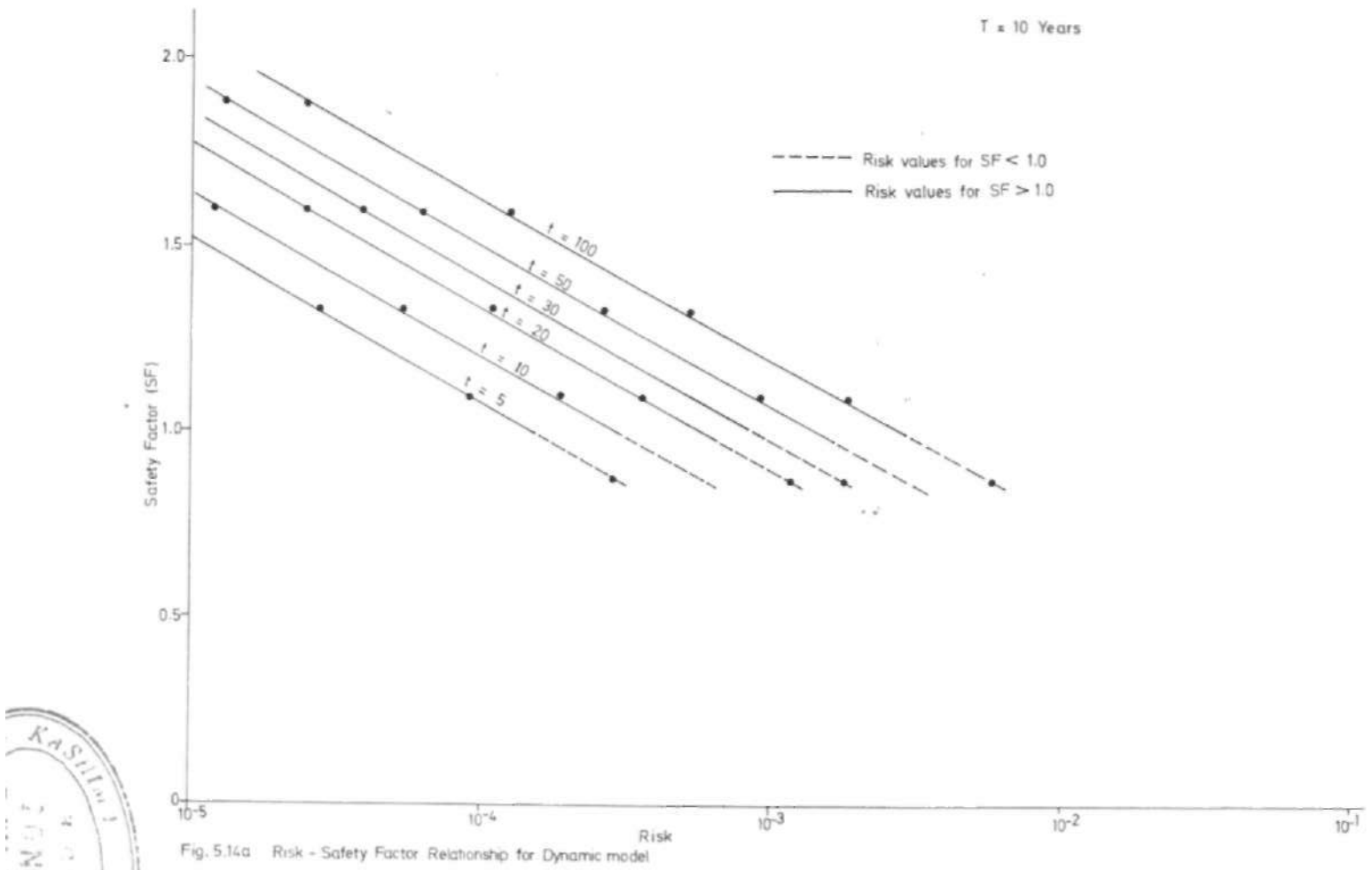

Fig. 5.14a Risk - Safety Factor Relationship for Dynamic model

KASETSART
21 NOV 1990
234041

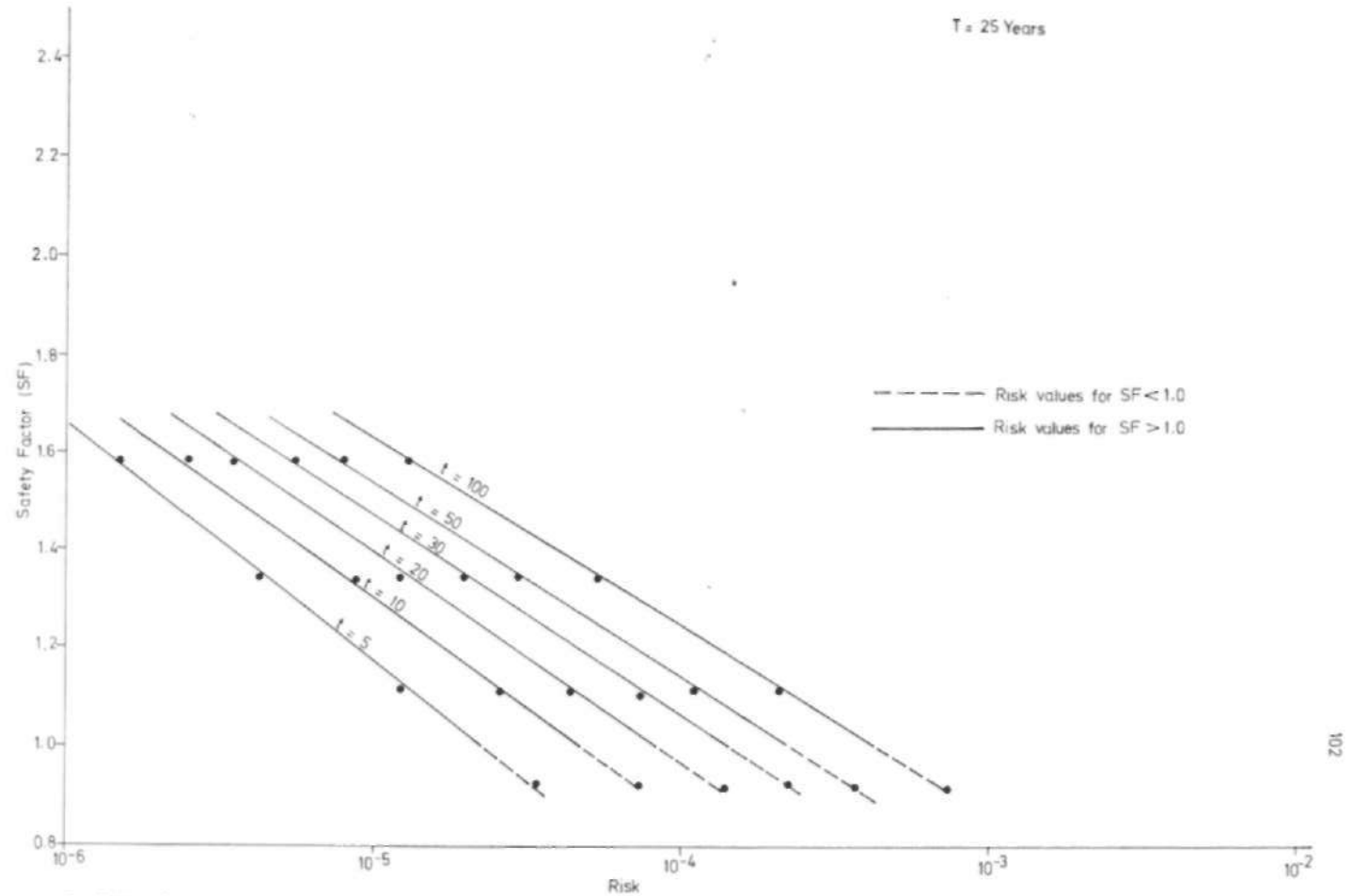

Fig. 5.14b Risk - Safety Factor Relationship for Dynamic model

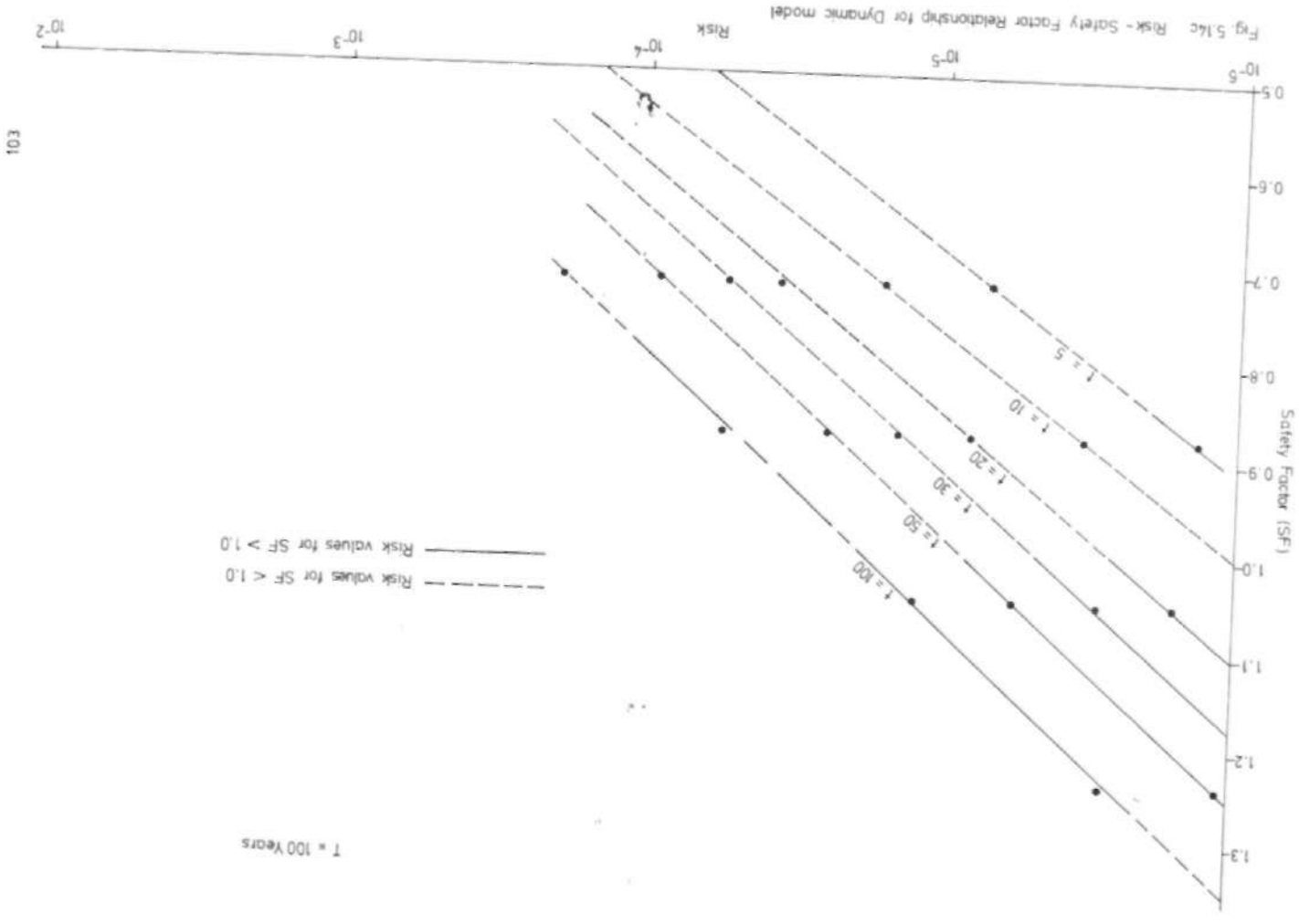
Fig 5.14c Risk - Safety Factor Relationship for Dynamic model
Risk
10^{-5}
10^{-4}
10^{-3}
10^{-2}
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
Safety Factor (SF)
t = 5
t = 10
t = 20
t = 30
t = 50
t = 100
Risk values for SF < 1.0
Risk values for SF > 1.0
T = 100 Years
103

5.6 Development of Hydrologic Risk Model.

The probability density function of theloading from eq. 4.5 is

$$f_{Q_d}(Q_d) = \frac{1}{\alpha_2} \exp\left[- \left(\frac{Q_d - \alpha_1}{\alpha_2} \right) - \exp \left(\frac{- Q_d - \alpha_1}{\alpha_2} \right) \right] . \qquad - 5.36$$

where Q_d is the maximum annual flow, α_1 and α_2 are defined as before .

The reliability of the structure based on only hydrologic uncertainty is expressed as :

$$R = \int_o^{Q_c} f_{Q_d}(Q_d)\, dQ_d \qquad - 5.37$$

The hydrologic risk is computed using

$$\bar{R} = 1 - R.$$

$$\bar{R} = 1 - \int_o^{Q_c} f_{Q_d}(Q_d) dQ_d \qquad - 5.38$$

But $\int_o^{Q_c} f_{Q_d}(Q_d) dQ_d = F_{Q_d}(Q_c)$,

where $F_{Q_d}(Q_c)$ = cummulative density function

Thus eq. 5.38 can be re-written as :

$$\bar{R} = 1 - F_{Q_d}(Q_c) \qquad - 5.39$$

But from eq. 5.29 ; $F_{Q_d}(Q_c) = \exp\left\{ - \exp \left(- \frac{Q_c - \alpha_1}{\alpha_2} \right) \right\}$.

Thus eq. 5.39 becomes :

$$\bar{R} = 1 - \exp\left\{-\exp\left(\frac{-Q_c - \alpha_1}{\alpha_2}\right)\right\}. \qquad — 5.40.$$

5.6.1 Computation of Hydrologic Risk.

The hydrologic risk is computed using eq. 5.40.

The parameters of the Gumbel distribution are α_1 = 415.52 and α_2 = 145.9. Thus eq. 5.40 is expressed as :

$$\bar{R} = 1 - \exp\left\{-\exp\left(\frac{-Q_c - 415.52}{145.9}\right)\right\} \qquad — 5.41$$

For a deterministic spillway capacity,

Q_c is calculated using the equation 4.8 i.e.,

$$Q_c = CL_n H^{\frac{3}{2}} \quad \text{where} \quad L_n = L_t - 4K_p H.$$

$$Q_c = C(L_t - 4K_p H)H^{\frac{3}{2}} \qquad — 5.42$$

C = Coefficient of discharge for Dadin - Kowa Dam. It is obtained from Table 5.3.

K_p = Pier contraction coefficient obtained from Table 5.6

The result of the computation of Q_c from eq. 5.42 is shown in Table 5.14. Also shown in the table is the risk values computed for various head ratio. The risk values are computed using eq. 5.41.

Table 5.14: Results of Hydrologic Risk with Various Head Ratio.

Head ratio H_r	Total Head H(m)	Pier Contraction Coefficient K_p	Eff. Crest length L_n(m)	Coeff.of Dis. D.Kowa (C)	Disch. Cap. Q_c m^3/sec	Risk $\bar{R}$
0.3	3.0	0.070	17.16	1.95	174	0.995
0.4	4.0	0.053	17.15	2.02	277	0.92
0.5	5.0	0.037	17.82	2.04	406	0.66
0.6	6.0	0.030	17.28	2.09	531	0.36
0.7	7.0	0.023	17.36	2.14	688	0.14
0.8	8.0	0.017	17.53	2.16	856	0.047
0.9	9.0	0.010	17.64	2.19	1043	0.014
1.0	10.0	0.003	17.88	2.20	1244	0.0034
1.1	11.0	-0.007	18.18	2.24	1486	0.0006
1.2	12.0	-0.008	18.38	2.26	1727	0.000125

5.7 Comparison of Hydrologic Risk and Composite Risk.

Comparing the results of Table 5.12 and 5.14. It is observed that the values of hydrologic risk obtained is smaller than the composite risk. The percentage contribution of the hydraulic risk to the composite risk is shown in Table 5.15.

Table 5.15: Comparison of Hydrologic Risk and Composite Risk,

Head ratio H_r	Composite Risk	Hydrologic Risk	% Contr. of Hydraulic Risk
0.3	0.996	0.995	0.1
0.4	0.94	0.92	2.1
0.5	0.71	0.66	7.0
0.6	0.39	0.36	7.7
0.7	0.16	0.14	12.5
0.8	0.054	0.047	12.96
0.9	0.016	0.014	12.5
1.0	0.0040	0.0034	15.0
1.1	0.0007	0.0006	14.3
1.2	0.000200	0.000125	37.5

CHAPTER SIX

DISCUSSION

6.1 Selection of Hydrologic Probability Distribution

At first, flood frequency analysis is performed to account for the stochastic process of flood flow. Three distributions, Log - Pearson Type III, Lognormal, and Gumbel distributions are selected to test their validity to fit the flood events. The three distributions all plotted as straight lines on probability distribution papers. This shows that the three are all potential candidates to fit the model. When subjected further to hydrologic model varification using Kolmogorov - Smirnov test at significance level = 0.10, all of them have maximum deviations less than critical value C = 0.219 determined from Kolmogorov - Smirnov table at α = 0.10. However, Gumbel distribution shows the least maximum deviation of 0.05 compared to log-Pearson and lognormal which have maximum deviation of 0.056 and 0.053 respectively. Based on the K - S test, all of the distributions can be accepted.

Confidence band was then constructed for the three fitted distributions around the flood frequency curves to determine the reliability of points representing the annual maximum discharge for a recurrence interval. The confidence band was constructed at

significance level = 0.10. The lognormal distribution has one event magnitude lying on the lower band and one on the upper band, while log- Pearson distribution has two event magnitudes outside the band and four event magnitudes lying directly on the lower band. Gumbel distribution has all the even magnitudes lying within the band. On the basis of this confidence band drawn around the three frequency curves for the three distributions, Gumbel distribution is considered to best fit the flood flow.

6.2 Determination of Spillway Discharge Equation Variables.

The computation of the values of the variables of the spillway discharge equation was performed. The coefficient of discharge was calculated from various sources. The results showed that under the same operating condition (negligible depth of approach, and negligible effect of spillway submergence) and standard vertical upstream face of the spillway, there was variation in the values of the coefficients of discharge. For example, while the coefficient of discharge C=1.92 for head ratio H_r = 0.40 Creager (1964), it was 1.93 , 1.95 and 2.02 for the same head ratio from Chow (1959), USBR(1974), and I. Iowa respectively. This confirms the uncertainty in the use of the discharge coefficient variable in determining the capacity of the structure. This is expected because hydraulic conditions for flow through or over flood conveyance structure is complicated, and highly unsteady, it is therefore possible to have different discharge coefficients under the supposed operating flow conditions.

Equations were fitted to determine the boundary values for the coefficients of discharge. For head ratio between 0.3 and 1.2, the lower boundary value of C varies from 1.85 and 2.21, the probable value varies between 1.90 and 2.24; and the upper boundary value varies between 1.95 and 2.26. Fig. 5.10. These values are all within the limits of coefficient of discharge given by Hwang (1981).

Equations were also fitted to determine the boundary values for the pier contraction coefficients. For head ratio between 0.3 and 1.2, the lower boundary value of pier contraction coefficient varies between 0.063 and - 0.012, the probable value for b. Kowa pier nose shape, varies between 0.068 and - 0.005; and the upper boundary value varies between 0.075 and - 0.0005. Table 5.7.

6.3 Static Composite Risk Model.

First order analysis was performed to compute the means and coefficients of variation of spillway discharges for different heads. The computed mean discharge was used together with flood magnitudes for various return periods to determine the safety factor. The computed safety factor was found to increase for increasing capacity of the structure and decreases with increasing return period Fig. 5.13. For example, for a mean spillway capacity of 523.36 m^3/sec, the safety factor was 1.14, while it was 3.70 for a capacity of 1700.08 m^3/sec for a return period of T_r = 2 years. Table 5.10. For a return

period of 100 years, for the same spillway capacity, the safety factor was 0.45 and 1.45 respectively. This result is expected because safety factor is inversely proportional to flood magnitudes or return periods.

Composite risk values consisting of hydrologic and hydaulic risks, for the static model were computed using numerical integration method. The integration is performed using a computer. Risk - sefety factor relationships for various return periods and spillway discharge capacity were established. The risk values were found to decrease with increasing capacity of spillway Fig. 5.13. For example, for a spillway capacity of 387.82 m^3/sec and 1459.07 m^3/sec, the composite risk values were 0.71 and 0.0002 respectively. The decreasing risk value was true in engineering practice, for an increasing spillway capacity, because increasing the capacity of the structure results in a more safe or reliable structure. The established risk - safety factor relationships with return periods as parameters will enhance the choice of any of these parameters in engineering design and planning especially in optimal design of spillway structure.

6.4 Dynamic Composite Risk Model.

In the development of the Static Model for composite risk, a single (worse) flooding was imposed in the hydraulic structure. This model has a serious shortcoming because it does not take into account

the repeated application or occurrence of flooding (loading). It is also not known when the worse loading (flooding) occurs during the service life, of the project. In otherwords, the model can not relate risk values to the expected service life of the structure. It is therefore not possible to ascertain the behaviour of risk levels with increasing or decreasing service life of the structure.

A hydraulic structure is subjected to a repeated occurrence of flooding over its expected service life. To overcome the shortcomings of the Static Model, the use of Time - Dependent or a Dynamic Model which takes into account the flooding and resistance (spillway capacity) random variables with respect to time and flooding cycles is preferrable. Table 5.12, shows that they will tend towards zero but not exactly zero.

6.5 Comparison of Hydrologic Risk and Composite Risk.

The hydrologic risk was computed and compared with the composite risk values. The risk was found to be underestimated when only hydrologic risk was considered. The percentage of the underestimation increases with increase in spillway capacity. For low capacity, the risk is dominated by the hydrologic risk, but as the capacity increases, the contribution of hydraulic risk into the composite model becomes significant, Table 5.13. For a spillway capacity of 277 m^3/sec, the percentage contribution of hydraulic risk is 2.1%. But for a spillway capacity of 1727 m^3/sec, the contribution of the hydraulic risk to the composite risk is as much as 37.5%. This latter example implies

that the risk is underestimated as much as 37.5% when only hydrologic risk is considered for large spillway capacity.

This result has significant importance :

(i) For small spillway capacity, hydrologic risk can be considered and the spillway capacity can be treated as deterministic i.e. the hydraulic risk can be neglected. But for large spillway capacity, the incorporation of the hydraulic risk into the design procedure may be justified.

(ii) Considering only the hydrologic risk in the design of spillway structure of small capacity may significantly underestimate the economic consequences of flood hazards.

The result of risk values computed for Dynamic Model Table 5.13 has confirmed the fact that they increase with increase service life of structure and decrease with increase in return period. For example, for return period of 10 years, mean discharge $\bar{Q}_c$ = 675.87 m^3/sec, service life of 10, and 50 years, the risk values are 1.9×10^{-4} and 9.4×10^{-3} respectively. For a return period of 25 years, the same mean discharge of 675.87 m^3/sec, the same service life of 10, and 50 years, the risk values are 2.6×10^{-5} and 1.1×10^{-4} respectively.

The established risk- safety factor relationships, return priods, and expected service life, will facilitate the development of optimal risk based design of spillway. In such optimisation

model, the minimum cost which may include the construction and damage costs can be incorporated to obtain a cost - benefit model.

The similarities that can be drawn between Static and Dynamic Models are that they both exhibited the fact that the composite risk of structures decreases with the increase in the capacities of the structures and that risk values decrease with increase in return period.

In the computation of the composite risk for the Dynamic Model, some risk values obtained in Table 5.13 are zero. In practical reality there can not be a structure that can be 100% safe. The reason for the zero values are due to the low accuracy of the computer model type used to compute the risk. It is therefore not possible to ascertain the behaviour of risk values at high discharge capacity for large return periods and service life. But a closer look at the results the risk valuves are under-estimated.

From the discussions, in summary, there are significant advantages provided by the composite risk design of spillway structure :

(i) The establishment of risk - safety factor relationships, return periods, and service life of the project which can enhance further studies in the optimisation model.

(ii) The approach of computing the risk can be extended to compute the expected damage costs and thereby incorporated into optimal risk based design procedures. This will provide proper evaluation of economic flood hazards and minimum cost spillway systems.

CHAPTER SEVEN

Conclusions and Suggestions

Based on the study reported in this thesis the following conclusions and suggestions are made:

7.1 Conclusions

1. The conventional return period method of design of spillway structures which only considers hydrologic risk underestimates the risk of failure of the structure. This may underestimate significantly the economic losses that can be associated with flood hazards.

2. The risk theory has been applied to integrate the hydrologic and hydraulic risks in the design of spillway structures. Both Static and Dynamic Models have been considered to compute the composite risk of the structure.

3. The percentage contribution of hydraulic risk to the composite risk ranges between 0.1% and 37.5% for the range of safety factor between 1.04 and 3.70 and return period between 2 and 100 years.

4. Risk - Safety factor, service life of the structure, and return periods relationships have been established which can facilitate the optimal design of spillway structures.

5. Extreme value Type 1 distribution (Gumbel) best fits the flood magnitudes of River Gongola at Dadin - Kowa.

7.2. Suggestions.

1. Because of the underestimation of risk when only hydrologic risk is considered, it is suggested that a composite risk be considered in the design of spillway structures especially for flood with large return period and high safety factor.

2. The Static Model has a short-coming since it does not take into account the repeatability of flood events. It is therefore suggested that Dynamic Model be used in the risk design of spillway structure.

3. The risk - safety factor relationships may be sensitive to different probability distributions when fitted to the flood events. It is suggested that a composite probability distribution such as using Bayesian approach be used as an alternative to the unique one used in this study.

4. Single station flood frequency analysis is associated with large sampling error. This can underestimate the parameters of the population distribution. Regional flood frequency analysis is therefore suggested to reduce this error.

5. The models considered in this study are not all inclusive in that they do not consider structural and economic links. It is suggested that further studies be carried out to integrate them for optimal risk based design of spillway structures.

REFERENCES.

America Water Works Association, (1966). Spillway Design Practise In : AWWA Design Manual, No. M13, New York.

Beard, L.R., (1962). Statistical Methods in Hydrology, In: U.S Army Corps of Engineers; Civil Works Investigation, Sacramento District, PP. 180 - 187.

Benjamin, J.R., and C.A. Cornell, (1970). Probability, Statistics, and Decision for Civil Engineers, Mc Graw-Hill Book Co., Inc., New York. PP.256 - 280.

Chow, V.T., (1959). Open Channel Hydraulics, Mc Graw Hill Book Co., N. York PP. 365 - 377.

Chow, V.T., and N. Takase,(1977). Design Criteria for Hydrologic Extremes, Journal of Hydraulics Division, ASCE, Vol. 103 (HY4) PP. 425 - 436.

Cicioni, G., G. Guiliano., and F.M. Spaziani (1973). Best Fitting of Probability Functions to a Set of Data for Flood Frequency Studies. Proc., In: Second International Symposium in Hydrology, Floods and Droughts, Water Resources Publications, Fort Collins, Colorado, PP. 304 - 314.

Creager, W.P., J. Hinds., and J.D. Justin (1964). Engineering for Dams, John Willey and Sons Book Co., Inc., Vol. II, New York. PP. 357 - 400.

Gumbel, E.J., (1955). The Calculated Risk in Flood Control,, In: Applied Science Research, The Hague, Vol. 5A, PP. 272 - 280.

Hall, W.A., and D.T. Howell, (1963). Estimating Flood Probability within Specific Time Interval, Journal of Hydrology, 1(1), PP. 265 - 271.

Hwang, N.H.C; (1981). Fundamentals of Hydraulic Engineering Systems, Prentice - Hall, Book Co., Inc., New York. P. 245.

Kapur, K.C., and L.B. Lamberson, (1977). Reliability in Engineering Design John Willey Book Co., N. York. PP. 189 - 205.

Mays, L.W., (1979). Optimal Design of Culverts under Uncertainties, Journal of Hydraulics Division, ASCE, Vol. 105 (HY6), PP. 443 - 460.

Rao, N. S. L., (1975). Theory of Weirs, Advances in Hydroscience, Vol. 10, PP. 309 - 406.

Tang, W.H., and Yen, B.C., (1972). Hydrologic and Hydraulic Design under Uncertainties, Proc., In: International Symposium on Uncertainties in Hydrologic and Water Resources Systems, Tucson, Ariz, Vol. 2. PP. 868 - 882.

Tang, W.H., L.W. Mays, and B.C. Yen., (1975). Optimal Risk - Based Design of Storm Sewer Networks, Journal of the Environmental Engineering Division, ASCE, Vol. 101 (EE 3), Proc. Paper 11360, PP. 381 - 398.

Tung Y.K., (1985). Models for Evaluating Flow Conveyance Reliability of Hydraulic Structures, Water Resources Research, Vol. 21 No. 10, PP. 1436 - 1468.

Tung, Y.K., and L.W. Mays, (1980a). Risk Analysis for Hydraulic Design, Journal of Hydraulics Division, ASCE,, Vol. 106 (HY 5), PP. 893 - 913.

Tung, Y.K., and L.W. Mays, (1981a). Risk Models for Flood Levee Design, Water Resources Research, Vol. 17, No. 4, PP. 833 - 841.

Tung, Y.K. and L.W. Mays, (1981b). Optimal Risk - Based Design of Flood Levee Systems, Water Resources Research, Vol, No.4, PP. 843 - 852.

United State Bureau of Reclamation (1974). Design of Small Dams. US Govt. Printing Office, Washington D.C. PP. 375 - 382.

Wood, E.F., and I. Rodrigues - Iturbe., (1975a). Bayesian Inference and Decision Making for Extreme Hydrologic Events, Water Resources Research, Vol. II, No. 4, PP. 533 - 542.

Yen, B.C., (1970). Risks in Hydrologic Design of Engineering Projects, Proc. Journal of Hydraulic Division, ASCE, Vol.96, (HY 4), PP. 959 - 966.

Yevjevich, V., (1978). Probability and Statistics in Hydrology, Water Resources Publications, Fort Collins, Colorado. PP. 119 - 154.

Young, G.K., M.R. Childery, and R.E. Trent, (1974). Optimal Design of Hyghway Drainage Culverts, Journal of the Hydraulic Division, ASCE, Vol. 100 (HY 7), Proc. paper 10676, PP. 971 - 996.

LIBRARY UNIVERSITY