STRUCTURAL SYNTHESIS USING STOCHASTIC PROGRAMMING

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OCTOBER, 1997

DECLARATION

I hereby declare that this thesis is my own research work and has been composed by myself and it has not been previously presented for a higher degree.

All sources of information which are not my own original work are specifically acknowledged by references.

Khanna Rachna

Octobert, 1997

CERTIFICATION

This project thesis entitled: **"STRUCTURAL SYNTHESIS USING STOCHASTIC PROGRAMMING"** by Khanna Rachna (Miss), meets the regulations governing the award of the degree of Master of Science (Structural Engineering) of Ahmadu Bello University, Zaria and is approved for its contribution to knowledge and literary presentation.

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DEDICATION

To my loving parents, Mr. G.P.Khanna & Mrs. Sudha Khanna, Uncle, Mr. S.S. Tulpule, sister, Reshma Khanna and brother, Ripul Khanna.

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I am greatly indebted to my main supervisor, Dr. J.O. Afolayan for tirelessly going through my work over and over again. His detail knowledge of the work, technical assistance to arrive at the results, his personal library and keen interest has made his guidance very helpful. The contributions in form of encouragement from my Co-supervisor and the Head of Department, Dr. S.P.Ejeh is sincerely acknowledged.

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ABSTRACT

This paper reports the investigation on the reliability levels implicit in the BS8110 requirements, probability-based optimum design and sensitivity tests for singly reinforced continuous beams. These are accomplished with the aid of the modern reliability analysis (First-Order Reliability Integration Scheme- F0RM5, Gollwitzer, et al,1988) and a Nonlinear Programming Algorithm. The design variables are the geometrical dimensions, material strengths and loading with their respective probabilistic nature considered. The limit state equations based on Ultimate and Serviceability limit states are derived and the implied safety levels for the various states computed. The results show that the criterion for bending is satisfactory with safety levels falling within an economical range earlier proposed (Moses, et al, 1990). However, the criterion for shear is inadequate yielding very low safety levels, while deflection requirement is unduly conservative.

In the optimum design the objective function was taken to be the total expected cost subjected to bending and shear constraints. Numerical values for the design variables at optimum points are proposed and the influence of the variabilities in concrete quality and live load on the objective function and the design variables is noted. It is established that valid estimates of the design parameters can be ensured if the relevant data is under control.

TABLE OF CONTENTS

CHAPTER		PAGE
Title Page		i
Declaration		ii
Certification		iii
Dedication		iv
Acknowledgement		v
Abstract		vi
Table of Contents		vii
List of Tables		x
List of Figures		xi
Notations		xiv
CHAPTER ONE		
INTRODUCTION		1
1.1	General	1
1.2	Background and Study Justification	2
1.3	Aim and Objectives of Research	3
1.4	Scope of Research	3
CHAPTER TWO		
LITERATURE REVIEW		5
2.1	Drawbacks of Structural Codes	5
2.2	Significance of Structural Reliability in Structural Engineering	6
2.3	Fundamentals of Structural Reliability Theory	8
2.4	Optimization in Structural Engineering	11
CHAPTER THREE		
METHODOLOGY		14
3.1	The Basic Reliability Problem	14
3.1.1	Normal Random Variables	16

3.2 First – Order Reliability Method 18
3.2.1 Introduction 18
3.2.2 Basic Variables and Failure Surfaces 19
3.2.3 Formulation of FORM 20
3.3 Optimum Design Concepts 25
3.3.1 Design Vector 26
3.3.2 Design Constraints 26
3.3.3 Objective Function 27
3.4 Classification of Optimization Problems 28
3.4.1 Classification based on the nature of Design 28
3.4.2 Classification based on the nature of Equations involved 29
3.4.3 Classification based on the number of Objective functions 31
3.5 NLPQL: a Fortran Subroutine Solving Constrained Nonlinear Programming Problems 32

CHAPTER FOUR

LIMIT STATE FUNCTIONS FOR SINGLY REINFORCED CONCRETE BEAMS 34
4.1 Introduction 34
4.2 Establishment of Performance Functions 35
4.2.1 Bending Criterion 35
4.2.2 Shear Criterion 37
4.2.3 Deflection Criterion 39
4.3 Statistical Characteristics of Relevant Design Parameters 40
4.4 Uncertainties in the Basic Design Variables 41
4.4.1 Variability in Strength of steel, f_y 43
4.4.2 Variability in Strength of Concrete, f_{cu} 43
4.4.3 Variabilities in member geometry 43
4.4.4 Uncertainties in Loading 44
4.5 Cost Optimization Problem 45

CHAPTER FIVE

FAILURE PROBABILITY LEVELS AND COST OPTIMIZATION 47

5.1 Introduction 47

5.2 Implied Safety Levels 47

5.3 Cost optimization and Sensitivity test 50

CHAPTER SIX

CONCLUSION AND RECOMMENDATIONS 76

6.1 Conclusion 76

6.2 Recommendations 77

REFERENCES 78

APPENDICES 82

Appendix-A : Computer programs for Reliability estimation and Optimization 82

Appendix-B : Sample Tables from computer Optimation results 92

LIST OF TABLES

4.1 Table of essential design parameters 41

LIST OF FIGURES

2.1	Optimum design process	11
3.1	Basic R - S problem: $F_R( )f_S( )$representation	15
3.2	Basic R - S problem: $f_R( )f_S( )$representation	16
3.3	Distribution of safety margin	17
3.4	A two dimensional case of limit state surface	20
3.5	Constraint and objective function surface	27
3.6	Interactive design optimization process	33
4.1	Loading patterns	34
4.2	Stress block diagram for a singly reinforced concrete beam	36
5.1	Implied safety levels with varying load ratios	49
5.2a	Breadth versus load ratio for varying Vfcu (l=3000mm and Q=2.5KN/m^2)	54
5.2b	Depth versus load ratio for varying Vfcu (l=3000mm and Q=2.5KN/m^2)	54
5.2c	Total cost versus load ratio for varying Vfcu (l=3000mm and Q=2.5KN/m^2)	54
5.3a	Breadth versus load ratio for varying Vfcu (l=4000mm and Q=2.5KN/m^2)	55
5.3b	Depth versus load ratio for varying Vfcu (l=4000mm and Q=2.5KN/m^2)	55
5.3c	Total cost versus load ratio for varying Vfcu (l=4000mm and Q=2.5KN/m^2)	55
5.4a	Breadth versus load ratio for varying Vfcu (l=5000mm and Q=2.5KN/m^2)	56
5.4b	Depth versus load ratio for varying Vfcu (l=5000mm and Q=2.5KN/m^2)	56
5.4c	Total cost versus load ratio for varying Vfcu (l=5000mm and Q=2.5KN/m^2)	56
5.5a	Breadth versus load ratio for varying Vfcu (l=6000mm and Q=2.5KN/m^2)	57
5.5b	Depth versus load ratio for varying Vfcu (l=6000mm and Q=2.5KN/m^2)	57
5.5c	Total cost versus load ratio for varying Vfcu (l=6000mm and Q=2.5KN/m^2)	57
5.6a	Breadth versus load ratio for varying Vfcu (l=7000mm and Q=2.5KN/m^2)	58
5.6b	Depth versus load ratio for varying Vfcu (l=7000mm and Q=2.5KN/m^2)	58
5.6c	Total cost versus load ratio for varying Vfcu (l=7000mm and Q=2.5KN/m^2)	58
5.7a	Breadth versus load ratio for varying Vq (l=3000mm and Q=2.5KN/m^2)	59
5.7b	Depth versus load ratio for varying Vq (l=3000mm and Q=2.5KN/m^2)	59
5.7c	Total cost versus load ratio for varying Vq (l=3000mm and Q=2.5KN/m^2)	59
5.8a	Breadth versus load ratio for varying Vq (l=4000mm and Q=2.5KN/m^2)	60

5.8b Depth versus load ratio for varying Vq (l=4000mm and Q=2.5KN/m²) 60
5.8c Total cost versus load ratio for varying Vq (l=4000mm and Q=2.5KN/m²) 60
5.9a Breadth versus load ratio for varying Vq(l=5000mm and Q=2.5KN/m²) 61
5.9b Depth versus load ratio for varying Vq (l=5000mm and Q=2.5KN/m²) 61
5.9c Total cost versus load ratio for varying Vq (l=5000mm and Q=2.5KN/m²) 61
5.10a Breadth versus load ratio for varying Vq(l=6000mm and Q=2.5KN/m²) 62
5.10b Depth versus load ratio for varying Vq (l=6000mm and Q=2.5KN/m²) 62
5.10c Total cost versus load ratio for varying Vq (l=6000mm and Q=2.5KN/m²) 62
5.11a Breadth versus load ratio for varying Vq(l=7000mm and Q=2.5KN/m²) 63
5.11b Depth versus load ratio for varying Vq (l=7000mm and Q=2.5KN/m²) 63
5.11c Total cost versus load ratio for varying Vq (l=7000mm and Q=2.5KN/m²) 63
5.12a Breadth versus load ratio for varying lengths (load ratio = 0.2) 64
5.12b Depth versus load ratio for varying lengths (load ratio = 0.2) 64
5.12c Total cost versus load ratio for varying lengths(load ratio = 0.2) 64
5.13a Breadth versus load ratio for varying lengths (load ratio = 0.5) 65
5.13b Depth versus load ratio for varying lengths (load ratio = 0.5) 65
5.13c Total cost versus load ratio for varying lengths(load ratio = 0.5) 65
5.14a Breadth versus load ratio for varying lengths (load ratio = 1.0) 66
5.14b Depth versus load ratio for varying lengths (load ratio = 1.0) 66
5.14c Total cost versus load ratio for varying lengths(load ratio = 1.0) 66
5.15a Breadth versus load ratio for varying lengths (load ratio = 1.5) 67
5.15b Depth versus load ratio for varying lengths (load ratio = 1.5) 67
5.15c Total cost versus load ratio for varying lengths(load ratio = 1.5) 67
5.16a Breadth versus load ratio for varying lengths (load ratio = 2.0) 68
5.16b Depth versus load ratio for varying lengths (load ratio = 2.0) 68
5.16c Total cost versus load ratio for varying lengths(load ratio = 2.0) 68
5.17a Breadth versus Vq for varying lengths (load ratio = 0.2) 70
5.17b Depth versus Vq for varying lengths (load ratio = 0.2) 70
5.17c Total cost versus Vq for varying lengths (load ratio = 0.2) 70
5.18a Breadth versus Vq for varying lengths (load ratio = 0.5) 71

5.18b	Depth versus Vq for varying lengths	(load ratio = 0.5)	71
5.18c	Total cost versus Vq for varying lengths	(load ratio = 0.5)	71
5.19a	Breadth versus Vq for varying lengths	(load ratio = 1.0)	72
5.19b	Depth versus Vq for varying lengths	(load ratio = 1.0)	72
5.19c	Total cost versus Vq for varying lengths	(load ratio = 1.0)	72
5.20a	Breadth versus Vq for varying lengths	(load ratio = 1.5)	73
5.20b	Depth versus Vq for varying lengths	(load ratio = 1.5)	73
5.20c	Total cost versus Vq for varying lengths	(load ratio = 1.5)	73
5.21a	Depth versus Vq for varying lengths	(load ratio = 2.0)	74
5.21b	Total cost versus Vq for varying lengths	(load ratio = 2.0)	74
5.21c	Total cost versus Vq for varying lengths	(load ratio = 2.0)	74

NOTATIONS

A_S = Cross-sectional area of main reinforcement

A_{sv} = Cross sectional area of shear reinforcement

B = Total benefits

b = Width of beam

C_f = Failure Cost

C_I = Initial Cost

C_T = Total Cost

COV = Coefficient of variation

D = Distance from the origin of the redced variate space to the failure surface

d = Effective depth of beam

E(X) = Expected value of random variable X

F_{cc} = Compressive force in concrete

f_{cu} = Characteristic strength of concrete

f_{st} = Characteristic strength of steel

f_s = Yield stress

$F_x(x)$ = Joint distribution function of x_i

$f_x(x)$ = Joint pdf of x_i

f_y = Characteristic strength of main reinforcement

f_{yv} = Characteristic strength of shear reinforcement

G_k = Ultimate dead load on beam

G' = Gradient function

G(X) = Limit state function / Safety margin

h = Overall depth of beam

L = Effective span of beam

M_{ai} = Maximum applied moment in bending

M_R = Ultimate moment of resistance

N_i = Design load on beam

p_f = Probability of failure

p_{fi}^T = Target failure probabilities of i criteria

Q_k = Live load on beam

R = Structural resistance or calculated resistance

S = Calculated load effect

S_v = Spacing of shear reinforcement

V = Critical shear force

V_{fcu} = Coefficient of variation of concrete

V_q = Coefficient of variation of live load

x = Depth of copressive stress block

x_i = Variates

x_i' = Reduced variates

Z = Lever arm

α = Ratio of dead load to live load

β = Safety Index

β_r = Redistribution factor

δ = Basic variability of observed data

Δ = Error in prediction of the mean value

γ = Ratio of intrisic shear strength of concrete to the applied shear stress

μ_i = Mean value of basic variable i

Φ = Standard normal interal

σ_i = Standard deviation of i

λ = Lagrange's multiplier

λ_F = Failure cost ratio

λ_T = Total cost ratio

υ = Applied shear stress

υ_c = Shear strength of concrete

Ω = Total uncertainity

ω = n-dimensional basic variable space

1
CHAPTER

INTRODUCTION

CHAPTER ONE

INTRODUCTION

1.1 General

A Structural Engineer basically aims at providing a structure which satisfies the criteria of safety and serviceability at reasonable cost of construction and to withstand dead and live loads, and some unforeseeable disturbances the structure will be subjected to during its intended useful life. This, however, must be achieved under certain economic and functional constraints.

Structural safety deals with the ability of a structure to withstand its weight and applied loads without unwarranted risks to human lives; while serviceability requirements of the structure insist on its fitness for use under working loads without excessive deflections, disturbing vibrations, noticeable cracking and other local failures, which may demand costly remedial work.

To ensure safety, it is necessary to provide a structure with sufficient strength and to safeguard its serviceability, with adequate stability. In order to assess the strength and the adequacy of its stability, mathematical models are developed for the analysis of load combinations and other structural design parameters which are stochastic by nature.

It is known that uncertainties exist in most areas of civil and structural engineering and rational design decisions cannot be made without modelling them and taking them into account. Some of these uncertainties include: uncertainties due to stochastic nature of loading, uncertainties associated with structural resistance, uncertainties due to human factor, modelling uncertainties, prediction uncertainties, statistical uncertainties etc. In a systematic manner, these uncertainties are not put into consideration, at least when designing simple structures because of the prescriptive and deterministic nature of codes of practice. This, however, is inappropriate because most loads and other structural design parameters are

rarely known with certainty and should be regarded as random even if in design calculations they are eventually treated as deterministic. A deterministic code employs factors that operate on the basis that the load and material strengths are constant, thus, inferring absolute safety. Absolute safety is of course unattainable; and such a goal is also undesirable, since absolute safety could be achieved only by deploying infinite resources. Thus, in modern engineering practice, engineers rely on mathematical and scientific methods to achieve some trade-off between safety and economy.

1.2 Background and Study Justification

In the past structural engineering was dominated by the use of deterministic thinking, characterized in design calculations by the use of specified minimum material properties, specified load intensities to compute stresses and deflections. Later, engineers and researchers observed several inadequacies with the deterministic design concept.

Firstly, the deterministic approach, to a large extent, has been codified using safety factors and as such there is a lack of feedback about the actual performance of structures. For example, actual stresses are rarely known, deflections are rarely observed or monitored, and since most structures do not collapse, the real reserves of strengths are generally not known.

Secondly, there has been a vast array of new structural materials from the aerospace industry which not only exhibited in the mean improved strength and stiffness or weight characteristics but also showed an increased variation as compared to conventional construction materials. This greater variation necessitated such high factors of safety that the improved properties of the materials were largely left unrealized. Also, there had been an increase in the extremes of the structural load environment. Nuclear power reactors, undersea and space exploration presented the designer with a broader load spectrum than the ones encountered. The luxury of picking the worst possible load condition and designing for that load was no longer

economically feasible under broad spectrum load conditions.

A third factor concerns the economy in terms of cost. The imprecise and sometimes erroneous factor of safety sometimes leads to low risk design, thus yielding uneconomical structures.

Consequently, the lack of information about the actual behaviour of structures combined with the use of codes embodying relatively high safety factors lead to the development of the more rational design approach known as the "Structural Reliability Theory."

In essence, to achieve an optimum structural design taking into consideration the uncertainties, risks involved and limited resources, a reliability based analysis is imperative to this study.

1.3 Aim and Objectives of Research

It is the intent of this research to develop a stochastic programming model for the synthesis of structural members. The research will be expected to achieve the following objectives:

1. Development of mathematical models to represent member behaviour.
2. Derivation of stochastic models for various modes of failure.
3. Computation of the safety indices for the failure modes, using appropriate solution techniques.
4. Verification of consistency of the safety indices and check whether they are within established design criteria.
5. Derivation of optimum structural dimensions at minimum cost and target safety level.

1.4 Scope and Methodology of Research

Structural synthesis using stochastic programming comprises the analysis and design of the various structural elements such as slabs, beams, columns, etc. when all the decision variables take values in a non-orderable

data space. This research work will be limited to the synthesis of continuous reinforced concrete beams. However, the given procedures or methodology can be used for other structural members.

The following procedure will be used to develop a stochastic programming model for the analysis of continuous reinforced concrete beams:

1. Limit state equations are derived for different failure modes.
2. These limit state equations are analyzed using the computer application programmes (First-Order Reliability Method, Gollwitzer, et al, (1988), and Non-Linear Programming Algorithm).
3. The safety indices are computed and checked / verified for consistency or otherwise.
4. Numerical values for the design variables at optimum point are proposed so that safety and economy are traded off.

2
CHAPTER
LITERATURE REVIEW

CHAPTER TWO

LITERATURE REVIEW

2.1 Drawbacks of Structural Codes

Most countries have standards that must be complied with by every designer in order to meet the strength, serviceability and other important load requirements. These standards are published in the form of Codes of Practice and at intervals changes or some major revisions are made to allow for: new types of structural forms (eg. reinforced masonry), the effects of improved understanding of structural behaviour (eg. of stiffened plated structures), the effects of changes in manufacturing tolerances or quality control procedures, a better knowledge of loads, etc. (Thoft-Christensen and Baker, 1982).

In Europe, the latest Structural code of Practice BS8110 (1985) is a revised version of CP110 (1972). Like other codes, BS8110 also follows the deterministic format in which the nominal strengths of the structural members designed to that code are governed by a number of partial coefficients or by equivalent means.

A number of drawbacks associated with structural codes have been discussed by Geschwindner (1988). Some of these drawbacks are listed below:

1. Structures are designed on the basis of codes which have known to be satisfactory by past experience and engineering judgement rather than by the understanding of the uncertainties that influence the strengths and loads of the structures.
2. The use of single-valued safety factors as incorporated in the deterministic codes imply that ability to predict loads and strengths are invariant. This is, however, untrue since some loads and strengths are more predictable than others (Viest, 1981).

3. Limit state design philosophy uses the concept of probability and therefore recognizes that there is a chance that a structure becomes unfit for use, that is, there is a small but finite probability that a particular limit state can be exceeded, however, there is no attempt to compute such a probability (Afolayan, 1992).
4. The imprecise and sometimes erroneous factor of safety gives uneconomical design in terms of cost.

2.2 Significance of Structural Reliability in Structural Engineering

The subject "Structural Reliability" has grown rapidly over the years and has evolved from academic research to a set of well-developed methodologies with a wide range of practical applications. This analysis is concerned with the rational treatment of uncertainties in structural engineering design and with methods for assessing the safety and serviceability of civil engineering and other structures. Its application to the design of single components is well established for both code writers and designers and it has also become an integral part of the risk assessment and management procedures for a wide variety of structures (Cazzulo, 1987) including such risk-sensitive structural systems as Nuclear Power Plants (Guerpinar and Gailiani, 1987; Jaeger, 1972).

Structural Reliability provides the following unique and significant advantages for safety analysis and design:

1. It provides a consistent basis for the systematic assessment and analysis of uncertainties, in a manner suitable and convenient for evaluating effects of uncertainty on safety and design; and
2. The probabilistic risk and reliability measures can be subjected to logical operations via the mathematical theory of probability, thus permitting a formal approach to the analysis of safety and performance of structures and structural components.

Ang and Amin (1969), studied the safety or reliability of a structure formulated in terms of an uncertainty factor combined with a generalized allowable risk. The uncertainty factor allows for the consideration of incomplete information and imperfect knowledge in the specification of a permissible risk level. Therefore, the real and significant roles of probability in structural engineering lie in its logical framework for uncertainty analysis and quantitative basis for risk and safety assessment.

Ott and Shah (1969), studied reliability under stochastic loads and resistances. Their aim was to develop mathematical tools to investigate the reliability of structural systems under probabilistic and stochastic loading. They found that derivation of the reliability distributions was fairly simple where only one parameter was a random variable. But it was not possible to find a useable analytical solution for the distributions chosen. Therefore, approximations were suggested.

Various studies have been carried out where the objective was to evaluate the maximum of the combined load effects during an anticipated useful life of the structure with different design criteria and constraints (Hilton and Fiegen, 1960; Moses and Kinser, 1967; Surahman and Rojiani, 1983; Frangopol, 1985 etc.) and a large number of applications are found, for example, in Thoft- Christensen and Morutsu, (1986) and Brebbia and Hernandez, (1989).

Apart from all the merits this method possesses, Shinozuka (1969) outlined some aspects which still need to be resolved. They include that:

1. It is not easy to obtain a consensus on the target levels of reliability even for traditional civil engineering structures.
2. Not everything is always probabilistic, although the reliability analysis methodology developed so far presumes that we can somehow formulate everything in terms of probability.
3. Even if it is agreed that everything can be interpreted as probabilistic, the casual fashion in which the source of the

variability is often divided into "uncertainty" could give the false impression that such a division is easy, while certainly it is not.

4. *The reliability methodology has so far been unable to properly* incorporate human, managerial as well as technical factors.

With time, in an effort to collect pertinent data, better statistical information will be available (although the uncertainty in the choice of a distribution function and the possibility of error in the determination of the parameters will remain for some time).

2.3 Fundamentals of Structural Reliability Theory

The principles of reliability analysis have been applied to a very large class of problems, ranging from the design of control systems for complex nuclear and chemical plants to the design of specific mechanical and structural systems, as well as more generally, in the field of electronics and aero-space. Its development in structural systems is towards its use particularly in the form of codified designs and primarily on methods of safety checking. These developments are readily available in the works of Freudenthal, 1947; Cornell, 1967; Shinozuka, et al, 1968; Moses and Stevenson, 1970; Ang, 1973; Ravindra, et al, 1974; etc.

Structural reliability, as previously defined, is the probability that a structure will not attain each specified limit state (ultimate or serviceability) during a specified reference period. Failure of a structure, generally means the realization of a specified limit state, including collapse. Structures or structural components fail when they encounter an extreme load, or when a combination of loads causes an extreme load effect of sufficient magnitude for the structure to attain failure state which may be an ultimate or serviceability condition. Part of the problem encountered above, is to predict the magnitude of these extreme events. The other part is to predict the strength or load-deflection characteristics of each structural component from the information

available at the design stage. In other words, it is necessary to synthesize probabilistic models for the two parts of the problem, including, on the one hand, all the uncertainties affecting the loading, and on the other, all the uncertainties affecting the strength or resistance of the structures.

Failure can be defined as:

$$R \leq S \tag{2.1}$$

where R = Structural resistance or calculated resistance, and

S = calculated load effect.

Both R and S are independent normally distributed variables, and the uncertainties are strictly those associated with the inherent randomness.

With simplification, the reliability index of a component can be calculated by defining the failure equation,

$$g = R - S \leq 0 \tag{2.2}$$

where g is known as the safety margin such that a realization of $g < 0$ is failure.

The reliability or safety index, β is defined as:

$$\beta = \frac{\mu_g}{\sigma_g} \tag{2.3}$$

where μ_g is the mean of safety margin, and

σ_g is the standard deviation of safety margin.

In other words, safety index, β is the number of standard deviations for which the mean margin of safety exceeds zero. In simple cases, where both R and S are normally distributed, β provides an exact risk (failure probability). In complex situations, several methods such as the First Order Reliability Method have been developed to obtain more accurate safety index and will be discussed in details in chapter 3.

In 1975, The Joint Committee on Structural Safety (an International body sponsored by such international organizations as CEB, CECM, IABSE, IASS, FIP and RILEM) set up a subcommittee to provide a broad classification

system for the different methods for checking the safety of structures and to establish the main differences between them. These methods of safety analysis grouped under the general heading of reliability theory, are categorized into three levels namely: levels 1, 2 and 3, depending on the degree of sophistication applied to the treatment of the various problems. These methods of structural analysis, as defined by Thoft-Christensen and Baker (1982) are:

LEVEL 1: Includes design methods in which appropriate degrees of structural reliability are provided on a structural element basis (occasionally on a structural basis) by the specification of partial safety factors, or partial coefficients, related to pre-defined characteristic or nominal values of the major structural and loading variables.

LEVEL 2: Includes methods involving certain approximate iterative calculation procedures to obtain an approximation to the failure probability of a structure or structural system, generally requiring an idealization of failure domain and often associated with a simplified representation of the joint probability distribution of the variables.

LEVEL 3: Includes methods in which calculations are made to determine the "exact" probability of failure for a structure or structural component, making use of a full probabilistic description of the joint occurrence of the various quantities which affect the response of the structure and taking into account the true nature of the failure domain.

The level 1 methods are those already in use for certain codes of practice. According to Thoft-Christensen and Baker (1982), level 1 methods are a discretisation of level 2 methods (i.e. giving identical designs to level 2 methods for only a few discrete sets of values of the structural design parameters). While level 2 methods are an approximation to level 3 methods. In this study, level 2 methods will be employed because they are efficient and they produce results with the desired degree of accuracy.

2.4 Optimization in Structural Engineering

It is a challenge for engineers to design efficient and cost-effective systems without compromising their integrity. The conventional design process depends on the designer's intuition, experience and skill. This overwhelming presence of a human element can sometimes lead to dangerous and erroneous results in the synthesis of complex systems.

Scarcity and need for efficiency in today's competitive world has forced engineers to evince greater interest in economical and better designs. With recent advances in computer technology affecting various disciplines of engineering, the design process can hardly remain untouched. Recently, the term computer-aided design optimization (CADO) has been used for summarizing all computer aids in design. Fig. 2.1 shows the optimum design process. Design is not only the more or less intuitively guided creation of new information but is comprised of analysis, presentation of results, simulation and optimization. These are essential constituents of an iterative process leading to a feasible and finally optimum design (Arora, 1989).

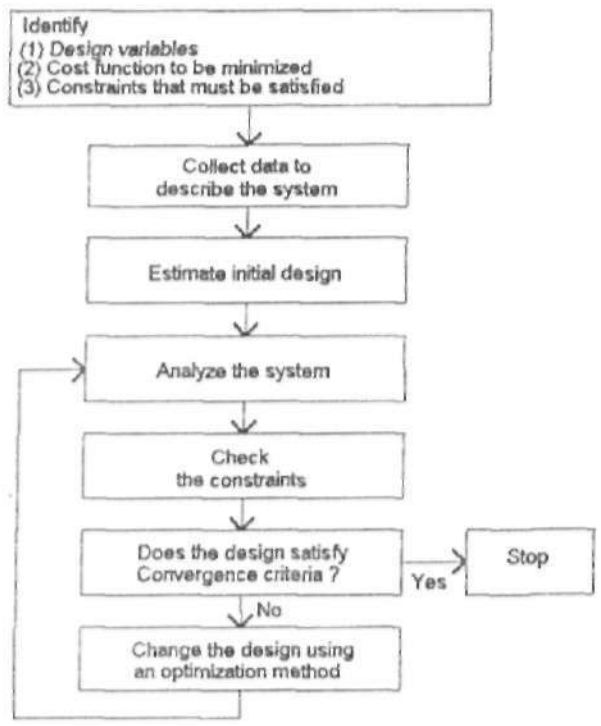


Fig. 2.1 : Optimum design process

Structural optimization can be achieved either within a deterministic design format or one based on reliability. The deterministic design philosophy uses some specified safety factors for both loads and material strengths against the failure of a structure. Using safety factors ensures a satisfactory performance under working loads, but only presupposes a reasonable margin against failure. Load factors, on the other hand, ensures a definite factor against failure and assumes a satisfactory performance under working loads. This approach has a number of limitations. For example, a major limitation is that, the safety factor, which can have a much greater effect on cost or weight than any of the subsequent results of mathematical programming techniques, is not included in the optimization procedure, either as a constraint or as part of the objective function. Also this method does not take into consideration the stochastic nature of loads and material strengths and safety is not specified in terms of risk value.

In the presence of statistical variabilities, as evidenced in loads (eg. live loads, wind loads, thermal loads, earthquakes, machine operational loads, etc.) and material strengths (eg. concrete and steel strength), a proper and more realistic evaluation of structural safety requires explicit consideration of probabilities. In other words, through a reliability based design. Some reliability based structural optimization schemes include:

1. Minimization of the cost or weight under the structural reliability constraint.
2. Minimization of the probability of failure for a fixed cost and
3. Minimization of the total overall cost (Surahman & Rojiani, 1983).

Most optimization studies consider the first two procedures due to the difficulties in determining the nature of failures and assigning monetary values to failure consequences.

In the past structural optimization was based on one constraint. Turner, (1969), developed a technique for determining relative proportion of selected elements of wing structures to attain a specified flutter speed with

minimum total mass. Ashley, et al, (1970), reported the one-dimensional structural optimization like beams under aeroelastic constraints. Kicher, (1968), obtained the minimum weight design of integrally stiffened cylindrical shells with constraints on buckling loads. Ding and Espieng, (1985), presented the minimum weight design problem for frame structures subject to stress and displacement constraints. Topping and Robinson, (1984), illustrated how the sequential linear programming technique can be used to design timber frames in accordance with the British code of practice, CP 112 (1971). They emphasize a practical method with results that comply with the code without alteration.

This research work includes the optimal design of statically indeterminate elastic beams for specified strength and stiffness requirements. The term optimal design, will depict the objective function which is the total design cost under the constraints on the failure probabilities of the critical sections in the continuous beam. The configuration of the loading conditions and material strengths together with their probabilistic nature are assumed to be known. Strength requirements are usually specified to be the maximum flexural and shear stresses within the beam. The allowable values of these stresses are indicated in the codes of practice. Likewise the stiffness requirements or serviceability requirements are prescribed to restrict the deflections of the deformed structure to values specified in the codes.

3
CHAPTER
METHODOLOGY

CHAPTER THREE

METHODOLOGY

3.1 The Basic Reliability Problem

As earlier mentioned, the basic reliability problem considers only one load effect, S, resisted by one resistance R. Each of S and R is described by a known probability density function, $f_s(s)$ and $f_r(r)$ respectively. A structure is considered to have failed if its resistance R is less than the load S (Melchers, 1987).

The probability, p_f of failure can be stated in any of the following ways:

$$p_f = p(R \leq S) \tag{3.1a}$$

$$= p(R - S \leq 0) \tag{3.1b}$$

$$= p(R/S \leq 1) \tag{3.1c}$$

$$= p(\ln R - \ln S \leq 0) \tag{3.1d}$$

or in general:

$$p_f = [G(R, S) \leq 0] \tag{3.1e}$$

where G( ) is the limit state function.

For a simple structural member with a known distribution function, F_R of ultimate strength, R, and F_S of the load effect S, equation (3.1b) is replaced by;

$$P_f = P(R-S \leq 0) = \int_{-\infty}^{+\infty} F_R(x)\, f_s(x)\, dx \tag{3.2}$$

under the condition that R and S are statistically independent. This integral is also known as a "Convolution Integral" which is easily understood by Fig. 3.1. (Melchers, 1987).

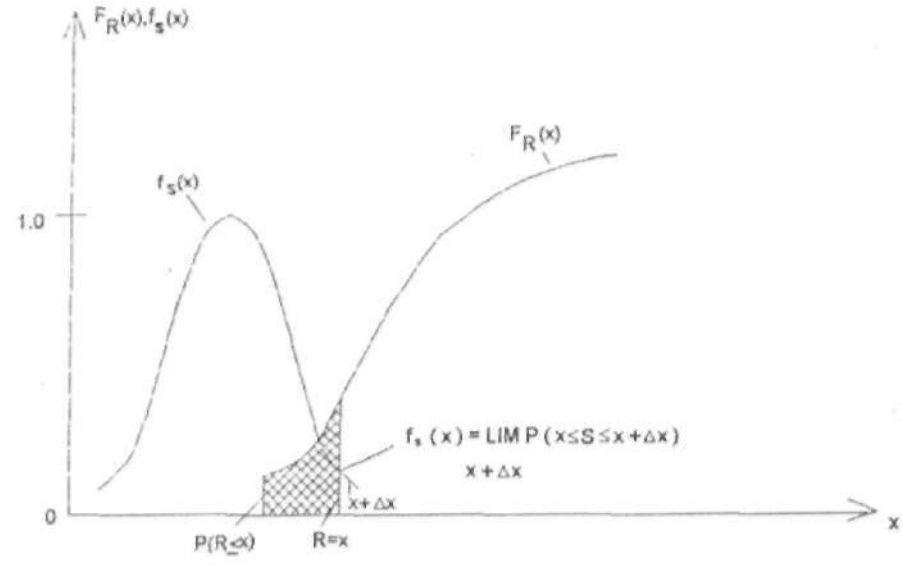


Fig. 3.1: Basic R-S problem: $F_R(\ )f_s(\ )$ representation

In equation (3.2), $F_R(x)$ is the probability that $R \le x$, or the probability that the actual resistance R of the member is less than some value x. Assuming this represents failure, $f_s(x)$ is the probability that the load effect, S acting in the member has a value between x and $x + \Delta x$ in the limit as $\Delta x \to 0$.

The total failure probability is obtained by considering all possible values of x, i.e. by taking the integral over all x. For a better understanding, the density functions of R and S are plotted in Fig. 3.2. It should be noted that R and S must necessarily have the same dimensions (e.g. loads and load-carrying capacities or bending moments and flexural strengths).

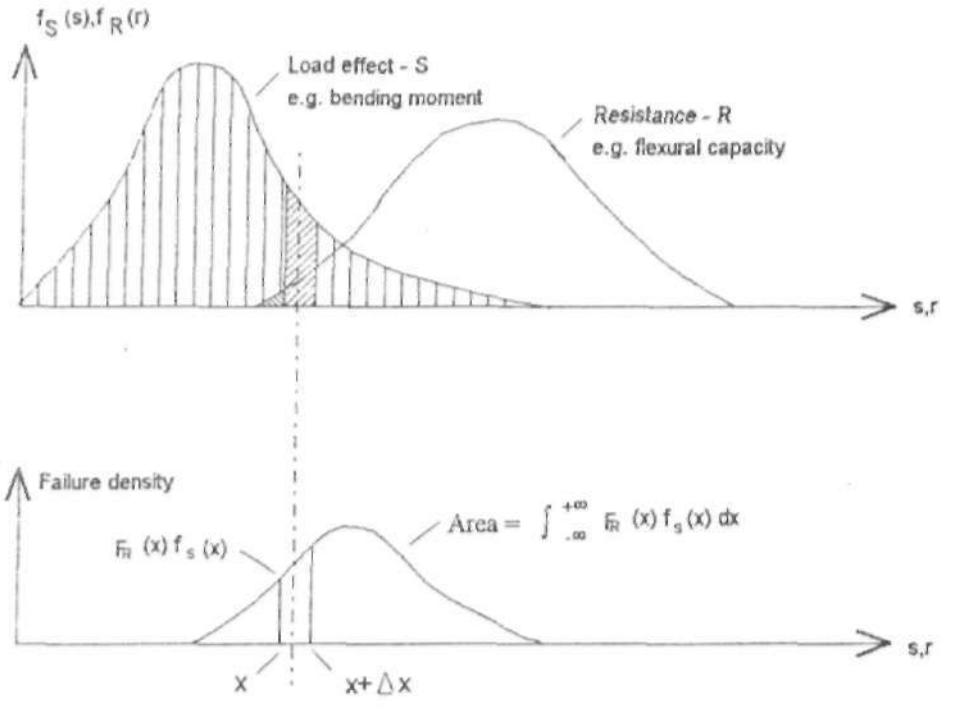


Fig. 3.2: Basic R-S problem: $f_R(\ ) f_S(\ )$ representation

An alternative to equation 3.2 will be

$$p_f = \int_{-\infty}^{+\infty} [1 - F_s(x)]\, f_R(x)\, dx \qquad (3.3)$$

which is simply the sum over all the cases of resistance for which the load exceeds the resistance.

3.1.1 Normal Random Variables

It is possible to integrate the convolution integral for a few distributions of R and S. The most notable example is when R and S are normal random variables with means μ_R and μ_S and variances σ_R^2 and σ_S^2 respectively. The safety margin will be:

$$M = R - S \qquad (3.4)$$

Then, the mean and variance will be given by:

$$\mu_M = E(M) = \mu_R - \mu_S \tag{3.5}$$

$$\sigma_M^2 = \mathrm{Var}(M) = \sigma_R^2 + \sigma_S^2 \tag{3.6}$$

giving.

$$\sigma_M = \sqrt{(\sigma_R^2 + \sigma_S^2)} \tag{3.7}$$

Since R and S are normal, M, a linear function of R and S, is also normally distributed and $(M - \mu_M)/\sigma_M$ is unit standard normal, giving

$$p_f = \Phi\left(\frac{0 - \mu_M}{\sigma_M}\right) = \Phi\left[\frac{\mu_S - \mu_R}{(\sigma_R^2 + \sigma_S^2)^{1/2}}\right] \tag{3.8}$$

where, Φ is the standard normal distribution function and μ_S, μ_R, σ_S and σ_R are respectively the means and standard deviations of S and R.

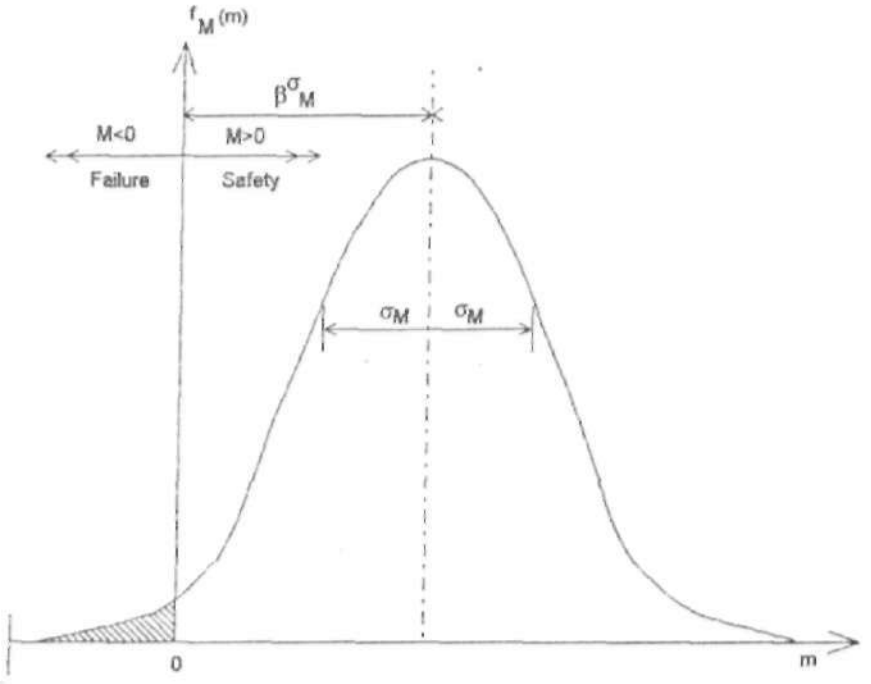


Fig. 3.3: Distribution of safety margin.

The reliability Index, β may now be defined as the ratio μ_M/σ_M or the number of standard deviations by which μ_M exceeds zero. See Figure 3.3. Then

$$p_f = \Phi\left(\frac{-\mu_M}{\sigma_M}\right) = \Phi(-\beta) \qquad (3.9)$$

Any increment made in either σ_s or σ_R or both, will reduce the term in the square brackets in equation 3.8, and hence p_f will increase, as might be expected. Similarly, if the resistance is reduced, p_f increases.

For many problems this simple formulation for p_f is not entirely adequate, since it may not be possible to reduce the structural reliability problem to a simple R versus S formulation with R and S independent random variables. In general, equation 3.2 needs to be evaluated by numerical methods.

3.2 First – Order Reliability Method

3.2.1 Introduction

In the above section an introduction to the fundamental concepts of structural reliability was given. In this section, one of the level 2 methods known as First-Order Reliability Method is treated, as it is one of the methods used in this research work to achieve the aim.

The First-Order Reliability Method, previously known as the Advanced Second Moment Method, was proposed in the late sixties. At first this method could efficiently calculate the probability of failure of single structural elements subjected to loads of which no more than one varies with time. But in recent years, its application has gained a wider spectrum of systems (e.g. offshore structures, portal frames) and load combinations. Examples of these applications can be found in publications by Rackwitz, 1976; Rackwitz and Fiessler, 1978; Fiessler et al, 1979; Manner, 1989; Thoft-Christensen and Baker, 1982; Madsen, Krenk and Lind, 1986; Ditlevsen and Bjerager, 1986; and many more.

3.2.2 Basic Variables and Failure Surfaces

Before the reliability of a given structure can be evaluated, it is essential to first decide upon which variables, (quantities or parameters) are of relevance. These variables, referred to as basic variables, can be geometrical quantities (e.g. the area of the cross-section of a beam), material strength (e.g. rupture or yield strength and external loads like traffic loads and wind loads).

For a given structure each variable x_i, $i = 1, 2, \ldots, n$ is considered a realization of a random variable X_i, $i = 2, \ldots, n$ and therefore the set of variables $x = (x_1, \ldots, x_n)$ is a realization of the random vector $\bar{X} = (X_1, \ldots, X_n)$. In other words, the variable x is a point in an n-dimensional basic variable space. In this method only the expectations

$$\mu_i = E(X_i), \qquad i = 1, 2, \ldots, n \tag{3.10}$$

and the covariances

$$C_{ij} = \mathrm{Cov}[X_i, X_j], \qquad i,j = 1, 2, \ldots, n \tag{3.11}$$

are used.

Also the $\mathrm{Var}[X_i] = \sigma_i^2 = \mathrm{Cov}[X_i, X_j]$.

In an n-dimensional basic variable space ω, with a set of basic variables $\bar{X} = (X_1, \ldots, X_n)$, a failure surface (or limit state surface) is defined as the surface dividing the basic variable space into two regions namely a failure region ω_f and a safe region ω_s. The failure region contains all realizations of $\mathbf{X}$ that would result in failure and the safe region contains all realizations of $\mathbf{X}$ that would not result in failure. Assuming the failure surface described by:

$$f(\bar{x}) = f(x_1, x_2, \ldots, x_n) \tag{3.12}$$

is chosen in such a way that positive values of f indicate safe sets of basic

variables (the safe region) and the negative values of f indicate unsafe set of variables (failure region). i.e.

$$f(\bar{x}) = \begin{cases} > 0 & when\ \bar{x} \in \omega_s \\ \leq 0 & when\ \bar{x} \in \omega_f \end{cases} \qquad (3.13)$$

A two dimensional case is illustrated in Fig. 3.4.

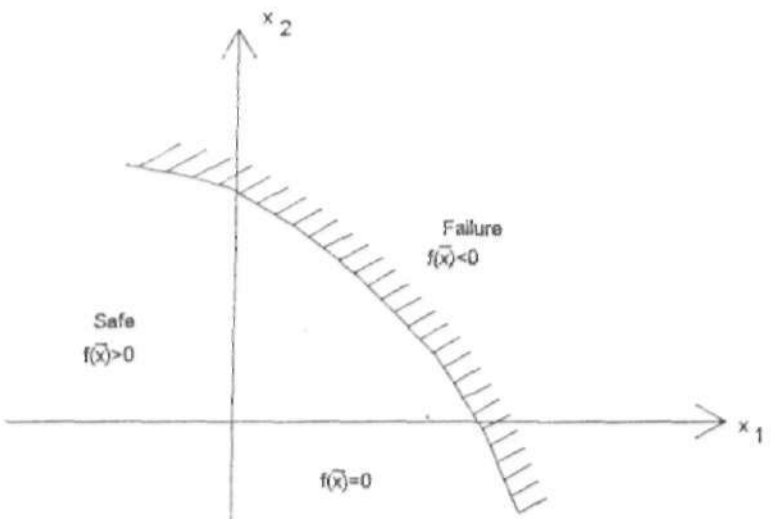


Fig. 3.4: A two dimensional case of limit state surface

According to Thoft-Christensen and Baker, (1982), reliability index, β, can be determined for linear and non-linear failure functions with normal, log-normal, uncorrelated etc. basic variables.

3.2.3 Formulation of FORM

In the First-Order Reliability Method, the performance function $G(X)$ is linearized at some point on the failure surface using the Taylor series expansion, rather than at the mean. The linearizing point is known as the design or checking point (Afolayan, 1994). Clearly, for non-linear failure functions,

calculation of the reliability index, $\beta = \mu_R/\sigma_R$ on the basis of a linearization, will depend on the choice of linearization point.

For a situation where G(X) is linear and uncorrelated, the reliability index, β can be calculated by the following:

1. Obtaining a reduced, normally distributed variable expression for each variate.

Let the variates be:

$$X_i = X_1, X_2, \ldots\ldots, X_n \tag{3.14}$$

Therefore the reduced variables will be

$$X_i' = X_1', X_2', \ldots\ldots, X_n' \tag{3.15}$$

where

$$X_i' = \frac{X_i - \mu_{X_i}}{\sigma_{X_i}}, \quad i = 1,2,\ldots,n \tag{3.16}$$

Therefore,

$$X_i = \sigma_{X_i} X_i' + \mu_{X_i} \tag{3.17}$$

2. Generating the limit state equation in terms of the reduced variates:

$$G(X) = G(\sigma_{X_1} X_1' + \mu_{X_1}, \sigma_{X_2} X_2' + \mu_{X_2}, .., \sigma_{X_n} X_n' + \mu_{X_n}) = 0 \tag{3.18}$$

3. Developing an expression to define the distance from the failure surface to the origin of the reduced variate space. Let the distance in question be D.

Then

$$D = \sqrt{(x_1')^2+(x_2')^2+\cdots\cdots+(x_n')^2} \tag{3.19}$$

In matrix form:

$$D = (x_i'^t\, x_i')^{1/2} \tag{3.20}$$

4. The point on the failure surface that has the minimum distance to the origin may by determined by minimizing the function D with the constraint $G(X) = 0$. For this purpose, the method of Lagrange's multiplier may be used. That is,

$$1' = D+\lambda G(x_i) \tag{3.21}$$

where λ = the Lagrange multiplier, then

$$\frac{\partial 1'}{\partial x_i'} = \frac{x_i'}{D}+\lambda\frac{\partial G(x_i)}{\partial x_i'} = 0 \tag{3.22}$$

and

$$\frac{\partial L}{\partial \lambda} = G\,(x_i) = 0 \tag{3.23}$$

The solution to equations (3.22) and (3.23) will give the most probable minimum distance point x'^* from the origin of the x'_i coordinate system.

5. The minimum distance is obtained by introducing the gradient vector

$$G' = \frac{\partial G}{\partial x_i'} \quad . \tag{3.24}$$

and together with appropriate matrix representations of D; in equation (3.20), equation (3.22) can be expressed for D in the form

$$D = \frac{-G'^t x'}{(G'^t G')^{1/2}} \tag{3.25}$$

where G'^t is the transpose of the gradient vector G', which in effect represents the reliability index as previously affirmed. Hence,

$$\beta = \frac{-G'^t * G'^*}{(G'^t * G'^*)^{1/2}} \tag{3.26}$$

The reliability index is also defined in terms of the multivariate performance function G as

$$\beta = \frac{\mu_G}{\sigma_G} \tag{3.27}$$

where μ_G and σ_G are the mean and standard deviation of the joint probability distribution of the variables of the performance function G(X).

Hence the probability of failure P_f can be represented as:

$$P_f = \Phi\ (-\beta) \tag{3.28}$$

where $\Phi(.)$ is the standard normal integral.

In general, the performance function $G(X)$ is nonlinear in the basic variables X_i, therefore the first order reliability method involves the linearization of $G(X)$ to arrive at the reliability index, β. This is achieved by expanding $G(X)$ as a First Order Taylor series about some point X^* on the failure surface and retaining only the linear terms (Thoft-Christensen and Baker, 1982).

$$G(X_1, X_2, \ldots, X_n) = G(X_1^*, X_2^*, \ldots, X_n^*) + \sum_{i=1}^{n} (X_i - X_i^*) \frac{\partial G(X_i)}{\partial X_i} \qquad (3.29)$$

but at the failure surface $G(X_1^*) = 0$, therefore,

$$G(X_1, X_2, \ldots, X_n) = \sum_{i=1}^{n} (X_i - X_i^*) \frac{\partial G(X_i)}{\partial X_i} \qquad (3.30)$$

Introducing the reduced variate,

$$X_i - X_i^* = (\sigma_{X_i} X_i' + \mu_{X_i}) - (\sigma_{X_i} X'^*_i + \mu_{X_i}) = \sigma_{X_i} (X_i' - X'^*_i) \qquad (3.31)$$

Also,

$$\frac{\partial G}{\partial X_i} = \frac{\partial G}{\partial X_i'} \cdot \left[\frac{\partial X_i'}{\partial X_i}\right] = \frac{1}{\sigma_{X_i}} \cdot \left[\frac{\partial G}{\partial X_i'}\right] \qquad (3.32)$$

$$G(X_i) = \sum_{i=1}^{n} (X_i' - X'^*_i) \frac{\partial G}{\partial X_i'} \qquad (3.33)$$

Noting that the parameters of $\mathbf{x}_i'$ are N(0,1),

$$\mu_G = -\sum_{i=1}^{n} x'^*_i \frac{\partial G}{\partial x_i'} \tag{3.34}$$

$$\sigma_G = \sum_{i=1}^{n} \sigma^2_{x_i'} \left(\frac{\partial G}{\partial x_i'} \right)^2 = \sum_{i=1}^{n} \left(\frac{\partial G}{\partial x_i'} \right)^2 \tag{3.35}$$

Even though the basic variables in equations (3.34) and (3.35) are assumed to be normally distributed and uncorrelated, the Taylor's expansion of $G(\mathbf{X})$ about the means yield quite accurate estimates of β. However, if $G(\mathbf{X})$ contains non-normally distributed and often correlated variables, the accuracy of the estimates of β will depend largely on the choice of the point of linearization. The coded algorithm employed, FORM5 (Gollwitzer, et al, 1988), solves this problem by introducing appropriate transformations, converting all variables into uncorrelated normal variables, subsequently linearizing the *emergent non-linear function* $G(X)$ *about a suitable point* $\mathbf{x}^*$ *selected through an* optimization procedure, and hence p_f is evaluated using the standard normal integral.

3.3 Optimum Design Concepts

Optimum design can be viewed as the minimization of some desired mathematical functions found in design problems. To achieve an intelligent design, the purpose of the design must be known in the face of any restrictions placed on it. To formulate an optimum design problem it is important to identify the set of variabilities that describe the system. Once the variables are given numerical values, we have a design of the system.

All systems are designed to perform within a set of constraints which include limitations on resources, material failure, response of the system,

member sizes and so on. The constraints must be influenced by the design variables of the system, because only then can they have a feasible (workable) system.

A criterion is needed to judge whether or not a given design is better than another. This criterion is known as the objective function or cost function. A valid objective function must be influenced by the variables of the design problem, i.e. it must be a function of the design variables. The following section elaborates on some basic terminologies used for optimization.

3.3.1 Design Vector

Any engineering system or component is described by a set of quantities of which some are viewed as variables during the formulation of the design problem. Some of the quantities are fixed at the outset and these are called preassigned parameters. All other quantities are treated as variables in the design process and are known as design or decision variables $x_i = 1,2,\ldots,n$. As far as possible, design variables must be independent of each other. One should be able to assign numerical value to any variable independent of any other variable. Collectively, these variables can be represented as a design vector.

$$X_i = \begin{vmatrix} X_1 \\ X_2 \\ \cdot \\ \cdot \\ \cdot \\ X_n \end{vmatrix} \tag{3.36}$$

3.3.2 Design Constraints

Design variables are chosen such that they satisfy certain specified functional and other requirements. The restrictions that must be satisfied to produce an acceptable design are known as constraints. Constraints that

represent behaviour or system performance limitations are termed as behaviour or functional constraints and those that represent physical limitations on the design variables like availability, fabricability and transportability are called geometric or side constraints.

3.3.3 Objective Function

There can be many feasible designs for a system and some are better than others. To make such a claim we must have some criterion to compare various designs. The criterion must be a scalar function whose numerical values can be obtained once a design is specified. That is, it must be a function of the design variables. Such a criterion is called an objective function or $f(\mathbf{x})$ to emphasize its dependence on design variable vector $\mathbf{x}$. A function that is to be minimized is also referred to as cost function.

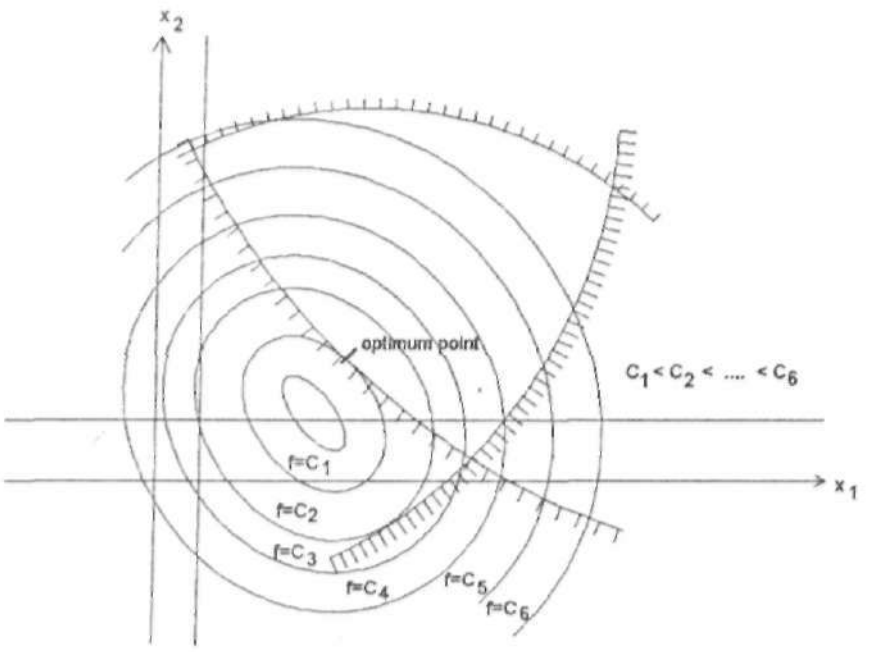


Fig. 3.5: Constraint and Objective Function Surface

As an illustration, we consider an optimization problem with only inequality constraints $G_i(X) < 0$. The set of X that satisfies the equation $G_i(X) = 0$ forms a hypersurface in the design space and is called a constraint surface. This is an (n-1) dimensional subspace where n is the number of design variables. The design space region is divided into two by the constraint surface; one in which $G_i(X) > 0$ are infeasible or unacceptable. The collection of all the constraint surfaces $G_i(X) = 0, i = 1,2,...m,$ which separates the acceptable region is called the composite constraint surface. The locus of all points satisfying $f(X) = C$ constraints forms a hypersurface in the design space and for each of C there corresponds a different member of a family of surfaces. These surfaces, called the objective function surfaces, are shown in the hypothetical two-dimensional design surface as in Fig. 3.5.

3.4 Classification of Optimization Problems

There are many ways of classifying optimization problems of which a few are described below:

3.4.1 Classification based on the nature of Design Constraints

a) Implicit Constraints: Some constraints are quite simple, such as minimum and maximum values of design variables, while more complex ones may be indirectly influenced by design variables. For example, deflection at a point in large structures depends obviously on its design. However, it is impossible to express deflection for very simple structures. These are called implicit constraints.

b) Linear and Nonlinear Constraints: Functions with only first-order terms in design variables are called linear constraints and can be solved using Linear Programming techniques, whereas more general problems have non-linear constraints.

c) Equality and Inequality Constraints: Design problems may have equality as well as inequality constraints. An example of an equality constraint is, to perform a desired operation, a machine component must move precisely by Δ. Examples of common inequality constraints are that calculated stresses must not exceed allowable stresses, deflections must not exceed specified limits, resources must not be exceeded, demands must be met, loads must be within limits so that failure does not occur and so on. Therefore, we have more feasible designs with respect to inequality constraints than equality constraints.

3.4.2 Classification based on the nature of Equations involved

This classification is very useful from the computational point of view since there are many methods developed solely for the efficient solution of a certain set of problem.

a) Non-Linear Programming: If any of the functions among the objective and constraint functions is non-linear, the problem is called a non-linear programming problem.

b) Geometric Programming Problem: A geometrical problem is one in which the objective function and constraints are expressed as posynomials in $\mathbf{x}$. Where posynomial can be defined as a function $h(\mathbf{x})$ expressed as the sum of power terms each of the form:

$$C_1 x_1^{a_{i1}} x_2^{a_{i2}} \ldots . x_n^{a_{in}} \tag{3.36}$$

where c_i and a_{ij} are constants with $c_i>0$ and $x_j>0$.

Thus, a posynomial can be expressed as:

$$h(x) = C_1 x_1^{a_{11}} x_2^{a_{12}} \ldots . x_n^{a_{1n}} + \ldots . + C_N x_1^{a_{N1}} \ldots x_n^{a_{Nn}} \tag{3.37}$$

Therefore, the geometric programming problem can be posed as follows:

Find **x** which minimizes

$$f(x) = \sum_{i=1}^{N_o} C_i \left(\prod_{k=1}^{n} X_k^{P_{ij}} \right) \leq 0 \ , \quad a_{ij} > 0 \tag{3.38}$$

subject to

$$g_j(x) = \sum_{i=1}^{N_j} a_{ij} \left(\prod_{k=1}^{n} X_k^{q_{ik}} \right) \leq 0, \quad a_{ij} > 0 \tag{3.39}$$

where N_o and N_j denote the number of posynomial terms in the objective and jth constraint functions respectively.

c) Quadratic Programming Problem: This is a non-linear programming problem with a quadratic objective function and linear constraints. It is formulated as follows:

Find **x**, which minimizes:

$$f(x) = C + \sum_{i=1}^{n} q_i X_i + \sum_{i=1}^{n} \sum_{j=1}^{n} Q_{ij} X_{ij} \tag{3.40}$$

subject to

$$\sum_{i=1}^{n} a_{ij} X_i = b_j, j = 1,2,\ldots,m, \ X_i => 0, \ i = 1,2,\ldots,n \tag{3.41}$$

where C, q_i, Q_{ij}, a_{ij} and b_j are constraints.

d) Linear Programming Problem: This is a linear programming problem with the objective function and the constraints as linear functions of the design variables.

Find

$$X_i = \begin{pmatrix} X_1 \\ X_2 \\ \cdot \\ \cdot \\ \cdot \\ X_n \end{pmatrix} \tag{3.42}$$

which minimizes

$$f(x) = \sum_{i=1}^{n} C_i X_i \sum_{k=1}^{n} a_{jk} X_j \tag{3.43}$$

subject to the constraints

$$\sum_{k=1}^{n} a_{jk} X_k = b_j; \quad j = 1,2,..,m; \quad X_i => 0; \quad i = 1,2,..,n \tag{3.44}$$

where c_i, a_{ix}, and b_j are constraints.

3.4.3 Classification based on the number of Objective functions

Depending on the number of objective functions to be minimized, optimization problems can be classified as single and multi-objective problems. A multi-objective problem can be stated as follows:

Find **x** which minimizes

$$f_1(x), f_2(x), \ldots, f_k(x) \tag{3.45}$$

subject to

$$g_j(x) < 0; \quad j = 1,2,\ldots,m \tag{3.46}$$

where f_1, f_2,...,f_k denote the objective functions to be minimized simultaneously.

3.5 NLPQL: a Fortran Subroutine Solving Constrained Nonlinear Programming Problems

The new design methodology is characterized by the phrase "model and analyze." Once the design problem is properly formulated, numerical methods can be used to optimize the system. The methods are iterative and generate a sequence of design points before converging to the optimum solution. They are best suited for computer implementation to exploit the speed of computers for performing repetitive calculations. (Schnittkowski, 1983). Fig. 3.6 is a conceptual flow diagram for the interactive design optimization process.

NLPQL is a FORTRAN implementation of a sequential quadratic programming method for solving nonlinearly constrained optimization problems with differentiable objective and constraint functions. At each iteration, the search direction is the solution of a quadratic programming subproblem.

This code was designed to solve constrained nonlinear programming problem:

min $f(x)$

subject to:

$$\begin{aligned} g_j(x) &= 0; \quad j = 1, \ldots, m_e \\ g_j(x) &\geq 0; \quad j = m_e+1, \ldots, m \end{aligned} \tag{3.47}$$

$x \in \mathbb{R}^n$: $\quad x_l \leq x \leq x_u$

The optimization method generates a sequence of quadratic subprograms which are to be solved successively. The algorithm is therefore known as the sequential quadratic programming (SQP) method. It was developed mainly by Han (1976) and Powell (1978) based on the initial work of Wilson (1963). The theoretical details and some convergence results are found in Schnittkowski (1983).

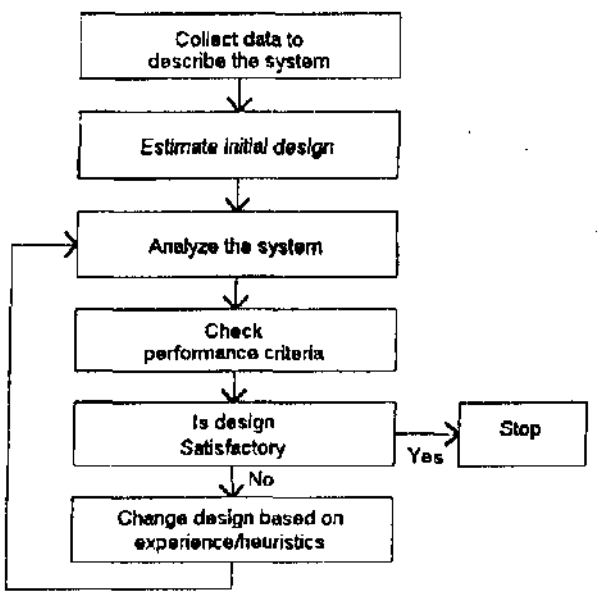


Fig. 3.6: Interactive design optimization process

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4

CHAPTER

LIMIT STATE FUNCTIONS FOR SINGLY REINFORCED CONCRETE BEAMS

CHAPTER FOUR

LIMIT STATE FUNCTIONS FOR SINGLY REINFORCED CONCRETE BEAMS

4.1 Introduction

In this chapter, models for the performance functions for the limit states are developed. The failure modes considered are the bending, shear and deflection. For each safety margins got by equating the resistance of the beam as proposed by BS8110 and the load effect calculated by elementary structural analysis, the basic variables are identified and their statistical characters suggested, from literature.

This work assesses interior beams of equal spans in an office building. To arrive at maximum values for moments and shear forces, two load combinations are studied.

Fig. 4.1 (a) and (b) show the arrangements of vertical loadings on the three span continuous beam to cause maximum sagging moment in alternate spans and maximum hogging moments at the supports, respectively.

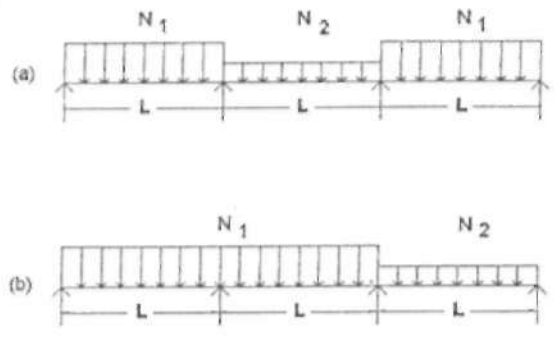


Fig. 4.1: Loading patterns

In Fig. 4.1. we have

N_1 = (1.4α + 1.6) x Q_k x Spacing of beams

N_2 = 1.0α x Q_k x Spacing of beams

Q_k = live load on beam

L = effective span of beam

α = G_k / Q_k

G_k = dead load on the beam.

4.2 Establishment of Performance Functions

4.2.1 Bending Criterion

The load effect in bending for maximum applied span moment, M_{a1}, is calculated using moment distribution method as :

$$M_{a1}=\frac{L^2}{768}(77N_1-19N_2) \qquad (4.1)$$

Similarly, the maximum applied moment at support, M_{a2}, is:

$$M_{a2}=\frac{L^2}{192}(22N_1-3N_2) \qquad (4.2)$$

At the ultimate limit state, it is important that sections subject to bending should be ductile, with failure occurring by the gradual yielding of tension steel rather than the catastrophic sudden crushing of the concrete in compression (Mosley and Bungey, 1989). For equilibrium of the beam section, the tensile force, F_{st}, in the reinforcement must be balanced by the compressive force, F_{cc}, in the stress block. See Fig. 4.2.

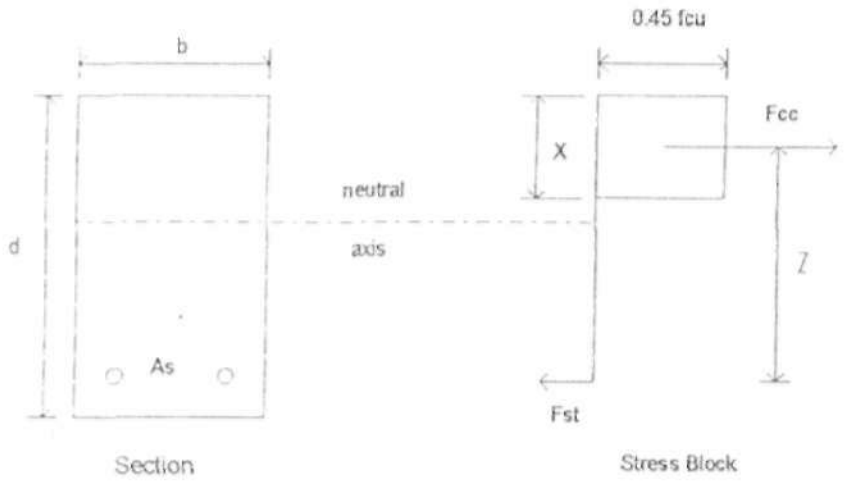


Fig. 4.2: Stress block diagram for a singly reinforced section

Thus,

$$F_{st} = F_{cc} \tag{4.3}$$

implying

$$0.87 f_y A_s = 0.45 f_{cu} b x \tag{4.4}$$

where f_y = characteristic strength of steel reinforcement

A_s = area of steel reinforcement

f_{cu} = characteristic strength of concrete

b = width of beam

x = depth of compressive stress block

Hence x can be determined as

$$x = \frac{0.87 f_y A_s}{0.45 f_{cu} b} \tag{4.5}$$

Therefore, the lever arm z, between F_{st} and F_{cc} is obtained as

$$Z = d - \frac{x}{2} \tag{4.6}$$

where d = effective depth of the beam section.

The moment of resistance M_R, for the section is evaluated as

$$M_R = F_{cc} . Z = 0.45 f_{cu} b . x . Z \tag{4.7}$$

The expression for the performance function G(X) in this limit state would then be

$$G(X) = M_R - M_{ai} \tag{4.8}$$

4.2.2 Shear Criterion

Having determined the magnitude of the critical shearing force V, using structural analysis, the ultimate shearing stress at a section of uniform depth d can be calculated from

$$U = \frac{V}{bd} \qquad (4.9)$$

Given that the loading is uniform and the spans do not differ by more than 15 percent of the longest span, the shearing force **v**, calculated from moment distribution is given as:

$$V = (0.55N_1 + 0.049N_2)L \qquad (4.10)$$

Since concrete itself offers some amount of resistance to shearing stresses equal to v_c, the resultant of the shearing stress created by **v** then equals $v - v_c$. However $v - v_c$ should not be greater than a certain v_{max} which is set by BS 8110 (1985) as being the smaller of $0.8\sqrt{f_{cu}}$ or $5N/mm^2$.

In the design of the beam for shear, the resistance to shear pitted against applied shearing stresses is estimated as

$$\frac{A_{sv}}{s_v} = \frac{(U-U_c)b}{0.87f_{yv}} \qquad (4.11)$$

This can be readjusted to

$$\frac{0.87A_{sv}.f_{yv}}{s_v.b} = \frac{V}{bd}[1-\gamma] \qquad (4.12)$$

where A_{sv} = Area of shear reinforcement

s_v = spacing of shear reinforcement

f_{yv} = characteristic strength of shear reinforcement

γ = v_c/v.

The performance function for the beam in shear is

$$G(X) = \frac{0.87 f_{yv} A_{sv}}{S_v b} - \frac{V}{bd}[1-\gamma] \qquad (4.13)$$

4.2.3 Deflection Criterion

Under the BS8110 requirement, the maximum vertical deflection may generally be assumed to be satisfied provided that the actual span to effective depth ratio are not greater than the limiting span to effective depth ratio.

Thus,

$$limiting \frac{L}{d} = 26 \; x \; m.f. \qquad (4.14)$$

where 26 = the basic span/effective depth ratio for continuous beams

The modification factor, m.f., for tensile reinforcement:

$$m.f. = 0.55 + \frac{477 - f_s}{120x(0.9 + M_{al}/bd^2)} \qquad (4.15)$$

where

$$f_s = 0.625 x f_y x \frac{A_{sreqd.}}{A_{sprov.}} x \frac{1}{\beta_r} \qquad (4.16)$$

in which β_r = the redistribution factor

Therefore, the performance function for deflection is

$$G(X) = limiting\frac{L}{d} - actual\frac{L}{d} \qquad (4.17)$$

4.3 Statistical Characteristics of Relevant Design Parameters

Techniques that can be utilized to obtain the probability distributions of basic variables include: direct inference from observed data, subjective assessment or their combination. Therefore, in practice, past observations and experience for similar structures can be validly used for the structure under assessment and the probability distribution functions can be obtained from physical models.

With limited information the probability distributions of the design variables were prescribed. However, it is important also to recognize that regardless the distribution of the individual variate, it is the distribution of the safety margin (R-S) that is important in the calculation of probability of failure, p_f (Ang and Cornell, 1974). It was also observed that if the risk level is high. (eg. $p_f > 10^{-3}$ or $\beta < 3.1$) the calculated failure probability is not much different regardless of the type of distribution. However, for a very small risk (eg. $p_f \leq 10^{-5}$ or $\beta \geq 4.25$) the failure probability could be sensitive to this distribution.

In second moment reliability method, the parameters of interest are first and second moments (mean and variance) for the computation of safety indices (Ayyub and Halder, 1984). It is also important to note that it is the COV (ratio of standard deviation to mean value) that are mostly published in literature (eg Galambos and Ravindra, 1978, Melchers, 1987). Values of relevant parameters for the basic variables used are shown in Table 4.1.

Table 4.1: Table of Essential Design Parameters

	BASIC VARIABLES	TYPE	NUM. CODE	MEAN	COEFFICIENT OF VARIATION
1	Compressive strength of concrete, f_{cu}	Log-Normal	3	25N/mm^2	0.15
2	Characteristic strength of steel, f_y	Log-Normal	3	460N/mm^2	0.15
3	Breadth, b	Normal	2	300mm	0.045
4	Effective depth, d	Normal	2	500mm	0.086
5	Live load, Q	Gumbel	7	0.0025N/mm^2	0.37
6	length, L	Normal	2	5000mm	0.044
7	Characteristic strength of links, f_{yv}	Log-Normal	3	250N/mm^2	0.15
8	Cross-sectional area of links, A_{sv}	Normal	2	157.1mm	0.036
9	Spacing of links, S,	Normal	2	250mm	0.089
10	Ratio v_c/v_s	Normal	2	0.6	0.3

A study carried out by Fiorato (1973) showed that all geometric variables are normally distributed while the concrete strength and strength of steel are assigned log normal distributions.

For the purpose of the application of FORM5, the following distributions have the following number code: Normal = 2, Lognormal = 3 and Gumbel = 7.

4.4 Uncertainties in the Basic Design Variables

It is seen in the formulation of the performance functions of the beam at the various limit states that the beam's capacity and the load effects on it are themselves functions of several intervening variables x_i. In engineering, uncertainties in quantities are the rule rather than the exception, therefore it is expected that all such basic variables should exhibit some degree of scatter.

The probability distribution describing this scatter, however, poses a problem in its identification chiefly because of a lack of sufficient data

permitting its unambiguous identification. In the absence of sufficient data, engineers are forced to propose such distributions with the benefit of mathematical back-up to physical reasoning (Ellingwood and Ang, 1974). Thus where the basic variable consists of the sum of several others e.g. dead loads, strength of concrete etc, the central limit theorem may be invoked to suppose that the normal distribution would be appropriate (Thoft- Christensen and Baker, 1982). Such a reasoning could be adopted also to suggest that geometrical dimensions could be normally distributed too.

The uncertainty of most variates is well established and given in literature (e.g. Ellingwood and Ang, 1974, Ravindra et al, 1974) in terms of the coefficient of variation. More generally, the total uncertainty is evaluated in terms of the basic variability of observed data and the error in the prediction of the mean value in the form:

$$\Omega = \sqrt{\delta^2+\Delta^2} \qquad (4.18)$$

where Ω = total uncertainty, δ= basic variability of observed data, Δ =error in the prediction of the mean value.

and

$$COV = \frac{\Omega}{\mu_i} \qquad (4.19)$$

where μ_i = mean value of the variate.

When practical values for μ_i are adopted, their corresponding variabilities could then be obtained with estimates of their total uncertainties.

4.4.1 Variability in Strength of steel, f_y

This has been found to be influenced by methods of analyses of data of various bar sizes and commercial testing procedures. Since the designer or code writer usually does not know beforehand which bar sizes might be required and in what combinations, uncertainties obtained from lump test data for bar sizes become appropriate (Ellingwood and Ang, 1974).

During commercial testing procedures, it is the unstable upper yield point rather than the stable lower yield point that is reported. Also, the characteristic mean strength of steel tends to increase due to the high strain rates applied. The uncertainties in the value of the mean resulting from these shortcomings suggest a total uncertainty Ω_{f_y} of 0.15.

4.4.2 Variability in Strength of Concrete, f_{cu}

The variability in the strength of concrete is affected by the method of mix and structural considerations. Evidently, f_{cu} obtained from test results differ from f_{cu} in the actual structure as a result of factors such as the strain rate, duration of loading, casting direction, curing of the member and the effects of creep, shrinkage and confinement. It has been shown that concrete proportioned for a nominal mix of 21N/mm^2 yield a mean strength of 23.8 N/mm^2 with a basic variability within the test results $\delta_{f_{cu}}$ of 0.12. The aforementioned factors are assumed to contribute a combined uncertainty of 0.18, hence a total uncertainty $\Omega_{f_{cu}}$ is obtained as being equal to 0.21 (Ellingwood and Ang. 1974).

4.4.3 Variabilities in member geometry

The width b, the overall depth h, of the beam and dimensions of reinforcement placements are deemed to be functions of the level of workmanship in construction. It is suggested that both Ω_b and Ω_h, and Ω_d and Ω_{sv} are equal respectively, since in the former case, errors arise from the laying out of dimensions, and in the latter case, errors arise from the

placement of reinforcement. Ω_b and Ω_h have been estimated as being equal to 0.045 while Ω_d and Ω_{sv} have been estimated as being about 0.086 (Ellingwood and Ang, 1974).

4.4.4 Uncertainties in loading

The loads N_1 and N_2, encountered in the formulations of the various performance functions, are considered in reinforced concrete design as being functions of characteristic dead loads, G_k and characteristic live loads Q_k.

$$N_1 = (1.4\alpha + 1.6)\ x\ Q_k\ x\ Spacing\ of\ beams;$$
$$N_2 = 1.0\alpha\ x\ Q_k\ x\ Spacing\ of\ beams \qquad (4.20)$$

When suitably practical values for α are adopted, the variations in design loads, N_1 and N_2 can be studied by considering those of the characteristic live loads chiefly.

Live loads in buildings arise from movable fixtures, occupants and other non-permanent installations and they depend on,

(1) The type of occupancy,

(2) The tributary area and

(3) The projected service life of the structure (Ellingwood and Ang, 1974).

Whereas physical reasoning could be evoked to suggest that dead loads are normally distributed, consideration is usually given to extreme-value distributions in proposing one for live loads. The application of extreme-value statistics to live loads effectively shunts the necessity for a time-dependent reliability approach, since the maximum load in each reference period is what is considered. Extreme-value type I (Gumbel) distribution has been found to be quite reasonable to model live loads (Melchers, 1987).

4.5 Cost Optimization Problem

Cost-benefit analyses seem to suggest that the desirable condition of any endeavour whatsoever can be expressed mathematically as max $(B-C_T)$, where B = Total benefits to be derived from the endeavour and C_T = Total cost of the project (Melchers, 1987).

Within the framework of the probabilistic assessment of the performance of the structure in question therefore, the total cost of the project can be expressed mathematically as:

$$C_T = C_I + p_f C_f \qquad (4.21)$$

where C_I = initial cost

p_f = probability of structural failure

C_f = cost of failure

C_I as above embodies the generality of costs associated with materials and labour C_i, provision of a number of quality assuarance measures, C_{QA}, and corrective actions taken in response C_c, and costs of maintenance, C_m, and insurance C_{ins}. In other words,

$$C_I = C_i + C_{QA} + C_c + C_m + C_{ins}. \qquad (4.22)$$

A major obstacle in the direct analysis of costs as presented lies in the fact that it is very impractical for C_I to be determined globally. It is obviously a function of the benefits derived from the structure by the client, the extent of damage of the structure, the failure mode, the injury caused etc. and usually aspects such as the cost of injury vary from society to society, if not from individual to individual.

One convenient way around the problem is to present these costs as non-dimensional quantities, expressing all costs as a ratio to the initial cost.

$$\frac{C_T}{C_I} = 1 + p_f \frac{C_f}{C_I} \tag{4.23}$$

Let

$$\frac{C_T}{C_I} = \lambda_T, \quad \frac{C_f}{C_I} = \lambda_F \tag{4.24}$$

represent the total and failure costs ratio respectively.

Hence, our objective function to be minimized is given by:

$$\lambda_T = 1 + \lambda_F \cdot p_f \tag{4.25}$$

Practical values of λ_T are specified in literature (e.g. Melchers, 1987). For beam elements, these values are said to range between 30 and 75, and in this study, the value 75 is used throughout for the simple reason that the worst effect on the cost of the beam is desired to be known. The design constraints to be satisfied are bending and shear.

5
CHAPTER

FAILURE PROBABILITY LEVELS AND COST OPTIMIZATION

CHAPTER FIVE

FAILURE PROBABILITY LEVELS AND COST OPTIMIZATION

5.1 Introduction

This chapter discusses the results of the reliability analysis and cost evaluation of continuous rectangular beams using the BS8110 (1985) design requirements. Alongside with the reliability-based optimum design, an investigation of the sensitivity of the optimized design to changes in the statistical parameters that define the loading and member strengths is carried out.

Firstly, with the various performance functions derived in section 4.3 and taking into consideration the variabilities in the design variables, the implied safety (reliability) levels associated with BS8110 criteria are computed. The computation follows the procedure of the First-Order Reliability Integration Method. Their results are presented in Fig. 5.1.

Secondly, the optimum design of a continuous singly reinforced concrete beam is studied with the objective function as the total cost ratio subjected to bending and shear constraints. This is accomplished with the aid of a coded Non-Linear Programming Algorithm (Schnittkwoski, 1986), and the First-Order Reliability Integration Scheme.

5.2 Implied Safety Levels

The adequacy of BS 8110 (1985) design criteria for beams is assessed. The implied β levels are examined and averaged over a suitable range of load ratio, α. Typically, β values in the range of 2 to 4 are used in establishing code safety margins for ultimate limit state and for serviceability limit state the range is 1 to 3 (Moses, 1990). Hence, β averages falling within this range reflect adequacy in the design criteria under study. A study by MacGregor (1981), showed that for gravity loads acting on reinforced concrete beams, the range of β was between 2.8 and 3.2.

Implied levels of safety inherent in strength predictions for bending, shear and deflection are studied. A suitable range of load ratios is consequently adopted within which subsequent analyses are performed.

Generally, it can be observed from Fig. 5.1 that the relationship of safety index, β, with load ratio, α, is polynomial by nature and β decreases uniformly with an increase in α for all the design criteria. Also, the β-values are more sensitive to α for bending and shear. This is illustrated by the fact that β-values change more in bending and shear than in deflection for corresponding change of α.

In bending, β ranges from 3.45 to 1.52 and has an average of 2.365 (p_t = 9.0155 x 10^{-3}) which is within the economical range of 2 to 4. This implies that the design equation for bending is well predicted. However, for a high α ratio, the adequacy becomes questionable as β values drop gradually. If β = 3.0 were taken as a target level, it can be seen that the beam would be sufficiently safe in bending at all α less than 0.5. An acceptable range for the ratio α for the beam in flexure could therefore be deduced as falling in the region $0.3 \leq \alpha \leq 0.5$.

The range of values for β for shear is between 2.66 and 0.95 with an average of 1.685 (p_t = 4.8491 x 10^{-3}) which is below the economical range. Thus, its requirements are seemingly critical in the code, and they require proper caution in application. Against the assumed target β level of 3.0, a suitable range of load ratios would be below α =0.2. While the beam could perform satisfactorily in bending at load ratios as high as 0.5, it can only perform similarly up to α = 0.2 in shear, indicating the inferiority of shear strength predictions. This discrepancy in performance levels suggests a non-uniformity in the governing design equations. This non-uniformity may be alleviated as suggested by Afolayan (1995).

By comparison, the deflection criteria is the most conservative, which is exemplified by its high average β of 5.70 (p_t =0.59907 x 10^{-8}). This is quite high and outside the economical range of 1 to 3. This result shows that the

satisfaction of the code requirement for deflection does not necessarily lead to economical designs.

However, an increment of 50% in load ratio will cause 17%, 20% and 3.5% reductions in the safety indices associated with bending, shear and deflection respectively. This shows that the shear requirement is the most sensitive while the condition for deflection is relatively insensitive though uneconomical. Therefore, it is indicative that the most probable cause of failure would be shear.

For illustrative purpose, let b = 300mm, d = 450mm, α = 2.0 and L = 5000mm, the M_R = 236.9KNm and M_{sl} = 73.44KNm. When designed to BS8110 requirements it will be a safe, singly reinforced concrete beam as the difference in M_{sl} and M_R is quite high. But when the design is subjected to a reliability analysis the safety index for bending is 1.52 ($p_f = 0.644 \times 10^{-1}$) and that for shear is 0.95 ($p_f = 0.171$). Thus, implying that the domain of failure is wide spread and the beam may not be as safe as it may appear. Also, despite the resistance being greater than the load effect it still falls within the failure domain. The pseudo-probabilistic code recognizes this small but finite probability but there has been no attempt to compute such a probability (Afolayan, 1992).

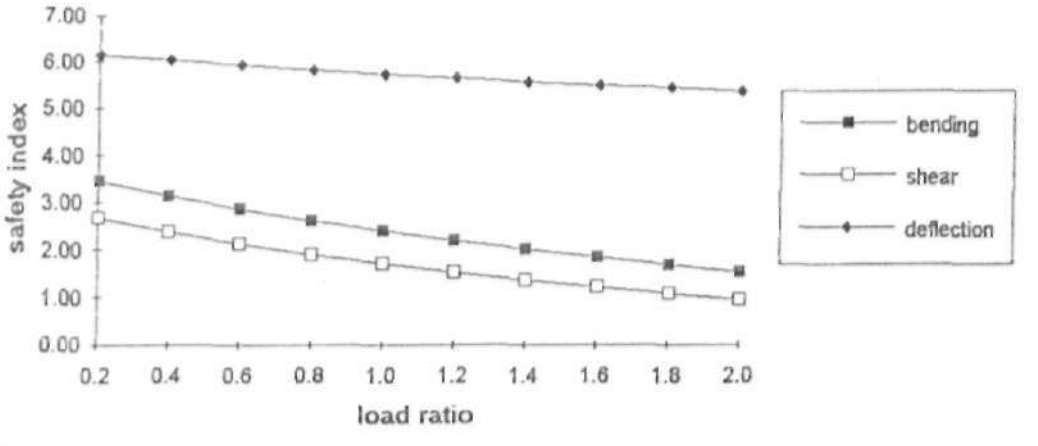


Fig.5.1 Implied Safety Levels with varying load ratios

In this investigation failure does not necessarily imply a total collapse but rather the violation of a limit state. The major concern is the margin between the resistance of a component and the load effect on it when variability inherent in the intervening variables are systematically quantified.

5.3 Cost Optimization and Sensitivity Tests

Reliability-based optimal designs have been treated by several authors (e.g., Morutsu et.al., 1984; Frangopol, 1985; Thoft-Christensen and Sorensen, 1987; Thoft-Christensen and Morutsu, 1986). The essential objective of sensitivity analysis of any system is to establish a measure of the way each response quantity varies with changes in the parameters that define the system. Some other important factors to be considered in sensitivity analysis are the influence on the reliability-based optimum solution of the approximations used in formulating the objective functions of the probabilistic methods for computing the system reliability, and of the optimization techniques themselves. This information is useful in reliability-based optimization for three principal reasons (Frangopol, 1985).

First, if the statistical parameters (eg., mean values, coefficient of variation, coefficient of correlation), or specified reliability levels are modified after the optimization is complete this will provide the designer with a measure of what effect such changes will have on the optimized structure. Limited statistical information on loads and strengths, lead to approximations in arriving at the statistical parameters associated with these random variables. Through sensitivity analysis these critical parameters can be identified, thereby, allowing for an efficient control of errors in specifying the required levels of approximation for each parameter of interest to achieve a reasonable compromise between reality and simplicity in reliability-based optimization.

Second, the information provided by sensitivity analysis of the optimum solution to changes in the objective functions, in the probabilistic methods to determine structural system reliability, or in the optimization techniques themselves, can be effectively used by the designer to rationalize the computational effort and to develop a reasonable level of confidence in the final design.

Third, the sensitivity analysis information is valuable for indicating new paths of research in reliability-based optimization.

In view of the above, sensitivity tests such as changes in coefficient of variation of live load and concrete strength with their effects on the objective function and the design parameters, are conducted.

In this study, the objective function for the constrained nonlinear problem is the total expected cost with bending and shear constraints. The deflection criterion has not been considered in this optimization procedure as it gives the most conservative and uneconomical results when checked for reliability. See Fig. 5.1.

The optimization problem for the beam element is given as:

$$\text{min.} \quad \lambda_T = 1 + \lambda_T \cdot p_{ft} \tag{5.1}$$

Subject to:

$$C_1 = p_{fb} - p^T_{fb} \leq 0 \tag{5.2}$$

$$C_2 = p_{fs} - p^T_{fs} \leq 0 \tag{5.3}$$

with

$$p_{ft} = p_{fb} + p_{fs} \tag{5.4}$$

for independent failure events; and

where λ_T is the total expected cost parameter

λ_F is the failure cost parameter

p_{ff} is the measure of inadequate performance of the designed element

C_1, C_2 are the bending and shear constraints

p_{fb}, p_{fs} are the probability of failures in bending and shear, respectively

p^T_{fb} p^T_{fs} are the target failure probabilities of the bending and shear criteria.

In this work, the bending and shear criteria are assumed to be mutually exclusive events. In other words, the member can fail in either bending or shear but both cannot occur at the same time.

The optimization results are shown in Figs. 5.2 to 5.21.

Figs. 5.2(a,b and c) are related as they illustrate the variations of breadth, b, depth, d and total cost ratio, λ_T, respectively, with α and coefficient of variation of concrete, V_{fcu}. The length, L, and coefficient of variation of live load, V_q are kept constant. The same trend is followed for Figs. 5.3 to 5.6.

Figs. 5.7(a,b,c) follow the same pattern as Figs. 5.2(a,b,c) with the exception that V_q is varied while V_{fcu} is kept constant. The same trend is followed for Figs. 5.8 to 5.11.

Figs. 5.12(a,b and c) show the relationship of b, d, and λ_T, respectively, with varying V_{fcu} and α for various lengths. V_q is kept constant. Same trend is followed by Figs 5.13 to 5.16.

Figs 5.17 to 5.21 show the relationship of b, d, and λ_T with varying V_q for different lengths. On scanning the figures some general trends were observed.

1. The breadth, b, depth, d, and total expected cost parameter, λ_T, increases with load ratio, α, for various V_{fcu}, V_q and L.

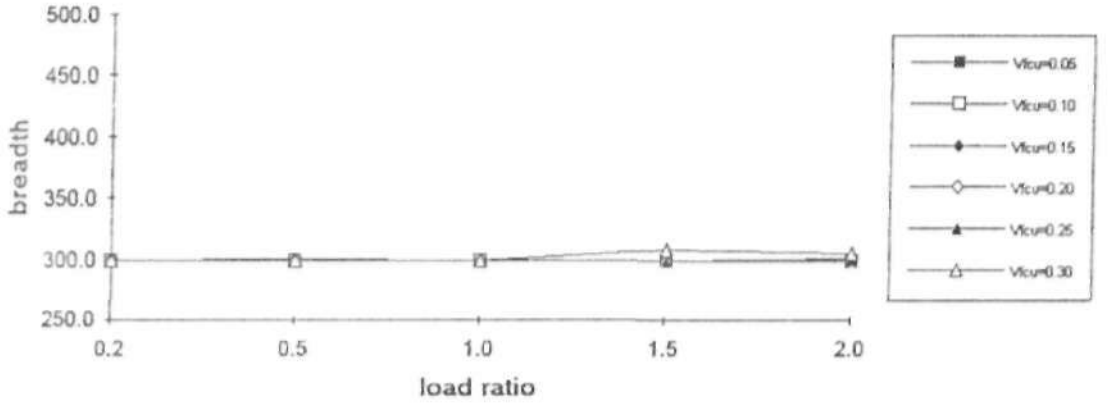
2. A rise in d, brings about a decline in λ_T by some percentage which depends on V_{fcu}, V_q, α, and L.

3. λ_T increases uniformly with α for all concrete variabilities (Figs. 5.2c, 5.3c.....5.6c). However, with respect to V_q, λ_T increases disproportionately (Figs. 5.7c, 5.8c.....5.11c).

4. In relation to V_{fcu}, λ_T is more or less constant for a constant length and load ratio but with V_q, λ_T increases disproportionately at each increment of 0.05 of V_q (Figs. 5.12c, 5.13c.....5.21c).

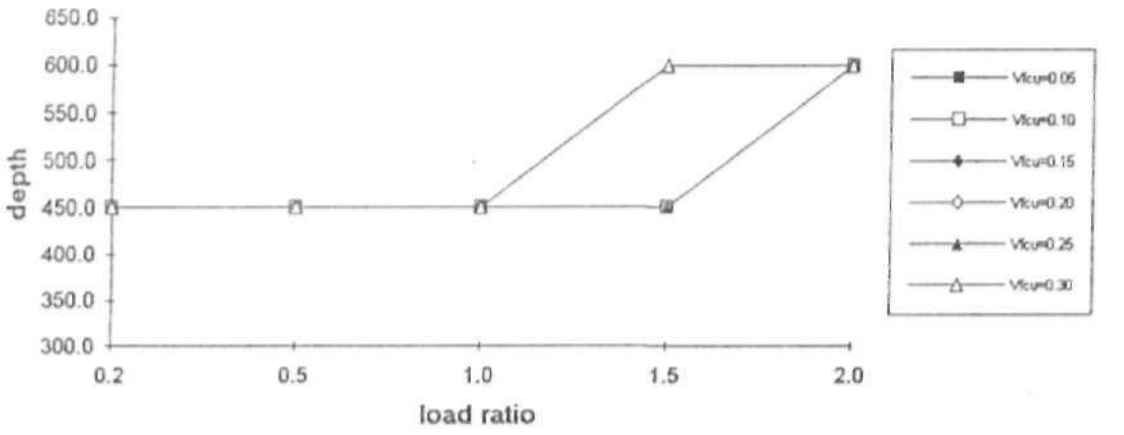
In normal deterministic design, when a section is inadequate for the applied load, the designer modifies the cross-sectional dimensions of the element to meet the requirements of safety and serviceability, but it is not known which of the parameters are more sensitive to the total cost. In this study, the effects of changes in b and d on λ_T are examined and it is observed that an increment of 33% in d, decreases λ_T in the range of 2% - 55%, in the various figures. Meanwhile, an increase in b, has no significant effect on λ_T. Accordingly, it can be argued that an increase in b only reduces the probability of failure marginally for both bending and shear criteria. Thus, a cumulatively high p_{tt} leads to an increase in λ_T. And for depth, the reverse is the case.

Figs. 5.2a, 5.3a,.....5.6a show that the change in b was irregular and inconsistent with α, L, and V_{fcu} especially for high V_{fcu} and L. and contrarily, Figs. 5.7a, 5.8a.....5.11a at first showed a constant b for lengths between 3000mm and 5000mm then there was a gradual and consistent increase in b for corresponding increase in α, L and V_q. The interpretation to this may be that



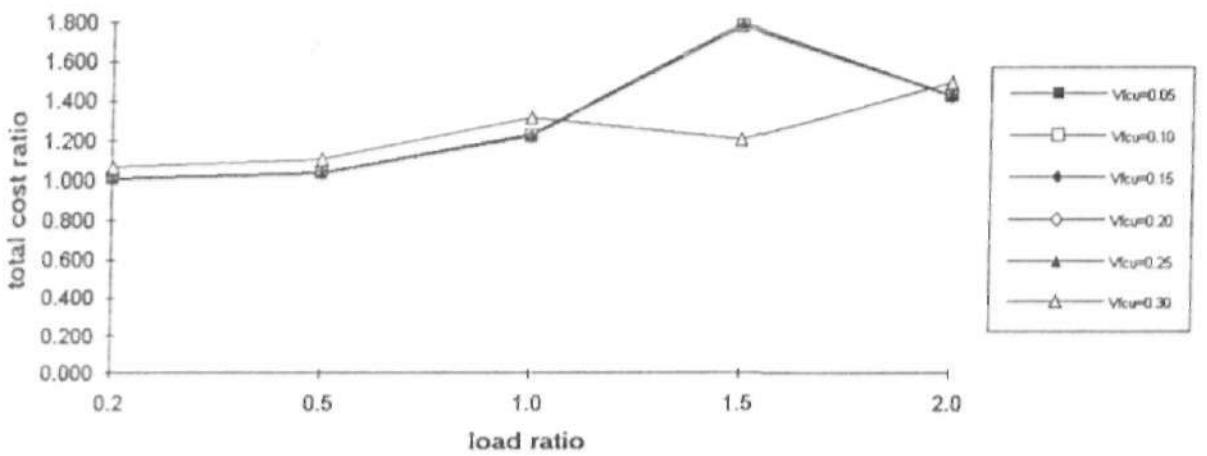
(l=3000mm and Q=2.5KN/m^2)

Fig.5.2a Breadth versus load ratio for varying Vfcu



(l=3000mm and Q=2.5KN/m^2)

Fig.5.2b Depth versus load ratio for varying Vfcu



(l=3000mm and Q=2.5KN/m^2)

Fig.5.2c Total cost ratio versus load ratio for varying Vfcu

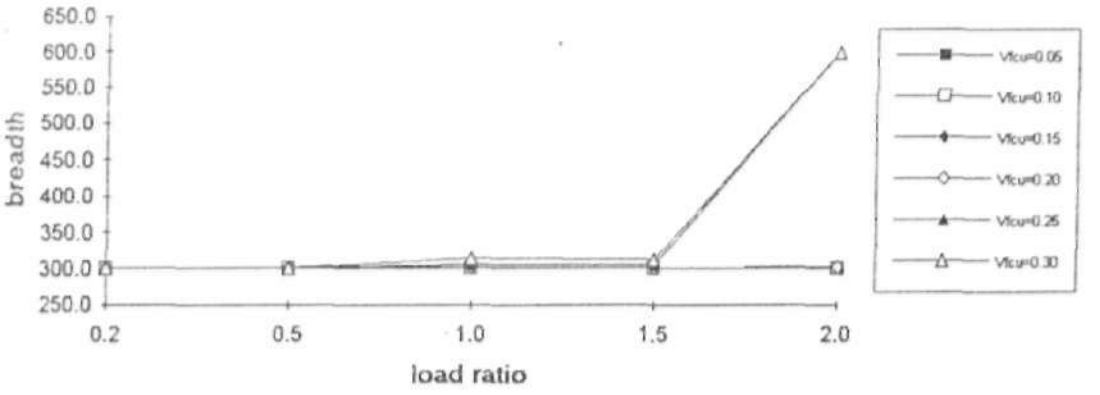


Fig.5.3a Breadth versus load ratio for varying Vfcu

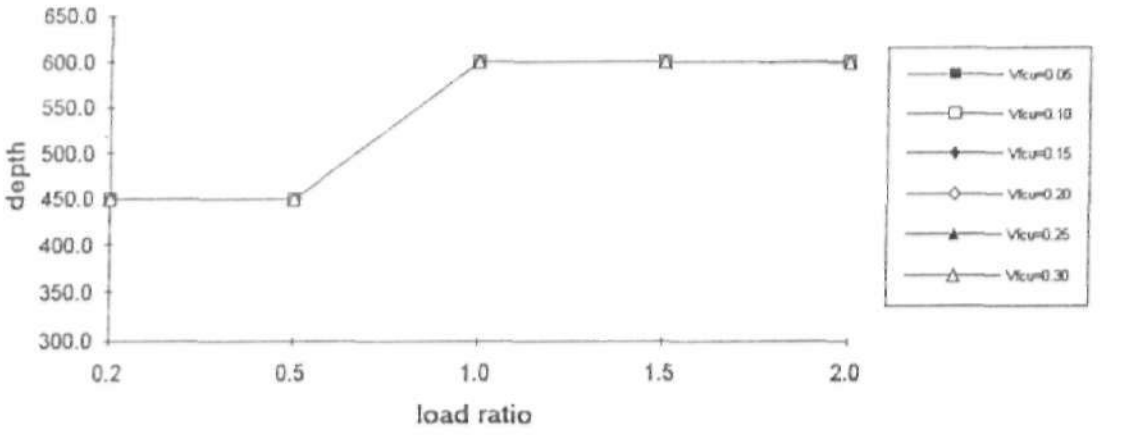
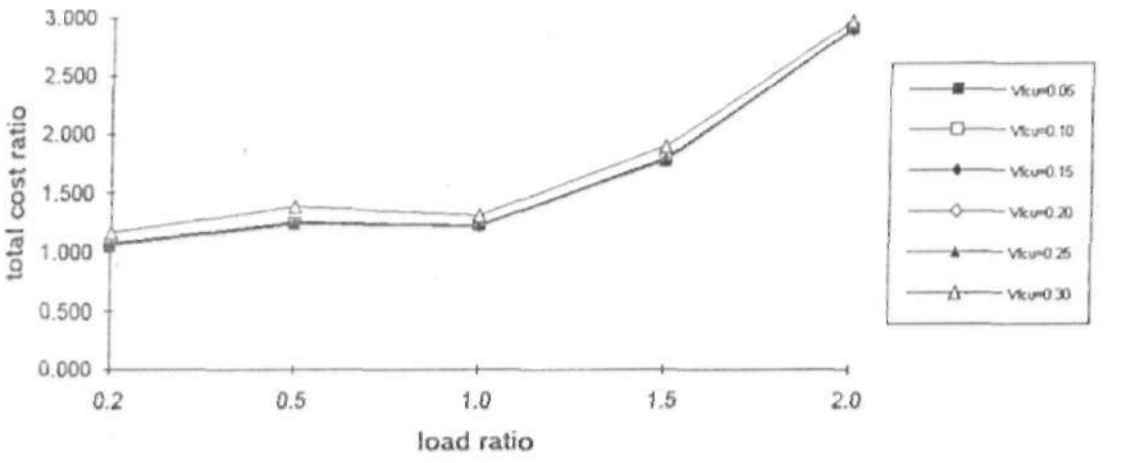
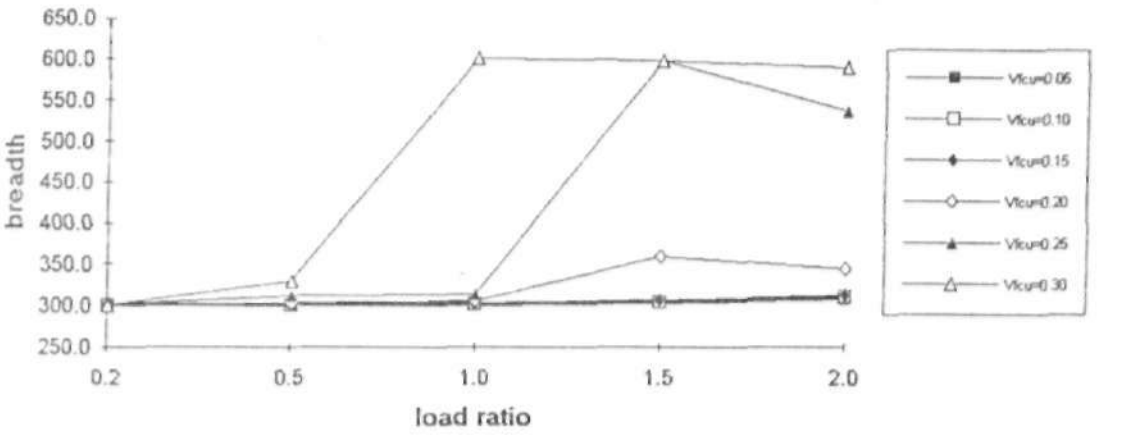


Fig.5.3b Depth versus load ratio for varying Vfcu



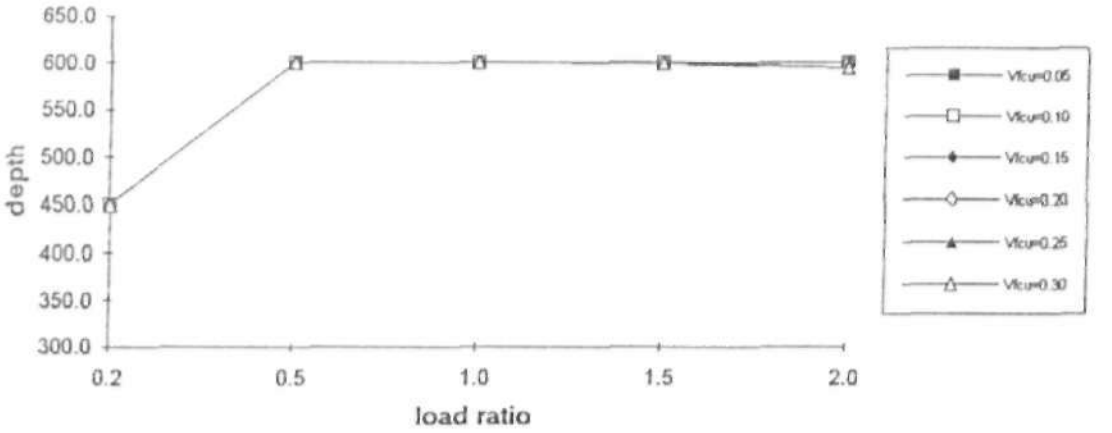
(l=4000mm and Q=2.5KN/m^2)

Fig.5.3c Total cost ratio versus load ratio for varying Vfcu



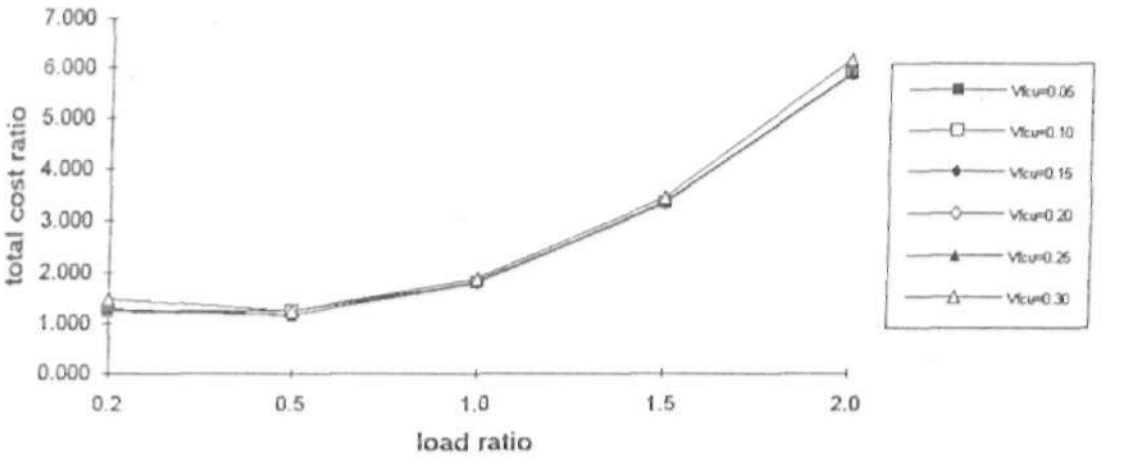
(l=5000mm and Q=2.5KN/m^2)

Fig.5.4a Breadth versus load ratio for varying Vfcu



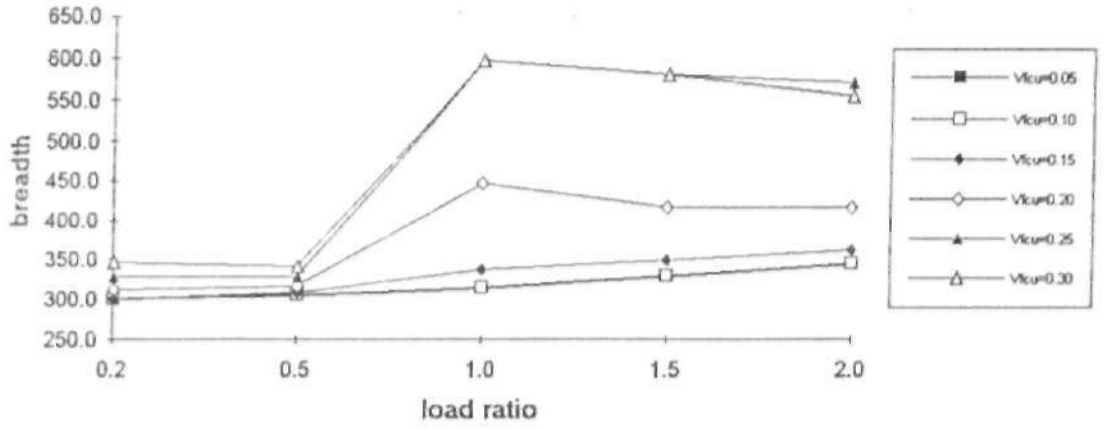
(l=5000mm and Q=2.5KN/m^2)

Fig.5.4b Depth versus load ratio for varying Vfcu



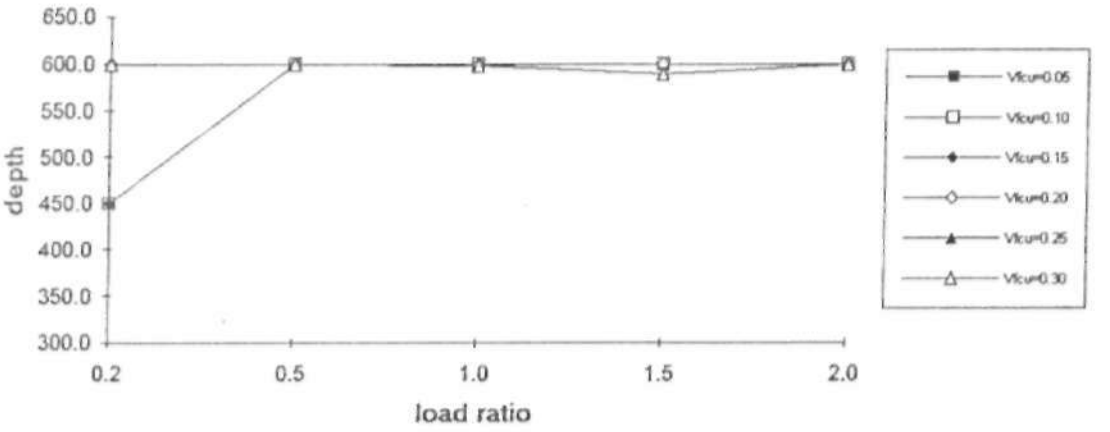
(l=5000mm and Q=2.5KN/m^2)

Fig.5.4c Total cost ratio versus load ratio for varying Vfcu



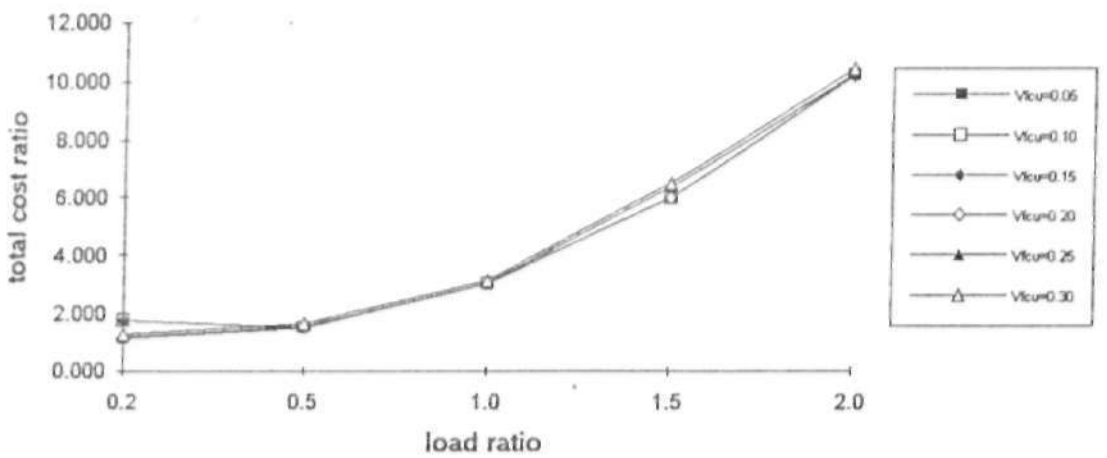
(l=6000mm and Q=2.5KN/m^2)

Fig.5.5a Breadth versus load ratio for varying Vfcu



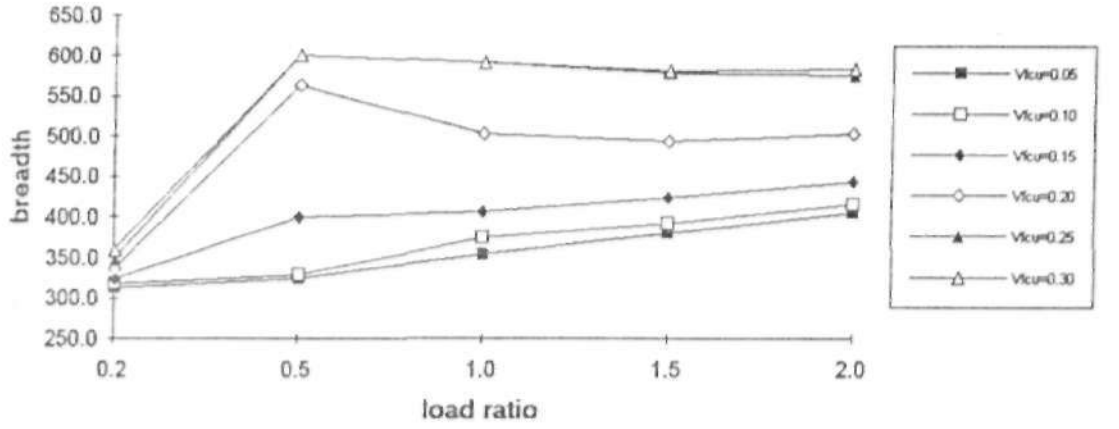
(l=6000mm and Q=2.5KN/m^2)

Fig.5.5b Depth versus load ratio for varying Vfcu



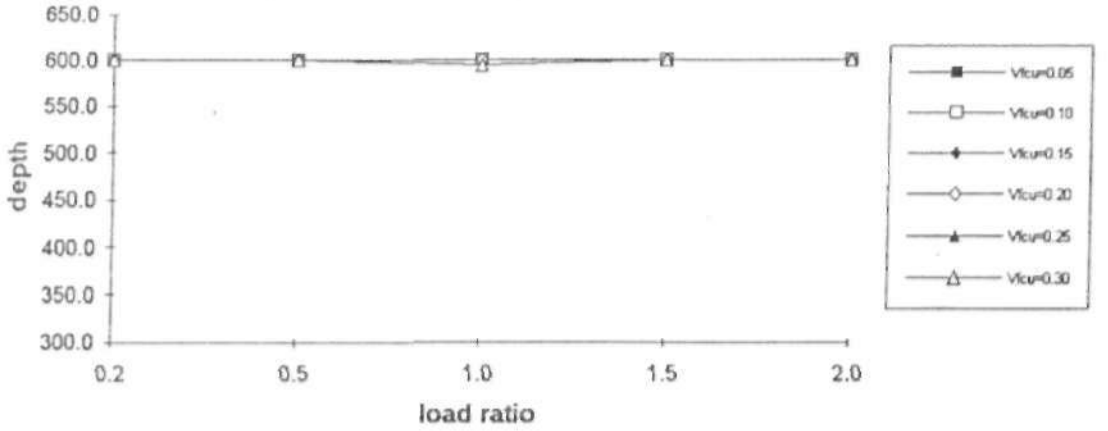
(l=6000mm and Q=2.5KN/m^2)

Fig.5.5c Total cost ratio versus load ratio for varying Vfcu



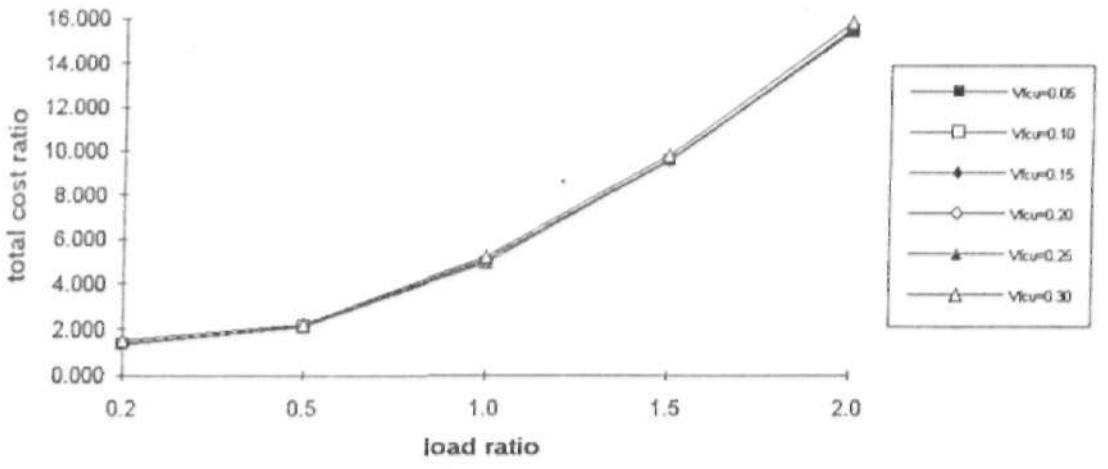
(l=7000mm and Q=2.5KN/m^2)

Fig.5.6a Breadth versus load ratio for varying Vfcu



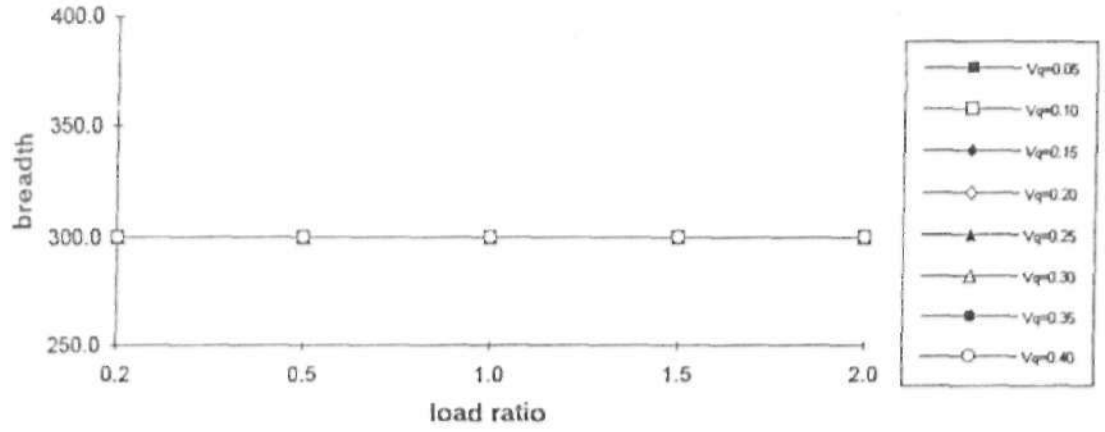
(l=7000mm and Q=2.5KN/m^2)

Fig.5.6b Depth versus load ratio for varying Vfcu



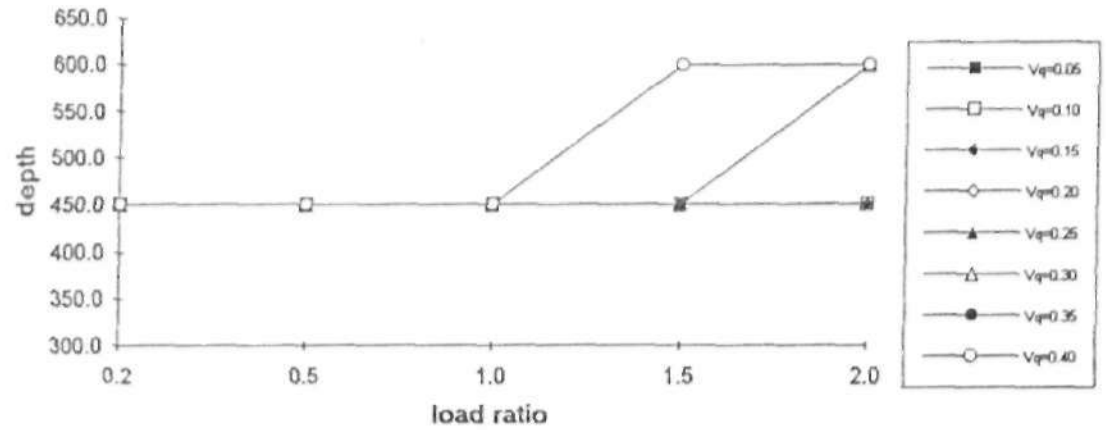
(l=7000mm and Q=2.5KN/m^2)

Fig.5.6c Total cost ratio versus load ratio for varying Vfcu



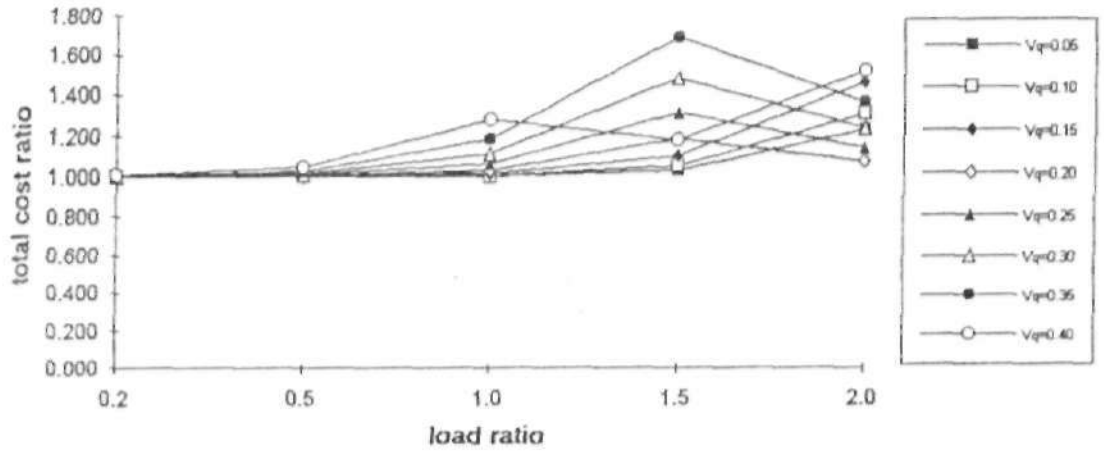
(l=3000mm and Q=2.5KN/m^2)

Fig.5.7a Breadth versus load ratio for varying Vq



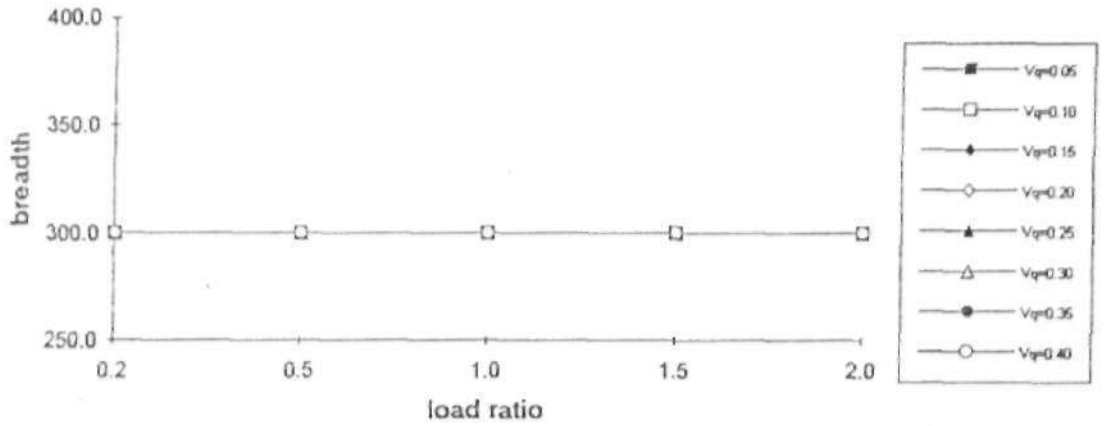
(l=3000mm and Q=2.5KN/m^2)

Fig.5.7b Depth versus load ratio for varying Vq



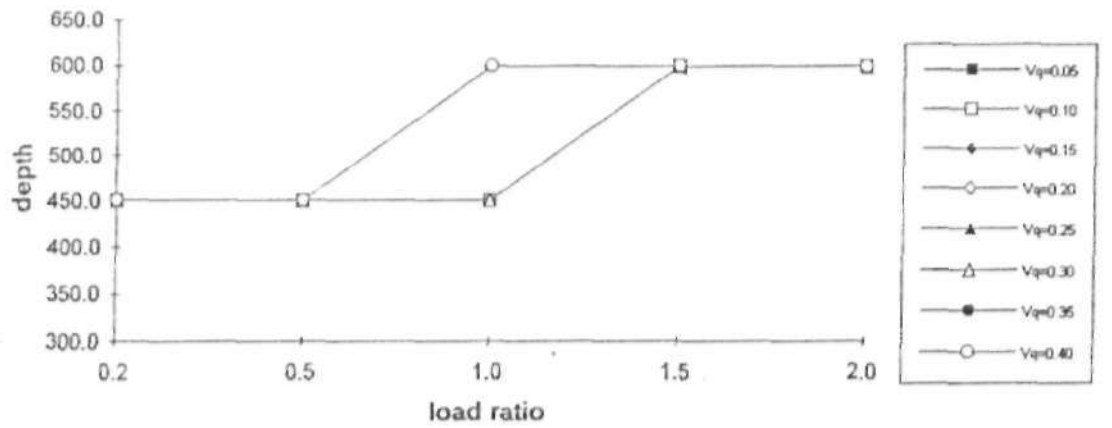
(l=3000mm and Q=2.5KN/m^2)

Fig.5.7c Total cost ratio versus load ratio for varying Vq



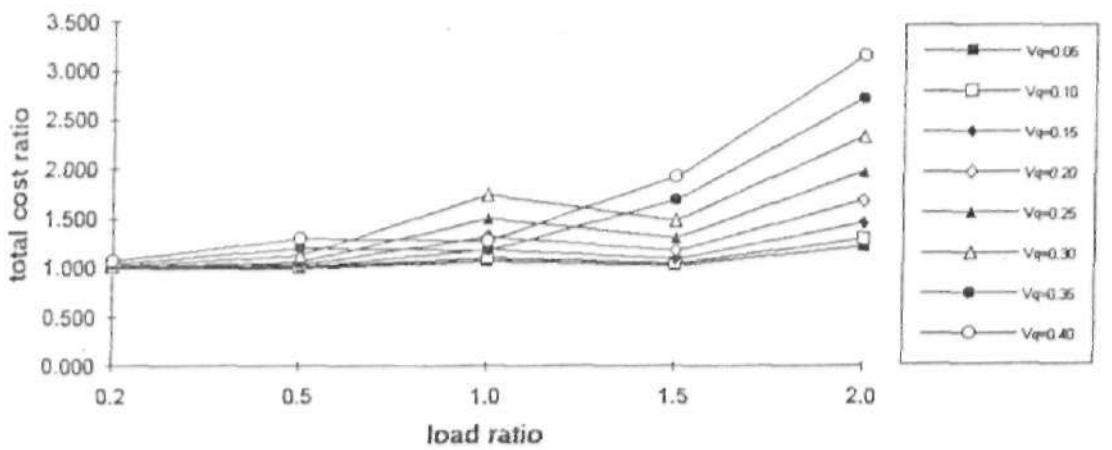
(l=4000mm and Q=2.5KN/m^2)

Fig.5.8a Breadth versus load ratio for varying Vq



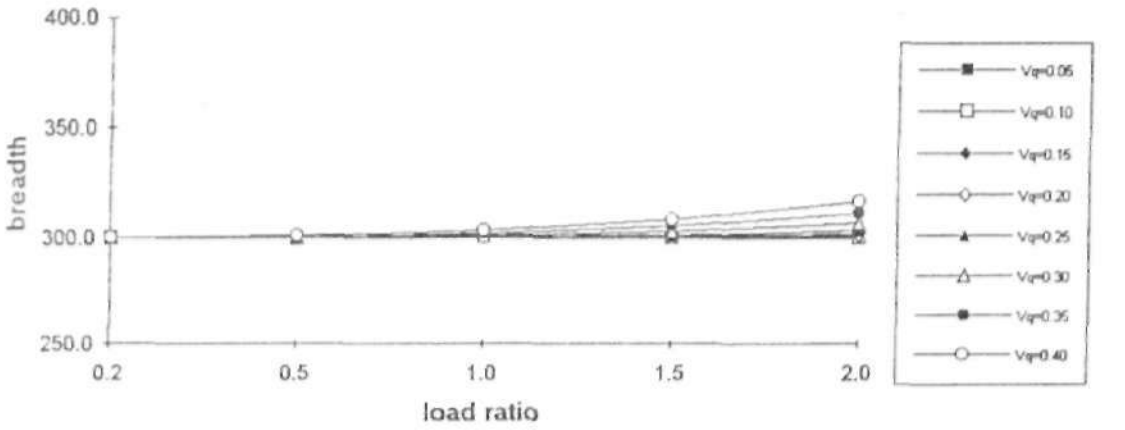
(l=4000mm and Q=2.5KN/m^2)

Fig.5.8b Depth versus load ratio for varying Vq



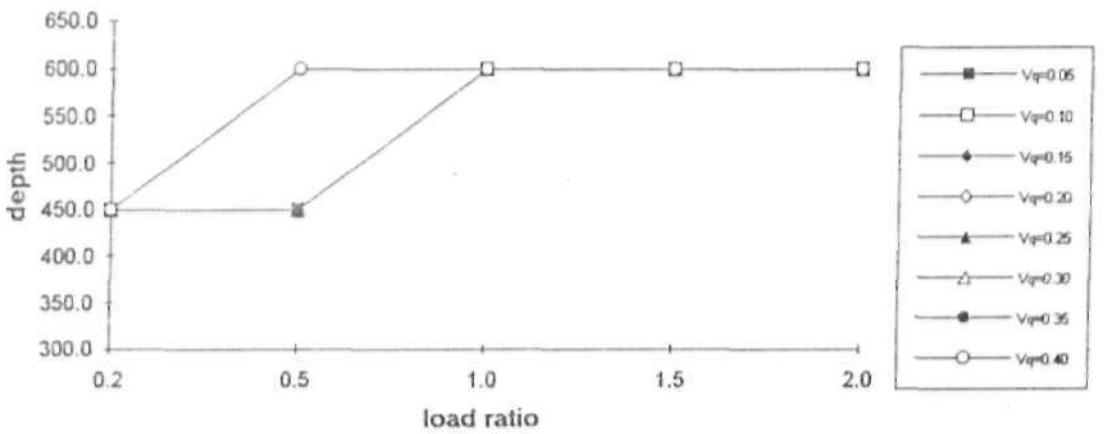
(l=4000mm and Q=2.5KN/m^2)

Fig.5.8c Total cost ratio versus load ratio for varying Vq



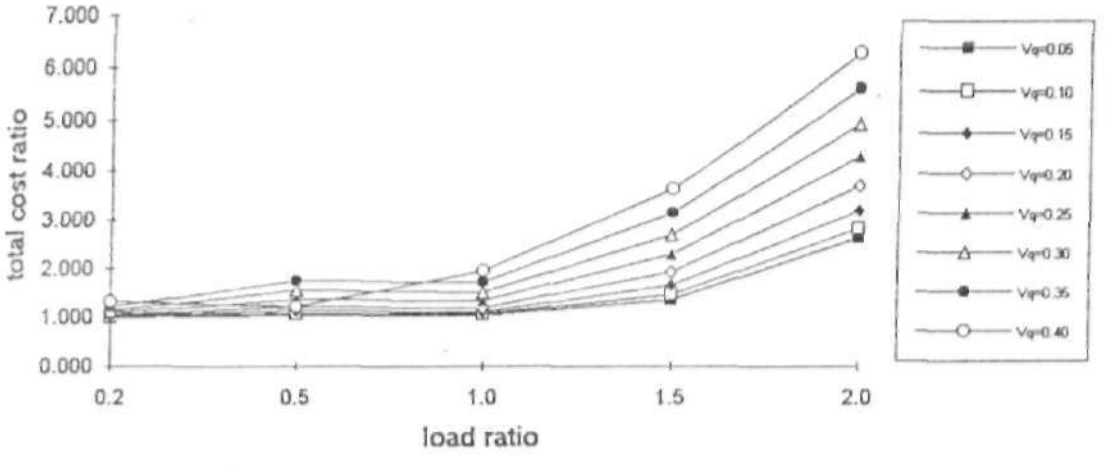
(l=5000mm and Q=2.5KN/m^2)

Fig.5.9 a Breadth versus load ratio for varying Vq



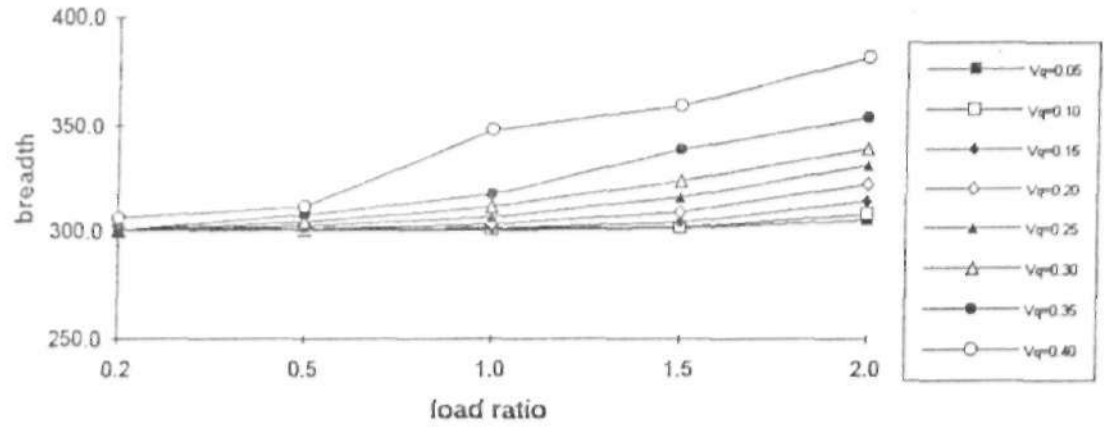
(l=5000mm and Q=2.5KN/m^2)

Fig.5.9b Depth versus load ratio for varying Vq



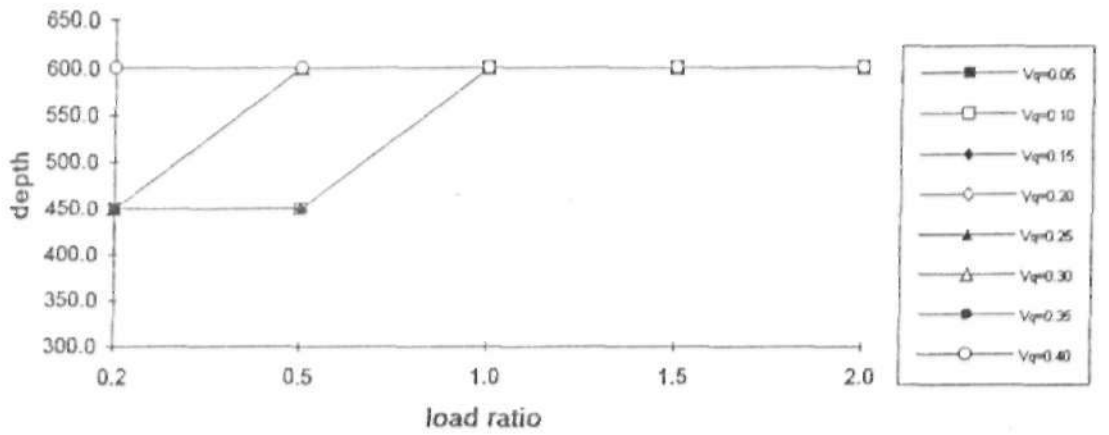
(l=5000mm and Q=2.5KN/m^2)

Fig.5.9c Total cost ratio versus load ratio for varying Vq



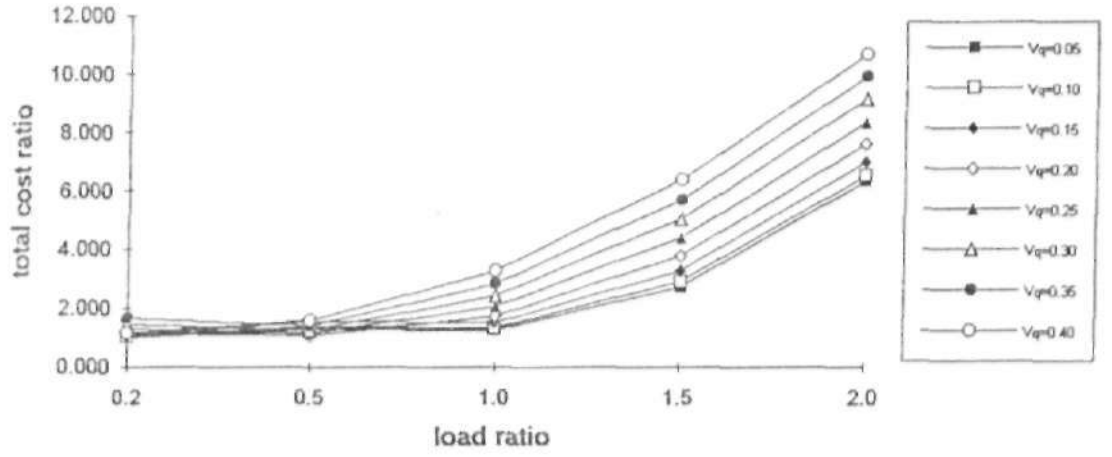
(l=6000mm and Q=2.5KN/m^2)

Fig.5.10a Breadth versus load ratio for varying Vq



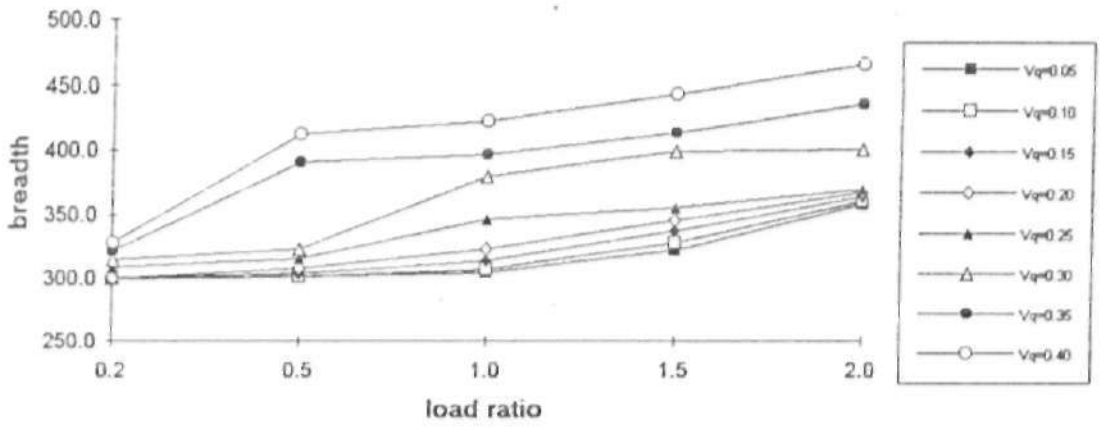
(l=6000mm and Q=2.5KN/m^2)

Fig.5.10b Depth versus load ratio for varying Vq



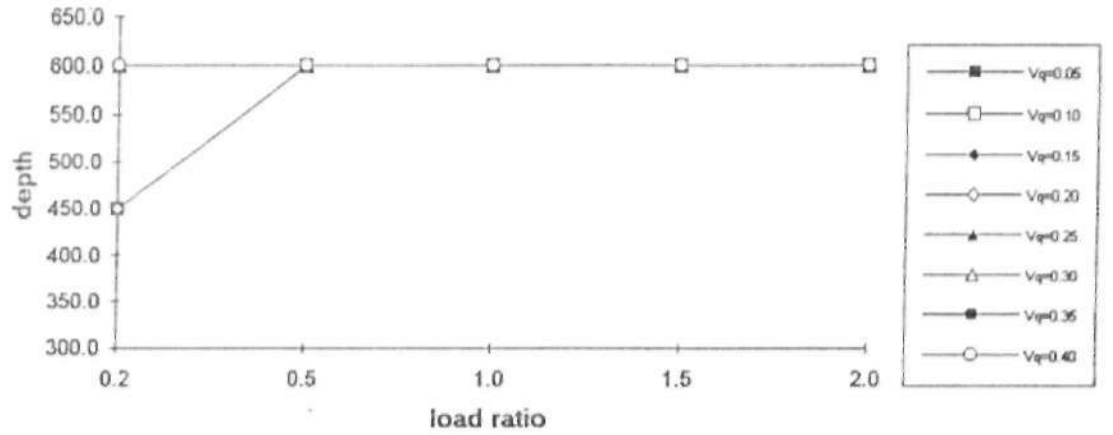
(l=6000mm and Q=2.5KN/m^2)

Fig.5.10c Total cost ratio versus load ratio for varying Vq



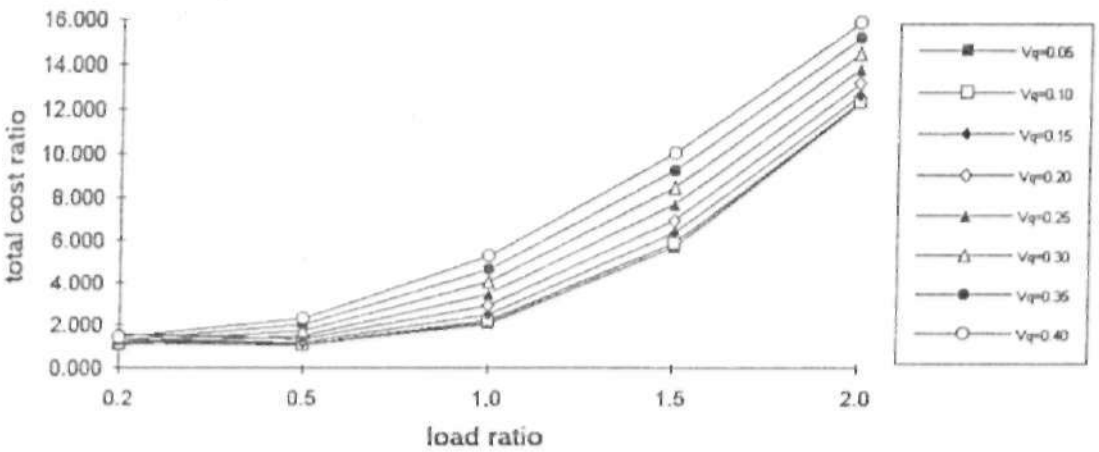
(l=7000mm and Q=2.5KN/m^2)

Fig.5.11a Breadth versus load ratio for varying Vq



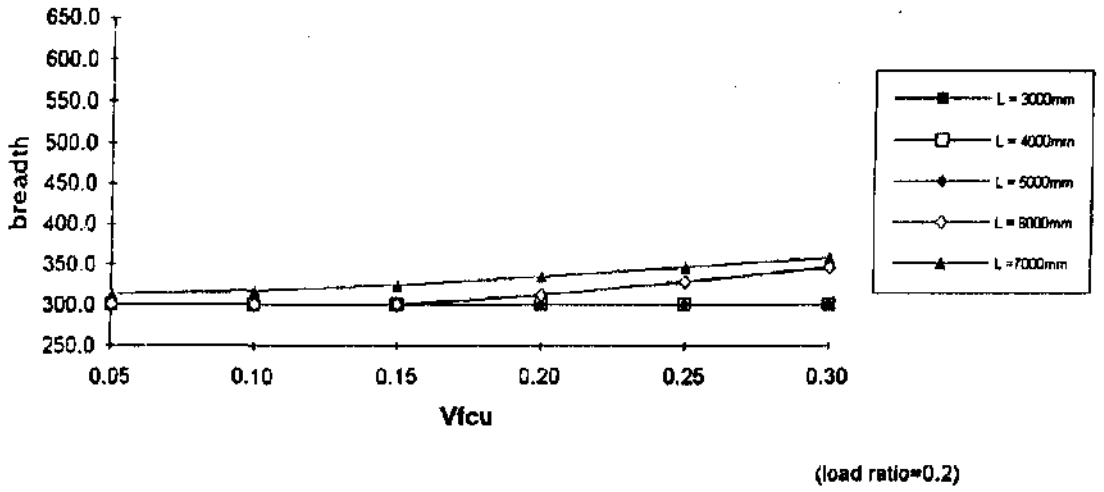
(l=7000mm and Q=2.5KN/m^2)

Fig.5.11b Depth versus load ratio for varying Vq



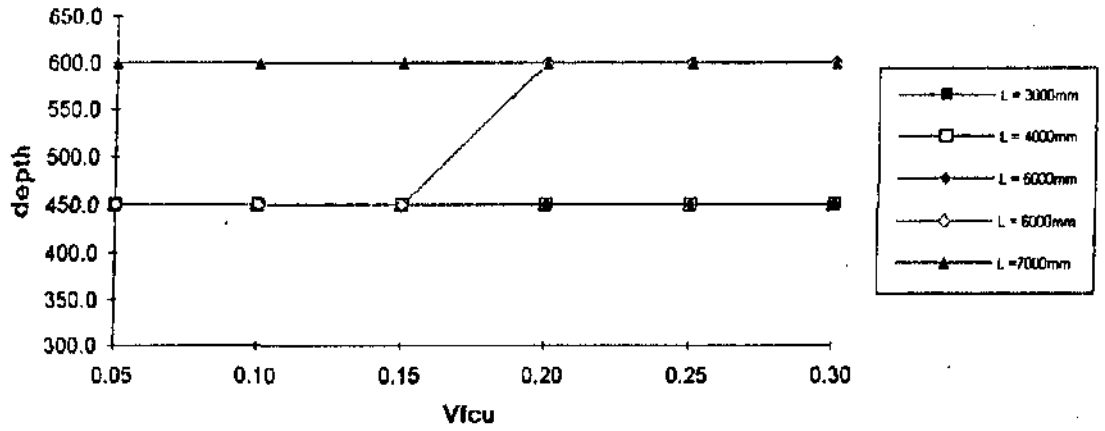
(l=7000mm and Q=2.5KN/m^2)

Fig.5.11c Total cost ratio versus load ratio for varying Vq



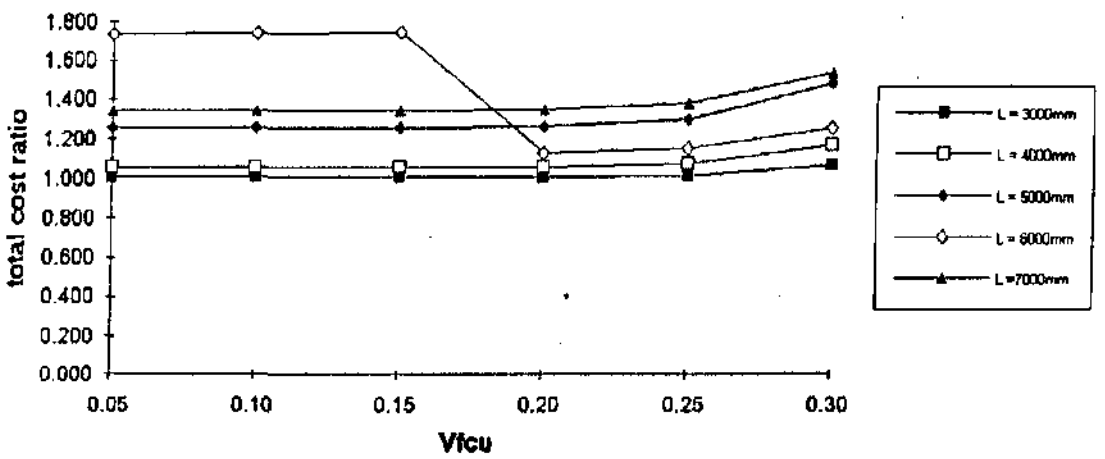
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Fig.5.12a Breadth versus Vfcu for varying lengths



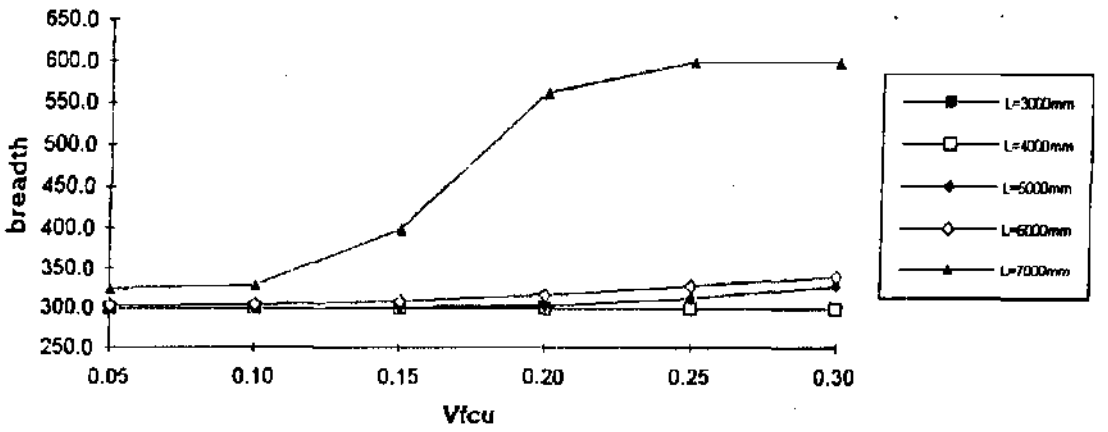
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Fig.5.12b Depth versus Vfcu for varying lengths



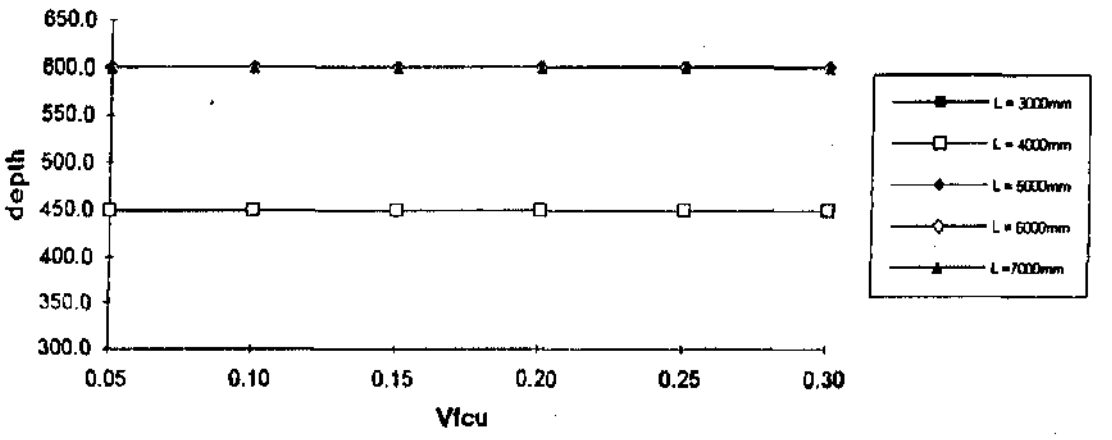
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Fig.5.12c Total cost ratio versus Vfcu for varying lengths



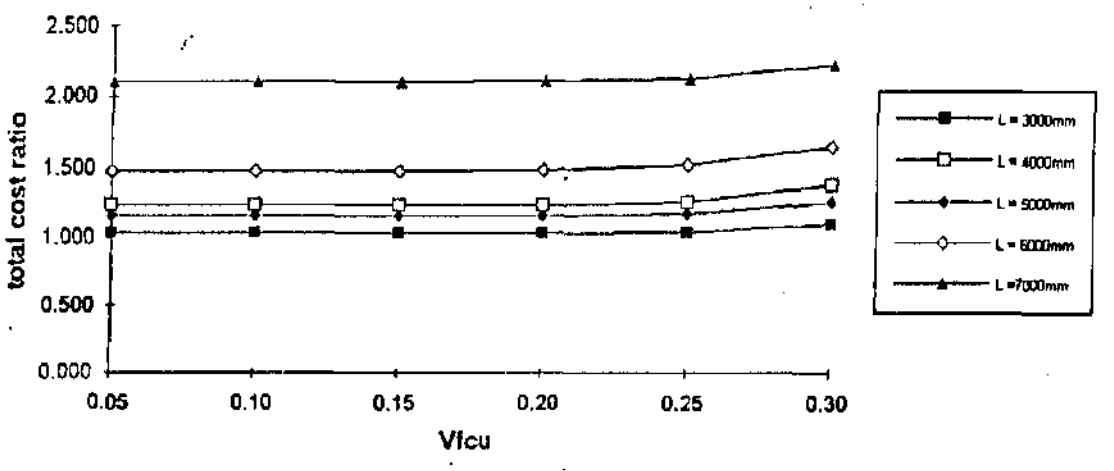
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Fig.5.13a Breadth versus Vfcu for varying lengths



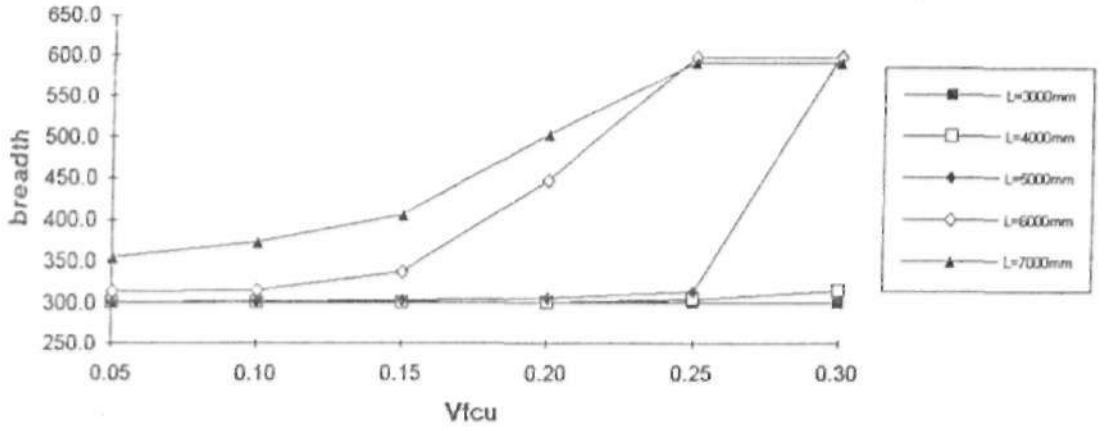
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Fig.5.13b Depth versus Vfcu for varying lengths



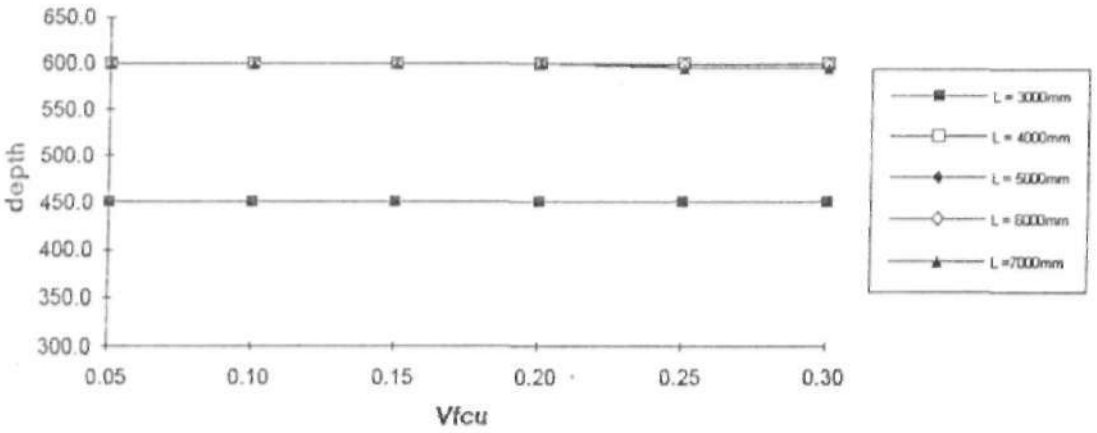
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Fig.5.13c Total cost ratio versus Vfcu for varying lengths



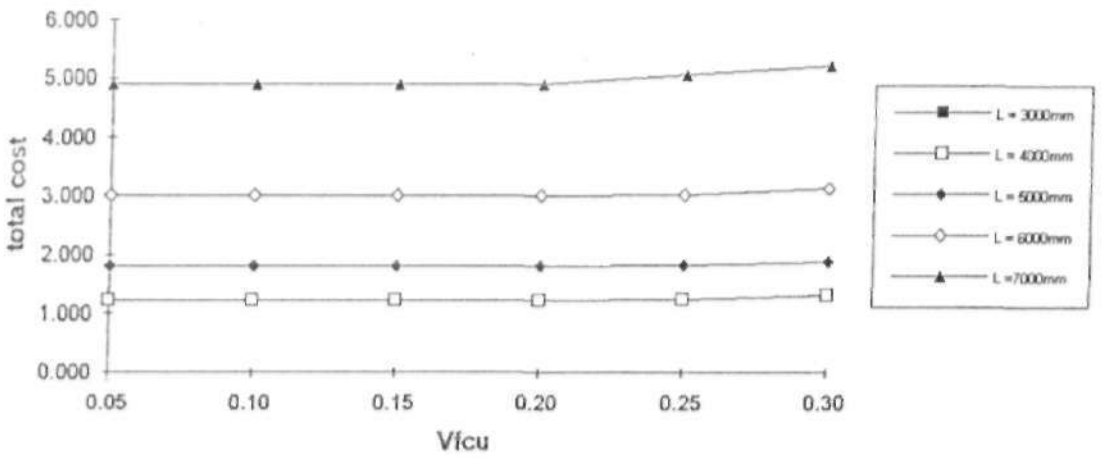
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Fig.5.14a Breadth versus Vfcu for varying lengths



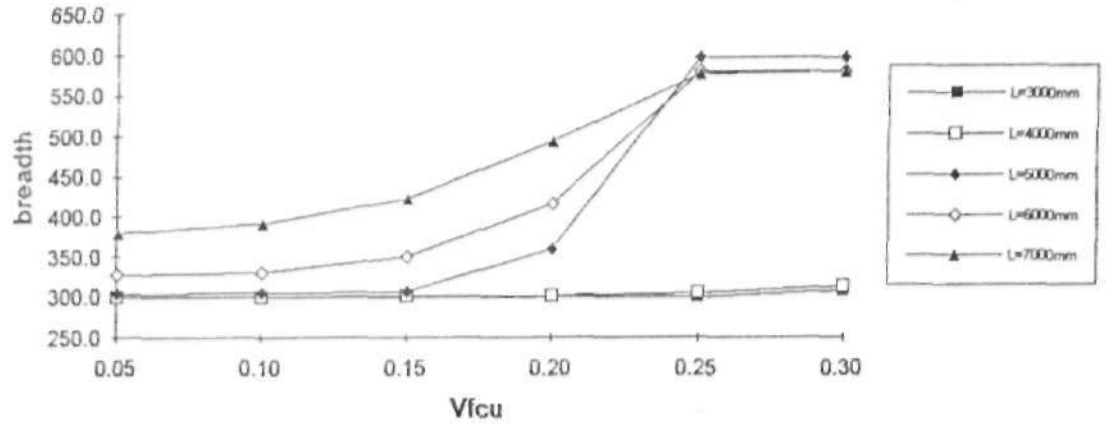
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Fig.5.14b Depth versus Vfcu for varying lengths



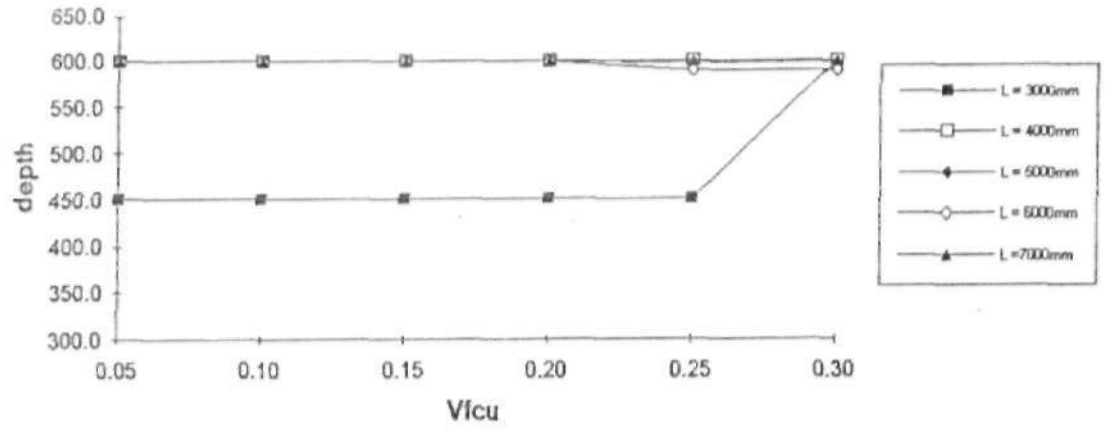
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Fig.5.14c Expected total cost versus Vfcu for varying lengths



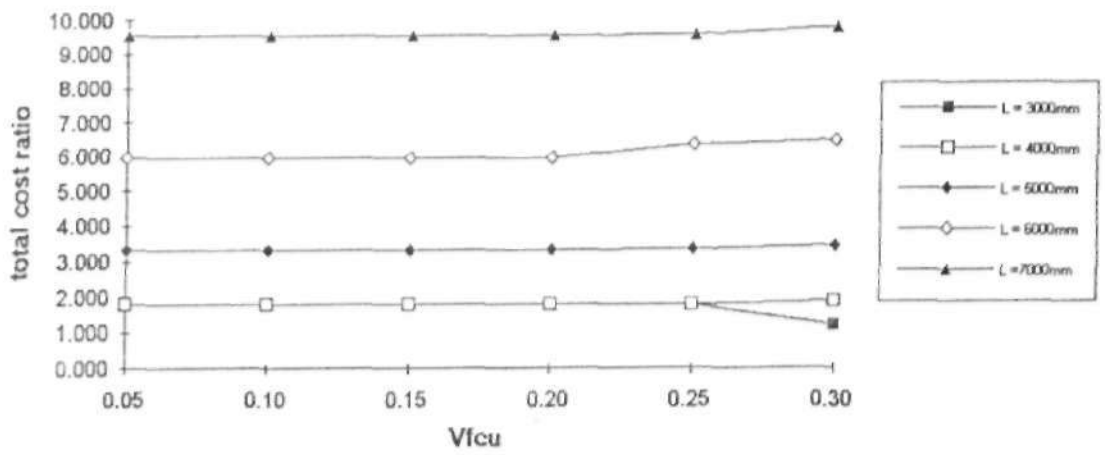
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Fig.5.15a Breadth versus Vfcu for varying lengths



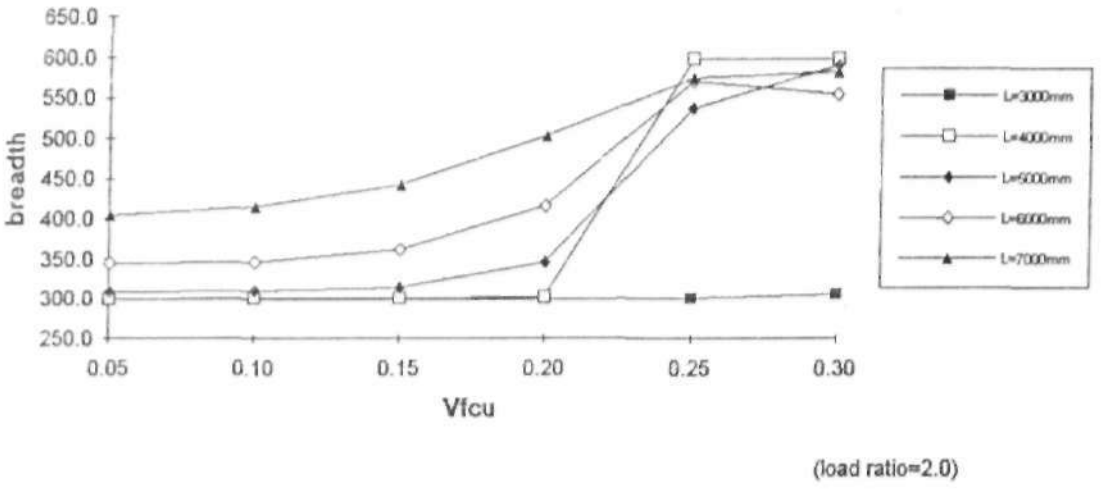
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Fig.5.15b Depth versus Vfcu for varying lengths



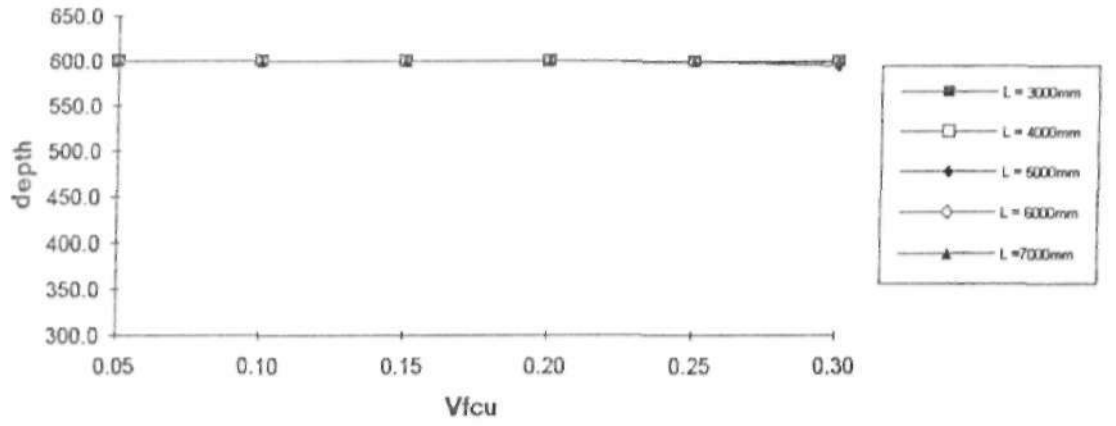
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Fig.5.15c Total cost ratio versus Vfcu for varying lengths



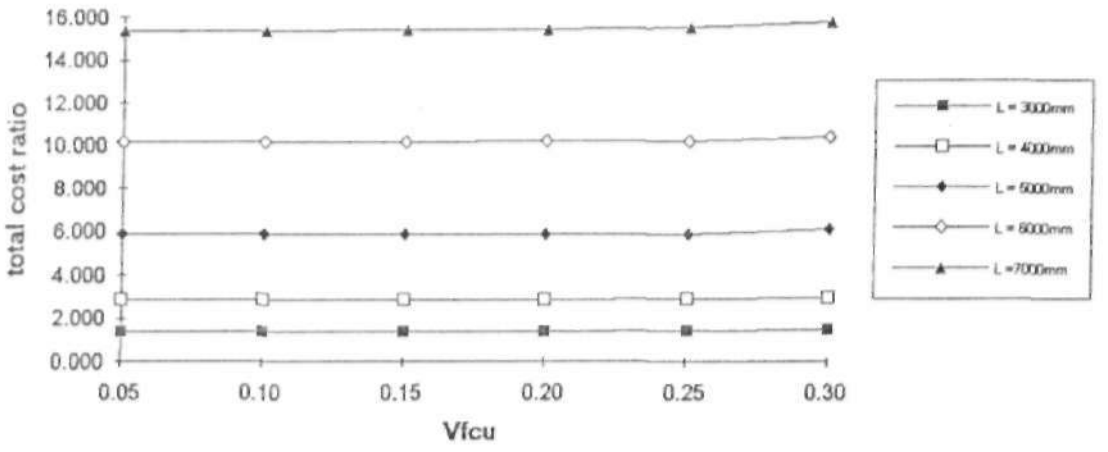
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Fig.5.16a Breadth versus Vfcu for varying lengths



(load ratio=2.0)

Fig.5.16b Depth versus Vfcu for varying lengths



(load ratio=2.0)

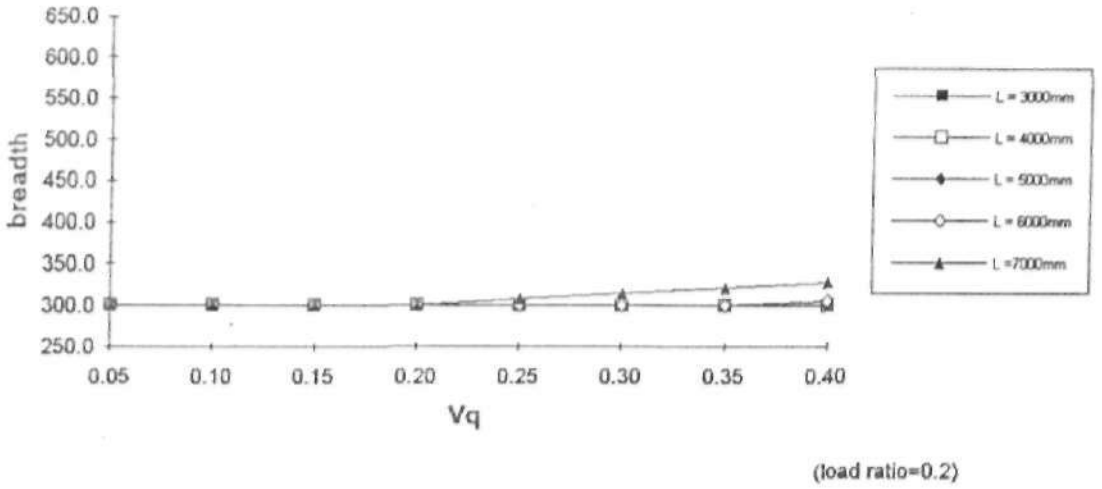
Fig.5.16c Total cost ratio versus Vfcu for varying lengths

breadth is more influenced by increase in V_{fcu} than V_q and accordingly, as the length and load ratio of the beam increase, b increases to counterbalance for the poor concrete quality to be able to accommodate the applied load and hence, reduce the probability of failure. The explanation for Figs. 5.7a, 5.8a.....5.11a, is that b is not affected by V_q for lengths less than 5000mm, after which b is dependent on V_q to compensate for inadequate estimation of applied loads.

A close look at Figs. 5.12 to 5.16 show that for $0.05 \leq V_{fcu} \leq 0.30$, at constant α, the total cost is constant with a variation of 0–15% between $0.25 \leq V_{fcu} \leq 0.30$ for all lengths. At the same time it can be observed that b increased gradually for low α, but then became inconsistent with increasing α, V_{fcu}, and L. The explanation to this may be that the cost saved in poor quality control is paid for if failure occurs. Hence, it appears that the quality of concrete and breadth do not significantly influence the total cost, and in the absence of good quality control or supervision it will be necessary to provide a greater dimension so as to compensate for the failure cost.

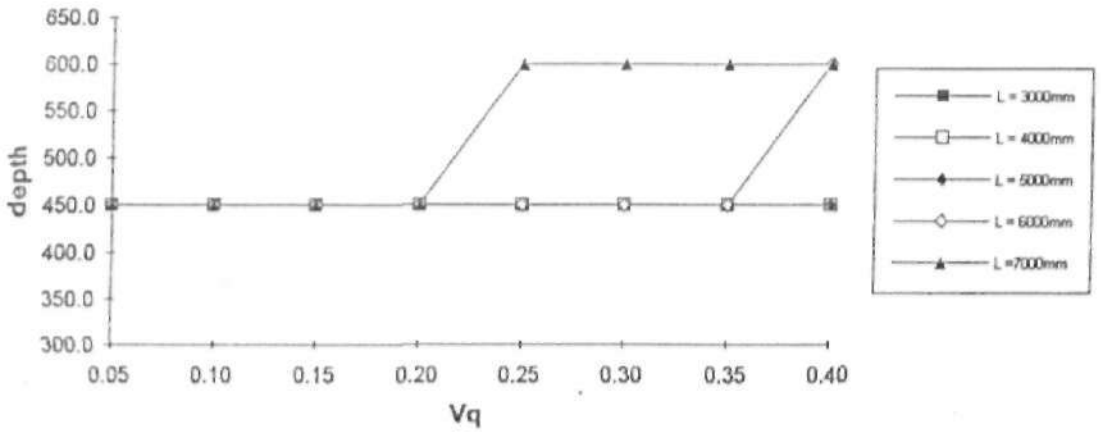
On the other hand, for $0.05 \leq V_q \leq 0.30$ (Figs 5.17 to 5.21) and at a constant α, the variation in λ_T is disproportionate and it ranges between 5% – 16% at each increment of 0.05 in V_q for all lengths. On the contrary, b increased gradually and consistently with increase in α, V_q, and L (Figs. 5.17 to 5.21). This is practical as the expected failure cost is reduced under well controlled loading. Thus, a designer's ability to predict loads accurately such that there is a low variability in load ratio will attract a more economical design than otherwise.

The range to which λ_T increases on 50% increment of load ratio is 40% – 135% with respect to V_{fcu} (Figs. 5.2c, 5.3c.....5.6c) and 5% – 150% with respect to V_q at constant lengths. (Figs. 5.7c, 5.8c.....5.11c). For example, in Fig 5.3c, the curve becomes steeper at $\alpha > 1.0$ and the difference in cost between $\alpha = 1.0$ and 1.5 and $\alpha = 1.5$ and 2.0 is 44% and 57%, respectively. This implies that in a situation where the self-weight of the structure is greater than the imposed



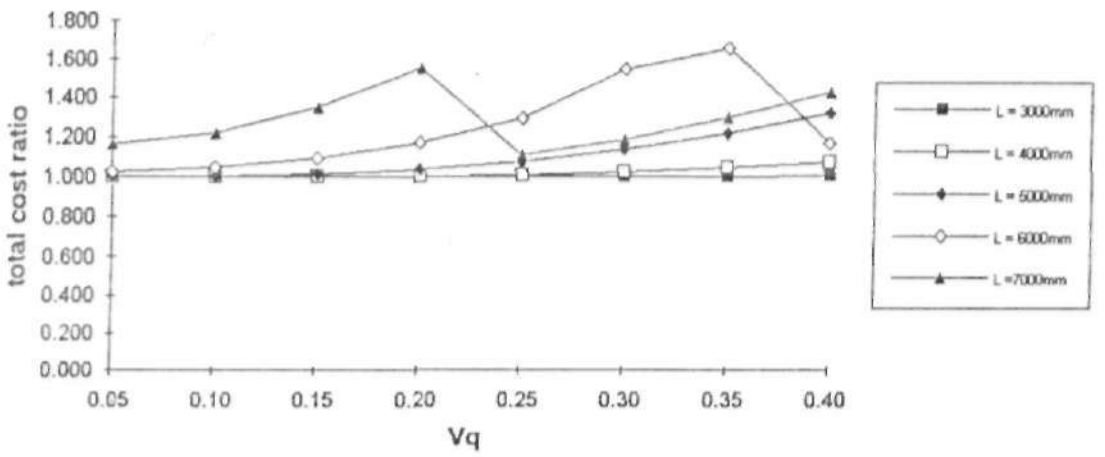
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Fig.5.17a Breadth versus Vq for varying lengths



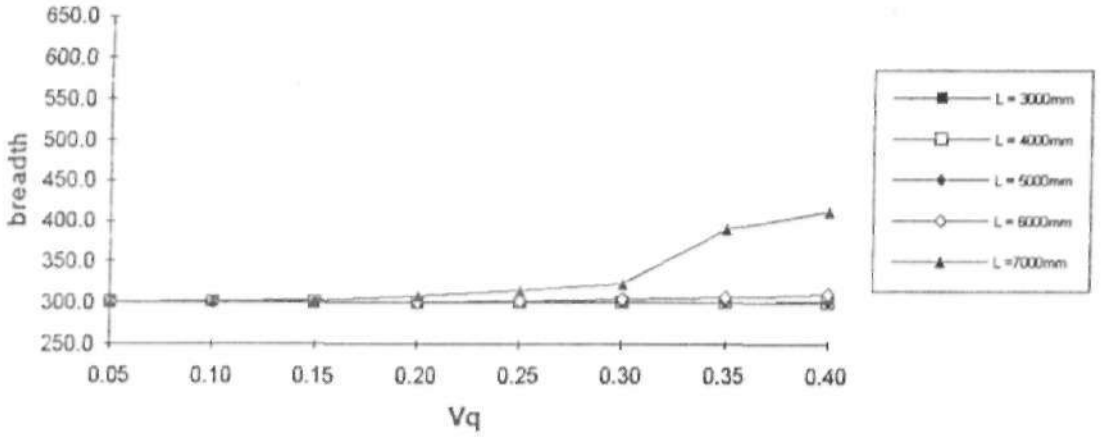
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Fig.5.17b Depth versus Vq for varying lengths



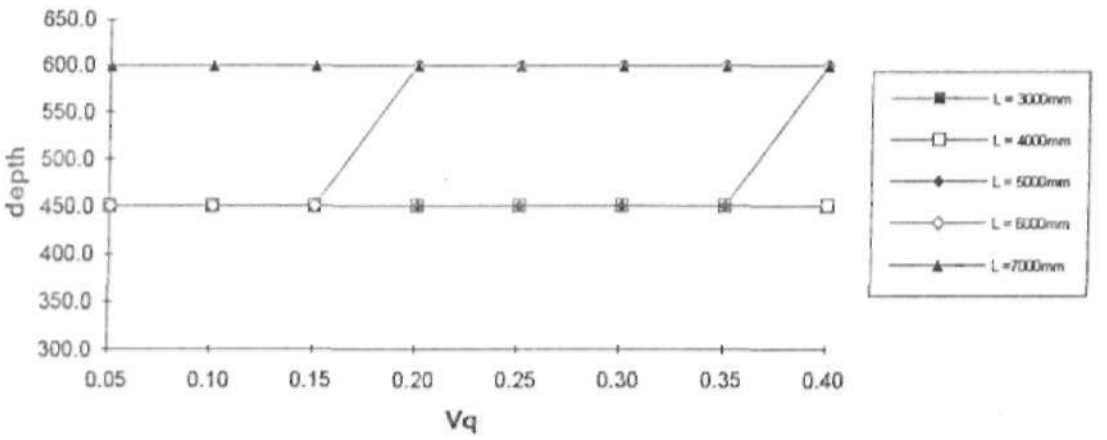
(load ratio=0.2)

Fig.5.17c Total cost ratio versus Vq for varying lengths



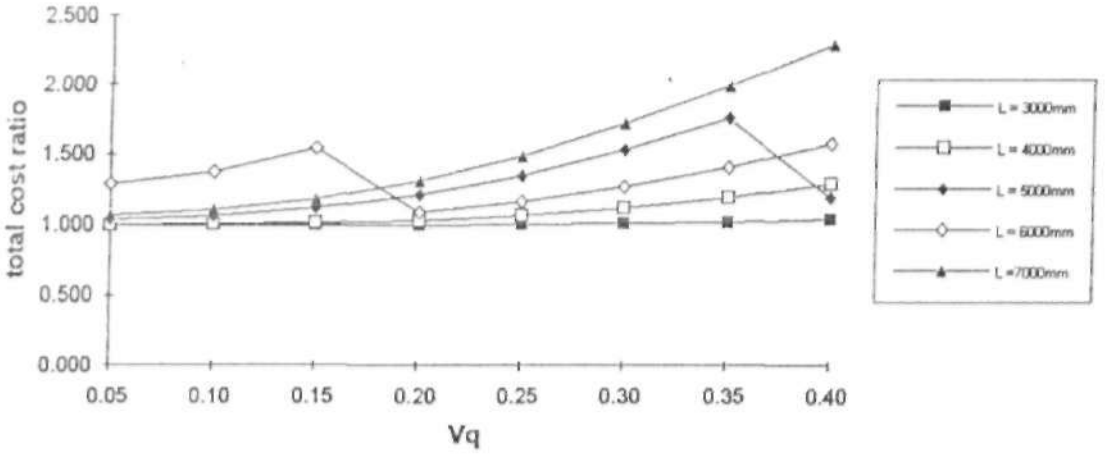
(load ratio=0.5)

Fig.5.18a Breadth versus Vq for varying lengths



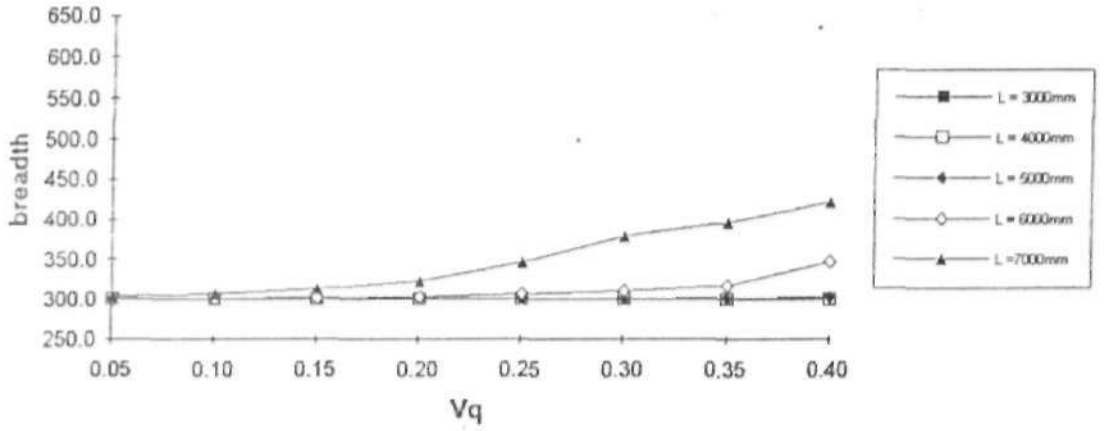
(load ratio=0.5)

Fig.5.18b Depth versus Vq for varying lengths



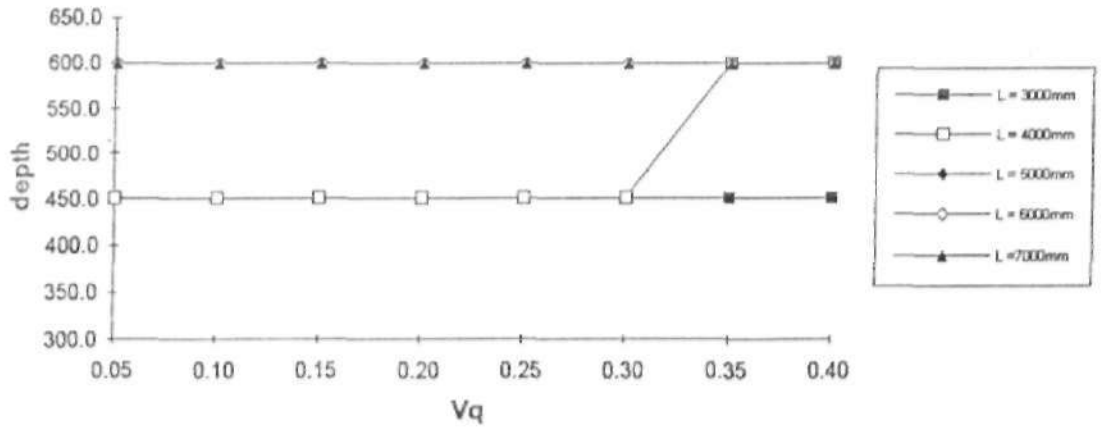
(load ratio=0.5)

Fig.5.18c Total cost ratio versus Vq for varying lengths



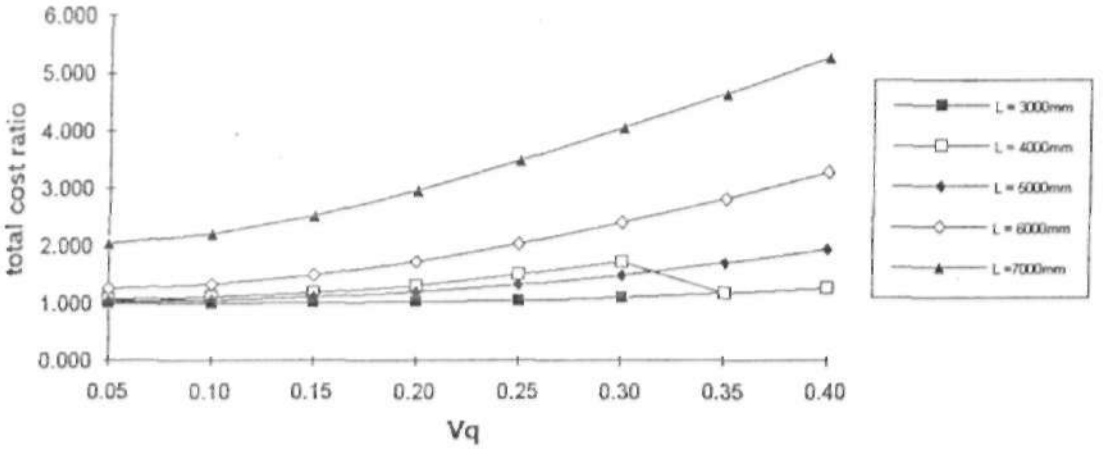
(load ratio=1.0)

Fig.5.19a Breadth versus Vq for varying lengths



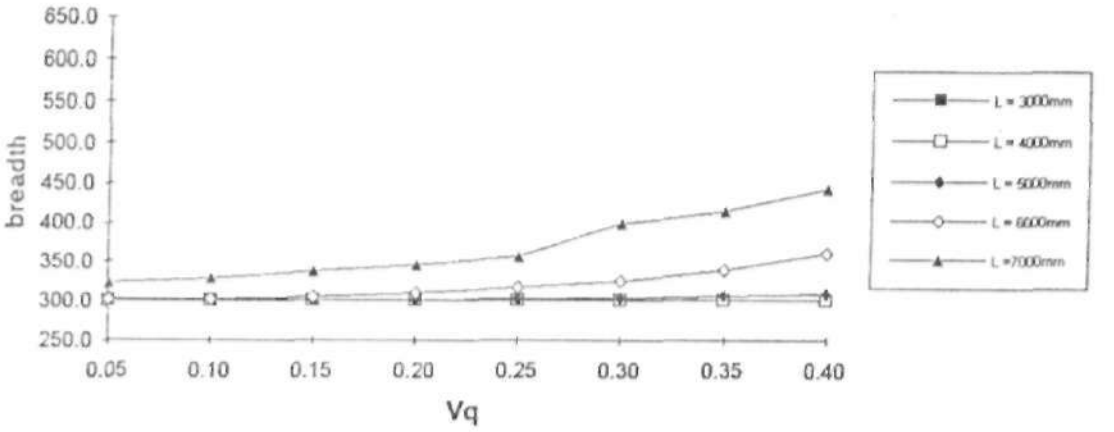
(load ratio=1.0)

Fig.5.19b Depth versus Vq for varying lengths



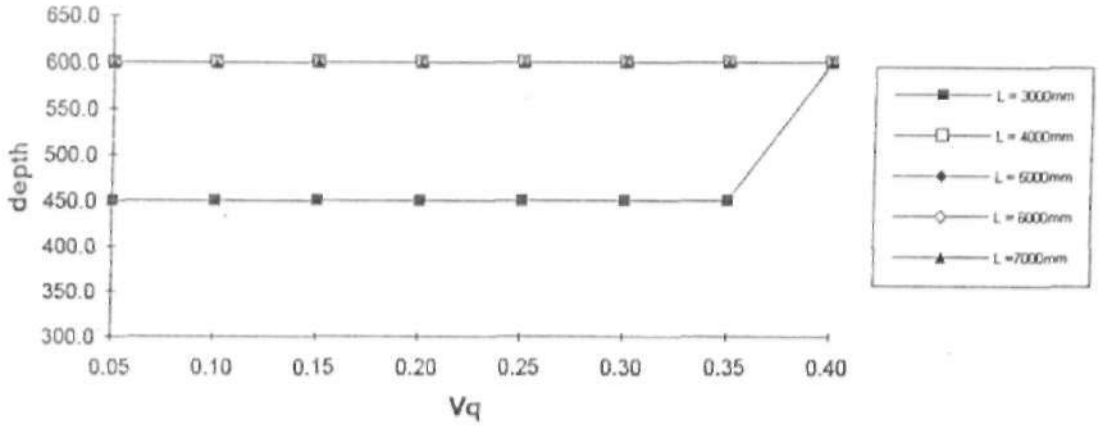
(load ratio=1.0)

Fig.5.19c Total cost ratio versus Vq for varying lengths



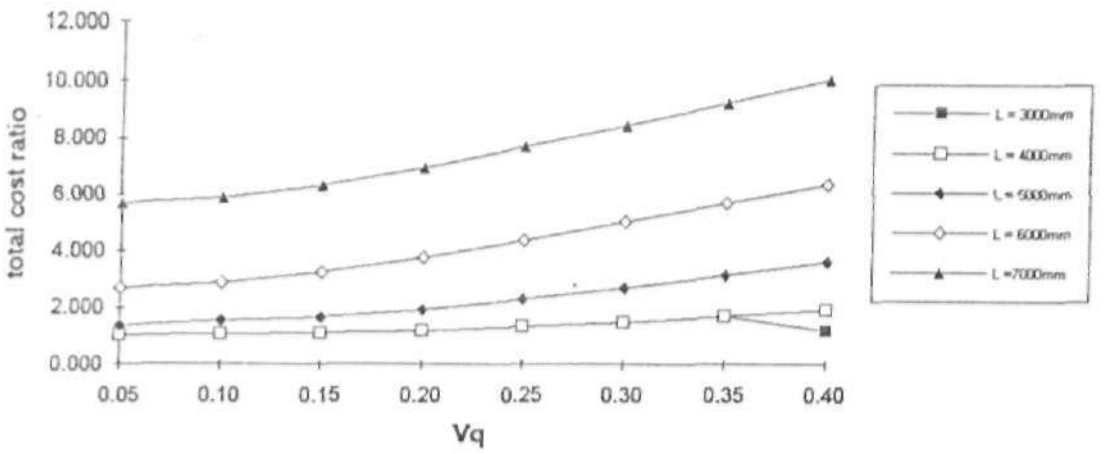
(load ratio=1.5)

Fig.5.20a Breadth versus Vq for varying lengths



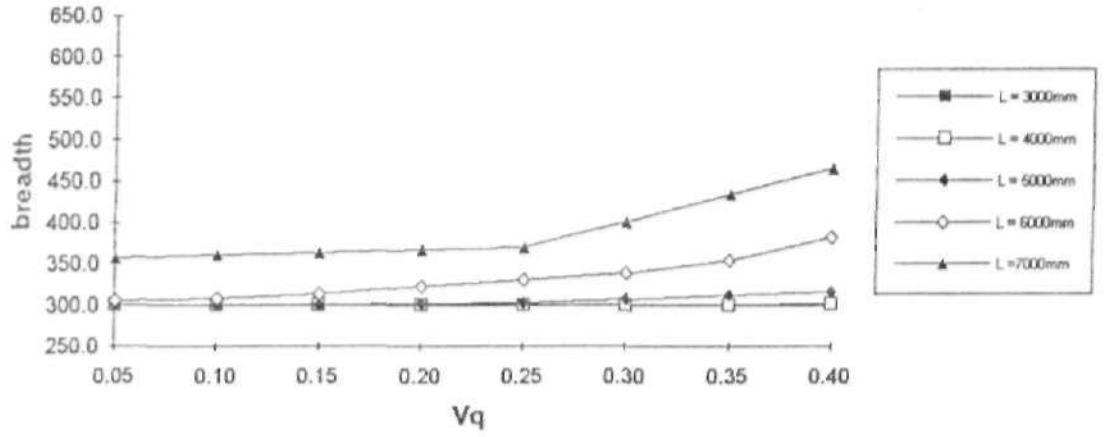
(load ratio=1.5)

Fig.5.20b Depth versus Vq for varying lengths



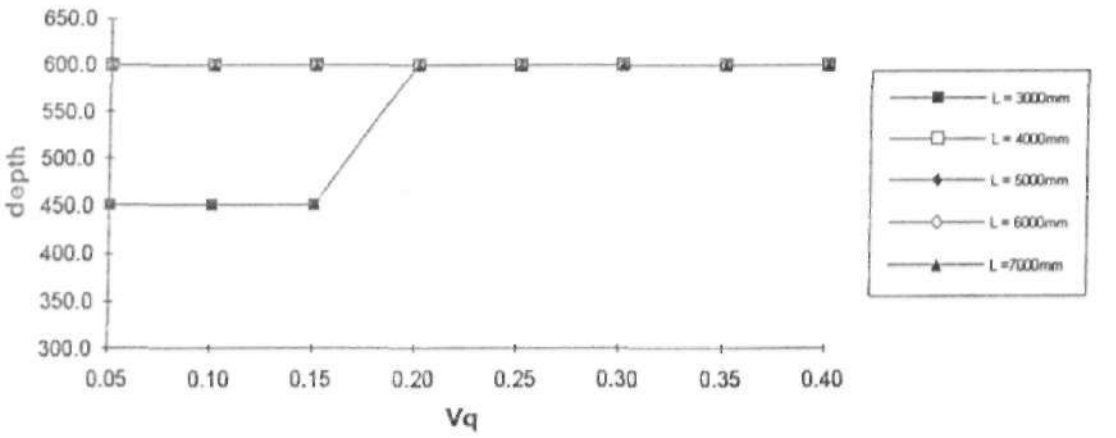
(load ratio=1.5)

Fig.5.20c Total cost ratio versus Vq for varying lengths



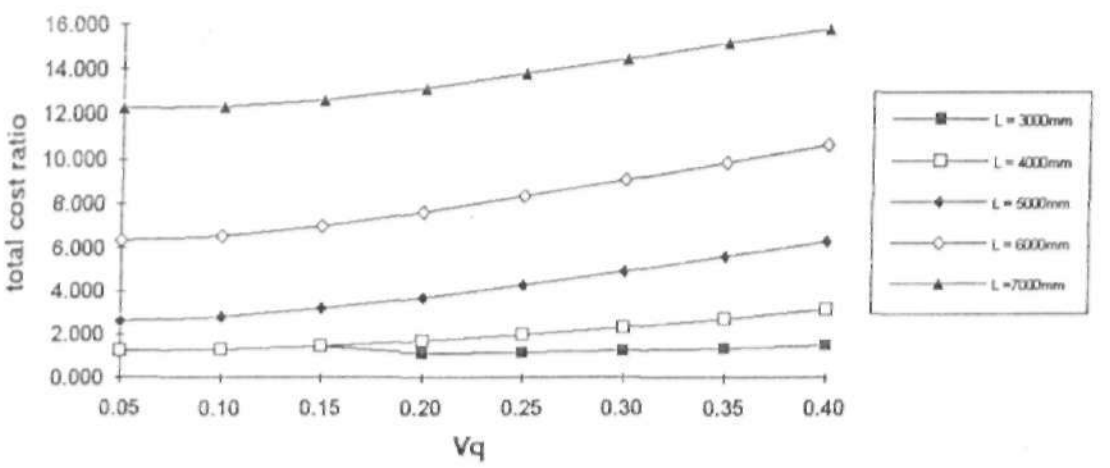
(load ratio=2.0)

Fig.5.21a Breadth versus Vq for varying lengths



(load ratio=2.0)

Fig.5.21b Depth versus Vq for varying lengths



(load ratio=2.0)

Fig.5.21c Total cost ratio versus Vq for varying lengths

load the cost incurred is ≥ 45% for poor quality control and ≥ 5% for incorrect predictions of loads. Moreover, Figs. 5.12c, 5.13c.....5.16c show the λ_T-bands for different lengths varying with V_{fcu} and with increasing α, the band becomes wider revealing the differences in cost for each length. For example, at α = 1.0, the percentage increment in cost per 1000mm increase in length, starting from 3000mm, are 0%, 48%, 66% and 63%, respectively (Fig. 5.14c). Furthermore, at α = 0.5, it is cheaper to have a beam of L = 5000mm than 4000mm, as its cost is 6.5% lower (Fig. 5.13c) and at α = 0.2 (Fig. 5.12c), a beam of length 6000mm would be more economical than 5000mm provided the variability of concrete exceeds 0.20 and the effective depth is appropriately augmented.

Summarily, the variation of optimum depth, breadth and minimum total cost parameter with beam span and load ratio leads to these observations:

1. Each chart illustrates an optimum parameter for each length and load ratio.

2. As anticipated, there is a general trend of increasing total expected cost parameter with increasing beam lengths and load ratios.

3. An increase in breadth has no direct effect on total cost ratio but an increase in depth reduces the total cost.

4. In relation to V_{fcu}, λ_T is more or less constant for a constant length and load ratio but with V_q, λ_T increases disproportionately at each increment of 0.05 of V_q.

6
CHAPTER

CONCLUSION AND RECOMMENDATIONS

CHAPTER SIX

CONCLUSION AND RECOMMENDATIONS

6.1 *Conclusion*

The reliability levels implicit in the BS8110 requirements for continuous reinforced concrete beams have been studied and optimization procedures are carried out to minimize the total expected cost under bending and shear criteria with some sensitivity tests. Limit state equations for the various design criteria are properly derived for probabilistic assessment.

The reliability levels implicit in each design criterion are computed using the First-Order Reliability Integration technique. The results exhibit the level of conservatism involved in each criterion. The average β value for bending is found to be adequate but the shear criterion signifies a criticality, while deflection is considered to be the most conservative and uneconomical criterion.

A nonlinear programming and modern reliability analysis procedures are effectively applied in determining the optimum values of the design variables. Numerical values for the design variables at optimum points are proposed. Sensitivity tests are carried out on the optimum values of breadth, b and depth, d when the variations in the live load and concrete quality are quantified. The influence of these variabilities on the total expected cost is also noted.

This investigation has highlighted the difficulties associated with the BS8110 (1985) design criteria for reinforced concrete beams to provide economical designs. Meanwhile an attempt has been made to outline some of the parameters with their optimum values, which influence the total expected cost of reinforced concrete beams using the conditions stipulated by BS8110. The methodology adopted can immensely assist design engineers in balancing safety and economy.

6.2 Recommendations

The concepts and methods of probability provide a proper basis for the evaluation of structural safety and performance, and the development of any design criterion. Based on these concepts and as a result of recent advances made in structural reliability and computer technology, it is highly recommended that probability based review of all authorized codes be enforced and efforts made to come up with more economical designs.

Further research can be carried out in any of the following areas:

1) the minimization of the overall probability of failure for a fixed cost,
2) minimization of the total cost or weight for a specified allowable overall failure probability,
3) the given procedure can be carried out for other structural members with various loading conditions and
4) optimization using other computer aided non-linear solving programs.

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APPENDIX-A

COMPUTER PROGRAMS FOR RELIABILITY ESTIMATION AND OPTIMIZATION

```
      PROGRAM BEAM
C     BENDING CONSIDERATION OF THE RELIABILITY ESTIMATION
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      EXTERNAL GBEAM
      DIMENSION X(5),EX(5),SX(5),VP(10,5),COV(5,5),ZES(3),
     +UU(5),EIVEC(5,5),IV(2,5)
C     X(1) = Fcu (CHARACTERISTIC STRENGTH OF CONC. - LOG NORMAL DISTRIBUTION)
C     X(2) = b (WIDTH OF BEAM - NORMAL DISTRIBUTION)
C     X(3) = h (DEPTH OF BEAM - NORMAL DISTRIBUTION)
C     X(4) = Q (LIVE LOAD - GUMBEL DISTRIBUTION)
C     X(5) = L (BEAM LENGTH - NORMAL DISTRIBUTION)
      CHARACTER * 10 PRT
      COMMON/RAC2/R
      DATA EX/2.5D1,3.00D2,4.5D2,0.0025D0,5.0D3/,
     +    N/5/,NC/5/,NE/5/,IRHO/1/
      SX(1)=0.150*EX(1)
      SX(2)=0.045*EX(2)
      SX(3)=0.086*EX(3)
      SX(4)=0.370*EX(4)
      SX(5)=0.040*EX(5)
      WRITE(*,*)'ENTER THE VALUE OF LOAD RATIO,R....>'
      READ(*,*)R
C     THE RESULTS ARE WRITTEN IN FILE No.7
      NAUS=7
C     PRINT TO SCREEN
      ICRT=0
      OPEN(7,FILE='RBEAM.DAT',STATUS='OLD',ERR=10)
      GO TO 20
   10 OPEN(7,FILE='RBEAM.DAT',STATUS='NEW')
C     SUBROUTINE YINIT IS USED TO PRESET VARIABLES VP,COV,AND IV
   20 CALL YINIT(N,IV,VP,IRHO,COV,NC)
      IV(1,1)=3
      IV(1,4)=7
      DO 100 I=1,N
      VP(5,I) = 1.0D0
      VP(6,I) = 1.0D0
  100 X(I)=EX(I)
      V1=0.945D0
      BETA=1.D0
      WRITE(NAUS,5000)
 5000 FORMAT(////,5X,70('*')/,30X,'FORM5'/,5X,
     +70('*')/,20X,'BENDING CONSIDERATION,5 VARIABLES:')
C     THE STOCHASTIC MODEL IS CALLED USING 'YKOPF'
      CALL YKOPF(NAUS,N,IV,EX,SX,VP,IRHO)
C     PRINT ALSO TO SCREEN
      WRITE(ICRT,*)'START OF FORM5'
      WRITE(ICRT,*)'STOCHASTIC MODEL'
      CALL YKOPF(ICRT,N,IV,EX,SX,VP,IRHO)
```

```
      CALL YKOPF(ICRT,N,IV,EX,SX,VP,IRHO)
C     THE MATRIX COV IS PRINTED USING 'YMAUS'
      PRT='  COV   '
      CALL YMAUS(NAUS,NC,N,COV,PRT)
C     THE MAIN SUBROUTINE IN FORM5 IS CALLED
      CALL FORM5(N,IV,EX,SX,VP,GBEAM,IRHO,COV,NC,EIVEC,
     +NE,V1,NAUS,BETA,X,UU,ZES,IER)
C     THE COORDINATES OF THE BETA-PT. ARE PRINTED WITH THE TITLE
C     VECTOR UU
      PRT='  UU   '
      CALL YFAUS(NAUS,N,UU,PRT)
C     THE VECTOR ZES IS PRINTED WITH THE TITLE VECTOR ZES
      PRT='  ZES   '
      CALL YFAUS(NAUS,3,ZES,PRT)
      WRITE(ICRT,*)'END OF FORM5:IER=',IER
      WRITE(ICRT,*)'FOR RESULTS SEE FILE RBEAM.DAT'
      STOP
      END


      SUBROUTINE GBEAM (N,X,FX,IER)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION X(N)
      COMMON/RAC2/R
      AN1= X(4)*(1.4*R+1.6)*3000.0
      AN2= X(4)*R*3000.0
      AM1= 0.156*X(3)*X(1)*X(2)**2
      AM2= (X(5)**2/192.)*(22.*AN1-3.*AN2)
      IF(AN1.Gt.0.AND.AN2.Gt.0.)THEN
      FX= AM1-AM2
      IER = 0
      ELSE
      FX = 1.D20
      IER = 1
      ENDIF
      RETURN
      END
```

****

****

```
      PROGRAM SHEAR
C     SHEAR CONSIDERATION OF THE RELIABILITY ESTIMATION
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      EXTERNAL GSHEAR
      DIMENSION X(9),EX(9),SX(9),VP(10,9),COV(9,9),ZES(3),
     +UU(9),EIVEC(9,9),IV(2,9)
      CHARACTER * 10 PRT
      COMMON/RAC2/R
C     X(6) = Fyv (CHARACTERISTIC STRENGTH OF LINKS - LOG NORMAL DISTRIBUTION)
C     X(7) = Asv (CROSS-SECTIONAL AREA OF LINKS - NORMAL DISTRIBUTION)
C     X(8) = Sv (SPACING OF LINKS - NORMAL DISTRIBUTION)
C     X(9) = K (RATIO VC/VS - NORMAL DISTRIBUTION)
      DATA EX/3.00D2,4.50D2,2.50D1, 0.0025D0,5.00D3, 2.50D2,1.571D2, 2.50D2,
     +6.00D-1/,
     +    N/9/,NC/9/,NE/9/,IRHO/1/
      SX(1)=0.045*EX(1)
      SX(2)=0.086*EX(2)
```

```
      SX(2)=0.086*EX(2)
      SX(3)=0.15*EX(3)
      SX(5)=0.04*EX(5)
      SX(4)=0.37*EX(4)
      SX(6)=0.15*EX(6)
      SX(7)=0.036*EX(7)
      SX(8)=0.09*EX(8)
      SX(9)=0.3*EX(9)
      WRITE(*,*)'ENTER THE VALUE OF LOAD RATIO,R.....>'
      READ(*,*)R
      NAUS=7
      ICRT=0
      OPEN(7,FILE='SHEAR.DAT',STATUS='OLD',ERR=10)
      GO TO 20
10    OPEN(7,FILE='SHEAR.DAT',STATUS='NEW')
20    CALL YINIT(N,IV,VP,IRHO,COV,NC)
      IV(1,3)=3
      IV(1,6)=3
      IV(1,4)=7
      DO 100 I=1,N
      VP(5,I) = 1.D0
100   X(I) = EX(I)
      V1 = 1.0D0
      BETA=1.D0
      WRITE(NAUS,5000)
5000  FORMAT(////,5X,70('*'),/,30X,'FORM5',/,5X,
     +70('*'),/,20X,'SHEAR CONSIDERATION,9 VARIABLES')
      CALL YKOPF(NAUS,N,IV,EX,SX,VP,IRHO)
      WRITE(ICRT,*)'START OF FORM5'
      WRITE(ICRT,*)'STOCHASTIC MODEL'
      CALL YKOPF(ICRT,N,IV,EX,SX,VP,IRHO)
      PRT=' COV  '
      CALL YMAUS(NAUS,NC,N,COV,PRT)
      CALL FORM5(N,IV,EX,SX,VP,GSHEAR,IRHO,COV,NC,EIVEC,
     +NE,V1,NAUS,BETA,X,UU,ZES,IER)
      PRT=' UU '
      CALL YFAUS(NAUS,N,UU,PRT)
      PRT=' ZES '
      CALL YFAUS(NAUS,3,ZES,PRT)
      WRITE(ICRT,*)'END OF FORM5:IER=',IER
      WRITE(ICRT,*)'FOR RESULTS SEE FILE SHEAR1.DAT'
      STOP
      END

      SUBROUTINE GSHEAR(N,X,FX,IER)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION X(N)
      COMMON/RAC2/R
      AN1 = X(4) * 3000.0 * (1.4 * R + 1.6)
      AN2 = X(4) * 3000.0 * R
      V = (0.55 * AN1 + 0.049 * AN2) * X(5)
      VS = V/(X(1) * X(2))
      VC = 0.8 * SQRT(X(3))
      IF (VS.LE.VC) THEN
      FX =( 0.87 * X(6) * X(7)) /(X(1) * X(8)) - VS * (1 - X(9))
      IER = 0
      ELSE
```

```
      ELSE
      FX = 1.D20
      IER = 1
      ENDIF
      RETURN
      END
*****************************************************************************
*****************************************************************************
      PROGRAM DEFLEC
C     DEFLECTION CONSIDERATION OF THE RELIABILITY ESTIMATION
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      EXTERNAL GBEAM3
      DIMENSION X(5),EX(5),SX(5),VP(10,5),COV(5,5),ZES(3),
     +UU(5),EIVEC(5,5),IV(2,5)
      CHARACTER * 10 PRT
      COMMON/RAC2/R
      DATA EX/3.00D2,4.5D2,2.5D1, 0.0025D0,5.00D3
     +    N/5/,NC/5/,NE/5/,IRHO/1/
      SX(1) = 0.045*EX(1)
      SX(2) = 0.086*EX(2)
      SX(3) = 0.150*EX(3)
      SX(4) = 0.370*EX(4)
      SX(5) = 0.040*EX(5)
      WRITE(*,*)'ENTER THE VALUE OF LOAD RATIO,R....>'
      READ(*,*)R
      NAUS=7
      ICRT=0
      OPEN(7,FILE='DBEAM.DAT',STATUS='OLD',ERR=10)
      GO TO 20
10    OPEN(7,FILE='DBEAM.DAT',STATUS='NEW')
20    CALL YINIT(N,IV,VP,IRHO,COV,NC)
      IV(1,3)=3
      IV(1,4)=7
      DO 100 I=1,N
      VP(5,I) = 1.D0
100   X(I)=EX(I)
      V1=1.0D0
      BETA=1.D0
      WRITE(NAUS,5000)
5000  FORMAT(////,5X,70('*')/,30X,'FORM5'/,5X,
     +70('*')/,20X,'DEFLECTION CONSIDERATION,5 VARIABLES:')
      CALL YKOPF(NAUS,N,IV,EX,SX,VP,IRHO)
      WRITE(ICRT,*)'START OF FORM5'
      WRITE(ICRT,*)'STOCHASTIC MODEL'
      CALL YKOPF(ICRT,N,IV,EX,SX,VP,IRHO)
      PRT=' COV  '
      CALL YMAUS(NAUS,NC,N,COV,PRT)
      CALL FORM5(N,IV,EX,SX,VP,GBEAM3,IRHO,COV,NC,EIVEC,
     +NE,V1,NAUS,BETA,X,UU,ZES,IER)
      PRT='  UU  '
      CALL YFAUS(NAUS,N,UU,PRT)
      PRT='  ZES  '
      CALL YFAUS(NAUS,3,ZES,PRT)
      WRITE(ICRT,*)'END OF FORM5:IER=',IER
      WRITE(ICRT,*)'FOR RESULTS SEE FILE DBEAM.DAT'
      STOP
```

```
      STOP
      END

      SUBROUTINE GBEAM3 (N,X,FX,IER)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION X(N)
      COMMON/RAC2/R
      AN1= X(4) * (1.4 * R + 1.6) * 3000.0
      AN2= X(4) * R * 3000.0
      AM= (X(5)**2 / 768.) * (77.* AN1 - 19.* AN2)
      CMF= 0.55 + 216./(120. * (0.9 + AM/(X(1) * X(2)**2)))
      IF(AN1.GT.0.AND.AN2.GT.0.)THEN
            IF(CMF.LE.2.0)THEN
            FX = 26.*CMF - X(5)/X(2)
            IER = 0
            ELSE
            FX= 26.*2 - X(5)/X(2)
            IER = 0
            ENDIF
      ELSE
            FX = 2.0D20
            IER = 1
      ENDIF
      RETURN
      END
*****************************************************************************
*****************************************************************************
THIS PROGRAM MINIMIZES THE TOTAL COST OF CONSTRUCTION AND FAILURE  BASED ON
                        F O R M   FORMULATION
*****************************************************************************
*****************************************************************************

      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/TAPES/ITAP1,ITAP2,ITAP3,ITAP4,ITAP5,ITAP6,ITAP7,ITAP8,
     -      ITAP9,ITAP10
      COMMON/BEAMS/ALPHA
      COMMON/RAC2/R,S,SL
      COMMON/CMACHE/EPS
      COMMON/NOTE/MAXFUN,MAXIT
      DIMENSION X(40),XL(40),XU(40),WA(4000),KWA(200)
      LOGICAL ACTIVE(200)
      ITAP6 = 6
      OPEN(ITAP6,FILE ='ARACO.RES',STATUS='OLD',ERR = 111)
      GOTO 112
111   OPEN(ITAP6,FILE='ARACO.RES',STATUS='NEW')
112   DO222 KK =1,5
      WRITE(*,*)' GIVE THE SPAN'
      READ(*,*) SL
      ALPHA = 2.0
      N = 2
      M = 2
      ME = 0
      IPRINT = 2
      MAXIT = 100
      MAXFUN = 15
      LWA=400
      LKWA=200
```

```
      LKWA=200
      LACTIV=200
      EPS=1.D-8
      XL(1) = 250.0
      X(1)  = 300.0
      XU(1) = 600.0
      XL(2) = 300.0
      X(2)  = 450.0
      XU(2) = 600.0
      CALL NLPQL0(M,ME,N,X,F,XL,XU,IPRINT,IFAIL,WA,LWA,KWA,LKWA)
      CALL AISI(5,X,PFB)
      CALL AISI2(9,X,PFS)
      WRITE(*,*)(X(I),I=1,9)
      WRITE(*,*)'OPTIMUM PFB =',PFB
      WRITE(*,*)'OPTIMUM PFS =',PFS
      PFF = PFB + PFS
      WRITE(*,*)'OPTIMUM TOTAL PF=',PFF
      WRITE(*,*)'OBJECTIVE FUNCTION =',F
222   CONTINUE
      END

      SUBROUTINE NLFUNC(M,ME,MMAX,N,F,G,X,ACTIVE)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION G(MMAX),X(N)
      LOGICAL ACTIVE(200)
      IF(M.GE.0)GOTO 10
      MM=IABS(M)
      GOTO (10),MM
10    CALL AISI(5,X,pfb)
      CALL AISI2(9,X,PFS)
      GAM = 75.0
      PFF = PFB + PFS
      F = 1.0 + GAM*PFF
      IF(M.EQ.0)GOTO 99
11    CALL AISI(5,X,PFB)
      C1 =-(PFB - 1.35D-3)
      IF(M.GT.0) G(1)=C1
12    CALL AISI2(9,X,PFS)
      C2 =-(PFS - 1.35D-3)
      IF(M.GT.0)G(2) = C2
      IF(M.EQ.0)GOTO 99
99    CONTINUE
      RETURN
      END

      SUBROUTINE NLGRAD(M,ME,MMAX,N,F,G,DF,DG,X,ACTIVE,WA)
C     NUMERICAL EVALUATION OF GRADIENTS BY FORWARD DIFFERENCES
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION G(MMAX),DF(N),DG(MMAX,N),X(N),ACTIVE(MMAX)
C     REAL F,G,DF,DG,X,ON,EPS,XEPS,XEPSI,FEPS,GJEPS,DMAX1,ABS
      LOGICAL ACTIVE
      ON=1.D0
      EPS=1.D-5
      DO1 I=1,N
      XEPS=EPS*(DMAX1(ON,ABS(X(I))))
      XEPSI=ON/XEPS
      X(I)=X(I) + XEPS
```

```
      X(I)=X(I) + XEPS
      CALL NLFUNC(M,ME,MMAX,N,FEPS,G,X,ACTIVE)
      DF(I)=(FEPS - F)*XEPSI
      IF(M.EQ.0) GOTO1
      DO2 J=1,M
      IF(.NOT.ACTIVE(J)) GOTO2
      CALL NLFUNC(-J,ME,MMAX,N,GJEPS,G,X,ACTIVE)
      DG(J,I)=(GJEPS - G(J))*XEPSI
2     CONTINUE
1     X(I)=X(I) - XEPS
      RETURN
      END

      SUBROUTINE    AISI (N,X,PFB)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      EXTERNAL GBEAM
      DIMENSION EX(5),SX(5),x(5)
      DIMENSION VP(10,5),COV(5,5),ZES(3),UU(5),EIVEC(5,5),IV(2,5)
      CHARACTER * 10 PRT
      COMMON/BEAMS/ALPHA
      COMMON/RAC2/R,S,SL
      N = 5
      EX(3) = 2.5D1
      EX(4) = .0025D0
      EX(5) = SL
      DO 444 I=1,2
444   EX(I) = X(I)
      SX(1) = 0.1D-6*X(1)
      SX(2) = 0.1D-6*X(2)
      SX(3) =0.15*EX(3)
      SX(4) = 0.37*EX(4)
      SX(5) = 0.04*EX(5)
      NC = N
      NE = NC
      IRHO = 1
      R=ALPHA
      S=3000.
      NAUS=7
      ICRT=0
      OPEN(NAUS,FILE='RBEAM.DAT',STATUS='OLD',ERR=10)
      GOTO 20
10    OPEN(NAUS,FILE='RBEAM.DAT',STATUS='NEW')
20    CALL YINIT(N,IV,VP,IRHO,COV,NC)
      IV(1,4)=7
      DO 100 I=1,N
      VP(5,I) = 1.0D0
      VP(6,I) = 1.0D0
      X(I)= EX(I)
100   CONTINUE
      V1 = 0.25D0
      BETA = 1.0D0

      WRITE(NAUS,5000)
5000  FORMAT(///,5X,70('*')/,30X,'FORM5'/,5X,
     +70('*')/,20X,'BENDING CONSIDERATION,5 VARIABLES:')
      CALL YKOPF(NAUS,N,IV,EX,SX,VP,IRHO)
      WRITE(ICRT,*)'START OF FORM5'
```

```
 WRITE(ICRT,*)'START OF FORM5'
 WRITE(ICRT,*)'STOCHASTIC MODEL'
 CALL YKOPF(ICRT,N,IV,EX,SX,VP,IRHO)
 PRT='  COV  '
 CALL YMAUS(NAUS,NC,N,COV,PRT)
 CALL FORM5(N,IV,EX,SX,VP,GBEAM,IRHO,COV,NC,EIVEC,
 +NE,V1,NAUS,BETA,X,UU,ZES,IER)
 PFB = YPHINF(-BETA)
 PRT='  UU  '
 CALL YFAUS(NAUS,N,UU,PRT)
 PRT='  ZES  '
 CALL YFAUS(NAUS, 3,ZES, PRT)
 WRITE(ICRT,*)'END OF FORM5:IER=',IER
 WRITE(ICRT,*)'FOR RESULTS SEE FILE RBEAM.DAT'
 RETURN
 END

 SUBROUTINE GBEAM (N,X,FX,IER)
 IMPLICIT DOUBLE PRECISION (A-H,O-Z)
 DIMENSION X(N)
 COMMON/RAC2/R,S,SL
 AN1 = X(4) * (1.4 * R + 1.6) * 3000.0
 AN2 = X(4) * R * 3000.0
 AM1 = 0.156*X(3)*x(1)*x(2)**2
 AM2 = (X(5)**2/192.)*(22.*AN1-3.*AN2)
 IF (AN1.GT.0.AND.AN2.GT.0.)THEN
 FX = AM1-AM2
 IER = 0
 ELSE
 FX = 1.0D20
 IER = 1
 ENDIF
 RETURN
 END


 SUBROUTINE AISI2 (N,X,PFS)
 IMPLICIT DOUBLE PRECISION (A-H,O-Z)
 EXTERNAL GSHEAR
 DIMENSION X(9),EX(9),SX(9),VP(10,9),COV(9,9),ZES(3),
 +UU(9),EIVEC(9,9),IV(2,9)
 CHARACTER * 10 PRT
 COMMON/BEAMS/ALPHA
 COMMON/RAC2/R,S,sl
 EX(3) = 2.5D1
 EX(4) = 0.0025D0
 EX(5) = SL
 EX(6) = 2.5D2
 EX(7) = 1.571D2
 EX(8) = 2.5D2
 EX(9) = 6.0D-1
 N= 9
 NC = N
 NE = NC
 IRHO = 1
 R = ALPHA
 S = 3000.
```

```
      DO 555 I = 1,2
555   EX(I) = X(I)
      SX(1) = 0.1D-6*X(1)
      SX(2) = 0.1D-6*X(2)
      SX(3)=0.15*EX(3)
      SX(4)=0.37*EX(4)
      SX(5)=0.04*EX(5)
      SX(6)=0.15*EX(6)
      SX(7)=0.036*EX(7)
      SX(8)=0.09*EX(8)
      SX(9)=0.3*EX(9)
      NAUS=7
      ICRT=0
      OPEN(7,FILE='SHEAR.DAT',STATUS='OLD',ERR=10)
      GO TO 20
10    OPEN(7,FILE='SHEAR.DAT',STATUS='NEW')
      IV(1,3)=3
      IV(1,6)=3
      IV(1,4)=7
      DO 100 I=1,N
      VP(5,I) = 1.D0
100   X(I) = EX(I)
      V1 = 1.0D0
      BETA=1.D0
      WRITE(NAUS,5000)
5000  FORMAT(////,5X,70('*'),/,30X,'FORM5',/,5X,
     +70('*'),/,20X,'SHEAR CONSIDERATION,9 VARIABLES:')
      CALL YKOPF(NAUS,N,IV,EX,SX,VP,IRHO)
      WRITE(ICRT,*)'START OF FORM5'
      WRITE(ICRT,*)'STOCHASTIC MODEL'
      CALL YKOPF(ICRT,N,IV,EX,SX,VP,IRHO)
      PRT=' COV  '
      CALL YMAUS(NAUS,NC,N,COV,PRT)
      CALL FORM5(N,IV,EX,SX,VP,GSHEAR,IRHO,COV,NC,EIVEC,
     +NE,V1,NAUS,BETA,X,UU,ZES,IER)
      PFS = YPHINF(-BETA)
      PRT=' UU  '
      CALL YFAUS(NAUS,N,UU,PRT)
      PRT=' ZES  '
      CALL YFAUS(NAUS,3,ZES,PRT)
      WRITE(ICRT,*)'END OF FORM5:IER=',IER
      WRITE(ICRT,*)'FOR RESULTS SEE FILE SHEAR.DAT'
      RETURN
      END

      SUBROUTINE GSHEAR(N,X,FX,IER)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION X(N)
      COMMON/RAC2/R,S,SL
      AN1 = X(4) * 3000.0 * (1.4 * R + 1.6)
      AN2 = X(4) * 3000.0 * R
      V = (0.55 * AN1 + 0.049 * AN2) * X(5)
      VS = V/(x(1)* x(2))
      VC = 0.8 * SQRT(X(3))
      IF (VS.LE.VC) THEN
      FX =( 0.87 * X(6) * X(7)) /(x(1) * X(8)) - VS * (1. - X(9))
      IER = 0
```

```
      ELSE
      FX = 1.D20
      IER = 1
      ENDIF
      RETURN
      END
```

APPENDIX-B

SAMPLE TABLES FROM COMPUTER OPTIMIZATION RESULTS

load ratio	safety index		
	bending	shear	deflection
0.2	3.45	2.66	6.14
0.4	3.13	2.37	6.02
0.6	2.86	2.12	5.91
0.8	2.61	1.89	5.81
1.0	2.39	1.70	5.71
1.2	2.19	1.52	5.63
1.4	2.00	1.36	5.55
1.6	1.83	1.21	5.48
1.8	1.67	1.07	5.42
2.0	1.52	0.95	5.35

Table for Fig.5.1

		breadth					
		Vfcu=0.0	Vfcu=0.1	Vfcu=0.1	Vfcu=0.2	Vfcu=0.2	Vfcu=0.30
	0.2	300.0	300.0	300.0	300.0	300.0	300.0
	0.5	300.9	300.0	300.0	300.0	300.0	300.0
load ratio	1.0	300.0	300.0	300.0	300.0	300.0	300.0
	1.5	300.0	300.0	300.0	300.0	300.0	308.1
	2.0	300.0	300.0	300.0	300.1	301.5	305.9

Table for Fig.5.2a (l=3000mm and Q=2.5KN/m^2
Breadth versus load ratio for varying Vfcu

		depth					
		Vfcu=0.0	Vfcu=0.1	Vfcu=0.1	Vfcu=0.2	Vfcu=0.2	Vfcu=0.30
	0.2	450.0	450.0	450.0	450.0	450.0	450.0
	0.5	450.0	450.0	450.0	450.0	450.0	450.0
load ratio	1.0	450.0	450.0	450.0	450.0	450.0	450.0
	1.5	450.0	450.0	450.0	450.0	450.0	600.0
	2.0	600.0	600.0	600.0	600.0	600.0	600.0

Table for Fig.5.2b (l=3000mm and Q=2.5KN/m^2
Depth versus load ratio for varying Vfcu

		total cost					
		Vfcu=0.0	Vfcu=0.1	Vfcu=0.1	Vfcu=0.2	Vfcu=0.2	Vfcu=0.30
	0.2	1.006	1.006	1.006	1.006	1.012	1.070
	0.5	1.032	1.032	1.032	1.032	1.040	1.106
load ratio	1.0	1.220	1.220	1.220	1.220	1.231	1.314
	1.5	1.778	1.778	1.778	1.778	1.794	1.204
	2.0	1.419	1.419	1.419	1.419	1.427	1.494

Table for Fig.5.2c (l=3000mm and Q=2.5KN/m^2
Total cost ratio versus load ratio for varying Vfcu

		breadth					
		Vfcu=0.0	Vfcu=0.1	Vfcu=0.1	Vfcu=0.2	Vfcu=0.2	Vfcu=0.30
	0.2	300.0	300.0	300.0	300.0	300.0	300.0
	0.5	300.0	300.0	300.0	300.0	300.0	300.0
load ratio	1.0	300.0	300.0	300.1	300.7	304.7	314.7
	1.5	300.0	300.1	300.3	301.3	304.9	312.2
	2.0	300.2	300.3	301.0	302.3	599.1	599.1

Table for Fig.5.3a (l=4000mm and Q=2.5KN/m^2
Breadth versus load ratio for varying Vfcu

		depth					
		Vfcu=0.0	Vfcu=0.1	Vfcu=0.1	Vfcu=0.2	Vfcu=0.2	Vfcu=0.30
	0.2	450.0	450.0	450.0	450.0	450.0	450.0
	0.5	450.0	450.0	450.0	450.0	450.0	450.0
load ratio	1.0	600.0	600.0	600.0	600.0	600.0	600.0
	1.5	600.0	600.0	600.0	600.0	600.0	600.0
	2.0	600.0	600.0	600.0	600.0	599.5	599.5

Table for Fig.5.3b (l=4000mm and Q=2.5KN/m^2
Depth versus load ratio for varying Vfcu

		total cost					
		Vfcu=0.0	Vfcu=0.1	Vfcu=0.1	Vfcu=0.2	Vfcu=0.2	Vfcu=0.30
	0.2	1.057	1.057	1.057	1.057	1.071	1.168
	0.5	1.237	1.237	1.237	1.238	1.259	1.381
load ratio	1.0	1.220	1.220	1.220	1.220	1.231	1.310
	1.5	1.778	1.778	1.778	1.778	1.793	1.892
	2.0	2.895	2.895	2.895	2.896	2.910	2.973

Table for Fig.5.3c (l=4000mm and Q=2.5KN/m^2
Total cost ratio versus load ratio for varying Vfcu

		breadth					
		Vfcu=0.0	Vfcu=0.1	Vfcu=0.1	Vfcu=0.2	Vfcu=0.2	Vfcu=0.30
	0.2	300.0	300.0	300.0	300.0	300.0	300.0
	0.5	300.1	300.2	300.8	303.5	312.9	328.9
load ratio	1.0	300.8	301.3	302.6	306.3	313.9	600.0
	1.5	303.3	304.3	306.7	359.9	598.0	598.1
	2.0	308.7	310.4	313.7	345.8	537.4	590.2

Table for Fig.5.4a (l=5000mm and Q=2.5KN/m^2
Breadth versus load ratio for varying Vfcu

		depth					
		Vfcu=0.0	Vfcu=0.1	Vfcu=0.1	Vfcu=0.2	Vfcu=0.2	Vfcu=0.30
	0.2	450.0	450.0	450.0	450.0	450.0	450.0
	0.5	600.0	600.0	600.0	600.0	600.0	600.0
load ratio	1.0	600.0	600.0	600.0	600.0	600.0	600.0
	1.5	600.0	600.0	600.0	600.0	598.9	598.9
	2.0	600.0	600.0	600.0	600.0	600.0	594.4

Table for Fig.5.4.b (l=5000mm and Q=2.5KN/m^2
Depth versus load ratio for varying Vfcu

		total cost					
		Vfcu=0.0	Vfcu=0.1	Vfcu=0.1	Vfcu=0.2	Vfcu=0.2	Vfcu=0.30
	0.2	1.255	1.255	1.255	1.259	1.298	1.482
	0.5	1.257	1.257	1.157	1.157	1.172	1.263
load ratio	1.0	1.808	1.808	1.808	1.810	1.834	1.883
	1.5	3.336	3.336	3.336	3.337	3.366	3.445
	2.0	5.874	5.874	5.875	5.880	5.891	6.163

Table for Fig.5.4c (l=5000mm and Q=2.5KN/m^2
Total cost ratio versus load ratio for varying Vfcu

		breadth					
		Vfcu=0.0	Vfcu=0.1	Vfcu=0.1	Vfcu=0.2	Vfcu=0.2	Vfcu=0.30
	0.2	300.0	300.0	300.0	312.4	328.4	347.0
	0.5	303.8	305.3	308.9	316.5	328.3	340.9
load ratio	1.0	313.0	315.4	337.1	447.2	598.3	598.3
	1.5	327.7	330.1	349.9	417.2	580.6	580.6
	2.0	344.2	345.8	362.4	417.0	571.4	555.4

Table for Fig.5.5a (l=6000mm and Q=2.5KN/m^2
Breadth versus load ratio for varying Vfcu

		depth					
		Vfcu=0.0	Vfcu=0.1	Vfcu=0.1	Vfcu=0.2	Vfcu=0.2	Vfcu=0.30
	0.2	450.0	450.0	450.0	600.0	600.0	600.0
	0.5	600.0	600.0	600.0	600.0	600.0	600.0
load ratio	1.0	600.0	600.0	600.0	600.0	599.0	599.0
	1.5	600.0	600.0	600.0	600.0	588.9	588.9
	2.0	600.0	600.0	600.0	600.0	600.0	600.0

Table for Fig.5.5b (l=6000mm and Q=2.5KN/m^2
Depth versus load ratio for varying Vfcu

		total cost					
		Vfcu=0.0	Vfcu=0.1	Vfcu=0.1	Vfcu=0.2	Vfcu=0.2	Vfcu=0.30
	0.2	1.738	1.740	1.746	1.130	1.149	1.255
	0.5	1.481	1.481	1.481	1.487	1.512	1.652
load ratio	1.0	3.005	3.005	3.005	3.006	3.034	3.129
	1.5	5.966	5.967	5.968	5.971	6.350	6.483
	2.0	10.210	10.210	10.210	10.220	10.240	10.420

Table for Fig.5.5c (l=6000mm and Q=2.5KN/m^2
Total cost ratio versus load ratio for varying Vfcu

		breadth					
		Vfcu=0.0	Vfcu=0.1	Vfcu=0.1	Vfcu=0.2	Vfcu=0.2	Vfcu=0.30
	0.2	313.5	317.9	323.8	334.8	347.8	359.0
	0.5	324.9	328.2	398.8	562.5	599.8	599.8
load ratio	1.0	354.4	373.6	405.6	502.5	591.1	591.1
	1.5	379.1	391.5	422.8	493.6	577.9	580.2
	2.0	404.8	414.7	443.4	502.9	575.0	583.4

Table for Fig.5.6a (l=7000mm and Q=2.5KN/m^2
Breadth versus load ratio for varying Vfcu

		depth					
		Vfcu=0.0	Vfcu=0.1	Vfcu=0.1	Vfcu=0.2	Vfcu=0.2	Vfcu=0.30
	0.2	600.0	600.0	600.0	600.0	600.0	600.0
	0.5	600.0	600.0	600.0	600.0	600.0	600.0
load ratio	1.0	600.0	600.0	600.0	600.0	594.9	594.9
	1.5	600.0	600.0	600.0	600.0	600.0	600.0
	2.0	600.0	600.0	600.0	600.0	600.0	600.0

Table for Fig.5.6b (l=7000mm and Q=2.5KN/m^2
Depth versus load ratio for varying Vfcu .

		total cost					
		Vfcu=0.0	Vfcu=0.1	Vfcu=0.1	Vfcu=0.2	Vfcu=0.2	Vfcu=0.30
	0.2	1.344	1.344	1.344	1.347	1.381	1.538
	0.5	2.110	2.110	2.110	2.111	2.127	2.228
load ratio	1.0	4.901	4.901	4.902	4.903	5.065	5.213
	1.5	9.546	9.548	9.543	9.553	9.593	9.801
	2.0	15.430	15.400	15.440	15.440	15.510	15.780

Table for Fig.5.6c (l=7000mm and Q=2.5KN/m^2
Total cost ratio versus load ratio for varying Vfcu

		breadth							
		Vq=0.05	Vq=0.10	Vq=0.15	Vq=0.20	Vq=0.25	Vq=0.30	Vq=0.35	Vq=0.40
	0.2	300	300	300	300	300	300	300	300
	0.5	300	300	300	300	300	300	300	300
load ratio	1.0	300	300	300	300	300	300	300	300
	1.5	300	300	300	300	300	300	300	300
	2.0	300	300	300	300	300	300	300	300

Table for Fig. 5.7a (l=3000mm and Q=2.5KN/m^2)
Breadth versus load ratio for varying Vq

		depth							
		Vq=0.05	Vq=0.10	Vq=0.15	Vq=0.20	Vq=0.25	Vq=0.30	Vq=0.35	Vq=0.40
	0.2	450	450	450	450	450	450	450	450
	0.5	450	450	450	450	450	450	450	450
load ratio	1.0	450	450	450	450	450	450	450	450
	1.5	450	450	450	450	450	450	450	600
	2.0	450	450	450	600	600	600	600	600

Table for Fig. 5.7b (l=3000mm and Q=2.5KN/m^2)
Depth versus load ratio for varying Vq

		total cost							
		Vq=0.05	Vq=0.10	Vq=0.15	Vq=0.20	Vq=0.25	Vq=0.30	Vq=0.35	Vq=0.40
	0.2	1.000	1.000	1.000	1.000	1.001	1.002	1.004	1.009
	0.5	1.000	1.000	1.000	1.001	1.005	1.013	1.025	1.045
load ratio	1.0	1.001	1.003	1.011	1.029	1.062	1.113	1.185	1.279
	1.5	1.030	1.050	1.099	1.186	1.312	1.480	1.685	1.180
	2.0	1.229	1.303	1.457	1.075	1.141	1.236	1.360	1.514

Table for Fig. 5.7c (l=3000mm and Q=2.5KN/m^2)
Total cost ratio versus load ratio for varying Vq

		breadth							
		Vq=0.05	Vq=0.10	Vq=0.15	Vq=0.20	Vq=0.25	Vq=0.30	Vq=0.35	Vq=0.40
	0.2	300	300	300	300	300	300	300	300
	0.5	300	300	300	300	300	300	300	300
load ratio	1.0	300	300	300	300	300	300	300	300
	1.5	300	300	300	300	300	300	300	300
	2.0	300	300	300	300	300	300	300	300

Table for Fig. 5.8a (l=4000mm and Q=2.5KN/m^2)
Breadth versus load ratio for varying Vq

		depth							
		Vq=0.05	Vq=0.10	Vq=0.15	Vq=0.20	Vq=0.25	Vq=0.30	Vq=0.35	Vq=0.40
	0.2	450	450	450	450	450	450	450	450
	0.5	450	450	450	450	450	450	450	450
load ratio	1.0	450	450	450	450	450	450	600	600
	1.5	600	600	600	600	600	600	600	600
	2.0	600	600	600	600	600	600	600	600

Table for Fig. 5.8b (l=4000mm and Q=2.5KN/m^2)
Depth versus load ratio for varying Vq

		total cost							
		Vq=0.05	Vq=0.10	Vq=0.15	Vq=0.20	Vq=0.25	Vq=0.30	Vq=0.35	Vq=0.40
	0.2	1	1	1.001	1.004	1.011	1.024	1.045	1.077
	0.5	1.002	1.004	1.013	1.032	1.068	1.123	1.2	1.3
load ratio	1.0	1.072	1.108	1.19	1.322	1.507	1.738	1.185	1.279
	1.5	1.03	1.05	1.099	1.186	1.312	1.48	1.685	1.926
	2.0	1.229	1.303	1.457	1.687	1.983	2.334	2.728	3.156

Table for Fig. 5.8c (l=4000mm and Q=2.5KN/m^2)
Total cost ratio versus load ratio for varying Vq

		breadth							
		Vq=0.05	Vq=0.10	Vq=0.15	Vq=0.20	Vq=0.25	Vq=0.30	Vq=0.35	Vq=0.40
	0.2	300	300	300	300	300	300	300	300
	0.5	300	300	300	300	300	300	300	301.2
load ratio	1.0	300	300	300	300.1	300.4	301	302.1	303.7
	1.5	300	300.1	300.1	300.5	301.3	302.9	305.5	308.8
	2.0	300.1	300.2	300.5	301.6	303.7	307.1	311.7	316.9

Table for Fig. 5.9a (*l=5000mm and Q=2.5KN/m^2)*
Breadth versus load ratio for varying Vq

		depth							
		Vq=0.05	Vq=0.10	Vq=0.15	Vq=0.20	Vq=0.25	Vq=0.30	Vq=0.35	Vq=0.40
	0.2	450	450	450	450	450	450	450	450
	0.5	450	450	450	450	450	450	450	600
load ratio	1.0	600	600	600	600	600	600	600	600
	1.5	600	600	600	600	600	600	600	600
	2.0	600	600	600	600	600	600	600	600

Table for Fig. 5.9b (l=5000mm and Q=2.5KN/m^2)
Depth versus load ratio for varying Vq

		total cost							
		Vq=0.05	Vq=0.10	Vq=0.15	Vq=0.20	Vq=0.25	Vq=0.30	Vq=0.35	Vq=0.40
	0.2	1.002	1.005	1.015	1.036	1.075	1.134	1.22	1.322
	0.5	1.038	1.062	1.118	1.216	1.356	1.54	1.736	1.202
load ratio	1.0	1.033	1.054	1.106	1.196	1.328	1.501	1.713	1.96
	1.5	1.361	1.456	1.648	1.93	2.286	2.695	3.146	3.628
	2.0	2.643	2.827	3.196	3.702	4.294	4.936	5.604	6.284

Table for Fig. 5.9c (l=5000mm and Q=2.5KN/m^2)
Total cost ratio versus load ratio for varying Vq

		breadth							
		Vq=0.05	Vq=0.10	Vq=0.15	Vq=0.20	Vq=0.25	Vq=0.30	Vq=0.35	Vq=0.40
	0.2	300.0	300.0	300.0	300.0	300.0	300.0	300.0	306.3
	0.5	300.0	300.0	300.0	300.8	302.0	304.2	307.4	311.4
total cost	1.0	300.2	300.4	301.1	302.9	306.4	311.4	317.4	348.3
	1.5	301.1	301.8	304.3	309.2	316.0	323.7	338.9	359.7
	2.0	305.3	308.0	314.1	322.6	331.4	339.3	354.0	382.0

Table for Fig. 5.10a (l=6000mm and Q=2.5KN/m^2)
Breadth versus load ratio for varying Vq

		depth							
		Vq=0.05	Vq=0.10	Vq=0.15	Vq=0.20	Vq=0.25	Vq=0.30	Vq=0.35	Vq=0.40
	0.2	450.0	450.0	450.0	450.0	450.0	450.0	450.0	600.0
	0.5	450.0	450.0	450.0	600.0	600.0	600.0	600.0	600.0
load ratio	1.0	600.0	600.0	600.0	600.0	600.0	600.0	600.0	600.0
	1.5	600.0	600.0	600.0	600.0	600.0	600.0	600.0	600.0
	2.0	600.0	600.0	600.0	600.0	600.0	600.0	600.0	600.0

Table for Fig. 5.10b (l=6000mm and Q=2.5KN/m^2)
Depth versus load ratio for varying Vq

		total cost							
		Vq=0.05	Vq=0.10	Vq=0.15	Vq=0.20	Vq=0.25	Vq=0.30	Vq=0.35	Vq=0.40
	0.2	1.026	1.045	1.091	1.172	1.292	1.453	1.655	1.168
	0.5	1.292	1.376	1.549	1.092	1.168	1.267	1.416	1.587
load ratio	1.0	1.259	1.338	1.500	1.745	2.056	2.422	2.831	3.274
	1.5	2.703	2.889	3.263	3.774	4.372	5.021	5.695	6.379
	2.0	6.349	6.537	6.986	7.615	8.344	9.118	9.902	10.670

Table for Fig. 5.10c (l=6000mm and Q=2.5KN/m^2)
Total cost ratio versus load ratio for varying Vq

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		breadth							
		Vq=0.05	Vq=0.10	Vq=0.15	Vq=0.20	Vq=0.25	Vq=0.30	Vq=0.35	Vq=0.40
	0.2	300.0	300.0	300.0	300.0	308.5	314.4	321.1	327.8
	0.5	300.9	301.5	303.6	308.4	315.3	323.3	390.7	412.3
load ratio	1.0	304.6	307.2	313.7	323.0	346.7	379.7	396.5	422.5
	1.5	322.4	328.1	337.1	346.0	355.9	398.8	413.6	443.4
	2.0	357.8	360.7	364.4	367.6	370.1	400.9	435.6	466.3

Table for Fig. 5.11a (l=7000mm and Q=2.5KN/m^2)
Breadth versus load ratio for varying Vq

		depth							
		Vq=0.05	Vq=0.10	Vq=0.15	Vq=0.20	Vq=0.25	Vq=0.30	Vq=0.35	Vq=0.40
	0.2	450.0	450.0	450.0	450.0	600.0	600.0	600.0	600.0
	0.5	600.0	600.0	600.0	600.0	600.0	600.0	600.0	600.0
load ratio	1.0	600.0	600.0	600.0	600.0	600.0	600.0	600.0	600.0
	1.5	600.0	600.0	600.0	600.0	600.0	600.0	600.0	600.0
	2.0	600.0	600.0	600.0	600.0	600.0	600.0	600.0	600.0

Table for Fig. 5.11b (l=7000mm and Q=2.5KN/m^2)
Depth versus load ratio for varying Vq

		total cost							
		Vq=0.05	Vq=0.10	Vq=0.15	Vq=0.20	Vq=0.25	Vq=0.30	Vq=0.35	Vq=0.40
	0.2	1.162	1.222	1.351	1.552	1.109	1.188	1.294	1.427
	0.5	1.068	1.103	1.183	1.313	1.494	1.722	1.992	2.298
load ratio	1.0	2.053	2.211	2.526	2.962	3.479	4.050	4.654	5.276
	1.5	5.689	5.885	6.338	6.965	7.692	8.457	9.239	10.010
	2.0	12.280	12.320	12.630	13.150	13.790	14.470	15.160	15.840

Table for Fig. 5.11c (l=7000mm and Q=2.5KN/m^2)
Total cost ratio versus load ratio for varying Vq

		breadth				
		L = 3000mm	L = 4000mm	L = 5000mm	L = 6000mm	L = 7000mm
	0.05	300.0	300.0	300.0	300.0	313.5
	0.10	300.0	300.0	300.0	300.0	317.9
Vfcu	0.15	300.0	300.0	300.0	300.0	323.8
	0.20	300.0	300.0	300.0	312.4	334.8
	0.25	300.0	300.0	300.0	328.4	347.0
	0.30	300.0	300.0	300.0	347.0	359.0

Table for Fig. 5.12a (load ratio=0.2)
Breadth versus Vfcu for varying lengths

		depth				
		L = 3000mm	L = 4000mm	L = 5000mm	L = 6000mm	L = 7000mm
	0.05	450.0	450.0	450.0	450.0	600.0
	0.10	450.0	450.0	450.0	450.0	600.0
Vfcu	0.15	450.0	450.0	450.0	450.0	600.0
	0.20	450.0	450.0	450.0	600.0	600.0
	0.25	450.0	450.0	450.0	600.0	600.0
	0.30	450.0	450.0	450.0	600.0	600.0

Table for Fig. 5.12b (load ratio=0.2)
Depth versus Vfcu for varying lengths

		total cost				
		L = 3000mm	L = 4000mm	L = 5000mm	L = 6000mm	L = 7000mm
	0.05	1.006	1.057	1.255	1.738	1.344
	0.10	1.006	1.057	1.255	1.740	1.344
Vfcu	0.15	1.006	1.057	1.255	1.746	1.344
	0.20	1.006	1.057	1.259	1.130	1.347
	0.25	1.012	1.071	1.298	1.148	1.381
	0.30	1.070	1.168	1.482	1.255	1.538

Table for Fig. 5.12c (load ratio=0.2)
Total cost ratio versus Vfcu for varying lengths

		breadth				
		L = 3000mm	L = 4000mm	L = 5000mm	L = 6000mm	L =7000mm
	0.05	300.0	300.0	300.1	303.8	324.9
	0.10	300.0	300.0	300.2	305.3	328.2
Vfcu	0.15	300.0	300.0	300.8	308.9	398.8
	0.20	300.0	300.0	303.5	316.5	562.5
	0.25	300.0	300.0	312.9	328.3	599.8
	0.30	300.0	300.0	328.9	340.9	599.8

Table for Fig. 5.13a (load ratio=0.5)
Breadth versus Vfcu for varying lengths

		depth				
		L = 3000mm	L = 4000mm	L = 5000mm	L = 6000mm	L =7000mm
	0.05	450.0	450.0	600.0	600.0	600.0
	0.10	*450.0*	*450.0*	*600.0*	*600.0*	*600.0*
Vfcu	0.15	450.0	450.0	600.0	600.0	600.0
	0.20	450.0	450.0	600.0	600.0	600.0
	0.25	450.0	450.0	600.0	600.0	600.0
	0.30	450.0	450.0	600.0	600.0	600.0

Table for Fig. 5.13b (load ratio=0.5)
Depth versus Vfcu for varying lengths

		total cost				
		L = 3000mm	L = 4000mm	L = 5000mm	L = 6000mm	L =7000mm
	0.05	1.032	1.237	1.157	1.481	2.110
	0.10	1.032	1.237	1.157	1.481	2.110
Vfcu	0.15	1.032	1.237	1.157	1.481	2.110
	0.20	1.032	1.238	1.157	1.487	2.111
	0.25	1.040	1.259	1.172	1.521	2.127
	0.30	1.106	1.381	1.263	1.652	2.228

Table for Fig. 5.13c (load ratio=0.5)
Total cost ratio versus Vfcu for varying lengths

		breadth				
		L = 3000mm	L = 4000mm	L = 5000mm	L = 6000mm	L = 7000mm
Vfcu	0.05	300.0	300.0	300.8	313.0	354.4
	0.10	300.0	300.0	301.3	315.4	373.6
	0.15	300.0	300.1	302.6	337.1	405.6
	0.20	300.0	300.7	306.3	447.2	502.5
	0.25	300.0	304.7	313.9	598.3	591.1
	0.30	300.0	314.7	600.0	598.3	591.1

(load ratio=1.0)

Table for Fig. 5.14a
Breadth versus Vfcu for varying lengths

		depth				
		L = 3000mm	L = 4000mm	L = 5000mm	L = 6000mm	L = 7000mm
Vfcu	0.05	450.0	600.0	600.0	600.0	600.0
	0.10	450.0	600.0	600.0	600.0	600.0
	0.15	450.0	600.0	600.0	600.0	600.0
	0.20	450.0	600.0	600.0	600.0	600.0
	0.25	450.0	600.0	600.0	599.0	594.9
	0.30	450.0	600.0	600.0	599.0	594.9

(load ratio=1.0)

Table for Fig. 5.14b
Depth versus Vfcu for varying lengths

		total cost				
		L = 3000mm	L = 4000mm	L = 5000mm	L = 6000mm	L = 7000mm
Vfcu	0.05	1.220	1.220	1.808	3.005	4.901
	0.10	1.220	1.220	1.808	3.005	4.901
	0.15	1.220	1.220	1.808	3.005	4.902
	0.20	1.220	1.220	1.810	3.006	4.903
	0.25	1.231	1.231	1.834	3.034	5.065
	0.30	1.314	1.310	1.883	3.129	5.213

(load ratio=1.0)

Table for Fig. 5.14c
Total cost ratio versus Vfcu for varying lengths

		breadth				
		L = 3000mm	L = 4000mm	L = 5000mm	L = 6000mm	L =7000mm
	0.05	300.0	300.0	303.3	327.7	379.1
	0.10	300.0	300.1	304.3	330.1	391.4
Vfcu	0.15	300.0	300.3	306.7	349.9	422.8
	0.20	300.0	301.3	359.9	417.2	493.6
	0.25	300.0	304.9	598.0	580.6	577.9
	0.30	308.1	312.2	598.1	580.6	580.2

Table for Fig. 5.15a (load ratio=1.5)

Breadth versus Vfcu for varying lengths

		depth				
		L = 3000mm	L = 4000mm	L = 5000mm	L = 6000mm	L =7000mm
	0.05	450.0	600.0	600.0	600.0	600.0
	0.10	450.0	600.0	600.0	600.0	600.0
Vfcu	0.15	450.0	600.0	600.0	600.0	600.0
	0.20	450.0	600.0	600.0	600.0	600.0
	0.25	450.0	600.0	598.9	588.9	600.0
	0.30	600.0	600.0	598.9	588.9	600.0

Table for Fig. 5.15b (load ratio=1.5)

Depth versus Vfcu for varying lengths

		total cost				
		L = 3000mm	L = 4000mm	L = 5000mm	L = 6000mm	L =7000mm
	0.05	1.778	1.778	3.336	5.966	9.546
	0.10	1.778	1.778	3.336	5.967	9.548
Vfcu	0.15	1.778	1.778	3.336	5.968	9.543
	0.20	1.778	1.778	3.337	5.971	9.553
	0.25	1.794	1.793	3.366	6.350	9.593
	0.30	1.204	1.892	3.445	6.483	9.801

Table for Fig. 5.15c (load ratio=1.5)

Total cost ratio versus Vfcu for varying lengths

		breadth				
		L = 3000mm	L = 4000mm	L = 5000mm	L = 6000mm	L =7000mm
	0.05	300.0	300.2	308.7	344.2	404.8
	0.10	300.0	300.3	310.4	345.8	414.7
Vfcu	0.15	300.0	301.0	313.7	362.4	443.4
	0.20	300.1	302.3	345.8	417.0	502.9
	0.25	301.5	599.1	537.4	571.4	575.0
	0.30	305.9	599.1	590.2	555.4	583.4

Table for Fig. 5.16a *(load ratio=2.0)*

Breadth versus Vfcu for varying lengths

		depth				
		L = 3000mm	L = 4000mm	L = 5000mm	L = 6000mm	L =7000mm
	0.05	600.0	600.0	600.0	600.0	600.0
	0.10	600.0	600.0	600.0	600.0	600.0
Vfcu	0.15	600.0	600.0	600.0	600.0	600.0
	0.20	600.0	600.0	600.0	600.0	600.0
	0.25	600.0	599.5	600.0	600.0	600.0
	0.30	600.0	599.5	594.4	600.0	600.0

Table for Fig. 5.16b (load ratio=2.0)

Depth versus Vfcu for varying lengths

		total cost				
		L = 3000mm	L = 4000mm	L = 5000mm	L = 6000mm	L =7000mm
	0.05	1.419	2.895	5.874	10.210	15.400
	0.10	1.419	2.895	5.874	10.210	15.400
Vfcu	0.15	1.419	2.895	5.875	10.210	15.440
	0.20	1.419	2.896	5.880	10.220	15.440
	0.25	1.427	2.910	5.891	10.240	15.510
	0.30	1.494	2.973	6.163	10.420	15.780

Table for Fig. 5.16c (load ratio=2.0)

Total cost ratio versus Vfcu for varying lengths

		breadth				
		L = 3000mm	L = 4000mm	L = 5000mm	L = 6000mm	L =7000mm
	0.05	300.0	300.0	300.0	300.0	300.0
	0.10	300.0	300.0	300.0	300.0	300.0
Vq	0.15	300.0	300.0	300.0	300.0	300.0
	0.20	300.0	300.0	300.0	300.0	300.0
	0.25	300.0	300.0	300.0	300.0	308.5
	0.30	300.0	300.0	300.0	300.0	314.4
	0.35	300.0	300.0	300.0	300.0	321.1
	0.40	300.0	300.0	300.0	306.3	327.8

Table for Fig.5.17a (load ratio=0.2)
Breadth versus Vq for varying lengths

		depth				
		L = 3000mm	L = 4000mm	L = 5000mm	L = 6000mm	L =7000mm
	0.05	450.0	450.0	450.0	450.0	450.0
	0.10	450.0	450.0	450.0	450.0	450.0
Vq	0.15	450.0	450.0	450.0	450.0	450.0
	0.20	450.0	450.0	450.0	450.0	450.0
	0.25	450.0	450.0	450.0	450.0	600.0
	0.30	450.0	450.0	450.0	450.0	600.0
	0.35	450.0	450.0	450.0	450.0	600.0
	0.40	450.0	450.0	450.0	600.0	600.0

Table for Fig.5.17b (load ratio=0.2)
Depth versus Vq for varying lengths

		total cost				
		L = 3000mm	L = 4000mm	L = 5000mm	L = 6000mm	L =7000mm
	0.05	1.000	1.000	1.002	1.026	1.162
	0.10	1.000	1.000	1.005	1.045	1.222
Vq	0.15	1.000	1.001	1.015	1.091	1.351
	0.20	1.000	1.004	1.036	1.172	1.552
	0.25	1.001	1.011	1.075	1.292	1.109
	0.30	1.002	1.024	1.134	1.543	1.188
	0.35	1.004	1.045	1.220	1.655	1.298
	0.40	1.009	1.077	1.322	1.168	1.427

Table for Fig.5.17c (load ratio=0.2)
Total cost ratio versus Vq for varying lengths

		breadth				
		L = 3000mm	L = 4000mm	L = 5000mm	L = 6000mm	L =7000mm
	0.05	300.0	300.0	300.0	300.0	300.9
	0.10	300.0	300.0	300.0	300.0	301.5
Vq	0.15	300.0	300.0	300.0	300.0	303.6
	0.20	300.0	300.0	300.0	300.8	308.4
	0.25	300.0	300.0	300.0	302.0	315.3
	0.30	300.0	300.0	300.0	304.2	323.3
	0.35	300.0	300.0	300.0	307.4	390.7
	0.40	300.0	300.0	301.2	311.4	412.3

Table for Fig.5.18a (load ratio=0.5)

Breadth versus Vq for varying lengths

		depth				
		L = 3000mm	L = 4000mm	L = 5000mm	L = 6000mm	L =7000mm
	0.05	450.0	450.0	450.0	450.0	600.0
	0.10	450.0	450.0	450.0	450.0	600.0
Vq	0.15	450.0	450.0	450.0	450.0	600.0
	0.20	*450.0*	*450.0*	*450.0*	*600.0*	*600.0*
	0.25	450.0	450.0	450.0	600.0	600.0
	0.30	450.0	450.0	450.0	600.0	600.0
	0.35	450.0	450.0	450.0	600.0	600.0
	0.40	450.0	450.0	600.0	600.0	600.0

Table for Fig.5.18b (load ratio=0.5)

Depth versus Vq for varying lengths

		total cost				
		L = 3000mm	L = 4000mm	L = 5000mm	L = 6000mm	L =7000mm
	0.05	1.000	1.002	1.038	1.292	1.068
	0.10	1.000	1.004	1.062	1.376	1.103
Vq	0.15	1.000	1.013	1.118	1.549	1.183
	0.20	1.001	1.032	1.216	1.092	1.313
	0.25	1.005	1.068	1.356	1.168	1.494
	0.30	1.013	1.123	1.54	1.276	1.722
	0.35	1.025	1.200	1.763	1.416	1.992
	0.40	*1.045*	*1.300*	*1.202*	*1.567*	*2.298*

Table for Fig.5.18c (load ratio=0.5)

Total cost ratio versus Vq for varying lengths

		breadth				
		L = 3000mm	L = 4000mm	L = 5000mm	L = 6000mm	L =7000mm
	0.05	300.0	300.0	300.0	300.2	304.6
	0.10	300.0	300.0	300.0	300.4	307.2
Vq	0.15	300.0	300.0	300.0	301.1	313.7
	0.20	300.0	300.0	300.1	302.9	323.0
	0.25	300.0	300.0	300.4	306.4	346.7
	0.30	300.0	300.0	301.0	311.4	379.7
	0.35	300.0	300.1	302.1	317.4	396.5
	0.40	300.0	300.1	303.7	348.3	422.5

Table for Fig.5.19a (load ratio=1.0)
Breadth versus Vq for varying lengths

		depth				
		L = 3000mm	L = 4000mm	L = 5000mm	L = 6000mm	L =7000mm
	0.05	450.0	450.0	600.0	600.0	600.0
	0.10	450.0	450.0	600.0	600.0	600.0
Vq	0.15	450.0	450.0	600.0	600.0	600.0
	0.20	450.0	450.0	600.0	600.0	600.0
	0.25	450.0	450.0	600.0	600.0	600.0
	0.30	450.0	450.0	600.0	600.0	600.0
	0.35	450.0	600.0	600.0	600.0	600.0
	0.40	450.0	600.0	600.0	600.0	600.0

Table for Fig.5.19b (load ratio=1.0)
Depth versus Vq for varying lengths

		total cost				
		L = 3000mm	L = 4000mm	L = 5000mm	L = 6000mm	L =7000mm
	0.05	1.001	1.072	1.033	1.259	2.053
	0.10	1.003	1.108	1.054	1.338	2.211
Vq	0.15	1.011	1.190	1.106	1.500	2.526
	0.20	1.029	1.322	1.196	1.745	2.962
	0.25	1.062	1.507	1.328	2.056	3.479
	0.30	1.113	1.738	1.501	2.422	4.050
	0.35	1.185	1.185	1.713	2.831	4.654
	0.40	1.279	1.279	1.960	3.274	5.276

Table for Fig.5.19c (load ratio=1.0)
Total cost ratio versus Vq for varying lengths

		breadth				
		L = 3000mm	L = 4000mm	L = 5000mm	L = 6000mm	L =7000mm
	0.05	300.0	300.0	300.0	301.1	322.4
	0.10	300.0	300.0	300.1	301.8	328.1
Vq	0.15	300.0	300.0	300.1	304.3	337.1
	0.20	300.0	300.0	300.5	309.2	346.0
	0.25	300.0	300.0	301.3	316.0	355.9
	0.30	300.0	300.1	302.9	323.7	398.8
	0.35	300.0	300.2	305.5	338.9	413.6
	0.40	300.0	300.4	308.8	359.7	443.4

Table for Fig.5.20a (load ratio=1.5)

Breadth versus Vq for varying lengths

		depth				
		L = 3000mm	L = 4000mm	L = 5000mm	L = 6000mm	L =7000mm
	0.05	450.0	600.0	600.0	600.0	600.0
	0.10	450.0	600.0	600.0	600.0	600.0
Vq	0.15	450.0	600.0	600.0	600.0	600.0
	0.20	450.0	600.0	600.0	600.0	600.0
	0.25	450.0	600.0	600.0	600.0	600.0
	0.30	450.0	600.0	600.0	600.0	600.0
	0.35	450.0	600.0	600.0	600.0	600.0
	0.40	600.0	600.0	600.0	600.0	600.0

Table for Fig.5.20b (load ratio=1.5)

Depth versus Vq for varying lengths

		total cost				
		L = 3000mm	L = 4000mm	L = 5000mm	L = 6000mm	L =7000mm
	0.05	1.030	1.030	1.361	2.703	5.689
	0.10	1.050	1.050	1.546	2.889	5.885
Vq	0.15	1.099	1.099	1.648	3.263	6.338
	0.20	1.186	1.186	1.930	3.774	6.965
	0.25	1.312	1.312	2.286	4.372	7.692
	0.30	1.480	1.480	2.695	5.021	8.457
	0.35	1.685	1.685	3.146	5.695	9.239
	0.40	1.180	1.926	3.628	6.379	10.010

Table for Fig.5.20c (load ratio=1.5)

Total cost ratio versus Vq for varying lengths

		breadth				
		L = 3000mm	L = 4000mm	L = 5000mm	L = 6000mm	L =7000mm
	0.05	300.0	300.0	300.1	305.3	357.8
	0.10	300.0	300.0	300.2	308.0	360.7
Vq	0.15	300.0	300.0	300.5	314.1	364.4
	0.20	300.0	300.0	301.6	322.6	367.6
	0.25	300.0	300.1	303.7	331.4	370.1
	0.30	300.0	300.2	307.1	339.3	400.9
	0.35	300.0	300.6	311.7	354.0	435.6
	0.40	300.0	301.2	316.9	382.0	466.3

Table for Fig.5.21a (load ratio=2.0)
Breadth versus Vq for varying lengths

		depth				
		L = 3000mm	L = 4000mm	L = 5000mm	L = 6000mm	L =7000mm
	0.05	450.0	600.0	600.0	600.0	600.0
	0.10	450.0	600.0	600.0	600.0	600.0
Vq	0.15	450.0	600.0	600.0	600.0	600.0
	0.20	600.0	600.0	600.0	600.0	600.0
	0.25	600.0	600.0	600.0	600.0	600.0
	0.30	600.0	600.0	600.0	600.0	600.0
	0.35	600.0	600.0	600.0	600.0	600.0
	0.40	600.0	600.0	600.0	600.0	600.0

Table for Fig.5.21b (load ratio=2.0)
Depth versus Vq for varying lengths

		total cost				
		L = 3000mm	L = 4000mm	L = 5000mm	L = 6000mm	L =7000mm
	0.05	1.229	1.229	2.643	6.349	12.280
	0.10	1.303	1.303	2.827	6.537	12.320
Vq	0.15	1.457	1.457	3.196	6.986	12.630
	0.20	1.075	1.687	3.702	7.615	13.150
	0.25	1.141	1.983	4.294	8.344	13.790
	0.30	1.236	2.334	4.936	9.118	14.470
	0.35	1.360	2.728	5.604	9.902	15.160
	0.40	1.514	3.156	6.284	10.670	15.840

Table for Fig.5.21c (load ratio=2.0)
Total cost ratio versus Vq for varying lengths