IMPACT OF METACOGNITIVE STRATEGY ON ATTITUDE, RETENTION AND PERFORMANCE IN CALCULUS AMONG COLLEGES OF EDUCATION STUDENTS IN NORTH-CENTRAL ZONE, NIGERIA

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DECLARATION

I Aliyu Alhaji ZAKARIYYA (PhD/EDUC/12301/2011-2012) declare that this thesis entitled "Impact of Metacognitive Strategy on Attitude, Retention and Performance in Calculus among Colleges of Education Students in North-central Zone, Nigeria" is my personal work. It has never been presented anywhere either wholly or partially for the purpose of the award of a higher degree. All the quotations and sources of information are duly acknowledged by means of references in this work.

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Signature Date

CERTIFICATION

This thesis titled "Impact of Metacognitive Strategy on Attitude, Retention and Performance in Calculus among Colleges of Education Students in North-central Zone" by Aliyu Alhaji ZAKARIYYA (PhD/EDUC/12301/2011-2012) has been read and approved as meeting the regulations governing the award of the degree of Doctor of Philosophy (Ph.D.) of Ahmadu Bello University, Zaria and is approved for its contributions to knowledge and literary presentation.

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DEDICATION

This thesis is dedicated to all my mathematics teachers and lecturers at all levels of my education and my parents whose effort and support made me what I am today.

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ABSTRACT

This study investigated the Impact of Metacognitive Strategy on Attitude, Retention and Performance in Calculus among Colleges of Education Students in North-central Zone, Nigeria. The study adopted a Pre-test, Post-test, post-post-test quasi-experimental design. The study used 135(83 males and 52 females) NCE II calculus students from two colleges of education, North-Central Zone, Nigeria. The experimental group consisted of 65 (42 males and 23 females) students, while the control group consisted of 70 (41 males and 29 females) students. The researcher developed and validated four instruments. These include: Calculus Pre-test (CPT), Calculus Achievement Test (CAT), an eight item theory questions with reliability coefficient of 0.78. Metacognitive Teaching Strategy (MTS), and Attitude Towards Calculus Inventory (ATCI), a thirty item adopted on a five point Likert-type scales with reliability coefficient of 0.72. The experimental period lasted for six weeks during which the control group was taught using lecture method while the experimental group was taught using Metacognitive Teaching Strategy. Seven research questions were answered and their corresponding seven research hypotheses were tested. The research questions were answered using descriptive statistics of mean and standard deviation. Hypotheses testing were done using inferential statistics of t-test for equality of means of independent sample, Mann-Whitney U test and Spearman's rank correlation test at $p \leq 0.05$, level of significance. There was a significant difference between the post-test mean scores of the experimental and control groups in favour of the experimental group. There was a significant difference in attitude change between the experimental and control groups. However, there was no significant difference in attitude change of the subjects in the control groups. There was a significant difference in the retention ability between the experimental and the control groups. While, there was no significant difference in retention ability of the subjects in the control groups. Strong and positive relationship exist between attitude and performance. It is recommended that lecturers should use metacognitive teaching strategy in teaching mathematics. Mathematics educators should appreciate metacognitive activity and develop ways to foster it within all students.

ABBREVIATIONS

ACI Attitude Calculus Inventory.

AMI Attitude Mathematics Inventory

CAT Calculus Achievement Test

LASSI Learning and Study Strategies Inventory

MAS Mathematics Attitudes Survey

MMQ Mathematics Meta-Cognition Questionnaire

MSQ Mathematics Self-Efficacy Questionnaire

MAI Metacognitive Awareness Inventory

MSA Metacognitive Skills and Knowledge Assessment

MSTM Metacognitive Strategy Teaching Model

NCE Nigeria Certificate in Education

OPERATIONAL DEFINITION OF TERMS

Metacognitive knowledge: One's knowledge or beliefs about the factors that control cognitive processes.

Metacognitive skills: These are the voluntary controls people have over their own cognitive processes.

Knowledge Monitoring: This is the ability of knowing what one knows and knowing what one doesn't know.

Reflection: Thisis a deliberate cognitive activity where learners connect thoughts, feelings, and experiences related to the learning activity in which they are involved in.

Cognitive goals: These describe the chosen path a learner chooses to manage the task at hand.

Declarative knowledge: This refers to knowledge about oneself as a learner and about the factors that influence learning.

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CHAPTER ONE

THE PROBLEM

1.1 Introduction

Students today are growing up in a world permeated by mathematics. The technologies used in homes, schools, and the workplaces are all built on mathematical knowledge. For students to participate fully in the society, they must learn mathematics with understanding, how to connect mathematical idea and how to reason mathematically (Hollebrands, Conner & Smith, 2010). Learning mathematics with understanding is the vision for school mathematics recommended by the National Council of Teachers of Mathematics (NCTM, 2014). It identified a number of principles of learning that provide the foundation for effective mathematics teaching. Specifically, it suggested that learners should have experiences that will enable them to:

i. engage with challenging tasks that involve active meaning making and support meaningful learning;

ii. develop metacognitive awareness of themselves as learners, thinkers, and problem solvers, and learn to also monitor their learning and performance.

However, these principles of learning are hardly been implemented in Nigerian schools and colleges of educations. Mathematics lecturers in Nigerian colleges of education and universities generally teach their students by means of the conventional or lecture method. Teachers select a set of mathematical problems, demonstrate the necessary steps leading to their solutions and their students then follow the same steps in finding solutions to similar problems (Agwagah, 2005; Leghara, 2008). Obioma (2011) observed that explanations for the low mathematics achievement of most students could be that they were not taught the appropriate strategies, they cannot self-regulate the study strategies, and do not understand how to apply these strategies.

According to Kaune (2006) students' abilities to think flexibly can be developed and enhanced by teachers modelling their thinking processes. This is by giving the students opportunity to solve problems and helping them become aware of their own thought processes as they solve mathematical problems. This process of analysing the thought processes of the learner is called metacognition.

Metacognition or awareness of the process of learning is a critical ingredient to successful learning. According to Flavell (1999) in Bergstresser (2013), metacognition refers to one's knowledge concerning one's own cognitive processes and products or anything related to them. Metacognition refers to learners' awareness of their own knowledge and their ability to understand, control and manipulate skills.

Calculus concepts are widely used in other high level mathematics courses such as Probability Theory, Optimisation, Ordinary Differential Equations, Analysis, Mechanics, Topology and Mathematical Modelling (David, 2013). Other disciplines such as engineering and economics also use calculus themes. However, it was widely acknowledged that calculus concepts are abstract and complex for students and that teaching and learning them can be challenging and even frustrating at times. Indeed, research has shown that many students have difficulties learning some of the key concepts of calculus (Stylianides, & Stylianides, 2013). Similarly, it can be observed that the average performance of the students in calculus from these colleges within the periods under consideration was poor. (see appendix x)

Attitude is viewed as a multidimensional construct, representing students' learned predispositions to respond positively or negatively, favourably or unfavourably to stimuli. It is said to have played a very important role in the development and progress of human identity and values (Huang, 2011; Miele, 2014). Attitude plays an important role in determining the individual's reaction to a particular entity. Attitude affects behaviour, influences what the learner selects from the environment, how he or she reacts towards teachers and the material being presented or concept being taught. Some researchers Clarke, Thomas & Vidakovic (2009) suggested that poor mathematics attitude develops primarily from the relationship between students and their teachers. Therefore, if teachers' mathematics attitudes are positive, then those beliefs may enhance their own global self-concept while affecting and benefiting their elementary pupils in their areas of mathematics learning.

Investigation of gender differences in the learning of mathematics is still an important issue in mathematics education (Isa & Balarabe, 2009; Ayodele, 2011). Many factors are associated with gender differences in mathematics learning and achievement. Those factors include: classroom interactions, students' attitudes, students' interest and self-esteem, gender and attitude of the teacher, curricular materials, beliefs, social and cultural norms (Sayid & Milad, 2011). Other important factors that influence learning are memory and retention. Memory is essential for individuals for it allows one to remember experiences and use them to respond to future events. Therefore, understanding retention and what facilitates it can help teachers select strategies to improve it and the retrieval of information.

Extensive research studies have been carried out on the effectiveness of some teaching methods on performance of primary and secondary school students in Nigeria, such as concept mapping (Eze, 2008); cooperative learning (Chianson, Kurumeh& Obida, 2011);

mastery learning (Benjamin& Clement, 2011) and computer animation (Gambari, Falode, & Adegbenro, 2014). However, corresponding research on the role of metacognition in calculus learning in colleges of education has been rare, especially in Nigeria.

National Policy on Education (2004) stated that Teacher education will continue to be given a major emphasis in all educational planning, because no education system can rise above the quality of its teachers. The policy emphasized that all teachers in the nation's educational institutions would be professionally trained. The policy also stated that the purpose of teacher education among others is to provide teachers with the intellectual and professional background adequate for their assignment and to make them adaptable to any changing situation, not only in the life of their country, but in the wider world. One of the major challenges to teacher education in the present dispensation according to Akindutire and Ekundayo (2012) is that of globalization. The world is now a global village, and for teachers to have currency and operate effectively and efficiently teachers need to imbibe the new technologies and methodologies of the advanced countries of the world. One of such methodologies is metacognitive teaching strategy. This study therefore, investigated the impact of metacognitive teaching strategy on attitude, retention and performance in calculus among Colleges of Education Students in North-central Zone, Nigeria.

1.2 Statement of the Problem

The quality of education in science, technology, engineering, and mathematics (STEM) fields is an issue of particular educational and economic importance. Similarly, the importance of mathematics in general and calculus in particular cannot be over-emphasized. Calculus is fundamental to further study of some mathematics courses at Colleges and Universities. However, the poor performances of students in calculus, over the years are of great concern to the students, parents and lecturers. One of the indications of incessant poor performance of students in calculus is that students find calculus concepts difficult.

Difficulties in learning calculus concepts has been associated with poor teaching methods lecturers use in teaching the concepts which failed to arouse the interests of the students in learning of the course (Agwagah, 2005; Leghara, 2008). In most of Nigerian colleges of education mathematics students are taught using lecture method. Lecturers tend to teach by doing standard type problems and solutions. As a result, students approach calculus problems by simply applying the steps they have memorized without a good grasp of the calculus concepts. The need for an alternate method of instruction to enhance teaching and learning of calculus is therefore essential.

Studies have shown that students' academic achievement is more likely to improve when they are given the opportunity to monitor and regulate their learning strategies. It is against these backdrops that this study was set to determine the impact of metacognitive strategy on attitude, retention and performance in calculus among colleges of education students in North-central Zone of Nigeria.

1.3 Objectives of the Study

The objective of the study is to determine the impact of metacognitive strategy on attitude, retention and performance in calculus among Colleges of Education Students in North-central Zone, Nigeria. Specifically, the exploration was carried out based on the following objectives to:

1. determine the performance of students taught calculus using metacognitive strategy and

those taught using lecture method.

2. compare the attitude of N.C.E II students taught calculus using metacognitive strategy and those taught using lecture method.
3. compare the retention ability of N.C.E II students taught calculus using metacognitive strategy and those taught using lecture method.
4. find the relationship between attitude and performance of N.C.E II students taught calculus using metacognitive strategy.
5. find out gender differences in performance of N.C.E II students taught calculus using metacognitive strategy.
6. determine gender difference in attitude toward calculus of N.C.E II students taught using metacognitive strategy.
7. find out gender differences in retention ability of N.C.E II students taught calculus using metacognitive strategy.

1.4 Research Questions

The following research questions were formulated and tested at $p \leq 0.05$ level of significance:

1. What is the performance of N.C.E II students taught calculus using metacognitive strategy compared to those taught using lecture method?
2. Is there an attitudinal change of N.C.E II students taught calculus using metacognitive strategy and those taught using lecture method?

3. What are the differences noticed in the retention ability of N.C.E II students taught calculus using metacognitive strategy and those taught using lecture method?

4. Is there relationship between attitude and performance of N.C.E II students taught calculus using metacognitive strategy?
5. Are there gender differences in performance of N.C.E II students taught calculus using metacognitive strategy?
6. Is there gender difference in attitude change of N.C.E II students taught calculus using metacognitive strategy?
7. Are there gender differences in retention ability of N.C.E II students taught calculus using metacognitive strategy?

1.5 Null Hypotheses

The following null hypotheses were formulated and tested at 0.05 significance levels:

$H0_1$: There is no significant difference in the performance between N.C.E II students taught calculus using metacognitive strategy and those taught using lecture method.

$H0_2$: There is no significant difference in the attitude between N.C.E II students taught calculus using metacognitive strategy and those taught using lecture method.

$H0_3$: There is no significant difference in the retention ability between N.C.E II students taught calculus using metacognitive strategy and those taught using lecture method.

$H0_4$: There is no significant relationship between attitude and performance of N.C.E II students taught calculus using metacognitive strategy.

$H0_5$: There is no significant difference in performance of male and female N.C.E II students taught calculus using metacognitive strategy.

$H0_6$: There is no significant gender difference in attitude change between N.C.E II students taught calculus using metacognitive strategy.

$H0_7$: There is no significant gender difference in retention ability between N.C.E II students taught calculus using metacognitive strategy.

1.6 Significance of the Study

This study was carried out at a period when there are calls for teaching methods that emphasize student-centred teaching that focuses on cognitive development of the student. The teacher's goal is to help students grasp the development of knowledge as a process rather than as a product. Therefore, this study is significant because of the following implications and applications:

Implications

Mathematics students in tertiary Institutions: Results of this study can assist students to be metacognitively aware learners. This is so because metacognitively aware learners are more strategic as they perform better than metacognitively unaware learners. Metacognitive awareness will help them to plan, sequence, and monitor their learning in a way that will directly help to improve their achievements. Knowledge of metacognitive strategies will enable students to develop a deeper understanding of the problem at hand. They will learn how to construct knowledge through a variety of methods and then recognize when they lack understanding and consequently, choose the right tools to correct their errors. Similarly, if students are taught metacognitive awareness concerning the purpose and usefulness of a strategy as they are taught the strategy, they are more likely to generalize the strategy to new situations. Metacognitive learning strategies afford students the needed opportunities to take their thinking to a higher level and express themselves clearly. Students with learning difficulties can benefit from instruction in metacognition because it helps them to become more aware of their thinking processes, to recognise when meaning breaks down, and to understand what strategies work best for them. It will be significant for students in that it will assist them in understanding their own learning in order to learn better and to enhance their personal and academic development. Training in metacognitive skills such as planning a course of action, selecting strategies, monitoring, evaluating, and revising has been shown to have supported students in developing a deeper understanding of mathematics.

Lecturers at the Tertiary Levels: Results of this study will assist teachers and lecturers who work to meet the diverse needs of learners and to address issues of equity and social justice. Students' achievement and understanding will be significantly improved when teachers and lecturers are aware of how students construct knowledge, are familiar with the intuitive solution methods that students use when they solve problems, and utilize this knowledge when planning and conducting instructions in mathematics. If the use of metacognitive strategies proves effective in improving mathematics performance, mathematical reasoning and metacognitive knowledge, teachers and lecturers in Nigeria will have additional instructional method that can be used to support students' learning with understanding.

It could also contribute to the improvement of instruction that would lead to high order thinking skills. It will assist instructors to know what skills are required to complete an assigned task and be able to influence students' self-efficacy with the design of lessons and activities. The results of this study will considerably aid teachers and lecturers as they strive to construct classroom environments that focus on strategic learning that is both flexible and creative.

Applications

Mathematics Educators: Result of this study will enable educators to incorporate metacognitive aspects of learning into mathematics instruction. If it can be shown that self-monitoring of progress during problem solving results in students successfully learning calculus concepts, mathematics educators could be better informed about the importance of identifying students' metacognitive thoughts. Similarly, it will also help educators in Nigeria in their search for an effective and efficient pedagogical strategy or model for improving learning with understanding. The findings of this study can afford educators the knowledge that will enable them to clinically evaluate a learner's ability to regulate, monitor and control his or her own cognitive processes.

The findings of this study will be significant to the Nigerian educational system since changing self-concept and attitude of students towards mathematics and improving the teaching procedures in the classroom is much easier to achieve by educators than changing background factors affecting students' performance.

Curriculum Developers: The National Commission for Colleges of Education (NCCE) and National Educational Research and Development Council (NERDC) who decide on course contents and recommend effective methods of instruction might wish to consider the result of this study with a view to recommend it as a teaching strategy in the colleges.

Professional bodies and Centres: Associations such as the Mathematical Association of Nigeria (MAN), Science Teachers Association of Nigeria (STAN) and National Mathematical Centre (NMC) which organize conferences, workshops, discuss and disseminate research findings can benefit from the result of this study by organizing workshops and training for its members.

Researchers: Researchers in mathematics education can benefit from this study by way of replicating the study using other mathematics courses such as Algebra, Trigonometry or

Geometry with this study serving as foundation. They can also investigate the effects of metacognitive strategies on other affective variables such as motivation, locus of control and anxiety. In addition, it could pave the way for more comprehensive research on the comparison of national and international research findings on the effects of metacognitive strategies on students' performance. Also, this will add new information to the existing literature.

1.7 Scope/Delimitation of the Study

In this study, NCE II mathematics students in government-owned colleges of education in North-central Zone of Nigeria are covered. The zone consists of six states namely: Benue, Kogi, Kwara, Nassarawa, Niger and Plateau states. There are thirteen government-owned colleges of education these are: Federal College of Education (FCE), Kontagora, Federal College of Education Okene and Federal College of Education, Pankshin. Others are Niger State College of Education(COE), Minna; Nassarawa State College of Education, Akwanga; Kwara State College of Education, Ilorin; Kwara State College of Education (Technical), Lafiagi; Kogi State College of Education (Technical), Kabba; College of Education, Oro; Kogi State College of Education, Ankpa; College of Education, Gindiri; Benue State College of Education, Katsina-Ala and Federal Capital Territory (FCT) College of Education, Zuba. Similarly, MAT 211: Integral Calculus was taught during the experimental period. The study did not covered the following colleges: City College of Education, Mararaba, Nassarawa State (private); Muhyideen College of Education, Ilorin, Kwara State (private) and College of Education, Offa, Kwara State (private).

1.8 Basic Assumptions of the study

The following assumptions are made:

1. The NCE II students have been taught by qualified lecturers.

2. The students have covered the contents of differential calculus which is a pre-requisite course to Integral calculus.

CHAPTER TWO

LITERATURE REVIEW

2.01 Introduction

The study investigates the impact of metacognitive strategy on attitude, retention and performance in calculus among Colleges of Education Students in North-central Zone, Nigeria. This chapter is devoted to the review of literatures relevant to the current research. Therefore, the review was discussed under the following sub-headings:

2.02 Theoretical Framework of the Study;

2.03 Conceptual Framework of the Study;

2.04 Mathematics Teaching Methods in Colleges of Education;

2.05 The Historical Perspective and Importance of Calculus;

2.06 The Concept and Models of Metacognition;

2.07 The Concept of Attitude;

2.08 Learning and Retention in Mathematics;

2.09 Gender and Mathematics Learning;

2.10 An Overview of Relevant Empirical Studies; and

2.11 Implications of the Reviewed Literature to the Study

2.02 Theoretical Framework of the Study

The constructivist theory based on early empirical and theoretical works of Piaget, Brunner and Vygotsky is the major theoretical component of metacognitive strategy of learning. The constructivists are of the view that knowledge must be constructed within the cognitive structure of every individual so that it is fundamentally personal while being dependent on experiences in the learning environment and on social interaction. They view learning as resulting from attempts by the individual to construct meaning into what he or she was taught and to make sense of the various concepts, principles, or facts presented by the teacher (Confrey & Kazak, 2006; Gupta, 2008).

A major theme in the theoretical framework of Brunner is that learning is an active process in which learners construct new ideas or concepts based on their current or past knowledge. The learner selects and transforms information, constructs hypotheses and makes decisions, relying on their cognitive structure to do so. Cognitive structure (schema and mental models) provides meaning and organization to experiences and allows the individual to go beyond the information given (Richard, 2015). Vygotsky's theory is one of unity between mental functioning and activity, with the development of the mind resulting from goal-oriented and socially-determined interaction between human beings, their tools, and environments. As for Piaget, knowledge is not information to be delivered at one end, and encoded, memorized, retrieved, and applied at the other end. Instead, knowledge is an experience that is acquired through interaction with the world, people and things (Kalpana, 2014).

Constructivism is an epistemology of learning fixed on the argument that the learner reflects on his or her experiences while constructing his or her understanding of the world. This allows learners to formulate a more concrete meaning of the subject matter. Vygotsky is often considered to be the father of social constructivism, while Piaget is often classified as the father of cognitive constructivism (Sjøberg, 2013). Learners cannot construct knowledge just by passively receiving, acquiring, or accepting it; or by inertly listening nor heeding. Knowledge is not formed during the transmission of it. Therefore, emphasis of instruction must be on the creation of meaning and understanding while encountering new information or new contexts.

Taber (2006) has summarized the core ideas of constructivist learning theorists to include:

- Knowledge is actively constructed by the learner, not passively received from the outside. Learning is something done by the learner, not something that is imposed on the learner.
- Learners come to the learning situation (in mathematics) with existing ideas about any phenomenon. Some of these ideas may be ad hoc and unstable; others may be more deeply-rooted and well developed.
- Learners have their own individual ideas about the world, but there are also many similarities and common patterns in their ideas. Some of these ideas are socially and culturally accepted and shared, and they are often part of the language, supported by say metaphors. They also often function well as tools to understand many phenomenon.
- These ideas are often at odds with accepted scientific ideas, just as some of them may be persistent and hard to change.
- Knowledge is represented in the brain as conceptual structures and it is possible to model and describe these in some details.
- Teachers have to take the learners' existing ideas seriously if they want to change or challenge these ideas.
- Although knowledge is in one sense personal and individual, but the learners construct their knowledge through their interaction with the physical world, collaboratively in social settings and in a cultural and linguistic environment.

Teachers' personal theories of learning have long been viewed as having considerable influence on virtually all aspects of their decisions about instruction. How one organizes, structures and sequences instruction is directly impacted by one's beliefs about learning.

Therefore, having correct beliefs about learning will guide teachers as they make decisions about desirable means of implementing and assessing instruction.

2.03 Conceptual Framework of the Study

The conceptual framework for this study combines ideas from works on the reasoning abilities and understandings students need to be successful in calculus Haripersad (2011), Social Cognitive Theory Bandura (2001), Self-regulation Zimmerman (2008), research on metacognitive training Moga (2012), Moghtaderi and Khanjani (2013) and research on the relationship between students' attitudes toward calculus and calculus achievement Alkhateeb and Mji (2005). The Social Cognitive Theory (SCT) holds that learners' behaviour is often predicted by what they believe they are capable of rather than the realization of their capabilities. In other words, students determine what to do with specific mathematical knowledge and skills by their self-efficacy rather than what they might actually understand mathematically. Self-Efficacy is the perceived ability to manage or cope with specific tasks and situations. Students who believe they are capable of completing or excelling in a required task are more apt to engage themselves in the task than students who believe their skills are inadequate.

Self-regulation refers to the ability to develop, implement, and flexibly maintain planned behaviour in order to achieve one's goals (Zimmerman, 2008). Self-regulated learners are those who have power over their actions and thoughts in order to successfully control their learning. Different subjects require the use of different metacognitive strategies and self-regulated learners are aware of and are able to control different strategies for acquiring knowledge for different subject areas more so than other students.

2.04 Mathematics Teaching Methods in Colleges of Education

The teaching method used in the class is one of the factors that can make students become either passive and have less interaction with one another in doing tasks, or become active in the classroom. According to Yousef (2015), there are many problems regarding the issue of teaching. However, when it comes to problems of students' low performance, teachers are among the first people that will be questioned. Teachers play an important role in educating the students. Teachers need to know their strength and weaknesses in teaching and should always try to improve on them. The following are some teaching methods used in the teaching of mathematics.

2.04.1 Lecture Method

In the lecture method, the lecturer spends the first half of the period talking about the theory, rule or formula involved in the topic at hand and then moves into examples of problems that are based on this theory. The material which is prepared beforehand is then written on the chalk or white board. Students strive to write down everything that is written. Since the concentration is often focused on getting everything written down, there is little time to listen or try to make sense of what is being said. Students rarely ask questions and the large lecture hall is simply too intimidating. There seemed not to be an emphasis on concepts and understanding. The atmosphere is not conducive for interaction between lecturer and students (Jaworski, 2006).

2.04.2 Cooperative Learning

Cooperative learning is the instructional use of small groups so that students work together to maximize their own and each other's learning. Zakaria, Solfitri, Daud and Zulkarnain (2013) view cooperative learning as a teaching strategy in which small teams, each with students of different ability levels, use a variety of learning activities to improve their understanding of a subject. Each member of a team is responsible not only for learning what is being taught, but also for helping his or her teammates learn, thus, creating an atmosphere of achievement. Within cooperative situations, individuals seek outcomes that are beneficial to themselves and all other group members.

Cooperative learning, according to the works of Zakaria, Chin and Daud (2010); David and Roger (2012), has the following benefits:

(i) Student achievement: The effects on student achievement are positive and long-lasting regardless of grade levels or subject matters.

(ii) It enhances student retention: Students are more apt to stay in school and not drop out when their contributions are solicited, respected and celebrated.

(iii) Improved relations: One of the most positive benefits is that students who cooperate with each other also tend to understand and like each other the more. This is particularly true for members of different ethnic groups. Relationships between students with learning disabilities and other students in the class improve dramatically as well.

(iv) Improved critical thinking skills: More opportunities for critical thinking skills are provided and students show a significant improvement in those skills.

(v) Oral communication improvement: Students improve in their oral communication skills with members of their peer group.

(vi) It Promotes social skills: Students' social skills are enhanced.

(vii) There is heightened self-esteem: When students' work is valued by team members, their individual self-esteem and respect escalate dramatically.

2.04.3 Mastery Learning

Mastery learning is an instructional strategy based on the principle that all students can learn a set of reasonable objectives with appropriate instruction and sufficient time to learn. Mastery Learning refers to a pedagogical approach that combines the qualities of conventional group-based teaching and one-to-one individual tutoring to achieve better academic performance in a more realistic and cost-effective manner (Wong & Kang,2012). In this type of learning environment, the challenge for the teacher becomes that of providing enough time and employing instructional strategies so that all students can achieve the same level of learning. Mastery learning uses differentiated and individualized instruction, progress monitoring, formative assessment, feedback, corrective procedures, and instructional alignment to minimize achievement gaps (Zimmerman & Dibenedetto, 2008). Inside the classroom the function of the teacher include the following:

i). Specify what is to be learned; ii). Motivate learners to learn it; iii). Provide them with instructional materials; iv). Administer these materials at a rate suitable for each pupil;

v). Monitor student's progress; vi). Diagnose difficulties and provide proper remediation for them; vii). Give praise and encouragement for good performance, and viii). Give review and practice that will maintain learners' learning over long periods of time.

2.04.4Problem-Based Learning (PBL)

Problem-based learning (PBL) has been defined as a teaching strategy that "simultaneously develops problem-solving strategies, disciplinary knowledge and skills by placing students in the active role as problem-solvers confronted with a structured problem which mirrors real-world problems. According to Savery (2006) Problem-based learning is an instructional (and curricular) learner-centred approach that empowers learners to conduct research, integrate theory and practice, and applies knowledge and skills to develop a viable solution to a defined problem.

Similarly, Sungur, Tekkaya and Geban (2006) are of the view that students want to make connections between what they learn and what they experience in their lives and that the problem-based learning (PBL) approach is a perfect way of marrying theory and practice in mathematics. The goal of PBL is to prepare students to be ready for true-to-life settings by increasing higher-level thinking skills by requiring them to think about problems critically and analysing data to find the solution. Students in PBL environments typically have greater opportunity to learn mathematical processes associated with communication, representation, modelling, and reasoning. Research suggests that effective use of problem-based learning methods can prepare students to be flexible thinkers who can work productively with others to solve problems (Hmelo-Silver, & Barrows, 2006).

Hung (2011) provides the following as some benefits of the Problem-Based Learning approach: (i) It allows the learners more responsibility and independence; (ii) It shows students that there are more than one way to solve a problem just as there may also be more than one answer to the problem; (iii) It promotes group work and collaboration in mathematics; (iv) It increases self-motivation and critical thinking; (v) It enables students to demonstrate their understanding and knowledge in a non- traditional way; and

(vi) It encourages life-long learning.

2.05 The Historical Perspective and Importance of Calculus

Calculus is a branch of mathematics that focuses on limits, functions, derivatives, integrals, and infinite series (Melissa, 2012). This branch of mathematics constitutes a major part of modern mathematics education. It has two major branches, differential calculus and integral calculus, which are related by the fundamental theorem of calculus. It provides a framework for modelling systems in which there is change and a way to deduce the predictions of such models. It provides a way to construct relatively simple quantitative models of change and to deduce their consequences. A course in calculus is a gateway to other more advanced courses in mathematics devoted to the study of functions and limits broadly called mathematical analysis. Calculus is used in every branch of the physical sciences, actuarial science, computer science, statistics, engineering, economics, business, medicine, demography, and in other fields wherever a problem can be mathematically modelled and an optimal solution is desired and can solve many problems for which algebra alone is insufficient (David, 2013).

The following are some areas of life where calculus can be applied as summarised by (Oghwephu, 2009; Melissa, 2012; and David 2013):

- Chemistry uses calculus in determining reaction rates and radioactive decay.
- Biologists use differential calculus to determine the exact rate of growth in a bacterial culture when different variables such as temperature and the food source has changed. This research can help increase the rate of growth of necessary bacteria, or decrease the rate of growth for harmful and potentially threatening bacteria.
- Statisticians use calculus to evaluate survey data to help develop business plans for different companies. Because a survey involves many different questions with a

range of possible answers, calculus allows a more accurate prediction for appropriate action.

- Physics makes particular use of calculus because all concepts in classical mechanics and electromagnetism are interrelated to each other through calculus. The mass of an object of known density, the moment of inertia of objects, as well as the total energy of an object within a conservative field can be found by the use of calculus. An example of the use of calculus in mechanics is Newton's second law of motion, Maxwell's theory of electromagnetism and Einstein's theory of general relativity are also expressed in the language of differential calculus.
- In economics, calculus allows for the determination of maximal profit by providing a way to easily calculate both marginal cost and marginal revenue.

  In economics, the term marginal denotes the rate of change of a quantity with respect to a variable on which it depends. For example, the cost of production C(x) in a manufacturing operation is a function of x, the number of units of product produced. The marginal cost production is the rate of change of C with respect to x, so it is: $\frac{dC}{dx}$

- In the realm of medicine, calculus can be used to find the optimal branching angle of a blood vessel so as to maximize flow. From the decay laws for a particular drug's elimination from the body, it is used to derive dosing laws. In nuclear medicine, it is used to build models of radiation transport in targeted tumor therapies.
- A medical statistician studies a drug that has been developed to lower blood pressure. The average reduction R in blood pressure from daily dosage of x mg will be recorded during the experiment. He will be interested to know the sensitivity of R to

dosage x at different dosage levels. The sensitivity of $R\ to\ x$ is $\frac{dR}{dx}$ which can be used to make inference about which dosage level works best to reduce blood pressure.

From the historical point of view, C*alculus as a branch of mathematics has received attention right from the period of Greek mathematics. Eudoxus (c. 408−355 BC) used the method of exhaustion, which foreshadows the concept of the limit, to calculate areas and volumes, while Archimedes (c. 287−212 BC) developed this idea further by inventing heuristics which resemble the methods of integral calculus (Grattan,2009). Similarly, in the 11th century, Ibn Al-Haytham (known as Alhacen in Europe) performed an integration in order to find the volume of a paraboloid and generalized his result for the integrals of polynomials up to the fourth degree* (Victor,1995). However, today it is generally agreed that Calculus was developed independently by two different men in the seventeenth century, Gottfried Wilhelm Leibniz (1646-1716) and Isaac Newton (1642-1727).

Newton's development of Calculus was based on the fluent or any quantity that constantly changed, which could be either geometric or physical. According to Reyes (2004), Newton wrote three accounts dealing with his development of Calculus, none of which was published until several years after its creation. The first account, *De analysi per aequationes numero terminorum infinitas (On Analysis by Equations with Infinitely Many Terms)* was written in 1669. In Newton's second text, *De Methodis Serierum et Fluxionum (A Treatise on the Methods of Series and Fluxions)* written in 1671, he was able to show that a maximum or minimum for a curve existed only when its fluxion, or derivative, was equal to zero. Newton's third text, *De Quadratura Curvarum was written in1676* where Newton came to define the present day derivative as the ultimate ratio of change, which he defined as the ratio between evanescent increments (the ratio of fluxions).

Leibniz focused on the tangent problem and came to believe that calculus was a metaphysical explanation of change. Leibniz discovered in 1673 that the tangent to any given curve was dependent on the difference between y-coordinates in a ratio with the difference between x-coordinates as such differences became infinitely small. Leibniz called these respective differences the differential of x, denoted by dx, and the differential of y, denoted by dy and their ratio, $\frac{dy}{dx}$ which Leibniz called the differential quotient, was the necessary quantity needed to find the tangent line.

After Newton and Leibniz had laid the foundation for defining what limit, continuity, derivative, and integral might mean, other mathematicians such as *Augustin-Louis Cauchy* (1789-1857), worked on definite integrals and calculus of residues, *Ferdinand Georg Frobenius (1849 – 1917) also contributed to the theory of elliptic functions, differential equations and group theory. Others include Abraham Robinson (1918 – 1974) who gave infinitesimal quantities in calculus a solid foundation through his development of non-standard analysis, a mathematically rigorous system whereby infinitesimal and infinite numbers were incorporated into mathematics.* Louis de Branges de Bourcia (1932-) has worked on calculus and gave it a more exact definition (Reyes, 2004; William, 2004; Michael, 2005).

2.06 The Concept and Models of Metacognition

At the heart of metacognitive instruction is the concept of metacognition which was introduced in cognitive psychology more than thirty years ago (Goh, 2008). Metacognition has been one of the most concentrated concepts among researchers in the field of psychology. Many definitions have been proposed for it. Stewart, Cooper and Moulding (2007) have conceptualized metacognition as one's knowledge concerning one's own cognitive processes and products or anything related to them. Metacognition is also viewed as a construct that refers to thinking about one's thinking or the human ability to be conscious of one's mental processes. It has also been described as what one knows about his or her cognitive processes and how he or she uses these processes in order to learn and remember (Pennequin, Olivier, Isabelle, & Roger, 2010). Similarly, it is looked at as the activity of monitoring and controlling one's cognition, knowledge about cognition and regulation of cognition (Ormrod, 2004). In summary, metacognition can be seen as all efforts a learner makes to monitor his or her knowledge, which is, remembering the entry behaviour of a given task, ability to recognize task demand and knowledge of appropriate strategy and self-regulation that is, continuous self-evaluation.

Metacognition describes one's knowledge about how he or she perceives, remembers, thinks, and acts. Metacognitive thought is an essential skill for learning. It ensures that the learner will be able to construct meaning from information. To accomplish this, the learner must be able to think about his or her own thought processes, identify the learning strategies that work best for him or her and consciously manage how he or she learns. Metacognition is essential to successful learning because it enables individuals to better manage their cognitive skills and to determine weaknesses that can be corrected by constructing new cognitive skills.

Therefore, developing metacognition can brings learners an awareness of the learning process and strategies that lead to success. When learners are equipped with this knowledge, they will understand their own thinking and learning process. Similarly, they are more likely to oversee the choice and application of learning strategies, plan how to proceed with a learning task, monitor their own performance on regular basis. Thus, finding solutions to problems encountered, and evaluate them upon task completion.

2.06.1 Models of Metacognition

A number of models have been proposed which are derived from different conceptualizations of metacognition. There are three general models that provide a theoretical framework for metacognition these are: Flavell's, Brown's, and Tobias and Everson's Model (Gama, 2004). Flavell's Model lays the foundations of the metacognition theory and, for the first time, attempts to define the components of metacognition and the interactions among these components. Similarly, Brown's Model was also presented because it makes an important distinction of two different categories of metacognition: knowledge of cognition and regulation of cognition. Finally, Tobias and Everson's Model was presented as they propose a modular model of metacognition, which was taken into account in the design of this study proposed model.

2.06.1.1 Flavell's Model of Cognitive Monitoring

In his classic article "Metacognition and Cognitive Monitoring", Flavell makes the first attempt to define the components of metacognition by creating a model of cognitive monitoring or regulation. His proposed model as summarised by London (2011) consists of four components: (a) metacognitive knowledge, (b) metacognitive experiences, (c) cognitive goals and (d) cognitive strategies. A person's ability to control a wide variety of cognitive enterprises depends on the actions and interactions among these components.

Metacognitive knowledge: Flavell states that metacognitive knowledge consists primarily of knowledge or beliefs about what factors or variables act and interact and in what ways to affect the course and outcome of cognitive enterprises. He also identifies three general categories of these factors: the person category, the task category, and the strategy category. The person category comprises the general knowledge one has about how human beings learn and process information, as well as individual knowledge of one's own learning processes. For example, the realization that one is better at calculus than at algebra. The tasks category considers the information available to solve a problem as well as information about the type of processing demanded of the specific cognitive tasks. This category takes cognizance of the way the information is presented. The strategy category consists of knowledge about which strategies are likely to be effective for achieving goals or sub-goals in various cognitive tasks. It also includes the conditional knowledge about when and where it is appropriate to use such strategies.

Metacognitive experiences: These are conscious cognitive or affective experiences that accompany a cognitive action. It functions to monitor strategic tasks related to decision making and cognitive processing as they take place. The feelings of familiarity, difficulty, and confidence are some examples of metacognitive experiences that contribute to the use of metacognitive strategies. For example, when a learner suddenly perceives that he or she does not understand what the teacher said, or when he or she experiences a feeling that certain task or question is difficult to understand, remember, or solve.

Cognitive goals: Cognitive goals describe the path one chosen to manage the task. Cognitive strategies refer to the utilization of specific techniques that assist in achieving cognitive goals.

Cognitive strategies: The cognitive strategies that students use influence how they will perform in school, as well as what they will accomplish outside of school. Strategy used can be influenced both by knowledge of what the strategy is and how to use it, and by belief in the effectiveness of the strategy (Moghtaderi & Khanjani, 2013). One reason why students may not use an effective strategy is that they do not know about it.

2.06.1.2 Brown's Model of Metacognition

Brown has described metacognition as an awareness of one's own cognitive activity; the methods employed to regulate one's own cognitive processes; and a command of how one directs, plans, and monitors cognitive activity (London, 2011). He divides metacognition into two broad categories: (1) knowledge of cognition, as activities that involve conscious reflection on one's cognitive abilities and activities; and (2) regulation of cognition, as activities regarding self-regulatory mechanisms during an on-going attempt to learn or solve problems. According to Brown, these two forms of metacognition are closely related, each feeding on the other recursively, although they can be readily distinguishable.

Knowledge of cognition: Knowledge of cognition refers to what individuals know about their own cognition and includes three different kinds of metacognitive knowledge: (a) declarative, (b) procedural, and (c) conditional. Declarative knowledge refers to knowing "about" things and includes knowledge about oneself as a learner and about the factors that influence learning. Procedural knowledge refers to knowledge about the execution of procedural skills. Conditional knowledge refers to knowing the "why" and "when" aspects of cognition and can be thought of as the declarative knowledge and about the relative utility of cognitive procedures. Knowledge of cognition allows individuals to plan, sequence and monitor their learning in a way that directly improves performance.

Regulation of cognition: Regulation of cognition is thought to refer to processes or mechanisms that help control and monitor thinking, performance, and subsequent learning. These processes include planning activities (predicting outcomes, scheduling strategies, and various forms of vicarious trial and error,) prior to undertaking a problem; monitoring activities (monitoring, testing, revising, and re-scheduling one's strategies for learning) during learning; and checking outcomes (evaluating the outcome of any strategic actions against criteria of efficiency and effectiveness). Students who use self-regulated strategies are intrinsically self-motivated and prove to be autonomous learners (Cleary & Chen 2009). These types of learners are motivationally and behaviourally active participants in their own learning process.

2.06.1.3 Tobias and Everson Hierarchical Model

Tobias and Everson (2009) proposed a four-tier model for metacognition. Metacognition is defined as the ability to monitor, evaluate, and make plans for one's learning. They designed a hierarchical metacognitive model. The model is organized into four ascending dependency levels. At the bottom appears the knowledge monitoring component. It is the ability of an individual for knowing what she knows and knowing what she does not know. At second level is the evaluation of learning. It holds criteria for determining the degree of satisfaction achieved according to former expectancies. At the third level is selection of strategies. It represents the attempt to set or adjust the course of action according to some guidelines. At the top of the hierarchy, the planning component is found. It defines the path of actions to be accomplished under the strategy's criteria to carry out the pending learning goals.

2.06.2 Metacognitive Strategy and Performance

Metacognition plays an important role in education for it helps learner to be capable of developing a plan, monitoring and evaluating how effective it is. That means metacognition helps the learner to be more involved in the learning process (Kocak & Bayaci, 2010). A lot of studies have reported that there is a difference in the metacognition of effective learners and ineffective learners. The effective users of metacognition are more strategic, more likely to use problem solving heuristics and better at predicating their test score (Uwazurike, 2010).

Researches have examined the relation between metacognition and academic achievement. They show that students with high academic achievements demonstrate high level of metacognitive awareness (Coutinho, 2007; Moga, 2012). And that students with good metacognition demonstrate good academic performance compared to students with poor metacognition.

Metacognitive strategies are sequential processes that one uses to control cognitive activities and to ensure that a cognitive goal, for example understanding a text or problem situation, has been met. These processes help to regulate and oversee learning. It consists of planning and monitoring cognitive activities, as well as checking the outcomes of those activities (Shannon, 2008). In a proposition to facilitate students in learning activities, many researchers have outlined a variety of metacognitive strategies that needed to be taught. It is widely agreed by these researchers that these strategies needed to be explicitly taught. Desoete (2007) also argues that teaching such strategies has twin benefits in that:

(a) it transfers responsibility for monitoring learning from teachers to students themselves, and (b) it promotes positive self-perceptions and motivation among students. In this manner, metacognition provides personal insight into one's own thinking and fosters independent learning.

Similarly, varieties of studies have also shown that there is a relationship between metacognition and teaching performance. Teachers with high level of metacognitive awareness were more active in the teaching process. Most studies in this area used training programmes and examined the impact of these programmes on teaching performance between two groups (control group - experimental group which has been taught using metacognitive strategies). The results have shown that experimental groups were more active in the teaching process, have confidence, interacted more socially with their students. They also used different and suitable teaching strategies and have the tendency to use and practice thinking skills in their classes (Gopinath, 2014; Abdellah, 2015).

These studies showed there is positive influence for training on metacognitive strategies and students with poor metacognition may benefit from these training to improve their metacognition and academic performance.

2.07 The Concept of Attitude

Attitudes are generally regarded as having been learnt. They predispose an individual to action that has some degree of consistency and can be evaluated as either negative or positive (Nicolaidou & Philippou, 2013). They are linked to beliefs and for each belief an individual would have a corresponding attitude. Attitudes have been linked to action and can be categorised according to their focus. Thus, behavioural attitudes indicate a person's judgment of performing the behaviour as good or bad or that the person was in favour of or against performing the behaviour. Clearly, other things being equal, the more favourable a person's attitude is toward behaviour, the more likely the person would intend to perform that behaviour. Asante (2012) defines an attitude towards mathematics as "a disposition towards an aspect of mathematics that has been acquired by an individual through his or her beliefs and experiences but which could be changed".

Attitudes can be seen as more or less positive. A positive attitude towards mathematics reflects a positive emotional disposition in relation to the subject. In a similar way, a negative attitude towards mathematics relates to a negative emotional disposition (Anderson, Teisl, Criner, Tisher, Smith & Hunter, 2007). These emotional dispositions have an impact on an individual's behaviour, as one is likely to achieve better in a subject that one enjoys, has confidence in or finds useful. For these reasons, positive attitudes towards mathematics are desirable since they may influence one's willingness to learn mathematics.

Educational change depends on what teachers do and think, as does the success or failure of the educational process. Teachers mediate between the learner and the subject to be learned. Consequently, teachers' beliefs, attitudes and expectations have a major impact on student achievement. Teacher attitudes impact their daily choices of activities, the amount of effort expended on each and their expectations of students' abilities to perform. Teachers who believe it is important for students to learn mathematics with understanding embrace the use of investigations, mathematical discourse and the appropriate mathematical notation and vocabulary (Maat & Zakaria, 2010). Because a teacher's beliefs influence his or her instructional decisions, pedagogical choices will differ among teachers, yielding varied student achievement results. A teacher's belief in a blend of whole class, individual work, and small-group work on challenging and interesting problems results in improved student achievement.

Teachers who believe in the importance of providing all students the opportunity to learn mathematics with understanding employ strategies that promote student engagement in problem solving. They encourage students to make, test, and revise conjectures, and to support their reasoning with evidence. In contrast, teachers who believe that computational prowess is the most important component of mathematics typically demonstrate procedures and provide students time in which to practice those steps.

2.07.1 Attitude and Performance in Mathematics

Understanding the relationship between the affective and cognitive domains of mathematics education is one important direction toward the improvement of mathematics education. The area of the affective domain that is of concern to this study is mathematics attitude. Attitude contributes substantially to the difficulties encountered by students in learning and understanding of mathematics. A learner's attitude relates to all the facets of his or her education. The attitude of a learner towards mathematics will determine his attractiveness or repulsiveness to mathematics. This, invariably, will influence the learner's achievement in the subject (Akinsola & Olowojaiye, 2008; Mohamed & Waheed, 2011). Attitude Toward Mathematics is an aggregated measure of a liking or disliking of mathematics, a tendency to engage in or avoid mathematical activities, a belief that one is good or bad at mathematics; and a belief that mathematics is useful or useless (Nicolaidou & Philippou 2013).

Students' achievement is one of the main determining factors in assessing the quality of education in any given society. Thus, identifying the factors that affect students' learning and achievement continues to be an important object of study of many educators. Studies of Köğce, Yıldız, Aydın and Altındağ (2009) have shown that factors that can influence mathematics performance and high achievement in mathematics are function of many interrelated variables related to students, families and schools. Among student variables, attitude is regarded as the most important factor to be taken into account. Researchers in mathematics education are thus, concern about the relationship of attitude and achievement in mathematics. They are also concerned with methods and approaches that will facilitate practice towards enhancing the quality and performance of students in mathematics (Mahanta, 2012).

Review of literature on attitudes and factors that influence their development in relation to differences between students had identified three groups of factors that play a vital role in influencing student attitudes. According to Mohamed and Waheed (2011) these factors include: those associated with the students themselves (that is, mathematical achievement, anxiety, self-efficacy and self-concept, motivation, and experiences at school); factors associated with the school, teacher, and teaching (that is, teaching materials, classroom management, teacher's knowledge, and attitudes towards mathematics, guidance and beliefs). Finally, there are factors from the home environment and society (that is, educational background and parental expectations). That could affect students' performance in mathematics.

It is generally believed that students' attitude towards a subject determines their success in that subject. In other words, favourable attitude results to good achievement in a subject. Research has, however, failed to provide consistent findings regarding relationship between positive attitude toward mathematics and achievement. Some studies found a positive relationship between the two variables Ma and Xu (2004); Mohammed and Mahmood (2011). Others such as White, Perry, Way and Southwell (2006); Evans (2007) did not. However, a positive attitude towards mathematics reflects a positive emotional disposition in relation to the subject. In a similar way, a negative attitude towards mathematics relates to a negative emotional disposition. These emotional dispositions have an impact on an individual's behaviour, as one is likely to achieve better in a subject that one enjoys, has confidence in or finds useful (Saha, 2007). For this reason, positive attitudes towards mathematics are desirable since they may influence one's willingness to learn.

Similarly, research has shown that teachers tend to shape their mathematics classroom practice based upon their own attitudes and beliefs, and thus transferring their own attitudes and beliefs to their students (Herrera, 2010). And those effective teaching strategies can create positive attitude on the students towards school subjects (Akinsola & Olowojaiye, 2008). It is important for teachers across all levels of mathematics instruction to exhibit positive attitudes and beliefs in order to allow their students to develop positive attitudes and beliefs towards mathematics. Thus, this study is aimed that finding out if NCE students' attitudes toward calculus could change positively as a result of metacognitive instruction.

2.08 Learning and Retention in Mathematics

Retention of knowledge denotes recalling or remembering pieces of knowledge, processes, or skills that were learned earlier in time. Knowledge retention is a significant goal of education. Narli (2011) has observed that the very existence of school rests on the assumption that people learn something of what is taught and later remember some part of it. Retention which is the process by which a long term memory preserves learning so that it can locate, identify and retrieve it in the future, has been studied by many researchers such as (Osemwinyen, 2009; Gambari, Falode and Adegbenro, 2014).

It was suggested that retention can be improved upon in several ways such as comprehensive learning of concepts and involving different instructional approaches. It has also been shown that knowledge and skills learned through understanding are retained and transferred better than that which is learned by rote memorization (Nejem and Muhanna (2014). Similarly, Chianson, Kurumeh, and Obida (2011) have called on mathematics teachers and lecturers to present to the learners mathematics concepts in a way that touches their sub-consciousness which can trigger quick recalling of the concept being taught or learnt. Therefore, it may prove to be important to examine the role of metacognitive strategy on retention ability of NCE II students when taught calculus.

2.09 Gender and Mathematics Learning

The issue of gender differences in mathematics achievement has been a source of worry to mathematics educators and researchers. Over the past three decades, a considerable number of inconsistent studies have been conducted in various countries. For example, Farayola (2011); Onyishi and Agwagah (2011) studies showed that there was no significant gender difference in mathematics achievement. Similarly, Habibollah, Rohani, Tengku Aizan and Jamaluddin (2009) found no gender differences on the overall factor scores of creativity. Arslan, Çanl and Sabo (2012) have in their study shown that female students have more positive attitudes than male students toward Mathematics and had higher mathematics grades.

However, several other studies Mendick (2005); McGraw, Lubienski and Strutchens (2006) have shown that, males tend to perform better than females in mathematics. For instance, Saito (2011) studies gender differences in learning achievement by exploring the Southern and Eastern African countries educational environment and curriculum contents, using (2000-2007) data from Southern and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ). He concluded that for the mathematics results, the general pattern was opposite that of reading. In none of the countries were there significant differences in favour of girls in all mathematics domains. The same trend was observed even in the developed countries where it was believed that gender inequality had been eliminated. Hall (2012) challenged the perception that gender issues on students' mathematics achievement and participation have been solved in developed countries. He observed that over time and across grade levels, boys generally showed more positive attitude toward mathematics and greater confidence in their mathematics abilities than girls.

Similarly, Vale (2008) conducted a meta- analysis of national and international large scale assessments of mathematics at the elementary and secondary school levels. The finding showed that boys' attitude, confidence and achievement were statistically higher than those of girls. However, females had higher mathematical anxiety scores than males. Finally, a study carried out by Popoola and Ajani (2011) revealed the following amazing results:

- There exists a significant difference in mathematics achievement of pupils taught by male and female teachers in favour of pupils taught by male teachers.
- There is a significant gender difference in mathematics achievement test scores of pupils taught by male teachers in favour of males.
- There is a significant gender difference in mathematics achievement test scores of pupils taught by female teachers in favour of males.
- There is no significant difference in the achievement test score of male pupils taught by teachers of different gender, although pupils in the male teachers' class had higher mean scores.
- There exists a significant difference in mathematics achievement of female pupils taught by male and female teachers in favour of girls taught by male teachers with higher mean score.

In the same vein it is a common observation that at the higher levels females take mathematics quite lesser than the male students. Most professions requiring higher level knowledge of mathematics are dominated by the male (Farooq & Shah, 2008).

Common explanations for gender disparities in mathematics achievement have been that girls showed more negative attitudes than boys. Compared with boys, girls lacked confidence, had debilitating causal attribution patterns and were anxious about mathematics. Females internalize failure and attribute it directly to perceived lower ability and they do not like competitive environments (Thompson & Dinnel, 2007; Effandi & Normah, 2009). Hall (2012) has observed that using a variety of pedagogical strategies that address different learning styles within instructional environments has been shown to encourage female achievement in mathematics classrooms. The present study thus examines the impacts of metacognitive strategy on gender of NCE II students.

2.10 An Overview of Relevant Empirical Studies

Several studies have been carried out on metacognition and its relationship with attitude, problem solving ability and performance. Ayub, Lian and Mukti (2005) have assessed attitudes toward calculus of 62 diploma students of University Putra Malaysia (UPM). Attitude was measured in cognitive, affective and behavioural domains. Affect refers to positive or negative feelings toward calculus. Cognition refers to how students perceive calculus, such as perceiving the usefulness of calculus in their lives or relating calculus to their daily lives, while behaviour reflects how students react to Calculus. The study used t-test to determine the attitude difference between males and females while MANOVA was carried out to determine the differences between males and females in terms of the three domains (affective, cognitive and behavioural) of attitudes.

The findings showed that a high percentage of students did not have positive attitudes toward Calculus. There was a statistically significant difference in the mean scores for males and females in the Calculus attitudes scale. Specifically, statistically significant differences were detected between males and females in two attitude domains: cognitive and behavioural. The correlation between students' attitudes toward Calculus and Mathematics grade was low and not statistically significant. The findings suggested that female participants had more positive perceptions with more favourable reactions toward Calculus when compared with the males. However, there was no significant difference between them when measured in terms of their feelings toward Calculus.

Ayub, Lian and Mukti (2005) study relates to the present study because it is concerned with diploma students' attitude towards calculus. The participants and the instrument are both similar to that of the present study. The study, however, differed from the present one in its adoption of survey technique. They also collected their data from just one University. The present study assessed the students' attitude toward calculus before and after instruction using metacognitive strategy.

Akinsola and Animasahun (2007) have investigated the effect of simulation-games environment on students' achievement and attitudes to mathematics in secondary schools in Osun-State, Nigeria. The study used 147 Senior Secondary (SSII) students and adopted pre-test, post-test quasi experimental design. The finding revealed that students' poor academic achievement in mathematics is partly due to the method of teaching used. Also, the findings revealed that the use of simulation-games environment led to improvement in achievement and positive attitude towards mathematics.

That study is relevant to the present one in that its participants are Nigerians and the content area is mathematics. Although the study used secondary school students, its findings have relevance to the present study because one of its aims is to determine the effects of instructional method that is metacognitive strategy on performance and attitude of students. This study differs from the study of Akinsola and Animasahun (2007) in that they used secondary school students, while the present study used NCE students.

Coutinho (2007) examined the relationship between achievement goals, metacognition, and academic success of 179 undergraduates (87 women, 92 men) of a Midwestern university. The participants completed a survey reporting their use of achievement goals, metacognition and Great Point Aggregate (GPAs). The instruments used are: Goals Inventory, the instrument comprises 12 items assessing mastery goals and five items assessing performance goals and Metacognitive Awareness Inventory (MAI). This is a comprehensive scale assessing various facets of metacognition. It has been shown to have good reliability and validity for metacognition assessment.

Results showed that mastery goals were related to GPA performance whereas performance goals were unrelated to GPA performance. That is to say that those students with the intent to deeply comprehend information tend to be successful in their academic performance. Similarly, students who seek to simply perform well on a test without understanding the information do not necessarily have good performance. Metacognition is also related to academic success and students with good metacognition have good GPAs. Mastery goals influence GPAs through metacognition as students with such goals may have superior metacognitive skills and strategies that they use to master information. The use of superior metacognition eventually leads to enhanced GPA.

The researcher has recommended support training programmes instructing students on how to adopt effective metacognitive skills and strategies and learn how to master information instead of simply seeking to perform well. He also called on teachers to use teaching techniques that present information to students in a way that encourages the use of mastery goals and metacognitive strategies.

The study of Coutinho (2007) examined the relationship between metacognitive awareness and academic success of undergraduate students. He also suggested that teachers should use teaching techniques that present information to students in a way that encourages the use of metacognitive strategies. The present study is in line with that suggestion since it wants to see the efficacy of teaching metacognitive strategies on Nigeria Certificate in Education students' performance. Coutinho (2007) study is related to the present study for the fact that both use post-secondary school students and metacognitive technique. However, the study differs from the present one in that Coutinho (2007) used expo-factor design in collecting his data, while the present study used pre-test, post-test, post-post-test quasi-experimental design. Also the two studies differed in the environments in which they were conducted.

Subocz (2007) examined the effects of metacognitive strategy instruction on the performance and attitude of community college students enrolled in pre-algebra courses. Students' metacognitive awareness was studied using the Metacognitive Awareness Inventory (MAI). Students' attitudes toward mathematics were measured through administration of a modified Fennema-Sherman, Mathematics Attitudes Survey (MAS) at the beginning and at the end of the course. Data were collected over a sixteen week period. Findings revealed that failure rates decreased for students who participated in the metacognitive skills programme. Findings also revealed that attitudes towards mathematics improved significantly more for students participating in the metacognitive skills programme. The study concluded that attitudes toward mathematics are correlated with metacognitive awareness and it improved student performance and attitudes toward mathematics. That study is relevant to the present one in that it uses post-secondary school students and the instrument of Metacognitive Awareness Inventory. The present study is aimed at confirming discrepancy or otherwise with the findings of Subocz (2007) for it wishes to determine the relationship between knowledge of metacognitive strategy and academic performance. Similarly, it also wished to find correlation between knowledge of metacognitive strategy and attitude towards calculus. However, this study differs from that of Subocz (2007) because his study used algebra while the present study used calculus.

Goldberg and Bush (2009) examined the impact of teaching metacognitive strategies on third-grade students' metacognitive and problem-solving skills. Forty four, eight and nine year old third-grade students are used. 21 students were in the Metacognitive class and 23 students in the Non-Metacognitive class. The teacher of the Metacognitive class sought to create a culture of thinking throughout the day by using different instructional strategies. These strategies encompassed a two-pronged approach: (a) strategies that focused on raising student self-awareness, especially of their own thinking; and (b) strategies that focused on planning, monitoring, and evaluating within problem-solving events. The Non-Metacognitive teacher did not attempt to create such a culture and made no overt attempts to develop her students' metacognitive skills. The study concluded that instruction in metacognitive skills increased the metacognitive skills used by third-grade students and thereby improved their performance in mathematical problem solving. In particular, students learned to monitor their thinking more often during problem-solving episodes as a result of the instruction. As a result of instruction in metacognition, students improved significantly in their attempts to understand the problem and slightly in their use of strategies and formulation of solutions.

The study of Goldberg and Bush (2009) is relevant to the present study for it uses similar metacognitive teaching strategies and has shown that metacognitive skills can be taught. Their findings showed that students improved significantly in their attempts to understand the problem but only slightly in their use of strategies and formulation of solutions. The slight improvement in the use of strategies and formulation of solutions by the pupils may not be unconnected with the development of metacognition, metacognitive knowledge precedes metacognitive skills. While metacognitive awareness may occur at the early age, metacognitive skills have been found to reach maturation during adolescence (Panaoura & Philippou, 2007).

Kevin (2009) investigated the relationship between academic performance and metacognitive development throughout the period of undergraduate study, as measured by longitudinal, Learning and Study Strategies Inventory (LASSI). The LASSI is a self-perception assessment of individual study and learning strategies and a measure of a student's thinking about their thinking. The tool consists of ten scales and eighty items which provide an assessment of students' awareness about and use of learning and study strategies related to the skill, will and self-regulation components of strategic learning. The study consists of a random sampling of 300 participants, which consists of 148 male and 152 female students from City University of Hong Kong. The LASSI entry measure was administered when students had been informed of acceptance at university but just prior to the start of their academic programme. The interim measure was taken after 15 months of undergraduate study and the exit measure was administered 3 months before graduation so as to avoid interfering with final examination revision periods. The results demonstrate a significant relationship between academic achievement and change in metacognition as measured by longitudinal LASSI assessment. The study concluded that students' awareness about and use of learning and study strategies related to the skill contributed significantly to successful study, and that they can be learned or enhanced through educational interventions such as learning and study skills courses.

Kevin (2009) study relates to the present one for its finding showed that there is a significant relationship between academic performance and change in metacognitive knowledge. The findings of that study is beneficial to the present study in that it showed that study strategies can be learned and enhanced through educational interventions. However, the study differs from the present one in that Kevin (2009) used expo-factor design in collecting his data, while this study used pre-test, post-test, post-post-test quasi-experimental design.

Cooper and Stewart (2009) tried to find out whether adults' metacognitive awareness and skills naturally increase with age. Experienced teachers who were working toward their Master's degrees at a Utah University were compared to undergraduate teachers-in-training using a paper-and-pencil self-evaluation instrument of Metacognitive Awareness Inventory (MAI) to assess metacognition. Sampled size was 91 participants from the undergraduate group and 123 participants from the graduate student group. The following are the findings of that study:

The scores for metacognitive knowledge, metacognitive regulation, and total MAI score were compared to detect differences between the two groups. In all three sets of scores, there were significant differences between the undergraduates and the graduates who participated in the study.

For the undergraduate students, age was significantly correlated with metacognitive regulation but not with metacognitive knowledge or with total MAI score. Within the group of experienced teachers, age was significantly correlated with metacognitive regulation score and with total MAI score, but not with knowledge score. For the pooled group of 214 subjects, metacognitive knowledge, regulation, and total MAI score were all strongly correlated with age.

Metacognitive knowledge did not show a significant correlation with years of teaching experience, but metacognitive regulation and total MAI score did show significant correlation with years of teaching experience. Metacognitive regulation appeared to be the component that contributed to increased metacognitive awareness among the experienced teachers. Finally, no significant gender differences were detected among the undergraduates, the graduate students or the pooled group.

The study suggested the inclusion of metacognitive awareness courses in college teacher training college curricula and in a variety of other fields. It concluded that promoting metacognitive awareness and skills could be a valuable method for improving learning and performance at all ages.

Cooper and Stewart (2009) study is relevant to the present one for it showed that metacognitive regulation significantly correlated with age and that metacognitive skills can be taught. Similarly, metacognitive awareness and skills could be a valuable method for improving learning and performance at all levels of education.

Özsoy and Ataman (2009) examined the effect of metacognitive strategy instruction on mathematical problem solving achievement of fifth grade primary school students. The study tried to find out whether metacognitive strategy instruction will have an impact on students' mathematical problem solving achievement and metacognitive knowledge and skills of the students. The experimental group (n=24) were instructed to improve their metacognitive skills. At the same time, the students in the control group (n=23) received no additional activities and continued their normal lessons. Students were pre- and post-tested with the Mathematical Problem Solving Achievement Test and the Turkish version of Metacognitive Skills and Knowledge Assessment (MSKA). Metacognitive Skills and Knowledge Assessment (MSKA) was used to measure the metacognitive knowledge and skills of students. The MSKA assesses two metacognitive components (knowledge and skills) including seven metacognitive parameters (declarative, procedural, conditional knowledge, prediction, planning, monitoring, and evaluation skills). The results showed that the metacognitive strategy instruction in the treatment group have led to a significant difference between the treatment and control group in terms of the level of metacognitive knowledge and skills. The results obtained indicated the group exposed to the instruction of metacognitive instruction are more effective than the group that have not been exposed to the instruction of metacognitive strategy in terms of the development of metacognitive skills. The conclusion of the study revealed that there was an increase in problem solving skills of the students who have been exposed to the instruction of metacognitive strategy. Accordingly, it was suggested that all instruction processes should include the instruction of metacognitive skills.

Jaafar and Ayub (2010) studied the relationship among students' self-efficacy, metacognition and academic performance of undergraduate students in calculus. Participants were 203 undergraduate university students taking a foundation mathematics calculus course. The researchers said they have chosen mathematics calculus because it is the most difficult mathematics course compared to other mathematics foundation courses such as algebra or statistics.

Three instruments that were used are: Mathematics Self-Efficacy Questionnaire (MSQ), Mathematics Meta-Cognition Questionnaire (MMQ) and Mathematics Performance. Items used to measure student's mathematics self-efficacy are related with what students believed in obtaining a good grade in calculus, their confidence in learning understanding mathematics concept and their confidence to master and accomplish the task given while taking the course. Mathematics Meta-Cognition Questionnaire consisted of 12 items. Items are related to the learning strategy that the students used while learning mathematics calculus.
The findings showed that there is a positive relationship between mathematics self-efficacy and mathematics performance. The findings also showed that there is a positive relationship between mathematics meta-cognition and mathematics academic achievement. They recommended that mathematics educators should give attention to mathematics self-efficacy and meta-cognition as important variables in mathematics education.

Tok, Özgan and Dös (2010) studied the effects of metacognitive awareness and learning strategies on students success in a distance English learning class. The study was undertaken with 126 students (70 females, 56 males) from the first year students of the faculty of Education in Gaziantep University. The data were collected through the use of Metacognitive Awareness Inventory and Learning Strategies Questionnaire. The study revealed that learning strategies has an important role on students' academic success in an online English course. Metacognitive awareness also has significant effect on academic success in an online course. It was recommended that students should be taught to use learning strategies and metacognitive awareness strategies effectively in their courses.

Huang (2011) investigated the attitudes toward calculus of engineering students in Taiwan. The participants were 792 first-year engineering students consisting of 257 females and 535 males from six technology universities in Taiwan. The study was aimed at: Assessing students' attitudes toward calculus; determining the differences in attitudes scores between males and females, and finding out if there was any correlation between students' attitudes toward calculus and calculus achievement. The study made use of two instruments, Students' Attitude Towards Calculus (SATC) and Calculus achievement. The SATC is an adapted instrument from the modified Fennema-Sherman Mathematics Attitude Scales based on the Tripartite Model. The instrument consisted of Section A that deals with students' gender, college major, calculus studying time, Internet time, and the frequency of asking calculus questions per week and calculus achievement of the last semester. Section B is based on the Tripartite Model, with five scales developed according to affective, cognitive and behaviour domains, respectively. Calculus achievement was measured in terms of students' mid-term examination. Students could achieve an A (score above 90), B (score between 80 and 89), C (score between 70 and 79), D (the score between 60 and 69) grade.

MANOVA test was conducted to determine if differences exist between male and female students in usefulness, self-efficacy, motivation, anxiety and learning habit domains. An independent-samples t-test was used to determine differences in achievement. The findings of that study show that a high percentage of students did not have positive attitudes toward calculus. It showed that 50% of the participants had negative (23.1%) and moderately positive (26.9%) attitudes. Statistical significant differences were detected between males and females in two attitude domains: cognitive and behaviour. The correlation between students' attitudes toward calculus and calculus achievement was statistically significant in the self-efficacy scale.

The study found that a high proportion of engineering students had negative and moderately positive attitudes toward calculus. This was attributed to the way calculus is taught in Taiwan. Calculus is currently taught using the traditional approach.

Rahman, Abdullah, Yasin, Meerah, Halim and Amir (2011) studied students' learning style and their preferences for the promotion of metacognitive development activities in a science class. The study sampled 161 science-stream students from six secondary schools in Malaysia. Data were collected to measure students' learning styles, perceptions of metacognitive development activities in the classroom and preferences regarding metacognitive development activities in the classroom.

Two instruments were used to (i) measure student's perceptions and preferences of strategies to promote metacognitive development in the classroom. This questionnaire contained seven components which are: Metacognitive reflection, Student-student discourse, student-teacher discourse, student voices, distributed control, teacher encouragement and motivation and emotional support. (ii) Another instrument was used to determine the respondents' learning styles. The questionnaire composed of 20 items. The items are related to the students' tendency towards visual, auditory, tactual or kinaesthetic.

The results of the study showed that the highest mean score reported for the implementation of metacognitive development strategies in the classroom were for activities related to emotional support, supporting students' voices and teacher encouragement and motivation. Further analysis showed that emotional support was rated the highest of all activities by all learning styles and rated the highest overall by kinaesthetic students, followed by tactical, visual and auditory students, in that order. They suggested that students should be encouraged to be actively involved in learning and should be taught how to plan, monitor and evaluate what they have learned in order to promote their metacognitive development, which is important for their lifelong learning process.

Javanmard, Hoshmandja and Ahmadzade (2012) studied relationship among self-efficacy, cognitive, metacognitive strategies and academic self-handicapping with academic achievement of 322 male high school students. The study utilized the instruments of Self-efficacy Questionnaire, Cognitive and Metacognitive Strategies Questionnaire and Self-handicapping Questionnaire, while Students' GPA was considered as the indicator of their academic achievement**.**

The findings revealed that there is a significant positive relationship between self-efficacy and academic achievement. However, there is a significant negative relationship between academic self-handicapping and academic achievement. In other words, students undertaking studies in different fields are similar with regard to academic self-efficacy, cognitive and metacognitive strategies. Another important finding revealed by the study is that there was a significant relationship between cognitive and metacognitive strategies and academic achievement. That is, the more cognitive and metacognitive strategies students use, the more their academic success was.

Moga (2012) investigated the effects of metacognitive training on students' mathematics performance from inclusive classrooms through different approaches of metacognitive training (individual or small groups), different age students (seventh and third grade students). The study used metacognitive training, based on the IMPROVE model offered by Kramarski and Mevarech. IMPROVE is the acronym of seven teaching steps: Introducing new concepts, Metacognitive questioning, Practicing, Reviewing, Obtaining mastery on the objective proposed, Verification, and Enrichment. The study concluded that students can improve their metacognitive skills through individual metacognitive training or a metacognitive training in small groups, both being effective. Metacognitive training leads to improvement in performance, increased metacognitive knowledge, developing, strengthening and automating of prediction, planning, monitoring and evaluation skills. The results also indicate that metacognition can be taught to elementary school children.

Bergstresser (2013) assessed the impact of students' motivation and beliefs about calculus and their ability to retain mathematical skills after having practiced and developed metacognitive skills. The study involved 27 students in two separate senior calculus classes. Two topics of calculus content, Functions and Function Relations and Limits, were treated during the experimental period. The students were all pre-tested and a post-tested on attitudes, learning styles and motivation. During the metacognitive training session, the students were asked the following questions about their mathematics strategy: What do you do first? What do you do while working on the problem? What do you do if you are having trouble? What do you do when you finish? The questions were to encourage the students to self-assess as they worked on calculus problem.

The results of the study showed that there was a positive correlation between learning metacognitive skills and retaining content. The study concluded, by saying that while some students naturally develop metacognitive skills over time, most of them do not. And that teaching students to reflect and analyse how they think and learn allows them to not only develop a deeper understanding of the content, but also gives them a deeper sense of personal achievement. Teaching the students how to individualize their learning and reflect and adjust as what they learn not only affected their content knowledge but also their attitude towards calculus class. Similarly, students are more likely to invest in the content if they feel they can learn the material.

Moghtaderi and Khanjani (2013) investigated the effectiveness of training cognitive and meta-cognition process on self-efficacy improvement and ability to solve mathematical problems of students with dyscalculia in secondary schools. The sample consisted of 60 students with learning disability in mathematics. They made use of three instruments: Problem-Solving Ability Questionnaire, Researcher Made Questionnaire of Dyscalculia, Mathematics Problem Test, Self-Efficacy Questionnaire and training cognitive/meta-cognition strategy for mathematics verbal problem Montague's method.

Findings showed that training cognitive/meta-cognition strategy for mathematics verbal problem solving affects and enhances performance of problem solving in students with dyscalculia. Another finding of that research is that training in cognitive and meta-cognition strategy had effects on positive attitude to mathematics and mathematics problem solving in students with dyscalculia. Other findings of the study suggested that training in cognitive and meta-cognition strategy effects on strategic application of mathematics problem solving strategy.

Oppong (2013) investigated senior high student's attitudes towards mathematics and gender differences in attitudes towards mathematics among students in Accra. A cross-sectional data containing demographic information and Attitude Towards Mathematics Inventory (ATMI) were collected from 181 students made up of 109 boys and 72 girls. The results revealed that there was a significant difference in attitudes shown towards mathematics between boys and girls. The magnitude of the gender differences appears to be large for the total (ATMI) scores (.80) and self-confidence (.80), medium size for enjoyment (.68) and Value (.56), and a small effect size of .35 for motivation. The school environment, teacher's attitudes and beliefs, teaching styles and parental attitudes were identified as explanation factors that account for student's attitudes towards mathematics. It was also observed that high school students become exposed to a greater number of male teachers and to a more competitive and unstructured learning which place girls at a disadvantage with regard to self-esteem and confidence.

Sırmacı and Tuncer (2013) investigated the effects of metacognition strategies applied in 7th grade mathematics course (Permutation and Probability) on students' achievement, metacognition skills, attitudes and permanence. The study used pre-test post-test control group experimental design to determine the difference between achievement, metacognitive skills, attitudes and permanence of control group and experiment group. The experiment group students were taught using a teaching method based on metacognitive strategies while control group students were taught using the traditional teaching method. Three instruments that were used are: metacognitive strategies, achievement test and Attitude scale for mathematics course.

SPSS version 16 was used for data analysis. Independent t-test, paired t-test and descriptive statistics were used for data analysis. The result indicated that teaching with metacognition strategies is more effective than the traditional teaching to increase the achievement of students in mathematics course (Permutation and Probability). Similarly, metacognitive skills are more effective than traditional teaching to affects the attitudes of students towards mathematics. The result also showed that there is a statistically significant difference between permanence test scores of experiment and control group students in favour of the experiment group.

The study concluded that metacognitive skills can be developed through education. It also suggested that since the metacognition teaching process can be realized and be more effective in the long-term, teaching methods based on metacognitive strategies should be applied from the first stages of education.

Smith (2013) studied the metacognitive levels of two classes of differential equations students to determine whether the metacognitive scores could predict course performance. The sample consisted of 70 students from spring 2010 (Class 1) and 105 from spring 2011 (Class 2). Students completed Metacognitive Awareness Inventory (MAI). The questions from the MAI were aimed at three components concerning the students' knowledge about their cognition: declarative knowledge, procedural knowledge, and conditional knowledge. The metacognition scores for all students in the study were broken into three sub-intervals indicating low, moderate, and high metacognition for the three categories of metacognition examined. Majority of students were in the moderate subinterval for all categories, over 40% of the students were in the high sub-interval for procedural metacognition (understanding how to use past strategies) and nearly 40% of the students were in the high category for conditional metacognition (knowing which strategy is most effective).

No significant differences were found in the mean and median metacognitive scores for students in Class 1 with differing grades. However, the statistical analyses showed that students with grades B and C in Class 2 differed significantly in their mean metacognitive scores in knowing what was important to learn (declarative knowledge), in knowing what strategy is most effective (conditional knowledge), with B students having higher means. However, the C and D students from Class 2 also had significantly different mean metacognitive scores in all categories of metacognition, with D students having the higher means. The study concluded that it would be difficult to predict a student's course performance, as measured by the grade in the course, by considering their scores in declarative, procedural and conditional metacognition.

Miele (2014) investigated the extent to which the study of number theory might enhance high school students' metacognitive functioning, mathematical curiosity and attitudes towards mathematics. Participants for the study were 40 high school students in the 11th or 12th grade from high schools in New York City, and high school in Dalian, China.

The study used a three-part written questionnaire for data collection, broken in to parts A,B and C. Part A of the questionnaire measured students' current metacognitive abilities as well as their attitudes towards mathematics and their propensities for inventing, discussing, and exploring mathematical problems and concepts. Part B of the questionnaire consisted of eight problems in the field of elementary number theory, which the students worked on independently, collaboratively, and/or using any research materials they chose. Some of the problems in Part B of the questionnaire were purposefully written to encourage the students to monitor their thought processes, engage in critical thinking, and rewrite the problems in a way that was more meaningful to them. Part C of the questionnaire was a revised version of the questions presented in Part A. Part C questions measured the students' metacognitive potential, mathematical attitudes, and levels of inquisitiveness in the context of the elementary number theory problems which they attempted in Part B. The Part A scores and the Part C scores of each student were compared to determine if the scores increased or decreased after the students had worked on the number theory problems.

Findings of the study revealed that most of the student participants had at least reasonable levels of metacognitive functioning at the start of that study, while some of them did not. It also showed that at the start of the study, less than one-tenth of the participating students had high levels of mathematical curiosity and positive mathematics attitudes. Finally, the results of the study indicated that all but one of the 40 participating students described or presented some evidence of metacognitive enhancement, greater mathematical curiosity, and/or improved attitudes towards mathematics after the students had the new experience of working on the Part B number theory problem set.

The studies reviewed were similar to the present study, in the sense that some studied the effects of instructional method on achievement and attitudes of students. However, these were done at primary and secondary school levels. Others used Metacognitive Awareness Inventory (MAI) to assess metacognitive knowledge of the participants and Grade Point Aggregate (GPAs). These studies were survey in nature and do not involve teaching of metacognitive skills. Although other studies examined the impacts of teaching metacognitive strategies on third-grade students' metacognitive and problem-solving skills, they were carried at third and fifth grade of primary schools. Therefore, this study was undertaken to fill the felt gaps by examining the impact of metacognitive strategy on attitude, retention and performance in calculus among colleges of education students in North-central Zone, Nigeria.

2.11 Implications of Reviewed Literature to the Present Study

Akinsola and Animasahun (2007) have investigated the effect of simulation-games environment on students' achievement and attitudes to mathematics in secondary schools in Osun-State, Nigeria. That study looked at the effects of instructional method on achievement and attitude of students. Akinsola and Animasahun's study is similar to the present study, in the sense that they studied the effects of instructional method on achievement and attitudes of students. That was done at the secondary school level, while the current study is looking at such effect at the college of education level. Huang (2011) investigated the relationship between students' attitudes toward calculus and achievement of engineering students from six technology universities in Taiwan. The study used the research instrument of Students' Attitude Towards Calculus (SATC). Although the study is similar to the current study, it made use of only two variables.

The studies of Coutinho (2007), Subocz (2007), Kevin (2009) and Smith (2013 examined the effects of a programme of metacognitive strategy instruction on the performance. They also made use of Metacognitive Awareness Inventory (MAI) to assess metacognitive knowledge of the participants. This is similar to the present study. However, Coutinho (2007) used Grade Point Aggregate (GPAs) which cannot be used to determine achievement in a particular course. While both Subocz (2007) and Kevin (2009) used two variables of attitude and achievement respectively. The above studies used only one group, while the present study used experimental and control groups.

Özsoy and Ataman (2009) examined the effects of metacognitive strategy instruction on mathematical problem solving achievement of fifth grade primary school students. Similarly, Goldberg and Bush (2009) examined the impacts of teaching metacognitive strategies on third-grade students' metacognitive and problem-solving skills. The two studies are carried at third and fifth grade of primary schools.

The studies of Jaafa and Ayub (2010), Tok, Özgan and Dös (2010), Javanmard, Hoshmandja and Ahmadzade (2012) and Rahman, Abdullah, Yasin, Meerah, Halim and Amir (2011) looked at relationship among students' self-efficacy, metacognition and academic performance of undergraduate students in calculus and learning styles respectively.

From the review of previous studies, some research gaps in this field were noticed. First, the research on the relationship between metacognition and calculus performance is still rare in Nigeria, most of the studies were outside Nigeria. Tobias and Everson (2009) hierarchical model metacognition and its modified version, Metacognitive Strategy Teaching (MST) needs proofs from other studies. To address this, the present study undertakes exploration of the impact of metacognitive strategy on attitude, retention and performance in calculus among colleges of education students in North-central Zone, Nigeria.

CHAPTER THREE

RESEARCH METHODOLOGY

3.01 Introduction

The study investigated the impacts of metacognitive strategy on attitude, retention and performance in calculus among colleges of education students in North-central Zone, Nigeria. This chapter incorporates a description of the research design and procedure for data collection. It is presented in the following sub-headings:

3.02 Research Design

3.03 Population of the Study

3.04 Sample and Sampling Procedure

3.05 Instrumentation.

3.06 Validity of Calculus Achievement Test (CAT)

3.07 Pilot Testing of the Instruments

3.08 Reliability of the Instruments

3.09 Method of Scoring the Instruments

3.10 Procedure for Data Collection

3.11 Procedure for Data Analysis

3.02 Research Design

This study adopted Pre-test, Post-test, post-post-test quasi-experimental design. The design was employed as it is one of the most common quasi-experimental designs in educational research (Cohen, Manion & Morrison, 2007). This type of design is according to

Best and Kahn (2009) often used in classroom experiments when experimental and control groups are such naturally assembled groups such as intact classes. The researcher manipulates the independent variable (metacognitive teaching strategy) and observed the effect on the dependent variables (attitude, retention and performance).

Below is the research design:

Group

(E) → 0_1 → X_1 → 0_2 → 0_3

(C) → 0_1 → X_0 → 0_2 → 0_3

Figure 3.1 Research Design Illustrations

(E)= Experimental group

(C)= Control group

X_1= Metacognitive; X_0= Lecture Method

0_1= Pre-test

0_2= Post-test

0_3= Post-post-test

3.03 Population

Population of the study comprised all the N.C.E II mathematics students in North-Central Zone Colleges of Education. Statistics from National Commission for Colleges of Education (NCCE, 2013) showed that there are thirteen state-owned Colleges of Education, three private Colleges of Education and three Federal Colleges of Education. These were shown in the table 3.1 below.

Table 3.1 Population of the Study

College	Year of Establishment	Male	Female	Total
FCE, Kontagora	1978	42	12	54
FCE, Okene	1974	45	27	72
FCE, Pankshin	1974	51	26	77
COE, Minna	1975	71	41	112
COE, Akwanga	1977	67	34	101
COE, Ilorin	1994	58	21	79
COE Technical, Lafiagi	1993	32	15	47
COE, Katsina-Ala	1976	47	18	65
COE, Oro	1981	74	31	105
COE, Ankpa	1981	68	29	97
COE, Gindiri	1980	58	29	87
COE Technical, Kabba	2010	23	05	28
COE, Zuba FCT	1996	43	25	68
CITY COE, Mararaba	2010	18	06	24
Muhyideen COE, Ilorin	2007	22	05	27
COE, Offa	1987	33	14	47
Total		**753**	**338**	**1090**

Source: Colleges of Education, North-central Zone (Departments of Mathematics, 2014)

3.04 Sample and Sampling Procedure

The Multi-stage sampling procedure was adopted. In the first stage, purposive non-probability sampling was used to select all the state colleges of education, as suggested by Teddlie and Yu (2007). These are: Niger State College of Education(COE), Minna; Nassarawa State College of Education, Akwanga; Kwara State College of Education, Ilorin; Kwara State College of Education (Technical), Lafiagi; College of Education Technical, Kabba; College of Education, Oro; College of Education, Ankpa; College of Education, Gindiri; College of Education, Katsina-Ala; and Federal Capital Territory (FCT) College of Education, Zuba. This was due to their homogeneity in terms of staffing, funding and facilities. Similarly, these colleges were similar regarding characteristics such as achievement in calculus and aspects of the teaching-and-learning situation such as time allocated to teaching, method of teaching, teachers' qualifications and experience as well as the college environment. In the second stage a random sample of two Colleges of Education was made using a table of random numbers. In the third stage one college was randomly assigned to experimental group, while the other as control group using flipping of coin as suggested by Bryman (2008).

Table 3.2 Sample Selected for the Study

Group	Male	Female	Total
Experimental	42	23	65
Control	41	29	70
Total	83	52	135

3.05 Instrumentation

The instruments that were used for data collection are as follow: Calculus Pre -Test (CPT), Calculus Achievement Test (CAT), Metacognitive Teaching Strategy (MTS), and Attitude Toward Calculus Inventory (ATCI), respectively.

3.05.1 Calculus Pre- Test (CPT)

Calculus Pre -Test (CPT) was administered at the beginning of the study and used as a pre-test measure of background knowledge of the participants in the domain of calculus. This consisted of two sections. Section A: bio-data of the participants, name of the College, Course Combination and Sex. Section B consisted of five essay questions developed by the researcher covering the contents of differential calculus. Differentiation, using product rule, quotient rule, implicit differentiation, trigonometric differentiation and introductory integration had one question each. Essay questions were adopted in order to assess the subjects' problem solving skill. (See appendix I)

3.05.2 Calculus Achievement Test (CAT)(post-test)

Calculus Achievement Test (CAT) was used to measure students' performance in calculus. This consists of two sections. Section A: bio-data of the participants, name of the College, Course Combination and Sex. Section B consists of eight essay questions developed by the researcher. The questions were broken into four questions covering areas of integration by substitution and four questions covering areas of integration by part. Integration by substitution and integration by part are the second and third topics of integral calculus for NCE II. This was used as post-test and post-post-test. Essay questions were adopted in order to assess the subjects' problem solving skill and to assess the experimental group use of metacognitive knowledge. (See appendix II)

Table 3.3 Table of Specification for a Ten- item Essay Test on Integration (Calculus)

Content Area	Powers	Roots	Objectives Fractions	Trig	Exponential	Total
Integration by substitution	1	1	2	-	-	4
Integration by parts	-	-	1	3	2	6
Total Items	1	1	3	3	2	10

3.05.3 Attitude Towards Calculus Inventory (ATCI)

The Scale of Calculus Attitude, developed by Tapia and Marsh (2004) was adapted with modification to determine participants' attitudes towards calculus. This consists of two sections. Section A: bio-data of the participant, name of the College, Course Combination and Sex. Section B consisted of 30 items written to represent four subscales: self-confidence (8 items), value of calculus (8 items), enjoyment of calculus (8 items), and motivation to learn calculus (6 items). Students were asked to respond to a series of statements using a scale from 1 (Strongly Disagree) to 5 (Strongly Agree). Negatively worded items were reverse scored and items were ranked, with higher rank indicating positive attitudes toward calculus. (See, appendix III).

3.05.4 Metacognitive Teaching Strategy (MTS)

The Metacognitive Teaching Strategy was developed based on Tobias & Everson's (2009) hierarchical model of metacognition. Their model, according to Erskine (2009), differs from other theoretical models because it supports the view that metacognition skills could be developed in an incremental way. This makes it more appropriate for training purposes. In their pyramidal model, knowledge monitoring is viewed as the most basic metacognitive skill, supporting the development of other metacognitive skills. Metacognitive Teaching Strategy (MTS) is a five stage non-linear, clockwise and anti-clockwise. The strategy started with knowledge monitoring, readiness, selecting strategy, problem solving and has reflection in the centre.

Knowledge Monitoring: Knowledge monitoring is the ability of knowing what one knows and knowing what one doesn't know. Monitoring of prior learning is a fundamental or a prerequisite metacognitive process. If a student cannot differentiate accurately between what he or she knows and do not know, he or she can hardly be expected to engage in advanced metacognitive activities such as evaluation of learning realistically, or making plans for effective learning. Learners who can accurately differentiate between what has been learned previously and what they are yet to learn are better able to focus attention on other cognitive resources on the material to be learned. This ability to monitor one's own knowledge is key to metacognitive and self-regulation processes during learning. Indeed, Tobias and Everson (2009), in their proposed hierarchy of metacognitive processes, placed monitoring knowledge as the foundation. Tobias and Everson (2009) said that other metacognitive processes, such as selecting strategies, evaluating learning and planning are dependent upon accurate knowledge monitoring.

At this stage, the facilitator tries to assess the students' entry behaviour by posing some questions or problems on concepts that are related to the concept to be learnt. At the start of a learning activity, the facilitator should ask learners what they know and what they do not know about the task. Further questions include whether they understand the question posed and whether they can link the main concept in the question to other concepts and to prior knowledge. The connections between prior knowledge and new concepts should also be sought during the learning activity since the integration of prior knowledge and new concepts enables learners to understand the unified and interconnected nature of knowledge.

Readiness: At this stage, direct instruction was provided to introduce participants to both the problem-solving heuristic and metacognitive strategies. Problem solving strategies consist of five different strategies of problem solving approaches:

(i) One lets his or her mind wander freely and try to produce as many ideas as possible, by not evaluating them at once. One considers each idea and then begins to analyse, judge and choose the best ones.

(ii) One tries to recall problems successfully solved in the past which are similar to the current problem. One looks for previous situations which shares some aspects, elements or features with the current problem so that one can transfer some ideas from the former ones to the current one.

(iii) One tries to go systematically and look for the sequence of steps or phases which are needed to reach the solution gradually. That is, by trying to break the whole problem into sub- problems, to identify intermediate goals, plan and to order hierarchically the operations to be carried out.

(iv) One tries to visualise the problem; that is, to represent it in one's mind through images. One tries to see the situation with one mind's eye, draws pictures, schemas and graphs.

(v) One tries to combine different aspects of the problem. One tries to associate, perhaps randomly, some elements of the problem so that one can reach any result.

Here, the facilitator introduces the concept to be learned and links it to the previously learnt concept.

Selecting Strategy: Students decide which strategy is useful for a given task, which is dependent on declarative knowledge that is, knowledge of the concepts of a given task and conditional knowledge, that is, an awareness of when and why one strategy may be superior to another or more appropriate to use. The facilitator goes on to pose some problems and urge students to suggest which of the problem solving strategy will be more appropriate and why. The facilitator then goes ahead to solve the problem, however, engaging the students throughout the session.

Problem Solving: Participants are engaged in solving a wide variety of non-routine calculus problems. They were to practice, on their own, the use of various cognitive and metacognitive strategies. They are prompted to maintain conscious and periodic self-checking of their strategies, reflect upon and evaluate them.

During the planning phase; learners were encouraged to ask questions such as;

What am I supposed to learn?

What prior know-ledge will help me with this task?

What should I do first?

What should I look for in this problem?

How much time do I have to complete this?

Where do I direct my thinking?

During the monitoring phase: learners were encouraged to ask:

How am I doing?

Am I on the right track?

How should I proceed?

What information is important to remember?

Should I move in a different direction?

Should I adjust the pace because of the difficulty?

What can I do if I do not understand?

During the evaluation phase: learners were encouraged to ask;

How well did I do?

What did I learn?

Did I get the results I expected?

What could I have done differently?

Can I apply this way of thinking to other problems or situations?

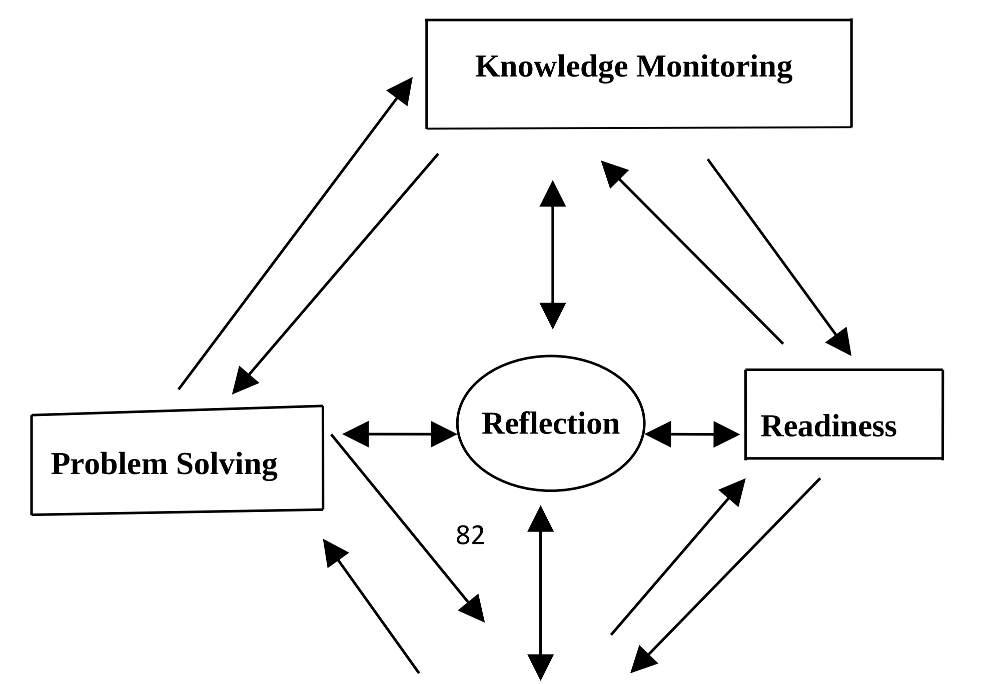
Is there anything I don't understand or any gaps in my knowledge?

Do I need to go through the task again to fill in any gaps in my understanding?

How might I apply this line of thinking to other problems?

Reflection: This is a deliberate cognitive activity where learners connect thoughts, feelings and experiences related to the learning activity in which they were involved. It is an act of looking back in order to process experiences. It involves activities such as setting goals beforehand, monitoring progress as students work, and evaluating the outcome compared to the original goals. Reflection thus forms the important link between processing the new information and integrating it with the existing understanding. It also linked the other four stages.

Figure 3.2: The Metacognitive Strategy Teaching Model (MSTM)



Selecting
Strategy

Source: Tobias & Everson (2009), (modified)

Figure 3.3: Tobias & Everson's hierarchical model of metacognition (2009)

3.06 Validity of Calculus Achievement Test (CAT)

Validity is to establish if an instrument actually provides a measure of what it purports to measure. To do that the drafted sample questions of ten items were distributed to three senior lecturers from mathematics department, Niger State College of education Minna, with master's degree in mathematics for validation of the instrument. They were to look at the test items and their corresponding weights as to the suitability of the items for the purposes of the test, thus, giving the test content validity. The test items were also presented to two senior lecturers with Ph.D in mathematics education and Ph.D in mathematics from Science

Education Department of Federal University of Technology, Minna and Ahmadu Bello University, Zaria respectively, thus giving the test face validity. The necessary changes were done according to their suggestions and remarks. These include the inclusion of questions (3b) $\int \frac{xdx}{2x^2+3}$ and (5a) $\int tanxsec^2$ while question (6) $\int (\sqrt[3]{2x^2+3})x$ dx was modified and question (7(ii)) was removed. (See appendix VII)

3.07 Pilot Testing of the Instruments

Pilot testing is a preliminary test or trial run of instruments carried out before a research design is finalised to assist in defining the research question or to test the feasibility, reliability and validity of the proposed study design. Pilot study can be used to determine the feasibility of conducting a large scale study. Therefore, a pilot testing of the instruments were conducted with college of education, Minna, with 57 students, comprising of 42 males and 15 females. Metacognitive Teaching Strategy (MTS), Attitude Towards Calculus Inventory (ATCI) and Calculus Achievement Test (CAT) were administered. The testing was conducted in order to achieve the following objectives:

(i) to establish the appropriate timing for the instruments,

(ii) to establish the characteristics of the instruments,

(iii) to establish the reliability of the instruments; and

(iv) to simulate the scene.

After the respondents had completed the testing instruments, some of them were invited to provide feedbacks regarding the instruments. For example, they were asked how long it took them to fill in the questionnaire and answer the test questions; and whether they found the items on the questionnaire clear or confusing. After collecting respondents' feedbacks, these include the timing and number of items. The researcher carefully read the suggestions provided by them and made corresponding adjustments. The pilot testing revealed that the researcher had to be friendly and let the participant feel that he was part of them, act as a facilitator and not as a lecturer.

Table 3.4: Results of the Pilot Study

Comparison of the Pre-test and Post-test Mean Scores of Pilot Group

Group	N	Mean	SD	df	t-cal	t-cri	p-value	decision
Pre-test	57	45.69	9.54	112	0.48	1.96	0.72	**
Post-test	57	47.42	9.96					

** Not significant at $p \leq 0.05$

Spearman Rank Correlation for Attitude and Performance of Pilot Group

Variables	N	Mean	Std. De	df	r	p-value	Decision
Attitude	57	50.44	15.34	112	0.788	0.001	*
Performance	57	50.92	15.17				

* Correlation is significant

3.08.1 Reliability of Calculus Achievement Test (CAT)

To ensure the test reliability of Calculus Achievement Test (CAT), the test was administered to a pilot sample of (57) students who were not part of the study sample. After the tests had been administered and scored, a post hoc analysis was performed in order to evaluate the effectiveness of the test. Item difficulty analysis of the individual items on the test was conducted as suggested by Boopathiraj and Chellamani (2013). The difficulty index for Calculus Achievement Test (CAT) was from 0.34 to 0.42 respectively. This is within acceptable level according to Taib and Yusoff (2014). Similarly, the reliability co-efficient of the Calculus Achievement Test (CAT) was determined using the split-half method. The scores on the odd and even items were subjected to Pearson Product Moment Correlation Coefficient (PPMCC) using Statistical Package for Social Sciences (SPSS). The instrument was found to have reliability coefficient of 0.78.

3.08.2 Reliability of Attitude Towards Calculus Inventory (ATCI)

The Attitude Towards Calculus Inventory (ATCI) consisted of 30 items representing four subscales: self-confidence (8 items), value of calculus (8 items), enjoyment of calculus (8 items) and motivation to learn calculus (6 items). The instrument was adapted from Huang (2011). Croasmun (2011) suggested that when using the Likert-type scales, it is essential that a researcher calculates and reports Cronbach's alpha coefficient for internal consistency reliability. The Cronbach -alpha coefficient was computed to determine (ATCI) reliability. The Cronbach -alpha coefficient of the items was found to be 0.72, which according to Gay, Mills, and Airasian (2009) is acceptable.

3.09 Method of Scoring the Instruments

Calculus Pre-test (CPT) and Calculus Achievement Test (CAT) were marked over 50. As for the Attitude Towards Calculus Inventory (ATCI), the participants were asked to respond to a series of statements using a scale from 1(Strongly Disagree) to 5(Strongly Agree). Negatively-worded items were reversed. Since attitude as a construct is qualitative in nature, however, it can be transformed to become a quantitative variable by using the Likert scale. This will enable calculations to be done to establish the relationship between attitude and achievement using Spearman's rank correlation. According to Rebekić, Loncaric, Petrovic & Maric, (2015) Spearman's coefficient assesses how well an arbitrary monotonic function can describe a relationship between two variables, without making any assumptions about the frequency distribution of the variables. It does not require the assumption that the relationship between the variables is linear, nor does it require the variables to be measured on interval scales; it can be used for variables measured at the ordinal level.

3.10 Procedure for Data Collection

Before starting the experiment participants were assessed to ascertain their based knowledge by giving them a pre-test, Calculus Pre- Test (CPT). They were also given Attitude Towards Calculus Inventory (ATCI) with the help of research assistants. At the completion of the experiment they were post- tested with Calculus Achievement Test (CAT) and Attitude Towards Calculus Inventory after 4 weeks of treatment. Two weeks after that, Calculus Achievement Test (CAT) was re-administered as post-post-test to assess their retention ability. Two topics in integration, that is, integration by substitution and integration by part were taught during the experimental period. The content of activities for the two groups was the same but it differed in its structure of instruction.

3. 10.1 Teaching the Experimental Group

Treatment session for the experimental group was conducted by trained research assistant with the aid of lesson plans prepared by the researcher and lesson conducted under the supervision of the researcher. The lesson was presented using the following steps:

Knowledge Monitoring: At this stage, the facilitator tried to assess the students' entry behaviour by posing some questions or problems on concepts that were related to the concept to be learnt.

At the start of a learning activity, the facilitator asked learners what they knew and what they did not know about the task. Further questions included whether they understood the question posed, whether they could link the main concept in the question to other concepts and to their prior knowledge.

Readiness: In this stage, direct instruction was provided to introduce participants to both the problem-solving heuristic and metacognitive strategies. Here the facilitator introduced the concept to be learnt and linked it to the previously learnt concept.

Selecting Strategy: The facilitator went on to pose some problems and urged the students to suggest which of the problem solving strategy will be more appropriate and why. The facilitator also went ahead to solve the problem, however, engaging the students throughout the session.

Problem Solving: Participants were engaged in solving a wide variety of non-routine calculus problems. They were to practice, on their own, the use of various cognitive and metacognitive strategies. They were prompted to maintain conscious and periodic self-checking of their strategies, reflect upon and evaluate them. The facilitator asked participants questions such as: What prior knowledge do you think will help you with this task? What should you do first? What should you look for in this problem? How much time do you have to complete this?

Reflection Phase: Participants were encouraged to:

(1) Reflect on what they had learned and how it could affect their future teaching activities.

(2) Reflect on the degree of familiarity and other qualities of the solution process.

(3) Reflect on the entire solution process.

(4) Identifying critical features in the process.

(5) Evaluate the solution process for adaptability in other situations and different ways of solving it.

(6) Reflect on the mathematical rigour involved, one's confidence in handling the process, and the degree of satisfaction.

Details of the lesson plan is presented in appendix IV

3.10.2 Teaching the Control Group (Lecture method)

The actual treatment session for control group was conducted by research assistant using lesson plans prepared by the researcher. Lecture method was used to teach the group. In each period, the lecturer spent the first few minutes of the period talking about the concept or the topic at hand by writing some formulas that may be used in solving the problem within the concept to be learnt. He then went on to solve some examples based on the concept. Thereafter, the material was then written on the (white) board. The students listened and took notes. Difficult concepts were explained in a more generalized way to the students just some exercises were given for students to practice. Each period lasted for two hours. On the whole, the exercise lasted for six weeks, after which a post-test was administered.

3.11 Procedure for Data Analysis

Seven research questions were answered using descriptive statistics in the form of mean and standard deviation. Hypotheses testing were done using t-test for equality of means of independent sample test at $p \leq 0.05$, level of significance. Spearman's rank coefficient was used to describe the relationship between attitude and performance.

CHAPTER FOUR

DATA PRESENTATION, ANALYSIS AND DISCUSSION

4.1 Introduction

The study investigated the Impact of Metacognitive Strategy on Attitude, Retention and Performance in Calculus among Colleges of Education Students in North-central Zone, Nigeria. The chapter incorporated discussions on data analysis and results. The data collected were analysed using the Statistical Package for Social Sciences (SPSS). The chapter is presented using the following sub-headings:

4.2 Data Presentation

4.3 Data Analysis

4.4 Summary of the Major Findings

4.5 Discussions

4.2 Data Presentation

The study was conducted among N.C.E II mathematics students in North-Central Zone. A sample size of One Hundred and Thirty Five (135) students was used. In this study seven research questions with their corresponding seven hypotheses were formulated, answered and tested. Each of the research questions and hypotheses were restated and answered. The data collected were used to test the stated hypotheses. The results are presented in Tables 4.01- 4.13

4.3 Data Analysis

4.3.1 Post-test for Experimental and Control Groups

Data on Post-test scores of N.C.E II students taught calculus using Metacognitive strategy and those taught using lecture method.

Table 4.01: Results of the Mean and Standard Deviation of Post-test for Experimental and Control Groups

Group	N	Mean	Std. Dev.	Std. Error.	Mean Diff.
Experimental	65	50.92	15.31	1.90	17.23
Control	70	33.69	13.63	1.63	

Result of Table 4.01 showed that the means and standard deviations of the experimental group was 50.92 and 15.31 and that of the control group was 33.69 and 13.63. The mean difference is 17.23. In order to establish if the mean difference is statistically significant, inferential statistics was used to test the null hypothesis.

Hypothesis One:

$\boldsymbol{H0_1}$: There is no significant difference in the performance of N.C.E II students taught calculus using metacognitive strategy and those taught using lecture method.

To test this hypothesis the post-test scores of the subjects in the experimental and control groups were compared using t-test statistics. Table 4.02 showed the result obtained

Table 4.02: Summary of t-test for Post-test of Experimental and Control Groups

Group	N	Mean	SD	df	t-cal	t-cri	p-value	decision
Experimental	65	50.92	15.31	133	6.92	1.96	.001	*
Control	70	33.69	13.63					

* significant at $p \leq 0.05$

From Table 4.02 the p-value obtained was 0.001which is < 0.05, the value of t-calculated of 6.92 which is found to be higher than t- critical of 1.96 further confirms the result. The null hypothesis was thus rejected. This means that there was significant difference between the performance of the experimental with mean of 50.92 and the control groups with mean of 33.69. The result thus showed that metacognitive strategy is more effective in improving students' performance in calculus concepts taught than the lecture method

4.3.2 Post-test on Attitude of Experimental and Control Groups

Data on attitudinal change of N.C.E II students taught calculus using Metacognitive strategy and those taught using lecture method.

Table 4.03: Results of the Mean Ranks and Sum of Ranks of Post-test on Attitude of Experimental and Control Groups

	Group	N	Mean Rank	Sum of Rank
Attitude	Experimental	65	91.09	5921.00
	Control	70	46.56	3259.00
	Total	135		

Table 4.03 revealed that the mean and sum of ranks of attitude of the experimental group was 91.09 and 5921.00, while that of the control group was 46.56 and 3259.00. In order to establish if the differences in the mean ranks is statistically significant, inferential statistics of Mann-Whitney U test was used to test the null hypothesis.

Hypothesis Two:

$H0_2$: There is no significant difference between the attitude of N.C.E II students taught calculus using metacognitive strategies and those taught using lecture method.

To test this hypothesis the attitude ranked scores of both experimental and control groups were subjected to Mann-Whitney U test. The result is shown in Table 4.04.

Table 4.04: Mann-Whitney U test on Attitude Change of N.C.E II Students Based on Methods of Teaching used

Test Statistics

Group	**N**	Mean Rank	Sum of Ranks	U	Z	P
Experimental	65	91.09	5921.00	774.000	-6.614	.000
Control	70	46.56	3259.00			

An examination of the findings in Table 4.04 showed that the results of the Mann Whitney U test applied to the attitude scores of students in the experimental and control groups revealed a statistically significant difference at the level of $p<.05$ ($Z=-6.614$; $p=.000<.05$). The rank average of the attitude scores of the experimental group students was 91.09, while the students in the control group had attitude score rank average of 46.56. The result thus showed that metacognitive strategy is more effective in improving students' attitude towards calculus than the lecture method.

4.3.3 Post-test on Retention of Experimental and Control Groups

Data on the retention ability of N.C.E II students taught calculus using metacognitive strategy and those taught using lecture method.

Table 4.05: Results of the Mean and Standard Deviation on Retention of Experimental and Control Groups

Group	N	Mean	Std. Deviation	Std. Error	Mean Diff
Experimental	65	49.91	15.38	1.91	19.42
Control	70	30.49	14.36	1.72	

Table 4.05 revealed that the means and standard deviations of retention scores of the experimental group was 49.19 and 15.38, while that of the control group was 30.49 and 14.36 with mean difference of 19.42. In order to establish if the difference in retention is statistically significant, inferential statistics was used to test the null hypothesis.

Hypothesis Three:

$\boldsymbol{H0_3}$**:** There is no significant difference in the retention ability of N.C.E II students taught calculus using metacognitive strategy and those taught using lecture method.

To test the null hypothesis independent sample t-test statistics was carried out to compare the effect of the teaching methods on students' retention of calculus concept. The result is shown in Table 4.06

Table 4.06: Summary of t-test for Retention of Experimental and Control Groups

Group	N	Mean	SD	df	t-cal	t-cri	p-value	decision
Experimental	65	49.91	15.38	133	7.59	1.96	.001	*
Control	70	30.49	14.36					

* significant at $p \leq 0.05$

From Table 4.06 the p-value obtained was 0.001which is < 0.05. The value of t-calculated of 7.59 which was found to be higher than t- critical of 1.96 further confirms the result. The null hypothesis was thus rejected. This means that there was significant difference between the retention of the experimental and the control groups. The result thus shows that metacognitive strategy with the mean 49.91 is more effective in improving students' retention in calculus concepts taught than the lecture method which has mean 30.49.

4.3.4: Relationship between Attitude and Performance of N.C.E II Students Taught Using Metacognitive Strategy

Spearman's rank correlation coefficient which is a nonparametric (distribution-free) rank statistic was used to measure the strength of the association between attitude and performance. It is a measure of a monotone association that is used when the distribution of data makes Pearson's correlation coefficient undesirable or misleading. It does not require the assumption that the relationship between the variables is linear, nor does it require the variables to be measured on interval scales; it can be used for variables measured at the ordinal level.

Hypothesis Four:

$\boldsymbol{H0_4}$**:** There is no significant relationship between attitude and performance of N.C.E II students taught calculus using metacognitive strategies.

To test the null hypothesis Spearman's rank correlation was conducted to measure the strength of association between students' attitude toward calculus and their performance in calculus. This is shown in table 4.07.

Table 4.07: Spearman's Rank Correlation for Attitude and Performance of N.C.E II Students Taught Using Metacognitive Strategy

Correlation			Attitude	Performance
Spearman's rho	Attitude	Correlation Coefficient	1.000	.889*
		Sig. (2-tailed)	.	.000
		N	65	65
	Performance	Correlation Coefficient	.889**	1.000
		Sig. (2-tailed)	.000	.
		N	65	65

*Correlation is significant

Table 4.07 revealed the correlation between attitude and performance of the experimental group in calculus. The Spearman's rank correlation coefficient revealed that the relationship between attitude and performance is high, positive and significant ρ =0.889. This means that students who are taught using metacognitive strategy have more favourable attitudes toward calculus tend to perform better.

4.3.5 Test for Gender Differences in the Mean Scores of Experimental group

Data of performance by gender of N.C.E II students taught calculus using metacognitive strategy.

Table 4.08: Results of the Mean and Standard Deviation on Gender Performance of Experimental Group

Gender	N	Mean	Std. Deviation	Std. Error	Mean Diff
Male	42	51.21	16.13	2.49	0.82
Female	23	50.39	14.04	2.93	

Table 4.08 showed the means and standard deviations of performance by gender of experimental group. It revealed that male subjects have the mean of 51.21 and standard deviation of 16.13, while female subjects have the mean of 50.39 and standard deviation of

14.04 with the mean difference of 0.82. However, to establish if the differences is statistically significant, inferential statistics was used to test the null hypothesis.

Hypothesis Five:

$H0_5$ There is no significant difference on gender performance of N.C.E II students taught calculus using metacognitive strategy.

To test the hypothesis the post-test scores of male and female N.C.E II students in the experimental group were compared using t-test statistics. The results obtained from the analysis of the relevant data are summarized in Table 4.09

Table 4.09: Results of t-test on Performance Scores of Male and Female N.C.E II Students in Experimental Group

Group	Gender	N	Mean	SD	df	t-cal	t-cri	p-value	decision
Exp	M	42	51.21	16.13	63	0.21	1.96	0.84	**
	F	23	50.39	14.04					

** Not significant at $p > 0.05$

From table 4.09, the p-value obtained was 0.84 which is > 0.05. The value of t-calculated of 0.21 which is found to be less than t- critical of 1.96 further confirms the result. The null hypothesis was thus accepted. This means that there is no significant difference between the performance of male and female N.C.E II students of the experimental group. Therefore, the performance scores of male and female N.C.E II students exposed to metacognitive strategy do not differed significantly, mean difference = 0.82.

4.3.6 Test for Gender difference in Attitude of N.C.E II Students in the Experimental Group

Data on Gender difference in attitude of N.C.E II students taught calculus using metacognitive strategy.

Table 4.10: Results of the Mean Rank and Sum of Ranks on Gender in Attitude of Experimental Group

	Gender	N	Mean Rank	Sum of Rank
Attitude	Male	42	32.68	1372.50
	Female	23	33.58	772.50
	Total	65		

Table 4.10 revealed that the mean and sum of ranks of attitude of the male was 32.68 and 1372.50, while that of the female was 33.59 and 772.50. In order to establish if the differences in the mean ranks is statistically significant, inferential statistics of Mann-Whitney U test was used to test the null hypothesis.

Hypothesis Six:

$\boldsymbol{H0_6}$: There is no significant gender difference in attitude of N.C.E II students taught calculus using metacognitive strategies.

To test the hypothesis the post-test attitude scores of male and female N.C.E II students in the experimental group were compared using t-test statistics. The results obtained from the analysis of the relevant data are summarized in Table 4.11

Table 4.11: Mann-Whitney U test on Attitude of Male and Female N.C.E II Students in Experimental Group

Test Statistics

Gender	**N**	Mean Rank	Sum of Ranks	U	Z	P
Male	42	32.68	1372.50	469.500	-0.185	0.853
Female	23	33.59	772.50			

An examination of the findings in Table 4.11 revealed the results of Mann Whitney U test for the male and female attitude scores of the students in the experimental group did not show any statistical difference (Z=-.185; p=.853>.05). The rank average of the male attitude scores was 32.68, while the female attitude ranks average of 33.59. The close rank averages of the male and female attitude scores of the students in the experimental group indicates that the attitude of male and female subjects exposed to metacognitive strategy do not differed significantly.

4.3.7 Test for Gender difference in Retention of N.C.E II Students in the Experimental Group

Data on gender difference in retention ability of N.C.E II students taught calculus using metacognitive strategy.

Table 4.12: Results of the Means and Standard Deviations on Gender in Retention of Experimental Group

Gender	N	Mean	Std. Deviation	Std. Error	Mean Diff
Male	42	49.38	15.51	2.39	1.49
Female	23	50.87	15.43	3.22	

The result of Table 4.12 showed that the male obtained a mean score of 49.38 and standard deviation of 15.51 in the retention test. Similarly, the female obtained a mean score of 50.87

and a standard deviation of 15.43, with mean difference of 1.49. In order to establish if the difference is statistically significant, inferential statistics was used to test the null hypothesis.

Hypothesis Seven:

$H0_7$: There is no significant gender difference in retention ability of N.C.E II students taught calculus using metacognitive strategy.

To test the null hypothesis the post-test and post-post-test mean scores of the male and female N.C.E II students in the experimental group were subjected to t-test statistical analysis. The result was shown in Table 4.13

Table 4.13: Results of t-test on the Retention Ability using post-post-test Mean Scores of Male and Female in Experimental Group

Group	Gender	N	Mean	SD	df	t-cal	t-cri	p-value	decision
Exp	M	42	49.38	15.51	63	-0.37	1.96	0.71	**
	F	23	50.87	15.43					

** Not significant at $p > 0.05$

Table 4.13 showed the result of the t-test analysis of the post-post-test mean scores of males and females in the experimental group. The p-value obtained was 0.71 which is> 0.05. The value t-calculated of -0.37 was found to be lower than the t-critical value of 1.96, showed that there was no significant difference. This indicates that the retention level of the subjects when taught calculus using metacognitive strategy is not affected by gender. Thus, the null hypothesis was retained.

4.4 Summary of Major Findings

The major findings of the study are presented as follows:

1. There was a significant difference between the post-test mean of the experimental and control groups in favour of the experimental group.
2. There was a significant difference in attitude between the experimental and control groups. However, there was no significant difference in attitude of the subjects in the control groups.
3. There was a significant difference in the retention ability between the experimental and the control groups. However, there was no significant difference in retention ability of the subjects in the control groups.
4. Strong and positive relationship exist between positive attitude and performance. Subjects who have more favourable attitudes toward calculus perform better on calculus concept than those who have less favourable attitudes.
5. There was no significant difference in the mean performance scores of male and female subjects in the experimental group. The results showed that metacognitive strategy is gender friendly.
6. The study revealed that there was no significant difference in attitude of subjects in the experimental group. Thus, metacognitive strategy enhance attitude of male and female subjects in similar way.
7. Finally, the study revealed that there was no significant difference in the retention ability of male and female subjects in the experimental group. Based on the finding

male and female subjects retain equivalent concepts when exposed to metacognitive strategy.

4.5 Discussions

The discussions of the findings of the present study are here been presented in relation to both research questions and hypotheses. The result from statistical analysis relating to hypothesis one as shown in Table 4.02, revealed that the experimental group that was exposed to metacognitive strategy perform better than the control group that was exposed to lecture method.

The finding of this study agreed with that of study Özsoya and Ataman (2009), who examined the effect of metacognitive strategy instruction in mathematical problem solving and students' achievement, the study tried to find out whether metacognitive strategy instruction will have an impact on students' mathematical problem solving achievement and metacognitive knowledge and skills of the students. They concluded that there was an increase in problem solving skills of the students who have been exposed to the instruction of metacognitive strategy. Similarly, Moga (2012), investigated the effects of metacognitive training on students' mathematics performance, showed that students improved their metacognitive skills through metacognitive training. Further studies of Ozsoy and Ataman (2009); Jaafar and Ayub (2010) on learner's metacognition and achievement in mathematics have also established a correlation between learner's metacognition and mathematics achievement. In support of this study is the finding of Goldberg and Bush (2009), who examined the impact of teaching metacognitive strategies on third-grade students' metacognitive and problem-solving skills. Goldberg and Bush (2009) concluded that instruction in metacognitive skills increased the metacognitive skills used of students and thereby improved their performance in mathematical problem solving. In particular, students learned to monitor their thinking more often during problem-solving episodes as a result of the instruction.

The result of Table 4.04 showed that there was significant difference in the mean attitude scores between students exposed to metacognitive strategy and those exposed to lecture method. That is, N.C.E II Students exposed to metacognitive strategy showed more positive attitude toward calculus after exposure to metacognitive strategy. In support of this finding are the studies of (Subocz,2007; and Sırmacı and Tuncer, 2013). Subocz (2007), examined the effects of metacognitive strategy instruction on performance and attitude of community college students enrolled in pre-algebra courses. The study concluded that attitudes toward mathematics are correlated with metacognitive awareness and it improved students' performance and attitude towards mathematics. Similarly, Sırmacı and Tuncer (2013) investigated the effects of metacognition strategies applied in 7th grade mathematics course of Permutation and Probability on students' achievement, metacognition skills, attitudes and permanence.

The result indicated that teaching with metacognition strategies is more effective than traditional teaching to increase the achievement of students in Permutation and Probability. So also, metacognitive skills are more effective than traditional teaching to effects the attitudes of students towards mathematics. In a related work by Huang (2011) who investigated the relationship between students' attitudes toward calculus and achievement of engineering students in Taiwan. The results revealed that a high proportion of engineering students have negative and moderately positive attitudes toward calculus. This according to the researcher could possibly be due to the way calculus was taught in Taiwan, using the traditional approach.

Table 4.06 result indicates that there was a significant difference in the student retention ability between the experimental and the control group. This is, to say that subjects exposed to metacognitive strategy retained learnt concepts more than their counterpart who are exposed to lecture method. That is, the participants recalled or remembered pieces of knowledge, processes, or skills that were learned earlier in time. In concurrent with this finding is the finding of Narli (2011); Chianson, Kurumeh, and Obida (2011) and Bergstresser (2013). Narli (2011) investigates the long-term effects of instructing Cantor set theory using constructivist learning approach on student knowledge retention. Analyses of the data revealed that the students in the constructivist learning environment showed better retention of almost all of the concepts related to Cantor set theory than the students in the traditional class.

In their study, Chianson, Kurumeh, and Obida (2011) investigated the effect of cooperative learning method compared with the conventional learning method in order to find out the retention level of students' in circle geometry. Their findings showed that students who were subjected to the cooperative learning strategy were able to retain the concepts of circle geometry more than those students who were taught using the conventional learning approach. It should be noted that cooperative learning strategy is based on the same philosophical paradigm as metacognitive strategy, that is, constructivism. Similarly, Bergstresser (2013) assessed the impact of a student's motivation and beliefs about calculus and their ability to retain mathematical skills after having practiced and developed metacognitive skills. The results of the study showed that there is a positive correlation between learning metacognitive skills and retaining content.

The findings of the study are in agreement with the constructivist theory which emphasis that instruction must be on the creation of meaning and understanding while encountering new information or new contexts. Similarly, it also in line with the Social Cognitive Theory (SCT) which holds that learners' behaviour is often predicted by what they believe they are capable of rather than the realization of their capabilities. That is to say learners who believe that they are capable of completing or excelling in a required task are more apt to engage themselves in the task than students who believe their skills are inadequate.

Table 4.07 reveals that there is correlation between attitude and performance of the experimental group. The Spearman Ranks correlation coefficients revealed that the relationship between attitude and performance is high, positive and significant. This finding is in line with the findings of Akinsola and Animasahun (2007) and Huang (2010). Their results indicated significant differences in attitudes among different groups of mathematics students. Also the study showed that attitude is directly linked with students' performance in mathematics. Akinsola and Animasahun (2007) used simulations-games method of instruction which are experiential exercises that transport learners to another world. There they apply their knowledge, skills, and strategies in the execution of their assigned roles.

Table 4.09 results revealed that there was no significant difference in the mean performance scores of male and female subjects in the experimental group. This indicates that metacognitive strategy is gender friendly. This finding is in agreement with the finding of Adamu (2014) who reported no significant gender difference in Calculus achievement of engineering students. Similarly, the studies of Etukudo and Utin (2006); Farayola (2011); Onyishi and Agwagah (2011) laid support to this finding. Their studies showed that there was no significant gender difference in mathematics achievement. However, the finding is in disagreement with the findings of Arslan, Çanlı and Sabo (2012) which showed that female students have more positive attitudes than male students toward Mathematics and had higher mathematics grades. It also disagree with the findings of Mendick (2005); McGraw, Lubienski and Strutchens (2006) that showed that males tend to perform better than females in mathematics.

In a classroom situation where girls are often marginalized, given subordinate status and competitive mathematics class boys are likely to perform better. Similarly in mathematics where the emphases are on rote learning, hard work and perseverance girls are likely to perform better. However boys and girls preferred a mathematics environment that enabled them to work at their own pace, think and develop their own ideas as well as their understanding (Ajai & Imoko, 2015).

Table 4.11 results shows that there was no significant difference in the attitude of male and female subjects in the experimental group. This shows that the subjects exposed to metacognitive strategy exhibit similar attitude regardless of their gender. Metacognitive strategy-based instructional programme showed that performance in calculus is a function of orientation rather than gender.

However, the study of (Ayub, Lian & Mukti, 2005) that assessed attitudes toward calculus of 62 diploma students of University Putra Malaysia showed that female participants had more positive perceptions with more favourable reactions toward Calculus when compared with the males. It should be noted that (Ayub, Lian & Mukti, 2005) adopted a survey technique in collecting their data and no instruction was involved. Similarly, (Huang, 2011) study showed that female students have more positive attitudes toward calculus. Specifically, females perceive calculus as more important in their everyday lives compared to males. Female students also put forth more effort, such as working more calculus exercises than those given by lecturers throughout the calculus course.

Table 4.13 results revealed that there was no significant difference in the retention ability of the subjects in the experimental group. That is the male and the female subjects that are exposed to metacognitive strategy retained the learnt concept in proportional manner. This conform to the findings of (Chianson, Kurumeh,and Obida, 2011; and Nejem and Muhanna,2014) which showed that when participants become both visually and physically engaged in a collaborative learning environment, both male and female retain learnt concept equally. The reason for the equal performance of male and female students in the metacognitive teaching strategy may not be unconnected with the fact that both see themselves as equals and capable of actively collaborating in classroom activities.

CHAPER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1 Introduction

The study investigated the Impact of Metacognitive Strategy on Attitude, Retention and Performance in Calculus among Colleges of Education Students in North-central Zone, Nigeria. The chapter is presented using the following sub headings:

5.2 Summary

5.3 Major Findings

5.9 Conclusions

5.10 Recommendations

5.11 Contribution to knowledge

5.12 Limitation of the study

5.13 Suggestions for further study

5.2 Summary

The study adopted Pre-test, Post-test, post-post-test quasi-experimental design. Intact classes were used to avoid disruption of normal classes. The researcher manipulated the independent variable (metacognitive teaching strategy) and observed the effect on the dependent variables (performance, attitude and retention). Multi-stage sampling procedure was adopted. In the first stage, purposive non-probability sampling was used to select all the state colleges of education. This was due to their homogeneity in terms of staffing, funding and facilities. Similarly, these colleges were similar in terms of characteristics such as achievement in calculus, and aspects of the teaching-and-learning situation such as time allocated to teaching, method of teaching, teachers' qualifications and experience, as well as the college environment. In the second stage a random sample of two Colleges of Education was made using a table of random numbers. In the third stage one college was randomly assign as experimental group, while the other as control group using flipping of coin.

The total sample of One Hundred and Thirty Five (135) N.C.E II students, consisting of Sixty Five (65) students in the experimental group and Seventy (70) students in the control group were used. Four instruments were used for data collection, these include: Calculus Pre- Test (CPT), Calculus Achievement Test (CAT), Metacognitive Teaching Strategy (MTS), and Attitude Toward Calculus Inventory (ATCI).

At the beginning of the experiment, the participants were pre-tested using Calculus Pre- Test (CPT) and Attitude Towards Calculus Inventory (ATCI). At the completion of the

experiment the participants were post- tested using Calculus Achievement Test (CAT) and Attitude Towards Calculus Inventory (ATCI). Two weeks after Calculus Achievement Test (CAT) was re-administered as post-post-test to assess the participants' retention ability. Two topics in integration, that is, integration by substitution and integration by part were taught during the experimental period.

The content of activities for the two groups was the same but differed in its structure of instruction. Treatment session for the experimental group was conducted by trained research assistant with the aid of lesson plans prepared by the researcher based on Metacognitive Teaching Strategy (MTS). Similarly, actual treatment session for control group was conducted by research assistant using lesson plans prepared by the researcher. Lecture method was used to teach the group.

Statistics of mean, standard deviation, Spearman's rank correlation coefficient, Mann-Whitney U test and t-test were used to answer research questions and test the null hypotheses at 0.05 levels of significance for accepting or rejecting the hypothesis.

5.3 Major Findings

The following are the summary of the major findings of the study:

(i) The result of the study showed that Metacognitive Teaching Strategy does have positive effects on the performance of N.C.E II students in calculus. The students used metacognitive strategy and skills in their problem-solving activities after the training.

(ii) The data suggest that the Metacognitive Teaching Strategy instruction has a positive influence on attitude towards calculus held by N.C.E II students. The significant gains of the attitude towards calculus scales suggest that the increased use of metacognitive strategy in calculus classes can be fruitful in terms of the improvement of N.C.E II students' attitude towards calculus.

(iii) This study has also indicated that relationship between metacognitive instruction and N.C.E II students' attitude towards calculus is positive and significant. In addition, this approach has not effectiveness on students' performance based on their gender.

(iv) The Spearman's rank correlation coefficient conducted on relationship between attitude and academic performance revealed that there existed strong and positive relationship between attitude and academic performance.

(v) Male and female N.C.E II students exposure to metacognitive strategy exhibit similar attitude towards calculus.

(vi) Finally male and female N.C.E II students exposure to metacognitive strategy showed similar retention ability.

5.4 Conclusions

Based on the findings from the study, the following conclusions are drawn:

1. That due to the fact that students exposure to metacognitive strategy scored higher in the calculus achievement tests, a conclusion can be made that the N.C.E II students' performance can be enhanced by receiving explicit instruction in metacognitive strategy.

2. Having metacognitive knowledge for selecting and using relevant strategies means that, N.C.E II students are not only able to think but are also able to consciously decide about the process of learning. Therefore, the explicit instruction of metacognitive strategy seems to have contributed to the students' ability in learning of the calculus concepts.

3. The results of statistical analysis indicated that the experimental group that received the explicit instruction, outperformed the control group, and employed monitoring and assessment strategies more frequently after the training sessions.

4. From this study, it is evident that the students taught using metacognitive strategy develop better learning skills compared to those who were not taught using metacognitive strategy. It can be noted that the students in the experimental group recorded better achievements in calculus concepts taught due to the strategic approaches that they used.

5. The result of this study also shows that gender has no significant influence on performance in calculus of N.C.E II students based on their use metacognitive skills. The results revealed that male and female N.C.E II students are similar in achievement scores.

6. This study has also provided empirical evidence there is relationship between learner's metacognitive skill and attitude towards calculus.

7. This study shows that teaching using metacognitive strategy is effective in enhancing N.C.E II students' retention ability.

8. Finally, the results of this short-term instruction of metacognitive strategy showed that metacognitive skills are teachable and learnable. Thus, lecturers can learn such skills themselves and teach them to their students. This way they can improve their students' achievement.

5.5 Recommendations

According to the backgrounds and the findings of the current study and analysis of related studies of which most of them verify the role of metacognitive strategy training in the academic progress, the following recommendation are made:

1. Explicit attention to the enhancement of metacognitive behaviour in mathematics learning should be given in the pre-service and in-service training of mathematics teachers. 2. There should be specific programs, similar to the one proposed by this study, that provide direct instruction and training on various cognitive and metacognitive aspects of mathematical behaviour. Such instruction should be also carried over into all areas of mathematics teaching at colleges of education.

3. Mathematics educators need to be aware of and attentive to students' metacognition. They need to appreciate metacognitive activity and develop ways to foster it within all students.

4. Lecturers should endeavour to enhance the following learner-related aspects: understanding of their strengths and weaknesses in calculus; knowledge of what kind of information is the most important to learn calculus; ability to remember calculus facts and principles; ability to control how well they learn calculus; ability to judge how well they understand different aspects of mathematics, and their attitudes towards calculus.

5. Other studies can make used of triangulated data collection techniques where data are collected using: pretest, posttest, students' written feedback, questionnaire, and interview.

5.6 Contribution to Knowledge

The study has contributed to knowledge in the following ways:

1. This study contributes to the research on metacognitive strategies by providing an applicable teaching model for teaching calculus concepts. The teaching model proved to be effective in improving the students' use of metacognitive strategies and provided empirical support for other research findings in that area.

2. The current study provides an initial step towards assessing the impact of metacognitive teaching strategy in N.C.E. mathematics programme in the Zone. This will enable other researchers to replicate the study in other Zones, thus further ascertaining the effectiveness of the instructional process.

3. This study also lead support to the claim that gender differences gap in mathematics education can be bridged through proper instructional method.

4. The results of this study can provide evidence about the effectiveness of teaching metacognitive skills.

5. The modified instruments used in this study will be handy for future researchers.

5.7 Limitation of the Study

The study investigated the impact of metacognitive strategy on students' attitude retention and performance, of N.C.E II students in calculus in North-Central Zone, Nigeria. While there are a number of strengths resulting from this study, limitations should be noted with consideration for future directions. When conducting quasi-experiment there are numbers of potentially influential factors that are outside the control of the experimenter, and while efforts were made to ameliorate these confounds, the following limitations are acknowledged:

1. The findings of this study only describe what was observed during a six week period within the course environment.
2. Limited generalizability because of the small sample size and limited amount of time during which the study was conducted. Since findings have not been replicated in many

cases, the findings are not as strong as they could be.

3. There is the possibility of interactions between the metacognitive intervention and students' initial aptitudes and abilities that is not accounted for in this study.
4. In this study the external validity was controlled by the researcher, but because the instructors of the experimental and control group were different, the internal validity cannot be guaranteed.
5. The study used Metacognitive Teaching Strategy (MTS), hence, findings are limited to those variables within the used procedures.

5.8 Suggestions for Further Study

1. Implementing longitudinal studies in order to compare the growth of metacognitive skill acquisition in the students' different academic levels.
2. Analysis of different models of metacognitive skill training should be carried out.
3. Future studies could explore differences or similarities in larger samples of like students and their awareness of and abilities to explain metacognitive processes.
4. Future studies could explore the relationship between students' metacognitive skills and their cognitive styles.

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Appendix I

Calculus Pre-test (CPT)

Instruction: answer all the questions **Time: 1 Hour**

1. Find y' and y'' given that $x^3y + xy^3 = 2$ at $x = 1, y = 1$
2. Differentiate (a) y= $\sqrt[\frac{3}{2}]{3x^2 - 5x + 4}$ (b) y= $(x^2 + 4)(2x^3 - 1)$
3. Find the $\frac{dy}{dx}$ given that $y = \left(\frac{x^3-1}{2x^3+1}\right)^4$

4. Differentiate (a) $y = tan^2(3x - 2)$ (b) y= $ln^2(x + 3)$

5. Integrate (a) $\int(x^3 - 5x^2 + 7x - 15)dx$ (b) $\int(\sqrt[3]{x} - \frac{1}{\sqrt[4]{x}})dx$

Solution to Calculus Pre-test (CPT)

1. Find y' and y'' given that $x^3y + xy^3 = 2$ at $x = 1, y = 1$

$\rightarrow 3x^2y + x^3\, y' + y^3 3xy^2\, y' = 0$ ……………………………….1 $^1/_2$marks

$\rightarrow y'\,(x^3 + 3xy^2) = -(y^3 + 3x^2y)$ ………………………….…1 $^1/_2$marks

$\rightarrow y' = \frac{-(y^3+3x^2y)}{(x^3+3xy^2)}$ ……………………………………………………..1mark

$at\ x = 1, y = 1;\ y' = \frac{-(1+3)}{(1+3)} = -1$ ……………………………………1mark

Now differentiate again

$\rightarrow 6xy + 3x^2y' + 3x^2y' + x^3y'' + 3y^2y' + 6xy(y')^2 + 3xy^2y'' = 0$ …….1 $^1/_2$marks

$\rightarrow 6xy + 6x^2y' + x^3y'' + 6y^2y' + 6xy(y')^2 + 3xy^2y'' = 0$ ……………..1 $^1/_2$marks

$at\ x = 1, y = 1;\ y' = -1$

$\rightarrow 6 - 6 + y'' - 6 + 6 + 3y'' = 0$ ……………………………………………1mark

$\rightarrow 4y'' = 0\ ;\ \therefore\ y'' = 0$………………………………………………………1mark

$(10marks)$

2.Differentiate

$(a)\ (3x^2 - 5x + 4)^{3/2}$

$dy/_{dx} = {}^3/_2\ (3x^2 - 5x + 4)^{3/2 - 1} \times (6x - 5)$ ……………………2marks

$= {}^3/_2\ (3x^2 - 5x + 4)^{1/2} \times (6x - 5)$ ……………………………..2marks

(b) $y = (x^2 + 4)(2x^3 - 1)$

$Let\ u = x^2 + 4, \frac{du}{dx} = 2x$ …………………………………………………1mark

$v = 2x^3 - 1, \frac{dv}{dx} = 6x^2$ ……………………………………………………1mark

Now

$\frac{dy}{dx} = (2x^3 - 1).2x + (x^2 + 4)\ 6x^2$ ………………………………………..2marks

$= 4x^4 - 2x + 6x^4 + 24x^2$ …………………………………………………1mark

$= 10x^4 + 24x^2 - 2x\ or\ 2x(5x^3 + 12x - 1)$ ……………………………1mark

$(10marks)$

3. $y = \left(\frac{x^3-1}{2x^3+1}\right)^4$

$let\ u = (x^3 - 1)^4; du = 4(x^3 - 1)^3.3x^2$ …………………………1mark

$v = (2x^3 + 1)^4; dv = 4(2x^3 + 1)^3.6x^2$ …………………………1mark

Now

$\frac{dy}{dx} = \frac{(2x^3+1)^4.12x^2(x^3-1)^3 - (x^3-1)^4.24x^2(2x^3+1)^3}{(2x^3+1)^8}$…………………………2marks

$= \frac{12x^2(2x^3+1)^3(x^3-1)^3[2x^3+1-2(x^3-1)]}{(2x^3+1)^8}$ ………………………………….2marks

$= \frac{12x^2(2x^3+1)^3(x^3-1)^3[2x^3+1-2x^3+2]}{(2x^3+1)^8}$…………………………………….2marks

$= \frac{12x^2(2x^3+1)^3(x^3-1)^3[3]}{(2x^3+1)^8}$…………………………………………………...1mark

$= \frac{36x^2(x^3-1)^3}{(2x^3+1)^5}$ ……………………………………………………….....1mark

$(10 marks)$

4. (a) $y = tan^2(3x-2)$

$let\ u = 3x-2; du = 3$ ……………………………………..$^1/_2\ mark$

Now $y = tan^2 u = (\tan u)^2$ …………………………………$^1/_2\ mark$

$\frac{dy}{du} = 2\tan u. sec^2 u$ ………………………………………2marks

Then $\frac{dy}{dx} = 2\tan(3x-2). sec^2(3x-2). 3$ …………………2marks

= $6\tan(3x-2). sec^2(3x-2)$ ……………………1mark

(b) $y = ln^2(x+3)$

$\frac{dy}{dx} = 2\ln(x+3). \frac{d}{dx}\ln(x+3)$ ………………………………$1\ ^1/_2\ marks$

= $2\ln(x+3)\ \frac{1}{x+3}\frac{d}{dx}(x+3)$ ………………………………$1\ ^1/_2\ marks$

$= \frac{2\ln(x+3)}{x+3}$ ……………………………………………………1mark

$(10 marks)$

5(a) $\int(x^3 - 5x^2 + 7x - 15)dx$

= $\int x^3\, dx - 5\int x^2 dx + 7\int x dx - 15$ …………………………2marks

$=\frac{x^4}{4} - \frac{5}{3}x^3 + \frac{7}{2}x^2 - 15x + c$ …………………………………..2marks

(b) $\int(\sqrt[3]{x} - \frac{1}{\sqrt[4]{x}})dx$

$= \int x^{\frac{1}{3}}\ dx - \int x^{-\frac{1}{4}}\ dx$ ……………………………………………2marks

$=\frac{x^{4/3}}{4/3} - \frac{x^{3/4}}{3/4}$+c ………………………………………………..2marks

$=\frac{3}{4}x^{4/3} - \frac{4}{3}x^{3/4} + c$ ………………………………………………..2marks

$(10 marks)$

Appendix II

Calculus Achievement Test (CAT) Post-test

Instruction: answer all the questions Time: 1½ Hours

1. $\int(2x-7)^3\,dx$ (5marks)

2. $\int(\sqrt[3]{2x^2+3})x$ dx (5marks)

3. $\int\frac{4x-8}{x^2-4x+5}$ dx (b) $\int\frac{xdx}{2x^2+3}$ (5marks)

4. $\int xsecx^2dx$ (5marks)

5(a) $\int tanxsec^2 x$ (b)$\int sin^2 xcosxdx$ (5marks)

6 $\int\frac{x}{\sqrt{x^2-4}}$ dx (5marks)

7.$\int x^2\,e^{3x}$dx (10marks)

8. $\int x^2\,tan^{-1}x$dx (10marks)

Solution to Calculus Achievement Test (Post-test)

Solution

1. $\int(2x-7)^3\,dx$

$let\ u=(2x-7);\frac{du}{dx}=2;dx=\frac{1}{2}du$ ……………………………1mark

Now

$\int(2x-7)^3\,dx=\frac{1}{2}\int u^3\,du=\frac{1}{2}\frac{u^4}{4}+c$ ……………………………….2marks

$=\frac{1}{8}(2x-7)^4+c$ ……………………………………………………2marks

$(5marks)$

2. $\int(\sqrt[3]{2x^2+3})xdx$

$let\ u=2x^2+3;then\ \frac{du}{dx}=4x\ and\ dx=\frac{du}{4x}$ ………………………..1mark

Now the integral becomes

$\int u^{1/3}\frac{du}{4x}.x = \frac{1}{4}\int u^{1/3}du = \frac{1}{4}\frac{u^{4/3}}{\frac{4}{3}} + c$ ……………………………………2marks

$= \frac{3}{16}(2x^2+3)^{\frac{4}{3}} + c$ ……………………………………………………2marks

$(5marks)$

3a. $\int \frac{4x-8}{x^2-4x+5}dx$

$let\ z = x^2 - 4x + 5; dz = 2x - 4$ ……………………………………$^1/_2\ mark$

Now $\int \frac{4x-8}{x^2-4x+5}dx = 2\int \frac{dz}{z} = 2\ln(x^2-4x+5) + c$ ………………2marks

b. $\int \frac{xdx}{2x^2+3}$

$=\frac{1}{4}\int \frac{4xdx}{2x^2+3}$ ………………………………………………………………1mark

$=\frac{1}{4}\log(2x^2+3) + c$ ……………………………………………………1 $^1/_2\ marks$

$(5marks)$

4. $\int xsec^2xdx$

$let\ u = x^2;\ \frac{du}{dx} = 2x\ ;\ dx = \frac{du}{2x}$ ……………………………………………1mark

Now $\int xsec^2xdx = \int xsec^2u\frac{du}{2x} = \frac{1}{2}\int sec^2udu$ …………………………2marks

$=\frac{1}{2}tanx^2 + c$ ……………………………………………..2marks

$(5marks)$

5a. $\int tanxsec^2xdx$

$let\ u = tanx; then\ du = sec^2x$ ……………………………………$^1/_2\ mark$

Now $\int tanxsec^2xdx = \int udu$ ………………………………………1mark

$= \frac{u^2}{2} + c = \frac{tan^2x}{2} + c$ ………………………………………………1mark

b. $\int sin^2xcosxdx$

$let\ u = sinx; then\ du = cosx$ ………………………………………… $^1/_2\ mark$

Now $\int sin^2xcosxdx = \int u^2\,du = \frac{u^3}{3} + c$ ……………………………1mark

$= \frac{sin^3x}{3} + c$ ……………………………………………………1mark

$(5marks)$

6. $\int \frac{x}{\sqrt{x^2-4}}dx$

$let\ u = x^2 - 4; du = 2xdu$ ……………………………………………1mark

Now $\int \frac{x}{\sqrt{x^2-4}}dx = \frac{1}{2}\int \frac{du}{u^{1/2}} = \frac{1}{2}\int u^{-1/2}\,du = \frac{1}{2}\frac{u^{1/2}}{\frac{1}{2}} = u^{1/2+c}$………..2marks

$= \sqrt{x^2-4} + c$ ……………………………………………………..2marks

$(5marks)$

7. $\int x^2e^{3x}\,dx$

$let\ u = x^2; du = 2x; and\ dv = e^{3x}; v = \frac{e^{3x}}{3}$ …………………………1mark

Now $\int x^2e^{3x}\,dx = \frac{x^2e^{3x}}{3} - \int \frac{e^{3x}}{3}.2xdx$ ………………………………..1mark

$= \frac{x^2e^{3x}}{3} - \frac{2}{3}\int xe^{3x}dx$ ………………………………………………..1mark

But $\int xe^{3x}dx = \frac{e^{3x}}{3} - \int \frac{e^{3x}}{3}dx$

$= \frac{xe^{3x}}{3} - \frac{e^{3x}}{9}$ …………………………………………………………2marks

$\therefore\ \int x^2e^{3x}\,dx = \frac{x^2e^{3x}}{3} - \frac{2}{3}\left[\frac{xe^{3x}}{3} - \frac{e^{3x}}{9}\right] + c$ ………………………………2marks

$= \frac{x^2e^{3x}}{3} - \frac{2xe^{3x}}{9} + \frac{2e^{3x}}{9.3} + c$ …………………………………………..2marks

$= \frac{e^{3x}}{3}\left[x^2 - \frac{2x}{3} + \frac{2}{9}\right] + c$ ……………………………………………1mark

$(10marks)$

8. $\int x^2tan^{-1}x\,dx$

$let\ u = tan^{-1}x; du = \frac{1}{1+x^2}; dv = x^2; \frac{x^3}{3}$ ………………………………$1\ ^1/_2\ marks$

Now $\int x^2 tan^{-1}x\, dx = \frac{x^3}{3} tan^{-1}x - \int \frac{x^3}{3} . \frac{1}{1+x^2} dx$ ……………………$1\ ^1/_2\ marks$

$=\frac{x^3}{3} tan^{-1}x - \frac{1}{3}\int \frac{1}{1+x^2} dx$ ……………………………………………1mark

$=\frac{x^3}{3} tan^{-1}x - \frac{1}{3}\left[\int xdx - \int \frac{1}{1+x^2} dx\right]$ ……………………………………2marks

$=\frac{x^3}{3} tan^{-1}x - \frac{1}{3}\left[\frac{x^2}{2} - \frac{1}{2}\ln(1+x^2)\right]$ ……………………………………2marks

$=\frac{x^3}{3} tan^{-1}x - \frac{x^2}{6} + \frac{1}{6}\ln(1+x^2) + c$ …………………………………..2marks

$(10marks)$

APPENDIX III

Attitudes Towards Calculus Inventory (ATCI)

Each of the statement of this inventory expresses a feeling towards calculus. Against each statement there are five items. Strongly Agree (SA), Agree (A), Undecided (U), Disagree (D) Strongly Disagree (SD). Please tick only one item for each statement that best represent your personal feeling.

Name of the college----------------------------------............... Course Combination------------------------

Sex: Male ( ) Female ( )

S/N		SA	A	U	D	SD
1	Calculus is a very worthwhile and necessary course.					
2	I want to develop my calculus skills.					
3	I get a great deal of satisfaction out of solving calculus problem.					
4	Calculus helps develop the mind and teaches a person to think.					
5	Calculus is important in everyday life.					
6	Calculus is one of the most important subjects for people to study.					
7	Calculus course would be very helpful no matter what I decide to study in future.					
8	I can think of many ways that calculus can be used outside of school.					
9	Calculus is one of my most dreaded courses.					
10	Attending calculus lesson makes me feel nervous.					
11	Calculus makes me feel uncomfortable.					
12	I am always under a terrible strain in calculus class.					
13	Calculus does not scare me at all.					
14	I have a lot of self-confidence when it comes to calculus.					
15	I can solve calculus problems without too much difficulty.					
16	I expect to do fairly well in calculus class.					
17	I am always confused in calculus class.					
18	I feel a sense of insecurity when attempting problems in calculus.					
19	I learn calculus easily.					
20	I am confident that I could learn more advanced calculus courses.					
21	I usually enjoyed studying calculus.					
22	Calculus is dull and boring.					
23	I like to solve new problems in calculus.					
24	I would prefer to do an assignment in calculus to other mathematics courses.					
25	I am happier in a calculus class than in any other class.					
26	Calculus is a very interesting subject.					
27	The challenge of calculus appeals to me.					
28	I think studying advanced calculus is useful.					
29	I believe I am good at solving calculus problems.					
30	I am comfortable answering questions in calculus class					

Appendix IV

Lesson plan for Experimental Group

Lesson I: Integration by substitution

Integration (Anti-derivative) is the inverse operation of differentiation.

A function F(x) is an Anti-derivative of a function f(x) if F'(x)=f(x) for all x in the domain of f. The set of all anti-derivatives of f is called the indefinite integral of f with respect to x, denoted by $\int f(x)\,dx$

The symbol $\int$ is an integral sign. The function f is called the integrand of the integral and x is the variable of Integration.

Knowledge Monitoring Stage: At this stage the facilitator tries to assess the students' entry behaviour by posing some questions or problems on concepts that are related to the concept to be learnt.

Facilitator:

Integrate (i) $(\int x^5 + 3x^4 - 6x^3 + 7x^2 - 11x + 13)dx$

(ii) $(\int \sqrt[3]{x} - \frac{1}{\sqrt[4]{x}})dx$ (iii) $\left(\int \frac{4x^{10} - 2x^4 + 15x^2}{x^3}\right)dx$ (iv) $\int \frac{1}{x}\,dx$

Solution

$(i)(\int x^5 + 3x^4 - 6x^3 + 7x^2 - 11x + 13)dx = \int x^5 + \int 3x^4 - \int 6x^3 + \int 7x^2 - \int 11x + \int 13$

$= \frac{x^6}{6} + \frac{3}{5}x^5 - \frac{3}{2}x^4 + \frac{7}{3}x^3 - \frac{11}{2}x^2 + 13x + c$

(ii) $(\int \sqrt[3]{x} - \frac{1}{\sqrt[4]{x}})dx = \int x^{1/3} - \int x^{-1/4}$

$= \frac{3}{4}x^{4/3} - \frac{4}{3}x^{3/4} + c$

(iii) $\left(\int \frac{4x^{10}-2x^4+15x^2}{x^3}\right)dx = \int \frac{4x^{10}}{x^3} - \int \frac{2x^4}{x^3} + \int \frac{15x^2}{x^3}$

$$=\int (4x^7 - 2x + \frac{15}{x})dx$$

$$=\frac{1}{2}x^8 - x^2 + 15Inx + c$$

(iv) $\int \frac{1}{x}dx = Inx$

Readiness Stage: In this stage, direct instruction is provided to introduce participants to both problem-solving heuristic and metacognitive strategy.

Sometimes we may be required to integrate functions like those above, but where x is replaced by a linear function of x. Such integration can be solved by substitution.

1.Choose *u(x)* and compute *du/dx*

2. Locate *v(u)* times *du/dx* times *dx,* or *v(u)* times *du*

3**.** Integrate $\int v(u)$ *du* to find *f(u)* + *C*

4. Substitute *u(x)* back into the antiderivative *f.*

5.The choice of *u* must be right, to change everything from *x* to *u.*

Selecting Strategy Stage: The facilitator goes on to pose some problems and urge students to suggest which of the problem solving strategy will be more appropriate and why. The facilitator then goes ahead to solve the problem, however, engaging the students throughout the session.

Example I

$\int 7x(3x^2 + 1)^4\ dx$

Metacognitive questions:

(i) what is this problem about?
(ii) how is this problem similar or different from those we have encounter before?
(iii) what strategy will be appropriate to solve this problem?

Let try and go systematically and to look for the sequence of steps or phases which are needed to reach the solution gradually. That is, by trying to decompose the whole problem into sub-problems, to identify intermediate goals, plan and to order hierarchically the operations to be carried out.

When the substitution is complicated, it is a good idea to get *du/dx* where you need it. Here $3x^2 + 1$ needs $6x$:

$\int 7x(3x^2+1)^4\,dx$ can be written as $\frac{7}{6}\int(3x^2+1)^4 6x\,dx$

Now

Let u= $3x^2+1$

$\rightarrow \frac{du}{dx}$=6x, and du= 6xdx

Now $\frac{7}{6}\int(3x^2+1)^4 6x\,dx$ can be written as $\frac{7}{6}\int u^4\ du$

$= \frac{7}{6}\frac{u^5}{5}$+c

now substitute back the value of u gives

$= \frac{7}{6}\frac{(3x^2+1)^5}{5} + c$

Example II

$\int \frac{2t^3+1}{(t^4+2t)^3}dt$

Metacognitive questions:

(i) what is this problem about?
(ii) how is this problem similar or different from those we have encounter before?
(iii) what strategy will be appropriate to solve this problem?

It can be observed that when t^4+2t is differentiated it yield $4t^3+2$,

$which\ is = 2(2t^3+1)$

Now let $u = t^4 + 2t$, $du = (4t^3 + 2)dt \; = 2(2t^3 + 1)dt$

This gives $(2t^3 + 1)dt = \frac{1}{2}du$

The question can now be rewritten as

$$\int \frac{2t^3+1}{(t^4+2t)^3}dt = \frac{1}{2}\int \frac{1}{u^3}du$$

$$= \frac{1}{2}\int u^{-3}\,du$$

$$= \frac{1}{2}(-\frac{1}{2})u^{-2}$$

$$= -\frac{1}{4}(t^4 + 2t)^{-2} + c$$

Example III

$$\int \frac{xdx}{x^4+3}$$

Metacognitive questions:

(i) what is this problem about?
(ii) how is this problem similar or different from those we have encounter before?
(iii) what strategy will be appropriate to solve this problem?

Differentiating $x^4 + 3$ $gives$ $4x^3$, this is not related to xdx

However, if we

Let u=x^2 then $\frac{du}{dx} = 2x$, and dx= $\frac{du}{2x}$

Now $\int \frac{xdx}{x^4+3}$ can be written as $\int \frac{x\frac{du}{2x}}{(x^2)^2+(\sqrt{3})^2}$

$$= \frac{1}{2}\int \frac{1}{u^2+(\sqrt{3})^2}du$$

$$=\frac{1}{2}\frac{1}{\sqrt{3}}tan^{-1\frac{x^2}{\sqrt{3}}+c}$$

Problem Solving Stage: Participants will be engaged in solving a wide variety of non-routine

Problems involving Integration by substitution, they were to practice, on their own, the use of various cognitive and metacognitive strategies. They will be prompted to maintain conscious and periodic self-checking of their strategies, and reflect upon and evaluate them.

Integrate (i) $\int \frac{6x^2}{x^3-4}dx$ (ii) $\int \frac{3x}{5x^2+4}dx$ (iii) $\int \frac{x^2}{\sqrt{1-x^6}}dx$

The problem solving phase has three stages: planning, monitoring and evaluation stage.

(i) During planning stage, the participant will be encouraged to ask themselves the following metacognitive questions;

What am I supposed to solve?

What prior know-ledge will help me with this task?

What should I do first? What should I look for in this problem?

How much time do I have to complete this task? In what direction do I direct my thinking?

(ii) During monitoring stage, the participant will be encouraged to ask themselves the following metacognitive questions;

How am I doing?

Am I on the right track?

How should I proceed?

What information is important to remember?

Should I move in a different direction?

Should I adjust the pace because of the difficulty?

What can I do if I do not understand?

(iii) During evaluation stage, the participant will be encouraged to ask themselves the following metacognitive questions;

How well did I do?

What did I learn?

Did I get the results expected?

What could I have done differently?

Can I apply this way of thinking to other problems or situations?

Is there anything I don’t understand, any gap in my knowledge?

Do I need to go back through the task to fill in any gap in understanding?

How might I apply this line of thinking to other problems?

Reflection Stage: Participants will be asked the following questions:

1. What did you do well in this lesson?
2. What did you struggle with in this lesson?
3. Did you learn anything? Why or why not?
4. What would help you next time?

The Participants will answer these questions on a paper and submit for the teacher’s evaluation against the next lesson.

Lesson II

Knowledge Monitoring Stage: At this stage the facilitator tries to assess the students' entry behaviour by posing some questions or problems on concepts that are related to the concept to be learnt.

Facilitator:

Integrate (i) $\int sinxdx$ (ii) $\int cosxdx$ (iii) $\int tanxdx$ (iv) $\int secxdx$ (v) $\int cotxdx$

(i) $\int sinxdx = -cosx + c$

(ii) $\int cosxdx = sinx + c$

(iii) $\int tanxdx = \int \frac{sinx}{cosx} dx = -\int \frac{-sinx}{cosx} dx = -Incosx + c$

(iv) $\int secxdx = In \left| secx + tanx \right| + c$

(v) $\int cotxdx = Insinx + c$

Readiness Stage: In this stage, direct instruction is provided to introduce participants to both problem-solving heuristic and metacognitive strategy.

At times we often see the integral such as $\int \tan x \sec^2 x dx$ observe that the $\left(\sec^2 x\right)$ of the product is the differential coefficient of the other function $\left(\tan x\right)$

Now let $u = \tan x$ *so that* $du = \sec^2 x$

The integral becomes

$$\int u du = \frac{u^2}{2} = \frac{\tan^2 x}{2} + c$$

Let us consider integrals of the form $\int sin^k x cos^n x dx$, where k and n are nonnegative integers. If the exponent of $sinx$ is odd, let $u = cosx$ and if the exponent of $sinx$ is even, you let $u = sinx$

Selecting Strategy: The facilitator goes on to pose some problems and urge students to suggest which of the problem solving strategy will be more appropriate and why. The facilitator then goes ahead to solve the problem, however, engaging the students throughout the session.

Example I

Integrate $\int tan^3 2xsec2xdx$

Metacognitive questions:

(i) what is this problem about?
(ii) how is this problem similar or different from those we have encounter before?
(iii) what strategy will be appropriate to solve this problem?

Let try and go systematically and to look for the sequence of steps or phases which are needed to reach the solution gradually. That is, by trying to decompose the whole problem into sub-problems, to identify intermediate goals, plan and to order hierarchically the operations to be carried out.

Let z= sec2x; in that case $\frac{dz}{dx}$ = 2sec2xtan2x

Then dz= 2sec2xtan2xdx

How will the original question looks like now?

Now $\int tan^3 2xsec2xdx$ = $\int tan^2\, 2xtan2xsec2xdx$

What is

$sin^2x + cos^2$ =

$sec^2x - tan^2x =$

$cosec^2x - cot^2x =$

Now $sec^2x - tan^2x = 1 \rightarrow tan^2x = \ sec^2x - 1$ and 2sec2xtan2xdx=dz

$\int tan^2\, 2xtan2xsec2xdx$ = $\int (sec^2\, 2x - 1)\frac{dz}{2}$

$=\frac{1}{2}\int(z^2-1)dz$

$= \frac{1}{2}[\frac{z^3}{3}-z]+c$ = $\frac{1}{2}$x $\frac{1}{3}(z^3-3z)$ + c

Now put z= sec2x

$= \frac{1}{6}\,(sec^3 2x-3sec2x)+c$

Example II

Integrate

$\int sin^3 xcos^2 xdx$

Metacognitive questions:

(i) what is this problem about?
(ii) how is this problem similar or different from those we have encounter before?
(iii) what strategy will be appropriate to solve this problem?

The question is of the form $\int sin^k xcos^n xdx$ and $sinx\ is\ odd$

So let $u=cosx, then\ du=\ -sinxdx$

$$\int sin^3 xcos^2 xdx\ can\ be\ written\ as \int sin^2 xcos^2 xsinxdx$$

$but\ sin^2=1-cos^2 x$

Now $\int sin^3 xcos^2 xdx=\int(1-cos^2 x\ )\,cos^2 xsinxdx$

$=-\int(1-u^2)u^2 du$

$=\int(u^4-u^2)du$

$=\frac{u^5}{5}-\frac{u^3}{3}+c$

$=\frac{cos^5 x}{5}-\frac{cos^3 x}{3}+c$

Example III

Integrate

$\int sin^4 xcos^7 xdx$

Metacognitive questions:

(i) what is this problem about?
(ii) how is this problem similar or different from those we have encounter before?
(iii) what strategy will be appropriate to solve this problem?

The question is of the form $\int sin^k xcos^n xdx$ and $sinx\ is\ even$

So let $u = sinx, then\ du = \ cosxdx$

$\int sin^4 xcos^7 xdx\ can\ be\ written\ as$

$\int sin^4 xcos^6 xcosxdx$

$but\ cos^2 x = 1 - \ sin^2,$

$cos^6 x = (cos^2 x)^3 = (1 - sin^2)^3 = (1 - u^2)^3$

Now

$\int sin^4 xcos^7 xdx = \int sin^4 xcos^6 xcosxdx$

$= \int u^4 (1 - u^2)^3 du$

But $(1 - u^2)^3 = (1 - u^2)^2 (1 - u^2)$

$= (1 - 2u^2 + u^4)(1 - u^2)$

$= 1 - u^2 - 2u^2 + 2u^4 + u^4 - u^6$

$= 1 - 3u^2 + 3u^4 - u^6$

$\int u^4 (1 - u^2)^3 du = \int u^4 (1 - 3u^2 + 3u^4 - u^6) du$

$= \int u^4 - 3u^6 + 3u^8 - u^{10})$

$= \frac{1}{5} u^5 - \frac{3}{7} u^7 + \frac{1}{3} u^9 - \frac{1}{11} u^{11}$

$= \frac{1}{5} sin^5 x - \frac{3}{7} sin^7 x + \frac{1}{3} sin^9 x - \frac{1}{11} sin^{11} x + c$

Problem Solving Phase: Participants will be engaged in solving a wide variety of non-routine

Problems involving Integration by substitution, they were to practice, on their own, the use of various cognitive and metacognitive strategies. They will be prompted to maintain conscious and periodic self-checking of their strategies, and reflect upon and evaluate them.

Integrate

(i) $\int sec^9 x tan^5 x dx$ (ii) $\int sin^5 x cos^2 x dx$ (iii) $\int sin^5 x cos^9 x dx$

The problem solving phase has three stages: planning, monitoring and evaluation stage.

(i) During planning stage, the participant will be encouraged to ask themselves the following metacognitive questions;

What am I supposed to solve?

What prior know-ledge will help me with this task?

What should I do first? What should I look for in this problem?

How much time do I have to complete this task? In what direction do I direct my thinking?

Reflection Stage: Participants will be asked the following questions:

1. What did you do well in this lesson?

2. What did you struggle with in this lesson?

3. Did you learn anything? Why or why not?

4. What would help you next time?

Lesson III

Knowledge Monitoring Stage: At this stage the facilitator tries to assess the students' entry behaviour by posing some questions or problems on concepts that are related to the concept to be learnt.

Facilitator:

Makes the following algebraic expression a perfect square

(i) $x^2 + 4x$ (ii) $x^2 - 7x$ (iii) $x^2 + {}^{5}/_{2}\, x$ (iv) $5x^2 - 2x$

$x^2 + 4x + 4$

$x^2 - 7x + \frac{49}{4}$

$x^2 + {}^{5}/_{2}\, x + \frac{25}{16}$

$x^2 - {}^{1}/_{5}\, x + {}^{1}/_{25}$

Readiness Stage: In this stage, direct instruction is provided to introduce participants to both problem-solving heuristic and metacognitive strategy.

At times we do come across integration of the form

$\int \frac{dx}{x^2+4x+5}$

Here, our earlier discussed method of substitution will not work, for if we let

$u = x^2 + 4x, and\ du = 2x + 4.$ There is no manipulation we will do with these values to give us the original question. However, the problem can be solved by making $x^2 + 4x$ a perfect square then manipulate 5. For example , $x^2 + 4x + 5\ can\ be\ written\ as\ (x + 2)^2 + 1$. Therefore, to solve integration of this form, make the expression a perfect square.

Recall $\int \frac{1}{a^{2+x^2}} dx = \frac{1}{a} tan^{-1} \frac{x}{a} + c$

$$\int \frac{1}{a^2-x^2}\, dx = \frac{1}{a} tanh^{-1}\frac{x}{a} + c \; or \; \frac{1}{2a} In \left| \frac{a+x}{a-x} \right|$$

$$\int \frac{1}{x^2-a^2}\, dx = \frac{1}{a} coth^{-1}\frac{x}{a} + c \; or \; \frac{1}{2a} In \left| \frac{x-a}{x+a} \right|$$

$$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a} sec^{-1}\frac{x}{a} + c$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \frac{1}{a} sin^{-1}\frac{x}{a} + c$$

Selecting Strategy: The facilitator goes on to pose some problems and urge students to suggest which of the problem solving strategy will be more appropriate and why. The facilitator then goes ahead to solve the problem, however, engaging the students throughout the session.

Example I

Integrate

$$\int \frac{dx}{x^2+4x+5}$$

Metacognitive questions:

(i) what is this problem about?
(ii) how is this problem similar or different from those we have encounter before?
(iii) what strategy will be appropriate to solve this problem?

Complete the square

$$x^2 + 4x + 5 = (x+2)^2 + 1$$

Think of a number $x + \left(\frac{4}{2}\right)^2 = 5$

Observed that $x + 4 = 5 \rightarrow x = 1$

Now $\int \frac{dx}{x^2+4x+5} = \int \frac{dx}{(x+2)^2+1}$

Let $u = x + 2, then\ du = dx$ so

$\int \frac{dx}{(x+2)^2+1} = \frac{du}{u^2+1}$

$= tan^{-1}u + c$

$=tan^{-1}(x+2) + c$

Example II

Integrate

$\int \frac{1}{x^2-10x+18} dx$

Complete the square

What will be added to 25 which is the square of half of 10 to give18?

-7

Now

$x^2 - 10x + 18 = \ (x-5)^2 - 7$

Think of a number $x + \left(\frac{10}{2}\right)^2 = 18$

$\int \frac{1}{x^2-10x+18} dx = \int \frac{1}{(x-5)^2-7} dx$

$\int \frac{1}{(x-5)^2-7} dx = \int \frac{1}{(x-5)^2-(\sqrt{7})^2} dx$

Compare with $\int \frac{1}{x^2-a^2} dx = \frac{1}{a} coth^{-1} \frac{x}{a} + c$

$\int \frac{1}{(x-5)^2-(\sqrt{7})^2} dx = \frac{1}{\sqrt{7}} coth^{-1} \frac{x-5}{\sqrt{7}} + c$

Example III

Integrate

$\int \frac{dx}{2+3x-x^2}$

$2 + 3x - x^2 = 2 - \left(x^2 - 3x + \frac{9}{4}\right) + \frac{9}{4}$

What is $(\frac{3}{2})^2$ and $2 + \frac{9}{4}$

$= \frac{17}{4} - (x - \frac{3}{2})^2$

Now

$\int \frac{dx}{2+3x-x^2} = \int \frac{dx}{\frac{17}{4}-(x-\frac{3}{2})^2}$

Compare with $\int \frac{1}{a^2-x^2} dx = \frac{1}{a} tanh^{-1} \frac{x}{a} + c$

$\frac{17}{4} = (\frac{\sqrt{17}}{2})^2$

$\int \frac{dx}{2+3x-x^2} = \int \frac{dx}{\frac{17}{4}-(x-\frac{3}{2})^2} = \int \frac{dx}{(\frac{\sqrt{17}}{2})^2-(x-\frac{3}{2})^2} = \frac{2}{\sqrt{17}} tanh^{-1} \frac{x-{}^3/_2}{\frac{\sqrt{17}}{2}} + c$

$= \frac{4}{\sqrt{17}} tanh^{-1} \frac{x-{}^3/_2}{\sqrt{17}} + c$

Problem Solving Stage: Participants will be engaged in solving a wide variety of non-routine

Problems involving Integration by substitution, they were to practice, on their own, the use of various cognitive and metacognitive strategies. They will be prompted to maintain conscious and periodic self-checking of their strategies, and reflect upon and evaluate them.

Integrate

(i) $\int \frac{dx}{2x^2+4x+3}$ (ii) $\int \frac{dx}{x^2+6x+2}$ (iii) $\int \frac{dx}{\sqrt{20+8x-x^2}}$

The problem solving phase has three stages: planning, monitoring and evaluation stage.

(i) During planning stage, the participant will be encouraged to ask themselves the following metacognitive questions;

What am I supposed to solve?

What prior know-ledge will help me with this task?

What should I do first? What should I look for in this problem?

How much time do I have to complete this task? In what direction do I direct my thinking?

Reflection Stage: Participants will be asked the following questions:

1. What did you do well in this lesson?

2. What did you struggle with in this lesson?

3. Did you learn anything? Why or why not?

4. What would help you next time?

Lesson IV: Integration by part

Knowledge Monitoring Stage: At this stage the facilitator tries to assess the students' entry behaviour by posing some questions or problems on concepts that are related to the concept to be learnt.

Integrate

(i) $\int tanxsec^2x\, dx$ (ii) $\int sin^2xcosxdx$ (iii) $\int(x^2-7x-4)(2x-7)dx$

(i) $\int tanxsec^2x\, dx = \frac{tan^2x}{2} + c$

(ii) $\int sin^2xcosxdx = \frac{sin^3x}{3} + c$

(iii) $\int(x^2-7x-4)(2x-7)dx = \frac{(x^2-7x-4)^2}{2} + c$

Readiness Stage: In this phase, direct instruction is provided to introduce participants to both problem-solving heuristic and metacognitive strategies.

Step I

We often need to integrate a product where neither of the functions is the differential coefficient of the other. In that case we use the method of integration by parts. This technique is useful when you can write the integrand as the product of two separate integrable functions, and can be derived directly from the product rule for differentiation. The method is $\int udv$ =uv- $\int vdu$ for indefinite integrals, where *u* and *v* are functions of *x*, and *du* and *dv* are the derivatives of those functions.

The following can serve as guide line for choosing u:

1. If one of the functions is a logarithmic function, that logarithmic function should be taken as u

2. If there is no logarithmic function, but a power of x that becomes u

3. If is neither a logarithmic nor power of X, then the exponential function is taken as u

Selecting Strategy Stage: The facilitator goes on to pose some problems and urge students to suggest which of the problem solving strategy will be more appropriate and why. The facilitator then goes ahead to solve the problem, however, engaging the students throughout the session.

Example I

Evaluate

$\int x^2 Inx dx$

Metacognitive questions:

(i) what is this problem about?
(ii) how is this problem similar or different from those we have encounter before?
(iii) what strategy will be appropriate to solve this problem?

Let try and go systematically and to look for the sequence of steps or phases which are needed to reach the solution gradually. That is, by trying to decompose the whole problem into sub-problems, to identify intermediate goals, plan and to order hierarchically the operations to be carried out.

Here one of the functions is a logarithmic function therefore, we shall take it as our u

Let u = Inx, then $du = \frac{1}{x}$; also if $dv = x^2$ then integrating will give v = $\frac{x^3}{3}$

Now using $\int udv$ =uv- $\int vdu$

$\int x^2 Inx dx = \frac{x^3\ Inx}{3} - \int \frac{x^3}{3}\frac{1}{x}dx$

$= \frac{x^3\ Inx}{3} - \int \frac{x^2}{3}dx$

$= \frac{x^3\ Inx}{3} - \frac{1}{3}\int x^2 dx$

$= \frac{x^3\ Inx}{3} - \frac{1}{3}.\frac{x^3}{3} = \frac{x^3\ Inx}{3} - \frac{x^3}{9} + c$

Example II

Evaluate

$\int x^3 e^{2x}\, dx$

Here one of the functions is a power of X therefore, we shall take it as our U

Let $u = x^3$ then $du = 3x^2$, so also $dv = e^{2x}$ and

Let us $\int e^{2x}\, dx$

Let $u = 2x$ then $du = 2$ and $dx = \frac{du}{2}$

Now $\int e^{2x}\, dx$ become $\frac{1}{2}\int e^{u}\, du = \frac{e^u}{2}$

$\int e^{2x} = \frac{e^{2x}}{2}$

$v = \frac{e^{2x}}{2}$

Now using $\int udv = uv - \int vdu$

$\int x^3 e^{2x}\, dx = \frac{x^3 e^{2x}}{2} - \int \frac{3x^2}{2} . e^{2x} dx$

$= \frac{x^3 e^{2x}}{2} - \frac{3}{2}\int x^2 . e^{2x} dx$

Integrate

$\int x^2 . e^{2x} dx$ Following the earlier procedure

Let u= $x^2, du = 2x; dv = . e^{2x}, and\ v = \frac{1}{2} . e^{2x}$

$\int x^2 . e^{2x} dx = x^2 \frac{1}{2} . e^{2x} - \int 2x. \frac{1}{2} . e^{2x} dx$

$= \frac{x^2}{2} . e^{2x} - \int e^{2x}\, dx$

$= \frac{x^2}{2} . e^{2x} - \frac{1}{2} e^{2x}$

Now

$\int x^3 e^{2x}\, dx = \frac{x^3 e^{2x}}{2} - \frac{3}{2}\int x^2 . e^{2x} dx$

$= \frac{x^3 e^{2x}}{2} - \frac{3}{2}\left[\frac{x^2}{2} . e^{2x} - \frac{1}{2} e^{2x}\right]$

$= \frac{x^3 e^{2x}}{2} - \frac{3}{4}[x^2 e^{2x} - e^{2x}] + c$

Example III

$\int x^2 \ln x\, dx$

Let $u = \ln x,\ so that\ du = 1/x\ \ and\ \ dv = x^2\ then\ v = \frac{x^3}{3}$

Then

$$\int x^2 \ln x\, dx = \frac{x^3 \ln x}{3} - \int \frac{x^3}{3}\frac{1}{x} dx$$

$$= \frac{x^3 \ln x}{3} - \int \frac{x^2}{3} dx$$

$$= \frac{x^3 \ln x}{3} - \frac{1}{3}\int x^2 dx$$

$$= \frac{x^3 \ln x}{3} - \frac{1}{3} \cdot \frac{x^3}{3}$$

$$= \frac{x^3 \ln x}{3} - \frac{x^3}{9} + c \quad or \quad \frac{x^3}{9}[3\ln x - 1] + c$$

Problem Solving Stage: Participants will be engaged in solving a wide variety of non-routine

Problems involving Integration by substitution, they were to practice, on their own, the use of various cognitive and metacognitive strategies. They will be prompted to maintain conscious and periodic self-checking of their strategies, and reflect upon and evaluate them.

Evaluate

1. $\int x^2 sinxdx$ 2. $\int e^{3x} cos3xdx$ 3. $\int x^2 e^{-3x}dx$

The problem solving phase has three stages: planning, monitoring and evaluation stage.

(i) During planning stage, the participant will be encouraged to ask themselves the following questions;

What am I supposed to solve? What prior know-ledge will help me with this task? What should I do first? What should I look for in this problem? How much time do I have to complete this task? In what direction do I direct my thinking?

(ii) During monitoring stage, the participant will be encouraged to ask themselves the following questions;

How am I doing? Am I on the right track? How should I proceed? What information is important to remember? Should I move in a different direction? Should I adjust the pace because of the difficulty? What can I do if I do not understand?

(iii) During evaluation stage, the participant will be encouraged to ask themselves the following questions;

How well did I do? What did I learn? Did I get the results expected? What could I have done differently? Can I apply this way of thinking to other problems or situations? Is there anything I don't understand, any gap in my knowledge? Do I need to go back through the task to fill in any gap in understanding? How might I apply this line of thinking to other problems?

Reflection Stage: Participants will be asked the following questions:

1. What did you do well in this lesson?

2. What did you struggle with in this lesson?

3. Did you learn anything? Why or why not?

4. What would help you next time?

The Participants will answer these questions on a paper and submit for the teacher's evaluation against the next lesson.

Appendix V

Lesson plan for Control Group

Lesson I: Integration by Substitution

Integration (Anti-derivative) is the inverse operation of differentiation.

A function F(x) is an Anti-derivative of a function f(x) if F'(x)=f(x) for all x in the domain of f. The set of all anti-derivatives of f is called the indefinite integral of f with respect to x, denoted by $\int f(x)\, dx$

The symbol $\int$ is an integral sign. The function f is called the integrand of the integral and x is the variable of Integration.

Integration by substitution

Step I

The lecturer introduces the lesson by posing the following questions to the students:

Integrate (i) $(\int x^5 + 3x^4 - 6x^3 + 7x^2 - 11x + 13)dx$

(ii) $(\int \sqrt[3]{x} - \frac{1}{\sqrt[4]{x}})dx$ (iii) $\left(\int \frac{4x^{10} - 2x^4 + 15x^2}{x^3}\right) dx$ (iv) $\int \frac{1}{x} dx$

Step II

Sometimes we may be required to integrate functions like those in the standard list, but where x is replaced by a linear function of x. Such integration can be solved by substitution.

1.Choose *u(x)* and compute *du/dx*

2. Locate *v(u)* times *du/dx* times *dx,* or *v(u)* times *du*

3**.** Integrate $\int v(u)$ *du* to find *f(u)* + *C*

4. Substitute *u(x)* back into this antiderivative *f.*

5.The choice of *u* must be right, to change everything from *x* to *u.*

Example I

$\int 7x(3x^2 + 1)^4\, dx$

When the substitution is complicated, it is a good idea to get *du/dx* where you need it. Here *$3x^2$* + 1 needs *6x:*

$\int 7x(3x^2 + 1)^4\, dx$ can be written as $\frac{7}{6}\int (3x^2 + 1)^4 6x\, dx$

Now

Let u= $3x^2 + 1$

$\rightarrow \frac{du}{dx}$=6x, and du= 6xdx

Now $\frac{7}{6}\int (3x^2 + 1)^4 6x\, dx$ can be written as $\frac{7}{6}\int u^4\ du$

$\frac{7}{6}\frac{u^5}{5}$+c now substitute back the value of u

$= \frac{7}{6}\frac{(3x^2+1)^5}{5} + c$

Example II

Integrate $\int \frac{2t^3+1}{(t^4+2t)^3} dt$

When you differentiate $t^4 + 2t$ it yield $4t^3 + 2, which\ is$

$= 2(2t^3 + 1)$

Now let $u = t^4 + 2t, du = (4t^3 + 2)dt = 2(2t^3 + 1)dt$

This gives $(2t^3 + 1)dt = \frac{1}{2}du$

The question can now be rewritten as

$\int \frac{2t^3+1}{(t^4+2t)^3} dt = \frac{1}{2}\int \frac{1}{u^3} du$

$= \frac{1}{2}\int u^{-3}\, du$

$= \frac{1}{2}(-\frac{1}{2})u^{-2}$

$= -\frac{1}{4}(t^4 + 2t)^{-2} + c$

Example III

$\int \frac{xdx}{x^4+3}$

Let u=x^2 then $\frac{du}{dx} = 2x$, and dx= $\frac{du}{2x}$

Now $\int \frac{xdx}{x^4+3}$ can be written as $\int \frac{x\frac{du}{2x}}{(x^2)^2+(\sqrt{3})^2}$

= $\frac{1}{2}\int \frac{1}{u^2+(\sqrt{3})^2}$du

=$\frac{1}{2}\frac{1}{\sqrt{3}} tan^{-1\frac{x^2}{\sqrt{3}}+c}$

Do the following exercises

Integrate (i) $\int \frac{6x^2}{x^3-4}$dx (ii) $\int \frac{3x}{5x^2+4} dx$ (iii) $\int \frac{x^2}{\sqrt{1-x^6}} dx$

Lesson II: Integration by Substitution

The lecturer introduces the lesson by posing the following questions to the students:

Integrate

(i) $\int sinxdx$ (ii) $\int cosxdx$ (iii) $\int tanxdx$ (iv) $\int secxdx$ (v) $\int cotxdx$

At times we often see the integral such as $\int \tan x \sec^2 xdx$ observe that the $\left(\sec^2 x\right)$ of the product is the differential coefficient of the other function $\left(\tan x\right)$

Now let $u = \tan x$ *so that* $du = \sec^2 x$

The integral becomes

$$\int udu = \frac{u^2}{2} = \frac{\tan^2 x}{2} + c$$

Let us consider integrals of the form $\int sin^k xcos^n xdx$, where k and n are nonnegative integers. If the exponent of $sinx$ is odd, let $u = cosx$ and if the exponent of $sinx$ is even, you let $u = sinx$

Step I

Example I

Integrate $\int tan^3 2xsec2xdx$

Let z= sec2x; in that case $\frac{dz}{dx}$ = 2sec2xtan2x

Then dz= 2sec2xtan2xdx

How will the original question looks like now?

Now $\int tan^3 2xsec2xdx$ = $\int tan^2 2xtan2xsec2xdx$

What is

$sin^2 x + cos^2$ =

$sec^2 x - tan^2 x =$

$cosec^2x - cot^2x =$

Now $sec^2x - tan^2x = 1 \rightarrow tan^2x = sec^2x - 1$ and 2sec2xtan2xdx=dz

$\int tan^2 2xtan2xsec2xdx = \int (sec^2 2x - 1)\frac{dz}{2}$

$=\frac{1}{2}\int (z^2 - 1)dz$

$= \frac{1}{2}[\frac{z^3}{3} - z] + c = \frac{1}{2}$x $\frac{1}{3}(z^3 - 3z)$ + c

Now put z= sec2x

$= \frac{1}{6}(sec^3 2x - 3sec2x) + c$

Example II

Integrate

$\int sin^3 xcos^2xdx$

The question is of the form $\int sin^k xcos^n xdx$ and $sinx\ is\ odd$

So let $u = cosx, then\ du = -sinxdx$

$$\int sin^3 xcos^2xdx\ can\ be\ written\ as \int sin^2 xcos^2xsinxdx$$

$but\ sin^2 = 1 - cos^2x$

Now $\int sin^3 xcos^2xdx = \int (1 - cos^2x)\, cos^2xsinxdx$

$=-\int (1 - u^2)u^2du$

$=\int (u^4 - u^2)du$

$=\frac{u^5}{5} - \frac{u^3}{3} + c$

$=\frac{cos^5x}{5} - \frac{cos^3x}{3} + c$

Example III

Integrate $\int sin^4 xcos^7 xdx$

The question is of the form $\int sin^k xcos^n xdx$ and $sinx\ is\ even$

So let $u = sinx, then\ du = \ cosxdx$

$\int sin^4 xcos^7 xdx\ can\ be\ written\ as$

$\int sin^4 xcos^6 xcosxdx$

$but\ cos^2 x = 1 - \ \ sin^2,$

$cos^6 x = (cos^2 x)^3 = (1 - sin^2)^3 = (1 - u^2)^3$

Now

$\int sin^4 xcos^7 xdx = \int sin^4 xcos^6 xcosxdx$

$= \int u^4 (1 - u^2)^3 du$

But $(1 - u^2)^3 = (1 - u^2)^2 (1 - u^2)$

$= (1 - 2u^2 + u^4)(1 - u^2)$

$= 1 - u^2 - 2u^2 + 2u^4 + u^4 - u^6$

$= 1 - 3u^2 + 3u^4 - u^6$

$\int u^4 (1 - u^2)^3 du = \int u^4 (1 - 3u^2 + 3u^4 - u^6) du$

$= \int u^4 - 3u^6 + 3u^8 - u^{10})$

$= \frac{1}{5} u^5 - \frac{3}{7} u^7 + \frac{1}{3} u^9 - \frac{1}{11} u^{11}$

$= \frac{1}{5} sin^5 x - \frac{3}{7} sin^7 x + \frac{1}{3} sin^9 x - \frac{1}{11} sin^{11} x + c$

Do the following:

Integrate

(i) $\int sec^9 xtan^5 xdx$ (ii) $\int sin^5 xcos^2 xdx$ (iii) $\int sin^5 xcos^9 xdx$

Lesson III: Integration by substitution

Step I

The lecturer introduces the lesson by asking the students on solving of quadratic equation using square method

Makes the following algebraic expression a perfect square

(i) $x^2 + 4x$ (ii) $x^2 - 7x$ (iii) $x^2 + {}^5/_2\, x$ (iv) $5x^2 - 2x$

At times we often come across integration of the form

$\int \frac{dx}{x^2+4x+5}$

Here, our earlier discussed method of substitution will not work, for if we let

$u = x^2 + 4x, and\ du = 2x + 4$. There is no manipulation we will do with these values to give us the original question. However, the problem can be solved by making $x^2 + 4x$ a perfect square then manipulate 5.

For example ; $x^2 + 4x + 5\ can\ be\ written\ as\ (x + 2)^2 + 1$. Therefore, to solve integration of this form, make the expression a perfect square.

Recall $\int \frac{1}{a^{2+x^2}} dx = \frac{1}{a} tan^{-1} \frac{x}{a} + c$

$$\int \frac{1}{a^{2-x^2}} dx = \frac{1}{a} tanh^{-1} \frac{x}{a} + c\ or\ \frac{1}{2a} In \left| \frac{a+x}{a-x} \right|$$

$$\int \frac{1}{x^{2-a^2}} dx = \frac{1}{a} coth^{-1} \frac{x}{a} + c\ or\ \frac{1}{2a} In \left| \frac{x-a}{x+a} \right|$$

$$\int \frac{dx}{x\sqrt{x^2}-a^2} = \frac{1}{a} sec^{-1} \frac{x}{a} + c$$

$$\int \frac{dx}{\sqrt{a^2}-x^2} = \frac{1}{a} sin^{-1} \frac{x}{a} + c$$

Step II

Integrate

$\int \frac{dx}{x^2+4x+5}$

Complete the square

$x^2 + 4x + 5 = (x+2)^2 + 1$

Think of a number $x + \left(\frac{4}{2}\right)^2 = 5$

Observed that $x + 4 = 5 \rightarrow x = 1$

Now $\int \frac{dx}{x^2+4x+5} = \int \frac{dx}{(x+2)^2+1}$

Let $u = x + 2, then\ du = dx$ so

$\int \frac{dx}{(x+2)^2+1} = \frac{du}{u^2+1}$

= $tan^{-1}u + c$

=$tan^{-1}(x+2) + c$

Example II

Integrate

$\int \frac{1}{x^2-10x+18} dx$

Complete the square

What will be added to 25 which is the square of half of 10 to give18?

-7

Now

$x^2 - 10x + 18 = (x-5)^2 - 7$

Think of a number $x + \left(\frac{10}{2}\right)^2 = 18$

$\int \frac{1}{x^2-10x+18} dx = \int \frac{1}{(x-5)^2-7} dx$

$\int \frac{1}{(x-5)^2-7} dx = \int \frac{1}{(x-5)^2-(\sqrt{7})^2} dx$

Compare with $\int \frac{1}{x^2-a^2} dx = \frac{1}{a} coth^{-1} \frac{x}{a} + c$

$\int \frac{1}{(x-5)^2-(\sqrt{7})^2} dx = \frac{1}{\sqrt{7}} coth^{-1} \frac{x-5}{\sqrt{7}} + c$

Example III

Integrate

$\int \frac{dx}{2+3x-x^2}$

$2 + 3x - x^2 = 2 - \left(x^2 - 3x + \frac{9}{4}\right) + \frac{9}{4}$

What is $(\frac{3}{2})^2$ and $2 + \frac{9}{4}$

$= \frac{17}{4} - (x - \frac{3}{2})^2$

Now

$\int \frac{dx}{2+3x-x^2} = \int \frac{dx}{\frac{17}{4}-(x-\frac{3}{2})^2}$

Compare with $\int \frac{1}{a^2-x^2} dx = \frac{1}{a} tanh^{-1} \frac{x}{a} + c$

$\frac{17}{4} = (\frac{\sqrt{17}}{2})^2$

$\int \frac{dx}{2+3x-x^2} = \int \frac{dx}{\frac{17}{4}-(x-\frac{3}{2})^2} = \int \frac{dx}{(\frac{\sqrt{17}}{2})^2-(x-\frac{3}{2})^2} = \frac{2}{\sqrt{17}} tanh^{-1} \frac{x-{}^3/_2}{\frac{\sqrt{17}}{2}} + c$

$= \frac{4}{\sqrt{17}} tanh^{-1} \frac{x-{}^3/_2}{\sqrt{17}} + c$

Exercise III

Integrate

(i) $\int \frac{dx}{2x^2+4x+3}$ (ii) $\int \frac{dx}{x^2+6x+2}$ (iii) $\int \frac{dx}{\sqrt{20+8x-x^2}}$

Lesson IV: Integration by part

Integrate

(i) $\int tanxsec^2x\,dx$ (ii) $\int sin^2xcosxdx$ (iii) $\int(x^2-7x-4)(2x-7)dx$

(i) $\int tanxsec^2x\,dx = \frac{tan^2x}{2} + c$

(ii) $\int sin^2xcosxdx = \frac{sin^3x}{3} + c$

(iii) $\int(x^2-7x-4)(2x-7)dx = \frac{(x^2-7x-4)^2}{2} + c$

We often need to integrate a product where neither of the functions is the differential coefficient of the other. In that case we use the method of integration by parts. This technique is useful when you can write the integrand as the product of two separate integrable functions, and can be derived directly from the product rule for differentiation. The method is $\int udv$ =uv- $\int vdu$ for indefinite integrals, where *u* and *v* are functions of *x*, and *du* and *dv* are the derivatives of those functions.

The following can serve as guide line for choosing u:

1. If one of the functions is a logarithmic function, that logarithmic function should be taken as u

2. If there is no logarithmic function, but a power of X that becomes u

3. If is neither a logarithmic nor power of X, then the exponential function is taken as u

Example I

Evaluate

$\int x^2Inxdx$

Here one of the functions is a logarithmic function therefore, we shall take it as our u

Let u = Inx, then $du = \frac{1}{x}$; also if $dv = x^2$ then integrating will give v = $\frac{x^3}{3}$

Now using $\int udv$ =uv- $\int vdu$

$\int x^2 Inxdx = \frac{x^3\ Inx}{3}$ -$\int \frac{x^3}{3}\frac{1}{x}dx$

$= \frac{x^3\ Inx}{3} - \int \frac{x^2}{3}dx$

$= \frac{x^3\ Inx}{3} - \frac{1}{3}\int x^2 dx\ =\ \frac{x^3\ Inx}{3} - \frac{1}{3}.\frac{x^3}{3}$

$= \frac{x^3\ Inx}{3} - \frac{x^3}{9}$ + c

Example II

Evaluate

$\int x^3 e^{2x}\, dx$

Here one of the functions is a power of X therefore, we shall take it as our U

Let $u = x^3$ then du =3x^2, so also $dv = e^{2x}$ and

Let us $\int\ e^{2x}\ dx$

Let $u = 2x$ then $du = 2$ and $dx = \frac{du}{2}$

Now $\int\ e^{2x}\ dx$ become $\frac{1}{2}\int e^u\ du = \frac{e^u}{2}$

$\int\ e^{2x} = \frac{e^{2x}}{2}$

$v = \frac{e^{2x}}{2}$

Now using $\int udv$ =uv- $\int vdu$

$\int x^3 e^{2x}\, dx = \frac{x^3 e^{2x}}{2} - \int \frac{3x^2}{2}. e^{2x}dx$

$= \frac{x^3 e^{2x}}{2} - \frac{3}{2}\int x^2 . e^{2x}dx$

Integrate

$\int x^2 . e^{2x}dx$ Following the earlier procedure

Let u= $x^2, du = 2x; dv = . e^{2x}, and\ v = \frac{1}{2}. e^{2x}$

$\int x^2 . e^{2x} dx = x^2 \frac{1}{2} . e^{2x} - \int 2x . \frac{1}{2} . e^{2x} dx$

$= \frac{x^2}{2} . e^{2x} - \int e^{2x}\, dx$

$= \frac{x^2}{2} . e^{2x} - \frac{1}{2} e^{2x}$

Now

$\int x^3 e^{2x}\, dx = \frac{x^3 e^{2x}}{2} - \frac{3}{2} \int x^2 . e^{2x} dx$

$= \frac{x^3 e^{2x}}{2} - \frac{3}{2} \left[\frac{x^2}{2} . e^{2x} - \frac{1}{2} e^{2x} \right]$

$= \frac{x^3 e^{2x}}{2} - \frac{3}{4} [x^2 e^{2x} - e^{2x}] + c$

Example III

$\int x^2 \ln x\, dx$

Let $u = \ln x,\ so that\ du = 1/x\ \ and\ \ dv = x^2\ then\ v = \frac{x^3}{3}$

Then

$$\int x^2 \ln x\, dx = \frac{x^3 \ln x}{3} - \int \frac{x^3}{3} \frac{1}{x} dx$$

$$= \frac{x^3 \ln x}{3} - \int \frac{x^2}{3} dx$$

$$= \frac{x^3 \ln x}{3} - \frac{1}{3} \int x^2 dx$$

$$= \frac{x^3 \ln x}{3} - \frac{1}{3} \cdot \frac{x^3}{3}$$

$$= \frac{x^3 \ln x}{3} - \frac{x^3}{9} + c \quad or \quad \frac{x^3}{9} [3 \ln x - 1] + c$$

Do the following exercises IV

Evaluate

1. $\int x^2 sinx dx$ 2. $\int e^{3x} \cos 3x dx$ 3. $\int x^2\, e^{-3x} dx$

Appendix VI

Calculus Pre-test (CPT) Pre-test (Draft)

Instruction: answer all the questions

1 Find y' and y'' given that $x^3y + xy^3 = 2$ at $x = 1, y = 1$

2 Differentiate y= $(x^2 + 4)(2x^3 - 1)$

3 Find the $\frac{dy}{dx}$ given that

$$y = \left(\frac{x^3 - 1}{2x^3 + 1} \right)^4$$

4.a Differentiate y= sin^2x (b) Find the $\frac{d^2y}{dx^2}$, given that x=t+$1/t$, y= t+1

5. Differentiate

(a) $y = tan^2(3x - 2)$ (b) y= $ln^2(x + 3)$ (c) $\frac{x^2}{cos2x}$

Appendix VII

Calculus Achievement Test (CAT) **Post-test (Draft)**

Instruction: answer all the questions

5. $\int (2x-7)^3\,dx$ (5marks)

6. $\int (\sqrt[3]{2x^2+3})x$ dx (5marks)

7. $\int \frac{4x-8}{x^2-4x+5}$ dx (5marks)

8. $\int xsecx^2dx$ (5marks)

5(i) $\int tanxsec^2 x$ (ii)$\int sin^2 xcosxdx$ (5marks)

6 $\int \frac{3x+1}{2x-3}$ dx (5marks)

7.$\int x^2 e^{3x}$dx (ii) $x^3 logxdx$ (10marks)

8. $\int x^2 tan^{-1}x$dx (10marks)

Appendix VIII

Pre-Experimental Treatment Data

To control any differences between experimental and control groups at the beginning of the study, pre-test was administered. Calculus Pre-Test (CPT) was used as a pre-test to determine the academic levels of both experimental and control groups. Pre-test data for the groups were analyzed using t-test statistics. The results of the analysis are presented in the table below. Similarly, attitude questionnaire was also administered as a pre-test.

Test of Homogeneity of Variance

Groups	N	Mean	SD	df	t-cal	t-cri	p-value	decision
Experimental	65	31.97	16.38	13	-0.05	1.96	0.96	**
Control	70	32.11	16.62	3				

** Not significant at $p \leq 0.05$

From the table above, the p-value obtained is 0.96 at $p \leq 0.05.$ this is not significant. The value of t-calculated of -0.05 which is found to be less than t- critical of 1.96 further confirms the result. The null hypothesis is thus accepted. This means that there is no significant difference between the performance of the experimental and the control group. The result indicated that the two groups are academically equivalent before the experiment started.

Before beginning the experiment the subjects were assessed to ascertain their based attitude towards calculus by asking the subjects to complete Attitude Towards Calculus Inventory. The attitude pre-test data for the groups were analysed using independent sample t-test statistics. The results of the analysis are presented in the table below

Result of the t-test analysis of Pre-test on Attitude of Experimental and

Control Groups

Groups	N	Mean	SD	df	t-cal	t-cri	p-value	decision
Experimental	65	32.95	13.03	133	0.42	1.96	0.67	**
Control	70	32.09	10.86					

** Not significant at $p \leq 0.05$

From the table above, the p-value obtained is 0.67 at $p \leq 0.05.$ this is not significant. The value of t-calculated of 0.42 which is found to be less than t- critical of 1.96 further confirms the result. The null hypothesis is thus accepted. This means that there is no significant difference between the attitude towards calculus of the experimental and the control group. This indicated that the two groups are equivalent in relation to their attitude towards calculus before the experiment.

Appendix IX

Students' Performance in Calculus at some Colleges of Education, North-Central Zone (2009/10-2013/14)

Year	Colleges	Pass	%	Fail	%	Total
2009/10	COE Minna	74	37.95	121	62.05	195
	FCE Okene	40	41.24	57	58.76	97
	COE Gindiri	41	39.05	64	60.95	105
	COE Ilorin	46	40.35	68	59.65	114
	COE Katsina-Ala	25	34.72	47	65.28	72
	COE (FCT) Zuba	24	38.10	39	61.90	63
2010/11	COE Minna	56	31.46	122	68.54	178
	FCE Okene	33	39.29	51	60.71	84
	COE Gindiri	39	34.82	73	65.18	112
	COE Ilorin	40	40.82	58	59.18	98
	COE Katsina -Ala	28	41.18	40	58.82	68
	COE (FCT) Zuba	27	37.50	45	62.50	72
2011/12	COE Minna	58	40.85	84	59.15	142
	FCE Okene	32	42.11	44	57.89	76
	COE Gindiri	35	40.70	51	59.30	86
	COE Ilorin	35	39.33	54	60.67	89
	COE Katsina -Ala	32	38.10	52	61.90	84
	COE (FCT) Zuba	24	35.29	44	64.71	68
2012/13	COE Minna	48	41.74	67	58.26	115
	FCE Okene	34	40.96	49	59.04	83
	COE Gindiri	36	39.13	56	60.86	92
	COE Ilorin	32	31.68	69	68.32	101
	COE Katsina -Ala	25	36.23	44	63.77	69
	COE (FCT) Zuba	24	32.43	50	67.57	74
2013/14	COE Minna	58	32.22	122	67.78	180
	FCE Okene	28	38.89	44	61.11	72
	COE Gindiri	40	45.45	48	54.55	88
	COE Ilorin	30	37.97	49	62.03	79
	COE Katsina -Ala	23	35.38	42	64.62	65
	COE (FCT) Zuba	26	41.27	37	58.73	63

Source: Departments of Mathematics, Colleges of Education, North-Central Geopolitical Zone (2014)