

THERMAL PROPERTIES OF THE LANDAU - CLUSTER IN THE INTEGRAL
QUANTUM HALL EFFECT.

BY

ABDULKADIR AJUJI

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Physics Department
Faculty of Science
Ahmadu Bello University,
ZARIA - NIGERIA.

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Dedicated to my fiancée Rakiya Abdullahi Makarfi.


DECLARATION

It is hereby declared that:

1. The report presented in this thesis is the results of research carried out by the author.
2. This thesis has not been submitted in whatever form to any other institution, organisation or body for the award of any degree.
3. All inclusion from the work of others have been duly acknowledged.

A. Ajuji
(Author)

Date



Dr. S.B. Elegba
(Supervisor)

14/2/89

Date

Co-supervisor

Date

CERTIFICATION

This thesis entitled; Thermal Properties of the Landau-cluster, in the integral Integral Quantum Hall Effect (IQHE); by Abdulkadir Ajuji meets the regulations governing the award of the degree of Master of Science of Ahmadu Bello University, Zaria, and is approved for its contribution of knowledge and literary presentation.



 Chairman, Supervisory Committee

14/2/89.

 Date



 Member, Supervisory Committee

14/2/89

 Date



 Head of Department

14/2/89

 Date



 Dean, Postgraduate School

28/6/89

 Date

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ABSTRACT

The thermal transport properties of two-dimensional electron gas system in a uniform magnetic field, specifically the thermopower is calculated using **the** thermal response coefficients in which the effect of disorder has not been considered.

In addition it has also been shown that;the IQHE is not due to the effect of impurities, but as an independent electron phenomena in which coulomb interaction is ignored.

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ABBREVIATIONS AND SYMBOLS

J_i	Particle current density ($A.m^{-2}$)
L_{ij}	Response coefficient
S_{ij}	Thermopower tensor
$f(E)$	Fermi function
μ	Chemical potential
T	Temperature
∇V	Potential gradient
A	Vector potential
B	Magnetic induction (T or $wb.m^{-2}$)
ω_c	classical cyclotron frequency
ϵ	$= (N + \frac{1}{2})\hbar\omega_c$ Harmonic oscillator eigenvalue
K	Wavevector in K-space ($K = \frac{2\pi n}{L}$)
H	Hamiltonian
σ_{ij}	Conductivity tensor
K	Thermal conductivity
∇	Del operator
$-e$	Electronic charge
K_B	Boltzman constant ($8.62 \times 10^{-5} \text{ evk}^{-1}$)
P	momentum

E	Electric field
δ_{ij}	Kronecker delta
β	$= (k_e T)^{-1}$ Inverse temperature.
2DEG	Two-dimensional electron gas
QHE	Quantum Hall Effect.
FQHE	Fractional Quantum Hall Effect
1eV	$= 1.602 \times 10^{-19} \text{J} = 1.605 \times 10^4 \text{K}$
$\frac{k}{e}$	$= 86.2 \times 10^{-6} \text{VK}^{-1} = 86.2 \mu\text{VK}^{-1}$.

CHAPTER 1

INTRODUCTION

In the last fifteen years or so, there has been a lot of interest in the study of two-dimensional electron systems. Such systems were recently reviewed by Ando and co-workers (Ando, et al, 1982). This interest has been motivated partly by the relevance of two-dimensional systems to the properties of surfaces and thin films. The inversion layers of semi-conductor surfaces are a good example of two-dimensional systems. When a sufficiently strong electric field is applied across the interface of an insulator and a P-type semiconductor, and the conduction band edge is bent below the fermi level in the bulk, an n-type inversion layer is formed. The motion of the carrier in the direction perpendicular to the surface is quantized and the energy levels of the electrons are grouped in electric sub-bands, each of which corresponds to a quantized level of motion in the Z-direction, with a continuum for motion in the xy-plane parallel to the surface. It is possible for only the lowest electric sub-bands to be occupied at sufficiently low temperatures, and one can regard the inversion layer essentially as a two-dimensional system.

Typical behaviour of the two-dimensional system appears especially when a strong magnetic field is applied perpendicular to the surface. The orbital motion of the electrons is perfectly quantized and the energy spectrum becomes discrete and are called Landau levels. The transport properties of such systems have attracted even a lot more interest since the experimental discovery by von Klitzing, Dorda and Pepper (von Klitzing et al, 1980) that the Hall conductivity of a metal oxide semi-conductor interface (MOSFET) is quantized. This is the integral Quantum Hall effect (IQHE) by which the Hall conductivity assumes the value .

$$\sigma_{xy} = \frac{ie^2}{h}$$

where i is a non zero integer

Thus, the quantized values depend only on the fundamental constants.

Although some aspects of this phenomenon had been predicted in 1975 by Ando, Matsumoto and Uemura (Ando et al, 1975), the precision of the quantized Hall plateaus was completely unexpected. This new interest has led to a flurry of theoretical and experimental activities, some of which include Aoki and Ando, 1981; Baraff and Tsui, 1981; Girvin et al, 1982, Glasser 1983; Niizaki et al, 1983; Laughlin 1981, Elegba 1987, Apel, 1987; and many others. Most aspects of the effect, which occur in particular when the Fermi level is between two Landau levels of the sample, can be understood with an essentially single-electron description, in which the electron-electron interaction has only a secondary role. The approach followed by most of these authors (except Laughlin 1981) is that the intergral quantum Hall effect is due to the effect of impurities which result in the localization of the electrons.

Subsequent to the experimental discovery of von Klitzing et al (1980), Tsui, Stormer and Gossard (Tsui et al, 1982) used high-carrier-mobility heterojunctions, GaAs-Al_xGa_{1-x}As and discovered σ_{xy} plateaus similar to those of von Klitzing et al but at fractional, instead of integral values of i ,

$$\text{i.e. } i = P/q$$

where P is a small integer 1,2,3, ... and q is an odd integer 1,3,5,7, This is the fractional quantum Hall effect (FQHE). Here again, there have been a lot of theoretical effort to explain the phenomenon but none yet is conclusive. Some of the works on the FQHE include Laughlin, 1983b, Haldane, 1983; Halperin, 1983; Girvin and Jach, 1984; Priusken, 1984; Levin et al, 1984a; 1984b; 1984c; Rasolt and MacDonald, 1986 Girvin et al,

1985; 1986; Apel, 1987a, 1987b, Lee et al 1987, Gafels, 1987; Chui, 1987; Kivelson et al, 1981 and many more others.

Although the thrust of the present work is on the thermal properties of the integral quantum Hall effect, the comment on the fractional quantum Hall has been added only for the sake of completeness to highlight the present state of difficulty in the development of a theoretical explanation of the quantum Hall effect. Before going to the subject matter of this work it is vital to state here that the theoretical model used for the explanation of the integral quantum Hall effect here is conceptually different from all those cited above in the sense that the phenomenon does not require the presence of impurities. Rather, it is based on a mass-symmetry breaking operation for a finite number of electrons, still without including electron-electron interaction. This model was first proposed by Elegba et al, 1987; and will be discussed in greater details in chapter 3.

This is the basis of the formation of the Landau electron clusters. This result is then used to study thermoelectric effect which has always been used as a sensitive probe of transport mechanisms in metals and semiconductors.

Based on the relationship between the various transport coefficients (Luttinger, 1964; Mahan, 1981), the thermopower was calculated and details of this calculation are presented in chapter 3. With these results, other important informations can be obtained (at least in principle). Information about the structure of the energy spectrum, can be deduced from measurements of thermodynamic properties such as entropy and heat capacity.

The calculated thermopower provides a measure of the entropy as long as electronic contribution dominates - (i.e. when phonon contributions are neglected). Experimental measurements, however, still seem to be

scanty (Obloh et al, 1984; Smith et al, 1984; Fletcher et al, 1985). This is not say that the theory has been well developed but there is the need for comparison with experiment. In this regard, some theoretical results have been reported by Oji, (1984); Girvin and Jonson (1982); Jonson and Girvin (1984) and by MacDonald et al, (1986). Although the results of the present work agree qualitatively with those obtained by Jonson and Girvin (1984), differences still remain on the role of impurities in the system, which in our case play no role. Furthermore, the results of the present work agrees quantitatively with the experimental results of Fletcher et al, (1985). The results of the present work are discussed in Chapter 4.

CHAPTER 2

MOTION OF AN ELECTRON IN A MAGNETIC FIELD

2.1 GENERALISED COORDINATES AND LAGRANGES' EQUATIONS

The basis of classical non-relativistic mechanics in Newton's law of motion for a single particle,

$$\hat{F} = m\ddot{r} \quad 2.1$$

relating the forces acting on the particle to the acceleration.

For conservative forces, the force is derivable from a potential,

$$\hat{F} = -\hat{\nabla}V \quad 2.2$$

where V is a function of coordinates possibly of time. Eqn (2.1) is a second-order differential equation (actually, because it is a vector equation, it is equivalent to three equations obtained by resolving the vectors along three orthogonal axes) whose solution in terms of the initial particle position and velocity specifies the motion of the particle for all future and past time. The resolution of the vector equation in terms of three cartesian coordinates is straight forward. However, in many cases the symmetries of the problem or the constraints imposed indicated that it would be convenient to use some other set of orthogonal coordinates. For this purpose, one considers a system consisting of N particles, it thus has $3N$ degrees of freedom. One may choose any convenient set of "generalised coordinates" q_i ($i = 1, 2, 3, \dots, 3N$) to describe the system.

These will be related to the $3N$ cartesian coordinates describing the particle by the equations

$$\begin{aligned}x_i &= x_i (q_1, q_2, \dots, q_{3N}, t) \\y_i &= y_i (q_1, q_2, \dots, q_{3N}, t) \\z_i &= z_i (q_1, q_2, \dots, q_{3N}, t)\end{aligned}\quad 2.3$$

or simply

$$r_i = r_i (q_i, t) \quad 2.4$$

while the coordinate transformation of Eqn (2.3) can be substituted directly into Eqn (2.1), and for conservative forces into Eqn (2.2) the resulting equations are in general complex and difficult to solve. In mechanics, it is usual to consider a function L of the generalised coordinates q_i , generalised velocities \dot{q}_i and time t .

It is assumed that the functions $q_i(t)$ are so chosen as to make the integral W , defined by,

$$W = \int_{t_1}^{t_2} L dt \quad 2.5$$

an extremum. Here t_1 and t_2 are to be considered as fixed times. This condition on the functions $q_i(t)$ can be expressed by saying that an arbitrary small variation δq_i in the function $q_i(t)$ does not alter the value of the integral W . The variations δq_i are assumed to vanish at t_1 and t_2 . In calculus of variation,

$$\begin{aligned}\delta W &= \delta \int_{t_1}^{t_2} L dt \\&= \int_{t_1}^{t_2} \left(\frac{\partial L}{\partial q_i} \delta q_i + \frac{\partial L}{\partial \dot{q}_i} \delta \dot{q}_i \right) dt = 0\end{aligned}\quad 2.6$$

The functions \dot{q}_i are not independent of q_i , and hence

$$\delta \dot{q}_i = \frac{d}{dt} (\delta q_i) \quad 2.7$$

The second term in the parentheses of Eqn (2.6) can be integrated by parts using Eqn (2.7)

$$\int_{t_1}^{t_2} \frac{\partial L}{\partial \dot{q}_i} \delta \dot{q}_i dt = \int_{t_1}^{t_2} \frac{\partial L}{\partial \dot{q}_i} \frac{d}{dt} \delta q_i dt$$

$$= \left. \frac{\partial L}{\partial q_i} \delta q_i \right|_{t_1}^{t_2} - \int_{t_1}^{t_2} \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) \delta q_i dt \quad 2.8$$

since the variations δq_i vanish at t_1 and t_2 ,

$$\int_{t_1}^{t_2} \frac{\partial L}{\partial \dot{q}_i} \delta \dot{q}_i dt = - \int_{t_1}^{t_2} \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) \delta q_i dt \quad 2.9$$

using this expression, Eqn (2.6) can be written as

$$\sum_i \int_{t_1}^{t_2} \left[\frac{\partial L}{\partial q_i} - \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) \right] \delta q_i dt = 0 \quad 2.10$$

since the variations δq_i were assumed to be arbitrary, this equation can be valid only if the bracketed expression vanishes:

$$\frac{\partial L}{\partial q_i} - \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) = 0 \quad 2.11$$

This equation is known as the Euler equation of the calculus of variation. It represents a set of differential equations determining the functions $q_i(t)$ in such a way as to minimize (or maximize) the integral W of Eqn (2.5). The derivatives in (2.11) are to be formed as though q_i and \dot{q}_i are independent variables.

To obtain the equations of motion for conservative forces, let us assume that

$$L = T - V \quad 2.12$$

For this choice of the function L , the Euler's equation becomes Lagrangian equations and the function L is known as the Lagrangian.

In the case of non-conservative systems, if the velocity-dependent forces can be related to a generalised potential function U by

$$F_i = - \frac{\partial U}{\partial q_i} + \frac{d}{dt} \left(\frac{\partial U}{\partial \dot{q}_i} \right) \quad 2.13$$

the Lagrangian function then takes the form

$$L = T - U$$

An (extremely) important example of this kind of velocity-dependent force that fits this specification is that of the (Lorentz) force acting on a charge particle in an electromagnetic field. The

force is given by,

$$\vec{F} = q \left[\vec{E} + \frac{1}{c} (\vec{v} \times \vec{B}) \right] \quad 2.15$$

the fields are expressed in terms of the scalar potential and the vector potential \vec{A} .

$$\begin{aligned} \vec{E} &= -\vec{\nabla}\phi - \frac{1}{c} \frac{\partial \vec{A}}{\partial t} \\ \vec{B} &= \vec{\nabla} \times \vec{A} \end{aligned} \quad 2.16$$

These equations do not uniquely specify ϕ and \vec{A} . Maxwell's equations expressed in terms of ϕ and \vec{A} take their simplest form when the scalar and vector potential are related to each other through the Lorentz condition

$$\vec{\nabla} \cdot \vec{A} + \frac{1}{c} \frac{\partial \phi}{\partial t} = 0 \quad 2.17$$

On substituting Eqn(2.16) into Eqn(2.15), one obtains

$$\vec{F} = q \left[-\vec{\nabla}\phi - \frac{1}{c} \left[\frac{\partial \vec{A}}{\partial t} - \vec{v} \times (\vec{\nabla} \times \vec{A}) \right] \right] \quad 2.18$$

From the Poisson bracket for \vec{A} one obtains

$$\begin{aligned} \frac{d\vec{A}}{dt} &= \frac{\partial \vec{A}}{\partial t} + \sum_i \frac{\partial \vec{A}}{\partial q_i} \dot{q}_i + \frac{\partial \vec{A}}{\partial p_i} \dot{p}_i \\ &= \frac{\partial \vec{A}}{\partial t} + \frac{\partial \vec{A}}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial \vec{A}}{\partial y} \frac{\partial y}{\partial t} + \frac{\partial \vec{A}}{\partial z} \frac{\partial z}{\partial t} \\ &= \frac{\partial \vec{A}}{\partial t} + \vec{v} \left(\frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right) (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \\ &= \frac{\partial \vec{A}}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \vec{A} \end{aligned} \quad 2.19$$

But

$$\vec{v} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla} (\vec{v} \cdot \vec{A}) - (\vec{v} \cdot \vec{\nabla}) \vec{A} \quad 2.21$$

Eqn (2.18) can be written as

$$\vec{F} = q \left[-\vec{\nabla} \left(\phi - \frac{1}{c} \vec{v} \cdot \vec{A} \right) - \frac{1}{c} \frac{d\vec{A}}{dt} \right] \quad 2.22$$

comparing this equation with Eqn (2.13) one sees that the generalised potential U is

$$U = q \left(\phi - \frac{1}{c} \vec{v} \cdot \vec{A} \right) \quad 2.23$$

$$\text{since, } \frac{d}{dt} \left(\frac{\partial U}{\partial \vec{v}} \right) = - \frac{1}{c} \frac{d}{dt} \vec{A}$$

Now ϕ and \vec{A} are not dependent on velocity, thus the proper

Lagrangian is

$$L = T - q\phi + \frac{q}{c} \vec{v} \cdot \vec{A} \quad 2.24$$

2.2 HAMILTON'S EQUATION FOR A PARTICLE IN AN ELECTROMAGNETIC

FIELD.

A Hamiltonian function is defined by

$$H = \sum_i (p_i \dot{q}_i) - L \quad 2.25$$

Where P_i is the generalised momentum defined by

$$P_i = \frac{\partial L}{\partial \dot{q}_i} \quad 2.26$$

$$\text{and } L = L(q_i, \dot{q}_i, t) \quad 2.27$$

The Hamiltonian H can thus be expressed as a function of canonical variables.

$$H = H(P_i, q_i, t) \quad 2.28$$

Upon differentiation, one obtains

$$dH = \sum_i \left(\frac{\partial H}{\partial q_i} dq_i + \frac{\partial H}{\partial P_i} dP_i \right) + \frac{\partial H}{\partial t} dt \quad 2.29$$

from equation (2.25).

But

$$dH = \sum_i \left(p_i d\dot{q}_i + \dot{q}_i dp_i - \frac{\partial L}{\partial q_i} dq_i - \frac{\partial L}{\partial \dot{q}_i} d\dot{q}_i \right) - \frac{\partial L}{\partial t} dt \quad 2.30$$

hence, the first and fourth terms cancel because of (2.26)

thus

$$dH = \sum_i \left(\dot{q}_i dp_i - \frac{\partial L}{\partial q_i} dq_i \right) - \frac{\partial L}{\partial t} dt \quad 2.31$$

Comparing the coefficients of the independent differentials dp_i , dq_i and dt in Eqns. (2.29) and (2.31) gives the canonical equations of motion:

$$\dot{q}_i = \frac{\partial H}{\partial P_i}, \quad \dot{P}_i = -\frac{\partial H}{\partial q_i}, \quad \frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t} \quad 2.32$$

If one eliminates the momentum variables p_i that have been introduced into the Hamiltonian formalism, the result is not unexpectedly Lagrange's equations.

From (2.29)

$$\frac{dH}{dt} = \sum_i \left(\frac{\partial H}{\partial q_i} \dot{q}_i + \frac{\partial H}{\partial p_i} \dot{p}_i \right) + \frac{\partial H}{\partial t} \quad 2.33$$

From the canonical Eqns (2.32) this reduces to

$$\frac{dH}{dt} = \frac{\partial H}{\partial t} \quad 2.34$$

Therefore, if the Hamiltonian function is not an explicit function of time, then H is a constant of motion.

In the case of a dynamical system and a coordinate system such that the time does not appear in the equations defining the generalised coordinates, the kinetic energy T is a homogeneous quadratic function of the \dot{q}_i ;

$$T = \sum_{ij} \alpha_{ij} \dot{q}_i \dot{q}_j \quad \alpha_{ij} = \alpha_{ji} \quad 2.35$$

From this, it is seen that

$$\begin{aligned} \frac{\partial T}{\partial \dot{q}_i} &= 2 \sum_j \alpha_{ij} \dot{q}_j \\ \sum_i \dot{q}_i \frac{\partial T}{\partial \dot{q}_i} &= 2 \sum_i \alpha_{ii} \dot{q}_i^2 \\ &= 2 T \end{aligned} \quad 2.36$$

and assuming that the system is conservative ($L = T - V$)

$$p_i = \frac{\partial L}{\partial \dot{q}_i} = \frac{\partial T}{\partial \dot{q}_i} \quad 2.37$$

using (2.36) and (2.37) in Eqn (2.25) yields

$$\begin{aligned} H &= \sum_i \frac{\partial T}{\partial \dot{q}_i} \dot{q}_i - L \\ &= 2 T - (T - V) \\ H &= T + V \end{aligned} \quad 2.38$$

In this case the Hamiltonian function can be physically interpreted as the sum of the kinetic and potential energies of the system expressed as a function of the canonical variables.

Considering the motion of a charged particle moving in an

electromagnetic field. From Eqn (2.24) the Langrangian is

$$L = T - q\phi + \frac{q}{c} \vec{v} \cdot \vec{A} \quad 2.39$$

The generalised momenta, from Eqn (2.26) are given by

$$P_i = \frac{\partial L}{\partial \dot{q}_i} = \frac{\partial T}{\partial \dot{q}_i} + \frac{q}{c} \frac{\partial}{\partial \dot{q}_i} (\vec{v} \cdot \vec{A}) \quad 2.40$$

If the generalised coordiantes do not depend explicitly on time

$$\vec{v} \cdot \vec{A} = \sum_j \dot{q}_j \cdot A_j \quad 2.41$$

where A_j is not necessarily a cartesian component of \vec{A} . Hence

$$\frac{\partial}{\partial \dot{q}_i} (\vec{v} \cdot \vec{A}) = \sum_j A_j \cdot \delta_{ij} = A_i \quad 2.42$$

and

$$P_i = \frac{\partial T}{\partial \dot{q}_i} + \frac{q}{c} A_i \quad 2.43$$

where

$$\sum_i \dot{q}_i \frac{\partial T}{\partial \dot{q}_i} = 2T$$

as before

The Hamiltonian is then given by

$$\begin{aligned} H &= \sum_i \left(\frac{\partial T}{\partial \dot{q}_i} + \frac{q}{c} A_i \right) \dot{q}_i - [T - q\phi + \frac{q}{c} \vec{v} \cdot \vec{A}] \\ &= 2T + \frac{q}{c} \vec{v} \cdot \vec{A} - T + q\phi - \frac{q}{c} \vec{v} \cdot \vec{A} \\ &= T + q\phi \end{aligned} \quad 2.45$$

In this case it is seen that the Hamiltonian is just the total energy, since $q\phi$ is the potential energy of the particle.

For cartesian coordinates, the canonical momenta of Eqn (2.43) are given by

$$P_x = mv_x + \frac{q}{c} A_x \quad \text{e.t.c.} \quad 2.46$$

or vectorially by

$$\vec{P} = m\vec{v} + \frac{q}{c} \vec{A} \quad 2.47$$

Thus the canonical momentum no longer is the ordinary linear(kinetic) momentum $m\vec{v}$. From this, the Hamiltonian Eqn (2.45) is

$$H = \frac{1}{2m} \left(\vec{P} - \frac{q}{c} \vec{A} \right)^2 + q\phi \quad 2.48$$

This is the Hamiltonian of a non-relativistic charged particle in an electromagnetic field.

This is the starting point for all dynamical problems of a nonrelativistic charged particle in an electromagnetic fields.

2.3

TWO-DIMENSIONAL ELECTRON GAS IN A UNIFORM MAGNETIC FIELD

The non-relativistic Hamiltonian for a charged particle in an electromagnetic field derived in the last section is given by

$$H = \frac{1}{2m} (\vec{P} + \frac{e}{c} \vec{A})^2 + e\phi \quad 2.49$$

where \vec{P} is the canonical momentum and is related to the ordinary linear (Kinetic) momentum, P

$$\vec{P} = m\vec{v} + \frac{e}{c} \vec{A} \quad 2.50$$

and $-e$ is the charge of the electron ($e > 0$).

The Hamiltonian may be substituted into the Schrodinger equation

$$H\psi = E\psi$$

Since we shall treat the fields as externally applied entities, \vec{A} and ϕ are simply ordinary function of position and time.

$$H = \frac{1}{2m} (\vec{P} + \frac{e}{c} \vec{A})^2 \quad 2.52$$

Now,

$$\vec{B} = \vec{\nabla} \times \vec{A} \quad 2.53$$

$$\vec{F} = e [\vec{E} + \frac{1}{c} (\vec{v} \times \vec{B})] \quad 2.54$$

The fields are expressed in terms of the scalar potential and the vector potential \vec{A} .

$$\vec{E} = -\vec{\nabla}\phi - \frac{1}{c} \frac{\partial \vec{A}}{\partial t} \quad 2.55$$

The Lorentz condition Eqn (2.17)

$$\vec{\nabla} \cdot \vec{A} + \frac{1}{c} \frac{\partial \phi}{\partial t} = 0$$

$$\text{reduces to } \vec{\nabla} \cdot \vec{A} = 0 \quad 2.56$$

$$\text{Since } [f(x), P_x] = i\hbar \frac{\partial f(x)}{\partial x} \quad 2.57$$

$$\text{then } [A, p] = i\hbar \vec{\nabla} \cdot \vec{A} \quad 2.58$$

$$= 0 \quad \text{from (2.56) above.}$$

Thus the momentum operator and the vector potential commute with each other.

Expanding Eqn (2.52) and making use of the commutation relation (2.58) we obtain

$$H = \frac{1}{2m} (P^2 + \frac{2e}{c} \vec{P} \cdot \vec{A} + \frac{e^2}{c^2} A^2) \quad 2.59$$

$$= \frac{P^2}{2m} + \frac{e}{mc} \vec{A} \cdot \vec{P} + \frac{e^2}{2mc^2} A^2 \quad 2.60$$

Furthermore in the case of a uniform magnetic field, the vector potential \vec{A} can be written as

$$\vec{A} = \frac{1}{2} \vec{B} \times \vec{r} \quad 2.61$$

Assuming that the uniform magnetic field is in the Z direction (0, 0, B)

The symmetric gauge

$$\vec{A} = (-\frac{By}{2}, \frac{Bx}{2}, 0) \quad 2.62$$

shall be used.

In this gauge the Schroedinger equation

$$H \psi = E \psi \quad 2.63$$

reduces to

$$\begin{aligned} & -\frac{\hbar^2}{2m} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \psi + \frac{1}{2m} \frac{eB\hbar}{ic} \left(y \frac{\partial}{\partial x} - x \frac{\partial}{\partial y} \right) \psi \\ & + \frac{1}{2m} \frac{e^2 B^2}{4c^2} (x^2 + y^2) \psi = E \psi \\ & -\frac{1}{2} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \psi + \frac{ieB}{2\hbar c} \left(x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x} \right) \psi + \frac{e^2 B^2}{8\hbar^2 c^2} (x^2 + y^2) \psi \\ & = E \psi \\ & -\frac{1}{2} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \psi + \frac{i\alpha}{2} \left(x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x} \right) \psi + \frac{\alpha^2}{8} (x^2 + y^2) \psi = E \psi \quad 2.64 \end{aligned}$$

where

$$\alpha = \frac{eB}{\hbar c} = \frac{1}{l^2} \quad \text{and } w_c = \frac{eB}{mc}$$

Eqn (2.64) can be solved in cylindrical coordinates, by using the associated Laguerre functions, and has a general solution (Elegba, 1985),

$$\psi_{nm}(\vec{r}) = \chi_{nm}(r) \frac{e^{im\phi}}{\sqrt{2\pi}} \quad 2.65$$

The function

$$\chi_{nm}\left(\frac{\alpha r^2}{2}\right) = \left(\frac{\alpha N!}{2^N (N+|m|)!}\right)^{1/2} e^{-\frac{\alpha r^2}{4}} \left(\frac{\alpha r^2}{2}\right)^{\frac{|m|}{2}} L_N^{|m|}\left(\frac{\alpha r^2}{2}\right) \quad 2.66$$

is the radial part of the eigenfunction in terms of Laguerre function,

$$N = n - \frac{(m + |m|)}{2}, \quad -\infty < m \leq n \quad 2.67$$

$$\alpha = \frac{eB}{\hbar c} = \frac{1}{l^2} \quad 2.68$$

l is the classical radius of the ground state Landau orbit, and is given by

$$l = \left(\frac{\hbar c}{eB}\right)^{1/2} \quad 2.69$$

n = the principal quantum number

m = the magnetic quantum number

The radial wavefunctions $\chi_{nm}\left(\frac{\alpha r^2}{2}\right)$ form an orthonormal set

$$\int_0^\infty r dr \chi_{nm}\left(\frac{\alpha r^2}{2}\right) \chi_{n'm}\left(\frac{\alpha r^2}{2}\right) = \delta_{n'n} \quad 2.70$$

The corresponding eigenvalues of the Schrödinger equation (2.63) are called the Landau levels and are given by

$$E_n = \hbar \omega_c \left(n + \frac{1}{2}\right) \quad 2.71$$

where $n = 0, 1, 2, \dots$

The quantities expressed by Equations 2.65 and 2.71 represent the standard solution for a single two-dimensional electron in strong magnetic field.

CHAPTER 3

TRANSPORT COEFFICIENTS OF FEW ELECTRON SYSTEM

The present approach to the study of the integral quantum Hall effect (IQHE) is based on a consideration of a small number of electrons in the system without the inclusion of the coulombic interaction. In earlier theoretical efforts, notably by Laughlin (1982), the noninclusion of the coulombic interaction has been justified. The approach of Laughlin (1982, 1983) is however conceptually different from the present one. Laughlin proposed a variational wave function on the basis of which the electronic density was shown to exhibit some integral quantization. There have been other theoretical attempts to demonstrate IQHE such as the field-theoretic approach of Pruisken et al, (1984), the group-theoretic approach of Brezin et al, (1984), and the many-body formalism of Girvin et al, (1985, 1986), all of which are essentially based on the trial wavefunction of Laughlin.

The hypothesis of the transformation of the Landau - Cluster of two-dimensional electron in a magnetic field was first proposed by Elegba et al (1988) to explain the integral quantum Hall effect. According to this hypothesis, it was shown that only odd-number electrons can form energetically stable clusters.

For this purpose a finite number of electrons is confined to the X-y plane under the influence of a magnetic field B perpendicular to the plane containing the electrons.

The Hamiltonian of such a system is of the form

$$H = \frac{1}{2m} (\vec{P} + \frac{e}{c} \vec{A})^2 + e^2 \sum_{i < j}^Q \frac{1}{r_{ij}} \quad 3.1$$

where Q is the number of electrons in the system.

Considering a system of two electrons, the Hamiltonian becomes,

$$H = \frac{1}{2m} (\vec{P}_1 + \frac{e}{c} \vec{A}_1)^2 + \frac{1}{2m} (\vec{P}_2 + \frac{e}{c} \vec{A}_2)^2 + \frac{e^2}{r_{12}} \quad 3.2$$

In the centre-of-mass and internal coordinates, given by

$$Z_a = \frac{Z_1 + Z_2}{2} \quad \text{and} \quad Z_b = \frac{Z_1 - Z_2}{\sqrt{2}} \quad 3.3$$

the momenta and vector potentials transform as

$$\begin{aligned} \vec{P}_a &= \vec{P}_1 + \vec{P}_2, & \vec{P}_b &= \frac{\vec{P}_1 - \vec{P}_2}{\sqrt{2}} \\ \vec{A}_a &= \vec{A}_1 + \vec{A}_2, & \vec{A}_b &= \frac{\vec{A}_1 - \vec{A}_2}{\sqrt{2}} \end{aligned} \quad 3.4$$

We have

$$\begin{aligned} \vec{P}_1 &= \frac{1}{2} (\vec{P}_a + \sqrt{2} \vec{P}_b), & \vec{A}_1 &= \frac{1}{2} (\vec{A}_a + \sqrt{2} \vec{A}_b) \\ \vec{P}_2 &= \frac{1}{2} (\vec{P}_a - \sqrt{2} \vec{P}_b), & \vec{A}_2 &= \frac{1}{2} (\vec{A}_a - \sqrt{2} \vec{A}_b) \\ P_1^2 + P_2^2 &= \frac{1}{4} (2P_a^2 + 4P_b^2) \\ A_1^2 + A_2^2 &= \frac{1}{4} (2A_a^2 + 4A_b^2) \\ \vec{P}_1 \vec{A}_1 + \vec{P}_2 \vec{A}_2 &= \frac{1}{4} (2\vec{P}_a \vec{A}_a + 4\vec{P}_b \vec{A}_b) \end{aligned}$$

Eqn (3.2) becomes

$$\begin{aligned} H &= \frac{1}{2m} [P_1^2 + P_2^2 + \frac{2e}{c} (\vec{P}_1 \vec{A}_1 + \vec{P}_2 \vec{A}_2) + \frac{e^2}{c^2} (A_1^2 + A_2^2)] + \frac{e^2}{\sqrt{2}|Z_b|} \\ &= \frac{1}{2m} [\frac{1}{4} (2P_a^2 + 4P_b^2) + \frac{2e}{c} \cdot \frac{2}{2} (\vec{P}_a \vec{A}_a + 2\vec{P}_b \vec{A}_b) + \frac{e^2}{4c^2} (2A_a^2 + 4A_b^2)] \\ &\quad + \frac{e^2}{\sqrt{2}|Z_b|} \\ &= \frac{1}{2m} [\frac{1}{4} (2P_a^2 + \frac{4e}{c} \vec{P}_a \vec{A}_a + \frac{2e^2}{c^2} A_a^2) + (P_b^2 + \frac{2e}{c} \vec{P}_b \vec{A}_b + \frac{e^2}{c^2} A_b^2)] \\ &\quad + \frac{e^2}{\sqrt{2}|Z_b|} \\ &= \frac{1}{2m} [\frac{1}{2} (\vec{P}_a + \frac{e}{c} \vec{A}_a)^2 + (\vec{P}_b + \frac{e}{c} \vec{A}_b)^2] + \frac{e^2}{\sqrt{2}|Z_b|} \\ &= \frac{1}{2(2m)} (\vec{P}_a + \frac{e}{c} \vec{A}_a)^2 + \frac{1}{2m} (\vec{P}_b + \frac{e}{c} \vec{A}_b)^2 + \frac{e^2}{\sqrt{2}|Z_b|} \quad 3.5 \end{aligned}$$

$$H = \frac{1}{2m_a} (\vec{P}_a + \frac{e}{c} \vec{A}_a)^2 + \frac{1}{2m_b} (\vec{P}_b + \frac{e}{c} \vec{A}_b)^2 + \frac{e^2}{\sqrt{2}|Z_b|} \quad 3.6$$

where $m_a = 2m$ and $m_b = m$.

The Hamiltonian separates into that of two particles with masses $m_a = 2m$ and $m_b = m$.

Neglecting the coulomb term in Eqn (3.6), this is the eigenvalue problem of two non interacting particles with masses m_a and m_b .

The corresponding eigenvalues for the two pseudo-particles are given by Eqn (2.71) as

$$E_a = \frac{\hbar\omega_c}{2} (N_a + \frac{1}{2}) \text{ and } E_b = \hbar\omega_c (n_b + \frac{1}{2})$$

From the energy spectra of the two subsystems (fig. 3.1) it is not possible to achieve a state of the system in which the subsystems are isoenergetic and therefore the entire system will not be at equilibrium. This implies that the two subsystems form a non-interacting system in which the energies never overlap. The relevance of this comment will be clearer in the next chapter.

In the case of a 3-electron system the Hamiltonian is given by

$$H = \frac{1}{2m} (\vec{P}_1 + \frac{e}{c} \vec{A}_1)^2 + \frac{1}{2m} (\vec{P}_2 + \frac{e}{c} \vec{A}_2)^2 + \frac{1}{2m} (\vec{P}_3 + \frac{e}{c} \vec{A}_3)^2 + \frac{e^2}{r_{12}} + \frac{e^2}{r_{13}} + \frac{e^2}{r_{23}} \quad 3.7$$

this can be transformed using the coordinate transformation

$$\begin{aligned} Z_a &= \frac{Z_1 + Z_2 + Z_3}{3} \\ Z_b &= \frac{1}{\sqrt{6}} (Z_1 + Z_2 - 2Z_3) \\ Z_c &= \frac{Z_1 - Z_2}{\sqrt{2}} \end{aligned} \quad 3.8$$

The momenta and vector potential transform as

$$\begin{aligned} \vec{P}_a &= \vec{P}_1 + \vec{P}_2 + \vec{P}_3 \\ \vec{P}_b &= \frac{\vec{P}_1 + \vec{P}_2 - 2\vec{P}_3}{\sqrt{6}} \\ \vec{P}_c &= \frac{\vec{P}_1 - \vec{P}_2}{\sqrt{2}} \end{aligned} \quad 3.9$$

that is

$$\begin{aligned} \vec{P}_1 &= \frac{1}{6} (2\vec{P}_a + \sqrt{6} \vec{P}_b + 3\sqrt{2} \vec{P}_c) \\ \vec{P}_2 &= \frac{1}{6} (2\vec{P}_a + \sqrt{6} \vec{P}_b - 3\sqrt{2} \vec{P}_c) \\ \vec{P}_3 &= \frac{1}{3} (\vec{P}_a - \sqrt{6} \vec{P}_b) \end{aligned}$$

Thus

$$P_1^2 + P_2^2 + P_3^2 = \frac{1}{36} [12P_a^2 + 36P_b^2 + 36P_c^2]$$

$$A_1^2 + A_2^2 + A_3^2 = \frac{1}{36} [12A_a^2 + 36A_b^2 + 36A_c^2]$$

$$\vec{P}_1 \vec{A}_1 + \vec{P}_2 \vec{A}_2 + \vec{P}_3 \vec{A}_3 = \frac{1}{36} [12\vec{P}_a \vec{A}_a + 36\vec{P}_b \vec{A}_b + 36\vec{P}_c \vec{A}_c]$$

Thus Eqn (3.7) becomes

$$H = \frac{1}{2m} [P_1^2 + P_2^2 + P_3^2 + \frac{2e}{c} (\vec{P}_1 \vec{A}_1 + \vec{P}_2 \vec{A}_2 + \vec{P}_3 \vec{A}_3) + \frac{e^2}{c^2} (A_1^2 + A_2^2 + A_3^2)] + \frac{e^2}{r_{12}} + \frac{e^2}{r_{23}} + \frac{e^2}{r_{31}} \quad 3.10$$

$$= \frac{1}{2m} \cdot \frac{1}{3} (P_a + \frac{e}{c} A_a)^2 + \frac{1}{2m} (P_b + \frac{e}{c} A_b)^2 + \frac{1}{2m} (P_c + \frac{e}{c} A_c)^2 + \frac{e^2}{r_{12}} + \frac{e^2}{r_{23}} + \frac{e^2}{r_{31}}.$$

$$H = \frac{1}{2m_a} (P_a + \frac{e}{c} A_a)^2 + \frac{1}{2m_b} (P_b + \frac{e}{c} A_b)^2 + \frac{1}{2m_c} (P_c + \frac{e}{c} A_c)^2 + \frac{e^2}{r_{12}} + \frac{e^2}{r_{23}} + \frac{e^2}{r_{31}} \quad 3.12$$

and

$$r_{12} = |z_1 - z_2| = \sqrt{2} |z_c|$$

$$r_{23} = |z_2 - z_3| = \sqrt{2} |-\frac{1}{2} z_c + \frac{\sqrt{3}}{2} z_b| \quad 3.13$$

$$r_{31} = |z_3 - z_1| = \sqrt{2} |-\frac{1}{2} z_c - \frac{\sqrt{3}}{2} z_b|$$

Therefore,

$$H = \frac{1}{2m_a} (P_a + \frac{e}{c} A_a)^2 + \frac{1}{2m_b} (P_b + \frac{e}{c} A_b)^2 + \frac{1}{2m_c} (P_c + \frac{e}{c} A_c)^2 + \frac{e^2}{\sqrt{2}} \left[\frac{1}{|z_b|} + \frac{1}{|-\frac{1}{2} z_c + \frac{\sqrt{3}}{2} z_b|} + \frac{1}{|-\frac{1}{2} z_c - \frac{\sqrt{3}}{2} z_b|} \right] \quad 3.14$$

where,

$$m_a = 3m, m_b = m_c = m. \quad \text{Again neglecting coulomb interaction}$$

the corresponding eigenvalues for the 3-particle cluster are;

$$E_a = \frac{\hbar\omega_c}{3} (N_a + \frac{1}{2}) \quad \text{and} \quad E_b = E_c = \hbar\omega_c (n_b + \frac{1}{2}). \quad 3.15$$

In this case of 3-particle system it is possible to achieve a state of the entire system in which the two subsystems are iso-energetic, Figures 3.2 and 3.3. Hence such configuration will be energetically more stable than the 2-particle system.

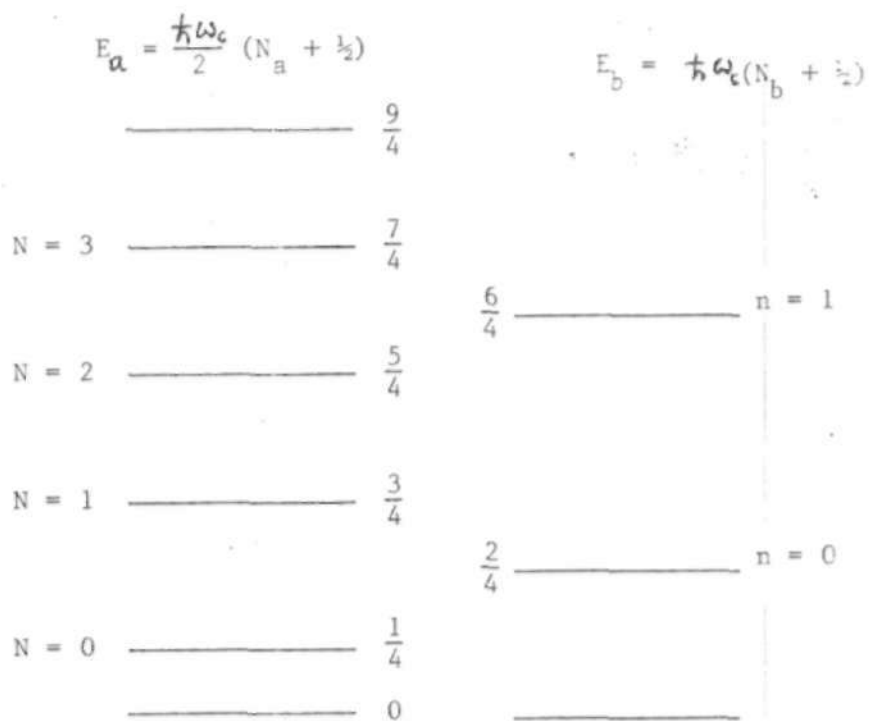


Fig. 3.1: Energy spectrum for the 2 particle Landau cluster

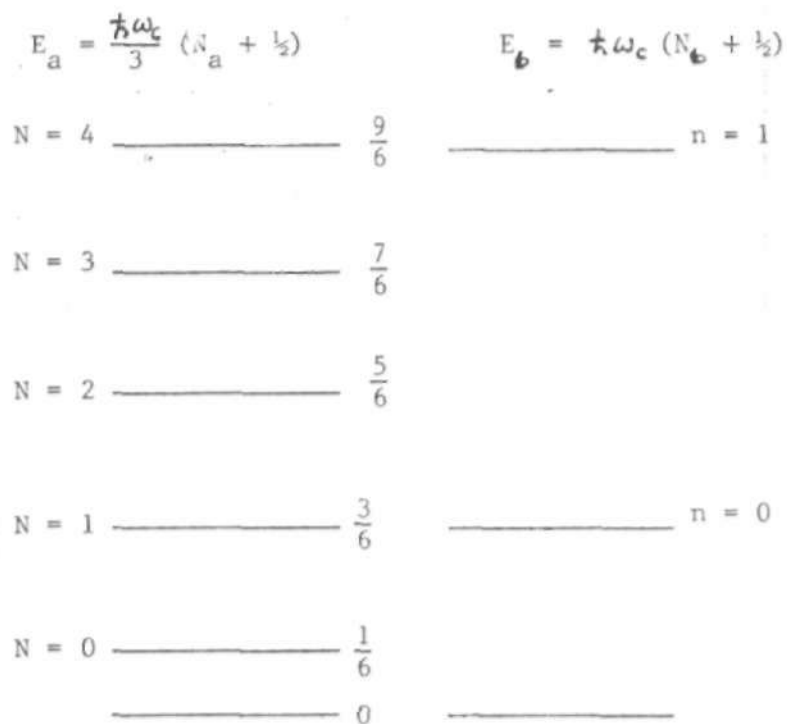


Fig. 3.2: Energy spectrum for the 3 particle Landau cluster

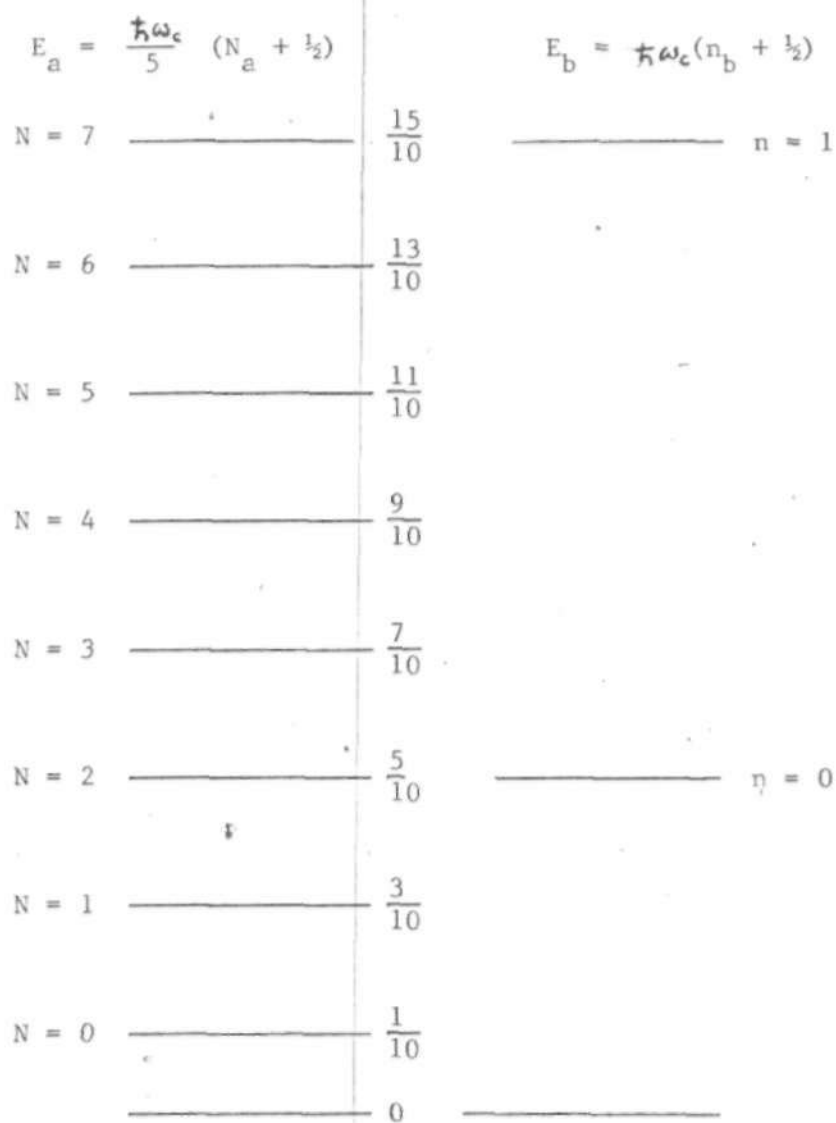


Fig. 3.3: Energy spectrum for the 5 particle Landau cluster.

The result can be extended to the case of more than 3-particles (Appendix C) and it has been shown in an earlier work by Elegba et al, that only odd-number particles can form energetically stable clusters. And that partial splitting of the degenerate Landau levels by the more massive particle is responsible for the integral quantum Hall effect which occurs whenever the Fermi energy crosses the energy levels of the massive particles. Hence further discussion will be restricted to only odd-number electron system. The transformation could be achieved for any odd-number particles. In general, the energy levels of the cluster are given by

$$E_N = \frac{\hbar\omega_c}{Q} (N + \frac{1}{2}) \quad Q = 3, 5, 7 \dots$$

$$N = 0, 1, 2 \dots \quad 3.16$$

for the massive particle.

and

$$E_{n_c} = \hbar\omega_c (n + \frac{1}{2}) \quad n = 0, 1, 2 \dots \quad 3.17$$

for the single particle.

In the next section the theory of transport coefficients will be considered particularly the coefficients that relates conductivity and thermopower in the presence of a strong magnetic field.

3.1 ONSAGER'S RECIPROCITY RELATION

Many physical phenomena show a certain symmetry which is often called the reciprocity. When a signal is sent out from a point A and is received at another point B, it is just as equally receivable as a signal sent in the reversed way. The transmission of signal is then said to be reciprocal. In irreversible processes, however we usually are dealing with various kinds of flow such as heat current, electric current, particle flow and so forth. These currents are driven by forces which are produced by the general tendency of nature to recover equilibrium once a system is shifted from it. So far as the state is not far from

equilibrium one may assume linear relations between flows and forces namely

$$J_i = \sum_j Z^{ij} X_j \quad 3.18.$$

where J_i ($i = 1, 2, 3, \dots$) denotes flows of various sorts and the X_i 's are forces, the coefficients Z^{ij} are called kinetic coefficients, examples of which are heat conductivity, diffusion constant, electric conductivity and so on. Generally, the off-diagonal elements of the matrix (Z^{ij}) will not vanish; they are responsible for cross effects. It has also been recognised empirically that reciprocal relations such as $Z^{ij} = Z^{ji}$ exist. For example, the tensor of heat conductivity. This is however not true in the presence of magnetic field. In general, the forces may be temperature gradient ∇T , or electric field $E = -\nabla V$, or concentration gradients which are expressed as gradients of the chemical potential $\nabla \mu$.

In an attempt to derive the coefficients, one starts with the kinetic equation

$$\frac{\partial S}{\partial t} = \sum_i J_i \cdot X_i \quad 3.19.$$

Suppose J refer to particle current, while J_E is the energy current.

For a free particle it is

$$\begin{aligned} \vec{j}_E &= \sum_p \vec{v}_p \epsilon_p n_p \\ \vec{J}_E &= \langle \vec{j}_E \rangle \end{aligned} \quad 3.20.$$

which is just the velocity of the particle times the energy times the number of particles. For a system with only these two currents, the forces are (Luttinger, 1964).

$$\begin{aligned} X_1 &= \frac{-e}{T} \vec{\nabla} V - \vec{\nabla} \left(\frac{\mu}{T} \right) \\ X_2 &= \vec{\nabla} \left(\frac{1}{T} \right) \end{aligned} \quad 3.21.$$

These currents and the forces are then substituted into Eqn (3.19).

The rate of entropy production is then

$$\begin{aligned} \frac{\partial S}{\partial t} &= \sum_i \vec{J}_i \cdot \vec{X}_i \\ &= \vec{J}_1 \cdot \vec{X}_1 + \vec{J}_2 \cdot \vec{X}_2 \\ &= -\vec{J} \left[\frac{e}{T} \vec{\nabla} V + \vec{\nabla} \left(\frac{\mu}{T} \right) \right] + \vec{J}_E \cdot \vec{\nabla} \left(\frac{1}{T} \right) \end{aligned} \quad 3.23.$$

By Eqn (3.18) and Eqn (3.19)

$$\frac{\partial S}{\partial t} = -\vec{J} \cdot \left[\frac{e}{T} \vec{\nabla} V + \vec{\nabla} \left(\frac{\mu}{T} \right) \right] + \vec{J}_E \cdot \vec{\nabla} \left(\frac{1}{T} \right)$$

the linear response equations are,

$$\vec{J}_x = -M_{xy}^{11} \left[\frac{e}{T} \vec{\nabla} V + \vec{\nabla} \left(\frac{\mu}{T} \right) \right] + M_{xy}^{12} \vec{\nabla} \left(\frac{1}{T} \right) \quad 3.24$$

$$\vec{J}_{E,x} = -M_{xy}^{21} \left[\frac{e}{T} \vec{\nabla} V + \vec{\nabla} \left(\frac{\mu}{T} \right) \right] + M_{xy}^{22} \vec{\nabla} \left(\frac{1}{T} \right) \quad 3.25$$

With these definitions one has the relationship $M_{xy}^{12} = M_{xy}^{21}$.

The current which describes thermal conductivity is

$$\vec{J}_Q = \vec{J}_E - \mu \vec{J} \quad 3.25$$

which for free particle has the form

$$\begin{aligned} \vec{J}_Q &= \sum_p n_p \vec{v}_p (\epsilon_p - \mu) \\ \vec{J}_Q &= \langle \vec{j}_Q \rangle \end{aligned} \quad 3.26$$

So that the reference energy is the chemical potential μ . This definition makes sense only for particles with a positive chemical potential μ . For example electrons above the Fermi surface in metals. If one takes an electron below the chemical potential and moves it down to the other end of the sample, one has moved energy. But when the electron arrives at the new location, it finds locally filled Fermi distribution, so that the only states available to it are above the Fermi energy. But the energy gain required to increase its energy to

get it above the Fermi energy must come from the surroundings. The energy gained from the surroundings must cool the locality, so that the electron has actually brought coldness with it. Only electrons which arrive with energy above the Fermi energy and give energy to the surroundings bring heat. Thus with this choice of thermal current, the forces are

$$\begin{aligned}\vec{X}_1 &= -\frac{1}{T} \vec{\nabla} \bar{\mu} \\ \vec{X}_2 &= \vec{\nabla} \left(\frac{1}{T} \right)\end{aligned}\quad 3.27$$

where we have taken

$$\bar{\mu} = \mu + eV \quad 3.28$$

Just as in (3.21) and (3.22) the rate of entropy production is

$$\begin{aligned}\frac{\partial S}{\partial t} &= -\vec{J} \cdot \left[\frac{e}{T} \vec{\nabla} V + \vec{\nabla} \left(\frac{\mu}{T} \right) \right] + \vec{J}_E \cdot \vec{\nabla} \left(\frac{1}{T} \right) \\ &= -\vec{J} \cdot \left[\frac{e}{T} \vec{\nabla} V + \frac{1}{T} \vec{\nabla} \mu + \mu \vec{\nabla} \left(\frac{1}{T} \right) \right] + \vec{J}_E \cdot \vec{\nabla} \left(\frac{1}{T} \right) \\ &= -\vec{J} \cdot \left[\frac{1}{T} \vec{\nabla} (eV + \mu) + \mu \vec{\nabla} \left(\frac{1}{T} \right) \right] + \vec{J}_E \cdot \vec{\nabla} \left(\frac{1}{T} \right) \\ &= -\vec{J} \cdot \left[\frac{1}{T} \vec{\nabla} \bar{\mu} \right] + (\vec{J}_E - \mu \vec{J}) \cdot \vec{\nabla} \left(\frac{1}{T} \right) \\ \frac{\partial S}{\partial t} &= \frac{-\vec{J}}{T} \cdot \vec{\nabla} \bar{\mu} + \vec{J}_Q \cdot \vec{\nabla} \left(\frac{1}{T} \right)\end{aligned}\quad 3.29$$

Thus the linear response equations are

$$\vec{J}_x = -\frac{1}{T} L_{xy}^{11} \vec{\nabla} \bar{\mu} + L_{xy}^{12} \vec{\nabla} \left(\frac{1}{T} \right) \quad 3.30a$$

$$\vec{J}_{Qx} = -\frac{1}{T} L_{xy}^{21} \vec{\nabla} \bar{\mu} + L_{xy}^{22} \vec{\nabla} \left(\frac{1}{T} \right) \quad 3.30b$$

where

$$L_{xy}^{12} = L_{xy}^{21}. \quad \text{These equations are consistent with the set (3.21)}$$

(3.23) and (3.24).

Similarly one can operate on the current equation 3.23 in order to find the relationship between the coefficients M^{ij} and L^{ij} .

Thus from (3.24)

$$\vec{J}_x = -M_{xy}^{11} \left[\frac{e}{T} \vec{\nabla} V + \frac{1}{T} \vec{\nabla} \mu + \mu \vec{\nabla} \left(\frac{1}{T} \right) \right] + M_{xy}^{12} \vec{\nabla} \left(\frac{1}{T} \right)$$

and

$$J_{E,x} = -M_{xy}^{21} \left[\frac{e}{T} \vec{\nabla} V + \frac{1}{T} \vec{\nabla} \mu + \mu \vec{\nabla} \left(\frac{1}{T} \right) \right] + M_{xy}^{22} \vec{\nabla} \left(\frac{1}{T} \right)$$

$$\begin{aligned} J_x &= -\frac{1}{T} M_{xy}^{11} \vec{\nabla} (ev + \mu) + (M_{xy}^{12} - \mu M_{xy}^{11}) \vec{\nabla} \left(\frac{1}{T} \right) \\ &= -\frac{1}{T} M_{xy}^{11} \vec{\nabla} \bar{\mu} + (M_{xy}^{12} - \mu M_{xy}^{11}) \vec{\nabla} \left(\frac{1}{T} \right) \end{aligned} \quad 3.31$$

Similarly,

$$\begin{aligned} J_{E,x} &= -\frac{1}{T} M_{xy}^{21} \vec{\nabla} (ev + \mu) + (M_{xy}^{22} - \mu M_{xy}^{21}) \vec{\nabla} \left(\frac{1}{T} \right) \\ &= -\frac{1}{T} M_{xy}^{21} \vec{\nabla} \bar{\mu} + (M_{xy}^{22} - \mu M_{xy}^{21}) \vec{\nabla} \left(\frac{1}{T} \right) \end{aligned} \quad 3.32$$

Now consider the following:

$$\begin{aligned} J_{Qx} &= J_{Ex} - \mu J_x \quad 3.33 \\ &= -\frac{1}{T} (M_{xy}^{21} - \mu M_{xy}^{11}) \vec{\nabla} \bar{\mu} + (M_{xy}^{22} - \mu M_{xy}^{21} - \mu M_{xy}^{12} + \mu^2 M_{xy}^{11}) \vec{\nabla} \left(\frac{1}{T} \right) \end{aligned} \quad 3.34$$

These equations for J_x and $J_{Q,x}$ now have the same form as the current in (3.30).

Comparing (3.30a) with (3.31). Also (3.30b) with (3.34) one has (Luttinger, 1964 and Mahan, 1981).

$$\begin{aligned} L_{xy}^{11} &= M_{xy}^{11} \\ L_{xy}^{12} &= M_{xy}^{12} - \mu M_{xy}^{11} \\ L_{xy}^{21} &= M_{xy}^{12} - \mu M_{xy}^{11}, \text{ thus } L_{xy}^{12} = L_{xy}^{21} \\ L_{xy}^{22} &= M_{xy}^{22} - \mu M_{xy}^{21} - \mu M_{xy}^{12} + \mu^2 M_{xy}^{11} \\ &= M_{xy}^{22} - 2\mu M_{xy}^{12} + \mu^2 M_{xy}^{11} \quad \text{since } M_{xy}^{12} = M_{xy}^{21} \end{aligned} \quad 3.35$$

The coefficients M^{ij} and L^{ij} are the correlation functions of current operators. The coefficients are similar to the standard Kubo formula for electrical conductivity

$$\sigma_{xy}(\omega) = \int_0^{\infty} e^{-i\omega t} dt \int_0^{\beta} d\beta' \text{Tr} \rho_0 [J_y(-i\hbar\beta') J_x(t)] \quad 3.36$$

and they are given by

$$L_{xy}^{11} = -\frac{1}{\beta} \int_0^{\infty} e^{-i\omega t} dt \int_0^{\beta} d\beta' \text{Tr} \rho_0 [J_x(-i\hbar\beta') J_y(t)] \quad 3.37$$

$$L_{xy}^{12} = -\frac{1}{\beta} \int_0^{\infty} e^{-i\omega t} dt \int_0^{\beta} d\beta' \text{Tr} \rho_0 [J_{Qx}(-i\hbar\beta') J_y(t)] \quad 3.38$$

$$L_{xy}^{22} = -\frac{1}{\beta} \int_0^{\infty} e^{-i\omega t} dt \int_0^{\beta} d\beta' \text{Tr} \rho_0 [J_{Qx}(-i\hbar\beta') J_{Qy}(t)] \quad 3.39$$

where the J_x and J_Q are the particle and heat currents respectively.

3.2 ELECTRICAL CONDUCTIVITY

The electrical conductivity is usually defined when there is no temperature gradient, $\vec{\nabla}(T) = 0$ and no concentration gradient $\vec{\nabla}\mu = 0$. In this case, the important operator is the electrical current, which is the summation over all particles and their velocity. Each group of particles contain identical particles with the same charge e .

Let J_x be the particle current for each kind of particle. Then the electrical current operator is

$$J_{E,x} = e J_x \quad 3.40$$

using J_x from Eqn (3.24).

$$\begin{aligned} J_{E,x} &= -eM^{11} \left[\frac{e}{T} \vec{\nabla}V + \vec{\nabla} \left(\frac{\mu}{T} \right) \right] + M^{12} \vec{\nabla} \left(\frac{1}{T} \right) \\ &= -\frac{e^2}{T} M^{11} \vec{\nabla}V - eM^{11} \left[\left(\frac{1}{T} \right) \vec{\nabla}\mu - \frac{\mu}{T^2} \vec{\nabla}T \right] - \frac{M^{12}}{T^2} \vec{\nabla}(T) \end{aligned} \quad 3.41$$

But

$$\vec{\nabla}T = 0, \text{ and } \vec{\nabla}\mu = 0$$

Therefore

$$J_{E,x} = -\frac{e^2}{T} M^{11} \vec{\nabla}V \quad 3.42$$

From Eqn (3.35)

$$L_{xy}^{11} = M_{xy}^{11} \quad 3.43$$

$$\vec{J}_{E,x} = \frac{e^2}{T} L_{xy}^{11} \vec{E}, \text{ where } \vec{E} = -\vec{\nabla}V \quad 3.44$$

Therefore the electrical conductivity is

$$\sigma_{xy} = \frac{e^2}{T} L_{xy}^{11} \quad 3.45$$

In our case the Hall conductivity σ_{xy} is first calculated and the thermopower thereafter.

Starting with the Kubo formula,

$$\sigma_{xy}(\omega) = \frac{1}{S} \int_0^\infty dt e^{-i\omega t} \int_0^\beta d\lambda \langle j_y(-i\hbar\lambda) j_x(t) \rangle \quad 3.46$$

The equilibrium average is defined as

$$\langle \hat{A} \rangle = \text{Tr} \{ \hat{\rho} \hat{A} \} \quad 3.47$$

The grand canonical ensemble density operator $\hat{\rho}$ is

$$\hat{\rho} = \frac{1}{Z} e^{-\beta(\hat{H} - \mu \hat{N})} \quad 3.48$$

where μ is the chemical potential, \hat{N} the particle number operator, and \hat{H} the Hamiltonian of the system.

Also,

$$Z = \text{Tr} \{ \hat{\rho} \} = \text{Tr} e^{-\beta(\hat{H} - \mu \hat{N})} \quad 3.49$$

Thus,

$$\hat{\rho} = \frac{e^{-\beta(\hat{H} - \mu \hat{N})}}{\text{Tr} e^{-\beta(\hat{H} - \mu \hat{N})}} \quad 3.50$$

Also,

$$\hat{j}(t) = e^{(\frac{i}{\hbar})\hat{H}t} \hat{j}(0) e^{-(\frac{i}{\hbar})\hat{H}t} \quad 3.51$$

Now changing the limit of integral in Eqn (3.46) from 0 to $+\infty$ to $-\infty \rightarrow +\infty$ the Heaviside theta function can be introduced.

$$\begin{aligned} \theta(t) & \text{ is defined by} \\ \theta(t) & = \frac{i}{2\pi} \int_{-\infty}^{\infty} dz \frac{e^{-izt}}{z+i\delta} = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases} \end{aligned} \quad 3.52$$

where δ , is small and goes to zero.

i.e.,

$$\begin{aligned} \sigma_{xy}(\omega) & = \frac{1}{\Omega} \int_{-\infty}^{\infty} dt \theta(t) e^{i\omega t} \int_0^{\beta} d\lambda \langle j_y(0) j_x(t+i\lambda) \rangle \\ & = \frac{i}{2\pi\Omega} \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dz \frac{e^{-izt}}{z+i\delta} e^{i\omega t} \int_0^{\beta} d\lambda \langle j_y(0) j_x(t+i\lambda) \rangle \end{aligned}$$

Now

$$\langle j_y(0) j_x(t) \rangle = \text{Tr} \{ \rho j_y(0) j_x(t) \} \quad 3.53$$

Therefore,

$$\langle j_y(0) j_x(t+i\lambda) \rangle = \text{Tr} \left\{ \rho j_y(0) e^{\frac{i}{\hbar}H(t+i\lambda)} j_x(0) e^{-\frac{i}{\hbar}H(t+i\lambda)} \right\}$$

But,

$$\rho = \frac{1}{Z} \text{Tr} e^{-\beta(H-\mu)} \quad 3.54$$

$$\langle j_y(0) j_x(t+i\lambda) \rangle = \frac{1}{Z} \text{Tr} \left\{ e^{-\beta(H-\mu)} j_y(0) e^{\frac{i}{\hbar}H(t+i\lambda)} j_x(0) e^{-\frac{i}{\hbar}H(t+i\lambda)} \right\} \quad 3.55$$

$$= \sum_{n,m} \rho_n j_y j_x e^{\frac{i}{\hbar}(t+i\lambda)(E_m-E_n)}$$

where

$$\hat{H}|n\rangle = E_n|n\rangle$$

$$\rho_n = \langle n|\rho|n\rangle$$

Thus one can write

$$\begin{aligned} \sigma_{xy}(\omega) &= \frac{i}{2\pi\hbar} \sum_{n,m} \rho_n j_y j_x \int_{-\infty}^{\infty} \frac{dz}{z+i\delta} \int_0^{\beta} \int_0^{\beta} e^{i(\omega-z)t} e^{\frac{i}{\hbar}(t+i\lambda)(E_m-E_n)} dt d\lambda \\ &= \frac{i}{2\pi\hbar} \sum_{n,m} \rho_n j_y j_x \int_{-\infty}^{\infty} \frac{dz}{z+i\delta} \int_{-\infty}^{\infty} dt e^{it[\omega-z+\frac{E_m-E_n}{\hbar}]} \int_0^{\beta} d\lambda e^{-\lambda(E_m-E_n)} \end{aligned} \quad 3.56$$

But,

$$\int_{-\infty}^{\infty} dt e^{it[\omega-z+\frac{E_m-E_n}{\hbar}]} = 2\pi\delta(\omega-z+\frac{E_m-E_n}{\hbar}) \quad 3.57$$

And,

$$\int_0^{\beta} d\lambda e^{-\lambda(E_m-E_n)} = \frac{1 - e^{-\beta(E_m-E_n)}}{E_m-E_n} \quad 3.58$$

$$\begin{aligned} \sigma_{xy}(\omega) &= \frac{i}{2\pi\hbar} \sum_{n,m} \rho_n j_y j_x \int_{-\infty}^{\infty} \frac{dz}{z+i\delta} \left[2\pi\delta(\omega-z+\frac{E_m-E_n}{\hbar}) \right] \left[\frac{1 - e^{-\beta(E_m-E_n)}}{E_m-E_n} \right] \\ &= \frac{i}{\hbar} \sum_{n,m} \rho_n j_y j_x \left[\frac{1 - e^{-\beta(E_m-E_n)}}{E_m-E_n} \right] \int_{-\infty}^{\infty} \frac{dz}{z+i\delta} \delta(\omega-z+\frac{E_m-E_n}{\hbar}) \end{aligned} \quad 3.59$$

From the property of Dirac delta function,

the above integral, on letting $z \rightarrow \omega + \frac{E_m-E_n}{\hbar}$, reduces to

$$\int_{-\infty}^{\infty} \frac{dz}{z+i\delta} \delta(\omega-z+\frac{E_m-E_n}{\hbar}) = \frac{\hbar}{E_m-E_n+\hbar(\omega+i\delta)} \quad 3.60$$

Therefore,

$$\sigma_{xy}(\omega) = \frac{i\hbar}{\hbar} \sum_{n,m} \rho_n j_y j_x \frac{1 - e^{-\beta(E_m-E_n)}}{(E_m-E_n)(E_m-E_n+\hbar(\omega+i\delta))} \quad 3.61$$

But,

$$\rho_n [1 - e^{-\beta(E_m-E_n)}] = \rho_n - \rho_m$$

where $\rho_m = \rho_n e^{-\beta(E_m-E_n)}$ 3.62

$$\begin{aligned}
\sigma_{xy}(\omega) &= \frac{i\hbar}{S} \sum_{nm} \frac{(\rho_n - \rho_m) j_y j_x}{[E_n - E_m][E_n - E_m + \hbar(\omega + i\delta)]} \\
&= \frac{-i\hbar}{\hbar S(\omega + i\delta)} \sum_{nm} (\rho_n - \rho_m) \frac{j_y j_x}{n_m n_m} \left\{ \frac{1}{E_n - E_m - \hbar(\omega + i\delta)} - \frac{1}{E_n - E_m} \right\} \\
&= \frac{i}{S(\omega + i\delta)} \left\{ \sum_{nm} \frac{(\rho_n - \rho_m) j_y j_x}{n_m n_m} \frac{1}{E_n - E_m + \hbar(\omega + i\delta)} - \sum_{nm} \frac{(\rho_n - \rho_m) j_y j_x}{n_m n_m} \frac{1}{E_n - E_m} \right\} \quad 3.63
\end{aligned}$$

$$= \frac{1}{S(\omega + i\delta)} \left\{ F_{xy}(\omega) - F_{xy}(0) \right\} \quad 3.64$$

where,

$$F_{xy}(\omega) = \sum_{nm} \frac{(\rho_n - \rho_m) j_y j_x}{n_m n_m} \frac{1}{E_n - E_m - \hbar(\omega + i\delta)} \quad 3.65$$

And,

$$F_{xy}(0) = \sum_{nm} \frac{(\rho_n - \rho_m) j_y j_x}{n_m n_m} \frac{1}{E_n - E_m} \quad 3.66$$

For the case $\omega \rightarrow 0, \delta \rightarrow 0$, and in term of the advanced and retarded Green functions,

$$G_{nm}^{R,A} = \langle n | \frac{1}{E - H + i\delta} | m \rangle \quad 3.67$$

$$\begin{aligned}
\text{and density of state } \rho_n &= \rho(E_n) = \int_{-\infty}^{\infty} dE \rho(E) \delta(E - E_n) \\
&= \int_{-\infty}^{\infty} dE f(E) \delta(E - E_n) \quad 3.68
\end{aligned}$$

where $f(E)$ is the Fermi distribution function.

The result obtained for the static conductivity $\sigma_{xy}(0)$ is

$$\sigma_{xy}(0) = \frac{i\hbar}{\pi S} \int_{-\infty}^{\infty} dE f(E) \text{Tr} \left\{ j_x I_m G(E) j_y \frac{dG^A(E)}{dE} - j_x \frac{dG^R(E)}{dE} j_y I_m G(E) \right\} \quad 3.69$$

Using the matrix element of the velocity operators,

$$\langle n' m' | v_x | n m \rangle = \frac{1}{i} \frac{\hbar}{m} \left(\frac{\alpha}{2} \right)^k \left\{ n^k \delta_{n',n-1} \delta_{m',m-1} - (n+1)^k \delta_{n',n+1} \delta_{m',m+1} \right\} \quad 3.70$$

and,

$$\langle n' m' | v_y | n m \rangle = \frac{\hbar}{m} \left(\frac{\alpha}{2} \right)^k \left\{ n^k \delta_{n',n-1} \delta_{m',m-1} + (n+1)^k \delta_{n',n+1} \delta_{m',m+1} \right\} \quad 3.71$$

Eqn (3.69) gives

$$\sigma_{xy}(0) = -\frac{e^2}{h} \int_{-\infty}^{\infty} dE f(E) \sum_{n,N} \delta(E - \epsilon_n - E_N) \quad 3.72$$

where E_N corresponds to the energy of the massive particle, and E_n that of the lighter particle Eqn (3.16) and (3.17) respectively. Here the conductivity is considered to be due to the effect of odd-electron clusters, considered before.

$$\text{Now, } \int_{-\infty}^{\infty} dE f(E) \delta(E - \epsilon_n - E_N) = - \int_{-\infty}^{\infty} dE \frac{df}{dE} \theta(E - \epsilon_n - E_N)$$

$$\text{In the limit } T = 0, \quad \frac{df}{dE} = -\delta(E - E_F)$$

Therefore

$$\sigma_{xy}(0) = -\frac{e^2}{h} \sum_{n,N} \theta(E_F - \epsilon_n - E_N) \quad 3.73$$

In the extreme Quantum limit (EQL) ($\hbar\omega_c \geq E_F$), $n = 0$ we have

$$\sigma_{xy}(0) = -\frac{e^2}{h} \sum_{N=0} \theta(E_F - \frac{\hbar\omega_c}{2} - E_N) \quad 3.74$$

This was the main result obtained by Elegba and co-workers (1987). Its form as given by (3.72) will be the starting point of the present work. It will be used to calculate the correlation coefficients L_{xy}^{11} , L_{xy}^{12} and then investigate the effect of clustering on the thermopower S_{xy} .

3.3 THERMAL CONDUCTIVITY

The thermal conductivity is usually measured under the condition of no particle current $J = 0$. From (3.24a) dropping the subscript x,y we have for $J = 0$;

$$0 = M^{11} \left[\frac{e}{T} \vec{\nabla}V + \vec{\nabla} \left(\frac{\mu}{T} \right) \right] + M^{12} \vec{\nabla} \left(\frac{1}{T} \right) \quad 3.75$$

$$\frac{e}{T} \vec{\nabla}V + \vec{\nabla} \left(\frac{\mu}{T} \right) = -\frac{M^{12}}{M^{11}} \vec{\nabla} \left(\frac{1}{T} \right)$$

This gives from (3.24b) the energy current as

$$J_E = -M^{21} \frac{M^{12}}{M^{11}} \vec{\nabla} \left(\frac{1}{T} \right) + M^{22} \vec{\nabla} \left(\frac{1}{T} \right) \quad 3.76$$

Now thermal conductivity K is usually defined as

$$J_E = -k \nabla T \quad 3.77$$

from 3.37

$$J_E = \frac{-1}{T^2} \left[M^{22} - \frac{(M^{12})^2}{M^{11}} \right] \nabla(T)$$

where we have used $\nabla \left(\frac{1}{T} \right) = -\frac{1}{T^2} \nabla(T)$

$$\text{Thus } K = \frac{1}{T^2} \left[M^{22} - \frac{(M^{12})^2}{M^{11}} \right] \nabla(T) \quad 3.78$$

Also from Eqn (3.30a) if there is no particle flow, then $J_Q = J_E$. Thus one should be able to obtain the same expression for thermal conductivity from this equation.

The restriction that no current flows (i.e. equilibrium) gives from Eqn (3.30a)

$$\begin{aligned} \frac{1}{T} L^{11} \nabla \mu &= L^{12} \nabla \left(\frac{1}{T} \right) \\ \frac{1}{T} \nabla \mu &= \frac{L^{12}}{L^{11}} \nabla \left(\frac{1}{T} \right) \end{aligned} \quad 3.79$$

and the heat current is

$$J_Q = -\frac{L^{12}}{L^{11}} L^{21} \nabla \left(\frac{1}{T} \right) + L^{21} \nabla \left(\frac{1}{T} \right) \quad 3.80$$

$$\begin{aligned} J_Q &= \left[L^{21} - \frac{(L^{12})^2}{L^{11}} \right] \nabla \left(\frac{1}{T} \right) \\ &= -\frac{1}{T^2} \left[L^{22} - \frac{(L^{12})^2}{L^{11}} \right] \nabla(T) \end{aligned} \quad 3.81$$

again, the thermal conductivity is

$$K = \frac{1}{T^2} \left[L^{22} - \frac{(L^{12})^2}{L^{11}} \right] \quad 3.82$$

The identities in Eqn (3.35) can also be used to show that Eqn (3.82) is consistent with Eqn (3.78).

That is,

$$\begin{aligned} K &= \frac{1}{T^2} \left[M^{22} - 2\mu M^{12} + \mu^2 M^{11} - \frac{(M^{12} - \mu M^{11})^2}{M^{11}} \right] \\ &= \frac{1}{T^2} \left[M^{22} - \frac{(M^{12})^2}{M^{11}} \right] \end{aligned}$$

3.4 THERMOPOWER

A temperature gradient in a thin bar of metal is accompanied by an

electric field directed opposite to the temperature gradient. This field is known as the thermoelectric field. It is conventionally written as

$$\vec{E} = S \vec{\nabla} T$$

where the electric field $\vec{E} = \vec{E}' + \frac{1}{e} \vec{\nabla} \mu$ (This is because the chemical potential gradient leads as a result of temperature gradient to a diffusion current, in addition to the current driven mechanically by the electric field \vec{E}'). S is known as the thermopower, and is the proportionality constant between the contribution of the metal to the electric field and the temperature gradient in the metal.

L_{xy}^{12} is the thermoelectric coefficient, which arises as a result of temperature gradient, and can be used to calculate the thermopower. In which case if a solid has a temperature gradient $\vec{\nabla} T$ and no particle current ($J_x = 0$) and no concentration gradient ($\vec{\nabla} \mu = 0$), then the voltage difference VV is measured which is proportional to ∇T and is given using Eqn (3.79) as

$$\frac{1}{T} \vec{\nabla} \mu = \frac{L^{12}}{L^{11}} \vec{\nabla} \left(\frac{1}{T} \right) \quad 3.83$$

$$\frac{1}{T} \vec{\nabla} (\mu + eV) = \frac{L^{12}}{L^{11}} \cdot -\frac{1}{T^2} \vec{\nabla} (T) \quad 3.84$$

$$\frac{e}{T} \vec{\nabla} V = -\frac{L^{12}}{L^{11}} \frac{1}{T^2} \vec{\nabla} (T)$$

$$\vec{\nabla} V = -\frac{1}{eT} \frac{L^{12}}{L^{11}} \vec{\nabla} (T) \quad 3.85$$

This equation is the definition of the thermoelectric coefficient S , which is also called the thermopower,

$$S = \frac{\vec{\nabla} V}{\vec{\nabla} T} \quad 3.86$$

$$S_{xy} = -\frac{1}{eT} \frac{L_{xy}^{12}}{L_{xy}^{11}}$$

3.87

The thermopower S may have either sign. In ion diffusion models in solids the sign of the thermopower provides a direct determination of the sign of the charge of the diffusion ion (Girvin, 1978).

CHAPTER 4

RESULTS AND DISCUSSIONS

A generalisation to $T \neq 0$ of the result shown in Eqn (3.72) shows that the Hall conductivity of a two-dimensional electron system can be written as

$$\sigma_{xy} = \frac{e^2}{h} \sum_{N,n} \int_{-\infty}^{\infty} dE \frac{df(E)}{dE} \theta(E - \epsilon_n - \epsilon_N) \quad 4.1$$

$$= \frac{e^2}{h} \sum_{N,n} \int_{\epsilon_n + \epsilon_N}^{\infty} dE \frac{df(E)}{dE} \quad 4.2$$

comparing with Eq (3.45) i.e

$$\sigma_{xy} = \frac{e^2}{T} L_{xy}^{11} \quad 4.3$$

$$L_{xy}^{11} = \frac{T}{h} \sum_{N,n} \int_{\epsilon_n + \epsilon_N}^{\infty} dE \frac{df(E)}{dE}$$

where $f(E)$ is the Fermi-Dirac distribution function.

$$L_{xy}^{11} = -\frac{T}{h} \sum_{N,n} f(\epsilon_n + \epsilon_N) \quad 4.4$$

The coefficient L_{xy}^{12} can be calculated using the standard Kubo formula involving the particle-current-heat-current correlation function with the heat-current operator defined by (Girvin and Jonson, 1984);

$$J_Q = \frac{1}{2} (H_0 J + J H_0) - \mu J \quad 4.5$$

where μ is the chemical potential and H_0 is the unperturbed Hamiltonian

$$H_0 = \frac{1}{2m} (\vec{P} + \frac{e}{c} \vec{A})^2 + V \quad 4.6$$

with eigenvalue E .

thus,

$$J_Q = (E - \mu) J \quad 4.7$$

With this definition of heat current, and using

$$\sigma_{xy}(\omega) = \frac{1}{S} \int_0^{\infty} dt e^{-i\omega t} \int_0^{\beta} d\lambda \langle J_{Qx}(-i\lambda) J_y(t) \rangle \quad 4.8$$

$$\text{where } \langle J_Q(0) J_y(t + ih\lambda) \rangle = \text{Tr} \left\{ \rho J_Q(0) e^{\frac{i}{\hbar} H(t+i\hbar\lambda)} J_y(t) e^{-\frac{i}{\hbar} H(t+i\hbar\lambda)} \right\} \quad 4.9$$

$$= \text{Tr} \left\{ \rho(E-\mu) J_x(0) e^{\frac{i}{\hbar} H(t+i\hbar\lambda)} J_y(0) e^{-\frac{i}{\hbar} H(t+i\hbar\lambda)} \right\} \quad 4.10$$

Substituting Eqn (4.10) into Eqn (4.8), and carrying out the integration

before one obtains

$$\sigma_{xy}(0) = \frac{-e^2}{h} \int_{-\infty}^{\infty} (E-\mu) dE f(E) \sum_{N,n} \delta(E-E_n-E_n) \quad 4.11$$

thus,

$$L_{xy}^{12} = -\frac{T}{h} \sum_{N,n} \int_{-\infty}^{\infty} (E-\mu) dE f(E) \delta(E-E_n-E_n) \quad 4.12$$

$$= \frac{T}{h} \sum_{N,n} \int_{-\infty}^{\infty} (E-\mu) dE \frac{df(E)}{dE} \delta(E-E_n-E_n)$$

$$= \frac{T}{h} \sum_{N,n} \int_{E_n+E_n}^{\infty} (E-\mu) dE \frac{df(E)}{dE} \quad 4.13$$

where, $f(E)$ is the Fermi distribution function:

$$f(E) = \frac{1}{e^{\beta(E-\mu)} + 1}$$

$$\frac{df(E)}{dE} = \frac{-\beta}{4 \cosh^2 \frac{\beta}{2}(E-\mu)} \quad 4.14$$

Eqn(4.14) is substituted into Eqn (4.13) to obtain

$$L_{xy}^{12} = \frac{-\beta T}{4h} \sum_{N,n} \int_{E_n+E_n}^{\infty} \frac{(E-\mu) dE}{4 \cosh^2 \frac{\beta}{2}(E-\mu)}$$

By introducing new variables,

$$x = E - \mu, \quad a = \beta/2 \quad \text{where } \beta = \frac{1}{kT} \quad \text{one obtains}$$

$$L_{xy}^{12} = \frac{-\beta T}{4h} \sum_{N,n} \int_{E_n+E_n}^{\infty} \frac{x dx}{\cosh^2 ax} \quad 4.15$$

Integrating Eqn (4.15) by parts gives

$$L_{xy}^{12} = \frac{\beta T}{4ha^2} \sum_{N,n} \left[-\ln 2 - \ln(\cosh a(E_n+E_n-\mu)) + a(E_n+E_n-\mu) \tanh a(E_n+E_n-\mu) \right] \quad 4.16$$

Now substituting L_{xy}^{12} and L_{xy}^{11} into the expression for the thermopower

$$S_{xy} = -\frac{1}{eT} \frac{L_{xy}^{12}}{L_{xy}^{11}} \quad 4.17$$

we have,

$$S_{xy} = \frac{1}{eT} \frac{\beta T}{4ha^2} \frac{h}{T} \sum_{N,n} \frac{\left[-\ln 2 - \ln(\cosh a(E_n+E_n-\mu)) + a(E_n+E_n-\mu) \tanh a(E_n+E_n-\mu) \right]}{\sum_{N,n} \left[e^{\beta(E_n+E_n-\mu)} + 1 \right]^{-1}}$$

$$= \left(\frac{k}{e}\right) \sum_{N,n} \frac{[-\ln 2 - \ln(\cosh a(E_n + E_n - \mu) + a(E_n + E_n - \mu) \tanh a(E_n + E_n - \mu))]}{\sum_{N,n} [e^{\beta(E_n + E_n - \mu)} + 1]^{-1}} \quad 4.19$$

It is pertinent here to restrict our consideration to the Extreme Quantum Limit case where $n = 0$, and therefore the summation is only over N .

Furthermore, it is also instructive to consider the result for σ_{xy} at $T = 0$ in which case even the summation over N terminates at some finite value.

To determine the number of terms to be summed up, it is important to consider the results of Eqn (3.74) which is the case of $T = 0$.

that is,

$$\sigma_{xy}(0) = -\frac{e^2}{h} \sum_{N=0} \theta(E_F - \frac{\hbar\omega_c}{2} - E_N)$$

with $E_N = \frac{\hbar\omega_c}{Q} (N + \frac{1}{2})$.

$$\sigma_{xy}(0) = -\frac{e^2}{h} \sum_{N=0} \theta\left[\eta_F - \frac{1}{2} - \frac{1}{Q}(N + \frac{1}{2})\right]$$

where, $\eta_F = \frac{\mu_F}{\hbar\omega_c}$ (since $E_F = \mu_F$), is the reduced chemical potential.

In the case of $Q = 3$ particles,

$$\sigma_{xy}(0) = -\frac{e^2}{h} \sum_{N=0} \theta\left[\eta_F - \frac{1}{2} - \frac{1}{3}(N + \frac{1}{2})\right] \quad 4.20$$

at EQL, $\frac{1}{2} \leq \eta_F \leq \frac{3}{2}$ 4.21

The sum in Eqn (4.20) terminates at exactly $N = 2$.

Since for $N > 3$ the theta function is zero.

Thus for $Q = 3$, $N = 2$. Similarly for the case of $Q = 5$ particles the summation terminates at $N = 4$. With an idea of how the summation terminates Eqn (4.19) was further simplified to give

$$S_{xy} = \left(\frac{k}{e}\right) \sum_N \frac{[-\ln 2 - \ln \cosh\left(\frac{\eta_F - \frac{1}{2} - \frac{1}{Q}(N + \frac{1}{2})}{2\eta_T}\right) + \left(\frac{\eta_F - \frac{1}{2} - \frac{1}{Q}(N + \frac{1}{2})}{2\eta_T}\right) \tanh\left(\frac{\eta_F - \frac{1}{2} - \frac{1}{Q}(N + \frac{1}{2})}{2\eta_T}\right)]}{\sum_N [e^{-\frac{\eta_F - \frac{1}{2} - \frac{1}{Q}(N + \frac{1}{2})}{\eta_T}} + 1]^{-1}} \quad 4.22$$

with $Q = 3$,

$$S_{xy} = \left(\frac{k}{e}\right) \sum_{N=0}^2 \frac{[-\ln 2 - \ln \cosh\left(\frac{\eta_F - \frac{(N+1)}{3}}{2\eta_T}\right) + \left(\frac{\eta_F - \frac{(N+1)}{3}}{2\eta_T}\right) \tanh\left(\frac{\eta_F - \frac{(N+1)}{3}}{2\eta_T}\right)]}{\sum_N [e^{-\frac{(N+1)}{3\eta_T}} + 1]^{-1}} \quad 4.23$$

Eqn (4.23) expresses the thermopower as a universal function of the reduced temperature ($\eta_T = \frac{k_B T}{\hbar \omega_c}$) and reduced chemical potential ($\eta_F = \frac{\mu}{\hbar \omega_c}$).

A computer program was developed (see Appendix B) to calculate the thermopower from this expression, at a fixed value of $\eta_T = 0.05$. This corresponds to $T = 4.3\text{k}$ at $B = 12\text{T}$ in a $\text{Si}(110)$ inversion layer (Jonson and Girvin, 1984), and with $\eta_T = 0.01$ for the sake of further comparison. η_F was then varied from 0.5 to 2.0.

The results calculated are displayed in fig 4.3, and fig. 4.6. Similarly the numerator and the denominator which are just L_{xy}^{11} and L_{xy}^{12} respectively except for the factor T/h were similarly plotted against η_F in figures 4.1, 4.2, 4.4 and 4.5.

A computer program was similarly developed for the case of $Q = 5$ particles using the expression Eqn (4.22) and the result for the thermopower tensor S_{xy} , L_{xy}^{11} and the thermal response function L_{xy}^{12} were calculated at fixed value of $\eta_T = 0.015$ which correspond to $T = 1.29\text{k}$ at $B = 12\text{T}$ in a $\text{Si}(110)$ inversion layer. The results are displayed in figures 4.7, 4.8 and 4.9.

All the graphs show remarkable features worth pointing out. As a function of chemical potential, the thermopower has a series of large peaks for values of the chemical potential near the Landau-level positions ($\frac{1}{Q} (N + \frac{1}{2})$). Also the thermopower have very large values at small values of η_F .

Another noteworthy point is that S_{xy} is in terms of reduced temperature and reduced chemical potential, independent of the magnetic field, the effective mass etc.

At low temperature the response function L_{xy}^{11} show steplike increase with plateaus at values of the chemical potential corresponding to the

Landau-level of the massive particle ($\frac{1}{Q} (N + \frac{1}{2})$). The effect of temperature on L_{xy}^{11} can be seen significantly in figures 4.1, 4.4 and 4.7.

The results of the thermopower tensor calculated in this work seem to agree quite well with the results of (Jonson and Girvin, 1984). Even though the effect of disorder have not been considered (which they did in their own case).

Our analysis is simple and has the benefit of demonstrating the effect of temperature.

In conclusion we have studied the behaviours of a two-dimensional electron gas in a strong uniform magnetic field with the full compliment of temperature. We have shown that the formation of clusters lead to steplike character of the transport coefficients studied even at $T \neq 0$.

The result of this work has also demonstrated that the steplike character of the integral quantum Hall effect is not due to impurities as has been suggested by some authors notably, Girvin and Jonson, 1984.

For sometime only limited experimental results for the thermopower of a two-dimensional system in a quantizing magnetic field exist. Further experimental work would be a valuable source of information about transport properties of such systems.

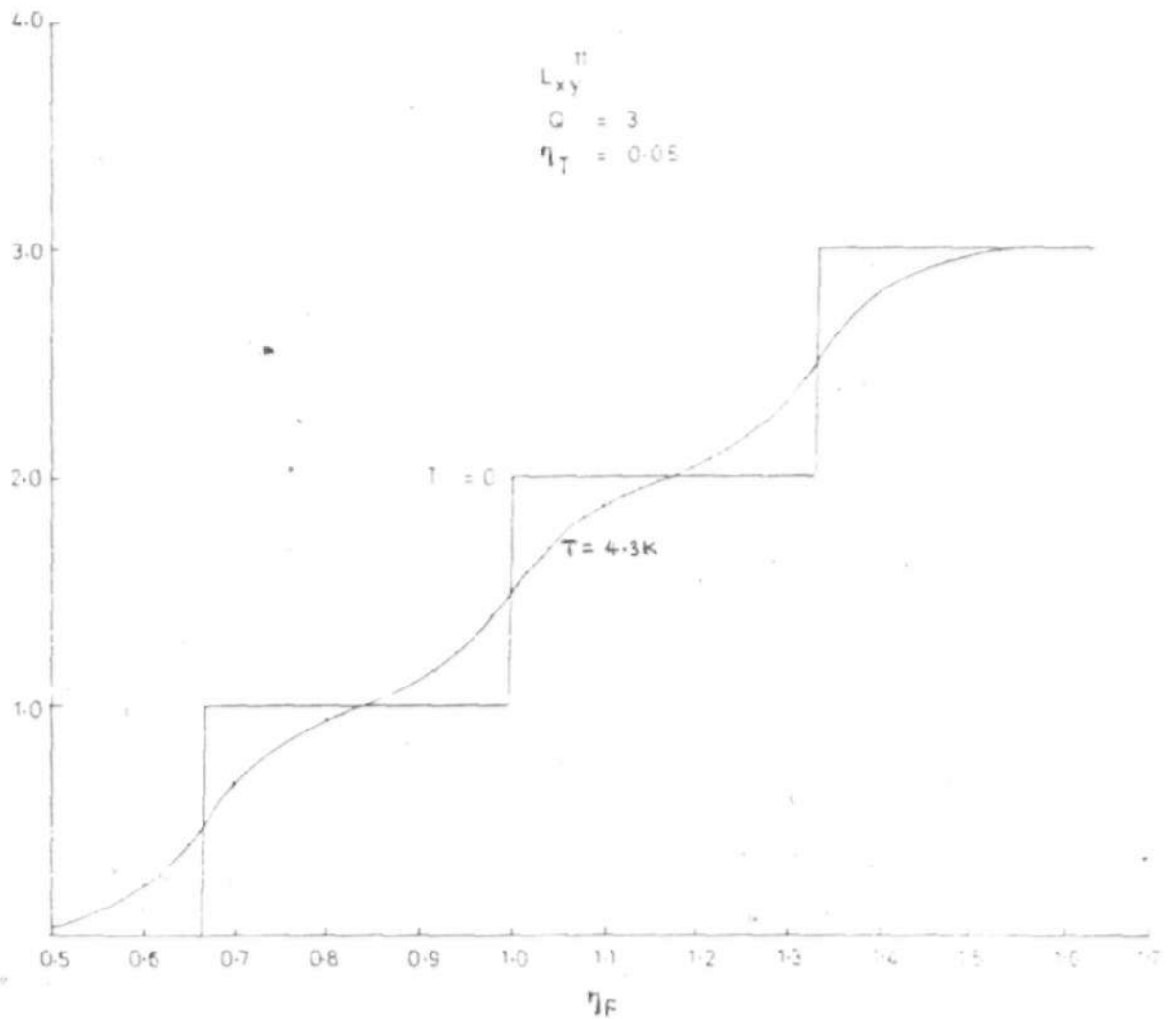


Fig. 4.1: Off-diagonal component of the response coefficient L_{xy}^{11} vs reduced chemical potential where the reduced temperature $\eta_T = 0.05$. This corresponds to $T = 4.3K$ at $B = 12T$ in a Si (110) inversion layer. $Q = 3$ particles.

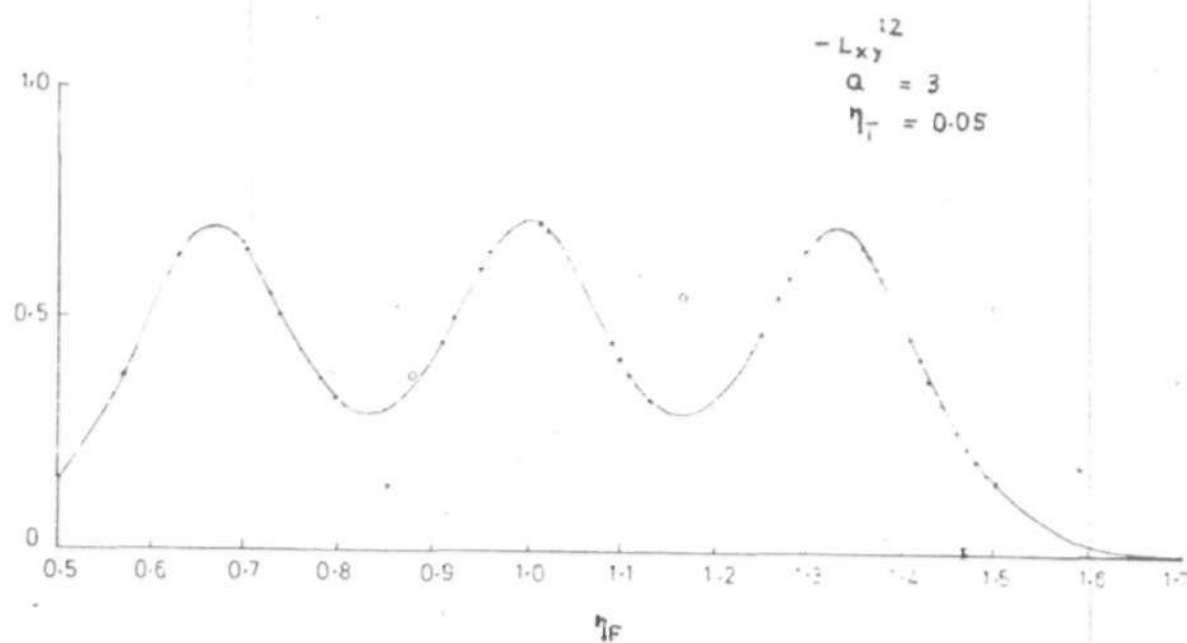


Fig. 4.2: Off-diagonal component of the thermal response coefficient L_{xy}^{12} vs chemical potential where the reduced temperature $\eta_T = 0.05$. This corresponds to $T = 4.3k$ at $B = 12T$ in a Si (110) inversion layer. $Q = 3$ particles.

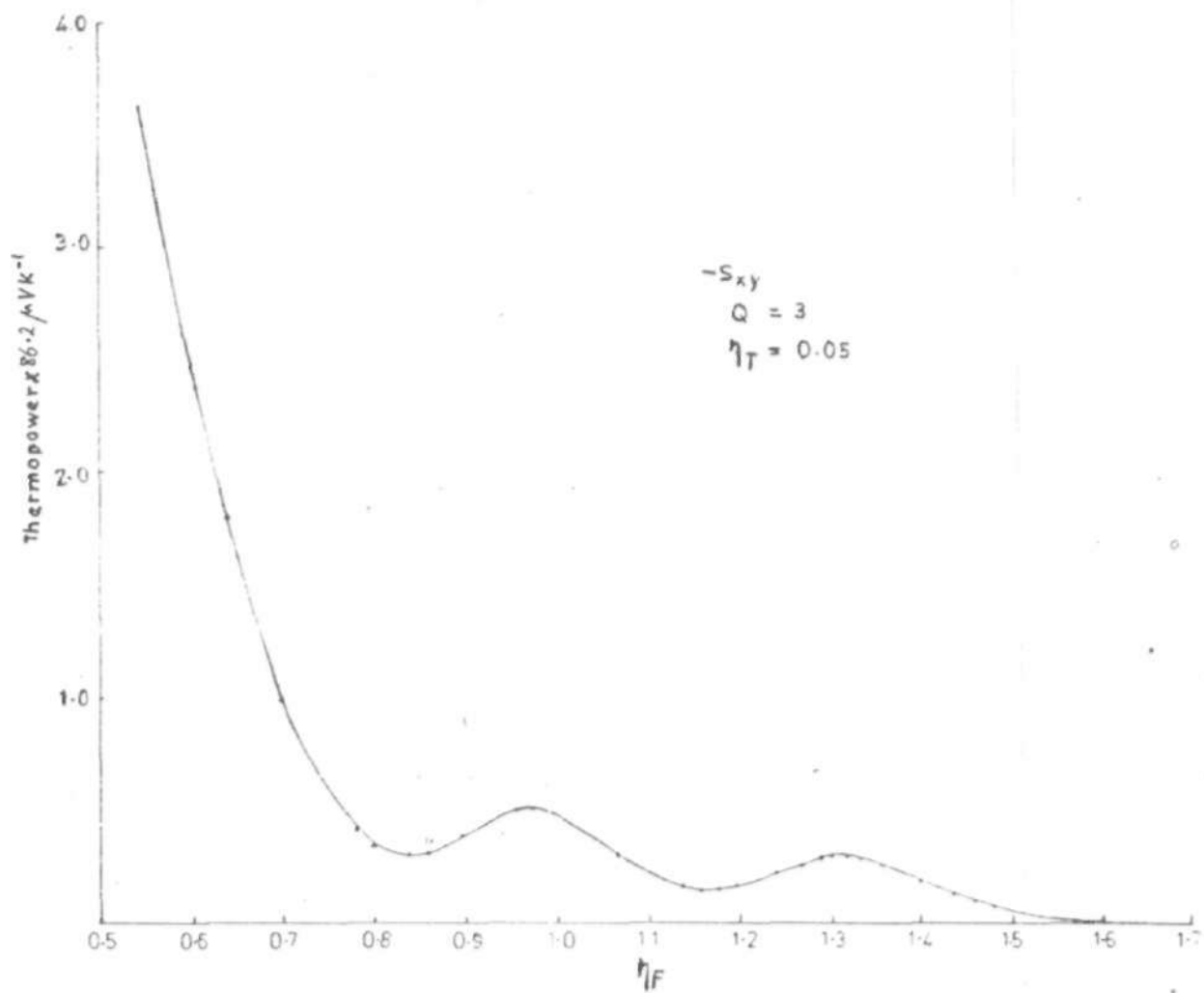


Fig. 4.3: Off diagonal component of the thermopower vs chemical potential where the reduced temperature $\eta_T = 0.05$. This corresponds to $T = 4.3\text{k}$ at $B = 12\text{T}$ in a Si (110) inversion layer. $Q = 3$ particles.

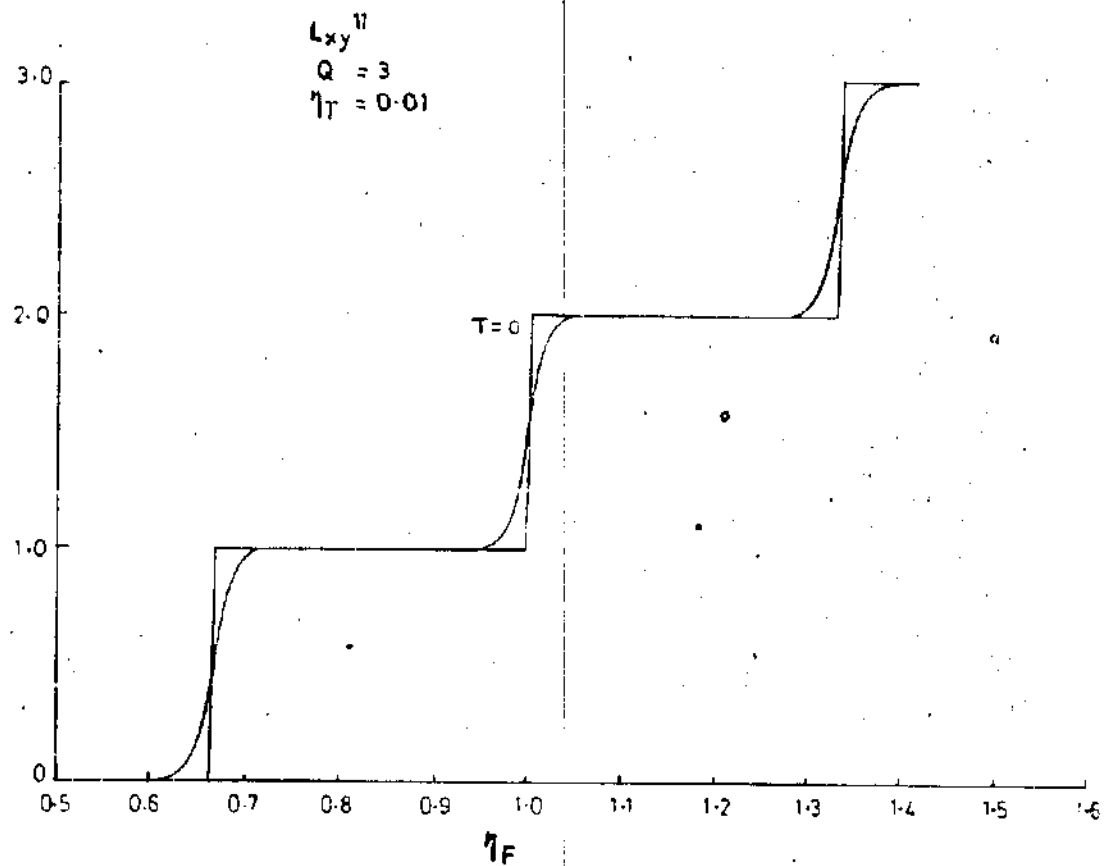


Fig. 4.4: Off-diagonal component of the response coefficient L_{xy}^{11} vs reduced chemical potential, where the reduced temperature $\eta_T = 0.01$. This corresponds to $T = 0.86k$ at $B = 12T$ in a Si (110) inversion layer. $Q = 3$ particles

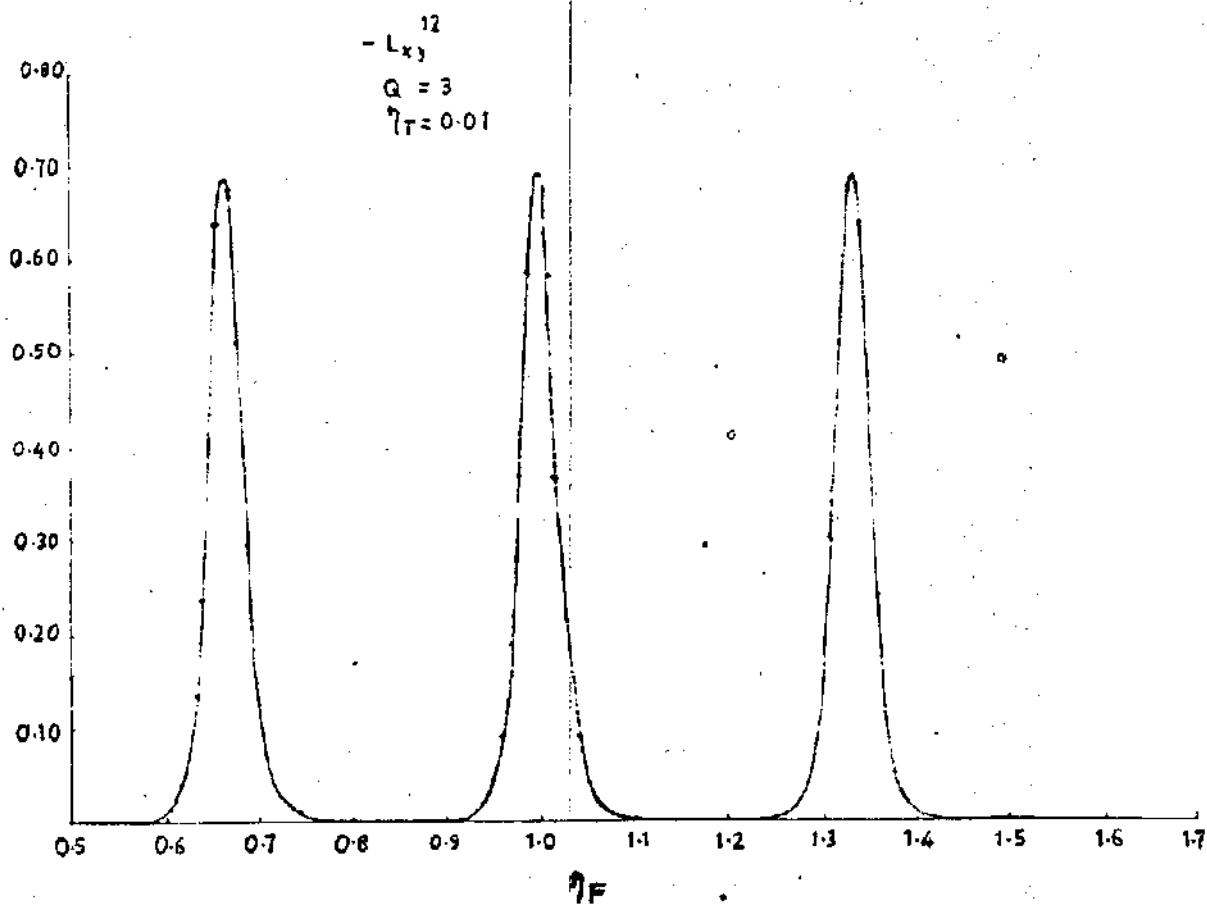


Fig. 4.5: Off-diagonal component of thermal response coefficient L_{xy}^{12} vs chemical potential where the reduced temperature $\gamma_T = 0.01$. This corresponds to $T = 0.86k$ at $B = 12T$ in a Si (110) inversion layer. $Q = 3$ particles.

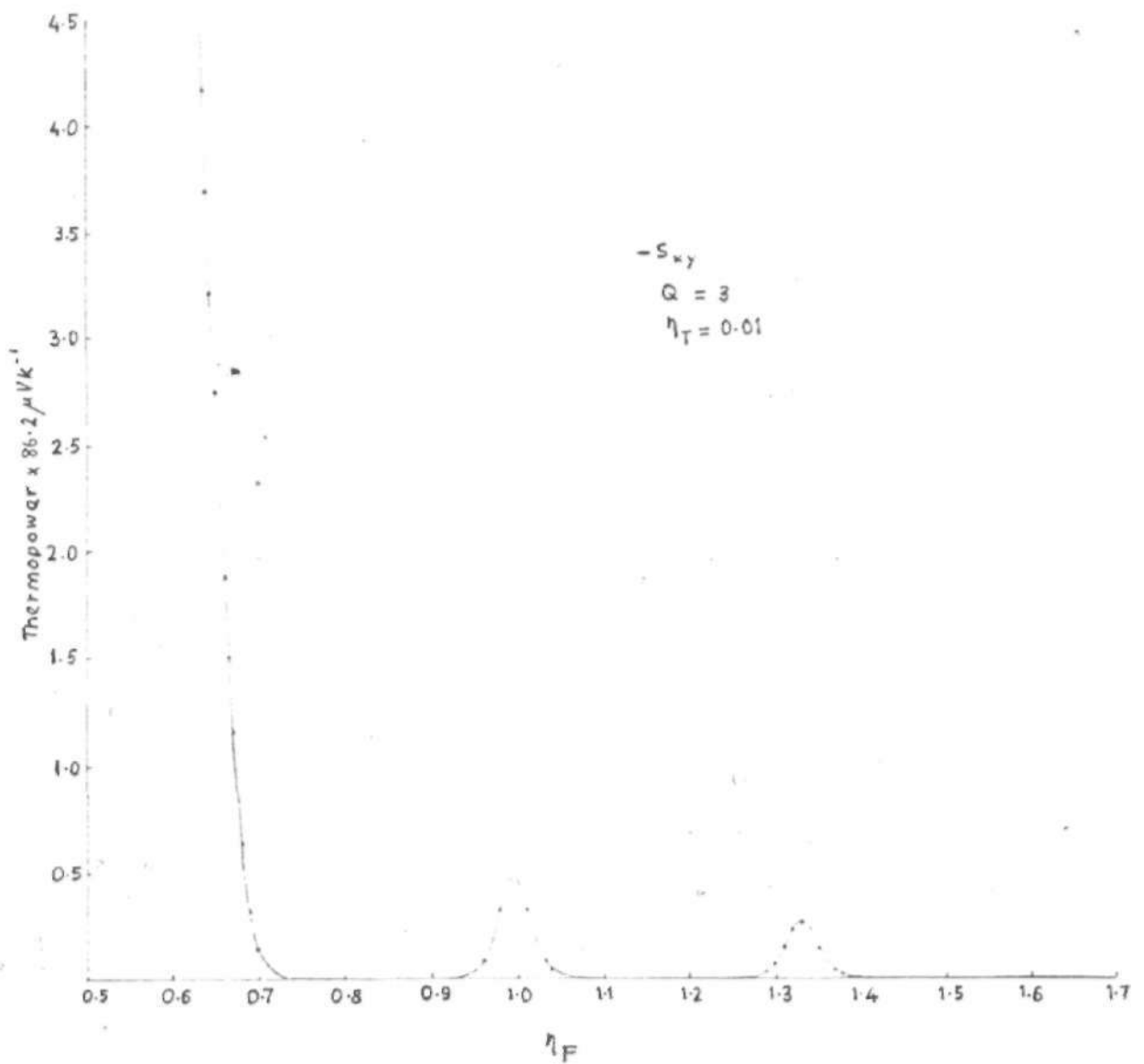


Fig. 4.6: Off-diagonal component of the thermopower vs chemical potential where the reduced temperature $\eta_T = 0.01$. This corresponds to $T = 0.86\text{k}$ at $B = 12\text{T}$ in a Si (110) inversion layer. $Q = 3$ particles.

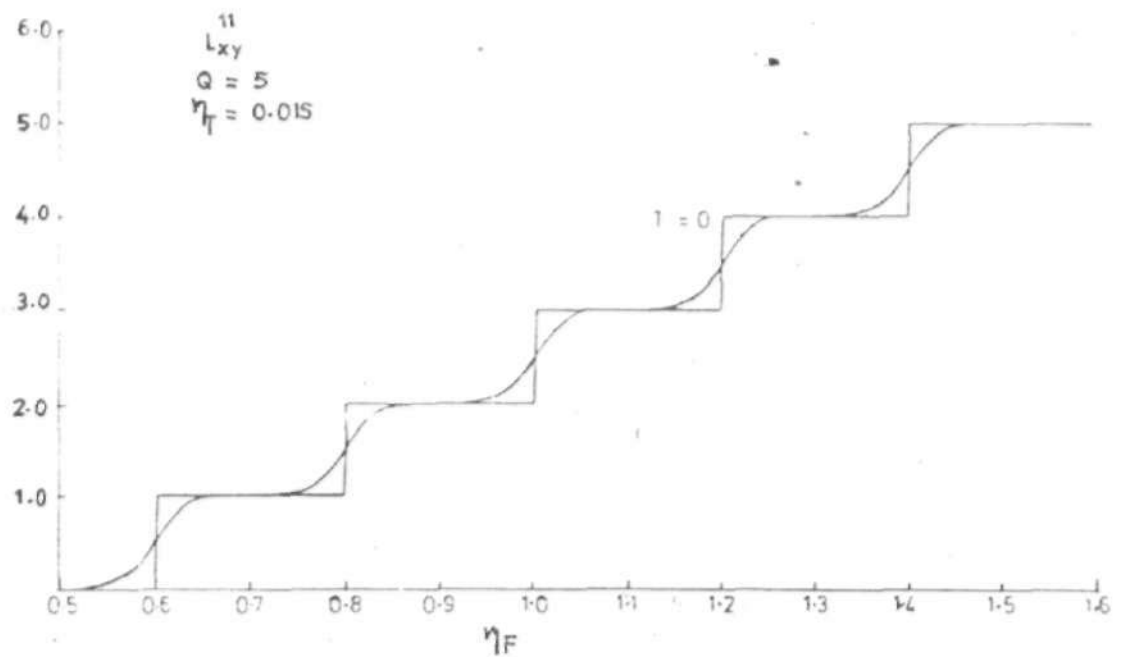


Fig. 4.7: Off-diagonal component of the response coefficient L_{xy}^{11} vs reduced chemical potential η_F where the reduced temperature $\eta_T = 0.015$. This corresponds to $T = 1.29k$ at $B = 12T$ in Si (110) inversion layer. $Q = 5$ particles.

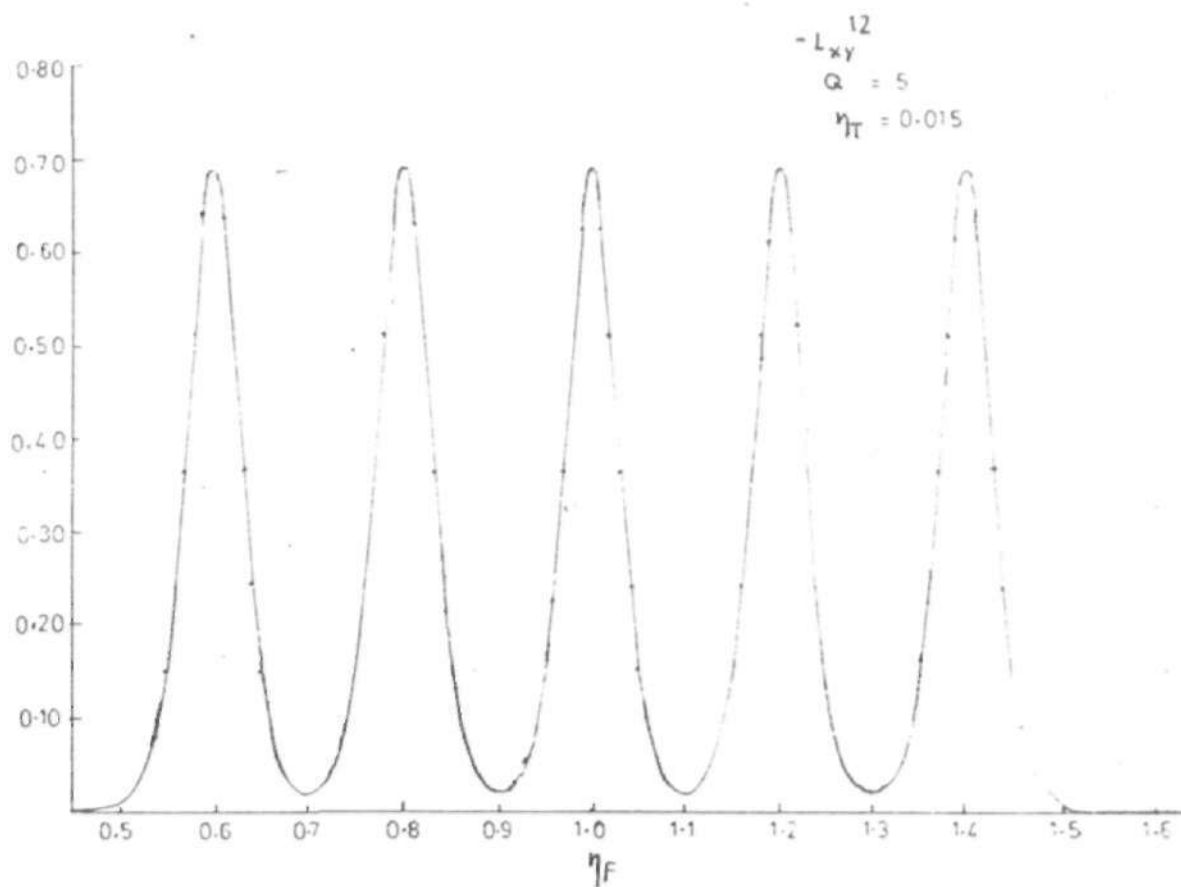


Fig. 4.8: Off-diagonal component of the thermal response coefficient vs chemical potential where the reduced temperature $\eta_T = 0.015$. This corresponds to $T = 1.29k$ at $B = 12T$ in a Si (110) inversion layer. $Q = 5$ particles.

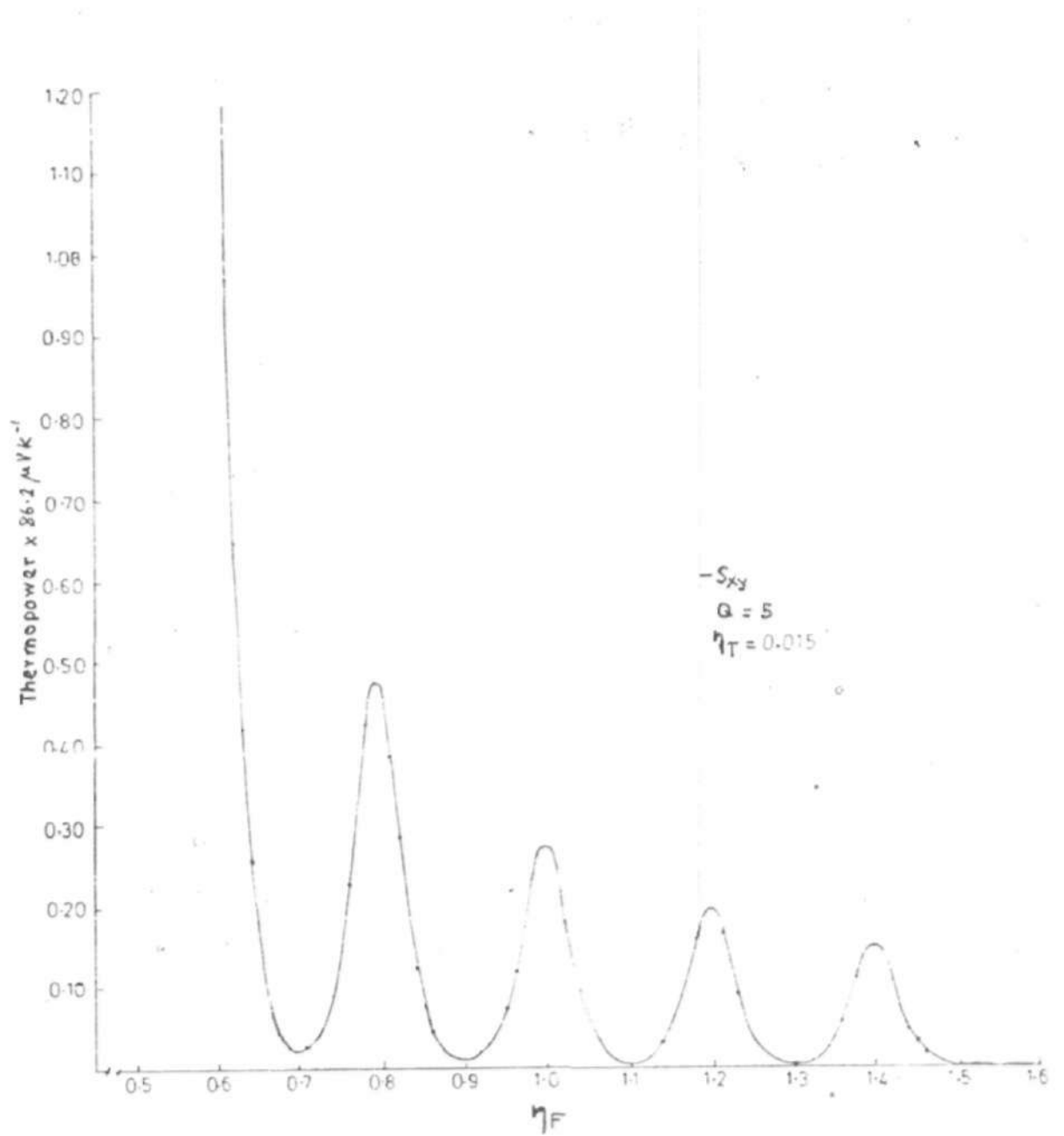


Fig. 4.9: Off-diagonal component of the thermopower vs chemical potential where the reduced temperature $\eta_T = 0.015$. This corresponds to $T = 1.29\text{k}$ at $B = 12\text{T}$ in a Si (110) inversion layer. $Q = 5$ particles.

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APPENDIX A

Properties of associated Laguerre Polynomials $L_n^m(\rho)$.

$$\Psi_n^m(\rho) = e^{-\rho/2} \cdot \rho^{m/2} L_n^m(\rho) \quad \text{A.1}$$

$$L_n^m(\rho) = \frac{1}{n!} \rho^{-m} e^{\rho} \frac{d^n}{d\rho^n} (\rho^{m+n} e^{-\rho}) \quad \text{A.2}$$

where $\Psi_n^m(\rho)$ is a solution to the differential equation

$$\rho y'' + y' + \left(\frac{m+1}{2} + n \frac{-\rho}{4} - \frac{m^2}{4\rho} \right) y = 0 \quad \text{A.3}$$

for $n = 0, 1, 2 \dots$

$$\int_0^{\infty} \Psi_n^m(\rho) \Psi_{n'}^m(\rho) d\rho = \frac{\Gamma(n+m+1)}{n!} \delta_{nn'} \quad \text{A.4}$$

Recurrence Relations

$$1. \quad \rho \Psi_n^m(\rho) = - (n+1) \Psi_{n+1}^m(\rho) + (2n+m+1) \Psi_n^m(\rho) - (n+m) \Psi_{n-1}^m(\rho) \quad \text{A.5}$$

$$2. \quad \rho^{1/2} \Psi_n^{m+1}(\rho) = (n+m+1) \Psi_n^m(\rho) - (n+1) \Psi_{n+1}^m(\rho) \quad \text{A.6}$$

$$= (\rho - n) \Psi_n^m(\rho) - (n+m) \Psi_{n+1}^m(\rho) \quad \text{A.7}$$

$$3. \quad \rho^{1/2} \Psi_n^{m-1}(\rho) = \Psi_n^m(\rho) - \Psi_{n-1}^m(\rho) \quad \text{A.8}$$

$$= \frac{\rho - n - 1}{n+m} \Psi_n^m(\rho) + \frac{n+1}{n+m} \Psi_{n+1}^m(\rho)$$

$$4. \quad \rho \frac{d}{dx} \Psi_n^m(\rho) = \left(n + \frac{m}{2} - \frac{\rho}{2} \right) \Psi_n^m(\rho) - (n+m) \Psi_{n-1}^m(\rho) \quad \text{A.9}$$

APPENDIX B

```

PROGRAM S1
C THIS PROGRAM CALCULATES THE TRANSPORT COEFFICIENTS AND
C THE THERMOPOWER OF A THREE PARTICLE LANDAU CLUSTER.
C
C THE REDUCED TEMPERATURE,  $\eta_t$ , IS 0.01.
C THE NUMBER OF PARTICLES, Q, IS 3.
C THE REDUCED CHEMICAL POTENTIAL,  $\eta_f$ , IS VARIED FROM
C 0.5 TO 2.0.
C L12 IS THE THERMAL RESPONSE COEFFICIENT.
C L11 IS THE RESPONSE COEFFICIENT.
C S IS THE THERMOPOWER.
C
REAL L11,L12
INTEGER Q
ETAT=0.01
Q=3
A=ALOG(0.5)
M=0
WRITE(6,101)ETAT
ETAF=0.5
10 L11=0.
L12=0.
NMAX=Q-1
DO 99 N=M,NMAX
RQ=(N+0.5)/Q
BN=(ETAF-0.5-RQ)/(ETAT*2.)
CC=COSII(BN)
T=TANH(BN)
B=ALOG(CC)
C=BN*T
DD=EXP(-(BN*2.))+1.
D=1./DD
L11=L11+D
L12=L12+(A-B+C)
99 CONTINUE
S=L12/L11
WRITE(6,102)Q,ETAF,L12,L11,S
IF(ETAF.GE.2.0)GO TO 100
ETAF=ETAF+0.005
GO TO 10
100 WRITE(6,103)
101 FORMAT (/5X,' $\eta_t$  =',F4.2/5X,66(' ')/5X,':',1X,':',
$11X,':',14X,':',14X,':',14X,':' /5X,':',3X,'Q',3X,':',
$4X,' $\eta_f$ ',3X,':',6X,'L12',5X,':',6X,'L11',5X,':',7X,'S',
$6X,':' /5X,66(' '))
102 FORMAT (5X,':',3X,11,3X,':',3X,F5.3,3X,':',3X,F8.4,
$3X,':',3X,F8.4,3X,':',3X,F8.4,3X,':')
103 FORMAT (5X,66(' '))
STOP
END

```

```

PROGRAM S3
C THIS PROGRAM CALCULATES THE TRANSPORT COEFFICIENTS AND
C THE THERMOPOWER OF A FIVE PARTICLE LANDAU CLUSTER.
C
C THE REDUCED TEMPERATURE,  $\eta_t$ , IS 0.015.
C THE NUMBER OF PARTICLES, Q, IS 5.
C THE REDUCED CHEMICAL POTENTIAL,  $\eta_f$ , IS VARIED FROM 0.5
C TO 2.0.
C L12 IS THE THERMAL RESPONSE COEFFICIENT.
C L11 IS THE RESPONSE COEFFICIENT.
C S IS THE THERMOPOWER.
C
REAL L11,L12
INTEGER Q
ETAT=0.015
Q=5
A=ALOG(0.5)
M=0
WRITE(6,101)ETAT
ETAF=0.5
10 L11=0.
L12=0.
NMAX=Q-1
DO 99 N=M,NMAX
RQ=(N+0.5)/Q
BN=(L11-0.5-RQ)/(ETATRQ.)
CC=COSH(BN)
T=TANH(BN)
B=ALOG(CC)
C=BN*T
DD=EXP(-(BN*2.))+1.
D=1./DD
L11=L11+D
L12=L12+(A-B+C)
99 CONTINUE
S=L12/L11
WRITE(6,102)Q,ETAF,L12,L11,S
IF(ETAF.GE.2.0)GO TO 100
ETAF=ETAF+0.01
GO TO 10
100 WRITE(6,103)
101 FORMAT(/5X,' $\eta_t$  =',F4.2/5X,66(' ')/5X,':',7X,':',
$11X,':',14X,':',14X,':',14X,':'/'5X,':',3X,'Q',3X,':',
$4X,' $\eta_f$ ',3X,':',6X,'L12',5X,':',6X,'L11',5X,':',
$7X,'S',6X,':'/'5X,66(' '))
102 FORMAT(5X,':',3X,11,3X,':',3X,F5.3,3X,':',3X,F8.4,
$3X,':',3X,F8.4,3X,':',3X,F8.4,3X,':')
103 FORMAT(5X,66(' '))
STOP
END

```

```

PROGRAM S4
C THIS PROGRAM CALCULATES THE TRANSPORT COEFFICIENTS AND
C THE THERMOPOWER OF A THREE-PARTICLE LANDAU-CLUSTER.
C
C THE REDUCED TEMPERATURE,  $\eta_t$ , IS 0.05.
C THE NUMBER OF PARTICLES, Q, IS 3.
C THE REDUCED CHEMICAL POTENTIAL,  $\eta_f$ , IS VARIED FROM
C 0.5 TO 2.0.
C L12 IS THE THERMAL RESPONSE COEFFICIENT.
C L11 IS THE RESPONSE COEFFICIENT.
C S IS THE THERMOPOWER.
C
REAL L11,L12
INTEGER Q
ETAT=0.05
Q=3
A=ALOG(0.5)
M=0
WRITE(6,101)ETAT
ETAF=0.5
10 L11=0.
L12=0.
NMAX=Q-1
DO 99 N=M,NMAX
RQ=(N+0.5)/Q
BN=(ETAF-0.5-RQ)/(ETAT*2.)
CC=COSH(BN)
T=TANH(BN)
B=ALOG(CC)
C=BN*T
DD=EXP(-(BN*2.))+1.
D=1./DD
L11=L11+D
L12=L12+(A-B+C)
99 CONTINUE
S=L12/L11
WRITE(6,102)Q,ETAF,L12,L11,S
IF(ETAF.GE.2.0)GO TO 100
ETAF=ETAF+0.005
GO TO 10
100 WRITE(6,103)
101 FORMAT(/5X,' $\eta_t$  =',F4.2/5X,66('-'))/5X,':',7X,':',
$11X,':',14X,':',14X,':',14X,':',14X,':',5X,':',3X,'Q',3X,':',
$4X,' $\eta_f$ ',3X,':',6X,'L12',5X,':',6X,'L11',5X,':',7X,'S',
$6X,':',5X,66('-'))
102 FORMAT(5X,':',3X,11,3X,':',3X,F5.3,3X,':',3X,F8.4,
$3X,':',3X,F8.4,3X,':',3X,F8.4,3X,':')
103 FORMAT(5X,66('-'))
STOP
END

```


APPENDIX C

The separation of the Hamiltonian for N-electron Interaction.

Two particles

N = 2. The Hamiltonian is

$$H = \frac{1}{2m} \left(\vec{P}_1 - \frac{e}{c} \vec{A}_1 \right)^2 + \frac{1}{2m} \left(\vec{P}_2 - \frac{e}{c} \vec{A}_2 \right)^2 + \frac{e^2}{r_{12}} \quad 1$$

define relative and centre of mass (c.o.m) coordinates;

$$Z_a = \frac{Z_1 + Z_2}{2} \quad \text{centre of mass} \quad 2$$

and

$$Z_b = n_1 Z_1 + n_2 Z_2 \quad \text{Rel. co-ord.} \quad 3$$

subject to the condition

$$Z_1^2 + Z_2^2 = 2Z_a^2 + Z_b^2 \quad 4$$

The Hamiltonian is thus separable (P_1, P_2, A_1, A_2 are quadratic on expansion

$P_1^2 + P_2^2$ and $A_1^2 + A_2^2$ should be separable).

Rearranging (2) + (3)

$$Z_1 + Z_2 = 2Z_a$$

$$n_1 Z_1 + n_2 Z_2 = Z_b$$

solve for Z_1 and Z_2 above and apply condition (4) we obtain

$$n_1 = \frac{1}{\sqrt{2}}$$

$$n_2 = -n_1 = -\frac{1}{\sqrt{2}}$$

therefore the coordinates are given by

$$Z_a = \frac{Z_1 + Z_2}{2}$$

$$Z_b = \frac{Z_1 - Z_2}{\sqrt{2}}$$

The Hamiltonian is then separable to;

$$H = H_{int} + H_{ext}$$

where,

$$H_{\text{ext}} = H_{\text{c.m.}} = \frac{1}{2M} (\vec{P}_a - \frac{e}{c} \vec{A}_a)^2, \quad M = 2m$$

and,

$$H_{\text{int}} = \frac{1}{2m} (\vec{P}_b - \frac{e}{c} \vec{A}_b)^2 + \frac{e^2}{\sqrt{1} r_{12}}$$

$$r_{12} = |z_1 - z_2|$$

Three particles

One write the 2-selected coordiantes for the two particle system and add a third one viz (Z_n is extended to 3-particles, Z_b goes to Z_c , and the new Z_b).

$$Z_a = \frac{Z_1 + Z_2 + Z_3}{3} \quad \text{c.o.m.}$$

$$Z_b = m_1 Z_1 + m_2 Z_2 + m_3 Z_3 \quad \text{one type of relative coordiante.}$$

$$Z_c = \frac{Z_1 - Z_2}{2} \quad \text{- relative coordiante.}$$

The Hamiltonian for the 3-particles is

$$H = \frac{1}{2m} (\vec{P}_1 - \frac{e}{c} \vec{A}_1)^2 + \frac{1}{2m} (\vec{P}_2 - \frac{e}{c} \vec{A}_2)^2 + \frac{1}{2m} (\vec{P}_3 - \frac{e}{c} \vec{A}_3)^2 + \frac{e^2}{r_{12}} + \frac{e^2}{r_{23}} + \frac{e^2}{r_{13}} \quad 1.$$

impose the condintion that

$$Z_1^2 + Z_2^2 + Z_3^2 = 3Z_a^2 + Z_b^2 + Z_b^2 + Z_c^2 \quad 2.$$

Rearranging the expension for the coordiantes we obtain;

$$Z_1 + Z_2 + Z_3 = 3Z_a$$

$$m_1 Z_1 + m_2 Z_2 + m_3 Z_3 = Z_b$$

$$\frac{1}{\sqrt{2}} Z_1 - \frac{1}{\sqrt{2}} Z_2 = Z_c$$

Solve for Z_1 , Z_2 , Z_3 and impose (2) one obtains

$$m_1 = \frac{1}{\sqrt{6}} = m_2, \quad m_3 = -\frac{\sqrt{2}}{3}$$

Thus the coordiantes are;

$$Z_a = \frac{Z_1 + Z_2 + Z_3}{3}$$

$$Z_b = \frac{Z_1 + Z_2 - Z_3}{\sqrt{6}}$$

and

$$Z_c = \frac{Z_1 - Z_2}{\sqrt{2}}$$

The Hamiltonian again separates into

$$H = H_{\text{ext}} + H_{\text{int}}$$

Where,

$$H_{\text{ext}} = \frac{1}{2M} (\vec{P}_a - \frac{e}{c} \vec{A}_a)^2, \quad M = 3m$$

and,

$$H_{\text{int}} = \frac{1}{2m} (\vec{P}_b - \frac{e}{c} \vec{A}_b)^2 + \frac{1}{2m} (\vec{P}_c - \frac{e}{c} \vec{A}_c)^2 +$$

$$\frac{e^2}{r_{12}} + \frac{e^2}{r_{23}} + \frac{e^2}{r_{21}}$$

$$r_{31} = |Z_3 - Z_1|$$

$$r_{12} = |Z_1 - Z_2|$$

$$r_{23} = |Z_2 - Z_3|$$

Four particles

As usual we write the 3-particle coordinates and select a fourth.

i.e

$$Z_a = \frac{Z_1 + Z_2 + Z_3 + Z_4}{4} \quad 1.$$

Retain,

$$Z_b = \sqrt{\frac{1}{6}} (Z_1 + Z_2 - 2Z_3) \quad 2.$$

$$Z_c = \frac{Z_1 - Z_2}{\sqrt{2}} \quad 3.$$

but, add

$$Z_d = n_1 Z_1 + n_2 Z_2 + n_3 Z_3 + n_4 Z_4 \quad 4.$$

Rearranging the equation to;

$$Z_1 + Z_2 + Z_3 + Z_4 = 4Z_a$$

$$Z_1 + Z_2 - 2Z_3 = \sqrt{6} Z_b$$

$$Z_1 - Z_2 = \sqrt{2} Z_c$$

$$n_1 Z_1 + n_2 Z_2 + n_3 Z_3 + n_4 Z_4 = Z_d$$

Solve for Z_1, Z_2, Z_3, Z_4

and impose the condition;

$$z_1^2 + z_2^2 + z_3^2 + z_4^2 = 4z_a^2 + z_b^2 + z_c^2 + z_d^2$$

5

one obtain on solving;

$$z_1 = -\frac{2n_4}{\sqrt{3}} z_a + \frac{1}{2\sqrt{2}} (n_3 - n_4) z_b + \frac{1}{2\sqrt{6}} z_c (2n_2 + n_3 - 3n_4) + \frac{1}{2\sqrt{3}} z_d$$

$$z_2 = -\frac{2n_4}{\sqrt{3}} z_a + \frac{1}{\sqrt{2}} (n_3 - n_4) z_b + \frac{1}{2\sqrt{6}} (n_2 - n_1) z_c + \frac{1}{2\sqrt{3}} z_d +$$

$$\frac{1}{2\sqrt{6}} (n_2 - n_1) z_b + \frac{1}{2\sqrt{3}} z_c$$

$$z_3 = -\frac{2}{\sqrt{3}} n_4 z_a + \frac{1}{2\sqrt{2}} (2n_4 - n_1 - n_2) z_b + \frac{1}{2\sqrt{6}} (n_2 - n_1) z_c + \frac{1}{2\sqrt{3}} z_d$$

$$z_4 = \frac{2(n_1 + n_2 + n_3)}{\sqrt{3}} z_a + \frac{1}{2\sqrt{2}} (n_1 + n_2 - 2n_3) z_b + \frac{1}{2\sqrt{6}} (3n_1 - 3n_2) z_c - \sqrt{\frac{3}{4}} z_d$$

imposing condition (5) on the coefficients of z_a one obtain

$$n_4 = -\frac{\sqrt{3}}{2}$$

$$n_3 = \frac{1}{2\sqrt{3}}$$

$$n_2 = n_1 = \frac{1}{2\sqrt{3}}$$

$$z_1 = z_a + \frac{1}{\sqrt{6}} z_b + \frac{1}{\sqrt{2}} z_c + \frac{1}{2\sqrt{3}} z_d$$

$$z_2 = z_a + \frac{1}{\sqrt{6}} z_b - \frac{1}{\sqrt{2}} z_c + \frac{1}{2\sqrt{3}} z_d$$

$$z_3 = z_a - \sqrt{\frac{2}{3}} z_b + \frac{1}{2\sqrt{3}} z_d$$

$$z_4 = z_a - \frac{\sqrt{3}}{2} z_d$$

therefore the coordiante transformations are;

$$z_a = \frac{z_1 + z_2 + z_3 + z_4}{4}$$

$$z_b = \sqrt{\frac{2}{3}} \left(\frac{z_1 + z_2}{2} - z_3 \right)$$

$$z_c = \frac{1}{\sqrt{2}} (z_1 - z_2)$$

$$z_d = \frac{\sqrt{3}}{2} \left(\frac{z_1 + z_2 + z_3}{3} - z_4 \right)$$

Thus the 4-particle Hamaltonian can be separated into,

$$H_{\text{ext}} = \frac{1}{2M} (\vec{P}_a - \frac{e}{c} \vec{A}_a)^2, \quad M = 4m$$

$$\text{and, } H_{int} = \frac{1}{2m} (\vec{p}_b - \frac{e}{c} \vec{A}_b)^2 + \frac{1}{2m} (\vec{p}_c - \frac{e}{c} \vec{A}_c)^2 \\ + \frac{1}{2m} (p_b - \frac{e}{c} A_b)^2 + \frac{e^2}{r_{12}} + \frac{e^2}{r_{23}} + \frac{e^2}{r_{14}} + \frac{e^2}{r_{23}} + \frac{e^2}{r_{24}} + \frac{e^2}{r_{34}}$$

COMMENT

For a general N-particle system we can generate the coordinate representation for the (N-1) particle system, and add the Nth representation, i.e

we impose the condition

$$\sum_{i=1}^N Z_i^2 = NZ^2 + \sum_{i=1}^{N-1} Z_{ai}^2$$

where the Z_{ai} ($i = 1, 2 \dots N - 1$) are the representation for the N-1 particle system and

$$Z = \sum_{i=1}^N Z_i / N$$

the Z_{ai} , $i = N - 1$ is the added representation.

APPENDIX D

Calculation of the conductivity tensor $\sigma_{xy}^{(0)}$

To calculate the velocity matrix element;

$$\langle N^1, M^1; n^1, m^1 | v_x^b | n_1, m_1, N_1, M_1 \rangle$$

where

$$H_b | n m \rangle = \epsilon_n | n m \rangle \quad 1.$$

one observes that this matrix element equals

$$\langle n', m' | v_x^b | n, m \rangle \delta_{N'N} \delta_{M'M}$$

in the expression for the Green's functions, it should be noted that

$$G^{R,A}(E) = \sum_{nm} \frac{\psi_{NM}^{n,m} \psi_{NM}^{n,m}}{E - \epsilon_n - \epsilon_m \pm i\delta} \quad 2.$$

where

$$\psi_{NM}^{n,m} \equiv \varphi_{NM}(z_a) \varphi_{nm}(z_b) \quad 3.$$

so that,

$$\begin{aligned} & \text{Tr} \left\{ v_x I_m G(E) v_y \frac{dG^A}{dE} \right\} \\ &= \sum \langle v_\mu | v_x | v, \mu \rangle (-\pi \delta [E - \epsilon_n - \epsilon_m]) \langle v, \mu | v_y | v, \mu \rangle \frac{-1}{(E - \epsilon_n - \epsilon_m)^2} \end{aligned}$$

$$\text{where } v = \left\{ \begin{array}{l} n_1 N \end{array} \right\}$$

$$\mu = \left\{ \begin{array}{l} m_1 M \end{array} \right\}$$

But

$$\langle v_\mu | v_x^b | v, \mu \rangle \equiv \langle n m | v_x^b | n, m \rangle \delta_{N'N} \delta_{M'M} \quad 4.$$

we have

$$\begin{aligned} & \sum \langle n m | v_x^b | n, m \rangle \langle n, m | v_y^b | n m \rangle \delta_{N'N} \delta_{M'M} \frac{\pi \delta(E - \epsilon_n - \epsilon_m)}{(E + \epsilon_n - \epsilon_n - \epsilon_m)^2} \\ &= \sum_{\substack{N, M \\ n, m; n m}} \frac{\pi}{i} \left(\frac{\hbar}{M} \right)^2 \left(\frac{\alpha}{2} \right) \delta_{N'N} \delta_{M'M} \delta_{N,N} \delta_{M,M} \\ & \quad \times \left\{ (n+1)^k \delta_{n, n+1} \delta_{m, m+1} - n^k \delta_{n, n-1} \delta_{m, m-1} \right\} \\ & \quad \times \left\{ n^k \delta_{n, n-1} \delta_{m, m-1} + (n+1)^k \delta_{n, n+1} \delta_{m, m+1} \right\} \frac{\delta(E - \epsilon_n - \epsilon_m)}{\dots} \end{aligned}$$

$$\begin{aligned}
&= \sum_{\substack{n, m \\ n+1, m}} \frac{\pi}{i} \left(\frac{\hbar}{M}\right)^2 \left(\frac{\alpha}{2}\right) \frac{\delta[E - E_{n+1} - E_n]}{(E_{n+1} - E_n)^2} \times \left\{ (n+1) \delta_{n, n+1} \delta_{m, m+1} - n \delta_{n, n-1} \delta_{m, m-1} \right\} \\
&= \frac{\pi}{i} \left(\frac{\hbar}{M}\right)^2 \left(\frac{\alpha}{2}\right) \sum_{\substack{NM \\ nm}} \frac{(n+1) \delta[E - E_{n+1} - E_n]}{(\hbar \omega_c)^2} - \frac{n \delta[E - E_{n-1} - E_n]}{(\hbar \omega_c)^2} \\
&= \frac{\pi}{i} \left(\frac{\hbar}{M}\right)^2 \left(\frac{\alpha}{2}\right) \frac{M}{\hbar^2 \alpha} \sum_{\substack{NM \\ nm}} (n+1) \delta[E - E_{n+1} - E_n] - n \delta[E - E_{n-1} - E_n] \\
&= \frac{\pi}{i} \frac{1}{2M} \sum_{\substack{NM \\ nm}} \left\{ (n+1) \delta[E - E_{n+1} - E_n] - n \delta[E - E_{n-1} - E_n] \right\} \quad 5
\end{aligned}$$

Similarly, $\text{Tr} \left\{ v_x \frac{dG^R(E)}{dE} v_y \text{Im} G(E) \right\} = \sum \langle n, m | v_x | n, m \rangle \left[\frac{1}{(E - E_{n-1} - E_n)^2} \right]$
 $\times \langle n, m | v_y | n, m \rangle \left[-\pi \delta(E - E_n - E_n) \right]$
 $= \sum \langle n, m | v_x^\dagger | n, m \rangle \langle n, m | v_y^\dagger | n, m \rangle \frac{-\pi \delta(E - E_n - E_n)}{(E_n - E_n)^2}$

So that

$$\text{Tr} \left\{ v_x \frac{dG^R(E)}{dE} v_y \text{Im} G(E) \right\} = -\pi \sum_{\substack{NM \\ nm}} \langle n, m | v_x^\dagger | n, m \rangle \langle n, m | v_y^\dagger | n, m \rangle \times \left\{ \frac{\delta[E - E_{n+1} - E_n]}{(E_{n+1} - E_n)^2} - \frac{\delta[E - E_{n-1} - E_n]}{(E_{n-1} - E_n)^2} \right\}$$

But

$$\begin{aligned}
\langle n, m | v_x^\dagger | n, m \rangle \langle n, m | v_y^\dagger | n, m \rangle &= \left\{ (n+1)^{\frac{k}{2}} \delta_{n, n+1} \delta_{m, m+1} - n^{\frac{k}{2}} \delta_{n, n-1} \delta_{m, m-1} \right\} \\
&\times \left\{ n^{\frac{k}{2}} \delta_{n, n-1} \delta_{m, m-1} + (n+1)^{\frac{k}{2}} \delta_{n, n+1} \delta_{m, m+1} \right\} \\
&= \left\{ (n+1) \delta_{n, n+1} \delta_{m, m+1} - n \delta_{n, n-1} \delta_{m, m-1} \right\}
\end{aligned}$$

$$\text{Tr} \left\{ v_x \frac{dG^R(E)}{dE} v_y \text{Im} G(E) \right\} = \frac{-\pi}{2iM} \sum_{\substack{N, M \\ n, m}} \left\{ (n+1) \delta(E - \epsilon_{n+1} - \epsilon_N) - n \delta[E - \epsilon_{n-1} - \epsilon_N] \right\} \quad 6.$$

Substitute Eqn (5) and (6) into

$$\sigma_{xy}(0) = \frac{ie^2}{\pi S} \int_{-\infty}^{\infty} dE f(E) \text{Tr} \left\{ v_x \text{Im} G(E) \frac{dG^A}{dE} - v_x \frac{dG^R}{dE} v_y \text{Im} G(E) \right\}$$

and Simplifying one gets,

$$\sigma_{xy}(0) = -\frac{e^2}{h} \int_{-\infty}^{\infty} dE f(E) \sum_{\substack{N \\ n, m}} \delta(E - \epsilon_n - \epsilon_N)$$