A STOCHASTIC MODEL FOR THE PUBLIC TRANSPORT LINKAGE BETWEEN THE NEW AND OLD CAMPUSES OF BAYERO UNIVERSITY, KANO.

BY

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DECLARATION

I hereby declare that this thesis has been composed by myself and that it is a record of my own research. It has not been submitted in any previous application for a higher degree.

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1996

CERTIFICATION.

This thesis entitled "A STOCHASTIC MODEL FOR THE PUBLIC TRANSPORT LINKAGE BETWEEN THE NEW AND OLD CAMPUSES OF BAYERO UNIVERSITY, KANO " by HASHIM MOHAMMED ALHASSAN meets the regulations governing the award of the degree of Master of Science (Civil Engieering, HIGHWAY AND TRANSPORTATION) OF Ahmadu Bello University and is approved for its contribution to knowlegde and literary presentation.

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A C K N O W L E D G E M E N T S

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To all those whose contributions made this project possible, I say GOD BLESS.

ABSTRACT

This study evaluates the performance measures of the queueing systems emanating from the Bayero University, Kano, new campus access area as a result of the demand for transportation to its old campus. Analysis of the queueing systems reveal that transient behaviour is more appropriate in describing the system than conventional steady-state analysis. A computer simulation package using Runge-Kutta numerical methods was used to solve the set of linear differential equations developed for the three transport modes serving the accesss area. The simulation models were tested with field data collected from the access area on all the trips made during the study period. The probabilities of delay predicted by the simulation models agreed closely with field observations. The chi-squared test indicated that there were no significant differences between the models and the field values at the 5% level of significance. Passengers at the access area experienced an average delay of 16.4minutes, 12.4minutes and 23.5minutes for the taxi, minibus and university transport respectively. The methodology could be useful in analysing the queueing systems in other universities or areas having similar trip demand characteristics as Bayero university, Kano.

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CHAPTER ONE

INTRODUCTION

1.1 General

There are two campuses of Bayero University, Kano. These are the old campus, situated at Kofar Kabuga, and the the new campus at Rimin Gata village as shown in figure 1.0. The two campuses are about 5km apart. Every morning students from both campuses arrive at an access area shown in (figure 1.1) to take buses to their destinations and choose one of two options; free university bus service or commercial transport. Of the commercial transport, two modes are available. These are the taxi mode and the mini-bus mode.

Prospective passengers arriving at the new campus access area either join the mode of their choice or wait for service if service is not immediately available. Thus a queueing situation arises. However, the only group of customers studied are those whose prefered mode of access to the old campus is a commercial mode (ie taxi or minibus) Interestingly, even for this group if there are no commercial modes available and a university bus departs before the next commercial mode, then the queued would-be commercial mode riders is cleared at random times which depends upon the frequency of university bus service.

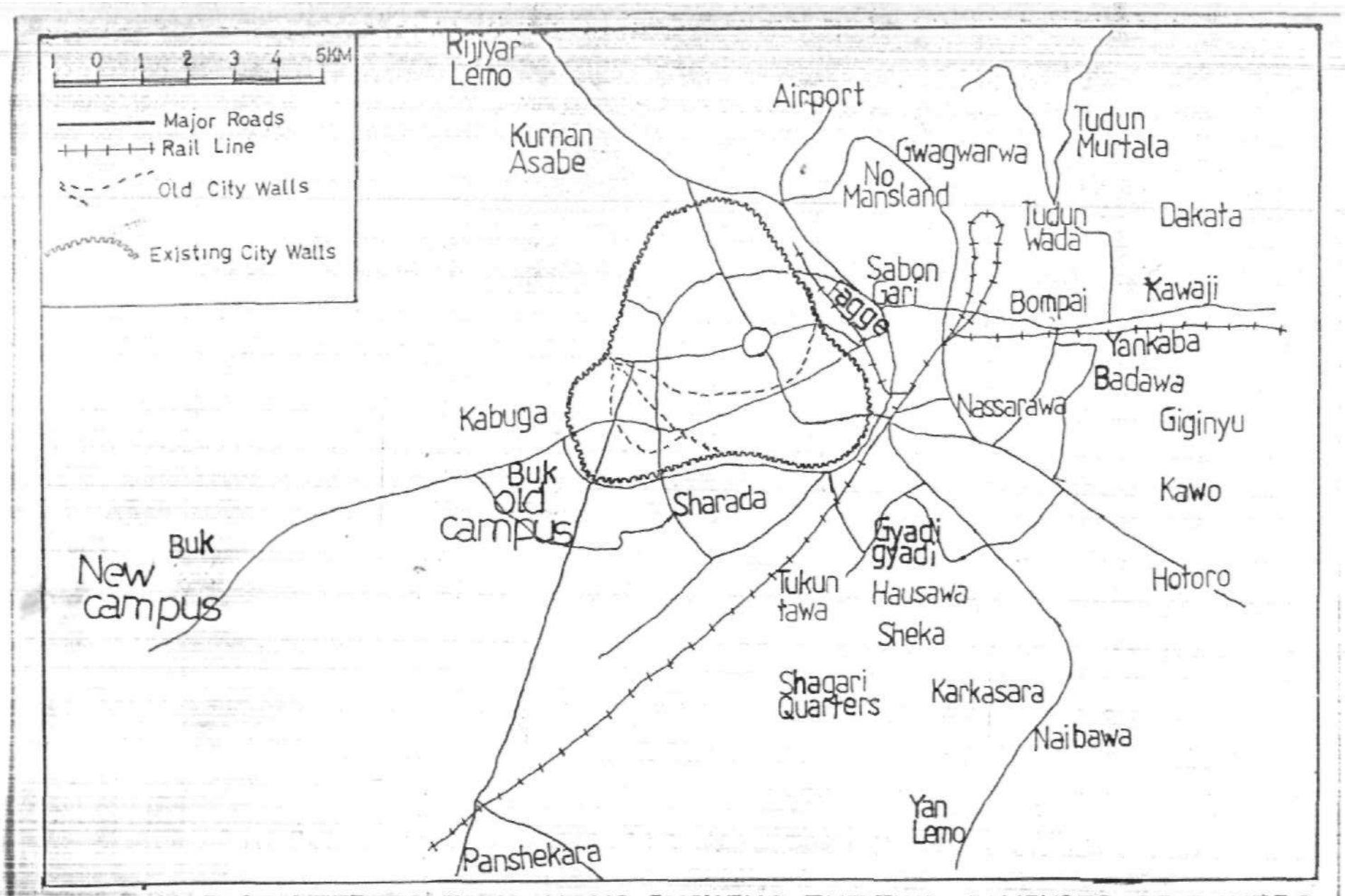

Fig. 1·0:-MAP OF METROPOLITAN KANO SHOWING THE TWO CAMPUSES OF BAYERO UNIVERSITY.

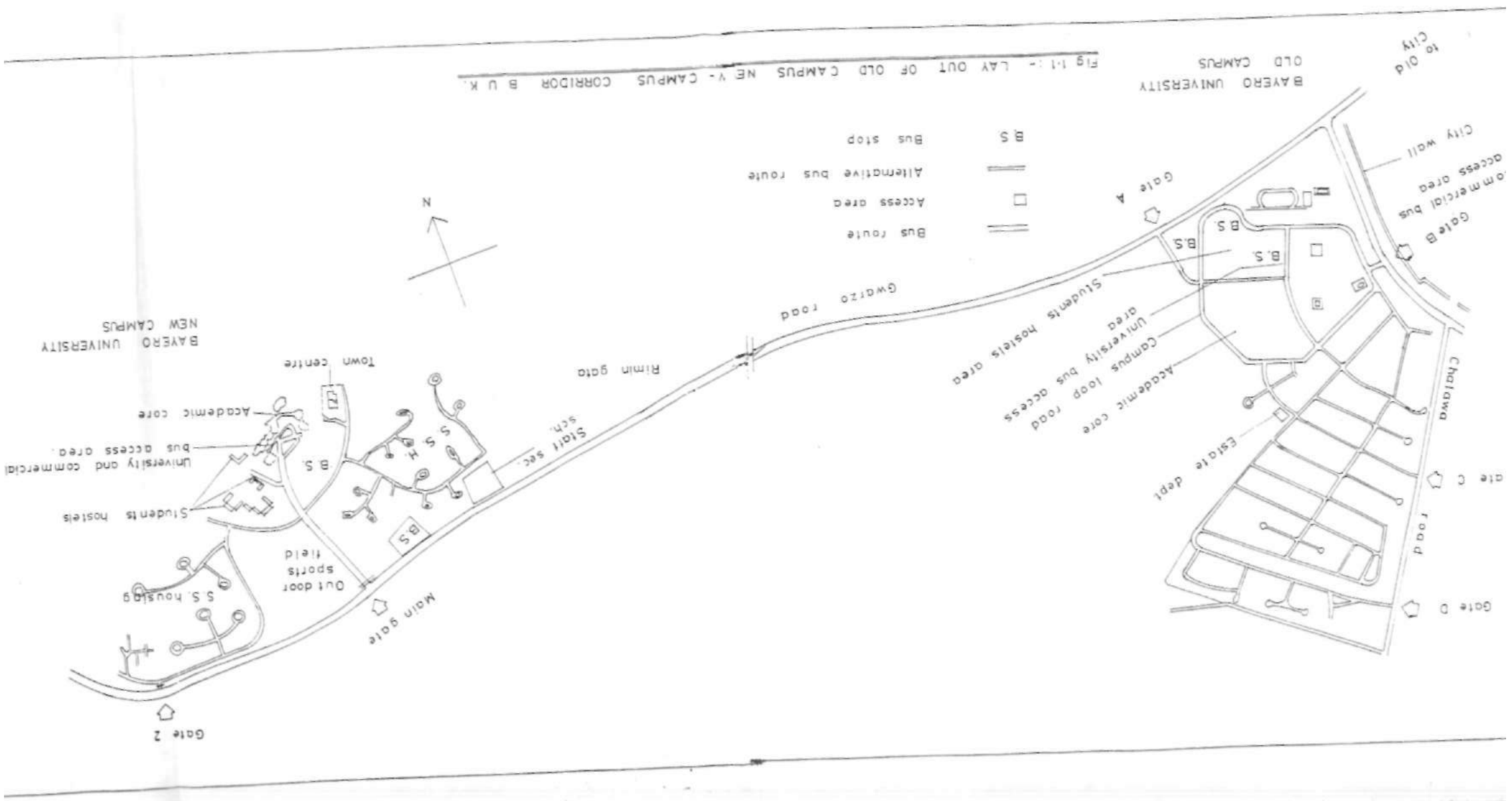

Fig 1.1 :- LAY OUT OF OLD CAMPUS NEW CAMPUS CORRIDOR B.U.K.

The specific problem this thesis seeks to tackle is that of generating policy options on effective solution to the problem of transporting students and staff between the two campuses of Bayero University, Kano (BUK).

1.2 BACKGROUND TO TRANSPORTATION PROBLEMS IN THE NEW CAMPUS

The need to provide transportation to ply between the two campuses of Bayero University, arose when the first set of students in the faculties of technology and Law, and the department of mass communications began taking lectures for the 1983/84 academic session. Due to their small number, they constituted too small a market to attract conventional public transport service to the old campus. Moreover, a bus (mini-bus) was made available by the university authority to convey them to the old campus for recreational activities. In addition, some of the students had personal transport facilities for their movements. This situation posed no problem to the transportation demand at the new campus at that time.

Eventually, the number of faculties in the new campus increased, students number soared and it became apparent that a deliberate effort was needed to curb the emerging transportation problems characterised by long queues and unreliable service. During the 1989/90 academic session, the

university administration moved to the new campus and a final reshuffling of faculties took place, resulting in even larger student population in the new campus. The university administrative staff-both senior and mostly junior staff rely on university transportation to commute between the two campuses. Visitors to the university (non-vehicle owners) also depend on university transportation to commute between the campuses. In addition, most students must commute between the two campuses daily for lectures. This situation stretched the transportation system much further.

Measures taken to tackle the transportation problems were;

(1) Increase in the number of university buses commuting between the two campuses.

(2) Collection of token fares to offset the cost of maintenance, which was at first twenty kobo,but later increased to fifty kobo and is now one naira per ride.

These two measures failed to improve the transport situation because university buses scheduled for the operation were often arbitrarily diverted and assigned to other university business. The collection of the token fares did not endure either. Often tickets were not available, and it soon became known that the cost of producing the tickets

far outweighed the revenue generated from the buses operation. Students unwillingness to pay, often with the excuse that they had no coins, contributed to the poor revenue generated.

While the demand to commute between the two campuses increased, the buses operation became unreliable as they did not adhere to any schedule of operation. Finally, the university authority invited private transport proprietors to ply between the campuses. The taxi mode and the mini-bus mode responded. Despite these, however, congestion has not been eradicated and the long queues and delays still persist.

1.3 OBJECTIVE AND SIGNIFICANCE OF STUDY

The objectives of the research are:

(1) To develop a stochastic model suitable for predicting the trends in transport demand and the interaction between the various modes of transport between the campuses.

(2) To use the model developed to investigate the implications of varying the number of private taxis, mini-buses and the university bus on the level of service.

1.4 SCOPE

The scope of this research include:

(1) Statistical analysis of data on passenger arrival, vehicle arrival and service characteristics.

(2) Development of a model to predict the demand of customers departing the new campus and the number of vehicle trips within each mode required to serve the demand and

(3) A stochastic model (queueing) suitable for studying the queueing systems arising at each transport mode at the access area in the new campus.

(4) Development of a computer program to aid in the analysis of the queueing model developed and hence computer simulation of the studied situation.

1.5 RESEARCH METHODOLOGY

This research involved basically field data collection and mathematical modelling. Data relating to trips between the campuses were collected to generate information on:

(i) Trip demand by time of day.

(ii) Number of customers carried daily.

(iii) Number of vehicle trips of each transport mode operating daily.

(iv) Arrival pattern of customers to the access area for

each mode per some chosen unit of time.

(v) The service times of each transport mode operating at the access area.

Statistical analysis was carried out on items (iv) and (v) to determine which standard probability distributions fit the data. This was with a view to properly defining the arrival and service patterns of the queueing process. The transient behaviour was also analysed to determine whether a stationary process or transient character is more critical. Appropriate mathematical models were then developed and a computer program developed for the analysis.

CHAPTER TWO

LITERATURE REVIEW.

2.1 INTRODUCTION

Queueing is a phenomenon that is observed in everyday life. Due to increasing congestion and urbanisation in towns and cities as a result of large populations, there is hardly a situation in life in which a queue does not arise. For instance,we wait in line at traffic jams or toll booths, at road junctions to enter major traffic streams, at transportation access points to enter buses or taxis to our destinations, at post offices, at banks, at supermarkets, at petrol filling stations, to mention a few.

These waits or queues arise because there is more demand for service than there is facility for service Gross and Harris,(1972). There may be several reasons why the demand for service exceeds the facility available; first there may be a shortage of available servers, or it may be infeasible economically to provide the level of service necessary to prevent waiting or there may be a limit to the amount of space that can be provided. The proprietors or managers at the establishments at which we wait (queue) might be interested in such questions as: How much equipment or facility must be provided so that the

proportion of customers experiencing delays will be below an acceptable level? The answer to such a question will be based in part on such diverse considerations as politics, economics and technical knowledge. But it possesses an important characteristic; The times at which requests for service will occur and the lengths of times that these requests will occupy facilities cannot be predicted except in a statistical sense, Cooper, (1978). The branch of mathematics amenable to such prediction and analysis is probability theory under which falls the theory of stochastic processes.

The term stochastic implies that a probability function is generating an ordered sequence of events. In the queueing context it is common to relate that ordering to time Giffin,(1978). For instance, a time series plot of the number of customers waiting at a bus stop at the time each bus arrives for pick-up represents a stochastic process.

2.1.1 DESCRIPTION OF A QUEUEING SYSTEM

A queueing system can be described as customers arriving for service; waiting for service if it is not immediate, and if having waited for service, leaving the system after being served. A queueing process is therefore basically described by:

(1) the arriving pattern of customers.

(2) service pattern of servers.

(3) queue discipline.

The *arrival pattern* or input to a queueing system is often measured in terms of the average number of arrivals per some unit of time (mean arrival rate) or by the average time between successive arrivals (mean inter-arrival time). Another factor of interest concerning the arrival pattern is the possibility that arrivals come in batches instead of one at a time. In this case, the batch size and the time between successive arrivals of the batches are random. A customer arriving at a queue may decide not to enter, that is *BALK* or may leave after a long time in the queue, that is *RENEGE*. In the event that, there are two or more parallel waiting lines, customers may switch from one to another that is *JOCKEY* for position. Finally, the arrival pattern may change with time or may be constant over time. In the latter, it is said to be a stationary arrival pattern whilst in the former, it is a non-stationary arrival pattern. Any or all of these characteristics may be best described by random variables.

The *service pattern*, like the arrival pattern, is also measured by the number of customers served per some unit of time or as a time required to service a customer. However, service rate or time are conditioned on the fact that the

system is not empty. If the system is empty, the service facility is idle. Service may also be batch or single. The service rate may depend on the number of customers waiting for service, in which case, it is referred to as state-dependent service. The service pattern, like the arrival pattern may be also stationary or non-stationary. All the characteristics of the service pattern can also be best described by random variables.

The *queue discipline* defines the rules of behaviour within the queue. The most common rule that can be observed in everyday life is first come, first served or first in, first out (FIFO). Some others in common usage are last in ,first out (LIFO), selection for service in random order (SIRO), and a variety of priority schemes. Priority disciplines may be pre-emptive or non pre-emptive. Pre-emptive queues may again be pre-emptive resume or pre-emptive repeat.

2.1.2 QUEUEING NOTATION.

In describing a queueing process, a notation has evolved due, for the most part, to Kendall (1951), which is now rather standard throughout queueing literature. A queueing process is described by a series of symbols and slashes such as A/B/X/Y/Z, where A indicates the inter arrival time distribution,B the service pattern as described

by the probability for service time, X the number of parallel service channels, Y the restriction on the system capacity, and Z the queue discipline. For example, the notation M/D/2/ /FIFO, indicates a queueing process with exponential inter-arrival times, deterministic service times, two parallel channels, no restriction on the maximum number allowed in the system, and first in, first out queue discipline (Gross and Harris,1974).

2.1.3 APPLICATIONS OF QUEUEING THEORY

The first work of any great significance that dealt with a real queueing situation was the work of A.K.Erlang, (1909) in his paper "The theory of probability and telephone conversations". Erlang was an employee of the Danish Telephone Company. Since then telephone remained the sole recipient of the principal application of the theory till about 1950. Numerous other applications have been found since then. Galliter and Wheeler,(1958) applied the theory to the landing of aircraft. Koopman,(1972) also studied aircraft landing patterns in his paper "Air-terminal queues under Time dependent conditions".

Several other applications have been reported in the fields of operations research, management sciences, statistics, inventory control, manufacturing processes, production lines, computer sciences etc.. Cooper, (1978) and

Giffin (1978) gave an extensive bibliography on the subject. Similarly, papers to which this work is related have been reported as well. The work of Curry et.al.,(1979) to which this thesis draws heavily is worth citing. In their paper, Curry et.al. studied a queueing situation at an airport terminal in which passengers disembarking from a flight choose among alternative means of transportation to their destination in the Central Business District.

Uber, (1978) also used queueing model to study start-up times and queue acceptance of large gaps at T-junctions, while Egurube, A.J. (1990) studied priority intersections under queueing situations. Feng-Bor and Donald C, (1986) evolved a simulation model to study queue dissipation characteristics for the analysis of signal controls. Eddie, (1956) used queueing theory to study queue lengths at Toll-Booths and a similar work has been reported in the literature for the Lagos-Ibadan expressway.

2.2 BIRTH AND DEATH PROCESSES IN QUEUES.

A birth in queueing theory generally refers to the arrival of a customer to be served, while a death refers to the departure of a customer after service or for other reasons. Birth and death models are a special type of Markovian models. That is, the conditional probability of finding the system in state j at time $t+\Delta t$, given that it

occupied state i at time t, depends only on the starting state and the transition time Δt. It is independent of the path used to reach state i. In its most general form the birth postulates can be formulated as follows;

(1) If the system is in state n at time t, the probability of a single arrival in the interval $(t,t+\Delta t)$ for small Δt is $\lambda n(t)\Delta t$ where

$$\lim_{\Delta t \to o} \frac{\lambda n(t)\Delta t}{\Delta t} = \lambda n(t) \quad \text{.....2.1}$$

(2) The probability of more than a single arrival during a small interval of time Δt is $0(\Delta t)$ where

$$\lim_{\Delta t \to o} \frac{0(\Delta t)}{\Delta t} = 0 \quad \text{.....2.2}$$

The death postulates are:

3) If the system is in state n at time t, the probability of a single service in the interval $(t,t+\Delta t)$ for small Δt is $\mu n(t)\Delta t$. where

$$\lim_{\Delta t \to o} \frac{\mu n(t)\ \Delta t}{\Delta t} = \mu n(t) \quad \text{.....2.3}$$

(4) The probability of more than a single service during a small interval of time Δt is $0(\Delta t)$ where

$$\lim_{\Delta t \to o} \frac{0(\Delta t)}{\Delta t} = 0 \quad \text{.....2.4}$$

For stationary processes in which the arrival and service rates do not change with time, the functional argument t in

the birth and death postulates is dropped such that

$$\lambda n(t) = \lambda n \quad \text{for all } t \qquad \text{.....2.5}$$

$$\mu n(t) = \mu n \quad \text{for all } t$$

Let $P_{ij}(\Delta t)$ be the probability of moving from state i to state j during a time Δt. Beginning with state zero the system can move only to a higher order state or stay in state zero. It will remain in state zero if no customers arrive or if a single customer arrives and is served in $(t,t+\Delta t)$. The probability of these events is

$$P_{00}(\Delta t) = (1-\lambda_o \Delta t) + \lambda_o \Delta t \mu_o \Delta t \qquad \text{.....2.6}$$

The system will move from state zero to state one if a single customer arrives and none are served.

$$P_{01}(\Delta t) = \lambda_o \Delta t(1-\mu_o \Delta t) \qquad \text{.....2.7}$$

Multiple arrivals would be necessary to move the system from zero to any state $j>1$. But by the birth postulates the probability of such events for small Δt is zero. That is

$$P_{0j}(\Delta t) = 0 \qquad j > 1 \qquad \text{.....2.8}$$

From higher order initial states , transitions can occur in either a positive or negative direction. The system can stay in state i with no births or deaths, move from i to i-1 with a death, or move from i to i+1 when a birth occurs in the interval $(t,t+\Delta t)$. The postulates of these events are;

$$P_{i,i-1}(\Delta t) = \mu_i \Delta t(1-\lambda_i \Delta t)$$

$$P_{ii}(\Delta t) = \lambda_i \Delta t \mu_i \Delta t + (1-\lambda_i \Delta t)(1-\mu_i \Delta t) \qquad \text{.....2.9}$$

$$P_{i,i+1}(\Delta t) = \lambda_i \Delta t(1-\mu_i \Delta t)$$

Since multiple arrivals or services are not admissible in our current system, $P_{ij}(\Delta t) = 0$ for $|i-j| > 1$. The complete transition matrix can now be written as

$$T = \begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & \lambda_o\mu_o\Delta t^2+(1-\lambda_o\Delta t) & \lambda_o\Delta t(1-\mu_o\Delta t) & \cdots\cdots \\ 1 & \mu_1\Delta t(1-\lambda_1\Delta t) & \lambda_1\mu_1\Delta t^2+(1-\lambda_1\Delta t)(1-\mu_1\Delta t) & \lambda_1\Delta t(1-\mu_1\Delta t) \\ 2 & 0 & \mu_2\Delta t(1-\lambda_2\Delta t) & \lambda_2\mu_2\Delta t^2+(1-\lambda_2\Delta t)(1-\mu_2\Delta t) \end{array}$$

.....2.10

In vector notation,

$$P(t+\Delta t) = P(t)T$$

and multiplication leads to the following set of equations.

$$P_0(t+\Delta t) = P_0(t)\left[\lambda_o\mu_o\Delta t^2+(1-\lambda_o\Delta t)\right] + P_1(t)\left[\mu_1\Delta t(1-\lambda_1\Delta t)\right]$$

$$P_n(t+\Delta t) = P_{n-1}(t)\left[\lambda_{n-1}\Delta t(1-\mu_{n-1}\Delta t)\right] + P_n(t)\left[\lambda_n\mu_n\Delta t^2 + (1-\lambda_n\Delta t)(1-\mu_n\Delta t)\right] + P_{n+1}(t)\left[\mu_{n+1}\Delta(t)(1-\lambda_{n+1}\Delta t)\right]$$

$n \geq 1$ ..2.11

By transferring $P_n(t)$ to the left side, dividing by Δt and taking the limit as Δt goes to zero, we are led to the set of differential difference equations

$$\frac{dp_o(t)}{dt} = -\lambda_o p_o(t)+\mu_1 p_1(t)$$

2.12

$$\frac{dp_n(t)}{dt} = -(\lambda_n+\mu_n)p_n(t)+\lambda_n p_{n-1}(t)\mu_{n+1} p_{n-1}(t) \qquad n > 1$$

If we reintroduce time dependent arrival and service rates, the birth-death equations then becomes the non-homogeneous

set $$\frac{dp_n(t)}{dt} = -\lambda_o(t)p_o(t)+\mu_1(t)p_1(t)$$

$$\frac{dp_n(t)}{dt} = -\left[\lambda_n(t) + \mu_n(t)\right]P_n(t) + \lambda_{n-1}(t)P_{n-1}(t)$$

$$+ \mu_{n+1}(t)P_{n+1}(t) \qquad n \geq 1 \quad .....2.13$$

2.3 STEADY-STATE SOLUTIONS.

In most queueing processes, the long-run behaviour of the system is of great interest. In such analysis,the systems are assumed to have reached steady state in the sense that their probabilistic behaviour becomes independent of time. This

approach is often justified on the basis that most design problems are more concerned with a long-term average behaviour than they are with temporary perturbations caused by start-up conditions or short interruptions in system operation. However, an even more important, if often unarticulated, reason for emphasising steady-state solutions is that the transient solutions are simply too difficult to obtain (Giffin,1978).

To assume that the system has reached steady-state is equivalent to stating that the probability distribution is no longer changing with time. Mathematically,this means that the derivative of $P_n(t)$ with respect to time must be zero. Rewriting the birth-death equations for steady-state conditions we have

$$0 = -\lambda_o P_o + \mu_1 P_1$$

$$0 = -(\lambda_n + \mu_n)P_n + \lambda_{n-1} P_{n-1} + \mu_{n+1} P_{n+1} \quad ; \qquad n \geq 1$$

The above set of equations is most easily solved by iteration. Beginning with the first equation

$$P_1 = \frac{\lambda_o}{\mu_1} P_o$$

using that result in the second equation yields :

$$P_2 = \frac{1}{\mu_2} (\lambda_1+\mu_1)P_1 - \frac{\lambda_o}{\mu_2} P_o$$

$$= \frac{1}{\mu_2}(\lambda_1+\mu_1)\frac{\lambda_0 P_0}{\mu_1} - \frac{\lambda_0 P_0}{\mu_2}$$

$$= \frac{\lambda_0 \lambda_1 P_0}{\mu_1 \mu_2}$$

and in general

$$P_n = \prod_{i=0}^{n-1} \lambda_i \Big/ \prod_{i=1}^{n} \mu_i \qquad 2.14$$

where the Pi notation represents the product of indicated terms. Invoking the definition of a probability distribution to determine P_0 and hence all P_n. We have

$$\sum_{n=0}^{\infty} P_n = 1$$

it follows that

$$P_0 + \sum_{n=0}^{\infty} (\Pi\lambda_i / \Pi\mu_i) P_0 = 1 \qquad \text{.....2.15}$$

from which

$$P_0 = \left[1 + \sum_{n=1}^{\infty} \left(\prod_{i=0}^{n-1} \lambda_i \Big/ \prod_{i=1}^{n} \mu_i \right) \right]^{-1} \qquad \text{.....2.16}$$

Equations 2.15 and 2.16 then constitute the general steady state solutions for the birth-death process. These equations can be used to model a great number of system configurations including finite capacity, infinite capacity, single-channel,

multi-channel, infinite source,finite source,state-dependent service rate and many other pure and mixed characteristics. All that is necessary is to define the proper set of arrival rate λ_n and service rate μ_n coefficients for the configuration being considered,then solve the resulting equations. Detail derivations of the state probabilities and the measures of performance for the different system configurations can be found in any standard queueing text.

2.4 TRANSIENT SOLUTION OF QUEUEING SYSTEMS.

In the analysis and design of many service systems, it may be vital to know how the system will respond to sudden changes in input or service capabilities. For example, if one seeks to study the flow of air traffic in a major terminal area, it is the time varying nature of arrivals and service which are important in analysing system performance. Similarly, in studying the arrival and service patterns in a transportation access area, the time varying demand for transportation is important in analysing system performance. The flow of arrivals at 10.00am could be much higher than that at 2.00pm. Furthermore,service interruptions may occur in the form of vehicle breakdowns, traffic jams, route bottlenecks, etc, which may temporarily restrict the ability of the access area to process traffic.

Steady-state analysis is inadequate for both surface

and air traffic control problems. Each requires a consideration of the time-varying nature of the queue and its associated delays. If these systems can be modeled as finite Markov chains it is a simple matter to extract the desired time-varying performance by harnessing the computational power of modern digital computer.

The reasons one does not often find transient analysis used in most queuing systems are that very few transient solutions exist and those solution which are available are extremely awkward to manipulate. It is therefore common practice to resort to simulation studies in an attempt to analyse time-varying behaviour. However, simulation is an experimental tool with all of the statistical pitfalls of decision making from experimental data. In addition, it is very expensive to obtain sufficiently large samples of simulation results to have confidence in the conclusions drawn.

The transient solutions of queueing systems are therefore obtained by numerical solution to systems of differential equations and by iterative calculation of state probabilities using basic Markov chain arguments. In these cases, the output is not an equation but rather a table of numbers which enumerates the numerical values of the state distribution. But after all it is the set of numbers

which is most important in the analysis and design of queueing systems.

2.5 BULK QUEUES.

In most queueing systems, it is assumed that customers arrive singly and are served singly.Such assumptions may not be reasonable for some special classes of queueing systems. For example, at an airline terminal, each arriving flight delivers a batch of passengers who will require service at the baggage pick-up area. In this case, one must account for bulk arrivals coupled with single-channel service in order to develop a reasonable queueing model.

Bulk or batch operations are also common on the service side of some systems. Consider, for example,passenger pickup at a city bus stop. Passengers normally arrive singly and form a queue. However, when service occurs, an entire group of the waiting passengers, equal to the maximum of the number present or the available passenger space on the bus, is served. A single-service model would be a poor choice for analysing such a bulk-service problem.

2.5.1 Poisson Bulk Service

This system can be analysed by the same techniques employed for the birth-death process. The only difference is that we must now admit jumps to states other than immediately adjacent states. Using the usual rate of change

arguments we can write the appropriate set of differential difference equations as

$$\frac{dPo(t)}{dt} = \mu P_1(t) - \lambda Po(t)$$

$$\frac{dPn(t)}{dt} = \mu Pn+1(t) - \lambda Pn(t) - \mu Pn(t) \qquad o<n<b$$

$$\frac{dPn(t)}{dt} = \mu Pn+1(t) - \lambda Pn(t) - \mu Pn(t) + \lambda Pn-b(t) \qquad n\geq b$$

Where b is the batch size. Due to the inherent complexity of the transient solution, let us proclaim interest only in the equilibrium condition which exists as long as $b\lambda<\mu$. Rewriting for the steady-state by setting the derivatives equal to zero we have,

$$\lambda Po = \mu P_1$$

$$(\lambda+\mu)Pn=\mu Pn+1 \qquad o<n<b$$

$$(\lambda+\mu)Pn = \mu Pn+1 + \lambda Pn-b \qquad n\geq b$$

The resulting set of difference equations is most easily solved by using geometric transforms. This is beyond the scope of this thesis. Giffin (1978) however gives the probability distribution for the Poisson bulk arrival system as

$$Pn = (\mu-b\lambda)\sum_{i=1}^{b} Ci\,zi^{-(n+1)}$$

2.5.2 Random Bulk Arrival Size

Here the batch size b is distributed by the probability

mass function

$$\Pr(\text{batch} = b) = g_b$$

The arrival stream over time is a compound Poisson process. We could view such an arrival process as a combination of separate Poisson streams with the arrival rate for the ith stream being λg_i. This means that transitions into state j due to arrivals can occur from any state below it since an arriving batch may be of any size including zero. A zero batch size implies that the system remains in its current state. Transition into state j due to service occurs only from state (j+1) as we still assume a single-channel system. Transition out of j due to service goes only to (j-1). Transition out of j due to arrivals may lead to any higher order state. However, the total rate of departure flow from state j is still λ due to

$$\sum_{i=0}^{\infty} \lambda g_i = \lambda \sum_{i=0}^{\infty} g_i = \lambda$$

Putting this information into the usual rate of change equations leads to

$$\frac{dP_n(t)}{dt} = \mu P_1(t) - \lambda P_0(t) + \lambda g_0 P_0(t)$$

$$\frac{dP_n(t)}{dt} = \mu P_{n+1}(t) - \lambda P_n(t) - \mu P_n(t) + \sum_{i=0}^{n} \lambda g_{n-1} P_i(t) \qquad n \geq 1$$

By setting the derivatives equal to zero we have the

equilibrium equations.

$$\lambda P_0 = \mu P_1 + \lambda g_0 P_0$$

$$(\lambda+\mu)P_n = \mu P_{n+1} + \sum_{i=0}^{n} \lambda g_{n-1} P_i \qquad n \geq 1$$

Again geometric transforms are used to solve the above system of equations. The final form of the transform obtained is

$$P(z) = \frac{(\mu-\lambda E(b))(1-z)}{\mu(1-z)-\lambda z(1-G(z))}$$

The transform can be used directly to obtain the moments of the state distribution or inverted to obtain any desired probabilities. The performance measures are as follows.

$$L = \frac{\lambda(\sigma_b^2 + E(b) + (Eb))^2)}{2(\mu-\lambda E(b))}$$

In terms of the utilisation factor, it is

$$L = \frac{\rho(1+E(b)+\sigma_b^2/E(b))}{2(1-\rho)}$$

where $\rho = \lambda E(b) / \mu$

$$L_q = L - \lambda E(b) / \mu$$

Given an effective arrival rate of $\lambda_e = \lambda E(b)$, the waiting times are given by

$$W = L / \lambda_e$$

$$= \frac{1+E(b)+\sigma_b^2/E(b)}{2\mu(1-\rho)}$$

$$W_q = W - 1/\mu$$

2.5.3 Exponential Bulk Service

In this system, customers arrive singly in Poisson fashion but are served in bulk. When service takes place an entire group of C may be served. The duration of service for the group is assumed to be exponentially distributed.

Two variations in such service mechanisms may be encountered. In the first version, the server will not act until a group of C customers has accumulated. He will then serve the entire group.In the second version service takes on groups which cannot be larger than C. If less than C customers are present, all are served. If more than C are present a group of C will be served.

2.8.4 Constant Size Service Batch

Flow into state n occurs with an arrival from state (n-1) or bulk service from state (n+c). Flow out of state n goes to (n+1) with an arrival or to (n-c) with a service whenever n>c. The proper differential-difference equations can then be formulated as

$$dP_0(t) = \mu P_c(t) - \lambda P_0(t)$$

$$\frac{dP_n(t)}{dt} = \mu P_{n+c}(t) + \lambda p_{n-1}(t) - \lambda p_n(t) \qquad 1 \leq n < c$$

$$\frac{dP_n(t)}{dt} = \mu P_{n+c}(t) + \lambda p_{n-1}(t) - \mu P_n(t) - \lambda P_n(t) \qquad n \geq c$$

By setting the derivatives equal to zero we obtain the usual steady-state balance equations.

$$\lambda P_o = \mu P_c$$

$$\lambda P_n = \mu P_{n+c} + \lambda P_{n-1} \qquad 1 \leq n < c$$

$$(\lambda+\mu) P_n = \mu P_{n+c} + \lambda P_{n-1} \qquad n \geq c$$

To solve these equations we again resort to transform techniques. The final form of the state distribution for the fixed bulk size (c) exponential service, Poisson arrival system is

$$P_n = \begin{cases} \dfrac{1-(1/z_o)^{n+1}}{c} & o \leq n < c \\ (1-(1/z_o)^{c})(1/z_o)^{n-c+1} & n \geq c \end{cases}$$

This is solved by first locating the root of the equation.

$$\rho z^{c+1} - (\rho+1) z^{c} + 1 = 0$$

which lies outside the unit circle, say z_o. This value is then used in the above equation to find the necessary probabilities. The performance measures are given as

$$L = (c-1)/2 + 1/(z_o-1)$$

$$L_q = L-\rho$$

where $$E(s) = C(1 - \sum_{i=0}^{c-1} P_i) = \rho$$

corresponding waiting times are

$$W = 1/\lambda \left((c-1)/2 + 1/(z_o-1)\right)$$

$$W_q = W-1/\mu$$

2.5.5 Variable Service Batch Size.

In this system whenever the server is free it will

accept C customers for bulk service if they are available. If less than C are present when the server becomes free it will serve in bulk all who present. We will assume Poisson arrivals and an exponentially distributed time to serve a batch, regardless of its size.Following the usual flow rate arguments, one can quickly construct the proper set of differential-difference equations as

$$\frac{dP_o(t)}{dt} = \mu \sum_{i=1}^{c} P_i(t) - \lambda P_o(t)$$

$$\frac{dP_n(t)}{dt} = \mu P_{n+c}(t) + \lambda P_{n-1}(t) - \mu P_n(t) - \lambda P_n(t) \quad n \geq 1$$

The equilibrium equations obtained by setting $dP_n(t)/dt = 0$ are then

$$\lambda P_o = \mu \sum_{i=1}^{c} P_i$$

$$(\lambda+\mu)P_n = \mu P_{n+c} + \lambda P_{n-1} \qquad n \geq 1$$

Resorting to transform techniques, the final form of transform is

$$P(z) = (1-z_o)/(z-z_o)$$

$$= (1-1/z_o)/(1-z/z_o)$$

The inverse of this transform gives the distribution for the number of customers in this bulk service system and is

$$P_n = (1-1/z_o)(1/z_o)^n$$

where z_o is the root of the equation

$$\rho z^{c+1} - (1+\rho)z^c + 1 = 0$$

which lies outside the unit circle.The performance measures

are

$$L = \frac{1/z_o}{1-1/z_o}$$

$$= 1/(z_o-1)$$

and

$$L = \frac{1}{\lambda(z_o-1)}$$

CHAPTER THREE.

DATA COLLECTION AND ANALYSES.

3.1 FIELD SURVEYS.

In order to decide on the suitable locations to collect data for this study, field surveys were carried out at both the old campus and the new campus of Bayero University, Kano. Though this study concerns the analysis of departures from the new campus, it is obvious that all trips to the new campus originate from the old campus and data related to trips between the two campuses can as well be taken at the old campus. The layout of the old campus together with the new campus is shown in figure 1.1.

The transport route runs through an arterial highway that connects the two campuses. The arterial roadway is a two-lane two-way highway with the traffic lanes each about 3.5m wide and a shoulder width of about 2m on either side. The physical roadway (carriageway) conditions are good and the road could be rated as very good. Access to the highway is not restricted.

University buses departing the new campus to the old campus have an alternative route through gate A, as shown in figure 1.1. Normally, buses,taxis and mini-buses all follow the route through gate B. Thus an observer stationed at the

university bus access area inside the old campus will record inconsistent arrival times as a result of differences in route lengths. Also the access areas for the three modes are not located at the same area and therefore it would be impossible to observe passenger characteristics such as *Balking, Reneging, Jockeying,* and *Switching.*

Figure 1.1 shows an access area outside the main gate B. This access area is used by the taxi and mini-bus modes but occasionally the university bus stops for boarding and alighting of passengers, so that passengers waiting at the access area inside the campus are delayed unduly. For these reasons, it was found convenient to collect all data from the new campus access area since only one road leads to the new campus access area and all arrivals and departures take place there, during and after lectures.

3.2 LOCATION OF OBSERVATION POINTS AT THE NEW CAMPUS.

The location of the data collection points are shown in fig.3.0. The criteria for selection of observation points were such that the site conditions reflected the study objectives and factors known to influence passenger behaviour at the access area were isolated. For instance, it was important that the drivers did not notice the observers recording their arrival and departure times as well as taking their plate numbers. This could give the

feeling that the university authority was monitoring the

work c

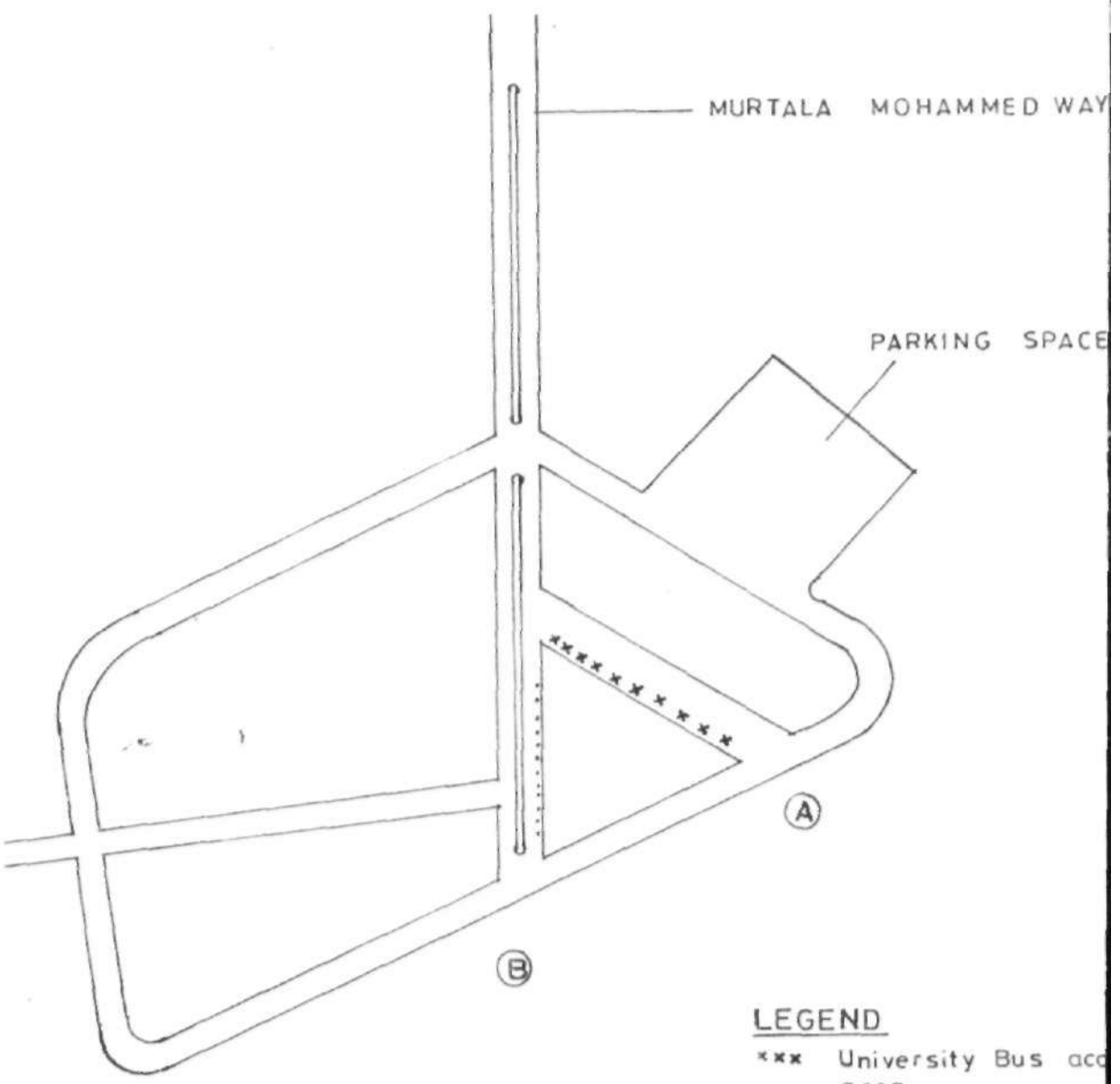

Fig 3.0:- NEW CAMPUS ACCESS AREA OBSERVATION POINTS

feeling that the university authority was monitoring the work of the drivers and subsequently influence the arrival pattern of the university buses to the access area.

Two sites were selected for observation. Site A was monitored by two observers. One observed and recorded the university buses' arrival and departure times at the access area. The other counted and recorded the number of passengers arriving at the access area every five minutes. Site B was monitored by four observers. One observer each was assigned to note the arrival and departure times of the taxi and mini-bus modes respectively. The remaining two observers individually monitored the arrival pattern of passengers for each of the two modes. The location of site B was such that it was glaring to them if *balking*, *reneging*, or *jockeying* took place.

In order to fine tune the designed data collection process and enhance the efficiency of data collection, two days were first used for trial data collection. These were the 6th and 7th of January 1994. The full-scale data collection commenced on Monday January 10 and stretched to Saturday January 15. No data was collected on Sunday January 16. This is because no lectures take place on Sundays and, as a result, the access area is normally deserted completely on Sundays.However, the university bus service and the other

modes offer a kind of door-step service to passengers. Passengers arriving from the old campus are transported to their respective hostels and the buses taxis and mini-buses, in turn pick their clients from the hostels to the old campus. This form of service constitutes a deviation from the service under study, and as such no data was collected on that day. All data collected spanned from 7.00am to 7.00pm each day for all the six days that were studied.Three types of data collection sheets were used. These are namely,

(i) Trip demand sheet.

(ii) Passenger arrival pattern and count sheet.

(iii)Elapsed trip time sheet.

On the trip demand sheet, records were made of the arrival times, departure times and the capacities of the different vehicles that served passengers. By this , it was possible to estimate the number of passengers carried by each mode on daily basis. The second sheet contained the number of passengers every five minutes. This also gave the cummulative number of passengers arriving at the respective loading points and facilitated an estimation of the total trip demand for each mode on daily basis as obtained. The observers used quartz timers and manual counters for purposes of collecting data. Passengers arriving at the respective points of access for the different modes were

counted by depressing the knob on the manual counter which reading increased by one with each depression. At the end of every five minutes the number indicated gave the cumulative arrivals up to that period and the difference in reading between the most recent reading and the immediately previous reading gave the number of passengers in that particular period.

The elapsed trip time sheet was used to record the origin and time of trip, enroute stoppages and their durations as well as the trip destination and destination time. From this the average trip time of each transport mode is calculated.

3.30 SOME CHARACTERISTICS OF THE ARRIVAL TIMES.

Throughout the observation period, the taxi mode was the first to arrive at the new campus access area. This was followed by the university bus mode , then the mini-bus mode. There was a difference of about 41 minutes between the times of the first taxi and university bus to reach the new campus. The mini-bus mode does not start operation till much later and this phenomenon was observed throughout the period of study. Due to the relatively early arrival times of the taxi mode, students wishing to catch the first lectures of the day tended to rely on the taxi mode to be on time. Similarly, staff who must report early to work tended

also to opt for the taxi mode. Whether the taxi mode is the most reliable of the three modes is, however, open to question. The arrival times abstracted from the raw data, for the three modes, for a typical day are shown in Tables 3.1, 3.2, and 3.3.

Another interesting observation on the arrival times was the headways between the university buses. Due to the high demand for university buses and the resulting long queues, the university buses seemed to operate in platoons especially during peak hours. On most occassions, all the university buses operating between the two campuses travel with very small headways between them. Table 3.1, shows this clearly. This is presumably for two reasons. First, university buses which arrived singly when passengers have lost patience or when passengers arriving most recently into the system attempt to shunt the queue ahead of the earlier arrivals on the queue, disrupts the FIFO discipline commonly observed prior to bus arrivals . Consequently, passengers board the buses through the windows, the drivers seat and some hang on the buses to make the trip to the old campus. The buses also get vandalised in the process. Secondly, by arriving in platoons, a large number of passengers are carried to either clear the queue or reduce it considerably. Yet another reason is that passengers

TABLE 3.1 UNIVERSITY BUS ARRIVAL TIMES

(Times are in hours, minutes and seconds)

8 38 16	9 54 16	12 40 18	13 28 30	16 31 35
8 41 58	9 54 20	12 40 25	13 28 33	16 31 42
8 48 36	9 54 24	12 40 28	14 37 05	16 31 47
8 51 35	9 55 59	12 40 32	14 37 08	16 58 55
9 03 50	10 16 38	12 40 37	14 37 10	17 07 09
9 07 15	10 16 42	13 05 13	14 37 15	17 08 22
9 08 57	10 16 46	13 05 17	14 37 20	17 23 38
9 12 20	10 20 34	13 05 25	15 05 40	17 32 30
9 29 56	10 42 55	13 05 28	15 05 45	17 32 35
9 30 06	10 42 57	13 27 18	15 05 50	18 16 21
9 30 13	10 42 59	13 27 21	15 06 15	18 31 17
9 33 50	11 53 22			18 33 43

TABLE 3.2 MINIBUS ARRIVAL TIMES

(Times are in hours,minutes and seconds)

8 53 15	11 58 05	13 00 25	16 54 15
8 55 05	12 10 30	13 13 13	17 57 54
9 55 05	12 12 25	13 19 21	
10 23 10	12 18 35	13 24 28	
10 26 08	12 18 40	13 35 05	
10 37 25	12 25 35	13 35 41	
10 54 35	12 35 40	13 39 31	
11 11 20	12 43 37	13 43 53	
11 16 45	12 44 16	14 58 05	
11 20 00	12 47 21	15 00 03	
11 25 00	12 51 18	15 44 40	
11 28 06	12 54 20	16 44 35	

TABLE 3.3 TAXI ARRIVAL TIMES

(Times are in hours,minutes and seconds)

7 57 15	8 35 15	8 42 05	9 02 34	9 41 08
7 58 30	8 36 06	8 45 35	9 03 55	9 57 55
7 59 20	8 36 15	8 48 45	9 10 05	9 58 00
8 02 40	8 36 20	8 49 00	9 12 00	9 58 20
8 07 15	8 36 25	8 50 00	9 16 45	9 59 15
8 07 35	8 37 10	8 52 05	9 20 10	10 02 08
8 11 10	8 38 07	8 52 25	9 21 05	10 06 35
8 17 15	8 39 05	8 52 55	9 28 50	10 12 33
8 19 45	8 40 10	8 53 00	9 28 52	10 12 45
8 20 05	8 40 30	8 54 15	9 31 10	10 13 21
8 24 05	8 41 55	9 01 06	9 40 35	10 19 00
8 28 40	8 42 00	9 01 20	9 41 05	10 23 45
10 36 56	11 27 55	12 46 11	14 56 04	
10 37 00	11 34 25	12 50 00	15 21 00	
10 38 00	11 35 00	13 20 32	15 27 05	
10 39 10	11 41 08	13 21 10	15 43 20	
10 51 06	12 00 00	13 33 31	16 00 00	
10 57 00	12 07 01	14 00 05	16 01 35	
10 57 00	12 07 04	14 01 05	16 05 05	
11 10 00	12 12 27	14 03 05	16 11 07	
11 19 00	12 30 15	14 19 20	16 12 35	
11 19 16	12 31 45	14 49 45	16 44 18	
11 22 10	12 32 05	14 54 10	16 51 30	
11 26 00	12 36 40	14 54 12	17 01 05	

carried from the old campus to the new campus do not often get the opportunity to alight when buses arrive singly and on account of stampede by entering passengers,such passengers run the risk of being returned to the old campus. Platoon arrivals on the other hand reduce the rush on a particular bus and passengers can afford to alight. Thus very small headways are observed between the university buses when they arrive in platoons. Headways of two seconds have been observed. These small headways are interspersed with long headways normally not less than the round trip time for the university buses. When the platoon behaviour of the university buses breakdown, the headways become erratic, following the pattern of the other modes of transport.

There was no definite pattern in the observed taxi arrival times. Smaller headways have equally been observed in the range 2-5 seconds, longer headways of sizes generally greater than 1000 seconds were more common. The mini-bus mode starts operation much later than the other two modes, during the period of study. Though it starts late, it captures a sizeable portion of the customers leaving the new campus and competes favourably with the taxi mode in terms of convenience. See Tables 3.1 to 3.3. and Table 3.6 for comparison. It also enjoys greater patronage because of

lower fares. They run the smallest number of trips between the two campuses as compared to the other modes, and observations reveal that headways less than 50 seconds were uncommon. See(Table 3.2).

3.40 SERVICE TIME CHARACTERISTICS.

Service time in the context of this work, is defined as the *difference between the arrival and departure times of* the three modes of transport. The service time distribution for the three modes obtained from aggregated data is shown in table 3.4. For ease of presentation, three categories of service times are identified.They are short service times (less than or equal to 60 secs), medium service (60 secs to 110 secs), long service times (greater than 110 secs). The service times of the university buses consists of 20% short, 36% medium, and 44% long. Short service times are those up to but less than 60 secs and indicates a breakdown of queue discipline.

Passengers arriving from the old campus have little or no opportunity to alight from the buses. In such situations only passengers who could hang on the buses or force their way into the buses are able to obtain service at the access area. Medium service times are those greater than 60 secs but less than or equal to 110 secs. These represent an orderly queueing situation. Service times greater than 110 secs are

considered long and represent situations in which the queue discipline is largely FIFO wherein nearly all passengers patiently await their turn to board the university bus in an orderly manner. Service times much longer than 110 seconds are indicative of the absence of queues at the access area

TABLE 3.4 SERVICE TIME DISTRIBUTION FOR THE THREE MODES.

(AGGREGATED DATA)

Duration of service time (seconds)	Number of observations.		
	Uni.bus mode	Mini-bus mode	Taxi mode.
<10	-	-	76
10+ - 20	2	7	160
20+ - 30	13	24	141
30+ - 40	8	19	140
40+ - 50	23	22	83
50+ - 60	21	25	77
60+ - 70	20	23	43
70+ - 80	23	10	21
80+ - 90	30	7	25
90+ - 100	31	7	10
100+ - 110	16	8	7
>110	145	46	40

and the drivers as a result, wait for passengers till the bus gets filled to capacity.In some cases, the long service times arise because passengers outside the access area flag the driver to wait or stop. During peak hour times, long service times occured because arriving passengers are unable to alight in good time. Yet still, in some instances, the drivers motives for excessive waiting at the access area cannot easily be speculated upon.

For all the cases considered, it was not possible to distinguish between the alighting process and the boarding process due mainly to the fact that on most occasions the passengers have lost patience by the arrival time of the university bus.

The taxi mode offered a much smaller service time than both the university bus mode and the taxi mode. This is mainly as a result of the smaller number of passengers carried by the taxi mode. The percentage service times offered for the three categories of service times are 82%, 13%, and 5% respectively. For service times in the first category, it is commonly observed that an orderly queueing situation obtains, and the number of passengers waiting is not up to taxi capacity or a simultaneous arrival of a taxi with one or both of the other two modes occured. Situations typical of medium service times are non-arrival of a taxi

with the other modes simultaneously. Service times in the last category is as a result of a taxi waiting for passengers and excessive waiting on the part of the driver for no apparent reason.

The proportion of service times for the mini-bus mode in the three categories are 49%, 28%, and 23% respectively. It would be seen that for the three categories considered and for the three modes of transport, the mini-bus mode offers service times in between the university bus mode and the taxi mode. In other words the service times are smaller than the university bus mode but longer than the taxi mode. When a mini bus arrival coincides with a university bus arrival, often the mini-bus is ignored till the departure of the university bus. In situations like this , the mini-bus either moved away in which case smaller service times are observed or waits till the departure of the university bus in the hope of getting passengers left behind. In such cases, longer service times occured. Thus reneging of passengers to the university bus mode is frequently observed when a university bus mode arrives before the departure of the waiting mini-bus. Like the other two modes, long service times are indicative of waiting for no good reasons. Table 3.5 summarises the service time characteristics for the three modes for the duration of

TABLE 3.5 SERVICE TIME CHARACTERISTICS FOR THE THREE MODES OF TRANSPORT

DATE	MODE	SAMPLE SIZE	MEAN SERVICE	STANDARD DEVIATION	COEFFICIENT OF VARIATION
10/1/94	UNIV.BUS	62	120.05	66.10	0.55
	MINIBUS	50	80.90	54.60	0.67
	TAXI	165	48.15	43.18	0.90
11/1/94	UNIV.BUS	53	111.00	52.67	0.47
	MINIBUS	39	79.26	51.79	0.65
	TAXI	144	51.67	30.81	0.60
12/1/94	UNIV.BUS	52	92.30	48.85	0.53
	MINIBUS	34	107.26	78.80	0.73
	TAXI	138	47.40	48.16	1.02
13/1/94	UNIV.BUS	58	96.00	35.97	0.37
	MINIBUS	38	106.74	70.00	0.66
	TAXI	153	31.51	20.83	0.66
14/1/94	UNIV.BUS	67	100.75	54.61	0.54
	MINIBUS	32	56.97	43.49	0.76
	TAXI	134	32.63	22.16	0.68
15/1/94	UNIV.BUS	40	110.58	59.09	0.53
	MINIBUS	15	91.67	62.82	0.69
	TAXI	86	38.73	33.84	0.87
Overall Aggreg. Data	UNIV.BUS	332	105.02	54.07	0.51
	MINIBUS	208	87.10	61.85	0.71
	TAXI	820	42.41	43.06	1.02

study. The results in table 3.5 reveal the wide variation in the service times of the three modes of transport. Of the

three modes, the university bus exhibited the smallest degree of unevenness in its service times. Even in global terms, (bottom of table 3.5) the university bus mode offered the least variation in its service times. The taxi mode had a coefficient of variation of 1.02 for aggregated data. This indicates that randomness is a constant feature of the taxi mode.

3.50 PASSENGER CHARACTERISTICS.

Figure 3.1 shows the distribution over time of passenger arrivals on the three modes. A clear indication from the results is that the passenger arrival patterns for the three modes show similar characteristics. For all the modes,the arrival patterns at the access areas indicates time-varying arrivals, as shown in Figure 3.1. The university bus mode attracted the highest number of passengers throughout the observation period. For much of the time, single arrivals were observed but surges of arrivals often occured prior to the arrivals of the university bus mode. This was observed to be as a result of the practice of the university bus drivers who regularly signal their arrivals to the access areas by horning. Passengers not immediately available at the access areas are alerted and rush out to the access areas in a bid to catch the buses.

This situation creates a problem. First the single

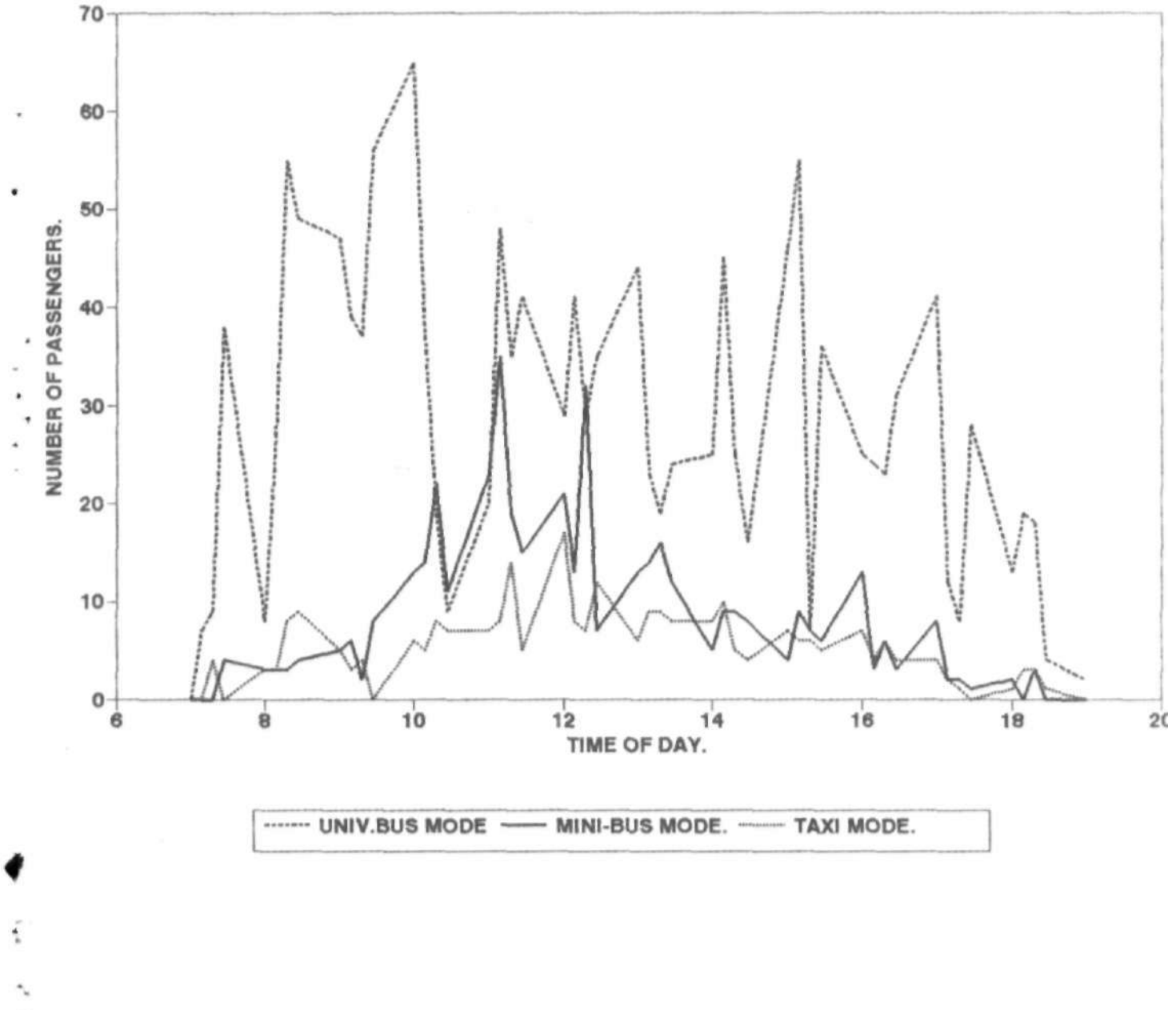

Fig.3.1 Passenger Arrival Pattern (13/1/94)

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arrival pattern characteristic of the passengers often collapses and it then becomes very difficult to count the numbers of passengers as a result of the arrival surges. The consequence of this is that departures far outnumber arrivals and this gives a false impression that passengers do not experience any delays. There may be several reasons why passengers tend to delay their arrivals at the access area to just when the university buses arrive but two of them are obvious.First, the access area lack shade where intending passengers could protect themselves from the sun, and secondly passengers who are captive to the university bus mode are apprehensive of a much longer delay should they fail to catch the buses. They therefore come out in their numbers and abandon whatever they are doing upon hearing the signals from the buses. Many do this at the expense of their reading times and lectures.

The university bus passenger arrivals (figure 3.1) are characterised by three peaks except for Fridays and Saturdays whem only two peaks are prominent. No such peaks are observed for the other two modes as Figure 3.1 reveals. The number of passengers carried by each mode and the number of vehicles arriving for each mode for the aggregated data are shown in Figures 3.2a and b to 3.4a and b.The cummulative daily number of passengers carried by each mode and the

number of vehicles arriving for each mode per day, together with the number of passengers carried per trip for each mode, for both the disaggregate, as well as , the aggregated data for the duration of study are given in Table 3.6. The table reveals that for the aggregated data, the university bus mode carried an average of 27 passengers per trip, the mini-bus mode 12 per trip and the taxi mode 1 per trip. It is noteworthy, that the taxi mode is the most under utilised transport mode judging from the number of trips made.

TABLE 3.6 AVERAGE NUMBER OF PASSENGERS CARRIED PER TRIP.

DATE	Transport Mode	Cumulative no of veh.observ	Cumulative Passengers Carried	Average no. of Pass/trip
10/1/94	UNIV.BUS	62	1529	32.81
	MINIBUS	50	541	10.82
	TAXI	165	286	1.73
11/1/94	UNIV.BUS	53	1418	26.75
	MINIBUS	39	463	11.87
	TAXI	144	179	1.24
12/1/94	UNIV.BUS	52	1300	25.00
	MINIBUS	34	423	12.44
	TAXI	138	155	1.12
13/1/94	UNIV.BUS	58	1501	26.36
	MINIBUS	38	463	12.18
	TAXI	153	251	1.64
14/1/94	UNIV.BUS	67	2034	30.36
	MINIBUS	32	404	12.63
	TAXI	134	150	1.12
15/1/94	UNIV.BUS	40	1022	25.55
	MINIBUS	15	164	10.93
	TAXI	86	103	1.23
Overall Aggreg. Data	UNIV.BUS	332	8804	26.52
	MINIBUS	208	2477	11.91
	TAXI	820	1124	1.37

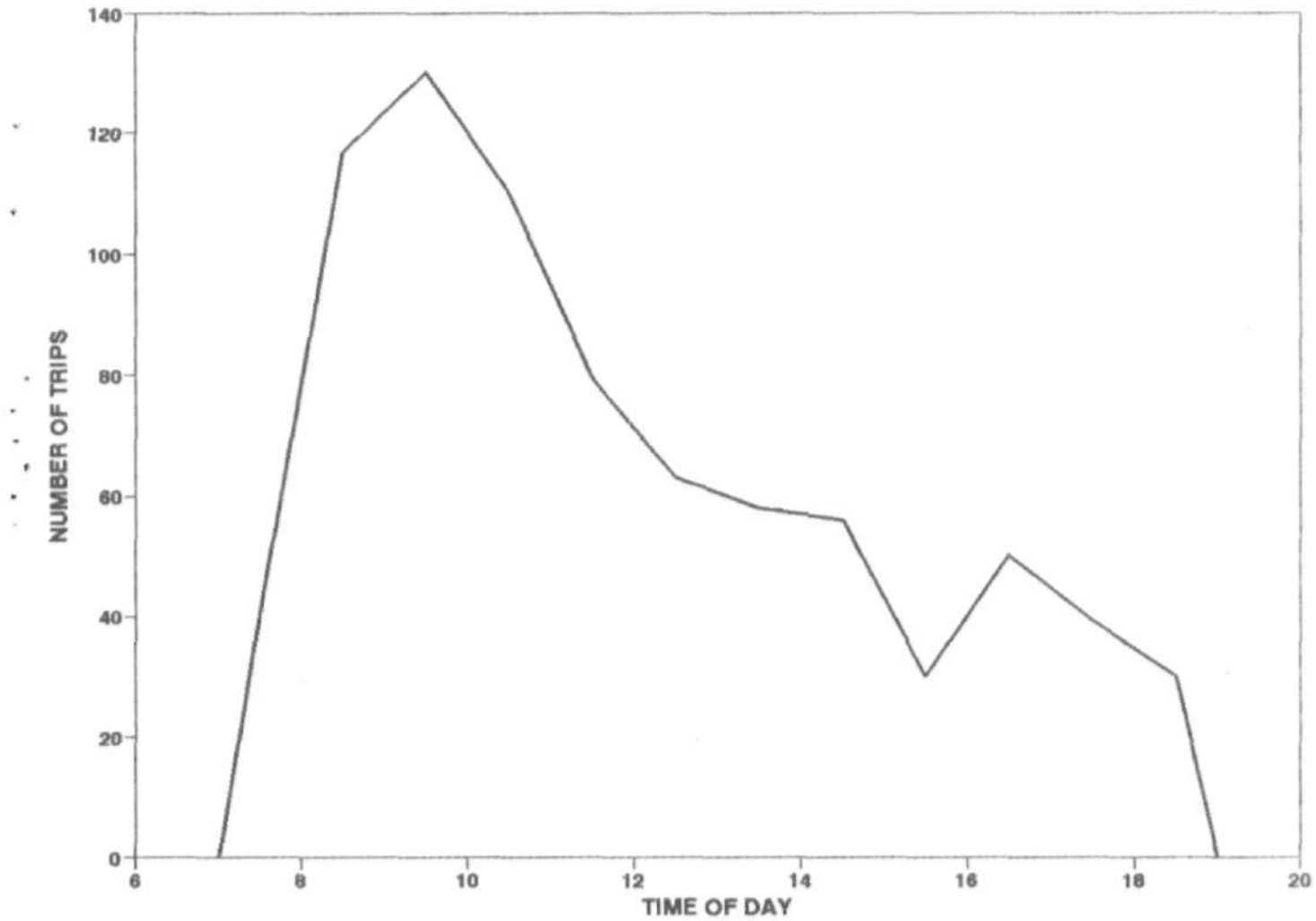

Fig.3.2a Taxi Trip Distribution

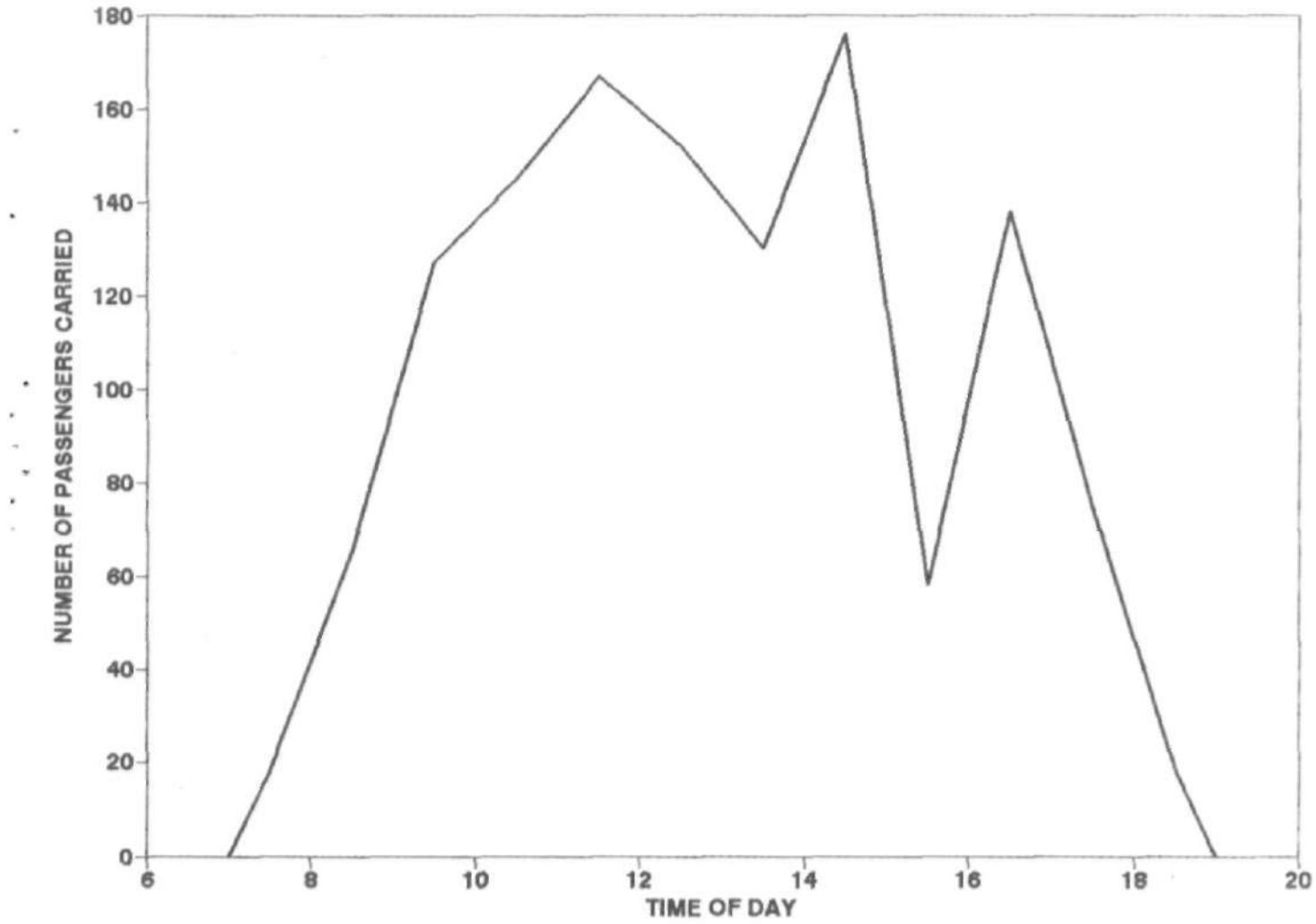

Fig 3.2b Taxi Passenger Distribution

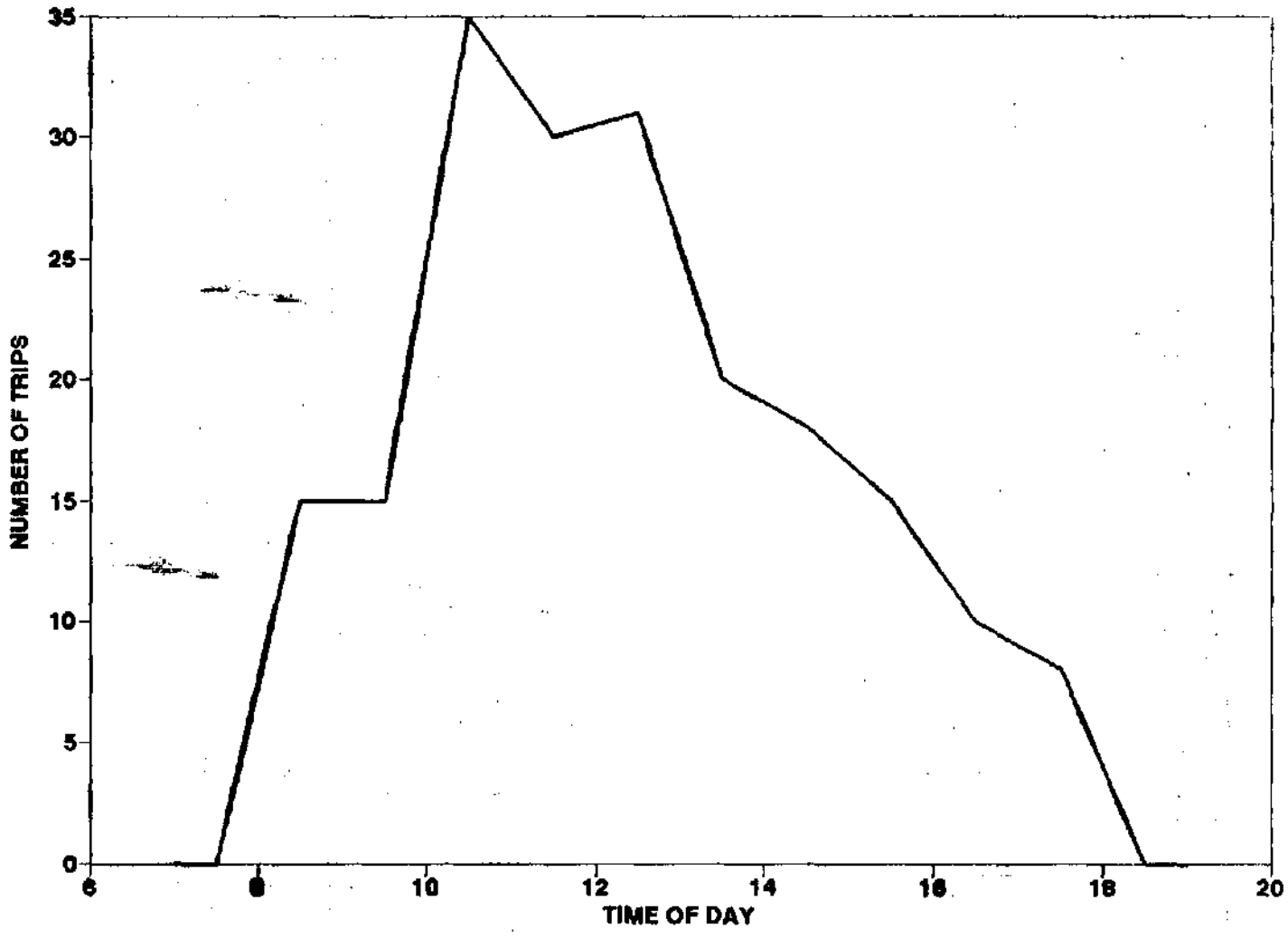

Fig.3.3a Minibus Trip Distribution

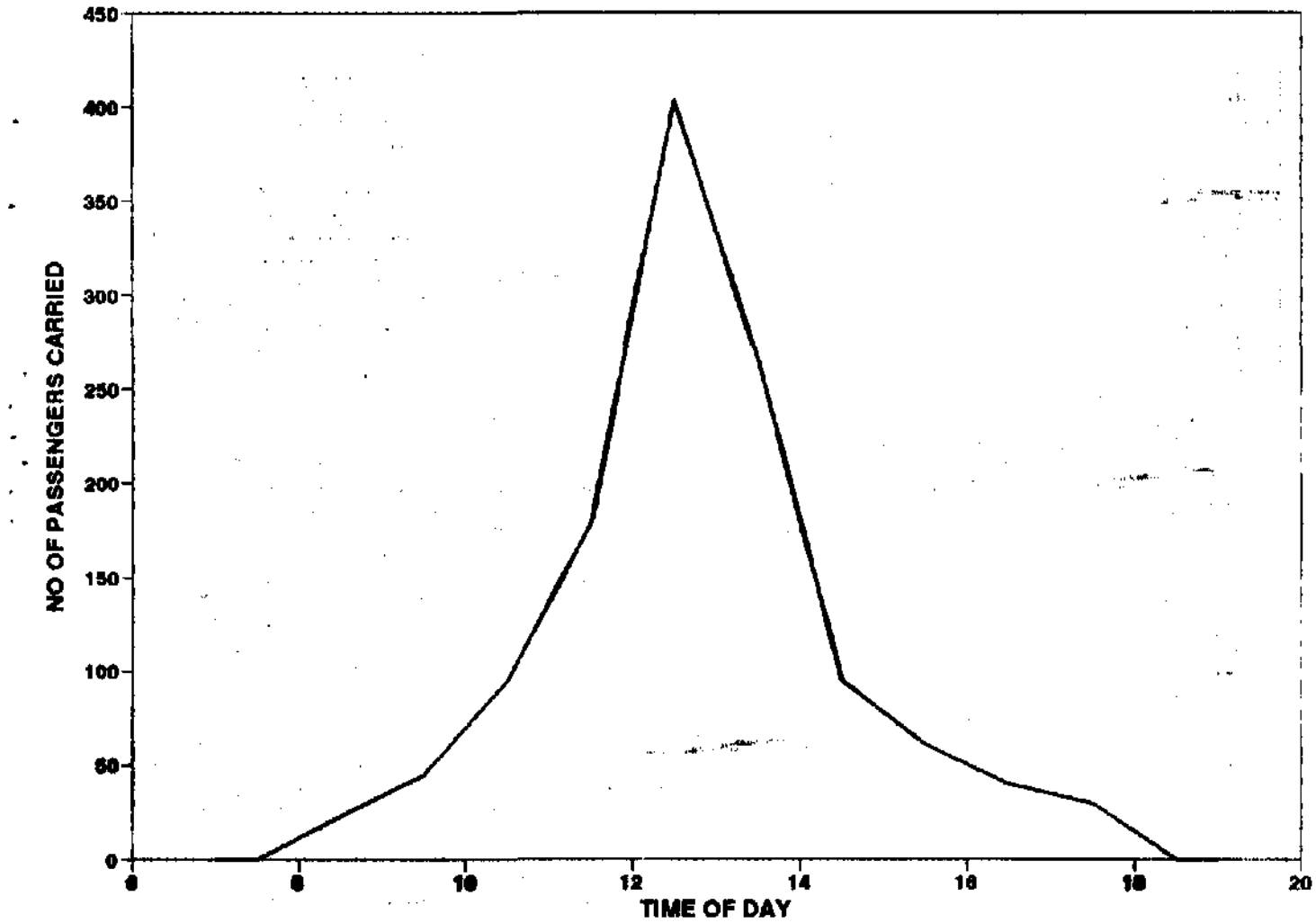

Fig.3.3b Minibus Passenger Distribution

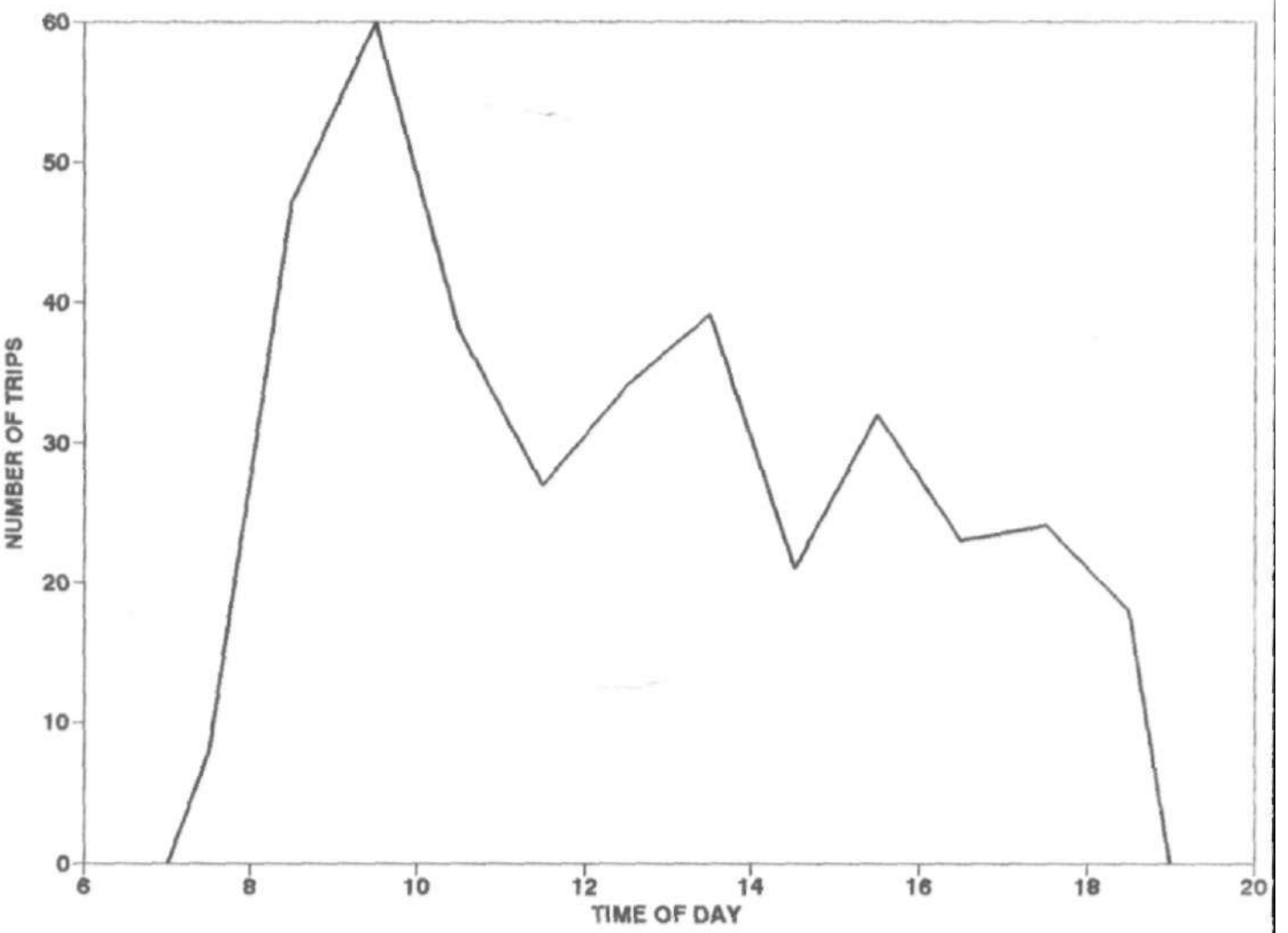

Fig.3.4a University Bus Trip Distribition

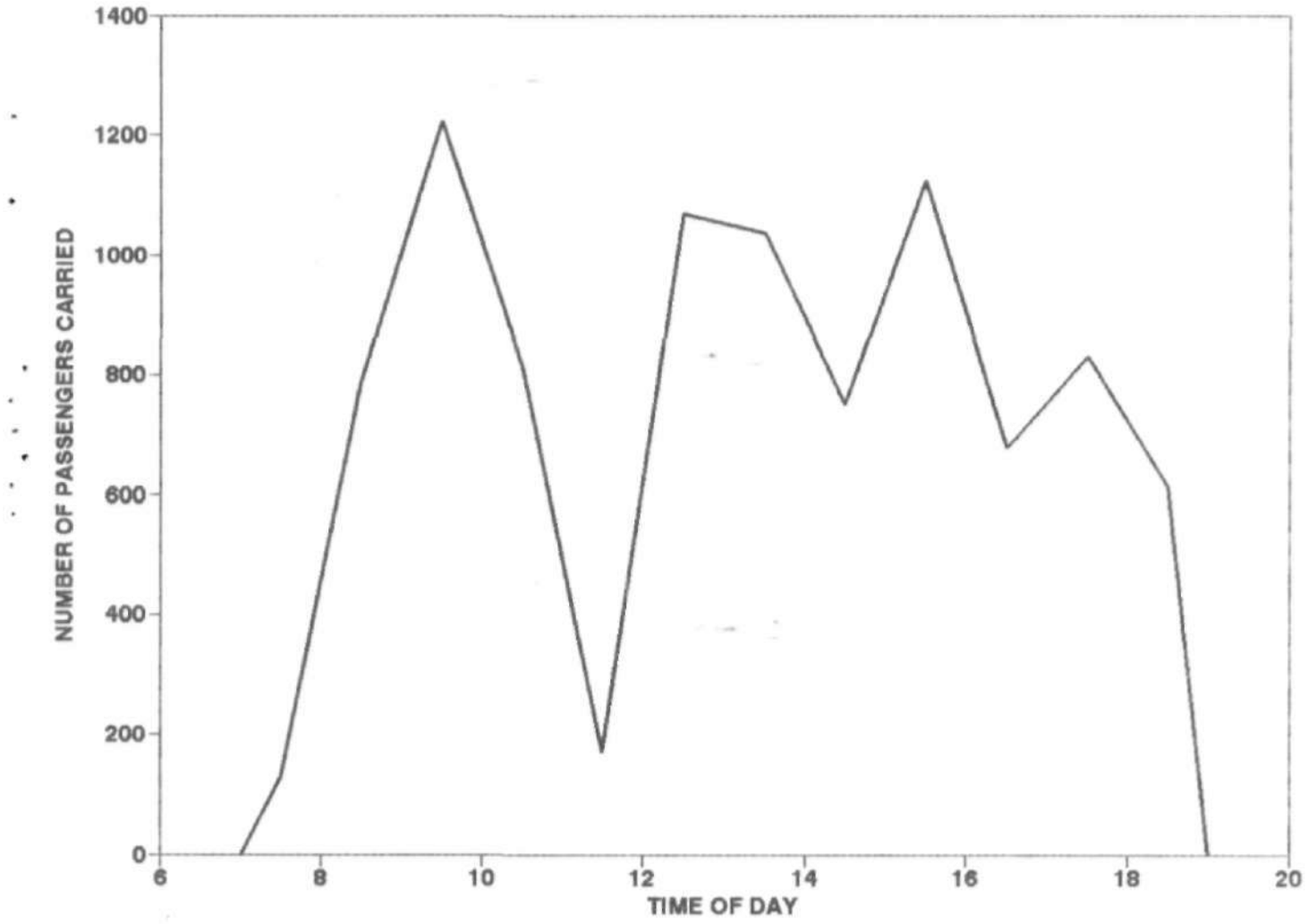

Fig.3.4b University Bus Passenger Distribution

ANALYSIS

In the modelling of most queueing systems, the important parameters to specify are the arrival rate λ and the service rate μ. However, two principal statistical problems also need to be tackled. One problem area is the assumption that we know the general form of the underlying service and demand distributions, but must estimate their parameters. From the estimates, we then need to make confidence interval statements concerning system performance measures under alternative design configurations.

The second major problem area, which really is the more critical of the two, concerns the initial selection of a model to describe an observed system. The problem here is to select probability density functions for the service and inter arrival times on the basis of observed data. Both problems are extremely important. In this work, systematic evaluations of the two problems involved will be pursued. In many queueing situations,(Giffin,Gross and Harris,Cooper) suggest that the gamma family of density functions is the candidate choice to describe the arrival and service distributions of the system. The gamma is a two-parameter distribution (a,b) of the form

$$f(t) = \frac{b(bt)^a}{\Gamma(a+1)} e^{-bt} \qquad t>0.$$

whose mean and variance are given by

$$\mu = (a+1)/b$$

$$\sigma^2 = (a+1)/b^2$$

Since the exponential distribution is a member of this family of curves in which a=0, we shall begin our exploits with this member. To begin however, it is important to decide whether one can use a steady-state model or go to a time-varying representation of a queueing system. It may be thus desirable to test whether the arrival rate and service rate fluctuate over time. To do this , one could employ a test for homogeneity of variance such as Bartletts tests. To apply the test, consider kr ordered event times, such as the arrival times of the first kr passengers of the day which have been arranged in k groups each of size r. The assumption is that the mean arrival rate is constant within each group. The object is to test whether it fluctuates from group to group. We must first calculate the total of all inter arrival times (ti) within each group where:

$$T1 = T(t_{ir}) = \sum_{i=1}^{r} ti \qquad .....3.$$

$$Tj = T(t_{jr} - t_{j(r-1)}) = \sum_{i=r(j-1)+1}^{jr} ti \qquad ...3.2$$

$$T = T(t_{kr}) = \sum_{j=1}^{k} T_j \qquad ....3.3$$

The variables T_j are mutually independent and $2\lambda T_j$ is for each group distributed as $X^2(2r)$ while $2\lambda T$ is distributed as $X^2(2rt)$. The test statistic

$$c = 2rk \{ \ln(T/k) - 1/k \sum_{i=1}^{k} \ln T_i \} / (1 + (k+1)/6rk) \qquad ...3.4$$

will be approximately distributed as $X^2(k-1)$. The hypothesis that λ is the same over all groups is rejected at the significant level α if $C > X^2(1-\alpha)(k-1)$where

$$Pr[X^2(k-1) > X^2_{1-\alpha}(k-1)] = \alpha$$

To illustrate the test, consider the number of passengers arriving as extracted from passenger arrival and count sheet for the university bus mode in Table 3.8 Each row represents arrivals for one hour, thus considering three groups each of twenty minutes, the total inter arrival number of passengers for each group are

$$T1 = \sum_{i=1}^{20} ni = 6$$

$$T2 = \sum_{i=21}^{40} ni = 13$$

$$T3 = \sum_{i=41}^{60} ni = 39$$

Therefore T = T1+T2+T3

= 58

TABLE 3.8 NUMBER OF ARRIVALS WITHIN TIME SLOTS ON THE UNIVERSITY BUS MODE

slot with Hr	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	.55	1.0	Tot/ Hr
0	0	2	4	5	2	2	4	12	21	3	3	58
2	8	9	9	0	11	44	5	19	25	20	15	167
12	9	13	16	7	9	21	23	6	27	6	18	167
41	23	9	5	7	5	7	5	0	4	7	7	120
6	14	18	16	0	16	19	25	11	5	12	6	148
10	12	13	16	10	9	9	19	7	9	13	9	136
21	8	12	3	8	4	7	8	8	8	5	5	97
15	17	13	14	8	8	9	5	5	6	3	21	124
22	13	15	27	3	0	3	0	0	36	5	16	140
4	13	6	5	7	8	7	10	9	12	5	15	101
21	6	0	6	7	0	1	16	3	7	4	5	76
4	7	4	7	5	6	7	3	0	2	0	0	45

From equation 3.4, k=3, r=4

c= 2.3.4 {ln(58/3)- 1/3 [ln6+ln13+ln39])/ 1+(3+1)/6.3.4

= 6.56

From the X^2 tables with two degrees of freedom we find the critical point for the 5 percent significance level at

$$X^2_{.95}(2)=5.99$$

Since c=6.56 is greater than 5.99 we must reject the hypothesis that the arrival rate is constant over these groups. A stationary model would not be appropriate for this system. Similar analysis for the passenger arrival rate of the taxi and the mini-bus modes indicate that a stationary model would be inappropriate.

In queueing systems, in which a stationary model is inappropriate only transient solutions have meaning. Even if $\lambda(t)$ and $\mu(t)$ are smooth well-behaved functions it may be impossible to find a compact expression for Pn(t). When these rate functions are described by curves developed from empirical data a complete formal solution is totally out of reach. We will circumvent these problems by following the approach suggested by Koopman.

(Koopman, 1972) suggests that one approximates the time-varying arrival and service rate functions by a series of step functions. The width of the step being selected on the basis of the rate of change of $\lambda(t)$ and $\mu(t)$. If these functions are changing very slowly one can tolerate a fairly long step size. If they are changing rapidly, shorter steps are necessary to achieve a reasonable approximation.

Within the time period covered by a single step, the arrival and service rates are assumed to be constant. The finite set of differential equations with constant coefficients are then developed and solved step-wise.

A second alternative is to find the solution within each time interval using numerical methods for solving systems of linear differential equations with constant coefficients. There are a number of standard numerical methods available for solving simultaneous ordinary initial value differential equations. Two of such are the predictor-corrector method and the Runge-Kutta method. The former provides the user with estimates of the error bound of the solution while the Runge-Kutta method does not. In comparing the output from the two routines over several prototype time-varying queueing problems (Hartman,1971) found that the accuracy of the two methods for the problems tested was identical. Furthermore, the predictor-corrector routine required approximately 50 percent longer execution time than the Runge-Kutta routine for the same problem. Consequently he recommended that the Runge-Kutta method be used to solve the equations for the time-varying queueing model. The Runge-Kutta method will therefore be used in this thesis.

It has often been argued that neither the constant

service time nor the exponential service time assumptions are very realistic. More probably, a real-world service distribution will fall somewhere between these two extremes. There is further reason to argue that the exponential and constant service time assumptions bound most other reasonable service distributions (Reza, 1961) for instance.

(Koopman, 1972) and (Hartman, 1971) have also compared results for real-world system time-varying demand data run against both constant and exponential service distributions. The result was that the envelope formed for most performance measures by these two models is extremely tight. Since the performance measure bounds are so narrow and since it has been argued that most real time service distributions should fall between the two extremes of constant and exponential service, it seems reasonable to use the model which is most easily manipulated with little regard to the actual form of the service distribution. (Koopman, 1972) and (Hartman, 1971) then suggested that the exponential service distribution model, which is a very robust tool and is easily solved by numerical methods, is the proper choice for analysis and design of most time-varying queueing systems.

Thus we are in a position to specify the parameters of our queueing model to be developed in the next chapter. We have a time-varying demand $\lambda(t)$, exponential service $\mu(t)$

and a first-in first-out queue discipline

TRIP TIMES OF THE TRANSPORT MODES

The trip times for the three transport modes were observed for three periods of the day. These were the morning, afternoon, and evening periods. For each period, three random samples of the trip times were observed, and averaged to obtain the daily trip times for the six day study period. Also the values for the six days were averaged to obtain a global average for the three modes. Table 3.7 shows that, mean global trip times for the University bus, Mini-bus, and Taxi modes are 22.07, 18.93 and 12.98 minutes respectively. The standard deviations obtained though not included in the table are 0.50, 0.35 and 0.47 respectively for the University bus, Mini-bus and Taxi modes. This gives a coefficient of variation of 0.022, 0.018 and 0.036 for the three modes respectively.The fares for the three are one naira for the university bus, five naira for the Mini-bus mode and ten naira for the Taxi mode.

TABLE 3.7 AVERAGE TRIP TIMES FOR THE THREE TRANSPORT MODES.

DATE	MODE	MORNING TRIP TIME (Mins)	AFTERNOON TRIP TIME (Mins)	EVENING TRIP TIME (Mins)	DAILY AVERAGE (Mins)
10/1/94	UNIV.BUS	23.15	22.49	22.19	22.48
	MINIBUS	18.27	19.02	20.14	19.14
	TAXI	13.55	13.12	12.29	12.99
11/1/94	UNIV.BUS	21.28	22.16	24.06	22.37
	MINIBUS	19.13	19.06	18.33	18.84
	TAXI	14.18	13.56	14.11	13.95
12/1/94	UNIV.BUS	23.37	21.44	22.58	22.46
	MINIBUS	19.12	19.24	18.44	18.93
	TAXI	12.44	13.33	13.38	13.05
13/1/94	UNIV.BUS	21.01	21.14	21.18	21.11
	MINIBUS	18.53	18.22	18.57	18.44
	TAXI	12.19	13.27	12.48	12.65
14/1/94	UNIV.BUS	20.37	22.43	22.32	21.70
	MINIBUS	19.07	18.59	18.41	18.69
	TAXI	12.51	12.15	13.09	12.58
15/1/94	UNIV.BUS	20.18	23.11	23.52	22.27
	MINIBUS	20.13	19.22	19.26	19.54
	TAXI	13.11	12.47	12.43	12.67
GLOBAL AVERAGE	UNIV.BUS	21.56	22.13	22.64	22.07
	MINIBUS	19.04	18.89	18.85	18.93
	TAXI	12.99	12.98	12.96	12.98

CHAPTER FOUR

MODELLING

4.1 MODEL DEVELOPMENT AND VALIDATION

The description of the modelling process has been explained in chapter one. Also the parameters for the queueing model have been shown to be, a time-varying arrival rate $\lambda(t)$ with exponential service distribution $\mu(t)$ and a queue discipline of FIFO in chapter three. The taxi and mini-bus elements of the transportation system is described by the traditional exponential bulk service system encountered in section 2.5.3 of chapter two.

In this system, service takes place on groups which cannot be larger than C. If less than C customers are present, all are served but if more than C are present a group of C will be served. The state of the system is defined by the number of customers in the queue and the number of taxis and mini-buses available for service. Incorporating the university buses into the system requires that the M/ M^{X} / K system be extended to a model with random clearing of all the queued taxi and mini-bus customers. Random clearing is a form of mechanical reneging which clears the entire queue as distinguished from the single customer reneging characteristic of customers.

The assumptions and notations of the model are as follows.

$\lambda_T(t)$= mean taxi customer arrival rate, at time t.

$\lambda_{MB}(t)$= mean mini-bus customer arrival rate, at time t.

$\lambda_{UB}(t)$= mean university bus arrival rate, at time t

$1/\Upsilon_{UB}$ =mean time between university buses.

$1/\mu_T$ =mean round trip time per taxi.

$1/\mu^*_T$ =one way mean taxi trip time.

$1/\mu_{MB}$ =mean round trip time per mini-bus.

$1/\mu^*_{MB}$ =one way mean mini-bus trip time.

$1/\Upsilon^*_{UB}$ =one way mean university bus trip time.

N_T = number of taxis operating daily (daily average).

N_{MB} =number of mini-buses operating daily (daily average).

W_T =taxi fare.

W_{MB} =mini-bus fare.

W_{UB} =university bus fare.

V_T =value of time of taxi customers.

V_{MB} =value of time of mini-bus customers.

V_{UB} =value of time of university bus customers.

It is assumed that customers plan their mode of travel on the basis of the expected full Π price which is defined as follows.

$$\Pi_T = W_T + V_T E[T_T] \qquad \text{taxi mode} \qquad 4.1$$

$$\Pi_{MB} = W_{MB} + V_{MB} E[T_{MB}] \qquad \text{mini-bus mode} \qquad 4.2$$

$$\Pi_{UB} = W_{UB} + V_{UB}E[T_{UB}] \quad \text{univ.bus mode} \quad 4.3$$

where $E[T_i]$, i=T, MB, UB is the expected transit time including waiting for taxi, mini-bus and university bus respectively, assuming no modal switch. This is because traditionally different components of transit time are valued differently. From field observation it is assumed that

$$E[T_T] < E[T_{MB}] < E[T_{UB}] \text{ and } W_T > W_{MB} > W_{UB}.$$

as it is evidenced in chapter three.Customers who value time will give preference to the taxi and mini-bus modes respectively since

$$\Pi_T \leq \Pi_{MB}$$

$$\Pi_T \leq \Pi_{UB} \quad \text{.....4.4}$$

$$\Pi_{MB} \leq \Pi_{UB}$$

and the value of time will at least satisfy:

$$V_1^* = \frac{W_T - W_{UB}}{E[T_{UB}]-E[T_T]}$$

$$V_2^* = \frac{W_{MB} - W_{UB}}{E[T_{UB}]- E[T_{MB}]} \quad 4.5$$

$$V_3^* = \frac{W_T - W_{MB}}{E[T_{MB}]- E[T_T]} \quad 4.6$$

Now to see if a queued taxi and mini-bus customer will switch to a university bus if one arrives before a taxi or mini-bus, we consider the conditional expected full price given that the customer is the n^{th} customer in either of the

taxi and mini-bus queues, that is:

$$\Pi_T^* = W_T + V_T E[T_T/n] \qquad \text{taxi mode}$$

$$= W_T - V_T[1/\mu_T + n/c_T\mu_T] \qquad 4.8$$

$$\Pi_{MB}^* = W_{MB} + V_{MB} E[T_{MB}/n] \qquad \text{mini-bus mode}$$

$$= W_{MB} - V_{MB}[1/\mu_{MB} + n/c_{MB}\mu_{MB}] \qquad 4.9$$

Also it is possible for a queued taxi customer to switch to a mini-bus if one arrives before a taxi, when a university bus is not available. Where $c_T\mu_T$ and $c_{MB}\mu_{MB}$ are the expected taxi and mini-bus service rates respectively. If a university bus is ready to depart, then the conditional full prices are

$$\Pi_{UB}^* = W_{UB} + V_T[1/Y_{UB}] \qquad \text{for taxi customers} \qquad 4.10$$

$$\Pi_{MB}^* = W_{MB} + V_T[1/Y_{MB}] \qquad \text{for taxi when a mini bus is departing} \qquad 4.11$$

$$\Pi_{UB}^* = W_{UB} + V_{MB}[1/Y_{UB}] \qquad \text{for mini-bus customers} \qquad 4.12$$

The largest number n_T^* of customers who would remain in the taxi queue, given that a university bus is departing immediately is

$$n_T^* = \left[W_{UB} - W_T + V_T/Y_{UB} - V_T/\mu_T\right] c_T\mu_T / V_T \qquad 4.13$$

$$n_T^{**} = \left[W_{MB} - W_T + V_T/Y_{MB} - V_T/\mu_T\right] c_T\mu_T / V_T \qquad 4.14$$

Similarly for the mini- bus mode

$$n_{UB}^* = \left[W_{UB} - W_{MB} + V_{MB}/Y_{UB} - V_{MB}/\mu_{MB}\right] c_{MB}\mu_{MB}/V_{MB} \qquad 4.15$$

We can therefore define n_T and n_{MB}, the maximum number of taxi and mini-bus customers who will remain in the queue given that a university bus is leaving as

$$n_T = \begin{cases} [n_T^*] + [n_T^{**}] & \text{for } n_T^* + n_T^{**} \geq 0 \\ 0 & \text{for } n_T^* < 0 \\ & n_T^{**} < 0 \end{cases} \qquad 4.16$$

$$n_{MB} = \begin{cases} [n_{MB}^*] & \text{for } n_{MB}^* \geq 0 \\ 0 & \text{for } n_T^* + n_T^{**} < 0 \end{cases} \qquad 4.17$$

where $[n_T^*]$, $[n_T^{**}]$, and $[n_{MB}^*]$ denotes the largest integer less than or equal to n_T^*, n_T^{**} and n_{MB}^* respectively. In the subsequent analysis it is assumed that $n_T = 0$, which is implied whenever

$$W_T - W_{MB} \geq V_T[1/\gamma_{UB} - 1/\mu_T] \qquad 4.18$$

$$W_T - W_{MB} \geq V_{MB}[1/\gamma_{MB} - 1/\mu_T] \qquad 4.19$$

and

$$W_{MB} - W_{UB} \geq V_{MB}[1/\gamma_{UB} - 1/\mu_{MB}] \qquad 4.20$$

whenever all the three modes immediately available, n_T and n_{MB} will be zero when the fare premium on the taxi exceeds the value of the time saved by taking the taxi instead of the university bus, and the mini-bus. Similarly for the

mini-bus mode. This limiting case is interesting because it focuses attention on the convenience of taxis and mini-buses versus the university buses rather than on their enroute speed.

4.2 BULK SERVICE SOLUTION

The equations for the Bulk service problem encountered in section 2.5.3 of chapter two becomes:

$$\frac{dP_o(t)}{d(t)} = -\lambda(t)P_o(t) + \mu_1(t)P_1(t)$$

$$\frac{dP_n(t)}{d(t)} = \lambda_{n-1}(t)P_{n-1}(t) - \left[\lambda_n(t)+\mu_n(t)\right]P_n(t) + \mu_{n+1}(t)P_{n+1}(t) \qquad 0 < n < k$$

$$\frac{dP_k(t)}{d(t)} = \lambda_{k-1}(t)P_{k-1}(t) - \mu_k(t)P_k(t) \qquad \text{.....4.21}$$

when the system is truncated at n=k.

Equation (1) of 4.21 arises when an empty system is considered. Equation (2) of 4.21 represents the system when there are n of the possible k customers requiring service at the access area. Finally equation (3) of 4.21 represents the system when all k customers are present.

The equations are too complex for analytical manipulations and, hence, the resort to computer simulations. The simulation process of the queueing model as well as the decision criteria for each transport mode is shown in the

flow chart of figure 4.1. On arrival at the access area, customers join the mode of their choice on the basis of the expected full price shown by equations 4.1 to 4.3.

The queues arising from the three modes are analysed using the system of equations 4.21. The set of equations 4.21 are ordinary differential equations with variable coefficients. The fourth order Runge-Kutta method is used to solve the finite set of differential equations 4.21. The classical fourth order method requires four evaluations of the first derivative to obtain a Taylor series approximation through terms of order h^4. Let $\bar{y}_n = \bar{y}(to+nh)$ and $t_n = (to+nh)$, then for the single variable problem, the fourth order Runge-Kutta method calculates

$$yn+1 = yn+1/6\left[k1 + 2k2 + 2k3 + k4\right]$$

where

$$k1 = h.f(tn, \bar{y}n)$$

$$k2 = h.f(tn+h/2,\ \bar{y}n+k1/2)$$

$$k3 = h.f(tn+h/2,\ \bar{y}n+k2/2)$$

$$k4 = h.f(tn+h/2,\ \bar{y}n+k3)$$

In the programs presented in appendix B, the set of equations 4.21 is solved by Runge-Kutta within each interval. An interval is defined by the width of the step used to approximate the arrival and service rates. Starting at the beginning of an interval Tj the equations are solved in increments h until the end of the interval is reached.

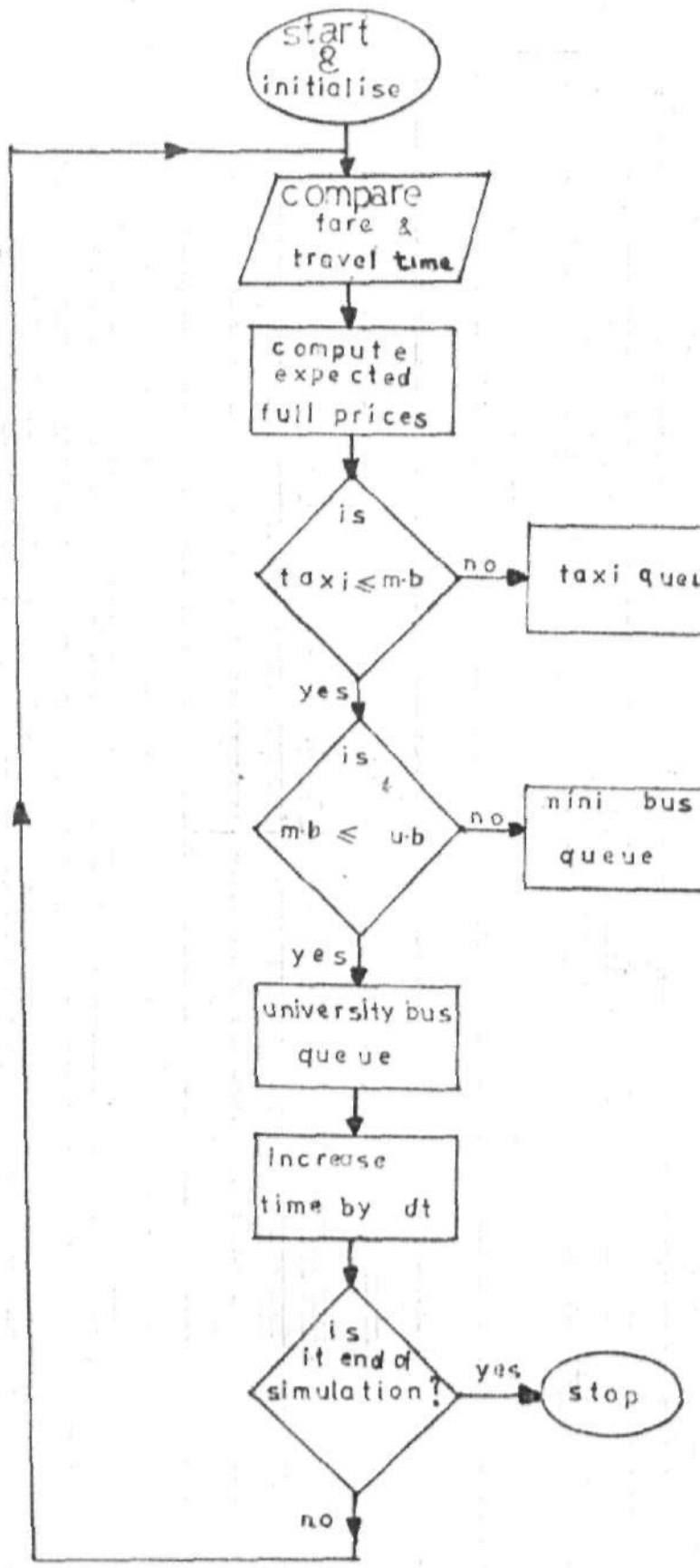

fig 4.1 flow chart for the queueing process at the new campus access area

The values at the end of the interval becomes the initial conditions for the next interval. Operating statistics are calculated and printed at the end of each interval of time T_j. The resulting computer program is listed in Appendix A after its full development.

4.2.1 PERFORMANCE MEASURES

The measures of performance for the truncated time-varying queue offered in the program in appendix B are all related to the state probability distribution. The probability distribution Pn(t) is printed at the end of each time interval Tj. From that distribution the program calculates the expected number in the queue, the standard deviation for number in the queue, and the expected total customer minutes delay time in that interval. The program also calculates an approximation to the expected number of rejects, ie. customers who cannot gain access to the system because of saturation.

The expected total delay time within a computation interval is calculated as the product of the average number of customers in the queue during that interval and the length of the interval. If one assumes that the expected number in the queue varies in linear fashion over the interval, the average number present can be approximated

from $$E(Q) = \frac{E(Qj-1)+E(Qj)}{2}$$

where E(Qj) is the expected number in queue at the end of interval j as calculated from Pn(t). The total expected delay time within an interval of length T is then

$$E(WAIT) = E(Q).T$$

$$= \frac{E(Qj-1)+E(Qj)}{2}.T \qquad ....(1)$$

These waiting times are summed over the entire execution period for the convenience of the analyst and calculated from

$$E(TWAIT)=\sum_{j=1}^{n} E(WAITj) \qquad .....(2)$$

where E(WAITj) is the expected value of customer-minutes delay in period j. The cumulative number of rejects, ie customers denied service, is approximated by multiplying Pk(t), the probability of saturation, by $\lambda(t)$, the arrival rate and the time during which the system is saturated. This calculation takes place at each step in the Runge-Kutta approximation rather than just at the end a major subdivision.

4.2.2 DATA INPUT.

The data required for successful running of the computer programs in Appendix B are the 48 values each for the arrival rate and service rate of the queueing systems. These values represent 48 fifteen-minute time periods in a

12-hour period. If a different number of periods is desired then it is necessary to change the corresponding dimension statements. Also data for the batch size KI, maximum number of customers permitted in the system MM, and the total length of the time period being divided into 48 subperiods TIMPER is desired. All the data can be found at the end of the computer package in Appendix B for the three modes.

The program solves the finite set of linear differential equations for each subperiod using Runge-Kutta numerical methods. It assumes that the system is initially empty, ie Po(0)=1 though this is in conformity with field observations. The time-varying solution over the first subperiod, usually taken to be fifteen minutes, is obtained from the constant coefficient differential equations where the arrival rate and service rate are the step values read in for that subperiod. At the end of the first period various operating statistics are calculated and printed along with the entire state probability distribution at that point in time. The distribution at the end of the first subperiod becomes the initial value vector for the next subperiod. These values are then used together with new constant arrival and service rates to find the time-varying solution for the next subperiod. the solution proceeds in this manner untill the entire 48 subperiods in TIMPER have

been evaluated.

4.3 MODEL VALIDATION

One method which may be useful in establishing the worth of numerical methods such as those advocated in this thesis is to compare the results of the time-varying queueing model to those obtained by other researchers when both are driven by the same input data. Koopman, (1972) carried out a similar analysis from data generated from an FAA study in the U.S.of landings and departures at La Guardia airport during a one-month period in 1968 . His model was a single channel, time-varying queueing model with a constant service rate. The program in Appendix B was fed with the data as reported by Koopman. The results obtained replicated what Koopman reported and is shown in Appendix C. Also the simulation model developed was tested with field observations on all the trips made during the study period. The probabilities of delay as obtained from field observations for the three modes were compared with the model results and are shown in figures 4.2 to 4.4. In addition, some of the parameters for both field and model results are compared in table 4.1 Using the validated model, the performance measures of the queueing systems generated by the three modes of transport was analysed and are presented in Figures 4.5 to 4.7.

The number of trips and the demand for each mode as generated from the data collection sheets were

TABLE 4.1 :VALIDATION OF SIMULATION MODEL

PARAMETER	MODE	FIELD VALUE	MODEL VALUE	DIFFERENCES (%)
PROBABILITY OF DELAY	TAXI	0.069	0.112	+4.3
	MINIBUS	0.073	0.069	-0.4
	UNIV.BUS	0.064	0.088	-2.4
AVERAGE DELAY (mins)	TAXI	12.514	12.439	-7.5
	MINIBUS	16.337	16.401	+6.4
	UNIV.BUS	23.518	23.462	-5.6
CUMULATIVE DELAY (mins)	TAXI	660.278	662.345	+6.7
	MINIBUS	443.371	444.432	-6.1
	UNIV.BUS	1132.569	1231.628	+5.9

analysed statistically and shown in Table 3.6. When the number of trips is regressed on the demand, it is possible to predict the number of trips needed for a given demand. This is shown in Figure 4.8, for the three modes. It shows that for each mode some trips will be generated even though there is no demand. This is in agreement with what is observed on site. Because trips to the new campus originate from the old campus some trips will be observed even though there is no demand for transportation at the new campus access area. Hence for a typical day 72 taxi trips will

be observed for which there is no demand at the new campus access area. Similarly, 17 university bus trips will be observed for which there is no demand. However, the situation is different for the mini-bus mode and a negative mini-bus trip of 1 will occur. Physically, this means that no trips are observed for which a demand does not exist. Detail results for the regression output is in Appendix B.

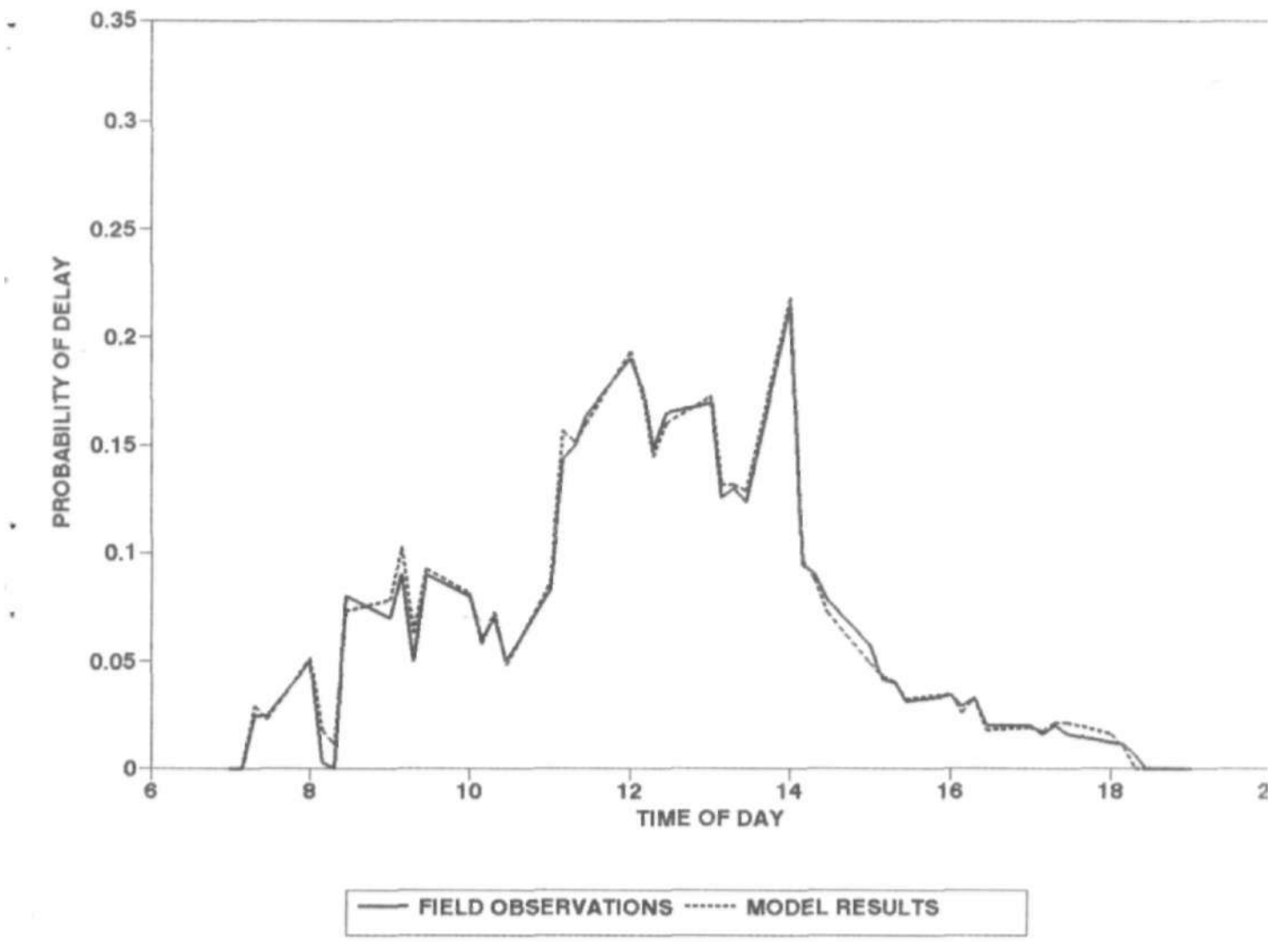

Fig. 4.2-TAXI MODE:PROBABILITIES OF DELAY

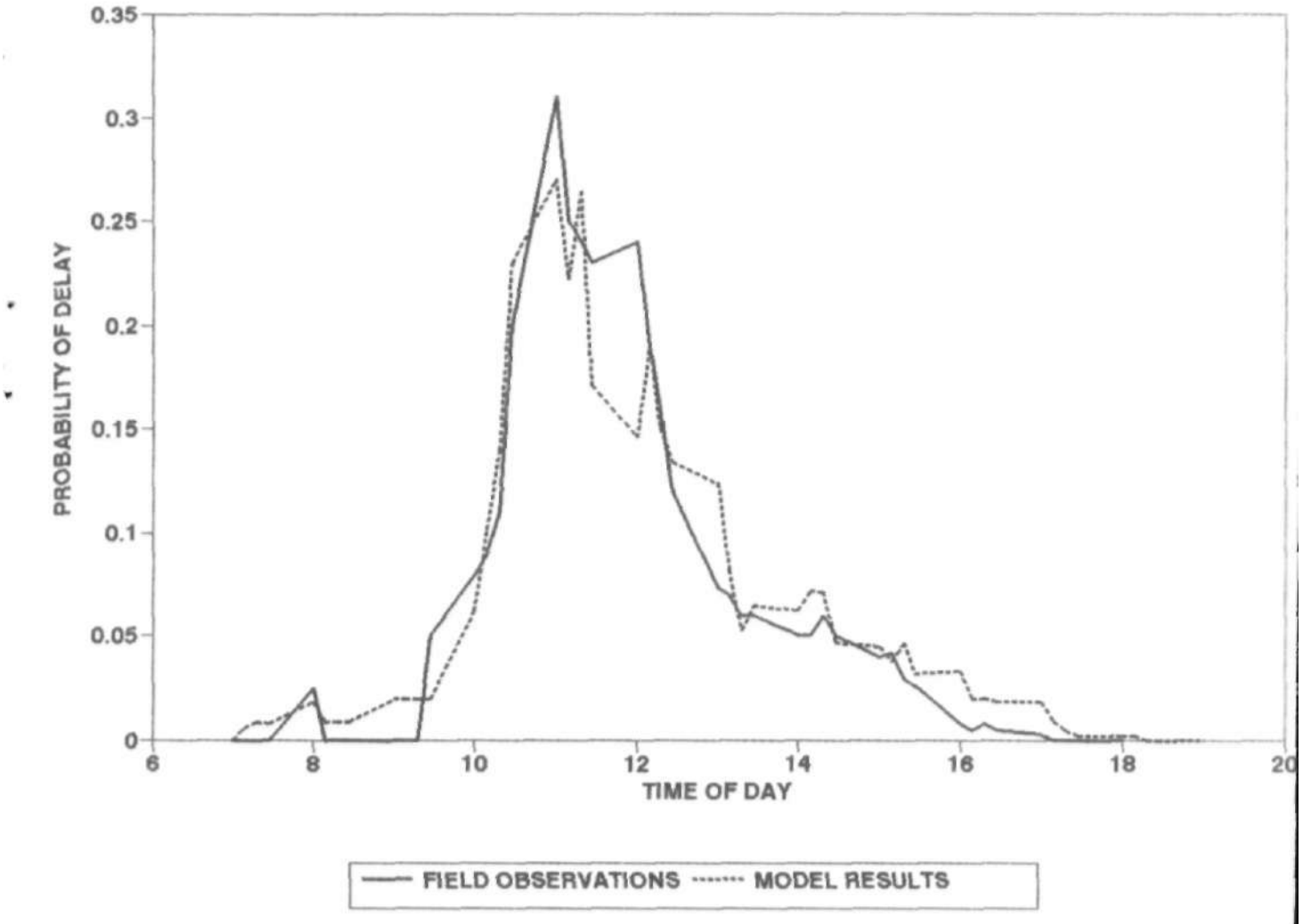

Fig.4.3-MINIBUS MODE:PROBABILITIES OF DELAY

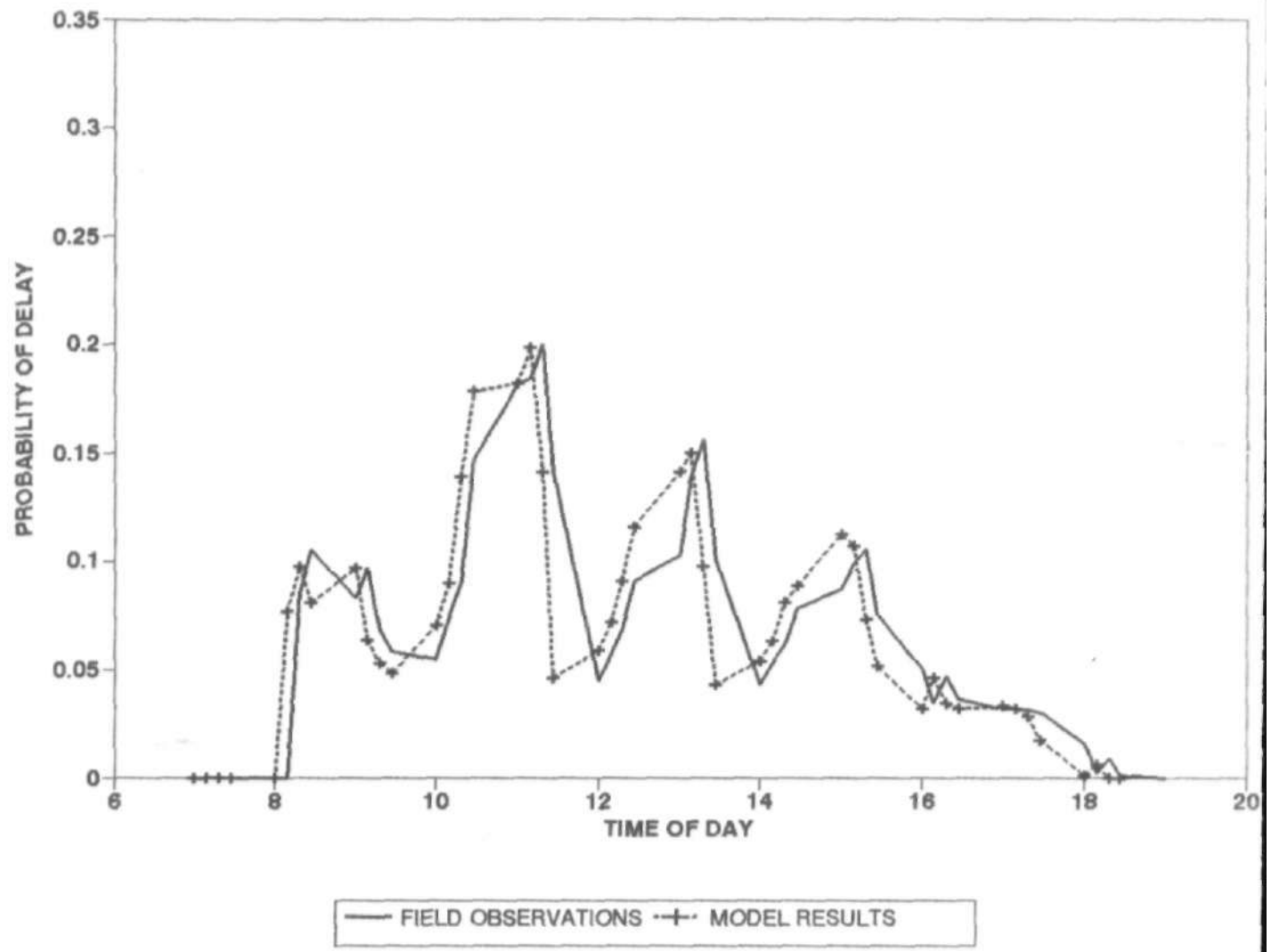

Fig.4.4-UNIVERSITY MODE:PROBABILITIES OF DELAY.

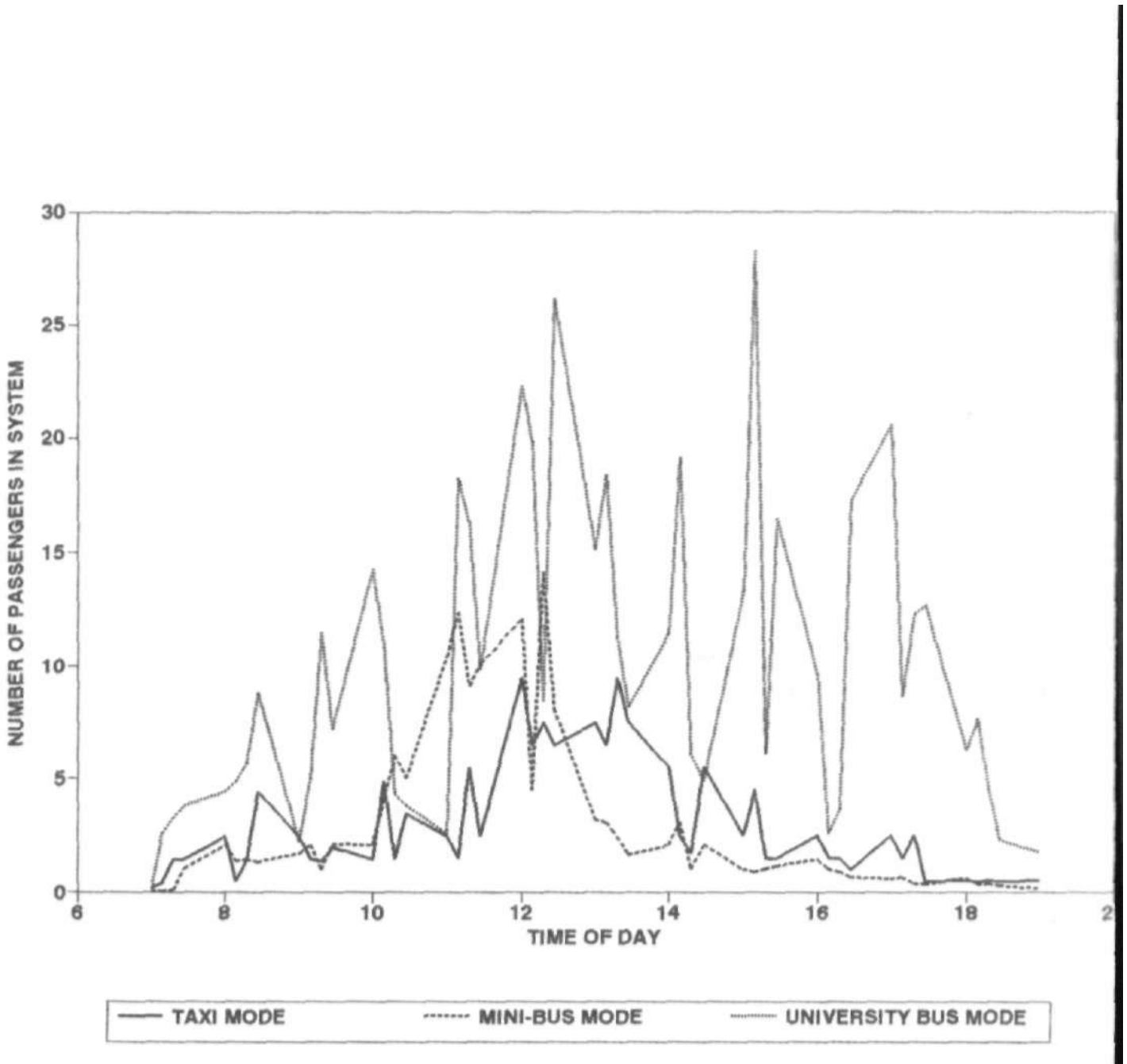

Fig.4.5-MEAN NUMBER OF PASSENGERS IN THE SYSTEM

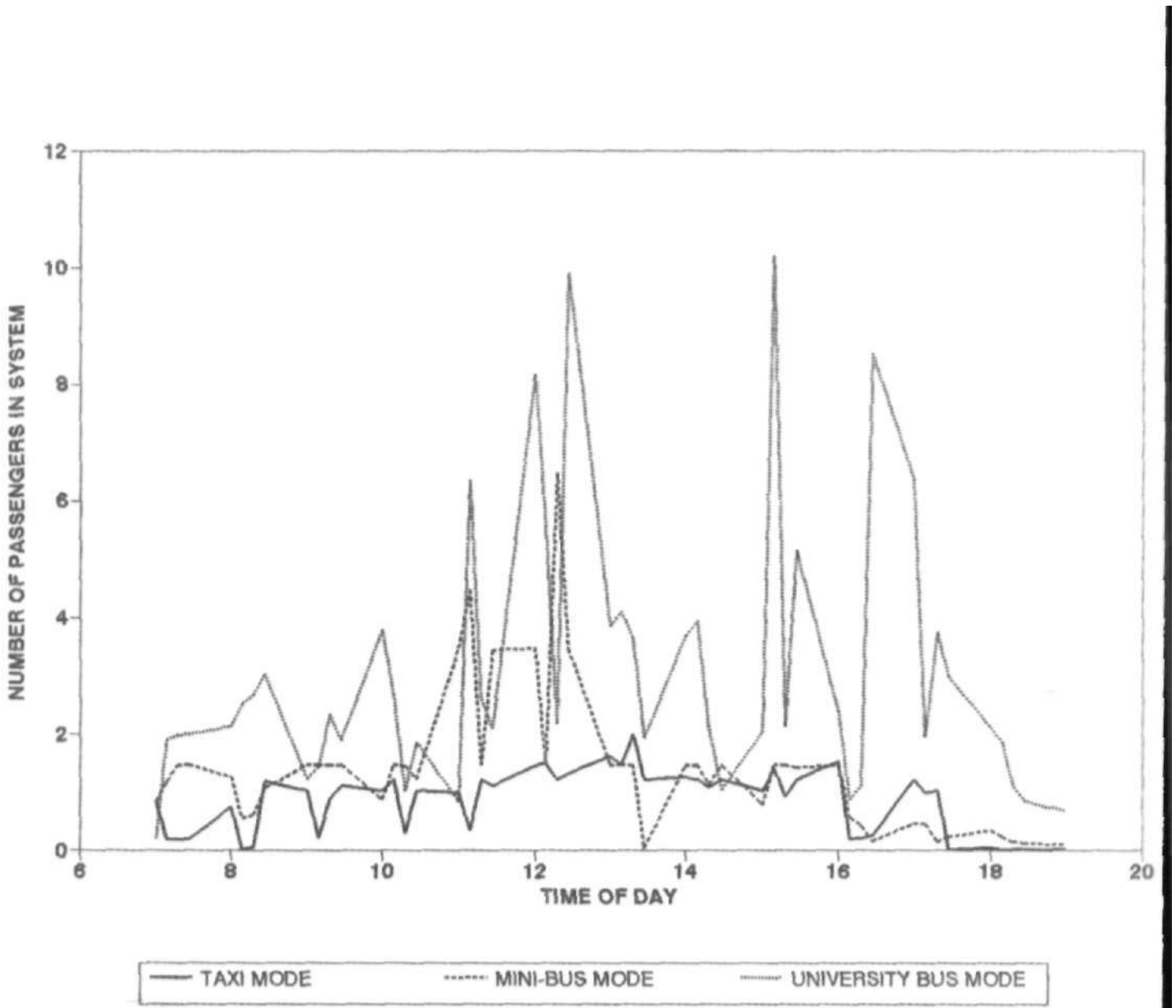

Fig.4.6-STANDARD DEVIATION OF NUMBER OF PASSENGERS IN THE SYSTEM

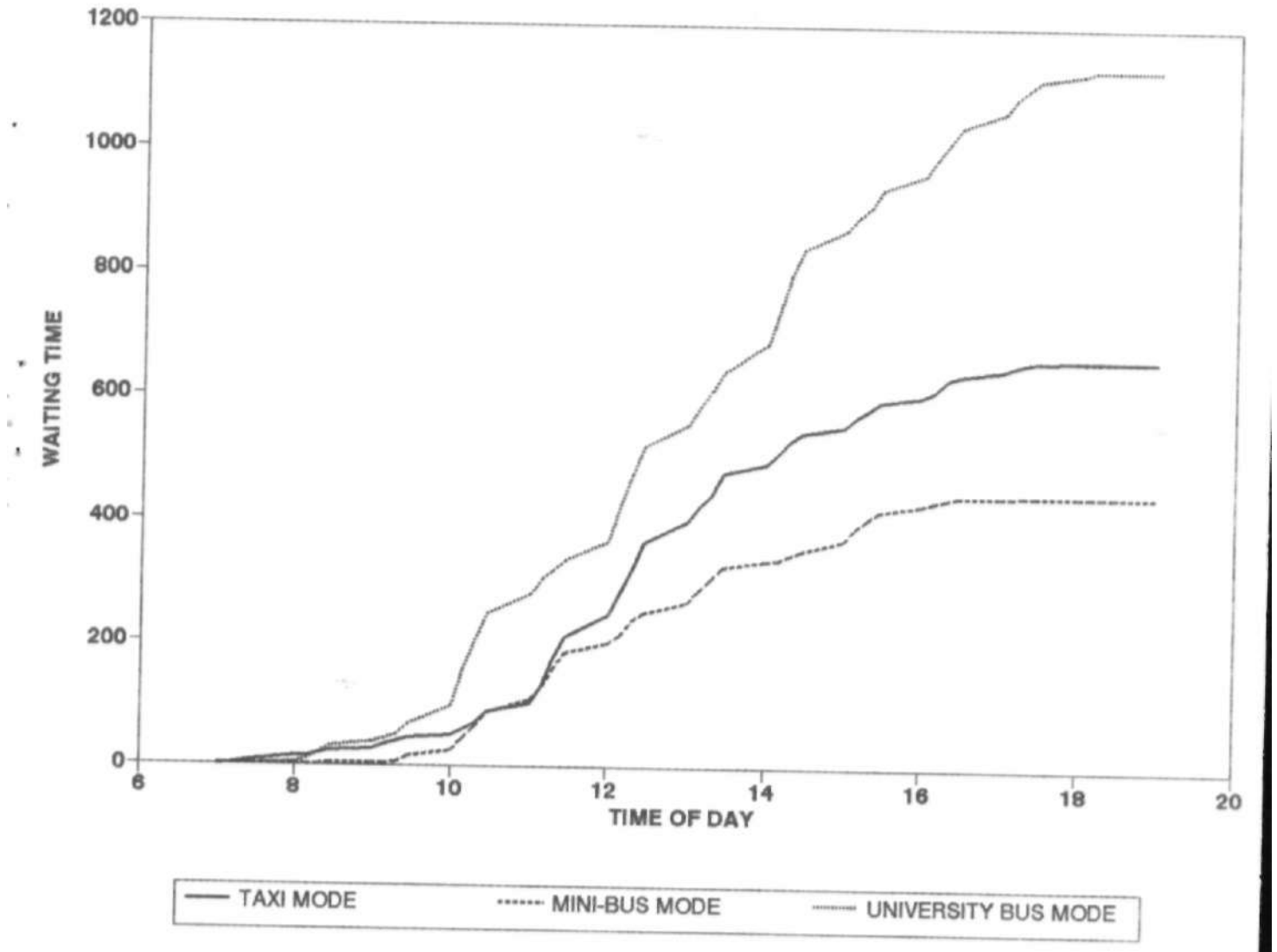

Fig.4.7-CUMULATIVE WAITING TIME

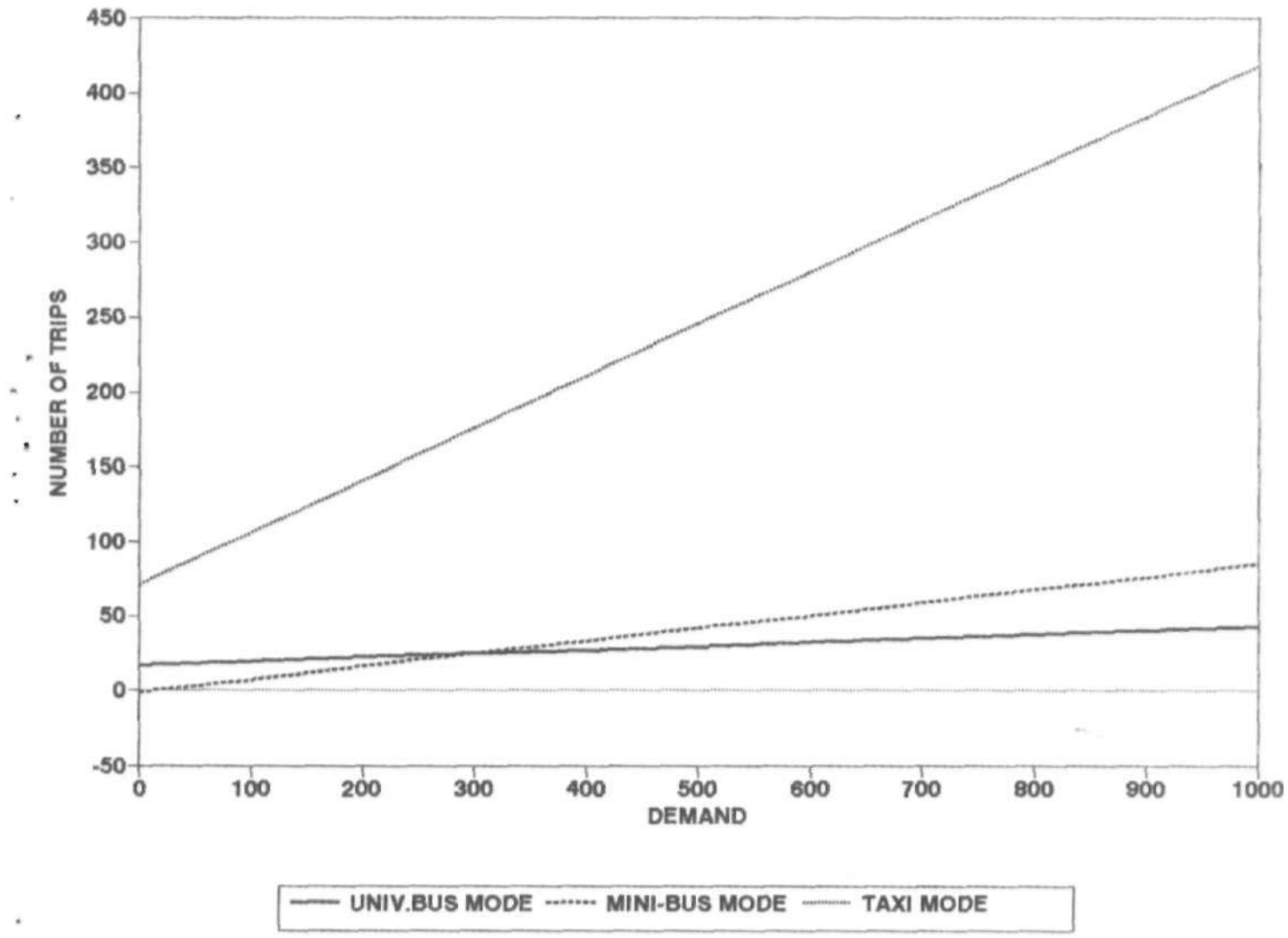

Fig.4.8-TRIP DEMAND MODEL

CHAPTER FIVE

5.1 CONCLUSIONS AND RECOMMENDATIONS

A queueing model describing the queueing situation at BUK new campus access area has been developed and shown to effectively model the interplay between the commercial modes and university transport.

Several conclusions emerge from the analysis made with the model. First, for each level of demand of each mode, there is a corresponding number of trips of each transport mode required to capture all the would-be customers at the access area. Secondly, from the measures of performance of the queueing systems it is easy to see which times of the day will see the most traffic congestion and at what times arriving customers are most likely to be delayed. Figures 4.2 to 4.4 shows the probabilities of delay for the taxi, minibus and the university bus modes. It reveals that delays are most likely to occur from 11.00am to 2.00pm for the taxi mode. Minibus passengers suffer longer delays in the period 10.30am to 12.30pm, whilst university bus passengers experience more delays at three distinct periods. These are 10.30am to 11.30am, 1.00pm and 3.00pm. Delays are longer from 11.00am for all the modes because university buses have cleared the queues at the old campus as a result of which they wait for longer times to fill the buses. At the new

campus on the other hand, queues build up to peak levels and therefore passengers get apprehensive and would make the trip by any available mode. The conclusion that can be drawn is that the university transportation system could be relied upon to effectively transport its customers between the two campuses.This is based on observations during the study period.

5.2 RECOMMENDATIONS

These results carry a number of implications for the new campus access policy. To maintain a steady level of servvice, it is necessary to register all operators of private commercial transport that commute between the two campuses. This category of transport should be made to carry identification marks on them for easy identification by security personnel at the main gate. This will give on the spot knowledge of the number of private taxis and mini-buses registered and operating daily and hence a ready assessment of the level of service offered.

The present situation whereby taxis and mini-buses go to the new campus only on demand at the old campus adds to the anxiety in looking for transportation at the new campus. In addition, the proprietors of taxis and mini-buses could withdraw their services without notice. This could aggravate the situation. When the university authority registers them, it is possible to replace the ones who

withdraw their services and hence maintain the level of service.

Another policy implication concerns the arrival pattern of passengers to the access area. It has been observed in previous chapters that surges of arrivals is a common feature of the university bus passengers especially when the buses horn to signal their arrivals. The chaotic situation created as a result defies any attempt to promote orderliness. This leads to vandalisation of the buses. It is therefore recommended that the university should regularly inform its customers of its dailly operating schedule to and from the new campus and should also announce cancelled trips promptly to prevent unnecessary waiting. A weekly or monthly schedule of the university buses could be displayed in advance and anticipated cancellations could then be announced before scheduled times. When sudden cancellation of trips become inevitable it should be announced at the access area during the preceding trips since not all of the buses are withdrawn at any given time. This would go a long way in shaping the arrival pattern of its passengers to coincide with the arrival times of the university buses. In addition to this, it is recommended that the access area be provided with a structure with sitting arrangement for customers to wait. This will

encourage customers to wait at the access area and thus prevent rushing upon the arrival of the buses.

A third policy implication concerns the number of trips required to capture all the passengers for a given level of demand. For the present level of demand, of 1467, 413, and 187 passengers respectively for the university bus mode, mini-bus mode and the taxi mode an average of 55, 35, and 137 trips respectively are needed for the three modes to capture all the would be customers at the access area. This is adequate for the taxi and mini-bus modes, even for the peak hour demand. This is because each mode carried less customers than they would normally have carried. For instance, the taxi mode made an average of 137 trips corresponding to about 685 passengers but it carried only 187 of them. What is required is a policy to shift customers from the university bus mode to the other modes. This can be achieved by an upward review of the university bus fare from the current one naira to two naira. The taxi and mini-bus modes charges ten and five naira respectively.

It must be added that the number of trips have been specified rather than the number of taxis or mini-buses or university buses for good reasons. For the taxi mode and the mini-bus mode it is observed that 90 percent of taxis and mini-buses did not make a second trip. Thus specifying

the number of taxis or mini-buses would mean that a large number of the various modes would be needed to transport passengers from the new campus. Specifying the number of trips has an advantage in that knowing the round trip time could help in determining an optimum number of taxis or mini-buses to be assigned to the new campus corridor.

Despite that the University bus service seems capable of meeting the demand for transportation, the other two modes are still very relevant to the efficient transportation of passengers between the two campuses. During occassional breakdown of the University buses, students holidays, hint of students riots, the University buses are withdrawn leaving customers to rely heavily on the other modes to commute between the two campuses. Also the findings in this thesis may assume a transient character so that the system might not be consistent in its performance. In addition, some customers are now captive to both the Taxi and the Mini-bus modes. For these reasons, it is recommended that the present policy of inviting modes other than the University bus should be maintained so that when situations like those mentioned above occur, passengers could depend on those modes to commute between the two campuses.

To ensure continuity, reliability and effective service, it is recommended that the university transport service be

made autonomous. This will minimise the frequent interruptions in its operations, make it revenue generating and thus enable it to maintain its fleet.

The results of this study still requires fine tuning, for confidence to be built into them most probably in the following ways.

(a) It is necessary to repeat the analysis periodically to capture structural changes that may affect the arrival pattern of passengers. For example, annual time-table changes for students lectures might give rise to different arrival patterns.

(b) Develop a prototype computer package for planning applications. This would make it easier to test the performance of the models in tracking the effects of actual transportation system changes and would be of use to planners.

(c) Estimate the same (or similar) models on data for other areas or campuses and time periods. This would provide evidence on the generalisability of the findings reported here.

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APPENDIX A

```
      OPROGRAM TAXI
C THIS PROGRAM CALCULATES TRANSIENT STATE PROBABILITIES AND THE RELATE
C BEHAVIOURAL CHARACTERISTICS FOR A KI BATCH TRUNCATED QUEUEING
C SYSTEM WITH VARIABLE POISSON ARRIVAL AND EXPONENTIAL SERVICE RATES.
C A TABULAR STEP-FUNCTION IS USED TO APPROXIMATE THE MEAN ARRIVAL AND
C SERVICE RATES.

C SURROUTINE RKQC IS USED TO SOLVE THE SIMULTANEOUS DIFFERENTIAL
C EQUATIONS WHICH DESCRIBE THE SYSTEM.THIS SUBROUTINE USES THE RUNGE-KU
C METHOD FOR THE SOLUTION OF INITIAL VALUE PROBLEMS,WITH ADAPTIVE STEP-
C CONTROL.

C THE QUEUE STATISTICS CALCULATED ARE:
C   1.EXPECTED NUMBER IN THE QUEUE = EQ
C   2.STD.DEV. OF NUMBER IN THE QUEUE = SDQ.
C   3.EXPECTED TOTAL WAITING TIME FOR A SUB-DIVISION = EWAIT.
C   4.SUMMATION OF WAITING TIMES = TWAIT.
C   5.APPROXIMATE NUMBER OF CUMMULATIVE REJECTS = RJECTS.
C   6.EXPECTED NUMBER IN SYSTEM = ES.
C   7.STD.DEV.OF NUMBER IN SYSTEM = SDS.
C   8.PROBABILITY OF DELAY = PW.

C THE VARIABLES USED ARE:
C   TIMPER = THE TOTAL LENGTH OF TIME FOR WHICH THE EQUATIONS ARE TO BE
C   SOLVED.
C       KI = BATCH SIZE.
C       MM = MAXIMUM NUMBER ALLOWED IN THE SYSTEM.
C        M = MM+1  = NUMBER OF POSSIBLE STATES OF THE SYSTEM.
C      NST = THE NUMBER OF SUB-DIVISIONS OF TIMPER FOR BOTH DATA INPUT
C   AND OUTPUT OF RESULTS.
C       DT = THE LENGTH OF THE TIME-PERIOD SUB-DIVISION.
C        PSCAL = AN ERROR WEIGHT USED BY RKQC.
C     R(I) = INPUT VECTOR OF MEAN ARRIVAL RATES FOR THE TIME-PERIOD
C  UNDER CONSIDERATION.  I =1,2,3,....NST.
C     L(I) = INPUT VECTOR OF MEAN SERVICE PER CHANNEL FOR PERIOD UNDER
C  CONSIDERATION.  I =1,2,3,....NST.
C     P(J) = PROBABILITY OF J-1 IN THE SYSTEM AT A GIVEN TIME; J = 1,2
C  DERY(J) = DERY IS THE VECTOR OF DERIVATIVES OF P AT A POINT T.
C  T1 = LOWER BOUND OF THE SOLUTION INTERVAL FOR ONE ITERATION.
C  T2 = UPPER BOUND OF THE SOLUTION INTERVAL FOR ONE ITERATION.
C  H1 = INITIAL STEP-SIZE OF INTEGRATION.
C  EPS = UPPER ERROR BOUND ACCEPTABLE ON VALUES OF P.
C  DQ = EXPECTED NUMBER IN THE QUEUE AT THE END OF CURRENT SUB-DIVISIO
C  DQL = EXPECTED NUMBER IN THE QUEUE AT THE END OF LAST SUB-DIVISION.
C  MX = STATE WITH LARGEST PROBABILITY.MAY BE ADJUSTED TO ACCOMMODATE
C       NUMERICAL ERRORS.
C NOTE:  THE PROGRAM IS COMPLETELY GENERAL WITH THE FOLLOWING EXCEPTIO
C       1.IF NST IS CHANGED,THE DIMENSIONS ON R AND L MUST BE CHANGED.
C       2.THE CALCULATION OF EXPECTED WAITING TIME FINDS THE AVERAGE
C         EXPECTED NUMBER IN THE QUEUE FOR A SUB-DIVISION AND MULTIPLI
C         THIS BY THE NUMBER OF MINUTES IN A SUB-DIVISION TO OBTAIN THE
C         TOTAL EXPECTED WAITING TIME FOR THAT SUB-DIVISION.

      DIMENSION P(60),DERY(60),PSCAL(60)
      REAL L(48)
      COMMON I,R(48),L,M,MM,TIMLIM,DT,RJECTS,TSAV,KI
      COMMON DQ,DQL,TWAIT
      PARAMETER (N=60)
```

```
      NST=48
      DQ=0.
       DQL=0.
      TWAIT=0.
      OPEN( 5,FILE='TAXI.DAT',STATUS='OLD')
      OPEN( 6,FILE='TAXIOUT',STATUS='NEW')
      READ(5,100)(R(I),I=1,NST)
      READ(5,100)(L(I),I=1,NST)
      READ(5,50) KI,MM,TIMPER
   50 FORMAT(2I5,F4.0)
      TSAV=0.
      M=MM+1
      RJECTS=0.
      DT=TIMPER/FLOAT(NST)
      Z=1./FLOAT(M)
      WRITE(6,101) MM,KI
C       INITIALIZE P(I) AT TIME=0
      DO 1 J=1,60
    1 P(J)=0.0
      P(1)=1.
C  ESTABLISH STEP-SIZE FOR INTEGRATION AND ERROR BOUND
C
      H1=0.03
      EPS=1.0E-3
      DO 4 JIK=1,60
          PSCAL(JIK)=Z
    4 CONTINUE

C SOLVE THE SYSTEM OF EQUATIONS FOR THE NST TIME PERIODS IN TIMPER
C
      DO 3 I=1,NST
      TIMLIM=DT*FLOAT(I)
      T1=TIMLIM-DT
      T2=TIMLIM
      DO 2 K=1,M
      DERY(K)=Z
    2 CONTINUE
      CALL RKQC(P,DERY,N,T1,H1,EPS,PSCAL,HDID,HNEXT)
      CALL OUTP(T,P)
    3 CONTINUE
  100 FORMAT(10F5.0)
  101 FORMAT('-',5X,'RESULTS FOR THE TRUNCATED KI BATCH QUEUEING SYS'
     1 ,/,10X,'TIME VARYING POISSON ARRIVALS AND EXPONENTIAL SERVICE'
     2 ,/,15X,'MAXIMUM QUEUE SIZE=',I4,5X,'NUMBER OF SERVERS=',I4)
      CLOSE (5)
      CLOSE (6)
      STOP
      END

      SUBROUTINE DERIVS(P,DERY)
C
C THIS SUBROUTINE EVALUATES THE RIGHT HAND SIDE OF THE DIFFERENTIAL
C       EQUATIONS WHICH DESCRIBE THE SYSTEM
C
C T= TIME, THE INDEPENDENT VARIABLE
C
C NEITHER T NOR P SHOULD BE CHANGED BY THIS SUBROUTINE
      DIMENSION P(60),DERY(60)
      REAL L(48)
```

```
      COMMON I,R(48),L,M,MM,TIMLIM,DT,RJECTS,TSAV,KI
      COMMON DQ,DQL,TWAIT
      PRINT*,'enter derivs'
      KK=KI+1
      DERY(1)=L(I)*P(2)-R(I)*P(1)
      DERY(M)=R(I)*P(M-1)-FLOAT(KI)*L(I)*P(M)
      DO 1 J=2,KI
      Q=FLOAT(J)
    1 DERY(J)=Q*L(I)*P(J+1)+R(I)*P(J-1)-((Q-1.0)*L(I)+R(I))*P(J)
      DO 2 J=KK,MM
      QQ=FLOAT(KI)
    2 DERY(J)=QQ*L(I)*P(J+1)+R(I)*P(J-1)-(QQ*L(I)+R(I))*P(J)
      PRINT*,'exit derivs'
      RETURN
      END

      SUBROUTINE OUTP(T1,P)
C
C
C THIS SUBROUTINE CALCULATES QUEUE STATISTICS AND OUTPUTS THE RESULTS
C     AT THE END OF EACH OF THE NST TIME PERIODS.
C
C THE VARIABLES USED INCLUDE:
C     1. SUM=THE SUM FROM 1 TO M OF THE STATE PROBABILITIES,P,
C        (SHOULD EQUAL 1.0)
C     2. EQ=E(# IN THE QUEUE) AT TIME = T
C     3. SDQ=STD . DEV. OF # IN THE QUEUE
C     4. EWAIT=E(WAITING TIME), NOT INCLUDING SERVICE TIME
C     5. TWAIT = CUMULATIVE TOTAL EXPECTED WAITING TIME
C     6. RJECTS= APPROXIMATE # OF CUMULATIVE REJECTS
C     7. ES=E(# IN THE SYSTEM) AT T
C     8. SDS=STD DEV OF # IN THE SYSTEM
C     9. PW= PROBABILITY THAT A CUSTOMER WILL BE DELAYED
      DIMENSION P(60)
      REAL L(48)
      COMMON I,R(48),L,M,MM,TIMLIM,DT,RJECTS,TSAV,KI
      COMMON DQ,DQL,TWAIT

C OUTP IS CALLED BY MAIN AT EACH INCREMENT OF INTEGRATION. TO AVOID
C     UNNECESSARY OUTPUT, CONTROL IS RETURNED TO MAIN, UNLESS THE END
C     OF ONE OF THE NST SUB-DIVISIONS OF TIMPER IS REACHED.  IF ADDITI
C     OUTPUT IS DESIRED, THIS 'IF' STATEMENT MAY BE CHANGED ACCORDINGL
C     PRINT*,'enter outp'
      PM=P(M)
      IF(PM.LT.0.)PM=0.
      RJECTS=RJECTS+R(I)*PM*(TIMLIM-T1)
      TSAV=T1
      IF(T1+.001.GT.TIMLIM)RETURN
      SUM=0.0
      PMAX=0.
      MX=0
      DO 4 J=1,M
      IF(P(J).LT.0.)P(J)=0.
      IF(P(J).LT.PMAX)GO TO 4
      PMAX=P(J)
      MX=J
    4 SUM=SUM+P(J)
      IF(SUM.EQ.1.)GO TO 8
      DELT=SUM-1.
```

```
      P(MX)=P(MX)-DELT
    8 CONTINUE
      PPW=0.
      DO 10 J = 1,KI
   10 PPW = PPW + P(J)
      PW =1.- PPW
      KI2=KI+2
      EQ=0.
      VQ=0.
      SDQ=0.
      DO 12 J=KI2,M
      EQ=EQ+FLOAT(J-1-KI)*P(J)
   12 VQ=VQ+( (FLOAT(J-1-KI) )**2)*P(J)
      IF(VQ.LT.EQ**2)GO TO 13
      SDQ=SQRT(VQ-EQ**2)
   13 DQ=EQ
      EWAIT=30.*DT*(DQ+DQL)
      DQL=DQ
      TWAIT=TWAIT+EWAIT
      ES=0.
      VS=0.
      SDS=0.
      DO 14 J=2,M
      ES=ES+FLOAT(J-1)*P(J)
   14 VS=VS+( (FLOAT(J-1) )**2)*P(J)
      IF(VS.LT.ES**2)GO TO 15
      SDS=SQRT(VS-ES**2)
   15 WRITE(6,101) TIMLIM,SUM,EQ,SDQ,ES,SDS,R(I),L(I),MX
      WRITE(6,111) PW,EWAIT,TWAIT,RJECTS
      WRITE(6,102) (J,J=1,14),(P(J),J=1,15)
      WRITE(6,103) (J,J=15,29),(P(J),J=16,30)
      WRITE(6,103) (J,J=30,44),(P(J),J=31,45)
      WRITE(6,103) (J,J=45,59),(P(J),J=46,60)
  101 FORMAT('-',1X,'TIME=',F6.3,3X,'SUM=',F6.3,3X,'EQ=',F7.4,3X,
     1'SDQ=',F7.4,3X,'ES=',F7.4,3X,'SDS=',F7.4,3X,
     2'ARR=',F3.0,3X,'SER=',F3.0,3X,'MX=',I3,/)
  111 FORMAT('-',15X,'PW=',F8.6,3X,'EWAIT=',F6.2,'MIN',3X,'TWAIT=',
     1F8.2,3X,'CUM RJECTS=',F6.2,///)
  102 FORMAT(' ',1X,'STATE # :',9X,'0',14I7,/,1X,'P(N) : ',6X,15F7.2)
  103 FORMAT(' ',1X,'STATE # :',3X,15I7,/,1X,'P(N) :',7X,15F7.2)
      PRINT*,'exit outp'
      RETURN
      END


      SUBROUTINE RKQC(P,DERY,N,T1,H,EPS,PSCAL,HDID,HNEXT)
      PARAMETER (NMAX=60,FCOR=.0666666667,
     *    ONE=1.,SAFETY=0.9,ERRCON=6.E-4)
      DIMENSION P(60),DERY(60),PSCAL(60),PTEMP(NMAX),PSAV(NMAX),
     * DPSAV(NMAX)
      PRINT*,'enter rkqc'
      PGROW=-0.20
      PSHRNK=-0.25
      TSAV=T1
      DO 11 J=1,60
        PSAV(J)=P(J)
        DPSAV(J)=DERY(J)
   11 CONTINUE
    1 HH=0.5*H
```

```
      CALL RK4(PSAV,DPSAV,N,HH,PTEMP)
      T1=TSAV+HH
      CALL DERIVS(PTEMP,DERY)
      CALL RK4(PTEMP,DERY,N,HH,P)
      T1=TSAV+H
      IF( T1.EQ.TSAV )PAUSE  'Stepsize not significant in RKQC.'
      CALL RK4(PSAV,DPSAV,N,H,PTEMP)
      ERRMAX=0.
      DO 12 J=1,60
        PTEMP(J)=P(J)-PTEMP(J)
        ERRMAX=MAX(ERRMAX,ABS(PTEMP(J)/PSCAL(J)))
12    CONTINUE
      ERRMAX=ERRMAX/EPS
      IF(ERRMAX.GT.ONE) THEN
        H=SAFETY*H*(ERRMAX**PSHRNK)
        GOTO 1
      ELSE
        HDID=H
        IF(ERRMAX.GT.ERRCON)THEN
          HNEXT=SAFETY*H*(ERRMAX**PGROW)
        ELSE
          HNEXT=4.*H
        ENDIF
      ENDIF
      DO 13 J=1,60
        P(J)=P(J)+PTEMP(J)*FCOR
13    CONTINUE
      PRINT*,'exit rkqc'
      RETURN
      END

      SUBROUTINE RK4(P,DERY,N,H,POUT)
      PARAMETER (NMAX=60)
      DIMENSION P(60),DERY(60),POUT(60),PT(NMAX),DPT(NMAX),DPM(NMAX)
      PRINT*,'enter rk4'
      HH=H*0.5
      H6=H/6.
      DO 11 J=1,N
        PT(J)=P(J)+HH*DERY(J)
11    CONTINUE
      CALL DERIVS(PT,DPT)
      DO 12 J=1,N
        PT(J)=P(J)+HH*DPT(J)
12    CONTINUE
      CALL DERIVS(PT,DPM)
      DO 13 J=1,N
        PT(J)=P(J)+H*DPM(J)
        DPM(J)=DPT(J)+DPM(J)
13    CONTINUE
      CALL DERIVS(PT,DPT)
      DO 14 J=1,N
        POUT(J)=P(J)+H6*(DERY(J)+DPT(J)+2.*DPM(J))
14    CONTINUE
      PRINT*,'exit rk4'
      RETURN
      END
```

```
 STATE # :        15      16      17      18      19      20      21      22      23
  24      25      26      27      28      29
P(N) :              .00     .00     .00     .00     .00     .00     .00     .00     .00
.00     .00     .00     .00     .00     .00
 STATE # :        30      31      32      33      34      35      36      37      38
  39      40      41      42      43      44
P(N) :              .00     .00     .00     .00     .00     .00     .00     .00     .00
.00     .00     .00     .00     .00     .00
 STATE # :        45      46      47      48      49      50      51      52      53
  54      55      56      57      58      59
P(N) :              .00     .00     .00     .00     .00     .00     .00     .00     .00
.00     .00     .00     .00     .00     .00
- TIME=   .750   SUM= 1.000   EQ=   .327   SDQ=   .9848   ES=   .423   SDS= 1.1905
 ARR= 4.   SER= 3.   MX=  1

-              PW= .02900   EWAIT=   3.6MIN   TWAIT=     4.10   CUM RJECTS=    .
00
```

```
 STATE # :         0       1       2       3       4       5       6       7       8
   9      10      11      12      13      14
P(N) :              .94     .03     .02     .01     .00     .00     .00     .00     .00
.00     .00     .00     .00     .00     .00
 STATE # :        15      16      17      18      19      20      21      22      23
  24      25      26      27      28      29
P(N) :              .00     .00     .00     .00     .00     .00     .00     .00     .00
.00     .00     .00     .00     .00     .00
 STATE # :        30      31      32      33      34      35      36      37      38
  39      40      41      42      43      44
P(N) :              .00     .00     .00     .00     .00     .00     .00     .00     .00
.00     .00     .00     .00     .00     .00
 STATE # :        45      46      47      48      49      50      51      52      53
  54      55      56      57      58      59
P(N) :              .00     .00     .00     .00     .00     .00     .00     .00     .00
.00     .00     .00     .00     .00     .00
- TIME= 1.000   SUM= 1.000   EQ=   .328   SDQ=   .0180   ES=  1.424   SDS= 0.1920
  ARR= 0.   SER= 0.   MX=  1

-              PW= .02300   EWAIT=   3.45MIN   TWAIT=     7.55   CUM RJECTS=    .
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 STATE # :          0       1       2       3       4       5       6       7       8
  9      10      11      12      13      14
P(N) :            .89     .09     .02     .00     .00     .00     .00     .00     .00
.00     .00     .00     .00     .00     .00
 STATE # :         15      16      17      18      19      20      21      22      23
 24      25      26      27      28      29
P(N) :            .00     .00     .00     .00     .00     .00     .00     .00     .00
.00     .00     .00     .00     .00     .00
 STATE # :         30      31      32      33      34      35      36      37      38
 39      40      41      42      43      44
P(N) :            .00     .00     .00     .00     .00     .00     .00     .00     .00
.00     .00     .00     .00     .00     .00
 STATE # :         45      46      47      48      49      50      51      52      53
 54      55      56      57      58      59
P(N) :            .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
- TIME= 1.250   SUM= 1.000   EQ=  .329   SDQ=  .2872   ES= 2.427   SDS= 0.7353
 ARR= 3.   SER= 1.   MX=  1

-              PW= .05100  *EWAIT=   6.89MIN   TWAIT=    14.44   CUM RJECTS=
.00


 STATE # :          0       1       2       3       4       5       6       7       8
  9      10      11      12      13      14
P(N) :            .87     .05     .02     .05     .01     .00     .00     .00     .00
.00     .00     .00     .00     .00     .00
 STATE # :         15      16      17      18      19      20      21      22      23
 24      25      26      27      28      29
P(N) :            .00     .00     .00     .00     .00     .00     .00     .00     .00
.00     .00     .00     .00     .00     .00
 STATE # :         30      31      32      33      34      35      36      37      38
 39      40      41      42      43      44
P(N) :            .00     .00     .00     .00     .00     .00     .00     .00     .00
.00     .00     .00     .00     .00     .00
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 STATE # :        45      46      47      48      49      50      51      52      53
  54      55      56      57      58      59
P(N) :             .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
- TIME= 1.500   SUM= 1.000   EQ=   .330   SDQ=   .0083   ES=   .429   SDS= 0.0149
 ARR= 3.   SER= 6.   MX=  1

-                PW= .01800   EWAIT=    .04MIN   TWAIT=   14.48   CUM RJECTS=    .0
0


 STATE # :         0       1       2       3       4       5       6       7       8
                  11      12      13      14
P(N) :             .90     .03     .02     .01     .02     .00     .01     .01     .00
 .00     .00     .00     .00     .00     .00
 STATE # :        15      16      17      18      19      20      21      22      23
 24      25      26      27      28      29
P(N) :             .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
 STATE # :        30      31      32      33      34      35      36      37      38
 39      40      41      42      43      44
P(N) :             .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
 STATE # :        45      46      47      48      49      50      51      52      53
 54      55      56      57      58      59
P(N) :             .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
- TIME= 1.750   SUM= 1.000   EQ=   .337   SDQ=   .0089   ES=  1.432   SDS= 0.0537
  ARR= 8.   SER= 8.   MX=  1

-                PW= .01100   EWAIT=   3.63MIN   TWAIT=    182.11   CUM RJECTS=
 .00


 STATE # :         0       1       2       3       4       5       6       7       8
  9      10      11      12      13      14
P(N) :             .00     .04     .05     .01     .03     .02     .01     .23     .11
```

```
 .00     .00     .00     .00     .00     .00
  STATE # :         15      16      17      18      19      20      21      22      23
  24      25      26      27      28      29
 P(N) :              .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
  STATE # :         30      31      32      33      34      35      36      37      38
  39      40      41      42      43      44
 P(N) :              .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
  STATE # :         45      46      47      48      49      50      51      52      53
  54      55      56      57      58      59
 P(N) :              .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
- TIME= 2.000   SUM= 1.000   EQ=  1.330   SDQ=  .9894   ES=  4.434   SDS= 1.1964
   ARR= 9.   SER= 6.   MX=  1

-              PW= .07300   EWAIT=   4.20MIN   TWAIT=    22.31   CUM RJECTS=
.00
```

```
  STATE # :         0       1       2       3       4       5       6       7       8
   9      10      11      12      13      14
 P(N) :              .43     .07     .02     .09     .11     .22     .01     .03     .03
 .00     .00     .00     .00     .00     .00
  STATE # :         15      16      17      18      19      20      21      22      23
  24      25      26      27      28      29
 P(N) :              .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
  STATE # :         30      31      32      33      34      35      36      37      38
  39      40      41      42      43      44
 P(N) :              .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
  STATE # :         45      46      47      48      49      50      51      52      53
  54      55      56      57      58      59
 P(N) :              .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
- TIME= 2.250   SUM= 1.000   EQ=  .831   SDQ=  .9899   ES=  2.436   SDS= 1.0131
  ARR= 5.   SER= 4.   MX=  1
```

PW= .007800 EWAIT= 5.31MIN TWAIT= 27.62 CUM RJECTS=
.00

STATE # :	0	1	2	3	4	5	6	7	8
9	10	11	12	13	14				
P(N) :	.79	.13	.02	.02	.01	.02	.01	.00	.00
.00	.00	.00	.00	.00	.00				
STATE # :	15	16	17	18	19	20	21	22	23
24	25	26	27	28	29				
P(N) :	.00	.00	.00	.00	.00	.00	.00	.00	.00
.00	.00	.00	.00	.00	.00				
STATE # :	30	31	32	33	34	35	36	37	38
39	40	41	42	43	44				
P(N) :	.00	.00	.00	.00	.00	.00	.00	.00	.00
.00	.00	.00	.00	.00	.00				
STATE # :	45	46	47	48	49	50	51	52	53
54	55	56	57	58	59				
P(N) :	.00	.00	.00	.00	.00	.00	.00	.00	.00
.00	.00	.00	.00	.00	.00				

TIME= 2.500 SUM= 1.000 EQ= .451 SDQ= .0904 ES= 1.437 SDS= 0.1977
ARR= 3. SER= 4. MX= 1

PW= .10200 **EWAIT= 6.20MIN TWAIT= 33.82** CUM RJECTS=
.00

STATE # :	0	1	2	3	4	5	6	7	8
9	10	11	12	13	14				
P(N) :	.49	.16	.10	.17	.03	.04	.01	.00	.00
.00	.00	.00	.00	.00	.00				
STATE # :	15	16	17	18	19	20	21	22	23
24	25	26	27	28	29				
P(N) :	.00	.00	.00	.00	.00	.00	.00	.00	.00
.00	.00	.00	.00	.00	.00				
STATE # :	30	31	32	33	34	35	36	37	38
39	40	41	42	43	44				
P(N) :	.00	.00	.00	.00	.00	.00	.00	.00	.00

```
 .00     .00     .00     .00     .00     .00
  STATE # :        45      46      47      48      49      50      51      52      53
  54      55      56      57      58      59
 P(N) :             .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
- TIME= 2.750   SUM= 1.000   EQ=  .331   SDQ=  .2210   ES= 1.384   SDS= 0.8814
    = 1.   SER= 3.   MX= 1

-                PW= .06300   EWAIT=   6.10MIN   TWAIT=    39.92   CUM RJECTS=
.00


  STATE # :          0       1       2       3       4       5       6       7       8
   9      10      11      12      13      14
 P(N) :             .36     .20     .13     .19     .12     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
  STATE # :         15      16      17      18      19      20      21      22      23
 24      25      26      27      28      29
 P(N) :             .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
  STATE # :         30      31      32      33      34      35      36      37      38
  39      40      41      42      43      44
 P(N) :             .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
  STATE # :         45      46      47      48      49      50      51      52      53
  54      55      56      57      58      59
 P(N) :             .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
- TIME= 3.000   SUM= 1.000   EQ=  .332   SDQ=  .7915   ES= 1.934   SDS= 1.0990
 ARR= 2.   SER= 3.   MX= 1

-                PW= .09215   EWAIT=   6.60MIN   TWAIT=    46.52   CUM RJECTS=
.00


  STATE # :          0       1       2       3       4       5       6       7       8
    9      10      11      12      13      14
```

P(N) :		.29	.09	.18	.13	.09	.11	.04	.07	.00	.00
.00	.00	.00	.00	.00	.00						
STATE # :		15	16	17	18	19	20	21	22	23	24
25	26	27	28	29							
P(N) :		.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
.00	.00	.00	.00	.00	.00						
STATE # :		30	31	32	33	34	35	36	37	38	39
40	41	42	43	44							
P(N) :		.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
.00	.00	.00	.00	.00	.00						
STATE # :		45	46	47	48	49	50	51	52	53	54
55	56	57	58	59							
P(N) :		.00	.00	.00	.00	.00	.00	.00	.00	.00	

MINIBUS MODE

RESULTS FOR THE TRUNCATED KI SERVER QUEUEING SYS
TIME VARYING POISSON ARRIVALS AND EXPONENTIAL SERVICE
MAXIMUM QUEUE SIZE= 50 NUMBER OF SERVERS= 12

TIME= .250 SUM= 1.000 EQ= .0122 SDQ= .5599 ES= .0210 SDS= .8410
ARR= 0. SER= 0. MX= 1

PW= .00000 EWAIT= .09MIN TWAIT= .09 CUM RJECTS= .0
0

STATE # :	0	1	2	3	4	5	6	7	8
9	10	11	12	13	14				
P(N) :	1.00	.00	.00	.00	.00	.00	.00	.00	.00
.00	.00	.00	.00	.00	.00				

STATE # :	15	16	17	18	19	20	21	22	23
24	25	26	27	28	29				
P(N) :	.00	.00	.00	.00	.00	.00	.00	.00	.00
.00	.00	.00	.00	.00	.00				

STATE # :	30	31	32	33	34	35	36	37	38
39	40	41	42	43	44				
P(N) :	.00	.00	.00	.00	.00	.00	.00	.00	.00
.00	.00	.00	.00	.00	.00				

STATE # :	45	46	47	48	49	50	51	52	53
54	55	56	57	58	59				
P(N) :	.00	.00	.00	.00	.00	.00	.00	.00	.00
				00					

TIME= .500 SUM= 1.00 EQ= .0244 SDQ= .7916 ES= .0420 SDS= 1.1890
ARR= 0. SER= 0. MX= 1

PW= .00641 EWAIT= .27MIN TWAIT= .37 CUM RJECTS= .00

STATE # :	0	1	2	3	4	5	6	7	8
9	10	11	12	13	14				
P(N) :	.99	.01	.00	.00	.00	.00	.00	.00	.00
.00	.00	.00	.00	.00	.00				

STATE # :	15	16	17	18	19	20	21	22	23
24	25	26	27	28	29				

```
P(N) :          .00     .00     .00     .00     .00     .00     .00     .00     .00
.00     .00     .00     .00     .00     .00
 STATE # :        30      31      32      33      34      35      36      37      38
 39      40      41      42      43      44
P(N) :          .00     .00     .00     .00     .00     .00     .00     .00     .00
.00     .00     .00     .00     .00     .00
 STATE # :        45      46      47      48      49      50      51      52      53
 54      55      56      57      58      59
P(N) :          .00     .00     .00     .00     .00     .00     .00     .00     .00
.00     .00     .00     .00     .00     .00
- TIME=   .750   SUM= 1.000   EQ=   .0367   SDQ=   .9692   ES=   .0631   SDS= 1.4558
   ARR= 0.   SER= 0.   MX=  1

-              PW= .00927   EWAIT=    .46MIN   TWAIT=      .82   CUM RJECTS=    .00


 STATE # :        0       1       2       3       4       5       6       7       8
  9      10      11      12      13      14
 .00     .00     .00     .00     .00     .00
 STATE # :       15      16      17      18      19      20      21      22      23
 24      25      26      27      28      29
P(N) :          .00     .00     .00     .00     .00     .00     .00     .00     .00
.00     .00     .00     .00     .00     .00
 STATE # :        30      31      32      33      34      35      36      37      38
 39      40      41      42      43      44
P(N) :          .00     .00     .00     .00     .00     .00     .00     .00     .00
.00     .00     .00     .00     .00     .00
 STATE # :        45      46      47      48      49      50      51      52      53
 54      55      56      57      58      59
P(N) :          .00     .00     .00     .00     .00     .00     .00     .00     .00
.00     .00     .00     .00     .00     .00
- TIME= 1.000   SUM= 1.000   EQ=   .0367   SDQ=   .9700   ES=   .0634   SDS= 1.4570
   ARR= 4.   SER= 3.   MX=  1

-              PW= .00811   EWAIT=    .55MIN   TWAIT=     1.37   CUM RJECTS=    .00


 STATE # :        0       1       2       3       4       5       6       7       8
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```
  9     10      11      12      13      14
P(N) :          .75     .13     .05     .03     .03     .01     .00     .00     .00
.00     .00     .00     .00     .00     .00
 STATE # :        15      16      17      18      19      20      21      22      23
 24      25      26      27      28      29
P(N) :          .00     .00     .00     .00     .00     .00     .00     .00     .00
.00     .00     .00     .00     .00     .00
 STATE # :        30      31      32      33      34      35      36      37      38
 39      40      41      42      43      44
P(N) :          .00     .00     .00     .00     .00     .00     .00     .00     .00
.00     .00     .00     .00     .00     .00
 STATE # :        45      46      47      48      49      50      51      52      53
 54      55      56      57      58      59
P(N) :          .00     .00     .00     .00     .00     .00     .00     .00     .00
.00     .00     .00     .00     .00     .00
- TIME= 1.250   SUM= 1.000   EQ=   .0368   SDQ=   .9707   ES=   .0636   SDS= 1.4582
   ARR= 3.   SER= 1.   MX=  1

-              PW= .01936   EWAIT=    .55MIN   TWAIT=    1.93   CUM RJECTS=    .00
```

```
 STATE # :        0       1       2       3       4       5       6       7       8
  9     10      11      12      13      14
P(N) :          .57     .15     .12     .13     .02     .01     .00     .00     .00
.00     .00     .00     .00     .00     .00
 STATE # :        15      16      17      18      19      20      21      22      23
 24      25      26      27      28      29
P(N) :          .00     .00     .00     .00     .00     .00     .00     .00     .00
.00     .00     .00     .00     .00     .00
 STATE # :        30      31      32      33      34      35      36      37      38
 39      40      41      42      43      44
P(N) :          .00     .00     .00     .00     .00     .00     .00     .00     .00
.00     .00     .00     .00     .00     .00
 STATE # :        45      46      47      48      49      50      51      52      53
 54      55
P(N) :          .00     .00     .00     .00     .00     .00     .00     .00     .00
.00     .00     .00     .00     .00     .00
- TIME= 1.500   SUM= 1.000   EQ=   .0368   SDQ=   .9714   ES=   .0639   SDS= 1.4593
   ARR= 3.   SER= 5.   MX=  1
```

PW= .01938 EWAIT= .55MIN TWAIT= 2.48 CUM RJECTS= .00

STATE # :	0	1	2	3	4	5	6	7	8
9	10	11	12	13	14				
P(N) :		.41	.17	.10	.21	.11	.00	.00	.00
.00	.00	.00	.00	.00	.00				
STATE # :		15	16	17	18	19	20	21	22	23
24	25	26	27	28	29				
P(N) :		.00	.00	.00	.00	.00	.00	.00	.00	.00
.00	.00	.00	.00	.00	.00				
STATE # :		30	31	32	33	34	35	36	37	38
39	40	41	42	43	44				
P(N) :		.00	.00	.00	.00	.00	.00	.00	.00	.00
.00	.00	.00	.00	.00	.00				
STATE # :		45	46	47	48	49	50	51	52	53
54	55	56	57	58	59					
P(N) :		.00	.00	.00	.00	.00	.00	.00	.00	.00
.00	.00	.00	.00	.00	.00					

TIME= 1.750 SUM= 1.000 EQ= .0369 SDQ= .9722 ES= .0641 SDS= 1.4604
ARR= 3. SER= 3. MX= 1

PW= .01941 EWAIT= .55MIN TWAIT= 3.03 CUM RJECTS= .00

STATE # :	0	1	2	3	4	5	6	7	8
9	10	11	12	13	14				
P(N) :		.21	.17	.13	.20	.12	.09	.05	.03	.00
.00	.00	.00	.00	.00	.00				
STATE # :		15	16	17	18	19	20	21	22	23
24	25	26	27	28	29				
P(N) :		.00	.00	.00	.00	.00	.00	.00	.00	.00
.00	.00	.00	.00	.00	.00				
STATE # :		30	31	32	33	34	35	36	37	38
39	40	41	42	43	44				
P(N) :		.00	.00	.00	.00	.00	.00	.00	.00	.00
.00	.00	.00	.00	.00	.00				
STATE # :		45	46	47	48	49	50	51	52	53

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 54      55      56      57      58      59
P(N) :          .00     .00     .00     .00     .00     .00     .00     .00     .00
.00     .00     .00     .00     .00     .00
- TIME= 2.000   SUM= 1.000   EQ=  .0369   SDQ=  .9729   ES=  .0643   SDS= 1.4616
   ARR= 3.   SER= 2.   MX=  1

-              PW= .01945   EWAIT=   .55MIN   TWAIT=    3.58   CUM RJECTS=    .00


 STATE # :          0       1       2       3       4       5       6       7       8
  9      10      11      12      13      14
P(N) :          .11     .19     .12     .09     .23     .08     .13     .05     .00
.00     .00     .00     .00     .00     .00
                                                 19      20      21      22      23
 24      25      26      27      28      29
P(N) :          .00     .00     .00     .00     .00     .00     .00     .00     .00
.00     .00     .00     .00     .00     .00
 STATE # :         30      31      32      33      34      35      36      37      38
 39      40      41      42      43      44
P(N) :          .00     .00     .00     .00     .00     .00     .00     .00     .00
.00     .00     .00     .00     .00     .00
 STATE # :         45      46      47      48      49      50      51      52      53
 54      55      56      57      58      59
P(N) :          .00     .00     .00     .00     .00     .00     .00     .00     .00
.00     .00     .00     .00     .00     .00
- TIME= 2.250   SUM= 1.000   EQ=  .0370   SDQ=  .9736   ES=  .0646   SDS= 1.4627
   ARR= 4.   SER= 5.   MX=  5

-              PW= .01947   EWAIT=   .55MIN   TWAIT=    4.14   CUM RJECTS=    .00


 STATE # :          0       1       2       3       4       5       6       7       8
  9      10      11      12      13      14
P(N) :          .00     .00     .00     .12     .11     .17     .26     .32     .02
.00     .00     .00     .00     .00     .00
 STATE # :         15      16      17      18      19      20      21      22      23
 24      25      26      27      28      29
P(N) :          .00     .00     .00     .00     .00     .00     .00     .00     .00
.00     .00     .00     .00     .00     .00
```

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  STATE # :        30      31      32      33      34      35      36      37      38
   39      40      41      42      43      44
                                                                                    00
  .00     .00     .00     .00     .00     .00
  STATE # :        45      46      47      48      49      50      51      52      53
   54      55      56      57      58      59
 P(N) :           .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
- TIME= 2.500   SUM= 1.000   EQ=  .0370   SDQ=  .9741   ES=  .0649   SDS= 1.4636
   ARR= 6.   SER= 4.   MX=  8

-             PW= .01950   EWAIT=   .56MIN   TWAIT=    4.69   CUM RJECTS=    .00


  STATE # :        0       1       2       3       4       5       6       7       8
   9      10      11      12      13      14
 P(N) :           .07     .23     .14     .10     .19     .20     .04     .05     .00
 .00     .00     .00     .00     .00     .00
  STATE # :       15      16      17      18      19      20      21      22      23
   24      25      26      27      28      29
 P(N) :           .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
  STATE # :       30      31      32      33      34      35      36      37      38
   39      40      41      42      43      44
 P(N) :           .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
  STATE # :       45      46      47      48      49      50      51      52      53
   54      55      56      57      58      59
 P(N) :           .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
   ARR= 2.   SER= 4.   MX=  2

-             PW= .01952   EWAIT=   .56MIN   TWAIT=    5.25   CUM RJECTS=    .00


  STATE # :        0       1       2       3       4       5       6       7       8
   9      10      11      12      13      14
 P(N) :           .00     .02     .03     .05     .11     .30     .08     .01     .19
```

```
 .21     .00     .00     .00     .00     .00
  STATE # :        15      16      17      18      19      20      21      22      23
  24      25      26      27      28      29
P(N) :              .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
  STATE # :        30      31      32      33      34      35      36      37      38
  39      40      41      42      43      44
P(N) :              .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
  STATE # :        45      46      47      48      49      50      51      52      53
  54      55      56      57      58      59
P(N) :              .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
- TIME= 3.000   SUM= 1.000   EQ=  .0371   SDQ=  .9750   ES=  .0653   SDS= 1.4651
   ARR= 8.   SER= 5.   MX=  6

-              PW= .01954   EWAIT=   .56MIN   TWAIT=    5.81   CUM RJECTS=    .00
```

```
  STATE # :        0       1       2       3       4       5       6       7       8
   9      10      11      12      13      14
                .00     .00     .00     .00     .00     .00     .00     .03     .07
 .05     .05     .33     .21     .22     .04
  STATE # :        15      16      17      18      19      20      21      22      23
  24      25      26      27      28      29
P(N) :              .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
  STATE # :        30      31      32      33      34      35      36      37      38
  39      40      41      42      43      44
P(N) :              .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
  STATE # :        45      46      47      48      49      50      51      52      53
  54      55      56      57      58      59
P(N) :              .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
```

UNIVERSITY BUS MODE

RESULTS FOR THE TRUNCATED KI SERVER QUEUEING SYS

TIME VARYING POISSON ARRIVALS AND EXPONENTIAL SERVICE

MAXIMUM QUEUE SIZE= 50 NUMBER OF SERVERS= 25

TIME= .250 SUM= 1.000 EQ= .154 SDQ= .3018 ES= .210 SDS= .8410
ARR= 0. SER= 0. MX= 1

PW= .00000 EWAIT= .0MIN TWAIT= .0 CUM RJECTS= .00

STATE # :	0	1	2	3	4	5	6	7	8
9	10	11	12	13	14				
P(N) :	1.00	.00	.00	.00	.00	.00	.00	.00	.00
.00	.00	.00	.00	.00	.00				
STATE # :	15	16	17	18	19	20	21	22	23
24	25	26	27	28	29				
P(N) :	.00	.00	.00	.00	.00	.00	.00	.00	.00
.00	.00	.00	.00	.00	.00				
STATE # :	30	31	32	33	34	35	36	37	38
39	40	41	42	43	44				
P(N) :	.00	.00	.00	.00	.00	.00	.00	.00	.00
.00	.00	.00	.00	.00	.00				
STATE # :	45	46	47	48	49	50	51	52	53
54	55	56	57	58	59				
P(N) :	.00	.00	.00	.00	.00	.00	.00	.00	.00
.00	.00	.00	.00	.00	.00				

TIME= .500 SUM= 1.000 EQ= .254 SDQ= .3022 ES= 2.613 SDS= .8424
ARR= 6. SER= 6. MX= 1

PW= .00000 EWAIT= .4MIN TWAIT= .4 CUM RJECTS= .00

STATE # :	0	1	2	3	4	5	6	7	8
9	10	11	12	13	14				
P(N) :	.99	.00	.01	.00	.00	.00	.00	.00	.00
.00	.00	.00	.00	.00	.00				
[illegible]		[illegible]		17	18	19	20	21	22	23
24	25	26	27	28	29				
P(N) :	.00	.00	.00	.00	.00	.00	.00	.00	.00

```
 .00     .00     .00     .00     .00     .00
  STATE # :        30      31      32      33      34      35      36      37      38
  39      40      41      42      43      44
 P(N) :             .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
  STATE # :        45      46      47      48      49      50      51      52      53
  54      55      56      57      58      59
 P(N) :             .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
- TIME=   .750   SUM= 1.000   EQ= 1.654   SDQ=  .3025   ES=  3.215   SDS=  1.974
  ARR= 9.   SER= 7.   MX=  1

-              PW= .00000   EWAIT=   .8MIN   TWAIT=     1.2   CUM RJECTS=    .00


  STATE # :        0       1       2       3       4       5       6       7       8
   9      10      11      12      13      14
 P(N) :             .00     .00     .00     .01     .17     .08     .11     .40     .00
 .00     .00     .00     .00     .00     .00
  STATE # :        15      16      17      18      19      20      21      22      23
  24      25      26      27      28      29
 P(N) :             .00     .00     .00     .00     .00     .00     .00     .00     .17
 .06     .00     .00     .00     .00     .00
  STATE # :        30      31      32      33      34      35      36      37      38
  39      40      41      42      43      44
 P(N) :             .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
  STATE # :        45      46      47      48      49      50      51      52      53
  54      55      56      57      58      59
 P(N) :             .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
- TIME= 1.000   SUM= 1.000   EQ=  1.054   SDQ=  .3027   ES= 3.821   SDS= 1.993
 ARR=38.   SER=38.   MX=  8

-              PW= .00000   EWAIT=   .8MIN   TWAIT=     2.0   CUM RJECTS=    .00


  STATE # :        0       1       2       3       4       5       6       7       8
```

```
  9     10     11     12     13     14
P(N) :          .00    .00    .00    .00    .00    .00    .09    .07    .03
.11    .03    .04    .02    .01    .00
 STATE # :       15     16     17     18     19     20     21     22     23
 24     25     26     27     28     29
P(N) :          .00    .00    .00    .00    .00    .00    .00    .00    .00
.00    .00    .00    .00    .00    .00
 STATE # :       30     31     32     33     34     35     36     37     38
 39     40     41     42     43     44
P(N) :          .00    .00    .00    .00    .00    .00    .00    .00    .00
.00    .00    .00    .00    .00    .00
 STATE # :       45     46     47     48     49     50     51     52     53
 54     55     56     57     58     59
P(N) :          .00    .00    .00    .00    .00    .00    .00    .00    .00
.00    .00    .00    .00    .00    .00
- TIME= 1.250   SUM= 1.000   EQ=  2.054   SDQ=  1.3025   ES= 4.418   SDS=  2.124
   ARR= 8.   SER= 8.   MX=  1

-              PW= .00000   EWAIT=   .8MIN   TWAIT=     3.6   CUM RJECTS=    .00
```

```
 STATE # :       0      1      2      3      4      5      6      7      8
  9     10     11     12     13     14
P(N) :          .00    .94    .01    .03    .01    .01    .00    .00    .00
.00    .00    .00    .00    .00    .00
 STATE # :       15     16     17     18     19     20     21     22     23
 24     25     26     27     28     29
P(N) :          .00    .00    .00    .00    .00    .00    .00    .00    .00
.00    .00    .00    .00    .00    .00
 STATE # :       30     31     32     33     34     35     36     37     38
 39     40     41     42     43     44
P(N) :          .00    .00    .00    .00    .00    .00    .00    .00    .00
.00    .00    .00    .00    .00    .00
 STATE # :       45     46     47     48     49     50     51     52     53
 54     55     56     57     58     59
P(N) :          .00    .00    .00    .00    .00    .00    .00    .00    .00
.00    .00    .00    .00    .00    .00
- TIME= 1.500   SUM= 1.000   EQ=  1.054   SDQ=  .3026   ES= 4.920   SDS=  2.524
   ARR=26.   SER=21.   MX=  2
```

PW= .07700 EWAIT= 6.0MIN TWAIT= 9.6 CUM RJECTS= .00

STATE # :	0	1	2	3	4	5	6	7	8
9	10	11	12	13	14				
P(N) :		.00	.04	.02	.94	.00	.00	.00	.00	.00
.00	.00	.00	.00	.00	.00					
STATE # :		15	16	17	18	19	20	21	22	23
24	25	26	27	28	29					
P(N) :		.00	.00	.00	.00	.00	.00	.00	.00	.00
.00	.00	.00	.00	.00	.00					
STATE # :		30	31	32	33	34	35	36	37	38
39	40	41	42	43	44					
P(N) :		.00	.00	.00	.00	.00	.00	.00	.00	.00
.00	.00	.00	.00	.00	.00					
STATE # :		45	46	47	48	49	50	51	52	53
54	55	56	57	58	59					
P(N) :		.00	.00	.00	.00	.00	.00	.00	.00	.00
.00	.00	.00	.00	.00	.00					

TIME= 1.750 SUM= 1.000 EQ= 1.054 SDQ= .4325 ES= 5.722 SDS= 2.684
ARR=55. SER=52. MX= 4

PW= .09800 EWAIT= 8.0MIN TWAIT= 17.6 CUM RJECTS= .00

STATE # :	0	1	2	3	4	5	6	7	8
9	10	11	12	13	14				
P(N) :		.00	.01	.11	.00	.88	.00	.00	.00	.00
.00	.00	.00	.00	.00	.00					
STATE # :		15	16	17	18	19	20	21	22	23
24	25	26	27	28	29					
P(N) :		.00	.00	.00	.00	.00	.00	.00	.00	.00
.00	.00	.00	.00	.00	.00					
STATE # :		30	31	32	33	34	35	36	37	38
39	40	41	42	43	44					
P(N) :		.00	.00	.00	.00	.00	.00	.00	.00	.00
.00	.00	.00	.00	.00	.00					
STATE # :		45	46	47	48	49	50	51	52	53
54	55	56	57	58	59					

```
 P(N) :            .00    .00    .00    .00    .00    .00    .00    .00    .00
 .00    .00    .00    .00    .00    .00
- TIME= 2.000   SUM= 1.000   EQ=  2.214   SDQ=  6.325   ES=  8.824   SDS=  3.018
4   ARR=49.   SER=42.   MX=  5

-              PW= .08100   EWAIT=  13.0MIN   TWAIT=    30.6   CUM RJECTS=    .
00


  STATE # :        0      1      2      3      4      5      6      7      8
   9     10     11     12     13     14
 P(N) :            .00    .10    .00    .18    .00    .10    .55    .00    .00
 .07    .00    .00    .00    .00    .00
  STATE # :       15     16     17     18     19     20     21     22     23
  24     25     26     27     28     29
 P(N) :            .00    .00    .00    .00    .00    .00    .00    .00    .00
 .00    .00    .00    .00    .00    .00
  STATE # :       30     31     32     33     34     35     36     37     38
  39     40     41     42     43     44
 P(N) :            .00    .00    .00    .00    .00    .00    .00    .00    .00
 .00    .00    .00    .00    .00    .00
  STATE # :       45     46     47     48     49     50     51     52     53
                  56
 P(N) :            .00    .00    .00    .00    .00    .00    .00    .00    .00
 .00    .00    .00    .00    .00    .00
- TIME= 2.250   SUM= 1.000   EQ=  1.053   SDQ=  .7025   ES=  2.215   SDS=  1.236
   ARR=47.   SER=50.   MX=  7

-              PW= .09700   EWAIT=  7.0MIN   TWAIT=     37.6   CUM RJECTS=   .0
1


  STATE # :        0      1      2      3      4      5      6      7      8
   9     10     11     12     13     14
 P(N) :            .00    .05    .06    .02    .00    .00    .00    .00    .00
 .00    .00    .00    .00    .00    .00
  STATE # :       15     16     17     18     19     20     21     22     23
  24     25     26     27     28     29
 P(N) :            .00    .00    .00    .00    .00    .00    .00    .00    .00
```

```
 .00     .00     .00     .00     .00     .00
  STATE # :         30      31      32      33      34      35      36      37      38
  39      40      41      42      43      44
 P(N) :              .00     .87     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
  STATE # :         45      46      47      48      49      50      51      52      53
  54      55      56      57      58      59
 P(N) :              .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
- TIME= 2.500   SUM= 1.000   EQ=  1.112   SDQ=  .4125   ES=  5.017   SDS=  1.424
   ARR=38.   SER=34.   MX=  32

-              PW= .06400   EWAIT=  6.0MIN   TWAIT=    43.6   CUM RJECTS=    .01


  STATE # :          0       1       2       3       4       5       6       7       8
   9      10      11      12      13      14
 P(N) :              .00     .00     .00     .00     .10     .02     .03     .04     .07
 .17     .00     .00     .00     .00     .00
  STATE # :         15      16      17      18      19      20      21      22      23
  24      25      26      27      28      29
 P(N) :              .00     .11     .10     .09     .22     .05     .00     .00     .00
 .00     .00     .00     .00     .00     .00
  STATE # :         30      31      32      33      34      35      36      37      38
  39      40      41      42      43      44
 P(N) :              .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
  STATE # :         45      46      47      48      49      50      51      52      53
  54      55      56      57      58      59
 P(N) :              .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
- TIME= 2.750   SUM= 1.000   EQ=  .054   SDQ=  .3025   ES=  .228   SDS=  .8449
 ARR=37.   SER=21.   MX=  20

-              PW= .05300   EWAIT=  6.0MIN   TWAIT=     49.6   CUM RJECTS=    .0
1


  STATE # :          0       1       2       3       4       5       6       7       8
```

```
  9     10      11      12      13      14
P(N) :              .00     .00     .00     .01     .00     .00     .00     .00     .00

  STATE # :         15      16      17      18      19      20      21      22      23
  24      25      26      27      28      29
P(N) :              .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .47     .00     .00     .00     .00
  STATE # :         30      31      32      33      34      35      36      37      38
  39      40      41      42      43      44
P(N) :              .00     .00     .00     .00     .00     .00     .33     .00     .00
 .00     .00     .00     .00     .00     .00
  STATE # :         45      46      47      48      49      50      51      52      53
  54      55      56      57      58      59
P(N) :              .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
- TIME= 3.000   SUM= 1.000   EQ=  2.054   SDQ=  .6225   ES=  7.230   SDS=  1.885
   ARR=56.   SER=52.   MX=  26

-              PW= .04800   EWAIT=   17.5MIN   TWAIT=     67.1   CUM RJECTS=
 01

  STATE # :         0       1       2       3       4       5       6       7       8
   9     10      11      12      13      14
P(N) :              .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
  STATE # :         15      16      17      18      19      20      21      22      23
  24      25      26      27      28      29
P(N) :              .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
  STATE # :         30      31      32      33      34      35      36      37      38
  39      40      41      42      43      44
P(N) :              .18     .14     .08     .10     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
  STATE # :         45      46      47      48      49      50      51      52      53
  54      55      56      57      58      59
P(N) :              .05     .09     .10     .12     .14     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
```

APPENDIX B

UNIVERSITY BUS MODE

Regression Output

Constant	16.6843
Std Err of Y Est	3.6886
R squared	0.87612
No of Observations	6
Degree of Freedom	4
X Coefficients(s)	0.02634
Std Err of Coef.	0.00495

MINIBUS MODE.

Regression Output

Constant	-0.6521
Std Err of Y Est	2.79073
R Square	0.95275
No.of Observations	6
Degree of Freedom	4
X Coefficients	0.08555
Std Err of Coef.	0.00953

TAXI MODE

Regression Output

Constant	71.817
Std Err of Y Est	14.9505
R Squared	0.75832
No.of Observations	6
Degree of Freedom	4
X Coefficient(s)	0.34617
Std Err of Coef.	0.09771

APPENDIX C-- SAMPLE OUTPUT

```
-     RESULTS FOR T-E TRUNCATED KI BATCH QUEUEING SYS
          TIME VARYING POISSON ARRIVALS AND EXPONENTIAL SERVICE
               MAXIMUM QUEUE SIZE=  50     NUMBER OF SERVERS=   1
- TIME=   .250   SUM= 1.000   EQ= 0.0138   SDQ= 0.1307   ES=  0.1249   SDS=  0.37
47   ARR= 5  SER= 45  MX=  1

-              PW=0.111058   EWAIT=    .10MIN   TWAIT=      .10   CUM RJECTS=   .0
0

  STATE # :          0      1      2      3      4      5      6      7      8
   9     10      11     12     13     14
 P(N) :             .89    .10    .01    .00    .00    .00    .00    .00    .00
 .00    .00     .00    .00    .00    .00
  STATE # :         15     16     17     18     19     20     21     22     23
  24     25      26     27     28     29
 P(N) :             .00    .00    .00    .00    .00    .00    .00    .00    .00
 .00    .00     .00    .00    .00    .00
  STATE # :         30     31     32     33     34     35     36     37     38
  39     40      41     42     43     44
 P(N) :             .00    .00    .00    .00    .00    .00    .00    .00    .00
 .00    .00     .00    .00    .00    .00
  STATE # :         45     46     47     48     49     50     51     52     53
  54     55      56     57     58     59
 P(N) :             .00    .00    .00    .00    .00    .00    .00    .00    .00
 .00    .00     .00    .00    .00    .00
- TIME=   .500   SUM= 1.000   EQ=  0.0139   SDQ=  0.1310   ES=  0.1250   SDS= 0.3
750   ARR= 5  SER= 45  MX=  1

-              PW= .111110   EWAIT=    .21MIN   TWAIT=      .31   CUM RJECTS=    .
00


  STATE # :          0      1      2      3      4      5      6      7      8
   9     10      11     12     13     14
```

```
 P(N) :            .89     .10     .01     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
  STATE # :          15      16      17      18      19      20      21      22      23
  24      25      26      27      28      29
 P(N) :            .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
  STATE # :          30      31      32      33      34      35      36      37      38
  39      40      41      42      43      44
 P(N) :            .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
  STATE # :          45      46      47      48      49      50      51      52      53
  54      55      56      57      58      59
 P(N) :            .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
- TIME=   .750   SUM= 1.000   EQ=  0.0139   SDQ=  0.1310   ES=  0.1250   SDS= 0.3
750   ARR= 5  SER= 45  MX=  1

-              PW= .111111   EWAIT=    0.21MIN   TWAIT=      0.52   CUM RJECTS=
 .00


  STATE # :          0       1       2       3       4       5       6       7       8
   9      10      11      12      13      14
 P(N) :            .89     .10     .01     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
  STATE # :          15      16      17      18      19      20      21      22      23
  24      25      26      27      28      29
 P(N) :            .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
  STATE # :          30      31      32      33      34      35      36      37      38
  39      40      41      42      43      44
 P(N) :            .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
  STATE # :          45      46      47      48      49      50      51      52      53
  54      55      56      57      58      59
 P(N) :            .00     .00     .00     .00     .00     .00     .00     .00     .00
 .00     .00     .00     .00     .00     .00
- TIME= 1.000   SUM= 1.000   EQ=  0.0139   SDQ=  0.01310   ES=  0.1250   SDS= 0.
```

APPENDIX C-- KOOPMANS RESULTS

RESULTS FOR THE TRUNCATED KI BATCH QUEUEING SYS
TIME VARYING POISSON ARRIVALS AND EXPONENTIAL SERVICE
MAXIMUM QUEUE SIZE= 50 NUMBER OF SERVERS= 1

- TIME= .250 SUM= 1.000 EQ= 0.0138 SDQ= 0.1307 ES= 0.1249 SDS= 0.37
47 ARR= 5 SER= 45 MX= 1

- PW=0.111058 EWAIT= .10MIN TWAIT= .10 CUM RJECTS= .0
0

STATE # :	0	1	2	3	4	5	6	7	8
9	10	11	12	13	14				
P(N) :	.89	.10	.01	.00	.00	.00	.00	.00	.00
.00	.00	.00	.00	.00	.00				
STATE # :	15	16	17	18	19	20	21	22	23
24	25	26	27	28	29				
P(N) :	.00	.00	.00	.00	.00	.00	.00	.00	.00
.00	.00	.00	.00	.00	.00				
STATE # :	30	31	32	33	34	35	36	37	38
39	40	41	42	43	44				
P(N) :	.00	.00	.00	.00	.00	.00	.00	.00	.00
.00	.00	.00	.00	.00	.00				
STATE # :	45	46	47	48	49	50	51	52	53
54	55	56	57	58	59				
P(N) :	.00	.00	.00	.00	.00	.00	.00	.00	.00
.00	.00	.00	.00	.00	.00				

- TIME= .500 SUM= 1.000 EQ= 0.0139 SDQ= 0.1310 ES= 0.1250 SDS= 0.3
750 ARR= 5 SER= 45 MX= 1

- PW= .111110 EWAIT= .21MIN TWAIT= .31 CUM RJECTS= .
00

STATE # :	0	1	2	3	4	5	6	7	8
9	10	11	12	13	14				

```
 P(N) :                .89      .10      .01      .00      .00      .00      .00      .00      .00
 .00      .00      .00      .00      .00      .00
  STATE # :             15       16       17       18       19       20       21       22       23
  24       25       26       27       28       29
 P(N) :                .00      .00      .00      .00      .00      .00      .00      .00      .00
 .00      .00      .00      .00      .00      .00
  STATE # :             30       31       32       33       34       35       36       37       38
  39       40       41       42       43       44
 P(N) :                .00      .00      .00      .00      .00      .00      .00      .00      .00
 .00      .00      .00      .00      .00      .00
  STATE # :             45       46       47       48       49       50       51       52       53
  54       55       56       57       58       59
 P(N) :                .00      .00      .00      .00      .00      .00      .00      .00      .00
 .00      .00      .00      .00      .00      .00
- TIME=   .750   SUM= 1.000   EQ=   0.0139   SDQ=  0.1310   ES=  0.1250   SDS= 0.3
750   ARR= 5  SER= 45  MX=  1
-                FW= .111111   EWAIT=    0.21MIN   TWAIT=      0.52   CUM RJECTS=
 .00

  STATE # :             0        1        2        3        4        5        6        7        8
   9       10       11       12       13       14
 P(N) :                .89      .10      .01      .00      .00      .00      .00      .00      .00
 .00      .00      .00      .00      .00      .00
  STATE # :             15       16       17       18       19       20       21       22       23
  24       25       26       27       28       29
 P(N) :                .00      .00      .00      .00      .00      .00      .00      .00      .00
 .00      .00      .00      .00      .00      .00
  STATE # :             30       31       32       33       34       35       36       37       38
  39       40       41       42       43       44
 P(N) :                .00      .00      .00      .00      .00      .00      .00      .00      .00
 .00      .00      .00      .00      .00      .00
  STATE # :             45       46       47       48       49       50       51       52       53
  54       55       56       57       58       59
 P(N) :                .00      .00      .00      .00      .00      .00      .00      .00      .00
 .00      .00      .00      .00      .00      .00
```