IMPACT OF COMPUTER MEDIATED SYSTEMS TEACHING APPROACH ON MATHEMATICS PERFORMANCE, ATTITUDE AND ANXIETY OF NIGERIAN POLYTECHNIC ENGINEERING STUDENTS

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AUGUST, 2014.

DECLARATION

I, Iliyasu YUSUF, declare that the work in this dissertation entitled Impact of Computer Mediated Systems Teaching Approach (CMSTA) on Mathematics Performance, Attitude and Anxiety of Nigerian Polytechnic Engineering Students has been carried out by me in the Department of Science Education. The information derived from the literature has been duly acknowledged in the text and a list of references provided. No part of this thesis was previously presented for another degree or diploma at this or any institution.

Iliyasu Yusuf		**28 August, 2014**
Name of Student	Signature	Date

CERTIFICATION

This dissertation entitled IMPACT OF COMPUTER MEDIATED SYSTEMS TEACHING APPROACH (CMSTA) ON MATHEMATICS PERFORMANCE, ATTITUDE AND ANXIETY OF NIGERIAN POLYTECHNIC ENGINEERING STUDENTS by Iliyasu YUSUF meets the regulations governing award of the degree of Doctor of Philosophy in Mathematics Education of the Ahmadu Bello University, and is approved for its contribution to knowledge and literary presentation.

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DEDICATION

This dissertation is dedicated to my late mother, Hajiya Habiba Yusuf, who tried all within her powers and means to see that I have the best in life. May Allah (SWT) reward her with Jannatul Firdaus. Ameen.

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ABSTRACT

This study investigated the impact of computer mediated systems teaching approach (CMSTA), used to teach mathematics to Nigerian Polytechnic engineering students, on performance, attitude and anxiety. Specifically, the study determined whether the mean mathematics performance scores, attitude towards mathematics dispositions and mathematics anxiety levels of (a) experimental and control groups differed significantly and (b) experimental group differed significantly gender wise. Six research questions and six null hypotheses were formulated. The experimental group were taught mathematics using the CMSTA while the control group were taught with the conventional teaching approach (CTA) for 12 weeks. 485 subjects, with 274 and 211 in the experimental and control groups respectively, were used. Three instruments, Mathematics Performance Tests (MPTs), Attitude Towards Mathematics Inventory (ATMI) and Mathematics Anxiety Rating Scale (MARS), were used to gather data which were analysed using descriptive and inferential statistics. Hypothesis testing was done at the 95% level of confidence. Significant differences were found between (a) the mathematics performance levels, attitude towards mathematics dispositions and mathematics anxiety levels of the experimental and control groups (b) the mathematics performances of male and female students in the experimental group. However, significant differences were not found between the attitude towards mathematics dispositions and mathematics anxiety levels of the male and female students in the experimental group. Conclusion reached was that the treatment has enhanced the performance, fostered their attitude towards mathematics positively and lowered their mathematics anxiety. Consequently, it was recommended amongst others that the CMSTA should be adopted or adapted widely by mathematics teachers/lecturers in Nigerian Polytechnics in particular and other tertiary institutions in general. The contributions and implications of the study, areas for further studies as well as limitations of the study were also stated.

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ABBREVIATIONS

- ATMI: Attitude Towards Mathematics Inventory
- CMC: Computer Mediated Communication
- CMSTA: Computer Mediated Systems Approach
- FGN: Federal Government of Nigeria
- MARS: Mathematics Anxiety Rating Scale
- MPT: Mathematics Performance Test
- NBTE: National Board for Technical Education
- NPE: National Policy on Education
- RMARS: Revised Mathematics Anxiety Rating Scale
- TVE: Technical and Vocational Education
- TVET: Technical and Vocational Education and Training
- ICT: Information and Communications Technology
- HND: Higher National Diploma
- ND: National Diploma

OPERATIONAL DEFINITION OF TERMS

In this study, the following terms are to be taken to represent the definitions which have been ascribed to them:

- **Attitude Towards Mathematics**: A person's perception of the importance, use and difficulty level of mathematics A relational mental state connecting a person to a presupposed view of mathematics/
- **Computer - Mediated Systems Teaching Approach**: Any Teaching activity carried by means of two or more networked computers.
- **Computer Mediated Communication**: Any communicative transaction which occurs through the use of two or more networked computers.
- **Mathematics Anxiety:** A feeling of tension, apprehension, or fear that interferes with mathematics understanding and performance.
- **Mathematics Performance**: The level of attainment in any or all mathematics skills or concepts based on an agreed measurement or assessment criteria.

CHAPTER ONE

INTRODUCTION

1.1 Background to the Study

All over the world, computers are put to use in industry, commerce and education. In the early days of the invention of computers, their size, cost and complexity made them only available in large, rich corporations and well-endowed universities and research establishments. Now that smaller, cheaper and more reliable ones are being made and improved upon, they have become available to small businesses, schools, colleges, and in homes especially in developed countries (Commonwealth Secretariat, 1991). It is this

budget, some still allocate resources for the introduction of computers into their schools and colleges. This may be on account of the central role computers have been established to be capable of playing in the promotion of education qualitatively and quantitatively. However, there may be other countries with scarce resources and urgent social priorities for development who may consider computers in schools a privilege of rich countries.

Whether or not a country allocates enough, meagre or no resources for the introduction or use of computers into its schools and colleges, it is without controversy that the introduction or use of computers in the curriculum of developing countries is an essential step in the struggle to close the technological gap between the developed and developing countries. Against this background, it is hereby posited that the introduction or use of computers in the curriculum of all levels of education in all developing countries is a sine qua non for their technological breakthrough.

It has been observed that in many developing countries, most schools use 19th century technologies and methods to teach, despite the fact that students can now have access to vast quantities and varieties of information which, formerly, would have been available only in printed form and to a privileged few (Commonwealth Secretariat, 1991). Contrastingly, in developed countries, students have unlimited access to computers, where these computers are not only being used for knowledge propagation but as powerful thinking aids for cognitive development and resolution of affective issues that might interfere with attainment of educational goals. This situation becomes perturbing when seen from the perspective that in the midst of what amounts to a global information revolution era; schools are no longer expected to continue to offer a curriculum based on a view of information which dates back to the time when knowledge seem stable and bounded when we used to think it was enough for students to learn from books or course notes, and to be able to produce the facts or opinions they had been taught (Commonwealth Secretariat, 1991).

The above contention has to be examined in the light of existing education practices in

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teaching situation in many developing countries is very much prevalent in Nigeria. In order to reverse this situation, there is need for intervention by all stake holders in the education sector. This need provides the basis whereby this study derives its elixir. The Computer Mediated Systems Teaching Approach (CMSTA) used to teach mathematics to Nigerian Polytechnic engineering students, whose effect was investigated in this study, is hinged on the premise that computers can be used to enhance students

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as well lower their mathematics anxiety.

There are different reasons for advocating use of computers in the teaching/learning

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society and possibly futuristic years ahead. For this reason, it is the believe of the researcher that the need for computer technology and literacy in the educational system will continue to be increasingly very relevant. Therefore, computers should be widely used in schools. Computers can be used to diversify, develop and improve the pedagogical relation of teaching and learning (McCormick, 1993). Computer has been found to be an effective device for presenting an instructional programme (Bada, et. al., 2010). Infact, they (Bada et. al., 2010) posit that the computer is a tireless, relentless, evaluating teacher which has several modes of instruction at its disposal such as sound, sight and tough. According to Baugher (1999), the patience, memory and endless capacity for details of the computer are assets that defy competition from ordinary teacher.

The computer might also be used to handle the extremely complex programmes that are necessary for more individualized learning (Roblyer & Elkhalm, 2000). In this regard, the computer can be used for diagnostic test which in turn will provide branched progrmames to accommodate individual needs and give prescriptive assignments that might refer the student to a textbook, a laboratory experiment or a consultation with the instructor. Fajola (2001) asserts that the computer is diligent and consistent in its mode of operation, as it does not suffer from tiredness or lack of concentration like human beings. Computer performs multi-functional roles in teaching and learning processes at all levels.

Another use of the computer in the teaching/learning process as identified by Adekomi (2001) is that information can be stored in manual files in the computer magnetic disc and retrieved when needed. Therefore, the computer can provide a convenient technique

for designing and developing a course of instruction and as well provide dynamic interaction between students and instructional programme. This might not be easily not possible with most media. Abimbade (1997) identified other potentials of computer in the teaching/learning to include:

- Its ability to help students learn at their own pace thereby significantly making time saving over conventional classroom instruction possible.
- G over the rate and sequence of their learning. It gives appropriate feedback.
- It promotes individualized instruction through personalized responses to ,
- It provides a more positive affective climate especially for slower learners. It provides appropriate record- ,
- It puts more information in the hands of teachers.

These citations on the usefulness of computers in the teaching/learning process may be considered as an indication that computers can be used to address the already highlighted concerns on the prevailing teaching situations in Nigeria. On this basis, a research on the impact of CMSTA was considered to have become pertinent. This was particularly pertinent to the current situation regarding the teaching/learning of *

bedevilled mathematics performance status, apparent negative d high anxiety levels; amongst other problems. It was believed that the opportunities afforded by new, improved and emerging technological developments in field of Information and Communications Technologies (ICTs) have the potential of providing the much needed impetus for addressing these problems. This

forms the motivation for this study.

1.2 Statement of the Problem

The speedy growth rate of scientific and technological information continues to pose challenges to the teaching/learning process. While the education system of advanced nations are continuously being made to respond to these challenges through the development and use of computer application packages to assist both teachers and learners cope with the demands of new technologies, same cannot be claimed to be the

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observes that new technologies are failing to penetrate the Nigerian education system because the Colleges of Education have failed to equip prospective teachers with the

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observation that this situation has not changed despite the National Policy on

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techniques shall be increasingly used and improved upon at all levels of the educations

, titutes a major loss of opportunity that would have been used to tackle some of the problems encumbering the teaching/learning of mathematics

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implement the above aspect of NPE (FGN, 2004) is a problem which can be addressed by the use of a CMSTA to teach mathematics.

Despite the fact that the use of multimedia courseware in engineering has rapidly gained

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using traditional approaches of written course notes combined with formal lectures. This approach is lagging behind that of other nations using new technologies. This

observed difference between the mathematics teaching approaches to engineering

addressed.

Another problem observed by the researcher is that in Nigerian Polytechnics, mathematics teaching is lagging behind the teaching of other subjects like physics, chemistry and biology in terms of experimentation. For, it is highly uncommon to see mathematics teachers carrying out mathematical teaching experiments despite the fact that it has been established that experiments help students in their learning (Bourne et al, 1996). This problem needs to be addressed. It is hereby believed that a CMSTA, which makes simulation of many physical phenomena governed by mathematical laws possible, could be easily used to address it.

It was further observed that other problems associated with the teaching of mathematics

seemingly negative attitude towards the subject and their high mathematics anxiety levels which might have been due to the shortcomings earlier mentioned. The lack of use of modern technological facilities in mathematics teaching, negative attitude towards mathematics and high mathematics anxiety level can constitute stumbling

engineering profession.

From the discussion so far, it could be seen that there exists the need to create appropriate computer application packages that can be used to address the identified

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increasing students mathematics performance, fostering their attitude towards mathematics positively, help lower their mathematics anxiety levels, and provide a

window of opportunity for tackling some other problems facing the teaching/learning of mathematics in Nigeria; this research was undertaken. Thus, this research was undertaken to investigate the impact of CMSTA on mathematics performance, attitude and anxiety of Nigerian Polytechnic engineering students.

1.3 Objectives of the Study

The main purpose of carrying out this research work was to take advantage of new and emerging technologies in the field of Information and Communications Technology (ICT) to study the impact of Computer Mediated Systems Teaching Approach (CMSTA) on mathematics performance, attitude and anxiety of Nigerian polytechnic engineering students. Specifically, the study sought to determine:

1. If the difference between the mean mathematics performance scores of the students taught mathematics using the Computer Mediated Systems Teaching Approach (CMSTA) and that of those taught with the Conventional Teaching Approach (CTA) will differ significantly.
2. Whether the mean Mathematics Performance scores of the students taught mathematics using the Computer Mediated Systems Teaching Approach (CMSTA) differs significantly by gender.
3. If difference between the attitude towards mathematics dispositions of the students taught mathematics using the Computer Mediated Systems Teaching Approach (CMSTA) and that of those taught with the Conventional Teaching Approach (CTA) will differ significantly.
4. Whether the gender difference in attitude towards mathematics dispositions of the students taught mathematics using the Computer Mediated Systems Teaching Approach (CMSTA) will be significant.

5. If difference between the mathematics anxiety levels of the students taught mathematics using the Computer Mediated Systems Teaching Approach (CMSTA) and that of those taught with the Conventional Teaching Approach (CTA) will differ significantly.
6. Whether the mathematics anxiety levels of the students taught mathematics using the Computer Mediated Systems Teaching Approach (CMSTA) will differ significantly by gender.

1.4 Research Questions

The following research questions were raised in order to achieve the above objectives:

1. Will the mean mathematics performance score the students taught mathematics using the CMSTA be higher than that of those taught with the CTA and to what extent will this difference be?
2. Which gender will have higher mean mathematics performance score amongst the students taught mathematics using the CMSTA and to what extent will this difference be?
3. Will the mean attitude towards mathematics disposition of the students taught mathematics using the CMSTA be more favourable than that of those taught with the CTA and to what extent will this difference be?
4. Which gender will exhibit more positive attitude towards mathematics dispositions amongst the students taught mathematics using the CMSTA and to what extent will this difference be?
5. Will the mean mathematics anxiety level of the students taught mathematics using the CMSTA be lower than that of those taught with the CTA and to what extent will this difference be?

6. Which gender will exhibit lower mathematics anxiety level amongst the students taught mathematics using the CMSTA and to what extent will this difference be?

1.5 Research Hypotheses

In order to answer the above research questions, the following null hypotheses were formulated:-

Ho_1: There is no significant difference between the mean mathematics performance scores of the students taught mathematics using the CMSTA and that of those taught with the CTA.

Ho_2: There is no significant gender difference between the mean mathematics performance scores of the students taught mathematics using the CMSTA.

Ho_3: There is no significant difference between the mean attitude towards mathematics dispositions of the students taught mathematics using the CMSTA and that of those taught with the CTA.

Ho_4: There is no significant gender difference in attitude towards mathematics dispositions of the students taught mathematics using the CMSTA.

Ho_5: There is no significant difference between the mean mathematics anxiety levels of the students taught mathematics using the CMSTA and that of those taught with the CTA.

Ho_6: There is no significant gender difference in the mathematics anxiety levels of the students taught mathematics using the CMSTA.

1.6 Significance of the Study

Data available to the researcher through literature search revealed that none of the researches in Nigeria bordering on the use of computers in our education system for content delivery was focused on their impact on achievement, attitude and anxiety. For instance, Onyemchukwe (2003) only set out a costing for equipping a model laboratory in the local government headquarters of each state. Iyamu and Aduwa Ogiegben (2005) examined major obstacles militating against the use of ICT in Nigerian secondary education. In the Polytechnic sector, no work was found on the deployment of computers for curriculum content delivery in mathematics and other subjects, let alone their impact on mathematics performance, attitude and anxiety. The above situation is in contrast to what obtains in the other countries, where the works of Thalmann (1983), Bastien, St Pierre and Cantin (1997), Miller (2001), Paulsen (1997) and Vassigh (2005) can be cited as examples.

This re , during literature survey, any research work in Nigeria on the deployment of computers for mathematics curriculum content delivery and their impact on mathematics performance, attitude and anxiety of Nigerian polytechnic engineering students makes it imperative for a research of this nature to be undertaken. This is particularly necessary as the developed nations of the world had already advanced in researches and implementation of the use of computers in the teaching/ learning process. Therefore the significance of this study lies in the fact that, firstly, it is an effort to close the existing dearth of research work on deployment of computers to enhance teaching/learning of mathematics in Nigeria, in general and the Polytechnic sector in particular. Secondly, it is considered as a contributory work in the field of developing indigenous CMSTA framework for teaching and learning mathematics at the Polytechnic level in Nigeria. Researchers in other disciplines can replicate, built and

even improve upon the CMSTA for the advancement of our education system. Thirdly, it is an attempt to respond to declarations made in sections 9(c), 9(d), 9(h), 9(i), 19(m), 30(f), 73 and 103(d) of the NPE (FGN, 2004).

Fourthly, mathematics lecturers teaching mathematics to engineering students in Nigerian Polytechnics will benefit from the study in that it will provide them with a CMSTA framework which is potentially capable of helping them improve their students mathematics performance, positively change their attitude towards mathematics and as well help lower their mathematics anxiety level. Fifthly, engineering students in Nigerian Polytechnics will benefit from the outcome of this study. This is because it is hoped that the developed CMSTA, when used to teach mathematics, will go a long way to enhance the quality of mathematics content delivery by Polytechnic mathematics teachers and by extension will facilitate the rate at which learning takes place.

Other researchers could replicate this study in other subject matter areas. Therefore other subject matter areas may benefit from the product of this study. Psychologist interested in the impact of man/machine interaction on variables such as attitude and anxiety will also benefit. Education Policy makers in Nigeria may use the experiences gleaned from this study to come up with new policies on computer usage in the teaching/learning process.

1.7 Scope of the Study

This study was undertaken to determine the impact of the CMSTA on mathematics performance, attitude and anxiety of Nigerian polytechnic engineering students. Therefore, mathematics curricula of both the National Diploma (ND) and Higher National Diploma (HND) levels in the Nigerian Polytechnic engineering disciplines

were covered. Engineering mathematics curricula of other levels such as those of post HND, Pre ND and non engineering disciplines were excluded in this study. A * B F B performance, attitude towards mathematics and mathematics anxiety levels were focused on. The mathematics performance, attitude towards mathematics and mathematics anxiety level other engineering students, such as those of post HND, Pre ND and non engineering disciplines were excluded in this study.

1.8 Basic Assumptions of the Study

In the course of conducting this study, the following basic assumptions were made:

1. Nigerian polytechnic engineering students are taught mathematics based on the guidelines stipulated by the National Board for Technical Education (NBTE), the supervisory agency responsible for regulatory of technical/vocational education in Nigeria.
2. The learning environment available to the polytechnic engineering students in Nigeria is the same in terms of the adequacy of teaching/learning materials.
3. The mathematics lecturers teaching mathematics to the polytechnic engineering students in Nigeria possess the minimum academic requirement for carrying out their job functions.

CHAPTER TWO

REVIEW OF RELATED LITERATURE

2.1 Introduction

In this chapter, a review of some related literature relevant to the subject matter of this study is presented. The literature reviewed is organized under the following headings:

- Conceptual Framework
- Computers and the Teaching/Learning Process
- Mathematics Performance/Achievement
- Attitude Towards Mathematics
- Mathematics Anxiety
- Relationships between the Study Variables
- A Review of Some Related Studies
- Implications of the Reviewed Literature to the Study

2.2 Conceptual Framework

2.2.1 Concept of computer-mediated systems teaching approach (CMSTA)

Literature search reveals that no single definition for the concept of Computer Mediated Systems Teaching Approach (CMSTA) exists. However definition exists for a related term, Computer Mediated Communication (CMC), which by extension can also be considered a related concept. Therefore, it is believed that any attempt to define concept of CMSTA will be based on that of CMC. This is because teaching, in all its ramifications, is an activity which is basically communicative in nature. Based on this reason, the concept of CMSTA is discussed herein with substantial recourse to that of the CMC.

CMC has been defined as any communicative transaction which occurs through the use of two or more networked computers (McQuail, 2005). Traditionally, the term CMC was used in reference to communications that occur via computer mediated formats such as instant messages, e-mails, chat rooms. Later on, the term was applied to other forms of text-based interaction such as text messaging (Thurlow et. al., 2004). Popular forms of CMC include e-mail, video, audio or text chat (text conferencing including instant messaging), bulletin boards, list-servs, Massively Multiplayer Online Games (MMOs) and Weblogs (blogs).

From the above, the concept of CMSTA is herein considered to signify the ways in which telecommunication technologies could be merged with computers and computer networks to give teachers new tools to support teaching. In other words, CMSTA describes the ways teachers could use computer systems and networks to transfer, store and retrieve information for the purpose of supporting the teaching process. Implied in this concept is the notion that computers and computer networks is a taken as a single entity which is primarily a medium for supporting the teaching/learning process rather than being the teacher. In order to support the teaching/learning process, CMSTA entails the use electronic mail and real-time chat capabilities to deliver instructions and facilitate student-to-student as well as student-to-teacher interactions across a desk, classroom or across the world.

2.2.2 Concept of mathematics performance

Literature encountered in the course of literature search reveals that the concept of mathematics performance is substantially dependent on that of academic performance. This is basically due to the fact that mathematics is an academic discipline and all discussions found in literature on it were within the purview of academic performance.

Therefore, this review will start by first considering the concept of academic performance, which will form the basis for the discussion on the concept of mathematics performance that will ensue.

Academic performance, also referred to as academic achievement, has been defined as the extent to which a student, teacher or institution has

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However, while academic performance is the expected outcome within a short time, academic achievement is for a longer period. Therefore academic performance could be conceptualized as the degree of attainment of the objective(s) specified in an academic discipline based on some agreed measurement criteria. Therefore, academic achievement is the degree of attainment of the goal(s) specified in an academic discipline based on some agreed measurement criteria. In relation this definition,

mathemat 8 , , -achievement) or declarative knowledge. It is in furtherance of this concept, that mathematics performance should be conceptualized in this study to signify level of attainment in any or all mathematics skills or declarative knowledge.

2.2.3 Concept of attitude towards mathematics

As far back as 1945, Strauss (1945) posited that attitude has profound influence on

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other people, exposure to and comprehension of information, choice of friends. Literature search reveals that the concept of attitude has been differently comprehended by different researchers. It is therefore not surprising that while some researchers have defined it conceptually, others defined it operationally. The Online Business Dictionary

(www.businessdictionary.com) defines attitude as a predisposition or a tendency to respond positively or negatively towards a certain idea, object, person or situation. Other definitions of attitude encountered in the course of literature search include those of Thomas and Znaniecki (1927), Bain (1928), Warren (1931), Allport (1935), Katz and Scotland(1959), Krech and Crutchfield (1984).

According to Thomas and Znaniecki (1927), attitude is a process of individual consciousness which determines real or possible activity of the individual in the social world. Bain (1928) defines attitude as the relatively stable overt behavior which affects

, /731) asserts that when a certain type of experience is constantly repeated, a change of set is brought about which affects many central neurons and tends to spread over other parts of the central nervous system. These changes in the general set of the central nervous system temper the process of reception which leads to the formation of certain general modes of receiving and integrating stimuli. In terms of the subjective life, these general sets are called attitudes.

Allport (1935) defines attitude as a mental and neural state of readiness, organized

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response to all objects and situations with which it is related. While Katzand and Scotland (1959) defines it as a tendency or a predisposition to evaluate an object or symbol of that object in a certain way evaluation consists of attributing goodness/badness or desirable/undesirable qualities to an object; Krech and Crutchfield (1984) defines attitude as an enduring organization of motivational, emotional, perceptual and cognitive processes with respect to some aspect of the

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Sharma (2011) asserts that two most common distinct aspects used to define attitude are: as set of readiness and as effect and evaluation. According to Sharma (2011), as set of readiness, attitude will be conceived as a tendency to respond to some social object. For those who hold this conception, their definitions of attitude had the component of readiness or disposition to act. Those who conceive of attitude as effect and evaluation define the term on the basis of its effects and influences on evaluation. The definitions of Krech and Crutchfield (1984) and Katz and Scotland (1959) above are in this category.

It is clear from the above that whatever conception of attitude one holds, this psychological construct influences our choice of action, responses to challenges, incentives and rewards. It is therefore not surprising that Neale (1969) declared that in the field of mathematics education, research on attitude has been motivated by the belief that it plays a crucial role in learning mathematics. The conceptions of attitude towards mathematics encountered in literature are in tandem with the ones held on attitude, i.e. just like attitude has been differently comprehended by different researchers, so has attitude towards mathematics.

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or disliking of mathematics, a tendency to engage in or avoid mathematics activities, a belief that one is good or bad at mathematics, and a belief that mathematics is useful or

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sponses to the

easy/difficult dimension as well as the important/unimportant dimension of mathematics.

In general, attempts have been made to explicitly or by inference define attitude towards mathematics in one of the three following ways, with each based on the definition of attitude:

- Haladyna et. al., (1983) and McLeod (1992) are of the opinion that attitude towards mathematics is just a positive or negative emotional disposition towards mathematics. This view point is based on the simple definition which describes attitude as the positive or negative degree of affect associated with a certain subject.
- Based on a bi-dimensional definition, in which behaviours do not appear explicitly Daskalogianni and Simpson (2000) defines attitude towards mathematics as the pattern of beliefs and emotions associated with mathematics.
- From a multidimensional perspective which takes emotional response, beliefs regarding the subject and behaviour related to the subject as the three components of attitude into cognisance, emerges the point of view where attitude towards mathematics is defined in a more complex way by the emotions that an individual associates with mathematics (which, however, have a positive

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  behaves (Hart, 1989).

Kulm (1980) posits that it is probably not possible to offer a definition of attitude towards mathematics that would be suitable for all situations and that even if one were agreed on, it would probably be too general to be useful. In this way, the definition of

(Daskalogianni and Simpson, 2000). This position implies that attitude towards mathematics should be seen as functional to the res -posed problems. As

long as the definition of attitude towards mathematics is not simply borrowed from the context in which it appears in social psychology, but is rather outlined as an instrument capable of taking into account peculiar problems in mathematics education; then it may be considered useful in the context of mathematics education. This is in line with the

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2.2.4 Concept of mathematics anxiety

According to Barnes (1984), anxiety is a term generally used to depict any psychologically perceived threat to an individual, which may be real or imagined. He further asserts that feelings of distress, confusion and fear are experienced by those who are unable to avoid the threat. Spielberger (1972) conceptualised anxiety as a state, trait and a process. According to him, anxiety is a result of a chain reaction that consists of a stressor, a perception of threat, a state of reaction, cognitive reappraisal and coping. Research on mathematics anxiety has shown that it is a multidimensional psychological construct that involves complex factors, such as feelings of pressure, performance inadequacy and test anxiety (McMorris, 2004; Kazelskis, 1998; Rounds and Hendel, 1980). Mathematics anxiety is a phenomenon that is often considered when examining

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One of the earliest definitions of mathematics anxiety was that suggested by Richardson and Suinn (1972) as feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations. According to Cemen (1987) in Trujillo et. al., (1999), mathematics anxiety can be described as a state of anxiety which occurs in response to situations involving mathematics that are perceived as threatening to self-esteem. While Ashcraft and Faust (1994) defines mathematics anxiety as the feeling of tension,

helplessness, mental disorganization and dread one has when required to manipulate numbers and shapes and the solving of mathematical problems; May (1977) describes it

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of tension, apprehension, or fear that

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Other authorities defined mathematics anxiety in terms of a chain reaction or cycle. For example, Mitchell (1987) described a mathematics anxiety cycle and stated that mathematics anxiety experienced in the present has its roots in the past. Anxiety is perpetuated through negative self-talk manifesting in beliefs which cause anxiety. This leads to physical symptoms, an inability to think and avoidance which, in turn, leads to the inability to perform, causing anxiety and more negative self-talk, and the continuation of the mathematics anxiety cycle (Mitchell, 1987). This cycle leads to negative educational and societal mathematics attitudes which often become a self-fulfilling prophecy, and generally leads to mathematics avoidance (Williams, 1988).

From the above it can be noted that mathematics anxiety can be described as a combination of factors. As described by Mitchell (1987), mathematics anxiety is a combination of physical, cognitive and psycho-behavioural components. Physical aspects of mathematics anxiety are biological, consisting of hormonal, chemical and muscular changes in the body which results in a disability to think (Mitchell, 1987). In conclusion one may infer that mathematical anxiety can be considered as anxiety about

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Recognizing the complexity of mathematics anxiety is the key to understanding the phenomenon. Mathematics anxiety is not a discrete condition but rather it is a

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/776' ration this intricacy by encompassing both the affective and the cognitive domains of learning. In line with this, mathematics anxiety has been at various times defined summarily as a feeling of intense frustration mathematics, and can be described as a learned emotional response to participating in a mathematics class, listening to a lecture, working through problems, and/or discussing mathematics to name but a few examples (Hembree, 1990; Le Moyne College, 1999).

2.3 Computer and the Teaching/Learning Process

The use of computers to enhance the teaching/learning process dates back to the 1950s * /761' ,

computer-orientation to the very flexible ones. Along this trend, terms such as Computer Assisted Training (CAT), Computer Based Training (CBT), Computer Assisted Instruction (CAI), Computer Assisted Learning (CAL), etc have come to be generously used to portray various efforts in the deployment of computer to enhance the teaching/learning process. A common feature of all these approaches is the development of programmes that computers use in the processes. Summarily, Leinonen (2005) categorizes the use of computers in education into the following five major phases:

- The Programming, Drill and Practice phase
- The Computer Based Training (CBT) with Multimedia phase
- The Internet Based Training (IBT) phase
- The e-Learning phase
- The Social Software plus Free and Open Content phase

The Programming, Drill and Practice phase was from the late 1970s to the early 1980s. According to Leinonen (2005), the pedagogical reason to teach programming was not to

skills. At some points during this period, some educational software were developed which provided very simple drill and practice exercises for mathematics and language learning. In most instances, these exercises did not considerably help students to reach any deeper unde

The CBT with Multimedia phase was popular from the late 1980s to the early 1990s. Within this period, multimedia computers with advanced graphics and sound were massly marketed as it was believed that the drill and practice exercises failed to teach much because they did not contain multimedia. It was argued that students would learn if they could watch animations in colours, small video clips and then do the exercises. This was the golden era of CD-ROMs and multimedia computers, a combination that was seriously expected to have a huge impact on the ways students learn. The pedagogical basis for advocating CBT with multimedia was that human beings are different and some students learn better by watching movies/animations and listening to audios whereas some learn better by reading or watching still images.

Even with the above reason for adopting the CBT with multimedia, it was realized after some time that multimedia CD-ROMs could not also get people to deep learning and understanding. This thus set the stage for the third phase. This phase, which came with the rise of the World Wide Web (WWW), was the IBT phase. It began in the early 1990s. The failure of CD-ROMs were claimed to be related to the challenges to update the content in the CD-ROMs. The promoters of the new paradigm claimed that information changes so fast that one should update it almost every day. The Internet and

the IBT was therefore believed to be the solution. At this point computer-based training was brought to Internet, but again without the multimedia. All that could be done on the Internet, that time, was text and pictures and some early experiments with animations, video and audio. It was quickly realized that clicking and reading e-learning course materials online did not make people very smart. And again some people claimed that the problem was the lack of multimedia. The educational ideas behind IBT were not pedagogical. Rather, it was the belief that it is cost-efficient as there were no more travelling to training or absence from workplace. However, it was found not to be that cost-efficient and therefore there was very little under the bottom line i.e. people did not learn much.

The IBT got transformed in the late 1990s and early 2000s in a form of e-learning. The hype around e-learning was a kind of classical example of creating needs. Thousands of websites, articles and companies made it clear for all somehow related to education that this is something people must be involved with. The pedagogical thinking around e-learning is closely related to the CBT. The point is to deliver courses for students. Later on, the learning platform developers became more aware that learning requires social activities among the learners themselves as well as between the learner and the teacher(s). Still, the user interfaces of the Learning Management Systems (LMS) are at least implicitly indicating that the learner should first read the content and if there is something he/she did not understand, he/she may ask his/her peers or teacher. On the other hand, the e-learning field is nowadays so wide that it is hard to say what the pedagogical thinking behind it is. E-learning is no more one. It could be said that all the earlier paradigms live inside the e-leaning plus some clues of the present: social software and open content.

The Social Software plus Free and Open content phase became popular as from the late 2000s. As blogs and wikis have already brought web back to its original idea of a simple tool for personal notes that are easily accessible and even editable by peers and potential peers, it was believed that the social software plus free and open content phase will make a real breakthrough in the field of educational technology. Such projects as the GNU-GPL, Creative Commons, Wikipedia and Open courseware have shown that free content benefits all and that people are willing to contribute to the common good. The pedagogical thinking behind the social software plus the free and open content can be located in the social constructivist theory and cultural-historical psychology. Learning with computers is not about programming or drill and practice, nor about multimedia, nor about fast updating or cost-efficiency it is all about people sharing ideas.

2.3.1 Curriculum issues on deployment of education technology in the teaching/learning process

The changing phases of the use of computers to enhance the teaching/learning process chronicled above conforms to observations from literatures on curriculum studies that a major area of concern in the deployment of technology in education is how best to utilize technological advancements to prepare students for a society that is increasingly being technologically driven. In the present age of ICT, the main issue of focus is how much computerization should be introduced into the curriculum of the education system of a nation. Seidel, Anderson and Hunter (1982) discuss many arguments for the universal education of students in computer skills which will presumably increase their chances of effective function in any information age (Brent and Anderson, 1990). While these arguments tend to be generally accepted within educational circles, considerable disagreements can be found over which computer related skills and insights are

sufficiently important to justify replacing any existing subject in the curriculum. Brent and Anderson (1990) opined that the level of usage of computers in the education system of a country should be specific to particular social or occupational roles which that education system seeks to promote.

The speedy growth rate of scientific, technological and other information poses a problem to teaching/learning process intensification, which many countries are trying to solve by means of the computer aided instruction (CAI) systems. It is now possible for students to learn mathematics, physics, chemistry, foreign languages and other subjects with the aid of a computer. The computer aided instruction system has given rise to a qualitatively new variety of teaching/learning process e.g. the on line tutorials. Heretofore, the students can only obtain required information from books, journals, films and from lectures.

Despite the vast possibilities of the computer in all spheres of human activity, there is no ground whatsoever to revise the man/machine relationship, explicitly defined in philosophical and sociological respects. Man is the maker of computer, its master and user. No matter what rudiments of intelligence computers might have, man shall never be their slave. It is from this perspective that the use of computers in the teaching/learning process should be understood as being meant to facilitate the process and as well complement the efforts of the teacher/learner in the process. This is the perspective adopted in this study i.e. the CMSTA employed to teach mathematics is not meant to replace the role of the mathematics teacher; rather it should be capable of making the work of the teacher more effective and efficient.

In many countries, attempts have been made to improve the quality and efficiency of the teaching/learning process in their domain by introducing computer facilities which

include both the traditional information systems and on line tutorials, new interactive systems of computer managed task solution and expert systems with elements of artificial intelligence. Part of the current study is an endeavour in this direction. Towards this extent, an attempt was made to integrate the possibilities inherent in the on line tutorial process with the interactive systems of computer managed task solutions to develop a flexible CMSTA, which was used to teach mathematics to Polytechnic engineering students.

The efforts of the former socialist countries in the above direction are worth citing. These countries attempted to tackle these problems within the framework of the Council for Mutual Economic Assistance (CMEA) comprehensive programme for scientific and technological progress, involving the then Soviet Union, Bulgaria, Hungary, the GDR, Poland, Czechoslovakia, as well as Cuba and Mongolia. Their solution was located in the design of a unified Computer Technology of Instruction (CTI) designed for all levels of education. It was an attempt to unite the efforts of teachers, psychologists, physiologists, programmers and system engineers of many countries in an international research group that led to the development of the technology of design and multiplication of respective software intended for boosting cognitive activity of students at all levels. The CTI tasks may be summarized as follows:

- Wide use of computers in all educational activities such as teaching process, research work, management, occupational guidance, etc.;
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  teaching/learning methods so as to stimulate their creative capabilities and develop skills and aptitudes for independent work and acquisition of knowledge;
- Individualization of instruction under the conditions of collective activity within

a single teaching and educational process.

In this study, all the three tasks above were focused upon. In this wise, the developed CMSTA, which is primarily for aiding as well as complementing the mathematics

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cognitive activity in o

status. The individualization of instruction within the framework of collective activity in any given teaching session, which was an integral part of the CMSTA, meant to address issues related to

anxiety. Arising from this, it is quite natural for one to expect the integration of the computer technology in the teaching/learning process to rely, above all, on the achievements of pedagogic psychology and on the system of continuous training in computer use. To this extent, three major uses of computer technology in the teaching process were focused upon in this study. These are:

- * f algorithm presentation and programming for solution of problems);
- As an object of application (application in various fields of occupational activity)
- As a means of instruction (on line tutorials, expert systems, etc) for studying a variety of subjects.

It should be noted that as an object of study, the computer was used for basic instruction; as an object of application, it was used for specialized instruction of various kinds. However, using the computer as a means of instruction was a special goal which can be hopefully attained in the process of teacher training, retraining and upgrading. This last level forms part of the motivation for this study. Therefore, it is being posited

that in all pre service teacher training programmes in our tertiary institutions, courses on using the computer as a means of instruction ought to feature prominently on account of the considerable amount of literature found on the importance of computer supported education technology in the teaching/learning process. The next section deals with a brief review of this aspect.

2.3.2 Computer supported education technology and the teaching/learning process

Two major computer supported education technology found in literature that is considered relevant to the focus of this study were the Computer Supported Collaborative Work (CSCW) and Computer Supported Intentional Learning Environment (CSILE). CSCW is the combination of communications and computer technologies to support various activities in groups of varying size, permanence and structure (Olson, Olson, and Kraut, 1992). CSCW helps groups to structure work through group decision support systems, project management tools, electronic conferencing systems and shared editors. CSCW technologies can support groups across a distributed environment. For example, collaborative problem solving in classrooms can be supported by the Issue-Based Information System (IBIS) hypertext environment (Conklin and Begeman, 1987), which provides an argument structure including issues, positions and argument nodes. Users can add their comments about the problem under discussion so that a logical discourse can be produced that will result in an effective and acceptable solution. These environments help collaborative groups construct a common understanding of the problem being solved and negotiate the most appropriate solution to that problem.

Research laboratories involved in CSCW have explored the use of video transmissions to support informal communication in the workplace (Heath and Luff, 1992). This type

of technology has interesting ramifications for both classroom and distance education. Two way real time video transmission of information implies a new definition of real world context. Although video mediated, the learning environments could potentially include the actual environment or a close facsimile with which the learner could remotely interact. These collaborative problem-solving situations enhance knowledge construction through the addition of visual information and remote interaction with other learners. The video transmission of authentic, realistic contexts adds a significant dimension to anchored instruction and situated learning environments.

Computer-Supported Intentional Learning Environments (CSILEs) are another computer - supported education technology that can assist learners in knowledge construction. CSILEs are educational knowledge media systems that allow different types of information (text, drawings, graphs, timelines, etc.) to be entered into a common database where they are available for retrieval, review and contribution (Scardamalia et al., 1989). CSILEs, which can be implemented both at the classroom level and at a distance, promote intentional control over learning by providing an environment that requires students to plan, monitor, set goals and solve problems. CSILEs require learners to reflect on their personal knowledge, state learning intentions and publish ideas to a communal database, thus producing cumulative, progressive results for the group (Scardamalia and Bereiter, 1993/1994).

CSILEs promote knowledge construction through the procedural facilitation process and the building of a collective database that provides open access to the learning context, facts and information needed for solving specific problems. Procedural facilitation also provides a scaffolding effect by providing learners with temporary support processes until they are prepared for more complex strategies and information. Making knowledge construction activities overt contributes to this process through

internal negotiation and deliberate actions such as goal setting, identifying and solving problems of understanding, and connecting old and new knowledge (Scardamalia et al. 1989).

The last two sections above provide the bases for believing that the use of computers in the teaching/learning process can be potentially rewarding. It is thus pertinent that a review of some issues bordering on the use of the CMSTA be undertaken. The next section deals with this aspect.

2.3.3 Some issues on use of CMSTA

CMSTA can be compared to other approaches of teaching by examining a number of aspects which are universal to all forms of teaching. These include (but not limited to) synchronicity, persistence or recordability and anonymity. For example, instant messaging is prototypically synchronous, but rarely persistent since contents can be lost when the dialog box is closed unless a message log had been set up or had been manually copy-pasted the conversation. E-mail and message boards are similar; both are prototypically low in synchronicity since response time varies, but high in persistence since messages sent and received are saved. Properties that separate CMSTA from other approaches also include transience, its multimodal nature and its relative lack of governing codes of conduct, which is similar to the case of the CMC as noted by McQuail (2005). Therefore, the CMSTA can be considered as capable of overcoming physical and social limitations of other forms of teaching approaches and thus allow the interaction of people who are not physically sharing the same space.

From the above, it can be seen that the CMSTA can be carried out synchronously or asynchronously or by a combination of both modes. The synchronous mode of using the CMSTA entails that all participants (both students and teacher) are online at the same

time, while the asynchronous mode of usage occurs without time constraints In more advanced economies of the world where computers and computer network are being deployed to support the teaching/learning process, these uses are enabling and promoting the paradigm shifts from teacher centered teaching system approach to that of the student-centered learning system as well as the merging of informal dialogues, invisible colleges, oral presentations, and scholarly publications into a kind of dialogic or even multilogic virtual university.

Two issues of importance found in literature which are considered central to the use of the CMSTA revolves around how to incorporate existing and emerging knowledge about teaching/learning and how educators can use CMSTA productively as we move into an era that is becoming more technological laden. In literature, these issues were discussed from various perspectives relating to both classroom and distance learning. These perspectives include accommodation of different learning styles, empowerment of learners, regardless of physical challenge

use of the same tools and methods that professionals use. A general conclusion arising from these discussions was that teachers using CMSTA must adopt an interdisciplinary, project-oriented approach to teaching/learning. All of these can potentially create authentic practice. On this basis, it was posited by some (e.g. McQuail, 2005; Thurlow et al., 2004) that computer usage in teaching/learning process has the potential of changing the process in two main ways, viz:

- generating improved technological tools that allow students to use a fuller range of interactive methodologies, and
- encouraging teachers and administrators to pay more attention to the instructional design of courses.

Both factors above can improve the quality and quantity of the materials students practice during learning and this was what was envisaged when the CMSTA was being deployed to teach mathematics to the Polytechnic engineering students.

2.3.4 Benefits of the CMSTA

Educators often categorize the use of computer for instructional purposes in three ways. These are for conferencing, informatics and Computer-Assisted Instruction (CAI). The use of computers for conferencing entails the use of e-mail, interactive messaging, and small and large group discussion; for informatics (repositories or maintainers of organized information), it entails the use of library Online Public Access Catalogs (OPACs), interactive access to remote databases, program/data archive sites (e.g., archives of files for pictures, sound, text, movies), Campus-Wide Information Systems (CWIS), Wide-Area Information Systems (WAIS) and information managers, such as Gopher and Veronica; and for Computer-Assisted Instruction (CAI), the computer is used to structure and manage both the presentation of information and the possible responses available to the human user. In general, computer conferencing, informatics, and CAI can be used for any or combinations of the following activities which have direct bearing on the teaching/learning process:

- mentoring, such as advising and guiding students
- project-based instruction, either within the classroom or in projects involving community, national, or international problem solving
- guest lecturing, which promotes interaction between students and persons in the larger community
- didactic teaching, that is, supplying course content, posting assignments, or other information germane to course work

- retrieval of information from online information archives, such as OPACs, Educational Resources Information Center (ERIC) and commercial databases
- course management, for example, advising, delivery of course content, evaluation, collecting and returning assignments
- public conferencing, such as discussion lists using main frame Listserv software
- interactive chat, used to brainstorm with teachers or peers and to maintain social relationships
- personal networking and professional growth and such activities as finding persons with similar interests on scholarly discussion lists
- facilitating collaboration
- individual and group presentations
- peer review of writing, or projects involving peer learning, groups/peer tutorial sessions and peer counseling
- practice and experience using emerging technologies that may be intrinsically useful in today's society
- computer-based instruction, such as tutorials, simulations and drills.

From the above, it becomes clear that there are many benefits to using CMSTA. Other benefits include its ability to liberate the teaching/learning process from the constraints of time and distance. The convenience of access from home, school or office permits many students and teachers to better meet travel, job and family responsibilities. CMSTA can promote self-discipline because it requires students to take more responsibility for their own learning. Using CMSTA, teachers may vary a course's instructional design to include everything from structured projects to open projects where students are free to work on "messy" but authentic problem solving. It is however the opinion of the researcher that this may also be a constraint. Because students must

manage their own learning, this newfound independence may be a hindrance to those students who need more structure.

In this information revolution era, it cannot be denied that power comes to those who have information and know how to access it. If we consider which factors will be most important to education in the information age, it seems that our goals should be to develop self-motivated learners and help people learn to find and share information. If designed well, CMSTA can be used effectively to facilitate collaboration among students as peers, teachers as learners and facilitators, and guests or experts from outside the classroom.

One very important aspect of CMSTA is that it depends wholly on the use of CMC, which is text- , D ,
Since communication cannot occur on a computer network without writing, it therefore follows that if used effectively; the CMSTA can encourage and motivate students to become involved in authentic projects and to write for a real audience of their peers or persons in the larger world community, instead of merely composing assignments for teachers. Thus CMSTA can promote the equalization of learning opportunities. Here again, it is the researchers believe that this may be a source of shortcoming if not properly handled. In this wise, it must be recognized that not all students can express themselves well in writing and even for those who can, the act of writing and using online text-based applications can be a time-consuming struggle.

All that has been enumerated so far points to the fact that the CMSTA cannot be denied as a powerful teaching tool. However, it must always be kept in mind that responsible use of CMSTA means using it in addition to other approaches, not as a replacement. For, our job as educators is to provide options to fit a variety of learning styles. It is in

this regard that the CMSTA can help the most. There are technical benefits to using the CMSTA of teaching. These include the ease of circulating and archiving files and

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hand, the learning curve, with regard to learning the system and the technical "how tos" of the computer and telecommunications, can be steep. The cost of buying and supporting systems or accessing other networks is a significant "overhead" item in schools and colleges, as is the cost and inconvenience of upgrading, repairing, or replacing hardware. Further, computer systems are not 100% reliable, a fact that adds to inconvenience and wasted time. With so many systems to learn and sources to tap, information overload may become a problem as some users are likely to struggle with the lack of criteria to help them decide what to keep and what to discard from the swiftly flowing stream of incoming information.

Since teachers, information designers and instructional developers can use CMSTA to promote collaboration, cooperation, the sharing of ideas and as an equalizing teaching strategy, it means that the roles of students and teachers will have to change. Teachers could no longer be perceived as the sole experts and information providers; they have to become facilitators and guides. Conversely, students can no longer be passive learners, attempting to mimic what they see and hear from the expert teacher. Rather, they are to become participants and/or collaborators in the creation of knowledge and meaning.

While the above advantages are real and can be made manifest, care must be taken not to reproduce or augment the problems associated with the gap between technology "haves" and "have nots" when the CMSTA is being designed and put to use. In using the CMSTA therefore, this fact must be taken into cognizance so that the designer create and use it with innovations that allow for multiplicity, change and difference. For, successful integration of the CMSTA into the curriculum depends on one's ability

to design and use CMC applications that meet course goals, delivery goals or both. Therefore, as part of course planning, teachers must address issues such as course goals, , J s on CMC tools for the classroom will not help unless teachers understand how the technology helps fulfil the goals of the course.

For many students, CMSTA can provide a new avenue for learning one not reliant on time, location, or instructor that allowed them to access information in an exploratory fashion. Further, CMSTA can afford many students a chance to use electronic mail, computer conferencing and synchronous communication with their peers to independently build their own useful knowledge structures. These are additional benefits derivable when the CMSTA is employed. Also, the flexibility and variety inherent in the CMSTA allows teachers to meet their students numerous learning and personal needs, especially when working with individuals with special needs and those who are less mobile or shy.

From the discussions so far, it can be concluded that the CMSTA possesses great potentials in enhancing the teaching/learning process. It is thus on this basis that, at the end of the study, recommendations were put forward on the possibility of its adoption and wide use in the teaching of mathematics. It is believed that for the recommendations to be implementable, the technologies to be deployed should be amenable for usage in the development of a CMSTA. It therefore becomes necessary that an examination of how CMC technologies can be deployed to effectively implement the CMSTA be undertaken. The next section addresses this aspect.

2.3.5 The role of computer mediated communication (CMC) technologies in the CMSTA

Computer Mediated Communication (CMC), which refers to the use of networks of

computers to facilitate interaction between spatially separated persons, employs technologies such as electronic mail, computer conferencing and on-line databases. The most prominent of these technologies electronic mail and computer conferencing has direct bearing on the CMSTA, in that they can be easily used to facilitate sophisticated synchronous (real-time) or asynchronous (delayed) communication during any teaching/learning process.

The power of computer conferencing and electronic mail as learning tools and environments lie in their capabilities to support conversation and collaboration. With the aid of computer conferencing and electronic mail, groups can work together to solve problems, argue about interpretations, negotiate meaning or engage in other educational activities such as coaching, modeling and scaffolding of performance. During conferencing, the learner is electronically engaged in discussions and interactions with peers and experts in a process of social negotiation.

From the above, it could be said that when used in the teaching/learning process, computer conferencing and electronic mail provide the fulcrum for knowledge construction. This occurs when students explore issues, take positions, discuss those positions in an argumentative format, reflect on and re-evaluate their positions. As a result of contact with new or different perspectives, these activities may contribute to a higher level of learning through cognitive restructuring or conflict resolution, leading to new ways of understanding the material (Harasim, 1990). Sharing knowledge through an electronic medium also aids the overt exchange of naturally covert processes and strategies with other on-line learners in order to solve collective or individual problems. These exchanges can be viewed by all learners and contribute to the formation of a collaborative mental model in a specific subject area.

In contrast to the traditional classroom, where the teacher contributes up to 80% of the verbal exchange (Dunkin and Biddle, 1974; McDonald and Elias, 1976), it has been observed that in on- *

10-15 % of the message volume (Harasim, 1987; Winkelmans, 1988). This type of interaction pattern exemplifies the constructivist design model of reciprocal teaching through the use of written rather than verbal dialogue. Originally designed as a procedure for teaching poor readers to approach text as successful readers do (Palincsar et. al, 1989), reciprocal teaching method is easily adaptable to the on-line environment through the fundamental principle of systematically alternating control between teacher and students. Allowing learners to generate questions, summarize content, clarify points and predict upcoming events is also applicable to other educational tasks. When performed online, these activities can facilitate the discussion of various structural relationships within the subject matter. Social negotiation of the structure of ideas represented in the written content may also induce knowledge construction.

Electronic mail, news groups and computer conferencing support the development of discourse communities, groups of individuals who share and discuss common interests and goals. When focused on learning outcomes and scaffolded by different discourse structures, these discourse communities can become more purposive communities of learners and thinkers (Brown and Campione, 1990) or knowledge building communities focusing on problems and depth of understanding, decentralized control, and a broader knowledge community (Scardamalia and Bereiter, 1993/1994). Critical to the knowledge construction process is the articulation of a meaningful purpose for learning. This is the intentional, goal-oriented behavior of the learner while performing the database search that facilitates and strengthens connections between elements of information that results in higher-order thinking and meaningful learning. At this

juncture, it is worth noting that the impact of CMC Technologies in the CMSTA in general and mathematics in particular discussed thus far can be realized only when ICT are properly applied and used. In the next section, an attempt was made to present a review of those applications and uses relevant to the context of this study.

2.3.6 ICT applications and uses in the CMSTA

There are many different kinds of ICT applications and uses. Means et. al. (1993) classified ICT applications not in terms of their base technologies but in terms of how they can be used with students. This classification scheme encompasses tutorial, exploratory, tools and communications use. Tutorial uses of ICT involve using it for drill and practice programs, tutoring systems, satellite transmission of lectures. It is this

mathematics teaching. For, these uses, in essence, entails using the ICT tools to do the same things that teachers have been doing traditionally for more students perhaps more systematically and efficiently. Exploratory programs can be exciting adjuncts to an instructional program, but rarely carry a major part of the core curriculum in practice (Levin and Meister, 1985).

Based on the consideration that if the goal of teaching is to provide students with a different kind of education which will be structured around the provision of challenging tasks that can prepare them for a technology laden world, then the most relevant uses of ICT should be as tools and communication channels. Giving students experiences in selecting appropriate ICT tools and in applying technologies such as word processors, spreadsheets, hypermedia, and network search tools to their work can readily support the performance of complex, authentic tasks and as well provide experiences that prepare them for the world outside of school. It is believed that the CMSTA meets this

aspiration.

Using ICT as tools and communication channels has been referred to as authentic use because students will be using these technologies for the same kinds of purposes and in the same ways that professionals would use them outside the school walls. That is, they would use word processing and desktop publishing facilities in the same way that any practicing professional in a job requiring writing might do. They will search for information resources on the internet using the same kinds of resources and for the same kinds of reasons that practicing researchers or an informed individual seeking broad information might. Authentic uses of technology have advantages over more didactic uses simply because they are flexible. Many tutorial or exploratory ICT applications may not be adopted by schools and teachers if the content they convey does not match the objectives of the particular subject or topic. The consequence of this is that technology application will be used only with few students as "enrichment" or is never adopted at all. In contrast, ICT applications that can be used as a tool or a communication vehicle (e.g., word processing and spreadsheet software, drawing programs, networks) can support any curriculum and can be fully assimilated into a teacher's ongoing core practice (Means et al., 1993).

The following features of authentic uses of ICT is being hoped to be the gains derivable from the deployment of the CMSTA to teach mathematics to the Polytechnic engineering students:

- B GA
  mathematics tasks.
- Integration of ICT into activities that are a core part of the mathematics curriculum.

- Use of ICT as a tool to help accomplish complex tasks rather than a subject of study for its own sake.

Because ICT and mathematics continues to play an important role in modern industrial society, integrating ICT into the mathematics education curricular will help prepare students to succeed in a technologically rapidly changing world. To ensure that ICT is effectively integrated into the mathematics education curricular, mathematics educators, ICT experts and community members must collaborate to create a formal plan. The development of a plan to realize the above, meant more than providing for the acquisition of computers and software. This study may be considered as an effort in that direction.

2.3.7 Pedagogical basis for the use of computers in the teaching/learning process

Increasing recognition of the potential of computer usage in the teaching/learning process has encouraged innovative approaches to the design of both classroom and distance learning. These innovative approaches have contributed to a movement away from the duplication of traditional instructional methods, both in the classroom and at a distance (Turoff 1995) towards more resource-based ones that no longer emphasizes the teacher as the main source of knowledge (Smith and Kelly, 1987; Beaudoin, 1990; Gunawardena, 1992). Garrison (1993) and Crotty (1994) held that this perspective within technology use in education aligns itself with the principles of constructivism. It could thus be said that the pedagogical basis for the use of computers to enhance the teaching/learning process can be located within the principles of constructivism. In this section therefore, an attempt has been made to discuss the principles of constructivism as it relates to the deployment of computers to enhance the teaching/learning process.

Constructivism

Constructivism is a theory based on observation and scientific study about how people learn. It holds that people construct their own understanding and knowledge of the world through experiencing things and reflecting on those experiences, i.e. people are active creators of their own knowledge. Fosnot (1989) describes constructivism by reference to the following four principles:

- learning, in an important way, depends on what we already know
- new ideas occur as we adapt and change our old ideas
- learning involves inventing ideas rather than mechanically accumulating facts
- meaningful learning occurs through rethinking old ideas and coming to new conclusions about new ideas which conflict with our old ideas

The consequences of the above are twofold. These are:

- Teachers have to focus on the learner when thinking about and planning how to structure learning materials rather than the subject/lesson to be taught
- There is no knowledge independent of the meaning attributed to experience (constructed) by the learner, or community of learners

Von Glasersfeld (1995) observes that constructivism is often misconstrued as a learning theory that compels students to "reinvent the wheel." He therefore clarifies that rather than compelling students to reinvent the wheel, constructivism taps into and triggers the student's innate curiosity about the world and how things work. In other words, Von Glasersfeld (1995) is of the view that constructivism does not make students to reinvent the wheel but helps them understand how it turns, how it functions. According to the constructivist, students learn by applying their existing knowledge and real-world experience, learning to hypothesizing, testing their theories, and ultimately drawing

conclusions from their findings.

Knowledge, its nature and how we come to know, are essential considerations for constructivists. In the constructivist perspective, knowledge is constructed by the individual through his interactions with his environment. Constructivist teaching is meant to foster critical thinking which will ultimately create active and motivated learners. Zemelman et. al., (1993) are of the opinion that learning in all subject areas involves inventing and constructing new ideas. They therefore suggest that constructivist theory be incorporated into the curriculum, and advocate that teachers should always create environments in which students can construct their own understandings. Fosnot (1989) recommends that a constructivist approach should always be used to create learners who are autonomous, inquisitive thinkers who question, investigate and reason. For, a constructivist approach frees teachers to make decisions that will enhance and enrich students' development in these areas.

In the classroom, the constructivist view of learning can point towards a number of different teaching practices. In the most general sense, it usually means encouraging students to use active techniques (experiments, real-world problem solving) to create more knowledge and then to reflect on and talk about what they are doing and how their understanding is changing. The teacher makes sure he/she understands the students' preexisting conceptions, and guides the activity to address them and then build on them. Von Glasserfeld (1995) holds that a constructivist teacher and a constructivist classroom exhibit a number of discernible qualities markedly different from a traditional or direct instruction classroom. He further explains that a constructivist teacher is able to flexibly and creatively incorporate ongoing experiences in the classroom into the negotiation and construction of lessons with small groups and individuals. The environment is democratic, the activities are interactive and student centered, and the students are

empowered by a teacher who operates as a facilitator/consultant.

Constructivist classrooms are structured so that learners are immersed in experiences within which they may engage in meaning-making inquiry, action, imagination, invention, interaction, hypothesizing and personal reflection. In this perspective, the teacher is no longer to be considered as an expert, who knows the answers to the

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constructions rather than to construct their own meanings. In a constructivist classroom, students are encouraged to use prior experiences to help them form and reform interpretations.

From the above, it can be said that in a constructivist classroom, the teacher and the student share responsibility in the teaching/learning process and decision making and there is demonstrable mutual respect. This is a democratic and interactive process which allows students to be active and autonomous learners. A constructivist classroom is therefore a student-centered classroom. This consists of learner-centered, active instruction. In such a classroom, the teacher provides students with experiences that allow them to hypothesize, predict, manipulate objects, pose questions, research, investigate, imagine, and invent. The teacher's role is to facilitate this process.

Negotiation is an important aspect of a constructivist classroom. It unites teachers and students in a common purpose. Smith (1993) confirms that negotiating curriculum

-building classes every day to fit ,

(1992) explains that negotiating the curriculum means deliberately planning to invite students to contribute and to modify the educational program, so that they will have a real investment both in the learning journey and the outcomes. Cook (1992) explains why negotiating the curriculum with students is important. Learners will work harder

and better, and what they learn will mean more to them if they are discovering their own ideas, asking their own questions, and fighting hard to answer them for themselves. Out of negotiation comes a sense of ownership in learners for the work they are to do, and therefore a commitment to it. A constructivist teacher therefore is expected to offer his/ her students options and choices in their work. Rejecting the common practice of telling students what to do, he/she engages their trust and invites them to participate in a process which will allow them to be involved in decisions about their learning. Gray (1997) summarily provides the characteristics of a constructivist classroom as follows:

- the learners are actively involved
- the environment is democratic
- the activities are interactive and student-centered
- the teacher facilitates a process of learning in which students are encouraged to be responsible and autonomous

Furthermore, in the constructivist classroom, students work primarily in groups and learning and knowledge are interactive and dynamic. There is a great focus and emphasis on social and communication skills, as well as collaboration and exchange of ideas. This is contrary to the traditional classroom in which students work primarily alone, with learning being achieved through repetition, and the subjects are strictly adhered to and are guided by a textbook.

Central to constructivism is its conception of learning. Von Glasersfeld (1995) argues that from the constructivist perspective, learning is not a stimulus-response phenomenon. It requires self-regulation and the building of conceptual structures through reflection and abstraction. Fosnot (1996) adds that rather than behaviours or skills as the goal of instruction, concept development and deep understanding are the

foci. For educators, the challenge is to be able to build a hypothetical model of the conceptual worlds of students since these worlds could be very different from what is intended by the educator (von Glasersfeld, 1996).

Constructivist beliefs have recently been applied to teaching/learning in the classroom. Because all learning is filtered through pre-existing schemata, constructivists suggest that learning is more effective when a student is actively engaged in the learning process rather than attempting to receive knowledge passively. A wide variety of methods claim to be based on constructivist learning theory. Most of these methods rely on some form of guided discovery where the teacher avoids most direct instruction and attempts to lead the student through questions and activities to discover, discuss, appreciate and verbalize the new knowledge.

Constructivist approaches can also be used in the CMSTA of teaching. In this case, tools such as discussion forums, wikis and blogs can enable learners to actively construct knowledge. What is essentially involved in constructivist strategies and

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than emphasizing characteristics of the final products, process-oriented instruction focuses on the language and problem-

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In a process approach, Langer and Applebee (1987) explain, a context is created within which students are able to explore new ideas and experiences. Within this context, the teacher's role in providing information decreases and is replaced by a strengthened role

-making abilities. In a

process approach to learning, ideas are allowed to develop in th

through a series of related, supportive activities; where taking risks and generating

hypotheses are encouraged by postponing evaluation; and where new skills are learned in supportive instructional contexts. (Langer and Applebee, 1987)

Constructivist activities in any subject area can range from very simple to sophisticated and complex, depending on the teacher's learning objectives. The possibilities for constructivist activities are limitless. It is important, however, regardless of subject area,

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questions that will lead them to develop their own conclusions on the subject. Thus, Jonassen (1999) identified three major roles for teachers, as facilitators, to support students in constructivist learning environments. These are:

- Modeling
- Coaching
- Scaffolding

While constructivism has clearly gained popularity as a new paradigm for learning, however many still question how the philosophy can be operationalized. They argue that it does not provide a method, approach or particular pedagogy. Because there are those who argue that constructivism does not provide a model for implementation; numerous researchers, educators and authors have actively engaged themselves in using constructivist principles to design and implement new learning environments. Towards this end, technology is increasingly being touted as an optimal medium for the application of constructivist principles to learning. It is against this backdrop that the CMSTA employed to teach mathematics to Polytechnic engineering students adopted the constructivist philosophy to produce the pedagogical basis for this study. For,

numerous online environments and technology-based projects are showing that the theory can effectively guide educational practice.

With regards to the above, Jonassen (1999) proposed a model for developing Constructivist Learning Environments (CLEs) around a specific learning goal, which may take one of the following several forms (from least to most complex):

- Question or issue
- Case study
- Long-term Project
- Problem (multiple cases and projects integrated at the curriculum level)

Jonassen (1999) recommends making the learning goals engaging and relevant but not overly structured. Learning is driven in CLEs by the problem to be solved; students learn content and theory in order to solve the problem. This is different from traditional objectivist teaching where the theory would be presented first and problems would be used afterwards to practice theory. This has been taken into consideration during the use of the CMSTA to teach mathematics to the engineering students.

The Role Constructivism in the CMSTA

In an effort to supplement or replace live face to face instruction, technologically mediated learning has more often than not merely replicated the ineffective methods that limit learning in traditional classroom situations (Turoff, 1995). Too often, potentially interactive technologies are used to present one-way lectures to students. This is contrary to the constructivist principles of teaching where the teacher is expected to fully involve the students in the process. In this study, the developed CMSTA is meant to address this shortcoming. Towards this end, it is hoped that the CMSTA will provide ample opportunity for the students to interact together (synchronous aspect of

the system) in order to build a community of scholars and practitioners of mathematics users in their respective engineering disciplines.

The above position agrees with Selfe and Eilola (1989), Bates (1990), Seaton (1993), Nalley (1995) amongst others, that the most valuable activity in a classroom of any kind is the opportunity for students to work and interact together and to build and become part of a community of scholars and practitioners. For, a good learning experience is one in which a student can master new knowledge and skills, critically examine assumptions and beliefs, and engage in an invigorating, collaborative quest for wisdom and personal, holistic development (Eastmond and Ziegahn 1995). Technology used in education should facilitate these good learning experiences in an extended classroom model rather than broadcast teacher-centered lectures and demonstrations (Burge and Roberts 1993). These are based on the constructivist principles and it is from this pedestal that the use of the CMSTA in this study relies heavily on.

A significant consequence of the above goal (i.e. the constructivist approach) is that the CMSTA used to teach mathematics in this study attempts to address the impediments inherent in the fact that many teachers and instructional designers approach technology use in education from traditional backgrounds, bringing with them assumptions about teaching/learning that are not theory-based and do not translate well to technologically mediated instruction (Schieman et. al., 1992). The main purpose of technology use in education is to promote well-designed, mediated instruction that moves the teacher from podium to sideline, from leader to coach, from purveyor of knowledge to facilitator of personal meaning making (Romiszowski and de Haas, 1989; Beaudoin 1990; Gunawardena 1992; Burge and Roberts 1993). This is another pedagogical derivation from the constructivist principles which was considered and adhered to in the employment of the CMSTA to teach mathematics to engineering students. Burge and

Roberts (1993) asserts that technology can be used to create communities of learners and practitioners and can facilitate the interactions and activities necessary for solving real-world problems. This is what has been targeted at by the current study.

From the above, it could be seen that constructivism being resorted to as a pedagogical principle for this study, is meant to help the study reconceptualize the teaching/learning of mathematics so as to significantly alter how we conduct the teaching/learning of mathematics. The means of achieving this, it is hoped will be through the development and use of the CMSTA. The set of guiding principles of constructivism was therefore used to help the researcher create a CMSTA which was believed to be learner centred and having collaborative environments that support reflective and experiential processes. With this, it is hoped that students and teachers can then build meaning, understanding and relevant practice together and go far beyond the mere movement of

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Although some authors have promoted constructivist approaches to technology use in education, none has recommended any specific strategy for accomplishing that goal. In this research, an attempt has been made to offer a number of recommendations for employing CMSTA to support constructive mathematics learning. A constructivist approach to knowledge construction and learning, it is believed, can be well supported in CMSTA through a variety of technologies. It is further believed that these constructivist environments and tools can replace the deterministic, teacher controlled model of teaching with contextualized work environments, thinking tools, and conversation media that support the knowledge construction process in different settings.

Having considered the pedagogical basis for use of computers in the teaching/learning process, which, to a very large extent can also be considered as the psychological consideration that support the use of computers in the teaching/learning process; it becomes necessary to also look at the sociological considerations. This aspect is reviewed in the next section.

2.3.8 Sociological issues in the use of computers for teaching/learning

This section is concerned with the sociological issues that should be taken into consideration when ICT is to be integrated into mathematics teaching sessions in a manner that it becomes the catalyst for better promotion of the acquisition of higher-order mathematics skills by all students. It is believed that if properly done, this integration can have a particularly significant impact on the schooling of even economically disadvantaged students, whose educational experiences have largely been on repetitious rote and drill on lower-order skills, with relatively little attention to the areas of comprehension, problem solving and mathematical reasoning that will support both higher education and effective functioning in the real world.

When a teacher decides to deploy the computer for content delivery, one of his overriding objectives must be to ensure that all members of his learning community have equitable access to the use of the technology that will support the deployment. This entails that the teacher must make plans which will assist him achieve this objective. This is predicated on the line of reasoning that if students do not have equal opportunities to learn, they will be unable to profit most from high-technology applications which in effect will result in unequal opportunities to use these applications. Without equitable access, the gap between the technology "haves" and the "have nots" will widen, further exacerbating the already troubling disparity in the

quantity and quality of educational resources that are available to different populations because of location or socioeconomic conditions.

According to Means and Olson (1995), equitable access to educational technology at school can give students from low-income homes, where there is little or no access to technology, the needed edge to compete with children coming from more affluent homes, where technology is commonplace. Guaranteeing equitable access for all students to affordable education technology in order to achieve curricular goals makes it possible to begin to address the inequities that exist in schools in terms of the availability of instructional resources.

It must however be quickly pointed out that the mere presence of educational technology in schools does not guarantee that it will be or is being used to provide academic benefits to students. For, in some schools, available educational technologies

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students in authentic learning experiences. DeVillar and Faltis (1991) point out that when computers are present in schools serving at-risk students, they are usually used for drill-and-practice programs on basic skills rather than as tools to support students in designing their own projects. Ensuring equity means that all students regardless of class, race, culture, gender, socioeconomic status or physical limitations have equal opportunities to participate in meaningful and authentic applications of educational technology. To realize the benefits of education technology, note Kozma and Croninger (1992), teachers, school administrators, and policymakers must ensure that all students have access to these technologies, that the technologies are used effectively, and that other aspects of schooling also promote high levels of student learning.

To promote student learning, technology must be used in effective, engaging ways. Becker (1992) is of the opinion that the critical questions about equity are about equal access to effective uses of technologies. It therefore follows that to promote meaningful, engaged learning through authentic uses of technology; students must be provided with opportunities to interact with a wealth of resources, materials and data sets. In this study, it is believed that educational technology applications such as the Internet and CD-ROMs which were used at the classroom level helped achieve challenging educational standards, provide powerful alternatives for creating more effective mathematics learning environments which led to more productive learning opportunities.

Equitable access is one of several indicators of high technology performance that teachers can use to guide their technology implementation efforts. To begin their technology planning efforts, teachers should endeavour to make plans for providing adequate technology access and equalizing technology access, as well as strategies for allocating computers to ensure that all students have equal access. This strategy which happens to be within the powers of the teacher was strictly adhered to in this study.

The second strategy for determining equitable access and use of education technology is appropriate funding. This strategy does not lie within the powers of the teacher but he can advise the school to enlist community support in developing funding strategies for education technology policy that recognizes technology as an ongoing investment. It is on this basis that at the end of this study report, suggestions were put forward to educational institutions wishing to adopt/adapt the CMSTA on strategies they are likely to adopt to enlist appropriate funding for the technology that will be used to support the use of CMSTA. Just as appropriate funding contributes to equitable technology use, Terrett (1995) notes that the use of technology itself contributes to equitable use of

funding. According to him (Terret, 1995):

It has been my experience that the effective use of technology allows a more equitable expenditure of school district funds. The use of technology, even though viewed by some as expensive and unnecessary, creates a cost-efficient mechanism that gives students access to materials and resources that were previously unavailable.

A final strategy in ensuring equitable use of education technology is continous professional development of teachers in the technologies and their applications. Teachers must have knowledge and experience with the vast range of educational technology and must learn strategies for using it effectively in the classroom (Hilliard, 1992). Ramirez and Bell (1994) note that professional development must be put in place to support teachers as they attempt to implement technology in the most efficient way possible as well as help them identify the most effective curricular models for their classrooms.

Professional development issues require institutions to consider how they will help teachers learn about technology equipment and its applications. To ensure that students have equitable access to technology, teachers must become knowledgeable about the te

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support curricular goals and meaningful instruction for all students, it reflects the indicators of engaged learning. Teachers need a reason to use the technology in the classroom and therefore they should be given opportunities to collaborate in developing projects that apply technology to student learning. To ensure the equitable implementation of technology throughout the curriculum, professional development should include strategies for involving a majority of teachers in technology use and providing technical support for technology use and maintenance. Equal learning

opportunities for students rely in part on the commitment of schools and government to invest in teachers by providing ongoing professional development and access to on-site support personnel responsible for troubleshooting and assistance after the technology and lessons are in place.

2.3.9 Problems and prospects of the CMSTA

Over the years, there have been several calls from various quarters for the desirability and need to promote and facilitate the use of ICT in teaching. Today, ICT tools like the Integrated Virtual Learning Environment (IVLE), webcast lectures and PowerPoint presentations are commonly used by lecturers in countries with developed and emerging economies. As these are tools employed in the CMSTA, it follows that the problems and prospects that accompany their usage are also applicable to the case of their usage in the CMSTA. In this section, an attempt has been made to review the main problems and prospects of the CMSTA.

The four main problems that might likely confront the deployment of the CMSTA as identified from literature bothers on technical, pedagogical, economic and strategic issues. Two main technical issues that are most likely to arise when the CMSTA is to be used to enhance learning are the problems associated with using multimedia resources (primarily, sound); and the emergence of new ways for enhancing the interactivity of Electronic Course Delivery (ECD) materials. On using audio files to explore augmentation issues relating to online lectures, Barker and van Schaik (1998) reports that the use of sound and motion video can be problematic due to the large file sizes that are required and the significant bandwidth that is needed to transmit these. While it is possible to move a sound file over a fast intranet link in a reasonable time-scale, the time involved in transmitting the same file over the Internet could prove unacceptable

depending upon the other traffic that is simultaneously flowing through the network.

Another problem identified by Barker and van Schaik (1998) in relation to the augmentation of electronic lectures was the limited interactivity that could be achieved with presentation application packages such as PowerPoint. However, the researcher has discovered from the works of others that to a certain extent, many of the limitations could be overcome by using Hyper Text Markup Language (HTML) for creating online presentations.

There are five main pedagogic issues to be considered when the CMSTA is to be deployed for content delivery. These are those concerned with:

- the nature and role of augmentation
- the amount of face-to-face contact that should be provided
- monitoring and assessment
- the cost effective use of hypermedia
- how to employ network resources to promote new approaches to learning based on collaboration both in the classroom and at a distance.

Augmentation of lectures is an important issue with respect to using lecture materials in a self-study context. There are variety of different ways of achieving this - such as using digital audio narratives, pop-up notes and lecture transcripts (Tan, 2007). It is the resources available on the internet, for instance, are to be best utilized in conjunction with the other facilities that the network infrastructure (such as email, chat and conferencing) provides. According to Tan (2007), one important decision that has to be made relates to the amount of face-to-face contact that should be provided for students. Unfortunately, the researcher is

unaware of any well-

make a decision on this. This may be an area of future research.

Monitoring and assessment tools are important feedback mechanisms for both staff and students within any ICT environment. Because of the substantial improvements that

eriences, it is the

investigation in the future. The use of hypermedia techniques within the context of augmenting electronic lectures is an important issue that has to be considered. There is substantial evidence to show that, when correctly used, hypermedia can bring substantial benefits to online and offline teaching/learning processes. However, the more hypermedia that is used, the more expensive it becomes to develop the resources. The question of optimality therefore arises. That is, given that only a limited amount of time and effort can be used to develop hypermedia materials, we need to consider strategies for using the available resources in an optimal way. This is an important issue which teachers intending to use the CMSTA should explore in some depth.

The availability of web-based teaching/learning facilities opens up many new possibilities for the development of new techniques to support collaborative learning both in the classroom and at a distance. The Virtual Classroom (VC) paradigm can be employed to explore these possibilities for fairly simple task domains (such as questionnaire design and analysis) based on the use of electronic mail. The economic production of CMSTA is an issue which must be taken into consideration if the adoption of the approach is meant to be sustainable. Naturally, creating quality ECD materials require quite a substantial investment in time and effort and the provision of an appropriate in-house technical support infrastructure. Therefore, institutions wishing to adopt the CMSTA must take this variable into consideration when making its budget.

Once ICT resources for the CMSTA have been mounted, they can easily be made available globally via the World Wide Web. It is possible to augment available teaching resources by providing students with access to such facilities as an electronic library and various (simulated) laboratory environments. In addition, the provision of automated management, teletutoring and administration facilities means that it is feasible to construct a VC environment that only exists within the confines of Internet servers (e.g. the World Lecture Hall at http://www.utexas.edu/lecture). As discussed by Barker et. al., (1998a; 1998b), ECD materials, used either alone or within the framework of a VC environment, can be employed in order to make educational opportunities more widely available to all through the provision of lifelong learning facilities.

On the part of the teachers who may be charged with the responsibility for translating the CMSTA into practical reality, the following challenges they are most likely to face have to be also strategically resolved:

- Teachers often reluctance to invest the time required to convert materials to an appropriate ICT format.
- Appropriate use of ICT involves using its features to enhance the teaching/learning experience and this may be daunting to some who do not wish to spend significant time developing their understanding of the pedagogical opportunities and limitations offered this technology
- There is always reluctance, particularly among the older and less technology-orientated teachers, to adopt new technologies.

In addition to the above, it is important that the following are also taken into consideration:

- There may be a shortage of some key technological elements (e.g. computers).

This is a significant resource issue for many institutions in Nigeria.

- There are several risk factors associated with using educational technology. The main ones being: equipment failure, file corruption caused by magnetic or physical damage so that the presentation will not run, incompatible media and lack of appropriate training in both the programme and the technology.

The resolution of the above will guarantee that the following prospects of the CMSTA can be derived when it is deployed to facilitate the teaching/learning process:

- It renders uniform the level of knowledge by reaching all types of students whatever their knowledge or background and location.
- It enables the creation of tools which permit the visualization of physical phenomena and concepts that might otherwise be difficult to grasp.
- Appropriate use of ICT can enhance and enrich teaching/learning experience for both teachers and students.
- It provides encouragement and support to teachers by facilitating the structuring of their presentation in a professional manner.
- It can be made appealing to a number of different learning styles and be made more stimulating. It allows distribution and modification of the ECD materials for/by students unable to be present
- Editing of the ECD materials is very easy with no associated reprinting costs.
- The portability of the files, especially on compact disks (CDs) with their large capacity, allows presentations to be given wherever the technology is available or distributed where appropriate. Presentations can also be set up to run automatically if required e.g. as demonstrations/instructions within a laboratory.

So far, the potential utility of the CMSTA as an educational access and delivery mechanism has been discussed. The techniques involved in using it for teaching/learning have not been illustrated because it is outside the scope of this section. The researcher is very optimistic about the future potential that the CMSTA offers when implemented by means of intranet technology - and supported by the appropriate use of the Internet and the World Wide Web. Indeed, used in the correct ways, the CMSTA offers the potential for creating significant enhancements with respect to improving the possibly, at substantially reduced cost compared with the conventional methods of presenting course material. This, in effect, can guarantee the achievement of the goals of sustainable development.

2.4 Mathematics Performance/Achievement

Several research efforts have been and perhaps, is being undertaken to examine issues bordering on the factors affecting mathematics performance/achievement, reasons for poor mathematics performance/achievement, methods of improving student achievement in mathematics, measurement of mathematics performance/achievement, the nature of the relationship between mathematics performance/achievement and various mathematics related psychological constructs such as mathematics anxiety, attitude towards mathematics, teaching methods and learning styles; amongst others. In as much as one of the focuses of this study is on the relationship between mathematics performance/achievement and a particular teaching method (i.e. the CMSTA), the next sections will attempt to review the above issues in order to enable us fully explore the impact of the CMSTA on mathematics performance/achievement of the subjects of the study at the end of the study.

2.4.1 Factors influencing mathematics performance/achievement

A A , , s (1966) report, extensive research bordering on in and out of school variables affecting students' mathematics performance/achievement has been carried out. Research findings encountered in literature show that mathematics performance/achievement is influenced by factors such E I * 0. . 49 A -Petruso et. al., 2004; Sprigler and Alsup, 2003; Fennema and Sherman, 1977; Janson, 1996), attitude towards mathematics (Papanastasiou, 2002; Schreiber, 2000; Alrwais, 2000; Genshaft; 1982), mathematics anxiety (Ma, 1999; Goleman, 1996; Ashcraft and Faust, 1994; Hembree, 1990; Wither, 1988), mathematics self-concept (Hamachek, 1995; Franken, 1994; Marsh, 1993; Wilhite, 1990; Bryen and Shavelson, 1987; Campbell, Connolly and Pizzo, 1986), teaching methods (Stuart, 2000; Russell, 1995; Barnard, 1995; Gudan, 1995; Dossel, 1993), schools climate and culture (Blum and Libbey, 2004; Fullan, 2001), home background and environment (Alomar, 2006; Engheta, 2004; Fullarton, 2004; Blum and Libbey, 2004; Mullis et. al., 2004; Marjoribanks, 2002; Janson, 1996; Robitialle and Garden, 1989; Beane and Lipka, 1986),

Gender difference, as a factor affecting mathematics performance/achievement, has , D (1977) early research on this issue indicated that males outperformed females in mathematics performance/achievement at the junior high and high school levels. Sprigler and Alsup (2003) reported discrepancies in mathematics performance/achievement among girls and boys in school. Janson (1996) is of the opinion that the history of gender difference in mathematics performance/achievement indicated that male superiority in this area is a universal phenomenon. Gallagher and Kaufman (2006) recognized that the mathematics performance/achievement and interest

of boys are better than the g , A -Petruso et. al., (2004) have shown that gender differences in mathematics performance/achievement become apparent at the secondary level when female students begin to exhibit less confidence in their mathematics ability and perform lower than males on problem solving and higher level mathematics tasks.

There are however research findings which did not support male superiority or any gender difference in mathematics performance/achievement, B , , 0. . 5' findings from a longitudinal study of gender differences in mathematics showed equivalent growth trend in mathematics performance/achievement among both genders during the study times, which they contend, implies that there is no difference among boys and girls in mathematics performance/achievement. Also, Mullis et al. (2004) reports that findings from an international study conducted by IEA showed that averagely, there was no significant difference in achievement between boys and girls at either the eighth or fourth grade across all countries studied. Furthermore, Kiamanesh (2006) reported that findings of two consecutive international studies conducted by TIMSS (1999 and 2003) on Iranian educational system also confirmed that there is no significant difference between boys and girls in mathematics performance/achievement.

While the controversy on the existence or not of gender difference in mathematics performance/achievement is on-going and may probably not be conclusive, advocates of the existence of this difference have gone further to attempt identifying variables that account for the gender difference. Their purpose was that if these variables are known, it may be possible to reduce gender inequality in mathematics performance/achievement A -Petruso and Miranda, 2004). In this regard, different theories have been advanced to account for gender difference in mathematics performance/achievement. Research evidences encountered during literature search show that gender differences in

mathematics performance/achievement are due to factors such as biological differences between both genders (Geary et al., 2000), mathematics learning strategies (Carr and Jessup. 1997), sex hormones on brain organization (Kimura, 2002) and symbolic gender (Nielsen, 2003).

The biological theorists argue that innate differences in spatial ability, higher order thinking, or brain development produce a gap in achievement. Societal explanations focus on how girls are socialized into believing that mathematics is not important, useful, doable, or part of the identity of a girl (Wilder and Powell, 1989). In addition to the above-mentioned theories, gender similarities hypothesis argues that boys and girls are similar in most, but not all, psychological variables (Hyde, 2005). The gender stratification hypothesis proposes that in a society with more societal stratification based on gender, and more inequality of opportunity, girls will report less positive attitudes and more negative affect and will perform less well on mathematics performance/achievement tests than will their boy peers. Gender stratification C /761' -value theoretical model. According to this model, cultural inequities in educational or career opportunities have , * /757' * * of positive attitude toward and participation in mathematics.

The relation between mathematics attitude and mathematics performance/achievement has been extensively investigated. Schreiber (2000) found that students who performed better on a mathematics test tended to have positive attitude toward mathematics. Alrwai 0. . . ' predictor of mathematics performance/achievement. Based on TIMSS data, Papanastasiou (2002) reported a direct relation between mathematics attitude and

mathematics performance/achievement. Hadfield et. al., (1992) found a strong correlation between the variable of persistence and degree of mathematics anxiety, thus

thought. Genshaft (1982) notes that more positive attitude accompany lower level of anxiety which can enhance increased gains in the future.

Many researches undertaken to establish the nature of the relationship between mathematics anxiety and mathematics performance/achievement have shown that mathematics anxiety level of students is a strong determiner of mathematics performance/achievement. Hembree (1990) noted that mathematics anxiety seriously constrains performance in mathematical tasks and reduction in anxiety is consistently associated with improvement in achievement. Ashcraft and Faust, (1994) posits that highly mathematics anxious individuals will be less fluent in computation, less knowledgeable about mathematics and less likely to have discovered special strategies and relationships within the mathematics domain.

Quantifying the potential improvement in mathematics achievement when mathematics anxiety is reduced, Ma (1999) found that the relationship between mathematics anxiety is significant in mathematics achievement from grade 4 and that once mathematics anxiety takes shape; its relationship with mathematics achievement is consistent across grade levels. Goleman (1996) opines that the influence of anxiety on performance, including mental performance is such that too little anxiety results into apathy or too little motivation to try hard enough to do well, while too much anxiety sabotages any attempt to do well. According to Wither (1988), number manipulation anxiety and test anxiety showed significant inverse relationships with respect to mathematics achievement. This implies that the achievement of learners who are anxious about tests and number manipulation techniques are likely to be poor. All these research points to

one direction, i.e. the presence of high mathematics anxiety lead to low mathematics achievement

Studies on the relationship between self-

educational settings have been a major focus of research for many years (Hamachek, 1995). Most of these researches have supported the belief that there is a persistent and significant relationship between self-concept and academic achievement and that a change in one seems to be associated with a change in the other. In a major longitudinal study Brookover, Erikson and Joiner as cited in Hamachek (1995), investigated the relationship between self-concept of ability and academic performance of more than 1000 male and female students from the time they started seventh grade to the time they completed 10th grade. They found out that self-concept was a significant factor in achievement at each grade level studied. Low self-concept tends to appear together with students' underachievement. Most findings in this area showed that those who have higher self-concept, i.e., having more confidence in mathematics, gain higher scores in mathematics (Wilhite, 1990). Not only self-concept influences students' mathematics achievement (Bryen and Shavelson, 1987, Campbell, Connolly and Pizzo, 1986), but also as Franken (1994) concludes, it forms the basis of all motivated behaviors.

Marsh (1993) found a very specific relationship between self-concept and academic achievement. Marsh (1993) reports that general self-concept and non-academic aspects were found not to be related to academic achievement while general academic self-concept was related moderately to academic achievement measures. Specific measures of subject-related self-concepts were found to be highly related to academic success in that content area. Research has also supported the view that academic self-concept and academic achievement are mutually reinforcing to each other to the extent that a positive (or negative) change in one facilitates a commensurate change in the other. This

relationship has been described as interactive and reciprocal. Achievement at school -concept (Hamachek, 1995). According to Hamachek, (1995) students who have positive self-concepts are more likely to establish realistic, achievable goals, tend to be task-persistent, take school work more seriously, are able to work more independently, have a higher degree of curiosity, prefer challenging work and have had few school failures than their low self-concept counterparts.

Another determinant of mathematics performance/achievement found in literature is

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approach has an effect on learners achievement. Dossel (1993) opines that the atmosphere in the classroom, including perceived warmth, may lower anxiety and improve mathematics performance. According to Stuart (2000), students like to do what they are good at and to make them feel good about mathematics, teachers need to build up the self-confidence and refine the skills required to be successful at mathematics. These postulations border heavily on teaching methods. Teaching methodology has been shown by various researchers to have positive effects on mathematics performance/achievement. For instance, Russell (1995) and Barnard (1995) in two separate studies, showed that the Kumon method, which is a specific teaching method, has positive effects on mathematics performance/achievement. In another study by Gudan (1995), the effectiveness of an individualised format of teaching versus the traditional format was investigated. It was found that the individualised approach provided a more positive experience and had an effect on decreasing mathematics anxiety. This implies that different teaching methodologies affect mathematics performance/achievement.

Research has shown that schools climate and culture are also factors influencing achievement in general. Bulach et. al., (1995) found that schools climate and students

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morale and improve their achievement (Freiberg, 1998). It was earlier reported by some researchers that ability and family backgrounds of students (Coleman et al., 1966; Jencks et al., 1972) are factors that determine academic achievement and that the influence of schools climate is dependent on this factor. However, Raudenbush and Willms (1991), in their criticisms of these early works point out that the procedures employed was not able to separate out accurately school, student and classroom factors because account was not taken of the hierarchical nature of the data used. Later school effectiveness research has used multi-level modelling techniques to account for the clustering effects of different types of data. Bosker and Witziers (1996), in their meta-analysis of school effectiveness research using multi-level modelling techniques, showed that school effects account for approximately 8 to 10 per cent of the variation in

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language.

In support of schools climate as a factor in academic achievement, other researches (Lamb, 1997; Mortimore et al., 1988; Nuttall et al., 1989; Smith and Tomlinson, 1989) showed that there are substantial variations between schools in terms of academic achievement. For instance, Scheerens (1993) and Scheerens et. al, (1989) concluded that classrooms as well as schools are important and that teacher and classroom variables account for more variance than school variables. Schmidt et al. (1997) in their comparison of achievement across countries using TIMSS data reported that classroom-level differences accounted for a substantial amount of variation in several countries. Further research into the schools climate variables affecting achievement indicate that these differences are due to differences in teacher factors (i.e their qualifications, teaching methods, anxiety, attitude etc.), classroom organisation, pupil management

, K 0. . . ' -level modelling techniques to modelled the impact teachers had on achievement growth and reported that over 30 per cent of the variance in pupil progress was due to teachers. The report concluded that rather than other classroom, school and student factors; teacher quality and effectiveness are la ,

Hill et al. (1996) gave an account of a three-year longitudinal study of educational effectiveness when they examined student, class/teacher and school differences in mathematics and English achievement using multi-level modeling procedures. In the first phase of the study, they found that at the primary level, 46 per cent of the variation in mathematics was due to differences between classrooms, whereas at secondary level the rate was almost 39 per cent. Further analyses showed that between-class differences were also important in examining student growth in mathematics achievement and that differences in achievement progress located at the classroom level ranged from 45 to 57 per cent (Hill et al., 1996; Hill and Rowe, 1998). In explaining the large classroom-level differences in student achievement in mathematics, Hill et. al. (1996) highlighted the role of teacher quality and teacher effectiveness. They contended that, although not fully confirmed, evidences can be found of substantial differences between teachers and between schools on teacher attitudes to their work and in particular their morale, which they said supported the view that it is primarily through the quality of teaching that effective schools make a difference.

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(Hammouri, 2004, Antonijevic, 2005). According to Butty (2001) instructional practices has impact on mathematics' achievement as well as attitude toward mathematics. A supportive classroom and suitable teaching motivate students to become better

mathematics learners (House, 2004). Teachers who do not receive support in their work may be less motivated to teach and perform well in the classroom (Ostroff, 1992). Some

mathematics achievement and attitude toward mathematics (Butty, 2001).

School connectedness, which is the belief held by students that whether adults in the school care about their learning and about them as individuals (Blum and Libbey, 2004), was reported to also be a key factor in academic achievement. According to House (2005), if students know that their teachers care about them and have been, they usually have clear and reasonable expectation and can get better scores. Supportive teachers play a significant role in student's engagement in school (House, 2005).

* ate and culture (Fullan, 2001) and school connectedness or engagement (Blum and Libbey, 2004) are two key factors for students' success. Freiberg (1998) points out that a positive school climate can promote higher morale, and improve student achievement. Bulach et. al. (1995) have also shown that there is a significant correlation between student's achievement and school climate. According to Klem and Connel (2004) students engaged in school are more likely to get higher test scores. Schools with high SES students are more likely to have greater support from parents, fewer disciplinary problems, and more chance to attract talented and motivated teachers (Ma and Willms, 1999). Students connected to their school report more cohesion and less friction among classmates and less likely to engage in conduct problem.

The home background variable may be broken down into components such as the

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status. Many studies indicate that the educ C *

2004; Beaton et al., 1996; Robitialle and Garden, 1989), home educational resources (Mullis et al. 2000), socioeconomic status of the family (Marjoribanks, 2002), home

language versus language of test (Howie, 2002) and providing quality homework assistance by parents (Engheta, 2004) are among factors that can explain variance in

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and influences them both directly and indirectly through the kind of relationship the family members have among themselves as well as through helping them to get in contact with the society (Weiss and Krappmann, 1993). Fullarton (2004) asserts that

evement in mathematics.

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performance, found that insufficient parental income, family type and lack of funding by governments are factors influencing students' academic performance. Using the concepts of financial capital, human capital, and social capital, Chow (2000) attempted to disentangle the major factors which affected the academic performance of 368 recent Hong Kong immigrant students attending 26 different public high schools in Toronto. Results of the ordinary-least square regression analysis used indicated that presence of father in Canada, higher self-rated socio-economic status, immigration to Canada being politically motivated, and higher level of English proficiency were significantly and positively related to academic performance.

From the review of the factors influencing academic achievement in general and mathematics achievement in particular, it can be concluded that good or poor achievement in mathematics can be regarded as a function of how those factors are

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the one most readily available to teachers who are interested in improving their

ment. It is from this perspective that an attempt was made in this study to develop a CMSTA framework which was used to teach the subjects of the study engineering mathematics.

2.4.2 Measurement of mathematics performance/achievement

As in the measurement of academic performance/achievement, mathematics performance/achievement is usually measured or estimated by performance on a test, examinations or continuous assessment. No general agreement was found in literature on how best to measure mathematics performance/achievement or which aspects (i.e. procedural knowledge such as skills or declarative knowledge such as facts) of it are the most important to be measured. It was however found that one source of information should not be used to assess it. For, a student may demonstrate knowledge on one aspect and not on another. Using good strategies to assess mathematics performance/achievement from multiple sources will ensure good information and the best possible educational planning.

2.5 Attitude Towards Mathematics

2.5.1 Classifications of attitude towards mathematics

Zan and Di Martino (2007) observe that the term attitude is used in both practice and research together with the adjectives positive/negative and that the dichotomy between positive/negative attitude pervades mathematics education research, both implicitly and

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mathematics clearly depends on the definition of attitude itself. They further observe that in most studies the choice of a definition for attitude, and consequently a

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often it is not made at all, and the assessment/measurement instruments used by the researcher implicitly end up by continuously wavering between various definitions

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negative is in most cases simply the result of a process of measurement, performed through instruments such as the Thurstone or Likert attitude scales or the semantic differential technique. This process ends up in a score - attitude - obtained by summing points relating to the single items. The choice of scores to be assigned to the items naturally leads to a positive/negative evaluation of each one.

Depending on the criteria used to evaluate attitude, different results may be obtained. D * * * , oblem is only apparently overcomed when the algebraic sum of the two components results in a single evaluation. Furthermore, as observed by Zan and Di Martino (2007), beliefs are often used to assess the significance of the emotional dimension or evaluated according to * ,

choices of assessment/measurement instruments: it also triggers a different formulation of the research problem to be dealt with. For example, the problem of identifying how * *

completely different approach depending on whether the positive attitude refers only to the emotional component or it refers to a particular pattern of beliefs and emotions, to be assumed as a model.

2.5.2 Factors

e toward mathematics

are:

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  Nason, 2004; Wilkins and Brand, 2004; Swan et. al., 2000; Grouws and Cramer, 1989; Schoenfeld, 1985),
- Teaching style and behavior (Harkness et. al., 2006; Chesebro, 2003; Stage, 2000; Wanzer and McCroskey, 1998; Thompson and Thompson, 1989; Adams, 1989; Midgley et. al., 1989),
- Teaching techniques (Whitin, 2007; Anderson, 2005; Elliott et. al., 2001; Kinney, 2001; Raymond and Leinenbach, 2000; Pearce et. al., 1999; Mitchell, 1999; Yusof and Tall, 1999; Townsend et. al., 1998; Higgins, 1997),
- Parent attitudes and beliefs,
- Assessment and Achievement (Hannula 2002, Tapia and Marsh 2001, Lopez et. al, 1997, Midgley et. al., 1989).

There are many relationships among these five factors. The first four factors are external

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have been building and been influenced throughout their lives. It is this

attitudes towards mathematics.

2.5.3 Measurement of attitude towards mathematics

According to Leder (1992), while researchers in recent times may have acknowledged that attitude is a multi - dimensional construct which requires a multidimensional

measure, there is however some dispute as to which components of attitude, such as anxiety, enjoyment, self-concept and belief in the usefulness or value of mathematics, provide the best indicators of attitude.

Several scales have been developed to assess attitude toward mathematics in adults and adolescents. Generally, each scale includes different factors. For example, Sandman (1973) as well as Fennema and Sherman (1976) developed and administrated questionnaires to secondary school students which evaluate a variety of feelings and beliefs pertaining to mathematics. Factors assessed in their scales include attitudes about success in mathematics, perceptions of encouragement by parents, perceptions of

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of mathematics, mathematics anxiety, motivation in mathematics and utility of mathematics. Another widely used measure is the Attitudes Toward Mathematics Instrument (ATMI) developed by Tapia and Marsh (2004). The report of a confirmatory factor analysis which they (Tapia and Marsh, 2004) undertook indicates that there are four main factors that determine ATMI. These factors, which are self-confidence, value, enjoyment and motivation, were included in the ATMI. Their (Tapia and Marsh, 2004) use of the instrument in a study led to their conclusion that the ATMI is valid for high school students and college students as well. This accounts for its adaptation for use in this study.

2.6 Mathematics Anxiety

2.6.1 Forms of mathematics anxiety

Two forms of mathematics anxiety identified by Gierl and Bisanz (1995) are:

- mathematics test anxiety, which is defined as feelings of nervousness associated with past, present and future mathematical test taking situations.
- mathematics problem-solving anxiety, which is defined as feelings of nervousness associated with situations in and out of school that require learners to solve mathematics problems and use the solutions in some way.

Gierl and Bisanz (1995) reports that results of studies conducted pertaining to mathematics anxiety have shown that learners become more anxious about mathematics testing situations as they progress through school. What then are the causes of mathematics anxiety? The next section is devoted to a discussion on the causes of mathematics anxiety.

2.6.2 Causes of mathematics anxiety

A number of different factors have been ascribed to be the causes of mathematics anxiety. Norwood (1994) attributes the causes of mathematics anxiety to include the inability to handle frustration, excessive school absences, poor self-concept, parental and teacher attitudes towards mathematics and emphasis on learning mathematics through drill without understanding. Stuart (2000) believes that a lack of confidence when working in mathematical situations is the cause of mathematics anxiety. Dossel (1993) identified the following factors as those leading to the creation of mathematics anxiety:

- Personality factors (the belief that success cannot be attributed to effort

feelings associated with lack of control).

- Pressure from perceived authority figures such parents, teachers etc.
- Time pressure (to answer quickly and verbally).
- Effect of public failure (asking to perform in front of a class).
- Right wrong dichotomy.

Some authorities believe that the beginnings of mathematics anxiety can often be traced to negative classroom experiences and the teaching of mathematics (Stodolsky 1985; Williams 1988). Therefore Newstead (1998) advocates the importance of examining classroom practices so as to establish whether the causes of mathematics anxiety may be in instructional methods and in the quality of mathematics teaching. Greenwood (1984) states that the principal cause of mathematics anxiety lies in the teaching methodologies , F practice ,

He went on to explain that teachers create anxiety by placing too much emphasis on memorizing formulae, learning mathematics through drill and practice, applying rote-memorized rules and setting out work in the traditional way. As for Butterworth (1999), a lack of understanding is the cause of anxiety and avoidance and that understanding based learning is more effective than drill and practice.

Another source of mathematics anxiety that has been identified is word problems. Tobias (1978) believes that word problems are the heart of mathematics anxiety. According to Tobias (1978), learners need higher levels of reasoning and if not taught strategies to solve these problems, they may grow up avoiding mathematics and science. Ashcraft and Faust (1994) identified the degree of accuracy and ease at which numbers can be manipulated as a cause of mathematics anxiety. According to them mathematics

anxiety is the result of nervousness about the required manipulation of numbers in mathematics classes including tests, homework or in-class instruction. Martinez (1987) identified a significant component of mathematics anxiety to be the fear of failure.

anxiety for learners. Martinez (1987) states that a

likely to be transmitted to their students. Martinez (1987) reports a study undertaken to determine the underlying anxieties of teacher trainees which found that many had gaps in their mathematics knowledge or an awareness of imperfectly learned concepts which in turn, can be transmitted to the learners they teach.

Another source of mathematics anxiety has been found to be attributable to the effect of having to perform and provide explanations in front of teachers or peers. In a study focusing on mathematics anxiety between the ages of nine and eleven, Newstead (1998) found that learners reported a significant amount of anxiety about the social, public aspects of doing mathematics in the presence of teachers and peers. There were some learners who expressed the fact that this was the only cause of anxiety for them, and that doing sums and working with numbers was not a source of anxiety (Newstead 1998).

2.6.3 Consequences of mathematics anxiety

According to McCulloch Vinson et. al. (1997), some of the consequences that result from being mathematics-anxious as opposed to mathematics-confident include:

- the fear to perform tasks that are mathematically related to real life incidents, such as dividing a restaurant bill amongst diners or developing a household budget,
- avoidance of mathematics classes,
- the belief that it is alright to fail/dislike mathematics,

- feelings of physical illness, faintness, fear or panic e)
- an inability to perform in a test or test-like situations, and
- the utilization of tutoring sessions that provide little success.

Some commonly held beliefs about mathematics identified by Kogelman and Warren (1978) still hold true today and these are associated with mathematics-anxiety and mathematics avoidance (Frank, 1990). According to Sam (1999) these are; (a) Inherited

must always know how you got the answer, (c) there is one best way to do a mathematics problem, (d) mathematics requires a good memory (e) men are better at mathematics than women (f) it is always important to get the answer exactly right (g) mathematicians solve problems quickly in their heads (h) it is bad to count on your fingers. The implication of such negative beliefs and negative school mathematics experiences on many primary teacher education students has resulted in the continuity of the mathematics-anxiety phenomenon. Of concern is the persistent argument found in the research literature for the transference of mathematics-anxiety from teacher to

According to Trujillo et al. (1999), feelings of anxiety can lead to panic, tension, helplessness, fear, distress, shame, and an inability to cope, as well as physiological symptoms such as sweaty palms, stomach difficulty relating to nervousness, breathing difficulty and the loss of the ability to concentrate. Ashcraft (2002) observes that students with high mathematics anxiety will avoid situations in which they have to perform mathematical equations. He further asserts that mathematics avoidance leads to less mathematics competency, exposure and practice. These make students more anxious and mathematically unprepared to achieve. In college and university,

mathematics anxious students take fewer mathematics courses and tend to feel negatively towards mathematics. In fact, Ashcraft (2002) found that the correlation between mathematics anxiety and variables such as confidence and motivation are strongly negative. According to Ashcraft (2002), because mathematics anxiety can cause mathematics avoidance, an empirical dilemma arises. For instance, when a highly mathematics-anxious student performs disappointingly on a mathematics question, it could be due to mathematics anxiety or the lack of competency in mathematics occasioned by mathematics avoidance. Ashcraft (2002) reports that by administering a test that becomes increasingly more mathematically challenging, he noticed that even highly mathematics-anxious individuals do well on the first portion of the test measuring performance. However, on the latter and more difficult portion of the test, there was a stronger negative relationship between accuracy and mathematics anxiety.

Research studies (Ikegulu, 1998) have found that mathematics anxiety is related to test anxiety because it surfaces most dramatically when students are under evaluation or perceives that they are under evaluation. Although early researches suggest that the term

phenomenon (Olson and Gillingham, 1980), more recent researches into mathematics anxiety has recognized it not only to be more complex than general anxiety but also mor G */776',

Research findings have shown that mathematics anxiety can have a negative effect on the ability of a learner to learn. One of the consequences of mathematics anxiety as stated by Goleman (1996) is that learners who are anxious cannot take in information efficiently or deal with it well, resulting in not being able to learn. According to E /774'

present and the learner is unable to hold in mind all information relevant to the task in

hand which results in him/her not being able to think straight. Skemp (1986) similarly states that anxiety becomes debilitating in terms of performance and higher mental activities and perceptual processes. Strong emotion blocks reasoning and learners under pressure try to remember rather than understand, causing them to be handicapped mathematically (Wells, 1994).

Ashcraft and Faust (1994) noted that highly mathematics anxious learners tend to avoid the distressing mathematics stimuli. This has far reaching consequences as highlighted by Hembree (1990) who stated that when otherwise capable learners avoid the study of

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base in science and technology. In a study by Chipman et. al. (1992), it was found that at every level of mathematical skill, mathematics anxiety had a negative correlation with interest in scientific careers.

Research evidences have shown that the speed and accuracy at which learners complete mathematics tasks is dependent on the anxiety that they experience. Ashcraft and Faust (1994) reported that a group of low anxious learners were consistently more rapid and accurate than a medium anxious group who were consistently the slowest. The most anxious group were the most prone to errors. High Mathematics anxious learners were willing to sacrifice accuracy in order to just finish the task (Ashcraft and Faust 1994). This perspective informs the choice of mathematics anxiety as one of the variables this study intends to focus on.

2.6.4 Treatment of mathematics anxiety

This section gives an overview of some research findings dealing with the treatment of

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as psychological treatments. Gentile and Monaco (1988) state that the teacher is in a

position to influence development of learned helplessness by changing beliefs about the causes of success and failure. In this regard, Gentile and Monaco (1988) posited that the teacher can prevent learned helplessness by:

- Providing learners with success experiences.
- Providing learners early with attributions about effort, persistence and strategy.
- Deemphasizing attributions that would defy native ability and aptitude.

Different teaching methodologies can be used to reduce mathematics anxiety. Vacc (1993) believes that a personal and process-orientated teaching method which emphasizes understanding rather than drill and practice reduces anxiety. According to Greenwood (1984), problem solving and the discussion of various strategies for solving problems are important for the prevention of mathematics anxiety. Martinez (1987) provides the following guidelines to teachers for creating an anxiety-free mathematics class:

- Match instruction to cognitive levels

  cognitive development, frustration and anxiety can be prevented.
- Make numbers real by exploring and using examples from everyday living.
- Mastery learning by reducing the risks and consequences of failing by competing only with themselves, working at the individuals own pace.
- Teaching through play. Mathematics games and puzzles motivate learning. Early learning experiences that are creative and unpressured reduce early anxiety.

Maree (1992) provides the following guidelines for reducing mathematics anxiety:

- Provision of learning relaxation techniques.
- Re-evaluating test situations by changing attitudes from negative to positive.

Poor results do not mean a learner is a failure and will never pass.

- Realise that results have no bearing on the learner as a person.
- Setting realistic goals.
- Developing good study methods.

Martinez (1987) believes that mathematics anxiety often begins with the teacher. According to him, mathematics anxious teachers are very much likely to produce mathematics anxious students, and helping teachers confront and control their own fears and feelings of insecurities when faced with numbers is essential to stop the spread of anxiety to learners. Another aspect where teachers can assist students reduce their ude to errors. Barnes (1984) noted that a positive attitude to errors is one of the most important steps in reducing anxiety in the classroom. Learners need to understand that errors are an essential part of the learning process and that mistakes help the learner to discover what it is they do not understand and are essential for progress (Barnes, 1984). To actualize this, Frakenstein (1984) advices that teachers should emphasise that no attempted solution is all wrong, and that all solutions involve some correct, logical reasoning. This helps learners realize they are not stupid if they make a mistake.

Another approach that teachers could use with respect to errors is to remove some of the threat and mystery of evaluation by having learners correct their own work, using an answer key (Martinez, 1987). The teacher should have learners correct their own errors to show the causes of their mistakes and point out a pattern of errors (Gentile and Monaco, 1988). In this respect, Martinez (1987) believes that by allowing learners control the evaluation process, they are in effect controlling their own knowledge.

In agreement with the above discussions, National Council of Teachers of Mathematics

(NCTM) (1989, 1995) suggestions for teachers seeking to prevent mathematics anxiety include:

- Accommodating for different learning styles
- Creating a variety of testing environments
- Designing positive experiences in mathematics classes
- Refraining from tying self-esteem to success with mathematics
- Emphasizing that everyone makes mistakes in mathematics
- Making mathematics relevant
- Allowing students have some input into their own evaluations
- Allowing for different social approaches to learning mathematics
- Emphasizing the importance of original, quality thinking rather than rote manipulation of formulas

Emotionally, a low self- ,
(1990) believes that when a learner has a firm belief in his competency, he can be protected from the sabotage of uncertainty and concern about failure. This can be accomplished by starting with easy problems and praising each step that is correct (Skiba, 1990). When learners realize that they can take control of their mathematics performance, they become encouraged to take control of other aspects of their lives (Kogelman 1981). This increases their self-confidence. Psychological treatments have also been used in the treatment of mathematics anxiety. In this and the next four paragraphs, some psychological approaches that have been used by researches are outlined. Hembree (1990) found that psychological treatments such as systematic desensitization and anxiety management training are highly successful in reducing mathematics anxiety levels. Sharp et al. (2000) maintains that relaxation training can

significantly lower mathematics anxiety and significantly increase mathematics performance. Schneider and Nevid (1993) found that systematic desensitisation and stress management training with college students lowered mathematics anxiety but did not change performance on an aptitude test.

Maree (1992) suggests consulting a psychologist if anxiety persists and the learner

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interviews can provide insight into a learners experience regarding past difficulties in mathematics (Long and Ben-Hur, 1991). By verbalizing fears and frustrations, some hostility towards the subject can be overcome and the situation can be reviewed more logically (Skiba, 1990). Eisenberg (1992) advocates the use the Transactional Analysis (TA) model as a psychological model in treating mathematics anxiety. He maintains that internal dialogues that develop using the TA model will help to point out critical issues in understanding mathematics anxiety and show learners how to get past any resentment and limiting beliefs in mathematics. Another psychological tool for the treatment of mathematics anxiety worth mentioning is anchoring. According to Thalgott (1986), anchoring is a technique that can be used effectively to reduce an anxiety reaction or a mental block to a task such as mathematics. The use of this tool entails that an association is made to a positive experience which is then used as an anchor when an anxious reaction becomes disabling, allowing for recall of previously learned information.

Another psychological tool that can be used in the treatment of mathematics anxiety is journal writing. Frakenstein (1984) states that keeping a journal can serve as a vent for a learners feelings about mathematics putting fears down on paper and then re-reading can relieve tension and help put those feelings into perspective. A journal also helps learners to see themselves as competent in mathematics because they will have a

concrete record of their mathematics progress (Frakenstein, 1984). Journals can also be used for students to write informally, using writing as an extension of their thinking to clarify new concepts and question ideas that remain unclear (Stuart, 2000). In the same vein, Skiba (1990) believes that

own words helps overcome anxiety. Dodd (1992) notes that the use of peer tutoring in conjunction with writing exercises is an effective way to deal with anxieties. Miller (1991) notes that writing in journals offers learners an opportunity to privately seek assistance if something is not understood.

A final psychological approach that can be discussed is one based on behaviour modification. Harcum (1989) outlined a comprehensive teaching-therapeutic programme where desired behaviour is rewarded (instrumental conditioning) and good performance in mathematics associated with pleasant experiences (classical conditioning). In support of this tool, Harcum (1989) reports the success obtained from a specific case study in the treatment of mathematics anxiety where the above approach was employed including family therapy and giving the learner a mild tranquilizer to help him relax and reduce negative behaviour and mathematics anxiety

Research has shown that cognitive factors should be taken into account in the treatment of mathematics anxiety. In this and the next three paragraphs, some of these factors are outlined. Ma (1999) notes that treatments that help students overcome their cognitive difficulties in mathematics learning may be associated with an appreciable reduction in mathematics anxiety. Handler (1990) in Ma (1999) suggests that a cognitive process should include making knowledge work for the learner, joining skills and content, linking motivation to cognition and using social communities. Hadfield and Maddux (1988) similarly believe that cognitive style is associated with mathematics anxiety. Miller and Mitchell (1994) describe a successful treatment of mathematics anxiety as

involving two phases of activities. In one phase the learner receives special tutoring with emphasis on manipulatives whilst attempting to make the content meaningful by relating it to interest and career goals. The second phase is aimed at a different method of evaluation where credit is given for demonstrating understanding through tutoring sessions.

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bad at mathematics needs to be distinguished from having a poor history and not being any good. Skiba (1990) suggests dispelling myths such as the belief that a mathematics Intelligent Quotient (IQ) exists and that learners who have enjoyed past success in mathematics should be aware that the talent still exists. Another approach is in using group activities. Group activities have been found to be helpful in reducing anxieties (Dodd, 1992). Similarly, Vacc (1993) noted that encouraging learners to work with peers in groups that are co-operative reduces mathematics anxiety. Not all researchers have found that learners experience positive outcomes while working in groups and some learners will be reluctant to interact with others in a setting that could be potentially embarrassing. Newstead (1998) reported that some learners experienced anxiety in the performance of mathematics in the presence of peers. Greenwood (1984) suggests problem solving and the discussion of various strategies for solving these problems is important for the prevention of mathematics anxiety.

Wells (1994) argues that learners should have the freedom to make their own choices such as being allowed to work by themselves, with a friend or a group of friends. Learners should have a broader and deeper understanding of how professionals feel and behave towards mathematics, including the knowledge that adult mathematicians have a lot of choice in how they work and with whom they work (Wells, 1994). As can be seen

from the discussions in this section, methods of treating mathematics anxiety are many and varied. In the design and development of the CMSTA used in this study, many of these methods were taken into consideration as they were incorporated into it. In the next section the measurement of mathematics anxiety will be discussed.

2.6.5 Measurement of mathematics anxiety

The measurement of mathematics anxiety is important for understanding the nature of this construct and the degree of its presence for intervention planning and instructional delivery (McMorris, 2004; Newstead, 1998; Hembree, 1990). There have been an appreciable number of instruments developed and used to measure Mathematics anxiety.

The first mathematics anxiety measuring instrument encounter in the course of literature search is the Mathematics Anxiety Rating Scale (MARS), developed by Richardson and Suinn in 1972. It is a 98-item instrument made up of a 5-point Likert scale designed to measure college students' mathematics anxiety. Several researchers (e.g. Hopko et. al., 2003b; Dew et. al, 1983; Richardson and Woolfolk, 1980) believe that MARS was once regarded as the best available measure of mathematics anxiety with the highest reliability and validity.

Following the publication of MARS, several revisions were developed as a result of the examinations undertaken by several researchers on mathematics anxiety in empirical studies. As pointed out by Suinn et. al. (1989), these revisions include the Fennema and

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adolescents (MARS-A), developed by Suinn and Edwards (1982) and the Mathematics Anxiety Rating Scale for Elementary School elementary school: see school. Students (MARS-E) developed by Suinn et. al. (1988). Of these, the MARS-A and MARS-E are

the most popular.

According to McMorris (2004), Suinn and Edwards (1982) MARS-A consists of 98 questions with a 5-point Likert scale. The Cronbach's alpha i.e., internal consistency reliability coefficients, ranged from .89 to .96, but no validity information was clearly reported. McMorris (2004) also re , , /766' K -E has 26 items with a 5-point Likert scale, and its Cronbach's alpha was .88. McMorris (2004) observes that the two primary factors, mathematics test anxiety and mathematics performance adequacy anxiety of MARS-E, accounted for less than 30% of the variance. In his review of MARS, MARS-A, and MARS-E, McMorris (2004) suggested that additional or revised measures need to be developed to improve the measurement of mathematics anxiety.

To reduce the test length, Plake and Parker (1982) modified MARS and developed the Mathematics Anxiety Rating Scale-Revised (MARS-R), a shorter 24-item version of the 98-item MARS. Although it yielded a high reliability estimate of .98, MARS-R was demonstrated to be a remarkably poor fit to a large sample of undergraduates (Hopko, 2003). Thus Hopko et al.'s (2003a) came up with the Abbreviated Mathematics Anxiety Scale (AMAS), which is an adapted version of MARS-R with only nine items. According to Tapia and Marsh (2004), AMAS was reported to have two factors, learning mathematics anxiety and mathematics evaluation anxiety, which accounted for 70% of the variance from an exploratory factor analysis using principal components extraction with varimax rotation. However, some factor loadings were found to be non-significant, and the varimax rotation was also reported as being dubious because its orthogonality assumes no correlation between the factors, which runs counter to the concept that the two factors should be related with each other (Tapia and Marsh, 2004).

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anxiety scale for children (MASC) for children from grades four through to eight. The focus in this research was to identify South African learners in grade eight with mathematics anxiety who could participate in a case study. The study orientation questionnaire in Mathematics (SOM) was designed for South African pupils from grade seven to grade twelve and was used to identify study orientation including Mathematics anxiety (Maree et. al, 1997).

Another line of research independent of the MARS family was initiated by Fennema and Sherman (1976). They developed the Mathematics Attitudes Scale, a 12-item, 5-point Liker-scale instrument used for high school students. This brief scale was intended to assess the feelings of anxiety, dread, and nervousness associated with mathematics. However, there was little research about the validity and reliability of the scale, which seriously hindered its use for research purposes (Rounds and Hendel, 1980).

The above mentioned mathematics anxiety measures were all developed with unidimensional representation of negative affect consistent with anxiety. However, Kazelskis (1998) identified positive affect toward mathematics as one of the six oblique factors in mathematics anxiety: mathematics test anxiety, numerical anxiety, negative affect toward mathematics, worry, positive affect toward mathematics, and mathematics course anxiety. Watson (1988) pointed out that positive affect reflected a degree of pleasurable interaction with the environment, whereas negative affect reflected an adverse response to the environment. The items of negative affect toward mathematics indicate feelings of discomfort, restlessness, uneasiness, and confusion; while the items of positive affect toward mathematics indicate a sense of ease, a lack of discomfort, and the absence of fear (Kazelskis, 1998). To better understand the construct of mathematics

anxiety so as to find better intervention strategies to reduce anxiety, the inclusion of the positive affect of mathematics anxiety was argued to be helpful (Bai, et.al., 2009).

Betz (1978) intended to measure mathematics anxiety from both positive and negative affects and created the Mathematics Anxiety Scale (MAS) by adapting Fennema and Sherman's (1976) Mathematic Attitudes Scale. MAS is a 10-item scale with five items positively worded and the other five items negatively worded. Pajares and Urdan (1996) pointed out that MAS has acceptable stability and internal consistency. MAS was reported to have a Cronbach's alpha of 0.72 and test-retest reliability test-retest reliability of 0.87 for undergraduates (Dew et. al., 1983). However, when examining the two-factor structure of MAS, Pajares and Urdan (1996) found that Item 6 did not load on either of the two factors, positive affect and negative affect; and Item 1 failed to load or loaded weakly across different samples, such as middle school, high school, and college. He therefore concludes that MAS has some construct validity problems.

It is probably in response to shortcoming leveled against the above measures that Suinn and Winston (2003) designed the Revised Mathematics Anxiety Rating Scale (RMARS). RMARS is a 24-item instrument designed to measure the degree of mathematics anxiety of respondents. RMARS is a shorter version of the 98-item Mathematics Anxiety Rating Scale (MARS). Suinn and Winston (2003) stated that despite the usefulness of MARS, which has been a major scale used for research and clinical studies since 1972, RMARS was developed meet researchers quest for a shorter version of the scale partly to reduce the administration time of the original 98-item scale.

Like in the MARS, items of the RMARS address brief everyday life and academic situations pertaining to mathematics. The instrument uses a five point Likert scale

format with the following categories: 1 for not at all, 2 for a little, 3 for a fair amount, 4 for much and 5 for very much. For each RMARS, the total score ranges from 24 to 120, with the higher scores correlating with higher levels of mathematics anxiety. The RMARS is the scale adapted for use in this study.

2.7 Relationships between the Study Variables

2.7.1 Relationship between attitude towards mathematics and mathematics performance/achievement

The relation between mathematics attitude and mathematics performance/achievement has been extensively investigated. Schreiber (2000) found that students who performed better on a mathematics test tended to have positive attitude toward mathematics.

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predictor of mathematics performance/achievement. A direct relation between mathematics attitude and mathematics performance/achievement was also reported by Papanastasiou (2002), based on TIMSS data.

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performance/achievement. Hadfield et. al. (1992) found a strong correlation between the variable of persistence and degree of mathematics anxiety, thus concluding that a

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similarly been noted that more positive attitudes accompany lower levels of anxiety and are conducive to increased gains in the future (Genshaft 1982). Wells (1994) believes in telling a class beforehand that the subject matter is difficult, giving learners a truthful picture of mathematics as something difficult and challenging, but which they can do successfully, as opposed to leaving learners to draw conclusions from their own failure that mathematics is difficult.

There is evidence that different cultural groups have different attitudes to mathematics performance/achievement. Stevenson (1987), in a study showing the difference between American and Asian approaches to mathematics, shows that Americans believe one is either born with a mathematics ability or not. Asians believe mathematics success is a result of hard work, perseverance and hours of study and believe the virtue of effort is the avenue for accomplishment (Stevenson, 1987). In a study, Wither (1988) showed a significant relationship between perseverance and mathematics achievement.

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achievement. Tobias (1978) states that the differences in how learners cope with uncertainty, whether they can tolerate a certain amount of floundering, whether they are willing to take risks, what happens to their concentration when an approach fails, and how they feel about failure determine how well they will achieve. Tobias (1978) further asserts that attitude and self-image, particularly during adolescence when the pressures to conform are important, can result in negative attitudes that can inhibit intellect and

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Gierl and Bisanz (1995) showed that problem solving anxiety related negatively to intrinsic value, confidence and attitude towards success. Positive experiences and a

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Wither (1988) states that the influence of the enjoyment and value that a learner places on mathematics will significantly affect achievement. In a study by Le Fevre et. al. (1992) it was shown that learners with positive views towards mathematics rated their skill as high whilst those with negative views indicated that they were anxious about and avoided mathematics. Stuart (2000), in a study, gave students the opportunity to relate their best and worst experiences with mathematics. Positive experiences included good grades, awards, teacher praise and overcoming mathematical difficulty. Worst

experiences included failure, criticism and difficulties yet to be overcome.

It can be concluded from the above contributions of various researchers that attitudes,

mathematics. It is therefore on this basis that attitude towards mathematics is one of the variables of focus in this study.

2.7.2 Relationship between mathematics anxiety and mathematics performance/achievement

Several research projects have been undertaken to establish the nature of the relationship between mathematics anxiety and mathematics performance/achievement. Hembree (1990) noted that mathematics anxiety seriously constrains performance in mathematical tasks and reduction in anxiety is consistently associated with improvement in achievement. As such, it is to be expected that highly mathematics anxious individuals will be less fluent in computation, less knowledgeable about mathematics, and less likely to have discovered special strategies and relationships within the mathematics domain (Ashcraft and Faust, 1994).

Quantifying the potential improvement in mathematics achievement when mathematics anxiety is reduced, Ma (1999) found that the relationship between mathematics anxiety and mathematics achievement is significant from grade 4 and that once mathematics anxiety takes shape; its relationship with mathematics achievement is consistent across grade levels. Ma (1999) also found that the relationship was consistent across gender groups, ethnic groups and instruments used to measure anxiety.

Goleman (1996) describes the relationship between anxiety and performance, including mental performance in terms of an upside-down U. At the peak of the inverted U is the optimal relationship between anxiety and performance. Too little anxiety, the first side

of the U, brings about apathy or too little motivation to try hard enough to do well, while too much anxiety, the other side of the U, sabotages any attempt to do well. Number manipulation anxiety and test anxiety showed significant inverse relationships with respect to mathematics achievement. Learners who are anxious about forthcoming tests and number manipulation techniques are likely to perform at a lower level (Wither, 1988).

From the above it can be noted that the presence of mathematics anxiety has a negative effect on mathematics performance/achievement. It is from this view point that in this study, an attempt has been made to focus on mathematics anxiety as one of the variables under consideration.

2.7.3 Relationship between teaching methods and mathematics performance/achievement

Previous research encountered in the course of literature review with respect to the relationship between mathematics performance/achievement and teaching methods has

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classroom, including perceived warmth, may lower anxiety and improve mathematics performance. Stuart (2000) states that students like to do what they are good at. Therefore, to make them feel good about mathematics, teachers need to build up the self-confidence and refine the skills required to be successful at mathematics. These postulations border heavily on teaching methods. Teaching methodology has been shown by various researchers to have positive effects on mathematics performance/achievement. For instance, the Kumon method is a specific teaching methodology that has been shown by various researchers to have positive effects on mathematics achievement.

Russell (1995) researched three schools that had followed the Kumon method over a , /773' h are:

- At the Cleora public school in Oklahoma, the Kumon method was introduced in January 1990 for approximately 130 students. A comparison of the Iowa test of basic skills scores from early 1990 to April 1993 shows that the average for the group in focus (i.e. grades 1-8) in total mathematics including concepts, computation and problem solving rose from the 63rd percentile to the 82nd percentile.
- In Dundee, Illinois, grade 3 students at St.Catherine of Siena school also took the Iowa test of basic skills at the beginning of 1990 and again in grade 5 in 1992. After two years of Kumon study, average scores increased from 39% in computation to 81% and in total mathematics average scores increase from 56% to 77%.
- In 1990, the Christ Episcopal school in Texas chose two groups, grade two and grade five for a trial programme in using the Kumon method in its regular mathematics courses. Results were assessed after one year by comparing their Stanford mathematics achievement test results and significant difference was recorded in favor of the Kumon method.

It can be concluded from the above results that after introduction of the Kumon method, the mathematics achievement increased in all cases. A similar result was obtained by Barnard (1995) in an empirical investigation into the effect of the Kumon method on achievement in mathematics by learners in two private schools in Johannesburg, South Africa. He concluded that learners taught by means of the Kumon method outperformed eighty five percent of the learners taught by means of the traditional method and fared

significantly better than learners taught by the means of the problem centred approach (Barnard, 1995).

Further studies involving the effectiveness of an individualised format of teaching versus the traditional format was conducted by Gudan (1995). An outline of research with respect to teaching methodology as described by Gudan (1995) follows. The experimental format consisted of a diagnostic test, individualised work plan and learners working at their pace to master topics. The materials were sequenced in a mathematics workbook with answers provided. The traditional format was lecture and demonstration based. Students that were pre-tested and received individualised instruction benefited the most in terms of post-test scores. Surveys administered to the students indicated that the individualised approach was a positive experience and had a small effect on decreasing mathematics anxiety.

It can be noted from the above discussion that different teaching methodologies affect mathematics performance/achievement. In particular, mathematics skills need to be

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a particular teaching method, referred to herein as the CMSTA, is being studied in relation to mathematics performance/achievement of the subjects upon whom the method was used.

2.8 Related Studies

Many studies bothering on the use of computers to facilitate the teaching/learning process have been conducted since the commencement of the use of the modern day computers in education. These studies covered a wide range of issues with some having direct bearing on the subject matter of this study. In this section, some of these studies will be reviewed.

Literature search reveals that one of the early efforts made on the development and use of computers to facilitate the teaching/learning process was that reported by Harasim /765' , G , , /760'

s to be learned about the educational and psychological effects of technological approaches to instruction, Harasim (1987) reports the development and use of an on line course activities to deliver two graduate course at the Ontario Institute for Studies in Education (OISE). The programme was such that each course, delivered entirely on line, lasted 12 weeks. Students were expected to log on at least two times per week and to spend a total of 2 3 hours per week participating in the on-line programme. The course design was built upon the distinctive character of the conferencing system to develop a learner-centered, group learning approach. The on-line educational environment was designed to take advantage of key features of the conferencing system (in particular, multiple conference spaces and branching capabilities) to create a rich and varied learning context.

Harasim (1987) reports that rather than employ a single conference strand to house all the seminar sessions and learning activities, a branchi

employed to provide spaces for core and optional learning activities, for plenary, small group and working group discussions, and for informal (social) interactions. On-line

actions during the session. A number of educational activities were redesigned for the on-line medium, such as plenary discussions, debates, small-group discussions, working groups, class presentations, and group feedback and critiques. There were three phases to the on-line course, these are the four one-week electronic seminars, on-line working groups and class presentations and final plenary sessions for feedback and reflection over the course topic and process. While the course was carried entirely by computer conferencing, two face to face

meetings were held to provide training and a printed learning kit with course guides, readings and technical documentation on computer conferencing was provided to each student at the outset.

For the purpose of impact assessment for the programme, both qualitative and quantitative data were collected from participants in the two graduate level courses which were offered entirely on-line. The qualitative methods employed to obtain user reactions to the use of computer conferencing as an educational delivery medium included semi-structured interviews, participant observation, and analysis of selected conferences. The quantitative data gathered were in the form of usage statistics and responses to on-line polls and questionnaires.

Harasim (1987) reported that the goal of facilitating active learner participation was successfully achieved. Analysis of usage data showed that students in the courses participated actively on- , ses to open-ended questions on-line provide insight into their impressions of participation in an on-line medium. The great majority indicated very positive experiences. Student assessment of the effectiveness of learning in the on-line course was very positive.

Commenting on the relevance of this study, Harasim (1987) posits that analysis of the

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increased interaction in both quantity and intensity; access to group knowledge and support; democratic environment; convenience of access in the form of a 24-hour class; increased user control over the learning interaction; motivational aspect; and text-based communication. Some of the difficulties which the students reported in relation to learning on-line in the study were those attributed to information overload, asynchronicity (delayed responses), inconvenience of increased access, following on-

line discussion threads, loss of visual cues and health concerns. The relevance of F /765'

number of issues related to how computer conferencing can be utilized to support effective, active teaching/learning in graduate level courses. In this regard, the first issue reported by Harasim (1987) was the determination of whether on-line courses could serve as more than an emulation of the process of face-to-face communication in the traditional classroom, where an affirmative answer was obtained. The second issue was to determine if computer conferencing can support a different, more active and involved style of learning on the part of students and it was found that computer conferencing has the potential to offer both quantitatively and qualitatively different, and in many cases, better learning experiences than that of the classroom. The third issue addressed was how to utilize computer conferencing to support effective, active learning, whereby the conclusion arrived at was that one approach found very effective for achieving these goals on-line is the use of a collaborative group learning design based upon group discussions and interactions among learners, with the instructor as the learning facilitator.

The above issues are of direct bearing to the main thrust of this study. For, these are what are being hoped to be partly achieved in the study. Based on the difficulties reported earlier, there exists the need to realize that there are some important design implications that need to be addressed when using a collaborative on-line model. Some of the more positive aspects of on- -edged. For example, increased participation and interaction can lead to information overload, both in terms of the volume of input and discontinuity of discussion threads. Related to this is the need to manage and focus the discussions, particularly within the seminar

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to and build upon one another.

Another work related to the above was that reported by Bastien et. al. (1997) whereby a pedagogical tool was created for the teaching/learning of applied engineering mathematics at the Faculty of Engineering, Laval University, Sainte-Foy (Quebec). The tool, which uses the World Wide Web (WWW) as a means of dissemination, is a didactic software made up of three hypertext link modules. Bastien et. al. (1997) report that the tool resembles more a piece of software than a Web site as it was created in such a way that any student using it could feel he is not navigating on the internet but is in a special environment where he can learn mathematics. The three basic criteria followed in the creation of the tool were the design of the Graphical User Interface (GUI), the structure of the multimedia documents and the environment in which the users will navigate (browser). Based on this design criteria, the basic structure of the tool was made up of three main integrated modules, viz: the tutorial, exerciser and simulator modules.

The tutorial module was meant to complement written course notes and formal lectures.

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and simulator. Within this module, a topic by topic condensed account of theory was displayed. The interaction of this module is achieved with the use of quizzes. Examples and advanced notions displayed in a theory page highlighted with hypertext links. The use of hypertext links were meant to change the rhythm of reading of the user and draw

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particular theory section, the reading of the material is finished, the user is invited to perform exercises within the exerciser module and to use the simulator module related to the theory.

The exerciser module was meant to assist students within their level of learning. Exercises are presented according to skill levels which are from 1 1* /

1 , se exercises are constructed in an interactive fashion so that the system is able to correct a student if a wrong answer is given. The simulator module is the only one of the three modules that can be used equally by a student or a teacher and is composed of a series of interactive tools intended for demonstration. The main idea is to allow the visualization of physical phenomena and mathematical concepts that are difficult to grasp.

Other works similar to the two cited above which have direct bearing on the main focus of this study, as found in literature, include those by Catarino et. al. (2006); Alonso et. al. (2005); Verster (2008) and Nordin et. al. (2005). Catarino et. al. (2006) reported the development of a blended multimedia system that consists of two modules, one for online learning and the other for offline or classroom learning. The system, designed to work with any mathematics course, is amenable to function in a multilingual learning

, uctured lessons with a final quiz exercise that will test the acquired knowledge, the offline module is an interactive DVD that works as an e-book. The offline module is a comprehensive repository with rich multimedia content and a powerful interactive interface. The core part of the system was developed with adobe technology and XML language. Adobe Director was used to develop the rich interactive multimedia visual interfaces that appear on the e-book and online lessons. The system was then integrated in a Moodle platform that was being used at Superior School of Technology of Setúbal, Potugal.

Alonso et. al. (2005) propose a psychopedagogical instructional model based on content structure, the then latest research into information processing psychology and social constructivism, and defines a blended approach to the learning process. Technologically

speaking, the instructional model was supported by learning objects, a concept inherited from the object-oriented paradigm. The instructional model Alonso et. al. (2005) proposed was supported by a blended and eclectic view of learning theories (i.e. what one thinks is knowledge determines the approach to learning). Their e-lesson design was based on psychopedagogical prescriptions inspired by different learning approaches. First, they looked at the content structure and structured the content according to information types and performance goals. The aim was to develop coherent

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Second, they focused on the cognitive process of understanding how the mind works during the learning process and what factors determine and condition the success of the process. The aim of the psychopedagogical instructional model was to guide the transformation of the information received by the learner in the sensory memory into structured knowledge that is stored in the long-term memory. Thirdly, in the design and development phases, they included activities designed to create a social environment that act as a scaffold for collaborative learning. During the execution phase, the instructor nurtures and promotes a sense of learning community between participants. Thus the instructional e-

personalised teaching, adopting a blended approach to the learning process.

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instruction and learning, made up of seven phases, viz: analysis, design, development, implementation, execution, evaluation, and review. The analysis phase defines what to teach. Here, the learner and the educational contents to be taught are analysed. Its

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environment in which the learning is to take place and the available resources. It outputs two documents, viz:

- Learning objectives, which define the primary and secondary objectives, as well as their hierarchical structure.
- Educational contents, which defines the knowledge and skills to be learned and the tasks to be developed to acquire this knowledge. Here, knowledge is considered as the set of facts, concepts, and processes that the learner has to use properly to solve future problems related to the subject to be studied.

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defining the learning approach, the structure and granularity of the information to be delivered (i.e. facts, concepts, processes, procedures and principles), standards to be used, execution criteria, and achievement expected of the learner. The presentation of information depends on the type of contents to be taught and should differ depending on whether the aim is to teach facts, concepts, procedures, or principles. The method includes setting activities and exercises to evaluate the knowledge acquired. The development phase describes the real learning process, including the tools that are to be used to teach. Here, the materials, strategies, event sequences, and necessary resources are prepared. The implementation phase involves building the software of the e-learning process using an authoring tool, and its location in a Learning Management System (LMS) platform.

In the execution phase, the learner uses the learning process. This is the execution phase which provides information on the problems encountered and the knowledge acquired. In the evaluation phase, information output during execution, which is stored in the learner log within the LMS platform, is gathered and the results are analysed on the basis of the learning objectives. The log can be used for monitoring purposes to determine successes and ascertain the learning product quality. The review phase serves

to refine the learning process by analysing the results of the evaluation. Any instructional model phases can undergo review.

The e-lesson was divided into six sections. These are:

- Presentation, which describes the subject that will be developed throughout the sections of which the e-lesson is composed.
- Objectives, which indicates what the result of the learning will be, briefly describing the tasks that learners will be able to perform.
- Necessary knowledge, which provides the information and instructions associated with the steps required to perform the task to be learned in the e-lesson.
- Learning task, which teaches the skills and tasks to be learned.
- Practice, which consolidates what was taught in the last two sections by putting it into practice on a real case. Discussions and group activities drive community learning.
- Conclusion, which reinforces and recalls the key points learned throughout the e- , G

  about whether or not they have achieved the proposed objectives with regard to the key issues of the e-lesson.

These six sections of which the e-lesson is composed within the instructional model are divided into two categories: content and context categories. The content category is composed of the necessary knowledge and the tasks to be learned, which make up the body of the e-lesson. The presentation, objectives, practice, and conclusion are context category, which serve to provide the learner with guidance about the content of the e-lesson. The e-lesson structure provides a consistent framework that covers the needs of

the instructional method for learning. In justifying the relevance of their work, Alonso et. al. (2005) opined that the instructional learning model provides instructors with the capability of generating personalised e-learning processes focused on some educational objectives and on the characteristics and needs of learners.

Verster (2008) reports the development of an online intervention aimed at helping teachers cope with the implementation of Mathematical Literacy (ML), a new subject for the Further Education and Training (FET) band in South Africa (introduced in 2006). The intervention specifically focused on the use of various online tools to make the intervention a reality with the intention of to create a ML Community of Practice

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intervention can support ML teachers through the creation of a virtual space where the teachers can support one another as a CoP, exchange resources, and share tacit knowledge and experiences. To achieve this, an online platform was developed, where the ML teachers can join, participate and create their own learning ecology, using the chosen online tools.

Vester (2008) reports that this approach made it possible to not only study the effect of

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learning ecology consists of two strands that impact on it, namely the practice of ML and the use of technology, which makes it possible for teachers to interact online within the Learning Ecology and within the CoP. The focus of the study was to investigate how an online learning ecology consisting of a CoP as well as the online tools that supports the community activities, can address the real life problems that the ML teachers were facing as they implement the new subject. The CoP comprises of the researcher actively working alongside the teachers. The CoP was made up of active and

non-active members (lurkers). Lurkers are members who do not actively participate, but

community. Lave and Wenger (1991) described this as legitimate peripheral participation. Members could be novice teachers, experienced teachers, material developers, textbook publishers, other researchers, ML experts, nongovernmental organisation personnel, learning area managers (curriculum advisors) and other interested parties.

Nordin et. al. (2005) reported a study undertaken to develop a suitable instructional design model using the eLearning approach. The use of such approach, they hope, would enhance literacy and motivate interest towards mathematics. The elements of this instructional design model were made up of Mathematical Approaches, Hierarchy of Teaching, Interactive Multimedia, Systematically Designed and Developed Teaching/learning Modules and Sub Modules, and Holistic Development of Students. Nordin et. al. (2005) claimed that the online materials used in the design were developed based on the pedagogical, androgogical and technological aspects of teaching/learning to ensure the effectiveness of the teaching/learning activities received by the students. They further assert that the instructional design of the teaching of mathematics embedded in the Strategy component of the eLearning Model is designed and developed with the integration of the following elements:

- Mathematical Approaches
- Lesson Sequencing
- Teaching Modules
- Holistic Development of Students.

There are six mathematics teaching approaches embedded in the instructional design with the main one being the expository approach. The second was the modeling approach which was meant to show how and why an expert does a task. The third teaching approach is coaching, which observes students as they work and correct their performance immediately as the students go through the sub modules provided. The next teaching approach used was articulation, meant to get the students to articulate

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instructional design adopted the exploration approach to push students into the mode of trying to do the activities better on their own.

There were three aspects of lesson sequencing embedded in the instructional design. The first aspect is scaffolding, which allows students to develop greater autonomy as they master subsets of the required skills. The next aspect of lesson sequencing is increasing complexity, as the students become more competent in a topic. The final aspect of the lesson sequencing is increasing diversity, which will enable the students to learn when and where the skills they learned are relevant and useful.

The teaching modules built in the mathematics course materials were based on the following foundational aspects of students learning process: Teacher Module; Revision Module; Comprehensive Exercise Module and Smart Reinforcement Module. The Teacher Module attempts to give introduction to each subtopic by presenting advance organizers such as video clips and related activities prior to the actual lessons. This module was considered very important because it involves explanation of the mathematical concepts, the operations involved, the steps and examples of how to understand the concepts and solve the problems in a more detailed manner. The Teacher Module involves steps and examples of how to understand certain concepts and to solve

problems in the required manner. The module consists of four sub modules namely: Explanation, Example, Exercise and Topical Test. Explanation sub module includes the expository learning approach to introduce a topic that is to be taught with the aim to attract interest of the student. Example sub module includes steps and examples on how to solve problems related to the topic discussed. Exercise sub module involves

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module, the questions are randomized and the given is specific. Topical Test sub module has the objective of testing students on the understanding of the specific topic in the system.

The Revision Module has the same objective as the Teacher Module but it was designed specially for the slow learners who cannot follow the Teacher Module. This module involves steps and examples of how to understand certain concept and to solve problems in more detailed manner meant for slow learners. The Revision Module has one extra sub module, the Guided Example sub module. This sub module uses more interactivity element of multimedia, so that the slow learner can interact more with the system. Students can work on the module for as long as they wish without the feeling of embarrassment from others, before they continue on to a new topic in the Teacher Module.

The Comprehensive Exercise Module was built in the system with the aim of testing students on the understanding of the overall topics taught in the system (summative evaluation). This module includes two sections, which are objective tests and subjective or structural test. The Smart Reinforcement Module enables fast learners who are good in certain topic to be given a more enriching and challenging test to motivate them further. The module consists of objective questions.

On the Holistic Development of students, Nordin et. al. (2005) claimed that because the mathematics teaching/learning process involved in the eLearning environment is based on the multimedia technology, which integrates the multimedia elements such as texts, sounds, voices, animations, video and audio; the technology has the capability to touch various mod

in the learning process. The process of delivering mathematical concepts, generalizations and applications, which are presented in all the modules, will be able to develop the cognitive, affective and psychomotor aspects in the following manner. The

thinking process on the mathematical concept that they are learning. The thinking process would indirectly help in the development of the cognitive aspect of the students. The development of the affective aspect, which involves emotional responses, is embedded mostly in Teacher and Comprehensive Exercise modules. In Teacher module, the affective aspect is embedded in the advance organizer in the form of video clips. In this study, this instructional design model was adapted to develop the CMSTA used to teach mathematics to the engineering students.

So far, the review undertaken borders on the development and use of computers by individual researchers or group of researchers which were intended to facilitate and improve on the teaching/learning process. However, there are other works encountered in literature where computer application packages already developed were used to also facilitate and/or improve the teaching/learning process. The next paragraphs will address this aspect.

Some works/studies on the Use of Computer Aided Teaching/learning Packages relevant to the current study found in literature include those by Tan (2007), Kosheleva et. al. (2007), Mohler (2001), Lamp and Godwin (1999) and Atlas and Eustace (1995).

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Environment (IVLE), webcast lectures and PowerPoint presentations effectively in

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it for their modules because it is easy to create and good to have one. However, few of such lecturers actually make full use of the forum. After creating a forum, most lecturers leave students to interact by themselves and never visit it again. He (Tan, 2007) therefore advises that it is good practice for lecturers to be present in the forum and interact with students. In this regards, he asserts:

Lecturers may also use the forum to post additional problems for students to discuss.

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students may post undesirable remarks and the lecturer should step in to remind these

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are also lecturers who respond almost instantaneously to questions students post on the forum. Such a practice may discourage other students from contributing to the

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Tan (2007) further observes that if not properly used, IVLE forums can easily turn into a Question and Answer platform. This, he contends, defeats the very purpose of having

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excellent IVLE tools that can be used to create assessments and surveys to supplement teaching, the two tools should be less frequently used because performing these tasks

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should be used to design simple self-assessment quizzes for students to check their progress and understanding graded quizzes as summative assessments. Though it is time

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analyse results of the assessments. Furthermore, the questions can be saved

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customised questionnaires throughout the semester to supplement the student evaluation exercise done at the end of each semester. Customised questionnaires allow the lecturer to gather feedback from students on specific areas and make necessary adjustments to either the curriculum or his/her teaching. Like the assessment tool, there is also a

ating future online

surveys less time-consuming as the questions can be retrieved and reused.

On Webcast lectures, Tan (2007) is of the view that it can enhance student learning, especially in large classes because they not only allow students who have missed a lecture the opportunity to catch up but also permits others, especially slower learners, to review difficult concepts. He expresses concern that many lecturers do not like to webcast their lectures because of the misconception that doing a webcast recording is a complicated and tedious process while others do not feel comfortable teaching in front of a video camera. These concerns, he believes, are unfounded since the lecturer just needs to lecture as usual and let the IT staff take care of the technical part. He also observes that there are lecturers who do not like what they say to be recorded. On this, he asserts that if it was something that should not be said, then the lecturer should not have said it all. Even if the lecture was not recorded for webcast, students can always use their own recording devices (e.g. hand phones, personal digital assistants (PDAs) to capture what a lecturer says and does in the classroom. Other than IVLE, there are many channels for recorded videos to be published on the Internet.

Tan (2007) opines that a more valid concern is that webcasting lectures will affect class attendance. If that were to happen as a result of the webcast, it proves that the lecturer has done a good job. He should not worry too much about low attendance as long as

students are still following his lectures through a different channel. A lecturer will have reason to be concerned if class attendance dropped even though he does not webcast his lectures. To encourage students to attend lectures, the lecturer can incorporate activities into their lectures that cannot be replaced by viewing the recorded lectures.

On PowerPoint or similar software for classroom presentation, he believes it is necessary to employ them in order to do a good webcast lecture. His expression of concern and advice of how to use this IT tool is reproduced below:

> However, many lecturers think PowerPoint is only good for presenting information in point form, and cannot be used to deliver detailed information. These lecturers only s ,
> PowerPoint effectively, one needs to make good use of the animation features. Incorporating animation into PowerPoint slides adds f lair to presentation and makes students more attentive in class. Instead of delivering the content in a straight forward manner, graphics, movies, web links and other multimedia objects can be included to enhance the presentation and help students visualise difficult concepts. However, incorporating animation into a PowerPoint presentation is time consuming. Though it is possible to get someone to help put a presentation together, it would be better if the lecturer can work on the content and animation himself. In my opinion, the content and animation are all part of a presentation.

In his concluding remarks, Tan (2007) asserts that:

> The IVLE, webcast lectures and PowerPoint are just some IT tools amongst many that can be used to supplement our teaching. It is important to choose the right tools to do the right job. This, and the question of how we should make full use of IT tools to make our teaching more effective, require serious thinking and careful planning.

Kosheleva et. al. (2007) gave a report of their study which focuses on the use of innovative Tablet Personal Computer (PC) technology in learning and teaching mathematics. Their study specifically analysed the effects of incorporating Tablet PC

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significant impact of technology use in mathematics education, they evaluated and compared the preservic ,

two groups of students. The treatment group consisted of fifteen students that regularly met in a professional development school where they were provided with 14 Tablet PCs for use in their classes. The control group consisted of 23 students who were enrolled in the same courses with the same instructors but met at different times and location. In the control group Tablet PC technology was not used. The standard inquiry method was used in teaching pre-service teachers from the control group.

Kosheleva et. al. (2007) further report that the effectiveness of the proposed technology enhancement for teaching mathematics was statistically compared to the standard inquiry based method. This comparison was based on the results of two distinct items:

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on all the investigations throughout the semester. According to Kosheleva et. al. (2007), the outcome shows that the technology enhanced group achieved significantly higher scores than the control group. This outcome, they contend, indicates a greater improvement in the treatment grou

that of the control group. They thus concluded that the technology-enhanced courses proved to provide a truly transformational experience for the pre-service teachers. In particular, they observed that the greatest transformation occurred in the affective domain. They cited the two examples reproduced below to support their claim of the effectiveness of their treatment:

One good example was a student who came to our class with a fear of mathematics. At the end of class, she successfully taught a mathematical lesson using Tablet PCs in her internship classroom. This is just one example of a successful story. Here is another one. Typically, pre-service teachers after their graduation have to take the standardized test to acquire teaching license certification. This test proved to be somewhat difficult

for some of the students. Sometimes pre-service teachers take the test more than one time. All the pre-service teachers from the treatment group were able to pass the test immediately after they graduated. The average total score for the treatment group was 259, while the score for the mathematical domain questions was 269.5. The situation was different for the control group students. Several of them took the test more than one time (one pre-service teacher had to take the test four times). The average total score for the control group (after everybody passed the test) was 232, and the score for the mathematical domain questions was 238.5. These results once again confirm our conclusion that treatment group achieved a greater understanding of mathematics content when compared to the control group.

This approach of using a treatment and control group was adopted in this study. Also adopted was the pre-test/post-test approach to determine the impact of the CMSTA which was used to teach mathematics to the subjects of the study.

In his work entitled Using Interactive Multimedia Technologies to Improve Student Understanding of Spatially-Dependent Engineering Concepts, Mohler (2001) describes an interactive multimedia application that was designed to better communicate spatially based materials science and engineering concepts to students. The interactive multimedia program was developed for engineering students by McGraw-Hill publishers and Purdue University in an effort to enhance the understanding of materials science concepts. In this effort, McGraw-Hill publishers worked with the Department of Computer Graphics at Purdue University to create an ancillary CD-ROM called Materials in Focus to accompany one of its materials science texts. The CD-ROM was designed to provide interactive multimedia components that would enable students to better understand the minute details and interactions of materials on which the discipline focuses. The CD was also designed so that instructors could use the assets in

lecture and demonstration sessions to better engage students in active learning. Using Virtual Reality (VR) technology, the CD-ROM provides the ability to navigate a variety of structures and elements. Rather than presenting linear video clips or static images, the CD provides interactive components that the student directly manipulates.

On the post implementation follow up results obtained, Mohler (2001) reports that:

> Although not statistically validated, comments received concerning the VR components in Materials in Focus indicated that students were better able to build a cognitive model of the content being presented. Students were not only able to visualize a structure, but also manipulate it and view it from a variety of locations. Relationships between particles, as well as an overall understanding of a composition, were more readily understood, as shown in Figure 3. Students could visually compare the arrangement of atoms and molecules, something that was mentally dependent when only static orthogonal views were used. An added benefit for the student was the ability to compare the two-dimensional drawings found in the text with the three-dimensional arrangement displayed in t AB ,
> 2D arrangement and then compare their mental image with the true 3D arrangement in the VR clip.

In another study, Lamp and Godwin (1999) report the use of Computer Mediated Communications (CMC) to enhance the teaching of team based project management at Deakin University. Their main aim for incorporating CMC into the learning environment for Information Systems Project was to provide students with an alternative to face to face meetings so that they will able to communicate more

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used were:

- group (one to many) communication
- private (one to one) communication
- both asynchronous and synchronous (chat) communication
- access control for all functionality on an individual user basis
- file repository - built in upload and download functions

- access to Web based resources through hot linked URLs
- dynamic structure and access control to allow for changing needs during the project

Lamp and Godwin (1999) report that because the project was the first time that CMC

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opinions on the use of CMC. A mix of open and Likert scale questions were used. Statistics on the number and location of messages posted to CMC by the groups were also collected. The last lecture in the unit was on post implementation reviews. Rather than stick solely to a traditional lecture, a post implementation review of the unit and the use of CMC was conducted by the lecturer, using brainstorming. They further report that there was very good acceptance of the use of CMC. All students participated in its use and the vast majority used it more than once a week. These results were confirmed by the responses to the open ended questions. This report of good acceptance by the students was a source of hope that the engineering students of Nigerian Polytechnics will also accept the CMSTAin a good manner.

Atlas and Eustace (1999) illustrate the use of on-line hypermedia on the World Wide Web (WWW) in the teaching of university and high school mathematics at Charles Sturt University (CSU), Australia. They report that in this university, educators have to develop techniques to balancing distance teaching and on-campus teaching methods. The computing technologies of hypermedia and the WWW, in conjunction with the use of self-instruction learning methods, computer software and new methods in mathematics teaching, were employed to arrest any imbalance with dynamic teaching methods. They examined the solution of inequalities amongst various calculus concepts to demonstrate the merits of a WWW site and discuss the use of this medium to balance

distance and on-campus teaching.

Considering the progress made so far in the development and use of various computer application packages to facilitate the teaching/learning process, it is thus not surprising that Majewski (1999) made the following remark:

> A broad range of new computer packages have been developed in recent years, which are intended to help in teaching subjects in various areas. This is particularly prevalent in the teaching of mathematics. For most teachers, these novel teaching environments are very attractive. With all their advantages, an unwanted feature common among them is that they bring a lot of confusion in many respects vital to teaching. Therefore, two essential questions that should naturally arise in the use of these packages are how teachers can really use these new tools to teach mathematics if they are confusing, while being so powerful in solving any mathematical problem? The second is what will the student learn if the program is capable of doing anything and everything for him or her?

He (Majewski, 1999) then went ahead to pin-point some of the major traps that teachers may encounter in using mathematical software for teaching. He also identified the major instances where a computer program may expand both our teaching abilities and teaching environment. Finally he shows how the teacher and student alike may benefit from using mathematical packages for teaching.

Garofolo et. al. (2000) presented guidelines for technology utilization by mathematics educators, teachers, and students based on the NCTM Principals and Standards. These guidelines include using technology in context, applying technology to meaningful problems, employing technology to improve pedagogy instead of replacing existing pedagogical techniques, and taking advantage of technology to connect various mathematical representations. Some of the excellent examples they present make use of spreadsheets.

Özgün-Koca (2003) outlined NCTM standards that can be tackled by using spreadsheets to teach mathematics. Chief among these are mathematical concepts of plotting,

exploring functions, investigating relationships between different representations, and allowing reasoning uninhibited by calculations and mechanics. Özgün-Koca (2003) was able to show that spreadsheets are particularly adept at displaying multiple representations simultaneously, which is an advantage over most graphing calculators. Academics, educational organizations, and commercial concerns have contributed many lessons and suggestions for the use of spreadsheets to teach mathematics. A good sample of some of these o J 0. . /' *
which lists many Web links to spreadsheet assignments, along with the NCTM principals and standards addressed.

In their efforts to extend upon some existing works in mathematics teaching, Looms (1993) developed some hypermedia learning tools to assist in teaching first year mathematics at Colorado State University (CSU). Looms effort was based on Bruce et. , /771' * *
communications technology is required to give students a mathematical and technological experience that will form a more dynamic way of learning. The opportunity afforded by the use of software tools such as Maple or Mathematica, to test mathematical ideas such as the new methods in calculus and at the same time develop an awareness of the wider learning arena provided by access to the Internet was highly relied upon in the work. Hypertext links were used to learn mathematics visually, by experiencing not only the worked examples of the WWW tutorials, but also by motivating the student to experiment with the Maple software in the computer laboratory.

Research evidences encountered during literature search show that many students feel that traditional mathematics subjects lack application to real situations that are meaningful and relevant to their own ideas. Thus many on-line teaching of mathematics

are being developed to inscribe mathematical concepts as part of the student's own experience, improving their overall understanding, logical thought processes, communication skills, confidence and the enjoyment in learning mathematics.

Among the numerous on-line mathematics teaching projects in existence, the Harvard A A A ignificant contribution in this area (Hughes-Hallet and Gleason, 1995). Project CALC (Calculus as a Laboratory Course) is another project at Duke University that has developed a new curriculum for calculus instruction based on laboratory experiments, discovery learning, real-world applications, cooperative learning, writing, revision of writing, and high expectations of students (Moore and Smith, 1995). The Calculus and Mathematica Distance Education Program (C&M DEP) at the University of Illinois at Urbana-Champaign and The Ohio State University has some features which are closely related to the initiative at CSU (Uhl and Woods, 1995). The C&M DEP runs a wide area network connecting rural and inner city high school students to the university and sends top university undergraduates to the schools - forming a collaborative community devoted to mathematics learning. The Calculus and Mathematica Distance Education Team consists of a homogeneous group of high school students, high school teachers, undergraduate students and mathematics faculty members.

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translations from one representation of a function to another can be assisted by the use of computers, Hall and Elliott (1993) combined the use of a computer-mediated environment with a metacognitive teaching approach and found that the gains in mathematical achievement are greater in the computer-mediated environments, even when the more traditional learning environments are metacognitively enriched. Eustace (1994) states that the two issues which should be borne in mind in the use of on-line

educational resources in university teaching are:

- Merging distance teaching and on-campus teaching methods requires a delicate balance of computer technology and user interface design, in which the learning medium is not forced upon those who prefer other learning styles, but is made attractive to the learner
- Educators need a high level of computer literacy and skills to provide and maintain a domain of interactive CD-ROM and linked Internet services such as the WWW and a multimedia MOO.

Bogacki (1995) reports that the Computer - based calculus project group at Old Dominion University conducted an evaluation of their Mathcad based mathematics teaching medium with 54 students and concluded from the responses that:

- students found the computer-based calculus interesting and enjoyable;
- the windows version of Mathcad was easy to use;
- the use of Mathcad improved the learning of calculus;
- computer labs improve students' perception of their calculus skills.

From the review of related studies so far, it could be said that computer based applications meant to facilitate the teaching / learning process generally provide some unique avenues for the teaching / learning of concepts in mathematics. It is thus not surprising that most mathematical materials, which were until recently still paper-based, are being transformed by more and more educators into publisher-generated materials or customised self-developed digital utilities into their curricula. This researcher believes that it is vital that mathematics educators continue to integrate digital tools into their classrooms because of the unique avenue they can provide for active student learning

opportunities and presentation of mathematics contents in ways that are not possible with traditional methods. Therefore, in this research, the importance of spatial ability in mathematics curricula was taken into consideration and an attempt has been made to integrate interactive multimedia applications in the CMSTA so as to better communicate spatially based mathematics materials and concepts to students.

The sole limiting factor of paper-based materials is that they provide a calloused or distant means of user interaction with the information being presented. They also give a shallow and somewhat blurred view of intended meaning since they utilize only one human sense through obscure characters and motionless graphics, which often interferes with both transfer and retention. This type of media, also known as "monomedia" (Lindstrom, 1994), has low aesthetic value due to the static nature of the printed page. It presents a monotonous world to humans who are multimedia communicators desiring motion and sound and who thrive on interaction. Similarly with the mass of

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engineering students, it is questionable whether traditional presentations and traditional learning materials are the most efficient means to obtain critical information.

Although mathematics contents lend themselves well to interactive multimedia techniques for delivery, in Nigerian Polytechnics, most of these materials are still traditionally based. Thus they lack user interaction and are hindered by low sensory attractiveness. This is especially true of traditional classroom presentations delivered to the engineering students. These students are growing up in the information age and may be easily bored with traditional presentations. They may fail to grasp an important concept just because they are disenchanted with the method of presentation. Thus, our mathematics and engineering educators must begin to utilize interactive multimedia more broadly in their curriculum materials. Although interactive multimedia use is

increasing, the rate of student expectations for technology integration has exceeded the

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proactively find, review, evaluate and implement interactive technologies within the classroom when such materials are found to be reliable, valid, and therefore valuable. This study is an effort in this direction.

2.9 Implications of the Reviewed Literature to the Study

It could be seen that the review of related literature relevant to this study have been undertaken from many perspectives. Consequently, there could be many implications of it to this study. However, the implications considered most pertinent to the study are bothering on the variables of the study. Therefore, implications discussed in this section are focused on the use of computers in the teaching/learning process and its effect on mathematics performance/achievement, attitude and anxiety.

The literature reviewed related to the use of computers in the teaching/learning process has shown that there is enormous benefits derivable when ICTs are deployed facilitate the process. Many educators have embraced the idea that students today learn better using the new technologies that can provide a dynamic representation of information and concepts. This implies that in teaching mathematics, considerable effort should continued to be made to push forward the development of user-friendly systems, important to a deeper understanding of mathematical concepts, particularly in undergraduate students. For, according to Bruce et. al. (1993), a combination of mathematics, computer graphics, and communications technology is required to give students a mathematical and technological experience that is a more dynamic way of learning. Indeed the on-line teaching of mathematics can inscribe mathematical concepts as part of the student's own experience, improving their overall understanding,

logical thought processes, communication skills, confidence and the enjoyment in learning mathematics. In this regard, Kaput (1992) has suggested that the mathematical thinking ability to recognise translations from one representation of a function to another can be assisted by the use of computers.

In the literature review, it could be deduced that the use of technology in learning is often controversial, not favoured by all students and has problems associated with gender, equity and ethics. These issues must be addressed by the future researchers on the use of computers to facilitate the teaching/learning process. In this regard, two issues Eustace (1994) contend that must be borne in mind in the use of on-line educational resources in university teaching:

- Merging distance teaching and on-campus teaching methods requires a delicate balance of computer technology and user interface design, in which the learning medium is not forced upon those who prefer other learning styles, but is made attractive to the learner (Kearsley, 1992).
- Educators need a high level of computer literacy and skills to provide and maintain a domain of interactive CD-ROM and linked Internet services such as the WWW and a multimedia MOO.

The above does not imply that using the technology means giving up some traditional teaching/learning tools which have already proved effective over the years. Rather, it implies that in order to get best out of technology in teaching/learning it should be complementary to the skills of students. That means teachers should teach their students how to solve the problem with technology, confirm the solution with their skills; solve the problem with their skills, confirm the solution by using technology.

An implication arising from the literature review concerning method of teaching most

suitable when computers are to be deployed to facilitate the teaching/learning process is that the classroom should be democratic and dialogical. This means that students should have a say in how their learning is structured and their values should be respected. This is an area of endeavour in Nigerian schools that has not been observed to be put in practice.

Overall, the literature reviewed provided little insight into clarifying what teachers should know and be able to do with regard to use of computers to facilitate the teaching/learning process. Nor does this literature offer guidance on how to design and implement instruction that supports the use of computers to facilitate the teaching/learning process. Teaching standards, teacher competencies, and state policies do not answer the question of what teachers should know and be able to do with regard to use of computers to in order to support student learning. This implies that there should be:

- A systematic approach to documenting and describing the computer usage demands of teacher-student interactions, teacher talk, textbooks, and assessments. Such an approach could result in a commonly accepted framework that could be used to guide stakeholders in the education sector in making instructional, professional development and assessment decisions on the use of computers to facilitate the teaching/learning process.
- Research that identifies the specific demands computer usage places on the teaching/learning process and the ways in which computers and ICT tools could be used in different school settings.
- Studies of the impact of different approaches to computer usage in instruction on student academic achievement and other affective domain of educational

objectives.

- More rigorous documentation and evaluation of professional development programs designed to develop teacher knowledge and skill in the use of computers to facilitate the teaching/learning process.

Whilst it would be difficult to transform the nature of engineering mathematics curricula offered in the Polytechnic in Nigeria, at least in the short term, the reviewed literature does imply that it is possible to put in place classroom activities that enhance task value which might make significant contributions to how the quality of students experience might be improved. The literature reviewed points out the need for continued

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attitude and anxiety in the Nigerian education sector. Also, more research is needed regardi

achievement, attitude and anxiety. As much work was not found in this area in Nigeria and many developing countries, in contrast to what obtains in the advanced economies of the world, it means that gaps exist which must be filled. Therefore, this means that concerted efforts from all stake holders in the education sector must made to put all hands on deck to ensure that much research on deployment of ICT tools to facilitate the teaching/learning process and their implementations are made continuous.

CHAPTER THREE

METHODOLOGY

3.1 Introduction

This study was undertaken to examine the impact of computer mediated systems teaching approach (CMSTA), which was used to teach mathematics to Nigerian * performance, attitude towards mathematics and mathematics anxiety. In this chapter, the methodology adopted to conduct the study is presented under the following subheadings:

- Research design
- Population of the Study
- Sample and Sampling Techniques
- Instrumentation
- Pilot Study Report
- Validity and Reliability of Research Instruments
- Treatment and Treatment Procedure
- Data Collection and Analyses Procedures

3.2 Research Design

The quasi experimental research design was adopted to carry out this study. This design was considered suitable for the study for two main reasons. Firstly, the study sought to evaluate the impact of CMSTA (i.e. the treatment) on Nigerian polytechnic eng mathematics performance, attitudes towards mathematics and mathematics anxiety. This implies that the study involved the determination of cause and effect relationship between a dependent variable and three independent variables

where the independent variables will be manipulated. According to Gribbons and Herman (1997), quasi experimental research designs are especially useful in studies involving the determination of cause and effect relationships where the independent variables are to be manipulated. In this study, the CMSTA (i.e. the treatment) was the mathematics performance, attitudes towards mathematics and mathematics anxiety were the dependent variables. It was envisaged that cause and effect relationship could be established when questions about the impact mathematics performance, attitudes towards mathematics and mathematics anxiety have been addressed.

The second reason for adopting this design was that the subjects used were members of intact groups where total randomization was impossible to achieve, i.e. the subjects were students in various engineering departments and levels of a polytechnic and as such their complete random assignment to either experimental or control group was , E F /775'* experimental designs are commonly employed in the evaluation of educational programs when total random assignment of subjects to both experimental and control groups are almost practically impossible. In other words, quasi experimental designs are commonly employed when both experimental and control groups are selected from intact groups. With respect to this study, the students used as subjects in both the experimental and control groups were members of specific pre existing classes of engineering courses in Kaduna Polytechnic, thereby making them members of intact groups. The notation for the adopted research design is as given below:

$$EG: O_1 \longrightarrow X_1 \longrightarrow O_2$$

$$CG: O_1 \longrightarrow X_2 \longrightarrow O_2$$

Key: EG: Experimental Group

CG: Control Group

O_1: Pretest

$O_{2:}$ Posttest

X_1: Computer Mediated Systems Teaching Approach (Treatment).

X_2: Conventional Teaching Approach (Non-Treatment).

In the adoption of this quasi experimental research design, the pre-test/post-test Non Equivalent Groups Design (NEGD) approach was used in line with guidelines and steps outlined by Gall et.al. (1996). Thus, the under listed steps were followed to carry out the study:

Step 1: Random assignment of the intact group subjects to experimental and control groups.

Step 2: Administration of the research instruments as pre-tests to both experimental and control groups.

Step 3: Administration of treatment (i.e. CMSTA) to the experimental group but not to the control group, i.e while the experimental group was taught mathematics for 12weeks using the CMSTA, the control group was taught same mathematics contents using the Conventional Teaching Approach (CTA) for the same period of time.

Step 4: Administration of the research instruments as post-tests to both experimental and control groups

Step 5: Analyses and interpretation of data collected

In line with the above steps, an empirical assessment of whether there were pre-existing differences between the two groups (i.e. experimental and control) in each major segments (i.e. HND and ND) as well as between the minor segments (i.e. Male and Female) in each major segments of each group in relation to the dependent variables (i.e. mathematics performance, attitude towards mathematics and mathematics anxiety) of interest were undertaken. This was done immediately data from pre-tests were obtained. Table 3.1 that follows gives the results of the independent sample t-test analysis of the pre-tests data obtained.

Table 3.1: Independent Sample t-Test Analysis Between Experimental and Control Group's Pretest MPT, ATMI and MARS Scores

Variable	Level	Group	N	Mean	Std. Dev	Std. Error Mean	*df*	*t*	*p*
MPT Scores	HND	Experimental	139	62.29	9.86	0.84	240	1.55	0.12
		Control	103	60.47	7.87	0.78			
	ND	Experimental	117	63.59	8.37	0.77	201	-0.12	0.91
		Control	86	63.71	5.54	0.60			
ATMI Scores	HND	Experimental	139	136.53	29.27	2.48	240	-0.17	0.86
		Control	103	137.18	28.87	2.84			
	ND	Experimental	117	137.06	29.21	2.70	201	-0.25	0.80
		Control	86	138.05	24.99	2.69			
MARS Scores	HND	Experimental	139	46.17	18.58	1.58	240	-0.58	0.56
		Control	103	47.57	18.57	1.83			
	ND	Experimental	117	45.96	18.53	1.71	201	-0.10	0.92
		Control	86	46.22	18.06	1.95			

From Table 3.1, it could be seen that none of the p-values is less than 0.05. It therefore follows that there were no pre existing significant difference between the pre-test MPTs, ATMI and MARS scores of the experimental and control groups. In other words, it was empirically established that there were no pre-existing significant differences between the two groups (i.e. experimental and control) in each corresponding major segments (i.e. HND and ND).

Also, Table 3.2 which follows shows the independent sample t-Test analysis between the minor segments (i.e. Male and Female) within the major segments (i.e. HND and ND) of each group.

Table 3.2: Independent Sample t-Test Analysis Between Male and Female Pretest MPT, ATMI and MARS Scores

Level	Group	Variable	Gender	N	Mean	Std. Deviation	Std. Error Mean	df	t	p
HND	Experimental	MPTs Scores	Male	111	61.64	10.87	1.03	137	-1.55	0.12
			Female	28	64.86	2.75	0.52			
		ATMI Scores	Male	111	136.74	29.06	2.76	137	0.17	0.87
			Female	28	135.71	30.62	5.79			
		MARS Scores	Male	111	46.16	18.58	1.76	137	-0.00	0.99
			Female	28	46.18	18.91	3.57			
	Control	MPTs Scores	Male	95	59.78	7.82	0.80	101	-3.19	0.00*
			Female	8	68.63	0.92	0.32			
		ATMI Scores	Male	95	136.83	29.23	3.00	101	-0.43	0.67
			Female	8	141.38	25.48	9.01			
		MARS Scores	Male	95	47.48	18.90	1.94	101	-0.17	0.87
			Female	8	48.63	15.16	5.36			
ND	Experimental	MPTs Scores	Male	97	65.45	7.91	0.80	115	6.07	0.00*
			Female	20	54.55	2.65	0.59			
		ATMI Scores	Male	97	136.87	29.51	3.00	115	-0.16	0.88
			Female	20	138.00	28.45	6.36			
		MARS Scores	Male	97	46.11	18.65	1.89	115	0.20	0.84
			Female	20	45.20	18.38	4.11			
	Control	MPTs Scores	Male	71	64.42	5.82	0.69	84	2.69	0.01
			Female	15	60.33	1.50	0.39			
		ATMI Scores	Male	71	138.44	26.28	3.12	84	0.31	0.76
			Female	15	136.20	18.25	4.71			
		MARS Scores	Male	7	4.14	18.72	0.22	84	-0.09	0.93
			Female	15	46.60	15.17	3.92			

*P<0.05

From Table 3.2, it could be seen that the only pre existing significant gender difference detected were on MPT scores of HND control group and ND experimental group students. These two cases could be considered as isolated cases, since literature reviewed has shown that there is inconclusive evidence of existence or not of real significant gender difference in mathematics achievement. As for the other values, it could be seen that none of the *p*-values is less than 0.05. This implies that there were no pre existing significant gender difference within the HND and ND students in the experimental as well as control groups pre-test MPT, ATMI and MARS scores. Here also, it was empirically established that there were no pre-existing significant gender differences in each of the major segments (i.e. HND and ND) of each group.

Conduct of the above tests and analyses was considered necessary in order to satisfy the condition of homogeneity before treatment administration. When it was found that there were no pre-existing significant differences between the groups in each major segment as well as within minor segments of ea *

mathematics performance, attitude towards mathematics and mathematics anxiety; treatment administration was carried out. The treatment was in the form of teaching mathematics to the experimental group for 12weeks using the CMSTA while the control group was taught same mathematics contents using the CTA. After the treatment administration, another assessment of whether differences had emerged between the two groups and between major segments within each group in relation to the dependent variables of interest was then carried out. Thereafter, analyses and interpretations of the data collected were done. Figure 3.1 that follows is a flow chart depicting the steps followed in the conduct of the research.

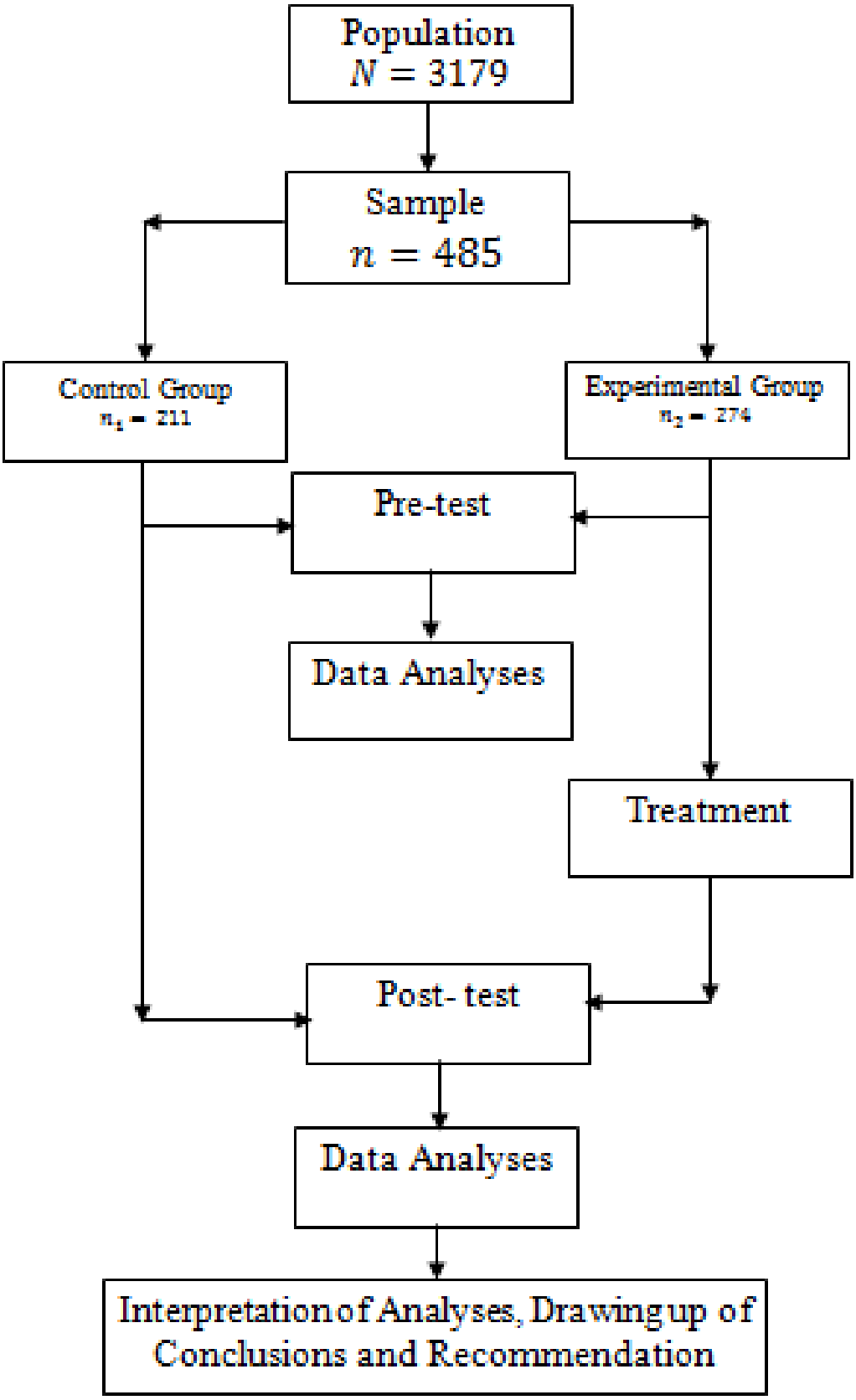


Figure 3.1: Flow Chart Showing Research Steps

3.3 Population of the Study

The population of this study was made up of all the Nigerian Polytechnic engineering students. These students are basically categorized into two levels, viz: National Diploma (ND) and Higher National Diploma (HND) levels.

The ND students, before being admitted into any engineering programme in any Nigerian Polytechnic, must have the following minimum entry requirements as stipulated by the National Board for Technical Education (NBTE) the governing body responsible for regulating all science and technology and related courses offered in all technology institutions in the country apart from the universities:

- SSCE/GCE Ordinary Level or its equivalent with credit level passes in five subjec
- Five credit level passes in relevant subjects obtained at the final examination of an NBTE recognized preliminary ND programme offered in a Polytechnic or Similar Post-Secondary Technical Institution.
- The National Technical Certificates (NTC), National Business Certificate (NBC) or National Vocational Certificate-Final (NVC-Final) with Credit level passes in five subjects relevant to the programme.

In addition to 1 - 3 above, candidates must obtain a satisfactory score in the Joint Admissions and Matriculation Board (JAMB) University and Tertiary Matriculation Examination (UTME) which is conducted annually.

A holder of the Advanced National Technical Certificate (ANTC) may be given advanced placement in the ND Programme provided the candidate has also met the minimum entry requirements into the ND Programme and obtained a minimum of credit

level grades in the ANTC Examinations.

From the above ND minimum entry requirements stipulated by NBTE, it follows that ND engineering programme students have credit level passes in English Language, Mathematics, Physics and Chemistry and one subject from: Biology/Agricultural Science, Technical Drawing, Basic Electronics/Basic Electricity, Auto Mechanics, Metal Work, Woodwork (for Civil Engineering Tech.), Further Mathematics, Geography, Economics/Commerce.

The general admissions requirements for HND, as stipulated by the NBTE, are:

- The National Diploma with a minimum of lower credit pass in the discipline obtained from an NBTE accredited programme plus a minimum of one-year Post-ND cognate work experience in the field of specialisation.
- Candidates with ND at pass level and a minimum of two-year post-ND work experience may also be considered.

As at the time the research was carried out, there were 75 Polytechnics in Nigeria. In terms of ownership spread, 20 of these Polytechnics were owned by the federal government, 37 by various states governments and the remaining 18 owned by private individuals or organizations. As for geographical spread, the northwest had 10 of these Polytechnics, northeast had nine, northcentral has 11, southwest has 24, southeast had nine and southsouth had eleven. The remaining one was located in the Federal capital Territory (FCT), Abuja. Table 3.3 that follows shows the distribution of these institutions by zone and ownership.

Table 3.3: Distribution of Nigerian Polytechnics by Zone and Ownership

Zone	Ownership			Total
	Federal	State	Private	
North West	4	6	0	**10**
North East	4	5	0	**9**
North Central	4	6	1	**11**
South west	4	10	10	**24**
South East	4	2	3	**9**
South South	0	8	3	**11**
FCT	0	0	1	**1**
Total	**20**	**37**	**18**	**75**

Source: NBTE (2011): Directory of Accredited Programmes in Polytechnics, Similar Tertiary Institutions, Technical Colleges, and Vocational Enterprise Institutions in Nigeria (16th edition)

3.3.1 Target population

Since the study was experimental in conception and design, which made it practically impossible to use many engineering students from different Polytechnics, both the control and experimental groups were drawn from Kaduna Polytechnic. The choice of Kaduna polytechnic is based on the convenience it affords the researcher in the conduct of the research because the researcher is a staff of the institution. It is worth noting that on account of the basic assumptions of the study as stated in section 1.7, the generalization of the findings of the study to other engineering students in Nigerian Polytechnics suffices.

As at the time the study was carried out (i.e. during the second semester of the 2010/2011 academic session), the total enginee I Polytechnic stood at 3179. These students were spread unevenly across the two schools that made up the college of engineering of the Polytechnic. These are the schools of

Industrial Engineering and Natural Resources Engineering. The school of Industrial Engineering had four departments, viz: Chemical, Computer, Electrical and Mechanical engineering departments while the school of Natural Resources Engineering had the following three departments: Agricultural Mechanization, Civil and Mineral Resources engineering departments. In each of these departments, there were four levels of study, viz: Higher National Diploma One (HNDI), Higher National Diploma Two (HNDII), National Diploma One (NDI) and National Diploma Two (NDII) levels. Table 3.4 that follows shows the distribution of the target population of the study by schools, departments, levels and gender.

Table 3.4: Distribution of Target Population of the Study

School	Department	Levels								Total
		HND I		HND II		ND I		ND II		
		M	F	M	F	M	F	M	F	
Industrial Engineering	Chemical Engineering	114	31	113	27	113	24	113	33	**568**
	Computer Engineering	45	15	24	6	105	24	122	25	**366**
	Electrical Engineering	212	16	139	7	114	8	131	11	**638**
	Mechanical Engineering	131	4	138	2	108	0	143	3	**529**
Natural Resources Engineering	Agricultural Mechanization Engineering	56	12	61	10	61	13	69	16	**298**
	Civil Engineering	109	11	123	11	133	12	94	14	**507**
	Mineral Resources Engineering	27	5	53	5	74	17	76	16	**273**
Total		**694**	**94**	**651**	**68**	**708**	**98**	**748**	**118**	**3179**

3.4 Sample and Sampling Techniques

The study was carried out during the second semester of the 2010/2011 academic session. Thus, only second semester mathematics courses being taught at the Polytechnics could be used. Only ND I and HND I students are taught mathematics courses in second semester. Consequently, the study sample for both experimental and control groups were drawn from the ND I and HND I students. Therefore, ND I and HND I students constitute the sampling frame from which both the experimental and control groups were drawn. Table 3.5 that follows shows the distribution of sampling frame.

Table 3.5: Distribution of Sampling Frame

School	Department	Levels				Total
		HND I		ND I		
		M	F	M	F	
Industrial Engineering	Chemical Engineering	114	31	113	24	**282**
	Computer Engineering	45	15	105	24	**189**
	Electrical Engineering	212	16	114	8	**350**
	Mechanical Engineering	131	4	108	0	**243**
Natural Resources Engineering	Agricultural Mechanization Engineering	56	12	61	13	**142**
	Civil Engineering	109	11	133	12	**265**
	Mineral Resources Engineering	27	5	74	17	**123**
Total		**694**	**94**	**708**	**98**	**1594**

From Tables 3.4 and 3.5*

population of 3179, the ND I and HND I students were 1594. This implied that the samples were drawn from no less than 50% of the target population, which was considered adequate for valid generalization of results to be obtained

Members of the experimental group were drawn from two departments in the School of Industrial Engineering while those of the control group were also drawn from two departments in the School of Natural Resources Engineering. This action was considered necessary because of the desire to eliminate the possibility of both experimental and control groups interacting during the treatment period, which can possibly interfere unduly with the results of the study. The selection of the intact groups (i.e. subjects) was done randomly. In each school, one HNDI and one NDI class were randomly chosen. This process produced HNDI Chemical Engineering and NDI Computer Engineering students as members of the experimental group while HND I Civil Engineering and NDI Mineral Resources Engineering students were selected as subjects in the control group. Table 3.6 that follows depicts the detail breakdown of subjects in both the experimental and control groups.

Table 3.6: Distribution of Subjects in Experimental and Control Groups

Group	School	Department	Levels				Total
			HND I		ND I		
			M	F	M	F	
Experimental	Industrial Engineering	Chemical Engineering	114	31	0	0	**145**
		Computer Engineering	0	0	105	24	**129**
	Sub Total		**114**	**31**	**105**	**24**	**274**
Control	Natural Resources Engineering	Civil Engineering	109	11	0	0	**120**
		Mineral Resources Engineering	0	0	74	17	**91**
	Sub Total		**109**	**11**	**74**	**17**	**211**
Grand Total			**223**	**42**	**179**	**41**	**485**

From table 3.6, it could be seen that the sample size for the study was 485, with 274 being in the experimental group and 211 in the control group. A combination of the convenience, purposive and random sampling procedures was therefore adopted to select the samples used in the study. The convenience sampling procedure was adopted when Kaduna Polytechnic was chosen as the institution where the study was conducted. Purposive sampling procedure was employed in the determination of the levels whose students were used as subjects in either the experimental or control groups because only students that were being taught mathematics courses in the semester in which the experiment was carried out could be used. The random sampling procedure was used to assign the schools and levels to both the control and experimental groups.

3.5 Instrumentation

Three instruments were used in the collection of data in this study. These are:

- Mathematics Performance Tests (MPTs)
- Attitude Towards Mathematics Inventory (ATMI)
- Mathematics Anxiety Rating Scale (MARS)

3.5.1 Mathematics Performance Tests (MPTs)

Mathematics Performance Tests (I and II) were the instruments used to gather data on performance. Since the study was conducted in the second semester, the subjects first semester mathematics courses examinations were used as their pre-test mathematics performance tests (i.e. MPT I) while those of their second semester were used as their post-test mathematics performance tests (MPT II). These performance in the following five spheres of ability: recall, process, application, speed and accuracy.

Details of the MPTs (I and II) are attached as appendix I (A and B).

3.5.2 Attitude towards mathematics inventory (ATMI)

The Attitude Towards Mathematics Inventory (ATMI), developed by Tapia and Marsh (2004) to investigate the underlying dimensions of attitude towards mathematics was adapted and , K G a 40-item instrument constructed to address confidence, value, enjoyment and motivation. According to Tapia and Marsh (2004), these are factors reported in literature to be important in shaping of attitude towards mathematics.

-concept of their

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beliefs on the usefulness, relevance and worth of mathematics in their life now and in the future. The enjoyment of mathematics category measures the degree to which students enjoy working mathematics and mathematics classes. The motivation category measures interest in mathematics and desire to pursue studies in mathematics. Each category has 10 items. All items in the ATMI were constructed using a Likert-scale format with the following anchors: 1 for strongly disagree, 2 for disagree, 3 for neutral, 4 for agree and 5 for strongly agree. For each ATMI, the total score ranged from 40 to 200, with the higher scores correlating with higher levels of positive attitude towards mathematics. Details of the ATMI are attached as appendix II.

3.5.3 Mathematics anxiety rating scale (MARS)

The Mathematics Anxiety Rating Scale (MARS) is a 24-item instrument adapted from the Revised Mathematics Anxiety Rating Scale (RMARS) designed by Suinn and Winston (2003) to measure the degree of mathematics anxiety of respondents. This

instrument was used to gather data on the mathematics anxiety levels. Like the RMARS, items of the MARS address brief everyday life and academic situations pertaining to mathematics. The instrument uses a five point Likert scale format Likert scale A subjective scoring system that allows a person being surveyed to quantify likes and preferences on a 5-point scale, with 1 being the least important, relevant, interesting, and 5 being most excellent etc with the following categories: 1 for not at all, 2 for a little, 3 for a fair amount, 4 for much and 5 for very much. For each MARS, the total score ranged from 24 to 120, with the higher scores correlating with higher levels of mathematics anxiety. Details of the MARS are attached as appendix III.

3.6 Pilot Study Report

Two of the instruments (i.e ATMI and MARS) used for the study were pilot-tested before final administration to the samples. The pilot test was carried out during the first semester of the 2010/2011 academic session using HNDII and NDII Mechanical engineering students of Kaduna Polytechnic. Despite being in the same school of Industrial Engineering with some of the students used as samples in the study, it should be noted that the subjects used for the pilot study still differ from those used for the real

leaked to

the subjects of the real study was ensured. The subjects used for the pilot study differ from those used for the real study in three major respects. The first is that both sets of subjects are in different levels, i.e. while the subjects used for the pilot study were HND II and ND II students, those used for the real study were students of HNDI and NDI. The second is that all of them were from different departments, i.e. while the subjects used for the pilot study were in Mechanical Engineering Department, those used for the real study were students of Chemical Engineering and Computer Engineering Departments. Thirdly, while the pilot study was carried out during the first semester, the

real study was carried out during the second semester.

Both the treatment and instruments administered on the pilot study subjects were personally carried out by the researcher. Specific instructions guiding how the students should respond to the instruments were read and explained to them to ensure that they know what was expected of them. Difficulties and problems that arose during the treatment and instruments administration were carefully noted for improvement in the final instruments.

The purposes of the pilot study were to:

- Identify problems or difficulties that the researcher may encounter during treatment administration with a view to eliminating them during the final treatment administration;
- Determine the approximate time duration which the researcher may need during treatment administration;
- Assess the feasibility of the study before administration;
- Identify problems or difficulties that the subject may encounter when responding to the instruments with a view to eliminating them in the final instrument;
- Determine the approximate time duration which the subjects would need to respond to the instruments;
- Determine the difficulty and discrimination levels of the items in each instrument;
- Determine reliability of the instruments before administration.

Despite the fact the developers of the original ATMI and RMARS reported that both instruments have high validity and reliability, the data obtained from the pilot study

were analysed to determine the difficulty and discrimination indices of each item in each of the instrument as well as the reliability coefficient of each as a whole in relation to the study.

3.6.1 Item analyses

In order to determine the indices of difficulty and discrimination of each item in both the ATMI and MARS as well as the reliability coefficient of each instrument as a whole in relation to the study, item analyses of the students responses to each of the 40 items of the ATMI and 24 items of the MARS were undertaken. The Facility Indices, which gave the difficulty level or the ease of answering an item in both ATMI and MARS, were obtained by use of the formula:

$$\text{Facility Index} = \frac{R}{T}$$

Where

R = the number of correct responses.

T = the total number of students.

Also, the Discrimination Indices, which indicated the discriminating power of an item in both ATMI and RMARS i.e. discriminating between good and not so good performers, were calculated using the formula:

$$D.I = P_1 \quad P_2$$

Where

P_1 = Proportion in top 50% responding correctly

P_2 = Proportion in bottom 50% responding correctly

Using the above formulae, the facility and discrimination indices of all the 40 items of ATMI and the 24 items of MARS, which are presented in table 3.6 were calculated.

Table 3.7: Item Difficulty (F.I) and Discrimination Index (D.I) ($P_1 – P_2$) for ATMI and MARS.

Item Difficulty (F.I)				Discrimination Index (D.I)			
ATMI		MARS		ATMI		MARS	
ITEM	FACILITY INDEX (F.I)	ITEM	FACILITY INDEX (F.I)	ITEM	D.I (P1 P2)	ITEM	D.I (P1 P2)
1	0.57	1.R	0.33	1.R	0.2	1	0.47
2	0.63	2.R	0.77	2.R	0.27	2	0.53
3	0.47	3.R	0.3	3.R	0.27	3.R	0.2
4	0.37	4.R	0.3	4	0.33	4.*	0.07
5	0.53	5	0.37	5	0.4	5.R	0.2
6	0.57.	6	0.47	6	0.47	6.*	0.13
7	0.4	7.R	0.27	7	0.53	7.R	0.2
8	0.43	8.R	0.33	8.R	0.2	8.R	0.27
9.*	0.17	9.*	0.07	9.*	0.07	9.R	0.27
10.*	0.17	10.*	0.13	10.*	0.2	10.R	0.33
11	0.23	11	0.37	11.*	0.07	11	0.45
12.R	0.23	12	0.47	12.R	0.2	12.*	0.17
13	0.2	13	0.53	13.*	0	13	0.4
14	0.37	14.R	0.33	14.*	0.2	14	0.47
15	0.27	15.R	0.27	15.*	0	15	0.4
16.R	0.23	16.R	0.3	16.R	0.2	16	0.42
17	0.43	17.*	0.1	17.*	0.07	17.*	0.09
18.R	0.67	18.R	0.33	18.R	0.34	18	0.52
19	0.43	19.R	0.33	19	0.47	19.*	0.1
20.R	0.3	20.*	0.15	20.R	0.33	20	0.39
21	0.53	21	0.57.	21	0.4	21	0.53
22.R	0.33	22	0.4	22.R	0.27	22	0.47
23.R	0.27	23	0.43	23.R	0.27	23.*	0.05
24.R	0.33	24.*	0.17	24	0.4	24	0.2
25.*	0.07			25.*	0.13		
26.R	0.33			26	0.53		
27.R	0.77			27	0.33		
28.R	0.3			28	0.47		
29	0.3			29.*	0.07		
30	0.37			30.R	0.2		
31	0.47			31.*	0.07		
32.R	0.27			32	0.4		
33.*	0.13			33.*	0.27		
34	0.37			34	0.33		
35	0.47			35.R	0.27		
36	0.3			36.*	0.07		
37.*	0.1			37.*	0.07		
38.*	0.33			38.R	0.27		
39	0.33			39.*	0.13		
40.*	0.15			40	0.27		

KEY:
* Discarded for being too difficult
R Revised and retained

After analyses, any item with difficulty index below 0.2 was discarded for being too difficult while those with indices ranging from 0.2 to 0.35 were selected for inclusion in the final instrument with some modification or reframing. Items with indices ranging from 0.36 to 0.64 were selected without any modification, while those with indices above 0.65 were modified and accepted.

C /753' dices were used to select items for the final ATMI and MARS. Accordingly, items with discrimination indices of: (i) 0.40 and above were adjudged very good, (ii) 0.30 0.39 were adjudged reasonably good, (iii) 0.20 0.29 were considered marginally items that need improvement and (iv) below 0.19 were considered poor items which were discarded. On the basis of this

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reframed and accepted. Thereafter, new items were drafted to replace and make up the 40 items and 24 items of the ATMI and MARS respectively.

3.7 Validity and Reliability of Research Instruments

3.7.1 Validity and reliability of MPTs

The MPTs used were obtained from mathematics examinations which were developed for the students by their various mathematics lecturers. By a process of external moderation of the questions usually adopted by the institution, the items were adjudged to be of high content validity and reliability.

3.7.2 Validity and reliability of ATMI

Tapia and Marsh (2004) reported that to estimate internal consistency of the ATMI scores, Cronbach alpha coefficient was calculated. For scores on the 40 items, Cronbach alpha was 0.97, indicating a high degree of internal consistency for group analyses. They further reported that each of the 40 items of the ATMI had an item-to-total correlation above 0.50, with the highest being 0.82. This suggested that all items contributed significantly and that they were homogeneous, which was interpreted to mean that they tend to measure a common trait.

Tapia and Marsh (2004) further reported that the Pearson correlation coefficient was used for test-retest reliability in a four-month follow-up of the 40-item inventory, administered to 64 students who had previously taken the survey. The coefficient for test-retest for the total scale was 0.89, and coefficients for the subscales were as follows: Self-confidence 0.88; Value 0.70; Enjoyment 0.84; and Motivation 0.78.These data indicate that the scores on the inventory and the subscales were stable over time.

Despite the high degree of internal consistency reported for the ATMI by Tapia and Marsh (2004) as recounted above, the need to determine new estimates for its validity and reliability coefficient in relation to this study became pertinent since some of its items were modified while others were discarded and replaced. Three experts with each in psychology, measurement and evaluation and mathematics education were given the revised ATMI for face and content validation. The reliability coefficient of the adapted ATMI was determined by subjecting the scores obtained from the pilot study to the Kuder-Richardson Formula 21, where 0.90 was obtained as the reliability coefficient, suggesting that the adapted ATMI was quite a reliable measuring instrument for the study.

3.7.3 Validity and reliability of MARS

Suinn and Winston (2003) reported that the RMARS has a Cronbach alpha of 0.96, indicating high internal consistency and a test-retest reliability coefficient of 0.90 ($p<.001$). Like in the case of the ATMI, the need to determine the new estimate for the reliability coefficient of the MARS in relation to this study became pertinent since some of its items were also modified while others were discarded and replaced. Like with the ATMI, three experts with each in psychology, measurement and evaluation and mathematics education were also given the revised MARS for face and content validation. The reliability coefficient was determined by also subjecting the scores obtained from the pilot study to the Kuder-Richardson Formula 21, where a reliability coefficient of 0.94 was obtained. This also suggested that the MARS was quite a reliable measuring instrument for this study.

3.8 Treatment and Treatment Procedure

3.8.1 Treatment

As stated in section 3.2, the pre-test/post-test Non Equivalent Groups Design (NEGD) approach was used in line with the guidelines and steps outlined by Gall, et.al, (1996). This approach demands that a control and an experimental group be formed. While the control group was taught mathematics using the CTA, the experimental group was taught with the CMSTA. A lecture manual was designed for CMSTA. The CMSTA lecture manual which was designed to provide general tips on teaching each topic and specific teaching, examination and CMSTA resources for each topic, is divided into two parts for each level (i.e. HNDI and NDI levels). Part I contains lecture contents suggestions, classroom exercises and questions for each topic and Part II is a framework on how you can use the CMSTA. The CMSTA lecture manual is attached as appendix

IV.

3.8.2 Treatment procedure

The following four stage procedure was adopted in order to administer the treatment:

- Assignment of research participants by departments and levels to experimental and control groups.
- Administration of the research instruments as the pre-tests to both control and experimental groups. This was to enable the researcher to empirically assess whether there were pre-existing differences between the two groups and between the major segments with each group in relation to the variables (i.e. mathematics achievement, attitude towards mathematics and mathematics anxiety level) of interest.
- Administration of the treatment to the experimental group for 12 weeks but not to the control group.
- Administration of the research instruments as the post-tests to both control and experimental groups after the administration of the treatment.

3.9 Data Collection and Analyses Procedures

3.9.1 Data collection procedures

As noted in 3.5.1, the data pertaining to MPTs (I and II) scores were obtained by the researcher from the examination offices of the departments used. The first and second semester mathematics courses examination result scores of the subjects in the 2010/2011 academic sessio , 1,3,/* semester mathematics course examination results were considered as their pre-test mathematics performance tests scores while their second semester mathematics courses

examination results were considered as their post-test mathematics performance tests scores.

The ATMI and MARS were each administered to both the control and experimental group at the beginning and end of the second semester of the 2010/2011 academic session. The data obtained from the administration of these instruments at the beginning of the second semester served as the pre-test scores while that obtained at the end of the same semester served as the post- itude towards mathematics and their mathematics anxiety level variables. The researcher personally administered these instruments.

The MPTs, ATMI and MARS were used as matched pre-test and post-test instruments to collect data on the research participants performance, attitudes towards mathematics, mathematics anxiety levels respectively. While the independent variable was the CMSTA, the dependent variables were mathematics performance, attitudes towards mathematics and mathematics anxiety levels as measured by the MPTs, ATMI and MARS respectively.

3.9.2 Data analyses procedures

As stated in section 3.2, the research was conducted in line with the pre-test/post-test Non Equivalent Group Design (NEGD). Therefore, the first step in analyzing data from the study was the computation of descriptive statistics on all of the variables measured by the instruments. Consequently, the Mean, Standard Deviation and Standard Error of the Mean were computed for the pre-tests and post-tests scores obtained from both the experimental and control groups. The method of t-test statistics was used to analyze the data obtained where the means of the scores of the experimental and control groups pertaining to each of the dependent variables were compared to determine whether the

experimental and control groups differ on mathematics performance, attitudes towards mathematics and mathematics anxiety levels. Also, the means of the scores of the male and female students taught mathematics using the CMSTA were compared to detect whether gender differences have emerged after the subjects were given the treatment. The hypothesis testing was done at the 95% confidence level.

The raw data produced from the responses generated by the instruments were coded and keyed into the Statistical Package for the Social Sciences (Version 17.0), which was used to compile and analyze the descriptive and inferential statistics. The analyses obtained were used to address the research questions and hypotheses.

CHAPTER FOUR

RESULTS, ANALYSES AND DISCUSSIONS

4.1 Introduction

The purpose of this study was to investigate the impact of CMSTA on mathematics performance, attitude and anxiety of Nigerian Polytechnic engineering students. Consequently, data bothering on mathematics performance, attitude and anxiety of the students taught mathematics using the CMSTA and that of those taught mathematics using the CTA were generated. In this chapter, the data generated, results obtained from data analyses and interpretations of the analyses as well as discussion of the results are presented under the following subheadings:

- Data Presentation
- Analyses
- Summary of Major Findings
- Discussions

4.2 Data Presentation

The data obtained were from all the different segments of the study population. These included Pre-test and Post-test MPT, ATMI and MARS scores of male and female HND as well as ND students taught mathematics with the CMSTA (i.e. the experimental group) and that of those taught mathematics with the CTA (i.e. the control group). As stated in section 3.5; MPTs, ATMI and MARS were used to gather data bothering on mathematics performance, attitude towards mathematics and mathematics anxiety respectively.

The lowest and highest MPT scores obtainable were 0 and 100 respectively. An MPT score between 0 and 29 was adjudged very poor performance; 30 and 39, poor performance; 40 and 49, weak performance; 50 and 59, fair performance; 60 and 69, good performance; 70 and 79, very good performance and 80 and 100, excellent performance. This categorization implied that the minimum score for good performance by a student was 60. In the case of ATMI, the total score obtainable ranged from 40 to 200. An ATMI score of between 40 and 79 was adjudged totally negative attitude towards mathematics; 80 and 119, just negative attitude towards mathematics; 120 and 159, positive attitude towards mathematics and 160 and 200, totally positive attitude towards mathematics. Also, this categorization implied that the minimum score for a student to be adjudged to have positive attitude towards mathematics was 120. As for the MARS, the total score obtainable ranged from 24 to 120. The mathematics anxiety level of subjects with total a MARS score of between 24 and 47 was adjudged to be very low; 48 and 71, low; 72 and 95, high and 96 and 120,very high. This categorization also implied that the minimum score for a student to be adjudged to have exhibited high mathematics anxiety was 72. These bench mark scores were chosen so as to provide a basis for answering the research questions in categorical and articulate terms.

The data presented in this section has been divided into two main subsections, viz: demographic and research questions/hypotheses data. While the demographic data focuses on the demographic characteristics of the subjects used during the pre-tests and post-tests, the research questions/hypotheses data represent the data obtained in respect of students in each category of the samples after the pre-tests and post-tests administration of the instruments used to gather data.

4.2.1 Respondents demographic data

As stated in section 3.4, the sample size for the study was 485, with 274 in the experimental group while the remaining 211were in the control group. However, as at the time the pre-test aspect of the study was conducted, not all members took the test. Table 4.1 below shows the distribution of members of the experimental group who took the pre-tests.

Table 4.1: Distribution of Members of Experimental Group who took the Pretests

Group	Department	Sex	Instrument		
			MPT	**ATMI**	**MARS**
Experimental	Chemical Engineering	M	113	111	111
		F	31	28	28
		Total	**144**	**139**	**139**
	Computer Engineering	M	99	97	97
		F	24	20	20
		Total	**123**	**117**	**117**
	Group Total		**267**	**256**	**256**

From Table 4.1, it could be seen that 267 students in the experimental group took the MPT while 256 responded to both the ATMI and MARS. This information is further depicted in Figure 4.1 below.

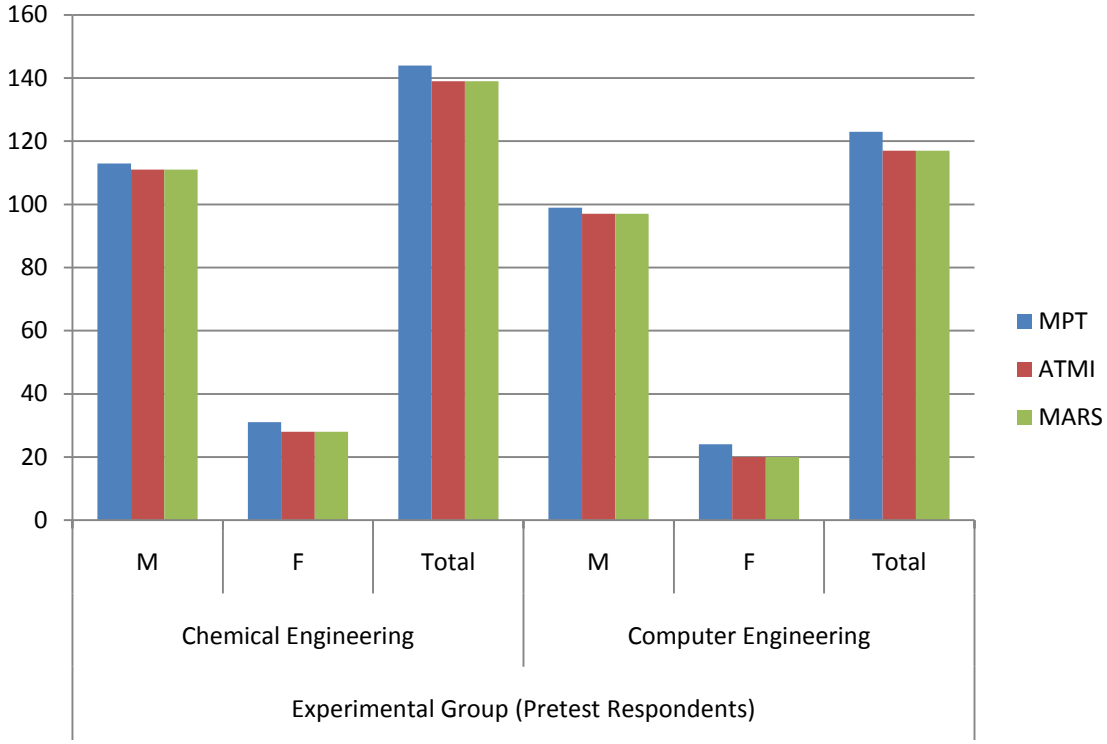


Figure 4.1: Members of Experimental Group who took Pre-tests

Table 4.2 below shows the distribution of members of the control group who took the pre-tests.

Table 4.2: Members of Control Group who took the Pre-test

Group	Department	Sex	Instrument		
			MAT	ATMI	RMARS
Control	Civil Engineering	M	105	95	95
		F	10	8	8
		Total	**115**	**103**	**103**
	Mineral Resources Engineering	M	71	71	71
		F	17	17	17
		Total	**88**	**88**	**88**
	Group Total		**203**	**191**	**191**

From Table 4.2, it could be seen that in the control group, 203 students took the MPT while 191 responded to both the ATMI and MARS. This information is further depicted in Figure 4.2 below.

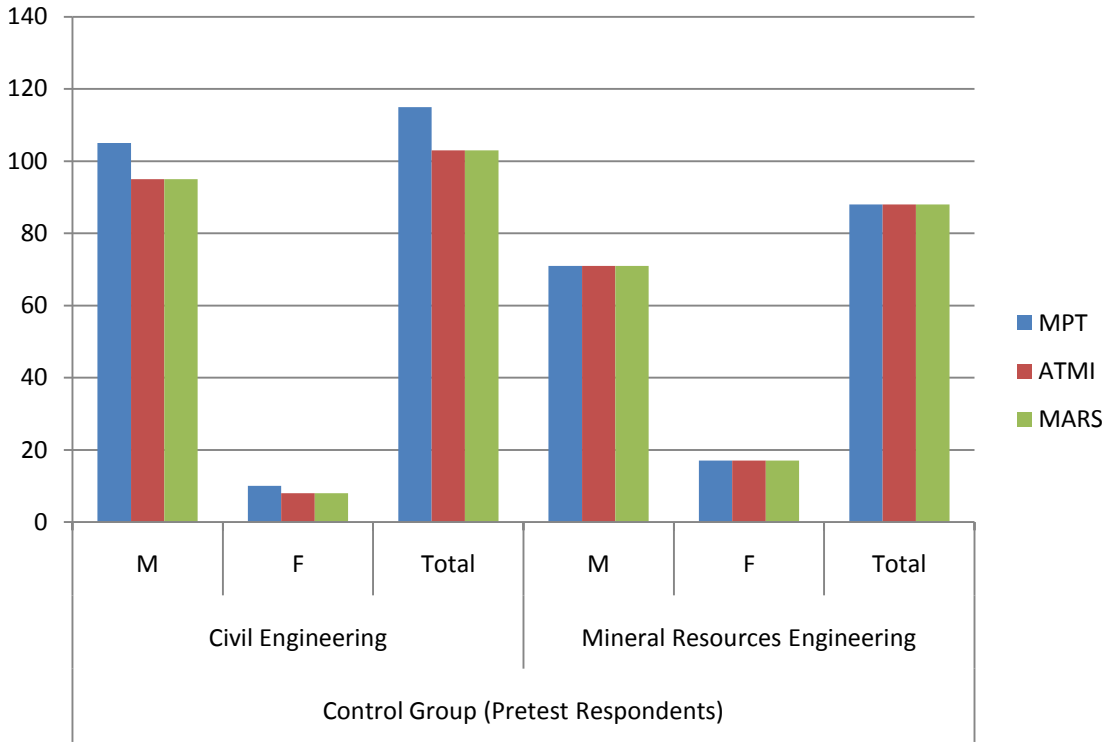


Figure 4.2: **Members of Control Group who took Pre-tests**

From Tables 4.1 and 4.2, it therefore follows that during the pre-test, the number of students that took to the MPT was 470 while 447 students responded to both the ATMI and MARS.

Table 4.3 below shows the distribution of members of the experimental group who took the post-tests.

Table 4.3: **Member of Experimental Group who took the Post-tests**

Group	Department	Sex	Instrument		
			MPT	**ATMI**	**RMARS**
Experimental	Chemical Engineering	M	113	113	113
		F	31	31	31
		Total	**144**	**144**	**144**
	Computer. Engineering	M	99	99	99
		F	24	24	24
		Total	**123**	**123**	**123**
	Group Total		**267**	**267**	**267**

From Table 4.3 above, it could be seen that during the post-test aspect of the study, 267 students in the experimental group responded to all the three instruments. This information is further depicted in Figure 4.3 below.

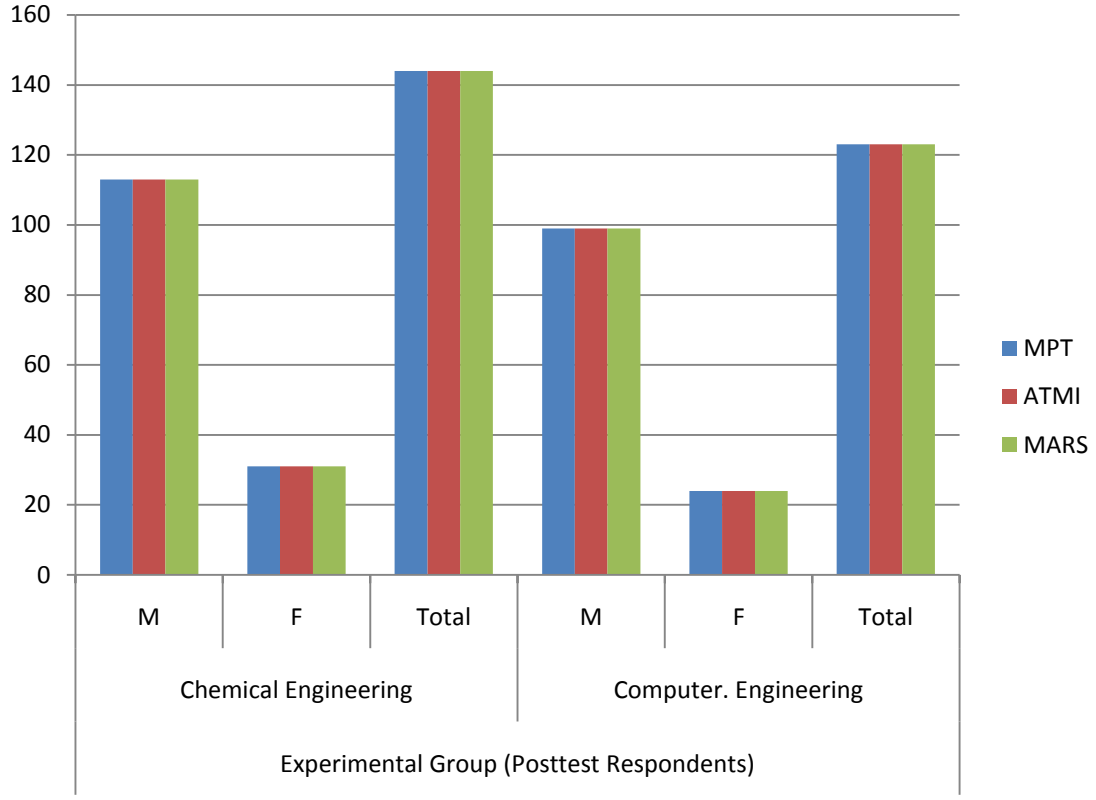


Figure 4.3: **Members of Experimental Group who took Post-tests**

Table 4.4 below shows the distribution of members of the control group who took the post-tests.

Table 4.4: **Members of Control Group who took the Post-tests**

Group	Department	Sex	Instrument		
			MPT	**ATMI**	**MARS**
Control	Civil Engineering	M	105	105	105
		F	10	10	10
		Total	**115**	**115**	**115**
	Mineral Resources Engineering	M	71	71	71
		F	17	15	15
		Total	**88**	**86**	**86**
	Group Total		**203**	**201**	**201**

From Table 4.4 above, it could be seen that during the post-tests, 203 students in the control group took the MPT while 201 responded to both the ATMI and MARS. This information is further depicted in Figure 4.4 below.

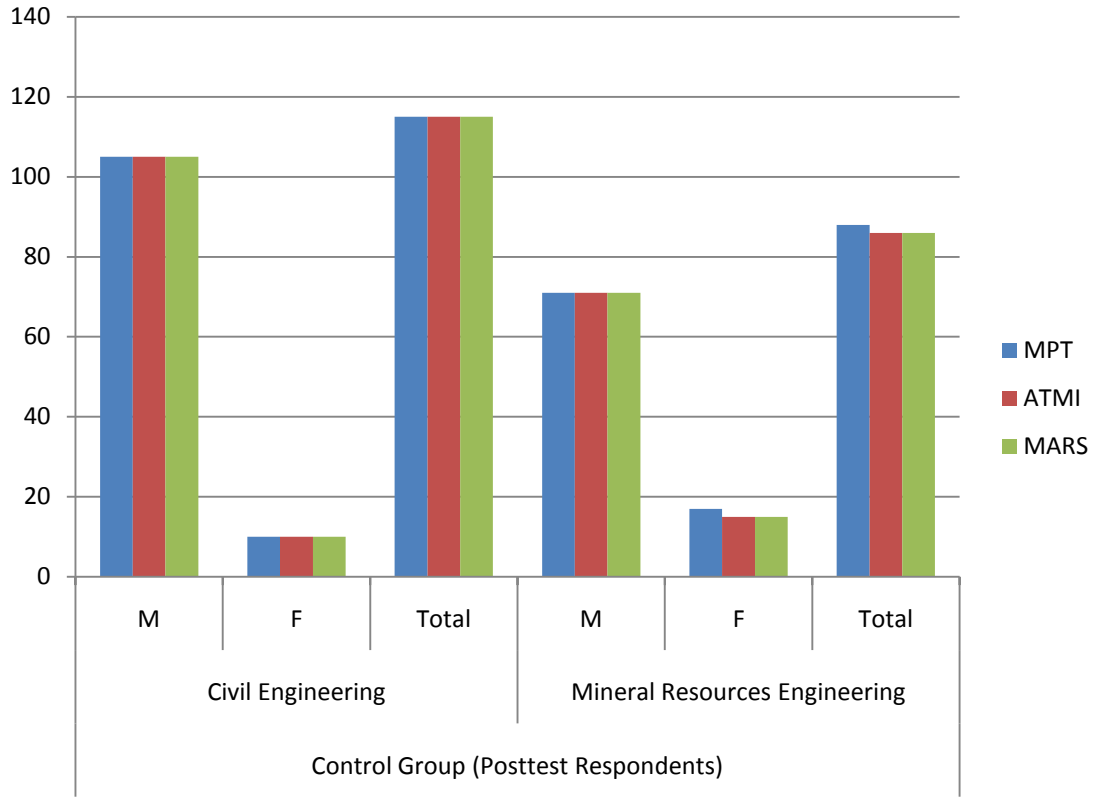


Figure 4.4: Members of Control Group who took Post-tests

From Tables 4.3 and 4.4, it could be seen that during the post-test, the number of students that took the MPTs were 470 while 468 students responded to both the ATMI and MARS.

From Tables 4.1, 4.2, 4.3 and 4.4, it could be seen that in both the pre-tests and post-tests aspects of the study, there existed differences in the number of the subjects who took MPTs on the one hand and those who responded to ATMI and MARS on the other. This was attributed to the fact that the MPTs were collected immediately after subjects have finished responding each time the tests were administered, whereas for both ATMI

and MARS, the subjects were allowed to take the instruments home and return two days later and as a result not all respondents returned their responses.

However, it could be seen that while approximately 92% of the subjects targeted to be used as samples responded to both the ATMI and MARS during the pre-test, during the post-test the percentage increased by approximately four. As for the MPTs, approximately 97% of the subjects responded both during the pre-tests and post-tests. This implied that the minimum response rate to the instruments used for this study was approximately 92%. Nworgu (1991) posits that where all the response rates are less than 70%, the result could differ considerably. Therefore the response rates obtained for this study is considered adequate for valid analysis.

4.2.2 Research questions/hypotheses data

The data generated after pre-test and post-test administration of the instruments to the respondents are provided in Tables 6.1, 6.2, 6.3, 6.4, 6.5 and 6.6 in Appendix V.

4.3 Analysis

The analyses of the data generated were undertaken from two main perspectives (i.e. the descriptive and inferential statistics perspectives). From the descriptive statistics perspective, the mean, standard deviation and standard error of the mean were calculated for each set of the data obtained. These were used to answer the research questions. From the perspective of the inferential statistics, the t-test statistics was employed to ascertain whether significant differences existed between the variables of interest as stated in the research hypotheses in section 1.5.

4.3.1 Research questions data analyses and answers

Research Question One: Will the mean mathematics performance score the students taught mathematics using the CMSTA be higher than that of those taught with the CTA, and to what extent will this difference be?

In order to answer this research question, descriptive statistics of MPT scores of the students taught mathematics using the CMSTA (experimental group) and that of those taught with the CTA (control group) were computed. Table 4.5 below shows the results obtained.

Table 4.5: Descriptive Statistics of MPT Scores of Experimental and Control Groups

Variable	Level	Group	N	Mean	Std. Deviation	Std. Error Mean
MPTs Scores	HND	Experimental	139	74.53	10.105	0.857
		Control	103	60.75	8.093	0.797
	ND	Experimental	117	75.75	9.756	0.902
		Control	86	63.67	5.904	0.637
	Both	Experimental	256	75.09	9.946	0.622
		Control	189	62.08	7.310	0.532

From Table 4.5, it could be seen that:

1. with a mean MPT score of 74.53, the HND mean mathematics performance score was higher than that of their counterparts in the control group whose mean MPT score was 60.75 by 14.78. It was therefore concluded that mean mathematics performance scores of HND students taught mathematics using the CMSTA was higher than that of those taught with the CTA.

2. with a mean MPT score of 75.75, the ND mean mathematics performance score was higher than that of their counterparts in the control group whose mean MPT score was 63.67 by 12.08. It was therefore concluded that mean mathematics performance score of ND students taught mathematics using the CMSTA was higher than that of those taught with the CTA.
3. with a mean MPT score of 75.09, both HND and ND members of the experimental counterparts in the control group whose mean MPT score was 62.08 by 13.01. It was therefore concluded that mean mathematics performance score of the students taught mathematics using the CMSTA was higher than that of those taught with the CTA.

Research Question Two: Which gender will have higher mean mathematics performance score amongst the students taught mathematics using the CMSTA and to what extent will this difference be?

In order to answer this research question, descriptive statistics of the MPT scores of male and female students taught mathematics using the CMSTA (i.e. experimental group) were computed. Table 4.6 below shows the results obtained.

Table 4.6: Descriptive Statistics of MPT Scores of Male and Female Students taught Mathematics Using the CMSTA

Variable	Level	Gender	N	Mean	Std. Deviation	Std. Error Mean
MPT Scores	HND	Male	111	75.41	11.060	1.050
		Female	28	71.04	2.769	0.523
	ND	Male	97	76.72	10.264	1.042
		Female	20	71.05	4.548	1.017
	Both	Male	208	76.02	10.691	0.741
		Female	48	71.04	3.573	0.516

Table 4.6 shows that:

1. with a mean MPT score of 75.41, the mean mathematics performance score of the male subjects in the HND experimental group was higher than that of their female counterpart whose mean MPT score was 71.04 by 4.37. It was therefore concluded that mean mathematics performance score of the HND male students taught mathematics using the CMSTA was higher than that of their female counterpart.
2. with a mean MPTs score of 76.72, the mean mathematics performance score of the male subjects in the HND experimental group was higher than that of their female counterparts whose mean MPT score was 71.05 by 5.67. It was therefore concluded that mean Mathematics Performance score of the ND male students taught mathematics using the CMSTA was higher than that of their female counterparts.
3. with a mean MPT score of 76.02, the mean mathematics performance score of the male subjects in the experimental group was higher than that of their female counterparts whose mean MPT score was 71.04 by 4.98. It was therefore concluded that mean mathematics performance score of the male students taught mathematics using the CMSTA was higher than that of their female counterparts.

Research Question Three: Will the mean attitude towards mathematics disposition of the students taught mathematics using the CMSTA be more favourable than that of those taught with the CTA, and to what extent will this difference be?

In order to answer this research question, descriptive statistics of ATMI scores of the students taught mathematics using the CMSTA (experimental group) and than that of those taught with the CTA (control group) were computed. Table 4.7 below shows the results obtained.

Table 4.7: Descriptive Statistics of Experimental and Control Groups' ATMI scores

Variable	Level	Group	N	Mean	Std. Deviation	Std. Error Mean
ATMI Scores	HND	Experimental	139	147.33	20.544	1.743
		Control	103	137.02	28.994	2.857
	ND	Experimental	117	145.86	20.232	1.870
		Control	86	138.27	24.715	2.665
	Both	Experimental	256	146.66	20.375	1.273
		Control	189	137.59	27.067	1.969

From Table 4.7, it could be seen that:

1. the mean ATMI score (i.e. 147.33) of the HND students taught mathematics using the CMSTA (i.e. experimental group) was higher than that of those taught with the CTA (i.e. control group) whose mean ATMI score was 137.02, by 6.31. It was therefore concluded that the attitude towards mathematics dispositions of the HND students taught mathematics using the CMSTA was more favourable than that of those taught with the CTA.

2. the mean ATMI score (i.e. 145.86) of the ND students taught mathematics using the CMSTA (i.e. experimental group) was higher than that of those taught with the CTA (i.e. control group) whose mean ATMI score was 138.27, by 7.59. It was therefore concluded that the attitude towards mathematics dispositions of the ND students taught mathematics using the CMSTA was more favourable than that of those taught with the CTA.
3. the mean ATMI score (i.e. 146.66) of the both HND and ND students taught mathematics using the CMSTA (i.e. experimental group) was higher than that of those taught with the (i.e. control group) whose mean ATMI score was 137.59, by 9.07. It was therefore concluded that the attitude towards mathematics dispositions of the students taught mathematics using the CMSTA was more favourable than that of those taught with the CTA.

Research Question Four: Which gender will exhibit more positive attitude towards mathematics dispositions amongst the students taught mathematics using the CMSTA, and to what extent will this difference be?

In order to answer this research question, descriptive statistics the ATMI scores of male and female students taught mathematics using the CMSTA (experimental group) were computed. Table 4.8 below shows the results obtained.

Table 4.8: **Descriptive Statistics ATMI Scores of Male and Female Students taught Mathematics using CMSTA**

Variable	Level	Gender	N	Mean	Std. Deviation	Std. Error Mean
ATMI Scores	HND	Male	111	147.76	20.844	1.978
		Female	28	145.64	19.581	3.700
	ND	Male	97	146.39	19.501	1.980
		Female	20	143.30	23.859	5.335
	Both	Male	208	147.12	20.192	1.400
		Female	48	144.67	21.255	3.068

Table 4.8 shows that:

1. with a mean score of 147.76, the mean ATMI score of the male subjects in the HND experimental group was higher than that of their female counterparts whose mean ATMI score was 145.64 by 2.12. It was therefore concluded that male subjects in the HND experimental group exhibited more favourable attitude towards mathematics disposition than their female counterparts.
2. with a mean score of 146.39, the mean ATMI score of the male subjects in the ND experimental group was higher than that of their female counterparts whose mean ATMI score was 143.30 by 3.09. It was therefore concluded that male subjects in the ND experimental group exhibited more favourable attitude towards mathematics disposition than their female counterparts.
3. with a mean score of 147.12, the mean ATMI score of the male subjects in the experimental group was higher than that of their female counterparts whose mean ATMI score was 144.67 by 2.45. It was therefore concluded that male subjects in the experimental group exhibited more favourable attitude towards mathematics disposition than their female counterparts.

Research Question Five: Will the mean mathematics anxiety level of the students taught mathematics using the CMSTA be lower than that of those taught with the CTA, and to what extent will this difference be?

In order to answer this research question, descriptive statistics of the MARS scores of students taught mathematics using the CMSTA (experimental group) and that of those taught with the CTA (control group) were computed. Table 4.9 below shows the results obtained.

Table 4.9: **Descriptive Statistics of Mars Scores of Experimental and Control Groups**

Variable	Level	Group	N	Mean	Std. Deviation	Std. Error Mean
MARS scores	HND	Experimental	139	34.35	14.214	1.206
		Control	103	47.55	18.628	1.835
	ND	Experimental	117	34.41	14.238	1.316
		Control	86	46.02	18.122	1.954
	Both	Experimental	256	34.38	14.197	0.887
		Control	189	46.86	18.366	1.336

From Table 4.9, it could be seen that:

1. the mean MARS score (i.e. 34.35) of the HND students taught mathematics using the CMSTA (i.e. experimental group) was lower than that of those taught with the CTA (i.e. control group) whose mean MARS score was 47.55, by 13.20. It was therefore concluded that the mathematics anxiety levels of the HND students taught mathematics using the CMSTA was lower than that of those taught with the CTA.
2. the mean MARS score (i.e. 34.41) of the ND students taught mathematics using the CMSTA (i.e. experimental group) was lower than that of those taught with the CTA

(i.e. control group) whose mean MARS score was 46.02, by 11.61. It was therefore concluded that the mathematics anxiety levels of the ND students taught mathematics using the CMSTA was lower than that of those taught with the CTA.

3. the mean MARS (i.e. 34.38) score of the students taught mathematics using the CMSTA (i.e. experimental group) was lower than that of those taught with the CTA (i.e. control group) whose mean MARS score was 46.86, by 12.48. It was therefore concluded that the mathematics anxiety levels of the students taught mathematics using the CMSTA was lower than that of those taught with the CTA.

Research Question Six: Which gender will exhibit lower mathematics anxiety level amongst the students taught mathematics using the CMSTA, and to what extent will this difference be?

In order to answer this research question, descriptive statistics of MARS scores of male and female students taught mathematics using the CMSTA (experimental group) were computed. Table 4.10 below shows the results obtained.

Table 4.10: **Descriptive Statistics of RMARS Scores of Male and Female Students taught Mathematics using the CMSTA**

Variable	Level	Gender	N	Mean	Std. Deviation	Std. Error Mean
MARS Scores	HND	Male	111	34.22	14.214	1.349
		Female	28	34.89	14.464	2.733
	ND	Male	97	34.19	14.231	1.445
		Female	20	35.50	14.591	3.263
	Both	Male	208	34.20	14.187	0.984
		Female	48	35.15	14.365	2.073

Table 4.10 shows that:

1. with a mean score of 34.22, the mean MARS score of the male subjects in the HND experimental group was lower than that of their female counterparts whose mean MARS score was 34.89 by 0.67. It was therefore concluded that male subjects in the HND experimental group exhibited lower mathematics anxiety levels than their female counterparts.
2. with a mean score of 34.19, the mean MARS score of the male subjects in the ND experimental group was lower than that of their female counterparts whose mean MARS score was 35.50 by 1.31. It was therefore concluded that male subjects in the ND experimental group exhibited lower mathematics anxiety levels than their female counterparts.
3. with a mean score of 34.20, the mean MARS score of the male subjects in the experimental group was lower than that of their female counterparts whose mean MARS score was 35.15 by 0.95. It was therefore concluded that male subjects in the experimental group exhibited lower mathematics anxiety levels than their female counterparts.

4.3.2 Hypotheses testing

Ho_1: There is no significant difference between the mean mathematics performance scores of the students taught mathematics using the CMSTA and that of those taught with the CTA.

In order to test this null hypothesis, independent sample t-test analysis between the mean MPT scores of the students taught mathematics using the CMSTA (experimental group) and that of those taught with the CTA (control group) was undertaken. Table 4.11 below shows results obtained.

Table 4.11: Independent Sample t-Test Analysis between the MPT Scores of Experimental and Control group

Variable	Level	Group	N	Mean	Std. Deviation	Std. Error Mean	df	t	p
MPTs Scores	HND	Experimental	139	74.53	10.105	0.857	240	11.391	0.000*
		Control	103	60.75	8.093	0.797			
	ND	Experimental	117	75.75	9.756	0.902	201	10.188	0.000*
		Control	86	63.67	5.904	0.637			
	Both	Experimental	256	75.09	9.946	0.622	443	15.199	0.000*
		Control	189	62.08	7.310	0.532			

*$p < 0.05$

From Table 4.11, it could be seen that

1. with $p = 0.000 < 0.05$, the difference between the experimental and control MPT scores of the HND students is significant. In other words, the difference between the mean mathematics performance scores of the HND students taught mathematics using the CMSTA and that of those taught with the CTA is significant.
2. with $p = 0.000 < 0.05$, the difference between the experimental and control MPTs scores of the ND students is significant. In other words, the difference between the mean mathematics performance scores of the ND students taught mathematics using the CMSTA and that of those taught with the CTA is significant.
3. with $p = 0.000 < 0.05$, the difference between the experimental and control K scores is significant. In other words, the difference between the mean mathematics performance scores of the students taught mathematics using the CMSTA and that of those taught with the CTA is significant.

Ho_2: There is no significant gender difference between the mean mathematics

performance scores of the students taught mathematics using the CMSTA.

In order to test this null hypothesis, independent sample t-test analysis between the mean MPT scores of the male and female students taught mathematics using the CMSTA (experimental group) was undertaken. Table 4.12 below shows the results obtained.

Table 4.12: Independent Sample t-Test Analysis between the MAT Scores of Experimental Group's Male and Female students

Variable	Level	Gender	N	Mean	Std. Deviation	Std. Error Mean	df	t	p
MPTs Scores	HND	Male	111	75.41	11.060	1.050	137	2.069	0.040*
		Female	28	71.04	2.769	0.523			
	ND	Male	97	76.72	10.264	1.042	115	2.416	0.017*
		Female	20	71.05	4.548	1.017			
	Both	Male	208	76.02	10.691	0.741	254	3.181	0.002*
		Female	48	71.04	3.573	0.615			

*$p < 0.05$

From Table 4.12, it could be seen that

1. with $p = 0.040 < 0.05$, the difference between the MPT scores of male and female HND students taught mathematics using the CMSTA (experimental group) is significant. In other words, there was significant difference between the mean mathematics performance scores of the male and female HND students taught mathematics using the CMSTA.
2. with $p = 0.017 < 0.05$, the difference between the MPT scores of male and female ND students taught mathematics using the CMSTA (experimental group) is significant. In other words, the difference between the mean mathematics performance scores of the male and female ND students taught mathematics using the CMSTA is significant.

3. With $p = 0.002 < 0.05$, the difference between the MPT scores of male and female students taught mathematics using the CMSTA (experimental group) is significant. In other words, the difference between the mean mathematics performance scores of the male and female students taught mathematics using the CMSTA is significant.

Ho_3: There is no significant difference between the mean attitude towards mathematics dispositions of the students taught mathematics using the CMSTA and that of those taught with the CTA.

In order to test this null hypothesis, independent sample t-test analysis between the mean ATMI scores of the students taught mathematics using the CMSTA (experimental group) and that of those taught with the CTA (control group) was undertaken. Table 4.13 below shows the results obtained.

Table 4.13: **Independent Sample t-Test Analysis between ATMI Scores of Experimental and Control Group**

Variable	Level	Group	N	Mean	Std. Deviation	Std. Error Mean	df	t	p
ATMI Scores	HND	Experimental	139	147.33	20.544	1.743	240	3.238	0.001*
		Control	103	137.02	28.994	2.857			
	ND	Experimental	117	145.86	20.232	1.870	201	2.405	0.017*
		Control	86	138.27	24.715	2.665			
	Both	Experimental	256	146.66	20.375	1.273	443	4.034	0.000*
		Control	189	137.59	27.067	1.969			

*$p < 0.05$

Table 4.13 shows that:

1. with $p = 0.001 < 0.05$, the difference between the experimental and control K G F B , G * re was significant difference between the attitude towards mathematics dispositions of the HND students taught mathematics using the CMSTA and that of those taught with the CTA.
2. with that $p = 0.017 < 0.05$, the difference between the experimental and K G B , G * the difference between the attitude towards mathematics dispositions of the ND students taught mathematics using the CMSTA and those taught with the CTA is significant.
3. with that $p = 0.017 < 0.05$, the difference between the experimental and K G , G * difference between the attitude towards mathematics dispositions of the students taught mathematics using the CMSTA and that of those taught with the CTA is significant.

Ho_4: There is no significant gender difference in attitude towards mathematics dispositions of the students taught mathematics using the CMSTA.

In order to test this null hypothesis, the independent sample t-test analysis between the mean ATMI scores of the male and female students taught mathematics using the CMSTA (experimental group) was undertaken. Table 4.14 below shows results obtained.

Table 4.14: **Independent Sample t-Test Analysis between the ATMI Scores of Experimental Group's Male and Female Students**

Variable	Level	Gender	N	Mean	Std. Deviation	Std. Error Mean	df	t	p
ATMI Scores	HND	Male	111	147.76	20.844	1.978	137	0.485	0.628
		Female	28	145.64	19.581	3.700			
	ND	Male	97	146.39	19.501	1.980	115	0.621	0.536
		Female	20	143.30	23.859	5.335			
	Both	Male	208	147.12	20.192	1.400	254	0.751	0.453
		Female	48	144.67	21.255	3.068			

From Table 4.14 it could be seen that:

1. with $p = 0.628 > 0.05$, the difference between the ATMI scores of male and female HND experimental group students is not significant. In other words, gender difference in attitude towards mathematics disposition of the HND students taught mathematics using the CMSTA is not significant.
2. with $p = 0.536 > 0.05$. This means that the difference between the ATMI scores of male and female ND experimental group students is not significant. In other words, gender difference in attitude towards mathematics disposition of the ND students taught mathematics using the CMSTA is not significant.
3. with $p = 0.453 > 0.05$. This means that the difference between the ATMI scores of male and female experimental group students is not significant. In other words, gender difference in attitude towards mathematics disposition of the students taught mathematics using the CMSTA is not significant.

Ho_5: There is no significant difference between the mean mathematics anxiety levels of the students taught mathematics using the CMSTA and that of those taught with the CTA.

In order to test this null hypothesis, independent sample t-test analysis between the mean MARS scores of the students taught mathematics using the CMSTA (experimental group) and that of those taught with the CTA (control group) was undertaken. Table 4.15 below shows the results obtained.

Table 4.15: **Independent Sample T-Test Analysis between the MARS Scores of Experimental and Control Group**

Variable	Level	Group	N	Mean	Std. Deviation	Std. Error Mean	df	t	p
MARS scores	HND	Experimental	139	34.35	14.214	1.206	240	-6.253	0.000*
		Control	103	47.55	18.628	1.835			
	ND	Experimental	117	34.41	14.238	1.316	201	-5.111	0.000*
		Control	86	46.02	18.122	1.954			
	Both	Experimental	256	34.38	14.197	0.887	443	-8.082	0.000*
		Control	189	46.86	18.366	1.336			

$* p < 0.05$

Table 4.15 shows that:

1. with $p = 0.000 < 0.05$, K
   of the HND students is significant. In other words, there is significant difference between the mathematics anxiety levels of the HND students taught mathematics using the CMSTA and that of those taught with the CTA.
2. with that $p = 0.000 < 0.05$, the difference between the experimental and
   mean MARS scores of the ND students is significant. In other words, there is significant difference between the mathematics anxiety levels of the ND students taught mathematics using the CMSTA and that of those taught with the CTA.
3. with that $p = 0.000 < 0.05$* K
   scores is significant. In other words, there is significant difference between the

mathematics anxiety levels of the students taught mathematics using the CMSTA and that of those with the CTA.

Ho_6: There is no significant gender difference in the mathematics anxiety levels of the students taught mathematics using the CMSTA.

In order to test this null hypothesis, independent sample t-test analysis between the mean MARS scores of the male and female students taught mathematics using the CMSTA was undertaken. Table 4.16 below shows independent sample t-test analysis between the mean MARS scores of the male and female students taught mathematics using the CMSTA.

Table 4.16: **Independent Sample T-Test analysis between RMARS Scores of Experimental Group's Male and Female Students**

Variable	Level	Gender	N	Mean	Std. Deviation	Std. Error Mean	df	t	p
MARS Scores	HND	Male	111	34.22	14.214	1.349	137	-0.224	0.823
		Female	28	34.89	14.464	2.733			
	ND	Male	97	34.19	14.231	1.445	115	-0.375	0.709
		Female	20	35.50	14.591	3.263			
	Both	Male	208	34.20	14.187	0.984	254	-0.415	0.679
		Female	48	35.15	14.365	2.073			

Table 4.16 shows that:

1. with $p = 0.823 > 0.05$, the difference between the MPT scores of male and female HND experimental group students is not significant. In other words, there is no significant gender difference in the mathematics anxiety levels of the HND students taught mathematics using the CMSTA.

2. with $p = 0.709 > 0.05$, the difference between the MPT scores of male and female ND experimental group students is not significant. In other words, there is no significant gender difference in the mathematics anxiety levels of the ND students taught mathematics using the CMSTA.
3. with $p = 0.679 > 0.05$, the difference between the MPT scores of male and female experimental group students is not significant. In other words, there is no significant gender difference in the mathematics anxiety levels of the students taught mathematics using the CMSTA.

4.4 Summary of Major Findings

The following are the major findings of this study:

1. Mathematics performances of the students taught mathematics using the CMSTA were significantly higher than the performances of those taught with the CTA.
2. Mathematics performances of male students taught mathematics using the CMSTA were significantly higher than those of their female counterparts.
3. Attitude towards mathematics dispositions of the students taught mathematics using the CMSTA were significantly more favourable than the attitude towards mathematics dispositions of those taught with the CTA.
4. The male students taught mathematics using the CMSTA exhibited more favourable attitude towards mathematics dispositions than their female counterparts. However, these differences were not significant.
5. Mathematics anxiety levels of the students taught mathematics using the CMSTA were significantly lower than the mathematics anxiety levels of those taught with CTA.
6. Male students taught mathematics using the CMSTA exhibited lower mathematics

anxiety levels than their female counterparts. However, these differences were not significant.

4.5 Discussions

That the mathematics performances of the students taught mathematics using the CMSTA were significantly higher than the mathematics performances of those taught with the CTA may be attributed to the effect of treatment on the experimental group. This line of reasoning was supported by the fact that, unlike for the post-tests (see Tables 4.11), no significant difference was found between the experimental and control groups pre-test mean MPT scores (see Table 3.1). In fact, it was found that the pre-test mean MPT scores of both groups were good whereas the post-test mean MPT score of the experimental group was found to be very good as against that of the control group that was still good. Furthermore, it could be seen that while the difference between the pre-test and post-test mathematics performance of the control group was not significant, this difference was significant in the case of the experimental group.

This finding is similar to that obtained by Taylor (2008) and Sánchez et. al. (2004). 0. . 6' a web-based, computer-assisted curriculum in remedial mathematics classes as compared to classes that use the traditional lecture method of instruction. He (Taylor, 2008) reported that mathematical achievement of the students in the web-based, computer-assisted curriculum classes improved when their mean scores went from 16.56 to 20.56 (an increase that was statistically significant), with a Cohen's d of about 0.611. Sánchez et. al. (2004) who were initially interested in studying gender differences in attitudes towards mathematics and mathematics taught with computers, found a positive correlation between high marks and a positive attitude towards mathematics taught with

computers was found for boys while a positive correlation between high marks and self-confidence was found for girls.

The finding that the mathematics performances of the male students taught mathematics using the CMSTA were significantly higher than the mathematics performances of their female counterpart could not be attributed to the effect of treatment on the experimental group. This line of reasoning was supported by the fact that both male and female students taught mathematics using the CMSTA were exposed to the same treatment at the same time and under the same condition throughout the treatment period. Therefore, any gender difference in level performance could not be attributed to treatment effect, but possibly to other factors which were not examined in the study. This did not appeared as a surprise as it was in conformity with many studies (e.g. Fennema and Sherman, 1977; Sprigler and Alsup, 2003; Janson, 1996; Gallagher and Kaufman, 2006; A -Petruso et. al., 2004; Entwistle et. al., 1994; Gamoran, 1992; Callahan and Clements, 1984; Dossey et. al., 1988) that reported male superiority over female in mathematics achievement.

Some of these studies (e.g. Callahan and Clements, 1984; Dossey et al., 1988) hold that due to the fact that gender differences seem to appear from puberty stage, the differences may be attributed to innate biological differences, social factors and anxiety among females rather than the subject matter or teaching methodology. This position is supported by the line of reasoning that because both males and females learn under the same conditions, yet with the females having lower scores, these other factors must be the intervening factors responsible for the difference. As in previous studies (e.g. Fennema and Sherman, 1977; Sprigler and Alsup, 2003; Janson, 1996; Gallagher and I *0. . 49 A -Petruso et. al., 2004; Entwistle et. al., 1994; Gamoran, 1992; Callahan and Clements, 1984; Dossey et. al., 1988), the results in this study suggested

that the male students seemed to have performed better than their female counterparts both at the pre-tests and post-tests levels. It is observed that this situation appears to be the same for all the subgroups in each group that made up the study subject except at the pre-test level of one of the subgroup.

However, there are other studies that contradict the above results which tend to claim male superiority in mathematics performace. Studies such as those of Ding et. al., (2007), Mullis et al. (2004), Kiamanesh (2006), Taylor (2008) reported the absence of gender difference in mathematics achievement. It this researchers conviction that this contradiction had also gone further to support the belief that perhaps the issue of the existence or not of gender difference in mathematics performance will remain inconclusive for a very long time to come, as it is today. Therefore, the need exists for continued research in this area until an incontrovertible conclusive position is attained.

The finding that the attitude towards mathematics dispositions of the students taught mathematics using the CMSTA were significantly more favourable than the attitude towards mathematics dispositions of those taught with the CTA may be attributed to the effect of treatment on the experimental group. This line of reasoning was supported by the fact that no significant difference was found between the experimental and control groups pre-test mean ATMI scores (see Table 3.1) whereas in the case of the post-test, the difference was significant (see Tables 4.13). These results contradict that obtained by Taylor (2008).

Taylor (2008) set forth to examine the effects each treatment had on students' mathematics anxiety and mathematics attitude in addition to the main purpose of her study which was to look at differences in student achievement in a web-based, computer-assisted curriculum in remedial mathematics classes as compared to classes

that use a traditional lecture method of instruction. She (Taylor, 2008) reported no statistically significant differences for the experimental group from the pre-test to the post-test, statistically significant differences for the control group from the pre-test to post-test, and no statistically significant differences between both (experimental and control) groups from the pre-test to post-test. She however reported that while statistical significant difference did not occur for the experimental group, their attitudes toward mathematics did improve. She further reported that on the other hand, the control group showed statistical significance, but their scores showed that their attitude toward mathematics was not as good at the end of the semester. This also contradicts the results

subjects in both the experimental and control groups are positive. Another area of disagreement between the result of this study and that of Taylor (2008) is in this study both the pre-tests and post-tests range of scores of the attitude towards mathematics inventory for the experimental group were less than those of the control group.

That male students taught mathematics using the CMSTA exhibited more favourable attitude towards mathematics disposition than their female counterparts even though these differences were not significant, may be attributed to the effect of treatment on the experimental group. The effect of the treatment on the experimental group seems to be homogenous as can be seen from the rejection of the fifth and sixth null hypotheses. This line of reasoning was supported by the fact that both male and female subjects in the experimental group were exposed to the same treatment at the same time and under the same condition throughout the treatment period. Therefore, it is only natural that since the treatment has impacted on the experimental group as could be seen from the rejection of the fifth and sixth null hypotheses above, both genders would have contributed approximately equally towards the result. In this regard, the result suggested

that while no significant difference was found between the pre-test and post-test mean ATMI scores of both male and female subjects in the control group; in the case of the male and female subjects in the experimental group the difference was significant.

The above results, as obtained in this study, are in affirmation with similar findings from recent research such as by Sánchez et. al, (2004); Pajares and Graham (1999) and Ma and Kishor (1997). Sánchez et. al., (2004), who were interested in studying gender differences in attitudes towards mathematics and mathematics taught with computers, detected a general tendency to a positive attitude towards mathematics and towards mathematics taught with computers but they found no significant gender differences in this. Earlier, Pajares and Graham (1999) as well as Ma and Kishor (1997) found that although males possess higher Self-efficacy, have more positive attitude towards mathematics and tend to obtain better results than girls; there is no significant difference among gender in terms of attitude towards mathematics.

The finding that the mathematics anxiety levels of the students taught mathematics using the CMSTA were significantly lower than the mathematics anxiety levels of those taught with the CTA, may be attributed to the effect of treatment on the experimental group. This line of reasoning was supported by the fact that while no significance between the experimental and control groups pre-test mean MARS scores, while for the post-test scores this difference was significant.

These results are similar to that obtained by Taylor (2008), who set forth to also examined the effects a web-based, computer-assisted curriculum in remedial mathematics classes mathematics anxiety and in addition to the main purpose of her study which was to look at differences in student achievement in a web-based, computer-assisted curriculum in remedial mathematics classes as compared to classes

that use a traditional lecture method of instruction. She reported statistically significant differences from pre-test to post-test for the experimental group, no statistically significant differences from pre-test to post-test for the control group, and statistically significant differences from pre-test to post-test for both (experimental and control) groups. She further reported that the anxiety level of the experimental group and the control group both decreased, but the anxiety level of the experimental group decreased more than that of the control group. In view of the results she obtained, she posits that mathematics anxiety decreased at a greater rate in the experimental group than the control group; therefore, the anxiety of the experimental group was less than the anxiety in the control group.

Finally, the finding that the male students taught mathematics using the CMSTA exhibited lower mathematics anxiety levels than their female counterparts even though these differences were found not to be significant, might be attributed to the effect of treatment on the experimental group. The effect of the treatment on the experimental group seemed to be homogenous as could be seen from the rejection of the sixth null hypotheses. This line of reasoning was supported by the fact that both male and female subjects in the experimental group were exposed to the same treatment at the same time and under the same condition throughout the treatment period. Therefore, it is only natural that since the treatment has impacted on the experimental group as could be seen from the rejection of the sixth null hypotheses, both genders had contributed approximately equally towards the result. In this regard, the result showed that while no significant difference was found between the pre-test and post-test MARS scores of both male and female subjects in the control group; in the case of the male and female subjects in the experimental group the difference was significant. Furthermore, the result indicated no significant difference the male and female pre-test as well as post-

test MARS scores of the experimental group.

These findings are only similar to that obtained by Gierl and Bisanz (1995) who found reported no significant difference in the mathematics anxiety in either gender in the early grades and different from that obtained by Bernstein et. al. (1992) as well as A C /775'

anxiety in secondary school and in college. The belief that this finding is attributed to the treatment is further supported by research findings which suggested that teachers can prevent and reduce mathematics anxiety by designing better teaching practices (Cornell, 1999; Steele and Arth, 1998), creating a comfortable atmosphere (Jackson and Leffingwell, 1999; Steele and Arth, 1998), providing encouragement (Godbey, 1997; Jackson and Leffingwell, 1999), etc.

Research on the effects of computer-based instruction in the past 10 years encountered during literature search revealed that the positive effects of the use of a computer for teaching to supplement regular instruction includes (a) Students learned more in less time, (b) had slightly higher grades on post-tests, and (c) had improved attitudes toward learning (Kinney et. al., 2004; Kulik and Kulik, 1986). In addition it was found that students benefited from computer-mediated learning because all students met at the same time with the same instructor, which gave them a sense of community, and the students were more likely to meet the course objectives on schedule (Kinney and Robertson, 2003). Mediated learning offers students an alternative to the direct-instruction approach learned in traditional lecture classes (Lundell et. al., 2001).

Based on the discussions so far and findings of the study, drawing inference of causality between mathematics performance of the subjects, their attitude towards mathematics and mathematics anxiety levels is outside the scope of this study. However, since some

research have reported direct relationship between mathematics performance and attitude towards mathematics while others have reported a reciprocal relationship between mathematics performance and mathematics anxiety levels; it is believed that the CMSTA used to teach mathematics had positive effect on the students performance, attitude towards mathematics and mathematics anxiety.

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 Introduction

In this chapter, a summary of the whole study is presented. This is followed by conclusion derived from the summary and then some recommendations based on the observations and findings made during the study. Also discussed are implications of the

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chapter ends with a section suggesting areas for further research. These issues are discussed under the following subheadings:

- Summary
- Major Findings
- Conclusions
- Recommendations
- Implications of the Findings
- Contributions to Knowledge
- Limitations of the Study
- Suggestions for Further Research

5.2 Summary

In this study, the impact of a Computer Mediated Systems Teaching Approach (CMSTA) on the mathematics performance, attitude and anxiety of Polytechnic engineering students was investigated. The rational for undertaking the study stems

developing countries is an essential step in the struggle to close the technological gap

between the developed and developing countries. Further, it is believed that computer usage to compliment and supplement the teaching/learning process is a sine qua non for the technological breakthrough of developing countries. Reckoning on the different reasons put forward by advocates of computers use in the teaching/learning process, it was posited that computers can be used to address many challenges confronting mathematics teaching situations in Nigeria. It is believed that the opportunities afforded by new, improved and emerging technology developments, especially in the field of ICTs, have the potentials of providing the much needed impetus for addressing these problems. This forms the motivation for this study.

The problem of the study stemmed from the fact that despite the National Policy on C DE *0. . 2'

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Nnoli and Sulaiman (2001) observed that new technologies were failing to penetrate the Nigerian education system. It was observed that teaching of mathematics to Polytechnic engineering students in Nigeria is a largely performed using traditional approach which is lagging behind that of other nations using new technologies. Since the use of multimedia courseware in engineering has rapidly gained popularity (Reagan and Shepard, 1996), there is the need to make efforts to catch up with international best practices.

It was also observed that in many Polytechnics in Nigeria, mathematics teaching is lagging behind other subjects in term of experimentation. Mathematics teachers hardly carry out mathematical teaching experiments despite the fact that it has been established that experiments help students in their learning (Bourne et. al., 1996). This problem could be addressed by a CMSTA whereby many physical phenomena governed by

mathematical laws could be easily simulated. Other observed problems associated with the teaching of mathematics to Polytechnic engineering students in Nigerian emanates from their negative attitude towards the subject and high anxiety level. Since it has been established that computer usage in the teaching/learning process can positively change

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might be useful in providing the way forward.

The study found its significance in its attempt to close the existing dearth of research work on deployment of computers to enhance teaching/learning of mathematics in the

m in particular and other sectors in general. Secondly, it was considered as a contributory work in the field of developing indigenous CMSTA framework for teaching/learning mathematics at the Polytechnic level. Thirdly, it was an attempt to respond to declarations made in sections 9(c), 9(d), 9(h), 9(i), 19(m), 30(f), 73 and 103(d) of the National Policy on Education (NPE, 2004). Fourthly, it was posited that the study could provide mathematics lecturers teaching mathematics to Polytechnic engineering students in Nigeria with a CMSTA framework which is potentially capable of helping them improve their students mathematics performance, positively foster their attitude towards mathematics and as well help lower their mathematics anxiety level. Fifthly, it could assist the students facilitate the rate at

content delivery. Finally, education policy makers in Nigeria may use the experiences gleaned from this study to come up with new policies on computer usage in the teaching/learning process.

Review of relevant literature related to the subject matter of the study was undertaken from six perspectives. These were conceptual framework, issues related to the use of

computers in the teaching/learning process, issues related to mathematics performance, issues related to attitude towards mathematics, issues related to mathematics anxiety and related studies perspectives. Also undertaken was a review of the relationship between the study variables which was organized under the following subheadings: relationship between Attitudes Towards Mathematics and Mathematics Achievement, relationship between Mathematics Anxiety and Mathematics Achievement and relationship between Teaching Methods and Mathematics Achievement. Finally, a review of some studies directly related to the subject matter of this research was undertaken before the literature review was concluded with a discussion on implications of the reviewed literature to the study

In the course of the study, only mathematics curricula of both the National Diploma (ND) and Higher National Diploma (HND) levels in the engineering disciplines were , A * B F B mathematics performance, attitude towards mathematics and mathematics anxiety level were focused on. The quasi experimental research design was adopted to carry out the study. This design was considered suitable for the conduct of the study because the study sought to evaluate the impact of AK , , ' mathematics performance, their attitudes towards mathematics and mathematics anxiety levels. The pre-test/post-test Non Equivalent Groups Design (NEGD) approach was used in line with the guidelines and steps outlined by Gall et.al, (1996). Consequently, the control group was taught mathematics using the conventional approach while the experimental group was taught with the CMSTA.

The Three instruments used to gather data in the study were Mathematics performance Tests (MATs), Attitude Towards Mathematics Inventory (ATMI) and Mathematics

Anxiety Rating Scale (MARS). MATs, ATMI and MARS were used to gather data on mathematics performance, attitude towards mathematics and mathematics anxiety respectively. A combination of the convenience, purposive and random sampling techniques was adopted to select 487 students used for data analyses, which were undertaken from the descriptive and inferential statistics perspective.

As the study was experimental in nature, a one semester educational programme A Mediated Systems Teaching Approach AK ' , implementation of the adopted research design was: assignment of research participants to experimental and control groups, administration of the research instruments as pre-tests to both groups, administration of the treatment to only experimental group for one semester and administration of the research instruments as post-test to both groups after treatment administration.

5.3 Major Findings

The following are the major findings of this study:

1. Mathematics performances of the students taught mathematics using the CMSTA were significantly higher than the performances of those taught with the CTA.
2. Mathematics performances of male students taught mathematics using the CMSTA were significantly higher than those of their female counterparts.
3. Attitude towards mathematics dispositions of the students taught mathematics using the CMSTA were significantly more favourable than the attitude towards mathematics dispositions of those taught with the CTA.
4. The male students taught mathematics using the CMSTA exhibited more favourable attitude towards mathematics dispositions than their female counterparts. However,

these differences were not significant.

5. Mathematics anxiety levels of the students taught mathematics using the CMSTA were significantly lower than the mathematics anxiety levels of those taught with CTA.
6. Male students taught mathematics using the CMSTA exhibited lower mathematics anxiety levels than their female counterparts. However, these differences were not significant.

5.4 Conclusion

In view of the results obtained in this study and similar results from related studies, it

mathematics performance, foster their attitude towards mathematics positively and lower their mathematics anxiety.

5.5 Recommendations

Based on the findings of the study and other related studies, the following recommendations are put forward:

1. The CMSTA should be adopted or adapted by mathematics teachers/lecturers in Nigerian Polytechnics in particular and other tertiary institutions in general to teach engineering students mathematics.
2. Mathematics teachers/lecturers in Nigerian Polytechnics in particular and other tertiary institutions in general should endeavour to understand how technology, particularly ICT, helps the goals of teaching mathematics in these institutions.
3. ICT should be integrated into the mathematics education curricular of all levels of education in Nigeria.

4. There should a policy that will compel mathematics educators, ICT experts and community members to collaborate to create a formal plan to help achieve the goals of integrating ICT into the mathematics education curriculum of Nigeria.
5. In view of the limited resources available for technology implementation efforts, authorities of Nigerian Polytechnics wishing to adopt/adapt the CMSTA should enlist community support in developing funding strategies for funding the requirement of the technology.
6. Mathematics teachers/lecturers in Nigerian Polytechnics in particular and other tertiary institutions in general should be supported by their institutions authorities to continuously engage in professional development in the area of computer usage and applications in teaching. This support could be in the form of (i) conduct seminars to make them aware of the different mathematics software available which can be used to help in the teaching process, (ii) making sure that they are convinced of the benefits of using these packages, (iii) training of the mathematics teachers on these software packages and making sure that they incorporate the use of these packages in the teaching process in such a way that the pupils use them in a systematic manner for learning, revising and practicing
7. An awareness program should be developed for reluctant students which will make them become aware of the benefits of using computers, software and the internet in the teaching/learning process.

5.6 Implications of the Findings

Based on the findings of the study which showed that that the use of the CMSTA to teach mathematics to Polytechnic engineering students can increase the students mathematics performance, foster the students attitudes towards mathematics positively

and lower their mathematics anxiety levels, it implies that all stake holders in the Nigerian education system in general and the Polytechnic sector in particular should put all hands on deck to begin to ensure that the full benefits of ICT integration into the teaching/learning process are fully harnessed.

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stakeholders and having positive beliefs concerning using technology could guarantee the realization of the full benefits of ICT integration into the teaching/learning process. Since teacher beliefs and experiences can positively impact their use of technology in the classroom, therefore careful planning and mathematics teacher technology training can lead to successful integration of technology in the mathematics classroom. With the integration of technology into the mathematics classroom, more students will be able to experience the positive results indicated in this study.

Mathematics teachers in general need to be aware of the effects of technology

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attitude and anxiety. Therefore, their capacity should be empowered to enable them be able to continuously develop teaching strategies which will incorporate ICT usage that

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towards the subject they teach and lower their anxiety. With all these efforts it hoped that more students will better comprehend what they are being taught. This, in effect, can prevent the whole education system from being dysfunctional.

5.7 Contributions to Knowledge

Arising from the findings of this study, the following are considered as the contributions of this study to knowledge:

1. A CMSTA framework has been developed which was used to teach mathematics to Polytechnic engineering students.
2. mathematics performance, attitude towards mathematics and mathematics anxiety was examined where it was

   mathematics performance*

   positively and lowering their mathematics anxiety.
3. The study has added to the body of knowledge on deployment of computers to enhance teaching and learning of mathematics in Nigeria thereby reducing the existing dearth of research work in this area.
4. The study is considered as a contributory work in the field of developing indigenous CMSTA framework for teaching and learning mathematics at the Polytechnic level.
5. It is believed that the study has provided mathematics teachers/lecturers teaching mathematics to engineering students in Nigerian Polytechnics with a framework which will assist them develop their customised computer application package for the purpose of helping to improve their students mathematics performance, foster their attitude towards mathematics positively and as well help lower their mathematics anxiety level.
6. It is believed that the study has contributed to literature which education policy makers in Nigeria may use to come up with new policies on computer usage in the teaching/learning process.

5.8 Limitations of the Study

There may be many possible limitations of this study. The following are considered worthy of noting:

1. The students preconceived technology bias might have affected their reaction to the treatment,

   mathematics performance attitude and anxiety.
2. In each of the intact groups, some of the students could have been influenced by the mathematics attitudes of a small number amongst them. Therefore, individual mathematics attitude as measured by the ATMI may not be totally independent of that of the others within the group.

5.9 Suggestions for Further Research

1. A similar research on the impact of the CMSTA on mathematics performance, attitude and anxiety of (a) other science and technology students and (b) social science and business studies students in Nigerian Polytechnics should be undertaken.
2. Another similar research on the impact of the CMSTA on mathematics performance, attitude and anxiety of (a) Nigerian Polytechnics students, (b) Colleges of Education students and (c) University students should be undertaken.
3. There should be a study geared towards producing a well-founded scientific or pedagogic guideline that can be used to make a decision on the amount of face-to-face contact vis-à-vis online contact that should be provided for students when the CMSTA is being deployed for usage.
4. Monitoring and assessment tools are important feedback mechanisms for both staff and students within any ICT environment. Because of the substantial improvement that developments in this area can bring to students learning experiences, this issue needs further investigation.

5. With the large number of students enrolling and attending tertiary institutions in Nigeria, research needs to continue to investigate learning best practices for these students so that they will receive the help that they so desperately need.

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APPENDICES

Appendix I

Mathematics Performance Tests (I and II)

1.0 Mathematics Performance Test I: Pretest for ND I (MTH 112)

Kaduna Polytechnic, Kaduna

College of Science and Technology

Department of Mathematics, Statistics & Computer Science

1st/3rd Semester Examination, 2010/2011 Session

Class: NDI Civil, Chemical, Mechanical, Electrical, Computer, Agric & Mineral Res. Engineering

Course: Algebra & Elementary Trigonometry. **Code**: MTH 112

Time: Two (2) Hours

Instructions: Answer any four (4) Questions

1. (a) Solve the following equations simultaneously: $\begin{matrix} 8 = 2^{(x+y)} \\ 1 = 3^{(x-y)} \end{matrix}$

   (b) Simplify without using tables or calculator:

   $\frac{1}{3}\log\left(\frac{125}{8}\right) - 2\log\left(\frac{2}{5}\right) + \log\left(\frac{80}{125}\right)$

2. (a) Express $\sqrt{32} + \frac{6}{\sqrt{2}}$ as a single surd, hence simplify $\frac{7}{\sqrt{2}}\left(\sqrt{32} + \frac{6}{\sqrt{2}}\right)$

   (b) Solve the following equations by Completing the square $3x^2 - 4x - 5 = 0$, correct to 2 d.p.

3. The difference between the third and first terms of a geometric progression (G.P) is 42. If the fourth term is greater than the second term by 168, find the: (i) first term (ii) Fourth term.

   (b) Find the value of x in the expression $\frac{3^{(x+1)}}{3^{(x-4)} \times 3^{(-7x)}} = 27^{x}$

4. (a) A committee of 6 is to be selected from a group of 5 men and 7 women to include exactly 4 women. Determine the number of committees possible.

(b) How many subsets each having 4 elements can be obtained from the set $\{p, q, r, s, t, z\}$?

(c) Expand completely $(x - 3y)^7$

5. (a) If $A = \{a, b, c, d\}$, $B = \{d, e\}$, $C = \{b, d, f\}$, and the universal set $U = \{a, b, c, d, e, f\}$, write down the following subsets: (i) $(A \cap B) \cap C$ (ii) $A \cup (B \cap C)$ (iii) $A' \cup B$ (iv) $A \cap \phi$ (v) $B \cup U$

(b) Use a Venn diagram to represent the relation between the sets: $U = \{1, 2, 3, 4, x, y, z, p\}$, $A = \{3, 4, x, y\}$, $B = \{2, 3, 4, xy, z\}$, $A \cup B$, $A \cap B$.

6. (a) A group consists of 5 boys and 3 girls. In how many ways can they be seated on a single row if:

   i. There is no restrictions
   ii. The girls must stay together
   iii. The girls must stay together, and the boys stay together also

(b) Security Codes consists of two letters followed by three digits taken from the sets $K = \{A, I, J, M\}$ and $L = \{3, 5, 7, 9\}$. If repetition is allowed only among the digits, how many codes are possible? How many of the codes consist of the same digits?

2.0 Mathematics Performance Test I: Pretest for HND I (MTH 311)

Kaduna Polytechnic, Kaduna

College of Science and Technology

Department of Mathematics, Statistics & Computer Science

1st/3rd Semester Examination, 2010/2011 Session

Class: HNDI Agric , Civil , Mechanical, Electrical, Chemical, Computer , &MR Engineering.

Course: Advanced Algebra. **Code**: MTH 311

Time: Two (2) Hours

Instructions: Answer any four (4) Questions

1 (a) Solve the following set of equations using Matrix Inversion Method.

$$2x_1 - x_2 + x_3 = 5$$

$$x_1 + 3x_2 - 2x_3 = 4$$

$$x_1 - 2x_2 + 2x_3 = 1$$

(b) Determine the eigenvalues of the matrix below $\begin{pmatrix} 2 & 0 & 1 \\ -1 & 4 & -1 \\ -1 & 2 & 0 \end{pmatrix}$

2 ' K e^x, e^{-x}, $\cosh x$, and $\sinh x$, show that:

i. $\cosh x = \frac{1}{2}(e^x + e^{-x})$ (ii) $\sinh x = \frac{1}{2}(e^x - e^{-x})$

b) Show that if $y = \tanh x$, then $\frac{dy}{dx} = \sec h^2 x$

c) If $y = \cosh^{-1} x$, obtain $\frac{dy}{dx}$

3. ' ' ' K ,

b) B K 2th term: (i) e^x (ii) $\cos x$.

4. (a)Prove by the Principle of Mathematical Induction that:

$$1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$$

b) Evaluate each of the following limits using J F Rule

i. $\lim_{x\to 0}\left\{\frac{\sinh x-\sin x}{x^3}\right\}$ (ii) $\lim_{x\to 0}\left\{\frac{e^x+e^{-x}-2}{2\cos 2x-2}\right\}$

5. (a) If $(8+i)(-9-6i)=11x-19y$, simplify (i) $(-7x+6iy)(-3x+5iy)$ (ii) $\frac{3x+8iy}{9x+5iy}$

   b) If $(a+b)+(a+b)i=\overset{\frown}{(}+5iy\overset{\frown}{)}^2+\overset{\frown}{(}-3i\overset{\frown}{)}$, find the values of *a* and *b*.

   c) Find the three cube roots of $8(\cos 225^\circ+i\sin 225^\circ)$ and express the principal root in the form *a+bi*.

6. (a) Find the value of *x* if the vectors $2\mathbf{i}+\mathbf{j}-2\mathbf{k}, \mathbf{i}+\mathbf{j}+3\mathbf{k}, x\mathbf{i}+\mathbf{j}$ are coplanar.

   b) Determine the volume of whose adjacent sides are $\mathbf{i}+\mathbf{j}+\mathbf{k}, -\mathbf{i}+2\mathbf{k}, 2\mathbf{i}+\mathbf{j}+2\mathbf{k}$.

   c) Given the vector $\mathbf{v}=\mathbf{i}xy^2-\mathbf{j}yz^2+\mathbf{k}x^2yz$. Show that div curl **v=0**

3.0 Mathematics Performance Test II: Posttest for ND I (MTH122)

Kaduna Polytechnic, Kaduna

College of Science and Technology

Department of Mathematics, Statistics & Computer Science

$2^{nd}/4^{th}$ Semester Examination, 2010/2011 Session

Class: NDI Civil, Chemical, Mechanical, Electrical, Computer, Agric & Mineral Res. Engineering

Course: Trigonometry & Analytical Geometry. **Code**: MTH 122

Time: Two (2) Hours

Instructions: Answer any four (4) Questions

1. a) Find the acute angle between the two straight lines $2x + y = 6$ and $3x - 2y = -2$

b) Find the equation of the line passing through the point (2, 3) and perpendicular to the line $3x + 2y = -5$

C) The straight line AB makes intercepts of $\frac{3}{4}$ on the x-axis and 1 on the y-axis, the straight line CD has a gradient of -3 and passing through points (1,-1). Calculate the point of intersection of AB and CD.

2. a) Triangle ABC has vertices A(1,1), B(3,2), C(2,5). D is the mid point of AB. Write down the co-ordinate of D. hence sow that $AC^2 + CB^2 = 2CD^2 + 2AD^2$

b) Find the area of a pentagon ABCDE given A(0,3), B(6,1), C(4,3), D(2,7), (-1,1).

c) Find the equation of a straight line which passes through (-3, 5) and parallel to the line $3y - 5x + 2 = 0$.

3. a) Convert i) $\frac{2\pi}{5}$ rad ii) $\frac{7\pi}{4}$ rad to degree.

b). Convert i) $22\frac{1}{2}^{\ 0}$ ii) 540^0 to radians leaving π in your answer.

C). a chord PQ subtends a right angle at the centre of the circle of radius r. QR is a chord in the minor segment incline at 15^0 to QP. Show that the area bounded by the two chords and the arc PR is $\frac{1}{2}$ r^2 $(\frac{1}{6}\ \pi + \frac{\sqrt{3}}{2}$ -1).

4. a). Find the Cartesian equation of i) $r = a(1 - \sin\theta)$ $ii)\ r = (b + \cos\theta \tan\theta)$

b). Find the polar equation of i) $9x^2 - 16y^2 = 144$ ii) $2x - 3y + 1 = 0$

A' , 1 ,3

tip in 40 minutes

5. a). Show that the point (-1, 2) is equidistance from the lines with the equations $3x + 4y = 20$ and $5x - 12y = 10$.

b). Find the equation of the tangent to the parabola $y^2 = -12x$ and parallel to the line $x + y = 5$

c). Find the equation of the normal to the hyperbola $9x^2 - 4y^2 = 36$ at the point (-2, 0).

6. a). Find the equation of the tangent to the parabola $y^2 = 12x$ at the point P ($3t^2$,6t).

Find also the co-ordinate of M given that M is the mid point between P and Q which is the intercept of the tangent on the x- axis.

b). show that $2x^2 + 3y^2 - 4x + 12y - 20 = 0$ is an ellipse. Find the foci, eccentricity, centre and equation of the directrices.

c). find the centre and radius of the circle $36x^2 + 36y^2 - 72x - 108y - 108 = 0$

4.0 Mathematics Performance Test II: Posttest for HND I (MTH312)

Kaduna Polytechnic, Kaduna

College of Science and Technology

Department of Mathematics, Statistics & Computer Science

2nd/4th Semester Examination, 2010/2011 Session

Class: HNDI Agric , Civil , Mechanical, Electrical, Chemical, Computer , &MR Engineering.

Course: Advanced Calculus. **Code**: MTH 312

Time: Two (2) Hours

Instructions: Answer any four (4) Questions

1. An inductor of 2 henrys, a resistor of 16 ohms and a capacitor of 0.02 farads are connected in series with an e.m.f. of 300volts. At t=0 the charge on the capacitor and current in the circuit are zero. Find the charge and current at any time t>0.

   (Hints $L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = E$)

2. Solve the following differential equations

   a) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = x^2$

   b) $\frac{d^2y}{dt^2} = 3sint - 4y,\ at\ t = 0\ \ y = 0\ \&\ \frac{dy}{dt} = 1$

3. Solve the simultaneous linear differential equations

   a) $\frac{dx}{dt} = -2x + y\ \ \&\ \ \frac{dy}{dt} = -4x + 3y + 10cost$

   b) $\frac{dx}{dt} + \frac{dy}{dt} = 4cos2t\ \&\ \ \frac{dx}{dt} - \frac{dy}{dt} = 4sin2t, at\ t = 0, x = 1.2, \& y = 0$

4. a) Given that $\mathcal{L}\{f(t)\} = F(s)$show that $\mathcal{L}\{f''(t)\} = s^2F(s) - sf(0) - f'(0)$.

   Hence find the Laplace transform of $\cos\omega t$.

   b) Find the inverse Laplace transform of i) $\frac{2s+1}{s^2+4}$ ii) $\frac{n\pi L}{L^2s^2+n^2\pi^2}$

5. Apply Laplace transform derivatives to solve the equation

   $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = 0,\ at\ t = 0, y = 3\ \&\ \frac{dy}{dt} = 1$

6. Find the Fourier series of the function

$$f(x) = \begin{cases} x, & if \quad -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \pi - x, & if \quad \frac{\pi}{2} < x < \frac{3\pi}{2} \end{cases}$$

Appendix II

Attitudes Toward Mathematics Inventory (ATMI)

Dear Sir/Madam,

I am a Postgraduate Student in the Mathematics Education Section of the Department of Education, Ahmadu Bello University, Zaria currently working on a research entitled **Impact of a Computer Mediated Systems Teaching Approach (CMSTA) on Mathematics performance, Attitudes and Anxiety of Polytechnic Engineering Students**. One of the objectives of this study is to determine the attitude of respondents towards mathematics. The attached questionnaire is meant to solicit information that will assist me in achieving this objective. I assure you that all information provided will be treated with utmost confidentiality and will only be used for the purpose of this research only.

Thank you.

ILIYASU YUSUF
Ph.D/Educ/46217/2012-2013
Mathematics Education Section
Department of Science Education
Faculty of Education
Ahmadu Bello University
Zaria

SECTION A: Personal Data

Instruction: Please fill or tick as appropriate

1. ,,
2. Reg. ,
3. Department.
   a. Agricultural Mechanization Engineering [ ]
   b. Chemical Engineering [ ]
   c. Civil Engineering [ ]
   d. Computer Engineering [ ]
   e. Electrical/Elecronics Engineering [ ]
   f. Mechanical Engineering [ ]
   g. Mineral Resources Engineering [ ]
3. Level
   a. 100 level [ ]
   b. 200 level [ ]
   c. 300 level [ ]
   d. 400 level [ ]
4. Sex
   a. Male [ ]
   b. Female [ ]

SECTION B: Attitudes Towards Mathematics Inventory (ATMI)

Directions: This inventory consists of statements about your attitude toward mathematics. There are no correct or incorrect responses. Read each item carefully. Please think about how you feel about each item. Using the response scale corresponding to each item, tick the box that most closely corresponds to how the statements best describes your feelings.

Key to Responses:

SD - Strongly Disagree

D - Disagree

N - Neutral

A - Agree

SA - Strongly Agree

S/No.	Items	Responses				
		SD	D	N	A	SA
1	Mathematics is a very worthwhile and necessary subject.					
2	I want to develop my mathematical skills.					
3	I get a great deal of satisfaction out of solving a mathematics problem.					
4	Mathematics helps develop the mind and teaches a person to think.					
5	Mathematics is important in everyday life.					
6	Mathematics is one of the most important subjects for people to study.					
7	Polytechnic mathematics courses would be very helpful no matter what I decide to study.					
8	I can think of many ways that I use mathematics outside of the Polytechnic.					
9	Mathematics is one of my most dreaded subjects.					
10	My mind goes blank and I am unable to think clearly when working with mathematics.					
11	Studying mathematics makes me feel nervous.					
12	Mathematics makes me feel uncomfortable.					
13	I am always under a terrible strain in a mathematics class.					
14	When I hear the word mathematics, I have a feeling of dislike.					
15	It makes me nervous to even think about					

	having to do a mathematics problem.					
16	Mathematics does not scare me at all.					
17	I have a lot of self-confidence when it comes to mathematics					
18	I am able to solve mathematics problems without too much difficulty.					
19	I expect to do fairly well in any mathematics class I take.					
20	I am always confused in my mathematics class.					
21	I feel a sense of insecurity when attempting mathematics.					
22	I learn mathematics easily.					
23	I am confident that I could learn advanced mathematics.					
24	I have usually enjoyed studying mathematics in school.					
25	Mathematics is dull and boring.					
26	I like to solve new problems in mathematics.					
27	I would prefer to do an assignment in mathematics than to write an essay.					
28	I would like to avoid using mathematics in the Polytechnic.					
29	I really like mathematics.					
30	I am happier in a mathematics class than in any other class.					
31	Mathematics is a very interesting subject.					
32	I am willing to take more than the required amount of mathematics.					
33	I plan to take as much mathematics as I can during my education.					
34	The challenge of mathematics appeals to me.					
35	I think studying advanced mathematics is useful.					
36	I believe studying mathematics helps me with problem solving in other areas.					
37	I am comfortable expressing my own ideas on how to look for solutions to a difficult problem in mathematics.					
38	I am comfortable answering questions in mathematics class.					
39	A strong mathematics background could help me in my professional life.					
40	I believe I am good at solving mathematics problems.					

Appendix III

Mathematics Anxiety Rating Scale (MARS)

Dear Sir/Madam,

I am a Postgraduate Student in the Mathematics Education Section of the Department of Education, Ahmadu Bello University, Zaria currently working on a research titled **Impact of a Computer Mediated Systems Teaching Approach (CMSTA) on Mathematics performance, Attitudes and Anxiety of Polytechnic Engineering Students**. One of the objectives of this study is to determine the mathematics level of respondents. The attached questionnaire is meant to solicit information that will assist me in achieving this objective. I assure you that all information provided will be treated with utmost confidentiality and will only be used for the purpose of this research only.

Thank you.

ILIYASU YUSUF
Ph.D/Educ/46217/2012-2013
Mathematics Education Section
Department of Science Education
Faculty of Education
Ahmadu Bello University
Zaria

SECTION A: Personal Data

Instruction: Please fill or tick as appropriate

1. ,,

2. , ,

3. Department.

   h. Agricultural Mechanization Engineering [ ]

   i. Chemical Engineering [ ]

   j. Civil Engineering [ ]

   k. Computer Engineering [ ]

   l. Electrical/Electronic Engineering [ ]

   m. Mechanical Engineering [ ]

   n. Mineral Resources Engineering [ ]

3. Level

   e. 100 level [ ]

   f. 200 level [ ]

   g. 300 level [ ]

   h. 400 level [ ]

4. Sex

   c. Male [ ]

   d. Female [ ]

SECTION B: Mathematics Anxiety Rating Scale

Directions: The items in this questionnaire refer to things and experiences that may cause fear or apprehension. Answer each item below to indicate how you feel today using the following code:

1 = not at all
2 = a little
3 = moderate
4 = much
5 = very much

Work quickly and be sure to consider each item individually.

S/No.	Items	Ratings				
		1	2	3	4	5
1	Watching a teacher work on a mathematical equation on the blackboard.					
2	,					
3	Being given a mathematics homework / assignment of many difficult problems which is due the next class meeting.					
4	Thinking about an upcoming mathematics test on the day before.					
5	Solving a mathematics problem.					
6	Reading and interpreting mathematical graphs and charts.					
7	Signing up for a course in mathematics.					
8	Listening to another student explain a mathematics formula.					
9	Walking into a mathematics class.					
10	Looking through the pages of a mathematics book.					
11	Starting a new chapter in a mathematics book.					
12	Walking on campus and thinking about a mathematics course.					
13	Picking up a mathematics textbook to begin working on a home work/assignment.					
14	Taking an test/examination in a mathematics course.					
15	,					
16	Working on an abstract mathematical problem.					
17	Reading a formula in mathematics					
18	Taking a final examination in a mathematics class					

19	Getting ready to study for a mathematics test/examination.					
20	,					
21	Waiting to get a mathematics test returned on which I am expecting to do well.					
22	Listening to a lecture in a mathematics class.					
23	Having to use mathematics tables in the back of a mathematics book.					
24	Being told how to interpret mathematics statements.					

Appendix IV

Computer Mediated Systems Teaching Approach (CMSTA) Lecture Manual

Appendix IV (a): Advanced Calculus CTA Lecture Manual for HND Polytechnic Engineering Students

1.0 Second – Order Linear Ordinary Differential Equations (ODE)

1.1 Introduction

The general form of the second order linear ODE is given by

$$\left.\begin{array}{c} p(x)y'' + q(x)y' + r(x)y = g(x) \\ \text{or} \\ p(x)\frac{d^2y}{dx^2} + q(x)\frac{dy}{dx} + r(x)\,y = g(x) \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$

where the coefficients $p(x), q(x), r(x)$ and $g(x)$ can be zero or non-zero functions, constant or non-constant functions, linear or non-linear functions. Only the function $y(x)$ and its derivatives are used in determining if a differential equation is linear.

Example 1.1: Each of the following is an example of a second order linear ODE

1. $2y'' + 6y' + 2y = 0$
2. $y'' - 5y' + 6y = 0\ ;\quad y(1)\ e^2, y'(1) = 3e^2$
3. $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = 0$
4. $2\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 0$
5. $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 5y = 0$
6. $y'' + y' + y = 3x^2$
7. $y'' - 2y' + 5y = \text{Sin}2x$
8. $2y'' - 2y' + 6y = 4x - 9$
9. $4\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 2y = \text{Cost}$
10. $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 5y = 4e^{3x} + 3e^{x}\ ;\ y(0) = 3, y'(0) = 11$

The second order linear ODE can be classified as homogeneous or non homogeneous. It can also be classified in terms of the coefficients $p(x), q(x), r(x)$ and $g(x)$ as having constant or non constant coefficients. Each type is examined in the sections that follow.

1.2 Second – Order Homogenous Linear ODE

The Second Order Homogeneous linear ODE is obtained when $g(x) = 0$, in (1). This implies that the general form of the second order homogeneous linear ODE is given by

$$\left.\begin{array}{c} p(x)y'' + q(x)y' + r(x)y = 0 \\ \text{or} \\ p(x)\frac{d^2y}{dx^2} + q(x)\frac{dy}{dx} + r(x)\,y = 0 \end{array}\right\} \quad \text{..... (2)}$$

Example 1.2: Each of the following is an example of a second order homogeneous linear ODE

a) $2y'' + 6y' + 2y = 0$

b) $y'' + y' + y = 0$

c) $y'' - 2y' + 5y = 0$

d) $2y'' - 2y' + 6y = 0$

e) $y'' - 5y' + 6y = 0 \quad y(1) = e^2, y'(1) = 3e^2$

f) $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = 0$

g) $4\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 2y = 0$

h) $2\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 0$

i) $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 5y = 0$

j) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 5y = 0, y(0) = 3, y'(0) = 11$

1.3 Second – Order Non-Homogenous Linear ODE

The Second Order Non-Homogeneous linear ODE is obtained when $g(x) \neq 0$, in (1). This implies that the general form of the second order non-homogeneous linear ODE is given by

$$\left.\begin{array}{c} p(x)y'' + q(x)y' + r(x)y = g(x) \\ \text{or} \\ p(x)\frac{d^2y}{dx^2} + q(x)\frac{dy}{dx} + r(x)\,y = g(x) \end{array}\right\} \quad \text{..... (3)}$$

where $g(x) \neq 0$

Example 1.3: Each of the following is an example of a second order non - homogeneous linear D.E

a) $2y'' + 6y' + 2y = 7e^{4x}$

b) $y'' + y' + y = 3x^2$

c) $y'' - 2y' + 5y = \text{Sin}2x$

d) $2y'' - 2y' + 6y = 4x - 9$

e) $y'' - 5y' + 6y = -3e^{2x} \quad y(1) = e^2, y'(1) = 3e^2$

f) $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = 5\text{Cosh}2t$

g) $4\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 2y = \text{Cost}$

h) $2\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 4\text{Sin}3x + 3\text{Cos}4x$

i) $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 5y = 7\text{Cost}9t$

j) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 5y = 4e^{3x} + 3e^{x} \quad y(0) = 3, y'(0) = 11$

1.4 Second – Order Linear ODE with Constant or Non-Constant Coefficients

In (1), if the coefficients $p(x)$, $q(x)$, $r(x)$ and $g(x)$ are constant then second order linear ODE is said to be with constant coefficients. Otherwise, it is called a second order linear ODE with non-constant or variable coefficients.

From this definition, it follows that called a second order linear ODE can be homogeneous with constant coefficients or homogeneous with non constant or

variable coefficients. Similarly, a second order linear ODE can be non homogeneous with constant coefficients or homogeneous with non constant or variable coefficients.

Example 1.4

1. Each second order homogeneous linear ODE given in **example 1.2** above is an example of a second order homogeneous linear ODE with constant coefficients
2. Each second order non homogeneous linear ODE given in **example 1.3** above is an example of a second order non homogeneous linear ODE with constant coefficients

Take Home Exercises: Write down five examples of a second order homogeneous linear ODE with:

1. non constant or variable coefficients
2. non constant or variable coefficients

1.5 Solutions of Second – Order Homogeneous Linear ODE with Constant Coefficients

From our knowledge of the solution of the first order linear ODE;
solutions to (4) will be of the form $y = e^{mx}$

$\rightarrow \quad y' = me^{mx}$ and $y'' = m^2e^{mx}$

Substituting these in (4) gives:

$am^2e^{mx} + bme^{mx} + ce^{mx} = 0 \quad \rightarrow \quad (am^2 + bm + c)e^{mx} = 0$

$\rightarrow \quad am^2 + bm + c = 0$ since $e^{mx} \neq 0$ for all values of m.

$am^2 + bm + c = 0$ is typically called the **auxiliary equation (AE)** for (4).

Observe that the AE is quadratic and so will have two roots, m_1 and m_2. These roots have three possible forms. These are

1. Real, distinct roots, m_1, m_2
2. Complex root, $m_1 = \alpha + i\beta$ and $m_2 = \alpha - i\beta$.
3. Double roots, $m_1 = m_2 = m$

A solution type will be generated for each of these three possible forms of the roots of the AE. The next sections address these aspects.

1.5.1 General Solution for Real, Distinct Roots, m_1, m_2

If $m = m_1$ and m_2 then $y_1 = e^{m_1 x}$ and $y_2 = e^{m_2 x}$ are the solutions.

Thus, the general solution will then be given by a linear combination of y_1 and y_2

$\rightarrow \quad y = c_1 y_1 + c_2 y_2 = c_1 e^{m_1 x} + c_2 e^{m_2 x}$

$\rightarrow \quad y = Ae^{m_1 x} + Be^{m_2 x}$,, , /7'

where $A = c_1$ and $B = c_2$

Example 1.5.1: Solve $2\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 2y = 0$

Solution

Given $2\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 2y = 0$

A. E is $2m^2 - 5m + 2 = 0 \rightarrow (2m-1)(m-2) = 0 \rightarrow m = \frac{1}{2}$ or 2

Hence the general equation is $y = Ae^{x/2} + Be^{2x}$

1.5.2 General Solution for Complex Roots, $m_1 = \alpha + i\beta$ and $m_2 == \alpha - i\beta$.

If $m_1 = \alpha + i\beta$ and $m_2 == \alpha - i\beta$ then we have

$y_1 = e^{(\alpha+i\beta)x}$ and $y_2 = e^{(\alpha-i\beta)x}$

$\rightarrow \quad y = c_1 y_1 + c_2 y_2 = c_1 e^{(\alpha+i\beta)x} + c_2 e^{(\alpha-i\beta)x}$

$= c_1 e^{\alpha x} e^{i\beta x} + c_2 e^{\alpha x} e^{-i\beta x}$

$= (c_1 e^{i\beta x} + c_2 e^{-i\beta x}) e^{\alpha x}$

C $* \quad e^{i\theta} = \cos\theta + i\sin\theta \quad \therefore \quad e^{-i\theta} = \cos\theta - i\sin\theta$

Using these in $y = (c_1 e^{i\beta x} + c_2 e^{-i\beta x}) e^{\alpha x}$ gives

$y = \{c_1(\text{Cos}\beta x + i\text{Sin}\beta x) + c_2(\text{Cos}\beta x - i\text{Sin}\beta x)\}e^{\alpha x}$

$= \{(c_1 + c_2)\text{Cos}\beta x + i(c_1 - c_2)\text{Sin}\beta x\}e^{\alpha x}$

$= \{A\text{Cos}\beta x + B\text{Sin}\beta x\}e^{\alpha x}$,, 0. '

where $A = c_1 + c_2$ and $B = i(c_1 - c_{2)}$

Example 1.5.2: Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

Solution

A.E is $m^2 - 2m + 2 = 0 \rightarrow m = 1 \pm i$

Hence the general solution is $y = e^x(A\text{Cos}x + B\text{Sin}x)$

1.5.3 General Solution for Double Roots, $m_1 = m_2 = m$

If $m_1 = m_2 = m$ then $y_1 = e^{m_1 x} = e^{mx}$ and $y_2 = e^{m_2 x} = e^{mx}$

$\rightarrow y = y_1 = y_2$

Since these are the same solution, it is therefore not enough to form a general solution.

* * , J

second solution be the product of f(x) (i.e. any function of x) and e^{mx}.

i.e. Let $y_2 = f(x)e^{mx}$ $\rightarrow$ $y_2' = mf(x)e^{mx} + f'(x)e^{mx}$

$\rightarrow$ $y_2'' = m^2f(x)e^{mx} + mf'(x)e^{mx} + mf'(x)e^{mx} + f''(x)e^{mx}$

$= m^2f(x)e^{mx} + 2mf'(x)e^{mx} + f''(x)e^{mx}$

Substituting these values in (4) gives

$a(m^2f(x)e^{mx} + 2mf'(x)e^{mx} + f''(x)e^{mx}) + b(mf(x)e^{mx} + f'(x)e^{mx}) + cf(x)e^{mx} = 0$

$\rightarrow$ $a\big(m^2f(x) + 2mf'(x) + f''(x)\big) + b\big(mf(x) + f'(x)\big) + cf(x) = 0$

$\rightarrow \quad (am^2 + bm + c)f(x) + (2am+b)f'(x) + af''(x) = 0$

$\rightarrow \quad af''(x) = 0$

Because $am^2 + bm + c = 0$ and $2am + b = 0$ (since $m = -\frac{b}{2a}$)

$\rightarrow \quad f''(x) = 0 \quad \rightarrow \quad f'(x) = C \quad \rightarrow \quad f(x) = Cx + D$

$\therefore \; y_2 = (Cx + D)e^{mx}$

Hence, $y = c_1y_1 + c_2y_2 = c_1e^{mx} + c_2(Cx + D)e^{mx}$

$= (c_1 + c_2Cx + c_2D)e^{mx}$

$= \{(c_1 + c_2D) + c_2Cx\}\, e^{mx}$

$\rightarrow \quad y = \{A + Bx\}\, e^{mx}$ 0/'

where $c_1 + c_2D = A$ and $c_2C = B$

Example 1.5.3: Solve $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$

Solution

Given $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$

A. E is $m^2 + 4m + 4 = 0 \quad \rightarrow \quad (m + 2)(m + 2) = 0 \quad \rightarrow \quad m = -2$ (twice)

Hence the general equation is $y = e^{-2x}(Ax + B)$

Take Home Exercises: Obtain the general solutions of

1. $2y'' + 6y' + 2y = 0$
2. $y'' + y' + y = 0$
3. $y'' - 2y' + 5y = 0$
4. $2y'' - 2y' + 6y = 0$

5. $y'' - 5y' + 6y = 0$

   $y(1) = e^2, y'(1) = 3e^2$

6. $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = 0$

7. $4\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 2y = 0$

8. $2\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 0$

9. $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 5y = 0$

10. $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 5y = 0$

    $y(0) = 3, y'(0) = 11$

1.6 Solutions of Second – Order Non-Homogeneous Linear ODE with Constant Coefficients

The general solution of the second order non - homogeneous linear ODE (5) is given by

$$y = y_c + y_p$$

where y_c is the general solution to the associated second order homogeneous linear ODE

$$ay'' + by' + cy = 0$$

and y_p is the particular solution to the second order homogeneous linear ODE

$$ay'' + by' + cy = g(x)$$

Of these two parts of the solution, y_c is called the **Complementary Function (CF)** while y_p is called the **Particular Integral (PI).**

The determination of the particular integral depends on the type of function represented by g(x). For each type of the function represented by g(x), a particular type of general function is suggested to be used as P.I. The suggested P.I is usually substituted into the D.E to determine the constants. The table below gives some suggested P.I for each type of g(x).

$\mathbf{g(x)}$	**Suggested Particular Integral**
A	K
$A_1e^{m_1x} + A_2e^{m_2x} + A_3e^{m_3x} + \cdots + A_ne^{m_nx}$	$K_1e^{m_1x} + K_2e^{m_2x} + K_3e^{m_3x} + \cdots + K_ne^{m_nx}$
$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_{n-1}x + a_n$	$K_0x^n + K_1x^{n-1} + K_2x^{n-2} + \cdots + K_{n-1}x + K_n$
$ACos\alpha x$, $ASin\alpha x$ or $ACos\alpha x + BSin\alpha x$	$KCos\alpha x + NSin\alpha x$
$ACosh\alpha x$, $ASinh\alpha x$ or $ACosh\alpha x + BSinh\alpha x$	$KCosh\alpha x + NSinh\alpha x$

Example 1.6.1: Solve $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = x^2$

Solution

Given $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = x^2$

The Auxiliary Equation (AE) is given by $m^2 - 5m + 6 = 0$

$\rightarrow \quad (m-2)(m-3) = 0 \quad \rightarrow \quad m = 2 \text{ or } m = 3$

Therefore, Complementary Function (CF) is $y_c = Ae^{2x} + Be^{3x}$

To Obtain the Particular Integral (P.I), let $y_p = Cx^2 + Dx + E$ so that

$\frac{dy_p}{dx} = 2Cx + D$ and $\frac{d^2y_p}{dx^2} = 2C$

Substituting y_p, $\frac{dy_p}{dx}$ and $\frac{d^2y_p}{dx^2}$ in $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = x^2$

$\rightarrow \quad 2C - 5(2cx + D) + 6(Cx^2 + Dx + E) = x^2$

$\rightarrow \quad 6cx^2 + (6D - 10C)x + 2C - 5D + 6E = x^2$

Equating the coefficient of x^2 term: $6C = 1 \rightarrow C = \frac{1}{6}$

Equating coefficient of x term: $6D - 10C = 0 \quad \rightarrow \quad 6D = 10C$

$\rightarrow \quad 6D = \frac{10}{6}$ (since $C = \frac{1}{6}$) $\rightarrow D = \frac{10}{36} = \frac{5}{18}$

Equating constant terms: $2C - 5D + 6E = 0 \quad \rightarrow \quad \frac{2}{6} - \frac{25}{18} + 6E = 0$ (since $D = \frac{5}{18}$ and $C = \frac{1}{6}$) $\rightarrow \quad E = \frac{19}{108}$

$$\therefore y_p = \frac{1}{6}x^2 + \frac{5}{18}x + \frac{19}{108}$$

Hence the General Solution (G.S) is $y = y_c + y_p = Ae^{2x} + Be^{3x} + \frac{1}{6}x^2 + \frac{5}{18}x + \frac{19}{108}$

Example 1.6.2: Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = x^3$

Solution

Given $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = x^3$

A. E is $m^2 - 4m + 3 = 0 \quad \rightarrow \quad (m-3)(m-1) = 0 \rightarrow \ m = 3, 1$

Hence C.F, $y_c = Ae^{5x} + Be^{x}$

For P.I, let $y_p = ax^3 + bx^2 + Cx + d \quad \rightarrow \quad y'_p = 3ax^2 + 2bx + c$

$$\rightarrow \quad y''_p = 6ax + 2b$$

Substituting in the given D.E, we have

$$6ax + 2b - 4(3ax^2 + 2bx + c) + 3(ax^2 + bx^2 + cx + d) = x^3$$

$$\rightarrow \quad 6ax + 2b - 12ax^2 - 8bx - 4c + 3ax^2 + 3bx^2 + 3cx + 3d = x^3$$

$$\rightarrow \quad 3ax^2 + (3b - 12a)x^2 + (3c - 8b + 6a)x + 2b - 4c + 3d = x^3$$

Equating coefficient of x^3: $3a = 1 \quad \rightarrow \quad a = \frac{1}{3}$

Equating coefficient of x^2: $3b - 12a = 0 \ \rightarrow b - 4a = 0 \ \rightarrow \quad b = 4a = \frac{4}{3}$

Equating coefficient of x: $3c - 8b + 6a = 0 \ \rightarrow \ 3c - \frac{32}{3} + 2 = 0 \ \rightarrow \ 3c = \frac{26}{3}$

$$\therefore c = \frac{26}{9}$$

Equating constant terms: $2b + 3d - 4c = 0 \quad \rightarrow \quad \frac{8}{3} + 3d - 4c = 0$

$\rightarrow \quad \frac{8}{3} + 3d - 4 \times \frac{26}{9} = 0 \quad \rightarrow \quad \frac{24-104}{9} + 3d = 0 \quad \rightarrow 3d = \frac{80}{9} \quad \therefore d = \frac{80}{27}$

$\therefore \quad y_p = \frac{1}{3}x^3 + \frac{4}{3}x^2 + \frac{26}{9}x + \frac{80}{27}$

$\therefore$ General Solution, $y = y_c + y_p = Ae^{3x} + Be^x + \frac{x^3}{3} + \frac{4}{3}x^2 + \frac{26}{9}x + \frac{80}{27}$

Example 1.6.3: $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = Sin2x$

Solution

Given $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = Sin2x$

A.E $m^2 + 2m + 5 = 0 \quad \rightarrow \quad m = -1 \pm 2i$

Hence C.F, $y_c = e^{-x}(ACos2x + BSin2x)$

For P.I, let $y_p = aCos2x + bSin2x$

$\rightarrow \quad \frac{dy}{dx} = -2aSin2x + 2bCos2x$ and $\frac{d^2y}{dx^2} = -4bSin2x - 4bSin2x$

Substituting in the given D.E

$-4aCos2x - 4bSin2x + 2(-2aSin2x + 2bCos2x) + 5(aCos2x + bSin2x) = Sin2x$

$-4aCos2x - 4bSin2x \pm 4aSin2x + 4bCos2x + 5aCos2x + 5bSin2x = Sin2x$

$aCos2x + bSin2x + 4bCos2x - 4aSin2x = Sin2x$

$(a + 4b)Cos2x + (b - 4a)Sin2x = Sin2x$

Equating coefficients of Sin2x and Cos2x give

$b - 4a = 1$ and $a + 4b = 0 \rightarrow b = \frac{1}{17}$ and $a = -\frac{4}{17}$

$\therefore \quad y_p = -\frac{4}{17}Cos2x + \frac{1}{17}Sin2x$

General Solution, $y = y_c + y_p = e^{-x}(ACos2x + BSin2x) - \frac{4}{17}Cos2x + \frac{1}{17}Sin2x$

Take Home Exercises: Obtain the general solutions of

1. $2y'' + 6y' + 2y = 7e^{4x}$
2. $y'' + y' + y = 3x^2$
3. $y'' - 2y' + 5y = \text{Sin}2x$
4. $2y'' - 2y' + 6y = 4x - 9$
5. $y'' + y = \text{Sin}x$
6. $y'' - y' - 2y = 4x^2$
7. $y'' - y = \text{Cos}x$
8. $y'' - 3y' - 10y = 6e^{4t}$
9. $y'' + 4y = 3\text{Sin}5x$
10. $y'' - 3y = x - 4x^2$
11. $-y'' + 2y' + 5y = xe^{3t}$
12. $y'' + y' = x \qquad y(0) = 1,\ y'(0) = 0$
13. $y'' - 5y' + 6y = -3e^{2x} \qquad y(1) = e^2, y'(1) = 3e^2$
14. $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = 5\text{Cosh}2t$
15. $4\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 2y = \text{Cos}t$
16. $2\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 4\text{Sin}3x + 3\text{Cos}4x$
17. $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 5y = 7\text{Cos}t9t$
18. $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 5y = 4e^{3x} + 3e^{x} \quad y(0) = 3, y'$

2.0 Laplace Transforms

2.1 Introduction

The Laplace transform of a function $f(t)$, denoted by $L\{f(t)\}$ is defined as the function $F(s)$ where $F(s)$ is given by the equation $F(s) = \int_0^\infty e^{-st} f(t)dt$.

Thus, we can write $L\{f(t)\} = \int_0^\infty e^{-st} f(t)dt = F(s)$

2.2 A Basic Property of the Laplace Transforms

If $L\{f(t)\} = F(s)$ and $L\{g(t)\} = G(s)$ then

$L\{Kf(t) \pm Mg(t)\} = KF(s) \pm MG(s)$,

where K and M are constants

Proof

If $L\{f(t)\} = F(s)$ and $L\{g(t)\} = G(s)$, from definition of the laplace transforms, it follows that

$L\{f(t)\} = \int_0^\infty e^{-st} f(t)dt = F(s)$ and $L\{g(t)\} = \int_0^\infty e^{-st} g(t)dt = G(s)$

Then

$$L\{Kf(t) \pm Mg(t)\} = \int_0^\infty e^{-st}\big(Kf(t) \pm Mg(t)\big)dt$$

$$= \int_0^\infty \big(e^{-st}Kf(t) \pm e^{-st}Mg(t)\big)dt$$

$$= \int_0^\infty (e^{-st}Kf(t))dt \pm \int_0^\infty (e^{-st}Mg(t))dt$$

$$= K\int_0^\infty (e^{-st}f(t))dt \pm M\int_0^\infty (e^{-st}g(t))dt$$

$$= KF(s) \pm MG(s)$$

2.3 Laplace Transforms of Some Standard Functions

2.3.1 $L\{K\} = \frac{K}{s}$ where K is a constant

Proof: From the definition of the Laplace transform, we have

$$L\{K\} = \int_0^\infty e^{-st}Kdt = K\int_0^\infty e^{-st}dt = K\left[-\frac{e^{-st}}{s}\Big|_0^\infty\right] = -K\left[0 - \frac{1}{s}\right] = \frac{K}{s}$$

Example 2.3.1

a) $L\{4\} = \frac{4}{s}$

b) $L\{-7\} = -\frac{7}{s}$

c) $L\{\frac{23}{7}\} = \frac{23}{7s}$

2.3.2 $\mathbf{L\{e^{at}\} = \frac{1}{s-a}}$

Proof: From the definition of the Laplace transform, we have

$$L\{e^{at}\} = \int_0^\infty e^{-st}e^{at}dt = \int_0^\infty e^{-(s-a)t}dt = \left[-\frac{e^{-(s-a)t}}{s-a}\Big|_0^\infty\right] = -\left[0 - \frac{1}{s-a}\right] = \frac{1}{s-a}$$

Example 2.3.2

a) $L\{e^{5t}\} = \frac{1}{s-5}b$

b) $L\{e^{-9t}\} = \frac{1}{s+9}$

c) $L\left\{e^{-\frac{9}{7}t}\right\} = \frac{1}{s+\frac{9}{7}} = \frac{7}{7s+9}$

2.3.3 $L\{Coshat\} = \frac{S}{s^2-a^2}$ and $L\{Sinhat\} = \frac{a}{s^2-a^2}$

Proof

Recall that $Coshat = \frac{e^{at}+e^{-at}}{2}$ and $Sinhat = \frac{e^{at}-e^{-at}}{2}$

Therefore $\{Coshat\} = L\left\{\frac{e^{at}+e^{-at}}{2}\right\} = \frac{1}{2}L\{e^{at} + e^{-at}\} = \frac{1}{2}\left\{\frac{1}{s-a} + \frac{1}{s+a}\right\}$

$$= \frac{1}{2}\left\{\frac{2s}{s^2-a^2}\right\} = \frac{s}{s^2-a^2}$$

In the same way, we obtain $L\{Sinhat\} = \frac{a}{s^2-a^2}$

Example 2.3.3:

a) $L\{Cosh\ 3t\} = \frac{s}{s^2-9}$

b) $L\{Sinh\ 3t\} = \frac{3}{s^2-9}$

c) $L\left\{Sinh\frac{3}{5}t\right\} = \frac{\frac{3}{5}}{s^2-(\frac{3}{5})^2} = \frac{15}{25s^2-9}$

2.3.4 $L\{Cosat\} = \frac{S}{s^2+a^2}$ and $L\{Sinat\} = \frac{a}{s^2+a^2}$

Recall that $e^{it} = Cost + iSint$ C D ' → $e^{-it} = Cost - iSint$

where $i^2 = -1$

Therefore $e^{iat} = Cosat + iSinat$,, /'

and $e^{-iat} = Cosat - iSinat$ 0'

Solving (1) & (2) for Cosat and Sinat gives

$Cosat = \frac{e^{iat}+e^{-iat}}{2}$ and $Sinat = \frac{e^{iat}-e^{-iat}}{2i}$

Therefore $L\{Cosat\} = L\left\{\frac{e^{iat}+e^{-iat}}{2}\right\} = \frac{1}{2}L\{e^{iat} + e^{-iat}\}$

$$= \frac{1}{2}\left\{\frac{1}{s-ia} + \frac{1}{s+ia}\right\}$$

$$= \frac{1}{2}\left\{\frac{2s}{s^2+a^2}\right\} = \frac{s}{s^2+a^2}$$

In the same way, we obtain $L\{Sinat\} = \frac{a}{s^2+a^2}$

Example 2.3.4

a) $L\{Cos\ 3t\} = \frac{s}{s^2+9}$

b) $L\{\text{Sin } 3t\} = \frac{3}{s^2+9}$

c) $L\left\{\text{Sin}\frac{3}{5}t\right\} = \frac{\frac{3}{5}}{s^2+(\frac{3}{5})^2} = \frac{15}{25s^2+9}$

2.3.5 $L(t^n) = \frac{n!}{s^{n+1}}$ where n is a positive integer.

Proof

From the definition of the Laplace transform, we have $L\{f(t)\} = L\{t^n\} = \int_0^\infty t^n e^{-st}\,dt$

Therefore $L\{f(t)\} = \int_0^\infty e^{-st}t^n\,dt$

$\rightarrow L\{t^n\} = \left[t^n\left(\frac{e^{-st}}{-s}\right)\right]_0^\infty + \frac{n}{s}\int_0^\infty e^{-st}t^{n-1}\,dt$

$= -\frac{1}{s}[t^n e^{-st}]_0^\infty + \frac{n}{s}\int_0^\infty t^{n-1}e^{-st}\,dt$

$= -\frac{1}{s}[0-0] + \frac{n}{s}\int_0^\infty t^{n-1}e^{-st}\,dt = \frac{n}{s}\int_0^\infty t^{n-1}e^{-st}\,dt$

Therefore, $L\{t^n\} = \frac{n}{s}\int_0^\infty t^{n-1}e^{-st}\,dt = \frac{n}{s}L\{t^{(n-1)}\}$,, /'

By the same process, we will obtain

$L\{t^{(n-1)}\} = \frac{n-1}{s}\int_0^\infty t^{n-2}e^{-st}\,dt = \frac{n-1}{s}L\{t^{(n-2)}\}$

$L\{t^{(n-2)}\} = \frac{n-2}{s}\int_0^\infty t^{n-3}e^{-st}\,dt = \frac{n-2}{s}L\{t^{(n-3)}\}$

$L\{t^{(n-(n-1))}\} = \frac{n-(n-1)}{s}\int_0^\infty t^0 e^{-st}\,dt = \frac{n-(n-1)}{s}L\{t^0\}$

Substituting these in (1) gives

$L\{t^n\} = \frac{n}{s}.\frac{n-1}{s}.\frac{n-2}{s}.\frac{n-3}{s}\ldots\frac{n-(n-1)}{s}L\{t^0\} = \frac{n!}{s^n}L\{t^0\}$ 0'

But $L\{t^0\} = \int_0^\infty t^0 e^{-st}\,dt = \int_0^\infty e^{-st}\,dt = -\frac{1}{s}\left[e^{-st}\Big|_0^\infty\right] = -\frac{1}{s}[0-1] = \frac{1}{s}$

Substituting $L\{t^0\} = \frac{1}{s}$ in (2) gives $L\{t^n\} = \frac{n!}{s^n}.\frac{1}{s} = \frac{n!}{s^{n+1}}$

Therefore $L(t^n) = \frac{n!}{s^{n+1}}$

Observe that by putting $n = 0$, we obtain $L(t^0) = \frac{0!}{s^{0+1}} \rightarrow L(1) = \frac{1}{s}$ since $0! = 1$. This result is the same obtained for $L\{K\} = \frac{K}{s}$ when $K = 1$.

Example 2.3.5

$$L(t^3 + 2t^2 - 4t + 1) = \frac{3!}{s^4} + 2.\frac{2!}{s^3} - 4.\frac{1!}{s^2} + \frac{1}{s} = \frac{6}{s^4} + \frac{4}{s^3} - \frac{4}{s^2} + \frac{1}{s}$$

$$= \frac{1}{s^4}(s^3 - 4s^2 + 4s + 6)$$

Exercise 2.1: Obtain the Laplace transforms of the following functions

a. $2Sin3t + Cos3t$

b. $4e^{2t} + 3Cosh4t$

c. $2Sin3t + 4Sinh3t$

d. $5e^{4t} + Cosh2t$

Solutions

a. $L(2Sin3t + Cos3t) = 2L(Sin3t) + L(Cos3t)$

$$= 2.\frac{3}{s^2+9} + \frac{s}{s^2+9} = \frac{S+6}{s^2+9}$$

b. $L\{4e^{2t} + 3Cosh4t\} = 4L(e^{2t}) + 3L(Cosh4t)$

$$= 4.\frac{1}{s-a} + 3.\frac{s}{s^2-16} = \frac{4}{s-a} + \frac{3s}{s^2-16} = \frac{7s^2-6s-64}{(s-2)(s^2-16)}$$

c. $L(2Sin3t + 4Sinh3t) = 2.\frac{3}{s^2+9} + 4.\frac{3}{s^2+9}$

$$= \frac{6}{s^2+9} + \frac{12}{s^2+9} = \frac{18(s^2+3)}{s^2-81}$$

d. $L(5e^{4t} + Cosh2t) = \frac{5}{s-4} + 4.\frac{s}{s^2-4} = \frac{6s^2-45-20}{(s-4)(s^2-4)}$

2.4 The Shift Theorem

If $L\{f(t)\} = F(s)$ then $L\{e^{at}f(t)\} = F(s-a)$

Proof: From definition, $L\{f(t)\} = \int_0^\infty e^{-st}f(t)dt = F(s)$

Therefore $L\{e^{at}f(t)\} = \int_0^\infty e^{-st}e^{at}f(t)dt = \int_0^\infty e^{-(s-a)t}f(t)dt = F(s-a).$

Exercise 2.2: Determine the following

1. $L(e^{-2t}Cosh2t)$
2. $L(2e^{3t}Sin3t)$
3. $L(2e^{2t}Cost)$
4. $L(e^{3t}Sinh2t)$
5. $L(t^3e^{-4t})$

Solutions

1. Since $L(Cosh2t) = \frac{s}{s^2-4}$

   Therefore $L(e^{-2t}Cosh2t) = \frac{s+2}{(s+2)^2-4} = \frac{s+2}{s(s+4)}$

2. Since $L(Sin3t) = \frac{3}{s^2+9}$

Therefore $L(2e^{3t}Sin3t) = 2L(e^{3t}Sin3t) = 2\left[\frac{3}{(s-3)^2+9}\right] = \frac{6}{s^2-6s+18}$

3. Since $L(Cost) = \frac{s}{s^2+1}$

Therefore $L(2e^{2t}Cost) = 2L(e^{2t}Cost) = 2\left[\frac{s-2}{(s-2)^2+1}\right] = \frac{2s-4}{s^2-4s+5}$

4. Since $L(Sinh2t) = \frac{2}{s^2-4}$

Therefore $L(e^{3t}Sinh2t) = \frac{2}{(s-3)^2-4} = \frac{2}{s^2-6s+5}$

5. Since $L(t^3) = \frac{3!}{S^{3+1}} = \frac{6}{S^4}$, therefore, $L(t^3e^{-4t}) = \frac{6}{(S+4)^4}$,

Take Home Exercises: Determine the following

1. $L(4te^{-t})$
2. $L(t^3e^{-2t})$
3. $L(2e^{-3t}Sin4t)$
4. $L(Cos2t - Sin2t)$
5. $L(3 + 2tCos3$

2.5 Laplace Transforms of Derivatives

$$L\{y'\} = L\left\{\frac{dy}{dt}\right\} = \int_0^\infty e^{-st}\left(\frac{dy}{dt}\right)dt = \int_0^\infty e^{-st}dy = e^{-st}y\Big|_0^\infty + s\int_0^\infty e^{-st}ydt$$

$$= 0 - y(0) + sL(y)$$

i.e $L\{y'\} = sL(y) - y(0)$, /'

$$L\{y''\} = L\left\{\frac{d^2y}{dt^2}\right\} = \int_0^\infty e^{-st}\left(\frac{d^2y}{dt^2}\right)dt = \int_0^\infty e^{-st}\frac{d}{dt}\left(\frac{dy}{dt}\right)dt$$

$$= \int_0^\infty e^{-st}d\left(\frac{dy}{dt}\right)$$

$$= e^{-st}\frac{dy}{dt}\Big|_0^\infty + s\int_0^\infty e^{-st}\frac{dy}{dt}dt$$

$$= 0 - y'(0) + sL(y')$$

$\rightarrow L\{y''\} = sL(y') - y'(0)$,, 0'

Substituting (1) in (2) gives

$L\{y''\} = s^2L(y) - sy(0) - y'(0)$ 1'

$$L\{y'''\} = L\left\{\frac{d^3y}{dt^3}\right\} = \int_0^\infty e^{-st}\left(\frac{d^3y}{dt^3}\right)dt = \int_0^\infty e^{-st}\frac{d}{dt}\left(\frac{d^2y}{dt^2}\right)dt$$

$$= \int_0^\infty e^{-st}d\left(\frac{d^2y}{dt^2}\right)$$

$$= e^{-st}\frac{d^2y}{dt^2}\Big|_0^{\infty} + s\int_0^{\infty} e^{-st}\frac{d^2y}{dt^2}dt$$

$$= 0 - y''(0) + sL(y'')$$

$\rightarrow L\{y'''\} = sL(y'') - y''(0)$,, 2'

Substituting (3) in (4) gives

$L\{y'''\} = s^3L(y) - s^2y(0) - sy'(0) - y''(0)$ 3'

Continuing in this manner, we can obtain

$$L\{y^{(n)}\} = s^nL(y) - s^{n-1}y(0) - s^{n-2}y'(0) - \cdots y^{(n-1)}(0)$$

Example 2.5: Express the transforms of the given expressions in terms of s and L(y).

1. $y'' + y', y(0) = 0, y'(0) = 0$
2. $y'' + 3y',\ y(0) = 2,\ y'(0) = 1$
3. $2y'' - y' + y, f(0) = 1, f'(0) = 0$
4. $2y'' - 3y' + 2y,\ \ f(0) = 1, f'(0) = 0$

Solutions

1. $L\{y'' + y'\} = s^2L(y) - sy(0) - y'(0) + sL(y) - y(0)$

Since $y(0) = 0$ and $y'(0) = 0$

Therefore, $L\{y'' + y'\} = s^2L(y) - 0 - 0 + sL(y) - 0$

$$= s^2L(y) + sL(y) = s(s+1)L(y)$$

2. $L\{y'' + 3y'\} = S^2L(y) - Sy(0) - y'(0) - 3[SL(y) - y(0)]$

$$= S^2L(y) - Sy(0) - y'(0) - 3SL(y) + 3y(0)$$

But $y(0) = 2,\ y'(0) = 1$

Therefore, $L\{y'' - 3y'\} = S^2L(y) - 2S - 1 - 3SL(y) + 6$

$$= S^2L(f) - 3SL(f) - 2S += S(S-3)L(y) - 2S + 5$$

3. $L\{2y'' - y' + y\} = 2\left(S^2L(f) - Sf(0) - f'(0)\right) - [SL(f) - f(0)] + L(f)$

$= 2S^2L(f) - 2Sf(0) - 2f'(0) - SL(f) + f(0) + L(f)$

But $f(0) = 1, f'(0) = 0$

Therefore $L\{2y'' - y' + y\} = 2S^2L(f) - 2S - 0 - SL(f) + 1 + L(f)$

$= 2S^2L(f) - SL(f) + L(f) - 2S + 1$

$= (2S^2 - S + 1)L(f) - 2S + 1$

4. $L\{2y'' - 3y' + 2y\} = 2[S^2L(f) - Sf(0) - f'(0)] - 3[SL(f) - f(0)] + 2L(f)$

$= 2S^2L(f) - 2Sf(0) - 2f'(0) - 3SL(f) + 3f(0) + 2L(f)$

But $f(0) = 1, f'(0) = 0$

Therefore, $L\{2y'' - 3y' + 2y\} = 2S^2L(f) + 2S - 0 - 3SL(f) - 3 + 2L(f)$

$= 2S^2L(f) - 3SL(f) + 2L(f) + 2S - 3$

$= (2S^2 - 3S + 2)L(f) + 2S - 3$

Table of Laplace Transforms

$\mathbf{F(x)}$	$\mathbf{L\{F(x)\}}$
K	$\frac{K}{s}$ $s > 0$
e^{ax}	$\frac{1}{s-a}$ $s > a$
Sinhax	$\frac{a}{s^2-a^2}$ $s > 0$
Coshax	$\frac{s}{s^2-a^2}$ $s > 0$
Sinax	$\frac{a}{s^2+a^2}$ $s > 0$
Cosax	$\frac{s}{s^2+a^2}$ $s > 0$
x^n, (where $n = 1,2,3, \dots$)	$\frac{n!}{s^{n+1}}$ $s > 0$
$Y'(x)$	$sL\{Y(x)\} - Y(0)$
$Y''(x)$	$s^2L\{Y(x)\} - sY(0) - Y'(0)$
$Y^{(n)}(x)$, (where $n = 1,2,3, \dots$)	$s^nL\{Y(x)\} - s^{(n-1)}Y(0) - s^{(n-2)}Y'^{(0)} - \cdots Y^{(n)}(0)$

2.6 Inverse Laplace Transform

If the Laplace Transform of a function is known, then it is possible to find the function by finding the Inverse Laplace Transform. For example, we know that

$L\{\text{Cosat}\} = \frac{s}{(s^2+a^2)} = F(s)$, therefore $\text{Cosat} = L^{-1}\left\{\frac{s}{(s^2+a^2)}\right\} = L^{-1}\{F(s)\}$.

Thus, if $F(s) = \frac{s}{(s^2+a^2)}$, then $L^{-1}(F) = L^{-1}\frac{s}{(s^2+a^2)} = \text{Cosat} = f(t)$

In general, if $L\{f(t)\} = F(s)$ then $f(t) = L^{-1}\{F(s)\}$ where L^{-1} denotes the inverse Laplace transform.

$F(s)$ is the Laplace Transform of $f(t)$ while f(t) is the Inverse Laplace Transform of $F(s)$

In finding Inverse Laplace Transform, we have to reverse process of finding the Laplace Transform, i.e. given a laplace transform, we have to find the function of t to which it belongs.

Example 2.6

1. Since $L\{\text{Sinat}\} = \frac{a}{s^2+a^2}$ $\therefore L^{-1}\left(\frac{s}{s^2+a^2}\right) = \text{Sinat}$

2. Since $L\{e^{2t}\} = \frac{1}{s-2}$ $\therefore \quad L^{-1}\left(\frac{1}{s-2}\right) = e^{2t}$

3. Since $L\{\text{Cosh5t}\} = \frac{s}{s^2-25}$ $\therefore \quad L^{-1}\left(\frac{s}{s^2-25}\right) = \text{Cosh5t}$

4. Since $L\{4\} = \frac{4}{s}$ $\therefore \quad L^{-1}\left(\frac{4}{s}\right) = 4$

5. Since $L\{4\text{Sinh3t}\} = 4L\{\text{Sinh3t}\} = 4.\frac{3}{s^2-9} = \frac{12}{s^2-9}$

$$\therefore \quad L^{-1}\left(\frac{12}{s^2-9}\right) = L^{-1}\left(4.\frac{3}{s^2-9}\right) = 4L^{-1}\left(\frac{3}{s^2-9}\right) = 4\text{Sinh3t}$$

Therefore, given a transform, we can write down the corresponding function in t, provided we can recognize it from our table of transforms. In most cases, the given transform has to be rewritten in a form that can be easily recognized from table. Most

often the use of partial fraction is resorted to in order to rewrite the given transform in a form that corresponds with one from table.

Table of Inverse Transform

f(s)	f(t)
$\frac{a}{s}$	a
$\frac{1}{s+a}$	e^{-at}
$\frac{n!}{s^{n+1}}$	t^n (n is +ve integer)
$\frac{1}{s^n}$	$\frac{t^{n-1}}{(n-1)!}$ (n is +ve integer)
$\frac{a}{s^2+a^2}$	Sinat
$\frac{s}{s^2+a^2}$	Cosat
$\frac{a}{s^2-a^2}$	Sinhat
$\frac{s}{s^2-a^2}$	Coshat

Example 2.6.1: To find $L^{-1}\left[\frac{3s+1}{s^2-s-6}\right]$, $\frac{3s+1}{s^2-s-6}$ has to be rewritten in a standard form obtainable from the table of Laplace transform. Therefore, resorting to partial fraction we have $\frac{3s+1}{(s+2)(s-3)} = \frac{1}{s+2} + \frac{2}{s-3}$

Therefore, $L^{-1}\left[\frac{3s+1}{s^2-s-6}\right] = L^{-1}\left[\frac{1}{s+2} + \frac{2}{s-3}\right] = L^{-1}\left[\frac{1}{s+2}\right] + 2L^{-1}\left[\frac{1}{s-3}\right] = e^{-2t} + 2e^{3t}$

2.7 Rules of Partial Fractions

1. The numerator must be of lower degree than the denominator. If it is not, then we first divide out.
2. Factorize the denominator into its prime factors. These determine the shapes of the partial fractions.
3. Denominator with a linear factor $(s + a)$ gives a partial fraction $\frac{A}{s+a}$ where A is a constant to be determined.
4. Denominator with a repeated factor $(s + a)^2$ gives $\frac{A}{(s+a)} + \frac{B}{(s+a)^2}$ where A and B are constants to be determined.
5. Similarly, denominator with a repeated factor $(s + a)^3$ gives $\frac{A}{(s+a)} + \frac{B}{(s+a)^2} + \frac{C}{(s+a)^3}$ where A, B and C are constants to be determined.

6. Denominator with a quadratic factor $(s^2 + ps + q)$ gives $\frac{AS+B}{s^2+ps+q}$ where A and B are constants to be determined.
7. Denominator with a repeated quadratic factors $(s^2 + ps + q)^2$ gives $\frac{AS+B}{s^2+ps+q} + \frac{CS+D}{(s^2+ps+q)^2}$ where A, B, C and D are constants to be determined.

Example 2.7

1. $\frac{s-19}{(S+2)(s-5)} = \frac{A}{s+2} + \frac{B}{s-5} = \frac{A(s-5)+B(s+2)}{(s+2)(s-5)} \rightarrow s - 19 = A(s-5) + B(s+2)$

   Putting $s = -2$ gives $-21 = -7A \rightarrow A = 3$

   Putting $s = 5$ gives $-14 = 7B \rightarrow B = -2$

   Therefore $\frac{s-19}{(S+2)(s-5)} = \frac{3}{s+2} - \frac{2}{s-5}$
2. $\frac{3s^2-45+11}{(s+3)(s-2)^2} = \frac{A}{s+3} + \frac{B}{(s-2)} + \frac{C}{(s-c)^2}$

Example 2.7.1: Determine

1. $L^{-1}\left[\frac{5s+1}{s^2-s-12}\right]$
2. $L^{-1}\left[\frac{9s-8}{s^2-25}\right]$
3. $L^{-1}\left[\frac{s-5}{(s-3)(s-4)}\right]$
4. $L^{-1}\left[\frac{s-5}{s^2+45+20}\right]$
5. $L^{-1}\left[\frac{s^2-15s+41}{(s+2)(s-3)^2}\right]$
6. $L^{-1}\left[\frac{4s^2-5s+6}{(s+1)(s^2+4)}\right]$

Solutions

1. $\frac{5s+1}{s^2-s-12} = \frac{5s+1}{(s-4)(s+3)} = \frac{A}{5-4} + \frac{B}{s+3} \rightarrow 5s + 1 = A(s+3) + B(s-4)$

Putting $s = 4$ gives $21 = A(7) + B(0) \rightarrow A = 3$

Putting $s = -3$ gives $-14 = A(0) + B(-7) \rightarrow B = 2$

Therefore $\frac{5s+1}{s^2-s-12} = \frac{3}{5-4} + \frac{2}{s+3}$

Hence $L^{-1}\left[\frac{5s+1}{s^2-s-12}\right] = L^{-1}\left(\frac{3}{5-4}\right) + L^{-1}\left(\frac{2}{5+3}\right) = 3e^{4t} + 2e^{-3t}$

2. $\frac{9s-8}{s^2-25} = \frac{9s-8}{s(s-2)} = \frac{A}{s} + \frac{B}{s-2} = \frac{A(s-2)+Bs}{s(s-2)} \quad \rightarrow \quad 9s-8 = A(s-2)+Bs$

Putting $s = 0$: $-8 = A(-2) + B(0) \rightarrow A = 4$

Putting $s = 2$: $10 = A(0) + B(2) \rightarrow B = 5$

Therefore, $\frac{9s-8}{s^2-25} = \frac{4}{s} + \frac{5}{s-2}$

Hence $L^{-1}\left[\frac{9s-8}{s^2-25}\right] = L^{-1}\left(\frac{4}{s}\right) + L^{-1}\left(\frac{5}{5-2}\right) = 4 + 5e^{2t}$

3. $\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4} = \frac{A(s-4)+B(s-3)}{(s-3)(s-4)} \rightarrow s-5 = A(s-4)+B(s-3)$

Putting $s = 3$: $-2 = A(-1) + B(0) \rightarrow A = 2$

Putting $s = 4$: $-1 = A(0) + B(1) \rightarrow B = -1$

Therefore, $\frac{s-5}{(s-3)(s-4)} = \frac{2}{s-3} + \frac{1}{s-4}$

Hence $L^{-1}\left[\frac{s-5}{(s-3)(s-4)}\right] = L^{-1}\left(\frac{2}{s-3}\right) - L^{-1}\left(\frac{1}{s-4}\right) = 2e^{3t} + e^{4t}$

4. $L^{-1}\left[\frac{s-5}{s^2+45+20}\right] = L^{-1}\left[\frac{s-5}{(s+2)^2+16}\right] = L^{-1}\left[\frac{s+2}{(s+2)^2+16}\right] + L^{-1}\left[\frac{-7}{(s+2)^2+16}\right]$

$= e^{-2t}Cos4t - 7L^{-1}\left[\frac{1}{(s+2)^2+16}\right]$

$= e^{-2t}Cos4t - \frac{7}{4}L^{-1}\left[\frac{4}{(s+2)^2+16}\right]$

$= e^{-2t}Cos4t - \frac{7}{4}e^{-2t}Sin4t$

5. $\frac{s^2-15s+41}{(s+2)(s-3)^2} = \frac{A}{s+2} + \frac{B}{s-3} + \frac{C}{(s-3)^2} = \frac{A(s-3)^2+B(s+2)(s-3)+C(s+2)}{(s+2)(s-3)^2}$

$\rightarrow \quad s^2 - 15s + 41 = A(s-3)^2 + B(s+2)(s-3) + C(s+2)$

Putting $s = 3$: $5 = 5C \rightarrow C = 1$

Putting $s = -2$: $75 = 25A \rightarrow A = 3$

*

power of s on each side, i.e. the coefficients of s^2. This gives

$1 = A + B \ \therefore 1 = 3 + B \rightarrow B = -2$

Therefore, $\frac{s^2-15s+41}{(s+2)(s-3)^2} = \frac{3}{s+2} + \frac{2}{s-3} + \frac{1}{(s-3)^2}$

Hence $L^{-1}\left[\frac{s^2-15s+41}{(s+2)(s-3)^2}\right] = L^{-1}\left[\frac{3}{s+2}\right] - L^{-1}\left[\frac{2}{s-3}\right] + L^{-1}\left[\frac{1}{(s-3)^2}\right]$

$$= 3e^{-2t} - 2e^{3t} + L^{-1}\left[\frac{1}{(s-3)^2}\right]$$

But we recall $L^{-1}\left[\frac{1}{s^2}\right] = t$ and that by the shift theorem, we know that if $L[F(t)] = f(s)$ then $L[e^{at}F(t)] = f(s-a)$

$\therefore \frac{1}{(s-3)^2}$ is like $\frac{1}{s^2}$ with s replaced by $(s-3), i.e. a = 3$

$$\therefore L^{-1}\left[\frac{1}{(s-3)^2}\right] = te^{3t}$$

Finally, $L^{-1}\left[\frac{s^2-15s+41}{(s+2)(s-3)^2}\right] = 3e^{-2t} - 2e^{3t} + te^{3t}$

6. $\frac{4s^2-5s+6}{(s+1)(s^2+4)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+4} = \frac{A(s^2+4)+(Bs+C)(s+1)}{(s+1)(s^2+4)}$

$\rightarrow \quad 4s^2 - 5s + 6 = A(s^2+4) + (Bs+C)(s+1)$

Putting $s = -1$: $15 = 5A \quad \rightarrow \quad A = 3$

Equating coefficients of highest power, i.e. s^2: $4 = A + B \quad \therefore 4 = B + 3 \rightarrow B = 1$

Equating the constant terms: $6 = 4A + C \quad \therefore 6 = 12 + C \rightarrow C = -6$

Therefore, $\frac{4s^2-5s+6}{(s+1)(s^2+4)} = \frac{3}{s+1} + \frac{5}{s^2+4} - \frac{6}{s^2+4}$

Hence $L^{-1}\left[\frac{4s^2-5s+6}{(s+1)(s^2+4)}\right] = L^{-1}\left[\frac{3}{s+1}\right] + L^{-1}\left[\frac{5}{s^2+4}\right] - L^{-1}\left[\frac{6}{s^2+4}\right]$

$$= 3e^{-t} + Cos2t - 3Sin2t$$

Take Home Exercises: Find the inverse transform of

a. $\frac{1}{2s-3}$

b. $\frac{s}{(s-4)^3}$

c. $\frac{3s+4}{s^3+9}$

d. $\frac{22s+16}{(s+1)(s-2)(s+3)}$

e. $\frac{s^2-11s+6}{(s+1)(s-2)^2}$

f. $\frac{4s^2-17s-24}{s(s+3)(5-4)}$

g. $\frac{5s^2-4s-7}{(s-3)(s^2+4)}$

Answers

a. $\frac{1}{2}e^{3t/2}$

b. $5t^2e^{4t}$

c. $3Cos3t + \frac{4}{3}Sin3t$

d. $e^{-t} + 4e^{2t} + 5e^{-3t}$

e. $2e^{-t} - (1+4t)e^{2t}$

f. $2 + 3e^{-3t} - e^{4t}$

g. $2e^{3t} + 3Cos2t + \frac{5}{2}Sin2t$

2.8 Solution of Differential Equation by Laplace Transform

Example 2.8.1: Solve the differential equation $2y' - y = 0 \quad if \quad y(0) = 1$

Solution

Taking transform of each term and plugging in the initial conditions

$L(2y') - L(y) = L(0) \qquad \rightarrow \qquad 2L(y') - L(y) = 0$

$\rightarrow \quad 2[SL(y) - y(0)] - L(y) = 0 \qquad \rightarrow 2[SL(y) - 1] - L(y) = 0$

$\rightarrow \quad 2SL(y) - 2 - L(y) = 0 \qquad \rightarrow \quad 2SL(y) - L(y) = 2$

$\rightarrow \quad L(y)[2S - 1] = 2 \qquad \rightarrow \quad L(y) = \frac{2}{2s-1} = \frac{1}{s-\frac{1}{2}}$

$\rightarrow \quad y = L^{-1}\left[\frac{1}{s-\frac{1}{2}}\right] = e^{t/2}$

Therefore $y = e^{t/2}$

Example 2.8.2: Solve the differential equation $y'' + 2y' + 2y = 0, if\ y(0) = 0\ and\ y'(0) = 1$

Solution

Taking transform of each term of the equation

$y'' + 2y' + 2y \rightarrow L(y'') + 2L(y') + 2L(y) = L(0)$

$\rightarrow \quad L(y'') + 2L(y') + 2L(y) = 0$

$\rightarrow \quad S^2L(y) - 2y(0) - y'(0) + 2[SL(y) - y(0)] + 2L(y) = 0$

$\rightarrow \quad S^2L(y) - 2y(0) - y'(0) + 2SL(y) - 2y(0) + 2L(y) = 0$

$\rightarrow \quad S^2L(y) - 0 - 1 + 2SL(y) - 0 + 2L(y) = 0 \ \ since\ y(0) = 0\ and\ y'(0) = 1$

$\rightarrow \quad S^2L(y) + 2SL(y) + 2L(y) = 1$

$\rightarrow \quad L(y)[S^2 + 2S + 2] = 1 \quad \rightarrow \quad L(y) = \frac{1}{S^2+2S+2} = \frac{1}{(s+1)^2+1}$

$\rightarrow \quad y = L^{-1}\left[\frac{1}{S^2+2S+2}\right] = L^{-1}\left[\frac{1}{(s+1)^2+1}\right] = e^{-t}Sint$

Therefore $y = e^{-t}Sint$

Example 2.8.3: Solve the differential equation $y'' + y = Cost, y(0) = 1, y'(0) = 2$

Solution

Taking transform of each term of the equation

$L(y'') + L(y) = L(Cost) \quad \rightarrow \quad [s^2L(y) - sy(0) - y'(0)] + L(y) = \frac{s}{S^2+1}$

$\rightarrow \quad [S^2L(y) - S - 2] + L(y) = \frac{s}{S^2+1} \quad since \;\; y(0) = 1, y'(0) = 2$

$\rightarrow \quad (s+1)^2L(y) = \frac{s}{S^2+1} + s + 2 \quad \rightarrow \quad L(y) = \frac{s}{(s+1)^2} + \frac{s}{(S^2+1)} + \frac{2}{(S^2+1)}$

$\rightarrow y = L^{-1}\left[\frac{s}{(s+1)^2} + \frac{s}{(S^2+1)} + \frac{2}{(S^2+1)}\right] = \frac{t}{2}Sint + Cost + 2Cost$

Therefore, $y = \frac{t}{2}Sint + Cost + 2Cost$

Example 2.8.4: An electric circuit contains a $1-H$ indicator, a $10-n$ resistors, and a battery whose voltage is $6V$. Find the current as a function of the time, if the initial current is zero. The differential equation for this circuit is $L\, di/dt + 10i = 6$

Solution

Taking transform of each term of the equation

$L\left(\frac{di}{dt}\right) + 10L(i) = L(6) \quad \rightarrow \quad [SL(i) - i(0)] + 10L(i) = \frac{6}{s}$

$\rightarrow L(i)[s + 10] = \frac{6}{s} \quad \rightarrow \quad L(i) = \frac{6}{s(s+10)} = \frac{0.6}{s} - \frac{0.6}{s+10}$

Therefore, $i = L^{-1}\left[\frac{0.6}{s} - \frac{0.6}{s+10}\right] = 0.6 - 0.6e^{-10t} = 0.6\,(1 - e^{-10t})$

Hence $i = 0.6\,(1 - e^{-10t})$

Example 2.8.5: A string is stretched 1m by a mass of 5kg (force of gravity of 50N). The medium resists the motion of the object with a force of 10V, where V is the velocity. The differential equation describing the motion is $5\frac{d_y^2}{dt^2} + 10\frac{dy}{dt} + 50y =$

0. Find y, the displacement of the object, as a function of the time if $y(0) = 1\ and\ \frac{dy}{dt} = 0\ when\ t = 0$.

Solution

Dividing through by 5 and denoting the derivative by y'' and y', the D.E becomes

$y'' + 2y' + 10y = 0$

Taking transform of each term

$L(y'') + 2L(y') + 10L(y) = L(0)$

$S^2L(y) - Sy(0) - L(y') + 2[SL(y) - y(0)] + 10L(y) = 0$

$S^2L(y) - S + 2SL(y) - 2 + 10L(y) = 0$

$S^2L(y) + 2SL(y) + 10L(y) = s + 2$

$L(y) = \frac{s+2}{S^2+2s+10} = \frac{s+2}{(S+1)^2+3^2} = \frac{s+1}{(S+1)^2+3^2} + \frac{1}{(S+1)^2+3^2}$

$\therefore y = e^{-t}Cos3t + \frac{1}{3}e^{-t}Sin3t$

$= e^{-t}(Cos3t + \frac{1}{3}Sin3t)$

Take Home Exercises: Solve by Laplace transforms method

i. $y' + y = 0,\ y(0) = 1$
ii. $2y' - 3y = 0,\ y(0) = -1$
iii. $y'' + 4y = 0,\ y(0) = 0,\ y'(0) = 1$
iv. $y'' - 4y' + 5y = 0,\ y(0) = 1,\ y'(0) = 2$
v. $y'' + 2y' + y = e^{-t},\ y(0) = 1,\ y'(0) = 2$
vi. A 2-H inductor, and 80-n resistor, and 8-V battery are connected in series. Find the current in the circuit as a function of time if the initial current is zero.

3.0 Solutions of Simultaneous Linear Ordinary Differential Equations

3.1 The Substitution Method

Example 3.1: Solve $3\frac{dx}{dt} = 6x + y, \frac{dy}{dt} = 3x + 2y$ given that $t = 0$ when $x = -1$ and $y = 9$

Solution

$3\frac{dx}{dt} = 6x + y$, ,, /'

$\frac{dy}{dt} = 3x + 2y$, ,,, 0'

From (1) $y = 3\frac{dx}{dt} - 6x$,,,, 1'

Substituting (3) in (2) gives

$3\frac{d^2x}{dt^2} - 6\frac{dy}{dt} = 3x + 6\frac{dx}{dt} - 12x \quad \rightarrow \quad \frac{d^2x}{dt^2} - 4\frac{dy}{dt} + 3x = 0$

A.E: $m^2 - 4m + 3 = 0 \quad \rightarrow \quad m = 1, 3$

Therefore $x(t) = Ae^t + Be^{3t}$ .. , 2'

Substituting (4) in (3) gives

$y = 3\frac{d(Ae^t + 3Be^{3t})}{dt} - 6(Ae^t + Be^{3t}) = 3Ae^t + 9Be^{3t} - 6Ae^t - 6Be^{3t}$

$\rightarrow \quad y = -3Ae^t + 3Be^{3t}$,, 3'

Since $x = -1$ and $y = 9$, when $t = 0$,

(4) becomes $-1 = A + B \rightarrow A + B = -1$,.,, ,, 4'

and (5) becomes $9 = -3A + 3B \rightarrow -A + B = 3$,, , 5'

Solving (6) and (7) simultaneously gives $A = -2$ and $B = 1$

Therefore $x = -2e^t + e^{3t}$ and $y = 6e^t + 3e^{3t}$

3.2 The D-Operator Method

Example 3.2.1: Solve $\frac{dx}{dt} + 2y + 3x = 0, \quad 3x + \frac{dy}{dt} - 2y = 0$

Solution

Let $D = \frac{d}{dt}$ so that $Dx = \frac{dx}{dt}$ and $Dy = \frac{dy}{dt}$

$\frac{dx}{dt} + 2y + 3x = 0$ becomes $(D+3)x + 2y = 0$,,, /'

and $3x + \frac{dy}{dt} - 2y = 0$ becomes $3x + (D-2)y = 0$,, 0'

Multiplying (1) by $(D-2)$ and (2) by 2 gives

$(D-2)(D+3)x + 2(D-2)y = 0$,,* , 1'

and $6x + 2(D-2)y = 0$, ,,, , 2'

Subtracting (4) from (3) gives

$(D^2 + D - 6)x - 6x = 0 \quad \rightarrow \quad (D^2 + D - 12)x = 0$

$\rightarrow \quad D^2x + Dx - 12x = 0 \quad \rightarrow \quad \frac{d^2x}{dt^2} + \frac{dx}{dt} - 12x = 0$

A .E: $m^2 + m - 12 = 0 \quad \rightarrow \quad m = 3, -4$

Therefore, $x(t) = Ae^{3t} + Be^{-4t}$

Substituting $x(t) = Ae^{3t} + Be^{-4t}$ into $\frac{dx}{dt} + 2y + 3x = 0$ gives

$\frac{dy}{dt}(Ae^{3t} + Be^{-4t}) + 2y + 3(Ae^{3t} + Be^{-4t}) = 0$

$\rightarrow \quad 3Ae^{3t} - 4Be^{-4t} + 2y + 3Ae^{3t} + 3Be^{-4t} = 0$

$\rightarrow \quad y(t) = -3Ae^{3t} + \frac{1}{2}Be^{-4t}$

Example 3.2.2: Solve $\frac{dx}{dt} - x + 2y = 0, \; 2x + \frac{dy}{dt} - y = 0$ given that when $t = 0, \; x = 4$ and $y = 2$

Solution

Let $D = \frac{d}{dt}$ so that $Dx = \frac{dx}{dt}$ and $Dy = \frac{dy}{dt}$

$\frac{dx}{dt} - x + 2y = 0$ becomes $Dx - x + 2y = 0$

$\rightarrow \quad (D-1)x + 2y = 0$,,, /'

and $2x + \frac{dy}{dt} - y = 0$ becomes $2x + Dy - y = 0$

$\rightarrow \quad 2x + (\mathrm{D}-1)y = 0$, 0'

Multiplying equation (1) by 2 and equation (2) by $(D-1)$ gives

$2(D-1)x + 4y = 0$,,, ,,,,, ,,, , 1'

and $2(D-1)x + (D-1)^2 y = 0$,, ,,, 2'

Subtracting (4) from (3) gives

$4y - (D-1)^2 y = 0 \quad \rightarrow \quad [4 - (D-1)^2]y = 0$

$\rightarrow \quad (4 - D^2 + 2D - 1)y = 0 \quad \rightarrow \quad (-D^2 + 2D + 3)y = 0$

$\rightarrow \quad -D^2 y + 2Dy + 3y = 0 \quad \rightarrow \quad -\frac{d^2 y}{dt^2} + 2\frac{dy}{dt} + 3y = 0$

A.E: $-m^2 + 2m + 3 = 0 \quad \rightarrow \quad m^2 - 2m - 3 = 0 \quad \rightarrow \quad m = 3, -1$

Therefore $y = Ae^{3t} + Be^{-t}$ 3'

Substituting $y = Ae^{3t} + Be^{-t}$ in $2x + \frac{dy}{dt} - y = 0$ gives

$2x + 3Ae^{3t} - Be^{-t} - Ae^{3t} - Be^{-t} = 0 \quad \rightarrow \quad x = Be^{-t} - Ae^{+3t}$ 4'

Since $x = 4$ and $y = 2$, when $t = 0$,

(5) becomes $4 = B - A$ i.e. $-A + B = 4$, ,, 5'

and (6) becomes $2 = A + B$,,, ,, 6'

Solving (7) and (8) simultaneously, we have $A = -1$, $B = 3$

$$\therefore x = 3e^{-t} + e^{3t}$$

$$y = e^{3t} + 3e^{-t}$$

3.3 The Laplace Transforms Method

Example 3.3: Solve the pair of simultaneous differential equations $\frac{dy}{dx} - x = e^t$, $\frac{dx}{dt} + y = e^{-t}$ given that at $t = 0$, $x = 0$ and $y = 0$

Solution

Taking the Laplace Transforms of $\frac{dy}{dx} - x = e^t$ and $\frac{dx}{dt} + y = e^{-t}$ gives

$sY(s) - y(0) - X(s) = \frac{1}{s-1}$ → $sY(s) - X(s) = \frac{1}{s-1}$, ,,,, /'

and $sX(s) - x(0) + Y(s) = \frac{1}{s+1}$ → $sX(s) + Y(s) = \frac{1}{s+1}$,, 0'

Since $y(0) = 0$ and $x(0) = 0$

From equation (1) $X(s) = sY(s) - \frac{1}{s-1}$ .. ,, 1'

Substituting (3) in (2), we have

$s\left[sY(s) - \frac{1}{s-1}\right] + Y(s) = \frac{1}{s+1}$ → $s^2Y(s) - \frac{s}{s-1} + Y(s) = \frac{1}{s+1}$

→ $(s^2 + 1)Y(s) = \frac{1}{s+1} + \frac{s}{s-1} = \frac{S^2+25-1}{(s+1)(s-1)}$

→ $Y(s) = \frac{S^2+25-1}{(s+1)(s-1)(s^2+1)}$,, 2'

Let $\frac{S^2+25-1}{(s+1)(s-1)(s^2+1)} = \frac{A}{s+1} + \frac{B}{s-1} + \frac{Cs+D}{s^2+1}$

→ $S^2 + 25 - 1 = A(s-1)(s^2+1) + B(s+1)(s^2+1) + (Cs+D)(s^2-1)$

Putting $s = 1$: $2 = 4B$ → $B = \frac{1}{2}$

Putting $S=-1$: $\quad -2=-4A \quad \rightarrow \quad A=\frac{1}{2}$

Putting $S=0$: $\quad -1=-A+B-D \quad \rightarrow \quad D=1$

Putting $S=2$: $\quad 7=5A+15B+(2C+D)3 \quad \rightarrow \quad C=-1$

$$Y(s)=\frac{S^2+25-1}{(s+1)(s-1)(s^2+1)}=\frac{\frac{1}{2}}{s+1}+\frac{\frac{1}{2}}{s-1}+\frac{-s+1}{s^2+1}$$

$$=\frac{1}{2}\left(\frac{1}{s+1}\right)+\frac{1}{2}\left(\frac{1}{s-1}\right)-\frac{s}{s^2+1}+\frac{1}{s^2+1}$$

Taking the inverse transform, we have $y(t)=\frac{1}{2}e^{-t}+\frac{1}{2}e^{t}-Cost+Sint.$

$$y(t)=\frac{1}{2}(e^{-t}+e^{t})-Cost+Sint=Cosht-Cost+Sint$$

From equations (3) and equation (4)

$$X(s)=s\left[\frac{S^2+25-1}{(s+1)(s-1)(s^2+1)}\right]-\frac{1}{s-1}=\frac{S^2-2s-1}{(s+1)(s-1)(s^2+1)}$$

Let $\quad \frac{S^2-2s-1}{(s+1)(s-1)(s^2+1)}=\frac{A}{s+1}+\frac{B}{s-1}+\frac{Cs+D}{s^2+1}$

$\rightarrow \quad S^2-2s-1=A(s-1)(s^2+1)+B(s+1)(s^2+1)+(Cs+D)(s^2-1)$

By solving, we have $\quad A=-\frac{1}{2},\ B=-\frac{1}{2},\ C=1$ and $D=1$

$$X(s)=-\frac{1}{2}\left(\frac{1}{s+1}\right)-\frac{1}{2}\left(\frac{1}{s-1}\right)+\frac{s}{s^2+1}+\frac{1}{s^2+1}$$

Taking inverse transform, we have

$$X(t)=-\frac{1}{2}e^{-t}-\frac{1}{2}e^{t}+Cost+Sint$$

$$=-\frac{1}{2}\left(e^{-t}+\frac{1}{2}e^{t}\right)+Cost+Sint$$

$$=-Cosht+Cost+Sint \quad \text{or} \quad Cost+Sint-Cosht$$

Take Home Exercises: Solve the following $D.E$

1. $x'=ax+by,\ y'=bx+ay$

2. $\frac{dx}{dt} = -y$, $\frac{dy}{dt} = x$, given that at $t = 0$, $x = 4.5$, $y = 0.8$

4.0 Fourier Series

4.1 Period Functions

A functions $f(x)$ is said to have a period T or to be periodic with period T if for all x, $f(x+T) = f(x)$, where T is a positive const.

Example 4.1: $Sinx$ has periods 2π, 4π, 6π* $Sin(x+2\pi) = Sin(x+4\pi) = Sin(x+6\pi)$ ..... $= Sinx$, and the period for $Sinn\pi$ or $Cosnx$, where n is a positive integer, is $\frac{2\pi}{n}$. Period for $tanx$ is π.

4.2 Definition: Fourier Series

Let $f(x)$ be defined in the interval $(-L, L)$ and outside this interval by $f(x+2L) = f(x)$, i.e. assume that $f(x)$ has the period $2L$. The Fourier series or Fourier expansion corresponding to $f(x)$ is given by;

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n Cos\frac{n\pi x}{L} + b_n Sin\frac{n\pi x}{L}\right) \quad ,, \quad /'$$

where the Fourier coefficients a_n and b_n are

$$\left.\begin{aligned} a_n &= \frac{1}{L}\int_{-L}^{L} f(x)Cos\frac{n\pi x}{L}dx \\ b_n &= \frac{1}{L}\int_{-L}^{L} f(x)Sin\frac{n\pi x}{L}dx \end{aligned}\right\} \quad 0'$$

$$n = 1,2,$$

If $f(x)$ has the period $2L$, the coefficients a_n, b_n can be determined equivalently from

$$\left.\begin{aligned} a_n &= \frac{1}{L}\int_{C}^{C+2L} f(x)Cos\frac{n\pi x}{L}dx \\ b_n &= \frac{1}{L}\int_{C}^{C+2L} f(x)Sin\frac{n\pi x}{L}dx \end{aligned}\right\} \quad , \quad 1'$$

where C is any real numbers

To determine a_0 in (1) we use (2) or (3) with $n = 0$ to get $a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$ or $a_0 = \frac{1}{L}\int_{C}^{C+2L} f(x)\,dx$

Example 4.2.1: If the series $A+\sum_{n=1}^{\infty}\left(a_nCos\frac{n\pi x}{L}+b_nSin\frac{n\pi x}{L}\right)$ converges uniformly to $f(x)$ in $(-L,L)$ show that for $n=0,1,2,3,\ldots$ show that

a. $a_n=\frac{1}{L}\int_{-L}^{L}f(x)Cos\frac{n\pi x}{L}dx$

b. $b_n=\frac{1}{L}\int_{-L}^{L}f(x)Sin\frac{n\pi x}{L}dx$

c. $A=\frac{a_0}{2}$

Solution

a. Multiply $f(x)=A+\sum_{n=1}^{\infty}\left(a_nCos\frac{n\pi x}{L}+b_nSin\frac{n\pi x}{L}\right)$ by $Cos\frac{n\pi x}{L}$ and integrating from $-L$ to L

$$\int_{-L}^{L}f(x)Cos\frac{n\pi x}{L}dx=A\int_{-L}^{L}Cos\frac{n\pi x}{L}dx+\sum_{n=1}^{\infty}\left[a_n\int_{-L}^{L}Cos\frac{n\pi x}{L}Cos\frac{n\pi x}{L}dx+b_n\int_{-L}^{L}Cos\frac{n\pi x}{L}Sin\frac{n\pi x}{L}dx\right]$$

$$=a_nL \text{ if } n\neq 0$$

Thus $a_n=\frac{1}{L}\int_{-L}^{L}f(x)Cos\frac{n\pi x}{L}dx$, if $n=1,2,3,\ldots$

b. Multiply $f(x)=A+\sum_{n=1}^{\infty}\left(a_nCos\frac{n\pi x}{L}+b_nSin\frac{n\pi x}{L}\right)$ by $Sin\frac{n\pi x}{L}$ and integrating from $-L$ to L.

$$\int_{-L}^{L}f(x)Sin\frac{n\pi x}{L}dx=A\int_{-L}^{L}Sin\frac{n\pi x}{L}dx+\sum_{n=1}^{\infty}\left[a_n\int_{-L}^{L}Sin\frac{n\pi x}{L}Cos\frac{n\pi x}{L}+b_n\int_{-L}^{L}Sin\frac{n\pi x}{L}Sin\frac{n\pi x}{L}dx\right]$$

$$=b_nL$$

Thus $b_n=\frac{1}{L}\int_{-L}^{L}f(x)Sin\frac{n\pi x}{L}dx$

c. Integrating $f(x)=A+\sum_{n=1}^{\infty}\left(a_nCos\frac{n\pi x}{L}+b_nSin\frac{n\pi x}{L}\right)$ from $-L$ to L and putting $m=0$

$$\int_{-L}^{L}f(x)dx=\int_{-L}^{L}A\,dx+\sum_{n=1}^{\infty}\left[a_n\int_{-L}^{L}Cos\frac{m\pi x}{L}dx+b_n\int_{-L}^{L}Sin\frac{m\pi x}{L}dx\right]=2AL$$

Thus $\int_{-L}^{L} f(x)dx = 2AL \qquad \therefore A = \frac{1}{2L}\int_{-L}^{L} f(x)dx$

$$A = \frac{a_0}{2}$$

Example 4.2.2: Find the Fourier coefficients corresponding to the function

$$f(x) = \begin{cases} 0 & -5 < x < 0 \\ 3 & 0 < x < 5 \end{cases} \text{ period } 10$$

Solution

Let Period $= 2L = 10 \rightarrow L = 5.$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)Cos\frac{n\pi x}{L}dx = \frac{1}{5}\int_{-5}^{5} f(x)Cos\frac{n\pi x}{L}dx$$

$$= \frac{1}{5}\left\{\int_{-5}^{0}(0)Cos\frac{n\pi x}{L}dx + \int_{0}^{5}(3)Cos\frac{n\pi x}{5}dx\right\}$$

$$= \frac{3}{5}\int_{0}^{5} Cos\frac{n\pi x}{L}dx = \frac{3}{5}\left(\frac{5}{n\pi}Sin\frac{n\pi x}{5}\right)\Big|_0^5 = 0 \text{ if } n \neq 0$$

If $n = 0, \quad a_n = a_0 = \frac{1}{L}\int_{-L}^{L} f(x)Cos\frac{n\pi x}{L}dx = \frac{1}{5}\int_{-5}^{5} f(x)Cos\frac{0\pi x}{5}dx$

$$\rightarrow \quad a_0 = \frac{1}{5}\int_{-5}^{5} f(x)dx = \frac{1}{5}[\int_{-5}^{0} f(x)dx + \int_{0}^{5} f(x)dx = \frac{1}{5}\int_{0}^{5} f(x)dx$$

$$\rightarrow \quad a_0 = \frac{3}{5}\int_{0}^{5} dx = \frac{3}{5}x\Big|_0^5 = 3$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)Sin\frac{n\pi x}{L}dx = \frac{1}{5}\int_{-5}^{5} f(x)Sin\frac{n\pi x}{L}dx$$

$$= \frac{1}{5}\left\{\int_{-5}^{0}(0)Sin\frac{n\pi x}{5}dx + \int_{0}^{5}(3)Sin\frac{n\pi x}{5}dx\right\}$$

$$= \frac{3}{5}\int_{0}^{5} Sin\frac{n\pi x}{5}dx = \frac{3}{5}\left(-\frac{5}{n\pi}Cos\frac{n\pi x}{5}\right)\Big|_0^5 \text{ if } n \neq 0$$

$$= \frac{3}{5}\left(-\frac{5}{n\pi}Cos\frac{n\pi x}{5}\right)\Big|_0^5 = \frac{3(1-Cosn\pi)}{n\pi}$$

The corresponding Fourier series is given by

$$\frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_nCos\frac{n\pi x}{L} + b_nSin\frac{n\pi x}{L}\right) = \frac{3}{2} + \sum_{n=1}^{\infty}\frac{3(1-Cosn\pi)}{n\pi}Sin\frac{n\pi x}{L}$$

$$= \frac{3}{2} + \frac{6}{\pi}\left(Sin\frac{\pi x}{5} + \frac{1}{3}Sin\frac{3\pi x}{5} + \frac{1}{5}Sin\,5\pi x + \cdots\right)$$

Example 4.2.3: Find the Fourier series expansion of $f(x) = x^2,\ 0 < x < 2\pi$ if the period is 2π

Solution

Period $= 2L = 2\pi \quad \rightarrow \quad L = \pi$

Using $a_n = \frac{1}{L}\int_C^{C+2L} f(x)Cos\frac{n\pi x}{L}dx$ and $b_n = \frac{1}{L}\int_C^{C+2L} f(x)Sin\frac{n\pi x}{L}dx$ while choosing $C = 0$ gives

$$a_n = \frac{1}{L}\int_C^{C+2L} f(x)Cos\frac{n\pi x}{L}dx = \frac{1}{\pi}\int_0^{2\pi} x^2 Cos\frac{n\pi x}{\pi}dx = \frac{1}{\pi}\int_0^{2\pi} x^2 Cosn\pi x\,dx$$

$$\rightarrow \qquad a_n = \frac{1}{\pi}x^2\frac{Sinnx}{n} - (2x)\frac{(-Cosnx)}{n^2} - 2\frac{(-Sinnx)}{n^3}\Big|_0^{2\pi} = \frac{4}{n^2}$$

If $n = 0,\ a_0 = \frac{1}{\pi}\int_0^{2\pi} x^2 dx = \frac{1}{\pi}\left(\frac{x^3}{3}\right)\Big|_0^{2\pi} = \frac{8\pi^2}{3}$

Also, $b_n = \frac{1}{L}\int_C^{C+2L} f(x)Sin\frac{n\pi x}{L}dx = \frac{1}{\pi}\int_0^{2\pi} x^2 Sin\frac{n\pi x}{\pi}dx = \frac{1}{\pi}\int_0^{2\pi} x^2 Sinnx\,dx$

$$\rightarrow \qquad b_n = \frac{1}{\pi}(x^2)\frac{(-Cosnx)}{n} - (2x)\frac{(-Sinnx)}{n^2} + (2)\frac{(Cosnx)}{n^3}\Big|_0^{2\pi} = \frac{-4\pi}{n}$$

Then $f(x) = x^2 = \frac{4\pi^2}{3} + \sum_{n=1}^{\infty}\left(\frac{4}{n^2}Cosnx - \frac{4\pi}{n}Sinnx\right)$

Example 4.2.4: Find the Fourier series expansion for the function

$$f(x) = \begin{cases} 1\ for - \pi \le x < 0 \\ x\ for \quad 0 \le x < \pi \end{cases}\ Period = 2\pi$$

Solution

Period $= 2L = 2\pi \quad \rightarrow \quad L = \pi$

Using $a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$, $a_n = \frac{1}{L}\int_{-L}^{L} f(x)Cos\frac{n\pi x}{L}dx$ and $b_n = \frac{1}{L}\int_{-L}^{L} f(x)Sin\frac{n\pi x}{L}dx$ gives

$a_0 = \frac{1}{\pi}\int_{-\pi}^{0} 1\,dx + \frac{1}{\pi}\int_{0}^{\pi} x dx = \frac{x}{\pi}\int_{-\pi}^{0} + \frac{x^2}{2\pi}\Big|_0^{\pi} = 1 + \frac{\pi}{2} = \frac{1+\pi}{2}$

$a_1 = \frac{1}{\pi}\int_{-\pi}^{0} Cosx\,dx + \frac{1}{\pi}\int_{0}^{\pi} x\,Cosx\,dx = \frac{1}{\pi}Sinx\Big|_{-\pi}^{o} + \frac{1}{\pi}(Cosx + xSinx)\Big|_0^{\pi} = -\frac{2}{\pi}$

$a_2 = \frac{1}{\pi}\int_{-\pi}^{0} Cos2x\,dx + \frac{1}{\pi}\int_{0}^{\pi} x\,Cos2x\,dx = \frac{1}{2\pi}Sin2x\Big|_{-\pi}^{o} + \frac{1}{4\pi}(Cos2x + 2xSin2x)\Big|_0^{\pi}$

$\rightarrow \quad a_2 = 0$

$a_3 = \frac{1}{2\pi}\int_{-\pi}^{0} Cos3x\,dx + \frac{1}{\pi}\int_{0}^{\pi} x\,Cos3x\,dx == \frac{1}{3\pi}Sin3x\Big|_{-\pi}^{o} + \frac{1}{9\pi}(Cos3x + 3xSin3x)\Big|_0^{\pi}$

$\rightarrow \quad a_3 = \frac{2}{9\pi}$

$b_1 = \frac{1}{\pi}\int_{-\pi}^{0} Sinx\,dx + \frac{1}{\pi}\int_{0}^{\pi} x\,Sinx\,dx = -\frac{1}{2\pi}Cosx\Big|_{-\pi}^{o} + \frac{1}{\pi}(Sinx - xCosx)\Big|_0^{\pi}$

$\rightarrow \quad b_1 = \frac{\pi-2}{\pi}$

$b_2 = \frac{1}{\pi}\int_{-\pi}^{0} Sin2x\,dx + \frac{1}{\pi}\int_{0}^{\pi} x\,Sin2x\,dx = -\frac{Cos2x}{2\pi}\Big|_{-\pi}^{o} + \frac{Sin2x - 2x\,Cos2x}{4\pi}\Big|_0^{\pi} = -\frac{1}{2}$

Thus $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n Cos\frac{n\pi x}{L} + b_n Sin\frac{n\pi x}{L}\right)$

$= \frac{2+\pi}{4} - \frac{2}{\pi}COsx - \frac{2}{9\pi}Cos3x + \frac{(\pi-2)}{\pi}Sinx - \frac{1}{2}Sin2x + \cdots$

Example 4.2.5: Find the Fourier Series for the square wave function

$$f(x) = \begin{cases} 0\ for - \pi \le x < 0 \\ 1\ for \quad 0 \le x < \pi \end{cases} \quad Period = 2\pi$$

Solution

Period $= 2L = 2\pi \quad \rightarrow \quad L = \pi$

Using $a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$, $a_n = \frac{1}{L}\int_{-L}^{L} f(x)Cos\frac{n\pi x}{L}dx$ and $b_n = \frac{1}{L}\int_{-L}^{L} f(x)Sin\frac{n\pi x}{L}dx$ gives

$a_0 = \frac{1}{2\pi}\int_{-\pi}^{0} 0\,dx + \frac{1}{2\pi}\int_{0}^{\pi} 1dx = \frac{1}{2}$

$a_n = \frac{1}{\pi}\int_{-\pi}^{0} 0\ dx + \frac{1}{\pi}\int_{0}^{\pi}(1)Cosnx\ dx = \frac{1}{\pi}(Sinnx)\Big|_0^{\pi} = 0$

$a_n = 0$ is valid for values of n since $Sinn\pi = 0$ i.e. if $n = 1$, $a_1 = 0$, etc.

$b_1 = \frac{1}{\pi}\int_{-\pi}^{0} 0\ dx + \frac{1}{\pi}\int_{0}^{\pi}(1)Sinx\ dx = \frac{-1}{\pi}(Cosx)\Big|_0^{\pi} = \frac{-1}{\pi}(-1-1) = \frac{2}{\pi}$

$b_2 = \frac{1}{\pi}\int_{-\pi}^{0} 0\ dx + \frac{1}{\pi}\int_{0}^{\pi} Sin2x\ dx = -\frac{1}{2\pi}(Cos2x)\Big|_0^{\pi} = -\frac{1}{2\pi}(1-1) = 0$

$b_3 = \frac{1}{\pi}\int_{-\pi}^{0} 0\ dx + \frac{1}{\pi}\int_{0}^{\pi} Sin3x\ dx = -\frac{1}{3\pi}(-1-1) = \frac{2}{3\pi}$

In general we can see that if n is even, $b_n = 0$ and if n is odd, $b_n = \frac{2}{n\pi}$.

Thus $f(x) = \frac{1}{2} + \frac{2}{\pi}Sinx + \frac{2}{3\pi}Sin3x + \cdots$

Example 4.2.6: Certain electric devices allow an electric current to pass through in only on direction. The result is that when an alternating current is applied, the current exists for only half cycle. (The current is a function of time). A device which causes this effect is called a half-wave rectifier. Derive the Fourier Series for the half of the rectifier wave for which $f(t) = Sint, \quad (0 \le t \le \pi)$ and $f(t) = 0$ for the other half of the cycle.

Solution

Period, $2L = 2\pi \quad \rightarrow \quad L = \pi$

Using $f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n Cos\frac{n\pi t}{L} + b_n Sin\frac{n\pi t}{L}\right)$ where $a_n = \frac{1}{L}\int_{-L}^{L} f(t)\,dt$, $a_n = \frac{1}{L}\int_{-L}^{L} f(t)Cos\frac{n\pi t}{L}dt$ and $b_n = \frac{1}{L}\int_{-L}^{L} f(t)Sin\frac{n\pi t}{L}dt$ gives

$a_0 = \frac{1}{\pi}\int_{0}^{\pi} Sint\ dt = \frac{1}{\pi}(-Cost)\Big|_0^{\pi} = \frac{1}{\pi}(1+1) = \frac{2}{\pi}$

$a_n = \frac{1}{L}\int_{0}^{L} f(t)Cos\frac{n\pi t}{L}dt = \frac{1}{\pi}\int_{0}^{\pi} Sint\ Cos\frac{n\pi t}{\pi}dt = \frac{1}{\pi}\int_{0}^{\pi} SintCosnt\ dt$

But $SinACosB = \frac{1}{2}[Sin(A+B) + Sin(A-B)]$

$$a_n = \frac{1}{\pi}\int_0^{\pi}\frac{1}{2}[Sin(t-nt)+Sin(t+nt)]dt = \frac{1}{2\pi}\left[Cos\frac{(1-n)t}{1-n}+Cos\frac{(1+n)t}{1+n}\right]_0^{\pi}$$

$$= -\frac{1}{2\pi}\left[Cos\frac{(1-n)\pi}{1-n}+Cos\frac{(1+n)\pi}{1+n}-\frac{1}{1-n}-\frac{1}{1+n}\right]$$

Valid for all n except $n = 1$

$$a_1 = \frac{1}{2\pi}\int_0^{\pi} Sint\ Cosnt\ dt = \frac{1}{2\pi}Sin^2t\Big|_0^{\pi} = 0$$

$$a_2 = -\frac{1}{2\pi}\left(\frac{-1}{-1}+\frac{-1}{3}-\frac{1}{-1}+\frac{1}{3}\right) = -\frac{2}{3\pi}$$

$$a_3 = -\frac{1}{2\pi}\left(\frac{-1}{-2}+\frac{1}{4}-\frac{1}{-2}-\frac{1}{4}\right) = 0$$

$$a_4 = -\frac{1}{2\pi}\left(\frac{-1}{-3}+\frac{-1}{5}-\frac{1}{-3}-\frac{1}{5}\right) = -\frac{2}{15\pi}$$

Also $b_n = \frac{1}{L}\int_{-L}^{L} f(t)Sin\frac{n\pi t}{L}dt = \frac{1}{\pi}\int_0^{\pi} Sint\ Sin\ nt\ dx$

But $SinASinB = -\frac{1}{2}[Cos(A+B)-Cos(A-B)]$

$$b_n = \frac{1}{\pi}\int_0^{\pi} -\frac{1}{2}[Cos(n+nt)-Cos(n-nt)dt = -\frac{1}{2\pi}\left[\frac{Sin(1+n)t}{1+n}-\frac{Sin(1-n)t}{1-n}\right]_0^{\pi}$$

$$= \frac{1}{2\pi}\left[\frac{Sin(1-n)\pi}{1-n}-\frac{Sin(1+n)\pi}{1+n}\right]$$

Valid for all n except $n = 1$

For $n = 1$, $b_1 = \frac{1}{\pi}\int_0^{\pi} Sint\ Sint\ dt = \frac{1}{\pi}\int_0^{\pi} Sin^2t\ dt$

But $Sin^2A = \frac{1}{2}[1-Cos2A]$

$$b_1 = \frac{1}{\pi}\int_0^{\pi} Sin^2t\ dt = \frac{1}{\pi}\int_0^{\pi}\frac{1}{2}[1-Cos2t]dt = \frac{1}{2\pi}\left(t-\frac{1}{2}Sin2t\right)\Big|_0^{\pi} = \frac{1}{2}$$

We see that $b_n = 0$ if $n > 1$, Since each is evaluated in terms of the $Sine$ of a multiple of π. Therefore the Fourier Series for the rectified wave is

$$f(t) = \frac{1}{\pi}+\frac{1}{2}Sint-\frac{2}{\pi}\left(\frac{1}{3}Cos2t+\frac{1}{15}Cos4t+\cdots\right)$$

Note: All the types of periodic functions such as this, may be seen on an oscilloscope when the proper signal is sent into it. In this way the oscilloscope may be used to analyze the periodic nature of such phenomena as second waves and electric currents.

Take Home Exercises: Obtain the Fourier series expansions of each of the following functions over the stipulated intervals:

1. $f(x) = x, -\pi < x < \pi$
2. $f(x) = 3x^2, -1 < x < 1$
3. $f(x) = \pi^2 - x^2, -\pi < x, \pi$
4. $f(x) = \begin{cases} x-1, & -\pi < x < 0 \\ x+1, & 0 < x < \pi \end{cases}, \quad -\pi < x < \pi$
5. $f(x) = \begin{cases} x+1, & -1 < x < 0 \\ x-1, & 0 < x < 1 \end{cases}, \quad -1 < x < 1$
6. $f(x) = \begin{cases} 1, & -2 < x < -1 \\ -x, & -1 < x < 0 \\ 1, & 1 < x < 2 \end{cases}, \quad -2 < x < 2$
7. $f(x) = Cos2x, -\pi < x < \pi$
8. $f(x) = Sin^2x, \ -\pi < x < \pi$
9. $f(x) = e^x, \ -\pi < x < \pi$
10. $f(x) = Cosh\frac{x}{2}, \ -\pi < x < \pi$

4.3 Even and Odd Functions

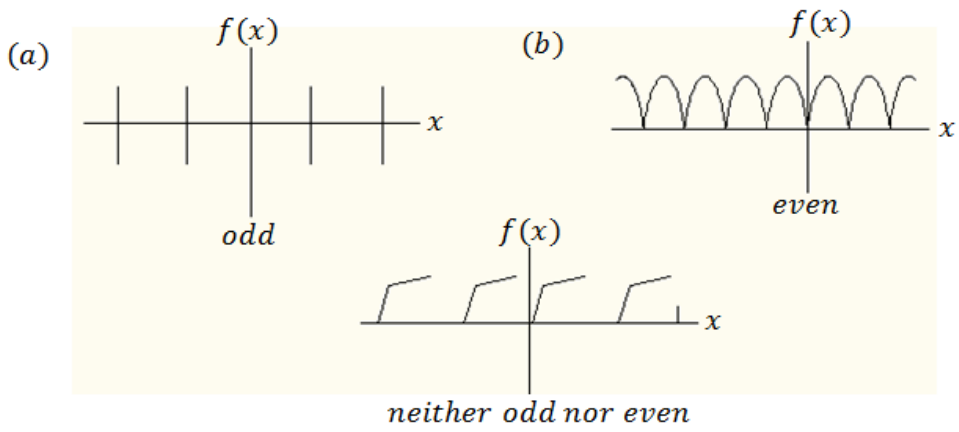
A function $f(x)$ is called

a) even if $f(-x) = f(x)$
b) odd if $f(-x) = -f(x)$

Example 4.3.1: $x^4, 2x^6 - 4x^2 + 3, Cosx, e^x, e^{-x}$ are even functions.

Example 4.3.2: $x^3, \ x^5 - 3x^3 + 2x, \ Sinx, \ tanx$ are odd functions.

Example 4.3.3: The following are some examples of graphs of even and od functions



Take Home Exercises: Determine whether each of the following functions is an even or odd function or neither.

1. $f(x) = Sin3x$
2. $f(x) = xCosx$
3. $f(x) = x(x+1)$
4. $f(x) = x(5x^2+7)$
5. $f(t) = 3e^{4t}$
6. $f(t) = |4t^3 - 3t^2 + 2t - 10|$
7. $f(x) = \begin{cases} x^2, & -1 < x < 0 \\ -x^2, & 0 < x < 1 \end{cases}$
8. $f(t) = \begin{cases} t+5, & -2 < t < 0 \\ -t+5, & 0 < t < 2 \end{cases}$
9. $f(x) = x^3, 0 \leq x \leq 2$
10. $f(t) = 2|t| - 1$

4.4 Properties of Even and Odd Functions

4.4.1. The sum (difference) of two even functions is even

Proof: Let $f(x)$ and $g(x)$ be two even functions.

Then by definition, $f(-x) = f(x)$ and $g(-x) = g(x)$.

Let $V(x) = f(x) + g(x)$.

Then $V(-x) = f(-x) + g(-x) = f(x) + g(x) = V(x)$

$V(-x) = V(x) \rightarrow$ the sum is even.

Let $R(x) = f(x) - g(x)$.

Then $R(-x) = f(-x) - g(-x) = f(x) - g(x) = R(x)$

$R(-x) = R(x) \rightarrow$ the difference is even.

4.4.2. The sum (difference) of two odd functions is odd

Proof: Let $f(x)$ and $g(x)$ be two odd functions.

Then by definition, $f(-x) = -f(x)$ and $g(-x) = -g(x)$.

Let $V(x) = f(x) + g(x)$.

Then $V(-x) = f(-x) + g(-x) = -f(x) - g(x) = -\{f(x) + g(x)\} = -V(x)$

$V(-x) = -V(x) \rightarrow$ the sum is odd.

Now, Let $R(x) = f(x) - g(x)$.

Then $R(-x) = f(-x) - g(-x) = -f(x) + g(x) = -\{f(x) - g(x)\} = -R(x)$

$R(-x) = -R(x) \rightarrow$ the difference is odd.

4.4.3. The product (quotient) of two even functions is even

Proof: Let $f(x)$ and $g(x)$ be two even functions.

Then by definition, $f(-x) = f(x)$ and $g(-x) = g(x)$.

Let $V(x) = f(x)g(x)$.

Then $V(-x) = f(-x)g(-x) = f(x)g(x) = V(x)$

$V(-x) = V(x) \rightarrow$ the product is even.

Now, Let $R(x) = \frac{f(x)}{g(x)}$.

Then $\mathrm{R}(-x) = \frac{f(-x)}{g(-x)} = \frac{f(x)}{g(x)} = R(x)$

$R(-x) = R(x) \rightarrow$ the quotient is even.

4.4.4. The product (quotient) of two odd functions is even

Proof: Let $f(x)$ and $g(x)$ be two odd functions.

Then by definition, $f(-x) = -f(x)$ and $g(-x) = -g(x)$.

Let $V(x) = f(x)g(x)$.

Then $V(-x) = f(-x)g(-x) = \{-f(x)\}\{-g(x)\} = f(x)g(x) = V(x)$

$V(-x) = V(x) \rightarrow$ the product is even.

Now, Let $R(x) = \frac{f(x)}{g(x)}$.

Then $R(-x) = \frac{f(-x)}{g(-x)} = \frac{-f(x)}{-g(x)} = \frac{f(x)}{g(x)} = R(x)$

$R(-x) = R(x) \rightarrow$ the quotient is even.

4.4.5. The product (quotient) of an even function and an odd function is odd

Proof: Let $f(x)$ be an even function and $g(x)$ bean odd functions.

Then by definition, $f(-x) = f(x)$ and $g(-x) = -g(x)$.

Let $V(x) = f(x)g(x)$.

Then $V(-x) = f(-x)g(-x) = \{f(x)\}\{-g(x)\} = -f(x)g(x) = -V(x)$

$V(-x) = -V(x) \rightarrow$ the product is odd.

Now, Let $R(x) = \frac{f(x)}{g(x)}$.

Then $R(-x) = \frac{f(-x)}{g(-x)} = \frac{f(x)}{-g(x)} = -\frac{f(x)}{g(x)} = -R(x)$

$R(-x) = -R(x) \rightarrow$ the quotient is odd.

4.4.6. If $f(x)$ is even, then $\int_{-L}^{L} f(x)\,dx = 2\int_{0}^{L} f(x)\,dx$

Proof: $\int_{-L}^{L} f(x)\,dx = \int_{-L}^{0} f(x)\,dx + \int_{0}^{L} f(x)\,dx = \int_{0}^{L} f(-x)\,dx + \int_{-L}^{L} f(x)\,dx$

Since $f(x)$ is even, then $f(-x) = f(x)$

Therefore, $\int_{-L}^{L} f(x)\,dx = \int_{0}^{L} f(-x)\,dx + \int_{-L}^{L} f(x)\,dx = \int_{0}^{L} f(x)\,dx + \int_{0}^{L} f(x)\,dx$

$= 2\int_{0}^{L} f(x)\,dx$

4.4.7. If $f(x)$ is odd, then $\int_{-L}^{L} f(x)\,dx = 0$

Proof: $\int_{-L}^{L} f(x)\,dx = \int_{-L}^{0} f(x)\,dx + \int_{0}^{L} f(x)\,dx = \int_{0}^{L} f(-x)\,dx + \int_{-L}^{L} f(x)\,dx$

Since $f(x)$ is odd, then $f(-x) = -f(x)$

Therefore, $\int_{-L}^{L} f(x)\,dx = \int_{0}^{L} f(-x)\,dx + \int_{-L}^{\mathrm{L}} f(x)\,dx = -\int_{0}^{L} f(x)\,dx + \int_{0}^{L} f(x)\,dx$

$= 0$

4.5 Half Range Fourier Sine or Cosine Series

A half range Fourier Sine or Cosine Series is a series in which only Sine terms or only Cosine terms are present respectively. When half range series corresponding to a given function is defined, the function is generally defined in the interval $(0, L)$ [which is half of the interval $(-L, L)$], in line with the name half range and then the function is specified as odd or even, so that it is clearly defined in the other half of the interval, namely $(-L, 0)$. In such case, we have;

$a_n = 0$, $b_n = \frac{2}{L}\int_{0}^{L} f(x) Sin\frac{n\pi x}{L}\,dx$ for half range $Sine$ Series

$b_n = 0$, $a_n = \frac{2}{L}\int_{0}^{L} f(x) Cos\frac{n\pi x}{L}\,dx$ for half range $Cosine$ Series

Example 4.5.1: Show that an even function can have no $Sine$ D expansion.

Solution

Method 1: No $Sine$ terms appear if $b_n = 0$, $n = 1,2,3, \ldots \ldots \ldots$ to show that, let us write

$b_n = \frac{1}{L}\int_{-L}^{L} f(x) Sin\frac{n\pi x}{L}\,dx = \frac{1}{L}\int_{-L}^{0} f(x) Sin\frac{n\pi x}{L}\,dx + \frac{1}{L}\int_{0}^{L} f(x) Sin\frac{n\pi x}{L}\,dx$, /'

Making the transformation $x = -u$ in the first integral on the right of (1) gives

$$\frac{1}{L}\int_{-L}^{0} f(x)Sin\frac{n\pi x}{L}dx = \frac{1}{L}\int_{-L}^{0} f(-u)Sin\frac{(-n\pi u)}{L}du = -\frac{1}{L}\int_{0}^{L} f(-u)Sin\frac{n\pi u}{L}du$$

$$\rightarrow \quad \frac{1}{L}\int_{-L}^{0} f(x)Sin\frac{n\pi x}{L}dx = -\frac{1}{L}\int_{0}^{L} f(x)Sin\frac{n\pi x}{L}du$$

So for an even function $f(-u) = f(u)$

$$b_n = -\frac{1}{L}\int_{0}^{L} f(x)Sin\frac{n\pi x}{L}dx + \frac{1}{L}\int_{0}^{L} f(x)Sin\frac{n\pi x}{L}dx = 0$$

Method 2: Given $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n Cos\frac{n\pi x}{L} + b_n Sin\frac{n\pi x}{L}\right)$

Then $f(-x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n Cos\frac{n\pi x}{L} - b_n Sin\frac{n\pi x}{L}\right)$

If $f(x)$ is even, $f(-x) = f(x)$. Hence

$$\frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n Cos\frac{n\pi x}{L} + b_n Sin\frac{n\pi x}{L}\right) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n Cos\frac{n\pi x}{L} - b_n Sin\frac{n\pi x}{L}\right)$$

$$\rightarrow \quad 2\sum_{n=1}^{\infty} b_n Sin\frac{n\pi x}{L} = 0 \quad \rightarrow \quad \mathrm{b}_n = 0$$

$$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n Cos\frac{n\pi x}{L}$$

Example 4.5.2 If $f(x)$ is even, show that

a. $a_n = \frac{1}{L}\int_{0}^{L} f(x)Cos\frac{n\pi x}{L}dx$

b. $b_n = 0$

Solution

a. $$a_n = \frac{1}{L}\int_{0}^{L} f(x)Cos\frac{n\pi x}{L}dx = \frac{1}{L}\int_{L}^{0} f(x)Cos\frac{n\pi x}{L}dx + \frac{1}{L}\int_{0}^{L} f(x)Cos\frac{n\pi x}{L}dx$$

Letting $x = -u$, $\frac{1}{L}\int_{L}^{0} f(x)Cos\frac{n\pi x}{L}dx = \frac{1}{L}\int_{-L}^{0} f(-u)Cos\frac{(-n\pi u)}{L}du$

$$= -\frac{1}{L}\int_{0}^{L} f(-u)Cos\frac{(-n\pi u)}{L}du$$

$$= \frac{1}{L}\int_{0}^{L} f(u)Cos\frac{n\pi u}{L}du$$

for $n = 1$, $a_1 = \frac{2}{\pi}\int_0^{\pi} Sinx\ Cosnx\ dx = \frac{2}{\pi}\ \frac{Sin^2x}{2}\Big|_0^{\pi} = 0$

for $n = 2$, $a_2 = \frac{2(1+Cos2\pi)}{\pi(2^2-1)} = \frac{4}{\pi(2^2-1)}$

for $n = 3$, $a_2 = \frac{2(1+Cos3\pi)}{\pi(3^2-1)} = \frac{4}{\pi(3^2-1)} = 0$

for $n = 4$, $\mathrm{a}_2 = \frac{2(1+Cos4\pi)}{\pi(4^2-1)} = \frac{4}{\pi(4^2-1)}$

for $n = 5$, $a_2 = \frac{2(1+Cos5\pi)}{\pi(5^2-1)} = \frac{4}{\pi(5^2-1)} = 0$

Therefore, $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n Cos\frac{n\pi x}{L} = \frac{2}{\pi} - \frac{2}{\pi}\sum_{n=2}^{\infty}\frac{(1+Cosn\pi)}{n^2-1}Cosnx$

$$= \frac{2}{\pi} - \frac{4}{\pi}\left(\frac{Cos2x}{2^2-1} + \frac{Cos4x}{4^2-1} + \frac{Cos6x}{6^2-1} + \cdots\right)$$

Example 4.5.4 Prove that $\int_{-L}^{L} Sin\frac{k\pi x}{L}dx = \int_{-L}^{L} Cos\frac{k\pi x}{L}dx = 0$ if $k =$ 1,2,3, … … …

Solution

$$\int_{-L}^{L} Sin\frac{k\pi x}{L}dx = -\frac{L}{k\pi}Cos\frac{k\pi x}{L}\Big|_{-L}^{L} = -\frac{L}{k\pi}\left\{Cos\frac{(k\pi)L}{L} - Cos\frac{(-k\pi)L}{L}\right\}$$

$$= -\frac{L}{k\pi}[Coskπ - Cos(-k\pi)]$$

Since $Cos(-\theta) = Cos\theta$

Therefore, $\int_{-L}^{L} Sin\frac{k\pi x}{L}dx = -\frac{L}{k\pi}[Coskπ - Cos(k\pi)] = 0$

Similarly

$$\int_{-L}^{L} Cos\frac{k\pi x}{L}dx = \frac{L}{k\pi}Sin\frac{k\pi x}{L}\Big|_{-L}^{L} = \frac{L}{k\pi}\left[Sin\frac{(k\pi L)}{L} - Sin\frac{(-k\pi L)}{L}\right]$$

$$= \frac{L}{k\pi}[Sin\ k\pi + Sin(-k\pi)]$$

Since $Sin(-\theta) = -Sin\theta$

Therefore $\int_{-L}^{L} Cos\frac{k\pi x}{L}dx = \frac{L}{k\pi}[Sin\, k\pi + Sink\pi] = \frac{2L}{k\pi}Sin\, k\pi = 0$

because $Sink\pi = 0$ for all values of k

$\therefore \quad \int_{-L}^{L} Sink\pi\, x\, dx = \int_{-L}^{L} Cos\frac{k\pi x}{L}\, dx = 0$

Example 4.5.5

Prove that $\int_{-L}^{L} Cos\frac{m\pi x}{L}Cos\frac{n\pi x}{L}dx = \int_{-L}^{L} Sin\frac{m\pi x}{L}Sin\frac{n\pi x}{L}dx = \begin{cases} 0 & if \quad m \neq n \\ L & if \quad m = n \end{cases}$

where m and n can assume any of the values 1,2,3, ...

Solution

Recall from trigonometry

$CosA\, CosB = \frac{1}{2}\{Cos(A+B) + Cos(A-B)\}$

$SinA\, SinB = -\frac{1}{2}\{Cos(A+B) - Cos(A-B)\}$

Then if $m \neq n$

$\int_{-L}^{L} Cos\frac{m\pi x}{L}Cos\frac{n\pi x}{L}dx = \int_{-L}^{L}[\frac{1}{2}\{Cos\frac{(m+n)\pi x}{L} + Cos\frac{(m-n)\pi x}{L}\}]dx$

$$= \frac{-L}{2(m+n)\pi}Sin\frac{(m+n)\pi x}{L}\Big|_{-L}^{L} + \frac{-L}{2(m-n)\pi}Sin\frac{(m-n)\pi x}{L}\Big|_{-L}^{L}$$

$$= -\frac{L}{2}\left[\frac{1}{(m+n)\pi}Sin\frac{(m+n)\pi L}{L} - \frac{1}{(m+n)\pi}Sin\frac{(m+n)\pi(-L)}{L} + \frac{1}{(m-n)\pi}\frac{Sin(m-n)\pi L}{L} - \frac{1}{(m-n)\pi}Sin\frac{(m-n)\pi(-L)}{L}\right]$$

$$= -\frac{L}{2}\left[\frac{1}{(m+n)\pi}Sin(m+n)\pi + \frac{1}{(m+n)\pi}Sin(m+n)\pi + \frac{1}{(m-n)\pi}Sin(m-n)\pi + \frac{1}{(m-n)\pi}Sin(m-n)\pi\right]$$

$$= -\frac{L}{2}\left[\frac{2}{(m+n)\pi}Sin(m+n) + \frac{2}{(m-n)\pi}Sin(m-n)\pi\right]$$

$$= 0$$

Similarly if $m \neq n$

$$\int_{-L}^{L} Sin\frac{m\pi x}{L} Sin\frac{n\pi x}{L} dx = -\frac{1}{2}\int_{-L}^{L}\left\{Cos\frac{(m+n)\pi x}{L} + Cos\frac{(m-n)\pi x}{L}\right\} dx = 0$$

$$= \frac{L}{2(m+n)\pi} Sin\frac{(m+n)\pi x}{L}\Big|_{-L}^{L} - \left[\frac{L}{2(m-n)\pi} Sin\frac{(m-n)\pi x}{L}\Big|_{-L}^{L}\right]$$

$$= 0$$

If $m = n$, we have $Cos\frac{m\pi x}{L} Cos\frac{n\pi x}{L} = Cos\frac{n\pi x}{L} Cos\frac{n\pi x}{L} = Cos^2\frac{n\pi x}{L}$

From Trigonometry, $Cos^2 x = \frac{1}{2}(1 + Cos2x)$

Therefore, $Cos^2\frac{n\pi x}{L} = 1 + Cos\frac{2n\pi x}{L}$

Hence $\int_{-L}^{L} Cos\frac{m\pi x}{L} Cos\frac{n\pi x}{L} dx = \frac{1}{2}\int_{-L}^{L}\left(1 + Cos\frac{2n\pi x}{L}\right) dx$

$$= \frac{1}{2}\left[x + \frac{L}{2n\pi} Sin\frac{2n\pi x}{L}\right]_{-L}^{L}$$

$$= \frac{1}{2}\left[L + L + \frac{1}{2n\pi}\left(Sin\frac{2n\pi xL}{L} - Sin\ 2n\pi(-L)\right)\right]$$

$$= \frac{1}{2}(2L + 0) = L$$

Similarly, If $m = n$, we have $Sin\frac{m\pi x}{L} Sin\frac{n\pi x}{L} = Sin\frac{n\pi x}{L} Sin\frac{n\pi x}{L} = Sin^2\frac{n\pi x}{L}$

From Trigonometry, $sin^2 x = \frac{1}{2}(1 - Cos2x)$

Therefore, $Sin^2\frac{n\pi x}{L} = 1 - Cos\frac{2n\pi x}{L}$

Hence $\int_{-L}^{L} Sin\frac{m\pi x}{L} Sin\frac{n\pi x}{L} dx = \frac{1}{2}\int_{-L}^{L}(1 - Cos2n\pi x)dx$

$$= \frac{1}{2}\left[x - \frac{1}{2n\pi}\left(Cos\frac{2n\pi x}{L}\right)\right]_{-L}^{L}$$

$$= \frac{1}{2}\left[2L - \frac{1}{2n\pi}\left(Sin\frac{2n\pi L}{L} - Sin\frac{2n\pi(-L)}{L}\right)\right]_{-L}^{L}$$

$$= \frac{1}{2}\left[2L - \frac{1}{2n\pi}(0)\right] = L$$

Example 4.5.6 Prove $\int_{-L}^{L} Sin\frac{m\pi x}{L} Cos\frac{n\pi x}{L} dx = 0$

Solution

Recall we have $SinA\ CosB = \frac{1}{2}\{Sin(A+B) + Sin(A-B)\}$

If $m \neq n$

$$\int_{-L}^{L} Sin\frac{m\pi x}{L} Cos\frac{n\pi x}{L} dx = \frac{1}{2}\int_{-L}^{L}\left\{Sin\frac{(m+n)\pi x}{L} + Sin\frac{(m-n)\pi x}{L}\right\} dx$$

$$= \frac{1}{2}\left[\frac{-L}{(m+n)\pi} Cos\frac{(m+n)\pi x}{L} + \frac{-L}{(m-n)\pi} Cos\frac{(m-n)\pi x}{L}\right]_{-L}^{L}$$

$$= \frac{1}{2}\left[\begin{array}{l} \frac{-L}{(m+n)\pi}\left\{Cos\frac{(m+n)\pi L}{L} - Cos\frac{(m+n)\pi(-L)}{L}\right\} \\ + \frac{-L}{(m-n)\pi}\left\{Cos\frac{(m-n)\pi L}{L} - Cos\frac{(m-n)\pi(-L)}{L}\right\} \end{array}\right]$$

$$= \frac{1}{2}\left[\begin{array}{l} \frac{-L}{(m+n)\pi}\left\{Cos\frac{(m+n)\pi L}{L} - Cos\frac{(m+n)\pi(L)}{L}\right\} \\ + \frac{-L}{(m-n)\pi}\left\{Cos\frac{(m-n)\pi L}{L} - Cos\frac{(m-n)\pi(L)}{L}\right\} \end{array}\right]$$

$$= \frac{1}{2}\left[\frac{-L}{(m-n)\pi}.0 + \frac{-L}{(m+n)\pi}.0\right] = \frac{1}{2}(0+0) = 0$$

If $m = n \quad Sin\frac{m\pi x}{L} Cos\frac{n\pi x}{L} = Sin\frac{n\pi x}{L} Cos\frac{n\pi x}{L}$

From Trigonometry, $SinACosA = \frac{1}{2}Sin2A$

Therefore, $Sin\frac{m\pi x}{L} Cos\frac{n\pi x}{L} = Sin\frac{n\pi x}{L} Cos\frac{n\pi x}{L} = \frac{1}{2}Sin\frac{2n\pi x}{L}$

Hence $\int_{-L}^{L} Sin\frac{m\pi x}{L} Cos\frac{n\pi x}{L} dx = \frac{1}{2}\int_{-L}^{L} Sin\frac{2n\pi x}{L} dx$

$$= \frac{1}{2}\left[\frac{-L}{2n\pi} Cos\frac{2n\pi x}{L}\right]_{-L}^{L}$$

$$= \frac{1}{2}\left[\frac{-L}{2n\pi}\left\{Cos\frac{2n\pi L}{L} - Cos\frac{(-2n\pi L)}{L}\right\}\right]$$

$$= \frac{1}{2}\left[\frac{-L}{2n\pi}\left(Cos\frac{2n\pi L}{L} - Cos\frac{2n\pi L}{L}\right)\right]$$

$$= 0$$

Take Home Exercises

1. Obtain the Fourier Cosine series for each of the following functions over the stipulated interval:

a. $f(x) = \begin{cases} 0, & 0 < x < \frac{1}{2} \\ 1, & \frac{1}{2} < x < 1 \end{cases}, \quad 0 < x < 1$

b. $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2' \end{cases} \quad 0 < x < 2$

c. $f(x) = Cos2x, \quad 0 < x < \pi$

d. $f(x) = Sin2x, \quad 0 < x < \pi$

e. $f(x) = Cosh2x, \quad 0 < x < \pi$

f. $f(x) = Sinh2x, \quad 0 < x < \pi$

g. $f(x) = x^4, \ 0 < x < 10$

h. $f(x) = x(10 - x), \ 0 < x < 10$

2. Obtain the Fourier Sine series for each of the following functions over the stipulated interval:

a. $f(x) = 1, \ 0 < x < 5$

b. $f(x) = \begin{cases} 1, & 0 < x < \frac{1}{2} \\ 0, & \frac{1}{2} < x < 1 \end{cases}, \quad 0 < x < 1$

c. $f(x) = \begin{cases} 0, & 0 < x < \frac{1}{2} \\ x - \frac{1}{2}, & \frac{1}{2} < x < 1 \end{cases}, \quad 0 < x < 1$

d. $f(x) = Cos2x, \quad 0 < x < \pi$

e. $f(x) = Sin3x, \quad 0 < x < \pi$

f. $f(x) = Cosh2x, \quad 0 < x < \pi$

g. $f(x) = Sinh2x, \quad 0 < x < \pi$

h. $f(x) = e^{-x}, \quad 0 < x < \pi$

3. Find the Fourier expansion of the electronic device known as full-wave rectifier. This is found by using as a function for the current $f(t) = -Sint$ for $-\pi \leq t \leq 0$ and $f(t) = Sint$ for $0 < t \leq \pi$.

Appendix V (b): A Framework on the use of the CMSTA for Teaching Advanced Calculus

Course: Advanced Calculus **Course Code:** MTH 312 **Contact Hours:** 2HRS/WK

Week 1	**Content Specification**	**Teacher's Activities**	**Learner's Activities**
	General Introduction	Introduce the Course and topics to be Learnt	Students allowed to interject with suggestions and discussions as well as questioning
		Set the rules of engagement	
		Assign students into five arbitrary groups	
		Listen to students comments (if any), responds appropriately and administers ATMI and RMARS as pre-tests	
Week 2	**Content Specification**	**Teacher's Activities**	**Learner's Activities**
Second – Order Linear Differential Equations	General Form of: • second order linear differential equation • second order homogeneous linear differential equation with constant coefficients • second order non-homogeneous linear differential equation with constant coefficients	State the general form of: • second order linear differential equation • Second order homogeneous linear differential equation with constant coefficients • Second order non-homogeneous linear differential equation with constant coefficients with good examples and notes where necessary Assess the students	Each student to give at least three examples of : • second order linear differential equation with constant coefficients • Second order homogeneous linear differential equation • Second order non-homogeneous linear differential equation with constant coefficients

	Condition for the existence and uniqueness of solutions to 2nd Order differential equations	Explain the existence and uniqueness of solutions to 2nd Order differential equations with good examples and notes where necessary Assess the students	Each student to give at least three examples of 2nd Order differential equations problems that satisfy the conditions
	Solutions of Second order homogeneous linear differential equation with constant coefficients.	Derive the general solutions for Second order homogeneous linear differential equation with constant coefficients when the auxiliary equation has real and distinct, equal and complex roots and apply each general solution type to solving specific problem Assess the students	Each student to find the general solutions of at least three Second order homogeneous linear differential equation with constant coefficients whose auxiliary equation has real and distinct, equal and complex roots.
Week 3 Second - order differential equations (Cont.)	**Content Specification**	**Teacher's Activities**	**Learner's Activities**
	Solutions of Second order non-homogeneous linear differential equation with constant coefficients	Establish that the general solutions for Second order non-homogeneous linear differential equation with constant coefficients is made up of the sum of complementary function and particular integral and apply each general solution type to solving specific problem Assess the students	Each student to find the general solution of at least three Second order non-homogeneous linear differential equation with constant coefficients.
	Solutions of Second order homogeneous and non-homogeneous linear differential equation (with constant coefficients) with given initial values	Solve many problems involving Second order homogeneous and non-homogeneous linear differential equation (with constant coefficients) with given initial values Assess the students	Each student to find the general solution of at least three Second order homogeneous and non-homogeneous linear differential equation (with constant coefficients) with given initial values
Weeks 2 – 3 Resources Materials for Teaching Second - order differential equations			
• Teacher – Made Lecture Notes • Computer Lab with fully networked systems. • Internet connectivity • tesconnect. Online: http://www.tes.co.uk/teaching-resource/Second-Order-Differential-Equation-6348540/ and			

http://www.tes.co.uk/teaching-resource/Second-Order-Inhomogeneous-Differential-Equations-6188283/ • MITOPENCOURSEWARE. Online: http://ocw.mit.edu/courses/mathematics/18-03sc-differential-equations-fall-2011/unit-ii-second-order-constant-coefficient-linear-equations/modes-and-the-characteristic-equation/ • Mathcentre. Online: http://www.mathcentre.ac.uk/students/courses/engineering/differential-equations/			
Week 4 Laplace Transform	**Content Specification**	**Teacher's Activities**	**Learner's Activities**
	Definition of Laplace transform	Define Laplace transform with good examples and notes where necessary Assess the students	Each student to use the definition of Laplace transform to write down the Laplace transforms of at least three simple functions without evaluating them
	Laplace transform of simple functions	Derive general expressions for the Laplace transform of simple functions and use them to find the Laplace transforms of specific simple functions and a combination of some of them	Each student to use the general expressions for the Laplace transform of simple functions to find the Laplace transforms of at least three specific simple functions and a combination of some of them Each student to: 1. use the definition of Laplace transform to write down the Laplace transforms of at least three simple functions and evaluate them. 2. use the general expressions for the Laplace transform of simple functions to find the Laplace transforms of at least three simple functions whose Laplace transforms were obtained in (1) above 3. compare the results of (1) and (2) above

Week 5 Laplace Transform (Cont.)	**Content Specification**	**Teacher's Activities**	**Learner's Activities**
	Definition of the inverse Laplace transform	Define the inverse Laplace transform with good examples and notes where necessary Assess the students	Each student to use the definition of the inverse Laplace transform to obtain the inverse Laplace transform of at least three specific expressions each of which are the Laplace transforms of known functions.
	Inverse Laplace transform of simple functions	Derive general expressions for the Laplace transform of simple functions and use them to find the inverse Laplace transforms of specific simple functions and a combination of some of them Assess the students	Each student to use the general expressions for the inverse Laplace transform of simple functions to find the Laplace transforms of at least three specific simple functions and a combination of some of them
Week 6 Laplace Transform (Cont.)	**Content Specification**	**Teacher's Activities**	**Learner's Activities**
	Laplace transform of derivatives	Use the Laplace transforms method to solve many problems involving first and second order homogeneous and non-homogeneous linear differential equation (with constant coefficients) with given initial values Assess the students	Each student to Use the Laplace transforms method to find the general solution of at least three first and second order homogeneous and non-homogeneous linear differential equation (with constant coefficients) with given initial values

Weeks 4 – 6 Resources Materials for Teaching Laplace Transform

- Teacher – Made Lecture Notes
- Computer Lab with fully networked systems.
- Internet connectivity
- tesconnect. Online: http://www.tes.co.uk/teaching-resource/Laplace-Transform-6390945/, http://www.tes.co.uk/teaching-resource/Laplace-Transform-1-6200581/
- MITOPENCOURSEWARE. Online: http://ocw.mit.edu/resources/res-6-007-signals-and-systems-spring-2011/video-lectures/lecture-20-the-laplace-transform/ and http://ocw.mit.edu/resources/res-18-008-calculus-revisited-complex-variables-differential-equations-and-linear-algebra-fall-2011/part-ii/lecture-7-laplace-transforms/
- KHANACADEMY. Online: https://www.khanacademy.org/math/differential-equations/laplace-transform/laplace-transform-tutorial/v/laplace-transform-1#!

Week 7 Methods of solving second - order differential equations	**Content Specification**	**Teacher's Activities**	**Learner's Activities**
	Systems of homogeneous linear differential equations of 1st order with constant coefficients	Illustrate with examples some systems of homogeneous linear differential equations of 1st order with constant coefficients Assess the students	Each student to give at least three examples systems of homogeneous linear differential equations of 1st order with constant coefficients
	Solutions of systems of homogeneous linear differential equations of 1st order with constant coefficients	Illustrate with examples three methods (i.e. substitution, operator-D and Laplace transforms methods) of solving some systems of homogeneous linear differential equations of 1st order with constant coefficients Assess the students	Each student to solve at least three systems of homogeneous linear differential equations of 1st order with constant coefficients using three methods (i.e. substitution, operator-D and Laplace transforms methods)
Week 8 Methods of solving simultaneous linear differential equations (Cont.)	**Content Specification**	**Teacher's Activities**	**Learner's Activities**
	Systems of non-homogeneous linear differential equations of 1st order with constant coefficients	Illustrate with examples some systems of non homogeneous linear differential equations of 1st order with constant coefficients Assess the students	Each student to give at least three examples systems of non homogeneous linear differential equations of 1st order with constant coefficients
	Solutions of systems of non homogeneous linear differential equations of 1st order with constant coefficients	Illustrate with examples three methods (i.e. substitution, operator-D and Laplace transforms methods) of solving some systems of non-homogeneous linear differential equations of 1st order with constant coefficients Assess the students	Each student to solve at least three systems of non-homogeneous linear differential equations of 1st order with constant coefficients using three methods (i.e. substitution, operator-D and Laplace transforms methods)
Weeks 7 – 8 Resources Materials for Teaching Methods of solving simultaneous linear differential equations			
• Teacher – Made Lecture Notes • Computer Lab with fully networked systems. • Internet connectivity • MITOPENCOURSEWARE. Online: http://ocw.mit.edu/courses/mathematics/18-03-differential-equations-spring-2010/video-lectures/lecture-25-homogeneous-linear-systems-with-constant-coefficients/			

Week 9 Fourier series	**Content Specification**	**Teacher's Activities**	**Learner's Activities**
	Periodic and non-periodic functions	Define of the periodic and non-periodic functions	Each student to use the definition of Periodic and non-periodic functions to give at least three examples of a period and non-periodic function.
		Examples of periodic and non-periodic functions	
		Graphical representation of periodic and non-periodic functions.	Each student to sketch the graph of the Periodic and non-periodic function he/she has cited as example.
	Even and odd functions	Define of even and odd functions	Each student to use the definition of even and odd functions to give at least three examples of an even and odd function.
		Examples of even and odd functions	
		Graphical representation of even and odd functions	Each student to sketch the graph of the even and odd function he/she has cited as example.
		Characteristics of even and odd functions	Each student to show that the even and odd function he/she cited as example fits into the characteristics discussed.
Week 10 Fourier series (Cont.)	**Content Specification**	**Teacher's Activities**	**Learner's Activities**
	Fourier series	Define of Fourier series	Each student to obtain the Fourier series expansion of some given functions.
	Fourier coefficients	Derive expression for the Fourier coefficients	
	Fourier series expansion of simple algebraic and trigonometric functions	Use of Fourier series to expand simple algebraic and trigonometric functions	
	Fourier series expansion of functions with arbitrary period	Use of Fourier series to expand functions with arbitrary period	

Week 11 Fourier series (Cont.)	Content Specification	Teacher's Activities	Learner's Activities
	Half range Fourier series	Derive the half range Fourier series for trigonometric functions	Each student to obtain the half range Fourier series expansion of some given functions.
	Application of Fourier series	Apply Fourier series to suitable engineering problems	

Weeks 9 – 11 Resources Materials for Teaching Fourier series

- Teacher – Made Lecture Notes
- Computer Lab with fully networked systems.
- Internet connectivity
- MITOPENCOURSEWARE. Online: http://ocw.mit.edu/resources/res-6-008-digital-signal-processing-spring-2011/video-lectures/lecture-8-the-discrete-fourier-series/
- tesconnect. Online: http://www.tes.co.uk/teaching-resource/Fourier-Series-Coefficients-6389354/ and http://www.tes.co.uk/teaching-resource/Fourier-Series-Representation-6388696/
- College Math Teaching. Online: http://collegemathteaching.wordpress.com/category/fourier-series/
- Classroom Tips and Techniques: Teaching Fourier Series with Maple. Online: http://www.maplesoft.com/applications/view.aspx?SID=4848
- Vidyarthiplus. Online: http://www.vidyarthiplus.in/2011/07/mathematics-3-fourier-series-problems_14.html#.U67EM_ldVAc
- WolframMathWorld. Online: http://mathworld.wolfram.com/FourierSeries.html
- WIZIQ education. Online: http://www.wiziq.com/tutorial/167725-Fourier-Series

Week 12	Content Specification	Teacher's Activities	Learner's Activities
	Summary and Conclusion	Summarise the Course and topics that has been taught	Students allowed to interject with suggestions and discussions as well as questioning
		Leads discussion on gray areas of the rules of engagement	
		Listen to students comments (if any), responds appropriately and administers ATMI and RMARS as post-tests	

Appendix IV (c): Trigonometry and Analytical Geometry CMSTA Lecture Manual for ND Polytechnic Engineering Students

1.0 Trigonometry – Sums and Differences of Sines and Cosines

1.1 Derivation of Sums and Differences of Sines and Cosines Formulae

From the Compound Angle formulae, recall that

$Cos(X+Y) = CosXCosY - SinXSinY$,, /'

$Cos(X-Y) = CosXCosY + SinXSinY$, ,, 0'

$Sin(X+Y) = SinXCosY + CosXSinY$,, ,, 1'

$Sin(X-Y) = SinXCosY - CosXSinY$,, ,, 2'

Adding (1) and (2) gives

$Cos(X+Y) + Cos(X-Y) = 2CosXCosY$,,, ,, 3'

Subtracting (2) from (1) gives

$Cos(X+Y) - Cos(X-Y) = -2SinXSinY$,, ,, 4'

Adding (3) and (4) gives

$Sin(X+Y) + Sin(X-Y) = 2SinXCosY$, ,, 5'

Subtracting (4) from (3) gives

$Sin(X+Y) - Sin(X-Y) = 2CosXsinY$,, 6'

Let $X+Y=A$ and $X-Y=B$

Solving for X and Y in terms of A and B will give $X = \frac{A+B}{2}$ and $Y = \frac{A-B}{2}$

Substituting $X = \frac{A+B}{2}$ and $Y = \frac{A-B}{2}$ in (5), (6), (7) and (8) will give

$CosA + CosB = 2Cos\frac{A+B}{2}Cos\frac{A-B}{2}$,, 7'

$CosA - CosB = -2Sin\frac{A+B}{2}Sin\frac{A-B}{2}$,,,,, .. /. '

$SinA + SinB = 2Sin\frac{A+B}{2}Sin\frac{A-B}{2}$, .. //'

$SinA - SinB = 2Cos\frac{A+B}{2}Sin\frac{A-B}{2}$ ... /0'

By formulas (9) (12), a sum or a difference of the sines or the cosines of two angles is transformed into a product. Hence these formulas, often called **product formulas**, are useful in adapting other formulas to the use of logarithms. By the converses of formulas (5) (8), a product involving sines or cosines or both is transformed into a sum or a difference of sines or cosines.

1.2 Application of Sums and Differences of Sines and Cosines Formulae

Example 1.2.1: Express each of the following as a sum or difference.

(a) $Sin40^0Cos30^0$ (b) $Cos110^0Sin55^0$ (c) $Cos50^0Cos35^0$ (d) $Sin55^0Sin40^0$

Solutions

(a) $Sin40^0Cos30^0 = \frac{1}{2}\{Sin(40^0 + 30^0) + Sin(40^0 - 30^0)\} = \frac{1}{2}\{Sin70^0 + Sin10^0\}$

(b) $Cos110^0Sin55^0 = \frac{1}{2}\{Sin(110^0 + 35^0) - Sin(110^0 - 35^0)\} = \frac{1}{2}\{Sin165^0 - Sin55\}$

(c) $Cos50^0Cos35^0 = \frac{1}{2}\{Cos(50^0 + 35^0) + Cos(50^0 - 35^0)\} = \frac{1}{2}\{Cos85^0 + Cos15^0\}$

(d) $Sin55^0Sin40^0 = \frac{1}{2}\{Cos(55^0 + 40^0) - Cos(55^0 - 40^0)\} = \frac{1}{2}\{Cos90^0 - Cos15^0\}$

Example 1.2.2: Express each of the following as a product.

(a) $Sin50^0 + Sin40^0$ (b) $Sin70^0 - Sin20^0$

(c) $Cos55^0 + Cos25^0$ (d) $Cos35^0 + Cos75^0$

Solutions

(a) $Sin50^0 + Sin40^0 = 2Sin\frac{50^0+40^0}{2}Cos\frac{50^0-40^0}{2} = 2Sin45^0Cos5^0$

(b) $Sin70^0 - Sin20^0 = 2Cos\frac{70^0+20^0}{2}Sin\frac{70^0-20^0}{2} = 2Cos45^0Sin25^0$

(c) $Cos55^0 + Cos25^0 = 2Cos\frac{55^0+25^0}{2}Cos\frac{55^0-25^0}{2} = 2Cos40^0Cos15^0$

(d) $Cos35^0 + Cos75^0 = -2Sin\frac{35^0+75^0}{2}Sin\frac{35^0-75^0}{2} = -2Sin55^0Sin(-30^0) = 2Sin55^0Sin30^0$

Example 1.2.3: Prove that $\frac{Sin4A+Sin2A}{Cos4A+Cos2A} = tan3A$

Solution

$\frac{Sin4A+Sin2A}{Cos4A+Cos2A} = \frac{2Sin\frac{1}{2}(4A+2A)Cos\frac{1}{2}(4A-2A)}{2Cos\frac{1}{2}(4A+2A)Cos\frac{1}{2}(4A-2A)} = \frac{2Sin3ACosA}{2Cos3ACosA} = \frac{Sin3A}{Cos3A} = tan3A$

Example 1.2.4: . Prove that $\frac{SinA-SinB}{SinA+SinB} = \frac{tan\frac{1}{2}(A-B)}{tan\frac{1}{2}(A+B)}$

Solution

$\frac{\sin A-\sin B}{\sin A+\sin B} = \frac{2Cos\frac{1}{2}(A+B)Sin\frac{1}{2}(A-B)}{2Sin\frac{1}{2}(A+B)Cos\frac{1}{2}(A-B)} = Cot\frac{1}{2}(A+B)tan\frac{1}{2}(A-B) = \frac{1}{tan\frac{1}{2}(A+B)}.tan\frac{1}{2}(A-B)$

$= \frac{tan\frac{1}{2}(A-B)}{tan\frac{1}{2}(A+B)}$

Example 1.2.5: Prove that $Cos^3xSin^3x = \frac{1}{16}(2Cosx - Cos3x - Cos5x)$

Solution

$Cos^3xSin^2x = Cosx(CosxSinx)^2$, , /'

Comparing $SinxCosx$ and $Sin\frac{A+B}{2}Cos\frac{A+B}{2} \rightarrow x = \frac{A+B}{2}$ and $x = \frac{A-B}{2} \rightarrow A = 2x$ and $B = 0$

Substituting $A = 2x$ and $B = 0$ in $2Sin\frac{A+B}{2}Cos\frac{A+B}{2} = SinA + SinB$ leads to

$SinxCosx = \frac{1}{2}Sin2x$, 0'

Substituting (2) in (1) leads to

$Cos^3xSin^2x = Cosx(CosxSinx)^2 = cosx(\frac{1}{2}Sin2x)^2 = \frac{1}{4}Sin2x(CosxSin2x)$

$= \frac{1}{4}Sin2x(Sin3x + Sinx)$

$= \frac{1}{4}\left[\frac{1}{2}(Sin2xSin3x + Sin2xSinx)\right]$

$= \frac{1}{8}\left[-\frac{1}{2}(Cos5x - Cosx) - \frac{1}{2}(Cos3x - Cosx)\right]$

$= \frac{1}{16}(2Cosx - Cos3x - Cos5x)$

Example 1.2.6: Prove that $1 + Cos2x + Cos4x + Cos6x = 4CosxCos2xCos3x$

Solution

$1 + Cos2x + Cos4x + Cos6x = 1 + (Cos2x + Cos4x) + Cos6x$

$= 1 + 2Cos3xCosx + Cos6x$

$= (1 + Cos6x) + 2Cos3xCosx$

$= 2Cos^2 3x + 2Cos3xCosx$

$= 2Cos3x(Cos3x + Cosx)$

$= 2Cos3x(2Cos2xCosx)$

$= 4CosxCos2xCos3x$

Example 1.2.7: Transform $4Cosx + 3Sinx$ in the form $KCos(x - \alpha)$

Solution

Setting $4Cosx + 3Sinx = KCos(x - \alpha) = KCosxCos\alpha + KSinxSin\alpha$

$\rightarrow 4Cosx = KCosxCos\alpha$ and $3Sinx = KSinxSin\alpha$

$\rightarrow 4 = KCos\alpha$ and $3 = KSin\alpha$

$\rightarrow\ 4^2 + 3^2 = K^2(Cos^2\alpha + Sin^2\alpha)$

$\rightarrow\ 25 = K^2$ Since $Cos^2\alpha + Sin^2\alpha = 1 \quad \rightarrow\ K = 5$ and -5

Using $K = 5\ \rightarrow Cos\alpha = \frac{4}{5}$ and $Sin\alpha = \frac{3}{5} \quad \rightarrow\ \alpha = 0.6435$rad

$\therefore\ 4Cosx + 3Sinx = 5Cos(x - 0.6435)$

Using $K = -5\ \rightarrow Cos\alpha = -\frac{4}{5}$ and $Sin\alpha = -\frac{3}{5} \quad \rightarrow\ \alpha = 3.7851$rad

$\therefore\ 4Cosx + 3Sinx = -5Cos(x - 3.7851)$

Example 1.2.8: Find the maximum and minimum values of $4Cosx + 3Sinx$ on the interval $0 \leq x \leq 2\pi$

Solution

Transforming $4Cosx + 3Sinx$ into the form K$Cos(x - \alpha)$ as done above gives

$4Cosx + 3Sinx = 5Cos(x - 0.6435)$ or $4Cosx + 3Sinx = -5Cos(x - 3.7851)$

On the interval $0 \leq x \leq 2\pi$, $Cos\theta = 1$, which is its maximum value when $\theta = 0$ and $Cos\theta = -1$, which is its minimum value when $\theta = \pi$.

Therefore, the maximum value of
$4Cosx + 3Sinx = 5Cos(x - 0.6435)$ or $5Cos(x - 3.7851)$
occurs when $x - 0.6435 = 0$ or $x - 3.7851 = 0$ i.e. when $x = 0.6435$ or 3.7851

When $x = 0.6435,\ 4Cosx + 3Sinx = 5$

Also when $x = 3.7851$, $4Cosx + 3Sinx = 5$

Similarly, the minimum value of
$4Cosx + 3Sinx = 5Cos(x - 0.6435)$ or $5Cos(x - 3.7851)$

Occurs when $x - 0.6435 = \pi$ or $x - 3.7851 = \pi$ i.e. when $x = 3.7851$ or 6.9267

When $x = 3.7851,\ 4Cosx + 3Sinx = -5$

Also when $x = 6.9267$, $4Cosx + 3Sinx = -5$

Example 1.2.9: Transform (using radians)

a) $4Cosx + 3Sinx$ in the form $KSin(x - \alpha)$

b) $4Cosx + 3Sinx$ in the form $KSin(x + \alpha)$

a) $Sinx - Cosx$ in the form $KSin(x - \alpha)$

b) $5Cos3t + 12Sin3t$ in the form $KSin(3t - \alpha)$

Solutions

a) Setting $4Cosx + 3Sinx = KSin(x + \alpha) = KSinxCos\alpha + KCosxSin\alpha$

$\rightarrow$ $4Cosx = KCosxSin\alpha$ and $3Sinx = KSinxCos\alpha$

$\rightarrow$ $4 = KSin\alpha$ and $3 = KCos\alpha$

$\rightarrow$ $4^2 + 3^2 = K^2(Sin^2\alpha + Cos^2\alpha)$

$\rightarrow$ $25 = K^2$ Since $Sin^2\alpha + Cos^2\alpha = 1$

$\rightarrow K = 5$ and -5

Using Using $K = 5$ $\rightarrow Sin\alpha = \frac{4}{5}$ and $Cos\alpha = \frac{3}{5}$ $\rightarrow$ $\alpha = 0.9273$rad

$\therefore$ $4Cosx + 3Sinx = 5Sin(x + 0.9273)$

Using $K = -5$ $\rightarrow Sin\alpha = -\frac{4}{5}$ and $Cos\alpha = -\frac{3}{5}$ $\rightarrow$ $\alpha = 4.0689$rad

$\therefore$ $4Cosx + 3Sinx = -5Cos(x + 4.0689)$

b) Setting $4Cosx + 3Sinx = KSin(x - \alpha) = KSinxCos\alpha - KCosxSin\alpha$

$\rightarrow$ $4Cosx = -KCosxSin\alpha$ and $3Sinx = KSinxCos\alpha$

$\rightarrow$ $4 = -KSin\alpha$ and $3 = KCos\alpha$

$\rightarrow$ $4^2 + 3^2 = K^2(Sin^2\alpha + Cos^2\alpha)$

$\rightarrow$ $25 = K^2$ Since $Sin^2\alpha + Cos^2\alpha = 1$

$\rightarrow K = 5$ and -5

Using Using $K = 5 \rightarrow Sin\alpha = -\frac{4}{5}$ and $Cos\alpha = \frac{3}{5} \rightarrow \alpha = 5.3559$rad

$\therefore\ 4Cosx + 3Sinx = 5Sin(x - 5.3559)$

Using $K = -5 \rightarrow Sin\alpha = \frac{4}{5}$ and $Cos\alpha = -\frac{3}{5} \rightarrow \alpha = 2.2142$rad

$\therefore\ 4Cosx + 3Sinx = -5Cos(x - 2.2142)$

c) Setting $Sinx - Cosx = KSin(x - \alpha) = KSinxCos\alpha - KCosxSin\alpha$

$\rightarrow\ Sinx = KSinxCos\alpha$ and $Cosx = KCosxSin\alpha$

$\rightarrow\ 1^2 + 1^2 = K^2(Sin^2\alpha + Cos^2\alpha)$

$\rightarrow\ 2 = K^2$ Since $Sin^2\alpha + Cos^2\alpha = 1$

$\rightarrow K = \pm\sqrt{2}$

Using $K = \sqrt{2} \rightarrow Cos\alpha = \frac{1}{\sqrt{2}}$ and $sin\alpha = \frac{1}{\sqrt{2}} \rightarrow \alpha = 0.7854$rad

$\therefore\ Sinx - Cosx = \sqrt{2}Sin(x - 0.7854)$

Using $K = -\sqrt{2} \rightarrow Cos\alpha = -\frac{1}{\sqrt{2}}$ and $sin\alpha = -\frac{1}{\sqrt{2}} \rightarrow \alpha = 3.9270$rad

$\therefore\ Sinx - Cosx = \sqrt{2}Sin(x - 3.9270)$

d) Setting $5Cos3t + 12Sin3t = KSin(3t - \alpha) = KSin3tCos\alpha - KCos3tSin\alpha$

$\rightarrow\ 5Cos3t = KSin3tCos\alpha$ and $12Sin3t = KCos3tSin\alpha$

$\rightarrow\ 5 = -KSin\alpha$ and $12 = KCos\alpha$

$\rightarrow\ 5^2 + 12^2 = K^2(Sin^2\alpha + Cos^2\alpha)$

$\rightarrow\ 169 = K^2$ Since $Sin^2\alpha + Cos^2\alpha = 1$

$\rightarrow K = \pm 13$

Using $K = 13 \rightarrow Sin\alpha = -\frac{5}{13}$ and $Cos\alpha = \frac{12}{13} \rightarrow \alpha = 5.8884$rad

$\therefore\ 5Cos3t + 12Sin3t = 13Sin(3t - 5.8884)$

Using $K = -13 \rightarrow Sin\alpha = \frac{5}{13}$ and $Cos\alpha = -\frac{12}{13} \rightarrow \alpha = 2.7468$rad

$\therefore\ 5Cos3t + 12Sin3t = -13Sin(3t - 2.7468)$

Example 1.2.10: By expressing each of the following in the form required, find its maximum and minimum value and the value of x or t between 0 and 2π at which each occurs.

a) $4Cosx + 3Sinx$ in the form $KSin(x - \alpha)$

b) $4Cosx + 3Sinx$ in the form $KSin(x + \alpha)$

a) $Sinx - Cosx$ in the form $KSin(x - \alpha)$

b) $5Cos3t + 12Sin3t$ in the form $KSin(3t - \alpha)$

Solution

a) From 9 (a) above, $\therefore\ 4Cosx + 3Sinx = 5Sin(x + 0.9273)$

Between 0 and 2π,

$Sin\theta = 1$, which is its maximum value when $\theta = \frac{\pi}{2}$ = 1.5708

and

$Sin\theta = -1$, which is its minimum value when $\theta = \frac{3\pi}{2} = 4.7124$.

Therefore, the maximum value of

$4Cosx + 3Sinx = 5Sin(x + 0.9273)$

occurs when $x + 0.9273 = 1.5708$ i.e. when $x = 0.6435$.

When $x = 0.6435$, $4Cosx + 3Sinx = 5$

Similarly, the maximum value of

$4Cosx + 3Sinx = 5Sin(x + 0.9273)$

occurs when $x + 0.9273 = 4.7124$ i.e. when $x = 3.7851$

When $x = 3.7851$, $4Cosx + 3Sinx = -5$

b) From 9 (b) above, $4Cosx + 3Sinx = 5Sin(x - 5.3539)$

Between 0 and 2π,

$Sin\theta = 1$, which is its maximum value when $\theta = \frac{\pi}{2}$ = 1.5708

and

$Sin\theta = -1$, which is its minimum value when $\theta = \frac{3\pi}{2} = 4.7124$.

Therefore, the maximum value of

$4Cosx + 3Sinx = 5Sin(x - 5.3539)$

occurs when $x - 5.3539 = 1.5708$ i.e. when $x = 6.9267$.

When $x = 6.9267$, $4Cosx + 3Sinx = 5$

Similarly, the maximum value of

$4Cosx + 3Sinx = 5Sin(x - 5.3539)$

occurs when $x - 5.3539 = 4.7124$ i.e. when $x = 10.0663$

When $x = 10.0663$, $4Cosx + 3Sinx = -5$

c) From 9 (c) above, $Sinx - Cosx = \sqrt{2}Sin(x - 0.7854)$

Between 0 and 2π,

$Sin\theta = 1$, which is its maximum value when $\theta = \frac{\pi}{2}$ = 1.5708

and

$Sin\theta = -1$, which is its minimum value when $\theta = \frac{3\pi}{2} = 4.7124$.

Therefore, the maximum value of

$$Sinx - Cosx = \sqrt{2}Sin(x - 0.7854)$$

occurs when $x - 0.7854 = 1.5708$ i.e. when $x = 2.3562$.

When $x = 2.3562$, $Sinx - Cosx = \sqrt{2}$

Similarly, the maximum value of $Sinx - Cosx = \sqrt{2}Sin(x - 0.7854)$ occurs when $x - 0.7854 = 4.7124$ i.e. when $x = 5.4978$.

When $x = 5.4978$, $Sinx - Cosx = -\sqrt{2}$

d) From 9 (d) above, $5Cos3t + 12Sin3t = 13Sin(3t - 5.8884)$

Between 0 and 2π,

$Sin\theta = 1$, which is its maximum value when $\theta = \frac{\pi}{2}$ = 1.5708

and

$Sin\theta = -1$, which is its minimum value when $\theta = \frac{3\pi}{2} = 4.7124$.

Therefore, the maximum value of $5Cos3t + 12Sin3t = 13Sin(3t - 5.8884)$ occurs when $3t - 5.8884 = 1.5708$ i.e. when $t = 2.4864$.

When $t = 2.4864$, $5Cos3t + 12Sin3t = 13$

Similarly, the maximum value of $5Cos3t + 12Sin3t = 13Sin(3t - 5.8884)$ occurs when $3t - 5.8884 = 4.7124$ i.e. when $t = 3.5336$.

When $t = 3.5336$, $5Cos3t + 12Sin3t = -13$

2.0 Half Angle Formulae

2.1 Derivation of the Half Angle Formulae

Recall that $Cos(A + B) = CosACosB - SinASinB$

Putting $A = B$ gives $Cos(A + A) = CosACosA - SinASinA$

$\rightarrow Cos2A = Cos^2A - Sin^2A$ /'

But $Cos^2A + Sin^2A = 1 \ \therefore \ \ Cos^2A = 1 - Sin^2A$ and $Sin^2A = 1 - Cos^2A$

$Cos2A = Cos^2A - (1 - Cos^2A) = 2Cos^2A - 1$,, ,, 0'

and

$Cos2A = (1 - Sin^2A) - Sin^2A = 1 - 2Sin^2A$,, 1'

From (2) and (3), it follows that

$Cos^2A = \frac{1}{2}(1 + Cos2A)$ and $Sin^2A = \frac{1}{2}(1 - Cos2A)$

Putting $2A = \theta \rightarrow A = \frac{1}{2}\theta$

$\therefore \ Cos^2A = \frac{1}{2}(1 + Cos2A)$ becomes $Cos^2\frac{1}{2}\theta = \frac{1}{2}(1 + Cos\theta)$

and

$Sin^2A = \frac{1}{2}(1 - Cos2A)$ becomes $Sin^2\frac{1}{2}\theta = \frac{1}{2}(1 + Cos\theta)$

$\rightarrow Cos\frac{1}{2}\theta = \pm\sqrt{\frac{1}{2}(1 + Cos\theta)}$, , 2'

and

$Sin\frac{1}{2}\theta = \pm\sqrt{\frac{1}{2}(1 - Cos\theta)}$,,, ,,, , (5)

Using (4) and (5), we get

$$tan\frac{1}{2}\theta = \frac{Sin\frac{1}{2}\theta}{Cos\frac{1}{2}\theta} = \pm\sqrt{\frac{1-Cos\theta}{1+Sin\theta}} = \pm\sqrt{\frac{(1-Cos\theta)(1+Cos\theta)}{(1+Cos\theta)(1+Cos\theta)}} = \pm\sqrt{\frac{1-Cos^2\theta}{(1+Cos\theta)^2}} = \pm\sqrt{\frac{Sin^2\theta}{(1+Cos\theta)^2}}$$

$$= \frac{Sin\theta}{1+Cos\theta}$$

Also

$$tan\frac{1}{2}\theta = \frac{Sin\frac{1}{2}\theta}{Cos\frac{1}{2}\theta} = \pm\sqrt{\frac{1-Cos\theta}{1+Sin\theta}} = \pm\sqrt{\frac{(1-Cos\theta)(1-Cos\theta)}{(1+Cos\theta)(1-Cos\theta)}} = \pm\sqrt{\frac{(1-Cos\theta)^2}{1-Cos^2\theta}} = \pm\sqrt{\frac{(1-Cos\theta)^2}{Sin^2\theta}}$$

$$= \frac{1-Cos\theta}{Sin\theta}$$

$\therefore tan\frac{1}{2}\theta = \frac{Sin\theta}{1+Cos\theta} = \frac{1-Cos\theta}{Sin\theta}$, 4'

$Sin\frac{1}{2}\theta = \pm\sqrt{\frac{1}{2}(1-Cos\theta)}$, $Cos\frac{1}{2}\theta = \pm\sqrt{\frac{1}{2}(1+Cos\theta)}$ and $tan\frac{1}{2}\theta = \pm\frac{Sin\theta}{1+Cos\theta} = \frac{1-Cos\theta}{Sin\theta}$ are respectively called the half angle formulae for $Sine$, $Cosine$ and $tangent$.

2.2 Applications of the Half Angle Formulae

Exercise 2.2.1: Using the half-angle formulae, find the exact values of

a) $Sin15°$ b) $Sin292.5°$ c) $Sin\frac{\pi}{8}$

Solutions

a) Given $Sin15°$, Let $\frac{1}{2}\theta = 15°$ so that $\theta = 30°$

Using $Sin\frac{1}{2}\theta = \pm\sqrt{\frac{1}{2}(1-Cos\theta)}$ gives

$Sin15° = \pm\sqrt{\frac{1}{2}(1-Cos30°)} = \pm\sqrt{\frac{1}{2}\left(1-\frac{\sqrt{3}}{2}\right)} = \pm\frac{1}{2}\sqrt{2-\sqrt{3}}$

Since $Sin\theta$ is positive when θ is in the first quadrant $\therefore Sin15° = \frac{1}{2}\sqrt{2-\sqrt{3}}$

b) Given $Sin\ 292.5°$, Let $\frac{1}{2}\theta = 292.5°$ so that $\theta = 583°$

Using $Sin\frac{1}{2}\theta = \pm\sqrt{\frac{1}{2}(1-Cos\theta)}$ gives

$Sin292.5° = \pm\sqrt{\frac{1}{2}(1-Cos583°)} = \pm\sqrt{\frac{1}{2}\left(1-\frac{\sqrt{2}}{2}\right)} = \pm\frac{1}{2}\sqrt{2-\sqrt{2}}$

Since $Sin\theta$ is negative when θ is in fourth quadrant $\therefore Sin292.5° = -\frac{1}{2}\sqrt{2-\sqrt{2}}$

c) $Sin\frac{\pi}{8}$, Let $\frac{1}{2}\theta = \frac{\pi}{8}$ so that $\theta = \frac{\pi}{4}$

Using $Sin\frac{1}{2}\theta = \pm\sqrt{\frac{1}{2}(1-Cos\theta)}$ gives

$Sin\frac{\pi}{8} = \pm\sqrt{\frac{1}{2}\left(1 - Cos\frac{\pi}{4}\right)} = \pm\sqrt{\frac{1}{2}\left(1 - \frac{\sqrt{2}}{2}\right)} = \pm\frac{1}{2}\sqrt{2-\sqrt{2}}$

Since $Sin\theta$ is positive when θ is in the first quadrant $\therefore Sin\frac{\pi}{8} = \frac{1}{2}\sqrt{2-\sqrt{2}}$

Exercise 2.2.2: Using the half-angle formula, find the exact values of

a) $Cos15°$ b) $Cos22\frac{1}{2}°$ c) $Cos67.5°$

d) $Cos105°$ e) $Cos112.5°$ f) $Cos157\frac{1}{2}°$

g) $Cos165°$

Solution

a).Given $Cos15°$, Let $\frac{1}{2}\theta = 15°$ so that $\theta = 30°$

Using $Sin\frac{1}{2}\theta = \pm\sqrt{\frac{1}{2}(1+Cos\theta)}$ gives

$Cos15° = \pm\sqrt{\frac{1}{2}(1+Cos30°)} = \pm\sqrt{\frac{1}{2}\left(1+\frac{\sqrt{3}}{2}\right)} = \pm\frac{1}{2}\sqrt{2+\sqrt{3}}$

Since $Cos\theta$ is positive in the first quadrant $\therefore Cos15° = \frac{1}{2}\sqrt{2+\sqrt{3}}$

b) .Given $Cos22\frac{1}{2}°$, Let $22\frac{1}{2}° = \frac{1}{2}\theta$ so that $\theta = 45°$

Using $Cos\frac{1}{2}\theta = \pm\sqrt{\frac{1}{2}(1+Cos\theta)}$ gives

$Cos22\frac{1}{2}° = \pm\sqrt{\frac{1}{2}(1+Cos45°)} = \pm\sqrt{\frac{1}{2}\left(1+\frac{\sqrt{2}}{2}\right)} = \pm\frac{1}{2}\sqrt{2+\sqrt{2}}$

Since $Cos\theta$ is positive in the first quadrant $\therefore Cos22\frac{1}{2}° = \frac{1}{2}\sqrt{2+\sqrt{2}}$

c).Given $67\frac{1}{2}°$, Let $\frac{1}{2}\theta = 57\frac{1}{2}°$ so that $\theta = 135°$

Using $Cos\frac{1}{2}\theta = \pm\sqrt{\frac{1}{2}(1+Cos\theta)}$ gives

$Cos67\frac{1}{2}° = \pm\sqrt{\frac{1}{2}(1 + Cos135°)} = \pm\sqrt{\frac{1}{2}\left(1 - \frac{\sqrt{2}}{2}\right)} = \pm\frac{1}{2}\sqrt{2 - \sqrt{2}}$

Since $Cos\theta$ is positive in the first quadrant $\therefore Cos67\frac{1}{2}° = \frac{1}{2}\sqrt{2 - \sqrt{2}}$

d). Given $Cos105°$, Let $\frac{1}{2}\theta = 105°$ so that $\theta = 210°$

Using $Cos\theta = \pm\sqrt{\frac{1}{2}(1 + Cos\theta)}$ gives

$Cos105° = \pm\sqrt{\frac{1}{2}(1 + Cos210°)} = \pm\sqrt{\frac{1}{2}\left(1 - \frac{\sqrt{3}}{2}\right)} = \pm\frac{1}{2}\sqrt{2 - \sqrt{3}}$

Since $Cos\theta$ is negative in the second quadrant $\therefore Cos105° = -\frac{1}{2}\sqrt{2 - \sqrt{3}}$

e).Given $Cos112.5°$, Let $\frac{1}{2}\theta = 112.5°$ so that $\theta = 225°$

Using $Cos\frac{1}{2}\theta = \pm\sqrt{\frac{1}{2}(1 + Cos\theta)}$ gives

$Cos112.5° = \pm\sqrt{\frac{1}{2}(1 + Cos225°)} = \pm\sqrt{\frac{1}{2}\left(1 - \frac{\sqrt{2}}{2}\right)} = \pm\frac{1}{2}\sqrt{2 - \sqrt{2}}$

Since $Cos\theta$ is negative in the second quadrant $\therefore Cos112.5° = -\frac{1}{2}\sqrt{2 - \sqrt{2}}$

f). Given $Cos157\frac{1}{2}°$, Let $\frac{1}{2}\theta = 157\frac{1}{2}°$ so that $\theta = 315°$

Using $Cos\frac{1}{2}\theta = \pm\sqrt{\frac{1}{2}(1 + Cos\theta)}$ gives

$Cos157\frac{1}{2}° = \pm\sqrt{\frac{1}{2}(1 + Cos315°)} = \pm\sqrt{\frac{1}{2}\left(1 + \frac{\sqrt{2}}{2}\right)} = \pm\frac{1}{2}\sqrt{2 + \sqrt{2}}$

Since $Cos\theta$ is negative in the second quadrant $\therefore Cos112.5° = -\frac{1}{2}\sqrt{2 - \sqrt{2}}$

g). Given $165°$, Let $\frac{1}{2}\theta = 165°$ so that $\theta = 330°$

Using $Cos\frac{1}{2}\theta = \pm\sqrt{\frac{1}{2}(1 + Cos\theta)}$ gives

$$Cos165° = \pm\sqrt{\frac{1}{2}(1 + Cos330°)} = \pm\sqrt{\frac{1}{2}\left(1 + \frac{\sqrt{3}}{2}\right)} = \pm\frac{1}{2}\sqrt{2 + \sqrt{3}}$$

Since $Cos\theta$ is negative in the second quadrant $\therefore Cos165° = -\frac{1}{2}\sqrt{2 + \sqrt{3}}$

Exercise 2.2.3: Use the half-angle formula to find the exact values of

a) $tan15°$ b) $tan22.5°$ c) $tan67\frac{1}{2}°$ d) $tan105°$ e)$tan112.5°$
f) $tan157.5°$ g) $tan165°$

Solution

a). Let $\frac{1}{2}\theta = 15°$ so that $\theta = 30°$

Using $tan\frac{1}{2}\theta = \frac{Sin\theta}{1+Cos\theta}$ gives

$$tan15° = \frac{Sin30}{1+Cos30} = \frac{\frac{1}{2}}{1+\frac{\sqrt{3}}{2}} = \frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}}.\frac{2-\sqrt{3}}{2-\sqrt{3}} = 2 - \sqrt{3}$$

Similarly, Using $tan\frac{1}{2}\theta = \frac{1-Cos\theta}{Sin\theta}$ gives

$$tan15° = \frac{1-Cos30°}{Sin30°} = \frac{1-\sqrt{3}/2}{1/2} = 2 - \sqrt{3}$$

b). Let $\frac{1}{2}\theta = 22.5°$ so that $\theta = 45°$

Using $tan\frac{1}{2}\theta = \frac{Sin\theta}{Sin\theta}$ gives

$$tan22.5° = \frac{1-Cos45°}{Sin45} = \frac{1-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = -1 + \sqrt{2}$$

c). Let $\frac{1}{2}\theta = 67\frac{1}{2}°$ so that $\theta = 135°$

Using $tan\frac{1}{2}\theta = \frac{1-Cos\theta}{Sin\theta}$ gives

$$tan67\frac{1}{2}° = \frac{1-Cos135°}{Sin135} = \frac{1+\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1 + \sqrt{2}$$

d). Let $\frac{1}{2}\theta = 105°$ so that $\theta = 210°$

Using $tan\frac{1}{2}\theta = \frac{1-Cos\theta}{Sin\theta}$ gives

$$tan105° = \frac{1-Cos210°}{Sin210°} = \frac{1+\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -2-\sqrt{3}$$

e). Let $\frac{1}{2}\theta = 112.5°$ so that $\theta = 225°$

Using $tan\frac{1}{2}\theta = \frac{1-Cos\theta}{Sin\theta}$ gives

$$tan112.5° = \frac{1-Cos225°}{Sin225°} = \frac{1+\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = -1-\sqrt{2}$$

f). Let $\frac{1}{2}\theta = 157.5°$ so that $\theta = 315°$

Using $tan\frac{1}{2}\theta = \frac{1-Cos\theta}{Sin\theta}$ gives

$$tan157.5° = \frac{1-Cos315°}{Sin315°} = \frac{1-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = 1-\sqrt{2}$$

g). Let $\frac{1}{2}\theta = 165°$ so that $\theta = 330°$

Using $tan\frac{1}{2}\theta = \frac{1-Cos\theta}{Sin\theta}$ gives

$$tan165° = \frac{1-Cos330°}{Sin330°} = \frac{1-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -2+\sqrt{3}$$

Exercise 2.2.4: Find the values of $Sin\frac{1}{2}\theta$, $Cos\frac{1}{2}\theta$ and $tan\frac{1}{2}\theta$ given that;

a) $Sin\theta = \frac{5}{13}$ and θ is in second quadrant

b) $Cos\theta = \frac{3}{7}$ and θ is in fourth quadrant

Solution

Using $Sin^2\theta + Cos^2\theta = 1$ gives

$Cos^2\theta = 1 - Sin^2\theta = 1 - \frac{25}{169} = \frac{144}{169} \quad \therefore Cos\theta = \pm\frac{12}{13}$

Since θ is in the second quadrant, therefore $Cos\theta$ is negative

Hence $Cos\theta = -\frac{12}{13}$

Using $Sin\frac{1}{2}\theta = \pm\sqrt{\frac{1}{2}(1 - Cos\theta)}$ gives $Sin\frac{1}{2}\theta = \pm\sqrt{\frac{1}{2}\left(1 - \frac{12}{13}\right)} = \pm\sqrt{\frac{25}{26}}$

Since θ is in second quadrant, $\frac{1}{2}\theta$ is in first quadrant, $\therefore Sin\frac{1}{2}\theta = \sqrt{\frac{25}{26}} = \frac{5\sqrt{26}}{26}$

Using $Cos\frac{1}{2}\theta = \pm\sqrt{\frac{1}{2}(1 - Cos\theta)}$ gives $Cos\frac{1}{2}\theta = \pm\sqrt{\frac{1}{2}\left(1 - \frac{12}{13}\right)} = \pm\sqrt{\frac{1}{26}}$

Since θ is in second quadrant, $\frac{1}{2}\theta$ is in first quadrant $\therefore Cos\frac{1}{2}\theta = \sqrt{\frac{1}{26}} = \frac{\sqrt{26}}{26}$

Using $tan\frac{1}{2}\theta = \frac{1 - Cos\theta}{Sin\theta}$ gives $tan\frac{1}{2}\theta = \frac{1 - \frac{12}{13}}{\frac{5}{13}} = 5$

b). Using $Cos^2\theta + Sin^2\theta = 1$ gives

$Sin^2\theta = 1 - Cos^2\theta = 1 - \frac{9}{49} = \frac{40}{49} \quad \therefore Sin\theta = \pm\sqrt{\frac{40}{49}} = \pm\frac{2\sqrt{10}}{7}$

Since θ is in fourth quadrant, $Sin\theta$ is negative, $\therefore Sin\theta = -\frac{2\sqrt{10}}{7}$

Using $Sin\frac{1}{2}\theta = \pm\sqrt{\frac{1}{2}(1 - Cos\theta)}$ gives $Sin\frac{1}{2}\theta = \pm\sqrt{\frac{1}{2}\left(1 - \frac{3}{7}\right)} = \pm\frac{\sqrt{2}}{7} = \pm\frac{\sqrt{14}}{7}$

Since θ is in fourth guardant, $\frac{1}{2}\theta$ is in second quadrant, $\therefore Sin\frac{1}{2}\theta = \frac{\sqrt{14}}{7}$

Using $Cos\frac{1}{2}\theta = \pm\sqrt{\frac{1}{2}(1 + Cos\theta)}$ gives $Cos\frac{1}{2}\theta = \pm\sqrt{\frac{1}{2}\left(1 + \frac{3}{7}\right)} = \pm\frac{\sqrt{5}}{7} = \pm\frac{\sqrt{35}}{7}$

Since θ is in fourth guardant, $\frac{1}{2}\theta$ is in second quadrant, $\therefore Cos\frac{1}{2}\theta = -\frac{\sqrt{35}}{7}$

Using $tan\frac{1}{2}\theta = \frac{1 - Cos\theta}{Sin\theta}$ gives $tan\frac{1}{2}\theta = \frac{1 - \frac{3}{7}}{-\frac{2\sqrt{10}}{7}} = -\frac{2}{\sqrt{10}} = -\frac{\sqrt{10}}{5}$

Exercise 2.2.5: Find a) $Cos\frac{1}{2}A$ b) $Sin\frac{1}{2}A$ c) $tan\frac{1}{2}A$ if

i. $CosA = \frac{3}{5} and\ 0 < A < 90°$

ii. $CosA = \frac{3}{5}\ and\ 270° < A < 360°$

iii. $CosA = \frac{3}{5}\ and\ 180° < A < 270°$

iv. $CosA = \frac{3}{5}\ and\ 90° < A < 180°$

v. $SinA = -\frac{4}{5}\ and\ 630° < A < 720°$

vi. $SinA = \frac{4}{5}\ and\ 450° < A < 540°$

Solution

i). Using $Cos^2A + Sin^2A = 1$ gives

$Sin^2A = 1 - Cos^2A = 1 - \frac{9}{25} = \frac{16}{25} \rightarrow SinA = \pm\frac{4}{5}$

If $\theta < A < 90°$, then $SinA$ is positive $\therefore SinA = \frac{4}{5}$

Using $Cos\frac{1}{2}A = \pm\sqrt{\frac{1}{2}(1 + CosA)}$ gives $Cos\frac{1}{2}A = \pm\sqrt{\frac{1}{2}\left(1 + \frac{3}{5}\right)} = \pm\sqrt{\frac{4}{5}} = \pm\frac{2\sqrt{5}}{5}$

Since $0 \leq A \leq 90°$, $Cos\frac{1}{2}A$ is positive $\therefore Cos\frac{1}{2}A = \frac{2\sqrt{5}}{5}$

Using $Sin\frac{1}{2}A = \pm\sqrt{\frac{1}{2}(1 - CosA)}$ gives $Cos\frac{1}{2}A = \pm\sqrt{\frac{1}{2}\left(1 - \frac{3}{5}\right)} = \pm\frac{\sqrt{5}}{5}$

Since $0 \leq A \leq 90°$, $Sin\frac{1}{2}A$ is positive $\therefore Sin\frac{1}{2}A = \frac{\sqrt{5}}{5}$

Using $tan\frac{1}{2}A = \frac{1-CosA}{SinA}$ gives $tan\frac{1}{2}A = \frac{1-\frac{3}{5}}{\frac{4}{5}} = \frac{1}{2}$

Since $0 \leq A \leq 90°$, $Sin\frac{1}{2}A$ is positive, $\therefore Sin\frac{1}{2}A = \frac{\sqrt{5}}{5}$

ii). Using $Cos^2A + Sin^2A = 1$ gives

$Sin^2A = 1 - Cos^2A = 1 - \frac{9}{25} = \frac{16}{25} \rightarrow SinA = \pm\frac{4}{5}$

Since $270° < A < 360°$, $SinA$ is negative, $\therefore SinA = -\frac{4}{5}$

Using $Cos\frac{1}{2}A = \pm\sqrt{\frac{1}{2}(1 + CosA)}$ gives $Cos\frac{1}{2}A = \pm\sqrt{\frac{1}{2}\left(1 + \frac{3}{5}\right)} = \pm\sqrt{\frac{4}{5}} = \pm\frac{2\sqrt{5}}{5}$

Since $270 \leq A \leq 360°$, then $135° < \frac{1}{2}A < 180°$ $\rightarrow Cos\frac{1}{2}A$ is negative.

$$\therefore Cos\frac{1}{2}A = -\frac{2\sqrt{5}}{5}$$

Using $Sin\frac{1}{2}A = \pm\sqrt{\frac{1}{2}(1 - CosA)}$ gives $Sin\frac{1}{2}A = \pm\sqrt{\frac{1}{2}\left(1 - \frac{3}{5}\right)} = \pm\frac{\sqrt{5}}{5}$

Since $270° < A < 360°$, then $135° < \frac{1}{2}A < 1800$ $\rightarrow Sin\frac{1}{2}A$ is positive

$$\therefore Sin\frac{1}{2}A = \frac{\sqrt{5}}{5}$$

Using $tan\frac{1}{2}A = \frac{1-CosA}{SinA}$ gives $tan\frac{1}{2}A = \frac{1-\frac{3}{5}}{-\frac{4}{5}} = -\frac{1}{2}$

iii). Using $Cos^2A + Sin^2A = 1$ gives

$Sin^2A = 1 - Cos^2A = 1 - \frac{9}{25} = \frac{16}{25}$ $\rightarrow$ $SinA = \pm\frac{4}{5}$

Since $180° < A < 270°$,then $90° < \frac{1}{2}A < 135°$ $\rightarrow Cos\frac{1}{2}A$ is negative

$$\therefore CosA = -\frac{2\sqrt{5}}{5}$$

Using $Sin\frac{1}{2}A = \pm\sqrt{\frac{1}{2}(1 - CosA)}$ gives $Sin\frac{1}{2}A = \pm\sqrt{\frac{1}{2}\left(1 - \frac{3}{5}\right)} = \pm\frac{\sqrt{5}}{5}$

Since $180 \leq A \leq 270°$, ten $90° < \frac{1}{2}A < 135°$ $\rightarrow Sin\frac{1}{2}A$ is positive.

$$\therefore Sin\frac{1}{2}A = \frac{\sqrt{5}}{5}$$

Using $tan\frac{1}{2}A = \frac{1-CosA}{SinA}$ gives $tan\frac{1}{2}A = \frac{1-\frac{3}{5}}{-\frac{4}{5}} = -\frac{1}{2}$

iv). Using $Cos^2A + Sin^2A = 1$ gives

$Sin^2A = 1 - Cos^2A = 1 - \frac{9}{25} = \frac{16}{25} \rightarrow SinA = \pm\frac{4}{5}$

Since $90° < A < 360°$, then $SinA$ is positive

$\therefore SinA = \frac{4}{5}$

Using $Cos\frac{1}{2}A = \pm\sqrt{\frac{1}{2}(1 + CosA)}$ gives $Cos\frac{1}{2}A = \pm\sqrt{\frac{1}{2}\left(1 + \frac{3}{5}\right)} = \pm\sqrt{\frac{4}{5}} = \pm\frac{2\sqrt{5}}{5}$

Since $90° < A < 180°$, then $45° < \frac{1}{2}A < 90°$ $\rightarrow Cos\frac{1}{2}A$ is positive.

$\therefore Cos\frac{1}{2}A = \frac{2\sqrt{5}}{5}$

Using $Sin\frac{1}{2}A = \pm\sqrt{\frac{1}{2}(1 - CosA)}$ gives $Sin\frac{1}{2}A = \pm\sqrt{\frac{1}{2}\left(1 - \frac{3}{5}\right)} = \pm\frac{\sqrt{5}}{5}$

Since $90° < A < 180°$, then $45° < \frac{1}{2}A < 90°$ $\rightarrow Sin\frac{1}{2}A$ is positive

$\therefore Sin\frac{1}{2}A = \frac{\sqrt{5}}{5}$

Using $tan\frac{1}{2}A = \frac{1-CosA}{SinA}$ gives $tan\frac{1}{2}A = \frac{1-\frac{3}{5}}{\frac{4}{5}} = \frac{1}{2}$

v). Using $Cos^2A + Sin^2A = 1$ gives

$Cos^{-2}A = 1 - Sin^2A = 1 - \frac{16}{25} = \frac{9}{25} \rightarrow CosA = \pm\frac{3}{5}$

Since $630° < A < 720°$, then $CosA$ is positive, $\therefore CosA = \frac{3}{5}$

Using $Cos\frac{1}{2}A = \pm\sqrt{\frac{1}{2}(1 + CosA)}$ gives $Cos\frac{1}{2}A = \pm\sqrt{\frac{1}{2}\left(1 + \frac{3}{5}\right)} = \pm\frac{2\sqrt{5}}{5}$

Since $630° < A < 720°$, then $315° < \frac{1}{2}A < 360°$ $\rightarrow Cos\frac{1}{2}A$ is positive.

$\therefore Cos\frac{1}{2}A = \frac{2\sqrt{5}}{5}$

Using $Sin\frac{1}{2}A = \pm\sqrt{\frac{1}{2}(1 - CosA)}$ gives $Sin\frac{1}{2}A = \pm\sqrt{\frac{1}{2}\left(1 - \frac{3}{5}\right)} = \pm\frac{\sqrt{5}}{5}$

Since $630° < A < 720°$, then $315° < \frac{1}{2}A < 360°$ $\rightarrow Sin\frac{1}{2}A$ is negative

$$\therefore Sin\frac{1}{2}A = -\frac{\sqrt{5}}{5}$$

Using $tan\frac{1}{2}A = \frac{1-CosA}{SinA}$ gives $tan\frac{1}{2}A = \frac{1-\frac{3}{5}}{-\frac{4}{5}} = -\frac{1}{2}$

vi). Using $Cos^2A + Sin^2A = 1$ gives

$Cos^2A = 1 - Sin^{-2}A = 1 - \frac{16}{25} = \frac{9}{25} \rightarrow CosA = \pm\frac{3}{5}$

Since $450° < A < 540°$, then $CosA$ is negative $\therefore CosA = -\frac{3}{5}$

Using $Cos\frac{1}{2}A = \pm\sqrt{\frac{1}{2}(1 + CosA)}$ gives $Cos\frac{1}{2}A = \pm\sqrt{\frac{1}{2}\left(1 - \frac{3}{5}\right)} = \pm\frac{\sqrt{5}}{5}$

Since $450° < A < 540°$, then $225° < \frac{1}{2}A < 270°$ $\rightarrow Cos\frac{1}{2}A$ is negative.

$$\therefore Cos\frac{1}{2}A = -\frac{\sqrt{5}}{5}$$

Using $Sin\frac{1}{2}A = \pm\sqrt{\frac{1}{2}(1 - CosA)}$ gives $Sin\frac{1}{2}A = \pm\sqrt{\frac{1}{2}\left(1 + \frac{3}{5}\right)} = \pm\frac{2\sqrt{5}}{5}$

Since $450° < A < 540°$, then $225° < \frac{1}{2}A < 270°$ $\rightarrow Sin\frac{1}{2}A$ is negative

$$\therefore Sin\frac{1}{2}A = -\frac{2\sqrt{5}}{5}$$

Using $tan\frac{1}{2}A = \frac{1-CosA}{SinA}$ gives $tan\frac{1}{2}A = \frac{1+\frac{3}{5}}{\frac{4}{5}} = 2$

2.3 Derivation Half-Angle Formulae (in terms of the sides of a triangle)

If the three sides of a triangle are given, the following formulae are suitable for work with logarithms.

In any triangle ABC

$$Sin\frac{1}{2}A = \sqrt{\left\{\frac{(s-b)(-c)}{bc}\right\}},\ Cos\frac{1}{2}A = \sqrt{\left\{\frac{s(s-a)}{bc}\right\}},\ tan\frac{1}{2}A = \sqrt{\left\{\frac{(s-b)(s-c)}{s(s-a)}\right\}}$$

Where $s = \frac{a+b+c}{2}$

Proof

Recall that

$Sin\frac{1}{2}A = \pm\sqrt{\frac{1}{2}(1-CosA)}$, ,, /'

and

$Cos\frac{1}{2}A = \pm\sqrt{\frac{1}{2}(1+CosA)}$,, 0'

By the Cosine formula, $CosA = \frac{(b^2+c^2-a^2)}{2bc}$. Therefore

$$\frac{1}{2}(1-CosA) = \frac{1}{2}\left(1-\frac{b^2+c^2-a^2}{2bc}\right) = \frac{1}{2}\left(\frac{2bc-(b^2+c^2-a^2)}{2bc}\right)$$

$$= \frac{a^2-(b-c)^2}{4bc} = \frac{(a-b+c)(a+b-c)}{4bc}$$,, 1'

and

$$\frac{1}{2}(1+CosA) = \frac{1}{2}\left(1+\frac{b^2+c^2-a^2}{2bc}\right) = \frac{1}{2}\left(\frac{2bc+b^2+c^2-a}{2bc}\right)$$

$$= \frac{1}{2}\left(\frac{(b+c)^2-a^2}{2bc}\right) = \frac{(b+c+a)(b+c-a)}{4bc}$$, 2'

NB: Based on use of difference of two squares i.e. $X^2 - Y^2 = (X+Y)(X-Y)$ we had

$a^2-(b-c)^2 = (a-b+c)(a+b-c)$ in (3)

and

$(b+c)^2 - a^2 = (b+c+a)(b+c-a)$ in (4)

Since $s = \frac{1}{2}(a+b+c)$, it follows that

$s-a = \frac{b+c-a}{2}$, $s-b = \frac{a-b+c}{2}$ and $s-c = \frac{a+b-c}{2}$

Substituting $s-a = \frac{b+c-a}{2}$, $s-b = \frac{a-b+c}{2}$ and $s-c = \frac{a+b-c}{2}$ in (3) and (4) yields

$\frac{1}{2}(1-CosA) = \frac{(s-b)(s-c)}{bc}$ and $\frac{1}{2}(1+CosA) = \frac{s(s-a)}{bc}$

Therefore, $Sin\frac{1}{2}A = \pm\sqrt{\frac{1}{2}(1-CosA)} = \pm\sqrt{\frac{(s-b)(s-c)}{bc}}$

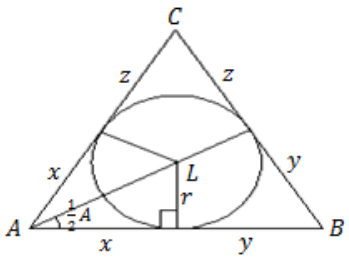
and

$Cos\frac{1}{2}A = \pm\sqrt{\frac{1}{2}(1+CosA)} = \pm\sqrt{\frac{s(s-a)}{bc}}$

The formula for $tan\frac{1}{2}A$ follows at once since

$tan\frac{1}{2}A = \frac{Sin\frac{1}{2}A}{Cos\frac{1}{2}A} = \sqrt{\left\{\frac{(s-b)(s-c)}{bc}\right\}}.\sqrt{\frac{bc}{s(s-a)}} = \sqrt{\left\{\frac{(s-b)(s-c)}{s(s-a)}\right\}}$

At this point it is appropriate to obtain another formula for $tan\frac{1}{2}A$



L is the centre of the inscribed circle of triangle ABC and the lengths of the tangents from A, B, C are x, y, z respectively. The angle $LAB = \frac{1}{2}A$

$$\therefore tan\frac{1}{2}A = \frac{r}{x}$$

Now the circumference of triangle ABC is $2x + 2y + 2z = a + b + c$

$$\therefore x + y + z = \frac{1}{2}(a+b+c)$$

but we write $s = \frac{1}{2}(a+b+c)$ and from the figure $y + z = a$

$$\therefore x = s - a$$

$$Tan\frac{1}{2}A = \frac{r}{s-a}$$

We may easily obtain an expression for r, the radius of the inscribed circle, by equating the two formulae for $tan\frac{1}{2}A;\ thus$

$$\frac{r}{s-a} = \sqrt{\left\{\frac{(s-b)(s-c)}{s(s-a)}\right\}}$$

$$\therefore \quad r = \sqrt{\left\{\frac{(s-a)(s-b)(s-c)}{s}\right\}}$$

3.0 SOLUTION OF TRIANGLES

3.1 Introduction

An oblique triangle is one which does not contain a right angle. Such a triangle contains either three acute angles or two acute angles and an obtuse angle. The convention of denoting the angles by A, B and C and the lengths of the corresponding sides by a, b and c will be used here. (see fig. 1)

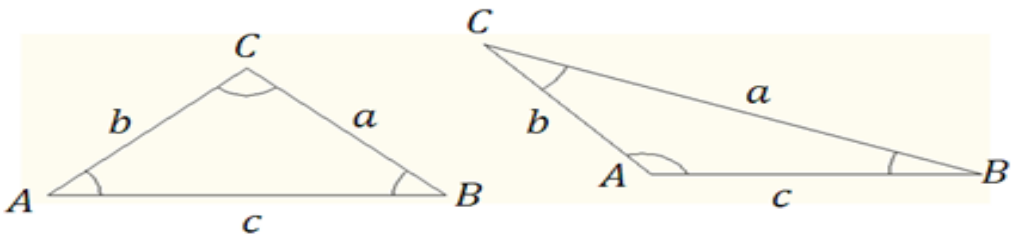


Figure 1

There are to relationships which the sides of any triangle bear to the *Sines* and *Cosines* of its angles. These relationships, which are fundamental in the study and solution of triangles, are called the laws of *Sines* and the laws of *Cosines* respectively.

3.2 Laws of *Sines*

The sides of a triangle are proportional to the *Sines* of their opposite angles. In other words in any triangle ABC, the ratio of a side and the *Sine* of the opposite angle is constant mathematically, this law can be expressed thus;

$$\frac{a}{Sin\ A} = \frac{b}{Sin\ B} = \frac{c}{Sin\ C} \qquad \text{or} \qquad \frac{Sin\ A}{a} = \frac{Sin\ B}{b} = \frac{Sin\ C}{c}$$

Proof:

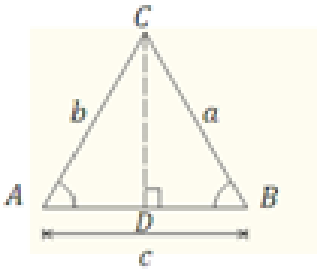


Figure 2

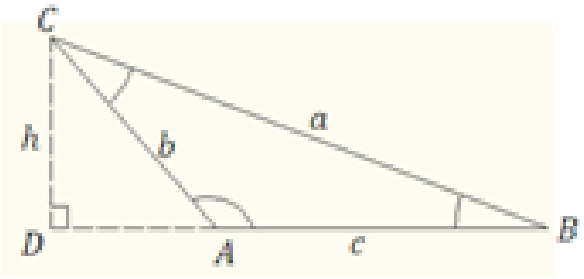


Figure 3

Let ABC be any oblique triangle. In fig.2, angles A and B are acute while in fig.3, angle A is obtuse. Draw CD perpendicular to AB or AB extended and denote its length by h.

In the right triangle BCD in either figure,

$Sin\ B = \frac{h}{a} \rightarrow h = aSinB$, ,,, , /'

In the right triangle ACD (fig.2), $h = bSinA$

In the right triangle ACD (fig.3), $h = bSin\ (180 - A) = bSinA$, , 0'

Equating (1) and (2) yields $aSinB = bSinA \rightarrow \frac{a}{SinA} = \frac{b}{SinB}$

In a similar manner (i.e. by drawing a perpendicular from B to AC or a perpendicular from A to BC), we obtain $\frac{a}{SinA} = \frac{c}{SinC}$ or $\frac{b}{SinB} = \frac{c}{SinC}$

Thus, finally $\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$

3.3 Law of *Cosine*

In any triangle ABC, the square of the any side is equal to the sum of the squares of the other two sides minus the product of these two sides and the $Cosine$ of their included angle.

Mathematically, this law can be expressed thus;

$a^2 = b^2 + c^2 - 2bc\ CosA$

$b^2 = a^2 + c^2 - 2ac\ CosB$

$c^2 = a^2 + b^2 - 2ab\ CosC$

Proof:

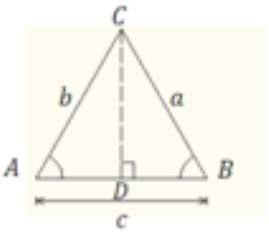


Figure 2

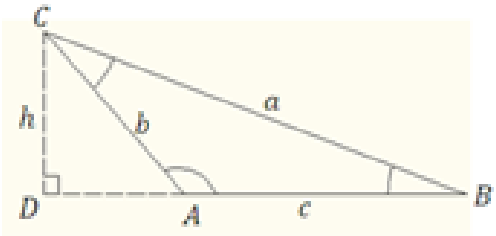


Figure 3

Figure 2 and 3 as reproduced above will be used

In the right triangle ACD in fig.2

$h = bSinA$ and $AD = bCosA$

But $AD + DB = AD \rightarrow DB = C - bCosA$

$a^2 = h^2 + DB^2 = b^2Sin^2A + (c - bCosA)^2$

$= b^2Sin^2A + c^2 - 2bcCosA + b^2Cos^2A$

$= b^2(Sin^2A + 2bcCosA) + c^2 - 2bc\ CosA$

In the right triangle ACD in fig.3,

$h = bSin(180 - A) = bSinA$

$AD = bCos(180 - A) = -bCosA$

$a^2 = h^2 + DB^2 = h^2+(AD - DB)^2$

$= h^2+(-bCosA + c)^2$

$= b^2Sin^2A + b^2Cos^2A - 2bcCosA + c^2$

$= b^2(Sin^2A + Cos^2A) + c^2 - 2bc\ CosA$

$$= b^2 + c^2 + 2bc\ CosA$$

3.4 Applications of the Laws of Sines and Cosines – Solutions of Oblique Triangles

When three parts of a triangle, not all angles, are known then all other parts can be determined except in one case which will be noted below. There are five cases in which the three parts of a triangle can be given for each of these cases, the laws of $Sines$ or $Cosines$ can be used to determine the other parts. The table below gives the five cases, the first part that should be determined and the law to be used.

Case	Description	First part to find	Law to Use
1.	When two angles and the side opposite one of them are given	Side opposite second angle given	$Sines$
2.	When two angles and the included side are given	Third angle, then either of remaining sides	$Sines$
3.	When two sides and the angle opposite one of them are given	Angle opposite second side given	$Sines$
4.	When two sides and the included angle are given	Third side	$Cosines$
5.	When the three sides are given	Any angle can be found	$Cosines$

NOTE: In case 3, there is always a unique solution. It is possible to have no solution for the angle, one solution for the angle, two solutions an angle and its supplements.

1. **Case 1**: Given two angles and the side opposite one of them, i.e. suppose b, B and C are given.

- To find C, use $\frac{c}{SinC} = \frac{b}{SinB}$ then $c = \frac{bSinC}{SinB}$
- To find A, use $180° - (B + C)$
- To find a, use $\frac{a}{SinA} = \frac{b}{SinB}$, then $a = \frac{bSinA}{SinB}$

2. **Case 2**: Given two angles and the included side, suppose a, B and c are given.

- To find A, use $A = 180° - (B + C)$
- To find b, use $\frac{b}{SinB} = \frac{a}{SinA}$ then $b = \frac{aSinB}{SinA}$
- To find c, use $\frac{c}{SinC} = \frac{b}{SinB}$, then $c = \frac{bSinC}{SinB}$

3. **Case 3**: Given two sides and the angle opposite one of them, suppose b, c and B are given.

- To find c, use $\frac{b}{SinB} = \frac{c}{SinC}$, then $SinC = \frac{cSinB}{B}$

Note

1. If $SinC > 1$, no angle is determined
2. If $SinC = 1$, $C = 90°$ and a right triangle is determined
3. If $SinC < 1$, to angles are determined: an acute angle C and an obtuse angle $C' = 180° - C$. Thus, there may be one or two angles triangles determined if $C' + B \geq 180°$, then the angle C' is not a solution.
4. When the given angle is acute, there will be;

a. One solution if the side opposite the given angle is equal to or greater than the other given side.

b. No solution, one solution (right triangle), or two solutions if the side opposite the given angle is less than the other given side when the given angle is obtuse, there will be

c. No solution when the side opposite the given angle is less than or equal to the other given side.

d. One solution if the side opposite the given angle is greater than the other given side.

4. **Case 4**: Given two sides and the included angle, suppose a, b and c are given

- To find C, use $c^2 = a^2 + b^2 - 2abCosC$
- To find A, use $SinA = \frac{aSinC}{C}$
- To find B, use $SinB = \frac{bSinC}{C}$ or $B = 180° - (A + C)$

5. **Case 5**: Given the three sides, suppose a, b and c are given

- To find A, use $CosA = \frac{b^2+c^2+a^2}{2bc}$
- To find B, $CosB = \frac{c^2+a^2+b^2}{2ac}$
- To find C $CosC = \frac{a^2+b^2+c^2}{2ab}$

Example 4.1: Solve the following triangles

a) $a = 13.2, A = 21.3°, B = 61.4°$

b) $a = 28.5, A = 30.0°, B = 65°$

c) $c = 12.5, A = 13.2°, B = 49°$

d) $c = 5.0,\ \ A = 100°, B = 45°$

e) $b = 6.35,\ \ A = 4.29, B = 42.3°$

f) $c = 5.0,\ \ b = 8.0, B = 45°$

g) $a = 4,\ \ b = 22, B = 30°$

h) $b = 0.452,\ \ b = 0.606, B = 91.8°$

i) $a = 4.38,\ \ b = 3.79, B = 5.22°$

j) $a = 832,\ \ b = 623, B = 345°$

Solutions

a) $\alpha = 13.2,\ A = 21.3°,\ B = 61.4°$

$c = 180° - (21.3 + 61.4)° = 97.3°$

Using $\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$ yields

$b = \frac{aSinB}{SinA} = \frac{13.2Sin61.4°}{Sin21.3°} = 31.9$ and $c = \frac{aSinC}{SinA} = \frac{13.2Sin97.3°}{Sin21.3°} = 36.0$

b) $\alpha = 28.5, b = 30.0, c = 65°$

Using $c^2 = a^2 + b^2 - 2abCosC$

$c^2 = 28.5^2 + 30.0^2 - 2(28.5)(30.0)Cos65° = 989.6 \quad \therefore C = 31.5$

Using $\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$ yields

$SinA = \frac{aSinC}{c} = \frac{28.5Sin65°}{31.5} = 0.82 \quad \therefore A = 55.1 \approx 55°$

$SinB = \frac{bSinC}{C} = \frac{30Sin65°}{31.5} = 0.86 \quad \therefore B = 59.6 \approx 60°$

c) $c = 12.5,\ \ b = 13.2,\ \ B = 49°$

Using $\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$ yields

$SinC = \frac{cSinB}{b} = \frac{12.5Sin49°}{13.2} = 0.71 \quad \therefore C = 45.2$

$A = 180° - (B + C) = 180° - (49° + 45.2) = 85.8°$

$a = \frac{bSinA}{SinB} = \frac{13.2Sin85.8^0}{Sin49^0} = 17.4$

d) $c = 5.0, A = 100°, B = 45°$

$c = 180° - (A + B) = 180° - (100 + 45)° = 35°$

Using $\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$ yields

$a = \frac{cSinA}{SinC} = \frac{5.0Sin100°}{Sin35°} = 8.6$

$b = \frac{cSinB}{SinC} = \frac{5.0Sin45°}{Sin35°} = 6.2$

e) $b = 6.35, c = 4.29, C = 42.3°$

Using $\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$ yields

$SinB = \frac{bSinC}{C} = \frac{6.35Sin42.3°}{4.29} = 0.996 \quad \therefore B = 84.9$

Using $A = 180° - (B + C)$ yields $A = 180° - (84.9 + 42.3) = 52.8°$

Finally, $a = \frac{bSinA}{SinB} = \frac{6.35Sin52.8°}{Sin84.9} = 5.078$

f) $c = 5.0, b = 8.0, C = 45°$

Using $\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$ yields

$SinB = \frac{bSinC}{C} = \frac{8.0Sin45°}{5.0} = 1.13$

Since $SinB = 1.13 > 1$, the triangle has no solution

g) $a = 4, b = 22, A = 30°$

Using $\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$ yields

$SinB = \frac{bSinA}{a} = \frac{22Sin30°}{4} = 2.75$

Since $SinB = 2.75 > 1$, the triangle has no solution.

h) $b = 0.452, \quad c = 0.606, \quad C = 91.8°$

Using $\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$ yields

$SinB = \frac{bSinC}{c} = \frac{0.452Sin91.8°}{0.606} = 0.746 \qquad \therefore B = 48.2°$

Using $A = 180° - (B + C)$ gives $A = 180 - (48.2° + 91.8°) = 40°$

Finally, $a = \frac{bSinA}{SinB} = \frac{0.4525Sin40°}{Sin48.2} = 0.390$

i) $a = 4.38, b = 3.79, C = 5.22$

Using $a^2 = b^2 + c^2 - 2bcCosA$ gives

$CosA = \frac{b^2+c^2+a^2}{2bc} = \frac{3.79^2+5.22^2+4.38^2}{2(3.79)(5.22)} = 0.5668$

$\therefore \quad A = 55.5°$

Similarly $CosB = \frac{a^2+c^2+b^2}{2ac} = \frac{4.38^2+5.22^2-3.79^2}{2(4.38)(5.22)} = 0.7013$

$\therefore B = 45.5°$

Finally $CosC = \frac{a^2+b^2+c^2}{2ab} = \frac{4.88^2+3.79^2-5.22^2}{2(4.38)(3.79)} = 0.896$

$\therefore \quad C = 79.1°$

j) $a = 832, b = 623, C = 345$

Using $a^2 = b^2 + c^2 - 2bcCosA$ gives

$CosA = \frac{b^2+c^2+a^2}{2bc} = \frac{623^2+345^2-832^2}{2(623)(345)} = -0.4305$

$\therefore \quad A = 115.5°$

Similarly $CosB = \frac{a^2+c^2+b^2}{2ac} = \frac{832^2+345^2-623^2}{2(832)(345)} = 0.7370$

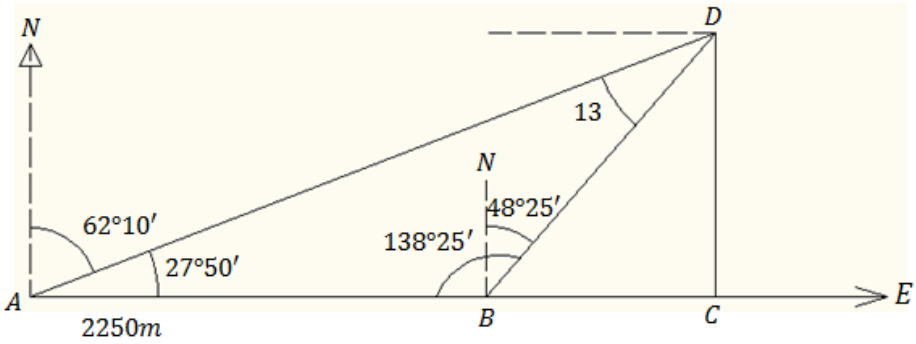
$\therefore B = 42.5°$

Finally $CosC = \frac{a^2+b^2+c^2}{2ab} = \frac{832^2+623^2-345^2}{2(832)(623)} = 0.9273$

$\therefore C = 22.0°$

Example 4.2: A ship is sailing due cast when a light is observed on a bearing of $N62° 10'E$. After the ship has travelled $2250m$, the bearing of the light is now observed to be $N48°25$. If the course is continued, what will be the closest distance between the ship and the light?

Solution



From ΔABD,

$A = 27°50' = 27.8°$

$B = 90° + 48°25' \ = 138°25' \ = 138.4'$

$\therefore D = 180 - (27.8 + 138.4) = 13.8°$

Using the law of $Sines$, $\frac{BD}{Sin27.8} = \frac{2250}{Sin13.8} \rightarrow \quad BD = \frac{2250Sin27.8}{Sin13.8} = 4399.26m$

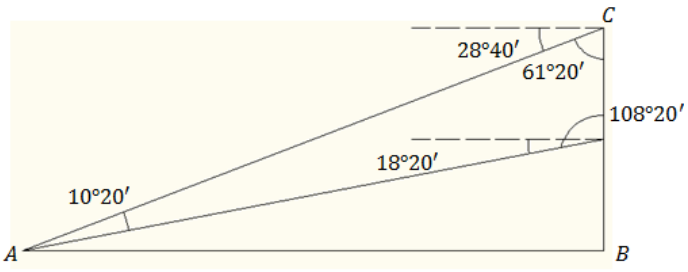
From $\Delta\, BDC$,

$B = 180 - 138°25' = 41.58°$ and $DC = BDSinB \ = 4399.26Sin41.58°$

$= 2919.64m \approx 2920m$

Example 4.3: A tower of height $125ft$, is on a cliff on the bank of a river. From the top of the towers the angle of depression of a point on the opposite shore is $28°40'$, and from the base of the tower, the angle of depression of the same point is $180°20'$. Find the width of river and the height of the cliff.

Solution



BC represents the tower. DB represents the cliff and A is the point on the opposite shore.

From $\Delta ABC, < ABC = 90 - 28°40' = 61°20' = 61.33°$

$< CBA = 90 + 18°20' = 108°20' = 108.33°$

$< BAC = 180°(< CBA + < ACB) = 10°20' = 10.33°$

Using the law of Sine, we get $\frac{AB}{Sin61.33} = \frac{BC}{Sin10.33}$

$\therefore AB = \frac{BCSin61.33}{Sin10.33} = \frac{125Sin61.33}{Sin10.33} = 611$

From $\Delta\, ABD,$

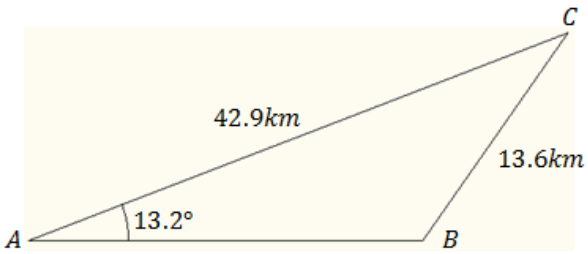
$DB = ABSin18.33 \;\; = 611Sin18.33 = 192$

$AD = ABCos18.33 \;\; = 611Cos18.33 = 580$

$\therefore$ Width of river $= 580ft$ and Height of cliff $= 192ft$

Example 4.4: A ground-based radar at a point A determines that he angle of elevation to an aircraft $42.9km$ away is 13.2°. Another radar at point B eastward of A determines the same aircraft to be $13.6km$ away. Find the distance between A and B.

Solution



Using the law of $Sines$, we get

$\frac{13.6}{Sin13.2} = \frac{42.9}{SinB} \quad \rightarrow \quad SinB = \frac{42.9Sin13.2}{13.6} = 0.72 \quad \therefore B = 46.1°$

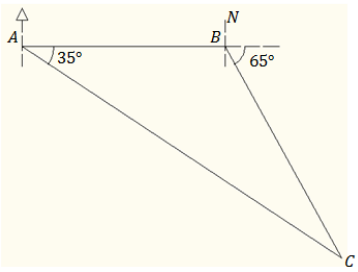
$C = 180° - (13.2 + 46.1) = 120.7°$

Using the laws of Sines,

$$\frac{42.9}{Sin46.1} = \frac{AB}{Sin120.7} \rightarrow AB = \frac{42.9Sin120.7}{Sin46.1} = 51.2km$$

Example 4.5:. A man lost in the desert travels $8km$ due east from the point he discovered he has lost his way. He then turns towards the south through an angle of $65°$ and proceeds until he crosses a line of right from the starting position to itself that is $35°$ south of east. At this point, how far is the man from his starting point?

Solution



From $\Delta\, ABC$,

$A = 35°,\ B = 180 - 65 = 115°$ and $C = 180 - (35 + 115) = 30°$

Using the law of $Sines$, $\frac{8}{Sin30°} = \frac{AC}{Sin115°} \rightarrow AC = \frac{8Sin115°}{Sin30°} = 14.5$

∴ The man is $14.5km$ from his starting point

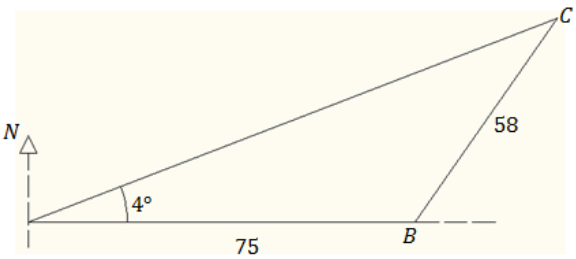
Example 4.6:. Two towns are $75km$ apart. An aircraft between the two towns is

, n

angle of elevation of $40°$*

the city B is $58km$ and that the angle of elevation there is less than $40°$. What is the slant distance from the plane to town A?

Solution



Using the law of $Sines$

$$\frac{AC}{Sin81.78} = \frac{58}{Sin40} \quad \rightarrow \quad SinC = \frac{75Sin40}{58} = 0.83312 \qquad \therefore C = 58.22°$$

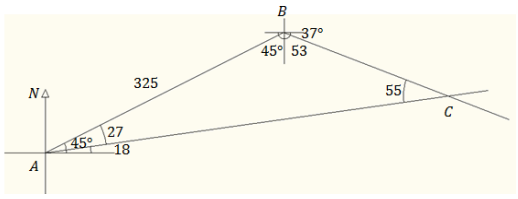
$$B = 180 - (40 + 58.22) = 81.78$$

$$\frac{AC}{Sin81.78} = \frac{58}{Sin40} \quad \rightarrow \quad AC = \frac{8Sin81.78}{Sin40} = 89.30 \approx 89$$

$\therefore$ The slant distance from the plane to town A is $89km$.

Example 4.7: An army observation post is $325m$ northeast of a second post. At this point, a tank is sighted on a straight line of sight $37°$ south of east. The same tank is sighted at the second post along a line of sight $18°$ north of east. How far is the tank from the first observation posts?

Solution



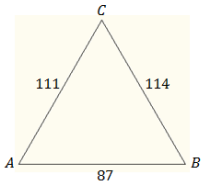
Using the laws of $Sines$, $\frac{325}{Sin55} = \frac{AC}{Sin98} \rightarrow AC = \frac{325Sin98}{Sin55} = 392.89 \approx 393$

Similarly, $\frac{325}{Sin55} = \frac{BC}{Sin27} \rightarrow BC = \frac{325Sin278}{Sin55} = 180.12 \approx 180$

Therefore, the tank is $393km$ and $180km$ from the observation posts.

Example 4.8: Two observers are $87m$ apart and a range finder shows that a certain building is $111m$ from one observer and $114m$ from the other. What is the angle formed by the two lines of right from the building to the observers?

Solution



Using the law of $Cosines$, $\quad 87^2 = 111^2 + 114^2 - 2(111)(114)\ CosC$

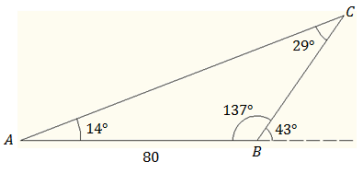
$\rightarrow\ Cos\ C = \frac{111^2+114^2-87^2}{2(111)(114)} = 0.7013 \quad \rightarrow \quad C = 45.47°$

Example 4.9: Ahmad walks $80ft$ along the sidewalk next to a field. Then he turns at an angle of $45°$ to the sidewalk and leads across the field. When he stops, he looks back at the starting point, finding a $29°$ angle between his path across the field and the direct route back to the starting point. Find how far

i. Across the field he had walked

ii. Does he have to walk to get directly back to his starting point?

Solution



From $\Delta\ ABC, \quad B = 80 - 43 = 137° \qquad \therefore\ A = 180 - (137 + 29) = 14°$

Using the law of $Sines$

$\frac{80}{Sin29} = \frac{AC}{Sin137} \quad \rightarrow \quad AC = \frac{80Sin137}{Sin29} = 112.54$

Similarly $\frac{80}{Sin29} = \frac{BC}{Sin14} \quad \rightarrow \quad BC = \frac{80Sin29}{Sin14} = 39.92$

Example 4.10: The points $(5,3), (-2,1)$ and $(1,-4)$ form a triangle. Find the measure of the smallest angle in this triangle.

Solution

Let the point $(5,3), (-2,1)$ and $(1,-4)$ be represented by A, B and C respectively, then:

The distance $AB = c = \sqrt{(5+2)^2 + (3-1)^2} = \sqrt{53}$

The distance $AC = b = \sqrt{(5-1)^2 + (3+4)^2} = \sqrt{65}$

The distance $BC = a = \sqrt{(-2-1)^2 + (1+4)^2} = \sqrt{34}$

Using the laws of $Cosine$,

$$CosA = \frac{b^2+c^2-a^2}{2bc} = \frac{65+53-34}{2(\sqrt{65})(\sqrt{53})} = 0.7156. \quad \therefore A = 44.31°$$

Also, $CosB = \frac{a^2+c^2-b^2}{2ac} = \frac{34+53-65}{2(\sqrt{34})(\sqrt{53})} = 0.2591 \quad \therefore B = 74.98°$

Finally $CosC = \frac{a^2+b^2-c^2}{2ab} = \frac{34+65-53}{2(\sqrt{34})(\sqrt{65})} = 0.4893 \quad \therefore C = 60.71°$

5.0 Area of a Triangle

5.1 Introduction

The Area X of any triangle equals one-half the product of its base and altitude. Generally if enough information about a triangle is available to the extent that it can be solved, then its area can easily be found.

Recall that it was noted in section 2.3 that there are five cases in which three parts of a triangle can be given and for each of these cases, the laws of $Sines$ and $Cosines$ that can be used to determine the other parts were also noted.

In the same vein the area of a triangle can be found when any of these case is known. The formula to compute the area of a triangle corresponding to each of the five cases and other derivation are given as follows.

5.2 Cases 1 and II: When two angles and a side of the triangle is given, the third angle is found using $A + B + C = 180°$. In this case, the area of the triangle is given by

$$X = \frac{a^2 SinBSinC}{2SinA} \text{ or } \frac{b^2 SinASinC}{2SinB} \text{ or } \frac{c^2 SinASinB}{2SinC}$$

In other words, given two angles and a side of triangle ABC, the area of the triangle equals product of square of the given side, and $Sines$ of the angles including the side divided by twice the $Sine$ of the angle opposite the given side

Proof

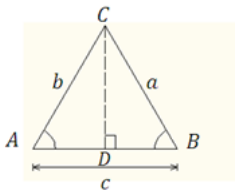


Figure 1

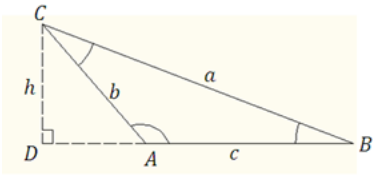


Figure 2

From definition, Area X of any triangle X is given by $X = \frac{1}{2}(base)(height)$

Therefore, in either triangle, $X = \frac{1}{2}bh$

In fig 1 $h = bSinA$ or $aSinB$

In fig 2, $h = bSin(180 - A)$ or $aSinB = bSinA$ or $aSinB$

Since in either triangle, $h = bSinA$ or $aSinB$

$$\therefore X = \frac{1}{2}bcSinA \text{ or } \frac{1}{2}acSinB$$

From the law of $Sines$ (ie. $\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$), $b = \frac{cSinB}{SinC}$ and $a = \frac{cSinA}{SinC}$

$$\therefore X = \frac{c^2 SinASinB}{2SinC} \text{ or } \frac{a^2 SinBSinC}{2SinA}$$

In general $X = \frac{a^2 SinBSinC}{2SinA} = \frac{b^2 SinASinC}{2SinB} = \frac{c^2 SinASinB}{2SinC}$

5.3 Case III: When two sides of a triangle and the angle opposite one of them is given.

In this case, a second angle can be found using the law of $Sines$ and then the appropriate formula from Case I can then be used.

NOTE: Since sometimes there are two solution for the second angle, it follows that there will be times when the area of two triangles must be found.

5.4 Case IV: When two sides of a triangle and the included angle is given.

In this case, the area of the triangle is given by $X = \frac{1}{2}abSinC = \frac{1}{2}acSinB = \frac{1}{2}bcSinA$

Proof

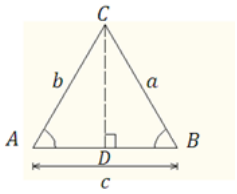


Figure 1

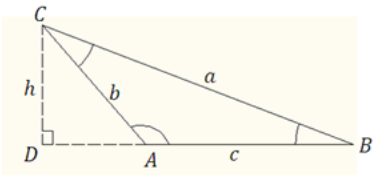


Figure 2

In either triangle, $X = \frac{1}{2}hc$

From fig. 1, $h = bSinA$ or $aSinB$

From fig. 2, $h = bSin(180 - A)$ or $aSinB = bSinA$ or $aSinB$

From either triangle, $h = bSinA$ or $aSinB$

$\therefore X = \frac{1}{2}bcSinA$ or $\frac{1}{2}acSinB$

In general, $X = \frac{1}{2}abSinA = \frac{1}{2}acSinB = \frac{1}{2}bcSinA$

5.5 Case V: When the three sides of a triangle is given.

In this case, the area of the triangle is given by $X = \sqrt{s(s-a)(s-b)(s-c)}$

Where $s = \frac{1}{2}(a + b + c)$

This formula is known as F F '

Proof

By the law of $Cosines$ $CosA = \frac{b^2+c^2-a^2}{2bc}$

$1 - CosA = 1 - \frac{b^2+c^2-a^2}{2bc} = \frac{2bc-b^2-c^2+a^2}{2bc} = \frac{2bc-(b-c)^2}{2bc} = \frac{(a-b+c)(a+b-c)}{2bc}$

and

$1 + CosA = 1 + \frac{b^2+c^2-a^2}{2bc} = \frac{2bc+b^2-c^2+a^2}{2bc} = \frac{(b+c)^2-a^2}{2bc} = \frac{(b+c+a)(b+c-a)}{2bc}$

Let $2s = a + b + c$

$\therefore a - b + c = (a + b + c) - 2b = 2s - 2b = 2(s - b)$

$a + b - c = (a + b + c) - 2c = 2s - 2c = 2(s - c)$

$b + c - a = (a + b + c) - 2a = 2s - 2a = 2(s - a)$

$\therefore 1 - CosA = \frac{2(s-b).2(s-c)}{2bc} = \frac{2(s-b)(s-c)}{bc}$,, ,,, ,,, (1)

and

$1 + CosA = \frac{2s.2(s-a)}{2bc} = \frac{2s(s-a)}{bc}$,,, , 0'

Multiplying (1) and (2) yields

$(1 - CosA)(1 + CosA) = \frac{4s(s-a)(s-b)(s-c)}{b^2c^2}$

$\rightarrow \quad 1 - Cos^2A = \frac{4s(s-a)(s-b)(s-c)}{b^2c^2} \quad \rightarrow \quad Sin^2A = \frac{4s(s-a)(s-b)(s-c)}{b^2c^2}$

$\therefore \quad SinA = \frac{2\sqrt{s(s-a)(s-b)(s-c)}}{bc}$

Substituting $SinA = \frac{2\sqrt{s(s-a)(s-b)(s-c)}}{bc}$ in $X = \frac{1}{2}bcSinA$ yields

$X = \sqrt{s(s-a)(s-b)(s-c)}$

Example 5.1: Find the areas of each of the given triangle ABC, if

a) $c = 23cm, A = 20°$ and $C = 15°$

b) $c = 23cm, A = 20°$ and $B = 15°$

c) $a = 112m, A = 219m$ and $A = 20°$

d) $b = 27m, c = 14m$ and $A = 43°$

e) $a = 14.27cm, c = 17.2cm$ and $B = 86°14'$

f) $a = 5m, b = 7m$ and $C = 10m$

g) $a = 1.017m, b = 2.032m$ and $c = 2.055m$

h) $b = 2.86cm, c = 4.43cm$ and $B = 23.3°$

i) $a = 225cm, b = 290cm$ and $c = 4.9cm$

j) $b = 444.8, A = 110°16'$ and $B = 30°10'$

Solution

a) $c = 23cm, A = 20°$ and $C = 15°$

$\therefore B = 180° - (20 + 15)° = 145°$

Using $X = \frac{c^2 SinASinB}{2SinC}$ gives

$$X = \frac{23^2 Sin20°Sin145°}{2Sin15°} = \frac{529(0.3420)(0.5736)}{0.2588} = 400.98cm^2 = 401m^2$$

b) $c = 23cm, A = 20°$ and $B = 15°$

$C = 180° - (20 + 15)° = 145$

Using $X = \frac{c^2 SinASinB}{2SinC}$ gives

$$X = \frac{23^2 Sin20°Sin15°}{2Sin145°} = \frac{529(0.3420)(0.2588)}{0.5736} = 81.63cm^2 = 82m^2$$

c) $a = 112m, A = 219m$ and $A = 20°$

Using $\frac{a}{SinA} = \frac{b}{SinB}$ gives $SinB = \frac{bSinA}{a}$

$$\rightarrow SinB = \frac{219Sin20°}{112} = \frac{219(0.3420)}{112} = 0.6689$$

$B = 41.9668° \approx 42°$ or $138°$

$C = 180° - (A + B)$

$= 180 - (20 + 42)$ or $180 - (20 + 138)$ $= 118°$ or $22°$

Using $X = \frac{1}{2}abSinC$ gives

$X = \frac{1}{2}(112)(219)Sin118°$ or $\frac{1}{2}(112)(219)Sin22°$

$= \frac{1}{2}(112)(219)(0.8829)$ or $\frac{1}{2}(112)(219)(0.3746)$

$= 10827.8856$ or $4594.0944 \approx 10828m^2$ or $4594m^2$

d) $b = 27m, c = 14m$ and $A = 43°$

Using $X = \frac{1}{2}bcSinA$ yields

$\therefore \quad X = \frac{1}{2}(27)(14)Sin43° = \frac{1}{2}(27)(14)(0.6820) = 128.898 \approx 129m^2$

e) $a = 14.27cm, c = 17.2cm$ and $B = 86°14'$

Using $\frac{a}{SinA} = \frac{b}{SinB}$ gives $SinA = \frac{bSinB}{a}$

$$\therefore SinA = \frac{14.27Sin86°14'}{17.2} = \frac{14.27Sin86.23°}{17.2}$$

$$SinA = \frac{14.27S(0.9978)}{17.2} = 0.8287$$

$A = 55.97° \approx 42°$ or $124.03°$

$C = 180° - (A + B)$

$= 180 - (55.97 + 86.23)$ or $180° - (124.03 + 86.23)$

$= 37.8°$ or $-30.26°$

$C \neq -30.26 \quad \therefore A = 124.03°$ is not feasible

Hence $A = 55.97°,\ B = 86.23°$ and $C = 37.8°$

Using $X = \frac{1}{2}abSinC$ gives

$X = \frac{1}{2}(14.27)(17.2)Sin37.8° = \frac{1}{2}(14.27)(17.2)(0.6129)$

$= 71.22 \approx 71cm^2$

f) $a = 5m, b = 7m$ and $c = 10m$

Using $X = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a+b+c)$ gives

$s = \frac{1}{2}(5+7+10) = 11$

and $X = \sqrt{11(11-5)(11-7)(11-10)}$

$= \sqrt{264} = 16.25 \approx 16m^2$

g) $a = 1.017m, b = 2.032m$ and $c = 2.055m$

Using $X = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a+b+c)$ gives

$s = \frac{1}{2}(1.017 + 2.032 + 2.055) = 2.552$

and $X = \sqrt{2.552(2.552-1.017)(2.552-2.032)(2.552-2.055)}$

$= \sqrt{1.0124} = 1.0062 \approx 1.006m^2$

h) $b = 2.86cm, c = 4.43cm$ and $B = 23.3°$

Using $\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$ yields $\frac{a}{SinA} = \frac{2.86}{Sin23.3°} = \frac{4.43}{sinC}$

$\rightarrow SinC = \frac{44.3Sin23.3°}{2.86} = 0.6127 \rightarrow c = 37.8°$

Using $A + B + C = 180°$ gives $A + 23.3° + 37.8° = 180° \rightarrow A = 118.9°$

Using $\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$ yields $\frac{a}{Sin118.9°} = \frac{2.86}{Sin23.3°} = \frac{4.43}{sin37.8°}$

$\rightarrow a = \frac{2.86Sin118.9°}{Sin23.3°} = 6.33$

Using $X = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a+b+c)$ gives

$s = \frac{1}{2}(6.33 + 2.86 + 4.43) = 6.81$

and $X = \sqrt{6.81(6.81-6.33)(6.81-2.86)(6.81-4.43)}$

$= \sqrt{30.73} = 5.5435 \approx 5.54cm^2$

i) $a = 225cm, b = 290cm$ and $c = 4.9cm$

Using $X = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a+b+c)$ gives

$s = \frac{1}{2}(225 + 290 + 419) = 467$

and $X = \sqrt{467(467-225)(467-290)(467-419)}$

$= \sqrt{960166944} = 30986.5607 \approx 30987m^2$

j) $b = 444.8, A = 110°16'$ and $B = 30°10'$

Using $A + B + C = 180°$

Gives $C = 180 - (110.18 + 30.17) = 39.65°$

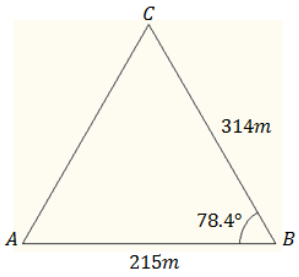
Using $X = \frac{b^2 SinASinC}{SinB}$ gives

$$X = \frac{444.8Sin110.18°Sin39.65°}{Sin30.17°} = \frac{444.8(0.9386)(0.6381)}{0.5026} = 530.0436cm^2$$

$$= 530m^2$$

Example 5.2: A man walks from a point A to another point B which is 215m due east of A. He then turns through an angle of °78.4° and walks $314m$ before returning to his starting point. What area of land has he covered?

Solution

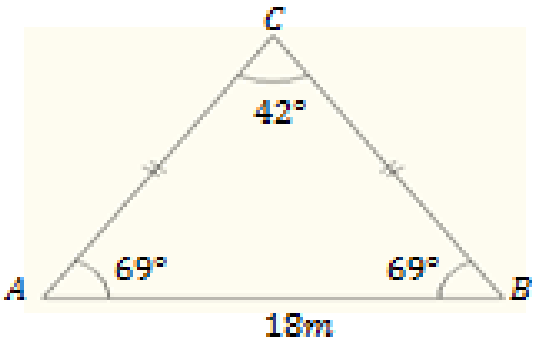


Using $X = \frac{1}{2}acSinB$ gives $X = \frac{1}{2}(215)(314)Sin78.4° = \frac{1}{2}(215)(314)(0.9796)$

$= 33066.398 \approx 33066m^2$

Example 5.3: An artist wishes to make a sign in the shape of an isosceles triangle with a 42° vertex angle and a base of $18m$. What is the area of the triangle?

Solution

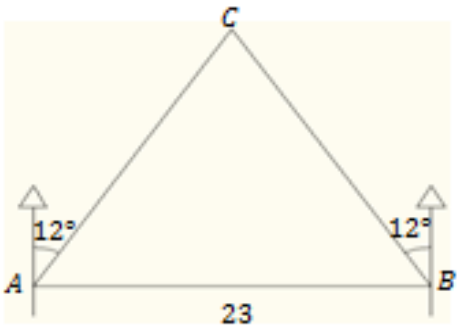


Using $X = \frac{c^2 SinASinB}{SinC}$ gives $X = \frac{18^2 Sin69°Sin69°}{Sin42°} = \frac{324(0.9336)^2}{2(0.6691)}$

$$= 211.031cm^2 \approx 211m^2$$

Example 5.4: Point C has a bearing of $N28°E$ from point A and a bearing of $N12°W$ from point B. What is the area of triangle ABC if B is $23km$ due East of A?

Solution



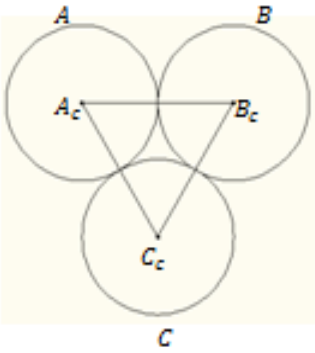
From triangle ABC, $A = 90 - 28° = 62°, B = 90 - 12 = 78$ and $C = 180 - (A + B) = 180 - (62 + 78) = 40°$

Using $X = \frac{c^2 SinASinB}{SinC}$ gives $X = \frac{23^2 Sin62°Sin78°}{Sin40°} = \frac{23(0.8829)(0.9781)^2}{2(0.6428)}$

$$= 335.34 \approx 355km^2$$

Example 5.5: Three circles have radii 7.72, 4.84 and 11.4cm. If they are externally tangent, what is the area of the triangle formed by joining their centres?

Solution



Let the circles be represented by A, B and C so that their radii are 7.72, 4.84 and 11.4cm respectively.

Distance from centre of A to centre of B $= 7.72 + 4.84 = 12.56cm$

Distance from centre of B to centre of C $= 4.84 + 11.4 = 16.24cm$

Distance from centre of C to centre of A $= 11.4 + 7.72 = 19.12cm$

Therefore, the triangle are of sides $12.56cm, 16.24cm$ and $19.12cm$.

Using $X = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a+b+c)$ gives

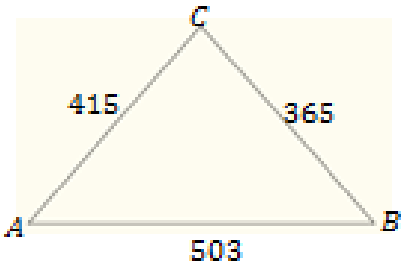
$s = \frac{1}{2}(12.56 + 16.24 + 19.12) = 23.96$

$\therefore X = \sqrt{23.96(23.96 - 12.54)(23.96 - 16.24)(23.96 - 19.12)}$

$= \sqrt{10223.87614} = 101.113 \approx 101cm^2$

Example 5.6: A woman hikes 503m, turns and jogs 415m, turns again and runs 365m returning to her starting point. What is the area of the triangle formed by her path?

Solution



Using $X = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a+b+c)$ gives

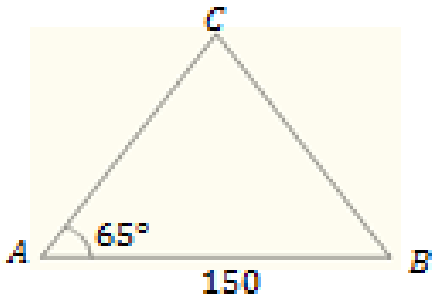
$s = \frac{1}{2}(503 + 365 + 415) = 641.5$

$\therefore X = \sqrt{641.5(641.5-503)(641.5-365)(641.5-415)}$

$= \sqrt{5564290251} \ = 74594.17036 \approx 74.594m^2$

Example 5.7: Two streets intersect at an angle of 65°. A third street connecting these two is to be constructed so that the distance between the first two streets and the third one will be 150m and 200m. What area of land will be bound by the three streets.

Solution

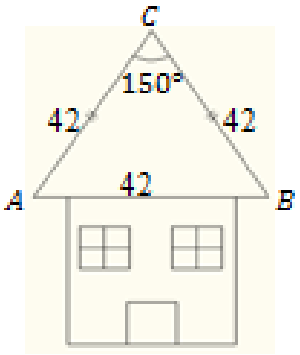


Using $X = \frac{1}{2}bcSinA$ gives

$X = \frac{1}{2}(200)(150)Sin65° \ = \frac{1}{2}(200)(150)(0.9063) \ = 13594.5 \ \approx 13595$

Example 5.8: A painter wishes to find the area of the gable end of a house. What is the area of the gable if it is a triangle with two sides 42.0ft that meets at a $105°$ angle?

Solution

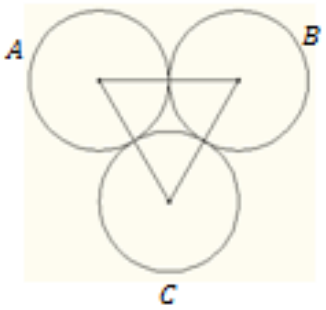


Using $X = \frac{1}{2}abSinA$ gives $X = \frac{1}{2}(42)(42)Sin105° = \frac{1}{2}(42)(42)(0.9659)$

$= 851.9238 \approx 852ft^2$

Example 5.9: Three circles of radii 3.0, 5.0 and 9.0cm are tangent to each other externally. Find the area of the triangle formed by joining their centres

Solution



Let the circles be represented by A, B and C so that their radii are 3.0, 5.0 and 9.0cm respectively.

Distance from centre of A to centre of B $= 3.0 + 5.0 = 8.0cm$

Distance from centre of B to centre of C $= 5.0 + 9.0 = 14.0cm$

Distance from centre of C to centre of A $= 9.0 + 3.0 = 12.0cm$

Therefore, the triangle are of sides $8.0cm, 14.0cm$ and $12.0cm$.

Using $X = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a+b+c)$ gives

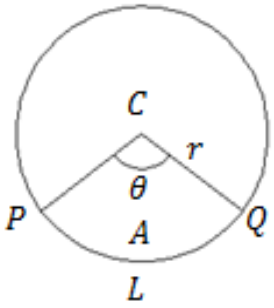
$s = \frac{1}{2}(8.0 + 14.0 + 12.0) = 17.0$ and

$X = \sqrt{17(17-8)(17-14)(17-12)} = \sqrt{2295} = 47.906 \approx 48m^2$

6.0 Mensurations

6.1 Circular Measure

The mathematical relationships regarding the area (A) of the sectorPCQ, length $PQ(L)$ and the subtended angle (Q) of the arc of a circle can be simply derived using the ratios shown below.



C centre of the circle

r radius of the circle

PQ the line joining the pts P and Q is called the chord of the circle.

$$\frac{Subtended\ angle}{total\ angle\ at\ centre} = \frac{length\ of\ arc}{circumference\ of\ circle} = \frac{area\ of\ sector}{area\ of\ circle}$$

i.e. $\frac{\theta}{2\pi} = \frac{\mu}{2\pi r} = \frac{A}{\pi r^2}$

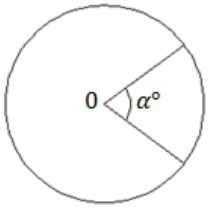
From these ratios, we can deduce the following

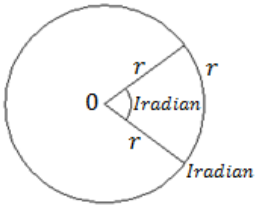
i. $l = r\theta$

ii. $A = {}^{lr}/_{2}$

iii. $A = \frac{1}{2}r^2\theta$

6.2 Relation between radians and Degree





Arc length $r = 1radian$

$$\therefore 2\pi r = 360°$$

$$360° = \frac{2\pi r}{r} radians$$

$$360° = 2\pi \rightarrow 180° = \pi rad.$$

Example 6.1

a) $60° = \frac{60}{180}\pi rads = \frac{1}{3}\pi rads$

b) $270° = \frac{270}{180}\pi rads = \frac{3}{2}\pi rads$

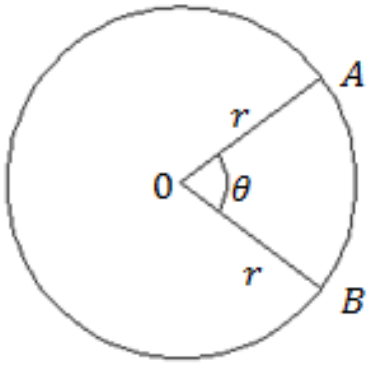
c) $150° = \frac{150}{180}\pi rads = \frac{5}{6}\pi rads$

d) $1080° = \frac{1080}{180}\pi rads = 6\pi rads$

e) $\frac{\pi}{2} rads = \frac{1}{2}(180)° = 90°$

f) $\frac{4}{3}\pi rads = \frac{4}{3}(180) = 240°$

6.3 Arc Length and Area of Sector



6.3.1 Arc Length

$\frac{Lenght\ of\ Arc\ AB}{2\pi r} = \frac{\theta}{360^0} \rightarrow$ Length of Arc $AB = \frac{2\pi r\theta}{360^0}$

Since $360° = 2\pi$rads,

Therefore, Length of Arc $AB = \frac{2\pi r\theta}{2\pi} = r\theta$, where θ is in radians

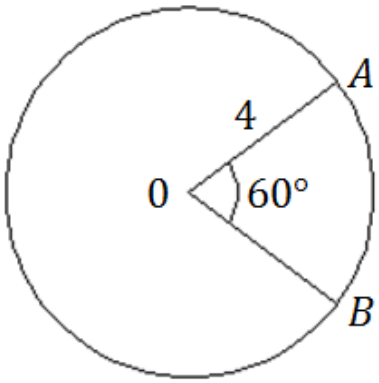
6.3.2 Area of Sector

$\frac{Area\ of\ sector\ A0B}{\pi r^2} = \frac{\theta}{360^0} \rightarrow$ Area of sector $A0B = \frac{\pi r^2\theta}{360^0}$

Since $360° = 2\pi$rads,

Therefore, Area of sector $A0B = \frac{\pi r^2\theta}{2\pi} = \frac{1}{2}r^2\theta$, where θ is in radians

Example 6.2.1: Find the length of the arc AB and area of the sector AOB in the figure below.



Solution

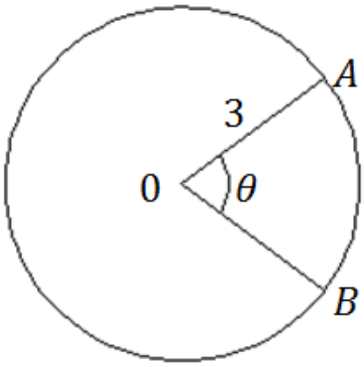
$60° = \frac{\pi.60}{180} radians \ = \frac{\pi}{3}$ rads

Therefore, Arc length $AB = r\theta = 4(\frac{\pi}{3}) = \frac{4\pi}{3}$ units

Area of sector $A0B = \frac{1}{2}r^2\theta = \frac{1}{2}(4^2)\left(\frac{\pi}{3}\right) = \frac{8\pi}{3} sq.units$

Example 6.2.2: The radius of a circle is $3cm$, what is the angle contained by a sector of area $18cm^2$?

Solution



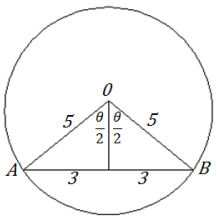
Area $= 18cm^2, \theta =$?

Area of sector $A0B = \frac{1}{2}r^2\theta \rightarrow 18 = \frac{1}{2} \times 3 \times 3 \times \theta \rightarrow \theta = \frac{18}{9} \times 2 = 4 \rightarrow \theta = 4rads$

Since $1rad = 57.296° \therefore \theta = 4 \times 57.296°$

Example 6.2.3: A segment is cut off a circle of radius $5cm$ by chord AB, $6cm$ long. What is the length of the minor arc AB.

Solution

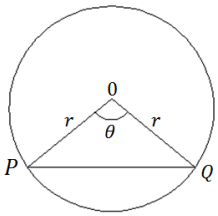


From the diagram, $Sin\frac{\theta}{2} = \frac{3}{5} = 0.6 \rightarrow \frac{\theta}{2} = 36°49' \ \rightarrow \ \theta = 73°38'$

$\therefore$ arc $AB = \frac{\theta}{360}(2\pi r) = \frac{73.63 \times 5\pi}{180}$

Example 6.2.4: A chord PQ of a circle with radius$r$, subtends an angle θ at the centre. Show that area of the minor segment $PQ = \frac{1}{2}r^2(\theta - Sin\theta)$ and write down the area of the major segment PQ in terms of r and θ.

Solution



To show that area of minor segment PQ $= \frac{1}{2}r^2(\theta - Sin\theta)$, recall that

Area of sector $P0Q = \frac{1}{2}r^2\theta$ and Area of $\Delta POQ = \frac{1}{2}r^2 Sin\theta$

Therefore, Area of the minor segment $PQ = \frac{1}{2}r^2\theta - \frac{1}{2}r^2 Sin\theta = \frac{1}{2}r^2(\theta - Sin\theta)$

and

Area of major segment $= \pi r^2 - \frac{1}{2}r^2(\theta - Sin\theta) \ = r^2(\pi - \frac{1}{2}\theta + \frac{1}{2}Sin\theta)$

$= \frac{1}{2}r^2(2\pi + Sin\theta)$

Take Home Exercises

1. Convert to degree

   (a) $\frac{3\pi}{2}rad$ (b) $4\pi rad$ (c) $\frac{7\pi}{2}rad$ (d) $\frac{5\pi}{2}rad$ (e) $5\pi rad$ (f) $\frac{\pi}{36}rad$

   (g) $\frac{7\pi}{12}rad$ (b) $\frac{7\pi}{2}rad$

2. Convert to radians, leaving π in your answer

(a) $15°$ (b) $540°$ (c) $45°$ (d) $360°$ (e) $90°$ (f) $37°30'$

(g) $12\frac{1}{2}°$ (h) $12'$

6.3 Measuration of Solid Shapes

1. Volume of a cube $= L \times L \times L = L^3$ or $L \times B \times H$
   $= area \times height$, surface area $= 6L^2$
2. Volume of a cuboid $L \times B \times H =$ Area $\times$ height
   Surface area $= 2(Lb + Lh + bh)$
3. Volume of triangular prism $= A \times h$ (area of Δface $\times$ height)
   Or $V = \frac{1}{2}bh \times$ height of prism
4. Volume of cylinder $\pi r^2 h$, Area $= 2\pi rh$
   Curved surface area $= 2\pi rh$
   Total surface area $= 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$
5. Volume of Cone $= \frac{1}{3}\pi r^2 h$
   Curved surface $= \pi rL$
   Total surface area $= \pi r^2 + \pi rL$
6. Volume of Sphere $= \frac{4}{3}\pi r^3$
   Surface area $4\pi r^2$

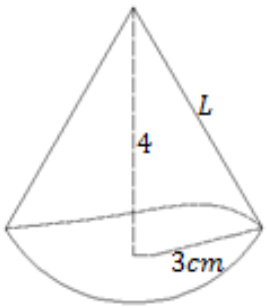
Example 6.3.1: Calculate the vol. of a cuboid whose length is $3cm$, breadth $5cm$ and height $4cm$.

Solution:

Volume of Cuboid $= L \times B \times H$

$$= 3 \times 5 \times 4 = 60cm^3$$

If the height of a cone is $4cm$ with base radius $3cm$, calculate the total surface area of the cone and its volume.



$L^2 = 4^2 + 3^2 \rightarrow L^2 = 16 + 9 \rightarrow L = \sqrt{25} = 5cm$

Total surface area $= \pi r(L + r)$

Volume of the cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 3^2 \times 4 = \frac{264}{7} = 37.71cm^3$$

Example 6.3.2: If a cylinder has an end radius of $3.5cm$ and a height of $5cm$ find the area of the cylinder and its volume.

Solution

Area $= 2\pi rh = 2 \times \frac{22}{7} \times 3.5 \times 5 = 44 \times 0.5 \times 5 = 110cm^2$

Volume of cylinder $= \pi r^2 h$

$$= 2 \times \frac{22}{7} \times 3.5 \times 3.5 \times 5 = 22 \times 0.5 \times 3.5 \times 5 = 192.5cm^3$$

Example 6.3.3: A cylindrical tank has a capacity of $3080cm^3$, find its depth and total surface area if the diameter of the base is $14cm$. $\left[\pi = \frac{22}{7}\right]$

Solution

$V = 3080, D = 14 \rightarrow r = 7$

Using $V = \pi r^2 h$, we have $3080 = \frac{22}{7} \times 7 \times 7 \times h \rightarrow h = \frac{3080}{154} = 20cm$

$\therefore Depth = 20cm$

Total surface area $= \pi r^2 + 2\pi rh \ = \pi r(2h + r)$

$$= \frac{22}{7} \times 7(2 \times 20 \times 7) = 22 \times 47 = 1034 cm^2$$

Example 6.3.4: Calculate the total surface area of solid hemisphere of radius $6.8cm$. ($\pi = 0.4971$)

Solution

Total surface area = curved surface area + plane surface area

∴Total surface area $= 2\pi r^2 + \pi r^2 = 3\pi r^2 = 3 \times 0.4971 \times 6.8^2 = 680.96 cm^2$

7.0 Analytical Geometry

7.1 Cartesian Coordinate System

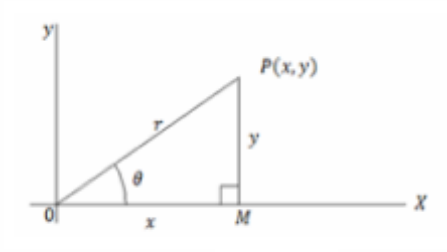


Fig.1

In Fig.1, let P be the point whose horizontal and vertical distances from the origin are x and y respectively. Then P can be described in terms of x and y as P(x, y). This method of describing the position of a point in a plane is called the Cartesian (or rectangular) Coordinates system. The horizontal distance $OM = x$ is called the abscissa and vertical distance $PM = y$ is the ordinate.

7.2 Polar Coordinate System

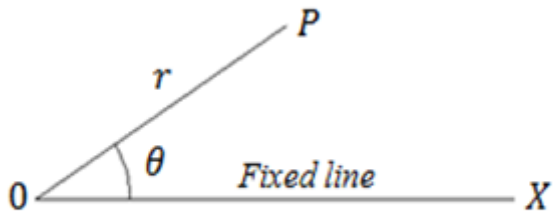


Fig.2

Let $\mathbf{O}$ be a fixed point and OX a fixed line, then if P is any point, let $OP = r$ and angle $POX = \theta$, then r and θ are called the polar coordinates of the point P. The coordinates, r and θ, of the point P is usually written as (r, θ). O is called the origin and OX the initial line.

NOTE: Bearing is usually measured in a clockwise sense from north. In mathematics the polar coordinates θ is normally represented in an anticlockwise sense.

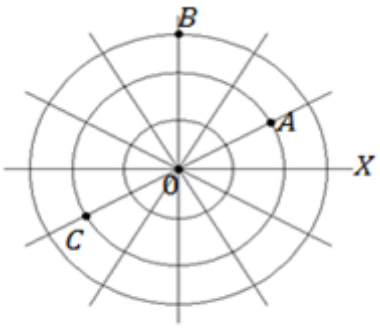


Fig.3

Thus the coordinates of A are $(2,30°)$ and B is $(3,90°)$, A point may be described in different ways. For instance, C may be written as $(2,210°)$, $(2,-150°)$, $(-2,30°)$ and so on.

If for any reason, a unique way of referencing to each point is required, r may be taken to be positive and θ to lie in the range $-180° < \theta < 180°$.

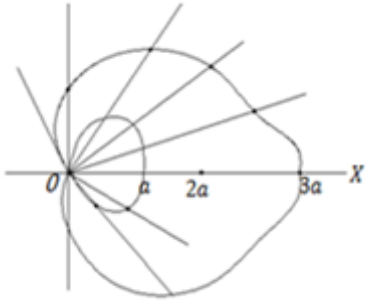
Example 7.1: Sketch the curve whose polar equation is $r = a(1 + 2Cos\theta)$

Solution

Take values of θ, and calculate $1 + 2Cos\theta$ as follows.

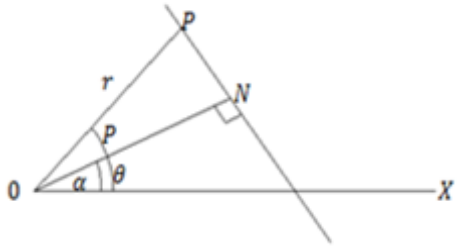
θ	0°	30°	45°	60°	90°	120°	135°	150°	180°
$2Cos\theta$	2	1.732	1.414	1	0	-1	-1.414	-1.732	-2
$1 + 2Cos\theta$	3	2.732	2.414	2	1	0	-0.414	-0.732	-1

Now if α is any angle, $Cos(-\alpha) = Cos\alpha$, therefore the same values of r will be obtained for negative values of θ. Thus the curve may be completed.



Example 7.2: Find the polar equation of a line such that the perpendicular to it from the origin is of length p and makes an angle α with the initial line.

Solution



In the ΔONA, N is a right angle and angle $PON = \theta - \alpha$ (or $\alpha - \theta$)

$\therefore rCos(\theta - \alpha) = P$ or $rCos(\alpha - \theta) = P$

$\therefore$ in either case, the polar equation of the line is $rCos(\theta - \alpha) = P$

7.3 Relations between Polar and Cartesian Coordinates

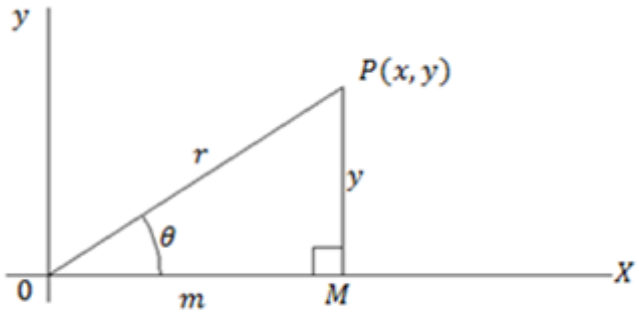


Fig. 4

In Fig.6, P is the point (x, y) in Cartesian coordinates and (r, θ) in polar coordinates, and PM is an ordinate.

Now by the definition of Cosine and Sine,

we have $Cos\theta = \frac{x}{y}$ and $Sin\theta = \frac{y}{r}$

$\rightarrow \quad x = rCos\theta$ /'

and

$$y = rSin\theta \qquad 0'$$

Therefore, x and y has been given in terms of r and θ by equations (1) and (2)

Squaring (1) and (2) and adding them, we get

$x^2+y^2 = r^2Cos^2 + r^2Sin^2\theta \quad \rightarrow \quad x^2 + y^2 = r^2(Cos^2\theta + Sin^2\theta)$

$\rightarrow \ r^2 = x^2 + y^2$ (Since $Cos^2\theta + Sin^2\theta = 1$)

$\therefore \ r = \sqrt{x^2+y^2}$,, , 1'

Dividing (1) by (2) gives $\frac{rSin\theta}{rCos\theta} = \frac{y}{x} \leftrightarrow tan\theta = \frac{y}{x}$

$\therefore \ \theta = tan^{-1}\frac{y}{x}$,, 2'

Therefore r and θ can also be given in terms x and y by the equations.

(1), (2), (3) and (4) give the relationship between Polar and Cartesian coordinates.

Examples 7.3.1: Find the Cartesian equations equivalent to the given polar equations

i. $r = a(1 + 2Cos\theta)$ ii. $rCos(\theta - \alpha) = P$

Solutions

i. Given $r = a(1 + 2Cos\theta)$, the presence of $Cos\theta$ suggests the relation $x = rCos\theta$, so multiplying $r = a(1 + 2Cos\theta)$ through by r gives

$r^2 = a(r + 2rCos\theta) \quad \rightarrow \quad x^2 + y^2 = a\left(\sqrt{(x^2 + y^2)} + 2x\right)$

$\rightarrow \quad x^2+y^2 - 2ax = a\sqrt{x^2 + y^2} \quad \rightarrow \quad (x^2 + y^2 - 2ax)^2 = a^2(x^2+y^2)$

$\therefore$ The Cartesian equation of $r = a(1 + 2Cos\theta)$ is $(x^2 + y^2 - 2ax)^2 = a^2(x^2+y^2)$

(ii). $rCos(\theta - \alpha) = P \leftrightarrow rCos\theta \ Cos\alpha + rSin\theta \ Sin\alpha = P$

Since $rCos\theta = x$ and $rSin\theta = y \therefore \ rCos\theta \ Cos\alpha + rSin\theta \ Sin\alpha = P$ becomes

$xCos\alpha + ySin\alpha = P$

Hence, the Cartesian equation of $rCos(\theta - \alpha) = P$ is $xCos\alpha + ySin\alpha = P$

Examples 7.3.2: Find the polar equation of the circle whose Cartesian equation is $x^2 + y^2 = 4x$

Solution

Putting $x = rCos\theta,\ y = rSin\theta$ in $x^2 + y^2 = 4x$ gives

$r^2Cos^2\theta + r^2Sin^2\theta = 4rCos\theta \qquad \rightarrow r^2(Cos^2\theta + Sin^2\theta) = 4rCos\theta$

$\rightarrow\ r^2 = 4rCos\theta \qquad \rightarrow \qquad r = 4Cos\theta$

$\therefore$ The polar equation of the circle $x^2 + y^2 = 4x$ is $r = 4Cos\theta$

Take Home Exercises

1. Sketch the curves given by the following polar equations
   (i) $r = a(1 + Cos\theta)$ (ii) $r = aCos2\theta$
   (iii) $r = a(1 - Sin\theta)$ (iv) $r = aSin3\theta$
2. Obtain the polar equations of the following loci
   (i) $x^2 + y^2 = a^2$ (ii) $x^2 - y^2 = a^2$ (iii) $x^2 + y^2 - 2y = 0$
3. Obtain the Cartesians equations of the following loci
   (i) $r = 2$ (ii) $r = a(1 + Cos\theta)$
   (iii) $r = rCos\theta$ (iv) $r = atan\theta$

8.0 The Straight Line Equation

The equation of a straight line can be given in four different forms, viz: Point Slope form, Slope Intercept form, Double Intercept Form and Two Point form (Double Point form). Each of these depends on the particular information about the line made available.

8.1 Point – Slope Form

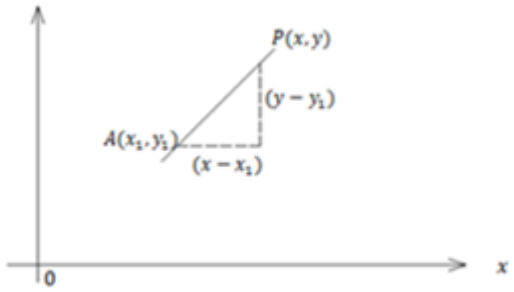


Fig. 5

In Fig.5, let $P(x, y)$ be any point and $A(x_1, y_1)$ be a particular point on a straight line of Slope m.

Then $m = \frac{y-y_1}{x-x_1}$ or $y - y_1 = m(x - x_1)$.

The equation of a straight line with slope M and which passes through $pt_A(x_1, y_1)$ is $y - y_1 = m(x - x_1)$

Example 8.1: Find the equation of the line passing through

(i) (4,3) with slope $\frac{1}{4}$ (ii) (0,5) with slope -3

Solutions

(i) Let $p(x, y)$ be any point other on of the line passing through (4,3) with slope $\frac{1}{4}$

Then $y - 3 = \frac{1}{4}(x - 4) \leftrightarrow 4y - 12 = x - 4 \leftrightarrow 4y - x - 8 = 0$

(ii) Let $p(x, y)$ be any point other on of the line passing through $(0,3)$ with slope -3

Then $y - y_1 = m(x - x_1) \rightarrow y - 5 = -3(x - 0) \rightarrow y - 5 = -3x \rightarrow y + 3x - 5 = 0$

8.2 Slope – Intercept form

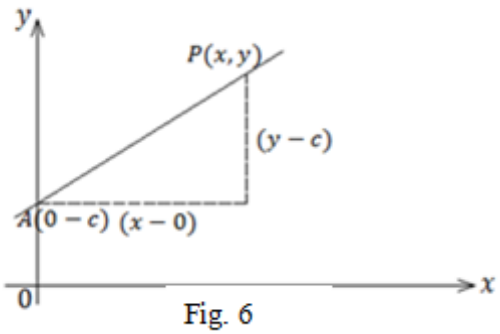


Fig. 6

In Fig.6 let the y intercept be $A(0, c)$, $P(x, y)$ be any other point on the line and the slope of the line through (x, y) and (o, c) be m.

Then $m = \frac{y-c}{x-o}$ i.e. $y - c = mx \leftrightarrow y = mx + c$

Hence, the equation of the straight line having slope m and intercept of c on the y-axis is $y = mx + c$

Example 8.2.1: Find the equation of the straight line of gradient 4 which passes through the origin.

Solution

Let $p(x, y)$ is any point on the line, other than the origin $O(0,0)$.

The gradient of the line may be written as $\frac{y-0}{x-0} = \frac{y}{x}$

$\therefore \frac{y}{x} = 4 \rightarrow y = 4x \leftrightarrow y - 4x = 0$ is the required equation.

Example 8.2.2: Find the equation of straight line of gradient 3 which cuts the y-axis at $(0,1)$

Solution

Let $p(x,y)$ be any point on the line.

Then the required equation is given by $\frac{y-1}{x-0} = 3 \rightarrow\ y = 3x + 1$

NB:

- $y = mx + c$ is a line of gradient m, passing through (o, c)
- $y = mx$ is a line of gradient m, passing through origin
- $y = c$ is a line of zero gradient (i.e. parallel to the x-axis)
- $x = k$ is a line parallel to the y-axis

Example 8.2.3: Find the gradient of the straight line $7x + 4y + 2 = 0$ and its intercepts on the axis

Solution

$7x + 4y + 2 = 0 \quad \rightarrow \quad 4y = -7x - 2 \quad \rightarrow \quad y = -7x - \frac{1}{2}$

Observe that $y = -7x - \frac{1}{2}$ is in the form $y = mx + c$, where $m = -\frac{7}{4}$ and $c = -\frac{1}{2}$

Therefore the gradient is $m = -\frac{7}{4}$ and the intercepts on y-axis is $-\frac{1}{2}$.

Alternative Method

To find intercept on y-axis, put $x = 0$ in $7x + 4y + 2 = 0$, then $4y = -2 \leftrightarrow y = -\frac{1}{2}$

To find intercept on x-axis, put $y = 0$ in $7x + 4y + 2 = 0$, then $7x = -2 \leftrightarrow x = -\frac{2}{7}$

Example 8.2.4: Find the equation of the straight line of gradient $-\frac{2}{3}$, which passes through $(-4,1)$

Solution

Let $p(x,y)$ be any point on the line. The gradient is $\frac{y-1}{x-(-4)} = \frac{y-1}{x+4}$

But gradient is given as $-\frac{2}{3}$ $\therefore \frac{y-1}{x+4} = -\frac{2}{3}$

$\rightarrow$ $3(y-1) = -2(x+4)$ $\rightarrow$ $2x+3y+5=0$

Take Home Exercises

Find the equations of the straight lines joining of given gradients, passing through the points:

(i) $4, (1,3)$, (ii) $3, (-2,5)$, (iii) $-\frac{5}{3}, \left(\frac{1}{4}, -3\right)$ (iv) $\frac{1}{3}, (2,-5)$

8.3 Double Intercept Form

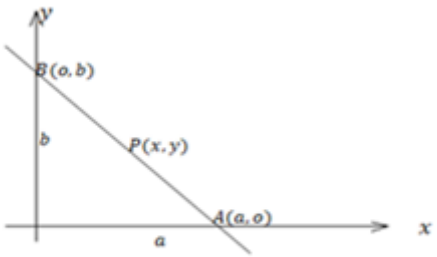


Fig. 7

In Fig.7 let $P(x,y)$ be any point on the line AB, where $A(a,0)$ and $B(0,b)$ are points on the same line. Since the three points A, B, P are collinear.

Then gradient AP = gradient PB

i.e. $\frac{y-0}{x-a} = \frac{b-y}{0-x}$ $\rightarrow -xy = (b-y)(x-a)$

$\rightarrow$ $0 = xb - ab + ay$ $\rightarrow$ $bx + ay = ab$

Dividing both sides of $bx + ay = ab$ by ab gives $\frac{x}{a} + \frac{y}{b} = 1$

The equation of the straight line whose intercept on the x-axis and y-axis are $(a, 0)$ and $(0, b)$ is $\frac{x}{a} + \frac{y}{b} = 1$

Example 8.3.1: Find the equation of the line which intercepts the axes at $(-8,0)$ and $(0,5)$

Solution

Putting $a = -8$ and $b = 5$ in $\frac{x}{a} + \frac{y}{b} = 1$ gives

$\frac{x}{-8} + \frac{y}{5} = 1 \quad \rightarrow \quad -5x + 8y - 40 = 0$

Example 8.3.2: Find the equation of the lines through $(1, -6)$ if the product of the intercepts for each line is 1

Solution

$\frac{x}{a} + \frac{y}{b} = 1$,,, , /'

Since $ab = 1,\ b = \frac{1}{a}$, , 0'

Putting equation (2) in (1) we have

$\frac{x}{a} + \frac{y}{\left(\frac{1}{a}\right)} = 1 \ \rightarrow \frac{x}{a} + ay = 1$ 3)

Since point $(1, -6)$ lies on the line, then we substitute $x = 1,\ y = -6$ into equation (3) to give

$\frac{1}{a} - 6a = 1 \rightarrow 6a^2 + a - 1 = 0 \ \rightarrow (3a - 1)(2a + 1) = 0 \leftrightarrow a = \frac{1}{3}, -\frac{1}{2}$

Putting $a = \frac{1}{3}$ or $-\frac{1}{2}$ into equation (1) give

$3x + \frac{1}{3}y = 1 \rightarrow 9x + y = 3$ or $-2x - \frac{1}{2}y = 1 \rightarrow 4x + y = -2$

8.4 Two – Point form (Double – Point form)

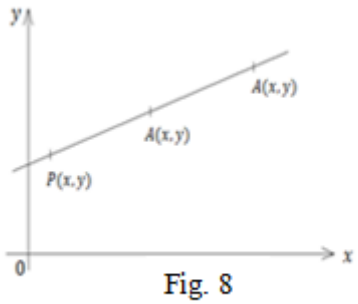


Fig. 8

In Fig.8 let $P(x, y)$ be any other point on the line passing through the points $A(x_1, y_1)$ and $B(x_2, y_2)$.

Slope of line through $P(x, y)$ and $A(x_1, y_1) =$ Slope of line through $A(x_1, y_1)$ and $B(x_2, y_2)$.

Hence $\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1} \rightarrow y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x - x_1)$

Therefore, the equation of the straight line through any two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x - x_1)$

Example 8.4.1: Determine the equation of the line passing through points $(-4,1)$ and $(3,-5)$

Solution

Using $y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x - x_1)$ gives

$y - 1 = \frac{-5-1}{3-(-4)}[x - (-4)] \rightarrow y - 1 = \frac{-6}{7}(x + 4) \rightarrow 7y + 6x + 17 = 0$

Example 8.4.2: Find the equation of the straight line joining the points $(-5,2)$ and $(3,-4)$

Solution

Gradient $= \frac{2-(-4)}{-5-3} = \frac{6}{-8} = -\frac{3}{4}$, /'

If $P(x, y)$ is any point on the line other than $(3, -4)$, the gradient may be written as

$\frac{y-(-4)}{x-3} = \frac{y+4}{x-3}$, 0'

Equating RHS of (1) and (2) yields

$\frac{y+4}{x-3} = -\frac{3}{4}$ → $4(y+4) = -3(x-3)$

→ $4y + 16 = -3x + 9$ → $3x + 4y + 7 = 0$

Therefore, the required equation is $3x + 4y + 7 = 0$

Alternative Method

Since gradient of the line through $(3, -4)$ is $m = -\frac{3}{4}$

Therefore $y = mx + c$ becomes $y = -\frac{3}{4}x + c$ /'

Since the line through $(3, -4)$, then we substitute $x = 3$ and $y = -4$ in (1) to obtain the value of c.

Thus, $-4 = -\frac{3}{4}(3) + c \leftrightarrow -4 + \frac{9}{4} = +c \leftrightarrow -\frac{7}{4} = c$

Hence the required equation is $y = -\frac{3}{4}x - \frac{7}{4}$ i.e. $3x + 4y + 7 = 0$

Take Home Exercises

Find the equations of the straight lines joining the following pairs of points

(i) $(1,6)$ and $(5,9)$ (ii) $(3,2)$ and $(7, -3)$ (iii) $(-3,4)$ and $(8,1)$

(iv) $\left(\frac{1}{2}, 2\right)$ and $\left(3, \frac{1}{2}\right)$ (v) $(-1, -4)$ and $(4, -3)$

8.5 Perpendicular form of the equation of a straight line

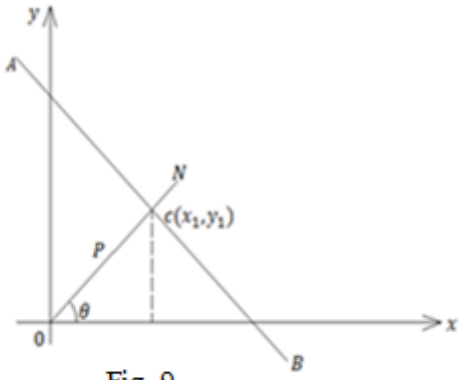


Fig. 9

In Fig. 9, let AB be given line and the coordinates of the point C be (x_1, y_1), P = length of the perpendicular line from the origin $(0,0)$ to the line concerned and $\theta =$ angle subtended by the perpendicular line with the positive end of the x-axis.

Draw ON perpendicular to AB so that $x_1 = PCos\theta$ and $y_1 = PSin\theta$

Slope of $AB = -\frac{1}{slope\ of\ ON} = -\frac{1}{\tan\theta} = -\frac{Cos\theta}{Sin\theta}$

If (x, y) is any other point on AB, then by the point-slope formula

$y - y_1 = -\frac{Cos\theta}{Sin\theta}(x - x_1) \quad \rightarrow \quad y - PSin\theta = -\frac{Cos\theta}{Sin\theta}(x - PCos\theta)$

$\rightarrow \quad ySin\theta - PSin^2\theta = -xCos\theta + PCos^2\theta$

$\rightarrow \quad xCos\theta + ySin\theta - P(Sin^2\theta + Cos^2\theta) \rightarrow xCos\theta + ySin\theta - P = 0$

$\therefore \quad xCos\theta + ySin\theta - P = 0$ is the perpendicular form of the equation of a straight line.

Example 8.5: Reduce the equation of the line $3x - 4y - 6 = 0$ to its normal form. Find P and θ.

Solution

From the given equation of the line, $A = 3,\ B = -4$

$$\sqrt{A^2 + B^2} = \sqrt{3^2 + (-4)^2} = \sqrt{25} = 5$$

Since $C = -6$ is positive, $\sqrt{A^2 + B^2}$ is assigned the positive sign.

∴ The equation normal form is

$$\frac{3}{\sqrt{3^2+(-4)^2}}x - \frac{4}{\sqrt{3^2+(-4)^2}}y - \frac{6}{\sqrt{3^2+(-4)^2}} = 0 \quad \text{i.e.} \quad \frac{3}{5}x - \frac{4}{5}y - \frac{6}{5} = 0$$

Consequently, $Cos\theta = \frac{3}{5} = 0.6,\ Sin\theta = -\frac{4}{5}$ and $P = \frac{6}{5} \rightarrow \theta = Cos^{-1}(0.6)$

Since $Cos\theta$ is positive and $Sin\theta$ is negative, θ is in a fourth quadrant angle.

Hence $\theta = 360^0 - Cos^{-1}(0.6) \ \rightarrow\ \theta = 360^0 - 53°8' \ \rightarrow\ \theta = 306°52'$

8.6 The Distance of a Point from a Line

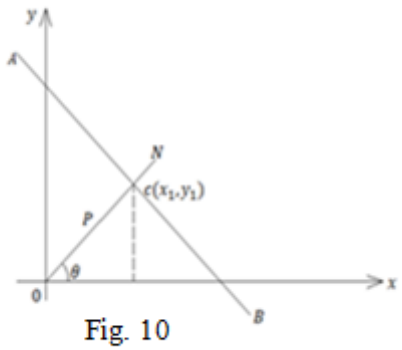


Fig. 10

In Fig. 10, given a point $P_1(x_1, y_1)$ and the line $ax + by + c = 0$, the perpendicular distance of (x_1, y_1) from the line $ax + by + c = 0$ is $\pm\frac{ax_1+by_1}{\sqrt{(a^2+b^2)}}$

Example 8.6.1: Find the distances of the points (i) $(1,3)$, (ii) $(-3,4)$, (iii) $(4,-2)$ from the line $2x + 3y - 6 = 0$

Solutions

The distance of (x_1, y_1) from the line $ax + by + c = 0$ is $\pm\frac{ax_1+by_1}{\sqrt{(a^2+b^2)}}$

$\therefore$ the distance of $(1,3)$, $(-3,4)$, $(4,-2)$ from $2x + 3y - 6 = 0$ are respectively

i. $\pm\frac{2\times1+3\times3-6}{\sqrt{(2^2+3^2)}} = \pm\frac{2+9-6}{\sqrt{13}} = \pm\frac{5}{\sqrt{13}}$

ii. $\pm\frac{2\times(-3)+3\times4-6}{\sqrt{2^2+3^2}} = \pm\frac{-6+12-6}{\sqrt{13}} = \pm\frac{0}{\sqrt{13}} = 0$

iii. $\pm\frac{2\times4+3(-2)-6}{\sqrt{2^2+3^2}} = \pm\frac{8-6-6}{\sqrt{13}} = \pm\frac{-4}{\sqrt{13}}$

$\therefore$ the distances are $\frac{5}{13}\sqrt{13}$, and $\frac{4}{13}\sqrt{13}$ respectively.

Note: The necessity for the sign in the formula is illustrated by the example. The line $2x + 3y - 6 = 0$ divides the plane into two parts above the line $y > \frac{-2}{3}x + 2$ and the expression $2x + 3y - 6$ is $negative$ below the line.

$y > \frac{-2}{3}x + 2$, and $2x + 3y - 6$ is $negative$

Note: The perpendicular distance is zero if the point lies on the line.

Example 8.6.2: Find the distance d from line $6x - 8y + 12 = 0$ to point $(-1,7)$

Solution

$$d = \frac{6(-1)-8(7)+12}{\sqrt{(6^2+(-8)^2)}} = \frac{-50}{-10} = 5units$$

Since d is positive, the point $(-1,7)$ and the origin are on the opposite sides of the line.

Note: The minus sign in front of the square root appears because of the $+ve$ sign with the constant p i.e. $p = +12$.

Take Home Exercises: Find the distances of the given points from the following lines

i. $(3,2), 3x - 4y + 4 = 0$

ii. $(2,-1), 5x + 12y = 0$

iii. $(2,5), x + y - 1 = 0$

iv. $(p,q), 3x + 4y - 3p = 0$

v. $(x,y), 12x - 5y + 7 = 0$

8.7 Distance from a Line to a Point

The perpendicular distance d from a given line to a given point with coordinates (x_1, y_1) is given by

$d = x_1 Cos\theta + y_1 Sin\theta - P$ or $d = \frac{x_1 A + y_1 B - P}{\sqrt{A^2 + B^2}}$

Note: If (x_1, y_1) and the origin are on opposite sides of the given line L, the distance d is positive, if they are on the same side of the line, d is negative.

Proof

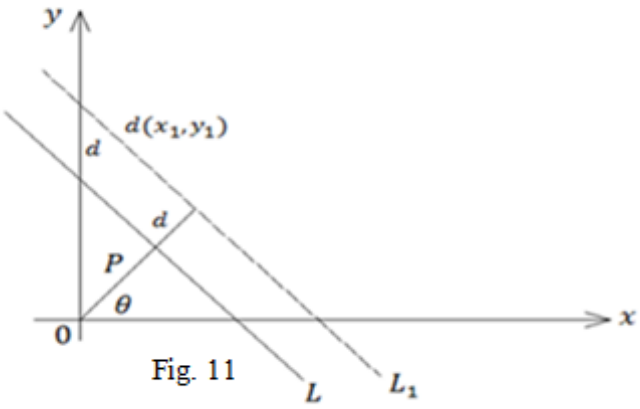


Fig. 11

In Fig.11, draw line L_1 through the given point (x_1, y_1) and parallel to L. The perpendicular distance from line L to point (x_1, y_1) is d.

The equation of L is $xCos\theta + ySin\theta - P = 0$, , /'

The equation of L_1 is $xCos\theta + ySin\theta - (P + d) = 0$ .. ,, 0'

Since the point (x_1, y_1) lies on L_1, then we can put $x = x_1$ and $y = y_1$ in (2) to get

$x_1 Cos\theta + y_1 Sin\theta - (P + d) = 0$, 1'

Solving for d in (3) gives $d = x_1 Cos\theta + y_1 Sin\theta - P$

Example 8.7.1: Calculate the distance d from line $6x - 8y + 12 = 0$ to $P(-1,7)$

Solution

Using the formula, $d = \frac{x_1 A + y_1 B - P}{\sqrt{A^2 + B^2}}$ yields

$$d = \frac{6(-1) - 8(7) + 12}{\sqrt{6^2 + (-8)^2}} = \frac{-50}{-10} = 5units$$

Since d is positive, the point $(-1,7)$ and the origin are on the opposite sides of the line.

Note that the minus sign in front of the square root appears because of the positive sign with the const P i.e. $P = +12$

Example 8.7.2: Find the equation of the line passing through the point of intersection of the lines $3x - 5y + 9 = 0$ and $4x + 7y - 28 = 0$ and through.

Solution

$3x - 5y + 9 + k(4x + 7y - 28) = 0$ is the equation of the system of lines passing through the intersection of the two given lines.

Since the required line passes through $(-3, -5)$ then

$3(-3) - 5(-5) + 9 + k(4(-3) + 7(-5) - 28) = 0$ i.e. $25 - 75k = 0 \therefore k = \frac{1}{3}$

The required line is $3x - 5y + 9 + \frac{1}{3}(4x + 7y - 28) = 0$

i.e. $9x - 15y + 27 + 4x + 7y - 28 = 0 \ \therefore 13x - 18y - 1 = 0$

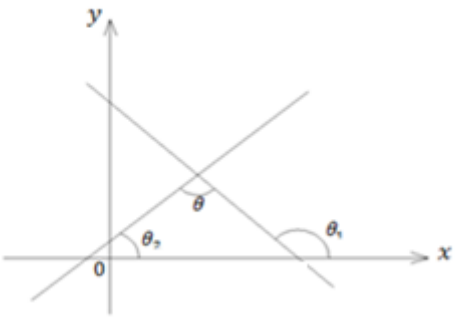
8.8 Determination of Angle between two lines of Slopes m_1 and m_2

The angle (θ) between two lines of slopes m_1 and m_2 is given by

$$\theta = tan^{-1}\frac{(m_2 - m_1)}{1 + m_1, m_2}$$

Example 8.8: Find the acute angle between the lines $2x + y - 6 = 0$ and $3x - 2y + 2 = 0$

Solution



If lines $2x + y - 6 = 0$ and $3x - 2y + 2 = 0$ make angles θ_1 and θ_2 with the x-axis then it follows that one of the angles between two lines $\theta = \theta_1 - \theta_2$ (because exterior angle of a triangle is equal to the sum of the two interior opposite angles).

Gradients of $2x + y - 6 = 0 \rightarrow y = -2x + 6$ and $3x - 2y + 2 = 0 \rightarrow y = \frac{3}{2}x + 1$

are $m_1 = -2$ and $m_2 = \frac{3}{2}$ respectively.

Using $\theta = tan^{-1}\frac{(m_2 - m_1)}{1 + m_1, m_2}$ yields

$$\theta = tan^{-1}\frac{-2 - \frac{3}{2}}{1 + \left(-2 \times \frac{3}{2}\right)} \rightarrow \theta = tan^{-1}\left(\frac{7}{4}\right) = tan^{-1}1.75 = 60°21'$$

Take Home Exercises: Find the angles between the following pairs of straight lines use tan^{-1} in your answers.

(i) $y = x - 6,\ y = 3x + 1$ (ii) $2x + y + 3 = 0,\ x + y = 0$

(iii) $4x - 2y + 3 = 0,\ 6x - 2y + 1 = 0$ (iv) $3x + 4y - 2 = 0,\ 2x - 2y - 4 = 0$

8.9 Relationship between Slopes of Parallel Lines

If two lines L_1 and L_2 are parallel, then their slopes m_1 and m_2 respectively are equal i.e. $m_1 = m_2$.

8.10 Relationship between Slopes of Perpendicular Lines

If the lines L_1 and L_2 are perpendicular, the product of their slopes is minus unity i.e. $m_1 m_2 = -1$. In general, if two lines are perpendicular, the product of their gradient is -1. In other words, if the gradient of one is m, the gradient of the other is $-\frac{1}{m}$.

Example 8.9: Write down the gradients of the lines perpendicular to the lines of gradient

(i) 3 (ii) $\frac{1}{4}$ (iii) -6 (iv) $-\frac{2}{3}$ (v) $2m$ (vi) $-\frac{h}{a}$ (vii) $-\frac{m}{2}$

Solution

(i) grad = $-\frac{1}{3}$ (ii) -4, $\frac{1}{6}$, $\frac{3}{2}$, $-\frac{1}{2m}$, $\frac{a}{b}$, $\frac{2}{m}$

9.0 Equation of Locus of Points

9.1 Definition: A locus is represented by an equation such that;

i. Every point whose coordinates satisfies the equation lies on the locus.

ii. The coordinates of every point which lies on the locus satisfy the equation.

Let A and B are the points (x_1, y_1) and (x_2, y_2) and let $P(x, y)$ be the coordinate of a point which is equidistance from A and B.We may readily find the equation connecting the coordinates.

Expressed geometrically, the condition to be satisfied by P is $PA = PB \rightarrow PA^2 = PB^2$

Since $PA^2 = (x - x_1)^2 + (y - y_1)^2$ and $PB^2 = (x - x_2)^2 + (y - y_2)^2$

Therefore, the equation which must be satisfied by the coordinates of P is

$(x - x_1)^2 + (y - y_1)^2 = (x - x_2)^2 + (y - y_2)^2$

i.e $2x(x_2 - x_1) + 2y(y_2 - y_1) + ({y_1}^2 - {y_2}^2) = 0$

Example 9.1.1: Find the locus of the points equidistant from $(3,2)$ and $(5, -1)$.

Solution

Let $P(x, y)$ be any point which is equidistance from $A(3,2)$ and $B(5, -1)$

Then $PA = PB \rightarrow PA^2 = PB^2$

Since $PA^2 = (x - 3)^2 + (y - 2)^2$ and $PB^2 = (x - 5)^2 + (y + 1)^2$

Therefore, the equation which must be satisfied by the coordinates of P is

$$(x - 3)^2 + (y - 2)^2 = (x - 5)^2 + (y + 1)^2$$

i.e. $x^2 - 6x + 9 + y^2 - 4y + 4 = x^2 - 10x + 25 + y^2 + 2y + 1$

$\rightarrow$ $4x - 6y - 13 = 0$

Hence, the equation of the locus of points equidistant from $(3,2)$ and $(5, -1)$ is

$4x - 6y - 13 = 0$

Example 9.1.2: Find the equation of the circle of radius two units with it centre at the origin.

Solution:

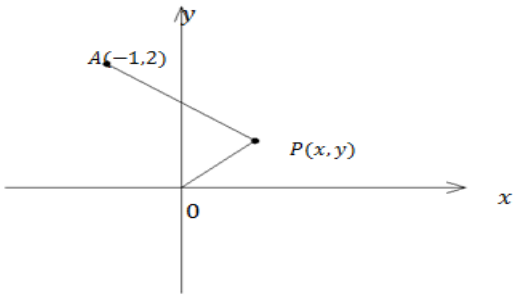
Let (x, y) be any point on the locus

Then by Pythagoras $x^2 + y^2 = 4$

Therefore, the equation of the circle is $x^2 + y^2 = 4$

Example 9.1.3: Find the locus of a point P, whose distance from the point $A(-1,2)$ is twice its distance from the origin.

Solution:



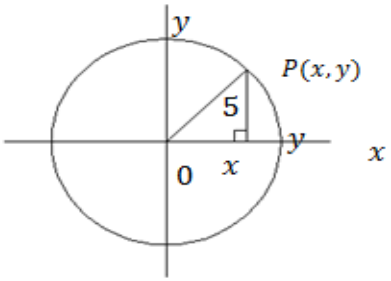
Let $P(x, y)$ be a point on the locus, then $PA = 2PO \rightarrow PA^2 = 4PO^2$

$\rightarrow (x-3)^2 + (y-2)^2 = 4(x^2 + y^2) \rightarrow x^2 + 2x + 1 + y^2 - 4y + 4 = 4x^2 + 4y^2$

Therefore, the locus of P is $3x^2 + 3y^2 - 2x + 4y - 5 = 0$

Example 9.1.4: Find the equation of a circle with centre at the origin and radius 5 units

Solution



Let (x, y) be any point on the locus, then by Pythagoras $x^2 + y^2 = 5^2$

Therefore, the equation of the circle is $x^2 + y^2 = 25$

Example 9.1.5: A point P moves so that its distance from $A(2,1)$ is twice its distance from $B(-4,5)$. What is the locus of P.

Solution

Let $P(x, y)$ be any point on the locus, then $PA = 2PB \rightarrow \quad PA^2 = 4PB^2$

i.e. $(x-2)^2 + (y-1)^2 = 4\{(x+4)^2 + (y-5)^2\}$

$\rightarrow \qquad x^2 - 4x + 4 + y^2 - 2y + 1 = 4\{x^2 + 8x + 16 + y^2 - 10y + 25\}$

$\rightarrow \qquad x^2 - 4x + y^2 - 2y + 5 = 4x^2 + 32x + 64 + 4y^2 - 40y + 100$

$\rightarrow \quad 3x^2 + 3y^2 + 36x - 38y + 159 = 0$

9.2 Equation of Circles

By definition, a circle is the locus of a point equidistance from a fixed point.

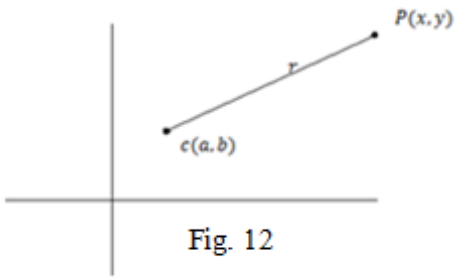


Fig. 12

To find the equation of a circle interior of the coordinates of its centre and radius, in the figure, observe that $PC^2 = r^2 \rightarrow (x-a)^2 + (y-b)^2 = r^2$

$\rightarrow x^2 - 2ax + a^2 + y^2 - 2yb + b^2 = r^2$

$\rightarrow\ x^2 + y^2 - 2ax - 2by + a^2 + b^2 - r^2 = 0$

The above can be written in the form $x^2 + y^2 + 2gx + 2fy + c = 0$

This is called the general equation of a circle.

Note

To find the coordinates of the centre and the radius from the equation of the circle $x^2 + y^2 + 2gx + 2fy + \text{c} = 0$, we do the following:

1. Write the equation $x^2 + y^2 + 2gx + 2fy + c = 0$ /'

   as $x^2 + 2gx + y^2 + 2fy = -c$, 0'

2. Add $g^2 + f^2$ to both sides of (2) to get

   $x^2 + 2gx + g^2 + y^2 + 2fy + f^2 = -c + g^2 + f^2$,, 1'

3. (3) can now be written as

   $(x+g)^2 + (y+f)^2 = g^2 + f^2 - c$ 2'

Therefore, coordinates of the centre = $(-g,-f)$ and Radius = $\sqrt{g^2+f^2-c}$

When the radius of a circle is 0, the circle is said to be a point circle. And when the radius is less than 0, the circle is said to be imaginary circle because the square roots of such number do not exist. If the equation represents the circle x-coordinate and y-coordinates must be the same.

The equation $x^2+y^2=r^2$ represent a circle with centre at the origin i.e. the point $(0,0)$ and radius r.

Example 9.2.1: Find the radius and the coordinates of centre of the following circle

$36x^2+36y^2-24x-36y-23=0$

Solution

Recall that the general equation of a circle is given by

$x^2+2gx+g^2+y^2+2fy+f^2=-c+g^2+f^2$,, /'

Transforming $36x^2+36y^2-24x-36y-23=0$ into the same form as (1) gives

$x^2+y^2-\frac{24}{36}x-\frac{36}{36}y-\frac{23}{36}=0$ 0'

Comparing RHS of (1) with RHS of (2) gives

$$2gx=-\frac{24x}{36}\leftrightarrow g=-\frac{1}{3},\quad 2fy=-y\ \leftrightarrow\ f=-\frac{1}{2}\text{ and }c=-\frac{23}{36}$$

∴ Centre $\left(-\frac{1}{3},-\frac{1}{2}\right)$ and Radius = $\sqrt{g^2+f^2-c}$ = $\sqrt{\left(-\frac{1}{3}\right)^2+\left(-\frac{1}{2}\right)^2+\frac{23}{36}}=1unit$

Example 9.2.2: Find the equation of the circle which has centre $(2,-1)$ and passes through $(4,2)$

Solution

Let the equation of the circle be $x^2+y^2+2gx+2fy+c=0$, /'

→ $-g=2$ or $g=-2$ and $-f=-1$ or $f=1$

Since it passes through $(4,2)$, we put $x=4$ and $y=2$ in (1) to get

$4^2 + 2^2 + 2g(4) + 2f(2) + c = 0 \rightarrow 20 + 8g + 4f + c = 0$,, 0'

Since $g = -2$ and $f = 1$, (2) becomes $20 - 16 + 4 + c = 0 \rightarrow c = -8$

∴ The required equation of the circles is

$x^2 + y^2 + 2x(-2) + 2y(1) - 8 = 0 \rightarrow x^2 + y^2 - 4x + 2y - 8 = 0$

Alternative Method

Let $P(x, y)$ be any point on the circle, A be the centre $(2, -1)$ and B be the point $(4, 2)$.

Then radius of the circle = PA = BA $\rightarrow PA^2 = BA^2$

Since $PA^2 = (x-2)^2 + (y+1)^2$ and $BA^2 = (4-2)^2 + (2+1)^2$

$\therefore (x-2)^2 + (y+1)^2 = (4-2)^2 + (2+1)^2 \rightarrow x^2 - 4x + 4 + y^2 + 2y + 1 = 13$

$\rightarrow x^2 + y^2 - 4x + 2y - 8 = 0$

Example 9.2.3: Find the equation of the circle which passes through the points $(1,1), (2,4), (3,2)$.

Solution

Let the equation of the circle be $x^2 + y^2 + 2gx + 2fy + c = 0$

Since its passes through $(1,1)$,

then $1 + 1 + 2g + 2f + c = 0 \rightarrow 2g + 2f + c = -2$,, ... , ,, /'

It passes through $(2,4)$,

then $4 + 16 + 4g + 8f + c = 0 \rightarrow 4g + 8f + c = -20$,..,, 0'

It passes through $(3,2)$,

then $9 + 4 + 6g + 4f + c = 0 \rightarrow 6g + 4f + c = -13$,,, 1'

Subtracting equation (1) from equation (2) i.e. $(2) - (1)$ yields

$2g + 6f = -18 \rightarrow g + 3f = -9$,, , 2'

Subtracting equation (2) from equation (3) i.e. $(3)-(2)$ gives

$2g-4f=7 \rightarrow g-2f=\frac{7}{2}$,,, ,,,,, 3'

Subtracting equation (4) from equation (5) i.e. $(5)-(4)$ gives

$-5f=\frac{25}{2} \rightarrow f=-\frac{5}{2}$, ,,,,, 4'

Substituting for f in equation (5) gives $g+5=\frac{7}{2} \rightarrow g=-\frac{3}{2}$

Substituting in $f=-\frac{5}{2}$ and $g=-\frac{3}{2}$ any of the equation (1) yields

$2(-\frac{3}{2})+2(-\frac{5}{2})+c=-2 \rightarrow c=6$

∴ Required equation of the circle is

$x^2+y^2+2x-\frac{3}{2}x+2x-\frac{5}{2}y+6=0 \rightarrow x^2+y^2-3x-5y+6=0$

Example 9.2.4: Find the equation of the circle with given centre and radius: Centre (2,3), radii 1

Solution

Using $(x-a)^2+(y-b)^2=r^2$ yield

$(x-2)^2+(y-3)^2=1^2 \rightarrow x^2+y^2-4x-6y+12=0$

Take Home Exercises

1. Find the equation of the circles with following centres and radii.

   i. Centre $(-3,4)$, radius 5, ii. Centre $(0,-5)$, radius 5

   iii. Centre $(3,0)$, radius $\sqrt{2}$ iv. Centre $\left(-\frac{1}{4},\frac{1}{3}\right)$, radius $\frac{1}{2}\sqrt{2}$

   v. Centre $\left(\frac{2}{3},-\frac{1}{3}\right)$, radius $\frac{2}{3}$

2. Find the radii and the coordinates of the centre of the following circles.

i. $x^2 + y^2 + 4x - 6y + 12 = 0$ ii. $x^2 + y^2 - 2x - 4y + 1 = 0$

iii. $x^2 + y^2 - 3x = 0$ iv. $x^2 + y^2 + 3x - 4y - 6 = 0$

3. Find the equation of the locus of a point such that

(i) Its distance from $(-1,2)$ and $(3,0)$ are equal

(ii) Its distance from $(-1,2)$ is double its distance from $(3,0)$.

(ii) State which of the two Loci is a circle is and find its centre and radius.

4. The coordinate of two points A and B are $(3,2)$ and $(1,-1)$ respectively. Two straight lines, drawn through A and B intersect at right angles at a point $P(x,y)$. Find out and simplify the equation of the locus of P. Show that this equation is that of a circle and calculate its radius and coordinates of its centre. Show that the origin lies within the circle.

5. Find the centre and radius of the circle $4x^2 + 4y^2 + 12x - 8y - 5 = 0$. Show that the point $P\left(-\frac{9}{2}, 1\right)$ lies outside this circle.

6. A circle passes through the points $P(2,-4)$ and $Q(-1,5)$ and has its centre on the x-axis obtain the equation of the circle.

7. Show that if $a^2 + c^2 > 4ad$ and $a \neq 0$ the equation $ax^2 + ay^2 + bx + cy + d = 0$ defines a circle with centre $\left(-\frac{b}{2a}, -\frac{c}{2a}\right)$ and radius $\frac{\sqrt{b^2+c^2-4cd}}{4a^2}$. Why is the condition $a \neq 0$ and $b^2 + c^2 > 4ad$ necessary for the equation?

9.3 Orthogonal Circles

If the tangents to two circles at their points of intersection are perpendicular, the circles are said to be orthogonal.

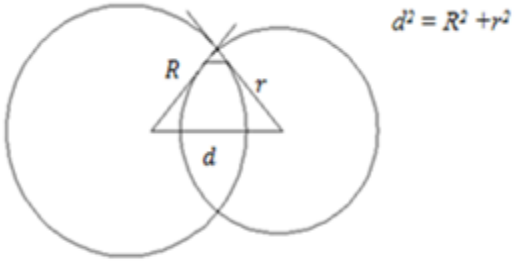


Fig. 13

Example 9.3: Show that the circles $x^2 + y^2 - 6x + 4y + 2 = 0$ and $x^2 + y^2 + 8x + 2y - 22 = 0$ are orthogonal

Solution

Comparing $x^2 + y^2 - 6x + 4y + 2 = 0$ with $x^2 + y^2 + 2gx + 2fy + c = 0$ gives $2gx = -6x$, $2fy = 4y$ and $c = 2 \rightarrow g = -3$, $f = 2$ and $c = 2$

Therefore, the circle $x^2 + y^2 - 6x + 4y + 2 = 0$ has centre $(3, -2)$ and radius $\sqrt{(3)^2 + 2^2 - 2} = \sqrt{11}$

Also, Comparing $x^2 + y^2 + 8x + 2y - 22 = 0$ with $x^2 + y^2 + 2gx + 2fy + c = 0$ gives $2gx = 8x$, $2fy = 2y$ and $c = -22 \rightarrow g = 4$, $f = 1$ and $c = -22$

Therefore, the circle $x^2 + y^2 + 8x + 2y - 22 = 0$ has centre $(-4, -1)$ and radius $\sqrt{4^2 + 1^2 + 22} = \sqrt{39}$

The sum of the squares of the radii is $\left(\sqrt{11}\right)^2 + \left(\sqrt{39}\right)^2 = 50$

The square of the distance between the centres = $(3 + 4)^2 + (-2 + 1)^2 = 50$

Since the sum of the squares of the radii = the square of the distance between the centres,

Therefore, the circles are orthogonal.

Alternative Method

The relation $2(g_1g_2 + f_1f_2) = c_1 + c_2$ can also be used to prove that two circles $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and $x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ are orthogonal.

Thus, from the given circles, we have $g_1 = -3$, $f_1 = 2$, $c_1 = 2$ and $g_2 = 4$, $f_2 = 1$, $c_2 = -22$.

Substituting these values in $2(g_1g_2 + f_1f_2) = c_1 + c_2$ we get

LHS = $2(g_1g_2 + f_1\mathrm{f}_2) = 2\{(-3)(4) + (2)(1)\} = -20$

RHS = $c_1 + c_2 = 2 - 22 = -20$

Since LHS = RHS, the circles are orthogonal.

Take Home Exercises: Show that the following pairs of circles are orthogonal

i. $x^2 + y^2 - 4x + 2 = 0, x^2 + y^2 + by - 2 = 0$
ii. $x^2 + y^2 + 10x - 4y - 3 = 0,\ x^2 + y^2 - 2x - 6y + 5 = 0$
iii. $x^2 + y^2 - 6x + 8y + 9 = 0,\quad x^2 + y^2 = 9$

9.4 Tangents and Normals

Example 9.4.1: Verify that the point $(3,2)$ lies on the circle $x^2 + y^2 - 8x + 2y + 7 = 0$ and find the equation of the tangent at this point.

Solution

Substitute the coordinates $(3,2)$ into the equation $x^2 + y^2 - 8x + 2y + 7 = 0$

LHS: $9 + 4 - 24 + 4 + 7 = 0 = RHS$ $\therefore (3,2)$ lies on the circle

$x^2 + y^2 - 8x + 2y + 7 = 0 \ \rightarrow\ x^2 - 8x\ + y^2 + 2y = -7$

$\rightarrow\ x^2 - 8x + 16 + y^2 + 2y + 1 = -7 + 16 + 1$ (by adding $\frac{1}{2}$ coefficient of x and y on both sides)

$\rightarrow (x - 4)^2 + (y + 1)^2 = 10$

$\therefore$ The centre of the circle is $(4,-1)$.

Hence the gradient of the radius through $(3,2)$ is $\frac{-1-2}{4-3} = -3$

$\therefore$ the gradient of the tangent is $\frac{1}{3}$

To obtain the equation of the tangent to the circle at $(3,2)$, we use $y - y_1 = m(x - x_1)$, to get $y - 2 = \frac{1}{3}(x - 3) \leftrightarrow 3y - 6 = x - 3 \rightarrow x - 3y + 3 = 0$

Therefore, the equation of the tangent to the circle at $(3,2)$ is $x - 3y + 3 = 0$

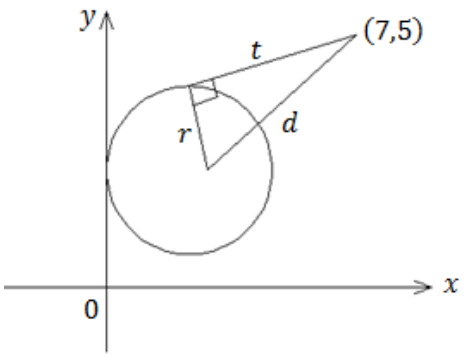
Since the gradient of the tangent is $m = \frac{1}{3}$, therefore gradient of the normal is $-\frac{1}{m} = -3$

To obtain the equation of the normal to the circle at $(3,2)$, we use $y - y_1 = -\frac{1}{m}(x - x_1)$, to get $y - 2 = -3(x - 3) \rightarrow 3x + y - 11 = 0$

Therefore, equation of normal to the circle at $(3,2)$ is $3x + y - 11 = 0$

Example 9.4.2: Find the length of the tangents from the point $(7,5)$ to the circle $x^2 + y^2 - 4x - 6y + 9 = 0$

Solution



From the diagram, we see that $d^2 = r^2 + t^2$ or $t^2 = d^2 - r^2$

$x^2 + y^2 - 4x - 6y + 9 = 0 \quad \rightarrow \quad x^2 - 4x + 4 + y^2 - 6y + 9 = 4$

$\rightarrow \quad (x-2)^2 + (y-3)^2 = 2^2$

$\therefore$ Centre is $(2,3)$ and radius is 2

Now by Pythagoras theorem $d^2 = (7-2)^2 + (5-3)^2 = 25 + 4 = 29$

Since $r^2 = 4 \quad \therefore t^2 = 29 - 4 = 25 \rightarrow t = 5$

Therefore, the length of the tangent from $(7,5)$ to the circle is 5

9.5 The Intersection of Two Circles

Example 9.5.1: Find the equation of the common chord of the circles $x^2 + y^2 - 4x - 2y + 1 = 0$ and $x^2 + y^2 + 4x - 6y - 10 = 0$

Solution

The coordinates of the points of intersection A and B of the circles satisfy the two equations. Therefore, at the points of intersection

$x^2 + y^2 - 4x - 2y + 1 = x^2 + y^2 + 4x - 6y - 10$

$\rightarrow \ -8x + 4y + 11 = 0$

But this equation i.e. $-8x + 4y + 11 = 0$ represents a straight line, and it is satisfied by the coordinates of A and B, $\therefore$ it is the equation of the common chord.

Appendix IV (d): A Framework on the use of the CMSTA for Teaching Trigonometry and Analytical Geometry

Course: Trigonometry And Analytical Geometry **Course Code:** MTH 122 **Contact Hour:** 2HRS/WK

Week 1	**Content Specification**	**Teacher's Activities**	**Learner's Activities**
	General Introduction	Introduce the Course and topics to be Learnt	Students allowed to interject with suggestions and discussions as well as questioning
		Set the rules of engagement	
		Assign students into five arbitrary groups	
		Listen to students comments (if any), responds appropriately and administers ATMI and RMARS as pre-tests	
Week 2 Trigonometry	**Content Specification**	**Teacher's Activities**	**Learner's Activities**
	Relationship Between sums and differences of trigonometric ratios and their products	Derive expressions for conversion of sums and differences of trigonometric ratios to their products	Each student to use expressions for conversion of sums and differences of trigonometric ratios to their products to solve at least three trigonometric equation and identity problems
		Use expressions for conversion of sums and differences of trigonometric ratios to their products to solve trigonometric equation and identity problems Assess students	
Week 3 Trigonometry (Cont.)	**Content Specification**	**Teacher's Activities**	**Learner's Activities**
	Half angle formulae for sine, cosine and tangent	Derive half angle formulae for sine, cosine and tangent	Each student to use half angle formulae for sine, cosine and tangent to solve at least three trigonometric equation and identity problems
		Use half angle formulae for sine, cosine and tangent to solve trigonometric equation and identity problems Assess students	

Week 4 Trigonometry (Cont.)	Content Specification	Teacher's Activities	Learner's Activities
	Sine and Cosine formulae	Derive of the sine and cosine formulae of triangles	Each student to solve at least five triangles using the sine and cosine formulae
		Solve of triangles using the sine and cosine formulae	
		Calculation of angles of elevation and depression using trigonometric ratios Assess students	Each student to solve at least three problems involving the calculation of angles of elevation and depression using trigonometric ratios
		Computation of bearings, heights and distances of inaccessible objects and projections Assess students	Each student to solve at least three problems involving the computation of bearings, heights and distances of inaccessible objects and projections
Weeks 1 – 4 Resources Materials for Teaching Trigonometry			
• Teacher – Made Lecture Notes • Computer Lab with fully networked systems. • Internet connectivity • Mathcentre. Online: http://www.mathcentre.ac.uk/students/topics/trigonometry/			
Week 5 Mensuration	**Content Specification**	**Teacher's Activities**	**Learner's Activities**
	Circular measure	Explains concept of circular measure	Each student to use the expression for the relation between radians and degrees to convert at least five angles expressed in degrees into radians and vice versa
	Relationship between radians and degrees	Establish the relation between radians and degrees and use the relationship to convert angles expressed in degrees into radians and vice versa Assess students	
	Arc length and area of a sector	Derive the formulae for arc length and area of a sector and use them to solve problems involving the determination of arc length and area of a sector of a circle Assess students	Each student to use the formulae for arc length and area of a sector to solve at least three problems involving the determination of arc length and area of a sector of a circle

	Length of a chord and Area of a segment	Identify a chord and segment of a circle Derive expressions for the length of a chord and area of a segment of a given circle and use them to solve problems involving the determination of the length of a chord and area of a segment of a given circle Assess students	Each student to use the expressions for the length of a chord and area of a segment of a given circle to solve at least three problems involving the determination of the length of a chord and area of a segment of a given circle
Week 6 Mensuration (Cont.)	**Content Specification** Surface areas and volumes of simples shapes such as cylinder, sphere and cone	**Teacher's Activities** State and explain formulae for calculating the surface areas and volumes of simple shapes such as cylinder, sphere and cone. Use formulae for calculating the surface areas and volumes of simple shapes such as cylinder, sphere and cone to solve problems involving the determination of surface areas and volumes of simple shapes such as cylinder, sphere and cone. Assess students	**Learner's Activities** Each student to use formulae for calculating the surface areas and volumes of simple shapes such as cylinder, sphere and cone to solve at least three problems involving the determination of surface areas and volumes of simple shapes such as cylinder, sphere and cone
Weeks 5 – 6 Resources Materials for Teaching Mensuration • Teacher – Made Lecture Notes • Computer Lab with fully networked systems. • Internet connectivity			
Week 7 Analytical geometry	**Content Specification** Cartesian and Polar-coordinate systems. Plotting and sketching of graphs.	**Teacher's Activities** Concept of two dimensional coordinate systems: Cartesian and Polar-coordinate systems. Explain plotting and sketching of graphs w.r.t. the two coordinate systems.	**Learner's Activities** Each student to use the expression for the relationship between Cartesian and Polar coordinates to solve at least three problem involving conversion of Cartesian coordinates to Polar Coordinates and vice versa.

	Relationship between Cartesian and Polar coordinates.	Relate Cartesian coordinate to polar coordinates, derive expression for the relationship between Cartesian and Polar coordinates and solve problem involving conversion of Cartesian coordinates to Polar Coordinates and vice versa.	
	Slope of a line	Explain the slope of a line in relation to the above concepts	
Week 8 Analytical geometry	**Content Specification**	**Teacher's Activities**	**Learner's Activities**
	Intercept of a line	Concept of the intercept of a line	
	Gradient of line passing through two points.	Derive the formula for the gradient of line passing through two points and use it to solve relevant problems	Each student to use the formula for the gradient of line passing through two points to solve at least three relevant problems
	Equation of a straight line given the gradient and the co-ordinates of a point.	Derive equation of a straight line given the gradient and the co-ordinates of a point and use it to solve relevant problems.	Each student to use the equation of a straight line given the gradient and the co-ordinates of a point to solve at least three relevant problems.
	Equation of a straight line in the intercept form.	Reduce a given linear equation to the intercept form and use it to solve relevant problems.	Each student to reduce at least three linear equations to the intercept form.
	Coordinates of the point of intersection of two straight lines.	Determine the coordinates of the point of intersection of two straight lines.	Each student to solve at least three problems involving the determination of coordinates of the point of intersection of two straight lines.
Week 9 Analytical geometry	**Content Specification**	**Teacher's Activities**	**Learner's Activities**
	Locus	Define locus	
	Equation of a straight line in slope-intercept form	Derive the slope-intercept form of the equation of a straight line and use it to solve relevant problems.	Each student to solve at least three problems involving the use of the slope-intercept form of the equation of a straight line.
	Equation of a straight line in point - slope form	Derive the point - slope form of the equation of a straight line and use it to solve relevant problems.	Each student to solve at least three problems involving the use of the point - slope form of the equation of a straight line.
	Equation of a straight line in the double - point form	Derive the double - point form of the equations of the straight line and use it to solve relevant problems.	Each student to solve at least three problems involving the use of the double - point form of the equations of the straight line.
	Perpendicular form of the equation of a straight line	Derive the perpendicular form of the equation of a straight line and use it to solve relevant problems.	Each student to solve at least three problems involving the use of the perpendicular form of the equation of a straight line.

Week 10 Analytical geometry	**Content Specification**	**Teacher's Activities**	**Learner's Activities**
	Angle (Q) between two lines whose slopes, (m_1, and m_2) are Known	State and explain the formula for finding the angle (Q) between two lines whose slopes, (m_1, and m_2) are Known and use it to solve relevant problems	Each student to solve at least three problems involving finding the angle (Q) between two lines whose slopes, (m_1, and m_2) are Known.
	Conditions for two lines to be parallel and to be perpendicular.	Explain the conditions for two lines to be parallel and to be perpendicular and use it to solve relevant problems.	Each student to solve at least three problems involving the use of the conditions for two lines to be parallel and to be perpendicular.
	Expression for the perpendicular distance from a point to a line.	Derive the expression for the perpendicular distance from a point to a line and use it to solve relevant problems.	Each student to solve at least three problems involving use of the expression for the perpendicular distance from a point to a line.
	Equation of a circle with centre at the origin and radius r.	Draw a circle and derive the equation of a circle with centre at the origin and radius and use it to solve relevant problems	Each student to solve at least three problems involving the use of the equation of a circle with centre at the origin and radius.
Week11 Analytical geometry	**Content Specification**	**Teacher's Activities**	**Learner's Activities**
	Equation of a circle with centre outside the origin.	Derive the equation of a circle with centre outside the origin and use it to solve relevant problems.	Each student to solve at least three problems involving the use of the equation of a circle with centre outside the origin.
	General equation of a circle. Coordinates of the centre of a circle from a given equation of a circle.	State general equation of a circle. Determine the coordinates of the centre of a circle from a given equation of a circle and use it to solve relevant problems	Each student to solve at least three problems involving the determination the coordinates of the centre of a circle from a given equation of a circle.
	Orthogonal circles	Draw orthogonal circles and explain the conditions for two circles to be orthogonal and solve relevant problems involving orthogonal circles.	Each student to solve at least three problems involving the use of the conditions for two circles to be orthogonal.
	Equations of the tangent and the normal at a point circle	Find the equations of the tangent and the normal at a point on a circle	Each student to solve at least three problems involving the determination of the equations of the tangent and the normal at a point on a circle

<table>
<tr><td colspan="4">Weeks 7 – 11 Resources Materials for Teaching Analytical geometry

Teacher – Made Lecture Notes
Computer Lab with fully networked systems.
Internet connectivity
Mathcentre. Online: http://www.mathcentre.ac.uk/students/topics/geometry/
GeometryBits.com. Online: http://mathbits.com/GeometryBits/
MITOPENCOURSEWARE. Online: http://ocw.mit.edu/resources/res-18-006-calculus-revisited-single-variable-calculus-fall-2010/part-i-sets-functions-and-limits/lecture-1-analytic-geometry/
The Math Forum@ Drexel. Online: http://mathforum.org/library/topics/analytic/
</td></tr>
<tr><td>Week 12</td><td>Content Specification</td><td>Teacher's Activities</td><td>Learner's Activities</td></tr>
<tr><td rowspan="3"></td><td rowspan="3">Summary and Conclusion</td><td>Summarise the Course and topics that has been taught</td><td rowspan="3">Students allowed to interject with suggestions and discussions as well as questioning</td></tr>
<tr><td>Leads discussion on gray areas of the rules of engagement</td></tr>
<tr><td>Listen to students comments (if any), responds appropriately and administers ATMI and RMARS as post-tests</td></tr>
</table>

Appendix V

Data Generated From Administration of the Instruments

Table 6.1: Pre-tests and Post-tests MPTs scores of Experimental Group

Respondents	HND I Chemical Engineering Students				ND I Computer Engineering Students			
	Male		Female		Male		Female	
	Pre-test Scores	Post-test Scores	Pre-test Scores	Post-test Scores	Pre-test Scores	Post-test Scores	Pre-test Scores	Post-test Scores
Respondent1	45	60	60	68	54	63	51	60
Respondent2	45	60	60	68	54	63	51	62
Respondent3	45	61	61	68	55	63	51	64
Respondent4	46	60	61	68	55	63	51	69
Respondent5	46	61	61	68	55	63	53	70
Respondent6	46	62	62	68	55	63	53	70
Respondent7	47	62	62	69	56	64	53	70
Respondent8	47	61	62	69	56	64	53	70
Respondent9	47	62	64	69	55	64	53	71
Respondent10	47	62	64	69	56	64	55	71
Respondent11	48	61	64	69	56	64	55	72
Respondent12	48	63	65	70	56	64	55	72
Respondent13	49	63	65	70	56	64	55	74
Respondent14	49	63	66	70	56	64	55	74
Respondent15	50	63	66	70	56	66	57	75
Respondent16	50	63	66	71	57	65	57	75
Respondent17	50	63	66	71	57	65	57	75
Respondent18	50	63	66	71	57	66	58	75
Respondent19	52	64	66	71	57	66	59	76
Respondent20	52	64	67	74	57	67	59	76
Respondent21	51	64	67	74	57	67	59	76
Respondent22	52	64	67	74	57	67	60	77
Respondent23	52	64	67	74	57	67	60	77
Respondent24	52	64	67	74	57	68	64	77
Respondent25	52	64	68	74	57	68		
Respondent26	52	64	68	76	58	68		
Respondent27	52	66	69	76	58	68		
Respondent28	53	65	69	76	58	68		
Respondent29	53	65	70	77	59	68		
Respondent30	53	66	70	77	59	69		
Respondent31	53	66	70	77	59	69		
Respondent32	53	67			60	69		
Respondent33	53	67			60	69		
Respondent34	53	67			60	69		

Respondent35	53	67	61	70
Respondent36	53	68	61	70
Respondent37	53	68	62	70
Respondent38	54	68	62	70
Respondent39	54	68	62	71
Respondent40	54	68	62	71
Respondent41	55	68	62	71
Respondent42	55	69	62	71
Respondent43	55	69	62	72
Respondent44	56	69	62	72
Respondent45	56	69	62	72
Respondent46	56	69	63	72
Respondent47	57	70	63	73
Respondent48	57	70	63	73
Respondent49	58	70	63	73
Respondent50	58	70	63	73
Respondent51	58	71	64	73
Respondent52	58	71	64	74
Respondent53	58	71	65	74
Respondent54	58	71	65	75
Respondent55	58	72	66	78
Respondent56	58	72	68	81
Respondent57	58	72	69	83
Respondent58	59	72	71	83
Respondent59	59	73	71	84
Respondent60	59	73	71	84
Respondent61	59	73	71	84
Respondent62	59	73	71	85
Respondent63	60	73	71	85
Respondent64	60	74	72	85
Respondent65	62	74	72	85
Respondent66	63	75	72	86
Respondent67	64	78	72	86
Respondent68	67	81	72	86
Respondent69	69	83	72	86
Respondent70	70	84	72	86
Respondent71	70	85	72	86
Respondent72	70	85	72	87
Respondent73	70	85	73	87
Respondent74	70	85	73	87
Respondent75	70	85	73	88
Respondent76	71	85	73	88
Respondent77	71	85	73	88
Respondent78	71	86	73	88
Respondent79	72	86	74	88
Respondent80	72	86	74	88

Respondent81	72	86	74	88
Respondent82	72	86	74	90
Respondent83	73	86	74	90
Respondent84	73	88	74	90
Respondent85	73	88	74	90
Respondent86	73	88	77	90
Respondent87	73	88	77	90
Respondent88	74	88	77	90
Respondent89	74	88	77	91
Respondent90	74	88	77	91
Respondent91	74	88	78	91
Respondent92	75	88	78	91
Respondent93	75	88	78	91
Respondent94	75	90	78	91
Respondent95	75	90	78	91
Respondent96	76	90	78	92
Respondent97	76	90	78	94
Respondent98	76	90	78	95
Respondent99	76	90	78	95
Respondent100	76	90		
Respondent101	77	91		
Respondent102	77	91		
Respondent103	77	91		
Respondent104	77	91		
Respondent105	78	91		
Respondent106	78	91		
Respondent107	78	91		
Respondent108	78	93		
Respondent109	79	93		
Respondent110	79	93		
Respondent111	79	93		
Respondent112	80	95		
Respondent113	80	95		

Table 6.2: Pre-tests and Post-tests ATMI scores of Experimental Group

Respondents	HND I Chemical Engineering Students				ND I Computer Engineering Students			
	Male		Female		Male		Female	
	Pre-test Scores	Post-test Scores	Pre-test Scores	Post-test Scores	Pre-test Scores	Post-test Scores	Pre-test Scores	Post-test Scores
Respondent1	58	85	60	91	59	85	68	86
Respondent2	60	91	71	95	59	91	92	100
Respondent3	68	91	84	113	68	91	98	124
Respondent4	69	91	96	132	71	91	115	131
Respondent5	70	92	101	135	71	93	127	132
Respondent6	71	93	109	135	71	101	130	136
Respondent7	78	101	126	140	79	104	135	138
Respondent8	79	104	127	143	90	105	137	140
Respondent9	90	105	130	145	92	105	140	140
Respondent10	92	105	136	145	95	122	143	143
Respondent11	95	122	136	146	95	129	143	144
Respondent12	96	129	141	147	96	131	143	145
Respondent13	96	130	142	149	98	132	144	145
Respondent14	98	131	143	149	99	135	146	149
Respondent15	98	132	143	150	99	135	147	150
Respondent16	99	135	143	150	100	136	157	158
Respondent17	99	135	146	150	103	136	161	164
Respondent18	100	136	146	150	115	137	171	175
Respondent19	101	136	147	150	115	137	179	180
Respondent20	108	137	148	150	119	137	184	186
Respondent21	115	137	149	150	121	140		188
Respondent22	115	137	150	155	124	140		189
Respondent23	120	138	158	163	125	142		189
Respondent24	123	138	161	167	127	142		189
Respondent25	124	138	171	168	127	143		
Respondent26	125	142	177	169	127	145		
Respondent27	127	142	178	170	128	145		
Respondent28	127	143	181	171	129	145		
Respondent29	127	145		175	130	145		
Respondent30	127	145		180	130	145		
Respondent31	128	145		182	130	145		
Respondent32	129	145			135	145		
Respondent33	129	145			136	146		
Respondent34	129	145			136	146		
Respondent35	129	145			136	146		
Respondent36	134	145			136	147		
Respondent37	135	146			137	147		
Respondent38	136	146			138	147		

Respondent39	136	146	140	148
Respondent40	136	146	141	149
Respondent41	136	147	141	149
Respondent42	137	148	142	149
Respondent43	138	148	142	149
Respondent44	140	148	143	149
Respondent45	140	149	143	150
Respondent46	141	149	143	150
Respondent47	141	149	143	150
Respondent48	142	149	143	150
Respondent49	142	149	143	150
Respondent50	142	149	143	150
Respondent51	143	150	143	150
Respondent52	143	150	143	150
Respondent53	143	150	143	150
Respondent54	143	150	143	150
Respondent55	143	150	143	150
Respondent56	143	150	143	150
Respondent57	143	150	143	150
Respondent58	143	150	144	150
Respondent59	143	150	144	150
Respondent60	143	150	144	150
Respondent61	143	150	144	150
Respondent62	143	150	144	150
Respondent63	143	150	145	150
Respondent64	143	150	145	150
Respondent65	143	150	145	150
Respondent66	144	150	145	150
Respondent67	144	150	146	150
Respondent68	144	150	146	150
Respondent69	144	150	146	150
Respondent70	144	150	146	151
Respondent71	145	150	147	151
Respondent72	145	150	147	161
Respondent73	145	150	147	161
Respondent74	145	150	155	162
Respondent75	146	150	157	162
Respondent76	146	150	157	162
Respondent77	146	150	157	162
Respondent78	146	150	159	162
Respondent79	146	150	161	163
Respondent80	146	151	161	163
Respondent81	147	151	163	163
Respondent82	147	161	164	164

Respondent83	147	161	166	164
Respondent84	148	161	168	164
Respondent85	157	165	173	165
Respondent86	157	167	173	165
Respondent87	157	167	174	166
Respondent88	157	167	177	166
Respondent89	157	167	177	166
Respondent90	159	168	179	167
Respondent91	160	168	179	170
Respondent92	161	169	181	171
Respondent93	162	169	181	172
Respondent94	163	169	182	174
Respondent95	165	169	184	175
Respondent96	166	169	184	175
Respondent97	171	170	185	176
Respondent98	171	170		177
Respondent99	172	171		178
Respondent100	170	171		
Respondent101	176	171		
Respondent102	176	172		
Respondent103	178	172		
Respondent104	178	176		
Respondent105	181	176		
Respondent106	181	176		
Respondent107	182	179		
Respondent108	183	180		
Respondent109	184	180		
Respondent110	184	181		
Respondent111	185	182		
Respondent112		182		
Respondent113		183		

Table 6.3: Pre-tests and Post-tests RMARS scores of Experimental Group

Respondents	HND I Chemical Engineering Students				ND I Computer Engineering Students			
	Male		Female		Male		Female	
	Pre-test Scores	Post-test Scores	Pre-test Scores	Post-test Scores	Pre-test Scores	Post-test Scores	Pre-test Scores	Post-test Scores
Respondent1	100	79	93	75	100	78	92	72
Respondent2	95	75	89	64	95	75	78	61
Respondent3	93	71	76	59	93	70	68	56
Respondent4	91	70	70	53	91	69	59	50
Respondent5	91	69	66	52	90	67	53	46
Respondent6	90	67	58	48	86	63	53	44
Respondent7	86	63	53	46	85	59	50	40
Respondent8	85	62	53	43	79	59	49	33
Respondent9	79	59	52	39	79	58	48	30
Respondent10	79	59	51	34	73	57	44	30
Respondent11	74	58	49	32	72	57	43	28
Respondent12	73	56	49	30	70	56	34	26
Respondent13	71	54	49	30	69	55	33	26
Respondent14	70	54	46	29	68	54	31	24
Respondent15	69	52	43	26	65	51	31	24
Respondent16	68	52	40	26	65	50	31	24
Respondent17	65	52	34	26	63	49	30	24
Respondent18	65	51	33	25	61	47	29	24
Respondent19	64	50	33	24	58	47	24	24
Respondent20	63	49	31	24	54	47	24	24
Respondent21	59	48	31	24	53	47		24
Respondent22	57	47	31	24	53	46		24
Respondent23	53	46	31	24	53	44		24
Respondent24	53	45	31	24	53	44		24
Respondent25	53	45	29	24	53	44		
Respondent26	53	45	24	24	53	42		
Respondent27	53	45	24	24	53	41		
Respondent28	53	43	24	24	53	39		
Respondent29	53	42		24	52	39		
Respondent30	53	42		24	52	38		
Respondent31	53	41		24	51	35		
Respondent32	53	40			51	32		
Respondent33	52	39			51	32		
Respondent34	52	37			51	32		
Respondent35	51	35			49	31		
Respondent36	51	33			49	31		
Respondent37	51	33			49	31		
Respondent38	51	33			49	30		
Respondent39	50	32			49	30		

Respondent40	49	31	49	30
Respondent41	49	31	49	30
Respondent42	49	31	49	30
Respondent43	49	30	48	29
Respondent44	49	30	48	27
Respondent45	49	30	48	27
Respondent46	49	30	47	27
Respondent47	49	30	46	26
Respondent48	49	30	46	26
Respondent49	49	29	44	26
Respondent50	48	29	43	26
Respondent51	48	27	43	26
Respondent52	48	27	43	26
Respondent53	47	27	40	25
Respondent54	46	26	40	25
Respondent55	46	26	40	25
Respondent56	44	26	40	25
Respondent57	43	26	34	24
Respondent58	43	26	34	24
Respondent59	43	26	33	24
Respondent60	43	26	33	24
Respondent61	40	26	33	24
Respondent62	40	25	33	24
Respondent63	40	25	33	24
Respondent64	40	25	33	24
Respondent65	35	25	32	24
Respondent66	34	25	32	24
Respondent67	33	24	31	24
Respondent68	33	24	31	24
Respondent69	33	24	31	24
Respondent70	33	24	31	24
Respondent71	33	24	31	24
Respondent72	33	24	31	24
Respondent73	33	24	31	24
Respondent74	33	24	31	24
Respondent75	32	24	31	24
Respondent76	32	24	31	24
Respondent77	31	24	31	24
Respondent78	31	24	31	24
Respondent79	31	24	31	24
Respondent80	31	24	31	24
Respondent81	31	24	31	24
Respondent82	31	24	30	24
Respondent83	31	24	30	24
Respondent84	31	24	30	24
Respondent85	31	24	29	24

Respondent86	31	24	29	24
Respondent87	31	24	29	24
Respondent88	31	24	29	24
Respondent89	31	24	24	24
Respondent90	31	24	24	24
Respondent91	31	24	24	24
Respondent92	31	24	24	24
Respondent93	31	24	24	24
Respondent94	31	24	24	24
Respondent95	30	24	24	24
Respondent96	30	24	24	24
Respondent97	29	24	24	24
Respondent98	29	24		24
Respondent99	29	24		24
Respondent100	29	24		
Respondent101	29	24		
Respondent102	24	24		
Respondent103	24	24		
Respondent104	24	24		
Respondent105	24	24		
Respondent106	24	24		
Respondent107	24	24		
Respondent108	24	24		
Respondent109	24	24		
Respondent110	24	24		
Respondent111	24	24		
Respondent112		24		
Respondent113		24		

Table 6.4: Pre-tests and Post-tests MATs scores of Control Group

Respondents	HND I Civil Engineering Students				ND I Mineral Resources Engineering Students			
	Male		Female		Male		Female	
	Pre-test Scores	Post-test Scores	Pre-test Scores	Post-test Scores	Pre-test Scores	Post-test Scores	Pre-test Scores	Post-test Scores
Respondent1	44	45	68	68	55	55	57	56
Respondent2	45	45	68	68	55	55	58	56
Respondent3	45	46	68	68	55	56	59	57
Respondent4	45	44	68	69	56	56	60	57
Respondent5	45	44	68	69	56	56	60	58
Respondent6	46	74	69	69	56	56	61	58
Respondent7	46	46	70	70	57	56	61	58
Respondent8	47	46	70	70	57	56	62	63
Respondent9	47	47	72	71	57	56	60	64
Respondent10	48	47	73	73	58	57	60	61
Respondent11	49	48			58	57	60	60
Respondent12	49	48			58	58	62	61
Respondent13	49	49			59	58	62	62
Respondent14	50	49			59	58	61	62
Respondent15	51	50			60	60	62	61
Respondent16	51	50			60	60	63	62
Respondent17	52	51			60	61	62	62
Respondent18	52	52			60	60		
Respondent19	52	52			62	61		
Respondent20	53	52			62	62		
Respondent21	53	53			61	62		
Respondent22	54	54			62	61		
Respondent23	54	54			62	62		
Respondent24	54	54			62	62		
Respondent25	54	55			62	61		
Respondent26	54	55			62	63		
Respondent27	55	55			62	63		
Respondent28	55	55			63	63		
Respondent29	55	55			63	63		
Respondent30	55	56			63	63		
Respondent31	56	56			63	63		
Respondent32	56	56			63	63		
Respondent33	56	56			63	64		
Respondent34	57	56			63	64		
Respondent35	57	56			63	64		
Respondent36	57	56			63	64		
Respondent37	58	57			63	64		
Respondent38	58	57			64	64		
Respondent39	58	58			64	64		
Respondent40	59	58			64	64		

Respondent41	59	58	65	66
Respondent42	60	60	65	65
Respondent43	60	60	65	65
Respondent44	60	61	66	66
Respondent45	60	60	66	66
Respondent46	62	61	66	67
Respondent47	62	62	67	67
Respondent48	61	62	67	67
Respondent49	62	61	68	67
Respondent50	62	62	68	68
Respondent51	62	62	68	68
Respondent52	62	61	68	68
Respondent53	62	63	68	68
Respondent54	62	63	68	68
Respondent55	63	63	68	68
Respondent56	63	63	68	69
Respondent57	63	63	68	69
Respondent58	63	63	69	69
Respondent59	63	63	69	69
Respondent60	63	64	69	69
Respondent61	63	64	69	70
Respondent62	63	64	69	70
Respondent63	63	64	70	70
Respondent64	63	64	70	70
Respondent65	64	64	72	71
Respondent66	64	64	73	73
Respondent67	64	64	74	75
Respondent68	65	66	77	78
Respondent69	65	65	79	80
Respondent70	65	65	80	81
Respondent71	66	66	80	80
Respondent72	66	66		
Respondent73	66	67		
Respondent74	67	67		
Respondent75	67	67		
Respondent76	68	67		
Respondent77	68	68		
Respondent78	68	68		
Respondent79	68	68		
Respondent80	68	68		
Respondent81	68	68		
Respondent82	68	68		
Respondent83	68	69		
Respondent84	68	69		
Respondent85	69	69		
Respondent86	69	69		

Respondent87	69	69
Respondent88	69	70
Respondent89	69	70
Respondent90	70	70
Respondent91	70	70
Respondent92	72	71
Respondent93	73	73
Respondent94	74	75
Respondent95	77	78
Respondent96	79	80
Respondent97	80	81
Respondent98	80	80
Respondent99	81	81
Respondent100	83	83
Respondent101	84	85
Respondent102	85	85
Respondent103	85	86
Respondent104	86	86
Respondent105	87	87

Table 6.5: Pre-tests and Post-tests ATMI scores of Control Group

Respondents	HND I Civil Engineering Students				ND I Mineral Resources Engineering Students			
	Male		Female		Male		Female	
	Pre-test Scores	Post-test Scores	Pre-test Scores	Post-test Scores	Pre-test Scores	Post-test Scores	Pre-test Scores	Post-test Scores
Respondent1	58	58	93	94	81	82	87	88
Respondent2	60	61	125	126	87	86	111	110
Respondent3	68	68	138	137	86	86	121	123
Respondent4	69	69	143	143	90	91	132	130
Respondent5	71	71	143	144	91	92	134	135
Respondent6	78	77	148	147	92	93	137	139
Respondent7	79	79	161	162	96	97	141	142
Respondent8	90	81	180	180	97	98	143	143
Respondent9	92	92		181	98	98	144	143
Respondent10	96	95		184	98	99	144	143
Respondent11	96	96			99	99	143	143
Respondent12	98	98			102	101	145	146
Respondent13	98	99			108	107	145	147
Respondent14	99	99			115	116	156	157
Respondent15	100	101			122	123	160	161
Respondent16	101	101			126	127	176	
Respondent17	108	107			128	129	181	
Respondent18	115	114			128	128		
Respondent19	115	115			128	128		
Respondent20	120	120			129	130		
Respondent21	126	125			130	130		
Respondent22	127	127			130	132		
Respondent23	127	126			135	133		
Respondent24	127	127			135	134		
Respondent25	127	128			136	137		
Respondent26	127	127			137	136		
Respondent27	128	128			137	137		
Respondent28	129	129			140	139		
Respondent29	130	130			140	140		
Respondent30	135	134			141	144		
Respondent31	135	135			142	141		
Respondent32	136	135			142	142		
Respondent33	136	137			143	144		
Respondent34	136	135			143	144		
Respondent35	136	136			143	145		
Respondent36	137	136			143	144		
Respondent37	137	136			143	143		
Respondent38	140	140			143	142		

Respondent39	140	141	143	145
Respondent40	141	141	143	144
Respondent41	142	140	143	145
Respondent42	142	141	143	145
Respondent43	142	142	144	145
Respondent44	143	143	144	145
Respondent45	143	142	145	145
Respondent46	143	143	145	145
Respondent47	143	144	145	145
Respondent48	143	143	146	145
Respondent49	143	143	146	145
Respondent50	143	143	146	146
Respondent51	143	143	146	146
Respondent52	143	143	147	149
Respondent53	143	143	147	150
Respondent54	143	143	147	149
Respondent55	143	143	156	154
Respondent56	143	143	156	155
Respondent57	144	143	156	156
Respondent58	144	144	159	158
Respondent59	144	144	160	161
Respondent60	144	144	162	160
Respondent61	145	145	165	164
Respondent62	145	145	172	170
Respondent63	145	144	172	171
Respondent64	145	145	175	173
Respondent65	145	144	177	176
Respondent66	146	146	179	178
Respondent67	146	146	179	179
Respondent68	146	146	182	180
Respondent69	147	147	184	183
Respondent70	147	147	185	186
Respondent71	147	148	186	186
Respondent72	148	149		
Respondent73	157	158		
Respondent74	157	158		
Respondent75	157	157		
Respondent76	157	158		
Respondent77	157	157		
Respondent78	157	156		
Respondent79	161	160		
Respondent80	161	162		
Respondent81	163	162		
Respondent82	166	167		

Respondent83	171	171
Respondent84	171	171
Respondent85	172	172
Respondent86	177	177
Respondent87	177	177
Respondent88	179	178
Respondent89	179	179
Respondent90	181	180
Respondent91	181	181
Respondent92	182	182
Respondent93	184	183
Respondent94	184	184
Respondent95	187	187
Respondent96		187
Respondent97		187
Respondent98		187
Respondent99		187
Respondent100		187
Respondent101		188
Respondent102		188
Respondent103		188
Respondent104		188
Respondent105		188

Table 6.6: Pre-tests and Post-tests RMARS scores of Control Group

Respondents	HND I Civil Engineering Students				ND I Mineral Resources Engineering Students			
	Male		Female		Male		Female	
	Pre-test Scores	Post-test Scores	Pre-test Scores	Post-test Scores	Pre-test Scores	Post-test Scores	Pre-test Scores	Post-test Scores
Respondent1	101	101	76	77	100	99	81	81
Respondent2	96	95	58	57	92	91	67	69
Respondent3	95	95	53	54	91	90	63	61
Respondent4	94	94	53	50	91	91	52	54
Respondent5	90	90	48	48	86	85	52	53
Respondent6	89	89	40	40	80	80	50	51
Respondent7	87	86	31	31	75	75	49	50
Respondent8	84	85	30	31	73	74	48	47
Respondent9	77	77		30	70	71	44	36
Respondent10	75	76		24	69	69	35	34
Respondent11	74	73			65	65	34	33
Respondent12	71	71			65	64	32	32
Respondent13	70	72			61	60	31	30
Respondent14	68	69			58	57	31	32
Respondent15	69	69			53	54	30	31
Respondent16	66	66			53	53	24	
Respondent17	64	65			54	53	24	
Respondent18	65	64			53	53		
Respondent19	62	62			53	53		
Respondent20	56	56			53	53		
Respondent21	53	53			52	51		
Respondent22	53	54			52	51		
Respondent23	53	53			51	52		
Respondent24	53	53			51	52		
Respondent25	53	54			50	51		
Respondent26	53	53			49	50		
Respondent27	53	53			50	51		
Respondent28	52	53			49	50		
Respondent29	52	52			50	50		
Respondent30	51	51			49	48		
Respondent31	51	51			49	49		
Respondent32	51	51			48	48		
Respondent33	50	50			48	48		
Respondent34	50	50			46	46		
Respondent35	49	49			46	46		
Respondent36	49	49			44	45		
Respondent37	49	49			43	43		
Respondent38	49	48			43	42		
Respondent39	48	48			40	39		

Respondent40	48	48	40	39
Respondent41	49	48	39	40
Respondent42	48	49	35	34
Respondent43	48	49	33	32
Respondent44	49	48	33	32
Respondent45	48	48	33	32
Respondent46	47	47	32	33
Respondent47	47	47	33	32
Respondent48	46	46	32	32
Respondent49	48	48	31	31
Respondent50	45	44	31	31
Respondent51	44	45	31	31
Respondent52	44	44	31	31
Respondent53	43	43	31	31
Respondent54	43	42	31	31
Respondent55	42	42	31	31
Respondent56	42	43	31	31
Respondent57	43	42	31	31
Respondent58	41	40	31	31
Respondent59	40	41	31	31
Respondent60	40	40	31	31
Respondent61	39	39	30	30
Respondent62	33	32	29	29
Respondent63	33	32	29	28
Respondent64	32	33	29	28
Respondent65	32	33	28	25
Respondent66	31	30	24	24
Respondent67	32	31	24	24
Respondent68	31	30	24	24
Respondent69	32	32	24	24
Respondent70	33	33	24	24
Respondent71	31	32	24	24
Respondent72	31	30		
Respondent73	31	30		
Respondent74	31	30		
Respondent75	31	33		
Respondent76	31	31		
Respondent77	31	31		
Respondent78	31	31		
Respondent79	31	30		
Respondent80	31	30		
Respondent81	30	31		
Respondent82	30	30		
Respondent83	30	30		
Respondent84	30	30		
Respondent85	29	29		

Respondent86	29	29
Respondent87	27	27
Respondent88	26	26
Respondent89	26	26
Respondent90	25	25
Respondent91	25	25
Respondent92	24	24
Respondent93	24	24
Respondent94	24	24
Respondent95	24	24
Respondent96		24
Respondent97		24
Respondent98		24
Respondent99		24
Respondent100		24
Respondent101		24
Respondent102		24
Respondent103		24
Respondent104		24
Respondent105		24