This is an authorized facsimile
and was produced by microfilm-xerography
in 1980 by
UNIVERSITY MICROFILMS INTERNATIONAL
Ann Arbor, Michigan, U.S.A.
London, England

71-20,488

KRAFT, Donald Harris, 1943-
THE JOURNAL SELECTION PROBLEM IN A UNIVERSITY LIBRARY SYSTEM.

Purdue University, Ph.D., 1971
Information Services, information storage and retrieval systems

University Microfilms, A XEROX Company, Ann Arbor, Michigan

THE JOURNAL SELECTION PROBLEM IN A UNIVERSITY LIBRARY SYSTEM

A THESIS

Submitted to the Faculty

of

Purdue University

Donald Harris Kraft

In Partial fulfilment of the Requirements for the Degrree

of

Doctor of Philosophy

January, 1971

Grad School
Form No. 9
Revised

PURDUE UNIVERSITY

Graduate School

This is to certify that the thesis prepared

By Donald Harris Kraft

Entitled THE JOURNAL SELECTION PROBLEM IN A UNIVERSITY LIBRARY SYSTEM

Complies with the University regulations and that it meets the accepted standards of the Graduate School with respect to originality and quality

For the degree of:

Doctor of Philosophy

Signed by the final examining committee:

Thomas W. Hill, Jr., chairman

Adrian H. Leenhouts

Marcel F. Neuts

Norman R. Baker

Approved by the head of school or department:

December 30 1970 A. H. Leenhouts

To the librarian:

This thesis is not to be regarded as confidential

Thomas W. Hill, Jr.
Professor in charge of the thesis

PLEASE NOTE:

Some pages have small
and indistinct type.
Filmed as received.

University Microfilms

"Mathematics possesses not only truth but extreme beauty -- a beauty cold and austere ... Mathematics takes us into the region of absolute necessity, to which not only the actual world, but every possible world, must conform."

Bertrand Russell

The Study of Mathematics

ACKNOWLEDGMENTS

The author wishes to express his appreciation and gratitude to the many persons who have contributed to the completion of this research effort. In particular, a special measure of thanks belong to:

Professor T. W. Hill, Jr., Chairman of the author's Graduate Advisory Committee for his personal guidance, assistance, valuable suggestions, humor, encouragement, and friendship throughout this study and throughout the author's graduate program.

Professors F. F. Leimkuhler and W. R. Baker for their instruction, advice, support and encouragement, and for their personal interaction and critical commentary during all stages of the research.

Professors M. Neuts, W. Studden, R. Korfhage, C. Moodie, I. Burr, for providing time, encouragement, and advice.

Professor J. H. Moriarty, Mr. J. Borden and Mr. R. Simon of the Purdue University Libraries, for providing the opportunity to discuss the author's research efforts and for help in understanding the operation of a university library.

The Office of Science Information Services of the National Science Foundation for supporting this research under Grant NSF GN-759.

And finally the author especially wishes to thank his wife for her constant love and support, and Miss Joyce Hinds for putting up with all the difficulties associated with the typing of the manuscript.

TABLE OF CONTENTS

	Page
LIST OF TABLES	vii
LIST OF FIGURES	viii
ABSTRACT	x
INTRODUCTION	1
Purpose of the Research Effort	1
Scope	1
Library Goals and the Objectives of this Research	4
The Library in a State of Crisis	5
A Contrast to the Inventory Control Problem	9
Current Journal Selection Procedures	10
Some General Comments on the Nature of Acquisitions	13
AN OBJECTIVE FUNCTION - MEASURING THE PERFORMANCE OF THE LIBRARY SYSTEM	17
The Objective Function	18
The University Library as a Service Organization	26
The Users	27
The Funders	30
The Servicers	30
The Library -- An Information Retrieval System	31
Evaluating the Library System	35
Library Objectives	38
Performance Measures	39
Relevance and Retrieval -- A Contingency Table	42
A Scheduling Model Format	47
CONSTRAINTS - DETERMINING THE SET OF FEASIBLE SOLUTIONS FOR THE JOURNAL SELECTION PROBLEM	48
Constraints on the Decision Variable $Y_{j,t,q}$	49
Costs and Budget Constraints	52
The Cost Model	56
Other Constraints on Scarce Resources	58

TABLE OF CONTENTS (Continued)

	Page
The Total Model in Review	60
A Scheduling Model Format	62
ONE MEASURE OF A JOURNAL'S WORTH: USAGE	65
Usage Studies - Past Models	65
A Markovian Model	70
A Modification of the Markovian Model	73
The "Pn" Model	79
Revisions in the "Pn" Model	81
Estimation of Parameters	84
The Markovian Model	84
The Modified Markovian Model	87
Generalized Parameters for the Modified Markovian Model	88
The Morse-Elston Data Analysis	94
The "Pn" Model	96
ANOTHER CRITERION FOR MEASURING A JOURNAL'S WORTH: PRODUCTIVITY	101
A General Model of Productivity	104
Literature Growth -- Epidemic Models	107
The Bradford Distribution	112
Estimation of the Parameter β Over Time	114
A Markovian Analysis of Growth and Scattering	117
Relating Relevancy to Usage	120
Data Analysis and Results	123
A SOLUTION ALGORITHM FOR THE JOURNAL SELECTION MODEL	128
A Dynamic Programming Formulation	129
Midler's Lemma on Optimal Control	131
Midler's Lemma Applied to the Journal Selection Model	138
A Suboptimal Computational Algorithm	139
When Is Suboptimal Optimal?	148
A Lagrangian Approach	156
Updating the Multipliers	159
The Stopping Rule	163
The Lagrangian Model	168
An Upper Bound Algorithm	172
Final Improvements	179
An Example Problem	184
Sensitivity Analysis	202

TABLE OF CONTENTS (Continued)

Page

SUMMARY, CONCLUSIONS, AND IMPLICATIONS FOR FUTURE RESEARCH 218

BIBLIOGRAPHY 229

APPENDICES

Appendix A: An Annotated Bibliography 236
Appendix B: Some Interesting Aspects of the Duality of an Integer Programming Problem 262
Appendix C: Computer Listings 267

VITA 286

LIST OF TABLES

Table		Page
1.	A Modified Presentation of Morse and Elston's Parameter Estimates	95
2.	Parameter Estimates for the Modified Markovian Model	96
3.	The Parameter Estimate $\hat{B}_{o,q}$ Over Time	124
4.	Initial Values and Parameters for the Example Problem	189
5.	Initial Feasible Solutions Generated by the Dynamic Programming Algorithm	192
6.	Some Lagrangian Upper Bound Algorithm Solutions	194
7.	Some Other Solutions	199
8.	Some Solutions Illustrating Continuity	201
9.	Solutions With Different Budget Parameters -- Increasing $\lambda_{j,o,q}$ versus q	203
10.	Solutions With Different Budget Parameters -- Decreasing $\lambda_{j,o,q}$ versus q	207
11.	Solutions Formed by Varying One $\lambda_{j,o,q}$ by Some Fixed Percentage	213

LIST OF FIGURES

Figure		Page
1.	Purdue University Libraries Organization Chart 1967	28
2.	An Information Storage and Retrieval System	34
3.	Swets Contingency Table of Pertinence and Retrieval	43
4.	The Density Functions of Swet's Index z	49
5.	The Total Journal Selection Model	61
6.	A Graph of $\bar{\alpha}(t)$ and $\bar{a}\bar{c}^{t}$ Versus Time	97
7.	$E_{t,q}$ as a Function of s and of Dispersion	116
8.	A Computational Algorithm for Finding a Feasible Solution to the Journal Selection Problem	143
9.	The Upper-Bound Algorithm	176
10.	The Final Improvement Algorithm	185

Appendix Figure

C1.	A Computer Program to Estimate the Parameters a and c for the Modified Markovian Model of Usage	269
C2.	A Computer Program to Convert the Raw Data on Literature Growth and Scattering to Time Series Data	272
C3.	A Computer Program to Find Estimates for the Bradford Parameter B_t Over Time	274
C4.	A Computer Program to Evaluate the Limits of the Statistics E_t and to Calculate the Corresponding Estimates of B_t	275

LIST OF FIGURES (Continued)

Appendix Figure		Page
C5.	A Computer Program to Test the Goodness-of-Fit of the Epidemic Models of Literature Growth	276
C6.	A Computer Program of the Dynamic Programming Computational Algorithm (Steps 2 through 4)	278
C7.	A Computer Program of the Lagrangian Upper Bound Algorithm	281

ABSTRACT

Kraft, Donald Harris. Ph.D., Purdue University, January 1971. The Journal Selection Problea In A University Library System. Major Professor: Thomas W. HilT Jr

This dissertation is concerned with the problem of selecting the proper Journals to be acquired by a university library. A general model is formulated as a zero-one linear programing problea with an objective function that evaluates various selection policies in terms of the net worth of selecting specific Journals and rejecting others, journal usage and Journal productivity are two Important means of Measuring the worth of acquiring a specific Journal; and models are developed to describe both measures as dynamic, time-dependent variables. Journal usage is adopted as the better measure of Journal worth, using a modified Markovian approach to describe expected usage patterns over time.

The constraints are primarily related to cost restrictions that arise due to budgetary controls. The relevant costs are those involved with the ordering, storage, and circulation of acquired Journals. The only other type of constraint deals with continuity in that once an item has been acquired, it reseing a permanent part of the library's collection.

The optimitation of the Journal selection model is considered from two different approaches since its large size does not allow for a fit to one of the now existing computational algorithms for sero-one linear programing problems. A dynamic programing formulation allovi us to decompose the large problem into several smaller ones of the knapsack variety and generate a feasible solution if at least one exists. Special cases are found, some of which seem quite realistic In the light of practical library selection procedures, where the solution la also optimal. A Lagrangian formulation gives an upper bound on the optimal value of the objective function and also generates several near-optical solutions, scne of which nay be feasible. A Knapsack approach is then again taken to try to improve the best feasible solution at hand to yield a better, near-optimal or optimal, feasible solution.

A small example problem, illustrating how the algorithms operate, is presented and discussed. The computational results show that a solution can be generated in a reasonable amount of time and computer storage space, for at least small and medium-sized problems.

CHAPTER I

INTRODUCTION

Purpose of the Research Effort

The purpose of this research effort is to investigate the journal selection problem facing university libraries. Specifically, the question is what scientific journals should be purchased given a particular demand pattern exhibited by the users of a university library, and the limited resources available to the library.

Scope

The scope of this research effort is limited specifically to the selection of scientific journals and periodicals in a university library system.

It is important to note that the information system the library represents is primarily a document retrieval system. This thesis restricts its emphasis to hard copy in the form of journals, with little regard to microfilm or other forms of information storage.

In a report of the serials cataloging policy committee of the Purdue University Libraries, Borden, Farris, Hausser and Ferris [8] define a serial as "a publication issued in numbered or dated parts,

usually at regular intervals, and intended to continue indefinitely". They also define a pseudo-serial as "a frequently re-issued and revised publication which quite properly may be, and on first publication generally is, considered to be a monograph. After it has been re-issued several times, however, it may be conveniently regarded as a serial, no matter whether the library keeps merely the latest issue or back file as well"; and a monographic series as "a number of separate works issued in succession and related to one another by a collective title, normally issued by the same publisher and frequently in a numbered sequence".

The report also lists as examples of serials the following:

1. Abstracts
2. Advances
3. Annuals
4. Bulletins
5. Circulars
6. Contributions
7. Directories
8. Essays
9. Journals
10. Lectures
11. Loose-leaf services
12. Manuals
13. Memoirs
14. Newspapers
15. Numbered monographic series
16. Occasional papers
17. Papers
18. Periodicals
19. Proceedings
20. Progress
21. Publications
22. Reports
23. Reviews
24. Studies
25. Transactions
26. Who's Who
27. Yearbooks
28. Handbooks

(or equivalent words in other languages)

When this author refers to the journal selection problem, he is making use of the first definition, that of a serial, with special emphasis upon those journals which would be used by people involved in specific fields of scientific endeavor.

The journal selection problem is in essence a problem of formulating policies to guide selection decisions. A policy will determine what issues of what journals should be acquired by the library system, and at what time they will be acquired. Other supporting decision such as the vendor or the degree to which record keeping is automated will not be considered here.

The constraints which limit the region of acceptable journal selection policies, are of three types:

i. scarce resource limitations, especially cost constraints due to limited budgets;

ii. dimensionality constraints due to the structure of the decision variables; and

iii. arbitrary constraints imposed on the model by librarians who perceive various interrelationships and priorities to exist among journals.

Those factors which help determine selection decisions and are more or less uncontrollable by the librarian are the user demand patterns, which are determined by user needs, and the budgets and controls set by the funders of the library organization. Feedback techniques by which librarians can influence user needs or funder controls are largely ignored in this study.

The performance of a specific policy is evaluated by the determination of the worth of all journals, in terms of satisfying user needs, i.e., the relevance of the journals. This worth is related to user needs through two variables, journal usage and journal productivity.

Library Goals and the Objectives of this Research

A primary goal of many libraries is to obtain optimum library performance in terms of their perceptions of user satisfaction. This research effort will contribute to this goal by modeling the journal selection problem and by manipulating such a model in order to optimize a general criteria for good library performance.

Before a decision problem such as journal selection can be modeled, it is necessary to first obtain a clear understanding of the general organization of a university library. This means that a model can then be built to realistically reflect the environment in which journal selection decisions are made. Even more important than organization are the needs and goals of the people, especially those other than the librarians, who are involved in the functioning and usage of a library.

In reaching the goal of constructing a model which will yield an optimal journal selection policy, the following objectives will be met:

The first objective of this research effort is to develop a suitable objective function with which to evaluate various journal selection and acquisition policies in a university library system. Measures of a journal's worth or relevance are developed, using both journal usage and journal productivity. A second objective is to develop constraints representing the restrictions under which a librarian must operate. It is shown that costs are the most frequently constraining items due to limited budgets. Finally, the third objective will be to develop algorithms to solve the journal selection model as a mathematical programming problem.

The Library in a State of Crisis

The university library is an integral part of the higher educational system and is in a state of crisis. It faces a widespread dilemma known as the information deluge [44]. The chief factor behind the crisis is growth, which is evidenced by the fact that technical literature is continuing to expand exponentially. In fact, the journal selection problem is part of the problem of growth which has made it all but impossible for a library to accumulate all published serials.

Metcalf [32] points out that growth seems to be the underlying cause for nearly all of the library's financial problems. Meier [31] discusses how dire the consequences of growth are, or what he terms "communications overload", to systems such as libraries. If the system is not designed to handle such an overload, it can eventually have breakdowns in service.

Hacker [27] states that in spite of rising demands for information and the rising costs of providing it, funds are limited and outside competition for the use of those funds is increasing. One of the means that he suggests to remedy the situation is to make every effort to improve the management of the library system. In order to improve the quality of management, several authors have suggested systems analysis techniques as a way in which better policies can be formulated and better procedures instituted into library systems.[1]

For example, consider the problems surrounding the decisions as to what titles should be acquired as new additions to the collection.

---

1 An annotated bibliography containing several articles which discusses issues relevant to various aspects of the journal selection problem can be found in Appendix A.

Dean R. Downs [17] of the University of Illinois Graduate Library School, feels that a complete record of human thoughts, emotions, and actions, without restriction, would be ideal. However, he also points out that such a goal hardly seems realistic due to restrictions of time, space, and available funds which prevent a library from acquiring all materials that are available.

On a different level, however, a library should not really try to accumulate all materials. It seems that a library can become too bulky and cumbersome to be useful. Rather than choosing the policy of acquiring everything, a choice that is somewhat intellectually appealing, the policy should be to select only those materials which are relevant to the users of the library and which the library can afford to acquire.

Morse [57] feels that a research science library would be a much better one if rarely used or nonrelevant books were not acquired. The inference is that a library should be usage oriented, existing for the convenience of the user, as opposed to being operated according to an arbitrarily set group of rules based on the aesthetics of the librarian, or on buying for its own sake. The author feels that the worth of a journal lies in its being used as opposed to its merely existing and being available.

Thus the librarian, or someone in a decision-making position, must make decisions as to which materials are to be acquired. Hence, a selective policy must be set forth that can discriminate and determine

which materials are most relevant to the library's users. Downs [17] further states that the basis for such a decision should be the nature of the clientele being served. For example, an undergraduate library will probably require fewer titles and cover more general areas than a library serving people who are doing fundamental research.

The importance of journals in a library is pointed out by Wulfekoetter [82] who states that serials, or journals, are different than other library materials and as such, they are handled separately in many libraries. The importance of journals, especially in relation to support of research activities, is also illustrated by Straus, Strieby, and Brown [73 , p. 124]:

> "Periodicals constitute the most important part of a science library's resources because they publish the immediate results of experimental research and announcements of technical developments. A continuous record of scientific advance is presented in these journals, though not as a story that is easily read since related information is frequently scattered in numerous papers. The larger the collection of periodicals in one library, particularly of long runs of important titles, the more of the story there will be at hand to be put together, and it is for this reason that periodicals, in contrast to most books, appreciate in value with time. Consequently they merit the most careful selection and subsequent preservation. Recognition of this is evidenced in the policy of many libraries of spending a greater portion of the budget for subscriptions to periodicals and purchase of back files than for large numbers of books. Major subscriptions should never be affected by a cut in the budget with the expectation of their being purchased in future more prosperous times. It is often impossible to acquire certain issues at any price. Books, on the other hand, may not go out of print so rapidly and are more likely to be available through the used book market. The acquisition and servicing of periodicals is regarded as a serious responsibility by those who are fully aware of their significance. The periodical files available in a science library may well be the most accurate indication of the worth of the whole collection".

There is another aspect of the selection problem that seems interesting. The librarian has two main options open to him, either to select or reject the item being considered. Rejection means that a user desiring that particular item must either be turned away empty-handed in terms of the item or be forced to wait while a copy of the item is obtained from another source. Centralization has been viewed as one way to reduce the time and hence the severity of the consequences of a forced delay. As access to alternative sources becomes easier, one might even be able to reduce the quantity of items selected and maintained in an individual collection as opposed to a central lending library.

Several authors such as Kent [32] have looked at centralization of library efforts and holdings, as opposed to decentralization. It is felt that a centralized technical processing group has the potential of reducing costs and of eliminating duplicated effort. Bryant's [10] tradeoff of the users of resources between acquisition and processing versus borrowing from other sources becomes important since a central bureau would make access to such sources easier. A centralized set of holdings makes access even easier still, which implies that the need to acquire everything is now lessened.

Centralization can take place on several levels. Branch libraries may resort to a central administrative body for technical processing and management. On a higher level, cooperation on a regional, state, or even national basis could lead to better modes of operation. Some librarians have been bold enough to suggest international cooperation and centralization. The journal selection problem seems to present an interesting study in the case for centralization at any level.

A Contrast to the Inventory Control Problem

The journal selection problem can be compared to the general inventory control problem found in industry. The questions of how much to buy, when to buy, and the restrictions of time, space, and limited capital are present in both cases. Moreover the costs of acquisition, storage, and retrieval for use are a part of both problems.

However, the university library has several characteristics that make the journal selection problem a difficult one worthy of independent study. The librarian must also consider what to buy, a decision that is not in the realm of inventory control theory. There is a very large number of journals to be considered. Each volume, in fact each issue, of each journal is a separate entity which must be handled individually when acquired, processed, or circulated. Eventually issues will usually be bound together in volumes, but requests will still be for specific issues of a journal, at least part of the time. Moreover, the user is usually interested in specific articles. To indicate how much work is involved due to the number of items, most university libraries have a large portion of their professional staff involved in classification and cataloguing of serials or journals.

The demand for knowledge, in the form of printed documents at least, is much more erratic and difficult to predict than sales or part demands in an industrial operation. The user does not always have a clear picture of exactly what information is being sought. Moreover, the sheer number of library items which might be relevant to some user makes prediction very hard. Another cause of difficulty

is the unfamiliarity of the librarian with the intricate details, technicalities, terminology, and latest developments of the various fields of scientific endeavor in which the library is involved. The fact that such fields of endeavor are dynamic and may expand, contract, or change radically within the span of a few years adds to the problem. Since subscription lengths often run over a period of more than one year, sometimes as much as five years, a planning horizon of several subscription intervals can be quite lengthy. Thus, time also makes the decision problem quite difficult to analyze.

Continuity is the last major difference between journals and general inventory items. It is important that long runs of very important journals not be interrupted or else a major portion of the history of the technical fields being developed in the journal would be missing.

Current Journal Selection Procedures

In order to devise a better method of selecting journals in a university library, it seems desirable to explore how journals are currently being selected and placed into the system. This is an example of the environment in which journal selection is taking place before any decision models are used. The Purdue University Serials Unit of the Library is responsible for all journal acquisitions and its policies seem to exemplify current journal selection procedures.[2]

---

2 The cooperation of the Head of the Serials Unit, Mr. Borden, and his staff must be acknowledged. Mr. Simon of the AA&ES and IE Library and Prof. Moriarty, Director of the Purdue Library, must also be thanked for their helpful comments.

The technical procedures can best be seen by outlining the flow of a journal through the Serials Unit. A request for a journal originates from one of the branch librarians or department liason officers or from the central library's reference desk. The requests are then checked to see if the library already has a copy on hand, especially in the branch requesting the journal. If the branch does not have a copy, the request is examined to insure that the bibliographic information (title, publisher, volumes available, etc.) is correct. Also, information as to price and addresses of publishers or purchasing agents is ascertained. The accounting ledgers are then checked to see if funds are available for the requested journal's subscription price. If not, the item will have to be held up until next year when more funds are available. If funds are available, the order is placed.

An order clerk now types out an order card and sends one copy to the publisher or purchasing agent. Apparently there is not too much concern as to which agent to buy from since there is usually only one to consider. The account of the branch library is adjusted to allow for the new acquisition. When the issues arrive, they are checked and sent to the proper branch of the library. If any of the issues of the journal do not arrive on time, the clerk may have to claim the issues by notifying the publisher of their non-arrival.

A new acquisition is also sent the cataloger who designates a number by which reference can be made to the journal (such as a Dewey Decimal or a Library of Congress number), according to its content.

A journal retains this number even in the event of subject matter changes in its contents. A card containing bibliographic information on this latest addition to the collection is placed in the file. A notice recording the receipt of the journal is kept on file to be referred to when the library is billed by the publisher or agent for the journal.

When a complete volume has been accumulated, those journal issues must be bound into one hard-cover item. The issues are gathered together and replacements found for any missing or incomplete issues. These issues are sent to the bindery after being checked for completeness and returned and checked again, this time for proper binding.

This final check completes the cycle for a journal in the Serials Unit of the Purdue University Library. All reference work, circulation, and shelving is done within the branch library, outside the realm of the Serials Unit. One final point that should be made is that it is very hard to terminate a subscription once it has been established. However, a branch library may terminate a journal without too much trouble if another branch is or will be subscribing to that same journal. Budgets are usually assigned so as to automatically include all current subscriptions. This means that since an acquired journal usually becomes a fixed part of the collection, a decision to acquire means an expenditure of a good deal of time, effort, and capital. Thus, the journal selection decision becomes increasingly important in terms of using the scarce resources properly to optimize a library system's performance.

Some General Comments on the Nature of Acquisitions

In discussing the general characteristics of special libraries, Strable [72] notes that an acquisitions unit must at least collect and receive publications, check for relevance and duplication, keep abreast of user interests and needs, receive announcements of new publications, and establish order records. The unit may also contact dealers, establish special collections, maintain order files, and review and evaluate collections. The unit may even prepare written selection policies, contact dealers of unusual items, maintain special collections, and select material in anticipation of future needs. Strable also points out the advantages of centralized acquisitions.

While studying the acquisition policies of several different libraries, Chitwood [12] notes a reasonable similarity in the types of functions performed. He also notes that faculty requests are used by many departmental branches of university libraries as a basis for the selection decision. Lists of available publications are also consulted to suggest new titles to improve a collection. Coffin [13] discusses blanket ordering as another possibility, buying all publications in a certain field.

Downs [17] worries about the over-dependence upon faculty recommendations and suggests concentrating upon building up and intensifying whole fields of endeavor rather than worrying about individual titles. Welch [80] discusses difficulties of acquiring foreign publications, and Sanders [70] notes the extreme difficulties of acquiring back issues and out-of-print items, which heighten the importance of making the proper journal selection decisions.

Stevens [71] discusses the acquisitions unit in relation to the total library system. He sees the main functions as identifying and procuring new material, maintaining of records, and paying bills. He also considers the role of the acquisitions department in assisting technical processing units, assisting the users, assisting the library administrators, as well as working with the collection to build, weed, and evaluate it. Stevens also feels a collection policy dealing with whole fields is preferred to a title selection policy.[3]

Another method of acquiring materials for the library is through the Gifts and Exchange Unit. Often times new journals are sent as gifts to encourage future purchases. More likely, however, are the gifts or donations of materials which may contain back issues desired by the library. Moreover, duplicate copies can be exchanged for such back issues, when available, from other libraries.[4]

---

3 Wulfekoetter [82], in a book that presents in great detail the work involved in acquisition units, agrees with the concept of field rather than title selection.

4 The interested reader can consult Wulfekoetter [82] for a detailed description of the work involved in a Gifts and Exchange Unit.

In Chapter II we shall develop the objective function of the model as a mathematical statement of the library's primary overall objectives in terms of the decision variables. This will be done in order to evaluate library system performance under various journal selection policies. In order to do this successfully, the library is viewed both as a service organization and as an information retrieval system. Evaluation techniques for information systems are discussed to show how well the objective function incorporates the relevant factors for evaluation.

The constraints associated with journal selection decisions are developed in Chapter III. The discussion centers around the cost constraints and the constraints directly associated with how the decision variables are defined. However, possibilities for other types of constraints are not excluded entirely.

The main topic of Chapter IV is journal usage as a criterion for measuring the worth of acquiring a journal. Several models of usage are considered, with special emphasis on a Markovian model, a modified Poisson process, and additional modifications of these two. Estimation of parameters is discussed and the method of moments is used to provide such estimates for the various models.

As an alternative criterion of journal worth, journal productivity, the number of relevant articles in a journal, is considered in Chapter V. Epidemic models of literature growth and a scattering model based on the Bradford distribution are discussed in detail. These models are tested, with parameters being estimated, using data from a body of queueing theory literature.

In Chapter VI, we shall develop algorithms to help solve the resulting integer programming problem formed by the journal selection model. A dynamic programming formulation is used in conjunction with stochastic control theory to yield a feasible suboptimal solution if at least one exists. A Lagrangian multiplier technique is then employed to generate more solutions, some of them feasible, and to yield an upper bound to the optimal value of the objective function. These solutions can be improved to yield a near-optimal feasible solution. A small example problem is presented to illustrate how the algorithms work.

The summary of the research, conclusions, and recommendations for further research are included in Chapter VII. Appendix A contains an annotated bibliography of additional material deemed relevant to the journal selection problem. Appendix B contains computer listings for the various algorithms and parameter estimation routines. Finally, the solutions for several problems consisting of variations of the input data for the original example problem are presented in Appendix C.

CHAPTER II

AN OBJECTIVE FUNCTION - MEASURING THE PERFORMANCE OF THE LIBRARY SYSTEM

In order to determine which journals should be acquired, a mathematical model of the journal selection problem will be constructed. The model, an abstraction of the real decision problem, will allow us to generate, consider, and evaluate various feasible alternative policies and choose the best one. The key element of the model is an objective function which states the system's overall objectives or goals and evaluates the performance of the system under each policy.

The objective function is constructed using a measure of the worth of each journal with respect to the overall library system. In order to determine what measures are suitable, a clear understanding of the library, its participants and their needs, goals, and interactions is necessary. In this chapter we will attempt to review some of the literature describing the library and its environment, and also to model the form of the objective function.

The first section is concerned with developing the general objective function for the journal selection problem in a university library system, stated in mathematical form. However, the development

of specific measures of journal worth is deferred to the next few chapters. In the second and third sections we deal briefly with the library as a service organization and as an information system, respectively. The fourth section is a survey of the literature on evaluation of information systems in terms of performance measures. Thus the objective function of the first section is shown to encompass all the relevant factors involved in evaluation of library performance. In the final section, we consider a slightly modified objective function set up as a scheduling problem to show the relationship of the journal selection problem to other models.

The Objective Function

To gain a better understanding of the problem and to provide some insights into methods of solution, a model representing the library system must be constructed. Morris [54] points out that model building is an art, best described as intuitive in nature. He suggests beginning with a simple model and enrichening it as one confronts the model with data and the inherent assumptions used in deriving results from the model.

Consider first a planning horizon for the librarian decision-maker; that is, the span of time over which a servicer is specifically interested and capable of making decisions. The time span runs from time point zero, usually representing the current situation, to some future time point T; with the span being specified as (0,T].[1] We

---

1 Note that the interval (a,b] implies a continuous segment running from points greater than but not including point a, up to and including point b.

shall view time as a discrete variable, implying that the time span can be divided into r intervals of equal length, T/r. The subscripts t and q will be used to designate specific time periods. Time period t designates the time interval $((t-1)T/r, tT/r]$ while time period q designates a similar interval when $t=1,2,\ldots,r-q$ and $q=1,2,\ldots,r$.[2] If t or q are specified as equal to zero, initial conditions that must be specified empirically are implied.

The set, S, of journals being considered for acquisition is the next item of discussion. It shall be assumed that the set is complete at any point in time when it contains all journals which might have been, are now, or will be relevant in the planned future, to some specific user needs. By constantly updating the model, new journals can be added to the set as they are needed. Questionning the users, especially ranking members of the faculty, will also help insure that the university librarian is considering most if not all of the relevant journals. The use of bibliographies, citation lists, abstracting journals, and the like, as well as consulting other libraries, can also be used to help insure that the set is complete. Let j be the subscript used to indicate a specific journal with s being the total number of journals in the set, so that $j=1,2,\ldots,s$.

The journal selection problem is whether or not to acquire specific issues of specific journals at given points in time. To represent this problem in a model, decision variables are needed for all alternatives

---

2 The notation $t=1,2,\ldots,r$ implies that variable t may take on any of the integers up to and including r.

under consideration. The basic decision variable of this model is $Y_{j,t,q}$, a binary-valued variable which takes on the value of one if the library system has acquired the issues of journal j that were published in period q as of t periods after publication, the value of zero if it has not. Note that this implies that the quantity $(1-Y_{j,t,q})$ also is a binary-valued variable that is one if the issues have not been acquired and zero if they have; a mirror image of the original decision variable $Y_{j,t,q}$. The decisions of whether or not to acquire certain issues of a specific journal at a certain point in time have many far-reaching implications for library organization. We will explore many of these in an effort to better understand and improve the performance of the university library system.

If a journal is to be selected for acquisition, it should be because it adds to the overall performance of the system by helping to better attain the objectives of the servicers in terms of user satisfaction. Satisfaction can be related to either a single set or to several sets of users in terms of the types of needs users have. Thus one must define a measure of a journal's worth in terms of these objectives. The measure must include a reward if the acquired journal is pertinent and a penalty if the journal is not; but the concept of pertinence must be expanded and made more general than it is when applied to just one specific request for information. Let $G_{j,t,q}$ represent a weighting factor which measures the worth of having acquired the issues of journal j published in time period q as of t periods after publication; and in like manner, let $H_{j,t,q}$ represent a weighting factor which measures the worth of not having acquired those issues.

With the decision variables and the worth of acquiring or not acquiring pertinent journals defined, a general objective function for journal selection can be derived. All journals in the set, all time periods in the planning horizons, all issues of these journals published within the planning horizon [3], and all opportunities to purchase these issues must be considered. The worth of purchasing the issues of journal j that were published in time period q as of t periods after publication is $Y_{j,t,q}G_{j,t,q}$; the worth of not purchasing the issues is $(1-Y_{j,t,q})H_{j,t,q}$. Summing over time and over the set of journals yields the following objective function:

$$\text{II-1)} \quad Z' = \sum_{j=1}^{s} \sum_{q=o}^{r} \sum_{t=o}^{r-q} [Y_{j,t,q}G_{j,t,q} + (1-Y_{j,t,q})H_{j,t,q}].$$

This expression represents the sum total of the library system's worth in terms of its journal collection. As such, Z' should be maximized.

Rearranging terms, we have

$$\text{II-1a)} \quad Z' = \sum_{j=1}^{s} \sum_{q=o}^{r} \sum_{t=o}^{r-q} [Y_{j,t,q}(G_{j,t,q} - H_{j,t,q}) + H_{j,t,q}]$$

$$= Z'' + \sum_{j=1}^{s} \sum_{q=o}^{r} \sum_{t=o}^{r-q} H_{j,t,q} \ ,$$

---

[3] The case may arise where a library has the opportunity to purchase older issues of a journal, issues published before the current planning horizon began. These issues can be considered separately, as a journal entity unto themselves, so that one can use the model to make journal selection decisions. Of course, the time scale will have to be shifted for these specific issues.

where

$$\text{II-2)} \quad Z'' = \sum_{j=1}^{s} \sum_{q=0}^{r} \sum_{t=0}^{r-q} Y_{j,t,q}(G_{j,t,q} - H_{j,t,q}) .$$

Since the expression $\sum_{j=1}^{s} \sum_{q=0}^{r} \sum_{t=0}^{r-q} H_{j,t,q}$ is independent of the decision variables $Y_{j,t,q}$, maximization of Z" accomplishes the desired maximization of Z', the objective function.

Consideration must be given to the relationship between $G_{j,t,q}$ and $H_{j,t,q}$. If they can be easily related mathematically, the objective function can then be further simplified. In order to better see such a relationship, let us consider an information retrieval system approach towards evaluation of performance. $G_{j,t,q}$ and $H_{j,t,q}$ seem to be analogous to contingency table variables, seen in Figure 3 and discussed in detail below in a separate section of this chapter, which represent the consequences of pertinence, V_1 if acquired and K_2 if not, respectively. There are, however, no direct analogies to the variables K_1 if acquired or V_2 if not, which represent the consequences of nonpertinence. The worth of not acquiring nonpertinent journals or of acquiring nonpertinent journals seems quite difficult to measure, especially in terms of determining which journals are nonpertinent, as opposed to those that are only slightly pertinent. It is far easier to determine which journals are pertinent in terms of a variable such as usage. The librarian is usually more concerned with pertinance in that he wants to satisfy user needs for relevant items as best he can. Moreover, pertinence is usually defined with respect to a user and a specific demand for information; while a servicer must base decisions on some overall measure of worth.

Two points can be made to show that the variables measuring journal worth can be construed so as to consider at least some aspects of nonpertinence. The cost constraints, to be discussed in detail later, become involved with nonpertinence in that acquiring a nonpertinent journal uses up scarce funds that might be put to better use toward the purchase of more pertinent items. Secondly, the variables should be related to nonpertinence in terms of general usage patterns. As a journal becomes less desirable and thus, less used or more nonpertinent, its worth if acquired should decrease while the worth of not acquiring it should increase. Note that this conceptualization tends to view all journals in the set as having at least some pertinence and some value and thus being worthy of consideration.

With the servicer primarily interested in the satisfaction of user needs, the library's main function being to select, acquire, maintain, and disseminate relevant information, and with $G_{j,t,q}$ representing the worth of a specific acquisition towards these ends, it seems reasonable to assume that $G_{j,t,q}$ is a nonnegative variable. In like manner, it seems reasonable to assume that $H_{j,t,q}$, representing a deficit to potential user satisfaction, should be less than $G_{j,t,q}$ and might normally but not necessarily be considered as a nonpositive variable for all values of j, t, and q. If $H_{j,t,q}$ is nonpositive, the worth of not acquiring a journal becomes a negative worth, that is, a cost or deficit. The validity of these assumptions is reinforced by the notion that availability of relevant library materials is a very important evaluation criteria. While this thesis will consider only cases where these assumptions hold, one may revert to equation II-1) for a more general formulation of the objective function.

One way of relating $H_{j,t,q}$ to $G_{j,t,q}$ would be to let $H_{j,t,q}$ be a decreasing function of $G_{j,t,q}$ for all values of j, t, q. Certain multiple choice tests that have their final grade computed on the basis of the number of correct answers minus a portion of the number of incorrect answers can be considered as an analogy to this type of situation. This subtraction is performed to remove bias. One function of interest is the linear case where

II-2a) $H_{j,t,q} = -\rho_1 G_{j,t,q}$ for all values of j, q, and t .

Then the objective function may be modified to yield

$$\text{II-3)} \quad Z'' = \sum_{j=1}^{s} \sum_{q=0}^{r} \sum_{t=0}^{r-q} Y_{j,t,q}(G_{j,t,q} - H_{j,t,q})$$

$$= \sum_{j=1}^{s} \sum_{q=0}^{r} \sum_{t=0}^{r-q} Y_{j,t,q}[G_{j,t,q} - (-\rho_1 G_{j,t,q})]$$

$$= \sum_{j=1}^{s} \sum_{q=0}^{r} \sum_{t=0}^{r-q} Y_{j,t,q} G_{j,t,q} (1+\rho_1)$$

$$= (1+\rho_1) \sum_{j=1}^{s} \sum_{q=0}^{r} \sum_{t=0}^{r-q} Y_{j,t,q} G_{j,t,q} .$$

The assumption that $G_{j,t,q} \geq H_{j,t,q}$ implies that $\rho_1 \geq -1$ or $1+\rho_1 \geq 0$. However, if we further assume that $\rho_1 > -1$ so $(1+\rho_1) > 0$, we can eliminate the trivial case where $\rho_1 = -1$ and a servicer is indifferent to all selection policies. This means that it is always at least slightly more desirable to acquire than to not acquire a specific journal in terms of the objective function. Using this assumption, one can re-write the objective function as

$$\text{II-3a)}\quad Z = \sum_{j=1}^{s}\sum_{q=0}^{r}\sum_{t=0}^{r-q} Y_{j,t,q}G_{j,t,q} = Z'/(1+\rho_1)\ .$$

If $H_{j,t,q} \leq 0 \leq G_{j,t,q}$, which seems to be the most plausible situation, $\rho_1 \geq 0$. If ρ_1 is in the range (-1,0), this means that not buying a specific journal could have a small positive but lesser worth than if one did buy the journal. However, it does not seem as though one would normally assume this to be the case.

Lister [48], when considering the use of compact storage for the less active items in a library collection noted that the most sensitive variable was the cost to the user of not having an item immediately available. An analogy to this cost is represented here by the expression $H_{j,t,q}(1-Y_{j,t,q})$, the worth of not being able to provide a service to the users. Using the relationship of $H_{j,t,q}$ and $G_{j,t,q}$ suggested by (II-2a) implies that the cost of unavailability is expressable in terms of the variable ρ_1. However, one need not concern himself with specific values of ρ_1, only the boundary restriction that ρ_1 cannot be less than -1. Thus, Lister's problem of determining a specific cost for delay of service can be circumvented somewhat in this model since we do not need to specify the cost of not being able to provide service directly.

With the general objective function for the journal selection problem defined, the question arises as to the nature of various measures of journal worth that might be used. The measures must be capable of incorporating the factors of system performance as discussed above; but they must also be simple enough to be estimable.

In actuality, the objective function has been modeled with certain possible measures of journal worth in mind -- journal usage and journal productivity. Both measures will be discussed in succeeding chapters and their relative merits and disadvantages pointed out in some detail.

However, before that can occur, we must consider the library in the context of a service organization and as an information system and must see how the performance of such systems is to be evaluated.

The University Library as a Service Organization

In order to establish the merit of the objective function in terms of its capability to realistically evaluate journal selection policies, the university library system, seen as a service organization, and the nature of the participants and elements of such an organization must be considered.

The university library has been analyzed as a service organization by Baker [1,2] who developed a descriptive model to present a theoretical framework in which decisions can be made. A "service organization" is "one whose prime beneficiary is that part of the public in direct contact with the organization, with whom and on whom its members work -- in short, an organization whose basic function is to serve clients" [1].

The general participants of the service organization are identified by Baker as

i) users,

ii) funders, and

iii) servicers.

Examples of these categories, in a university library system, are, respectively:

i) the faculty and student body,

ii) the administrators of the university,[4] and

iii) the staff of the library.[5]

Baker also considers each participant as having a separate set of decision problems. These problems have a direct bearing on the interaction of these participants in the library environment. What factors influence the decisions of the users, funders, and servicers?

The Users

The users, either those who are now using or those who might in the future use the services of the university library, are seen by Baker as people reacting to a set of personal needs. A user decision

---

4 The state legislature and federal government, or even beyond that to the taxpayers, the people who pay tuition and fees, and perhaps even the donors of various gifts to the library can be considered as funders, depending on the breadth of the analysis. Within the context of the university library, the administration is the library's primary source of funds and funder interaction.

5 The Director of the Library, his administrative staffs, and professional librarians, all of whom are involved in policy decision-making functions; these are the servicers considered within the scope of this research effort. An organization chart of the Purdue University Libraries [65] can be seen in Figure 1. The library administration, like that of a factory or of any business firm, has decision problems that bear investigation.

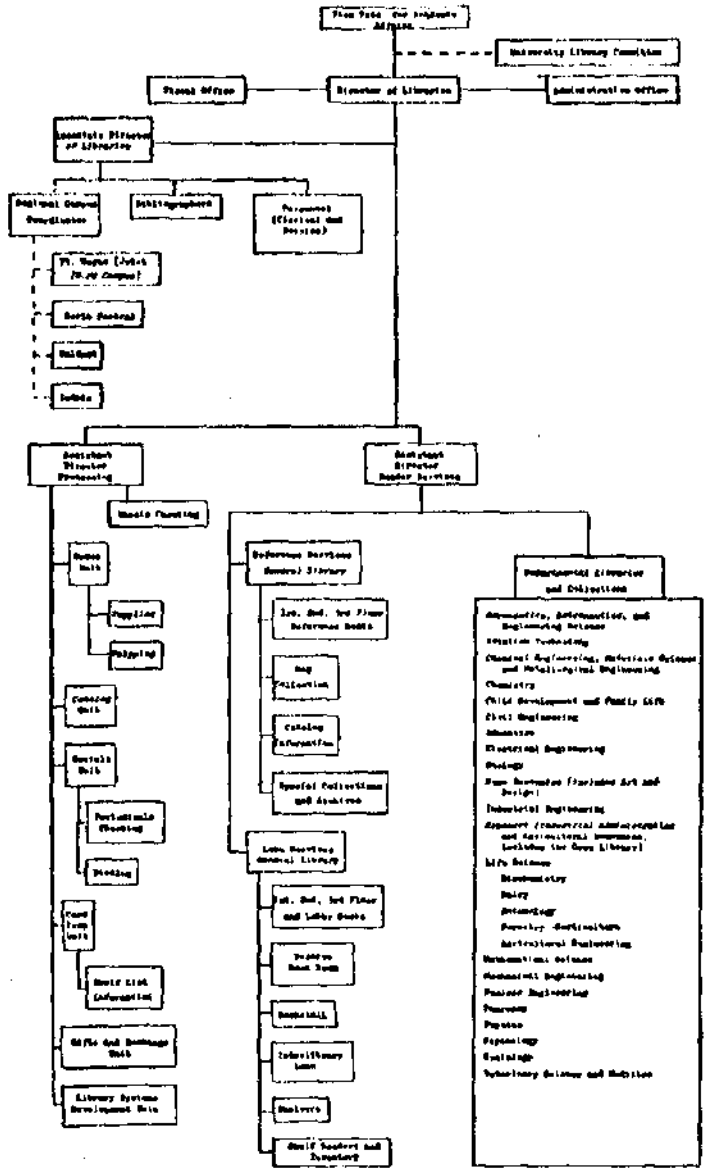


Figure 1. Purdue University Libraries Organization Chart 1967

model based on a reward-cost structure of library evaluation is developed by Baker in terms of seeing how users determine their own patterns of library usage.

The users, as clients of the service organization, base their decisions concerning usage upon past evaluations of the library performance, be it this specific library or accounts of friends' experiences in a library environment and upon present perceptions of current library policies. From these evaluations and perceptions come expectations regarding service which may or may not be reinforced by interaction with the library. Finally a decision comes in terms of a selection of a set of sources from where needed services can be obtained. This implies alternative sources such as personal collections, collections of colleagues, and other libraries, which must be considered and evaluated by the user. In a sense, these sources are competitive, at least from the librarian's point of view. The librarian must be able to satisfy his users' needs; he must make the library the best source of desired information or else alternative sources of information may be used. Also inherent in this structure is the decision to let the need go unsatisfied. This last alternative seems especially relevant when dealing with commodities such as information and its related services.

The specific behavior patterns of users of a library have been studied quite extensively. Most of the work has been in the area of identifying the user or determining the quantity of usage attributable to various library materials. Jain [30] presents several early references and their summaries (up to 1965). The bibliography of user studies compiled by DeWeese [16] extends Jain's work.

The Funders

The funders, those people who provide the capital to operate and maintain the service organization, may be at least in some cases, the same people who use the services. However, in a university library system, the sets of users and funders may be quite separate entities, verging almost on mutual exclusiveness.

The funders seem to operate on the principle that "he who pays the piper calls the tune". The concept of investing capital to support a service organization carries with it an implication of control, or at least of influence, over the policies of the organization. Even if no controls are specifically placed upon the library, as Baker points out, the budgeting of available funds can in itself impose critical restrictions on the feasible courses of action open to a servicer.

The funders, according to Baker's model, base the decision as to what controls are to be placed upon the organization on previous experiences and present perceptions of current library policy. Of course, it is expected that part of the comparative evaluation of the library by the funders will take user satisfaction into account.[6]

The Servicers

The third set of people involved in the service organization is the servicers. Within the context of a college library, this includes the library staff; however, the scope of the model is limited to those

---

6 The reader may consult Rzasa's [63] thesis which discusses some cost-benefit analysis aspects of university library funding decisions.

library administrators and professionals directly involved in decision making. As Baker points out, the librarian is considered a professional,[7] and is expected to have the welfare of the clients as his primary concern. Moreover, the professional will, at least in the ideal case, be penalized if self-interests are promoted at the expense of service to those clients.

While a user is generally concerned with satisfaction of his own personal needs at a minimum of cost and inconvenience, the servicers are more interested in aggregate user satisfaction and overall measures of system performance and cost. Baker sees the servicer as influenced by users and their needs and subjected to various modes of funder control. Moreover, the servicers have goals for the library and for themselves as individuals. Their decision problems center around trying to best achieve these goals while satisfying user needs and yielding to funder controls. Baker also points out that the servicer must find the proper balance between becoming too subservient to the users or to the funders, virtually becoming a "captive" in the decision-making process at the expense of the other participants, and becoming overly preoccupied with status and career or administrative goals. Otherwise, system performance will suffer.

The Library -- An Information Retrieval System

In order to demonstrate how well the objective function can measure library system performance under a specific journal selection policy, we shall consider the library as an information retrieval

---

7 It may be argued that librarianship is, in fact, not a true profession. However, it may also be true that a librarian, highly sensitive to such matters, is compelled to be more professional, along stereotyped lines, than members of more established professions.

system. We shall investigate the literature on evaluation of such systems and on what vital factors are involved in evaluation, and shall see that our objective function does incorporate those factors in a suitable fashion.

The university library is many things: Trueswell [76] notes that it is both a study hall and a social center as well as an information retrieval system . While it does not necessarily have to be a library, an information retrieval system (sometimes referred to as an information system or as an information storage and retrieval system) can take on the functional appearance of a library.

Korfhage and DeLutis [36] consider three major pieces which combine to form an information retrieval system:

i) the endosystem -- the combination of computer and communication equipment, programming support, and personnel necessary to achieve the given storage and retrieval task;

ii) the ectosystem -- that portion which directly affects the endosystem, including users, data, sources, and financial support; and

iii) the environment -- everything else.

They describe the system by listing its functions as

a) storing and maintaining data suitable for retrieval,

b) provide services to users inquiring about the stored data or about these services, and

c) establish and maintain interfaces between the endosystem, the users, and the data sources.

Salton [69] lists the following functions for libraries as information systems: acquisition, storage, reference searching, retrieval, provision of hard copy or microcopy, dissemination, abstract dissemination, and preparation of bibliographies. He suggests that the additional functions of abstract preparation and index preparation and dissemination belong solely to abstracting and indexing services while answering of technical questions and preparation and dissemination of analytical studies solely to information analysis centers.

To illustrate, Korfhage [37] uses the model of Figure 2 as a representation of the ideal information retrieval system. The user, seen as one of two major inputs, enters with needs for information which must be related to a servicer. These needs are translated into queries which can be analyzed. The files are searched, after the index search has revealed what relevant items or documents are available, for the specific queries. The user is then given the available items he desires.

The other major input, the items, or documents, which enter the system, are analyzed, indexed, filed, and stored until needed. The usual form of such documents are printed matter such as books or periodicals.[8]

---

8 Discussions of information retrieval systems and their functions in general terms can be found in Vickery [78], Salton [69], and Becker and Hayes [7]. Some of these functions, as carried out in a library, are discussed in detail in Eaton [18], Goldhor [25], and Wulfekoetter [82].

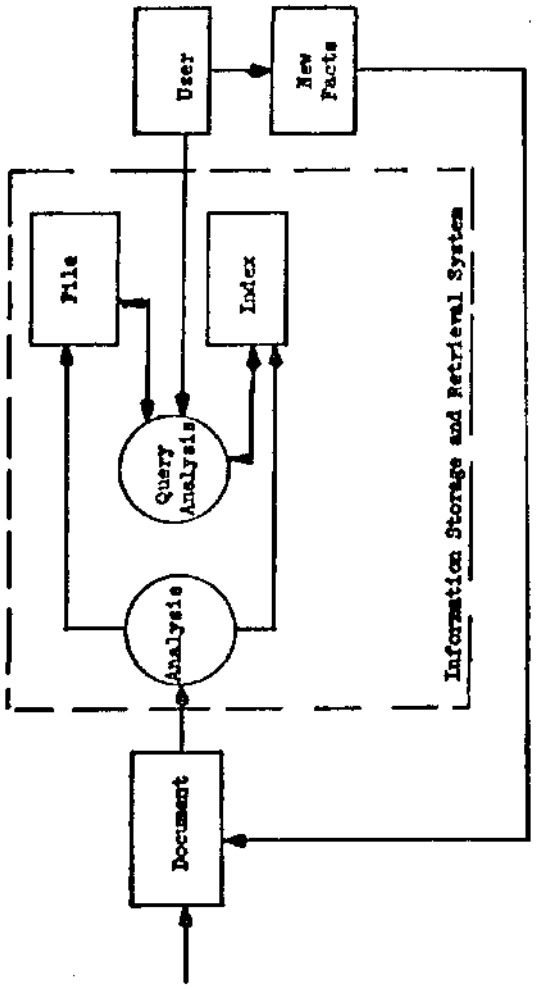


Figure 2. An Information Storage and Retrieval System

Techniques to evaluate performance of information retrieval systems can thus be used for library systems. We shall now see what techniques are available and how well our objective function incorporates these techniques.

Evaluating the Library System

Tauber [75] lists several "facts-of-life" about information systems that must be faced if one is to honestly evaluate such systems:

i) present systems such as libraries do work and function reasonably well,

ii) economic justification of any system or modification is necessary,

iii) information systems are characterized by light activity levels so a system must be relatively inexpensive -- usage provides revenues in either funds or benefits and usage is usually low,

iv) availability of desired items is crucial -- no one really likes an information system,

v) most queries are simple in structure and require simple answers, and

vi) the scattering of item usage is such that a small portion of the file -- say 20% -- gets most of the usage -- say 80% -- while the rest of the file is used less frequently.

Murdock and Liston [10] list three separate categories of measures used to determine system characteristics:

i) benefits or consequences of the system in terms of the value of the system's output -- how other systems and how peoples' behavior have been affected,

ii) costs -- the capital expenditures required to operate the system; and

iii) performance measures -- the qualities or assets of the system that are controllable, such as accuracy, speed, quality, or extent of coverage.

Consider now what the possible benefits would be of having a library system able to provide its users with materials relevant to their needs for information. Some of these benefits, those directly associated with information retrieval systems, must be related to user satisfaction and usefulness of the information. King [34] cites studies to suggest that:

i) information is valuable to the extent that it contributes to the achievement of goals,

ii) information has value in terms of dollars or man-hours saved,

iii) information can have both private and social value, and

iv) the exact value of information is complex and difficult to determine in precise terms.

These last studies also suggest that all material has the potential of being relevant to someone at some time so that all materials should be acquired if possible with minimization of the cost of provision being a prime goal.

It is felt that the benefits of information-oriented services are more closely aligned with user decisions concerning what sources of information to use. The funder may also use these benefits to justify his investments in the system. The servicer, on the other hand, realizes that these benefits exist, but must concern himself with costs and performance measures of his organization. The librarian, it seems, should be directly involved with the provision of relevant items to the client at a minimum of cost to either the user or the

service organization. This means that service should be given quite democratically, with all customers being treated equally, with little regard as to the perceived benefits of providing information to a specific individual user.

Cost measures are usually associated with the operations of the information system. While there can also be costs associated with the usage of such a system, in general one is specifically referring to the operational costs. It will be assumed that usage costs are, to the best of the servicer's ability, held to a minimum, making relevant materials as easily accessible as possible. It is very important that the servicer become conscious of costs and attempt to minimize operational costs while maintaining a high level of service. Even more important is the fact that there is very little that has been written on the costs of performing the various functions of an information system.

Since the servicer should be primarily interested in how well his organization responds to its clients' needs, it is felt that costs should enter the decision model in the form of budgetary constraints, as imposed by the funder. Williams, et al [81] use cost as the objective function to be minimized in order to determine how large the demand for a specific journal must be before it should be acquired. It is felt that this work does not use the proper objective function; cost minimization is not the prime goal of the librarian, but maximization of good service is. In a discussion of

production functions Baumol [6] points out that cost minimization subject to constraints on minimal production levels can lead to identical results if one maximizes production subject to budget constraints. However, it is not necessarily so that both cases are equivalent, for the journal selection problem, especially in the light of the dynamic nature of the problem.

As Meier [30] points out, public service is the library's raison d' etre. Since, except for minor service charges and excises, it operates as a free public utility, the criteria by which the library should be evaluated must be based on performance measures. Such values as the efficiency with which the system operates and its effectiveness in supplying the user with desired, relevant material are the measures of importance.

In order to determine what policies can best satisfy the requirements of a university library system, the librarian must evaluate how well the system works under any given set of policies and procedures.

Library Objectives

The remaining category of measures used to determine system characteristics are those related to system performance. There has been a good deal of literature written on such performance measures, with a major concern being standards of service to the users. Of course, these measures can never really be isolated from considerations of costs or benefits of service functions. Before considering what factors are used to measure performance, the goals and objectives of a university library should be established. Performance can then be measured in terms of how well these have been accomplished.

As Rzasa [65] points out, the main goal of the Purdue University Libraries has been expressed as "the provision of the material and information needed by its patrons". He goes on to show that the two critical factors in this goal are the availability of the material and/or information sought by a user and the time required to provide such to the user.

Kent [33] lists some possible general objectives of the servicer and/or systems designer of an information retrieval system, objectives such as profit or revenue maximization, enhancement of the servicer's reputation, provision of novel or attractive services in a competitive situation, or fulfillment of an obligation, e.g., the promotion of the free flow of information among members of a professional society. On a more altruistic plane, one can consider the provision of information or documents in response to requests with such quality, scope, speed, and economy as to justify the system's establishment and maintenance to its environment (especially the funders).

Performance Measures

With the objectives of a library defined, various factors can be identified for use in measuring information system performances. Of course these factors will be related to the objectives so that performance implies progress towards goal attainment. In other words, this discussion of performance measures for an information system will provide a background against which the objective function measuring how well a selection policy contributes toward the accomplishment of the libraries' overall aims, can be compared. In terms of the journal

selection problem, in terms of the objective function, the emphasis is on the effectiveness of the system in satisfying demands for information rather than on the efficiency of the system in terms of time or costs involved in the handling of materials.

Using the number of materials and information items requested, the number of satisfactorily answered queries, and the number of users, Rzasa [65] develops some measures of effectiveness for university library performance. His intended use of these measures is for cost-effectiveness analyses to aid in the program budgeting procedures of the Purdue University Libraries.

Accessibility as the ability for a library to easily provide access to desired materials can be considered as a function of comprehensiveness, or the depth and breadth of the collection. As comprehensiveness begins to be increased by the acquisition of materials, accessibility and user convenience increases since there are more articles with which to satisfy user demand. However, if the collection becomes too large, the convenience with which the user can obtain desired materials is impeded. Thus we see O'Neill's [62] tradeoff between accessibility and convenience in terms of comprehensiveness. O'Neill agrees with Morse's [57] inference that if a collection becomes too large, as comprehensiveness increases, it also becomes cumbersome so that convenience or ease of use is lessened. However, it must be pointed out that availability, the second component of accessibility increases with comprehensiveness in that more items are present and ready to be used. The decision

problem should include provisions for these factors and the trade-off itself when evaluating performance of the system.

Convenience, however, is a function not of a single journal but of an overload, if it exists, of several journals and it is a somewhat subjective factor in terms of the decisions of individual users. Thus the objective function of a selection decision model for a specific journal cannot be based directly on convenience.

Constraints on the number of titles selected can be introduced, if desired, to include factors of convenience. The considerations of space and capability of the technical processing unit to properly catalog and shelve large numbers of journals becomes crucial here if such a constraint is used.

Meier [50] suggests four levels on which a library can judge its performance:

i) the quality of its holdings -- based perhaps on the frequency users cite them,

ii) adequacy of reference services -- based on the number of queries successfully answered in a reasonable length of time,

iii) a general performance index -- such as one suggested by Bush, Galliher, and Morse [11], the ratio of the quantity of materials requested to the quantity of materials provided in a reasonable amount of time, and

iv) participation -- the degree by which the library personnel and materials can serve and become an intrinsic part of the library environment.

He notes that these criteria must include weightings for relevance to public welfare.

Meier develops a unit, the item-use-day, to measure the amount of material charged out to users within a standard amount of time. This measure was developed to be used in interlibrary comparisons of performance.

Maizell [49] agrees that norms should be set up for interlibrary comparisons; he bases such norms on median values of the libraries' statistics. He suggests that quality of the collection can be based on comparisons with general listings of available materials in specific fields of endeavor, and that the adequacy of the reference services should include the factors of cost and total number of queries made. He also adds to the list of variables relevant to evaluation of library system what he believes to be the most important factor, though very difficult to quantify and measure; this is the impact of the library system on its environment. Maizell does consider quantification of this factor in discussing the extent to which a library can save funds and effort, or contribute to the solution of difficult research problems.

Relevance and Retrieval -- A Contingency Table. John Swets [74] references and summarizes several measures of the effectiveness (accuracy, sensitivity, discrimination) of an information retrieval system. He uses a 2x2 contingency table of pertinence (relevance to the users) and retrieval as represented in Figure 3. The symbols V_1 and V_2 denote the values of retrieving (R) a pertinent (P) item and not retrieving ($\bar{R}$) a nonpertinent ($\bar{P}$) item, respectively; the symbols K_1 and K_2 denote the costs of retrieving a nonpertinent item and not retrieving a pertinent item, respectively. The symbols a, b, c, and d represent the frequencies of occurrence, simple or weighted, of the four conjunctions.

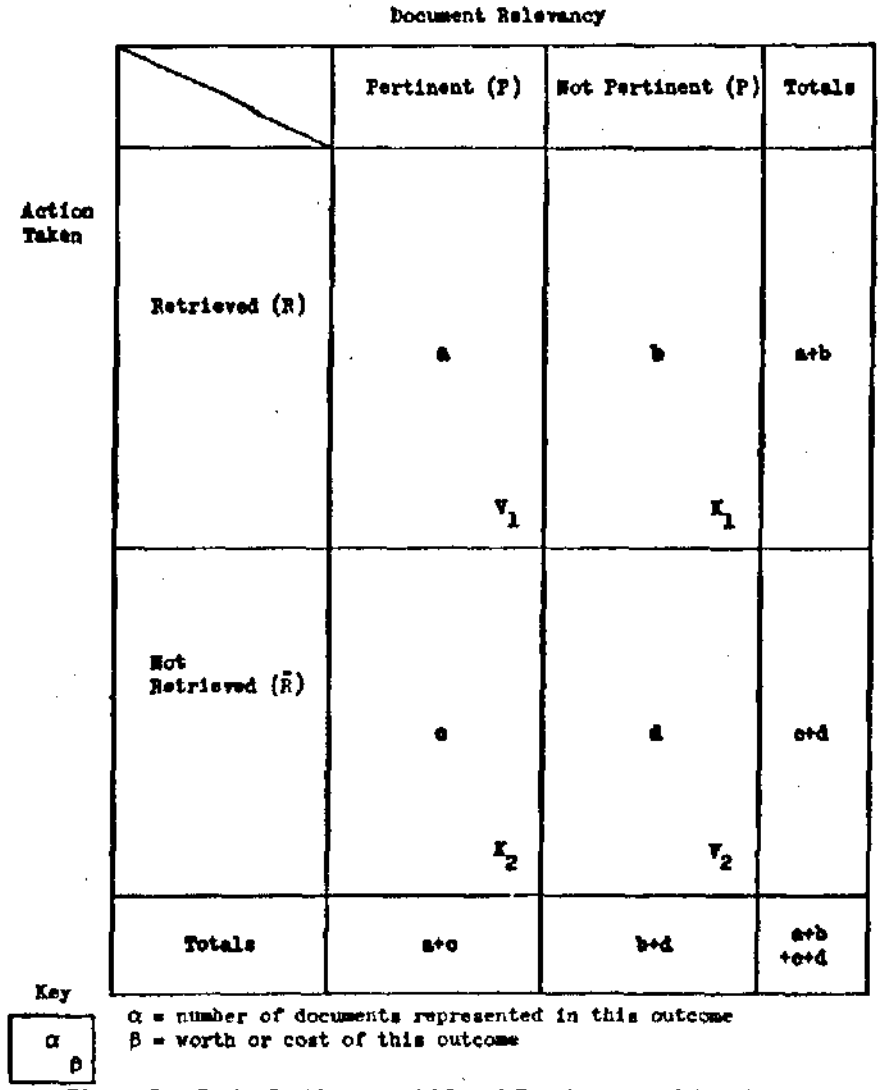


Figure 5. Swets Contingency Table of Pertinence and Retrieval

Swets proposes an analysis based on statistical decision theory, similar to work he has done in signal detection theory. One must assume the existence of a fallible index Z to determine the pertinence of an item relative to a specific query. The variable q representing the probability of relevance of the item to the query, is an example of such an index. One then determines a critical value Z_c such that the item is retrieved if $Z > Z_c$. Figure 4 represents a Swet's comparison of the conditional probability functions of retrieval given either pertinence or nonpertinence.

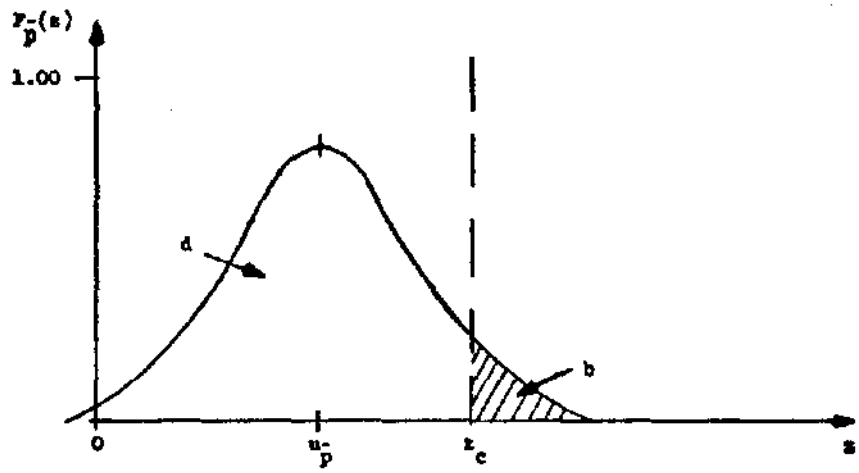
Swets defines the probability density function of the index under the condition of pertinance as $f_p(Z)$ and under the condition of nonpertinence as $f_{\overline{P}}(Z)$. Let μ_p and $\mu_{\overline{P}}$ represent their respective means, σ_p and $\sigma_{\overline{P}}$ represent their respective standard deviations. Assuming normality, he defines a measure of effectiveness as the normal deviate,

II-4) $$E = (\mu_p - \mu_{\overline{P}})/\sigma_{\overline{P}} .$$

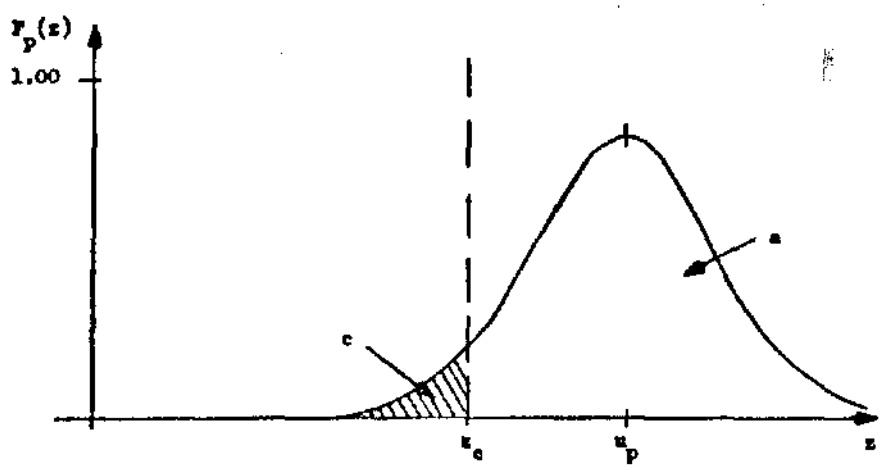
He considers the cases of both constant and increasing variances; the latter with respect to the value of the mean. He finally uses the liklihood ratio $f_p(Z)/f_{\overline{P}}(Z)$ to conclude that the optimal Z_c should be

II-5) $$Z_c = [(V_2 + K_1)/(V_1 + K_2)] \cdot [Pr(\text{nonpertinence})/Pr(\text{pertinence})]$$

if the cost and value factors are known and a priori estimates of the probabilities can be made.



The distribution of z given nonpertinence



The distribution of z given pertinence

Figure 4. The density Functions of Swet's Index z

After delving into the literatures on information retrieval systems evaluation criteria, it becomes apparent that very little has been done to evaluate selection and acquisition policies. One cannot discuss retrieval of pertinent items without consideration as to what items have been placed in the store prior to a request.

Consider the contingency table analysis just discussed. To fully evaluate a library system, statistics gathered on items not retrieved for a specific query (c and d) should be measured not just in terms of what items in the store have not been retrieved but in terms of all documents that exist within the range of the consideration by the information system, i.e., the coverage of the system. However, with the immense number of items that exist, and this number is increasing exponentially, it is virtually impossible to determine the pertinence of all these items.[9] With such statistics too difficult to gather empirically, this research effort must limit itself to measures of effectiveness involving selection and acquisition without consideration to retrieval efforts once in the university library system. Thus, criteria involving the efficiency of retrieval equipment will not be considered at this time.

If the concept of retrieval is replaced by acquisition in the contingency table, one can evaluate selection policies by considering the consequences of the four conjunctions: acquisition of pertinent items, acquisition of nonpertinent items, nonacquisition of pertinent

---

9 Thus recall, the percentage of relevant documents thatare retrieved cannot be realistically measured due to the lack of a measure for the number of relevant documents.

items, and nonacquisition of nonpertinent items. Of course, pertinence will have to be considered separately for each query. Here again the realities of considering all documents not retrieved forces one into a different form of analysis.

A Scheduling Model Format

In order to see the total model from another viewpoint, let us consider it in a scheduling format. Let $t_{j,q}$ = the first period after publication in which the library acquires the issues of journal j published in period q. The range of this variable is normally $0,1,2,\ldots,r-q$; but if the acquisition of the specific issues never occurs in the planning horizon, we can let $t_{j,q} = r-q+1$. With this definition $Y_{j,t,q} = 1$ if $t \geq t_{j,q}$; 0 otherwise. Thus

$$\text{II-6)} \quad Z = \sum_{j=1}^{s} \sum_{q=0}^{r} \sum_{t=0}^{r-q} Y_{j,t,q} C_{j,t,q} = \sum_{j=1}^{s} \sum_{q=0}^{r} \sum_{t=t_{j,q}}^{r} C_{j,t-q,q}$$

is the objective function of the scheduling model. The problem is when to schedule the acquisition of the journals.

The purpose of this reformulation is to first illustrate its relationship to standard problems such as production scheduling so that we see that library problems are in some ways similar to those of other organizations, and second, to show that solutions to the journal selection problem have implications for some problems of these other organizations. The objective function as represented in (II-3) can be seen as analogous to that for the general scheduling problem. The reader may consult Conway, Maxwell and Miller [15] for further references to the scheduling problem.

CHAPTER III

CONSTRAINTS - DETERMINING THE SET OF FEASIBLE SOLUTIONS FOR THE JOURNAL SELECTION PROBLEM

With the objective function defined in terms of journal worth, the next step is to consider what constraints or restrictions are imposed upon the decision maker. These constraints are imposed by the nature of the model itself, by the harsh realities of limited resources, and by the additional considerations of the funders, servicers, and users of the system.

The first section is concerned with constraints on the decision variables $Y_{j,t,q}$. In the second section, we deal with costs of operation in relation to budget constraints, the main set of constraints in the model. The third section contains other constraints representing other scarce resources such as space. In the final sections we present the total model in review and consider the scheduling model format as originally discussed in Chapter II.

Constraints on the Decision Variable $Y_{j,t,q}$

By definition, the decision variable, $Y_{j,t,q}$ can be either one or zero, depending upon whether or not the issues of journal j published in time period q have been acquired as of period t+q. Thus, we have the obvious constraints:

III-1) $Y_{j,t,q} = 0$ or 1 for every j, t, and q as defined.

This leads us to the zero-one integer programming problem format to be discussed in Chapter VI.

Another set of constraints comes from the definition of the decision variable in terms of the subscript t, representing age. Since we are considering acquisition of a journal as of t periods after publication, we assume that once acquired, a journal stays acquired in terms of remaining within the library system, or that

III-2) $Y_{j,t,q} \leq Y_{j,t+1,q}$ for all j, t, and q as defined.

Thus, lost, defaced, or stolen items will be replaced as soon as possible. The value of the journal along with the costs involved usually determine whether a replacement for a missing or defaced volume will be purchased directly or ordered through gifts and exchange channels. This decision procedure can be looked upon as a separate acquisition problem all its own.

There are initial conditions that are specified by whether or not the journal is being currently acquired as of the start of the planning horizon. For any journal j being so acquired, $Y_{j,o,o} = 1$.

The final type of constraint for the variable $Y_{j,t,q}$ arises from the desire for continuity. This means that a journal is ordered continuously with no gaps so that one does not order, discontinue, and re-order subscriptions in some more or less random fashion. Thus, one cannot obtain issues of a journal published in periods q and q+2 without obtaining the issues published in period q+1 if continuity is to be preserved. Let us define a variable $Y_{j,\cdot,q} \equiv \prod_{\omega=0}^{r-q} (1-Y_{j,\omega,q})$ = a dual valued function equalling unity if the issues of journal j published in period q are selected in the planning horizon, zero if not. The continuity constraints could be set up as follows:

III-3d) $\quad Y_{j,\cdot,q+1} \geq Y_{j,\cdot,q} + Y_{j,\cdot,q+2} - 1 \quad .$

Several modifications on this basic theme can be considered as well.

However, it is not reasonable to have to consider the four latter constraint forms, III-3a, b, c, and d, on the decision variable $Y_{j,t,q}$ in this journal selection model. The librarian, as has been mentioned before, is not necessarily an expert in all the areas covered by the library. Thus, without prior knowledge, all journals become independent in terms of importance and benefits to the user group, and can be purchased separately. The issues themselves can be treated independently by letting ρ_j, the subscription length beyond one period, be equal to zero. This makes the model much more tractable and does not arbitrarily force continuity. Further, as far as continuity is concerned, it is to be expected that most journals will be selected by the model's solution algorithm within the period in which they were published. The expected benefits such as usage are highest and the costs are lowest when a

Weingartner [79] has shown how one can form constraints that illustrate desirable interrelationships among journals. For example, if we have the situation where the issues of journal j must always be acquired when the issues of journal k have been acquired, for all time periods, we can represent this situation as follows:

III-3a) $Y_{j,t,q} \geq Y_{k,t,q}$ for all t and q as defined.

Note that this means separate constraints for all pairs of journals so related. A different example of these constraint forms is the instance when the issues of both journals j and k cannot both be acquired but at least one of them must be for all time periods, we have

III-3b) $Y_{j,t,q} + Y_{k,t,q} = 1$ for all t and q as defined.

By using and manipulating these types of representations, one can build up a network of a priori relationships among the journals.

Another type of constraint, of a similar form, is the one reflecting the fact that a subscription may last for several periods. This means that if journal j is purchased with the issues published in period q being acquired in period q ($Y_{j,o,q} = 1$) and ρ_j is one period less than the duration of the subscription, we have

III-3c) $Y_{j,o,q} \leq Y_{j,o,q+1} \leq \ldots \leq Y_{j,o,q+\rho_j'}$

with $\rho_j' = \begin{cases} \rho_j & \text{for } q \leq r-\rho_j \\ r-q & \text{for } q > r-\rho_j \end{cases}$

so that all issues in the subscription period are acquired. The subscription length is usually around five years.

journal is acquired as soon as published. Thus, in the light of how the criteria of journal worth (usage or productivity) vary over time and the additional costs of backordering journals, it will be seen that this seems reasonable. It is highly doubtful that a library would accept a selection model if it were not to allow for such behavior.

It is of note that when such constraints are initially ignored, we are implying that either the initial optimal solution will not violate such constraints, or that such constraints are artificially imposed by past rules of thumb or intuition without rational justification and only serve to hinder the library's attainment of its goals. The model is, after all, a prescriptive one, describing how decisions should be made rather than how they are currently being made.

Costs and Budget Constraints

As has been mentioned before, the most restricting factor in library operations is the scarcity of capital to suitably handle the deluge of information and of user demands. In fact, it can be said that many of the other constraints, such as lack of space, can be related to scarce capital in that expansion is prohibited when there is no money to pay for expansion. The lack of infinite resources plus the desire for the funder to set controls means that budgets for various phases of operation will be set for the library system.

The setting of budgets is not merely a factor of control or of scarce resources, however; in practice, operating costs and performance also count. The final budget figure usually becomes a negotiated sum with library administrators working with university officials [65].

Thus, the model can show the effects of various budgeting policies on the library system's ability to provide service. In the case of journal selection, one can demonstrate a return on investment in terms of the objective function representing journal worth, and so perhaps justify higher budgets.

A study of library costs has been made by Williams, et.al. [61] for the purpose of determining optimal selection policies. It is from this study, based on a survey of four major university libraries, that the cost constraints shall be derived.

The study found that the major activities of a library system associated with journal selection costs were acquisition and subscription functions, cataloging, check-in, claiming, binding and bindery contracts, marking, administration and training functions and others. The authors divide these activities into seven cost categories:

1. initial costs -- cost of selection and ordering, cataloging, and all costs that only occur if the journal is ordered initially in the given period;

2. annual recurring costs -- subscription, check-in, claiming, binding, marking, and bindery costs;

3. storage or maintenance costs;

4. disposition or weeding costs;

5. internal use costs -- circulation, reshelving and shelf maintenance costs as a function of demand;

6. borrowing costs -- personal services, photo duplication and mailing costs associated with items not acquired by the library; and

7. salvage value -- the value or negative cost of the journal at the end of the planning period in terms of dollars and usually assumed to be zero.

Note that the borrowing costs are not really relevant to this study since they relate to alternatives to acquisition. Williams, et.al. also consider adding on 50% of all direct labor costs to cover overhead.

Williams' study found that mid-range costs for his sample were: $60.60 for initial costs, $19.80 for annual recurring costs excluding subscriptions, $0.194 for storage costs, $0.52 for disposition costs, $1.48 for internal use costs, and $6.49 for borrowing costs.

As Leimkuhler [40] points out, this boils down to a cost model of the form

$$\text{III-4)} \qquad K_t = c_1 + c_2 t + c_3 u(t)$$

where K_t is the cost of owning one item for t years, c_1 is the initial cost of acquisition, c_2 is the annual cost of storage and retention, and $c_3 u(t)$ is the usage cost which is proportional to the number of usages made of the item in those t years. According to Williams' [65] study, c_1 = $19.80 + subscription cost, c_2 = $0.194, c_3 = $1.48.[1]

There may be reason to doubt the ability of the cost model to fully reflect exactly how expenditures should be allocated. The major difficulty lies in the cost accounting problem of specifying how much additional capital is actually spent through the acquisition of a given journal. As Leimkuhler and Cooper [45] point out, the cost of interest is the marginal cost for providing the next unit of service.

---

1 Note that for journals, the $60.60 initial cost is only relevant when the journal is initially ordered. The $19.80 initial cost refers to the initial cost of purchasing a specific set of issues of an ongoing journal.

The point is that this cost is to be used to determine whether or not it is feasible or desirable to acquire this journal. Since a library collection consists of several items to be stored and loaned out upon demand, a new journal may be selected, acquired, cataloged, stored, and circulated by mechanisms already in existence so that it is hard to discern exactly what additional effort is required. It is only when several new items come in that expansion of facilities causes a direct increase in fixed expenditures. In other words, it is hard to specify what costs are fixed and what costs are variable in terms of the number of new journals acquired, or in terms of usage, or in terms of the total number of items stored. The quasi-variable status of costs that increase only at discrete steps causes difficulties.

It will be assumed that budgeting and control of basic facilities such as space and manpower are handled externally and independently of selection decisions. It will be seen that this could imply further constraining the journal selection model. It all boils down, then, to assuming that when one assigns a cost to a specific journal above its subscription price, he implies that this cost represents resources of time, space, manpower, capital, and the like that could be put to use elsewhere if the journal is not acquired as opposed to using existing facilities.

As an example, consider a circulation assistant, whose salary is being allocated as a cost to each item checked out, who must be able to do other things when not busy at his station. If he is idle when not checking out library materials, the problem arises in that he must be

there anyway so each individual item should not be charged with an equal portion of his salary. Note that acquiring an additional journal means little in increasing costs in terms of this particular assistant. However, if several thousand journals are acquired, an additional assistant may have to be hired, thereby increasing circulation costs significantly. Thus, hiring circulation assistants is assumed beyond the control of the selection decision and his salary is a variable cost only if he can be used elsewhere when idle, as otherwise it is a fixed cost of the system.

It is felt that while Williams, et.al. largely ignore this problem, their results can indicate ranges in which the cost parameters exist.

The Cost Model

Let c_1 represent any additional cost associated with initially adding the issues of a journal to the collection in a time period excluding subscription costs. Let c_2 represent the periodic (annual) storage and recurring costs including some expected amount to replace lost, stolen, and defaced journals. Next, define c_3 as the cost per usage.[2] The cost per usage will be related to expected rather than actual usage. It seems reasonable to assume that budgets are set up for expected usage with allowances for occasional excesses. This allows us to construct a much simpler model of the problem. Now, let $K_{j,t,q}$ represent the subscription cost associated with acquiring the

---

2 Note that slight modifications in the definitions of c_1, c_2 and c_3 have been made in order to model the cost constraint. We shall use these new definitions from here on.

issues of journal j published in period q in period t+q, and let c_4 represent the cost of initially acquiring a journal into the system. These costs are, of course, the marginal increases in total cost due to the acquisition of the issues of the specific journal in question. We shall assume that these costs are nonnegative. This leads to the following total cost expression for period q, TC_q,

$$\text{III-5)} \quad TC_q = \sum_{j=1}^{s} Y_{j,o,q}(K_{j,o,q} + c_1) + \sum_{j=1}^{s} \sum_{i=o}^{q-1} (Y_{j,q-i,i} - Y_{j,q-i-1,i})$$

$$(K_{j,q-i,i} + c_1) + \sum_{j=1}^{s} \sum_{i=o}^{q} Y_{j,q-i,i}(c_2 + \lambda_{j,q-i,i}c_3)$$

$$+ \sum_{j=1}^{s} c_4 \left(\prod_{i=o}^{q-1} (1-Y_{j,q-1-i,i}) \right)\left(1 - \prod_{i=o}^{q} (1 - Y_{j,q-i,i})\right)$$

where $\lambda_{j,t,q}$ = the expected use, t periods after publication, of the issues of journal j published in period q.

The expression for total cost represents the sum of the costs of subscribing and initially acquiring both current and past issues of the various journals, plus the costs of storing and using such journals.

Letting $B_q \geq 0$ for all q represent the total budget for time period q allotted for journal selection and maintenance, we have the constraint set

$$\text{III-5a)} \quad TC_q \leq B_q, \quad q=1,\ldots,r .$$

The last expression in the cost constraint, the cost to initially order a journal, involves the product of several decision variables, which implies a nonlinear condition. In order to keep the constraint

set linear, let us assume that $c_4=0$. This means that the fixed portion of the cost to initially catalog and store a document is budgeted in total separately, so that this cost will not be considered in the journal selection problem. Thus, there is no marginal increase in cost due to the initial selection of a journal. Note that one can made similar arguments for allowing c_1, c_2, and c_3 to be zero. In terms of the operation of one specific university library system, the Purdue University Libraries, already discussed in some detail in Chapter I, this assumption that $c_4=0$ is realistic.

Other Constraints on Scarce Resources

The librarian, faced with the decision of which journals to acquire, may be limited by factors other than capital. It may be, as this author feels, that capital is the only really vital constraint to be considered in this decision problem. However, let us examine some other possibilities.

One very interesting possibility is due to limited space in which to store journals. In actuality, this is really not a vital consideration due to two facts. One is that, as Lister [48] has shown, the collection can be divided and journals that are not used very often or that are very old can be stored compactly and separately. Moreover, with microfilm and other such miniaturization devices at hand, space can become a relatively small problem. Of course, costs and ease of converting to the use of these devices can still present problems, but these are really outside the realm of the journal selection decision. In any case, limited resources such as space can become restrictive and can be modeled as additional constraints.

Letting $M^K_{j,t,q}$ = the amount of scarce resource K used up by having journal j's issues published in time period q acquired as of period t+q, and B^K_q = the budget or available level of resource K in period q, we have the constraint set

$$\text{III-6)} \qquad \sum_{j=1}^{s} \sum_{\ell=0}^{q} Y_{j,q-\ell,\ell} M^K_{j,q-\ell,\ell} \leq B^K_q \qquad \forall\ q,K .$$

It can very well be that $M^K_{j,q-\ell,\ell}$ will be dependent on several factors such as usage. Besides space, manpower and time are two other very precious and all too scarce resources; both can be modeled with expression III-6.

Perhaps the most important reason for presuming that such factors as space, manpower, and time are *outside* the range of consideration for the journal selection problem is the idea that there will be no unit increase in usage of these scarce resources. As mentioned above, the acquisition of specific issues of a specific journal may add little or no cost to the total. In like manner, the resources may already be there, budgeted on an a priori basis, as fixed assets so that the journal issues in question do not require any additional effort. Thus the capacities of a library may become inadequate by exceeding bounds on the number of new acquisitions rather than by the purchase of any specific item.

One special case of a constraint on scarce resources that can be important is the restriction on the number of journals selected in a period. Since one is interested in the use of scarce resources only

to the extent that the selection of a new journal exhausts a portion of those resources, the capacity of the library system may limit the total number of selections made. Thus we have

$$\text{III-7a)} \quad \sum_{j=1}^{s} \sum_{\ell=0}^{q} Y_{j,q-\ell,\ell} \leq B_q^1 \qquad \forall\ q.$$

where B_q^1 is the total number of selections as of period q allowed. If we want to limit the number of selections made in period q, consider

$$\text{III-7b)} \quad \sum_{j=1}^{s} \left[\sum_{\ell=0}^{q-1} (Y_{j,q-\ell,\ell} - Y_{j,q-\ell-1,\ell}) + Y_{j,o,q} \right] \leq B_q^2$$

where B_q^2 is the total number of selections in period q allowed. Note that we can produce III-7a) from III-6) by assumming that $M^1_{j,q-\ell,\ell} = 1$ for all j, ℓ, and q. This means that if each new acquisition uses up the same amount of a scarce resource, we merely restrict the number of new journals to cover the constraint limitations.

The Total Model in Review

The total model as it will be when analyzed in Chapter VI is presented in Figure 5. Note that in Figure 5, formula III-8) is a re-work of III-5).

Thus we have a zero-one integer programming problem that will be discussed in much more detail in Chapter VI of this thesis. Chapters IV and V will deal with measures of a journal's worth, that is to say, methods of specifying values for $C_{j,t,q}$.

$$\text{Max } z = \sum_{j=1}^{s} \sum_{q=0}^{r} \sum_{\ell=0}^{q} Y_{j,q-\ell,\ell}\, a_{j,q-\ell,\ell}$$

s.t.

III-8) $$\sum_{j=1}^{s} \sum_{\ell=0}^{q} Y_{j,q-\ell,\ell}\,(c_1 + c_2 + c_3\,\lambda_{j,q-\ell,\ell} + K_{j,q-\ell,\ell}) + \sum_{j=1}^{s} \sum_{\ell=0}^{q-1} Y_{j,q-\ell-1,\ell}\,(-c_1 - K_{j,q-\ell,\ell}) \leq B_q \quad \forall\ q=1,\ldots,r$$

III-1) $Y_{j,q-\ell,\ell} = 0$ or 1 $\quad \forall\ j=1,\ldots,s;\ \forall\ q=1,\ldots,r;\ \forall\ \ell=0,1,\ldots,q$

III-2) $Y_{j,q-\ell,\ell} \geq Y_{j,q-\ell-1,\ell}$ $\quad \forall\ j=1,\ldots,s;\ \forall\ q=1,\ldots,r;\ \forall\ \ell=0,1,\ldots,q-1$

Figure 5. The Total Journal Selection Model

A Scheduling Model Format

The reformulation of the journal selection problem constraints into a scheduling problem format again yields a zero integer programming format in that one ends up with the identical constraints as those of III-8, III-1, and III-2. This means that one has to define a zero-one decision variable, $Y_{j,t,q}$, as before. Here $Y_{j,t,q}$ $= [t+q-t_{j,q}]^+/(t+q-t_{j,q})$ where $[a]^+ = \begin{cases} a \text{ if } a \geq 0. \\ 0 \text{ if } a < 0. \end{cases}$

Thus the relationship of the journal selection problem to scheduling problems is still appropriate but no new insights are gained by such relationships.

CHAPTER IV

ONE MEASURE OF A JOURNAL'S WORTH: USAGE

In considering what criterion to use when measuring the worth of acquiring a specific journal, one factor that consistently comes to mind is the usage of or demand for that journal. Williams, et.al. [81] suggest using estimates of demand as a basis for journal selection in order to minimize the total cost of operation. Letting $C_{j,t,q}$ represent the number of demands for the issues of journal j published in time period q made t periods after publication has several interesting implications. First of all, this means that the objective function now represents the number of demands that the library has the potential to directly satisfy. Moreover, the expression

$$\sum_{j=1}^{s} \sum_{q=0}^{r} \sum_{t=0}^{r-q} (1-Y_{j,t,q})R_{j,t,q}$$

now represents the total worth of not acquiring the proper journals in terms of the number of demands that cannot be even potentially satisfied by the university library without resorting to external sources through facilities such as interlibrary loan agencies. The

variable $H_{j,t,q}$, as expressed in II-2a), takes the position of a corrective factor; a portion of the unsatisfied demand is deducted from the potentially satisfied demand in the objective function of II-3a) to compensate for bias.

Finally, it is to be expected that relevance or pertinence in terms of a contingency table analysis and usage are directly related. As demand decreases, one would expect relevance to decrease so that $Q_{j,t,q}$ becomes somewhat representative of the smaller worth of acquiring a nonpertinent journal. On the other hand, as usage decreases, $H_{j,t,q}$ increases and becomes somewhat representative of the lesser detriment of not acquiring a nonpertinent journal.

Some difficulties do arise, however, when considering usage as a criterion for measuring a journal's worth. Usage alone cannot account for user convenience nor the unavailability of desired items due either to prior circulation or to nonselection if no user petitions the library. Moreover, as O'Neill [62] and Jain point out, usage can be divided into many categories, such as external use or circulation as opposed to in-library use, so that exact measurement of total usage is difficult. Usage does not imply utility or usefulness of items in the collection to specific users. On the other hand, usage provides an advantage in that it measures, to some degree, both the availability of sought items and the comprehensiveness of the collection.

This chapter is concerned with models of usage so that demand patterns can be predicted and put to use in a journal selection model. Demands for library materials are treated as random variables, changing over time. In the first section we consider past studies and models of usage over time with most of the emphasis being on

parametric representations of expected usage. The second section contains a Markovian model and modifications of that model which make it more realistic. It is this model that will be used in the journal selection decision model. In order to consider other possibilities, the "Pn" model, a modified Poisson distribution of usage, is investigated in the third section, with some attention given to a few parametric forms that could be used in a modified version of a "Pn" model. Note that the second and third sections are involved with probabilistic models, although the emphasis is on expected usage over time. In the fourth section we examine the estimation of parameters of such usage models. The "Pn" model is used to illustrate how parameter estimation difficulties can limit the complexity one can allow a parametric form to take. The final section deals with the fitting of data to the modified Markovian model.

Usage Studies - Past Models

Jain [30] estimated that as of 1967 there were approximately 700 articles written on various studies of usage in a library system. He gives a brief review of several usage studies and discusses many of the mathematical models involved in describing user behavior. Many of the studies were concerned with identification of the user, how long he spent in the library, how many tasks he performed in the library, what tasks he performed in the library, and what materials he used in performing these tasks.

There are a number of studies concerned with circulation statistics, which can be used to indicate what items have been supplied in answer to user demands for information. Another point brought forth is that

expected circulation of a particular item should decline over time as the item ages and becomes obsolescent. Williams, et.al. [81] found a seven percent decline annually in circulation as a journal aged. Leimkuhler [38] used an exponential decay model to suggest that

IV-1) λ_t = the average annual circulation for an item t years old

$= \lambda_o(1-\beta)^t, \quad 0 < \beta < 1.$

This implies that

IV-2) C_t = the average cumulative circulation for the first t years

$= \sum_{i=1}^{t} \lambda_i = C_o(1-\beta)(1-(1-\beta)^t)/\beta$ and that $\lambda_o = C_o$.

Leimkuhler also considered an exponential growth rate,

IV-3) N_t = the size of the library as of the end of year t in terms of the number of items acquired

$= N_o(1+\alpha)^t,$

and combines IV-1) and IV-3) to form a statement as to the average circulation of the total library collection in year t:

IV-4) $\lambda(N_t) = \alpha C_o N_t / (\alpha+\beta) + \beta C_o (1-\beta)^t N_o/(\alpha+\beta).$

He also suggests a decision rule that would relocate all items d or more years old into areas of compact storage. The proportion of the collection to be relocated as of year t, n_d, and the proportion of total circulation due to these relocated items, c_d, are shown to be

IV-5a) $n_d = (1+\alpha)^{-d}$

and

IV-5b) $c_d = [(1-\beta)/(1+\alpha)]^d$.

Jain, based on studies of circulation in various branches of a university library system, estimates that $\alpha = 0.06$ and that $\beta = 0.06$ or $\beta = 0.045$, depending on whether age is defined as the time since acquisition or since publication, respectively.

Cole [14] considered usage of journals as a function of age and examined the following equation:

IV-6) $R_x = R_T e^{-\lambda x}$

where R_x = the number of references older than x years out of a total of R_T references. Letting $R_x/R_T \times 100 = w$ and $K = \lambda \log_{10} e$, Cole rewrites IV-6) as

IV-6a) $\log_{10} w = 2 - K_x$.

Cole also discusses the median age of a collection and unsatisfied demands that are due to having never acquired certain journals and due to age for journals acquired but removed from the collection. Jain shows that if N represents the maximum age under consideration, Cole's model can be re-written as

IV-7) $C_t = C_N [1-e^{-\lambda t}]$ or

IV-7a) $\lambda_t = C_N e^{-\lambda t}(e^{\lambda} - 1)$.

Jain's dissertation cites several other studies which consider various parametric forms to describe circulation over time. One of particular interest is Shaffer and Ernst's

IV-8) $\lambda_t = Ae^{-at} + Be^{-bt}$

where they found the best fit to be A = 10, a = 0.9, B = 1.1, b = 0.015. Bush, Galliher, and Morse [11] use a similar form to describe the number of attendees staying in the library longer than t hours. They assume two groups of users, one with a mean stay of t_1 = 50 minutes and another with a mean stay of t_2 = 100 minutes. Using n_1 = 105 and n_2 = 170 as a best fit, they assumed a curve

IV-9) $n(t) = n_1e^{-t/t_1} + n_2e^{-t/t_2}$.

Jain also cites Jacob who considers usage versus age. Letting a = age = 1962 - year of publication and f(a) = density function of used journals, he suggested that

IV-10) $f(a) = 0.039(0.935)^a + 0.188(0.53)^a$.

This parametric form, repeated here three times, will also occur later in a major modification of the Markovian model of usage.

Jain considers an exponential class of functions

IV-11) $\lambda_t = \alpha\beta^a$

and a logarithmic class of functions

IV-12) $\lambda_t = B \log(1+1/t-1+b)$.

He discusses a square root class of functions and shows examples of studies that examine this form. He also notes that λ_t and $\Delta C_t = C_t - C_{t-1}$ were monotonically increasing in t for all classes of functions considered.

Williams, et.al. suggest a lognormal probability distribution of demand,

IV-13) $$f(x)dx = (2\pi)^{-1/2} \cdot 1/\sigma \cdot e^{-(\ln x - u)^2/2\sigma^2} \, dx/x$$

where f(x)dx is the fraction of titles with a demand per title between x and x+dx. This implies that the fraction of titles with d or more demands is given by

IV-14) $$F_S(d) = \int_d^\infty f(x)dx = \int_{(\ln d - u)/\sigma}^\infty (2\pi)^{-1/2} \cdot e^{-y^2/2} \, dy$$

$$= 1 - \Phi\left(\frac{\ln d - u}{\sigma}\right) = \Phi\left(\frac{u - \ln d}{\sigma}\right)$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standardized normal distribution. A value of 1.94 is suggested for σ.

If a decision rule is used that says that a library will purchase all journals having an estimated demand greater than or equal to d, Williams, et.al., show that the fraction of requests that can be potentially satisfied is given by

IV-15) $$F_R(d) = \Phi\left(\frac{u - \ln d}{\sigma} - \sigma\right)$$

which is inversely related to d. All fractions and proportions in this study are with respect to the total number of journals, or titles, with nonzero, thus positive, demand. The study also suggests that the fraction of total demand for a specific title whose age is less than or equal to A years is given by

IV-16) $F(A) = 1 - c^{A}$

where c is estimated to be equal to 0.93.

A Markovian Model

In order to better understand how demand for a specific item varies over time, Morse [58] analyzes demand within the framework of a dynamic random variable. He uses a Markovian model, basing current demand on the previous time period's demand structure. Thus, his model, as stated in our terms, is built around the following probability statement:

IV-17) $Pr[G_{j,t,q} = n \mid G_{j,t-1,q} = m] = e^{-\lambda' j,t,q} \lambda'^{n}_{j,t,q}/n!$

with n=1,2,3,...; for all j, t, and q as defined before. Since the form of IV-17) implies the Poisson distribution, one can say that

IV-18) $E(G_{j,t,q} \mid G_{j,t-1,q} = m) = \lambda'_{j,t,q}$ for all j, t, and q with $\lambda'_{j,t,q}$ being a function of m. One can also show that the following holds true:

IV-19) $$Pr[G_{j,t,q} = n] = \sum_{m=0}^{\infty} Pr[G_{j,t,q} = n \mid G_{j,t-1,q} = m] \; Pr[G_{j,t-1,q} = m]$$

$$= \sum_{m=0}^{\infty} Pr[G_{j,t-1,q} = m] \; e^{-\lambda' j,t,q} / n!$$

for all j, t, and q.

Morse assumes a linear form for $\lambda'_{j,t,q}$,

IV-20) $\lambda'_{j,t,q} = \alpha + bm$

and concludes that

IV-21) $\lambda_{j,t,q} = E(G_{j,t,q}) = \alpha + b\, E(G_{j,t-1,q}) = \alpha + b\, \lambda_{j,t-1,q}$

$= (\lambda_{j,o,q} - \alpha/1-b)b^t + \alpha/1-b$,

and

IV-21a) $Var(G_{j,t,q}) = \alpha\, [(1-b^t)(1-b^{t+1})/(1-b)(1-b^2)] +$

$[(1-b^t)/(1-b)]b^t\, \lambda_{j,o,q} + b^{2t}\, Var(G_{j,o,q})$.

It is also assumed that b lies in the interval [0,1) which implies an exponentially decreasing demand. It is of interest to write IV-21) as

IV-21b) $E(G_{j,t,q}) = \lambda_{j,t,q} = \lambda_{j,o,q}\, b^t + (\alpha/1-b)(1-b^t)$

$= \lambda_{j,o,q}\, b^t + \alpha \sum_{i=o}^{t-1} b^i$.

Thus expected demand consists of two portions, both of an exponential decay form. The first expression, $\lambda_{j,o,q}\, b^t$, represents the usual concept of exponential decay. The second represents a more unusual form in terms of a partial series of exponential decays.

It can be shown that the cumulative expected demand for t periods is given by:

IV-22a) $C_{j,t,q} = \sum_{\ell=o}^{t} \lambda_{j,\ell,q} = \sum_{\ell=o}^{t} [(\lambda_{j,o,q} - \alpha/1-b)b^\ell + \alpha/1-b]$

$= (\lambda_{j,o,q} - \alpha/1-b)(1-b^{t+1})/(1-b) + (\alpha/1-b)(t+1)$;

= the expected demand for the issues of journal j published in period q as of the first t periods after publication,

and differential demand, as given by

IV-22b) $\Delta \lambda_{j,t,q} = \lambda_{j,t,q} - \lambda_{j,t-1,q} = -(\lambda_{j,o,q} - \alpha/1-b)(1-b)b^{t-1}$.

Jain defines other statistics and shows, for the Markovian model, what form they take in order to compare various usage models. Consider one such statistic, *obsolescence*:

IV-22c) $O_{j,q}(t) = -\Delta \lambda_{j,t,q}/\lambda_{j,t-1,q} = (\lambda_{j,o,q} - /1-b)(1-b)b^{t-1} /$

$$[(\lambda_{j,o,q} - \alpha/1-b)b^{t-1} + \alpha/1-b] .$$

Considering $\lambda_{j,t,q}$ as a continuous function of t, Jain then looks at the derivatives:

IV-22d) $\lambda^{o}_{j,t,q} = \partial\lambda_{j,t,q}/\partial t = (\lambda_{j,o,q} - \alpha/1-b)b^{t}\ \ln b$, and

IV-22e) $\rho_{j,q}(t) = -\lambda^{o}_{j,t,q}/\lambda_{j,t,q} = (\lambda_{j,o,q} - \alpha/1-b)b^{t}\ \ln b /$

$$[(\lambda_{j,o,q} - \alpha/1-b)b^{t} + \alpha/1-b], \text{ and}$$

IV-22f) $\partial[C_{j,t,q}]/\partial t = (b/1-b)\lambda^{o}_{j,t,q} + \alpha/1-b$.

Jain points out that $\lambda_{j,t,q}$ decreases monotonically, remains constant, or increases monotonically, as does $\frac{\partial C_{j,t,q}}{\partial t}$ and $-\lambda^{o}_{j,t,q}$, with respect to t depending upon whether $\lambda_{j,o,q} > \alpha/1-b$, $\lambda_{j,o,q} = \alpha/1-b$ (so that $\lambda_{j,t,q} = \alpha/1-b$, $C_{j,t,q} = (\alpha/1-b)t$, $\partial C_{j,t,q}/\partial t = \alpha/1-b$, and $\lambda^{o}_{j,t,q} = 0 = \rho_{j,q}(t) = O_{j,q}(t) = \Delta \lambda_{j,t,q}$), or $\lambda_{j,o,q} < \alpha/1-b$, respectively.

Morse and Elston [59] consider the z-transform of IV-19) which yields

IV-23)
$$P_{j,t+1,q}(z) = \sum_{j=0}^{\infty} \Pr\{G_{j,t+1,q} = n\} z^n$$

$$= \sum_{n=0}^{\infty} z^n \sum_{m=0}^{\infty} \Pr\{G_{j,t+1,q} = n \mid G_{j,t,q} = m\} \Pr\{G_{j,t,q} = m\}$$

$$= \sum_{n=0}^{\infty} z^n \sum_{m=0}^{\infty} \Pr\{G_{j,t,q} = m\} e^{-(\alpha+bm)} (\alpha+bm)^n / n!$$

$$= P_{j,t,q}\left[e^{b(z-1)}\right] e^{\alpha(z-1)}$$

which is an exceedingly difficult transform to invert explicitly. They have also looked at this recursive relationship and solved for $P_{j,t,q}(z)$ for small values of t.

A Modification of the Markovian Model

Morse and Elston present estimates of the values for the parameters of their model based on circulation data. Their results and the results of a study to fit the parameters of the modified model below can be found later in this chapter. It should be noted that for all the various scientific fields of endeavor, the parameter α is decreasing over time instead of remaining constant as originally assumed by Morse and Elston. They found that overall the best estimate for α was 0.37 and for b was 0.48, but that α ranged from 0.37 to 0.15 over time for all books in the sample under observation.

To account for the discrepancy in the parameter α, consider the following change:

IV-24) $\alpha_t = ac^t$ or

IV-24a) $\lambda'_{j,t,q} = ac^t + bx$,

where a and c are nonnegative parameters and c is in the interval (0,1]. Thus, in a manner similar to the one before, we have

IV-25) $\lambda_{j,t,q} = ac^t + b\lambda_{j,t-1,q} = (\lambda_{j,o,q} - ac/c-b)b^t + (ac/c-b)c^t$

$= \lambda_{j,o,q}\, b^t + ac[c^t-b^t]/(c-b)$

for all values of j, t, and q as defined before. Of course, this assumes that $c \neq b$. Notice the similarity of this form as compared to the models represented by equations IV-8), IV-9), and IV-10). If $c=b$, we have the special case

IV-25a) $\lambda_{j,t,q} = ab^t + b\lambda_{j,t-1,q} = (\lambda_{j,o,q} + at)b^t$;

however, it can be seen that, generally speaking, parameter estimates should indicate that $c \neq b$. Note also that when $c=1$, the original case, as presented in equation IV-21), is obtained. One can thus generalize expression IV-23) to yield

IV-23a) $P_{j,t+1,q}(z) = P_{j,t,q}\,[e^{b(z-1)}]\; e^{ac^t(z-1)}$.

Now, consider the various expressions used by Jain to compare various parametric forms of usage models for this modification of the Markovian model:

IV-26a) $$C_{j,\ell,q} = (\lambda_{j,o,q} - ac/c-b)(1-b^{t+1})/(1-b) + (a/1-b)(1-c^{t+1})/(1-c)$$

IV-26b) $$\Delta \lambda_{j,t,q} = -(\lambda_{j,o,q} - ac/c-b)(1-b)b^{t-1} - (ac/c-b)(1-c)c^{t-1}$$

IV-26c) $$O_{j,q}(t) = [(\lambda_{j,o,q} - ac/c-b)(1-b)b^{t-1} - (ac/c-b)(1-c)c^{t-1}] \; /$$

$$[(\lambda_{j,o,q} - ac/c-b)b^{t} - (ac/c-b)c^{t}]$$

IV-26d) $$\lambda^{o}_{j,t,q} = (\lambda_{j,o,q} - ac/c-b)b^{t} \ln b + (ac/c-b)c^{t} \ln c$$

IV-26e) $$\rho_{j,q}(t) = [-(\lambda_{j,o,q} - ac/c-b)b^{t} \ln b - (ac/c-b)c^{t} \ln c] /$$

$$[(\lambda_{j,o,q} - ac/c-b)b^{t} + (ac/c-b)c^{t}]$$

IV-26f) $$\partial[C_{j,\ell,q}]/\partial t = -[(\lambda_{j,o,q} - ac/c-b)b^{t} \ln b/1-b]$$

$$-[(a/c-b)c^{t+1} \ln c/1-b]$$

To further illustrate the properties of the modified Markovian model, let us consider the following lemma.

Lemma

The expression $\lambda_{j,t,q}$ is a decreasing function of t for large values of t if the parameters a, b, and c, as used in equation II-25) to define $\lambda_{j,t,q}$ satisfy the following:

i) $b \in (0,1)$,

ii) $c \in (0,1)$,

iii) $b \neq c$,

iv) $a > 0$.

Proof:

i) $\Delta \lambda_{j,t+1,q} = \lambda_{j,t+1,q} - \lambda_{j,t,q} = -(\lambda_{j,o,q} - ac/c-b)(1-b)b^t - \left(\frac{ac}{c-b}\right)(1-c)c^t$ from equation II-26b). However, if $\lambda_{j,t+1,q}$ is a decreasing function of t, we must have $\lambda_{j,t+1,q} \leq \lambda_{j,t,q}$, or $\Delta \lambda_{j,t+1,q} = \lambda_{j,t+1,q} - \lambda_{j,t,q} \leq 0$, which implies that

ii) $-(\lambda_{j,o,q} - ac/c-b)(1-b)b^t - (ac/c-b)(1-c)c^t \leq 0$ or $-(\lambda_{j,o,q} - ac/c-b)(1-b)b^t \leq (ac/c-b)(1-c)c^t$, which leads to

iii) $-(\lambda_{j,o,q} - ac/c-b)\left(\frac{1-b}{1-c}\right) \leq (c/b)^t (ac/c-b)$.

Case 1: $\lambda_{j,o,q} \geq ac/c-b$ and $c > b$ which implies that the left hand side of iii) is nonpositive. With $c > b$, $ac/c-b > 0$, so we have

iv) $-(\lambda_{j,o,q} - ac/c-b)\left(\frac{1-b}{1-c}\right)/(ac/c-b) \leq (c/b)^t$ where the left hand side of iv) remains nonpositive. By the nature of exponentials $(c/b)^t \geq 0$ so that iv) always holds for all t as defined.

Case 2: $\lambda_{j,o,q} \geq ac/c-b$ and $c < b$.

v) $(c/b)^t \leq -(\lambda_{j,o,q} - ac/c-b)(1-b/1-c)/(ac/c-b)$ or

$t \log(c/b) \leq \log[-(\lambda_{j,o,q} - ac/c-b)(1-b)/(1-c)(ac/c-b)]$ or

$t \geq \log[(-\lambda_{j,o,q} - ac/c-b)(1-b)/(1-c)(ac/c-b)]/\log(c/b)$.

Note that with $c < b$, $\log(c/b) < 0$, $ac/c-b < 0$, and $\lambda_{j,o,q} \geq 0 > ac/c-b$.

Case 3: $\lambda_{j,o,q} < ac/c-b$ which leads to the following statement:

vi) $t \log(c/b) \geq \log[-(\lambda_{j,o,q} - ac/c-b)(1-b)/(1-c)(ac/c-b)]$.

Since this case can only arise if $c > b$, which implies that $\log(c/b) > 0$, one has the following result:

vii) $t \geq \log[-(\lambda_{j,o,q} - ac/c-b)(1-b)/(1-c)(ac/c-b)]/\log(c/b)$.

These results can be interpreted in the following way: $\lambda_{j,t,q}$ is monotonically decreasing with respect to time if $t \geq t^*_{j,q}$, and is increasing if $t < t^*_{j,q}$ with $t^*_{j,q}$ defined as follows:

$$t^*_{j,q} = \begin{cases} < \log[-(\lambda_{j,o,q} - ac/c-b)(1-b)/(1-c)(ac/c-b)]/\log(c/b) >^+ \\ \quad \text{if } \lambda_{j,o,q} < ac/c-b \text{ and } c > b \text{ or if } c < b \text{ and} \\ \quad \lambda_{j,o,q} > ac/c-b \\ 0 \quad \text{otherwise} \end{cases}$$

where $< a >^+ = \begin{cases} \text{the largest integer} \leq a & \text{if } a > 0 \\ 0 & \text{if } a \leq 0 \end{cases}$

We restrict $t^*_{j,q}$ to integer values due to the discrete time parameter in our model.

Note that $t^*_{j,q} \geq 1$ only if $\lambda_o \leq \left(\frac{ac}{c-b}\right)\left[1 - \frac{c(1-c)}{b(1-b)}\right]$ which can only occur when $c(1-c) \leq b(1-b)$. It should also be pointed out that while this rule holds for all t in the interval $(0,1,2,\ldots,r-q)$, it may be that $r-q<t^*_{j,q}$ so that t never reaches $t^*_{j,q}$. This causes no real problems, however, so that the proof is completed. One final point is that this same kind of analysis can be used on Jain's other expressions, as illustrated in equations IV-26a) through IV-26f) to show that this modification of Morse's model still exhibits characteristics similar to those of the original model at least for sufficiently large t.

We can now conclude several things about the modified Markovian model. First of all, the time dependence of all parameters is considered. Secondly, the revised model, which resembles the form of other models of library usage studies, also can account for initial increases in demand by having a suitably low initial usage rate. This is true since $\lambda_{j,o,q}$ is now an observable quantity rather than a parameter.

Let us consider the scheduling model objective function

$$\max z = \sum_{j=1}^{s} \sum_{q=1}^{r} \sum_{\ell=t_{j,q}}^{r-q} \lambda_{j,\ell,q}$$

formed from II-6). Using the modified form of Morse's model,

$$\lambda_{j,t,q} = (\lambda_{j,o,q} - ac/c-b)b^t + (ac/c-b)c^t$$

we have

$$z = \sum_{j=1}^{s} \sum_{q=0}^{r} \sum_{\ell=t_{j,q}}^{r-q} [(\lambda_{j,o,q} - ac/c-b)b^\ell + (ac/c-b)c^\ell]$$

$$= \sum_{j=1}^{s} \sum_{q=0}^{r} \left[\left(\lambda_{j,o,q} - \frac{ac}{c-b} \right) b^{t_{j,q}} \left(1-b^{r-q-t_{j,q}+1}\right)/(1-b) \right.$$

$$\left. + (ac/c-b)c^{t_{j,q}} \left(1-c^{r-q-t_{j,q}+1}\right)/(1-c) \right] .$$

This can be simplified to

$$z = \sum_{j=1}^{s} \sum_{q=0}^{r} \left[(\lambda_{j,o,q} - ac/c-b)b^{t_{j,q}}/(1-b) + (ac/c-b)c^{t_{j,q}}/(1-c) \right]$$

$$+ \sum_{j=1}^{s} \sum_{q=0}^{r} \left[-(\lambda_{j,o,q} - ac/c-b)b^{r-q+1}/(1-b) - (ac/c-b)c^{r-q+1}/(1-c) \right]$$

where only the first term is relevant. Here it can be shown that with $b \leq 1$ and $c \leq 1$, we have z as a decreasing function of $t_{j,q}$ for all j and q. This implies that we could optimize by allowing $t_{j,q}$ to be as small as possible, zero if the constraints will allow.

The "Pn" Model

Jain [30] proposes another usage model because preliminary studies have convinced him that the probability of an item not being used was higher than predicted with other models and should be accounted for separately. The "Pn" model, as it is called, is presented here as a contrast to the simpler Markovian model of the previous section. We shall consider revisions of this Pn usage model in order to make it applicable to a broader class of situations. We shall also see that difficulties can arise due to the complexity of the parametric forms involved in these revised models. Jain's model is based on the following probability statement:

IV-27) $$\Pr\{G_{j,t,q}=n\} = \begin{cases} p_{j,t,q} & n=0 \\ (1-p_{j,t,q})\phi_{j,t,q}(n)/(1-\phi_{j,t,q}(0)) & n=1,2,3,\ldots, \end{cases}$$

and he examines in detail the case where

IV-27a) $$\phi_{j,t,q}(n) = e^{-u_{j,t,q}} u_{j,t,q}^{n}/n!\ .$$

This implies that

IV-28a) $$\lambda_{j,t,q} = E(G_{j,t,q}) = (1-p_{j,t,q})u_{j,t,q}/(1-e^{-u_{j,t,q}}),$$

IV-28b) $$\sigma^2_{j,t,q} = (1-p_{j,t,q})u_{j,t,q}/(1-e^{-u_{j,t,q}}) = \mathrm{Var}(G_{j,t,q})$$

IV-28c) $$E(G_{j,t,q} \mid G_{j,t,q} > 0) = u_{j,t,q}/(1-e^{-u_{j,t,q}}),$$

and

IV-28d) $$\mathrm{Var}(G_{j,t,q} \mid G_{j,t,q} > 0) = u_{j,t,q}/(1-e^{-u_{j,t,q}}).$$

Letting $p_{j,t,q} = e^{-u_{j,t,q}}$ generates $\Pr\{G_{j,t,q}=n\}$ as the Poisson distribution. However, as pointed out by Jain, preliminary studies indicate that $p_{j,t,q} > e^{-u_{j,t,q}}$. After considering several parametric forms, he settles on

IV-29a) $$p_{j,t,q} = (\beta+t)/(\beta+\gamma+t) \quad \text{with } \beta>0,\ \gamma>0$$

and

IV-29b) $u_{j,t,q} = u \geq 0$ for all t. [1]

Note that $p_{j,t,q}$, as all probabilities must be, is in the interval [0,1]; and that $p_{j,t,q}$ is an increasing function of t, going to 1 as t grows large, which implies an obsolescence of literature as it ages, assuming $r > 0$. Further,

IV-30) $$p_{j,t,q}(z) = \sum_{n=0}^{\infty} z^n \Pr[C_{j,t,q} = n]$$

$$= p_{j,t,q} + (1-p_{j,t,q})/(1 - \phi_{j,t,q}(0)) \sum_{n=1}^{\infty} \phi_{j,t,q}(n) z^n$$

$$= p_{j,t,q} + (1-p_{j,t,q}) e^{-u_{j,t,q}} (e^{u_{j,t,q} z} - 1)/(1-e^{-u_{j,t,q}})$$

Revisions in the "Pn" Model

Consider now a more general functional form that seems to have some desirable characteristics

IV-31) $$p_{j,t,q} = e^{-u_{j,t,q}/t + \delta} \quad \text{with } \delta \geq 0.$$

The following lemma enunciates the properties this form does have.

Lemma

$p_{j,t,q}$, as defined in IV-31) has the following properties:

i) $p_{j,t,q}$ lies in the interval [0,1] for all j, t, and q;

---

1 A slight difference is seen between IV-29c) and Jain's original formulation due to a difference in the use of the time subscript t.

ii) $p_{j,t,q}$ is increasing in t for large t and approaches one in the limit for all j and q;

iii) $p_{j,t,q}$ is a decreasing function of $u_{j,t,q}$ for all forms of $u_{j,t,q}$ over time for large t and for all j and q; and

iv) $p_{j,t,q} > e^{-u_{j,t,q}}$.

Proof:

i) $p_{j,t,q} = e^{-u_{j,t,q}/t+\delta} \geq 0$ by the nature of the exponential function. $p_{j,t,q} \leq 1$ if $\ln p_{j,t,q} = -u_{j,t,q}/t+\delta \leq 0$; or that $u_{j,t,q}/t+\delta \geq 0$, which always holds true.

ii) In order for $p_{j,t,q}$ to be an increasing function of t $p_{j,t+1,q} \geq p_{j,t,q}$ or $\ln (p_{j,t+1,q}/p_{j,t,q}) = -u_{j,t+1,q}/(t+1+\delta) + u_{j,t,q}/(t+\delta) \geq 0$. Thus $u_{j,t+1,q} \leq u_{j,t,q}\left(\frac{t+1+\delta}{t+\delta}\right) = u_{j,t,q}(1+1/t+\delta)$ or $\Delta u_{j,t+1,q} = u_{j,t+1,q} - u_{j,t,q} \leq u_{j,t,q}/t+\delta = \ln(1/p_{j,t,q})$. If $\Delta u_{j,t+1,q} \leq 0 \leq u_{j,t,q}/t+\delta$, as will be the case for at least large t, our case is proven. If $u_{j,t,q}$ approaches zero in the limit as t approaches infinity, $p_{j,t,q}$ can be easily shown to approach 1.

iii) If $t+\delta > 0$, which always holds, $p_{j,t,q}$ is seen as being a decreasing function of $u_{j,t,q}$ for at least large t. This is shown by the following argument:

$\partial p_{j,t,q}/\partial u_{j,t,q} = -1/t+\delta \; e^{-u_{j,t,q}/t+\delta} \leq 0$.

iv) $p_{j,t,q} = e^{-u_{j,t,q}/t+\delta} > e^{-u_{j,t,q}}$ implies that $\ln p_{j,t,q} = -u_{j,t,q}/t+\delta > -u_{j,t,q}$. Since $u_{j,t,q} \geq 0$, we have the following result

$$1/t+\delta < 1 \quad \text{or}$$

$$t > 1-\delta$$

which holds for large t. If $\delta > 1$, it holds for all t; and if $0 < \delta < 1$, it holds for $t > 0$. Thus the lemma has been proven.

Note that while the conditions hold for large t, t is restricted to the values (0,1,2,...,r-q) so that at times t may never actually be allowed to become large. However, the point to be made is that the functional form of $p_{j,t,q}$ is such that the desired properties hold if t does become large.

Jain suggests a constant $u_{j,q}$ for $u_{j,t,q}$ over time for all t in order to successfully fit the parameters of his "Pn" model. Jain has allowed $p_{j,t,q}$ to absorb all of the time-dependent properties, freeing $u_{j,t,q}$ from any functional dependence on t. This seems to be one of the few forms that lends itself somewhat readily to parameter estimation.

A second possibility is to use Morse's model as modified by this author in IV-25):

IV-29c) $$u_{j,t,q} = (u_{j,o,q} - ac/c-b)b^t + (ac/c-b)c^t.$$

However, the estimation of parameters becomes exceedingly difficult using this form. Thus Jain's initial form seems to be a reasonable compromise between functional complexity and ease of estimating parameters in trying to reflect reality. As Jain points out, the "Pn" model yielded very good performances when used to fit data.

Estimation of Parameters

The estimation of parameters is a vital part of the modeling process, especially when the model is used to predict variables used in a decision problem. An example of such variables is the future demand rates for library materials. The degree of difficulty of parameter estimation can limit the complexity of a model.

Jain investigates the method of moments in order to estimate the parameters of usage models. The k^{th} moment, defined in our terms, is $\sum_{\ell=0}^{t} \lambda_{j,\ell,q}\, \ell^k$ for the issues of journal j published in time period q in the first t years after publication, t=0,1,2,...,r-q. The method will be used to calculate parameter estimates for the Markovian and "Pn" models.

The Markovian Model

For the original Markovian model where $\lambda_{j,\ell,q} = (\lambda_{j,o,q} - \alpha/1-b)b^{\ell} + \alpha/1-b$, it has been shown that

$$C_{j,t,q} = \sum_{\ell=0}^{t} \lambda_{j,\ell,q} = (\lambda_{j,o,q} - \alpha/1-b)(1-b^{t+1})/(1-b) + \alpha(t+1)/(1-b).$$

It can also be shown that

IV-32) $$E_{j,t,q} = \sum_{\ell=0}^{t} \ell\, \lambda_{j,\ell,q} = (\lambda_{j,o,q} - \alpha/1-b)(1-b^{t} - tb^{t} + tb^{t+1})/(1-b)^2 + \alpha t(t+1)/2(1-b).$$

Paralleling Jain's work, we can obtain as an estimate of $\alpha_{j,q}(t)$,

IV-33a) $$\alpha_{j,q}(t) = F_1(b) = \bar{C}_{j,t,q}(1-b)^2 - \bar{\lambda}_{j,o,q}(1-b)(1-b^{t+1})/(b^{t+1} - b - b^t + t)$$

and a recursive relationship with which to solve for $b_{j,q}(t)$,

IV-33b) $$- \bar{E}_{j,t,q} + \bar{\lambda}_{j,o,q}\, b(1-b^t - tb^t + tb^{t+1})/(1-b)^2 - \frac{F_1(b)b(1-b^t - tb^t + tb^{t+1})}{(1-b)^3} + F_1(b)t(t+1)/2(1-b) = 0$$

where $\bar{E}_{j,t,q} = \sum_{\ell=0}^{t} \ell\, \bar{\lambda}_{j,\ell,q}$,

$$\bar{C}_{j,t,q} = \sum_{\ell=0}^{t} \bar{\lambda}_{j,\ell,q}$$

and $\bar{\lambda}_{j,\ell,q}$ = the observed value of $C_{j,t,q}$. The subscripts on $\alpha_{j,q}(t)$ and $b_{j,q}(t)$ as well as the parameter estimates being a function of t imply that the parameter describes the expected usage of only the specific item in question -- the issues of journal j published in period q based on usage in the first t periods after publication, rather than describing the usage of a whole set of items. Once IV-33b) has been used to solve for $b_{j,q}(t)$, IV-33a) can be used to calculate

$\alpha_{j,q}(t)$. [2] Requirements for a sufficient statistic demand that t-r-q to yield $\alpha_{j,q}(r-q) = \alpha_{j,q}$ and $b_{j,q}(r-q) = b_{j,q}$, statistics now independent of the time variable t.[3]

Some of the differences occurring between Jain's original formulation of parameter estimates and this current one are due to the slight difference in the use of the time subscript t in the definition of the variable $G_{j,t,q}$. Specifically, $G_{j,o,q}$, or $\bar{\lambda}_{j,o,q}$ as observed, is considered here as an observable commodity rather than as a parameter.

It is important to point out that $\alpha_{j,q}(t)$ and $b_{j,q}(t)$ are functions of the variables j, q, and t so that each volume of each journal can have different parameters which can also vary over time.

---

2 One method of approximating $b_{j,q}(t)$ that can be used is known as Newton's method and is discussed in Hadley [28]. Note that the subscripts of $b_{j,q}(t)$ have been left off in IV-33a) and IV-33b) for the sake of convenience. This deletion of subscripts will be used throughout the paper when the b and c parameters are estimated in complex recursion formulas.

3 By sufficient statistic we imply a random variable that can be used to estimate an unknown population parameter and has the properties that it is not dependent on any unknown parameters and that it exhausts all information available from a given random sample drawn from the population in question. This information is related to the values of the unknown parameters [29]. If we draw a random sample of size n from a population with a normal probability distribution of unknown mean μ and known variance σ^2, $\bar{x} = \sum_{j=1}^{n} x_j/n$ is a sufficient statistic in terms of the parameter μ.

Both Morse and Jain, however, consider $b_{j,q}(t)$ to be always equal to $b_{j,q}$ in that b is independent of time. This implies that $b_{j,q} = b_{j,q}(r-q)$ is the best statistic to use. One might want to calculate $b_{j,q}(t)$ for all t and test for dependence with respect to time using the statistic $x^2 = \sum_{t=0}^{r-q} [b_{j,q}(t) - b_{j,q}]^2 / b_{j,q}$, a chi-square statistic, with the hypothesis of independence being rejected at the α level of significance if $x^2 > x^2_\alpha$ [63].

The Modified Markovian Model

The parameter $\alpha_{j,q}(t)$ has been observed by Morse and Elston to be a time-dependent function which decreases over time. To account for this, consider the parametric form $\alpha_t = ac^t$. Using the method of moments, we can obtain as the estimate of $a_{j,q}(t)$,

$$\text{IV-33c)} \quad a_{j,q}(t) = (1-c) \sum_{t=0}^{t} \alpha_{j,q}(t) / (1-c^{t+1}) ;$$

and a recursive relationship with which to calculate $c_{j,q}(t)$,

$$\text{IV-33d)} \quad -\sum_{t=0}^{t} t\, \alpha_{j,q}(t) + c(1-c^t - tc^t + tc^{t+1}) \sum_{t=0}^{t} \alpha_{j,q}(t)/(1-c)(1-c^{t+1}) = 0.$$

Here again $a_{j,q}(t)$ and $c_{j,q}(t)$ have subscripts and are functions of t so that each item has its own set of usage parameter values. Sufficiency requires that IV-33c) and IV-33d) use t=r-q yielding $a_{j,q} = a_{j,q}(r-q)$ and $c_{j,q} = c_{j,q}(r-q)$, both independent of time. These assumptions of independence could be tested using chi-square statistics as before. Another statistical test of interest is whether or not $c_{j,q}$ is significantly

smaller than zero; establishing whether or not the estimator $\alpha_{j,q}(t)$ is really a function of time.

Generalized Parameters for the Modified Markovian Model

While the estimates of $a_{j,q}$, $b_{j,q}$, and $c_{j,q}$ are assumed to be independent of journal age, they may still be functions of volume and journal variables. Each volume of each journal may have unique demand parameters associated with it which could force a decision-maker to have to estimate an enormous number of parameters. However, both Jain's and Morse's studies imply that more general statistics were used to try to estimate demand parameters for library materials concerned with whole fields of scientific endeavor rather than individual parameters for each item. This is evidenced by the fact that the probability of an item t periods old being used n times is measured by Jain as the ratio of the number of items t periods old observed having exactly n uses to the number of items t periods old. This means that the parameters are not common only to one specific item but to a set of items in a scientific field.

Let us consider the following generalized moments

$$A_{j,t} = \sum_{q=0}^{r-t} \lambda_{j,t,q}$$

= the expected number of demands for the issues of journal j t periods after publication

$$= \sum_{q=0}^{r-t} \lambda_{j,o,q} + a(r+1)/(1-b) - at/(1-b) - a(r+1)b^t/(1-b) + atb^t/(1-b),$$

$$C_{j,t} = \sum_{\ell=0}^{t} A_{j,\ell} = \sum_{\ell=0}^{t} \sum_{q=0}^{r-\ell} \lambda_{j,\ell,q}$$

$$= (1-b^{t+1})/(1-b) \sum_{q=0}^{r-t} \lambda_{j,o,q} + \sum_{y=0}^{t} b^{y} \sum_{q=r-t+1}^{r-y} \lambda_{j,o,q}$$

$$+ a(r+1)(t+1)/(1-b) - at(t+1)/2(1-b) - a(r+1)(1-b^{t+1})/$$

$$(1-b)^2 + ab[1-b^t - tb^t + tb^{t+1}]/(1-b)^3 ,$$

and

$$E_{j,t} = \sum_{\ell=0}^{t} \ell A_{j,\ell} = \sum_{\ell=0}^{t} \ell \sum_{q=0}^{r-\ell} \lambda_{j,\ell,q}$$

$$= \sum_{\ell=0}^{t} \ell b^{\ell} \sum_{q=0}^{r-\ell} \lambda_{j,o,q} + a(r+1)t(t+1)/2(1-b)$$

$$- at(t+1)(2t+1)/6(1-b) - a(r+1)b[1-b^t - tb^t + tb^{t+1}]/(1-b)^3$$

$$+ 2ab[1-b^t - tb^t + tb^{t+1}]/(1-b)^4 - ab(1-b^t)/(1-b)^3$$

$$- at^2 b^{t+1}/(1-b)^2 .$$

Again using the method of moments, we have

IV-34a)
$$\alpha_j(t) = \frac{\sum_{\ell=0}^{t} \sum_{q=0}^{r-\ell} \lambda_{j,\ell,q} - (1-b^{t+1})/(1-b) \sum_{q=0}^{r-t} \bar{\lambda}_{j,o,q} + \sum_{y=0}^{t} b^{t} \sum_{q=r-t+1}^{r-y} \bar{\lambda}_{j,o,q}}{(r+1)(t+1)/(1-b) - t(t+1)/2(1-b) - (r+1)(1-b^{t+1})/(1-b)^2}$$

$$+ b[1-b^t - tb^t + tb^{t+1}]/(1-b)^3$$

$$= F_2(b)$$

and the recursive relationship

$$\text{IV-34b)} \quad -\sum_{\ell=0}^{t}\ell\sum_{q=0}^{r-\ell}\bar{\lambda}_{j,\ell,q}+\sum_{\ell=0}^{t}\ell b^{\ell}\sum_{q=0}^{r-\ell}\bar{\lambda}_{j,0,q}$$

$$+[F_2(b)][(r+1)t(t+1)/2(1-b)-t(t+1)(2t+1)/6(1-b)$$

$$-(r+2)b(1-b^t-tb^t+tb^{t+1})/(1-b)^3$$

$$+2b(1-b^t-tb^t+tb^{t+1})/(1-b)^4-b(1-b^t)/$$

$$(1-b)^3-t^2b^{t+1}/(1-b)^2]=0$$

with which to solve for $b_j(t)$.

Again one can solve for α_j and b_j independently of t by letting t=r for sufficiency. Moreover, one can use IV-34c) and IV-34d) to solve for a_j and c_j if desired:

$$\text{IV-34c)} \quad a_j=[\sum_{\ell=0}^{r}\alpha_j(\ell)](1-c_j)/(1-c_j^{r+1})$$

$$\text{IV-34d)} \quad -\sum_{\ell=0}^{r}\ell\alpha_j(\ell)+[\sum_{\ell=0}^{r}\alpha_j(\ell)](1-c_j^{\ell}-tc_j^{\ell}+tc_j^{r+1})/$$

$$(1-c_j)(1-c_j^{t+1})=0$$

with sufficiency requirements taken into account.

In like manner we can obtain estimates independent of journal j but dependent on date of publication, q, using

$$\alpha_q(t) = (1-b) \sum_{\ell=0}^{t} \sum_{j=1}^{s} \bar{\lambda}_{j,\ell,q}$$

$$- \Big(\sum_{j=1}^{s} \bar{\lambda}_{j,o,q} \Big)(1-b)(1-b^{t+1})/(b^{t+1} - b - b^{t} + t)$$

and

$$a_q = (1-c) \sum_{\ell=0}^{r-q} \alpha_q(\ell)/(1-c^{r-q+1})$$

in conjunction with the recursive relationships

$$- \sum_{j=1}^{s} \sum_{\ell=0}^{r-q} \ell \, \bar{\lambda}_{j,\ell,q} + \Big(\sum_{j=1}^{s} \bar{\lambda}_{j,o,q} \Big) b[1-b^{r-q} - (r-q)b^{r-q}$$

$$+ (r-q)b^{r-q+1}]/(1-b)^2$$

$$- [\alpha_q(r-q)]b[1-b^{r-q} - (r-q)b^{r-q} + (r-q)b^{r-q+1}]/(1-b)^3$$

$$+ [\alpha_q(r-q)](r-q)(r-q+1)/2(1-b) = 0$$

and

$$- \sum_{j=1}^{s} \sum_{\ell=0}^{r-q} \ell \, \alpha_q(\ell) + c[1-c^{r-q} - (r-q)c^{r-q} + (r-q)c^{r-q+1}] \sum_{\ell=0}^{r-q} \alpha_q(\ell)/$$

$$(1-c)(1-c^{r-q+1}) = 0$$

with which to solve for b_q and c_q, respectively.

To now remove the dependence of a, b, and c upon any specific item classification, let us examine the following moments

$$B_\ell = \sum_{j=1}^{s} \sum_{q=o}^{r-\ell} \lambda_{j,\ell,q}$$

= the expected number of demands for issues of journals exactly ℓ periods after publication

$$= b^{\ell} \sum_{j=1}^{s} \sum_{q=o}^{r-\ell} \lambda_{j,\ell,q} + as(r+1)/(1-b) - as\ell/(1-b) - as(r+1)b^{\ell}/(1-b)$$

$$+ as\ell b^{\ell}/(1-b),$$

$$C_t = \sum_{\ell=0}^{t} B_\ell$$

$$= (1-b^{t+1})/(1-b) \sum_{q=0}^{r-t} \sum_{j=1}^{s} \lambda_{j,o,q} + \sum_{y=0}^{t} b^y \sum_{q=r-t+1}^{r-y} \sum_{j=1}^{s} \lambda_{j,o,q}$$

$$+ as(r+1)(t+1)/(1-b) - ast(t+1)/2(1-b)$$

$$- as(r+1)(1-b^{t+1})/(1-b)^2 + asb(1-b^t - tb^t + tb^{t+1})/(1-b)^3 ,$$

and

$$E_t = \sum_{\ell=0}^{t} \ell B_\ell$$

$$= \sum_{\ell=0}^{t} \ell \sum_{q=0}^{r-\ell} \sum_{j=1}^{s} \lambda_{j,o,q} + as(r+1)t(t+1)/2(1-b)$$

$$- ast(t+1)(2t+1)/6(1-b) - as(r+1)b(1-b^t - tb^t + tb^{t+1})/(1-b)^3$$

$$+ 2asb(1-b^t - tb^t + tb^{t+1})/(1-b)^4 - asb(1-b^t)/(1-b)^3$$

$$- at^2 b^{t+1}/(1-b)^2 .$$

Again using the method of moments, we have

IV-35a)

$$\alpha(t) = \frac{\sum_{\ell=0}^{t} \sum_{j=1}^{s} \sum_{q=0}^{r-\ell} \bar{\lambda}_{j,\ell,q} - (1-b^{t+1})/(1-b) \sum_{q=0}^{r-\ell} \sum_{j=1}^{s} \bar{\lambda}_{j,o,q} + \sum_{y=0}^{t} b^y \sum_{q=r-t+1}^{r-y} \sum_{j=1}^{s} \bar{\lambda}_{j,o,q}}{s[(r+1)(t+1)/(1-b) - t(t+1)/2(1-b) - (s+1)(1-b^{t+1})/(1-b)^2 + b(rb^t - tb^t + tb^{t+1})/(1-b)^3]}$$

$$= F_3(b) ,$$

and the recursive relationship

$$\text{IV-35b)} \quad -\sum_{\ell=0}^{t}\ell\sum_{q=0}^{r-\ell}\sum_{j=1}^{s}\bar{\lambda}_{j,\ell,q}+\sum_{\ell=0}^{t}\ell b^{\ell}\sum_{q=0}^{r-\ell}\sum_{j=1}^{s}\bar{\lambda}_{j,o,q}$$

$$+[F_3(b)]s[(r+1)t(t+1)/2(1-b)-t(t+1)(2t+1)/6(1-b)$$

$$-b(1-b^t)/(1-b)^3-t^2b^{t+1}/(1-b)^2+2b(1-b^tt-b^t+tb^{t+1})/$$

$$(1-b)^4-(r+1)b(1-b^t-tb^t+tb^{t+1})/(1-b)^3]=0$$

with which to solve for b(t).

And once again, one can solve for b independent of t by letting t=r for sufficiency. Further

$$\text{IV-35c)} \quad a=[\sum_{\ell=0}^{r}\alpha(\ell)](1-c)/(1-c^{r+1})$$

and

$$\text{IV-35d)} \quad -[\sum_{\ell=0}^{r}\ell\alpha(\ell)+\sum_{\ell=0}^{r}\alpha(\ell)](1-c^r-rc^r+rc^{r+1})/(1-c)(1-c^{r+1})=0$$

will yield sufficient estimates of a and c. It will be these latter estimates of a, b, and c, parameters assumed to be constant for all items in a specific field of endeavor, that will be used in the journal selection model. Tests can be made to determine whether a and c are independent of journal j or period of publication q, or both, using chi-square statistics as before.

The Morse-Elston Data Analysis

Morse and Elston [59] present the results of a least squares fit for a small sample of book usage data from the M.I.T. Library System, giving them estimates of α and of b for their original model. They restricted themselves to books related to scientific subjects. Their results, revised somewhat for purpose of manipulation, are presented in Table 1, and will be used as the basis for fitting the parameters of the revised Markovian model. The data are divided into five categories according to the specific scientific field of endeavor with which each book is concerned. Note that α and b for each field are independent of time and of any specific item within a scientific field. Further, the time axis is segmented into three zones: years 1-4, years 8-10, and over 15 years. These zones represent the age of books.

Morse and Elston provide only one estimate of $\alpha(t)$ for each zone, even though each zone represents several years. To compensate for this, in order to make parameter estimation for our modified model easier, a step-function approach is taken. Thus, for each year in a particular time zone, the α parameter estimate is the same. One final point is that the last time zone is listed as over 15 years old, so that no upper limit to the zone is given. Again for ease in estimating the parameters of the modified model, an upper limit of 20 years is assumed so that this zone consists of years 16-20. The results of this analysis can be seen in Table 2 or graphically in Figure 6. Note that $\bar{c}$ is significantly less than unity for all scientific fields according to the t-test [63] at a level of 0.0005. Also consider the high values of R^2, implying good fit.

Table 1. A Modified Presentation of Morse and Elston's Parameter Estimates

Zones	Year	Parameter $\bar{\alpha}(t)$	Fields of Scientific Endeavor					Overall
			Biology	Chemistry	Geology	Mathematics	Physics	
1	1		0.4	0.6	0.4	0.3	0.5	0.37
	2		0.4	0.6	0.4	0.3	0.5	0.37
	3		0.4	0.6	0.4	0.3	0.5	0.37
	4		0.4	0.6	0.4	0.3	0.5	0.37
	5		-	-	-	-	-	-
	6		-	-	-	-	-	-
	7		-	-	-	-	-	-
2	8		0.3	0.5	0.2	0.25	0.4	-
	9		0.3	0.5	0.2	0.25	0.4	-
	10		0.3	0.5	0.2	0.25	0.4	-
	11		-	-	-	-	-	-
	12		-	-	-	-	-	-
	13		-	-	-	-	-	-
	14		-	-	-	-	-	-
	15		-	-	-	-	-	-
3	16		0.2	0.3	0.1	0.2	0.3	0.15
	17		0.2	0.3	0.1	0.2	0.3	0.15
	18		0.2	0.3	0.1	0.2	0.3	0.15
	19		0.2	0.3	0.1	0.2	0.3	0.15
	20		0.2	0.3	0.1	0.2	0.3	0.15
		$\bar{\bar{\alpha}}$	0.45	0.5	0.2	0.6	0.6	0.48

Table 2. Parameter Estimates for the Modified Markovian Model

Parameter	Fields of Scientific Endeavor				
	Biology	Chemistry	Geology	Mathematics	Physics
$\hat{a}$	0.4421	0.6806	0.4804	0.3168	0.5366
s	0.9575	0.9578	0.9157	0.9749	0.9685
R^2	0.96666	0.9420	0.9590	0.9657	0.9663

The "Pn" Model

Consider the original "Pn" model of

IV-28a) $\lambda_{j,t,q} = E(G_{j,t,q}) = (1-p_{j,t,q})u_{j,t,q}/(1-e^{-u_{j,t,q}})$

where

IV-29b) $u_{j,t,q} = u_{j,q}$

for all t, and

IV-29a) $p_{j,t,q} = (t+\beta_{j,q})/(t+\beta_{j,q}+\gamma_{j,q})$.

Jain uses a transformation such that

IV-37a) $z_{j,q} = \gamma_{j,q}\, u_{j,q}/(1-e^{-u_{j,q}})$

and

IV-37b) $x_{j,q} = \gamma_{j,q} + \beta_{j,q}$

so that

IV-38) $\lambda_{j,t,q} = (1 - t+\beta_{j,q}/t+\beta_{j,q}+\gamma_{j,q})u_{j,q}/(1-e^{-u_{j,q}})$

$= \gamma_{j,q}\, u_{j,q}/(t+\beta_{j,q}+\gamma_{j,q})(1-e^{-u_{j,q}}) = z_{j,q}/(t+x_{j,q})$.

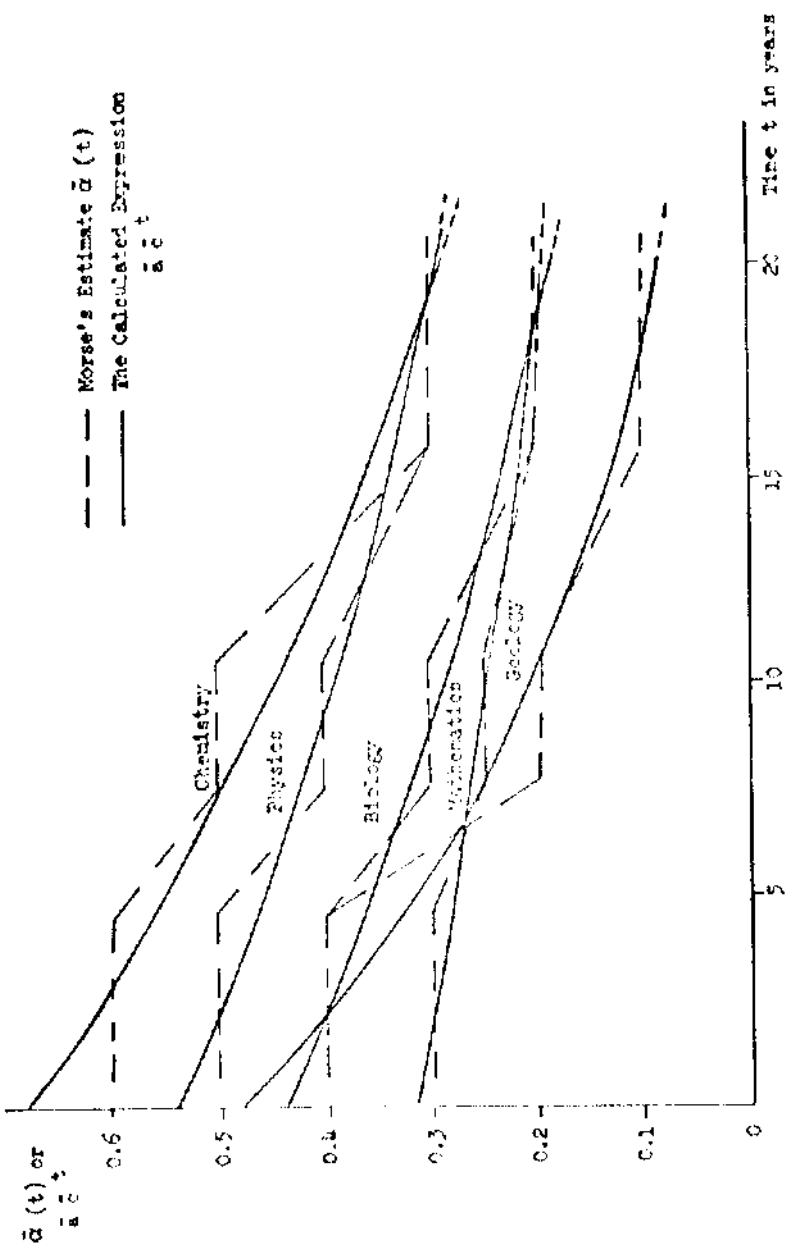


Figure 5. A Graph of ā (t) and ā e^t Versus Time

Again, with Jain's method of moments we have

$$\text{IV-39a)} \quad z_{j,q}(t) = \bar{C}_{j,t,q} \Big/ \sum_{\ell=0}^{t} (\ell + x_{j,q}(t))^{-1}$$

and a recursive relationship with which to solve for $x_{j,q}(t)$:

$$\text{IV-39b)} \quad \bar{C}_{j,t,q} \sum_{\ell=0}^{t} (\ell\,\ell + x_{j,q}(t)) \Big/ \sum_{\ell=0}^{t} (\ell + x_{j,q}(t))^{-1} - \bar{F}_{j,t,q} = 0.$$

Inverting to solve for the original parameters, we have

$$\text{IV-40a)} \quad r_{j,q}(t) = z_{j,q}(t)(1-e^{-u_{j,t,q}})/u_{j,t,q}$$

$$= \bar{C}_{j,t,q} (1-e^{-u_{j,t,q}})/u_{j,t,q} \sum_{\ell=0}^{t} (\ell + x_{j,q}(t))^{-1}$$

and

$$\text{IV-40b)} \quad \beta_{j,q}(t) = x_{j,q}(t) - r_{j,t,q} = x_{j,q}(t) - \bar{C}_{j,t,q} (1-e^{-u_{j,t,q}})/$$

$$u_{j,t,q} \sum_{\ell=0}^{t} (\ell + x_{j,q}(t))^{-1}.$$

Sufficiency requirements demand that one let t=r-q to solve for $\beta_{j,q}$ and $r_{j,q}$. It is, moreover, desirable that more generalized parameters be found as before. The estimates for the more generalized parameters, r_j, β_j, r_q, β_q, r, and β, will be found by summing over j, q, or both as before.

The estimation of $u_{j,q}$ is quite another matter.

$$\text{IV-41a)} \quad \text{Letting: } \epsilon_{j,t,q} = \begin{cases} 1 & \bar{x}_{j,t,q} > 0 \\ 0 & \bar{x}_{j,t,q} = 0 \end{cases}$$

we have that

$$\text{IV-41b)} \quad E(G_{j,t,q} \mid G_{j,t,q} > 0) = \sum_{\ell=0}^{t} \bar{\lambda}_{j,\ell,q} \Big/ \sum_{\ell=0}^{t} \epsilon_{j,\ell,q}$$

Using IV-28c) we can find the following recursive relationship with which to solve for $u_{j,q}$,

$$\text{IV-41c)} \quad u_{j,q}/(1-e^{-u_{j,q}}) - \sum_{\ell=0}^{r-q} \bar{\lambda}_{j,\ell,q} \Big/ \sum_{\ell=0}^{r-q} \epsilon_{j,\ell,q} = 0.$$

Since only journals with some expected usage will be considered for selection, it is not realistic to allow $\epsilon_{j,t,q} = 0$ for all t so that IV-41c) is always defined. Moreover, one can form more generalized parameter estimates of u_j, u_q, or u if desired.

Estimation of the parameters for the revised "Im" models becomes quite difficult. In fact, this is a perfect case of how estimation problems can limit the complexity of parametric expressions considered when modeling demand over time. Essentially we have two relationships which must be solved simultaneously for $\theta_{j,q}$ and $u_{j,\ell,q}$

$$- \sum_{\ell=0}^{r-q} \bar{\lambda}_{j,\ell,q} + \sum_{\ell=0}^{r-q} u_{j,\ell,q}(1-e^{-u_{j,\ell,q}/\ell+\theta_{j,q}})/(1-e^{-u_{j,\ell,q}}) = 0,$$

and

$$- \sum_{\ell=0}^{t} \bar{\lambda}_{j,\ell,q} \Big/ \sum_{\ell=0}^{t} \epsilon_{j,\ell,q} + u_{j,t,q}/(1-e^{-u_{j,t,q}}) = 0$$

for every t, $t=0,1,\ldots,r-q$. One must then use relationships similar to those of IV-33a) through IV-33d) to find $a_{j,q}$, $b_{j,q}$ and $c_{j,q}$ as a function of the calculated values of $u_{j,t,q}$ if we use IV-29c). It does indeed seem silly to have to deal with such difficult estimation techniques when more simple forms are at hand that can be fitted quite well to available data.

Thus we have seen that expected usage can be used as a criterion for journal worth. Moreover, while there are several models to help predict and describe usage patterns, a modified form of the Markovian model seems the best due to ease of parameter estimation, accuracy in describing usage, and rationale behind the derivation and results of the model.

CHAPTER V

ANOTHER CRITERION FOR MEASURING A JOURNAL'S WORTH: PRODUCTIVITY

The worth of a journal can be considered in terms of a somewhat different measure than total usage. Instead of the number of demands for a specific journal, consider productivity, the number of articles contained in the journal. We are especially concerned with articles that are relevant to a specific, narrowly-defined field of scientific endeavor. While we shall see that journal usage is a better criterion for measuring the worth of acquiring a journal, productivity presents a good contrast and is a reasonable alternative to usage. Furthermore, it may be easier to count relevant articles than to estimate usage. Finally, the model used to describe productivity over time illustrates many important points about the journal selection decision, such as the relationship of article relevancy to journal usage.

The librarians must be able to specify what fields of endeavor are of interest to the users of their particular library system. This is in line with the concept of a policy based on a selection of all material relevant to a general field, termed a collection policy, rather

than on selecting only specific library materials with a complex individual evaluation of each item, as discussed in Stevens [71]. The problems of defining the composition and scope of a scientific discipline are discussed by O'Neill [62].

Let $g_{j,t,q}$ be the number of articles in the issues of journal j published in period q that are relevant to a library's users in period t+q. Note that lower case letters will be used to denote journal productivity so as to not become confused with journal usage. This means that the objective function now represents the total number of relevant articles that the library system can potentially put before the users. The variable $h_{j,t,q}$ now represents the penalty, in terms of number of articles, for not acquiring the journal in which some relevant articles exist.

While the number of articles is not directly related to usage, it will be seen that one can define relevance of an article in terms of usage. Moreover, relating relevancy of a journal to the number of articles deemed relevant in the journal can be interesting, in terms of the contingency table analysis of Chapter II. As the number of relevant articles decreases, the total relevancy of the journal declines so that $g_{j,t,q}$ now becomes somewhat representative of the smaller worth of acquiring a nonpertinent journal. Moreover, as $g_{j,t,q}$ decreases, $h_{j,t,q}$ increases and becomes somewhat representative of the lesser detriment of not acquiring a nonpertinent journal.

Article relevancy is defined here as being an all or nothing, one or zero, concept; either an article is totally relevant or not relevant at all with respect to a specific field of scientific endeavor.

One could consider partial relevancy, perhaps a weight w between zero and one for each article over time. However, as O'Neill points out, partial relevancy is subjective and dependent upon the current interests of a specific user. It is analogous to the utility of an item's being used in that it is hard to measure from a librarian's standpoint, and is more relevant to a specific user than to the servicer; therefore, partial relevancy will not be considered in this study.

It must be assumed that the librarian, as a decision-maker, can specify what fields are of interest to his users. A field can be a small subset of knowledge such as queueing theory, a more general subset such as the physical sciences, or an area anywhere in the range between the two. O'Neill discusses this point in detail and considers methods for describing the breadth and boundaries of a discipline or field. Moreover, the librarian must be able to determine which articles are relevant to these fields of endeavor. Of course, the completeness of the set of journals to be considered is a crucial assumption that must be made. All journals worthy of consideration must be included in the set.

The number of relevant articles, like usage, can be related to comprehensiveness and availability of desired items but not to convenience which is more dependent upon indexing, classification and other storage and retrieval aspects of the library. Moreover, there is difficulty in obtaining lists of relevant articles; citation lists tend to consider only research oriented articles while abstract journals tend to be too narrow in their definition of a field [6]. Thus like usage, productivity measures have certain limitations.

In the first section of this chapter we will look at a general model of productivity, relating journal growth to literature growth and to literature scattering, the distribution of articles throughout various journals. In the second section, an epidemic growth model with various refinements is considered to describe literature growth patterns. The third section deals with scattering functions and Leimkuhler's [29] Bradford distribution. Both of the latter two sections deal with parameter estimation problems. In the fourth section we consider a Markovian analysis of growth while in the fifth section we try to relate productivity to usage through an interesting definition of relevancy. The final section, a study of queueing theory literature, leads us to the conclusion that productivity is a poorer measure of journal worth than is usage, primarily because we are unable to accurately predict expected productivity over time.

A General Model of Productivity

With $\varepsilon_{j,t,q}$ defined, a general model can be constructed to describe how journals collect relevant articles. Let $n_{t,q}$ represent the total number of articles written in period q and relevant in period t≥q in the set of journals. Assuming that the set of journals is complete, we can let $n_{t,q}$ represent the number of relevant articles that are published within the specific field of endeavor in time period q and relevant in period t≥q. Thus,

V-1) $$n_{t,q} = \sum_{j=1}^{s} \varepsilon_{j,t,q} .$$

Now, cumulative statistics must be defined in order to be able to discuss how many articles have been accumulated over a period of time:

V-2) $$g^{*}_{j,t,q} = \sum_{\ell=0}^{q} g_{j,t+q-\ell,\ell}$$

= the number of articles written in the issues of journal j that were published in the first q time periods and relevant in period t+q;

V-3) $$n^{*}_{t,q} = \sum_{j=1}^{s} g^{*}_{j,t,q} = \sum_{j=1}^{s} \sum_{\ell=0}^{q} g_{j,t+q-\ell,\ell} = \sum_{\ell=0}^{q} n_{t+q-\ell,\ell}$$

= the number of articles written in the journals published in the first q time periods and relevant in period t+q.

The four variables $g_{j,t,q}$, $n_{t,q}$, $g^{*}_{j,t,q}$, and $n^{*}_{t,q}$ all have as their range the set of nonnegative integers.

Next, one can define the scattering functions

V-4a) $$\alpha_{j,t,q} = g_{j,t,q}/n_{t,q} = g_{j,t,q}/\sum_{j=1}^{s} g_{j,t,q}$$

= the proportion of articles written in period q and relevant in period t+q and found in journal j; and

V-4b) $$\alpha^{*}_{j,t,q} = g^{*}_{j,t,q} / n^{*}_{t,q} = g^{*}_{j,t,q} \Big/ \sum_{j=1}^{s} g^{*}_{j,t,q}$$

$$= \sum_{\ell=0}^{q} g_{j,t+q-\ell,\ell} \Big/ \sum_{j=1}^{s} \sum_{\ell=0}^{q} g_{j,t+q-\ell,\ell}$$

= the proportion of articles written in the first q time periods and relevant in period t+q and found in journal j.

Using the facts that

V-5a) $$g^{*}_{j,t,q} - g^{*}_{j,t+1,q-1} = g_{j,t,q}\,,$$ and

V-5b) $$n^{*}_{t,q} - n^{*}_{t+1,q-1} = n_{t,q}\,,$$

we can show that

V-6) $$g^{*}_{j,t,q} = g^{*}_{j,t+1,q-1} + \alpha_{j,t,q}\, n_{t,q}\,.$$

If $n_{t,q} = 0$, let $\alpha_{1,t,q} = 1$ and $\alpha_{j,t,q} = 0$ for all $j \neq 1$. In like manner, if $n^{*}_{t,q} = 0$, let $\alpha^{*}_{1,t,q} = 1$ and $\alpha^{*}_{j,t,q} = 0$ for all $j \neq 1$. This convention allows the variables $\alpha_{j,t,q}$ and $\alpha^{*}_{j,t,q}$ to be consistent.

Lemma

The variables $\alpha_{j,t,q}$ and $\alpha^{*}_{j,t,q}$ have the properties of probability density functions in that they are always nonnegative and their sum over j equals one for all values of t and q.

Proof

1) By definition, $n^*_{t,q}$, $n_{t,q}$, $\varepsilon^*_{j,t,q}$, and $\varepsilon_{j,t,q}$ are nonnegative. Thus $\alpha_{j,t,q} = \varepsilon_{j,t,q}/n_{t,q}$ and $\alpha^*_{j,t,q} = \varepsilon^*_{j,t,q}/n^*_{t,q}$ are nonnegative since ratios of nonnegatives must remain nonnegative.

2) $n_{t,q} = \sum_{j=1}^{s} \varepsilon_{j,t,q}$ so that $\sum_{j=1}^{s} \alpha_{j,t,q} = \sum_{j=1}^{s} \varepsilon_{j,t,q}/n_{t,q}$

$= n_{t,q}/n_{t,q} = 1.$

In like manner it can be shown that $\sum_{j=1}^{s} \alpha^*_{j,t,q} = 1$.

3) If $n_{t,q} = 0$, we have $\alpha_{1,t,q} = 1$ and $\alpha_{j,t,q} = 0$ for all $j \neq 1$ so that $\alpha_{j,t,q} \geq 0$ and $\sum_{j=1}^{s} \alpha_{j,t,q} = 1$. The results are also consistent for $\alpha^*_{j,t,q}$ when $n^*_{t,q} = 0$. Thus, the proof is complete.

Since $\varepsilon_{j,t,q}$ is the parameter that will be used to evaluate the journal's worth, consideration of how to provide estimates of productivity must be given. One obvious technique is to view the variable as a time series. However, this means that one must treat separate time series simultaneously and this can lead to difficulty. Thus, we use equation V-(6), and try to predict literature growth and scattering over time.

Literature Growth -- Epidemic Models

In trying to predict journal productivity over time, we have related such productivity to literature growth and to literature scattering. Let us now consider growth as an independent phenomenon.

In Chapter I of this thesis, it was pointed out that studies show libraries to be growing exponentially over time. It seems reasonable that underlying this might be an exponential growth of literature. Assuming that relevant articles do not become obsolescent, we could model the growth of journal literature as follows:

V-7) $$\bar{n}^{*}_{t,q} = \bar{n}^{*}_{t,o}\, e^{\alpha q} \quad , \qquad q=0,1,\ldots,r-t$$

for all $t=0,1,2,\ldots,r$ where $\bar{n}^{*}_{t,q}$ indicates the expected value of $n^{*}_{t,q}$. However, this seems to be unrealistic in that articles do become obsolete as time since publication, t, increases. To account for this, let us consider literature as an epidemic. Since one good article can lead to several more, and since one can find scientific fields that seem almost saturated so that growth rates may be declining or at least leveling off, epidemic theory is a logical choice.

Coffman [21] uses an epidemic process to model the spread of scientific ideas based on the following differential equations:

V-8) $$d\,\bar{S}_t/dt = (-\beta\bar{S}_t\bar{I}_t - \delta\,\bar{S}_t + \mu)$$

$$d\,\bar{I}_t/dt = (\beta\bar{S}_t\bar{I}_t - \gamma\,\bar{I}_t + \nu)$$

$$d\,\bar{R}_t/dt = (\delta\,\bar{S}_t + \gamma\,\bar{I}_t)$$

where

$\bar{S}_t$ = the expected number of susceptibles at time t,

$\bar{I}_t$ = the expected number of infectives at time t,

and

$\bar{R}_t$ = the expected number of removals at time t.

Letting

V-8a) $\bar{N}_t$ = the total expected population at time t

$= \bar{R}_t + \bar{I}_t + \bar{S}_t$

we have

V-8b) $d\,\bar{R}_t/dt = (\mu + \nu)\,.$

A susceptible is a person who does not yet have, but could get, the disease; an infective is a person who has the disease and is spreading it to others with whom he has contact; and a removal is a person who has either recovered or died, but who can no longer spread or contract the disease. Goffman defines a susceptible as an author who has not as of time t but who may contribute to the literature in the future; an infective as an author who is contributing to the literature by having at least one article published by time t with more articles coming; and a removal as an author who has ceased to contribute and has published his last article relevant to some specific scientific field of endeavor before time t.

Note that time is a discrete variable but Goffman uses a continuous approximation. This model is fitted to data with good results by Goffman who feels it has extensive predictive power. In other articles, Goffman [27, 28] shows that stability occurs when $d\,\bar{I}_t/dt_t$ = a constant, and that for an epidemic to begin or to continue, $d\,\bar{I}_t/dt_t > 0$.

A modified version of Goffman's model can be used to incorporate exponential literature growth as well as a degree of obsolescence in the variable $\bar{n}_{t,q}$ as follows:

V-9a) $$\frac{d\,\bar{n}_{o,q}}{dq} = \alpha\,\bar{n}_{o,q} + \gamma \qquad [1]$$

or

V-9b) $$\bar{n}_{o,q} = \bar{n}_{o,o}\, e^{\alpha q} + \gamma/\alpha\,(e^{\alpha q} - 1)$$

$$= (\bar{n}_{o,o} + \gamma/\alpha)\, e^{\alpha q} - \gamma/\alpha\,.$$

The model reflects a linear epidemic growth rate of relevant articles plus a factor that allows the field of endeavor to contract by obsolescence if it becomes saturated. This factor is proportional to the current level of relevant articles. Note the striking resemblence of equation V-9b) to Morse's [58] Markovian model of equation IV-21).

However, obsolescence, a factor that should increase with age, is not suitably taken into account with the previous model. Suppose that growth is considered to act somewhat independently of obsolescence so that we have

V-10a) $$d\,\bar{n}_{t,q}/dt = -\beta\,\bar{n}_{t,q} - \delta$$

and

V-10b) $$d\,\bar{n}_{t,q}/dq = (\alpha\,\bar{n}_{t,q} + \gamma)$$

---

1 This parameter α is not to be confused with the scattering factors, $\alpha_{j,t,q}$ or $\alpha^{*}_{j,t,q}$.

or

V-10c) $$\bar{n}_{t,q} = (n_{o,o} + \gamma/\alpha)\, e^{-\beta t}\, e^{\alpha q} - \gamma/\alpha\ .$$

Note that a solution exists only when $\alpha\delta = \beta\gamma$ or $\delta = \beta\gamma/\alpha$. This model combines exponential growth over time with obsolescence that increases with age. If obsolescence is to be ignored, equation V-10c) can be used with $\beta = 0$, yielding the same result as V-9b) for any t such as $t = 0$.

In general if $\delta = \beta\gamma/\alpha$ so that a solution exists, we have

V-10d) $$\bar{s}_{j,t,q} = (s_{j,o,o} + \gamma/s\alpha)\, e^{-\beta t}\, e^{\alpha q} - \gamma/\alpha s$$

V-10e) $$\bar{n}^{*}_{t,q} = (n_{o,o} + \gamma/\alpha) e^{-\beta t} (e^{\alpha+\beta}\, e^{\alpha q} - e^{-\beta q})/(e^{\alpha+\beta} - 1) - \gamma(q+1)/\alpha$$

V-10f) $$\bar{s}^{*}_{j,t,q} = (s_{j,o,o} + \gamma/s\alpha)\, e^{-\beta t}\, (e^{\alpha+\beta}\, e^{\alpha q})/(e^{\alpha+\beta} - 1) - \gamma(q+1)/s\alpha$$

V-10g) $$\bar{\alpha}_{j,t,q} = [(s_{j,o,o} + \gamma/s\alpha)\, e^{-\beta t}\, e^{\alpha q} - \gamma/\alpha s\,]/$$

$$[(n_{o,o} + \gamma/\alpha)\, e^{\alpha q}\, e^{-\beta t} - \gamma/\alpha\,]$$

and

V-10h) $$\bar{\alpha}^{*}_{j,t,q} = \frac{(s_{j,o,o} + \gamma/s\alpha)\, e^{-\beta t}\, (e^{\alpha+\beta}\, e^{\alpha q} - e^{-\beta q})/[(e^{\alpha+\beta} - 1) - \gamma(q+1)/s\alpha]}{(n_{o,o} + \gamma/\alpha)\, e^{-\beta t}\, (e^{\alpha+\beta}\, e^{\alpha q} - e^{-\beta q})/[(e^{\alpha+\beta} - 1) - \gamma(q+1)/s\,]}$$

The implication is that if $\gamma = 0$, $\alpha_{j,t,q}$ and $\alpha^{*}_{j,t,q}$ are independent of both t and q with this formulation.

Jain's [30] method of moments can be employed to yield estimates for α, β, and γ of V-10c). However, since the data currently available, as discussed later in this chapter, do not allow for obsolescence, we consider only the variable $\bar{n}_{o,q}$ with β no longer a measurable parameter but equal to zero. We are left with a recursive relationship based on moments derived from V-9b) with which to solve for α,

$$\text{V-11a)} \quad (\bar{n}_{o,o}+\gamma/\alpha)e^{\alpha}\,(1-e^{\alpha r}-re^{\alpha r}+re^{\alpha(r+1)})/(1-e^{\alpha})^{2} - \gamma\, r(r+1)/2\alpha - \sum_{q=o}^{r} q\,\bar{n}_{o,q} = 0,$$

where $\bar{n}_{o,q}$ are the observed values of $n_{o,q}$; and an equation with which to solve for γ,

$$\text{V-11b)} \quad \gamma = \alpha\left[\sum_{q=o}^{r}\bar{n}_{o,q} - \left(\frac{1-e^{\alpha(r+1)}}{1-e^{\alpha}}\right)\bar{n}_{o,o}\right] \Bigg/ \left[\left(\frac{1-e^{\alpha(r+1)}}{1-e^{\alpha}}\right) - (r+1)\right].$$

The Bradford Distribution

In order to describe the scattering of relevant literature throughout various journals, Leimkuhler [39] derives the Bradford distribution. The underlying assumption, known as Bradford's law, is that a geometric series can be used to describe the scattering of articles throughout the ranked journals. Ranking the journals in decreasing order with

respect to $g^{*}_{j,o,q}$ so that the journal with a rank of one is the most productive, Bradford then grouped the journals into zones by going down the ranked list, with each zone being equally productive, having the same number of relevant articles. He noticed that each successive zone needed an increasing number of journals and that he could represent this phenomenon as a geometric series. Thus, dropping the subscripts t and q for the time being, we have

V-12) n_K = the number of journals in zone K

$$= n_{K-1} a = n_1 a^{K-1}$$

where $a > 1$ is the scattering factor.

Leimkuhler shows that

V-13) $F(x)$ = the proportion of relevant articles in the top x percent of the ranked journals

$$= \log(1+\beta s\, x/n_1)/\log(1+\beta s/N_1),$$

V-14) $p(x)$ = $F(x) - F(x-1/s)$

= the proportion of relevant articles in the journal with rank $s\,x$

$$= \log(1+\beta/n_1 + (s\,x-1)/\log(1+\beta s/n_1),$$

and

V-15) $E(n)$ = the expected number of journals needed to be searched in order to find n relevant article when going down the list of ranked journals

$$= \sum_{n=1}^{s} n\, p(n/s)$$

$$= s - \log\left[\sum_{n=1}^{s-1} (1+\beta n/n_1)\right]/\log(1+\beta s/n_1).$$

Note that s represents the total number of journals in the set being considered, n_1 represents the number of journals in zone 1, and that $\beta = s-1$. With all considerations being for r time periods, it is the function p(x) of equation V-14) that best represents the scattering function $\alpha_{j,t,q}$ with $x = \frac{\text{rank of journal } j}{s}$.

O'Neill [62] gives a lengthy discussion of the derivation of the Bradford distribution, its properties, and how well it can be fitted to data. He shows that while the Bradford distribution can be fitted to data, it can produce results that are not valid for a large population of journals when a small, more productive subgroup is used as a sample on which the parameters for the whole population are estimated. This seems due to a larger-than-predicted segment, or tail, of relatively unproductive journals occurring. O'Neill also shows that other distributions exist that can, to some extent, describe the scattering processes of literature.

Estimation of the Parameter β Over Time

In order to facilitate estimation procedures for the parameter β, Leimkuhler considered a continuous analog to equation V-14) which is much simpler to use than the discrete form. Letting $B = \beta/n_1$, we have, for this continuous case,

$$\text{V-16)} \qquad E(n) = \int_0^1 x\, d[1/(x)] = [1/\log(1+B) - 1/B]n .$$

Leimkuhler also defines a statistic,

$$\text{V-17)} \qquad K_{t,q} = 1/r\, \bar{n}^*_{t,q} \sum_{j=1}^{s} \bar{r}_{j,t,q}\, \bar{R}^*_{j,t,q}$$

which represents the expected value as calculated from observed data, where $\bar{n}^*_{t,q} = \sum_{j=1}^{s} \bar{r}^*_{j,t,q}$, $\bar{r}^*_{j,t,q}$ is the observed value of $r^*_{j,t,q}$, and $\bar{r}_{j,t,q}$ is the observed rank of journal j based on values of $\bar{r}^*_{j,t,q}$.

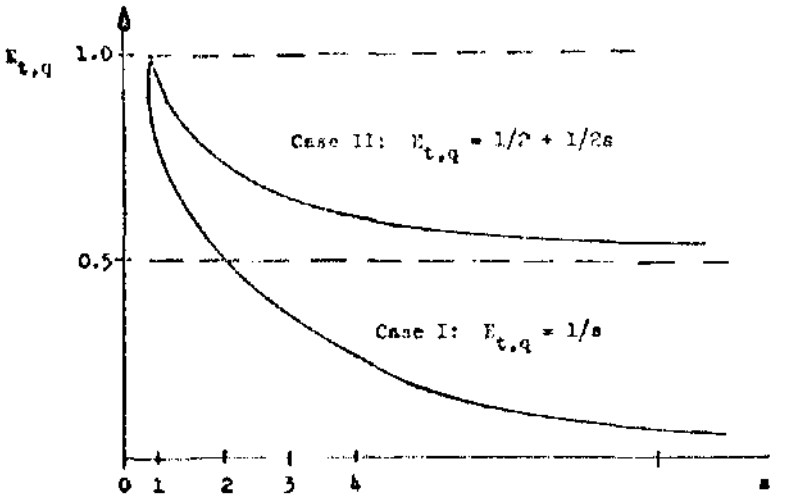
It can be shown that there are limits on $E_{t,q}$ due to the size of the set of journals, s, as seen by the following two extreme cases. First, suppose all articles are found in journal j_o so that $r_{j_o} = 1$, $\bar{r}^*_{j_o,t,q} = \bar{n}^*_{t,r}$, and $\bar{r}^*_{j,t,q} = 0$ for all $j \neq j_o$. This implies that $E_{t,q} = 1/s$. The second extreme case is when the articles are dispersed perfectly evenly, the exact opposite of case 1, so that $\bar{r}^*_{j,t,q} = \bar{n}^*_{t,q}/s$ for all j. Since in this case all journals are equally productive, we shall arbitrarily assign to journal j the rank of j. This implies that $E_{t,q} = 1/(\bar{n}^*_{t,q} \cdot s) \sum_{j=1}^{s} j\,(\bar{n}^*_{t,q}/s) = 1/s^2\,(\frac{s(s+1)}{2}) = 1/2 + 1/2s$. Note that for both cases, $E_{t,q}$ is a decreasing function of s; with

$$\lim_{s \to 1} E_{t,q} = 1, \text{ and } \lim_{s \to \infty} E_{t,q} = \begin{cases} 0 & \text{for case 1 - one productive journal} \\ 1/2 & \text{for case 2 - equal scattering among journals} \end{cases}.$$

This is illustrated graphically in Figure 7.

To find the estimate $\hat{B}_{t,q}$, we have the recursive relationship, found by equating V-16) and V-17):

V-17a) $$1/(\bar{n}^*_{t,q} \cdot s) \sum_{j=1}^{s} \bar{r}_{j,t,q}\, \bar{c}^*_{j,t,q} - 1/\log(1+\hat{B}_{t,q}) + 1/\hat{B}_{t,q} = 0.$$



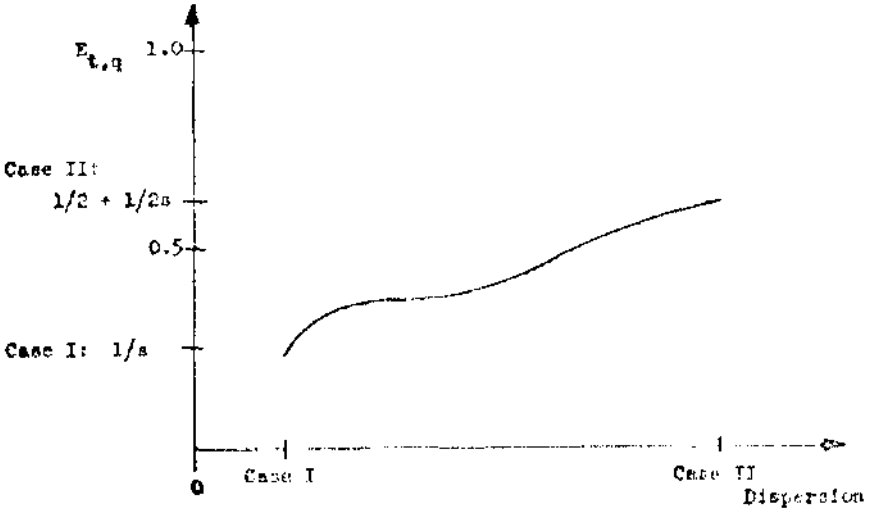


Figure 7. $E_{t,q}$ as a Function of s and of Dispersion

We see that for the first case $\hat{p}_{t,q}$ can be large, increasing as s increases. For the second case, $\hat{p}_{t,q}$ is smaller but is again an increasing function of s.

If only a portion of the top ranked journals are considered in the data base, say s journals out of the total of s, representing the best 100 s'/s percent of the journals, Leimkuhler has shown that the true value of $\hat{p}_{t,q}$ is equal to the $p_{t,q}$ of equation V-17) divided by (s'/s). Moreover, we shall assume here that $\hat{p}_{t,q}$ is independent of time t, letting t = 0 for convenience, since obsolescence cannot be taken into account with many types of data such as citation lists. Sufficiency implies that q=r as per the above formulae, if $p_{o,q}$ is also independent of q. However, we can find separate estimates of $\hat{p}_{o,q}$ for each q and test for independence with a chi-squared statistic.

A Markovian Analysis of Growth and Scattering

One way to analyze literature growth and scattering over time is by a Markovian probability model such as that used by Morse [58] to study usage patterns. The underlying assumption is that the number of relevant articles in a journal is a function only of the number of relevant articles in the previous period. As a basic model, we have

$$\text{V-18)} \quad \Pr(\sigma^{*}_{j,t,q} = b \mid \sigma^{*}_{j,t+1,q-1} = a) = \Pr(\sigma^{*}_{j,t+1,q-1} + \alpha_{j,t,q}\eta_{t,q} = b \mid \sigma^{*}_{j,t+1,q-1} = a) = \Pr(\alpha_{j,t,q}\eta_{t,q} = b-a \mid \sigma^{*}_{j,t+1,q-1} = a) \quad \text{for } b \geq a \geq 0.$$

This can be further reduced for $b>a$ to

V-18a) $$\Pr(\kappa^{*}_{j,t,q} = b \mid \kappa^{*}_{j,t+1,q-1} = a)$$

$$= \sum_{k=1}^{\infty} \Pr(n_{t,q} = k,\ \alpha_{j,t,q} = b-a/k \mid \kappa^{*}_{j,t+1,q-1} = a) \cdot 1/k .$$

For $b = a$ we have

V-18b) $$\Pr(\kappa^{*}_{j,t,q} = b = a \mid \kappa^{*}_{j,t+1,q-1} = a) = \Pr(\alpha_{j,t,q} n_{t,q} = b-a = 0 \mid \kappa^{*}_{j,t+1,q-1} = a)$$

$$= \Pr(\alpha_{j,t,q} = 0 \mid \kappa^{*}_{j,t+1,q-1} = a)$$

$$+ \Pr(n_{t,q} = 0 \mid \kappa^{*}_{j,t+1,q-1} = a] - \Pr(\alpha_{j,t,q} = n_{t,q} = 0 \mid \kappa^{*}_{j,t+1,q-1} = a).$$

Let us now make the assumption that $\alpha_{j,t,q}$ and $n_{t,q}$ are statistically independent. An example of this type of behavior was given by Neuts [61], who pointed out that if one had s independent Poisson renewal processes being pooled together, then the probability that a renewal occurring in time period q was from process j is independent of the number of renewals occurring in time period q. This assumption of independence implies that

V-19) $$\Pr(\alpha_{j,t,q} = b-a/k,\ n_{t,q} = k \mid \kappa^{*}_{j,t+1,q-1} = a)$$

$$= \Pr(\alpha_{j,t,q} = b-a/k \mid \kappa^{*}_{j,t+1,q-1} = a) \Pr(n_{t,q} = k \mid \kappa^{*}_{j,t+1,q-1} = a)$$

and that

$$\Pr(\alpha_{j,t,q} = n_{t,q} = 0 \mid \kappa^{*}_{j,t+1,q-1} = a)$$

$$= \Pr(\alpha_{j,t,q} = 0 \mid \kappa^{*}_{j,t+1,q-1} = a) \Pr(n_{t,q} = 0 \mid \kappa^{*}_{j,t+1,q-1} = 0] .$$

Note that

V-20) $$\Pr[n_{t,q} = k \mid g^{*}_{j,t+1,q-1} = a] = \Pr\{\sum_{j=1}^{s} g_{j,t,q} = k \mid \sum_{\ell=0}^{q-1} g_{j,t+q-\ell,\ell} = a\}.$$

The nature of such a relationship is too difficult to develop further due to the complexities of the summing over several time series variables.

Letting $x_{j,t,q} = r_{j,t,q}/s$, where $r_{j,t,q}$ is the rank of journal j based, in decreasing order, on $g^{*}_{j,t,q}$, we can use the Bradford distribution,

V-21) $$\alpha_{j,t,q} = \log(1+\beta/R_j + \beta\, x_{j,t,q} - \beta)/\log(1+\beta\, s/R_j),$$

by considering R_j to arbitrarily assume any value up to s, say unity. This implies that

$$\Pr(\alpha_{j,t,q} = x \mid g^{*}_{j,t+1,q-1} = a) = \Pr(x_{j,t,q} = 1/x[(1+\beta)^{x} - 1]$$

$$+ 1/s - 1/\beta\, s \mid g^{*}_{j,t+1,q-1} = a]$$

which yields the fact that as x goes to zero, $x_{j,t,q}$ must approach infinity.

One point that may simplify the model is that the librarian as a decision-maker considering relevant literature growth and scattering over time in order to evaluate journal worth, is primarily interested in the number of relevant articles as of period q in journal j.

$\beta^{*}_{j,o,q}$, and in the field, $\eta^{*}_{o,q}$, both now independent of age since many data bases may not reflect obsolescence. Note that this means a single subscript with which to track time. Basically, this means we can write V-18a) as

V-22) $$\Pr(\beta^{*}_{j,o,q} = b \mid \beta^{*}_{j,o,q-1} = a) = \Pr(\alpha_{j,o,q}\eta_{o,q} = b-a \mid \beta^{*}_{j,o,q-1} = a).$$

Realistically, this means little, however, in terms of being able to write functional forms for the Markovian probabilities.

Relating Relevancy to Usage

The most difficult problems associated with using productivity to measure journal worth are centered around determining which articles are relevant. One way would be to relate relevancy to usage and then to employ usage estimates to determine journal worth.

Suppose the number of articles published in journal j in time period q is $\epsilon_{j,q}$ and that this is a random variable with a Poisson distribution

V-23) $$\Pr(\epsilon_{j,q} = n) = e^{-\bar{\epsilon}_{j,q}}\,\bar{\epsilon}^{\,n}_{j,q}/n!\,, \qquad n=0,1,2,\ldots$$

with expected value $E(\epsilon_{j,q}) = \bar{\epsilon}_{j,q} > 0$. If desired, one could allow for the journal itself without regard to any articles within to be relevant, for purposes of browsing or keeping a collection intact, by allowing $\epsilon_{j,q}$ to include the journal itself in the count. This implies that the probability distribution is now

V-23a) $$\Pr(\epsilon_{j,q} = n) = e^{-\bar{\epsilon}_{j,q}}\,\bar{\epsilon}^{\,(n-1)}_{j,q}/(n-1)!\,(1-e^{-\bar{\epsilon}_{j,q}}), \quad n=1,2,\ldots$$

and that the expected value is now $E(\epsilon_{j,q}) = \bar{\epsilon}_{j,q} + 1$.

Let us now define $F_{j,t,q,k}$ as a dual valued variable, being one if the k^{th} article in journal j in the issues published in time period q is relevant in period t|q, zero if not. Also, let $u_{j,t,q,k}$ be the probability that $F_{j,t,q,k}=1$, that is, the k^{th} article is relevant. It is obvious that by definition

$$V\text{-}24) \qquad g_{j,t,q} = \sum_{k=1}^{\epsilon_{j,q}} F_{j,t,q,k} \ .$$

If we assume that all articles are statistically independent of each other with respect to relevance, and that each article has an equal chance, a priori, of being relevant, then for the original articles without consideration of the journal as an article, we have

$$V\text{-}25) \qquad \Pr(g_{j,t,q}=n) = e^{-\bar{\epsilon}_{j,q}\, u_{j,t,q}} (\bar{\epsilon}_{j,q}\, u_{j,t,q})^n/n! \ , \qquad n=0,1,\ldots$$

and

$$V\text{-}26) \qquad E(g_{j,t,q}) = \bar{\epsilon}_{j,q}\, u_{j,t,q} \ .$$

If the journal itself is considered as an independent article with an equal chance of being relevant, we have

$$V\text{-}26a) \qquad E(g_{j,t,q}) = (\bar{\epsilon}_{j,q}+1)\, u_{j,t,q} \ .$$

If the probability of the journal itself being relevant is equal to $u'_{j,t,q}$ and journal relevancy is again independent of the articles, we have

$$V\text{-}26b) \qquad E(g_{j,t,q}) = \bar{\epsilon}_{j,q}\, u_{j,t,q} + u'_{j,t,q} \ .$$

To define relevancy and to relate it to usage, let us consider an article relevant only if it will be used at least d times. Thus

V-27) $v_{j,t,q}(d)$ = Pr(an article of journal j published in period q was used at least d times in period t+q).

This means that $g_{j,t,q}=n$ implies that exactly n articles will have demand at least as large as d. The relationship of relevancy and usage also implies that $G_{j,t,q}$, the number of demands for the whole journal, can be no smaller than nd for n articles to be relevant. This model is related to Jain's [30] "Pn" model. If we assume that articles are statistically independent of usage, when d = 1, we have

V-28) $$P_{j,t,q}=[1-v_{j,t,q}(1)]^{e_{j,q}}.$$

Note that $[1-u'_{j,t,q}(1)]$ = the probability that the issues of journal j published in period q will have no uses in period t+q. While $G_{j,t,q} \geq nd$ is necessary for $g_{j,t,q}=n$, it is not sufficient. A few articles may have a high enough demand to make $G_{j,t,q}$ large but the number of articles is smaller than n. However, since the librarian is interested in journal rather than article worth, concentration of decision making efforts should be on whether or not $G_{j,t,q}$ is large enough to justify buying journal j.

Data Analysis and Results

Consider the testing of the models of literature growth and scattering in terms of how well they describe the dynamic properties of literature. Using the bibliography in Saaty's [66] textbook on queuing theory, a small data base was constructed. From the journal articles cited in the bibliography, it was found that over the period 1932-1961, $s = 157$ journals had a total of 681 articles with 36 additional articles published prior to 1932. Further, $s' = 72$ of these journals contained at least two articles, for a total of 556 articles in $100 \times 72/157 = 45.9\%$ of the most productive journals.

The data, since it comes from a citation list, has the property of allowing no compensation for an item obsolescing over time as the item ages. An article, if once cited by Saaty, is considered relevant from the time of publication on, and it remains relevant throughout the time horizon. Thus, we can ignore age and consider $\varepsilon^*_{j,t,q} = \varepsilon^*_{j,0,q}$ for all $t=0,1,\ldots,r-q$.

Note that in this specific case, the time subscript q represents the year of publication minus 1931. Each time period represents a calendar year, with 1931 being considered as a base year and all citations before and during 1931 being lumped into one initial condition. The growth of the literature as seen through Saaty's citations is charted as it moves through time until 1961, 30 years later.

In order to test the stability of the lemkoid distribution over time, we can find estimates, $\hat{b}_{0,q}$ using V-16) for each time period $q=0,1,2,\ldots,r$. The results of the study are given below in Table 3. In the table $f_{0,q}$ is the number of journals having at least one relevant

Table 3. The Parameter Estimate $\hat{B}_{0,q}$ Over Time

Time Period q	$E_{0,q}$	$n^*_{0,q}$	$n_{0,q}$	$f^*_{0,q}$	$f_{0,q}$	$\hat{B}_{0,q}$	$\hat{B}_{0,q}/(c'/c)$
0 (1931)	.04102	36	36	9	9	6.435×10^{10}	1.403×10^{11}
1	.04012	36	0	9	0	6.435×10^{10}	1.403×10^{11}
2	.04452	39	3	10	1	5.497×10^{9}	1.199×10^{10}
3	.04400	41	2	10	0	7.091×10^{9}	1.544×10^{10}
4	.04660	42	1	11	1	2.070×10^{9}	4.514×10^{9}
5	.05437	47	5	13	2	9.706×10^{7}	2.117×10^{8}
6	.06039	55	8	16	3	5.453×10^{6}	1.189×10^{7}
7	.06775	57	2	17	1	2.854×10^{6}	6.223×10^{6}
8	0.7040	58	1	18	1	1.474×10^{6}	3.213×10^{6}
9	.07650	61	3	19	1	4.772×10^{5}	1.050×10^{6}
10	0.8632	65	4	21	2	1.061×10^{5}	2.314×10^{5}
11	.09058	69	4	22	1	6.200×10^{4}	1.352×10^{5}
12	.09475	73	4	23	1	3.820×10^{4}	8.329×10^{4}
13	.09475	73	0	23	0	3.820×10^{4}	8.329×10^{4}
14	.09901	77	4	24	1	2.375×10^{4}	5.178×10^{4}
15	.10654	85	8	24	0	1.181×10^{4}	2.575×10^{4}
16	.11697	94	9	27	3	5.126×10^{3}	1.118×10^{4}
17	.13715	112	18	32	5	1.413×10^{3}	3.080×10^{3}
18	.15079	126	14	36	4	712.5	1.554×10^{3}
19	.15399	137	11	37	1	616.8	1.345×10^{3}
20	.16551	156	19	42	5	381.2	831.2
21	.18679	187	31	46	4	180.1	392.6
22	.20508	222	35	51	5	104.5	227.9
23	.22869	273	51	59	8	56.64	123.5
24	.25076	328	55	64	5	34.60	75.47
25	.25891	391	63	68	4	29.79	64.97
26	.25512	458	67	71	3	33.1	72.19
27	.25142	489	31	72	1	34.31	74.82
28	.24547	531	39	72	0	38.68	84.99
29	.23671	592	31	72	0	47.89	104.4
30 (1961)	.23404	596	4	72	0	49.04	106.9

article as of period q while $f^{*}_{o,q}$ is the number of journals possessing relevant articles for the first time in period q. Note that $\hat{B}_{o,q}$ is not independent of q at all but seems to be decreasing exponentially over time and that further, the values of $\hat{B}_{o,q}$ for small q are rather enormous. One plausible explanation for this phenomenon is the nature of the scientific field of endeavor of the data base. Queueing theory is a relatively new field, with a great bulk of articles not having appeared until well after World War II. Thus, one can see the scattering of literature in this field as it is in its infancy. Initially, scattering is quite small, with only a handful of journals having relevant articles, which accounts for the initially large values of $\hat{B}_{o,q}$.

The values of $\hat{B}_{o,q}$ towards the end of the 30-year study are more "reasonable" in that they are more in line with values calculated from other studies [39]. Even this, however, does not necessarily mean stability and steady-state behavior for the parameter. This leveling of $\hat{B}_{o,q}$ is offset, however, by a sudden decline in growth around period 27 representing year 1968 on to the end of the study. One plausible explanation for this contraction is that the author of the bibliography began writing his book, published in 1968, which implies that he might have had less time in the last few years prior to publication for searching the literature to find relevant articles. This is borne out somewhat in an additional list of citations in a later article by Saaty [67] which shows queueing theory to be growing as much as before.

In order to solve for the parameters of the epidemic model of literature growth, we use V-10a) and V-10b) in conjunction with the observed values of $n_{o,q}$ as listed in Table 3, which yields $\hat{\alpha} \sim 0.432$ and $\hat{\gamma} \sim 15.56$ to be used in equation V-9b). If one uses a χ^2 test for goodness of fit, he must reject the hypothesis that the epidemic model is an adequate description of literature growth at any reasonable level of significance [6]. One reason for this conclusion is due to the large initial value of $n_{o,o} = 36$, caused by the lumping of all articles published prior to 1932 into one initial value.

We can draw several conclusions from the results in Table 3. First of all, we note that citation lists are not a very firm foundation on which to base future journal selection decisions. The search effort may be lessened in the last few periods prior to publication if the list comes from bibliographies of books. Moreover, there is no indication that different sources of relevant articles and/or different scientific fields of endeavor would not have widely varying patterns of literature growth and scattering. Secondly, we see that there is no allowance for a field to contract or even die, nor is there any allowance for it to change and divide into independent subfields or merge with other areas of science. Since the librarian may be unfamiliar with the technical aspects and latest developments of the scientific fields of endeavor which his library services, he may not be able to determine the precise stages of development and growth for the fields in question.

The model of literature scattering is seen as a static tool with little capability to account for dynamic situations. One knows that scattering exists in that a small percentage of the most productive journals will yield a majority of the relevant articles in the field. However, the model cannot adequately describe which journals are in the top percentage over time. This is complicated further by the fact that the scattering parameter $\beta_{o,q}$ is highly time dependent. Allowing article relevancy to change over time, i.e., obsolescence, can complicate matters even further.

Finally, we see that the dynamic epidemic model cannot satisfactorily describe the growth of literature. Without proper growth or scattering functions being adequately described, we cannot readily use productivity as a measure of journal worth.

CHAPTER VI

A SOLUTION ALGORITHM FOR THE JOURNAL SELECTION MODEL

With the model constructed and journal worth defined, the final task is to determine methods of solving the model for the optimal journal selection policy. There are, of course, integer or zero-one programming methods [29] that can be brought to bear, but these are not equipped to handle, realistically speaking, problems of this magnitude. In a problem with a set of s journals and a planning horizon of r time periods, there are $s(r+3)r/2$ decision variables. Thus, if we have $s = 100$ journals and $r = 30$ time periods, there are 49,500 decision variables. In order to handle such a problem we shall incorporate a dynamic programming formulation, and invoke a theorem on optimal control of stochastic systems [53]. The subproblems then, as we shall see, take on the characteristics of the familiar knapsack problem which can be handled through the use of a ranking of simple ratios. We shall find, however, that only in special cases will this lead us to a global optimum. To account for this, a Lagrangian approach, outlined by Everett [19], will be used to solve or at least to bound this discrete optimization problem.

In the first section we shall derive a suboptimal computational algorithm based on a dynamic programming formulation of the problem. Special cases where the algorithm yields the optimal solution are discussed. The second section contains an algorithm based on a Lagrangian approach to the journal selection problem that will yield an upper bound on the optimal objective function. The theory behind this approach is also discussed in detail. Methods of improving the solution are presented to yield a near-optimal answer to the journal selection problem. In the final section we present a small example problem that will illustrate the algorithms in operation.

A Dynamic Programming Formulation

Let us consider the general model, as presented in Figure 5 of Chapter III, and formulate it as a dynamic programming problem. This will enable us to break down the problem into smaller, less complex stages and allow us to develop solution algorithms.

We first define the state variable as follows:

VI-1) $$x_q = B_q - \sum_{j=1}^{s} \sum_{t=0}^{q-1} Y_{j,q-t-1,t}(c_2 + c_3 \lambda_{j,q-t,t})$$

= the amount of money left in period q for new journal acquisitions after paying for the storage and circulation in period q of former acquisitions.

Note that we can derive the dynamics of the state equation as

$$x_{q+1} = x_q + (E_{q+1} - B_q) - \sum_{j=1}^{n} \Big[Y_{j,0,q}(c_2 + c_3 \lambda_{j,1,q}) + \sum_{\ell=0}^{q-1} (Y_{j,q-\ell,\ell}(c_2 + c_3 \lambda_{j,q+1-\ell,\ell}) - Y_{j,q-\ell-1,\ell}(c_2 + c_3 \lambda_{j,q-\ell,\ell}))\Big]$$

We now employ the principle of optimality to define the recursive relationship that represents the objective function over time, and will allow us to break down the journal selection model.

VI-2) Let $F_q^*(x_q) = \max_{\{Y_{j,q-\ell,\ell}\}} \Big[\sum_{j=1}^{n} \sum_{\ell=0}^{q} Y_{j,q-\ell,\ell} \lambda_{j,q-\ell,\ell} + F_{q+1}^*(x_{q+1}) \Big]$

= the optimal return at stage or time period q as a function of the state variable x_q and the decision variables $Y_{j,q-\ell,\ell}$, over which the return function is to be maximized.

Note that decisions made in period q influence returns in period q+1 so that a recursive relationship is needed to take these effects into account.

The constraints on the journal selection model then become, for period q:

VI-3c) $Y_{j,q-\ell,\ell} = 0$ or 1 $\quad \forall\ j=1,2,\ldots,n;\ \ell=0,1,\ldots,q$

VI-3b) $Y_{j,q-\ell,\ell} \geq Y_{j,q-\ell-1,\ell} \qquad \forall\, j=1,2,\ldots,s;\ \ell=0,1,\ldots,q-1$

VI-3c) $$\sum_{j=1}^{s} [Y_{j,0,q}(c_1 + K_{j,0,q} + c_2 + c_3\,\lambda_{j,0,q}) + \sum_{\ell=0}^{q-1} (Y_{j,q-\ell,\ell} - Y_{j,q-\ell-1,\ell})(c_1 + K_{j,q-\ell,\ell} + c_2 + c_3\,\lambda_{j,q-\ell,\ell})] \leq x_q$$

VI-3d) $$x_{q+t} = B_{q+t} - \sum_{j=1}^{s} \sum_{\ell=0}^{q+t-1} Y_{j,q+t-\ell-1,\ell}(c_2 + c_3\,\lambda_{j,q+t-\ell,\ell}) \geq 0$$

$$\forall\ t=1,2,\ldots,r-q\ .$$

We note that VI-3d) enforces the feasibility of future budget constraints by not allowing us to acquire more items in period q than the budgets of future periods can accommodate in terms of storage and circulation of those items. We are assuming that $B_q \geq 0$ for all q, which seems reasonable for a budget figure. We also assume that the cost figures, c_1, c_2, c_3, and $K_{j,q-\ell,\ell}$, and expected usage, $\lambda_{j,q-\ell,\ell}$, are non-negative for all j, q, and ℓ.

Midler's Lemma on Optimal Control

The optimal selection policy can be found independently of the state variable. That is the surprising result Midler [53] found for a general class of discrete time stochastic systems that are linear in the state. We shall see that the journal selection problem is analogous to a special case of this general class and that his formulation can aid us in finding a solution.

Midler first develops his model as follows (note that we have modified his notation to fit conventions already adopted in this dissertation).

We have the state equations

$$\text{VI-4)} \qquad x_{q+1} = A_q x_q + G_q(u_q) + \eta_q \qquad q=0,1,\ldots,r$$

$$x_0 = \xi$$

where x_q is the n-dimensional state vector, u_q is the m-dimension control or decision vector, A_q is a random matrix of order n x n, and η_q is a random n-dimensional vector. Also, U_q is the set of feasible controls, a subset of E^m (Euclidean m-space), so that $G_q(u_q)$ is a random mapping[1] from U_q to E^n. Thus, the state dynamics are linear in the state with random coefficients and general random functions of the controls with additional random perturbances.

Midler considers the loss function

$$\text{VI-5)} \qquad L = \sum_{q=1}^{r} [Q_{q+1} x_{q+1} + \varphi_q(u_q)]$$

where Q_q is a random n-dimensional vector and φ_q is random function of u_q, mapping U_q into E^1. We note that both the loss function and the state equations are linear in form. The goal or objective is to minimize E(L), the expected loss, by choosing (u_q^*), the optimal feasible

---

1 We shall infer that Midler means that a random mapping is a mapping from a set of points A, the domain, to a set of points B, the range, so that associated with a point x in A is a point or subset to points in B. The random mapping implies that for a specific point x in A, there is no definite mapping from x to subsets in B, but only a probabilistic statement about which subset in B to which x will be mapped.

controls over time for $q=1,2,\ldots,r$. By feasibility, we mean that $u_q^* \in U_q$ for all q; that is, the optimal control vectors are elements of the set of feasible controls. Note that E, the expected value operator, is with respect to the joint distribution of the random vectors, matrices, and functions. Further note that the use of expectations gives us a certainty equivalence for the problem, so that we can deal with it as one would deal with a deterministic control problem.

Midler assumes that

i) all random functions have known, finite expectations, denoted as bars, e.g., $E(A_q) = \bar{A}_q$;

ii) the functions $\bar{G}_q(u_q)$ and $\bar{c}_q(u_q)$ are continuous functions of u_q over a collection of open sets for which U_q is closed and bounded;

iii) the random variables are independent over time.[2]

---

2 Midler points out that this assumption does not rule out nonindependence of a linear Markovian type for the additive disturbances η_q. For example, consider the case of $\eta_q = R_q \eta_{q-1} + \zeta_q$ where R_q and ζ_q are random matrices, independent over time, with finite known expectations. Incidentally, η_t and the other components of the state vector need not be observable. One may adjoin the relations describing the generation of η_q to the state vector dynamics while still retaining the form given in VI-4).

Midler's Lemma: For a stochastic control system whose dynamics are given by VI-4) and whose loss function is given by VI-5) and which is to be minimized, let us suppose that assumptions i), ii), and iii) hold. Then the recursive relationship becomes

VI-6) $$F_q^*(x_q) = p_q x_q + k_q \qquad q=1,2,\ldots,r$$

where we define for convenience:

VI-7a) $$p_q = (p_{q+1} + \bar{Q}_{q+1})\,\bar{A}_q \qquad q=1,2,\ldots,r-1$$

VI-7b) $$p_r = \bar{Q}_r\,\bar{A}_{r-1}$$

VI-7c) $$k_q = k_{q+1} + (p_{q+1} + \bar{Q}_{q+1})(\bar{\eta}_q + G_q(u_q^*)) + \varphi_q(u_q^*) \qquad q=1,2,\ldots,r-1$$

VI-7d) $$k_r = \bar{Q}_{r+1}\,\bar{G}_r(u_r^*) + \varphi_r(u_r^*) + \bar{Q}_r\,\bar{\eta}_r$$

and where, by convention

VI-7e) $$F_{r+1}^* = 0 .$$

Note the recursive relationship of the k_q and p_q variables; but also note that one calculates them by starting with p_r and k_r and working back towards p_1 and k_1. Here u_q^*, the optimal control variable in period q, is the point in U_q that minimizes the expression

VI-8) $$F_q(x_q) = E(G_{q+1}x_{q+1} + \varphi_q(u_q) + F^*_{q+1}(x_{q+1}))$$

so that, for $q=0,1,\ldots,r-1$,

VI-6a) $$F_q^*(x_q) = \min_{x_q \in X_q} F_q(x_q) = \min_{x_q \in X_q} (p_q x_q + k_q)$$

Since the recursive relationship of VI-6a) involves p_q, which is independent of x_q and u_q, and k_q, which is independent of x_q, the optimal policy u_q^*, $q=1,\ldots,r$ is independent of the state vector x_q. In fact, one need to merely minimize k_q as a function of u_q in order to solve for u_q^*.

Nidler's proof depends on a dynamic programming formulation of the problem. He uses an inductive proof of equation VI-(6a) through the recursive equation that depicts the form of the optimal return function. We question the need for assuming that $\bar{G}_q(u_q)$ and $\bar{r}(u_q)$ are continuous functions of u_q, since continuity is never used in his proof. Note that the variation of $\bar{G}_q(u_q)$ and $\bar{\varphi}_q(u_q)$ must be bounded. The minimization process merely becomes discrete as opposed to continuous if we add a condition such as the restriction of the control or decision variables to specific integers. Also, $G_q(u_q)$ and $\varphi_q(u_q)$ may now take on only certain discrete values in terms of the discrete control variable u_q.

In an example problem, Nidler assumes that U_q is a convex polyhedral given by

VI-9) $$U_q = (u_q \mid C_q u_q \leq b_q,\ u_q \geq 0)$$

for nonstochastic matrices C_q of order $m' \times n$ and b_q of order $m' \times 1$. He further assumes that $\varphi_q(u_q) = h_q u_q$ and $G_q(u_q) = D_q u_q$, where h_q and

E_q are appropriate stochastic matrices, of order $1 \times n$ and $n \times n$ respectively. One must not confuse the notation for matrix E_q as used here with the expression for the budget in period q as used elsewhere. Note that the state vector dynamics are still in the very general terms in which they were defined, and without any specific qualification. This yields a series of r linear programming problems to be solved sequentially and given by

VI-10) $$\text{Min } [(p_{q+1} + \bar{Q}_{q+1})\bar{E}_q + \bar{h}_q]u_q$$

$$\text{s.t. } c_q u_q \leq b_q$$

$$u_q \geq 0$$

for each $q=1,2,\ldots,r$.

This result is obtained by considering

$$\begin{aligned} F_q^* &= p_q x_q + k_q \\ &= p_q x_q + k_{q+1} + (p_{q+1} + \bar{Q}_{q+1})(\bar{E}_q + \bar{G}_q(u_q^*)) + \bar{r}_q(u_q^*) \\ &= p_q x_q + k_{q+1} + (p_{q+1} + \bar{Q}_{q+1})(\bar{E}_q + E_q u_q^*) + h_q u_q^* \\ &= p_q x_q + k_{q+1} + (p_{q+1} + \bar{Q}_{q+1})\bar{E}_q + [(p_{q+1} + \bar{Q}_{q+1})E_q + h_q]\, u_q^*. \end{aligned}$$

This means that one must minimize the expression

$$[(p_{q+1} + \bar{Q}_{q+1})E_q + h_q]u_q$$

over the set of feasible controls, U_q, in order to find the optimal policy u_q^*. This is true since the expression $p_q x_q + k_{q+1} + (p_{q+1} + \bar{Q}_{q+1})\bar{E}_q$ is a constant, independent of u_q. Note that this differs slightly from Midler's reported findings.

In this example, we see that no difficulty arises if we do restrict the elements of the control vector to integer values. This additional restriction means that one now has a series of linear integer programming problems. More specifically, one can easily go to a zero-one linear programming problem as in the journal selection problem by proper choice of the feasible control set U_q. Thus we have demonstrated the maintenance of the validity of Nider's Lemma when $G_q(u_q)$ and $\varphi_q(u_q)$ are not continuous functions of u_q.

In Nidler's formulation, he indicated that A_q, $G_q(u_q)$, $\pi_q\zeta_\eta$, and $\varphi_q(u_q)$ were random matrices by the device of their being functions of a random variable w_q, a variable not used in our notation for convenience's sake. To extend the system to the case where the random parameters are not independent over time, Nidler replaces w_q with w_q^1 and w_q^2, two random variables. He assumes that w_q^1 is independent over time while w_q^2 is dependent over time and appears only in $G_q(u_q)$ and in $\varphi_q(u_q)$. He further assumes that the realizations of w_q^2 are observable. The one change necessary in the result is that all expectations in period q now must become conditional upon all relevant past realizations and are denoted as $E^c(\cdot)$. The recursive relationship to be minimized at each stage in order to solve for u_q^* now becomes

$$[(p_{r-q-1} + \bar{\zeta}_{q+1})E^c(G_q(u_q)) + E^c(\varphi_q(u_q)) + c_q]$$

where c_q is a constant independent of u_q. It is here that u_q becomes dependent on past realizations of w_q^2 and thus upon an extended notion of the state, so that one must choose the optimal policy sequentially.

Finally, Midler points out that if w_q is governed by a Markov chain with observable states, w_q^* is still independent of x_q and $F_q^*(x_q)$ is still linear in the state. However, the control variable must now be calculated for each possible realization of w_q^1, since the coefficients of $F_q^*(x_q)$ are now dependent upon the realization in the previous period.

Midler's Lemma Applied to the Journal Selection Model

In order to better understand the journal selection model and to legitimize the decomposition of the overall problem via the principle of optimality, let us apply Midler's results to our problem. Recalling the state variable of the selection model,

$$\text{VI-1)} \qquad x_q = B_q - \sum_{j=1}^{s} \sum_{t=0}^{q-1} Y_{j,q-t-1,t}(c_2 + c_3 \lambda_{j,q-t,t}) ,$$

we find that the dimension of the state vector is $n=1$, and that A_q and η_q are zero so that $\bar{A}_q$ and $\bar{\eta}_q$ are also zero for all q. Further $G_q(u_q) = x_q = B_q - \sum_{j=1}^{s} \sum_{t=0}^{q-1} Y_{j,q-t-1,t}(c_2 + c_3 \lambda_{j,q-t,t})$ with u_q representing the decision variables $Y_{j,q-t,t}$ for $j=1,2,\ldots,s$ and $t=0,1,\ldots,q$. Recalling the recursive relationship that states our objective function, which is to be maximized, we have

$$\text{VI-2)} \qquad F_q^*(x_q) = \max_{\{Y_{j,q-t,t}\}} \left[\sum_{j=1}^{s} \sum_{t=0}^{q} Y_{j,q-t,t} \lambda_{j,q-t,t} + F_{q+1}(x_{q+1}) \right]$$

we see that if we let Q_q, and thus $\bar{Q}_q$, equals zero, this implies

that $\varphi_q(u_q) = \sum_{j=1}^{s} \sum_{\ell=1}^{q} Y_{j,q-\ell,\ell} \lambda_{j,q-\ell,\ell}$, for all q.

Using Midler's Lemma, we find that

$$p_q = 0 \text{ for all } q$$

and that

$$K_q = K_{q-1} + \varphi_q(u_q^*)$$

or

$$\text{VI-11)} \quad K_q = \sum_{\gamma=0}^{q} \sum_{j=1}^{s} \sum_{\ell=0}^{\gamma} Y^*_{j,\gamma-\ell,\ell} \lambda_{j,\gamma-\ell,\ell} .$$

Thus we now have the problem of determining $Y^*_{j,\gamma-\ell,\ell}$ so as to maximize K_q for all q. Note that the constraint set, as defined by VI-3a), VI-3b), VI-3c), and VI-3d), defines the feasible region of controls, U_q. While the feasible region is described by linear forms, these forms are not independent of time, and it is this dependence that will cause us some problems, as we shall see.

A Suboptimal Computational Algorithm

Midler's work, as outlined above, has been used to illustrate how the overall journal selection model can be decomposed into smaller subproblems. The subproblems can then be viewed as a set of knapsack problems, which can be solved by conventional means.

A computational algorithm, illustrated in Figure 8, is outlined below.

1) The journal selection problem must be set up as outlined in Chapter III and the first section of this chapter. This means specifying the costs, $(c_1, c_2, c_3, K_{j,q-\ell,\ell})$ expected journal usage $(\lambda_{j,q-\ell,\ell})$, budgets, $(B_q \geq 0)$ and the setting of $Y_{j,o,o}$ to one for all journals purchased prior to the planning horizon.[3] One must check to see if a feasible solution exists. If $B_q \geq \sum_{j=1}^{s} Y_{j,o,o}(c_2 + c_3 \lambda_{j,q,o})$ for all $q=1,\ldots,r$ a feasible solution does exist and we can proceed to step 2. If at least one q exists such that the condition is violated, no feasible solution exists and we must terminate our algorithm.

2) Set $q=1$. Let $Y_{j,1,o} = 1$ if $Y_{j,o,o} = 1$ for all $j=1,2,\ldots,s$.

3) Calculate the cost functions

VI-12) $$C_{j,q-\ell,\ell} = \begin{cases} c_1 + c_2 + c_3 \lambda_{j,o,q} + K_{j,o,q} & \text{if } q=\ell \\ (1-Y_{j,q-\ell-1,\ell})(c_1 + K_{j,q-\ell,\ell}) + c_2 + c_3 \lambda_{j,q-\ell,\ell} & \text{if } q>\ell \end{cases}$$

for all $j=1,2,\ldots,s$ and for all $\ell=0,1,\ldots,q$. Also define

$$x_q = B_q - \sum_{j=1}^{s} \sum_{\ell=0}^{q-1} Y_{j,q-\ell-1,\ell}(c_2 + c_3 \lambda_{j,q-\ell,\ell}).$$

---

3 This means one must define a planning horizon and divide it into r consecutive periods; and one must choose a complete set of s journals and label them with a number j, $j=1,2,\ldots,s$. One must also estimate any parameters used to depict expensive usage or any other variable over time.

4) Calculate the ratio $R_{j,q-\ell,\ell}$ for all $j=1,2,\ldots,s$ and $\ell=0,1,\ldots,q$ where

VI-13) $$R_{j,q-\ell,\ell} = \begin{cases} \lambda_{j,0,q}/c_{j,0,q} & \text{if } \ell=q \\ \lambda_{j,q-\ell,\ell}(1-\gamma_{j,q-\ell-1,\ell})/c_{j,q-\ell,\ell} & \text{if } \ell<q \end{cases}$$

5) Rank the ratios $R_{j,q-\ell,\ell}$ in decreasing order. Define the variable $\Theta_{j,q-\ell,\ell}$ as the rank of the item having the ratio $R_{j,q-\ell,\ell}$. Thus the item with the largest ratio will have a Θ value of 1.

6) Define $C(\Theta_{j,q-\ell,\ell}) = c_{j,q-\ell,\ell}$ $\forall$ j, q, and ℓ and p_q as the largest integer such that

VI-14) $$\sum_{\Theta_{j,q-\ell,\ell}=1}^{p_q} C(\Theta_{j,q-\ell,\ell}) \le x_q .$$

Thus, we sum over the ranks of the items, in order, to select the highest ranked items and to acquire as many as possible under the constraint limitations.

7) If VI-14) is an equality, we have optimized the return at stage q. Go to step 9.

8) Here VI-14) is a strict inequality, so we must use branch and bound techniques, such as those outlined in [25] and [26], to achieve some desired level of near-optimality if not optimality at stage q.

9) Calculate $x_{q+t} = B_{q+t} - \sum_{j=1}^{s} \sum_{\ell=0}^{q+t-1} Y_{j,q+t-\ell-1,\ell}(c_j + \lambda_{j,q+t-\ell,\ell})$ for $t=1,2,\ldots,r-q$.

10) If $x_{q+t} \geq 0$ for all t, we are feasible for all future time periods in terms of the policy for period q. Go to step 12.

11) If $x_{q+t} < 0$ for at least one $t=1,2,\ldots,r-q$ we have violated the budget constraint for period q+t through the storage and circulation of items accumulated before period q+1. To restore feasibility, begin to set $Y_{j,q-\ell,\ell} = 0$ for enough variables to satisfy the violated constraint. This must be done by beginning at the lowest value of $R_{j,q-\ell,\ell}$ for which $Y_{j,q-\ell,\ell} = 1$, and $Y_{j,q-\ell-1,\ell} = 0$ if $q > \ell$, going up the scale as the ratio increases, and setting each $Y_{j,q-\ell,\ell} = 0$ as needed.

12) Set q equal to the value q+1.

13) If $q \leq r$, we must re-cycle to continue the process for new time periods. We must first update our decision variables by letting $Y_{j,q-\ell,\ell} = 1$ for all $Y_{j,q-\ell-1,\ell} = 1$ for all $j=1,\ldots,s$ and $\ell=0,1,\ldots,q$. Now go to step 5.

14) Here $q > r$, so we have completed the solution and must terminate. Calculate $\sum_{j=1}^{s} \sum_{q=0}^{r} \sum_{\ell=0}^{q} Y_{j,q-\ell,\ell}\, \lambda_{j,q-\ell,\ell}$ for the expected usage from our selection policy.

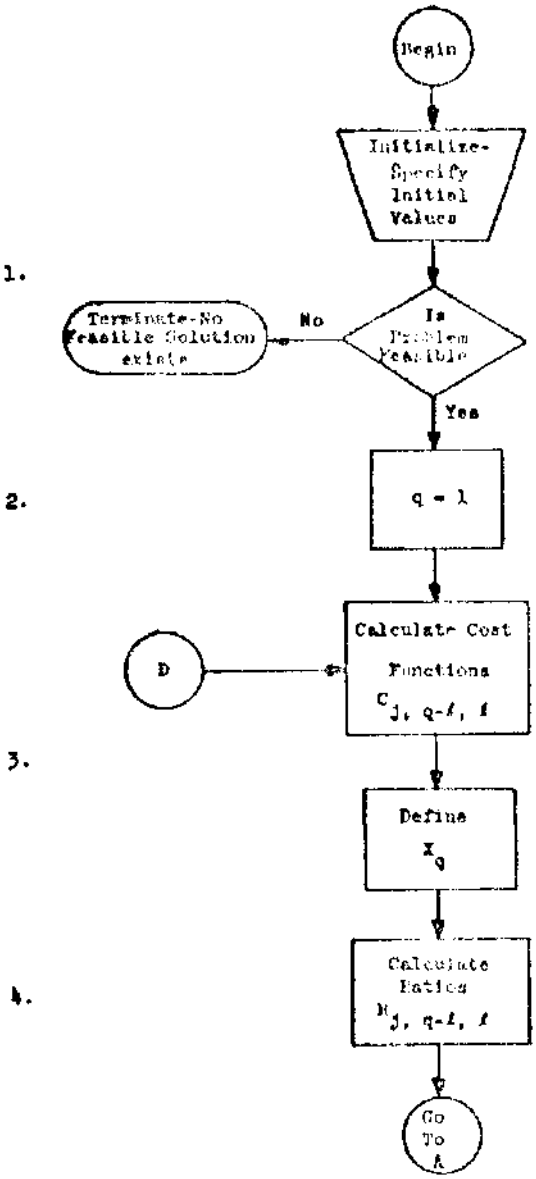


Figure 6. A Computational Algorithm for Finding a Feasible Solution to the Journal Selection Problem

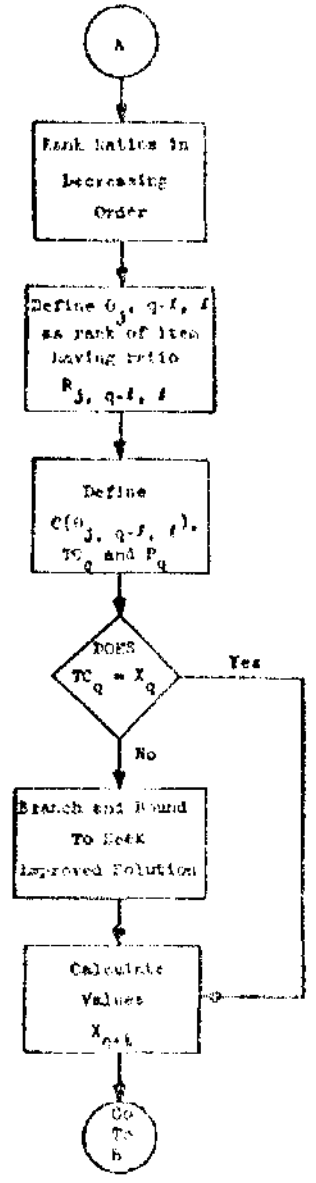


Figure 5. (Continued)

10.

11.

12.

13.

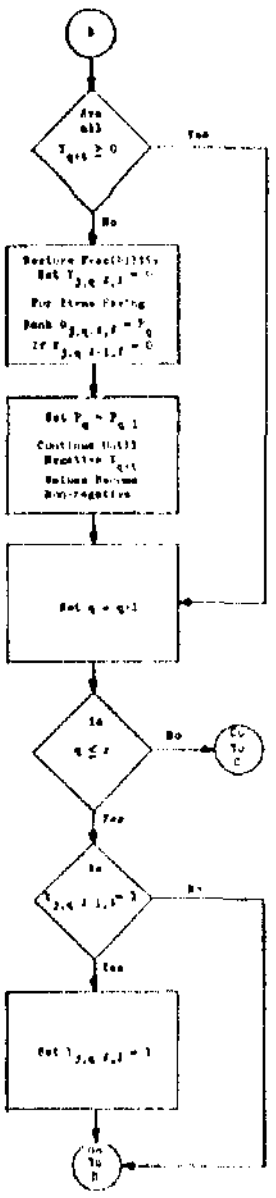


Figure 8. (Continued)

14.

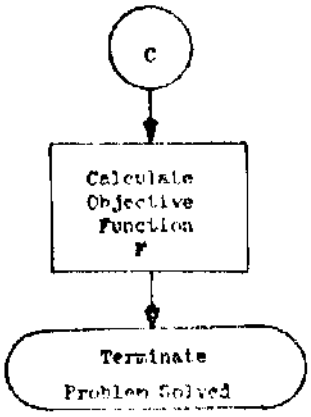


Figure 8. (Continued)

The conditions for the existence of a feasible solution (step 1) are that the budgets for each period can cover the cost of storing and circulating the items bought previously to the planning horizon as described by the given initial selection variables $Y_{j,o,o}$. The satisfaction of this condition implies that at the very least the policy of acquiring no new journals is feasible. If this is the case, one must then seek better solutions; if this is not the case, no solution is feasible so we must terminate the solution algorithm.

We note that the algorithm is iterative in nature; one starts with q=1 and proceeds by increasing q by 1 each time until we have solved all r problems. Further, by starting with q=1 and working up to q=r, we can maintain the relationship that $Y_{j,q-\ell,\ell} \geq Y_{j,q-\ell-1,\ell}$ (step 2 and step 13) and can properly calculate the increased cost due to acquisition in period q (step 3) as well as the increased usage (step 4). Note that in the calculation of the ratios (step 4), we have the multiplication of the original benefit-over-cost function by $(1 - Y_{j,q-\ell-1,\ell})$ to set that ratio to zero, and virtually eliminate the ratio from consideration, if $Y_{j,q-\ell-1,\ell} = 1$ which means that acquisition of that item will not occur in period q but has already taken place. The calculation of these ratios, the ranking of these ratios in descending order (step 5), the selection of those items with the highest ratios to be acquired (step 6), the branch-and-bounding to check for better solutions if the budget figure is not met exactly (step 8), and the termination of the q^{th} problem if the budget is met exactly (step 7) are all part of the knapsack problem and can be

solved accordingly [76]. To keep the solution feasible for all future time periods, we then must proceed with a routine (steps 9-11) to insure that x_{t+q} is nonnegative for all t=1,2,...,r-q.

When Is Suboptimal Optimal?

The algorithm is obviously suboptimal in that each stage or time period is optimized separately and independently with no regard to future effects of a current policy decision. This is exemplified by equation IV-31), which shows that items currently selected have circulation and storage costs in all future periods which can influence future decisions. Thus, in essence, the recursiveness of the dynamic programming problem formulation has been dropped to yield a feasible, suboptimal solution so that VI-2) becomes

$$F_q^* = \max_{(Y_{j,q-\ell,\ell})} \sum_{j=1}^{s} \sum_{\ell=0}^{q} Y_{j,q-\ell,\ell} \lambda_{j,q-\ell,\ell} .$$

However, the worth of this solution in terms of a global optimal answer is in general unknown.

Let us now assume that a feasible solution exists (the conditions for which have been discussed above). In order to investigate some special cases where optimizing each stage can lead to the global optimal solution, we shall prove the following proposition.

Proposition: The dynamic programming computational algorithm, optimizing each stage, can lead to the global optimal solution if one of the following cases hold:

i) $c_2 = c_3 = 0$

ii) $c_3 = 0$ and $K_{j,q-\ell,\ell} = K_q$ $\quad \forall\ j,\ \ell,$ and q

iii) $K_{j,q-\ell,\ell} = K_q$ and $\lambda_{j,q-\ell,\ell} = \lambda_q$ $\quad \forall\ j,\ \ell,$ and q

In cases ii) and iii), other special conditions must also hold if the solution is to be recognized as a global optimal one.

Proof:

Case i: $c_2 = c_3 = 0$.

In this case, we can see that the state equation of VI-1) becomes

$$x_q = B_q \qquad \forall\ q,$$

and that the budget constraints of VI-3c) become

$$\sum_{j=1}^{s} \sum_{\ell=0}^{q} Y_{j,q-\ell,\ell}(c_1 + K_{j,q-\ell,\ell}) - \sum_{j=1}^{s} \sum_{\ell=0}^{q-1} Y_{j,q-\ell-1,\ell}(c_1 + K_{1,q-\ell,\ell}) \leq x_q \qquad \forall\ q$$

Of course, all other relationships in the journal selection model, as discussed above, remain the same. With the link between each stage or time period having been broken in terms of the future budgets being unaffected by the storage and circulation costs of current selections, the computational algorithm yields not only a feasible but a global optimal solution. Thus, the journal selection model decomposes, via Kidler's Lemma, into a series of knapsack problems that can be solved independently. These problems, for each $q=1,2,\ldots,r$, are of the form

$$\text{Max} \quad P_q = \sum_{j=1}^{s} \sum_{\ell=1}^{q} Y_{j,q-\ell,\ell}\, \lambda_{j,q-\ell,\ell}$$

$$\text{s.t.} \quad Y_{j,q-\ell,\ell} = 0 \text{ or } 1 \qquad \forall\ j=1,\ldots,s;\ \forall\ \ell=0,1,\ldots,q$$

$$Y_{j,q-\ell,\ell} \geq Y_{j,q-\ell-1,\ell} \qquad \forall\ j=1,\ldots,s;\ \forall\ \ell=0,1,\ldots,q-1$$

$$\sum_{j=1}^{s} \sum_{\ell=0}^{q} Y_{j,q-\ell,\ell}(c_1 + K_{j,q-\ell,\ell})$$

$$- \sum_{j=1}^{s} \sum_{\ell=0}^{q-1} Y_{j,q-\ell-1,\ell}(c_1 + K_{j,q-\ell,\ell}) \geq P_q .$$

Case ii: $c_2 = 0$ and $K_{j,q-\ell,\ell} = K_q$ $\forall$ j, ℓ, and q.

This implies that VI-1) becomes

$$x_q = B_q + c_2 \sum_{j=1}^{s} \sum_{\ell=0}^{q-1} Y_{j,q-\ell-1,\ell} \qquad \forall\ q$$

and that the cost constraint set of VI-3c) now becomes

$$(c_1 + c_2 + K_q)\Big[\sum_{j=1}^{s} Y_{j,0,q} + \sum_{j=1}^{s} \sum_{\ell=0}^{q-1} (Y_{j,q-\ell,\ell} - Y_{j,q-\ell-1,\ell})\Big] \leq x_q \qquad \forall\ q$$

Again, all other relationships in the model remain unchanged. While we have not yet eliminated the dynamic relationship between stages, the process which established the computational algorithm as one yielding the optimal solution such as in Case i, we can form conditions which will accomplish this end.

Since $\sum_{j=1}^{s} Y_{j,0,q} + \sum_{j=1}^{s} \sum_{\ell=0}^{q-1} (Y_{j,q-\ell,\ell} - Y_{j,q-\ell-1,\ell})$ is a series of additions and subtractions of zero-one variables, it must be an integer. Thus, with $c_1 + c_2 + K_q > 0$, we can re-write the cost constraints as

$$\sum_{j=1}^{s} Y_{j,0,0} + \sum_{j=1}^{s}\sum_{t=0}^{q-1} (Y_{j,q-t,t} - Y_{j,q-t-1})$$

$$s < x_q/c_1 + c_2 + K_q >$$

$$= < \frac{B_q}{c_1 + c_2 + K_q} - \frac{c_2 \sum_{j=1}^{s}\sum_{t=0}^{q-1} Y_{j,q-t-1,t}}{c_1 + c_2 + K_q} >$$

where $< x >$ = the largest integer $\leq x$.

In the special case where $< x_q/(c_1 + c_2 + K_q) > = < B_q/(c_1 + c_2 + K_q) >$ $\forall$ q, we can eliminate the dynamic relationship between cost constraints. To obtain this phenomenon, we note that $< a-b > = < a >$ implies that $< a > \leq a-b$ for nonnegative a and b. Thus, we have

$$< B_q/(c_1 + c_2 + K_q) > \leq b_q/(c_1 + c_2 + K_q) - c_2 \sum_{j=1}^{s}\sum_{t=0}^{q-1} Y_{j,q-t-1,t}/(c_1 + c_2 + K_q)$$

or,

$$\sum_{j=1}^{s}\sum_{t=0}^{q-1} Y_{j,q-t-1,t} \leq B_q/(c_1 + c_2 + K_q) - (c_1 + c_2 + K_q) < B_q/(c_1 + c_2 + K_q) > /c_2$$

for all q. Note that this occurs by maintaining a large enough budget to account and compensate for any and all past acquisitions in terms of current storage and circulation costs. Of course, we still have the feasibility requirements that

$$x_{q+t} + I_{q+t} - c_2 \sum_{j=1}^{s}\sum_{t=0}^{q+t-1} Y_{j,q+t-t-1,t} \geq 0 \quad \text{for all } t=1,\ldots,r-q$$

which implies that

$$\sum_{j=1}^{s} \sum_{\ell=0}^{q+t-1} Y_{j,q+t-\ell-1,\ell} \leq B_{q+t} / c_2 > \qquad \text{for all } t=1,\ldots,r-q.$$

Thus, our special case implies that the computational algorithm has yielded an optimal solution to the journal selection model when all the conditions have been met.

Case iii: $\lambda_{j,q-\ell,\ell} = \lambda_q$ and $K_{j,q-\ell,\ell} = K_q$ for all j, ℓ, and q.

In a situation parallel to that of Case ii, we have

$$x_q = B_q - (c_2 + c_3\lambda_q) \sum_{j=1}^{s} \sum_{\ell=0}^{q-1} Y_{j,q-\ell-1,\ell}$$

and

$$(c_1 + c_2 + c_3\lambda_q + K_q) \left[\sum_{j=1}^{s} Y_{j,0,q} + \sum_{j=1}^{s} \sum_{\ell=0}^{q-1} (Y_{j,q-\ell,\ell} - Y_{j,q-\ell-1,\ell}) \right]$$

$$\leq x_q$$

for all q with all other relationships in the journal selection model remaining constant. Note that the objective function is now to maximize $\sum_{q=0}^{r} \lambda_q \sum_{j=1}^{s} \sum_{\ell=0}^{q} Y_{j,q-\ell,\ell}$. As before, since

$$\sum_{j=1}^{s} Y_{j,0,q} + \sum_{j=1}^{s} \sum_{\ell=0}^{q-1} (Y_{j,q-\ell,\ell} - Y_{j,q-\ell-1,\ell})$$

is an integer and $c_1 + c_2 + c_3\lambda_q + K_q$ is positive, we have

$$\sum_{j=1}^{s} Y_{j,o,q} + \sum_{j=1}^{s}\sum_{t=o}^{q-1} (Y_{j,q-t,t} - Y_{j,q-t-1,t})$$

$$\leq \langle x_q/c_1 + c_2 + c_3\rangle_q + \Sigma_q >$$

$$= \left\langle \frac{\left[B_q - (c_2 + c_3\lambda_q) \sum_{j=1}^{s}\sum_{t=o}^{q-1} Y_{j,q-t-1,t} \right]}{(c_1 + c_2 + c_3\lambda_q + K_q)} \right\rangle$$

As before, the conditions for global optimality are

$$\langle x_q/(c_1 + c_2 + c_3\lambda_q + K_q)\rangle = \langle B_q/(c_1 + c_2 + c_3\lambda_q + K_q)\rangle$$

or

$$\sum_{j=1}^{s}\sum_{t=o}^{q-1} Y_{j,q-t-1,t} \leq B_q/(c_2 + c_3\lambda_q)$$

$$= \frac{(c_1 + c_2 + c_3\lambda_o + K_q)}{(c_2 + c_3\lambda_q)} \langle B_q/(c_1 + c_2 + c_3\lambda_q + K_q)\rangle$$

Feasibility further requires that

$$x_{q+t} = B_{q+t} - (c_2 + c_3\lambda_{q+t}) \sum_{j=1}^{s}\sum_{t=o}^{q+t-1} Y_{j,q+t-t-1,t} \geq 0$$

or

$$\sum_{j=1}^{s}\sum_{t=o}^{q+t-1} Y_{j,q+t-t-1,t} \leq \langle B_{q+t}/(c_2 + c_3\lambda_{q+t})\rangle$$

for all $t=1,\ldots,r-q$.

This completes the proof.

As discussed in Chapter V, the cost coefficients must represent the actual amount of additional expenditure and effort required by the selection of a new journal. With journals being selected in an environment such as the Purdue University Libraries, as described in Chapter I, it seems logical to let c_2 and c_3 be zero as in Case i. That is to say, in an environment where circulation and storage of library materials are considered and budgeted separately relative to the acquisitioning functions, the relevant costs for selection decisions are subscription and ordering costs only. In fact, in the Purdue University Libraries, one could also let $c_1 = 0$ since only the actual subscription costs are used in making journal selection decisions, with ordering costs handled separately.

Case ii also has some very interesting implications for the librarian. First of all we see that the additional restrictions on the number of items bought previous to period q and on the total cumulative number of acquisition made as of all future periods in terms of the budgets for period q and future budgets, respectively, leads to an optimal solution via the computational algorithm. It seems quite reasonable to find situations where one would desire restrictions on the number of items selected. This idea seems reinforced when assuming that certain costs are zero in terms of selection decisions; for certain functions such as ordering, storing, or circulating journals the library may have enough resources to handle only a fixed amount of material. If one does not exceed the limits on selections, no additional costs are incurred by new selections in terms of these functions.

Secondly, if one can increase the budget enough in each period so that these additional requirements are satisfied in all periods, we can guarantee an optimal solution. Note that since

$$\sum_{j=1}^{s} \sum_{t=0}^{q-1} Y_{j,q-t-1,t}$$

and

$$\sum_{j=1}^{s} \sum_{t=0}^{q+t-1} Y_{j,q+t-t-1,t}$$

are increasing functions of q, B_q must also be an increasing function of q to satisfy all conditions. One difficulty with this case is that the assumption that $K_{j,q-t,t} = K_q$, which implies that in period q all items have a constant subscription cost regardless of age or of which journal or specific volume is being selected. Another problem is that there is no a priori way to determine a sufficient increase in the values of the budgets over time.

The same comments made in Case ii regarding the restrictions on the number of items selected can be made for Case iii. The solution obtained from the computational algorithm is optimal if the conditions are met. The implications of budgets needing to be increased enough again indicates the relevance of this result to people who are interested in understanding the consequences of setting budgets. One may now be able to define how much money is adequate to maintain a certain collection and to be able to update a library by acquiring recent volumes of ongoing journals so as to retain continuity.

It is not unreasonable to consider a new library as being in such a situation as indicated by these latter cases. With circulation, acquisition, and storage departments not already established, each new journal may mean incremental costs. Further, one may be able to purchase several items at a relatively constant subscription price from dealers or from gifts and exchange departments of other libraries. Finally, with no past usage data, one may assume a relatively constant usage for several items purchased in a specific period. However, it is the assumptions of constant subscription price and usage for all items selected in a specific time period that could be the great weakness of this case in general situations.

A Lagrangian Approach

The dynamic programming formulation yields a computational algorithm that generates a feasible solution. While this solution is optimal for several special cases, some of which are very realistic in terms of the journal selection problem in a university library system, one may desire an optimal solution to the general mathematical programming problem as stated in the journal selection model. At the very least, one may require bounds on the optimal solution. To do so, we will consider a Lagrangian formulation.

By expanding the classical Lagrangian multiplier technique to cover a broader class of problems, Everett [19] has provided us with a basis with which to solve the journal selection problem. Rather than solve by differentiation of the Lagrangian function (which requires a continuous, twice-differentiable function) for the necessary and

sufficient conditions for a stationary or saddle-point solution [28]. Everett views the goal of Lagrangian analysis to be optimization against constraints.

Consider the problem

VI-15) $\underset{x \in X}{\text{Max}} \; f(x)$

$$\text{s.t.} \quad g_i(x) \leq b_i \qquad \text{for } i=1,2,\ldots,m$$

where x is an n-dimensional vector and an element of the feasible region X, which is itself a subset of Euclidean n-space. Everett selects m non-negative constants, δ_i, i=1,2,...,m, and investigates the following problem:

VI-16) $$\underset{x \in X}{\text{Max}} \; L = f(x) - \sum_{i=1}^{m} \delta_i g_i(x) \; .$$

He proves that if x^* maximizes the unconstrained Lagrangian function of VI-16) over all $x \in X$, then it maximizes f(x) over all $x \in X$ such that $g_i(x) \leq g_i(x^*)$ for all i. This means that if the optimal solution to the unconstrained Lagrangian function can be found, it is also a solution to the problem of maximizing the original objective function f(x) over all $x \in X$ subject to the newly generated constraints of the form $g_i(x) \leq g_i(x^*)$ [25].

The crux of the problem is to select proper values for the multipliers δ_i. Unfortunately the best choices are unknown; yet these values directly influence the generated constraints. If the generated constraints do not satisfy the original constraints, i.e., at least one $g_j(x^*) > b_j$, we must revise the initial estimates of δ_i and

resolve the Lagrangian function for a new x^*. Yormark [83] points out that since $g_i(x^*)$ is a monotone decreasing function of δ_i, one can decrease $g_i(x^*)$ by increasing δ_i. Everett shows this by proving that if $\delta^{(1)}$ and $\delta^{(2)}$ are two m-dimensional vectors representing the Lagrangian multipliers for two separate solutions with Lagrangian objective functions $L^{(1)}$ and $L^{(2)}$ and with solution vectors $x^{(1)}$ and $x^{(2)}$ respectively, and if the constraint levels $g_K(x^{(1)})$ and $g_K(x^{(2)})$ generated for both solutions are equal for all $i \neq k$, and if $g_K(x^1) > g_K(x^2)$, then

VI-17) $$\delta_K^{(1)} \geq (L^{(1)} - L^{(2)})/[g_K(x^{(1)}) - g_K(x^{(2)})] \geq \delta_K^{(2)} .$$

Thus $\delta_K^{(2)} \geq \delta_K^{(1)}$ but $g_K(x^{(2)}) < g_K(x^{(1)})$. However, one must use care, for the constraints are not necessarily independent; meaning that often, many of the multipliers must change simultaneously to produce better solutions.

This iterative approach guarantees an optimal solution when it converges; however, there may exist gaps in the solution space due to nonconcavities in the response surface of maximal f(x) versus generated constraints. An optimal solution lying in such a gap cannot be generated by this Lagrangian method. We note that the journal selection problem, with its linear form, is always concave and will not produce such gaps.

To provide a bound for his algorithm, Everett shows that if x' comes within ϵ of maximizing the Lagrangian function; i.e., if

VI-18) $$L' = f(x') - \sum_{i=1}^{m} \delta_i g_i(x') \geq f(x) - \sum_{i=1}^{m} \delta_i g_i(x) - \epsilon$$

for all $x \in X$,

then x' is a solution of the original problem with newly generated constraints of the form $g_i(x') \leq b_i$ for all i, and that this solution is within ϵ of the maximum for the original problem with these newly generated constraints. Thus, Everett shows that even if the recursive technique does not converge, one can find bounds for the optimal solution.

Updating the Multipliers

Brooks and Geoffrion [9] suggest a linear programming approach to solve for the needed multipliers in Everett's generalized Lagrangian optimization techniques. The authors state that one must initially find a first estimate by one means or another for the multipliers. They denote f^t as the original objective function and $g_i{}^t$ as the i^{th} constraint value associated with the optimal Lagrangian function using the t^{th} set of multipliers. Of course, b_i is the amount of resource i available. Their algorithm then proceeds to find the k^{th} set of multipliers based on the values of the prior k-1 sets and the associated values of f^t, $g_i{}^t$, and b_i for all $t=1,2,\ldots,k-1$.

Brooks and Geoffrion consider maximizing a convex combination of the objective functions generated by prior multipliers. They formulate the linear programming problem

$$\text{VI-19)} \quad \text{Max} \sum_{t=1}^{k-1} \lambda_t f^t$$

$$\text{s.t.} \quad \sum_{t=1}^{k-1} \lambda_t = 1$$

$$\sum_{t=1}^{k-1} \lambda_t g_i^t \leq b_i \qquad \forall\ i=1,\ldots,m$$

$$\lambda_t \geq 0 \qquad \forall\ t=1,\ldots,k-1 .$$

The algorithm uses the dual variables σ_i^k for $i=1,\ldots,m$ as the new multipliers for the k^{th} iteration. We see that the dual problem to the above linear program is

$$\text{VI-20)} \quad \text{Min} \quad u_o^k + \sum_{i=1}^{m} \sigma_i^k b_i$$

$$\text{s.t.} \quad u_o + \sum_{i=1}^{m} \sigma_i g_i^t \geq f^t \qquad \forall\ t=1,\ldots,k-1$$

$$u_i \geq 0 \qquad \forall\ i=1,\ldots,m$$

u_o is unrestricted in sign.

Note that this implies that for dual feasibility and primal optimality,

$$\text{VI-20a)} \quad L^{(t)} = f^{(t)} - \sum_{i=1}^{m} \sigma_i^{(k)} g_i \leq u_o^k \qquad \forall\ t=1,\ldots,k-1.$$

To see why Brooks and Geoffrion's method is valid, we shall consider the integer programming problem in a different light. Restating it in general terms, we get

VI-21) $$\text{Max } f = \sum_{j=1}^{n} c_j x_j = cx$$

$$\text{s.t. } g_i = \sum_{j=1}^{n} a_{ij} x_j \leq b_i \qquad \forall\, i=1,\ldots,m$$

$$x_j \geq 0 \quad x_j \text{ integer} \quad \forall\, j=1,\ldots,n.$$

We note that in cases such as the journal selection problem, where one has a discrete optimization problem with the decision variables allowed to take on a finite number of values, we can generate only a finite, although sometimes very large, number of alternative solutions. For example, if we have M zero-one integer decision variables, the number of alternatives, n, is equal to 2^M.

Suppose f^t = the objective function as evaluated for the t^{th} alternative, and $c_q{}^t$ = the q^{th} constraint as evaluated for the t^{th} alternative, where $t=1,2,\ldots,n$. We can now write, as an equivalent problem to VI-21) the following, based on maximizing a convex combination of all n possible solutions, subject to the fact that a convex combination of all n alternatives for constraint i must be feasible for all i.

VI-22) $$\text{Max } \sum_{t=1}^{n} \lambda_t f^t$$

$$\text{s.t. } \sum_{t=1}^{n} \lambda_t = 1$$

$$\sum_{t=1}^{n} \lambda_t c_i{}^t \leq b_i \qquad \forall\, i=1,\ldots,m$$

$$\lambda_t \geq 0 \quad \lambda_t \text{ integer} \qquad \forall\, t=1,\ldots,n.$$

We note that problem VI-22) and the problem of using a restricted basis linear programming approach to approximate problems involving separable functions [28] are quite similar. The similarities between VI-19) and VI-22) are also obvious. but there are differences that lie in the integrality of λ_t and in the fact that problem VI-22) is involved with all possible alternatives. Note that $f^t = cx^t = \sum_{j=1}^{n} c_j x_j^t$ and $g_i^t = \sum_{j=1}^{n} a_{ij} x_j^t$; where x_j^t is the j^{th} element of x^t, the t^{th} alternative solution. Since λ_t must be a nonnegative integer and $\sum_{t=1}^{n} \lambda_t = 1$, this implies that $\lambda_p = 1$ and $\lambda_t = 0 \ \forall \ t \neq p$ for some p. This means that $\sum_{t=1}^{n} \lambda_t g_i^t = g_i^p \geq b_i$, or that the p^{th} alternative must be feasible. The objective function is now $\sum_{t=1}^{n} \lambda_t f^t = f^p$. Since it is a maximizing problem, one must choose p to yield the last feasible alternative as the one that maximizes the objective function while remaining feasible, which is the goal of problem VI-21). Thus, VI-21) and VI-22) are equivalent problems.

Brooks and Geoffrion fail to mention it, but in order to use their linear programming technique, one needs at least one trial to be feasible for each constraint. We shall show this by proving the following lemma.

Lemma:

If $g_i^t > b_i \ \forall \ t$, then no convex combination of the g_i^t's ($\sum_{t=1}^{k} \lambda_t g_i^t$, where $\sum_{t=1}^{k} \lambda_t = 1$ and $\lambda_t \geq 0 \ \forall \ t=1,2,\ldots,k$) can be feasible in terms of being less than or equal to b_i.

Proof:

If $g_i^t > b_i$ and $\lambda_t > 0 \ \forall\ t$, we can say that $\lambda_t g_i^t > b_i \lambda_t$ and so $\sum_{t=1}^{k} \lambda_t g_i^t > \sum_{t=1}^{k} \lambda_t b_i = b_i \sum_{t=1}^{k} \lambda_t = b_i \cdot 1 = b_i$. We note that for $\lambda_t g_i^t$ to equal $\lambda_t b_i$, λ_t must be zero, and that λ_t cannot be zero for all t since $\sum_{t=1}^{k} \lambda_t = 1$. Thus the strict inequality $\sum_{t=1}^{k} \lambda_t g_i^t > b_i$ is seen to be proper. The method of proof was suggested by Gass [20, p. 51].

It seems that the simplest way to insure feasibility is to generate a totally feasible solution if one exists. If one does not exist, there is no need to proceed further. We shall see that the Lagrangian formulation can be used to generate just such an initial feasible solution.

Another point is that the initial linear programming problem has only one initially nonbasic variable associated with it, formed from the one known iteration which yielded f^1 and $g_q^1 \ \forall\ q$. This implies that $\delta_q^2 = 0 \ \forall\ q$, so that the next iteration will yield a solution of selecting everything available. If this is not feasible, the algorithm will use convex combinations of past solutions to try to find a feasible and optimal solution.

The Stopping Rule

Brooks and Geoffrion have developed a stopping rule to determine when an optimal solution has been obtained. Since it is computationally infeasible to try to solve problem V(-??) due to the large values of n (in the journal selection problem n could be as large as $2^{40,500}$

for 100 journals over a 30 period horizon), consider dropping the integer constraint to yield the linear programming problem

$$\text{VI-23)} \quad \text{MAX} \sum_{t=1}^{n} \lambda_t r^t$$

$$\text{s.t.} \sum_{t=1}^{n} \lambda_t = 1$$

$$\sum_{t=1}^{n} \lambda_t g_i^t \leq b_i \qquad \forall\, i=1,\ldots,m$$

$$\lambda_t \geq 0 \quad \forall\, t=1,\ldots,n\ .$$

This means that they do not optimize the original problem but consider a slightly different case. Thus, by deleting the integer constraint, their optimal solution will provide an upper bound to the optimal value of the objective function of the original problem. Note that adding an integrality constraint to VI-19) will only yield the best feasible solution generated as of iteration k.

However, even here n is too large to be handled in any reasonable fashion. Brooks and Geoffrion show that the dual problem VI-20) can be used in conjunction with Everett's technique to generate enough alternatives to optimize problem VI-23). The dual to VI-23) is

$$\text{VI-24)} \quad \text{MIN} \quad u_o + \sum_{i=1}^{m} \delta_i b_i$$

$$u_o + \sum_{i=1}^{m} \delta_i g_i^t \geq r^t \qquad \forall\, t=1,\ldots,n$$

$$\delta_i \geq 0 \quad \forall\, i=1,\ldots,m\ .$$

Thus, the stopping rule forces us to terminate the process at the k^{th} step, which optimizes VI-23), where

VI-24a) $L^t = f^t - \sum_{i=1}^{m} v_i^k r_i^t \leq u_o^k \qquad \forall\ t=1,\ldots,k-1$

and

VI-24b) $L^k = f^k - \sum_{i=1}^{m} v_i^k r_i^t \geq u_o^k$.

The algorithm actually optimizes problem VI-23) using the Lagrangian formulation of the journal selection problem. The value of the objective function of problem VI-23), at its maximal point is the upper bound to the optimal value of the objective function of the journal selection model.

If the final solution yields one $\lambda_p = 1$, so $\lambda_t = 0 \quad \forall\ t \neq p$, the p^{th} alternative, which must now be feasible, is also optimal, not only for problem VI-22), but for the journal selection problem itself. This occurs because all requirements needed for problem VI-22) to be optimal have been met in this particular case.

Note that problem VI-19) is a special subset, more or less, of problem VI-23) in that the former generates only k alternatives of the possible n existing ones but optimizes both problems. Note also that problem VI-23) must eventually converge since we have a finite linear program. Further, we can have only m+1 basic, positive variables in the latter linear programming solution and some of these may be good approximations to the solution of the original journal selection

problem. The authors compare VI-23) to a mixed strategy generalization of the original problem with λ_p equal to the probability of choosing alternative p. They also note one need not store each alternative but only each f^p and g_i^p if one is attempting to computerize the algorithm.

The comparison of Brooks and Geoffrion's dual problem of VI-24) to that of Balas' [5] dual to the general integer programming problem (see Appendix B) reveals that the variable u_o is equivalent to the Balas' expression -vx. Balas states the primal problem as

$$\underset{x,y}{\text{Max}} \; [f = cx \mid Ax+y = g+y \leq b,\ y \geq 0,\ x \geq 0,\ x \text{ integer}]$$

and shows that the dual problem is

$$\underset{x}{\text{Max}}\ \underset{\delta}{\text{Min}} \; [\delta b - vx \mid \delta A - v = c,\ x \geq 0,\ \delta \geq 0,\ x \text{ integer},\ v \text{ unconstrained}].$$

Assuming primal feasibility ($g = Ax \geq b$ or $g+y = b$, $x \geq 0$, $y \geq 0$, x integer), which is the only case of interest, Brooks and Geoffrion show that

$$L^t = f^t - \delta g^t = cx^t - \delta g^t \leq u_o \qquad \forall\ t=1,\ldots,n;\ \delta \geq 0$$

as the conditions for dual feasibility. This implies that

$$L'^t = f^t + \delta(b-g^t) = f^t + \delta y^t \leq u_o + \delta b.$$

The authors also claim that at optimality,

$$L^t = f^t - \delta^k g^t \leq u_o^k \qquad \forall\ t=1,\ldots,n;$$

$$\delta^k \geq 0;$$

and that

$$L^k = f^k - \delta^k g^k = u_o^k \quad \text{and} \quad \delta^k y^k = 0.$$

This means that $L'^k = f^k + \delta(b-g^k) = u_o^k + \delta^k b = f^k + \delta^k y^k = f^k$.

Balas achieves a similar result with his dual feasibility consideration

$$\delta A - v = c, \quad \delta \geq 0 \quad (x \geq 0, \ x \text{ integer}).$$

This implies that

$$\delta Ax - vx = \delta g - vx = cx = f.$$

If we are primal feasible

$$\delta b \geq \delta g = \delta Ax$$

which implies that

$$\delta b - vx \geq \delta Ax - vx = f.$$

If one is at an optimal point, where primal and dual feasibility coexist,

$$L^k = f^k \ \delta^k g^k = -v^k x^k.$$

The final point made by Brooks and Geoffrion is that we can extend Everett's initial complementary slackness conditions to those of Balas', that is

$$\delta_i(b_i - g_i) = 0 \qquad \forall \ i=1,\ldots,r$$

at optimum. However, since the problem they solve is not the exact original problem, it is not necessary to have to impose these conditions.

The reason this does not bring about extra problems is that Everett's technique centers about the primal optimization problem. Thus, the concern is not for finding the optimal dual variables but only in finding those variables that will be suitable Lagrangian multipliers. As pointed out before, by establishing g_q^k as a generated right-hand-side value of the constraint $g_i \leq g_i^k$, Everett's technique does allow for complementary slackness in terms of $\delta_i^k(g_i^k - g_i) = 0$. Since $g_i = g_i^k$ for the k^{th} iteration, δ_i^k can be any positive number.

The Lagrangian Model

The Lagrangian function for the journal selection model can be formulated as follows:

VI-25) Let $$L = \sum_{j=1}^{s}\sum_{q=0}^{r}\sum_{t=0}^{q} Y_{j,q-t,t}\,\lambda_{j,q-t,t}$$

$$- \sum_{q=1}^{r} \delta_q \sum_{j=1}^{s}\sum_{t=0}^{q} Y_{j,q-t,t}(c_1 + c_2 + c_3\lambda_{j,q-t,t}$$

$$+ K_{j,q-t,t}) + \sum_{q=1}^{r} \delta_q \sum_{j=1}^{s}\sum_{t=0}^{q-1} Y_{j,q-t-1}(c_1 + K_{j,q-t,t})$$

$$= \sum_{j=1}^{s}\sum_{q=0}^{r}\sum_{t=0}^{q} Y_{j,q-t,t}\,\lambda'_{j,q-t,t} \,,$$

where

$$\text{VI-26)}\quad \lambda'_{j,q-\ell,\ell} = \begin{cases} \lambda_{j,t-\ell,\ell} - \delta_r(c_1+c_2+c_3\lambda_{j,r-\ell,\ell}+K_{j,r-\ell,\ell}) \\ \qquad \forall\ j=1,\ldots,S;\ \ q=r;\ \ \forall\ \ell=0,1,\ldots,q \\ \\ \lambda_{j,q-\ell,\ell} - \delta_q(c_1+c_2+c_3\lambda_{j,q-\ell,\ell}+K_{j,q-\ell,\ell}) \\ \qquad + \delta_{q+1}(c_1+K_{j,q-\ell+1,\ell}) \\ \qquad \forall\ j=1,\ldots,S;\ \ \forall\ q=1,\ldots,r-1;\ \ \forall\ \ell=0,1,\ldots,q \\ \\ \lambda_{j,o,o}+\delta_1(c_1+K_{j,1,o}) \quad \forall\ j=1,\ldots,S;\ \ q-\ell=0 \end{cases}$$

Of course, the Lagrangian function is to be maximized subject to the constraints that

$$Y_{j,q-\ell,\ell} = 0 \text{ or } 1 \qquad \forall\ j=1,\ldots,S;\ q=0,1,\ldots,r;\ \ell=0,1,\ldots,q$$

VI-26a) and

$$Y_{j,q-\ell,\ell} \geq Y_{j,q-\ell-1,\ell} \qquad \forall\ j=1,\ldots,S;\ q=1,\ldots,r;\ \ell=0,1,\ldots,q-1.$$

Note that

$$\text{VI-26b)}\quad f = \sum_{j=1}^{S}\sum_{q=0}^{r}\sum_{\ell=0}^{q} Y_{j,q-\ell,\ell}\,\lambda_{j,q-\ell,\ell}$$

and

$$\text{VI-26c)}\quad g_q = \sum_{j=1}^{S}\sum_{\ell=0}^{q-1} Y_{j,q-\ell,\ell}(c_1+c_2+c_3\lambda_{j,q-\ell,\ell}+K_{j,q-\ell,\ell}) - \sum_{j=1}^{S}\sum_{\ell=0}^{q-1} Y_{j,q-\ell-1,\ell}(c_1+K_{j,q-\ell,\ell}) \quad \forall\ q.$$

In order to solve our Lagrangian problem, we must prove the following lemma:

Lemma:

If we have the optimization problem

$$\text{VI-27)} \quad \text{Max } z = \sum_{j=1}^{s} \sum_{t=o}^{r} Y_{j,t} \lambda'_{j,t}$$

$$\text{s.t. } Y_{j,t} \geq Y_{j,t-1} \qquad \forall\ j=1,\ldots,S;\ \forall\ t=1,\ldots,r$$

$$Y_{j,t} = 0 \text{ or } 1 \qquad \forall\ j=1,\ldots,S;\ \forall\ t=1,\ldots,r,$$

the optimal decision rule is

$$\text{VI-28)} \quad Y_{j,t} = \begin{cases} 1 & \text{if } Y_{j,t-1} = 1 \\ 1 & \text{if } Y_{j,t-1} = 0 \text{ but } \sum_{\ell=t}^{k} \lambda'_{j,\ell} \geq 0 \quad \forall\ k=t,t+1,\ldots,r \\ 0 & \text{otherwise} \end{cases}$$

for all j and t.

The problem is solved iteratively, starting with t=1 and continuing to increase t until t=r is reached. $Y_{j,o}$ is a given parameter of the problem, not a decision variable.

Proof:

Since $Y_{j,t}$ is a zero-one binary valued variable and is constrained sequentially by the constraint $Y_{j,t} \geq Y_{j,t-1}$ $\forall$ t, one really has a scheduling problem of when to begin to take the action represented by

$Y_{j,t}$ being equal to one. This is a parallel to the scheduling model format discussed in Chapter II and III of this thesis. Once begun, $Y_{j,t}$ remains unity for all t larger than the initial stage when it is set equal to one.

Since there are no constraints of any kind on the number of decision variables allowed to be positive, we can decompose the original problem into s subproblems of the form

$$\text{Max} \quad z_j = \sum_{t=o}^{r} \lambda^j_{j,t} Y_{j,t}$$

$$\text{s.t.} \quad Y_{j,t} \geq Y_{j,t-1} \quad \forall\, t=1,\ldots,r$$

$$Y_{j,t} = 0 \text{ or } 1 \quad \forall\, t=1,\ldots,r .$$

Note that z, the original objective function, is equal to $\sum_{j=1}^{s} z_j$.

Let p_j = the point at which $Y_{j,t}$ becomes unity; i.e., $Y_{j,t} = 1$ if and only if $t \geq p_j$. Note that p_j is a nonnegative integer in the range [0,r+1]. Thus $z_j(p_j) = \sum_{t=o}^{r} \lambda_{j,t} \lambda^j_{j,t} = \sum_{t=p_j}^{r} \lambda^j_{j,t}$. If no $Y_{j,t}$ is greater than zero, p_j is set equal to r+1. Let $K_j + 1$ represent another possible decision point, where $K_j = p_j, p_j+1, \ldots, r$.

Then, $z_j(K_j+1) = \sum_{t=K_j+1}^{r} \lambda^j_{j,t} = \sum_{t=p_j}^{K_j} \lambda^j_{j,t} + \sum_{t=K_j+1}^{r} \lambda^j_{j,t} - \sum_{t=p_j}^{K_j} \lambda^j_{j,t}$

$= \sum_{t=p_j}^{r} \lambda^j_{j,t} - \sum_{t=p_j}^{K_j} \lambda^j_{j,t} = z_j(p_j) - \sum_{t=p_j}^{K_j} \lambda^j_{j,t}$. If p_j is the optimal

decision point, $z_j(p_j) \geq z_j(K_j+1)$ or $\sum_{t=p_j}^{K_j} \lambda^j_{j,t} \geq 0$

$\forall\ K_j = p_j, p_j+1, \ldots, r$.

If $Y_{j,t-1} = 1$, $Y_{j,t}$ must be equal to one by the constraint VI-29a).

If $Y_{j,t-1} = 0$ but $\sum_{\ell=t}^{K_j} \lambda^j_{j,\ell} \geq 0$ $\forall\ K_j = t, t+1, \ldots, r$, set $Y_{j,t} = 1$.

Otherwise, set $Y_{j,t} = 0$.

Note that we solve the problem iteratively, starting with t=1 and increasing t until t=r. This insures feasibility as well as optimality, and the proof is complete.

This lemma implies that the Lagrangian formulation can be optimized for a specific set of δ_q values by calculating $\lambda^j_{j,q-\ell,\ell}$ for all j, q, and ℓ via VI-26) and by using the following decision rule iteratively

VI-29) $$Y_{j,q-\ell,\ell} = \begin{cases} 1 & \text{if } Y_{j,q-\ell-1,\ell} = 1 \\ 1 & \text{if } Y_{j,q-\ell-1,\ell} = 0 \text{ but } \sum_{t=0}^{K} \lambda^j_{j,q-\ell+t,\ell} \geq 0 \\ & \qquad \forall\ K = 0, 1, \ldots, r-q \\ 0 & \text{otherwise.} \end{cases}$$

An Upper Bound Algorithm

Using the Lagrangian formulation of the journal selection model, we now have a device for obtaining a good upper bound for the optimal objective function. Everett's generalized Lagrangian method, incorporating Brooks and Geoffrion's technique to up-date the multipliers via linear

programming and their stopping rule to detect the optimal solution to their problem, can be applied here. The complete algorithm, illustrated in Figure 9, is described in detail below. A computerized version of this algorithm is presented in Appendix C.

1) The journal selection problem must be set up as outlined in Chapter III. This means specifying s, r, c_1, c_2, c_3, B_q $\forall$ q, and $K_{j,q-\ell,\ell}$ and $\lambda_{j,q-\ell,\ell}$ $\forall$ j, q, and ℓ. One must also specify $Y_{j,o,o}$ $\forall$ j. The check for feasibility, $B_q \geq \sum_{j=1}^{s} Y_{j,o,o}(c_2+c_3\lambda_{j,q,o})$ $\forall$ q must be carried out. If a feasible solution exists, proceed to step 2 to find such a solution. If no feasible solution exists, the algorithm terminates here.

2) Set switch IDON=0 if we first desire an initial solution. Set IDON=1 is we are seeking an optimal solution and already have an initial solution.

3) Set k=1. If IDON=1, one must specify δ_q $\forall$ q. Set $\delta_q = 0$ $\forall$ q if IDON=0.

4) Calculate $\lambda'_{j,q-\ell,\ell}$ $\forall$ j, q, ℓ according to VI-26).

5) Use the optimization procedure of VI-29) to iteratively find $Y_{j,q-\ell,\ell}$ $\forall$ j, q, and ℓ.

6) Calculate $g_q^k = \sum_{j=1}^{s} \sum_{\ell=0}^{q} Y_{j,q-\ell,\ell}^k \, (c_1 + c_2 + c_3 \lambda_{j,q-\ell,\ell} + K_{j,q-\ell,\ell})$

$$- \sum_{j=1}^{s} \sum_{\ell=0}^{q-1} Y_{j,q-\ell-1,\ell}^k \, (c_1 + K_{j,q-\ell,\ell}) \quad \forall\, q$$

and $f^k = \sum_{j=1}^{s} \sum_{q=0}^{r} \sum_{\ell=0}^{q} Y_{j,q-\ell,\ell}^k \, \lambda_{j,q-\ell,\ell}^{\prime}$.

7) If IDON=1, go to step 10.

8) If $g_q > B_q$ for any q, set $\delta_q = \delta_q + 0.001$ (or some other desired value). Set k=k+1 and go to step 4.

9) Here $g_q \leq B_q \quad \forall\, q$, so we have a feasible solution. Set IDON=1 and go to step 3.

10) If $g_q > B_q$ for any q, we have an infeasible solution. Go to step 13.

11) If k=1, go to step 13.

12) If $f^t = \sum_{q=1}^{r} \delta_q^k \, g_q^t \leq u_o^k \quad \forall\, t=1,\ldots,k-1$, and

$f^k = \sum_{q=1}^{r} \delta_q^k \, g_q^k = u_o^k$ we have an upper bound answer, so we calculate the upper bound $= \sum_{t=1}^{k} \lambda_t f^t$ and terminate the algorithm. If not, continue to step 13.

13) Find the new multipliers, u_o^{k+1}, $s_1^{k+1},\ldots,s_r^{k+1}$ by solving for the optimal dual variables with the following linear programming problem:

$$\text{Max} \quad \sum_{t=1}^{k} d_t r^t$$

$$\text{s.t.} \quad \sum_{t=1}^{k} d_t = 1, \quad \sum_{t=1}^{k} d_t s_i^t \leq B_i \quad \forall\, i=1,\ldots,r, \quad d_t \geq 0 \quad \forall\, t=1,\ldots,k.$$

Set k=k+1 and go to step 4.

The algorithm starts by specifying initial values to all parameters and checking for the existence of a feasible solution (step 1). We note that the algorithm has the capability of finding a "reasonable" initial feasible solution (steps 2, 3, 4, 5, 6, 7, 9). One may, however, use any feasible solution that is at hand, such as the one investigated by step 1 of buying no new journals or the initial feasible solution found by the use of the dynamic programming computational algorithm (see Figure 8).

The use of the Lagrangian formulation and subsequent optimization procedure is illustrated in steps 5, 6, and 7. The use of the linear programming technique to up-date the multipliers is illustrated in step 13, while the stopping rule is used to advantage in step 12. Note that the upper bound, the optimal objective function of VI-23) is also calculated in step 12.

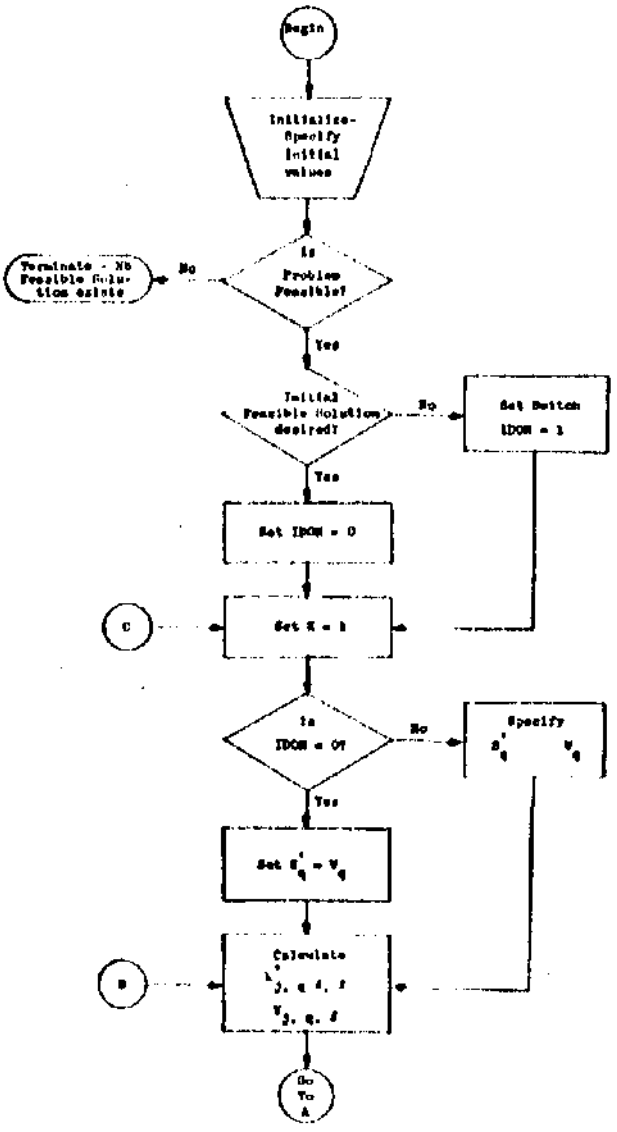
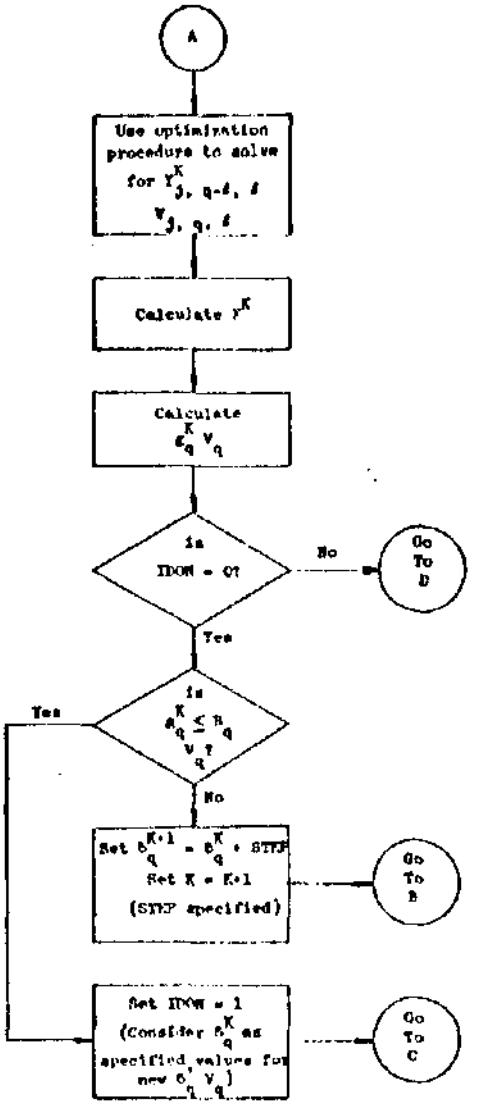


Figure 9. The Upper-Bound Algorithm



5.

6.

7.

8.

9.

Figure 9. (Continued)

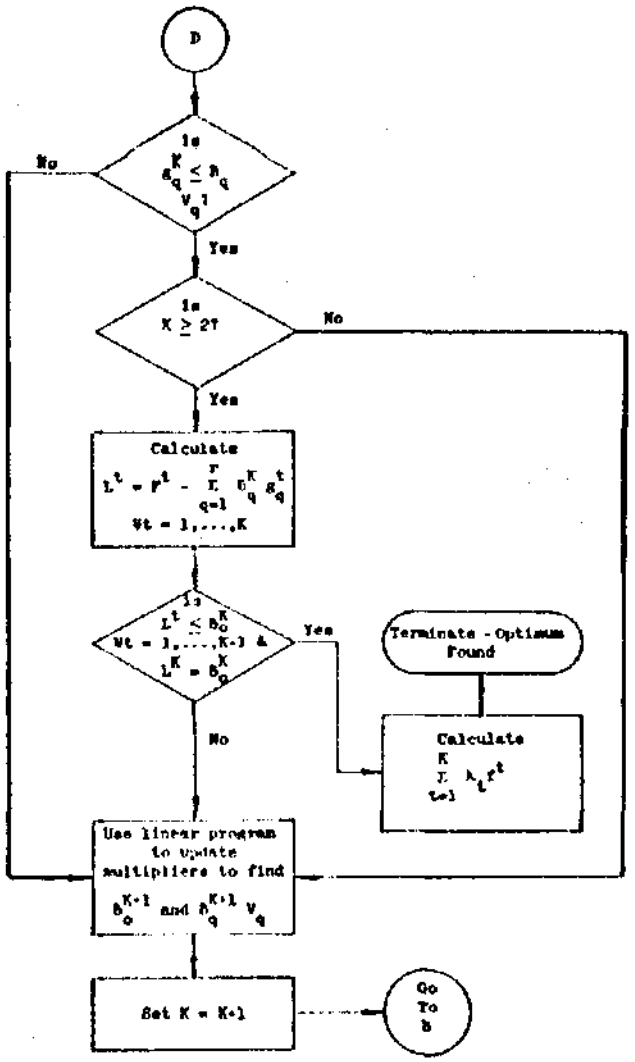


Figure 9. (Continued)

With an upper bound generated, we can now select the best feasible solution available and see how near it is to the bound. Or we can try to improve the solution by methods discussed below.

Final Improvements

While a set of feasible solutions and an upper bound on the optimal value of the objective function have been generated, we may be able to improve the solutions already generated via a knapsack approach, to yield a better feasible solution. Brooks and Geoffrion note that the set of solutions involved in the optimal basis of their linear programming problem are reasonable approximations of the optimal solution to the original journal selection problem. We shall consider these solutions as well as several other solutions generated by the Lagrangian algorithm as candidates for improvement. The object is to generate the best possible feasible solution.

We shall improve a specific solution by treating it as a knapsack problem. These are similar to the sub-problems formed by the decomposition of the overall journal selection model using dynamic programming. Any solutions having infeasible budgets in certain periods will be driven to feasibility by reducing the number of items selected for acquisition in that or former periods. Of course the items with least expected usage per unit cost should be the ones deleted from this set of selected journal issues. After feasibility has been obtained, we can now add desirable but unselected items to any solution set of journals with large enough slacks in its budgets.

The computational algorithm is illustrated in Figure 10 and discussed below. The similarities of this procedure and the dynamic programming algorithm of Figure 8 above will be obvious since both do employ the knapsack problem approach. However, there are differences that will arise since this improvement technique becomes involved with more than one time period when additions and deletions are made from the list of selected items.

1) Specify for each of the L solutions to be considered the selection policy $Y^{t}_{j,q-\ell,\ell}$ for all $j,q-\ell,\ell$. Here the superscript t represents the t^{th} solution, $t=1,\ldots,L$.

2) Calculate $f^{t} = \sum_{j=1}^{s} \sum_{q=0}^{r} \sum_{\ell=0}^{q} Y^{t}_{j,q-\ell,\ell} \lambda_{j,q-\ell,\ell}$ and

$$g_q^{t} = \sum_{j=1}^{s} \sum_{\ell=0}^{q} Y^{t}_{j,q-\ell,\ell} \left(c_1 + c_2 + c_3 \lambda_{j,q-\ell,\ell} + K_{j,q-\ell,\ell}\right) - \sum_{j=1}^{s} \sum_{\ell=0}^{q-1} Y_{j,q-\ell-1,\ell} \left(c_1 + K_{j,q-\ell,\ell}\right)$$

for all q and for all t.

3) Set t=1.

4) Set q=1.

5) If $g_q^{t} \leq B_q$, go to step 12.

6) Here we have an infeasible solution. Calculate the ratios

$$R^{t}_{j,p-\ell,\ell} = \begin{cases} Y^{t}_{j,q-\ell,\ell}\,(1-Y^{t}_{j,q-\ell-1,\ell})\lambda_{j,q-\ell,\ell} \,/ \\ \quad (c_1+c_2+c_3\lambda_{j,q-\ell,\ell}+K_{j,q-\ell,\ell}) \\ \quad \text{if } p=q, \ \forall\ \ell=0,\ p-1 \\ \\ Y^{t}_{j,o,q}\,\lambda_{j,o,q}\,/\,(c_1+c_2+c_3\lambda_{j,o,q}+K_{j,o,q}) \\ \quad \text{if } p=q=\ell \\ \\ Y^{t}_{j,q-\ell,\ell}\,Y^{t}_{j,o,\ell}\,\sum_{\gamma=\ell}^{q-1}\lambda_{j,\gamma-\ell,\ell}\,/ \\ \quad (c_2+c_3\lambda_{j,q-\ell,\ell}) \quad \ell=p \ \ \forall\ p=1,\ldots,q-1 \\ \\ Y^{t}_{j,q-\ell,\ell}\,(Y^{t}_{j,p-\ell,\ell}-Y^{t}_{j,p-\ell-1,\ell}) \\ \\ \quad \sum_{\gamma=\ell}^{q-1}\lambda_{j,\gamma-\ell,\ell}\,/\,(c_2+c_3\lambda_{j,q-\ell,\ell}) \\ \quad \forall\ p=1,\ldots,q-1 \quad \forall\ \ell=0,1,\ldots,p-1\,. \end{cases}$$

7) Rank the journals on the basis of the values of $R^{t}_{j,p-\ell,\ell}$ in ascending order for all values that are strictly positive. Since we have already established the fact that a feasible solution exists in order to even consider improvements, some $R^{t}_{j,p-\ell,\ell}$ must be positive.

8) Set the decision variable corresponding to the highest ranked item, $Y^t_{j,q-\ell,\ell}$, equal to zero. Set $Y_{j,q-\ell+b,\ell}=0$ for all b=1,...,r-q+ℓ.

9) Calculate the revised values of f^t and of g^t_{q+b} for all b=0,1,...,r-q+ℓ. Go to step 5.

10) Set q equal to the value q+1. If q ≤ r, go to step 5. Otherwise, set q=1 and go to step 11.

11) Calculate the following ratios for all j and q:

$$R^t_{j,q-\ell,\ell} = \begin{cases} (1-Y^t_{j,q-\ell-1,\ell}) \sum_{\tau=0}^{r-q} \lambda_{j,q-\ell+\tau,\ell} \Big/ \\ \qquad c_1 + c_2 + c_3\lambda_{j,q-\ell,\ell} + K_{j,q-\ell,\ell} \\ \qquad\qquad \forall\ \ell=0,1,\ldots,q-1 \\ \sum_{\tau=0}^{r-q} \lambda_{j,\tau,q} \Big/ c_1 + c_2 + c_3\lambda_{j,o,q} + K_{j,o,q} \quad \ell=q \end{cases}.$$

12) Calculate $R''^t_{j,q-\ell,\ell} = R^t_{j,q-\ell,\ell}(1-Y^t_{j,q-\ell,\ell})$ ∀ j, q, and ℓ. Rank the journals on the basis of the values of $R''^t_{j,q-\ell,\ell}$ in descending order.

13) Consider the item with the highest rank with its associated decision variable $Y_{j,q-\ell,\ell}$. Calculate $x^t_{q+c} = g^t_{q+c} + c_2 + c_3\lambda_{j,q-\ell+c,\ell}$ for c=1,2,...,r-q+ℓ and $x^t_q = c_1 + c_2 + c_3\lambda_{j,q-\ell,} + K_{j,q-\ell,\ell}$.

14) If $x^{t}_{q+c} \leq B_{q+c}$ for all c=0,1,...,r-q+ℓ set $Y^{t}_{j,q-\ell+c,\ell} = 1$ for all c=0,1,...,r-q+ℓ.

15) Now consider the next journal down the list of ranked items and go to step 13. If no more items exist, go on to step 16.

16) Set q equal to the value of q+1. If q ≤ r, go to step 11; otherwise, go on to step 17.

17) Set t equal to the value of t+1. If t ≤ L, go to step 4. Otherwise, stop.

Note that steps 1 and 2 are the initial input where the data previously generated as the L candidate solutions are specified. In steps 4 through 9, we seek to generate feasible solutions by deleting some of the items selected for acquisition so that all budget constraints will no longer be violated. In step 6 we calculate ratios of the total usage of a specific item up to period q, where the budget for period q has been violated, to the cost in period q to acquire and/or store and circulate that item. The items with lowest positive ratios will be deleted from the list of selections (steps 7 and 8). No item selected prior to period q will be deleted in period q since such a policy would violate the constraint that once an item is acquired, it remains within the library system.

The assignment of new items to the list is achieved in steps 10 through 16. Here we consider the ratio of all future usage as of period q to the cost in period q, where period q has some slack in

its budget (steps 11 and 12). Of course, the selection of a new item cannot violate the cost constraints of period q or of any future budget (steps 13, 14, and 15).

The set of improved feasible solutions, consisting of the r+1 solutions corresponding to the optimal bases of the linear programming problem, other feasible solutions generated by the Lagrangian algorithm, the feasible solution generated by the dynamic programming algorithm, and any other solutions generated by the Lagrangian algorithm one might desire to improve, now yields the desired near-optimal solution as the one among the set with the largest value for its objective function. The true optimum is then bounded from below by this value, and bounded from above by the upper bound generated by the Lagrangian algorithm.

An Example Problem

In order to demonstrate the algorithms and to illustrate how they work, an example is presented below. The size of the problem will be kept small to maintain an understanding of the computations.

Suppose we consider the purchasing of a few journals to add breadth and depth to an ongoing collection. This means that past usage data is easily accessible, costs and facilities are fixed and relatively stable, and the decision in question is the selection of some subset of a small number of available journals over a small time horizon. To be specific, let us consider the purchase of s=4 journals related to some field of scientific endeavor, such as operations

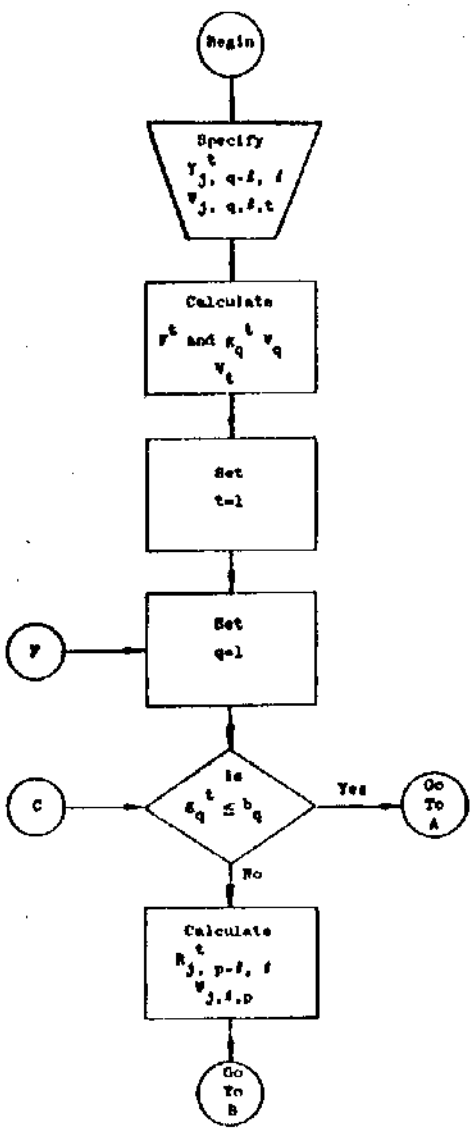


Figure 10. The Final Improvement Algorithm

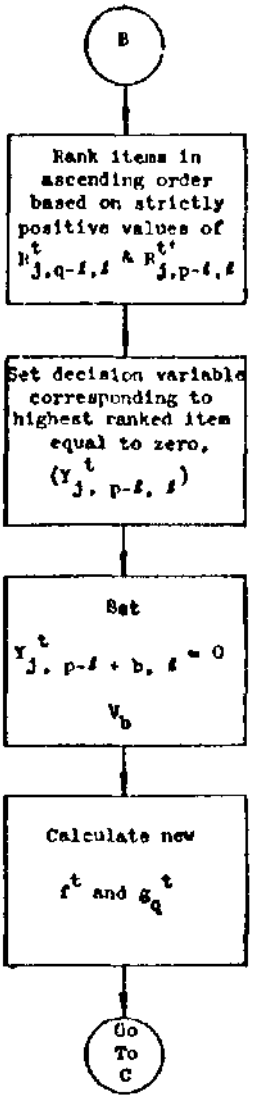
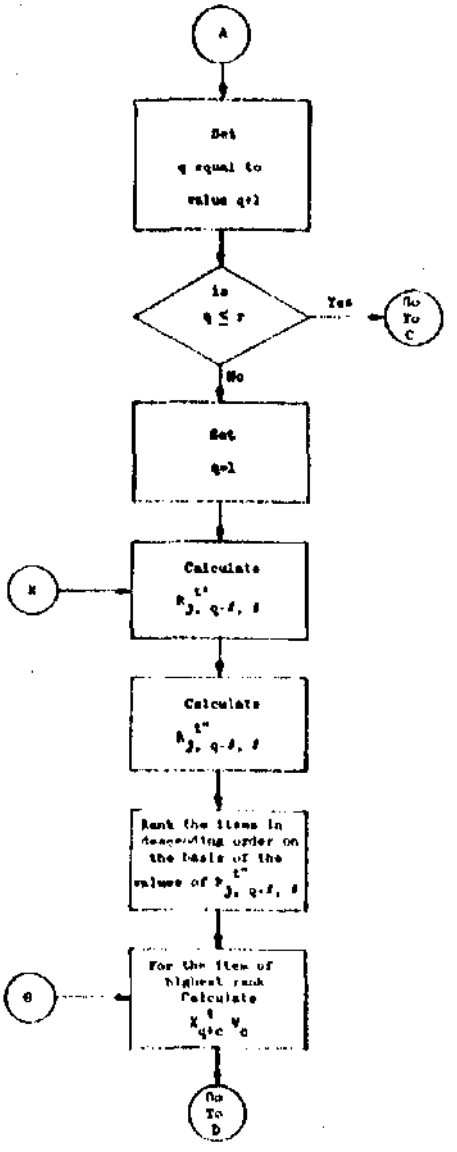


Figure 10. (Continued)



10.

11.

12.

13.

Figure 10. (Continued)

14.

15.

16.

17.

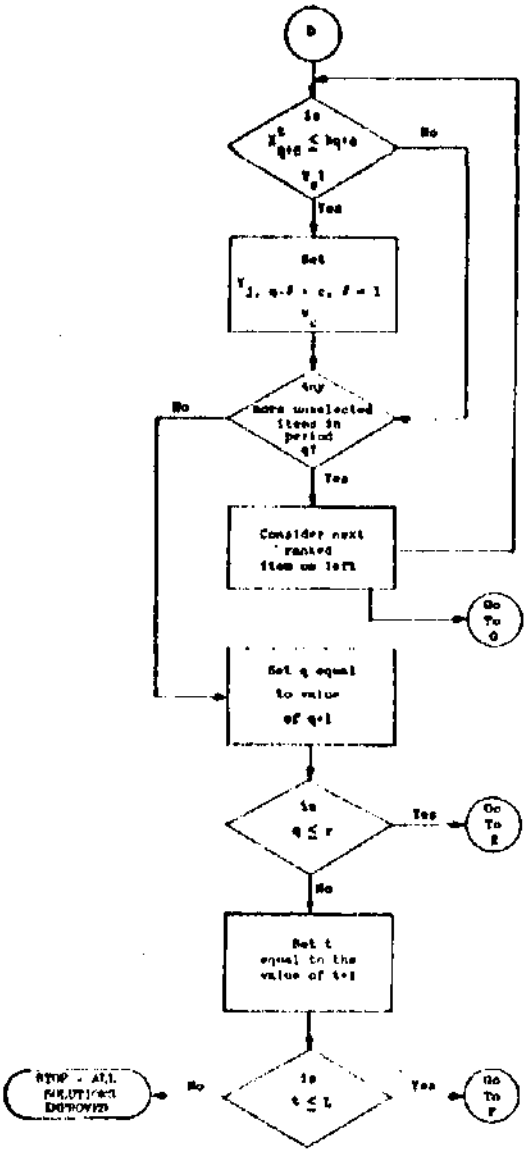


Figure 10. (Continued)

research, over a r=5 period planning horizon. We shall assume that we are interested in a yearly subscription cycle for each year so that each time period represents a one-year period.

The initial input data are presented in Table 4 below.

Table 4. Initial Values and Parameters for the Example Problem

Usage Parameters

a	b	σ
0.60	0.50	0.95

Costs

c_1	c_2	c_3
19.80	0.194	1.48

Budgets

B_q	q=	1	2	3	4	5
		115	125	130	140	150

Initial Usage

$A_{j,o,q}$	q =	0	1	2	3	4	5
j = 1		1.50	2.00	2.00	2.50	2.50	3.00
2		3.00	4.00	5.00	6.00	7.50	8.00
3		1.00	1.25	1.50	1.75	2.00	2.25
4		2.00	2.00	2.00	2.00	2.00	2.00

Subscription Costs

$K_{j,t,o}$	t =	0	1	2	3	4	5
j = 1		10	11	12	13	14	15
2		15	15	15	20	20	25
3		5	5	10	10	10	10
4		10	11	12	13	14	15

Initial Acquisitions

$Y_{j,o,o}$	j =	1	2	3	4
		0	1	1	0

The parameters a, b, and c are used to calculate expect usage, $\lambda_{j,q-s,s}$, according to the revised Markovian model described in equation IV-25). This model uses the initial usage estimates, $\lambda_{j,o,q}$, which were assigned to the journals randomly but as non-decreasing functions of publishing date. The cost figures for c_1, c_2, and c_3 are taken from estimates in the study by Williams, et.al. [81]. The budgets, increasing over time, were selected in order to allow for a solution that, a priori, seemed reasonable in terms of continuity of ongoing journals. The initial subscription costs, assigned randomly to the journals, were set up as increasing functions of an item's age. In this example, it was assumed that subscription costs were not a function of an item's date of publication. The final item in the initial data set is the specification of journals already being purchased as of the initial period, i.e., initial acquisitions.

It is not to be implied that this example problem is at all representative of any large general class of problems, but it will be seen that this example does illustrate the mechanics of the algorithms. It is noteworthy to mention that for even a small problem such as this, a lot of input data is required, since this can be a major roadblock to implementation.

The solutions generated by the algorithms are presented, in part, below in Tables 5 through 11. Since the Lagrangian upper bound algorithm generated 27 solutions, in addition to two from the computational algorithm of the dynamic programming formulation, one new solution from the final improvement algorithm, and one additional solution with

a "full" policy for comparison is generated, we shall only consider those solutions deemed relevant or good. The tables list each solution with its objective function, the budgets for the problem (B_q), expenditures in each time period (g_q), the Lagrangian multipliers where appropriate (δ_q), and $t_{j,q}$, the period in which the issues of journal j published in period q are acquired. If $t_{j,q} = r+1 = 6$, the item is never acquired in the five-period planning period. Note that $t_{j,q} \geq q$ by definition. Also note that each solution is feasible or infeasible only in terms of the cost constraints. The constraints on continuity, of the form $Y_{j,t,q} \leq Y_{j,t-1,q}$, are maintained through the solution procedures of the various algorithms, as are the constraints on the decision variables $Y_{j,q-\ell;\ell}$ as being zero or one in value.

The first answer, illustrated in Table 5 as solution 1, is the one formed by considering an "empty" policy of buying no new journal issues at all. The feasibility of this answer establishes the existence of at least one feasible solution to the example problem. Solution 2, also in Table 5, is the feasible solution generated by the computational algorithm of the dynamic programming formulation.

The initial feasible solution generated by the first portion of the Lagrangian upper bound algorithm is presented as solution 3, the first answer in Table 6. The next r+1=6 solutions in the table are those corresponding to the optimal basis of the linear programming problem. These solutions are numbered 4 through 9 inclusively. The optimal u_o is 40.5336, the optimal λ_t's are 0.6344, 0.0527, 0.0682, 0.1051, 0.0489, and 0.0907, in that order, and the upper bound to the objective function is 123.0564. Solutions 10 and 11 are additional

Table 5. Initial Feasible Solutions Generated by the Dynamic Programming Algorithm

Solution 1 Objective Function = 16.3095

q =	0	1	2	3	4	5
B_q	-	115.0	125.0	130.0	140.0	150.0
ε_q	-	5.0352	4.3144	3.8739	3.5775	3.3570
$t_{j,q}$						
j=1	6	6	6	6	4	5
2	0	6	6	6	4	5
3	0	6	6	6	4	5
4	6	6	6	6	6	6

Solution 2 Objective Function = 120.5742

q =	0	1	2	3	4	5
B_q	-	115.0	125.0	130.0	140.0	150.0
ε_q	-	111.8572	121.6492	126.0104	135.7799	144.9031
$t_{j,q}$						
j=1	6	1	2	3	4	5
2	0	1	2	3	4	5
3	0	6	6	3	4	5
4	6	1	2	6	6	6

feasible solutions generated by the Lagrangian algorithm, while number 12 is the answer generated by using the optimal dual variables of the linear programming problem as multipliers. These latter solutions are also part of Table 6.

Solution 13, found in Table 7 together with number 14, comes from the final improvement algorithm. Solutions 3, 5, 6, 7, 8, and 9 yielded number 13 upon improvement; while solutions 1, 4, 10, 11, and 12 yielded solution 2 upon improvement. We note that branch and bound techniques could be employed on solution 13, specifically in period q=2 to make $Y_{4,0,2}$ equal to unity and $Y_{3,0,2}$ equal to zero, with subsequent corrections in all future periods, to yield solution 2. Solution 14 is a "full" policy, consisting of buying all items as soon as possible. This policy, having the largest value of the objective function of any set of zero-one decision variables for this example, can be used as a comparison for the other solutions. The Lagrangian algorithm generated this solution as the second answer in the latter portion of the algorithm.

We see that the upper bound is 123.0564 and that the best feasible solution generated is solution number 2 which has 120.5742 for its objective function value. This means that the true optimum lies somewhere between or on one of these two extremes. Any further improvements can mean an increase of at most only (123.0546 - 120.5742)/ 120.5742 x 100% = 2.0587% in the objective function.

The "full" policy yielded a value of 133.7011 as the maximal upper limit to the objective function but it is not a valid comparison since this policy is completely infeasible. We see that the optimal

Table 6. Some Lagrangian Upper Bound Algorithm Solutions

Solution 3 Objective Function = 103.2835

q =	0	1	2	3	4	5
B_q	-	115.0	125.0	130.0	140.0	150.0
g_q	-	111.8572	82.9552	95.5388	103.1510	109.9673
δ_q	0	0.178	0.161	0.146	0.121	0.08
$t_{j,q}$						
j=1	6	1	6	3	4	5
2	0	1	2	3	4	5
3	0	6	2	6	6	6
4	6	1	6	6	6	6

Solution 4 Objective Function = 125.7109

q =	0	1	2	3	4	5
B_q	-	115.0	125.0	130.0	140.0	150.0
g_q	-	111.8572	148.8632	128.1580	137.7521	146.7476
δ_q	0	0.1847	0.1598	0.1437	0.1127	0.0623
$t_{j,q}$						
j=1	6	1	2	3	4	5
2	0	1	2	3	4	5
3	0	6	2	3	4	5
4	6	1	2	6	6	6

Table 6 (Continued)

Solution 5 Objective Function = 119.1325

q =	0	1	2	3	4	5
B_q	-	115.0	125.0	130.0	140.0	150.0
ε_q	-	111.8572	82.9552	123.1228	166.3916	118.3453
δ_q	0	0.1840	0.1660	0.1484	0.0946	0.0544
$t_{j,q}$						
j=1	6	1	6	3	4	5
2	0	1	2	3	4	5
3	0	6	2	3	4	5
4	6	1	6	6	4	5

Solution 6 Objective Function = 121.2611

q =	0	1	2	3	4	5
B_q	-	115.0	125.0	130.0	140.0	150.0
ε_q	-	138.7012	84.9178	125.0025	155.2358	177.5470
δ_q	0	0.1705	0.1640	0.1394	0.1084	0.0578
$t_{j,q}$						
j=1	6	1	6	4	4	5
2	0	1	2	4	4	5
3	0	1	2	4	4	5
4	6	1	6	6	6	5

Table 6 (Continued)

Solution 7 Objective Function = 118.4590

q =	0	1	2	3	4	5
B_q	-	115.0	125.0	130.0	140.0	150.0
ε_q	-	111.8572	82.9552	156.0768	135.9552	145.0309
δ_q	0	0.1805	0.1643	0.1355	0.1042	0.0618
$t_{j,q}$						
$j=1$	6	1	6	3	4	5
2	0	1	2	3	4	5
3	0	6	2	3	4	5
4	6	1	6	3	6	6

Solution 8 Objective Function = 122.8311

q =	0	1	2	3	4	5
B_q	-	115.0	125.0	130.0	140.0	150.0
ε_q	-	138.7012	84.9178	125.0025	168.1898	147.1106
δ_q	0	0.1787	0.1641	0.1382	0.1027	0.0631
$t_{j,q}$						
$j=1$	6	1	6	3	4	5
2	0	1	2	3	4	5
3	0	1	2	3	4	5
4	6	1	6	6	4	6

Table 6 (Continued)

Solution 9 Objective Function = 113.5625

$q =$	0	1	2	3	4	5
B_q	-	115.0	125.0	130.0	140.0	150.0
ε_q	-	111.8572	82.9552	123.1228	133.4376	142.8737
δ_q	0	0.1807	0.1641	0.1382	0.1045	0.0615
$t_{j,q}$						
j=1	6	1	6	3	4	5
2	0	1	2	3	4	5
3	0	6	2	3	4	5
4	6	1	6	6	6	6

Solution 10 Objective Function = 94.1224

$q =$	0	1	2	3	4	5
B_q	-	115.0	125.0	130.0	140.0	150.0
ε_q	-	45.9492	50.7060	116.6607	127.5915	137.4517
δ_q	0	0.2048	0.1765	0.1373	0.1172	0.0615
$t_{j,q}$						
j=1	6	6	6	3	4	5
2	0	1	1	3	4	5
3	0	6	6	3	4	5
4	6	6	6	6	6	6

Table 6 (Continued)

Solution 11 Objective Function = 108.4258

q =	0	1	2	3	4	5
B_q	-	115.0	125.0	130.0	140.0	150.0
g_q	-	111.8572	55.7412	120.9752	131.4654	141.0292
δ_q	0	0.1812	0.1666	0.1399	0.1062	0.0653
$t_{j,q}$						
j=1	6	1	6	3	4	5
2	0	1	2	3	4	5
3	0	6	6	3	4	5
4	6	1	6	6	6	6

Solution 12 Objective Function = 141.8760

q =	0	1	2	3	4	5
B_q	-	115.0	125.0	130.0	140.0	150.0
g_q	-	138.7012	150.8258	162.9917	175.0219	186.0958
δ_q	0	0.1799	0.1655	0.1367	0.1037	0.0607
$t_{j,q}$						
j=1	6	1	2	3	4	5
2	0	1	2	3	4	5
3	0	1	2	3	4	5
4	6	1	2	3	4	5

Table 7. Some Other Solutions

Solution 13 Objective Function = 119.6567

q =	0	1	2	3	4	5
B_q	-	115.0	125.0	130.0	140.0	150.0
$t_{j,q}$						
j=1	6	1	2	3	4	5
2	0	1	2	3	4	5
3	0	6	2	3	4	5
4	6	1	6	6	6	6

Solution 14 Objective Function = 153.7011

q =	0	1	2	3	4	5
B_q	-	115.0	125.0	130.0	140.0	150.0
ε_q	-	204.9664	154.9553	166.7731	178.5332	189.4297
δ_q	0	0.0	0.0	0.0	0.0	0.0
$t_{j,q}$						
j=1	1	1	2	3	4	5
2	0	1	2	3	4	5
3	0	1	2	3	4	5
4	1	1	2	3	4	5

solution, as surmised, must have a value of its objective function that lies between the value for the "empty" policy, 16.3093, and the "full" policy. However, these limits are obviously superceded by the tighter bounds discussed above.

One point is clear, that in none of the solutions presented except the last one corresponding to the "full" policy did the algorithms allow older items to be selected over new material. Thus, no $Y_{j,t,q}=1$ for $t>0$ unless $Y_{j,t-1,q}=1$ for all but the last solution. Another point is that there is some loss of continuity in the best feasible solution, number 2. The initial conditions specify that journals 2 and 3 are being acquired. In the first two time periods, we select current issues of journals 1, 2, and 4; while in the last three time periods, we select current issues of journals 1, 2, and 3. In other words, journal 3 is dropped for the first two time periods in favor of journal 4. Then, the situation reverses itself and stays so throughout the remaining periods. If continuity is desired, a librarian might examine this solution and decide to purchase only journals 1, 2, and 3 throughout the planning horizon. This revised plan is illustrated below as solution 15, the first answer in Table 8. The second solution, number 16, consists of a policy of acquiring journals 1, 2, and 4 throughout the horizon and provides a contrast to the previous answer. Note that solution 15 is feasible while solution 16 is not.

Table 8. Some Solutions Illustrating Continuity

Solution 15 Objective Function = 118.1835

q =	0	1	2	3	4	5
B_q	-	115.0	125.0	130.0	140.0	150.0
δ_q	-	105.7472	115.3543	125.3628	135.4561	144.7412
$t_{j,q}$						
j=1	6	1	2	3	4	5
2	0	1	2	3	4	5
3	0	1	2	3	4	5
4	6	6	6	6	6	6

Solution 16 Objective Function = 120.7617

q =	0	1	2	3	4	5
B_q	-	115.0	125.0	130.0	140.0	150.0
δ_q	-	111.8572	121.6492	131.3803	140.9649	149.6187
$t_{j,q}$						
j=1	6	1	2	3	4	5
2	0	1	2	3	4	5
3	0	6	6	6	6	6
4	6	1	2	3	4	5

Sensitivity Analysis

Consider changing some of the input parameters, specifically the budgets B_q and initial expected demands $\lambda_{j,0,q}$. This is done to demonstrate the fact that the algorithms can work properly under different, and perhaps more adverse, conditions than those presented in the original example. We shall use the best feasible solution generated by the Lagrangian upper bound algorithm and attempt to improve it via the final improvement algorithm. The improved solution, the upper bound on the optimal value of the objective function, and the maximal percentage improvement = (the upper bound - the best feasible value of the objective function) x 100% / (the best feasible value) are presented below in Tables 9, 10, and 11.

Solutions A, B, C, and D are presented below in Table 9 as four cases illustrating the affects of changes in the budgets. Solution A represents the condition of dropping B_2 from 125.0 to 75.0 and increasing B_4 from 140.0 to 200.0, with all other parameters remaining fixed. We note a decrease of selections in period 2 and an increase in period 4 as expected. We also note that the upper bound has been decreased from 123.0564 to 120.2918. This implies that a set of budgets increasing properly over time is better than a sudden drop of one budget to be compensated for in a later period. Solution B represents the original problem with B_3 decreased from 130.0 to 35.0. The results of changing the original problem by decreasing B_5 from 130.0 to 15.0 is presented in solution C. In both of these latter two solutions, no new items were selected in period 3. With a budget of only 15.0

Table 9. Solutions With Different Budget Parameters -- Increasing $\lambda_{j,o,q}$ versus q

Solution A Objective Function 114.8923

q =	0	1	2	3	4	5
B_q	-	115.0	75.0	130.0	200.0	150.0
g_q	-	111.8472	55.7412	120.9752	197.7370	145.7040
$t_{j,q}$						
j=1	6	1	6	3	4	5
2	0	1	2	3	4	5
3	0	6	6	3	4	5
4	6	1	6	4	4	6

Upper bound = 120.2918 Maximal improvement = 3.62%

Solution B Objective Function = 99.9442

q =	0	1	2	3	4	5
B_q	-	115.0	125.0	35.0	140.0	150.0
g_q	-	111.8472	115.9092	20.0129	136.6920	137.3040
$t_{j,q}$						
j=1	6	1	2	6	4	5
2	0	1	2	4	4	5
3	0	6	2	6	6	5
4	6	1	6	6	6	6

Upper bound = 106.2531 Maximal improvement = 6.32%

Table 9 (Continued)

Solution C Objective Function = 76.9262

q =	0	1	2	3	4	5
B_q	-	115.0	125.0	15.0	140.0	150.0
s_q	-	111.8572	13.3472	11.0856	129.8240	131.6127
$t_{j,q}$						
j=1	6	1	6	6	4	5
2	0	1	6	4	4	5
3	0	6	6	6	6	6
4	6	1	6	6	6	6

Upper bound = 88.6179 Maximal improvement = 15.2%

Solution D Objective Function = 115.1777

q =	0	1	2	3	4	5
B_q	-	115.0	125.0	120.0	140.0	150.0
s_q	-	111.8572	115.9092	98.0564	133.2622	142.7459
$t_{j,q}$						
j=1	6	1	2	3	4	5
2	0	1	2	3	4	5
3	0	6	2	6	4	5
4	6	1	6	6	6	6

Upper bound = 121.6133 Maximal improvement = 4.25%

for period C, the solution C selections in period 2 were also null since there are not enough funds to cover storage and circulation in period 3. With a budget of 35.0 for period 3, we can allow for some selections being made in period 2 as in solution B. Solution D has a change only in B_3 decreasing from 130.0 to 120.0 with all other parameters remaining as they were in the original problem. Here we can test the tradeoff of current versus past selections as the best answer for the original problem (solution 2 of Table 5) requires an expenditure of 126.0104 for period 3. We see in solution D little change in selections made prior to period 3 and a reduction of selections made in period 3 itself. One final observation is that the percent improvement possible as indicated in Tables 9 and 10 increases as we deviate from the original problem. One reason may be that in cases such as solution C, it is hard to reconcile not selecting any new items in period 2 with a budget of $B_2 = 125.0$.

The initial values for $\lambda_{j,o,q}$ as presented in Table 4 and used in the original example problem reveal that $\lambda_{j,o,q}$ is a nondecreasing function of q. It seems that this phenomenon might help explain why all tradeoffs occurred in favor of the most recent selection. That is to say, when any period had a low enough budget to force a choice between acquiring a current journal issue and selecting a previous issue of a journal, the current item was always selected. To test this, the initial values of $\lambda_{j,o,q}$ were rearranged to yield

$\lambda_{j,o,q}$	q = 0	1	2	3	4	5
j=1	3.00	2.50	2.50	2.00	2.00	1.50
2	8.00	7.50	6.00	5.00	4.00	3.00
3	2.25	2.00	1.75	1.50	1.25	1.00
4	2.00	2.00	2.00	2.00	2.00	2.00

We note that now $\lambda_{j,o,q}$ is a nonincreasing function of publication date q. Using the various budgets of the last four solutions plus the original budget, we have the following answers, presented in Table 10:

Solution E, the first solution in Table 10 and the one using the budgets of the original problem, yielded a somewhat different solution than that of solution 2 of Table 5. The reason for this seems to be that the higher uses that are now found in the first few periods are more than the lower budgets of those periods can handle. This is seen in period 1 quite clearly. Furthermore, as illustrated in Table 4, journal 3 is a low usage-low cost item while journal 4 has both higher uses and uses that remain constant over publication date and higher costs. In the original problem, as demand for journal 3 increased until it surpassed the demand for journal 4 and so journal 3 replaced journal 4 in the list of selected items. However, in the case of solution E that now confronts us, the high demand for journal 3 soon subsides and makes way for journal 4 in the latter time periods. These phenomena occur throughout the solutions of Table 10. We note the higher upper bound and lower objective function value of solution E which implies a higher maximal percentage of improvement in comparison to solution 2.

Solution F has the same input parameters as solution B of Table 9. The phenomena of fewer items being selected in period 1 and the issues of journal 4 selected over those of journal 3 in the latter periods

Table 10. Solutions With Different Budget Parameters -- Decreasing $\lambda_{j,o,q}$ versus q

Solution E — Objective Function = 119.0709

q =	0	1	2	3	4	5
B_q	-	115.0	125.0	130.0	140.0	150.0
ϵ_q	-	83.7082	120.8841	124.6394	133.3160	135.4220
$t_{j,q}$						
j=1	6	6	2	6	4	5
2	0	1	2	3	4	5
3	0	1	2	3	6	6
4	6	6	6	3	4	5

Upper bound = 128.3079 Maximal improvement = 7.75%

Solution F — Objective Function = 101.5829

q =	0	1	2	3	4	5
B_q	-	115.0	125.0	35.0	140.0	150.0
ϵ_q	-	83.7082	120.8841	21.834	130.028	
$t_{j,q}$						
j=1	6	6	2	6	6	5
2	0	1	2	4	4	5
3	0	1	2	6	6	6
4	6	6	6	6	4	5

Upper bound = 113.2302 Maximal improvement = 11.6%

Table 10 (Continued)

Solution G Objective Function = 82.9228

q =	0	1	2	3	4	5
B_q	-	115.0	125.0	15.0	140.0	150.0
E_q	-	55.7542	57.0885	14.7000	124.1529	147.0649
$t_{j,q}$						
j=1	6	6	6	6	5	6
2	0	1	2	4	4	5
3	0	6	6	6	5	6
4	6	6	6	6	4	5

Upper Bound = 89.2204 Maximal improvement = 7.6%

Solution H Objective Function = 115.0304

q =	0	1	2	3	4	5
B_q	-	115.0	125.0	120.0	140.0	150.0
E_q	-	83.7082	120.8841	97.4254	131.1684	133.4350
$t_{j,q}$						
j=1	6	6	2	3	4	5
2	0	1	2	3	4	5
3	0	1	2	6	6	6
4	6	6	6	6	4	5

Upper bound = 126.9202 Maximal improvement = 10.34%

Table 10 (Continued)

Solution I Objective Function = 112.2641

q =	0	1	2	3	4	5
B_q	-	115.0	75.0	130.0	200.0	150.0
s_q	-	83.7082	59.6061	125.1592	183.0707	135.5606
$t_{j,q}$						
j=1	6	6	6	3	4	3
2	0	1	2	3	4	3
3	0	1	6	4	4	6
4	6	6	6	3	4	3

Upper bound = 124.2172 Maximal improvement = 10.6%

again occurs in F. We also note that solution F has a higher upper bound, a higher value of the objective function, and a higher maximal percentage of improvement than solution B.

Solution G corresponds to solution C of Table 9, while solution H corresponds to solution D of Table 8. The usual phenomena as discussed above again occurs. Solution G has a higher upper bound, a higher value of the objective function, and lower maximal percentage of improvement; while solution H has a lower value of the objective function, a higher upper bound, and a higher maximal percentage of improvement. We note that in solution G, the latter periods seem to prefer items published in previous periods but not yet acquired in the latter periods to current items. This is due to the higher expected usage of earlier material taking precedence, even with age, over newer items. This unusual phenomenon occurs nowhere else and is quite contrary to what we expected.

Solution I corresponds to solution A of Table 9. Again the usual situation as observed throughout Table 10 occurs in terms of the switching of positions of journals 3 and 4 and the selection of only two journals in the first period. Other than that, the two analogous solutions are quite similar. One point of interest is that solution I could not be improved at all using the final improvement algorithm so that solution I represents the best feasible solution possible for the problem and in this situation, it is unique.

Several observations can be made in terms of the contrast of solution 2 of Table 5, the best feasible solution of the original example problem, and the solutions of Tables 9 and 10 with changes

in budget and/or initial usage parameters. First of all, the number of feasible solutions generated by the Lagrangian upper bound algorithm is limited to one, the initial one, in all cases except the original example problem which had a total of four feasible solutions. Secondly, none of the latter solutions had as low a maximal percentage of improvement as did solution 2. Lower percentages imply that the value of the objective function is quite close to its upper bound, with little room for improvement. One reason for this could be that several other "good" solutions generated by the Lagrangian algorithm were not improved; specifically, those solutions associated with the optimal bases of the linear programming problems of the algorithm. However, even in the most adverse case, the percentage never exceeded 16%. In our limited computational experience we found that generally, the percentage was below 11%. In a few cases it was below 5%, with the best percentage coming from the original problem, 2.0587%. Thus we can conclude that the algorithms work reasonably well in solving the mathematical programming problem as exhibited in the journal selection model.

Computational effort was not deemed to be a limiting factor for this example problem. The whole Lagrangian algorithm required from 14 to 30 seconds, using the CDC 6500 computer, and from 24 to 37 iterations. The simplex algorithm, used for the linear programming segment, required from 1 to 19 iterations for the few times in which this statistic was observed, and used from 5 to 10 seconds of computer time. The other algorithms were manipulated without the aid of a computer so that computational effort cannot be specified.

In order to test the sensitivity of the model to small changes in some of the initial expected usage estimates, we varied, for all q, $\lambda_{3,o,q}$ and $\lambda_{4,o,q}$ by plus or minus 20% and $\lambda_{3,o,q}$ by plus or minus 10%. Since usage is an input data requirement, the librarian must be able to come up with a reasonable estimate that has some margin for error in the estimation procedure. In a library system there most likely are very limited funds for such activities and few records of vital statistics where accurate estimates could be obtained easily.

The results of these final perturbations of the original data set are presented below in Table 11. Solution J, the first one in the table, was formed by increasing $\lambda_{3,o,q}$ by 20% for all q, holding all other parameters constant. We can see that in the first two periods, journal 3 is now preferred to journal 4, as opposed to solution 2 of Table 5. However, since we did not examine and improve all possible candidate solutions, there is a good chance that journal 4 may still supercede journal 3 for these early periods in the true optimal solution. In any case, a 20% variation seems big enough to at least indicate the possibility of some changes in the optimal solution.

Solution K corresponds to a decrease of 20% in $\lambda_{3,o,q}$ for all q; while solutions L and M correspond to an increase and a decrease of 20% in $\lambda_{4,o,q}$ for all q, respectively, with no other parameters being allowed to vary. We note that there is no change in the final answer from that of solution 2 in Table 5. Of course, the values of the objective function and upper bound changed as did the parameter that was increased or decreased. One point of interest was that when we

Table 11. Solutions Formed by Varying One $\lambda_{j,o,q}$ by Some Fixed Percentage

Solution J Objective Function = 121.2867

q =	0	1	2	3	4	5
B_q	-	115.0	125.0	130.0	140.0	150.0
g_q	-	106.2652	118.0572	126.2524	136.4829	145.9216
$t_{j,q}$						
j=1	6	1	2	3	4	5
2	0	1	2	3	4	5
3	0	1	2	3	4	5
4	6	6	6	6	6	6

Upper bound = 125.5027 Maximal improvement = 3.48%

Solution K Objective Function = 118.5014

q =	0	1	2	3	4	5
B_q	-	115.0	125.0	130.0	140.0	150.0
g_q	-	111.7092	121.5752	126.4554	134.8808	143.8075
$t_{j,q}$						
j=1	6	1	2	3	4	5
2	0	1	2	3	4	5
3	0	6	6	3	4	5
4	6	1	2	6	6	6

Upper bound = 121.2931 Maximal improvement = 2.34%

Table 11 (Continued)

Solution L Objective Function = 131.1305

q =	0	1	2	3	4	5
B_q	-	115.0	125.0	130.0	140.0	150.0
E_q	-	113.4852	123.9752	128.9334	139.4613	149.1119
$t_{j,q}$						
j=1	6	1	2	3	4	5
2	0	1	2	3	4	5
3	0	6	6	3	4	5
4	6	1	2	6	6	6

Upper bound = 131.9078 Maximal improvement = 0.52%

Solution M Objective Function = 110.0180

q =	0	1	2	3	4	5
B_q	-	115.0	125.0	130.0	140.0	150.0
E_q	-	110.2292	119.3752	123.0874	132.0951	140.2144
$t_{j,q}$						
j=1	6	1	2	3	4	5
2	0	1	2	3	4	5
3	0	6	6	3	4	5
4	6	1	2	6	6	6

Upper bound = 114.2049 Maximal improvement = 3.81%

Table 11 (Continued)

Solution M Objective Function = 120.9058

q =	0	1	2	3	4	5
B_q	-	115.0	125.0	130.0	140.0	150.0
$\mathcal{E}_q$	-	111.9312	116.1676	126.0288	136.0371	145.3772
$t_{j,q}$						
j=1	6	1	2	3	4	5
2	0	1	2	3	4	5
3	0	6	2	3	4	5
4	6	1	6	6	6	6

Upper bound = 124.2053 Maximal improvement = 2.78%

Solution O Objective Function = 119.5461

q =	0	1	2	3	4	5
B_q	-	115.0	125.0	130.0	140.0	150.0
$\mathcal{E}_q$	-	111.8572	121.6122	125.7329	135.3451	144.3527
$t_{j,q}$						
j=1	6	1	2	3	4	5
2	0	1	2	3	4	5
3	0	6	6	3	4	5
4	6	1	2	6	6	6

Upper bound = 122,0482 Maximal improvement = 2.09%

increased $\lambda_{4,o,q}$, the utilization of all budgets was so high as to yield a maximal percentage of improvement of only 0.52% in solution L.

Solution N corresponds to an increase of 10% in $\lambda_{3,o,q}$ for all q. Here again as in solution J, journal 3 replaces journal 4, although only in period 2 for solution N. Since all possible candidates have not been examined for improvement, we cannot be sure if this is the best feasible solution. However, we have demonstrated the extreme sensitivity of the model to $\lambda_{3,o,q}$ and perhaps to other variables as well. Solution O corresponds to a decrease of 10% in $\lambda_{3,o,q}$ for all q and remains unchanged from the answer of solution 2. Furthermore, solution O could not be improved at all using the improvement algorithm, implying that it represents the best feasible solution possible for the problem.

We note that this sensitivity analysis of changes in initial expected usage estimates yielded results that stemmed primarily from its effects upon the objective function. While the usage variables also appear in the constraints, the small value of its cost coefficient c_3 no doubt reduced the effects of small perturbations in the usage parameters. Moreover, this analysis showed that in several cases, in the latter periods journal 4 was preferable to journal 3 but was too expensive so that journal 3 was selected while journal 4 was not. We note that changes as large as 20% in a parameter did not produce large changes in the value of the best objective function obtained for each problem, even when such changes indicated policy changes. In general, the objective function changed less than 5%, with no change being larger than 10%. Finally, throughout the example problem, an interesting fact was seen to occur in that the optimization segment of the Lagrangian algorithm was quite sensitive to small changes in

the values of the generated multipliers. Since the decision rules are based on the nonnegativity of a series of sums, it seems that the multipliers attempted to just make some of these sums become negative or positive by only a very small amount so that any change in the multipliers will mean quite a difference in the generated solution. It was noted that variations in the fourth decimal place of the multipliers could be quite significant.

CHAPTER VII

SUMMARY, CONCLUSIONS, AND IMPLICATIONS FOR FUTURE RESEARCH

In the preceeding chapters we have developed the journal selection model as a zero-one linear programming problem. An objective function has been constructed, based on the library being both a service organization and an information retrieval system; and the objective function has been compared to various means of evaluating the performance of information systems. Ways of measuring journal worth, necessary for use in the objective function, have been investigated. The constraints were generated primarily from budget and cost considerations, with the other constraints originating from the definition of the decision variables. Finally, we have developed two algorithms, one based upon a dynamic programming formulation, and the other based upon a Lagrangian formulation of the model. These algorithms both generate feasible solutions, if at least one exists; and the latter one gives us an upper bound on the optimal value of the objective function. A final improvement algorithm may then be used to improve the solutions generated by these algorithms in order to produce a feasible, near-optimal or optimal solution. Thus, the journal selection problem can be solved.

Two important implications for the university librarian can be seen from the specific form of the objective function. First of all, the worth of a journal when acquired is the key element of the objective function. Relating this worth to the worth of not acquiring the item, we get a simple form for our model in terms of a binary-valued decision variable that represents whether or not an item is acquired by the library. This form leads to a linear objective function and seems quite adaptable to other library problems. For example, consider the decision to form a centralized library as opposed to a decentralized or branch system. Moreover, we see how we can avoid such sticky issues as the determination of the cost of user delay, which Lister [48] faced and found to be quite important in terms of the sensitivity of the model to such costs. Secondly, we see that such a form, in conjunction with the cost constraints, can represent the conjunction of pertinence or nonpertinence and acquisition or nonacquisition, as discussed by Swets [74] in terms of the evaluation of information retrieval systems. In general, we find that the objective function is quite representative of the important factors involved in measuring the performance of such systems.

The cost constraints were formed primarily to reflect the harsh restraint imposed by the scarcity of capital and to show the influence of funder control on a library organization. We use the linear form as described by Leimkuhler [40] and by Williams, et.al. [81], who also provide us with estimates of the magnitude of these costs. The relevant costs are the initial ordering, subscription, storage, and circulation costs. Other scarce resources that could restrain library

activities could be included in the model using inequalities that are quite similar to those in the cost constraints. However, we believe that, in terms of the journal selection decision, the chief factor is the scarcity of capital. We can employ additional means to handle all other scarce resources; means such as to cope with restrictions on available space.

The definitional constraints specify, first of all, that the decision variables can take on only the values zero or one. Secondly, the constraints specify that once a particular item has been acquired, it remains within the library system. This implies that items that have been lost, stolen, or defaced must be replaced as soon as possible.

The journal selection problem takes the form of a zero-one linear programming problem. Linearity is maintained by allowing one of the cost factors, the cost associated with the first acquisition of a new journal, to be zero. Further research could be done to see how one would handle this general, nonlinear (bilinear) case. The model can be re-written as a scheduling model of the kind used in industrial operations. While this does not in itself yield a new method of solution, it does establish a link between a specific model of a library problem and an established set of important problems of an entirely different sort.

Journal worth, the prime element of the objective function, which is to be maximized over all journals and over time, can be measured by two different criteria. The first, journal usage, fits in quite nicely with the concept of information system performance evaluation.

Moreover, the modified Markovian model can be used to adequately describe expected usage over time. There are other models, especially the "Pn" model that can be used instead, but modifications can lead to grave complications in terms of parameter estimation. We see that the literature on usage tends to view parametric forms as having parameters common to all items relevant to a specific field of endeavor. This causes no problems at all for the modified Markovian model. One problem that does arise is that most work in this area has dealt only with technical literature, with emphasis more on curve fitting than on behavioral reasons behind the suitability of such parametric forms. Thus, more research could be done to expand usage models to cover nontechnical literature, to compare these models with the others, and to find explanations that reveal why a particular form can describe usage over time.

Journal productivity, the number of relevant articles in a journal, is the alternate measure of journal worth. While it too fits in quite well with the concept of the evaluation of an information system, it cannot be described quite so nicely over time. Epidemic models of literature growth are considered, as is Leimkuhler's [39] Bradford distribution, for use in describing the scattering of relevant articles. This implies that a librarian must be able to specify what fields of endeavor are important to the users of his library, and that he must be able to specify which articles are relevant to those fields. Furthermore, he must be able to consider the complete set of journals which might include relevant articles

Using data to test the models, we find that the epidemic model fails to adequately describe the growth of literature. The Bradford distribution directly establishes the fact that a majority of relevant articles are situated in a very few of the most productive journals, a point quite relevant to librarians. It does not, however, lend itself for use in deciding which journals are in the set of top-ranked items and should be acquired. The set is a dynamic entity, changing over time so that the question of which journals are in the set at any particular moment cannot be determined by the Bradford distribution. Furthermore, for the body of queueing theory literature cited in the bibliography of a textbook on the subject [66], which is used as the data base, the Bradford parameter changes drastically, declining over time, revealing still another side of the dynamic nature of the process. Since the field was relatively new, perhaps the growth of the field from infancy explains the variance; however, the librarian cannot be expected to be so familiar with each field of scientific endeavor so as to be able to accurately describe each field's growth and scattering characteristics.

The establishment of a relationship between journal productivity and usage leads us to the inference that we can use either criterion and still maintain the advantages of both in terms of measuring journal worth. Thus, usage, which can be modeled and described over time with relative ease and accuracy, should be selected as the best criterion. One final point here is that a Markovian process was developed to aid in the modeling of the decision problem of selecting

journals in a university library system. Even if epidemic theory and Bradford distribution models were adopted, the process is so complex as to render the whole concept impractical.

The solution algorithms attempt to decompose the zero-one programming problem into smaller, more workable forms. The size of the original problem renders the existing algorithms impractical. It goes without saying that better techniques for exact solutions to these general optimization problems are needed. However, as our example problem demonstrates, these algorithms can find a near-optimal or optimal, feasible solution for the journal selection problem and an upper bound on the optimal value of the objective function. Moreover, several interesting implications stem from the formulation and solution of the problem.

First of all, the dynamic programming formulation allows us to break down the large problem into a series of subproblems at each stage or time period in the planning horizon. The link between the stages is the cost of storage and circulation in future periods of items acquired now, since these costs can affect the acquisition policies of these future periods. Ignoring these effects, we can generate the subproblems as a series of knapsack problems, and can solve them to produce a feasible, suboptimal solution. We employ a theorem dealing with the control of a stochastic system [53], which uses expected values to yield a deterministic equivalence, to aid us in properly structuring the subproblems.

The special cases when the algorithm provides us with an optimal solution tell us quite a bit about the structure of the problem. The first case is concerned with the situation where the cost of storing and circulating an item is set to zero so that the link between stages is broken and the resulting solution is optimal. The costs used in a selection decision model must be those associated with the incremental cost if a particular item is acquired. In a library system such as Purdue University's, the only relevant cost in the model is the subscription cost, since all other associated activities have been budgeted for separately. This implies that there could be restrictions on the number of items acquired due to a fixed, restrictive capacity of the system to perform some required activity. It also implies that more research needs to be done on the determination of incremental costs of library activities.

The second and third cases are concerned with situations that cause the budget constraints to be reduced to limitations on the number of items that can be selected. The underlying assumption is that all items have a constant subscription cost and/or demand in a time period, regardless of age or journal. In the context of a new library attempting to start a collection, with no past usage data and with dealers that might sell sets of journals to the library at a fixed price, this assumption might be realistic. However, in a more general case, this assumption can be quite restrictive.

The Lagrangian algorithm deals with a classical approach to continuous optimization problems as applied to a discrete variable problem. It involves linear programming, scheduling theory, and duality

theory, especially in relation to discrete problems. It is quite interesting that, using Everett's [19] approach, we can generate an easily solvable scheduling type of problem. Moreover, we can use Brooks and Geoffrion's [9] linear programming technique to update the Lagrangian multipliers, and use their stopping rule to indicate when an upper bound on the optimal value of the objective function has been reached. We see that this approach, especially with respect to the stopping rule, is quite analogous to Balas' [5] theory of duality for discrete problems.

The example problem demonstrated the applicability of our algorithms to the journal selection problem. The size of the example problem is quite small and yet it requires a good deal of computer storage and time. While this may be a function of mediocre computer programming skills, it does indicate a large amount of storage space and time may be required for a real library to apply these results. However, this approach still seems more practical than attempting to use existing integer programming routines to find an exact solution or abandoning the model and resorting to existing rules of thumb.

The most important implication of the example, in the light of the sensitivity analysis where budget and initial usage parameters were perturbed, is that the "nicest" solutions occur when problems are set up with budgets that the funders had the foresight or good fortune to make large enough. By large enough, we mean that there is enough money to cover storage and circulation costs for past acquisitions and to maintain on-going journals. In some cases, there may be enough money left over to begin acquiring new journals or to go back and fill

in the gaps in existing journal sets. This concept is reinforced by the latter two special cases discussed above where the dynamic programming algorithm generates an optimal solution. Thus, the model can be used by a funder to determine a priori what are the effects of various levels of funding and to determine some optimal level at which to operate.

The final improvement algorithm again points out how closely the journal selection problem is related to the knapsack problem with respect to the structure of the two problems [26]. Moreover, we see that the solutions generated, especially by the Lagrangian algorithm, while not all feasible, are subject to improvement. Candidates for improvement can be any solution available; however, the solutions of interest include all feasible ones available plus those that correspond to the optimal basis of the linear programming segment of the Lagrangian algorithm. We can then use the best feasible solution now at hand as the near-optimal, or optimal, solution. However, in general there is no way of knowing if we have serendipitously reached an optimal solution.

The data requirements for the selection model seem to pose the most severe roadblock to successful implementation of this research effort in a general university library system. The data must be taken with care and accuracy if a meaningful solution is desired. The quantity of data needed is rather large, but with the exception of initial demand and demand parameter estimates as well as incremental costs, the data can be easily obtained from standard sources such as accounting figures. Usage seems to be the largest problem since there is a good deal of data required for this item and since unbiased estimates may be very difficult to obtain. Since usage will determine

journal worth and hence policy, a decision-maker may bias estimates in favor of those items he decides a priori are good for one reason or another. Research must be done to determine methods of obtaining good, unbiased estimates that can be had with relative ease and accuracy, and they must be reasonably inexpensive to obtain.

We note that variations in some of the initial expected usage estimates of as little as 10% indicate the possibility of slight changes in the optimal solution. While this occurred for only one case, as was the situation for a 20% variation, this degree of sensitivity did reinforce the need for accurate estimates. It also showed that some variables are more stable than others, so that a librarian may use the model to investigate alternative solutions for different levels of the more sensitive parameters. More research is needed to provide an analytical base for determining the sensitivity of the model to parameter perturbations in a discrete programming situation.

One possible way to reduce the amount of data required is to establish simple parametric forms for $\lambda_{j,o,q}$ (expected usage in period q of the issues of journal j published in period q) and for $K_{j,q,t}$ (subscription cost of the issues of journal j published in period t if acquired in period t+q) versus the variable q. Of course, this will then add to the problem of finding good estimates for all of the parameters used in the model.

The implementation of the journal selection model to produce policy decisions in an actual library environment seems to be the next required step for further research. We have taken usage studies and developed

the decision model. As pointed out before, the model performs very well under favorable conditions such as reasonable size. However, there are other problems besides size that must be considered. First of all, the present model allows for random selection in that there is no restriction on which issue of a journal must be acquired first, second, or in any specific order whatsoever. Thus, there is no guarantee that all issues of a journal will be acquired in sequence, or even that all issues will even be acquired. One could add more constraints to account for continuity of on-going journals. However, some of these constraints may be nonlinear. One could instead use the approach taken by Williams, et.al. [81], who select journals rather than issues of journals. They obtain continuity by making it inherent in the structure of their model. We rejected this approach for academic reasons, hoping to see what sort of policies would be generated if additional freedom was granted the decision-maker. However, research should be done to see which approach is best for a given practical situation.

Finally, the librarian must never allow this model, or any other, to replace him as the ultimate decision-maker. This implies that the responsibility for decisions must rest with the person, not the tool. The model is just that, a tool to be used as a guide and an aid to the decision-maker. With this in mind, it is our hope that general uses may be found for the model as applied to the journal selection decision in a university library system, and perhaps to be applied under other circumstances as well.

BIBLIOGRAPHY

BIBLIOGRAPHY

[1] Baker, N. R., "A Descriptive Model of Library/User/Funder Behavior in a University Environment", Drexel Library Quarterly, Vol. 4, No. 3, January 1968.

[2] _____, "Toward An Analytical Framework for Service Organizations", XIV International Meeting of TIMS, Mexico City, August 1967.

[3] _____, Notes for Library Seminar, Unpublished notes at Purdue University Library Seminar, Lafayette, Indiana, March 1966.

[4] _____, and R. E. Nance, "The Use of Simulation in Studying Information Storage and Retrieval Systems", 32nd National ORSA Meeting, Chicago, 1967.

[5] Balas, Egon, "Duality in Discrete Programming", Technical Report No. 67-5, Department of Operations Research, Stanford University, Stanford, California, July 1967.

[6] Baumol, W., Economic Theory and Operations Analysis, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1963.

[7] Becker, J., and R. Hayes, Information Storage and Retrieval: Tools, Elements, Theories, John Wiley & Sons, Inc., N.Y., 1963.

[8] Borden, J. C., R. C. Farris, D. P. Hammer, and H. D. Ferris (Chairman), Serials Cataloging Policy Committee Report, 1968, Purdue University Library, Lafayette, Indiana.

[9] Brooks, R., and A. Geoffrion, "Finding Everett's Lagrange Multiplier by Linear Programming", Operations Research, Vol. 14, 1966.

[10] Bryant, E. C., "Modeling in Document Handling", in Electronic Handling of Information Testing and Evaluation by A. Kent, O. Taulbee, J. Belzer, and G. Goldstein (Ed.), Academic Press, London, and Thompson Book Company, Washington, D.C., 1967.

[11] Bush, O. C., N. P. Gilliher, and P. Morse, "Attendance and Use of the Science Library at MIT", American Documentation, Vol. 7, pp. 87-109, (1956).

[12] Chitwood, J., "T Mechanisms of Book Selection", in H. Goldhor (Ed.), Selection and Acquisition Procedures in Medium-Sized and Large Libraries, Illini Union Bookstore, Champaign, Illinois, 1963.

[13] Coffin, L. C., "Blanket Book Ordering", in H. Goldhor (Ed.), Selection and Acquisition Procedures in Medium-Sized and Large Libraries, Illini Union Bookstore, Champaign, Illinois, 1963.

[14] Cole, P., "Journal Usage Versus Age of a Journal", Journal of Documentation, Vol. 19, pp. 1-11, (1963).

[15] Conway, R. W., W. L. Maxwell, and L. W. Miller, Theory of Scheduling, Addison-Wesley Publishing Company, Reading, Massachusetts, 1967.

[16] DeWeese, L. C. III, "Use Studies Bibliography", Unpublished report, School of Industrial Engineering, Purdue University, Lafayette, Indiana, 1967.

[17] Downs, R. B., "The Implementation of Book Selection Policy in University and Research Libraries", in H. Goldhor (Ed.), Selection and Acquisition Procedures in Medium-Sized and Large Libraries, Illini Union Bookstore, Champaign, Illinois, 1963.

[18] Eaton, Thelma, Cataloging and Classification, 4th Edition, Edwards Brothers, Inc., Ann Arbor, Michigan, 1967.

[19] Everett, H., "Generalized Lagrange Multiplier Method for Solving Problems of Optimum Allocation of Resource", Operations Research, Vol. 11, 1963.

[20] Gass, S. I., Linear Programming, McGraw-Hill Book Company, N.Y., 1969, 3rd Edition.

[21] Goffman, W., "Mathematical Approach to The Spread of Scientific Ideas -- The History of Mast Cell Research", Nature, Vol. 212, No. 5061, pp. 449-452, October 29, 1966.

[22] ____, "Stability of Epidemic Processes", Nature, Vol. 210, No. 5038, May 1966.

[23] Goffman, W., "Probabilistic Models in Information Retrieval", Information Retrieval in Action, Center for Documentation and Communications Research, Western Reserve University, Cleveland, Ohio, 1963.

[24] _____, and U. A. Newell, "Communication and Epidemic Processes", Proceedings of the Royal Society, Vol. 298, pp. 316-34, (1967).

[25] Goldhor, H. (Ed.), Selection and Acquisition Procedures in Medium-Sized and Large Libraries, Illini Union Bookstore, Champaign, Illinois, 1963.

[26] Greenberg, H., and R. L. Hegerich, "A Branch Search Algorithm for the Knapsack Problem", Management Science, Vol. 16, No. 5, January 1970.

[27] Hacker, H. L., "Financial Problems of the Large Library", The Library Quarterly, Vol. 38, No. 1, January 1968, pp. 41-57.

[28] Hadley G., Nonlinear and Dynamic Programming, Addison-Wesley Publishing Company, Reading, Massachusetts, 1964.

[29] Hogg, R. V., and A. T. Craig, Introduction to Mathematical Statistics, 2nd Edition, The Macmillan Company, New York, 1965.

[30] Jain, Aridiman, "A Statistical Study of Book Usage", Ph.D. Thesis, School of Industrial Engineering, Purdue University, Lafayette, Indiana, 1967.

[31] Kaplan, Seymour, "Solution of the Lorie-Savage and Similar Integer Programming Problems by the Generalized Lagrange Multiplier Method", ORSA, Vol. 14, No. 6, November-December 1966.

[32] Kent, A., "Centralization, Decentralization, and Specialization: A Problem in Resource Allocation", in Electronic Handling of Information Testing and Evaluation, A. Kent, O. Taulbee, J. Belzer, and G. Goldstein (Ed.), Academic Press, London, 1967.

[33] _____, "The Information Retrieval Game", in Electronic Information Handling, A. Kent, and O. Taulbee (Ed.), Spartan Books, Washington, D.C., 1965.

[34] King, D. W., "Design and Evaluation of Information Systems", Annual Review of Information Science and Technology, C. A. Cuadra (Ed.), Interscience Publishers, 1968.

[35] Kolesar, P. J., "A Branch and Bound Algorithm for the Knapsack Problem", Management Science, Vol. 13, 1967.

[36] Korfhage, R. R., and T. G. DeLutis, "A Basis for Time and Cost Evaluation of Information Systems", Research Memorandum 69-6, School of Industrial Engineering, Purdue University, Lafayette, Indiana, June 1969.

[37] Korfhage, R. R., Class notes, CS 560, Information Storage and Retrieval Systems, Spring 1968.

[38] Leimkuhler, F. F., "Systems Analysis in University Libraries", College and Research Libraries, Vol. 27, No. 1, January 1966.

[39] _____, "The Bradford Distribution", Journal of Documentation, Vol. 23, No. 3, September 1967.

[40] _____, "Storage Policies for Information Systems", Research Memorandum 69-8, School of Industrial Engineering, Purdue University, Lafayette, Indiana, June 1969.

[41] _____, and N. R. Baker, "Library Design and Operations Study", Report to National Science Foundation for fiscal year 1967-68.

[42] _____, "Operational Analyses of Information Systems", Proposal to National Science Foundation, January 1968.

[43] _____, and G. Cox, "Compact Book Storage in Libraries", Operations Research, XII, pp. 419-27, May-June 1964.

[44] _____, and A. Neville, "The Uncertain Future of the Library", The Johns Hopkins University Magazine, Fall 1967.

[45] _____, and M. D. Cooper, "Cost Accounting and Analysis for University Libraries", P-2 paper, Office of Vice President-Planning and Analysis, University of California, Berkeley, California, January 1970.

[46] _____, and M. D. Cooper, "Analytical Planning for University Libraries", P-1 paper, Office of Vice President-Planning and Analysis, University of California, Berkeley, California, January 1970.

[47] _____, "Library Search", Working paper FL-1, School of Industrial Engineering, Purdue University, Lafayette, Indiana, August 1966.

[48] Lister, W. C., "Least Cost Decision Rules for the Selection of Library Materials for Compact Storage", Ph.D. Thesis, School of Industrial Engineering, Purdue University, Lafayette, Indiana, 1967.

[49] Maizell, "Standards for Measuring the Effectiveness of Technical Library Performance", IRE Transactions on *Engineering Management*, *EM-7*, *pp. 69-72*, *(1960)*.

[50] Meier, R., "Efficiency Criteria for a Large Library", Library Quarterly, Vol. 31, pp. 215-34, (1961).

[51] _____, "Overload and Growth in Communications-Oriented Institutions", in Mathematical Explorations in Behavioral Science, F. Massarik and P. Ratoosh, Irwin, Homewood, Ill., 1965.

[52] Metcalf, K. D., "Why We Need to be Investigated", College and Research Libraries, October 1954.

[53] Midler, J. L., "Optimal Control of a Discrete Time Stochastic System Linear in the State", J. Math. Analysis and Applications, Vol. 25, No. 1, pp. 114-120, January 1969.

[54] Morris, W. T., "On the Art of Modeling", Management Science, Vol. 13, No. 12, August 1967.

[55] _____, Management Science: A Bayesian Introduction, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1968.

[56] _____, The Analyses of Management Decisions, Richard D. Irwin, Inc., Homewood, Illinois, 1964.

[57] Morse, P. M., "Probabilistic Models for Library Operations", Minutes of 63rd Meeting of the Association of Research Libraries, Chicago, pp. 9-13, January 1964.

[58] _____, Library Effectiveness, MIT Press, Cambridge, Massachusetts, 1968.

[59] _____, and C. Elston, "A Probabilistic Model for Obsolescence", J.O.R.S.A., Vol. 17, No. 1, 1969.

[60] Murdock, J. W., and D. M. Liston, Jr., "A General Model of Information Transfer", Theme paper, 1960 Annual ADI Convention, American Documentation, Vol. 18, October 1967.

[61] Neuts, Marcel, Personal communication and class notes, Purdue University, Lafayette, Indiana, 1967.

[62] O'Neill, E. T., "Journal Usage Patterns and Their Implications in the Planning of Library Systems", Ph.D. Thesis, School of Industrial Engineering, Purdue University, Lafayette, Indiana, 1970.

[63] Ostle, Bernard, Statistics in Research, Iowa State University Press, Ames, Iowa, 1963.

[64] Purdue University Libraries, Annual Report of the Assistant Director for Processing Services, July 1, 1967 - June 30, 1968.

[65] Rzasa, P. V., "The Development of Measures of Effectiveness for a University Library", M.S. Thesis, School of Industrial Engineering, Purdue University, Lafayette, Indiana, 1969.

[66] Saaty, T., Elements of Queueing Theory, McGraw-Hill Book Company, New York, 1961.

[67] _____, "Seven More Years of Queues", Naval Research Logistics Quarterly, Vol. 13, No. 4, pp. 447-477, December 1966.

[68] _____, Optimization on Integers and Related Extremal Problems, McGraw-Hill Book Company, New York, 1970.

[69] Salton, G., Automatic Information Organization and Retrieval, McGraw-Hill Book Company, New York, 1968.

[70] Sanders, H., "The Mechanics of Book Ordering", in H. Goldhor (Ed.), Selection and Acquisition Procedures in Medium-Sized and Large Libraries, Illini Union Bookstore, Champaign, Illinois, 1963.

[71] Stevens, R. E., "The Relationship of the Acquisitions Department to the Libraries Total Program", in Goldhor, H. (Ed.), Selection and Acquisition Procedures in Medium-Sized and Large Libraries, Illini Union Bookstore, Champaign, Illinois 1963.

[72] Strable, E. G., (Ed.), "Special Libraries: A Guide for Management", Special Libraries Association, New York, 1966.

[73] Strauss, L. J., I. M. Stricby, and A. L. Brown, Scientific and Technical Libraries: Their Organization and Administration, Interscience Publishers, John Wiley and Sons, Inc., New York, 1964.

[74] Swets, John A., "Information Retrieval Systems", in M. Kochen, The Growth of Knowledge, John Wiley and Sons, Inc., New York, 1967.

[75] Tauber, A. S., "The Facts of Life About the Man-Information Interaction", in Electronic Handling of Information Testing and Evaluation by A. Kent, O. Taulbee, J. Belzer, and G. Goldstein (Ed.), Academic Press, London, 1967.

[76] Trueswell, R. W., "User Behavior Patterns and Requirements and Their Effect on the Possible Applications of Data Processing and Computer Techniques in a University Library", Ph.D. Thesis, Northwestern University, Evanston, Illinois, 1964.

[77] _____, "Two Characteristics of Circulation Control and Their Effect on the Implementation of Mechanized Circulation Control Systems", College and Research Libraries, Vol. 25, 1964.

[78] Vickery, B. C., On Retrieval System Theory, Butterworths, London, 2nd edition, 1965.

[79] Weingartner, "Capital Budgeting of Interrelated Projects: Survey and Synthesis", Management Science, Vol. 12, March 1966.

[80] Welch, H. M., "Selection and Acquisition of Foreign Publications", in H. Goldhor (Ed.), Selection and Acquisition Procedures in Medium-Sized and Large Libraries, Illini Union Bookstore, Champaign, Illinois, 1963.

[81] Williams, G., E. C. Bryant, R. R. V. Wiederkehr, and U. G. Palmour, "Library Cost Models: Owning Versus Borrowing Serial Publications", Report, Center for Research Libraries, Chicago, 1968.

[82] Wulfekoetter, Gertrude, Acquisition Work: Processes Involved in Building Library Collections, University of Washington Press, Seattle, 1961.

[83] Yormark, Jonathan S., "A Two-Dimensional Resource Allocation Problem", Ph.D. Dissertation, Purdue University, School of Industrial Engineering, Lafayette, Indiana, 1970.

APPENDICES

APPENDIX A

AN ANNOTATED BIBLIOGRAPHY

APPENDIX A

AN ANNOTATED BIBLIOGRAPHY

The Library Crisis

1. Bourne, C. P., Methods of Information Handling, John Wiley & Sons, Inc., New York, 1963.

A citing of Library of Congress estimates as to the existence of approximately 30,000 scientific and technical journals with other estimates going as high as 100,000 journals. A growth rate of 900,000 to over 2.1 million articles per year is also estimated.

2. Danton, J. P., Book Selection And Collection: A Comparison of German and American Libraries, Columbia University Press, New York, 1963.

Research materials are printed in many languages and the attainment of bibliographic information concerning them may be costly. In addition, it may be difficult to forecast the degree to which these materials will actually, if ever, be used.

3. de Solla Price, D., "The Scientific Foundations of Science Policy", Nature, Vol. 206, pp. 233-38.

A growth rate of seven percent annually for an exponential growth curve of library size is noted. This appears to have been in effect since the 17th century when libraries were started in this country.

4. de Solla Price, D., "Networks of Scientific Papers", in Kochen, M., The Growth of Knowledge, John Wiley & Sons, Inc., New York, 1967.

Literature growth is seen, in terms of citation listings, as an expanding network.

5. Dunn, O. C., W. F. Siebert, and J. A. Scheuneman, The Past and Likely Future of 58 Research Libraries; 1951-1980: A Statistical Study of Growth and Change, Purdue University Library, Lafayette, Indiana, 1967.

A study of the Purdue University Library system that shows that an exponential growth of a library with a doubling interval of 16 years on the average with little deviation still holds.

6. Garvey, W. D., and E. B. Compton, "The Flood and How to Survive It", The Johns Hopkins University Magazine, Fall 1967.

A brief look at the crisis of ever expanding literature is given. They note that literature grows exponentially doubling every 10 or 15 years.

7. Graves, E. C. (Ed.), Ulrich's Periodicals Directory: A Classified Guide to A Selected List of Current Periodicals, Foreign and Domestic, 8th Edition, P. R. Booker Company, New York, 1959.

A selective directory of periodicals that lists over 16,000 foreign and domestic journals that are available.

8. Hacker, H. S., "Financial Problems of the Large Library", The Library Quarterly, Vol. 38, No. 1, January 1968, pp. 41-57.

The author points out that in spite of rising demands and rising costs, funds are limited and outside competition for the use of these funds is increasing. He suggests three means with which to remedy the situation: participation in regional centralization, investigation of new sources of income, and making every effort to improve the management of the library system.

9. Leimkuhler, F. F., and A. Neville, "The Uncertain Future of the Library", The Johns Hopkins University Magazine, Fall 1967.

Four major problem areas faced by the university library are pointed out:

the physical problem - space requirements due to the bulk of acquired material

the operational problem - time and cost considerations associated with information storage and retrieval

the intellectual problem - the descriptions of user queries and library materials for retrieval

the behavioral problem - the changes in user needs and in fields of endeavor which provide documented information.

10. Rider, F., The Scholar and the Future of the Research Library, Hadham Press, New York, 1944.

A library growth curve of an exponential form, doubling every 16 years is suggested. Four factors supporting these growth rates are listed: the continually increasing realization of the importance of libraries in the educational process, the continual addition of new schools to universities, the increased desire and means to acquire vast ranges of previously published materials in already established fields, and the enormous increase in contemporary output. He also points out that usage of research materials in a library has characteristics which add to the difficulty of being able to forecast the degree to which such materials are actually used. The items often have a very low usage rate, users are often limited to those who have obtained a certain level of intellectual competence in a specific area, and these materials are often cited as references but not read extensively or in depth.

11. Swank, R. C., "The Cost of Keeping Books", in Problems and Prospects of the Research Library, E. Williams (Ed.), Scarecrow Press, New Jersey, 1955.

The growth of libraries in the past 50 years is partially attributed to research. The author points out that scholarship has certain qualities which contribute to expansion: knowledge is elastic, dynamic, and ever expanding; research has an inconclusive quality about itself so that a terminal state is never reached; faculty members, while not all are great scholars, are encouraged to contribute -- "we have reared a race of paper writers"; and creative activity is unpredictable.

12. Weiss, P., "Knowledge: A Growth Process", in The Growth of Knowledge, M. Kochen, John Wiley & Sons, Inc., New York, 1967.

Expansion of knowledge is compared to organic growth processes. "Our knowledge grows the way a living body does ... like an organic tree, not as a compilation of collectors' items".

The User/Servicer/Funder Models of the Library as a Service Organization

13. Baker, N. R., Notes for Library Seminar, Unpublished notes at Purdue University Library Seminar, Lafayette, Indiana, March, 1966.

A set of seminar notes that initially sets forth the idea of a library being a service organization. The participants-users, funders, and servicers - are also discussed and put together in a schematic flow model of the library.

14. Baker, N. R., "Toward An Analytical Framework for Service Organizations", Presented at the XIV International Meeting of TIMS, Mexico City, August 1967.

A revised discussion of the library as a service organization is discussed. Examples of the various decision problems facing each participant are examined and models of their respective decision processes viewed.

15. Baker, N. R., and R. E. Nance, "The Use of Simulation in Studying Information Storage and Retrieval Systems", Presented at the 32nd National ORSA Meeting, Chicago, 1967.

A general discussion of the use of simulation techniques in order to study information storage and retrieval systems is presented.

16. Nance, R. E., "Strategic Simulation of a Library/User/Funder System", Ph.D. Thesis, School of Industrial Engineering, Purdue University, Lafayette, Indiana, June 1968.

An industrial dynamics simulation of a university library system is presented in this doctoral dissertation. The basis for the general model is Baker's user/servicer/funder flow model.

17. Nance, R.E., "Systems Analysis and the Study of Information Systems", Proceedings of the ADI Annual Meeting, Vol. 4, October 1967, New York.

A general discussion of how systems analysis could be used to study and improve performance of information systems.

18. Fairthorne, R. A., A Morphology of "Information Flow", J. of ACM, Vol. 14, October 1967.

Information flow relationships are developed between sources (as authors), messages, designations (as topics), codes, destinations (as users), and channels.

19. Jain, A., "A Statistical Study of Book Usage", Ph.D. Thesis, School of Industrial Engineering, Purdue University, Lafayette, Indiana, 1967.

The author discusses some of the functions of an information retrieval system. He views proper selection of materials as a prerequisite to acquisition. He also considers indexing functions, as identified by classifying and cataloguing functions, and circulation functions since the user is usually given materials on a loan only basis.

20. Kent, A., "The Information Retrieval Game", in A. Kent and O. Taulbee (Ed.), Electronic Information Handling, Spartan Books, Washington, D.C., 1965.

The major functions or unit operations of a servicer of an information retrieval system are listed as follows:

i) acquisition of source documents,

ii) analyses of source documents,

iii) control of terminology and source headings,

iv) recording of results of analysis on a searchable medium,

v) storage of source documents, extracts, abstracts, and/or bibliographic references,

vi) analysis of questions and development of search strategy,

vii) conducting of search,

viii) delivery of results of search, and

ix) utilization of results of search.

21. King, D. W., "Design and Evaluation of Information Systems", in C. A. Cuadra (Ed.), Annual Review of Information Science and Technology, Interscience Publishers, Vol. 3, 1968.

The list of information retrieval functions is extended by considering announcements of collection additions and changes. He also groups the other functions as follows:

i) surrogation - cataloguing, abstracting, and indexing;

ii) document management - storage, retrieval, and dissemination;

iii) index operations - storage, search, retrieval, dissemination; and

iv) correlation - creation of information, bibliographic preparation, answering technical questions.

22. Vickery, B. C., On Retrieval System Theory, 2nd Edition, Washington, Butterworth, 1965.

In a book on information retrieval systems, the author notes that a lot of feedback relationships are characteristic of such systems.

23. Wall, E., "A Rationale for Attacking Information Problems", American Documentation, Vol. 18, April 1967.

The classical functions are labeled "processing variables" to separate them from utilization variables such as economic, sociopolitical, and technical variables (such as user needs with respect to time and quality of service) and from origination variables.

24. Baker, N. R., and F. F. Leimkuhler, "Library Design and Operations Study", Report to NSF for Fiscal Year 1967-68.

A brief review of some of the library system studies undertaken at Purdue University is presented.

25. Borko, H., "Design of Information Systems and Services", in C. A. Cuadra (Ed.), Annual Review of Information Science and Technology, Vol. 2, Interscience Publisher, 1967.

He defines systems analysis as "a formal procedure for examining a complex process or organization, reducing it to its component parts, and relating these parts to each other and to the whole in accordance with an agreed upon performance criterion. A bibliography of other studies and a brief discussion of their contents concerning systems analysis in information systems is also given.

26. Jain, op. cit.

Library materials can be classified as research materials -- those materials that extend the frontiers of knowledge and constitute contributions to it or which enable the scholar to make such a contribution, such as monographs and research studies; and non-research materials -- those materials which do not make such contributions, such as texts and introductory works.

27. King, op. cit.

An extensive discussion of the available literature on the topic of systems analysis used in evaluating the performance of information systems.

28. Rees, A. M., "Evaluation of Information Systems and Services", in C. A. Cuadra (Ed.), Annual Review of Information Science and Technology, Vol. 2, Interscience Publisher, 1967.

An extensive bibliography and brief discussion of studies on systems analysis in information systems.

29. Taylor, R., "The Interface Between Librarianship and Information Science and Engineering", Special Libraries, Vol. 58, January 1967.

Systems analysis can provide new insights about information systems.

Library Evaluation

30. Altmann, B., "A Multiple Testing of the Natural Language Storage and Retrieval ABC Method: Preliminary Analysis of Test Results", American Documentation, Vol. 18, January 1967.

Estimates of the probabilities of an item being relevant with respect to a specific query are discussed.

31. Bednarik, A. B., and Jack Belzer, "A System Performance Function", in A. Kent, O. Taulbee, J. Belzer, and G. Goldstein, Electronic Handling of Information: Testing and Evaluation, Academic Press, London, 1967.

Regression analysis is used in considering service time versus user requirements, growth rate of queries, manpower requirements or manning levels, operating costs, precision, versus user requirements, recall, file relevance, and implementation manpower.

32. Brooks, B. C., "The Measures of Information Retrieval Effectiveness Proposed by Swets", J. of Documentation, Vol. 24, No. 1, March 1968.

Swets' measures of effectiveness are discussed in great detail. Some modifications are suggested.

33. Bryant, E. C., "Modeling in Document Handling", in A. Kent, O. Taulbee, J. Belzer, and G. Goldstein (Ed.), Electronic Handling of Information Testing and Evaluation, Academic Press, London, 1967.

Information systems must maintain some sort of trade-off between acquisitioning and associated processing of materials prior to requests, and borrowing items from other sources when needed. Time is important and so is the proportion of users unwilling to wait more than t days for a requested item which the author suggests follows an exponential distribution, $1-e^{-Kt}$. Other factors to be considered are system costs and efficiency.

34. Cheydleur, B. F., "Indexing Depth, Retrieval Effectiveness, and Time Sharing", in A. Kent, O. Taulbee, J. Belzer, and G. Goldstein (Ed.), Electronic Handling of Information Testing and Evaluation, Academic Press, London, 1967.

Evaluation of the indexing elements, one small part of the total system, is considered. This study shows how detailed one of many components can be.

35. Creager, W. A., and D. E. Sparks, "A Serials Data Program for Science and Technology", Final Report to NSF, Contract NSF C-413, (1965).

An interesting bibliography of the literature concerning evaluation of information systems is presented.

36. Cuadra C., and R. Katter, "The Relevance of Relevance Assessment", Proceedings of the 30th Annual Meeting of the ADI, New York, October 1967.

Users can and do discriminate on the basis of estimated pertinence to their specific needs; pertinence or relevance being a relationship between an item and a specific need for information.

37. Downs, R. B., A Survey of the Libraries of Purdue University, Purdue University Libraries, Lafayette, Indiana, 1967.

Quality, not quantity, and usefulness of a collection contribute to its value. Downs' report on a study conducted at Purdue University listed 11 major reasons for using a library:

i) find and read material required for a course,

ii) read library material for self-improvement,

iii) read for pleasure or fun,

iv) borrow library material for further reading,

v) do research for a term paper,

vi) do research for graduate exams or thesis work,

vii) do research for a publishable paper or book,

viii) get some material copied,

ix) return materials to the library,

x) do homework with his own books, and

xi) seek information which does not require borrowing of library materials -- reference services.

38. Dym, E. D., "Relevance Predictability I: Investigation Background and Procedures", in A. Kent, O. Taulbee, J. Belzer, and G. Goldstein (Ed.), Electronic Handling of Information Testing and Evaluation, Academic Press, London, 1967.

Factors such as user motivation, user population, and tools utilized are considered in the context of an information theory analysis to see how much a representation of an item reduces the uncertainty about the state of the item.

39. Fairthorne, R. A., Towards Information Retrieval, Butterworth, London, 1961.

Recall and precision ratios can vary without retrieval characteristics (such as indexing, selection, or criteria of acceptance) having been changed. Further, an arbitrary removal of pertinent items retrieved would drastically alter these ratios, which could be misleading. Finally, the author points out that there are several statistical problems that might occur.

40. Fairthorne [18], Op. cit.

Fairthorne identifies information flow relationships between sources (as authors), messages, designations (as topics), codes, destinations (as users), and channels.

41. Fasana, Paul J., and James E. Fall, "Processing Costs for Science Monographs in the Columbia University Libraries", Library Resources and Technical Services, II, pp. 97-114, Winter 1967.

The authors present a study of how they estimated various processing costs for science monographs. This includes selection and ordering (87.4¢), acquisition searching and ordering (125.2¢), receipt of materials and processing of invoices (26.6¢), miscellaneous activities (65.6¢) and materials (28.5¢), pre-cataloging (110.14¢), cataloging (397.0¢), card production (75.65¢), post-cataloging (67.03¢), bindery processing (13.7¢), and receiving procedures (29.0¢), for a total of $10.26 costs per book excluding the original cost of the book.

42. Gotterer, M. H., "Identification of Performance Criteria of an Electronic Information System", in A. Kent, O. Taulbee, J. Belzer, and G. Goldstein (Ed.), Electronic Handling of Information Testing and Evaluation, Academic Press, London, 1967.

The economic theory of supply and demand is stressed in evaluating the performance of an information system. The author points out that the determination of the price a user is willing to pay for service is necessary for evaluation purposes.

43. Jain [19], Op. cit.

Jain adds to the list of library functions by considering selection as a prerequisite to acquisition, identifying the indexing functions as classifying and cataloging functions, and by pointing out that the user is usually given such materials on a circulation or loan basis.

44. Kent, A., "Evaluating Procedures for Processing and Searching Metallurgical Literature", Communications of the ACM, Vol. 3, No. 11, 1960.

Qualitative factors (such as subject coverage, depth of analysis of source documents in a retrieval system, precision of retrieval service) are distinguished from quantitative factors (such as precision of retrieval service, speed).

45. Kent, A., Textbook on Mechanized Information Retrieval, 2nd Ed., Interscience Publishers, New York, 1966.

Some factors involved in measuring the performance of information systems are: physical requirements (equipment, form and type of input and output, and special conditions such as space limitations or security restrictions) and environmental variables (human acceptance and resistance to change; and changes in the system parameters such as interests of the clientele, complexity of the subject matter, available tools, file size and file use, and operating funds).

46. King [21], Op. cit.

A good discussion of some of the literature on the topic of testing and evaluation of information systems is presented. Some of the articles deal with cost studies of operations in a library environment. Studies are also cited that show a majority of items in an information retrieval system existing as either pertinent items retrieved in answer to a specific query or as nonpertinent items not retrieved, as viewed in Swets [58] contingency table analyses, and especially in the latter case. Overall correlation of pertinence and retrieval is high. Tests of correlation versus random scattering have been suggested. It seems that changes in the statistics in any of the pertinent-retrieved, pertinent-not retrieved, or nonpertinent-retrieved categories will alter the values concerning the benefits of the system but will not deter correlation.

Some studies of the relationship between recall and precision ratios imply that the usual hypothesized relationship as seen by Cleverdon is invalid in the limiting values. Other studies imply that these ratios should only be used to indicate when a system is faring poorly, requiring failure analysis. There are also several statistical estimation problems encountered when using these ratios to evaluate information systems.

Simulation studies are cited that investigate operating characteristics such as volume and frequency of work requirements, operating speeds, cost of equipment, personnel allocation, availability of failure rates of equipment, job scheduling, and queue-unloading algorithms. Finally, the bibliography should be consulted for these and other studies.

King cites studies showing that users rely on several sources of information, many of them informal, which is partly due to ignorance of the extent of services offered by formal systems. Further, the single most important factor may be accessibility of information. This implies that user convenience may exceed expected amount of information retrieved as a criteria for the user in determining his sources of information. He also cites Pollock who shows that users require various numbers of items depending upon the specific need.

King also extends the list of library functions by adding announcement and by grouping the functions:

i) surrogation -- cataloging, abstracting, and indexing;

ii) document management -- storage, retrieval, and dissemination;

iii) index operation -- storage, search, retrieval, dissemination; and

iv) correlation -- creation of information.

47. Korfhage, R. R., and T. G. DeLutis, "A Basis for Time and Cost Evaluation of Information Systems", Research Memorandum 69-6, School of Industrial Engineering, Purdue University, Lafayette, Indiana, June 1969.

Time and cost measures are used to evaluate internal design of information systems.

48. Lancaster, F., and W. P. Climenson, "Evaluating the Economic Efficiency of a Document Retrieval System", J. of Documentation Vol. 24, No. 1, March 1968.

Economic efficiencies of information retrieval systems are discussed. Usability, the value of literature included in a specific topic in terms of age, reliability, and comprehensibility, is added as an additional factor. Mention is made of the importance of selection criteria. Citation counts and circulation statistics are listed as data sources for the selection decision.

49. Lancaster, F. W., and J. Mills, "Testing Indexes and Index Language Devices: The ASLIB Cranfield Project", American Documentation, Vol. 15, No. 1, 1964.

Cleverdon's list of relevant evaluation factors is cited as including: time lag, effort required by users, presentation of output, extent of coverage of the collection, recall - the portion of pertinent articles that are retrieved, and precision - the portion of retrieved articles that are pertinent.

50. Maron, M. E., and J. L. Kuhns, "On Relevance Probabilistic Indexing and Information Retrieval", J. of the Assoc. for Computing Machinery, July 1960, Vol. 7, No. 3.

Estimates of the probability of an item being relevant with respect to a specific request are developed using a Bayesian approach. Termination of search efforts is not discussed, however.

51. McCarn, D. B., and C. R. Stein, "Intelligence Systems Evaluation", in A. Kent, O. Taulbee, J. Belzer, and G. Goldstein (Ed.), Electronic Handling of Information Testing and Evaluation, Academic Press, London, 1967.

The distribution of recall ratios yield different results than those obtained from an average value alone.

52. Mearns, D. C., The Story Up to Now, The Library of Congress, 1800-1946, Washington, D.C., p. 211-12.

Mearns cites the objectives of The Library of Congress as follows:

i) the library should own and make available such books, periodicals, and other library materials as are required by the students and faculties of the university in the performance of their research work;

ii) the library should own and make available such books, periodicals, and other library materials as are related to the instructional and research programs of the university;

iii) the library should own and make available those parts of the past and present graphic records of those cultures and peoples whose experiences are related in any way to the educational and research programs of the university; and

iv) the library should own all materials relating to the history, development, and character of the university.

53. Meier, R., "Efficiency Criteria for a Large Library", Library Quarterly, Vol. 31, pp. 215-34, (1961).

The author, interested in time losses to users, developed the unit item-use-day to measure the amount of materials charged out to users within a standard period of time. He noted that for every minute of time saved in getting library materials, faculty members gained two or three minutes acquiring and reading more literature. Moreover, he noted that faculty members spent 15 minutes while students spent seven or eight minutes, on the average, to travel to the library and attempt to obtain desired materials.

54. Rees, [28], Op. cit.

Systems evaluated in terms of comparisons with similar systems are discussed in this article which also investigates how well comparison techniques have been used.

55. Rzasa, P. V., "The Development of Measures of Effectiveness for a University Library", M.S. Thesis, School of Industrial Engineering, Purdue University, Lafayette, Indiana, 1969.

Some performance factors that cannot be controlled by the servicer are new departments, rapid increases in existing departments, technological development, government policies, industrial policies, and funder controls. Rzasa assumes the funders are user need oriented and develops some measures of effectiveness.

He designs a primary measure of effectiveness (PME) in terms of the total population served by the library (N), the number of users per a specific time period (n), the number of non-material information items sought per time period (n_i), the number of non-material information items provided per time period (n_a), the total number of users studying in the library with their own books or using the library for social purposes (n_s), and the total number of material items used per time period. His measure is

1.1) $$PME = w_m\left[\frac{n}{N}\left(1+\frac{n}{N}\right)\right] + w_a\left[\frac{n_a}{N}\left(1+\frac{n_i}{N}\right)\right] + w_s\left(\frac{n_s}{N}\right)$$

where w_m, w_a, and w_s are weighting factors. He then defines k subpopulations into which one can partition the users. His secondary measure of effectiveness for subpopulation j (SME(j)) is defined in terms of the number of individuals from group j who use the library facilities per time period (n_j), the number of times group j used the library facilities per time period (n'_j), the activity of group j or the number of times users came more than once to the library ($A_j = n'_j - n_j \geq 0$), the number of individuals

in group j (N_j), and the number of individuals in group j who do not use the library ($N_j - n_j$). He notes that

$$N = \sum_{j=1}^{K} N_j \quad \text{and} \quad n = \sum_{j=1}^{K} n_j .$$ Rzasa's secondary measure is

$$SME(j) = [(n_j' - n_j)/N_j]\left(\frac{n_j}{N_j}\right) = A_j n_j / N_j^2 .$$

56. Salton, G., Automatic Information Organization and Retrieval, McGraw-Hill Book Company, New York, 1968.

The following factors are listed as being relevant to the performance of information systems: user population -- type of users and rate which users enter the information system; indexing -- type, level, and accuracy of the indexing system; collection coverage; analysis and search -- type, power, and complexity as well as effort required; type of equipment and form of input and output; and operating efficiency -- cost, time lag, response time, and maintenance or service problems.

The author suggests that measures of performance and efficiency are becoming increasingly important as information systems are being used more and more. Further, the fact that automatic systems encounter fewer human errors but require more complex evaluation procedures is pointed out.

A very heavy mathematical treatment of recall and precision rates is also given. Methods for estimating these measures, presenting results, and using them in designing information systems are discussed in some detail.

57. Snyder, M. B., "Some Behavioral Aspects of Test and Evaluation", in A. Kent, O. Taulbee, J. Belzer, and G. Goldstein (Ed.), Electronic Handling of Information Testing and Evaluation, Academic Press, London, 1967.

Human factors are an important factor in the testing and evaluating processes.

58. Swets, John A., "Information Retrieval Systems", in M. Kochen The Growth of Knowledge, John Wiley & Sons, Inc., New York, 1967.

A contingency table analysis of pertinence versus retrieval is set up and discussed. The proportion of items (a, b, c, d) and the value or cost (V_1, X_1, X_2, V_2) for the four categories (pertinent-retrieved, nonpertinent-retrieved, pertinent-nonretrieved nonpertinent-nonretrieved), respectively, are defined. Several studies concerning the use of these variables to define measures of system performance are discussed in detail.

Bourne, et.al., proposed an "omnibus" measure based not only on the contingency table variables associated with retrieval (a and b) but also on delay, ease of communications, complexity of search logic accommodated, form in which items are delivered, and degree of assurance that items or a given subject do not exist. Some of the latter elements are measures of efficiency rather than of effectiveness, concentrating on factors such as time, convenience, operating cost, and product form. The measure attempts to combine aspects of system performance and related user requirements; however, it is not clear that a single metric can successfully combine all these various elements together.

Bornstein divides pertinence into four categories (pertinent as represented by the variable a, partially pertinent, peripherally pertinent, and nonpertinent as represented by b) in discussing what has been retrieved. He feels the responses to queries, the time spent by users in examining the responses or retrieved materials, the substitution for "hard copy", and actual coincidence and uniqueness of responses can be compared in each of the four categories of pertinence. He also suggests consideration of interaction terms and defends this relative measure by saying that absolute measures require excessive effort and introduce biases.

Willys introduced a sequential search routine and proposed a restriction ratio as the reduction in the number of potentially pertinent items effected by such a search. He also considered the cost of using this method with provisions for time, money, inconvenience, and indexing costs, the variable 'c', and the loss function 'K_2' with provisions for degrees of pertinence.

Verhoeff, Goffman, and Belzer propose a measure which when translated into our terms becomes

$$M = aV_1 - bK_1 - cK_2 + dV_2 .$$

It is assumed that V_1, V_2, K_1, and K_2 are nonnegative and that each user will define his own concept of pertinence. To maximize this measure, items would be retrieved only if it had a probability of pertinence, calculated for each item relative to each query, greater than a critical probability

$$P_c = (V_2 + K_1)/(K_1 + K_2 + V_1 + V_2).$$

While this measure is actually more germane to the design rather than the analysis and evaluation of an information retrieval system, it still bears mention here.

Swanson has suggested a slightly different performance measure, expressed in our terms as follows:

$$M = R - K_1[(a+b) - (a+c)R] = R[1 + K_1(a+c)] - K_1(a+b),$$

where R denotes the ratio of the sum of the relevance weights of all items retrieved in answer to a specific query to the sum of the relevance weights of all items in the store. An example of a relevance weight would be the estimate of an item's pertinence, q. Borko modifies the measure somewhat to obtain the following:

$$M = R - [b/(a+b)].$$

Mooers proposes three ratios calculated as the number of items retrieved divided by the number of items in the store, the differences coming in terms of the pertinence of the items retrieved for any specific query. The first ratio considers only crucially pertinent items, is considered an important ratio, and is indicative of a good system if near unity. The second ratio considers crucially pertinent and pertinent items, is somewhat less important than the first, and should also be near unity if the system is effective. The last ratio considers nonpertinent items, is as important as the first, and a good system will possess a value for this ratio near zero.

Perry and Kent concentrate on the variables a/a+c and a/a+b. Cleverdon calls these variables the "recall ratio" and the "relevance ratio" or "precision" respectively. He hypothesizes an inversely proportional relationship between the two ratios. He also considers coverage of the collection, time lag, presentation of output, and effort required of user. As one increases the restrictiveness of responses, the relevance ratio increases while the recall ratio decreases and vice versa if restrictiveness is decreased. Swanson recommended that a/a+c be plotted against b to yield a positive relationship, the recall ratio increasing as b does but with a decreasing slope. Swets, on the other hand, has suggested plotting recall ratios against portions of nonpertinent articles retrieved, b/b+d. This will yield a curve similar to Swanson's, but will also incorporate all terms of the contigency table and assumes the form of an operating curve, such as used by statisticians.

59. Treu, S., "Testing and Evaluation Review", in A. Kent, O. Taulbee, J. Belzer, and G. Goldstein (Ed.), Electronic Handling of Information Testing and Evaluation, Academic Press, London, 1967.

Studies concerning testing as well as evaluating information systems are discussed. The author gives a lengthy bibliography of such studies.

60. Tukey, John W., "A Statistician's Comment", in A. Kent, O. Taulbee, J.Belzer, and G. Goldstein (Ed.), Electronic Handling of Information Testing and Evaluation, Academic Press, London, 1967.

It is difficult to make precise estimates of factors used in performance measures, but they must somehow be made. Furthermore, they must be made adequately enough so that improper conclusions will not be formed.

61. Wall [23], Op. cit.

Wall calls the technical processing functions "processing variables" as opposed to utilization variables such as economic, sociopolitical and technical variables (such as user needs with respect to time and quality of service) or origination variables.

62. Welch, John T., Jr., "Measures and Criteria for Evaluation", in A. Kent, O. Taulbee, J. Belzer, and G. Goldstein (Ed.), Electronic Handling of Information Testing and Evaluation, Academic Press, London, 1967.

Service time, handling costs, manpower requirements, and file relevance are suggested as the important factors in measuring the efficiency of an information retrieval system.

63. Downs [37], Op. cit.

A study conducted at Purdue lists 11 major reasons for using a university library:

1) to find and read materials required for a course,

2) to read library materials for self improvement,

3) to read for pleasure or fun,

4) to borrow library materials for further reading,

5) to do research for a term paper,

6) to do research for graduate exams or thesis work,

7) to do research for a publishable paper or book,

8) to get some materials copied,

9) to return materials to the library,

10) to do homework with one's own books, or

11) to seek information which does not require borrowing of library materials -- reference services.

64. Herner, Saul, and Mary Herner, "Information Needs and Users in Science and Technology", in C. A. Cuadra (Ed.), Annual Review of Information Science and Technology, Vol. 2, Interscience Publishers, 1967.

References on usage of library materials are presented.

65. Jain [19], Op. cit.

Appendices A and B and Chapter I of his doctoral dissertation give several examples of previous studies of user behavior. Brief descriptions of the work, data, models if used, and results are given (up to 1965). Most of the work has been in the area of determining who the user is and what quantity of usage can be attributed to various library materials.

66. King [21], Op. cit.

A good discussion of several articles related to the testing and evaluation of information systems. Studies are cited that show that users rely on many sources for information, many of them informal, which is partly due to ignorance of the extent of services. The single most important factor may be accessibility of information which implies that user convenience may exceed expected amounts of information retrieved as a criteria for the user in determining his sources of information.

67. Paisley, W., "Information Needs and Uses", in C. A. Cuadra (Ed.), Annual Review of Information Science and Technology, Vol. 3, Interscience Publishers, 1968.

References on usage of library materials are presented.

68. Pollock, S. M., "Measures for the Comparison of Information Retrieval Systems", American Documentation, October 1968.

Users require various numbers of items depending upon the specific need.

69. Trueswell, R. W., "User Behavior Patterns and Requirements and Their Effect on the Possible Applications of Data Processing and Computer Techniques in a University Library", Ph.D. Thesis, Northwestern University, Evanston, Illinois, 1964.

User needs are shown to be very important in terms of understanding how to improve library service. User needs are categorized by users being divided, in a university library environment, into faculty and student body components. With the faculty having both teaching and research needs, the undergraduate students have course work needs, and the graduate students have both course work and research needs. He also states that a university library has three functions. It is a social center, a study hall, and an information retrieval system.

Journal Productivity

70. Bradford, S. C., Documentation, Crosly Lockwood & Son, Ltd., London, 1953.

The Bradford law, stating that journals can be ranked and divided into equally productive zones, with the number of journals in each successive zone being represented as a geometric series, is formulated and discussed.

71. Brookes, B. C., "The Derivation and Application of the Bradford-Zipf Distribution", Journal of Documentation, Vol. 24, No. 4, December 1968.

Criticizing the Bradford distribution as developed by Leimkuhler, the author derives a similar Bradford-Zipf law, viewing the relative frequency of ranked items as being proportional to the inverse of the rank.

72. Brookes, B. C., "Bradford's Law and the Bibliography of Science", Nature, Vol. 224, December 6, 1969.

The formulation of Bradford's law is divided into two segments to account for the tail,

$R(n)$ = the number of articles in the top n ranked journals

$$= \begin{cases} \alpha n^{\beta} & 1 \leq n \leq c \\ K \log n/s & c \leq n \leq N. \end{cases}$$

Data are used to show how realistic his model is.

73. Cezairlyan, A. O., P. S. Lykoudis, and Y. S. Touloukian, "A New Method for the Search of Scientific Literature Through Abstracting Journals", J. of Chem. Documentation, Vol. 2, No. 86, 1962.

A comprehensive source of scientific journals is conducted for useful articles on thermophysical properties.

74. Cole, "A New Look at Reference Scattering", J. of Documentation, Vol. 18, No. 2, June 1962.

A linear relationship between the cumulative fraction of total titles and cumulative fraction of total references is obtained.

75. Good, J. J., "The Population Frequencies of Species and the Estimation of Population Parameters", Biometricka, Vol. 40, December 1953.

Population frequencies of species are discussed and a scattering function shown that is similar to the general scattering distributions.

76. Groos, "Bradford's Law and the Keenan-Atherton Data", American Documentation, Vol. 19, No. 1, 1967.

The fact that only approximately 4 percent of the journals had 50 percent of the articles is noted. Also 50 percent of the journals, the top 50 percent carried approximately 93 percent to 95 percent of the articles. He poses an interesting proposition that service to the user is evaluated by the user in terms of titles of journals produced and not in terms of the number of citations or articles produced from a small fistful of titles.

77. Haight, F. A., "Some Statistical Problems in Connection with Word Association Data", Journal of Mathematical Psychology, Vol. 3, No. 1, February 1966.

Several distributions such as the Yule distribution, Borel distribution, Fisher distribution, Zeta distribution and Zipf conjecture are discussed. He also lists several phenomena which have scattering characteristics: word lists, words in a given book, scientists and a number of published papers, size and color of cars, counties in states, abundance of chemical elements, size of cities, income of individuals, length of congressional speeches, number of species of genera, area of countries, and frequency of surnames.

78. Holt, C., and W. Schrank, "Growth of the Professional Literature in Economics and Some Other Fields and Some Implications", American Documentation, January 1958.

Literature growth is discussed and exponential growth models presented.

79. Kendall, M. G., "The Bibliography of Operational Research", Operational Research Quarterly, Vol. 11, No.s 1 and 2, (1960).

The bibliographic data of articles on operational research topics was used to demonstrate Bradford's law. Comparisons to similar scattering functions were also made.

80. O'Neill, E. T., "Journal Usage Patterns and Their Implications in the Planning of Library Systems", Ph.D. Thesis, School of Industrial Engineering, Purdue University, Lafayette, Indiana, January 1970.

O'Neill discusses several distributions which can be used to describe scattering such as the Yule distribution, the Borel distribution, the Fisher distribution, and the Zeta distribution. O'Neill also points out that Brookes [71] has misused the statement of Bradford's law and inadvertently come up with an approximation that is incorrectly claimed as exact.

81. Simon, Herbert, "On A Class of Skew Distribution Functions", Biometrica, Vol. 42, 1955.

The scattering functions exhibited in various sociological, biological, and economical phenomena are discussed in detail. They are usually 'J' shaped or at least highly skewed with very long upper tails. The tails are approximated by the form $f(i) = (a/i^k)b^i$ where a, k, and b are constants and b is just less than unity. The constant k is much greater than 1 and may sometimes be very close to 2. Usually the functional form also describes the distribution for small i with $f(2)/f(1)$ being about 1/3 and rarely reaching 1/2 and $\sum_{i=1}^{\infty} f(i)$ being about 1/2. Examples of such distributions are the negative binomial, Fisher logarithmic series, and Yule model.

A stochastic model is developed for scattering of word frequencies in works of literature. Generalization of the model and explanations of why the model holds for such phenomena are also given.

82. Stevens, R. E., "Characteristics of Subject Literatures", American College and Reasearch Libraries, Monograph Series, VI, 1953.

Data are collected and analyzed to verify Bradford's law.

83. Vickery, B. C., "Bradford's Law of Scattering", J. of Documentation, Vol. 4, 1948.

Bradford's initial article is clarified and a correct method of fitting a theoretical curve to observed data is developed. The law is seen to be independent of the number of zones chosen although this does affect the value of the ratio multiplier.

84. Zipf, Human Behavior and the Principle of Least Effort, Addison-Wesley Publishing Company, 1949.

In a book showing how humans behave as if minimizing effort, the author develops a scattering relationship. The proportion of articles in a journal should be inversely proportional to the relative rank (rank divided by the total number of journals in the set) of the journal.

[5]. Greenberg, H., and R. L. Hegerich, "A Branch Search Algorithm for the Knapsack Problem", Management Science, Vol. 16, No. 5, January 1970.

The authors develop a branch and bound algorithm, based on Kolesar's work, for the knapsack problem, stated as

$$\text{Max}\left[\sum_{i=1}^{n} v_i x_i \;\middle|\; \sum_{i=1}^{n} w_i x_i \leq W, \quad x_i = 0 \text{ or } 1 \quad \forall\, i=1,\ldots,n\right].$$

They point out that there is no loss of generality in assuming that the parameters W and w_i and v_i $\forall\, i$ are nonnegative.

Glover provides transforms of the general case to the totally nonnegative case. The authors also assume that the variables are numbered so that $v_1/w_1 \geq v_2/w_2 \geq \ldots \geq v_n/w_n$.

1. An initial feasible solution is formed as follows:

$$x_i = \begin{cases} 1 & \text{if } i < r \\ 0 & \text{if } i > r \\ (W - \sum_{i=1}^{r-1} w_i)/w_r & \text{if } i = r \end{cases}$$

where r is the smallest integer such that $\sum_{i=1}^{r} w_i \geq W$. If $r=0$, no feasible solution exists. If all parameters are nonnegative, this cannot occur. If $r = n-1$, the trivial solution $x_i = 1$ $\forall\, i$ is optimal. Otherwise, we go on to step 2a).

2a. If x_r is fractional, i.e., $x_r \in (0,1)$, the initial objective function is $z(1) = \sum_{i=1}^{r-1} v_i = v_r x_r$, where $z(n)$ is the value of the objective function at node n of our decision tree. Note here that $n=1$. One always branches to the node with the largest value of $z(n)$.

2b. If, at node n, all values of x_j are integer, we have reached the optimum solution.

3. If all values of x_j are not integer at node n, calculate the objective function for the $n+1^{st}$ node by setting the fractional variable, x_t, equal to zero and resolving the original problem via step 1) with this additional constraint. Then, set x_t equal to unity and resolve the problem, yielding $z(n+2)$. Then recycle to step 2a).

The authors also develop a modification of the algorithm, using what they term a branch search technique. An example and computational results are presented.

36. Kolesar, P. J., "A branch and bound Algorithm for the Knapsack Problem", Management Science, Vol. 13, 1967.

The author develops a branch and bound algorithm for the knapsack problem. He assumes that the parameters are all non-negative and notes that this implies the feasibility of the trivial solution where $x_i = 0 \ \forall i$. He further points out that, under the non-negativity of parameters assumption, the trivial solution where $x_i = 1 \ \forall i$ is optimal if it is feasible

$$\left(\sum_{i=1}^{n} w_i \leq W \text{ where } \left[\sum_{i=1}^{n} w_i x_i \leq W, \ x_i = 0 \text{ or } 1 \ \forall i=1,\ldots,n \right] \right.$$

is the constraint set).

His algorithm is not very different than the one developed by Greenberg and Hegerich, which is based on Kolesar's work. However, Kolesar works with the constrained variable with the smallest index. Kolesar assumes that the variables are ordered in decreasing order on the ratio v_j/w_j, where the objective function is

$$\max \sum_{i=1}^{n} v_i x_i \ .$$

APPENDIX B

SOME INTERESTING ASPECTS OF THE DUALITY OF AN INTEGER PROGRAMMING PROBLEM

APPENDIX A

SOME INTERESTING ASPECTS OF THE DUALITY OF AN INTEGER PROGRAMMING PROBLEM

Balas[1] discusses a duality theory for discrete programming. In order to better understand how Everett's technique works, we shall now explore this theory. He considers as the primal problem

$$\max_{x,y} \; [f=cx \mid Ax+y=b,\; x \geq 0,\; y \geq 0,\; x \text{ integer}]$$

and states that its dual is

$$\max_{x} \min_{\delta} [\delta=vr \mid vA-v=c,\; x \geq 0,\; \delta \geq 0,\; x \text{ integer},\; v \text{ unconstrained}].$$

He also proves the following properties for the dual problem:

a. $f^* = \max_x cx = \min_\delta \max_x (cx+\delta y)$ for all feasible x and y.

b. The dual of the dual problem is the primal problem.

Further, x integer $\longleftrightarrow$ v unconstrained

$y \geq 0 \longleftrightarrow \delta$ constraints.

---

1. Balas, E., "Duality in discrete programming", Technical Report No. 67-5, Department of Operations Research, Stanford University, Stanford, California, July 1967.

c. If feasible solutions exist for the primal problem, then there exists a $\bar{\pi} \geq 0$, with $\bar{\pi}A \geq c$, such that $cx \leq \bar{\pi}b$ for all feasible x.

d. If $\bar{x}$ is an optimal primal solution, there exists a $\bar{\pi}$ such that $(\bar{\pi},\bar{y})$ is an optimal dual solution with $F(\bar{x},\bar{\pi}) = F(\bar{x},\bar{y}) = F(\bar{\pi},\bar{y})$, implying a saddle-point solution. Note that $F(x,\pi) = cx + \pi(b-Ax)$.

e. If $\bar{x}$ is the optimal primal solution, there exists a $\bar{\pi} \geq 0$ such that $\bar{\pi}\bar{y} = 0$. If $(\bar{\pi},\bar{x})$ is the optimal solution to the dual problem, $\bar{\pi}\bar{y} = 0$. Here $\bar{y}$ is the optimal primal slacks corresponding to $\bar{x}$. Further, $f^* = c\bar{x} = \min_{\pi \geq 0} \pi b - \nu\bar{x}$.

f. x is an extreme solution to the primal problem if x is an extreme point of the convex hull of the primal feasible region. Thus, π is an extreme solution to the dual problem if π is an extreme point of the convex hull of the dual feasible region. x is a nondegenerate solution if the number of positive elements of y equals the number of constraints; degenerate if the number of positive elements is less.

g. If $\bar{x}$ is a unique, nondegenerate, optimal primal solution, $(\bar{\pi},\bar{x})$ is a unique, optimal dual solution.

h. If the primal feasible region is an empty set, both objective functions are equal to $+\infty$. If the dual feasible region is an empty set, both objective functions are equal to $-\infty$. If both the primal and dual are empty

sets, both objective functions are undefined. If both feasible regions are not empty sets, both objective functions are finite and they are equal. Of course, one and only one of the above cases holds for both the primal and dual problems.

i. If $\bar{x}$ is the optimal primal solution and $(\bar{\pi},\bar{x})$ is the optimal dual solution, $(\bar{\pi},\bar{v})$ is the optimal solution to $[\min_{\pi,v} [\pi b - v\bar{x} \mid \pi A - v \geq c, \pi \geq 0, v \text{ unconstrained}]]$.

j. When a primal variable x_j is an integer by constraint, the associated dual constraint is 'relaxed' by the introduction of an unconstrained slack v_j. The dual objective function tries to maximize vx with respect to x while minimizing with respect to π (or v). Thus we cannot find a primal feasible solution that leads to a dual feasible solution as in linear programming. Here, we come as close as possible to dual feasibility and the correction $-v_j$ measures the gap by which we fail. Note that we might also map by x_j and attempt to maximize it with respect to v. In discrete programming we do not necessarily have the 'nice' result that $\bar{v}\,\bar{x} = 0$ as in linear programming.

k. If we can generate the convex hull of the primal feasible solutions given by the linear programming problem $\max_{x} [cx \mid Ax = b, x \geq 0]$, then we can solve the dual to that integer programming problem by solving $\min_{\pi,v} [\pi b - vx \mid \pi A - v \geq c, \pi \geq 0, \pi_i \text{ unr.}]$, where we use the solution $\bar{\pi}$, $\bar{v}$ where

holds for the optimal solution to this linear programming problem. Further, if $(\bar{\delta},\bar{\eta},\bar{\xi})$ optimizes this dual problem, it optimizes $\min_{\delta,\eta,\xi}[\delta b + \eta\bar{v}' \mid \delta A + \eta\rho - \xi = c,\ \delta \geq 0,\ \eta \geq 0,\ \xi \geq 0]$ with $\bar{x}$ as a basic optimal primal solution. Thus $\bar{v} = \bar{\eta}\rho + \bar{\xi}$ and $\bar{\xi}_j = 0$ if $\bar{x}_j > 0$. However, in general one cannot easily generate the necessary constraints to describe the convex hull of the primal feasible region.

1. If $\bar{x}$ is an optimal solution to the original primal problem and $(\bar{\delta},\bar{\eta})$ is an optimal solution to the original dual problem, then $\bar{x}$ is an optimal solution to $\max_{x}[(c+s)x' \mid Ax \leq b,\ x \geq 0]$ where

$$s_j = \begin{cases} \bar{v}_j & \text{if } \bar{x}_j > 0 \\ \min[0,\bar{v}_j] & \text{if } \bar{x}_j = 0 \end{cases}$$

Thus we have the facts that $\bar{\delta}A \geq c+s$, $(\bar{\delta}A-c-s)\bar{x} = 0$, $\bar{\delta}(b-A\bar{x}) = 0$, $(c+s)\bar{x} = \bar{\delta}b$, and $\bar{x}$ and $\bar{\delta}$ are unique if $\bar{x}$ is a unique, optimal primal solution.

m. We see that $\bar{v}$ need not be optimal to the linear programming problem formed by dropping the intensity constraints from the primal problem. We see that the changing of c to $c+s$ changes the slope of the objective function hyperplane to make it parallel to one of the facets of the convex hull of the primal feasible region. We also see that $\bar{\delta}_i$ is the change in the linear programming objective function due to a unit increase in b_i. In discrete time queuing, $\bar{\eta}_i \geq 0$

means the objective function will not change due to a unit change in b_i, but it does not mean that the i^{th} constraint is not binding. By binding, we mean that the removal of the i^{th} constraint renders $\bar{x}$ no longer optimal. Finally, we see that shadow prices alone cannot induce an optimal primal solution since other noninteger solutions lie on the same facet of the convex hull of the feasible region which satisfy the optimality requirements.

In order to sum up these properties, let us consider the Lagrangian formulation of the primal problem,

$$\begin{array}{lll} \text{Min} & \text{Max} & L(x,\pi) = cx + \pi(b-Ax-y) \quad cx + \pi b - \pi Ax - \pi y \\ \pi \geq 0 & x \geq 0 & \\ & x \text{ integer} & \leq F(x,\pi) - \pi y \\ & y \geq 0 & \end{array}$$

We note that since $b-Ax-y = 0$ for primal feasibility, $L(x,\pi) = cx$ for all feasible x. Further, we note that $\pi A - v = c$, or $v = \pi A - c$, for dual feasibility; so that we can re-write that v is unconstrained. Finally, we note that, as Balas points out, $\bar{\pi}\,\bar{y} = 0$, implying that $L(\bar{x},\bar{\pi}) = c\bar{x} = F(\bar{y},\bar{v})$, where $\bar{x}$, $\bar{y}$, $\bar{\pi}$, and $\bar{v}$ imply optimal solutions.

APPENDIX C

COMPUTER LISTINGS

The first program (Figure C1) is used to evaluate the parameters a and c for the modified Cobb-Douglas model of IV-2(b). It uses the original estimates of α_t from the cross-section results, which are discussed in Chapter IV. The second program (Figure C2) uses the raw data from the Social Sciences Literature Data Base, discussed in Chapter V, and converts it into the time series n_t, n_t^*, $\delta_{j,t}$, $\delta^*_{j,t,a}$, $\alpha_{j,t,q}$, and $\alpha'_{j,t,q}$, corresponding to the variables of literature growth and scattering. The third program (Figure C3) uses the statistics $\bar{x}_t$, as calculated in the second program, and calculates estimates for the "radius" parameter over time, R_t. The fourth program (Figure C4) calculates the limits on E_t as a function of s as per Chapter V, and finds estimates for i_t corresponding to these limits. The fifth program (Figure C5) uses the time series n_t and n_t^* to estimate parameters for the epidemic model of literature and to calculate predictions to test the goodness-of-fit. The sixth program (Figure C6) [illegible] the basic calculations of Steps 2 through 4) of the Gozinto procedure, as outlined in Chapter VI. Using [illegible] the [illegible]

are calculated and displayed. Finally, the [illegible] (Figure 27) performs all calculations of the bayesian algorithm, also discussed in Chapter VI, including the linear programming portions. All solutions generated are displayed in the output.

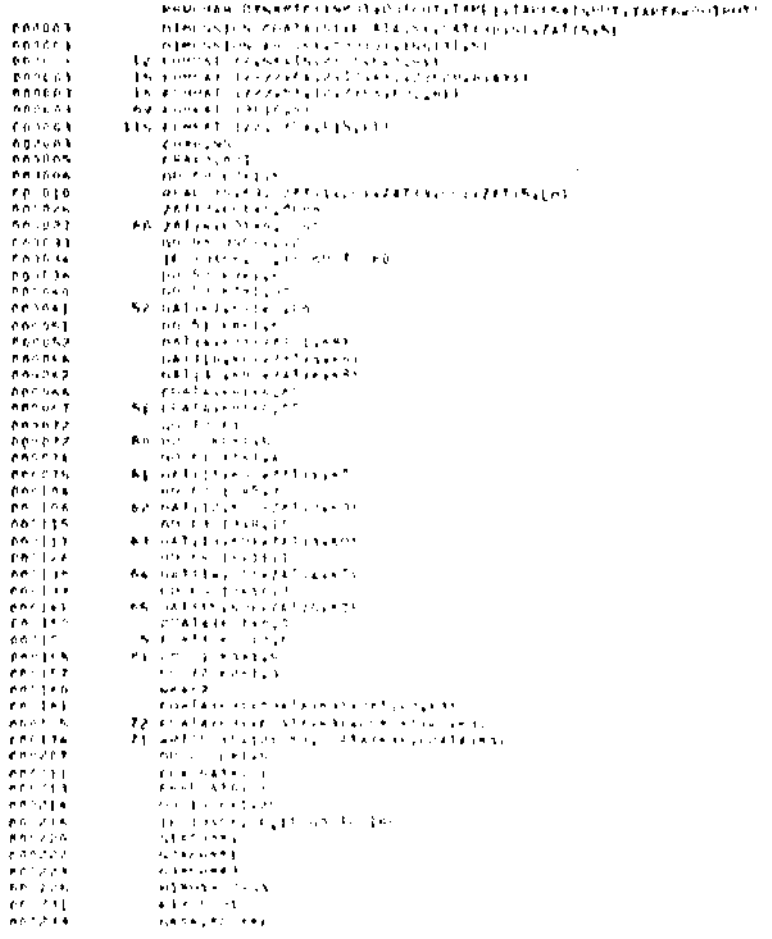


Figure 61.

```
0012f0
0012f4
0012f8
0012fc
001300
001304
001308
00130c
001310
001314
001318
00131c
001320
001324
001328
00132c
001330
001334
001338
00133c
001340
001344
001348
00134c
001350
001354
001358
00135c
001360
001364
001368
00136c
001370
001374
001378
00137c
001380
001384
001388
00138c
001390
001394
001398
00139c
0013a0
0013a4
0013a8
0013ac
0013b0
0013b4
0013b8
0013bc
0013c0
0013c4
0013c8
0013cc
0013d0
0013d4
0013d8
0013dc
```

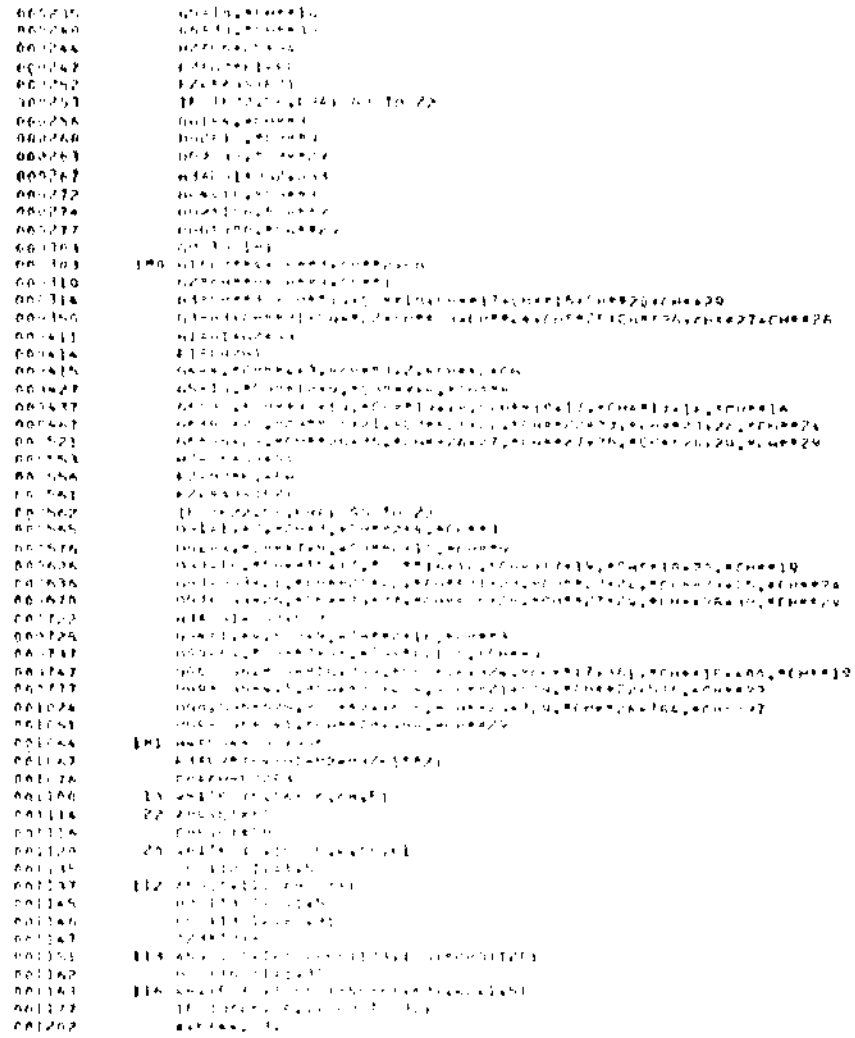


Figure C1. (illegible)

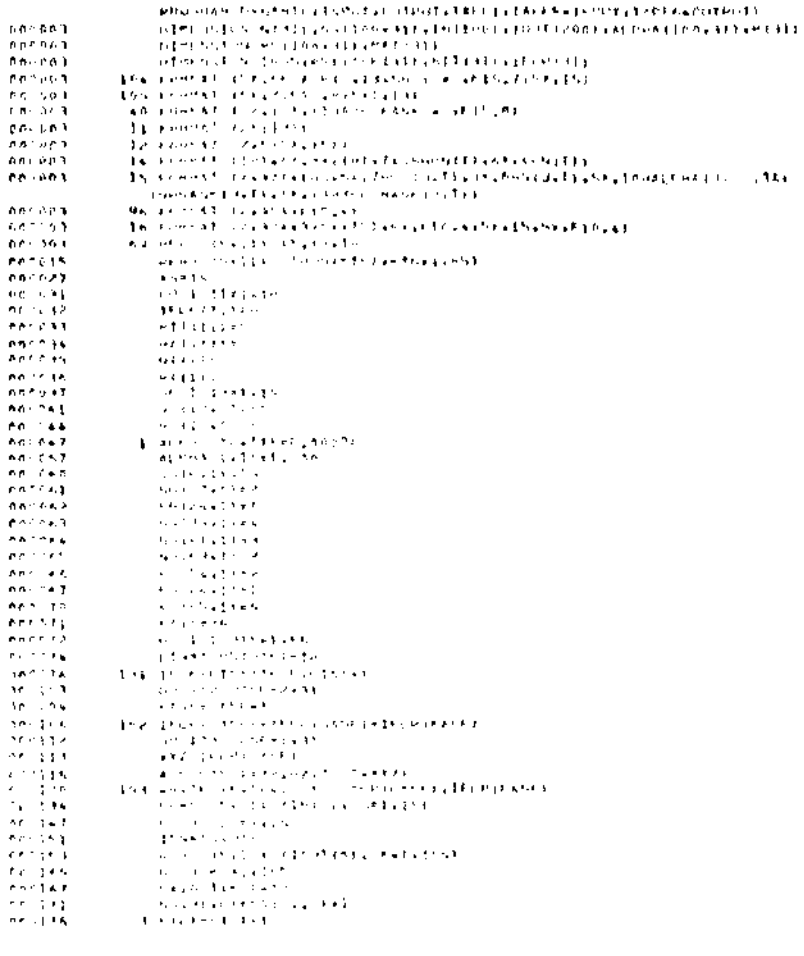


Figure 42. A [illegible] Concept the [illegible]
Listed [illegible] and [illegible] to [illegible]

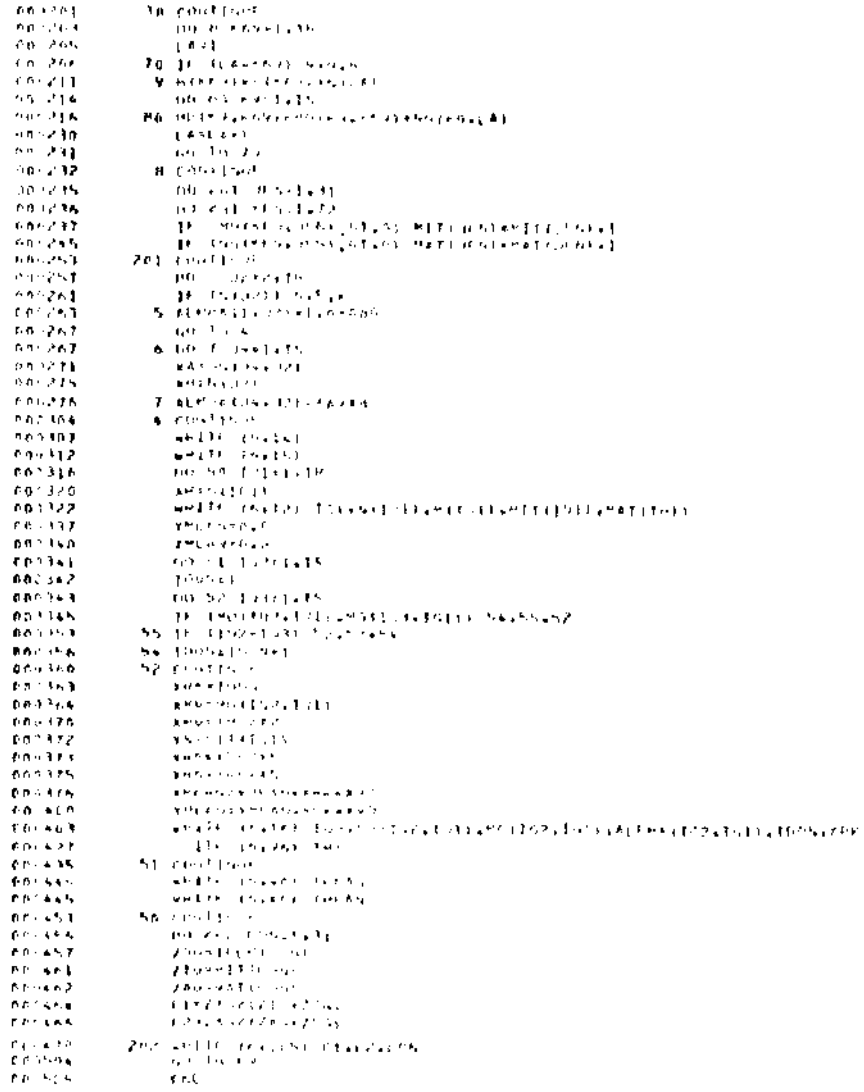


Figure (9. (C .11 ..1)

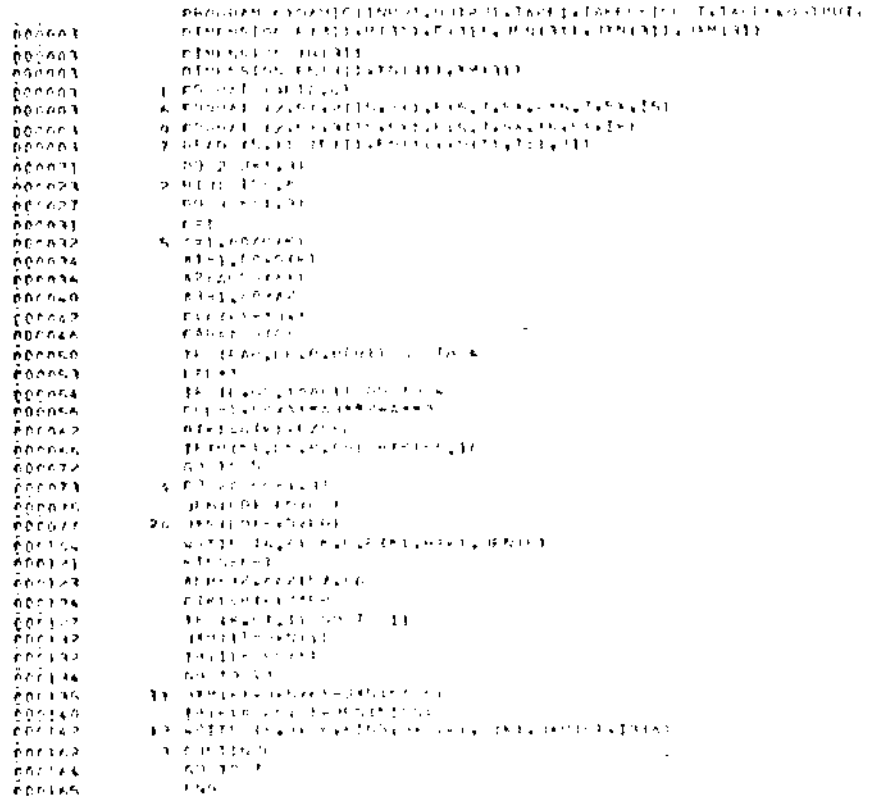


Figure 3. [illegible]

```
      PROGRAM CONTINUED FRACTION EXPANSION OF PI
      DIMENSION A(20),P(20),Q(20)
    1 FORMAT (I2,2(F10.5,I5),F20.15,F20.15)
   10 FORMAT (I2,F10.5,F20.15)
      A(0)=3.
      N=1
      S=3.
      DO 20 I=1,5
      P(0)=3.*A(0)+1.
      DO 20 J=1,2
      I=I
      N(K)=10.
    4 X=1./P(I)
      A(I)=INT(X)
      P(I+1)=10.*(X-A(I))
      A2=A(I)
      P=1.
      DO 3 K=1,I
      IF(A(K).EQ.A(K-1)) GO TO 4
      I=I+1
      IF (I.GE.10001) GO TO 4
      Q(I)=A(I)*Q(I-1)+Q(I-2)
      N(K)=N(K)+1
      IF(N(K).LT.A(K)+1) N(K)=0
      GO TO 3
    4 C(K)=Q(I-1)/Q(I)
      WRITE (6,1) K,A(K),N(K),C(K),P
   20 CONTINUE
      IF (I.GT.21) STOP
      S=1.*(I-1)
      J=I
      GO TO 10
      END
```

Figure 6. A Computer Program to Evaluate the Limits of the Sequences $\{p_n\}$, $\{q_n\}$ and to Calculate the Corresponding Estimates of π

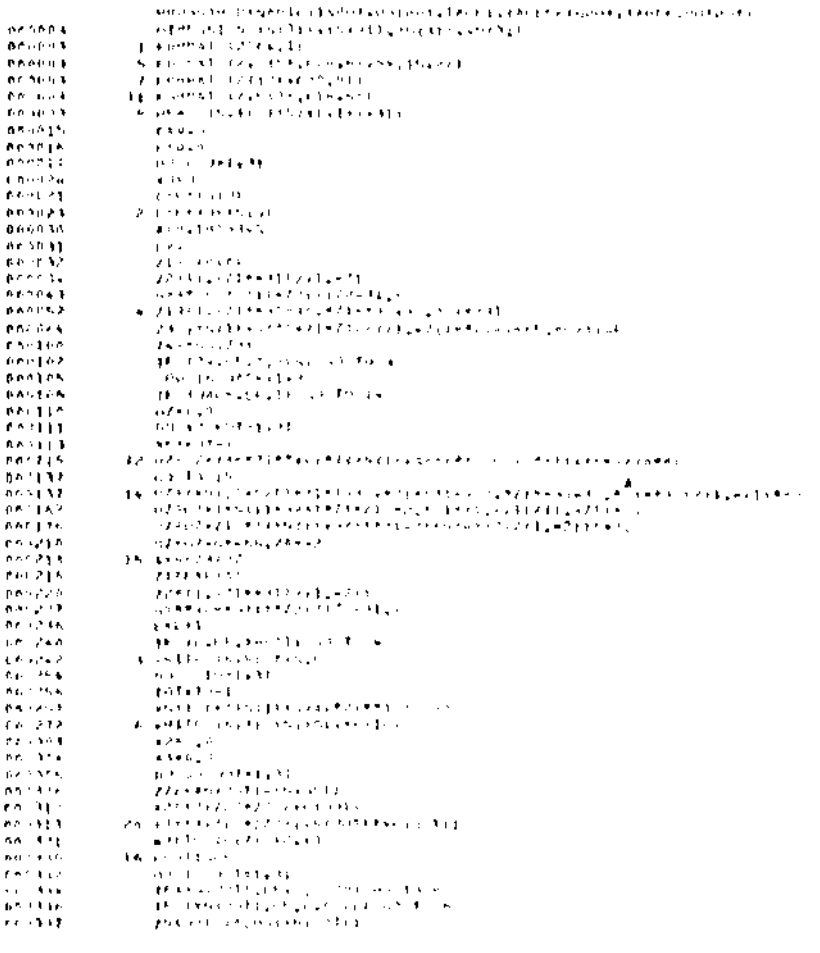


Figure 6. [illegible]

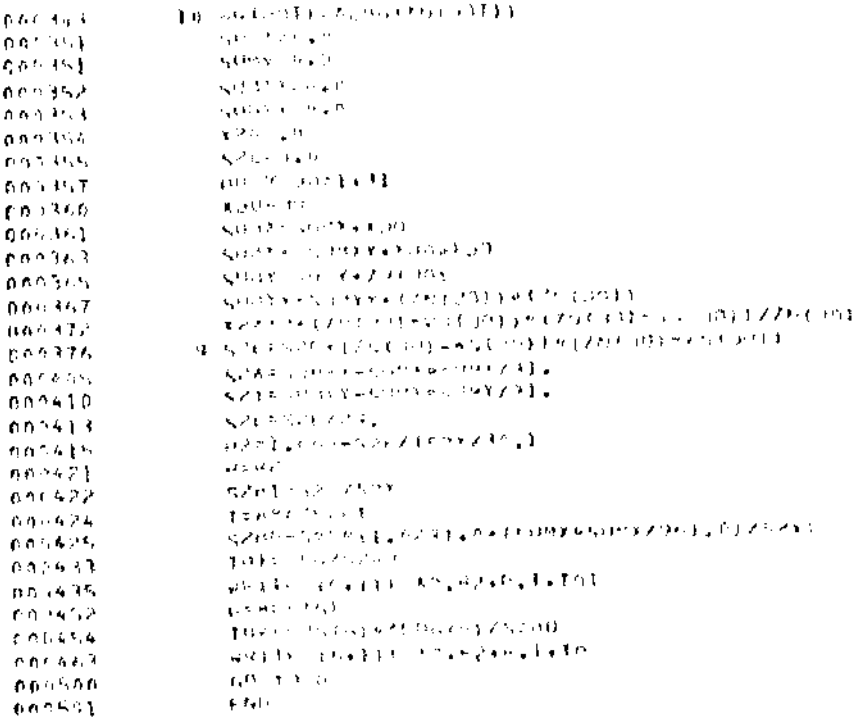


Figure C. (Continued)

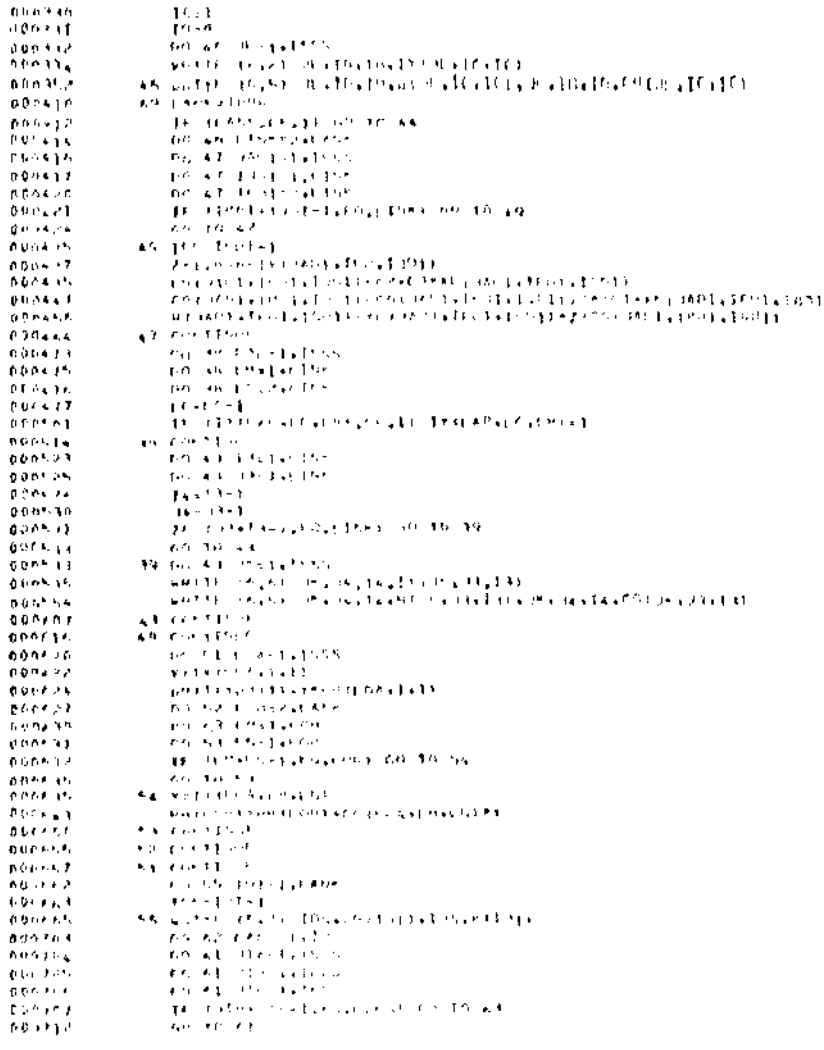


Figure C1. (See transcript)

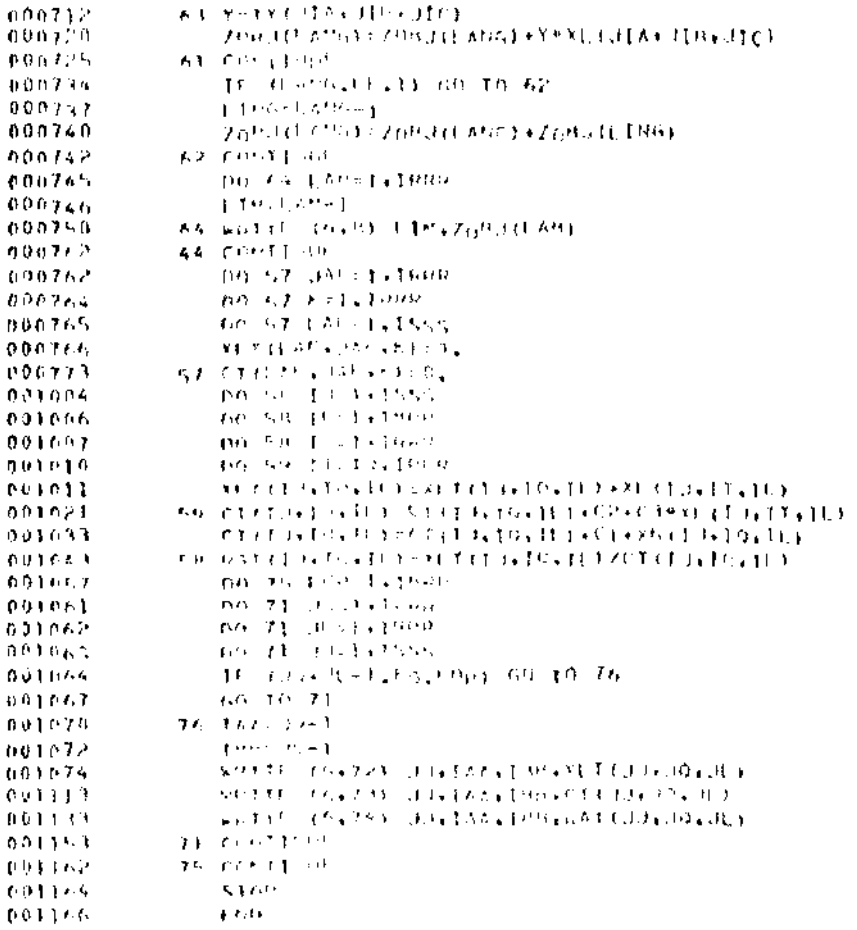


Figure 8. (Continued)

```
[illegible]
```

Figure [illegible]. [illegible]
[illegible]

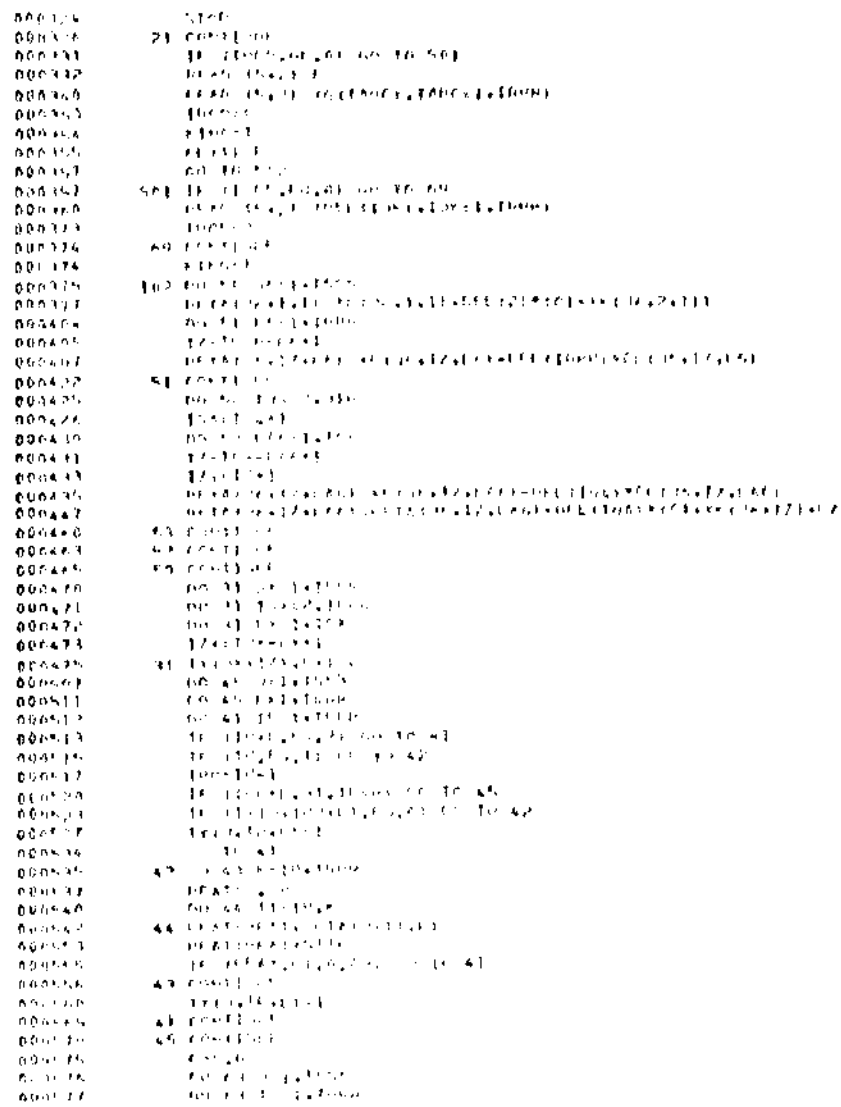


Figure 77. (Continued)

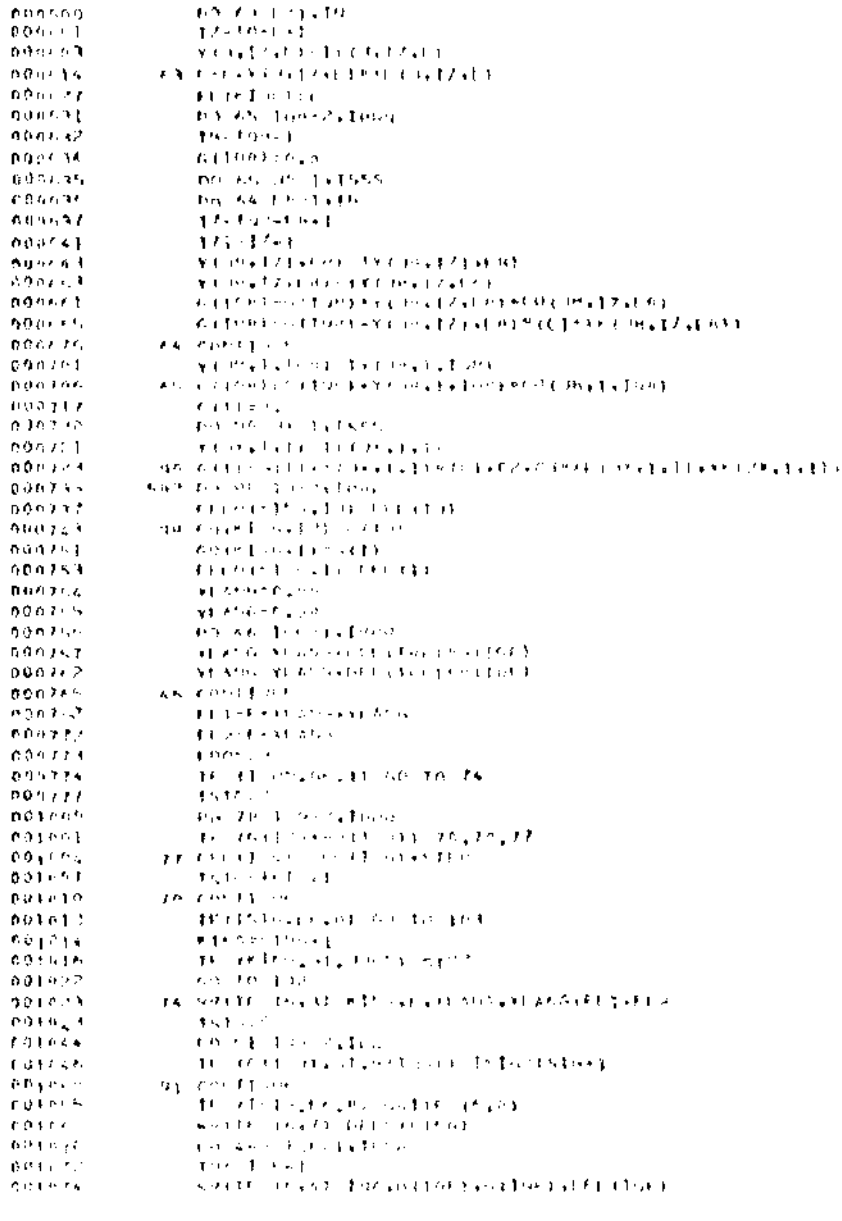
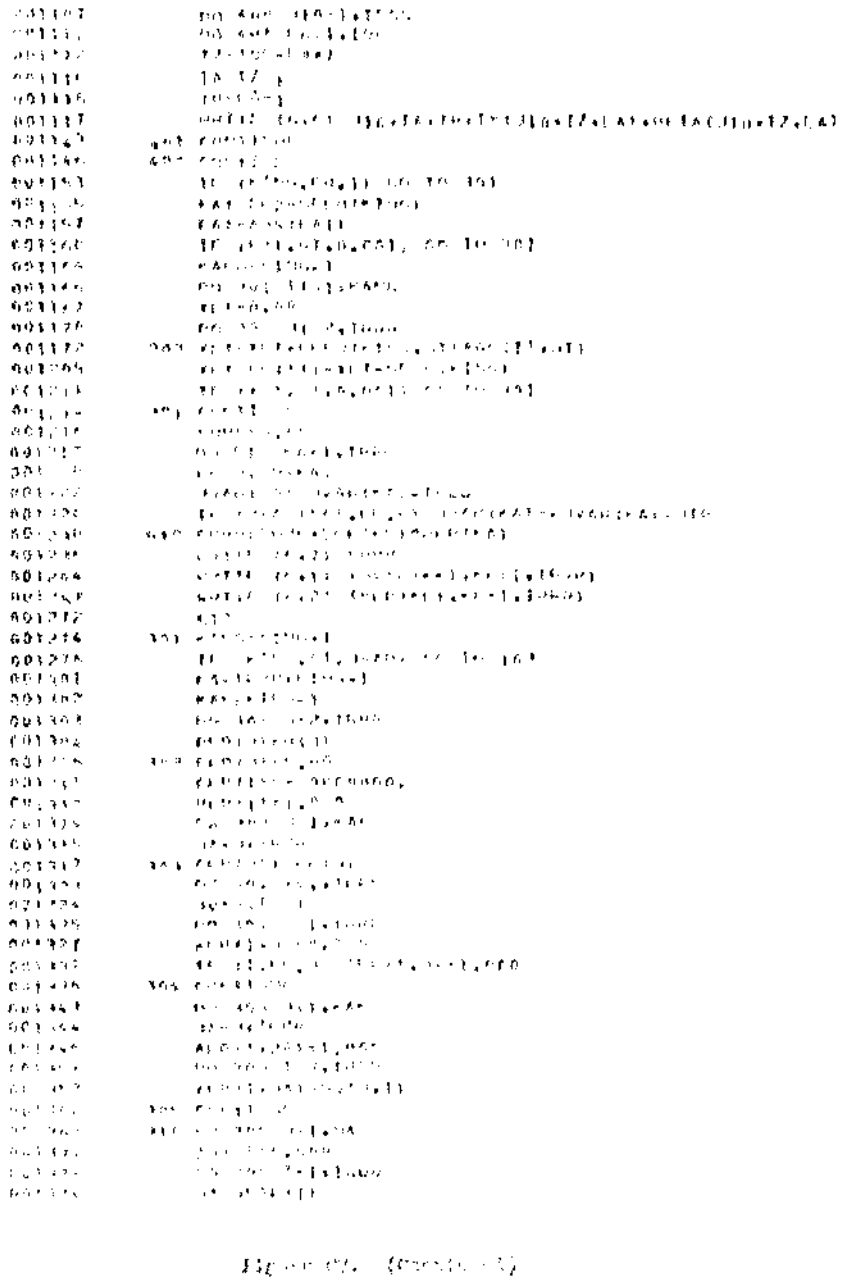


Figure 67. (Continued)



[illegible]

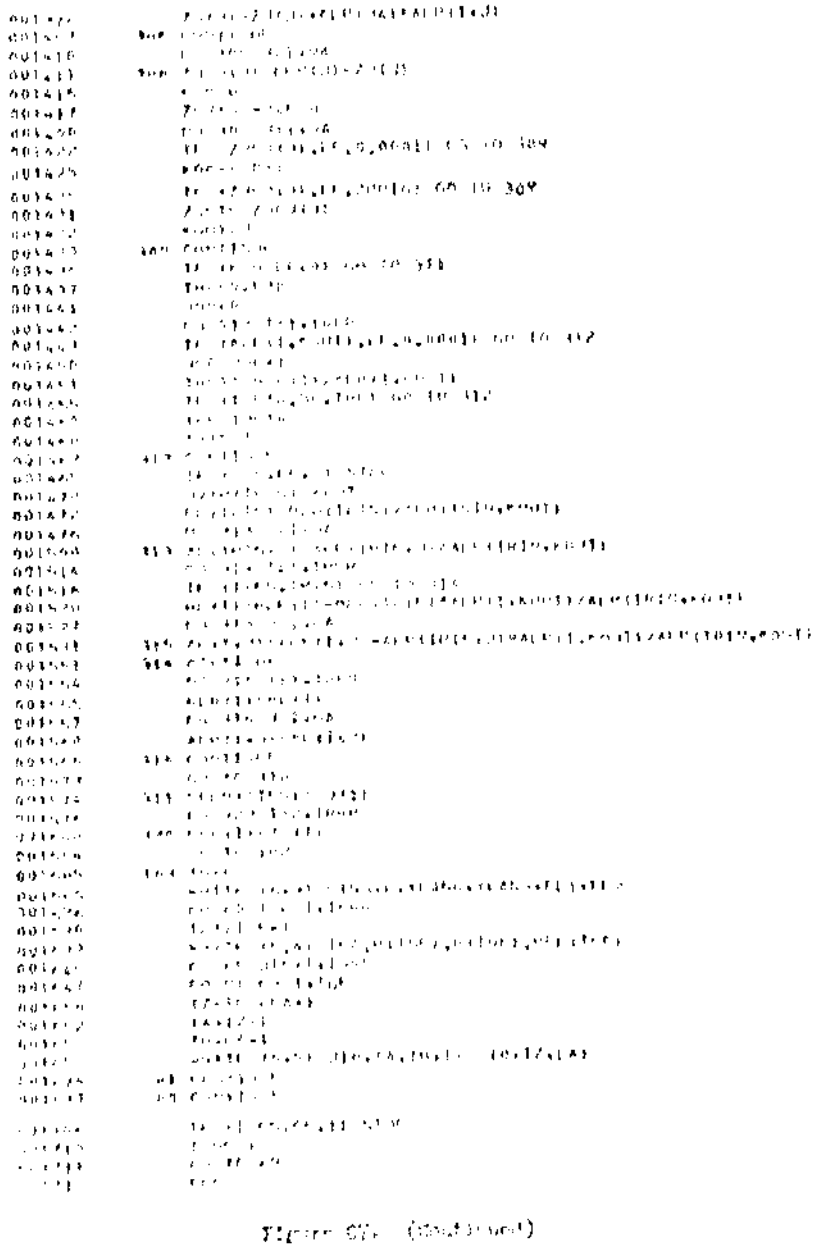


Figure 6.1. (illegible)

VITA

VITA

The author was born on December 21, 1943, in Omaha, Nebraska, and is a United States citizen. In June, 1961, he graduated from Omaha Central High School. He entered Purdue University in September, 1961, and received a Bachelor of Science in Industrial Engineering in June, 1965, and a Master of Science in Industrial Engineering in June, 1966.

His industrial experience includes summer employment with the following firms: IBM, Allison Division of General Motors, and Western Electric. From September, 1968, to June, 1970, he was an Instructor at Purdue University.

The author currently holds membership in the following professional and honorary societies: The American Institute of Industrial Engineers, The Operations Research Society of America, The Institute of Management Science, The American Society for Engineering Education, Alpha Pi Mu, and Phi Eta Sigma.

He is married to the former Linda Gail Ohlbaum of New York City. He has accepted a position as Assistant Professor in the School of Library and Information Services at the University of Maryland.