A STOCHASTIC LINEAR PROGRAMMING APPROACH FOR THE PRODUCTION, DISTRIBUTION AND INVENTORY SYSTEMS OF THE NIGERIAN BOTTLING COMPANY (NBC) : A CASE STUDY OF THE NORTH-WESTERN ZONE OF NIGERIA

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FEBRUARY , 2015

DECLARATION

I, Njoku, Christian Onyewuchi declare that the work in the thesis entitled "A Stochastic Linear Programming Approach For the Production, Distribution and Inventory Systems of the Nigerian Bottling Company (NBC) : A Case Study of North-Western Zone of Nigeria" has been carried out by me in the Department of Mechanical Engineering under the supervisions of Dr. D. S. Yawas and Dr. I. M. Dagwa.

The information derived from the literature has been duly acknowledged in the text and a list of references provided. No part of this project was previously presented for another degree or diploma at this or any other institution.

Name of Student	Signature	Date

CERTIFICATION

This project thesis entitled "A Stochastic Linear Programming Approach For the Production, Distribution and Inventory Systems of the Nigerian Bottling Company (NBC) : A Case Study of North-Western Zone of Nigeria" by Christian Onyewuchi NJOKU meets the regulations governing the award of the degree of Master of Science in Mechanical Engineering of the Ahmadu Bello University, and is approved for its contribution to knowledge and literary presentation.

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ABSTRACT

In the Nigerian Bottling Company, Production planning has a fundamental role to play. In this study, the particular scenario considered concerns the Nigerian Bottling Company (NBC) with many production facilities and multi-products production systems. The Products are being distributed to a number of depots at which the demand for each product is known. The problem of interest involves determining what products should be made, how much of each product should be produced, and where production should take place. The objectives of the company are to minimize the total cost of operations as well as maximizing the total sales revenue based on the set of decisions, including demands, capacity restriction and budget constraints. The model, which consists of eighty-eight (88) variables and fifty-four (54) constraints is solved using a linear programming software known as Linear Programming Solver. The results show that production without the optimization principle gives a profit margin of five billion, forty two million, four hundred and thirty one thousand, two hundred naira (-N-5,042,431,200.00K) while production with the optimization principle gives a profit margin of five billion, sixty six million, eight hundred and ninety thousand naira (-N-5,066,890,000.00K). The model improved the profit of the company under study by -N-24,458,800 and reduced the Production, Inventory and Distribution (PID).

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Chapter One
INTRODUCTION

1.1 BACKGROUND OF THE STUDY

For nearly three decades, multi-echelon supply chains have constituted a focal research area. As a result, models for the control of supply chain of several forms and operating disciplines are now available. Due to the shear volume and variety of these models, surveys of varying scope or focus often appear (Inderfurth, 1994; Van Houtum et al., 1996; Diks et al., 1996; de Kok and Fransoo, 2003 ; Mula et al., 2006). Manufacturing and Production companies are profit-oriented; as a result, a well-defined mathematical model should be established and formulated, so that solving this model can generate an optimal strategy. In recent years, the mathematical theory of production, inventory and distribution has been extended to cover many of the situations that arise in practice. Mathematical programming has been applied frequently and successfully to a wide variety of production, inventory and distribution problems for a variety of industries. For example, Camm and Moni (1997) used integer programming and network to improve Procter and Gamble's distribution system; Arntzen and Luka (1995) used mixed integer linear programming to determine Digital Equipment Corporation's distribution strategy: Martin et al (1993) used linear programming to assist in distribution operations for Libbey-Owens-Ford; Robinson et al (1993) used optimization in designing a distribution decision support system for Dow Brands, Inc; Mehring and Gutterman (1990) used

linear programming to plan distribution at Almoco (U.K.) Limited. Most consumer products flow through a pipeline that begins with production at a plant, finished products moved down to the warehouse located at the plant site (if there is any), followed by transportation/distribution to a depot or customer outlets. Most companies manage these three functions independently with little or no integration among production, inventory and distribution planning. Companies will need to make the necessary organizational changes that will facilitate coordination of these major functions and develop an ability to make more decisions within the structure. Many research works have dealt with the coordination of production, inventory and distribution Pyke (1987) studied integrated production/distribution systems under stochastic demand. He developed an analytical model of a simple three-node system (factory, finished goods stockpile and single retailer) and examined the properties of the cost functions arising from this model for a single product case.

1.2 STATEMENT OF THE PROBLEM

This project work will integrate the coordination of production, inventory and distribution of multi-products, multi-facilities in the Nigerian Bottling Company (NBC), with several plant locations that produce a number of products over time. Finished goods (or products) are maintained at each plant warehouse, followed by the distribution of these products by

a fleet of trucks to a number of depots/customers. The demand for each product at each depot is known for each period horizon and such products are subsequently distributed. There are about 13 bottling plants and 59 depots across Nigeria namely: Abuja, Apapa, Asejire, Benin, Challawa, Enugu, Ikeja, Ilorin, Jos, Kaduna, Maiduguri Owerri and PortHarcourt. For example, Kaduna plant has two production lines namely: line 1 which produces 35cl with filler capacity cases of 1,083 cases per day and also 50clwith filler capacity cases of 833 cases per day while line 3 produces only 50cl with filler capacity cases of 1000 cases per day. Conservatively, the plant sales volume ranges from an average of 22,000 – 24,000 cases per day. The unit production time of products at any production line is 8 hours per day while the maximum available time at each plant is 20 hours per day. The Capacity restriction on production is 540 cases averagely while the maximum storage capacity at warehouse ranges from 100,000 units – 150,000 units. Similarly, Challawa plant has two production lines namely, line 1 which produces 35cl with filler capacity cases of 1,123 cases per day and also 50cl withfiller capacity cases of 944 cases per day while line 2 produces only 50cl with filler capacity cases of 1024 cases per day. The sales volume range for Challawa plant ranges from an average of 23,500-24,500 cases per day. The unit production time of products at any production line is 9 hours per day while the maximum available time at each plant is 22 hours per day. The capacity restriction on production is 600 cases averagely while the maximum storage capacity at warehouse ranges from 150,000 - 200,000 units. According

to the Nigerian Bottling Company balance sheet Management and Assessment (2013), the company's profit stood at about ₦5,024,242,323.74K in 2013. Based on the dynamic nature of the Nigerian Bottling Company (NBC), the following problems are identified:

- There is a seasonal demand for the NBC products which create a chain of intricate and far reaching effects that could not be responded to so adequately.
- There is a serious erosion or reduction of the NBC profit as a result of a steeply escalating production, inventory and distribution costs.
- There is a web of interacting influences which spanned through the NBC principal activities - production, inventory and distribution which require an integrated computer-based planning system to uncover the appropriate decisions.
- There is a need for optimal product mix where NBC produces different products each competing for the available scarce resources.
- There is period capacity restriction on production of NBC products.
- There is facility restriction on production of NBC products.
- There is storage capacity restriction of NBC products.

(Extracted from the NBC Balance Sheet Management And Assessment Records).

1.3 THE PRESENT RESEARCH

To improve the production and supply chain's performance under demand uncertainty (which happens by chance) in the Nigerian Bottling Company (NBC), a review of mathematical programming models for supply chain production and transport planning is inevitable. The purpose of this review is to identify current and future researches in this field and to propose a taxanomy framework based on the following elements: supply chain structure, decision level, modeling approach, purpose, shared information, limitations, novelty and application.

1.4 AIM AND OBJECTIVES OF THE STUDY

The overall aim of the study is to optimize the production, inventory and distribution of the Nigerian Bottling Company products so as to maximize profit.

The objectives of the study include the following:

a) To carry out an assessment of the NBC production, inventory and distribution systems.

b) To identify the decision variables, parameters and constraints necessary for formulating a model of the NBC production, inventory and distribution operations

c) To fit a linear programming model relating to the production, inventory and distribution of the NBC products.

d) To evaluate the profit margin in the production plant of the NBC.

e) To identify production activities that will achieve high efficiency in the production processes of the NBC.

1.5 SIGNIFICANCE OF THE STUDY

In this work, a solution was developed to forecast the ordering quantity in a given period of time, where every entity was allowed to use different inventory systems. A constraint network was proposed and incorporated with agent technology to coordinate the supply chain. Based on experimental results, this approach showed great promise in searching for a desired demand ordering quantity and globally optimized the total cost of the supply chain. According to Lucey (1996), the inventory model will help companies.

1.6 JUSTIFICATION OF THE STUDY

The seasonal demand for beverages is creating a chain of intricate and far reaching effects that could not be responded to adequately. As a result, the company's profit was being seriously eroded by steeply escalating production, inventory and distribution such as the NBC to reduce the variation in demand and production of their products by disposing goods at each finite time intervals to cushion the effect of excess demand which could be taken care of by an initial quantity of goods in the warehouse. Furthermore, it will give the NBC the leverage to take advantage of bulk purchasing discount by enjoying some price reductions on account of quantity order of raw materials. It will also help the NBC to meet up with possible shortages in future so as to enable production processes flow smoothly and efficiently. Finally, it will help the NBC to

cushion the effects of serious reduction of the company's profit by formulating a linear programming model that will determine the best combination of products quantities that would minimize cost or maximize profit.

1.7 SCOPE OF THE STUDY

For the sake of clarity and ease of validation, the model presented in this work is limited to 2-plants: Kaduna and Challawa plants, 2-products: Coke and Fanta, 3-depots: Zaria, Katsina and Dutse depots and 4-periods: Period 1, Period 3 and Period 4. However, the model can extend to any length of time depending on the capacity of the software. costs. The web of interacting influences which spanned the company's principal activities – Production, inventory and distribution, required an integrated computer-base planning system to uncover the appropriate decisions. The integrated framework allows the system to consider the relevant impacts of all decisions simultaneously, thereby equipping it to provide analyses for planning and operational decisions.

1.8 COMPANY PROFILE

This thesis work was carried out in Nigerian Bottling Company (NBC). Coca-cola was first made on 18th May, 1886 by Dr. John Pembeton in his home town Atlanta, Georgia in USA. Coca-cola first came to Nigeria in the year 1953, when the Nigerian bottling Company opened its first plant in Lagos. Nigerian bottling Company PLC was then

incorporated in November 1951 as a subsidiary of the A.G Leventis group with the franchise to bottle and sell products of the Coca-cola company in Nigeria. Two years later in 1953, the production of Coca-cola began at a bottling facility in Ebute-metta, Lagos state. In the same year, the company opened its first plant in Apapa and its second bottling facility at Ibadan, Oyo state. There is also a Quality Assurance department whose responsibility is to ensure that only quality products that meet the requirements are forwarded to the warehouse as finished products. Finished products are distributed/transported to the depots using a fleet of trailers. Not all the products are produced in all the plants, e.g. the production of the bottled coke and other beverages is at Kaduna plant, the production of the plastic coke is at the Lagos plant only, while the production of Five alive is at the Benin plant only. The finished products such as coke, Fanta, Sprite, Lemon Five-alive, Eva water, etc are transferred to the warehouse located at each plant site. A fleet of trucks are stationed at each plant site to deliver products from the plant to a number of depot/customer locations. The demand for each product at each depot is known. The goal of the company is to coordinate production, inventory and distribution to maximize profit without violating other environmental restrictions.

Chapter Two
LITERATURE REVIEW

2.1 INTRODUCTION

Supply chain management (SCM) is an emerging field that has commanded attention and support from the industrial community. Demand forecasting and taking inventory into consideration is an important issue in SCM. There are many diverse inventory systems, in theory or practice, which are operated by entities (companies) in a supply chain. In order to increase supply chain effectiveness, minimize total cost, and reduce the bullwhip effect, integration and coordination of these different systems in the supply chain (SC) will be required using information technology and effective communication. This research will develop a multi-agent system to simulate a supply chain, where agents operate these entities with different inventory systems. Agents are coordinated to control inventory and minimize the total cost of a supply chain by sharing information and forecasting knowledge. The demand will be forcasted with a genetic algorithm (GA) and the ordering quantity is offered at each echelon incorporating the perspective of "systems thinking". By using this agent-based system, the result will show that the total cost decreases and the ordering variation curve becomes smooth. According to New and Payne (1995), Supply chain is defined as the chain linking each entity of the manufacturing and supply process from raw materials through to the end user. In the words of Yung and Yang (1999), a supply chain comprises many systems, including

various manufacturing, storage, transportation, and retail systems. Gavirneni et al. (1999) showed that managing any one of these systems involves a series of complex trade-offs between different business function costs. For example, to efficiently run a manufacturing operation, the cost must compromise with the costs of inventory and raw materials Chen et al. (2000). According to Lee et al. (1992), to integrate different supply chain systems, entities must be coordinated incorporating the "system thinking" perspective.

2.2 A SUPPLY CHAIN MODEL

A supply chain model may be considered as an integrated process in which a group of several organizations, such as suppliers, producers, distributors and retailers, work together to acquire raw materials with a view to converting them into end products which they distribute to retailers (Beamon,1998). Simultaneously considering supply chain production and transport planning problems greatly advances the efficiency of both processes. The literature in the field is vast, so an extensive review of existing research on the topic is presented in an attempt to gain a better understanding of the mathematical modeling methods used in supply chain production and transport planning so as to provide a basis for future research. Given the globalization of operations, new models and tools for improving the forecasting, replenishment and production plans along supply chains, manufacturing companies increasingly need to integrate production and

transport planning in order to optimize both these processes simultaneously. Multi echelons including suppliers, manufacturers, distributors and retailers are allowed to use different inventory systems as shown below in figures 1.1 and 1.2 respectively.

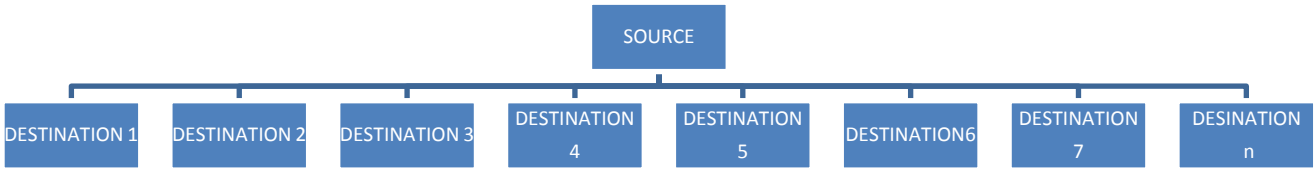


Fig 1.1 The pattern of flow of products to various destinations.

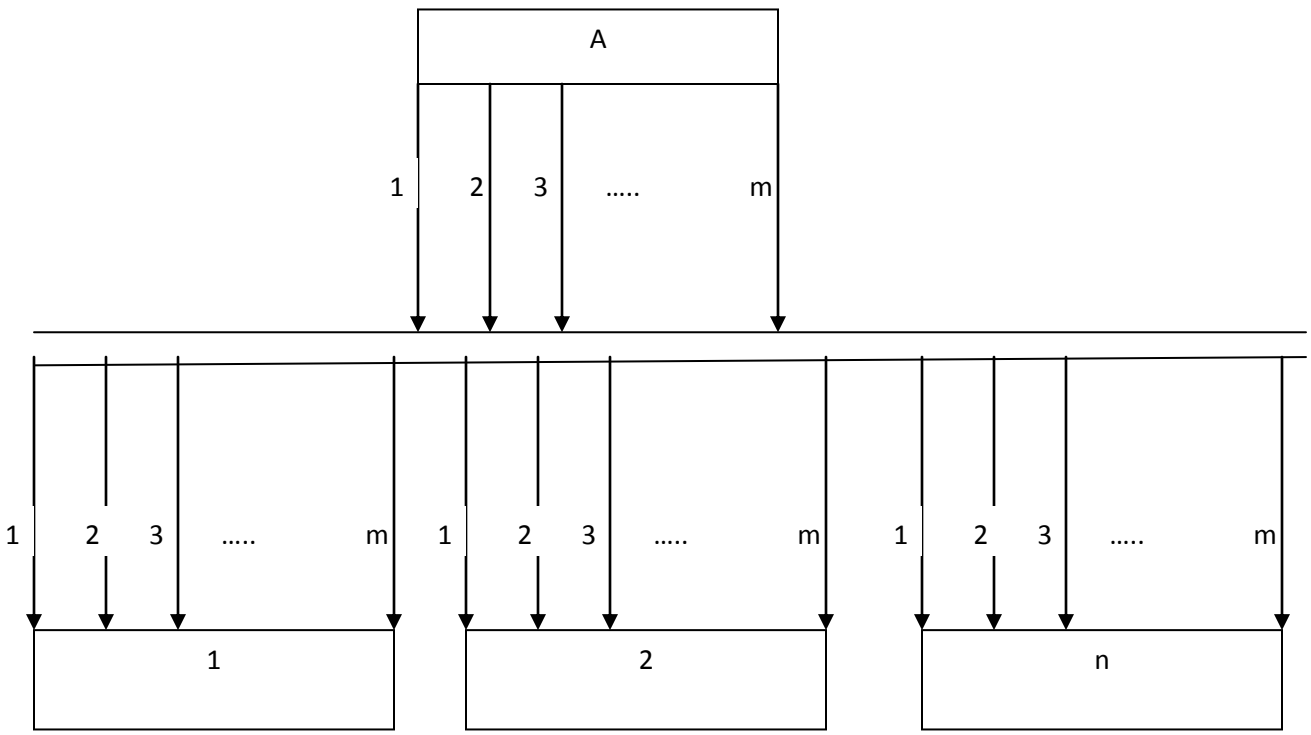


Fig 1.2 Flow of multiple products to destinations.

2.3 OPTIMIZATION

Optimization techniques have been widely used as the most suitable methods to choose the best one from a set of alternatives according to defined objective criteria. Several efforts have been directed towards the formulation of a coordinated planning model of production, inventory and distribution. Production, inventory and distribution planning are optimization problems and care is required to be sure that the sub optimization techniques described contribute to the overall optimization of the activities of which they form a part. Mathematical programming or constrained optimization is a mathematical procedure for determining optimal allocation of resources. Mathematical programming, and its most popular special form, Linear Programming (LP) has found practical applications in almost every facets of business, from advertising to production planning. Global optimization is the central issue of systems thinking. All managers' best interest is to ensure that the overall cost is reduced and operations among various systems are integrated through coordination.

2.4 STOCHASTIC LINEAR PROGRAMMING

Linear programming is a technique that allows a decision maker to solve maximization and minimization problems where there are certain constraints that limit what is achievable. It is a mathematical model concerned with the allocation of scarce resources. Linear programming was developed by a Russian mathematician L.V. Kantorovich in 1939 and extended by the American mathematician G.B. Dantzig in 1947 (Agbadudu, 1996). Linear programming as a mathematical tool in decision-making Procedure, has become useful because organizations big or small face many constraints in achieving their goals of profit maximization, cost minimization or other objectives. A stochastic linear programming is a procedure which offers a decision maker an opportunity to collect, analyze and allocate his/her scarce resources in a probabilistic or random manner (random sampling) while taking into considerations the constriants that limit what is achievable. Sabri and Beamon (2002), put forward a multi-objective stochastic model for strategical sub-model act as the inputs of the operational sub-model which incorporates demand uncertainty of production and distribution whose results once again feedback the strategical sub-model in an iterative process. This model will be solved by transforming the bi-level stochastic model into an equivalent deterministic model with multi-objective non-linear programming to which linear iteration techniques will be applied to solve it. Linear programming (LP) is a mathematical technique for determining the best possible allocation of resources to meet certain specifications. The

goal is determining a permissible allocation of resources that will maximize or minimize some numerical quantities such as:

- Maximizing profit.

b) Minimizing cost.

c) Minimizing labour turn over rate.

d) Maximizing revenue.

e) Minimizing machine down time.

f) Maximizing personnel utilization, and so on.

Limited resources, objective either not to violate or minimize the violation of resources restriction such as:

- ✓ Limited time.

ii. Limited budget.

iii. Limited manpower.

iv. Limited storage capacity.

v. Limited raw materials, etc.

Any other legal restriction such as:

i. A physical requirement that a variable(s) be non-negative.

ii. A contractual requirement that specifies that a variable(s) must equal to or exceed a certain minimum value.

In linear or mathematical Programming, there is usually a single objective function, which is either to be maximized or minimized subject to a number of constraint functions. However, there are cases of multi-objective functions which may also be investigated.

2.5 COMPONENTS OF THE LINEAR PROGRAMMING MODEL

A standard linear programming problem is made up of three parts namely:

- The objective function – The objective underlying the linear Programming could either be to maximize a given desirable e.g. Profit, contribution, that is the difference between selling price and Variable cost or to minimize an undesirable usually cost. The Objective function captures the objective to be optimized.

- The constraint function – A constraint is any factor which has the Capability of impacting on the extent to which the stated objective can be optimized. For example, the ability of a firm to maximize Profit is restricted by its market share and sales potential as well as Maximum quantities that can be produced, given the available labour, Materials and other resources.

- The non – negative constraint function – This function is usually stated to the effect that a firm cannot produce a negative output or use a negative quantity of any input.

2.6 CONDITIONS FOR USING THE LINEAR PROGRAMMING TECHNIQUE

The linear programming technique is not amenable to all management problems. To be useful, the following assumptions or conditions must hold:

- The problem must be capable of being stated in numeric terms, that is, it can only lend itself to problems with variables which can take numerical values.

- There should be one objective at a time, that is, we must either be talking of a maximization or a minimization problem.

- The problem must permit a choice or choices between alternative courses of action.

- There must be one or more restrictions or constraints on the objective, as the essence of management is actually to utilize scarce resources.

- The relationship between the variable or factors are linear, that is, the objective function which the organization seeks to maximize or minimize as well as the constraints that it faces are linear and can be represented graphically by a straight line.

2.7 APPLICATIONS OF LINEAR PROGRAMMING TECHNIQUES

Linear Programming has been applied in the following problem areas:

- Optimal process selection, that is to select the best process of Producing a given product where a number of alternative Processes with different technology and input combination requirements exist.

- Optimal product mix, that is, where a company produces different products each competing for the available scarce resources.

- Transportation problem, that is, the routing of goods to different destinations at minimum cost.

- Assignment problem, that is, the allocation of available personnel to various activities, jobs, e.t.c, at a minimum cost.

2.8 ALTERNATIVES TO LINEAR PROGRAMMING TECHNIQUES

It is useful to note that where the linear Programming tool is jettisoned, the following tools could still be used to achieve the same result:

- Differential Calculus, that is obtaining the minimum or Maximum point of an equation using differentiation.
- Lagrange multipliers, that is a sophisticated method of solving constrained optimization problem especially where the constraints are numerous.
- The use of isocost and isoquant, that is a line depicting the various quantities of either or both of two inputs that can be acquired with a given budget.

2.9 THE GENERAL LINEAR PROGRAMMING MODEL

The general form of this LP model is denoted as:

$$Min(Max)Z_i = f_i(X_j)$$

Subject to:

$$\sum a_{ij} X_j \leq b_i$$

$$Or \quad \sum a_{ij} X_j \equiv b_i$$

$$Or \quad \sum a_{ij} X_j \geq b_i$$

Where a_{ij} and b_i are constants and $X_j's$ are the system variables. The objective function is to minimize or maximize a function.

2.10 MODELLING

For successful implementation of Linear Programming, the models constructed must be realistic and adequately represent the performance of the various plants units. The assumption inherent in the model development must be noted and respected. The assumptions are linear, while most are not also linear. These are not necessary insuperable, some functions are piecewise linear, and many may be approximated to linear functions within limits. Difficulties also arise in the definition of objective function Objectives often conflict and it is not sufficient to think in terms of such generalities as "maximize profits" and "minimize cost". Also for an effective implementation, the input parameters must be reliable. It is important to note that the input parameters are not invariant and might change from time to time.

The basic issues involved in modeling are:

a) Formulating the model.

b) Gathering the data.

Formulating the model lies on the effectiveness of the model

a) Determining the time horizon for the effectiveness of the model.

b) Determining the decision variable and the parameters of the model.

c) Specifying the constraints as inequalities/equalities with each inequality/ equality for a distinct constraint.

d) Expressing the objective function of the problem as an equation.

e) Specifying the non-negative function also as an inequality.

Among the advantages of mathematical modeling over verbal description of the problem is that it describes a problem more concisely and this tends to make the overall structure of the problem more comprehensive. It also helps to reveal important cause and effect relationships. In this way, it indicates more clearly what additional data are relevant to the analysis. It also facilitates dealing with the problem in its entirety and considering all its interrelationships simultaneously. In addition, a mathematical model forms a bridge to the use of high-powered mathematical techniques and a computer to analyze the problem.

2.11 REVIEW OF PAST WORK

Richard (1997) considered a production and inventory problem with stochastic seasonal demand. A scarce resource limits production in each time, and setup time was assumed negligible. Heuristics were used in calculating the optimal solutions. Gerald et al. (2001) discussed in their paper how Kellogg Company has been using a large scale linear program to guide its operational (weekly) production, inventory and distribution decisions for breakfast cereals and other foods. The model considered tactical decisions on budgeting, capacity expansion, capacity reassignment, and other similar issues. Linet and Tulin (1999) developed a production-distribution model involving production and transportation decisions in a central factory and its warehouses. The model was based on the operating system of a multi-national company producing detergents in a central

factory from which products were distributed to geographically distant warehouses. The overall system cost was optimized considering factory and warehouse inventory costs and transportation costs. Constraints include production capacity, inventory balance and fleet size integrity. Thus, a model that involved an aggregation of products, demand, capacity and periods were solved. Kim et al. (2000) considered a multi-period inventory distribution planning problem (MPIDP) in a one warehouse multi-retailer distribution system where a fleet of heterogeneous vehicle moves products from a warehouse to several retailers. The objective was to minimize transportation costs for product delivery and inventory holding costs at retailers over the planning horizon. The problem was formulated as a mixed integer linear programming and solved by a Largrangian relaxation approach. Jukka et al. (2002) proposed a framework by which the risks related to a customer-supplier relationship, the service requirements by the customers and the strategies of the supplier company were included in production capacity allocation and supply chain design model. The target was to prepare sales plan where the limited production capacity was allocated to the customers based on their strategic importance and the risk involved. Furthermore, the supply chain was designed based on customers' strategic importance and service requirements. The model framework was demonstrated with a numerical example and it was based integrating the analytical hierarchy process (AHP) and mixed integer programming (MIP). Palekar and Luka (2003) developed mathematical models and algorithms for multi-products supply chains that share

common parts as well as common sources. The model was a mixed integer program and used for assembly products with several thousand models in the bill of materials. The basic model was extended to consider the joint optimization of production, transportation and inventory costs. The model assumed a base-stock policy and determines the optimal placement of safety stocks and the allocation of production activities to various locations. Andrew and Graeme (2001) describe the formulation and development of a supply-chain optimization model. The model, known as Paper Industry Value Optimization Tool (PIVOT), was a large mixed integer program that finds an optimal allocation of supplier to mill, product to paper machine, and paper machine to customer, while at the same time modeling many of the supply chain details. Ketzenberg et al (2001) developed a heuristic for a common production/inventory problem: multiple products with stochastic, seasonal demand, lost sales, and a constraint on overall production. The proposed heuristic was compared with those in current use as well as exact optimal solutions under a variety of conditions. The proposed heuristic was near optimal, deviating from optimality by average of 1.7% over a two-product experimental design, compared with existing practitioner heuristics, which deviated from optimally by averages ranging from 4.5-10.6%. Palekar and Luka (2003) considered the problem of designing the manufacturing supply chain for complex assembled products. Mathematical models and algorithms were developed for multi-products supply chains that shared common processes as well as common resources. The model was a mixed integer

program and was used for assembly products with several thousand modes in the bill of materials. The model was extended to consider the joint optimization of production. Agbadudu (1996) developed a production-distribution model involving production and transportation decisions in a central factory and its warehouses. The model was based on the operating system of a multi-national company producing detergents in a central factory from which products were distributed to geographically distant warehouses. The overall system costs was optimized considering factory and warehouse inventory costs and transportation costs. Constraints include production capacity, inventory balance and fleet size integrity. Thus, a model that involved an aggregation of products, demand, capacity and periods were solved. Ogunrinde and Ishola (2002) dealt with the spread of technological innovations in a one-machine, two-product manufacturing system with set-up changes, operating in a continuous time dynamic environment in a deterministic system. When production was switched from one product to the other, a known constant set-up time and a set-up cost were incurred. Each product had specified constant processing time and constant demand rate, as well as an infinite supply of raw material. The problem was formulated as a feedback control problem. The objective was to minimize the total backlog, inventory and set-up costs incurred over a finite horizon. The optimal solution provided the optimal production rate and set-up switching epochs as a function of the state of the system.

Chapter Three

METHODOLOGY AND MODELING

3.1 METHODOLOGY

Having highlighted the importance of optimization techniques in the previous chapter, the comprehensive model for Nigerian Bottling Company, (NBC) will be developed and all parameters substituted as required.

3.2 LOCATION OF STUDY AREAS

The data used in this project work was collected from Nigerian Bottling Company (NBC) Kaduna plant in Kaduna and Challawa plant in Kano. All in the North-western Zone of the country.

3.3 SOURCES OF DATA

Data for this project work was collected from Nigerian Bottling Company (NBC). This data includes: estimated demand for products at depot in a given time interval, maximum shipping weight of truck, unit weight of each product depending on bottle type, unit production time of products at plant, maximum available time for production of products at plant, fixed set up cost of production facilities, transportation cost from plant to depot, unit cost of production of products at plant in a given time interval, inventory level at

their warehouse and depot, maximum storage capacity for each product at warehouse, fixed vehicle cost, etc.

3.4 METHOD OF DATA COLLECTION

The type of data collected is as follows:

- Production-related:

a. Unit production time of products at plant.

b. Maximum unit time available at each plant.

c. Fixed set-up cost of production facilities.

d. Unit production cost of products.

- Warehouse-related:

a. Products demanded at depot.

b. Inventory level.

c. Inventory holding cost

d. Maximum storage capacity of products.

3. Distribution/Sales-related:

a. Cost of direct travel from plant to plant.

b. Fixed vehicle cost per route travelled.

c. Unit weight of product.

d. Maximum shipping weight of truck.

e. Total budget.

f. Sales revenue.

3.5 SOFTWARE APPLICATION

Linear Programming Solver, LiPs, (version: 1.10.1.zip-Win RAR) is a convenient and powerful tool for solving linear, integer, and quadratic programming problems. These problems occur in areas of business, industry, research and government. Specific application areas where Linear Programming Solver has proven to be of great use include product distribution, ingredient blending, production and personnel scheduling and inventory management. The size of model that can be conveniently solved depends mainly on the speed of the machine used, memory limitations of the machine, and how well the model is formulated.

3.5.1 The Working Principles Of the Linear Programming Solver, LiPs

Linear Programming Solver, (LiPS) is an optimization package intended for solving linear, integer and goal Programming problems. The main features of the Lips are

- LiPs solver is based on the efficient implementation of the modified Simplex method that solves large scale problems.
- LiPs provides not only an answer but a detailed solution process as a sequence of simplex tables, so you can use it in studying linear programming.
- LiPs provides the procedures of sensitivity analysis which enables us to study the

behavior of the model when you change its parameters including: analysis of changes in the right sides of constraints, analysis of changes in the coefficients of the objective function and analysis of changes in the column/row of the technology matrix. Such information may be extremely useful in the practical application of LP models.

- LiPs also provides the method of goal programming including the Lexicographic and weighted GP methods. Goal programming Methods are intended for solving multi-objective optimization problems.

3.5.2 Model Editor

Model editor provides functions to create, edit, save and load models. The system supports a standard format model programming solver, MPS, but also has its own input format linear programming solver, LPX. When using the Linear Programming Solver, LPX format, it provides the user two options for input models: algebraic and tabular.

3.5.3 Model Solver

To run the solution process, use the menu LiPs >> Solver model. Model solver has two modes: basic and advanced. In the basic mode (which is suitable for learning the simplex method), LiPs provides not only an answer but a detailed solution process as a sequence of simplex tables. At each iteration, the output includes: corresponding simplex table, variable to be made basic, variable out of the basic set, etc. The form of the computer solution simulates the process of manual solution of the problem. In advance model, LiPS provides a set of methods for solution of large-scale problems: a modified primal

and dual simplex method based on LU-decomposition, branch-and-bound method for MILP.

3.5.4 Sensitivity Analysis

Sensitivity analysis provides efficient computational techniques to study the dynamic behavior of the optimal solution resulting from making changes in the parameters of the model. Lips provides the following types of sensitivity analysis:

- Analysis of changes in the right hand sides of the constraints.
- Analysis of changes in the coefficients of the objective function.
- Analysis of changes in the column of the technology matrix.
- Analysis of changes in the row of the technology matrix.

3.6 LINEAR PROGRAMMING MODELLING

The Linear Programming model has three basic components:

1. Decision variables to be determined.
2. Objective (goal or aim) to optimize.
3. Constraints that need to be satisfied.

The parameters, variables and constraints that need to be determined are as follows:

Notations

Index

J = Total number of products.

I = Total number of plants.

K = Total number of depots.

T = Total number of periods.

Parameters

S_{jk} = unit selling price of product, j at depot, k.

h_{jk} = unit-holding cost of product, j at depot, k.

V_{jik} = unit distribution cost of product, j from plant, i to depot, k.

I_{jit} = inventory of product, j at plant, i in period, t.

A_i = maximum storage capacity at plant, i.

W_{ji} = unit production time (cycle time) of product, j at plant, i.

T_i = maximum available production time at plant, i.

B_u = upper budget limit.

B_l = lower budget limit.

C_{jit} = unit production cost of product, j at plant, i in period, t.

D_{jkt} = demand for product, j at depot, k in period, t.

Variables

Q_{jit} = quantity of product, j produced at plant, i in period, t.

X_{jikt} = quantity of product, j from plant, i supplied to depot, k in period, t .

INV_{jkt} = inventory of product, j at depot, k in period, t.

3.6.1 Assumptions

For simplicity of the problem, some assumptions are proposed as follows:

1. Ready market for all products.
2. All plant locations have same standard operating process.
3. Enough personnel (skilled and knowledge) exist.
4. Efficient transport system is in place.
5. Demand estimation can be made.

3.6.2 Constraints

1. Capacity restriction on production in every period.
2. Inventory balance at production facility.
3. Inventory balance at depot.
4. Total demand for each item at customer location.
5. Total budget.

3.6.3 Model Development

Component formulation

Profit = Sales Revenue – Total cost

$$\text{Sales revenue} = \sum_{1}^{J} \sum_{1}^{I} \sum_{1}^{K} \sum_{1}^{T} S_{jk} X_{jikt} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Total Cost = production cost + inventory cost + distribution cost.

$$\text{Production cost} = \sum_{1}^{J}\sum_{1}^{I}\sum_{1}^{T} C_{jit}\, Q_{jit} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Inventory cost = average sum of starting inventory and ending inventory cost

$$= 0.5\, h_{jk} \sum_{1}^{J}\sum_{1}^{K}\sum_{1}^{T} (INV_{jkt-1} + INV_{jkt}) \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$\text{Distribution cost} = \sum_{1}^{J}\sum_{1}^{I}\sum_{1}^{K}\sum_{1}^{T} V_{jik}\, X_{jikt} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Objective function

Maximize:

$$\text{Profit} = \sum\sum\sum\sum S_{jikt}\, X_{jikt} - \sum\sum\sum C_{jit}\, Q_{jit} - 0.5\, h_{jkt} \sum\sum\sum (INV_{jkt-1} + INV_{jkt}) -$$

$$\sum\sum\sum\sum V_{jik}\, X_{jikt} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Constraints

Subject to:

Period capacity restriction on production

$\sum\sum\sum W_{jit}\, Q_{jit} \leq T_i$, for all i and t ……(i)

Production facility inventory balance

$\sum\sum\sum\sum (I_{jit} + Q_{jit} - X_{jikt}) \leq A_i$ ……..……..(ii)

Depot inventory balance

$\sum\sum\sum\sum X_{jikt} - (\sum\sum\sum INV_{jkt} - \sum\sum\sum INV_{jkt-1}) = \sum\sum\sum D_{jkt}$ …… ….(iii)

Production-demand balance

$\sum\sum\sum Q_{jit} + \sum\sum\sum I_{jit-1} \leq \sum\sum\sum D_{jkt}$ …….....(iv)

Budgetary constraint

$\sum\sum\sum C_{jit}\, Q_{jit} - \sum\sum\sum 0.5\, h_{jkt}\, (INV_{jkt-1} + INV_{jkt}) - \sum\sum\sum\sum V_{jikt}\, X_{jikt} \leq B_u$ ……(Va)

$\sum\sum\sum C_{jit}\, Q_{jit} - \sum\sum\sum 0.5\, h_{jkt}\, (INV_{jkt-1} + INV_{jkt}) - \sum\sum\sum\sum V_{jikt}\, X_{jikt} \geq B_l$ ……..……(Vb)

(Obtained from Ekoko, (1999)). Thus, the model developed above will be used in the next chapter and from the available data, parameters would be substituted and the Linear Programming Solver Software would be used to run the model and solve the software.

Chapter Four
DATA PRESENTATION AND ANALYSIS

For ease of identification and Simplification, the variables used are redefined as follows

Table 4.1: Quantity of product 'j' produced at plant 'i' in period 't' in year 2013.

Q_{jit}	j	i	t
Q_1	1	1	1
Q_2	2	1	1
Q_3	1	2	1

Where,

Q_1 is quantity of product 1 made at plant 1 in period 1,

Q_2 is quantity of product 2 made at plant 1 in period 1,

Q_3 is quantity of product 1 made at plant 2 in period 1.

Product 1 is **'**Coke' while Product 2 is 'Fanta'.

(See Table A.1 in Appendix A for details)

X_{jikt} is quantity of product 'j' produced at plant 'i' distributed to depot k in period 't', eg,

X_1 is quantity of product 1 produced at plant 1 distributed to depot 1 in period 1.

(See Table B.2 in Appendix B for details)

I_{jikt} is inventory of product ‘j’ at depot ‘k’ in period ‘t’, e.g. I_1 is inventory of product 1 at depot 1 in period 1.

(See Table B.3 in Appendix B for details)

Table 4.2: Demand for product 'j' at depot 'k' in period 't' (quarters), D_{jkt} (unit) in year 2013.

Period, t (Quarter)	Depot, k	Product, j 1	Product, j 2
	1	75,500	250,000
1	2	125,500	400,000
	3	199,000	640,000
Total		400,000	1,290,000
	1	50,000	205,000
2	2	100,000	300,000
	3	140,000	435,000
Total		340,000	1,040,000
	1	40,000	105,000
3	2	75,000	260,000
	3	125,000	425,000
Total		290,000	890,000
	1	100,000	385,000
4	2	180,000	560,000
	3	260,000	805,000
Total		440,000	1,650,000

Starting inventory of product ‘j’ at plant ‘i’ in period ‘t’ = 1, assumed to be zero, Similarly, starting inventory of product ‘j’ at depot ‘k’ in period ‘t’ = 1, assumed to be zero.

Table 4.3: Unit production time, ‘W_{ji}’ (second, s) of product in year 2013.

Plant, i	Product, j	
	1	2
1	4.32	4.32
2	2.4	2.4

Maximum available time at each plant:

Plant 1 = 20hrs

Therefore, the maximum available time (in seconds) in each period or quarter

(1 period = a quarter = 3 months)

= 60mins/1hr x 60secs/1min x 20hrs/1day x 20days/1mnth x 3mnths/1 period

= 4,320,000 seconds/quarter

Similarly,

Plant 2 = 20hrs

Therefore, the maximum available time (in seconds) in each period or quarter

(1 period = a quarter = 3 months)

= 60mins/1hr x 60secs/1min x 20hrs/1day x 20days/1mnth x 3mnths/1 period

= 4,320,000 seconds/quarter

Table 4.4: Total unit cost of producing product 'j' at plant 'i', (₦/unit) in year 2013.

Plant, 'i'	Production Cost of 'j' (₦/unit)	
	1	2
1	830	830
2	830	830

Table 4.5: Selling price per unit of product 'j' at depot 'k', (₦/unit) in year 2013.

Product, 'j'	Depot, 'k'		
	1	2	3
1	980	980	980
2	980	980	980

Selling price per unit product (crate) = ₦1,210 (50cl)

Selling price per bottle (liquid Content Only) = ₦70 (50cl)

Selling price per unit product (crate) = ₦980 (35cl)

Selling price per bottle (liquid Content Only) = ₦50 (35cl)

Table 4.6: Distribution cost per unit of product, (₦/unit) in year 2013.

Plant, 'i'	Depot, 'k'		
	1	2	3
1	7	9.5	11
2	11	9	6

Table 4.7: Inventory holding cost at each depot, ₦/unit (assumed constant Over a long range) in year 2013.

Depot, 'k'	Product, 'j'	
	1	2
1	13.5	15
2	14.5	14
3	15	13

Maximum storage capacity at warehouse:

Plant 1 = 100,000 units

Plant 2 = 150,000 units

Substituting all the parameters above into the model developed in chapter three (3):

Objective Function to Optimize:

MAXIMIZE

$980 X_1 + 980 X_2 + 980 X_3 + 980 X_4 + 980 X_5 + 980 X_6 + 980 X_7 + 980\text{X}8 X_8 + 980$

$X_9 + 980 X_{10} + 980 X_{11} + 980\text{X} X_{12} + 980 X_{13} + 980 X_{14} + 980 X_{15} + 980 X_{16} + 980$

$X_{17} + 980 X_{18} + 980 X_{19} + 980 X_{20} + 980 X_{21} + 980 X_{22} + 980 X_{23} + 980 X_{24} + 980$

$X_{25} + 980 X_{26} + 980 X_{27} + 980 X_{28} + 980 X_{29} + 980 X_{30} + 980 X_{31} + 980 X_{32} + 980$

$X_{33} + 980 X_{34} + 980 X_{35} + 980 X_{36} + 980 X_{37} + 980 X_{38} + 980 X_{39} + 980 X_{40} + 980$

$X_{41} + 980 X_{42} + 980 X_{43} + 980 X_{44} + 980 X_{45} + 980 X_{46} + 980 X_{47} + 980 X_{48} - 830 Q_1 -$

$830 Q_2 - 830 Q_3 - 830 Q_4 - 830 Q_5 - 830 Q_6 - 830 Q_7 - 8300 Q_8 - 830 Q_9 - 830 Q_{10} - 830 Q_{11}$

$- 830 Q_{12} - 830 Q_{13} - 830 Q_{14} - 830 Q_{15} - 830 Q_{16} - 13.5 INV_1 - 14.5\text{INV}2 INV_2 - 15 INV_3 -$

$15 INV_4 - 14 INV_5 - 13 INV_6 - 13.5 INV_7 - 14.5 INV_8 - 15 INV_9 - 15 INV_{10} - 14 INV_{11} - 13$

$INV_{12} - 13.5 INV_{13} - 14.5 INV_{14} - 15 INV_{15} - 15 INV_{16} - 14 INV_{17} - 13 INV_{18} - 13.5 INV_{19} -$

$14.5 INV_{20} - 15 INV_{21} - 15 INV_{22} - 14 INV_{23} - INV_{24} - 7 X_1 - 9.5 X_2 - 11 X_3 - 11 X_4 - 9$

$X_5 - 6 X_6 - 7 X_7 - 9.5 X_8 - 11 X_9 - 11 X_{10} - 9 X_{11} - 6 X_{12} - 7 X_{13} - 9.5 X_{14} - 11 X_{15} - 11$

$X_{16} - 9 X_{17} - 6 X_{18} - 7 X_{19} - 9.5 X_{20} - 11 X_{21} - 11\text{X} X_{22} - 9 X_{23} - 6 X_{24} - 7 X_{25} - 9.5 X_{26}$

$$- 11 X_{27} - 11 X_{28} - 9 X_{29} - 6 X_{30} - 7 X_{31} - 9.5 X_{32} - 11 X_{33} - 11 X_{34} - 9 X_{35} - 6 X_{36} - 7$$

$$X_{37} - 9.5 X_{38} - 11 X_{39} - 11 X_{40} - 9 X_{41} - 6 X_{42} - 7 X_{43} - 9.5 X_{44} - 11 X_{45} - 11 X_{46} - 9$$

$$X_{47} - 6 X_{48}$$

It can further be reduced to:

$$973 X_1 + 970.5 X_2 + 969 X_3 + 969 X_4 + 971 X_5 + 974 X_6 + 973 X_7 + 970.5 X_8 + 969$$

$$X_9 + 969 X_{10} + 971 X_{11} + 974 X_{12} + 973 X_{13} + 970.5 X_{14} + 969 X_{15} + 969 X_{16} + 971$$

$$X_{17} + 974 X_{18} + 973 X_{19} + 970.5 X_{20} + 969 X_{21} + 969 X_{22} + 971\text{X } X_{23} + 974 X_{24} + 973$$

$$X_{25} + 970.5 X_{26} + 969 X_{27} + 969 X_{28} + 971 X_{29} + 974 X_{30} + 973 X_{31} + 970.5 X_{32} + 969$$

$$X_{33} + 969 X_{34} + 971 X_{35} + 974 X_{36} + 973 X_{37} + 970.5 X_{38} + 969 X_{39} + 969 X_{40} + 971$$

$$X_{41} + 974 X_{42} + 973 X_{43} + 970.5 X_{44} + 969 X_{45} + 969 X_{46} + 971 X_{47} + 974 X_{48} - 830$$

$$Q_1 - 830 Q_2 - 830 Q_3 - 830 Q_4 - 830 Q_5 - 830 Q_6 - 830 Q_7 - 830 Q_8 - 830 Q_9 - 830 Q_{10}$$

$$- 830 Q_{11} - 830 Q_{12} - 830 Q_{13} - 830 Q_{14} - 830 Q_{15} - 830 Q_{16} - 13.5 INV_1 - 14.5 INV_2 - 15$$

$$INV_3 - 15 INV_4 - 14 INV_5 - 13 INV_6 - 13.5 INV_7 - 14.5 INV_8 - 15 INV_9 - 15 INV_{10} - 14$$

$$INV_{11} - 13 INV_{12} - 13.5 INV_{13} - 14.5 INV_{14} - 15 INV_{15} - 15 INV_{16} - 14 INV_{17} - 13 INV_{18}$$

$$- 13.5 INV_{19} - 14.5 INV_{20} - 15 INV_{21} - 15 INV_{22} - 14 INV_{23} - 13\text{INV24 } INV_{24}$$

Constraints to satisfy:

SUBJECT TO

Period Capacity Restriction on Production

$$4.32\,Q_1 + 4.32\,Q_2 \leq 6264000 \quad (1a)$$

$$2.4\,Q_3 + 2.4\,Q_4 \leq 6264000 \quad (1b)$$

$$4.32\,Q_5 + 4.32\,Q_6 \leq 6264000 \quad (1c)$$

$$2.4\,Q_7 + 2.4\,Q_8 \leq 6264000 \quad (1d)$$

$$4.32\,Q_9 + 4.32\,Q_{10} \leq 6264000 \quad (1e)$$

$$2.4\,Q_{11} + 2.4\,Q_{12} \leq 6264000 \quad (1f)$$

$$4.32\,Q_{13} + 4.32\,Q_{14} \leq 6264000 \quad (1g)$$

$$2.4\,Q_{15} + 2.4\,Q_{16} \leq 6264000 \quad (1h)$$

Production Facility Inventory Balance

$$Q_1 + Q_2 - X_1 - X_2 - X_3 - X_7 - X_8 - X_9 \leq 100000 \quad (2a)$$

$$Q_3 + Q_4 - X_4 - X_5 - X_6 - X_{10} - X_{11} - \mathrm{X}\,X_{12} \leq 150000 \quad (2b)$$

$$Q_1 + Q_2 + Q_5 + Q_6 - X_1 - X_2 - X_3 - X_7 - X_8 -$$

$$X_9 - X_{13} - X_{14} - X_{15} - X_{19} - X_{20} - X_{21} \leq 100000 \quad (2c)$$

$$Q_3 + Q_4 + Q_7 + Q_8 - X_4 - X_5 - X_6 - X_{10} - X_{11} -$$

$$X_{12} - X_{16} - X_{17} - X_{18} - X_{22} - X_{23} - \mathrm{X24}\,X_{24} \leq 150000 \quad (2d)$$

$$Q_1 + Q_2 + Q_5 + Q_6 + Q_9 + Q_{10} - X_1 - X_2 - X_3 -$$

$$X_7 - X_8 - X_9 - X_{13} - X_{14} - X_{15} - X_{19} - X_{20} -$$

$$X_{21} - X_{25} - X_{26} - X_{27} - X_{31} - X_{32} - X_{33} \leq 100000 \quad (2e)$$

$$Q_3 + Q_4 + Q_7 + Q_8 + Q_{11} + Q_{12} - X_4 - X_5 - X_6 -$$

$$X_{10} - X_{11} - X_{12} - X_{16} - X_{17} - X_{18} - X_{22} - X_{23} -$$

$$X_{24} - X_{28} - X_{29} - X_{30} - X_{34} - X_{35} - X_{36} \leq 150000 \quad (2f)$$

$$Q_1 - Q_2 - Q_5 - Q_6 - Q_9 - Q_{10} - Q_{13} - Q_{14} - X_1 -$$

$$X_2 - X_3 - X_7 - X_8 - X_9 - X_{13} - X_{14} - X_{15} - X_{19} -$$

$$X_{20} - X_{21} - X_{25} - X_{26} - X_{27} - X_{31} - X_{32} - X_{33} -$$

$$X_{37} - X_{38} - X_{39} - X_{43} - X_{44} - X_{45} \leq 100000 \quad (2g)$$

$$Q_3 + Q_4 + Q_7 + Q_8 + Q_{11} + Q_{12} + Q_{15} + Q_{16} - X_4 -$$

$$X_5 - X_6 - X_{10} - X_{11} - X_{12} - X_{16} - X_{17} - X_{18} - X_{22} -$$

$$X_{23} - X_{24} - X_{28} - X_{29} - X_{30} - X_{34} - X_{35} - X_{36} -$$

$$X_{40} - X_{41} - X_{42} - X_{46} - X_{47} - X_{48} \leq 150000 \quad (2h)$$

Depot Inventory Balance

$$X_1 + X_4 - 0.5\,INV_1 = 75500 \quad (3a)$$

$$X_2 + X_5 - 0.5\,INV_2 = 125500 \quad (3b)$$

$$X_3 + X_6 - 0.5\,INV_3 = 199000 \quad (3c)$$

$$X_7 + X_{10} - 0.5\,INV_4 = 250000 \quad (3d)$$

$$X_8 + X_{11} - 0.5\,INV_5 = 400000 \quad (3e)$$

$$X_9 + X_{12} - 0.5\,INV_6 = 640000 \quad (3f)$$

$$X_{13} + X_{16} - 0.5\,INV_1 - 0.5\,INV_7 = 50000 \quad (3g)$$

$$X_{14} + X_{17} - 0.5\,INV_2 - 0.5\,INV_8 = 100000 \quad (3h)$$

$$X_{15} + X_{18} - 0.5\,INV_3 - 0.5\,INV_9 = 140000 \quad (3i)$$

$$X_{19} + X_{22} - 0.5\,INV_4 - 0.5\,INV_{10} = 205000 \quad (3j)$$

$$X_{20} + X_{23} - 0.5\,INV_5 - 0.5\,INV_{11} = 300000 \quad (3k)$$

$$X_{21} + X_{24} - 0.5\,INV_6 - 0.5\,INV_{12} = 435000 \quad (3l)$$

$$X_{25} + X_{28} - 0.5\,INV_7 - 0.5\,INV_{13} = 40000 \quad (3m)$$

$$X_{26} + X_{29} - 0.5\,INV_8 - 0.5\,INV_{14} = 75000 \quad (3n)$$

$$X_{27} + X_{30} - 0.5\,INV_9 - 0.5\,INV_{15} = 125000 \quad (3o)$$

$$X_{31} + X_{34} - 0.5\,INV_{10} - 0.5\,INV_{16} = 105000 \quad (3p)$$

$$X_{32} + X_{35} - 0.5\,INV_{11} - 0.5\,INV_{17} = 260000 \quad (3q)$$

$$X_{33} + X_{36} - 0.5\,INV_{12} - 0.5\,INV_{18} = 425000 \quad (3r)$$

$$X_{37} + X_{40} - 0.5\,INV_{13} - 0.5\,INV_{19} = 100000 \quad (3s)$$

$$X_{38} + X_{41} - 0.5\,INV_{14} - 0.5\,INV_{20} = 180000 \quad (3t)$$

$$X_{39} + X_{42} - 0.5\,INV_{15} - 0.5\,INV_{21} = 260000 \quad (3u)$$

$$X_{43} + X_{46} - 0.5\,INV_{16} - 0.5\,INV_{22} = 385000 \quad (3v)$$

$$X_{44} + X_{47} - 0.5\,INV_{17} - 0.5\,INV_{23} = 560000 \quad (3w)$$

$$X_{45} + X_{48} - 0.5\,INV_{18} - 0.5\,INV_{24} = 805000 \quad (3x)$$

Production Demand Balance

$$Q_1 + Q_3 \leq 400000 \quad (4a)$$

$$Q_2 + Q_4 \leq 1290000 \quad (4b)$$

$$Q_1 + Q_3 + Q_5 + Q_7 - X_1 - X_2 - X_3 - X_4 - X_5 - X_6 \leq 290000 \quad (4c)$$

$$Q_2 + Q_4 + Q_6 + Q_8 - X_7 - X_8 - X_9 - X_{10} - X_{11} - X_{12} \leq 940000 \quad (4d)$$

$$Q_1 + Q_3 + Q_5 + Q_7 + Q_9 + Q_{11} - X_1 - X_2 - X_3 - X_4 - X_5 - X_6 - X_{13} - X_{14} - X_{15} - X_{16} - X_{17} - X_{18} \leq 240000 \quad (4e)$$

$$Q_2 + Q_4 + Q_6 + Q_8 + Q_{10} + Q_{12} - X_7 - X_8 - X_9 - X_{10} - X_{11} - X_{12} - X_{19} - X_{20} - X_{21} - X_{22} - X_{23} - X_{24} \leq 790000 \quad (4f)$$

$$Q_1 + Q_3 + Q_5 + Q_7 + Q_9 + Q_{11} + Q_{13} + Q_{15} - X_1 - X_2 - X_3 - X_4 - X_5 - X_6 - X_{13} - X_{14} - X_{15} - X_{16} - X_{17} - X_{18} - X_{25} - X_{26} - X_{27} - X_{28} - X_{29} - X_{30} - X_{37} - X_{38} - X_{39} - X_{40} - X_{41} - X_{42} \leq 540000 \quad (4g)$$

$$Q_2 + Q_4 + Q_6 + Q_8 + Q_{10} + Q_{12} + Q_{14} + Q_{16} - X_7 - X_8 - X_9 - X_{10} - X_{11} - X_{12} - X_{19} - X_{20} - X_{21} - X_{22} - X_{23} - X_{24} - X_{31} - X_{32} - X_{33} - X_{34} - X_{35} - X_{36} -$$

$$X_{43} - X_{44} - X_{45} - X_{46} - X_{47} - X_{48} \leq 1750000 \text{ (4h)}$$

Budgetary Constraint

$$830Q_1 + 830Q_2 + 830Q_3 + 830Q_4 + 830Q_5 + 830Q_6 +$$

$$830Q_7 + 830Q_8 + 830Q_9 + 830Q_{10} + 830Q_{11} +$$

$$830Q_{12} + 830Q_{13} + 830Q_{14} + 830Q_{15} + 830Q_{16} +$$

$$13.5INV_1 + 14.5INV_2 + 15INV_3 + 15INV_4 +$$

$$14INV_5 + 13INV_6 + 13.5INV_7 + 14.5INV_8 +$$

$$15INV_9 + 15INV_{10} + 14INV_{11} + 13INV_{12} +$$

$$13.5INV_{13} + 14.5INV_{14} + 15INV_{15} + 15INV_{16} +$$

$$14INV_{17} + 13INV_{18} + 13.5INV_{19} + 14.5INV_{20} +$$

$$15INV_{21} + 15INV_{22} + 14INV_{23} + 13INV_{24} \leq 3060000000 \text{ (5a)}$$

$$830Q_1 + 830Q_2 + 830Q_3 + 830Q_4 + 830Q_5 +$$

$$830Q_6 + 830Q_7 + 830Q_8 + 830Q_9 + 830Q_{10} +$$

$$830Q_{11} + 830Q_{12} + 830Q_{13} + 830Q_{14} + 830Q_{15} +$$

$$830Q_{16} + 13.5INV_1 + 14.5INV_2 + 15INV_3 +$$

$$15INV_4 + 14INV_5 + 13INV_6 + 13.5INV_7 +$$

$$14.5INV_8 + 15INV_9 + 15INV_{10} + 14INV_{11} +$$

$$13INV_{12} + 13.5INV_{13} + 14.5INV_{14} + 15INV_{15} +$$

$15\,INV_{16} + 14\,INV_{17} + 13\,INV_{18} + 13.5\,INV_{19} +$

$14.5\,INV_{20} + 15\,INV_{21} + 15\,INV_{22} + 14\,INV_{23} +$

$13\,INV_{24} \geq 2900000000$ (5b)

The variables can further be separated into their individual component parts as given by the model developed in chapter three.

For X_{jikt} : Quantity of product 'j' from plant 'i' supplied to depot 'k' in period 't'

MAXIMIZE PROFIT

$973\,X_1 + 970.5\,X_2 + 969\,X_3 + 969\,X_4 + 971\,X_5 + 974\,X_6$

SUBJECT TO

$-X_1 - X_2 - X_3 \leq 100000$

$-X_4 - X_5 - X_6 \leq 150000$

$X_1 + X_4 = 75500$

$X_2 + X_5 = 125500$

$X_3 + X_6 = 199000$

$-X_1 - X_2 - X_3 - X_4 - X_5 - X_6 \leq 290000$

$-X_1 - X_2 - X_3 - X_4 - X_5 - X_6 \leq 240000$

$-X_1 - X_2 - X_3 - X_4 - X_5 - X_6 \leq 540000$

$X_1, X_2, X_3, X_4, X_5, X_6 \geq 0$

For Q_{jit} : Quantity of product ‘j’ produced at plant ‘i’ in period ‘t’

MAXIMIZE PROFIT

$$980\,Q_1 + 980\,Q_2 + 980\,Q_3 + 980\,Q_4$$

SUBJECT TO

$$4.32\,Q_1 + 4.32\,Q_2 \leq 6264000$$

$$2.4\,Q_3 + 2.4\,Q_4 \leq 6264000$$

$$Q_1 + Q_2 \leq 100000$$

$$Q_3 + Q_4 \leq 150000$$

$$Q_1 + Q_3 \leq 400000$$

$$Q_2 + Q_4 \leq 1290000$$

$$Q_1 + Q_3 \leq 290000$$

$$Q_2 + Q_4 \leq 940000$$

$$Q_1 + Q_3 \leq 240000$$

$$Q_2 + Q_4 \leq 790000$$

$$Q_1 + Q_3 \leq 540000$$

$$Q_2 + Q_4 \leq 1750000$$

$$Q_1,\ Q_2,\ Q_3,\ Q_4 \geq 0$$

For INV_{jkt} : Inventory of product 'j' at depot 'k' in period 't'

MAXIMIZE PROFIT

$13.5\,INV_1 + 14.5\,INV_2 + 15\,INV_3 + 15\,INV_4 + 14\,INV_5 + 13\,INV_6$

SUBJECT TO

$0.5\,INV_1$	=	75500
$0.5\,INV_2$	=	125500
$0.5\,INV_3$	=	199000
$0.5\,INV_4$	=	250000
$0.5\,INV_5$	=	400000
$0.5\,INV_6$	=	640000
$INV_1, INV_2, INV_3, INV_4, INV_5, INV_6$	$\geq$	0

The values of the variables: $X_1, X_2, X_3, X_4, X_5, X_6 \ldots X_{48}, Q_1, Q_2, Q_3, Q_4 \ldots Q_{16}$,

$INV_1, INV_2, INV_3, INV_4, INV_5, INV_6 \ldots INV_{24}$, are computed as given in appendix C.

4.1.1 Profit analysis of year 2013 (without Optimization Principle)

Selling Price per unit product (35cl) = ₦980

Average Sales Volume range per day = 24,000 cases per day

Sales Revenue = ₦980 X 24,000 X 365 = ₦8,584,800,000

Unit Production time of Products = 8hrs per day

Filler Capacity Cases (35cl) = 1,083 cases per hour

Unit Production Cost per Case (35cl) = ₦830

Production Cost = ₦830 X 1,083 X 8 X 365 = ₦2,624,758,800

Maximum Storage Capacity at the warehouse = 150,000 units

Inventory holding Cost at the warehouse per unit product = ₦15

Inventory Cost = ₦15 X 150,000 X 365 = ₦821,250,000

Sales Volume Range per day = 24,000 cases per day

Distribution Cost per unit case (35cl) = ₦11

Distribution Cost = ₦11 X 24,000 X 365 = ₦96,360,000

Profit = Sales Revenue – Total Cost

Where Total Cost = Production Cost + Inventory Cost + Distribution Cost

Profit = Sales Revenue – Production Cost – Inventory Cost – Distribution Cost

Profit = ₦8,584,800,000 – ₦2,624,758,800 – ₦821,250,000 – ₦96,360,000

Profit = ₦5,042,431,200.00K

4.1.2 Profit analysis of year 2013 (with Optimization Principle)

Profit = Sales Revenue – Total Cost

Where Total Cost = Production Cost + Inventory Cost + Distribution Cost

Profit = Sales Revenue – Production Cost – Inventory Cost – Distribution Cost

Profit = ₦389,148,000 + ₦1,255,010,000 + ₦282,110,000 + ₦914,455,000 +

₦233,495,000 + ₦768,575,000 + ₦525,320,000 + ₦1,702,440,000 --

₦207,500,000 – ₦207,500,000 – ₦207,500,000 – ₦207,500,000 --

₦46,988,000 – ₦34,310,000 – ₦28,485,000 – ₦63,880,000

Profit = ₦5,066,890,000.00K

Where: 389148000, 1255010000, 282110000, 914455000, 233495000, 768575000, 525320000, 1702440000, 207500000, 207500000, 207500000, 207500000, 46988000, 34310000, 28485000, 63880000.

are the values of the Optimum Solutions generated from the Linear Programming Solver Software.

Chapter Five

RESULTS AND DISCUSSION

The result of the linear model is as shown in Appendix C.

5.1 INTERPRETATION AND DISCUSSION OF RESULTS

The following summarizes the information from the results of the formulated model:

5.1.1 The Optimum solution

The Objective function gives a maximum profit value of ₦5,066,890,000.00K (target year – 2013) and the inventory level is kept at minimum as against the existing profit margin of ₦5,042,431,000.00K without Optimization principle. Hence, the Optimization method employed in the profit analysis of year 2013 has yielded a significant extra profit increment of ₦24,458,800.00K as against the profit margin recorded in year 2013 without Optimization principle.

For example, in period 2:

125,500 units (X_5) of product 1 from plant 1 should be supplied to depot 2 while 250,000 units (X_7) of product 1 from plant 2 should be supplied to depot 2; that is the demand for depot 2 of product 1 should be met from plants 1 and 2 in this order. Similarly, 0 unit (X_{22}) of product 2 from plant 1 should be supplied to depot 3 while 435,000 units (X_{24}) of product 2 from plant 2 should be supplied to depot 3; that is the demand for depot 3 of product 2 should be met from plants 1 and 2 in this order.

Furthermore, the products being seasonal products, excess demands at peak periods are met.

5.1.2 Sensitivity Analysis

The Optimal Solution of a Linear Programming is mostly based on a snapshot of the conditions that prevail at the time the model is formulated and solved. In reality, the decision environments rarely remain static. Hence, it is essential to determine how the optimal solution changes when the model parameters are changed. Sensitivity analysis provides efficient computational techniques to study the dynamic behavior of the optimal solution resulting from making changes in the parameters of the model.

5.1.3 Slacks/Surplus

This tells how close the solution is to the right-hand side limit on the constraints. For example, Constraint 13: (equation 2e) i.e. constraint on production facility inventory balance with a slack of 0, shows that the constraint is exactly satisfied. Similarly, Constraint 53: (equation 4e) i.e. constraint on Production Demand Balance also with a slack of 0, shows that the constraint is exactly satisfied.

5.1.4 Duality/Dual Price (Shadow Price)

This is the amount by which the objective could improve given a unit of increase in the right-hand side of the constraint. It tells how much one should be willing to pay for additional units of resources. For example, in Constraint 25, any additional units of the

input will increase the objective by 973 units. Similarly, in Constraint 9, any additional units of the input will increase objective by460 units. Conversely, any additional input in constraint 1, will have no effect on the objective function.

5.1.5 Reduced Cost

This is the amount of penalty to be paid for introducing a variable into the solution. For example, introducing a unit of variable X_3 or X_9, will increase the profit by 5 units. Similarly, introducing a unit of variable X_4 or X_{10}, will increase the profit by 4 units. Similarly, introducing a unit of variable X_2 or X_8, will increase the profit by 0.5 units. Similarly, introducing a unit of variable Q_5 or Q_{10}, will increase the profit by 0 unit. Conversely, introducing a unit of variable INV_6 or INV_7, will decrease the profit by 0 unit.

Chapter Six

SUMMARY, CONCLUSION AND RECOMMENDATIONS

6.1 SUMMARY

This Project work studied a manufacturing Company (case Study – Nigeria Bottling Company), with many production facilities and multi-products production system. In the system, products are distributed to a number of depots at which the demand for each product is known. Chapter one gives the introduction of the project work where the study objectives, statement of the problem, justification of study, limitation and sources of data are all stated. Chapter two deals mainly with the literature review; where past related work and the contributions of other previous researchers are carefully studied. Chapter three comprises the method of work, data collection, software application, and model development while the data is presented and analyzed in chapter four and the results are discussed in chapter five.

6.3 CONCLUSION

The Production, inventory and distribution systems of the NBC have been assessed and the potentials for using Linear Programming model in managing a large scale Production, inventory and distribution problems subsequently identified. The decision variables, parameters and constraints necessary for formulating a model of the

company's Production, inventory and distribution operations so identified have been solved using the linear Programming Solver, Lips. A linear Programming model which consists of eighty-eight (88) variables and fifty-four (54) constraints has been fitted and subsequently formulated. The Optimal Production, inventory and distribution of NBC products has been analyzed and the model improved the profit of the company under study by about ₦24,458,800 and also enhanced the Production, inventory and distribution (PID) strategy used by the company. The model developed resulted insignificant profit margin of about ₦24,458,800 with the Optimization principle (a profit margin of ₦5,066,890,000.00K – year 2013) in contrast with the existing profit of ₦5,042,431,200.00K without the Optimization principle in year 2013 and a profit increment of ₦42,647,676.26K without the Optimization principle in year 2012. The result of the work has shown that optimization of different NBC products can be achieved using LiPs software (linear programming) and is highly sensitive to changes putting into considerations the constraints that limit what is achievable. It is therefore highly recommended that the Optimization software should be adopted and used for effective planning and profit maximization.

6.4 RECOMMENDATIONS

There are some limitations in using the linear programming for planning operations in the NBC. There are also some fluctuations in market demand and price. A possible remedy to linear programming problems is to update model continuously based on plant data . There are many other software packages such as ASPEN PLUS (ASPEN FIMS), HYSIS etc, that can also be used to achieve similar results. Further work can still be done on this project work especially on the review of multi-objective optimization for the decision model and the effects of the addition of new constraints should also be studied. The addition of a new economic activity (introduction of a new variable) provided it is profitable (that is, if it improves the optimal value of the objective function) should be studied.

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APPENDIXES

Appendix A

The Coca-cola Company

Data Collection and analysis for project work

Kaduna Plant

Data for production Line 1

Type of drink produced

35cl beverage drink: Rated filler capacity cases/day = 1,083 cases

50cl beverage drink: Rated filler capacity cases/day = 833 cases

Data for production Line 3

Type of drink produced

50cl beverage drink only: Rated filler capacity cases/day = 1000 cases

Unit production time of products at any production Line = 8 hours/day

Maximum available time at each plant = 20 hours/day

Capacity restriction on production = 540 cases (on the average)

Maximum storage capacity at warehouse = 100,000 – 150,000 units

Plant Sales volume range = 22,000 – 24,000 cases/day (on the average)

Challawa Plant

Data for production Line 1

Type of drink produced

35cl beverage drink: Rated filler capacity cases/day = 1,123 cases

50cl beverage drink: Rated filler capacity cases/day = 944 cases

Data for production Line 2

Type of drink produced

50cl beverage drink only: Rated filler capacity cases/day = 1,024 cases

Unit Production time of products at any production line = 9 hours/day

Maximum available time at each plant = 22hours/day

Capacity restriction on production = 600 cases (on the average)

Maximum storage capacity at warehouse = 150,000 – 200,000 units

Plant sales volume range = 23,500 – 24,500 cases/day (on the average)

Unit production time of products in year 2013

Unit Production time at plant '1' = 4.32secs

Unit Production time at plant '2' = 2.4secs

Maximum available time at each plant = 20hrs

Therefore, the maximum available time (in secs) at each plant per quarter (1 period = a quarter = 3 months) = 60mins/1hr X 60secs/1min X 20hrs/1day X 20days/1month X 3months/1 period = 4,320,000secs/qr

Period Capacity restriction on production at each plant = 62,640,000sec

Selling price of products in year 2013

Selling price per unit product (crate) = ₦1,210 (50cl)

Selling price per bottle (liquid content only) = ₦70 (50cl)

Selling price per unit product (crate) = ₦980 (35cl)

Selling price per bottle (liquid content only) = ₦50 (35cl)

Production Cost per unit product (crate) = ₦830 (35cl)

Distribution Cost of products in year 2013

Cost of distribution per unit of product from plant '1' to depot '1' = ₦7

Cost of distribution per unit of product from plant 1 to depot '2 = ₦9.5

Cost of distribution per unit of product from plant 1 to depot '3' = ₦11

Cost of distribution per unit of product from plant '2' to depot 1 = ₦11

Cost of distribution per unit of product from plant '2' to depot '2' = ₦9

Cost of distribution per unit of product from plant '2' to depot '3' = ₦6

Inventory holding cost at each depot in year 2013

Inventory holding cost of product '1' at depot '1' = ₦13.50K

Inventory holding cost of product '1' at depot '2' = ₦14.50K

Inventory holding cost of product '1' at depot '3' = ₦15.00K

Inventory holding cost of product '2' at depot '1' = ₦15.00K

Inventory holding cost of product '2' at depot '2' = ₦14.00K

Inventory holding cost of product '2' at depot '3' = ₦13.00K

Quantity of products demanded from different depots per period in year 2013

Note – 1 period = a quarter = 3 months

	Depot, K	Product, J1	Product, J2
	1	75500	250000
1	2	125500	400000
	3	199000	640000
Total		400000	1290000
	1	50000	205000
2	2	100000	300000
	3	140000	435000
Total		290000	940000
	1	40000	105000
3	2	75000	260000
	3	125000	425000
Total		240000	790000
	1	100000	385000
4	2	180000	560000
	3	260000	805000
Total		540000	1750000

Production-demand balance of product ‘1’ based on the table is as follows:

In period ‘1’ and at depots ‘1’, ‘2’ and ‘3’ = 75,500 + 125,500 + 199,000 = 400,000units

In period ‘2’ and at depots ‘1’, ‘2’ and ‘3’ = 50,000 + 100,000 + 140,000 = 290,000units

In period ‘3’ and at depots ‘1’, ‘2’ and ‘3’ = 40,000 + 75,000 +125,000 = 240,000units

In period ‘4’ and at depots ‘1’, ‘2’ and ‘3’ = 100,000 + 180,000 + 260,000 = 540,000units

Production-demand balance of product ‘2’ based on the table is as follows:

In period ‘1’ and at depots ‘1’, ‘2’ and ‘3’ = 250,000 + 400,000 + 640,000 = 1,290,000units

In period ‘2’ and at depots ‘1’, ‘2’ and ‘3’ = 205,000 + 300,000 + 435,000 = 940,000units

In period ‘3’ and at depots ‘1’, ‘2’ and ‘3’ = 105,000 + 260,000 + 425,000 = 790,000units

In period ‘4’ and at depots ‘1’, ‘2’ and ‘3’ = 385,000 + 560,000 + 805,000 = 1,750,000units

Budgetary constraint according to the NBC balance sheet Management and Assessment record in year 2013 ranges from ₦2,900,000,000 to ₦3,060,000,000.

The profit analysis from the NBC balance sheet Management and Assessment in year 2013 stood at about ₦5,024,242,323.74K.

Appendix B

Table B.1: Quantity of Product ‘j’ produced at plant ‘i’ in period ‘t’ in year 2013.

Q_{jit}	j	i	t
Q_1	1	1	1
Q_2	2	1	1
Q_3	1	2	1
Q_4	2	2	1
Q_5	1	1	2
Q_6	2	1	2
Q_7	1	2	2
Q_8	2	2	2
Q_9	1	1	3
Q_{10}	2	1	3
Q_{11}	1	2	3
Q_{12}	2	2	3
Q_{13}	1	1	4
Q_{14}	2	1	4

Q_{15}	1	2	4
Q_{16}	2	2	4

Table B.2: Quantity of Product 'j' produced at plant 'i' and shipped to depot 'k' in period 't' in year 2013.

X_{jikt}	j	i	k	t
x_1	1	1	1	1
x_2	2	1	1	1
x_3	1	2	1	1
x_4	2	2	1	1
x_5	1	1	2	2
x_6	2	1	2	2
x_7	1	2	2	2
x_8	2	2	2	2
x_9	1	1	3	3
x_{10}	2	1	3	3
x_{11}	1	2	3	3

x_{12}	2	2	3	3
x_{13}	1	4	4	4
x_{14}	2	1	1	4
x_{15}	1	2	1	4
x_{16}	2	2	1	4
x_{17}	1	1	2	1
x_{18}	2	1	2	1
x_{19}	1	2	2	1
x_{20}	2	2	2	1
x_{21}	1	1	3	2
x_{22}	2	1	3	2
x_{23}	1	2	3	2
x_{24}	2	2	3	2
x_{25}	1	1	1	3
x_{26}	2	1	1	3
x_{27}	1	2	1	3
x_{28}	2	2	1	3
x_{29}	1	1	2	4
x_{30}	2	1	2	4

x_{31}	1	2	2	4
x_{32}	2	2	2	4
x_{33}	1	1	3	1
x_{34}	2	1	3	1
x_{35}	1	2	3	1
x_{36}	2	2	3	1
x_{37}	1	1	1	2
x_{38}	2	1	1	2
x_{39}	1	2	1	2
x_{40}	2	2	1	2
x_{41}	1	1	2	3
x_{42}	2	1	2	3
x_{43}	1	2	2	3
x_{44}	2	2	2	3
x_{45}	1	1	3	4
x_{46}	2	1	3	4
x_{47}	1	2	3	4
x_{48}	2	2	3	4

Table B.3 : Inventory level of Product ‘j’ at depot ‘k’ in period ‘t’ in year 2013.

I_{jkt}	j	k	t
I_1	1	1	1
I_2	1	1	2
I_3	2	1	3
I_4	2	1	4
I_5	1	2	1
I_6	1	2	2
I_7	2	2	3
I_8	2	2	4
I_9	1	3	1
I_{10}	1	3	2
I_{11}	2	3	3
I_{12}	2	3	4
I_{13}	1	1	1
I_{14}	1	1	2
I_{15}	2	1	3

I_{16}	2	1	4
I_{17}	1	2	1
I_{18}	1	2	2
I_{19}	2	2	3
I_{20}	2	2	4
I_{21}	1	3	1
I_{22}	1	3	2
I_{23}	2	3	3
I_{24}	2	3	4

Appendix C

OBJECTIVE FUNCTION VALUE

0.506689E + 10

VARIABLE	VALUE	REDUCED COST
x_1	75500.00	0.00
x_2	0.00	0.50
x_3	0.00	5.00
x_4	0.00	4.00
x_5	125500.00	0.00
x_6	199000.00	0.00
x_7	250000.00	0.00
x_8	0.00	0.50
x_9	0.00	5.00
x_{10}	0.00	4.00
x_{11}	400000.00	0.00
x_{12}	640000.00	0.00
x_{13}	50000.00	0.00
x_{14}	0.00	0.50
x_{15}	0.00	5.00

x_{16}	0.00	4.00
x_{17}	100000.00	0.00
x_{18}	140000.00	0.00
x_{19}	205000.00	0.00
x_{20}	0.00	0.50
x_{21}	0.00	5.00
x_{22}	0.00	4.00
x_{23}	300000.00	0.00
x_{24}	435000.00	0.00
x_{25}	40000.00	0.00
x_{26}	0.00	0.50
x_{27}	0.00	5.00
x_{28}	0.00	4.00
x_{29}	75000.00	0.00
x_{30}	125000.00	0.00
x_{31}	105000.00	0.00
x_{32}	0.00	0.50
x_{33}	0.00	5.00
x_{34}	0.00	4.00

x_{35}	260000.00	0.00
x_{36}	425000.00	0.00
x_{37}	100000.00	0.00
x_{38}	0.00	0.50
x_{39}	0.00	5.00
x_{40}	0.00	4.00
x_{41}	180000.00	0.00
x_{42}	260000.00	0.00
x_{43}	385000.00	0.00
x_{44}	0.00	0.50
x_{45}	0.00	5.00
x_{46}	0.00	4.00
x_{47}	560000.00	0.00
x_{48}	805000.00	0.00
Q_1	90000.00	0.00
Q_2	100000.00	0.00
Q_3	150000.00	0.00
Q_4	0.00	0.00
Q_5	90000.00	0.00

Q_6	1 00000.00	0.00
Q_7	150000.00	0.00
Q_8	0.00	0.00
Q_9	90000.00	0.00
Q_{10}	100000.00	0.00
Q_{11}	150000.00	0.00
Q_{12}	0.00	0.00
Q_{13}	100000.00	0.00
Q_{14}	0.00	0.00
Q_{15}	150000.00	0.00
Q_{16}	0.00	0.00
INV_1	150000.00	0.00
INV_2	250000.00	0.00
INV_3	398000.00	0.00
INV_4	500000.00	0.00
INV_5	800000.00	0.00
INV_6	1280000.00	0.00
INV_7	100000.00	0.00
INV_8	200000.00	0.00

INV_9	280000.00	0.00
INV_{10}	410000.00	0.00
INV_{11}	600000.00	0.00
INV_{12}	870000.00	0.00
INV_{13}	80000.00	0.00
INV_{14}	150000.00	0.00
INV_{15}	250000.00	0.00
INV_{16}	210000.00	0.00
INV_{17}	520000.00	0.00
INV_{18}	850000.00	0.00
INV_{19}	200000.00	0.00
INV_{20}	360000.00	0.00
INV_{21}	520000.00	0.00
INV_{22}	770000.00	0.00
INV_{23}	1120000.00	0.00
INV_{24}	1610000.00	0.00

ROW	SLACK OR SURPLUS	DUAL PRICES
1.	5832000.00	0.00
2.	5904000.00	0.00
3.	5832000.00	0.00
4.	5904000.00	0.00
5.	5832000.00	0.00
6.	5904000.00	0.00
7.	5832000.00	0.00
8.	5904000.00	0.00
9.	175500.00	0.00
10.	474500.00	0.00
11.	350000.00	0.00
12.	1190000.00	0.00
13.	150000.00	0.00
14.	390000.00	0.00
15.	305000.00	0.00
16.	885000.00	0.00
17.	140000.00	0.00
18.	350000.00	0.00
19.	205000.00	0.00
20.	835000.00	0.00
21.	200000.00	0.00
22.	590000.00	0.00
23.	485000.00	0.00

24.	1515000.00	0.00
25.	0.00	973.00
26	0.00	971.00
27.	0.00	974.00
28.	0.00	973.00
29.	0.00	971.00
30.	0.00	974.00
31.	0.00	973.00
32.	0.00	971.00
33.	0.00	974.00
34.	0.00	973.00
35.	0.00	971.00
36.	0.00	974.00
37.	0.00	973.00
38.	0.00	971.00
39.	0.00	974.00
40.	0.00	973.00
41.	0.00	971.00
42.	0.00	974.00
43.	0.00	973.00
44.	0.00	971.00
45	0.00	974.00
46.	0.00	973.00
47.	0.00	971.00

48.	0.00	974.00
49.	160000.00	0.00
50.	12890000.00	0.00
51	50000.00	0.00
52.	930000.00	0.00
53.	0.00	0.00
54.	780000.00	0.00
55.	300000.00	0.00
56.	1740000.00	0.00