RELIABILITY EVALUATION OF IN-SITU STRENGTH OF REINFORCED CONCRETE MEMBERS

BY

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August, 1999.

DECLARATION.

I hereby declare that this thesis has been prepared by me and that it is a record of my own Research work. It has not been presented in any previous publication for a higher degree. All quotations are indicated and all sources of information are specially acknowledged by means of references.

Amartey.

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August, 1999

CERTIFICATION.

This thesis entitled "Reliability evaluation of in-situ strength of reinforced concrete members," by Yusuf Dada Amartey meets the regulations governing the award of the degree of Master of Science in Civil Engineering Construction Materials, of Ahmadu Bello University, Zaria and is approved for its contribution to knowledge.

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DEDICATION.

This work is dedicated to my loving wife Maryam Yusuf Dada Amartey. The love of my heart!!!

ACKNOWLEDGMENT.

My utmost thanks goes to God Almighty, the creator of the heavens and the earth, for His infinite Mercy and guardian.

I also do realize the it has been a great privilege for me to work with a renown long standing scholar, Dr. J.O. Afolayan, who in his very tight schedule has shown so much interest in my work.

The encouragement, advice and support of Prof. O. Adebisi & family, Dr. S.P.Ejeh & family, Dr. K.J. Osinubi & family and Mr S.S. Letcha & family are memorable.

My sincere thanks goes to my parent; Alhaji And Hajia Muhammadu Baba Amartey for their parental care and understanding. The brotherly and sisterly love of my junior Brothers and sister; Ndakolo, Bologi, Yawo, Ndazo and Ndanusa Amartey is also memorable.

I appreciate from the depth of my heart, the support and love of Mr.& Mrs Aluko, Mike Omotosho and Mr. Bala Achika & family.

Y.D. Amartey.

ABSTRACT

A description is presented for extending reliability analysis concepts or probabilistic approach to the formation of design equations and compliances on sites to the equations. The design equations for concrete structural elements were reviewed.

The test gave very reasonable prediction of how reliably the design equations are and the compliance of workmanship to the design equations requirements. The First Order Reliability Method (FORM) was reviewed as a basis for the study in which the inherent uncertainties in material and geometric properties together with loading conditions were explicitly considered.

A FORTRAN 77 computer program was followed to compute the reliability levels implicit in the design equations in respect of field tested values of structural elements like slabs, beams and short columns. The statistical values for the design variables were based on information obtained from sites.

The results obtained showed that with all the basic variables assuming normal distribution, the implicit reliability levels, β, range from 4.96541 to 7.2113 for short columns, the interpretation of this high value of reliability level is due to the fact that on all sites there are good quality control and good workmanship, which results in small coefficient of variation.

The variations of reliability level with span, varying compressive strength, yield strength were compared.

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NOTATION

A^{1}_{s}:	Area of compression Reinforcement
b:	Breath of the structural element
CDF, F_R (r):	Cumulative Distribution Function
COV:	Coefficient of Variation
d:	Effective depth of structural element
d^{1}:	Depth to the compressive reinforcement
D:	Dead load
F_{cu}:	Mean compressive strength of concrete structural element
f_y:	Yield strength of steel
$g(x_i)$:	State function of basic variables
L:	Length of structural element
M_A:	Applied moment
M_u:	Moment of resistant
Q:	Live load.
β:	Safety index or reliability level
α:	Reinforcement ratio
R:	Resistance capacity.
S:	Load effect
x^i:	basic variate
x^i:	Reduced basic variate
M:	Mean value
σ:	Standard deviation.
w:	Total load on the element; uniformly distributed

CHAPTER ONE

INTRODUCTION

1.1 Background of the Study.

In recent years, a probabilistic approach to structural design has been proposed. Since both the load and structural resistance are subject to inevitable variations, the probabilistic approach to structural design enables the structural safety problem to be treated in a more rational manner. One group of necessary information in this approach is the variation in the structural resistance of finished structures compared to the specified resistance[1]. In particular, when the structural members are manufactured on construction sites, as is the case with concrete columns, slabs,and beams,etc.

In current concrete practice, control of quality of concrete for acceptance purposes in structural members is based on the compressive strength of standard 150mm cubes, cast, cured and tested in accordance with standard established procedures. If test results based on the 150mm cubes indicate compliance with specified strength and the concrete has been placed, consolidated, and cured in accordance with the accepted practices, it is assumed that the concrete represented by the specimens and cast as part of the structure meets the design requirements. Unfortunately, in practice, these assumptions are not always correct and more often than not, placing, vibrating and curing conditions leave a lot to be desired. This, combined with the fact that reinforced concrete members are considerably larger and more massive in size, bring doubts whether the 150mm cubes do really represent the strength of concrete in structures. This is not to say that the practice of accepting concrete on the basis of 28-day compressive strength of standard-cured cubes has not served the construction industry well. In fact, it has, and thousands of reinforced concrete structures in our cities today are examples, to the acceptance of the quality of concrete on the above basis.

Notwithstanding this, the problem often arises when the strength of the Reinforced Concrete members deviates greatly from the 28-day strength of the standard cubes.

Field data are important for two main reasons: it gives information on how reliable and realistic the design strength of a building is and hence lead to a more rational basis of estimating the safety factor for a building. Secondly, it give an indication of the effect actual site conditions have on the quality of concrete compared to preparing the concrete product under laboratory conditions [2].

1.2 AIM AND OBJECTIVES.

The aim and objectives of this work include the following:

1. To make available additional practical data on in-situ concrete.
2. To assess the degree of consistency of workmanship at sites or under site conditions.

3. To suggest a practical minimum value of compressive strength reinforced concrete members for site conditions.
4. It is also intended that the information provided should serve as control data to assist in the interpretation of non-destructive test results aimed at evaluating the strength of existing buildings.
5. The work is also aimed at the evaluation of final probability of failure levels for in-situ columns, beams and slabs on per site basis.

1.3 JUSTIFICATION OF THE PRESENT STUDY.

In Nigeria, application of reliability analysis has not kept pace with other development in the Construction Industry. Safety provisions have at times been crude and have seldom been consistent. However, recently, efforts have been made to place reliability considerations on a rational basis by considering member strengths and load values as probabilistic quantities. Progress in this direction will lead to more realistic and more consistent safety provisions.[6]

The application of reliability theory in design has been hampered by the lack of data required to define the parameters as random variable. In this work, the variability of concrete quality in existing reinforced concrete structural elements (e.g. columns, slabs and beams) will be described by using hardness test (rebound hammer) as a non-destructive measure of concrete quality. The information provided will also serve as control data to assist in the interpretation of non-destructive test results aimed at evaluating the strength of existing buildings.

1.4 METHODOLOGY.

To make a reasonably accurate and relevant assessment of concrete quality in existing structures, it is decided to use a non-destructive testing system in order to collect a large amount of data without affecting the structure in any way.

Data on structural elements, i.e. columns, beams and slabs were collected from four construction sites located in Kaduna and Zaria.

The following procedures are adopted in the collection of data.

1. Ensuring the correct functioning of the device from time to time and especially prior to data collection.
2. Selecting smooth, clean and dry surface of the concrete structure for test. Loosely adhering scale,is rubbed off with a stone. Rough surfaces resulting from incomplete compaction, loss of grout, spalled or tooled surfaces which do not give reliable results are avoided. [4]
3. At least 12 valid readings were taken from each structural element. B.S. 4408 Part 4:1971 [4] recommends that at least nine valid readings are necessary to obtain a good representative mean, and that the significance of a mean is not improved substantially by taking more than 25 readings.
4. To reduce any bias, the readings are confined and a grid of

lines are drawn at about 20mm to 50mm apart and the intersections of these lines are taken as test points.

5. If the circumstances had justify it, more than one test whoud have been carried out on one structure or unit. The differences between the tests whoud then be used as a measure of the variability of the concrete within that structure or unit.
6. The results were used to calculate the reliability of each structural elements, (the first order reliability method was used to compute the reliability level).

1.4.1 Description of the test program.

Hardness tests (using Rebound Hammer) were carried out through 4 construction sites located in Zaria and Kaduna.

The buildings are described below and are chosen to provide a wide range of types. Most are 3 years and above old. All used in - situ mixed concrete. The specifications were not available for all the buildings. However, practice would suggest that except for building No 4 a medium slump concrete (50 to 100 mm) with a maximum aggregate size of 19 mm and a characteristic strength range of 24 to 32 N/mm^2 would be used . In every case, a design firm is responsible for the supervision of construction:

1.4.2 Description of selected buildings.

1. Professorial offices :- It is a four story office building including the ground floor. The building is to be used

as an administrative building and professors' offices at the new Ahmadu Bello University Teaching Hospital.

2. New Agricultural Engineering complex :- It is a three story building including the ground floor. The building has workshop and laboratories attached. Laboratories and classrooms are interspace in the ground floor while the upper floors are used as offices and lecture rooms.

3. National Electric Power Authority (NEPA) Building, Kaduna:- A proposed eight story building, just at the fourth floor at the time of study. The first three floors has been completed for four years now. They are mostly administrative and office blocks.

4. Engineering Complex of the Nigerian Defence Academy, Kaduna:- There are three sets of blocks, the first is a three story administrative blocks, the second is also a three story building to be used as class rooms and offices, the third is a two story building to be used as laboratories and students' work - shop rooms.

1.4.3 Methods of Testing Existing Structural Elements.

Test on existing structural elements can be broadly classified into Mechanical Tests to destruction and non-destructive tests (which allow repeated testing of the same element and thus make possible; a study of the variation in properties with time).

The destructive testing technique involves obtaining samples from existing structures by a coring technique and assessing the strength of such specimens in the laboratory. It has been realized that strengths obtained from cores are generally lower than that of standard cylinders produced and cured in the laboratory as prescribed for standard tests specimens. Malhotra [12] suggested that a strength reduction of up to 15% for $40N/mm^2$ could occur because of the damage no matter the efficiency of the coring procedure.

1.4.4 Non-Destructive Testing Technique

This is the method employed in the present study. It is simply a science of examining materials or building components in order to ascertain their fitness for certain purposes without impairment of their existing properties. Various methods of non-destructive testing exist and these include: visual examination, hammer sounding, Rebound Hammer technique, pressure methods, thermal, radiography, magnetic, etc [14]. The Rebound Hammer technique has been employed in the present study and the development, advantages and applications are well documented in chapter 2.

1.5 Scope of study.

The present study focuses on the following operations :

- Field testing of 4 construction sites to study the variation of concrete strength in actual structures.
- The use of hardness method.

- A statistical analysis of the field data to determine Means, standard deviations and, coefficients of variation.
- Computation of Reliability index for each element on per site basis,using the mean first - Order second moment procedure.

CHAPTER TWO

LITERATURE REVIEW

2.1 Probability in Engineering.

Many problems in engineering involve natural processes and phenomena that are inherently random; the states of such phenomena are naturally indeterminate and thus cannot be described with definiteness. Decisions required in the process of engineering planning and design invariably must be made, and are made, under conditions of uncertainty. [11]

The effect of inherent uncertainty on design and planning is important. However, quantification of such uncertainty and evaluation of its effect on the performance and design of engineering system should include concepts and methods of probability.

In structural design, uncertainties result from random nature of the loading and structural resistance, as well as imperfections in the load and resistance prediction models. The effect of such uncertainties is included in design through the use of safety factors which are based on engineering judgements and past experience with similar structures. Recognizing that safety involves a consideration of random variables (design parameters), it is generally accepted that the rational approach to the analysis of structural safety (and indeed safety factors) is through the use of probabilistic methods [11].

Thus, under conditions of uncertainty, the design and planning of engineering systems involve risks and formulation of the related decisions requires risk-benefit trade off which are within the realm of applied probability. It is therefore glaring that the role of probability is quite pervasive in Engineering. It ranges from the description of information to the development of basis for design and decision making.

The failure probability serves as a common and logical basis for the evaluation of safety and performance (of components and systems, as well as of structures subjected to single load application and repeated load applications) and the development of rational design equations (including the evaluation of the proper safety or load factors). Failure probabilities need not, however, appear explicitly in the routine design equation.

2.2 COMPRESSIVE STRENGTH OF CONCRETE.

Under current design, production, testing and quality control procedures, the strength of concrete in a structure may differ from its specified design strength and may not be uniform throughout the structure.

As already mentioned, the major sources of variation in concrete strength are the variations in material properties and proportions of the concrete mix, the variations in mixing, transporting, placing and curing methods, the variations in testing procedures and variations due to concrete being in a structure rather than in control specimens [15].

Not much work has been done in estimation of concrete strength by non-destructive test of site concrete, but the rebound hammer estimates compressive strength just as compressive strength is estimated from cylinder cube specimens. Most of the work documented on compressive strength of site concrete has been based on cylinder and cube strength from precast, ready-mix in-situ and site-mix in-situ concrete, the cylinders and cubes are field-cast, field-cured and all other quality-control are assumed to be as near as practicable to site conditions [12].

The variability of concrete strength depends mainly on the quality control of all the concreting operations. In most construction, the coefficient of variation of field-cast laboratory-cured specimens is, in many cases, between 15% and 20%, which suggests that 20% is a reasonable maximum value for average controls. An examination of some previous works (6,13) indicates that the standard deviation or coefficient of variation is not constant for different strength levels. Erntray [2] concluded on the basis of his data that it appears that the average coefficient of variation can be taken as roughly constant at 10%, 15% and 20% for strength levels below $24N/mm^2$ for excellent, average and poor control, respectively. For concrete with an average strength above $24N/mm^2$, the standard deviation remains approximately constant with values of $2.75\ N/mm^2$, $4.12\ N/mm^2$ and $5.49\ N/mm^2$ for the three levels of control listed above. This is expected, since the greater control required for the production of higher strength concrete contributes to smaller variability.

An analysis of concrete cylinders by Allen [20] from nearly 300 jobs across Canada showed that the coefficients of variation of cylinder strength were approximately 12%, 15% and 18% for precast, ready-mix in-situ and site-mix in-situ concrete, respectively. The total variation in concrete strength measured by control cylinders and cubes includes in most cases the variation in concrete strength within a single batch. Thus, the in-batch coefficients of variation of laboratory tests varied from 0.5%-8.1% with an overall average value of 3.6%. American Concrete Institute (ACI) Committee 214 [5] recommended that the level of control for within-batch tests could be divided into three classes with corresponding coefficients of variation as follows: 4%-5% for good control, 5%-6% for average control and above 6% for poor control.

2.2.1 In-Situ Strength Versus Control Cylinder Strength

The strength of concrete in a structure tends to be somewhat lower than the strength of control cylinders or cubes molded from the same concrete. This difference is due to, as earlier mentioned, to the effects of different in placing and curing procedures, the effects of vertical migration of water during the placing of concrete in deep members, the effects of difference in size and shape, and the effects of different stress regimes in the structure and the specimens [9]. The difference in directions of casting and loadings of the structure and the specimens may also influence the strength.

The average ratios of core strength to standard cylinder strength from the study by Malhotra [12] varied from 0.74-0.96 with an overall average of 0.87. He also showed that the ratio of the strength of concrete in a structure to the strength of the same concrete in a standard cylinder is not constant and decreases as the strength level increases and that the difference in-situ strengths for minimum acceptable and good curing standards could be approximated by a factor of 0.9.

The concrete in higher portions of deep members tends to be weaker than the concrete in the lower parts of some members, this may be due to the increased water-cement ratio at the top due to water migration after concrete has been placed and by the greater compaction of concrete near the bottom due to the weight of the concrete higher in the form.

In a detailed study of the relationship between strength and height, Malhotra[12] derived the equation :

$$\sigma = 1.12 - \sqrt{0.03h - 0.001} \quad for\, h > 0.05$$

where σ = strength factor

h = relative height. = $\dfrac{\text{height at any point}}{\text{Total height}}$

The relative strengths at h equal to 0.1 and 0.9 are 1.08 and 0.96 respectively, which gives a bottom to top ratio of 1.13.

2.3 The use of Rebound Hammer to Evaluate Concrete Quality.

Non-destructive testing has several distinct advantages and has proved to be of real economic importance in many industries. In-situ tests are possible and tsts can be made on structural members in service. Non-destructive testing methods can be used to assess a given property of concrete provided an empirical relationship can be found between that property and a non-destructive parameters.

The measurement of surface hardness by the use of rebound hammer developed by Ernst Schmidt has been found to be very suitable for extensive studies of variability of concrete quality in existing structures [17]. Extensive documentation of theoretical and practical aspects is available in some research papers and reviews [8,16,17]. Efforts to derive a general relationship between rebound number and concrete strength have not been very successful. But a lot of researchers have stressed on the importance of the rebound hammer, notable among them is Korek [17], who gave the following conclusion in his study of relation between cube crushing strength and rebound number. According to him, it is often found that cast cube results may convey too optimistic pictures of the actual strength in the structure and in the evaluation of the actual strength, the non-destructive test with the Schmidt rebound hammer is of great value and in some circumstances, gives a more realistic result than the compression machine, and it is often the only test applicable. Korek [17] also gave two distinct applications of the Schmidt rebound hammer which are envisaged for the concrete engineer as follows:

1. in quality control, in precast works and on building sites as

a measure of uniformity of the concrete;

2. as a non-destructive test for strength and other properties of concrete.

For the first, the user is advised to derive his own standards based on past experience. A chart based on the rebound number of, say, a number of units of known reliable quality can serve as a standard future production.

For the second application, a self-derived calibration curve, linking the rebound number with say, strength or some other property of the concrete is a prerequisite of maximum accuracy, but where only an estimate of the strength is required, curves supplied with the hammer or devised by other experimenters can be used.

2.4 Comparative Review of Advantages of Non-destructive tests Over Destructive tests.

2.4.1 Advantages of Non-destructive Tests

1. Tests are made directly upon the objects to be used in service. Consequently, there is no doubt that the tests are made on representative test objects.
2. Tests can be made on every unit to be used in service if economically justified.
3. Tests may be made on the entire production part or in all critical regions of it. Consequently, the evaluation applies to all part as a whole. Many critical sections of the part may be examined simultaneously or sequentially as convenient.
4. Many non-destructive tests, each sensitive to different properties or regions of the material or part, may be applied

simultaneously or in sequence. In this way, it is feasible to measure as many different properties correlated with service performance as desired.

5. Non destructive tests permit repeated checks of a given unit over a period of time. In this way, the rate of service damage is detectable and its correlation with service failure may be established clearly.
6. Little or no specimen preparation is required for many forms of non-destructible tests. Several forms of non-destructive testing equipments are pmrtable. Many are capable of rapid testing or sorting and in some cases, may be made fully automatic. The cost of non-destructive tests is less in most cases, both per object tested and for overall testing than the cost of adequate destructive tests.
7. Most non-destructive test methods are rapid and require far fewer man-hours or actual hours than typical destructive tests. Consequently, they are often suitable for testing all the production units at a cost normally less than, or comparable to the cost of inspecting destructively only a minor percentage of the units in production lots[16].

2.4.2 Limitations of Destructive Test.

1. Tests are not made on the objects actually used in service. Consequently, the correlation or similarity between the objects tested and those used in service must be proven by other means.

2. Tests can be made on only a fraction of the production lot to be used in service. They may have little value when the properties vary unpredictably from unit to unit.
3. Tests often cannot be made on complete production parts. The tests are often limited to test bars cut from production parts or from special material specimens processed to simulate the properties of the parts to be used in service.
4. A single destructive test may measure only one or a few of the properties that may be critical under service conditions.
5. Destructive tests are not usually convenient to apply to parts in service. Generally, service must be interrupted and the part permanently removed from service.
6. Cumulative change over a period of time cannot readily be measured on a single unit. If several units from the same lot or service are tested in succession over a period of time, it must be proven that the units were initially similar. If the units are used in service and removed after various periods of time, it must be proven that each was subject to similar condition of service before valid data can be obtained.
7. Many destructive tests require extensive machining or other preparation of the test specimens. Often, massive precision - testing machines are required. In consequence, the cost of destructive testing may be very high, and the number of samples that can be prepared and tested maybe severely limited. In addition, such preparations and tests may make severe demands upon the time of highly skilled workers.

8. The time and man-hour requirements of many destructive tests are very high. Excessive production costs may be incurred if adequate and extensive destructive tests are used as the primary method of production quality control.

2.5 Significance of Reliability Methods.

The theoretical concepts of risk and reliability analysis have been available for some time and the significance of such concepts in structural safety and design (formation of factors of safety, or load factors) has been well recognized [11]. The implementation of these concepts in the practice of engineering has been limited and at best been lacking in Nigeria. Recently, there has been a resurgence of efforts (restricted to paper) aimed at the implementation of reliability theory in design. These include the development of statistically consistent design formats [6] which emphasized the consistency of probability-based design formats. Other developments [7] have been concerned with the classical reliability model, emphasizing the roles of engineering judgement in risk calculation and the problems associated with the implementation of risk measures in design.

Ang [7] listed the principal advantages afforded by the probabilistic approach to structural safety and performance are as follows:

1. that it provides a consistent basis for the systematic assessment and analysis of uncertainty; and
2. it permits logical or mathematical operations of reliability

measures and thus provides a theoretical basis for the analysis of safety and reliability.

He also made the following comments on the practical basis for implementing probability concepts to structural evaluation and design.

1. That with a prescribed probability distribution, the failure probability can be expressed as a function of only the mean *values and uncertainties of the loads and resistance; thus,* for a chosen distribution, the problem of structural risk evaluation involves essentially the assessment of uncertainties, in addition to the prediction of the mean values of the design variables.
2. That in consistent with the probabilistic measure of risk, all uncertainties should be quantified in terms of coefficients of variation.
3. That from the standpoint of risk and risk-based design, it is most convenient and practically desirable to designate or describe the resistances and loads in terms of mean values and associated uncertainties.

CHAPTER THREE

THEORETICAL FORMULATION

3.1 RELIABILITY METHOD

3.1.1 Safety Index Concept

The problems of reliability of engineering systems can be essentially reduced to problem of supply to meet up with demand. *In other words, problems of engineering reliability may be* formulated as the determination of the [supply] capacity of an engineering element [system] to meet certain [demand] requirements. In the consideration of the safety of a structure, we are concerned with ensuring that the strength of the structure [supply] is sufficient to withstand the lifetime maximum applied load [demand].

The determination of the available supply as well as the determination of the maximum demand are not simple problems. Estimation and prediction are invariably necessary for these purposes; implying that uncertainties are unavoidable for the simple reason that engineering information is invariably *incomplete. In light of such uncertainties, the available supply* and actual demand cannot be determined precisely; they may be described as belonging to the respective ranges [or population] of possible supply and demand. In order to explicitly represent or reflect the significance of uncertainty, the available supply and required demand may be modelled as random variables[11]. Not unlike other theoretical concepts, the practical implementation of reliability theory requires the introduction of certain approximations and assumptions [7].

Let a structural system or element with Moment Resistance R or resistance capacity R [i.e. the supply] be acted upon by the load effect *S* [i.e. the demand]. The objective of the reliability analysis is to ensure the event [$M_R<M_S$ or R<S] throughout the useful life of the system. Conversely, the probability of the complimentary event [$M_R<M_S$ or R<S] is the corresponding measure of unreliability or failure.

Lets assume, for the moment, that the necessary probability distributions of R and S are available, [though rarely possible] that, the Cumulative Probability Function [CDF] or Probability Density Function [PD], $F_R(r)$ or $f_R(r)$ and $F_S(s)$ or $f_S(s)$ respectively.

The required probabilities is given by [11]

$$P_F=P(R<S \text{ or } M_R<M_S) \quad \sum P(R<S/S=s)P(S=s) \tag{3.1}$$

If the supply and demand, R and S are statistically independent, that is,

$$P(R<S/S=s)=P(R<s) \quad (3)$$

for continuous R and S we have

$$P_F=\int_0^\infty F_R(s)f_S(s)ds \tag{3.2}$$

This is diagrammatically represented in Fig. (3.1)

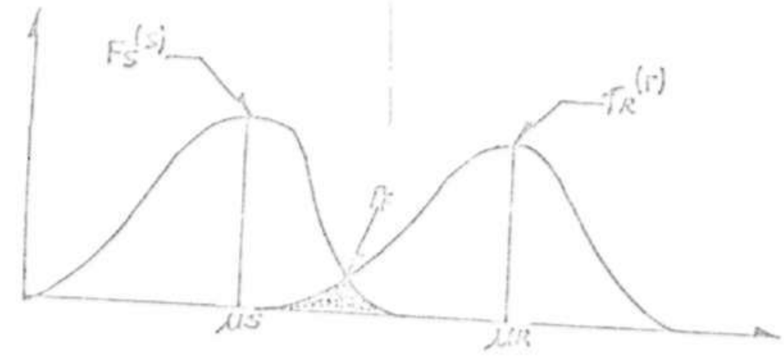


Fig. 3.1: Basic Resistance/Load Effect Problem $f_R(r)$, $f_S(s)$ representation.

As portrayed in Fig. 3.1, the overlapping region of the curves $f_R(r)$ and $f_S(s)$ represent a qualitative measure of the failure probability P_f. This region is dictated by the positions of their respective means μ_R and μ_S and also the degree of dispersions in $f_R(r)$ and $f_S(s)$ called their respective standard deviations σ_R and σ_S. The R and S problem may also be formulated in terms of the safety margin such that:

$$M = R - S \tag{3.3}$$

As R and S are random variables, M is also a random variable with corresponding PD $F_M(m)$. In this case, failure is clearly the event (M<0) and the probability of failure is

Graphically, this is represented by the area under $f_M(m)$ below o, as shown in Fig. 3.2

$$P_F = \int_{-\infty}^{0} f_M(m)\,dm = F_M(0) \tag{3.4}$$

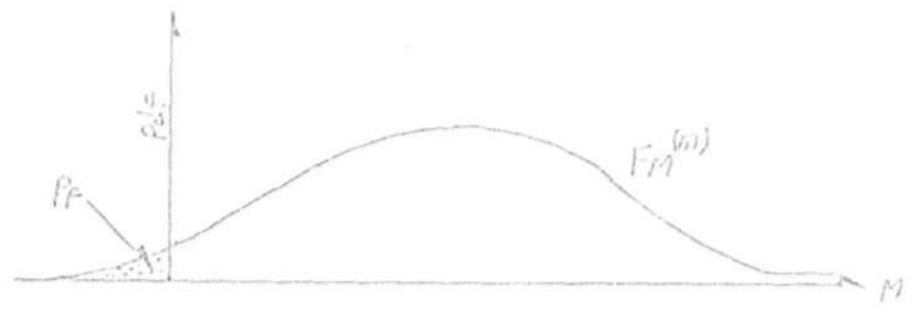


Fig. 3.2: Distribution of safety margin.

Now consider a structure whose resistance capacity R, is a normal random variable $N(\mu_R, \sigma_R)$. Similarly, the load S is also a normal random variable $N(\mu_S, \sigma_S)$ [12]. The probability distribution of the safety margin M = R - S is also normal $N(\mu_M, \sigma m)$, in which

$$\mu_M = \mu_R - \mu_S \tag{3.5}$$

and for statistically independent R and S, the variance of M is

$$\sigma_M^2 = \sigma_R^2 + \sigma_S^2 \tag{3.6}$$

If M is transformed into standard normal distribution, i.e. $(M-\mu_M)/\sigma_M$ is N(0,1) [since R and S are statistically independent], then

$$P_F = F_M(0) = \phi[(\frac{M-\mu_M}{\sigma_M})] = 1 - \phi(\frac{\mu_M}{\sigma_M}) \tag{3.7}$$

It is now shown that reliability is a function of the ratio μ_M/σ_M, which is the safety margin expressed in μ_m and σ_M and is called reliability index or safety index, denoted by Ps.

That is:

$$\beta = \frac{\mu_M}{\sigma_M} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \tag{3.8}$$

The probability of failure becomes:

$$P_F = \phi(-\beta)$$

or

$$P_F = 1 - \phi(\beta) \tag{3.9}$$

In which ϕ represents the normal distribution function common in statistical tables.

It is important to note that R and S are functions of other variables [e.g. material properties, dimensions, loading, etc.] and these design parameters are termed the basic variables.

3.2 SECOND-MOMENT FORMULATION

The calculation of the probability of safety or probability of failure requires the knowledge of the distributions $f_R(r)$ and $f_S(s)$, or the joint distribution $f_{R,S}(r,s)$. In practice, this information is often unavailable or difficult to obtain for reasons of insufficient data. Even when the required distributions can be specified, the exact evaluation of the probabilities, generally requiring the numerical integration of equations, may be impractical, alternative, equivalent normal distributions may be resorted to in approximation.

Most often than not, the available information or data may be sufficient only to evaluate the first and second moments, namely; the mean values and variances of the respective random variables and practical measures of safety or reliability, must often be limited to functions of these first two moments. Thus, the implementation of reliability concepts must necessarily be limited to a formulation based on the first and second moments of the random variables, i.e. restricted to the second moment formulation [1]. With the second-moment approach, the reliability may be measured entirely with a function of the first and second moments of the design variables, namely the reliability index, β, when there is no information on the probability distributions.

Recall the safety margin M = R-S. In this term, the safe state of a system is defined as M > 0, whereas the failure state is M < 0. The boundary separating the safe and failure states is the limit-state defined by

$$M=R-S=0$$

Introducing the reduced variate

$$R'=\frac{R-\mu_R}{\sigma_R}$$

$$then \quad R=\sigma_R R^{\prime}+\mu_R \quad (3.10)$$

Similarly

$$S^{\prime}=\frac{S-\mu_S}{\sigma_S}$$

and

$$S=\sigma_S\, S^{\prime}+\mu_S \tag{3.11}$$

Substituting equations (3.10) and (3.11) into the limit-state equation, gives

$$\sigma_R\, R^{\prime}-\sigma_S\, S^{\prime}+\mu_R-\mu_S=0 \tag{3.12}$$

Equation (3.12) is a linear equation of the reduced variables and the graphical representation shown in Fig. 3.3.

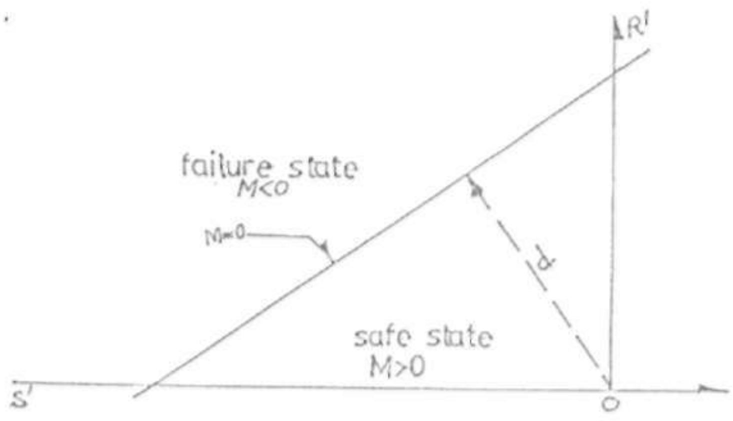


Fig. 3.3: Limit State Boundary.

The x-y Coordinate, the graphical representation of equation 3.12, becomes

$$Y = \sigma_R/\sigma_S\, x + \frac{(\mu_R - \mu_S)}{\sigma_S} \quad (3.13)$$

The perpendicular to line M=0 and passing through origin is

$$y = \frac{\sigma_S}{\sigma_R}\, x \tag{3.14}$$

Solving for x and y from equations (3.13) and (3.14) gives

$$x = \frac{\sigma_R(\mu_R - \mu_S)}{(\sigma_R^2 + \sigma_S^2)} \tag{3.15}$$

and

$$y = -\frac{\sigma_S(\mu_R - \mu_S)}{(\sigma_R^2 + \sigma_S^2)} \tag{3.16}$$

The line distance, d, from the origin to m = 0 in a general form is

$$d = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2} \tag{3.17}$$

Substituting equations (3.15) and (3.16) in (3.17), we have:

$$d = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \tag{3.18}$$

As it was earlier stated, the distance from the [linear] failure line to the origin, O, d is a measure of reliability as shown in (3.18), i.e.

$$d = \beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \tag{3.19}$$

Thus reliability is $P_s = \Phi(d)$

3.1.3 Generalization as a function of Basic Variables

The reliability of an engineering system involve many variables, i.e. R and S are functions of some basic variables. In a broader sense, the reliability of an engineering system may be defined as the probability of performing its intended function or mission. The level of performance of a system will depend on the properties of the system or the basic variables. In this context and for the purpose of a generalized formulation in terms of the basic design variables, we define a performance function or state function.

$$g(X) = g(X_1, X_2, \ldots X_n) \tag{3.20}$$

where $X = (X_1, X_2, \ldots, X_n)$ is a vector of basic state (or design) variables of the system and the function g(X) determines the performance or state of the system. The limit state of the system is:

$$g(X_i) = 0 \tag{3.21}$$

and $[g(X_i) > 0]$ implies safe state and $[g(X_i) < 0]$ corresponds to failure state.

The basic variables $(X_1, X_2, \ldots X_n)$ in the equation may be correlated or uncorrelated. So introducing first the set of uncorrelated reduced variates

$$X_i = \frac{X_i - \mu_{X_i}}{\sigma_{X_i}}; i=1,2....,n. \quad (3.22)$$

The safe state and failure state may also be represented graphically in the space of the reduced variate (equation (3.22)) separated by the appropriate limit state equation. In terms of the reduced variate the limit state equation is,

$$g(\sigma_{xi}X'_1 + \mu_{xi}, \sigma_{x2} X'_2 + \mu_{x2}, \ldots, \sigma_{xn}X'_n + \mu_{xn}) = 0 \quad (3.23)$$

The position of the failure surface relative to the origin of the reduced variates should determines the safety or reliability of the system. The position of the failure surface may be represented by the minimum distance from the surface $g(x) = 0$ to the origin of the reduced variates[18]. It has also been shown that the point on the failure surface with minimum distance to the origin is the most probable failure point[19], i.e. this minimum distance may be used as a measure of reliability and will be determined as follows:

The distance from a point $X' = (X'_1, X'_2, \ldots, X'_n)$ on the failure surface $g(X) = 0$ to the origin of X' is of course:
In matrix form, we have

$$D=\sqrt{X'^2_1+...,X'^2_n} \tag{3.24}$$

$$D = [X'_1 X'_2 ... X'_n] \begin{vmatrix} X'_1 \\ X'_2 \\ \cdot \\ \cdot \\ \cdot \\ X'_n \end{vmatrix} = [X'^t_i X]^{1/2} \tag{3.25}$$

$$x_i = 1,2,...n$$

The point on the failure surface, $(x'_1, x'_2, ..., x'_n)$, having the minimum distance to the origin may be determined by minimizing the function D, subject to the constraint $g(X) = 0$; that is,

Minimize D

Subject to $g(X_i) = 0$

For this purpose, the method of Lagrange's Multiplier may be used. Let

$$L=D+\lambda g(X_i) \tag{3.26}$$

or

$$L=(X_i^{\prime t}\ X_i')^{1/2}+\lambda g(X_i) \tag{3.27}$$

where λ is the value of the multiplier.

In Scalar notation,

In which $X_i = \sigma_{xi} X'_i + \mu_{xi}$

$$L = \sqrt{X_1'^2 + X_2'^2 + \ldots + X_n'^2} + \lambda g(X_1, X_2, \ldots X_n) \tag{3.28}$$

Minimizing L, we obtain n+1 equations with n+1 unknown as

$$\frac{\delta L}{\partial X_i} = \frac{X'_i}{\sqrt{X_1'^2 + X_2'^2 + \ldots + X_n'^2}} + \lambda \frac{\delta g}{\partial X'_i} = 0 \tag{3.29}$$

and

$$\frac{\delta L}{\partial \lambda} = g(X_1, X_2, \ldots X_n) = 0 \tag{3.30}$$

The solution of the above set of equations should yield the most probable failure point (x_1^*, x'^*_2, ..., x'^*_n)

Introducing the gradient vector

$$G = \left(\frac{\partial g}{\partial X'_1}, \frac{\partial g}{\partial X'_{2},} \ldots \frac{\partial g}{\partial X'_n}\right)$$

in which

$$\frac{\partial g}{\partial X'_i} = \frac{\partial g}{\partial X'_i} \frac{dX_i}{dX'_i} = \sigma_{x_i} \frac{\partial g}{\partial X_i}$$

Equation (3.29) can be written in matrix notation as [10].

$$\frac{X'}{(X'^t X')^{1/2}} + \lambda G = 0$$

From which

$$X' = -\lambda DG \tag{3.31}$$

Therefore, from equation (3.25)

and thus,

$$D=[(\lambda DG^t)(\lambda DG)]^{1/2} = \lambda D(G^tG)^{1/2}$$

$$\lambda = (G^tG)^{-1/2} \qquad (3.32)$$

Substituting equation (3.32) in (3.31) gives

$$X' = \frac{-GD}{(G^tG)^{1/2}} \qquad (3.33)$$

Multiply both sides of equation (3.33) by G^t gives

$$G^tX^1 = \frac{-G^tGD}{(G^tG)^{1/2}}$$

Implying that

$$G^tX^1=-(G^tG)^{1/2}D$$

and

$$D=\frac{-G^tX'}{(G^tG)^{1/2}} \qquad (3.34)$$

Recall that at the failure point, the minimum distance from the origin to the point equals ps and therefore equation (3.34) becomes

$$\beta = \frac{-G^{*t}X'^{*}}{(G^{*t}G^{*})^{1/2}} \qquad (3.35)$$

in which G^* is the gradient vector at the most probable failure point $(x'^*_1, x'^*_2, \ldots, x'^*_n)$.

In scalar form Equation (3.35) is
where the derivatives (g/ x'_i). are evaluated at x'^*_1, x'^*_2, ...,

$$\beta = \frac{-\sum_i x'^{*}_i \left(\frac{\partial g}{\partial X'_i}\right)_*}{\sqrt{\sum_i \left(\frac{\partial g}{\partial X'_i}\right)^2_*}} \tag{3.36}$$

x'^{*}_n) and the most probable point on the failure surface for minimum ps from equation (3.33) is

$$X'^{*} = \frac{-G^{*}\beta}{(G^{*t}G^{*})^{1/2}} \tag{3.37}$$

and in scalar form is

$$x'^{*}_i = \alpha^{*}_i \beta;\ i=1,2,\ldots,n \tag{3.38}$$

in which

$$\alpha^{*}_i = -\frac{\left(\frac{\partial g}{\partial X'_i}\right)_*}{\sqrt{\sum_i \left(\frac{\partial g}{\partial X'_i}\right)^2_*}} \tag{3.39}$$

are the direction cosines along the x_i axes.

3.3.1 First-Order Interpretation

When the performance function g(X) is expanded in a Taylor Series at a point x^*, which is on the failure surface, i.e.

$$g(X^{*})=0 \tag{3.40}$$

This gives

where the derivatives are evaluated at (x_1^*, x_2^*, ... x_n^*) but g(x_1^*,

$$g(X_1,X_2,...X_n) = g(x_1^*,x_2^*,...x_n^*)+\sum_{i=1}^{n}(X_i-x_i^*)(\frac{\partial g}{\partial X_i}) + \frac{\sum_{j=1}^{n}\sum_{i=1}^{n}(X_i-x_i^*)(X_j-x_j^*)}{(\frac{\partial^2 g}{\partial X_i\,\partial x_i})} +... \tag{3.41}$$

x_2^*, ... x_n^*) = 0 on the failure surface; therefore,

$$g(X_1,X_2,...X_n)=\sum_{i=1}^{n}(X_i-x_i^*)(\frac{\partial g}{\partial X_i})_* + \frac{\sum_{j=1}^{n}\sum_{i=1}^{n}(X_i-x_i^*)(X_j-x_j^*)}{(\frac{\partial^2 g}{\partial x_i\,\partial x_j})_*} \tag{3.42}$$

Introducing the concept of reduced variate, we have,

$$X_i-x_i^*=(\sigma_{xi}X'_i+\mu_{xi}-\sigma_{xi}x_i^*+\mu_{xi}=\sigma_{xi}(X_i-x_i^*)$$

and

$$\frac{\partial g}{\partial X_i}=\frac{\partial g}{\partial X'_i}(\frac{\partial X'_i}{\partial X_i})=\frac{1}{\sigma X_i}(\frac{\partial g}{\partial X'_i})$$

Truncating equation (3.42) at the first order linear term and substituting the reduced variate into it, gives

$$g(x_i)=\sum_{i=1}^{n}(X'_i-X'^*_i)(\frac{\partial g}{\partial X'_i})_* \tag{3.43}$$

Applying the properties of expectation and variance [13], the mean value of equation (3.43) is
whereas the corresponding first-order approximate variance is
Thus the ratio

$$\mu_g \approx -\sum x'^*_i \left(\frac{\partial g}{\partial X'_i}\right)_* \tag{3.44}$$

$$\sigma_g^2 \approx \sum_{i=1}^{n} \sigma_{xi}^2 \left(\frac{\partial g}{\partial X'_i}\right)_*^2 = \sum_{i=1} \left(\frac{\partial g}{\partial X'_i}\right)_*^2$$

$$\frac{\mu_g}{\sigma_g} = \frac{-\sum_{i=1}^{n} x'^*_i \left(\frac{\partial g}{\partial X_i}\right)_*}{\sqrt{\sum_{i=1}^{n} \left(\frac{\partial g}{\partial X'_i}\right)_*^2}} \tag{3.45}$$

This equation (3.45) obtained by this method conforms to equation (3.36) derived earlier, therefore

$$\beta = \frac{\mu_g}{\sigma_g} \tag{3.46}$$

3.3.1 LINEAR PERFORMANCE FUNCTIONS

Consider a specialised class of performance functions, namely the linear performance function. It is useful and can be used as the basis for an approximation to non-linear performance functions.

A linear performance function may be represented as

$$g(X) = a_o + \sum_i a_i X_i$$

where a_o and a_i's are constants. The corresponding limit state equation therefore, is

$$a_o+\sum_i a_i x_i=0 \tag{3.47}$$

In terms of the reduced variates, the limit-state equation becomes

$$a_o+\sum_i a_i(\sigma_{xi}X_i+\mu_{xi})=0 \tag{3.48}$$

In two or three dimensions, Equation (3.48) is

$$a_{o_{a_1}}(\sigma_{x_1}X_1+\mu_{x_1})+a_2(\sigma_{x_2}X_2+\mu_{x_2})+a_3(\sigma_{x_3}X_3+\mu_{x_3})=0 \tag{3.49}$$

which is a plane surface in the x'_1, x'_2, x'_3 space.

The distance of the failure plane equation (3.48) to the origin of the reduced variate X' is

$$\beta=\frac{a_o+\sum_i a_i\mu_{x_i}}{\sqrt{\sum_i (a_i\sigma_{x_i})^2}} \tag{3.50}$$

3.4 STRUCTURAL ELEMENTS

The safety or reliability of structural elements may be evaluated on the basis of a linear performance function of the form:

$$g(x) = M_u - M_A \tag{3.51}$$

where M_u = Moment of resistance

M_A = Applied moment or load effect.

So that, for a singly reinforced concrete slab or beam element;

$$g(x) = 0.15\ f_{cu}\ bd^2 - \frac{W'L^2}{8} \quad (3.52)$$

with f_{cu} = Mean compressive strength of the structural element;

b = Breadth of the structural element;

d = Effective depth of structural element;

w^1 = The total load on the element, uniformly distributed;

and L = Length of structural element.

For a combination of dead and live loads;

W = $(1.6\ Q + 1.4\ D)$ in which;

Q = The live load;

and D = Dead load.

Therefore:

$$g(x) = 0.15 f_{cu}\ bd^2 - \frac{Q(1.6 + 1.4\ D/Q)L^2}{8}$$

$$= 0.15\ f_{cu}\ bd^2 - \frac{Q(1.6 + 1.4\ \alpha)\ L^2}{8} \quad (3.53)$$

where α = Dead to live load ratio.

Applying Taylor Series expansion to Eq. (3.53); that is;

$$\frac{\partial g(x)}{\partial (f_{cu})} = 0.15bd^2 \quad (3.54a)$$

$$\frac{\partial g(x)}{\partial (b)} = 0.15 f_{cu} d^2 \quad (3.54b)$$

$$\frac{\partial g(x)}{\partial (d)} = 0.30 f_{cu} bd \tag{3.54c}$$

$$\frac{\partial g(x)}{\partial (Q)} = \frac{(1.6+1.4\alpha)L^2}{8} \tag{3.54d}$$

Using Eq. (3.45), the safety index, β, can be written as

$$\beta = \frac{M_u - M_A}{\sqrt{\sigma_{M_u}{}^2 - \sigma_{M_A}{}^2}} \tag{3.55}$$

Substituting Eq. (3.54a) to (3.54d) in Eq. (3.55) we obtain:

$$\beta = \frac{0.15 f_{cu} bd^2 - \dfrac{Q(1.6+1.4\alpha)L^2}{8}}{\sqrt{A_0 + A_1 + A_2 + A_3}} \tag{3.56}$$

where:

$$A_0 = (0.15bd^2)^2 \sigma(_{f_{cu}})^2 \tag{3.56a}$$

$$A_1 = (0.15 f_{cu} d^2)^2 \sigma_b^2 \tag{3.56b}$$

$$A_2 = (0.3 f_{cu} bd)^2 \sigma_d^2 \tag{3.56c}$$

$$A_3 = \frac{-(1.6+1.4\alpha)L^2)^2}{8} \sigma_Q^2 \tag{3.56d}$$

For a doubly Reinforced Concrete Beam element, the performance function g(x) is given by

$$g(x) = 0.15f_{cu}bd^2 + 0.72F_yA_s(d-d') - \frac{Q(1.6+1.4\alpha)L^2}{8} \qquad (3.57)$$

where

f_{cu}	=	Mean compressive strength of the structural element;
b	=	Breadth of the structural element;
d	=	Effective depth of structural element;
f_y	=	Yield strength of steel;
d'	=	Depth to the compression reinforcement;
Q	=	Live load
A'_s	=	Area of compression reinforcement in N/mm^2
and L	=	Length of the structural element in mm

Applying Taylor Series expansion to Eq. (3.57), that is:

$$\frac{\partial g(x)}{\partial f_{cu}} = 0.15bd^2 \qquad (3.58a)$$

$$\left(\frac{\partial g(x)}{\partial (b)}\right) = 0.15f_{cu}d^2 \qquad (3.58b)$$

$$\left(\frac{\partial g(x)}{\partial (d)}\right) = 0.3f_{cu}bd - 0.72f_yA_s^1d^1 \qquad (3.58c)$$

$$(\frac{\partial g(x)}{\partial d^{1}}) = 0.72 f_y A^{1} d \quad (3.58d)$$

$$(\frac{\partial g(x)}{\partial (Q)}) = -\frac{(1.6+1.4\alpha)L^2}{8} \quad (3.58e)$$

Substituting Eqs (3.58a) to (3.58e) in Eq. (3.45), we obtain:

$$\beta = \frac{0.15 f_{cu} b d^2 + 0.72 f_y A_s^1 (d-d^1) - \frac{Q(1.6+1.4\alpha)L^2}{8}}{\sqrt{A_4+A_5+A_6+A_7+A_8}} \quad (3.59)$$

$$A_4 = (0.15 b d^2)^2 \sigma_{f_{cu}}^2 \quad (3.59a)$$

$$A_5 = (0.15 f_{cu} d^2)^2 \sigma_b^2 \quad (3.59b)$$

$$A_6 = (0.3 f_{cu} b d - 0.72 f_y A^{1} d^{1})^2 \sigma_d^2 \quad (3.59c)$$

$$A_7 = (0.72 f_y A^{1} d)^2 \sigma_{d^1}^2 \quad (3.59d)$$

$$A_8 = \frac{-(1.6+1.4\alpha)L^2}{8} \sigma_Q^2 \quad (3.59e)$$

For an Axially loaded short concrete column, the performance function is given by:

$$g(x) = 2bh\ 0.4 f_{cu} + 0.75 \rho f_y - 0.4 \rho f_{cu} - Qj(\alpha+1) \quad (3.61)$$

where:

f_{cu} = Mean compressive strength of the structural element;

b = Breadth of the structural element;

d = Effective depth of the structural element;

h = Overall depth of section;

f_y = Yield strength of steel;

Q = Live load effect;

α = Reinforcement ratio;

j = the number of floors.

Applying Taylor series expansion to Eq.(3.61), we obtain the following:

$$\frac{\partial g(x)}{\partial(b)} = 0.8hf_{cu} + 1.5h\rho f_y - 0.8\rho f_{cu} \tag{3.61a}$$

$$\frac{\partial g(x)}{\partial(h)} = 0.8bf_{cu} + 1.5\rho f_y - 0.8\rho f_{cu} \tag{3.61b}$$

$$\frac{\partial g(x)}{\partial(f_{cu}} = 0.8bh - 0.8bh\rho \tag{3.61c}$$

$$\frac{\partial g(x)}{\partial(\rho)} = 1.5bhf_y - 0.8bhf_{cu} \tag{3.61d}$$

$$\frac{\partial g(x)}{\partial(f_y)} = 1.5bh\rho \tag{3.61e}$$

$$\frac{\partial g(x)}{\partial(Q)} = -j(\alpha + 1) \tag{3.61f}$$

Substituting Eqs. (3.61a) to (3.61f) in Eq. (3.45) the safety index; β; for an Axially loaded short concrete column can be written as:

$$\beta = \frac{2bh\ 0.4f_{cu}+0.75\rho f_y-0.4\rho f_{cu}-Qj(\alpha+1)}{\sqrt{A_9+A_{10}+A_{11}+A_{12}}} \tag{3.62}$$

A computer program in FORTRAN 77, is used in this study to compute the implied safety levels β (Eq. 3.56, 3.59 and 3.62) This is

$$A_9 = (0.8hf_{cu}+1.5h\rho f_y-0.8h\rho f_{cu})^2\sigma_b^2 \tag{3.62a}$$

$$A_{10} = (0.8bf_{cu}+5b\rho f_y-0.8\rho f_{cu})^2\sigma_h^2 \tag{3.62b}$$

$$A_{11} = (0.8bh-0.8bh\rho)^2\sigma_{f_{cu}}+(1.5bhf_y-0.8bhf_{cu})^2\sigma_\rho^2 \tag{3.62c}$$

$$A_{12} = (1.5bh\rho)^2\sigma_{fy}^2+-j(\alpha+1)^2\sigma_Q^2 \tag{3.62d}$$

A computer program in FORTRAN 77, is used in this study to compute the implied safety levels β (Eq. 3.56, 3.59 and 3.62) This is shown in Appendix 1.

CHAPTER FOUR

COMPUTATION OF SAFETY INDICES

4.1 Acquisition of Data:

Hardness tests using the Rebound hammer were carried out. At least nine valid readings and usually not more than twenty-five such valid readings were taken to obtain a good representative mean. These nine to twenty-five individual readings were confined to an area not exceeding about 300mm x 300mm in order to reduce biasness by the operator. A grid of lines were drawn at about 20mm to 50mm apart and the intersections of these lines were taken as test points. The point of impact were also at least 20mm away from any edge or sharp discontinuity. Normal sizes and covers of reinforcing steel in concrete have no effect on the hardness readings[4]. Horizontal and Vertical directions of test were used.

4.2 Analysis of Data:

In all calculations, all readings (including abnormally high and abnormally low) were used. The coefficient of variation of individual reading within one test is of the order of 12% but can be as low as 7% and as high as 15%. The coefficient of variation decreases with increase in the strength of concrete and increase with increase in the size and amount of coarse aggregate [4]. The statistical description for the four sites are given in Tables 4.1-4.3.

The mean slab dimensions are breadth b = 1600mm, depth d = 105mm, and a mean length l = 4000mm. For doubly reinforced beam, breath b = 300mm, depth d = 600mm, and a mean length L = 400mm. Reinforcement is mostly of Grade 40, with a mean Yield strength f_y = 410N/mm^2. The mean area of compression reinforcement A^1_s is 628mm^2. In the course of the study, the reinforcement ratio, $\rho = (A_s+A^1_s)/bh$, is allowed to vary from 0.01-0.07 and a mean value of ρ equal 0.03, was used in this work. This covers the range of many column designs. Mean depths of the tensile and compressive reinforcement are d = 510mm,and d^1 = 50mm respectively.

As explained previously, in second moment reliability method, the parameters of interest are first and second moments (mean and variance) for the computation of safety indices [11]. It is important to note that it is the coefficient of variation that are mostly published in literature [9]. The values of some relevant parameters for the basic variables used in this study are shown in Table 4.4

Table 4.1: Mean value of Compressive Strength

Sites Number	Slabs (N/mm^2)	Beams (N/mm^2)	Short columns (N/mm^2)
1	29.908	24.827	26.580
2	28.102	23.772	22.294
3	31.341	24.733	24.506
4	29.431	25.037	23.826

Table 4.2: Mean Standard deviation

Sites Number	Slabs (N/mm^2)	Beams (N/mm^2)	Short columns (N/mm^2)
1	4.326	3.459	3.0955
2	3.967	2.730	1.771
3	3.685	2.730	3.019
4	3.622	3.533	3.020

Table 4.3: Coefficient of Variation (COV)

Sites	Coefficient of Variation for		
Number	Slabs	Beams	Short columns
1	0.1446	0.1393	0.1165
2	0.1412	0.1148	0.0774
3	0.1176	0.125	0.1232
4	0.1231	0.1411	0.1268

Table 4.4: Statistical values of design parameters.

s/no	Basic Variables	Mean	Coeff.Variation
1.	Live load		0.2-0.35
2.	Dead load		0.05-0.10
3.	Breadth	300mm	0.06
4.	Overall depth of section	400mm	0.04-0.06
5.	Compressive strength of concrete	$24N/mm^2$	0.15
6.	Yield strength of steel	$410N/mm^2$	0.15

In Table 4.4 all mean values are practical values in the design of an adequate slab, beam or column element, with compliance to BS8110 Provisions. Computed safety indices are shown in Tables 4.5-4.11.

Table 4.5: Variation of Safety Index With CompressiveStrength For Singly, Doubly Reinf.Concrete Beams and Short Columns.

Compressive Strength (N/mm^2)	Safety Index		
	Singly Reinf. Beams	Doubly Reinf.Beams	Short Columns
20	4.9651	6.9303	7.1536
22	4.9653	6.7444	7.1827
24	4.9656	6.5906	7.2006
26	4.9659	6.4613	7.2095
28	4.9661	6.3511	7.2113
30	4.9663	6.256	7.2074
32	4.9665	6.1731	7.199
34	4.9666	6.1002	7.1871
36	4.9668	6.0357	7.1726
38	4.9669	5.9781	7.156
40	4.967	5.9264	7.13

Table 4.6: Variation of Safety Index With Span L (m) atload Ratio of 2.5 (For Singly Reinforced Element)

Span (m)	Safety Index			
	Site 1	Site 2	Site 3	Site 4
2.5	4.9677	4.9677	4.9578	4.9677
3.0	4.9671	4.9671	4.9673	4.9672
3.5	4.9665	4.9664	4.9667	4.9665
4.0	4.9657	4.9656	4.9659	4.9657
4.5	4.9648	4.9647	4.9581	4.9649
5.0	4.9636	4.9636	4.9642	4.9639
5.5	4.9627	4.9625	4.9632	4.9628
6.0	4.9615	4.9612	4.9620	4.9614
6.5	4.9601	4.9599	4.9608	4.9603
7.0	4.9587	4.9584	4.9595	4.9589
7.5	4.9572	4.9568	4.9581	4.9574
8.0	4.9556	4.9552	4.9566	4.9580
8.5	4.9538	4.9534	4.9550	4.9541
9.0	4.9520	4.9515	4.9533	4.9522
9.5	4.9500	4.9494	4.9515	4.9503

Table 4.7: Variation of Safety Index With Span L(m) at load Ratio of 2.5 (For Singly Reinforced Element)

Span L (m)	Safety Index			
	Site 1	Site 2	Site 3	Site 4
2.5	6.5365	6.605	6.5431	6.5234
3.0	6.5364	6.6049	6.543	6.5234
3.5	6.5363	6.6048	6.5429	6.5233
4.0	6.5362	6.6047	6.5428	6.5232
4.5	6.5361	6.6046	6.5427	9.5231
5.0	6.536	6.6045	6.5426	6.523
5.5	6.5359	6.6043	6.5425	6.5229
6.0	6.5357	6.6042	6.5423	6.52227
6.5	6.5356	6.604	6.5422	6.5226
7.0	6.5354	6.6038	6.5442	6.5224
7.5	6.5352	6.6037	6.5418	6.5222
8.0	6.535	6.6035	6.5416	6.522
8.5	6.5348	6.6032	6.5414	6.5218
9.0	6.5346	6.6032	6.5412	6.5216
9.5	6.5344	6.6028	6.541	6.5214

Table 4.8: Variation of Safety Index with Span L(m) at different load Ratio (Singly Reinforced Beam) Site 2

Span (m)	Safety Index at Different Load Ratio					
	0.5	1	1.5	2	2.5	3
2.5	4.9684	4.9681	4.9679	4.9677	4.9675	4.9673
3.0	4.9681	4.9678	4.9674	4.9671	4.9668	4.9665
3.5	4.9677	4.9667	4.9665	4.9665	4.9661	4.9656
4.0	4.9673	4.9661	4.9662	4.9657	4.9651	4.9646
4.5	4.9668	4.9662	4.9655	4.9648	4.9641	4.9634
5.0	4.9663	4.9655	4.9646	4.9638	4.9629	4.9621
5.5	4.9657	4.9647	4.9637	4.9627	4.9617	4.9606
6.0	4.9651	4.9639	4.9627	4.9615	4.9603	4.959
6.5	4.9644	4.963	4.9616	4.9601	4.9587	4.9573
7.0	4.9637	4.962	4.9604	4.9587	4.9571	4.9554
7.5	4.9629	4.961	4.96591	4.9572	4.9553	4.9534
8.0	4.962	4.9599	4.9577	4.9556	4.9534	4.9512
8.5	4.9611	4.9587	4.9562	4.9538	4.9514	4.9489
9.0	4.9601	4.9574	4.9547	4.952	4.9492	4.9465
9.5	4.9591	4.953	4.953	4.95	4.9469	4.9439

Table 4.9: Variation of Safety Index with Span L(m) at different Load Ratio (Doubly Reinforced Beam) Site 2.

Span (m)	Safety Index at Different Load Ratio					
	0.5	1	1.5	2	2.5	3
2.5	6.5365	6.5365	6.5365	6.5365	6.5365	6.5364
3.0	6.5365	6.5365	6.5365	6.5364	6.5364	6.5364
3.5	6.5365	6.5364	6.5364	6.5364	6.5363	6.5363
4.0	6.5364	6.5364	6.5363	6.5363	6.5362	6.5362
4.5	6.5364	6.5363	6.5363	6.5362	6.5361	6.636
5.0	6.5363	5.5363	6.5362	6.5361	6.536	6.5359
5.5	6.5363	6.5362	6.5361	6.535	6.5359	6.5358
6.0	6.5362	6.5361	6.536	6.5359	6.5357	6.5356
6.5	6.5361	6.536	6.5359	6.5357	6.53.56	6.5354
7.0	6.5361	6.5359	6.5367	6.5356	6.5354	6.5352
7.5	6.5359	6.5358	6.536	6.5354	6.5352	6.535
8.0	6.5358	6.5357	6.5355	6.5352	6.535	6.5348
8.5	6.5356	6.5356	6.5353	6.5351	6.5348	6.5346
9.0	6.5357	6.5354	6.5352	6.5349	6.5346	6.5343
9.5	6.5356	6.5353	6.535	6.5347	6.5344	6.5341

Table 4.10: Variation of Safety Index With Yield Strength fy(N/mm^2) (Doubly Reinforcement Beam and Short Column) Site 2

Yield Strength	Doubly Reinforced Beams	Short column
231	5.8369	7.0654
257	5.9371	7.1143
283	6.0577	7.1512
308	6.136	7.1771
334	6.2366	7.1955
360	6.3387	7.2064
386	6.4412	7.2111
411	6.5403	7.2106
437	6.64381	7.2059
463	6.7477	7.1974
488	6.8481	7.1865

Table 4.11: Variation of Safety Index With Reinforcement Ratio (Short Columns)

Reinforcement Ratio	Safety Index
0.01	6.7419
0.02	7.1318
0.03	7.2107
0.04	7.1524
0.05	7.04444
0.06	6.925
0.07	6.8095

Fig 4.1 **Variation of Safety Index With Compressive Strength For Singly Reinf**

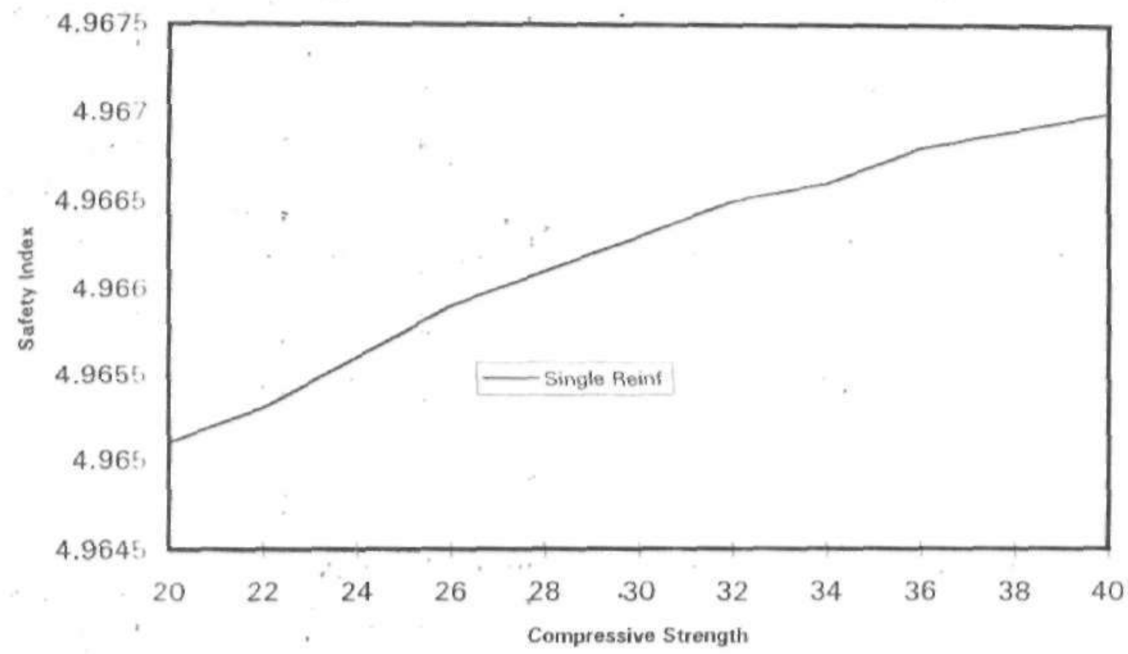
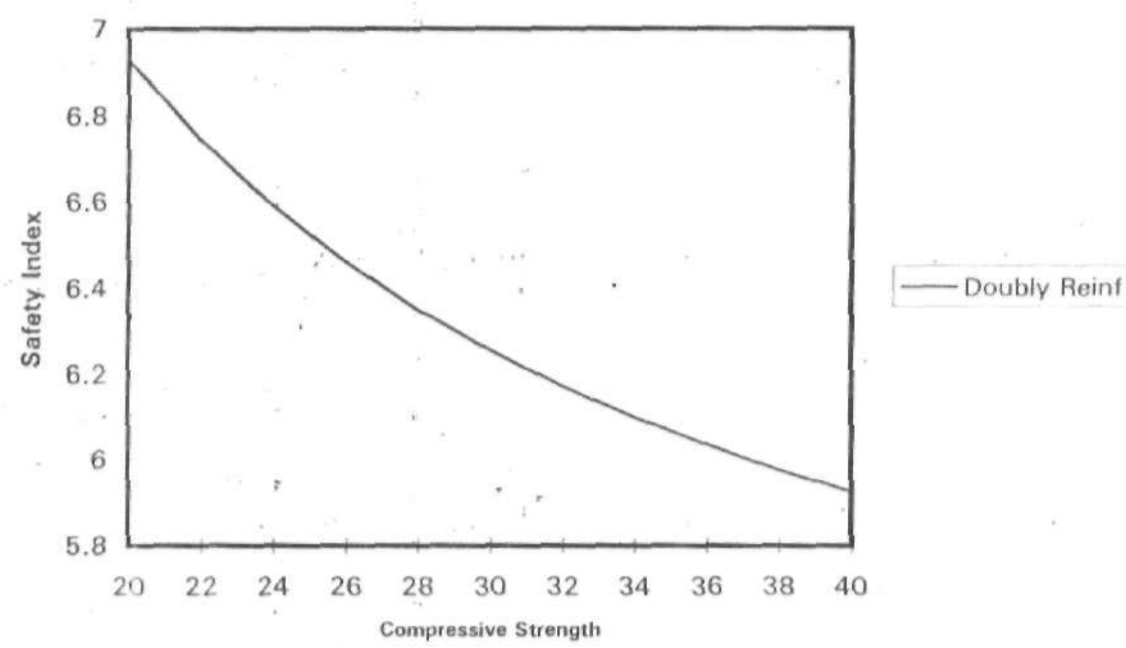


Fig 4.2 **Variation of Safety Index With Compressive Strength For Doubly Reinforced Element.**



4.3 DISCUSSION OF RESULTS

4.3.1 Rebound Hammer Reading

The range of mean rebound hammer reading shown is from 19 N/mm^2 for building 2 to 40 N/mm^2 for building 4. However, this need not imply that building 2 has the lowest concrete strength, of more interest is the variation of concrete quality expressed as standard deviation, which ranged from rebound hammer reading of 2.7 N/mm^2 to 5.5 N/mm^2. A standard deviation for laboratory condition cubes would rarely be greater than 5.2 N/mm^2 [7]. However, larger variations are to be expected in existing buildings because of greater differences in curing conditions, compaction and different ages of the concrete. Hence, higher values could still indicate good quality control or good concrete.

Definite trend toward lower rebound hammer readings (compressive strength) at higher floor levels is evident. The interpretation of this general decrease in rebound hammer readings with height could logically be attributed to deterioration in quality control associated with working conditions for placing and curingofconcrete at greater heights.

4.3.2 Variability of concrete strength

Although records of concrete cylinders tested at the time of construction provided some information on the type of concrete used, it is suggested that such tests give little if any indication of the local variations in strength which occur in existing structures. The non-destructive method of testing concrete in

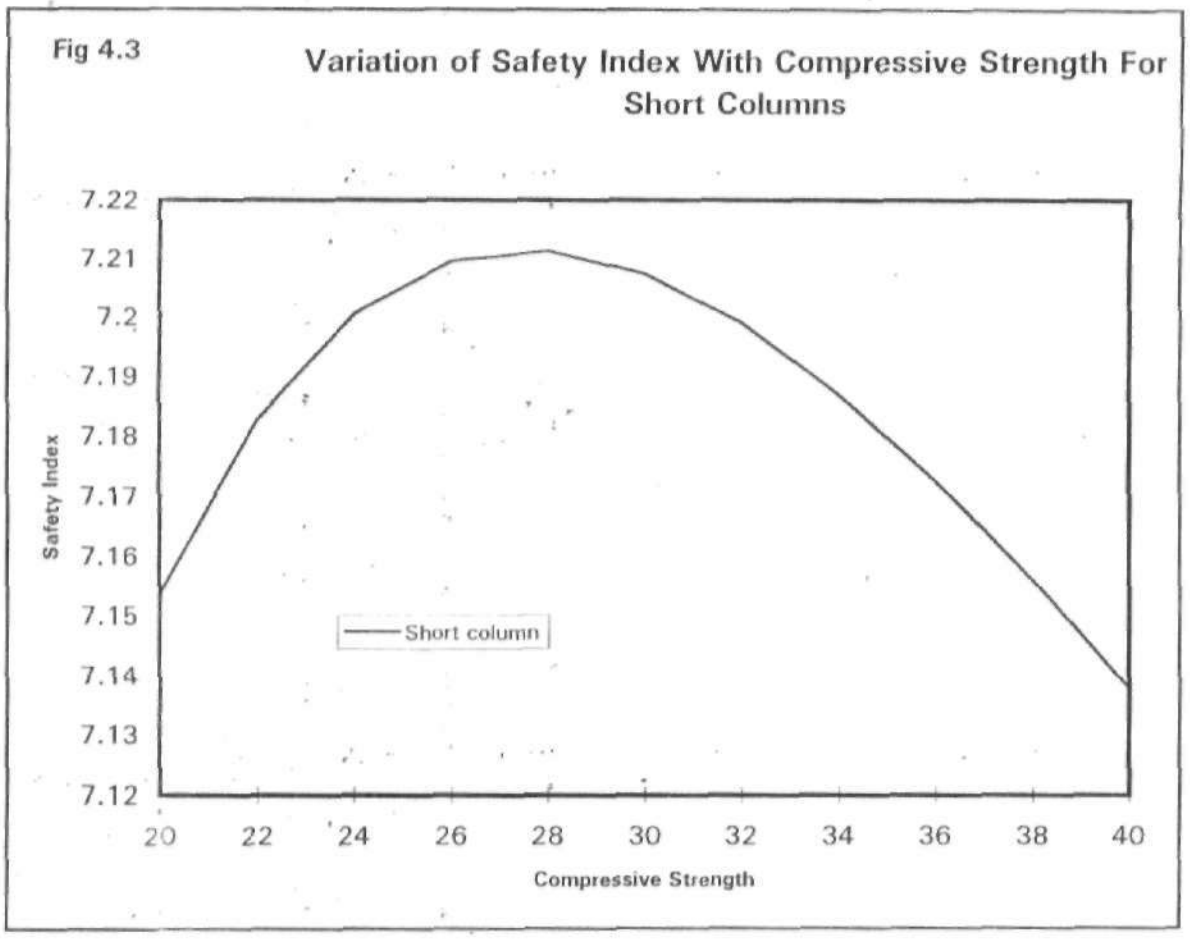


Fig 4.3
Variation of Safety Index With Compressive Strength For Short Columns
7.22
7.21
7.2
7.19
7.18
7.17
7.16
7.15
7.14
7.13
7.12
Safety Index
Short column
20
22
24
26
28
30
32
34
36
38
40
Compressive Strength

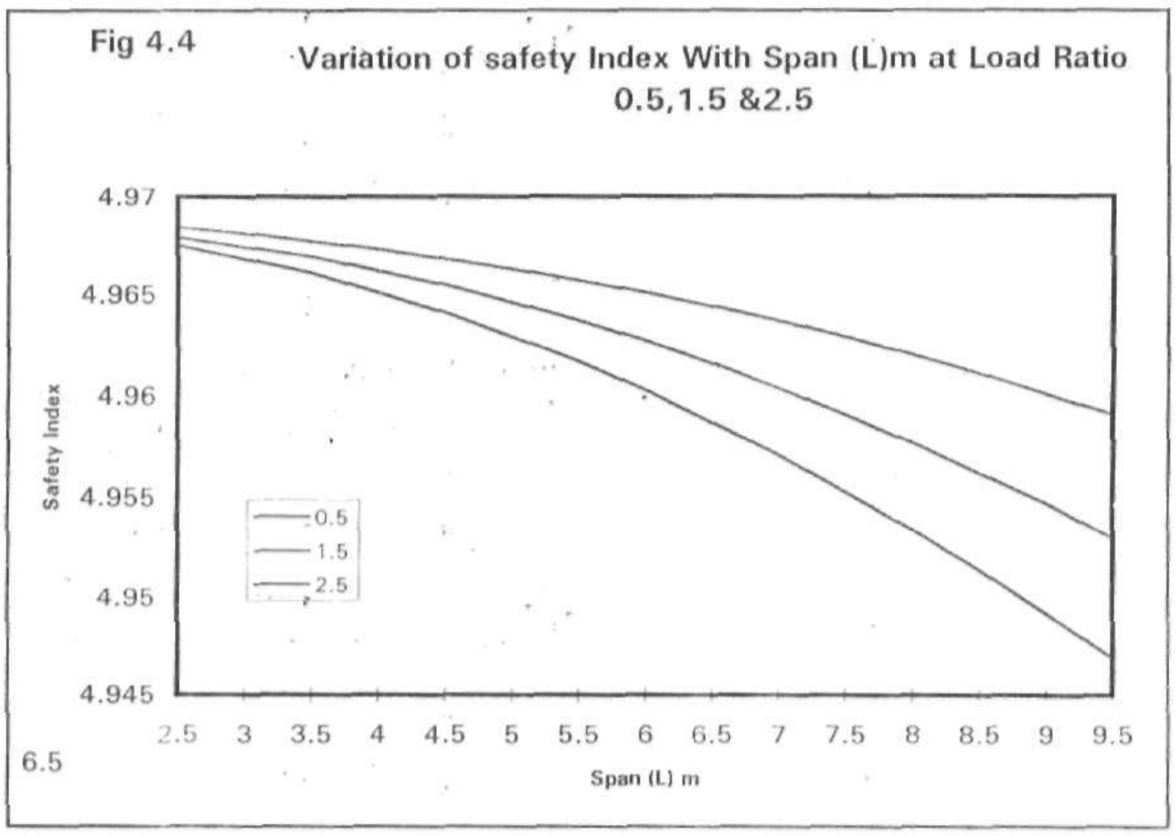


Fig 4.4
Variation of safety Index With Span (L)m at Load Ratio 0.5,1.5 &2.5
4.97
4.965
4.96
4.955
4.95
4.945
Safety Index
0.5
1.5
2.5
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
6.5
Span (L) m

Fig 4.5 Variation of Safety Index With Span (L)mm at Load Ratio 1, 2 & 3.

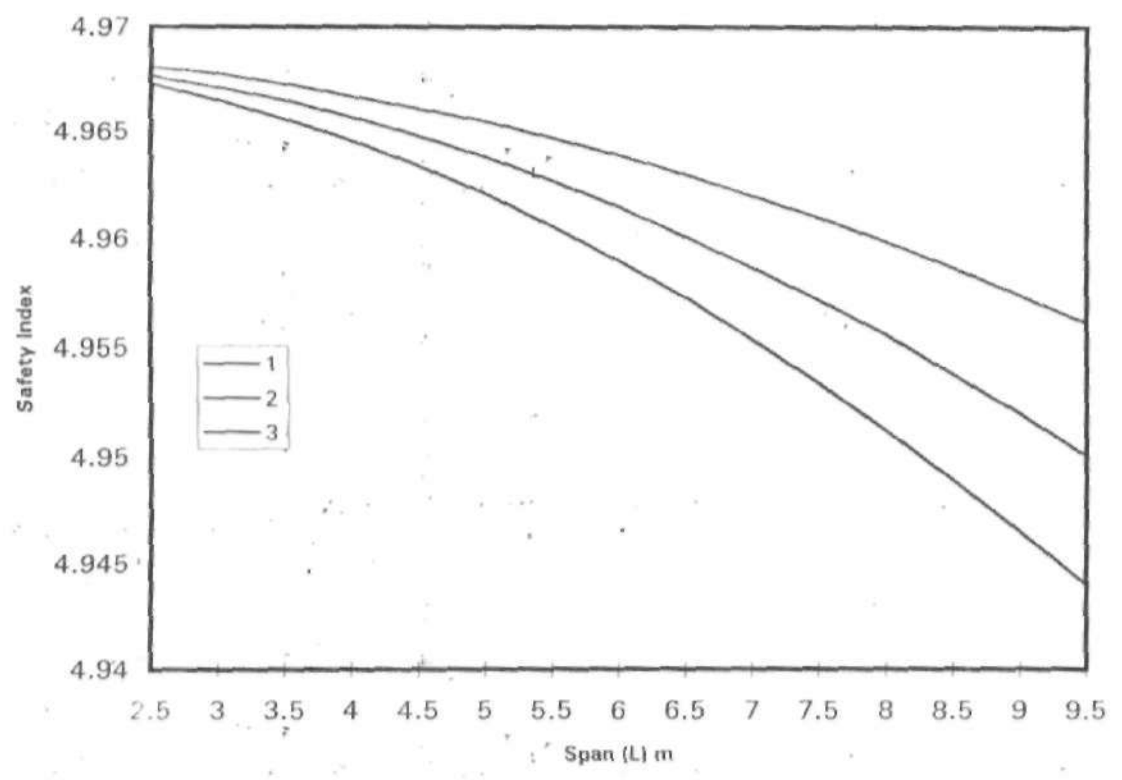
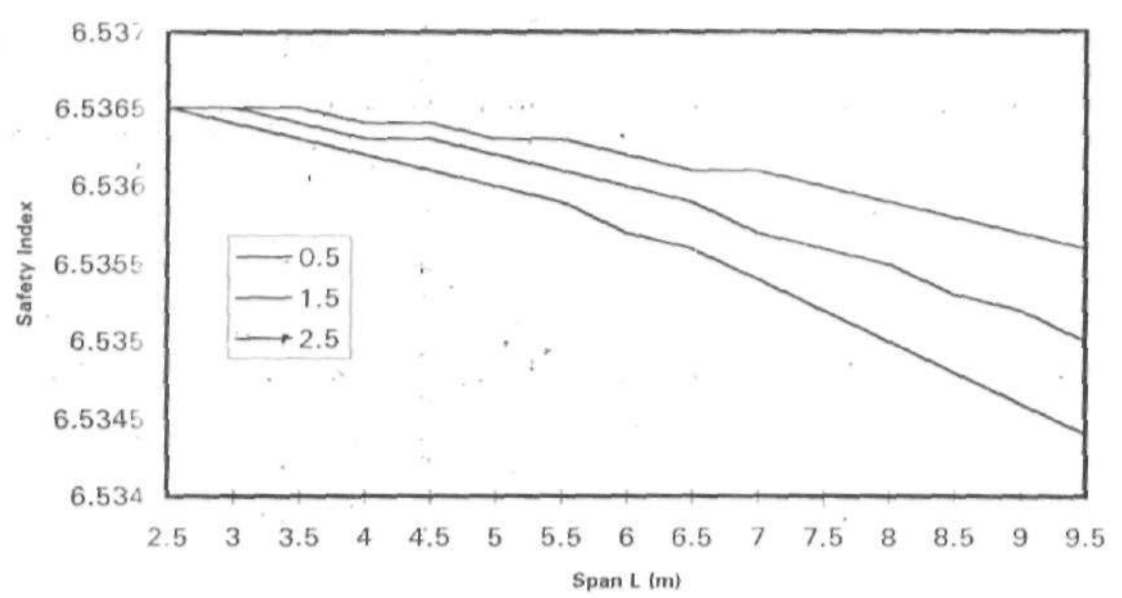


Fig 4.6 Variation of Safety Index With Span L (m) at Load Ratio 0.5, 1.5 & 2.5. For Doubly Reinf. Element.



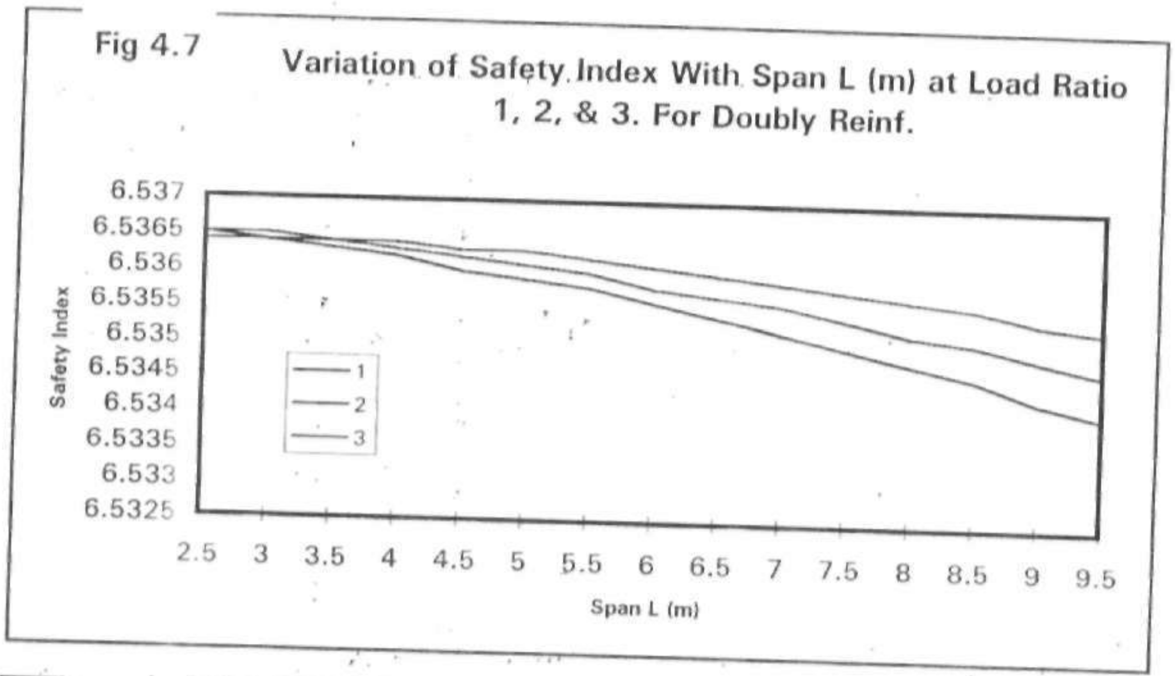


Fig 4.7
Variation of Safety Index With Span L (m) at Load Ratio 1, 2, & 3. For Doubly Reinf.
Safety Index
6.537
6.5365
6.536
6.5355
6.535
6.5345
6.534
6.5335
6.533
6.5325
1
2
3
2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8 8.5 9 9.5
Span L (m)

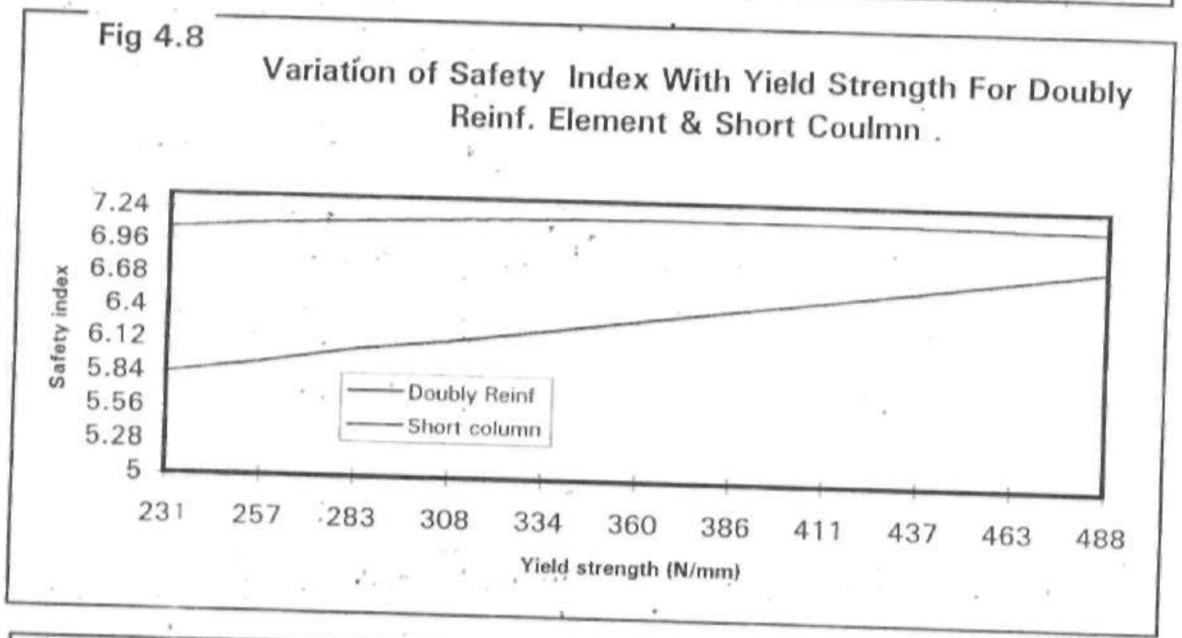


Fig 4.8
Variation of Safety Index With Yield Strength For Doubly Reinf. Element & Short Coulmn .
Safety index
7.24
6.96
6.68
6.4
6.12
5.84
5.56
5.28
5
Doubly Reinf
Short column
231 257 283 308 334 360 386 411 437 463 488
Yield strength (N/mm)

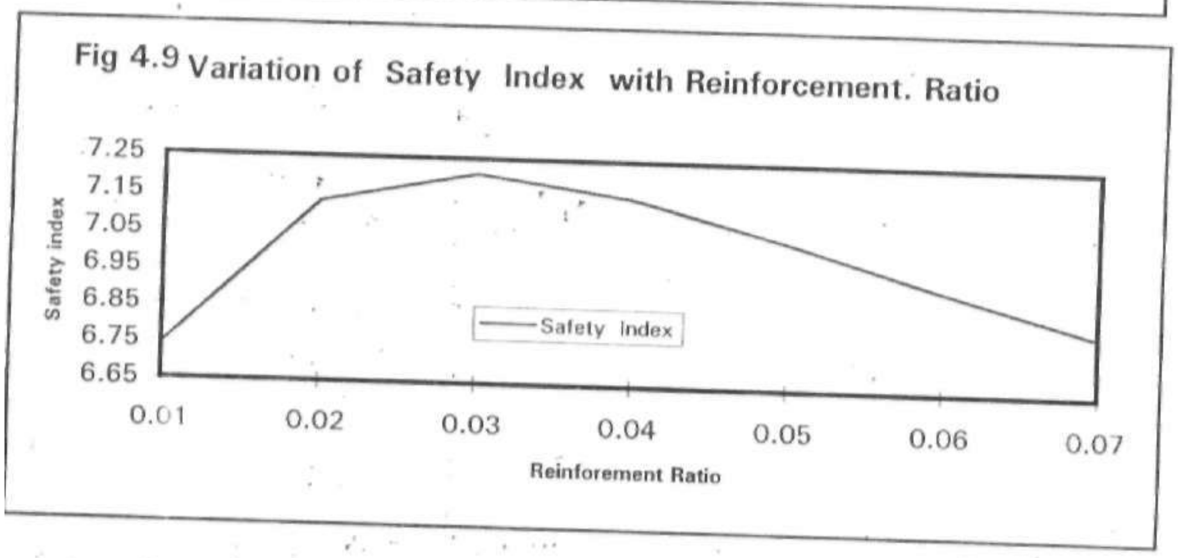


Fig 4.9 Variation of Safety Index with Reinforcement. Ratio
Safety index
7.25
7.15
7.05
6.95
6.85
6.75
6.65
Safety Index
0.01 0.02 0.03 0.04 0.05 0.06 0.07
Reinforement Ratio

4.3 DISCUSSION OF RESULTS

4.3.1 Rebound Hammer Reading

The range of mean rebound hammer reading shown is from 19 N/mm^2 for building 2 to 40 N/mm^2 for building 4. However, this need not imply that building 2 has the lowest concrete strength, of more interest is the variation of concrete quality expressed as standard deviation, which ranged from rebound hammer reading of 2.7 N/mm^2 to 5.5 N/mm^2. A standard deviation for laboratory condition cubes would rarely be greater than 5.2 N/mm^2 [7]. However, larger variations are to be expected in existing buildings because of greater differences in curing conditions, compaction and different ages of the concrete. Hence, higher values could still indicate good quality control or good concrete.

Definite trend toward lower rebound hammer readings (compressive strength) at higher floor levels is evident. The interpretation of this general decrease in rebound hammer readings with height could logically be attributed to deterioration in quality control associated with working conditions for placing and curingofconcrete at greater heights.

4.3.2 Variability of concrete strength

Although records of concrete cylinders tested at the time of construction provided some information on the type of concrete used, it is suggested that such tests give little if any indication of the local variations in strength which occur in existing structures. The non-destructive method of testing concrete in structures using the rebound hammer with corrections by [5]; (although a high degree of accuracy is not claimed), provide a good indication of the local variation of strength that occur in existing structures. The mean coefficient of variation ranged from 0.0774 to 0.1446. On the basis of results of control cylinders, [9] found a coefficient of variation of 0.10 in a case of good control and a value of 0.26 in a case of poor control. The results of coefficients of variation from this work lie within the region of satisfactory control quoted by [9,12]. According to thire classifications, short columns from building number 2 could be considered to have had excellent quality control (coefficient of variation less than 0.10), all other group of element from other sites also have good control (0.10 to 0.15)

4.3.3 Variation of reliability index

Table 4.5 presents the general variation of compressive strength with safety index for singly reinforced slabs, doubly reinforced beams and short columns. The reliability level is generally high, ranging from a value of 4.9651 at compressive strength of 20 N/mm^2 to 7.2113 at compressive strength of 28 N/mm^2 for short column. The interpretation of this high value of reliability level may be due to the fact that on all sites; there are good quality control which results in small coefficient of variation. The reliability levels increase as the compressive strength increase for singly reinforced elements. The reliability levels decrease as the compressive strength increase for doubly reinforced member and for short columns the variation with compressive strength is parabolic in nature with maximum of 7.2113 at a compressive strength of 28 N/mm^2 . These trends are shown in figures 4.1,4.2 and 4.3 respectively.

The variation of safety index with span L (m) at different load ratio for single reinforced beam or slab are shown in Table 4.6 and 4.7 respectively. The reliability levels decrease as the L(m) increase. This is clearly shown in figs. 4.4 and 4.5 for singly and doubly reinforced elements respectively.

The relationship could be due to the facts that in the design and construction of these elements; the same level of workmanship may have been given at all lengths which will result in a small coefficient of variation (range of 0.0774 - 0.1496) for shorter lengths and higher reliability levels. For doubly reinforced elements, the reliability levels are less affected by changes in span as compared to the singly reinforced case (Table 4.7)

It is interesting to note that at any span, the reliability level decreases as the load ratio increases. The variation of reliability level with load ratio is almost negligible at lower span (i.e. 2.5m to about 4.0m) for doubly reinforced beam, but the variation is quite significant at higher span (i.e 7.0m to 9.5m) as shown in Table 4.8 and 4.9.

The variation of the computed reliability level with yield strength, fy (N/mm^2) is shown in Table 4.10. For doubly reinforced concrete element, the reliability level for yield strength of 231 (N/mm^2) is 5.8361 and for yield strength of 488 (N/mm^2) 6.8481 which shows an increasing linear relationship with yield strength. This is shown in Fig. 4.8. For axially loaded short column, the variation of reliability levels with yield strength is parabolic with a pick value of $\beta = 7.21$ at a yield strength of 386 (N/mm^2); also shown in Fig. 4.8.

Under varying reinforcement ratio, the computed reliability levels are shown in Table 4.11. At reinforcement ratio of 0.01, β value of 6.7419 is obtained. The values of β increase to 7.2107 at a reinforcement ratio of 0.03 and decreased to 6.8095 at reinforcement ratio of 0.07. This could be interpreted as follows; that all things been equal, there is a peak reinforcement ratio for each load ratio or some other variation of parameters. This relationship is also shown in Fig. 4.9.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

5.1 CONCLUSION

In this thesis, the adequacy of existing reinforced concrete elements with regards to the design criteria for reinforced concrete elements was assessed. Rebound hammer was used as a non-destructive device to collect hardness readings from four sites. Computer program based on the Mean First - Order Second Moment Method of reliability analysis was developed to compute the reliability levels.

After a critical analysis and evaluation of the collected data and computed reliability levels, the following conclusions have been reached.

1. Poor- quality concrete can be located easily by the use of Rebound hammer measurements, where low readings simply indicate low strength.

2. An evaluation of concrete strength variation in a building can be obtained from rebound hammer reading without performing a core (i.e; destructive) test on the concrete. This will be sufficient for many purposes like checking the uniformity of concrete quality with respect to mean quality.

3. The coefficient of variability of concrete strength for the elements in a building usually falls in the range of 0.10 to 0.20, but has been found in this work to range from 0.077 to 0.145. This

agree with previous range of estimates for very good quality control.

4. The information on variability of concrete strength is intended *to assist in the evaluation of the safety* level of existing structural elements. The average of the computed safety index in this study for site 1 is 5.112, site 2 is 5.201, site 3 is 5.141 and site 4 is 5.081. These values are quite high and show a high level of workmanship and conservativeness in favor of safety in the formation of design equations.

5. However, the findings provide other useful information as follows:

a. The design criteria for reinforced concrete elements are consistent with regard to the basic design variables because the implied reliability levels do not change significantly with changes in the values of the basic design variables.

b. The study has also revealed that, depending on the manner in which a section resists external load, individual uncertainties may contribute disproportionately to the uncertainty in resistance. For example, in the design of short columns, the reinforcement ratio, ($\rho = (A_s + A'_s)/bh$), has a peak value in it variation with safety index.

5.2 *SUGGESTIONS AND RECOMMENDATIONS*

It is recommended that as a non destructive test for strength and other properties of concrete, the schmidt rebound hammer testing during or after inspection could be an efficient and effective method of inspection as an alternative to the destructive strength tests.

It is important to remark that the probabilistic approach employed in this study provides an extremely useful procedure for evaluating those aspects of safety of a structure that are direct functions of material properties; when the basic decision variable are random in nature.

A detail sensitivity analysis should be conducted to ascertain the actual contribution of each of the basic variable to the reliability level of existing structural elements.

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Appendix 1: Computer program for the computation Of safety indices

```
b        PROGRAM   STEEL
C
       OPEN(UNIT=1,FILE='STREL1.DAT')
       OPEN(UNIT=2,FILE='STEEL.OUT')
       OPEN(UNIT=3,FILE='STREL2.DAT')
       OPEN(UNIT=7,FILE='STREL3.DAT')
C      ITYPE = 01 - SINGLE REINFORCED CONCRETE BEAM OR SLAB
C              02 - DOUBLY REINFORCED CONCRETE BEAM
C              03 - AXIALLY LOADED SHORT CONCRETE COLUMN
C
       WRITE(*,*)'GIVE VALUE OF:ITYPE'
       READ (*,*)ITYPE
       IF(ITYPE.EQ.1)THEN
         READ(1,*)NV
         DO 11 K=1,NV
        READ(1,*)A,B,D,E,Q,A1
C
         G1=0.15*A
         G2=0.06*B
         G3=0.06*D
         G5=0.3*Q
         D2=D*D
         E2=E*E
         G12=G1*G1
         G22=G2*G2
         G32=G3*G3
         G52=G5*G5
C        A1=C/Q
         A2=(1.6+(1.4*A1))*E2/8.0
         U=0.15*A*B*D2-Q*A2
         V1=(0.15*B*D2)*(0.15*B*D2)*G12
         V2=(0.15*A*D2)*(0.15*A*D2)*G22
         V3=(0.3*A*B*D)*(0.3*A*B*D)*G32
         V4=A2*A2*G52
         V=SQRT(V1+V2+V3+V4)
         BETA=U/V
         WRITE(2,61)K,BETA
   11    CONTINUE
       ELSEIF(ITYPE.EQ.2)THEN
         READ(3,*)NV
         DO 21 K=1,NV
         READ(3,*)A,B,D,E,F,G,H,Q,A3
C
         G1=0.15*A
```

```
      G2=0.06*B
      G3=0.06*D
      G4=0.06*H
      G5=0.3*Q
      D2=D*D
      E2=E*E
      G12=G1*G1
      G22=G2*G2
      G32=G3*G3
      G42=G4*G4
      G52=G5*G5
C     A3=C/Q
      A2=(1.6+(1.4*A3))*E2/8.0
      U=0.15*A*B*D2+0.72*F*G*(D-H)-Q*A2
      V1=(0.15*B*D2)*(0.15*B*D2)*G12
      V2=(0.15*A*D2)*(0.15*A*D2)*G22
      V3=0.3*A*B*D-0.72*F*G
      V3=V3*V3*G32
      V4=(0.72*F*G)*(0.72*F*G)*G42
      V5=A2*A2*G52
      V= SQRT(V1+V2+V3+V4+V5)
      BETA =U/V
      WRITE(2,61)K,BETA
 21   CONTINUE
     ELSEIF(ITYPE.EQ.3)THEN
      READ(7,*)NV
      DO 31 K=1,NV
      READ(7,*)A,B,F,P,T,XN,Q,A3
C
      G1=0.15*A
      G2=0.06*B
      G5=0.30*Q
      G6=0.06*T
      G7=0.15*F
      G8=0.06*P
      A1=XN*(A3+1.0)
      A2=0.4*A
      F2=0.75*F
      AF=A2+(F2-A2)*P
      GQ=-A1
      GB=2.0*T*A2+2.0*T*P*F2-2.0*T*P*A2
      GH=2.0*B*A2+2.0*B*P*F2-2.0*B*P*A2
      GA=0.8*B*T
      GA=(1-P)*GA
C     GA=Q*A1*(0.8*B*T-0.8*B*T*P)/GA
      GF=1.5*B*T*P
C     GF=GF*GF
C     GF=1.5*Q*A1*B*T*P/GF
      GP=2.0*B*T
      GP=(F2-A2)*GP
C     GP=2.0*Q*A1*B*T*(F2-A2)/GP
      U=2.0*B*T*AF
```

```
      U=U-Q*A1
      V1=GQ*GQ*G5*G5
      V2=GB*GB*G2*G2
      V3=GH*GH*G6*G6
      V4=GA*GA*G1*G1
      V5=GF*GF*G7*G7
      V6=GP*GP*G8*G8
      V=SQRT(V1+V2+V3+V4+V5+V6)
      BETA=U/V
      WRITE(2,61)K,BETA
31    CONTINUE
     ENDIF
61   FORMAT(/,10X,'BETA(',I2,')=',F12.4)
     STOP
     END
```