Comparative Study Of Principal Components And Factor Analytical Techniques

By

Reuben, BenhamZangaluka

Department Of Mathematics, Faculty Of Science,

Ah mad u Bello University,

Zaria

October, 2013

Comparative Study of Principal Components and Factor Analytic Techniques

By

Reuben, Benham Zangaluka

M.Sc/Scien/01928/2009-2010

A Thesis Report Submitted to the Postgraduate School, Ahmadu Bello University, Zaria in Fulfillment for the Award of M.Sc Degree in

Statistics

Department of Mathematics

DECLARATION

I Reuben, Benham Zangaluka do hereby declare that this research titled Comparative Study of Principal Components and Factor Analytic Techniques was conducted by me in the Department of Mathematics under the supervision of Professor S.U. Gulumbe and Dr. A. Yahaya, and that no part of this work has been presented for the award of any degree. The information derived from the literature has been duly acknowledged in the text and list of references provided.

Reuben, Benham Zangaluka	[signature]	21-10-2013
Name of Student	Signature	Date

CERTIFICATION

This Thesis entitled "Comparative Study of Principal Component and Factor Analytic Techniques" by Benham Zangaluka REUBEN (M.Sc./Scie/01928/2009-2010) meets the regulations governing the award of the degree of Master of Science of Ahmadu Bello University, Zaria and is approved for its contribution to knowledge and literary presentation.

Professor S. U. Gulumbe Chairman, Supervisory Committee	9/1/2014 Date
Dr. A. Yahaya Member, Supervisory Committee	17/01/2014 Date
Dr. A. D. Koko External Examiner	23/5/2013 Date
Dr. B. Sani Head of Department	17/01/14 Date
Professor A. A. Joshua Dean, School of Postgraduate Studies	27 APR 2014 Date

DEDICATION

This thesis is dedicated to my father, W02 Reuben Zangaluka Belwa and to my brothers, Mr. David Aljibi Zangaluka and Mr. Makun Zangaluka all of blessed memories, for their kindness, guidance and unwavering support. May their gentle souls continue to rest in the bosom of the Lord, Amen.

ACKNOWLEDGEMENTS

I would like to thank Professor S.U. Gulumbe, who helped edit and supervised this research, and whose invaluable insights played a critical role in the shape and content of this research. I remain thankful to Dr. Abubakar Yahaya who also supervised this research, for recognizing the merit of early manuscript and making erudite suggestions for improvement. My regards go to Dr. H.G. Dikko for igniting the interest in this write up.

I would like to appreciate the Zangalukas, whom I am part of, for their endless support and patience. They are the cause for this endless quest for knowledge. I must also not forget to pay tribute to Michael Adejo Chenemi for his encouragement

I would also like to thank Weng Chollom, who lent me his laptop and connected me to the web where most of the materials to this research were sourced. Without him this research would not have been easy. In addition, my indebtedness goes to Mrs. Cecilia Ene Audu, the Headmistress of Krystal International School, Kano, for the permission given to me to study and work.

ABSTRACT

Principal component and factor analytic techniques take large number of variables and reduce them to much smaller number of coherent subset such that variables within a subset are related to one another but independent to those in other subsets. These methods summarize patterns of correlation between observed variables. In this research work, Principal Components and Factor Analytic Techniques are compared using data from Nigerian Consumption Pattern 2009/2010. The results revealed that factor analytic techniques preserve correlation more than principal components, while on the other hand, principal components preserve variance more than factor analytic techniques. We therefore conclude that factor analysis should be used when we are interested in making statements about the factors that are responsible for a set of observed responses, and principal component analysis should be used when we are simply interested in performing data reduction.

TABLE OF CONTENTS

APPENDICES

LISTS OF TABLES

CHAPTER ONE

GENERAL INTRODUCTION

1.1 BACKGROUND TO THE STUDY

Factor analysis (FA) is often confused with principal component analysis (PCA), a similar statistical procedure. However, there are significant differences between the two: factor analysis and principal component analysis will provide somewhat different results when applied to the same data. The purpose of PCA is to derive a relatively small number of components that can account for the variability found in a relatively large number of measures. This procedure, called data reduction, is typically performed when a researcher does not want to include all the original measures in analysis but still wants to work with the information that they contain (DeCoster, 1998). On the other hand, the primary objectives of factor analysis are to determine the number of common factors influencing a set of measures, and the strength of the relationship between each factor and each observed measure.

Because it is a variable reduction procedure, principal component analysis is similar in many respects to exploratory factor analysis. In fact, the steps followed when conducting a PCA, are virtually identical to those followed when conducting an exploratory factor analytical techniques. However, there are significant conceptual differences between the two procedures, and it is important that we do not mistakenly claim that we are performing factor analysis when actually performing principal component analysis. The most important conceptual difference between the procedures deals with the assumption of an underlying causal structure: factor analysis assumes that the co-variation in the observed variables is due to the presence of one or more latent variables (factors) that exert causal influence on those observed variables (Kline,1994). In contrast, PCA makes no assumption about the underlying causal model. PCA is simply a variable reduction procedure that (typically) results in a relatively small number of components that account for most of the variance in a set of observed variables.

Both factor analysis and principal component analysis have important roles to play in social science research, but their conceptual foundations are quite distinct. Hence, the consequence of using both techniques interchangeably should be addressed by putting them in the right perspective.

1.2 RESEARCH MOTIVATION

Principal components analysis is sometimes confused with factor analysis, and this is understandable, because there are many important similarities between the two procedures: both are variable reduction methods that can be used to identify groups of observed variables that tend to hang together empirically. Both procedures examine the patterns of correlation. Because of these ties, there is a growing concern among researchers (Brown, 2009) who use these procedures on whether there are different assumptions or data formalities that must be true to use one of the two techniques over the other.

Nonetheless there are some important conceptual differences between PCA and FA that should be understood at the outset. This research attempts to accentuate some of these differences by highlighting how each tool behaves at each stage of computation.

1.3 STATEMENT OF THE PROBLEM

In general terms, both principal component analysis and factor analysis can be seen as approaches for summarizing and uncovering any patterns in a set of multivariate data, essentially by reducing the complexity of the data. The details behind each method are quite different. Factor analysis, like principal component analysis, attempts to explain a set of data in terms of a smaller number of dimensions that one begins with, but the procedures used to achieve this goal are essentially different in the two methods. Factor analysis unlike principal component analysis, begins with a hypothesis about the covariance (or correlational) structure of the variables (Landau and Everitt, 2004).

Apart from the descriptive statistics and correlation matrix every other result is dependent upon the numerous techniques of extraction and methods of rotation. Some of the limitation of this study are the use of only one (maximum likelihood method) of the several methods of extraction in factor analysis; the use of 25 as maximum number of iterations for convergence, the use of varimax out of other methods of rotation among others.

Timm (2002) concluded that the biggest difference between principal components and factor analysis comes from model philosophy. Factor analysis imposes a strict structure of a fixed number of common (latent) factors whereas the principal component analysis determines a given number of components in decreasing order of importance.

Although much work has been done in comparing principal component analysis and factor analysis, sufficient attention has not been paid to the comparative edge of the two statistical procedures – one over the other. More importantly, to the best of our knowledge, only little has been done on accentuating the computational differences between the solutions of the two statistical procedures that would be enough not to confuse the tools to be used interchangeably. The purpose of this study is to compare principal component and factor analytic techniques with a view to determine specific areas of application of each of the procedures.

1.4 AIM AND OBJECTIVES OF THE STUDY

This research examines the association/dissociation between principal component and factor analytic techniques used in data reduction. The aim is to ascertain the extent to which data structure can be fitted into appropriate model. To this end, the objectives of this study are to find:

i. the degree of numeric association/dissociation between the solutions of principal component and factor analytic techniques,

ii. the strength of each of the techniques in terms of computational efficiency with a view to further deepen the discrepancies between the two techniques so as to assuage the illusion of using both tools interchangeably.

1.5 SIGNIFICANCE OF THE STUDY

In order to have thorough understanding on specific areas of applying each of the two statistical procedures, this research will provide answers to the following questions:

i. What are the numeric similarities and dissimilarities between the solutions of principal component and factor analytic techniques?

ii. To what extent are any of these procedures more appropriate to use than the other?

This research should be able to tell us which technique to use when we are interested in making statements about the factors that are responsible for a set of observed responses, and which technique to use when we are interested in performing data reduction.

1.6 SCOPE AND LIMITATION OF THE STUDY

This research is circumscribed by the use of discretion to determine the number of iterations for convergence during extraction and the little attention accorded confirmatory factor analysis. This research only gives an overview of each of the procedures in an attempt to highlight the similarities as well as the differences between the two procedures with respect to data reduction.

1.7 DEFINITION OF TERMS

- Communality: This refers to the percent of variance in an observed variable that is accounted for by the retained components (or factors) (DeCoster, 1998).

- Construct: This refers to an idea formed by combining several pieces of information or knowledge (Timm, 2002).

- Correlation: This is a measure of association between two variables of great importance (Suhr, 2009).

- Correlation Coefficient: This is a numerical measure of the degree of agreement between set of score. It runs from +1 to -1. +1 indicates full agreement, 0 no relationship and – 1 complete disagreement (Jollife, 2002).

- Covariance: This is the expression in the relationship between two variables for X (Timm, 2002).

- Eigen values: These indicate the amount of variance explained by each principal component or each factor (Brown, 2009).

- Eigenvectors: These are weights in a linear transformation when computing principal component scores (Brown, 2009).

- Latent: Something that is latent (dormant) is present but hidden, and may develop or become more noticeable in the future (Jollife, 2002).

- Oblique: This means other than 90 degree angle (Field, 2004).

- Orthogonal: This means at a 90 degree, perpendicular (Field, 2004).

CHAPTER TWO

LITERATURE REVIEW

2.1 INTRODUCTION

In chapter one, a number of principles and concepts underlying this study were introduced. It was pointed out that the study is concerned with ascertaining the degree of association or otherwise of principal components analysis and factor analysis with a view to make statements about the factors that are responsible for a set of observed responses, and which technique to use when we are interested in performing data reduction. To this end, the discussion in this chapter will focus on the following:

i. Principal component analysis: An overview.

ii. Factor analysis: An overview.

iii. Principal components analysis versus factors analysis.

2.2 AN OVERVIEW OF PRINCIPAL COMPONENT ANALYSIS

This section gives a brief of the concept of principal component analysis, uses and characteristics of principal components and possible areas of application of the technique.

2.2.1 Concept of Principal Components Analysis

Principal components analysis as noted by Smith (2002) is a way of identifying patterns in data, and expressing the data in such a way as to highlight their similarities and differences. The main advantage of PCA as a powerful tool for analyzing data is that once you have found these patterns in the data, and you compress the data, that is, by reducing the number of dimensions with very little or no information lost.

Landau and Everitt (2004) have shown that principal component analysis is a multivariate technique for transforming a set of related (corrected) variables into set unrelated (uncorrelated) variables that account for decreasing proportions of the variation of the original observations. The rationale behind the method is an attempt to reduce the complexity of the data by decreasing the number of variables.

The central idea of principal components analysis (PCA) is to reduce the dimensionality of a data set consisting of a large number of interrelated variables, while retaining as much as possible the variation present in the data set. This is achieved by transforming to a new set of variables, the principal components (PC's), which are uncorrelated, and which are ordered so that the few retained components account for most of the variation present in the original variables (Jollife, 2002).

Bilodeau and Brenner (1999) assumed that $\mathbb{X} \in \mathbf{R}^p$ with $E(X) = \mathbf{U}$ and $var(X) = \sum = \sigma ij$. When the dimension p is too large, the principal components methods seek to replace $\mathbb{X}$ by $\mathbb{Y} \in \mathbf{R}^k$, where $k < p$ (and hopefully much smaller), without losing too much 'information'.

2.2.2 Uses of Principal Components

Timm (2002) pointed out that in PCA a set of p-correlated variables is transformed to a smaller set of uncorrelated hypothetical constructs called principal components.

The Principals Components are used to:

i. discover and interpret the dependencies that exist among variables,

ii. examine relationships that may exist among individuals, and

iii. stabilize estimates use to evaluate multivariate normally, and to detect outliers.

2.2.2 Characteristics of Principal Components

- The first components extracted in a principal component analysis accounts for a maximal amount of total variance in the observed variables. Under typical conditions, this means that the first component will be correlated with at least some of the observed variables. It may be correlated with many.
- The second component extracted will have two important characteristics. First, this component will account for a maximal amount of variance in the data set that was not accounted for by the first component. Again under typical conditions, this means that the second component will be correlated with some of the observed variables that did not display strong correlations with component one.
- The remaining components that are extracted in the analysis display the same two characteristics: each component accounts for maximal amount of variance in the observed variables that was not accounted for the preceding components, and correlated with some of the observed variables.
- A PCA proceeds in this fashion, with each new component accounting for progressively smaller and smaller amounts of variance (this is why only the first few components are usually retained and interpreted).
- When the analysis is complete, the resulting components will display varying degrees of correlations with the observed variables, but are completely uncorrelated with one another.

2.2.3 Applications of Principal Components

There are many important applications in which data under study can naturally be modeled as low-rank plus a sparse contribution. All the statistical applications, in which robust principal components are sought, of course fit our model. Below, Candes *et al.* (2009) gave examples inspired by contemporary challenges in computer science, and noted that depending on the applications, either the low-rank (L_o) component or the sparse (S_o) component could be the object of interest:

i. Video Surveillance: Given a sequence of surveillance video frames, we often need to identify activities that stand out from the background. If we stack the video-frames as columns of matrix M, then the low-rank component L_o naturally corresponds to sparse component S_o captures the moving objects in the foregrounds pixels, and each video fragment contains hundreds or thousands of frames. It would be impossible to decompose M= (L_o + S_o) in such a way unless we have a truly scalable solution to this problem.

ii. Face Recognition: Images of a human's face can be well-approximated by a low-dimensional subspace. Being able to correctly retrieve this subspace is crucial in many applications such as face images which often suffer from self-shadowing or saturations in brightness, which make this a difficult task and subsequently compromise the recognition performance.

iii. Latent Semantic indexing: Web search engines often need to analyze and index the content of an enormous corpus of documents. A popular scheme is the latent semantic indexing (LSI). The basic idea is to gather a document-versus-term matrix M whose entries typically encode the relevance of a term (or a word) to a document. PCA or Spectral Variable Decomposition (SVD) has traditionally been used to decompose the matrix as a low-rank part plus a residual, which is not necessarily sparse (as we would like). If we were able to decompose M as a sum of low-rank component L_o and a sparse component S_o, then L_o could capture common words used in all the documents while S_o captures the few key words that best distinguish each document from others.

v. Ranking and Collaborative filtering: The problem of anticipating user tastes is gaining increasing importance in online commerce and advertisement. Companies now routinely collect user ranking for various products, e.g., movie, books, games, or web tools, among which the Netflix Prize for movie ranking is the best known. The problem is to use incomplete ranking provided by the users on some of the products to predict the preference of any given user on any of the products. This problem is typically cast as a low-rank matrix completion problem. However, as the data collection process often lacks control or is sometimes even ad-hoc a small portion of the available rankings could simultaneously complete the matrix and correct the errors. We need to infer a low-rank matrix L_o from a set of incomplete and corrupted entries.

ı summary, several researchers have studied principal component analysis as a variable duction procedure (Simar and Hardle, 2007), the assumption underlying PCA (Marques ; sa, 2007), Candes *et al* (2009) went as far as studying the robustness of principal ›mponent analysis while Nwabueze (2009) analyzed the robustness of principal ›mponent as an approach to estimating parameters in factor analysis under different stribution.

nith (2002) study is interesting, especially in its categorization of the procedure as a ›werful tool for analyzing data since patterns in data can be hard to find in data of high dimension, where luxury of graphical representation is not available. Indeed PCA is arguably the most widely used statistical tool for data analysis and dimensionality reduction today (Candes *et al.*, 2009).

In conclusion, PCA is a powerful tool for reducing a number of observed variables into a smaller number of artificial variables that account for most of the variance in the data set. It is particularly useful when you need a data reduction procedure that makes no assumptions concerning an underlying causal structure that is responsible for covariation in the data. When it is possible to postulate the existence of such an underlying causal structure, it may be appropriate to analyze the data using exploratory factor analysis.

2.3 AN OVERVIEW OF FACTOR ANALYTICAL TECHNIQUES

This section deals with the concept, types and objectives of factor analysis. It also gives the uses of factor analysis with regards to data reduction and areas of possible application of the technique.

2.3.1 Concept of Factor Analysis

Factor analysis is a multivariate statistical approach commonly used in psychology, education, and more recently in health related professions (Williams *et al.*, 2010). Factor analysis consists of a number of statistical techniques the aim of which is to simplify complex sets of data. In the social sciences factor analysis is usually applied to correlations between variables (Kline, 1994).

Factor analysis is a generic term that we use to describe a number of methods designed to analyze interrelationship within a set of variables. Although the various techniques differ greatly in their objectives and in the mathematical model underlying them, they all have one feature in common, the construction of a few hypothetical variables, called factors, that are supposed to contain the essential information in a large set of observed variable. The factors are constructed in a way that reduces the overall complexity of the data by taking advantage of inherent interdependencies. As a result, a small number of factors will usually account for approximately the same amount of information as do the much larger set of the original observations. Thus, factor analysis, in this one sense, a multivariate method of data reduction (Reyment and Joreskog, 1996).

Reyment and Joreskog (1996) also noted how psychologists had developed, and had made extensive use of factor analysis in their studies of human mental ability. The method was primarily devised for analyzing the observed scores of many individuals on a large battery of psychological tests. Tests of aptitude and achievement were designed to measure various aspects of mental ability but it soon became apparent that they often displayed a great deal of correlation with each other. Factor analysis attempts to "explain" these correlations by an analysis, which when carried out successfully, yields a small number of underlying factors, which contain all the essential information about the correlations among the tests. Interpretation of factors has led to the theory of fundamental aspects of human ability.

Factor analysis (more properly exploratory factor analysis) is concerned with whether the covariances or correlations between a set of observed variables can be explained in terms of a smaller number of unobservable constructs known either as latent variables or common factors. In general terms, exploratory factor analysis is concerned with whether the covariances or correlations between a set of observed variables $\chi_1, \chi_2, \ldots, \chi_p$ can be 'explained' in terms of a small number of unobservable latent variables or common factor, $f_1, f_2, \ldots, f_k$, where $k<p$ and hopefully much less (Landau and Everritt., 2004).

Fabrigar *et al.* (1999) reviewed the major designs and analytical decisions that must be made when conducting a factor analysis and noted that each of these decisions has important consequences for the obtained results, discussed recommendations that have been made in the methodological literature and analysis of 3 existing empirical data sets were used to illustrate how questionable decisions in conducting factor analyses can yield problematic results. Faleye (2008) had shown how 24-item Teacher Efficacy Scale (TES) in its present form was capable of effectively measuring Teacher Efficacy (TE) among secondary school teachers in Nigeria using factor analysis.

2.3.2 Types of Factor Analytical Techniques

DeCoster (1998) noted that there are basically two types of factor analysis: exploratory and confirmatory.

i. Exploratory factor analysis (EFA) attempts to discover the nature of the constructs influencing a set of responses.

ii. Confirmatory factor analysis (CFA) tests whether a specified set of constructs is influencing responses in a predicted way.

2.3.3 Objectives of Factor Analysis

The primary objectives of an exploratory factor analysis as expressed by DeCoster are to determine:

i. The number of common factors influencing a set of measures.

ii. The strength of the relationship between each factor and each observed measure.

2.3.4 Characteristics of Factor Analysis

Rummel (1970) stressed the characteristics of factor analysis that distinguishes it from other related statistical procedures as;

i. Factor analysis can analyze such a large number of phenomena with the assistance of an electronic computer that 100-variable analyses become routine.

ii. It disentangles complex interrelationship among the phenomena into functional unities or separate or independent patterns of behaviour and identifies the independent influences or causes at work.

iii. It handles social phenomena in the situation. There is no need to abstract phenomena to a laboratory setting or to select only certain variables and assume that others are constant. The interrelationship between behaviour and environment can be analyzed as they exist in real life.

iv. Factor analysis is a flexible instrument applicable to a wide range of research designs (by hypothesis-testing), (concept-mapping, and case studies) and to a variety of data (time series, voting results, sample survey responses).

2.3.5 Uses of Exploratory Factor Analysis

Some common uses of EFA as elaborated by DeCoster are to:

i. Identify the nature of the constructs underlying responses in a specific content area.

ii. Determine what sets of items "hang together" in a questionnaire.

iii. Demonstrate the dimensionality of a measurement scale. Researchers often wish to develop scales that respond to a single characteristic.

iv. Determine what features are most important when classifying a group of items.

v. Generate "factor scores" representing values of the underlying constructs for use in other analysis

2.3.6 Applications of Factor Analysis

Some areas of application for which factor analysis can be used as expressed by Rummel (1970) are as follow:

i. Interdependency and Pattern delineation: If a scientist has a table of data-say, UN votes, personality characteristics, or answers to a questionnaire-and if he suspects that these data are interrelated in a complex fashion, then factor analysis may be used to untangle the linear relationship into the separate patterns. Each pattern will appear as a factor delineating a distinct cluster of interrelated data.

ii. Parsimony or data reduction: Factor analysis can be useful for reducing a mass of information to an economical description. For example, data on fifty characteristics for 300 nations are unwieldy to handle, descriptively or analytically. The management, analysis, and understanding of such data are facilitated by reducing them to their common factor patterns. These factors concentrate and index the dispersed information in the original data and can therefore replace the fifty characteristics without much loss of information. Nations can be more easily discussed and compared on economic development, size, and politics dimensions, for example, than on the hundreds of characteristics each dimension involves.

iii. Structure: Factor analysis may be employed to discover the basic structure of a domain. As a case in point a scientist may want to uncover the primary independent lines of dimensions-such as size, leadership, and age-of variation in group characteristics and behaviour. Data collected on a large sample of group and factor analyzed can help disclose this structure

iv. Mapping: Factor analysis enables a scientist to map the social terrain. By mapping it is meant that a systematic attempt to chart major empirical concepts and sources of variation. These concepts may then be used to describe a domain or to serve as inputs to further research. Some social domains, such as international relations, family life and public administration, have yet to be charted. In some other areas, however, such as personality, ability, attitudes, and cognitive meaning, considerable mapping has been done.

In summary, a number of researchers have studied the uses of factor analysis (Gorsuch, 1983; Rummel, 1970; Kline 1994), Thurstone (1947) specified formal criteria for simple structure, but essentially, each factor should be represented by a distinct subset of manifest variable (MV's) with large factor loadings, subsets of MV's defining different factors should overlap minimally, and each MV should be influenced by only a subset of common factor. Woods and Edwards (2003) claimed that even though some of the methods used in exploratory and confirmatory factor analysis (EFA and CFA) are distinct, the boundary between them is often blurred. Rather than imagining them as completely separate techniques, it is useful to think of EFA and CFA as opposite ends of the same continuum.

Since this research work is particularly interested in comparing principal component analysis and factor analysis, our concern is to use factor analysis in place of exploratory factor analysis (as it is the statistical procedure which is often confused with principal component analysis).

2.4 PRINCIPAL COMPONENTS AND FACTOR ANALYTICAL TECHNIQUES

Factor analysis and principal component analysis use the set of mathematical tools (spectral decomposition, projections, etc.). One could conclude, on first sight, that they share the same view and strategy and therefore yield very similar results. That is not true. There are substantial differences between these two data analysis techniques.

Simplistically, though, factor analysis derives a mathematical model from which factors are estimated, whereas PCA merely decomposes the original data into a set of linear variate (Dunteman, 1989). Guadagnoli and Velicer (1988) concluded that the solutions generated from principal component analysis differ little from those derived from factor analytic techniques.

Brown (2009) shown what PCA and EFA are, and in part, how they should be presented and interpreted, In the process, he had defined and exemplified loadings, communalities, proportion of variance, components, factors, PCA and EFA. He also went further to explore the basic mathematical and conceptual differences between PCA and EFA, and discussed how researchers decide on whether to use PCA or EFA.

From an implementation point of view, the PCA is based on a well-defined, unique algorithm (spectral decomposition), whereas fitting a factor analysis model involves a variety of analysis procedure which opens the door for subjective interpretation and yields therefore a spectrum of results. This data analysis philosophy makes factor analysis difficult especially if the model specification involves cross-validation and a data-driven selection of the number of factors (Simar and Hardle, 2007).

PCA solved a problem similar to the problem of common factor analysis, but different enough to lead to confusion (Richard, 2004). Osborne and Fitzpatrick (2012) confirm that in the 21st century, exploratory factor analysis remains commonly used (and commonly misused) technique that is widely available.

In summary, FA assumes that the variance in the measured variables can be decomposed into that accounted for by common factors and that accounted for by unique factors. The principal components are defined simply as linear combinations of the measurements, and so will contain both common and unique variance.

2.5 COMPONENTS/FACORS RETENTION

In principal component analysis (PCA) and factor analysis (FA), the number of components/factors extracted is equal to the number of variables being analyzed, giving rise to how many of these components are truly significant and worthy of being retained for rotation and interpretation. It is generally expected that only the first few components/factors account for reasonable amount of variance and that subsequent components/factors will be considered to account for only frivolous variance. The next step of the analysis, therefore, is to determine how many relevant components/factors should be retained for interpretation.

Jackson (1993) categorized approaches to determining the number of components to interpret from PCA into Heuristic procedures (which include; retaining component with eigenvalues $\lambda s>1$ (i.e. Kaiser-Guttman Criterion); the screen plot; the broken-stick model; and components with λs totaling to a fixed amount of total variance) and statistical approaches (such as Bartlett's test of sphericity; Bartlett's test of homogeneity of the correlation matrix; Lawley's test of the second λ; bootstrapped confidence limited on successive λs (i.e., significant differences between λs); and bostrapped confidence limits on successive. He concluded that the most consistent results were obtained from the broken – stick model and a combined measure using boot strapped λs and associated eigenevector coefficients.

Franklin *et al.* (1995) reported that of several methods proposed to determine the significance of principal components, Parallel Analysis (PA) has proven consistently accurate in determining the threshold for significance of principal components, variable loadings, and analytical statistics when decomposing a correlation matrix. Franklin *et al.* (1995) also noted that in this procedure, eigenvalues from a data set prior to rotation are compared with those from a matrix of random values of the same dimensionality (P variables and n samples). PCA eigenvalues from the data greater than PA eigenvalues from the corresponding random data can be retained.

Pare-Neto *et al.* (2005) compared a total of 20 stopping rules and proposed a two – step approach that appears to be highly effective. Dray (2007) based his work on similarity measurements, singular value decomposition and permutation procedures. He conducted a simulation study evaluating the relative merits of the proposed approaches and the results obtained showed, that one method based on the PV coefficient is very accurate and seems to be more efficient than other existing approaches.

Ladesma and Velero-Mora (2007) emphasized Parallel Analysis as a superior alternative to commonly used methods for determining the number of components to retain in PCA and FA such as the scree test or the Kaiser's eigenvalue greater-than one rule where he described and illustrated how to apply Parallel Analysis with an easy-to-use computer program called Vista-PARAN.

The commonest rule for selecting a number needed for perfect reconstruction is the Kaiser's rule which sets m equals to the number of eigenvalues greater than 1. This rule is often used in common factor analysis as well as in PCA. Several lines of thought lead to Kaiser's rule, but the simplest is that since an eigenvalue is the amount of variance explained in one more factor, it does not make sense to add a factor that explains less variance than is contained in one variable. Since a component analysis is supposed to summarize a set of data, to use a component that explains less than a variance of 1 is something like writing a summary of a book in which one section of the summary is longer than the book section it summarizes – which make no sense (Richard, 2004).

Cangelosi and Goriely (2007) used Shannon entropy to provide an estimate of the number of interpretable components in a principal component analysis. In addition, they reviewed several ad hoc stopping rules for dimension determination and presented modification of the broken stick model which incorporates a test for the presence of an "effective degeneracy" among the subspaces spanned by the eigenvectors of the correlation matrix of the data set then allocates the total variance among subspaces.

CHAPTER THREE

MATERIALS AND METHODOLOGY

3.1 INTRODUCTION

In chapter two we have seen that principal component analysis is sometimes confused with factor analysis, and this is understandable, because there are many important similarities between the two procedures: both are variable reduction methods that can be used to identify groups of observed variables that tend to hang together empirically. Both procedure were calculated with the aid of an electronic computer package, precisely, Special Package for Social Sciences (SPSS). Therefore, consistent with this fact, this chapter attempts to outline detail of:

i. Source of Data;
ii. Method of data analysis;
iii. Model definition and assumption underlying PCA;
iv. Model definition and assumption underlying factor analysis and approaches of parameter estimation;
v. Principal component analysis and factor analysis using SPSS.

3.2 SOURCE OF DATA

The data used in this research were secondary data on non-food commodity expenditure for 36 states of Nigeria and the Federal Capital Territory, Abuja. The data were sourced from the National Bureau of Statistics, Preliminary Report of Consumption Pattern in Nigeria for the year 2009/2010 (Appendix I).

The variables contained in the data from appendix are defined as follows:

X_1= Clothing and footwear

X_2 = Rent

X_3 = Fuel/Light

X_4 = Household Goods

X_5 = Health Expenditure

X_6 = Transport

X_7 = Education Expenditure

X_8= Entertainment

X_9 = Water

X_{10} = other services

3.3 METHOD OF DATA ANALYSIS

Principal component analysis and factors analysis were conducted on the data using SPSS. The variables were measure on the same experimental unit of percentages of non-food commodity expenditure of 36 states of the federal republic of Nigeria and Abuja. Correlations between variables were obtained. Kaiser criterion was used as the method for determining the optimal number of factors or components for inclusion. Principal components extraction and Maximum likelihood extraction were used to extract components and factors respectively. Varimax orthogonal rotation which produces uncorrelated components/factors was used to rotate factors or components to obtain final solution that aid interpretation.

After obtaining the output of both analyses, the solutions were used to compare the computational efficiency of principal components analysis and factor analysis based on the following criteria:

(i) Data Fitness: Residuals of the model are the differences between the matrix based on the model and matrix based on observed data. SPSS produces these residuals in the lower table of the reproduced matrix and it is expected to be relatively few of these values to be greater than 5% (Field, 2004). The higher the residuals the less fit a dataset is to the model.

(ii) Variance Maximization: The eigenvalue associated with each component (factor) represent the variance explained by that particular component/factor. A model is consistent if the cumulative value of the retained components (factors) is the same before and after rotation (Field, 2004). Rotation has the effect of optimizing the factor structure and its consequence is that the relative importance of the retained factors (components) is equalized.

To generalize the results in this analysis beyond the data collected, test of normality was conducted and it was found that most variables are normally distributed as their significances are greater than 5% (see Appendix II).

Communalities (h^2) are obtained using:

$$comm(Xj) = \sum_{i=1}^{m} \lambda^2 ij$$

Where λ_{ij} is the amount of variance accounted for by the i^{th} component (factor) in the j^{th} variable and j= 1, 2… 10 and m = 3.

In other words, it can be thought of as the sum of squared multiple-coefficients between the Xj and the factors (Garret-Mayer, 2006).

$$Var(X) = Var(F) + Var(e)$$

$$Var(X) = Communalities + Uniqueness$$

$$Communality = Var(F)$$

$$Uniqueness = Var(e)$$

$$Uniqueness(Xj) = 1 - Comm(Xj).$$

3.4 MODEL DEFINATION AND ASSUMPTIONS UNDERLYING PRINCIPAL COMPONENTS

3.4.1 Model Of Principal Components

The aim of principal component analysis is to summarize a p-dimensional feature vectors by projecting down into a q-dimensional subspace. The summary will be the projection of original vectors into a q-directions, the principal components to get components scores

$$\mathbb{Y} = \mathbb{X}\mathbb{W} \qquad 3.01$$

This turns the $n\ x\ p$ matrix $\mathbb{X}$ into $n\ x\ q$ matrix $\mathbb{Y}$ by multiplying from right $p\ x\ q$ matrix $\mathbb{W}$ of eigenvectors. Suppose $\mathbb{W}^T$ the transpose is a $q\ x\ p$ matrix. This is because the ij^{th} entry in $\mathbb{W}^T\mathbb{W}$ is the dot product of the i^{th} row of $\mathbb{W}^T$ with the j^{th} column of $\mathbb{W}$, i.e. the dot product of two eigenvectors of $\mathbb{V}$ if the data come from IID sample of a distribution with covariance matrix $\mathbb{V}$, then the sample covariance matrix $\mathbb{V}=\frac{1}{n}\mathbb{X}^T\mathbb{X}$ will converge on $\mathbb{V}_0$ as $n \to \infty$ (Timm, 2002)

The largest variance Y_1 as noted by Timm (2002) is the largest root λ_1 of equation (3.01). To determine the second principal component, the combination

$$Y_2 = \mathbb{X}\mathbb{W}_2 \qquad 3.02$$

is constructed such that it is uncorrelated with Y_1 and has maximal variance. For Y_2 to be uncorrelated with Y_1, the covariance between Y_2 and Y_1 must be zero. However, $\mathbb{V}\mathbb{W}_1 = \mathbb{W}_1\lambda_1$ so that the

$$cov(Y_2 Y_1) = \mathbb{W}_2\mathbb{V}\mathbb{W}_1 = \mathbb{W}^T{}_2\mathbb{W}_2\lambda_1 = 0 \qquad 3.03$$

implies that $\mathbb{W}_2\mathbb{W}_1 = 0$. Furthermore, if $\mathbb{W}_2$ is the second eigenvector of equation (3.01) then $\mathbb{V}\mathbb{W}_2 = \mathbb{W}_2\lambda_2$ and the

$$var(Y_2) = \mathbb{W}^T{}_2\mathbb{V}\mathbb{W}_2 = \lambda_2 = 0 \qquad 3.04$$

Where $\lambda_1 \geq \lambda_2$. More generally, by Spectral Decomposition Theorem there exist an orthogonal matrix $\mathbb{W}(\mathbb{W}^T\mathbb{W} = 1)$ such that

$$\mathbb{W}^T\mathbb{V}\mathbb{W} = \Lambda = diag(\lambda_i) \qquad 3.05$$

Where $\lambda_1 \geq \lambda_2 \dots\dots\dots \geq \lambda_p \geq 0$.

3.4.2 Assumptions Underlying Principal Components

Because a PCA is performed on a matrix of Pearson Correlation Coefficients, the data should satisfy the assumptions for this statistic. These assumptions are:

i. Interval-level measurement: All analyzed variables should be assessed on an interval or ratio level of measurement.

ii. Random Sampling: Each subject will contribute one score on each observed variable. These sets of scores should represent a random sample drawn from the population of interest.

iii. Linearity: The relationship between all observed variables should be linear.

iv. Normal Distributions: Each observed variable should be normally distributed. Variables that demonstrate marked skewness or kurtosis may be transformed to better approximate normality.

v. Bivariate Normal Distribution: Each pair of observed variables should display a bivariate normal distribution.

3.5 MODEL DEFINITION AND ASSUMPTIONS UNDERLYING FACTOR ANALYSIS AND APPROACHES OF PARAMETER ESTIMATION

3.5.1 Factor Analytical Model

Pison *et al.* (2001) accentuated that classical factor tries to describe the correlation matrix or covariance matrix of n observations between the original variables $X1, X2, \dots \dots, Xn$ by a small number $m \leq n$ of new variable $F1, F2 \dots \dots, Fm$ called factors. These factors are unobserved. In particular, with n observed variables and m factors, the orthogonal factor analysis model says

$$\mathbb{X}_{nx1} = \Lambda_{nxm}\mathbb{F}_{mx1} + \mathbb{E}_n \qquad 3.06$$

Where $\mathbb{X}$=$(X_1 - \mu_1, \dots, X_n - \mu_n)^T$, is the deviation from the mean vector, Λ is the nxm matrix of factor loadings (this represent degree to which each of the variables correlates with each of the factor), $\mathbb{F} = (F_1, \dots, F_m)^T$ is the derived factor vector, and the error term is$\mathbb{E} = (\mathcal{E}_1, \dots, \mathcal{E}_n)^T$. Assume that the random vector $\mathbb{F}$ and $\mathbb{E}$ are independent,$E(\mathbb{F}) =$

$O, cov(\mathbb{F}) = \mathbb{I}$, $E(\mathcal{E}) = O$ and $cov(\mathcal{E}) = diag(\Psi)$ with $\Psi = (\psi_1, .., \psi_n)$, under these assumptions we obtain

$$\Sigma = \Lambda\Lambda^T + diag(\Psi) \qquad 3.07$$

3.5.2 Assumptions Underlying Factor Analysis

As elaborated by Landau and Everitt, the assumptions underlying factor analysis are:

i. Since the factors are unobserved, we can fix their location and scale arbitrarily, so we assume they are in standardized form with mean zero and standard deviation one.

ii. We assume that the residual (error) or specific terms are uncorrelated with each other and with the common factors. This implies that, given the values of the factors, the manifest variables are independent so that the correlations of observed variables arise from their relationships with the factors.

iii. Since the factors are unobserved, the factor loading cannot be estimated in the same way as regression coefficients in multiple regressions.

3.5.3 Approaches to Parameter Estimation in Factor Analysis

There are several approaches to parameter estimation in factor analysis, these are principal (axis) factoring image factoring, maximum likelihood factoring , alpha factoring, weighted least-square, generalized least square etc. The different techniques vary according to the criterion against which factor extraction is judge and which determines the ending of the iteration process;

(i) Principal factoring like PCA, extracts maximum orthogonal variance with each other.

(ii) Alpha factoring is an iterative communality estimates that maximize coefficient alpha (a psychometric measure of factor reliability).

(iii) Least squares considers only off-diagonal elements of correlation matrix (communalities not estimated) and minimized squares residual correlations (either unweighted or variables weighted by amount of squares variance with other variables).

(iv) Maximum likelihood estimates population values of factor loadings that have greatest probability of yielding observed correlation matrix.

(v) Image factoring is a non-iterative technique which uses an image score variance covariance matrix as basis for factor extraction (rather than the observed correlation matrix).

This research used the most popular of the aforementioned approaches to estimation which is maximum likelihood factoring (Laudan and Everitt, 2004). Unless there is a serious lack of multivariate normally in the measure, maximum likelihood is generally the best method of extraction in factor analysis (DeCoster, 1998).

3.6 BASIC STEPS FOR PERFORMING PCA AND FA

The following are basic steps in performing Principal Component Analysis (PCA) and Factor Analysis (FA) as outlined by Suhr (2009):

(i) Measure observed variables

(ii) Compute correlation matrix.

(iii) Extract factor (components).

(iv)Rotate factor (components to aid interpretation.

(v) Interpret the results.

CHAPTER FOUR

ANALYSIS OF DATA

4.1 INTRODUCTION

In this chapter, we performed principal component analysis and factor analysis using SPSS version 16 on the same data. The objectives are:

- To reduce the $p = 10$ observed variables $x1, x2, \dots, x10$ and find combinations of these to produce components $y1, y2, \dots, y10$ that are uncorrected in the order so that $y1$ displays the largest amount of variation, $y2$ display second largest amount of variation not accounted for $y1$ and so on, that is, $var(y1) > var(y2) > \dots, > var\ (y10)$, *where* $var\ (yi)$ denotes the variance of y_i in the data set being considered. *The yi's* are called principal component.
- To determine the nature of the $p = 10$ observed variables, $x1, x2, \dots, x10$ that can be explained in terms of a smaller number of unobservable latent variables or common factors$f1, f2 \dots, fk$ *where* $k < p$.

4.2 DATA ANALYSIS AND INTERPRETATION

Combining two correlated variables into one factor, illustrate the basic idea of factor analysis and principal components analysis. If the combination of two variables is extended to multiple variables, the computation becomes more involved but the basic principles of expressing two or more variables by a singles factor or component remain the same. Having looked at the assumptions and model derivation of principal component analysis on one hand and factor analysis on the other hand and the steps for computing the two techniques on chapter three, the results are therefore interpreted as follows:

4.2.1 Descriptive Statistics

Principal component analysis and factor analysis (using maximum likelihood factor extraction) display the same result for descriptive statistics as both analyses are performed by examining the patterns of correlations or covariation between the observed measures.

Table 4.1 Descriptive Statistics

	Mean	Std. Deviation	Analysis N	Missing N
X_1	13.8019	6.81294	37	0
X_2	35.4843	9.07343	37	0
X_3	12.4495	4.08699	37	0
X_4	14.9611	6.80913	37	0
X_5	2.0954	1.20005	37	0
X_6	7.7849	4.77627	37	0
X_7	1.7981	1.27075	37	0
X_8	.8049	.72490	37	0
X_9	.4442	.34128	37	4
X_{10}	10.5238	5.15803	37	0

a. For each variable, missing values are replaced with the variable mean.

Table 4.1 simply shows the mean, standard deviation and sample size for each variable as it summarizes the data.

Table 4.2: Eigenvalues and Communalities

variables	PCA				FA			
	Comp1	Comp2	Comp3	h^2	Factor1	Factor2	Factor3	h^2
X_1	-.121	**.909**	.194	.878	-.072	**.978**	-.193	.99
X_2	-.227	**-.836**	**.340**	.865	-.224	**-.821**	**-.524**	.99
X_3	-.135	-.046	**-.841**	.727	-.046	-.096	**.686**	.48
X_4	**-.916**	.152	-.161	.889	**-961**	.160	.222	.99
X_5	.131	**.696**	**.514**	.766	.129	**.586**	-.166	.38
X_6	**.827**	.026	-.206	.727	**.772**	-.033	.385	.74
X_7	.286	**-.563**	.359	.528	.221	-.435	-.137	.25
X_8	**.543**	.333	-.420	.586	**.526**	.229	.210	.37
X_9	-.285	-.057	**.656**	.515	-.159	-.037	**-.487**	.26
X_{10}	**.941**	.007	.031	.886	**.923**	-.017	.055	.85
% of variance	.3044	.2458	.1866	.7368	.2798	.2253	.1311	.63

Table 4.2 shows PCA and FA analyses with Varimax rotation, Eigenvalue ≥ 1 and the resulting loadings for the percentage of non-food commodities in Nigeria for the year 2009/2010. Observe that the first column has 10 variables as earlier defined, then the next three columns show the results for a PCA of the data, and the last three columns show analogous results for an FA of the same data. Observe that the actual loadings differ for the PCA and FA. Note also that the pattern of relatively strong loadings are the same for both analyses, so in that sense, it made little difference which analysis was used. However, loadings, higher communalities, and ultimately the proportion of variance accounted for in the aggregate is 73.68% in PCA as opposed to 63.62% in FA. This is because FA excludes unique variances which are used in the PCA to contribute to higher loadings with the components in ways that are not present in FA.

PCA, we get components that are outcomes built from linear combinations of the variables. Component scores are calculated as follows:

$$Y_1 = -o.121x_1 - 0.227x_2 - 0.135x_3 - 0.916x_4 + 0.131x_5 + 0.827x_6 + 0.286x_7 + 0.543x_8 - 0.285x_9 + .0941x_{10}$$

$$Y_2 = 0.909x_1 - 0.836x_2 - 0.046x_3 + 0.152x_4 + 0.696x_5 + 0.026x_6 - 0.563x_7 + 0.338x_8 - 0.057x_9 + 0.007x_{10}$$

$$Y_3 = o.194x_1 + 0.340x_2 - 0.841x_3 - 0.161x_4 + 0.514x_5 - 0.206x_6 + 0.359x_7 - 0.420x_8 + 0.656x_9 + 0.031x_{10}$$

In FA, we get factors that are thought to be the cause of the observed variables and can b calculated as follows:

$$X_1 = -0.072F_1 + 0.978F_2 - 0.193F_3 + e_1$$

$$X_2 = -0.224F_1 - 0.821F_2 - 0.524F_3 + e_2$$

$$X_3 = -0.046F_1 - 0.096F_2 + 0.686F_3 + e_3$$

$$X_4 = -0.961F_1 - 0.160F_2 + 0.222F_3 + e_4$$

$$X_5 = 0.129F_1 + 0.586F_2 - 0.166F_3 + e_5$$

$$X_6 = 0.772F_1 - 0.033F_2 + 0.385F_3 + e_6$$

$$X_7 = 0.221F_1 - 0.435F_2 - 0.137F_3 + e_7$$

$$X_8 = 0.526F_1 + 0.229F_2 + 0.210F_3 + e_8$$

$$X_9 = -0.159F_1 - 0.037F_2 - 0.487F_3 + e_9$$

$$X_{10} = 0.923F_1 - 0.017F_2 + 0.055F_3 + e_{10}$$

From Table 4.2, Communality is sum of square across row.

In PCA, proportions of variance accounted for by the variables are:

$$comm(X_1) = -0.121^2 + 0.909^2 + 0.194^2 = 0.878$$

$$comm(X_2) = -0.227^2 - 0.836^2 + 0.340^2 = 0.865$$

$$comm(X_3) = -0.135^2 - 0.046^2 - 0.841^2 = 0.727$$

$$comm(X_4) = -0.916^2 + 0.152^2 - 0.161^2 = 0.889$$

$$comm(X_5) = 0.131^2 + 0.696^2 + 0.514^2 = 0.766$$

$$comm(X_6) = 0.827^2 + 0.026^2 - 0.206^2 = 0.727$$

$$comm(X_7) = 0.286^2 - 0.563^2 + 0.359^2 = 0.528$$

$$comm(X_8) = 0.543^2 + 0.338^2 - 0.420^2 = 0.586$$

$$comm(X_9) = -0.285^2 - 0.057^2 + 0.656^2 = 0.515$$

$$comm(X_{10}) = 0.941^2 + 0.007^2 + 0.031^2 = 0.886$$

In FA, proportions of variance accounted for by the variables are as follows:

$$comm(X_1) = -0.072^2 + 0.978^2 - 0.193^2 = 0.999$$

$$comm(X_2) = -0.224^2 - 0.821^2 - 0.524^2 = 0.999$$

$$comm(X_3) = -0.046^2 - 0.096^2 + 0.686^2 = 0.482$$

$$comm(X_4) = -0.961^2 + 0.160^2 + 0.222^2 = 0.999$$

$$comm(X_5) = 0.129^2 + 0.586^2 - 0.166^2 = 0.387$$

$$comm(X_6) = 0.772^2 - 0.033^2 + 0.385^2 = 0.745$$

$$comm(X_7) = 0.221^2 - 0.435^2 - 0.135^2 = 0.257$$

$$comm(X_8) = 0.526^2 + 0.229^2 + 0.210^2 = 0.373$$

$$comm(X_9) = -0.156^2 - 0.037^2 - 0.487^2 = 0.264$$

$$comm(X_{10}) = 0.923^2 - 0.017^2 + 0.055^2 = 0.856$$

In PCA, the proportions of variance explained by the component are:

$$Component1 = (-0.121^2 - 0.227^2 - 0.135^2 - 0.916^2 + 0.131^2 + 0.827^2 + 0.286^2 + 0.543^2 - 0.285^2 + 0.941^2)/10 = 0.3044$$

$$Component2 = (0.909^2 - 0.836^2 - 0.046^2 + 0.152^2 + 0.696^2 + 0.026^2 - 0.563^2 + 0.338^2 - 0.057^2 + 0.007^2)/10 = 0.2458$$

$$Component3 = (0.194^2 + 0.340^2 - 0.841^2 - 0.161^2 + 0.514^2 - 0.206^2 + 0.359^2 - 0.420^2 + 0.656^2 + 0.031^2)/10 = 0.1866$$

In FA, the proportions of variance explained by the factors are:

$$Factor1 = (-0.072^2 - 0.224^2 - 0.046^2 - 0.961^2 + 0.129^2 + 0.772^2 + 0.221^2 + 0.526^2 - 0.159^2 + 0.923^2)/10 = 0.2798$$

$$Factor2 = (0.978^2 - 0.821^2 - 0.096^2 + 0.160^2 + 0.586^2 - 0.033^2 - 0.435^2 + 0.229^2 - 0.037^2 + 0.055^2)/10 = 0.2253$$

$$Factor3 = (-0.193^2 - 0.524^2 + 0.686^2 + 0.222^2 - 0.166^2 + 0.385^2 - 0.137^2 + 0.210^2 - 0.487^2 + 0.055^2)/10 = 0.1311$$

Table 4.3: Total Variance Explained by Retained Components

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	3.044	30.444	30.444	3.044	30.444	30.444	2.969	29.689	29.689
2	2.458	24.579	55.023	2.458	24.579	55.023	2.470	24.702	54.391
3	1.866	18.656	73.679	1.866	18.656	73.679	1.929	19.288	73.679
4	.753	7.526	81.204						
5	.637	6.372	87.576						
6	.545	5.450	93.025						
7	.336	3.357	96.382						
8	.250	2.500	98.882						
9	.111	1.112	99.994						
10	.001	.006	100.000						

Extraction Method: Principal Component Analysis.

Table 4.4: Total Variance Explained by Retained Factors

Factor	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	3.044	30.444	30.444	2.291	22.914	22.914	2.798	27.975	27.975
2	2.458	24.579	55.023	2.722	27.216	50.130	2.253	22.534	50.509
3	1.866	18.656	73.679	1.349	13.487	63.617	1.311	13.108	63.617
4	.753	7.526	81.204						
5	.637	6.372	87.576						
6	.545	5.450	93.025						
7	.336	3.357	96.382						
8	.250	2.500	98.882						
9	.111	1.112	99.994						
10	.001	.006	100.000						

Extraction Method: Maximum Likelihood.

Table 4.3 and Table 4.4 show the importance of the ten principal components (factors). Only the first three have eigenvalues over 1.00, and together these explain over 73% of the total variability in the data. While this figure remains the same after extraction using PCA, it differs by about 10% after rotation using FA (maximum likelihood) extraction.

Appendix III and Appendix IV show the reproduced correlation of PCA and FA respectively. The reproduced correlations matrix differs from those in the observed matrix because they stem from the model. Therefore, to assess the fit of the model we can look at the differences between the observed correlations and the correlations based on the model. The difference can be calculated as follows:

Residual = observed correlation – correlation from model.

Note that this difference is the value quoted in the lower half of the reproduced matrix (labeled residuals). Therefore, the lower half of the reproduced matrix contains the differences between the observed correlation coefficients and the ones predicted from the model. For a good model these values will all be small. In fact, most values should be less than 0.05 significant levels (Field, 2004). Rather than scan this huge matrix, SPSS provides a footnote summary, which states how many residuals have absolute value greater than 0.05. For these data values there are 14 (31%) absolute values greater than 0.05 in FA and 23(51%) absolute values greater than 0.05 in PCA. There are no hard and fast rules about what proportion of residuals should be below 0.05; however, if more than 50% are greater than 0.05 we probably have grounds for concern (Field, 2004).

4.2.2 Discussion of Results

In most stages except at the reproduced correlation stage where FA appears to be more of a fitted model to the data than PCA (Appendices II and IV), principal component analysis has produced results that was intended, which is to say it has been more successful in virtually all stages of the computation than factor anaysis. Therefore, principal component analysis has displayed more efficiency than factor analysis.

Principal component analysis has been more consistent in meeting the conditions in achieving stability in the component (factor) solution than factor analysis. The findings in this research have shown that principal component analysis has observed all the necessary conditions for attaining component (factor) solution than factor analysis eventhough the later can be used as a predictive model because it fit distribution to the data more adequately than the earlier model and can decompose thevariancw from the observed measures in to that accounted for by common factors and that accounted for by unique factor.

Principal component analysis has as much as it is needed for reducing the dimension in a set of data. After ascertaining the fitness of the data to the models eventhough unsatisfactory with principal component analysis model, every condition, requirement, and creteria has been adhered to by principal component analysis in the course of this research which makes it more sufficient than factor analysis which fail to meet certain conditions for attaining component (factor) solution.

In sum, the primary differences between PCA and FA are that PCA is appropriate when researchers are just exploring for patterns in their data without a theory and therefore want to include unique and error variances in the analysis, and FA is appropriate when researchers are working from a theory drawn from previous research about the relationships among the variables and therefore want to include only the variance that is accounted for in an analysis (thereby excluding unique and error variances) in order to see what is going on in the co-variation, or common variance structure.

In summary, researchers tend to used PCA if they are trying to find patterns in their data and have no theory to base the analysis on, or use FA if they have a well-established theory to base their analysis on.

In relation to the data used, policy can be formulated, prioritized and assigned through exploration of data pattern. Components one, two and three show the different segments of non-food commodity expenditure in Nigeria for the year 2009. It shows that the variables in each component are correlated with one another but completely uncorrelated with variables in other components.

Since X_4, X_6, X_8 and X_{10} cluster together into component one, it means that household goods, transport, entertainment and other services are correlated and should be dealt with together in terms of policy formulation. More so, since they account for most of the variance in the observed variables, they should be given the top most priority. Component two has variables X_1, X_2, X_5 and X_7 which are clothing/wear, rent, health expenditure and education expenditure in its subset and like component one should also be treated as a whole by policy makers in determining what to produce, in what quantity and quality. The remaining variables X_3 and X_9 belong to component three and its shows that fuel/light and water expenditure should be treated together because of their inclusion in the subset.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER STUDY

5.1 INTRODUCTION

The study is an attempt to characterize differences in the patterns of robustness between principal component analysis and factor analysis. There are different statistical packages that can be used to compute the two procedures. This study used SPSS which has the following factors (components) extraction techniques: principal component analysis, principal (axis) factoring, image factoring, maximum likelihood factoring, alpha factoring, unweighted least squares and generalized least squares. Apart from principal component analysis, the rest are factor analytic techniques. This research adopted two approaches to factors (components) extraction (principal component extraction and maximum likelihood factoring). These approaches are in fact similar in some respect and differ in a number of areas.

Before embarking on the analysis, a number of questions were raised. This chapter attempts to assess the extent to which issues raised in these questions have been met by the results obtained in the analysis of data.

5.2 SUMMARY

Principal component analysis works on the initial assumption that all variance is common; therefore, before extraction the communalities are all 1 while in factor analysis, the initial communalities show that not all the variances associated with a variable are assumed to be common variance. Since principal components are linear combinations of the measurements, that contains both common and unique variance, all variances of observed variables are analyzed, and therefore, communalities play little or no part in principal component analysis. Factor analysis assumes that the variance in the measured variables can be decomposed into that accounted for by common factors and that accounted for by unique factors. Since communality is the proportion of common variance within a variable, it can be reflected easily in factor analysis than in principal component analysis.

In both techniques, the initial eigenvalues are the same. The middle part of Table 4.2 shows the eigenvalues and percentage of variables explained for just the three factors of the initial solution that are regarded as important. In principal component analysis, the three rotated components together explain for the variability in the data as the three components in the initial solution (73.68%). While in factor analysis, the two rotated factors together explain for 63.617% of the variability in the data. This is different from the two factors in the initial solution which together explain for 73.68% of the variability in the data (Table 4.4).

Since more than 50% (precisely 51%) of the differences between observed correlation coefficients and the ones predicted from the model are greater than 0.05 for principal component analysis, implying that the model is not a good fit for the data. On the other hand, factor analysis is a good fit for the data since only 31% of the differences between the observed correlation coefficients and the ones predicted from the model called residuals are greater than 0.05 (Appendices III and IV).

While the variance of component 1 was minimized to maximize the variance of components 2 and 3 in principal components analysis, the variance of factors 2 and 3 were minimized to maximize the variance of factor 1 with some information lost as a result of discarding unimportant factors.

These similarities and differences show the extend and instances to which the two procedures are related and / or unrelated.

5.3 CONCLUSIONS

Based on the findings in this study, it can be concluded that:

(a) factor and principal component solutions are similar since the number of factors (components) extracted and retained, the step for carrying out the two procedures, and conditions for attaining factor (component) solutions are the same,

(b) the conspicuous differences found in the study are:

(i) 51% of the residuals in principal component analysis are more than 0.05, hence makes the model fairly fit for the data while only 31% of the residuals in factor analysis are more than 0.05, hence makes the model adequately fit for the data,

(ii) in both analyses the initial eigenvalues are the same but differ after rotation. For principal components, the three components after rotation together explained for over 73% of the variability in the data while in factor analysis, the two factors after rotation together explained for 63% of the variability in the data, and

(c) comparison in computational efficiency suggest that principal component analysis should not be used if a researcher wishes to obtain parameters reflecting latent construct.

5.4 RECOMMENDATIONS FOR FURTHER STUDY

In accordance with the limitation discussed in chapter one, we suggest the following as area of further research. In this research the method of rotation used to obtain final solutions is orthogonal rotation which assumed that any underlying factors/components are independent and the factor loading is the correlation between the factor and the variable, but is also the regression coefficient put another way, the values of the correlation coefficients are the same as the values of the regression coefficients. However there are situations in which the underlying factors are assumed to be related or correlated to each other. In these situations, oblique rotation can be used to rotate the solution for interpretation in furtherance of this study. And ascertaining the most appropriate method of components/factors retention is another area to explore in the build up to this research.

5.5 CONTRIBUTION TO KNOWLEDGE

Even though a lot of work has been done to show the distinction between principal component and factor analytical techniques as observed in the literature review, to the best of my knowledge, very little has been done to accentuate the numeric similarities or dissimilarities between the two multivariate techniques which this research has been able to uncovered, therefore, findings in this report have shown that:

i. Factor analytical techniques preserve correlations more than principal component analysis with a small residuals of 31% which show that there is a very little difference between the reproduced correlations and the correlations actually observed between the variables against the residuals of PCA which stands at 51%,

ii. PCA preserve more variability of the original data set at 73.68% over FA which preserves variability of the original data set at 63.62%.

While FA preserves more correlations than PCA. the later accounts for more variance in the observed variables than the former. Therefore, when it is needful to do a data reduction, PCA should be used; but when a statement about the underlying causal structure is desirable, FA should be used.

This research has further deepened the discrepancies between PCA and FA, thereby providing assistance to researchers to ease their decision making as to which technique to use apriori.

REFERENCES

Bilodeau, M. and Brenner, D. (1999). Theory of Multivariate Statistics. Springer-Verlag. New York:Page. PP 180.

Brown, J.D. (2009): Principal Component Analysis and Exploratory Factor Analysis- Definition, Differences and Choices : JALT Testing and Evaluation. SIG Newsletter. Vol. 13. Number 1, PP 26-30. Retrieved April 16, 2012. Retrieved from Shiken database.

Cances, E.J., Li, X, Ma, Y., and Wright, J. (2009). Robust Principal Component Analysis. PP 2 and 3. Retrieved September 28, 2011, from http://www.stat.stanford.edu.

Cangelosi, R. and Goriely, A. (2007): Component Retention in Principal Component Analysis with application to cDNA microarray data. Biology Direct. Retrieved 17 January 2007, from htt://www.biology-direct.com/content/2/1/2

Danda, M. (2008). A Chronicle of Crime Data Using Some Multivariate Techniques. Unpublished Msc. Thesis, Ahmadu Bello University, Zaria, Nigeria.

DeCoster, J. (1998). Overview of Factor Analysis. PP 1-3. Retrieved June 14, 2011, from http://www.stat-help.com

Dray S. (2007). On the number of principal components: of test of dimensionality based on measurements of similarity between Matrices. Computational Statistics and Data Analysis. Vol.52, PP 2228-2237.

Dunteman, G. H. (1989). Principal Component Analysis: Quantitative Applications in the Social Sciences. Newbury park, California: Sage Publication. PP 7

Fabrigar, L.R., Wegener, D.T., Madallum, R.C., and Strahan, E. J. (1999): Evaluating the used of exploratory factor analysis in psychological research. Psychological Methods, Vol.4, Number 3, PP 272-299.

Faleye, B. A. (2008): Reliability and Factor Analyses of Teacher Efficacy Scale for Nigerian Secondary School Teacher. Journal of Research in Educational Psychology. Vol.6 Number 3, PP 823-846.

Field, A. (2004). Discovering Statistics Using SPSS, Second Edition. : Sage Publications. London. PP 641-665

Franklin S.B.; Buibson D.J.; Robertson P.A.; Pohlmann J.T.; Fralish J.S. (1995). Parallel Analysis: a method for Determining significant principal components. Journal of Vegetation Science . Vol.6, Number 1, PP 99-106.

Garrett-Mayer, E. (2006). Factor Analysis. Statistics in Psychosocial Research. A lecture presented at John Hopkins Bloomberg School of Public Health, Washington DC.

Gorsuch, R.L.(1983). Factor Analysis: Lawrence Erlbaum Associate,Inc. New Jersey. PP 123.

Guadognoli, E. and Velicer, W.F. (1988). Relation of Sample Sizes to the Stability of Component Patterns. Psychological Bulletin. PP 265-275.

Jackson D.A. (1993). Stopping Rules in Principal Components Analysis; Comparison of Heuristical and Statistical Approaches. Ecological Society of America Journal, Vol.74, Number 8, PP 2204-2214.

Haris, R.J. (2001). A Primer of Multivariate Statistics, Third Edition: Lawrence Erlbaum Associates, Inc. London. PP 397

Jollife, I. T.(2002). Principal Component Analysis, Second Edition : Springer-Verlag Inc. New York. PP 1,2 and 63.

Kline, P. (1994). An Easy Guide to Factor Analysis. London: Routledge.

Landau, S. and Everitt, B.S. (2004). Statistical Analysis Using SPSS. Chapman and Hall/CRC Press LLC. London. PP 213.

Ledesma R.D and Velers-Mora P. (2007). Determining the Number of Factor to Retain in EFA: An Easy-to-use computer program for camping-out-parallel Analysis. Practical Assessment, Research and Evaluation: A peer reviewed electronic journal. Vol 12, Number 2.

Marques, J.P. (2007). Applied Statistics Using SPSS, STATISTCA, MATLAB and R: Springer-Verlag. Berlin Heidelberg. PP 330

National Bureau of Statistics (2010). Consumption Pattern in Nigeria (2009/2010). Retrieved March 20, 2010, from www nigerianstat.gov.ng

Nwabueze, J.C. (2009). Principal Component Procedure in Factor Analysis and Robustness. African Journal of Mathematics and Computer Science Research; Vol.3, Number 3, PP 049-053.Retrieved March 6, 2010, from http://www.academicjournals.org/AJMCSR.

Osborne, J. W. and Fitzpatrick, D. C. (2012): Replication Analysis in Exploratory Factor Analysis: What it is and why it makes your analysis better. Practical Assessment, Research and Evaluation (A peer-reviewed electronic journal). Vol.17, Number 15. Retrieved November 2012.

Peres-Neto P.R., Jackson D.A. and Somes K.M. (2005): How many Principal Components? Stopping rule for determining the number of non-trivial axes.

Computational Statistics and Data Analysis. Vol.49. PP 974-9 97. Retrieved July 4, 2005 from www.sciencedirect.com

Pison, G., Rousseeuw, P. J., and Croux, C. (2001): Robust Factor Analysis. Retrieved August 1, 2001, from http://lirias.kuleuven.be/../1/paper A.pdf

Richard, B. D. (2004): Factor Analysis. Retrieved July 24, 2004, from http://www2.chass.ncsu.edu/garson/pa765/factor.htm

Rencher, A. C. (2002). Method of Multivariate Analysis: John Wiley and Sons, Inc. Publication. Canada. PP 380 and 408.

Reyment, R.A., and Joreskog, K.G. (1996). Applied Factor Analysis in Natural Science: Cambridge University Press. London. Page 71 and 79.

Rummel, R.J. (1970). Understanding Factor Analysis. Retrieved May 6, 2011 from http://www.hawaii.ed/powerkill

Simar, L. and Hardle, W. (2007). Applied Multivariate Statistical Analysis. Berlin Heidelberg: Springer-Verlag. PP 265

Smith, L I. (2002). A Tutorial on Principal Component Analysis. Retrieved August 13, 2011, from (http://www.cs.otago.ac.nz) . PP 12

Suhr, D. (2009): Principal Component Analysis versus Exploratory Factor Analysis. Statistical and Data Analysis. SUGI 30 proceedings. Retrieved 5 April 2012. PP 203-30

Thurstone, L.L. (1947). Multiple Factor Analysis: University of Chicago Press. Chicago. PP 158

Timm, N.H. (2002). Applied Multivariate Analysis: Springer-Verlag inc. New York. Page 445.

Williams, B., Onsmen, A., and Brown, T. (2010): Exploratory Factor Analysis: A five-step guide for novices. Journal of Emergency Primary Health Care (JEPHC). Vol.8. Retrieved April 3, 2010, from www.jephc.com

Woods,C.M. and M.C., Edwards (2003). Epidemiology and Medical Statistics. Handbook of Statistics, Vol. 27, PP 367

APPENDICES

Appendix I: Expenditure of Non-Food Commodity items for 2009/2010

STATES	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
ABIA	10.39	37.33	12.71	10.81	2	8.23	3.03	1.12	0.61	13.76
ADAMAWA	19.43	32.09	8.36	19.72	3.64	6.67	0.67	0.32	1.01	8.1
AKWA IBOM	17.17	30.68	9.63	12.38	3.17	7.81	1.98	1.31	0.28	15
ANAMBRA	12.6	41.6	8	9.5	3.7	4.2	2.9	0.5	1	16.2
BAUCHI	16.7	36.54	11.34	24.83	1.99	2.14	0.32	0.51	0.43	5.19
BAYELSA	29.12	29.41	7.18	11.41	1.47	4.74	1.04	1.3	0.02	14.31
BENUE	14.03	40.79	8.25	20.15	2.07	6.06	1.79	0.41	0.21	6.25
BORNO	15.15	32.9	7.76	22.42	2.67	11.61	1.76	0.41	0.5	4.8
C/RIVERS	12.85	43.52	8.06	12.51	1.7	5.54	5.57	0.21	1.05	8.98
DELTA	9.45	39.82	7.6	8.32	2	12.14	2.07	0.97	0.19	17.44
EBONYE	2.24	50.06	14.69	20.32	1.28	2.55	4.05	0.4		4.41
EDO	2.22	46.27	10.18	9.29	0.35	11.05	2.16	0.54	0.13	17.81
EKITI	14.05	32.29	10.99	7.97	1.74	14.54	3.55	0.71	0.03	14.14
ENUGU	3.42	59.38	8.4	6.62	1.68	2.78	4.07	0.26	0.39	11.05
GOMBE	27.27	32.71	5.47	20.8	5.11	4.11	0.04	0.79	0.5	3.2
IMO	10.02	34.6	14.07	9.89	4.41	7.7	3.29	1.17	0.4	14.45
JIGAWA	17.92	33.38	10.7	29.06	1.43	1.19	0.85	0.11	0.6	4.75
KADUNA	19	22.42	15.93	14.36	3.15	9.38	1.84	1.97	0.04	11.92
KANO	25.6	24.99	14.43	17.43	2.84	4.81	0.55	0.39	0.89	8.08
KATSINA	19.36	32.29	14.62	24.41	1.71	3.38	1.1	0.39	0.49	2.26
KEBBI	10.06	39.67	20.32	24.53	0.95	0.31	0.06	0.22	0.02	3.86
KOGI	16.46	42.17	9.57	10.38	1.56	6.74	0.93	0.16	1.02	11
KWARA	18.25	24.8	12.88	9.06	4.16	15.84	0.62	0.87		13.51
LAGOS	6.34	41.27	12.34	6.59	0.65	14.24	0.68	0.77	0.52	16.6
NASARAWA	15.52	31.72	12.37	18.24	2.06	7.98	1.53	0.42	0.14	10.01
NIGER	3.13	37.66	15.52	18 12	0.5	12.24	1.87	1.55	0.06	9.36
OGUN	5.35	43.76	10.23	6.87	1.83	13.44	0.97	1.42	0.46	15.67
ONDO	11.16	31.2	13.79	11.13	1.44	13.11	3.16	0.74	0.16	14.11
OSUN	16.18	22.23	14.25	10.37	3.15	12.48	1.99	0.58		18.78
OYO	13.03	20.87	17.35	8.6	1.44	19.61	1.75	0.52	0.01	16.33
PLATEAU	17.96	31.88	12.03	19.99	2.75	5.57	2.79	0.9	0.4	7.74
RIVERS	17.31	36.17	9.72	8.47	2.65	8.34	2.25	1.2	1.01	14.05
SOKOTO	21.49	27.16	14.84	22.84	3.36	2	0.58	0.96	0.56	6.21
TARABA	8.7	33.51	17.86	17.52	0.44	8.53	2.02	1.41	0.05	9.95
YOBE	1.85	58.3	15.3	16.39	0.29	4.16	1.59	0.11	1.23	1.67
ZAMFARA	9.12	36.33	22.34	28.29	0.31	0.67	1.01	0.07		1.85
FCT,ABUJA	13.77	21.15	21.05	3.97	1.88	12.15	0.1	4.09	0.25	16.58

Source: National Bureau of statistics (Consumption Pattern in Nigeria 2009/2010)

Appendix II: Tests of Normality

Var.	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	*Df*	*Sig.*	*Statistic*	*Df*	*Sig.*
X1	.095	33	.200*	.977	33	.684
X2	.116	33	.200*	.935	33	.049
X3	.100	33	.200*	.957	33	.209
X4	.145	33	.074	.945	33	.097
X5	.170	33	.016	.953	33	.167
X6	.100	33	.200*	.967	33	.406
X7	.113	33	.200*	.940	33	.069
X8	.165	33	.024	.754	33	.000
X9	.119	33	.200*	.904	33	.007
X10	.163	33	.026	.939	33	.065

a. Lilliefors Significance Correction

*. This is a lower bound of the true significance.

There are a number of different extraction methods in factor analysis, unless there is a serious lack of multivariate normality, maximum likelihood extraction is the best among these method (DeCoster, 1998), hence the need to normality test.

The Kolmogorov-Smirnor and Shapiro-Wilk tests compare the scores in the sample to a normally distributed set of scores with the same mean and standard deviation. If the test is non-significant ($p > 0.05$) it tells us that the distribution of the sample is not significantly different from a normal distribution (Field, 2004).

The Appendix II includes the test statistic itself, the degrees of freedom and the significance value of the test. A significant value less than 0.05 indicate a deviation from normality. Both tests are highly non-significant, indicating that the distribution is normal.

Appendix III: Reproduced Correlations (PCA)

Var	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10
X1	.878[a]	-.666	-.189	.217	.717	-.117	-.477	.160	.110	-.101
X2	-.666	.865[a]	-.217	.027	-.437	-.279	.528	-.549	.335	-.209
X3	-.189	-.217	.727[a]	.252	-.482	.060	-.314	.264	-.511	-.153
X4	.217	.027	.252	.889[a]	-.097	-.721	-.406	-.379	.147	-.866
X5	.717	-.437	-.482	-.097	.766[a]	.020	-.171	.091	.261	.144
X6	-.117	-.279	.060	-.721	.020	.727[a]	.148	.544	-.372	.772
X7	-.477	.528	-.314	-.406	-.171	.148	.528[a]	-.186	.186	.277
X8	.160	-.549	.264	-.379	.091	.544	-.186	.566[a]	-.449	.501
X9	.110	.335	-.511	.147	.261	-.372	.186	-.449	.515[a]	-.248
X10	-.101	-.209	-.153	-.866	.144	.772	.277	.501	-.248	.886[a]
X1		-.020	-.035	-.034	-.122	-.047	.061	-.014	-.041	.008
X2	-.020		-.055	-.059	.013	-.068	-.149	.132	-.014	-.013
X3	-.035	-.055		-.072	.104	-.042	.093	.008	.230	.010
X4	-.034	-.059	-.072		.030	.059	.093	-.044	-.108	-.012
X5	-.122	.013	.104	.030		-.018	.090	.024	-.051	-.041
X6	-.047	-.068	-.042	.059	-.018		-.080	-.184	.040	-.067
X7	.061	-.149	.093	.093	.090	-.080		.025	-.134	-.065
X8	-.014	.132	.008	-.044	.024	-.184	.025		.114	-.066
X9	-.041	-.014	.230	-.108	-.051	.040	-.134	.114		.013
X10	.008	-.013	.010	-.012	-.041	-.067	-.065	-.066	.013	

Extraction Method:
Principal Component
Analysis.

a. Reproduced
communalities

b. Residuals are computed between observed and reproduced correlations. There are 23 (51.0%) non redundant residuals with absolute values greater than 0.05.

Appendix IV: Reproduced Correlations (FA)

Var	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10
X1	999[a]	-.686	-.223	.183	.596	-.163	-.415	.146	.070	-.093
X2	-.686	999[a]	-.271	-.033	-.423	-.347	.380	-.416	.321	-.222
X3	-.223	-.271	.482[a]	.181	-.176	.232	-.063	.098	-.323	-.003
X4	.183	-.033	.181	.999[a]	-.067	-.662	-.313	-.422	.039	-.878
X5	.596	-.423	-.176	-.067	.387[a]	.016	-.203	.167	.039	.100
X6	-.163	-.347	.232	-.662	.016	.745[a]	.133	.479	-.309	.734
X7	-.415	.380	-.063	-.313	-.203	.133	.257[a]	-.012	.047	.204
X8	.146	-.416	.098	-.422	.167	.479	-.012	.373[a]	-.195	.493
X9	.070	.321	-.323	.039	.039	-.309	.047	-.195	.264[a]	-.173
X10	-.093	-.222	-.003	-.878	.100	.734	.204	.493	-.173	.856[a]
X1		-2.325E-6	.000	-2.538E-6	.000	.000	.000	6.052E-5	.000	.000
X2	-2.325E-6		.000	-2.873E-6	.000	.000	.000	-4.266E-5	.000	.000
X3	.000	.000		-.001	-.201	-.213	-.159	.174	.043	-.140
X4	-2.538E-6	2.873E-6	-.001		.000	.000	-7.064E-5	.000	.000	-1.997E-5
X5	.000	.000	-.201	.000		-.014	.123	-.053	.171	.002
X6	.000	.000	-.213	.000	-.014		-.064	-.118	-.023	-.029
X7	.000	.000	-.159	-7.064E-5	.123	-.064		-.148	.004	.007
X8	6.052E-5	-4.266E-5	.174	.000	-.053	-.118	-.148		-.141	-.058
X9	.000	.000	.043	.000	.171	-.023	.004	-.141		-.061
X10	.000	.000	-.140	-1.997E-5	.002	-.029	.007	-.058	-.061	

Extraction Method: Maximum Likelihood.

a. Reproduced communalities

b. Residuals are computed between observed and reproduced correlations. There are 14 (31.0%) non redundant residuals with absolute values greater than 0.05.

Appendix V: Correlation Coefficients

Correlation Matrix

Variables	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
X_1	1.000	-.668	-.181	.325	.579	-.336	-.371	.108	.074	-.247
X_2	-.668	1.000	-.339	-.153	-.360	-.245	.326	-.449	.352	-.100
X_3	-.181	-.339	1.000	.052	-.347	.147	-.245	.406	-.311	-.039
X_4	.325	-.153	.052	1.000	.104	-.608	-.393	-.398	.043	-.872
X_5	.579	-.360	-.347	.104	1.000	-.228	-.038	.074	.232	-.069
X_6	-.335	-.245	.147	-.608	-.228	1.000	.163	.350	-.372	.661
X_7	-.371	.326	-.245	-.393	-.038	.163	1.000	-.160	.055	.297
X_8	.108	-.449	.406	-.398	.074	.350	-.160	1.000	-.343	.435
X_9	.074	.352	-.311	.043	.232	-.372	.055	-.343	1.000	-.262
X_{10}	-.247	-.100	-.039	-.872	-.069	.661	.297	.435	-.262	1.000

The correlation coefficients show the relationship between the observed measures. The correlation matrix of principal components is the same to those of factor analytical techniques. It is from this matrix that reproduced correlations for both techniques are estimated (see Appendix III and Appendix IV).