DEVELOPMENT OF A FUZZIFIED-TREND MAPPING AND IDENTIFICATION (FTMI) MODEL FOR FUZZY TIME SERIES FORECASTING

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DECEMBER, 2014

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M.Sc/Eng/7386/2011-2012

A THESIS SUBMITTED TO THE SCHOOL OF POSTGRADUATE STUDIES,

AHMADU BELLO UNIVERSITY, ZARIA

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE AWARD OF MASTER OF SCIENCE (M.SC) DEGREE IN ELECTRICAL ENGINEERING

DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING,

FACULTY OF ENGINEERING

AHMADU BELLO UNIVERSITY, ZARIA

NIGERIA

DECEMBER, 2014

DECLARATION

I declare that the work in this Thesis entitled DEVELOPMENT OF A FUZZIFIED-TREND MAPPING AND IDENTIFICATION (FTMI) MODEL FOR FUZZY TIME SERIES FORECASTING has been carried out by me in the department of Electrical and Computer Engineering. The information derived from literatures has been duly acknowledged in the text and a list of references provided. No part of this dissertation was previously presented for another degree or diploma at this or any other Institution.

Christopher M. OGBONNA ______________________ ______________________

CERTIFICATION

This Thesis entitled DEVELOPMENT OF A FUZZIFIED-TREND MAPPING AND IDENTIFICATION (FTMI) MODEL FOR FUZZY TIME SERIES FORECASTING by Christopher Muoneke OGBONNA meets the regulations governing the award of the degree of Master of Science (M.Sc) Degree in Engineering of the Ahmadu Bello University, and is approved for its contribution to knowledge and literary presentation.

Dr. M. B. Mu'azu ______________________ ______________________

(Chairman, Supervisory Committee) (Signature) Date

Dr. O. Akinsanmi ______________________ ______________________

(Member, Supervisory Committee) (Signature) Date

Dr. M. B. Mu'azu ______________________ ______________________

(Head of Department) (Signature) Date

Prof A. Z. Hassan ______________________ ______________________

(Dean Postgraduate School) (Signature) Date

DEDICATION

This thesis work is dedicated to the Almighty God, who is the truth and source of all pure knowledge, and who in his munificence has granted humans both the desire and capacity to seek for the truth.

ACKNOWLEDGEMENT

My perpetual gratitude goes to the almighty God whose loving kindness, revelations and faithfulness to me during this work boggles my mind, to say the least.

My unreserved thankfulness goes to my esteemed supervisor Dr. M. B. Mu'azu, who besides the demanding administrative engagements provided timely corrections, brandished valuable analyses and directions and gave inspiring motivations. He is indeed committed to the core. I also humbly appreciate Dr. O. Akinsanmi for his fatherly advice and moral support during this work.

I bow in gratitude to Dr. T. H. Sikiru for his hearty goodwill, patience and intellectual mentorship. Undeniably, his professional and scholarly contributions elevated the academic stance of this work.

I particularly acknowledge the contributions of Dr. S. M. Sani, Prof. B. G. Bajoga and Dr A. D. Usman. Their constructive criticisms and suggestions improved the quality of this thesis work.

I am thankful to these lecturers who in one way or the other have contributed to this work: Dr. S. Garba, Engr Malam Y. Jibril, Dr. Jimoh, Dr. D.D.Dajab, Dr. J.Y.Oricha, Engr. Kabir Ahmad Abu-Bilal, Engr E Okafor, Engr S. A Adamu, and all those whose names cannot be captured for want of space.

My heartfelt appreciation goes to my loving brother, Mr. Ogbonna I.C. His moral, financial, and spiritual supports during this work are unprecedented.

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LIST OF ABBREVIATIONS

Acronym	Meaning
FTS	Time Series Forecasting
FTMI	Fuzzified Trend Mapping and Identification
RPD	Re-Partitioning Discretizaation
OoD	Order of Difference
TV	Trend Values
AR	Auto-Regressive
MA	Moving Average
ARMA	Auto-Regressive Moving Average
ARIMA	Auto-Regressive Integrated Moving Average
QoS	Quality of Service
APE	Average Percentage Error
MAPE	Mean Average Percentage Error
MSE	Mean Squared Error
ICT	Information Communication Technology
FLR	Fuzzy Logic Relations

FLRG	Fuzzy Logic Relationship Groups
PSO	Particle Swarm Optimization
TFA	Trapezoidal Fuzzification Appraoch
CPDA	Cumulative Percentage Distribution Appraoch
IFS	Intuitsionistic Fuzzy Set
ACF	Auto-Correlation Function
PACF	Partial Auto-Correlation Function

LIST OF SYMBOLS

μ	Mean
U	Universe of Discourse
∈	An Element of
Δ	Delta (Change in a variable)
Σ	Summation
%	Percentage
σ	Standard Deviation

ABSTRACT

Fuzzy time series (FTS) forecasting is a technique based on time series and fuzzy logic theory developed for the purpose of analysis and prediction of time series events. The proposed Fuzzified Trend Mapping and Identification (FTMI) model uses a Re-Partitioning Discretization (RPD) approach to optimize the partitioning of the interval lengths and high-order fuzzy relations to construct the trend values. In the proposed model, the mapped out trends are fuzzified into classes both in linguistic and numeric terms to capture both the uncertainty and fuzziness inherent in the trends. Each trend class is given distinct ordinal position for ease of identification during deffuzzification and forecasting. The proposed model is tested on three time series data of different structural and statistical characteristics using mean average percentage error (MAPE) as statistical performance measure. The adaptability of the proposed model to different time series events is also tested using statistical measure of dispersion (variance). Empirical result shows an increase of over 50% in forecast accuracy over pioneer and recent models. Also, the statistical variance of the forecast errors of the proposed model from the MAPE were 0.12, 0.488 and 1.267 compared to 0.58, 8.037 and 4.915 of Shah's (2012) model for the three time series data respectively. These results demonstrate both the superiority of the proposed FTMI model in accuracy of prediction and its robustness in adaptation to time series of different structural and statistical characteristics when compared to existing models. The effect of increasing the order of difference on both the data trend and the accuracy of forecast are also investigated. Results obtained show that it does not necessarily increase the forecast accuracy regardless of the structure of the time series. The FTMI model is also applied to forecast the short term Internet traffic data of ABU, Zaria. The empirical result shows a MAPE of 0.27 for the Internet traffic, indicating a good accuracy of prediction considering the large size of these traffics.

CHAPTER ONE

INTRODUCTION

1.1 Preamble

Time series is simply a collection of quantitative variables at regular intervals of time. Whether discrete or continuous, time series is always both non linear and non stationary since they are sample functions realized from processes that are always stochastic (Subanar and Abadi, 2011). Time series analyses and forecasting play a vital role in planning, equipment maintenance and optimization, efficient quality of service (OoS), and even anomaly detection in diverse fields such as: engineering, medicine, stock market, information and communication technology (ICT) (Sah and Konstantin, 2005; Klevecka, 2011; Zhani *et al.*, 2011; Cortez *et al.*, 2012). Time series forecasting has been widely studied and investigated for the past three decades or so (Box and Jenkins, 1976; Song and Chissom, 1993a; Huarng and Yu, 2003; Wang *et al.*, 2008; Singh and Borah, 2013). In simple terms, time series forecasting involves the analyses of historical time series data and prediction of future variables from the analyzed data (Box and Jenkins, 1976; Hassan *et al.*, 2012). Traditionally, time series forecasting problems are being solved using a class of statistical linear autoregressive (AR), moving average (MA) and their hybrid (ARMA) models. These models and their subsequent extensions such as auto-regressive integrated moving average (ARIMA) and other linear models assume that the time series are both linear and stationary. A viable alternative to these linear techniques are soft computing techniques which are capable of approximating any real continuous function without making assumptions about the structure of the data (Subanar and Abadi, 2011). Among these techniques such as neural network, evolutionary algorithm etc, fuzzy logic has received a much greater attention because of its over-riding advantages (Song and Chissom, 1993a; Chabaa and Zeroual, 2009; Shah, 2012). Of many critical issues in fuzzy time series (FTS) models, trend mapping and identification has not been fully exploited from the research community. This research is focused on the development of a robust FTS forecasting model from trend mapping and identification approach.

1.2 Motivation

Time series analyses and forecasting is one of the ways humans try to exert some form of control in the future in order to avoid catastrophes, build capacities and to efficiently maintain and maximize scarce resources. Although, time series analyses and forecasting has arguably received the greatest attention in engineering, it spans in almost all the fields of human endeavor (Sah and Konstantin, 2005). The complexities involved in the analyses and training of large amount of historical time series data is of great concern in time series forecasting (Hassan *et al.,* 2012). Another major challenge in addressing time series forecasting problems is the non linear nature of time series in addition to its attendant causative factors that are highly unpredictable. Hence, linear techniques and models have often fallen short of the basic requirements needed for effective time series analyses and forecasting, such as prediction accuracy. More so, because linear models assume that the underlying generation process of time series is time invariant, not all time series can be analyzed by these conventional models (Subanar and Abadi, 2011). A major way to overcome these shortcomings in linear techniques is by the use of non linear techniques which do not require large amount of training data and which can analyze time series events in ways humans think, namely linguistic terms. Fuzzy logic, more than any other non linear techniques, has proved robust and efficient in dealing with time series forecasting issues (Shahida *et al.,* 2003; Chabaa and Zeroual, 2009; Shah, 2012). Being a universal approximator, fuzzy systems have advantages that the developed models are characterized by both numeric and linguistic interpretability and the generated rules can be understood, verified, extended and incorporate expert knowledge (Shah, 2012; Subanar and Abadi, 2011). Hence, fuzzy time series (FTS) developed in the past two decades (Song and Chissom, 1993b; Song and Chissom, 1993a), has commanded unprecedented attention from the research community to date. However, the issue of accuracy of forecasting is still a major challenge in FTS since prediction accuracy reported in the open literature is not enough.

Another issue to contend with in time series analyses and prediction is the fact that times series has varying structural and statistical characteristics depending on both the data source and time of collection. Although the inherent characteristics of some time series data vary negligibly with source and time, such as internet traffic (Zhani *et al.*, 2011), others vary significantly. As a result, long-term prediction is often associated with variability of errors (Papadopouli *et al.*, 2006). These have led researchers to favor short-term prediction in place of long-term prediction. Also, to improve accuracy of forecasting, researchers have tried to adapt time series models to the structure of time series under analyses. Nevertheless, this has led to the problem of over fitting of models (Shah, 2012) - another challenge bedeviling time series forecasting. Consequently, models developed may be effective in predicting the time series under study, but erratic in prediction of other time series of different or even similar structure. It has become imperative therefore, to develop robust FTS model that will adapt to the varying dynamics and characteristics of time series data while simultaneously improving the forecasting accuracy.

1.3 Aim and Objectives

The aim of the research is the development of a fuzzified-trend mapping and identification (FTMI) based fuzzy time series model.

This aim is achieved with the following objectives:

I. Development of a Fuzzified-Trend Mapping and identification (FTMI) fuzzy time series forecasting model using the Re-Partitioning Discretization (RPD) approach (Singh and Borah, 2013) for optimizing the partitioning of the interval lengths and using the Order of Difference (OOD) for trend mapping and identification.
II. Validation of the developed model using the standard data of the students' enrolment of the University of Alabama and two other time series data with different structural characteristics; and comparing its performance with pioneer and existing models.
III. Determination of effect of increment of the Order of Difference (OoD) on the forecast accuracy.
IV. Application of the model on the Internet traffic data of Ahmadu Bello University, Zaria.

1.4 Statement of the Problem

Many FTS models have been developed and used to forecast time series data in various domains. Regardless of the targeted domains, the main focus of these variant models is to improve the accuracy of forecasting relative to the pioneer model (Song and Chissom, 1993a; Song and Chissom, 1994), and any prevailing model(s) as the case may be. However, a fundamental limitation of the existing fuzzy time series models in the open literature has been the problem of over fitting of models (Shah, 2012). That is, the models are adapted to a particular time series data only and therefore, not effective in predicting other time series of different structure or even similar structure, in some cases. Many researchers have used first or second-order difference to see how the accuracy of forecasting may be improved. But the problem of how increasing the orders of difference affect both the data trend of a time series and the accuracy of forecasting has not been investigated. It is crucial, therefore, to develop robust fuzzy forecast model which will adapt to the varying dynamics (non-linearity, non-stationarity) of different time series data while simultaneously improving the forecast accuracy.

In view of this, a novel FTS model is developed in this research using an adaptive optimizing approach namely, Re-Partitioning Discretization (RPD) to optimize the determination of its interval length and high-order trend relation to properly identify the general trend(s). The general trends identified are fuzzified to capture the uncertainty or fuzziness inherent in them. Also, this research extended the order of difference to fourth-order and investigated the effect of increasing the order of difference on both the data trend and accuracy of forecasting.

1.5 Methodology

Like most fuzzy time series systems, the Trend Mapping and Identification technique has two main components: Fuzzification and defuzzification. These components have many parts and an inference system which is rule based. The general methodology adopted in this model is as follows:

I. Apply the Re-Partitioning Discretization (RPD) approach presented in Section 2.5 based on the collected data to optimize both the partitioning of data sets and the universe of discourse.
II. Define linguistic terms for each of the intervals.
III. Establish the Fuzzy Membership Functions and the fuzzy sets.
IV. Fuzzify the time series data set and establish the high-order fuzzy relations (FLRs).
V. Find the Order of Difference (OoD) between successive data to map out the trends.
VI. Fuzzification of the trends. The trends are fuzzified based on the number of classes of the historical data.

VII. Development of a fuzzy Inference System of rules based on the fuzzified trends for defuzzification and forecasting.

VIII. Validation of the developed model using the standard data of the students' enrolments of the University of Alabama and two time series data of different structural characteristics by comparison with the results obtained using the Song and Chissom's and Shah's methods on the bases of MAPE and variance statistic.

IX. Determination of the effect of increasing the Order of Difference on the forecast accuracy.

X. Application of the model on the Internet traffic data obtained from Ahmadu Bello University, Zaria.

1.6 Significant Contribution of the Developed FTMI Model

The contributions of the developed FTMI Model in time series forecasting are as follows:

1. The developed FTMI model is able to capture the trend's vagueness and has significantly improved the forecast accuracy (over 50% when compared to recent models such as Shah (2012) model.
2. The developed FTMI model can trap both the fuzziness and uncertainty inherent in the trend, and has significantly eliminated noise in all the time series tested.
3. The developed FTMI model is robust and adapts well to time series of different statistical and structural characteristics.The statistical measure of dispersion (variance) is introduced in the thesis to validate the proposed model and those of previous models considered in this paper. The variance of the proposed FTMI model for the three time series data tested are 0.12, 0.488 and 1.267 and are all less than their respective MAPE of

0.37, 1.054 and 2.400 unlike other models. Hence, the proposed FTMI model does not have high variability in forecasting time series of different structural and statistical characteristics. Therefore, it is suitable for different time series events

4. Using three different time series of different structural characteristics, this research extended the order of difference to fourth-order and demonstrated that though, increasing the order of difference affected the data trend, it did not increase the accuracy of forecasting.

1.7 Scope of the Research

The research borders on the development of a fuzzy time series (FTS) model based on the trend mapping and identification method for short-term prediction of time series data based purely on a univariate approach. The scope of this research did not cover the following:

- Effect of multi-variables in time series.
- Seasonal time series.
- Long-term prediction.
- Residual and stationarity control.
- Post-sample prediction

1.8 Thesis Organisation

The summary of how the rest of the chapters are organised is presented thus:

Having presented the introduction with the general background of the thesis in chapter one, chapter two presents the literature review consisting of systematic review of some fundamental concepts theories relevant to FTS and the proposed FTMI Model. Also in this chapter is a comprehensive review of works similar to this investigation. The development of the proposed

FTMI Model is clearly analysed in chapter three. Prior to this, the general structure of the conventional fuzzy trend mapping models is briefly highlighted in the chapter. Chapter four presents the validation and application of the proposed FTMI Model. Three time series of varying structural characteristics are all used in the validation. Subsequently, the proposed FTMI Model is applied to forecast the internet traffic of Ahmadu Bello University, Zaria. Chapter five presents the conclusions of the study. Also in this chapter are the summary of findings and the suggestions for further work. Finally, all the cited references are presented at the end of this thesis work.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

The first part of this chapter is dedicated to the comprehensive review of the fundamental concepts on which the proposed FTMI Model is built. Time series models and the methods of analyses, fuzzy set theory and fuzzy time series, Re-Partitioning Discretisation (RPD) approach of interval length partitions and the order of difference are the fundamental theories highlighted in the first part. The second part x-rays the systematic review of works related to this thesis as obtained from the open literature.

2.2 Review of Fundamental Concepts

These sections describe the general concepts fundamental to the study.

2.2.1 Time Series Models and Analyses

A time series, conventionally represented as a set of discrete values $x_1, x_2, x_3, \ldots, x_n$, is a sequence of observations of a physical quantity or variable ordered sequentially usually at regular intervals of time or space. Whichever phenomenon that is observed or measured is indexed by time as the only parameter (independent variable); therefore, the name time series is used (Shrivastav and Ekata, 2012). There are two kinds of time series:

- Continuous – where there is a collection of observation at every instance of time. In this time series, the observation at time t is denoted at $X(t)$.
- Discrete – where there is an observation at usually regular spaced intervals. Each observation at time (kT) is denoted at $X_{(t)}$ (Mua'zu and Adeola, 2008).

Time series is represented mathematically as:

$\{X_t: t = 1, 2, 3, \ldots, N\}$

Where t is the time index, N is the total number of observations and X_t is the function of components represented as:

$$X_t = f\left(T_t, S_t, C_t, I_t\right) \tag{0.1}$$

where T_t, S_t, C_t, I_t represent the trend, seasonal, cyclic and irregular components respectively (Subanar and Abadi, 2011). These components are the features or characteristics that define a time series. They are briefly defined as:

I. Trend Components: When allowance has been made for other components, trend is a long-term movement of a time series in a particular direction. It is the general tendency of a time series to move upward or downward. A simple way of detecting trend in seasonal data is to take averages over a certain period. If these averages change with time, then there is evidence of the trend in the series (Mua'zu and Adeola, 2008).

II. Seasonal Components: This is the component of a time series that describes regular fluctuation with a period of less than one year. It is the component of a variable which is dependent on time of the year and occurs in weekly or monthly data. Seasonal dependency is formally defined as correlational dependency of order k between each i^{th} element of the series and the $(i\text{-}k)^{th}$ element and measured by autocorrelation i.e., a correlation between the two terms; k is usually called the lag (Box and Jenkins, 1976).

III. Cyclical Components: In weekly or monthly data, cyclic components describe any regular fluctuations. It is a non seasonal component which varies in recognizable cycle.

IV. Irregular Components: When other components (trend, seasonality and cyclical) have been accounted for, whatever is left over is the irregular components (Mua'zu and Adeola, 2008).

Time series are analyzed to study these inherent structural characteristics (components) with a view to understanding the dynamic processes by which the time series data are generated. The main aim of this analyses is to characterize, model and forecast time series (Subanar and Abadi, 2011). Traditionally, time series analysis is defined as a branch of statistics that generally deal with the structural dependencies between the observation data of random phenomena and the related parameters (Shrivastav and Ekata, 2012). Generally, the methods for analyzing time series are grouped into two namely: statistical conventional methods (linear models) and soft computing methods (non linear models). These two methods will be described in the next two sections.

2.2.1.1 Statistical Conventional Methods

Traditional models used in analyzing and forecasting time series assume that they are both stationary and linear. These conventional linear models are autoregressive (AR), moving average (MA). If the time series data contain considerable error, then the first step in the process of trend identification is smoothing. It involves some form of local averaging of data such that the nonsystematic components of individual observations cancel each other out. The most common technique is moving average smoothing which replaces each element of the series by either the simple or weighted average of n surrounding elements, where n is the width of the smoothing window (Box and Jenkins, 1976). In trend analyses, moving average provides a set of very powerful indicator for tracking trend and trend reversals. Moving average is a lagging indicator, or trend following formula, that smoothens the volatile swings in observations. It attempts to tone down the fluctuations of observations to a smoothed trend, so that distortions are reduced to a minimum (Mua'zu and Adeola, 2008).

Seasonal dependency, another general component of the time series pattern, is the association or mutual dependence between values of the same time series and different time lags and is measured by autocorrelation. If the measurement error is not too large, seasonality can be visually identified in the series as a pattern that repeats every k elements, and seasonal patterns of time series can be examined through autocorrelograms (Kendall, 1976). Autocorrelation is also a statistical method of forecasting which is the correlation (relationship) between members of a time series of observations, such as weekly share prices or interest rates, and the same value at a fixed time interval (Mua'zu and Adeola, 2008). The orders of simple autoregressive and moving average models can be determined by the autocorrelation function (ACF) and partial autocorrelation function (PACF) plots of the time series data and the models (Subanar and Abadi, 2011). Another model that assume that time series is stationary is the hybrid AR and MA (ARMA) models. An ARMA model with order (p,q) is expressed as ARMA(p, q) where p is order of moving average (MA) and q is order of autoregressive (AR). The serial dependency for a particular lag of k can be removed by differencing the series, that is converting each i^{th} element of the series into its difference from the $(i\text{-}k)^{th}$ element (Box and Jenkins, 1976). One reason for this removal is to make the series stationary which is necessary for ARMA and other techniques.

In real-life research and practice, patterns of the data are unclear, individual observations involve considerable error, and it is not only required to uncover the hidden patterns in the data but also to generate forecasts. In this case, the integrated ARMA or autoregressive integrated moving average (ARIMA) is used (Box and Jenkins, 1976). This technique is a univariate approach whose model parameters are the autoregressive parameters (q) the number of differencing passes

(r) and the moving average parameter(p). In this notation, models are summarized as ARIMA (prq) (Box and Jenkins, 1976). So, for example, a model described as $(0,3,2)$ means that it contains zero autoregressive (p) parameters and three moving average (q) parameters which were computed for the series after it was differenced twice (r). However, ARIMA is a complex technique, the results obtained depend on the researcher's level of expertise and the method is appropriate only for series that is truly stationary (Mua'zu and Adeola, 2008).

2.2.1.2 Non Conventional Methods

Non conventional models, otherwise known as soft computing techniques, is a term applied to a field within computer science and engineering which is characterized by the use of inexact solutions to computationally-hard tasks (Subanar and Abadi, 2011). Soft computing techniques include fuzzy systems, neural networks, genetic algorithms and their hybrids. As universal approximators, they have the capability of modelling the complexities and non-linearities in natural processes without making assumptions in the structure of the processes or data. These techniques deal with fuzziness, complexities, imprecision, uncertainty, partial truth, and approximation to achieve predictability, tractability, robustness and low solution cost (Subanar and Abadi, 2011). Hence, soft computing techniques have been used to model the complexity of relationships in nonlinear time series to a relatively higher degree of accuracy.

2.2.2 Fuzzy Set Theory

The Fuzzy sets theory, advanced in 1965 (Zadeh, 1965), can be defined as a mathematical formulation of variable sets that enables the elimination of indefiniteness and deal with incomplete, inaccurate information of both qualitative and quantitative nature (Mua'zu and Adeola, 2008). The theory expands the notion of purely crisp sets by assigning membership degrees to set elements so the transition from membership to non-membership is gradual rather than abrupt (Poulsen, 2009). The theory of fuzzy sets excludes any precise description of the task and doing this gives a solution scheme to the problem such that subjective reasoning and evaluation plays a principal role. Normally, the membership degree of a set element is a real number between 0 and 1. The closer the membership degree is to 1, the more an element belongs to a given set. A membership degree of zero (0) means that an element is clearly not a member of a particular set. Elements with a membership degree between 0 and 1 are more or less members of a particular set (Poulsen, 2009).

Mathematically conceived, a fuzzy set is a class of objects with a continuum of grade of membership. Let U be the universe of discourse with $U = \{u_1, u_2, u_3, \dots, u_n\}$ where u_i are possible linguistic values of U, then fuzzy set $A_i \; of \; U$ is defined by:

$$A_i = \frac{\mu_{A_i}(u_1)}{u_i} + \frac{\mu_{A_i}(u_2)}{u_2} + \ldots + \frac{\mu_{A_i}(u_n)}{u_n} \tag{0.2}$$

Where μ_{A_i} is the membership function of the fuzzy set A_i, such that:

$$\mu_{A_i} : U \rightarrow [0,1]$$

The Fuzzy Logic System is a computing framework based on the concepts of Fuzzy Set theory, Fuzzy IF-THEN, and Fuzzy reasoning (Mua'zu and Adeola, 2008). Fuzzy Logic was designed to mathematically represent uncertainty and vagueness and to provide formalized tools for dealing with the imprecision intrinsic to many problems. It has applications in areas such as automatic control, data analysis, time series prediction, robotics and pattern recognition. The fuzzy logic system is a system that is developed in essentially three (3) steps as shown in Figure 2.1.

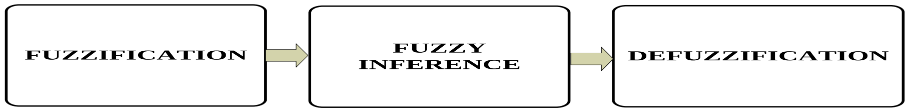

Figure 2.1 Block Diagram of Fuzzy Logic System

Fuzzification: This is using fuzzy sets to translate real (numerical) values into linguistic variable values and using membership functions to represent the variables graphically. It is therefore a mapping from the observed numerical input space to the fuzzy sets defined in the corresponding universe of discourse.

Fuzzy membership functions such as: triangular, Gaussian, trapezoidal, bell etc, can be mathematically represented as piecewise linear functions. The membership degree of a variable is computed using the following algorithm:

I. the membership function is defined as a set of points $(P(x_i, \mu_i))$ with:

$x_1 < x_i < x_n$ for $i = 1, ..., n$

Where $\mu_i \ \varepsilon \ [0,1]$ and $[x_i, \dots, x_n]$ is the Base Variable range. The current operational value is:

$X \in [x_i, ... x_n]$

II. Evaluate the valid Base Variable interval with $x_k < X < x_k + 1$

III. Compute membership degree by:

$$\mu = \frac{\mu_i + (X - x_i) \times ([\mu_i + 1] - \mu_i)}{(x_i + 1) - x_i} \quad (0.3)$$

where μ = Membership Degree, $\mu(x)$ = Membership Function, X = Universe of Discourse and x = Element of X (Mua'zu and Adeola, 2008).

Fuzzy inference: This is the fuzzy logic reasoning process that determines the output corresponding to fuzzified inputs. The fuzzy rules that describe relationships at a high level (in a linguistic sense) are typically written as antecedent-consequent pairs of IF-THEN statements. Basically, there are four approaches to the developing fuzzy rules:

I. Extract from expert experience and control engineering knowledge.

II. Observe the behavior of human operators

III. Use a fuzzy model of a process

IV. Learn relationships through experience or simulation with a learning process.

These approaches do not have to be mutually exclusive. Due to the use of linguistic variables and fuzzy rules, the system can be made understandable to a non-expert operator. The basic role of a linguistic variable is that it encapsulates the properties of approximate or imprecise concepts in a systematic and computationally useful way.

Fuzzy Inference is a calculus consisting of three steps:

I. Aggregation (Computation of the IF part of the rules):

I. Use Minimum operator for AND aggregation:

$$\mu_{IF} = \min(\mu_i) \tag{0.4}$$

Use Maximum operator for OR aggregation:

$$\mu_{IF} = \max(\mu_i) \tag{0.5}$$

II. Composition (Computation of the THEN part of the rules): This step computes the degree of truth for the rule. Standard methods include MAX/MIN (also called MAX/PROD)

III. Result Aggregation: after the degree of truth for the rules are compared, this step determines which rules will contribute to the defuzzified result. The MAX operator can be used for result aggregation (Mua'zu and Adeola, 2008).

Defuzzification: This maps output Fuzzy Sets defined over an output Universe of Discourse to crisp outputs with the aim of producing the non-fuzzy output that best represents the distribution of an inferred fuzzy output. It is a process to extract an easily comprehensible answer from the solution set (Mua'zu and Adeola, 2008).

Defuzzification means dropping a “plumb line” to some point on the underlying domain. At the point where this line crosses the domain axis, the expected value of the fuzzy set is read. Underlying all the defuzzification functions is the process of finding the best place along the surface of the fuzzy set to drop this line. This generally means that defuzzification algorithms are a compromise with or a tradeoff between the need to find a single point result and the loss of information such a process entails. The two most frequently used defuzzification methods are composite moments (centroid) and composite maximum.

2.2.2.1 Fuzzy Time Series

Fuzzy time series (FTS) is the mathematical amalgamation of fuzzy set theory and time series for the purpose of analyses and forecasting of time series data. Among other factors, the motivation for introducing this new forecasting framework was the need to model time series problems when historical data are defined as linguistic values (Song and Chissom, 1993b; Song and Chissom, 1993a). Briefly, some foundational definitions of FTS framework as they were originally conceived in pioneering papers (Song and Chissom, 1993b; Song and Chissom, 1993a; Song and Chissom, 1994) and other subsequent models (Chen, 1996; Chen, 2002) are given below:

Definition 1: Fuzzy Time Series

Let $Y(t)$ $(t = \cdots, 0,1,2,\ldots)$, a subset of real numbers, be the universe of discourse on which fuzzy sets $f_i(t)$ $(i = 1,2,\ldots)$ are defined. If $F(t)$ is a collection of $f_i(t)$ $(i = 1,2,\ldots)$, then $F(t)$ is called a fuzzy time series on:

$$Y(t)\left(t = \ldots, 0, 1, 2, \ldots\right)$$

Definition 2: Nth-Order Fuzzy Relation

Let $F(t)$ be a fuzzy time series. If $F(t)$ is caused by $F(t-1)$, $F(t-2)$, ... , $F(t-n)$, then this fuzzy relationship is represented by $F(t-n)$, ... , $F(t-2)$, $F(t-1)$ $\rightarrow F(t)$ and is called nth -order fuzzy time series.

Definition 3: Time-Invariant Fuzzy Time Series

Suppose $F(t)$ is caused by $F(t-1)$ only and is denoted by $F(t-1) \rightarrow F(t)$, then there is a fuzzy relationship between $F(t)$ and $F(t-1)$ which is expressed as follows:

$$F(t) = F(t-1) \times R(t-1, t) \tag{0.6}$$

The relationship R is referred to as a first order model of $F(t)$ If $R(t-1, t)$ is independent of time t, that is, for different times t_1 and t_2:

$$R(t_1, t_1 - 1) = R(t_2, t_2 - 1) \tag{0.7}$$

Then $F_{(t)}$ is called a time-invariant fuzzy time series. Otherwise it is called a time-variant fuzzy time series

Definition 4: Fuzzy Logic Relationship Group (FLRG)

Relationships with the same fuzzy set on the left hand side can be further grouped into a

relationship group. Relationship groups are also referred to as fuzzy logical relationship groups (FLRG). Suppose there are relationships such that:

$A_i \rightarrow A_{j1},$

$A_I \rightarrow A_{j2},$

...

$A_i \rightarrow A_{jn}$

then they can be grouped into a relationship group as follows:

$A_i \rightarrow A_{j1}, A_{j2} \rightarrow A_{jn}$

The same fuzzy set cannot appear more than once on the right hand side of the relationship group.

In summary, the fuzzy based approach consists of three steps:

I. The first step is the fuzzification of observations.

II. In the second step, IF-THEN fuzzy rules and relationships are established.

III. Finally, the results are defuzzified into crisp value.

Most important aspect for any fuzzy based model is right selection of fuzzy sets and membership function, realistic fuzzy inference rules and right selection of defuzzification tool (Shah, 2012).

2.2.3 Re-Partitioning Discretization (RPD) Approach

Ever since Huarng (2001a) demonstrated that the choice of interval lengths affected the accuracy of prediction, the choice of interval lengths had become a critical issue in any fuzzy time series model, including the number of interval lengths (Mua'zu and Adeola, 2008). The optimization of the interval lengths to unequal interval and the universe of discourse are also important issue since they affect the accuracy of the prediction. Clustering, an unsupervised classification technique, is an optimization tool that classifies data, items etc into groups or clusters. However, clustering is largely data driven and so a clustering method that works for a particular dataset may not work for another dataset. Re-Partitioning Discretization (RPD) approach has been proposed (Singh and Borah, 2013), which optimized both the interval lengths and the universe of discourse. The work demonstrated that the RPD approach optimized the universe of discourse and the interval lengths effectively. Therefore, in this work, the RPD approach is adopted. The main steps involved in this approach include:

Step 1. Compute range (R) of sample S:

$$R = Max_{value} - Min_{value} \tag{0.8}$$

Step 2. Split the data R into M equally spaced classes:

$$M = 1 + \log_2{}^{n} \tag{0.9}$$

Where n is the size of the sample S.

Step 3. Obtain the width of an interval H:

$$H = \frac{R}{M} \tag{0.10}$$

Step 4.define the universe of discourse U of the sample S:

$$U = [l_b, u_b] \tag{0.11}$$

Where $l_b = Min_{value} - H \ and \ u_b = Max_{value} + H$.

Step 5. Compute the mid-point U_{mid} of the universe of discourse U as:

$$U = \frac{l_b + u_b}{2} \tag{0.12}$$

Step 6. Find the subsets of sample S such that:

$$A = \{x \in S \mid x \le U_{mid}\} \tag{0.13}$$

$$B = \{x \in S \mid x \ge U_{mid}\} \tag{0.14}$$

Step 7. Determine the deciding factors for $A \ and \ B$ as:

$$D_{FA} = \frac{A_{\max} - A_{\min}}{N_A} \tag{0.15}$$

$$D_{FB} = \frac{B_{\max} - B_{\min}}{N_B} \tag{0.16}$$

Where $A_{min}, A_{max} \ and \ B_{min}, B_{max}$ are the boundaries of $A \ and \ B$ respectively.

Step 8. Define the sub-boundaries of $A \ and \ B$ as:

$$U_A = [A_{\min}, A_{\max}] \tag{0.17}$$

$$U_B = [B_{\min}, B_{\max}] \tag{0.18}$$

Step 9. Partition the sub-boundaries U_A and U_B into different length intervals:

$$u_i = \left[L(i), U(i)\right], i = 1, 2, 3, \ldots; 1 \leq U(i) < A_{\max}; u_i \in U_A \tag{0.19}$$

Where:

$L(i) = A_{\min} + (i-1) \times D_{FA}$ and $U(i) = A_{\min} + i \times D_{FA}$, and

$$v_i = \left[M(i), V(i)\right], i = 1, 2, 3, \ldots,; 1 \leq V(i) < B_{\max}; v_i \in U_B \tag{0.20}$$

Also,

$M(i) = B_{\min} + (i-1) \times D_{FB}$ and $V(i) = B_{\min} + i \times D_{FB}$, (Singh and Borah, 2013).

2.2.4 The Order of Difference (OD)

This is simply the difference between the successive values in the time series. The order of the difference depends on the number of times it is differenced. This differencing is basically to identify the hidden pattern or trend in the series. In the linear models, this differencing is also used to make the time series stationary, that is, to have constant mean, variance and autocorrelation through time (Box and Jenkins, 1976). In FTS, the order of difference is used to identify and map out the trends. Mathematically, the trend(s) are computed from the difference between the successive values in the series. The first and second order difference is computed as follows (Shah, 2012):

First Order Difference:

$$\Delta(t-1) = \{f(t-1) - f(t-2)\} \tag{0.21}$$

For all t. Set first level fuzzy rule to identify change: IF $\Delta(t-1) > 0$ THEN change is positive ELSEIF $\Delta(t-1) < 0$ THEN change is negative ELSE constant change.

Second Order Difference:

$$\Delta^2(t-1) = \Delta(t-1) - \Delta(t-2) \tag{0.22}$$

For all $t = 1,2, \dots, n-2$. IF $\Delta^2(t-1) > 0$ THEN trend is upward ELSEIF

$\Delta(t-1) < 0$ THEN trend is downward ELSE trend is constant.

In this proposed model, the order of difference is extended to fourth-order.

Third Order Difference:

$$\Delta^3(t-1) = \Delta^2(t-1) - \Delta^2(t-2) \tag{0.23}$$

For all $t = 1,2,3, \dots, n-3$. IF $\Delta^3(t-1) > 0$ THEN trend is high ELSEIF

$\Delta(t-1) < 0$ THEN trend is low ELSE trend is constant.

Fourth Order Difference:

$$\Delta^4(t-1) = \Delta^3(t-1) - \Delta^3(t-2) \tag{0.24}$$

For all $t = 1,2,3, \dots, n-3$. IF $\Delta^4(t-1) > 0$ THEN trend is high ELSEIF

$\Delta(t-1) < 0$ THEN trend is low ELSE trend is constant.

2.3 Review of Similar Works

One major goal in analyzing any time series event or data is to forecast future values of the time series variable. Time series forecasting problems are being solved using a class of statistical linear autoregressive (AR), moving average (MA), hybrid (ARMA) models and their subsequent extensions. Popular among these conventional techniques is the methodology developed by Box and Jenkins (1976) called auto regressive integrated moving average (ARIMA) that analyzed time series data and then generated forecasts. The method was a univariate approach that involved differencing the input series to make it stationary, and then the parameters were estimated using function minimization procedures. The estimated parameters were then used in the forecasting. The method produced relatively satisfactory results, but it involved a great deal of linearity assumptions and affected the accuracy greatly since time series events are non-linear in nature. Also, the method is not easy to use because it is a complex technique, and the results obtained depended largely on the researcher's level of expertise. Another setback of this method and other traditional time series approaches is that when the given datum is little or in linguistic terms, the statistical methods fail (Song and Chissom, 1993b; Song and Chissom, 1993a). The problem was to develop a non-linear technique which would analyze time series events, in numeric or linguistic terms, to generate forecasts.

Inspired by the great work of Zadeh (1965), Song and Chissom (1993a, 1993b) introduced the concept of fuzzy time series in a series of papers to forecast student enrollments at the University of Alabama. They proposed a new forecasting framework, a time-variant model and time-invariant model based on fuzzy set theory. Max-min and min-max fuzzy operations were used to construct the fuzzy relations in the two models respectively, and showed conclusively that their results were better than those obtained using linear regression methods. However, the basic limitation of the FTS models developed by Song and Chissom (1993a,1993b) was that they were associated with high computational overheads due to complex matrix operations involved in both max-min and min-max operations. Also, the defuzzification processes in the model required many calculations.

Chen (1996) proposed a fuzzy time series method to increase the accuracy of Song and Chissom's work, with emphasis on reducing the computational overheads of the time-variant and time-invariant models. The methodology employed used a simplified model including only simple arithmetic operations and introduced a method called Fuzzy Logic Relationship Groups (FLRG). Although the work improved significantly on the pioneer work of Song and Chissom, however, it did not address many issues such as interval length optimization, high order fuzzy relations, trend mapping and identification, which are vital to fuzzy time series prediction accuracy. The basic steps of FTS model namely: fuzzification, derivation of fuzzy logical relationships and defuzzification all play essential role in forecasting performance.

The challenge to the research community had been the choice and effect of Universe of Discourse, Membership Function, Interval Length, Fuzzification, Fuzzy relationships and Defuzzification type on the prediction accuracy. Huarng showed that the accuracy of prediction depended on the manipulation and hence the choice of interval lengths (Huarng, 2001b; Huarng, 2001a). Huanrg adjusted Chen's model by proposing an effective length of interval which was based on frequency density of the distribution and average based distribution correspondingly. The effectiveness and superiority of the model over the conventional model showed itself as many researchers adopted it in their various models to successfully predict times series data in various domains (Chen and Hsu, 2004; Shah, 2007; Jilani *et al.*, 2008; Jilani and Burney, 2008; Stevenson and Porter, 2009; Shah, 2012). In a similar vein, Mua'azu and Adeola demonstrated in their work that odd number interval lengths produced better results than even number interval lengths (Mua'zu and Adeola, 2008). Many researchers (Shah, 2007; Jilani *et al.*, 2008; Jasim *et al.*, 2012; Shah, 2012) have also successfully used odd number interval lengths. Effective as these works were, one of the limitations of their studies was that the interval length was not optimized and the selection of the interval lengths and universe of discourse was largely subjective. To optimize the interval length, researchers (Chang *et al.*, 2009; Abd Elaal *et al.*, 2010; Chen and Tanuwijaya, 2011; Alpaslan *et al.*, 2012) have used various clustering techniques to effectively partition the interval lengths to unequal interval lengths. However, clustering is highly data sensitive and its validity is difficult to ascertain as most clustering methods are dependent on the specific applications (Khalilian and Mustapha, 2010). Also, its optimizing capability has been questioned as it can only guarantee a local optimal solution, but not global solution (Ghaemi *et al.*, 2011). Recently, a new optimizing approach has been proposed namely, Re-Partitioning Discretization (RPD) approach (Singh and Borah, 2013), which successfully optimized both the interval lengths and the universe of discourse of time series with different structural characteristics. All of these studies produced better prediction accuracies than the traditional ones which optimized neither the interval lengths nor the universe of discourse.

In the meantime, a high order fuzzy time series models have been proposed (Tsai and Wu, 2000; Chen, 2002). Both argued and pointed out that a time series event did not just depend on the preceding value but also on the previous observations. Their forecast results predicted better than the first order fuzzy time series model and hence justified their claims. Subsequently, many researchers successfully used high order fuzzy time series model to improve forecasting in different domains (Huarng and Yu, 2003; Aladag *et al.*, 2009; Singh and Borah, 2013). But one major setback in high order fuzzy time series model is its complexity, and so one of the studies included a model that used a feed-forward neural network to define fuzzy relations in order to reduce the complexities involved (Aladag *et al.*, 2009).

Meanwhile, as the research in fuzzy time series evolved, some researchers developed hybrid models to optimize two or more of the processes in FTS. A hybrid model with two advance methods has been developed (Teoh *et al.*, 2008), namely: Cumulative Probability Distribution Approach (CPDA) and Rough Set Rule Induction to forecast stock markets. A hybrid method of forecasting based on fuzzy time series and Intuitionistic Fuzzy Sets (IFS) has also been presented (Joshi and Kumar, 2012). Their model used the degree of nondeterminacy to establish fuzzy logical relations on time series data. Trapezoidal Fuzzification Approach (TFA) and Particle Swarm Optimization (PSO) techniques have been successfully applied to optimize the fuzzification and defuzzification processes respectively (Poulsen, 2009; Eleruja *et al.*, 2012). Their models utilizes aggregation and particle swarm optimization to reduce the mismatch between forecasts and actual.

Because the pioneering work of Song and Chissom (1993a, 1993b) used the historical data of enrolments for the University of Alabama to test their model, most of these literatures used the University enrolments as their forecasting targets. Others used various domains such as temperature, Internet traffic, voice traffic, video traffic, stock exchange, sales volume, Gross Domestic Capital, road traffic etc. All these papers have improved accuracy in their various degrees.

However, one critical factor in all time series prediction problems had been the proper identification and mapping out of its trends since it has been observed that trend identification provide essential information concerning different forecast specifications and permits making predictions about future occurrences (Ding *et al.*, 1996; Rosenberg, 1997). The problem then is the development of effective time series model that will effectively map out the trends and then use it in forecasting. In the past decade, many models have been developed to exploit these advantages.

Huarng (2001b) proposed a novel method that employed a heuristic fuzzy logic relationship group. The first Order Difference between successive data was used to form a two-variable heuristic trend and was extended to a three-variable trend by introducing a threshold. The heuristic trends were up or down depending on whether the difference is positive or negative. The fuzzy logic relationship groups formed from these trends were used for forecasting of Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX). The results showed better performance compared to existing models and that three-variable heuristic model forecasted better than a two-variable heuristic trend. However, this model will not work in time series data that represents a growth pattern since the entire heuristic trend will be upward and therefore redundant.

Shahida *et al* (2003) presented an approach to modeling and identification of dynamic systems based on fuzzy logic and applied it to time series prediction. The methodology used was identification based on table-lookup scheme to form a system model using a description language based on fuzzy logic with fuzzy predicates. The task was to generate set of fuzzy if-then rules from the desired input-output pairing which involved a five-step table look-up identification scheme. Simulation results showed that the generated fuzzy model system is capable of approximating non linear continuous function, and that prediction can be greatly improved by dividing the domain interval into finer regions. The method is simple as it just performed one-pass operation on the training data, but a great price paid for this was that the partitioning of the domain intervals and the selection of membership functions were determined in overly ad hoc manner, and this makes their approach too simplistic for time series prediction problems.

Chen and Hsu (2004) proposed a first order time variant fuzzy time series model. The method utilized the effective length of interval introduced by Huarng (2001a). The trend of the forecasting was determined through the Order of Difference between successive values. The first and third quartile points of the interval corresponding to the fuzzified enrollment were used to forecast downward and upward trend respectively. The results improved the existing works, but the model is deficient in twofold: first, the interval length was not optimized and second, it limited the fuzzy relationships to first order only.

However, **Yu (2005)** developed a fuzzy time series model that assigned trend weights to fuzzy logical relationships to account for reoccurrence and chronology. Yu (2005) argued that fuzzy logical relationships that repeated themselves should not be ignored, as was the case in previous studies. The reoccurrence represented a kind of trend and was accounted for by assigning increasing weights to fuzzy logic group which was arranged in chronological order. The model outperformed previous ones that did not consider the trend in the fuzzy logical relationship groups. But, the weights were chosen arbitrarily with no explanation and the work used the first order fuzzy relations only, this is a disadvantage for time series events.

In a similar vein, **Sah and Konstantin (2005)** introduced a new (modified) time-invariant method to forecast university enrolment. Unlike the conventional time-invariant method proposed by Song-Chissom and Chen, the method utilized variational trends of the available historical data as fuzzy time series instead of direct usage of numeric values. The first Order Difference between successive data was used to determine variations. The model also investigated the effect of change in the number of fuzzy sets on forecast accuracy. The average forecasting error of the model was less than that of Song-Chissom and Chen’s model. Their work demonstrated that the average forecasting error decreased with the number of fuzzy sets. But the trend identification was not evident in the work and the interval length was not optimized.

Jilani *et al* (2008a) developed a simple time-variant quantile based fuzzy time series forecasting method. The model was based on forecasting using prediction of future trend of the data, and the trend prediction was based on the third order fuzzy relationships. The model introduced a ‘trend’ parameter that determined the direction of the series and the trend value was computed using the last three forecasted values. Three forecast rules were then formed based on a threshold value. Simulation results showed a satisfactory performance compared with the existing models. However, the selections of the threshold for the forecast rules were done by trial and error approach and the universe of discourse and the interval length were not optimized. These issues inevitably affect the forecast results.

Subsequently, **Jilani and Burney (2008b)** modified the earlier model (Jilani *et al.*, 2008), by introducing a heuristic fuzzy metric to use the frequency based partitions of the universe of discourse. The model sub-partitioned the equal length intervals based on frequency density. A heuristic value that predicts the expected trend direction was introduced which was computed using a selected criterion of weights with a set threshold and trend predictor to calculate forecasts. The results obtained showed improvement in their earlier work and other existing models, but nevertheless still used equal length intervals and the threshold while also the weights were selected arbitrarily.

In a similar vein, **Lee *et al* (2009)** proposed a model that improved the earlier method of trend weghts (Yu, 2005). Their work was based on the development of weights on FTS with a collection of fuzzy logical relations (FLRS) in fuzzy logic group (FLG) arranged chronologically. Unlike Yu's (2005) model, the weights assigned were not assumed to be increasing gradually and model also reversed the transpose matrix elements on forecast rules. Simulation results showed greater accuracy compared to Yu's (2005) work, but also shared some of the weaknesses of the work. Also, the model involved complex matrix operations to implement forecast rules.

Jasim *et al* (2012) introduced a novel algorithm to forecast enrolment for the University of Alabama. The proposed algorithm used average based length method to partition the dataset and used first order fuzzy relations. The method used both first Order Difference and second Order Difference to map out the trend which will either go up or down. Forecast rules, with a threshold, were formed along with the trend to predict the University of Alabama enrolment. A weight of 0.25 and 0.75 were given to downward and upward trend respectively. The proposed model presented better forecast results compared to existing models. However, like most existing models, the partitioning of the interval length was not optimized and the model did not map out the general trend implicit in the series nor capture the point of transition.

Then, **Shah (2012)** proposed a new fuzzy logic based approach to capture the trend of a time series as well as the point of transition and use same in prediction. The methodology used the second Order of Difference (OoD) between consecutive data to map out the trends inherent in the series. The second Order Difference (OoD) was used to capture the trend in the time series which is either upward or downward trend. The result showed a good accuracy of prediction and is closer to real life data since the proposed method can capture the points of transition, giving room to incorporate a subject's expert opinion to improve the prediction accuracy. However, only first and second order differences between consecutive data were used to capture the general trend of the series and the partitioning of the dataset was not optimized, as it relied on the traditional method of equal interval length partition. This will inevitably affect the accuracy of the forecast and the uncertainty in the whole trend was not captured, as it limited the trend identification process to either more increasing or less decreasing. The work did not also investigate the effect of increasing the order of difference on the accuracy of the prediction.

2.3.1 Summary

This chapter has presented the fundamental concepts in this work and a systematic review of the evolution of works in FTS with special emphases on the works that involve trend mapping and identification approach. While some literatures have unarguably developed an improved method for fuzzy time series forecasting by trend mapping and identification method, none have provided a model that effectively mapped out and identified the prevailing trend(s) and its vagueness. At best, the existing models identified the trends in two-fold only, such as upward or downward, increasing or decreasing, and the weights selected to form forecast rules were chosen arbitrarily. The problem with these approaches was that they did not really capture the vagueness implicit in the trends. For example, a positive change of 54 and 1019 were both seen as upward or increasing trend and a negative change of -110 and -22918 were both seen as downward or decreasing trend in most models. While these trends or differences in observations provided better forecasting accuracy, differences alone lacked context for which the increase or decrease occurred (Stevenson and Porter, 2009), and these approaches clearly undermined the theory of fuzzy logic as laid down by Zadeh (1965). Also, the arbitrary selection of weights and thresholds in these models make them fitted only to the time series data under study and the models adaptability to other time series data of different structural and statistical characteristics have not been demonstrated in any way. To overcome these drawbacks, this research proposes a novel FTS method based on the trend mapping approach, namely fuzzified trend mapping and identification (FTMI), which used the Re-Partitioning Discretization (RPD) approach to optimize the partitioning of the interval length. The model identified the trends and the trend values (T_V) in the time series using the Order of Difference (OoD) between successive data and high-order trend relations respectively. The identified trends were fuzzified in order to capture their fuzziness or uncertainty so that the accuracy of prediction may be improved. In addition, the effect of increasing the Order of Difference on the accuracy of the research process is also investigated in this research. The general structure of this FTS works based on the trend mapping approach and the structure of the proposed FTMI model are presented in the next chapter.

CHAPTER THREE

DEVELOPMENT OF THE NEW FTMI MODEL

3.1 Introduction

Trend mapping and identification approach in FTS is the main focus of this chapter. First, the structure of the conventional fuzzy trend mapping models is highlighted. The significant limitations of these models followed in this chapter. Then, the developed FTMI model is presented along with its structure, algorithm and the data flow diagram. How the limitations of the conventional models were overcome by this proposed model is shown briefly in the chapter's summary.

3.2 Conventional Fuzzy Trend Mapping Models

The Song and Chissom's models and other subsequent fuzzy models involved series of methods that analyse and train historical data. The training of the historical data is aimed at identifying some pattern or trend inherent in the historical data. The trend(s) identified in these models were somewhat 'hidden' in the fuzzy logical relationship (FLR) that were constructed from the trained historical data from which fuzzy inference rules were created for defuzzification and forecasting. Identifying and mapping out of 'hidden' trends in the historical data provide essential information concerning different forecast specifications and helps in improving prediction accuracy (Ding *et al.*, 1996; Rosenberg, 1997), and so many models (Huarng, 2001b; Sah and Konstantin, 2005; Jilani *et al.*, 2008; Jasim *et al.*, 2012; Shah, 2012) have been developed to exploit these advantages. The general methodology and structure of these models in deffuzification and forecasting are summarised thus:

I. Determine the trend by finding the first and/or second-order difference between successive values. Positive value indicated upward/high/increasing trend while negative value indicated downward/low/decreasing trend.

II. Construct set of rules with some threshold (chosen arbitrarily) for the upward/high/increasing trend and downward/low/decreasing trend.

III. Impose trend weights (also selected arbitrarily) on the upward/high/increasing and downward/low/decreasing trend.

IV. Use the rule along with the trend weights for defuzzification and forecasting.

Like other time series models, one of the general limitations of these models was that they are just fit to a particular time series data. The models are sensitive to the time series data under study and hence effective in the prediction of the time series data only. Even for the models that were validated on more than one time series data, the model's adaptability on the different time series data were not in any way demonstrated. The models being over fit to particular time series data is partly due to the arbitrary selection of weights or thresholds since weights that are appropriate to a time series will not be effective for another time series data of different structural and statistical characteristics.

Another limitation of these models was that they introduced a lot of 'noise' in the forecasted values. That is, the models have the same predicted values for different times. Indeed, some models (Song and Chissom, 1993a; Song and Chissom, 1994; Chen and Hsu, 2004; Sah and Konstantin, 2005; Shah, 2007; Lee *et al.*, 2009; Shah, 2012), introduced noise in the forecasted values even in data that did not have noise in the original historical data. This introduction of noise clearly undermined the performance of these models in accuracy. The trends identified in these models were only two-fold (upward/downward, high/low etc), and therefore devoid of the trend's vagueness since differences alone lacked context for which the increase or decrease occurred (Stevenson and Porter, 2009). This resulted in making the effect of the identified trend in the forecasting weak.

3.3 The New FTMI Model

A fuzzified-trend mapping and identification (FTMI) has been developed, which mapped out the trends using the order of differences and the trend values (T_V), and identified the trend's vagueness by fuzzifying it into trend classes. The trend classes are presented both in numeric and linguistic terms to reflect the true sense of fuzzy logic developed by (Zadeh, 1965). Each trend class is given an ordinal position for ease of identification during defuzzification and forecasting. A fuzzy inference rule based on the trend values (T_V) and fuzzified trend is developed without any arbitrary imposition of threshold or selection of weights. The rule is very simple, yet more comprehensive in identifying trend's uncertainty and hence more accurate in forecasting.

The new FTMI model has the following structure:

I. Determine the trend by finding the 1^{st}, 2^{nd}, 3^{rd} and 4^{th}-order difference between successive values.

II. Construct trend values (T_V) of the 1^{st}, 2^{nd}, 3^{rd} and 4^{th}-order difference using high-order fuzzy relation (third-order).

III. Fuzzify the trend values of the order of difference into trend classes. The number of the trend classes is determined by the number of the classes of the main dataset.

IV. Use model's rule along with the trend values (T_V) and its ordinal positions on the fuzzified trend for deffuzification and forecasting.

3.3.1 Algorithm of the New FTMI Model

The algorithm for the developed FTMI Model is presented as follows:

Step 1: Select the time series dataset.

Step 2: Apply the Re-Partitioning Discretization (RPD) approach presented in Section 2.5 of chapter two on the time series data to optimize both the partitioning of its interval lengths and the universe of discourse.

Step 3: Defining Fuzzy Terms

Step 3.1. Define linguistic terms for each intervals obtained in Step 2

Step 3.2 Establish fuzzy membership function and define fuzzy sets

Step 4: Fuzzify the time series data set and establish the high-order fuzzy relations (FLRs).

Third-order fuzzy relation is used based on its definition (Chen, 2002).

Step 5: Find the Order of Difference to determine the trends. This is determined using any of the equations presented in Section 2.6 of chapter two.

Step 6: Establish third-order trend relations and compute trend values (T_V).

Third-order trend relations is computed using equation of Section 3.3.2.

Step 7: Fuzzification of the trends is undertaken in this step.

The trends are fuzzified based on the number of classes of the historical dataset. The fuzzified trends are presented both in linguistic and numeric terms and each given an ordinal position for ease of identification during defuzzification and forecasting.

Step 8: Development of a fuzzy Inference System of rule based on the fuzzified trends and trend values (T_V) for defuzzification and forecasting.

The following rule is used to capture the vagueness inherent in the trend(s) for defuzzification and prediction:

If the trend's value at a time $\boldsymbol{t}$ falls at the $\boldsymbol{x_i}^{\boldsymbol{th}}$ ordinal position on the fuzzified trend, then the forecasted output (value) is the point at the $\left(\boldsymbol{x}/\boldsymbol{N_T}\right)^{\boldsymbol{th}}$ position on the corresponding fuzzified interval, where $\boldsymbol{1 \leq x \leq N_T}$, $\boldsymbol{i = 1, 2, 3, \dots, N_T}$ and $\boldsymbol{N_T}$ is the number of the fuzzified trend.

The proposed FTMI model data-flow diagram is illustrated in Figure 3.1.

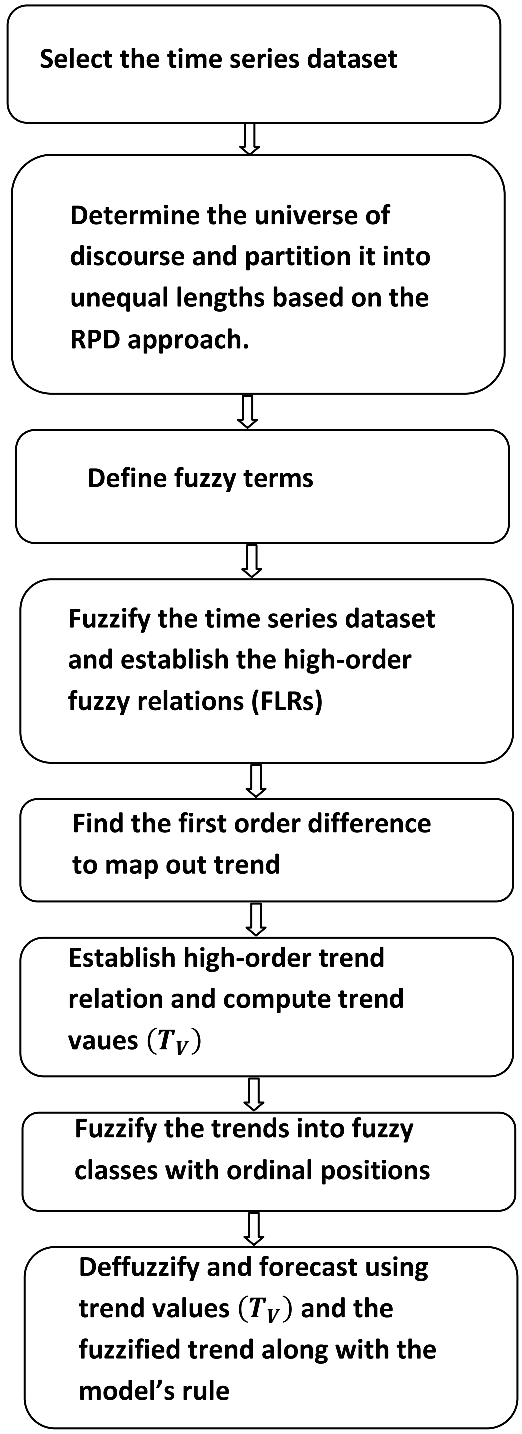

Figure 3.1 Proposed FTMI Forecasting Model

3.3.2 Trend Values (T_V) and Trend Fuzzification

In this proposed FTMI model, the trend values of each of the observation are computed using the results of the order of difference and the high-order fuzzy relations. Previous studies have only considered first order fuzzy relation in determining trend values, that is, it has always been assumed that the trend at any instant is caused by the previous value only. Since many studies have shown that high-order fuzzy relation improves forecasting regardless of the method (Tsai and Wu, 2000; Chen, 2002; Aladag *et al.*, 2009; Singh and Borah, 2013), the third-order fuzzy relation are used to compute the trend value for each observation in this model. The assumption here is that the trend at any instant is caused by the previous three values. These will be represented as third-order trend relations as follows:

$T_V(t-3), T_V(t-2), T_V(t-1) \rightarrow T_V(t)$ for all t. where $T_V(t)$ is the trend value for the current time (predicted time) and $T_V(t-3), T_V(t-2)\ and\ T_V(t-1)$ are the trend values for the three previous times in chronological order. Therefore, each trend values will be computed thus:

$$T_V(t) = \frac{T_V(t-3) + T_V(t-2) + T_V(t-1)}{3}. \qquad (0.25)$$

The trend values obtained from the order of difference are fuzzified. This unusual approach is taken for two reasons:

I. First, to capture the vagueness that is inherent in these trends so as to maximize the forecast accuracy. With this trend fuzzification, the transition from one trend value to the next is gradual rather than abrupt as represented in the other models. However, this fuzzification of the trend does not follow the normal fuzzification process in fuzzy logic; as the approach is taken chiefly to capture the vagueness inherent in the trend data. As a result, the trend classes obtained depends completely on the number of fuzzy classes of the main dataset.

II. Second, the trend values are fuzzified so that the effect of the trend on prediction may be strengthened. The 'hidden' trend values captured from the trend fuzzification are now able to participate in the forecasting process which makes the predicted values more reflective of the general data trend. Each of the trend class is given an ordinal position for ease of identification during deffuzzification and forecasting.

3.4 Summary

The structure of the conventional fuzzy trend mapping models as well as their limitations has been presented in this chapter. Among others, these limitations include over fitting of the models to a particular time series and introduction of noise to the predicted values. A new trend mapping approach namely, the FTMI model was presented in this chapter to overcome these limitations as well as improve the forecasting accuracy. This is achieved in this model by:

I. Optimizing the determination of the interval lengths.
II. Using high-order trend relation to determine the trend values.
III. Fuzzifying the identified trends both in linguistic and numeric terms.
IV. Using a new defuzzification method in FTS.

The next chapter presents both the validation and application of the proposed FTMI model.

CHAPTER FOUR

VALIDATION AND APPLICATION OF THE NEW FTMI MODEL

4.1 Introduction

This chapter presents the validation of this proposed FTMI model with the pioneer and existing FTS models and its application on a new data with regard to tackling time series problems. The areas of validation considered here are the forecasting accuracy and the model adaptability to different time series structures. To demonstrate this, three time series data of different sources and structure, including the standard data of students' enrolment of Alabama, and variance statistics are used. Later in this chapter, the effect of increasing the order of difference on forecast accuracy and the application of the proposed FTMI model on internet traffic of ABU, Zaria are demonstrated.

4.2 Description of the Time Series Datasets

The first time series data considered in this research is the standard data of the students' enrolment for the University of Alabama from 1971 to 1992. This data is shown in Table 4.1. This data was the forecast target of Song and Chissom's (1994) model and also the forecast target of many subsequent FTS models, hence the choice of this data for validation of this model.

Table 4.1 Enrolments of the Alabama University (Song and Chissom, 1994)

Year	Enrolment	Year	Enrolment
1971	13055	1982	15433
1972	13563	1983	15497
1973	13867	1984	15145
1974	14696	1985	15163
1975	15460	1986	15984
1976	15311	1987	16859
1977	15603	1988	18150
1978	15861	1989	18970
1979	16807	1990	19328
1980	16919	1991	19337
1981	16388	1992	18876

The second data considered is the Sales Volume of Propylene Company. Table 4.2 shows the Sales Volume of product for a Propylene Manufacturing Company from the month of January to December. This type of time series data has high variability with unpredictable outliers and the previous years' pattern are not always shown.

Table 4.2 Sales Volume of Propylene Company (Shah, 2012).

Months	Actual Sales Volume
Jan.	26658
Feb.	29216
March	30035
April	30846
May	29171
June	21068
July	28416
Aug.	25651
Sept.	22122
Oct.	26423
Nov.	22467
Dec.	23274

Finally, the third data considered is the Gross Domestic Capital of India is shown in Table 4.3. This type of time series represents an increasing growth pattern that rarely returns to the same fuzzy classes unlike the two previous data. Statistically, the trend of this data is always on the increase, with some remarkable uncertainty.

Table 4.3 Gross Domestic Capital of India (Shah, 2012).

Year	Actual Value	Year	Actual Value
1981	12105	1992	57633
1982	16986	1993	63977
1983	20139	1994	70834
1984	21265	1995	88206
1985	25600	1996	90977
1986	29990	1997	96187
1987	34772	1998	100653
1988	33757	1999	114545
1989	40136	2000	134484
1990	46405	2001	131505
1991	53099		

The practical example of applying the FTMI model on these time series is as follows:

Using the algorithm of the developed FTMI model presented in Section 3.3.1 of chapter three, the new FTMI model is applied in Students' enrolment data as follows:

Step 1: The Students Enrolment data in Table 4.1 is selected.

Step2: The interval length is optimized using the Re-Partitioning Discretization (RPD) approach.

Step 2.1 Computation of range (R) of the sample $S = \{x_1, x_2, \dots, x_n\}$

The maximum and minimum value for the data set S from Table 4.1 are 19337 and 13055.

Using equation (1) above:

$$R = 19337 - 13055 = 6282.$$

Step 2.2 Split the data range R into M equally spaced classes, from Equation (0.9):

$$M = 1 + \log_2^{22} = 1 + 4.46 \cong 5.$$

Step 2.3 Obtain the width of an interval H by using Equation (0.10):

$$H = \frac{6282}{5} = 1256.$$

Step 2.4 Define the universe of discourse U of the sample S , obtained from Equation (0.11) as:

$$U = [13055 - 1256, 19337 + 1256] = [11799, 20593].$$

Step 2.5 Compute the midpoint U_{mid} of the universe of discourse U , using Equation (0.12) as

$$U_{mid} = \frac{11799 + 20593}{2} = 16196.$$

Step 2.6 Find the subsets of the sample S (A and B), using Equations (0.13) and (0.14) as:

$$A = \{13055, 13563, 13867, 14696, 15145, 15163, 15311, 15433, 15460, 15497, 15603, 15861, 15984\}$$
$$B = \{16388, 16807, 16859, 16919, 18150, 18876, 18970, 19328, 19337\}$$

Step 2.7 Define sub-boundaries for (A and B), based on Equations (0.17) and (0.18) as:

$$U_A = [13055, 15984].$$

$$U_B = [16388, 19337].$$

Step 2.8 Determine the deciding factors for (A and B). From Equations (0.15) and (0.16):

$$D_{FA} = \frac{15984 - 13055}{13} = 225.31 \cong 226.$$

$$D_{FB} = \frac{19337 - 16388}{9} = 327.67 \cong 328.$$

Step 2.9 Partition the sub-boundaries U_A and U_B into different length intervals based on equations (0.19) and (0.20) as:

$$u_{a_1} = [13055, 13281], u_{a_2} = [13281, 13507], u_{a_3} = [13507, 13733], ..., u_{a_{13}} = [15767, 15993].$$

$$u_{b_1} = [16388, 16716], u_{b_2} = [16716, 17044], u_{b_3} = [17044, 17372], ..., u_{b_9} = [19012, 19340].$$

Step 3: Defining Fuzzy Terms

Step 3.1to 3.2 Linguistic Terms for the Intervals, membership function and fuzzy sets are computed in this step. The linguistic terms are defined for intervals obtained in step 2.9 of the RPD approach in Section 3. Assuming that the historical data is distributed among the intervals of Section 3 (step 2.9) i.e. $u_1, u_2, u_3, ..., u_n$. Therefore, n linguistic variables $R_1, R_2, R_3, ..., R_n$ are defined in this model. It is assumed that the linguistic variable $students\ enrolment$ can take the fuzzy values $R_1, R_2, R_3, ..., R_n$ .Using triangular membership function with $[0, 0.5, 1]$ as membership, each of these fuzzy sets are defined as follows:

$$R_1 = \frac{1}{u_1} + \frac{0.5}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \cdots + \frac{0}{u_{11}} + \frac{0}{u_{12}} + \frac{0}{u_{13}}$$

$$R_2 = \frac{0 \cdot 5}{u_1} + \frac{1}{u_2} + \frac{0 \cdot 5}{u_3} + \frac{0}{u_4} + \cdots + \frac{0}{u_{11}} + \frac{0}{u_{12}} + \frac{0}{u_{13}}$$

$$R_3 = \frac{0}{u_1} + \frac{0 \cdot 5}{u_2} + \frac{1}{u_3} + \frac{0 \cdot 5}{u_4} + \cdots + \frac{0}{u_{11}} + \frac{0}{u_{12}} + \frac{0}{u_{13}}$$

$$\vdots \qquad \qquad \vdots$$

$$R_{13} = \frac{0}{u_1} + \frac{0}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \cdots + \frac{0}{u_{11}} + \frac{0 \cdot 5}{u_{12}} + \frac{1}{u_{13}}$$

Step 4 Fuzzification of Data Set and High-order Fuzzy Relations

Before fuzzification of the data set, each of the elements in the historical data will be allocated to the interval where it falls. This implies that the elements of A, B will be assigned to their corresponding intervals obtained after partitioning the sub-boundaries U_A, U_B. These intervals along with their corresponding elements are shown in Table 1. Note that the intervals which do not cover the historical data are removed from the list.

After, the degree of membership of each year's enrolment value belonging to each R_i are obtained, i.e. where $i = 1, 2, 3, \dots, n$. Here the maximum degree of membership of fuzzy set R_i occurs at interval u_i and $1 \leq i \leq n$. To fuzzify the historical data, if one year's datum belong to the interval u_i, then it is fuzzified into R_i. That is, if the enrolment of year t is $x \epsilon u_i$, and there is a value represented by a fuzzy set R_i with maximum membership value occurring on u_i, then x is fuzzified as R_i. The results are shown in Table 4.4 for the fuzzified enrolment.

Table 4.4 Intervals with Corresponding Elements and the Fuzzified Enrolments

Interval for U_A	*Corresponding Element(s)*	*Fuzzified Enrolment*
[13055,13281]	(13055)	R_1
[13507,13733]	(13563)	R_2
[13733,13959]	(13867)	R_3
[14637,14863]	(14696)	R_4
[15089,15315]	(15145,15163,15311)	R_5
[15315,15541]	(15433,15497,15460)	R_6
[15541,15767]	(15603)	R_7
[15767,15993]	(15861,15984)	R_8
Interval for U_B	*Corresponding Element(s)*	*Fuzzified Enrolment*
[16388,16716]	(16388)	R_9
[16716,17044]	(16807,16859,16919)	R_{10}
[18028,18356]	(18150)	R_{11}
[18684,19012]	(18876,18970)	R_{12}
[19012,19340]	(19328,19337)	R_{13}

High-Order Fuzzy Relations

Based on the definition of a high-order fuzzy time series, a third-order fuzzy relation has been established. For example, the fuzzified yearly enrolment values for years 1971, 1972 and 1973 are $R_1, R_2, and\ R_3$ respectively. Here, to establish the third-order FLR among these fuzzified values, it is considered that R_4 is caused by the previous three fuzzified values $R_1, R_2, and\ R_3$. Hence, the third-order FLR is represented in the following form:

$$R_1, R_2, R_3 \rightarrow R_4.$$

Table 4.5 shows all the years' enrollment along with the third order fuzzy relation.

Table 4.5 University of Alabama Students' Enrolment Showing High-Order Fuzzy Relations

Year	Enrolment	Fuzzified Enrolment	3rd-Order Fuzzy Relation
1971	13055	R_1	
1972	13563	R_2	
1973	13867	R_3	
1974	14696	R_4	$R_1, R_2, R_3 \rightarrow R_4$
1975	15460	R_6	$R_2, R_3, R_4 \rightarrow R_6$
1976	15311	R_5	$R_3, R_4, R_6, \rightarrow R_5$
1977	15603	R_7	$R_4, R_6, R_5 \rightarrow R_7$
1978	15861	R_8	$R_6, R_5, R_7 \rightarrow R_8$
1979	16807	R_{10}	$R_5, R_7, R_8 \rightarrow R_{10}$
1980	16919	R_{10}	$R_7, R_8, R_{10} \rightarrow R_{10}$
1981	16388	R_9	$R_8, R_{10}, R_{10} \rightarrow R_9$
1982	15433	R_6	$R_{10}, R_{10}, R_9 \rightarrow R_6$
1983	15497	R_6	$R_{10}, R_9, R_6 \rightarrow R_6$
1984	15145	R_5	$R_9, R_6, R_6 \rightarrow R_5$
1985	15163	R_5	$R_6, R_6, R_5 \rightarrow R_5$
1986	15984	R_8	$R_6, R_5, R_5 \rightarrow R_8$
1987	16859	R_{10}	$R_5, R_5, R_8 \rightarrow R_{10}$
1988	18150	R_{11}	$R_5, R_8, R_{10} \rightarrow R_{11}$
1989	18970	R_{12}	$R_8, R_{10}, R_{11} \rightarrow R_{12}$
1990	19328	R_{13}	$R_{10}, R_{11}, R_{12} \rightarrow R_{13}$
1991	19337	R_{13}	$R_{11}, R_{12}, R_{13} \rightarrow R_{13}$
1992	18876	R_{12}	$R_{12}, R_{13}, R_{13} \rightarrow R_{12}$

Step 5: Find the Order of Difference

Using Equation (0.21), the first order difference between the successive values is calculated to determine the trends and they are shown in Table 4.6.

Step 6 High-order Trend relations and Computation of Trend Values (T_V)

The trend relations and trend values for each time observation will be computed using the results of the high-order fuzzy relations and the order of difference correspondingly. The third-order trend relation is used to compute the trend value for each of enrolment year in this model. For instance, to compute the trend value for year 1976 (considering the first order difference), the order of difference values for year 1973, 1974 and 1975 are 304, 829 and 764 respectively. This is computed as follows:

$$T_V(1976) = \frac{304 + 829 + 764}{3} = \frac{1897}{3} \cong 632.$$

Using equation (0.25), trend values are computed for first-order difference. Table 4.6 shows the first-order differences along with their trend values (T_{V_1}). Note that only two values (508 and 304) were used to compute for year 1974.

Table 4.6: First-Order Difference with their Corresponding Trend Values (T_V)

Year	**Enrolment**	**1st Order Difference**	T_{V_1}
1971	13055		
1972	13563	508	
1973	13867	304	
1974	14696	829	406
1975	15460	764	525
1976	15311	-149	632
1977	15603	292	481
1978	15861	258	302
1979	16807	946	134
1980	16919	112	499
1981	16388	-531	439
1982	15433	-955	176
1983	15497	64	-458
1984	15145	-352	-474
1985	15163	18	-414
1986	15984	821	90
1987	16859	875	162
1988	18150	1291	571
1989	18970	820	996
1990	19328	358	995
1991	19337	9	823
1992	18876	-461	396

Step 7 Fuzzification of the Trend

In this proposed FTMI model, the number of the fuzzy classes of the main dataset determines the number of the fuzzified trend. Therefore, since the number of fuzzy classes of main dataset is thirteen (13) as in Table 4.4, the trend values are fuzzified into thirteen classes. Each of the classes is given an ordinal position for ease of identification during deffuzzification and forecasting. From the first order difference, the highest trend value is 1291 while the lowest trend value is -955. The trend is fuzzified as shown in Table 4.7:

Table 4.7: Fuzzification of the Trend and their Ordinal Positions

Trend Classes	*Trend Intervals*	*Ordinal Positions*
Exceptionally Low Trend	$-955\ to -782$	1^{st}
Extremely Low Trend	$-782\ to -609$	2^{nd}
Extra Low Trend	$-609\ to -436$	3^{rd}
Very Very Low Trend	$-436\ to -263$	4^{th}
Very Low Trend	$-263\ to -90$	5^{th}
Low Trend	$-90\ to\ 83$	6^{th}
Medium Trend	$83\ to\ 256$	7^{th}
High Trend	$256\ to\ 429$	8^{th}
Very High Trend	$429\ to\ 602$	9^{th}
Very Very High Trend	$602\ to\ 775$	10^{th}
Extra High Trend	$775\ to\ 948$	11^{th}
Extremely High Trend	$948\ to\ 1121$	12^{th}
Exceptionally High Trend	$1121\ to\ 1294$	13^{th}

Step 8 Defuzzification and forecasting

The proposed model's rule is now applied to forecast the standard data of students' enrolment in the University of Alabama. In this case, the number of the fuzzified trend is thirteen (13).

To predict for year 1975, the trend value (T_{V_1}) from third-order fuzzy relation is 525 as in Table 4.6. This falls on the 9^{th} position (Very High Trend) of the fuzzified trend in Table 4. The predicted value is then at $\left(9/13\right)^{th}$ position of the corresponding fuzzified interval. The fuzzified interval is $[15315 - 15541]$, as in Table 4.4. That is:

$$\left(9/13\right)^{th}\ position\ on\ [15315 - 15541]$$

$$\Rightarrow 15315 + \left(\frac{9}{13} \times 226\right) = 15315 + 156 \cdot 46 \cong 15471.$$

Similarly, to predict for year 1984, the trend value is -474, which falls on the 3rd position (Extra Low Trend) of the fuzzified trend. Therefore, the predicted value is at $\left({}^{3}/_{13}\right)^{th}$ position on the fuzzified interval $(15089 - 15315)$. That is:

$$\Rightarrow 15089 + \left(\frac{3}{13} \times 226\right) = 15089 + 52 \cdot 15 \cong 15141\,.$$

To predict for 1989, the trend value is 996, which falls on the12th position (Extra High Trend) of the fuzzified trend. Therefore, the predicted value is at $\left({}^{12}/_{13}\right)^{th}$ position on the fuzzified interval $(18684 - 19012)$. That is:

$$\Rightarrow 18684 + \left(\frac{12}{13} \times 328\right) = 18684 + 302 \cdot 77 \cong 18987$$, and so on.

Sales Volume of Propylene Dataset

The FTMI model is applied on the Sales Volume dataset of Table 4.2 for prediction. Using the algorithm of this model, the processes is as follows:

Steps 1-2: The Re-Partitioning Discretisation (RPD) approach is applied on the Sales Volume dataset to partition it into unequal interval. Applying steps 1-6 of the RPD approach of Section 2.4, the elements of subsets $A\ and\ B$ are:

$A = \{21068, 22122, 22467, 23274, 25651\}$

$B = \{26423, 26658, 28416, 29171, 29216, 30035, 30846\}$

Applying steps 7-9 of the RPD approach of Section 2.4, the following intervals are gotten for the sub-boundaries $U_A\ and\ U_B$:

Intervals for U_A:

$$u_1 = [21068, 21985], u_2 = [21985, 22902], u_3 = [22902, 23819], ..., u_5 = [24736, 25653]$$

Intervals for U_B:

$$v_1 = [26423, 27055], v_2 = [27055, 27687], v_3 = [27687, 28319], ..., v_7 = [30215, 30847]$$

Each elements of the historical Dataset are allocated to the intervals where it falls, discarding the intervals that do not cover the historical dataset.

Steps 3-6: Applying Steps 3-6 of the FTMI algorithm, Tables 4.8 and 4.9 are computed.

Table 4.8 Intervals of Sales Volume with their Corresponding Elements

$Interval\ for\ U_A$	$Corresponding\ Element(s)$	$Fuzzified\ Sales$
[21068,21985]	(21068)	R_1
[21985,22902]	(22122, 22467)	R_2
[22902,23819]	(23274)	R_3
[24736,25653]	(25651)	R_4
$Interval\ for\ U_B$	$Corresponding\ Element(s)$	$Fuzzified\ Sales$
[26423,27055]	(26423, 26658)	R_5
[28319,28951]	(28416)	R_6
[28951,29583]	(29171, 29216)	R_7
[29583,30215]	(30035)	R_8
[30215,30847]	(30846)	R_9

The third-order fuzzy relation, order of difference and trend values follow the same step as was the case in the previous dataset (students' enrollment). Table 4.9 shows the results of the computations.

Table 4.9 Third-Order Fuzzy Relations, First-Order Difference and Trend Values

Month	Sales Volume	Fuzzified Sales	**3rd-order Relation**	1st Order Difference	T_{V_1}
Jan.	26658	R_5			
Feb.	29216	R_7		2558	
March	30035	R_8		819	
April	30846	R_9	$R_5, R_7, R_8 \rightarrow R_9$	811	1689
May	29171	R_7	$R_7, R_8, R_9 \rightarrow R_7$	-1675	1396
June	21068	R_1	$R_8, R_9, R_7 \rightarrow R_1$	-8103	-15
July	28416	R_6	$R_9, R_7, R_1 \rightarrow R_6$	7348	-2985
Aug.	25651	R_4	$R_7, R_1, R_6 \rightarrow R_4$	-2765	-810
Sept.	22122	R_2	$R_1, R_6, R_4 \rightarrow R_2$	-3529	-1173
Oct.	26423	R_5	$R_6, R_4, R_2 \rightarrow R_5$	4301	351
Nov.	22467	R_2	$R_4, R_2, R_5 \rightarrow R_2$	-3956	-664
Dec.	23274	R_3	$R_2, R_5, R_2 \rightarrow R_3$	807	-1061

Step 7: Fuzzification of the trends

The trend values (1st-order difference) are then fuzzified to capture its vagueness. Since the number of fuzzy classes of the main Dataset is nine (9), the trend values are fuzzified into nine classes. From the first-order difference, -8103 and 7348 are the least and highest trend values respectively. They are fuzzified as in Table 4.10:

Table 4.10 Trend intervals of Sales Volume with their Ordinal Positions

Trend Classes	*Trend Intervals*	*Ordinal Positions*
Extra Low Trend	$-8103\ to -6386$	1st
Very Very Low Trend	$-6386\ to -4669$	2nd
Very Low Trend	$-4669\ to -2952$	3rd
Low Trend	$-2952\ to -1235$	4th
Medium Trend	$-1235\ to\ 482$	5th
High Trend	$482\ to\ 2199$	6th
Very High Trend	$2199\ to\ 3916$	7th
Very Very High Trend	$3916\ to\ 5633$	8th
Extra High Trend	$5633\ to\ 7350$	9th

Step 8: Defuzzification and Forecasting

The proposed algorithm for defuzzification and forecasting rule in Section 6.9 is used to predict the sales volume. Observe that here the number of fuzzified trend is nine (9), as in Table 4.8. Each month sales' volume is predicted thus:

To predict for April, the trend value is 1689 which falls on the 6th position (High Trend) of the trend class; the predicted value is then at the $\left(6/_9\right)^{th}$ position on the corresponding fuzzified interval $[30215 - 30847]$, as in Table 4.8.

The forecast value is thus:

$$30215 + \left(\frac{6}{9} \times [30847 - 30215]\right) = 30215 + \left(\frac{6}{9} \times 632\right)$$
$$\Rightarrow 30215 + 421 \cdot 33 \cong 30636$$

Similarly, to predict for Nov. the trend value is -664 which falls on the 5th position (Medium Trend) of the trend class. The predicted value is then at the $\left(5/_9\right)^{th}$ position on the corresponding fuzzified interval $[21985 - 22902]$

The predicted value is thus:

$$21985 + \left(\frac{5}{9} \times [22902 - 21985]\right) = 21985 + \left(\frac{5}{9} \times 917\right)$$
$$\Rightarrow 21985 + 509 \cdot 44 \cong 22494.$$

And so on.

The Gross Domestic Capital of India dataset

The FTMI model is applied on the Sales Volume dataset of Table 4.2 for prediction. Using the algorithm of this model, the processes is as follows:

Steps 1-2: The Re-Partitioning Discretisation (RPD) approach is also applied on the Gross Domestic data of table 4.3 to partition it into unequal interval. Applying steps 1-6 of the RPD approach of Section 2.4, the elements of subsets $A\ and\ B$ are:

$$A = \begin{Bmatrix} 12105, 16986, 20139, 21265, 25600, 29990, 33757, 34772, 40136, 46406, \\ 53099, 57633, 63977, 70834 \end{Bmatrix}$$

$$B = \{88206, 90977, 96187, 100653, 114545, 131505, 134484\}$$

Applying steps 7-9 of the RPD approach of Section 2.4, the following intervals are gotten for the sub-boundaries $U_A\ and\ U_B$:

Intervals for U_A:

$$u_1 = [12105, 16300], u_2 = [16300, 20495], u_3 = [20495, 24690], ..., u_{14} = [66640, 70835]$$

Intervals for U_B:

$$v_1 = [88206, 94817], v_2 = [94817, 101428], v_3 = [101428, 108039], ..., v_7 = [127872, 134484]$$

Each elements of the historical Dataset are allocated to the intervals where it falls, discarding the intervals that do not cover the historical dataset.

Steps 3-6: Applying Steps 3-6 of the FTMI algorithm, Tables 4.11 and 4.12 are computed.

Table 4.11 Intervals with Corresponding Elements of Gross Domestic Product Data of India

$Interval\ for\ U_A$	$Corresponding\ Element(s)$	$Fuzzified\ Product$
$[12105, 16300]$	(12105)	R_1
$[16300, 20495]$	$(16986, 20139)$	R_2
$[20495, 24690]$	(21265)	R_3
$[24690, 28885]$	(25600)	R_4
$[28885, 33080]$	(29990)	R_5
$[33080, 37275]$	$(33757, 34772)$	R_6
$[37275, 41470]$	(40136)	R_7
$[45665, 49860]$	(46405)	R_8
$[49860, 54055]$	(53099)	R_9
$[54055, 58250]$	(57633)	R_{10}
$[62445, 66640]$	(63977)	R_{11}
$[66640, 70835]$	(70834)	R_{12}
$Interval\ for\ U_B$	$Corresponding\ Element(s)$	$Fuzzified\ Product$
$[88206, 94817]$	$(88206, 90977)$	R_{13}
$[94818, 101428]$	$(96187, 100653)$	R_{14}
$[108039, 114650]$	(114545)	R_{15}
$[127872, 134484]$	$(131505, 134484)$	R_{16}

The third-order fuzzy relation, order of difference and trend values follow the same step as was the case in the previous two datasets. Table 4.12 shows the results of these computations.

Table 4.12 Order of Difference, Trend Value and Third-Order Fuzzy Relation for Gross Domestic Data of India

Year	Actual value	Fuzzified value	1st-order difference	Trend values (T_{V_1})	3rd-Relation.
1981	12105	R_1			
1982	16986	R_2	4881		
1983	20139	R_2	3153		
1984	21265	R_3	1126	4017	$R_1R_2R_2 \rightarrow R_3$
1985	25600	R_4	4335	3053	$R_2R_2R_3 \rightarrow R_4$
1986	29990	R_5	4390	2871	$R_2R_3R_4 \rightarrow R_5$
1987	34772	R_6	4782	3283	$R_3R_4R_5 \rightarrow R_6$
1988	33757	R_6	-1015	4502	$R_4R_5R_6 \rightarrow R_6$
1989	40136	R_7	6379	2719	$R_5R_6R_6 \rightarrow R_7$
1990	46405	R_8	6269	3382	$R_6R_6R_7 \rightarrow R_8$
1991	53099	R_9	6694	3878	$R_6R_7R_8 \rightarrow R_9$
1992	57633	R_{10}	4534	6447	$R_7R_8R_9 \rightarrow R_{10}$
1993	63977	R_{11}	6344	5832	$R_8R_9R_{10} \rightarrow R_{11}$
1994	70834	R_{12}	6857	5857	$R_9R_{10}R_{11} \rightarrow R_{12}$
1995	88206	R_{13}	17372	5912	$R_{10}R_{11}R_{12} \rightarrow R_{13}$
1996	90977	R_{13}	2771	10191	$R_{11}R_{12}R_{13} \rightarrow R_{13}$
1997	96187	R_{14}	5210	9000	$R_{12}R_{13}R_{13} \rightarrow R_{14}$
1998	100653	R_{14}	4466	8451	$R_{13}R_{13}R_{14} \rightarrow R_{14}$
1999	114545	R_{15}	13892	4149	$R_{13}R_{14}R_{14} \rightarrow R_{15}$
2000	134484	R_{16}	19939	7856	$R_{14}R_{14}R_{15} \rightarrow R_{16}$
2001	131505	R_{16}	-2979	12766	$R_{14}R_{15}R_{16} \rightarrow R_{16}$

Step 7: Fuzzification of the trend

Sixteen (16) trend classes are created to reflect the number of classes of the main dataset which is sixteen, as in Table 4.11. From the 1st-order difference of Table 4.12, -2979 and 19939 are the least and highest trend values respectively. They are fuzzified as in Table 4.13.

Table 4.13 Fuzzified Trend Values of the Gross Domestic Data of India

Trend Classes	*Trend Intervals*	*Ordinal Positions*
Ultra Low Trend	$-2979\ to - 1546$	1st
Exceptionally Low Trend	$-1546\ to - 113$	2nd
Extremely Low Trend	$-113\ to\ 1320$	3rd
Extra Low Trend	$1320\ to\ 2753$	4rd
Very Very Low Trend	$2753\ to\ 4186$	5th
Very Low Trend	$4186\ to\ 5619$	6th
Low Trend	$5619\ to\ 7052$	7th
Medium Trend	$7052\ to\ 8485$	8th
High Trend	$8485\ to\ 9918$	9th
Very High Trend	$9918\ to\ 11351$	10th
Very Very High Trend	$11351\ to\ 12784$	11th
Extra High Trend	$12784\ to\ 14217$	12th
Extremely High Trend	$14217\ to\ 15650$	13th
Exceptionally High Trend	$15650\ to\ 17083$	14th
Ultra High Trend	$17083\ to\ 18516$	15th
Super High Trend	$18516\ to\ 19949$	16th

Step 8: Defuzzification and forecasting

The proposed algorithm for defuzzification and forecasting rule in Section 6.9 is used to predict the Gross Domestic Capital of India. Here the number of fuzzified trend is sixteen (16), as in Table 4.11. Each years' capital is predicted thus:

To predict for 1984, the trend value 4017 which falls on the 5th position (Very Very Low Trend) of the trend class; the predicted value is then at the $\left(5/_{16}\right)^{th}$ position on the corresponding fuzzified interval $[20495 - 24690]$, as in Table 5.13.

The forecast value is thus:

$$20495 + \left(\frac{5}{16} \times [24690 - 20495]\right) = 20495 + \left(\frac{5}{16} \times 4195\right)$$
$$\Rightarrow 20495 + 1310.94 \cong 21806$$

Similarly, to predict for year 2000, the trend value is 7856 which falls on the 8th position (Medium Trend) of the trend class. The predicted value is then at the $\left({}^{8}/_{16}\right)^{th}$ position on the corresponding fuzzified interval $[127872 - 134484]$

The predicted value is thus:

$$127872 + \left(\frac{8}{16} \times [134484 - 127872]\right) = 127872 + \left(\frac{8}{16} \times 6611\right)$$
$$\Rightarrow 127872 + 3305.5 \cong 131178.$$

And so on.

4.2.1 Performance of FTMI Model Vs Pioneer and Recent Models in Accuracy

In order to show the superiority of this proposed FTMI model, its forecasting performance is compared to the pioneer model (Song and Chissom, 1993a) and recent FTS model (Shah, 2012). The absolute percentage error (APE) and the mean absolute percentage error (MAPE) are the two statistical performance measures used to test the prediction accuracy of this model relative to existing ones. The following equations are used to determine these measures:

$$APE = \left(\frac{|actual\ value - forecast\ value|}{actual\ value}\right) \times 100 \qquad (0.26)$$

For all $i = 1,2, \dots, n$.

$$MAPE = \frac{1}{N} \times \left(\sum \frac{|actual\ value - forecast\ value|}{actual\ value}\right) \times 100 \qquad (0.27)$$

For all $i = 1,2, \dots, n$. where N is the number of the computed observations.

Table 4.14 shows the comparison of the result of the proposed model with that of Song and Chissom (1993b) and Shah (2012) model using absolute percentage error (APE) and mean absolute percentage error (MAPE) as performance measures:

Table 4.14 Comparison of Proposed FTMI Model with Song and Chissom (1993b) & Shah's (2012) Model

	Actual	**Proposed FTMI Model**		**Shah (2012) Model**		**Song and Chissom (1993b) Model**	
Year	Enrolment	Predicted value	Error (%)	Predicted value	Error (%)	Predicted value	Error (%)
1971	13055						
1972	13563						
1973	13867						
1974	14696	14776	0.54	14250	3.03	14000	4.74
1975	15460	15471	0.07	15315.5	0.95	15500	0.26
1976	15311	15263	0.31	15315.5	0.01	16000	4.50
1977	15603	15697	0.60	15626	0.14	16000	2.54
1978	15861	15906	0.28	15875	0.09	16000	0.88
1979	16807	16893	0.51	16916.7	0.65	16000	4.80
1980	16919	16943	0.14	16833.3	0.51	16813	0.63
1981	16388	16615	1.39	16500	0.68	16813	2.60
1982	15433	15437	0.03	15375	0.38	16789	8.79
1983	15497	15367	0.84	15375	0.79	16000	3.25
1984	15145	15141	0.03	15125	0.13	16000	5.67
1985	15163	15159	0.03	15125	0.25	16000	5.52
1986	15984	15889	0.59	15875	0.68	16000	0.10
1987	16859	16893	0.20	16833.3	0.15	16000	5.10
1988	18150	18 255	0.58	18500	1.93	16813	7.37
1989	18970	18987	0.09	18750	1.16	19000	0.16
1990	19328	19315	0.07	19250	0.40	19000	1.70
1991	19337	19290	0.24	19250	0.45	19000	1.74
1992	18876	18886	0.05	19250	1.98		
TOTAL ERROR			**7.11**		**14.36**		**60.24**
MAPE			**0.37**		**0.76**		**3.35**

The proposed model performed better than the recent model of Shah (2012) and Song and Chissom(1993b), as evident from MAPE results displayed in Table 4.14.

The comparison of the MAPE of the proposed FTMI model with those of recent models for the prediction of the Sales Volume and the Gross Domestic data of India respectively are as shown in Tables 4.15 and 4.16.

Table 4.15 Comparison of Proposed FTMI Model with Shah's (2012) for the Sales Volume Data

Month	Actual Sales Volume	**Proposed FTMI Model**		**Shah (2012) Model**	
		Predicted Value	Error (%)	Predicted value	Error (%)
Jan.	26658				
Feb.	29216				
March	30035				
April	30845	30636	0.681	30500	1.122
May	29171	29372	0.689	29500	1.128
June	21068	21577	2.416	21250	0.864
July	28416	28530	0.401	27000	4.983
Aug.	25651	25245	1.583	23500	8.386
Sept.	22122	22494	1.682	22500	1.709
Oct.	26423	26774	1.328	24500	7.278
Nov.	22467	22494	0.120	22500	0.147
Dec.	23274	23411	0.589	22500	3.326
TOTAL ERROR			**9.489**		**28.943**
MAPE			**1.054**		**3.216**

The developed model exhibited a much lower error compared to Shah (2012) model.

Table 4.16 Comparison of New FTMI Model with Shah's (2012) for Gross domestic Capital of India

Year	Actual value	**Developed FTMI Model**		**Shah (2012) Model**	
		Predicted Value	Error (%)	Predicted value	Error (%)
1981	12105				
1982	16986				
1983	20139				
1984	21265	21806	2.539	21250	0.071
1985	25600	26000	1.563	26250	2.539
1986	29990	30195	0.684	28750	4.135
1987	34772	34391	1.096	33750	2.939
1988	33757	34635	2.654	33750	0.021
1989	40136	38324	4.515	41666.667	3.814
1990	46405	46976	1.230	45000.002	3.028
1991	53099	51171	3.631	51666.67	2.697
1992	57633	55890	3.024	55000.00	4.569
1993	63977	64280	0.474	65000.00	1.599
1994	70834	68475	3.033	70000.00	1.177
1995	88206	91098	3.279	85000.00	3.635
1996	90977	92338	1.496	97500	7.170
1997	96187	98536	2.442	92500.00	3.833
1998	100653	98123	2.514	108333.33	7.631
1999	114545	110105	3.876	108333.33	5.423
2000	134484	131178	2.458	125000.00	7.052
TOTAL ERROR			**40.805**		**61.333**
MAPE			**2.400**		**3.608**

4.2.2 Result Discussion

The empirical results demonstrate that the proposed FTMI model has significantly reduced the forecast errors in the three time series by 51.3%, 67.2% and 33.5% for the university of Alabama student enrolment, Sales volume of a manufacturing company and the Gross domestic capital of India data respectively. The results obtained with the proposed FTMI model have lower errors compared to both the pioneer model and the recent model (Shah, 2012).

4.3 Increment of the Order of Difference (OoD)

In this research, the order of difference is extended to the fourth order in order to study whether or not the increment of the order of difference has effect on the data trend and accuracy of forecasting. Using Equation 2.15 and 2.16 of Section 2, third and fourth-order difference are computed along with first and second-order difference as shown in Table 4.17 for the Students' Enrolment data of University of Alabama. Trend values are also computed for each of these order of difference as shown in Table 4.17.

Table 4.17 1st, 2nd, 3rd and 4th-Order Difference with their Corresponding Trend Values (T_V)

Year	Actual Enrolment	**1**st Order Diff.	**2**nd Order Diff.	**3**rd Order Diff.	**4**th Order Diff	T_{V_1}	T_{V_2}	T_{V_3}	T_{V_4}
1971	13055								
1972	13563	508							
1973	13867	304	-204						
1974	14696	829	525	729		406			
1975	15460	764	-65	-590	-1319	525			
1976	15311	-149	-913	-848	-258	632	85		
1977	15603	292	441	1354	2202	481	-151	-236	
1978	15861	258	-34	-475	-1829	302	-179	-28	208
1979	16807	946	688	722	1197	134	-169	10	38
1980	16919	112	-834	-1522	-2244	499	365	534	523
1981	16388	-531	-643	191	1713	439	-60	-425	-959
1982	15433	-955	-424	219	28	176	-263	-203	222
1983	15497	64	1019	1443	1224	-458	-634	-371	-168
1984	15145	-352	-416	-1435	-2878	-474	-16	618	988
1985	15163	18	370	786	2221	-414	60	76	-542
1986	15984	821	803	433	-353	90	324	265	189
1987	16859	875	54	-749	-1182	162	252	-72	-337
1988	18150	1291	416	362	1111	571	409	157	229
1989	18970	820	-471	-887	-1249	996	424	15	-141
1990	19328	358	-462	9	895	995	0	-425	-440
1991	19337	9	-349	113	104	823	-172	-172	252
1992	18876	-461	-470	-121	-234	396	-427	-255	-95

For the sake of forecasting, each of the order of difference is fuzzified as shown in Table 2. Note that they are all fuzzified into thirteen classes to reflect the number of the classes of the main dataset.

Table 4.18 Fuzzification of Trends for 1st, 2nd, 3rd and 4th-Order Difference

1st –Order difference		**2nd –Order difference**		**3rd –Order difference**		**4th –Order difference**	
Trend Intervals	*Ord. Post.*	*Trend Intervals*	*Ord. Post.*	*Trend Intervals*	*Ord. Post.*	*Trend Intervals*	*Ord. Post.*
$-955\ to - 782$	1st	$-913\ to - 764$	1st	$-1522\ to - 1293$	1st	$-2878\ to - 2485$	1st
$-782\ to - 609$	2nd	$-764\ to - 615$	2nd	$-1293\ to - 1064$	2nd	$-2485\ to - 2092$	2nd
$-609\ to - 436$	3rd	$-615\ to - 466$	3rd	$-1064\ to - 835$	3rd	$-2092\ to - 1699$	3rd
$-436\ to - 263$	4th	$-466\ to - 317$	4th	$-835\ to - 606$	4th	$-1699\ to - 1306$	4th
$-263\ to - 90$	5th	$-317\ to - 168$	5th	$-606\ to - 377$	5th	$-1306\ to - 913$	5th
$-90\ to\ 83$	6th	$-168\ to - 19$	6th	$-377\ to\ 148$	6th	$-913\ to - 520$	6th
$83\ to\ 256$	7th	$-19\ to\ 130$	7th	$-148\ to\ 81$	7th	$-520\ to - 127$	7th
$256\ to\ 429$	8th	$130\ to\ 279$	8th	$81\ to\ 310$	8th	$-127\ to\ 266$	8th
$429\ to\ 602$	9th	$279\ to\ 428$	9th	$310\ to\ 539$	9th	$266\ to\ 659$	9th
$602\ to\ 775$	10th	*428 to577*	10th	*539 to 768*	10th	*659 to 1052*	10th
775 to 948	11th	*577 to 726*	11th	*768 to 997*	11th	*1052 to 1445*	11th
948 to1121	12th	*726 to 875*	12th	*997 to 1226*	12th	*1445 to 1838*	12th
1121 to 1294	13th	*875 to 1029*	13th	*1226 to 1455*	13th	*1838 to 2231*	13th

4.3.1 Effect of Increment of the Order of Difference (OoD)

Using the trend values computed on table 4.17, the fuzzified trends in Table 4.18 and the proposed defuzzification and forecasting algorithm of the proposed FTMI model, the time series data is predicted based on each of the order of difference. Table 4.19 shows the predicted results of the students' enrolment from the first to fourth order.

Table 4.19 MAPE Comparison for 1^{st}, 2^{nd}, 3^{rd} and 4^{th}-Order difference of Students Enrolment Data of University of Alabama

Year	Actual Enrol.	**1^{st}–Order difference** Predicted	Error (%)	**2^{nd} –Order difference** Predicted	Error (%)	**3^{rd}–Order difference** Predicted	Error (%)	**4^{rd}–Order difference** Predicted	Error (%)
1971	13055								
1972	13563								
1973	13867								
1974	14696	14776	0.54						
1975	15460	15471	0.07						
1976	15311	15263	0.31	15211	0.65				
1977	15603	15697	0.60	15654	0.27	15645	0.27		
1978	15861	15906	0.28	15854	0.04	15889	0.18	15906	0.28
1979	16807	16893	0.51	16842	0.21	16893	0.51	116918	0.66
1980	16919	16943	0.14	16943	0.14	16943	0.14	16943	0.14
1981	16388	16615	1.39	16539	0.92	16514	0.77	16514	0.77
1982	15433	15437	0.03	15402	0.20	15419	0.09	15454	0.14
1983	15497	15367	0.84	15350	0.95	15419	0.50	15437	0.39
1984	15145	15141	0.03	15211	0.44	15263	0.78	15263	0.78
1985	15163	15159	0.03	15211	0.32	15211	0.32	15193	0.20
1986	15984	15889	0.59	15923	0.38	15906	0.49	15906	0.49
1987	16859	16893	0.20	16918	0.35	16893	0.20	16893	0.20
1988	18150	18255	0.58	18255	0.58	18230	0.44	18230	0.44
1989	18970	18987	0.09	18911	0.31	18861	0.57	18861	0.57
1990	19328	19315	0.07	19189	0.72	19138	0.98	19189	0.72
1991	19337	19290	0.24	19138	1.03	19163	0.90	19214	0.64
1992	18876	18886	0.05	18788	0.48	18835	0.22	18886	0.54
TOTAL ERROR			**7.11**		**7.86**		**7.36**		**6.69**
MAPE			**0.37**		**0.47**		**0.46**		**0.46**

In a similar way, the order of difference is extended to fourth-order and the corresponding trend values computed for Sales Volume data and Gross Domestic Capital of India. Using the devloped defuzzification and forecasting algorithm of the proposed FTMI model, the two time series data are also predicted based on each of the order of difference. Tables 4.20 and 4.21 show summary of the predicted results of the different order of difference.

Table 4.20 MAPE Comparison for 1st, 2nd, 3rd and 4th-Order Difference of Sales Volume Data of Propylene Company

	1st–Order difference	**2nd–Order difference**	**3rd–Order difference**	**4th–Order difference**
TOTAL ERROR	**9.489**	**6.959**	**6.065**	**5.171**
MAPE	**1.054**	**0.994**	**1.011**	**1.034**

Table 4.21 MAPE Comparison for 1st, 2nd, 3rd and 4th-Order Difference of Gross Domestic Data of India

	1st–Order difference	**2nd–Order difference**	**3rd–Order difference**	**4th–Order difference**
TOTAL ERROR	**40.805**	**44.390**	**35.532**	**33.724**
MAPE	**2.400**	**2.774**	**2.369**	**2.409**

4.3.2 Result Discussion

Results obtained show that increasing the order of difference affects the general data trend of the time series in varying degrees; decreasing the ratio of positive-to-negative trends and introducing noise in the forecasted values of all the time series tested.

Also, results show that increasing the order of difference does not necessarily increase the forecast accuracy regardless of the structure of the time series. However, as the order of difference increases beyond second-order, the data trend of the time series become fairly constant, with most of the trend values falling in the same trend class. Hence, the third and fourth-order difference have approximately the same MAPE values for all the time series tested, namely, 0.5 & 0.5; 1.0 & 1.0 and 2.4 & 2.4 for the three time series respectively. The error is increasing in most cases probably because of the increase in the uncertainty of the data trend accessioned by increase in the order of difference.

4.4 The Model Adaptability

The statistical measure of dispersion is used to test the proposed model adaptability to different time series and compared with that of a recent model (Shah, 2012). The variance measures how far each percentage error in the set are from each other and the mean absolute percentage error (MAPE). Since variance measures the variability (volatility) from an average or mean, and volatility is a measure of risk, the variance statistic can help determine the 'risk' of using a model in forecasting a particular time series data. A high variance indicates that the percentage errors in the set are far from the MAPE and from each other. On the other hand, a lower variance indicates the opposite, that is, the variability of error is minimal in such predications

The following equation is used in calculating variance:

$$\sigma^2 = \frac{\sum (X - \mu)^2}{N} \tag{0.28}$$

Where σ^2 is the variance, X is the individual percentage errors, μ is the MAPE and N is the number of values in the set. Variance is computed using equation (0.28) for the proposed approach and that of Shah (2012). The results are illustrated in Tables 4.22 – 4.24.

Table 4.22: Variance of Forecast Errors from MAPE of Developed Model and Shah's (2012) Model for University of Alabama Students' Enrolment Data

Year	**Proposed FTMI Model** ($MAPE = 0.37$) Error (X)	($X - \mu$)	($X - \mu)^2$	**Shah (2012) Model** ($MAPE = 0.76$) Error (X)	($X - \mu$)	($X - \mu)^2$
1971						
1972						
1973						
1974	0.54	0.17	0.03	3.03	2.27	5.15
1975	0.07	-0.3	0.09	0.95	0.19	0.04
1976	0.31	-0.06	0.00	0.01	-0.75	0.56
1977	0.60	0.23	0.05	0.14	-0.62	0.38
1978	0.28	-0.09	0.01	0.09	-0.67	0.45
1979	0.51	0.14	0.02	0.65	-0.22	0.01
1980	0.14	-0.23	0.05	0.51	-0.25	0.06
1981	1.39	1.02	1.04	0.68	-0.08	0.01
1982	0.03	-0.34	0.12	0.38	-0.38	0.14
1983	0.84	0.47	0.22	0.79	0.03	0.00
1984	0.03	-0.34	0.12	0.13	-0.63	0.40
1985	0.03	-0.34	0.12	0.25	-0.51	0.26
1986	0.59	0.22	0.05	0.68	-0.08	0.01
1987	0.20	-0.17	0.03	0.15	-0.61	0.37
1988	0.58	0.21	0.04	1.93	1.17	1.37
1989	0.09	-0.28	0.08	1.16	0.40	0.16
1990	0.07	-0.3	0.09	0.40	-0.36	0.13
1991	0.24	-0.13	0.02	0.45	-0.31	0.10
1992	0.05	-0.32	0.10	1.98	1.22	1.49
TOTAL			**2.28**			**11.09**
VARIANCE:			**0.12**			**0.58**

Table 4.23 Variance of Forecast Errors from MAPE of Developed Model and Shah's (2012) Model for Gross Domestic Product Data of India

	Proposed FTMI Model **($MAPE = 1.054$)**			**Shah (2012) Model** **($MAPE = 3.216$)**		
Month	Error (X)	$(X-\mu)$	$(X-\mu)^2$	Error (X)	$(X-\mu)$	$(X-\mu)^2$
Jan						
Feb						
March						
April	0.681	-0.373	0.139	1.222	-2.094	4.385
May	0.689	-0.365	0.133	1.128	-2.088	4.360
June	2.416	1.362	1.855	0.864	-2.352	5.532
July	0.401	-0.653	0.426	4.983	1.787	3.122
Aug	1.583	0.529	0.280	8.386	5.170	26.729
Sept	1.682	0.628	0.394	1.709	-1.507	2.271
Oct	1.328	0.274	0.075	7.278	4.062	16.499
Nov	0.120	-0.934	0.872	0.147	-3.069	9.419
Dec	0.589	-0.465	0.216	3.326	0.11	0.012
TOTAL			**4.390**			**72.329**
VARIANCE:			**0.488**			**8.037**

Table 4.24 Variance of Forecast Errors from MAPE of Developed Model and Shah's (2012) Model for Gross Domestic Product Data of India

Year	**Proposed FTMI Model** **($MAPE = 2.400$)** Error (X)	$(X-\mu)$	$(X-\mu)^2$	**Shah (2012) Model** **($MAPE = 3.608$)** Error (X)	$(X-\mu)$	$(X-\mu)^2$
1981						
1982						
1983						
1984	2.539	0.139	0.019	0.071	-3.537	12.150
1985	1.563	-0.837	0.701	2.539	-1.069	1.143
1986	0.684	-1.716	2.945	4.135	0.527	0.278
1987	1.096	-1.304	1.700	2.939	-0.669	0.458
1988	2.654	0.254	0.065	0.021	-3.587	12.867
1989	4.515	2.115	4.473	3.814	0.206	0.042
1990	1.230	-1.170	1.369	3.028	-0.580	0.336
1991	3.631	1.231	1.515	2.697	-0.911	0.830
1992	3.024	0.624	0.389	4.569	0.961	0.924
1993	0.474	-1.926	3.709	1.599	-2.009	4.036
1994	3.330	0.930	0.864	1.177	-2.431	5.910
1995	3.279	0.879	0.773	3.635	0.027	0.001
1996	1.496	-0.904	0.817	7.170	3.562	12.688
1997	2.442	0.042	0.002	3.833	0.225	0.051
1998	2.514	0.114	0.013	7.631	4.023	16.185
1999	3.876	1.476	2.179	5.423	1.815	3.294
2000	2.458	0.058	0.003	7.052	3.444	11.861
TOTAL			**21.536**			**83.054**
VARIANCE:			**1.267**			**4.886**

4.4.1 Result Discussion

The variances based on Shah (2012) model are 0.58, 8.037 and 4.886 against 0.76, 3.216 and 3.608 of the MAPE for the same data set. It is important to note that two of the variance values are much greater than the respective MAPE values; indicating that the model has high variability in forecasting those time series data. The variance of the proposed FTMI model are 0.12, 0.488 and 1.267 and are all less than the MAPE of 0.37, 1.054 and 2.400 obtained for the same data based on the proposed model for the three cases considered in this study. Hence, the proposed FTMI model does not have high variability in forecasting time series of different structural and statistical characteristics. Therefore, it is suitable for different time series events.

4.5 Application of the FTMI Model on Internet Traffic

The FTMI model is applied to forecast the Internet traffic data obtained from the Information Communication Centre of Ahmadu Bello University, Zaria. The traffic data was obtained on the 25th June, 2014, using PRTG Network Monitor between 7:00AM to 10:00AM (at 5 minutes interval of time). Table 4.25 shows the total Internet traffic in measured in Kilo Bytes per time internal.

Table 4.25 Internet Traffic Data of ABU, Zaria

Time	Total Traffic (KB)	Time	Total Traffic (KB)
7:00AM	3158693	8:35AM	3406543
7:05AM	3148968	8:40AM	3332099
7:10AM	3280850	8:45AM	3136563
7:15AM	3303967	8:50AM	3231460
7:20AM	3213234	8:55AM	3111881
7:25AM	3184992	9:00AM	3150864
7:30AM	3335498	9:05AM	2976227
7:35AM	3252818	9:10AM	2936645
7:40AM	3111826	9:15AM	2986517
7:45AM	3096102	9:20AM	3002113
7:50AM	3221155	9:25AM	3318305
7:55AM	3152457	9:30AM	3141824
8:00AM	3242188	9:35AM	3038214
8:05AM	3215426	9:40AM	2950191
8:10AM	3207729	9:45AM	3059472
8:15AM	3253233	9:50AM	3221348
8:20AM	3202208	9:55AM	3114575
8:25AM	3331101	10:00AM	2982801
8:30AM	3352544		

The proposed FTMI model is applied on the internet traffic data using the model algorithm.

Steps 1-2: The Re-Partitioning Discretisation (RPD) approach is applied on the Internet traffic data of Table 6 to partition it into unequal interval. Here, 73% of the dataset are used for the training data (from 7:00AM to 9:10AM) and 27% of the dataset are used as the testing data (between 9:15AM to 10:00AM). Applying steps 1-6 of the RPD approach of Section 2.4, the elements of subsets $A \text{ and } B$ are:

$$A = \begin{Bmatrix} 2936645, 2976227, 3096102, 3136563, 3111881, 3111826, 3148968, 3150864, \\ 3152457, 3158693, \end{Bmatrix}$$

$$B = \begin{Bmatrix} 3184992, 3202208, 3207729, 3213234, 3215426, 3221155, 3231460, 3242188, \\ 3252818, 3253233, 3280850, 3303967, 3331101, 3335498, 3332099, \\ 3352544, 3406543 \end{Bmatrix}$$

Applying steps 7-9 of the RPD approach of Section 2.4, the following intervals are gotten for the sub-boundaries $U_A \text{ and } U_B$:

Intervals for U_A:

$$u_1 = [293664 \ , 2958850 \ ], u_2 = [2958850 \ , 2981055 \ ], ..., u_{10} = [3136490 \ , 3158695 \ ].$$

Intervals for U_B:

$$v_1 = [3184992 \ , 3198025 \ ], v_2 = [3198025 \ , 3211058 \ ], ..., v_{17} = [3393520 \ , 3406553 \ ].$$

Each elements of the historical Dataset are allocated to the intervals where it falls; however, the intervals that do not cover the historical dataset will not be discarded as previously done for prediction using testing data.

Steps 3-6: Applying Steps 3-6 of the FTMI algorithm, Tables 6 and 7 are computed.

Table 4.26 Intervals with Corresponding Elements of Internet Traffic

$Interval\ for\ U_A$	$Corresponding\ Element(s)$	$Fuzzified\ Trafficc$
[2936645,2958850]	(2936645)	R_1
[2958850,2981055]	(2976227)	R_2
[2981055,3003260]		
[3003260,3025465]		
[3025465,3047670]		
[3047670,3069875]		
[3069875,3092080]		
[3092080,3114285]	(3096102,3111826,3111881)	R_3
[3114285,3136490]		
[3136490,3158695]	(3136563,3148968,3150864, 3152457,3158693)	R_4
$Interval\ for\ U_B$	$Corresponding\ Element(s)$	$Fuzzified\ Traffic$
[3184992,3198025]	(3184992)	R_5
[3198025,3211058]	(3202208,3207729)	R_6
[3211058,3224091]	(3213234,3215426,3221155)	R_7
[3224091,3237124]	(3231460)	R_8
[3237124,3250157]	(3242188)	R_9
[3250157,3263190]	(3252818,3253233)	R_{10}
[3263190,3276223]		
[3276223,3289256]	(3280850)	R_{11}
[3289256,3302289]		
[3302289,3315322]	(3303967)	R_{12}
[3315322,3328355]		
[3328355,3341388]	(3331101,3335498,3332099)	R_{13}
[3341388,3354421]	(3352544)	R_{14}
[3354421,3367454]		
[3367454,3380487]		
[3380487,3393520]		
[3393520,3406553]	(3406543)	R_{15}

The third-order fuzzy relation, order of difference and trend values follow the same step as was the case in the previous two datasets. Table 4.27 shows the results of these computation

Table 4.27 Order of Difference, Trend Value and Third-Order Fuzzy Relation for Internet Traffic

Time	Total Traffic (KB)	Fuzzified value	1st-order difference	Trend values (T_{V_1})	3rd –Order Relation.
7:00AM	3158693	R_4			
7:05AM	3148968	R_4	-9725		
7:10AM	3280850	R_{11}	131882		
7:15AM	3303967	R_{12}	23117		$R_4R_4R_{11} \rightarrow R_{12}$
7:20AM	3213234	R_7	-90733	48425	$R_4R_{11}R_{12} \rightarrow R_7$
7:25AM	3184992	R_5	-28242	21422	$R_{11}R_{12}R_4 \rightarrow R_5$
7:30AM	3335498	R_{13}	150506	-31953	$R_{12}R_7R_5 \rightarrow R_{13}$
7:35AM	3252818	R_{10}	-82680	10510	$R_7R_5R_{13} \rightarrow R_{10}$
7:40AM	3111826	R_3	-140992	13195	$R_5R_{13}R_{10} \rightarrow R_3$
7:45AM	3096102	R_3	-15724	-24389	$R_{13}R_{10}R_3 \rightarrow R_3$
7:50AM	3221155	R_7	125053	79799	$R_{10}R_3R_3 \rightarrow R_7$
7:55AM	3152457	R_4	-68698	-10554	$R_3R_3R_7 \rightarrow R_4$
8:00AM	3242188	R_9	89731	13544	$R_3R_7R_4 \rightarrow R_9$
8:05AM	3215426	R_7	-26762	48695	$R_7R_4R_9 \rightarrow R_7$
8:10AM	3207729	R_6	-7697	-1910	$R_4R_9R_7 \rightarrow R_6$
8:15AM	3253233	R_{10}	45504	18424	$R_9R_7R_6 \rightarrow R_{10}$
8:20AM	3202208	R_6	-51025	3682	$R_7R_6R_{10} \rightarrow R_6$
8:25AM	3331101	R_{13}	128893	-4406	$R_6R_{10}R_6 \rightarrow R_{13}$
8:30AM	3352544	R_{14}	21443	41124	$R_{10}R_6R_{13} \rightarrow R_{14}$
8:35AM	3406543	R_{15}	53999	33104	$R_6R_{13}R_{14} \rightarrow R_{15}$
8:40AM	3332099	R_{13}	-74444	68112	$R_{13}R_{14}R_{15} \rightarrow R_{13}$
8:45AM	3136563	R_4	-195536	333	$R_{14}R_{15}R_{13} \rightarrow R_4$
8:50AM	3231460	R_8	94897	-71994	$R_{15}R_{13}R_{14} \rightarrow R_8$
8:55AM	3111881	R_3	-119579	-58361	$R_{13}R_{14}R_8 \rightarrow R_3$
9:00AM	3150864	R_4	38983	-73406	$R_{14}R_8R_3 \rightarrow R_4$
9:05AM	2976227	R_2	-174637	4767	$R_8R_3R_4 \rightarrow R_2$
9:10AM	2936645	R_1	-39582	-85078	$R_3R_4R_2 \rightarrow R_1$

Step 7: Fuzzification of the trend

Fifteen (15) trend classes are created to reflect the number of classes of the main dataset which is fifteen, as in Table 4.26. From the 1st-order difference of Table 4.27, -195536 and 150506 are the least and highest trend values respectively. They are fuzzified as in Table 4.28.

Table 4.28 Fuzzified Trend Values of the Internet Traffic

Trend Classes	*Trend Intervals*	*Ordinal Positions*
Ultra Low Trend	$-195536\ to -172466$	1^{st}
Exceptionally Low Trend	$-172466\ to -149396$	2^{nd}
Extremely Low Trend	$-149396\ to -126326$	3^{rd}
Extra Low Trend	$-126362\ to -103256$	4^{rd}
Very Very Low Trend	$-103256\ to -80186$	5^{th}
Very Low Trend	$-80186\ to -57116$	6^{th}
Low Trend	$-57116\ to -34046$	7^{th}
Medium Trend	$-34046\ to -10976$	8^{th}
High Trend	$-10976\ to\ 12094$	9^{th}
Very High Trend	$12094\ to\ 35164$	10^{th}
Very Very High Trend	$35164\ to\ 58234$	11^{th}
Extra High Trend	$58234\ to\ 81304$	12^{th}
Extremely High Trend	$81304\ to\ 104374$	13^{th}
Exceptionally High Trend	$104374\ to\ 127444$	14^{th}
Ultra High Trend	$127444\ to\ 150514$	15^{th}

Step 8: Deffuzification and forecasting

The proposed algorithm for deffuzification and forecasting rule is used to predict the Internet traffic data. Here the number of fuzzified trend from the trained data is sixteen (15).

To predict for 19:10AM, the trend value as computed with the trained data is -58412 which falls on the 6th position (Very Low Trend) of the trend class; the predicted value is then at the $\left(6/_{15}\right)^{th}$ position on the corresponding fuzzified interval $[2981055 - 3003260]$, as in Table 4.26.

The forecast value is thus:

$$2981055 + \left(\frac{6}{15} \times [3003260 - 2981055]\right) = 2981055 + \left(\frac{6}{15} \times 22205\right)$$
$$\Rightarrow 2981055 + 8882 \cong 2989937$$

Similarly, to predict for 9:45AM, the trend value is 16777 which falls on the 10th position (Very High Trend) of the trend class. The predicted value is then at the $\left(10/_{15}\right)^{th}$ position on the corresponding fuzzified interval $[3047670 - 3069875]$

The predicted value is thus:

$$3047670 + \left(\frac{10}{15} \times 22205\right) = 3053591$$

And so on. Table 4.29 shows the result of the predicted Internet values.

4.5.2 Comparison of the Actual and Predicted Traffic

The actual and the predicted traffic are compared using the testing data with MAPE as the performance measure.

Table 4.29 Prediction from the Trained Data of the Internet Traffic

Time	Trend Values (T_{V_1})	1st-order difference	predicted Traffic (KB)	
9:15AM	-58412	53292	2989937	
9:20AM	-53642	1480	2991417	
9:25AM	5063	331725	3323142	
9:30AM	-70913	-164447	3158695	
9:35AM	128832	-116946	3041749	
9:40AM	56253	-90301	2951448	
9:45AM	16777	102143	3053591	
9:50AM	-123898	163549	3217140	
9:55AM	-51701	-85095	3132049	
10:00AM	58464	-133230	2998819	

Table 4.30 Comparison of Actual Traffic and the Predicted Traffic

Time	Actual Traffic (KB)	Predicted Traffic (KB	Error (%)
9:15AM	2986517	2989937	0.11
9:20AM	3002113	2991417	0.36
9:25AM	3318305	3323142	0.15
9:30AM	3141824	3158695	0.54
9:35AM	3038214	3041749	0.12
9:40AM	2950191	2951448	0.04
9:45AM	3059472	3053591	0.19
9:50AM	3221348	3217140	0.13
9:55AM	3114575	3132049	0.56
10:00AM	2982801	2998819	0.54
TOTAL ERROR			**2.74**
MAPE			**0.27**

Figure 4.1 shows the graphical representation of the actual traffic vs the predicted traffic.

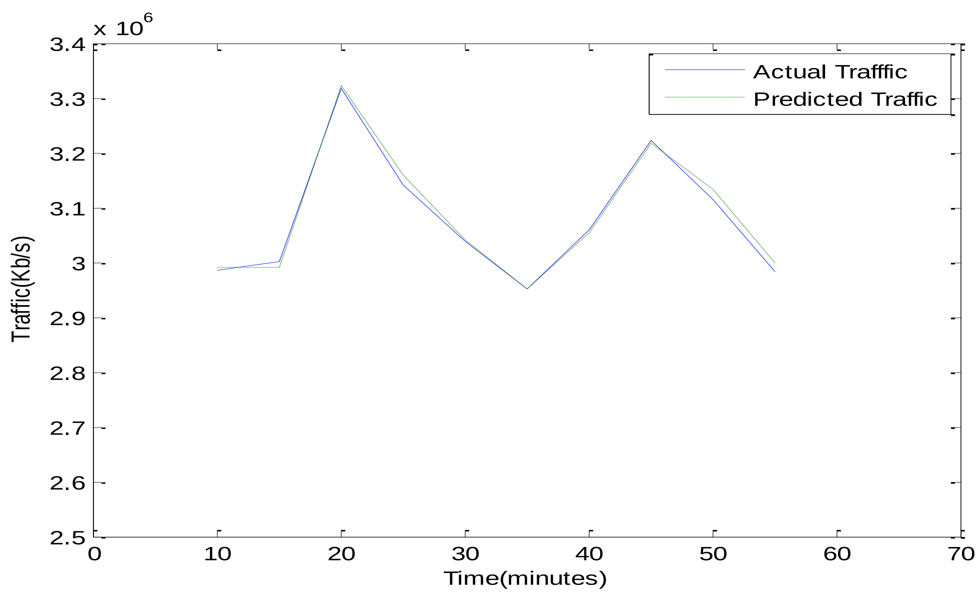

Figure 4.1 Graphical Representation of Actual and Predicted Traffic

The empirical result shows that when the error is averaged across the forecasting horizon, it gives a forecasting error 0.27 percent relative to the actual traffic for the Internet traffic, showing a good accuracy of prediction. From the graphical representation, it can be seen that the predicted traffic follows the trend of the actual traffic.

CHAPTER FIVE

CONCLUSION AND SUGGESTIONS FOR FURTHER WORK

5.1 Introduction

This chapter concludes this research thesis. Brief highlights of the main research findings are concisely presented. The conclusions of the research are presented next, followed by a summary of the limitations of this research work. Finally, further researches that could be carried out in this domain are suggested.

5.2 Summary of Findings

Here are the summary of this research findings:

I. The use of high-order trend relations in forming the trend values is more representative of the general data trend in any time series than the use of first order trend relation.

II. The fuzzification of general data trend helps to capture the fuzziness and uncertainty inherent in the data trend and makes the approach more reflective of the true sense of fuzzy set theory proposed by Zadeh in 1965. This trend fuzzification also strengthens the effect of the trend in the forecasting process; increasing the accuracy of forecasting.

III. Increasing the order of difference (OoD) between the consecutive values does not necessarily increase the accuracy of forecasting regardless of the structure of the time series. However, this increment in the order of difference affects the general data trend in various degrees, depending on the structural characteristics of the time series, and this often lead to the introduction of noise in the forecasted values.

IV. The use of any statistical measure of dispersion (variance, standard deviation and coefficient of variation) shows how erratic a model is at predicting time series, and therefore, can be used to measure the adaptability of a model at predicting a particular time series.

5.3 Conclusions

Based on the itemized deductions below, this research concludes as follows:

I. Inherent vagueness in the data trend of a time series can be fully captured by using high order trend relation and fuzzifying the identified trend. This has been developed effectively in this research model namely, the fuzzified trend mapping and identification (FTMI) based FTS forecasting. RPD was used to successfully optimize the determination of its interval lengths; order of difference (OoD) and trend values were used for trend mapping and identification respectively.

II. Identifying and mapping out trends in time series data help to improve the accuracy of forecasting. The trends identified in the previous models published in the open literature are only two-fold (upward/downward, high/low), and therefore does not capture the trend's vagueness since differences alone lacked context for which the increase or decrease occurred. The developed FTMI model has the ability to capture the trend's vagueness and has significantly improved the forecast accuracy (over 50% when compared to recent models such as model. This has been validated with three different time series of varying statistical characteristics including the standard data of students' enrolment in the University of Alabama.

III. More so, a challenge to the existing models is the introduction of noise in the forecasted values. That is, the models have the same predicted values for different times. Indeed, most models introduced noise even in the time series that did not have noise in the original dataset. The proposed FTMI model can trap both the fuzziness and uncertainty inherent in the trend, and has totally eliminated noise in all the time series tested.

IV. The nature and method of application of fuzzy rules affect the performance of a model. However, the existing models were fitted to a particular time series due to the arbitrary selection of weights and thresholds in forming and applying fuzzy rules. Therefore, they are only suitable or adaptable to forecast the time series specific to a data set understudy. Even models that were validated on more than one time series, the adaptability of the model on the time series and other time series of different structural and statistical characteristics were not in any way demonstrated. Analyses shows that a model may have a low MAPE or mean squared error (MSE) but erratic in prediction. The proposed FTMI model uses a very simple rule with no inclusion of weights or thresholds in its application. The proposed FTMI model is therefore robust and adapts well to time series of different statistical and structural characteristics. The statistical measure of dispersion (variance) is introduced in the thesis to validate the proposed model and those of previous models considered in this thesis.

V. It is a conventional practice to identify and map out the trends inherent in time series using first or second-order difference between consecutive values. Using three different time series of different structural characteristics, this research extended the order of difference to fourth-order and demonstrated that though, increasing the order of difference affect the data trend, it does not increase the accuracy of forecasting.

VI. This research demonstrated the successful application of the FTMI model to forecast internet traffic of ABU, Zaria – a time series data that has unique statistical characteristics which includes but are not limited to self-similarity, short and long range dependence.

5.4 Limitations

These are the limitations of this research work:

I. The basic limitation of this work, like other FTS models, is the assumption that the fuzzified class or classes of the forecasted value is known. In real life, this is not known.

II. In studying the effect of increment of the order of difference, this research limited it to fourth order. This order may not be enough to draw valid conclusions of this effect.

5.5 Suggestions for Further Work

Accurate analyses and forecasting of time series data is one of the challenges that confront engineers and mathematicians. This is partly due to the fact that time series is both non linear and non stationary and also that forecasting plays significant role in effective management and utilisation of resources in almost all fields of human endeavour. The successes recorded and the limitations encountered so far in the analyses and forecasting of time series show that there are high prospects for improvements. The following suggestions are made as to the improvement of research in this area:

I. In addition to prediction of the future values from the trained data, the fuzzified class of the future values of data in FTS may also be predicted. This will make FTS models more representative of the reality of natural processes and help in the post-sample prediction. Owing to the non linear nature of time series, non linear techniques such evolutionary algorithm (GA), particle swam optimization and artificial fish swam may be considered for prediction of the fuzzified class.

II. Because of the critical nature of interval length in FTS, further research may look at a way of modifying the Re-Partitioning Discretization (RPD) approach or development of new optimizing algorithm to further improve the accuracy of forecasting.

III. Trend values and the order of difference may be computed beyond third-order and fourth-order respectively, to see whether the accuracy forecasting may be improved.

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