IMPACT OF POLYA'S PROBLEM SOLVING STRATEGY ON ATTITUDE, PERFORMANCE AND RETENTION IN ALGEBRA AMONG JUNIOR SECONDARY SCHOOL STUDENTS IN ZAMFARA STATE, NIGERIA

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DECLARATION

I, Nasiru Anka HARUNA with registration number P15EDSC9021 declare that this thesis titled "Impact of Polya's Problem Solving Strategy on Attitude, Performance and Retention in Algebra among Junior Secondary School Students in Zamfara State, Nigeria" has been conducted by me and it is my personal work. It has never been presented in any research work. All information and materials that are not original to this research work were fully cited and reference lists are provided.

_____	_____	_____
Nasiru Anka HARUNA	Signature	Date

CERTIFICATION

This thesis titled "Impact of Polya's Problem Solving Strategy on Attitude, Performance and Retention in Algebra among Junior Secondary School Students in Zamfara State, Nigeria" by Nasiru Anka HARUNA P15EDSC9021 has been read and approved as meeting the regulations governing the award of the degree of Doctor of Philosophy (Ph. D) of Ahmadu Bello University, Zaria and is approved for its contributions to knowledge and literary presentation.

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DEDICATION

This study titled “Impact of Polya’s Problem Solving Strategy on Attitude, Performance and Retention in Algebra among Junior Secondary School Students in Zamfara State, Nigeria” is dedicated to my parents, Malam Haruna Abubakar and Hafsatu Muhammad who were unable to reap of their labour. It is also dedicated to my brothers, Ibrahim Haruna and Musa Haruna, my wife, Zarau Salihu and my children, Unaiza, Hafsat, Zainab and Amina for their encouragement and prayers towards the successful completion of this research work.

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ABBREVIATIONS

SLEPT	Simultaneous Linear Equation Performance Test
PJSS	Public Junior Secondary School
ASAIQ	Algebraic Students' Attitude Inventory Questionnaire
ZSTSB	Zamfara State Teachers' Service Board

OPERATIONAL DEFINITION OF TERMS

Algebra: A branch of mathematics in which numbers and letters are used to express mathematical ideas.

Attitude: A positive or negative feelings that students have towards learning of algebra.

Retention: The process by which students hold in their memories the mathematical facts and concepts they learnt and readily retrieve them in examinations.

Performance Test: Set of items that have been designed to deal only with a small portion, or sample of the entire collection of both knowledge and skills in a particular domain such as the algebra, for achieving the desired objectives.

Heuristics: These are general procedures or methods for solving problems that employ principles or rules of thumb that usually lead to a solution. Heuristics are certain type of questions and suggestions which teachers present to students to help them solve problems.

Problem Solving Strategy: Are general strategies or rules of thumb that may help solve a given problem.

Linear Equation: Linear equation refers to algebraic statement which describes relationship between two variables that produces a stright line graph.

Simultaneous Linear Equations: Simultaneous linear equations are two equations in two variables in which the values of the unknown variables are found such that they satisfy the equations at the same time.

Simultaneous Linear Equations by Substitution: This refers to two equations containing two unknowns which are obtained by writing one of the equations in terms of one unknown, and then substituting the expression in the second equation to obtain the other unknown.

Simultaneous Linear Equations by Elimination: This refers to two equations containing two unknowns in which one of the unknowns in the two equations is discarded and the other is solved, which you can then back-substitute into one of the original equations to solve for the other variable.

Conventional method: A procedure for explaining subject matter knowledge systematically to accomplish learning objectives.

Direct Translation Worded Problems: These are problems presented in English which are to be translated into the language of mathematics by using key words in the verbal description.

Geometry Worded Problems: These refer to problems presented in English which must be translated into the language of mathematics through geometric formulas.

Mixture Worded Problems: These are problems presented in English which involve two quantities combined in some fashion to be translated into the language of mathematics.

Uniform Motion Worded Problems: These refer to problems involving an object that travels at a constant speed.

Work Worded problems: These refer to problems in which two or more people join forces to complete a job.

ABSTRACT

This study investigated the Impact of Polya's Problem Solving Strategy on Attitude, Performance and Retention in Algebra among Junior Secondary School Students in Zamfara State, Nigeria. Quasi-experimental and Control group design involving pretest, posttest and post-post-test was used for the study. The population for this study consisted of all the public junior secondary school three students in one hundred and eighty (180) schools under ZSTSB that are in four Education Zones of Zamfara State. The total enrollement figure in the schools consisted of 27,357 students. Eight out of one-hundred and eighty schools in the four Education Zones were selected by balloting method. Four schools were assigned to experimental groups and the other four as control groups. One intact class was drawn from each of the eight selected schools by balloting method, giving a total of 463 JSSIII students used for the study. The experimental group had 235 students while the control group had 228 students. The experimental groups were taught by Polya's problem solving strategy and the control groups were taught by conventional method. The experimental period lasted for six weeks. The instruments used for data collection were SLEWPPT, SLEPT and ASAIQ. These instruments were validated by five specialists in mathematics education and one specialist in language education. The reliability coefficients of SLEPT and ASAIQ were computed and found to be 0.792 and 0.826 respectively. Six research questions and six corresponding null hypotheses were formulated and used to test the significance at $\alpha = 0.05$. The independent sample t-test and Mann-Whitney U test were used for data analysis. Findings from the study showed that students in the experimental group performed significantly better than those in the control group. It also revealed that experimental group significantly achieved positive attitude than the control group; also the retention level of students in the experimental group significantly exceed those of the control groups. The results also indicated that males in the experimental group had better attitude and performed significantly better than females. From the findings of this study, it was concluded that Polya's problem solving strategy was effective in promoting students' attitude and performance in algebra than the conventional method. In addition, students taught algebra with Polya's problem solving strategy in small groups have better attitude and performance than students taught algebra with conventional method. Based on the findings of the study, it was recommended among other things that State Government should train mathematics teachers in the effective use of Polya's problem solving strategy in teaching various mathematical topics through series of workshops and seminars in order to improve the attitude performance and retention of students in mathematics at the secondary level.

CHAPTER ONE

INTRODUCTION

1.1 Background to the Study

Education is essentially concerned with the growth and development of individual throughout life (Sharma, 2008). It is the bedrock upon which development revolves, and it is an invaluable tool for human growth and development (Abolade, 2009). This development can be achieved only when individuals are equipped with essential knowledge, skills and attitudes for survival and societal progress. Thus, to achieve this goal, the knowledge of mathematics is essentially needed because mathematics trains the mind and fosters creative thinking in people in order to solve problems that often occurred in everyday situations. Yusuf (2009) provides support for this view in which he stated that mathematics is not only considered important in its own right as a field of study and research, but also essential to almost every field of endearvour.

Similarly, Adetunde (2009) noted that mathematics forms the foundation of a solid education, adding that the overall national development of any nation and the building of healthy, happy and prosperous nation cannot be successfully achieved without mathematics. Therefore the position of mathematics in our national curriculum, and its role towards technological and industrial development put it in compulsory position in primary, secondary and tertiary levels of education (Madu & Hogan-Bassey, 2010). Despite the importance accorded mathematics in the educational system of Nigeria many secondary school students achieve very poorly in the subject. Many stakeholders such as the mathematics educators, researchers, governments, parents and the like have expressed their concern about students' persistent poor performance in mathematics especially at the

external examinations. Nevertheless, in the past few years several research studies were conducted to determine the causes of students' poor performance in mathematics with a view to improve their performance (Saritas & Akdemir, 2009; Eniayeju & Azuka, 2010; Farayola, 2011; Obomanu & Adaramola, 2011; Mbugua, *et al.,* 2012; Salman, *et al.,* 2012; Useni, *et al.,* 2012). Findings of these studies identified several factors responsible for this poor performance such as poor teaching method, lack of qualified teachers, lack of instructional materials, attitude, interest and motivation, mental ability, large classes and curriculum among others. Although the last decade, or so, have witnessed attempts on how best to improving the teaching of mathematics education in Nigeria, there still seems to be very little evidence of students' improvement in the subject. This is evident by the results obtained by secondary students at the senior school certificate examinations every year. This poor performance was reflected in their ten years of senior secondary certificate examinations reported by the West Africa Examination Council (2009 – 2018):

Table 1.1: Summary of WSSCE May/June General Mathematics Results from 2009 – 2018

Year	No. Candidates Sat for Exam.	Pass at Credit (A1-C6)	Ordinary Pass (D7-E8)	Fail (F9)
2009	1,348,528	634,382 (47.04%)	344,635 (25.56%)	315,738 (23.41%)
2010	1,306,535	548,065 (41.95%)	363,920 (27.85%)	355,382 (27.20%)
2011	1,508,965	608,866 (40.34%)	474,664 (31.45%)	421,412 (27.92%)
2012	1,658,357	838,879 (50.58%)	478,519 (28.85%)	298,742 (18.01%)
2013	1,656,527	897,655 (54.18%)	462,176 (27.90%)	245,263 (14.80%)
2014	1,632,377	766,945 (46.95%)	463,271 (28.38%)	402,161 (24.64%)
2015	1,629,126	801,127 (49.17%)	452,460 (27.77%)	375,539 (23.05%)
2016	1,552,758	822,496 (52.97%)	86,728 (5.59%)	643,534 (42.26%)
2017	1,559,083	923,486 (59.22%)	203,445 (13.05%)	432,152 (27.72%)
2018	1,572,396	786,016 (49.98%)	94,607 (6.02%)	691,773 (43.99%)

Source: Test Development Division, WAEC, Lagos.

Several research studies have identified poor methods of teaching as the major factor contributing to students' poor performance in mathematics (WAEC, 2009; NMC,

2009; Olunye, 2010; Abakpa & Iji, 2011; Obioma, 2011; Nizoloman, 2013). Moreover, Ali, *et al.,* (2010) observed that mathematics is one of the most poorly taught, widely hated and poorly understood subject in our schools. Indeed, one aspect of mathematics that seems to contribute immensely to this poor performance is algebra because it requires higher level of abstraction that involves the use of symbols or letters to represent unknown quantities. These letters are used as mathematical objects in problem solving which makes it difficult for students to understand what these objects represent and how they can be related to realistic situations. Algebra is a branch of mathematics upon which all other branches of mathematics are based. This subject encompasses a wide range of topics, all of which engage the learner in critical thinking during problem solving. Simultaneous linear equation is one of the several topics in algebra that help students engage in critical thinking and model real-life situations. It is used to describe relationships between variables, for example, the relationship between distance, rate and time; length and width; principal, rate and time; plane, trains and automobiles; cost and demand; number of people engage in a job and number of days taken to complete the job.

In spite of the numerous applications, research has shown that many students encounter difficulties in word problems involving simultaneous linear equations in two variables by substitution and elimination methods (Widyastuti, Mardiyana & Saputro, 2016). Therefore to teach mathematics effectively, the teacher needs to employ teaching strategies that will engage students in significant activities in the class to make the subject more meaningful and understandable for the students. This will enable them to learn more, retain more and apply what is learned. In response to the deplorable situation, Ogunyemi (2010) and Agwagwah (2013) suggested that problem solving strategy should be used in

teaching mathematics to improve students' achievement and retention in knowledge and skills. Therefore, more research is needed on instructional methods for helping students learn mathematics with understanding. It is believed that problem solving is one of those strategies. Through an emphasis on effective instructional methods such as the problem solving strategy, students can develop positive attitude, improve their performance, and retain more of what they have learned by engaging in significant activities. In the context of this study, attitude is defined as positive or negative feelings students may have towards learning of mathematics especially in algebra. Olarewaju and Popoola (2009) noted that through problem solving strategy, students developed positive attitude and construct knowledge for themselves rather than simply receive knowledge from the instructor. Thus to help students create positive attitude, it is desirable to structure lessons such that students become actively involved with content through manipulation of materials and social interactions.

Performance is also a very important factor in learning mathematics in general and algebra in particular. Nneji (2013) described performance as the gain in knowledge of students as a result of taking part in learning activity or programme. Performance in this study refers to the students' cognitive achievement level and academic skills which are measured in terms of marks scored in a test of algebra aspect of mathematics. It can also be described as the student's level of performance in algebra. Various studies have indicated the effectiveness of problem solving strategy on students' performance in mathematics (Perveen, 2010; Suleiman, 2010a; Suleiman, 2010b; Fajemidagba & Suleiman, 2012; Nneji, 2013; Nfon, 2013; Salman and Ameen, 2014). The results showed that students who were taught by problem solving strategy performed better than students who were taught

by conventional method of teaching. Retention is fundamental to students' learning. Hornby (2010) defined retention as the ability to remember things. In this study, retention is described as a process by which students hold in their memories the mathematical facts and concepts they learnt and readily retrieve them in examinations. According to Mangal (2012) the power of retention and retrieval are amongst other cognitive factors that affect both the processes and products of a person's problem-solving behavior. The use of effective teaching strategy which is stimulating and interesting, can enable students to retain and use the concepts they learnt to a wide variety of situations. Research studies carried out by Nneji (2013) found that students exposed to problem solving strategy retained better than those taught using conventional method. This is because problem solving strategy facilitates better understanding of mathematics concepts which in turns leads to higher performance and retention. Various innovative methods aimed at making the learning of mathematics more meaningful and interesting to students are being used by educators and teachers. One of them is problem solving strategy.

Problem solving is the process of finding an appropriate way to attain a goal when the goal is not readily available (King, 2008). There are several types of problem solving models developed by researchers to explain problem solving processes, including Polya (1957), Greeno (1973), Rubeinstein (1975), Lester (1978), Schoenfeld (1979), Bransford & Stein (1984), Gick (1986), Woods (1987), Krulik and Rudnik (1987), Kapa (2001) and Rusbult (2005) problem solving models. However, this study focused on Polya's (1957) problem solving model. The researcher has chosen Polya's model among other models of problem solving because it has received a wide acceptance and provided a foundation for other strategies (Aljaberi, 2015). Moreover, the model was used in this study because

according to Aljaberi (2015), “all recommendations and strategies in studies that deal with problem solving in mathematics are editable and can be accommodated one way or another to fit the Polya’s model”. So, there is the need to evaluate its effect on students’ attitude, performance and retention in algebra at the junior secondary school level. According to Mangal (2012) there are four basic steps needed in the solution of a problem outlined by Polya: understanding the problem which involves asking relevant questions that often helps students to draw a diagram representing the problem and the given information. In devising a plan, students are guided by the teacher to choose appropriate plan for solving the problem at hand. Then, they carry out their plan independently, checking each step to ensure that it is being properly implemented. Looking back means examining the solution to see whether it is correct or there is any other means of solving the problem. Against t his background, this study therefore, examined the impact of Polya’s problem solving strategy on attitude, performance and retention in algebra among junior secondary school students in Zamfara State, Nigeria.

1.2 Statement of the Problem

Mathematics is a subject that is used in many fields, including natural science, engineering, medicine as well as the social science (Gbolagade, Waheed & Sangoniyi, 2012). Therefore, the importance of mathematics in the development of science, technology and other fields of knowledge cannot be overemphasized. Notwithstanding, poor performance of secondary school students in the subject has become a thorny problem world over, particularly in Nigeria. For example, the data from the West African Examinations Council on students’ performance in mathematics (WAEC, 2015) revealed that for the years 2010 and 2011, the percentage of Nigerian students who obtained credit

pass in West African Senior School Certificate Examinations in mathematics was below average. Similar results were also recorded in the years 2014 and 2015 (WAEC, 2015). Also, the performance of students in internal and public examinations in mathematics has been attributed to many factors which include teachers' method of teaching, students' attitudes, and unavailability of learning materials amongst others. Nevertheless over the past two decades, a number of stakeholders have continued to take serious measures that would improve the learning of mathematics in Nigeria. These measures include organizing workshops, seminars, conferences, mathematics competitions and provision of adequate textbooks. Investigation also revealed that students were taught mathematics using approaches such as target task, Jigsaw and delayed formalization among others for the purpose of improving their performance in the subject.

Despite these, students still performed poorly in mathematics. In fact, simultaneous linear equation is one of the several topics in algebra that poses problems to students in the learning of mathematics. Research has shown that many students encounter difficulties in word problems involving simultaneous linear equations in two variables by substitution and elimination methods (Widyastuti, Mardiyana & Saputro, 2016). Similarly, literature consistently shows that most mathematics teachers in secondary schools in Nigeria still teach using the conventional method which typically involves overemphasis on rote memorization, drill and practice, exercise, and the representation of worked examples in textbooks that do not teach the effective processes of problem solving in mathematics. There is the need for effective teaching strategies that are leaner-centred where responsibility is placed on the learner to take initiative to achieve results for the various mathematical tasks during the learning process. Thus, the problem solving strategy has

been reported to be one of the active learning methods that have been found to be effective elsewhere in the world but little attention has been paid to its use in Nigeria. Therefore this study was to determined the impact of Polya's problem solving strategy on attitude, performance and retention in algebra among junior secondary school students in Zamfara State, Nigeria.

1.3 Objectives of the Study

The main objective of this study was to investigate the impact of Polya's problem solving strategy on attitude, performance and retention in algebra among junior secondary school students in Zamfara State, Nigeria. Specifically, the objectives of this study were to:

1. compare the attitude of students taught algebra using Polya's problem solving strategy and conventional method.
2. determine the performance of students taught algebra using Polya's problem solving strategy and those taught using conventional method.
3. compare the retention level of students taught algebra using Polya's problem solving strategy and those taught using conventional method.
4. determine gender difference on attitude of students taught algebra using Polya's problem solving strategy.
5. determine gender difference in performance of students taught algebra using Polya's problem solving strategy.
6. determine gender difference on retention level of students taught algebra using Polya's problem solving strategy.

1.4 Research Questions

The main objective of this study was to investigate the impact of Polya's problem solving on attitude, performance and retention in algebra among junior secondary school students in Zamfara State, Nigeria. Specifically, the research questions that guided the study were:

1. What is the difference in attitude if any between students taught algebra using Polya's problem solving strategy and those taught using conventional method?
2. What is the difference in performance if any between students taught algebra using Polya's problem solving strategy and those taught using conventional method?
3. What is the difference in retention level between students taught algebra using Polya's problem solving strategy and those taught using conventional method?
4. To what extent does gender has any influence on attitude of students taught algebra using Polya's problem solving strategy?
5. To what extent does gender influence the performance of students taught algebra using Polya's problem solving strategy?
6. What influence does gender has on retention level of students taught algebra using Polya's problem solving strategy?

1.5 Null Hypotheses

Based on the objectives and research questions of the study, the following null hypotheses were formulated and tested at $p \leq 0.05$ level of significance.

Ho_1: There is no significant difference in attitude between students taught algebra using Polya's problem solving strategy and those taught using lecture method.

Ho_2: There is no significant difference in the performance between students taught algebra using Polya's problem solving strategy and those taught using conventional method.

Ho_3**:** There is no significant difference between the retention level of students taught algebra using Polya's problem solving strategy and those taught using conventional method.

Ho_4**:** There is no significant difference in the attitude of male and female students taught algebra using Polya's problem solving strategy.

Ho_5: There is no significant difference in performance of male and female students taught algebra using Polya's problem solving strategy.

H_{O6}**:** There is no gender difference in the retention level of students taught algebra using Polya's problem solving strategy.

1.6 Significance of the Study

Mathematics plays a fundamental role in our everyday life (Akanmu, Otunla & Fajemidabga, 2015; Santos, Belecina & Diaz, 2015; Acharya, 2017; Rohmah & Sutiarso, 2018). Nonetheless, the existence of poor performance in mathematics among secondary students makes it imperative to search for better teaching strategies for effective learning of the subject. Such teaching strategies require active engagement to promote students' cognitive growth. Problem solving has been identified as one of the effective and efficient teaching strategies that enhances mental activities of students in the learning of mathematics (Rodrigues, 2015). Therefore, the findings from this study could hopely lead to other studies in mathematics in general and algebra in particular. These are explained as follows:

Students in secondary schools: The study would help secondary students to become good problem solvers. This is because good problem solvers have a repertoire of cognitive learning strategies that help them learn how to take control of their own mathematics learning; whereas poor problem solvers lack knowledge of strategies to perform learning tasks. Thus the knowledge of problem solving strategy would immensely assists students learn how to control and direct their thinking processes to facilitate learning. They would learn how to ask themselves questions, organize their thoughts, justify their ideas, makes sense of complex situations, and learn new ideas both collaboratively and independently.

They would also learn how to connect and integrate the new concepts they are trying to learn with knowledge they already possessed. Similarly, if students are taught mathematics with problem solving strategy in small groups, group members will be inclined to generate new and innovative solution to the problem being solved. Such group participation may actually enable students to produce more ideas than those generated by individuals working alone, simply because every member is actively involved in their learning. Moreover, through the use of problem solving strategy students will have greater opportunity to acquire variety of thinking skills and apply them to various new situations. These skills may include comparing, interpreting, observing, decision making, creating and criticizing. The results of this study would also assist students to know what methods are necessary for effective learning of mathematics in general and algebra in particular.

Teachers at the secondary level: The findings from this study would immensely assist teachers of mathematics in the secondary schools to understand the applications of mathematics in practical ways through problem solving approach and believe that mathematics is useful in our everyday lives. As a result, teachers will be equipped with

numerious heuristic activities such as brainstorming, open-ended questions, and discussions in which ideas for consideration are examined. This can easily be achieved by carefully choosing appropriate tasks relative to the students' current abilities. As students work to solve problems, the teacher provides guidance and encouragement to help them attain the desired outcomes. Also, through the use of problem solving method, secondary teachers would hoply enhance their teaching effectiveness on algebraic word problems which inturn improve students' attitude and performance in mathematics.

Guidance counselors: The guidance councilors who work with secondary school students would be enlightened on the different theories of learning (e.g., constructivist theories of learing) that lead to various innovative approaches to teaching such as the problem solving method for motivating students to learn mathematics. In addition, guidance councilors may wish to know about the way a student feels toward the learning of particular school subjects such as mathematics and sciences. The results of this study, then, can assist councilors in identifying and understanding the different learning strategies that help students develop positive attitude toward these subjects. This can be achieved as councilors organize enlightment programme on the importance of various psychological activities in which students are encouraged to talk to themselves and ask themselves about the mathematics problems being solved. Such examples of mathematics thinking and self-questioning are "What is missing"? "What am I to find"? As students are encouraged to talk to themselves, they will develop their own independence and success in learning mathematics and in monitoring their thinking about mathematics. Through these strategies, students can examine the methods that are used in handling problems. This will help them to eliminate fear of learning mathematics in general and algebra in particular.

School Administrators: The findings of this study would serve as a useful guide to the school administrators to know the efficient methods that increase the quality of instruction and encourage teachers to adopt those methods in their schools for improved performance.

Researchers: It is hope that findings of this study would help other researchers in mathematics education with useful information about various innovative approaches such as the problem solving strategy that are efficient and productive in various teaching situations. This information would probably equip researchers with more knowledge and skills to carry out more investigation into other areas of problem solving in mathematics that would contribute to the development of research for improving the quality of instruction in schools.

Textbooks writers: Textbook writers that are charged with responsibility of books production in the area of mathematics education may need to know the new approaches to the teaching and learning of mathematics such as the problem solving technique to be incorporated in to their work.

National Education Bodies: The Nigeria Educational Research and Development Council along with other educational bodies such as Science Teachers Association of Nigeria, Mathematics Association of Nigeria, and National Teachers Institute charged with the responsibility of developing and improving curriculum at both the primary and secondary levels would hopefully benefit from the findings of this study. The result of this study would provide the bodies with useful information on how to handle problems. This would help to enrich the curriculum to ease the understanding of the subject.

1.7 Justification of the Study

This study investigated the impact of Polya's problem solving strategy on attitude, performance and retention in algebra among junior secondary school students in Zamfara State, Nigeria. To provide a realistic understanding of students' learning in mathematics for improve performance, there is the need for clear instructional methods that would focuse on learner-centered and activity-based approaches. Problem solving strategy has been reported as an effective process for enhancing students performance (Rodrigues, 2015 & Lasak, 2017). A review of literature, however, has shown that there are a considerable number of studies that explored students' problem solving performance in simultaneous linear equations (Fajemidagba, Salman & Ayinla 2012; Tampuor 2012; Lagasse 2012; Hood 2013; Ogdol & Lapinid 2013; Esan 2015; Adisa & Abiola 2017). However, none of these researches was exclusively sets out to explore students' attitude, performance and retention in algebra using Polya's problem solving strategy especially at the junior secondary school level.

Additionally, most of the methodology adopted by the previous researchers in the field have not sufficiently and adequately address students' attitude, performance and retention in word problems involving simultaneous linear equations using Polya's problem solving strategy. Most researches relied on using approaches that did not involve students in collaborative investigations during problem solving. It is believed that students' performance is more likely to improve when they are given the opportunity to share their ideas and to become more actively engaged intellectually. Problem solving as an important component of mathematics curriculum is a process that involves the active construction of students' knowledge which happens in a social environment. Furthermore, students have

constructs which are essential to their mathematical understanding. These include what they know and how is this knowledge organised and represented; they have knowledge and beliefs about group work, and other inter-personal relationships in class; they have personal feelings about various aspects of mathematics. Through Polya's problem solving strategy and with a teacher faciliting, guiding and scaffolding during the learning process, students can develop a deeper understanding of simultaneous linear equations. The junior secondary school is the stage upon which firm foundation of basic mathematical knowledge is built in order to support the understanding of more complex concepts and skills that are necessary for future learning. Simultaneous linear equation is an aspect of algebra that deals with numerous concepts and skills needed in learning of other aspects of mathematics. Such valuable concepts like variables, expressions and equations are the basic building blocks in algebra, and word problems introduce a context where the above three components can link to a solution model.

Moreover, the Federal Government of Nigeria has outlined five important objectives of the JSS curriculum. Among them are the two objectives which mainly focus on giving students the opportunity to develop the understanding and application of mathematics skills and concepts necessary to thrive in the ever changing technological world; and to develop the essential elements of problem solving, communication, reasoning and connection within the study of mathematics, Federal Ministry of Education (FME) (2012). These expressions have indicated the need for developing mathematics skills and knowledge which are essential in raising individuals who would become productive citizens in the world work. Further, in a bid to achieve its goals in national development, the Federal Government of Nigeria in the National Policy on Education

(2004), clearly stated that the innovative educational techniques such as the problem solving approach should increasingly be used and improved upon at all levels of the education systems. There is therefore the need for unlimited research efforts geared towards improving the quality of mathematics teaching in Nigerian schools especially at the junior secondary level.

1.8 Scope/Delimitation of the Study

This study investigated the impact of Polya's problem solving on attitude, performance and retention in algebra among junior secondary school students in Zamfara State, Nigeria. Although various models of problem solving have been developed by educators to solve problems in mathematics such as Polya's (1957), Greeno (1973), Rubeinstein (1975), Lester (1978), Schoenfeld (1979), Bransford & Stein (1984), Gick (1986), Woods (1987), Krulik & Rudnik (1987) and Rusbult (2005) problem solving models, this study was delimited to only Polya's (1957) model of problem solving. The study was also delimited to Junior Secondary Schools (JSS III) students in eight local governments within the four Education Zones of Zamfara State.These comprised of Anka, Bukkuyum, Talata Mafara, Maradun, Gusau and Kaura-Namoda. The Junior Secondary School III students were considered appropriate for this study because the researcher wants to get terminal performance of students to prepare them for the senior secondary school level. The samples used for this study were eight public single-sex junior secondary schools in the four Education Zones of Zamfara State. The study was delimited to only three major themes on the JSSIII Mathematics Curriculum, namely; Number and Numeration, Geometry and Measurement. Number and Numeration contains number problems which include word problems involving sums and differences of whole numbers,

word problems involving sums and products of whole numbers, two-digit number problems, age word problems, word problems involving fractions, ratio problems, simple interest problems, rate of work problems and mixuture problems; Geometry contains rectangle problems and angle measures problems which include solving problems involving dimensions of rectangles, and problems involving complementary and supplementary angles; Measurement consists of uniform motion problems which includes motion affected by wind and by water current. These topics were treated under algebraic word problems involving simultaneous linear equations in two variables by substitution and elimination methods from JSS III scheme of work. The instruments used for this study are: (1) Simple Linear Equation and Word Problem Pre-Test (SLEWPPT) which was used as a measure of background knowledge and homogeneity of students before the - intervention, (2) Simultaneous Linear Eqution Performance Test (SLEPT) which consists of 10 essay items to measure students' performance in algebraic word problems on simultaneous linear equations and (3) Algebraic Students' Attitude Inventory Questionnaire (ASAIQ) which was used to assess the attitude of students toward algebra at the beginning and the end of the intervention.

1.9 Basic Assumptions of the Study

The following basic assumptions guided the planning and conduct of the study.

1. Students had sufficient knowledge and skills for solving algebraic tasks at the Junior Secondary School.
2. Students in the sample for this study have been exposed to the same national curriculum.
3. Teachers have similar background knowledge and skills in mathematics.

CHAPTER TWO

REVIEW OF RELATED LITERATURE

2.1 Introduction

This study examined the impact of Polya's problem solving strategy on attitude, performance and retention in algebra among junior secondary school students in Zamfara state, Nigeria. To achieved this purpose, various literature related to the study were reviewed. These include the Theoretical Framework, Conceptual Framework, Historical Overview of Algebra, The Concept of Simultaneous Linear Equations, Teaching of Simultaneous Linear Equations in Schools, Simultaneous Linear Equation by Substitution Method in Schools, Simultaneous Linear Equations by Elimination Method in Schools, Gender differences in Algebra Performance, Students' Performance in Algebra, Conceptual Definition of Attitude, Attitude and Students' Performance in Algebra, Retention and Students' Performance in Algebra, Review of Empirical Studies, and Implications of the Reviewed Literature.

2.2 Theoretical Framework

Learning is a complex process and one which involves continous interaction between development and experience throughout ones life (Byth, 1984 in Polland *et al.,* 2008). Early childhood theorists like Dewey, 1916; Bruner, 1960; Vygotsky, 1978 and Piaget, 1980 observed that children learn as they participate in social settings, and point that groups and learning communities are essential aspects of childrens' learning. These theorists have stressed the importance of classroom settings in enhancing authentic learning, where students are involved in problem solving that helps them construct their own meaning of the world. Dewey (1916) was considered the most influencial theorist of

the twentieth century and one whose philosophy has been described as progressivism, pragmatism, as well as experimentalism. According to him, the main goal of education is to help individuals to grow, understand and control their environment. He further stressed that knowledge is not an inert body of facts to be committed to memory, but one which consists of experiences that should be used to help solve immediate problems. According to Dewey, student's needs and interests have significant influence on learning, and that teachers must organize their lessons to reflect hands-on, learning by doing, and work within the students' past experiences.

From Dewey's view, children learn from social interaction, and like adults, they should learn how to structure their learning and develop independent thinking. Therefore, the teacher's role is to organize classrooms in a way that permits students to be working in small groups, moving about and talking to one another freely in order to solve problems so as to ensure that the desired goals have been truly achieved. Today, Dewey's writings and ideas continue to influence the study of many disciplines especially science and mathematics. Bruner (1960) is another influential educator who believed that a key to readiness for learning is how a child views the world. Bruner stressed that readiness depends more on an effective mix of three modes of learning: Enactive, Iconic and Symbolic learning modes than upon waiting until some emergined time when children are capable of learning certain ideas. The Enactive learning involves hands-on which enables the child to have direct contact with the real objects. The Iconic mode of learning is concerned with the use of visual materials such as films, pictures, diagrams and the like. And Symbolic learning mode is the one that requires the use of abstract symbols to represent the reality. Bruner further states that "what is most important for teaching basic

concepts is that the child be helped to pass progressibly from concrete thinking to the utilization of more conceptually adequate modes of thought." He also believed that children actively construct their own knowledge and that this construction of knowledge happens in a social environment and therefore suggested that during the learning process teachers should build on the children's experiences and provide moderately challenging tasks, and then provide the intellectual scaffolding to help them learn and progress through the different stages of development.

Also, Vygotsky (1978) believed that the intellect develops as children encounter new and challenging experiences, and as they strive to resolve discrepancies caused by these experiences. In the search for understanding, individuals link new knowledge to prior knowledge. In so doing, they construct new meaning. Vygotsky's ideas did not only focus mainly on the child's need to construct meaning, but also emphasizes the importance of social interaction for the development of his intellect. He claimed that new knowledge is built through involvement in social settings. To Vygotsky, cultural context is an essential element for cognitive development. To explain the social nature of cognitive development and the role that interpersonal relationships play in one's ability to solve problems, Vygotsky proposed that, all learning takes place in what he called the "zone of proximal development", which is the difference between what the child can do alone and what he can do with the assistance of adult or more capable peers. Unlike Vygotsky, Piaget (1980) believed that the basis of all learning is the child's own ability as he interacts with his physical and social environment. The child's mental activity is organized into structures or forms that enable him to adapt to his environment. Piaget hypothsised that two basic processes are involved in adaptation, these include assimilation and accommodation.

According to Piget, Assimilation is the process of taking in new experience into shemes. Thus, the child interprets new experience that he encounters with regards to the past experience and makes associations between new information and what is already known, Obodo (2004). Obodo further explained that the incorporation of the new experience into the existing knowledge modifies them inevitably. Accommodation is the adjustment of schemas to fit new information and experiences. In Piaget's view, assimilation and accommodation work mutually to develop a child's thinking and to create what he called cognitive equilibrium which is a balance between what the child has already known and the new situation or experience he encountered. Although accommodation to the environment results in a continuous modification of the child's schemes or patterns of behavior, the change is not merely quantitative. Rather, the mental structures of the child undergo series of qualitative changes as well.

In this way, Piaget hypothesized that children progress from the sensorimotor stage of the first two years to more complex stages of thinking. In the second stage of intellectual development, which he called the preoperational period, children are egocentric and are unable to perform simple logical operations. At about age 6 or 7 children enter the concrete operational stage and can perform concrete operations, such as those required to comprehend the principle of conservation. Finally, at about age 12, children enter the formal operational stage, in which they can think abstractly and hypothetically. Piaget has made tremendous performances in intellectual development by demonstrating how individuals' thinking processes change radically, though slowly, from birth to maturity. The contribution of Vygotsky's ideas to mathematics in general and mathematics education in particular is clear. Learning takes place through social interaction with

teachers and peers. With appropriate challenges and support from teachers or more able peers, students are moved forward into their zone of proximal development where new learning takes place. Thus, Vygotsky ideas on learning are similar to those of Dewey and Bruner in that both of them emphasized that teachers use the students' funds of knowledge and to provide the required support for students to acquire new knowledge, skills and strategies. Therefore, this study used Vygotsky's theory of social constructivism to examine the impact of Polya's problem solving strategy on attitude, performance and retention in algebra among junior secondary school students in Zamfara State.

2.3 Conceptual Framework

The Polya's (1957) framework for problem-solving that breaks down problem into four distinguishable steps and higlight the use of process in solving problems rather than focusing on getting the solution was adopted in this study. In the process of solving a problem, teacher guides the students to break the problem down and creatively and mindfully approach each step of the problem toward the desired solution. The four-step process includes understanding the problem, devising a plan, carrying out the plan and looking back. The first, often considered the hardest, and still the most fundamental step is the *understanding the problem*. This step requires the teacher to provide guidance to students to read carefully through the problem several times to obtain all the important information and ascertain exactly what the problem asked them to find, and then leads them to define terms and identify extraneous information and concentrate on certain basic elements of the problem. With the help of the teacher students can make sketch or draw diagram, where necessary, to show connections and relationships and look for similarities with previous problems. Having read the problem and know what the students want to

find, the teacher then leads them to *make a plan* by using the relevant information obtained from the first stage. During this stage, teacher encourages students to think about mathematical strategies and skills they have used previously in solving similar problems that could be helpful in solving the problem. At the third stage, *carrying out the plan*, the teacher asks the students to execute the plan made in the second stage. This involves them in computing and in explaining their procedure until the solution is obtained. At the final stage, *looking back*, the students with the help of the teacher, reflect back to see whether the problem has been completely solved. These four stages are linked in a cyclical process. According to Wilson (1993) the Polya's four-steps of problem solving are not linear as found in various textbooks. The steps are dynamic and cyclical.

Wilson futher emphasized that the problem solving process in mathematics cannot be described by the outer, one-directional arrows alone. This is because there are situations in which the first stage may reoccur during the second and third stages. For example, a student may begin with a problem and engages in critical thinking and variety of activities in order to comprehend what the problem is all about. The student then develops a plan of attack to solve the problem but may be unable to do so. Thus, the student makes a new plan or goes back to develop a new understanding of the problem or create a new but related problem to work on until the problem is completely solved. It is therefore worthnoting that the Polya's framework focusis mainly on discussing various pedagogical, curricular, instructional, and learning issues involved with the goals of mathematical problem solving in our schools. The cyclical pattern of a genuine mathematical problem solving can be depicted as shown in Figure 2.1.

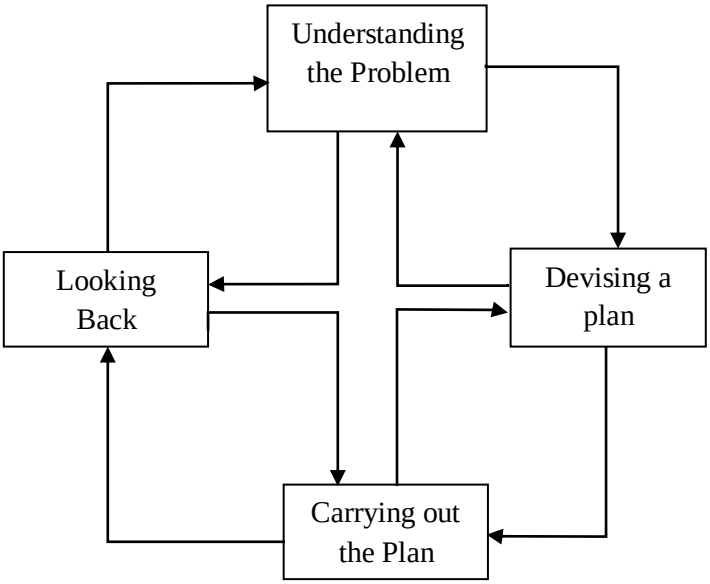


Figure 2.1: Dynamic and Representation of Polya's Problem Solving Model (Wilson *et al.*, 1993; Kolb, *et al.*, 1995; and McMillan, 2007).

2.4 Historical Overview of Algebra

Records in the history of human civilization have shown that algebra is one of the oldest branches of mathematics, and it plays a very significant role in this direction. Other components of mathematics that contribute to the development of mankind include among others arithmetic, geometry, statistics, trigonometry, calculus, logic, analytic geometry and mathematical modeling. Historically, algebra was developed in the ninth century by an Arab scholar Mohammed ibn Musa al-Khwarizmi, a native of Khwarizm. Manougian (1981) stated that Al-Khwarizmi named his book dealing with algebra as Ilm al-Jabr Wa'l-Muqabalah, which means the book of 'restoration' and 'comparison'. The word 'al-Jabr' means 'restoration', which implies adding something to a given sum or multiplying it so that it becomes equal to another. The word 'Muqabalah' means comparison, which is used for the comparison of the two sides of an equation. Algebra is a branch of mathematics that

uses symbols, usually letters of the alphabet to represent unknown quanties and to express relationships that holds for all members of a given set. According to Kesianye, Durwaarder and Sichinga (2001), the symbol system of algebra has developed through three main stages: rhetoric stage, syncopated stage and abstract stage. The rhetoric stage was characterized by using ordinary language to describe the solution to problems. That means no symbols or signs were used in describing the solutions to problems. The syncopated stage was the period in which abbreviated words rather than the complete words were used to explain solutions to problems. While in the abstract stage, the modern symbolic letters were used not only to represent unknown but also to represent given quantities. Kesianye, Durwaarder and Sichinga further stated that, the use of symbols made it possible, among others, to give general solutions to problems and to generalize patterns. Therefore the role that algebra plays in the development of knowledge cannot be over emphasized.

In analyzing the importance of language to the life of every individual and to the community as a whole, Olaoye (2007) remarked that language is a distinctively system of communication, based on oral and written symbols. Olaoye further noted that language of whichever form, is, "fundamentally a series of sounds which become meaningful only when these sounds are grouped together in certain definite arrangement". Thus, algebra as a language of mathematics also uses carefully defined terms and concise symbolic representations which enhance precision to communication. The language of algebra is systematic which consists of sounds, words and patterns that are commonly used in communicating mathematical ideas by mathematicians and other related professionals.

In fact, most of the advances that have been taken place in science and mathematics are as a result of algebra, and many great scholars in mathematics in various parts of the world had contributed immensely to these performances, particularly those from Greece, Egypt, Arabian, Babylonian, and Pasian among others. It is also interesting to note that historical records have indicated that initially, words were only used for communicating ideas in mathematics. In Egypt for example, Rhind Papyrus, written by the Egyptian priest Ahmes about 1650 B. C. E. was used in recording information on Egyptian mathematics, but with the recent advancements in mathematics, word problems are being translated into symbols. Commenting on the importance of algebra, Bennet, Burton and Nelson (2010) declare:

> Algebra is powerful tool for representing information and solving problems. It originated in Babylonia and Egypt more than 4000 years ago. At first there were no equations, and words rather than letters were used for variables. The Egyptians used words that have been translated as *heap* and *Aha* for unknown quantities in their word problems.

Bennet, Burton and Nelson went on to say that in the ancient times words rather than symbols were used by the Egyptians in representing information. For example: Heap and one-seventh of heap is 19. What is heap? This explains to us the extent to which Egyptians had developed their knowledge of mathematics. But today, a letter is used for unknown quantity and expresses the given information in an equation. For example, $x + \frac{1}{7}x = 19$. Actually, algebra describes the techniques of solving equations. According to Orji and Anaduaka (2010), algebra involves putting real life problems into equations and solving them. They further emphasized that the most important outcomes in the study of algebra include the ability to think logically, to use principles, to see relationships, to pick out essentials, to analyse and organize. Algebra introduces the concept of variables

representing numbers. The statements based on these variables are manipulated using the rules of operations that are applied to real numbers. As the language of mathematics, algebra consists of words, terms, letters or symbols, notation and signs that are used to express relationships in an algebraic expression and equation. However, algebra makes use of all these devices in accordance with a generally accepted rule or convention which is codified into a grammar that clarifies meanings. These meanings are usually derived from symbols. Barmby, Bilsborough, Harries and Higgins (2010) indicate that symbol is a powerful tool to the mathematical understanding. Due to its nature, algebra, the language of mathematics which uses abstract symbols, has been considered to have a variety of applications. Because of the prominent position that algebra occupies in mathematics and other fields of knowledge, the issue of competence as well as performance matters a lot. This is because it is only when one is competent in algebra that he can perform other functions. Thus, it is enough to conclude that competence in algebraic language can lead to understanding of other mathematical domains.

2.5 Concept of Simultaneous Linear Equations

The concept of linear equation has been explained in different ways by many researchers and mathematics educators. However, to have proper understanding of the concept "linear equation" in mathematical sense, there is the need to shed light on the meanings of both "linear" and "equation" as they complement each other. The term "linear" literary means a line which signifies a straight path that extends on and on in both directions without ever ending Sidhu (2006). Therefore in this study, the term "linear" refers to a set of points which formed a straight line to show axact relationship between two variables. While an equation is simply defined as an open number sentence consisting

of two expressions which are set equal to one another (Papaieronymou, 2007). The term 'relationship', implies a pattern in which, as the scores on one variable change, the corresponding scores on the other variable change in a consistent manner (Heiman, 2011). Therefore, linear equation in its simplest form, can be defined as the mathematical statement which describes relationship between two variables, and whose graph is a stright line. Linear equation is a broad umbrella term that encomphases several conceptual areas that have a wide range of applications in every day life. These include linear equations involving one variable and linear equations in two variables (such as simultaneous linear equations and simultaneous linear and quadratic equations). On the other hand, simultaneous linear equations have been defined by Mathematical Association of Nigeria (MAN) (2010) as equations in which the unknown values are found such that they satisfy the equations at the same time.

Thus, simultaneous linear equations are those equations which have exponent of degree one. These types of equations, therefore, will give straight line graphs. Various methods of solving simultaneous linear equations have been devised. These include graphical method, substitution method, elimination method, cross multiplication method, determinant method, and Gaussian elimination method. Of these, only three methods are basically used to solve simultaneous linear equations in Nigerian secondary schools as stipulated in the revised edition of the 9-Year Basic Education and Senior Secondary School Mathematics Curricula (FME, 2012). These include graphical method, substitution method and elimination method. However, for the purpose of this study, only the substitution and elimination methods would be considered. They were explained under the conventional methods of teaching simultaneous linear equations.

2.6 Teaching of Simultaneous Linear Equations in Schools

Conventional method, also called traditional method of teaching simultaneous linear equations, as referred to, in this study, is simply a procedure for explaining subject matter knowledge systematically to accomplish learning objectives. The technique is used in explaining content and skills deem necessary for learners to know and understand during the teaching and learning process by the teacher who is assumed to possess the knowledge. In this mode of teaching, students often remain passive while the teacher explains and demonstrates the aspects of the topic; they also take notes as the lesson progresses; and respond to questions as well. Because the way mathematics has traditionally been taught, students do not have opportunity to have deeper understanding of what they learnt. In other words, the students have not been given the opportunity to develop their powers of thinking, understanding and retention in what they learnt, Sidhu (2006).

One thing not often emphasized in the conventional methods is the active involvement of students in the learning process. Teachers focus almost entirely on practicing routine procedures, with virtually no emphasis on understanding of core mathematics concepts that might help students create connections among numerous mathematical procedures when solving problems. It is well known fact that the methods used to teach mathematics in most Nigerian schools are on routine procedures. The term "routine procedures" are solution methods that the learner is quite familiar with and is able to apply them correctly to routine problems (problems that the learner knows how to solve using past knowledge). These procedures are more often presented as step-by-step actions that students must memorise, not as moves that make sense mathematically. Thus, for students to develop higher order thinking that lead to understanding, analysis, exploration

and application of mathematics concepts, teachers must explicitly teach them how to solve non-routine problems. The term non-routine problems refer to problems for which a person does not immediately know the method of solution. They require productive thinking and reasoning on the part of the problem solver. The routine problems may be similar to other problems, but the solution often requires the problem solver knowing what features of the previous situation can be applied to the current problem. In order to convince students that mathematics makes sense, it is necessary for teachers to teach mathematics as a coherent and tightly related system of ideas and procedures that are logically linked together. One method which is considered vital in achieving this goal is problem solving. Normally, effective teaching requires good questioning and clear explanations of the steps used in solving a problem so that students do not learn by rote.

Atovigba, Okwu and Kurumeh (2010) point out that every step in problem-solving needs to be explained to student, in order to ensure replication and permanent understanding of what the learner has learned. This study, therefore, focuses on mathematical processes used in handling mathematical content, simultaneous linear equations and not the teaching process. Simultaneous linear equation in two variables is one of the topics in algebra that receives considerable attention in secondary school curriculum because of its important applications in real life situations. Therefore, handling of its content requires sound understanding of the methods used in teaching it, especially at the secondary level where a solid background of algebraic concepts, skills and techniques are needed for mathematical proficiency. Researchers and mathematics educators have devised several methods for solving simultaneous linear equations.

However, three of the methods that are commonly used for solving simultaneous linear equations in two variables at the secondary level are graphical, substitution and elimination methods. These methods are contained in both the revised edition of the 9-year Basic Education for junior secondary school and senior secondary school mathematics curricula that have been developed by the Federal Ministry of Education (FME, 2012). And also used in various textbooks written by Nigerian experts in mathematics education. They are broadly divided into two, namely, the coordinate approach and the algebraic approach. The coordinate approach involves solving simultaneous linear equations by graphing, while the algebraic approach is concerned with solving simultaneous linear equations algebraically. Though, graphing technique is one of the three methods of solving simultaneous equations in two variables at the secondary level, it is not of interest to this study because the approach provides inaccurate results as reported by (MAN, 2010). More so, by using the graphing technique, it is impossible for the researcher to generalize from the findings of the study. Because of the limitations of the method, researchers and mathematics educators suggested the use of substitution and elimination methods which produce exact solutions in simultaneous linear equations with two unknowns. These methods are discussed as follows:

2.6.1 Simultaneous Linear Equation by Substitution Method in Schools

The substitution method provides an accurate result than the graphical method when solving simultaneous linear equations in two variables. This is because the graphical technique does not always produce lines that intersect at integer value on the graph which makes it difficult to determine the solution of the simultaneous equations graphically. Therefore solving a pair of linear equations by substitution method involves finding the

value of one variable in terms of the other from the first equation and substitutes its value in the second equation to get linear equation in one variable. Then, solve the equation obtained and get the value of one variable (Central Board for Secondary Education, n.d). According to Mathematical Association of Nigeria (2010), substitution method is an algebraic method for solving simultaneous linear equations which involves writing one of the equations in terms of one unknown, and then substituting the expression in the second equation to obtain the other unknown. The term "substitution", according to Hood (2013) refers to replacement of one equation into the other in order to form a single variable equation. Therefore, the main idea of the method of substitution is first to find the value of one of the unknowns and then use this to find the value of the second unknown.

2.6.2 Simultaneous Linear Equations by Elimination Method in Schools

This method is also called elimination by addition or subtraction method. Like the substitution method, the elimination method of solving simultaneous linear equations in two variables does not depend on looking at graphs of equations. The elimination method, according to Sullivan III and Struve (2007) is a method in which one of the unknowns in the two equations is discarded and the other is solved. And this is achieved by getting the coefficients of one of the variables to be additive inverses by multiplying (or dividing) both sides of one (or both) equation(s) by a nonzero constant and adding the two equations in order to eliminate the variable whose coefficient has become an additive inverse. Solving this equation for one of the variables will produce the solution of the variable in question, which you can then back-substitute into one of the original equations to solve for the other variable. Therefore, the ultimate goal of elimination method is to add the equations and eliminate one of the variables. This means that it is possible for one to

eliminate one of these variables if the coefficients of one of them in the equations are opposite of each other. The main advantage of this method is that it is quicker and more accurate than the graphical method. A number of steps have been devised by researchers and mathematics educators for solving simultaneous linear equations by both substitution and elimination methods. However, all the different steps that have been provided follow the same pattern in arriving at a solution. This study, therefore, adapted the general stepts in solving word problems that result in simultaneous linear equations in two variables developed by Blitzer (1998) for teaching the control groups. The steps are as follows:

Step 1: Carefully work your way through the problem until you can let x and y (or any variables) represent the unknown quantities.

Step 2: Write simultaneous linear equations (in x and y) that describes the verbal conditions of the problem.

Step 3: Solve the equations written in step 2 using the method of addition (elimination) or substitution, and answer the problem's questions.

Step 4: Check the answers in the original wording of the problem, not in the equations.

2.7 Gender differences in Algebra Performance

Gender being a cognitive and social construct involves several characteristics and behaviours that are judged to be appropriate for males and females in accordance to cultural norms of a society. These cognitive and attitudinal behaviours have always been associated with many factors that have been identified by researchers as the causes of gender differences in mathematics performance. Woolfolk, Hughes and Walkup (2008) pointed out that gender stereotyping, gender bias in curriculum, gender discrimination in classrooms, as well as gender differences in mental abilities have been the major causes of

geder differences in school subjects especially in mathematics and science. In the same vein, Oteze (2011) noted that gender differences in mathematics performance have been associated with several veriables, including biological variables, environmental variables (such as school, teacher, peer group, parent and society), learner-related variables and cognitive variables (such as intelligence, spatial abilities, confidence, fear of success, attribution and persistence). Slavin (2009), however, has considered the causes of gender differences in two principal dimensions. One dimension is biological differences, and the other is sociological dfferences. The biological differences refers mainly to the inherent differences between males and females in general cognitive abilities (Arends, 2012); while the sociological differences are the differences caused by cultural expectations and norms of the society, such as parental expectations, peers' socialistion, differential treatment of males and females, low expectations of girls, low interest and confidence.

Empirically, research on gender differences in mathematics performance have not been consistent. For example, Bassey, Joshua and Asim (n.d) made a study on gender differences in mathematics performance at the senior secondary level. The focus was to determine whether parental socio-economic status as well as school proprietorship have any significant influence on students' performance in mathematics. A sample of 2000 subjects was drawn through stratefied and simple random sampling techniques. Also, a 30-item four multiple choice mathematics performance test was constructed and used for the study. The findings revealed gender inequality in the entire sample in which male students in the rural areas performed better than their female counterparts. However, these differences was only found among students of low socio-economic status families and those in the public schools. Similarly, at the other levels (high and private schools) of the

variables, they reported that no statistically significant difference was found between male and female students. In a similar study, Orji and Anaduaka (2010) examined the effectiveness of webbing instructional strategy on the academic performance of students in algebraic word problems. The sample consisted of 118 senior secondary school Two (SS2) students in two private co-educational schools that were purposively sampled. The data of the study showed that performance in algebra was not related to sex and hence neither the male nor the female students did significantly better than the other in the study. McFarland, Benson and McFarland (2011) compared performance scores of students in gender specific classrooms with students in traditional classrooms. The purpose of the research study was to provide information about gender specific classrooms at the elementary school level. The sample of the study consisted of 48 fifth-graders from mid-western public elementary school. The participants were divided into three sub-groups-boys only ($n = 8$); girls only ($n = 8$) and boys and girls ($n = 32$).

Archival data which included scores of reading and mathematics composites were collected from schools of the students. Two-way analyses of variance were used for data analysis regarding the MAP Test Scores obtained by the students. The results indicated that the scores on mathematics for females in the gender specific classroom were greater than scores from females in the traditional classroom. Moreover, females in the gender specific outperformed males in gender specific classroom as well as those in the traditional classrooms. It was, however, found from the study that traditional classrooms enhance reading performance of pupils better than gender specific classrooms. In a related study, Awofala (2011) tested one thousand seven hundred and eighty (880 males and 900 females) senior secondary three students in southern part of Nigeria based on single-sex,

coeducational, urban and rural settings. The results of the study indicated that male students performed significantly ($p < 0.05$) better than female students. It was, however, found from the study that the performance of males and females differed significantly only for those in single-sex and rural settings. And that the difference was infavour of males in all the two settings. Furthermore, the results showed that the performance of males and females did not differ significantly in either co-educational schools or urban schools. Inspite of the inconsistent results identified by many researchers on mathematics performance of students in single-sex schools and coeducational schools, there are still many advantages that the single-sex schools have over the coeducational schools.

More recently, Joseph and Kurumeh (2011) investigated the effect of two problem solving models on the academic performance of 260 Junior Secondary III Students in algebraic word problems using Greeno and Metes et al models. The findings of the study revealed that both male and female students had performed equally well when taught algebraic word problems using the two models. They then concluded that gender has no significant influence on male and female students' performance in algebraic word problems as measured by Mathematics Performance Test. Ogden (2011) used GCRCT to compare how the instructional setting influences student academic performance in mathematics. Four schools consisting of two single-gender and two coeducational middle schools were randomly selected from an urban school district in Georgia. The result of the study showed that males in the sixth grade in coeducational single-sex schools and females in the seventh grade in coeducational schools in the sample group were more likely to pass the GCRCT in mathematics than their counterparts in the other instructional settings. Furthermore, the study revealed that sixth and eighth grades of cohort females in

coeducational schools were more likely to pass the GCRCT in mathematics than their counterparts in the other instructional settings. Finally, the findings of the study showed that over the past three years, females in coeducational classes and females of the cohort group in a single-sex school had the greatest performances on the GCRCT in mathematics. Doris, Neill and Sweetman (2012) studied a representative sample of Irish 9 year old children from coeducational and single-sex schools to examine the determinants of gender gap in mathematics, focusing on the single-sex schooling. The survey sample was generated through the national primary school system in which 910 schools were randomly selected to participate in the study; and 6923 subjects with 3578 of the sample were girls and 3345 of the sample were boys used. They found that boys performed better than girls in mathematics. Further, the boys in single-sex schools did better than those in coeducational schools, whilst no corresponding difference was found for girls. Moreover, the study found that the difference in mathematics performance was larger for children educated in single-sex schools. They concluded that single-sex school does not reduce gender gap in mathematics.

Whalen III (2012) made a comparative study of mathematics performance differences between students in single-gender classes and those in mixed-gender classes. The results of the study indicated that there was no statistically significant difference in academic performance between students in single-gender classes and those in mixed-gender classes. Students in both single-gender classes were found to have a higher level of self-concept in their mathematics class, but that was more pronounced in boys' single-gender classes when compared to the mixed–gender classes. Moreover, there was no existence of any statistical difference between discipline referrals in the single-gender

classrooms and those in the mixed-gender classrooms. A study was carried out by Alshindi (2013) to investigate the relationship between single-sex schooling and academic performance in mathematics for eighth-graders in four countries in the Middle East. These were Iran, Syria, Jordan and Oman. The descriptive and regression analyses were used for data analysis. The results on descriptive statistics revealed that overall, moderate significant sex differences in favour of girls were found in mathematics academic performance. Furthermore, students attending segregated schools had moderately outperformed those attending mixed schools, but only one country- Oman had shown a reverse gap in favour of mixed schools. The overall results showed that mathematics performance scores were higher for boys and girls attending segregated schools compared to their counterparts in mixed schools. This difference was, however, moderately smaller for girls than boys.

Also, the regression analysis results showed that gender segregation was associated with a moderate and statistically insignificant increase in mathematics performance scores. Moreover, there was moderate negative significant effect on mathematics scores for females attending segregated schools, but the interaction had a negative effect across countries. While a negative but statistically significant results was also found for the interaction effect among three other countries- Iran, Jordan and Syria. Amatobi and Amatobi (2013) also carried out a study to determine the influences of gender and attitude differences in mathematics performance of final year senior secondary school students that comprised 30 males and 30 females. The data for the study was collected by using three instruments which included Student's Academic Ability Test, Students' Attitude to Mathematics Questionnaire and Structured Interviewed. The results indicatedd that attitude

of students towards mathematics in the studied schools, has a significant impact on the performance of students in mathematics. On the other hand, it was reported that gender difference in mathematics performance was in favour of boys but was found to be statistically insignificant. The difference that occured was reported to be attributed to positive attitude that male students exhibited towards the mathematics than their female counterparts. Therefore, it is a well known fact that the more interest and positive attitude a student shows towards a subject the higher the performance. In another development, Josiah and Adejoke (2014) investigated the effects of gender, age and mathematics anxiety of college students on their achievement in algebra. The study adopted an expost-factor design. The participants of the study were mathematics teacher trainees in the Federal Colleges of Education in Lagos and Ogun states of Nigeria. A questionnaire, a mathematics anxiety scale and participants' achievement scores in algebra course coded MAT 111 were the instruments used for data collection. The data collected were analysed using mean, standard deviation, independent t-test and one-way analysis of variance. The findings of the study indicated the average performance in algebra course. Moreover, the results showed non-significant differences in achievement across gender, age and mathematics anxiety groupings in terms of low, medium and high abilities.

In a meta-analysis of data from 184 countries that represented 1.6 million students in Grades k-12 from 21 countries for multiple outcomes involving mathematics attitudes and performance, Pahlke, Hyde and Allison (2014) found that, across controlled studies, there were positive effects of single-sex schooling in comparison to coeducational schooling on students' academic performance in mathematics among boys and girls. This study, however, points out that the results could have been negligible (non-significant)

when proper controls were included since the effect sizes were very small. Therefore, the study concluded that there was little evidence of gain of single-sex schooling for boys and girls for any of the results. Despite the fact that single-sex education has been acknowledged as the solution to coeducational schooling, having substantial benefits by research, it has some limitations. One is that educating children in a segregated setting has been criticized for decreasing their opportunity for full and meaningful contributions in future life. When boys and girls are placed in separate classes or schools, their chance of cooperating with one another has been denied; and this would eventually leads to academic and social problems of the sexes. It is believed that when boys and girls are put in the same learning environment, they would naturally learn to work together and this helps them reduce their feelings of isolation as well as improve their academic learning outcomes.

2.8 Students' Performance in Algebra

The place of algebra in school curriculum cannot be over emphasized. This is because algebra is one of the branches of mathematics that provides tools in understanding other areas in mathematics, as well as providing services to other disciplines such as physics, chemistry, biology, economics and agriculture that are dependent on its concepts, laws and principles, used in communication process for scientific and economic progress. Nevertheless, many students are still performing poorly in algebra aspect of mathematics. This is evident by the poor performance that students display in both the internal and external examinations every year. This poor performance can be ascribed largely due to poor teaching techniques that do not provide students the opportunity to explore and understand what is being taught in the classroom. According to West Africa Examination Council (WAEC, 2010) Mathematics Chief Examiner's report:

> In 2009, the Chief Examiner reported that the question on angle of elevation and depression was unpopular, and that candidates' performance was generally poor. In question on mensuration, the candidates' performance was reportedly poor. In question involving trigonometric equations and graphs, candidates had performed poorly because majority of them could neither complete the table nor draw the graph correctly. In 2010, candidates' performance regarding question on tangent to a circle were said to have performed poorly (2009: 276-285; 2010: 323).

From these statements, one observes that the scenario of students' difficulties remain the same almost every year. Thus, it is quite reasonable for one to deduce that these difficulties might be the result of students' poor knowledge in algebra, which in turn, leads to poor performance in school mathematics. This is because the knowledge of these concepts can only be built on proper understanding of algebra. This poor performance has also been ascribed to difficulties students face with learning algebra (Garo, n.d). Additionally, one could also believe that poor techniques used in handling the mathematical content might result to students' misconceptions of what they learned, and consequently lead to their poor performance.

Odili (2006) points out that secondary school students' performance in mathematics is very low particularly at the senior certificate examinations in Nigeria as a result of poor knowledge in mathematics. Therefore, students at the secondary level must be well grounded in algebra through appropriate using of techniques (e.g., questioning, reasoning and explanation strategies) to help them learn new concepts, emphasizing their significant aspects at each stage of instruction as well as to support students in their problem solving activities. This is especially important because in teaching of algebra the emphasis is always on techniques of manipulating expressions and solving equations. This implies that these techniques are necessary because they clarify important concepts and ideas that are essentials in the learning of algebra. More recently, the detailed analysis of

each question on students' performance in mathematics exanimations over a three year period have also been reported by the West Africa Examination Council (WAEC, 2011; 2012; 2013) Mathematics Chief Examiner. The report signified that:

> In 2011, the Chief Examiner reported that candidates' attempt on solution of linear equation in one unknown involving root and power of ten was poor. In question on change of subject of formula, the candidates performed poorly. In addition, they were unable to solve geometrical problems involving angles and were consistently performing poorly in it every year. Besides, they do not know how to use curve and straight lines to solve a quadratic equation. In 2012, the candidates generally demonstrated weakness in word problems which affected many of them to translate word problems to equivalent mathematical expressions. Likewise, most candidates could not manipulate algebraic fractions correctly. They were also unable to solve problems on probability correctly. Similarly, the Chief Examiners' Report of 2013 still reported general weakness translating word problems into equivalent algebraic expressions, which led to poor result. Moreover, most of them could not read the answer from the graph correctly.

In the reports above, one observes that the difficulties that students faced in mathematics for the three consecutive years is similar to those of the previous years. In fact, students' poor performances in Senior School Certificate Examinations continue to deteriorate every year. More recently, the evidenced-based report of this poor performance has been pointed out by Ambali (2014) in a keynote address on "mathematics education for sustainable development", at the opening ceremony of the 51th annual conference of the mathematical association of Nigeria. According to him, the report of the West Africa Examinations Council (2014) revealed that the results were generally poor. The report signified that about 70 per cent of the Nigerian candidates were unable to obtain the required credit pass in five subjects including Mathematics and English language. Similarly, the result showed that only 529,425 candidates, representing 31.28% of the 1,605,613 candidates obtained credit pass. The report further indicated that students' performance has been getting worse since 2012. In 2012, 61% failed; in 2013; about 63%

failed and in 2014, about 69% failed. All these poor performances in mathematics examinations could be due to poor knowledge in algebra. This observation had earlier been confirmed by Chief Examiner's reports of the West African Examination Council (WAEC) (2011). The Chief Examiner's report indicated that the percentage of students who failed in obtaining a credit pass in mathematics at the Senior Secondary Certificate Examination was due to their poor attempt at all algebra related questions. Similar observation had also been made by Atovigba, Vershima, O'Kwu and Ijenkeli (2012) who reported that there was generally low performance of students in algebra.

2.9 Conceptual Definition of Attitude

The word "attitude" has been defined in different ways by different psychologists and educators. Atkinson, Atkinson and Hilgard (1983) defined attitudes as, "likes and dislikes; affinities for and aversion to situations, objects, persons, groups; and other identifiable aspect of the environment, including abstract ideas and social policies." Thus, attitudes can simply be described as expressions of one's taste and distaste about entities in the social world. Attitudes have been noted to be closely related to cognitive, behavioural and affective components which relate to beliefs, actions, and likes and dislikes respectively. In the same vein, Horowitz and Bordens (1995) viewed attitude as, "feelings that are either positive or negative." According to these authors, attitude is exclusively dependent upon three components: Cognitive, Affective (emotional) and Behavioural components. The cognitive component relates to people's beliefs; the affective component defines what a person likes or dislikes; and the behavioural component refers to persons' tendencies to act towards these things in different ways. Notwithstanding, categorizing attitude into separate cognitive, behavioural and affective characteristics does not mean

that one component is not needed for attaining the other components. For example, it is not possible for one to think without having some feeling about what he is thinking or to feel something without thinking. Similarly much thinking involves actions that an individual takes with respect to the attitude object. This indicates that the cognitive, affective and behavioural responses are indeed interactive and intedependent. Although psychologists and researchers repeatedly have demonstrated a link between cognitive, affective and behavioural components, the affective response seems to dominate the human mind. This is because the attitude, whether positive or negative, of an individual towards a particular object is largely controlled by the emotional response of that individual. As an example, the development of students' attitude in learning of mathematics is greatly influenced by the emotional response.

Baron (2001) simply defined attitudes as our evaluations of different aspects of the social world which include people, objects, events, ideas and situations. In school, students normally change their attitudes toward mathematics depending on the way in which it is taught. For example, engaging students in significant and applealing activities in mathematics may lead to a change of attitude by students from negative to positive which in turn determine their success in the subject. On the other hand, when students are less active in the class they may manifest negative attitude to mathematics, which may in turn affect their academic performance negatively. It is therefore important that mathematics instruction be geared towards making students develop the right attitude for problems. In a similar vein, Myers (2001) described attitude as a belief and feeling that influences one to respond in a particular way to objects, people, and events. This definition is in line with the one provided by Baron (2001) in that both definitions centered on attitude object. The fact

is that people react to situation based on their feelings toward that situation. Amoo and Efunbajo (2004) simply described attitudes "as a state of readiness, a tendency to act or react in a certain way." More generally, they referred to attitudes as "a learned disposition or tendency on the part of individual to respond positively or negatively to a situation or another person." This implies that the attitudes of an individual can be favourable or unfavourable depending on the encountered situation. Therefore, to develop the right type of attitude, teachers need to use variety of suitable techniques and procedures to arouse and maintain students' interest in mathematics in general and algebraic word problems in particular. In an excellent discussion of the principles of meaningful learning, Sidhu (2006) noted that failure to adopt appropriate methods of teaching would inevitably bring about setback in creating and maintaining among others motivation, proper attitude, interest, attention among students for achieving higher learning outcomes.

One goal of teaching mathematics is to solve problems, and one way of achieving this goal is by making learning interesting and appealing to learners through adopting promising techniques and procedures. Considering these definitions, it is important, therefore, to note that attitudes are always likely to change depending on how a person encounters the object or situation; and in any case the attitudes of the person to react to the situation is determined by the conception of what he considered desirable (what he value) for him at a particular time. Thus, for example, a student with desirable attitude to mathematics would always enjoy studying the subject. But one who have unfavorable attitude towards the subject would certainly pay less attention to it and consequently, will not be able to do well. This shows that affective response is a significant factor in learning of mathematics in general and algebra in particular.

In the same vein, Gray (2007) described attitude as any belief or opinion that has an evaluative component; which means a judgment or feeling that something is good or bad, likable or unlikable, moral or immoral, attractive or repulsive. Gray (2007) further explained that attitudes link individuals cognitively and emotionally to the whole of their social world. In other words, attitude involves cognitive and emotional characteristics that interact with one another to explain individuals' feelings and actions in relation to other things in the social world. These include people, objects, events, ideas and situations that the individuals encounter in everyday life. Morgan, King, Wiesz and Schopler (2010) defined attitudes as simply expressions of how much individuals like or dislike various things. They further explained that attitudes represent our evaluations or preferences toward a wide variety of objects of attitude. Sarfo, Amartei, Adentwi and Brefo (2011) also defined attitude as a mental and neural state of readiness, organized through experience, exerting a directive or dynamic influence upon the individual's response to all objects and situations to which it is related. In a similar vein, Dodo (2012) defined attitude as the extent to which some ones feeling towards an object such as individual, group, school subject, idea, policy, institution or any thing is favourable or unfaviourable. From the various definitions considered so far, it is clear that attitude plays a significant role in learning in general and mathematics in particular. Therefore for effective learning to occur, various means of developing positive attitudes of students in mathematics is imperative. One of these ways to be considered in this study is the problem solving method. This is because problem solving develops positive attitudes and beliefs toward mathematics which result in deepening students' understanding of relationships and connections between the various mathematical concepts being learned.

2.10 Attitude and Students' Performance in Algebra

Attitude is the extent to which an individual's feeling towards certain phenomenon, which can be a person, group, school subject, idea, policy, institution, or anything, is faviourable or unfaviourable. Several studies were conducted with regard to relation between attitude and performance in various domains of mathematics particularly in algebra, but the result was that attitudes were found to have significant influence on students' performance. Though, many of these researches have reported that both attitudes and performance are reciprocal of each other in students' learning. Atnafu (n.d) carried out a study on sixteen tenth grade sections selected from Addis Ababa secondary schools. The purpose of the study was to examine the relationship between the attitudes and components of attitude of tenth grade students towards algebra with their algebra performances. Five dimensions of attitude namely, confidence, usefulness, enjoyment, subject perceived as a male domain, and teacher expectations were used in the study.

The participants of the study were made up of students and teachers drawn from sixteen tenth grade sections from twenty two Addis Ababa secondary schools using multistage sampling. The sample consisted of 632 students and 8 teachers. The instruments used for data collection include Algebra Attitude Scale, Algebra Performance Test, Closed-ended Questionnaire, Interview questions and Document. Two research questions and their corresponding hypotheses were formulated. The results of the study showed that there were weak relationships between algebra performance and each of the five components of attitudes including the attitude in algebra. The results further indicated that the contribution of all components of attitude in algebra as a whole significantly affected (but low) the performance of students in algebra. However, it was found that the

contribution of confidence was significant on the variable algebra performance, but the other components – usefulness, enjoyment, subject is perceived as a male domain and teacher expectations were not significant on the variable algebra performance. Also, it was found that the highest contribution for the knowledge, comprehension and application aspects of algebra performance was by confidence. In a related study, SattarGasim (n.d) investigated eighth grade students' attitude towards algebra in Maldivian schools. The research used a sample of 84 eighth grade students randomly selected from two major schools of Male'. The sample included 39 boys and 45 girls. One of the schools was a boy's school and the other one was girl's school.

One component of Fennema-Sherman Mathematics Attitude Scale and three components of Attitude towards Mathematics Inventory (ATMI) by Tapia and Mash (1996) were used in the study. These include confidence, value, enjoyment and teacher expectations. The results showed that the students have a neutral attitude for the components of attitude towards algebra as well as for the total attitude. The results further indicated that there was no gender difference in their attitude. Albay (2015) made a study on problem solving approach to the performance and attitude in college algebra of college students. The study employed the pretest-postest control group design involving experimental and control groups. The experimental group was taught selected topics in college algebra using problem solving approach while the control group was taught the same topics using traditional approach. The achievement test, attitude scale questionnaire in form 138, and college admission test results sheet were the instruments used for data collection. While the data gathered were analised using frequency counts, percentages, means, paired and t-test. The results of the analyses showed that the implementation of the

problem solving approach in teaching College Algebra had positively affected and improved the performance of the respondents. It was also found out that the utilization of the problem-solving approach in the delivery of instruction in College Algebra had positively contributed to the development and reinforcement of a favorable attitude towards the subject. In a related development, Zakaria (2009) conducted a study to investigate the attitudes and problem solving skills in algebra among Malaysian matriculation college students. In other words, the study examined whether there were differences in attitudes towards solving algebra problems and problem solving skills in algebra based on gender and course of study. The sample used for the study comprised of students from life sciences (154), physical sciences (141) and accounting (141). The total number of subjects involved in the study consisted of 436 matriculation college students in which 155 were males and 281 were females.

The instruments used to collect data include a problem solving test and a questionnaire on attitudes towards algebra problem solving. The data were analysed using a t-test and one way analysis of variance (ANOVA). The finding showed that there were no significant difference in attitudes and problem solving skills between male and female students. However, the findings revealed that matriculation students had moderately faviourable attitudes towards algebra problem solving. Similarly, the findings indicated that there were significant differences in attitudes with regards to self-confidence and problem solving skills between students in different courses of study. In another study, Leong and Alexander (2013) explored students' attitudes and performance in developmental algebra in a community college at the United States of America. The researchers intended to blend the web-based homework within the traditional framework in

order to improve students' performance in mathematics. There were 78 students who participated in the study. The mixed-methods approach was used to analyze the data. Findings from the study indicated mixed responses from the students on the benefits and disadvantages of the wed-based homework. The results showed that students with lower and average mathematics performance had a more positive attitude towards using web-based homework system compared to the high achieving students. The findings the study, therefore, suggested that there was need for redevelopment of positive attitudes about mathematics using web-based homework in online learning environments. A study was made by Yee Dat and Leng (n.d) on the effects of a portable computer algebra system on students' attitudes towards computer algebra system. Four subscales that included anxiety, confidence, liking, and usefulness were used in the study. One intact class, consisted of 22 students, comprising 10 females and 12 males, of the second year pre-university students in Singapore participated in the study.

The subjects were each given access to computer algebra system calculator for a period of six months. And the participants were given intervention programme. The computer algebra system attitude scale was then administered on three different occasions to the participants to measure their attitudes towards computer algebra system. There were four subscales of 10 items used to measure students' sense of Anxiety, Confidence, Liking, and Usefulness regarding computer algebra system. The t-test for paired samples was used to analyse the data. And the results of the study were not statistically significant.

2.11 Retention and Students' Performance in Algebra

According to Bichi (2002), retention is the ability to retain and consequently remember things experienced or learned by an individual later. Retention of concepts not

only in mathematics learning, but also in other subjects, is essential for achieving future leaning. What students have learned in the class is normally retained and remembered when they see meaning for learning it. This understanding would enable the student to transfer what he has learnt to other situations. In a learning environment, one factor that has been identified to influence retention of the material being learned by students is the method. This because it is believed that a method that provides complete involvement of learner in the learning process would significantly have effect on his learning. Anyamene and Nwokolo (2012) made an investigation into the effect of computer-assisted packages on the performance of senior secondary students in mathematics in Awka, Anambra state of Nigeria. The study was conducted on 20 males and 20 females drawn using stratified random sampling. Algebra performance test made up of 50 items multiple-choice objective type was used for data collection.

The analysis of the result was carried out using t-test statistic to test the hypotheses formulated. The findings of the study revealed that students taught by the computer-assisted package outperformed their counter parts taught using conventional method of instruction. Furthermore, students taught computer-assisted instructional method performed better than those in the control group in the retention test. Also, the result indicated that there was no significant difference in the post-test mean scores of male and female students taught using computer-assisted instruction. In a similar vein, Nneji (2013) compared Polya's problem solving model with the expository method to determine the performance and retention rates of secondary students in Ishielu local government area of Ebonyi State. The study used Quasi-experimental design with pretest-posttest non-equivalent control group. Four schools were purposively selected with eight intact classes,

four of which were randomly assigned to experimental and the other four to control were used. The samples of the study consisted of 220 senior secondary two (SSII) students. The Algebra Performance Test was used for collection of data. Also, the mean and standard deviation were used to answer the research questions while the Analysis of Covariance was used to test the hypotheses at the 0.05 level of significance. Findings of the study indicated that the experimental groups performed significantly better than the control groups both in the performance and retention tests. However, there was no significant difference in performance between male and female students in the study. This indicates that method of teaching that is interesting and appealing to students would always produce higher performance and retention in learning. More so, the results showed that retention is largely dependent on the type of method used by the teacher.

In another study by Wushishi, Danjuma and Usman (2013), investigated the effects of two modes of concept mapping instructional strategies on secondary school students' retention level in mathematics had revealed significant gains in learning of algebra. The study adopted a pretest-posttest randomized group design. Two hundred and four senior secondary school one (SSI) students were randomly selected from six co-educational secondary schools in Niger states. The spider mode of concept mapping strategy was used as treatment for control group while the hierarchy mode of concept mapping instructional strategy was used as treatment for experimental group. The algebra retention test on spider and hierarchy modes of concept mapping was used as data collection instrument. And one way analysis of variance was used to test the null hypotheses at 0.05 significant levels. The findings of the study showed that there was no significant difference in the retention level of the experimental and control groups. The results also revealed that both spider and

hierarchy modes of concept mapping instructional strategies were not gender bias at the retention level. Imoko and Ajai (2015) conducted a study to find out gender differences in mathematics performance and retention scores senior secondary one students using problem based learning method. The sample of the study consisted of 261 males and 167 females selected using multistage sampling technique. The Algebra Performance Test was used for data collection. Two research hypotheses were raised and tested at 0.05 significant levels. Findings of the study revealed that there was no significant difference between male and female students in performance scores. Zakariya, Ibrahim and Adisa (2016) compared the impacts of problem-based learning and conventional method on the mathematics performance and retention of junior secondary school students. The research design adopted was quasi-experimental design involving pretest, posttest, post-posttest, non equivalent control and experimental groups.

A sample of 149 students was randomly selected from a population of 664 JSS II students within Sabon-Gari local government area of Kaduna State. The sample consisted of 68 students (39 males and 29 females) in the control group and 81 students (53 males and 28 females) in the experimental group. Mathematics Performance Test I (MPT 1) was used for pretest and Mathematics Performance Test 2 was used for posttest and post-posttest (MPT 2). The two instruments were in multiple choice formats. The results indicated that students in the problem-based learning group performed better than students in the traditional group. Moreover, students taught with problem-based learning were able to retain the content taught better than their counter parts in the traditional group. Finally, the study found that problem-based learning was gender insensitive and therefore the method is applicable to both male and female students. More recently, Ugwuanyi (2016)

investigated the effect of algebraic factorization game instructional strategy on students' retention in algebra. The study employed quasi-experimental design. A sample of 180 students randomly selected from four co-educational schools in Enugu State participated in the study. An algebraic retention test and lesson plans were used for the study. The data collected were analyzed using mean scores and standard deviation for research questions and analysis of covariance for hypotheses. Findings from the study revealed that students who were taught algebra with algebraic factorization game retained more of the concept taught than those taught with conventional approach. Similarly, the game favoured both male and female students equally in promoting their retention in algebra.

2.12 Review of Empirical Studies

Simultaneous linear equation is an important component of algebra in secondary school mathematics curriculum. Despite its importance, the empirical research that have been conducted on this topic in mathematics teacher education is still limited, particularly at the junior secondary school level. Therefore, some research studies that have been found relevant to this study were reviewied. For example, Perveen (2009) investigated the effectiveness of expository strategy and problem solving strategy on the academic achievent of secondary school students in mathematics. The purpose of the study was to determine whether problem solving strategy was effective on academic achievement, attitude and retention of students in mathematics; as well as the achievement of high and low achieving students. The pretest-posttest equivalent group design was used for the study. The population of the study was all the secondary school students studying mathematics. Forty eight (48) girls students of 10^{th} class of Government Pakistan Girls Higher School Melad Nugar Rawalpindi participated in the study. The sample was divided

into experimental and control groups, and each group with twenty four students. The instruments used for the study were Mathematics Achievement Test (MACT) and Mathematics Interest Test (MIT). The MACT consisted of essay and objectives items containing four units of 10^{th} class mathematics namely, Algebraic Processes, Elimination, Practical Geometry and Trigonometry. While the MIT consisted of twenty items that measured students' interest in mathematics. The mean and standard deviation were used in answering the research questions, while the t-test for non-idependent samples was used in testing the null hypotheses at 0.05 alpha level. The results indicated that the experimental group performed significantly better than the control group on the pre-test, post-test and the retention test. Similarly, the performance of high achievers and low achievers of experimental group was significantly better than those of the control group. Thus, the results of the study indicated that problem-solving approach was more effective in terms of achievement, attitudes and retention in the subject of mathematics as compared to the expository strategy.

These studies are similar because the types of data analysed in the two studies are identical. However, the studies differ in the environments to which they were conducted; the statistical tools used, one used t-test for independent samples, the other used non-independent samples t-test; this study adopted quasi-experimental design in which intact classes were used, while the previous study employed equivalent control group design in which both the experimental and control groups were formed by random assignment. Further, the subjects for this study were both male and female students, whereas the subjects for the previous research were only female students. Suleiman (2010) studied the effects of problem solving models on secondary school students' performance in statistics

concepts of mathematics in Zamfara State. 200 SSII students from four co-educational private secondary schools in Gusau metropolis were purposively selected. Three schools were selected as the experimental groups and one school as the control group using simple random method. The experimental schools were assigned to instructional models of Polya, Gick, Bransford and Stein respectively using simple random teachnique, while one shool was assigned to lecture method. Each group contained 50 subjects that were classified as males and females as well as science and non-science. The Mathematics Performance Test (MPT) and two parallel Statistics Performance Tests were used for data collection. The findings of the study indicated that there was significant difference between the performance of the experimental and control groups F (3; 199) = 65.67, $P < 0.05$. The Polya model group with a mean gain score of 11.56 performed best, followed by the Gick model group with a mean gain of 5.30. Moreover, the Bransford and Stein model group had a mean gain score of 3.99, while the control group had the least mean gain score of 2.36.

The study also revealed that there was significant interaction effect of: instructional model by subject combination, gender by subject combination, instructional models by gender and by subject combination on the students' performance in the posttest administration. Also, it was found out that there was no significant difference in the performance of male and female students as well as science and non-sciene students. Suleiman's study is related to this study in that they both used Polya's problem solving model and Zamfara State. The study differs from the present one since it consisted of statistics concepts, whereas the present study examined students' performance in algebraic word problems involving simultaneous linear equations in two variables. They also vary in

assessing students' attitude and retention level. Moreover, the present study varies by using JSS III students while the previous study used SSII students.

Ali, Hukamdad and Khan (2010) employed quasi experiment with pre-test post-test design to study the effect of using problem solving method in teaching mathematics on the achievement of mathematics students at elementary level. Results were analyzed using mean, standard deviation and t-test. The items in the test were developed based on the 8th grade mathematics textbook in the area of set, information handling and geometry was used for data collection. The test consisted of 8 questions containing multiple choice questions, matching questions, fill in the blanks with correct answer and practical solutions to mathematical problems to measure students 'academic achievement and performance skills respectively. From the findings it was observed that the use of problem solving method enhanced the achievement of the students in mathematics. It was also found that the academic achievement of the students taught through problem solving method was better as compared to those who were taught through traditional method. This study is similar to the present study in methods used, but they differ with regards to country, instruments, and samples.

Bowman (2010) studied the processes that English Language Learners used to think about word problems. The study was specifically designed to explore how Polya's problem solving strategy affects English Language Learners' (ELLs) ability to communicate thinking, with both oral and written fluency, when approaching mathematical word problems. In other words, Bowman wanted to know whether Polya's problem solving strategy is an effective tool for improving the fluency of written and oral communication skills of ELLs when solving word problems. The study used qualitative

research design for data collection. The population of the study comprised of all 6th grade ELL students at middle school in the upper Midwest of America. Six students were selected using convenience sampling that comprised of 5 Latino and 1 Oromo, in which 4 were females and 2 were males. These students were from different ethnic groups that included Caucasian, Latino, African and African American, and were exposed to the same curriculum as their native English speaking peers as well. The study was conducted in an English as a Second Language (ESL) collaboration mathematics classroom and during student lunch times in an ESL classroom. The data for the study were collected through a series of MCA-II Mathematics constructed response practice problems and retrospective reports, which were recorded as students explained their mathematical thinking out loud. Results showed that Polya's problem solving strategy had improved ESL students' ability to solve word problems. This study is related to the preceeding one because both used Polya's problem solving strategy. These studies, however, differ in many ways including country of study, participants, design, content area, as well as the instruments for data collection.

Fajemidagba, Salman and Ayinla (2012) investigated the effect of teachers' instructional strategy pattern on the performance of senior secondary school students in mathematics word problems. The sample of the study consisted of 125 (61 experimental and 64 control) senior secondary Two (SSII) students purposively drawn from two schools in Ondo town of Ondo state, Nigeria. The subjects were categorized into three levels namely, high, medium and low scorers. The study was quasi-experimental type involving a 2×3 Factorial design. The Mathematics Performance Test was used as the instrument for data collection, while the t-test for independent samples and Analysis of Covariance were

used to test the hypotheses set for the study at 0.05 level of significant. Findings from the study showed that the experimental group that was exposed to Instructional Strategy Pattern performed significantly better in Mathematics word problem-solving involving simultaneous linear equations than their counterparts in the control group. It was also found that the Instructional Strategy Pattern was beneficial to all ability levels. This study differs from the present study because it was conducted in senior secondary two (SSII) students with different ability levels, while the present study was carried out on junior secondary Three (JSS III) students. Also, these studies differ in other ways in that the previous study focused only on students' performance in word problems involving simultaneous linear equations but the present study was designed to examine the impact of problem solving on attitude, performance and retention in algebra among junior secondary students in Zamfara State, Nigeria.

Tampuor (2012) conducted a study by using Microsoft excel to improve students' performance in solving simultaneous linear equations in two variables using graphical method. A sample of 50 students was selected from 350 second year students comprising 30 boys and 20 girls using students' registers. The study used action research; pretest and posttest, as well as questionnaires for data collection. Data was analysed using tables and charts to compare the pretest and posttest scores of the students and to determine the overall picture of their preferred method to solve simultaneous linear equations. Similarly, teachers' opinions were sought as to the best methods used in teaching simultaneous linear equations. The results revealed significant means differences between the pre-test and pos-test using paired-comparison t-test. The results of the study also indicated that teachers did not consider Microsoft Excel as vital instrument for effective teaching; as such they did not

integrate it in the teaching of mathematics in secondary schools. The study, therefore, concluded that Information and Communication Technology (ICT) be integrated in the teaching and learning of mathematics in senior high schools despite the fact that substitution is the students' preferred method for solving simultaneous equations. This will go a long way in creating learning opportunity for students to learn how to use Microsoft excels to improve their performance in solving simultaneous linear equations by graphical method. These studies are similar in their instruments for data collection. In spite of the similarities these studies have in determining students' performance in mathematics problem solving involving simultaneous linear equations, they also have differences in research designs, methods, school type and countries of their investigations.

Iyekekpolor and Michael (2012) wrote a position paper on teaching post-secondary linear equations. The purpose of the paper was to discuss the use of mathematica as an alternative strategy to solving simultaneous linear equations. In other words, the paper explained how mathematical programme could be used to investigate systems of linear equations of two variables at post secondary school level. The illustrations described involve singular, non-singular as well as ill-conditioned systems with an application approach to explain the importance of the programme in learning of mathematics. The concept of Information and Communication Technology was introduced and its significance in classroom instruction was emphasized and some limitations in visualization using the programme were stated. This research differs from the present study in that it described how mathematical programme could be used in investigating systems of linear equations of two variables in post secondary schools to improve students' performance, but the present study was aimed at examining the impact of problem solving strategy not only

on performance, but also on the retention and attitude to solving simultaneous linear equations in junior secondary schools.

Lagasse (2012) made an analysis of differences in approaches to systems of linear equations problems given multiple choice answers. The objectives of the study were to determine which approach to solving systems of linear equations in two variables students used most when given multiple choice answers, and to examine if students tend to solve problems differently depending on their major. There were essentially four approaches considered in the study. These included substitution, elimination, graphing and backwards that involved plugging in x and y values from each multiple choice. Participants were drawn from a midsize public university in the north of America. The main data collection instruments were questionnaire and semi-structured clinical interview. Participants solved systems of linear equations problems and answered questions based on their methods in a structured clinical interview. Each participant also filled out a questionnaire. It was shown from the results of this study that the majority of a student does not change the approach used to solve a problem by very much. Most students in the study chose to use substitution to solve the problems, usually because this was the method students remembered most and was deemed the "easiest" method by the students. This study is similar to the present study because both focused on solving systems of equations in two variables. However, the studies differ in design, participants as well as the country in which they were conducted.

Samuel (2012) made an investigation on students' preference of two methods of solving simultaneous equations. Three research questions and three hypotheses were formulated. The research questions were answered using the descriptive statistics of percentages, while, the hypotheses were tested using the chi-square. Survey design was

used for the study. The total population of SS2 students in Sapele local government area in 2007/2008 Session was 776. Proportionate stratified random sampling technique was used to obtained equitable sample size in the three-urban, semi-urban and rural areas. The total sample size used for the study was 458. The instrument for data collection was a Structured Test Questions (STQ) for the SS2 students taught by the researcher. The STQ given to the SS2 students was named Students' Preference of Method Assessment Test Questions (SPMATQ). The SPMATQ was face-validated by eight mathematics teachers. Kudder Richardson – 21 was applied to test the reliability of SPMATQ to obtained r = 0.73. The findings revealed that SS2 students' preferred substitution method irrespective of their gender for solving simultaneous equations. A recommendation was that students' preference order (substitution method) should be encouraged or followed in teaching and learning of simultaneous equations. This study is similar to the present study with regard to the topic and the attitude questionnaire that sought for students' opinion on preferred method of solution. However, they difer in design used, subjects and study area.

Another study by Fatade (2012) who investigated the effectiveness of Problem-based learning (PBL) in the Further Mathematics classrooms in Nigeria. The study employed pre-test-post-test non-equivalent control group quasi-experimental design. The population consisted of all Further Mathematics students in the S S I in Ijebu division of Ogun State, Nigeria. Two schools were selected from eight schools that were taking Further Mathematics using purposive and simple random sampling techniques. One school was randomly assigned as the experimental while the other as the control school. Intact classes were used and in all, 96 students participated in the study in which 42 were in the experimental group taught through the PBL and 54 in the control group taught using the

Traditional Method (TM)). Four research questions and four research hypotheses were raised, answered, and tested in the study. Four research instruments namely pre-test manipulated at two levels: Researcher-Designed Test (RDT) and Teacher- Made Test (TMT); post-test manipulated at two levels: RDT and TMT; pre-treatment survey of Students Beliefs about Further Mathematics Questionnaire (SBFMQ); and post-treatment survey of SBFMQ were developed for the study. The study lasted thirteen weeks (three weeks for pilot study and ten weeks for main study) and data collected were analysed using Mean, Standard deviation, Independent Samples t-test statistic, and Analysis of Variance. Findings revealed that there were statistically significant differences in the mean post-test achievement scores on TMT *(t=-3.58, p<0.05)*, mean post-test achievement scores on RDT *(t=-5.92, p<0.05)* and mean post-treatment scores on SBFMQ *(t=-6.22, p<0.05)* between students exposed to the PBL and those exposed to the TM, all in favour of the PBL group.

It also revealed that there was statistically significant difference in the post-test achievement scores on TMT at knowledge *(t= -23.97, p<0.05)* and application (t= -11.41, p<0.05) but not at comprehension (*t= -0.50, p>0.05, ns*) levels of cognition between students exposed to the PBL and the TM. Based on the results, the study recommended that the PBL should be adopted as alternative instructional strategy to the TM in enhancing meaningful learning in Further Mathematics. Fatade's study is akin to the present study because both employed problem solving strategy. Nevertheles, their point of departure included place where the study was conducted, the population of the study and the instruments used for data collection.

In the study carried out by Salman, Ayinla, Adeniyi, Ogundele and Ameen, (2012) on the effects of problem solving instructional strategy on attitude of senior secondary school students towards mathematics in Ondo, Nigeria. The study employed quasi experiment consisting of pretest posttest control group design involving a 2 × 3 factorial design. The population for the study comprised of all SS II students in Ondo having a sample of 173 students that was purposively selected by simple random sampling technique. The idependent variables were the problem solving instructional strategy and the scoring levels, while the dependent variable was students' attitude to mathematics. The test scores were analysed using mean scores, standard deviations, t-test and Analysis of variance on the two null hypotheses at $\alpha = 0.05$ to determine the level of significance. The research findings indicated that the experimental group performed significantly better in mathematics than the control group. Study recommended among others that, mathematics teachers should adopt the use of problem solving instructional strategy in teaching mathematics at all levels of education. The study has similarity with the present study with regards to the independent variable and the design. However, it differs from the present research in the area of study, instruments and participants.

Hood (2013) studied the solution technique used by eight grade mathematics students while solving systems of equations. The purpose of the study was in three folds: (1) to test whether the students could find the solutions to the problems given, (2) to record the preference for the technique used by students when solving systems of equations, and (3) to use the information obtained from both assessment and survey to come up with the most efficient curriculum for teaching how to find the solution of simultaneous linear equations in the secondary schools. Thus, 58 out of a total population of approximately

900 students in kindergarten through twelfth grade participated in the study. Of this number, 36 were males and 22 were females. The sample comprised four classes that were divided into two groups during the experiment. The first group of students was introduced to substitution method first, while the second group was introduced to elimination method first. Six hypotheses were formulated and tested at 5% significance level. The first three were tested using analysis of variance (ANOVA) while the last three were tested using Chi-Square test statistic. The results of the study showed that there were no statistical differences infavour of the six hypotheses formulated. Though there was no significant differences between the variables of interest in the study, it was however, found in the study among others that students that were taught substitution first preferred elimination over substitution by small margin, while those taught elimination first preferred substitution by large margin.

Similarly, the study revealed that students found problems on words, usually called verbal problems or story problems as the most difficult ones on the assessment. Therefore the study concluded that students should be encouraged to learn variety of methods to choose so that each student could use the one that suit him to solve a problem; be encouraged to learn the fundamentals of both arithmetic and algebra that serve as the foundation for learning complex concepts in mathematics, and teachers should connect the teaching of mathematics to real-life situations to eliminate fear of the subject among students. Certainly, Hood's research is very relevant to the present research because there are some important similarities between these researches. Three of the most important ones involve the content, research design, and the instruments for data collection. Despite these similarities, the two studies have however, some striking differences which include the

type of participants involved in each of the studies, and the countries' of their conduct. Other striking differences between that research and the present one is that students' retention and attitude to learning of mathematics have not been investigated by the previous research, which the present research intended to achieve in this respect.

Michelli (2013) studied the relationship between attitudes and achievement in mathematics among fifth grade students. The population of the study was 404 students in which 266 students participated in the study. Survey research design was used for the research study. Questionnaire was the instrument used for data collection, which consisted of two parts. The first part of the constructed questionnaire consisted of the Aiken Revised Mathematics Attitude Scale that comprised 20 questions with 5 possible answers for each using a Likert scale, was used to measure students' attitudes towards mathematics. The instrument measured four traits namely, extroversion, self-control, intellectual efficiency and consientiouness. While the second part of the constructed questionnaire was made up of a mathematics assessment from EasyCBM (EasyCBM, 2011) that was used for data collection on student achievement in mathematics. The test consisted of 48 multiple choice questions and included problems from geometry, number operations, and algebra. The statistical tools used for data analysis include Pearson product moment correlation, multiple regression, and t-test for independent samples. The results indicated that there was significant relationship between attitudes toward and achievement in mathematics. The results also revealed that males had a more positive attitude towards mathematics compared to females, but both males and females scored approximately the same on the achievement test. Finally, extroversion was the only trait to have a significant relationship with achievement. This study is similar to the present study with regards to the t-test

employed for data analysis. However, the studies differ in some ways in regards to the country of study; the types of participants; the types of design employed; the idependent variables; measures of relationship and regression analysis tools used for data analysis.

Ogdol and Lapinid (2013) examined the developing students' mathematical understanding on linear equations in two variables using an Understanding by Design (UbD). The UbD is a curriculum framework that de-emphasises the teaching of content knowledge and skills to focus on more general understanding. The purpose of the study was to evaluate the extent of the development of mathematical understanding in terms of instrumental and relational understanding (procedural and conceptual knowledge) of the students using the Understanding by Design. The study was carried out in a heterogeneous class of 40 first year high school students during the second grading period of school year 2012-2013 in a private laboratory school in Manila, China. The study employed the descriptive research design in which Qualitative and quantitative data were gathered using a) self-assessment checklist, b) instrumental understanding checklist for written exercises and performance task, c) relational understanding (procedural knowledge) checklist for written exercises and performance task, and d) relational understanding (conceptual knowledge) for written exercises and performance task. The descriptive statistics was used to support the quantitative analysis of determining if there were significant differences between the perceptions of the students on their skills and proficiency before and after the implementation of the unit plan on simultaneous linear equations in two variables.

Results showed that the students have developed almost all the instrumental understanding, the rate of the students who developed relational understanding (procedural knowledge) was remarkably high, and the results on the students' developed relational

understanding (conceptual knowledge) were promising. The z-statistic for differences between two proportions at 95% confidence showed that there are minimal significant differences between the students' perceptions of their skills and proficiency before and after the implementation of the UbD unit plan, and between the number of students who perceived that they could perform a certain skill and the number of students who were able to carry out the skill. The study recommended the engagement to more real world problems for the students to see the significance of studying mathematics and reinforcement of meaningful performance task as another means to gauge students' understanding. Although both studies focused on systems of linear equations in two variables, there are still some striking differences between them. These include the types of participants used; the research design in which the earlier study employed descriptive research design, whereas this research used quasi-experimental design; also the environments in which they took place are different.

Ogdol and Lapinid (2013) focused soley on the acquisition of instrumental and relational understanding whereas this study examined the impact of Polya's problem solving strategy on students' attitude, performace and retention in algebraic word problems involving simultaneous linear equations in two variables at the junior secondary school level. Another point of discontinuity concerns the instruments. The previous study used four measures to determine students' instrumental and relational understandings which included self-assessment checklist, instrumental understanding checklist for written exercises and performance task, relational understanding (procedural knowledge) checklist for written exercises and performance task, and relational understanding (conceptual knowledge) for written exercises and performance task. While the present research study

used attitude scale and test to measure students' perceptions and performance in algebraic word problems. Karatas and Baki (2013) carried out a study on effect of learning environments based on problem solving on students' achievements of problem solving. This study's sample consisted of 53 7th grade students in which 27 students were assigned to experiment group and 26 as control group. The aim of the study was to create a problem solving based learning environment to enhance the students' problem solving skill. Students practiced activities and problems through Polya (1945)'s problem solving phases, and throughout the study, students' success in problem solving have been evaluated.

The instrument consisted of eleven problem solving activities that were given to the students at the beginning, middle and end of the study, and the students' performances were analyzed based on problem solving phases. The findings revealed that, the experimental group students' success in problem solving activities has significantly increased, while the control group students' success has not changed significantly. Both studies applied polya's problem solving strategy and quasi-experimental design. The studies have, however, diferred in some ways. These include countries, participants and instruments. Nneji (2013) investigated the effect of polya's problem solving model on students' achievement and retention. Quasi-experimental design with Pretest-posttest, non equivalent control group was used. Eight intact classes, four of which were randomly assigned to experimental and the other four to control groups were used for the study. Sample of the study consisted of 220 SSII students from Ishielu Local Government Area of Ebonyi State. Four research questions and four hypotheses guided the study. Algebra Achievement Test (ALAT) was used for data collection. The ALAT instrument was constructed by the researcher and validated by three research experts. Mean and standard

deviation were used to answer the research questions while the hypotheses were tested at .05 level of significance using Analysis of Covariance (ANCOVA). Experimental groups were taught by Polya George's Problem Solving Model (POGPROSMO) while control groups were taught the same topics using expository method. Findings of the study revealed that students taught Algebra with POGPROSMO achieved higher and retained more than those taught with expository method. There was no significant difference between the mean achievement and retention scores of male and female students in the study.

It was recommended that Mathematics teachers should adopt POGPROSMO in teaching Algebra. This study is related to the present study because both used quasi experimental design and Polya's problem solving model to investigate students' performance and retention. These research studies have their point of departure in regards to area of study, the instruments used and the participants as well. Ajai, Imoko and O'kwu (2013) examined the effect of problem-Based learning (PBL) approach on senior secondary school students' achievement in algebra. A quasi experimental design with pre-test, posttest control group. A total of 447 SS I students of six grant-aided and government schools sampled using multistage sampling were involved in the study. 211 students were assigned to experimental group while 236 students were assigned to the control group. The instrument used for data collection was Algebra Achievement Test (AAT) constructed by the researchers. Four null hypotheses were raised and tested at $\alpha = 0.05$ level using Analysis of Covariance (ANCOVA). The results of the study revealed that students exposed to PBL performed significantly higher in the post test than those exposed to conventional method. It was also shown that there were no significant interaction effects

on the achievement due to methods and gender. The study recommended that the strategy should be used by teachers of mathematics to improve students' achievement. The present study is similar to the previous one in the method of instruction, design and the content area. The studies differ in population, area of study, statistical tools, and the type of subjects used.

Popoola (2014) carried out a study to investigate the effect of problem solving teaching strategy on performance and attitude of students towards mathematics in colleges of agriculture in North-west zone of Nigeria. The study adopted quasi-experimental pre-test, post-test, control group design. The population of the study consisted of all the National Diploma One students in the thirty five Colleges of Agriculture in Nigeria, having a total of 11, 010 students. The population composed of students from three categories of agricultural institutions in the country namely, State Colleges of Agriculture, University Colleges of Agriculture and Federal Colleges of Agriculture. The State Collges of Agriculture consisted of 9413 students, the University Colleges of Agriculture consisted of 500 students, while the Federal Colleges of Agriculture consisted of 1097 students. Four out of seven Colleges of Agriculture in the North-west zone were selected for the study in which two colleges were in urban area and the other two were in the rural area. The sample for the study was 274 (232 males and 42 females) National Diploma students. the instruments used for the study were Mathematics Achievement Test (MACT) and Students' Mathematics Attitude Questionnaire (SMAQ). The MACT was used to measure students' understanding of mathematics after the teaching experiment. The test consisted of 40 objective questions drawn from the areas of Logic, matrix, Pascal triangle and Permutation. While the SMAQ consisted of 35 items that sought for students' perceptions

and feelings towards mathematics. Findings from the study showed that students that were taught mathematics with problem-solving teaching strategy outperformed their counterparts taught mathematics with lecture method. It was also revealed that students in experimental group had more favourable attitude towards mathematics, and retained mathematics more than the students in the control group. Nevertheless, the result indicated that there was no significant difference between the attitudinal mean scores of male and female respondents towards mathematics in colleges of agriculture. These studies are related in that both studied performance and attitude; employed the same research design and t-test for independent samples. Despite these similaries, the studies differ in some ways including the types of participants used; environments where the studies were conducted; one used three dependent variables and the other used only two dependent variables.

Lupahla (2014) used Polya's problem-solving model to map the level of development of the algebraic problem solving skills of Grade 12 learners from the Oshana Region in Northern Namibia. A sample of 210 Grade 12 learners was used for the study. The study employed computer aided algebraic problem solving assessment (CAAPSA) tool. The assessment framework of the learners' problem solving skills was based on the Trends in International Mathematics and Science Study (TIMSS), Schoenfeld's (1992) theory of metacognition and Polya's (1957) problem solving model. The study followed a mixed methods triangulation design, in which both quantitative and qualitative data were collected and analysed simultaneously. The data collection instruments involved a knowledge base diagnostic test, an algebraic problem solving achievement test, an item analysis matrix for evaluating alignment of examination content to curriculum assessment

objectives, a purposively selected sample of learners' solution snippets, learner questionnaire and task-based learner interviews. The study found that 83.8% of the learners were at or below TIMSS level 2 (low) of algebraic problem solving skills. There was a moderate correlation between the achievement in the knowledge base and algebraic problem solving test (Pearson r = 0.5). It was also found that there was a high correlation between the learners' achievement in the algebraic problem solving test and achievement in the final Namibia Senior Secondary Certificate (NSSC) examination of 2010 (Pearson r = 0.7). Most learners encountered difficulties in Polya's first step, which focuses on the reading and understanding of the problem. The algebraic strategy was the most successfully employed solution strategy. This study has similarity to the present study with respect to the method employed. It is, however, different from the present study in terms of study area, instruments, sample and design.

Mehmood (2014) made investigation on the effect of polya's problem solving method of teaching on achievement of revised bloom's taxonomy in mathematics at elementary level. The objectives of the study were to find out the effect of problem solving method on conceptual knowledge dimention sub-levels of remembering, understanding, applying, analyzing, evaluating, creating, as well as its effect on Higher Order Thinking skills of Revised Bloom's Taxonomy. Eight research questions and eight null hypotheses were formulated. The study used true experimental pretest posttest (double control) design. The population of the study consisted of all 6932, 8^{th} grade male students in Islamabad model schools. The sample for the study was 132 8th grade students that were selected using simple random sampling. Pre-test which consisted of 60 items of six cognitive processes was developed. The test was validated by experts and reliability coefficient of

the instrument was found to be 0.89. On the basis of pre-test scores, three groups were formed through multistage sampling. The Experimental Group was taught using Problem Solving Method while the other two groups were taught by Conventional Method. Eight weeks were used to conduct the teaching experiment, and the posottest was developed by changing the order of the questions of the pre-test. One Way ANOVA and t-test were used for data analysis. The results indicated that students taught mathematics through Problem Solving Method performed better on Revised Bloom Taxonomy as compared to Conventional method. Findings of the study revealed that students taught with problem solving method had significant improvement at all levels of knowledge dimension than their counterparts in the conventional method of teaching mathematics. Therefore, problem solving method was recommended for teaching of Mathematics at Elementary level.

This study is similar to the previous study because both used Polya's problem solving strategy and t-test for data analysis. Nontheless, they differed in their country of study, the participants of the study, the experimental design, and the type of instruments used for data collection. Salman and Ameen (2014) compared the effects of two problem solving models on senior secondary school students' performance in mathematics word problems. They purposively selected a sample of 180 senior secondary two students from six secondary schools in Illorin metropolis to participate in the study. They used pre-test, post-test, quasi experimental design. The instrument used for data collection in the study was Mathematics Performance Test (MPT). The experimental groups were taught by polya's problem solving and Bransford and Stain while the control groups were taught by lecture method. The findings of the study showed that there was a significant difference between the performance of experimental and control groups. Also, it was found that

Gender had no significantr significant influence on the performance of the students exposed to the instructional models. It was also revealed that subject combination of students had significant effect on the performance of students exposed to the polya's problem solving model with the science students having the highest mean score followed by the commercial students while the art students had the least mean gain score. The study recommended among other things that mathematics teachers and mathematics textbooks authors should endearvour to employ Polya and Bransford and Stain problem solving models to enhance meaningful learning in mathematics. This study is related to the present study in that they both used quasi-experimental design and Polya's problem solving strategy. They, however, differ in the study area, sampling method, instruments and the subjects employed for the study.

Esan (2015) examined the effects of cooperative problem-solving strategy on the learning outcomes of Nigerian secondary school students in algebraic word problems. A sample of 240 junior secondary school students in Ilesa township of Osun State of Nigeria. The study samples were randomely assigned to control and experimental groups. The control group was taught using conventional method, while the experimental group was taught using the cooperative proglem solving strategy. The instruments used for the study were the cooperative problem solving instructional modules for solving word problems in algebra, the mathematics achievement test and the problem solving attitude scale. The t-test statistic was used to analyze the data. Findings from the study indicated that the overall achievement of students exposed to cooperative problem solving strategy was better than those exposed to the conventional method. Esan's study is related to the present study in that it both employed algebraic word problems involving simultaneous linear equations

and Polya's problem solving heuristics at the junior secondary school level. However, the two studies are different in that while Esan's investigated students' achievement and attitude using sixteen item questionnaire based on modified Likert scale version, this study examined the impact of problem solving strategy on attitude , performance and retention in algebra. A study by Caleb (2015), investigated the effects of problem-solving and discussion teaching methods on students' achievement across ability levels in genetics. The study adopted a pretest-posttest quasi-experimental design with non-equivalent control group. The sample for the study comprised of 122 SS III biology students. Purposive sampling was used to select four schools in Gombe education zone. The instrument used for the study was the Genetics Achievement Test (GAT). The instrument contained 20 multiple-choice item and 2 essay questions. The data collected were analyzed using mean and standard deviation and ANCOVA statistical tools.

The results showed among others that te ability level of students significantly influence achievement in genetics, high ability students outperformed their medium and low ability students; gender was not a significant factor, the interaction effect of gender and method was not statistically significant, and there was a significant difference in the mean achievement scores of students taught genetics using problem-solving and those taught using discussion teaching methods. These studies have similarities by using problem solving strategy. While the previous study investigated the effects of problem solving and discussion teaching methods on students' achievement across ability levels in genetics in the area of Biology, the present study investigated the impact of Polya's problem solving strategy on attitude, performance and retention in algebra. They also differ in instruments and area they were carried out.

Olatide, Esther and LeviI (2015) investigate the Effect of Polya Problem-Solving Model on Senior School Students' Performance in Current Electricity. The study was conducted in Ilorin metropolis in Kwara State Nigeria. Purposive sampling was used to select two schools in the area. Sixty S S II students were used in each school, making a total of 120 students. The study employed quasi experimental study of non- randomized, non equivalent pre-test post-test control group design. The experimental group was exposed to Polya Problem-Solving Model while the control group was exposed to Lecture method. The two groups were pre-tested and post-tested using Performance Test in Current Electricity (PTCE). Data collected after Six weeks were analyzed using mean, standard deviation and analysis of covariance (ANCOVA), the hypotheses were tested at alpha level of 0.05. The findings showed that students exposed to Polya Problem-Solving Model performed better than those exposed to Lecture method. The educational implications of the study were highlighted and recommendations were made.

These studies have similarities in that both used Polya's problem solving model, design and descriptive statics of the mean and standard deviation to answer research questions raised for the studies. Nontheles, they are different in regards to subject areas investigated, study area, instruments, imprential procedures, and participants. Woollard (2015) carried out a study to examine the use of the elimination method in the teaching of simultaneous equations. Specifically, the study focused on the merits of the elimination method and preferences for this method. This topic was taught in a sequence of six lessons to year 10 students. Three methods were used. These included elimination method, substitution method and graph method. IPads and the method of SOLO taxonomy were used during the lessons. Students' preference of method of solution was analysed where

the question allowed for choice, and their ability to answer the question where the method was specified. The research findings showed that students developed an understanding of the elimination method in order to use it effectively, developed a preference for this method and struggled with the substitution and graph methods. Therefore, it was concluded that the elimination method is the most appropriate for year 10 students. This research is related to the present study because both used simultaneous linear equations by substitution and elimination methods. Despite this, they are dissimilar in regards to design, the study area, the subjects and instructional materials. Crowley (2015) investigated the effects of problem-based learning on mathematics achievement of students who demonstrated higher ability in the subject area than their comparable peers. A sample of 65 students from six south-central Kentucky elementary schools participated in Project Gifted Education in Math and Science (Project GEMS). The students were assigned to one of three conditions: PBL-Plus, PBL, or Control based upon school of attendance.

The participants were then administered baseline testing in the fall of the third-grade year using the Test of Mathematical Abilities for Gifted Students (TOMAGS). The TOMAGS was then re-administered each subsequent spring (grades 3-6) for growth data. The data analysis was carried out by a mixed two-factor ANOVA. Findings revealed that there was no significant interaction between the groups across time. Therefore, it was determined that PBL instruction did not result in a greater level of mathematics achievement compared to a traditional curriculum; in addition, quantity of PBL instruction did not impact mathematics achievement. Similarly it was found that all groups demonstrated significant gains in mathematics achievement regardless of treatment condition. As a result, the study recommends further research employing stricter fidelity

checks and larger sample sizes. The present study is related to the previous study in terms of the problem solving strategy adopted. Nevertheless, the studies differ in regards to countries where they were conducted; the instruments used for data gathering, and the type of participants involed in the study.

Usman and Musa (2015) investigated the Effect of inquiry teaching method on students' achievement in algebra in Bauchi algebra of Bauchi state, Nigeria. The population of the study consisted of all the 10,814 JSS II students in the 53 schools in Bauchi LGA of Bauchi State. The sample comprised of 5,903 males and 4,911 females. The sample used for the study was 135 students from two randomly selected secondary schools in Bauchi metropolis. the study was a quasi experimental of non-equivalent control group design in which pretest-pottest was used. One school was assigned to experimental treatment and the other as control. The subjects in th experimental school were taught by inquiry method and subjects in the control school were taught using conventional method. The Algebraic Achievement Test (AAT) was the instrument used for data collection. Two research questions and two hypotheses guided the study. The mean and standard deviations were used to answer the research questions while the analysis of covariance (ANCOVA) was employed to test the hypotheses. The results showed that the post test performance of the inquiry method was higher than the conventional method. The study recommende that teachers should be trained the use inquiry method to enhance students' academic achievement in mathematics. The present research is relevant to the previous research study in some respects since they used quasi experimental design as well as the area investigated. The two studies also have diferences in the study area, the sample involved in the research, and Analysis of Coverience employed to test the hypotheses.

Brijlall (2015) explored the stages of Polya's problem solving model in grade ten learners whilst working with fractions. The purpose of the study was to investigate the learners' experiences during the mathematics class in problem solving tasks. A qualitative research method was adopted in the study. The population of the study was all the 47 learners in a grade ten classroom in which 24 learners were selected to participate in the study. The 24 learners consisted of 12 boys and 12 girls whose ages ranged from 14 to 17 years. The 24 learners were divided into 6 groups of 4 and 2 and 3 groups worked individually. The instruments used for the study consisted of semi-structured questionnaire, interview and essay questions. The semi-structured questionnaire was used to establish what transpired in and out of the problem-solving class during the research period by grade ten learners. The interview involved six learners from each class to explain the Polya's four steps in the solution process, while the questions on word problems were presented for learners to solve and examine which stages of Polya's linear problem solving model are demonstrated in their written solutions. The results revealed that learners that worked in groups demonstrated most of the stages of the Polya linear problem-solving model. The findings also identified the stages of the model that promoted effective problem solving. The study recommended among others that mathematics teahers should always present problems which have information from everyday life experiences; allow learners to work collaboratively during problem solving tasks. They should also be allowed to use any language to communicate mathematical ideas, concepts, generalizations and thought processes. This study is related to the previous study because both used Polya's model of problem sloving in collaborative classrooms. The studies, have, however, differ in country of study, participants, design as well as the instruments.

Raoano (2016) examined learners' mathematical word problem solving skills and strategies in Intermediate Phase. The purpose of the study was to improve learners' word problem solving skills and strategies through intervention in collaborative teaching. The study adopted action research collaborative method involving 26 Grade 6 learners and their mathematics educator. Problem solving model by Polya (1957) guided the study in answering three research questions: What are the challenges faced by Grade 6 learners in solving word problems? What are Grade 6 learners' strategies in solving word problems? How can learners' problem solving skills and strategies focusing on word problems be improved? Data were collected in a routine structured process: pre-intervention phase, intervention phase and post-intervention phase. Analysis was made through the development of a system of categorisation of learners' responses. The four principles of problem solving by Polya (1957) namely, the way learners understand the problem, how they devise the plan, how they carry out the plan and the manner in which they look back guided the analysis. Findings of the study showed that at the pre-intervention phase, learners were not exposed to any procedure or strategy that they could use to solve word problems. However, findings from the study revealed that during the internention, the strategies introduced assisted learners in making sense of the word problems and finally proceeding towards an adequate solution. Also, the findings indicated that the knowledge acquired by learners during the intervention stage helped them to solve word problems with ease. During the post-intervention phase, learners also showed that they had gained confidence when reporting their solutions. It was also revealed that collaborative teaching and direct teaching of skills and strategies assisted in improving the learners' skills and strategies in solving mathematical word problem. Also the study found out that the learners lacked the ability to read with understanding; the problem being their lack of competence in the

language of learning and teaching. The skills which learners also lacked when solving word problems were identified as arithmetic skills and reflective skills. This study is relevant to the present study in that four steps of problem solving by Polya (1957), that is, understanding the problem, devising the plan, carrying out the plan and looking back, were taught to learners in an attempt to improve their skills and strategies to solve word problems. Despite this, the present study differs from the previous one in respect to the country it was carried out, the educational level of the subjects, the design used, the instruments, and the analysis procedure adopted.

Lasak (2017) compared compared mathematics problem solving skills and mathematics achievement before and after using Polya's Problem Solving Process of mathematics student teachers. The study adopted one group Pretest-Posttest Design. The samples consisted of 30 mathematics student teachers who were studying in the second semester, the academic year 2016 at UdonThaniRajabhat University, UdonThani, Thailand that selected by cluster random sampling. The research instruments used for data collection consisted of lesson plan Polya's Problem Solving Process on Statistics for teacher, mathematics problem solving skills test and mathematics achievement test. The data were analyzed for mean, standard deviation, percentage, and t-test for dependent samples. The results revealed that both mathematics problem solving skills of mathematics student teachers who were taught by Polya's Problem Solving Process and mathematics achievement of mathematics student teachers who were taught by Polya's Problem Solving Process had significantly performed better in the posttests than in the pretests. This study is similar to the present study because both employed Polya's problem solving strategy but they differ in design, participants and their educational level, content area, as well as the country of study.

Nordin, Tengah, Shahrill, Tan and Leong (2017) employed action research to investigate the effect of replacing the common symbolic representations with pictorial (static visual) representations in solving simultaneous equations. The sample was 38 low-performing Year 10 students. The data for the study was collected by means of pre- and post-tests and facilitated by three worksheets as the intervention instruments. During the lessons, pictorial representations were used in solving simultaneous equations questions. These entailed drawing several pictorials that were given in the equations. These included drawings of burgers, fries, cheese slices, gift boxes and sweets were used as these were generally relatable to the students in real-life. The students were then provided with fresh worksheets and subsequently translated their workings from the pictorial part to *x* and *y*. And thus forms the symbolic part of the intervention. From the students" pre-test and post-test scores, it was revealed that the pictorial representations helped the students in making significant improvements in the topic. These studies are related in the topic being used. However, they are different in research design, study area, method, samples, as well as the study country.

Osuafor and Joseph (2017) investigated the effect of problem solving techniques on secondary school students' academic achievement in algebra. Two research questions were formulated and three null hypotheses were tested at 0.05 level of significance. The study adopted a quasi-experimental design, with pre-test posttest non-randomized control group design. The sample consisted of 80 SS 2 mathematics students from two co-educational schools in Owerri West Local Government Area of Imo State, Nigeria. Achievement Test in Algebra (ATA) used for data collection. The reliability coefficient of 0.94 was obtained using Kudder-Richardson formula 20. Data were analyzed using mean and ANCOVA.

Findings showed that the students taught with problem solving technique achieved higher than those taught with lecture method. Gender was found to have no significant influence on achievement in mathematics though the females performed slightly higher than the males. In addition, no significant interaction effect was observed between gender and treatment. The researchers recommended among others, that problem solving technique should be adopted by mathematics teachers to help students learn mathematics more effectively. This study is similar to the present study because both used quasi experimental design as well as problem solving strategy. Nevertheles, they difer in participants, inferential statistics and instruments as well.

Astriani, Surya and Syahputra (2017) investigated the effect of problem based learning to students' mathematical problem solving ability. The study employed quasi-experimental design. The study population comprised all students of class VIII MTs Al-Yusriyah. The sample for the study was two classes, VIII-A which was used as the experimental class while class VIII-B was as the control group. The samples were selected the selection by purposive sampling techniques. The instrument used was a test in the form of descriptions on Pythagoras theorem. Findings from the study revealed that problem-based learning model had effect on students' mathematical problem solving ability. Also, the experimental group taught by problem-based learning model performed significantly better than the control group by conventional method. The present study is similar to the Astriani, Surya and Syahputra (2017) study in respect to strategy. However, the present study differs from the previous study with respect to participants, instruments, geographical area and classes.

Fagbohun and Oni (2017) have carried out a study on practical realization of simultaneous equations solutions with elimination method and its effects on students performance in some selected schools in Ekiti and Ondo States of Nigeria. The instruments used were simultaneous equations and matrices training kits consisting of a simultaneous stepped-faced base, with a sizeable number of multi-colored cuboid inserted-numbered modeled cube, counters stand, counting number plates and arithmetic signs standing plates was developed for the practical teaching of Simultaneous equations in Schools. The two distinct differently colored cubes forms characters x and y to represent physical systems been used to represent values which of course are distinct from one another. 8 Secondary Schools in Ekiti and Ondo States of the Federal Republic of Nigeria with a class range of 82 to 124, with same teacher administering the subject over years, were selected for evaluation of the impact of the developed device in giving a clearer picture of the principles of solving simultaneous equations and matrices in order to strengthen students knowledge of applying the subject to varied physical problems. The percentage response of students that attempted, accurately solved the problems with and without the use of the kits were determined with respect to time of contacts. The results from the sampled Schools in use of the kits, shows that the total number of students attempt to questions on the Simultaneous equations in the whole class range between 80% and 91%, with between 82% and 97% correct solutions obtained, with between 3 and 4 contact class teachings of 45 minutes per class in comparison to the earlier 4 to 5 class teachings without the use of practical kits with between 22% and 40% of the total population of the school class well grounded in the methods of its solutions. The results showed that the practical training kits improves the perception of the theory of Simultaneous equations, and strengthen the

students in interpreting the underline principles. The study is related to the present study because both used simultaneous linear equations in two variables. Moreover, the studies have differed in terms of method, school type, and students' levels.

Yanti, Amin and Sulaiman (2018) investigated the representation of students in solving simultaneous linear equation problems based on three multiple intelligence. Specifically, the purpose of the study was to investigate representation of students who have musical, logical-mathematic and naturalist intelligence in solving a problem. The study was carried out on senior high school students in Manyar Gresik Indonesia, with 84 participan and selected 3 students with each subject at different intelligences at musical, logical-mathematic and naturalist intelligence. The data validation uses triangulation time, so that subjects are given TPM and interviewing twice in differrent time and with equivalent problem. Subjects were selected on the basis of multiple intelligence tests (TPM) consisting of 108 statements, with 102 statements adopted from Chislet and Chapman and 6 statements equal to eksistensial intelligences. Data were analyzed based on problem-solving tests (TPM) and interviewing. The validity of the data was determined by experts, then problem-solving tests (TPM) and interviewing were given twice with an analyzed using the representation indicator and the problem solving step. The results showed among others that, the stage of presenting information known, stage of devising a plan, and stage of carrying out the plan, those three subjects were using same form of representation. While the stage of presenting information asked and stage of looking back, subject of logical-mathematic was using different forms of representation with subjects of musical and naturalist intelligence. From this research, it is expected to provide input to the teacher in determining the learning strategy that will be used by considering the

representation of students with the basis of multiple intelligences. This study is akin to the present study because both used four steps of problem solving by Polya (1957) on simultaneous linear equation. Nevertheles, the studies differ in countries where thay were conducted and the type of participants involved in the research studies.

In the same vein, Rosemary and Anthony (2019) have examined the effectiveness of Polya and Woods problem solving models on students' academic performance in simple linear equation in one variable among junior secondary school students in Obio Akpor local government area of Rivers State. The study adopted pre-test post-test quasi-experimental design. Three questions and three hypotheses were formulated and tested at 0.05 level of significance. The population of the study was 6,045 students in which a sample of 200 students were selected using simple random sampling technic. The instrument used for data collection was Simple Linear Equation Performance Test (SLEPT). The mean and standard deviation were used to answer the research questions while t-test statistic was used to test the null hypotheses. The result indicated that students who were taught linear equation in one variable using Polya problem solving model performed significantly better than their counter parts who were taught with Woods problem solving model. The findings also showed significance difference in the performance of male and female students taught simple linear equation in one variable using Polya problem solving model. Similarly, significance difference was found between low and high achieving student in the experimental groups. The study recommended that Polya problem solving model should be used by mathematics teachers especially in teaching linear equation.

This study is similar to the previous one in that both used Polya's problem solving model and quasi experimental design with intact classes. However, this study differs from the preceeding one in that it involved single sex junior secondary three (JSS3) students, while the previous study focused on co-educational junior secondary two (JSS2) students. Also, the study examined the impact of Polya's problem solving strategy on students' attitude, performance and retention in simultaneous linear equation in two variables, while the preceeding one examined the effect of Polya and Woods models on the academic performance of JSS two students in simple linear equation in one variable. Furthermore, this study was conducted in Zamfara State while the previous study was carried out in Rivers State.

Crue, (n.d) conducted study to investigate the effects of experiences with a specific problem-solving model on students' ability to solve multi-step word problems and students‟ beliefs about problem solving. The samples were asked to complete test of the same 16 free response word problems and a beliefs survey before and two months after learning and using the problem-solving strategy. The findings showed that the overall and subscore means for the word problems post-test were higher than the pre-test means; the difference was statistically significant. Also, the results showed that students‟ abilities to approach word problems improved; the data showed that the number of problems that scored two or less points on the four-point, grading rubric decreased. A Likert scale was used to gather quantitative data about four beliefs. The mean scores on the post-survey were slightly higher but not statistically significant. Responses to open-ended items on the post-survey indicated that students found the problem-solving strategy useful. These studies are similar because both employed problem strategy, but they differ with respect to

country of study, samples, and instruments. From the review of the literature so far, many important similarities between the early studies and the present study have been identified. Despite these similarities, none of the early studies had collectively studied attitude, performance and retention in algebra at the junior secondary school level. Therefore, this study investigated the impact of Polya's problem solving strategy on attitude, performance and retention in algebra among junior secondary schools students in Zamfara State, Nigeria.

2.13 Implications of the Reviewed Literature

In the literature examined so far, the theories of learning of Dewey, 1916; Bruner, 1960; Vygotsky, 1978 and Piaget, 1980 have revealed a lot of benefits to the present study. For Piaget, assimilation and accommodation are the basic processes involved in adaptation. Piaget points out that assimilation and accommodation work mutually to develop a child's thinking and to create what he called cognitive equilibrium, which is a balance between what the child has already known and the new situation or experience he encountered. Therefore teachers should always connect the children's experience to what they learn. Piaget also contends that though development can proceeds without social interaction, the social environment in which a child lives is a major source for cognitive development. As such, activities that provide social interactions among learners during the learning process are essential. Piaget also recommended that teachers should provide rich environments that allow children for active explorations and hands-on activities. Unlike Piaget's theory, Dewey, Bruner, and Vygotsky's; however, place more emphasis on the acquisition of mental functions through social interaction as the key to cognitive development. Therefore, one of the instructional implications from socio-cultural theory is that teachers should

design instruction to facilitate scaffolding and cooperation among learners for knowledge sharing, and create good environment that develop mutual respect as well. Moreover, teaching and learning should involve activities that are meaningful and interesting to learners. More importantly, the learner's socio-cultural background should be given consideration so as to serve as the basis on which learning is built. It was also pointed out by researchers and educators that language plays a prominent role in the teaching and learning of mathematics, and that this subject makes use of a special language. The special language refers to symbols and notations that serve as instruments for communicating mathematical ideas and thoughts. The teaching implication is that before students are required to use and manipulate algebraic expressions and equations, the meanings of symbols must be established as they appear in different situations. Similarly, teachers must always put greater emphasis in explaining the structural aspects of what they teach such as expressions and equations. Furthermore, we learned from the literature reviewed that the type of school students attend has significant influence in their learning.

Although each school type contributes substantially to student development, some studies indicated that single-sex schooling has many gains over the other types of schooling (e.g., Ogden, 2011). One benefit derived from single-sex schooling is that they provide teachers greater opportunities to address students' needs particularly females. Teachers in those schools are more caring, and they are concerned with social and moral lessons that their students need to acquire, as well as academic ones. These teachers have higher expectations for their students. They therefore, suggested that single-sex schooling should be adopted. On the other hand, studies in the literature reviewed also highlight that the style or method employ by the teacher has strong support and contributes to the total

teaching system. Moreover, the literature notes that each method is also subject to criticism. Therefore, teachers must have a certain degree of knowledge, skills, and expertise in each method so as to be able to choose the suitable one that matches the interest and ability of learners. In the same way, a teacher must also be aware of the shortcomings and limitations of each method. Teachers should not be over dependent on a single method to teaching. In other words, they should have a variety of instructional techniques at their disposal and be creative and flexible enough to adapt the techniques to the particular needs of the learners. Many educators and researchers have highlighted the need for using different teaching techniques for different purposes in teaching (e.g., Arends, 2012). This is because one method of teaching will not be suitable in all situations. The literature reviewed has also stressed the importance of attitude in students' academic performance and importance of attitudinal variables (confidence, usefulness, enjoyment, subject is perceived as a male domain and teachers expectations) in enhancing learning.

Moreover, many teachers have expressed their concerns about students' negative feeling towards algebra topics and this leads to a main issue in the attitudes of students towards the subject. If students have negative attitudes towards algebra, they will eventually have low performance because algebra is the key to success in the learning of any mathematical domain. In this regard, mathematics teacher have to develop positive attitudes to their students through various means. One way of achieving this is to use variety of methods capable of promoting active learning, group interaction and cooperation among students. He should also show love and have confidence in the subject. On the other hand, the curriculum experts should as well plan appropriate curriculum that provides opportunities for students to engage in handling classroom activities meaningfully.

In summary, the various research studies that have been reviewed so far focused on other aspects of algebra and failed to investigate the impact of Polya's problem solving strategy on attitude, performance and retention in algebra at the junior secondary school level. The present study, therefore, examined the impact of Polya's problem solving on attitude, performance and retention in algebra among junior secondary school students in Zamfara State, Nigeria.

CHAPTER THREE

RESEARCH METHODOLOGY

3.1 Introduction

This study investigated the impact of Polya's problem solving strategy on attitude, performance and retention in algebra among junior secondary school students in Zamfara State, Nigeria. Therefore, this chapter explained the main methodological concepts used in various stages of this study. These include the Research design, Population of the Study, Sample and Sampling Techniques, Instrumentation, Simple Linear Equation and Word Problem Pre-Test (SLEWPPT), Simultaneous Linear Equation Performance Test (SLEPT) and Algebraic Students' Attitude Inventory Questionnaire (ASAIQ), Validation of the Instruments, Pilot Testing, Reliability of Instruments, Item Analysis, Item Difficulty Index of SLEPT, Item Discrimination Index of SLEPT, Procedure for Data Collection and Procedure for Data Analysis.

3.2 Research Design

This study adopted quasi-experimental design consisting of pre-test, post-test control group. This design was used when random selection of groups can not be achieved, (Walliman, 2011). According to Ary, Jacob and Razavieh (2002) quasi-experiment is used when intact classrooms in school setting are used as the experimental and control groups. This design was used in this study because it permits the researcher to reach reasonable conclusions even though full control is not possible (Ary, Jacob and Razavieh, 2002). Two groups of subjects were, therefore, used in this study: the experimental group and the control group. The experimental group was taught using the Polya's (1957) problem solving model, while the control group was taught by conventional method of teaching.

There are four public schools in each of the experimental and control groups. A pretest was administered on the subjects in both the experimental and control groups to determine their background knowledge and homogeneity. After the pretest, both the experimental and control groups were given treatment for six weeks and then took a posttest to determine the effect of the treatment. Similarly, two weeks after the posttest, a postpost-test was administered to the subjects to measure their retention level. The diagram for the research design is presented as follows:

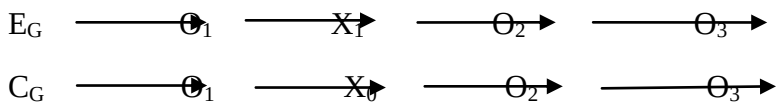


Figure 3.1: Illustration of Research Design

Where:

E_G = Experimental group

C_G = Control group

X_1 = Treatment (Polya's Problem Solving Strategy)

X_0 = Pseudo-treatment (Lecture Method)

O_1 = Pretest

O_2 = Posttest

O_3 = Retention test

3.3 Population of the Study

The target population for this study consisted of all the Public Junior Secondary School Three (JSSIII) students in one hundred and eighty (180) public junior secondary schools under Zamfara State Teachers Service Board (ZSTSB). These schools are located in the fourteen (14) local government areas of the state. The local government areas are divided into four Education Zones that comprise Anka, Talata Mafara, Gusau, and Kaura

Namoda. Public Junior Secondary Schools in Zamfara State have a total enrollment of twenty seven thousand three hundred and fifty seven (27,357) Junior Secondary School III students as at 2016/2017 academic year. Of this number, twenty one thousand and eighty five (21,085) were males while six thousand two hundred and seventy two (6,272) were females (Appendix O). The summary of the enrollment figures of public Junior Secondary School III students in each of the four Education Zones in Zamfara State is presented in Table 3.1

Table 3.1 Summary of Target Population of JSS III students' in Public Junior Secondary Schools under ZSTSB

S/N	Zone	Number of L.G.A	Number of Schools	Gender		Total
				Male	Female	
1.	Anka	3	28	2564	779	3343
2.	Talata Mafara	3	39	3544	924	4468
3.	Gusau	4	71	10007	3772	13779
4.	Kaura Namoda	4	42	4970	797	5767
Total		**14**	**180**	**21,085**	**6,272**	**27, 357**

Source: Zamfara State Post Primary Schools' Enrolment Report (2016/2017)

3.4 Sample and Sampling Techniques

There are ninety nine public junior secondary schools in the eight local government areas selected for this study. The sample for this study consisted of students from only the ninety eight single-sex junior secondary schools in eight local governments within the four Education Zones. These include eighty three male and fifteen female junior secondary schools. Two local governments were selected from each Education Zone using simple random sampling technique. Similarly, one school was selected from each local government area by simple random sampling technique, giving a total of eight (8) schools. The two schools that were selected from each Education Zone were of different gender (one male school and one female school). Therefore, a total of four schools (2 males and 2 females) were assigned to experimental treatment and the other four (2 males and 2

females) to control group by using balloting method. One intact class was drawn from each of the eight selected schools through simple random sampling method, giving a total of eight intact classes. The sample for this study was classified into three levels of data analysis: pretest, posttest and postpost-test. The sample size in each level was considered appropriate for this study according to Krejcie and Morgan (1971) table and the central limit theorem which stated that a minimum sample of 30 subjects is viable for experimental study as reported by Tuckman (1975) and Sambo (2008). Moreover, the population from which the sample was drawn is finite and normally distributed. Table 3.3a, 3.3b and 3.3c showed the samples used for the study at the pretest, posttest and retention levels respectively.

Table 3.3a Baseline Sample of the Study

Assessment	Group	Gender	Sample	Total
Pretest	Experimental	Male	120	235
		Female	115	
	Control	Male	116	228
		Female	112	
Total			**463**	**463**

At the beginning of the study, a pretest was given to 463 subjects. This number consisted of 235 (120 males and 115 females) subjects in the experimental group and 228 (116 males and 112 females) subjects in the control group. The purposes for given the pretest were to provide the researcher with background information of the subjects on knowledge and skills needed to solve algebraic word problems involving simultaneous linear equations. The pretest was also used to determine the initial equivalence of the subjects before the intervention. Thus the Simple Linear Equation and Word Problem Pre-Test (SLEWPPT) instrument comprising of ten items with five items on simple linear

equations and five on worded problems involving linear equations were used for data collection in the first phase of this study.

Table 3.3b Assignment of Subjects into Experimental and Control Groups

Assessment	Group	Gender	Sample	Total
Posttest	Experimental	Male	117	230
		Female	113	
	Control	Male	113	222
		Female	109	
Total			**452**	**452**

After the teaching experiment, a posttest was administered on 452 of the 463 subjects that took the pretest in the first phase of the study. The higher turnout of the subjects during the posttest was due to the headcount exercise that was conducted by Zamfara State Ministry of Education to determine the the actual enrollment figure of students in various secondary schools of the state as reported by the research assisstants. As a result, eleven (11) of the four hundred and sixty three (463) subjects that took the pretest in the first phase of the study did not participate in the posttest. Hence, four hundred and fifty two (452) subjects constituted the sample used for data analysis on the posttest for the second phase of the study. The sample comprised of 230 (117 males and 113 females) subjects in the experimental group and 222 (113 males and 109 females) subjects in the control group. The purpose of the posttest was to determine the effect of the teaching experiment in the content of algebraic word problems in simultaneous linear equations by comparing the performances of the subjects in the experimental and control groups. It was also used to compare the performances of gender subjects within the experimental group.

Table 3.3c Retention Sample of the Study

Assessment	Group	Gender	Sample	Total
Retention	Experimental	Male	115	225
		Female	110	
	Control	Male	104	203
		Female	99	
Total			**428**	**428**

In the third phase of this study, a post-posttest was administered on 428 out of 452 subjects that took the posttest in which 24 were the subjects mortality. This number consisted of 225 (115 males and 110 females) subjects in the experimental group and 203 (104 males and 99 females) subjects in the control group. The reason for having the post-posttest after a lapse of two weeks without interveining instruction was to determine the retention levels of the study subjects in the content of algebraic word problems involving simultaneous linear equation in two variables. In order to achieve this, the questions used in the posttest were shuffled and used for the post-posttest. This was especially necessary to further ascertain the effectiveness or otherwise of the treatment on the content of the algebraic word problems by the study subjects.

3.5 Instrumentation

There are three sets of instruments used in this study. These include Simple Linear Equation and Word Problem Pre-Test (SLEWPPT), Simultaneous Linear Equation Performance Test (SLEPT) and Algebraic Students' Attitude Inventory Questionnaire (ASAIQ). The SLEWPPT was administered at the beginning of the study and used to measure the background knowledge and skills as well as the initial equivalence of the subjects in simple linear equation and word problem involving one variable. The SLEPT was used to measure students' performance in algebraic word problems involving two variables after the teaching experiment, while the ASAIQ was used to measure students'

attitude towards algebra. All the three instruments were validated by five specialists in mathematics education and one language educator in both face and content, and their reliability coefficients were determined. These instruments are stated and explained as follows:

3.5.1 Simple Linear Equation and Word Problem Pre-Test (SLEWPPT)

To begin the teaching experiment in the sampled schools for this study a pre-test performance on simple linear equation and `word problem was administered to all the students. The purpose of the pre-test was to measure the students' entery skills and readiness levels for learning simultaneous linear equations, as well as their homogeneity on simple linear equation and word problem in linear equations in one variable. The test was divided into two sections (section A and B). Section 'A' contained five essay questions involving simple linear equations. The test consisted of five types of linear equations which represented the total content of linear equations in one variable included in the junior secondary school three mathematics classes. These include linear equation in standard form, linear equation in nonstandard form, linear equation containing parentheses, linear equation containing fractions and linear equation involving decimals. Section 'B' consisted of five essay questions involving algebraic word problems that require the subjects to translate word into its equivalent algebraic equation and then find the value of the variable for which the equation is true. The SLEWPPT was marked over 100. The test questions used in this study were adopted from New General Mathematics Text book for Junior Secondary School Three written by Macrae et al (2008). (Appendix B).

3.5.2 Simultaneous Linear Eqution Performance Test (SLEPT)

The SLEPT was an instrument constructed to measure the students' knowledge on algebraic word problems involving two variables. The test consisted of 10 essay items selected from Mathematical Association of Nigeria textbooks, New General Mathematics textbooks and Algebra textbooks. The test items were based on the five major categories of problems leading to simultaneous linear equations identified by Sullivan III and Struve (2007). These include direct translation problems, geometry problems, mixture problems, uniform motion problems and work problems. Two questions were constructed on each of the five major topics. The items for this test were specifically classified into two major cognitive categories of Bloom's taxonomy of educational objectives. They are Lower Order Thinking (LOT) and Higher Order Thinking (HOT). The LOT collapses Knowledge and Comprehension into one group while the HOT collapses Application, Analysis, Synthesis and Evaluation into another. According to Nitko and Brookhart (2007), the advantage of these condensations is that they eliminate the need for struggling to classify learning objectives into one of the top three categories of the taxonomy.

The Simultaneous Linear Eqution Performance Test (SLEPT) was used on two different occasions: first, as post-test to find out what the study subjects have gained on the content of algebraic word problems in simultaneous linear equations after the intervention. And second, as a post-posttest to determine the retained content of algebraic word problem in simultaneous linear equations that the students have learnt. The maximum score for each item in SLEPT is 10 marks while the minimum is zero. Since there are ten items, the maximum score is 100. A maximum of 2 hours 30 minutes was allocated on the test based

on the time spent on the instrument during the pilot study (Appendix E). The detail of item specification for SLEPT based on topics selected is presented in Table 3.4.

Table 3.4: Test Blueprint for Word Problems Leading to Simultaneous Linear Equations

s/n	Content	%	LOT	HOT	Total
1	Direct translation problems leading to SLE	20	1	1	2
2	Geometry Problems leading to SLE	20	1	1	2
3	Mixture Problems leading to SLE	20	1	1	2
4	Uniform Motion Problems leading to SLE	20	1	1	2
5	Work Problems leading to SLE	20	1	1	2
	Total	**100**	**5**	**5**	**10**

Source: Adapted from Nitko and Brookhart (2007).

Key: LOT = Lower Order Thinking; HOT = Higher Order Thinking; SLE= Simultaneous Linear Equations

Table 3.4 presented the SLEPT which consists of 10 questions drawn from five content areas in the Junior Secondary School Three (JSS III) mathematics curriculum to measure cognitive ability of the students ranging from the lower order thinking to the higher order thinking. Each column presented the questions chosen for each category while the total presents the number of questions chosen for each topic.

3.5.3 Algebraic Students' Attitude Inventory Questionnaire (ASAIQ)

The ASAIQ was the instrument used to measure the attitude of the subjects toward algebra before and after they were exposed to problem solving strategy. The ASAIQ was adapted from Fennema & Sherman (1976) mathematics attitude scale. It was divided into five sections. Section A was designed to collect participants' profile, while sections (B-E) were used to collect respondents' perception regarding their attitudes toward algebra. The scales measured students' attitude in: confidence in learning algebra; attitude toward success in algebra; algebra as a male domain; and usefulness of algebra. Each of the sub-scales had 12 items of which six measured positive attitudes while the other six measured negative attitudes towards learning of algebra. As a whole, there were forty eight items in

the attitude scale. Therefore, to determine feeling of the study subjects about learning of algebra, they were asked to indicate their perception about each statement on a 5-point Likert scale, ranging from strongly agree (SA), agree (A), Neutral (N), disagree (D), and strongly disagree (SD). The instrument was administered on both control and experimental groups to determine their change in attitude or behavior in learning of simultaneous linear equations of algebra components in particular and algebra in general both before and after using problem solving strategy. The scoring pattern for the responses is 5, 4, 3, 2, and 1 for Strongly Agree, Agree, Neutral, Disagree, and Strongly Disagree respectively for positive responses. And for the negatively worded statements, the scoring pattern is in the reverse order, from 1 to 5 points. Thus, the possible minimum score is 48 and the maximum score is 240 (Appendix G).

Table 3.5: Indicators of Students' Algebra Attitudinal change

	Expected Attitudinal changes			
Indicators of Attitude change in Algebra	**Cognitive change**	**Affective change**	**Behavioural change**	**Total**
Confidence in learning Algebra	4	4	4	12
Attitude toward success in Algebra	4	4	4	12
Algebra as a male domain	4	4	4	12
Usefulness of Algebra	4	4	4	12
Total	**16**	**16**	**16**	**48**

Source: Adapted from Nitko and Brookhart (2007)

Table 3.5, presented the expected attitudinal changes in the students' cognitive, affective and social components of algebra learning. The table also presents four indicators of attitudinal change with 48 items to be used in evaluating students' opinion toward learning of algebra. The items are distributed over the three domains of attitudinal change. Each column presents the number of items selected for each domain while the total presents the number of items selected for each indicator.

3.6 Validition of the Instruments

The SLEWPPT, SLEPT and ASAIQ were the three main instruments used for data collection in this study. Therefore in order to have valid conclusions from the data of the study, the instruments were secrutinized by the specialists in mathematics education, language education and psychology education. The validation of the instruments is decribed as follows:

3.6.1 Validation of Simple Linear Equation and Word Problem Pre-Test (SLEWPPT)

The validation of the (SLEWPPT) was carried out by six mathematics education specialists. Three of the specialists were mathematics teachers with first degree in mathematics education. The teachers had taught between ten and twelve years in secondary schools (Appendices AB to AD). The other three were Ph.D holders in mathematics education from Department of Science Education, Ahmadu Bello University Zaria. The draft sample questions of ten items comprising five in section A and five in section B was submitted to the subject specialists who inturn scrutinised the items by.looking at the content covered, the wording of the questions, and the adequacy of the sample of the items to measure the performance in question. As a result, the items for removal and inclusion in the two sections of the pre-test were identified and necessary changes were apprpriately made. Thus, questions 1, 2, 3(a) and 4(a) in section A of the initial draft were removed and replaced with appropriate questions that would elicite the actual behavior being measured, while question 5 was included in the final version (Appendices A & B). Similarly, questions 2 and 5 in section B were removed and replaced with suitable questions while questions 1, 3 and 4 were retained (Appendices A & B).

3.6.2 Validation of Simultaneous Linear Equation Performance Test (SLEPT)

To ensure the full realization of the objectives of this study, the researcher submitted twenty essay test items to six mathematics educators. Three were teachers from public Senior Secondary Schools with B. Ed degree in mathematics education and had taught between ten and twelve years (Appendices, AB to AD). The other three were senior lecturers with Ph. D in Mathematics Education from Science Education Department, Ahmadu Bello University Zaria. These experts were requested to scrutinize the items in terms of face and content validity. The experts were, however, guided by a table of specification that helped them relate to the objectives and content of the test (Table 3.4). After a careful study of the items, the experts recommended the exclusion of three out of the twenty items on the instrument due to their lengthiness in wordings and difficulty level. These included questions 2, 7 and 20. Nevertheless, these questions were substituted with equivalent ones that were in accordance with the ability level of the students. The final draft copy of the instrument was produced and used for pilot study (Appendices C, D & M).

3.6.3 Validation of Algebraic Students' Attitude Inventory Questionnaire (ASAIQ)

The ASAIQ instrument was submitted to four experts. One from the department of General Studies with Ph. D in Language Education, Umaru Ali Shinkafi Polytechnic Sokoto and three others from the Department of Science Education with Ph. D in Mathematics Education, Ahmadu Bello University Zaria. The validation was primarily conducted to increase the validity and practicality of the questionnaire. Specifically, it was aimed to: check the clarity of the questionnaire items, instructions and layout; eliminate ambiguities in wording; check the time taken to complete the questionnaire and identify

items that are too easy or too difficult from the respondents' experience. In the analysis, the expert in language eduction had found that items 1, 19 and 20, as well as 31 included in the subscales of confidence in learning algebra, attitude towards success in algebra and algebra as a male domain respectively were too difficult for the respondents to assimilate and this could lead to misinterpretations and inappropriate completion of the items. As such, five words which included "secure", "unpleasantly conspicuous, creep" and "genius" contained in each of the three items of the three subscales on the instrument were revised and appropriate words that included "safe", "uncomfortable and cheated", and "intelligent" were respectively used to replace them (Appendix, AE). Similar observations were also made by the mathematics education experts in regards to the content of the instrument. The experts suggested that the term 'mathematics' that appeared in every subscale be replaced with the term 'algebra' in order to reflect the objectives of the study (Appendices F & G). All the comments and suggestions provided by the experts were then incorporated into the final version before they were used for the study.

3.7 Pilot Study

The Simultaneous Linear Equation Performance Test (SLEPT) and Algebraic Students' Attitude Inventory (ASAIQ) were piloted using thirty (30) JSS III students of Unguwar Dan Baba Government Day Secondary School, Gusau. The students were selected using balloting method of sampling. The sample for the pilot study consisted of 15 male and 15 female students. The researcher with the help of two mathematics teachers of the school administered the two instruments. The SLEPT was administered to the students after an interval of two weeks between the two administrations as recommended by Turkman (1975) while the ASAIQ was administered once. Both SLEPT and ASAIQ were

administered on the same day. The reliability coefficient of SLEPT was determined using the Pearson Product Moment Correlation Coefficient, while the reliability coefficient of ASAIQ was determined using Cronbach's alpha method. The purpose of piloting the instruments was among others to determine the appropriate time to be used in responding to the items in the instruments; eliminate ambiguities or difficulties in wording; establish reliability and item characteristics of the instruments.

3.8 Reliability of the Instruments

The reliability of a measure is its ability to produce similar results when repeated measurements are made under identical conditions (Bordens & Abbott, 2008). Thefore, the reliabilities of SLEPT and ASAIQ are decribed as follows:

3.8.1 Reliability of Simultaneous Linear Equation Performance Test (SLEPT)

In this study, the reliability of Simultaneous Linear Equation Performance Test (SLEPT) was established through a test-retest method with a two-week period between the two testings. Using Pearson Product Moment Correlation Coefficient (r) in Microsoft Office Excel, a correlation coefficient of 0.792 was obtained for SLEPT. Rowntree (1991) indicates that a correlation coefficient ranging from 0.7 to 0.9 is relatively high and could be used in a research study. Moreover, Fraenkel and Wallen (2000) highlight that for research purposes; a useful rule of thumb is that reliability should be at least 0.70 and preferably higher. Therefore, having obtained a value of 0.792 after the retest, the researcher has considered the instrument reliable for this study (Appendix L).

3.8.2 Reliability of Algebraic Students' Attitude Inventory Questionnaire (ASAIQ)

The Cronbach's Coefficient Alpha was computed to determine the reliability of the survey items for Algebraic Students' Attitude Inventory Questionnaire (ASAIQ). The

alpha coefficient value was found to be 0.826. This estimate was obtained from twenty eight items of the questionnaire that were properly coded by the respondents, while the remaining two were excluded by the SPSS package during the analysis due to wrong coding. Sambo (2008) indicates that an estimated reliability value of above 0.7 is accepatable for an instrument to be used in a research study. Therefore, the reliability coefficient of 0.826 was considered satisfactory for this study (Appendix P).

3.9 Item Analysis of SLEPT

Item analysis is a procedure for determining the relationship between item scores and total scores in a test (Ogunleye, 2000). It is therefore important that in a research study using achievement test, items in the test be properly analyzed for the purpose of ascertaining their validity and reliability. To achieve the purpose of the study, an item analysis was conducted by using the result of the data collected from the pilot study to determine both the facility and discrimination indices of the test items. Items that were too easy or too difficult for the students as well as those that failed to discriminate well between the bright and poor students were discarded using item facility and item discrimination procedures. Based on these criteria, the 20 questions for the pilot study were finally reduced to 10 questions and used for posttest and post-posttest (Appendices D & E). The procedure for calculating item facility and item discrimination indices are described.

3.9.1 Item Facility Index

On like the multiple-choice response, essay test contains tasks with multipoint scoring. In an essay test marks are awarded at each stage in the solution process by using rubirics that involve method marks, marks for accuracy and independent accuracy marks not preceded by method mark. Thus, after the administration and scoring of the test items,

test scores were arranged in descending order of magnitude and the top 25% scorers and bottom 25% scorers of the total subjects that took the test were selected for the computation of the item difficulty indices. In computing item difficulty from the item analysis data, a formula developed by Nitko and Brookhart (2007) was employed. According to Nitko and Brookhart (2007), the difficulty of an essay question or performance task scored by a rubric or rating scale is simply defined as the average score.

Therefore to determine the item difficulty index for the individual test items, the difference between the class average score on each item and the minimum possible item score was divided by the difference between the maximum possible item score and minimum possible item score. Thus from the calculations, the test items with difficulty index values ranging from 0.20 to 0.80 are considered appropriate for this study. And items with difficulty indices below 0.20 and above 0.80 are considered difficult and easy respectively and hence they were discarded as recommended by Scopes (1973) and Kalejaiye (1985). Based on these criteria, the 20 questions for the pilot study were finally reduced to 10 questions and used for posttest and post-posttest (Appendix M). Items that have asterisk are those that did not fall within the acceptable range and were therefore discarded as recommended by Scopes (1973) and Kalejaiye (1985) who reported that any difficulty indix ranging from 0.20 to 0.80 is sufficiently accurate for use in a study.

3.9.2 Item Discrimination Index

According to Nitko and Brookhart (2007), the discrimination index of an essay question or performance task scored by a rubric or rating scale is simply defined as the difference between the upper and lower group averages. The discrimination index describes the extent to which a particular test item is able to differentiate the higher-scoring

students from the the lower-scoring students. The item discriminating powers of the SLEPT items were obtained by using the same procedure used in computing the item difficulty. That means the top 25% scorers and bottom 25% scorers of the total sample of subjects who actually sat for the test were selected. Thus item discriminating powers of the SLEPT items were obtained by subtracting the lower group average score on each item from the upper group average score on each item and dividing by the range of possible scores. On the basis of these calculations and the recommendation by Scopes (1973) and Kalejaiye (1985), the test items with discrimination indices ranging from 0.20 to 0.80 were retained. Items having negative indices are considerd useless and were therefore discarded (Appendix M). Therefore a total of 10 questions were selected for the study (Appendix E).

3.10 Procedure for Data Collection

At the beginning of the term, the researcher visited the eight sampled schools and administered Simple Linear Equation and Word Problem Pre-Test **(**SLEWPPT**)** and Students' Algebraic Attitude Inventory Questionnaire (ASAIQ) on all the subjects with the help of the research assistants. The administration of the instruments was done for two consecutive days, one on each day. The pretest given to all the groups was used to measure the homogeneity nature of the students in terms of their current knowledge in simple linear equations in one variable and their attitudes toward algebra before the treatment was applied. To conduct the quasi-experiment, the researcher gave five days training to four JSSIII mathematics teachers of the experimental schools on the application of problem solving strategy. The teacher's role was to serve as guide and facilitator of knowledge to the learners. The researcher trained these teachers on how to prepare activies for the learners during lessons and the manner each group of students should be handled. Each

teacher was given a copy of validated lesson plans involving Direct Translation Problems, Geometry Problems, Mixture Problems, Uniform Motion Problems and Work Problems leading to simultaneous linear equations. The treatment for the study was carried out in the normal school term and class periods within six weeks. The selection of six weeks for the treatment was based on the guidelines contained in the scheme of work for junior secondary school three published by Zamfara State Universal Basic Education Board (ZSUBEB, 2012).

During the teaching period, the researcher went round the sampled schools to monitor the administration of the instructional units. After each lesson, students were given opportunity to work out related problems both individually and in groups. At the end of the treatment, a posttest was given to the samples in order to ascertain the effect of the treatment on students' performance in algebraic word problems in simultaneous equations. Similarly, the ASAIQ was administered to the participants after the posttest to determine the effect of students' attitudes toward algebra. Two weeks after the posttest, the SLEPT was re-administered to both experimental and control groups to measure their retention level regarding word problems involving simultaneous linear equations. The scoring of the SLEPT was done by the researcher after collecting the scripts from the research assistants in the schools. Following are the steps used in conducting lessons in the experimental classes using problem solving stragy.

Step 1: Introduction of the lesson (2 minutes). At the beginning of every topic, teacher asked students variety of questions on their knowledge base that has link to the current topic.

Step 2: Dividing the class into manageable groups (2 minutes). At this stage, the teacher divided the class into atleast 5 groups, and each group comprised 10 students of varying ability levels who supported one another in multiple ways during problem solving process.

Step 3: Assigning roles to group members (2 minutes). Teacher assigned a specific role to each group member to encourage interactions and divergent thinking among students during problem solving. The typical roles included a group leader who facilitates group discussion and makes sure that group members work together on task; a monitor who checks time on task and ensures that everyone gets an equal opportunity to participate; an encourager who inspires reluctant and shy students to participate in group discussions; a reflector who keeps the group aware of progress; a quiet captain who monitors noise level; and a recorder who keeps a written activity or writes the final results for the group.

Step 4: Assigning activities to groups (18 minutes). After assigning specific roles to the members of each group in the class, the teacher pasted a cardboard paper on the wall/board which contained a worded problem on simultaneous linear equations and asked students to solve using Polya's problem solving strategy. The Polya's (1957) framework for problem-solving that breaks down problem into four distinguishable steps and higlight the use of process in solving problems rather than focusing on getting the solution was adopted in this study. In the process of solving a problem, teacher guides the students to break the problem down and creatively and mindfully approach each step of the problem toward the desired solution. The four-step process includes understanding the problem, devising a plan, carrying out the plan and looking back. These are explained as follows:

Stage 1: Understanding the problem: Teacher guides the students in reading the problem carefully several times; focusing on the key words; discussing their meanings; identifying extraneous information, and helping them know what they are asked to find.

Stage 2: Devising a plan: When the groups have jointly understand and extract the necessary information that is required to solve the problem, teacher asks all the groups to formulate mathematical models or two linear equations.

Stage 3: Carrying out the Plan: At this stage, the teacher instructs students to use appropriate strategy to solve the problem given to them. While students were working, teacher went round the class, listening to their discussions, observing their activities, assisting and encouraging them to participate in solving the problem.

Stage 4: Looking back: Teacher asks students' to reflect on their answers and the processes they use to arrive at the answers.

Step 5: Discussions of students' results (10 minutes). The task at this stage included comparing results at each stage of problem solving, discussing conclusions, and sharing experiences across groups. After the problem has been solved, each group was given 3 minutes to presents its results to the entire class. While doing so, other groups were listening and jotting down their observations. Afterwards, 5 minutes was given to the whole class to make comments on the results presented by each group. The discussion continued in this way until every group has presented its results. Each of the groups then incorporates into its work the observations made by members from other groups. Finally, teacher provides a summary of the important points learn in the lesson in 3 minutes.

Step 6: Evaluation of students' learning progress (4 minutes). Teacher asked students to return to their usual places after they have worked in groups. Then a problem is presented on the chalkboard and asked them to solve it individually.

Step 7: Conclusion (2 minutes): Teacher gives students homework to practice what has been learnt in the class.

3.11 Procedure for Data Analysis

In order to analyze the data that was collected from the field for the purpose of providing answers to the research questions and hypotheses in sections 1.4 and 1.5 of the study; the statistical techniques were employed:

Ho_1: There is no significant difference in attitude between students taught algebra using Polya's problem solving strategy and those taught using lecture method.

For, hypothesis 1, the descriptive statistics of the mean rank attitude scores, the sum of ranks, the median, the standard deviation and standard error of the mean were calculated to find the difference between the post-posttest mean scores of both the experimental and control groups; while the Mann-Whitney U test, at $\alpha = 0.05$ level of significance, was used to find the significant difference of their postpost-test scores. .

Ho_2: There is no significant difference in the performance between students taught algebra using Polya's problem solving strategy and those taught using conventional method.

To test hypothesis 2, the descriptive statistics of the mean performance scores, the standard deviation and standard error of the mean were calculated to find the difference between the posttest mean scores of both the experimental and control

groups. While t-test statistic for the independent samples, at $\alpha = 0.05$ level of significance, was used to find the significant difference of their posttest scores.

Ho_3**:** There is no significant difference between the retention level of students taught algebra Polya's using problem solving strategy and those taught using conventional method.

To test hypothesis 3, the mean, standard deviation and standard error of the mean were calculated to find the difference between the retention level of students taught algebra using Polya's problem solving strategy and those taught using conventional method. To find out whether there is any significant difference between the mean retention scores of students taught algebra using Polya's problem solving strategy and those taught using conventional method; the t-test statistic for the independent samples, at $\alpha = 0.05$ level of significance, was used.

Ho_4**:** There is no significant difference in the attitude of male and female students taught algebra using Polya's problem solving strategy.

To test hypothesis 4, the mean rank, the sum of ranks, the median, standard deviation and standard error of the mean were calculated to find the difference between the mean attitude scores of male and female students taught algebra using Polya's problem solving strategy. To find out whether there is any significant difference between the mean attitude scores of male and female students; the the Mann-Whitney U-test statistic for the independent samples, at $\alpha = 0.05$ level of significance, was used.

Ho_5: There is no significant difference in performance of male and female students taught algebra using Polya's problem solving strategy.

To test hypothesis 5, the descriptive statistics of the mean scores, the standard deviation as well as the standard error of the mean were computed to find the difference between the posttest mean scores of males and females. And the independent samples t-test was used at $\alpha = 0.05$ level of significance to find the significant difference between the mean posttest scores of males and females.

H_{O6}**:** There is no gender difference in retention level of students taught algebra using problem solving strategy.

To test hypothesis 6, the mean, standard deviation and standard error of the mean were computed to find the difference between the mean retention scores of male and female students taught algebra using Polya's problem solving strategy. While the t-test statistic for the independent samples, at $\alpha = 0.05$ level of significance, was used to find out whether there is any significant difference between the mean retention scores of male and female students.

CHAPTER FOUR

DATA PRESENTATION, ANALYSIS AND DISCUSSION

4.1 Introduction:

This study investigated the Impact of Polya's Problem Solving Strategy on Attitude, Performance and Retention in algebra among Junior Secondary School Students in Zamfara state, Nigeria. Therefore this chapter presented the results of the analysis and interpretations of the data collected for the study. This chapter is presented under the following subheadings: Data Presentation, Data Analysis, Summary of the Major Findings, and Discussions.

4.2 Data Presentation

The data collected for this study was classified into three levels for analysis namely, pretest, posttest and retention levels. At the pretest level, a sample of four hundred and sixty three (463) students who were present during the period was used for data analysis. This sample served as the baseline for determining the background knowledge and equivalence of both the experimental and control groups before the commencement of the experiment. After the treatment, four hundred and fifty two (452) students were given the posttest, while eleven (11) of the four hundred and sixty three (463) students that took the pretest had dropped out of the posttest. The purpose of the posttest was to determine the effect of the treatment on algebra. Two weeks after the posttest, a post-posttest was administered on a sample of four hundred and twenty eight (428) students to determine their retention level in algebra. While twenty four (24) of the four hundred and fifty two (452) students that took the posttest had dropped out of the post-posttest. The means and standard deviations were used to compute the data of the subjects, while the student t-test

statistic for the independent samples and Mann-Whitney U-test were used to test the six generated hypotheses. The results are presented in Tables 4.01- 4.18 respectively.

4.3 Data Analysis

The data analysis for this study was carried out on the pretest, posttest and post-posttest attitude and performance scores. The data were analysed using both descriptive and inferential statistics using Statistical Package for Social Sciences (SPSS) that applied statistical formulae and carried out computations. The descriptive statistics was used to report the differences between the experimental group and the control group on pretest, posttest and retention test scores. While the independent sample t-test and Mann-Whitney U-test were used to test the hypotheses at the 0.05 level of significance. Six null hypotheses were generated from the six objectives of the study. The results of the analysis are presented in Tables 4.01 – 4.18.

4.3.1 Pre-test Mean Ranks of Attitude Scores Analysis

There were a total of 463 subjects that were given the pretest on Algebraic Students' Attitude Inventory Questionnaire (ASAIQ). This number represented the initial sample for the study. The ASAIQ was used to measure wether the students' attitudes in both the experimental and control groups were similar before the commencement of the experiment. The means and standard deviations were computed on the data. The maximum obtainable score was 240 and the minimum score was 48. The results are presented in Table 4.03.

Table 4.01: Summary of Pretest Mean Rank Attitude Levels of Experimental and Control Groups

Group	N	Mean Rank	SD	Standard Error of the Mean	Mean Rank Difference
Experimental	235	234.66	7.16	0.47	5.4
Control	228	229.26	7.05	0.47	
Total	**463**	**463.92**	**14.21**	**0.94**	

Table 4.01 presented the mean ranks and standard deviation of the pretest attitudinal levels of students in the experimental and control groups. The results in Table 4.01 showed that the experimental group has the mean rank of 234.66 and standard deviation of, 7.16 while the control group has the mean rank of 229.26 and standard deviation of 7.05. The mean rank difference between the two groups was 5.4. In order to establish if the difference in the mean ranks is statistically significant, the Mann Whitney U test was used to test the null hypothesis at $\alpha = 0.05$ level of significance. The results are presented in Table 4.02.

Table 4.02: Summary of Mann-Witney U-test Attitudinal levels of Experimental and Control Groups

Group	N	Mean Rank	MD	U	Z	P
Experimental	235	234.66	94.00	26164.500	-0.436	.663
Control	228	229.26	93.50			
Total	**463**	**463.92**	**187.50**			

The results presented in Table 4.02, indicated that the Mann-Witney U-test revealed no significant difference between the mean attitude of experimental and control groups on pretest. From Table 4.02, the z value obtained is -0.436 with probability value of $p = .663$ which is greater than 0.05 alpha level of significance. Inspection of the two group mean ranks also revealed that the rank average attitude scores for experimental group (234.66) was higher than the score (229.26). Therefore the two groups were equivalent on ASAIQ scores prior to the intervention.

Table 4.03: Summary of Pre-test Mean Rank Attitudinal Levels of Gender in Experimental Group

Group	N	Mean Rank	SD	Standard Error of the Mean	Mean Rank Difference
Male	120	118.85	7.11	.65	1.73
Female	115	117.12	7.25	.68	
Total	**235**	**135.97**	**14.36**	**1.33**	

Table 4.03 showed the mean ranks and standard deviations of the pretest attitude scores of male and female students in the experimental group. The results in table 4.03 indicated that the males had the mean rank score of 118.85 and standard deviation of 7.11, while the females had the mean rank score of 117.12 and standard deviation of 7.25. The difference between the mean ranks was 1.73. In order to establish if the difference in attitude is statistically significant, the attitude ranked scores were subjected to Mann-Whiteny U test at $\alpha = 0.05$ level of significane. The summary of the analysis are presented in Table 4.04.

Table 4.04: Summary of Mann-Whitney U-test on Attitude Mean Rank of Gender in Experimental Group

Group	N	Mean Rank	MD	U	Z	P	d
Experimental	120	118.85	94.00	6798.500	-.195	.845	-.01
Control	115	117.12	94.00				
Total	**235**	**235.97**	**188.00**				

Table 4.04 showed the results of Mann-Whitney U-test on pretest attitude mean scores for male and female students in the experimental group. As seen from Table 4.04, the z value obtained is -.195 with probability value of $p = .845$ which is greater than 0.05 alpha level of significance. This implies that there was no statistically significant difference in the mean attitude scores of gender in the experimental group. Also, an observation of the means of the two groups revealed that the rank average attitude scores

for males (118.85) was higher than the score (117.12) for females, which indicated that the male and female attitude scores of the students in experimental group were equivalent on ASAIQ scores before the experimnt.

4.3.2 Pre-test Performance Scores Analysis

Before the beginning of the experiment, the pretest on Simple Linear Equations and Word Problem (SLEWPPT) was administered on all the students in the sampled schools. A total of 463 students were used for the pretest which served as the initial sample for the study. The purpose of the pretest was to measure whether the groups being compared have basic skills and knowledge that will aid in given the new instruction as well as to find out whether the groups are equivalent in their abilities before the beginning of the experiment. The results of the pre-test for both the experimental and control groups were computed in means and standard deviations. The maximum obtainable score in the test was 100. The results are presented in Table 4.05.

Table 4.05: Summary of Pretest Mean and Standard Deviation of Experimental and Control Groups

Group	N	Mean	SD	Standard Error of the Mean	Mean Difference
Experimental	235	11.23	6.36	.21	.14
Control	228	11.09	6.71	.22	
Total	**463**	**22.32**	**13.07**	**0.43**	

Table 4.05 presented the pretest performance scores of both the experimental and control groups. The result in Table 4.05 indicated that the experimental group has the mean score of 11.23 and standard deviation of 6.36. On the other hand, the control group has the mean score of 11.71 and the standard deviation of 6.71. The mean difference between the two groups was 0.14. Comparing the two groups on the basis of their mean difference, the groups were equivalent on the pretest since the variation between the means of the groups was only slight. In order to ensure the equality of the experimental and control groups,

preliminary analysis was carried out using independent *t*-test procedures that examined the equivalence of the groups on the SLEWPPT measure at pretest. The results of the independent *t*-test analysis are presented in Table 4.06.

Table 4.06: Summary of t-test pretest Mean and Standard Deviation Scores of Experimental and Control Groups

				Levenes's	Test
Group	N	Mean	SD	F	Sig.
Experimental	235	11.23	6.36	.86	.36*
Control	228	11.09	6.71		

***Not significant, $p > 0.05$, $p = .36$**

From Table 4.06, the Levene's test showed that the variences were not significantly different ($p = .36$). Therefore the test statistics of equal variences were assumed and it was found that there was no statistically significant difference between the mean scores of the students in the experimental group and those in the control group on the SLEWPPT.

4.3.3 Post-test Attitudinal levels of Experimental and Control Groups

This study presented the results of attitudinal change of JSS III students that were taught simultaneous linear equations with substitution and elimination methods using problem solving strategy and those taught using conventional method. Therefore, the means and standard deviations of students' attitudinal change were computed and presented as shown in Table 4.07.

Table 4.07: Summary of Post-test Mean Rank Attitudinal levels of Experimental and Control Groups

Group	**N**	**Mean Rank**	**SD**	**Standard Error of the Mean**	**Mean Rank Difference**
Experimental	230	244.30	26.03	1.72	36.25
Control	222	208.05	32.41	2.18	
Total	**452**	**452.35**	**58.44**	**3.90**	

Table 4.07 showed that, the mean rank and standard deviation of students in the experimental group were 244.30 and 26.03 respectively, while those of the control group

were 208.05 and 32.41 respectively. The difference in the posttest attitude mean rank scores of both the experimental and control groups was 36.25. This indicated that the posttest attitude of the average students in the experimental group was higher than that of the students in the control group. However, the spread of the scores around the average attitude of the control group was higher than that of the experimental group. The Mann-Whitney U-test statistic analysis was carried out to find out whether there was significant difference among the groups at $\alpha = 0.05$ level of significance.

Hypothesis One

There is no significant difference in attitude between students taught algebra using Polya's problem solving strategy and those taught using lecture method.

To test this hypothesis, the attitude ranked scores of both the experimental and control groups were subjected to Mann-Whitney U test at $\alpha = 0.05$ level of significance. The result is presented in Table 4.08

Table 4.08: Summary of Mann-Whitney U-test Attitudinal levels of Experimental and Control Groups

Group	N	Mean Rank	MD	U	Z	P	Effect size (**d**)
Experimental	230	244.30	160.00	21435	-2.950	.003	-.14
Control	222	208.05	151.50				
Total	**452**	**452.35**	**311.5**				

Table 4.08 revealed the posttest attitudes mean rank scores between experimental group and control group. As can be observed from table 4.08, the z value obtained is -2.950 with probability value of $p = .003$ which is less than 0.05 alpha level of significance. The null hypothesis was therefore rejected. This implies that there was significant difference between the mean attitude scores of experimental and control groups. Also, an examination of the means of the two groups revealed that the average attitude score for control group 208.05 is significantly lower than the average sore 244.30 for experimental

group. This implies that students in experimental group had significantly better attitude than students in the control group. The effect size, *d*, is approximately -.14 representing a low sized effect. The result, therefore, showed that Polya's problem solving strategy is more effective in enhancing students' attitude towards algebra than the conventional method.

4.3.4 Post-test Performance Scores of Experimental and Control Groups

This study comprised 463 students that were drawn from eight public junior secondary schools in the four Education Zones of Zamfara State. These were the students that served as the initial sample for the study. However, after the intervention a post-test was conducted on 452 students that were present during the period. This number of students constituted the sample used in analyzing the data on the post-test. The results of the students' performance in the posttest for the experimental and control groups are given in Table 4.09.

Table 4.09: Summary of Post-test Performance Scores for Experimental and Control Groups

Group	N	Mean	SD	Standard Error of the Mean	Mean Difference
Experimental	230	31.32	13.38	.88	9.43
Control	222	21.89	10.28	.69	
Total	**452**	**53.21**	**23.66**	**1.57**	

Table 4.09 presented the difference between mean performance scores of the experimental and control groups on posttest. The posttest mean and standard deviation scores of the experimental group were 31.32 and 13.38 respectively, while the mean and standard deviation scores for the control group were 21.89 and 10.28 respectively. The mean difference between the groups was 9.43. In order to establish whether the mean difference is statistically significant, t-test for the independent samples was used to test the null hypotheses at $\alpha = 0.05$ level of significance.

Null Hypothesis Two

There is no significant difference in the performance between students taught algebra using problem solving strategy and those taught using conventional method.

To test the null hypothesis t-test statistic for the independent samples was used to find whether there was significant difference between the experimental and control groups in the posttest scores at $\alpha = 0.05$ level of significance. The result is shown in Table 4.10.

Table 4.10: Summary of t-test on Performance Scores of Experimental and Control Groups

Group	**N**	**Mean**	**SD**	**Df**	**T**	**P**	Effect size (**d**)
Experimental	230	31.32	13.38	450	8.38	.001	0.92
Control	222	21.89	10.28				
Total	**452**	**53.21**	**23.66**				

****Significant at $p < 0.05$**

From the results of Table 4.10, it was observed that the t-value of 8.38 and the p-value of 0.001 were observed at the degree of freedom of 450. The p-value obtained was 0.001 which is less than $\alpha = 0.05$. Thus, the null hypothesis was rejected. This showed that there was significant difference between the performance of the experimental and the control groups. This implies that the students taught algebra using Polya's problem solving strategy performed significantly better than students taught algebra using conventional method as revealed in their mean scores. However, the effect size of the difference between the means of experimental and control groups was 0.92. This indicated that there was a very large effect size between the experimental and control groups using Cohen (1988) criteria.

4.3.5 Retention Levels of Experimental and Control Groups

This sub-section analysed the data collected on retention level of the Junior Secondary School Three (JSS III) students taught algebra usingPolya's problem solving strategy and those taught using conventional method. A total of four hundred and twenty eight (428) subjects who were present at the time of the post posttest were used to analyze data. Thus the means and standard deviations were calculated on retention scores of both the experimental and control groups two weeks after the posttest. The results are presented in Table 4.11.

Table 4.11: Summary of Retention levels of Experimental and Control Groups

Group	N	Mean	SD	Standard Error of the Mean	Mean Difference
Experimental	213	23.41	9.20	.63	5.59
Control	215	17.82	7.54	.51	
Total	**428**	**41.23**	**16.74**	**1.14**	

Table 4.11 indicated the means and standard deviations for retention scores of experimental and control groups in the Simultaneous Linear Equation Performance Test. The results from Table 4.11 revealed that the means and standard deviations for the performance scores of the retention test for experimental group were 23.41 and 9.20. While that of the control group were 17.82 and 7.54. It was also indicated in the table that the difference of 5.59 existed between the means of the two groups in the retention test, which showed that both the experimental and control groups were equivalent in their sustenance levels of the Simultaneous Linear Equations Performance Test. In order to establish whether the mean difference is statistically significant, the t-test statistic analysis for the independent samples was used to test the null hypothesis at $\alpha = 0.05$ level of significance.

Null Hypothesis Three

There is no significant difference between the retention level of students taught algebra using problem solving strategy and those taught using conventional method.

To test this hypothesis, the t-test for the independent samples was computed on the retention test scores for the experimental and control groups at $\alpha = 0.05$ level of significance. The summary of the results is presented in Table 4.12.

Table 4.12: Summary of t-test Retention levels of Experimental and Control Groups

Group	N	Mean	SD	Df	T	P	D
Experimental	213	23.41	9.20	426	6.87	.001	.74
Control	215	17.82	7.54				
Total	**428**	**41.23**	**16.74**				

From Table 4.12 the p-value obtained was .001 which is less than p = 0.05. The t-value with df = 426 was 6.87. Since the p-value obtained was .001 which is less than 0.05, Thus, the null hypothesis which stated that there was no significant difference was rejected. This implies that the experimental group differs significantly from the control group on simultaneous linear equation performance test. The effect size of the difference between the means of experimental and control groups was 0.74.

4.3.6 Attitude Levels of Gender in Experimental Group

A total of 463 students served as the initial sample for this study. After the intervention a post-test was administered on only 452 students that were present during the period. Thus, the means, standard deviations and Mann-Whitney U-test were computed on the posttest attitude scores of the male and female students. The results are represented in Table 4. 13.

Table 4.13: Summary of Posttest Mean Rank Attitudinal levels of Gender in Experimental Group

Group	N	Mean Rank	SD	Standard Error of the Mean	Mean Rank Difference
Male	117	127.79	20.19	1.87	25.02
Female	113	102.77	30.32	2.85	
Total	**230**	**230.56**	**50.51**	**4.72**	

Table 4.13 revealed that the mean rank score of male students was 127.79 with a standard deviation of 20.19, while their female counterparts had a mean rank score of 102.77 with a standard deviation of 30.32 in the posttest attitude. The difference in the posttest attitude mean rank scores of both the male and female students was 25.02 To establish if the mean difference is statistically significant, the Mann-Whitney U-test statistic analysis for the independent samples was used to test the null hypothesis at $\alpha = 0.05$ level of significance.

Null Hypothesis Four

There is no significant difference in the attitude of male and female students taught algebra using Polya's problem solving strategy.

To test this hypothesis, the posttest attitude scores of male and female JSS III students in the experimental group were compared using Mann-Whitney U-test statistic at 0.05 level of significance. Table 4.14 indicated the summary of the analysis.

Table 4.14: Summary of Mann-Whitney U-test Attitude levels of Male and Female Students in Experimental Group

Group	N	Mean Rank	MD	U	Z	p	Effect size (**d**)
Male	117	127.79	163.00	5172.5	-2.851	.004	-.19
Female	113	102.77	154.00				
Total	**230**	**230.56**	**317.00**				

Table 4.14 showed the average ranks for male and female students on attitude scores. As observed from table 4.14, the z value obtained is -2.851 with probability value

of $p = .004$ which is less than 0.05 alpha level of significance. Inspection of the two group means also revealed that the rank average attitude scores for female students (102.77) was significantly lower than (127.79) for males. This implies that the male students were significantly different from the female students on posttest attitude scores. Therefore, the null hypothesis was rejected. The effect size, *r*, is approximately $-.19$, representing a small effect size. Thus, the result showed that problem solving strategy is more effective in improving male students' attitude in solving simultaneous linear equations than the female students.

4.3.7 Post-test Performance Scores of Gender in Experimental Group

The data of performance by gender of the Junior Secondary School Three (JSSIII) students taught word problems involving simultaneous linear equations were computed. The analysis of the data involved the computation of the means, percentages and standard deviations. The results are presented in Table 4.15.

Table 4.15: Summary of Post-test Performance Scores of Gender in Experimental Group

Gender	N	Mean	SD	Standard Error of the Mean	Mean Difference
Male	117	25.28	13.27	1.23	3.43
Female	113	21.85	12.85	1.21	
Total	**230**	**47.13**	**26.12**	**2.44**	

Table 4.15 indicated the means and standard deviations of the students' performance by gender in the experimental group. The result in Table 4.15 showed that the males had the mean score of 25.28 and standard deviation of 13.27. Similarly, females had the mean score of 21.85 and standard deviation of 12.85. The difference in the posttest mean scores of males and females was 3.43. This indicated that the posttest performance of the average males was 3.43 higher than that of the females. Also, the spread of the scores around the average performance of males was a bit higher. The t-test statistic analysis was

carried out to find out whether there was significant difference among the groups at $\alpha = 0.05$ level of significance.

Null Hypothesis Five

There is no significant difference in performance of male and female students taught algebra using problem solving strategy.

To test the hypothesis, the independent samples t-test was used to find the significant difference between the mean of posttest scores of males and females at $\alpha = 0.05$ level of significance. The summary of the computations was presented in Table 4.16.

Table 4.16: Summary of t-test Performance Scores of Genders in Experimental Groups

Group	**N**	**Mean**	**SD**	**Df**	**T**	**P**	Effect size (**d**)
Male	117	25.28	13.27	228	1.99	.048	0.27
Female	113	21.85	12.85				
Total	**230**	**47.13**	**26.12**				

** **Significant at $p \leq 0.05$**

Table 4.16 showed that the t-value obtained, at 228 degree of freedom, was 1.99 with a p-value of 0.048. Since the p-value obtained was 0.048, which is less than $\alpha = 0.05$, thus, the null hypothesis was rejected. This implies that there was statistically significant difference in the performance of males and females in the experimental group taught algebra using Polya's problem solving strategy. However, the effect size, *d*, of the difference between the means of males and females was 0.27, which has a small effect size.

4.3.8 Retention Levels of Gender in Experimental Group

There were a total of 463 students that served as the initial sample for this study. However, after the intervention a post-test was administered on 452 students that were present. Two weeks after the completion of the posttest, a retention test was administered on only 428 students that were present during the period. The means, standard deviations and student t-test were computed on the retention scores of both the males and females in experimental group at $\alpha = 0.05$ level of significance. The results are represented in Table 4.17.

Table 4.17: Summary of Retention levels of Gender in Experimental Group

Group	N	Mean	SD	Standard Error of the Mean	Mean Difference
Experimental	115	29.97	11.90	1.11	2.83
Control	110	27.14	12.26	1.19	
Total	**225**	**57.11**	**24.16**	**2.30**	

Table 4.17 showed the descriptive statistics of the means and standard deviations for the male and female students on post-post-test scores. The mean and standard deviation of males were 29.97 and 11.90 respectively. Similarly the female students obtained the mean score of 27.14 and standard deviation of 12.26. The mean difference between the two groups was 2.83. In order to establish whether there is statistically significant difference between the two groups, the t-test for independent samples was used to test the null hypothesis at $\alpha = 0.05$ level of significance.

Null Hypothesis Six

There is no gender difference in retention level of students taught algebra using problem solving strategy.

To find out whether there is any significant difference between the mean retention scores of male and female students; the t-test statistic for the independent samples was

used at $\alpha = 0.05$ level of significance. The summary of the t-test retention scores of gender in experimental group is given in Table 4.18.

Table 4.18: Summary of t-test Retention levels of Gender in Experimental Group

Group	N	Mean	SD	Df	T	P
Male	115	29.97	11.90	223	1.76	.079
Female	110	27.14	12.26			
Total	**225**	**57.11**	**24.16**			

Table 4.18 showed that the t-value was 1.76 and the degree of freedoms was 223, and that the p-value was .079. Since the p-value of 0.079 was greater than $\alpha = 0.05$, the hull hypothesis was, therefore, not rejected. This means that, there was no significant difference in the retention levels of male and female students in algebra using Polya's problem solving strategy.

4.4. Summary of Major Findings

Based on the analysis of the data collected, the following are the major findings.

1. Students taught algebra using Polya's problem solving strategy had better attitude than those taught using lecture method. This implies that Polya's problem solving strategy was found more effective in promoting better attitude in algebra.
2. Students taught algebra using Polya's problem solving strategy performed better than their counterparts taught using conventional method in algebra posttest. Hence, Polya's problem solving strategy does result in a greater level of students' achievement in the learing of algebra compared to the conventional method of teaching.
3. Polya's problem solving strategy had produced significant difference in the retention level scores of students in algebra than the conventional method. This shows that teaching algebra word problems in JSS III classes with Polya's problem

solving strategy produced deeper learning and better retention on students' learning than the conventional method.

4. Significant difference exists in the attitude of male and female students taught algebra using Polya's problem solving strategy. This indicated that the male students had favourable attitude towards learning of algebra than their female counterparts.
5. Gender had great influence on students' performance with respect to the method used in teaching algebra word problems. This means that the Polya's problem solving strategy had great impact on the male students in the learning of algebra than their female counterparts.
6. Findings of the study revealed no difference in retention level between male and female JSS III students taught algebra using Polya's problem solving strategy. This means that both male and female students had similar retention level in the test of algebra using Polya's problem solving strategy.

4.5 Discussion

The main purpose of this study was to examine the impact of Polya's problem solving strategy on attitude, performance and retention in algebra among junior secondary school students in Zamfara State, Nigeria. Therefore this section discussed the results of data on the six null hypotheses tested in this study as well as the findings related to other previous research studies.

The findings from this study revealed that there was significant difference in attitude between students taught algebra using Polya's problem solving strategy and those taught using conventional method. This means that students taught algebra with Polya's

problem solving strategy had more favourable attitude than those taught with conventional method. The finding of this study is in line with those by Perveen (2009), Albay (2015) and Salman, Ayinla, Adeniyi, Ogundele and Ameem (2012). Perveen (2009) made the comparative effectiveness of expository strategy and problem solving approach of teaching mathematics at the secondary level. The results revealed that the posttest mean attitude scores of the experimental group were better than that of control group. Albay (2015) carried out a study to determine the effects of problem-solving approach on the performance in and attitude of freshmen college students towards College Algebra. The pretest-control group design was used which involved the experimental and control group. The experimental group was taught selected topics in college algebra using problem solving approach while the control group was taught the same topics using traditional approach. The results of the analysis showed that the application of problem solving approach in the delivery of instruction in College Algebra had positively contributed to the development and reinforcement of a favorable attitude towards the subject. Salman, Ayinla, Adeniyi, Ogundele and Ameem (2012) examined the effect of problem-solving instructional strategy on senior secondary school students' attitude towards mathematics in Ondo and found that the experimental group significantly performed better in mathematics than the control group. This difference in attitude between students taught using problem solving and those taught using lecture method has proved that the problem solving strategy is very effective than the conventional lecture method in helping students solve problems. This is really true because in problem solving strategy students are guided on how to analyse problems and to evaluate the effectiveness of the various steps that led to the

solutions, whereas in the conventional method students are only encouraged to memorise mathematical concepts with little understanding.

Furthermore, findings of the present study showed that there was significant difference in performance between students taught algebra using Polya's problem solving strategy and those taught using conventional method. This shows that Polya's problem solving has significant influence on students' learning and performance in algebraic word problems in particular and mathematics in general. This result is consistent with the findings of the studies by Suleiman (2010), Bowman (2010), Fatade (2012), Nneji (2013), Mehmood (2014), Esan (2015), Albay (2015), Brijlall (2015), Adisa and Abiola (2017), Rosemary and Anthony (2019) and Bassey, Joshua and Asim (n.d). Suleiman (2010) studied the effects of problem solving models on secondary school students' performance in statistics concepts of mathematics in Zamfara State. Findings from the study revealed that the experimental group that was exposed to Polya's problem solving strategy performed significantly better in statistics concepts than their counterparts in the control group. Bowman (2010) examined the influence of Polya's problem solving strategy on English Language Learners' (ELLs) ability to communicate thinking, with both oral and written fluency, when approaching mathematical word problems. The results showed that the ESL students had significantly improved their ability to solve mathematical word problems using polya's problem solving strategy. Fatade (2012) investigated the effectiveness of Problem-based learning (PBL) in the Further Mathematics classrooms in Nigeria. Findings revealed that there were statistically significant differences in the mean post-test achievement scores on TMT $(t=-3.58, p<0.05)$, mean post-test achievement scores on RDT $(t=-5.92, p<0.05)$ and mean post-treatment scores on SBFMQ $(t=-6.22,$

p<0.05) between students exposed to the PBL and those exposed to the TM, all in favour of the PBL group. Nneji (2013) investigated the effect of Polya's problem solving model on students'achievement and retention in algebra. The result showed that the students taught Algebra with Polya's problem solving strategy had achieved higher than those taught with expository method. Mehmood (2014) conducted a study to determine the effect of George Polya problem solving method of teaching on revised Bloom's Taxonomy of Educational Objectives in mathematics at elementary level, and found that students taught with problem solving method had significant improvement at all levels of knowledge dimension than their counterparts in the conventional method of teaching mathematics. Similarly, Esan (2015) examined the effects of cooperative problem-solving strategy on the learning outcomes of Nigerian secondary school students in algebraic word problems and found that the overall achievement of students exposed to cooperative problem solving strategy was better than those exposed to the conventional method. Albay (2015) conducted a study on problem Solving Approach to the Performance and Attitude in College Algebra of College Students in which the experimental group performed better in the posttest than the control group. Furthermore, Brijlall (2015) investigated the stages of Polya's problem solving model on fractions in mathematics class at a South African middle school, and found that students' performance on fractions problems had greatly improved. Similarly, the result of the study by Adisa and Abiola (2017) showed that students exposed to problem solving strategy achieved more in solving problems on quadratic equations than students who were exposed to conventional approach. In the same vein, Rosemary and Anthony (2019) have examined the effectiveness of Polya and Woods problem solving models on students' academic performance in simple linear equation in

one variable among junior secondary school students. The result indicated that students who were taught linear equation in one variable using Polya problem solving model performed significantly better than their counter parts who were taught with Woods problem solving model. Further, From the research by Bassey, Joshua and Asim (n.d) on gender differences in mathematics achievement at the senior secondary school level, it was found that male rural students performed better than their females counterparts.

There was significant difference between the retention level of students taught algebra using Polya's problem solving strategy and those taught using conventional method. The experimental group performed better than the control group. This finding supports the findings of Perven (2009) and Esan (2015) studies. Perven (2009) found that the retention of higher achievers that were taught using problem solving strategy was better than the retention level of higher achievers taught by convention method. Similarly, the study revealed that the retention rate of low achievers taught with problem solving was found better that those taught using conventional method. Esan (2015) examined the effects of cooperative problem-solving strategy on the learning outcomes of Nigerian secondary school students in algebraic word problems. Findings from the study indicated that the overall retention level of students exposed to cooperative problem solving strategy was better than those exposed to the conventional method.

The result of this study also found that there was significant difference in the attitude of male and female students taught algebra using Polya's problem solving strategy. This result showed that Polya's problem solving strategy is more effective in improving male students' attitude towards algebraic word problem than the female students. The finding of this study is in consonant with that of Michelli (2013) which revealed that male

students' attitude was higher than that of the female students in mathematics. However, the finding of this study disagree with that of Popoola (2014) who carried out a study on problem solving teaching strategy on students' academic performance and attitude towards mathematics in colleges of agriculture, north-west zone, Nigeria. The finding revealed that there was no significant difference between the attitudinal mean scores of male and female respondents towards mathematics in colleges of agriculture.

The result of this study showed that there was statistically significant difference in the performance of males and females in the experimental group taught algebra using Polya's problem solving strategy. That means, male junior secondary school three students exposed to Polya's problem solving strategy achieved significantly well in algebraic word problems than the females counterpart. This finding agrees with earlier findings of the studies by Awofala (2011) and Doris, Neill and Sweetman (2012). Awofala (2011) tested 1780 (880 males and 900 females) senior secondary three students in southern part of Nigeria based on single-sex, coeducational, urban and rural settings. The results of the study indicated that male students performed significantly ($p < 0.05$) better than female students. It was, however, found from the study that the performance of males and females differed significantly only for those in single-sex and rural settings. Similarly, Doris, Neill and Sweetman (2012) studied a representative sample of Irish 9 year old children from coeducational and single-sex schools to examine the determinants of gender gap in mathematics, focusing on the single-sex schooling. They found that boys performed significantly better than girls in mathematics. Nevertheless, the finding of this study is in contrast with the findings of Suleiman (2010), Orji and Anaduaka (2010), McFarland, Benson and McFarland (2011), Joseph and Kurumeh (2011), Michelli (2013) and Nneji

(2013). Suleiman (2010) investigated the effects of problem solving models on secondary school students' performance in statistics concepts of mathematics in Zamfara State. The study was set to find out whether Polya, Gick and Bransford and Stein approaches would have an impact on students' academic performance on statistics concepts of mathematics. The study found among others that there was no significant difference in the performance of male and female students. In their study, Orji and Anaduaka (2010) examined the effectiveness of webbing instructional strategy on the academic performance of SS II students in algebraic word problems. The data of the study showed that performance in algebra is not related to sex and hence neither the male nor the female students did significantly better than the other in the study. Moreover, McFarland, Benson and McFarland (2011) compared performance scores of students in gender specific classrooms with students in traditional classrooms at the elementary school level. The results of the study showed that females in the gender specific outperformed males in gender specific classroom as well as those in the traditional classrooms. Similarly, Joseph and Kurumeh (2011) investigated the effect of two problem solving models on the academic performance of 260 Junior Secondary III Students in algebraic word problems using Greeno and Metes et al models. The findings of the study revealed that both male and female students had performed equally well when taught algebraic word problems using the two models. They then concluded that gender has no significant influence on male and female students' performance in algebraic word problems as measured by Mathematics Performance Test. Michelli (2013) also studied the relationship between attitudes and achievement in mathematics among fifth grade students and found that both male and female students scored approximately the same on mathematics achievement test. In another study by

Nneji (2013) that compared Polya's problem solving model with the expository method to determine the performance and retention rates of secondary students in Ishielu local government area of Ebonyi State reveled that there was no significant difference in performance between male and female students in the study.

The finding showed that there was no significant difference in the retention levels of male and female students taught algebra using Polya's problem solving strategy. That means, gender does not significantly affect the retention ability of students in algebra. This finding agrees with the findings of the study by Nneji (2013) and Zakariya, Ibrahim and Adisa (2016). Nneji (2013), compared Polya's problem solving model with the expository method to determine the performance and retention rates of secondary students in Ishielu local government area of Ebonyi State. Findings of the study indicated that there was no significant difference between the mean retention scores of male and female students in the experimental group taught algebra using problem solving. Zakariya, Ibrahim and Adisa (2016) compared the impacts of problem-based learning and conventional method on the mathematics performance and retention of junior secondary school students. The results indicated that both male and female students had approximately scored the same in the algebra retention test.

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 Introduction

This study investigated the impact of Polya's problem solving strategy on attitude, performance and retention in algebra among junior secondary school students in Zamfara State, Nigeria. This chapter is summarized in the following sub-headings: Summary, Conclusion, Recommendations, Contribution to knowledge, Limitations of the Study, and Suggestions for Further Studyies.

5.2 Summary

This study investigated the impact of Polya's problem solving strategy on attitude, performance and retention in algebra among junior secondary school students in Zamfara State, Nigeria. The problem of this study emanated from the researcher's observations of the consistently poor performance in mathematics among students at the secondary school level especially at the Senior Secondary Certificate Examinations (SSCE). Several studies that were reviewed in this study have pointed out that the problem of poor performance has been created by, among other things, poor teaching methods. Most of these studies concluded that students' performance can be enhanced through effective instructional strategies which allow active participation of students. This study specifically focused on six objectives and their corresponding research questions and hypotheses. This research was also aimed to benefit various stakeholders that comprised students, teachers, parents, researchers, mathematics educators and the like. Furthermore, the study was delimited to only JSS III students in Zamfara State.

Several topics which are pertinent to this study were reviewed. These include the theoretical framework upon which the study was based, the conceptual framework that provided an explanation of the relationships and the interaction of the four stages of Polya's problem solving strategy as well as the historical overview of algebra. The conceptual meaning of linear equations relative to simultaneous linear equation in one and two variables and the conventional lecture method of teaching simultaneous linear equations by substitution and elimination methods were also highlighted. Gender differences in algebra performance, students' performance in algebra, conceptual definition of attitude, attitude and students' performance in algebra, retention and students' performance in algebra, empirical studies, and implications of the literature reviewed on the present study were stated and explained.

This study used pretest, posttest and post-posttest quasi-experimental design. Eight gender schools comprising four boys' schools and four girls' were randomly selected from four Education Zones of Zamfara State. Four schools were assigned to experimental treatment while the other four as control. A total of 463 students participated in the study. This number of subjects was the initial sample used for data analysis on the pretest scores. After the intervention a posttest was administered on 452 subjects. Two weeks later, a post-posttest test was given to 428 students. The instruments used for data collection include Simple Linear Equation and Word Problem Pre-Test (SLEWPPT), Simultaneous Linear Equation Performance Test (SLEPT) and Algebraic Students' Attitude Inventory Questionnaire (ASAIQ). The SLEPT consisted of 10 essay questions having a total score of 100 marks. The test was developed by the researcher and covered all the lessons in the module lesson plans. ASAIQ was adapted from Fennema and Sherman (1976)

mathematics attitude scale. This questionnaire consisted of 48 questions aimed at collecting in-depth information about the students' perception and feeling about algebra. The maximum score for questionnaire was 240 while the minimum score was 48. At the beginning of the experiment, the subjects in the sampled schools were administered a pretest on Simple Linear Equations and Word Problem Pre-Test (SLEWPPT) and Algebraic Students' Attitude Inventory Questionnaire (ASAIQ). At the end of the treatment the subjects were post-tested by using Simultaneous Linear Equations Performance Test (SLEPT) and Algebraic Students' Attitude Inventory Questionnaire (ASAIQ) to determine the effect of the treatment on students' performance in algebraic word problems in simultaneous equations as well as attitudinal change toward algebra. The data analysis procedures involved descriptive and inferential statistics. The means and standard deviations were used to answer the research questions, while the t-test for independent samples and Mann Whitney U test were used to test the null hypotheses at 5% level of significance that led to rejection or non-rejection of the null hypotheses.

Finally, the data collected from the field were presented, analysed and discussed. Thus, statistical tables were used to display information on the results obtained from the statistical analysis. These results provided answers to each of the six research questions and null hypotheses of the study. Based on the results obtained from testing the null hypotheses, the summary of the major findings were stated. Discussions were also made to corroborate the results of the findings with those of the other previous studies.

5.3 Conclusion

The main objective of this study was to investigate the impact of Polya's problem solving strategy on attitude, performance and retention in algebra among junior secondary school students in Zamfara State, Nigeria. Specifically, it focused on the algebraic word problems using substitution and elimination methods in Junior Secondary School Three (JSS III) students. A pretest, posttest and post-posttest quasi-experimental design was used and the data collected from the field were subjected to statistical test based on the objectives, research questions and hypotheses formulated. The descriptive and inferential statistics were used in the data analysis. Therefore, the findings of the study showed that:

1. students taught algebra using Polya's problem solving strategy had better attitude than those taught using lecture method.
2. students taught algebra using Polya's problem solving strategy performed better than their counterparts taught using conventional method in algebra posttest.
3. Polya's problem solving strategy had produced significant difference in the retention level scores of students in algebra than the conventional method.
4. significant difference exists in the attitude of male and female students taught algebra using Polya's problem solving strategy.
5. gender had great influence on students' performance with respect to the method used in teaching algebra word problems.
6. Findings of the study also revealed no difference in retention level between male and female JSS III students taught algebra using Polya's problem solving strategy.

7. Based on the findings of the study, mathematics teachers should employ Polya's problem solving strategy in teaching of algebra in particular and mathematics in general at the JSS levels of education to help students develop positive attitude and independence in order to improve their performance.

5.4 Recommendations

Based on the interpretations of the findings of this study, the following recommendations are made

1. Polya's problem solving strategy should be adopted in teaching mathematics at the secondary school level in Zamfara State. This will assist teachers to improve their skills and methodologies in teaching word problems in mathematics in general and word problems involving simultaneous linear equations in particular.
2. Students should be exposed sufficiently to upon ended algebraic expressions and word problems.
3. Teachers of mathematics should teach word problems that relate to students' real life situations or practical events to enhance their understanding and meaningful interpretation for successful problem solving.
4. Workshops and seminars should be organized by government on Polya's problem solving strategy for acquisition of problem solving skills and knowledge for teachers at all levels of education in Nigeria. This will enable teachers assist students to become proficient in solving word problems in mathematics in general and algebra in particular.

5.5 Contribution to Knowledge

Based on the findings of the study, it was established that:

1. The SLEPT and ASAIQ instruments developed and used in this study have provided data that could lead to the improvement in positive attitude, performance and retention in simultaneous linear equations among JSS III students.
2. This study has established the effectiveness of Polya's problem solving strategy on teaching algebraic word problems in simultaneous linear equations among JSS III students.
3. The study had established that students taught using Polya's problem solving strategy had higher retention ability than those taught using lecture method.

5.6 Limitation of the Study

Although the results of the findings of this study showed that problem solving strategy has had a direct impact on students' attitude, academic performance and retention in algebra at the junior secondary school level, it has the following limitations:

1. The study used Polya's Problem Solving Strategy. Hence, the results of the study are limited to those variables within the used procedures.
2. This study focused on only simultaneous linear equations, an aspect of algebra in the junior secondary school syllabus. This is because of the difficulty involved in managing the instructional progemme to extend the coverage to other topics of algebra.

5.7 Suggestions for Further Studies

This research work was conducted in order to encourage further research findings in various areas such as mathematics and other related areas. Therefore, the following suggestions were made for future research studies.

1. This study could be extendend to students in senior secondary schools; colleges of education to explore more mathematics areas such as calculus and geometry as well as other areas in mathematics at the polytechnics.
2. Further research should be carried out on impact of problem solving strategy on dependant variables such as self efficacy, self esteem and motivation.
3. The Polya's problem solving strategy used in this study could be used in other areas of secondary school mathematics curriculum such as statistics, trigonometry, geometry and mensuration.
4. Similar studies could also be carried out to compare the impact of Polya's problem solving model with other teaching models such as the Lester (1978) and Bransford and Stein (1984) models.

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Appendix A

Simple Linear Equation and Word Problem Pre-Test (SLEWPPT) Draft

Instruction: Answer all the questions **Time: 1hour 20minutes**

Section A

Solve the following equations:

1. a) $n-2=6$ b) $n+5=20$ c) $2x=10$
2. a) $2.6y=10.4$ b) $a-2.6=8.9$
3. a) $3x+5=11$ b) $3x+1=2(3x+5)$
4. a) $\frac{z}{4}=1$ b) $\frac{x}{2}+\frac{3x}{4}=5$

Section B

Construct a simple linear equation for each of the following problems and solve the equation:

1. A certain number is subtracted from 20 and the result is 7. Find the number.
2. Three more than twice a number k is 1. What is k ?
3. Three-fourth of some number r is twenty seven. Find the number.
4. I think of a number, add it to itself, the result is 2. What is the number?
5. The difference between seven times a number and five is sixteen. Find the number.

Solution to Simple Linear Equation and Word Problem Pre-Test (SLEWPPT) Draft

Section A

1. a) $n-2=6$

$n-2+2=6+2$ 2 marks

$n=8$ 2 marks

b) $n+5=20$

$n+5-5=20-5$ 2 marks

$n=5$ 2 marks

c) $2x=10$

$\frac{2x}{2}=\frac{10}{2}$ 2 marks

$x=5$ 2 marks

2. a) $2.6y=10.4$

$\frac{2.6y}{2.6}=\frac{10.4}{2.6}$ 2 marks

$y=4$ 2 marks

b) $a-2.6=6.8$

$a-2.6+2.6=8.9+2.6$ 2 marks

$a+0=11.5$ 2 marks

$a=11.5$ 2 marks

3. a) $3x+5=11$

$3x+5-5=11-5$ 2 marks

$3x+0=6$ 2 marks

$3x=6$ 2 marks

$\frac{3x}{3}=\frac{6}{3}$ 2 marks

$x=2$ 2 marks

b) $3x+1=2(3x+5)$

$3x+1=6x+10$ 2 marks

$3x-3x+1=6x-3x+10$ 2 marks

$1=3x+10$ 2 marks

$1-10=3x+10-10$ 2 marks

$-9=3x$ 2 marks

$\frac{-9}{3}=\frac{3x}{3}$ 2 marks

$-3=x$ 2 marks

4. a) $\frac{z}{4}=1$

$4\left(\frac{z}{4}\right)=1\times 4$ 2 marks

$z=4$ 2 marks

b) $\frac{x}{2}+\frac{3x}{4}=5$

$4\left(\frac{x}{2}+\frac{3x}{4}\right)=4\times 5$ 2 marks

$4\times\frac{x}{2}+4\times\frac{3x}{4}=4\times 5$ 2marks

$2x+3x=20$ 2 marks

$5x=20$ 2 marks

$x=4$ 2 marks

Total: 60 marks

Section B

1. Let the number be x $\frac{1}{2}$ mark

$20 - x = 7$ 2 marks

$20 - 20 - x = 7 - 20$ 2 marks

$-x = -13$ 2 marks

$x = 13$ 2 marks

2. $2k + 3 = 1$ 2 marks

$2k + 3 - 3 = 1 - 3$ 2 marks

$2k = -2$ 2 marks

$k = -1$ 2 marks

3. $\frac{3r}{4} = 27$ 2 marks

$4\left(\frac{3r}{4}\right) = 4 \times 27$ 2 $\frac{1}{2}$ marks

$3r = 108$ 2 marks

$r = 36$ 2 marks

4. Let the number be x $\frac{1}{2}$ mark

$x + x = 2$ 2 marks

$2x = 2$ 2 marks

$x = 1$

5. Let the number be x $\frac{1}{2}$ mark

$7x - 5 = 16$ 2 marks

$7x - 5 + 5 = 16 + 5$ 2 marks

$7x = 21$ 2 marks

$x = 3$ 2 marks

Total: 40 marks

Appendix B

Simple Linear Equation and Word Problem Pre-Test (SLEWPPT) Final Draft

Instruction: Answer all the questions **Time: 1hour 20minutes**

Section A

Solve the following equations:

1. $4b + 24 = 0$
2. $5a + 6 = 2a + 20$
3. $3x + 1 = 2(3x + 5)$
4. $\frac{x}{2} + \frac{3x}{4} = 5$
5. $0.8x + 0.5(400 - x) = 350$

Section B

Construct a simple linear equation for each of the following problems and solve the equation:

1. A certain number is subtracted from 20 and the result is 7. Find the number.
2. I add 24 to two times a certain number and the answer is 0. Find the number.
3. Three-fourth of some number r is twenty seven. Find the number.
4. I think of a number, add it to itself, the result is 2. What is the number?
5. I add 12 to a certain number and then double the result. The answer is 42. Find the number.

Solution to Simple Linear Equation and Word Problem Pre-Test (SLEWPPT)

Section A

1. $4b + 24 = 0$

$4b + 24 - 24 = 0 - 24$ 2 marks

$4b = -24$ 2 marks

$\frac{4b}{4} = \frac{-24}{4}$ 2 marks

$b = -6$ 2 marks

(8 marks)

2. $5a + 6 = 2a + 20$

$5a - 2a + 6 = 2a - 2a + 20$ 2 marks

$3a + 6 = 20$ 2 marks

$3a + 6 - 6 = 20 - 6$ 2 marks

$3a = 14$ 2 marks

$\frac{3a}{3} = \frac{14}{3}$ 2 marks

$a = 4\frac{2}{3}$ 2 marks

(12 marks)

3. $3x + 1 = 2(3x + 5)$

$3x + 1 = 6x + 10$ 2 marks

$3x - 3x + 1 = 6x - 3x + 10$ 2 marks

$1 = 3x + 10$ 2 marks

$1 - 10 = 3x + 10 - 10$ 2 marks

$-9 = 3x$ 2 marks

$\frac{-9}{3} = \frac{3x}{3}$ 2 marks

$-3 = x$ 2 marks

(14 marks)

4. $\frac{x}{2} + \frac{3x}{4} = 5$

$4\left(\frac{x}{2} + \frac{3x}{4}\right) = 4 \times 5$ 2 marks

$4 \times \frac{x}{2} + 4 \times \frac{3x}{4} = 4 \times 5$ 2 marks

$2x + 3x = 20$ 2 marks

$5x = 20$ 2 marks

$\frac{5x}{5} = \frac{20}{5}$ 2 marks

$x = 4$ 2 marks

(12 marks)

5. $0.8x + 0.5(400 - x) = 350$

$8x + 5(400 - x) = 3500$ 2 marks

$8x + 2000 - 5x = 3500$ 2 marks

$3x + 2000 = 3500$ 2 marks

$3x + 2000 - 2000 = 3500 - 2000$ 2 marks

$3x = 150$ 2 marks

$\frac{3x}{3} = \frac{150}{3}$ 2 marks

$x = 500$ 2 marks

(14 marks)

Total: 60 marks

Section B

1. Let x be the number

$20 - x = 7$ 1 mark

$20 - 20 - x = 7 - 20$ 2 marks

$-x = -13$ 2 marks

$(-1)(-x) = (-1)(-13)$ 1 mark

$x = 13$ 2 marks

(8 marks)

2. Let x be the number

$2x + 24 = 0$ 1 mark

$2x + 24 - 24 = 0 - 24$ 2 marks

$2x = -24$ 2 marks

$\frac{2x}{2} = \frac{-24}{2}$ 2 marks

$x = -12$ 2 marks

(9 marks)

3. $\frac{3r}{4} = 27$ 1 marks

$4\left(\frac{3r}{4}\right) = 27 \times 4$ 2 marks

$3r = 108$ 2 marks

$r = 36$ 2 marks

(7 marks)

4. Let x be the number

$x + x = 2$ 1 marks

$2x = 2$ 2 marks

$x = 1$ 2 marks

(5 marks)

5. Let x be the number

$2(x + 12) = 42$ 1 mark

$2x + 24 = 42$ 2 marks

$2x + 24 - 24 = 42 - 24$ 2 marks

$2x = 18$ 2 marks

$\frac{2x}{2} = \frac{18}{2}$ 2 marks

$x = 9$ 2 marks

(11 marks) **Total: 40 marks**

Appendix C

Simultaneous Linear Equation Performance Test (SLEPT) Draft

Instruction: **Answer all the questions**

Construct simultaneous linear equations in each of the following problems and solve the resulting pair of equations using substitution method or Elimination method.

1. The sum of the digits of a two-digit number is 9. The number is 27 more than the number obtained when the digits are reversed. Find the number.
2. A promoter sold ₦ 28, 500 tickets for a beauty contest. A ticket for standing costs ₦ 800 and a ticket for a seat costs ₦ 1250. Ticket sales produced a total of ₦ 28, 335, 000. How many of each type of tickets were sold?
3. Kola and Bukola can do a piece of work in 30 days. After they had worked together for 12 days, Bukola had an accident and Kola takes another 24 days to finish the rest of the work. How long would each take working alone?
4. A two-digit number is such that the sum of its digits is 11. The number is 27 greater than the number obtained by interchanging the digits. Find the number.
5. A nurse has two solutions that contain different concentration of a certain medication. One is 12% concentration, and the other is an 8% concentration. How many cubic centimeters (cc) of each should she mix together to obtain 20 cc of a 9% solution?
6. A business man sold 8 bags of mulch and 3 bags of fertilizer for ₦24. The next purchase was for 5 bags of mulch and 5 bags of fertilizer. The cost of that purchase was ₦25. Find the cost of a single bag of mulch and a single bag of fertilizer.
7. The perimeter of a badminton court is 128 feet. After a game of badminton, a player's coach estimates that the athlete has run a total of 444 feet, which is equivalent to six times the court's length plus nine times its width. What are the dimensions of a standard badminton court?
8. 3 boys and 1 girl can do a piece of work in 4 days while 1 boy and 2 girls can do it in 6 days. How long would each take working alone?
9. If 6 boys and 8 girls can do a piece of work in 15 days, while 7 boys and 4 girls can do the work in 20 days. How long would a boy take to finish the work single handed? When will a girl finish the work all alone?
10. Two angles are supplementary. The sum of half the measure of the larger angle and the measure of the smaller angle is 125^0. What is the measure of each angle?
11. A man rowed 20 kilometres downstream in 2 hours. Returning upstream, the man took 2.5 hours. Find his rate of rowing in still water, and the rate of the stream.
12. A man rows 18 kilometres downstream in 2 hours. He finds that it takes 6 hours to row back. What is his rate of rowing in still water, and what is the rate of the stream?
13. A nurse has two solutions that contain different concentrations of a certain medication. One is a 30% concentration and the other is a 15% concentration. How many cubic centimeters (cc) of each should she mix to obtain 45cc of a 20% solution?

14. One number is 1 more than another. The ratio of the smaller number increased by 2 to the larger number increased by 5 is $\frac{3}{7}$. What are the two numbers?
15. The perimeter of playing field is 1040feet and its length is 200feet longer than its width. Find the length and width of the playing field.
16. An aero-plane travels 2000 km with a tail wind in 2 hr. It then travels 2000 km with head in 4 hr. Find the speed of the aero-plane and the speed of the wind.
17. The sum of two angles is 180^0. One angle is 30^0 less than twice the other. Find the measures of the two angles.
18. Salman and Sikiru can do a piece of work in 12 hours. They start together but after 8 hours, Sikiru strikes and Salman finish the rest of the work in 10 more hours. How long would each take working alone?
19. A number is 3 more than another. The ratio of the smaller number increased by 1 to the larger number increased by 5 is $\frac{1}{2}$. What are the two numbers?
20. A man travels 10 km in 50 minutes if he runs for 8 km and walks for 2 km. If he runs for 4 km and walks 6 km, his time is 1 hour 15 minutes. Find his running and walking speeds.

Marking Scheme for SLEPT (Draft)

Substitution Method I

1. Solution

Let $x =$ Tens digit

$y =$ Units digit

Then

$10x + y =$ Original number

$10y + x =$ Number with digits reversed

From the first sentence of the question,

$x + y = 9$ (1)

From the second sentence of the question,

$10x + y = 10y + x + 27$

$10x + y - 10y - x = 27$

$9x - 9y = 27$

$x - y = 3$ (2)

From (1), $y = 9 - x$

Substitute $9 - x$ for y in (2) to obtain

$x - 9 + x = 3$

$2x - 9 + 9 = 3 + 9$

$2x = 12$

$x = 6$

Substitute 6 for x in (1) to obtain

$6 + y = 9$

$6 - 6 + y = 9 - 6$

$y = 3$

$\therefore$ the number is $10 \times 6 + 3$ or 63

2. Solution

$\frac{1}{y} =$ Fractional part of the work kola does per day

Let the number of tickets for standing be x

Let the number of tickets for a seat be y

Then,

$x + y = 28{,}500$ (1)

$800x + 1250y = 28{,}335{,}000$ (2)

From (1) $y = 28{,}500 - x$ (3)

Substitute $28{,}500 - x$ y in (2)

$800x + 1250(28{,}500 - x) = 28{,}335{,}000$

$800x + 35625000 - 1250x = 28{,}335{,}000$

$-450x = -729000$

$x = 16{,}200$

Substitute 16,200 for x in (1)

$16200 + y = 28500$

$y = 12{,}300$

There are 16,200 tickets for standing and 12,300 tickets for sitting.

3. Solution

Let

$x =$ Number of days for Bukola to complete the work

$y =$ Number of days for Kola to complete the work

Then

$\frac{1}{x} =$ Fractional part of the work Bukola does per day

If each works for 30 days, the entire work is done:

$\frac{30}{x} + \frac{30}{y} = 1$ (1)

When Bukola works only for 12 days, Kola has to work for an additional of 24 days or 36 days in all, to complete the entire work:

$$\frac{12}{x}+\frac{36}{y}=1 \qquad (2)$$

Let $u=\frac{1}{x}$ and $v=\frac{1}{y}$, the equations become:

$$30u+30v=1 \qquad (3)$$
$$12u+36v=1 \qquad (4)$$

From equation (3), $v=\frac{1-30u}{30}$

Substitute $\frac{1-30u}{30}$ for v in equation (4) to obtain

$$12u+36\left(\frac{1-30u}{30}\right)=1$$
$$360u+36-1080u=30$$
$$-720u=-6$$
$$u=\frac{1}{120}$$

Substitute $\frac{1}{120}$ for u in (3) to obtain

$$30\times\frac{1}{120}+30v=1$$
$$30+3600v=120$$
$$30-30+3600v=120-30$$
$$3600v=90$$
$$v=\frac{1}{40}$$

But $u=\frac{1}{x}$ and $v=\frac{1}{y}$,

$\frac{1}{120}=\frac{1}{x}$ and $\frac{1}{40}=\frac{1}{y}$

$\therefore\ x=120$ and $y=40$

So kola could do the work alone in 40 days, and Bukola could do it alone in 120 days.

4 Solution

Let $x=$ Tens digit

$y=$ Units digit

Then

$10x+y=$ Original number

$10y+x=$ Number with digits reversed

From the first sentence of the question,

$$x+y=11 \qquad (1)$$

From the second sentence of the question,

$$10x+y=10y+x+27$$
$$10x+y-10y-x=27$$
$$9x-9y=27$$
$$x-y=3 \qquad (2)$$

From (1) $y=11-x$

Substitute $11-x$ for y in (2)

$$x-(11-x)=3$$
$$x-11+x=3$$
$$2x=14$$
$$x=7$$

Substitute 7 for x in equation (2) to obtain

$$7-y=3$$
$$7-7-y=3-7$$
$$-y=-4$$
$$y=4$$

The number is $10\times7+4$ or 74

5. Solution

Let

$x=$ Number of cubic centimeters of 12% solution required

$y=$ Number of cubic centimeters of 8% solution required

The table below organizes the information as follows:

Summary table of Mixture Problem

	Number of CC	Percent of Concentration	Amount of Concentration
12% solut	x	0.12	$0.12x$
8% soluti	y	0.08	$0.08y$
9% solute	20	0.09	1.8

Since there are to be 20 cc in all, the total amount of solution is:

$$x + y = 20 \qquad (1)$$

The amount of concentration of the medication from the two components must equal the number of cc in the final mixture:

$$0.12x + 0.08y = 1.8 \qquad (2)$$

From equation (1),

$$y = 20 - x$$

Substitute $20 - x$ for y in equation (2) to obtain

$$0.12x + 0.08(20 - x) = 1.8$$

$$0.12x + 1.6 - 0.08x = 1.8$$

$$0.04x + 1.6 - 1.6 = 1.8 - 1.6$$

$$0.04x = 0.2$$

$$x = 5$$

Substitute 5 for x in (1) to obtain

$$5 + y = 20$$

$$y = 15$$

Therefore the nurse should mix 5cc of the 12% solution with 15cc of the 8% solution to give 20cc of the 9% solution.

6. Solution

Let $x =$ the cost of one bag of mulch

$y =$ the cost of one bag of fertilizer

The two different equations are:

$$8x + 3y = 24 \qquad (1)$$

$$5x + 5y = 25 \qquad (2)$$

From (1), $y = \dfrac{24 - 8x}{3}$

Substitute $\dfrac{24 - 8x}{3}$ for y in (2) to obtain

$$5x + 5\left(\frac{24 - 8x}{3}\right) = 25$$

$$15x + 120 - 40x = 75$$

$$120 - 25x = 75$$

$$25x - 25x + 120 = 75 + 25x$$

$$120 = 75 + 25x$$

$$120 - 75 = 75 - 75 + 25x$$

$$45 = 25x$$

$$x = 1.8$$

Substitute 1.8 for x in (2) to obtain

$$5 \times 1.8 + 5y = 25$$

$$9 + 5y = 25$$

$$9 - 9 + 5y = 25 - 9$$

$$5y = 16$$

$$y = 3.2$$

The coast of one bag of mulch is ₦ 1.80 and the cost of one bag of fertilizer is ₦ 3.20

7. Solution

Let the length of badminton court be x

Let the width of the badminton court be y

Then the equations are:

$2l + 2w = 128$ (1)

$6l + 9w = 444$ (2)

From equation (1),

$$l = \frac{128 - 2w}{2}$$

Substitute $l = \frac{128 - 2w}{2}$ for w in equation (2)

$$6\left(\frac{128 - 2w}{2}\right) + 9w = 444$$

$$768 - 12w + 18w = 888$$

$$6w = 120$$

$$w = 20$$

Substitute 20 for w in equation (1).

$$2l + 2 \times 20 = 128$$

$$2l + 40 = 128$$

$$2l = 88$$

$$l = 44$$

Length = 44 feet and width = 20 feet

8. Solution

Let $x =$ Number of days for one boy to finish the work

$y =$ Number of days for one girl to finish the work

Then

$\frac{1}{x} =$ Fractional part of the work one boy does per day

$\frac{1}{y} =$ Fractional part of the work one girl does per day

If 3 boys and 1 girl work 4 days, the entire work is done:

$$4\left(\frac{3}{x} + \frac{1}{y}\right) = 1$$

$$\frac{12}{x} + \frac{4}{y} = 1 \qquad (1)$$

If 1 boy and 2 girls can finish the work in 6 days, the entire work is done:

$$6\left(\frac{1}{x} + \frac{2}{y}\right) = 1$$

$$\frac{6}{x} + \frac{12}{y} = 1 \qquad (2)$$

Substitute $\frac{1}{x} = r$ and $\frac{1}{y} = s$ in equation (1) and (2) to get

$2r + 4s = 1$ (3)

$6r + 12s = 1$ (4)

From (4) $s = \frac{1 - 12s}{4}$

Substitute $\frac{1 - 12s}{4}$ for u in (4) to obtain

$$6r + 12\left(\frac{1 - 12r}{4}\right) = 1$$

$$24r + 12 - 144r = 4$$

$$-120r = -8$$

$$r = \frac{-86}{-120} = \frac{1}{15}$$

Substitute $\frac{1}{15}$ for r in (3) to obtain

$$12\left(\frac{1}{15}\right)+4s=1$$
$$12+60s=15$$
$$60s=3$$
$$s=\frac{3}{60}=\frac{1}{20}$$

But $\frac{1}{x}=u$ and $\frac{1}{y}=v$

$\frac{1}{15}=\frac{1}{x}$ and $\frac{1}{20}=\frac{1}{y}$

$x=15$ and $y=20$

A boy alone can do the work in 15days and a girl alone can do the work in 20 days.

9. Solution

Let $x=$ Number of days for one man to finish the work

$y=$ Number of days for one boy to finish the work

Then

$1/x=$ Fractional part of the work one man does per day

$1/y=$ Fractional part of the work one boy does per day

If 2 men and 1 boy work for 5 days, the entire work is done:

$$5\left(\frac{2}{x}+\frac{1}{y}\right)=1$$

$$\frac{10}{x}+\frac{5}{y}=1 \quad (1)$$

If 1 man and 2 boys can finish the work in 6 days, the entire work is done:

$$6\left(\frac{1}{x}+\frac{2}{y}\right)=1$$

$$\frac{6}{x}+\frac{12}{y}=1 \quad (2)$$

Substitute $\frac{1}{x}=u$ and $\frac{1}{y}=v$ in equation (1) and (2) to get

$$10u+5v=1 \quad (3)$$
$$6u+12v=1 \quad (4)$$

From (3), $v=\frac{1-10u}{5}$

Substitute $\frac{1-10u}{5}$ for v in (4) to obtain

$$6u+12\left(\frac{1-10u}{5}\right)=1$$
$$30u+12-120u=5$$
$$30u+12-12-120u=5-12$$
$$30u-120u=-7$$
$$-90u=-7$$
$$u=\frac{7}{90}$$

Substitute $\frac{7}{90}$ for u in (3) to obtain

$$10\times\frac{7}{90}+5v=1$$

$$\frac{7}{9}+5v=1$$
$$7+45v=9$$
$$7-7+45y=9-7$$

$$45v=2$$
$$v=\frac{2}{45}$$

$$\frac{7}{90}=\frac{1}{x} \text{ and } \frac{2}{45}=\frac{1}{y}$$

$$x=12\tfrac{6}{7} \text{ and } y=22\tfrac{1}{2}$$

10. Solution

Let x and y represent themeasures of the two angles.

Since the two angles are supplementary, then

$$x+y=180 \quad (1)$$

The sum of half the measure of the larger angle and the measure of the smaller is 125^0, then

$$\frac{x}{2}+y=125$$

$$x+2y=250 \quad (2)$$

From (1) $x=180-y$

Substituting $180-y$ for x in (2) we have

$$180-y++2y=250$$

$$y=250-180$$

$$y=70$$

Substituting 70 for y in (1) we have

$$x+70=180$$

$$x=180-70$$

$$x=110^0$$

Therefore $x=110^0$ and $y=70^0$

11. Solution

Let $x=$ The man' rate of rowing in still water

$y=$ The rate of the stream

Then

$x+y=$ The rate of boat downstream

$x-y=$ The rate of boat upstream

Summary table of Uniform Motion Problem

	d	r	t
Downstream	20	$x+y$	2
Upstream	20	$x-y$	2.5

$$d=rt$$

$$20=(x+y)2 \quad (1)$$

$$20=(x-y)2.5 \quad (2)$$

Simplify (1) and (2) to obtain

$$x+y=10 \quad (3)$$

$$x-y=8 \quad (4)$$

From (3), $y=10-x$

Substitute $10-x$ for y in (4) to obtain

$$x-(10-x)=8$$

$$x-10+x=8$$

$$2x-10=8$$

$$2x-10+10=8+10$$

$$2x=18$$

$$x=9$$

Substitute 9 for x in (3) to obtain

$$9+y=10$$

$$9-9+y=10-9$$

$$y=1$$

Thus the man's rate of rowing in still water is 9 km/h and the rate of the stream is 1km/h.

12. Solution

Let $x=$ The man's rate of rowing in still water

$y=$ The rate of the stream

Then

$x+y=$ The rate of the boat downstream

$x-y=$ The rate of the boat upstream

Summary table of Uniform Motion Problem

	d	r	t
Downstream	18	$x+y$	2
Upstream	18	$x-y$	6

$$d = rt$$

$$18 = (x+y)2 \quad (1)$$

$$18 = (x-y)6 \quad (2)$$

Dividing (1) by 2 and (2) by 6, we have

$$x+y=9 \quad (3)$$

$$x-y=3 \quad (4)$$

From equation (3),

$y = 9-x$

Substitute $9-x$ for y in (4) to obtain

$$x-9+x=3$$

$$2x=12$$

$$x=6$$

Substitute 6 for x in (3) to obtain

$$6+y=9$$

$$6-6+y=9-6$$

$$y=3$$

Therefore the man's rate of rowing is 6 km/h and the rate of the stream is 3 km/h.

13. Solution

Let $x=$ Number of cubic centimeters of the 30% solution

$y=$ Number of cubic centimeters of the 15% solution

The table below illustrates the information

Summary table of Mixture Problem

	Number of CC	Percent of Concentration	Amount of Concentration
30% Solution	x	0.30	$0.30x$
15% Solution	y	0.15	$0.15y$
20% Solution	45	0.20	9

Since there are to be 45cc in all, we must have the following equation:

$$x+y=45 \quad (1)$$

The amount of concentration of the medication from the two components must equal the number of cc in the final mixture:

$$0.30x+0.15y=9 \quad (2)$$

From (1), $y=45-x$

Substitute $45-x$ for y in (2) to obtain

$$0.30x+0.15(45-x)=9$$

$$0.30x+6.75-0.15x=9$$

$$0.15x+6.75=9$$

$$0.15x+6.75-6.75=9-6.75$$

$$0.15x=2.25$$

$$x=15$$

Substitute 15 for x in (1) to obtain

$$15+y=45$$

$$15-15+y=45-15$$

$$y=30$$

Therefore the nurse should mix 15cc of the 30% solution with 30cc of the 15% solution to obtain 45cc of the 20% solution.

14. Solution

Let

$x =$ the smaller number

$y =$ the larger number

Then

$x + 2 =$ the smaller number increased by 2

$y + 5 =$ the larger number increased by 5

One number is 1 more than another,

$$y = x + 1 \qquad (1)$$

The ratio of the smaller number is increased by 2 to the larger number increased by 5 is $\frac{3}{7}$,

$$\frac{x+2}{y+5} = \frac{3}{7}$$

$$7(x+2) = 3(y+5)$$

$$7x + 14 = 3y + 15$$

$$7x + 14 - 14 = 3y + 15 - 14$$

$$7x - 3y = 1 \qquad (2)$$

From (1), substitute $x + 1$ for y in (2) to obtain

$$7x - 3(x+1) = 1$$

$$7x - 3x - 3 = 1$$

$$4x - 3 + 3 = 1 + 3$$

$$4x = 4$$

$$x = 1$$

Substitute 1 for x in equation (1) to obtain

$$y = 1 + 1 = 2$$

Therefore, the numbers are 1 and 2

15. Solution

Let x and y represent the length and width of the playing field.

Then

$$2l + 2w = 1040 \qquad (1)$$

But the length is 200 feet longer than the width

Thus, $l = w + 200 \qquad (2)$

Substitute $w + 200$ for l in (1) to obtain

$$2(w + 200) + 2w = 1040$$

$$2w + 400 + 2w = 1040$$

$$4w + 400 = 1040$$

$$4w + 400 - 400 = 1040 - 400$$

$$4w = 640$$

$$w = 160$$

Substituting 160 for w in (2) we get

$$l = 160 + 200$$

$$l = 360$$

$\therefore$ Length = 360 and width = 160

16. Solution

This is a rate/distance problem. Thus we look for two different rates.

Let x = rate of the plane

y = rate of the wind

When the plane is travelling with the tail, it will be going faster.

When the plane is travelling with the head, it will be going slower.

Summary table of Uniform Motion Problem

	x	t	d
With tail wind	$x+y$	2	2000
With head wind	$x-y$	4	2000

$$rt = d$$

$$2(x+y) = 2000 \quad (1)$$

$$4(x-y) = 2000 \quad (2)$$

$$x+y = 1000 \quad (3)$$

$$x-y = 500 \quad (4)$$

From (3), $y = 1000 - x$

Substitute $1000 - x$ for y in (4)

$$x-(1000-x) = 500$$

$$x-1000+x = 500$$

$$2x-1000+1000 = 500+1000$$

$$2x = 1500$$

$$x = 750$$

Substitute 750 for x in (3) we have

$$750 + y = 1000$$

$$y = 1000 - 750 = 250$$

The speed of the aeroplane is 750 km/h and the speed of the wind is 250 km/h.

17. Solution

Let x and y represent the measures of the two angles.

Thus $x + y = 180 \quad (1)$

Since one angle is 30^0 less than twice the other,

$$y = 2x - 30 \quad (2)$$

Substitute $2x-30$ for y in (1) we have

$$70 + 2x - 30 = 180$$

$$2x + 40 = 180$$

$$2x + 40 - 40 = 180 - 40$$

$$2x = 140$$

$$x = 70$$

Substitute 70 for x in (2) to obtain

$$y = 2\times 70 - 30$$

$$y = 140 - 30$$

$$y = 110$$

$x = 70^0$ and $y = 110^0$

18. Solution

Le t

$x =$ Number of days for Sikiru to complete the work

$y =$ Number of days for Salman to complete the work

Then

$\frac{1}{x} =$ Fractional part of the work

Sikiru does per hour

$\frac{1}{y} =$ Fractional part of the work

Salman does per hour

If each works for 12 hours, the entire work is done:

$$\frac{12}{x} + \frac{12}{y} = 1 \quad (1)$$

When Sikiru works only for 8 hours, Salman has to work for an additional of 10 hours or 18 hours in all, to complete the entire work:

$$\frac{8}{x} + \frac{18}{y} = 1 \quad (2)$$

Let $r = \frac{1}{x}$ and $s = \frac{1}{y}$, the eqns. become:

$$12r + 12s = 1 \quad (3)$$
$$8r + 18s = 1 \quad (4)$$

From (3), $s = \dfrac{1-12r}{12}$

Substitute $\dfrac{1-12r}{12}$ for s in (4) to obtain

$$8r + 18\left(\frac{1-12r}{12}\right) = 1$$
$$96r + 18 - 216r = 12$$
$$-120r + 18 - 18 = 12 - 18$$
$$-120r = -6$$
$$r = \frac{1}{20}$$

Substitute $\dfrac{1}{20}$ for r in (3):

$$12 \times \frac{1}{20} + 12s = 1$$
$$12 + 240s = 20$$
$$12 - 12 + 240s = 20 - 12$$
$$240s = 8$$
$$s = \frac{8}{240} = \frac{1}{30}$$

But $r = \dfrac{1}{x}$ and $s = \dfrac{1}{y}$,

$$\frac{1}{20} = \frac{1}{x} \text{ and } \frac{1}{30} = \frac{1}{y}$$

$$\therefore \ x = 20 \text{ and } y = 30$$

So Sikiru could do the work in 20 hours and Salman could do it in 30 hours.

19. Solution

Let

$x =$ the smaller number

$y =$ the larger number

Then

$x + 1 =$ smaller number increased by 1

$y + 5 =$ smaller number increased by 5

A number is 3 more than another,

$$y = x + 3 \quad (1)$$

The ratio of the smaller number increased by 1 to the larger number increased by 5 is $\frac{1}{2}$,

$$\frac{x+1}{y+5} = \frac{1}{2}$$
$$2(x+1) = y + 5$$
$$2x + 2 = y + 5$$
$$2x + 2 - 2 = y + 5 - 2$$
$$2x - y = 3 \quad (2)$$

From (1), $y = x + 3$

Substitute $x + 3$ for y in (2) to get

$$2x - (x + 3) = 3$$
$$2x - x - 3 = 3$$
$$x - 3 = 3$$
$$x - 3 + 3 = 3 + 3$$
$$x = 6$$

Substitute 6 for x in equation (1) to obtain

$$y = 6 + 3$$
$$y = 9$$

Thus, the numbers are 6 and 9

20 Solution

Let the man's running and walking speeds be *x* and *y* respectively

The equations are:

$$8x + 2y = 50 \quad (1)$$

$4x + 6y = 75 \qquad (2)$

From (1) $x = \dfrac{50 - 2y}{8}$

Substitute $\dfrac{50 - 2y}{8}$ for x in (2)

$$\frac{4(50 - 2y)}{8} + 6y = 75 \qquad (3)$$

$$200 - 8y + 48y = 600$$

$$40y = 400$$

$$y = 10$$

Substitute 10 for y in (1)

$$8x + 2 \times 10 = 50$$

$$8x = 30$$

$$x = 3.75$$

Elimination Method II

1. Solution

Let $x =$ Tens digit

$y =$ Units digit

Then

$10x + y =$ Original number

$10y + x =$ Number with digits reversed

From the first sentence of the question,

$$x + y = 9 \qquad (1)$$

From the second sentence of the question,

$$10x + y = 10y + x + 27$$
$$10x + y - 10y - x = 27$$
$$9x - 9y = 27$$
$$x - y = 3 \qquad (2)$$

But, $x + y = 9$ (1)

Adding: $2x = 12$

$x = 6$

Substitute 6 for x in (1):

$6 + y = 9$

$6 - 6 + y = 9 - 6$

$y = 3$

The number is $10 \times 6 + 3$ or 63

2. Solution

Let the number of tickets for standing be x

Let the number of tickets for a seat be y

Then,

$x + y = 28{,}500 \qquad (1)$

$800x + 1250y = 28{,}335{,}000 \qquad (2)$

$(1) \times 800$: $1250x + 1250y = 28{,}500$

(2)×-1: $-800x - 1250y = -28{,}335{,}000$

Add: $450x = 7290000$

$x = 16200$

Substitute 16200 for x in (1)

$16200 + y = 28500$

$y = 12{,}300$

There are 16,200 tickets for standing and 12,300 for sitting.

3. Solution

Let

$x =$ Number of days for Bukola to complete the work

$y =$ Number of days for Kola to complete the work

Then

$\frac{1}{x} =$ Fractional part of the work Bukola does per day

$\frac{1}{y} =$ Fractional part of the work kola does per day

If each works for 30 days, the entire work is done:

$$\frac{30}{x} + \frac{30}{y} = 1 \qquad (1)$$

When Bukola works only for 12 days, Kola has to work for an additional of 24 days or 36 days in all, to complete the entire work:

$$\frac{12}{x} + \frac{36}{y} = 1 \quad (2)$$

Let $\frac{1}{y} = s$ and $v = \frac{1}{y}$, the equns become:

$$30u + 30v = 1 \quad (3)$$

$$12u + 36v = 1 \quad (4)$$

$(3) \times 12$: $360u + 360v = 12$

$(4) \times 30$: $\underline{360u + 1080v = 30}$

$(4) - (3)$: $720v = 18$

$$v = \frac{18}{720} = \frac{1}{40}$$

Substitute $\frac{1}{40}$ for v in (3) to obtain

$$30u + 30 \times \frac{1}{40} = 1$$

$$1200u + 30 = 40$$

$$1200u + 30 - 30 = 40 - 30$$

$$1200u = 10$$

$$u = \frac{10}{1200} = \frac{1}{120}$$

But $u = \frac{1}{x}$ and $v = \frac{1}{y}$,

$$\frac{1}{120} = \frac{1}{x} \text{ and } \frac{1}{40} = \frac{1}{y}$$

$\therefore \; x = 120$ and $y = 40$

So kola could do the work alone in 40 days, and Bukola could do it alone in 120 days.

4. Solution

Let $x =$ Tens digit

$y =$ Units digit

Then

$10x + y =$ Original number

$10y + x =$ Number with digits reversed

From the first sentence of the question,

$$x + y = 11 \qquad (1)$$

From the second sentence of the question,

$$10x + y = 10y + x + 27$$

$$10x + y - 10y - x = 27$$

$$9x - 9y = 27$$

$$x - y = 3 \qquad (2)$$

But, $\underline{x + y = 11} \qquad (1)$

Adding: $2x = 14$

$$x = 7$$

Substitute 7 for x in equation (1) to obtain

$$7 + y = 11$$

$$7 - 7 + y = 11 - 7$$

$$y = 4$$

The number is $10 \times 7 + 4$ or 74

5. Solution

Let

$x =$ Number of cubic centimeters of 12% solution required

$y =$ Number of cubic centimeters of 8% solution required

The table below organizes the information as follows:

Summary table of Mixture Problem

	Number of CC	Percent of oncentration	Amount of Concentration
12% Solu	x	0.12	$0.12x$
8% Solu	y	0.08	$0.08y$
9% Solu	20	0.09	1.8

Since there are to be 20 cc in all, the total amount of solution is:

$$x + y = 20 \qquad (1)$$

The amount of concentration of the medication from the two components

must equal the number of cc in the final mixture:

$0.12x + 0.08y = 1.8$ (2)

Multiply equation (1) by- 0.08 to make the coefficients of x opposites

$-0.08x - 0.08y = -1.6$ (1)

$0.12x + 0.08y = 1.8$ (2)

Add: $0.04x = 0.2$

$0.04x = 0.2$

$x = 5$

Substitute 5 for x in (1) to obtain

$5 + y = 20$

$y = 15$

Therefore the nurse should mix 5cc of the 12% solution with 15cc of the 8% solution to give 20cc of the 9% solution.

6. Solution

Let $x =$ the cost of one bag of mulch

$y =$ the cost of one bag of fertilizer

The two different equations are:

$8x + 3y = 24$ (1)

$5x + 5y = 25$ (2)

(1) × 5: $40x + 15y = 120$

(2) ×-3: $-15x - 15y = -75$

Add: $25x = 45$

$x = 1.8$

Substitute 1.8 for x in (1):

$8 \times 1.8 + 3y = 24$

$14.4 + 3y = 24$

$14.4 - 14.4 + 3y = 24 - 14.4$

$3y = 9.6$

$y = 3.2$

The coast of one bag of mulch is ₦ 1.80 and the cost of one bag of fertilizer is ₦ 3.20

7. Solution

Let the length of badminton court be x

Let the width of the badminton court be y

Then the equations are:

$2l + 2w = 128$ (1)

$6l + 9w = 444$ (2)

(1) × -3: $-6l - 6w = -384$

(2) $6l + 9w = 444$

Add: $3w = 60$

$w = 20$

Substitute 20 for l in equation (1)

$2l + 2 \times 20 = 128$

$2l + 40 = 128$

$2l = 88$

$l = 44$

Length = 44 feet and Width = 20 feet

8. (3)-(4) : $30r = 2$

$r = \frac{1}{15}$

Substitute $\frac{1}{15}$ for r in (3):

$$12\left(\frac{1}{15}\right)+4s=1$$

$$12+60s=15$$

$$60s=3$$

$$s=\frac{3}{60}=\frac{1}{20}$$

But $\frac{1}{x}=r$ and $\frac{1}{y}=s$

$$\therefore \quad \frac{1}{15}=\frac{1}{x} \text{ and } \frac{1}{y}=\frac{1}{20}$$

$x=15$ and $y=20$

A boy alone can do the work in 15days and a girl alone can do the work in 20 days.

9 Solution

Let $x=$ Number of days for one man to finish the work

$y=$ Number of days for one boy to finish the work

Then

$1/x=$ Fractional part of the work one man does per day

$1/y=$ Fractional part of the work one boy does per day

If 2 men and 1 boy work for 5 days, the entire work is done:

$$5\left(\frac{2}{x}+\frac{1}{y}\right)=1$$

$$\frac{10}{x}+\frac{5}{y}=1 \qquad (1)$$

If 1 man and 2 boys can finish the work in 6 days, the entire work is done:

$$6\left(\frac{1}{x}+\frac{2}{y}\right)=1$$

$$\frac{6}{x}+\frac{12}{y}=1 \qquad (2)$$

Substitute $\frac{1}{x}=u$ and $\frac{1}{y}=v$ in (1) and (2):

$$10u+5v=1 \qquad (3)$$

$$6u+12v=1 \qquad (4)$$

(3) × -6: $-60u-30v=-6$

(4) × 10: $\underline{60u+120v=10}$

Add: $90v=4$

$$v=\frac{2}{45}$$

Substitute $\frac{2}{45}$ for v in equation (3):

$$10u+5\left(\frac{2}{45}\right)=1$$

$$450u+10=45$$

$$450u=35$$

$$u=\frac{35}{450}=\frac{7}{90}$$

$$\frac{7}{90}=\frac{1}{x} \qquad \text{and} \qquad \frac{2}{45}=\frac{1}{y}$$

$x=12\frac{6}{7}$ and $y=22\frac{1}{2}$

One man alone can do the work in $12\frac{6}{7}$ days and one boy alone can do the work in $22\frac{1}{2}$ days.

10. Solution

Let x and y represent the measures of the two angles.

Since the two angles are supplementary, then

$$x + y = 180 \qquad (1)$$

The sum of half the measure of the larger angle and the measure of the smaller is 125^0, then

$$\frac{x}{2} + y = 125$$

$$x + 2y = 250 \quad (2)$$

But $\underline{x + y = 180}$ (1)

(2) – (1): $y = 70^0$

Substitute 70 for y in (1)

$x + 70 = 180$

$x + 70 - 70 = 180 - 70$

$x = 110^0$

$x = 110^0$ and $y = 70^0$

11. Solution

Let $x =$ The man' rate of rowing in still water

$y =$ The rate of the stream

Then

$x + y =$ The rate of boat downstream

$x - y =$ The rate of boat upstream

Summary table of Uniform Motion Problem

	d	r	t
Downstream	20	$x + y$	2
Upstream	20	$x - y$	2.5

$$d = rt$$

$$20 = (x + y)2 \qquad (1)$$

$$20 = (x - y)2.5 \quad (2)$$

Simplify equations (1) and (2):

$$x + y = 10 \qquad (3)$$

$$\underline{x - y = 8} \qquad (4)$$

Add: $2x = 18$

$$x = 9$$

Substitute 9 for x in (3):

$$9 + y = 10$$

$$9 - 9 + y = 10 - 9$$

$$y = 1$$

Thus the man's rate of rowing in still water is 9 km/h and the rate of the stream is 1km/h.

12. Solution

Let $x =$ The man's rate of rowing in still water

$y =$ The rate of the stream

Then

$x + y =$ The rate of the boat downstream

$x - y =$ The rate of the boat upstream

Summary table of Uniform Motion Problem

	d	r	t
Downstream	18	$x + y$	2
Upstream	18	$x - y$	6

$$d = rt$$

$$18 = (x + y)2 \qquad (1)$$

$$18 = (x - y)6 \qquad (2)$$

Divide (1) by 2 and (2) by 6:

$$x + y = 9 \quad (3)$$

$$\underline{x - y = 3} \quad (4)$$

Add: $2x = 12$

$$x = 6$$

Substitute 6 for x in (3):

$$6 + y = 9$$

$$6 - 6 + y = 9 - 6$$

$$y = 3$$

Therefore the man's rate of rowing is 6 km/h and the rate of the stream is 3 km/h.

13. Solution

Let

$x =$ Number of cubic centimeters of the 30% solution

$y =$ Number of cubic centimeters of the 15% solution

The table below illustrates the information

Summary table of Mixture Problem

	Number Of CC	Percent of Concentration	Amount of Concentration
30% Solution	x	0.30	$0.30x$
15% Solution	y	0.15	$0.15y$
20% Solution	45	0.20	9

The total amount of solution is:

$$x + y = 45 \quad (1)$$

The amount of concentration of the medication from the two components must equal the number of cc in the final mixture:

$$0.30x + 0.15y = 9 \quad (2)$$

(1)x -0.15:

$$-0.15x - 0.15y = -6.75 \quad (3)$$

$$\underline{0.30x + 0.15y = 9} \quad (2)$$

Adding: $0.15x = 2.25$

$$x = 15$$

Substitute 15 for x in (2):

$$0.30 \times 15 + 0.15y = 9$$

$$4.5 + 0.15y = 9$$

$$4.5 - 4.5 + 0.15y = 9 - 4.5$$

$$0.15y = 4.5$$

$$y = 30$$

Therefore the nurse should mix 15cc of the 30% solution with 30cc of the 15% solution to obtain 45cc of the 20% solution.

14. Solution

Let

$x =$ the smaller number

$y =$ the larger number

Then

$x + 2 =$ the smaller number increased by 2

$y + 5 =$ the larger number increased by 5

One number is 1 more than another,

$$y = x + 1 \quad (1)$$

The ratio of the smaller number is increased by 2 to the larger number increased by 5 is $\frac{3}{7}$,

$$\frac{x+2}{y+5}=\frac{3}{7}$$

$$7(x+2)=3(y+5)$$

$$7x+14=3y+15$$

$$7x+14-14=3y+15-14$$

$$7x-3y=1 \qquad (2)$$

From (1), $y-x=1$ (3)

(2): $7x-3y=1$

(3) $\times 3$: $-3x+3y=3$

Adding: $4x=4$

$$x=1$$

Substitute 1 for x in equation (1) to obtain

$$y=1+1=2$$

Therefore, the numbers are 1 and 2

15. Solution

Let x and y represent the length and width of the playing field.

Then $2l+2w=1040$ (1)

But the length is 200 feet longer than the width

Thus, $l=w+200$ or $l-w=200$ (2)

Multiply (2) by 2 to get'

$$2l-2w=400 \qquad (3)$$

Adding (1) and (2) we have

$4l=1440 \quad \therefore \quad l=360$

Substituting 360 for l in (1) we get

2(360) + 2w = 1040

2w = 1040 – 720

2w = 320 ∴ w = 160

Length = 360 and width = 160

16. Solution

This is a rate/distance problem. Thus we look for two different rates.

Let x = rate of the plane

y = rate of the wind

When the plane is travelling with the tail, it will be going faster.

When the plane is travelling with the head, it will be going slower.

Summary table of Unifom Motion Problem

	r	*t*	*d*
With tail wind	$x+y$	2	2000
With head wind	$x-y$	4	2000

Rate times time is equal to distance

$$2(x+y)=2000 \qquad (1)$$

$$4(x-y)=2000 \qquad (2)$$

$$x+y=1000 \qquad (3)$$

$$x-y=500 \qquad (4)$$

Adding: $2x=1500$

$$x=750$$

Substituting 750 for x in (3) we have

$$750 + y = 1000$$

$$y = 1000 - 750 = 250$$

The speed of the aeroplane is 750 km/h and the speed of the wind is 250 km/h.

17. Solution

Let x and y represent the measures of the two angles.

Thus $x + y = 180$ (1)

Since one angle is 30^0 less than twice the other,

$$y = 2x - 30$$

Or $2x - y = 30$ (2)

Adding (1) and (2) we have

$$3x = 210$$

$$x = 70$$

Substitute 70 for x in (1) we have

$$70 + y = 180$$

$$y = 180 - 70 = 110$$

Therefore $x = 70^0$ and $y = 110^0$

18. Solution

Le t

$x =$ Number of days for Sikiru to complete the work

$y =$ Number of days for Salman to complete the work

Then

$\frac{1}{x} =$ Fractional part of the work

Sikiru does per hour

$\frac{1}{y} =$ Fractional part of the work

Salman does per hour

If each works for 12 hours, the entire work is done:

$$\frac{12}{x} + \frac{12}{y} = 1 \qquad (1)$$

When Sikiru works only for 8 hours, Salman has to work for an additional of 10 hours or 18 hours in all, to complete the entire work:

$$\frac{8}{x} + \frac{18}{y} = 1 \qquad (2)$$

Let $r = \frac{1}{x}$ and $s = \frac{1}{y}$, the equations become:

$$12r + 12s = 1 \qquad (3)$$

$$8r + 18s = 1 \qquad (4)$$

(3) ×-8: $-96r - 96s = -8$

(4) ×12: $\underline{96r + 216s = 12}$

Adding: $120s = 4$

$$s = \frac{1}{30}$$

Substitute $\frac{1}{30}$ for s in equation (3) to obtain

$$12r + 12 \times \frac{1}{30} = 1$$

$$360r + 12 = 30$$

$$360r + 12 - 12 = 30 - 12$$

$$360r = 18$$

$$r = \frac{1}{20}$$

But $r = \frac{1}{x}$ and $s = \frac{1}{y}$,

$\frac{1}{20} = \frac{1}{x}$ and $\frac{1}{30} = \frac{1}{y}$

$\therefore\ x = 20$ and $y = 30$

So Sikiru could do the work in 20 hours and Salman could do it in 30 hours.

19. Solution

Let

$x =$ the smaller number

$y =$ the larger number

Then

$x+1 =$ smaller number increased by 1

$y+5 =$ smaller number increased by 5

A number is 3 more than another,

$$y = x+3$$

$$-x+y=3 \qquad (1)$$

The ratio of the smaller number increased by 1 to the larger number increased by 5 is $\frac{1}{2}$,

$$\frac{x+1}{y+5}=\frac{1}{2}$$

$$2(x+1)=y+5$$

$$2x+2=y+5$$

$$2x+2-2=y+5-2$$

$$2x-y=3 \qquad (2)$$

But, $-x+y=3 \qquad (1)$

$$x=6$$

Substitute 6 for x in equation (1):

$$-6+y=3$$

$$y=9$$

Thus, the numbers are 6 and 9

20. Solution

Let the man's running and walking speeds be x and y respectively

The equations are:

$$8x+2y=50 \qquad (1)$$

$$4x+6y=75 \qquad (2)$$

(1)×3 $\quad 24x+6y=150$

(2) $\quad 4x+6y=75$

(1) - (2): $\quad 20x=75$

$$x=3.75$$

Substitute 3.75 for x in (1)

$$8\times 3.75+2y=50$$

$$30+2y=50$$

$$2y=20$$

$$y=10$$

$x=3.75$ and $y=10$

Appendix D

Simultaneous Linear Eqution Performance Test (SLEPT) Pilot Study

Instruction: **Answer all the questions**

Construct simultaneous linear equations in each of the following problems and solve the resulting pair of equations using substitution method or Elimination method.

1. The sum of the digits of a two-digit number is 9. The number is 27 more than the number obtained when the digits are reversed. Find the number.
2. Three pencils and seven erasers cost ₦ 18.00. Four pencils and ten erasers cost ₦ 25.00; find the cost of a pencil and an eraser.
3. Kola and Bukola can do a piece of work in 30 days. After they had worked together for 12 days, Bukola had an accident and Kola takes another 24 days to finish the rest of the work. How long would each take working alone?
4. A two-digit number is such that the sum of its digits is 11. The number is 27 greater than the number obtained by interchanging the digits. Find the number.
5. A nurse has two solutions that contain different concentration of a certain medication. One is 12% concentration, and the other is an 8% concentration. How many cubic centimeters (cc) of each should she mix together to obtain 20 cc of a 9% solution?
6. A business man sold 8 bags of mulch and 3 bags of fertilizer for ₦24. The next purchase was for 5 bags of mulch and 5 bags of fertilizer. The cost of that purchase was ₦25. Find the cost of a single bag of mulch and a single bag of fertilizer.
7. The perimeter of a rectangle is 84cm and its length is 6cm longer than its breadth. Find the length and breadth of the rectangle.
8. 3 boys and 1 girl can do a piece of work in 4 days while 1 boy and 2 girls can do it in 6 days. How long would each take working alone?
9. If 2 men and 1 boy can do a piece of work in 5 days while 1 man and 2 boys can do it in 6 days. How long would the work take a man working single handed? When will a boy finish the work all alone?
10. Two angles are supplementary. The sum of half the measure of the larger angle and the measure of the smaller angle is 125^0. What is the measure of each angle?
11. A man rowed 20 kilometres downstream in 2 hours. Returning upstream, the man took 2.5 hours. Find his rate of rowing in still water, and the rate of the stream.
12. A man rows 18 kilometres downstream in 2 hours. He finds that it takes 6 hours to row back. What is his rate of rowing in still water, and what is the rate of the stream?
13. A nurse has two solutions that contain different concentrations of a certain medication. One is a 30% concentration and the other is a 15% concentration. How many cubic centimeters (cc) of each should she mix to obtain 45cc of a 20% solution?
14. One number is 1 more than another. The ratio of the smaller number increased by 2 to the larger number increased by 5 is $\frac{3}{7}$. What are the two numbers?

15. The perimeter of playing field is 1040feet and its length is 200feet longer than its width. Find the length and width of the playing field.
16. An aero-plane travels 2000 km with a tail wind in 2 hr. It then travels 2000 km with head in 4 hr. Find the speed of th+e aero-plane and the speed of the wind.
17. The sum of two angles is 180^0. One angle is 30^0 less than twice the other. Find the measures of the two angles.
18. Salman and Sikiru can do a piece of work in 12 hours. They start together but after 8 hours, Sikiru strikes and Salman finish the rest of the work in 10 more hours. How long would each take working alone?
19. A number is 3 more than another. The ratio of the smaller number increased by 1 to the larger number increased by 5 is $\frac{1}{2}$. What are the two numbers?
20. An aero-plane travels 3000 km with a tail wind in 3 hr. It then travels 3000 km with head in 4 hr. Find the speed of the aero-plane and the speed of the wind.

Marking Scheme for SLEPT (Pilot Study)

Substitution Method I

1. Solution

Let $x =$ Tens digit

$y =$ Units digit

Then

$10x + y =$ Original number

$10y + x =$ Number with digits reversed

From the first sentence of the question,

$x + y = 9$ (1)

From the second sentence of the question,

$10x + y = 10y + x + 27$

$10x + y - 10y - x = 27$

$9x - 9y = 27$

$x - y = 3$ (2)

From (1), $y = 9 - x$

Substitute $9 - x$ for y in (2) to obtain

$x - 9 + x = 3$

$2x - 9 + 9 = 3 + 9$

$2x = 12$

$x = 6$

Substitute 6 for x in (1) to obtain

$6 + y = 9$

$6 - 6 + y = 9 - 6$

$y = 3$

$\therefore$ the number is $10 \times 6 + 3$ or 63

2. Solution

Let one pencil cost x naira and one eraser cost y naira.

Then 3 pencils cost $3x$ naira and 7 pencils cost $7y$ naira.

Thus, $3x + 7y = 18$ (1)

$4x + 10y = 25$ (2)

From (1), $y = \dfrac{18 - 3x}{7}$ (3)

Substitute $\dfrac{18 - 3x}{7}$ for y in (2)

$$4x + 10\left(\frac{18 - 3x}{7}\right) = 25$$

$$28x + 180 - 30x = 175$$

$$-2x + 180 = 175$$

$$-2x + 180 - 180 = 175 - 180$$

$$-2x = -5$$

$$x = \frac{5}{2}$$

$$x = 2.5$$

Substitute 2.5 for x in (3)

$$y = \frac{18 - 3 \times 2.5}{7}$$

$$y = \frac{18 - 7.5}{7} = \frac{10.5}{7} = 1.5$$

A pencil cost ₦ 2.50 and an eraser cost ₦ 1.50

3. Solution

Let

$x =$ Number of days for Bukola to complete the work

$y =$ Number of days for Kola to complete the work

Then

$\dfrac{1}{x} =$ Fractional part of the work Bukola does per day

$\frac{1}{y}$ = Fractional part of the work kola does per day

If each works for 30 days, the entire work is done:

$$\frac{30}{x}+\frac{30}{y}=1 \qquad (1)$$

When Bukola works only for 12 days, Kola has to work for an additional of 24 days or 36 days in all, to complete the entire work:

$$\frac{12}{x}+\frac{36}{y}=1 \qquad (2)$$

Let $u=\frac{1}{x}$ and $v=\frac{1}{y}$, the equations become:

$$30u+30v=1 \qquad (3)$$

$$12u+36v=1 \qquad (4)$$

From equation (3), $v=\frac{1-30u}{30}$

Substitute $\frac{1-30u}{30}$ for v in equation (4) to obtain

$$12u+36\left(\frac{1-30u}{30}\right)=1$$

$$360u+36-1080u=30$$

$$-720u=-6$$

$$u=\frac{1}{120}$$

Substitute $\frac{1}{120}$ for u in (3) to obtain

$$30\times\frac{1}{120}+30v=1$$

$$30+3600v=120$$

$$30-30+3600v=120-30$$

$$3600v=90$$

$$v=\frac{1}{40}$$

But $u=\frac{1}{x}$ and $v=\frac{1}{y}$,

$\frac{1}{120}=\frac{1}{x}$ and $\frac{1}{40}=\frac{1}{y}$

$\therefore\ x=120$ and $y=40$

So kola could do the work alone in 40 days, and Bukola could do it alone in 120 days.

4. Solution

Let x = Tens digit

y = Units digit

Then

$10x+y=$ Original number

$10y+x=$ Number with digits reversed

From the first sentence of the question,

$$x+y=11 \qquad (1)$$

From the second sentence of the question,

$$10x+y=10y+x+27$$

$$10x+y-10y-x=27$$

$$9x-9y=27$$

$$x-y=3 \qquad (2)$$

From (1) $y=11-x$

Substitute $11-x$ for y in (2)

$$x-(11-x)=3$$

$$x-11+x=3$$

$$2x=14$$

$$x=7$$

Substitute 7 for x in equation (2) to obtain

$$7-y=3$$

$$7-7-y=3-7$$

$$-y=-4$$

$$y=4$$

The number is $10\times7+4$ or 74

5. Solution

Let

$x =$ Number of cubic centimeters of 12% solution required

$y =$ Number of cubic centimeters of 8% solution required

The table below organizes the information as follows:

Summary table of Mixture Problem

	Number of CC	Percent of Concentration	Amount of Concentration
12% solut	x	0.12	$0.12x$
8% soluti	y	0.08	$0.08y$
9% solute	20	0.09	1.8

Since there are to be 20 cc in all, the total amount of solution is:

$$x + y = 20 \qquad (1)$$

The amount of concentration of the medication from the two components must equal the number of cc in the final mixture:

$$0.12x + 0.08y = 1.8 \qquad (2)$$

From equation (1),

$$y = 20 - x$$

Substitute $20 - x$ for y in equation (2) to obtain

$$0.12x + 0.08(20 - x) = 1.8$$

$$0.12x + 1.6 - 0.08x = 1.8$$

$$0.04x + 1.6 - 1.6 = 1.8 - 1.6$$

$$0.04x = 0.2$$

$$x = 5$$

Substitute 5 for x in (1) to obtain

$$5 + y = 20$$

$$y = 15$$

Therefore the nurse should mix 5cc of the 12% solution with 15cc of the 8% solution to give 20cc of the 9% solution.

6. Solution

Let $x =$ the cost of one bag of mulch

$y =$ the cost of one bag of fertilizer

The two different equations are:

$$8x + 3y = 24 \qquad (1)$$

$$5x + 5y = 25 \qquad (2)$$

From (1), $y = \dfrac{24 - 8x}{3}$

Substitute $\dfrac{24 - 8x}{3}$ for y in (2) to obtain

$$5x + 5\left(\frac{24 - 8x}{3}\right) = 25$$

$$15x + 120 - 40x = 75$$

$$120 - 25x = 75$$

$$25x - 25x + 120 = 75 + 25x$$

$$120 = 75 + 25x$$

$$120 - 75 = 75 - 75 + 25x$$

$$45 = 25x$$

$$x = 1.8$$

Substitute 1.8 for x in (2) to obtain

$$5 \times 1.8 + 5y = 25$$

$$9 + 5y = 25$$

$$9 - 9 + 5y = 25 - 9$$

$$5y = 16$$

$$y = 3.2$$

The coast of one bag of mulch is ₦ 1.80 and the cost of one bag of fertilizer is ₦ 3.20

7. Solution

Let l and b represent the length and breadth of the rectangle respectively.

Then $2l + 2b = 84$ (1)

$l = b + 6$ (2)

Substituting $b + 6$ for l in (1) we have:

$2(b + 6) + 2b = 84$

$4b + 12 = 84$

$4b = 72$

$b = 18$

Substitute 18 for b in (2) we have

$l = 18 + 6 = 24$

∴ Length = 24cm and Breadth = 18

8. Solution

Let $x =$ Number of days for one boy to finish the work

$y =$ Number of days for one girl to finish the work

Then

$\frac{1}{x}$ = Fractional part of the work one boy does per day

$\frac{1}{y}$ = Fractional part of the work one girl does per day

If 3 boys and 1 girl work 4 days, the entire work is done:

$$4\left(\frac{3}{x}+\frac{1}{y}\right)=1$$

$$\frac{12}{x}+\frac{4}{y}=1 \qquad (1)$$

If 1 boy and 2 girls can finish the work in 6 days, the entire work is done:

$$6\left(\frac{1}{x}+\frac{2}{y}\right)=1$$

$$\frac{6}{x}+\frac{12}{y}=1 \qquad (2)$$

Substitute $\frac{1}{x}=r$ and $\frac{1}{y}=s$ in equation (1) and (2) to get

$$2r+4s=1 \qquad (3)$$

$$6r+12s=1 \qquad (4)$$

From (4) $s=\frac{1-12s}{4}$

Substitute $\frac{1-12s}{4}$ for u in (4) to obtain

$$6r+12\left(\frac{1-12r}{4}\right)=1$$

$$24r+12-144r=4$$

$$-120r=-8$$

$$r=\frac{-86}{-120}=\frac{1}{15}$$

Substitute $\frac{1}{15}$ for r in (3) to obtain

$$12\left(\frac{1}{15}\right)+4s=1$$

$$12+60s=15$$

$$60s=3$$

$$s=\frac{3}{60}=\frac{1}{20}$$

But $\frac{1}{x}=u$ and $\frac{1}{y}=v$

$$\frac{1}{15}=\frac{1}{x} \text{ and } \frac{1}{20}=\frac{1}{y}$$

$$x=15 \text{ and } y=20$$

A boy alone can do the work in 15days and a girl alone can do the work in 20 days.

9. Solution

Let $x=$ Number of days for one man to finish the work

$y=$ Number of days for one boy to finish the work

Then

$1/x=$ Fractional part of the work one man does per day

$1/y=$ Fractional part of the work one boy does per day

If 2 men and 1 boy work for 5 days, the entire work is done:

$$5\left(\frac{2}{x}+\frac{1}{y}\right)=1$$

$$\frac{10}{x}+\frac{5}{y}=1 \quad (1)$$

If 1 man and 2 boys can finish the work in 6 days, the entire work is done:

$$6\left(\frac{1}{x}+\frac{2}{y}\right)=1$$

$$\frac{6}{x}+\frac{12}{y}=1 \quad (2)$$

Substitute $\frac{1}{x}=u$ and $\frac{1}{y}=v$ in equation (1) and (2) to get

$$10u+5v=1 \quad (3)$$

$$6u+12v=1 \quad (4)$$

From (3), $v=\frac{1-10u}{5}$

Substitute $\frac{1-10u}{5}$ for v in (4) to obtain

$$6u+12\left(\frac{1-10u}{5}\right)=1$$

$$30u+12-120u=5$$

$$30u+12-12-120u=5-12$$

$$30u-120u=-7$$

$$-90u=-7$$

$$u=\frac{7}{90}$$

Substitute $\frac{7}{90}$ for u in (3) to obtain

$$10\times\frac{7}{90}+5v=1$$

$$\frac{7}{9}+5v=1$$

$$7+45v=9$$

$$7-7+45y=9-7$$

$$45v=2$$

$$v=\frac{2}{45}$$

$$\frac{7}{90}=\frac{1}{x} \text{ and } \frac{2}{45}=\frac{1}{y}$$

$$x=12\tfrac{6}{7} \text{ and } y=22\tfrac{1}{2}$$

10. Solution

Let x and y represent themeasures of the two angles.

Since the two angles are supplementary, then

$x+y=180 \quad (1)$

The sum of half the measure of the larger angle and the measure of the smaller is 125^0, then

$$\frac{x}{2} + y = 125$$

$$x + 2y = 250 \quad (2)$$

From (1) $x = 180 - y$

Substituting $180 - y$ for x in (2) we have

$$180 - y + +2y = 250$$

$$y = 250 - 180$$

$$y = 70$$

Substituting 70 for y in (1) we have

$$x + 70 = 180$$

$$x = 180 - 70$$

$$x = 110^0$$

Therefore $x = 110^0$ and $y = 70^0$

11. Solution

Let $x =$ The man' rate of rowing in still water

$y =$ The rate of the stream

Then

$x + y =$ The rate of boat downstream

$x - y =$ The rate of boat upstream

Summary table of Uniform Motion Problem

	d	r	t
Downstream	20	$x + y$	2
Upstream	20	$x - y$	2.5

$$d = rt$$

$$20 = (x + y)2 \quad (1)$$

$$20 = (x - y)2.5 \quad (2)$$

Simplify (1) and (2) to obtain

$$x + y = 10 \quad (3)$$

$$x - y = 8 \quad (4)$$

From (3), $y = 10 - x$

Substitute $10 - x$ for y in (4) to obtain

$$x - (10 - x) = 8$$

$$x - 10 + x = 8$$

$$2x - 10 = 8$$

$$2x - 10 + 10 = 8 + 10$$

$$2x = 18$$

$$x = 9$$

Substitute 9 for x in (3) to obtain

$$9 + y = 10$$

$$9 - 9 + y = 10 - 9$$

$$y = 1$$

Thus the man's rate of rowing in still water is 9 km/h and the rate of the stream is 1km/h.

12. Solution

Let $x =$ The man's rate of rowing in still water

$y =$ The rate of the stream

Then

$x + y =$ The rate of the boat downstream

$x - y =$ The rate of the boat upstream

Summary table of Uniform Motion Problem

	d	r	t
Downstream	18	$x + y$	2
Upstream	18	$x - y$	6

$$d = rt$$

$$18 = (x + y)2 \quad (1)$$

$18 = (x - y)6$ (2)

Dividing (1) by 2 and (2) by 6, we have

$$x + y = 9 \quad (3)$$
$$x - y = 3 \quad (4)$$

From equation (3),

$y = 9 - x$

Substitute $9 - x$ for y in (4) to obtain

$$x - 9 + x = 3$$
$$2x = 12$$
$$x = 6$$

Substitute 6 for x in (3) to obtain

$$6 + y = 9$$
$$6 - 6 + y = 9 - 6$$
$$y = 3$$

Therefore the man's rate of rowing is 6 km/h and the rate of the stream is 3 km/h.

13. Solution

Let $x =$ Number of cubic centimeters of the 30% solution

$y =$ Number of cubic centimeters of the 15% solution

The table below illustrates the information

Summary table of Mixture Problem

	Number of CC	Percent of Concentration	Amount of Concentration
30% olution	x	0.30	$0.30x$
15% Solution	y	0.15	$0.15y$
20% Solution	45	0.20	9

Since there are to be 45cc in all, we must have the following equation:

$$x + y = 45 \quad (1)$$

The amount of concentration of the medication from the two components must equal the number of cc in the final mixture:

$$0.30x + 0.15y = 9 \quad (2)$$

From (1), $y = 45 - x$

Substitute $45 - x$ for y in (2) to obtain

$$0.30x + 0.15(45 - x) = 9$$

$$0.30x + 6.75 - 0.15x = 9$$

$$0.15x + 6.75 = 9$$

$$0.15x + 6.75 - 6.75 = 9 - 6.75$$
$$0.15x = 2.25$$
$$x = 15$$

Substitute 15 for x in (1) to obtain

$$15 + y = 45$$

$$15 - 15 + y = 45 - 15$$
$$y = 30$$

Therefore the nurse should mix 15cc of the 30% solution with 30cc of the 15% solution to obtain 45cc of the 20% solution.

14. Solution

Let

$x =$ the smaller number

$y =$ the larger number

Then

$x + 2 =$ the smaller number increased by 2

$y+5=$ the larger number increased by 5

One number is 1 more than another,

$$y = x+1 \qquad (1)$$

The ratio of the smaller number is increased by 2 to the larger number increased by 5 is $\frac{3}{7}$,

$$\frac{x+2}{y+5} = \frac{3}{7}$$

$$7(x+2) = 3(y+5)$$

$$7x+14 = 3y+15$$

$$7x+14-14 = 3y+15-14$$

$$7x-3y = 1 \qquad (2)$$

From (1), substitute $x+1$ for y in (2) to obtain

$$7x-3(x+1) = 1$$

$$7x-3x-3 = 1$$

$$4x-3+3 = 1+3$$

$$4x = 4$$

$$x = 1$$

Substitute 1 for x in equation (1) to obtain

$$y = 1+1 = 2$$

Therefore, the numbers are 1 and 2

15. Solution

Let x and y represent the length and width of the playing field.

Then

$$2l + 2w = 1040 \qquad (1)$$

But the length is 200 feet longer than the width

Thus, $l = w + 200 \qquad (2)$

Substitute $w + 200$ for l in (1) to obtain

$$2(w + 200) + 2w = 1040$$

$$2w + 400 + 2w = 1040$$

$$4w + 400 = 1040$$

$$4w + 400 - 400 = 1040 - 400$$

$$4w = 640$$

$$w = 160$$

Substituting 160 for w in (2) we get

$$l = 160 + 200$$

$$l = 360$$

$\therefore$ Length = 360 and width = 160

16. Solution

This is a rate/distance problem. Thus we look for two different rates.

Let x = rate of the plane

y = rate of the wind

When the plane is travelling with the tail, it will be going faster.

When the plane is travelling with the head, it will be going slower.

Summary table of Uniform Motion Problem

	x	t	d
With tail wind	$x+y$	2	2000
With head wind	$x-y$	4	2000

$$rt = d$$

$$2(x+y) = 2000 \qquad (1)$$

$$4(x-y) = 2000 \qquad (2)$$

$$x+y = 1000 \qquad (3)$$

$x - y = 500$ (4)

From (3), $y = 1000 - x$

Substitute $1000 - x$ for y in (4)

$x - (1000 - x) = 500$

$x - 1000 + x = 500$

$2x - 1000 + 1000 = 500 + 1000$

$2x = 1500$

$x = 750$

Substitute 750 for x in (3) we have

$750 + y = 1000$

$y = 1000 - 750 = 250$

The speed of the aeroplane is 750 km/h and the speed of the wind is 250 km/h.

17. Solution

Let x and y represent the measures of the two angles.

Thus $x + y = 180$ (1)

Since one angle is 30^0 less than twice the other,

$y = 2x - 30$ (2)

Substitute $2x - 30$ for y in (1) we have

$70 + 2x - 30 = 180$

$2x + 40 = 180$

$2x + 40 - 40 = 180 - 40$

$2x = 140$

$x = 70$

Substitute 70 for x in (2) to obtain

$y = 2 \times 70 - 30$

$y = 140 - 30$

$y = 110$

$x = 70^0$ and $y = 110^0$

18. Solution

Le t

$x =$ Number of days for Sikiru to complete the work

$y =$ Number of days for Salman to complete the work

Then

$\frac{1}{x} =$ Fractional part of the work

Sikiru does per hour

$\frac{1}{y} =$ Fractional part of the work

Salman does per hour

If each works for 12 hours, the entire work is done:

$$\frac{12}{x} + \frac{12}{y} = 1 \qquad (1)$$

When Sikiru works only for 8 hours, Salman has to work for an additional of 10 hours or 18 hours in all, to complete the entire work:

$$\frac{8}{x} + \frac{18}{y} = 1 \qquad (2)$$

Let $r = \frac{1}{x}$ and $s = \frac{1}{y}$, the eqns. become:

$12r + 12s = 1$ (3)

$8r + 18s = 1$ (4)

From (3), $s = \frac{1 - 12r}{12}$

Substitute $\frac{1 - 12r}{12}$ for s in (4) to obtain

$$8r + 18\left(\frac{1 - 12r}{12}\right) = 1$$

$96r + 18 - 216r = 12$

$-120r + 18 - 18 = 12 - 18$

$-120r = -6$

$r = \frac{1}{20}$

Substitute $\frac{1}{20}$ for r in (3):

$$12 \times \frac{1}{20} + 12s = 1$$
$$12 + 240s = 20$$

$$12 - 12 + 240s = 20 - 12$$
$$240s = 8$$
$$s = \frac{8}{240} = \frac{1}{30}$$

But $r = \frac{1}{x}$ and $s = \frac{1}{y}$,

$$\frac{1}{20} = \frac{1}{x} \text{ and } \frac{1}{30} = \frac{1}{y}$$

$$\therefore \ x = 20 \text{ and } y = 30$$

So Sikiru could do the work in 20 hours and Salman could do it in 30 hours.

19. Solution

Let

$x =$ the smaller number

$y =$ the larger number

Then

$x + 1 =$ smaller number increased by 1

$y + 5 =$ smaller number increased by 5

A number is 3 more than another,

$$y = x + 3 \qquad (1)$$

The ratio of the smaller number increased by 1 to the larger number increased by 5 is $\frac{1}{2}$,

$$\frac{x+1}{y+5} = \frac{1}{2}$$

$$2(x+1) = y + 5$$
$$2x + 2 = y + 5$$
$$2x + 2 - 2 = y + 5 - 2$$
$$2x - y = 3 \qquad (2)$$

From (1), $y = x + 3$

Substitute $x + 3$ for y in (2) to get

$$2x - (x + 3) = 3$$
$$2x - x - 3 = 3$$
$$x - 3 = 3$$
$$x - 3 + 3 = 3 + 3$$
$$x = 6$$

Substitute 6 for x in equation (1) to obtain

$$y = 6 + 3$$
$$y = 9$$

Thus, the numbers are 6 and 9

20. Solution

This is a rate/distance problem. Thus we look for two different rates (speed).

Let x = rate of the plane

y = rate of the wind

When the plane is travelling with the tail, it will be going faster.

When the plane is travelling with the head, it will be going slower.

Summary table of Unifom Motion Problem

	r	t	d
With tail wind	$x + y$	3	3000
With head wind	$x - y$	4	3000

rt = d

$$3(x+y)=3000 \qquad (1)$$

$$4(x-y)=3000 \qquad (2)$$

$$x+y=1000 \qquad (3)$$

$$x-y=750 \qquad (4)$$

From (3), $y=1000-x$

Substitute $1000-x$ for y in (4):

$$x-(1000-x)=750$$

$$x-1000+x=750$$

$$2x-1000=750$$

$$2x-1000+1000=750+1000$$

$$2x=1750$$

$$x=875$$

Substituting 875 for x in (3):

$$875+y=1000$$

$$875-875+y=1000-875$$

$$y=125$$

Elimination Method II

1. Solution

Let $x =$ Tens digit

$y =$ Units digit

Then

$10x + y =$ Original number

$10y + x =$ Number with digits reversed

From the first sentence of the question,

$$x + y = 9 \qquad (1)$$

From the second sentence of the question,

$$10x + y = 10y + x + 27$$
$$10x + y - 10y - x = 27$$
$$9x - 9y = 27$$
$$x - y = 3 \qquad (2)$$

But, $\underline{x + y = 9}$ (1)

Adding: $2x = 12$

$$x = 6$$

Substitute 6 for x in (1):

$$6 + y = 9$$
$$6 - 6 + y = 9 - 6$$
$$y = 3$$

The number is $10 \times 6 + 3$ or 63

2. Solution

Let one pencil cost x naira and one eraser cost y naira.

Then 3 pencils cost $3x$ naira and 7 pencils cost $7y$ naira.

Thus, $3x + 7y = 18$ (1)

Similarly, $4x + 10y = 25$

(2) $(1) \times 4 : 12x + 28y = 72$

$(2) \times 3 : \underline{12x + 30y = 75}$

$(2) - (1) \qquad 2y = 3$

$$y = \frac{3}{2} = 1.5$$

Substitute 1.5 for y in (1)

$$3x + 7 \times 1.5 = 18$$
$$3x + 10.5 = 18$$
$$3x + 10.5 - 10.5 = 18 - 10.5$$
$$3x = 7.5$$
$$x = 2.5$$

A pencil cost ₦ 2.50 and an eraser cost ₦ 1.50

3. Solution

Let

$x =$ Number of days for Bukola to complete the work

$y =$ Number of days for Kola to complete the work

Then

$\frac{1}{x} =$ Fractional part of the work Bukola does per day

$\frac{1}{y} =$ Fractional part of the work kola does per day

If each works for 30 days, the entire work is done:

$$\frac{30}{x} + \frac{30}{y} = 1 \qquad (1)$$

When Bukola works only for 12 days, Kola has to work for an additional of 24 days or 36 days in all, to complete the entire work:

$$\frac{12}{x}+\frac{36}{y}=1 \quad (2)$$

Let $\frac{1}{y}=s$ and $v=\frac{1}{y}$, the equns become:

$$30u+30v=1 \quad (3)$$

$$12u+36v=1 \quad (4)$$

$(3)\times 12:\quad 360u+360v=12$

$(4)\times 30:\quad \underline{360u+1080v=30}$

$(4)-(3):\quad 720v=18$

$$v=\frac{18}{720}=\frac{1}{40}$$

Substitute $\frac{1}{40}$ for v in (3) to obtain

$$30u+30\times\frac{1}{40}=1$$

$$1200u+30=40$$

$$1200u+30-30=40-30$$

$$1200u=10$$

$$u=\frac{10}{1200}=\frac{1}{120}$$

But $u=\frac{1}{x}$ and $v=\frac{1}{y}$,

$$\frac{1}{120}=\frac{1}{x} \text{ and } \frac{1}{40}=\frac{1}{y}$$

$$\therefore\ x=120 \text{ and } y=40$$

So kola could do the work alone in 40 days, and Bukola could do it alone in 120 days.

4. Solution

Let $x=$ Tens digit

$y=$ Units digit

Then

$10x+y=$ Original number

$10y+x=$ Number with digits reversed

From the first sentence of the question,

$$x+y=11 \quad (1)$$

From the second sentence of the question,

$$10x+y=10y+x+27$$

$$10x+y-10y-x=27$$

$$9x-9y=27$$

$$x-y=3 \quad (2)$$

But, $\underline{x+y=11} \quad (1)$

Adding: $2x=14$

$$x=7$$

Substitute 7 for x in equation (1) to obtain

$$7+y=11$$

$$7-7+y=11-7$$

$$y=4$$

The number is $10\times 7+4$ or 74

5. Solution

Let

$x=$ Number of cubic centimeters of 12% solution required

$y=$ Number of cubic centimeters of 8% solution required

The table below organizes the information as follows:

Summary table of Mixture Problem

	Number of CC	Percent of oncentration	Amount of Concentration
12% Solu	x	0.12	$0.12x$
8% Solu	y	0.08	$0.08y$
9% Solu	20	0.09	1.8

Since there are to be 20 cc in all, the total amount of solution is:

$x + y = 20$ (1)

The amount of concentration of the medication from the two components must equal the number of cc in the final mixture:

$0.12x + 0.08y = 1.8$ (2)

Multiply equation (1) by- 0.08 to make the coefficients of x opposites

$$-0.08x - 0.08y = -1.6 \quad (1)$$
$$0.12x + 0.08y = 1.8 \quad (2)$$

Add: $0.04x = 0.2$

$0.04x = 0.2$

$x = 5$

Substitute 5 for x in (1) to obtain

$5 + y = 20$

$y = 15$

Therefore the nurse should mix 5cc of the 12% solution with 15cc of the 8% solution to give 20cc of the 9% solution.

6. Solution

Let $x =$ the cost of one bag of mulch

$y =$ the cost of one bag of fertilizer

The two different equations are:

$8x + 3y = 24$ (1)

$5x + 5y = 25$ (2)

(1) $\times 5$: $40x + 15y = 120$

(2) $\times$-3: $-15x - 15y = -75$

Add: $25x = 45$

$x = 1.8$

Substitute 1.8 for x in (1):

$8 \times 1.8 + 3y = 24$

$14.4 + 3y = 24$

$14.4 - 14.4 + 3y = 24 - 14.4$

$3y = 9.6$

$y = 3.2$

The coast of one bag of mulch is ₦ 1.80 and the cost of one bag of fertilizer is ₦ 3.20

7. Solution

Let l and b represent the length and breadth of the rectangle respectively. Then

$2l + 2b = 84$ (1)

But the length is 6cm longer than the breadth

Thus, $l = b + 6$

Or $l - b = 6$ (2)

(1): $2l + 2b = 84$

(2) $\times 2$: $2l - 2b = 12$

Adding: $4l = 96$

$$l = 24$$

Substituting 24 for b in (2):

$$24 = b + 6$$

$$24 - 6 = b + 6 - 6$$

$$b = 18$$

$\therefore$ Length = 24cm and Breadth = 18

8. Solution

Let $x =$ Number of days for one boy to finish the work

$y =$ Number of days for one girl to finish the work

Then

$\frac{1}{x}$ = Fractional part of the work one boy does per day

$\frac{1}{y}$ = Fractional part of the work one girl does per day

If 3 boys and 1 girl work 4 days, the entire work is done:

$$4\left(\frac{3}{x}+\frac{1}{y}\right)=1$$

$$\frac{12}{x}+\frac{4}{y}=1 \quad (1)$$

If 1 boy and 2 girls can finish the work in 6 days, the entire work is done:

$$6\left(\frac{1}{x}+\frac{2}{y}\right)=1$$

$$\frac{6}{x}+\frac{12}{y}=1 \quad (2)$$

Substitute $\frac{1}{x} = r$ and $\frac{1}{y} = s$ in (1) and (2) to get

$$12r + 4s = 1 \quad (3)$$

$$6r + 12s = 1 \quad (4)$$

$(3)\times 3$: $36r + 12s = 3$

(4): $\underline{6r + 12s = 1}$

(3)-(4): $30r = 2$

$$r = \frac{1}{15}$$

Substitute $\frac{1}{15}$ for r in (3):

$$12\left(\frac{1}{15}\right)+4s=1$$

$$12 + 60s = 15$$

$$60s = 3$$

$$s = \frac{3}{60} = \frac{1}{20}$$

But $\frac{1}{x} = r$ and $\frac{1}{y} = s$

$\therefore$ $\frac{1}{15} = \frac{1}{x}$ and $\frac{1}{y} = \frac{1}{20}$

$x = 15$ and $y = 20$

A boy alone can do the work in 15days and a girl alone can do the work in 20 days.

9. Solution

Let $x =$ Number of days for one man to finish the work

$y =$ Number of days for one boy to finish the work

Then

$1/x =$ Fractional part of the work one man does per day

$1/y =$ Fractional part of the work one boy does per day

If 2 men and 1 boy work for 5 days, the entire work is done:

$$5\left(\frac{2}{x}+\frac{1}{y}\right)=1$$

$$\frac{10}{x}+\frac{5}{y}=1 \qquad (1)$$

If 1 man and 2 boys can finish the work in 6 days, the entire work is done:

$$6\left(\frac{1}{x}+\frac{2}{y}\right)=1$$

$$\frac{6}{x}+\frac{12}{y}=1 \qquad (2)$$

Substitute $\frac{1}{x}=u$ and $\frac{1}{y}=v$ in (1) and (2):

$$10u+5v=1 \qquad (3)$$

$$6u+12v=1 \qquad (4)$$

(3) × -6: $-60u-30v=-6$

(4) × 10: $\underline{60u+120v=10}$

Add: $90v=4$

$$v=\frac{2}{45}$$

Substitute $\frac{2}{45}$ for v in equation (3):

$$10u+5\left(\frac{2}{45}\right)=1$$

$$450u+10=45$$

$$450u=35$$

$$u=\frac{35}{450}=\frac{7}{90}$$

$$\frac{7}{90}=\frac{1}{x} \quad \text{and} \quad \frac{2}{45}=\frac{1}{y}$$

$x=12\frac{6}{7}$ and $y=22\frac{1}{2}$

One man alone can do the work in $12\frac{6}{7}$ days and one boy alone can do the work in $22\frac{1}{2}$ days.

10. Solution

Let x and y represent the measures of the two angles.

Since the two angles are supplementary, then

$$x+y=180 \qquad (1)$$

The sum of half the measure of the larger angle and the measure of the smaller is 125^0, then

$$\frac{x}{2}+y=125$$

$$x+2y=250 \qquad (2)$$

But $\underline{x+y=180}$ (1)

(2) – (1): $y=70^0$

Substitute 70 for y in (1)

$$x+70=180$$

$$x+70-70=180-70$$

$$x=110^0$$

$x = 110^0$ and $y = 70^0$

11. Solution

Let $x =$ The man' rate of rowing in still water

$y =$ The rate of the stream

Then

$x + y =$ The rate of boat downstream

$x - y =$ The rate of boat upstream

Summary table of Uniform Motion Problem

	d	r	t
Downstream	20	$x + y$	2
Upstream	20	$x - y$	2.5

$$d = rt$$

$$20 = (x + y)2 \quad (1)$$

$$20 = (x - y)2.5 \quad (2)$$

Simplify equations (1) and (2):

$$x + y = 10 \quad (3)$$

$$x - y = 8 \quad (4)$$

Add: $2x = 18$

$$x = 9$$

Substitute 9 for x in (3):

$$9 + y = 10$$

$$9 - 9 + y = 10 - 9$$

$$y = 1$$

Thus the man's rate of rowing in still water is 9 km/h and the rate of the stream is 1km/h.

12. Solution

Let $x =$ The man's rate of rowing in still water

$y =$ The rate of the stream

Then

$x + y =$ The rate of the boat downstream

$x - y =$ The rate of the boat upstream

Summary table of Uniform Motion Problem

	d	r	t
Downstream	18	$x + y$	2
Upstream	18	$x - y$	6

$$d = rt$$

$$18 = (x + y)2 \quad (1)$$

$$18 = (x - y)6 \quad (2)$$

Divide (1) by 2 and (2) by 6:

$$x + y = 9 \quad (3)$$

$$x - y = 3 \quad (4)$$

Add: $2x = 12$

$$x = 6$$

Substitute 6 for x in (3):

$$6 + y = 9$$

$$6 - 6 + y = 9 - 6$$

$$y = 3$$

Therefore the man's rate of rowing is 6 km/h and the rate of the stream is 3 km/h.

13. Solution

Let

$x =$ Number of cubic centimeters of the 30% solution

$y =$ Number of cubic centimeters of the 15% solution

The table below illustrates the information

Summary table of Mixture Problem

	Number Of CC	Percent of Concentration	Amount of Concentration
30% Solution	x	0.30	$0.30x$
15% Solution	y	0.15	$0.15y$
20% Solution	45	0.20	9

The total amount of solution is:

$$x + y = 45 \quad (1)$$

The amount of concentration of the medication from the two components must equal the number of cc in the final mixture:

$$0.30x + 0.15y = 9 \quad (2)$$

(1)x -0.15:

$$-0.15x - 0.15y = -6.75 \quad (3)$$

$$0.30x + 0.15y = 9 \quad (2)$$

Adding: $0.15x = 2.25$

$$x = 15$$

Substitute 15 for x in (2):

$$0.30 \times 15 + 0.15y = 9$$

$$4.5 + 0.15y = 9$$

$$4.5 - 4.5 + 0.15y = 9 - 4.5$$

$$0.15y = 4.5$$

$$y = 30$$

Therefore the nurse should mix 15cc of the 30% solution with 30cc of the 15% solution to obtain 45cc of the 20% solution.

14. Solution

Let

$x =$ the smaller number

$y =$ the larger number

Then

$x + 2 =$ the smaller number increased by 2

$y + 5 =$ the larger number increased by 5

One number is 1 more than another,

$$y = x + 1 \quad (1)$$

The ratio of the smaller number is increased by 2 to the larger number increased by 5 is $\frac{3}{7}$,

$$\frac{x+2}{y+5} = \frac{3}{7}$$

$$7(x+2) = 3(y+5)$$

$$7x + 14 = 3y + 15$$

$$7x + 14 - 14 = 3y + 15 - 14$$

$$7x - 3y = 1 \quad (2)$$

From (1), $y - x = 1$ (3)

(2): $7x - 3y = 1$

(3) $\times 3$: $-3x + 3y = 3$

Adding: $4x = 4$

$$x = 1$$

Substitute 1 for x in equation (1) to obtain

$$y = 1 + 1 = 2$$

Therefore, the numbers are 1 and 2

15. Solution

Let x and y represent the length and width of the playing field.

Then $2l + 2w = 1040$ (1)

But the length is 200 feet longer than the width

Thus, $l = w + 200$ or $l - w = 200$ (2)

Multiply (2) by 2 to get'

$$2l - 2w = 400 \qquad (3)$$

Adding (1) and (2) we have

$$4l = 1440 \quad \therefore \quad l = 360$$

Substituting 360 for l in (1) we get

$$2(360) + 2w = 1040$$

$$2w = 1040 - 720$$

$$2w = 320 \therefore w = 160$$

Length = 360 and width = 160

16. Solution

This is a rate/distance problem. Thus we look for two different rates.

Let x = rate of the plane

y = rate of the wind

When the plane is travelling with the tail, it will be going faster.

When the plane is travelling with the head, it will be going slower.

Summary table of Unifom Motion Problem

	r	*t*	*d*
With tail wind	$x + y$	2	2000
With head wind	$x - y$	4	2000

Rate times time is equal to distance

$$2(x + y) = 2000 \qquad (1)$$

$$4(x - y) = 2000 \qquad (2)$$

$$x + y = 1000 \qquad (3)$$

$$\underline{x - y = 500} \qquad (4)$$

Adding: $2x = 1500$

$$x = 750$$

Substituting 750 for x in (3) we have

$$750 + y = 1000$$

$$y = 1000 - 750 = 250$$

The speed of the aeroplane is 750 km/h and the speed of the wind is 250 km/h.

17. Solution

Let x and y represent the measures of the two angles.

Thus $x + y = 180$ (1)

Since one angle is 30^0 less than twice the other,

$$y = 2x - 30$$

Or $2x - y = 30$ (2)

Adding (1) and (2) we have

$$3x = 210$$

$$x = 70$$

Substitute 70 for x in (1) we have

$$70 + y = 180$$

$$y = 180 - 70 = 110$$

Therefore $x = 70^0$ and $y = 110^0$

18. Solution

Le t

$x =$ Number of days for Sikiru to complete the work

$y =$ Number of days for Salman to complete the work

Then

$\frac{1}{x} =$ Fractional part of the work Sikiru does per hour

$\frac{1}{y} =$ Fractional part of the work Salman does per hour

If each works for 12 hours, the entire work is done:

$$\frac{12}{x} + \frac{12}{y} = 1 \qquad (1)$$

When Sikiru works only for 8 hours, Salman has to work for an additional of 10 hours or 18 hours in all, to complete the entire work:

$$\frac{8}{x} + \frac{18}{y} = 1 \qquad (2)$$

Let $r = \frac{1}{x}$ and $s = \frac{1}{y}$, the equations become:

$$12r + 12s = 1 \qquad (3)$$

$$8r + 18s = 1 \qquad (4)$$

(3) ×-8: $-96r - 96s = -8$

(4) ×12: $\underline{96r + 216s = 12}$

Adding: $120s = 4$

$$s = \frac{1}{30}$$

Substitute $\frac{1}{30}$ for s in equation (3) to obtain

$$12r + 12 \times \frac{1}{30} = 1$$

$$360r + 12 = 30$$

$$360r + 12 - 12 = 30 - 12$$

$$360r = 18$$

$$r = \frac{1}{20}$$

But $r = \frac{1}{x}$ and $s = \frac{1}{y}$,

$\frac{1}{20} = \frac{1}{x}$ and $\frac{1}{30} = \frac{1}{y}$

$\therefore \quad x = 20$ and $y = 30$

So Sikiru could do the work in 20 hours and Salman could do it in 30 hours.

19. Solution

Let

$x =$ the smaller number

$y =$ the larger number

Then

$x + 1 =$ smaller number increased by 1

$y + 5 =$ smaller number increased by 5

A number is 3 more than another,

$$y = x + 3$$

$$-x + y = 3 \qquad (1)$$

The ratio of the smaller number increased by 1 to the larger number increased by 5 is $\frac{1}{2}$,

$$\frac{x+1}{y+5} = \frac{1}{2}$$

$$2(x+1) = y+5$$

$$2x+2 = y+5$$

$$2x+2-2 = y+5-2$$

$$2x - y = 3 \qquad (2)$$

But, $-x + y = 3 \qquad (1)$

$$x = 6$$

Substitute 6 for x in equation (1):

$$-6 + y = 3$$

$$y = 9$$

Thus, the numbers are 6 and 9

20. Solution

This is a rate/distance problem. Thus we look for two different rates (speed).

Let x = rate of the plane

y = rate of the wind

When the plane is travelling with the tail, it will be going faster.

When the plane is travelling with the head, it will be going slower.

Summary table of Unifom Motion Problem

	r	t	d
With tail wind	$x + y$	3	3000
With head wind	$x - y$	4	3000

rt = d

$$3(x+y) = 3000 \qquad (1)$$

$$4(x-y) = 3000 \qquad (2)$$

Divide (1) by 3 and (2) by 4:

$$x + y = 1000 \qquad (3)$$

$x - y = 750 \qquad (4)$

Add: $2x = 1750$

$$x = 875$$

Substitute 875 for x in (3):

$$875 + y = 1000$$

$$y = 1000 - 875$$

$$y = 125$$

Appendix E

Simultaneous Linear Eqution Performance Test (SLEPT) Post-test

Instructions: Answer all the questions

Time: 2 $\frac{1}{2}$ Hours

Construct simultaneous linear equations in each of the following problems, and hence solve the problems by using either substitution or elimination method.

1. The sum of the digits of a two-digit number is 9. The number is 27 more than the number obtained when the digits are reversed. Find the number.
2. Three pencils and seven erasers cost ₦ 18.00. Four pencils and ten erasers cost ₦ 25.00; find the cost of a pencil and a eraser.
3. Kola and Bukola can do a piece of work in 30 days. After they had worked together for 12 days, Bukola had an accident and Kola takes another 24 days to finish the rest of the work. How long would each take working alone?
4. A nurse has two solutions that contain different concentration of a certain medication. One is 12% concentration, and the other is an 8% concentration. How many cubic centimeters (cc) of each should she mix together to obtain 20 cc of a 9% solution?
5. The perimeter of a rectangle is 84cm and its length is 6cm longer than its breadth. Find the length and breadth of the rectangle.
6. 3 boys and 1 girl can do a piece of work in 4 days while 1 boy and 2 girls can do it in 6 days. How long would each take working alone?
7. Two angles are supplementary. The sum of half the measure of the larger angle and the measure of the smaller angle is 125^0. What is the measure of each angle?
8. A man rows 18 kilometres downstream in 2 hours. He finds that it takes 6 hours to row back. What is his rate of rowing in still water, and what is the rate of the stream?
9. One number is 1 more than another. The ratio of the smaller number increased by 2 to the larger number increased by 5 is $\frac{3}{7}$. What are the two numbers?
10. An aero-plane travels 2000 km with a tail wind in 2 hr. It then travels 2000 km with head in 4 hr. Find the speed of the aero-plane and the speed of the wind.

Marking Scheme for SLEPT (Post-test)

Substitution Method I

1. **Solution**

Let $x =$ Tens digit

$y =$ Units digit

Then

$10x + y =$ Original number

$10y + x =$ Number with digits reversed

From the first sentence of the question,

$$x + y = 9 \qquad (1)$$

From the second sentence of the question,

$$10x + y = 10y + x + 27$$

$$10x + y - 10y - x = 27$$

$$9x - 9y = 27$$

$$x - y = 3 \qquad (2)$$

From (1), $y = 9 - x$

Substitute $9 - x$ for y in (2) to obtain

$$x - 9 + x = 3$$

$$2x - 9 + 9 = 3 + 9$$

$$2x = 12$$

$$x = 6$$

Substitute 6 for x in equation (1) to obtain

$$6 + y = 9$$

$$6 - 6 + y = 9 - 6$$

$$y = 3$$

Therefore the number is $10 \times 6 + 3$ or 63

2. **Solution**

Let one pencil cost x naira and one eraser cost y naira.

Then 3 pencils cost $3x$ naira and 7 pencils cost $7y$ naira.

Thus, $3x + 7y = 18 \qquad (1)$

Similarly, $4x + 10y = 25 \qquad (2)$

From equation (1), $y = \dfrac{18 - 3x}{7} \quad (3)$

Substitute $\dfrac{18 - 3x}{7}$ for y in (2)

$$4x + 10\left(\frac{18 - 3x}{7}\right) = 25$$

$$28x + 180 - 30x = 175$$

$$-2x + 180 = 175$$

$$-2x + 180 = 175$$

$$-2x + 180 - 180 = 175 - 180$$

$$-2x = -5$$

$$x = \frac{5}{2}$$

$$x = 2.5$$

Substitute 2.5 for x in (3)

$$y = \frac{18 - 3 \times 2.5}{7}$$

$$y = \frac{18 - 7.5}{7} = \frac{10.5}{7} = 1.5$$

A pencil cost ₦ 2.50 and an eraser cost ₦ 1.50

3. **Solution**

Let $x =$ Number of days for Bukola to complete the work

$y =$ Number of days for Kola to complete the work

Then

$\frac{1}{x} =$ Fractional part of the work Bukola does per day

$\frac{1}{y} =$ Fractional part of the work kola does per day

If each works for 30 days, the entire work is done:

$$\frac{30}{x} + \frac{30}{y} = 1 \qquad (1)$$

When Bukola works only for 12 days, Kola has to work for an additional of 24 days or 36 days in all, to complete the entire work:

$$\frac{12}{x} + \frac{36}{y} = 1 \qquad (2)$$

Let $u = \frac{1}{x}$ and $v = \frac{1}{y}$, the equations become:

$$30u + 30v = 1 \qquad (3)$$
$$12u + 36v = 1 \qquad (4)$$

From (3), $v = \frac{1 - 30u}{30}$

Substitute $\frac{1-30u}{30}$ for v in (4):

$$12u + 36\left(\frac{1-30u}{30}\right) = 1$$

$$360u + 36 - 1080u = 30$$

$$-720u = -6$$

$$u = \frac{1}{120}$$

Substitute $\frac{1}{120}$ for u in (3) to obtain

$$30 \times \frac{1}{120} + 30v = 1$$

$$30 + 3600v = 120$$

$$30 - 30 + 3600v = 120 - 30$$

$$3600v = 90$$

$$v = \frac{1}{40}$$

But $u = \frac{1}{x}$ and $v = \frac{1}{y}$,

$$\frac{1}{120} = \frac{1}{x} \text{ and } \frac{1}{40} = \frac{1}{y}$$

$$\therefore \; x = 120 \text{ and } y = 40$$

So kola could do the work alone in 40 days, and Bukola could do it alone in 120 days.

4. **Solution**

Let $x =$ Number of cubic centimeters of 12% solution required

$y =$ Number of cubic centimeters of 8% solution required

The table below organizes the information as follows:

Summary table of Mixture Problem

	Number of CC	Percent of CCtra	Amt of CCtra
12% St.	x	0.12	$0.12x$
8% St.	y	0.08	$0.08y$
9% St.	20	0.09	1.8

Since there are to be 20 cc in all, we must have the following equation relating x and y

$$x + y = 20 \qquad (1)$$

The amount of concentration of the medication from the two components must equal the number of cc in the final mixture:

$$0.12x + 0.08y = 1.8 \qquad (2)$$

From (1), $y = 20 - x$

Substitute $20 - x$ for y in (2):

$$0.12x + 0.08(20 - x) = 1.8$$

$$0.12x + 1.6 - 0.08x = 1.8$$

$$0.04x + 1.6 - 1.6 = 1.8 - 1.6$$

$$0.04x = 0.2$$

$$x = 5$$

Substitute 5 for x in (1):

$$5 + y = 20$$

$$y = 15$$

The nurse should mix 5cc of the 12% solution with 15cc of the 8% solution to give 20cc of the 9% solution.

5. **Solution**

Let l and b represent the length and breadth of the rectangle respectively.

Then $2l + 2b = 84 \qquad (1)$

But the length is 6cm longer than the breadth

Thus, $l = b + 6 \qquad (2)$

Substituting $b + 6$ for l in (1):

$$2(b + 6) + 2b = 84$$

$$4b + 12 = 84$$

$$4b = 72$$

$$b = 18$$

Substitute 18 for b in (2):

$$l = 18 + 6 = 24$$

∴ Length = 24cm and Breadth = 18

6. Solution

Let $x =$ Number of days for one boy to finish the work

$y =$ Number of days for one girl to finish the work

Then

$\frac{1}{x}$ = Fractional part of the work one boy does per day

$\frac{1}{y}$ = Fractional part of the work one girl does per day

If 3 boys and 1 girl work 4 days, the entire work is done:

$$4\left(\frac{3}{x}+\frac{1}{y}\right)=1$$

$$\frac{12}{x}+\frac{4}{y}=1 \qquad (1)$$

If 1 boy and 2 girls can finish the work in 6 days, the entire work is done:

$$6\left(\frac{1}{x}+\frac{2}{y}\right)=1$$

$$\frac{6}{x}+\frac{12}{y}=1 \qquad (2)$$

Substitute $\frac{1}{x}=r$ and $\frac{1}{y}=s$ in equation (1) and (2) to get

$$2r+4s=1 \qquad (3)$$
$$6r+12s=1 \qquad (4)$$
$$s=\frac{1-12s}{4}$$

From (4)

Substitute $\frac{1-12s}{4}$ for s in (4):

$$6r+12\left(\frac{1-12r}{4}\right)=1$$

$$24r+12-144r=4$$

$$-120r=-8$$
$$r=\frac{-86}{-120}=\frac{1}{15}$$

Substitute $\frac{1}{15}$ for r in (3) to obtain

$$12\left(\frac{1}{15}\right)+4s=1$$
$$12+60s=15$$
$$60s=3$$
$$s=\frac{3}{60}=\frac{1}{20}$$

But $\frac{1}{x}=u$ and $\frac{1}{y}=v$

$$\frac{1}{15}=\frac{1}{x} \text{ and } \frac{1}{20}=\frac{1}{y}$$

$$x=15 \text{ and } y=20$$

A boy alone can do the work in 15days and a girl alone can do the work in 20 days.

7. **Solution**

Let x and y represent themeasures of the two angles.

Since the two angles are supplementary, then

$$x+y=180 \qquad (1)$$

The sum of half the measure of the larger angle and the measure of the smaller is 125^0, then

$$\frac{x}{2}+y=125$$

$$x+2y=250 \qquad (2)$$

From (1) $x=180-y$

Substituting $180-y$ for x in (2):

$$180-y++2y=250$$

$$y=250-180$$

$y = 70$

Substituting 70 for y in (1):

$x + 70 = 180$

$x = 180 - 70$

$x = 110^0$

Therefore $x = 110^0$ and $y = 70^0$

8. **Solution**

Let $x =$ The man's rate of rowing in still water

$y =$ The rate of the stream

Then

$x + y =$ The rate of the boat downstream

$x - y =$ The rate of the boat upstream

Table of Uniform Motion Problem

	d	r	t
Downstream	18	$x + y$	2
Upstream	18	$x - y$	6

$d = rt$

$18 = (x + y)2$ (1)

$18 = (x - y)6$ (2)

Dividing equation (1) by 2 and equation (2) by 6, we have

$x + y = 9$ (3)

$x - y = 3$ (4)

From equation (3),

$y = 9 - x$

Substitute $9 - x$ for y in equation (4) to obtain

$x - 9 + x = 3$

$2x = 12$

$x = 6$

Substitute 6 for x in (3):

$6 + y = 9$

$6 - 6 + y = 9 - 6$

$y = 3$

Therefore the man's rate of rowing is 6 km/h and the rate of the stream is 3 km/h.

9. Solution

Let

$x =$ the smaller number

$y =$ the larger number

Then

$x + 2 =$ the smaller number increased by 2

$y + 5 =$ the larger number increased by 5

One number is 1 more than another,

$y = x + 1$ (1)

The ratio of the smaller number is increased by 2 to the larger number increased by 5 is $\frac{3}{7}$,

$\frac{x+2}{y+5} = \frac{3}{7}$

$7(x + 2) = 3(y + 5)$

$7x + 14 = 3y + 15$

$7x + 14 - 14 = 3y + 15 - 14$

$7x - 3y = 1$ (2)

From (1), substitute $x + 1$ for y in (2) to obtain

$7x - 3(x + 1) = 1$

$7x - 3x - 3 = 1$

$4x - 3 + 3 = 1 + 3$

$$4x = 4$$
$$x = 1$$

Substitute 1 for x in equation (1) to obtain

$$y = 1 + 1 = 2$$

Therefore, the numbers are 1 and 2

10. Solution

This is a rate/distance problem. Thus we look for two different rates.

Let x = rate of the plane

y = rate of the wind

When the plane is travelling with the tail, it will be going faster.

When the plane is travelling with the head, it will be going slower.

Summary table of Unifom Motion Problem

	r	t	d
With tail wind	$x + y$	2	2000
With head wind	$x - y$	4	2000

Rate times time is equal to distance

$$2(x + y) = 2000 \quad (1)$$

$$4(x - y) = 2000 \quad (2)$$

$$x + y = 1000 \quad (3)$$

$$x - y = 500 \quad (4)$$

From (3), $y = 1000 - x$

Substitute $1000 - x$ for y in (4)

$$x - (1000 - x) = 500$$

$$x - 1000 + x = 500$$

$$2x - 1000 + 1000 = 500 + 1000$$

$$2x = 1500$$

$$x = 750$$

Substituting 750 for x in (3):

$$750 + y = 1000$$

$$y = 1000 - 750 = 250$$

The speed of the aeroplane is 750 km/h and the speed of the wind is 250 km/h.

Elimination Method II

1. Solution

Let $x =$ Tens digit

$y =$ Units digit

Then

$10x + y =$ Original number

$10y + x =$ Number with digits reversed

From the first sentence of the question,

$$x + y = 9 \qquad (1)$$

From the second sentence of the question,

$$10x + y = 10y + x + 27$$

$$10x + y - 10y - x = 27$$

$$9x - 9y = 27$$

$$x - y = 3 \qquad (2)$$

But, $x + y = 9$ (1)

Adding: $2x = 12$

$$x = 6$$

Substitute 6 for x in equation (1) to obtain

$$6 + y = 9$$

$$6 - 6 + y = 9 - 6$$

$$y = 3$$

Therefore the number is $10 \times 6 + 3$ or 63

2. Solution

Let one pencil cost x naira and one eraser cost y naira.

Then 3 pencils cost $3x$ naira and 7 pencils cost $7y$ naira.

Thus, $3x + 7y = 18 \qquad (1)$

Similarly, $4x + 10y = 25 \qquad (2)$

$(1) \times 4 : \; 12x + 28y = 72$

$(2) \times 3 : \; \underline{12x + 30y = 75}$

$(2) - (1) \qquad 2y = 3$

$$y = \frac{3}{2} = 1.5$$

Substitute 1.5 for y in (1)

$$3x + 7 \times 1.5 = 18$$

$$3x + 10.5 = 18$$

$$3x + 10.5 - 10.5 = 18 - 10.5$$

$$3x = 7.5$$

$$x = 2.5$$

A pencil cost ₦ 2.50 and an eraser cost ₦ 1.50

3. Solution

Let

$x =$ Number of days for Bukola to complete the work

$y =$ Number of days for Kola to complete the work

Then

$\frac{1}{x} =$ Fractional part of the work Bukola does per day

$\frac{1}{y} =$ Fractional part of the work kola does per day

If each works for 30 days, the entire work is done:

$$\frac{30}{x} + \frac{30}{y} = 1 \qquad (1)$$

When Bukola works only for 12 days, Kola has to work for an

additional of 24 days or 36 days in all, to complete the entire work:

$$\frac{12}{x}+\frac{36}{y}=1 \qquad (2)$$

Let $\frac{1}{y}=s$ and $v=\frac{1}{y}$, the equations become:

$$30u+30v=1 \qquad (3)$$

$$12u+36v=1 \qquad (4)$$

$(3)\times 12:\quad 360u+360v=12$

$(4)\times 30:\quad \underline{360u+1080v=30}$

$(4)-(3):\quad 720v=18$

$$v=\frac{18}{720}=\frac{1}{40}$$

Substitute $\frac{1}{40}$ for v in equation (3) to obtain

$$30u+30\times\frac{1}{40}=1$$

$$1200u+30=40$$

$$1200u+30-30=40-30$$

$$1200u=10$$

$$u=\frac{10}{1200}=\frac{1}{120}$$

But $u=\frac{1}{x}$ and $v=\frac{1}{y}$,

$$\frac{1}{120}=\frac{1}{x} \text{ and } \frac{1}{40}=\frac{1}{y}$$

$$\therefore\ x=120 \text{ and } y=40$$

So kola could do the work alone in 40 days, and Bukola could do it alone in 120 days.

4. Solution

Let

$x=$ Number of cubic centimeters of 12% solution required

$y=$ Number of cubic centimeters of 8% solution required

The table below organizes the information as follows:

Summary table of Mixture Problem

	Number of CC	Percent of Concentration	Amount of Concentration
12% Solution	x	0.12	$0.12x$
8% Solution	y	0.08	$0.08y$
9% Solution	20	0.09	1.8

Since there are to be 20 cc in all, we must have the following equation relating x and y

$$x+y=20 \qquad (1)$$

The amount of concentration of the medication from the two components must equal the number of cc in the final mixture:

$$0.12x+0.08y=1.8$$

(2)

Multiply equation (1) by- 0.08 to make the coefficients of x opposites

$$-0.08x-0.08y=-1.6 \qquad (1)$$

$$\underline{0.12x+0.08y=1.8}$$

(2)

Adding: $0.04x = 0.2$

$0.04x = 0.2$

$x = 5$

Substitute 5 for x in equation (1):

$5 + y = 20$

$y = 15$

Therefore the nurse should mix 5cc of the 12% solution with 15cc of the 8% solution to give 20cc of the 9% solution.

5. Solution

Let l and b represent the length and breadth of the rectangle respectively.

Then $2l + 2b = 84$ (1)

But the length is 6cm longer than breadth

Thus, $l = b + 6$

Or $l - b = 6$ (2)

(1): $2l + 2b = 84$

$(2) \times 2$: $\underline{2l - 2b = 12}$

Adding: $4l = 96$

$l = 24$

Substituting 24 for b in (2) we have

$24 = b + 6$

$24 - 6 = b + 6 - 6$

$b = 18$

$\therefore$ Length = 24cm and Breadth = 18

6. Solution

Let $x =$ Number of days for one boy to finish the work

$y =$ Number of days for one girl to finish the work

Then

$\frac{1}{x} =$ Fractional part of the work one boy does per day

$\frac{1}{y} =$ Fractional part of the work one girl does per day

If 3 boys and 1 girl work 4 days, the entire work is done:

$$4\left(\frac{3}{x} + \frac{1}{y}\right) = 1$$

$$\frac{12}{x} + \frac{4}{y} = 1 \quad (1)$$

If 1 boy and 2 girls can finish the work in 6 days, the entire work is done:

$$6\left(\frac{1}{x} + \frac{2}{y}\right) = 1$$

$$\frac{6}{x} + \frac{12}{y} = 1 \quad (2)$$

Substitute $\frac{1}{x} = r$ and $\frac{1}{y} = s$ in equation (1) and (2):

$12r + 4s = 1$ (3)

$6r + 12s = 1$ (4)

$(3) \times 3$: $36r + 12s = 3$

(4): $\underline{6r + 12s = 1}$

Subtract (3) form (4) $30r = 2$

$$r = \frac{1}{15}$$

Substitute $\frac{1}{15}$ for r in (3):

$$12\left(\frac{1}{15}\right) + 4s = 1$$

$12 + 60s = 15$

$60s = 3$

$$s = \frac{3}{60} = \frac{1}{20}$$

But $\frac{1}{x} = r$ and $\frac{1}{y} = s$

$\therefore$ $\frac{1}{15} = \frac{1}{x}$ and $\frac{1}{y} = \frac{1}{20}$

$x = 15$ and $y = 20$

A boy alone can do the work in 15days and a girl alone can do the work in 20 days.

7. Solution

Let x and y represent the measures of the two angles.

Since the two angles are supplementary, then

$$x + y = 180 \qquad (1)$$

The sum of half the measure of the larger angle and the measure of the smaller is 125^0, then

$$\frac{x}{2} + y = 125$$

$$x + 2y = 250 \qquad (2)$$

But $\underline{x + y = 180} \qquad (1)$

(2) – (1): $y = 70^0$

Substitute 70 for y in (1)

$$x + 70 = 180$$

$$x + 70 - 70 = 180 - 70$$

$$x = 110^0$$

Therefore $x = 110^0$ and $y = 70^0$

8. Solution

Let $x =$ The man's rate of rowing in still water

$y =$ The rate of the stream

Then

$x + y =$ The rate of the boat downstream

$x - y =$ The rate of the boat upstream

Summary table of Uniform Motion Problem

	d	r	t
Downstream	18	$x + y$	2
Upstream	18	$x - y$	6

$$d = rt$$

$$18 = (x + y)2 \qquad (1)$$

$$18 = (x - y)6 \qquad (2)$$

Dividing equation (1) by 2 and equation (2) by 6, we have

$$x + y = 9 \qquad (3)$$

$$\underline{x - y = 3} \qquad (4)$$

Adding: $2x = 12$

$$x = 6$$

Substitute 6 for x in (3) to obtain

$$6 + y = 9$$

$$6 - 6 + y = 9 - 6$$

$$y = 3$$

Therefore the man's rate of rowing is 6 km/h and the rate of the stream is 3 km/h.

9. Solution

Let

$x =$ the smaller number

$y =$ the larger number

Then

$x + 2 =$ the smaller number increased by 2

$y + 5 =$ the larger number increased by 5

One number is 1 more than another,

$$y = x + 1 \qquad (1)$$

The ratio of the smaller number is increased by 2 to the larger number increased by 5 is $\frac{3}{7}$,

$$\frac{x+2}{y+5}=\frac{3}{7}$$

$$7(x+2)=3(y+5)$$

$$7x+14=3y+15$$

$$7x+14-14=3y+15-14$$

$$7x-3y=1 \qquad (2)$$

From (1), $y-x=1$ (3)

(2): $7x-3y=1$

(3) $\times 3$: $-3x+3y=3$

Adding: $4x=4$

$x=1$

Substitute 1 for x in (1):

$y=1+1=2$

Therefore, the numbers are 1 and 2

10. Solution

This is a rate/distance problem. Thus we look for two different rates.

Let x = rate of the plane

y = rate of the wind

When the plane is travelling with the tail, it will be going faster.

When the plane is travelling with the head, it will be going slower.

	r	t	d
With tail wind	$x+y$	2	2000
With head wnd	$x-y$	4	2000

Rate times time is equal to distance

$$2(x+y)=2000 \qquad (1)$$

$$4(x-y)=2000 \qquad (2)$$

$$x+y=1000 \qquad (3)$$

$$x-y=500 \qquad (4)$$

Adding: $2x=1500$

$x=750$

Substituting 750 for x in (3) we have

$750+y=1000$

$y=1000-750=250$

The speed of the aeroplane is 750 km/h and the speed of the wind is 250 km/h.

Summary table of Unifom Motion Problem

Appendix F

Mathematics Students' Attitude Inventory Questionnaire (ASAIQ) Draft

Dear Student,

This questionnaire is aimed at collecting information regarding a Ph.D. research study on the "Impact of Polya's Problem Solving Strategy on Attitude, Performance and Retention in Algebra among Junior Secondary School Students in Zamfara State, Nigeria". The instrument consists of forty eight questions that require you to indicate your opinion on each statement by making a tick (✓) in the bracket that correspond the statement. The instrument is constructed based on 5 points Likert scale ranging from strongly agree (SA), Agree (A), Neutral (N), Disagree (D), to Strongly Disagree (SD). Please answer the questions completely and as honestly as possible. All the information you have provided will be treated honestly and confidentially. Therefore, your kind cooperation is highly needed. Thank you.

Section A: Demographic Data

Sex: Male [ ] Female [ ]

School: ……..Class……

Confidence in Learning Mathematics Scale (C)
1. Generally I have felt secure about attempting Mathematics.
2. I am sure I could do advanced work in Mathematics.
3. I am sure that I can learn Mathematics.
4. I think I could handle more difficult Mathematics.
5. I can get good grades in Mathematics.
6. I have a lot of self-confidence when it comes to Mathematics.
7. I'm no good in math.
8. I don't think I could do advanced Mathematics.
9. I'm not the type to do well in math.
10 For some reason even though I study, math seems unusually hard for me.
11 Most subjects I can handle O.K., but I have a knack for messing up math.
12 Math has been my worst subject.
Attitude Toward Success in Mathematics Scale (AS)
13. It would make me happy to be recognised as an excellent student in Mathematics.
14. I'd be proud to be the outstanding student in math.
15. I'd be happy to get top grades in Mathematics.
16. It would be really great to win a prize in Mathematics.
17. Being first in a Mathematics competition would make me pleased.
18. Being regarded as smart in Mathematics would be a great thing.
19. Winning a prize in Mathematics would make me feel unpleasantly conspicuous.
20. People would think that I was some kind of creep if I got A's in math.
21. If I got the highest grade in math, I would try to hide it.
22. If I got the highest grade in math I'd prefer no one knew.
23. It would make people like me less if I were a really good math student.
24. I don't like people to think I'm smart in math.
Mathematics as a Male Domain Scale (MD)
25. Females are as good as males in Mathematics.
26. Studying Mathematics is just as appropriate for women as for men.
27. I would trust a woman just as much as I would trust a man to figure out important calculations.
28. Girls can do just as well as boys in Mathematics.
29. Males are not naturally better than females in Mathematics.
30. Women certainly are logical enough to do well in Mathematics.
31. It's hard to believe a female could be a genius in Mathematics.

32. When a woman has to solve a math problem, it is feminine to ask a man for help.
33. I would have more faith in the answer for a math problem solved by a man than a woman.
34. Girls who enjoy studying math are a bit peculiar.
35. Mathematics is for men, arithmetic is for women.
36. I would expect a woman mathematician to be a masculine type of person.
Usefulness of Mathematics Scale (U)
37. I'll need Mathematics for my future work.
38. I study Mathematics because I know how useful it is.
39. Knowing Mathematics will help me earn a living.
40. Mathematics is a worthwhile and necessary subject.
41. I'll need a firm mastery of Mathematics for my future work.
42. I will use Mathematics in many ways as an adult.
43. Mathematics is of no relevance to my life.
44. Mathematics will not be important to me in my life's work.
45. I see Mathematics as a subject I will rarely use in my daily life as an adult.
46. Taking Mathematics is a waste of time.
47. In terms of my adult life it is not important for me to do well in Mathematics in high school.
48. I expect to have little use for Mathematics when I get out of school.

Source: Fennema and Sherman (1976, pages 21-27)

Appendix G

Algebraic Students' Attitude Inventory Questionnaire (ASAIQ)
Final Draft

Dear Student,

This questionnaire is aimed at collecting information regarding a Ph.D. research study on the **Impact of Problem Solving Strategy on Attitude, Performance and Retention in Algebra among Junior Secondary School Students in Zamfara State, Nigeria**. The instrument consists of forty eight questions that require you to indicate your opinion on each statement by making a tick (✓) in the bracket that correspond the statement. The instrument is constructed based on 5 points Likert scale ranging from strongly agree (SA), Agree (A), Neutral (N), Disagree (D), to Strongly Disagree (SD). Please answer the questions completely and as honestly as possible. All the information you have provided will be treated honestly and confidentially. Therefore, your kind cooperation is highly needed. Thank you.

Section A: Demographic Data

Sex: Male [ ] Female [ ]

School: ……..Class……

Section: B Confidence in learning Algebra Scale

Item	Statement	SA	A	N	D	SD
1.	Generally I have felt safe about attempting algebra.					
2.	I am sure I could do advanced work in algebra.					
3.	I am sure that I can learn algebra.					
4.	I think I could handle more difficult algebra.					
5.	I can get good grades in algebra.					
6.	I have a lot of self-confidence when it comes to algebra.					
7.	I'm no good in algebra.					
8.	Idon't think I could do advanced algebra.					
9.	I'm not the type to do well in algebra.					
10.	For some reason even though I study, algebra seems unusually hard for me.					
11.	Most subjects I can handle O. K., but I have a knack for missing up algebra.					
12.	Algebra has been my worst subject.					
	Section: C Attitude toward Success in Algebra Scale					
Item	**Statement**	**SA**	**A**	**N**	**D**	**SD**
13.	It would make me happy to be recognized as an excellent student in algebra.					
14.	I'd be proud to be the excellent student in algebra.					
15.	I'd be happy to get top grades in algebra.					
16.	It would be really greater to win a prize in algebra.					
17.	Being first in an algebra competition would make me pleased.					
18.	Being regarded as smart in algebra would be a great thing.					
19.	Winning a prize in algebra would make me feel uncomfortable.					
20.	People would think that I cheated if I got A's in algebra.					
21.	If I got the highest grade in algebra, I would try to hide it					
22.	If I got the highest grade in algebra I'd prefer no one knew.					
23.	It would make people like me less if I were a really good algebra student.					
24.	I don't like people to think I'm smart in algebra.					
	Section: D Algebra as a Male Domain Scale					
Item	**Statement**	**SA**	**A**	**N**	**D**	**SD**

25.	Females are as good as males in algebra.					
26.	Studying algebra is just as appropriate for women as for men.					
27.	I would trust a woman just as much as I would trust a man to figure out important calculations.					
28.	Girls can do just as well as boys in algebra.					
29.	Males are not naturally better than females in algebra.					
30.	Women certainly are logical enough to do well in algebra.					
31.	It's hard to believe a female could be an intelligent in algebra.					
32.	When a woman has to solve a algebra problem, it is feminine to ask a man for help.					
33.	I would have more faith in the answer for a algebra problem solved by a man than a woman.					
34.	Girls who enjoy studying algebra are a bit peculiar.					
35.	Algebra is for me, arithmetic for women.					
36.	I would expect a woman mathematician to be a masculine type of person.					
	Section: E Usefulness of Algebra Scale					
Item	**Statement**	**SA**	**A**	**N**	**D**	**SD**
37.	I'll need algebra for my future work.					
38.	I study algebra because I know how useful it is.					
39.	Knowing algebra will help me earn a living.					
40.	Algebra is a worthwhile and necessary subject.					
41.	I'll need a firm mastery of algebra for my future work.					
42.	I will use algebra in my ways as an adult.					
43.	Algebra is of no relevance in my life.					
44.	Algebra will not be important to me in my life's work.					
45.	I see algebra as a subject I will rarely use in my daily life as an adult.					
46.	Taking algebra is a waste of time.					
47.	In terms of my adult life it is not important for me to do well in algebra in high school.					
48.	I expect to have little use for algebra when I get out of school.					

Source: Adapted from Fennema-Sherman Mathematics Attitude Scales (1976, pp. 21-27)

Appendix H

Lesson Plan for Experimental Group Using Polya's Problem Solving Strategy by Substitution and Elimination Methods

Substitution Method I

Week	**1 & 2**	**3**
Content	**Direct Translation Problems leading to simultaneous linear equations**	**Geometry Problems leading to simultaneous linear equations**
Objectives	Students should be able to: **i**. translate number word problems that lead to simultaneous linear equations. **ii**. solve word problems involving sums and differences of whole numbers that lead to simultaneous linear equations with substitution method using problem-solving strategy. **iii**. solve word problems involving differences and products of whole numbers that lead to simultaneous linear equations with substitution method using problem-solving strategy. **iv**. solve word problems involving two-digit numbers that lead to simultaneous linear equations with substitution method using problem-solving strategy. **v.** solve word problems involving ages that lead to simultaneous linear equations with substitution method using problem-solving strategy. **vi.** solve word problems involving fractions that lead to simultaneous linear equations with substitution method using problem-solving strategy. **vii**. solve word problems involving ratios that lead to simultaneous linear equations with substitution method using problem-solving strategy.	Students should be able to: **i**. translate geometry word problems that lead to simultaneous linear equations with substitution method using problem-solving strategy.. **ii**. solve word problems involving dimensions of a rectangle that lead to simultaneous linear equations with substitution method using problem-solving strategy. **iii**. solve word problems involving complementary angles that lead to simultaneous linear equations with substitution method using problem-solving strategy. **iv**. solve word problems involving supplementary angles that lead to simultaneous linear equations with substitution method using problem-solving strategy. **v.** solve word problems involving angles of an isosceles triangle that lead to simultaneous linear equations with substitution method using problem-solving strategy.

Instructional Materials	Calculators, charts of equations on sum and differences, differences and products of whole numbers, two-digit numbers, fractions and ratios involving word problems.	Calculators, charts of equations on dimensions of a rectangle, complementary angles and supplementary angles, angles of an isosceles triangle, word problems.
Step I: Introduction of the lesson (2 minutes)	At the beginning of each lesson, teacher will ask students variety of questions on their knowledge base that has link to the current topic.	At the beginning of each lesson, teacher will ask students variety of questions on their knowledge base that has link to the current topic.
Step 2: Dividing the class into manageable groups. (2 minutes)	At this stage, the teacher will divide the class into atleast 5 groups, and each group will contain 5 to 10 students of different ability levels who will support one another in various ways during problem solving process.	At this stage, the teacher will divide the class into atleast 5 groups, and each group will contain 5 to 10 students of different ability levels who will support one another in multiple ways during problem solving process.
Step3: Assigning roles to group members. (2 minutes)	Teacher will assign a specific role to each member of the group to encourage interactions and divergent thinking among students during problem solving. The typical roles will include a group leader who facilitates group discussion and makes sure that group members work together on task; a monitor who checks time on task and ensures that everyone gets an equal opportunity to participate; an encourager who inspires reluctant and shy students to participate in group discussions; a reflector who keeps the group aware of progress; a quiet captain who monitors noise level; and a recorder who keeps a written activity or writes the final results for the group.	Teacher will assign a specific role to each member of the group to encourage interactions and divergent thinking among students during problem solving. The typical roles will include a group leader who facilitates group discussion and makes sure that group members work together on task; a monitor who checks time on task and ensures that everyone gets an equal opportunity to participate; an encourager who inspires reluctant and shy students to participate in group discussions; a reflector who keeps the group aware of progress; a quiet captain who monitors noise level; and a recorder who keeps a written activity or writes the final results for the group.

Step 4: Assigning activities to groups. (18 minutes)	After assigning specific roles to the members of each group in the class, the teacher will paste a cardboard paper on the wall/board that contains a worded problem on simultaneous linear equations and ask students to solve using problem solving strategy as explained below.	After assigning specific roles to the members of each group in the class, the teacher will paste a cardboard paper on the wall/board that contains a worded problem on simultaneous linear equations and ask students to solve using problem solving strategy as explained below.
Stage 1:	*Understanding the problem:* Teacher will guide the students in reading the problem carefully several times; focusing on the key words; discussing their meanings; identifying extraneous information, and helping them know what they are asked to find. Then he leads students to represent the unknowns in each condition of the problem with two variables (say, x and y).	*Understanding the problem:* Teacher will guide the students in reading the problem carefully several times; focusing on the key words; discussing their meanings; identifying extraneous information, and helping them know what they are asked to find. Then he leads students to represent the unknowns in each condition of the problem with two variables (say, x and y).
Stage 2:	*Devising a plan:* When the groups jointly understand and extract the necessary information that is required to solve the problem, then teacher will asks each of the groups to formulate mathematical models or two linear equations.	*Devising a plan:* When the groups jointly understand and extract the necessary information that is required to solve the problem, then teacher will asks each of the groups to formulate mathematical models or two linear equations.
Stage 3:	*Carrying out the Plan:* At this stage the teacher will instructs students' to use appropriate strategy to solve the problem given to them. While students are working, teacher will be going round the class as a facilitator, listening to their discussions, observing their activities, assisting and encouraging them to participate in solving the problem.	*Carrying out the Plan:* At this stage the teacher will instructs students' to use appropriate strategy to solve the problem given to them. While students are working, teacher will be going round the class, listening to their discussions, observing their activities, assisting and encouraging them to participate in solving the problem.
Stage 4:	*Looking back:* Teacher asks students' to reflect on their answers and the processes they use to arrive at the answers.	*Looking back:* Teacher asks students' to reflect on their answers and the processes they use to arrive at the answers.
Step 5: Discussions of students'	The task at this stage will include comparing results at each stage of problem solving, discussing conclusions, and	The task at this stage will include comparing results at each stage of problem solving, discussing

results. (10 minutes)	sharing experiences across groups. After the problem has been solved, each group will be given 3 minutes to presents its results to the entire class. While doing so, other groups are listening and jotting down their observations. Afterwards, 5 minutes will be given to the whole class to make comments on the results presented by each group. The discussion will continue in this way until every group has presented its results. Each of the groups will then incorporate into its work the observations made by members from other groups. Finally, teacher provides a summary of the important points learn in the lesson in 3 mnutes.	conclusions, and sharing experiences across groups. After the problem has been solved, each group will be given 3 minutes to presents its results to the entire class. While doing so, other groups are listening and jotting down their observations. Afterwards, 5 minutes will be given to the whole class to make comments on the results presented by each group. The discussion will continue in this way until every group has presented its results. Each of the groups will then incorporate into its work the observations made by members from other groups. Finally, teacher provides a summary of the important points learn in the lesson in 3 minutes.
Step 6: Evaluation of students' learning progress. (4 minutes)	Teacher will ask students to return to their usual places after they have worked in groups. Then presents a problem on the chalkboard and asks them to solve it individually.	Teacher will ask students to return to their usual places after they have worked in groups. Then presents a problem on the chalkboard and asks them to solve it individually.
Step 7: Conclusion (2 mintes)	Teacher will give students homework assignment to practice what has been learnt in the class.	Teacher will give students homework assignment to practice what has been learnt in the class.

Week	**4**	**5**
Content	**Mixture Problems leading to simultaneous linear equations**	**Uniform Motion Problems leading to simultaneous linear equations**
Objectives	Students should be able to: **i**. translate mixture word problems that lead to simultaneous linear equations. **ii**. solve word problems involving costs of two quantities that lead to simultaneous linear equations with substitution method using problem solving strategy. **iii**. solve word problems involving liquid solutions that lead to	Students should be able to: **i**. translate motion word problems that lead to simultaneous linear equations. **ii**. solve word problems involving motion affected by wind that lead to simultaneous linear equations with substitution method using problem-solving strategy. **iii.** solve word problems involving speed that lead to simultaneous linear

	simultaneous linear equations with substitution method using problem-solving strategy. **iv**. solve word problems involving simple interest that lead to simultaneous linear equations with substitution method using problem-solving strategy.	equations with substitution method using problem-solving strategy. **iv**. solve word problems involving motion affected by water current that lead to simultaneous linear equations with substitution method using problem-solving strategy.
Instructional Materials	Calculators, charts of equations on costs of two quantities, liquid solutions and simple interest word problems.	Calculators, charts of equations on motion affected by wind, two travelers in the same direction and motion affected by water current.
Step I: Introduction of the lesson (2 minutes)	At the beginning of each lesson, teacher will ask students variety of questions on their knowledge base that has link to the current topic.	At the beginning of each lesson, teacher will ask students variety of questions on their knowledge base that has link to the current topic.
Step 2: Dividing the class into manageable groups. (2 minutes)	At this stage, the teacher will divide the class into atleast 5 groups, and each group will contain 5 to 10 students of varying ability levels who will support one another in multiple ways during problem solving process.	At this stage, the teacher will divide the class into atleast 5 groups, and each group will contain 5 to 10 students of different ability levels who will support one another in multiple ways during problem solving process.
Step3: Assigning roles to group members. (2 minutes)	Teacher will assign a specific role to each member of the group to encourage interactions and divergent thinking among students during problem solving. The typical roles will include a group leader who facilitates group discussion and makes sure that group members work together on task; a monitor who checks time on task and ensures that everyone gets an equal opportunity to participate; an encourager who inspires reluctant and shy students to participate in group discussions; a reflector who keeps the group aware of progress; a quiet captain who monitors noise level; and a recorder who keeps a written activity or writes the final results for the group.	Teacher will assign a specific role to each member of the group to encourage interactions and divergent thinking among students during problem solving. The typical roles will include a group leader who facilitates group discussion and makes sure that group members work together on task; a monitor who checks time on task and ensures that everyone gets an equal opportunity to participate; an encourager who inspires reluctant and shy students to participate in group discussions; a reflector who keeps the group aware of progress; a quiet captain who monitors noise level; and a recorder who keeps a written activity or writes the final results for the group.
Step 4: Assigning activities to groups.	After assigning specific roles to the members of each group in the class, the teacher will paste a cardboard paper on the wall/board that contains a	After assigning specific roles to the members of each group in the class, the teacher will paste a cardboard paper on the wall/board that contains a worded

(10 minutes)	worded problem on simultaneous linear equations and ask students to solve using Polya's Problem Solving Strategy as explained below.	problem on simultaneous linear equations and ask students to solve using Polya's Problem Solving Strategy.
Stage 1:	*Understanding the problem:* Teacher will guide the students in reading the problem carefully several times; focusing on the key words; discussing their meanings; identifying extraneous information, and helping them know what they are asked to find. Then he leads students to represent the unknowns in each condition of the problem with two variables (say, x and y).	*Understanding the problem:* Teacher will guide the students in reading the problem carefully several times; focusing on the key words; discussing their meanings; identifying extraneous information, and helping them know what they are asked to find. Then he leads students to represent the unknowns in each condition of the problem with two variables (say, x and y).
Stage 2:	*Devising a plan:* When the groups jointly understand and extract the necessary information that is required to solve the problem, then teacher will asks each of the groups to formulate mathematical models or two linear equations.	*Devising a plan:* When the groups jointly understand and extract the necessary information that is required to solve the problem, then teacher will asks each of the groups to formulate mathematical models or two linear equations.
Stage 3:	*Carrying out the Plan:* At this stage the teacher will instructs students' to use appropriate strategy to solve the problem given to them. While students are working, teacher will be going round the class, listening to their discussions, observing their activities, assisting and encouraging them to participate in solving the problem.	*Carrying out the Plan:* At this stage the teacher will instructs students' to use appropriate strategy to solve the problem given to them. While students are working, teacher will be going round the class, listening to their discussions, observing their activities, assisting and encouraging them to participate in solving the problem.
Stage 4:	*Looking back:* Teacher asks students' to reflect on their answers and the processes they use to arrive at the answers.	*Looking back:* Teacher asks students' to reflect on their answers and the processes they use to arrive at the answers.
Step 5: Discussions of students' results. (10 minutes)	The task at this stage will include comparing results at each stage of problem solving, discussing conclusions, and sharing experiences across groups. After the problem has been solved, each group will be given	The task at this stage will include comparing results at each stage of problem solving, discussing conclusions, and sharing experiences across groups. After the problem has been solved, each group will be given 2

	3 minutes to presents its results to the entire class. While doing so, other groups are listening and jotting down their observations. Afterwards, 5 minutes will be given to the whole class to make comments on the results presented by each group. The discussion will continue in this way until every group has presented its results. Each of the groups will then incorporate into its work the observations made by members from other groups. Finally, teacher provides a summary of the important points learn in the lesson in 3 minutes.	minutes to presents its results to the entire class. While doing so, other groups are listening and jotting down their observations. Afterwards, 5 minutes will be given to the whole class to make comments on the results presented by each group. The discussion will continue in this way until every group has presented its results. Each of the groups will then incorporate into its work the observations made by members from other groups. Finally, teacher provides a summary of the important points learn in the lesson in 3 minutes.
Step 6: Evaluation of students' learning progress. (4 minutes)	Teacher will ask students to return to their usual places after they have worked in groups. Then presents a problem on the chalkboard and asks them to solve it individually.	Teacher will ask students to return to their usual places after they have worked in groups. Then presents a problem on the chalkboard and asks them to solve it individually.
Step 7: Conclusion (2 minutes)	Teacher will give students homework assignment to practice what has been learnt in the class.	Teacher will give students homework assignment to practice what has been learnt in the class.

Week	**6**
Content	**Work Word Problems leading to simultaneous linear equations**
Objectives	Students should be able to: **i**. translate work word problems that lead to simultaneous linear equations. **ii**. solve word problems involving two people working together to complete a job that lead to simultaneous linear equations with substitution method using problem-solving strategy. **iii**. solve word problems involving more than two people working together to complete a job that lead to simultaneous linear equations with substitution method using problem-solving strategy.
Instructional Materials	Calculators, charts of equations on rate of work.
Step I: Introduction of the lesson (2 minutes)	At the beginning of each lesson, teacher will ask students variety of questions on their knowledge base that has link to the current topic.
Step 2: Dividing the	At this stage, the teacher will divide the class into atleast 5 groups, and each group will contain 5 to 10 students of varying ability levels who will support one

class into manageable groups. (2 minutes)	another in multiple ways during problem solving process.
Step3: Assigning roles to group members. (2 minutes)	Teacher will assign a specific role to each member of the group to encourage interactions and divergent thinking among students during problem solving. The typical roles will include a group leader who facilitates group discussion and makes sure that group members work together on task; a monitor who checks time on task and ensures that everyone gets an equal opportunity to participate; an encourager who inspires reluctant and shy students to participate in group discussions; a reflector who keeps the group aware of progress; a quiet captain who monitors noise level; and a recorder who keeps a written activity or writes the final results for the group.
Step 4: Assigning activities to groups. (18 minutes)	After assigning specific roles to the members of each group in the class, the teacher will paste a cardboard paper on the wall/board that contains a worded problem on simultaneous linear equations and ask students to solve using problem solving strategy as explained below.
Stage 1:	*Understanding the problem:* Teacher will guide the students in reading the problem carefully several times; focusing on the key words; discussing their meanings; identifying extraneous information, and helping them know what they are asked to find. Then he leads students to represent the unknowns in each condition of the problem with two variables (say, x and y).
Stage 2:	*Devising a plan:* When the groups jointly understand and extract the necessary information that is required to solve the problem, then teacher will asks each of the groups to formulate mathematical models or two linear equations.
Stage 3:	*Carrying out the Plan:* At this stage the teacher will instructs students' to use appropriate strategy to solve the problem given to them. While students are working, teacher will be going round the class, listening to their discussions, observing their activities, assisting and encouraging them to participate in solving the problem.
Stage 4:	*Looking back:* Teacher asks students' to reflect on their answers and the processes they use to arrive at the answers.
Step 5: Discussions of students' results. (10 minutes)	The task at this stage will include comparing results at each stage of problem solving, discussing conclusions, and sharing experiences across groups. After the problem has been solved, each group will be given 3 minutes to presents its results to the entire class. While doing so, other groups are listening and jotting down their observations. Afterwards, 5 minutes will be given to the whole class to make comments on the results presented by each group. The discussion will continue in this way until every group has presented its results. Each of the groups will then incorporate into its work the observations made by members from other groups. Finally, teacher provides a summary of the important points learn in the lesson in 3 minutes.
Step 6: Evaluation	Teacher will ask students to return to their usual places after they have worked in groups. Then presents a problem on the chalkboard and asks them to solve it

of students' learning progress. (4 minutes)	individually.
Step 7: Conclusion (2 minutes)	Teacher will give students homework assignment to practice what has been learnt in the class.

Elimination Method II

Weeks	**1 & 2**	**3**
Content	**Direct Translation Word Problems leading to simultaneous linear equations**	**Geometry Problems leading to simultaneous linear equations**
Objectives	Students should be able to: **i**. translate number word problems that lead to simultaneous linear equations. **ii**. solve word problems involving sums and differences of whole numbers that lead to simultaneous linear equations with elimination method using problem-solving strategy. **iii**. solve word problems involving differences and products of whole numbers that lead to simultaneous linear equations with elimination method using problem-solving strategy. **iv**. solve word problems involving two-digit numbers that lead to simultaneous linear equations with elimination method using problem-solving strategy. **v.** solve word problems involving ages that lead to simultaneous linear equations with elimination method using problem-solving strategy. **vi.** solve word problems involving fractions that lead to simultaneous linear equations with substitution method using problem-solving strategy. **vii**. solve word problems involving ratios that lead to simultaneous linear equations with elimination method using problem-solving instructional strategy.	Students should be able to: **i**. translate geometry word problems that lead to simultaneous linear equations with elimination method using problem-solving strategy.. **ii**. solve word problems involving dimensions of a rectangle that lead to simultaneous linear equations with elimination method using problem-solving strategy. **iii**. solve word problems involving complementary angles that lead to simultaneous linear equations with elimination method using problem-solving strategy. **iv**. solve word problems involving supplementary angles that lead to simultaneous linear equations with elimination method using problem-solving strategy. **v.** solve word problems involving angles of an isosceles triangle with elimination method using problem-solving strategy.
Instructional Materials	Calculators, charts of equations on sum and differences, differences and	Calculators, charts of equations on dimensions of a rectangle,

	products of whole numbers, two-digit numbers, fractions, and ratios involving word problems.	complementary angles, supplementary angles and isosceles angles word problems.
Step I: Introduction of the lesson (2 minutes)	At the beginning of each lesson, teacher will ask students variety of questions on their knowledge base that has link to the current topic.	At the beginning of each lesson, teacher will ask students variety of questions on their knowledge base that has link to the current topic.
Step 2: Dividing the class into manageable groups. (2 minutes)	At this stage, the teacher will divide the class into atleast 5 groups, and each group will contain 5 to 10 students of different ability levels who will support one another in multiple ways during problem solving process.	At this stage, the teacher will divide the class into atleast 5 groups, and each group will contain 5 to 10 students of different ability levels who will support one another in multiple ways during problem solving process.
Step3: Assigning roles to group members. (2 minutes)	Teacher will assign a specific role to each member of the group to encourage interactions and divergent thinking among students during problem solving. The typical roles will include a group leader who facilitates group discussion and makes sure that group members work together on task; a monitor who checks time on task and ensures that everyone gets an equal opportunity to participate; an encourager who inspires reluctant and shy students to participate in group discussions; a reflector who keeps the group aware of progress; a quiet captain who monitors noise level; and a recorder who keeps a written activity or writes the final results for the group.	Teacher will assign a specific role to each member of the group to encourage interactions and divergent thinking among students during problem solving. The typical roles will include a group leader who facilitates group discussion and makes sure that group members work together on task; a monitor who checks time on task and ensures that everyone gets an equal opportunity to participate; an encourager who inspires reluctant and shy students to participate in group discussions; a reflector who keeps the group aware of progress; a quiet captain who monitors noise level; and a recorder who keeps a written activity or writes the final results for the group.
Step 4: Assigning activities to groups. (18 minutes) *Stage 1:*	After assigning specific roles to the members of each group in the class, the teacher will paste a cardboard paper on the wall/board that contains a worded problem on simultaneous linear equations and ask students to solve using problem solving strategy as explained below. *Understanding the problem:* Teacher will guide the students in reading the problem carefully several times; focusing on the key words; discussing their meanings; identifying extraneous	After assigning specific roles to the members of each group in the class, the teacher will paste a cardboard paper on the wall/board that contains a worded problem on simultaneous linear equations and ask students to solve using problem solving strategy as explained below. *Understanding the problem:* Teacher will guide the students in reading the problem carefully several times; focusing on the key words; discussing their meanings; identifying

	information, and helping them know what they are asked to find. Then he leads students to represent the unknowns in each condition of the problem with two variables (say, x and y).	extraneous information, and helping them know what they are asked to find. Then he leads students to represent the unknowns in each condition of the problem with two variables (say, x and y).
Stage 2:	*Devising a plan:* When the groups jointly understand and extract the necessary information that is required to solve the problem, then teacher will asks each of the groups to formulate mathematical models or two linear equations.	*Devising a plan:* When the groups jointly understand and extract the necessary information that is required to solve the problem, then teacher will asks each of the groups to formulate mathematical models or two linear equations.
Stage 3:	*Carrying out the Plan:* At this stage the teacher will instructs students' to use appropriate strategy to solve the problem given to them. While students are working, teacher will be going round the class, listening to their discussions, observing their activities, assisting and encouraging them to participate in solving the problem.	*Carrying out the Plan:* At this stage the teacher will instructs students' to use appropriate strategy to solve the problem given to them. While students are working, teacher will be going round the class, listening to their discussions, observing their activities, assisting and encouraging them to participate in solving the problem.
Stage 4:	*Looking back:* Teacher asks students' to reflect on their answers and the processes they use to arrive at the answers.	*Looking back:* Teacher asks students' to reflect on their answers and the processes they use to arrive at the answers.
Step 5: Discussions of students' results. (10 minutes)	The task at this stage will include comparing results at each stage of problem solving, discussing conclusions, and sharing experiences across groups. After the problem has been solved, each group will be given 3 minutes to presents its results to the entire class. While doing so, other groups are listening and jotting down their observations. Afterwards, 5 minutes will be given to the whole class to make comments on the results presented by each group. The discussion will continue in this way until every group has presented its results. Each of the groups will then incorporate into its work the observations made by	The task at this stage will include comparing results at each stage of problem solving, discussing conclusions, and sharing experiences across groups. After the problem has been solved, each group will be given 3 minutes to presents its results to the entire class. While doing so, other groups are listening and jotting down their observations. Afterwards, 5 minutes will be given to the whole class to make comments on the results presented by each group. The discussion will continue in this way until every group has presented its results. Each of the groups will then incorporate into its work the

	members from other groups. Finally, teacher provides a summary of the important points learn in the lesson in 3 minutes.	observations made by members from other groups. Finally, teacher provides a summary of the important points learn in the lesson in 3 minutes.
Step 6: Evaluation of students' learning progress. (4 minutes)	Teacher will ask students to return to their usual places after they have worked in groups. Then presents a problem on the chalkboard and asks them to solve it individually.	Teacher will ask students to return to their usual places after they have worked in groups. Then presents a problem on the chalkboard and asks them to solve it individually.
Step 7: Conclusion (2 minutes)	Teacher will give students homework to practice what has been learnt in the class.	Teacher will give students homework to practice what has been learnt in the class.

Week	**4**	**5**
Content	**Mixture Word Problems leading to simultaneous linear equations**	**Uniform Motion Word Problems leading to simultaneous linear equations**
Objectives	Students should be able to: **i**. translate mixture word problems that lead to simultaneous linear equations. **ii**. solve word problems involving costs of two quantities that lead to simultaneous linear equations with elimination method using problem solving strategy. **iii**. solve word problems involving liquid solutions that lead to simultaneous linear equations with elimination method using problem-solving strategy. **iv**. solve word problems involving simple interest that lead to simultaneous linear equations with elimination method using problem-solving strategy.	Students should be able to: **i**. translate motion word problems that lead to simultaneous linear equations. **ii**. solve word problems involving motion affected by wind that lead to simultaneous linear equations with elimination method using problem-solving strategy. **iii.** solve word problems involving speed that lead to simultaneous linear equations with elimination method using problem-solving strategy. **iv**. solve word problems involving motion affected by water current that lead to simultaneous linear equations with elimination method using problem-solving strategy.
Instructional Materials	Calculators, charts of equations on costs of two quantities, liquid solutions and simple interest word problems.	Calculators, charts of equations on motion affected by wind, two travelers in the same direction and motion affected by water current.
Step I: Introduction of the lesson	At the beginning of each lesson, teacher will ask students variety of questions on their knowledge base that has link to the	At the beginning of each lesson, teacher will ask students variety of questions on their knowledge base

(2 minutes)	current topic.	that has link to the current topic.
Step 2: Dividing the class into manageable groups. (2 minutes)	At this stage, the teacher will divide the class into atleast 5 groups, and each group will contain 10 students of varying ability levels who will support one another in multiple ways during problem solving process.	At this stage, the teacher will divide the class into atleast 5 groups, and each group will contain 10 students of different ability levels who will support one another in multiple ways during problem solving process.
Step3: Assigning roles to group members. (2 minutes)	Teacher will assign a specific role to each member of the group to encourage interactions and divergent thinking among students during problem solving. The typical roles will include a group leader who facilitates group discussion and makes sure that group members work together on task; a monitor who checks time on task and ensures that everyone gets an equal opportunity to participate; an encourager who inspires reluctant and shy students to participate in group discussions; a reflector who keeps the group aware of progress; a quiet captain who monitors noise level; and a recorder who keeps a written activity or writes the final results for the group.	Teacher will assign a specific role to each member of the group to encourage interactions and divergent thinking among students during problem solving. The typical roles will include a group leader who facilitates group discussion and makes sure that group members work together on task; a monitor who checks time on task and ensures that everyone gets an equal opportunity to participate; an encourager who inspires reluctant and shy students to participate in group discussions; a reflector who keeps the group aware of progress; a quiet captain who monitors noise level; and a recorder who keeps a written activity or writes the final results for the group.
Step 4: Assigning activities to groups. (18 minutes) *Stage 1:*	After assigning specific roles to the members of each group in the class, the teacher will paste a cardboard paper on the wall/board that contains a worded problem on simultaneous linear equations and ask students to solve using problem solving strategy as explained below. *Understanding the problem:* Teacher will guide the students in reading the problem carefully several times; focusing on the key words; discussing their meanings; identifying extraneous information, and helping them know what they are asked to find. Then he leads students to represent the unknowns in each condition of the problem with two variables (say, x and y).	After assigning specific roles to the members of each group in the class, the teacher will paste a cardboard paper on the wall/board that contains a worded problem on simultaneous linear equations and ask students to solve using problem solving strategy as explained below. *Understanding the problem:* Teacher will guide the students in reading the problem carefully several times; focusing on the key words; discussing their meanings; identifying extraneous information, and helping them know what they are asked to find. Then he leads students to represent the unknowns in each condition of the problem with two variables (say, x and y).

Stage 2: *Stage 3:* *Stage 4:*	*Devising a plan:* When the groups jointly understand and extract the necessary information that is required to solve the problem, then teacher will asks each of the groups to formulate mathematical models or two linear equations. *Carrying out the Plan:* At this stage the teacher will instructs students' to use appropriate strategy to solve the problem given to them. While students are working, teacher will be going round the class, listening to their discussions, observing their activities, assisting and encouraging them to participate in solving the problem. *Looking back:* Teacher asks students' to reflect on their answers and the processes they use to arrive at the answers.	*Devising a plan:* When the groups jointly understand and extract the necessary information that is required to solve the problem, then teacher will asks each of the groups to formulate mathematical models or two linear equations. *Carrying out the Plan:* At this stage the teacher will instructs students' to use appropriate strategy to solve the problem given to them. While students are working, teacher will be going round the class, listening to their discussions, observing their activities, assisting and encouraging them to participate in solving the problem. *Looking back:* Teacher asks students' to reflect on their answers and the processes they use to arrive at the answers.
Step 5: Discussions of students' results. (10 minutes)	The task at this stage will include comparing results at each stage of problem solving, discussing conclusions, and sharing experiences across groups. After the problem has been solved, each group will be given 3 minutes to presents its results to the entire class. While doing so, other groups are listening and jotting down their observations. Afterwards, 5 minutes will be given to the whole class to make comments on the results presented by each group. The discussion will continue in this way until every group has presented its results. Each of the groups will then incorporate into its work the observations made by members from other groups. Finally, teacher provides a summary of the important points learn in the lesson in 3 minutes.	The task at this stage will include comparing results at each stage of problem solving, discussing conclusions, and sharing experiences across groups. After the problem has been solved, each group will be given 3 minutes to presents its results to the entire class. While doing so, other groups are listening and jotting down their observations. Afterwards, 5 minutes will be given to the whole class to make comments on the results presented by each group. The discussion will continue in this way until every group has presented its results. Each of the groups will then incorporate into its work the observations made by members from other groups. Finally, teacher provides a summary of the important points learn in the lesson in 3 minutes.
Step 6: Evaluation of students'	Teacher will ask students to return to their usual places after they have worked in groups. Then presents a	Teacher will ask students to return to their usual places after they have worked in groups. Then presents a

learning progress. (4 minutes)	problem on the chalkboard and asks them to solve it individually.	problem on the chalkboard and asks them to solve it individually.
Step 7: Conclusion (2 minutes)	Teacher will give students homework assignment to practice what has been learnt in the class.	Teacher will give students homework assignment to practice what has been learnt in the class.

Week	**6**
Content	**Work Problems leading to simultaneous linear equations**
Objectives	Students should be able to: **i**. interpret work word problems that lead to simultaneous linear equations. **ii**. solve word problems involving two people working together to complete a job that lead to simultaneous linear equations with elimination method using problem-solving strategy. **iii**. solve word problems involving more than two people working together to complete a job that lead to simultaneous linear equations with elimination method using problem-solving strategy.
Instructional Materials	Calculators, charts of equations on rate of work.
Step I: Introduction of the lesson (2 minutes)	At the beginning of each lesson, teacher will ask students variety of questions on their knowledge base that has link to the current topic.
Step 2: Dividing the class into manageable groups. (2 minutes)	At this stage, the teacher will divide the class into atleast 5 groups, and each group will contain 10 students of varying ability levels who will support one another in multiple ways during problem solving process.
Step3: Assigning roles to group members. (2 minutes)	Teacher will assign a specific role to each member of the group to encourage interactions and divergent thinking among students during problem solving. The typical roles will include a group leader who facilitates group discussion and makes sure that group members work together on task; a monitor who checks time on task and ensures that everyone gets an equal opportunity to participate; an encourager who inspires reluctant and shy students to participate in group discussions; a reflector who keeps the group aware of progress; a quiet captain who monitors noise level; and a recorder who keeps a written activity or writes the final results for the group.
Step 4: Assigning activities to groups. (18 minutes)	After assigning specific roles to the members of each group in the class, the teacher will paste a cardboard paper on the wall/board that contains a worded problem on simultaneous linear equations and ask students to solve using problem solving strategy as explained below.

Stage 1: *Stage 2:* *Stage 3:* *Stage 4:*	*Understanding the problem:* Teacher will guide the students in reading the problem carefully several times; focusing on the key words; discussing their meanings; identifying extraneous information, and helping them know what they are asked to find. Then he leads students to represent the unknowns in each condition of the problem with two variables (say, x and y). *Devising a plan:* When the groups jointly understand and extract the necessary information that is required to solve the problem, then teacher will asks each of the groups to formulate mathematical models or two linear equations. *Carrying out the Plan:* At this stage the teacher will instructs students' to use appropriate strategy to solve the problem given to them. While students are working, teacher will be going round the class, listening to their discussions, observing their activities, assisting and encouraging them to participate in solving the problem. *Looking back:* Teacher asks students' to reflect on their answers and the processes they use to arrive at the answers.
Step 5: Discussions of students' results. (10 minutes)	The task at this stage will include comparing results at each stage of problem solving, discussing conclusions, and sharing experiences across groups. After the problem has been solved, each group will be given 3 minutes to presents its results to the entire class. While doing so, other groups are listening and jotting down their observations. Afterwards, 5 minutes will be given to the whole class to make comments on the results presented by each group. The discussion will continue in this way until every group has presented its results. Each of the groups will then incorporate into its work the observations made by members from other groups. Finally, teacher provides a summary of the important points learn in the lesson in 3 minutes.
Step 6: Evaluation of students' learning progress. (4 minutes)	Teacher will ask students to return to their usual places after they have worked in groups. Then presents a problem on the chalkboard and asks them to solve it individually.
Step 7: Conclusion (2 minutes)	Teacher will give students homework assignment to practice what has been learnt in the class.

Appendix I

Sample Lesson Plan for Experimental Group Using Polya's Problem Solving Stratey by Substitution and Elimination Methods

Substitution Method I

Class: JSS III

Subject: Mathematics

Time: 40 minutes

Sex: Single Sex

Topic: Number word Problems

Sub-topic: Word problems involving sums and differences of whole numbers

Entry behavior: Students have known four basic operations and can solve simple linear equations in one and two variables.

Behavioural Objectives: At the end of the lesson students should be able to solve word problems involving sums and differences of whole numbers that lead to simultaneous linear equations using substitution method.

Instructional Materials: Calculators, charts of equations on sums and differences of whole numbers involving word problems.

Step 1: Introduction (2 minutes): Teacher introduces the lesson by asking students to solve linear equations in one and two variables algebraically.

Step 2: Dividing the class into manageable groups (2 minutes): Teacher will divide the class into atleast 5 groups each group will have 10 students of different ability levels.

Step 3: Assigning roles to group members (2 minutes): Teacher will assign different roles to groups' members that include among others a group leader, a monitor, an encourager, a reflector, a quiet captain, a recorder.

Step 4: Assigning activities to groups (18 minutes): Teacher will paste a cardboard paper on the wall/board that contains a word problem involving sums and differences of whole numbers and leads students to solve using problem solving strategy as follows:

Problem: The sum of two numbers is 17 and their difference is 3. Find the numbers.

Solution

Stage 1: Understanding the problem: Teacher leads the students to read the problem carefully and identify the key words and conditions required to solve it. "The conditions are 'the sum of two numbers is 17" and "their difference is 3". The problem is to find the two numbers. So let x = the first number and y = the second number.

Stage 2: Devising a Plan: Students translate the given conditions into two equations, as follows:

The sum of two numbers is 17

$$x + y = 17 \qquad (1)$$

Their difference is 3

$$x - y = 3 \qquad (2)$$

Stage 3: Carrying out the plan: Students solve the equations, as follows:

$$x + y = 17 \qquad (1)$$

$$x - y = 3 \qquad (2)$$

From (2), $x = 3 + y$ (3)

Substitute $3 + y$ for x in (1)

$$3 + y + y = 17$$

$$2y + 3 = 17$$

$$2y + 3 - 3 = 17 - 3$$

$$2y = 14$$

$$y = 7$$

Substitute 7 for y in (2)

$$x - 7 = 3$$

$$x - 7 + 7 = 3 + 7$$

$$x = 10$$

$$x = 10, \ \ y = 7$$

Stage 4: Looking Back: Teacher checks with the students to ensure whether the answer is reasonable, as follows:

The number, 17, is the sum of 10 and 7. The first condition is satisfied. Also, when the number 7 is subtracted from 10 the result is 3. The second condition is satisfied. Thus, the result is verified.

Step 5: Discussion of results (10 minutes): After the problem has been solved, each group will be given time to presents its results to the class. And thereafter the whole class will make comments on the results presented by each group. Finally, the teacher will provide a summary of the important points in the lesson.

Step 6: Evaluation of students' learning progress (4 minutes): Teacher asks the students to solve the following problem in the class individually.

Problem: The sum of two numbers is 19. Their difference is 5. Find the numbers.

Step 7: Conclusion (2 minutes): Teacher will ask students to solve the following problems as homework assignments:

1. The sum of two numbers is 26 and they differ by 28. Find the numbers.
2. The sum of two numbers is 20 and their difference is 6. Find the numbers.
3. The sum of two numbers is 7. Twice one of the numbers plus the other is 10. Find the numbers.

Elimination Method II

Class: JSS III

Subject: Mathematics

Time: 40 minutes

Sex: Single Sex

Topic: Number word Problems

Sub-topic: Word problems involving sums and differences of whole numbers

Entry behavior: Students have known four basic operations and can solve simple linear equations in one and two variables.

Behavioural Objectives: At the end of the lesson students should be able to solve word problems involving sums and differences of whole numbers that lead to simultaneous linear equations using elimination method.

Instructional Materials: Calculators, charts of equations on sums and differences of whole numbers involving word problems.

Step 1: Introduction (2 minutes): Teacher introduces the lesson by asking students to solve linear equations in one and two variables algebraically.

Step 2: Dividing the class into manageable groups (2 minutes): Teacher will divide the class into atleast 5 groups each group will have 10 students of different ability levels.

Step 3: Assigning roles to group members (2 minutes): Teacher will assign different roles to groups' members that include among others a group leader, a monitor, an encourager, a reflector, a quiet captain, a recorder.

Step 4: Assigning activities to groups (18 minutes): Teacher will paste a cardboard paper on the wall/board that contains a word problem involving sums and differences of whole numbers and leads students to solve using problem solving strategy as follows:

Problem: The sum of two numbers is 17 and their difference is 3. Find the numbers.

Solution

Stage 1: Understanding the problem: Teacher leads the students to read the problem carefully and identify the key words and conditions required to solve it. "The conditions are 'the sum of two numbers is 17" and "their difference is 3". The problem is to find the two numbers. So let x = the first number and y = the second number.

Stage 2: Devising a Plan: Students translate the given conditions into two equations, as follows:

The sum of two numbers is 17

$$x + y = 17 \qquad (1)$$

Their difference is 3

$$x - y = 3 \qquad (2)$$

Stage 3: Carrying out the plan: Students solve the equations, as follows:

$$\begin{array}{ll} x + y = 17 & (1) \\ \underline{x - y = 3} & (2) \\ \text{Add: } 2x = 20 & \\ x = 10 & \end{array}$$

Substitute 10 for x (1)

$$10 + y = 17$$
$$10 - 10 + y = 17 - 10$$
$$y = 7$$
$$x = 10, \ y = 7$$

Stage 4: Looking Back: Teacher checks with the students to ensure whether the answer is reasonable, as follows:

The number, 17, is the sum of 10 and 7. The first condition is satisfied. Also, when the number 7 is subtracted from 10 the result is 3. The second condition is satisfied. Thus, the result is verified.

Step 5: Discussion of results (10 minutes): After the problem has been solved, each group will be given time to presents its results to the class. And thereafter the whole class will make comments on the results presented by each group. Finally, the teacher will provide a summary of the important points in the lesson.

Step 6: Evaluation of students' learning progress (4 minutes): Teacher asks the students to solve the following problem in the class individually.

Problem: The sum of two numbers is 19. Their difference is 5. Find the numbers.

Step 7: Conclusion (2 minutes): Teacher will ask students to solve the following problems as homework assignments:

1. The sum of two numbers is 26 and they differ by 28. Find the numbers.
2. The sum of two numbers is 20 and their difference is 6. Find the numbers.
3. The sum of two numbers is 7. Twice one of the numbers plus the other is 10. Find the numbers.

Appendix J

Lesson Plan for Control Group Using Conventional Method of Teaching by Substitution and Elimination Methods

Substitution Method I

Weeks	**1 & 2**	**3**
Content	**Direct Translation Problems leading to simultaneous linear equations**	**Geometry Problems leading to simultaneous linear equations**
Objectives	Students should be able to: **i**. translate number word problems that lead to simultaneous linear equations. **ii**. solve word problems involving sums and differences of whole numbers that lead to simultaneous linear equations using substitution method. **iii**. solve word problems involving differences and products of whole numbers that lead to simultaneous linear equations using substitution method. **iv**. solve word problems involving two-digit numbers that lead to simultaneous linear equations using substitution method. **v.** solve word problems involving ages that lead to simultaneous linear equations using substitution method. **vi.** solve word problems involving fractions that lead to simultaneous linear equations using substitution method. **vii.** solve word problems involving ratios that lead to simultaneous linear equations using substitution method.	Students should be able to: **i**. translate geometry word problems that lead to simultaneous linear equations. **ii**. solve word problems involving dimensions of a rectangle that lead to simultaneous linear equations using substitution method **iii**. solve word problems involving complementary angles that lead to simultaneous linear equations using substitution method **iv**. solve word problems involving supplementary angles that lead to simultaneous linear equations using substitution method. **v.** solve word problems involving angles of an isosceles triangle that lead to simultaneous linear equations using substitution method.
Instructional Materials	Calculators, charts of equations on sum and differences, differences and products of whole numbers, two-digit numbers, fractions and ratios involving word problems.	Calculators, charts of equations on dimensions of a rectangle, complementary angles, supplementary angles and isosceles triangle's angles word problems.
Introduction	At the beginning of each lesson, teacher will ask students variety of questions on their knowledge base that has link to the current topic.	At the beginning of each lesson, teacher will ask students variety of questions on their knowledge base that has link to the current topic.
Instructional		

Procedure: **Step 1:** **Step 2:** **Step 3:** **Step 4:** **Step 5:** **Step 6:** **Step 7:** **Step8:** **Conclusion**	Teacher presents the lesson by announcing it to the class and then writes the topic on the chalkboard. Teacher writes the problem on the chalkboard, reads it aloud, explains the meanings of important words, and tells what the problem is all about to the students. Teacher represents one of the unknown quantities with x and the other with y Teacher writes two equations in x and y that describe the conditions. Teacher solves the equations on the chalkboard using substitution method. Teacher checks with the conditions of the problem. Teacher presents a problem on the chalkboard and asks the students to solve it in their notebooks. Teacher goes round the class to supervise work and assist students who have difficulties. He then writes the corrections on the chalkboard as chalkboard summary for the students to copy into their notebooks. Teacher presents some questions on the chalkboard and asks students to solve as homework assignment.	Teacher presents the lesson by announcing it to the class and then writes the topic on the chalkboard. Teacher writes the problem on the chalkboard, reads it aloud, explains the meanings of important words, and tells what the problem is all about to the students. Teacher represents one of the unknown quantities with x and the other with y Teacher writes two equations in x and y that describe the conditions. Teacher solves the equations on the chalkboard using substitution method. Teacher checks with the conditions of the problem. Teacher presents a problem on the chalkboard and asks the students to solve it in their exercise notebooks. Teacher goes round the class to supervise work and assist students who have difficulties. He then writes the corrections on the chalkboard as chalkboard summary for the students to copy into their notebooks. Teacher presents some questions on the chalkboard and asks students to solve as homework assignment.
Week	**4**	**5**
Content	**Mixture Problems leading to simultaneous linear equations**	**Uniform Motion Problems leading to simultaneous linear equations**
Objectives	Students should be able to: **i**. interpret mixture word problems that lead to simultaneous linear equations. **ii**. solve word problems involving costs of two quantities that lead to simultaneous linear equations using substitution method. **iii**. solve word problems involving liquid solutions that lead to simultaneous linear equations with substitution method. **iv**. solve word problems involving simple interest that lead to simultaneous linear equations using substitution method.	Students should be able to: **i**. interpret motion word problems that lead to simultaneous linear equations. **ii**. solve word problems involving motion affected by wind that lead to simultaneous linear equations using substitution method. **iii.** solve word problems involving speed that lead to simultaneous linear equations using substitution method. **iii**. solve word problems involving motion affected by water current that

		lead to simultaneous linear equations using substitution method.
Instructional Materials	Calculators, charts of equations on costs of two items, liquid solutions and simple interest word problems.	Calculators, charts of equations on motion affected by wind, speeds and motion affected by water current.
Introduction	At the beginning of each lesson, teacher will ask students variety of questions on their knowledge base that has link to the current topic.	At the beginning of each lesson, teacher will ask students variety of questions on their knowledge base that has link to the current topic.
Instructional Procedure:		
Step 1:	Teacher presents the lesson by announcing it to the class and then writes the topic on the chalkboard.	Teacher presents the lesson by announcing it to the class and then writes the topic on the chalkboard.
Step 2:	Teacher writes the problem on the chalkboard, reads aloud, explains the meanings of important words, and tells what it is all about to the students.	Teacher writes the problem on the chalkboard, reads it aloud, explains the meanings of important words, and tells what the problem is all about to the students.
Step 3:	Teacher represents one of the unknown quantities with x and the other with y	Teacher represents one of the unknown quantities with x and the other with y
Step 4:	Teacher writes two equations in x and y that describe the conditions.	Teacher writes two equations in x and y that describe the conditions.
Step 5:	Teacher solves the equations on the chalkboard using substitution method.	Teacher solves the two equations on the chalkboard using substitution method.
Step6:	Teacher checks with the conditions of the problem.	Teacher checks with the conditions of the problem.
Step 7:	Teacher presents a problem on the chalkboard and asks the students to solve it in their notebooks.	Teacher presents a problem on the chalkboard and asks the students to solve it in their notebooks.
Step 8:	Teacher goes round the class to supervise work and assist students who have difficulties. He then writes the corrections on the chalkboard as chalkboard summary for the students to copy into their notebooks.	Teacher goes round the class to supervise work and assist students who have difficulties. He then writes the corrections on the chalkboard as chalkboard summary for the students to copy into their notebooks.
Conclusion	Teacher presents some questions on the chalkboard and asks students to solve as homework assignment.	Teacher presents some questions on the chalkboard and asks students to solve as homework assignment.

Week	**6**
Content	**Work Problems leading to simultaneous linear equations**
Objectives	Students should be able to: **i**. translate work word problems that lead to simultaneous linear equations. **ii**. solve word problems involving two people working together to complete a job that lead to simultaneous linear equations using substitution method. **iii**. solve word problems involving more than two people working together to complete a job that lead to simultaneous linear equations using substitution method.
Instructional Materials	Calculators, charts of equations on rate of work.
Introduction	At the beginning of each lesson, teacher will ask students variety of questions on their knowledge base that has link to the current topic.
Instructional Procedure: **Step 1:** **Step 2:** **Step 3:** **Step 4:** **Step 5:** **Step 6:** **Step 7:** **Step 8:** **Conclusion**	 Teacher presents the lesson by announcing it to the class and then writes the topic on the chalkboard. Teacher writes the problem on the chalkboard, reads aloud, explains the meanings of important words, and tells what it is all about to the students. Teacher represents one of the unknown quantities with x and the other with y Teacher writes two equations in x and y that describe the conditions. Teacher solves the equations on the chalkboard using substitution method. Teacher checks with the conditions of the problem. Teacher presents a problem on the chalkboard and asks students to solve it in their notebooks. Teacher goes round the class to supervise work and assist students who have difficulties. He then writes the corrections on the chalkboard as chalkboard summary for the students to copy into their notebooks. Teacher presents some questions on the chalkboard and asks students to solve as homework assignment.

Elimination Method

Weeks	**1 & 2**	**3**
Content	**Direct Translation Problems leading to simultaneous linear equations**	**Geometry Problems leading to simultaneous linear equations**
Objectives	Students should be able to: **i**. translate number word problems that lead to simultaneous linear equations. **ii**. solve word problems involving sums and differences of whole numbers that lead to simultaneous linear equations using elimination method. **iii**. solve word problems involving	Students should be able to: **i**. translate geometry word problems that lead to simultaneous linear equations. **ii**. solve word problems involving dimensions of a rectangle that lead to simultaneous linear equations using elimination method.

	differences and products of whole numbers that lead to simultaneous linear equations using substitution method. **iv**. solve word problems involving two-digit numbers that lead to simultaneous linear equations using elimination method. **v.** solve word problems involving ages that lead to simultaneous linear equations using elimination method. **vi.** solve word problems involving fractions that lead to simultaneous linear equations using elimination method. **vi**. solve word problems involving ratios that lead to simultaneous linear equations using elimination method.	**iii**. solve word problems involving complementary angles that lead to simultaneous linear equations using elimination method. **iv**. solve word problems involving supplementary angles that lead to simultaneous linear equations using elimination method. **v.** solve word problems involving angles of an isosceles triangle.
Instructional Materials	Calculators, charts of equations on sum and differences, differences and products of whole numbers, two-digit numbers and ratios involving word problems.	Calculators, charts of equations on dimensions of a rectangle, complementary angles, supplementary angles and isosceles triangle's angles word problems.
Introduction	At the beginning of each lesson, teacher will ask students variety of questions on their knowledge base that has link to the current topic.	At the beginning of each lesson, teacher will ask students variety of questions on their knowledge base that has link to the current topic.
Instructional Procedure:		
Step 1:	Teacher presents the lesson by announcing it to the class and then writes the topic on the chalkboard.	Teacher presents the lesson by announcing it to the class and then writes the topic on the chalkboard.
Step 2:	Teacher writes the problem on the chalkboard, reads it aloud, explains the meanings of important words, and tells what it is all about to the students. meanings of important words, and tells what the problem is all about to the students.	Teacher writes the problem on the chalkboard, reads it aloud, explains the meanings of important words, and tells what the problem is all about to the students.
Step 3:	Teacher represents one of the unknown quantities with x and the other with y	Teacher represents one of the unknown quantities with x and the other with y
Step 4:	Teacher writes two equations in x and y that describe the conditions.	Teacher writes two equations in x and y that describe the conditions.
Step 5:	Teacher solves the equations on the chalkboard using elimination method.	Teacher solves the equations on the chalkboard using elimination method.
Step6:	Teacher checks with the conditions of the problem.	Teacher checks with the conditions of the problem.

Step 7: **Step 8:** **Conclusion**	Teacher presents a problem on the chalkboard and asks the students to solve it in their notebooks. Teacher goes round the class to supervise work and assist students who have difficulties. He then writes the corrections on the chalkboard as chalkboard summary for the students to copy into their notebooks. Teacher presents some questions on the chalkboard and asks students to solve as homework assignment.	Teacher presents a problem on the chalkboard and asks the students to solve it in their notebooks. Teacher goes round the class to supervise work and assist students who have difficulties. He then writes the corrections on the chalkboard as chalkboard summary for the students to copy into their notebooks. Teacher presents some questions on the chalkboard and asks students to solve as homework assignment.
Week	**4**	**5**
Content	**Mixture Problems leading to simultaneous linear equations**	**Uniform Word Problems leading to simultaneous linear equations**
Objectives	Students should be able to: **i**. translate mixture word problems that lead to simultaneous linear equations. **ii**. solve word problems involving costs of two quantities that lead to simultaneous linear equations using elimination method. **iii**. solve word problems involving liquid solutions that lead to simultaneous linear equations with substitution method. **iv**. solve word problems involving simple interest that lead to simultaneous linear equations using elimination method.	Students should be able to: **i**. translate motion word problems that lead to simultaneous linear equations. **ii**. solve word problems involving motion affected by wind that lead to simultaneous linear equations using elimination method. **iii.** solve word problems involving speed that lead to simultaneous linear equations using elimination method. **iii**. solve word problems involving motion affected by water current that lead to simultaneous linear equations using elimination method.
Instructional Materials	Calculators, charts of equations on costs of two items, liquid solutions and simple interest word problems.	Calculators, charts of equations on motion affected by wind, two travelers in the same direction and motion affected by water current.
Introduction	At the beginning of each lesson, teacher will ask students variety of questions on their knowledge base that has link to the current topic.	At the beginning of each lesson, teacher will ask students variety of questions on their knowledge base that has link to the current topic.
Instructional Procedure: **Step 1:** **Step 2:**	Teacher presents the lesson by announcing it to the class and then writes the topic on the chalkboard. Teacher writes the problem on the chalkboard, reads aloud, explains the meanings of important words, and tells	Teacher presents the lesson by announcing it to the class and then writes the topic on the chalkboard. Teacher writes the problem on the chalkboard, reads it aloud, explains the meanings of important words, and

	what it is all about to the students.	tells what the problem is all about to the students.
Step 3:	Teacher represents one of the unknown quantities with x and the other with y	Teacher represents one of the unknown quantities with x and the other with y
Step 4:	Teacher writes two equations in x and y that describe the conditions.	Teacher writes two equations in x and y that describe the conditions.
Step 5:	Teacher solves the equations on the chalkboard using elimination method.	Teacher solves the equations on the chalkboard using elimination method.
Step6:	Teacher checks with the conditions of the problem.	Teacher checks with the conditions of the problem.
Step 7:	Teacher presents a problem on the chalkboard and asks the students to solve in their notebooks.	Teacher presents a problem on the chalkboard and asks the students to solve it in their notebooks.
Step 8:	Teacher goes round the class to supervise work and assist students who have difficulties. He then writes the corrections on the chalkboard as chalkboard summary for the students to copy into their notebooks.	Teacher goes round the class to supervise work and assist students who have difficulties. He then writes the corrections on the chalkboard as chalkboard summary for the students to copy into their notebooks.
Conclusion	Teacher presents some questions on the chalkboard and asks students to solve as homework assignment.	Teacher presents some questions on the chalkboard and asks students to solve as homework assignment.

Week	**6**
Content	**Work Problems leading to simultaneous linear equations**
Objectives	Students should be able to: **i**. translate work word problems that lead to simultaneous linear equations. **ii**. solve word problems involving two people working together to complete a job that lead to simultaneous linear equations using elimination method. **iii**. solve word problems involving more than two people working together to complete a job that lead to simultaneous linear equations using elimination method.
Instructional Materials	Calculators, charts of equations on rate of work.
Introduction	At the beginning of each lesson, teacher will ask students variety of questions on their knowledge base that has link to the current topic.
Instructional Procedure:	
Step 1:	Teacher presents the lesson by announcing it to the class and then writes the topic on the chalkboard.
Step 2:	Teacher writes the problem on the chalkboard, reads aloud, explains the meanings of important words, and tells what it is all about to the students.
Step 3:	Teacher represents one of the unknown quantities with x and the other with y
Step 4:	Teacher writes two equations in x and y that describe the conditions.

Step 5:	Teacher solves the equations on the chalkboard using elimination method.
Step 6:	Teacher checks with the conditions of the problem.
Step 7:	Teacher presents a problem on the chalkboard and asks the students to solve in their notebooks.
Step 8:	Teacher goes round the class to supervise work and assist students who have difficulties. He then writes the corrections on the chalkboard as chalkboard summary for the students to copy into their notebooks.
Conclusion	Teacher presents some questions on the chalkboard and asks students to solve as homework assignment.

Appendix K

Sample Lesson Plan for Conrol Group Using Conventional Method of Teaching by Substitution and Elimination Methods

Substitution Method I

Class:	JSSIII
Subject:	Mathematics
Time:	40 minutes
Sex:	Single Sex
Topic:	Number Word problems
Sub-topic:	Word problems involving sums and differences of whole numbers
Entry behavior:	Students have known four basic operations and are able to solve problems on linear equations in one and two variables.
Behavioural Objectives	At the end of the lesson students should be able to solve word problems involving sums and differences of whole numbers
Instructional Materials:	Charts of equations on sums and differences of whole numbers
Introduction:	Teacher introduces the lesson by asking students to solve linear equations in one and two variables algebraically.
Procedure:	Teacher will use the following procedure to teach the topic
Step 1:	Teacher explains to the class that in real life there are problems that can be solve with one variable. However, such problems are easier to handle with two variables rather than one variable. For example, a problem with two verbal conditions can be easily transformed into

two equations having two unknowns. The following examples illustrate the situations.

Step 2: **Presentation of question and explanation of important words to students**

The sum of two numbers is 17 and their difference is 3. Find the numbers.

Step 3: **Representation of unknown quantities with variables**

Let $x =$ the first number and $y =$ the second number

Step 4: **Formation of equations in x and y that describe the conditions.**

$x + y = 17$ (1)

$x - y = 3$ (2)

Step 5: **Solving of equations**

$x + y = 17$ (1)

$x - y = 3$ (2)

From (2), $x = 3 + y$ (3)

Substitute $3 + y$ for x in (1)

$3 + y + y = 17$

$2y + 3 = 17$

$2y = 14$

$y = 7$

Substitute 7 for y in (2)

$x - 7 = 3$

$x = 10$

Step 6: **Checking of results**

The sum of two numbers $= x + y = 10 + 7 = 17$; their difference $= x - y = 10 - 7 = 3$. Therefore, the solutions are correct.

Step 7: **Assessment of students' progress**

Students to solve the following problems in the class:

1. The sum of two numbers is 19. Their difference is 5. Find the numbers.
2. The sum of two numbers is 16. Twice their difference increased by 3 is 7. Find the numbers.

Step 8: **Supervision of students' work in the class**

Teacher goes round the class to supervise work and assist students who have difficulties. And then writes the corrections on the chalkboard as chalkboard summary for the students to copy into their notebooks.

Conclusion: **Giving students assignment to do at home**

Teacher gives the following assignments to students to solve at home:

1. The sum of two numbers is 26 and they differ by 28. Find the numbers.
2. The sum of two numbers is 20 and their difference is 6. Find the numbers.
3. The sum of two numbers is 7. Twice one of the numbers plus the other is 10. Find the numbers.

Elimination Method II

Class: JSSIII

Subject: Mathematics

Time: 40 minutes

Sex: Single Sex

Topic: Number Word problems

Sub-topic: Word problems involving sums and differences of whole numbers

Entry behavior: Students have known four basic operations and are able to solve problems on linear equations in one and two variables.

Behavioural Objectives At the end of the lesson students should be able to solve word problems involving sums and differences of whole numbers

Instructional Materials: Charts of equations on sums and differences of whole numbers

Introduction: Teacher introduces the lesson by asking students to solve linear equations in one and two variables algebraically.

Procedure: Teacher will use the following procedure to teach the topic

Step 1: Teacher explains to the class that in real life there are problems that can be solve with one variable. However, such problems are easier to handle with two variables rather than one variable. For example, a problem with two verbal conditions can be easily transformed into two equations having two unknowns. The following examples illustrate the situations.

Step 2: **Presentation of question and explanation of important words to students**

The sum of two numbers is 17 and their difference is 3. Find the numbers.

Step 3: **Representation of unknown quantities with variables**

Let $x =$ the first number and $y =$ the second number

Step 4: **Formation of equations in x and y that describe the conditions.**

$x + y = 17$ (1)

$x - y = 3$ (2)

Step 5: **Solving of equations**

$$\begin{array}{rl} x + y = 17 & (1) \\ \underline{x - y = 3} & (2) \\ \text{Add: } 2x = 20 & \\ x = 10 & \end{array}$$

Substitute 10 for x (1)

$$\begin{array}{c} 10 + y = 17 \\ 10 - 10 + y = 17 - 10 \\ y = 7 \\ x = 10, \ \ y = 7 \end{array}$$

Step 6: **Checking of results**

The sum of two numbers $= x + y = 10 + 7 = 17$; their difference $= x - y = 10 - 7 = 3$. Therefore, the solutions are correct.

Step 7: **Assessment of students' progress**

Students to solve the following problems in the class:

1. The sum of two numbers is 19. Their difference is 5. Find the numbers.

2. The sum of two numbers is 16. Twice their difference increased by 3 is 7. Find the numbers.

Step 8: **Supervision of students' work in the class**

Teacher goes round the class to supervise work and assist students who have difficulties. And then writes the corrections on the chalkboard as chalkboard summary for the students to copy into their notebooks.

Conclusion: **Giving students assignment to do at home**

Teacher gives the following assignments to students to solve at home:

1. The sum of two numbers is 26 and they differ by 28. Find the numbers.
2. The sum of two numbers is 20 and their difference is 6. Find the numbers.
3. The sum of two numbers is 7. Twice one of the numbers plus the other is 10. Find the numbers.

Appendix L

Results of Pilot Test on Correlation Coefficient of SLEPT

S/No.	X	Y	X-square	Y-square	XY
1	38	39	1444	1521	1482
2	33.5	32	1122.25	1024	1072
3	31.5	31	992.25	961	976.5
4	30	29	900	841	870
5	30	29.5	900	870.25	885
6	30	31.5	900	992.25	945
7	29.5	30.5	870.25	930.25	899.75
8	29	32	841	1024	928
9	28	26	784	676	728
10	26	25	676	625	650
11	25.5	19	650.25	361	484.5
12	25.5	20	650.25	400	510
13	25	14	625	196	350
14	22	15	484	225	330
15	19	15	361	225	285
16	18	26	324	676	468
17	17	25	289	625	425
18	15	24	225	576	360
19	14.5	23	210.25	529	333.5
20	14	19	196	361	266
21	13.5	20	182.25	400	270
22	13	25	169	625	325
23	12	3	144	9	36
24	11.5	2	132.25	4	23
25	10.5	5	110.25	25	52.5
26	10	3	100	9	30
27	10	5	100	25	50
28	9.5	2	90.25	4	19
29	6.5	13	42.25	169	84.5
30	3	12	9	144	36
	600.5	595.5	14523.75	15052.75	14174.3
	r= 0.792467				

Appendix M

Results of Pilot Tests on Facility Indices and Discrimination Powers of SLEPT

S/No	P_1	D_1	P_2	D_2
1.	0.31	0.23	0.28	0.27
2.	0.25	0.23	0.25	0.36
3.	0.27	0.26	0.24	0.30
4.	0.17*	0.17	0.17*	0.23
5.	0.21	0.26	0.22	0.34
6.	0.21	-0.19*	0.15*	0.04*
7.	0.30	0.29	0.27	0.44
8.	0.29	0.29	0.27	0.33
9.	0.19*	0.24	0.17*	0.23
10.	0.31	0.27	0.29	0.39
11.	0.15*	0.25	0.20	0.15*
12.	0.21	0.35	0.22	0.33
13.	0.12*	0.21	0.13*	0.16*
14.	0.21	0.40	0.23	0.42
15.	0.10*	0.14*	0.17*	0.13*
16.	0.20	0.33	0.20	0.36
17.	0.12*	0.21	0.13*	0.20
18.	0.10*	0.19*	0.11*	0.20
19.	0.14*	0.21	0.15*	0.23
20.	0.10*	0.13*	0.11*	0.15*

Appendix N

Polya's Problem Solving Model

Stage	Component of the Model	Representative Questions
1.	Understand the problem	Read the problem and identify the unkwns.
2.	Devise a plan	Do you need a diagram? Restate the problem; make a representation. Do you use all the data, conditions etc?
3.	Carry out the plan	Carry out the plan. Check each step. Are the steps taken correct? Can your plan be proved correct?
4.	Look back	Check the solution obtained. Can you use the method for some other problems?

Source: Suleiman, B. (2010)

Appendix O

Summary of JSS III students' enrollment in Public Junior Secondary Schools under ZSTSB according to Education Zone

S/N	SCHOOL	JSS III	GENDER	ZONE
1	GGDSS GUMMI	444	Female	Anka
2	GGDSS BUKKUYUM	126	Female	Anka
3	GGDSS ANKA	209	Female	Anka
4	GUSS GUMMI	186	Male	Anka
5	GDSS GUMMI	610	Male	Anka
6	GDSS GAYARI	135	Male	Anka
7	GDSS BIRNIN TUDU GUMMI	96	Male	Anka
8	GDSS DAKI TAKWAS	106	Male	Anka
9	GDSS GYALANGE	51	Male	Anka
10	GDSS BARDOKI	52	Male	Anka
11	GDSS FALALE	83	Male	Anka
12	GDSS B/MAGAJI GUMMI	41	Male	Anka
13	GDSS GWALLI	42	Male	Anka
14	GDSS ZARUMMAI	81	Male	Anka
15	GDSS N/BURKULLU	143	Male	Anka
16	GDSS GWASHI	74	Male	Anka
17	GDSS RUWAN JEMA	55	Male	Anka
18	GDSS MASAMAR MUDI	83	Male	Anka
19	GDSS ADABKA	69	Male	Anka
20	GDSS ZUGU	18	Male	Anka
21	GDSS GADAR ZAIMA	56	Male	Anka
22	GSS ANKA	126	Male	Anka
23	GDSS ANKA	166	Male	Anka

24	GDSS BARAYAR ZAKI	34	Male	Anka
25	GDSS BAGEGA	60	Male	Anka
26	GDSS WARAMU	30	Male	Anka
27	GDSS WUYA	105	Male	Anka
28	GDSS S/BIRNIN BANAGA	62	Male	Anka
29	GGDSS MARADUN	205	Female	T/Mafara
30	GGDSS MAFARA	618	Female	T/Mafara
31	GGDSS BAKURA	101	Female	T/Mafara
32	MASS MARADUN	11	Male	T/Mafara
33	JSS SADO	66	Male	T/Mafara
34	GDSS MARADUN	304	Male	T/Mafara
35	JSS GIDAN KANO	37	Male	T/Mafara
36	GJSS JANBAKO	65	Male	T/Mafara
37	GJSS DANBAZA	39	Male	T/Mafara
38	GDSS G/GOGA	66	Male	T/Mafara
39	GDSS FARU	128	Male	T/Mafara
40	GJSS DAMAGA	56	Male	T/Mafara
41	GDSS G/NAMAYE	78	Male	T/Mafara
42	JSS DANMANAU	40	Male	T/Mafara
43	JSS MATUSGI	60	Male	T/Mafara
44	GDSS R/GORA	70	Male	T/Mafara
45	GDSS MAGAMI DIDDI	61	Male	T/Mafara
46	GDSS MORAI	111	Male	T/Mafara
47	JSS GAMA-GIWA	43	Male	T/Mafara
48	GDSS KAYA	68	Male	T/Mafara

49	S.I.E T/MAFARA	308	Male	T/Mafara
50	AGWARAGI SEC. SCH.	548	Male	T/Mafara
51	GDSS KAGARA	130	Male	T/Mafara
52	GDSS GARBADU	69	Male	T/Mafara
53	GDSS JANGEBE	241	Male	T/Mafara
54	GDSS GURBI	207	Male	T/Mafara
55	GDSS GWARAM	41	Male	T/Mafara
56	GDSS TAKE-TSABA	54	Male	T/Mafara
57	GDSS BAKURA	0	Male	T/Mafara
58	GDSS B/TUDU	143	Male	T/Mafara
59	JSS DANKAIWA	55	Male	T/Mafara
60	GDSS DAMBO	27	Male	T/Mafara
61	GDSS N/BURKULLU	55	Male	T/Mafara
62	JSS RINI	34	Male	T/Mafara
63	GDSS DAMRI	11	Male	T/Mafara
64	GDSS DAKKO	45	Male	T/Mafara
65	GDSS KWANAR-KALGO	46	Male	T/Mafara
66	GDSS GAMJI	75	Male	T/Mafara
67	GDSS YARKOFOJI	152	Male	T/Mafara
68	GGASS GUSAU	264	Female	Gusau
69	GGDSS GADA-BIYU	225	Female	Gusau
70	GGDSS GALADIMA	581	Female	Gusau
71	GGDSS SAMARU	961	Female	Gusau
72	GGDSS U/ZABARMA	210	Female	Gusau

73	GGDSS T/WADA	691	Female	Gusau
74	GGDSS TSAFE	195	Female	Gusau
75	GGDSS BUNGUDU	213	Female	Gusau
76	GGDSS MARU	201	Female	Gusau
77	GGUSS K/KOSHI	231	Female	Gusau
78	GDSS UNGUWAR DANBABA	98	Mixed	Gusau
79	GGDSS SARKIN KUDU	356	Male	Gusau
80	SCHOOL FOR SPECIAL EDUC.	231	Male	Gusau
81	GDSS FEGIN MAHE	34	Male	Gusau
82	GDSS MILLENIUM QTRS	214	Male	Gusau
83	GDSS R/BORE	58	Male	Gusau
84	GDSS KOTORKOSHI	481	Male	Gusau
85	UDSS BUNGUDU	402	Male	Gusau
86	GDSS FURFURI	75	Male	Gusau
87	GDSS SANKALAWA	86	Male	Gusau
88	GDSS KARAKKAI	42	Male	Gusau
89	GDSS BELA	70	Male	Gusau
90	GDSS KANOMA BIRNI	100	Male	Gusau
91	GDSS KANOMA	153	Male	Gusau
92	GDSS D/DADAN	250	Male	Gusau
93	GDSS D/GULBI	0	Male	Gusau
94	JSS DANKURMI	33	Male	Gusau
95	GDSS B/RUWA	302	Male	Gusau
96	GDSS S/GARI	880	Male	Gusau

97	GDSS WONAKA	27	Male	Gusau
98	GDSS U/GWAZA	217	Male	Gusau
99	SCE(W) GUSAU	35	Female	Gusau
100	GDSS SAMARU	405	Male	Gusau
101	GDSS MAGAMI	324	Male	Gusau
102	GDSS MADA	352	Male	Gusau
103	GDSS D/TURAI	401	Male	Gusau
104	GDSS WANKE	105	Male	Gusau
105	GDSS JANYAU	209	Male	Gusau
106	GDSS DAMBA	256	Male	Gusau
107	SAMBO SEC. SCHO GUSAU	133	Male	Gusau
108	GDSS SHEMORI	83	Male	Gusau
109	GDSS WANKE	149	Male	Gusau
110	GDSS Y/DAJI	356	Male	Gusau
111	GDSS KUCHERI	35	Male	Gusau
112	GDSS BILBIS	131	Male	Gusau
113	GDSS KETA	73	Male	Gusau
114	GDSS DANJIBGA	50	Male	Gusau
115	GDSS MAGAZU	90	Male	Gusau
116	GDSS TSAFE	74	Male	Gusau
117	GDSS CHEDIYA	28	Male	Gusau
118	GDSS TSAFE (SDP)	385	Male	Gusau
119	GDSS YANKUZO	126	Male	Gusau
120	GDSS K/KALGO	58	Male	Gusau

121	GSS TSAFE	145	Male	Gusau
122	GDSS BINGI	79	Male	Gusau
123	GDSS JABAKA	181	Male	Gusau
124	JSS MALELE	50	Male	Gusau
125	GDSS MAYANCHI	70	Male	Gusau
126	GDSS R/DORUWA	162	Male	Gusau
127	GDSS NAHUCHE	271	Male	Gusau
128	GDSS K/MOTA	85	Male	Gusau
129	GDSS GADA	51	Male	Gusau
130	GDSS RIBE	75	Male	Gusau
131	GDSS TOFA	140	Male	Gusau
132	GDSS YARKATSINA	42	Male	Gusau
133	GDSS RAWAYYA	228	Male	Gusau
134	GDSS DASHI	102	Male	Gusau
135	GDSS GULUBBA	65	Male	Gusau
136	GSS MARU	181	Male	Gusau
137	GDSS K/GANUWA	32	Male	Gusau
138	JSS BINDIN	81	Male	Gusau
139	GGDSS K/NAMODA	232	Female	K/Namoda
140	GGCSS MORIKI	146	Female	K/Namoda
141	GGDSS SHINKAFI	419	Female	K/Namoda
142	GSS KAURA	183	Male	K/Namoda
143	GDSS K/MADARO	166	Male	K/Namoda
144	GDSS KATSAURA	56	Male	K/Namoda

145	GDJSS D/ISAH	72	Male	K/Namoda
146	NSS K/NAMODA	388	Male	K/Namoda
147	GDSS M KAURA	63	Male	K/Namoda
148	GDSS MAILALLE	37	Male	K/Namoda
149	GDSS D/KADE	106	Male	K/Namoda
150	GDSS RUTUTU	36	Male	K/Namoda
151	GDSS BANGA	71	Male	K/Namoda
152	GDSS K/DAJI	147	Male	K/Namoda
153	GDSS S/G KAURA	200	Male	K/Namoda
154	GDSS SAKAJIKI	61	Male	K/Namoda
155	GDSS MORIKI	275	Male	K/Namoda
156	GASS ZURMI	330	Male	K/Namoda
157	JSS MASHEMA	19	Male	K/Namoda
158	GDSS KANWA	48	Male	K/Namoda
159	JSS MAYASA	28	Male	K/Namoda
160	GDSS G/BORE	396	Male	K/Namoda
161	GDSS BOKO	0	Male	K/Namoda
162	GDSS DAURAN	169	Male	K/Namoda
163	GSS B/MAGAJI	196	Male	K/Namoda
164	GDSS SABON BIRNIN D/ALI	51	Male	K/Namoda
165	GDJSS GUSAMI	63	Male	K/Namoda
166	GDSS MODOMAWA	130	Male	K/Namoda
167	GDSS B/TSABA	118	Male	K/Namoda
168	GDSS N/MAILAYI	141	Male	K/Namoda

169	GDSS N/GODEL	143	Male	K/Namoda
170	JSS GORA	50	Male	K/Namoda
171	GDSS B/MAGAJI	70	Male	K/Namoda
172	GDSS SHINKAFI	364	Male	K/Namoda
173	GDSS SHANAWA	224	Male	K/Namoda
174	GDSS JANGERU	111	Male	K/Namoda
175	GDSS BADARAWA	94	Male	K/Namoda
176	GDSS B/YERO	61	Male	K/Namoda
177	GDSS YANBUKI	80	Male	K/Namoda
178	GDSS GALADI	90	Male	K/Namoda
179	GDSS K/SHINKAFI	73	Male	K/Namoda
180	GDSS KATURU	60	Male	K/Namoda
	TOTAL	**27,357**		

Appendix P

ASAIQ reliability index

```
RELIABILITY
/VARIABLES= q1 q2 q3 q4 q5 q6 q7 q8 q9 q10 q11 q12 q13 q14 q14 q15 q16 q17 q18 q19 q20 q21 q22 q23 q24
q25 q26 q27 q28 q29 q30 q31 q32 q33 q34 q35 q36 q37 q38 q39 q40 q41 q42 q43 q44 q45 q46 q47 q48
/SCALE (Algebraic Students' Attitude Inventory ) ALL
/MODEL=ALPHA
/STATISTICS=DESCRIPTIVE SCALE CORR
SUMMARY=TOTAL CORR
```

Case Processing Summary

		N	%
Cases	Valid	28	93.3
	Excluded[a]	2	6.7
	Total	30	100.0

a. Listwise deletion based on all variables in the procedure.

Reliability Statistics

Cronbach's Alpha	Cronbach's Alpha Based on Standardized Items	N of Items
.826	.822	48

Appendix Q

Pretest Mean Rank Attitude Levels of Experimental and Control Groups

```
NPAR TESTS
  /M-W= Scores BY Group(1 2)
  /MISSING ANALYSIS.
```

NPar Tests

[DataSet0]

Mann-Whitney Test

Ranks

	Group	N	Mean Rank	Sum of Ranks
Attitude scores	Experimental	235	234.66	55145.50
	Control	228	229.26	52270.50
	Total	463		

Test Statistics[a]

	Attitude scores
Mann-Whitney U	26164.500
Wilcoxon W	52270.500
Z	-.436
Asymp. Sig. (2-tailed)	.663

a. Grouping Variable: Group

```
MEANS TABLES=Scores BY Group
  /CELLS MEAN COUNT STDDEV SEMEAN MEDIAN.
```

Means

[DataSet0]

Case Processing Summary

	Cases					
	Included		Excluded		Total	
	N	Percent	N	Percent	N	Percent
Attitude scores * Group	463	100.0%	0	0.0%	463	100.0%

Report

Attitude scores

Group	Mean	N	Std. Deviation	Std. Error of Mean	Median
Experimental	93.91	235	7.162	.467	94.00
Control	93.63	228	7.053	.467	93.50
Total	93.78	463	7.102	.330	94.00

Appendix R

Pre-test Mean Rank Attitudinal Levels of Gender in Experimental Group

```
NPAR TESTS
  /M-W= Scores BY sex(1 2)
  /MISSING ANALYSIS.
```

NPar Tests

[DataSet0]

Mann-Whitney Test

Ranks

	Sex	N	Mean Rank	Sum of Ranks
Pretest Attitude Scores	Male	120	118.85	14261.50
	Female	115	117.12	13468.50
	Total	235		

Test Statistics[a]

	Pretest Attitude Scores
Mann-Whitney U	6798.500
Wilcoxon W	13468.500
Z	-.195
Asymp. Sig. (2-tailed)	.845

a. Grouping Variable: sex

```
MEANS TABLES=Scores BY sex
  /CELLS MEAN COUNT STDDEV SEMEAN MEDIAN.
```

Means

[DataSet0]

Case Processing Summary

	Cases					
	Included		Excluded		Total	
	N	Percent	N	Percent	N	Percent
Pretest Attitude Scores * sex	235	50.8%	228	49.2%	463	100.0%

Report

Pretest Attitude Scores

Sex	Mean	N	Std. Deviation	Std. Error of Mean	Median
Male	93.98	120	7.107	.649	94.00
Female	93.84	115	7.249	.676	94.00
Total	93.91	235	7.162	.467	94.00

Appendix S

Pretest Mean and Standard Deviation of Experimental and Control Groups

```
T-TEST GROUPS=Group(1 2)
  /MISSING=ANALYSIS
  /VARIABLES=scores
  /CRITERIA=CI(.95).
```

T-Test

[DataSet0]

Group Statistics

	Group	N	Mean	Std. Deviation	Std. Error Mean
Pretest scores	1 Experimental	235	11.23	6.36	.415
	2 Control	228	11.09	6.71	.444

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Pretest scores	Equal variances assumed	.855	.356	.241	461	.810	.146	.607	-1.047	1.339
	Equal variances not assumed			.241	457.781	.810	.146	.608	-1.048	1.340

Appendix T

Mann-Whitney U-test Attitudinal Levels of Experimental and Control Groups

```
 NPAR TESTS
  /M-W= scores BY Group(1 2)
  /MISSING ANALYSIS.
```

NPar Tests

```
[DataSet0]
```

Mann-Whitney Test

Ranks

	Group	N	Mean Rank	Sum of Ranks
Attitude	1 Experimental	230	244.30	56190.00
	2 Control	222	208.05	46188.00
	Total	452		

Test Statistics[a]

	Attitude
Mann-Whitney U	21435.000
Wilcoxon W	46188.000
Z	-2.950
Asymp. Sig. (2-tailed)	.003

a. Grouping Variable: Group

```
MEANS TABLES=scores BY Group
  /CELLS MEAN COUNT STDDEV GMEDIAN SEMEAN.

DataSet0]
```

Case Processing Summary

	Cases					
	Included		Excluded		Total	
	N	Percent	N	Percent	N	Percent
Attitude * Group	452	100.0%	0	0.0%	452	100.0%

Report

Attitude

Group	Mean	N	Std. Deviation	Grouped Median	Std. Error of Mean
Experimental	158.66	230	26.033	159.78	1.717
Control	150.50	222	32.407	151.63	2.175
Total	154.65	452	29.587	156.56	1.392

Appendix U

Independent t-test Performance Scores for Experimental and Control Groups

```
T-TEST GROUPS=Group(1 2)
  /MISSING=ANALYSIS
  /VARIABLES=Score
  /CRITERIA=CI(.95).
```

T-Test

[DataSet0]

Group Statistics

	Group	N	Mean	Std. Deviation	Std. Error Mean
Algtest	Experimental	230	31.32	13.379	.882
	Control	222	21.89	10.279	.690

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Algtest	Equal variances assumed	13.681	.000	8.378	450	.000	9.425	1.125	7.215	11.636
	Equal variances not assumed			8.416	428.621	.000	9.425	1.120	7.224	11.627

Appendix V
Independent t-test Retention Levels of Experimental and Control Groups

```
T-TEST GROUPS=Group(1 2)
  /MISSING=ANALYSIS
  /VARIABLES=Score
  /CRITERIA=CI(.95).
```

T-Test

[DataSet0]

Group Statistics

	Group	N	Mean	Std. Deviation	Std. Error Mean
Algtest	Experimental	213	23.41	9.196	.630
	Control	215	17.82	7.540	.514

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Algtest	Equal variances assumed	11.043	.001	6.874	426	.000	5.585	.813	3.988	7.182
	Equal variances not assumed			6.867	408.781	.000	5.585	.813	3.986	7.184

Appendix W

Mann-Whitney U-test Attitude Levels of Male and Female Students in Experimental Group

```
NPAR TESTS
  /M-W= scores BY Group(1 2)
  /MISSING ANALYSIS.
```

NPar Tests

```
[DataSet0]
```

Mann-Whitney Test

Ranks

	Group	N	Mean Rank	Sum of Ranks
Attitude	Male	117	127.79	14951.50
	Female	113	102.77	11613.50
	Total	230		

Test Statistics[a]

	Attitude
Mann-Whitney U	5172.500
Wilcoxon W	11613.500
Z	-2.851
Asymp. Sig. (2-tailed)	.004

a. Grouping Variable: Group

```
MEANS TABLES=scores BY Group
  /CELLS MEAN COUNT STDDEV GMEDIAN SEMEAN.
```

Means

```
[DataSet0]
```

Case Processing Summary

	Cases					
	Included		Excluded		Total	
	N	Percent	N	Percent	N	Percent
Attitude * Group	230	50.9%	222	49.1%	452	100.0%

Report

Attitude

Group	Mean	N	Std. Deviation	Grouped Median	Std. Error of Mean
Male	163.21	117	20.194	163.44	1.867
Female	153.94	113	30.317	153.75	2.852
Total	158.66	230	26.033	159.78	1.717

Appendix X
Independent t-test Performance Scores of Genders in Experimental Groups

```
T-TEST GROUPS=Group(1 2)
  /MISSING=ANALYSIS
  /VARIABLES=Test
  /CRITERIA=CI(.95).
```

T-Test

[DataSet0]

Group Statistics

	Group	N	Mean	Std. Deviation	Std. Error Mean
AlgraTest	Male	117	25.28	13.267	1.227
	Female	113	21.85	12.850	1.209

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
AlgraTest	Equal variances assumed	.293	.589	1.992	228	.048	3.432	1.723	.037	6.828
	Equal variances not assumed			1.993	227.998	.047	3.432	1.722	.039	6.826

Appendix Y

Independent t-test Retention Levels of Gender in Experimental Group

```
T-TEST GROUPS=Group(1 2)
  /MISSING=ANALYSIS
  /VARIABLES=Test
  /CRITERIA=CI(.95).
```

T-Test

[DataSet0]

Group Statistics

	Group	N	Mean	Std. Deviation	Std. Error Mean
AlgraTest	Male	117	25.28	13.267	1.227
	Female	113	21.85	12.850	1.209

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
AlgraTest	Equal variances assumed	.293	.589	1.992	228	.048	3.432	1.723	.037	6.828
	Equal variances not assumed			1.993	227.998	.047	3.432	1.722	.039	6.826