ADAPTIVE CONTROL RULES FOR ECONOMIC PLANNING: AN APPLICATION TO THE NIGERIAN ECONOMY

A Dissertation
Presented to
the Faculty of the Graduate School
University of Missouri

In Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy

by
JOSEPH S. ODAMA
December 1973

Stanley R. Johnson Dissertation Supervisor

The undersigned, appointed by the Dean of the Graduate Faculty, have examined a thesis entitled

ADAPTIVE CONTROL RULES FOR ECONOMIC PLANNING: AN APPLICATION TO THE NIGERIAN ECONOMY

presented by Joseph S. Odama

a candidate for the degree of Doctor of Philosophy

and hereby certify that in their opinion it is worthy of acceptance.

Stanley R. Johnson

John P. Wolfe

Joseph C. Headley

ACKNOWLEDGEMENTS

The author wishes to express his appreciation to Dr. Stanley Johnson, dissertation supervisor, for generating his interest in the dissertation and for his counsel, guidance, and ecouragement throughout his study and development of the dissertation; and to Dr. John Doll for his inspiration as a teacher.

The author extends his gratitude to AHMADU BELLO UNIVERSITY, NIGERIA and USAID for making his study possible.

Finally, he appreciates the strain his study has had on his wife, Lillian, daughter Uninibile and the rest of his family. To all of them, he expresses his gratitude for their understanding.

TABLE OF CONTENTS

LIST OF TABLES vi

LIST OF FIGURES viii

CHAPTER PAGE

I. INTRODUCTION 1

MOTIVATION OF THE PROBLEM 1

A REVIEW OF ALTERNATIVE PLANNING MODELS 5

NATURE OF PROBLEM 15

THE PLAN AND ORGANIZATION 17

II. FORMULATION OF ECONOMIC POLICY MODEL . . . 19

THE GENERAL OBJECTIVE FUNCTION 19

The Theory of Social Reference Function 20

Specification of Explicit Functional Forms 27

Operational Treatments of the Welfare Function Problem 31

THE STRUCTURAL MODEL 35

Recursive Vs Non-Recursive Structures 36

A Structural Model for the Nigerian Economy 42

THE COMBINED STRUCTURAL COMPONENTS . . . 49

III. DATA, CLASSICAL ESTIMATION PROCEDURE AND RESULTS FOR THE NIGERIAN MODEL . . . 52

CHAPTER		PAGE
	DATA	52
	ESTIMATION PROCEDURE	60
	RESULTS OF ESTIMATION	72
IV.	THE DYNAMIC PROPERTIES OF THE NIGERIAN MODEL	83
	SIMULATION APPROACH	85
	ANALYTIC APPROACH	94
	EMPIRICAL RESULTS OF THE DYNAMIC PROPERTIES OF THE NIGERIAN MODEL . . .	107
V.	APPROACHES TO THE POLICY PROBLEM AND PROBLEMS OF COMPUTING OPTIMAL PLANS . .	133
	APPROXIMATE SOLUTIONS	136
	Certainty Equivalence and Optimal Feedback Rule	141
	Sequential Stochastic Controls and Adaptive Controls	145
	Criteria for Evaluating Control Rules	146
VI.	DETERMINISTIC CONTROL AND CERTAINTY EQUIVALENCE TECHNIQUE	150
	APPLICATION OF CERTAINTY EQUIVALENCE THEOREM	155
	OBTAINING THE DESIRED VALUES OF NON-CONTROLLED VARIABLES AND INSTRUMENTS .	163
	CERTAINTY EQUIVALENCE RESULTS	165
I.	STOCHASTIC CONTROLS	172
	STOCHASTIC SEQUENTIAL CONTROL STRATEGY .	173
	STOCHASTIC SEQUENTIAL CONTROL RESULTS .	184

CHAPTER PAGE

SEQUENTIAL VS ADAPTIVE CONTROL 186

SENSITIVITY TEST OF THE CONTROL TECHNIQUES 192

VIII. CONCLUSIONS 196

FOOTNOTES 202

BIBLIOGRAPHY 208

APPENDIX 220

LIST OF TABLES

TABLE		PAGE
3.1	The Nigerian Data: 1950-1969	54,55
3.2	Estimates of the Coefficients of the Structural Equations for 1951-1967 Period	73
3.3	Estimates of the Coefficient of the Structural Equations for 1951-1968 Period	74
3.4	Estimates of the Coefficients of the Structural Equations for 1951-1969 Period	75
3.5	Reduced Form Coefficients for 1951-1967 Period	79
3.6	Reduced Form Coefficients for 1951-1968 Period	80
3.7	Reduced Form Coefficients for 1951-1969 Period	81
4.1	Matrix C(L).	108
4.2	Matrix G(L).	110
4.3	Matrix of Parameters of Exogenous Variables (C_o^*)	111
4.4	Matrix A(L)	112
4.5	Matrix of Dynamic Multipliers	113
4.6	Power Spectra of Consumption, GDCF, Imports, Exports, and GNP Using Actual Data	116
4.7	Power Spectra of Consumption, GDCF, Imports, Exports, and GNP Implied by the Nigerian Model	117

TABLE		PAGE
4.8	Relative Peaks and Frequencies of Power for Simulated and Actual Data	118
4.9	Cross Spectra between Consumption and the Other Series	129
4.10	Cross Spectra between GDCF and Other Series	130
4.11	Cross Spectra between the Series	131
6.1	Desired Values of Non-controlled Variables and Instrument 1967-1969. ½ml.	166
6.2	Forecasts and Actual Decisions. ½ml. .	168
6.3	Decisions, Forecasts and Actual Levels of the Variables. ½ml.	169
6.4	Matrix K	171
7.1	Sequential Decision and Targets. ½ml. .	
7.2	Sensitivity Test of Certainty Equivalence Control and Sequential Control ½ml.	194

LIST OF FIGURES

FIGURE		PAGE
3.1	Consumption, GDCF, Imports, Exports and GNP 1951/69	56
4.1	Graph of the Power at each Frequency for Series 5 [GNP - Actual Data]	120
4.2	Graph of the Power at each Frequency for Series 5 [GNP - Simulated Data]	121
4.3	Graph of the Power at each Frequency for Series 1 [Personal Consumption]	122
4.4	Graph of the Power at each Frequency for Series 2 [GDCF]	123
4.5	Graph of the Power at each Frequency for Series 3 [Imports]	124
4.6	Graph of the Power at each Frequency for Series 4 [Exports]	125
4.7	Graph of the Frequency Window of the Power Spectrum Estimates	126
5.1	Feedback Mechanism	143

CHAPTER I

INTRODUCTION

MOTIVATION OF THE PROBLEM

The rise of nationalist movements and the attainment of independence in the fifties and sixties, most especially, in developing nations of Africa has been paralleled by an equally rapid growth in the number and scope of national development plans.[1] A development plan is many times viewed by these African states as one of the main characteristics of sovereignty, since the plan symbolizes an effort and a means of generating rapid improvement in the standard of living of the people. The very ubiquity of development plans in developing nations, however, raises questions as to the meaning of the term "plan". Even more serious doubts are raised by the frequent lack of meaningful economic goals. In 1959, for instance, the National Economic Council of Nigeria gave a vague directive that "a National Development Plan be prepared for Nigeria with the objective of the achievement and maintenance of the highest possible rate of increase in the standard of living and the creation of the necessary conditions to this end . . ."[2] The current Nigerian development plan (1970-1974) has an even more ardious task

of expressing the stated national objectives in realistic economic goals. The stated national objectives are: 1) a united, strong and self-reliant nation; 2) a great and dynamic economy; 3) a just and egalitarian society; 4) a land of bright and full opportunities for all citizens; and 5) a free and democratic society.

Jan Drewnowski[3] defines economic planning as "a policy which sets its objectives in the form of quantitative targets to be reached at a determined time . . . Targets set in the plan must be, of course, feasible. To make sure that they are so, they must be derived by confronting policy objectives with the available resources of the economy and their possible uses." In essence, policy formulation lies at the very core of development planning. While there are general discussions of political decisions or intentions in the development plans indicated above, there are no clear underlying economic strategies by which these intentions are to be realized. Individual projects and even sectorial programs are determined independently or without a clear strategy coordinating them. In most cases what is presented as a plan is no more than a shopping list of miscellaneous projects, which by themselves may be sound but may be at the expense of some policy targets. It is not surprising, therefore, that failure of plan objectives to correspond not only with actual results but with subsequent government action is

frequently reported.[4] A number of arguments have been advanced to explain the causes of the failure of some of these plans. Amongst such arguments are those based on a lack of such factors as capital and manpower and unrealistic plan goals. While these arguments are relevant in explaining the failure of the individual programs or projects, they do not adequately explain the failure of the plans to achieve the overall plan objectives. We submit that a major limitation of these plans is the wide gap between the policy objectives and program or project analysis. There is need to bridge this gap, by relating the project analysis to the overall objectives of the plans. Such a bridge may be provided by an aggregate econometric model. In the framework of planning, an aggregate econometric model may provide the linkage between political decisions and the individual programs. It is the location in the plan flow-chart where political intentions are given economic content. Furthermore, the implementation of this stage facilitates interaction between policy makers and planners on the one hand and between planners and project analysts on the other hand, thus matching political intentions with what are feasible and actual plan performance.

The realization of the importance of this linkage in economic plans can be traced back to the late 1940s and early 1950s in the United States. This awareness led to

the building of small econometric models like the Klein (1950), Klein-Goldberger (1957), and Suits (1961) models. As the economy of the United States developed and became more complex, these models were extended and strengthened. Nigeria is at a comparable state of development, as the United States was in the early 1950s, in that she can gainfully utilize similarly simple small scale econometric models. Furthermore, technology is now available for the analysis of such models at modest costs making them feasible alternatives. We, however, note the reluctance of planners in developing countries to employ (albeit in a modified form) policy models that have been developed and tested in other countries. The Nigerian plan (1970-1974, p. 43) points out that "the emphasis of this plan is on the development of a comprehensive and inter-related set of policies, projects and programs of action. Model building of theoretical growth process has not been given a pride of place, except to the extent that it may help to throw marginal light on some operational problems of development policy. National accounting has therefore been used, not so much as a prop for a particular growth model, but as a macro-economic frame of reference for checking the internal consistencies of the plan". Such statements reflect a lack of faith in the efficiency of model building as an important process in plan formulation and execution. On the other hand, the planners in

developing economies may be overlooking the full range and useful features of planning models that already exist.

A REVIEW OF ALTERNATIVE PLANNING MODELS

A number of macro-economic models have been developed and applied with some measure of success. Developing economies, such as Nigeria, can gainfully adapt some of these analytical models to provide an initial logical basis for studying the implications of alternative policies.

The list of econometric models for policy formulation from which they can choose range from the less restrictive General Systems computer Simulations technique of G. Johnson et al. (1973) to Theil's approach (1968) which is more restrictive in terms of application. A simulation is a means of computing the consequences through time--in the light of what is known--of contemplated courses of action. The General Systems Computer Simulation approach emphasizes a very close interaction between the planners and the decision makers. Its claimed strength lies in the flexibility of the model. It is flexible in the sense that it employs other techniques such as linear and non-linear programming, equilibrium simultaneous equations with parameters estimated statistically from time series data, input-output table analysis and cost and benefit analysis. Thus different sources of prior

restrictions are utilized. Unfortunately, however, this claimed flexibility is also a major source of weakness. Since many of the parameters are determined exogenously, no probability statements can be made about them. Furthermore, a huge amount of information is required, thus making the building of the model very expensive and time consuming. The developing economies are not likely to be in a position to devote the amount of resources required to such a model. The General Systems Computer Simulation model which is an outgrowth of the Consortium for the study of Nigerian Rural Development (CSNRD) experience (1969) is designed to indicate the effect or impact of selected programmes or policies on certain specified objectives. The model takes the general mathematical form:

$$\Psi_{t+1} = F\ [\Psi(t),\ \alpha(t),\ \beta(t),\ \gamma(t)] \qquad \ldots\ (1.1)$$

Where,

$\Psi(t)$ is a given state variable, $\alpha(t)$ is a set of parameters that define the structure of the system, $\beta(t)$ is a set of exogenous variables that influence the system's behavior and $\gamma(t)$ is a set of variables which can be controlled to alter the system's performance in various directions. The model as formulated is a building block or recursive system composed of inter-related functional relationships which can be broken into components and can be solved independently of each other. Further, prior

specification of a single performance criterion to be maximized or minimized is not required.

A more formal simulation model has been suggested by Naylor (1970). In the case of a linear econometric model, the form:

$$AX_t + BY_t + \sum_{j=1}^{p} B_j Y_{t-j} + CZ_t + D = U_t \qquad \ldots . (1.2)$$

as specified, where,

X_t is a vector of exogenous variables not under control of policy maker, Y_t is a vector of endogenous variables (target variables). Z_t is a vector of exogenous variables under control of the policy maker (policy instruments or controlled variables) and U_t a vector of stochastic disturbance terms. A, B, C, D are coefficient matrices containing parameters that can be estimated by various statistical methods from time series data. A target or non-controlled variable is a variable which although of interest to the policy maker, it is not controlled by him or at least beyond his control to a sufficiently appreciable degree. A policy instrument or controlled variable is a variable over which the decision maker may exercise discretionary power. It is a means by which he achieves or realizes the target. Essentially, Naylor's model is a forecasting model. We solve the set of simultaneous equations (1.2) for the reduced form[5] as:

$$Y_t = - B^{-1}AX_t - B^{-1} \sum_{j=1}^{p} B_j Y_{t-j} - B^{-1}CZ_t - B^{-1}D + B^{-1}U_t \quad ..(1.3)$$

and generate the time path of Y_t a vector of target variables. The exogenous variables X_t are read into the computer as data, Y_{t-j} have been generated in previous periods and fed back into the model in period t, the policy variables are specified by the planner and the stochastic disturbances are either suppressed or generated by an appropriate computer subroutine. Thus we only need to ask the policy maker what target variables are of particular interest to him and the sets of policy variables which appear politically feasible. In addition the planner may propose a few policies of his own for consideration by the policy maker, thus making for a healthy interaction between the planner and the decision maker. The policies are then put to similar test. The decision rule then becomes one of the policy maker selecting the policies most compatible with his preference function (which is unknown to the planner). The application of the simulation technique for planning purposes is limited by the non-linearity of economic systems which makes solving for the target variables or reduced forms difficult. Furthermore, the simulations may in no sense resemble the behavior of the system that they are designed to emulate. Additional problems arise in estimation if in a simulation experiment, the coefficients of an econometric model are random. Experience with Klein-Goldberger model (1957) indicates serious difficulties in solving

the model when this is the case. Moreover, the results for deterministic simulations are quite different from those of simulations with additive shocks.

Timbergen's (1952) approach to planning is for the policy maker to specify a fixed target value for each of the endogenous variables. For given values of exogenous, lagged endogenous, and stochastic variables, the equations of the model (1.2) are solved simultaneously for the set of values of the policy instruments, Z_t, that are consistent with the targets. One problem in the application of Timbergen's technique is that when the number of policy instruments is less than the number of targets, a solution is impossible. And when policy instruments are more in number than targets, an infinite number of solutions will be possible. Thus the problem of balancing the number of equations and the number of policy instruments may be a serious limitation of the Timbergen approach. An outgrowth of Timbergen's approach is Theil's approach (1968). Theil also assumes knowledge of the social welfare function, W, of the policy maker. But while only targets (flexible or fixed targets) appear in Timbergen's welfare function, both targets and policy instruments enter Theil's social welfare function. Thus with a criterion function of the form:

$$W_t = W_t(Y_t, Z_t) \qquad \ldots . (1.4)$$

the strategy is to find Y_t, Z_t that maximize W_t subject to the constraints imposed by the econometric model of the form (1.2). The question of existence, and the problem of a quantifiable approximation to the social welfare function is discussed later.

When several performance criteria can be transformed into a single basic objective along with necessary and appropriate constraints on resources, technology and institutions, R. Day (1963) has suggested the use of linear/non-linear recursive or dynamic programming models to determine the best policy from all the alternatives considered. Day's approach marked the beginning of sequential decision making in development planning. The most important use of programming in a policy model is that it allows, through duality theorems, the development of a set of "shadow prices" as implicit evaluators or "efficiency indicators" of fixed resources. An important limitation in the use of such procedures, however, is that the optimal solutions cannot, in general, be stated analytically in terms of the parameters of the model. Furthermore, non-linear programming methods are generally very complicated and computationally costly. As Chenery (1962) has emphasized in the context of developing economies, "the principal obstacle to the adoption of the programming solution is the lack of information and the high cost of increasing it. Given the existing lack of data, the mathematical programming

formulation does not correspond to the planner's view of his problem". Alternatively, Chenery (1962) has proposed a policy-oriented model similar to the regional input-output model used by Leotief (1953) and Chenery, Clark and CaoPina (1959) to explain regional production levels in different sectors. His supply-oriented model seeks to explain the sectorial and regional shares in Gross National Product (GNP) growth in terms of, annual growth in exports, average income elasticity of total demand in each sector, fraction of total supply of each commodity furnished by each region and regional transfers. Furthermore, GNP, national export and regional transfer are assumed exogenous to the model. Chenery's model suffers from the limitations that it focuses mainly on supply patterns, and that policy decisions are not derived with a framework of an explict preference function of the policy makers.

A model placing main emphasis on demand patterns has been suggested by Taylor (1969). He proposes a simple simulation model based on the assumption that steady Domar-type economic growth prevails in a three sector economy where only one sector (industry) directly produces investment goods.[6] The model fits cross-section regression equations into input-output accounting relationships from which it generates value-added shares of three aggregate producing sectors as functions of income and population.

The technique thus provides a useful means for evaluating the importance of the various final demand components (consumption, investment, etc.) in "causing" structural change and can be utilized in identifying the main factors influencing these demand components. The regression equations used take the form:

$$\ln X = a + b(\ln Y) + c(\ln Y)^2 + d(\ln N) \quad \ldots . (1.5)$$

where,

X is the dependent variable, Y is per capita income and N is population. These equations were then estimated by the ordinary least squares method (OLS). Taylor's results support the hypothesis that industrial shares in large countries have a declining elasticity with respect to per capita income, while the elasticity is constant in small countries. Furthermore, the service share of GNP tends to increase with per capita income at a fairly constant elasticity, indicating a demand shift towards service sector of the economy as per capita income increase. Taylor's partial equilibrium approach to development planning is inadequate in the sense that the model provides no way of finding out how well the model performs. The technique as applied merely indicates the presence (or absence) of structural changes while there is no indication as to how the magnitude and direction of these changes can be controlled using appropriate instruments.

Chenery and Taylor's approaches represent two competing views of development models in the literature. These views are, supply-oriented models of neoclassical type and demand-oriented of the Keynesian models. In the case of the former, since there is no explicit treatment of demand, the growth in output is a function mainly of supply parameters while in the latter, only demand parameters are important determining factors. Growth and sturctural change in an economy are, however, influenced by both supply and demand characteristics of factor and product markets. In the recognition of this fact, Kelley et al. (1972) propose a model of a dual economy within the framework of neoclassical assumptions. It is a generalization of Jorgerson (1961) and Fei and Ranis (1961) models and it is specifically adapted to low-income, developing economy characterized by dualism in production, consumption and demographic behavior. Kelley et al. incorporate the existence of rural-urban differences in consumer behavior, demographic behavior and production conditions in both their static equilibrium and disequilibrium models, thus providing a strong empirical evidence of rural-urban dichotomy. Furthermore, their models investigate inter-sectorial movement of factors from sectors of low productivity to those of higher productivity. A comparison of the results obtained from the two models highlights the relevance of disequilibrium

specifications of the growth process in developing economies. A simulation technique is employed in generating the line path of the endogenous variables with parameter values obtained from outside sources. For any given set of parameter values and initial conditions, however, a variety of time paths can be generated for the endogenous variables, thus limiting the possibilities for exploring further the qualitative properties of the system. Other limitations in the application of Kelley et al. approach include their assumption of a closed economy, whereas foreign trade is a very important sector in many developing economies. Furthermore, their models share in a criticism common to most development models in that they have not given an explicit recognition to the social welfare of the society in the determination of the policy decisions.

The conclusion drawn from the alternative models discussed above is that it is not easy to develop a single model that will capture all the relevant characteristics of an economy. The choice of a policy model will depend very much on what policy questions are to be answered. Irrespective of whether we give more emphasis to demand or supply characteristics, however, a development model should provide an explicit specification of preference function of the policy makers as Timbergen and Theil have done. Above all, for the model to be of any practical use, it should be relatively simple and analytically tractable.

NATURE OF PROBLEM

The suggestion is that within the framework of Theil's approach, Nigerian planners can evolve optimum economic policy decisions. Theil amongst others has stressed the need to specify a social welfare function in the steering phase of a development plan. The strategy is then to maximize (or minimize) this social welfare function subject to the constraints of the economy which he suggests should be of the form:

$$AY + BX = 0 \quad \ldots\ldots (1.6)$$

where,

A is the coefficient matrix of the endogenous variables (targets), B is the matrix of the predetermined variables (including policy instruments), Y and X are the corresponding vectors of variables. The argument is that if we simultaneously take account of the impact of the instruments on the targets and the social welfare, we inevitably select optimum policy instruments.

The issue on the use of a social welfare function for economic policy formulation has generated a lot of interest and controversies. As these controversies involve very substantive consequences, these controversies about a social welfare function must be understood if we are going to use it for analysis. Even when we are decided as to a welfare function, a justification for adopting a

particular structural model or constraint for Nigeria is needed. The estimated structural model may not be satisfactorily verified.[7] The possibility of this suggests we seek further information on the estimated model by determining whether the model's optimal economic behavior, with respect to some specified criterion function, is consistent with accepted tenets of economic theory. In particular, we need to investigate the dynamic properties of the estimated structural model prior to its utilization for the study of economic policy.

The actual form and representation of a structural model for Nigeria is not known. Moreover, the economy is conceivably characterized by frequent structural changes. As a consequence there is, in particular, uncertainty with respect to the values of the parameters of the structural model and, in fact, the structure itself. A sequential analysis of the Nigerian data as more information becomes available is, therefore, suggested. Furthermore, the need to design a flexible plan which has a learning process of adapting to changing circumstances cannot be over emphasized. Inventory theory[8] has indicated that a policy which incorporates the future possibilities of learning (and adjustments) will be qualitatively different from one which ignores them.

The general problems mentioned above are in the context of multiperiod forecasting where policy decisions

taken in one year affect the decisions of subsequent future years. The dynamic implication is, therefore, that optimization of policy decisions should be with respect to the overall planning horizon. Suppose we characterize the Nigerian economy by the set of linear equations of the form (1.6). We further assume that the policy makers' preference function can be represented by a quadratic loss function of the form:

$$Q = [y, z]'F[y, z] \qquad \ldots .(1.7)$$

where,

y and z are deviations of actual from desired levels of targets and controlled variables respectively, and F is a positive semi-definite matrix. The problem thus simplifies to maximizing the quadratic loss function Q subject to the linear constraints (1.6). This is equivalent to selecting optimum policy instruments which will maximize the derived target variables for each of the plan period. The instruments will be optimal in the sense that we simultaneously take account of the impact of the instruments on the non-controlled variables and the loss function of which both the targets and instruments are arguments.

THE PLAN AND ORGANIZATION

In Chapter II we analyze some of the controversies surrounding the use of a social welfare function in economic policy formulation. The justification for adopting

a particular structural model for Nigeria is also discussed. Chapter III discusses the sources of data and the estimation procedure employed for the structural model. In Chapter IV we investigate the dynamic properties of the model as estimated by the classical methods. The solutions to the economic planning problem and problems of computing optimal plans are discussed in Chapter V. The deterministic and Theil-type certainty equivalence solutions to the policy problem are taken up in Chapter VI. In Chapter VII, an alternative sequential estimating procedure suggested by Zellner (1971) and others is explored. We conclude by discussing the comparative use of the above techniques in the context of application to the Nigerian situation.

CHAPTER II

FORMULATION OF THE ECONOMIC POLICY MODEL

The existence, form and estimation of an aggregate social welfare function is discussed in this Chapter. A quadratic criterion function is recommended for use in the Nigerian study. Further, we develop a simple linear structural model (non-stochastic) for the Nigerian economy. This structural model is then integrated with the criterion function suggested as a basis for the theoretical formulation of an optimum economic policy decision.

THE GENERAL OBJECTIVE FUNCTION

In economic theory, we often assume a utility or preference function which depends only on the individual's own consumption of goods and services. Further, a "rational" consumer, is taken to maximize utility subject to his budget constraint [R.G.D. Allen (1959); Samuelson (1947)]. In translating this optimum behavior of a consumer to the national level, it is argued in Chapter I that the policy makers should, in order to attain an optimum economic decision, maximize a social welfare function subject to the constraints of the economic system. The problem then is whether such an aggregate

preference function for the community as a whole exists. If it does exist what is its relation to the individual welfare function and how is it characterized. Questions of the rational for arguments entering into the function and the form of the preference function also arise. Finally, there is the problem of estimating the functional relation amongst different arguments. That is, we need to know how to establish the weights or coefficients associated with the variables included as arguments of the preference function.

In Chapter I, we argued in favor of using Theil's approach to economic policy formulation. This approach is, however, often criticized because of the use of an arbitrary preference function. Such criticisms are mainly based on the questions of the appropriateness of aggregate social welfare function. In this Chapter, we propose to explore the various arguments and proposals that have been presented in the literature regarding these questions.

The Theory of Social Preference Functions

The problem of measuring and aggregating the utility of individuals is immense. Arrow's (1951, 1963) results are usually cited as the basis for not engaging in such a measurement or for that matter the optimization of models involving aggregate preference functions. Based on assumptions of individuality, Arrow explores the possibility of constructing a social ordering which will, in some way,

reflect all individual orderings of the members of a society. Clearly if we could find a way of passing from individual orderings to a social ordering, the task of specifying a welfare function would be greatly simplified. Arrow's conditions rule out social orderings that do not reflect the individual orderings of the members of the society, that give more weight to the orderings of one member than another or that are imposed upon the society by force. In his "Social Choice and Individual Values", Arrow seeks to construct a social welfare function based on value-free axioms but only succeeds in proving that any attempt to construct an innocuous (i.e. value-free) social ordering for three or more individuals will fail.

Arrow's work has thus served to cast doubt on the existence and relevance of measuring social welfare functions. Moreover, his work has been frequently cited by individuals not inclined to use optimizing models in applied policy analysis [Naylor (1969)]. Much of the dissatisfaction with Arrow's formulation of the problem of social choice rests on Arrow's imposition of the controversial "Independence of Irrelevant Alternatives" condition. Arrow's theorem, as revised in 1963 [1972 reprint; pp. 26-28], asserts that the Independence of Irrelevant Alternatives is inconsistent with universal domain (i.e. no restriction on individual preferences), non-dictatorship, and the principle that unanimity prevails.

The independence condition requires that social ranking of a set B of alternatives shall be fully determined by the individual rankings of B, and thus shall be independent of individual opinions concerning the "irrelevant" alternatives not in B. The main objections to this condition, and the other axioms are that they prevent the collective-choice rule from responding to intensity of individual preferences or interpersonal comparisons of utility. Moreover they ignore all but discrete rankings of preferences, thus making the choice rule unacceptable on a priori grounds [Kemp and Asimakopulos (1952); Hildreth (1953); Rothenberg (1961); Coleman (1966); Qirk and Saposnik (1968); Sen (1970); Kelly (1971); Blau (1971); Rausser and Johnson (1973)]. The general contention is that relative preference intensities are a vital characteristic of the choice-making process and that if properly taken into consideration, the possibility of obtaining surrogate preference functions is enhanced.

Hildreth has suggested the introduction of a cardinality concept as a way out of the impasse created by Arrow's axioms. The idea is that if Uninibile only barely prefers x to y whereas Itiafa desperately prefers y to x, the social system should rank y over x. Rothenberg in his contribution, postulates the "Pertinent Value Consensus" (PVC) view of public interest. He contends (1969, p. 29) ". . . it may be that interdependencies

between people and their interactions are more important than individual value structures . . . individuals voluntarily yield certain coercive power to a government which is somehow charged to discover, articulate and implement social priorities, or collective wants." This view, of course, assumes the existence of a decision-making group, elected by concensus, charged and always pursuing the 'good interest' of the society as a whole. If there exists such a group, then the focus should be on this smaller group for determining the prevailing values of the society. The existence of Rothenberg's pertinent value consensus means the estimation of an aggregate social welfare function is possible. As Rausser and Johnson have pointed out, however, "the resulting function, to provide a useful guide to decision making, should be consistent, permit the ranking of a large proportion of the policy alternatives, and be relevant in the sense of accurately portraying group values."

More recently there have been other attempts to dispel some of the gloom generated by Arrow's General Possibility Theorem. In his illuminating work, Sen (1970) evolves a 'possibility theorem' in terms of choice functions as against preference orderings. He shows that Arrow's 'impossibility' theorem about Social welfare functions does not carry over to social decision functions and that we can indeed find a social decision function

satisfying all the conditions of Arrow (unrestricted domain, Pareto principle, independence of irrelevant alternatives, and non-dictatorship). A preference ordering satisfies reflexivity, connectedness, and quasi-transitivity properties. The crucial difference, therefore, between Arrow's approach and Sen's approach is the requirement of full transitive and quasi-transitive property respectively. Inada (1970) has argued that quasi-transitivity is just as a rational individual behavior property as transitivity, although this fact has often been ignored. Quasi-transitivity is satisfied if an alternative x is more preferable to another alternative y, and y is more preferable to z, and x is more preferable to z. Then the following situation is not excluded: x is indifferent to y, y is indifferent to z, but x is preferable to z. Suppose the difference between x and y, and that between y and z are so small that they are not discernible, then it is possible that the difference between x and z is also so small that it is not discernible. In the case that the difference is large enough to be discernible, intransitivity of the indifference relation may arise. This type of choice does not reflect an irrational behavior on the part of the individual but rather shows that the ability of human beings to discern a small difference is imperfect. Thus, choice has stochastic components [Barten (1968)] which introduce fluctuations. The role played by these

stochastic components is important since their presence reduces the precision in choice.

The significance of a quasi-transitive social weak preference relation arises from the fact that the existence of Sen's choice function rests on the former. The existence of a choice function is defined by Inada (1970) as: for any finite non-empty subset of alternatives, if there exists at least one alternative preferred or indifferent to all other alternatives in the set we say that a choice function exists. Sen (1970) demonstrates that s sufficient condition for the existence of a choice function is quasi-transitivity. Earlier, Sen and Pattanaik (1969), and later Fishburn (1970), and Inada (1970) had established a sufficient condition for the quasi-transitivity of the social weak preference relation.[1]

More recently, there have been attempts by Fishburn (1970) Blan (1971), and Campbell (1973) to rescue Arrow's theorem from its critics. Fishburn, for instance, demonstrates that the only condition in which Arrow's other conditions are simultaneously compatible is if we allow infinite number of voters, and that as long as we have a finite set of voters, Arrow's theorem stands. In his own contribution, Blan explores the consequences of weakening the controversial "Independence of Irrelevant Alternatives" condition. This condition had been criticized as being too strong, and giving rise to the inconsistencies

in Arrow's theorem. When the independence condition is removed, Arrow's other conditions are consistent. Blau seeks a middle-ground by replacing the binary property of the controversial condition by ternary, and attempts to derive a ternary social welfare function satisfying Arrow's other conditions. His result is Arrow's conclusion. Hence to reject Arrow's results, the"Independence of Irrelevant Alternatives" condition has to be rejected entirely.

Campbell defends the above condition against the objection that it prevents the collective choice rule from responding to intensity of individual preference. He argues that this objection does not hold if intensity of preference is measured by an ability to make side payments in order to influence collective decision making.[2] Campbell shows that a modified version of this condition which takes explicit account of intensity of preference actually implies the original "Independence of Irrelevant Alternatives" condition for realistic environment sets.

The conclusion that emerges from these debates, based on the weight of the arguments, is that if we are prepared to make some non-trivial value judgements, we can find an aggregate social welfare function or surrogate which can be adequately measured. In particular, the specification of such an aggregate social welfare

function is determined by the problem we are dealing with. The function should, at least, have significant welfare connotations for different members of the society. In practical settings, we often fall back on simple specifications which are thought to provide reasonable approximation, in order to facilitate mathematical tractability.

Specification of Explicit Functional Forms

Variables entering as arguments of the welfare function should be those closely related to the variables considered important by the policy decision makers. Such variables should also be closely related to the economic welfare arguments of the groups affected by potential policy changes. Many[3] have argued that only the ends (targets) should enter into the welfare function. While others[4] favor the inclusion of both the ends and the means (targets and instruments). The contention of the latter is that the means and the ends are not always "neutral", that is, free from value judgements, and hence the means, and the ends may not actually be separable. Our position in this controvery is dictated by operational considerations. The inclusion of both targets and instruments as arguments of preference function is suggested. If some instruments in the function are relatively unimportant for welfare directly, we can always assume they have zero weights. This approach may, however, increase the already difficult problem of specifying the parameters of the social welfare

function.

Once we have determined the variables to be included as arguments of the aggregate preference function, the problem of functional form arises. Specification of an appropriate mathematical form involves formalizing assumptions regarding the substitutions between individual arguments. Additive, multiplicative, and discontinuous mathematical forms have been suggested. Van Eijik and Sandee (1959), for instance, suggest the use of a linear welfare function. Such a function implies a constant marginal utility and hence, constant marginal rates of substitution between the targets and the instruments. Theil (1968) amongst others has severely criticized a linear preference function mainly on ground that we ordinarily expect a decreasing marginal rate of substitution. Alternatively, Theil suggests the use of an explicit quadratic welfare (loss) function. Apart from yielding an expected decreasing marginal rate of substitution between the targets and instruments, it is relatively simple function which has a long standing tradition in applied mathematics and statistics. The concept of minimum variance and mean squared error are classic examples of applications of quadratic loss functions. A quadratic loss function also facilitates a sensitivity analysis with respect to variances in particular variables. On the other hand, however, a quadratic function suffers

from the disadvantage of assuming discrepancies of the same size but of opposite signs identical in consequences.

In a recent paper, Encarnation (1971) argues for a form of social welfare function which appears operationally more useful than Bergson-type or Theil-type functions. His hypothesis is based on the concept of lexicographic preferences which permits a natural representation of multiple social objectives in some order of importance or priority.[5] A social choice can be regarded as resulting from a series of separate decisions regarding social values and objectives. Encarnation's hypothesis implies that each separate decision can be determined by a simple majority rule. In the case when we have multiple goals Rausser and Johnson (1973) suggest the use of a scalar valued function to amalgamate the various dimensions of utility. This approach has computational appeal.

The usual assumption and use of continuous social preference orderings [e.g. Koopmans (1950) has employed a utility function constrained by a uniform continuity condition] has been challenged by Kelly (1971). He draws attention to the serious difficulties in the way of having continuous representations even when the preference aggregation conditions are met. He argues that continuous representations require severely restrictive set of conditions citing simple majority voting (single aggregation procedure) and the severe conditions that will guarantee a

social preference ordering with a continous representation as examples. Arrow's single-peaked preferences might ensure the existence of a social preference ordering but such a social ordering is not necessarily continuous. Kelly suggests the use of social preference functions which are neutral between generations as earlier suggested by Ramsey (1928).

Time preference presents additional complications in the specification of the mathematical form of a preference function. To begin, there are largely unresolved questions relating to social time preference rates and discount factors. The usual way of solving the time dimension problem is to incorporate it in an additive form by assuming:

1, a constant rate of marginal substitution between outcomes at different dates in the planning horizon; and

2, the preference relationship for each period to be stationary and independent of realized outcomes in previous periods. Some of these assumptions are unrealistic, in particular, the latter, but we gain in computational ease without seriously compromising the quality of our results.

The length of the planning horizon presents an additional problem in the specification of the mathematical form of a preference function. For practical purposes, however, the period preference functions over the entire

planning horizon, and the time preference discount factor are specified and estimated. The results for such estimates are readily obtainable in the simple one period case using "certainty equivalence" approach Theil (1968) . In the multiperiod case, however, allowance has to be given to the fact that present estimates affect estimates obtained in the future. In this case sequential and adaptive control results are probably more superior to certainty equivalence results.

The choice of an index of comparison between levels of preference function does not present a serious problem. Absolute levels of preference functions are not required, that is, it is not necessary to specify the preference function in cardinal terms. Normally a function which is unique only up to a linear transformation will serve adequately as an index for ranking alternative economic outcomes.

Operational Treatments of the Welfare Function Problem

Turning to the quantification of the coefficients associated with the variables in the arguments of the preference function, we find as in the case of an econometric model, a large number of techniques that can be employed. At times, a highly arbitrary method may be required to determine the values of these coefficients. In general, there are three approaches which might be considered in attempts to capture these coefficients.

These are the explicit, the implicit, and the arbitrary methods. In practice, however, some combination of these approaches might be utilized.

The explicit approach involves interviewing techniques to determine preferences. Gergen (1968) suggests a method of sequential interviewing of policy decision makers to determine the main issues of concern, and the individuals or groups who seem likely to significantly influence the final outcome of the bargaining process. The interview technique faces many problems. One of such problems is well expressed by Naylor (1970, p. 264). He contends that political decision makers are often not prepared to reveal their preference weights in detail since a successful bargaining in the political process places a premium on not revealing one's true preferences. Moreover, decision makers' perceptions of their preference functions as well as their choice possibility sets may be imperfect and may change in response to new information obtained during the bargaining process. In addition, the interview procedure is costly and it may be difficult to obtain access to central decision makers.

The implicit approach isolates the values of parameters of the preference function from historical data generated by past decisions. In this approach, it is implicitly assumed that previous decisions do reveal

the existing and future preferences of the policy makers. On the basis of these, Nijkamp (1970) has developed a procedure which, given the mathematical form and arguments of the preference function, the structural equations relating the endogenous to the controllable variables, infers the unknown coefficients of the preference function from actual decisions. Nijkamp's procedure has been critized by Rausser and Johnson (1973) on grounds that it presumes the policy maker is a rational and consistent preference function maximizer, that his preferences are stationary, and that the policy maker and the investigator have in mind the same structural model.

Alternatively, the investigator could specify arbitrarily the coefficients or preference weights. This approach incorporates the imaginary interviewing procedure suggested by van Eijk and Sandee (1959). "All available knowledge of private and public utterances of members of the government or its advisors must be used. Furthermore, one must interpret the political relations in parliament and in the Social Economic Council. In short, the presumable outcome of a real interview must be forecast." This arbitrary approach has an appeal. It is simple and inexpensive. Procedures of this sort have been utilized by Fromm (1969), Holt (1962), and Theil (1968). But it suffers from the disadvantage that two investigators could come up with very different

estimates. Moreover, the weights are only partially determined by imaginary interviewing because of the assumption of a separable preference function and the possibility of statistical bias in the method of interview.

To summarize, we emphasize the need to employ a preference function or surrogate in the analysis of policy decisions, while recognizing the difficulties involved in the construction of an aggregate preference function. In the words of Fromm (1969): "At this time, I would not advocate the rigorous application of utility functions for the evaluation of policies. Nevertheless, employing them in a limited fashion, especially when a range of arguments and weights are used, is helpful in acquiring perspective on the relative desirability of alternative policies". The mathematical form employed would depend on the assumptions made about the nature of marginal utility of the arguments. Both targets and decision variables should enter the argument of the preference function. When we have multi-dimensional goals, a scalar valued preference function is suggested where possible because of its simplicity in computation and presentation. We can incorporate time dimension in an additive fashion, while establishing the preference weights by "informed" arbitrariness would yield good results.

THE STRUCTURAL MODEL

A structural model forms the second segment of a macro economic policy analysis of the Theil type. The problem of constructing a structural model to simulate the entire economy of a country is immense. Such a model reflects technological or institutional constraints and behavioral relationships which exist in the economy. Further, it indicates the adjustment processes which occur when there is a disequilibrium in a particular market (such as markets for labor, money, etc.) and includes identities or definitions. The equations of the model bear these characteristics and are called structural equations. That is, each of these equations is supposed to describe some well-defined part of the structure of the economy. Thus the model embodies a theory or a set of theories as to the functioning of the economy.

K. A. Fox and E. Thorbecke (1965) have emphasised the technological basis for an important class of structural economic relationships. Firstly they argue that since from knowledge of interindustry flows technological change has a strong influence on the value of the input-output coefficients, there is need to update these coefficients in the light of such changes. Otherwise policy decisions derived with the aid of these coefficients will not be optimal; that is, there is a welfare implication of changes in technology and hence in quality of the coeffi-

cients. Secondly the input-output mechanism, is also involved in transmitting impacts from one sector of the economy to another over time. For instance, a technological innovation in a sector (say agriculture) of the economy would eventually affect other sectors of the economy (say industry).

Behavioral relations also play a crucial part in defining the structure of an economic system and the way in which changes originating in one sector are propagated into other economic sectors, both immediately and overtime. Essentially, the relations contain quantitative theories and hypotheses about empirical economic behavior.

Recursive Vs Non-recursive Structures

Variables and parameters make up the structural equations. However, organizing the variables into structural equations, (specification of the structural model) raises a number of fundamental questions. Firstly, it raises the question of whether the set of economic relationships to be included in the model is of a recursive type or an interdepencent (simultaneous) type. Structural equations which show a one-way flow of determination or causal influence are said to be recursive. By proper arrangement of the relationships a triangular matrix of coefficients, relating current values of the endogenous variables to one another, can be obtained so that only zero elements appear above the main diagonal. A causal

chain model developed by Wold (1957) has the character that the coefficient matrix of the endogenous variables is completely triangular with unit elements in the principal diagonal. Following Wold, a Purely Causal Chain (PCC) can be written as:

$$A_o Y_t + BY_{t-h} + CZ_t = U_t \qquad \ldots\ldots (2.1)$$

$h = 1, 2,$

where,

Y_t is column vector of endogenous variables, Y_{t-h} and Z_t are column vectors of lagged endogenous and exogenous variables respectively, U_t is a vector of residuals. The coefficient matrix A_o is assumed to be strictly triangular with unit element in the principal diagonal. The recursiveness in the coefficient of A_o allows one to treat in each equation the Y_t with the unit coefficient as the effect variable, and Y_{t-h} and Z_t as the cause variables.

The existence of causal relationships within and between the set of structural equations has a very important operational implications from the standpoint of the formulation of economic policy and the estimation procedure. Firstly, the behavior of the i^{th} decision-making unit can be given a stimulus-response or cause-effect interpretation in terms of the i^{th} equation of the system only. Secondly, predictive inference is always

possible in terms of the parameters of either the original system or the reduced form. The structural relations take the form of conditional expectations assuming that the residuals in each equation (whether lagged or non-lagged) are uncorrelated with the regressor variables and also uncorrelated mutually. Under such conditions, the estimation procedures are very simple.[6] A variant of PCC is 'vector causality' developed by H. Simon (1953). If in equation (2.1) the vector Y_t is partitioned into column vectors such that the coefficient matrix A_o can be conformably partitioned into a block triangular matrix, Simon showed that we have a relation of asymmetry or precedence in time that has a causal characteristic. Wold's concept of causal chain is more strict and explicit and may be viewed as a special case of Simon's concept of causal ordering in terms of vector causality.[7]

The issue of recursive versus interdependent structural equations has generated a number of controversies in the literature. Tinbergen (1952), Wold (1954, 1960), and Bentzel and Hansen (1954) have argued that structural models are, in general, of recursive nature. Wold, the main advocate of this view, conceptualizes a 'basic model' in which complete and explicit regard is taken of all actions of each single economic agent in the society. If the periods are sufficiently short to correspond to the actions of the economic agents, he

argues, this type of model is always recursive if it is constructed on the principles of the 'disequilibrium' method of the Stockholm School. On the other hand, T. Haavelmo (1943), amongst others, has justified the simultaneous character of the relationships in economic theory. His argument rests on the lack of knowledge of empirical values of ex-ante variables, the necessity of aggregation and simplification of different types to overcome the difficulties that arise from the fact that the time-periods of the empirical material at hand do not coincide with the unit-period of the dynamic theory. Other important sources of inter-dependency in this 'derived model' are the static equilibrium assumptions of the economic systems such as we have in Walrasian or Keynesian systems. These classical equilibrium systems are characterized by a complete inter-dependency between the variables.

The debate of recursive versus simultaneous is far from settled. In many cases, the choice of the model depends on what the model purports to explain and on the use to which it will be put. But if the case for simultaneous character of the relationships of economic theory rests on firm grounds, the ordinary least squares method for estimation of the set of equations cannot be relied upon. And if aggregative variables are used to describe the behavior of the economic system, an acceptable theory

of aggregation must be developed for the system, relating micro economic variables to their aggregative counterparts. Hurwicz (1952), Theil (1954), and Fisher (1962), have suggested solutions to this so-called aggregation problem in economics. Hurwicz, for instance, suggests using statistical decision theory as a guide to optimal aggregation procedures. Theil develops a normative concept of 'perfect aggregation' which is essentially a proposal to adjust the aggregating weights so that perfect consistency prevails between micro and macro models. Fisher, on the other hand, has developed a criteria for aggregation based on the notion of minimizing expected risk, such as minimizing bias in a macro-variable.

The second question that arises in the process of specifying a structural model is whether the relationship between the variables in the model is linear or non-linear (in parameters). It is often argued that a realistic structural model should be non-linear, citing non-linearity arising in the frequently used price times quantity terms in the identity defining GNP in current prices as an evidence. Sengupta (1965) advances basic advantages in utilizing a model with non-linear equations. Such a model, he argues, offers a wide range of choice of possible paths for a given linear relationship which is useful for analyzing economic instability in the form of cyclical fluctuations. Further, a non-linear model has the capacity

to shed considerable light on the sensitivity of solutions of the corresponding linearized model in respect to the stochastic process underlying the model. A non-linear model, however, presents problems in the context of structural specification. One of these problems is that there is not, in general, a one-to-one correspondence between the structural and the reduced form models. That is, the system cannot be solved uniquely for the endogenous variables in terms of the exogenous variables, lagged exogenous variables, and error terms. Further, the expected value of a particular endogenous variables as calculated from the structure will not be the same as its expected value calculated from the reduced form.[8]

A static model and a dynamic model present alternative specification of the structure of an economic system. A model is called dynamic if either at least one observable variable occurs in the structural equation with values taken at various points of time or at least one equation contains functions of time. A classic example of the latter is the treatment of capital stock as the sum of net investments in the past. Most structural models are dynamic. The reason for this popular choice of specification rests on the fact that a dynamic specification enables the checking of the nature of the time paths of the endogenous variables of the model. We need to know the stability and behavior of these variables in their

movements towards equilibrium before utilizing the model for policy formulation.

The three questions raised above are just a few (albeit important) questions which an investigator has to answer in the process of specifying a structural model. The choice amongst these alternative specifications is not unique. He can decide, for instance, on a linear dynamic simultaneous model specification. Furthermore, the number of equations which he considers adequate representation of the economic system will depend on a number of considerations including data availability, the level of aggregation he can tolerate, the ease of computation and, above all, the use to which the model is to be put. Where computer is available, the mere size of the model is no longer a serious barrier to the effective application of a large and detailed model but for purposes of exposition, a smaller, simpler model may be preferred.

A Structural Model for the Nigerian Economy

What we have discussed so far has been succintly expressed by Popper (1959, p. 109) who said "the choice of any particular theory is an art, a practical matter. The choice is decisively influenced by the application of the theory and the acceptance of the basic statements in connection with this application." Encouraged by Popper's words, we postulate a simple highly aggregative linear dynamic model for Nigeria. The model is, however,

sufficiently specific for the purposes we have in mind. The model in the deterministic form is:

$$C_t = \alpha_1 + \alpha_2 C_{t-1} + \alpha_3 GNP_t \quad \ldots\ldots(2.2)$$

$$I_t = \beta_1 + \beta_2 I_{t-1} + \beta_3 G_t \quad \ldots\ldots(2.3)$$

$$M_t = \gamma_1 + \gamma_2 M_{t-1} + \gamma_3 G_t \quad \ldots\ldots(2.4)$$

$$E_t = \phi_1 + \phi_2 \Delta YB_t \quad \ldots\ldots(2.5)$$

$$GNP_t = C_t + I_t - M_t + E_t + G_t \quad \ldots\ldots(2.6)$$

Where C, I, M, E, GNP are endogenous variables representing Private Consumption, Gross Domestic Capital Formation, Imports, Exports, and Gross National Income respectively, G is Federal, State and local government current and capital expenditure. YB is the net national income of Britain, G is a controlled variable, while YB, although an exogenous variable, is outside the control of the policy makers. α, β, γ, ϕ are unknown parameters to be estimated.

The system of equations (2.2) to (2.6) represent a Keynesian type model, and it is characterized by an open economy with an active public sector. The Nigerian structural model is linear in parameters, and the equations are inter-dependent in character. The model is also dynamic, although not in a unique way; that is, different lag structures are plausible.[9]

Equation (2.2) expresses the present private consumption expenditures as a function of last year's private consumption expenditures and this year's GNP. This consumption function can be interpreted in terms of either Permanent Income Hypothesis or Life Cycle hypothesis. According to Friedman (1957), consumption for the economy is a function of permanent income, where permanent income is defined as discounted, expected income. Thus,

$$C_t = a_1 + uY_t + urY_{t-1} + ur^2Y_{t-2} + ur^3Y_{t-3} + \cdots + ur^nY_{t-n} \quad (2.7)$$

where,

r is the rate of discount, C and Y are consumption and income respectively.

Similarly,

$$C_{t-1} = a_1 + uY_{t-1} + urY_{t-2} + \cdots + ur^{n-1}Y_{t-n-1} \quad (2.8)$$

We apply Koyck's transformation (1954). Multiplying (2.8) by r and subtracting the result from (2.7), we have that:

$$C_t = (1-r)a_1 + rC_{t-1} + uY_t \quad (2.8a)$$

(ignoring the last term ur^nY_{t-n})

We can rewrite (2.8a) as,

$$C_t = \alpha_1 + \alpha_2 C_{t-1} + \alpha_3 Y_t$$

Where,

$$\alpha_1 = (1-r)a_1, \quad \alpha_2 = r, \quad \alpha_3 = u$$

Modigliani and Brumbery (1963) suggest that consumption is a function of current and expected incomes and assets. Thus,

$$C_t = \alpha_1' + \alpha_2' Y_t + \alpha_3' Y_t^e + \alpha_4' A_{t-1} \quad \ldots . (2.9)$$

Where,

C and Y are defined as in above, Y^e is expected non-property income, and A is asset or wealth. We can combine Y^e and A and define the result as discounted expected future streams of income. Note that at present Y^e is not directly observable, but we can approximate the discounted expected future streams of income as:

$$Z_t = \sum_{t=1}^{\infty} \frac{Y_t}{(1+r)^t} \quad \ldots . (2.10)$$

where,

r is discount rate.

Substituting (2.10) in (2.9) and performing Koyck's transformation, we have (2.2). The only major difference between the permanent income hypothesis and the life cycle hypothesis, therefore, is the concept of the time period that is relevant to the definition of permanent income or expected non-property income and wealth. The two hypotheses attempt to build a simple model of rational consumption or saving behavior by assuming that the consumer attempts to smooth out (though not necessarily equalize) consumption over a planning period. In the Nigerian case, it should be

qualified that, for many individuals, we might have them consuming all their income at any time period. This would, in particular, be true of subsistence families. For the economy as a whole, however, the aggregate consumption function is heavily weighed with cases in which the marginal propensity to consume is high (very close to one).[10]

A number of factors explaining capital formation have been suggested in the literature.[11] These factors include income, government budget, stock of capital and interest rate. In Nigeria, as in many other developing economies, there is notably a large governmental supported investment promotion. Also the problem is not a lack of demand for output. On the contrary, in most sectors, the problem is supply failing to meet the ever growing demand. It is, therefore, expected that the existence of capacity will promote new fixed investment rather than hinder it. Besides, there are as yet many opportunities for profitable investment in Nigeria. This fact, coupled with the absence of a well developed capital market, means that the distribution of funds is accomplished almost entirely through non-price rationing. We note that there is a high correlation between GNP and government spending in Nigeria. We have, therefore, excluded GNP as an explanatory variable in the capital formation demand function. A further rationale for retaining government

spending in the function is that we need it as a policy variable. It takes time to complete investment projects and today's capital formation reflects government spending at different times in the past. The current level of capital formation is, therefore, regarded as a function of last period capital stock and last period government spending.

For the foreign trade sector, we postulate that current import of goods and services is a function of last year's level of import of goods and services and government spending for last year. A lag of one period is assumed to take care of the lag between order and delivery of merchandise. We rationalize our assumption that government expenditure helps to explain substantial fluctuations in imports on the grounds that at all governmental levels, items such as vehicles, machinery and equipment, chemicals, are required to execute development programs. The level of exports is, to a large extent, determined by outside factors. One of such factors is the prosperity of the consumers of Nigeria's main export products. In 1960, about fifty percent of Nigeria's export went to Britain and in 1965, this percentage was about forty, with the European Economic Community becoming an important market.[12] As a proxy index of prosperity of importers of Nigeria's product, therefore, we have employed change in the level of

personal, disposable income of British subjects.

Equation (2.6) is an identity. It shows the components of GNP.

The structural equations (2.2) to (2.6) deal with the analysis of final demand. An important part of Nigeria's economic activity is, however, concerned with intermediate demand. The analysis of such demand is the subject of input-output of the type first seriously studied by W. Leontief (1953) and employed by Chenery to the economy of Southern Italy (1962).

The Nigerian model is similar in many respects to other econometric models such as Klein, Klein-Goldberger, and Suits models developed at a comparable stage of development. For instance, the five equation model of Nigeria compares very closely with six equations Klein's model. The Nigerian model has four behavioral equations and one identity, while Klein's model has three behavioral equations and three identities. Both are highly aggregative dynamic and are characterized by the conspicuous exclusion of the monetary and labor sectors. The two models differ from the Klein-Goldberger and Suits models in size. Klein-Goldberger has twenty-five equations while Suits model has thirty-two equations. The differences in size are due to two main reasons. Firstly, there are more detailed specifications in each of the sectors in the Klein-Goldberger and Suits models. Consumption, for instance,

is sub-divided into consumption of automobiles, of other durables, of non-durables and of services. Secondly, the models include a detailed specification of the financial and labor sectors.

Klein's model was developed in 1950 followed by Klein-Goldberger model in 1957 and Suits model in 1962. The degree of complexity of the model is in the same order, starting with the very simple Klein model. It is also important to note that each of the models reflect the stage of development and complexity of the U.S. economy at the point of their formulation. Thus, the Nigerian model could be improved to reflect the complexity as the economy develops. Perhaps the improvement could be in the direction of Taylor's model (1969).

THE COMBINED STRUCTURAL COMPONENTS

Earlier in this Chapter, it was argued that a social welfare function or surrogate exists if we are prepared to make some non-trivial value judgements. In particular, it was suggested that a quadratic social welfare function of Theil's type is adequate to represent the form of aggregate preference functions experienced in practice. The optimization principle suggests we maximize this criterion function subject to the constraints of the economic system. We have suggested a linear constraint represented by equations (2.2) to (2.6).

With a view to formulating the aggregate social welfare function, we observe that the policy makers will not probably realize the desired levels of the targets and instruments. There are, thus, discrepancies between the levels of targets achieved and their desired levels on one hand and between the levels of instruments employed and their desired levels on the other. These discrepancies constitute welfare losses, if we assume the actual levels of targets and instruments are optimal levels. Thus, we re-formulate our problem as minimizing the welfare loss due to these discrepancies, subject to the constraints imposed by the economic system.

In terms of a quadratic welfare loss function and our structural model (2.2) to (2.6), the square of these discrepancies for one time period, i.e.:

$$Q_t = (C_t - C_t^*)^2 + (I_t - I_t^*)^2 + (D_t - D_t^*)^2 + (GNP_t - GNP_t^*)^2 + (G_t - G_t^*)^2$$

Where the starred variables represent the desired levels of the respective variables, and $D_t = E_t - M_t$, characterize our aggregate social welfare (loss) function for one time period. This criterion function does incorporate certain general targets of economic policy of Nigerian planners. The general targets include immediate welfare which we measure by private consumption C_t, the future welfare measured by I_t (or GDCF), increase

in GNP and finally, the achievement of balance of payments surplus.

For a multiple period (T) problem, we sum Q_t over all time periods T. If P_t and Z_t represent the sums of squared deviations of the optimal targets and instruments from their desired level respectively, the multiple period quadratic welfare (loss) function is given as:

$$Q\ (P,Z) = \sum_{t=1}^{T} (P_t + Z_t)$$

In conclusion we note an important feature of our criterion function, that is, both non-controlled and instrument variables appear as arguments. This is in conformity with our earlier discussion in which it was argued that both targets and instruments should enter the arguments of a criterion function.

CHAPTER III

DATA, CLASSICAL ESTIMATION PROCEDURE AND RESULTS FOR THE NIGERIAN MODEL

DATA

Suitable data for constructing an econometric model may be difficult to obtain. Further, errors of concept and coverage in a statistical series can be a major source of inaccuracy in the estimation and use of econometric policy models. Hence data collection is an important aspect of policy formulation.

The data employed in the Nigerian model are all time series. Further, they are secondary. A time series is a set of data collected and reported at specific intervals, say monthly or annually. The degree of inter-correlation among time series observations on an economy in which population, real income per capita, prices and wages are all increasing may be so high that the estimates of structural coefficients obtained may be absurd. Information derived from other sources such as cross-section data could be built into the policy model in the form of imposed coefficients.

The aggregate time series employed in the Nigerian model include private consumption expenditures, Gross

Domestic Capital Formation (GDCF), Government expenditures, export and import of goods and services, Gross National Product (GNP), and the disposable income of British subjects. With the exception of the latter series, all other time series are given in millions of Nigerian pound (£N1 = 2 NIRA = $3.04). The disposable income of the British subjects is in millions of British sterling. All series run from 1951 through 1969 and are at current prices. The use of series at current prices has been necessitated by the lack of reliable price deflators for most of the series. Table 3.1 and Figure 3.1 show all the series employed in the study.

Time series data for private consumption expenditures are not collected as such but are estimated as Gross Domestic Product (GDP) plus import of goods and services minus export of goods and services minus GDCF minus Government expenditures. Thus, the reliability of consumption expenditure data depends on the reliability of its components and the procedure employed in the derivation.

GDP data are among the weakest of the Nigerian data employed. It's evaluation is at factor cost. That is, it is the value of the output excluding indirect taxes (such as excise duties). Thus value of GDP equals the total payment to the factors of production plus depreciation. There are differences in concept and coverage in

TABLE 3.1. THE NIGERIAN DATA:[a] 1950-1969

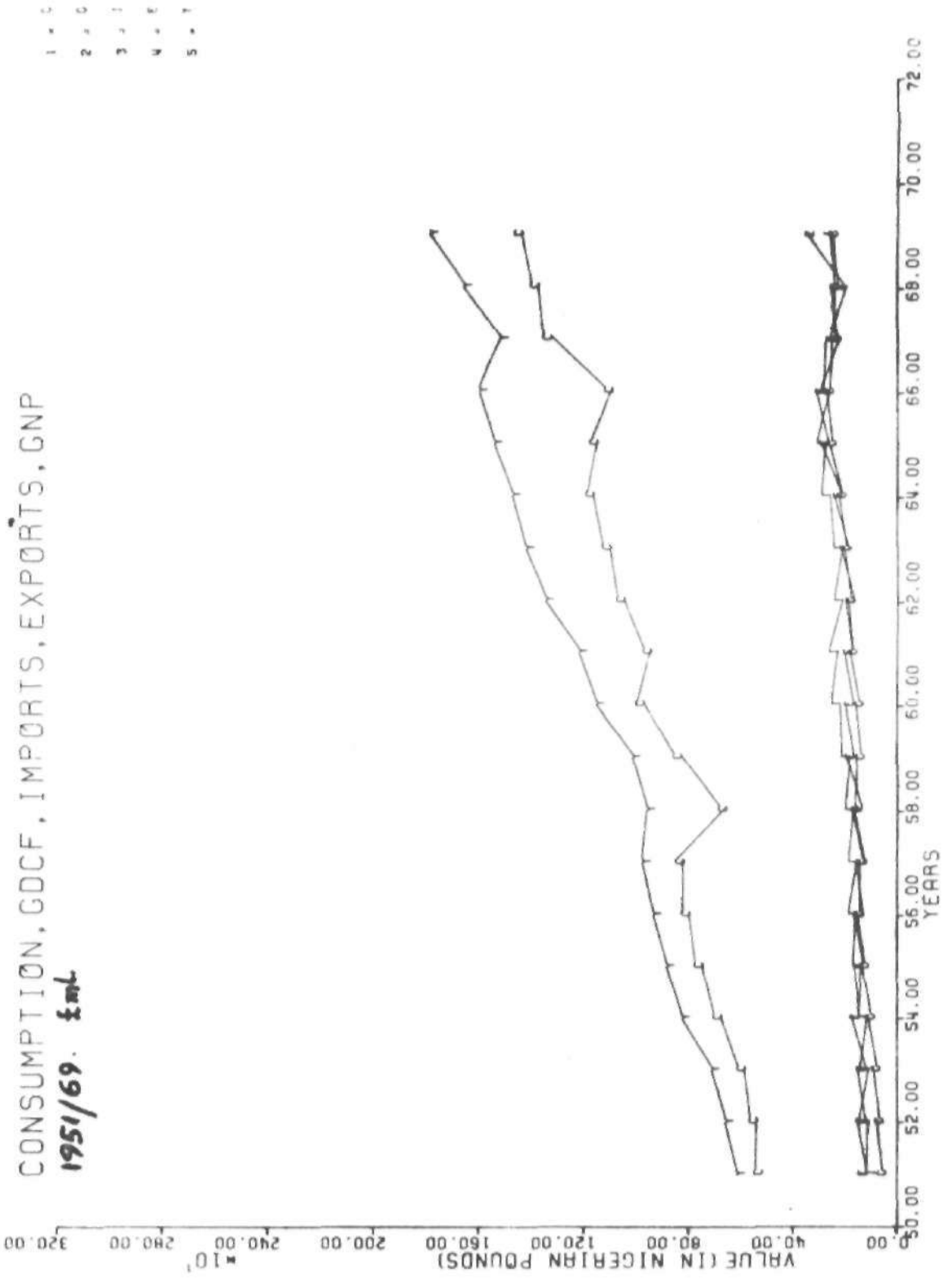
Year	Variables						
	Private Consumption	GDCF	Imports	Exports	GNP	Government Expenditure	ΔYB
1950	457.1	30.8	61.9	88.5	521.2	25.8	460
1951	514.9	41.8	84.6	116.6	582.5	33.9	760
1952	536.5	55.3	113.3	125.1	627.8	42.8	811
1953	580.3	63.6	108.3	120.9	674.7	58.1	747
1954	673.3	85.7	114.1	146.2	793.4	72.3	631
1955	743.1	108.8	136.1	129.8	850.6	87.5	1063
1956	792.7	128.3	152.8	132.3	900.9	90.0	995
1957	815.5	116.4	152.5	124.2	942.9	132.0	792
1958	650.9	146.4	166.4	135.6	925.0	133.4	637
1959	823.3	122.6	178.4	163.6	979.2	155.7	1019
1960	967.2	129.1	215.9	165.6	1117.5	190.9	1356
1961	938.2	152.5	225.5	170.1	1182.8	277.8	1305
1962	1041.8	159.8	206.0	163.9	1311.5	235.3	1104
1963	1095.7	177.0	207.7	184.9	1386.2	252.2	1301
1964	1158.6	195.0	253.9	210.4	1436.7	257.4	1622
1965	1144.7	234.1	273.4	263.2	1507.5	296.1	1634

TABLE 3.1.--CONTINUED

Year	Variables						
	Private Consumption	GDCF	Imports	Exports	GNP	Government Expenditure	ΔYB
1966	1087.4	242.6	257.0	278.7	1563.9	325.1	1488
1967	1321.9	219.4	223.5	238.1	1485.5	306.3	1143
1968	1364.6	223.2	193.2	206.5	1621.1	306.0	1797
1969	1428.8	228.6	248.7	319.7	1748.6	330.9	1851

[a]all variables except change in Disposable Income of British Citizen's (ΔYB) are measured in millions of Nigerian pound (£N1 = 2 NIRA = $3.04). ΔYB is measured in millions of British Sterling.

SOURCES: 1. Digest of Statistics, Vol. 20, No. 1, January 1971; FOS, Lagos, Nigeria.
2. Annual Abstract of Statistics, 1966, FOS, Lagos, Nigeria.
3. G. Helleiner: Peasant Agriculture, Government and Economic Growth in Nigeria.


CONSUMPTION, GDCF, IMPORTS, EXPORTS, GNP
1951/69. £m.
1 = C
2 = G
3 = I
4 = E
5 = Y
VALUE (IN NIGERIAN POUNDS)
YEARS
50.00
52.00
54.00
56.00
58.00
60.00
62.00
64.00
66.00
68.00
70.00
72.00
40.00
80.00
120.00
160.00
200.00
240.00
280.00
320.00

the series for the periods 1951 to 1957 and 1958 to 1969. These differences make the analysis of the GDP data difficult since the two periods are not really comparable. The 1951 to 1957 series are derived from Okigbo (1957) estimates while the 1958 to 1969 series are derived from the estimates of the Federal Office of Statistics, Nigeria (FOS) (1966). Okigbo estimates include data from Southern Cameroons (then a territory of Nigeria but now part of the Cameroon Republic) while this territory is excluded from FOS estimates. The derivation of GDP arising from agricultural sector has also presented quite a serious problem to National Accounts Statisticians in Nigeria. The problem assumes a rather high proportion in Okigbo estimates. It is not, therefore, surprising that many otherwise important activities, especially intra-household activities and land clearance by the Nigerian peasants were excluded from his estimates.[1] The first major agricultural survey was conducted during 1955 to 1960 period and it was from this survey estimates of the absolute level of non-exportable agricultural output in Okigbo study were based. Estimates from these surveys were not very reliable. Further, the population base for the projection of national aggregates was shaky. With over 50% of GDP derived from agricultural activities, therefore, the reliability of the 1951 to 1957 series is in doubt.

Improved coverage of activities by FOS since 1960 has increased the reliability of some of the data, notably those relating to non-exportable agricultural output, construction and manufacturing. The annual Industrial surveys conducted by FOS since 1963 have greatly increased the coverage and the quality of information available on Nigerian industries.

The problem of estimating capital formation in Nigeria is just as serious as the problem of estimating GDP. Although the problem has engaged the serious attention of many economists in and outside Nigeria, it remains largely unsolved.[3] GDCF is the expenditure of resources which results in the creation of new or improved physical assets meant for use in future productive processes. It includes the replacement of old physical assets but excludes repairs and maintenance which merely preserve the existing productive capacity. Physical assets include land (including agricultural and mining development), buildings, civil engineering works, plant, machinery and equipment and vehicles. The major problem areas in estimation are buildings, especially privately owned dwelling houses and agricultural land development. Most estimates have tended to ignore these sections as the easy way out of the problem.

GDCF series used also fall into two periods--1951 to 1957 estimates by Okigbo and 1958 to 1969 series by

FOS, with the FOS the more reliable because of its fuller coverage. The average annual rate of growth in GDCF for the period 1958 to 1969 was about 5% compared with its phenomenal rate of growth of about 30% in the period 1951 to 1957.

Perhaps the most reliable sets of data are the government expenditure by the federal, state, and local governments. Within the period of nineteen years covered in this study, there was a phenomenal increase in government expenditure from 25.8 million in 1950 to 330.9 million in 1969. As has been argued earlier the government of Nigeria plays a dominant role in economic development and this is reflected in its expenditures. A major item of expenditure not well reported in the time series between 1966 and 1969 inclusive is the military expenditure. Another weakness which is also true of all the Nigerian series employed is that there was no complete national coverage for the period 1966 to 1969 due to the internal crisis which affected data collection considerably. For instance, no data has yet been estimated for all the series for the areas physically affected by the Nigerian civil war.

There is also a high reliability on data on imports and exports. Fairly accurate figures of foreign trade sector have been kept since the British Colonial era. Favorable trade balances were reported in the period

1951 to 1954 and in particular during and immediately after the Korean War. This trade boom was, however, short lived. Between 1955 and 1965, Nigeria experienced trade deficit. 1966 was a turning point in Nigeria's foreign trade. Export of petroleum has since helped to usher in favorable trade balance.

ESTIMATION PROCEDURE

The deterministic specification of the structural model in Chapter II is incomplete for purposes of the estimation of the parameters. As Haavelmo (1943) has indicated, because of the very nature of economic behavior which depends upon enormous number of factors we cannot possibly take account explicitly for all the factors in practice. Thus we need a stochastic formulation to make simplified relations realistic enough for empirical applications.

Economic theory is concerned mainly with the properties of a deterministic specification of a model, while econometric analysis is concerned with both the properties of the 'exact' part and the stochastic aspect of the model. One way to make a deterministic model, such as the one given in (2.2) to (2.6) stochastic is to add error terms (U) to one or all of the behavioral equations (2.2) to (2.5). There are a number of other reasons why we need to add stochastic disturbance term to a macro

econometric model. As Howrey (1968) has pointed out, if the dynamic properties of an econometric model are to be investigated, "it may not be reasonable to disregard the impact of the distrubance terms on the time paths of the endogenous variables. Neither the characteristic roots nor the dynamic multipliers provide information about the magnitude or correlation properties of deviations from the expected value of the time path". Moreover, Howrey and Kelejian (1971) have demonstrated that, "the application of non-stochastic simulation procedures in econometric models that contain non-linearities in the endogenous variables yield results that are not consistent with the properties of the reduced form of the model". Finally, the inclusion of error terms enables us to replicate the simulation experiment, make statistical inferences, and test hypotheses about the behavior of the system being simulated. The analyst, although not knowing the values of these errors, is assumed to know the probability distribution of the random errors.

A number of assumptions about the probability distribution of the random errors are usually made in order to facilitate estimation. The 'classical' assumptions about these error terms are:[4]

1. $E(U) = 0$
2. $E(U_t U'_{t'}) = \Sigma, \; t = t' \quad t,t' = 1,2,\ldots, T$
   $= 0, \; t \neq t'$

3. $\text{plim } T^{-1}XU' = \text{plim } T^{-1}X_{-1}U' = \text{plim } T^{-1}Y_{-1}U' = 0$

where,

U is the error term, T is sample size, Σ is a positive definite matrix, X is a vector of exogenous variables, and Y is a vector of endogenous variables. Also U is often assumed to be a multinormal variate.

Based on the assumptions about the error terms, we then search for an appropriate procedure for estimating the unknown parameters of the model. In selecting amongst the available procedures, we should keep in mind that the estimation of the structural model is part of the total policy model, not independent of the decision making process. That is, the estimation of the structural model should be viewed as part of decision function theory and therefore should emphasize the optimization objective of a policy model. Thus the very purpose of estimation assumes a critical role [Fisher (1962)].

The unresolved issue of a recursive and simultaneity (inter-dependency) of economic systems was discussed earlier in Chapter II. It was emphasized that this issue has a far reaching statistical implications in respect of methods of estimating the unknown parameters of the structural model. Different estimation procedures may be indicated depending on whether the system is recursive or simultaneous. Haavalemo (1943), and Naylor (1971) for instance, have argued that if the system is inter-dependent it is not a satisfac-

tory method to determine each of the equations separately from the data without regard to the restrictions which the other equations might impose upon the same variables.[5] A number of methods are available for estimating the unknown parameters of the structural model.[6] These include, Ordinary Least Squares (OLS), Limited Information Maximum Likelihood (LIML), Two Stages Least Squares (2SLS), Three Stage Least Squares (3SLS), and Full Information Maximum Likelihood (FIML) (including their variants in restricted forms) methods.[7] OLS, LIML and 2SLS methods are employed on each of the equations of the model separately, hence they are called single equation methods. While 3SLS and FIML are known as simultaneous equation methods because in these methods all equations of the model are solved simultaneously. The classification of the estimation techniques into either single equation method or simultaneous equation method has a deep root in the identifiability of the parameters of the equations of the model.

The idea of identifiability refers to the issue of whether the model is appropriately restrictive so that when confronted with a sufficiently large sample of data just one hypothesis is consistent with both the model and the data. This concept is important and is in a sense logically prior to the issue of statistical estimation, because there is little point in attempting to estimate

structural parameters from a finite sample if even an infinite sample could not give the desired information about the parameters.[8] When a stochastic model is involved, limitations on the quantity and variety of the data used can permit more than one hypothesis to be consistent with both the data and the model. With only a finite sample, it is not possible to obtain exact knowledge of the probability distribution of the stochastic variables, because of sampling variation and because of the impossibility of covering the entire range of possible values of each variable. Therefore, with only a finite sample, we often cannot be sure whether a given hypothesis is consistent with the data or not.

Observational information cannot in general distinguish any equation of the model from any other linear combination of the equations of the model but can distinguish it from any equation not such a linear combination. Thus structural estimation is possible if and only if there is sufficient a priori information on the parameters of the model to distinguish a given equation from the set of all linear combinations of the model.

Suppose we have a general linear model of the form:

$$AX_t = U_t \qquad \ldots\ldots (3.1)$$

where,

A is an MxN matrix of constant parameters to be

estimated, X_t is an N-component column vector, and U_t is an M-component vector of unobservable disturbance term (with variance-covariance Σ) [Fisher (1966), pp. 23-24].

Let F be any MxN non-singular matrix. Premultiply (3.1) by F. Thus,

$$FAX_t = FU_t$$

or
$$A^*X_t = U_t^* \qquad \ldots . (3.2)$$

where,

$$A^* = FA, \quad U_t^* = FU_t$$

Observational information alone cannot distinguish equation (3.2) from the original equation (3.1).

Following Fisher (1966, p. 24) we define a linear transformation denoted by F as admissiable if and only if it has all three of the following properties:

1. F is non singular;
2. $A^* = FA$ satisfies all a priori restrictions on A; and
3. $\Sigma^* = F\Sigma A^*$ satisfies all a priori restrictions on Σ.

Then the i^{th} equation of the structural model is identifiable under a priori restrictions if and only if every admissible structure has as its i^{th} equation some scalar multiple of the true i^{th} equation. An equation which is not identifiable we say is under-identified. The i^{th} equation is over

identified if and only if there exists two different (but not necessarily disjoint) sets of a priori restrictions each of which just identify the i^{th} equation and the union of the two sets is an independent set of restrictions. Over-identification is, therefore, the situation in which there is more than enough independent information to distinguish the true equation from all others in the model. Such extra information is not, however, redundant, but does create estimation problems in the over-identified equations and this is the heart of all simultaneous equation estimations. This problem has been succintly expressed by Fisher [(1946), p. 46] who says "that the over-identification of any equation implies the existence of restrictions....on the parameters of each of the reduced form equations. Since indirect least squares estimate the reduced form without making use of such restrictions, it follows that the reduced form estimates so obtained will not in general be simultaneously compatible with all the prior restrictions on the structural equation to be estimated. The immediate consequence of this is that when some of these restrictions are used to go from the reduced form parameters to the structural equation in question, the structural parameters so derived will generally obey the unutilized restrictions." Fisher thus agrees with Haavalamo and Naylor on the importance of utilizing all the over-identifying restrictions in the

structural model.

The estimation of the parameters is not possible if an equation is under-identified. There are, however, ways of identifying an equation. Amongst such methods are linear restrictions on parameters and restrictions on variance-covariance matrix of the system [Fisher (1967)]. The choice of an estimation technique among the single equation methods and simultaneous equation methods is, thus, in the context of either just-identified, or over-identified equation. In the case of a just-identified equation (or equation systems), there is no preference among the various techniques since they all give the same results. Our discussion of the merits of the alternative estimation techniques will, therefore, be with reference to an over-identified equation.

The least squares method has the advantage of theoretical and computational simplicity, robustness and minimum variance among all methods discussed here. Its robust property is enhanced by the fact that its estimator is relatively less sensitive to the presence of multi-collinearity among the explanatory variables than either the 2SLS or LIML estimators, for instance. 2SLS and LIML methods have greater sensitivity to multicollinearity primarily because the parameters in the reduced form must be estimated first and then transformed back to the coefficients of the original model. The presence of lagged

endogenous variables as explanatory variables, however, destroys the BLUE property of the least squares method [Kendall and Durbin (1960)].

The LIML method developed by Anderson and Rubin in (1949) and (1950), provides estimates of the parameters of a single just-identified or over-identified structural equation on the maximum likelihood principle, the likelihood maximizing being done subject only to the prior identifying restrictions imposed on the equation being estimated. LIML estimators are consistent and asymptotically normal. An alternative to the LIML method for estimating an over-identified equation containing two or more jointly dependent variables-2SLS-was developed by Basmann (1957) and Theil (1958). The 2SLS method gives a consistent estimator.

The 3SLS and FIML are simultaneous equation methods. The 3SLS was developed by Zellner (1962) and it gives consistent estimators. It is efficient in that it utilizes all the over-identifying restrictions in all equations to derive the estimates. The FIML developed by Koopmanns, Rubin and Leipnik (1950) is consistent and asymtotically normal. Its efficiency also derives from the fact that all over-identifying restrictions of the model are employed in the estimation process.

The choice amongst the classical estimation techniques enumerated above is not clear cut. Apart from

the least squares methods, the techniques have largely only desirable large sample properties. This makes a comparison amongst the various techniques difficult since the small sample properties of LIML, 2SLS, 3SLS and FIML are not known yet. Whereas in the real-world situation we deal mostly with small samples.

There is limited evidence, however, about the relative merits of the different estimation methods when sample sizes are small. There is a set of evidences given by econometric studies of real-world data in which two or more estimation methods have been used for the same structural equation or equations. But in most cases, it is the least squares and LIML methods that have been used and again with no clear verdict. Comparisons of this kind cannot be expected to be conclusive, because the true values of the parameters are not known.

Monte Carlo experiments provide additional information about the relative merits of the different estimation methods when sample sizes are small. A Monte Carlo experiment is essentially an empirical technique of learning about the probability distribution of a statistic that one is interested in. It involves drawing a large number of samples from the population in question, computing the value of the statistic for each sample, and recording the empirically observed sampling distribution of that statistic. Results of Monte Carlo studies by

Wagner (1958), Basmann (1957), and Summers (1959) suggest that there will be many cases in which simultaneous equation methods of structural estimation will not give sufficient (if any) improvement over least squares to justify the extra cost involved in their use. As the services of high-speed computers become available, however, the cost advantage of least squares technique will diminish.

Some significant points have emerged from the debate on the relative merits of the estimation techniques. As has been emphasized by Klein (1969) comparing parameters individually does not serve much as a means of investigating the efficiency of a technique. What is important, in the final analysis, is how well the technique performs in estimating all the parameters. In this case the search for a criteria for comparison amongst the techniques should be a search for a loss function of a sort. It might well be that the overall performance of the model will be the most appropriate test of the techniques. Furthermore, every actual econometric problem is unique and a search for the best method of estimation should take this fact into consideration.

We have chosen the simultaneous approach and employed 3SLS as against FIML methods in estimating the Nigerian structural model. Equation (2.5) is over-identified while all other equations are just-identified. The 3SLS has been shown to possess a number of attractive

properties. It is much simplier computationally than FIML. Rothenberg and Leenders (1964) have shown that when the variance-covariance matrix (Σ) of the disturbance terms is unrestricted, the estimates have the same asymptotic variance-covariance matrix as the FMIL method. Further, under the same condition of unrestricted Σ matrix, Sargan (1964) has shown that the difference between the estimates by the 3SLS methods and FIML is of stochastic order $\frac{1}{T}$, where T is sample size.

The problem with 3SLS is that it involves the inversion of a moment matrix of a very large order. The difficulty in computing the inverse matrix accurately, especially when it may be near singular due to the presence of inter-correlation among the explanatory variables are well known [Klein and Nakamura (1962), Summers (1959)]. As a solution to this problem, Zellner and Theil have suggested that the equation system be divided into two groups, one consisting of all over-identified equations and the other consisting of all the exactly identified equations. They recommend the use of 3SLS for estimating the parameters of the former group and ordinary least squares for the second group, contending that the efficiency of the estimates is not impared.

When there is uncertainty in the parameters of the structural model, we can learn more about the distribution of the parameters by sequential updating estimation

procedure. This procedure is particularly useful in the Nigerian case where not much is yet known about the structure of the economy and where the economic structure is far from being stable. For the Nigerian example, estimation is first performed using data for the period 1951 to 1957. Then as more and more information (data) becomes available in subsequent years, the value of the parameters is updated accordingly by re-estimating the structural model, incorporating the data available in the current year.

RESULTS OF ESTIMATION

The 3SLS estimates of the coefficients of the structural equations, and their t-values for the periods 1951/67, 1951/68, 1951/69 are presented in Tables 3.2, 3.3, and 3.4. Comparison of our results for 1951/69 period with the results of models that were built at a comparable period of development in industrial countries discussed earlier--Vitz, Klein, Klein-Goldberger and Suits models--should be illuminating but unfortunatly cannot be intensive since these models differ in details and in the variables included from our model. A long-run marginal propensity to consume out of income of approximately 0.79 obtained for the Nigerian model compares favorable with 0.80 obtained in the Klein model, and 0.74 obtained in the Klein-Goldberger model. The coefficient associated

TABLE 3.2. STRUCTURAL ESTIMATES FOR 1951-1967 PERIOD

Dependent Variable	Explanatory Variable						
	Constant	C_{-1}	I_{-1}	M_{-1}	GNP	Government Expenditure	ΔYB
Private Consumption (C)	91.22 (1.66)	-0.152 (-0.5)			0.846 (3.85)		
GDCF (I)	26.23 (2.94)		0.672 (4.57)			0.173 (1.86)	
Imports (M)	34.62 (1.94)			0.784 (3.77)		0.075 (0.57)	
Exports	17.67 (0.76)						0.140 (6.79)

TABLE 3.3. STRUCTURAL ESTIMATES FOR 1951-1968 PERIOD

Dependent Variable	Explanatory Variable						
	Constant	C_{-1}	I_{-1}	M_{-1}	GNP	Government Expenditure	ΔYB
Private Consumption (C)	44.49 (0.85)	0.026 (0.11)			0.756 (3.99)		
GDCF (I)	24.28 (2.85)		0.749 (5.41)			0.121 (1.40)	
Imports (M)	18.11 (0.96)			1.05 (4.85)		-0.18 (-0.90)	
Exports	44.21 (1.90)						0.113 (5.73)

TABLE 3.4. STRUCTURAL ESTIMATES FOR 1951-1969 PERIOD

Dependent Variable	Explanatory Variable						
	Constant	C_{-1}	I_{-1}	M_{-1}	GNP	Government Expenditure	ΔYB
Private Consumption (C)	36.67 (0.75)	0.145 (0.58)			0.673 (3.49)		
GDCF (I)	25.56 (3.16)		0.713 (5.49)			0.143 (1.74)	
Imports (M)	53 (2.74)			0.608 (2.98)		0.148 (1.24)	
Exports	30.40 (1.26)						0.128 (6.48)

with the stock of capital in the capital formation equation for the Klein and Klein-Goldberger models were -0.146, and -0.073 respectively. Economic theory of capital adjustment would support the negative sign obtained in these two models. The theory of capital adjustment says that the more stock of capital there is, the less there is demand for more investment. There is thus an inverse relationship between capital stock and investment. In the Nigerian model, however, the sign of the capital stock coefficient is not only positive but highly statistically significant. As was argued earlier in Chapter II, however, this result is not as absurd as it seems. Our result supports Klein's (1965) argument that in a developing economy, where there are many opportunities for investment, the stock-adjustment form of investment function may not be realistic and that the existence of capacity may promote new investment rather than hinder it. It would seem reasonable to assume that the acquisition of vehicles would encourage the building of new roads, improvements of ports would call forth a net work of feeder roads, and consequently more plants and machinery.

Empirical results either from comparable Nigerian studies or from other less developed countries are scanty. In a paper on capital formation and fiscal policy in Nigeria, Pacard (1966) argues, although without any empirical evidence, that government fiscal policy,

particularly its expenditures have a significant positive impact on Gross Domestic Investment. The government expenditure coefficient in the capital formation function in our model is positive and statistically significant. Thus our result supports Packard's claim. In the import equation, the coefficients associated with lagged imports and government expenditures are positive and statistically significant, as will be expected. The governments of Nigeria import goods and services in large quantities, most especially vehicles, machinery and equipment, building materials and chemicals. In Oni's (1970) study of demand for Nigerian cocoa, he found an inverse relationship between cocoa demand and disposable income of British subjects (One unit increase in disposable income of British subjects reduces cocoa demand by 6.3 units). This is in sharp contrast to the result obtained in the Nigerian model for the Nigerian exports (One unit increase in disposable income of British subjects increases Nigerian exports by 0.128 units). Of course our exports include other items other than cocoa. It could be that some of these items are inferior goods and that cocoa falls in this category but there is no evidence that this is case.

The stochastic specification of equations (2.2) to (2.6) describe the five endogenous variables, Consumption, GDCF, Imports, Exports, and GNP, in terms of exogenous variables only in an implicit way. That is, the system is

in 'structural form', with each of its equations describing part of the structure of the economy. As it happens some of the equations describe an endogenous variables in terms of other endogenous variables. For instance, private consumption expenditures are functions of GNP. We can, however, express the system in the 'reduced form', that is, we solve the system with respect to endogenous variables, describing each of them in year t linearly in terms of the same variables lagged one year, of the exogenous variables in the same year, of the exogenous variables lagged one year and of the reduced form disturbances.[9]

In Tables 3.5, 3.6, 3.7 we show the reduced form coefficients for the periods 1951/67, 1951/68, and 1951/69. The coefficients relating to private consumption expenditures and GNP are very unstable, particularly for periods 1951/67 and 1951/68. This instability is probably partly explained by the internal crisis of the period 1966 to 1969 which seriously affected the economy, and partly by the high rate of inflation in the late sixties. The level of economic activity did decline at this period.[10] For instance, government expenditures dropped from 325.1 million in 1966 to 306.3 in 1968. In 1969, however, they got back to 330.9. Imports dropped from 257.0 millions in 1966 to 193.2 millions in 1968, while exports dropped from 278.7 millions to 206.5 millions in the same period.

TABLE 3.5. REDUCED FORM COEFFICIENTS FOR 1951-1967 PERIOD

Dependent Variable	Explanatory Variable					
	Constant	C_{-1}	I_{-1}	M_{-1}	Government Expenditure	ΔYB
Private Consumption (C)	647.3	-0.987	3.69	-4.31	6.03	0.769
GDCF (I)	26.2		0.672		0.173	
Imports (M)	34.6			0.784	0.075	
Exports	17.7					0.140

TABLE 3.6. REDUCED FORM COEFFICIENTS FOR 1951-1968 PERIOD

Dependent Variable	Explanatory Variable					
	Constant	C_{-1}	I_{-1}	M_{-1}	Government Expenditure	ΔYB
Private Consumption (C)	338.5	0.107	2.32	-3.25	3.84	0.350
GDCF (I)	24.3		0.749		0.121	
Imports (M)	18.1			1.050	-0.118	
Exports	44.2				0.113	

TABLE 3.7. REDUCED FORM COEFFICIENTS FOR 1951-1969 PERIOD

Dependent Variable	Explanatory Variable					
	Constant	C_{-1}	I_{-1}	M_{-1}	Government Expenditure	ΔYB
Private Consumption (C)	118.2	0.443	1.47	-1.25	2.05	0.263
GDCF (I)	25.7		.713		0.143	
Imports (M)	53.0			**0.608**	0.147	
Exports	30.4					0.128

In 1967, there was a drop of 78.4 millions in GNP compared with 1966. The coefficients for the other variables show considerable stability between these periods.

The interpretation of the reduced form coefficients of Table 3.7 is that a unit change in last period GDCF will change current period GDCF by 0.713 unit, while a unit change in Government Expenditure will change GDCF by a smaller amount of 0.143 unit. Similarly a unit change in Government Expenditure will change private consumption by over two units in the same direction. While a unit change in Government expenditure changes GDCF and Imports by only 0.143 unit and 0.147 unit respectively, and in the same direction.

CHAPTER IV

THE DYNAMIC PROPERTIES OF THE NIGERIAN MODEL

The estimated structural model will be employed to formulate policy decisions after we are certain that it is internally consistent and that it validly represents the system it is supposed to simulate. Information on the internal consistency and validity of our model representation involves the investigation of its dynamic properties. If the dynamic properties are consistent with established knowledge and theory then the internal consistency and hence the validity of our model are further strengthened.

The investigation of the dynamic properties of our model assumes particular relevance since the estimated structural equations include lagged endogenous variables among the set of predetermined variables. Following Dhrymes [1970, p. 542], "... it is not enough to estimate the parameters of a model, casually inspect them, and pronounce them believable. If the model is dynamic, as most such econometric models are likely to be, one must carefully investigate the dynamic properties and verify that they are not absurd and inadmissible."

In investigating the dynamic properties of our model, we have to know the sort of time path the stochastic

and non-stochastic equations of our model will generate in the absence of additional external constraints or shocks. In other words, do these equations offer an immediate explanation of our internally generated cyclical process? It is possible that the long-run predictive system will simulate the economy very well. On the other hand, however, it is also possible that this model is too simple to offer even a qualitative picture of the Nigerian economy. We also want to know whether the economy described by our system is stable under large exogenous displacements. That is, does the solution resume its unperturbed equilibrium growth trend even after a strong exogenous disturbance.

Following Samuelson (1965) a system is said to possess perfect stability of the first kind if in the entire region over which the system is defined, all of its motions converge to each other. Formally, a motion Y_t is said to converge to another motion Y_t^* if for any given positive vector ε whose elements are arbitrarily small, there exists some t^o such that:

$$|Y_t - Y_t^*| < \varepsilon \quad , \text{ for } t > t^o$$

Occasionally convergence of a sequence obeying above is said to be strong convergence. The investigation of the dynamic properties will, therefore, reveal whether, for the data we have, our model is non-explosive and oscillatory, explosive and oscillatory, non-explosive and non-

oscillatory, or explosive and non-oscillatory.

SIMULATION APPROACH

There are established procedures for investigating the dynamic implications of econometric models by analytical methods [Adleman and Adleman (1959), Goldberger (1964), Howrey (1971), Theil and Boot (1962), Zusman (1962), Dhrymes et al. (1972)]. In addition, Evans (1962), Fromm (1968), have proposed a simulation analysis approach for determining internal consistency of a model. The comparative appropriateness of the dynamic properties of econometric models discussed by the authors referenced above depends upon the nature of the structural model to be investigated.[1] In this section, we consider a limited stochastic linear structural model--an estimated structural model with parameter values treated as constants but with stochastic structural equations whose error terms are based on the estimated residual distributions--and the methods suggested in the literature for deriving the associated dynamic properties. In general, the question of when to use simulation rather than an analytical solution has not been settled. Howrey and Kelejian (1971) have, however, demonstrated that all of the information about validity and dynamic behavior of linear econometric models that can be obtained by the use of simulation methods can also be obtained from existing analytical

techniques. But simulation methods have the advantage of computational ease especially when high-order difference equation models are involved. Further, the interpretation of the simulated time paths of econometric models may prove to be more straightforward than the interpretation of the analytical solutions.

As a preliminary to the discussion of the simulation approach, we can write the reduced form of the stochastic specification of equations (2.2) to (2.6) as:

$$Y_t = AY_{t-1} + \Gamma Z_t + U_t \qquad \ldots (4.1)$$

where,

Y_t is a vector of endogenous variables in period t, Z_t is a vector of exogenous variables (including possibly lagged exogenous variables), and U_t is a vector of stochastic error terms. A and Γ are matrices of unknown coefficients.

Simulated values of Y_t, say $\hat{Y}_t$ may be generated from the reduced form (4.1). Thus we have:

$$\hat{Y}_t = \hat{A}\hat{Y}_{t-1} + \hat{\Gamma} Z_t + \hat{U}_t \qquad (4.2)$$

and $$\hat{Y}_o = Y_o$$

Under the usual assumptions regarding Y_{t-1}, Z_t, and Y_t [Goldberger (1964), pp. 295-302 and 336-338], and using either LIML, 2SLS, 3SLS or FIML, $\hat{A}$, $\hat{\Gamma}$ are consistent

estimators of A and Γ respectively. Thus we can write:

$$\hat{A} = A + \varepsilon_A \quad \text{and} \quad \hat{\Gamma} = \Gamma + \varepsilon_\Gamma$$

where the matrices ε_A and ε_Γ converge in probability to the null matrix as the sample size increases without limit, that is,

$$\text{plim } \varepsilon_A = 0 \quad \text{and plim} \quad \varepsilon_\Gamma = 0$$

We can generate the values of $\hat{U}_t$ from a multivariate normal distribution. The difference between the observed and simulated values of Y_t is obtained by subtracting (4.1) from (4.2).

We note here that the specification of our system is of one period lag. This makes our analysis simple. In the case the system is of higher order lags, we can condense it into a one period lag as has been demonstrated by Theil and Boot (1962).

Subtracting (4.1) from (4.2) we obtain,

$$\hat{Y}_t - Y_t = (\hat{Y}_{t-1} - Y_{t-1})A + Z_t\varepsilon_\Gamma + \hat{Y}_{t-1}\varepsilon_A + \hat{U}_t - U_t \qquad \ldots\ldots(4.3)$$

By appropriate backwards substitution and ignoring the errors in $\hat{A}$ and $\hat{\Gamma}$, the difference equation (4.3) can be written as:

$$Y_t = \hat{Y}_t + \sum_{j=1}^{t} (U_j - \hat{U}_j)A^{t-j} \qquad \ldots\ldots(4.4)$$

If we consider the large sample counterpart of equation (4.2), that is, set $\hat{A} = A$ and $\hat{\Gamma} = \Gamma$, and ignore the error terms in both (4.1) and (4.2) it is easy to see from (4.1) and (4.2) that the process generating $\hat{Y}_t$ is identical to the one generating the historical values Y_t. Hence an examination of the time paths of the elements of $\hat{Y}_t$ should yield information regarding the properties of the process generating the elements of Y_t. We can now asmptotically compare Y_t and the simulated $\hat{Y}_t$ by investigating the probability limit of the last term on the right hand side of (4.4), i.e. observing the characteristics of:

$$\text{plim} \sum_{j=1}^{t} (U_j - \hat{U}_j)A^{t-j} \qquad \ldots . (4.5)$$

But as Howrey and Kelejian [1969, pp. 212-213] have demonstrated (4.5) is in general not equal to zero, and it is also related to $\hat{Y}_t$. To show that (4.5) is in general not equal to zero, let

$$\text{plim} \sum_{j=1}^{t} (U_j - \hat{U}_j)A^{t-j}$$

$$= \text{plim} \sum_{j=1}^{t} (U_j - \bar{U} + \bar{U} - \hat{U}_j)A^{t-j}$$

$$= \text{plim} \sum_{j=1}^{t} (U_j - \bar{U})A^{t-j} + \text{plim} \sum_{j=1}^{t} (\bar{U} - \hat{U}_j)A^{t-j}$$

$$= 0 + \text{plim} \sum_{j=1}^{t} (\bar{U} - \hat{U}_j)A^{t-j}$$

The last term of the right hand side of the above equation does not vanish even when sample size increases except when $\bar{U} = \frac{1}{T} \sum_{j=1}^{T} \hat{U}_j$, where T is sample size. Hence (4.5) is not in general equal to zero.

Assume that the simulated endogenous values are independent of the errors associated with historical endogenous values; that is,

$$E\ [(\hat{Y}_t)'U_{t-j}] = 0\ ,\ j = 0,\ 1,\ldots,\ t-1 \quad \ldots(4.6)$$

Further, we ignore the small sample errors of $\hat{A}$ and $\hat{\Gamma}$. Then,

$$E\ [(\hat{Y}_t)'\hat{U}_{t-j}] = (\Gamma^j)'\hat{V},\ j = 0,\ 1,\ldots,\ t-1$$

where,

$$\hat{V} = E\ (\hat{U}_j'\hat{U}_j) \quad \ldots(4.7)$$

combining (4.6) and (4.7) we have the covariance matrix

$$E\ [(\hat{Y}_t)' \sum_{j=1}^{t} (U_j - \hat{U}_j)A^{t-j}]$$

$$= - \sum_{j=1}^{t} (A^{t-j})'\hat{V}A^{t-j} \quad \ldots(4.8)$$

If the econometric model is stable (i.e. the characteristic roots of the vector A are inside the unit circle in the complex plane), then the covariance matrix converges as

$$t \to \infty$$

$\hat{V}$ is a variance-covariance matrix, hence it must be positive definite. Therefore elements of the matrix (4.8), say, $-A'^{i}\hat{V}\,A^{i}$ must be negative. It follows that the diagonal elements of (4.8) are negative which in turn implies that each element of $\hat{Y}_t$ is negatively correlated with the corresponding element of:

$$\sum_{j=1}^{t} (U_j - \hat{U}_j)A^{t-j}$$

From these last two results and the results concerning errors of measurement [Johnston (1963, pp. 148-150)], Howrey and Kelejian concluded that if the elements of Y_t are regressed on the corresponding elements of $\hat{Y}_t$, the slope of the estimated line will be less than unity, and the constant term will be significantly different from zero. Their conclusion, therefore, was that even though the process generating $\hat{Y}_t$ is identical to the one generating Y_t, the elements of $\hat{Y}_t$, on the average, will fail to predict consistently the corresponding elements of Y_t; that is,

$$E\,[Y_t/\hat{Y}_t] \neq \hat{Y}_t$$

and hence in the case of linear limited stochastic models, simulation approach for describing the dynamic properties can be misleading. But Aigner (1972) has demonstrated that this conclusion is wrong. Suppose again we ignore the small sample errors on $\hat{A}$ and $\hat{\Gamma}$, we have

$$\hat{Y}_t = AY_{t-1} + \Gamma Z_t + \hat{U}_t$$

While U_j and $\hat{U}_j$ are independent, Y_t and $\hat{Y}_t$ are in general not. Observing their marginal distribution we find that Y_t and $\hat{Y}_t$ have equal variances. Thus,

$$V(Y_t) = V(\hat{Y}_t) = V[AY_{t-1} + \Gamma Z_t] + \sigma^2$$

with positive covariance

$$C(Y_t, \hat{Y}_t) = V[AY_{t-1} + \Gamma Z_t]$$

assuming,

$\hat{U}_t \sim N(o, \sigma^2)$ and independent of Y_t, Z_t and U_t. The scatter diagram of this distribution is an ellipse with 45^o line as its major axis. If Y_t and $\hat{Y}_t$ were independent, the covariance term will be zero and the scatter diagram will appear as a circle with its centre on the 45^o line.

Although the regression of either Y_t or $\hat{Y}_t$ on $AY_{t-1} + \Gamma Z_t$ is homogenous with slope equal to one, i.e.,

$$E[Y_t/AY_{t-1} + \Gamma Z_t] = AY_{t-1} + \Gamma Z_t ,$$

the regression of Y_t on $\hat{Y}_t$ or $\hat{Y}_t$ on Y_t is not. In fact, under bivariate normalty conditions, the conditional mean functions are linear with identical slopes:

$$\beta = \frac{C(\hat{Y}_t, Y_t)}{V(Y_t)} = \frac{C(Y_t, \hat{Y}_t)}{V(\hat{Y}_t)} = \frac{V(AY_{t-1} + \Gamma Z_t)}{V(AY_{t-1}+\Gamma Z_t)+\sigma^2} < 1 \quad \ldots (4.9)$$

and identical intercept, $(1-\beta)\mu$, where μ is the (unconditional) mean of both Y_t and $\hat{Y}_t$.

Thus for the case of stochastic simulation the (large sample) regression of actual on generated values (or vice versa) should not be expected to be homogenous with slope one in general as thought by Howrey and Kelejian. From (4.9) we note that the slope lies between zero and one. The intercept will be positive and bounded above by μ for $\mu > 0$, while it will be negative and bounded below by μ if $\mu < 0$. When $V[AY_{t-1} + \zeta_t]$ is, however relatively large compared with σ^2, the slope approaches one.

Aigner's argument stands if we accept the assumption of bivariate normality in the marginal distribution of Y_t and $\hat{Y}_t$. In any case, the verification of individual equations of a structural model may not be the best approach. As Aigner points out, "he must decide whether the simulated series are 'close enough' to reality (collectively) so that according to some loss function the simulated model will be taken to be a good proxy for the true structure" [1972, p. 619]. In this respect, Dhrymes [1970, p. 240] has suggested the use of a statistic analogous to the single-equation R^2, to test the hypothesis that $\beta = 0$, where β is the coefficient vector of the system of equations under consideration. He defines this statistic R^2 as:

$$R^2 = 1 - \frac{\hat{U}'\hat{U}}{(Y-e\bar{Y})'(Y-e\bar{Y})}$$

where,

$\hat{U}$ is the estimate of the error terms of a general linear model:

$$Y = X\beta + U$$

i.e. $$\hat{U} = Y - X\hat{\beta}$$

β is the OLS estimator of β given by

$$\beta = (X'X)^{-1}X'Y$$

and $e = (1,1,1,\ldots 1)'$, $\bar{Y} = \frac{1}{T}\sum_{t=1}^{T} Y_t$

Unfortunately, however, this statistic is dependent on the unknown covariance parameters of the joint distribution of the error terms of the system. Dhrymes (1970) has also derived an alternative test procedure regarding the goodness-of-fit of the reduced form model (the fraction of the generalized variance of the jointly dependent variables explained by the reduced form). But as he points out, this procedure involves the restriction that the number of variables in the model (endogenous plus predetermined) be less than the total number of observations. This restriction is generally not fulfilled, however. A potentially promising criterion for validating

an econometric model is suggested by Hooper (1962). It is called the 'trace correlation' statistic and it is based on the estimates of the canonical correlations. Currently, however, the distribution of 'trace correlation' is intractable.

Non-linear models present a major problem of validation. Because of the difficulties involved in obtaining analytical solutions to a system of non-linear equations and then in performing the integrations necessary to get the conditional expectations, the reduced-form equations, in general, will be unknown.[2] When the reduced-form of a non-linear model is known, Howrey and Kelejian (1972) have demonstrated that the elements of $\hat{Y}_t$ can be expected to diverge systematically from the corresponding elements of Y_t, even though the econometric model is properly specified. Furthermore, multiplier analysis based on non-stochastic simulation yields results that do not apply to the corresponding historical values [Evans and Kelin (1967), pp. 48-49 and chp. 5; Fromm and Taubman (1968, chp. 2)]. Thus non-linear models should not be validated in terms of comparisons between the elements of $\hat{Y}_t$ and Y_t.

ANALYTIC APPROACH

Analytic methods for describing the dynamic implications of stochastic, linear econometric models include

characterizing the stochastic response of the analytical solution to a system of difference equations (4.1) by the use of spectral (frequency domain) analysis and indirectly the derivation of the covariance matrix of the endogenous variables (time domain). Spectral (cross-spectral) analysis is a statistical technique that can be used to obtain a frequency decomposition of the variance (covariance) of a univariate (bivariate) stochastic process [Dhrymes et al. (1972)]. There are reasons why one might consider spectral analysis as a technique for investigating dynamic properties of a model.[3] Firstly, when we study a stochastic process, we are interested in, among other things, the average level of activity, deviations from this level, and how long these deviations last, once they occur. Spectral analysis provides this kind of information. Secondly, in spectral analysis we can construct confidence bands and test hypothesis for the purpose of comparing the simulated results of the use of two or more alternative economic policies. Furthermore, spectral analysis can be used to validate an econometric model. By comparing the estimated spectra of simulated data and corresponding real-world data one can infer how well the simulation results compare with the system it was designed to emulate. An important advantage of the spectral method over the simulation method is that the results of the

former are not subject to sampling variability because the spectra are expected values of the spectral power function. In addition we do not need to replicate stochastic model solutions as we do in the simulation approach.

The application of spectral method is, however, limited. Firstly, the length of available time series may give difficulties. When dealing with aggregative econometric models, it is not often that we have series of sufficient length to justify the use of spectral techniques. Secondly, the dependent variables will, generally, not have the property of (covariance) stationary, as required by spectral techniques unless they have been sufficiently detrended. That is, unless they have been 'cleared' of their (systematic) mean function. This requirement implies that in order to perform tests of comparison on the spectra estimates of the model and the actual series of observations for the endogenous variables we must also obtain the spectral matrix of the exogenous variables. The exogenous variables may then be decomposed into a deterministic function (say a time trend) and a (multivariate) covariance stationary process. Where this latter process and the process U_t are mutually independent, the spectral matrix of the exogenous variables Z_t may be easily obtained. Methods for detrending and removing a trending

mean as well as for handling a series whose variance and/or auto-covariance changes through time are well discussed by Granger and Hatanaka (1964), Fishman (1969), and Dhrymes (1970). These methods include polynomial regression, first differences and linear time detrending. Unfortunately, the spectrum we compute from observed time series is very much dependent on the transformation procedures applied. This has probably made spectral analysis less attractive as an alternative for investigating the dynamic properties of econometric models.

Granger and Hatanaka (1964), Nerlove (1964), Naylor et al. (1967), and Black and Tukey (1958) amongst others, have developed the method of spectral analysis as applied to econometric models. The basic idea underlying spectral representations is that the time series can be decomposed as a sum of uncorrelated components, each associated with a particular frequency. Hence the variance of the series is equal to the sum of the variances of the components. The power spectrum shows the variances of the components as a function of their frequencies and indicates the relative importance of the components in terms of their contribution to the overall variance. It is the squared amplitude associated with oscillations at different frequencies.

The definition of some useful concepts and related theorems are given in the appendix. In most parts, we follow Dhrymes (1970).

Consider the reduced form equation (4.1),

$$Y_t = AY_{t-1} + \Gamma Z_t + U_t$$

we can rewrite this equation in an operator representation form as:

$$C(L)Y_t = C^*(L)Z_t + U_t \qquad \ldots .(4.8)$$

Thus the final form of (4.8) is:

$$Y_t = [C(L)]^{-1}C^*(L)Z_t + [C(L)]^{-1}U_t \qquad \ldots .(4.9)$$

assuming the operator matrix C(L) is invertible. C(L) will have an inverse if and only if the roots of:

$$|C(\rho)| = 0$$

lie outside the unit circle [$|C(\rho)|$ is a polynomial in the real or complex variable ρ which is obtained by substituting ρ for the operator L in C(L)]. Hence the general structural model is basically a system of rational distributed lags in the exogenous variables. Furthermore from (4.8), it is clear that we are dealing with a system of stochastic difference equations with constant coefficients and 'forcing function' $C^*(L)Z_t$. From (4.9) the conditional expectation of the jointly dependent variables, given the exogenous variables of the system, is:

$$E\ (Y_t/Z_t) = [C(L)]^{-1}C^*(L)Z_t$$

The elements of:

$$[C(L)]^{-1}C^{*}(L)$$

are the elements of the matrix of dynamic multipliers. Dynamic multipliers indicate the response of the endogenous variables to changes in the exogenous variables. For short-term policy formulation, dynamic multipliers may provide a sufficient characterization of the dynamic properties of a structural model. But to repeat Haavelmo's warning, if the long-term properties of the model are to be investigated, it may not be reasonable to disregard the impact of the disturbance terms on the time paths of the endogenous variables as we do in deriving dynamic multipliers.

A property we are looking for in the Nigerian model is stability. By stability we mean if the values of the exogenous variables are specified to be finite, then the conditional expectation of the jointly dependent variables is also finite. Otherwise the Nigerian economy we purport to describe by (4.1) is unstable and will diverge to $+\infty$ or $-\infty$, depending on the initial conditions, or it will experience oscillations of increasing amplitude. It is necessary here to differentiate between global stability and local stability. An equilibrium position is said to be 'globally stable' if all the variables of the system approach this position in the limit as time passes, no matter what their initial values. On the other hand an equilibrium position is said to be 'stable locally or in

the small' if for sufficiently small deviations from equilibrium all the variables approach this position in the limit as time passes. Clearly if an equilibrium position is globally stable it is also stable locally too, but the converse is not necessarily true [Samuelson (1947, pp. 261-262)]. In an econometric model, however, we are more interested in local stability conditions since it is easier to test for these conditions than test for global stability conditions. Further, it is not clear whether we can expect anything more than local stability because our model is a simplification of the system to start with and our sample size is not large enough. Nevertheless, we do keep in mind that such tests on local stability conditions do not guarantee global stability, or stability over any given range of disturbances for that matter.

Stability for a system of difference equations such as (4.1) is insured if the roots of the polynomial

$$|C_o\phi^r - C_1\phi^{r-1} - C_2\phi^{r-2}, \ldots, - C_r| = 0 \quad \ldots(4.11)$$

are less than unity in absolute value. If ρi is the i^{th} root of (3) of Appendix 4 and ϕi is the i^{th} root of (4.11), then:

$$\phi i = \frac{1}{\rho i}$$

The following conclusions arise from theorem in Appendix 4.

(i) The final form (4.9) can be rewritten as:

$$Y_t = \frac{A(L)}{b(L)} Z_t + \frac{G(L)}{b(L)} U_t \qquad \ldots (4.12)$$

where,

$$b(L) = |C(L)| ; \; A(L) = G(L)C^*(L)$$

$G(L)$ is the adjoint of $C(L)$ and

$\frac{A(L)}{b(L)}$, $\frac{G(L)}{b(L)}$ are rational lag operators and admit of a power series representation.

(ii) The matrix of dynamic multipliers is given by:

$$\frac{A(1)}{b(1)}$$

(iii) We can write (4.12) as:

$$Y_t = \frac{A(L)}{b(L)} W_t + \frac{A(L)}{b(L)} \varepsilon_t + \frac{G(L)}{b(L)} U_t \qquad \ldots (4.13)$$

where,

$$Z_t = W_t + \varepsilon_t$$

Hence except for a deterministic mean, the jointly dependent variables form a covariance stationary stochastic process whose spectral matrix is the sum of the two spectral matrices associated, respectively, with the stochastic components in (4.13). Thus to determine the spectral

matrix of the jointly dependent variables of the system, we need only know the spectral matrices of the processes:

$$\frac{A(L)}{b(L)}\,\varepsilon_t \quad , \text{ and } \quad \frac{G(L)}{b(L)}\,U_t$$

which are:

$$\frac{A(\theta)\ F_{\varepsilon}(\theta)\bar{A}'(\theta)}{|b(\theta)|^2}$$

and

$$\frac{G(\theta)F_u(\theta)\bar{G}'(\theta)}{|b(\theta)|^2} \qquad \text{respectively.}$$

Hence the spectral matrix of the jointly dependent variables is given by:

$$F_Y(\theta) = \frac{A(\theta)F_{\varepsilon}(\theta)\bar{A}'(\theta)}{|b(\theta)|^2} + \frac{G(\theta)F_u(\theta)\bar{G}'(\theta)}{|b(\theta)|^2} \quad \ldots(4.14)$$

In particular if we make the assumption that the vector ε_t form a set of mutually independent, indentically distributed random variables with zero mean and covariance matrix Ω, then the spectral matrix of the endogenous variables becomes:

$$F_Y(\theta) = \frac{A(\theta)\Omega\bar{A}'(\theta)}{2\pi|b(\theta)|^2} + \frac{G(\theta)\Sigma\bar{G}'(\theta)}{2\pi|b(\theta)|^2} \quad \ldots(4.15)$$

An estimator for $F_Y(\theta)$ is given as:

$$\hat{F}_Y(\theta) = \frac{\hat{A}(\theta)\hat{\Omega}\hat{\bar{A}}'(\theta)}{2\pi|b(\theta)|^2} + \frac{\hat{G}(\theta)\hat{\Sigma}\hat{\bar{G}}'(\theta)}{2\pi|b(\theta)|^2} \quad \ldots .(4.16)$$

where,

$A(\theta)$, Ω, $G(\theta)$, Σ, and $b(\theta)$ are replaced by their consistent estimators [Dhrymes (1970, pp. 528-529), Naylor (1971, p. 313)]. We can now deduce the cyclical characteristics of the dependent variables of the system by studying the spectral densities:

$$f_{ii}(\theta) \ , \quad \forall i$$

which are the diagonal elements of $\hat{F}_Y(\theta)$.

Estimates of the cross-spectra between each of the endogenous variables may be obtained from the off-diagonal elements of $\hat{F}_Y(\theta)$. These off-diagonal elements indicate the coherence and the phase of the time series. Following Granger (1969) we define the coherence as the square of the correlation coefficient between corresponding frequency components of X_t and Y_t, a pair of stationary time series. The phase measures the phase difference between corresponding frequency components. When one variable is leading the other, the phase divided by θ gives the extent of the time lag. Thus, the coherence is used to measure the degree to which two series are related and the phase may be interpreted in terms of time lags. If X_t, Y_t and Z_t are three time series, there may be

misleading correlation and coherence values between two of them due to the influence on both of the third variable. Granger (1969) has demonstrated how this problem can be overcome by the use of partial cross-spectral methods. He gives the spectral, cross-spectral matrix between the three variables as:

$$F_{ij}(\theta)d\theta$$

where,

$$F_{ij}(\theta) = F_X(\theta) \ , \ i = j = X$$

$$= F_{XY}(\theta), \ i = X, \ j = Y$$

In general, however, the computation of cross-spectral densities present some problems since they involve complex numbers. [Granger (1969), Naylor et al. (1969), Smith (1971)]. To ease computation, Smith suggests we resolve the spectrum matrix of the system into real and complex parts. Thus,

$$F_Y(\theta) = F_1(\theta) + iF_2(\theta)$$

The diagonal elements of $F_1(\theta)$ are the power spectra at frequency θ (those of $F_2(\theta)$ are identically zero). The off-diagonal elements of $F_1(\theta)$ and $iF_2(\theta)$ are the co-spectrum and quadrature spectrum respectively. Hence the coherence statistic is:

$$F^2_{jk}(\theta) = \frac{F^2_{1jk}(\theta) + F^2_{2jk}(\theta)}{F_{1jj}(\theta) + F_{1kk}(\theta)}$$

Another interesting aspect of spectral analysis is revealed in the work of Blackman and Tukey (1958). Their work shows that the spectral estimates $F_Y(\theta)$ have desirable statistical properties, provided the process Z_t is normal. One property in particular, the statistical independence of spectral estimates at non-adjacent frequencies, allows the interpretation of confidence intervals. Blackman and Tukey further assume that the theoretical spectrum $F_{ii}(\theta)$ is reasonably smooth. Thus the distribution of:

$$\frac{\hat{F}_{ii}(\theta)}{F_{ii}(\theta)}$$

is approximately $\frac{\chi^2_k}{k}$, where k = degrees of freedom. Hence in order to gain some information about $F_{ii}(\theta)$ when the sample estimate is available, one may construct a $(100-\alpha)\%$ confidence interval for

$$F_{ii}(\theta)$$

Suppose $\alpha = .05$. Let $\chi^2_{.975,k}$ and $\chi^2_{.025,k}$ be the percentiles of the χ^2 distribution with k degrees of freedom. Then,

$$P_r \left\{ \frac{\chi^2_{.975,k}}{k} < \frac{\hat{F}_{ii}(\theta)}{F_{ii}(\theta)} < \frac{\chi^2_{.025,k}}{k} \right\} = .95$$

or

$$P_r \left\{ \frac{\hat{F}_{ii}(\theta)}{\chi^2_{.025,k/k}} < F_{ii}(\theta) < \frac{\hat{F}_{ii}(\theta)}{\chi^2_{.975,k/k}} \right\} = .95$$

We may be interested in constructing confidence bands to compare the estimated spectra of two series that are the result of the use of two different economic policies [Naylor et al. (1967)]. In this case, we proceed as above, setting a confidence interval for the ratio

$$\frac{F'_{ii}(\theta)}{F^2_{ii}(\theta)}$$

by the use of an F (variance-ratio) statistic.

In summary, spectral analysis is potentially a powerful tool for analyzing data generated by an econometric model. By constructing the appropriate confidence bands, spectral analysis enables one to validate a simulation model and to compare the results of alternative economic policies that are associated with different series generated by the model. The large sample size required for its appropriate application, however, constitutes an important barrier to economists using the technique for validating models whose data are of short duration.

EMPIRICAL RESULTS OF THE DYNAMIC PROPERTIES OF THE NIGERIAN MODEL

The matrix C(L) derived from the estimated structural coefficients of the Nigerian model is given in Table 4.1 for the period 1951 to 1969.

The equation of the associated determinant (3) Appendix 4 is:

$$|C(\rho)| = 0 = (1-.608\rho)(1-.713\rho)(.327-.145\rho) \quad \ldots(4.18)$$

The roots of this equation are:

$$\rho_1 = \frac{1}{.608}, \quad \rho_2 = \frac{1}{.713}, \quad \rho_3 = \frac{.327}{.145}$$

$$\text{i.e.} \quad \rho_1 = 1.64, \ \rho_2 = 1.40, \ \rho_3 = 2.26 \qquad \ldots(4.19)$$

We recall that C(L) will be invertible if and only if the roots of $|C(\rho)| = 0$ lie outside the unit circle.

Thus we conclude that our system is stable since its roots ρ_1, ρ_2, ρ_3 are all greater than 1. Stability is one of the properties we would require of a model of a real world economy. In addition, however, we want the long-run dynamic multipliers to be of reasonable magnitude. In this connection, we recall that the form of the system exists and takes the form:

$$Y_t = \frac{A(L)}{b(L)} Z_t + \frac{G(L)}{b(L)} U_t$$

TABLE 4.1. MATRIX C(L)

I - .145L	0	0	0	- .673I
0	I - .713L	0	0	0
0	0	I - .608L	0	0
0	0	0	I	0
-I	-I	I	-I	I

More over, it is know that :

$$|C(L)| = b(L)$$

From Table 4.1, the associated determinant of C(L) is:

$$C(L) = (1-.608L)(1-.713L)(.327-.145L)$$
$$= .327-.577L + .334L^2 - .063L^3$$

Thus,

$$b(L) = .327-.577L + .334L^2 - .063L^3$$

The matrix G(L) is the adjoint of C(L), that is, it is the transpose of the matrix of cofactors associated with the latter. The matrix G(L) is given in Table 4.2. The matrix A(L) is the product of matrices G(L) and C_o^*, i.e.

$$A(L) = G(L)C_o^*$$

C_o^* is the matrix of exogenous variables parameters and is given in Table 4.3, while A(L) is given in Table 4.4.

The estimated conditional expectation of the final form is obtained by dividing every element of the matrix A(L) by b(L), thus giving the matrix of dynamic multipliers. Thus the matrix of dynamic multipliers is:

$$[C(I)]^{-1}C_o^*(L) = \frac{A(L)}{b(L)} \qquad \ldots .(4.20)$$

Evaluating (4.20) after substituting 1 for L, we find that the matrix of dynamic multipliers is as given in Table 4.5. An interpretation of Table 4.5 is that if we assume

TABLE 4.2. MATRIX G(L)

(I-.713L)(I-.608L)	(-.673I)(I-.608L)	(-.673I)(I-.713L)	(-.673I)(I-.713L) X (I-.608L)	(.673)(I-.713L) X (I-.608L)
0	(I-.145L)(I-.608L) -.673I (I-.608L)	0	0	0
-(I-.713L)	0	(I-.145L)(I-.713L) -.673I (I-.713L)	0	0
0	0	0	(I-.145L)(I-.713L) -(I-.713L)(I-.608L)	0
(I-.713L)(I-.608L)	-(I-.145L)(I-.608L)	-(I-.145L) X (I-.713L)	-(I-.145L)(I-.713L) X (I-.608L)	(I-.145L) X (I-.713L) X (I-.608L)

TABLE 4.3. MATRIX OF PARAMETERS OF EXOGENOUS VARIABLES (C_o^*)

36.7	0	0
25.6	.143L	0
53.0	.148L	0
30.4	0	.128
0	1L	0

TABLE 4.4. MATRIX A(L)

$-36.7I+14.4L+7.0L^2$	$.477L-.759L^2+.292L^3$	$-.086I+.114L-.037L^2$
$8.4I-8.8L+2.3L^2$	$.327L-.344L^2+.088L^3$	0
$-19.4I+6.2L+5.5L^2$	$.484L-.559L^2+.015L^3$	0
$14.1L-10.1L^2$	0	$-2.179L-.331L^2$
$-72.9I+41.8L-10.9L^2+1.9L^3$	$.709L-1.231L^2+.597L^3+.063L^4$	$-.128I+.188L-.09L^2+.008L^3$

TABLE 4.5. MATRIX OF DYNAMIC MULTIPLIERS

Dependent Variables	Exogenous Variables		
	Constant	Government Expenditures	ΔYB[a]
Consumption	-729	.48	-.43
GDCF	90	3.38	0
Imports	-367	-2.86	0
Exports	190	0	.81
GNP	1910	6.57	-.57

[a]change in disposable income of British subjects.

government expenditures to be fixed at $\overline{G}$ (and ΔYB assumed zero), then consumption would tend to fluctuate about $-729 + .48\overline{G}$, *GDCF about* $90 + 3.38\ \overline{G}$, Imports about $-367 - 2.86\overline{G}$, Exports about 190 and GNP about $1910 + 6.57\overline{G}$. These values are read directly from Table 4.5. On basis of this analysis we observe that although the estimated equations as given in Table 3.4, appear reasonable, the matrix of dynamic multipliers of our model exhibits some undesirable properties. This result is probably due in part to the fact that we have not taken the effect of inflation in the economy into account (nominal values rather than real values employed in the analysis). If, however, we increase G by a unit, this increase maintained forever, will have the ultimate impact of increasing private consumption by .48, GDCF by 3.38, reducing imports by 2.86 and increasing GNP by 6.57. The latter result is consistent with a long run marginal propersity to consume of .82 obtained earlier for our model.[4]

As discussed earlier, the properties of dynamic multipliers may not provide a sufficient characterization of the dynamic properties of a model. For a more reliable technique, we turn to spectra representation. Furthermore, spectra representation may serve as a technique for validating an econometric model. The procedure suggested is to compare the power spectra implied by our model with the power spectra estimated directly from the series of

observations on the endogenous variables. Significant differences between the two might then be taken as an indication that the model is not correctly specified.

To obtain empirical estimates of the power spectra of consumption, GDCF, Imports, Exports, and GNP, we have used a computer program BMDX92 developed by Frank Massey III and Robert I. Jennrich (1966) of the Health Sciences Computing Facility, UCLA.

Estimates of the power spectrum are obtained for both actual values of the variables and the simulated values derived from one model. The two sets of estimates are given in tables 4.6 and 4.7 respectively. We observe that there is a close relationship between the two sets for corresponding endogenous variables with the exception of GNP series. In particular, the prominent peaks of the power are located at (or near) the same frequencies. For instance, the greatest power for consumption series (simulated) is located at a frequency of $\pi/7$, while the greatest power for the same series (actual) is concentrated at $\pi/6$. GDCF and Imports have their relative peaks centered around $\pi/7$, $\pi/6$ respectively for both actual and simulated series. Table 4.8 shows the frequencies at which the greater power are concentrated for each of the variables. Thus although there are differences in the magnitudes of the power of each variable (actual vs simulated), the general shapes of the power for the simulated series resemble the shapes of the power for the actual data.

TABLE 4.6. POWER SPECTRA OF CONSUMPTION, GDCF, IMPORTS, EXPORTS, AND GNP USING ACTUAL DATA

Frequency	Series				
θ	Consumption	GDCF	Imports	Exports	GNP
0	47.46	0.73	26.94	25.34	5.75
$\pi/8$	108.61	21.65	70.83	65.38	271.34
$\pi/7$	491.42	48.13	47.43	50.81	496.37
$\pi/6$	403.50	11.47	91.19	80.58	119.09
$\pi/5$	472.74	11.61	46.97	42.21	93.33
$\pi/4$	188.81	6.01	6.49	30.36	114.16
$\pi/3$	268.21	5.58	9.59	18.64	93.71
$\pi/2$	123.67	5.70	5.81	17.34	17.08
π	20.64	2.09	5.59	18.70	22.43

TABLE 4.7. POWER SPECTRA OF CONSUMPTION, GDCF, IMPORTS, EXPORTS, AND GNP IMPLIED BY THE NIGERIAN MODEL

Frequency	Series				
θ	Consumption	GDCF	Imports	Exports	GNP
0	127.69	0.26	12.63	3.75	247.25
$\pi/8$	351.48	7.99	22.85	20.30	508.42
$\pi/7$	379.31	15.82	10.17	21.47	464.17
$\pi/6$	381.90	7.31	28.96	94.44	776.88
$\pi/5$	374.43	11.12	24.12	129.95	988.60
$\pi/4$	102.38	4.98	2.46	9.21	152.81
$\pi/3$	38.37	4.65	2.71	12.65	79.14
$\pi/2$	128.96	3.06	0.38	17.90	232.28
π	202.83	0.58	0.22	18.51	312.58

TABLE 4.8. RELATIVE PEAKS AND FREQUENCIES OF POWER FOR SIMULATED AND ACTUAL DATA

Series	Simulated Data		Actual Data	
	Frequency	Greatest Power	Frequency	Greatest Power
Consumption	$\pi/6$	381.90	$\pi/7$	491.42
GDCF	$\pi/7$	15.82	$\pi/7$	48.13
Imports	$\pi/6$	28.96	$\pi/6$	91.19
Exports	$\pi/5$	129.95	$\pi/6$	80.58
GNP	$\pi/5$	988.60	$\pi/7$	496.37

Figures 4.1 and 4.2 depict this more clearly. The results thus further strengthen our claim that our model is a good representation of the Nigerian economic system.

The graphs of the power of our simulated series show concentration of power at low frequencies while tailing off at high frequencies. The graphs thus exhibit what Granger (1966, p. 150) calls the typical shape of the power of most economic variables. It should, however, be emphasized that these 'nice' results are probably a consequence of our employing a simple linear model. The spectral densities for consumption, GDCF, Imports, Exports, and GNP are estimated at points:

$$\theta = 0, \pi/8, \pi/7, \pi/6, \pi/5, \pi/4, \pi/3, \pi/2, \pi.$$

These points correspond respectively to components of period (in years) as, ∞,16, 14, 12, 10, 8, 6, 4, and 2. The period of a power, P, is the reciprocal of frequency, that is:

$$P = \frac{2\pi}{\theta}$$

The spectra are similar in some important respects as Table 4.7 and Figures 4.3, 4.4, 4.5, 4.6, 4.7 show. In the first instance, they all have relative peaks in the low frequency end. Further, consumption, GDCF, Imports, Exports, and GNP appear to have substantial cyclical component of period of about 12, 14, 12, 10, and 10

FIGURE 4.1

[GNP - Actual data]

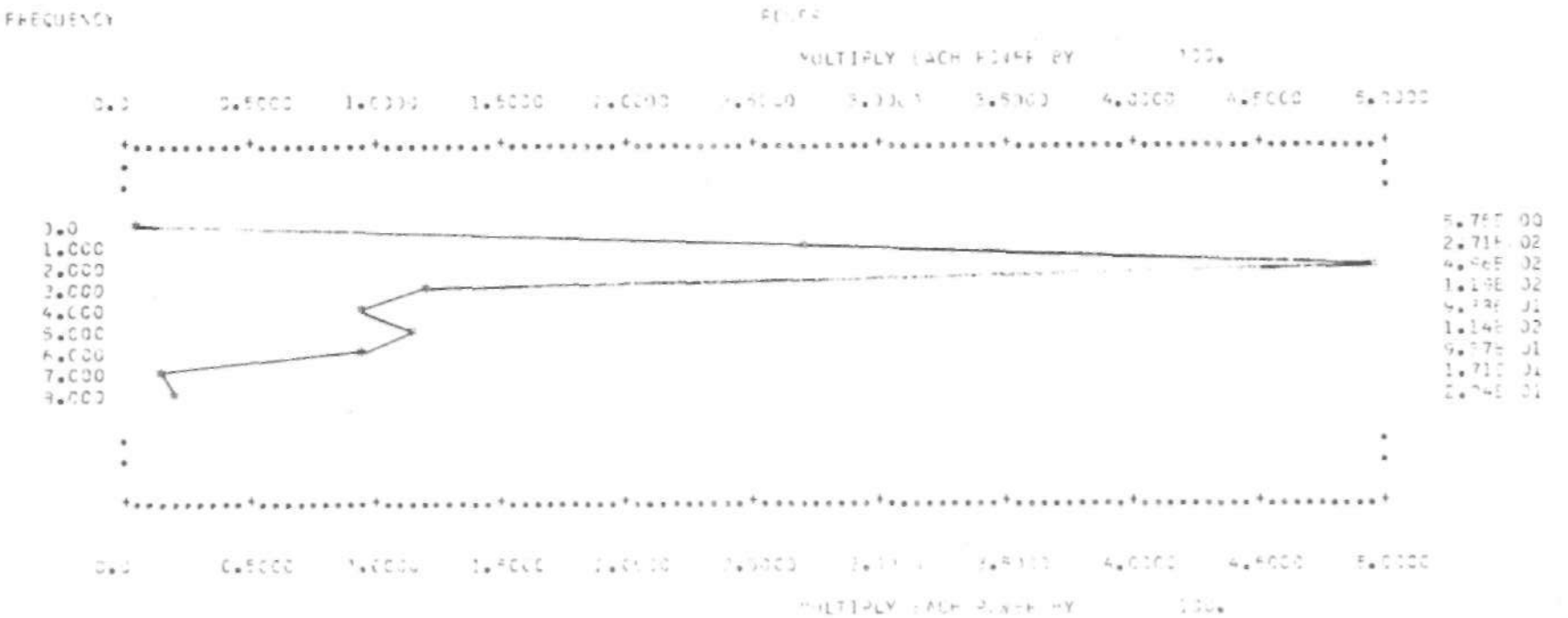


FIGURE 4.2

GRAPH OF THE POWER AT EACH FREQUENCY FOR SERIES 1 [GNP_Simulated data]

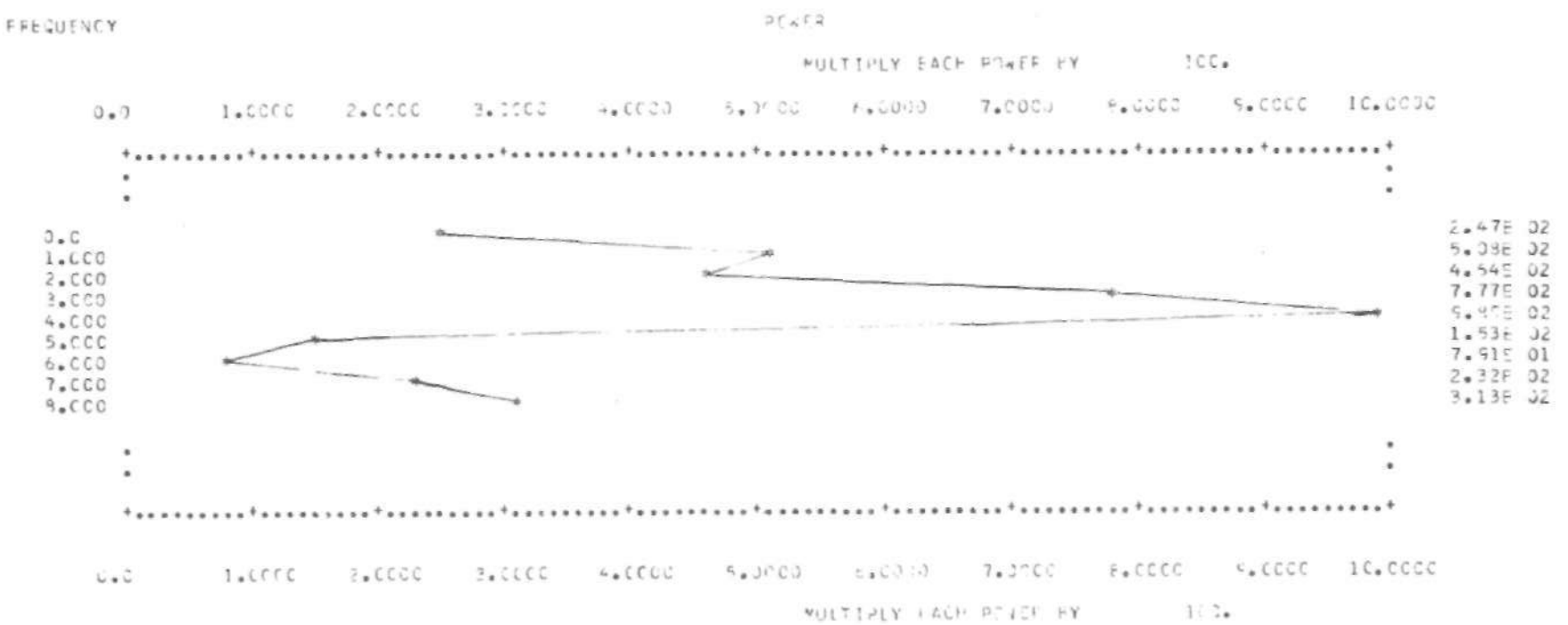


FIGURE 4.3

GRAPH OF THE POWER AT EACH FREQUENCY FOR SERIES 1 (PERSONAL CONSUMPTION)

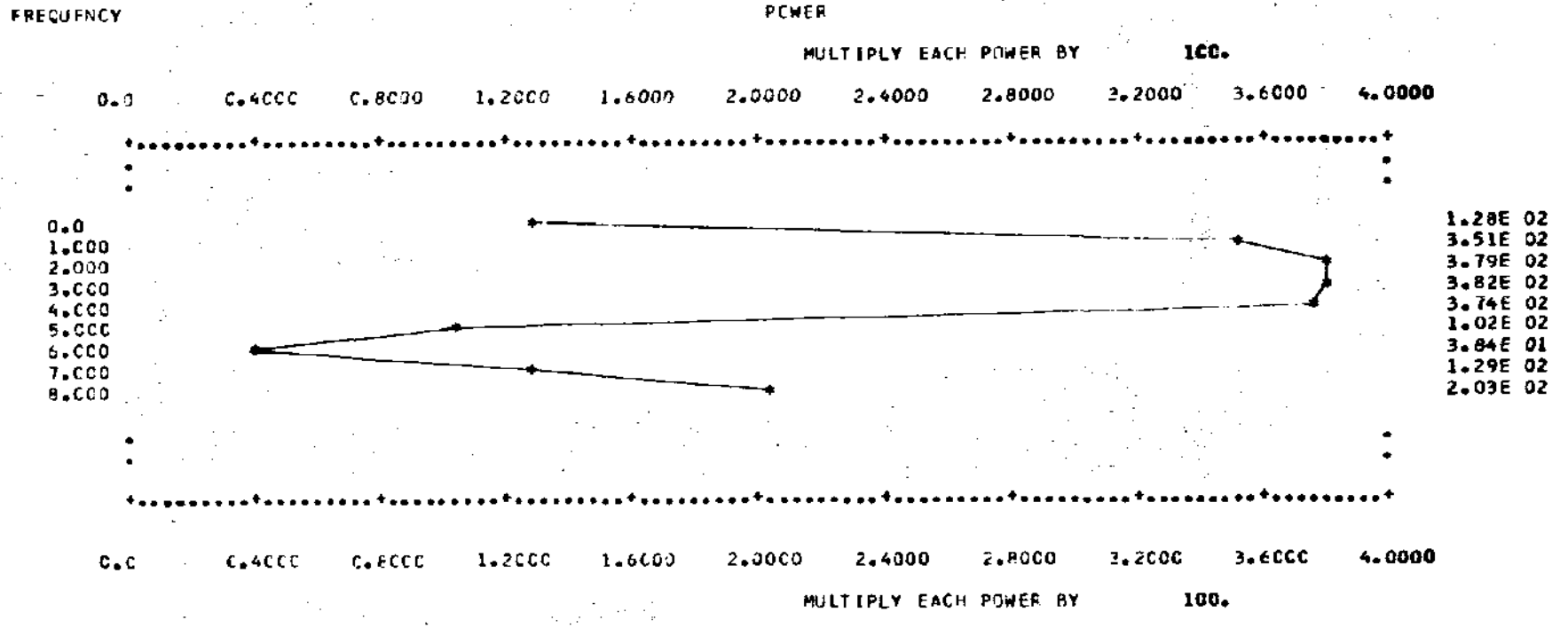


FIGURE 4.4

GRAPH OF THE POWER AT EACH FREQUENCY FOR SERIES 2 (GDCF)

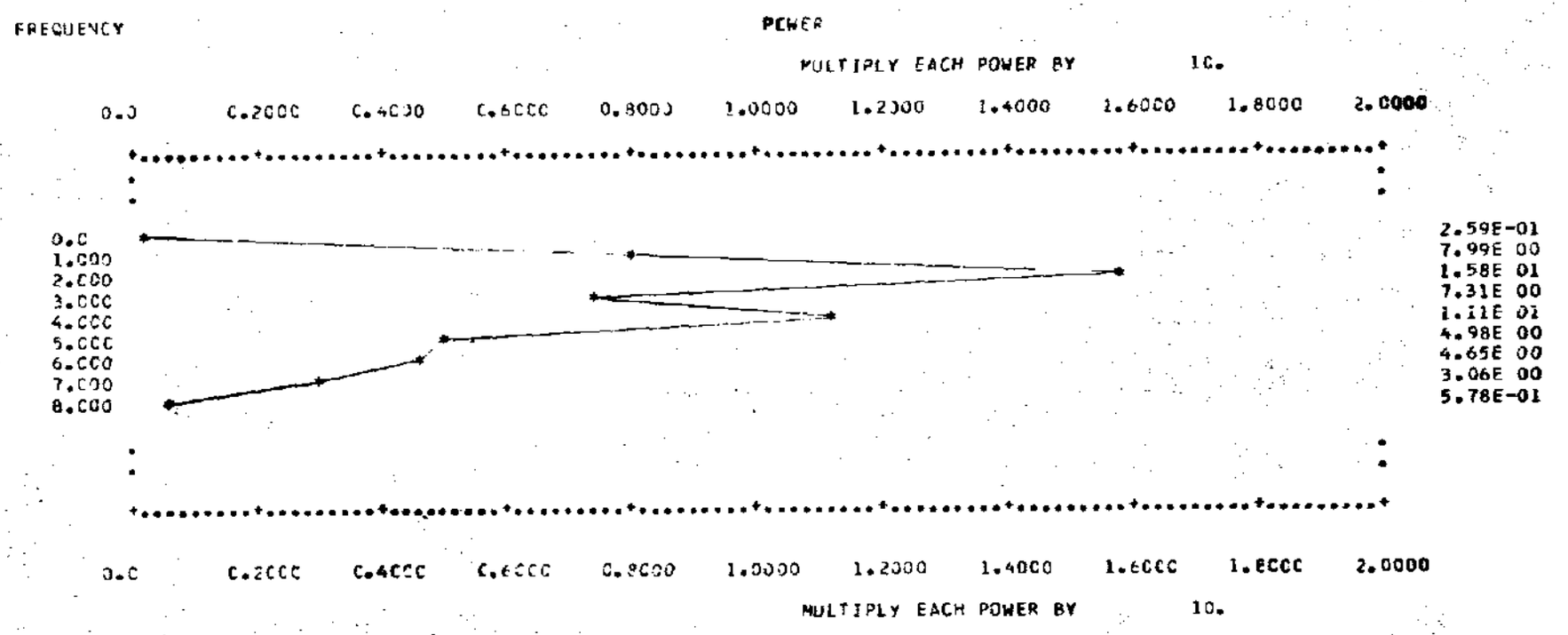


FIGURE 4.5

GRAPH OF THE POWER AT EACH FREQUENCY FOR SERIES 4 (IMPORTS)

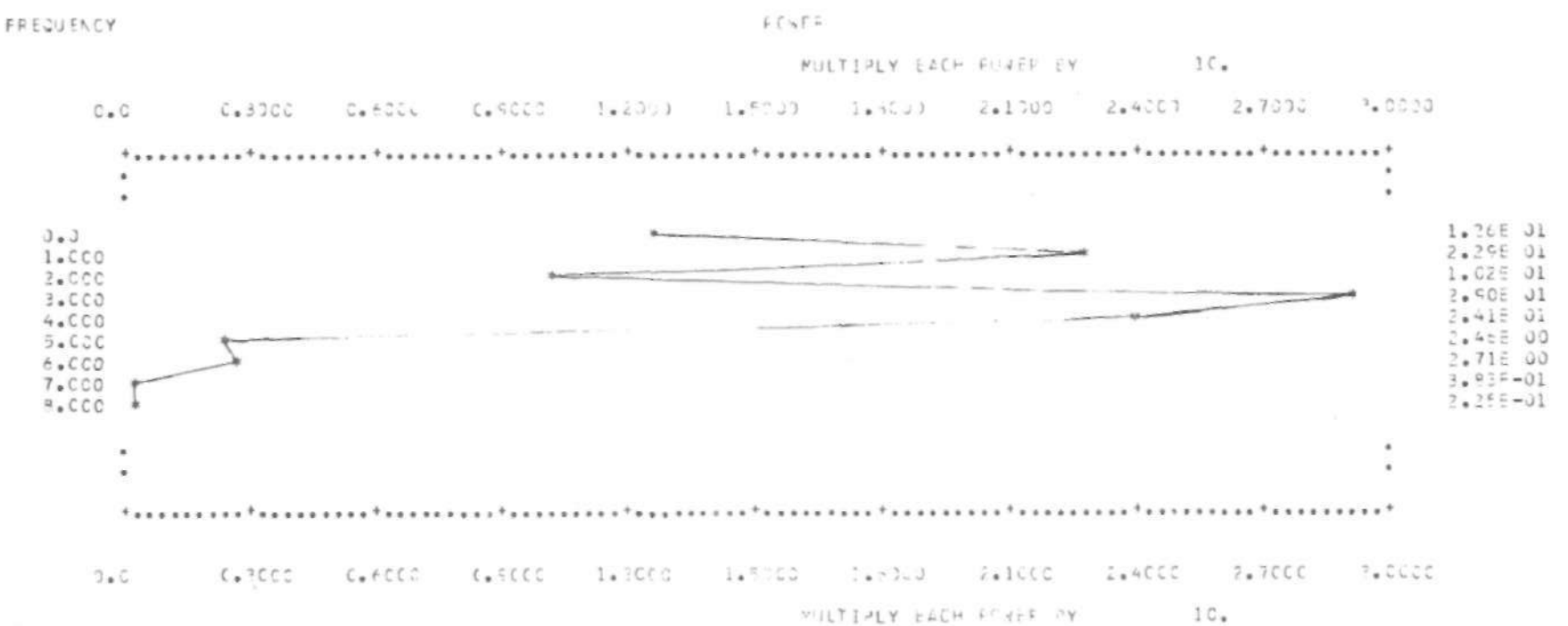


FIGURE 4.6

GRAPH OF THE POWER AT EACH FREQUENCY FOR SERIES 4 (EXPORTS)

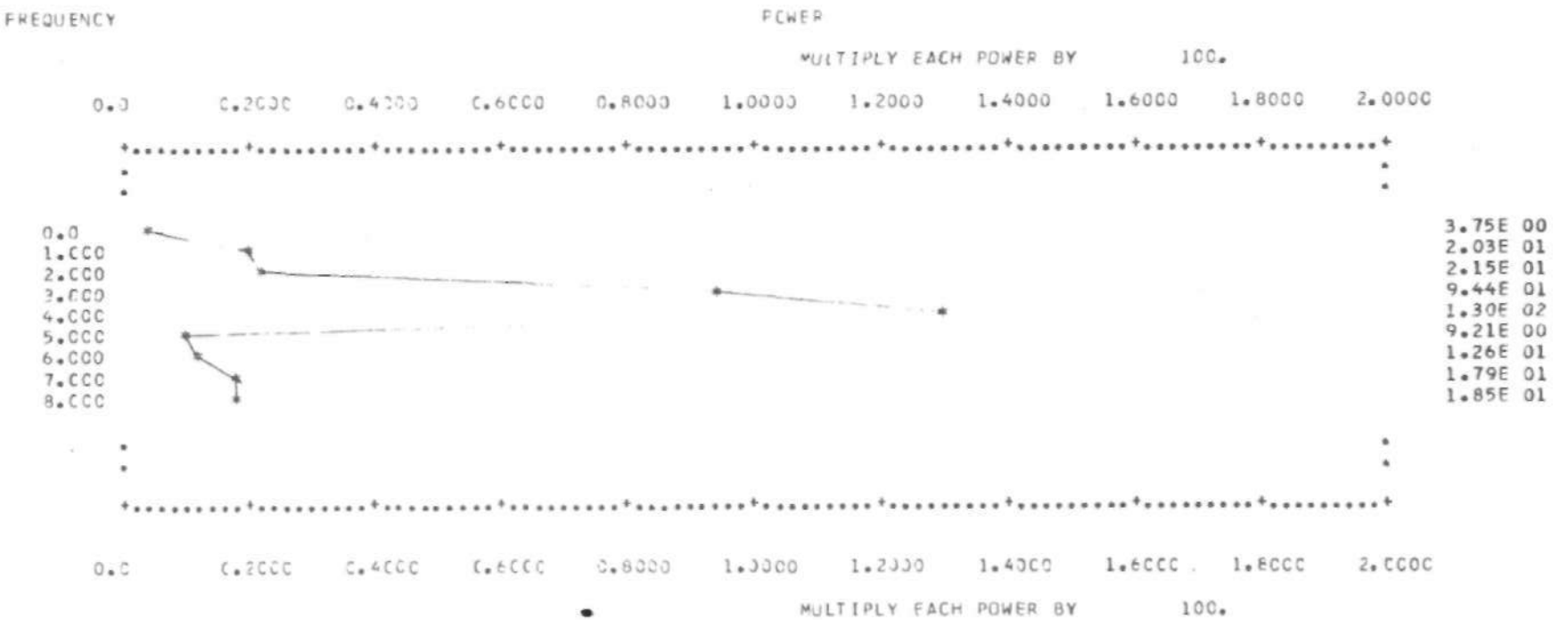
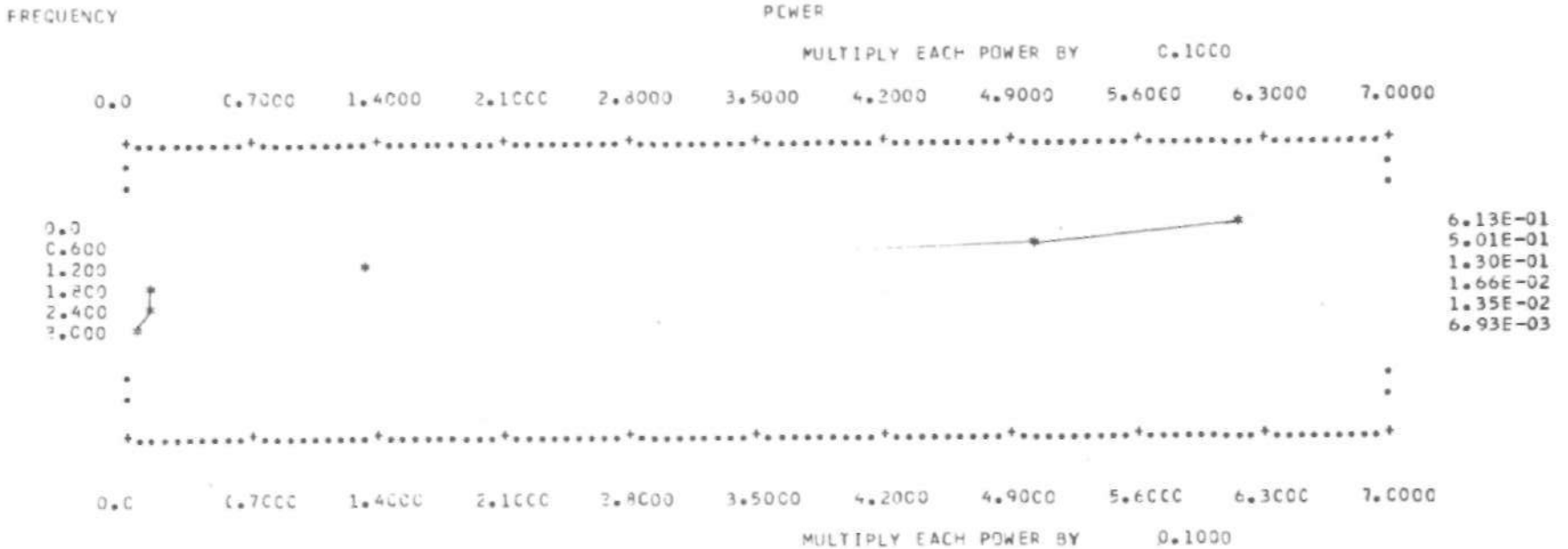


FIGURE 4.7

GRAPH OF THE FREQUENCY WINDOW OF THE POWER SPECTRUM ESTIMATES



respectively. The spectrum period is defined as the inverse of the frequency at which the power spectrum exhibits a relative peak (provided one exists). The substantial cyclical component of period and relative concentrations of power identified in our results indicate that there is tendency for the series to oscillate with some degree of regularity. This implies that the response of the endogenous variables of the system to random disturbances exhibits a fairly regular oscillation with a period of approximately 11 to 12 years per cycle.

An interpretation of a power spectrum is that if a band contributes a large proportion of the total variance, then it may be regarded as more important than a band where the power is less. A frequency is the point at which the power for a band is assumed to be concentrated. Thus the peak for consumption power is not as pronounced as the peak for Exports, and GNP power. For Exports, the frequency $\pi/5$ contributes over 30% of the total variance of the series, while for consumption the frequency $\pi/6$ contributes just about 20% of the total variance of the series. Thus it can be inferred that the export cycle does show a stronger regular tendency than consumption cycle. The relatively weak peak depicted by the consumption spectrum is, however, not surprising. It reflects the volatile nature of the various components of consumption expenditures--a phenomenum very familiar to economists.

Coherence values for each frequency are also estimated. The results are displayed in Tables 4.9, 4.10 and 4.11. The coherency function is very much like the square of the correlation coefficient between two random variables. It is a measure of association between any two variables at particular frequencies. One significant result is the high coherence values obtained for consumption and GNP at all frequency levels. This result is what is expected, however, since we expect the two variables to be positively highly correlated. Exports are also strongly related to GNP at all frequencies. While there is a high degree of fluctuation in the degree of correlation between other pairs of the economic variables considered.

Granger and Hatanaka (1964) advise that "the amount of data required before it becomes sensible to attempt to estimate a spectrum would seem to be greater than 100". We have employed a small sample of 19 observations. We cannot, therefore, claim very much from our results. In particular the rather limited number of observations used creates several problems in connection with the construction of significant tests for spectrum estimates, and significance tests of the oscillations in the economic time series identified in the spectral analysis. Howrey and Kelejian [Naylor (1971, chp. 12)] have effectively used spectral analysis to explore the

TABLE 4.9. CROSS SPECTRA BETWEEN CONSUMPTION AND THE OTHER SERIES

Frequency	Other Series			
θ	GDCF	Imports	Exports	GNP
0	0.984	0.993	0.980	0.682
$\pi/8$	0.524	0.692	0.949	0.707
$\pi/7$	0.688	0.519	0.542	0.764
$\pi/6$	0.474	0.418	0.497	0.807
$\pi/5$	0.842	0.675	0.545	0.684
$\pi/4$	0.976	0.563	0.559	0.224
$\pi/3$	0.961	0.411	0.391	0.529
$\pi/2$	0.793	0.751	0.701	0.631
π	0.242	0.342	0.671	0.975

TABLE 4.10. CROSS SPECTRA BETWEEN GDCF AND OTHER SERIES

Frequency	Other Series		
θ	Imports	Exports	GNP
0	0.969	0.968	0.683
$\pi/8$	0.550	0.614	0.946
$\pi/7$	0.508	0.449	0.986
$\pi/6$	0.925	0.751	0.770
$\pi/5$	0.710	0.876	0.910
$\pi/4$	0.374	0.517	0.069
$\pi/3$	0.145	0.438	0.371
$\pi/2$	0.268	0.497	0.276
π	0.826	0.557	0.221

TABLE 4.11. CROSS SPECTRA BETWEEN THE SERIES

Frequency θ	Imports/Exports	Imports/GNP	Exports/GNP
0	0.993	0.752	0.814
$\pi/8$	0.808	0.698	0.830
$\pi/7$	0.858	0.437	0.415
$\pi/6$	0.934	0.551	0.230
$\pi/5$	0.787	0.902	0.959
$\pi/4$	0.735	0.864	0.881
$\pi/3$	0.814	0.946	0.660
$\pi/2$	0.750	0.968	0.662
π	0.923	0.331	0.667

dynamic properties of Klein's model I for a sample size of 21 observations. And our results have shown that even for a relatively small sample size, spectral representation of an econometric model provides a useful analytical tool of validating the model and of checking its dynamic properties.

Simulation and spectral analysis techniques represent respectively the deterministic and stochastic approaches to investigating the dynamic properties of an econometric model. The choice between the two techniques is not an obvious one. The more important issue, however, is not which of the two techniques we employ, but it is that a model which is to be employed in policy formulation should be subjected to tests beyond 'casual observation'.

CHAPTER V

APPROACHES TO THE POLICY PROBLEM AND PROBLEMS OF COMPUTING OPTIMAL PLANS

In the previous chapters we developed an argument favoring the use of a social welfare function (or surrogate loss function) for economic policy decisions. A quadratic criterion function was suggested, while a linear structural model was assumed and estimated to represent the structure of the economic system. With knowledge of the criterion function and the constraints, policy evaluation is in principal, conceptually straight forward. The principle is to maximize (or minimize) the expected value[1] of the criterion function subject to the constraints formed by the structure of the economic system in order to achieve optimum control.

If it is possible to measure the state of the system exactly, that is, if the system we are dealing with is deterministic, the problem in obtaining the optimum control is considerably lessened. In the real world, however, many processes in economic systems are characterized by uncertainties and decision making (or control) has to be undertaken in such conditions.

Assuming certainty as to the real structural representation of our model, there are other sources of uncertainty in the relationships comprising the model which cannot be ignored because of their far reaching implications with respect to the final solutions to economic planning problems.

Uncertainty may arise from: 1) many approximations, including omitted variables, simplifying mathematical forms, and various forms of aggregation which lead to the specification of stochastic rather than deterministic relationships; 2) capturing only (small) sample estimates of the parameters entering the relationships; 3) structural changes; 4) imperfect information on the future values of the non-controlled exogenous variables; and 5) lack of knowledge of the way in which the marginal utilities are affected by changes in controlled and non-controlled variables.

One solution to the uncertainty phenomena is to specify the model as stochastic processes, thus allowing a characterization of the optimization problem, as one in stochastic optimal control theory. This procedure raises an immediate issue of whether or not there exists an optimal policy in the case of a linear dynamic system with stochastic coefficients as well as additive disturbances. An important result from stochastic control theory, however asserts that given a quadratic objective function,[2]

an optimal feedback policy for such a linear system will exist only if the amount of uncertainty associated with the coefficients is not too great [Wonham (1967, 1969), Turnovsky (1973)]. It is worth pointing out that this 'existence' problem does not arise if the only disturbances are additive; in this case the system will always posses an optimal solution.

A very important concept in stochastic control is the information pattern available to a policy maker at a specific time, for the purpose of decision making. Normally, as the decision process unfolds, additional information becomes available to the policy maker. Additional information could come through addition to sample. The use of sample additions allows learning to take place regarding, for example, the unknown coefficients of the constraint functions. Formal procedures for utilizing sample additions to update probability distribution specifications include Bayesian methods, Least Squares revision methods, and Kalman type filters.[3] Furthermore, additional information could arise from sample experiments through choosing a current policy which is less than optimal from a pure control point of view in order to gain more knowledge of the system. Thus, to quote Dreze (1972, p. 15) "a monoplist may wish to depart from the price which maximizes expected profit, simply to learn more about his demand function" Thus,

in stochastic optima control, 'learning' is present, whether 'accidental' or 'deliberate'. The information pattern available to the policy maker indicates not only what type of learning is possible at each point of time, but more importantly, whether further learning can be anticipated and how it could be influenced by present actions (by a built-in probing device). Since learning may improve overall control performance, the probing signal may indirectly help in controlling the stochastic system. Thus the inherent benefits of the additional information depend upon whether or not an improved representation of the structure results in superior future control. On the other hand, the benefits (promotion of learning) derived from probing should be carefully weighed against the cost of such probing. In general, probing will increase the expected cost of the system. There is, therefore, an interplay between learning and control. This interplay, called 'dual control problem.' is one of the most important aspects of stochastic control theory which is pointed out by Bellman (1961, pp. 198-209) and Fel'dbam (1965).

APPROXIMATE SOLUTIONS

Analytical solutions to dual control problems are unfortunately not yet possible. The intractable nature of the problem is due to the interaction between the

structure of the system and the built-in probing device. This interaction results in highly non-linear functions, the expected value of which can only be evaluated numerically. Moreover numerical solutions procedures rapidly encounter the 'curse of dimensionality' for even modest size control problems.[4] Florentin (1962), and Astrom (1965) have worked out simple examples indicating the dual role of learning and control. A dual control algorithm which is applicable to a family of large class of non-linear stochastic systems has been developed recently by Tse, et al. (1972). These studies present some simulation results indicating the differences between active and passive learning control strategies.

With respect to linear stochastic systems, the state of the art implies a resort to approximate solution procedures. These procedures involve some alterations or modifications of the original structure of the problem. The severity of these alterations dictates the degree of approximation of the control strategies to the optimal control solutions. The approximate solution procedures that have been advanced in the literature includes:[5] 1) the certainty equivalence controls; 2) the sequential stochastic controls; and 3) the adaptive controls.

The above procedures are enumerated in degree of complexity and approximation, with certainty equivalence controls relatively the easiest. The derivation of

analytical expressions for the certainty equivalence and sequential stochastic controls may be characterized by either the Pontryagin Maximum Principle (in its discrete form) or by dynamic programming. Pontryagin's (1961) Maximum Principle is a very important idea in control theory. Assume an optimum control exists and that we have a social welfare function denoted by:

$$J = \sum_{t=0}^{T} (1 + r)^{-t} U[Y_t, C_t, X_t] \qquad \ldots . (5.1)$$

Y_t, C_t, X_t represent state variables, control variables, and other exogenous variables respectively. r is an assumed rate of discount indicating time preference allowance. The system is described by the first order difference equations:

$$Y_t = \phi[Y_{t-1}, C_t, X_t], \quad t = 1, 2, \ldots, T$$

$$Y_o = \bar{Y}_o \qquad \ldots . (5.2)$$

The control theory problem then is to optimize the social welfare function J, subject to the constraints of the system characterized by (5.2). We form a Hamiltonian

$$H = (1 + r)^{-t}\{U[Y_t, C_t, X_t + \phi(Y_t)]\}$$

Maximizing J implies minimizing H; and from

$$\frac{\partial H}{\partial C} = 0$$

we determine the optimal control $\bar{C}_t$.

Dynamic programming is a procedure for solving these sequential decision making problems.[6] The essence of this technique is to break a multistage decision process into a series of single-stage problems so that the problem becomes soluable. At each of the stages of the decision system, it is assumed that there exists a transformation function describing the output state of the system as a function of the input state and decision, a set of decision variables that enable a decision maker to control the process, and a stage return that measures the utility in terms of input variables, output variables, and decisions. Under appropriate assumptions[7] we can decompose the objective function, and a multivariable optimization model can be solved by utilizing recursive optimization. That is, we repeatedly apply the optimality criteria which states that an optimal set of decisions has the property that regardless of the first decision, the subsequent decisions must be optimal with respect to that outcome which arises from the first decision.

In the case where we have a set of preference functions, the models described above permit sensitivity analysis of alternative policy strategies, thus providing useful and formal insights about the trade-offs implied by conflicting economic objectives. Sensitivity analysis

of the optimal solutions of a programming model may be approached in several ways. Saaty (1959), Barnett (1962), Courtillot (1962) and Williams (1963) approach the problem by analyzing the impact of the perturbation of the parameters on the extremum value of the objective function. Babbar (1955), and Tintner (1960) suggest analyzing the statistical distribution of the extrenum value of the objective function under alternative variations in the random components of the parameters of a programming problem. While Tintner investigates the variations of the spread between the optimum value of the objective function and any other value corresponding to a basic feasible solution vector.

The applicability of dynamic programming technique to policy situations has a number of limitations. First, not more than two instrumental variables can be considered, otherwise, the problem becomes untractable. Secondly, the assumptions of separability and monotonicity required in order to decompose the objective function, limit the application of the approach [Nemhauser, 1966, p. 56]. And thirdly, the many computations required tend to become rather burdensome, although if 'certainty equivalence' conditions discovered by Simon (1956) and generalized by Theil (1968) are satisfied, the number of computations required for the solution can be reduced.

Certainty Equivalence and Optimal Feedback Rule

Methods explicitly admitting uncertainty and learning processes are, however, available for the econometric representation. Among such methods are linear decision rules formulated for maximizing, in a dynamic situation, a quadratic preference function under linear constraints when uncertainty is involved [Holt, Modigliani, Muth and Simon (1960), and Theil (1957, 1964)].

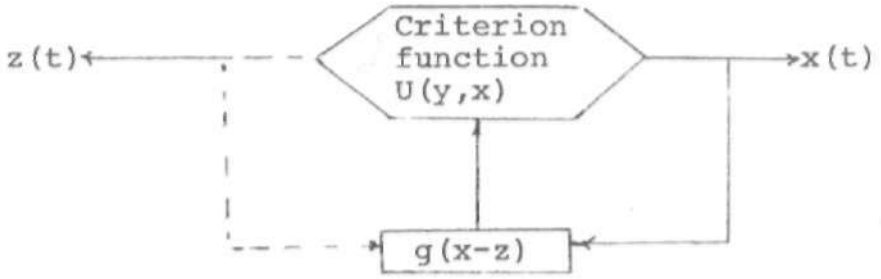
The basis of the theory of dynamic linear decision rules is the first period certainty equivalence [Simon (1956) and Theil (1957)]. An optimal strategy formulated according to the certainty equivalence theorem says that the first period decisions of the maximizing strategy are identical with the first-period decisions which are obtained by maximizing the quadratic preference function subject to linear constraints after the unknown additive random variables are replaced by their mathematical expectations as they are known at the start of the first period. The first-period certainty equivalence result generalizes to a dynamic problem. By reformulating the maximization problem in such a way that earlier decisions are taken as given, the decisions to be derived become the first-period decisions; that is, the second period becomes the first, the third the second, and so on. Information which becomes available about past periods is taken as given for the current period in which decisions

must be made. Thus decisions for successive periods are obtained by shifting the period of the problem. Theil (1968) has also proved that, in the static case, a first order certainty equivalence theorem still holds even when the coefficient vectors and matrices occurring in the quadratic function and in the linear constraints are stochastic. More recently Malinvaud (1969) demonstrated that the certainty equivalence property can be generalized into the dynamic case as well, except for singular cases, as long as the various functions involved are twice differentiable.

The above method is computationally efficient only if: 1) the coefficient vectors and matrices of the constraints have regular structure; that is, the corresponding coefficients for different periods are the same, apart from initial and final effects; and 2) the number of periods taken into account is infinite or very large; in which case the optimal decisions for the period immediately ahead are always the same linear functions of the mathematical expectations of the unknown variables for the periods. In most empirical applications, however, the number of periods taken into account does not approach infinity; typically a far shorter horizon is of interest and the structure of coefficient vectors and matrices may not be regular.

In the case when conditions 1, and 2, are not fulfilled, decisions generated by certainty equivalence methods cease to be optimal. We can, however, utilize this very deviation of the derived policies from their optimal level as a restoring force to guide subsequent choices of decisions. Schematically, we have:

Figure 5.1



The interpretation of Figure 5.1 is the following: We have a criterion function U(y,x) acted upon by a set of decisions x(t) which turn out to be sub-optimal when all information becomes available. z(t) is the set of decisions that would have been taken if all information were available at the time the decisions are taken. The two sets of decisions are then compared, resulting in a forcing influence g(x-z) which is exerted on the criterion function in a subsequent point of decision making. As a result of this feedback, the equation actually governing the behavior of the system over time takes the form:

$$\frac{dx}{dt} = h[x, g(x-z)],$$

$$x(o) = c \qquad \ldots . (5.3)$$

The way the feedback information is utilized determines the form of the function g, which in turn determines the form of the equation. In many cases the control is applied in such a way that equation 5.3, takes the simple form:

$$\frac{dx}{dt} = h(x) + g(x-z),$$

$$x(o) = c$$

The above is the basis of an algorithm developed by van de Panne (1965) as a numerical technique for obtaining optimal decisions when the certainty equivalence conditions (regularity and infinite time periods) are not fulfilled. Essentially, the 'linear feedback form' as the technique is called, recognizes and incorporates the fact that past decision errors cause decisions to differ from the decisions which would have been taken if no errors had been made. Van de Panne developed an expression which expresses the optimal decisions for period t, given the information available at the beginning of that period and given the decisions for periods before t as being equal to the optimal decisions for period t if all earlier decisions had been optimal plus a linear combination of decision errors made in earlier periods as perceived at the beginning of period t. Van de Panne (1965, p. 308) defines a decision error for period t-1 as the difference between the decisions for period t-1 based

on the information available at the beginning of period t-1 (given the decisions for periods before t-1) and the decisions for period t-1 which would have been optimal given the information available at the beginning of period t (given the decisions for periods before t-1). Computationally the difficulty of the feedback form increases linearly with increases in the number of controlled and uncontrolled variables, but remains manageable for problems of fairly large dimension.

Sequential Stochastic Controls and Adaptive Controls

Provided the criterion function is quadratic and the econometric model linear with known coefficients, the techniques described above provide a relatively simple means of finding optimal policy. Unfortunately, however, in most macroeconomic applications, the coefficients are not known with certainty and are normally estimated using limited number of historical observations. Besides the uncertainty about the coefficients arising from use of small samples, is the problem of structural changes. Kydland (1973), Lucas (1973), and Lucas and Prescott (1971) have investigated the effect of policy rule upon the parameters of structural equations and on the basis of their results argue that the expectations of future behavior of economic agents are not invariant to the rule being used for policy selection. Thus, as expectations of future behavior changes, the structure of the system shifts,

resulting in a drift in the parameters. This drift, in large part, reflects the adaptation of the economic agents to the changing character of policy makers' decisions. Hence any policy valuation ignoring this source of drift will lead to inferior policy control.

As noted earlier, Wonham (1967), and Turnosky (1973), have indeed shown that, for linear dynamic systems and quadratic objective functions, an optimum control exists provided the disturbances in the parameters are not too great and provided the parameters are independently distributed through time. Prescott (1971), Zellner (1971), and Tse et al. (1973) have investigated further the problem generated by disturbances in the parameters using Bayesian approach, while Taylor (1973) has looked at the same problem from a non-Bayesian point of view. Essentially, they all suggest sequential decision rules procedure as an approximate solution to the problem. This procedure consists of a sequence of optimizing actions (open-loop sub-problems). These sub-problems begin in each period of the planning horizon with an initial value. As policy decisions are made and as we proceed through time, however, additional information becomes available. Such additional information is utilized to 'update' the value and covariance of the parameters of the constraints. The revised estimates are then employed as prior information for the next open-loop sub-

problem; and so on. Provided experimentation is not a relevant consideration, it has been demonstrated [Prescott (1971), p. 373; Zellner (1971), p. 343] that sequential stochastic control will be optimal.

But as we have earlier discussed, we often design experiments in order to improve our knowledge of the parameters; that is, experimentation is at times a relevant consideration. In this case, sequential stochastic control procedures will not yield optimal decisions. Adaptative stochastic control is suggested as an approximate solution to this dual control problem [Zellner (1971, chp. 11)]. Adaptive control recognizes the fact that the setting of control variables in one period affects, in an important way, the information regarding parameter values for other time periods. Like the sequential control procedure, adaptive control provides a sequence of optimizing actions. The settings of the control variables through time take account of not only what we learn from new data, but also the combined and inter-dependent goals of control and learning about parameter values. Because the future actions embodied in the control solutions involves the adaptating of the policy makers actions to new data as they appear, the multiperiod control problem is referred to as an 'adaptive control problem.'

Criteria for Evaluating Control Rules

Depending on what assumptions we are prepared to

make regarding the probability distribution of the parameters and the additive error terms of our constraints, each of the techniques discussed above would yield optimal controls. The choice among the procedures, therefore, rests very much on our knowledge of the economic system in general and on the structure of the model, in particular. Analytical procedures for selecting among the techniques include observing and comparing the loss functions of the control rules [Prescott (1971), Zellner (1971)]. As a general rule, the smaller the loss function the better the control rule.

More recently, Taylor (1973) has developed another criterion for evaluating control rules in multiperiod problems with unknown parameters.[8] Taylor's criteria is defined as the limit, as the number of time periods approaches infinity, of the ratio of the sum of regrets in each period to an appropriate function of the number of time periods. Regrets are the deviation from the 'bliss level' of expected loss function due to the unknown parameters. The 'bliss level' is the minimum value of the expected loss in each period when the parameters are known. The criterion is closely related to the concepts of consistency and asmptotic efficiency of statistical estimation theory--a relationship that makes the Taylor's criteria operational. Applying this criterion, for example, Taylor (1973) shows, for a particular model, that the certainty

equivalence control rule performs nearly as well as any control rule designed for experimentation. If this result is shown to hold for more general models, then the practical use of certainty equivalence rule in models with long time horizon might be acceptable after all.

In the next chapter, we discuss the certainty equivalence technique in more detail and apply the technique to the Nigerian model. Although we shall not have the luxury of a long time horizon, we shall show that acceptable results can be obtained using the certainty equivalence technique.

CHAPTER VI

DETERMINISTIC AND CERTAINTY EQUIVALENCE TECHNIQUES

This chapter begins with a brief discussion of deterministic control techniques suggested in the literature. In particular, Tinbergen's and more recently Pindyck's and Chow's techniques are sketched. We then apply the certainty equivalence technique suggested by Simon and Theil as an approximate solution to optimal decision control to the Nigerian model. A three year (period) planning horizon is assumed.

Tinbergen's (1952) contribution to economic policy formulation was discussed earlier in Chapter I. The main features of his approach include a welfare function W, which is a scalar valued function of target variables, the instruments and random terms U. that is:

$$W = W\ (Y_1, Y_2, \ldots, Y_I;\ Z_1, Z_2, \ldots, Z_J;\ U)$$

where,

Y and Z represent targets and instrument, respectively. A linear relationship between these two variables, constituting the constraints of the economic system, is assumed to exist.

Assuming differentiability, the optimum set of

instruments by Tinbergen's approach is given by solving the following set of equations:

$$\frac{dW}{dZ_j} = \sum_i \frac{\partial W}{\partial Y_i} \frac{\partial Y_i}{\partial Z_j} + \frac{\partial W}{\partial Z_j} = 0 \qquad \ldots . \quad (6.1)$$

These equations are the necessary conditions for an extremum (optimum) provided that the random term U is identically zero and the second order condition for a maximum or minimum is fulfilled.

The term $\frac{\partial Y_i}{\partial Z_j}$ in equation (6.1) expresses the effectiveness of the instrument Z_j in inducing a change in the value of the target Y_i when all other instruments are held constant. Thus, the term:

$$\sum_i \frac{\partial Y_i}{\partial Z_j}$$

expresses the sum of all 'marginal' (i.e., partial) effects of a unit change in a given instrument Z_j on the variable Y_i. The efficiency[1] of the instrument Z_j is specified by the term $\frac{\partial W}{\partial Z_j}$ (and additionally by the second partial derivative) subject to the boundary constraints.

Tinbergen provides a relatively simple approach to policy formulation. His approach, however, suffers from the fact that unique values of instruments exist if and only if there are as many instruments as there are targets. Thus, in the Nigerian model where we specify five targets

but only one instrument, Tinbergen's approach has a limited application.

More recently, Pindyck (1971), Ku and Athens (1972), and Chow (1972) suggest a technique which avoids the restrictions inherent in Tinbergen's approach. Their technique is, however, similar to Tinbergen's technique in that they assume the error term is identically equal to zero and the parameters, and variables of the constraint equations are known with certainty. Following Pindyck (1972), we assume a quadratic loss function,

$$Q = 1/2 \sum_{t=1}^{T} [(Y_t - Y_t^*)K(Y_t - Y_t^*) + (X_t - X_t^*)R(X_t - X_t^*)] \quad ...(6.2)$$

and a linear econometric model of the form:

$$Y_t = AY_{t-1} + BX_t + b_t \quad ...(6.3)$$

and $\bar{Y}_o = Y_o$

where,

Y_t and X_t are target variables and instruments respectively (the star indicates desired levels). K is a T x T positive semi-definite matrix, and R is an r x r positive definite matrix. The diagonal elements of K, some of which may be zero, indicate the relative costs of deviating from the nominal path of each target variables--for example, the cost of deviating from GNP relative to the cost of deviating from balance of payment levels.

The diagonal elements of R (all of which must be non-zero) are similarly interpreted.[2] They give the relative costs of deviating from the paths of the control variables.

The optimal control problem is then to find a control sequence:

$$[\hat{X}_t,\ t=1,\ldots,\ T]$$

such that $\hat{X}_t$ and hence $\hat{Y}_t$ satisfy equations (6.2) and (6.3). The main idea is to steer Y_t close to the target Y_t^* by choosing appropriately the control variables X_t.

Introducing lagrange multipliers λ_t, we want to minimize the function:

$$L = 1/2 \sum_{t=1}^{T} [(Y_t-Y_t^*)K(Y_t-Y_t^*)+(X_t-X_t^*)R(X_t-X_t^*)] -$$

$$\sum_{t=1}^{T} \lambda_t(Y_t-AY_{t-1}-BX_t-b_t)$$

Differentiating the Lagrange expression with respect to X_t and Y_t we have,

$$\frac{\partial L}{\partial X_t} = R_t(X_t-X_t^*) + B_t'\lambda_t = 0,\ (t=1,\ldots,\ T) \quad \ldots .(6.4)$$

$$\frac{\partial L}{\partial Y_t} = K_t(Y_t-Y_t^*) - \lambda_t + A_{t+1}'\lambda_{t+1} = 0\ ,$$

$$(t=1,\ldots,\ T-1) \quad \ldots .(6.5)$$

From (6.4) and (6.5) and noting that $\lambda_{t+1} = 0$, we have:

$$R_T(X_T - X_T^*) + B_T'K_T(Y_T-Y_T^*) = 0 \quad \ldots .(6.6)$$

Substituting (6.3) into (6.6) we have,

$$R_T(X_T - X_T^*) + B_T'K_T(A_TY_{T-1} + B_TX_T + b_T - Y_T^*) = 0$$

i.e., $(R_T + B_T'K_TB_T)X_T = -B_T'K_TA_TY_{T-1} + R_TX_T^*$

$$-B_T'K_Tb_T + B_T'K_TY_T^*$$

$$\hat{X}_T = -(R_T+B_T'K_TB_T)^{-1}(B_T'K_TA_T)Y_{T-1}$$

$$-(R_T+B_T'K_TB_T)^{-1}[R_TX_T^*-B_T'K_Tb_T$$

$$+B_T'K_TY_T^*]$$

Thus,

$$\hat{X}_T = G_TY_{T-1} + g_T \qquad \ldots(6.7)$$

Where,

$$G_T = -(R_T+B_T'K_TB_T)^{-1}(B_T'K_TA_T)$$

$$g_T = -(R_T+B_T'K_TB_T)^{-1}[R_TX_T^*-B_T'K_Tb_T+B_T'K_TY_T^*]$$

To find the control for period T-1, we follow the above procedure replacing T with T-1 and so on. The result is a set of recursive equations, which begin with the last period of the decision horizon and end with the first decision period.

The result (6.7) is the so-called optimal closed-loop linear feedback control rule.[3] Each optimal decision $\hat{X}_t$ is expressed as a function of Y_{t-1} (which is already observed). This control rule tends to be 'self-correcting', that is, if random shocks are introduced into the system so that at different time Y_t moves away from its optimal value

Y_t^*, the resulting optimal control (in the following period) will force the state variables towards their optimal paths.

APPLICATION OF CERTAINTY EQUIVALENCE THEOREM

If we relax our assumptions and allow for an error term (additive term) in equation (6.3), we have a stochastic model, and in which case, the techniques suggested above by Tinbergen, Pindyck and others do not provide optimal decisions. In the case when the parameters of the system are known with certainty and the only stochastic elements are the additive terms, it has been shown that an optimal control is attainable by appealing to 'certainty equivalence' theorem [Simon (1956), Theil (1957)]. Certainty equivalence theorem was defined in Chapter V. Essentially, it suggests replacing stochastic additive terms with their mathematical expectations as known at the start of the period. Decisions for successive periods are obtained by shifting the periods of the problem forward.

The first step in an empirical application of the certainty equivalence technique is the formulation of an objective function (quadratic or linear-quadratic). In Chapter II we discussed the need and problems associated with an aggregate welfare function. A quadratic welfare function was suggested, while noting the limitations of such a function. The objective function should include the primary objectives of the policy makers. Such

objectives may, however, be conflicting in nature. But the maximizing principle adopted in certainty equivalence approach takes these conflicts into account and gives optimal solutions under regularity and infinite periods assumptions mentioned in Chapter V.

In the Nigerian example, we postulate general targets (objectives) which include increase in immediate welfare measured by private consumption, increase in future welfare measured by GDCF and GNP, and the achievement of favorable balance of payment. We assume then that our criterion function can be reasonably approximated by the sum of quadratic forms:

$$Q = (C-C^*)^2+(I-I^*)^2+(B-B^*)^2+(GNP-GNP^*)^2 + (G-G^*)^2 \quad \ldots (6.8)$$

where,

C and I are as defined earlier, G is government expenditures and B is trade balance. As before, stars on these variables indicate desired values. We note two important features of Q:

1. the function is quadratic to reflect that there is an increasing social cost of either over or under fulfilling our objective; and

2. both non-controlled variables and instruments are its arguments.

It is highly improbable that all the desired values will be realized exactly. The objective then is to

minimize the squared differences between actual and desired values subject to the constraints imposed by the economic system characterized by equation (6.3). The criterion function is therefore stochastic and impossible to minimize.[4] Hence the control problem is formulated as the minimization of the mathematical expectation of the criterion function.

The general form of Q can be written as:

$$Q(X,Y) = a'X + b'Y + 1/2(X'AX + Y'BY + X'CY + Y'C'X) \quad ..(6.9)$$

where,

a, b, A, B, C are column vectors and matrices of appropriate orders, and X and Y are instruments and non-controlled variables respectively. Expressing the variables in deviation form and comparing (6.8) and (6.9), the minimization of the mathematical expectation of Q is equivalent to the maximization of the mathematical expectation of:

$$Q(x,y) = - 1/2(xx + yy) \quad . . .(6.10)$$

Thus,

$$A = B = -I, \; C = 0$$

where,

I and O are unit and zero matrices respectively of appropriate orders. Since we define the instruments and non-controlled variables as deviations from desired levels,

$$a = b = o$$

We now turn to the structure of the Nigerian economic system as expressed in reduced form of the econometric model.

$$Y_t = AY_{t-1} + BX_t + b_t + u_t \qquad \ldots\ldots(6.11)$$

Thus,

$$Y_1 = AY_o + BX_1 + b_1 + u_1$$

and

$$Y_1 - Y_1^* = AY_o + B(X_1 - X_1^*) + b_1 + BX_1^* - Y_1^* + u_1$$

i.e.,

$$y_1 = Bx_1 + b_1 + AY_o + BX_1^* - Y_1^* + u_1$$

$$y_1 = BX_1 + S_1 \qquad \ldots\ldots(6.12)$$

Where,

y's and x's are in deviation (from desired values) forms, and,

$$S_1 = b_1 + AY_o + BX_1^* - Y_1^* + u_1$$

Similarly,

$$Y_2 = AY_1 + BX_2 + b_2 + u_2$$

$$y_2 = Ay_1 + Bx_2 + b_2 + BX_2^* + AY_1^* - Y_2^* + u_2 \qquad (6.13)$$

substituting (6.12) in (6.13)

$$y_2 = ABx_1 + Bx_2 + AS_1 + b_2 + BX_2^* + AY_1^* - Y_2^* + u_2$$

$$= ABx_1 + Bx_2 + S_2$$

where,

$$S_2 = AS_1 + b_2 + BX_2^* + AY_1^* - Y_2^* + u_2$$

We obtain y_3 in a similar way:

Thus,

$$y_3 = A^2Bx_1 + ABx_2 + Bx_3 + AS_2 + b_3 + BX_3^*$$

$$+ AY_2^* - Y_3^* + u_3$$

$$= A^2Bx_1 + ABx_2 + Bx_3 + S_3$$

where,

$$S_3 = AS_2 + b_3 + BX_3^* + AY_2^* - Y_3^* + u_3$$

Thus we have:

$$y_1 = Bx_1 + S_1$$

$$y_2 = ABx_1 + Bx_2 + S_2$$

$$y_3 = A^2Bx_1 + ABx_2 + Bx_2 + Bx_3 + S_3$$

i.e.

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} B & 0 & 0 \\ AB & B & 0 \\ A^2B & AB & B \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} S_1 \\ S_2 \\ S_3 \end{pmatrix} \quad \ldots\ldots(6.14)$$

It is easy to see that the matrix of coefficient associated with the vector x is composed of the final forms of the set of structural equations (2.2) to (2.6); i.e.

$$B,\ AB,\ A^2B,\ \text{etc.}$$

(6.14) can be written in a compact form as:

$$y = Rx + S \qquad \ldots\ldots(6.15)$$

where,

$$R = \begin{vmatrix} R_{11} & 0 & 0 \\ R_{21} & R_{22} & 0 \\ R_{31} & R_{32} & R_{33} \end{vmatrix}$$

$$= \begin{vmatrix} B & 0 & 0 \\ AB & B & 0 \\ A^2B & AB & B \end{vmatrix}$$

Thus,

$$R_{11} = R_{22} = R_{33} = B$$

$$R_{21} = R_{32} = AB$$

$$R_{31} = A^2B$$

R_{11} describes the effect of current instruments on current non-controlled variables. R_{21} shows the effect of current instruments on next period non-controlled variables. And so on. It is important to note that for certainty

equivalence theorem, R is assumed to be a non-stochastic matrix. The zero matrices occurring to the right of the diagonal blocks imply that none of the non-controlled variables depend on later instruments. For instance, present period consumption expenditure level is not expected to depend on next period government expenditures.

The stochastic column vector

$$S = \begin{pmatrix} S_1 \\ S_2 \\ S_3 \end{pmatrix}$$

is composed of: 1) the constant terms of the reduced form equation; 2) the initial value of the non-controlled variable; 3) the desired values of both non-controlled and controlled variables; 4) non-instrument exogenous variables; and 5) a linear combination of the error terms of the reduced form equation. The first four components are known and can be represented by a constant σ. Regarding the linear combination of the error terms, their coefficients are assumed constant overtime. Thus,

$$S = \sigma + su$$

and,

$E(S) = \sigma$, since $E(u)$ is assumed equal to zero.

Pooling together the criterion function (6.10) and the linear constraint (6.15), and applying certainty equivalence theorem, we obtain optimal decision controls by maximizing:

$$EQ(x,y) = -1/2\ E(x'x + y'y) \qquad \ldots\ldots(6.16)$$

subject to:

$$E(y) = Rx + E(S) \qquad \ldots\ldots(6.17)$$

substituting (6.17) into (6.16) and differentiating the result with respect to x, we have the first period optimal controls as:

$$\hat{x} = -(I+R'R)^{-1}R'E(s) \qquad \ldots\ldots(6.18)$$

$$= -K^{-1}h$$

where,

$K = (I + R'R)^{-1}$ and is a non-negative definite matrix,[5] and $h = -R'ES$

The result (6.18) implies that the optimal controls are functions of the S_i's. In particular:

$$\hat{x}_1 = f(S_1)$$

$$\hat{x}_2 = f(S_1, S_2)$$

$$\hat{x}_3 = f(S_1, S_2, S_3)$$

To obtain the second period optimal controls, we just shift the periods. The second period now becomes the first period, the third the second, and so on. The second period optimal controls are derived from:

$$\begin{bmatrix} y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} R_{22} & 0 \\ R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} S_2 + R_2\hat{x}_1 \\ S_3 + R_3\hat{x}_1 \end{bmatrix}$$

and the third period optimal controls are obtained from:

$$y_3 = R_{33}x_3 + S_3 + R_{31}\hat{x}_1 + R_{32}\hat{x}_2$$

OBTAINING THE DESIRED VALUES OF NON-CONTROLLED VARIABLES AND INSTRUMENTS

The 1966-1969 crisis in Nigeria did threaten the economy. Most economic indicators point towards a slack in the economy at this period.[6] It is, therefore, assumed in this exercise that the policy makers within the period 1967 to 1969 inclusive were determined to restore the economy to at least, its pre-crisis growth path or trend. We focus our attention to this period and formulate quantitative targets of economic policy in order to realize the policy makers objectives characterized by (6.8). It should, however, be emphasized that the quantitative targets are derived in a more or less arbitrary way, and as such other sets of desired values can be obtained based on different sets of assumptions.

In the Nigerian case, we assume the policy makers wanted to: 1) restore the pre-crisis average annual growth rate in private consumption of 5%; that is:

$$C_t^* = 1158.6(1 + .05)^t, \quad (t = 1,2,3)$$

2) devote 20% of GDP annual to capital formation as against 15% pre-crisis rate. A substantial amount of damage was done to the infrastructure such as roads, machinery and equipment, vehicles, and buildings during the crisis, hence the need to step up the level of capital formation. Thus,

$$I_t^* = .2\ GDP_t\ , \quad (t = 1,2,3)$$

3) with the import restrictions and favorable exports, due in particular to petroleum, 1967 to 1969 was a period of favorable trade balance. It is assumed in our example that the policy makers desired this level of trade surplus to be maintained, 4) realize an annual growth rate of 6% in GNP. The pre-crisis GNP of 1965 is taken as base value. Thus:

$$GNP_t^* = 1507.5(1 + 0.06)^t\ , \quad (t = 1,2,3)$$

Restoring the economy back to its pre-crisis trend in the period under consideration required a massive government effort. The main instrument for achieving the target formulated by the policy makers is, therefore, assumed to be government fiscal policy (government expenditures on goods and services). We formulate a simple exponential growth in government expenditures in this period. An annual rate of growth of 15% based on 1965 level is assumed to be desirable. Thus,

$$G_t^* = 296.1(1 + .15)^t\ , \quad (t = 1,2,3)$$

In Table 6.1, we show the desired values of both non-controlled variables and the instrument for the period 1967 to 1969.

CERTAINTY EQUIVALENCE RESULTS

Solution for a three-period planning horizon model is considered in our example. For analysis, a period is defined to coinside with one year.

When computing the first period (1967) decision of the maximizing strategy, we consider only the first component of the three-element vector:

$$\hat{\mathbf{x}} = -K^{-1}h$$

The other two components are forecasts for 1968, and 1969 decisions at the beginning of 1967. These forecasts are revised period after period (year after year). Thus,

$$\hat{g}_1 = G_{67} - G^*_{67} = 30.3$$

$$\hat{g}_2 = G_{68} - G^*_{68} = -21.6$$

$$\hat{g}_3 = G_{69} - G^*_{69} = -68.7$$

Reading the desired levels of G (Government expenditures) from the last row of Table 6.1, therefore, we have:

$$G_{67} = 421.9$$

$$G_{68} = 428.7$$

$$G_{69} = 448.3$$

TABLE 6.1. DESIRED VALUES OF NON-CONTROLLED VARIABLES AND INSTRUMENT 1967-1969. £ml.

Variables	Year		
	1967	1968	1969
Private Consumption	1341.2	1408.3	1478.7
GDCF	305.2	335.0	360.7
Imports	223.5	193.2	248.7
Exports	238.1	206.5	319.7
GNP	1693.9	1795.5	1903.2
Government Expenditures	391.6	450.3	517.8

In Table 6.2 we show the forecasts and actual decisions at the beginning of each year. Underlined values indicate actual decisions.

Table 6.2 shows that at the beginning of 1967, the decision (Government expenditures) for that year was £421.9 ml per year. In this same year a forecast of £429.9 ml government expenditures is made for the 1968 decision. This forecast turns out to over state the actual 1968 decision by only £1.2 ml. Surprisingly enough the forecast in 1967 for 1969 decision was very close to the actual decision in 1969. There was only a difference of £0.3 ml.

In Table 6.3, we show the decisions, forecasts of decisions at the first period and values actually realized for private consumption, GDCF, Imports and Government expenditures.

On the whole, the forecasts are rather accurate. The maximizing strategy implies a much higher level of government expenditures on goods and services than was actually realized in 1967-1969 period. The strategy leads to higher levels of consumption, capital formation and GNP. But imports rose higher than was desired in the first two years of the plan period. The desired lower imports were realized in 1969.

The first period loss associated with a first period decision error $G_{67} - G^{*}_{67}$ in the three years planning

TABLE 6.2. FORECASTS AND ACTUAL DECISIONS. £ml.

Estimates at beginning of	1967	1968	1969
1967	421.9	429.9	448.6
1968		428.7	435.6
1969			448.3

TABLE 6.3. DECISIONS, FORECASTS AND ACTUAL LEVELS OF THE VARIABLES. £m1.

Beginning of	Decision	Forecast	Actual
		Government Expenditure	
1967	421.9		306.3
1968	428.7	429.9	306.0
1969	448.3	428.7	330.9
		Private Consumption	
1967	1499.7		1321.9
1968	1625.7	1628.2	1364.6
1969	1728.3	1728.9	1428.8
		GDCF	
1967	259.0		219.4
1968	243.4	243.6	223.2
1969	248.9	243.4	228.6
		Imports	
1967	271.3		223.5
1968	251.9	252.1	193.2
1969	236.4	236.4	248.7

horizon is given by the quadratic form:

$$L_{67} = -1/2(G_{67} - G^{*}_{67})\ (-K_{11})\ (G_{67} - G^{*}_{67})$$

where,

K_{11} is the first element of the matrix K given in Table 6.4. From Table 6.4,

$$-K_{11} = -11.7$$

$$L_{67} = (G_{67} - G^{*}_{67})\ 5.9\ (G_{67} - G^{*}_{67})$$

$$= 5.9\ G^{2}$$

This implies that for every unit decision error committed in the first period the loss is ¢5.9 ml.

In summary, "whenever a real-world situation can be approximated by means of quadratic preference functions and linear constraints...linear decision rules provide a powerful tool for handling dynamic problems in the face of uncertainty" [Theil (1968)]. This rationalization provides the rationale upon which the analysis above is based.

TABLE 6.4. MATRIX K

11.7	4.8	1.9
4.8	11.2	3.9
1.9	3.9	9.4

CHAPTER VII

STOCHASTIC CONTROLS

In the last chapter we discussed the 'certainty equivalence' decision criterion and derived associated decision. The certainty equivalence solution presumes that the values of the parameters of the structural equations are known with certainty when infact they are not. However, as was discussed earlier, in most macro-economic applications, the coefficients are not known with certainty. They are normally estimated using historical data. Besides, Kydland (1973), Lucas (1973), and Kydland and Prescott (1973) have shown that policy rules have effect of shifting values of the parameters of structural equations by changing the expectations of economic agents. Thus, any policy valuation ignoring such a drift in the parameters will lead to sub-optimal control policy. The 'certainty equivalence' strategy, therefore, involves specification error. The example treated by Fisher (1962) and Prescott (1971) suggest that this specification error may be significant when ratios of the coefficient estimates to their associated standard errors (i.e. their t values) are less than four. Their results thus suggest that our results of Chapter VI

obtained by 'certainty equivalence' strategy may not be optimal.

In this chapter, we explore one approximate solution suggested in the literature--stochastic sequential control strategy--to improve on the shortcomings of certainty equivalence control strategy. An algorithm is developed for computing the three period decisions and a general period decision. Sequential decisions are optimal under special assumptions and in particular when experimentation is not a relevant consideration. We shall relax this assumption and discuss, theoretically, the effect of allowing for experimentation.

STOCHASTIC SEQUENTIAL CONTROL STRATEGY

Wonham (1967) and Turnovsky (1973) have shown that for linear dynamic systems, and quadratic objective functions, optimal controls can be found in the case where there is uncertainty in the parameters, provided the uncertainty in the parameters is not too large and provided the parameters are independently distributed through time. The problem arising from stochastic parameters has been investigated by Prescott (1971), Zellner (1971), MacRae (1972), Tse, et al. (1973) and Taylor (1973). A sequential decision strategy is suggested as an approximate solution to these more general problems.

Sequential decision procedures involve a sequence of optimizing actions (open-loop sub-problems). In each period of the planning horizon these sub-problems begin with an initial value. As policy decisions are effected and as we proceed through time, additional information becomes available. This additional information is used in improving knowledge about the parameters--in particular knowledge about their mean values and covariances. The revised or improved estimates are then employed as prior information for the next open-loop sub-problem; and so on. Thus, stochastic sequential control strategy at least partially recognizes the control and learning elements involved.

Operationally the sequential control procedure simply involves solving sequence of stochastic control problems, one for each period. In solving each of these problems, it is assumed that the probability distribution of the parameters is invariant to the sequence of new information (sample data). The first period controls are employed and the resulting sample information is utilized to update our knowledge about the mean and covariance of the parameters. The next period's controls are then determined by resolving a stochastic control problem for the updated parameters.

The stochastic control problem can be solved using the method of dynamic programming of Bellman (1957).

Bellman's principle of optimality states that: An optimal policy [here sequential rules for selecting values of the control vector, X_t's] has the property that whatever the initial state and initial decision are the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.

Consider the decision on X_T in the last period when the welfare cost will be:

$$Q_T = E_T \frac{1}{2} \{ (Y_T - Y_T^*)' K_T (Y_T - Y_T^*) + (X_T - X_T^*) R_T (X_T - X_T^*) \} \quad \ldots . (7.1)$$

where,

according to our previous notational conventions Y is a vector of non-controlled variables, and X is a vector of controlled variables. K is a positive definite quadratic form reflecting the trade-off for the non-controlled variables, while R is a positive semi-definite quadratic form reflecting the trade-off for the instrument variables. The stars on the variables Y and X indicate desired levels. E_T denotes expectation conditional on the information available at the beginning of period T.

We want to minimize the welfare cost (7.1) subject to the constraints imposed by the economic system. That is, subject to:

$$Y_T = A_T Y_{T-1} + B_T X_T + b_T + U_T \quad \ldots . (7.2)$$

where,

A, B, b and U are as defined earlier and Y and X are as defined above. To facilitate the generalization of (7.1) to other periods other than T, we re-write (7.1) by substituting for Y_T. Substituting (7.2) into (7.1) we have:

$$Q_T = E_T \{[A_T Y_{T-1} + B_T X_T + b_T - Y_T^*]' K_T [A_T Y_{T-1} + B_T X_T + b_T - Y_T^*] + [X_T - X_T^*]' R_T [X_T - X_T^*]\} + E_T U_T' K_T U_T \quad \ldots (7.3)$$

$$= X_T' B_T' K_T A_T Y_{T-1} +$$

$$\frac{1}{2} X_T' B_T' K_T B_T X_T +$$

$$X_T' B_T' K_T b_T -$$

$$X_T' B_T' K_T Y_T^* +$$

$$X_T' R_T X_T -$$

$$X_T' R_T X_T^* +$$

terms not containing X_T.

The necessary condition for a minimum is that:

$$\frac{\partial Q_T}{\partial X_T} = 0$$

Thus,

$$-[R_T + B_T' K_T B_T] X_T = B_T' K_Y A_T Y_{T-1} + B_T' K_T b_T - B_T' K_T Y_T^* - R_T X_T^*$$

$$\hat{X}_T = -\left[R_T + B_T' K_T B_T\right]^{-1}\{B_T' K_T A_T Y_{T-1} + B_T' K_T b_T - B_T' K_T Y_T^* - R_T X_T^*\} \quad \ldots (7.4)$$

(7.4) then gives the last period optimal control. It is clear that the last period control depends on all the information that is available at the beginning of the last period (i.e. Y_{T-1}). Substituting (7.4) for X_T in (7.3) gives the minimum expected cost for the last period. By the principle of optimality in dynamic programming, the optimal strategy for any period say, T-1, is obtained by minimizing the expected welfare cost from that period on under the assumption that future controls shall be optimally set. Thus to obtain decision for period T-1 given that decision for period T will be optimal, we minimize:

$$Q_{T-1} = E_{T-1}\frac{1}{2}\{\hat{Q}_T + (Y_{T-1} - Y_{T-1}^*)' K_{T-1}(Y_{T-1} - Y_{T-1}^*) + (X_{T-1} - X_{T-1}^*)' R_{T-1}(X_{T-1} - X_{T-1}^*)\} \quad \ldots (7.5)$$

subject to:

$$Y_{T-1} = A_{T-1} Y_{T-2} + B_{T-1} X_{T-1} + b_{T-1} + U_{T-1} \quad \ldots (7.6)$$

where,

$\hat{Q}_T$ is the optimal welfare loss for period T. Substituting (7.6) in (7.5) and replacing X_T by its optimal estimate $\hat{X}_T$, we have:

$$
\begin{aligned}
Q_{T-1} = E_{T-1}\{ & [A_T Y_{T-1} + B_T \hat{X}_T + b_T - Y_T^*]' K_T \\
& [A_T Y_{T-1} + B_T \hat{X}_T + b_T - Y_T^*] \\
& + (\hat{X}_T - X_T^*)' R_T (\hat{X}_T - X_T^*) + (Y_{T-1} - Y_{T-1}^*)' K_{T-1} \\
& (Y_{T-1} - Y_{T-1}^*) \\
& + (X_{T-1} - X_{T-1}^*)' R_{T-1} (X_{T-1} - X_{T-1}^*) \\
& + E_T U_T' K_T U_T + E_{T-1} U_{T-1}' K_{T-1} U_{T-1} \\
= \{ & [A_T (A_{T-1} Y_{T-2} + B_{T-1} X_{T-1} + b_{T-1} + U_{T-1}) \\
& + B_T \hat{X}_T + b_T - Y_T^*]' K_T \\
& [A_T (A_{T-1} Y_{T-2} + B_{T-1} X_{T-1} + b_{T-1} + U_{T-1}) \\
& + B_T \hat{X}_T + b_T - Y_T^*] + [A_{T-1} Y_{T-2} + B_{T-1} X_{T-1} \\
& + b_{T-1} - Y_{T-1}^*]' K_{T-1} [A_{T-1} Y_{T-2} + B_{T-1} X_{T-1} \\
& + b_{T-1} - Y_{T-1}^*] + (\hat{X}_T - X_T^*)' R_T (\hat{X}_T - X_T^*) \\
& + (X_{T-1} - X_{T-1}^*)' R_{T-1} (X_{T-1} - X_{T-1}^*) \\
& + E_T U_T' K_T U_T + E_{T-1} U_{T-1}' K_{T-1} U_{T-1} \}
\end{aligned}
$$

Thus,

$$
\begin{aligned}
Q_{T-1} = & X_{T-1}' B_{T-1}' A_T' K_T A_T A_{T-1} Y_{T-2} \\
& + X_{T-1}' B_{T-1}' K_{T-1} A_{T-1} Y_{T-2}
\end{aligned}
$$

$$+ \frac{1}{2} X'_{T-1} B'_{T-1} A'_T K_T A_T B_{T-1} X_{T-1}$$

$$+ \frac{1}{2} X'_{T-1} B'_{T-1} K_{T-1} B_{T-1} X_{T-1}$$

$$+ \frac{1}{2} X'_{T-1} R_{T-1} X_{T-1}$$

$$+ X'_{T-1} B'_{T-1} A'_T K_T A_T b_{T-1}$$

$$+ X'_{T-1} B'_{T-1} K_{T-1} b_{T-1}$$

$$+ X'_{T-1} B'_{T-1} A'_T K_T A_T B_T \hat{X}_T$$

$$+ X'_{T-1} B'_{T-1} A'_T K_T A_T b_T$$

$$- X'_{T-1} B'_{T-1} A'_T K_T A_T Y^*_T$$

$$- X'_{T-1} B'_{T-1} K_{T-1} Y^*_{T-1}$$

$$- X'_{T-1} R_{T-1} X^*_{T-1}$$

$$+ \text{terms not containing } X_{T-1}$$

Differentiating Q_{T-1} with respect to X_{T-1}, we have:

$$\frac{Q_{T-1}}{X_{T-1}} = 0$$

$$= [B'_{T-1} K_{T-1} A_{T-1} + B'_{T-1} A'_T K_T A_T A_{T-1}] Y_{T-2}$$

$$+ [R_{T-1} + B'_{T-1} K_{T-1} B_{T-1} +$$

$$B'_{T-1} A'_T A_T B_{T-1}] X_{T-1}$$

$$+ [B'_{T-1} K_{T-1} + B'_{T-1} A'_T K_T A_T] b_{T-1}$$

$$+ B'_{T-1}A'_T K_T A_T B_T \hat{X}_T$$

$$+ B'_{T-1}A'_T K_T A_T b_T$$

$$- B'_{T-1}A'_T K_T A_T Y^*_T$$

$$- B'_{T-1}K_{T-1}Y^*_{T-1}$$

$$- R_{T-1}X^*_{T-1}$$

Hence,

$$\hat{X}_{T-1} = - [R_{T-1} + B'_{T-1}K_{T-1}B_{T-1} + B'_{T-1}A'_T K_T A_T B_{T-1}]^{-1}$$

$$\{[B'_{T-1}K_{T-1}A_{T-1} + B'_{T-1}A'_T K_T A_T A_{T-1}]Y_{T-2}$$

$$+ [B'_{T-1}K_{T-1} + B'_{T-1}A'_T K_T A_T]b_{T-1}$$

$$+ B'_{T-1}A'_T K_T A_T B_T \hat{X}_T + B'_{T-1}A'_T K_T A_T b_T$$

$$- B'_{T-1}A'_T K_T A_T Y^*_T - B'_{T-1}K_{T-1}Y^*_{T-1} - R_{T-1}X^*_{T-1}\}$$

$\hat{X}_{T-1}$ has an interpretation similar to that for $\hat{X}_T$ and is the T-1 period stochastic sequential control.

To obtain sequential decision for T-2 period, we again invoke the principle of optimality. To get an optimal control for T-2 period given that the decisions for periods T and T-1 will be optimal, we minimize:

$$Q_{T-2} = E_{T-2}\frac{1}{2}\{\hat{Q}_T + \hat{Q}_{T-1} + (Y_{T-2} - Y^*_{T-2})'K_{T-2}(Y_{T-2} -$$

$$Y^*_{T-2}) + (X_{T-2} - X^*_{T-2})'R_{T-2}(X_{T-2} - X^*_{T-2})\} \quad ..(7.7)$$

subject to:

$$Y_{T-2} = A_{T-2}Y_{T-3} + B_{T-2}X_{T-2} + b_{T-2} + U_{T-2} \quad \ldots(7.8)$$

substituting (7.8) into (7.7) and replacing X_T and X_{T-1} with their optimal estimates, we have:

$$\begin{aligned}
Q_{T-2} = E_{T-2}\{&[A_T(A_{T-1}(A_{T-2}Y_{T-3} + B_{T-2}X_{T-2} + b_{T-2} \\
&+ U_{T-2}) + B_{T-1}\hat{X}_{T-1} + b_{T-1} + U_{T-1}) \\
&+ B_T\hat{X}_T + b_T - Y_T^*]'K_T[A_T(A_{T-1}(A_{T-2}Y_{T-3} \\
&+ B_{T-2}X_{T-2} + b_{T-2} + U_{T-2}) \\
&+ B_{T-1}\hat{X}_{T-1} + b_{T-1} + U_{T-1}) \\
&+ B_T\hat{X}_T + b_T - Y_T^*] \\
&+ [A_{T-1}(A_{T-2}Y_{T-3} + B_{T-2}X_{T-2} + b_{T-2} + U_{T-2}) \\
&+ B_{T-1}\hat{X}_{T-1} + b_{T-1} + U_{T-1} - Y_{T-1}^*]'K_{T-1}[A_{T-1} \\
&(A_{T-2}Y_{T-3} + B_{T-2}X_{T-2} + b_{T-2} + U_{T-2}) \\
&+ B_{T-1}\hat{X}_{T-1} + b_{T-1} + U_{T-1} - Y_{T-1}^*] \\
&+ [A_{T-2}Y_{T-3} + B_{T-2}X_{T-2} + b_{T-2} + U_{T-2} - Y_{T-2}^*]' \\
&K_{T-2}[A_{T-2}Y_{T-3} + B_{T-2}X_{T-2} + b_{T-2} + U_{T-2} - Y_{T-2}^*] \\
&+ (\hat{X}_T - X_T^*)'R_T(\hat{X}_T - X_T^*) + (\hat{X}_{T-1} - X_{T-1}^*)R_{T-1} \\
&(\hat{X}_{T-1} - X_{T-1}^*) + (X_{T-2} - X_{T-2}^*)'R_{T-2}(X_{T-2} - X_{T-2}^*)
\end{aligned}$$

Differentiating Q_{T-2} with respect to X_{T-2}, we have the first order condition,

$$\frac{\partial Q_{T-2}}{\partial X_{T-2}} = 0$$

Solving the first order condition gives:

$$\begin{aligned}\hat{X}_{T-2} = - &[R_{T-2} + B'_{T-2}K_{T-2}B_{T-2} + B'_{T-2}A'_{T-1}K_{T-1}A_{T-1}B_{T-2} \\ &+ B'_{T-2}A'_{T-1}A'_{T}K_{T}A_{T}A_{T-1}B_{T-2}]^{-1}\{[B'_{T-2}K_{T-2}A_{T-2} \\ &+ B'_{T-2}A'_{T-1}K_{T-1}A_{T-1}A_{T-2} + B'_{T-2}A'_{T-1}A'_{T}K_{T}A_{T}A_{T-1}A_{T-2}] \\ &Y_{T-3} + [B'_{T-2}K_{T-2} + B'_{T-2}A'_{T-1}K_{T-1}A_{T-1} + \\ &B'_{T-2}A'_{T-1}A'_{T}K_{T}A_{T}A_{T-1}]b_{T-2} + [B'_{T-2}A'_{T-1}K_{T-1}A_{T-1}B_{T-1} \\ &+ B'_{T-2}A'_{T-1}K_{T}A_{T}B_{T-1}]\hat{X}_{T-1} + [B'_{T-2}A'_{T-1}K_{T-1} + \\ &B'_{T-2}A'_{T-1}A'_{T}K_{T}A_{T}]b_{T-1} + B'_{T-2}A'_{T-1}A'_{T}K_{T}B_{T}\hat{X}_{T} + \\ &B'_{T-2}A'_{T-1}K_{T}b_{T} - B'_{T-2}K_{T}Y^{*}_{T} - B'_{T-2}A'_{T-1}K_{T-1}Y^{*}_{T-1} - \\ &B'_{T-2}K_{T-2}Y^{*}_{T-2} - R_{T-2}X^{*}_{T-2}\}\end{aligned}$$

Thus,

$\hat{X}_{T-2}$ is the optimum sequential decision for T-2 period.

Comparing expressions for $\hat{X}_{T}$, $\hat{X}_{T-1}$ and $\hat{X}_{T-2}$, that is, decisions for T, T-1, T-2 periods respectively, we

observe that a general pattern emerges. Utilizing this general pattern we can easily obtain the decision for, say $T-k^{th}$ period without having to find the necessary condition for a minimum (maximum) first and then solving for X_{T-k}. For instance, for $T-k^{th}$ period sequential decision, the first part (inverted element) of the expression for X_{T-k} will be given as:

$$- [R_{T-k} + B'_{T-k}K_{T-k}B_{T-k} + B'_{T-k}A'_{T-k+1}K_{T-k+1}A_{T-k+1}B_{T-k}$$

$$+ \ldots + B'_{T-k}A'_{T-k+1} \ldots A'_{T}K_{T}A_{T} \ldots$$

$$A_{T-k+1}B_{T-k}]^{-1}$$

As is evident, the solution for the decisions is obtained backward in time, starting from t = T. The whole problem is broken up into sub-problems and these sub-problems are then solved as a sequence of optimizing actions. These characteristics make the sequential procedure described above an 'open-loop feedback control' technique. The optimal open-loop feedback control is linear, and can be written in the general form:

$$\hat{X}_t = F_t Y_{t-1} + G_t \qquad \ldots (7.9)$$

where,

$\hat{X}_t$ is the optimal control for the t^{th} period, Y_{t-1} is the already observed information on non-controlled endogenous variables, F_t is the associated coefficient

derived from information available at the beginning of period t, and G_t includes the desired levels of both target and instrument variables.

For T^{th} period control,

$$F_T = - [R_T + B_T' K_T B_T]^{-1} B_T' K_T A_T$$

$$G_T = [R_T + B_T' K_T B_T]^{-1} (R_T X_T - B_T' K_T b_T + B_T' K_T Y_T^*)$$

For $T-1^{th}$ period control,

$$F_{T-1} = - [R_{T-1} + B_{T-1}' K_{T-1} B_{T-1} + B_{T-1}' A_T' K_T A_T B_{T-1}]^{-1}$$

$$\{[B_{T-1}' K_{T-1} A_{T-1} + B_{T-1}' A_T' K_T A_T A_{T-1}]\}$$

$$G_{T-1} = - [R_{T-1} + B_{T-1}' K_{T-1} B_{T-1} + B_{T-1}' A_T' K_T A_T B_{T-1}]^{-1}$$

$$\{[B_{T-1}' K_{T-1} + B_{T-1}' A_T' K_T A_T] b_{T-1} +$$

$$B_{T-1}' A_T' K_T A_T B_T X_T + B_{T-1}' A_T' K_T A_T b_T$$

$$- B_{T-1}' A_T' K_T A_T Y_T^* - B_{T-1}' K_{T-1} Y_{T-1}^* - R_{T-1} X_{T-1}^*\}$$

F_{T-2} and G_{T-2} are similarly obtained from $\hat{X}_{T-2}$.

STOCHASTIC SEQUENTIAL CONTROL RESULTS

We recall that the optimal decision for period T, $\hat{X}_T$ is given as:

$$\hat{X}_T = - [R_T + B_T' K_T B_T]^{-1} [B_T' K_T A_T Y_{T-1} + B_T' K_T b_T$$

$$- B_T' K_T Y_T^* - R_T X_T^*]$$

In the Nigerian example, T is the period relating to 1969, and,

$$R_T = K_T = - I$$

A, B, and b are the matrice and vectors of coefficients of endogenous and exogenous variables given in Table 4.5. The stars represent the desired levels of the target and instruments given in Table 6.1.

Deriving optimal decision for 1969, we find that:

$$B_T'B_T = 8.447$$

Hence,

$$[I + B_T'B_T]^{-1} = 0.1058$$

And,

$$B_T'A_T = [1.8163,\ 7.5845,\ -6.2861,\ 0,\ 0]$$

combining the results above with the given values of Y_{T-1}, b_T, Y_T^*, and X_T^*, we find that the sequential decision for 1969 is ₹431.8 ml. Results for 1968 and 1967 are similarly obtained. They are ₹488.0 ml, and ₹348.6 respectively. Comparing these results with the target set for Government Expenditures given in Table 6.1 we notice a feature of our sequential results of important policy implications. The target set envisages a simple exponential growth in government expenditures in the

three-year period. But what is suggested by the sequential control results is an easy policy in the first year, followed by a massive effort in the second year, then a cooling off again in the third period. On the whole, over the three-year planning horizon, much less total government expenditures is suggested by the stochastic sequential technique. In Table 7.1, we show the sequential decisions and the targets.

SEQUENTIAL VS ADAPTIVE CONTROL

As in evident from equations (7.1) to (7.9), stochastic sequential control treats the covariance of the parameters as if known and constant. But this may not be the case. The covariance matrices may themselves be stochastic. The randomness may arise from the fact that both the covariance and the conditional mean of the parameters are dependent on the random states and non-controlled exogenous variables. The problem created by this randomness in parameters is well discussed by Zellner (1971), MacRae (1972), and Rausser and Freebairn (1973), who have suggested 'adaptive control' procedure as an approximate solution by modelling the unknown parameters as Bayesian random variables with means and variances which change as additional information become available over time.

TABLE 7.1. SEQUENTIAL DECISIONS AND TARGETS. £ml.

Period	Sequential Decision	Targets
1967	348.6	391.6
1968	488.0	450.3
1969	431.8	517.8

An important characteristic of adaptive control is the interaction between direct control and experimentation. As has been recognized in the literature [Athans (1972); Zellner (1971); Taylor (1973); MacRae (1972)] learning and design considerations are involved in such multiperiod control problem. As we proceed into the future, we accumulate more information that permits us to learn more about the unknown parameters. This accumulation of information could arise out of a 'passive' or deliberate policy. Stochastic sequential control procedures rely only on information passively accumulated to update knowledge about the unknown parameters. Whereas, adaptive control recognizes the fact that apart from the passively accumulated information, we can also learn about the unknown parameters by setting the control variables, that is by 'designing an experiment'.

Following Zellner (1971) we sketch adaptive control procedure for obtaining a solution for a three-period problem. Suppose we have a loss function Q and we consider ourselves to be at the beginning of the third future period T. We can employ the conditional predictive probability density function for Y_T, given Y_{T-1}, X_{T-1}, Y_{T-2}, X_{T-2} and the given sample information, Y and X to evaluate

$$\underset{Y_T}{E}\ (Q/Y_{T-1},\ X_{T-1},\ Y_{T-2},\ X_{T-2},\ Y,\ X)$$

$$= g\ (Y_{T-1},\ X_{T-1},\ Y_{T-2},\ X_{T-2},\ X_T,\ Y,\ X) \qquad \ldots(7.10)$$

Minimizing (7.10) with respect to X_T we have:

$$\hat{X}_T = \hat{X}_T\ (Y_{T-1},\ X_{T-1},\ Y_{T-2},\ X_{T-2},\ Y,\ X) \qquad \ldots(7.11)$$

$\hat{X}_T$ thus depends on as yet un-observed values Y_{T-1} and Y_{T-2}, the as yet undetermined values of X_{T-1} and X_{T-2}, and on the given sample data.

Substituting $\hat{X}_T$ for X_T in (7.10) we obtain an expression:

$$g(Y_{T-1},\ X_{T-1},\ Y_{T-2},\ X_{T-2},\ \hat{X}_T,\ Y,\ X) \qquad \ldots(7.12)$$

we repeat the procedure by taken expectation of (7.12) with respect to Y_{T-1} to obtain the setting for X_{T-1}, say $\hat{X}_{T-1}$:

Where,

$$\hat{X}_{T-1} = \hat{X}_{T-1}(Y_{T-2},\ X_{T-2},\ Y,\ X) \qquad \ldots(7.13)$$

We then substitute $\hat{X}_{T-1}$ for X_{T-1} in

$\underset{Y_{T-1}}{E}\ g(Y_{T-1},\ X_{T-1},\ Y_{T-2},\ X_{T-2},\ \hat{X}_T,\ Y,\ X)$ to obtain

$$h(Y_{T-2},\ X_{T-2},\ \hat{X}_{T-1},\ \hat{X}_T,\ X,\ Y) \qquad \ldots(7.14)$$

We again take expectation of (7.14) with respect to Y_{T-2} to obtain:

$$\underset{Y_{T-2}}{E}\; h(Y_{T-2},\ X_{T-2},\ \hat{X}_{T-1},\ \hat{X}_{T},\ X,\ Y) \qquad \ldots(7.15)$$

Minimizing (7.15) with respect to X_{T-2}, we obtain optimal setting for X_{T-2}, say $\hat{X}_{T-2}$, where,

$$\hat{X}_{T-2} = \hat{X}_{T-2}(Y,\ X) \qquad \ldots(7.16)$$

As is clear from (7.16), $\hat{X}_{T-2}$ depends only on the given sample information, hence can be computed at the beginning of the first future period.

The procedure described above is such that the optimal setting for X_{T-2} in (7.16) takes account of how the future information Y_{T-2} can be employed in optimizing in the second and third future periods, T-1 and T; and how the first period setting for X_{T-2} will affect decisions in the second and third future periods. Further, the last future setting $X_T = \hat{X}_T$ incorporates all the sample information available at the beginning of T. One important feature of adaptive control technique that needs emphasizing is the fact that the choice of control in the first future period affects the future values of parameters. Thus we can learn more about the unknown parameters by appropriate designing of the first future control.

A close look at open-loop feedback sequential technique discussed earlier and the adaptive control technique sketched above reveals some similarities. Both

solutions are obtained by moving backward in time; and minimization of the loss function is accomplished on the assumption that the decisions of the periods ahead are optimally chosen. In Zellner's example actual computation of decisions can only follow after the first future decisions are derived from the sample information. This in practice is what will happen. In the Nigerian example where our future periods were 1967, 1968 and 1969, the data are already available. Hence future decisions can be computed as they are derived. By adopting the sequential approach, we have denied ourselves the option of experimenting with the first period control. It should be emphasized, however, that there is no inherent value in experimenting for its own sake in the decision making problem. The only value comes from improved future control, and the costs are those incurred by sacrificing current control for experimentation.

Existing results indicate that expected loss for adaptive control solutions is smaller than the expected loss for sequential control solutions [Zellner (1971, p. 197); MacRae (1972)]. However, the difference will often be small for long planning horizon and when the target values are not large. This is partly due to the fact that when planning horizon is long, adaptive control procedure tends to place too much emphasis on

experimentation; that is, there is excessive experimentation when adaptive control technique is employed in a long planning horizon model.

SENSITIVITY TEST OF THE CONTROL TECHNIQUES

One advantage of the certainty equivalence and sequential control techniques is their adaptability to sensitivity tests. In policy formulation, we are often not sure of the exact levels of targets. Frequently, therefore, plans fail to meet their objectives because the targets are set unrealistically. Since certainty equivalence and sequential control techniques are being recommended to Nigerian planners as a guide to taking optimal decisions, a sensitivity test of these techniques, involving levels of targets is carried out. That is, for each of the techniques, we seek:

$$\frac{\partial \hat{X}}{\partial Y^*} ,$$

where,

$\hat{X}$ is a vector of decisions and Y^* is a vector of targets variables. It might be the case, particularly in a developing economy, that there are a number of exogenous variables over which the policy makers have little or no control, but which have a substantial effect on the target variables. An example in the Nigerian model is the disposable income of British subjects,

which affects Nigerian exports.

Results of the sensitivity test based on the assumption that the Nigerian planners overestimated the original targets by 5% are given in Table 7.2. The results for certainty equivalence control show that for every unit decrease in the targets, there is 1.7 units, 1.8 units, and 1.2 units decrease in the first, second and third period decisions respectively. In the case of sequential control, a unit decrease in the targets leads to 0.94 unit decrease in the first period decision, a unit increase in the second period decision, and 1.7 units decrease in the third year decision. The conclusion based only on our results is that certainty equivalence technique is more sensitive to changes in the targets and hence more sensitive to uncontrollable 'outside' influences. This result probably arises from the fact that for certainty equivalence approach a steady state is not only assumed in the economic structure but the parameters are also assumed known with certainty. For a technique that is very sensitive to changes in non-controlled variables, caution should be exercised in its use as a planning tool.

Another interesting sensitivity test which will be useful to perform is the test on the sensitivity of decisions to values of parameters which are of interest. The knowledge of the response of decisions to parameters

TABLE 7.2. SENSITIVITY TEST OF CERTAINTY EQUIVALENCE CONTROL AND SEQUENTIAL CONTROL. £ml.

Period	Decisions			
	Certainty Equivalence on		Sequential Control on	
	Initial Targets	Revised Targets	Initial Targets	Revised Targets
1967	421.9	386.2	348.6	332.2
1968	429.9	389.8	488.0	510.6
1969	448.6	421.9	431.8	393.2

would help identify possible profitable areas and intensity of experimentation if and when adaptive control technique is to be employed to obtain optimal decisions. Thus, following from equation (7.9) we seek the solution to the equation:

$$\frac{\partial \hat{X}_t}{\partial \hat{\beta}_{ti}} = 0 = \frac{\partial}{\partial \hat{\beta}_{ti}} [F_t Y_{t-1} + G_t]$$

where,

$\hat{X}$, Y, F, G are as defined earlier, and $\hat{\beta}_{ti}$ is the estimate of the i^{th} parameter in the structural equations. If the decisions are sensitive to changes in particular parameters, the suggestion would be to experiment with such parameters.

We conclude by noting the difference in policy decisions implied by the certainty equivalence and the sequential control results obtained for the Nigerian model. The certainty equivalence procedures would imply a gradual growth in government expenditures over the three-year planning period, whereas the sequential control procedures would suggest an easy policy in the first year, then a massive government expenditures in the following year, followed by an easy policy in the third year.

CHAPTER VIII

CONCLUSIONS

The development of macro-econometric models over the past two decades has provided a means for studying the relationships between economic variables and their response to policy instruments, such as, government expenditures. Computer simulations of econometric models have been developed to compare the effects of different economic policies, test these policies and eliminate those whose economic effects are found undesirable [Naylor (1970), Glen Johnson et al. (1969)]. The more recent development of computationally tractable algorithms for efficient solution of sets of simultaneous difference equations has enhanced the usefulness of the computer simulation technique. Although it is an extremely useful tool for economic planning and analysis of stabilization policies, computer simulation does not provide a direct means of obtaining a policy that is optimal with respect to a given set of objectives over multi-period planning horizon.

Recently, there has been an interest in optimal control theory as a possible tool for economic planning. Given an econometric model which is acceptable as a

reasonable representation of the economy and given an objective function presumed to represent the goals and objectives of economic planners, then the design of an economic policy can be easily characterized as a problem in optimal control. The optimal control problem can be viewed either as a deterministic formulation or as a stochastic formulation. The former is a simplification which assumes a deterministic system. Since much of our work is often done under the guidance of simplification, we might choose to ignore any stochastic properties of our model or else make the necessary simplifying assumptions about them to facilitate application of the 'certainty equivalence' theorem in obtaining a solution.

But ignoring the randomness in estimated coefficients of econometric models has been shown to imply mis-specification of the model and consequently resulting in sub-optimal decisions [Prescott (1972)]. Stochastic sequential control technique is, therefore, suggested and has been shown to give optimal decisions when uncertainty in the parameters is not large. As we proceed in time new information acquired enable us to update the probability distribution of the parameters. This technique, thus allows for 'passive' learning. A technique allowing for both uncertainty in the parameters and 'active' learning is the 'adaptive control'. Adaptive control procedures incorporate experiments with levels

of decisions to learn more about the economic structure.

It should be emphasized that the techniques discussed here are only approximate solutions to the dual control problem arising from interaction between control and learning. In our application of the techniques, we have followed others [Simon (1956); Theil (1957); Zellner (1971); Prescott (1972); and MacRae (1972)] in assuming a linear econometric model, and a quadratic criterion function. Such formulations are necessarily restrictive but yield an attractive property of computational tractability. Attempts have been made to solve dynamic optimization problems with more general criterion function.[1] In the Nigerian case, however, we were willing to believe that a quadratic specification does not seem an unreasonable approximation to the definition of the policy makers' preception of society's economic goals and preferences.

Specification of a linear model probably presents a more serious restriction than a quadratic criterion function. Many econometric models are at least quasi-linear in structure, but sometimes the more interesting aspects of their dynamic behavior arise from the non-linearities. Livesey (1971) has solved the optimal decision problem with a continuous non-linear model of the U.K. The large amount of computational work involved in the conjugate gradient method he employed, however, raises the

question of whether non-linear optimization is worth all the computational difficulty it entails. The answer to this question depends very much on how much of the dynamic behavior of the economic system is explained by the non-linearities in the structure. This problem is made more complex because the existing analytical tools for dealing with the dynamics of non-linear systems are still meager.

The attractiveness of control techniques as economic planning or policy tools, with particular reference to developing economies, tends to depend on two important factors. Firstly, they must be computationally tractable. And secondly, it should be easy for the policy makers to experiment with different values for the parameters in the criterion function, with different forms of criteria function and different time-path for non-controlled exogenous variables.

Our theme is that adaptive control techniques are feasible tools of economic planning. Developments in these techniques have reached the stage at when they can be applied by developing economies. This we have tried to do. In particular we have applied the "certainty equivalence control' and 'stochastic sequential control' techniques to the Nigerian economy. The results obtained seem encouraging and do suggest that if these techniques are applied by the Nigerian planners, the pattern of

'failures' in Nigerian economic plans might be reversed. These failures we have argued, are partly due to the fact that there is, in most cases, no close relationship between national goals as expressed by the policy makers and the individual economic programmes as expressed in the plans. In order to achieve an internal consistency--that is, harmonizing policy targets and programmes--we need to confront policy objectives with the available resources of the economy and their possible uses. This is precisely what adaptive control techniques do.

In the Nigerian example we have characterized the policy objectives as a quadratic loss function, reflecting immediate increase in welfare, potential increase in welfare and balance of payments surplus. We have confronted this policy objective with a simple macro-economic model reflecting the contraints imposed by the Nigerian economic structure. While not much is claimed for the simple aggregate model, we believe it is adequate for the illustration of the techniques suggested for planning. The adequacy of the model is reflected by the values of the parameters obtained. In most cases, they are consistent with economic theory and available results from comparable models. The believability and adequacy (dynamic properties) of the Nigerian model is further shown by the 'nice' results obtained by the use of spectral analysis technique.

Finally, if the 'nice' results obtained in our analysis are due in part to the simplifying assumption we have made all the way, then these findings point to areas of future research. In particular, more work is needed on the form of loss function and the use of non-linear models as constraints implied by alternative economic structures.

FOOTNOTES

CHAPTER I

1. Ghana, Kenya, Nigeria, and Tanzania have each proposed a couple of development plans since the attaintment of independence. See R. H. Green (1965) for a comprehensive survey of Development Plans in the four countries.

2. 1962/68 Nigerian Development Plan.

3. J. Drewnowski, "Planning Economic Growth", in University Textbook for African Students, U.N.E.S.C.O., Paris, Quotation taken from R. H. Green, "Four African Development Plans: Ghana, Kenya, Nigeria, and Tanzania", Yale University Economic Growth Center, Centre Paper, No. 68, 1965.

4. This point is succintly made in R. H. Green's paper referenced above.

5. The reduced form of an equation expresses the endogenous variable as a function of its lagged values, lagged and non-lagged exogenous variables, and random errors.

6. Domar specifies a growth model of the type:

   $$Y = \sigma K$$

   where,

   K is the stock of capital, σ is the capital coefficient or marginal productivity of capital, and Y is output.

   E. D. Domar, "Capital Expansion, Rate of Growth, and Employment", Econometrica (April, 1946); and "Expansion and Employment," American Economic Review (March, 1947)

7. Such results have been reported by C. Holt (1962).

CHAPTER II

1. Sen and Pattanaik(1969) establish a sufficient condition--called 'value restriction'--in terms of restricted individual preferences for every binary, decisive, neutral, and non-negatively responsive social decision process to yield a quasitransitive social weak preference relation. While Fishburn (1970) establishes a sufficient condition for the quasitransitivity of the social weak preference relation under the method of majority decision.

2. Bergstrom (1970) has presented a set of assumptions which guarantee that such side payments will give pareto optimality..

3. Tinbergen (1952) is foremost in this school of thought.

4. Theil (1957) argues strongly for the inclusion of both targets and instruments in the welfare function on the grounds that there is an adjustment cost of deviating from the desired level of instruments.

5. Let $V'=(V'_1, V'_2, \ldots, V'_n)$ and $V^*=(V^*_1, V^*_2, \ldots, V^*_n)$ be any two n-dimensional vectors. Using L to mean "is lexicographically less than," the lexicographic ordering on points in n-space is defined by $V'LV^*$ if $V'_k < V^*_k$ for the smallest integer k $(0 < k < n)$ for which $V'_k \neq V^*_k$. Thus, L is clearly a complete strong ordering [Quirk and Saposnik (1968)].

6. Ordinary Least Squaresmethod can be employed.

7. Wold (1954) defines causality as : Z is a cause of y if, by hypothesis, it is or would be possible by controlling Z indirectly to control y, at least stochastically. But it may or may not be possible by controlling y indirectly to control Z. A causal relation is therefore in essence asymmetric, in that in any instance of its realization it is asymmetric.

8. See Naylor (1969) on validation of non-linear models.

9. See Dhrymes (1970) and Koyck (1954) for different lag structures.

10. Per-capital income in Nigeria is low (less than $400), hence we expect a high marginal propensity to consume.

11. See for instance, M. K. Evans, Macro-economic Activity: theory, forecasting, and control, Harper and Row, New York, 1969, pp. 73-198.

12. See Annual Abstract of Statistics 1966, published by Federal Office of Statistics, Nigeria.

CHAPTER III

1. See Helleiner's (1966) analysis of the problems and estimation of Nigerian National Accounts.

2. There was no complete coverage in the 1952 Nigerian population census.

3. I. I. U. Eke, 'The Nigerian National Accounts: A critical appraisal", Nigerian Journal of Economic and Social Studies, Vol. 8, November, 1966.

4. See Dhrymes (1970, p. 146) and Goldberger (1964, p. 162).

5. The disadvantage, however, is that if such restrictions are wrong, not only will the estimates of the parameters of the equations with the restrictions be biased but all other parameters will also be biased.

6. See Goldberger (1964, pp. 357-363) for a thorough discussion of the alternative structural estimation methods.

7. They all come under the general name of K-class estimators [Goldberger (1964, pp. 336-338, pp. 341-342)].

8. For full discussion on this see F. Fisher (1966).

9. We need to distinguish between restricted reduced form and unrestricted reduced form, since over-identification of any equation implies the existence of restrictions not only on the structural parameters but also on the parameters of the reduced form equations. If Indirect Least Squares method is used to estimate the parameters, we obtain unrestricted reduced form estimates because Indirect Least Squares method does not make use of the restrictions on the structural parameters. It follows that the reduced form estimates so obtained will not in general be simultaneously compatible with all the prior

restrictions on the structural equations [F. M. Fisher (1966; p. 46)].

CHAPTER IV

1. See Rausser and Johnson (1973) for a comprehensive discussion and comparison of the different approaches.

2. In general there is not a one to one correspondence between non-linear structural models and reduced form models. Furthermore, the expected value of a particular endogenous variable as calculated from the structure will not be the same as its expected value calculated from the restrictedreduced form and the reduced form residuals are not linear functions of the structural disturbances. For a rather complete discussion on non-linear structural models as well as some specific examples, see Goldfield and Quandt (1968).

3. See Naylor et al. (1969) for full discussion on this.

4. Marginal Propensity to consume derived from

$$\Delta GNP = \frac{1}{1 - MPC} \Delta G$$

CHAPTER V

1. Economic agents often make decisions under uncertainty. Alternative expectation models that have been suggested to capture the formation of their expection include extrapolative, adaptive, rational, implicit and Bayesian approaches. See Nerlove (1958), Koyck (1954), Muth (1961), and Mills (1962) for comprehensive discussion this problem.

2. An objective function in our analysis is a formal characterization of the policy of the Nigerian Planners.

3. For exposition of Bayesian methods see Raiffa and Schlaifer (1961), and Zellner (1971); for Least Squares revision methods, see Albert and Sitter (1965); for Kalman filters, see Kalman (1960) and Jazwinski (1970).

4. The simple pedagogic models investigated by Marschak (1963) and Ying (1967) illustrate the computational burden involved for numerical solutions.

5. See Rausser and Freebairn (1973) for detailed discussion of other approximate solutions.

6. For an excellent discussion of this approach see G. L. Nemhauser (1966).

7. See G. L. Nemhausser (1966) for the appropriate assumptions.

8. Taylor's criterion, called the 'bliss level criterion', is discussed in apprendix 5.

CHAPTER VI

1. Here efficiency is defined as the response of welfare W to changes in the level of the i^{th} instrument when other instruments remain unchanged.

2. Although the off diagonals of the matrix R are usually zero, in general, we can represent trade off by taking derivations.

3. The closed-loop property implies anticipation of how future learning will be accomplished and how it can be utilized. The feedback property implies solution is derived backward in time. As Tse (1973) has pointed out, the frequent interchange in the usage of closed-loop and feedback does not clearly bring out the dual characteristic of control. The fine distinction between feedback and closed-loop is in the information pattern available to the controller at each instant of time. Feedback control systems will ignore the possibility of future learning and perform control action in a cautious manner. Thus for a feedback system, learning is 'accidental'. While for the closed-loop, the controller can take account of possible future learning.

4. Since C, I, B, GNP and G are random, Q will be random. Q, therefore, will not have a maximum or minimum.

5. This is a required condition satisfying first and second order conditions for the existence of an optimal decision.

6. See Table 3.1 and figure 3.1 for Nigerian economic indicators.

CHAPTER VIII

1. See for instance, D. A. Livesey (1971).

BIBLIOGRAPHY

Adelman, I., and F. Adelman, "The Dynamic Properties of the Klein-Goldberger Model," Econometrica, 27: 596-625, October, 1959.

Aigner, D. J. A., "A note on Verification of Computer Simulation Models," Management Science, 18, No. 11, July, 1972.

Albert, A., and R. W. Sitter, "A Method of Computing Least Squares Estimators that Keep up with the Data," Journal of S.I.A.M. Control, Vol. 3, 1965.

Anderson, T. W., and H. Rubin, "Estimation of the Parameters of Single Equation in a Complete System of Stochastic Equations," Annals of Mathematical Statistics, Vol. 20, March, 1949.

_____, "The Asymptotic Properties of Estimates of the Parameters in a Complete System of Stochastic Equations," Annals of Mathematical Statistics, Vol. 21, December, 1950.

Aoki, M., Optimization of Stochastic System, New York: Academic Press, 1967.

Arrow, K. J., Social Choice and Individual Values, New York: John Wiley, Inc., 1951.

Athans, M., "The Discrete Time Linear-Quadratic-Gaussian Stochastic Control Problem," Annals of Economic and Social Measurement, October, 1972.

Babbar, M. M., "Distribution of Solutions of a Set of Linear Equations with Applications to Linear Programming," Journal of American Statistical Association, L, 1955.

Barnett, S., "Stability of the Solution to a Linear Programming Problem," Operational Research Quarterly, XIII, No. 3, September, 1962.

Barten, A. P., "Estimating Demand Equations," Econometrica, Vol. 36, No. 2, April, 1968.

Basmann, R. L., "A Generalized Classical Method of Linear Estimation of Coefficients in a Structural Equation," Econometrica, Vol. 25, 1957.

Bellman, R. E., Adaptive Control Process: A Guided Tour, Princeton, New Jersey: Princeton University Press, 1961.

Bentzel, R., and B. Hansen, "On Recursiveness and Interdependency in Economic Models," Review of Economic Studies, Vol. 22, 1954.

Black, R., and J. W. Tukey, The Measurement of Power Spectra, New York, Dover Publications, 1959.

Blau, J. "A Direct Proof of Arrows Theorem," Econometrica, Vol. 40, January, 1972.

Campbell, D. E., "Social Choice and Intensity of Preference," Journal of Political Economy, Vol. 81, 1973.

Chenery, H. S., "Development Policies for South Italy," Quarterly Journal of Economics, November, 1962.

Chenery, H. S., and P. Clark, Interindustrial Economics, John Wiley, New York, 1959.

Chow, G. C., "Optimal Control of Linear Econometric Systems with Finite Time Horizon," International Economic Review, Vol. 13, 1972.

Coleman, J. S., "The Possibility of a Social Welfare Function," American Economic Review, Vol. 56, December, 1966.

Consortium for the Study of Nigeria Rural Development: Preliminary Strategies and Recommendations for USAID Assistance to Nigeria Rural Development 1968-1985, East Lansing, Michigan, 1968.

Day, R. H., Recursive Programming and Production Response, Amsterdam, North-Holland Publishing Company, 1963.

Dhrymes, P. J., Econometrics: Statistical Foundations and Applications, New York, Harper and Row, 1970.

_____, Distributed Lags, San Francisco, Holden-Day, 1970.

Dhrymes, P. J., E. P. Howrey, S. H. Hymans, J. Kmenta, E. E. Leamer, R. E. Quandt, J. B. Ramsey, H. T. Shapiro and V. Zarnowitz, "Criteria for Evaluation of Econometric Models," Annals of Economic and Social Measurement, Vol. 1, No. 3, July, 1972.

Drewrowski, J., University Textbook for African Student, U.N.E.S.C.O., Paris.

Dreze, J. H., "Econometrics and Decision Theory," Econometrica, Vol. 40, 1972.

Encarnation, J., "On the Specification of a Social Welfare Function," Econometrica, Vol. 39, July, 1971.

Evans, M., "Computer Simulation of Non-linear Econometric Models," in The Design of Computer Simulation Experiments, T. H. Naylor (ed.), Durham: Duke University Press, 1969.

Evans, M. K., and L. R., Klein, "The Wharton Econometric Forecasting Model," Philadelphia Economics Research Unit, Department of Economics, Wharton School of Finance and Commerce, 1967.

Federal Office of Statistics, Annual Abstract of Statistics, Federal Office of Statistics, Lagos, Nigeria, 1966.

Federal Office of Statistics, Digest of Statistics, Federal Office of Statistics, Lagos, Nigeria, No. 1, January, 1971.

Feil, J. C., and G. Ranis, "A Theory of Economic Development," American Economic Review, Vol. 51, September, 1961.

Fel'dbam, A. A., Optimal Control Systems, New York, Academic Press, Inc., 1965.

Fishburn, P. C., "Utility Theory for Decision-making," Econometrica, Vol. 25, 1957.

_____, "Arrow's Impossibility Theorem: Course Proof and Infinite Voters," Journal of Economic Theory, Vol. 2, March, 1970.

Fisher, F. M., The Identification Problem in Econometrics, New York: McGraw-Hill Book Company, 1966.

Fisher, W. D., "Estimation in a Linear Decision Model," International Economic Review, Vol. 3, January, 1962.

Fishman, G. S., "Spectral Methods in Econometrics," R-453 PR, Rand Corporation, 1968.

Florentin, J. J., "Optimal, Probing, Adaptive Control of a Simple Bayesian System," J. Electronics and Control, Ser. 1, Vol. 13, No. 2, 1962.

Fox, K. A., and E. Thorbecke, "Specification of Structures and Data Requirement in Econometric Policy Models," in Quantitative Planning of Economic Policy, Hickman B. G. (ed.)

Fox, K. A., et al., The Theory of Quantitative Economic Policy, Chicago: Rand McNally, 1966.

Fromm, G., "Utility Theory and the Analysis of Simulation Output Data," in The Design of Computer Simulation Experiments, T. H. Naylor (ed.) Durham: Duke University Press, 1969.

Fromm, G., and P. Taubmann, Policy Simulation with an Econometric Model, Washington: The Brookings Institution, 1968.

Friedman, M., A Theory of the Consumption Function, Princeton University Press, 1957.

Goldberger, A. S., Econometric Theory, New York: John Wiley and Sons, 1964.

_____, Impact Multipliers and Dynamic Properties of the Klein-Goldberger Model, Amsterdam: North-Holland.

Goldfield, S. M., and R. E. Quandt, "Non-linear Simultaneous Equations, Estimation and Prediction," International Economic Review, vol. 9, February, 1968.

Granger, C. W. J., "The Typical Spectral Shape of an Economic Variable," Econometrica, Vol. 34, 1966.

_____, "Investigating Causal Relations by Econometric Models and Cross-spectral Methods," Econometrica, Vol. 37, July, 1969.

Granger, C. W. J., and M. Hatanaka, Spectral Analysis of Economic Time Series, Princeton: Princeton University Press, 1964.

Jazwinski, A., Stochastic Processes and Filtering Theroy, New York: Academic Press, 1970.

Johnson, G., et al., Strategies and Recommendations for Nigerian Rural Development, 1969-1985, CSNRD 33, Michigan State University, 1969.

Johnston, J., Econometric Methods, New York, McGraw-Hill Book Company, 1972.

Jorgerson, D. W., "The Development of a Dual Economy," Economic Journal, Vol. 71, June, 1961.

Kalman, R. E., "A New Approach to Linear Filtering and Prediction Problems," Journal of Basic Engineering, Trans. ASME, Ser. D, Vol. 82, 1960.

Kelley, A. C., J. G. Williamson, and R. T. Cheetham, Dualistic Economic Development: Theory and History, Chicago and London, 1972.

Kelly, J. S., "The Continuous Representation of a Social Preference Ordering," Econometrica, Vol. 39, May, 1971.

Kemp, M. C., and A. Asimakopulos, "A Note on Social Welfare Functions and Cardinal Utility," Canadian J. Economic and Political Science, Vol. 18, May, 1952.

Klein, L. R., "What Kind of Macro-economic Model for Developing Economies," Econometric Annual of the Indian Economic Journal, Vol. 13, No. 3, 1965.

Klein, L. R., "Estimation of Independent Systems in Macro-economics," Econometrica, April 1969.

_____, "Single Equation Vs Equation System Methods of Estimation in Econometrics," Econometrica, Vol. 28, October 1960.

Klein, L. R. and M. Nakamura, "Singularity in the Equation Systems of Econometrics: Some Aspects of the Problem of Multicollinearity," International Economic Review, Vol. 3, 274-299, 1962.

Koopmans, T. C., ed. Statistical Inference in Dynamic Economic Models, Cowles Commission monograph, No. 10 Willey, 1950.

Koopmans, T. C., H. Rubin, R. B. Leipnik, "Measuring the Equation Systems of Dynamic Economics," in Chapter II of Koopmans (ed.), Statistical Inference in Dynamic Models, New York: John Wiley & Sons, Inc., 1950.

Koyck, L. M., Distributed Lags and Investment Analysis, Amsterdam: North-Holland, 1954.

Kydland, F. E., and E. C. Prescott, "Optimal Stabilization: A New Approach," A paper read at the NBER-NSF Stochastic Control and Economic Systems Conference, Chicago, June 7-9, 1973.

Leontief, W. W., Studies in the Structure of the American Economy, Oxford Press, New York, 1953.

Livesey, D. A., "Optimizing Short-term Economic Policy," Economic Journal, September, 1971.

Lucas, R. E., Jr., "Econometric Policy Evaluation: A Critique," (Forthcoming) in Journal of Money, Credit and Banking, 1973.

Lucas, R. E., and E. C. Prescott, "Investment under Uncertainty," Econometrica, Vol. 39, 659-681, September, 1971.

MacRae, E. C., "Linear Decision with Experimentation," Annals of Economic and Social Measurement, Vol. 1, 1972.

Malinvaud, E., "First Order Certainty Equivalence," Econometrica, Vol. 37, October, 1969.

Marschak, J., "On Adaptive Programming," Management Science, 9, 1963.

Mills, E. S., Price, Output and Inventory Policy, New York: John Wiley and Sons, Inc., 1962.

Modigliani, F., and R. Brumberg, "Utility Analysis and the Consumption Function: An Interpretation of Cross-Section Data," in Kurihara, K. K., (ed.), Post-Keynesian Economies, George Allen and London, 1955.

Muth, J. F., "Rational Expectations and the Theory of Price Movement," Econometrica, XXIX, July, 1961.

Naylor, T. H., Computer Simulation Experiments with Models of Economic Systems, John Wiley and Sons, Inc., New York, 1971.

_____, "Policy Simulation Experiments with Macro-econometric Models: The State of the Art," American Journal of Agricultural Economics, May, 1970, pp. 263-271.

Naylor, T. H., et al., "Some Methods for Evaluating the Effects of Economic Policies Using Simulation Experiments," Review of the International Statistical Institute, Vol. 36: 184-200, 1968.

Naylor, T. H., et al., "Spectral Analysis of Data Generated by Simulated Experiments with Econometric Models," Econometrica, Vol. 37: 333-352, April, 1969.

Nemhauser, G. L., Introduction to Dynamic Programming, New York: John Wiley and Sons, Inc., 1966.

Nerlove, M., "Spectral Analysis of Seasonal Adjustment Procedures," Econometrica, July, 1964.

_____, The Dynamics of Supply: Estimation of Farmer's Responses to Price, Baltimore: Johns Hopkins Press, 1958.

Nigerian Development Plan, 1970/74; Ministry of Economic Development, Lagos, Nigeria, 1970.

Okigbo, P. N. C., Nigerian National Accounts: 1950-1957, Federal Ministry of Economic Development, December, 1961.

Oni, S. A., "The Demand for Nigerian Cocoa: A Statistical Analysis of Regional Markets and Projections," Nigerian Journal of Economic and Social Studies, Vol. 12, March 1970.

Packard, P. C., "A Note on Gross Domestic Investment and Fiscal Policy," Nigerian Journal of Economic and Social Studies, Vol. 8, July 1966.

Pindyck, R. S., "Optimal Stabilization Policies Via Deterministic Control," Annals of Economic and Social Measurement, October, 1972.

_____, "An Application of the Linear Quadratic Tracking Problem to Economic Stabilization Policy," IEE Trans. on Automatic Control, AC-17, 1972.

Pontryagin, L. S., "Optimal Regulation Processes," American Mathematical Society Translations, Series 2, XVIII, 295-339, 1961.

Popper, K. R., The Logic of Scientific Discovery, New York: Basic Books, 1959.

Prescott, E. C., "Adaptive Decison Rules for Macro-economic Planning," Western Economic Journal, Vol. 9, 1971.

Quirk, J., and R. Saposnik, Introduction to General Equilibrium Theory and Welfare Economics. New York: McGraw-Hill, 1968.

Raiffa, H., and R. Schlaifer, Applied Statistical Decision Theory, Division of Research, Harvard Business School, Boston, 1961.

Ramsey, F. P., "A Mathematical Theory of Saving," The Economic Journal, Vol. 38, 1928.

Rausser, G. C., and J. W. Freebairn, "Comparison of Approximate Adaptive Control Solution to U. S. Beef Trade Policy," Paper Read at NBER Conference on Stochastic Control, University of Chicago, June 7-9, 1973.

Rausser, G. C., and S. R. Johnson, "On the Limitations of Simulation as an Approach to Policy Problems Involving Econometric Models," (forthcoming) in Journal of Econometrics, 1973.

Rothenberg, J., The Measurement of Social Welfare, Englewood Cliff, Prentice-Hall, 1961.

Rothenberg, T. J., and C. T. Leenders, "Efficient Estimation of Simultaneous Equation Systems," Econometrica, Vol. 32, 57-76, 1964.

Saaty, T. L., "Coefficient Perturbation of a Contrained Extremum," Operations Research, VII, No.3, 1959.

Samuelson, P. A., Foundations of Economic Analysis, Cambridge, Harvard University Press, 1947.

Sargan, J. D., "Three-stage Least Squares and Full Information Maximum Likelihood Estimates," Econometrica, Vol. 32, 77-81, 1964.

Sen, A. K., "Interpersonal Aggregation and Partial Comparability," Econometrica, Vol. 38:393-409, May, 1970.

Sen, A. K., and P. K. Patternaik,"Necessary and Sufficient Conditions for Rational Choice under Majority Decision," J. Econ. Theory, Vol. 1, 1962.

Sengupta, J. K., "Optimal Stabilization Policy with a Quadratic Criterion Function," Review of Economic Studies, January, 1970.

Simon, H. A., "Dynamic Programming under Certainty with a Quadratic Criterion Function," Econometrica, Vol. 24, 1956.

Smith, V. K., et al., "The Dynamic Properties of a Macroeconomic Model of the U.S. with a Financial Sector," paper presented at the Winter Econometric Society Meeting at New Orleans, Louisiana, December 29, 1971.

Suits, D. B., "Forecasting and Analysis with an Econometric Model," American Economic Review, 1962.

Summers, R., "A Strategy for Appraising Various Simultaneous Equation Estimation Methods by means of Sampling Experiments,"(mimeographed), January, 1959.

Taylor, J. B., "A Criterion for Multiperiod Controls in Economic Models with Unknown Parameters," a paper presented at the second Wrokshop on Stochastic Control and Economic Systems, University of Chicago, June 7-9, 1973.

Taylor, L. J., "Development Patterns: A Simulation Study," The Quarterly Journal of Economics, No. 2, May, 1969.

Theil, H., "A Note on Certainty Equivalence in Dynamic Planning," Econometrica, Vol. 25, 1957.

Theil, H., Optimal Decision Rules for Government and Industry, Amsterdam: North-Holland, 1968.

_____, Linear Aggregation of Economic Relations, Amsterdam: North-Holland, 1954.

Theil, H., and J. C. G. Boot, "The Final Form of Econometric Equation Systems," Review of the International Statistical Institute, Vol. 30, 136-152, 1962.

Tinbergen, J., "On the Theory of Economic Policy," Comtributions to Economic Analysis I, Amsterdam: North-Holland, 1952.

Tintner, G., "What does Control Theory Have to Offer," American Journal of Agricultural Economics, Vol. 51, No. 2, May, 1969.

_____, G., "The Use of Stochastic Linear Programming in Planning," Indian Economic Review, Vol. V, August, 1960, pp. 159-167.

Tse, E., "Adaptive Dual Control Methods," a paper presented at the second Workshop on Stochastic Control and Economic Systems, University of Chicago, June 7-9, 1973.

Tse, E., Y. Bar-Shalom, and L. Meier, "Wide-sense Adaptive Dual Control for Non-linear Stochastic Systems," IEEE Trans. on Automatic Control, April, 1972, pp. 98-108.

Tse, E., and Y. Bar-Shalom, "An Actively Adaptive Control for Linear Systems with Random Parameters via the Dual Control Approach," IEEE Trans. on Automatic Control, April, 1972, pp. 109-116.

Turnovsky, S. J., "Optimal Choice of Monetary Instrument in a Linear Economic Model with Stochastic Coefficients," paper presented at the second Workshop on Stochastic Control and Economic System, University of Chicago, June 7-9, 1973.

van de Panne, C., "Optimal Strategy Decisions for Dynamic Linear Decision Rules in Feedback Form," Econometrica, Vol. 33:307-320, April, 1965.

van Eijik, C. J., and J. Sandee, "Quantitative Determination of an Optimum Economic Policy," Econometrica, Vol. 27, 1959.

Wagner, H. M., "A Monte Carlo Study of Estimates of Simultaneous Linear Structural Equations," Econometrica, Vol. 27, 1958, pp. 117-133.

Williams, A. C., "Marginal Values in Linear Programming," Journal of Society of Industrial and Applied Mathematics, XI, No. 1, 1973.

Wold, H., "Causality and Econometrics," Econometrica, Vol. 22, 1954.

_____, "Recursive Vs Non-recursive Systems: an attempt at Synthesis," Econometrica, Vol. 28, April, 1960.

_____, "Ends and Means in Econometric Model Building," in Probability and Statistics--The Harold Cramer Volume, New York, John Wiley and Sons.

Wonham, W. M., "Optimal Stationary Control of a Linear System with State-dependent Noise," SIAM Journal of Control, Vol. 5, 1967.

_____, "Random Differential Equations in Control Theory," in A. T. Bharucha-Reid (ed.) Probabilistic Methods in Applied Mathematics, Vol. II, Academic Press, New York, 1969.

Ying, C. C., "Learing by Doing: An Adaptive Approach to Multiperiod Decisions," Operations Research, Vol. 15, 1967.

Zellner, A., and H. Theil, "Three-stage Least Squares: Simultaneous Estimation of Simultaneous Equations," Econometrica, Vol. 30, pp. 54-78, 1962.

_____, Bayesian Inference in Econometrics, New York: John Wiley and Sons, 1971.

APPENDIX

APPENDIX 4

Following Thrymes [1970; pp. 382-393],

1. the stochastic process,

$[x(t):t_{\varepsilon} T]$

where, T is a linear index set, is said to be covariance stationary (or weakly stationary) if the covariance function exists; that is, if

$K(t,s), \quad t,s \ \varepsilon T$

is finite, and it is a function only of

$|t - s|$

Furthermore, the mean function of a covariance stationary process should be a constant over the index of the process for practical purposes.

2. Let,

$\{x(t):t_{\varepsilon}N, \ Y(t):t_{\varepsilon}N\}$

be two stochastic processes with mean function:

$M_X(\cdot), \ M_Y(\cdot),$

respectively. Their cross-covariance function (or kernel) is defined by

$$K_{XY}(t,s) = E\{[X(t) - M_X(t)]\ \overline{[Y(s) - M_Y(s)]}\}$$

3. Two stochastic processes

$\{X(t):t_{\varepsilon}N, \ Y(t):t_{\varepsilon}N\}$

are said to be jointly covariance stationary if each is covariance stationary and in addition,

$$K_{XY}(S + \gamma, S) = K_{XY}(\gamma)$$

4. The Lag operator L is defined by;

$$L^k X(t) = X(t-k) \qquad k = 0, 1, 2,$$

where, the operator corresponding to k = 0 is said to be the identity operator and is denoted by I. Similarly, we define a polynominal operator a(L) as:

$$\sum_{k=0}^{m} a_k L^k$$

Thus,

$$Y(t) = a(L)\ X\ (t)$$

has the meaning:

$$Y(t) = \sum_{k=o}^{m} a_k X_{t-k}$$

5. Consider the reduced form equation (4.1)

$$Y_t = AY_{t-1} + \Gamma Z_t + u_t$$

which can be rewritten in an operator representation form as:

$$C(L)Y_t = C^*(L)Z_t + u_t \qquad \ldots .(1)$$

If the operator matrix C(L) is invertible, then the final form of equation (4.1) may be explicitly expressed as:

$$Y_t = [C(L)]^{-1}C^*(L)Z_t + [C(L)]^{-1}U_t \qquad \ldots .(2)$$

C(L) will have an inverse if and only if the roots of

$$|C(\rho)| = 0$$

lie outside the unit circle [$|C(\rho)|$ is a polynomial in the real or complex variable ρ which is obtained by substituting ρ for the operator L in C(L)]

APPENDIX 5

Consider a control problem of the type:

$$Y_t = AY_{t-1} + BX_t + b = u_t, \quad t=1, 2, \ldots$$

where,

Y_t is a vector of endogenous variables, X_t is a vector of current control variables and u_t is a vector of unobservable random variables which is independently and normally distributed with zero mean and finite variance. A and B are unknown matrices of coefficients.

Assume a quadratic loss function with a mathematical expectation $W_t^*(X_t)$. The 'bliss level' for an infinite horizon model with known parameters can be obtained either by use of a constrained minimization procedure or a dynamic programming approach [Chow (1970), (1972)]. Both procedures lead to a bliss control rule $\{X_t^*\}$ found by minimizing $W_t^*(X_t)$.

If, however, the coefficients A and B are not known, another control rule is used. Let $\{\hat{X}_t\}$ represent any such alternative control rule with an expected loss function $\hat{W}_t(\hat{X}_t)$. It is possible that $\hat{X}_t$ might have the same form as X_t^*, with the least squares estimates of A and B replacing the unknown values. Alternatively, $\hat{X}_t$ might have an entirely different form than X_t, perhaps reflecting risk and experimentation to obtain more infor-

mation about the unknown parameters. However $\hat{X}_t$ is defined, its behavior will depend on the unknown parameters A and B and the unknown distribution of u_t.

Taylor (1973) suggests a criterion for evaluating the different control rules. He defines his criterion as a limit of a normalized sum of regrets defined by:

$$C(\{\hat{X}_t\}) = \lim_{T\to\infty} \frac{\sum_{t=1}^{T} E(\hat{W}_t - W_t^*)}{f(T)}$$

where,

f(T) is a positive increasing function of T defined so that the limit exists and is non-zero. The expectation is with respect to the random variables $\hat{X}_t$. Under this criterion $\{\hat{X}_t\}$ is preferred to

$$\hat{\hat{X}}_t \text{ , if } C(\{\hat{X}_t\}) < C(\{\hat{\hat{X}}_t\})$$

A basic requirement of the bliss criterion is that

$$E(\hat{W}_t - W_t^*) \to 0,$$

which implies the mean square covergence of:

$$(\hat{X}_t - X_t^*) \text{ to zero.}$$

The difficulty in meeting the above requirement should be noted. $E(\hat{W}_t - W_t^*)$ does not in general converge to zero quickly enough, most especially when the estimates of A and B upon which $\hat{X}_t$ is based are not consistent. This problem shows up, for instance, in the simulation results

obtained by Tse, et al. (1973). Furthermore, bliss control criterion might fail to distriminate between some control rules.

VITA

Joseph S. Odama was born in Ogori, Nigeria on October 28, 1938. He attended Government Secondary School, Okene and King's College Lagos from 1954-1961. He obtained a Bachelor of Science degree with a major in Economics in 1965 from the University of Ibadan, Nigeria. He then proceeded to London School of Economics and Political Science, London, after a two-year period with the Nigerian Government as a Statistician, on a British Council Fellowship, and obtained a Master of Science degree in Statistics in 1968. He took up appointment with the Department of Economics, Ahmadu Bello University, Zaria, Nigeria as an Assistant Professor the same year. In January, 1971 he obtained a U.S.A.I.D. Fellowship to study at the University of Missouri, Columbia, where he received a Doctor of Philosophy degree in Economics in December, 1973.