OPTIMUM DESIGN OF PRESTRESSED CONCRETE TRANSMISSION POLES

BY

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THESIS DECLARATION

I hereby declare that this thesis has been written by me and that it is a record of my own research work. It has not been accepted in any previous publication for a higher degree. All sources of information are specifically acknowledged by means of references.

UGA.B.W

MAY, 1994

THESIS APPROVAL

This thesis entitled "Optimum design of prestressed concrete transmission poles" by MR.UGA, BITRUS WUYANA meets the regulations governing the award of the degree of Master of Science (Structural engineering)-of Ahmadu Bello University, and is approved for its contribution to knowledge and literary presentation.

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D E D I C A T I O N

To the only wise one. Immortal, invincible,

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ABSTRACT

The minimum weight design problems for three different cross-sectional shapes of prestressed concrete transmission poles subject to stress displacement and buckling load restrictions is attempted. The poles are idealized using the class one prestressed conctrete equations. The optimization technique employed for the solution procedure utilizes the variable metric method by Davidon-Fletcher-Powell for unconstrained minimization and cubic interpolation method of one dimensional search, based on the interior penalty function method. It is a mathematical programming approach involving gradient based method. Partial derivatives are computed using a finite diference technique. The cross-sectional dimensions and tendon wires are used as design variables. Off-design charts have been obtained by performing sensitivity analysis about the final optimum design point. Numerical results for the three cross-sectional shapes considered are presented along with flow-charts, computer programs and program outputs.

NOMENCLATURE

$f(x)$	objective function, the weight of the pole
$g_j(x)$	jth constraint
l	length of pole
r_k	penalty parameter
Δx_k	small increment in design variable x_k
σ_t	induced tensile stress
σ_c	induced compressive stress
δ	deflection
λ	step length
λ^*	optimum step length
ϕ	phi function (penalty function)
ρ	material density
ε	small number used to detamine convergence
f'	first derivative of a function
x_i	ith design point
S_i	ith search direction
∇f	gradient of the function
tm	minimum thickness of section
t_f	flange thickness
tw	web thickness
A	area of section
B	breadth of section
d	depth of section
X(I)	Ith design variable
Pcr	critical buckling load

E modulus of elasticity

f_{max} maximum allowable stress in concrete at working condition

f_{min} minimum allowable stress in concrete at working condition

I second moment of area

J polar moment of area

Mo moment due to torsion

M_T moment due to total load

N number of cables

n number of tendon wires

n_1 loss percentage ratio at transfer

n_2 loss percentage ratio at working condition

P initial prestressing force

Uo prevailing velocity

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C H A P T E R O N E

1.0 INTRODUCTION

1.2 GENERAL

Man,s longing for perfection finds expression in the theory of optimization. It studies how to describe and attain what is best, once one knows how to measure and alter what is good or bad. Normally, one wishes the most, or maximum, and the least, or minimum. The word optimum, or minimum, meaning "best" is synonymous with "most" or "maximum" in the former case, and with "least" or "minimum" in the latter. Optimum has become a technical term connoting quantitative measurement and mathematical analysis, whereas "best" remains a less precise word more suitable for everyday affairs. The technical verb optimize, a stronger word than "improve", means to achieve the optimum, and optimization refers to the act of optimizing. Thus optimization theory encompasses the quantitative study of optima and methods for finding them.[1]

Optimization involves finding the best way to do things, this obvious reason makes it applicable to the practical world. Optimization theory has been known to mathematicians for centuries, but the tedious and voluminous computations

required prevented their practical application. The development of rapid, automatic computers in the middle of the twentieth century encouraged much new research on optimization. Today many important decisions are made by choosing a quantitative measure of effectiveness and then optimizing it. Deciding how to design, build, regulate or operate a physical or economics system ideally involves three steps; first, one should know, accurately and quantitatively, how the system variables interact. Second, one needs a single measure of system effectiveness expressible in terms of the system variables. Finally, one should choose those values of the system variables that yield optimum effectiveness.[2]. The first step, knowledge of the system, is of paramount importance, it is here that the professional skill and training as an engineer is brought to bear. There is little point in optimizing a model that does not describe what is truly happening in the system. Hence most of the work is devoted to understanding the system and describing it quantitatively in terms of tables, graphs, computer programs and mathematical equations.

Optimization methods have been applied to structural engineering problems for many years. However, these techniques have hardly been used by practising engineers in Nigeria. An attempt is made here to choose an optimum cross section for transmission poles for maximum wind speed load in Zaria area of Kaduna state Nigeria. The algorithm used is

implemented for the elastic design of prestressed concrete N.E.P.A. poles, an extension of Essien 1990 [3] work. The optimization scheme presented minimizes the weight of material used by varying the member cross sectional dimensions and prestressing wires. The scheme utilizes the variable metric programming approach as developed by Davidon-Fletcher-Powell for unconstrained minimization and the cubic interpolation method of one-dimensional search, based on the interior penalty function method. A mathematical programming approach involving gradient-based method which uses the derivative of the objective function and constraint functions as well as the values of function themselves.

For many reasons optimization theory is imperfect yet it must be used to predict the operating conditions of a system such that some performance criteria is satisfied. At best such theory can predict only that the system is near to the desired optimum. Optimization methods are then used to explore the local region of operation and predict the way that the system parameters should be adjusted to bring the system to optimum. The importance of optimization lies not in trying to find out all about a system but in finding out with the least possible effort, the best way to adjust the system. If this is carried out well, systems can operate more accurately or at less cost, and the system designer will have a better understanding of the effect of parameter interaction and variation on his design. For this reason sensitivity analysis

is conducted and presented.

1.2 PURPOSE AND SCOPE

The term 'optimization' has been used to describe many processes of improvement and we therefore begin by describing problems with which this thesis is and is not concerned.

The problem which we are concerned with is that of finding the (maximum or) minimum of a deterministic mathematical function, where there exist restrictions or constraints as to what are permissible values of the independent variables. We are not concerned with classical method or classical calculus of variations. Neither are we concerned with optimization of discrete variables or at discrete intervals instead of continuous. Our subject matter is in fact even more limited than has been indicated so far. It is not intended to describe optimization theory so much as it is an account of those principles of optimal design capable of precise description. The problem is modeled as optimizing an objective function subject to constraints. Mathematical rigor is avoided only employed sparingly, no more than needed to prove the design routes and circumscribe their applicability. The problem is that of choosing the best section and cross sectional dimensions for concrete transmission poles to resist the maximum wind load around

Zaria in terms of tensile stress, compressive stress, deflection and buckling.

C H A P T E R T W O

2.0 LITERATURE REVIEW

2.1 HISTORICAL BACKGROUND OF OPTIMIZATION

Linear optimization began formally in 1947 when under the compulsion of World warII a United States Air force project called SCOOP (scientific computation of optimum programs) was set up under the leadership of G.B.Dantzig. The simplex algorithm and much of the related theory was developed by Dantzing and his team in 1947 a further work on special problems and methods continued throughout the next decade by the Dantzing group in the U.S.A. and others in Europe. In the former U.S.S.R. L.V.Kantorovitch published in 1939 a monograph in which the possibilities of applying linear mathematical methods to increase the efficiency in organization and planning of production were suggested. Unfortunately the suggestions were not taken up otherwise much work might have been done in the then U.S.S.R. in linear programming before Dantzing. Contacts of the Dantzing team with John Von Neumann led to fundamental insight into the mathematical theory of linear programming. Neumann emphasized the importance of duality and could immediately see the connection between linear programming and the theory of Games on which he had done fundamental work in 1928. linear programming has become a very important tool in the hands of economists in the last quarter of the century.[1]

In the late 1940's Princeton became a focal point of studies leading to fundamental insight into the theory of mathematical programming. In addition to John Von Neumann,Tucker[1] and two of his former students, Gale and Kuhn[1] did some pioneering work. The Kuhn-Tucker theorem provided the basic link between programming problems and ideas of game theory and clarified the concept of duality. Kelley has proposed an algorithm for solving the problem of non-linear constraints based on the principle of approximating non-linear constraints with linear ones, and successively cutting out non-feasible solutions by additional linear constraints. Apostle [1969] gave a rigorous modern approach to the classical problem of extrema of functions of several variables. For function of two variables widder [1968] is sufficient.

Convex functions are discussed by Gass [1969][1] and in more sophisticated manner by Berge and Ghovila Houri [1965][1].

2.2 DEFINITION OF SOME TERMINOLOGIES

2.2.1 DESIGN VECTOR: Any engineering system or component is described by a set of quantities some of which are viewed as variables during the design process. Some of the quantities are fixed at the outset and these are called preassigned parameters. All the other quantities are treated as variables in the design process and are

called design or decision variables X_i, i=1,2,...,n. The design variables are collectively represented as a design vector.

$$X = \begin{pmatrix} X_1 \\ X_2 \\ \cdot \\ \cdot \\ \cdot \\ X_n \end{pmatrix}$$

2.2.2 DESIGN CONSTRAINTS Design variables are chosen such that they satisfy certain specified functional and other requirements. The restrictions that must be satisfied in order to produce an acceptable design are called constraints. The constraints which represent limitations on the behaviour or performance of the system are termed as behaviour or functional constraints. The constraints which represent physical limitations on the design variables like availability, fabricability and transportability are known as geometric or side constraints.

2.2.3 OBJECTIVE FUNCTION:

There is generally more than one acceptable design and the purpose of optimization is to choose the best one out of the many acceptable designs available. This makes it inevitable to choose a criterion for comparing the different alternate acceptable designs, and for selecting the best.

The criterion with respect to which the design is

optimized, when expressed as a function of the design variables is known as criterion or merit or objective function. The chose of objective function is governed by the nature of the problem.

FIG.1.1 CONSTRAINT AND OBJECTIVE FUNCTION SURFACE

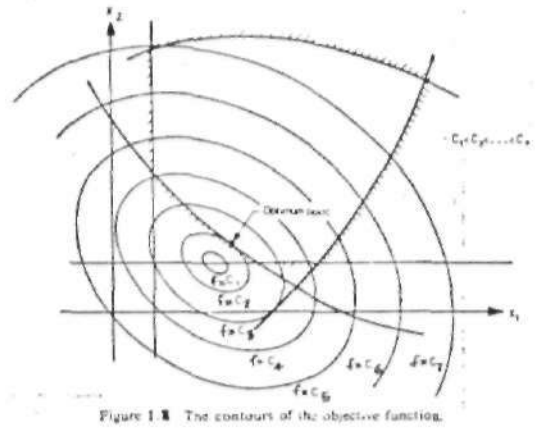


Figure 1.1 The contours of the objective function.

As an illustration, lets consider an optimization problem with only inequality constraints $g_i(x)<0$. The set of X that satisfy the equation $g_i(x)=0$ forms a hypersurface in the design space and is called a constraint surface. This is an (n-1) dimensional subspace where n is the number of design variables. The design space region is divided into two by the constraint surface; one in which $g^i(x)<0$. Thus, the point lying on the hypersurface will satisfy the constraint $g_i(x)$ critically whereas the point lying in the region where $g_i(x)>0$ are infeasible or unacceptable, and the point lying in the region where $g_i(x)<0$ are feasible or acceptable. The collection of all the constraint surfaces $g_i(x)=0, i=1,2,\ldots,m$ which separates the acceptable region is called the composite constraint surface. The locus of all points satisfying $f(x)=c$

Constraint forms a hypersurface in the design space and for each of c there corresponds a different member of a family of surfaces. These surfaces, called the objective function surfaces, are shown in the hypothetical two-dimensional design space in fig.1.1

2.3 CLASSIFICATION OF OPTIMIZATION PROBLEMS

Classification of optimization problems can be as described below.

2.3.1 CLASSIFICATION BASE ON THE EXISTENCE OF CONSTRAINTS:

Any optimization problem can be classified as a constrained or an unconstrained one depending upon whether the constraints exist or not in the problem.

2.3.2 CLASSIFICATION BASED ON THE NATURE OF DESIGN VARIABLES:

Optimization problem can be classified under two broad categories, based on the nature of the variables. In the first case the problem is to find values to a set of design parameters which make some prescribed function of these parameters minimum subject to certain constraints. Such problems are called parameter or static optimization problems. In the second category, the objective is to find a set of design parameters, which are all continuous functions of some other

parameters, that minimizes an objective function subject to the prescribed constraints. This type of problems, where the optimization depends not only on a set of variables but also on their trajectories through some sort of space, are known as trajectory or dynamic optimization problems.

2.3.3 CLASSIFICATION BASED ON THE PHYSICAL STRUCTURE OF THE PROBLEM: There can be optimal control or non optimal control problem depending upon the physical structure of the problem.

2.3.4 CLASSIFICATION BASED ON THE NATURE OF EQUATIONS INVOLVED: This is based on the nature of expressions for the objective function and the constraints. Optimization problems are classified here as linear, nonlinear, geometric and quadratic programming problems. The classification is extremely useful from the computational point of view since there are many methods developed solely for the efficient solution of a particular class of problems.

2.3.4.1 NONLINEAR PROGRAMMING PROBLEM -If any of the functions among the objective and constraint functions is nonlinear, the problem is called a nonlinear programming problem.

i. *GEOMETRIC PROGRAMMING PROBLEMS (GPM):*

Definition - A function h(x) is called a posynomial if it can be expressed as the sum of power terms each of the from

$$C_1 X_1^{ai1}\, X_2^{ai2} \ldots X_n^{ain}$$

where c_i and a_{ij} are constants with $c_i>0$ and $x_j>0$. Thus a posynomial can be expressed as:

$$h(x) = c_1 x_1^{a11} x_2^{a12} \ldots\ldots x_n^{a1n} + \ldots\ldots + c_N x_1^{aN1} x_2^{aN2} + \ldots\ldots + x_n^{aNn}$$

A geometrical problem is one in which the objective function and constraints are expressed as posynomials in x. Thus the GPM problem can be posed as follows:

Find X which minimizes

$$f(x) - \sum_{i-1}^{No} c_1 \left(\prod_{k-1}^{n} X_K^{Pij} \right) \leq 0, a_{ij} > 0$$

subject to

$$g_j(x) - \sum_{i-1}^{Nj} a_{ij}\left(\prod_{k-1}^{n} X_k^{gik}\right) \leq 0, a_{ij} > 0$$

Where No and N_j denote the number of posynomial terms in the objective and jth constraint function respectively.

ii QUADRATIC PROGRAMMING PROBLEM: This is a nonlinear programming problem with a quadratic objective function and linear constraints. It is formulated as follows:

Find X which minimizes

$$F(X) - C + \sum_{i-1}^{n} q_i X_i + \sum_{i-1}^{n}\sum_{j-1}^{n} Q_{ij} X_{i_j}$$

subject to

$$\sum_{i-1}^{n} a_{ij} X_i - b_j, j-1,2,..,m,\ x_i \geq 0, i-1,2,...,n$$

where c, q_i, Q_{IJ}, a_{ij} and b_j are constraints.

iii LINEAR PROGRAMMING PROGRAM:

When the objective function and the constraints are linear functions of the design variables, the problem is called a linear programming problem (LP). A (LP) is often stated as: Find:

$$x = \begin{pmatrix} X_1 \\ X_2 \\ . \\ . \\ . \\ X_n \end{pmatrix}$$

which minimizes

$$f(x) = \sum_{I=1}^{n} c_i x_i \sum_{K=1}^{n} a_{jk} X_j$$

subject to the constraints

$$\sum_{K=1}^{n} a_{jk} X_K = b_j, j = 1,2,..,M$$

and $X_i \geq 0, i = 1,2,...,n$

where c_i, a_{ik} and b_j are constraints.

2.3.5 CLASSIFICATION BASED ON THE PERMISSIBLE VALUES OF THE DESIGN VARIABLES:- Depending on the values permitted for the design variables optimization problems can be classified as integer-and real-valued programming problems.

2.3.6 CLASSIFICATION BASED ON THE DETERMINISTIC NATURE OF VARIABLES INVOLVED

Based on the deterministic nature of the variables involved, optimization problems can be classified as deterministic and stochastic programming problems in which all or some of the parameters (design variables and or preassigned parameters) are probabilistic (non deterministic ow stochastic).

2.3.7 CLASSIFICATION BASED ON THE SEPARABILITY OF THE FUNCTION:

Optimization problems can be classified as separatable and nonseparatable programming problems based on the separability of the objective and constraint function.

2.3.8 CLASSIFICATION BASED ON THE NUMBER OF OBJECTIVE FUNCTION

Depending on the number of objective functions to be minimized, optimization problems can be classified as single and multi-objective programming problem. A multi objective programming problem can be stated as

follows:

Find X which minimizes $f_1(x), f_2(X), \ldots, f_k(x)$

subject to

$$g_j(x) < 0, j=1,2,\ldots,m$$

where $f_1, f_2, \ldots, f_k$ denote the objective functions to be minimized simultaneously.

2.4.0 PROBLEM DEFINITION

For our problem and many others, the mathematical problem becomes that of minimizing or maximizing a function, f say, of n independent variables, $x_1, \ldots, x_n$ which is a general constrained optimization or mathematical programming problem, stated as:

Find

$$X = \begin{pmatrix} X_1 \\ X_2 \\ \cdot \\ \cdot \\ \cdot \\ X_n \end{pmatrix}$$

which minimizes f(x)

subject to the constraints

$$g_j(x) \leq 0, j=1,2,\ldots,m$$

and

$$L_j(x) = 0, \quad j=1,2,\ldots,p$$

where X is an n-dimensional vector called the design vector, f(x) is a scalar quantity called the "objective function" of vector x and $g_j(x)$ and $L_j(x)$ are

respectively, the inequality and the equality constraints, where, n, m and p need not be the same in number or even related.

2.5.0 REVIEW OF FLEXURAL FAILURES IN TRANSMISSION CONCRETE POLES

An investigation made by Essien [3] on the failure of N.E.P.A poles around Kaduna state, using Zaria as a case study because of the relatively high frequency of poles failure in that area. The failures where found to be in the form of cracking, tilting from foundation, exposure of reinforcements or tendon wire and wearing away of the concrete leading to total failure. These failures were in area of high wind speed, that was his reason for choosing Zaria, which has an average wind speed of 54m/s[NEPA]. The design was done for a prestressed concrete member. Of all the failure modes flexural failure was the most critical thus the investigation of that mode. He suspected inadequacy in the analysis:-that it was possible that all critical loads were not considered in the design. Fatigue failure due to cyclic wind loading was another suspected area. In carrying this task he collected some data and rechecked the following ;

I. Resonance (fatigue failure) on the poles

II. Structural analysis method used

III. Lifting stresses

His work was carried out on the I-section which he found to fail most. The hollowed cylindrical circular section was used as a check for comparison only. After carrying out the design he came out with the following findings:

At the design wind speed used (54m/s) the position of frequency holes determined by calculation did not correspond to the frequency holes of existing poles on site.

The sizes of the sections on site did not meet the stress requirement. They were smaller than the actual required sections.

Insufficient tendon wires.

A phase change in stress, the stresses from the bottom of the pole up to about 4.5m height kept decreasing. But from there they start to increase to about 6.5m height of the pole. Thus a range of weak point in the section between hight 4.5m and 6.5m is created.

The existing nominal stirrups spacing is too much.

From the conclusions and suggestions by Essien and looking through the results of his work we would rather think that what he considered to be the probable reason for the failure i.e critical loading due to cyclic wind loading was not that critical as to course failure. From his calculated results, the position of the frequency holes with the existing ones were not too far apart. This discrepancy could be due to the method of analysis used and

the degree of accuracy used by the designers. Moreover one of the purpose of prestressing is to resist cyclic loading. Cyclic loading may not be a critical load to be considered in the design. Also from the results the moment that remains the same throughout the poles' length is torsion. Cables are more likely to snap when the wind speed is high, causing torsional moment. For a tapered element like the pole, it is obvious that the thinner sections will have larger torsional stresses. This explains the frequent failure of the poles at areas of high wind speed as Essien(1990) discovered. In this study we also have the opinion that there will be a cross-sectional shape of pole that will resist this tensile stresses more even when tapered. This work will be set forth to explore this area in order to answer the riddle of phase change in stress.

Below is the method of analysis he used.

2.5.1 METHOD OF STRUCTURAL ANALYSIS

2.5.1.1 PRESTRESSED CONCRETE POLES

Based on the following facts and assumptions we shall set out to do the design as he did:

Prestressed columns are used when the eccentricities of load are high and so require high bending stresses to be resisted. They are defined in design code as having mean prestress of 2.5N/mm² or more. The analysis is the same as for reinforced concrete, and sections are design using the assumptions made for prestressed concrete beams. The primary objective in prestressing is to avoid excessive cracking and deflection whilst at the same time enabling

high strength materials, particularly high-tensile steel, to be used efficiently in construction. The main criteria governing the design of prestressed concrete are therefore characteristics of serviceability rather than ultimate strength [4]. Class one prestressed concrete is used for the design because class one is supposed to be free from cracking and flexural tensile stresses under service conditions. They also have additional advantage of resisting cyclic or dynamic loading under severe condition of exposure [4]. In assessing the condition of prestressing it will be assumed sufficient that the concrete deforms elastically under short-term loading, that creep of concrete can be treated by adopting an effective modulus for the concrete and that shrinkage is uniform across the section [BS8110]. Deflection checking is as for normal concrete section.

2.5.1.2 STRESS EQUATIONS

The tendons are usually tension to an initial stress equal to or less than 70 or up to 80% of their characteristic strength. The maximum compressive stress limit for concrete under loading at serviceability limit state is 0.33 cube strength (fcu) of concrete (fcu=60N/mm²) and maximum tensile stresses at transfer of concrete for class one equal to or less than 1N/mm² also at serviceability state.

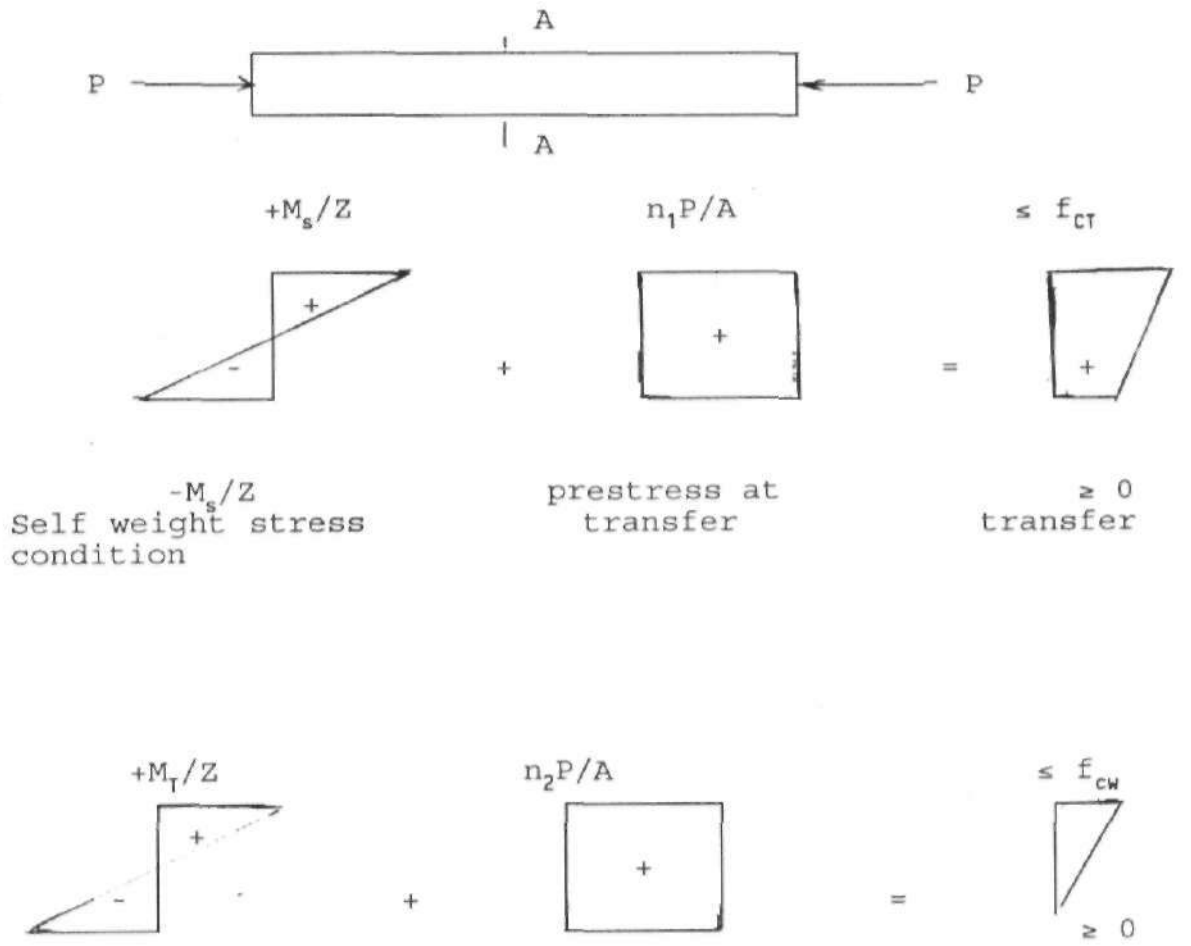


FIG.1.2 STRESSES IN MEMBER WITH AXIAL PRESTRESS FORCE.

The stress diagram shown in fig1.2 can be expressed algebraically as follows [5].

Transfer condition:

Top fibre

$$+\frac{M_S}{Z}+\frac{n_1P}{A}\leq f_{CT}\ldots.(2.1)$$

Bottom fibre

$$-\frac{M_S}{Z} \pm \frac{n_1 P}{A} \geq 0$$

.....(2.2)

i. SERVICE CONDITION

Top fibre

$$+\frac{M_T}{Z} + \frac{n_2 P}{A} \geq f_{cw}$$

.....(2.3)

Bottom fibre

$$-\frac{M_T}{Z} + \frac{n_2 P}{A} \geq 0$$

.....(2.4)

Where M_T=total load moment

M_S=moment due to self weight

f_{cT}= compressive stress at transfer

f_{cw}=compressive stress at working condition

Z = sectional modulus

n_1=loss percentage ratio at transfer

n_2=loss percentage ratio at working condition

P =initial prestressing force

Taking compressing as positive and tension as negative.In the case of members with torsion moment (Mo) the stresses are as follows

Top fibre:

$$+\frac{M_T}{Z_T} + \frac{M_O t_m}{J} + \frac{n_1 P}{A} \leq f_{ct}$$

.....(2.5)

Bottom fibre :

$$-\frac{M_T}{Z_T} - \frac{M_O t_m}{J} + \frac{n_1 P}{A} \geq 0$$

.....(2.6)

Service condition.

Top fibre:

$$+\frac{M_T}{Z_T} + \frac{M_O t_m}{J} + \frac{n_2 P}{A} \leq f_{cw}$$

.....(2.7)

Bottom fibre;

$$-\frac{M_T}{Z_T} - \frac{M_O t_m}{J} + \frac{n_2 P}{A} \geq 0$$

.....(2.8)

Where t_m=minimum thickness of section

M_0=Torsional moment

M_t=total bending moment

Z_T=sectional modulus

J= polar section moment of area of section

From equation (2.8) assuming equality sign

$$P = (\frac{M_T}{Z_T} + \frac{M_O t_m}{J}) \times \frac{A}{n_2}$$

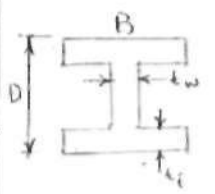
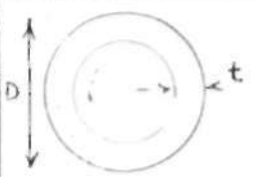
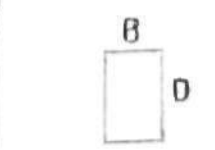
.....(2.9)

Substituting (2.9) into (2.7) we get

$$f_{cw} = 2\left(\frac{M_o t_m}{J} + \frac{M_z}{Z_T}\right)$$

.....(2.10)

TABLE 2.1 **SECTIONAL PROPERTIES**

	SECTION	J	Z_T
1		$\frac{1}{3}[2Bt_f^3 + (D-2t_f)t_\omega^3]$	$\frac{(D-2t_f)\,t_W^3 + 2t_f B^3}{6B}$
2		$\frac{\Pi[D^4-(D-2t)^4]}{32}$	$\frac{\Pi[D^4-(D-2t)^4]}{32D}$
3		$\frac{DB^3}{3}$	$\frac{BD^3}{6}$

2.5.1.3 LOAD ON THE POLE:

Poles are erected in a straight - line arrangement, but can change direction depending on the location of the point where electricity is to be transmitted [3] Poles in

the middle of the row are called intermediate poles and those at the ends are called end poles. Load on the pole is due to self weight, wind load and the weight of cables.

LOAD FROM CABLES. Maximum load from cables is transmitted when wind load on them is horizontal and transverse to the cables direction.

FIG: 2.3 POLES IN STRAIGHT LINE

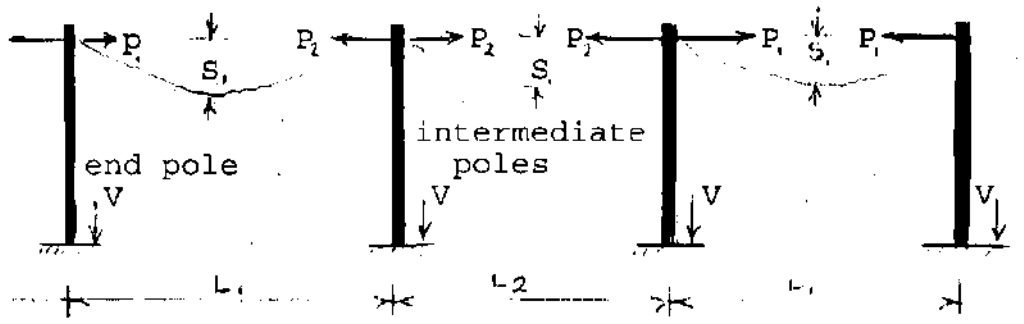


FIG: 2.4 VIEW IN THE DIRECTION OF LINE

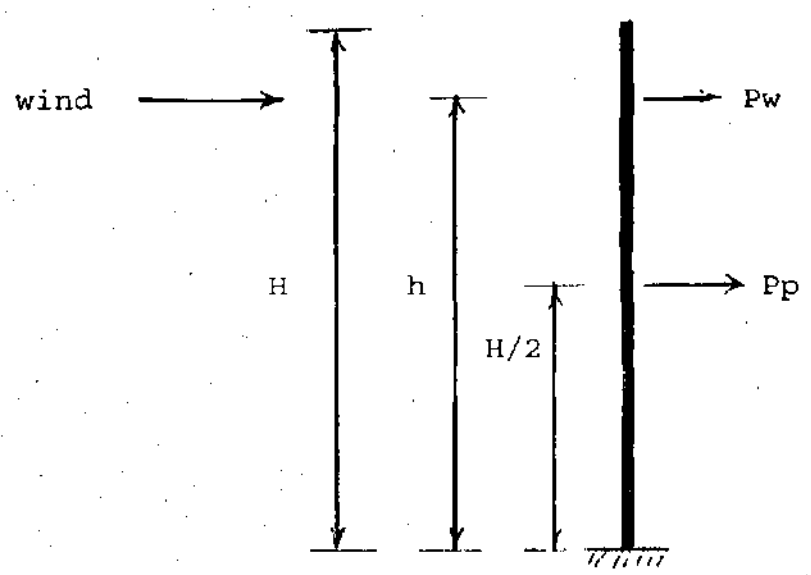


TABLE 2.2 **ULTIMATE LOAD ON POLE DUE TO SELF WEIGHT AND DEAD LOAD.**

	END POLE	INTERMIDIA-TE POLE	DESCRIPTIO N
1	$P_w = \frac{NP_C L_1 d_C}{24}$	$P = \frac{NP_C (L_1 + L_2) d_C}{24}$	Transverse force due to wind on conductor
2	$P_1 = \frac{NW_C L_1^2}{8S_1}$	$P_1 = \frac{NW_C L_1^2}{8S_1}$ $P_2 = \frac{NW_C L_2^2}{8S_2}$	Horizontal force in direction of line
3	$V = \frac{NW_C L_1 + W_D}{2}$	$V = \frac{NW_t (L_1 + L_2)}{2} + W_D$	Total vertical force
4	$P_P = P_P BH$	$P_P = P_P HB$	Resultant horizontal force due to wind on the pole

Where N = number of conductors

Wt = total vertical weight on the pole excluding self weight

B = breath or diameter of pole

dc = normal diameter of conductor

Wc = weight of conductor

WD = total weight of pole attachment etc

Pc = wind pressure on conductor

Pp = wind pressure on pole

If the sag is large compare to L_1 replace L_1 with

$$L_1 + (8S_1^2)/3L_1$$

and L_2 by

$$L_2 + 8S_2^2)/3L_2$$

Where

S = sag of the cables (maximum)

L = horizontal dimension of the spacing of the poles.

The maximum ultimate transverse loads are applied at height 'h' given by

h = H-600mm

and

H = total height of pole from ground level. The cables are suspended at the height h on the pole on a cross-tree of length 600mm [8.18.19]. The end poles carry maximum load and are anchored by means of cable wires. The anchorage relieves the high loading condition. We shall consider poles where the line is continuous. This is the most likely pole to fail [3]. This kind of pole must have maximum strength transverse to the direction of the line. The strength in the line generally considered is the same as that laid down in British standard specifications

for concrete pole (BS607) not less than 1/4 of the transverse strength. The load consists of or some of the following [3]

1. **Vertical loading from**

   a.self weight of pole

   b.cable weight from down-ward

   component of conductor tension

2.**Horizontal loading from**

   a.wind loading on the conductor

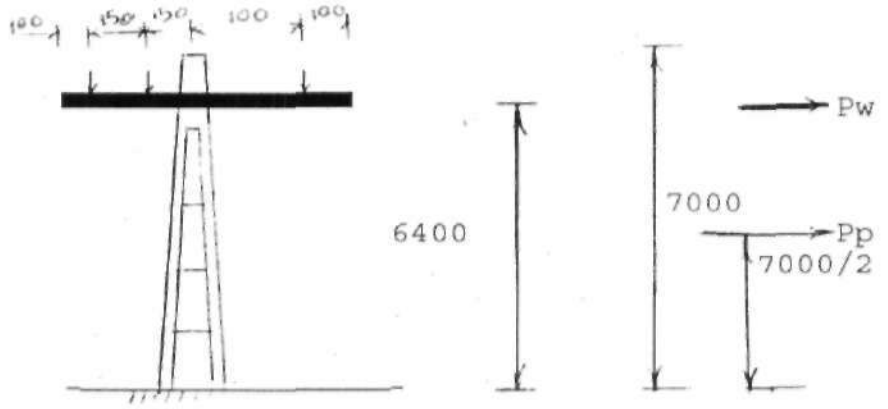
   b.wind loading on the pole

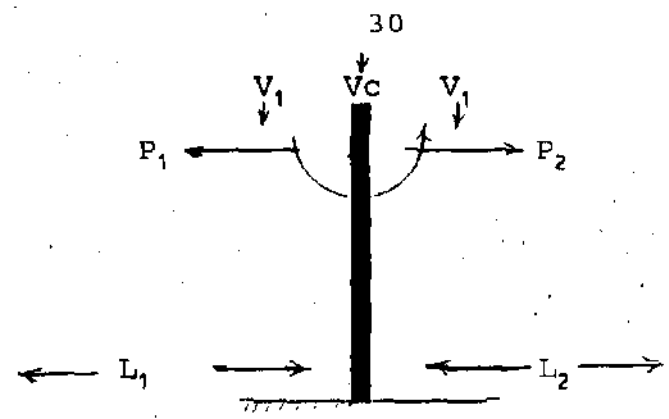
3.**Excess loading from**

   a.torsion due to skew snapping of wires

   b. collapse loading due to eccentric snapping of wires.

FIG:2.5 AXIAL LOAD ON POLES (FROM CABLES)





a. WIND PRESSURE ON CONDUCTORS

Velocity of wind Uo = 54m/s.

From table 5 NCP1 1973 part 3 wind pressure

by interpolation for 54m/s gives a velocity Pc = 101N/m^2.

Diameter of conductor dc =13.7mm [NEPA].

Spacing of poles L = 50 [NEPA].

Assuming no sag pressure on cables,

$Pw = NPc\ Ldc$

where N = number of cables = 3

$Pw = 3x101x50x13.7x10^{-3}$

=0.21KN

From ultimate load table, taking sag into consideration

and $L_1 = L_2 = 50M$

$PW = NPcdc(L_1+L_2)/24$

$= 4x1001(50+50)x13.7x10^{-3}$

$= 0.0234KN$

Take Pw = 1.0KN

VERTICAL FORCE ON POLE

Fom table 2.2

$Vc = NWt(L_1 + L_2)/2$

from appendix 7 Wt = 1.04Kg/m

N = Number of conductor wires = 3

$$Vc = 3x1.04x10(50+50)/2$$

$$Vc = 1.56KN$$

Say 2KN

b. HORIZONTAL COMPONENT OF THE WIRES

These forces are P_1 and P_2

From table of ultimate load

$$P_1 = (NW_cL_1^2)/8S_1$$

$$P_2 = (NW_cL_2^2)/8S_2$$

Since $L_1 = L_2$ and $S_1 = S_2$

Therefore $P_1 = P_2$ which cancels out having no effect on the pole.

c. WIND PRESSURE ON POLE

Design wind speed (CP3, part2, chap.V -1972)

$$Vs = Uo \times S_1 \times S_2 \times S_3$$

where $S_1 = 1$

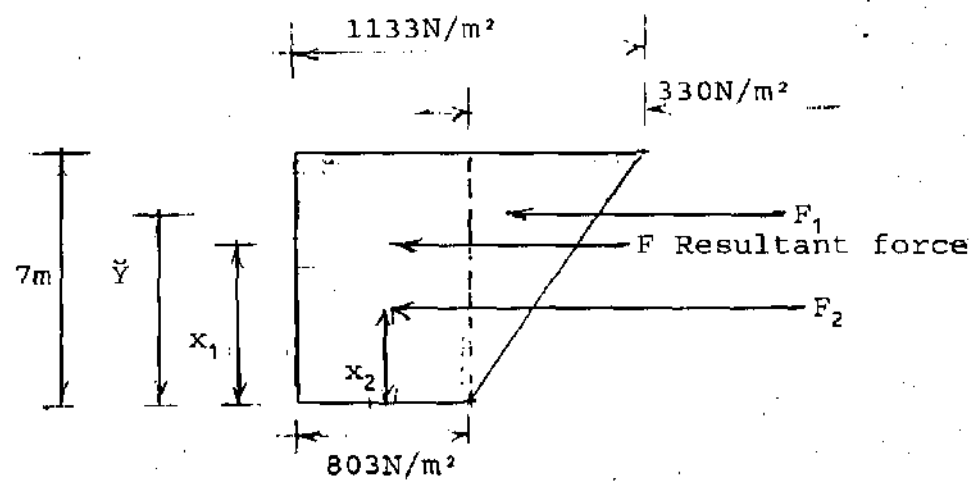
$S_3 = 1$

and S_2 depend on the height of pole. Uo is the wind speed (54m/s). From table 3 CP3 part3 chapter V-1972 wind pressure at various heights of the pole is tabulated and drawn bellow [7]

TABLE 2.3 **WIND PRESSURE ON POLE**

S_2	N/M^2	Hight/m
0.796	1133	7.0
0.768	1055	6.0
0.715	914	4.5
0.67	803	3.0
0.67	803	1.5
0.67	803	0.0

d.WIND PRESSURE DIAGRAM



To calculate leverarm $\bar{Y}$

$$\bar{Y} = (\Sigma A x X_1 I)/\Sigma A$$

$$= (25102)/6776$$

$= 3.7m$

Resultant force $F = b_1 \times$ (Area of the pressure diagram)

where

b_1 = maximum base dimension.

Area of trapezium $= 0.5(803 + 1133)x7$

$= 6776m$ sq

$F = b1x6676N$

$b_1 = 318mm$

$\therefore F = 0.318 \times 6776$

$= 2.155$

say 3KN

e . DETERMINATION OF MAXIMUM WIND MOMENT (MT)

This is at the ground level

$MT = Pw \times 6.4 + Pp \times 3.7$

$= 1.0 \times 6.4 + 3.0 \times 3.7$

$= 17.5KN$

Say 18KNM

f. TORSIONAL MOMENT COMPUTATION

Torsion in cables

Diameter of conductor dc = 13.7m

Area of conductor $Ac = (\pi dc^2)/4 = 148mm$ sq

Breaking load of conductor = 44.1KN (copper).

Factor of safety for the cables = 2

Tension in cables = (breaking load)/(factor of safety)

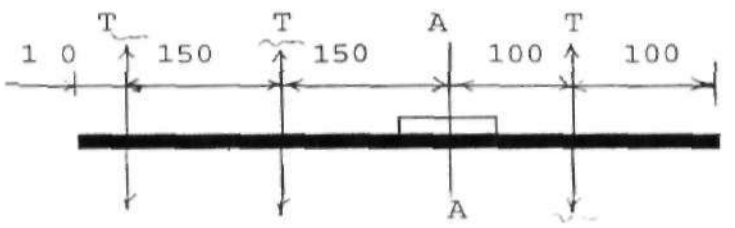
= (44100.0)/2

T = 22050N

take 22KN max. tension

(i) MOMENT CALCULATION

PLAN VIEW OF POLE AND CABLES



Taking moment about A-A for worst codition of failure of cables

$Mo = M_{(A-A)}$ = T x 0.3 + T x 0.25

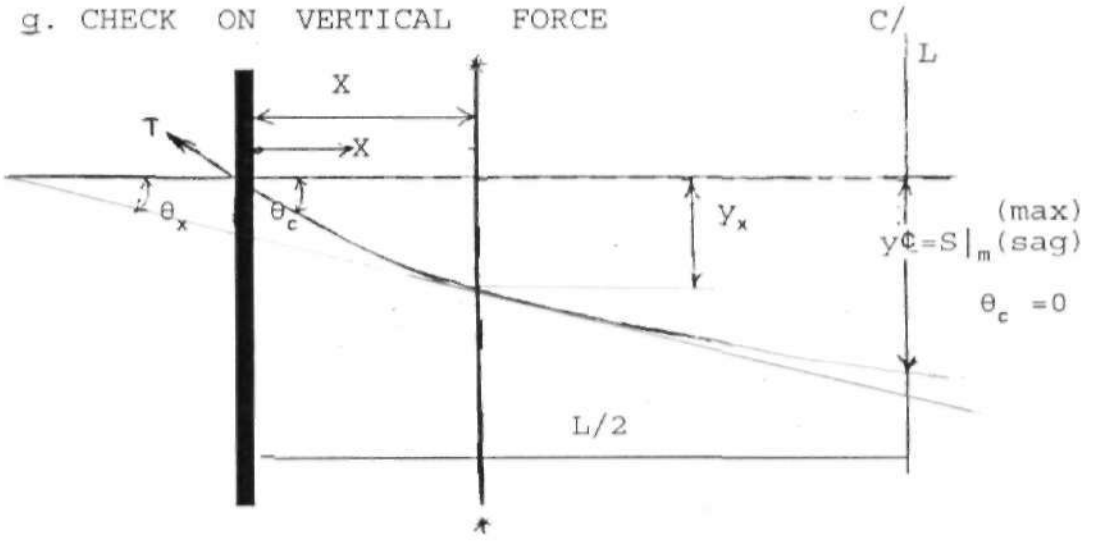
= 22 x 0.3 + 22 x 0.15

= 12.1KN

Take Mo = 15KNM

Any weakness of failure of any of the cables on either or both faces of the pole will induce torsional stress on the pole.

g. CHECK ON VERTICAL FORCE



Parabolic profile $y_x = (4S_xX(L-X))/L^2$

Slope

$$dy_x/dx = 4S_x/L - 8S_xX/L^2$$

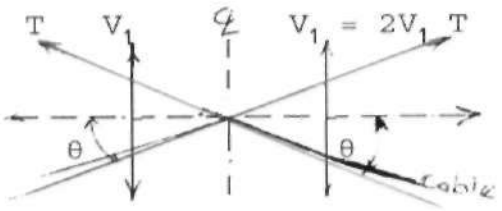
at x = 0 $dy_x/dx = \tan\theta_x = 0 = 4S_x/L$

Sm was calculated to be 500m [3]

L = 50m Sm = 0.5m

$\text{Tan}\theta = (4\text{x}0.5)/50 = 0.04$

$\theta = \tan^{-1}0.04 = 2.29^0$



V = Tsinθ

H = Tcosθ

For the conductor wires

V_T = 3 x 2 x 22 x Sin2.29

= 5.3 say 6KN this is used instead of 2KN

SOME EXISTING WORKS ON STRUCTURAL OPTIMIZATION

In the past structural optimization investigated and reported were base on one constraint. Turner [8] developed a technique for determining relative proportion of selected elements of wing structure to attain a specified flutter speed with minimum total mass. Ashley et al [9] reported the one-dimensional structure optimization like beams under aeroelastic constraints. Kicher [10] obtained the minimum weight design of integrally stiffened cylindrical shells with constraints on buckling loads. Moshe B.F and Maurice A.B [11] investigated continuous beams for optimum geometry with strain energy of the beam as the objective function. Rao [12] considered strength stability, frequency and flutter constraints for optimization of wing structures. Y.Ding and B:J.D.Esping [13] presented the minimum weight design problem for frame structures subject to stress and displacement constraints .B.H.V.Topping and D.J.Robinson [14] illustrated how the sequential linear programming technique can be used to design timber frames in accordance with the British code of practice.CP 112:part 2:1971. They emphasise a practical method having results that comply to the code.

C H A P T E R T H R E E

3.0 FORMULATION OF THE OPTIMIZATION PROBLEM

3.1 INTRODUCTION

Optimal design can be viewed as the optimization of some desired mathematical functions found in design problems [15]. To get an intelligent design, the purpose of the design must be known in the face of any restrictions placed on it. This goals and limitations are known as constraints. All structural members under load experience stresses and strains until, at certain loads the deformation and stresses become too large and the element yields or fails. This upper bound for stress and deformation must not be exceeded if the element must not fail. Limitation will be placed on the response of the pole under load, because the pole as any other structural element can fail if over stressed. This obvious reason makes the optimization problem a constrained optimization problem.

3.2 PROBLEM DEFINITION

The optimization problem can be stated as: Find the optimal cross-sectional dimensions,where the cross-sectional shape is given, that minimizes the structural weight or volume (since the density will be kept constant) and satisfies stress, displacement and stability

constraints. The problem considers an elastic pole exposed to wind flow over its body. It is required to determine the optimal cross sectional dimensions which:

should result in a pole of minimum weight for which the lateral displacement stresses and buckling load should not be more than permissible.

The wind speed parameter is to be held fixed during the optimization process and equal to the critical speed yielding critical stresses and displacement[16]. The problem considers only one design critical condition; that of wind load on the cables and on exposed face of the pole[17].

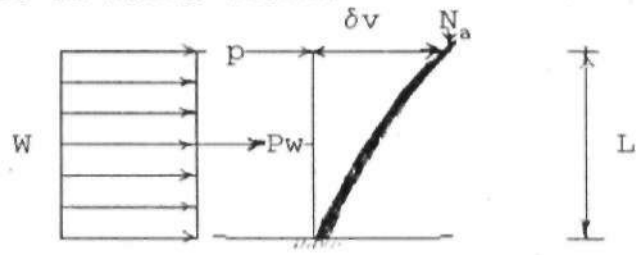
We can state the structural optimization in a standard mathematical programming problem as:

Find X to minimize F(X) subject to

$g_i(x) > 0,\ i = 1,2,....,m$ [18]

Where X is an n - dimensional vector of design variables and m number of constraints.

If we take the wind load to be W and, P to be wind load on cables while, N_a and δv are the weight of cables, and deflection of the pole respectively. Then the structural model will be as shown bellow



3.3 MATHEMATICAL FORMULATION

The first step in the analysis and synthesis of a system is the development of a suitable mathematical model, which is obtained from the structure of the system and the properties of the system, together, with fundamental physical laws. Difficulty may arise because of complexity of the system, and insufficient data.Oversimplified model may eliminate intrinsic characteristics of the structure and overcomplicated model will cause difficulty. A suitable model is a compromise between the mathematical difficulty attached to complicated equations and the accuracy desired in the final result.

3.3.1 OBJECTIVE FUNCTION

The objective of structural designs are always the reduction of some (operational or manufacturing) cost associated with the structure but can be very different according to its specific purpose [19]. The cost of manufacture of prestressed concrete poles can be said to be related to its structural weight. Thus weight minimization will be the goal of the numerical optimization to be studied, and the structure is to remain in the linear elastic region. Thus, by assuming that the structural weight is proportional to,

i the material density ρ_i

ii a geometric parameter L_i (length)

iii the design variable A_i (cross sectional area)

The functional to be minimized can be written generally as:

$$W = \rho_i L_i A_i$$

Where: W = weight and i = 1,...,ne

ne = number of elements [19]

Minimize weight (W) given as $W = \rho_i L_i A_i$,

$i = 1,2,.....,n_e$, n_e = number of elements.

Since we shall be dealing with one pole at a time ne = 1

i.e $W = \rho LA$

Where ρ, L and A as before defined.

3.3.2 CONSTRAINTS

The design criteria to be satisfied are the following:

3.3.2.1 FLEXIBILITY CONSTRAINTS (**Behaviour constraints**)

The elastic lateral displacement deformation of the pole under load must not exceed a certain limit.

The tensile and compressive stresses induced should not exceed a certain limit (the yield stresses of the materials). We shall assume that all the compressive stresses will be taken by the concrete and tensile stresses by the reinforcement.

Let the allowable stresses be:

σ_c = direct compressive

σ_{tp} = tensile

and the applied stresses be:

σ_a = maximum axial induce stress

σ_{ta} = maximum tensile induce stress

The constraint equations due to the stresses induced on the pole are given by:

$$g_1(x) \equiv \sigma_a/\sigma_c - 1 \leq 0$$

on compressive stresses

,and $g_2(x) \equiv \sigma_{ta}/\sigma_{tp} - 1 \leq 0$

on tensile stress.

3.3.2.2 STABILITY OF EQUILIBRIUM OR BUCKLING STRENGTH

Since the pole is a slender column a check is made on its stability. There should be no loss or even bifurcation of stability of equilibrium.It is a known fact that only short columns fail by crushing while failure occurs due to instability when certain load (called the critical or Euler load) is equal or is exceeded on slender columns. The end moments on a slender column cause it to deflect sideways and thus bring into play an additional moment.The moment causes a further lateral deflection and if the axial load exceeds a critical value, this deflection, and the additional moment becomes self-propagating until the column buckles [20]. Hence the strength of the column is not determine by the unit stress but by the mean load it can carry without becoming unstable. The condition of instability is characterized by disproportionally large increase in lateral deformation with slight increase in load. This may occur before the unit stress reaches the elastic limit.

The lateral displacement δv is restricted to a maximum value of δmax as:

$$g_3(x) \equiv \delta v/\delta max - 1 \leq 0$$

3.3.2.3 SIDE CONSTRAINTS

Side constraints will be imposed for convenience,and practicability so as to;

avoid difficulty in placing of concrete and

provide minimum cover for tendon wires (20mm)

Lower limits will be placed on the dimensions taken from site to:

avoid brittle failure due to insufficient prestressing steel in the section to sustain the tensile forces transferred to it on the development of cracks.Therefore minimum steel area As= 0.0013 x (Area of concrete section) [BS8110], will be observed as lower bound.

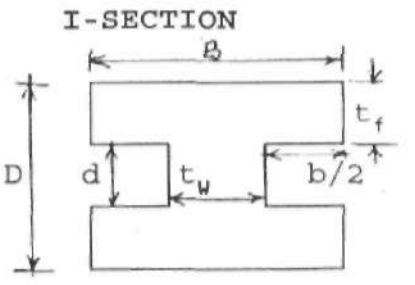
3.3.3 DESIGN VARIABLES

We shall limit yourself to transverse dimensional design variables and the amount of prestressing steel proportional to prestressing force only. The optimization of these variables leaves the geometry and material properties unchanged. From available knowledge,it is known that this order is roughly that of increasing complexity in an optimization program. Even change in geometric variables has essentially non linear repercussions on the response of the structure [19] which is not desirable, since the structure is supposed to be within linear elastic regime.

Part of the concrete dimensions will be assumed as preassigned parameters, thereby reducing the number of design variables and simplifying the optimal design problem. For example in this respect the length of the pole

will be kept constant while the shape will be varied but kept constant during the optimization process. Three shapes will be used I-section, circular tubular section and rectangular section.

3.3.4 I - SECTION EQUATIONS



From the above section $b=(B-t_w)$

$d= D-2t_f$

$$bd^3=(B-t_w)(D-2t_f)^3$$

Cross sectional area $A = BD - [(B-t_w)(D-2t_f)]$

$$= 2Bt_f - 2t_ft_w + Dt_w \quad .....(3.1)$$

If n = number of tendon wires.The area of tendons As is given by

$$\frac{n\Pi\Phi^2}{4}$$

.....(3.2)

where ϕ = diameter of wire.Which means that

$A_s = 0.785n\phi^2$

We shall for the sake of convenience use 5mm prestressing wires for all the sections varying only the number in the process &of the optimization.

Area of one wire = 19.6mm²

$\therefore A_s = 19.6x10^{-6}m^2$

3.3.4.1 OBJECTIVE FUNCTION

Minimizing weight $W = \rho \underline{l} A = 0.69\rho_1 + 19.6nL\rho_2$

$= 0.69\rho_1 \underline{l}(2Bt_f + Dt_w - 2t_f t_w) + 19.6n\rho_2 \underline{l}$

Where ρ_1 = weight of concrete ($24x10^{-6}N/mm^3$)

$\underline{l}$ = length of pole taken to be 7000mm

ρ_2 = weight of steel ($77x10^{-6}N/mm^3$

$L\rho_1 = 0.168$

$L\rho_2 = 0.539$

0.69 = ratio of reduction of weightof column due to linear tapering

The weight $W = 0.23184Bt_f + 0.11592Dt_w - 0.23184t_f t_w + 10.564n$ .....(3.3)

3.3.4.2 DESIGN VARIABLES

$$X = \begin{pmatrix} B \\ D \\ t_f \\ t_w \\ n \end{pmatrix} = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{pmatrix}$$

This is a 5-D design vector problem.

The objective function equation can be written as:

$OBJ = 0.23184X_1X_3 + 0.11592X_2X_4 - 0.23184X_3X_4 + 10.564X_5$

3.6.4.3 CONSTRAINTS

a. STRESSES (SERVICEABILITY LIMIT STATE)

(i) - TENSION

$$\frac{n_2 P}{A} - \frac{M_T}{Z_T} - \frac{M_o t_m}{J} \geq 0 \text{.....(3.4)}$$

$$\frac{AM_T}{n_2PZ_T} + \frac{AM_O t_m}{n_2PJ} - 1 \leq 0$$

.....(3.5)

From design code table (BS8II0) using $\phi 5$ tendon wires the characteristic strength is given as 30.8kN and n_2 assuming prestressing loss of 70% and n as before defined.

P = prestressing force

Therefore $n_2P = 0.7x30.8nx10^3 = 21.56x10^3n$ (prestressing force for n wires).

Sectional modulus :

$$Z_T = \frac{(D-2t_f)\,t_W^3+2t_fB^3}{6B} = \frac{Dt_W^3-2t_ft_W^3-2t_fB^3}{6B}$$

.....(3.6)

Polar sectional modulus:

$$J = \frac{2Bt_f^3+(D-2t_f)\,t_W^3}{3} = \frac{2Bt_f^3+Dt_W^3-2t_ft_W^3}{3}$$

.....(3.7)

Taking $t_m = t_w$ since t_w is going to be kept less than t_f; otherwise t_f

$$\frac{n_2P}{A} = \frac{21.56\times10^3n}{2Bt_f+Dt_W-2t_ft_W} \quad .....(3.8)$$

$$\frac{M_T}{Z_T} = \frac{6BM_T}{Dt_W^3 - 2t_f t_W^3 + 2t_f B^3}$$

.....(3.9)

$$\frac{M_O t_m}{J} = \frac{3M_O t_W}{2Bt_f^3 + Dt_W^3 - 2t_f t_W^3}$$

.....(3.10)

$$\frac{AM_T}{n_2 PZ_T} + \frac{AM_O t_m}{n_2 PJ} - 1 \leq 0$$

.....(3.11)

$$\frac{(2Bt_f + Dt_W - 2t_f t_W)M_T 6B}{21560n(Dt_W^3 - 2t_f t_W^3 + 2t_f B^3)} + \frac{(2Bt_f + Dt_W - 2t_f t_W)3M_O t_W}{21560n(2Bt_f^3 + Dt_W^3 - 2t_f t_W^3)} - 1 \leq 0$$

.....(3.12)

ii COMPRESSION

$$\frac{M_T}{Z_T} + \frac{M_O t_m}{J} + \frac{n_2 P}{A} \leq 19.8 N/mm^2$$

.....(3.13)

$$\frac{M_T}{Z_T} + \frac{M_O t_m}{J} + \frac{n_2 P}{A} - 19.8 \leq 0$$

.....(3.14)

$$\frac{M_T}{19.8Z_T} + \frac{M_O t_m}{19.8J} + \frac{n_2 P}{19.8A} - 1 \leq 0 \text{.....(3.15)}$$

$$\frac{M_T 6B}{19.8(Dt_W^3-2t_f t_W^3+2t_f B^3)}+\frac{3M_O t_W}{19.8(2Bt_f^3+Dt_W^3-2t_f t_W^3)}+\frac{0.7n(21560)}{19.8(2Bt_f+Dt_W-2t_f t_W)}-1\le 0$$

.....(3.16)

$$\frac{M_T 6X_1}{19.8(X_2X_4^3-2X_3X_4^3+2X_3X_1^3)}+\frac{3M_O X_4}{19.8(2X_1X_3^3+X_2X_4^3-2X_3X_4^3)}+\frac{21560X_5 c.7}{19.8(2X_1X_3+X_2X_4-2X_3X_4)}-1\le 0$$

.....(3.17)

Let this constraint equation (3.17) due to compressive stress, be represented by G(2).

b. DEFLECTION [appendix 8] due to:

i Resultant force from cables

$$a_{B_2}\frac{PL_3}{EI}$$

.....(3.18)

ii Resultant wind load

$$a_{B_2}\frac{WL^3}{EI}$$

.....(3.19)

Where a=6400/7000 = 0.914

$$a_{B_1} = \frac{a^2}{6}(3-a)$$

.....(3.20)

Therefore for force P

$$= \frac{(0.914)^2}{6}(3-0.914)$$

.....(3.21)

$$= 0.291$$

For the wind load W

$$a = 3700/700$$

$$= 0.529$$

$$a_{B_2} = \frac{(0.529)^2}{6}(3-0.529)$$

.....(3.22)

$$=0.115$$

Total deflection is given by

$$\frac{a_{B_1}PL^3 + a_{B_2}WL^3}{EI} = \frac{L^3(a_{B_1}P + a_{B_2}W)}{EI}$$

.....(3.23)

$$= \frac{7000^3(0.291P + 0.115W)}{40000I}$$

.....(3.24)

$$\therefore \frac{12.25\times7000(29.1P+11.5W)}{I} \le \frac{L}{350}=20$$

.....(3.25)

$$\frac{(29.1P+11.5W)4287.5}{I}-1\le 0$$

.....(3.26)

This gives the general equation that will be used for deflection constraint for the three sections.

For I-section, substituting for I gives the constraint equation G(3) as:

$$\frac{(29.1P+11.5W)4287.5\times12}{(BD^3-(B-t_w)(D-2t_f)^3}-1\le 0$$

.....(3.27)

$$\frac{(29.1P+11.5W)4287.5\times12}{X_1X_2^3-(X_1-X_4)(X_2-2X_3)^3}-1\le 0$$

.....(3.28)

c. STABILITY CONSTRAINT :- Buckling load should not exceed critical load N_{cr}

where

$$N_{cr}=\frac{\Pi^2EIP^*}{4L^2}$$

.....(3.29)

Where P^* is correction for critical load for tapered columns as given in appendix 6B [21].

$$Na \le Ncr$$

$$Na/Ncr - 1 \le 0$$

$$N_{cr} = \frac{3.142^2 \times 40,000I}{4 \times 7000^2} = 2.014 \times 10^{-3} I$$

.....(3.30)

Substituting for I in the above equation and writing the constraint equation G(4), we get:

$$\frac{N_a}{1.678 \times 10^{-4} (BD^3 - (B - t_w)(D - 2t_f)^3} - 1 \leq 0$$

.....(3.31)

$$\frac{N_a}{1.678 \times 10^{-4} (X_1 X_2^3 - (X_1 - X_4)(X_2 - 2X_3)^3} - 1 \leq 0$$

.....(3.32)

d. SIDE CONSTRAINTS.

$B \geq 210mm$

$G(5) \equiv 210/X_1 - 1 \leq 0$ .....(3.33)

$D \geq 300mm$

$G(6) \equiv 300/X_2 - 1 \leq 0$ .....(3.34)

$t_f \geq 40mm$

$G(7) \equiv 40/X_3 - 1 \leq 0$ .....(3.35)

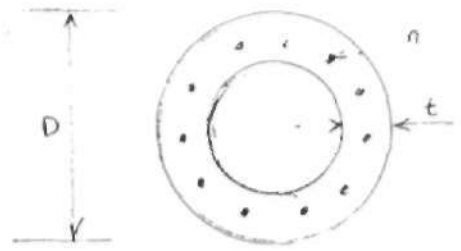
$t_w \geq 30mm$

$G(8) \equiv 30/X_4 - 1 \leq 0$ .....(3.36)

$$n \geq 30$$

$$G(5) \equiv 30/X_5 - 1 \leq 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.37)$$

3.3.5 CIRCULAR TUBULAR SECTION



$$\text{Area} \quad A = \pi t(D-t)$$

3.3.5.1 OBJECTIVE FUNCTION

$$W = \rho \underline{l} A = \underline{l}\{0.69\rho_1[\pi t(D-t)] + A_s\rho_2\}$$

Where ρ_1 and ρ_2 are as defined for I-section.

$$= 70000\{0.24[3.142t(D-t)] + 19.6n\rho_2\}$$

$$= [527.788(D-t)t + 10564.4n]0.69\times10^{-3}$$

a.DESIGN VARIABLES

$$X = \begin{pmatrix} D \\ t \\ n \end{pmatrix} = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}$$

The problem is 3- dimensional design vector problem from the above equation.

The objective function equation can be written as;

$$OBJ = 0.364182X_1X_2 - 0.364182X_2^2 + 10.564X_3$$

.....(3.38)

b. CONSTRAINTS

(i) TENSION

$$\frac{n_2P}{A} - \frac{M_T}{Z_T} - \frac{M_O t_m}{J} \geq 0$$

.....(3.39)

$$\frac{AM_T}{n_2PZ_T} + \frac{AM_O t_m}{n_2PJ} - 1 \leq 0$$

.....(3.40)

$$\frac{\Pi t(D-t)}{21.56nZ_T} + \frac{\Pi t(D-t)M_O t_m}{21.56nJ} - 1 \leq 0$$

.....(3.41)

Where

$$Z_T = \frac{\Pi[D^4-(D-2t)^4]}{32D} \quad , \quad J = \frac{\Pi(D^4-(D-2t)^4]}{32}$$

.....(3.42)

$$\frac{\Pi tM_T(D-t)32D}{21.56n\Pi[D^4-(D-2t)^4} + \frac{\Pi t^4M_0(D-t)32}{21.56n\Pi(D^4-(D-2t)^4} - 1 \leq 0$$

.....(3.43)

$$\frac{(32Dt-32t^2)(M_TD+M_Ot)}{21560[(D^4-(D-2t)^4]} - 1 \leq 0 \text{.....(3.44)}$$

$$\frac{(32X_1X_2-32X_2^2)\,(M_TX_1+M_OX_2)}{21560X_3(X_1^4-(X_1-2X_2)^4}-1 \le O = G(1)$$

.....(3.45)

(ii) COMPRESSION

$$\frac{M_T}{19.8Z_T}+\frac{M_Ot_m}{19.8J}+\frac{n_2P}{19.8A}-1 \le 0$$

.....(3.46)

Substituting for Z_T ,J and A;

$$\frac{0.514M_TD}{D^4-(D-2t)^4}+\frac{0.514M_Ot}{D^4-(D-2t)^4}+\frac{0.347n}{t(D-t)}-1 \le O$$

.....(3.47)

$$\frac{0.514M_TX_1}{X_1^4-(X_1-2X_2)^4}+\frac{0.514M_OX_2}{X_1^4-(D-2X_2)^4}+\frac{0.347X_3}{X_2(D-X_2)}-1\le 0 = G(2)$$

.....(3.48)

c.DEFLEXION

Substituting for I in the deflexion equation derived in the case of I-section,where I for circular section is given as;

$$I = \frac{\Pi t(D-t)^3}{8}$$

.....(3.49)

and

$$\frac{(29.1P+11.5W)4287.5\times56}{22t(D-t)^3}-1\le 0$$

.....(3.50)

$$\frac{(29.1P+11.5W)4287.5\times56}{22X_2(X_1-X_2)^3}-1\le 0 \;=\; G(3)$$

.....(3.51)

d.STABILITY (BUCKLING)

Na - Ncr ≤ 0

Where Na is the applied load and Ncr is the critical buckling load.

The equation can be written as;

$$\frac{N_a}{N_{cr}}-1\le 0$$

.....(3.52)

where

$$N_{cr} = \frac{\Pi^2 EI}{4L^2}P^*$$

.....(3.53)

Substituting I and E in the above equation we get;

$$N_{cr} = \frac{\Pi^3 t(D-t)^3}{8\times40000} = 0.7915\times10^{-3}t(D-t)^3$$

.....(3.54)

$$\frac{N_a}{0.7915\times10^{-3}X_2(X_1-X_2)^3} - 1 \leq 0 \quad = \quad G(4)$$

.....(3.55)

e. SIDE CONSTRAINTS

D ≥ 318mm

$G(5) \equiv 318/X_1 \leq 0$ .....(3.56)

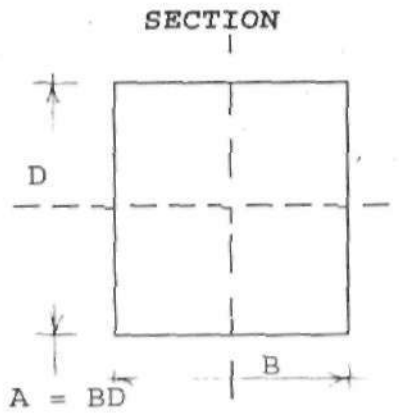
t ≥ 60mm

$G(6) \equiv 60/X_2 \leq 0$ .....(3.57)

n ≤ 25

$G(7) \equiv 25/X_3 \leq 0$ .....(3.58)

3.3.6 RECTANGULAR SECTION



Area A = BD

3.3.6.1 OBJECTIVE FUNCTION

Weight $W = \rho lA = 0.69(\rho_1 lBD) + A_s l\rho_2$

$\therefore \quad W = 0.11592BD + 10.564n$

$$X=\begin{pmatrix} B \\ D \\ n \end{pmatrix}=\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}$$

The problem is a 3 - dimensional design vector problem

The objective function equation is represented bellow

$$OBJ=0.11592X_1X_2+10.564X_3$$

.....(3.59)

a.CONSTRAINT

(i) TENSION

$$\frac{AM_T}{n_2PZ_T}+\frac{AM_Ot_m}{n_2PJ}-1\leq 0$$

.....(3.60)

$$\frac{BDM_T}{21.56\times10^3Z_T}+\frac{BDM_O}{21.56\times10^3J}-1\leq 0$$

.....(3.61)

Where

$$Z_T=\frac{BD^3}{6} \quad , \quad J=\frac{DB^3}{3}$$

.....(3.62)

Substituting for J and Z^T

$$\frac{0.278\times10^{-5}M_T}{D_2 n}+\frac{0.139\times10^{-5}M_O}{B^2 n}-1\le 0$$

.....(3.63)

$$\frac{0.278\times10^{-5}M_T}{X_2^2 X_3}+\frac{0.139\times10^{-5}M_O}{X_1^2 X_3}-1\le 0$$

.....(3.64)

This gives the constraint G(1)

(ii) COMPRESSION

$$\frac{M_T}{19.8Z_T}+\frac{M_O}{19.8J}+\frac{n_2 P}{19.8A}-1\le 0$$

.....(3.65)

Substituting for Z_T, J and A

$$\frac{0.303M_T}{BD^3}+\frac{0.152M_O}{DB^3}+\frac{1.089\times10^3 n}{BD}-1\le 0$$

.....(3.66)

$$\frac{0.303M_O}{X_1X_2^3}+\frac{0.152M_O}{X_2X_1^3}+\frac{1.089\times10^3}{X_1X_2}-1\le O=G(2)$$

.....(3.67)

b. DEFLECTION

The equation is as derived for the I- section

Substituting for I in the equation;

$$\frac{(29.1P+11.5W)4287.5\times12}{BD^3}-1\le0$$

.....(3.68)

$$\frac{(29.1P+11.5W)4287.5\times12}{X_1X_2^3}-1\le0 = G(3)$$

.....(3.69)

c. STABILITY (BUCKLING)

$$\frac{N_a}{N_{cr}}-1\le0$$

.....(3.70)

Where

$$N_{cr}=\frac{\Pi^2EIP^*}{4L^2}$$

.....(3.71)

This gives $N_{cr} = 7.0497\times10^{-3}I$

But $I = BD^3/12$

$$N_{cr}=2.014\times10^{-3}\frac{BD^3}{12} = \frac{\Pi^2}{40,000}\frac{BD^3}{12}$$

.....(3.72)

$$= 1.678\times10^{-4}BD^3$$

Substituting for N_{cr} in the constraint equation we get:

$$\frac{N_a}{1.678\times10^{-4}BD^3} - 1 \le 0$$

.....(3.73)

$$\frac{N_a}{1.678\times10^{-4}X_1X_2^3} - 1 \le 0 \equiv G(4)$$

.....(3.74)

d.SIDE CONSTRAINTS

$$B \ge 150mm$$

$$G(5) \equiv 150/x_1 - 1 \le 0 \quad .....(3.75)$$

$$D \ge 230mm$$

$$G(6) \equiv 230/X_2 - 1 \le 0 \quad .....(3.76)$$

$$n \ge 25$$

$$G(6) \equiv 25/X_3 - 1 \le 0 \quad .....(3.77)$$

C H A P T E R F O U R

4.0 OPTIMIZATION ALGORITHM AND SUBROUTINES

4.1 INTRODUCTION

The constrained optimization problem formulated in the last chapter is solved as a sequence of unconstrained minimization problems by using the interior penalty function approach [1] rather than the exterior penalty method. This is because the interior penalty method which restricts the search for the minimum to feasible region has proved proves rather more successful in practice.[22] In this method the objective function is augmented with a penalty term which is a function of the constraints and is small at points away from the constraints in the feasible region,but which blows up as constraints are approached. The Davidon-Flecher-Powell variable metric method is used for minimization, as this is known to be a powerful procedure for finding a local unconstrained minimum of a function of many variables. Cubic interpolation method of one dimensional search, which makes use of the gradient, is used for finding the step length[7]. The program consists of a main program and four subroutines called at various stages of the program to perform specific task as described below.A flow chart showing how the program works is shown in fig 4.3.

4.2 INTERIOR PENALTY FUNCTION METHOD

In the interior penalty function method a new function (ϕ- function) is constructed by augmenting a penalty term to the objective function. The penalty term is chosen such that its value will be small at points away from the constraint boundaries and will tend to infinity as the constraint boundaries are approached. Hence the value of the ϕ-function also "blow up" as the constraint boundaries are approached. This behaviour can be seen in fig 4.1. Thus once the unconstrained minimization of $\phi(X, r_k)$ is started from any feasible point X_1 the subsequent points generated will always lie within the feasible domain since the constrain boundaries act as barriers during the minimization process. This is the reason why the interior penalty function methods are also known as "barrier" methods. The ϕ-function defined originally by Carroll [7] is

$$\phi(X, r_K) = f(X) - r_k \sum_{j=i}^{m} \frac{1}{g_j(x)}$$

--------4.1

It can be seen that the value of the function ϕ will always be greater than f(x) since $g_i(x)$ is negative for all feasible

points X. If any constraint $g_i(x)$ is satisfied critically (with equality sign), the value of ϕ tends to infinity.

It is to be noted that the penalty term in eq. 4.1 is not defined if x is infeasible. This introduces serious shortcoming while using eq. 4.1. Since this equation does not allow any constraint to be violated it requires a feasible starting point for the search towards the optimum point. However, in many engineering problems, it may not be very difficult to find a point satisfying all the constraints $g_i(x)<0$, at the expense of large values of the objective function f(x). If there is any difficulty in finding a feasible starting point. Since the initial point as well as each of the subsequent points generated in this method lies inside the acceptable Hxgion of the design space, the method is classified as an " interior penalty function" formulated. The iteration procedure of this method can be summarized as follows:

(i) Start with an initial feasible point X, satisfying all the constraints with strict inequality sign, that is $g_j(x)<0$ for $j=1,2,....,m$ and an initial value of $r_1>0$, set k = 1.

(ii) Minimize $\phi(x,r_k)$ by using any of the unconstrained minimization methods and obtain the solution X_k^*

(iii) Test whether X_k^* is the optimum solution of the original problem. If X_k^* is found to be optimum, terminate the process, otherwise go to the next step.

(iv) Find the value of the next penalty parameter, r_{k+1}, as

$r_{k+1} = c_r k$ where c<1

(v) Set the value of k=k+1, take the new starting point as $X_1 = X_k^*$ and go to step (ii).

There are a number of points to be considered in implementing the method. These are:[7]

4.2.1 INITIAL VALUE OF THE PENALTY PARAMETER(r_k)

Since the unconstraint minimization of $\phi(x, r_k)$ is to be carried out for a decreasing sequence of r_k it might appear that by choosing a very small value of r_1, we can avoid an excessive number of minimization of the function ϕ. But, from computational point of view, it will be easier to minimize the unconstrained function $\phi(x, r_k)$ if r_k is large. This can be seen qualitatively from fig 4.1

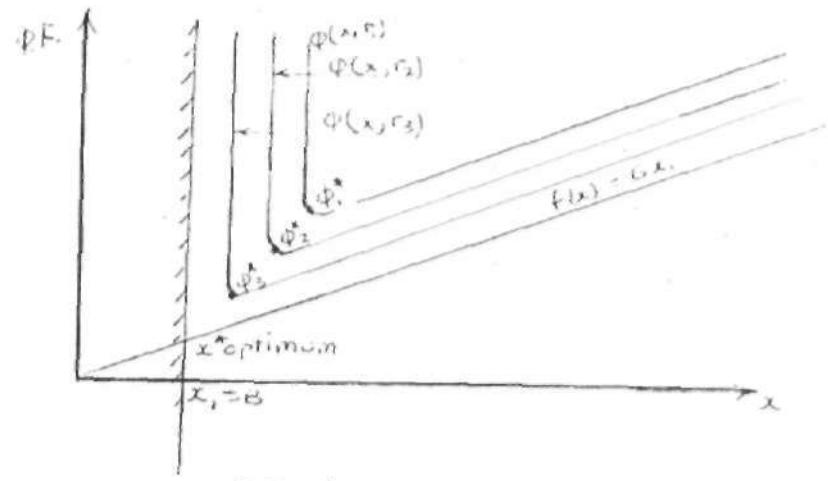


FIG 4.1

It can be seen that as the value of r_k becomes smaller, the value of the function ϕ changes more rapidly in the vicinity of the minimum ϕ_k^*. Since it is easier to find the minimum of a function whose graph is smoother the unconstrained

minimization of ϕ will be easier if r_k is large. However, the minimum of ϕ_k, X_k^*, will be farther away from the desired minimum X^* if r_k is large. Thus it requires an excessive number of unconstrained minimization of $\phi(x, r_k)$ (for several values of r_k) to reach the point X^* if r_k is selected to be very large. Thus a "moderate" value has to be chosen for the initial penalty parameter (r_k). In practice a value of r_1 which gives the value of $\phi(x_1, r_1)$ approximately equal to 1.1 to 2.0 times the value of $f(x_1)$ has been found to be quite satisfactory in achieving quick convergence of the process. Thus, for any initial feasible point X_1, the value of r_1 can be taken as [7]

$$r_1 = 0.1 \sim 1.0 \; \frac{f(x_i)}{-\sum_{j=1}^{m} \frac{1}{g_i(x_1)}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.3)$$

4.2.2 SUBSEQUENT VALUES OF THE PENALTY PARAMETER

Once the initial values of r_k is chosen, the subsequent value of r_k have to be chosen such that $r_{k+1} < r_k$. For convenience, the values of r_k are chosen according to the relation $r_{k+1} = cr_k$ where $c<1$. The value of c can be taken as 0.1 or 0.2 or 0.5 etc.

4.2.3 CONVERGENCE CRITERIA

Since the unconstrained minimization of $\phi(x, r_k)$ has to be carried out for a decreasing sequence of values r_k, it is necessary to use proper convergence criteria to identify the optimum point and to avoid unnecessary large number of unconstrained minimization. The process can be terminated whenever the following conditions are satisfied:-

(i) The relative difference between the values of the objective function obtained at the end of any two consecutive unconstrained minimization fall between a small value of ε_1

$$\frac{|f(x_K^*) - f(X_{K-1}^*)|}{|f(x_K^*)|} \leq \varepsilon_1 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.4)$$

The difference between the optimum point X_k^* and X_{K-1}^* becomes very small. This can be judged in several ways. Some of them are given below :

$$|(\Delta x)_i| \leq \varepsilon_2$$

where $\Delta x = x_k^* - x_{k-1}^*$ and $(\Delta x)_i$ is the i^{th} component of the vector Δx

$$\text{Max}|(\Delta x)_i)| \leq \varepsilon_3$$

$$|\Delta x| = [(\Delta x)^2_1 + (\Delta x)^2_2 + \ldots\ldots + (\Delta x)^2_n]^{1/2} \leq \varepsilon_4$$

Note that the values of ε_1 and ε_4 have to be chosen depending on the characteristics of the problem at hand. [7]

4.2.4 NORMALIZATION OF CONSTRAINTS

A structural optimization problem, for example might be having constraints on the deflection (δv) and the stress (σ)

$$g_1(x) = \delta(x) - \delta max < 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.5)$$

$$g_2(x) = \sigma(x) - \sigma max < 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.6)$$

Where the maximum allowable values are given by δmax = 0.5cm, and $\sigma max = 3{,}000 kg/cm^2$. If a design vector x_1 gives the values of g_1 and g_2 as -0.2and -2,000 the contribution of g_1 will be much larger than that of g_2 (by an order of 10^{-4}) in the ϕ - function equation. Thus it is advisable to normalize the constraints so that they vary between -1 and 0 as far as possible. For the constraints shown in equation 4.5 and 4.6 a suitable normalization is

$$g_1^1 - \frac{g_1(X)}{\delta_{max}} - \frac{\delta(X)}{\delta_{max}} - 1 \leq 0$$

$$\vdots$$

$$g_2^1 - \frac{g_2(X)}{\sigma_{max}} - \frac{\sigma(x)}{\sigma_{max}} - 1 \leq 0$$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.7)$

Where $r_1, r_2, \ldots . rm$ are selected such that the contributions of different $g_j(x)$ to the ϕ - function will be approximately the same at the initial point X_1. When the constrained minimization of $\phi(x, r_k)$ is carried for a decreasing sequence of values of r_k, the values of $r_1, r_2, \ldots . r_m$ will not be altered; however, they are expected to be effective in

contributions of the various constraints to the ϕ function.[7]

4.3 INTRODUCTION TO ONE DIMENSIONAL MINIMIZATION METHOD

If the optimization problem involves the objective function and or the constraints which are not stated as explicit functions of the design variables or which are too complicated to manipulate we cannot solve it by using the classical methods. In many engineering problems the objective function and the constraints cannot be written explicitly in terms of the decision variables. In such cases, only numerical methods of optimization can be used to solve the problem. The philosophy behind the numerical methods is to produce a sequence of improved approximations to the optimum according to the following scheme.

(i) Start with an initial trial point X_1

(ii) Find a suitable direction S_i (i=1 to start with) which points in the general direction of the minimum.

(iii) Find an appropriate step length λ_i^* for movement along the direction S_i.

(iv) Obtain the new approximation X_i+1 as

$$X_i+1 = X_i + \lambda_i^* S_i \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.8)$$

(v) Test whether $X_i + 1$ is optimum. If X_i+1 is optimum, stop the procedure. Otherwise, set new i=i+1, and repeat step (ii) onwards. The procedure is represented graphically for hypothetical two-variable problem below [7].

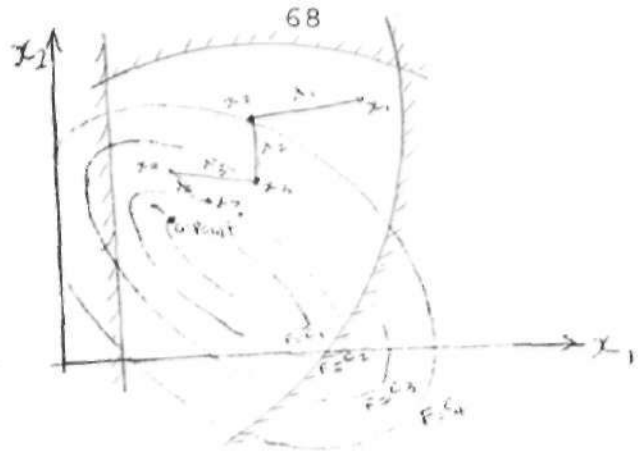


FIG 4.5

The iterative process of optimization:- from Equation 4.8 it can be seen that the efficiency of an optimization method depends on the efficiency with which the quantities λ_i^* and S_i are determined. If f(x) is the objective function to be minimized the problem of finding λ_i^* boils down to finding the $\lambda_i = \lambda_i^*$ which minimizes $f(X_i+1) = f(X_i+\lambda_i S_i) = f(\lambda_i)$ for fixed values of X_i and S_i. Since f becomes a function of one variable λ_i only, the methods of finding λ_i^* in equation 4.8 are called one- dimensional minimization methods. There are many methods for solving the one - dimensional minimization problems but we shall restrict ourselves to the numerical method of cubic interpolation.

4.4 SUBROUTINE UNCON - FOR IMPLEMENTING THE VARIABLE METRIC METHOD OF UNCONSTRAINED MINIMIZATION

The method was introduced by Davidon, and extended by Flecher and Powell in 1963.[7] This method is the best general purpose

unconstrained optimization technique. It is iterative, and given a starting point $X_{(1)}$ it generates a sequence of points $X_{(K)}$ (K=1,2....) that is intended to converge to the point at which fx is least. It can be stated as follows: [7]

(i) Start with an initial point X_1 and a nxn positive definite symmetric matrix H_1. Usually H_1 is taken as the identity matrix I. Set iteration number as i=1.

(ii) Compute the gradient of the function. ∇f_i at the point X_i and set $S_i = - H_{1\nabla} f_1$

(iii) Find the optimal step length λ^* in the direction S_i and set $X_{i+1} = X_i + \lambda_i^* S_i$

(iv) Test the new point X_{i+1} for optimality. If X_{i+1} is optimal terminate the iterative process. Otherwise, go to step (v).

(v) Update the H matrix as

$$H_{i+1} = H_i + M_i + N_i$$

Where $M_i = \lambda^*_i (S_i S^T_i) / (S^T_i Q_i)$

$$N_i = - (H_i Q_i) (H_i Q_i)^T / (Q^T_i H_i Q_i)$$

and $Q_i = \nabla f(X_{i+1}) - \nabla f(X_i) = \nabla f_{i+1} - \nabla f_1$

(vi) Set the iteration number l = i+1 and go to step (ii).

4.5 SUBROUTINE GRADT

This subroutine calculates the gradient of the function. In the algorithm, it is necessary to compute the gradients of the objective function and the constraints. The gradients of behaviour constraints can not be determined analytically because it is not convenient. Therefore the gradients are

determined by a finite difference method. The gradient at any point Xm can be evaluated as:

$$\nabla f|_{X_m} - \nabla f_m - \begin{pmatrix} \frac{df}{dx_1} \\ \frac{df}{dx_2} \\ . \\ . \\ \frac{df}{dx_n} \end{pmatrix}$$

n function evaluations are performed from a fixed point Xm by varying each variable X_i in turn, by a small amount ψ_i then df/dx_i is estimated from,

$$\frac{df}{dx_i}\Big|_{X_m} = \frac{f(x_m + \psi_i U_i) - f(X_m)}{\psi_i}, i = 1,2,\ldots,n$$

.....(4.9)

where ψ_i = small scalar quantity

U_i = vector of order n whose i_{th} component has a value of one and all other components have a value of zero. In practical calculations ψ_i has to be carefully chosen. If it is too small, the difference between the values of the

function evaluated at $(Xm + \psi_i U_i)$ and $(Xm - \psi_i U_i)$ may be very small and numerical round off error may predominate. On the other hand, if it is too large the truncation error may predominate in the calculation of the program which proved successful for the problems solved.[23]

4.6 SUBROUTINES ONEDIM

This subroutine implements the cubic interpolation method [7] This technique solves the problem of finding the minimum step length λ^* in four stages. It makes use of the derivative

$$f'(\lambda) = df/d\lambda = (d/d\lambda) f(\lambda + \lambda s) = S^T \nabla f(\lambda + \lambda S)$$

The first stage normalizes the S vector so that a step size $\lambda = 1$ is acceptable. The second stage establishes bound on λ^*, and the third stage finds the value of λ^* by approximating $f(x)$ using a cubic polynomial $h(\lambda)$. If the λ^* found in stage three does not satisfy the prescribed convergence criteria, the cubic polynomial is refitted in the fourth stage

(i) calculate $\Delta = \max_i |S_i|$. Where $|S_i|$ is the absolute value of the ith component of S, and divided each component of S by Δ .

Another type of normalization is to find

$$\Delta = (S^2_1 + S^2_2 + \ldots . S^2_n)^{1/2}$$

and divide each component of S by Δ.

(ii) To establish lower and upper bounds on the optimal step size λ^*, we have to find two points A and B at which the slope has different signs. We know that at $\lambda = 0$.

$$\frac{df}{d\lambda}|_{\lambda=0} = S^T \nabla f(x) < 0 \ldots\ldots (4.10)$$

Since S is presumed to be a direction of decent. Hence to start with, we can take A=0 and try to find λ=B at which the slope is positive. The point B can be taken as the first value out of to, 2to, 4to, 8to,....at which f' is nonnegative, where to is a preassigned initial step size. It then follows that λ^* is bounded in the interval $A < \lambda^* < B$

The minimum of f(x) lies between A and B

(iii) If the cubic equation

$$h(\lambda) = a + b\lambda + c\lambda^2 + d\lambda^3 \ldots\ldots\ldots\ldots\ldots\ldots (4.11)$$

is used to approximate the function f(x) between the points A and B we have to find the values

fA = f(λ=A),

f'A =df/dλ (λ = A),

fB=f(λ=B) and f'B =df/dλ(λ=B) in order to evaluate the constraints a,b,c, and d in equation 4.11. By assuming that A .≠.0 we can derive a general formula for λ^* from eq 4.11 we have

$$\left.\begin{aligned} f_A &= a + bA + cA^2 + dA^3 \\ f_B &= a + bB + cB^2 + dB^3 \\ f'_A &= b + 2cA + 3dA^3 \\ f'_B &= b + 2cB + 3dB^3 \end{aligned}\right\} \quad 4.12$$

Equations 4.12 can be solved to find the constants as

$$a = f_A - bA - CA^2 - dA^3 \quad \ldots\ldots\ldots\ldots\ldots\ldots (4.13)$$

Where

$$b = 1/(A - B)^2 [B^2 f'_A + A^2 f'_B + 2ABZ] \quad \ldots\ldots (4.14)$$

$$C = -1/(A - B)^2 [(A + B)Z + Bf_A' + Af_B']$$

And $$d = 1/3(A - B)^2 [2Z + f'_A + f'_B] \quad \ldots\ldots\ldots (4.15)$$

Where $$Z = 3(f_A + f'_B)/(B - A) + f'_A + f'_B \quad \ldots\ldots (4.16)$$

The necessary condition for the minimum of $h(\lambda)$ given by equation (4.11) is that

$$dh/d\lambda = b + 2c\lambda + 3d\lambda^2 = 0$$

$$i.e \quad \tilde{\lambda}^* = \frac{-C \pm (C^2 - 3bd)^{\frac{1}{2}}}{3d} \quad \ldots\ldots\ldots (4.12)$$

$$\ldots\ldots\ldots\ldots\ldots (4.17)$$

Application of the sufficiency condition for the minimum of $h(\lambda)$ leads to the condition

$$d^2h/d\lambda^2 | \lambda^* = 2C + 6d\lambda^* > 0 \quad \ldots\ldots\ldots\ldots\ldots (4.18)$$

By substituting the expression for b,c and d given by equations (4.14) into (4.17) and (4.18), we obtain that

$$\tilde{\lambda}^* = A + \frac{(f_A + Z \pm Q)}{(f_A + f_B + 2Z)} (B - A) \ldots\ldots (4.14)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots (4.19)$$

Where $$Q = (Z^2 - f'_A f'_B)^{1/2} \quad \ldots\ldots\ldots\ldots\ldots (4.20)$$

And $$2(B - A)(2Z + f'_A + f'_B)(f'_A + Z \pm Q)$$

$$- 2(B - A)(f'^2_A + Zf'_B + 3Zf'_A + 2Z^2)$$

$$- 2(B + A)f'_A > 0 \quad \ldots\ldots\ldots\ldots\ldots (4.21)$$

By specializing the equation (4.13) to (4.21) to the case where A = 0, we obtain

$$a = f_A$$

$$b = f'_A$$

$$c = -1/B(Z + f'_A)$$

$$d = 1/3B^2(2Z + f'_A + f'_B)$$

$$\tilde{\lambda}^* = B\frac{(f_A + Z \pm Q)}{(f_A + f_B + 2Z)}$$

$$Q = (Z^2 - f'_A f'_B)^{1/2} > 0 \qquad \ldots\ldots\ldots(4.23)$$

$$Z = (3(f_A - f_B))/B + f'_A + f'_B \qquad \ldots\ldots(4.24)$$

The two values of λ^* in eq 4.19 and 4.22 correspond to the two possibilities for the vanishing of $h'(\lambda)$, i.e at a maximum of $h(\lambda)$ and at a minimum.

In order to avoid imaginary values for Q, we should ensure the satisfaction of the condition

$Z^2 = f'Af'B > 0$ in equation 4.20.

This inequality is automatically ensured as we are assuming $f'<0$ Furthermore, sufficiency condition when A= 0 requires that Q>0, which is already satisfied. Now we calculate λ^* according to the formula 4.22 and proceed to the next stage. Stage (iv) . The value of λ^* found in stage (iii) is the true minimum of $f(\lambda)$. Hence the following convergence criteria can be used before taking λ^* approximately as $\tilde{\lambda}^*$

$$\frac{|h(\tilde{\lambda}^*) - f(\tilde{\lambda}^*)|}{|f(\tilde{\lambda}^*)|} \leq \varepsilon_1$$

.....(4.25)

$$|\frac{df}{d}\lambda|_{\tilde{\lambda}^*} - |s^T \nabla f|_{\tilde{\lambda}^*} \leq \varepsilon_2$$

.....(4.26)

Where ε_2 and ε_2 are small numbers whose values depend on the accuracy desired.The latter criterion can be stated in nondimensional form as.

$$\frac{|s^T \nabla f|}{|s||\nabla f|_{\tilde{\lambda}^*}} \leq \varepsilon_2$$

.....(4.27)

If the criterion stated in eqs 4.25 and 4.27 are not satisfied a new cubic equation $h'(\lambda) + a' + b'\lambda + c'\lambda^2 + d'\lambda^3$. can be used to approximate $f(\lambda)$. The constants a', b', c', and d' can be evaluated by using the function and derivative values

at the best two points out of the three points currently available, namely, A, B, and λ^* now the general formula given by eq 4.23 has to be used for finding the optimal step λ^*. If $f'(\lambda^*)<0$ the new points A and B are taken as λ^* and B respectively. Otherwise if $f'(\lambda^*)$ the new points A and B are taken as A and λ^* and eq 4.23 is applied to find the new value of λ^* equations 4.25 and 4.27 are again used to test the convergence of λ^*. If the convergence is achieved, λ^* is taken as $\tilde{\lambda}^*$ and the procedure is stopped. Otherwise the whole procedure is repeated until the desired convergence is achieved [7].

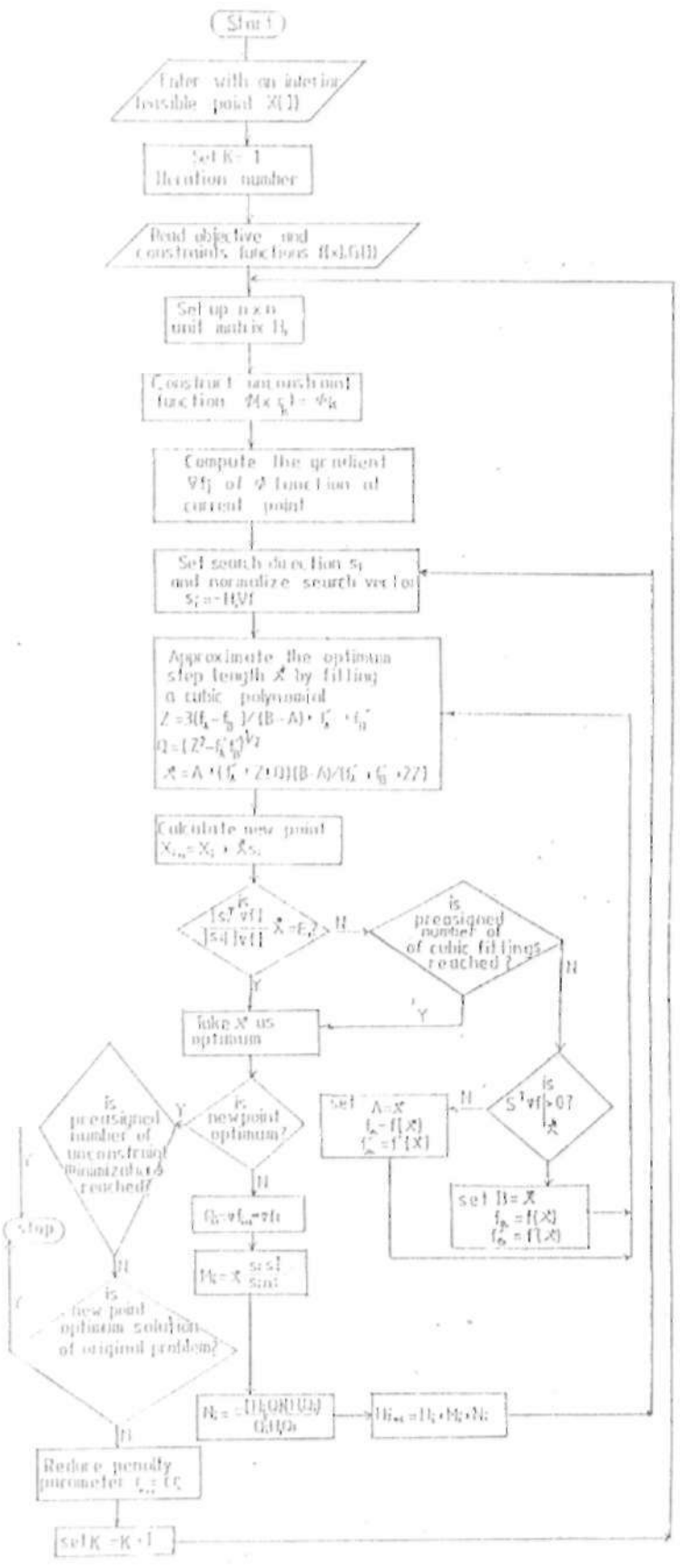


Fig4-3 VARIABLE MATRIX METHOD OF OPTIMIZATION FLOW

C H A P T E R F I V E

5.0 DISCUSSION OF RESULTS AND CONCLUSION

5.1 DISCUSSION OF RESULTS

The numerical problems presented give results which compare well with solutions from Essien [3]-see appendix - 9 transmission poles of I and circular tubular sections. A third section (rectangular) was also analyzed. The method of analysis is the same as that by Essien, using the same loading conditions. The wind load taken as the maximum for Zaria.

From the work it was discovered that the choice and use of the penalty function method proved to be advantageous as reported elsewhere [24].

Programming is easy and programs can be built on modular basis.The user only needs to formulate and program his own problem, while a general purpose optimization program can be used..." The problems solved converged in 6-60 iterations. [Fig5.0]. The optimization procedure results in the final cross-sectional dimensions instead of cross-sectional properties like area, moment of inertia and rotational moment of initia. It may be noticed that in the optimal designs buckling was considered which was omitted in the reference[3], and lifting moments which was considered in the reference[3] was not included in the optimal designs. It was considered sufficient that the lifting moments constraint were controlled to some extent by the lower limits of the design variables.

The designer may compare the results for different cross-

sectional shapes (Table 5.3) and select the best. We can see that the rectangular cross-sectional shape is the best with the least weight. We also notice that the I-section gives less iterations followed by the rectangular section while the tubular section gives the highest number of iterations.

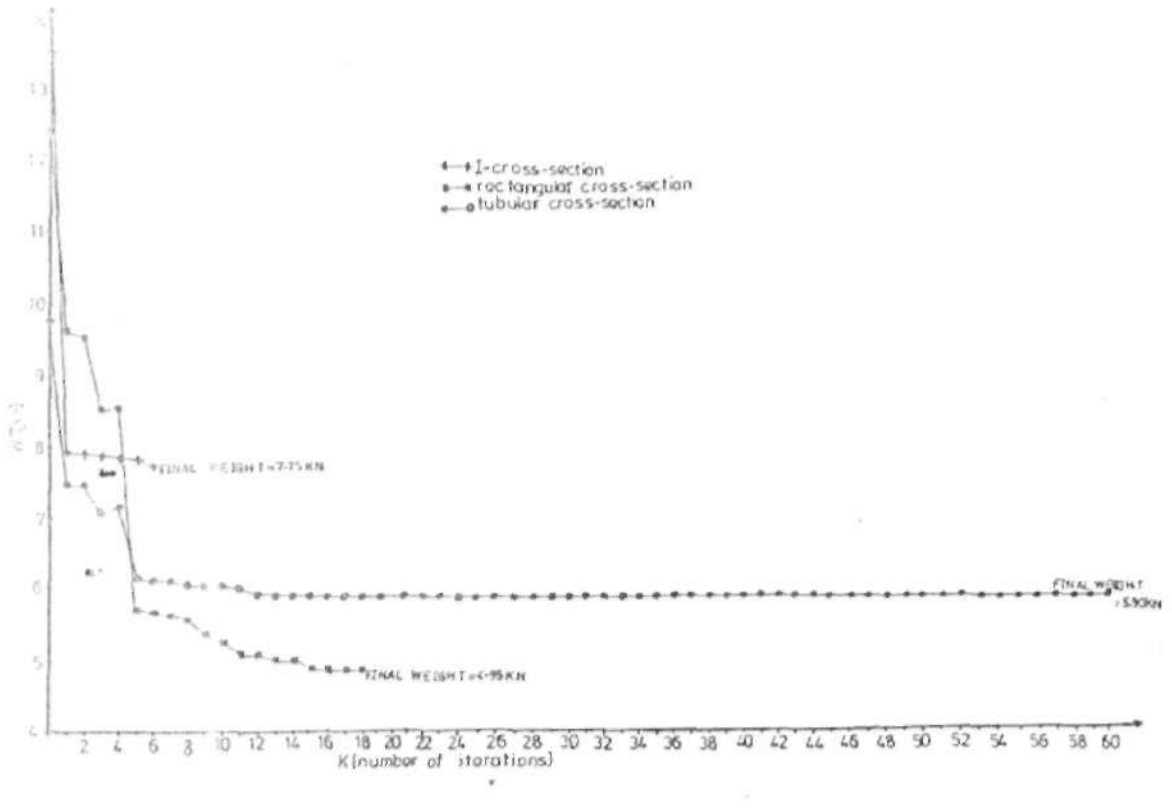


Fig 5.0 WEIGHT HISTORY OF POLE FOR THE THREE DIFFERENT CROSS SECTIONAL SHAPES

TABLE 5.1 OPTIMIZATION RESULTS FOR I-SECTION

BEHAVIOUR QUANTITIES AND DESIGN VARIABLES	LOWER BOUND	UPPER BOUND	INITIAL DESIGN	SEQUENTIAL UNCONSTRAINT MINIMAZATION	
				r=1.0	r=0.1
$\sigma_T (N/mm^2)$	-	0.0	-0.4939	-0.0691	-0.1351
$\sigma_C (N/mm^2)$	-	19.8	11.5452	19.7827	13.7171
$\delta_T (mm)$	-	20.00	1.9073	2.6245	2.0566
$P_{cr} (KN)$	6.000	-	5181.874	3765.809	3758.70
$X_1 (mm)$	210.00	-	350.00	318.6232	318.665
$X_2 (mm)$	300.00	-	450.00	444.8116	444.830
$X_3 (mm)$	40.00	-	150.00	87.4795	87.6451
$X_4 (mm)$	30.00	-	50.0000	33.6701	30.1888
X_5	30.00	-	40.00	30.8086	31.4781
PENALTY FUNCTION(KN)			13.481	9.058	8.790
WEIGHT (KN)			13.461	7.840	7.750
NUMBER OF ONE-DIMENSIONAL MINIMAZATION FOR EACH UNCONSTRAINED MINIMAZATION				5	1

σ_T = tensile stress
σ_C = compressive stress
P_{cr} = critical buckling load
X_1 = flange breath
X_2 = depth of section
X_3 = flange thickness
X_4 = web thickness
X_5 = tendon wires
δ_T = deflection

TABLE 5.2 OPTIMIZATION RESULTS FOR RECTANGULAR SECTION

BEHAVIOUR QUANTITIES AND DESIGN VARIABLES	LOWER BOUND	UPPER BOUND	INITIAL DESIGN	SEQUENTIAL UNCONSTRAINT MINIMIZATION	
				r=1.0	r=0.1
$\sigma_T (N/mm^2)$	-	0.0	-0.99998	-0.99994	-0.99994
$\sigma_C (N/mm^2)$	-	19.8	7.4052	13.0917	13.6291
$\delta_T (mm)$	-	20.00	4.5792	16.9638	19.9565
$P_{cr} (KN)$	6.000	-	215.328	582.619	495.249
$X_1 (mm)$	150.00	-	300.00	152.481	150.945
$X_2 (mm)$	230.00	-	350.00	283.439	269.404
$X_3 (mm)$	25.00	-	36.00	26.091	25.5457
PENALTY FUNCTION(KN)			12.489	5.357	5.022
WEIGHT (KN)			12.476	5.255	4.955
NUMBER OF ONE-DIMENSIONAL MINIMIZATION FOR EACH UNCONSTRAINED MINIMIZATION				10	8

σ_T = tensile stress
σ_c = compressive stress
δ_T = deflection
P_{cr} = critical buckling load

X_1 = breath
X_2 = depth
X_3 = tendon wires number

TABLE 5.3 OPTIMIZATION RESULTS FOR CIRCULAR TUBULAR SECTION

BEHAVIOUR QUANTITIES AND DESIGN VARIABLES	LOWER BOUND	UPPER BOUND	INITIAL DESIGN	SEQUENTIAL UNCONSTRAINT MINIMIZATION					
				r=1.0	r=0.1	r=0.01	r=0.001	r=0.0001	r=0.00001
$\sigma_T (N/mm^2)$	-	0.0	-0.7338	-0.3435	-0.3101	-0.3016	-0.2997	-0.2994	-0.2994
$\sigma_C (N/mm^2)$	-	19.8	2.0793	19.0320	18.9424	18.8740	18.8509	18.8480	18.8484
$\delta_T (mm)$	-	20.00	5.7255	11.6673	12.0603	12.1175	12.1244	12.1251	12.1252
$P_{cr} (KN)$	6.000	-	1727.173	847.574	819.959	8160.884	8156.205	815.0575	815.5704
$X_1 (mm)$	318.00	-	380.00	321.023	318.411	318.047	318.005	318.000	318.000
$x_2 (mm)$	60.00	-	85.00	60.6995	60.1174	60.0178	60.0014	60.0001	60.0000
X_3	25.00	-	60.00	26.4550	25.3632	25.0782	25.009	25.001	25.0001
PENALTY FUNCTION(KN)			9.782	6.253	6.059	6.010	6.014	6.011	6.017
WEIGHT (KN)			9.766	6.034	5.923	5.901	5.901	5.902	5.902
NUMBER OF ONE-DIMENSIONAL MINIMIZATION FOR EACH UNCONSTRAINED MINIMIZATION				10	10	10	10	10	10

σ_C = tensile stress
σ_T = compressive stress
δ_T = deflection
P_{cr} = critical buckling load
X_1 = overall sectional diameter
X_2 = tube thickness
X_3 = tendon wires number

5.2 SENSITIVITY ANALYSIS

For practical application a designer should be able to know how the response quantities vary with changes in the design variables. This sensitivity analysis will help manipulating the variables to suit some specific requirements. Furthermore in some cases the results obtained may have to be rounded off from the optimization procedure to the nearest practical values. Therefore a sensitivity analysis of the behaviour quantities is conducted about the final optimum design point. The program flow chart is shown in fig 5.15. And the computer program with an output is shown in the Appendix . Figure 5.1-5.14 show off-design charts from the sensitivity tests. Tables(5.4-5.6) shows a summary of the results in tabular form.

The sensitivity analysis is performed by gradually changing design variables and the function variation noted. This can be done by calculating the derivative of the functions with respect to the design variables (the gradient), which represent and characterize the trends of the function variation when design variables are changing. The structural response parameters (stress deflection and buckling loads) caused by external loading, are generally implicit functions of the design variables:there are no linear relations between responses and design variables. Because of this complication the conventional tool for calculating the function derivatives

by the finite difference technique is most of the time used. This information can simply be obtained by calling the routine GRADT which perform the function derivative computation using the finite deference method or by changing each variable gradually and reanalysing the whole problem. For the work presented the latter method was used because of convenience and simplicity. A program was designed which performed the test by keeping all other variables constant only varying the desired one from -50 to +50% and the percentage change in the functions studied. Each evaluation of the functions required a completely new analysis of the structure.

Detailed instruction on the required format for the input data for the two programs is given in the Appendix 1 and 2

Discussion of the results follows.

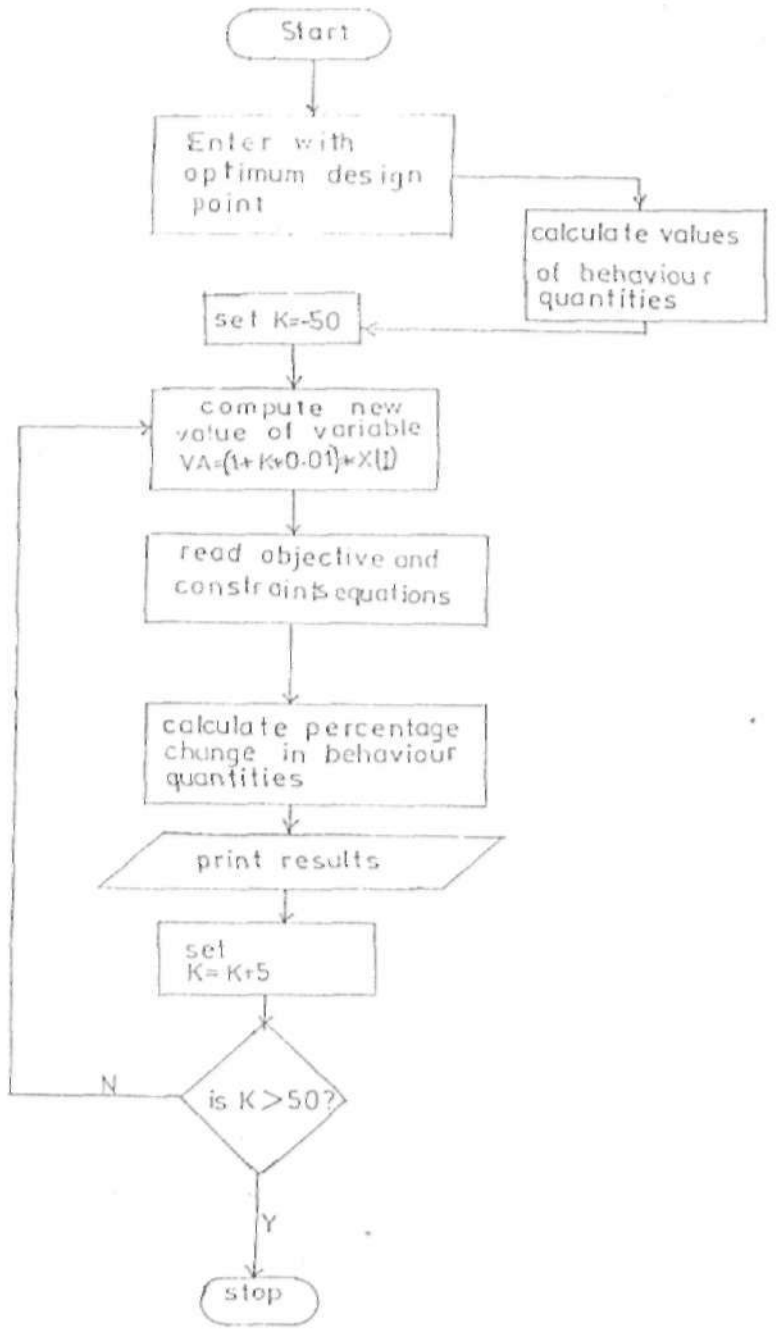


Fig 5.15 FLOW CHART FOR SENSITIVITY ANALYSIS

TABLE 5.4 I-SECTION PARAMETRIC STUDY RESULTS

PERCENTAGE CHANGE IN DESIGN VARIABLES (INITIAL VALUES)		PERCENTAGE CHANGE IN BEHAVIOUR QUANTITIES				
		f(x) (7.79)	σ_T (-0.14)	σ_c (13.72)	δ_T (2.06)	P_{cr} (3758.7)
	1	2	3	4	5	6
X_1(318.67)	+50	-41.77	-160.64	0.00	14.00	-48.68
	-50	41.77	545.18	0.00	-149.14	48.68
X_2(4444.83)	+50	-10.04	63.32	0.00	52.52	-169.28
	-50	10.04	-63.97	0.00	-723.96	84.48
X_3(87.65)	+50	-41.77	-155.44	0.00	-8.85	-22.50
	-50	41.77	773.94	0.00	-92.18	-22.45
X_4(30.19)	+50	-6.08	141.33	0.00	-26.19	-21.52
	-50	6.09	-140.37	0.00	-29.57	-18.88
X_5(31.48)	+50	-2.14	-213.31	0.00	-27.86	-20.20
	-50	2.15	639.92	0.00	-27.86	-20.20

TABLE 5.5 RECTANGULAR SECTION PARAMETRIC STUDY RESULTS

PERCENTAGE CHANGE IN DESIGN VARIABLES (INITIAL VALUES)		PERCENTAGE CHANGE IN BEHAVIOUR QUANTITIES				
		f(x) (4.95)	σ_T (-0.9999)	σ_c (13.61)	δ_T (19.96)	P_{cr} (495.21)
	1	2	3	4	5	6
X_1(15.94)	+50	48.15	0.002	0.00	-33.33	50.00
	-50	-46.99	-0.01	0.00	100.00	-50.00
X_2(269.40)	+50	48.15	0.002	0.00	-70.37	237.50
	-50	-46.99	-0.01	0.00	700.00	-87.50
X_3(25.55)	+50	3.30	0.002	0.00	0.00	0.00
	-50	-2.14	-0.01	0.00	0.00	0.00

TABLE 5.6 CIRCULAR SECTION PARAMETRIC STUDY RESULTS

PERCENTAGE CHANGE IN DESIGN VARIABLES (INITIAL VALUES)		PERCENTAGE CHANGE IN BEHAVIOUR QUANTITIES				
		f(x) (5.90)	σ_7 (-0.30)	σ_c (18.85)	δ_7 (12.12)	P_{cr} (815.57)
	1	2	3	4	5	6
X_1(318.00	+50	58.87	101.53	0.00	-76.32	322.23
	-50	-58.87	-461.83	0.00	1669.92	-94.35
X_2(60.00)	+50	31.10	-57.82	0.00	-3.40	3.42
	-50	-42.21	51.49	0.00	43.78	-30.45
X_3(25.00)	+50	2.24	78.00	0.00	0.00	0.00
	-50	-2.24	-234.01	0.00	0.00	0.00

I-section: Figure 5.1 shows that the optimum weight is sensitive to the flange width and flange thickness and to some extent affected by the web length and almost insensitive to changes in the tendon wires.Figure 5.2 indicate that the induced tensile stress is sensitive to the flange thickness, flange length,number of tendon wires and to a lesser extent to the web thickness and length. Figure 5.3 shows that the maximum displacement is sensitive to changes in web length flange length and width. Figure 5.4 indicate that the critical load is sensitive to changes in flange length ,web length and to a certain extent on the flange thickness.

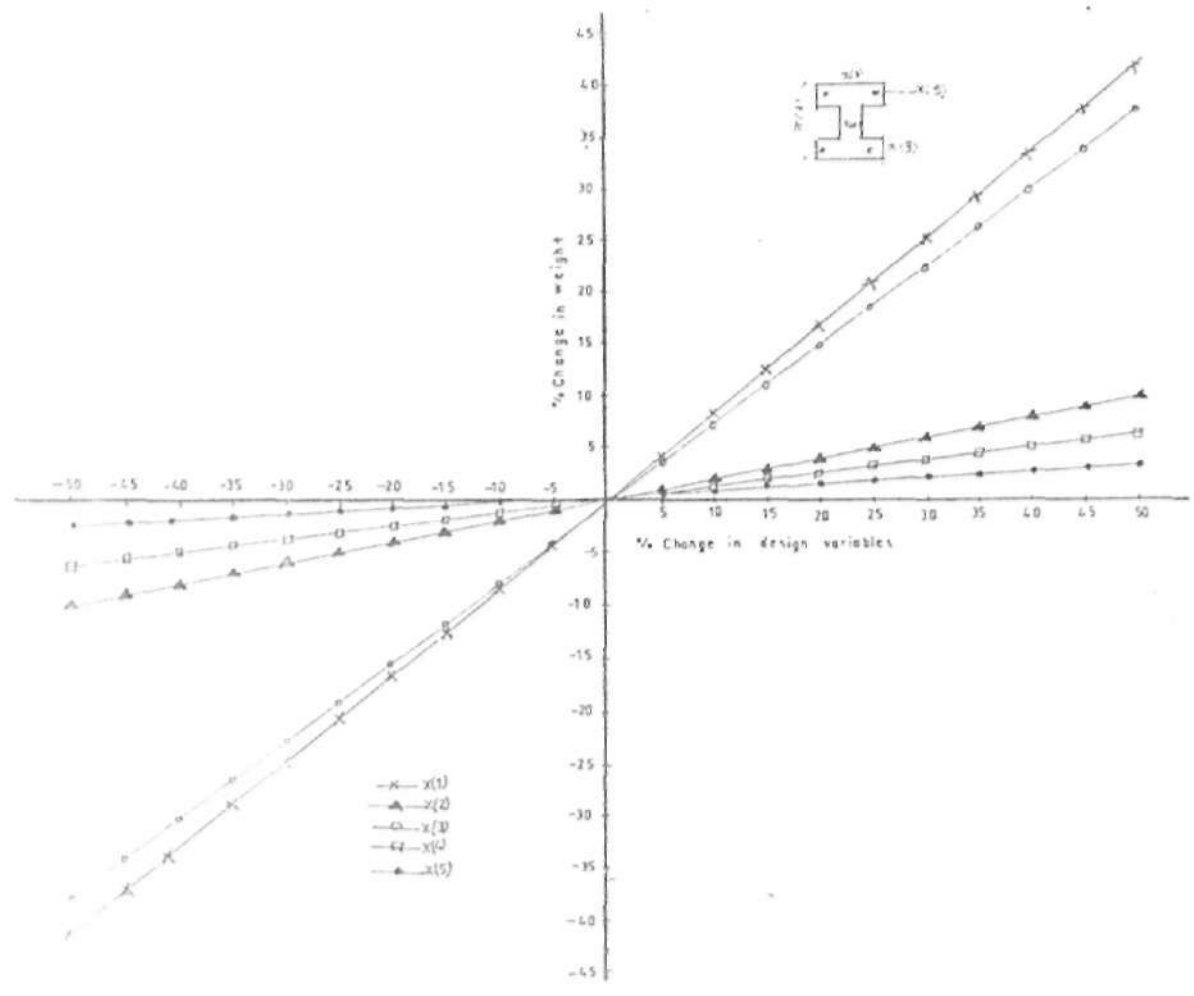


Fig5-1 VARIATION OF MAXIMUM WEIGHT

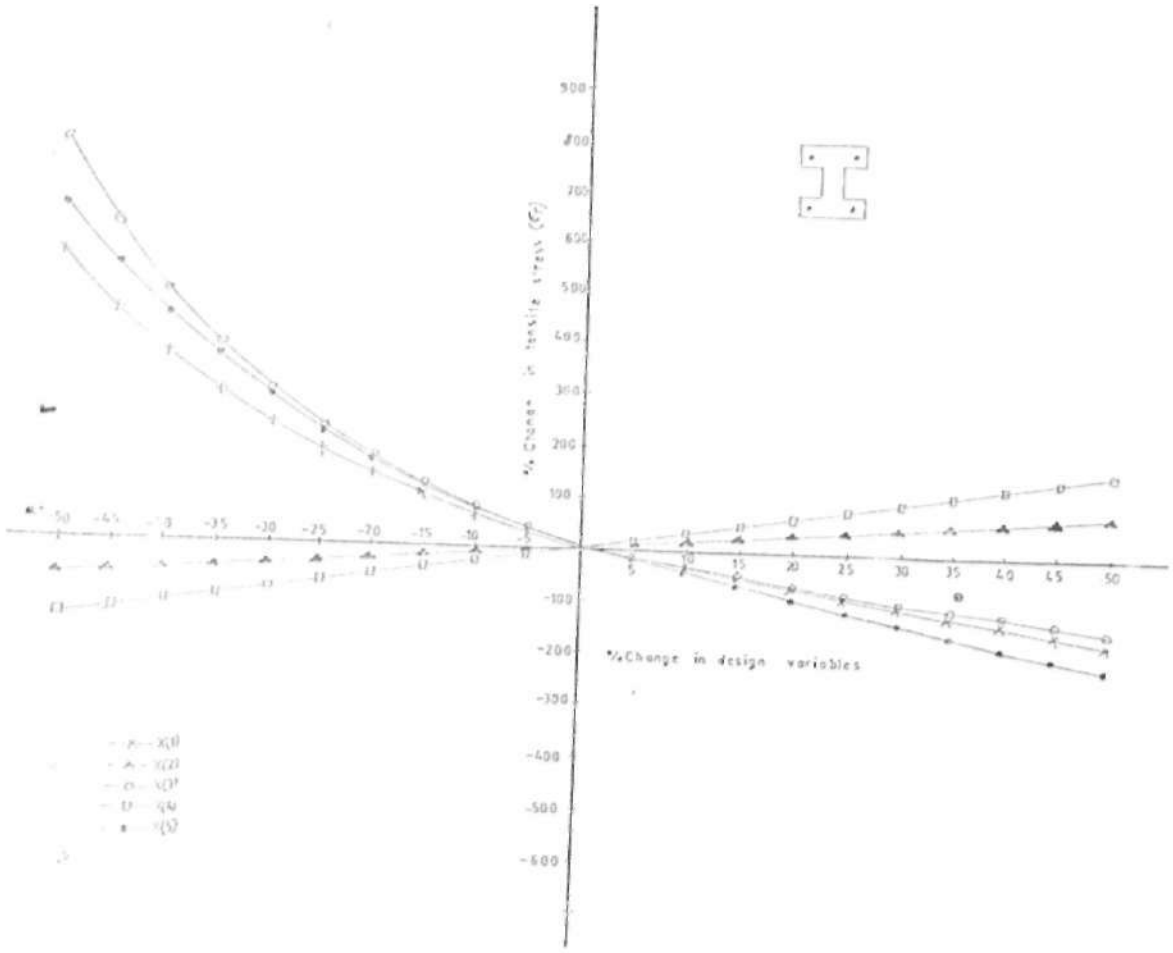


Fig 5.2 VARIATION OF INDUCED TENSILE STRESS

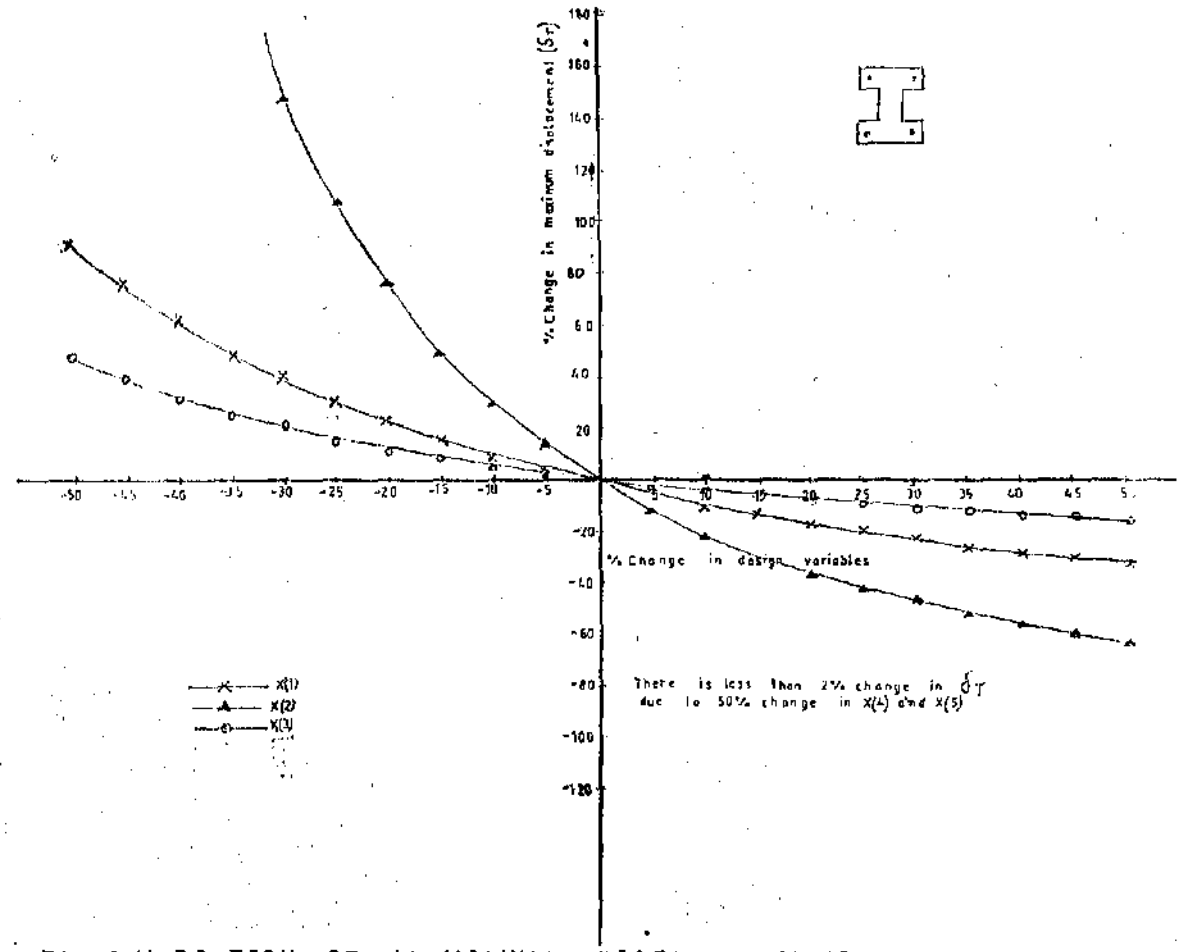


Fig5.3 VARIATION OF MAXIMUM DISPLACEMENT

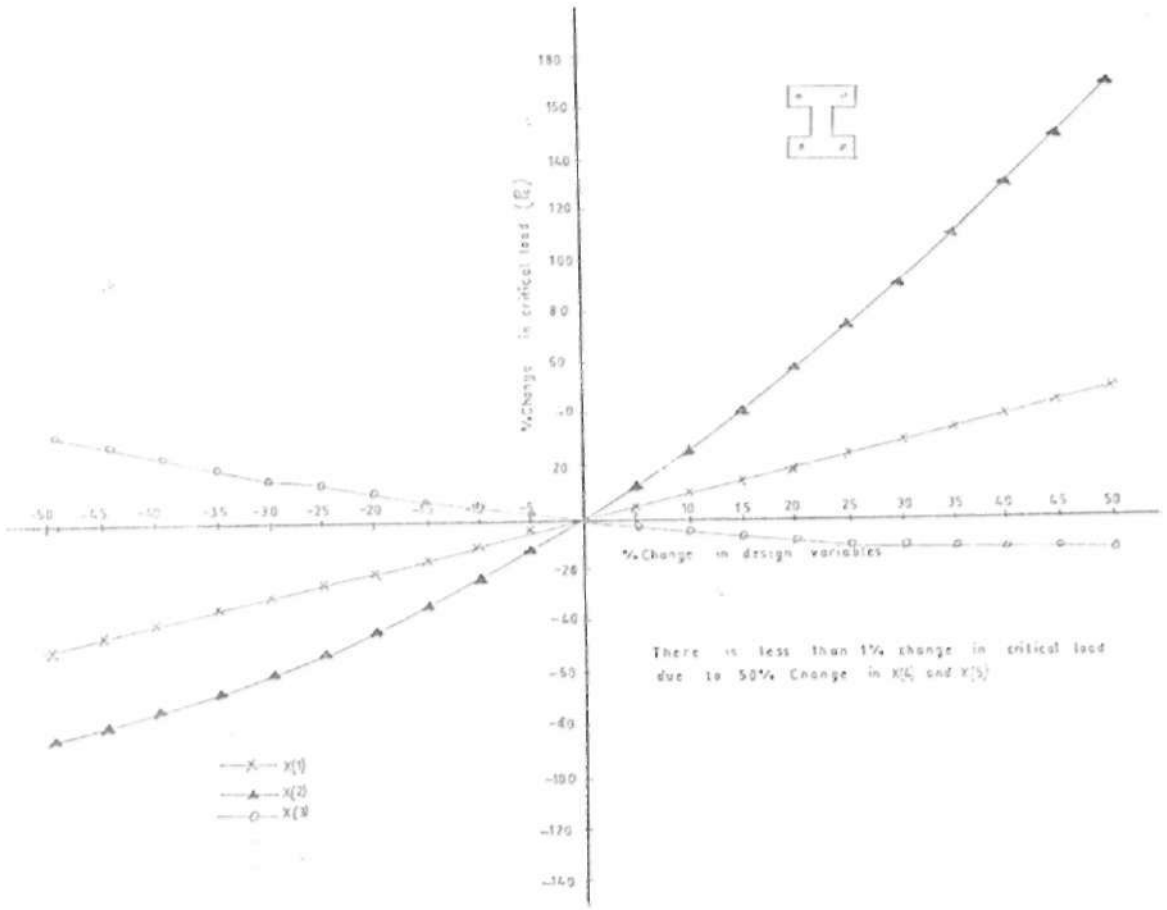


Fig 5.4 VARIATION OF CRITICAL LOAD

Circular tubular section: Figure 5.5 shows the maximum weight is sensitive to a variation on overall thickness of section and to a lesser extent on changes in core thickness and almost insensitive to tendon wire change. Figure 5.6 shows that the tensile strength is sensitive to overall thickness of tendon wires and to a lesser extend to the tube thickness changes. Figure 5.7 and 5.8 indicate that the changes in overall thickness of section causes a large displacement. A significant increase is also shown when the tube thickness is decreased.In both cases a gradual increase in the variables reduces the displacement very slightly until about 30% to 40% when the deflection starts to increase. This interesting behaviour shows that any attempt to increase the two variables beyond this range will be uneconomical. Figure 5.9 and 5.10 show that the critical load is sensitive to variation in overall thickness in the case of tube thickness more than 25% increase reduces the critical load therefore it should not be increase beyond 25% because it will be dangerous.

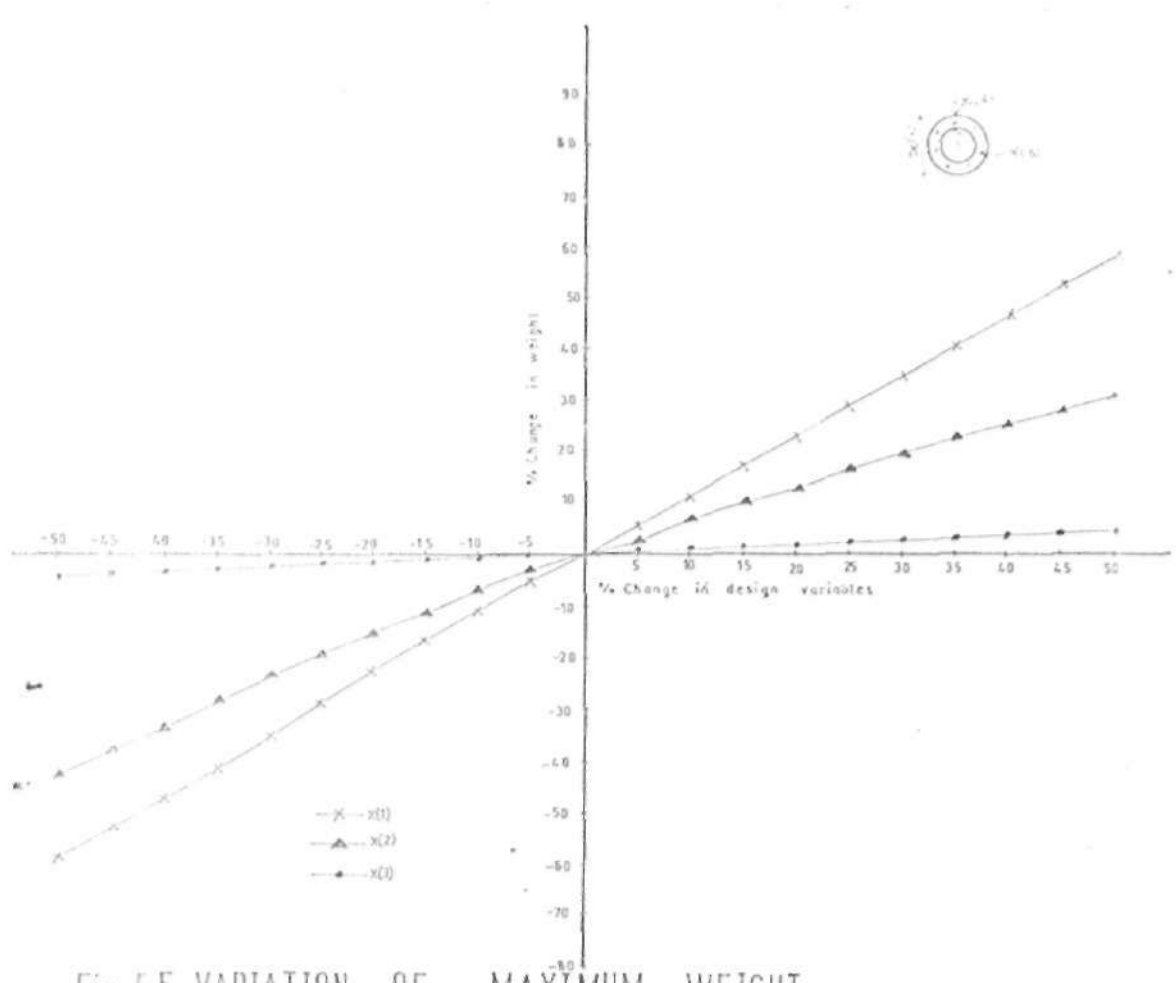


Fig 5.5 VARIATION OF MAXIMUM WEIGHT

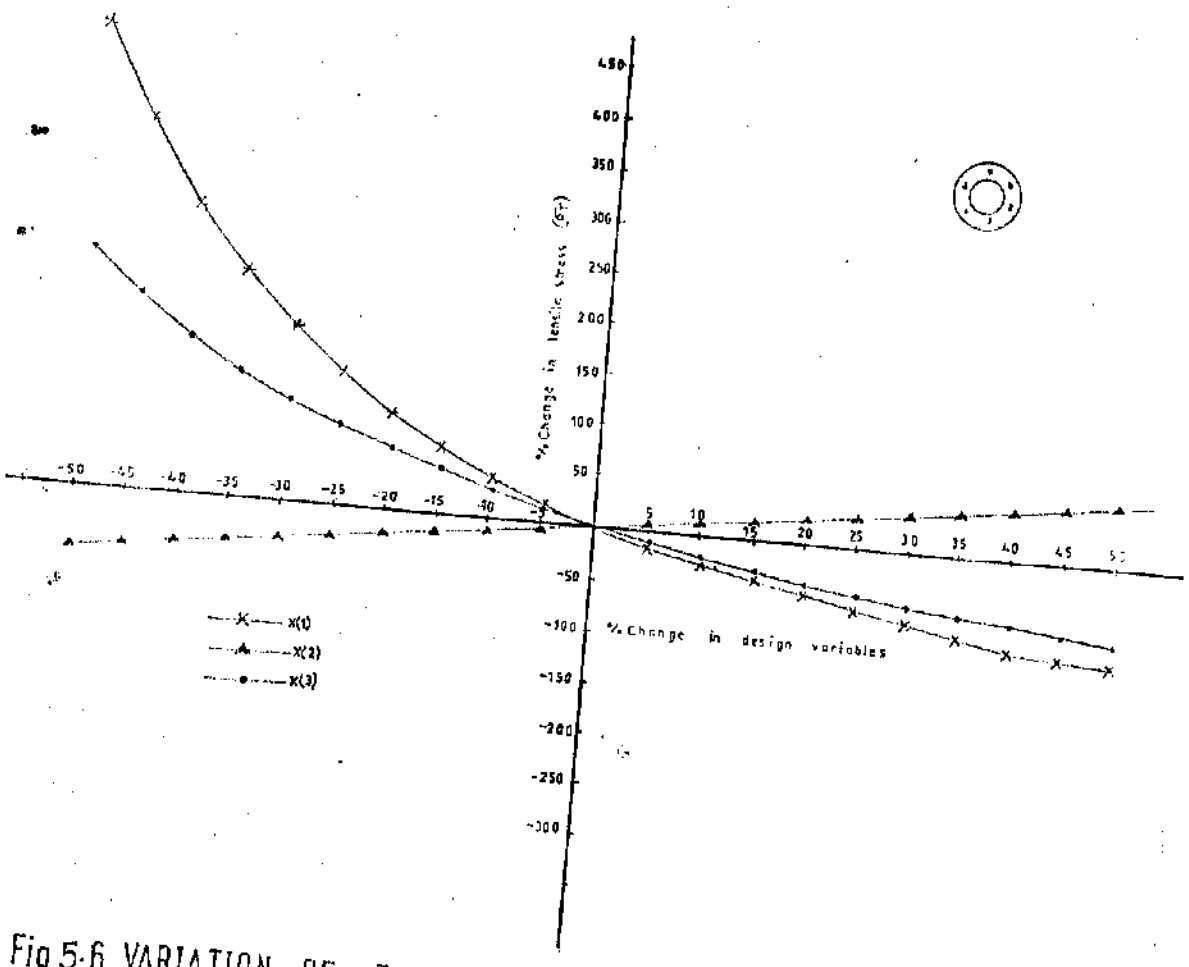


Fig 5.6 VARIATION OF TENSILE STRESS

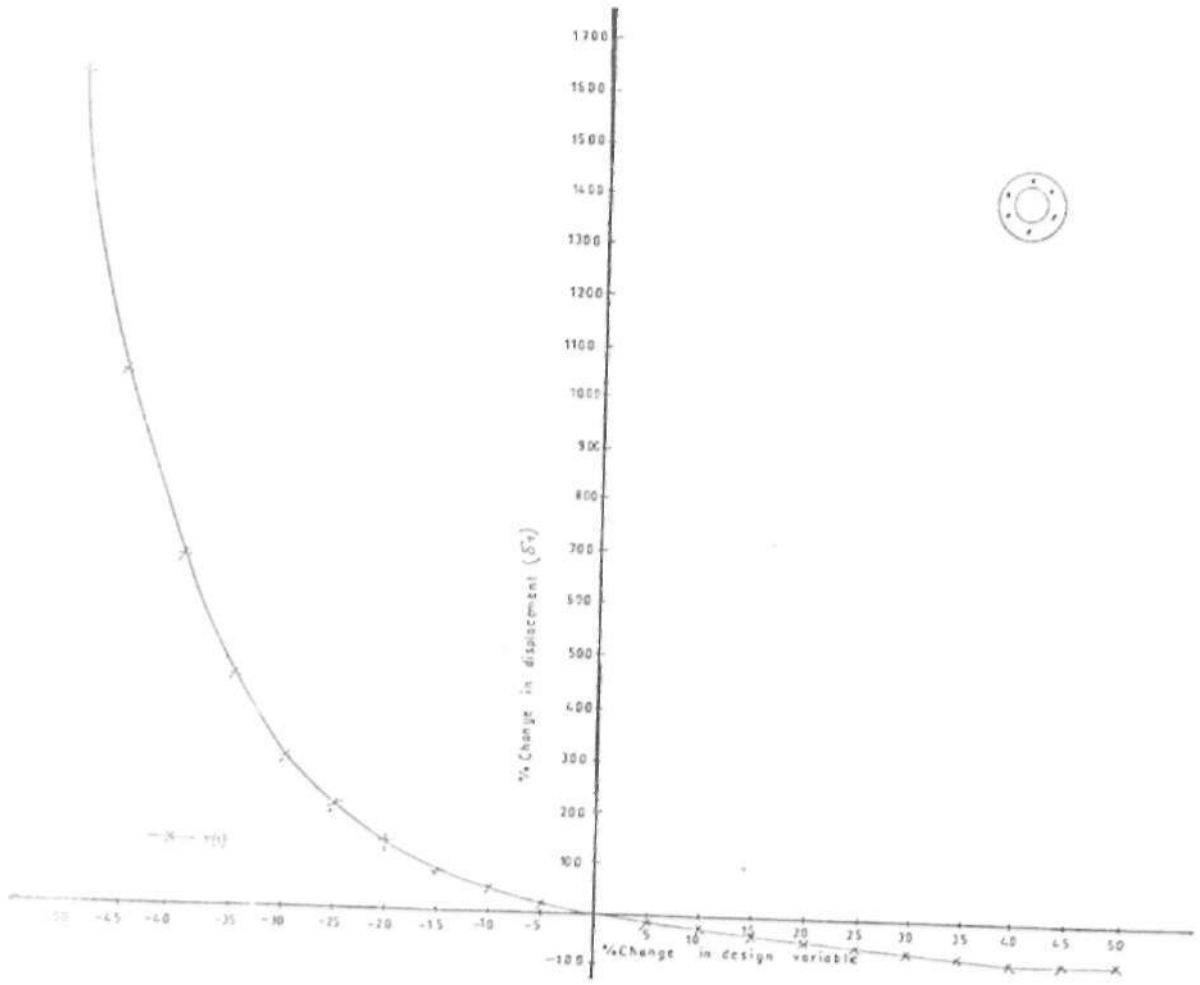


Fig5.7 VARIATION OF MAXIMUM DISPLACEMENT

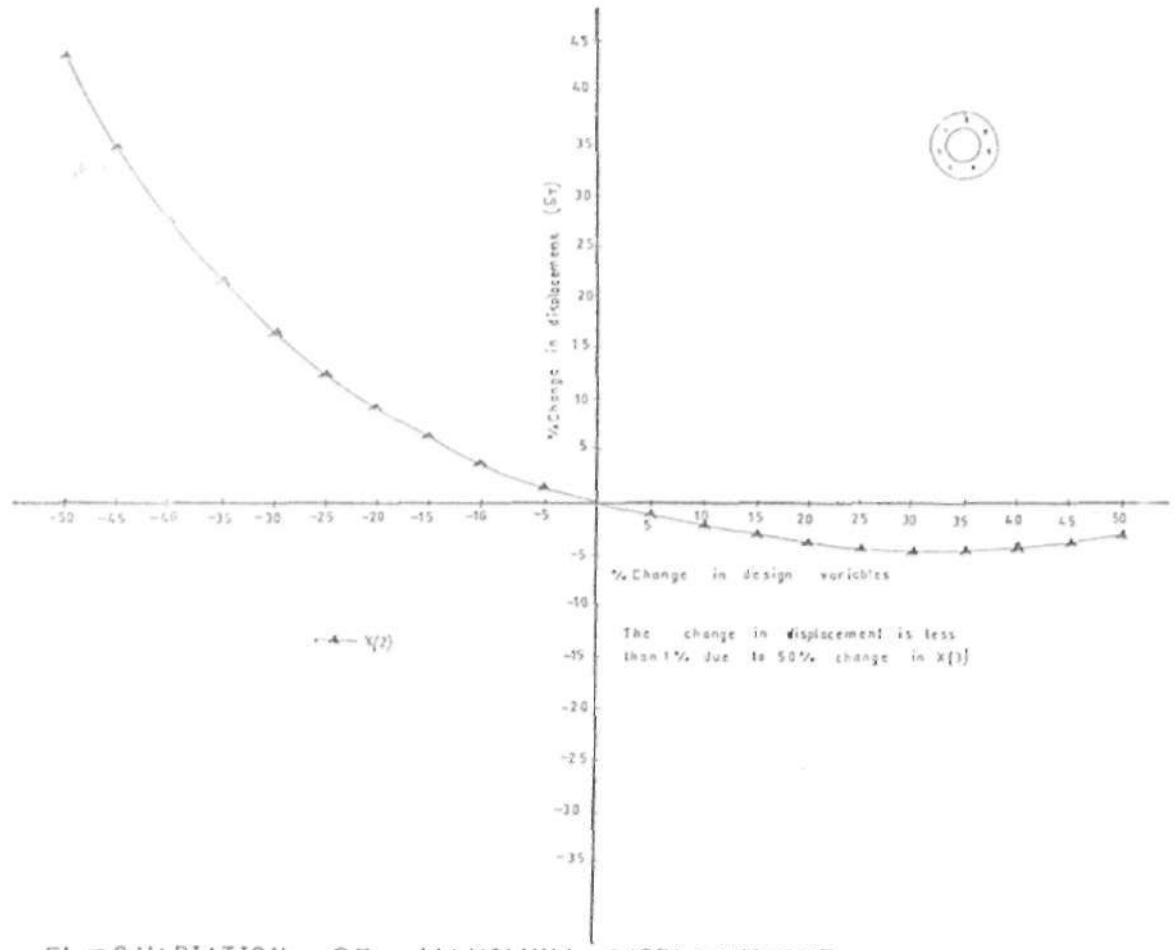


Fig5.8 VARIATION OF MAXIMUM DISPLACEMENT

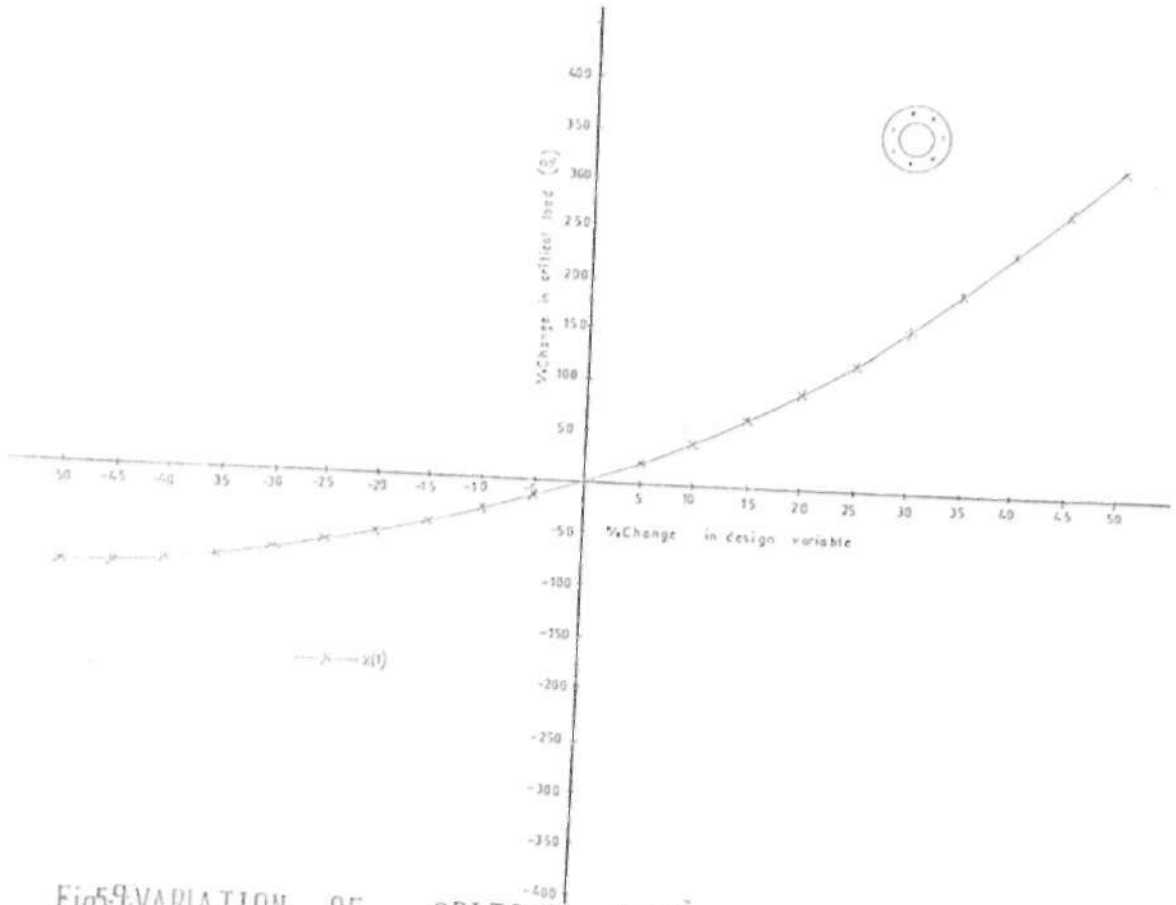


Fig5.9 VARIATION OF CRITICAL LOAD

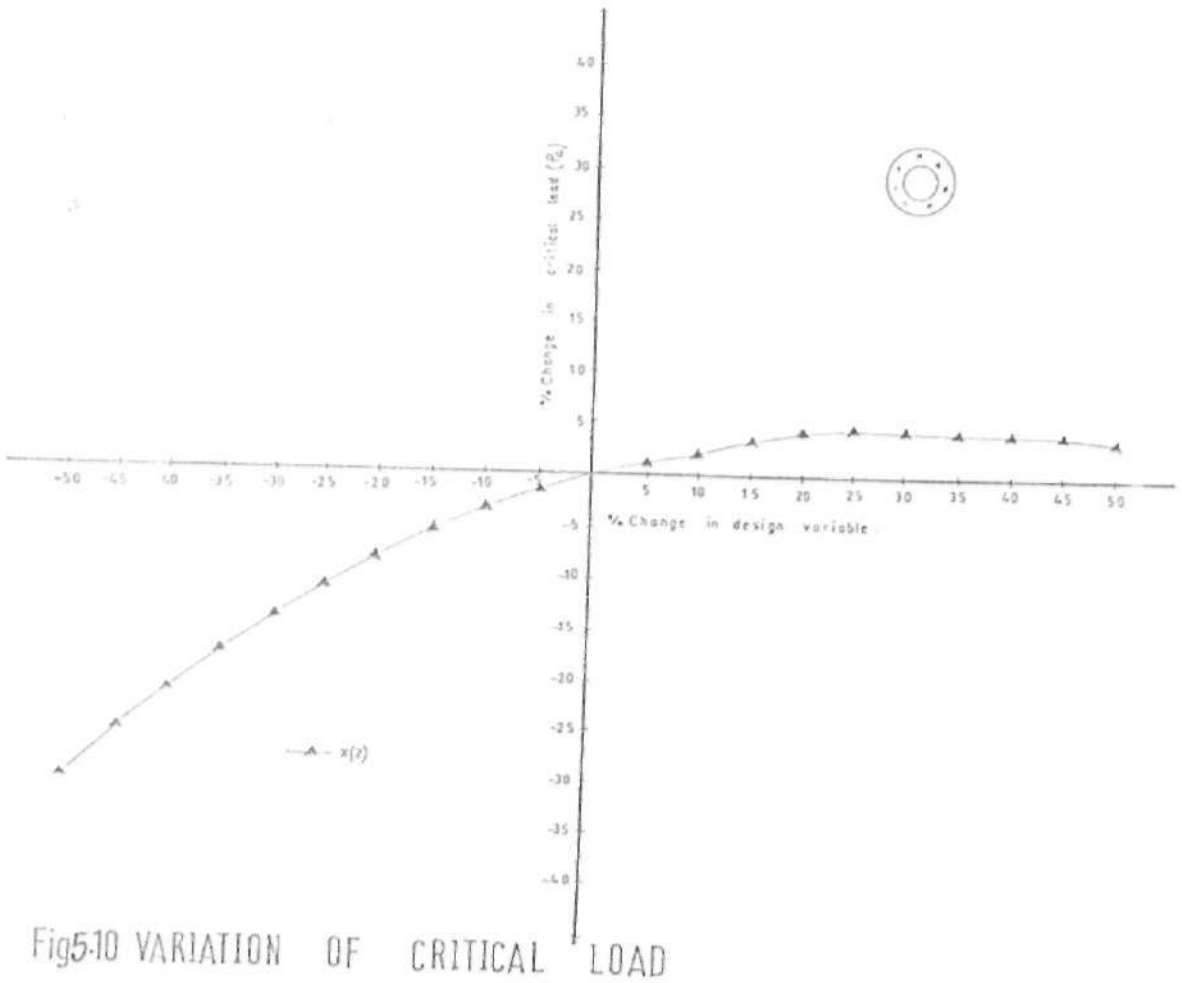


Fig5.10 VARIATION OF CRITICAL LOAD

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Rectangular section: Figure 5.11 shows that the weight is sensitive to the section depth D and breath B changes and almost insensitive to variation in tendon wires. Figure 5.12 shows that the tensile stress is insensitive to changes in the section breath, depth and number of tendon wires. From this result it can be concluded that tapering will not cause any significant change in flexural stress anywhere along the pole. Figure 5.13 shows that the deflection is sensitive to variation in breath and width of section and not sensitive to changes in tendon wires. Figure 5.14 shows that the critical load is sensitive to changes in depth and width of the section.



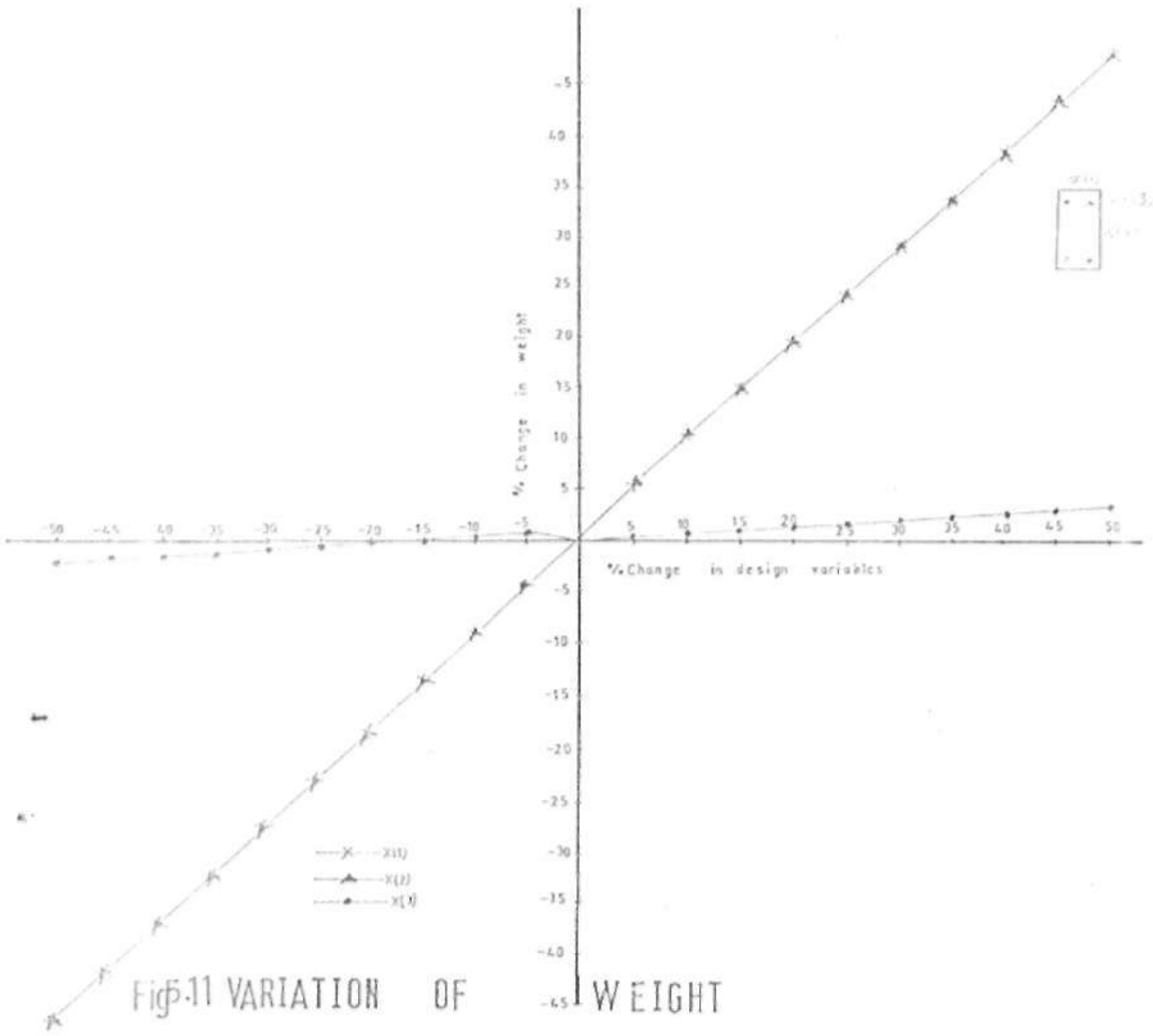


Fig 5.11 VARIATION OF WEIGHT

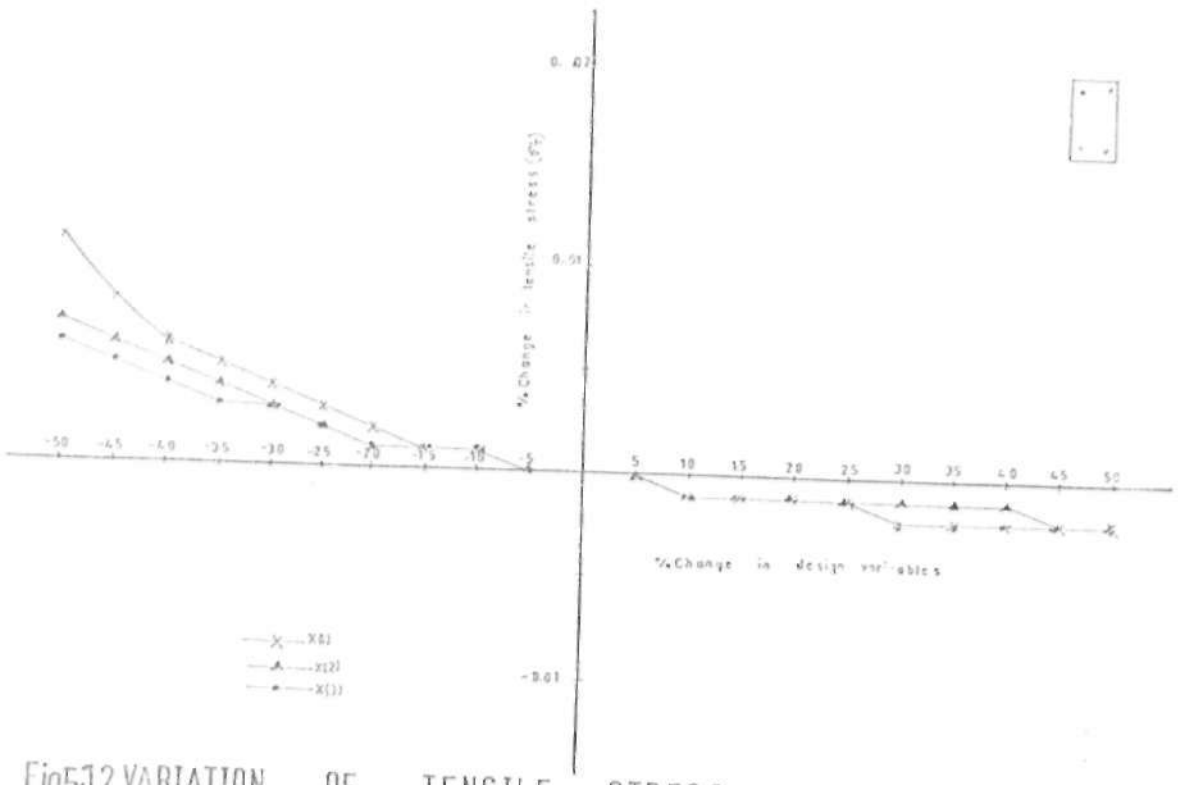


Fig5.12 VARIATION OF TENSILE STRESS

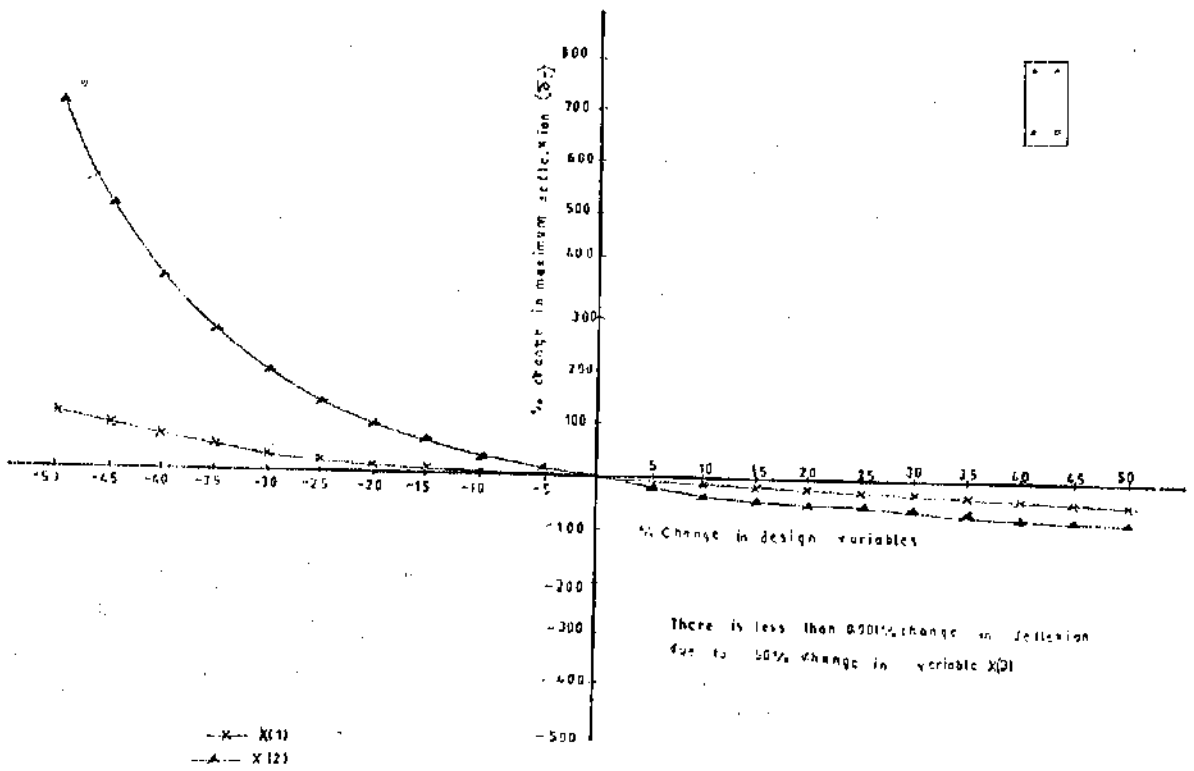


Fig 5.13 VARIATION OF MAXIMUM DISPLACEMENT

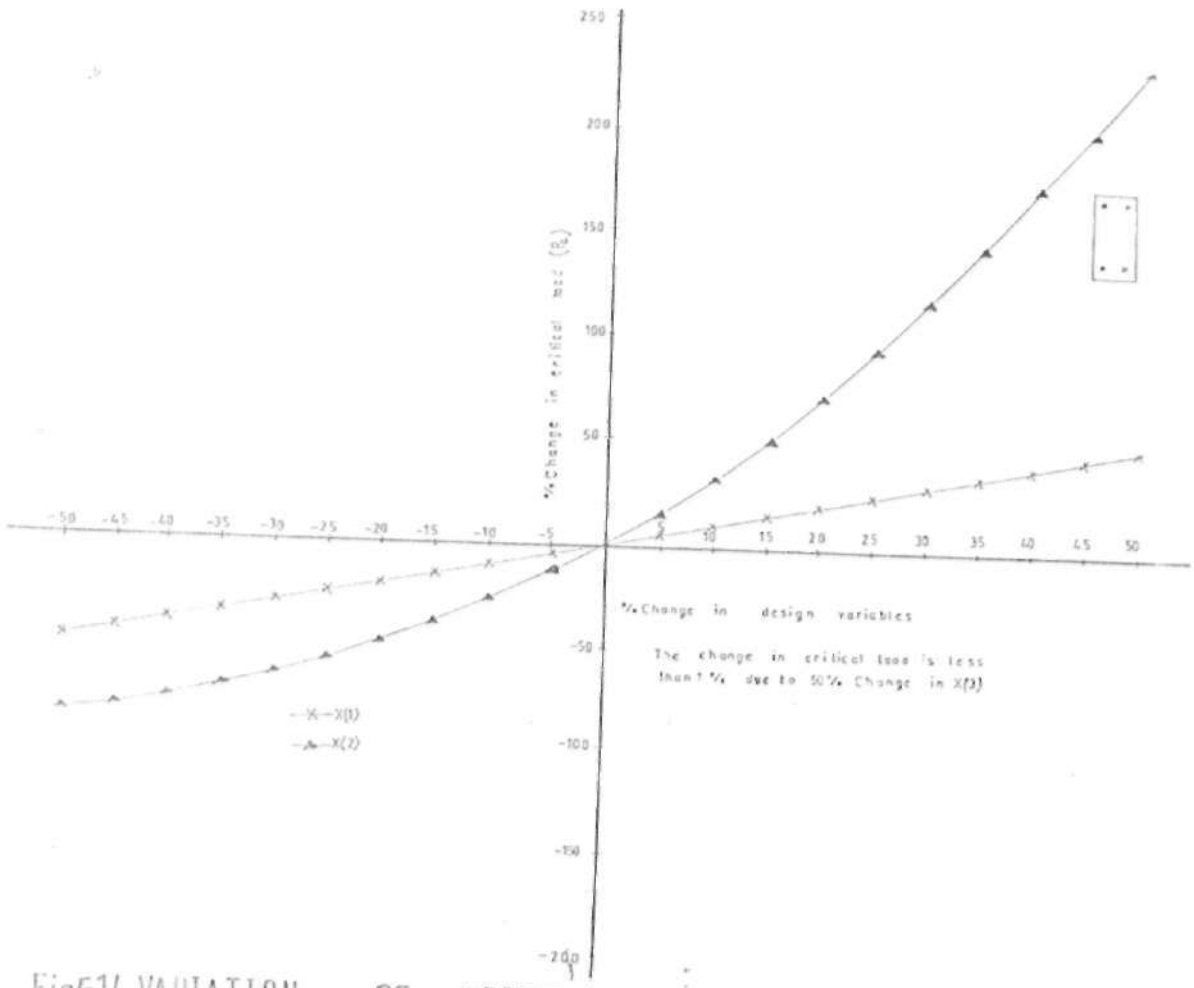


Fig5.14 VARIATION OF CRITICAL LOAD

From Fig 5.2 if the flange breath is increased by 30% the stress will be reduced by 100%, this will take care of any point along the changing cross section,and serve as a correction to phase change in induced stresses.Essiens [3] calculations showed that the cause of the phase change was due to torsional moment. This moment which is constant along the length of the pole causes increase in stress as the sectional area reduces. Thus with a taper section it is expected that the stress will increase as the section becomes smaller. ".. a constant cross section will be a better section although, naturally, there will be an increase in seismic and wind stresses but, for torsional resistance, the top must be as strong as the base. Additionally adoption of a constant cross section permits more effective use of prestressing and easier connections"[25].

5.3 CONCLUSION AND FURTHER RESEARCH

From the results of this study several conclusions can be stated.

1.The optimum weight of the I-section is approximately the same as that computed by Essien [14].

2.The optimum cross-section in terms of material economy is the rectangular section.

3.From the sensitivity analysis of the rectangular section, induced tensile stresses on the system do not depend

to some extent on variation of the cross-sectional dimensions.Thus it can be concluded that gradual tapering of this section will not have any appreciable stress increase as the sectional area is reduced.

4. From the results shown in Tables 5.11-5.3, it can be concluded that most of the constraints were active at the optimum point for the three cases.

From this work and previous ones the areas that may deserve further attention are: the effect of impact on the pole i.e. does the pole fall on an automobile, does it snap off suddenly when overloaded,or does it fail gradually, with large deformations and energy absorption? Another area is the development of positive means of insuring against brittle behaviour under impact. And the use of light weight concrete to ease transportation [25]. Optimization formulation can be made with the above recommendations. In the problems studied (fig 5.4 - 5.6) none of the design variables had any effect on the compressive stress, this could be due to the reason that prestressing force was not considered as a design variable. A further area of investigation can be an inclusion of prestressing force as a variable. Also more shapes can be studied so as to get the best. It will be noticed that dynamic loading was not considered in the study because from available literature it did not seem to be a critical load in designing or choosing the size of the sections. Apart from that "...compare with conventionally reinforced concrete,

prestressing increases the crack resistance, rigidity, and resistance to dynamic loads" [25]. And from Essiens work the position of the frequency holes was not exactly at the point of frequent failure of the poles, although close.

REFERENCES

1. K.V.Mital:[1977] Optimization method in operational reseach and systems analysis (wiley Eastern Ltd New Delhi)

2. Chars S.Beightler, Don T.Phillips and Douglas J.Wilde:[1979] Foundation of optimazation (2nd edition. prientice-Hill, inc, Englewood cliff N.J)

3. Essien, E.Bassey:[1990] Flecxural failures in transmission concrete poles (Msc thesis Dept.of civil engg.A.B.U. Zaria, Nigeria)

4. L.S.Blake: Civil engineering reference book 3rd edition.

5. S.S.Letcha: Msc lecture notes (Dept. of civil engg .A.B.U Zaria)

6. BS607;British standard [1960]: specification for concrete poles for electric transmission and traction systems. British standard institution; London.

7. S.S.Rao:[1978] Optimization, theory and application (wiley New Delhi)

8. M.J.Turner:[1969] Optimization of structures to satisfy flutter requrements (AIAAJ.7,944-957)

9. H.Ashley, S.C.McIntosh and W.H.Weatherill:[1970] Optimazation under aeroelastic 36,Turkey)

10. T.P.Kicher:[1968] Stractural synthesis of integrally stiffened cylinders (Journal spacecraft and Rockets 5,62-67)

11. Moshe B.Fudts and Maurice A.Brull:[1978] (Tel. Aviv University, Ramat Aviv- Tel.Aviv Isreal) A new strain energy

theorm and its use in the optimum design of continuous beams (comp.& str. vol.10 pp 647-657)

constraints.(symp.Stractural optimization.AGARD. Conf.proc.No 12. S.S.Rao:[1971] Automated optimum design of aircraft wings to satisfy strength ,stability, frequency and flutter requirement (Rep. No.49 Division of solid mechanics, structures and mechanical design ,case western Reserve University, Cleveland, Ohio)

13. Y .Ding and B.J.D.Esping:[1985] Optimal design of beams with different cross-sectional shapes (comp. & str.vol. 24 pp 707-726)

14. B.H.V.Topping and D.J.Robinson:[1984] Optimization of timber framed structures (comp.& str. vol.18 6 pp.1167-1177) -

15. Douglass J.Wide:[1978] Globally optimal Design (A wiley-interscience publication)

16. L.Librescu and L.Beiner:[1983] Aeroelastic optimazation of orthotropic rectangular panels(optimal control application & methods journal vol.4 No 2)

17. N.Krishna Raju:[1988] Advanced R.conc.Design(CBS publisher & distributors)

18. S.S Rao:[1980] Structural optimazation by chance constrained programming techniques (comp.& str.inter.journal vol 12 No6)

19. C.Fleary and M.Geradin:[1978] Optimality criteria and mathematical programming in structural weight.(Computers and structures {comp.& str.}international journal vol.8 PP.7-77

20. W.H.Mosley and J.H.Bungey:[1982] Reinforced concrete

21. James M.Gere and Winfred O.Carter:[1962]Critical Buckling loads for tapered columns.(Journal of the structural division,proc. of A.S.C.E. Feb.) design (Macmillan publishers)

22. R.L.Fox:[1977] Optimization method in engineering design (Addison wesley,Reading Massachusette)

23. Asgher Hussan and Kota Gangial:[1976] Optimazation techniques for chemical engineers.(Macmillan company of India Ltd)

24. Uri Kirsch:[1981] Optimal structural design (McGraw-Hill, Inc)

25. Ben.C.Gerwick,Jr.:[1971] Construction of prestressed concrete structures (Vol in the Willy & sons inc series)

26. R.I.Harless:[1980] A method for synthesis of optimal weight structures (comp.& str.journal vol. 12 No6)

27. Charles E.Reynolds and J.C.Steedman:[1976] Reinforced concrete designer,s Hand book (A viewpoint publication 8th edition)

28. J.Case and A.H.Chilver: Strength of materials and structures (E.L.B.S publishers Ltd London).

29. G.H.Wasish:[1975] Methods of optimization (A wiley-interscience publication)

30. Brian,J.B:[1964] Practical prestressed concrete design (Sir Isacc Pitman and son Ltd London pp 1-95)

31.Donald M.Monro:[1978] Computing with fortran IV.A practical

couse(Edward Arnold publishers)

32. V.R.Rao,N.G.R.Iyengar and S.S.Rao:[1978] Optimazation of wing structures to satisfy strength and frequency requirements (comp.& struc.vol.10,pp.669-674)

```
C   ************************************************************************
C   * DESIGN OPTIMIZATION OF PRESTRESSED CONCRETE TRANSMISSION POLES       *
C   ************************************************************************
C   *           Msc (structural engineering thesis)                        *
C   *           By UGA,B.WUYANA.                                           *
C   *           DEPT.OF CIVIL ENGINEERING                                  *
C   *           FACULTY OF ENGINEERING                                     *
C   *           A.B.U.ZARIA,1991.                                          *
C   ************************************************************************
C PROGRAM FOR SOLVING NONLINEAR PROBLEMS-prof.s.s.Rao.
C************************************************************************
C     THE PROGRAM SOLVES GENERAL NONLINEAR PROGAMMINGPROBLEMS OF THE TYPE:
C
C                ( X1 )
C                ( X2 )
C        Find X=( :  )which minimizes f(X)
C                ( :  )
C                ( XN )
C
C     subject to      gj(X)<Ø,j=1,2,.....,M
C
C
C    The solution procedure is base on the interior penalty function methc
C     couple with the Davidon-Fletcher-Powell method of unconstrained mini
C     ion and cubic interpolation method of one-dimensional search. The
C     program contains UNON,ONEDIM,GRADT,and FTN as subroutines.
C
C     The following input data is required by the program:
C  PARAMETERS
C **********
C     ITLIM - maximum number of iterations permitted in any one unconstrai
C             minimization of phi(any number between N and 2N is reasonabl
C
C     MAXIN - maximum number of cubic interpolations permitted in any one
C             one-dimensional search(any number between 3 and 6 is reasons
C
C     MAXGR - maximum number of gradient evaluations permitted in any one
C             one-dimensional search(any number between 3 and 6 is reasons
C
C     N-    number of design variables
C
C     M-  number of constraints
C
C     MAXPI- maximum number of phi-function to be minimized(any number bet
C            4 and 8 is reasonable
C
C     C - reduction factor for the penalty parameter(Ø.1 is commonly used)
C
C     R - initial value of the penalty parameter Rk(if no better value is
C         available, a value of 1.Ø can be used)
C
C        EPS - convergence requirment in unconstrained minimization(Ø.ØØ1
C              reasonable;smaller value can be used for more accurate sol
C
C     EPSS - convergence requirement in cubic interpolation (Ø.ØØ1 is reas
```

```
C             le; smaller value can be used for more accurate solution)
C
C     STEPO - trial step length to be used in cubic interpolation(Ø.Ø1 is
C             reasonable)
C     X(1) }
C     X(2) }
C     :    }} initial guess values of the design variable(X must be an ini
C     :    }  feasible point)
C     X(N) }
C
C   - The dimensions of the arrys X,XN,XOPT,GRAD,S,GRADN,Yand HY in the ma
C      program to be the number of design variables(N)
C   - The dimension of the arrys G and GG in the main program to be the nu
C     of constraints(M)
C   -The dimension of the arrys H,XMM and XNN in the main program to be (N
C   -Proper expressions to be given for the objective function(OBJ) and th
C     constraints G(1),G(2),.....,G(M) in the subroutine FTN
C
C   THE FOLLOWING QUANTITIES ARE OUTPUTED:
C
C     R=current value of the penalty parameter
C
C     PF=value of the phi-funtion
C
C     OBJ=value of the objective function(f)
C
C     X(I)=values of the design variables
C
C     G(I)=values of the constraints
C
C     S(I)=components of the search direction
C
C     ITER=iteration number in the unconstrained minimization
C
C     STEPO=optimal step length in the current one-dimensional search
C
C*************************************************************************
C
      PROGRAM NUMO(DPROB,SISECN,TAPE5=DPROB,TAPE6=SISECN)
      DIMENSION X(5),XN(5),XOPT(5),GRAD(5),S(5),GRADN(5),Y(5),HY(5),
     1G(9),GG(9),H(5,5),XMM(5,5),XNN(5,5)
      COMMON/COUNT/NFUN,NGRAD
      COMMON/MITER/ITLIM
      COMMON/CONVER/MAXIN,MAXGR
      NFUN=Ø
      NGRAD=Ø
      ITLIM=13
      MAXIN=5
      MAXGR=3
      DATA N,M,MAXPI,C,R,EPS,EPSS,STEPO
     1/5,9,6,Ø.1,1.Ø,Ø.ØØ1,Ø.ØØ1,1.Ø/
      X(1)=318.62319
      X(2)=444.81157
      X(3)=87.479513
      X(4)=33.67ØØ84
```

```
      X(5)=30.80859Ø
      K=1
30    CALL UNCON(N,M,R,EPS,EPSS,STEPO,X,XN,XOPT,H,GRAD,S,GRADN,Y,HY,XMM,
     1XNN,G,GG,F,OBJ,II)
      IF(K.EQ.MAXPI)GO TO 2Ø
      R=C*R
      DO 1Ø I=1,N
1Ø    X(I)=XOPT(I)
      K=K+1
      GO TO 3Ø
2Ø    STOP
      END

      SUBROUTINE UNCON(N,M,R,EPS,EPSS,STEPO,X,XN,XOPT,H,GRAD,S,GRADN,
     1Y,HY,XMM,XNN,G,GG,F,OBJ,II)
C     For implementing the Davidon-Fletcher-Powell method of
C     unconstrained minimization
      COMMON/COUNT/NFUN,NGRAD
      COMMON/MITER/ITLIM
      COMMON/CONVER/MAXIN,MAXGR
      DIMENSION X(N),XOPT(N),H(N,N),GRAD(N),S(N),GRADN(N),Y(N),HY(N),
     1XN(N),G(M),GG(M),XMM(N,N),XNN(N,N)
      DO 1Ø I=1,N
      DO 1Ø J=1,N
1Ø    H(I,J)=Ø.Ø
      DO 2Ø I=1,N
2Ø    H(I,I)=1.Ø
      II=1
      CALL FTN(G,X,F,OBJ,N,M,R)
      CALL GRADT(X,GG,N,M,R,GRAD,F)
      WRITE(6,111)R,F,OBJ
111   FORMAT(//,8X,"STARTING VALUES FOR UNCONSTRAINED MINIMIZATION"
     1,/,8X,"R=",E12.5,2X,"PF=",E15.8,2X,"OBJ=",E15.8)
      WRITE(6,112)(X(I),I=1,N)
112   FORMAT(11H  X(I) ARE,/,2X,3E15.8,/,2X,3E15.8)
      WRITE(6,113)(G(I),I=1,M)
113   FORMAT(11H  G(I) ARE,/,2X,3E15.8,/,2X,3E15.8,/,2X,3E15.8)
1ØØ   DO 3Ø I=1,N
      S(I)=Ø.Ø
      DO 3Ø J=1,N
3Ø    S(I)=S(I)+H(I,J)*GRAD(J)
      DO 4Ø I=1,N
4Ø    S(I)=-S(I)
      SUM=Ø.Ø
      DO 1 I=1,N
1     SUM=SUM+S(I)**2
      SUM=SQRT(SUM)
      DO 2 I=1,N
2     S(I)=S(I)/ABS(SUM)
      CALL ONED1M(N,M,R,EPSS,STEPO,SLAMDA,X,XN,S,F,FN,OBJ,GRAD,GRADN,G,
     1GG)
      DO 5Ø I=1,N
5Ø    X(I)=X(I)+SLAMDA*S(I)
      WRITE(6,114)II, STEPO,FN,OBJ,NFUN,NGRAD
114   FORMAT(/,8X,"ITER=",I3,2X,"STEPO=",E12.5,/,8X,"PF=",E12.5,2X,
```

```
     1"OBJ=",E12.5,2X,"NFUN=",I5,2X,"NGRAD=",I5)
       WRITE(6,112)(XN(I),I=1,N)
       WRITE(6,113)(G(I),I=1,M)
       SUM=Ø.Ø
       DO 6Ø I=1,N
6Ø     SUM=SUM+GRADN(I)**2
       SUM=SQRT(SUM)
       IF(SUM.LT.EPS)GO TO 11Ø
       IF(II.EQ.ITLIM)GO TO 11Ø
       DO 7Ø I=1,N
7Ø     Y(I)=GRADN(I)-GRAD(I)
       DEN=Ø.Ø
       DO 3 I=1,N
 3     DEN=DEN+S(I)*Y(I)
       FAC=SLAMDA/DEN
       DO 5 I=1,N
       DO 5 J=1,N
5      XMM(I,J)=S(I)*S(J)*FAC
       DO 4 I=1,N
       HY(I)=Ø.Ø
       DO 4 J=1,N
 4     HY(I)=HY(I)+H(I,J)*Y(J)
       DO 6 I=1,N
       DO 6 K=1,N
 6     XNN(I,K)=HY(I)*HY(K)
       DEN=Ø.Ø
       DO 7 J=1,N
 7     DEN=DEN+Y(J)*HY(J)
       DO 8 I=1,N
       DO 8 J=1,N
 8     XNN(I,J)=-XNN(I,J)/DEN
       DO 8Ø I=1,N
       DO 8Ø J=1,N
8Ø     H(I,J)=H(I,J)+XMM(I,J)+XNN(I,J)
       IF(II/N*N.NE.II)GO TO 39
       DO 37 I=1,N
       DO 37 J=1,N
37     H(I,J)=Ø.Ø
       DO 38 I=1,N
38     H(I,I)=1.Ø
39     CONTINUE
       II=II+1
       F=FN
       DO 9Ø I=1,N
9Ø     GRAD(I)=GRADN(I)
       GO TO 1ØØ
11Ø    DO 12Ø I=1,N
12Ø    XOPT(I)=X(I)
       RETURN
       END


       SUBROUTINE ONEDIM(N,M,R,EPSS,STEPO,SLAMDA,X,XN,S,F,FN,OBJ,GRAD,
      1GRADN,G,GG)
C      For implementing the cubic interpolation method of onedimensional
```

```
C       search
        COMMON/COUNT/NFUN,NGRAD
        COMMON/CONVER/MAXIN,MAXGR
        DIMENSION X(N),XN(N),S(N),GRAD(N),GRADN(N),G(M),GG(M)
        NNFUN=Ø
        NNGRAD=Ø
        WRITE(6,191)(S(I),I=1,N)
191     FORMAT(/,8X,"CUBIC INTERPOLATION STARTS",/,8X,"S(I)=",4E12.4,/,
       1(3X,4E12.4))
        SUM=Ø.Ø
        DO 1Ø I=1,N
 1Ø     SUM=SUM+GRAD(I)*S(I)
        A=Ø.Ø
        FA=F
        FAP=SUM
 4Ø     DO 2Ø I=1,N
 2Ø     XN(I)=X(I)+STEPO*S(I)
        CALL FTN(G,XN,FN,OBJ,N,M,R)
        NNFUN=NNFUN+1
        DO 21 J=1,M
        IF(G(J).GE.Ø.Ø)GO TO 22
 21     CONTINUE
        GO TO 41
 22     STEPO=STEPO*Ø.9
        GO TO 4Ø
 41     CONTINUE
        CALL GRADT(XN,GG,N,M,R,GRADN,FN)
        NNGRAD=NNGRAD+1
        SUM=Ø.Ø
        DO 3Ø I=1,N
 3Ø     SUM=SUM+GRADN(I)*S(I)
        FB=FN
        FBP=SUM
        IF(FBP.GT.Ø.Ø)GO TO 11Ø
        FA=FB
        FAP=FBP
        A=STEPO
        STEPO=STEPO*1.2
        GO TO 4Ø
11Ø     II=Ø
        B=STEPO
16Ø     Z=3.Ø*(FA-FB)/(B-A)+FAP+FBP
        Q=SQRT(Z*Z-FAP*FBP)
        SLAMDA=A+(FAP+Z+Q)*(B-A)/(FAP+FBP+2.Ø*Z)
        II=II+1
        DO 12Ø I=1,N
12Ø     XN(I)=X(I)+SLAMDA*S(I)
        CALL FTN(G,XN,FN,OBJ,N,M,R)
        NNFUN=NNFUN+1
        CALL GRADT(XN,GG,N,M,R,GRADN,FN)
        NNGRAD=NNGRAD+1
        SUM=Ø.Ø
        SUM1=Ø.Ø
        SUM2=Ø.Ø
        DO 13Ø I=1,N
```

```
      SUM=SUM+GRADN(I)*S(I)
      SUM1=SUM1+GRADN(I)**2
130   SUM2=SUM2+S(I)**2
      SUM1=SQRT(SUM1)
      SUM2=SQRT(SUM2)
      CONV=ABS(SUM/(SUM1*SUM2))
      IF(CONV.LE.EPSS)GO TO 140
      IF(II.GE.MAXIN)GO TO 140
      IF(SUM.GT.0.0)GO TO 150
      A=SLAMDA
      FA=FN
      FAP=SUM
      GO TO 160
150   B=SLAMDA
      FB=FN
      FBP=SUM
      GO TO 160
140   STEPO=SLAMDA
170   RETURN
      END

      SUBROUTINE GRADT(X,GG,N,M,R,GRAD,FF)
C     For evaluating the gradient of the phi-function using forward
C     finite difference method
      DIMENSION X(N),GRAD(N),GG(M)
      COMMON/COUNT/NFUN,NGRAD
      NGRAD=NGRAD+1
      DO 12 I=1,N
      XIP=X(I)
      X(I)=1.01*XIP
      IF(XIP.LE.0.1)X(I)=XIP+0.05
      CALL FTN(GG,X,FBP,OBJ1,N,M,R)
      X(I)=XIP
      GRAD(I)=(FBP-FF)/(0.01*X(I))
      IF(X(I).LE.0.1) GRAD(I)=(FBP-FF)/0.05
 12   CONTINUE
      RETURN
      END

      SUBROUTINE FTN(G,X,F,OBJ,N,M,R)
C     For providing the values of phi-,f, and gj, j=1,2,..,m
C     corresponding to any desging vector X
      DIMENSION X(N),G(M)
      COMMON/COUNT/NFUN,NGRAD
      NFUN=NFUN+1
C     OBJ=X(1)**3-6.0*X(1)**2+11.0*X(1)+X(3)
C     G(1)=X(1)**2+X(2)**2-X(3)**2
C     G(2)=4.0-X(1)**2-X(2)**2-X(3)**2
C     G(3)=X(3)-5.0
C     G(4)=-X(1)
C     G(5)=-X(2)
C     G(6)=-X(3)
C     F=OBJ
C     DO 10 I=1,M
C10   F=F-R/G(I)
```

```
C       RETURN
C       END
C
        OMT=Ø.18E+Ø8
        OMO=Ø.15E+Ø8
        ONA=Ø.6ØE+Ø4
        PA =Ø.9ØE+Ø3
        W  =Ø.27E+Ø4
C       I-Section equations
        A = (-2*X(3)*X(4)+2*X(1)*X(3)+X(2)*X(4))*(6*OMT*X(1))
C
        B = (2156Ø*X(5))*(X(2)*X(4)**3-2*X(3)*X(4)**3+2*X(3)*X(1)**3)
C
        C = (-2*X(3)*X(4)+2*X(1)*X(3)+X(2)*X(4))*(3*OMO*X(4))
C
        D = (2156Ø*X(5))*(2*X(1)*X(3)**3+X(2)*X(4)**3-2*X(3)*X(4)**3)
C
        E = OMT*6*X(1)
C
        T = Ø.198E+2*(X(2)*X(4)**3-2*X(3)*X(4)**3+2*X(3)*X(1)**3)
C
        Y = 3*OMO*X(4)
C
        H = Ø.198E+2*(2*X(1)*X(3)**3+X(2)*X(4)**3-2*X(3)*X(4)**3)
C
        O = 2156Ø*X(5)
C
        U = Ø.198E+2*(-2*X(3)*X(4)+2*X(1)*X(3)+X(2)*X(4))
C
        Q = (29.1*PA+11.5*W)*4287.5*12
C
        V = X(1)*X(2)**3-(X(1)-X(4))*(X(2)-2*X(3))**3
C
        S = 1.678E-4*(X(1)*X(2)**3-(X(1)-X(4))*(X(2)-2*X(3))**3)
C
        OBJ=-Ø.2318*X(3)*X(4)+Ø.2318*X(1)*X(3)+Ø.1159*X(2)*X(4)
       /+1Ø.564*X(5)
        G(1)=A/B+C/D-1
        G(2)=E/T+Y/H+O/U-1
        G(3)=Q/V-1
        G(4)=ONA/S-1
        G(5)=21Ø/X(1)-1
        G(6)=3ØØ/X(2)-1
        G(7)=4Ø/X(3)-1
        G(8)=3Ø/X(4)-1
        G(9)=3Ø/X(5)-1
C
C
C       Circular  section equations
C       A = (32*X(1)*X(2)-32*X(2)**2)*(OMT*X(1)+OMO*X(2))
C       B = 2156Ø*x(3)*(x(1)**4-(x(1)-2*x(2))**4)
C       C = Ø.514*OMT*X(1)
C       D = X(1)**4-(X(1)-2*X(2))**4
C       E = Ø.514*OMO*X(2)
C       T = X(1)**4-(X(1)-2*X(2))**4
```

```
C      Y = Ø.3466*X(3)
C      H = X(2)*(X(1)-X(2))
C      O = (29.1*PA+11.5*W)*4287.5*56
C      U = 22*X(2)*(X(1)-X(2))**3
C      Q = 7.915E-4*X(2)*(X(1)-X(2))**3
C      OBJ=Ø.364182*X(1)*X(2)-Ø.364182*X(2)**2+1Ø.564*X(3)
C      G(1)=A/B-1
C      G(2)=C/D+E/T+Y/H-1
C      G(3)=O/U-1
C      G(4)=ONA/Q-1
C      G(5)=318/X(1)-1
C      G(6)=6Ø/X(2)-1
C      G(7)=25/X(3)-1
C
C
C      Rectangular section equations
C      A = Ø.278E-5*OMT
C      B = X(2)**2*X(3)
C      WRITE(6,11Ø)B
C 11Ø FORMAT(/,2X,"B",E12.4)
C      C = Ø.139E-5*OMO
C      D = X(1)**2*X(3)
C      WRITE(6,12Ø)D
C12Ø  FORMAT(/,2X,"D",E12.4)
C      E = Ø.3Ø3*OMT
C      T = X(1)*X(2)**3
C      WRITE(6,13Ø)T
C13Ø  FORMAT(/,2X,"T",E12.4)
C      Y = Ø.152*OMO
C      H = X(2)*X(1)**3
C      WRITE(6,14Ø)H
C14Ø  FORMAT(/,2X,"H",E12.4)
C      O = Ø.1Ø89E+4*X(3)
C      U = X(1)*X(2)
C      Q =(29.1*PA+11.5*W)*4287.5*12
C      V = X(1)*X(2)**3
C      WRITE(6,15Ø)V
C15Ø  FORMAT(/,2X,"V",E12.4)
C      S = 1.678E-4*X(1)*X(2)**3
C      WRITE(6,16Ø)S
C16Ø  FORMAT(/,2X,"S",E12.4)
C      OBJ=Ø.1152*X(1)*X(2)+1Ø.564*X(3)
C      G(1)=A/B+C/D-1
C      G(2)=E/T+Y/H+O/U-1
C      G(3)=Q/V-1
C      G(4)=ONA/S-1
C      G(5)=15Ø/X(1)-1
C      G(6)=23Ø/X(2)-1
C      G(7)=25/X(3)-1
       F=OBJ
       DO 1Ø I=1,M
 1Ø    F=F-R/G(I)
       RETURN
       END
```

O U T P U T F O R R E C T A N G U L A R S E C T I O N

```
**********************************************************************
   STARTING VALUES FOR UNCONSTRAINED MINIMIZATION
   R=   .100000E+01  PF=   .12489391E+05  OBJ=   .12476304E+05
X(I) ARE,
  .30000000E+03   .35000000E+03   .36000000E+02

G(I) ARE,
 -.99998222E+00 -.62596328E+00 -.77104000E+00
 -.99722007E+00 -.50000000E+00 -.34285714E+00
 -.30555556E+00

      CUBIC INTERPOLATION STARTS
      S(I)=   -.7452E+00  -.6385E+00  -.1921E+00

      ITER=  1  STEPO=   .55810E+02
      PF=   .97271E+04  OBJ=   .96252E+04  NFUN=   133  NGRAD=    32
X(I) ARE,
  .25840895E+03   .31436297E+03   .25278973E+02

G(I) ARE,
 -.99996762E+00 -.66001840E+00 -.63315441E+00
 -.99554592E+00 -.41952475E+00 -.26836166E+00
 -.11035756E-01

      CUBIC INTERPOLATION STARTS
      S(I)=   -.7596E+00  -.6496E+00  -.3201E-01

      ITER=  2  STEPO=   .14621E+01
      PF=   .96763E+04  OBJ=   .95564E+04  NFUN=   175  NGRAD=    38
X(I) ARE,
  .25729834E+03   .31341319E+03   .25232175E+02

G(I) ARE,
 -.99996733E+00 -.65814031E+00 -.62821128E+00
 -.99548591E+00 -.41701916E+00 -.26614448E+00
 -.92015406E-02

      CUBIC INTERPOLATION STARTS
      S(I)=   -.7583E+00  -.6518E+00  -.8381E-02

      ITER=  3  STEPO=   .22654E+02
      PF=   .91300E+04  OBJ=   .85257E+04  NFUN=   263  NGRAD=    60
X(I) ARE,
  .24011932E+03   .29864727E+03   .25042314E+02

G(I) ARE,
 -.99996316E+00 -.61830428E+00 -.53955023E+00
 -.99440942E+00 -.37531058E+00 -.22986070E+00
 -.16897000E-02

      CUBIC INTERPOLATION STARTS
      S(I)=   -.1712E-01  -.1374E-01   .9998E+00

      ITER=  4  STEPO=   .15581E+01
```

```
C     SENSITIVITY ANALYSIS
C
C     PROGRAM TO DETERMINE HOW SENSITIVE THE BEHAVIOR
C     QUANTITIES ARE TO CHANGES IN DESGN VARIABLES
C     FOR CONCRETE TRANSMISSION POLES.(Msc thesis)
C
C        By UGA,B.W.(1991)
C        Dept.of civil engg.A.B.U.Zaria
C
C     THE ANALYSIS IS CONDUCTED AT THE FINAL OPTIMUM POINT
C
      PROGRAM SENS(ALBU,OUTPUT,TAPE5=ALBU,TAPE6=OUTPUT)
      DIMENSION X(5),G(5)
      COMMON OMT,OMO,ONA,PA,W
      X(1)=318.665
      X(2)=444.83
      X(3)=87.645
      X(4)=30.1888
      X(5)=31.478
      OBJ=7749.7
      OMT=0.18E +08
      OMO=0.15E+08
      ONA=0.60E+04
      PA=0.90E+03
      W=0.27E+04
      GN=-0.99993575
      CALL BEQUAN(GC,GD,GB)
      WRITE(6,10)OBJ,GN,GC,GD,GB
   10 FORMAT(8X,"OPTIMUM DESIGN POINT",//,14X,"OPT.WT=",E15.8,/,14X,
     /"TENSILE STRESS=",E15.8,/,14X,"COMP.STRESS=",E15.8,/,14X,
     /"DEFLEXION=",E15.8,/,14X,"BUCKLING LOAD=",E15.8)
      DO 30 K=-50,51,5
      VA=(1+K*0.01)*X(5)
      A = (2*X(1)*X(3)-2*X(3)*X(4)+X(2)*X(4))*(6*OMT*X(1))
      B = (21560*VA)*(X(2)*X(4)**3-2*X(3)*X(4)**3+2*X(3)*X(1)**3)
      C = (-2*X(3)*X(4)+2*X(1)*X(3)+X(2)*X(4))*(3*OMO*X(4))
      D = (21560*VA)*(2*X(1)*X(3)**3+X(2)*X(4)**3-2*X(3)*X(4)**3)
      E = OMT*6*X(1)
      T = 0.198E+2*(X(2)*X(4)**3-2*X(3)*X(4)**3+2*X(3)*X(1)**3)
      Y = 3*OMO*X(4)
      H = 0.198E+2*(2*X(1)*X(3)*3+X(2)*X(4)**3-2*X(3)*X(4)**3)
      O = 21560*VA
      U = 0.198E+2*(-2*X(3)*X(4)+2*X(1)*X(3)+X(2)*X(4))
      Q = (29.1*PA+11.5*W)*4287.5*12
      V = X(1)*X(2)**3-(X(1)-X(4))*(X(2)-2*X(3))**3
      S = 1.678E-4*(X(1)*X(2)**3-(X(1)-X(4))*(X(2)-2*X(3))**3)
      VOBJ=OBJ-(-0.2318*X(3)*X(4)+0.2318*X(1)*X(3)+0.1159*X(2)*X(4)
     /+10.564*VA)
      G(1)=GN-(A/B+C/D-1)
      G(2)=GC-(E/T+Y/H+O/U)
      G(3)=GD-(Q/V)
      G(4)=GB-S
C     CALCULATION OF PERCENTAGE CHANGE
      POBJ=(VOBJ/OBJ)*(-100)
      PT=(G(1)/GN)*100
```

44683

```
      PC=(G(2)/GC)*(-1ØØ)
      PD=(G(3)/GD)*(-1ØØ)
      PB=(G(4)/GB)*(-1ØØ)
      WRITE(6,25)K
      WRITE(6,2Ø)POBJ,PT,P,PD,PB
 2Ø   FORMAT(//,2X,"PERCENTAGE CHANGE IN BEHAVIOR QUANTITIES",//,
     /2X,"WEIGHT.",2X,"TENSILE ST.",2X,"COMP.ST.",
     /2X,"DEFLEXION.",2X,"BUCKLING LD.",/(2X,6F8.3))
 25   FORMAT(//,2X,"PERCENTAGE COMPUTED FOR",/(2X,513))
 3Ø   CONTINUE
      STOP
      END

      SUBROUTINE BEQUAN(GC,GD,GB)
      DIMENSION X(5),G(5)
      COMMON OMT,OMO,ONA,PA,W
      X(1)=318.665
      X(2)=444.83
      X(3)=87.645
      X(4)=3Ø.1888
      X(5)=31.478
      E = OMT*6*X(1)
      T = Ø.198E+2*(X(2)*X(4)**3-2*X(3)*X(4)**3+2*X(3)*X(1)**3)
      Y = 3*OMO*X(4)
      H = Ø.198E+2*(2*X(1)*X(3)**3+X(2)*X(4)**3-2*X(3)*X(4)**3)
      O = 2156Ø*X(5)
      U = Ø.198E+2*(-2*X(3)*X(4)+2*X(1)*X(3)+X(2)*X(4))
      Q = (29.1*PA+11.5*W)*4287.5*12
      V = X(1)*X(2)**3-(X(1)-X(4))*(X(2)-2*X(3))**3
      S = 1.678E-4*(X(1)*X(2)**3-(X(1)-X(4))*(X(2)-2*X(3))**3)
      GC=E/T+Y/H+O/U
      GD=Q/V
      GB=S
      RETURN
      END
```

TYPICAL SENSITIVITY ANALYSIS OUTPUT

OPTIMUM DESIGN POINT

(X1) OBJECTIVE .77697006E+06
[illegible] .13510414E+00
[illegible] .13774081E+02
DEFLEXION= .20565511E+01
BUCKLING LOAD= .37586998E+07

PERCENTAGE COMPUTED FOR
5

PERCENTAGE CHANGE IN BEHAVIOR QUANTITIES

WEIGHT	TENSIL ST	COMP.ST.	DEFLEXION	BUCKLING ST
-4.176	-23.639	0.000	-21.923	-4.868

PERCENTAGE COMPUTED FOR
10

PERCENTAGE CHANGE IN BEHAVIOR QUANTITIES

WEIGHT	TENSIL ST	COMP.ST.	DEFLEXION	BUCKLING ST
-8.353	-44.931	0.000	-16.515	-9.736

PERCENTAGE COMPUTED FOR
15

PERCENTAGE CHANGE IN BEHAVIOR QUANTITIES

WEIGHT	TENSIL ST	COMP.ST.	DEFLEXION	BUCKLING ST
-12.530	-64.207	0.000	-11.565	-14.604

PERCENTAGE COMPUTED FOR
20

PERCENTAGE CHANGE IN BEHAVIOR QUANTITIES

WEIGHT	TENSIL ST	COMP.ST.	DEFLEXION	BUCKLING ST
-16.707	-81.737	0.000	-7.020	-19.472

PERCENTAGE COMPUTED FOR
25

PERCENTAGE CHANGE IN BEHAVIOR QUANTITIES

WEIGHT TENSIL ST COMP.ST. DEFLEXION BUCKLING ST
-20.884 -97.746 0.000 -2.830 -24.340

PERCENTAGE COMPUTED FOR
30

PERCENTAGE CHANGE IN BEHAVIOR QUANTITIES

WEIGHT TENSIL ST COMP.ST. DEFLEXION BUCKLING ST
-25.061-112.423 0.000 1.045 -29.208

PERCENTAGE COMPUTED FOR
35

PERCENTAGE CHANGE IN BEHAVIOR QUANTITIES

WEIGHT TENSIL ST COMP.ST. DEFLEXION BUCKLING ST
-29.238-125.926 0.000 4.637 -34.076

PERCENTAGE COMPUTED FOR
40

PERCENTAGE CHANGE IN BEHAVIOR QUANTITIES

WEIGHT TENSIL ST COMP.ST. DEFLEXION BUCKLING ST
-33.415-138.389 0.000 7.979 -38.944

PERCENTAGE COMPUTED FOR
45

PERCENTAGE CHANGE IN BEHAVIOR QUANTITIES

WEIGHT TENSIL ST COMP.ST. DEFLEXION BUCKLING ST
-37.592-149.928 0.000 11.093 -43.812

PERCENTAGE COMPUTED FOR
50

PERCENTAGE CHANGE IN BEHAVIOR QUANTITIES

WEIGHT TENSIL ST COMP.ST. DEFLEXION BUCKLING ST
-41.769-160.641 0.000 14.004 -48.680

APPENDIX 5

SPECIFIED CHARACTERISTIC STRENGTH

Specified characteristic strengths of prestressing wire

Nominal size	Specified characteristic strength $A_{ps} f_{pk}$	Nominal cross-sectional area A_{ps}
mm	kN	mm²
2	6.34	3.14
2.65	10.3	5.5
3	12.2	7.1
3.25	14.3	8.3
4	21.7	12.6
4.5	25.7	15.9
5	30.8	19.6
7	60.4	38.5

Specified characteristic strengths of prestressing strand

Number of wires	Nominal size	Specified characteristic strength $A_{ps} f_{pk}$	Nominal cross-sectional area A_{ps}
	mm	kN	mm²
7	6.4	44.5	24.5
	7.9	69.0	37.4
	9.3	93.5	52.3
	10.9	125	71.0
	12.5	165	94.2
	15.2	227	138.7
19	18	370	210
	25.4	659	423
	28.6	823	535
	31.8	979	660

Specified characteristic strengths of prestressing bars

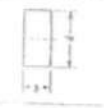
Nominal size	Specified characteristic strength $A_{ps} f_{pk}$	Nominal cross-sectional area A_{ps}
mm	kN	mm²
*20	325	314
22	375	350
*25	500	491
28	625	615
*32	800	804
35	950	961
*40	1250	1257

APPENDIX 6

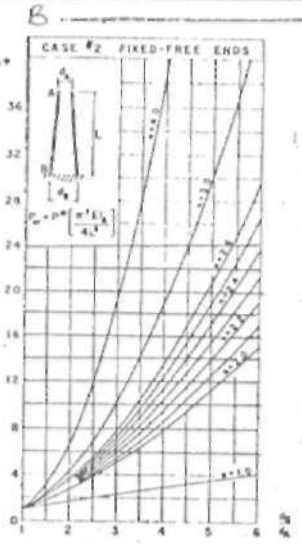
SHAPE FACTOR AND ALLOWABLE STRESSES

A

SHAPE FACTOR

SHAPE	SHAPE FACTOR n
 Wide-flange or I-section Constant dimensions b, t_w, t_f Varying depth d Buckling about horizontal axis	n = 2.1 to 2.6
 Closed box section Constant dimensions b, t_w, t_f Varying depth d Buckling about horizontal axis	n = 2.1 to 2.6
 Solid, rectangular section Constant width b Varying depth d Buckling about horizontal axis	n = 3
 Solid, rectangular section Constant width b Varying depth d Buckling about vertical axis	n = 1
 Open-web section Constant dimensions b, t_f Varying depth d Buckling about horizontal axis	n = 2
 Tower section Constant areas concentrated near corners Varying dimension d	n = 2
 Solid, circular section Varying diameter d	n = 4
 Solid, square section Varying dimension d	n = 4

B



COLUMN, FIXED-FREE ENDS

BUCKLING LOADS

C

Allowable compressive stresses at transfer

Nature of stress distribution	Allowable compressive stresses f_{ct}
Triangular or near triangular distribution of prestress	$0.5f_{ci}$
Uniform or near uniform distribution of prestress	$0.4f_{ci}$

Table 32. Compressive stresses in concrete for serviceability limit states

Nature of loading	Allowable compressive stresses f_{ce}
Design load in bending	$0.33f_{cu}$ In continuous beams and other statically indeterminate structures this may be increased to $0.4f_{cu}$ within the range of support moments
Design load in direct compression	$0.25f_{cu}$

APPENDIX 7

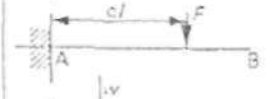
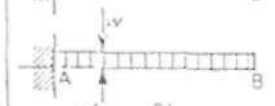
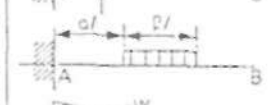
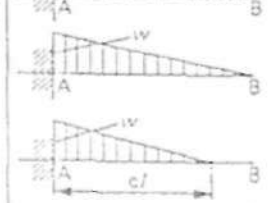
PROPERTIES OF CONDUCTORS

		Number of strands	Diameter of wire mm	Diameter of wire in	Overall diameter of conductor mm	Overall diameter of conductor in	Strength N	Strength lb	Weight kg/m	Weight lb/ft
Typical conductors	Copper	3	2.64	0.104	5.59	0.22	6,900	1,550	0.149	0.1
		3	2.74	0.108	9.91	0.39	18,900	4,250	0.446	0.3
		7	3.86	0.152	11.7	0.46	32,200	7,240	0.744	0.5
		7	4.57	0.180	13.7	0.54	44,100	9,920	1.04	0.7
		7	5.18	0.204	15.5	0.61	55,650	12,510	1.34	0.9
		19	3.66	0.144	18.3	0.72	77,300	17,380	1.79	1.2
	Steel-cored aluminium (*S* = steel; *A* = aluminium)	7	2.39	0.094	7.11	0.28	9,400	2,115	0.104	0.07
		7	4.09	0.161	12.2	0.48	25,300	5,690	0.312	0.21
		6*A* / 7*S*	5.28*A* / 1.75*S*	0.208*A* / 0.069*S*	15.7	0.62	40,550	9,120	0.491	0.33
		37	2.79	0.110	19.6	0.77	77,200	17,350	0.833	0.56
		37	3.18	0.125	22.4	0.88	98,300	22,100	1.09	0.73
		37	3.71	0.146	25.4	1.00	130,200	29,270	1.46	0.98
	Aluminium	3	3.35	0.132	7.37	0.29	4,350	975	0.073	0.05
		7	3.78	0.149	11.4	0.45	12,900	2,900	0.223	0.15
		7	4.90	0.193	14.7	0.58	20,300	4,570	0.357	0.24
		19	3.53	0.139	17.8	0.70	29,500	6,630	0.506	0.34
		19	3.99	0.157	20.1	0.79	37,000	8,310	0.655	0.44
		19	4.65	0.183	23.4	0.92	48,500	10,920	0.893	0.60

Cantilevers beams

Total load on cantilever = F. Length of cantilever = l.
Reaction at A = maximum shearing force = F.
Bending moment at A = (tabulated or calculated coefficient k_A) $\times F \times l$.
Bending moment at B = zero
Maximum deflection at B = (tabulated or calculated coefficient a_B) $\times \frac{Fl^3}{EI}$

Cantilevers

Loading	Total load F	Bending-moment coefficient k_A	Deflection coefficient a_B
	F	-1.0	$\frac{1}{3}$
	F	$-\alpha$	$\frac{\alpha^2}{6}(3-\alpha)$
	wl	$-\frac{1}{2}$	$\frac{1}{8}$
	$w\beta l$	$-\left(\alpha+\frac{1}{2}\beta\right)$	$\frac{1}{24}(-4\alpha^3+12\alpha^2-6\alpha^2\beta+12\alpha\beta-4\alpha\beta^2+4\beta^2-\beta^3)$
	$\frac{1}{2}wl$	$-\frac{1}{3}$	$\frac{1}{15}$
	$\frac{1}{2}w\alpha l$	$-\frac{\alpha}{3}$	$\frac{\alpha^2}{60}(5-\alpha)$
	$\frac{1}{2}wl$	$-\frac{2}{3}$	$\frac{11}{60}$
	$\frac{1}{2}wnl$	$-\left(1-\frac{\alpha}{3}\right)$	$\frac{1}{60}[20-10\alpha+\alpha^2]$
	$\frac{2}{3}wl$	$-\frac{1}{2}$	$\frac{7}{90}$
	$\frac{1}{2}wl$	$-\frac{1}{2}$	$\frac{11}{90}$