FAILURE COST ANALYSIS OF HUMAN ERRORS IN REINFORCED CONCRETE BEAM CONSTRUCTION

BY

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DECLARATION

I hereby declare that this thesis is my own research work and has been composed by me. It has not been previously presented for a higher degree.

All sources of information which are not originally mine are specifically acknowledged by references.



Joshua Sumbele

January, 1997.

CERTIFICATION

This project thesis titled "FAILURE COST ANALYSIS OF HUMAN ERRORS IN REINFORCED CONCRETE BEAM CONSTRUCTION" by Joshua Sumbele, meets the regulations governing the award of the degree of Master of Science (Structural Engineering) of Ahmadu Bello University, and is approved for its contribution to knowledge and literary presentation.

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DEDICATION

To my Daddy.

Mr. ABEL NKANE SUMBELE

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loving companionship I have known these past few years. With her around, the otherwise arduous task of carrying out this research became so much more enjoyable.

ABSTRACT

This study centres around the evaluation of the expected total cost of reinforced concrete beams designed to BS 8110 (1985) when gross errors *are* accounted for. Beam performance functions are obtained for bending, shear and torsion. The performance function for a beam performing in combined bending and torsion is adopted from CP 110 (1972), where it is explicitly given. Due to the multiplicity of uncertainties surrounding the performance of beams, they are only realistically assessed in terms of a probability. In this study, the reliability index, ft, and hence the probability of failure P_f, is used as a measure of beam performance. Consequently, the First Order Second Moment integration technique is employed to determine the values of ft or P_f implicit in the design criteria.

Since it is practically impossible to quantify the whole range of human errors likely to *occur* in reinforced concrete beam construction, consideration is therefore limited to workmanship error involving the transverse and longitudinal positioning of beam supports. In addition, the effects of perturbations in the design values of basic variables on the performance and the expected total cost of beams are examined.

The results show that the BS 8110 (1985) design criteria give fairly consistent and non-conservative designs. From structural design considerations only, the code's requirements are found to be

adequate.

The sensitivity analysis also shows that gross errors in the value of the effective depth, d, lead to the greatest associated failure cost among other design variables. On the other hand, the strength of concrete, f_{cu}, is found to show very little sensitivity to gross errors as far as the design equations are concerned. Associated failure costs *are* thus minimal. This assessment shows that for beams failing in bending, shear, torsion and combined bending and torsion, expected failure cost surcharges do not go higher than about 13%, 47%, 36.5% and 24.4% of the initial cost respectively.

The results of this investigation are only indicative trends of the BS 8110 (1985) design requirements since the scope of the work precludes experimental details that may give the actual distributions of the design parameters. It is therefore indicative from economic considerations only that the criteria for shear and torsion be reviewed.

TABLE OF CONTENTS

		Page
Title page		i
Declaration		ii
Certification		iii
Dedication		iv
Acknowledgments		v
Abstract		vii
Table of contents		ix
List of tables		xii
List of figures		xiv
Notations		xv
CHAPTER ONE		1
INTRODUCTION		1
1.1	General Introduction	1
1.2	Objectives of Study	4
CHAPTER TWO		6
LITERATURE REVIEW		6
2.1	The Causes and Effects of Human Errors	6
2.1.1	Human Error in Engineering	6
2.1.2	Socio-Political Aspects of Failure	16
2.1.3	Accidents and Traditional Beliefs	19

2.1.4	Methods of Structural Reliability Analysis	22
2.2	Problem Statement	23
2.3	Methodology	24
2.3.1	Theoretical Background To FORM5	24
2.3.1.1	Introduction	24
2.3.1.2	Determination of The Reliability Index β	26
2.3.1.3	Transformation of Non-normal Basic Variables	30
2.3.1.4	Transformation of Correlated Basic Variables	31
2.3.2	Evaluation of Expected Total Costs	33
CHAPTER THREE		35
LIMIT STATE FUNCTIONS FOR SINGLY REINFORCED CONCRETE BEAMS		35
3.0	Introduction	35
3.1	Establishment of Perfomance Functions	36
3.1.1	Bending Limitation	36
3.1.2	Shear Limitation	39
3.1.3	Torsion Limitation	40
3.1.4	Combined Bending and Torsion Limitation	44
3.2	Uncertainties In The Basic Design Variables	45
3.2.1	Variability In The Strength of Reinforcing steel f_y	47
3.2.2	Variability In The Strength of Concrete, f_{cu}	48
3.2.3	Variability In Member Geometry	48
3.2.4	Variability In The Area of Steel Reinforcement, A_s	49
3.2.5	Uncertainties In Loading	49
3.3	Summary of Statistical Characters of Basic Variables	50

CHAPTER FOUR 52

FAILURE PROBABILITY LEVELS AND COST ANALYSIS 52

4.0 Introduction 52

4.1 Implied Safety Levels 54

4.1.1 Bending Predictions 54

4.1.2 Shear Predictions 56

4.1.3 Torsion Predictions 57

4.1.4 Combined Bending and Torsion Predictions 59

4.2 Effects Of Gross Errors On Beam Performance 63

4.2.1 Failure In Bending 63

4.2.2 Failure In Shear 70

4.2.3 Failure In Torsion 76

4.2.4 Failure In Combined Bending and Torsion 80

4.3 Comparative Cost Analysis 87

4.4 Failure Cost Surcharges on The Expected Total Cost 93

4.5 Error Bounds For Code Adequacy 100

CHAPTER FIVE 104

SUMMARY AND CONCLUSION 104

REFERENCES 110

LIST OF TABLES

		Page
2.1	Multiple Causes For Structural Accidents.	14
3.1	Basic Random Variables In Bending failure and Their Statistical Characters.	51
3.2	Basic Random Variables In Shear Failure and Their Statistical Characters.	51
3.3	Basic Random Variables In Torsional Failure and Their Statistical Characters.	52
3.4	Basic Random Variables In Combined Bending and torsion Failure.	52
4.1	Implied β Levels For Varying Load Ratio ó, in Bending.	55
4.2	Implied β Levels For Varying Load Ratio ó, In Shear.	56
4.3	Implied β Levels For Varying Load Ratio ó, In Torsion.	58
4.4	Implied β Levels For Varying Load Ratio ó, In Combined Bending and Torsion.	60
4.5	Effects Of Gross Errors In d On Bending Failure Cost.	64
4.6	Effects Of Gross Errors In f_{cu} on Bending Failure Cost.	65
4.7	Effects Of Gross Errors In f_y or A_s On Bending Failure Cost.	66
4.8	Effects Of Gross Errors In L On Bending Failure Cost.	67
4.9	Effects Of Gross Errors In d, f_{yv} or A_{sv} On Shear Failure Cost.	71
4.10	Effects Of Gross Errors In S_v On Shear Failure Cost.	72
4.11	Effects Of Gross Errors In γ On Shear Failure Cost.	73
4.12	Effects Of Variation In e On Shear Failure Cost.	74
4.13	Effects Of Gross Errors In x, y, f_{yv} or A_{sv} On Torsional Failure Cost.	77
4.14	Effects Of Gross Errors In S_v or variation In e On Torsional Failure Cost.	78

4.15 Effects Of Gross Errors In d On The Failure Cost In Combined Bending and Torsion. 82

4.16 Effects Of Gross Errors In A_s or f_y On The Failure Cost In Combined Bending and Torsion. 83

4.17 Effects Of Gross Errors In x, y, A_{sv} or f_{yv} On The Failure Cost In Combined Bending and Torsion. 84

4.18 Effects Of Gross Errors In L On The Failure Cost In Combined Bending and Torsion. 85

4.19 Effects Of Gross Errors In S_v or Variation In e On The Failure Cost In Combined Bending and Torsion. 86

4.20 Effects Of Gross Errors In f_{cu} On The Failure Cost In Combined Bending and Torsion. 87

4.21 Total Cost Ratios C' With Gross Errors In L In Failure Mode I. 94

4.22 Total Cost Ratios C' With Gross Errors In L In Failure Mode IV. 95

4.23 Total Cost Ratios C" With Variation In e In Mode II. 95

4.24 Total Cost Ratios C" with Variation In e In Mode III. 96

4.25 Total Cost Ratios C" With Variation In e In Mode IV. 96

LIST OF FIGURES

		Page
2.1	Human Performance Function	10
3.1	Displaced Supports on A Floor-beam Gridwork.	36
3.2	Error In The Span of A Beam.	37
3.3	Torsional Reinforcement.	41
3.4	n Links Cut By One Diagonal Tension Crack.	41
3.5	Bending and Torsional Moments On Section.	44
4.1	Implied Safety Levels With Varying Load Ratios.	61
4.2	Sensitivity Of Basic Variables To Gross Errors In The Limit State For Bending.	68
4.3	Sensitivity Of Basic Variables To Gross Errors In The Limit State For Shear.	75
4.4	Sensitivity Of Basic Variables To Gross Errors In The Limit State For Torsion.	79
4.5	Sensitivity Of Basic Variables To Gross Errors In The Limit State For Combined Bending and Torsion.	81
4.6	Expected Total To Initial Cost Ratios With Gross Errors In d.	88
4.7	Expected Total To Initial Cost Ratios With Gross Errors In f_{cu}.	90
4.8	Expected Total To Initial Cost Ratios Variation In e.	91
4.9	Expected Total To Initial Cost Ratios With Gross Errors In L.	92
4.10	Total Cost Ratios For Gross Errors In L.	98
4.11	Total Cost Ratios With Variation In e.	99

NOTATION

A	-	Area of floor space acting on beam.
A_s	-	Cross sectional area of main steel reinforcement.
A_{sv}	-	Cross sectional area of shear reinforcement.
b_f	-	Breadth of Flange.
b_w	-	Width of web.
C^*	-	Ratio of expected total to initial cost of beam.
C'	-	Total cost ratio for gross errors in L.
C''	-	Total cost ratio with variation in e.
C_f	-	Cost of failure of beam.
C_I	-	Initial cost of beam.
C_T	-	Total cost of beam.
COV	-	Coefficient of variation.
C_X	-	Covariance matrix of basic variables x_i.
D	-	Distance from the origin of the reduced variate space to the failure surface.
d	-	Effective depth of beam.
e	-	Transverse error in alignment of beam supports.
E(X)	-	Expected value of random variable X.
F_{cc}	-	Compressive force in concrete.
f_{cu}	-	Characteristic strength of concrete.
F_{ST}	-	Tensile force in steel.
$F_X(x)$	-	Joint distribution function of X_i.
$f_X(x)$	-	Joint pdf of X_i.

f_y	-	Characteristic strength of main steel reinforcement.
f_{yv}	-	Characteristic strength of shear reinforcement.
G	-	Ultimate dead load.
G(X)	-	Limit state function.
G'	-	Gradient function.
L	-	Span of beam.
M_a	-	Applied span moment in bending.
M_u	-	Moment Capacity.
N_u	-	Applied direct thrust or pull on beam.
N_{uz}	-	Capacity of beam to resist direct compressive or tensile stresses.
P	-	Total ultimate dead and live loads on beam.
P[ ]	-	Probability of an event.
P_f	-	Probability of failure.
Q	-	Ultimate live load.
S	-	Depth of compressive stress block within flange.
S_v	-	Spacing of shear links.
T_a	-	Applied torsional moment.
T_u	-	Torsional capacity of beam.
V	-	Applied shear force in beam.
x	-	length of horizontal leg of links.
X_i	-	Basic variable.
X_i'	-	Reduced variate.
y	-	Length of vertical leg of links.
Z	-	Lever arm.

α	-	Ratio of live to dead loads.
β	-	Reliability index.
δ	-	Standard deviation of observed data.
Δ	-	Error in prediction of mean values.
γ	-	Ratio of intrinsic shear strength of concrete to the applied shear stress.
μ_X	-	Mean value of basic variable X.
$\Phi(.)$	-	Standard normal integral.
ρ	-	Correlation coefficient.
σ_x	-	Standard deviation of basic variable x.
λ	-	Lagrange's multiplier.
v	-	Applied shear stress.
v_c	-	Shear strength of concrete.
Ω	-	Total uncertainty of a variable.

CHAPTER ONE

INTRODUCTION

1.1 General Introduction

In the risk assessment exercise, fundamental questions on safety are bound to arise like 'How safe is 'safe enough'?', 'what financial contribution should make a structure safe enough?', and 'what are the consequences of failure?'. The concern for the safety of a structure arises from the realization by engineers of the numerous handicaps that have plagued the building industry literally from the dawn of time. This is chiefly manifested in the various ways the performance of a structure is affected by human involvement in the building process. Humans are generally known to exhibit an inherent fallible nature. Therefore it should be the concern of safety analysts to determine or measure the extent to which human involvement affects the safety of a structure. Moreover, since economic assessments form such an integral part of all contemporary engineering endeavours, the question of safety should decidedly be married to the economic implications of structural failure.

In reinforced concrete works, as in other forms of construction, the building process can be divided into three main parts: The design, the construction and the use of a structure. In all these, humans are actively involved with a myriad of ways in which man's fallibility could manifest. In this investigation all

of these ways shall summarily be referred to as human or gross error. In recent years, more and more interest is being generated towards identifying these errors and attempting to quantify their effect on the structure, with a view to developing more comprehensive and scientifically-based structural codes, embracing the probabilistic nature of design parameters including the uncertain effects of human factors on structural performance.

The importance of minimising the errors associated with a structure cannot be over-emphasized. As such errors only lead to a deviation from a desired condition, the attention paid to it should reflect the impact such errors are likely to have on all parties involved in the building process. The ubiquitousness and unpredictability of gross errors place enormous responsibilities on engineers and contractors alike, charged with the responsibility of ensuring proper performance of a structure. Society expects nothing less from them!

Engineers are frequently prone to errors in analysis and design, in evaluating environmental conditions as well as in the use of mathematical formulations to describe relevant phenomena. Generally, mathematical formulations are employed in an atmosphere of uncertainty, specifically characterized by the following [Melchers, 1987]:

* Statistical uncertainty
* Physical uncertainty
* Phenomenological uncertainty
* Modelling uncertainty
* Prediction uncertainty
* Decision uncertainty.

Uncertainties resulting from random environmental conditions can readily be felt within the design and construction stages of the building process e.g., soil type, wind conditions etc. Further, at the level of construction, more uncertainties are generated as a result of inclemency in the engineering 'climate' surrounding the project. Such adverse effects on the project have been studied broadly under the following headings: Political climate, Financial climate, Professional climate and Industrial climate with rather fuzzy distinction between them [Pugsley, 1974]. Thus with so much uncertainty involved in the building process, the assurance of the performance of a structure is realistic only in terms of a probability, in this case, its probability of failure [Blockley, 1980]. Failure is defined in this study as the violation of a prescribed limit state, where the limit state is the requisite performance criterion.

The evaluation of the performance of a structure subject to certain preconditions, in terms of its probability of failure, can be performed in several approved ways. For more complex systems, not necessarily of engineering bearing, quite reliable overall probabilities can be arrived at using probabilistic methods involving event-tree or fault-tree analyses, when sufficient data relating to the failure of its components are available [Mazur, 1980]. For characteristically one-off Civil Engineering structures however, where the evaluation of the performance of each member separately prevails, other techniques become more appropriate e.g.

the First Order Reliability Method (FORM), the Second Order Reliability Method (SORM) or simulation techniques. The technique adopted usually depends on the nature of the problem, the assumptions that need to be made and the quantity and quality of available information on the problem [Ayyub and Halder, 1984].

Engineering decision-making is greatly simplified when all outcomes are expressed in the same units permitting a global summation of costs and benefits. This permits the engineer to, for instance, balance safety with economy. On the whole, economy in a building process is achieved through evaluating the cost of alternatives, carrying out cost-benefit analyses between alternatives and selecting that alternative that proffers the maximum of benefits against total cost [Riggs, 1980].

The benefits of a structure in a wider sense encompass the credits that accrue to the client, but in the context of structural safety, it can be understood in the sense of proper performance of the structure. Costs on the other hand, as proposed by value analysts can be classified as being tangible or intangible [Afolayan, 1994]. Tangible costs are those to which some monetary value can be assigned while intangible costs are the contrary. For simplification purposes, these costs shall be expressed as a total cost, being the sum of the initial cost and the cost of failure.

1.2 Objectives of Study

Advanced structural requirements and good engineering practice

emphasize the need for improved structural efficiency. It therefore prevails upon proper workmanship that prescriptions of the design team are adhered to strictly. Knowing that this is not always the case due to faulty workmanship on reinforced concrete structures, an attempt is made at determining the probability of failure of a beam element due to measured human error of a particular nature.

The associated levels of reliability of such beam elements performing adequately under the limit states of bending, shear, torsion and combined bending and torsion is also investigated. Having the possibility of probabilistic assessment of the performance of a beam in these limit states has led to the search for answers to the following questions:

1. Which basic variables are particularly sensitive to the design and construction of beam elements?
2. What are their variation performance characteristics?
3. Which of the aforementioned failure modes incurs the most overall cost, taking into account the effect of human errors?
4. How does human error affect the total cost of a beam?

CHAPTER TWO

LITERATURE REVIEW

2.1 The Causes and Effects of Human Errors

Before undertaking the task of determining the effects of human errors on the performance of a reinforced concrete beam, it is desirable that a general appraisal of the concept and manifestation of human errors on construction be conducted. An overview of the effect of accidents in its social, political and industrial context, considered as the extremities of human errors on construction, with a view to assessing the effects of associated failure on all parties involved in an engineering project, is undertaken. Lastly, mention is made of the various methods of estimating the structural performance of the member in question followed by a qualitative analysis of the chosen method.

2.1.1 Human Error in Engineering

A number of surveys on recorded cases of structural failure show that human error is involved in the majority of these cases. Therefore no formal assessment of the safety of a structure can be complete without taking into consideration the effect of human error on it [Melchers, 1987]. In the past however, because of the subtlety of the presentation of human errors and the difficulty in their determination, there was a bias on the part of engineering scientists towards problems with marked repeatabilities and which

could be highly tested. Also, because structural engineering is a commercial endeavour where fiduciary relations are at stake, the acquisition of data on failure or near misses has proved to be quite a difficult, if not an impossible task [Blockley, 1980]. The result of these is a dearth of relevant information and data on human errors, with what little there is existing mainly in qualitative form [Melchers, 1987].

The various ways by which human factors are introduced into the building process are so vast that for convenience, they shall simply be categorized into [Melchers, 1987]:

(a) Human Errors

(b) Human Intervention

Human intervention generally describes all formal and informal strategies comprising both facilitative and control measures, adopted in order to curtail, minimize or dissuade the propagation of errors in engineering projects. Some of these measures include continuing professional education, simplification of complex tasks, personnel selection, quality control, self-checking, external checking and inspection, legal sanctions etc. A coordinated execution of all these strategies results in what is usually referred to as Quality Assurance (QA) [Melchers, 1987].

Errors in Engineering are those events associated with the design, construction and use of a structure that have the potentials of initiating or propagating a failure mechanism [Thoft-Christensen and Baker, 1982]. They are a direct consequence of

human involvement in the building process, and, since at all stages of this process, decisions on what to do, what is done and how it is done are effected by humans, it could be said that all error is human error [Blockley, 1980].

Human error can generally be broken down into [Melchers,1987]:

(a) Gross human errors

(b) Human variability in task performance

Gross human errors are essentially of two categories:

(i) Those occurring within the normal processes of the design, construction and use of the structure as a result of the engineering team's, the contractor's or the tenant's carelessness or negligence. Failure resulting from such is usually in a mode against which the structure is designed.

(ii) Those occurring as a result of ignorance of the fundamental behaviour of the structure, on the part of the engineer or the engineering profession, for which failure occurs in an unpredictable, previously unknown manner.

Gross human errors of category (i) represent those against which human intervention measures are likely to act. Errors of category (ii) cannot be recognized beforehand since it might not be clear to all parties concerned that an error is being committed in the first place.

Gross human error has been classified in several ways among which are [Thoft-Christensen and Baker, 1982, Blockley, 1980, Melchers, 1987]:

* Nature of error

* Type of failure associated with error

* Consequence of failure arising from error

* Responsibility for causing error

* Responsibility for not detecting error

* Subtlety of error

* Inadvertence in the error event

* Possibility of analytic representation etc.

None of these classes can in themselves be all-including, but usually within each class and between classes, there is much overlap [Blockley, 1980].

Human variability in task performance has been found to manifest on all stages of the building process. It is known to result from intrinsic responses to the levels of arousal in different individuals. It has been proposed that human beings perform best at an optimum level of arousal beyond which or below which the performance level deteriorates [Melchers, 1987]. This phenomenon can be presented diagrammatically as in Fig. 2.1. Also, at peak arousal, the performance of different individuals differ, a phenomenon to which an analogy could be seen in the results of IQ tests carried out on a class of school kids.

Within the building process, variability in task performance is strongly evident during construction when, it is suggested, the

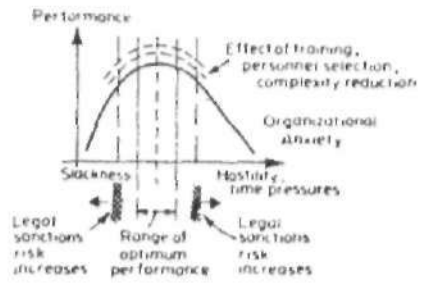


Fig. 2.1 Human Performance Function [Melchers, 1987].

performance of the engineer or the worker on site is governed by the engineering `climate' surrounding the project [Pugsley, 1974]. A close study of some major structural accidents within the past century provides clues to a number of important factors which could clearly be held responsible for the adverse atmosphere in which the engineering or the construction teams might be operating.

1. Public Pressures: An instance where public attitude to an engineering project resulted in catastrophe occurred when, by popular demand, fuelled by ostentatiousness, the British airship, the R.101, on her maiden flight to India, crashed in France in 1930. The engineering team had been pressured into hurriedly completing acceptance trials in order for her to take the air minister to India for the imperial conference.

2. Professional Climate: Adversity in this domain can be demonstrated by a paucity of knowledge within the profession or even an absence of rapport between engineers. While it is reiterated that the engineering team should exploit the state-of-the-art of the profession, engineering decisions are perhaps only a reflection of the degree of specialization and experience of the engineer. A survey carried out on 120 recorded cases of building failures show that over 60% of these cases were as a result of a lack of experience on the part of the personnel concerned [Thoft-Christensen and Baker, 1982]. Furthermore there has been a tendency on the part of designers towards engaging in radical extrapolations of current practice, making the insufficiency of engineering knowledge ever more pronounced.

Sometimes still, lack of professional liaison contributes to disastrous failures, as was demonstrated by the collapse of the Tacoma Narrows Bridge, 1940, due to violent oscillations in a 64 km/hr wind, in which it could be argued that bridge engineers had failed to benefit from the very much developed knowledge on aerodynamic oscillations from aeronautical engineers. On a more localized level, the lack of vital information flow between the engineers and workmen, especially when highly patented construction work is concerned, could be responsible for commission of some human errors. A possible example of this case is the scaffold failure at a power company cooling tower near the Ohio river, USA, on April 27th 1978 [Helander (ed.), 1981].

3. Industrial Climate: When the industrial atmosphere is characterized by dehumanized working conditions, or environments where personality or procedural clashes occur, with poor management strategies for the effective allotment of responsibilities, human intervention strategies are bound to go slack giving room for error to proliferate and propagate.

4. Financial Climate: Finances perhaps exert one of the strongest influences on the engineering climate of a project. Different projects are subject to differing degrees of funding and it is characteristic that most endeavours suffer from financial shortage. Among many other effects of financial inadequacy, the following have been known to contribute to structural failure [Pugsley, 1974]:

(i) Too little expenditure on research and development, as experienced at the first Quebec bridge failure where, among other factors, introduction was made of highly braced struts into the main cantilever structure without any prior checks on the behaviour of such structures through laboratory tests.

(ii) Parsimonious site supervision, as the enquiry into the collapse of the London block of flats at Ronan Point suggested.

Among the more subtle effects of financial inadequacy is the tendency for hasty, improperly engineered work, and especially in poor, developing nations where the laborers' pay is small comparatively, a lack of worker motivation [Helander (ed.), 1981].

Technically speaking, in addition to a combination of any of

the above engineering climatic factors, errors on the jobsite have had an increase in occurrence when construction methods have not been well tried and tested and/or when personnel involved in sitework and supervision are incapable of appreciating the technicalities of the design [Blockley, 1980]. Such human errors, by virtue of their source, are innumerable and quite difficult to predict. Some of those that occur frequently in building structures, for example, include placing the wrong amount of steel reinforcement, incorrect construction of member geometry, intended vertical columns are tilted or shifted, wrongful interpretation of designers' specifications etc, occuring because of inadequate inspection, poor quality control and an absence of communication between designers and contractors.

Unfortunately, when an accident occurs, most of the errors associated with it only come to light after disaster has struck. It is usually easier to identify the likelihood of error with the benefit of hindsight whereas the difficulty arises when the aim is to predict the likelihood of errors in structures yet unbuilt [Blockley, 1980]. Enquiries into famous accidents often show that no error singly brings a structure to collapse but usually, errors of different kinds and of differing severities combine to produce devastating effects. Table 2.1 shows the results of a few of documented enquiries. The numbers 1-10 represent contributory factors as are here defined:

1. Design error

2. Design/specification deficiency
3. Fabrication/construction error
4. Adverse engineering climate surrounding project
5. Misuse of structure
6. Misplaced management/technical responsibility
7. Procedural or management inconsistency
8. Use of inexperienced work force
9. Insufficient knowledge of methods used
10. Insufficient knowledge of material used

Table 2.1: Multiple Causes for Structural Accidents

S/No.	Event	Contributory Factors
1.	Collapse of roof and walls of ice-hockey arena at Listowel, Canada, February 1959.	3, 4, 6, 7, 8.
2.	Collapse of Officers' mess at Aldershot, United Kingdom, July 1963.	1, 2, 3, 9
3.	Collapse of the roof of the assembly hall at Camden School, London, July 1973.	1, 3, 10

Blockley (1980) suggests a number of QA measures of routine checks, comprising 25 questions of relevance to the job at hand,

designed to limit human errors in all its usual input channels. Broadly, they include, at the design stage, checks on loads and strengths, checks on the dependability of mathematical tools used, consideration of random hazards and their likely effect on the structure and the exhaustive appreciation of relevant scientific and professional know-how. At the level of construction they include the assessment of the capabilities of the design and construction team, an assessment of the familiarity of the construction methods and materials used, an assessment of the quality of interrelation between the designers and the contractors and an overview of the general engineering climate surrounding the project. Also included in these checks are a concern for the nature of contractual arrangements and the way they relate to the safety of the project, and lastly, the concern for the sensitivity of the structure to its use.

What are accidents and how does the society view the event, its causes and its effects? At present, no universally acclaimed definition can be given to the term. At best, it presents related ideas to different groups of people. To the safety engineer and largely to the structural engineer, accidents are viewed as the end product of a sequence of acts or events that result in some consequence that is judged to be undesirable [DeReamer, 1980]. Thus, as far as engineering projects are concerned, human error and environmental factors constitute the acts or the events that may result in violation of a limit state which could be described as an

undesirable consequence of the most benign form.

2.1.2 Socio-Political Aspects of Failure

On the socio-political front, the apprehension caused by accidents or failures in general could perhaps best be illustrated using society's view of technological advancements. While some proponents of the scientific thinking solely view technology as a tool to be used in the derivation of socially acceptable outcomes, a growing number of people, including those with no knowledge of the machinations or capabilities of technology, view it in terms of impending hazards and perils on the planet, and with good reason.

The rapid pace of scientific and technological advancements, their novelty and complexity and the uncertainty of their effect on the society emphasize the powerlessness of individuals to cope with such technological competence. Thus, responsibility for survival has automatically been shifted to political leadership [Collingridge, 1980]. Government is therefore charged with the grim task of intervening to prevent alarming trends becoming destiny, in setting up monitoring systems that provide either an extrapolation of preset trends or a data bank on technologies and their impacts. This is particularly true for governments of the developed nations. In the eyes of many more governments, scientific research and technological innovation are basic determinants of economic and social change. Previously, some

industrialized nations took a rather dim view of the interrelationship and complexity of the flow of technological innovation but more recently, the consequences of such technological advancements are fairly predictable with a thorough examination of all significant impact areas and their interrelationships [OECD, 1983].

It has been noted that wrongful interpretation of economic trends and lack of sufficient foresight in the introduction of technological innovations, or even the occurrence of structural or constructional accidents may topple whole governments. As an example, early last year, BBC radio carried the news that the people of South Korea had labelled their present government as 'accident prone', after a series of major structural and constructional accidents within the later part of 1994 and early 1995. Those accidents caused some economic distress and significant loss of life. Thus, the assurance of socially acceptable risks in terms of technological innovations and its attendant economic impacts generates much concern among political leadership.

Given the escalation of technology-induced problems, public interests require that technological developments be viewed and examined not only from the point of view of innovating bodies but also from that of the community, so as to achieve the best possible balance between the benefits of innovation and it implied risks [OECD, 1983]. In highly democratized societies, especially those

of western civilizations, much participatory decision-taking aimed at bringing about all-involving decisions are encouraged in the hope that public participation would evolve as a continuous social learning process for both the citizen and the policy maker [OECD, 1979]. The public participates in various degrees ranging from one-man lobbyists to institutionalized pressure/interest groups.

Hazards exist, most of which have been seen to evolve from the inadvertent effects of technology, and they arise not so much because of scientific knowledge and seldom from ill intent but rather from ignorance, human error and lack of imagination [Wenk, 1979]. History has a fine catalogue of industrial mishaps, two of the most serious being the Chernobyl disaster, Ukraine, 1986, where a nuclear reactor caught fire, and the chemical plant poison gas leakage in Bhopal, India, 1975, where hundreds of people died and whose effects are still being felt today. Such calamities are in themselves graphic reminders of what disasters could befall people today. Hence, the clamour by society for risk reduction if not risk elimination. The quest for safety, seen as the diminishing ability of political decision making bodies in producing compliance with such techno-economic and safety strategies and developments [Ronge, 1980], has developed into a separate problem of risk and its corresponding scientific field, risk assessment.

Risk assessment describes the total process of risk analysis embodying the quantification of risks or risk estimation; risk evaluation, concerned with the implications of technologies on

social values, and risk management, concerned with the institutional and political aspects of the integration of social and technical systems [Otway, 1980]. This exploration of the interaction between technical, social and political systems obviously means that risk research involves the concerted action of a flexible network of experts. While risk estimation is viewed as empirical scientific activity, risk evaluation and management are deemed as normative political activity. It is argued that while scientists are specifically equipped to measure the likelihood of failure and damage, they may not be knowledgeable in determining the value of such damage to the society [Johnston, 1980]. The "problem community" of risk research is constituted therefore not because it can offer scientific findings for the solution of problems at hand but because of the need in the society for scientific verification [Frederichs, 1980]. Scientists and engineers pose as mere counsellors to the socio-political public engaged in arriving at a judgement of absolute risk and its acceptability.

2.1.3. Accidents and Traditional Beliefs

A prime factor in determining how people react to accidents could be seen in the attitude people have towards these accidents. It has been the habit to judge accidents by the nature of the injury they cause. Consequently, any mishap that precipitates large loss of life or that causes the death of even one important

person in the society, or yet still, one that introduces a new or strange kind of injury receives much more attention than everyday fatalities like road accidents etc. The tendency here is that the injury rather than the accident gets prominence. It must be realized that such attitudes arise partly as a result of the artificial importance placed on some categories of accidents by the socio-political decision-making bodies and partly as a result of our own traditional beliefs on accident events. One of such beliefs tags Friday the 13th as an unlucky day in some societies. Others see accidents as acts of God and weave the doctrine of predestination around it. One of the prime objectives therefore of any safety program should be to get everyone to accept the following safety concepts [DeReamer, 1980]:

(1) That accidents are caused by us

(2) Steps must be taken to prevent accidents

(3) The same type of accident will recur if corrective action is not taken.

Summarily, recourse has been made to society's reaction to global hazards and much smaller scale industrial accidents in a bid to propose, by analogy, what the implications of human errors on reinforced concrete beam construction could mean to us. The extensive codification of the building process demonstrates the authority wielded by the several national and international professional bodies engaged in code writing, which could be compared to the authority of governments over technological trends.

The assurance of the proper interpretation and application of these codes rests with the engineering design team and, where structural accidents or failures do occur, it is usual that the structural engineer is taken greatly to task. Current trends involving the shift from traditional craft-based codes to scientifically based ones can be viewed as the result of the continuing process of education of both the scientists and engineers on the one hand and the socio-political public on the other hand. More understanding of the fundamental behaviour of structures is being achieved through continuing research on which the scientific codes are based. These trends also reflect a fair degree of public participation in the building process, as ideas are gathered in code writing practically from worldwide sources as when international symposia are organized.

Pertinent to this study, the following points need to be re-emphasized:

* Accidents, hazards or risks in general, be they of military or non-military character are undesirable to society.

* Socio-economic factors play a very important role in shaping the way society regards and treats such undesirable consequences.

* In the whole of the techno-risk assessment exercise, it is only the scientist or the engineer whose training particularly furnishes him with tools for risk estimation, and it is probable that more attention would be paid to, and more understanding attained of the process of judgement of risk when scientists and

engineers are allowed to become more engaged in this field.

2.1.4 Methods of Structural Reliability Analysis

Methods of arriving at associated levels of structural performance have been classified into 2 broad groups based on the type of calculations performed and the approximations made [Thoft-Christensen and Baker, 1982].

Level 3: Methods in which calculations are made to determine the 'exact' probability of failure for a structure or structural component, making use of a full probabilistic description of the joint occurrence of all the various quantities which affect the response of a structure and taking into account the true nature of the failure domain.

Level 2: Methods involving certain approximate iterative calculation procedures to obtain an approximation to the failure probability of a structure or structural system. The methods generally require an idealization of the failure domain and are often associated with a simplified representation of the joint probability distribution of all variables.

Because of their advantage of being computational fast and efficient, producing results with the desired degree of accuracy, [Thoft-Christensen and Baker, 1982], level 2 methods shall be employed in this study. Therefore the First Order Reliability Method (FORM) is employed.

2.2 Problem statement

In the design of straight-length continuous beams, it is normally assumed in analysis and design that all supports are collinear, with prescribed centre-to-centre spacing. A recurrent error during construction, usually blamed on faulty workmanship, is demonstrated in the construction of beam supports with a shift from the prescribed position.

These errors render the follow-up of detailing instructions more difficult and infringe on serviceability standards of providing adequate cover to reinforcement. They also add to the uncertainty of the true magnitude of the load effect by introducing stresses which may not be expressly accounted for in design. A measure of the performance of a beam should therefore take into account the effect of such load effects on the resistance to any failure mode.

The major problem in the determination of the levels of reliability in each of the limit states of bending, shear, torsion and combined bending and torsion involves 2 steps.

1. The modelling of the performance function, G(x), in each limit state.
2. The determination, or otherwise, of the statistical characters of all random variables present in each limit state function.

Once the statistical characters of the intervening variables are determined, the First Order Reliability Method (FORM5), a

specially adapted and versatile computer algorithm, presents itself as a powerful tool in determining the probability of failure of the beam, hence the expected total cost of the beam failing in each of the aforementioned modes can be predicted.

2.3 Methodology

This section of the study comprises 2 parts. The first contains the theoretical background to the procedures of FORM5, while the second explains the approach towards determining an expected total cost of a beam, taking into account its cost of failure.

2.3.1 Theoretical background to FORM5

2.3.1.1 Introduction

In carrying out a reliability assessment, efforts are directed towards determining the probability that the structure will not attain any of the known limit states likely to be violated within a specified period, in the face of such uncertainties as human error inputs, various environmental conditions, variation in material physical and engineering properties, prediction of future events etc. This probability of survival is the complement of the probability that the structure fails, thus

$$\mathfrak{R} = 1 - P_f \qquad (2.1)$$

The performance function of structures or their components are

usually modelled in terms of certain basic random variables X_1, X_2, ..., X_n, which must operate within certain limits for the structural component to be said to be performing adequately. Values of X_1, X_2,...,X_n, outside of these limits constitute the failure domain, and that surface within the n-dimensional space of basic variables X_i, that divides values of these variables into the failure domain and the domain for adequate performance is called the failure surface. A mathematical representation of this surface is known as the limit state equation [Ditlevson, 1981]. Summarily,

$G(X) > 0$ ..... adequate performance

$G(X) < 0$ ..... failure

$G(X) = 0$ ..... limiting state

$X = x_1, x_2, ..., x_n$, has a joint distribution function given by

$$F_x(x) = P\left[\bigcap_{i=1}^{n} (X_i \leq x_i)\right] \qquad (2.2)$$

and the probability of failure of this model is given by the n-fold integral

$$P_f = \int_{G(X)=0} f_x(x)\, dx \qquad (2.3)$$

where $f_x(x)$ is the joint pdf of x.

Because of a lack of full probability information, the difficulty involved in formulating a joint pdf is obvious and alternative measures of reliability i.e. the reliability indices, have evolved. One of the main objectives of FORM5 therefore is to

determine the value of the reliability index, β, which can be shown to be the minimum distance between the origin of the n-dimensional coordinate system of basic variables and the failure surface.

2.3.1.2 Determination of the Reliability Index β

When G(X) is linear and uncorrelated in the basic variables X_i, there are 5 basic steps needed to arrive at a value for the reliability index β.

1. Adopting a reduced, normally distributed variable expression for each variate.

Let

$$X = x_1, x_2, \ldots, x_n \tag{2.4}$$

then the reduced variate $X' = x_1', x_2', \ldots, x_n'$

where

$$x_i' = \frac{x_i - \mu_{x_i}}{\sigma_{x_i}} \tag{2.5}$$

Therefore,

$$x_i = \sigma_{x_i} x_i' + \mu_{x_i} \tag{2.6}$$

2. Developing an expression for the limit state function with the reduced variates:

$$G(x) = G(\sigma_{x_1} x_1' + \mu_{x_1}, \sigma_{x_2} x_2' + \mu_{x_2}, \ldots, \sigma_{x_n} x_n' + \mu_{x_n}) = 0 \tag{2.7}$$

3. Finding an expression of the distance from the failure surface to the origin of the reduced variate space:

Let the distance in question be D.Then,

$$D = \sqrt{(x_1')^2 + (x_2')^2 + \ldots\ldots + (x_n')^2} \qquad (2.8)$$

4. This expression is minimized subject to G(X) = 0 using the method of Lagrange's.

$$L = D + \lambda G(X_i) \qquad (2.9)$$

where λ = the Lagrange multiplier,

then

$$\frac{\partial L}{\partial X_i'} = \frac{X_i'}{D} + \lambda \frac{\partial G(X_i)}{\partial X_i'} = 0 \qquad (2.10)$$

and

$$\frac{\partial L}{\partial \lambda} = G(x_i) = 0 \qquad (2.11)$$

A solution of equations (2.10) and (2.11) gives the most likely minimum distance point $x^{*'}$ from the origin.

5. The minimum distance is obtained by introducing the gradient vector

$$G' = \frac{\partial G}{\partial X_i'}$$

and together with appropriate matrix representations of D; $D = (x_i'^{t}\, x_i')^{\frac{1}{2}}$, equation (2.10) is manoeuvred into the expression for

D in the form

$$D = \frac{-G'^{t}X'}{(G'^{t}G')^{1/2}} \tag{2.12}$$

where G'^{t} is the transpose of the gradient vector G'. Eq. (2.12) in effect represents the reliability index as previously claimed. Hence

$$\beta = \frac{-G'^{t*}X'^{*}}{(G'^{t*}G'^{*})^{1/2}} \tag{2.13}$$

The reliability index is also defined in terms of the multivariate performance function G as

$$\beta = \frac{\mu_G}{\sigma_G} \tag{2.14}$$

where μ_G and σ_G are the mean and standard deviation of the joint probability distribution of the variables of the performance function G(X). The probability of failure P_f can hence be determined as $P_f = \Phi(-\beta)$, where $\Phi(.)$ is the standard normal integral.

In general however, the performance function G(X) is nonlinear in the basic variables x_i, therefore the first order adaptation of the procedures to obtain the reliability index β involves that G(X) be linearized. This is achieved by expanding G(X) as a First Order Taylor series about some point X^* on the failure surface, and only the linear terms retained [Thoft-Christensen and Baker, 1982]:

$$G(X_1, X_2, \ldots, X_n) = G(X_1^*, X_2^*, \ldots, X_n^*) + \sum_{i=1}^{n} (X_i - X_i^*) \frac{\partial G(X_i)}{\partial X_i} \qquad (2.15)$$

But at the failure surface

$$G(X_i^*) = 0 \qquad (2.16)$$

therefore

$$G(X_1, X_2, \ldots, X_n) = \sum_{i=1}^{n} (X_i - X_i^*) \frac{\partial G(X_i)}{\partial X_i} \qquad (2.17)$$

Introducing the reduced variate,

$$X_i - X_i^* = (\sigma_{X_i} X_i' + \mu_{X_i}) - (\sigma_{X_i} X_i^{*\prime} + \mu_{X_i}) = \sigma_{X_i} (X_i' - X_i^{*\prime}) \qquad (2.18)$$

Also,

$$\frac{\partial G}{\partial X_i} = \frac{\partial G}{\partial X_i'} \cdot \frac{\partial X_i'}{\partial X_i} = \frac{1}{\sigma_{X_i}} \cdot \frac{\partial G}{\partial X_i'}$$

Therefore

$$G(X) = \sum_{i=1}^{n} (X_i' - X_i^{*\prime}) \frac{\partial G}{\partial X_i'} \qquad (2.19)$$

Noting that the parameters of X_i' are N(0,1),

$$\mu_G = -\sum_{i=1}^{n} X^{*\prime} \frac{\partial G}{\partial X_i'}$$

and

$$\sigma_G^2 = \sum_{i=1}^{n} \sigma_{X_i}^2 \left(\frac{\partial G}{\partial X_i'} \right)^2 = \sum_{i=1}^{n} \left(\frac{\partial G}{\partial X_i'} \right)^2$$

Thus the ratio

$$\frac{\mu_G}{\sigma_G} = \beta = -\frac{\sum_{i=1}^{n} x_i^{*\prime}\left(\frac{\partial G}{\partial x_i'}\right)}{\sum_{i=1}^{n}\left(\frac{\partial G}{\partial x_i'}\right)^2} \qquad (2.20)$$

In equation (2.20), where basic variables are assumed to be normally distributed and uncorrelated, the Taylor's expansion of G(X) about the means yields quite accurate estimates of β. Since, however, G(X) may contain non-normally distributed and often correlated variables, the accuracy of the estimates of β depends heavily on the choice of the point of linearization. FORM5 procedures solve this problem by introducing appropriate transformations, converting all variables into uncorrelated normal variables, subsequently linearizing the emergent non-linear function G(X) about a suitable point x^*, selected through an optimization procedure, and hence a probability of failure is evaluated using the standard normal integral.

2.3.1.3 Transformation of Non-normal Basic Variables

Non-normal basic variables are practically normalized by matching pointwise, the density functions and distribution functions of the original distributions with the standard normal distribution [Gollwitzer et al, 1988).

$$f_i(x_i) = \frac{1}{\sigma_{N,i}} \varphi\left(\frac{x_i - \mu_{N,i}}{\sigma_{N,i}}\right) \qquad (2.21)$$

$$F_i(x_i) = \Phi\left(\frac{x_i - \mu_{N,i}}{\sigma_{N,i}}\right) \qquad (2.22)$$

where $\mu_{N,i}$ and $\sigma_{N,i}$ are the mean and standard deviation of the equivalent normal distribution, and are evaluated from Equations (2.21) and (2.22) as:

$$\mu_{N,i} = x_i - \sigma_{N,i}\Phi^{-1}(F_i(x_i)) \qquad (2.23)$$

$$\sigma_{N,i} = \frac{\varphi(\Phi^{-1}(F_i(x_i)))}{f_i x_i} \qquad (2.24)$$

2.3.1.4 Transformation of Correlated Basic Variables

The variance of a probability distribution is usually given as

$$Var(x) = E(x-\mu_x)^2 = E(x^2) - \mu_x^2 \qquad (2.25)$$

In a joint distribution, when two or more random variables are correlated, i.e. when there exists a linear relationship between them, the covariance between any two of these variables x_1 and x_2 is given as

$$Cov(x_1, x_2) = E\left[(x_1 - \mu_{x_1})(x_2 - \mu_{x_2})\right] \qquad (2.26)$$

with the covariance matrix for n basic random variables given as:

$$\overline{C_x} = \begin{vmatrix} Var(x_1) & Cov(x_1, x_2) & \cdots & Cov(x_1, x_n) \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ Cov(x_n, x_1) & Cov(x_n, x_2) & \cdots & Var(x_n) \end{vmatrix} \qquad (2.27)$$

The ratio

$$\frac{Cov(x_1, x_2)}{\sigma_{x_1}\sigma_{x_2}}$$

is known as the correlation coefficient $\rho_{X1,X2}$ between any two variables x_1, x_2 and is subject to the limitation $-1 \leq \rho_{xixj} \leq +1$. When ρ_{xixj} is close to zero, it implies there exists no linear relationship between the pair of variables.

Recalling that the reduced variate

$$X_i' = \frac{X_i - \mu_{x_i}}{\sigma_{x_i}}$$

the covariance between any two reduced variates x_1' , x_2' could be obtained from

$$Cov(x_1', x_2') = E\left[\frac{(x_1-\mu_{x_1})}{\sigma_{x_1}} \frac{(x_2-\mu_{x_2})}{\sigma_{x_2}}\right]$$

$$= \frac{E(x_1-\mu_{x_1})(x_2-\mu_{x_2})}{\sigma_{x_1}\sigma_{x_2}} = \rho_{x_1,x_2} \qquad (2.28)$$

Thus the covariance matrix of x_i' may as well be represented by the correlation matrix of x_i.

No correlation between any pair of random variables exists if this matrix is a diagonal matrix. Such a matrix of uncorrelated variables, Y can be obtained by the transformation

$$\overline{Y} = \overline{A}\,\overline{X} \tag{2.29}$$

where A is an orthogonal matrix with column vectors equal to the orthonormal eigenvectors of the original correlation matrix C_x.

Thus

$$\overline{C}_y = \overline{A}^T \overline{C}_x \overline{A} \tag{2.30}$$

the diagonal elements of C_y being equal to the eigenvalues of C_x.

2.3.2 Evaluation of Expected total Cost.

Cost-benefit analyses seem to suggest that the desirable condition of any endeavour whatsoever can be expressed mathematically as max $(B-C_T)$, where B = Total benefits to be derived from the endeavour and C_T = Total cost of the project.

Within the framework of the probabilistic assessment of the performance of the structure in question therefore, the total cost of the project can be expressed mathematically as:

$$C_T = C_I + P_f C_f \tag{2.31}$$

where C_I = initial cost

P_f = probability of structural failure

C_f = cost of failure

C_I as represented above embodies the generality of costs associated

with materials and labor C_i, provision of a number of QA measures, C_{QA}, and corrective actions taken in response C_c, and costs of maintenance, C_m, and insurance C_{ins}. In other words,

$$C_I = C_i + C_{QA} + C_c + C_m + C_{ins}. \tag{2.32}$$

A major obstacle in the direct analysis of costs as presented lies in the fact that it is very impractical for C_f to be determined globally. It is obviously a function of the benefits derived from the structure by the client, the extent of damage of the structure, the failure mode, the injury caused etc and usually aspects such as the cost of injury vary from society to society, if not from individual to individual.

One convenient way around the problem is to present these costs as non-dimensional quantities, expressing all costs as a ratio to the initial cost. Thus equation (2.31) becomes:

$$\frac{C_T}{C_I} = 1 + P_f \frac{C_f}{C_I} \tag{2.33}$$

Practical values of C_f/C_I are specified in literature (e.g., Melchers, 1987).

For beam elements, these values are said to range between 30 and 75, and in this study, the value 75 is used throughout for the simple reason that the worst effect on the cost of the beam is desired to be known.

CHAPTER THREE

LIMIT STATE FUNCTIONS FOR SINGLY REINFORCED CONCRETE BEAMS.

3.0 Introduction

In this chapter, a model for the performance function in each of the limit states of bending, shear, torsion and a combined failure in bending and torsion would be developed. Basically, the resistance of the structure against each particular failure mode would be pitted against the load effects, including those resulting from human error, in such a way that the resultant expression describes a safety margin. The resistance model proposed by BS 8110 (1985), for each of the limit states shall be adopted and the load effects shall be arrived at with the aid of elementary structural analysis. Each limit state shall be considered separately, and after developing their performance functions, the basic variables corresponding to each shall be identified and their statistical characters suggested, having been obtained from literature.

Consider an office block having a high repeat of structural configuration with such a run of beams and supports for its floor system (see Fig.3.1).

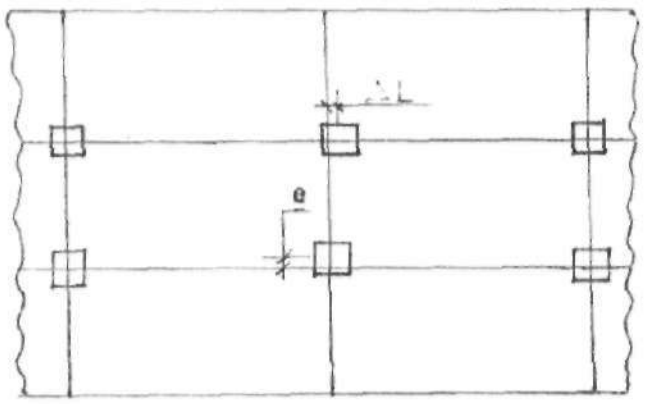


Fig. 3.1. Displaced supports on a floor-beam gridwork.

3.1 Establishment of Performance Functions

3.1.1 Bending Limitation

Consideration is given to interior spans of internal beams only, assuming equal spans and equal loading. For such cases, the load effect in bending represented by the span moment is proposed by BS 8110 (1985) as :

$$M_s = 0.07PL \qquad (3.1)$$

where M_s = Applied span moment

P = Total ultimate load on span

L = Effective span of beam

Additional load effects are introduced when the construction of longitudinal centre-to-centre spacing for the beam supports is

inaccurate. Thus with an effective design span of L, the actual constructed span is measured as L + ΔL.

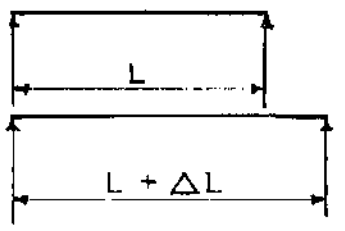


Fig. 3.2 Error in the Span of the a Beam.

Hence the model for the load effect can be expressed in statistical terms as a function of L + ΔL.

At the ultimate limit state, it is important that sections subject to bending should be ductile, with failure occurring by the gradual yielding of tension steel rather than the catastrophic sudden crushing of the concrete in compression.

For flanged sections, the design of the beam is preceded by either of the following conditions which need to be determined beforehand:

(a) The stress block lies entirely within the flange

(b) The stress block extends below the depth of the flange.

Only rarely does condition (b) occur in beams of regular building structures [Mosley and Bungey, 1989]. Therefore condition (a) shall be adopted for developing the resistance of the beam.

For equilibrium of the beam section, the tensile force in the

reinforcement must be balanced by the compressive force in the stress block. i.e.

$$F_{st} = F_{cc} \tag{3.2}$$

implying

$$0.87 f_y A_s = 0.45 f_{cu} b_f . S$$

where f_y = characteristic strength of steel reinforcement

A_s = area of steel reinforcement

f_{cu} = characteristic strength of concrete

b_f = width of flange

S = depth of compressive stress block within the flange.

Hence S can be determined as

$$S = \frac{0.87 f_y A_s}{0.45 f_{cu} b_f} \tag{3.3}$$

The lever arm Z, between F_{st} and F_{cc} is hence obtained as

$$Z = d - \frac{S}{2} \tag{3.4}$$

where d is the effective depth of the beam section.

The moment of resistance M_R, for the section is therefore evaluated as

$$M_R = F_{cc} . Z = 0.45 f_{cu} b_f . S . Z \tag{3.5}$$

The expression for the performance function G(X) in this limit state would then be

$$G(X) = M_R - M_a \qquad (3.6)$$

3.1.2 Shear Limitation

Having determined the magnitude of the critical shearing force V, using structural analysis, the ultimate shearing stress v at a section of uniform depth d can be calculated from

$$v = \frac{V}{bd} \qquad (3.7)$$

where b = supported width of the beam web

d = effective depth of the beam

As supports are displaced from a central position beneath the beam, the supported width of the beam tends towards the value b-e, where e is the magnitude of the shift from the central position. Thus,

$$v = \frac{V}{(b_w - e)\,d}$$

Given that the loading is uniform and the spans do not differ by more than 15% of the longest span, BS 8110 (1985) prescribes that the shearing force V be estimated as 0.55P, where P is the total ultimate load on the span.

Since concrete itself offers some amount of resistance to shearing stresses equal to v_c, the resultant of the shearing stress

created by V then equals $v - v_c$. However $v - v_c$ should not be greater than a certain v_{max} which is set by BS 8110 (1985) as being the smaller of $0.8\sqrt{f_{cu}}$ or $5N/mm^2$.

In the design of the beam for shear, the resistance to shear pitted against applied shearing stresses is estimated as

$$\frac{A_{sv}}{s_v} = \frac{(v-v_c)\,b_w}{0.87 f_{yv}} \qquad (3.8)$$

where A_{sv} = Area of shear reinforcement

s_v = spacing of shear reinforcement

f_{yv} = characteristic strength of shear reinforcement

This can be readjusted to

$$\frac{0.87 A_{sv}.f_{yv}}{s_v.b_w} = \frac{0.55P}{(b_w-e)\,d}\,[1-\gamma] \qquad (3.9)$$

where $\gamma = v_c/v$.

The performance function for the beam is therefore

$$G(X) = \frac{0.87 f_{yv} A_{sv}}{S_v b_w} - \frac{0.55P}{(b_w-e)\,d}\,[1-\gamma] \qquad (3.10)$$

3.1.3 Torsion Limitation

Torsion in a beam is a torque or a twist caused by unsymmetrical loading about its centre line. It is assumed in this failure mode, that the concrete is cracked in torsion, with tensile

vertical and horizontal components respectively, of this stress.

During failure, the reinforcement yields first, then the cracks on the soffit of the beam open up with crushing of the concrete on the topside occurring in complementary planes to the plane of the torsion cracks [Mosley and Bungey, 1989].

At the ultimate state, let the force F exist in the links.

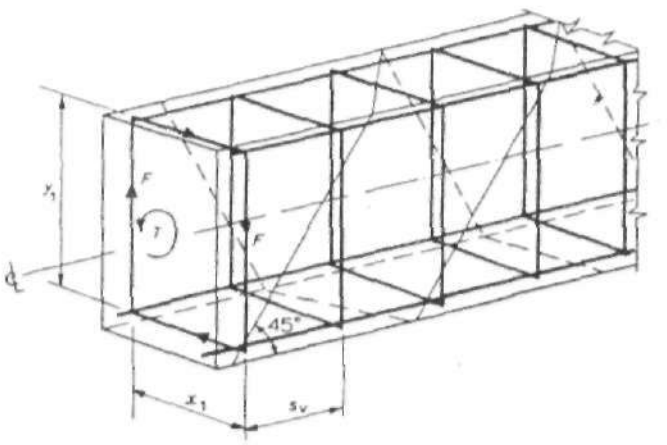


Fig. 3.3: Torsional Reinforcement

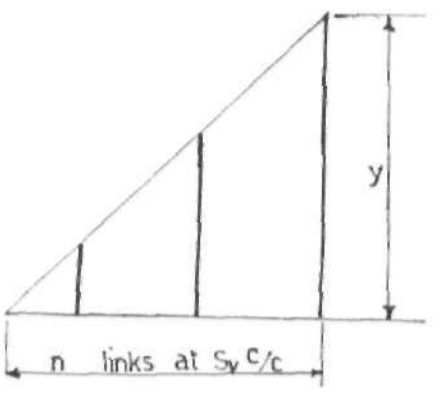


Fig. 3.4: n links cut by one diagonal torsion crack.

The moment about the centre line by the vertical links = F.x/2

The moment about the centre line by the horizontal links = F.y/2

The number of links n, with spacing S_v, cut by one di[illegible]l crack on the vertical face of [illegible]timated from

$$\frac{y}{nS_v} = \tan 45^0$$

which implies

$$n = \frac{y}{S_v} \qquad (3.11)$$

Similarly, on the horizontal face, $n = x/S_v$.

Therefore the moment of resistance to one torsion crack on the [illegible]tical face equals

$$2F\frac{x}{2}.\frac{y}{S_v},$$

and on the horizontal face,

$$2F.\frac{y}{2}.\frac{x}{S_v}.$$

Thus the total ultimate torsional moment of resistance T_R, equals

$$2F.\frac{x}{2}.\frac{y}{S_v}+2F.\frac{y}{2}.\frac{x}{S_v} = 2\frac{F.x.y}{S_v} \qquad (3.12)$$

Considering a partial f[illegible]fety for material strength for steel of 1.15 [Mosley and Bungey, 1989], the value of the force F is estimated as

$$F = \frac{f_{yv}}{1.15}.\frac{A_{sv}}{2} = 0.87 f_{yv}.\frac{A_{sv}}{2} \qquad (3.13)$$

where A_{sv} [illegible]a of both legs of links along one axis.

Therefore

$$T_R = 2\left[0.87 f_{yv} \frac{A_{sv}}{2} . \frac{x.y}{S_v}\right]$$

$$= 0.87 f_{yv}.x.y.\frac{A_{sv}}{S_v} \qquad (3.14)$$

BS 8110 (1985), introduces a factor of 0.8 to take care of error arising from the assumption of a truss-like behaviour for the torsional model. Thus,

$$T_R = \left(0.87 f_{yv}.x.y.\frac{A_{sv}}{S_v}\right)0.8$$

$$= 0.696x.y.f_{yv}\frac{A_{sv}}{S_v} \qquad (3.15)$$

The applied torsional moment T_a resulting from the eccentricity of loading on the span is determined as

$$T_a = P.\ e \qquad (3.16)$$

where P = total ultimate loading on the span

e = the eccentricity of their action

Hence, the expression for the performance function G(X) can be given as

$$G(X) = 0.696.x.y.f_{yv}.\frac{A_{sv}}{S_v} - P.e \qquad (3.17)$$

3.1.4 Combined Bending and Torsion Limitation

Practically, where torsion is present in beams, it mostly occurs in combination with bending and shearing forces and the possibility of the most critical condition being generated as a result of combined stresses exists. Bending and torsional actions occurring about mutually perpendicular axes on the beam section can be represented as shown in Fig. 3.5.

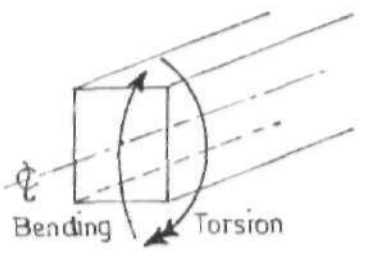


Fig. 3.5 Bending and Torsional moments on Section

The combined action can thus be resolved into a biaxial play of moments on the beam, and it is expected that diagonal tension cracks resulting from torsional stresses will develop from the top of flexural cracks.

CP110 (1972) proposes a limiting condition for such biaxial action as

$$\left(\frac{M_x}{M_{ux}}\right)^{\alpha_n} + \left(\frac{M_y}{M_{uy}}\right)^{\alpha_n} \leq 1 \qquad (3.18)$$

where M_x and M_y are the applied moments in the respective mutually perpendicular directions, and M_{ux} and M_{uy} are the corresponding moment capacities in these directions.

α_n is an exponent given by the expression

$$\alpha_n = \frac{2}{3}\left(1+\frac{5}{2}\cdot\frac{N}{N_{uz}}\right) \qquad (3.19)$$

where N is the applied direct thrust or pull on the beam and N_{uz} is the capacity of the beam to resist direct compressive or tensile stresses. It is required additionally that $1 \leq \alpha_n \leq 2$. For this study, α_n shall be used as equal to 1, for it is unlikely that any thrusts act on the beam.

The performance function for this failure mode can therefore be written as

$$G(X) = 1 - \left(\frac{0.07PL}{0.45f_{cu}b_fSZ}\right) - \left(\frac{P.e}{0.696xyf_{yv}.\frac{A_{sv}}{S_v}}\right) \qquad (3.20)$$

3.2 Uncertainties in the Basic Design Variables

It is seen in the formulation of the performance functions of the beam at the various limit states that the beam's capacity and the load effects on it are themselves functions of several intervening variables x_i. In engineering, uncertainties in

quantities are the rule rather than the exception, therefore it is expected that all such basic variables should exhibit some degree of scatter.

The probability distribution describing this scatter however posses a problem in its identification chiefly because of a lack of sufficient data permitting its unambiguous identification. In the absence of sufficient data, engineers are forced to propose such distributions with the benefit of mathematical back-up to physical reasoning [Ellingwood and Ang, 1974]. Thus where the basic variable consists of the sum of several others e.g. dead loads, strength of concrete etc, the central limit theorem may be invoked to suppose that the normal distribution would be appropriate [Thoft- Christensen and Baker, 1982]. Such a reasoning could be adopted also to suggest that geometrical dimensions could be normally distributed too.

The uncertainty of most variates is well established and given in literature (e.g. Ellingwood and Ang, 1974, Ravindra et al, 1974) in terms of the coefficient of variation, COV, which represents the ratio of the standard deviation to the mean value of the variate. More generally, the total uncertainty is evaluated in terms of the basic variability of observed data and the error in the prediction of the mean value in the form:

$$\Omega = \sqrt{\delta^2 + \Delta^2} \qquad (3.21)$$

where Ω = total uncertainty

δ = basic variability of observed data

Δ = error in the prediction of the mean value.

and

$$Cov = \frac{\Omega}{\mu_i} \qquad (3.22)$$

where μ_i = mean value of the variate.

When practical values for μ_i are adopted, their corresponding variabilities could then be obtained with estimates of their total uncertainties.

3.2.1 Variability in the Strength of Reinforcing, steel, f_y

This has been found to be influenced by the methods of analyses of data from various bar sizes and commercial testing procedures. Because the designer or code writer usually does not know beforehand which bar sizes might be required and in what combinations, uncertainties obtained from lumped test data for bar sizes become most appropriate [Ellingwood and Ang, 1974].

During commercial testing procedures, it is the unstable upper yield point rather than the stable lower yield point that is reported. Also, the characteristic mean strength of steel tends to increase due to the high strain rates applied. The uncertainties in the value of the mean resulting from these shortcomings seem to suggest a total uncertainty Ω_{fy} of 0.15.

3.2.2 Variability in the Strength of Concrete, f_{cu}

The variability in the strength of concrete is affected by the method of mix and structural considerations. Evidently, f_{cu} obtained from test results differ from f_{cu} in the actual structure as a result of factors such as the strain rate, duration of loading, casting direction, curing of the member and the effects of creep, shrinkage and confinement. It has been shown that concrete proportioned for a nominal mix of 21N/mm^2 yield a mean strength of 23.8 N/mm^2 with a basic variability within the test results δ_{fcu} of 0.12 [Ellingwood and Ang,1974]. The aforementioned factors are assumed to contribute a combined uncertainty of 0.18, hence a total uncertainty Ω_{fcu} is obtained as being equal to 0.21 [Ellingwood and Ang. 1974].

3.2.3 Variabilities in member geometry

The width b, the overall depth h, of the beam and dimensions of reinforcement placements are deemed to be functions of the level of workmanship in construction. It is suggested that both Ω_b and Ω_h, and Ω_d and Ω_{sv} are equal respectively, since in the former case, errors arise from the laying out of dimensions, and in the latter case, errors arise from the placement of reinforcement. Ω_b and Ω_h have been estimated as being equal to 0.045 while Ω_d and Ω_{sv} have been estimated as being about 0.086 [Ellingwood and Ang, 1974].

3.2.4 Variability in the area of Steel Reinforcement A_s

This may be obtained from the variability within individual bar areas, assuming that the individual areas are perfectly correlated within a member. The total uncertainty Ω_{As} in this respect, according to Ellingwood and Ang (1974) could be estimated as being about 0.035.

3.2.5 Uncertainties in loading

The load P, encountered in the formulations of the various performance functions, is considered in reinforced concrete design as being a function of characteristic dead loads, g_k, and characteristic live loads q_k.

$$P = k_1 g_k + k_2 q_k = G + Q \qquad (3.23)$$

which can also be expressed as

$$P = Q(\alpha + 1) \qquad (3.24)$$

where α = G/Q and K_1 and K_2 are partial factors of safety according to limit state design:

G and Q represent ultimate dead load and live loads respectively. When suitably practical values for α are adopted, the variations in P can be studied by considering those of the characteristic live loads chiefly.

Live loads in buildings arise from movable fixtures, occupants and other non-permanent installations and they depend on (1) The type of occupancy, (2) The tributary area and (3) The projected

service life of the structure [Ellingwood and Ang 1974].

Whereas physical reasoning could be evoked to suggest that dead loads are normally distributed, consideration is usually given to extreme-value distributions in proposing one for live loads. The application of extreme-value statistics to live loads effectively shunts the necessity for a time-dependent reliability approach, since the maximum load in each reference period is what is considered. Extreme-value type I (Gumbel) distribution has been found to be quite reasonable to model live loads [Melchers, 1987].

The total mean load P could be estimated from

$$\overline{P} = \overline{Q}(1 + \overline{\alpha})A \qquad (3.25)$$

where Q = mean live load

α = mean value of G/Q,

and A represents the area of floor space carried by the beam, held here to be deterministic.

A total uncertainty for the combined load effect has been estimated as being about 0.15 [Ellingwood and Ang, 1974].

3.3 Summary of Statistical Characters of Basic Variables

The statistics of the relevant variables in the considered failure modes are listed in Tables 3.1 to 3.4. The total uncertainties are based on data obtained from literature (e.g. Ellingwood and Ang, 1974).

Table 3.1: Basic Random Variables in Bending Failure and their Statistical Characters.

S/NO	Basic Variable	Distribution Type	Mean E(X)	Coefficient of Variation
1.	f_y	lognormal	425N/mm^2	0.15
2.	A_s	normal	402.1mm^2	0.036
3.	f_{cu}	normal	20N/mm^2	0.21
4.	b_f	normal	930mm	0.045
5.	d	normal	357mm	0.086
6.	Q	gumbel	0.0022N/mm^2	0.38
7.	L	normal	5000mm	0.04

Table 3.2: Basic Random Variables in Shear Failure and their Statistical Characters.

S/No.	Basic Variable	Distribution type	Mean E(X)	Coefficient of Variation
1.	f_{cu}	normal	20N/mm^2	0.21
2.	b_w	normal	230mm	0.045
3.	d	normal	357mm	0.086
4.	Q	gumbel	0.0022N/mm^2	0.38
5.	e	normal	50mm	0.1
6.	f_{yv}	lognormal	250N/mm^2	0.15
7.	A_{sv}	normal	157.1mm^2	0.036
8.	S_v	normal	250mm	0.086
9.	γ	normal	0.6	0.297

Table 3.3: Basic Random Variables in Torsional Failure and their Statistical Characters.

S/NO	Basic Variable	Distribution Type	Mean E(X)	Coefficient of Variation
1.	x	normal	180mm	0.086
2.	y	normal	350mm	0.086
3.	f_{yv}	lognormal	250 N/mm^2	0.15
4.	A_{sv}	normal	157.1mm^2	0.036
5.	S_v	normal	250mm	0.086
6.	Q	gumbel	0.0022N/mm^2	0.38
7.	e	normal	50mm	0.1

Table 3.4: Basic Random Variables in Combined Bending and Torsional Failure

S/NO	Basic Variable	Distribution Type	Mean E(X)	Coefficient of Variation
1.	f_y	lognormal	425N/mm^2	0.15
2.	A_s	normal	1006mm^2	0.036
3.	f_{cu}	normal	20N/mm^2	0.21
4.	b_f	normal	930mm	0.045
5.	d	normal	505mm	0.086
6.	Q	Gumbel	0.0022N/mm^2	0.38
7.	L	normal	5000mm	0.04
8.	x	normal	180mm	0.086
9.	y	normal	500mm	0.086
10.	f_{yv}	lognormal	250N/mm^2	0.15
11.	A_{sv}	normal	226.2mm^2	0.036
12.	S_v	normal	250mm	0.086
13.	e	normal	50mm	0.1

CHAPTER FOUR

FAILURE PROBABILITY LEVELS AND COST ANALYSIS

4.0 Introduction

In this Chapter, the results of the reliability determination process are discussed with meaningful trends identified. For each failure mode, measures of structural performance i.e., β, and therefore P_f are evaluated. The algorithm discussed in sec. 2.3 is used to compute their values. Since the evaluation of P_f does not take into account the effects of human intervention and also because some approximations are introduced into the computational process, the P_fs so obtained could be described as nominal. It has however been argued that the nominal P_f could be considered as an adequate measure of structural safety [Melchers, 1987].

The relative importance of the basic variables is incorporated into the reliability estimation by studying the effects of perturbations in their values. The performance and associated costs of a given beam section are observed as the mean values of the basic variables in each failure mode are varied from design values. This is done to simulate the effects of gross errors and the results are shown graphically for clarity and ease of study.

The ratios of the expected total cost of a beam subject to varying gross errors in the basic variables, to its expected total cost at an acceptable error level are investigated. The

perturbations are performed on those variables involving lateral and longitudinal positioning of columns only, represented by the basic variables e and L respectively.

4.1 Implied Safety Levels

The adequacy of BS 8110 (1985) design criteria in beams is assessed. This is done by examining implied β levels over a range of a suitable parameter such as the load ratio α, and averaging the βs obtained. Typically, βs in the range of 2 to 4 are used in establishing code safety margins (Raju et al, 1990). Hence β averages falling within this range reflect adequacy in the design criteria under study.

According to MacGregor (1980), studies on related structural codes in American practice within the early 1980s show that β fell between 2.8 and 3.2 as gravity loads acted on reinforced concrete beams. A target safety level of β= 3.0 was suggested when dead and live loads act on a beam. Against this standard, implied levels of safety inherent in strength predictions for bending, shear, torsion and combined bending and torsion are studied. A suitable range of load ratios is consequently adopted within which subsequent analyses are performed.

4.1.1 Bending Predictions

Beams are principally flexural members under gravity loads. Designs are therefore required to ensure appropriate levels of

safety under a practical range of loads. Safety levels in bending for varying load ratios a are obtained as shown in Table 4.1.

Table 4.1 Implied β levels for varying load ratio a, in bending

Load ratio, a	Implied β	Load ratio, a	Implied β
0.1	4.08	0.9	2.69
0.2	3.85	1.0	2.56
0.3	3.65	1.1	2.44
0.4	3.46	1.2	2.33
0.5	3.28	1.3	2.22
0.6	3.12	1.4	2.11
0.7	2.97	1.5	2.01
0.8	2.82		

Averaging implied βs given in Table 4.1 over the range of load ratios a = 0.1 to 1.5, a β value of 2.91 is obtained. This falls between β = 2.0 to β = 4.0 established for code safety margins. Therefore BS 8110 (1985) design criteria for bending are adequate over this range of a.

If β = 3.0 were taken as a target level, it can be seen in Table 4.1 that the beam would be sufficiently safe in bending at all a less than 0.7. An acceptable range for the ratio a for the beam in flexure could therefore be deduced as falling between a = 0.5 and a = 0.7. However when predicting for shear and

torsional strengths, this range leads to an over-estimation of the strength of the beam. Thus, under similar loading conditions, representative values of β may be expected to be higher in bending than in shear or torsion. This signifies the superiority of bending strength predictions to shear or torsional strengths.

4.1.2 Shear Predictions

Beams subject to flexure always encounter some shearing action. During testing for shear, implied β levels for varying a are obtained as shown on Table 4.2.

Table 4.2 Implied β levels for varying load ratio a, in shear

Load ratio a	Implied β	Load ratio a	Implied β
0.1	3.42	0.9	2.31
0.2	3.24	1.0	2.21
0.3	3.08	1.1	2.12
0.4	2.93	1.2	2.02
0.5	2.79	1.3	1.94
0.6	2.66	1.4	1.85
0.7	2.53	1.5	1.78
0.8	2.42		

When implied β levels given in Table 4.2 are averaged over the range of load ratios a=0.1 to 1.5, a β value of 2.34 is obtained. This approaches the lower bound of the typical range for code

safety index given as β=2.0 to 4.0. Thus while adequacy is suggested for BS 8110 (1985) design criteria for shear, gross errors in basic variables may easily render the performance of beams in shear unsatisfactory.

Against as assumed target β level of 3.0, it is seen on Table 4.2 that a suitable range of load ratios lies between α=0.2 and α=0.4. While the beam could perform satisfactorily in bending at load ratios as high as 0.7, it can only perform similarly up to α=0.4 in shear, confirming the inferiority of shear strength predictions. This discrepancy in performance levels suggests a non uniformity in the governing design equations. This non-uniformity may be alleviated as suggested by Afolayan (1995).

4.1.3 Torsional Predictions

Torsional effects in internal beams are usually ignored. They mainly arise out of sloppy workmanship during construction. Implied levels of safety during testing for torsional failure are shown on Table 4.3.

Table 4.3 Implied β levels for varying load ratio a, in torsion

Load ratio a	Implied β	Load ratio a	Implied β
0.1	3.62	0.9	2.35
0.2	3.41	1.0	2.23
0.3	3.32	1.1	2.12
0.4	3.05	1.2	2.01
0.5	2.89	1.3	1.98
0.6	2.74	1.4	1.81
0.7	2.60	1.5	1.72
0.8	2.47		

It is seen in Table 4.3 that within the range of load ratios a=0.1 to 1.5, implied βs range between 3.62 and 1.72. This gives an average β of 2.55, which falls within the range of β=2.0 to 4.0 established for code safety margins. This signifies that at mean observed error levels, BS 8110 (1985) design criteria for torsion in beams predicts strengths adequately.

For corresponding a, when a is small, implied β levels obtained in torsional predictions are superior to those obtained for shear prediction but inferior to those obtained for bending predictions (Compare Table 4.3 with Tables 4.1 and 4.2). This seems to suggest that shear and torsion failures are more likely to occur than failure in bending.

Assuming a target β level of 3.0, a beam is seen to perform satisfactorily in torsion as far as a load ratio of 0.4. Practical

load ratios adopted could more suitably range between 0.2 and 0.5.

4.1.4 Combined Bending And Torsion Predictions

Where torsional effects exist in beams, they are likely to occur simultaneously with bending action. It may be reasonable to expect that this combination of stresses would produce structural responses of more devastating effect than either bending or torsional effects alone.

In this study, in order for the performance of the beam to be at par with its performance in the other limit states, an adjustment of some design values are made. Because of this adjustment, it may not be reasonable to make direct comparison between the safety levels in this limit state and any other. It can only be conjectured that this limit state exhibits the least safety under practical loading conditions. Also, reference to CP 110 (1972) is made in developing an explicit limit state function for combined bending and torsion. Consequently, comparisons between this failure mode and any of the other three may not be rational since the bases of their formulation are different.

Implied levels of safety obtained under this consideration are as shown in Table 4.4.

Table 4.4 Implied β levels for varying load ratio α, in combined bending and torsion.

Load ratio α	Implied β	Load ratio α	Implied β
0.1	3.86	0.9	2.43
0.2	3.63	1.0	2.30
0.3	3.41	1.1	2.17
0.4	3.22	1.2	2.06
0.5	3.04	1.3	1.94
0.6	2.87	1.4	1.84
0.7	2.71	1.5	1.73
0.8	2.57		

With the adjusted section, implied βs range between 3.86 and 1.73 when load ratios α range between 0.1 and 1.5 respectively. An average value of β over this range equals 2.65 and is seen to fall within β=2.0 to 4.0, the range of βs typically used in establishing code safety margins. Thus for this design, CP 110 (1972) design criteria for combined bending and torsion seems to be adequate. Implied levels of safety are deemed satisfactory up to a load ratio of 0.5.

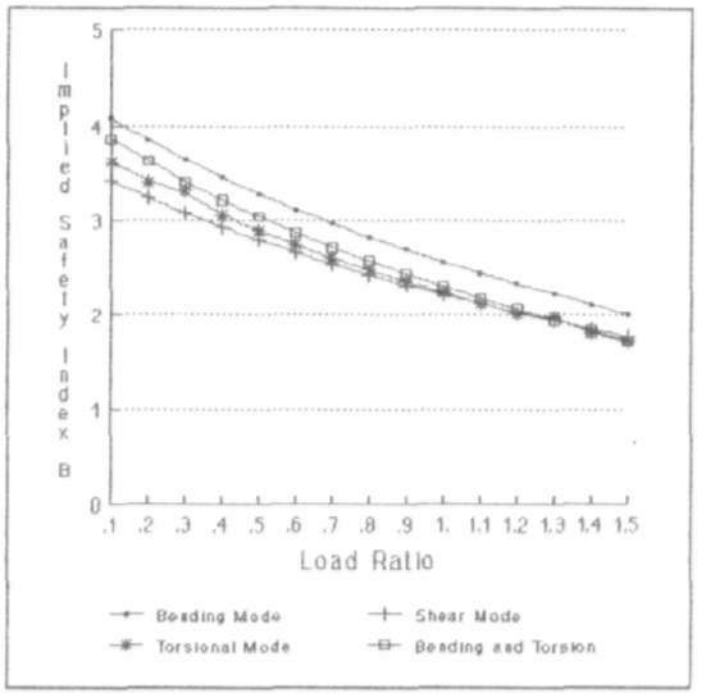


Fig. 4.1 Implied Safety levels with varying load ratios

It is seen in Fig. 4.1 that for all limit states considered, implied levels of safety reduce with increasing load ratio a. Because of its pseudo-probabilistic nature, an outright target β level is not imposed on BS 8110 (1985) design criteria. However, observation of the variation of β with a show that fairly uniform consistency in design criteria is achieved. Within the range a=0.1 to 1.5, implied β ranges between 4.08 and 2.02 in bending predictions, 3.42 and 1.78 in shear predictions, 3.62 and 1.72 in torsion predictions and 3.86 and 1.73 in combined bending and torsion predictions. It is noticed that implied βs vary less in

shear than in torsion hence we conclude that BS 8110 (1985) design criteria for shear are more consistent than those for torsion. It is observed that convergence in safety levels is achieved between these two design criteria when load ratios fall between 1.4 and 1.5.

Summarily, when the load ratio a is varied from 0.1 to 1.5, implied β levels range generally between 4.08 and 1.72 with an average β value of 2.60 ($P_f = 4.66 \times 10^{-3}$). This falls within the range of 2-4 established for code safety margins (Raju et al, 1990). Thus, the BS 8110 (1985) design criteria for beams performing in the limit states of bending, shear and torsion are deemed to be adequate at mean observed error levels. CP 110 (1972) design criteria for combined bending and torsion also show adequacy within a range of a of 0.1 to 1.5. BS 8110 (1985) design criteria for beam designs show fairly uniform consistency, with shear criteria being more consistent than criteria for torsion.

Values of a within which a beam performs satisfactorily with regard to an assumed target safety level β=3.0, do not exceed 0.7. An adopted practical range of a could therefore range between a = 0.2 and 0.5. Within this range, the hierarchy of strengths predicted by the code is in the order of shear, torsion and bending. No direct comparison could be made between the beam's performance in combined bending and torsion and its performance in the other three limit states. It is however assumed that combined bending and torsion action induces the worst performance in a beam.

4.2 Effect of Gross Errors On Beam Performance

Practical structural analysis generally ignores perturbations, implicitly relying on the assumption that the relative deviations of the actual structural response from the calculated one are of the same order as the difference between the actual and idealized structure and loading [Augusti et al, 1984]. Such expectations are often flouted because perturbations ostensibly modify main effects in structural members.

Gross errors affecting the strength parameters of a beam either tend to weaken or to further strengthen it. Those errors that further strengthen the beam have more significance on its initial cost while those that tend to weaken it affect its failure cost more. The latter effects lead to lower implied ß levels and thus can be observed as the sensitivity quotient β/β_0 gets smaller, where β_0 represents the implied safety level of the beam when basic variables are completely error-free. Subsequent analyses therefore centre around the determination of the degree of sensitivity with a view towards assessing the failure cost component to the total cost of a beam. The relative severities of gross errors on the performance of a beam are more clearly seen in sensitivity plots with gross errors in basic variables in each limit state. Generally, the more sensitive the basic variables are to gross errors, the more adverse their effect on a beam's performance.

4.2.1 Failure in Bending

The basic variables of interest according to the limit state equation for bending are as given on Table 3.1. Each of these, except the load Q, is varied by as much as 50% from mean or design values with attention paid to the following responses: The reliability index β, and hence the P_f, and the cost effect, non-dimensionalized as C_T/C_I and represented as C^*. The degree of error in variables is represented in terms of their freedom from error, where % error freedom greater than 100 signifies an augmentation in design values. All operations are performed within the adopted range of loads and the results are presented on Tables 4.5 to 4.8.

Table 4.5 Effects of gross errors in d on bending failure cost.

% Error free	a = 0.2		a = 0.5	
	β/β_0	C^*	β/β_0	C^*
50	0.535	2.49	0.460	5.93
60	0.655	1.44	0.604	2.81
70	0.758	1.13	0.723	1.67
80	0.849	1.041	0.826	1.25
90	0.927	1.013	0.918	1.098
100	1.0	1.0044	1.0	1.039
110	1.068	1.0015	1.076	1.016
120	1.127	1.00053	1.146	1.0065
130	1.184	1.00019	1.210	1.0027
140	1.236	1.000072	1.268	1.0012
150	1.286	1.000027	1.326	1.00051

Table 4.6 Effects of gross errors in f_{cu} on bending failure cost

% Error free	a = 0.2		a = 0.5	
	β/β_0	C^*	β/β_0	C^*
50	0.984	1.0056	0.982	1.048
60	0.990	1.0051	0.988	1.045
70	0.995	1.0049	0.994	1.042
80	0.997	1.0047	0.997	1.041
90	0.999	1.0045	0.998	1.040
100	1.0	1.0044	1.0	1.039
110	1.003	1.0043	1.003	1.038
120	1.004	1.0042	1.004	1.038
130	1.005	1.0042	1.005	1.037
140	1.006	1.0041	1.006	1.037
150	1.006	1.0041	1.007	1.036

Table 4.7 Effects of gross errors in f_y or A_s on bending failure cost.

% Error free	a = 0.2		a = 0.5	
	β/β_0	C^*	β/β_0	C^*
50	0.556	2.2	0.488	5.13
60	0.673	1.36	0.622	2.56
70	0.769	1.11	0.735	1.60
80	0.855	1.037	0.832	1.23
90	0.932	1.0081	0.921	1.094
100	1.0	1.0044	1.0	1.039
110	1.065	1.0016	1.073	1.016
120	1.122	1.00056	1.140	1.007
130	1.177	1.00022	1.201	1.0031
140	1.226	1.000087	1.259	1.0014
150	1.275	1.000035	1.311	1.00063

Table 4.8 Effects of gross errors in L on bending failure cost.

% Error free	a = 0.2		a = 0.5	
	β/β_0	C^*	β/β_0	C^*
50	1.491	1.00000036	1.558	1.000012
60	1.358	1.0000065	1.405	1.00015
70	1.247	1.000059	1.240	1.00098
80	1.153	1.00034	1.174	1.0044
90	1.073	1.0014	1.082	1.015
100	1.0	1.0044	1.0	1.039
110	0.938	1.012	0.927	1.089
120	0.878	1.027	0.860	1.18
130	0.826	1.058	0.799	1.32
140	0.779	1.10	0.744	1.56
150	0.732	1.18	0.692	1.86

Under the condition that the basic variables are 100% error-free, implied β levels are found to be β_0=3.85 and β_0=3.28, for load ratios a of 0.2 and 0.5 respectively. Whenever gross errors on basic variables strengthen the beam, the sensitivity quotient β/β_0 increases from unity, and reduces from unity as errors weaken the beam.

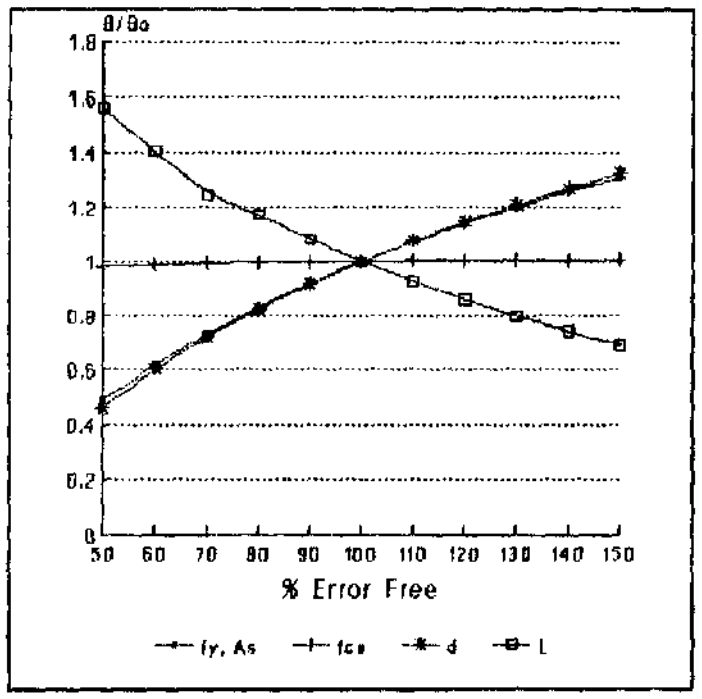


Fig. 4.2: Sensitivity of basic variables to gross error in the limit state of bending.

When gross errors reduce the span L of the beam by as much as half, the sensitivity quotient β/β_0 is found to be equal to 1.558 at α=0.5. Under similar loading and error in d for example, β/β_0 is found to be equal to 0.46. Similarly, errors in A_s and f_{cu} generate β/β_0 values equal to 0.488 and 0.982 respectively. When gross errors cause a 50% augmentation in the mean values of basic variables i.e. when the basic variables are 150% error free, β/β_0 values of 0.692, 1.326,1.311 and 1.007 are induced for errors in L, d, A_s or f_y and f_{cu} respectively. As seen in Fig.4.2 therefore, all

basic variables within the limit state of bending increase the strength of beams with augmentation in their design values, except the span L which reduces the strength of beams as it gets longer.

The sensitivity of the basic variables to errors in descending order is as follows (Fig. 4.2); d, A_s, L and f_{cu}. Both f_y and A_s are seen to show identical significance. Gross errors in d therefore generate the worst safety levels while the quality of concrete, f_{cu}, has very little effect on the performance of beams. f_y and A_s are very nearly as sensitive to gross errors as d, therefore structural responses due to matching gross error levels in them could be expected to be almost equal.

The ratio of the expected total cost of the beam to its initial cost C^* bears a striking relationship to the performance of the beam. Generally, the more adverse the beam's performance, the greater its expected total cost. Thus, gross errors in d inducing the most critical structural outcomes also generate the greatest failure costs and hence the greatest expected total cost of beams. Table 4.5 shows that when only half the required d is provided, the overall cost of beams could go as high as 2.5 and 5.9 times its initial cost at α = 0.2 and 0.5 respectively. On the other hand, the overall cost of the beam varies only slightly with f_{cu}, which shows little sensitivity to gross errors as shown in Table 4.6.

Similar overall cost values are realised with gross errors in f_y or A_s as with d. When corresponding gross error levels are realised, the overall costs are seen in Tables 4.5 and 4.7 to

compare in both sets of variables. This agrees well with the results of sensitivity analysis.

Gross errors causing a beam span shorter than its design span generate lesser overall costs than is generated for corresponding error in other basic variables. Therefore with such errors, changes in L are not likely to generate greater concern than changes in either d or A_s or f_y. When gross errors however cause positive changes in the design values of basic variables, errors in either of d or A_s or f_y generate lesser overall costs than errors in L.

4.2.2 Failure in Shear

Implied levels of safety with variation in α show that the beam exhibits one of its lowest safety levels, performing in shear. Subject to the requisite design values and at α=0.2 and α=0.5, implied β levels are obtained as β_0=3.24 and β_0=2.79 respectively. More comprehensively, the sensitivity quotients β/β_0 and the expected total cost to initial cost ratios, C^*, are presented in Tables 4.9 to 4.14.

Table 4.9: Effects of gross errors in d or f_{yv} or A_{sv} on shear failure cost.

% Error free	α = 0.2		α = 0.5	
	β/β_0	C^*	β/β_0	C^*
50	0.571	3.39	0.513	6.78
60	0.682	2.01	0.638	3.84
70	0.778	1.44	0.746	2.43
80	0.861	1.20	0.839	1.72
90	0.935	1.093	0.921	1.38
100	1.0	1.044	1.0	1.20
110	1.062	1.022	1.068	1.11
120	1.117	1.011	1.133	1.06
130	1.170	1.0056	1.190	1.033
140	1.219	1.0029	1.247	1.019
150	1.265	1.0016	1.297	1.011

Table 4.10: Effects of gross errors in S_y on shear failure cost.

% Error free	ó = 0.2		ó = 0.5	
	β/β_0	C^*	β/β_0	C^*
50	1.460	1.000085	1.520	1.00085
60	1.336	1.00057	1.380	1.0045
70	1.231	1.0024	1.262	1.016
80	1.145	1.0078	1.161	1.044
90	1.068	1.020	1.075	1.10
100	1.0	1.044	1.0	1.20
110	0.941	1.087	0.928	1.36
120	0.886	1.150	0.867	1.58
130	0.836	1.250	0.810	1.86
140	0.781	1.390	0.760	2.29
150	0.748	1.580	0.710	2.79

Table 4.11: Effects of gross errors in γ on shear failure cost.

% Error free	α = 0.2		α = 0.5	
	β/β_0	C^*	β/β_0	C^*
50	0.821	1.30	0.767	2.21
60	0.864	1.19	0.821	1.83
70	0.904	1.13	0.871	1.56
80	0.934	1.088	0.918	1.41
90	0.972	1.062	0.961	1.28
100	1.0	1.044	1.0	1.20
110	1.025	1.033	1.032	1.15
120	1.049	1.026	1.061	1.11
130	1.068	1.020	1.090	1.089
140	1.083	1.017	1.115	1.071
150	1.099	1.014	1.133	1.059

Table 4.12: Effects of variation in e on shear failure cost.

% Error free	a = 0.2		a = 0.5	
	β/β_0	C^1	β/β_0	C^1
50	1.086	1.012	1.097	1.084
60	1.071	1.020	1.075	1.10
70	1.052	1.024	1.057	1.12
80	1.032	1.030	1.039	1.14
90	1.019	1.036	1.018	1.17
100	1.0	1.045	1.0	1.20
110	0.981	1.055	0.978	1.23
120	0.963	1.068	0.957	1.29
130	0.944	1.084	0.935	1.34
140	0.923	1.10	0.910	1.41
150	0.901	1.13	0.889	1.50

The relative severities of gross errors on the relevant basic variables are observed in sensitivity plots on Fig. 4.3.

The degree of sensitivity governing the relative performance levels of beams performing in shear follows the order d, S_v, γ and e. A_{sv} and f_{yv} show identical significance with d, these three being the most sensitive basic variables. Lateral error e shows the least sensitivity. Recalling that implied β_0 levels are among the lowest in shear, the worst performances of beams may therefore be expected with gross errors in either of d, A_{sv} or f_{yv}.

Gross errors augmenting the design values of d or A_{sv} or f_{yv}

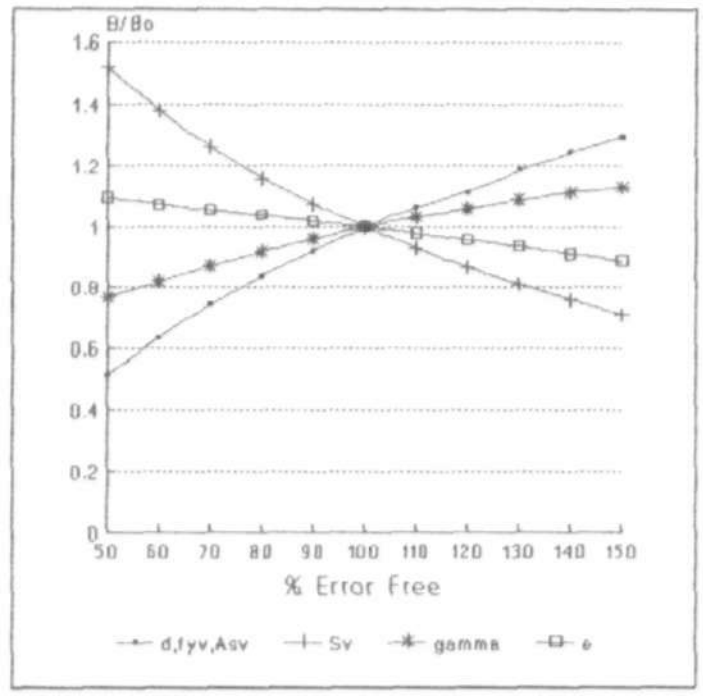


Fig. 4.3: Sensitivity of basic variables to gross error in the limit state for shear.

tend to strengthen beams while errors that cause a decrease in the design values of these variables weaken beams. On the other hand, increase in e or excessive S_v leads to weakening of beams, while a decrease in the value of these basic variables strengthens beams instead.

The variable γ is in actual fact an expression describing an interaction of other basic variables e.g. b, A_s, d, etc, and so is strictly speaking not a basic variable. It is seen in Table 4.11 that when the mean value of γ is 50% error free, its sensitivity

quotient depreciates to 0.821 and 0.767, at a = 0.02 and a = 0.5 respectively. Corresponding gross errors in d would depreciate this quotient to as much as 0.571 and 0.513. Thus, despite the involvement of very sensitive basic variables such as d, its interaction with other basic variables may generate expressions that are less sensitive to gross errors.

Variation in lateral error in seen to have the least effect on the beam's performance in shear. When such construction errors are envisaged, failure in shear is not likely to be of major concern.

The expected total costs and structural performance are related likewise in shear as in bending. Basic variables that severely affect a beam's performance with gross errors are also those on which gross errors generate severe failure costs. Therefore it can be deduced that for corresponding error levels, the severity in failure cost will follow the hierarchy in sensitivity of basic variables given earlier. Values of the expected total to initial cost ratio in Tables 4.9 to 4.12 agree with this observation.

4.2.3 Failure In Torsion

Sensitivity studies on basic variables involved in the limit state for torsion show that they all may be grouped under two significance levels: x, y, f_{yv}, A_{sv} and S_v,e.

Table 4.13: Effects of errors in x or y or f_{yv} or A_{sv} on torsion failure cost.

% Error free	a = 0.2		a = 0.5	
	β/β_0	C^*	β/β_0	C^*
50	0.531	3.63	0.450	8.3
60	0.654	1.97	0.595	4.72
70	0.752	1.37	0.720	2.24
80	0.848	1.14	0.824	1.66
90	0.927	1.058	0.917	1.30
100	1.0	1.024	1.0	1.14
110	1.064	1.010	1.076	1.07
120	1.126	1.0046	1.149	1.034
130	1.185	1.002	1.211	1.017
140	1.235	1.00094	1.273	1.0088
150	1.284	1.00044	1.329	1.0046

Table 4.14: Effects of gross errors in S_y and variation in e on torsional failure cost.

% Error free	a = 0.2		a = 0.5	
	β/β_0	C^*	β/β_0	C^*
50	1.493	1.000013	1.571	1.00021
60	1.361	1.00013	1.419	1.0016
70	1.249	1.00076	1.291	1.0073
80	1.155	1.0034	1.180	1.024
90	1.073	1.0095	1.087	1.064
100	1.0	1.024	1.0	1.14
110	0.935	1.054	0.924	1.28
120	0.877	1.11	0.855	1.51
130	0.821	1.19	0.792	1.83
140	0.771	1.32	0.734	2.28
150	0.724	1.51	0.678	2.88

Implied safety levels with varying a indicate safety levels of β_0 = 3.41 and β_0 = 2.89 at a = 0.2 and 0.5 respectively. More comprehensively, safety levels and the corresponding overall costs with gross errors in basic variables are given in Tables 4.13 and 4.14. These indicate that corresponding gross errors on both sets of basic variables affect the performance of beams in contrasting ways.

The overall costs of beams tend to be greater when gross errors decrease the design values of x,y, f_{yv} or A_{sv} than corresponding gross errors in S_y or variation in e. The reverse is

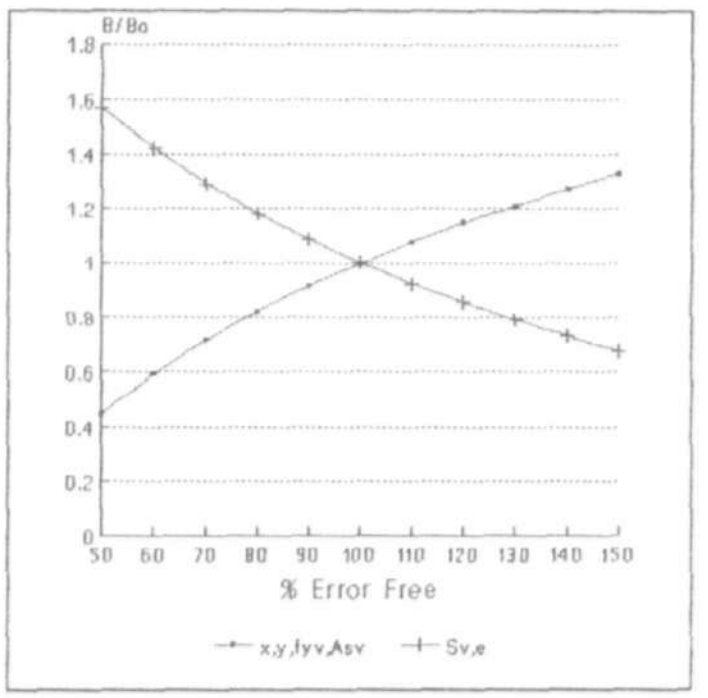


Fig. 4.4: Sensitivity of basic variables to gross errors in the limit state for torsion.

true when gross errors augment the design values of basic variables. The failure cost of beams when gross errors decrease the design values of x, y, f_{yv} or A_{sv} is quite significant. When the basic variable is 50% error free, a beam's total cost could rise as high as 3.63 times its initial cost under a loading of α=0.2. At a load ratio of 0.5, the total cost could rise as high as 8.3 times its initial cost (see Table 4.13). Gross errors which augment mean values of S_v and e by as much as 50% under similar loading conditions generate overall costs that may only go as high as 1.51

and 2.88 times its initial cost at α = 0.2 and α = 0.5 respectively. Therefore gross errors in x, y, f_{yv} or A_{sv} are likely to generate quite exorbitant failure costs while lateral error e and gross errors in S_v tend to generate much milder failure costs.

4.2.4 Failure In Combined Bending And Torsion

The implied β values with perturbations in basic variables presented here are arrived at after considering an adjusted section. Therefore no direct comparison may be made between the beam's performance in this limit state and any other previously discussed. Within this limit state itself, the relative effect of gross errors on the failure cost in the intervening basic variables is revealed by the sensitivity tests on them.

More clearly, the hierarchy in basic variables with the degree of sensitivity to gross errors shown in Fig. 4.5 is found to be as follows: d, f_y, f_{yv}, L, S_v, f_{cu}. Similarities in the structural significance of the basic variables are the same as for the basic variables in the bending and torsional failure modes.

Implied β levels for varying α, discussed in sec. 4.1 gave β values at α = 0.2 as β_0 = 3.63, and at α = 0.5 as β_0 = 3.04. The extent of deviation from these safety values for corresponding gross error levels in the basic variables follows the hierarchy in

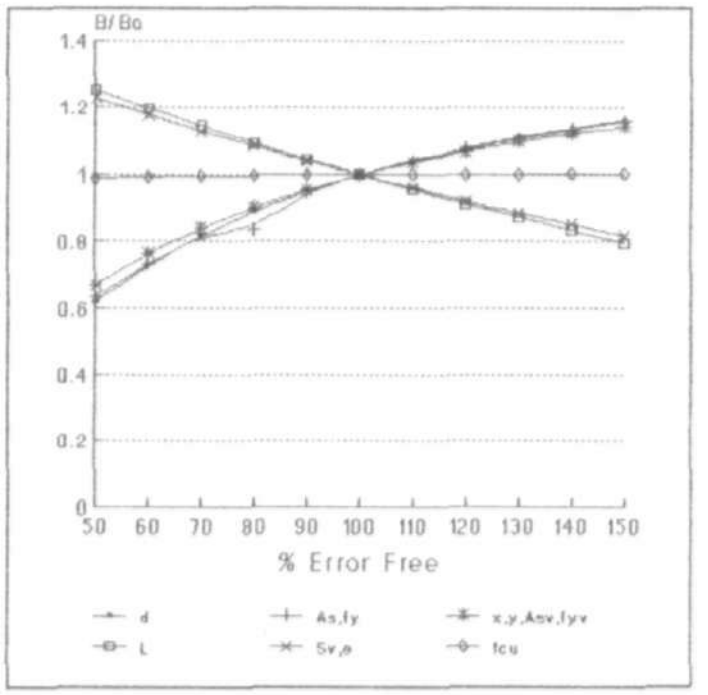


Fig. 4.5: Sensitivity of basic variables to gross errors in the limit state for combined bending and torsion.

sensitivity of these variables. The extents of deviation represent the relative effect of gross errors on the performance of beams. Therefore gross errors in d, A_s and f_y, A_{sv} and f_{yv} affect the performance of beams at corresponding error levels either advantageously or adversely as depicted on Fig. 4.5.

We recall that the performance of a beam is directly linked to its failure cost. Gross errors in d which induce the worst beam performance are therefore also likely to generate the most severe failure costs. (Compare Table. 4.15 with Tables 4.16 to 4.20).

Also, with positive changes in the design values, errors in d, A_s, f_y, A_{sv} or f_{yv} generate lesser failure costs than corresponding errors in S_v or L, or increase in e. The reverse holds true when gross errors cause a decrease in the design values of these basic variables.

Table 4.15: Effects of gross errors in d on the failure cost in Combined Bending and Torsion.

% Error free	a = 0.2		a = 0.5	
	β/β_0	C^*	β/β_0	C^*
50	0.678	1.52	0.622	3.2
60	0.771	1.19	0.730	1.98
70	0.846	1.081	0.819	1.48
80	0.906	1.038	0.891	1.26
90	0.956	1.019	0.951	1.15
100	1.0	1.011	1.0	1.089
110	1.036	1.0064	1.043	1.058
120	1.066	1.004	1.079	1.039
130	1.098	1.0027	1.112	1.028
140	1.118	1.0019	1.138	1.020
150	1.138	1.0014	1.164	1.015

Table 4.16:Effects of gross errors in A_s or f_y on the failure cost in Combined Bending and Torsion.

% Error free	a = 0.2		a = 0.5	
	β/β_0	C^*	β/β_0	C^*
50	0.691	1.45	0.638	3.0
60	0.780	1.17	0.734	1.89
70	0.851	1.075	0.826	1.45
80	0.909	1.036	0.895	1.25
90	0.959	1.019	0.951	1.14
100	1.0	1.011	1.0	1.089
110	1.036	1.0065	1.039	1.059
120	1.066	1.0042	1.076	1.040
130	1.091	1.0028	1.105	1.029
140	1.113	1.0020	1.135	1.021
150	1.135	1.0014	1.158	1.016

Table 4.17: Effects of gross errors in x, y, A_{sv} or f_{yv} on the failure cost in Combined Bending and Torsion.

% Error free	a = 0.2		a = 0.5	
	β/β_0	C^*	β/β_0	C^*
50	0.713	1.36	0.671	2.56
60	0.796	1.14	0.766	1.75
70	0.862	1.066	0.842	1.40
80	0.915	1.033	0.905	1.23
90	0.961	1.018	0.954	1.14
100	1.0	1.011	1.0	1.089
110	1.033	1.0067	1.036	1.061
120	1.061	1.0044	1.069	1.043
130	1.085	1.0030	1.099	1.032
140	1.107	1.0022	1.122	1.024
150	1.127	1.0016	1.145	1.019

Table 4.18: Effects of gross errors in L on the failure cost in Combined Bending and Torsion.

% Error free	a = 0.2		a = 0.5	
	β/β_0	C^*	β/β_0	C^*
50	1.209	1.00042	1.250	1.0055
60	1.165	1.00087	1.198	1.010
70	1.121	1.0017	1.145	1.019
80	1.080	1.0033	1.092	1.033
90	1.039	1.0061	1.046	1.056
100	1.0	1.011	1.0	1.089
110	0.961	1.018	0.954	1.14
120	0.926	1.029	0.914	1.21
130	0.890	1.046	0.872	1.30
140	0.857	1.069	0.836	1.42
150	0.826	1.10	0.796	1.58

Table 4.19: Effects of gross errors in S_y or variation in e on the failure cost in Combined Bending and Torsion.

% Error free	$a = 0.2$		$a = 0.5$	
	β/β_0	C^*	β/β_0	C^*
50	1.196	1.00053	1.227	1.0073
60	1.154	1.0011	1.178	1.013
70	1.113	1.0020	1.128	1.022
80	1.074	1.0036	1.086	1.037
90	1.036	1.0064	1.039	1.058
100	1.0	1.011	1.0	1.089
110	0.964	1.017	0.961	1.13
120	0.931	1.027	0.921	1.19
130	0.901	1.041	0.885	1.27
140	0.871	1.059	0.852	1.36
150	0.840	1.085	0.819	1.49

Table 4.20: Effects of gross errors in f_{cu} on the failure cost in Combined Bending and Torsion.

% Error free	a = 0.2		a = 0.5	
	β/β_0	C^*	β/β_0	C^*
50	0.992	1.012	0.990	1.098
60	0.994	1.0115	0.993	1.095
70	0.997	1.0112	0.997	1.093
80	0.997	1.0109	0.997	1.092
90	1.0	1.0108	0.997	1.090
100	1.0	1.0107	1.0	1.089
110	1.0	1.0106	1.0	1.088
120	1.0	1.0105	1.0	1.088
130	1.0	1.0104	1.0	1.088
140	1.003	1.0104	1.003	1.088
150	1.003	1.0104	1.003	1.087

4.3 Comparative Cost Analysis

The performance of a given beam cross section due to gross errors in the basic variables within each limit state is examined. A fair degree of non-uniformity in the governing design equations is established. This is noted in the differing implied levels of safety associated with the various limit state functions. The non-uniformity is translated through the performance-cost relationship to reflect on the total cost of the beam.

The non-dimensionalised total cost of the beam C^*, obtained from Tables 4.5 to 4.20 is plotted against gross error levels in the basic variables. These are presented in Figs. 4.6 to 4.11. Within these figures, the symbols I, II, III, IV refer to bending, shear, torsion and combined bending and torsion considerations respectively.

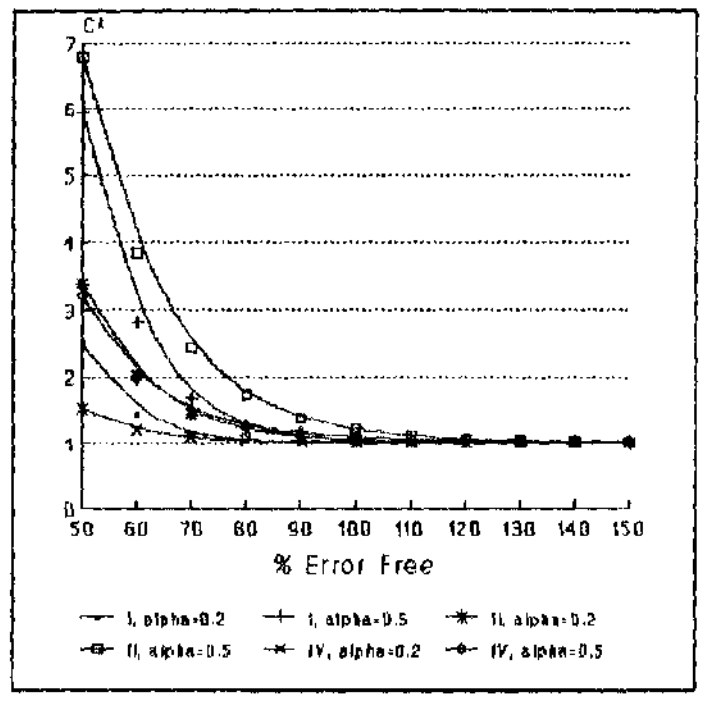


Fig. 4.6: Expected Total to initial cost ratios with gross errors in d.

At given design values, a greater failure cost component is realised with shear failure than with bending failure. This deduction is based on sec. 4.1 where implied levels of safety are computed, and is vividly portrayed when variation characteristics of very sensitive basic variables such as d are observed. Failure in shear should therefore generate more concern to the engineer than bending failure.

It should be recalled that the beam performs so well in combined bending and torsion because of its strengthened section. Compared to costs incurred in bending, corresponding error levels on the strengthened section are observed to be less significant. This may not mean lesser overall costs with failure in combined bending and torsion, as further strengthening of the beam increases its initial cost.

Fig.4.7 shows the variation of the overall cost with perturbations in the least sensitive basic variable f_{cu}. Only slight variation is observed in the total to initial cost ratios throughout the observed range of % error in the basic variables. The relative insensitivity of the beam to gross errors in f_{cu} is thus demonstrated. The insensitivity of the beam to gross errors in f_{cu} may be explained from the fact that the resistance equations are adequately factored [Afolayan,1995]. This means that the partial safety factor applied to f_{cu} is seemingly adequate.

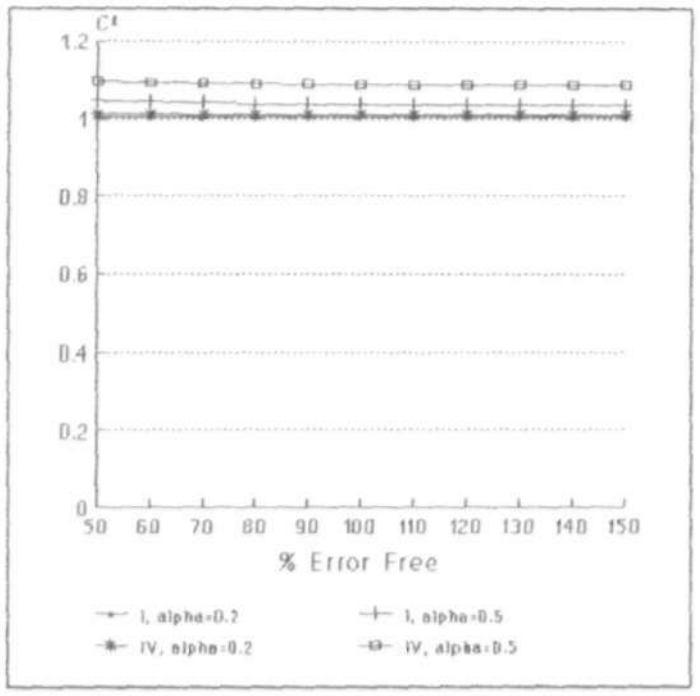


Fig. 4.7: Expected total to initial cost ratios with gross errors in f_{cu}.

The difference between the overall cost of the beam in bending and in combined bending and torsion failure modes may be construed as a measure of the difference between their implied levels of safety. When α is low, this difference in safety levels is not likely to be as great as when α is large, hence the discrepancy in cost differences.

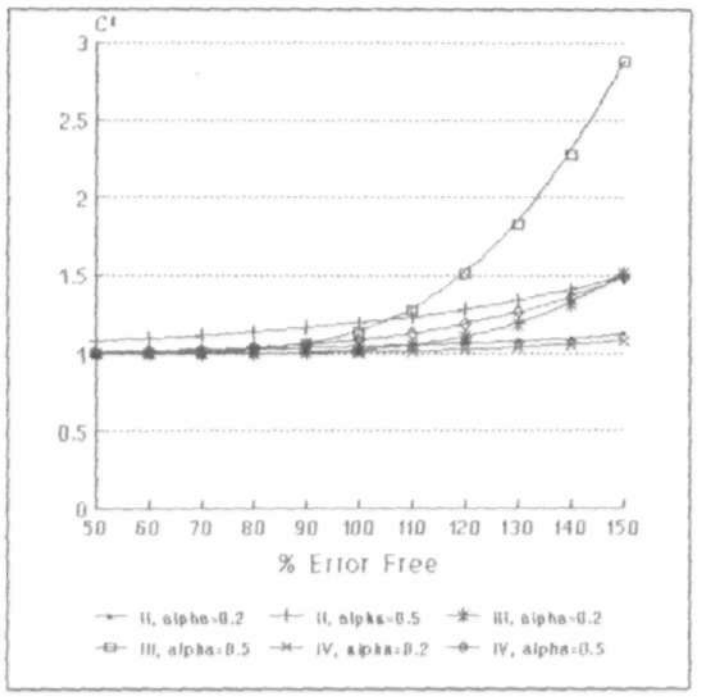


Fig. 4.8: Expected total to Initial cost ratios with variation in e.

The lateral error e is a basic variable which is only introduced into the limit state function because of human error during construction. It is observed in Fig. 4.8 that variations in it show the most significance with torsional failure of the beam. The shear and combined bending and torsion failure modes are relatively insensitive to variations in e. This suggests that where such errors are envisaged, adequate provisions for safety against torsional failure must be made.

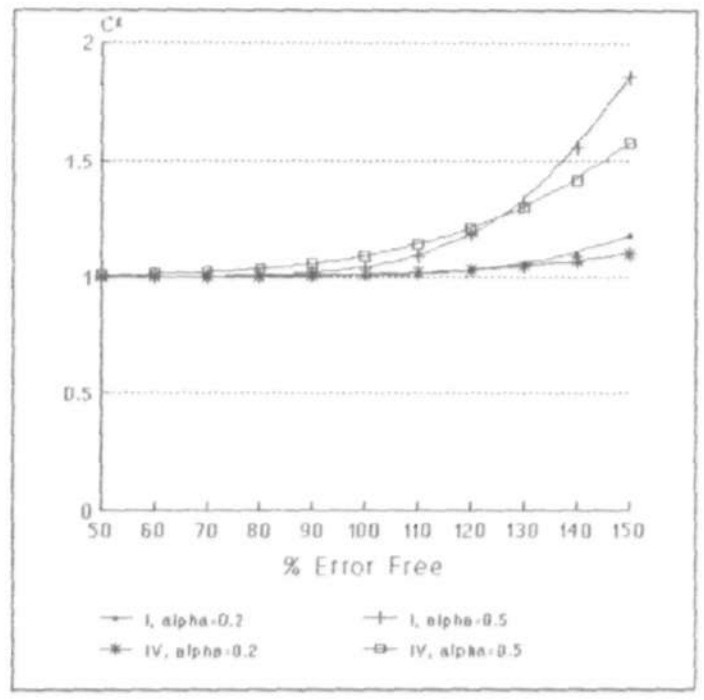


Fig. 4.9: Expected total to initial cost ratios with gross errors in L.

Errors in longitudinal placement of columns are measured in the basic variable L which effects only the limit states for bending and combined bending and torsion. However, a meaningful direct comparison in cost cannot be made since the associated initial costs are different.

Fig. 4.9 shows that when the assumed value of L is less than 130% error-free, the expected total to initial cost ratio with failure in combined bending and torsion tends to surpass that in flexural failure. This underlines the severity of failure in combined bending and torsion due to gross errors in construction

over bending failure when inadequate beam spans are provided. As the beam spans become unduly assigned, potential losses associated with modes I and IV also augment.

4.4 Failure Cost Surcharges on the Expected Total Cost:

When target reliability levels are intended, it is possible to predict a practical range for the expected total costs of beams. Gross errors that lead to a weakening of the beam inevitably introduce costs that go in excess of its expected total cost. This surcharge is introduced through an increased likelihood of failure generating increasingly exorbitant costs.

Let C' represent the ratio of the expected total cost C^{*}_{L} of the beam with varying gross errors in L, to its expected total cost $C^{*}_{[L=5000]}$, at a predefined value for L. Thus,

$$C' = \frac{C^{*}_{L}}{C^{*}_{[L=5000]}} \qquad (4.1)$$

Similarly, cost ratios associated with lateral placement e, of supports can be expressed as

$$C'' = \frac{C^{*}_{e}}{C^{*}_{[e=25]}} \qquad (4.2)$$

in which e is varied from 25mm to 75mm.

These ratios are investigated in all limit states where e or L occur, and the results are presented in Tables 4.21 to 4.25 and shown graphically in Figs. 4.10 and 4.11.

Table 4.21: Total Cost ratios C', with gross errors in L. in failure mode I.

% Error free	a=0.2	a=0.5
50	0.99563	0.96247
60	0.99563	0.96261
70	0.99567	0.96341
80	0.99597	0.96670
90	0.99702	0.97690
100	1.0	1.0
110	1.00756	1.04812
120	1.02251	1.13571
130	1.05038	1.27045
140	1.09518	1.50143
150	1.13028	1.79019

Table 4.22: Total Cost ratios C', with gross errors in L, in failure mode IV.

% Error free	α=0.2	α=0.5
50	0.98951	0.92336
60	0.99000	0.92747
70	0.99078	0.94100
80	0.99236	0.94859
90	0.99512	0.96965
100	1.0	1.0
110	1.00695	1.04680
120	1.01781	1.11111
130	1.03466	1.19374
140	1.05742	1.30395
150	1.08802	1.45096

Table 4.23: Total Cost ratios C", with variation in e, in mode II.

e(mm)	α=0.2	α=0.5
25	1.0	1.0
30	1.0081	1.0152
35	1.0121	1.0330
40	1.0173	1.0515
45	1.0235	1.0799
50	1.0331	1.1074
55	1.0428	1.1351
60	1.0548	1.1905
65	1.0707	1.2361
70	1.0870	1.3004
75	1.1682	1.3831

Table 4.24: Total Cost ratios C" with variation in e, in mode III.

e(mm)	a=0.2	a=0.5
25	1.0	1.0
30	1.0001	1.0010
35	1.0007	1.0070
40	1.0034	1.0235
45	1.0095	1.0638
50	1.0240	1.1402
55	1.0540	1.2804
60	1.1100	1.5106
65	1.1901	1.8282
70	1.3200	2.2779
75	1.5099	2.8818

Table 4.25: Total cost ratios C", with variation in e, in mode IV.

e(mm)	a=0.2	a=0.5
25	1.0	1.0
30	1.0006	1.0056
35	1.0015	1.0146
40	1.0031	1.0295
45	1.0059	1.0503
50	1.0105	1.0811
55	1.0165	1.1218
60	1.0264	1.1814
65	1.0404	1.2608
70	1.0584	1.3501
75	1.0844	1.4792

It is seen in Table 4.21 and 4.22 that the cost ratios C' for gross errors in L tend to be only slightly less than 1.0 when these errors only strengthen the beam. When gross errors cause a shortening of beams, a smaller load acts on them. Their chances of failure become slimmer and therefore the more their total costs tend towards initial costs. When these gross errors on the other hand cause the provision of longer than requisite lengths, the probability of failure of the beam increases. Hence, the expected total cost of the beam subject to varying gross errors increases with the error. These observations are vividly portrayed in Fig. 4.10.

At a 50% overestimation of L, when α=0.5, the overall cost realized could have a failure cost surcharge of as much as 79% and 45% on the expected total cost of the beam as it fails in bending, or combined bending and torsion respectively. However, these failure cost surcharges decrease in magnitude as the gross error leads to further strengthening of the beam, and as the failure load reduces. This suggests that where such gross errors are identified, the use of the structure should be restricted to sustaining acceptably low loading levels.

It could be wrongly deduced from these curves that gross errors that tend to strengthen beams imply a lower overall cost. If we recall that a greater strength of the beam may be associated with a high initial cost input, then along the cost profile, two

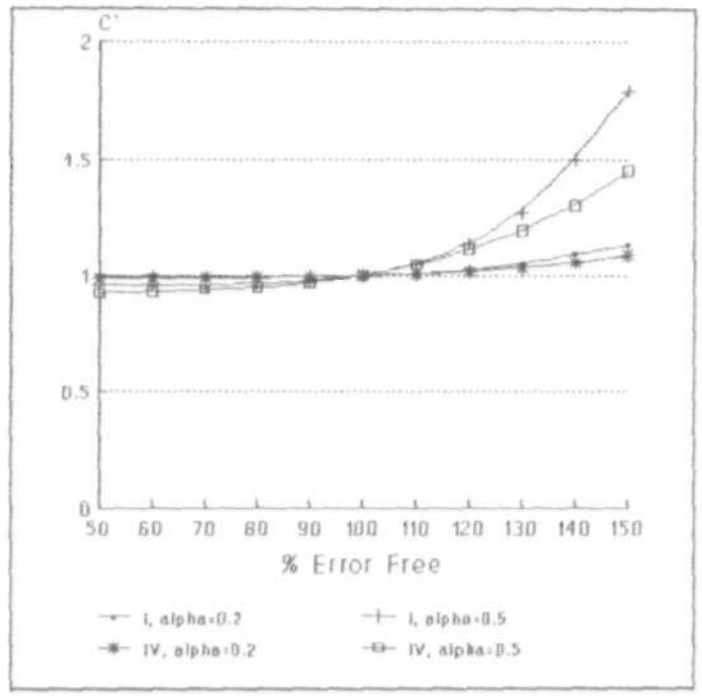


Fig. 4.10: Total cost ratios for gross errors in L

zones of undesirable costs are identified:

(i) At low probabilities of failure implying a high initial cost.

(ii) At high probabilities of failure implying a high failure cost.

Between these zones lies a zone of optimum structural performance, where the expected total cost of the beam is minimum. Proper economics applied to the design and construction of beams should therefore strive to locate performance levels and therefore their costs of failure within this zone.

More critical total cost ratios arise with errors in the lateral positioning of beam supports. BS 8110 (1985) permits an error of 0.05h, where h represents the larger dimension in mm, of the cross-section of the beam support. In this study, e = 25mm is adopted as the minimum eccentricity permitted by the code while the mean of the observed lateral error could be up to 50mm (see for example, Olusanya, 1980).

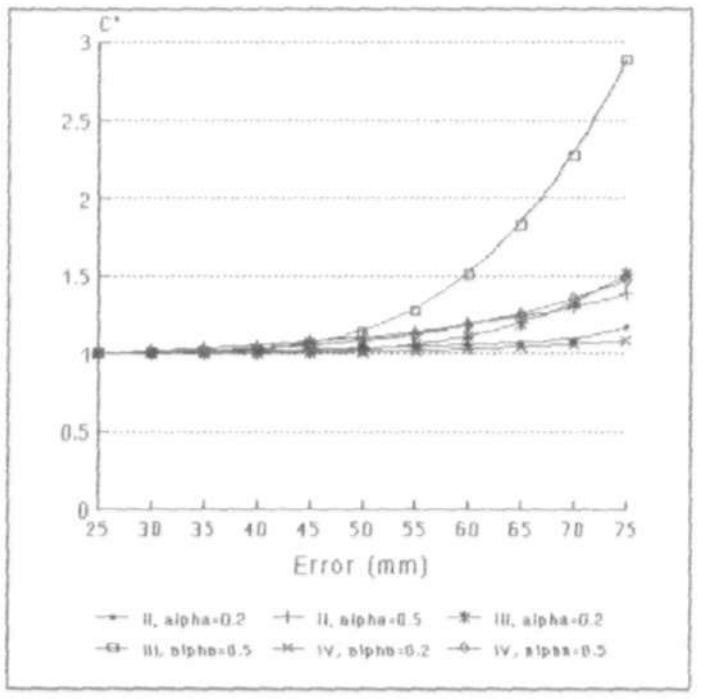


Fig. 4.11 Total cost ratios with variation in e.

Cost ratios with variation in e are relatively mild in mode II and mode IV. In these modes, a lateral error of 75mm, letting a = 0.5 for example, generate a total cost ratio of 1.38 and 1.48 respectively (See Tables 4.23 and 4.25). This represents a failure cost surcharge of 38% and 48% respectively on the expected total cost of beams at code-permissive error levels. In mode III however, a similar error on a beam under similar loading gives a total cost ratio of 2.88. Thus due to a lateral error of 75mm, the total cost of the beam is almost trebled. It is well known that the error in the lateral positioning of beam supports for particularly poor construction sometimes exceeds 75mm and may even go as large as 100mm or 150mm. At such error levels the beam may be likely to incur its most devastating failure cost when it fails in torsion.

4.5 Error Bounds For Code Adequacy

Structural codes (e.g BS 8110, 1985) are largely regarded as a reliable assurance of safety of structures when the structures are designed using them. These codes are subject to periodical review, with the goal to improving optimization. Revisions which however result in changes in safety levels greater than about 10% are generally considered to alarm practitioners [Melchers, 1987]. Similar apprehensions are expressed when human errors interfere with the strength predictions of current codes. Thus, with a 10% delimitation on the variability of ß, the tolerable error in basic

variables can be established.

Error bounds in basic variables occurring in the limit state function for bending prediction can be obtained subject to the restriction that $\beta/\beta_0 \geq 0.9$. It is noticed that at $\alpha = 0.5$, for example, a 12% error, in the negative sense, on the design value of d is deemed large enough to cause alarm (See Table 4.5). Similarly, the value of f_y or A_s should not be reduced by more than 12% (Table 4.7), and gross errors in L should not increase a beam's design length by more than about 15% (Table 4.8). When gross errors in any of these basic variables exceed these bounds, flexural strengths predicted by BS 8110 (1985) are deemed to be sufficiently tampered with to cause alarm.

The philosophy of limit state design does not assign partial factors of safety to sectional dimensions, hence their rather high sensitivity with perturbation. The concrete strength, f_{cu}, on the other hand, is adequately factored, and thus shows very little sensitivity to gross errors. The rather high sensitivity generated by gross errors in f_y suggests that partial factors of safety employed are seemingly inadequate.

Error bounds within the limit state for shear are obtained as is done for bending, with the limitation that $\beta/\beta_0 \geq 0.9$. Thus, at $\alpha = 0.5$, gross errors in d, A_{sv} or f_{yv} may not exceed a 12.5% reduction in design values (Table 4.9). Similarly, gross errors in S_v should not increase the spacing of links by more than 14.5% of the designed spacing. The insensitivity to variation of the

463147

lateral error e on the beam, as seen on Table 4.12 suggests that the limit state function for shear, according to BS 8110 (1985), can effectively accommodate a comparatively wide lateral error band. Thus, with respect to lateral error, the shear strength of beams in accordance with BS 8110 (1985) is quite adequate.

Under torsional considerations, similar limitations in performance levels imply that more than a 10% reduction in design values of x, y, A_{sv} or f_{yv} due to gross errors may cause alarm. These errors should not cause more than a 13.5% increment in S_v. Since S_v and e are identical in significance within the limit state for torsion, a 13.5% augmentation in the mean of observed lateral error is also deemed alarming. It can be noticed that the lateral error above which apprehension might be expressed is much smaller with torsion than with shear. In environments where such errors are rife, torsional failure is therefore more likely to occur.

Within the limit state for combined bending and torsion, it is noticed that gross errors on d, f_y, A_s, x, y, f_{yv} and A_{sv} may not be allowable beyond 18.5% reduction in their design values when α = 0.5, for instance. These error bounds are based on the limitations that $\beta/\beta_0 \geq 0.9$. Thus, similarly, gross errors in L and S_v may not increase design values by more than about 20% and 25% respectively.

Since S_v and e show identical structural significance, it can be deduced that practitioners can only tolerate about a 25% increase in e when mode IV considerations prevail. When observed lateral error exceeds this level, the code (CP 110, 1972)

predictions for the strength of beams in combined bending and torsion cannot be relied upon. These findings suggest that CP 110 (1972), is quite capable of taking care of errors in the span of beams while a fair degree of caution should be exercised with lateral positioning of columns.

Corresponding failure cost limitations may hence be deduced from these error bounds determined. A 10% delimitation on changes in β_0 imply that with failure in bending, failure cost surcharges do not go higher than about 13% of its initial cost. Similarly, beam failures in shear, torsion or combined bending and torsion do not incur cost surcharges that exceed about 47%, 36.5% and 24.4% respectively of its initial cost.

It is noticed that within these error bounds, failure of beams in shear incurs the most failure cost, followed by failure in torsion. Therefore, the necessity for error limitation notwithstanding, special care must be taken to safeguard against failure of beams in shear and torsion.

CHAPTER FIVE

SUMMARY AND CONCLUSION

The performance and the expected total cost of beams subject to gross errors inherent in reinforced concrete beam construction have been studied. It is practically impossible to quantify the whole range of possible errors in construction. The effects of errors involving the longitudinal and lateral positioning of beam supports are treated. In addition, the effects on the performance of beams, of other forms of design and construction variants, simulated herein by perturbations in the design values of basic variables, are also examined. Particular attention has been paid to the computation of the performance index β_0, and hence the probability of failure P_f, from which the expected total costs of beams are evaluated.

Although the CP 110 (1972) and BS 8110(1985) structural codes are pseudo-probabilistic in nature, a target β level is not assigned to their design criteria. It is found however that design criteria for beams in both codes show a fair degree of consistency. In accordance with BS 8110 (1985) criteria for bending, shear and torsion in beams, implied levels of safety range between 4.08 and 1.72 as the load ratio a varies from 0.1 to 1.5. Over this range of a, an average implied β value of 2.60 ($P_f = 4.66 \times 10^{-3}$) is obtained. This falls within the range of 2.0 - 4.0 established for code safety margins [Raju et al, 1990], implying that

BS 8110 (1985) design criteria are adequate at mean observed error levels. The low average implied ß means that design based on BS 8110 (1985) criteria tend to be non-conservative. The variation of implied β_s over this range of a is lesser in shear than in torsion. It therefore follows that BS 8110 (1985) shear criteria give more consistent designs than criteria for torsion. CP 110 (1972) design criteria for combined bending and torsion in beams lead to more critical designs. Therefore to achieve similar performance levels, adjustments of some key design variables are necessary.

Some structural codes in use in America within the early 1980s suggested a target performance index, $\beta = 3.0$, for beams carrying dead and live loads. Against this standard, load ratios, a, of dead to live loads are found to range most suitably between 0.2 and 0.5. Within this range, implied levels of safety for beams in bending, shear and torsion show that the least strength is predicted for shear mode while strength predictions for bending shows the highest safety levels.

In this study, a direct comparison is not made between the performance of the beam in combined bending and torsion and the other three modes for two reasons. Firstly, the strength predictions are based on different codes. The limit state function for combined bending and torsion is only obtainable from CP 110 (1972), while the rest are of BS 8110 (1985). Secondly, in order for their performance levels to be at par, a modification of some

key design variables is made under combined bending and torsion considerations.

Summarily, it is observed that the net effect of gross errors on reinforced concrete beams is either to weaken or further strengthen them. Relying on the notion that more recent codes are calibrated in such a manner as to suggest optimum designs, the effect of gross errors, either way on the beam, is to generate costs that may fall outside the range of optimum costs. Along the profile of the expected total cost of the beam, two zones of rather high and undesirable cost levels are realised. One occurring when probabilities of failure are low, meaning that a high initial cost may have been expended. The second is obtained where probabilities of failure are high, resulting in large failure costs. Between these extremes lies a zone of optimum probabilities of failure with minimum expected total costs.

The severity of the risk caused by gross errors depends on the affected quantity and on the magnitude of the error committed. It is observed that the most exorbitant total costs are realised when the error involves the basic variable which contributes most to the sensitivity of the beam to gross errors. Gross errors in the basic variables showing the least sensitivity incur the least total costs. Within the limit states of bending, shear and combined bending and torsion, the effective depth d, of the beam shows the most sensitivity to gross errors. On the other hand, concrete quality, f_{cu}, contributes least to the sensitivity of the beam

performance to gross errors.

The level of proper workmanship on construction is measurable by the basic variables e and L. These represent basic variables describing the errors in the transverse and longitudinal placement of beam supports respectively, and are investigated in terms of the overall cost of the beam subject to gross errors in them. While the risks in bending are not as severe as risks in the other limit states considered, gross errors that increase a beam's requisite design length by as much as half, could, in effect, double its expected total cost as it fails in bending when, for example, $a = 0.5$. At similar loading levels, a three-fold increase in transverse error, e, can treble the total expected cost of the beam when it fails in torsion. It is however noticed that transverse error has minimal effects on the strength of beams in shear and combined bending and torsion.

The integrity of a structural code is impaired when designs exhibit greater than about a 10% difference in the safety level of a structural member, resulting either from the review of the code or from human error. Such a discrepancy in safety indices generally alarms practitioners. Error bounds on the basic variables are hence computable within which no undue apprehensions are felt. Generally, when the least permissible error for the various basic variables is extracted from all limit states considered, d, A_s and f_y should not be reduced by more 12% of their design values while x, y, A_{sv} and f_{yv} should not reduce by more than

10%. Both S_v and e should not increase by more than 13.5% and L should not be increased by more than 15%. Concrete quality, f_{cu}, appears to be adequately factored therefore a rather large error margin on it can be tolerated by the code.

When these error bounds are observed, a beam failing in bending may incur failure cost surcharges of only 13% of its initial cost when, for example, $a = 0.5$. Similarly, beams failing in shear, torsion or combined bending and torsion may incur failure cost surcharges of as much as 47%, 36.5% and 24.4% respectively in their initial costs. Despite these bounds, cost surcharges resulting from failure of beams in shear or torsion are relatively high. Therefore, from economic considerations only, in addition to suggesting a review of BS 8110 (1985) design criteria for shear and torsion, emphasis should be placed on curbing gross errors that may lead to these failures.

In conclusion, emphasis should be placed on gross error limitation if code predictions are to be achieved. Since errors are cumulative, relatively harmless errors on a structure at different stages of the building process may collectively lead the structure to failure. Thus, at each stage of the building process, human intervention strategies should be established to curb gross errors and minimize its effects. The rather devastating implications of errors in basic variables such as d or A_s, on the performance and cost of beams, demonstrate the necessity for gross error limitation during design and construction. On no account

should economy be made on materials at the expense of the safety of the beam.

Adequate site supervision almost certainly means proper construction practice, with a minimum of gross errors realized. It has been demonstrated how, for instance, transverse error in the placement of beam supports could generate a critical beam performance in torsion, with matching exorbitant cost outcomes. Such errors may be eliminated or minimized when the quality of technical staff on site is high and adequate site supervision is ensured.

Generally, the full range of human intervention measures suggested by Blockley (1980), and outlined in Sec. 2.1 is strongly advocated for. Predictable errors can be curtailed if these measures are rigorously applied. When human errors influence the design criteria of BS 8110 (1985) only minimally, it is found that fairly consistent and non conservative designs of beams are achieved.

It must be emphasized that the results of this investigation are only indicative trends rather than strictly quantifying the code requirements.

REFERENCES

AFOLAYAN, J. O. (1994):Management value Analysis for Effective Quality control, Accepted for Publication in Quality Review, An International Journal of Quality Management Practice.

AFOLAYAN, J. O. (1995):Inequality Problems And their Applications in Structural Safety, Journal of Engineering and Applied Sciences, Vol. 14, No.2, pp. 11-16.

AUGUSTI,G. BARATTA, A. and CASCIATI, F. (1984):Probabilistic Methods in Structural Engineering, Chapman and Hall, London.

AYYUB, B. and HALDER, A. (1984):Practical Structural Reliability Techniques, Journal of Structural Engineering, ASCE, Vol. 110, No.8, pp. 1707-1724.

BLOCKLEY, D. I. (1980):The Nature of Structural Design and Safety, Ellis Horwood Ltd., Chichester.

British Standard Structural use of Concrete BS 8110 (1985): British Standards Institution, London.

COLLINGRIDGE, D. (1980): The Social Control of Technology, Frances Pinter Ltd, London

DeREAMER, R. (1980): Modern Safety and Health Technology, John Wiley and Sons, New York.

DITLEVSON, O. (1981): Uncertainty Modelling, with Applications to Multidimensional Civil Engineering Systems, McGraw Hill Inc. New York.

ELLINGWOOD, B. and ANG, A. H-S.(1974): Risk-Based Evaluation of Design Criteria, Journal of the Structural Division, ASCE, Vol. 100, No. ST9, pp 1771-1788.

FREDERICHS, G. (1980): Risk Research - A 'Problem Community' and its Role in Society, Proc. Society, Technology and Risk Assessment, Academic Press, London, pp 123-130.

GOLLWITZER, S., ABDO, T. and RACKWITZ, R. (1988): First Order Reliability Method (FORM) User's Manual, RCP-GmbH, Munich.

HELANDER, M(ed.) (1981): Human Factors/Ergonomics for Building and Construction, Wiley Interscience, New York.

JOHNSTON, R. (1980): The Characteristics of Risk Assessment Research, Proc. Society, Technology and Risk Assessment, Academic Press, London, pp. 105-122.

MacGREGOR, J. G. (1981): Load and Resistance Factors for concrete Design, Proc. Symposium on Probabilistic Methods in Structural Engineering, (ed.) M. Shinozuka and J. T. P. Yao, ASCE, New York, pp.14-26.

MAZUR, A. (1980): Societal and Scientific causes of the Historical Development of Risk Assessment, Proc. Society, Technology and Risk Assessment, Academic Press Inc. London, pp. 151-161.

MELCHERS, R. E. (1987): Structural Reliability Analysis and Prediction, Ellis Horwood, Chichester.

MOSLEY, W.H. and BUNGEY, J. H. (1989):Reinforced Concrete Design, Macmillan Educational Ltd, London.

OECD (1979): Technology on Trial, OECD Publications, France.

OECD (1983): Assessing the impacts of Technology on Society, OECD Publications, France.

OLUSANYA, O. S. (1980):Minimum Eccentricity in the ultimate strength Analysis, Thesis, A.B.U., Zaria.

OTWAY, H. J. (1980):Discussion Paper on A. Mazur's: Societal and Scientific causes of the Historical Development of Risk Assessment, Proc. Society, Technology and Risk Assessment, Academic Press Inc. London, pp. 163-164.

PUGSLEY, A. G. (1974):Engineering Climatology of structural Accidents,proc. International Conference On Structural Safety and Reliability, Pergamon Press, Oxford, pp. 335-340.

RAJU, S. K., MOSES, F. and SCHILLING, G.C. (1990): Reliability Calibration of Fatigue Evaluation and Design Procedures. Journal of Structural Engineering, ASCE, Vol. 116, No.5, pp. 1356-1369.

RAVINDRA, M. K., LIND, N. C. and SIU, W. (1974): Illustrations of Reliability-Based Design, Journal of the structural Division, ASCE, Vol. 100, No. ST9, pp. 1789-1811.

RIGGS, J. L. (1977): Engineering Economics, McGraw Hill Inc. New York.

RONGE, V. (1980):Theoretical Concepts of Political Decision Making, Proc. Society, Technology and Risk Assessment, Academic Press Inc. London, pp. 209-238.

The Structural use of concrete CP 110 (1972): Parts 1, 2 and 3, BSI. London.

THOFT-CHRISTENSEN, P. and BAKER, M. (1982):Structural Reliability Theory and its Applications, Springer-Verlag, Berlin.

WENK, E. (1979):Margins for survival, Pergamon Press Ltd. Oxford.