MONITORING AND IDENTIFICATION OF INFLUENTIAL PROCESS CHARACTERISTICS IN THE PRESENCE OF AUTOCORRELATION

BY

AKEEM AJIBOLA ADEPOJU

B.Sc. STATISTICS (UNIMAID, 2007)

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DECLARATION

I declare that the work in this thesis entitled Monitoring and Identification of Influential Process Characteristics in The Presence Of Autocorrelationhas been performed by me under the supervision of Dr. A. Yahaya, and Prof O. E. Asiribo in the Department of Mathematics. The information derived from the literature has been duly acknowledged in the text and a list of references provided. No part of this project thesis was previously presented for another degree or diploma at this or any other Institution.

______________________________ ______________________

ADEPOJU, Ajibola Akeem Date

CERTIFICATION

This thesis entitled "MONITORING AND IDENTIFICATION OF INFLUENTIAL PROCESS CHARACTERISTICS IN THE PRESENCE OF AUTOCORRELATION" by ADEPOJU Ajibola Akeem meets the regulations governing the award of the degree of Statistics Ahmadu Bello University, and is approved for its contribution to knowledge and literary presentation.

Dr. A. Yahaya	____________________	________________
Chairman, Supervisory Committee	Signature	Date
Prof. O.E Asiribo	____________________	________________
Chairman, Supervisory Committee	Signature	Date
Dr. B. Sani	____________________	________________
Head of Department	Signature	Date
Prof. Kabiru Bala	____________________	________________
Dean, School of Postgraduate Studies	Signature	Date

DEDICATION

This research work is dedicated to Almighty Allah for His sustenance and blessing over me, and also to my late parents Alhaji Abdulsalam Adepoju and Madam Sikirat Adepoju.

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All praises be to Almighty Allah, the most beneficent and the most merciful who in His extreme mercy sustain me up to this moment.

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ABSTRACT

The traditional methods of multivariate statistical process control (MSPC) are primarily based on the assumptions that the successive observation vectors are independent and normally distributed.However, some process observations are found to be dependent (known as autocorrelation or serial correlation) and if the autocorrelation is left untreated, this can consequently lead to wrong monitoring decision as well as wrong variable identification in the case of out-of-control, which consequently affect the performance of the control charts. This thesis looked into the problem of monitoring the mean vector of a process embedded with autocorrelation and failure of normality assumption. In order to remove the autocorrelation effect and normalized the original data, we proposed vector autoregressive model,VAR model whose residual is assumed to be independent and Johnson transformation(JT) to transform the original data to normality. We were able to show and compare the effect of applying traditional Hotelling's T^2 control chart on autocorrelated and non-normal data as against the residualsobtained from VAR (1) model and normally transformed data. However, since our intention is to achieve a better decision in industrial settings, we thereby complement this work by further adopted MYT model to decompose the overall contribution of the five process variables into individual contribution such that the influential variable(s) is/are identified. .

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CHAPTER ONE

INTRODUCTION

1.1 INTRODUCTION

Statistical Process Control (SPC) is involved with continuous monitoring of a process. In its capacity, it possesses the power of increasing products quality while at the same time reducing the cost of production by avoiding production of unconformable items. The origin of statistical process control (SPC) began with Walter Shewarts (1931). He observed that, in any production there exist variation in the quality of items, which could be due to two major causes: the random causes and the assignable causes. The random cause of variation is considered as common or unpredictable to the process thereby considered such as an in control process while a process with pattern or assignable cause to be out-of-control process.

Exposure to this Shewarts methodology, gave rise to various applications of Statistical Process Control in Engineering, Health, Economy and Sciences etc. This can be observed when quality professionals are monitoring some production process and the quality characteristics of produced items to ensure adherence to certain standard. An important tool for monitoring process/quality is the Control Chart (CC) introduced by Shewarts, which has become major topic of interest in different works of life.

1.2 ASSUMPTIONS OF STATISTICAL PROCESS CONTROL (SPC)

The assumptions in SPC are that the observed process values are normally,independently and identically distributed (iid) with mean (μ) and standard deviation σ, the assumption that the successive observation vectors are independent is very important so as avoid autocorrelation in the data.Autocorrelation in statistics can be described as the correlation between values of the process at different times. Before Multivariate Statistical Process Control (MSPC)can be implemented, the p variables must berelated to each other.Correlation analysis is a technique used to show the strength of the relationship betweenpairs of variables. Montgomery (2001), defined correlation as a degree to which twoor more quantities are associated.

1.3 STATEMENT OF RESEARCH PROBLEM

Research on Industrial Statistics has increasingly become the concern of scholars in statistics and other related fields. Industrial data are subjected to the traditional statistical process tools without verifying some important assumptions, and it has been noticed that often industrial data involve two or more related characteristics and there exist assumption of normality and independence that needed to be satisfied. Failure to satisfy the normality assumption can be resolved by data transformation; but failure to satisfy independence assumption indicates the existence of autocorrelation and partialautocorrelation among characteristics in the multivariate statistical process control. The research question is that: "is ignoring the non-normality, autocorrelation and the partialautocorrelation, while applying the traditional Hotelling's T^2 justifiable?"

1.4 AIM AND OBJECTIVES OF THE STUDY

The aim of this research work is to develop a methodology for implementing the multivariate statistical control to processes that contain significant autocorrelation and non normal in order to improve industrial process decision.

The objectives are to

a. Construct traditional Hotelling's T^2 chart to monitor the process using the original data containing the autocorrelation and non-normality

b. Subjectingthe original data to VAR(1) model in order to remove the autocorrelation and transform the data to satisfy the normality assumption, and then construct Hotelling's T^2 to monitor the residuals of the VAR(1) model.

c. Adopt the MYT decomposition approach to identify the influential variables among the five process characteristics under studies.

1.5 SCOPE AND LIMITATION

1.5.1 **Scope of the Research**

Towards highlighting the comparative position of the various charts, that is, control chart on original data and the control chart on residuals data of industrial observation. The data needed for this research has to be collected over time, embedded with autocorrelation and the variables must be correlated so as to be able to verified and ascertain the comparative position.

1.5.2 **Limitations of the Research**

In spite of best efforts to minimize all limitations that might creep in course of this research, there are certain constraints within which the research will be completed. These are discussed below –

a. The research is based on secondary data. The data was collected from the samples based in Ashaka Cement Plc. Ashaka Cement is one of the cement manufacturers in the Nigeria, samples selected from this industry cannot be considered as a proper representation of the process of the cement manufacturers in the country. Because every manufacturer has their peculiar production policy, however, the objective of this research was to evaluate some statistical assumptions with regards to the concept of industrial statistics.

b. Time constraint and problem of gathering the relevant data are part of the limitations

c. This research is limited to five related process characteristics. Although, this involved cumbersome calculation and the procedures are complex, but with the aid of Statistical softwares, such as R console, we were able to curtail the complexity.

1.6 SIGNIFICANCE OF THE STUDY

This research would point out the effects and the consequence of monitory industrial process with and without fulfilling the assumption of observations independence and normality by highlighting the comparative position of the various charts, that is, control chart on original data and the control chart on residuals data of the industrial observation which would help us to know where practitioners would stand with respect to handling industrial process monitoring.This research would also simplify the MYT decomposition for larger variables and deduce a graphical modification to clarify and categories the contribution of the influential variables in the process. This would help practitioners and researcher to understand the relative contributions of each and every variable in industrial process.

1.7 MOTIVATION OF THE STUDY

Nowadays, it is commonly required that we need to deal with several quality characteristics simultaneously. However, in statistical process control applications, monitoring these variables individually in separate control charts may not be very appropriate. Moreover, it is often expected that the consecutive measurements in statistical process control are somewhat serially dependent (autocorrelated) that is, observation taken overtime (second, minute, hour, day, weekly, monthly, yearly as so on) and influential effect of autocorrelation of a particular variable on the other variables, which may occur as a result of system dynamic and frequent sampling, and if appropriate measures are not taken to take care of the autocorrelation in the process, researcher may be driving towards wrong decision in the process monitoring.

1.8 OPERATIONAL DEFINITION OF TERMS

AUTOCORRELATION: the autocorrelation of a random process describes the correlation between values of the process at different times, as a function of the two times or of the time lag, also known as serial correlation, is the partialautocorrelation of a signal with itself

VECTOR AUTOREGRESSIVE: is an econometric model used to capture the linear interdependencies among multiple time series. A VAR model describes the evolution of a set of k variables (called *endogenous variables*) over the same sample period (t = 1, ...,T) as a linear function of only their past values

NORMALITY TEST: Normality tests are used to determine if a data set is well-modeled by a normal distribution and to compute how likely it is for a random variable underlying the data set to be normally distributed

MYT: Mason, Young and Tracy is a method of interpreting an out-of control signal by decomposing the Hotelling's T^2.

DATA TRANSFORMATION: is a method used to normalize a data set so that statistical tests can be performed to evaluate it properly or Data transformation can be referred to as application of a deterministic mathematical function to each point in a data set — that is, each data point z_i is replaced with the transformed value $y_i = f(z_i)$, where f is a function

ACF and PACF: Autocorrelation sample function and Partial autocorrelation sample function, in time series analysis this function aim at identifying the extent of the lag in an autoregressive model.

Q-Q plot: Quantile-Quantile plot is a graphical display of how well the normal distribution describes the data.

CORRELOGRAM: this is a visual test of autocorrelation which gives you an idea as to the order of autocorrelation as well as whether there exist autocorrelation in your regression equation.

TIME LAG: An event occurring at time t+k (k>0) is said to be lag behind event occurring at time t, and the extent of the lag being k. or period of time between one event and the other

CHAPTER TWO

LITERATURE REVIEW

2.1 INTRODUCTION

There exist two phases in Statistical Process Control (SPC), namely, phase 1 and phase 2. Phase 1 is considered as a retrospective phase, and it constitutes set of individual observations obtained from an in-control process whereby control limits are determined, which involve estimation of the unknown statistic(s), with the aim of achieving observations from an in-control process. Individual observation from such in-control process is then used as a reference data in phase 2. The phase I control limits for the Hotelling's T^2 control chart is as given below.

$$\text{UCL} = \frac{(m-1)^2}{m} \beta_{\alpha, \frac{p}{2}, \frac{m-p-1}{2}} \qquad (2.1)$$

$$\text{LCL} = 0$$

Where $\beta_{\alpha, \frac{p}{2}, \frac{m-p-1}{2}}$ is a $1-\alpha$ *th* percentile of Beta distribution with parameters $\frac{p}{2}$

and $\frac{m-p-1}{2}$,

UCL and LCL are the upper control limit, lower control limit respectively.

However, phase 2 is mainly used to monitoring future observations. The phase II control limits for the T^2 control chart is as given below.

$$\text{UCL} = \frac{p(m+1)(m-1)}{m(m-p)} F_{\alpha, p, m, m-p} \qquad (2.2)$$

$$\text{LCL} = 0$$

where $F_{\alpha,p,m-p}$ is a $1-\alpha$ th percentile of *F* distribution with parameters *p* and $m-p$ degrees of freedom , *m* is the size of base sample, *p* is the number of variables, X_i is the vector of individual observations and α is the chosen level of significance.

This research work will focus on phase 1 and phase 2 of control chart.

It is well known that, an efficient control chart must continue to sample as long as the process is in control and must give signal (out-of-control) to stop the sampling as fast as possible whenever the situation arises. Some of the widely used univariate control chart based on performance includes: Shewart chart, Cumulative Sum chart (CUSUM), Exponentially Weighted Moving Average chart (EWMA) to mention but few, which has their corresponding multivariate control chart such as Hotelling's T^2, Multivariate Cumulative Sum chart (MCUSUM) and Multivariate Exponential Weighted Moving Average chart (MEWMA) respectively.

2.2 UNIVARIATE CONTROL CHART

In some statistical control applications the process would have one particular observation known as quality or process characteristics. Univariate Control Charts involve monitoring individual process or quality characteristic with the aid of control chart, since such univariate control chart can only monitor one quality characteristic in a single chart, this chart is not limited to one quality characteristic alone, two or more quality characteristics can also be monitored but the users would have to look at each quality characteristic separately, that is independently, by doing this, any correlation among the quality characteristics would be considered unimportant and ignored.

2.2.1 SHEWART CONTROL CHART

The Shewart control chart is the most known chart, whose name emerges from Walter Shewart who established them in his pioneering work in 1931. Shewart control chart is used to detect assignable causes in a process. It uses X-bar chart to monitor and control the mean of a process while R and S chart are used to monitor and control the variability of the process.

Assuming $x_1, x_2 \ldots x_n$ is a sample of *n* independent, identically and normally distributed random variables with mean μ and standard deviation σ, (both known). Then the average of this sample $\bar{x}$ is distributed as a normal variable with mean μ and standard deviation $\frac{\sigma}{\sqrt{n}}$ then the control limits of the X-bar chart are as follows.

$$\begin{aligned} UCL &= \mu + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \\ LCL &= \mu - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \end{aligned} \tag{2.3}$$

Where UCL and *LCL* are the upper and lower control limits respectively. Limitation of the shewart charts includes its inability to consider the weight of the previous observations (it is memory less). Although, it is well known for its sensitivity in detecting large shift in the process mean, but very insensitive to small shift in the process mean. Once a shewart chart signals, it implies the presence of assignable cause that demands investigation. Champ and Woodall (1987),Reynolds *et al* (1988), Reynolds *et al* (1989),Prabhu *et al* (1994) to mention but a few had studied the Shewart chart and improved on it.

2.2.2 CUMULATIVE SUM CHART (CUSUM)

Cumulative Sum Chart (CUSUM) proposed by Page (1954) is a control chart designed to detect small shifts in the process by considering the Cumulative Sums of the deviations of successive samples from a target value. Cusum chart is recognized for its sensitivity to small shift in process mean, since its methodology accumulate information from successive observation, in this case it keeps memory of the previous observations. The cusum works by accumulating deviations from target value μ_0. The cusum chart incorporates all the information in the sequence of sample values by plotting the cumulative sums of the deviations of the sample values from a target value. The Cumulative Sum Control Chart is formed by plotting the quantity below:

$$C_i = \sum_{j=1}^{i} \bar{x}_j - \mu_0 \tag{2.4}$$

C_i is called the Cusum, μ_0 is the target value. Signal from a cusum chart could be traced to shift of a process away from the target value and an adjustment can be made to bring the process back to the target value, likewise a signal from a cusum chart could imply the presence of assignable cause that required investigation just like the shewhart control chart.

2.2.3 EXPONENTIALLY WEIGHTED MEAN AVERAGE CHART (EWMA)

Exponentially Weighted Moving Average Control Chart was designed by Roberts (1959), a better option to shewhart control chart in respect of sensitivity to small shift in the process mean. This chart uses current and historical data to detect small changes in the process mean. Its procedure typically gives the most recent data the most weight, and progressively gives smaller weights to the older data.

EWMA is defined as follows

$$Z_i = \lambda x_i + (1-\lambda)Z_{i-1} \tag{2.5}$$

Where λ is a smoothing constant such that $0 < \lambda \leq 1$, and starting value being the first sample means, is the process target such that $z_0 = \mu_0$

Sometimes the average preliminary data under study is used as the starting value of the EWMA such that $z_0 = \bar{x}$

and the control chart limits are as follows:

$$UCL = \mu_0 + L\sigma\left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2i}\right]$$

$$\text{Center line} = \mu_0 \qquad (2.6)$$

$$LCL = \mu_0 - L\sigma\left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2i}\right]$$

Where L is the width of the control limit, μ_0 is the mean and σ is the standard deviation of the sample under study respectively.

2.2.4 MULTIVARIATE CONTROL CHART

Multivariate control chart which is an extension of the univariate control chart is a typical control chart used to monitor and control multivariate observation, that is, more than one quality characteristics. There are some situations whereby individual univariate control chart may be suitable or applicable on each quality characteristics, when each quality characteristics are not dependant on each other, otherwise multivariate control chart would be a better control chart,since multivariate control chart take cognizance of the correlation among the quality characteristics.

2.2.5 HOTELLING'S T^2 STATISTIC

The first and widely used multivariate shewhart control chart is proposed by Hotelling's (1947), an extension to the univariate shewhart charts which was applied to bomb-sight data during World War II, namely Hotelling's T^2. This chart is basically used to monitor and control the mean changes in the p correlated characteristics of a process. This Hotelling's T^2 is defined as a generalized distance from a p-dimensional sample point $\boldsymbol{X} = (x_1, x_2, \ldots, x_p)$ with sample of size n, with the means vector $\bar{\mathbf{x}} = \bar{x}_1, \bar{x}_2, \ldots, \bar{x}_p$ whose distribution is multivariate normal distribution. Then the probability density function is given as

$$f(\mathbf{x}) = \frac{1}{2\pi^{p/2} |\Sigma|^{1/2}} e^{-\frac{1}{2} \mathbf{x}\text{-}\boldsymbol{\mu}' \Sigma^{-1} \mathbf{x}\text{-}\boldsymbol{\mu}} \tag{2.7}$$

with mean μ and Σ the covariance matrix, the control chart statistic is given as

$$\chi^2 = \bar{x}_i - \mu ' \Sigma^{-1} \bar{x}_i - \mu \sim \chi^2_{\alpha,p}$$

and the chart signal whenever $\chi^2 > \chi^2_{\alpha,p}$ where $\chi^2_{\alpha,p}$ is chi square distribution with p degree of freedom, where p is the number of process variables. When the population parameters are not available the unbiased sample estimator such as mean and covariance matrix and the control chart test statistic are given as $\bar{x}, \mathrm{Cov}\ x$ and the test statistic for subgroups data is given has

$$T^2 = n\ \bar{x} - \bar{\bar{x}}\ S^{-1}\ \bar{x} - \bar{\bar{x}}\ \square\ T^2_{\alpha,p,mn-p+1} \tag{2.8}$$

And the chart signals when

$$T^2 > T^2_{F_{\alpha,p,mn-p+1}}$$

Where $T^2_{\alpha,p,n-p}$ is the α percentage point of the *F*- distribution.

The lower control limit for the T^2 chart is zero and the upper control limit for the retrospective phase, that is phase I is given as

$$UCL = \frac{p(m-1)(n-1)}{mn-m-p+1} F_{\alpha,p,mn-p+1} \tag{2.9}$$

where $F_{\alpha,p,mn-p+1}$ is the $100(1-\alpha)^{th}$ percentile of the F distribution with the degree of freedom p and $mn-p+1$, where p is the number of process characteristics, m is the sample size, and n is the sample subgroups. In the case of monitoring phase, that is, phase II, the upper control limit for the T^2 chart is given as

$$UCL = \frac{p(m+1)(n-1)}{mn-m-p+1} F_{\alpha,p,mn-p+1} \tag{2.10}$$

For individual data, the Hotelling's T^2 statistic is given as

$$T^2 = (x-\bar{x})S^{-1}(x-\bar{x}) \sim T^2_{\alpha,p,m-p} \tag{2.11}$$

And the Phase I control limit is given as

$$\text{UCL} = \frac{m-1}{m}^{2} \beta_{\alpha,\frac{p}{2},\frac{m-p-1}{2}}$$
$$\text{LCL} = 0 \tag{2.12}$$

Where $\beta_{\alpha,\frac{p}{2},\frac{m-p-1}{2}}$ is a $1-\alpha$ th percentile of Beta distribution with parameters $\frac{p}{2}$ and $\frac{m-p-1}{2}$, UCL and LCL are the upper control limit, and lower control limit respectively.

However, phase 2 is mainly used to monitoring future observations. The phase II control limits for the T^2 control chart is as given below.

$$\begin{aligned} \text{UCL} &= \frac{p \;\; m+1 \;\; m-1}{m \;\; m-p} F_{\alpha,p,m,m-p} \\ \text{LCL} &= 0 \end{aligned} \tag{2.13}$$

where $F_{\alpha,p,m-p}$ is a $1-\alpha \;\; th$ percentile of F distribution with parameters p and $m-p$ degrees of freedom , m is the sample size, p is the number of process variables, X_i is the vector of individual observations and α is the chosen level of significance.

2.2.6 MULTIVARIATE CUMULATIVE SUM CHART (MCUSUM)

The Multivariate Cumulative Sum Control Chart (MCUSUM) proposed by Crosier (1988) is a procedure that uses the cumulative sum of deviations of each random vector previously observed compared to the nominal value to monitor the vector of means of a multivariate process. The Multivariate Cumulative Sum control chart (MCUSUM) in its capacity takes care of the correlation among the quality characteristics and it is well known for its sensitivity to small and moderate shift in the mean of a process. This chart is an extension of the univariate CUSUM control chart procedure. In this procedure, the scalar quantities are replaced by vectors.

$$C_i = \sqrt{\left[\;\; S_{i-1} + X_i - \mu_0 \;\; \Sigma^{-1} \;\; S_{i-1} + X_i - \mu_0 \;\; \right]} \tag{2.14}$$

the cumulative sums is given as

$$S_i = \begin{cases} 0, & if \;\; c_i \le k \\ S_{i-1} + X_i \left(1 - \frac{k}{c_i}\right), & if \;\; c_i > k \end{cases} \quad (2.15)$$

With S_i = 0 and the reference value *k > 0,* related to the magnitude of the change, where μ_0 is the target value. The covariance matrix Σ can be known or estimated.

The quantity to be plotted on the control chart is

$$Y_i = \sqrt{S_i^{'} \Sigma^{-1} S_i}$$

and the process gives an out of control signal if Y_i> h where h is the control limit.

2.2.7 MULTIVARIATE EXPONENTIAL WEIGHTED MOVING AVERAGE (MEWMA)

The MEWMA procedure proposed by Lowry *et al*. (1992) is an extension of EWMA chart, it takes cognizance of the multivariate characteristics. The MEWMA is designed and recognized for its sensitivity of small and moderate shift in the process means. The MEWMA is based on these statistics

$$Z = \lambda\bar{x} + \;\; I - \lambda \;\; Z_i \quad (2.16)$$

for group data

$$Z = \lambda x + \;\; I - \lambda \;\; Z_i, \quad i = 1, 2, ..., \quad (2.17)$$

for individual data

Where $Z_0 = 0$ and $\lambda = diag(\lambda_1, \lambda_2, \ldots, \lambda_p)$, for $0 < \lambda_j \leq 1$, $j = 1, 2 \ldots, p$.

The MEWMA chart gives an out of control signal when

$$T_i^2 = Z_i' \Sigma_{Z_i}^{-1} Z_i > h \tag{2.18}$$

Where h is the control limit

2.3 MULTIVARIATE NORMAL DISTRIBUTION

Generally the univariate statistical process control is depending on some statistical assumptions, which include normal distribution of observations. This assumption too, can be adopted for the multivariate case. Since multivariate normal distribution involved multiple variables. The probability density function for a p-dimensional observations multivariate normal distribution is as follows:

$$f(x_1, x_2, \cdots, x_p) = \frac{1}{(2\pi)^{p/2} |\Sigma|^{1/2}} e^{\frac{-(x-\mu)^T \Sigma (x-\mu)}{2}}$$

where, μ represents $p \times 1$ mean vector of p variables, Σ represents the $p \times p$ variance-covariance matrix of p variables, the diagonal elements of that matrix are the variances of each variables and the off-diagonal elements are the covariances. x represents the vector of $p \times 1$ random variable, $x = (x_1, x_2, \ldots x_p)'$. Thereby we have $x \sim N(\mu = 0, \Sigma = 1)$,

$$\mu = (\mu_1, \mu_2, \ldots \mu_p)'$$

and variance-covariance matrix generally is set to $p \times p$ diagonal matrix

$$\Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1p} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2p} \\ \vdots & \vdots & \vdots & \vdots \\ \sigma_{p1} & \sigma_{p2} & \cdots & \sigma_{pp} \end{pmatrix}$$

The main diagonal elements of Σ are the variances of the x's i.e. $\sigma_{ij,}\ i = 1,\ldots,p$ and $j = 1,\ldots,p,$ such that $i = j$ and the off-diagonal elements are the covariances $\sigma_{ij,}\ i = 1,\ldots,p$ and $j = 1,\ldots,p,$ $i \neq j$ indicates the correlations among the quality characteristics. When the diagonal elements are set as one, this matrix is equivalent to correlation matrix. If any off-diagonal element is different from zero, then it means corresponding variables are correlated. In addition, the covariance matrix should be symmetric positive definite for multivariate normality assumption to hold. It means that all eigenvalues of covariance matrix should be positive and must be symmetric.

2.3.1 ADVANTAGES OF MULTIVARIATE CONTROL CHART

Multivariate control chart detect symptoms of the process being out of control than the univariate control chart. For example the region in an ecliptical region of a bivariate control chart is smaller than the rectangle made by two individual control charts, since it takes care of the correlation between variables.

Multivariate control chart is easier for operator to assure quality of a process since it aggregate information from many variables on a chart, rather than as much the number of variables as many control chart to be monitored simultaneously.

2.3.2 DISADVANTAGES OF MULTIVARIATE CONTROL CHARTS

One of the most demerits of multivariate control charts is the identification of influential variable or interpretation of out-of-control situation. And this tends to be a very serious issue, since any out of control situation could be triggered by a variable and or relationship between two or more variables changing the mean vector and or changing the covariance matrix. This can be very complex.

Another demerit of the multivariate control charts is the mathematical complexity its entails, it requires sound mathematical approach such as matrix algebra, calculus, etc

2.4 MULTIVARIATE TESTS

A multivariate statistical tests are the tests performed to verify some underlying conditions about the parameters of the population from which sample was drawn. This implies that in modeling a multivariate statistics, it is assumed that the population distribution should follow a particular form of distribution such as normal distribution, poisson distribution, pareto distribution etc and also verify the hypotheses about the population parameters.

2.5 ASSUMPTIONS OF THE MULTIVARIATE TEST

The assumptions of the multivariate test plays very important role since the validity and reliability of any multivariate result is a function in which the strength of the assumptions lie: the most powerful tests are those having more extensive assumptions. (Chatfield & Collins, 1980) indicated that the multivariate assumption has to be fulfilled for Hotelling's T^2 to be adopted.

The assumptions governing the multivariate test includes:

a. The observations must be drawn from normally distributed population

b. The observation must be independent (not correlated).

c. Population must have variance-covariance matrix and mean vector.

d. Variables measurement values are expected to come from interval or ratio scale

Multivariate control is based on the assumptions of Normality and independence, but sometimes the characteristics measured exhibit correlation over time, then when the assumption of independence of at least one characteristic is violated, the construction of T^2 chart, directly on the data is inappropriate, autocorrelation effect may have impact on the control chart by increasing the false alarm rate and reducing the shift detection power. Another important issue is the possibility of existence of partial autocorrelation among variables.

Roberts (1988) indicated that using residuals from the ARIMA model to describe autocorrelated observations may be good enough to construct the control charts since the residuals of time series model of auto correlated process are independent and identically distributed with mean 0 and variance 1.

Harris and Ross (1991) fitted a time series model to the univariate observations, and then considered the effect of autocorrelation on the performance of CUSUM and EWMA chart by using residuals. Montgomery and Mastrangelo (1991) showed how useful and effective EWMA (exponentially weighted moving average) control charts may be for autocorrelated data when applying control charts to the residuals of time series model. Wardell,*et al* (1994), applied a general ARMA model to the autocorrelated observations and used that to show the sensitivity of EWMA charts in shift detection as compared to individual Shewhart charts when considering correlation on an ARMA (1,1) model. But they opined that the residual charts are sensitive to large shifts. Lu and Reynolds, (1999), used the AR (1) model to describe a univariate autocorrelated process, they showed how the EWMA control charts can be used to monitor the mean of autocorrelated process. They suggested that Shewhart control chart of observations will be better at detecting a shift in the process mean than a Shewhart chart of residuals for the low and moderate level of correlation. Also for a low and moderate shifts EWMA chart will be better than Shewhart chart. They also opined that applying time series model would be appropriate for the construction of control limits when there is high autocorrelation in the process. Schmid (1997a, 1997b) opined that Shewhart chart is better in sensitivity for a large shift in the process mean, while EWMA and CUSUM charts have better sensitivity to small and moderate shifts. Maragah and Woodall (1992) proposed an adjustment for the upper and lower control limits for monitoring autocorrelated univariate observations. But there is a need for tables to choose the critical value. Schmid (1995,1997a, 1997b) gives tables for the first order autoregressive process. However the residual charts required only one joint control limits which are based on independent and identically distributed case. Therefore, residual charts have an advantage on the construction of control limits than adjusting the control limits. Statistical process control widely channel to the residuals of univariate autocorrelated chart. However, the autocorrelation problem in univariate case also extends to multivariate cases. Hotelling's T^2 control charts, MEWMA (multivariate exponentially-weighted moving average) charts and MCUSUM (multivariate cumulative sum control) charts are most widely used control charts to monitor the mean shift in multivariate processes. In the presence of serial correlation, Pan and Jarret (2004) suggest the use of vector autoregressive model (VAR) to detect the shift in the mean of multivariate process by using the residues of the raw data of the model. In their search , they investigated the effects of shifts in the process parameters on the VAR residual chart. Kalgonda and Kulkarni (2004) suggested another control chart for monitoring the first order vector autoregressive (VAR (1)) process, named Z-chart. They also proposed using Z-chart to establish the identity of the variable causing the mean shift in the process mean. Pan and Jarret (2007) further monitored the serially correlated multivariate process using the residues from the vector autoregressive model (VAR (p)) on the Hotelling's T^2 control charts. They investigated and opined that in constructing control chart, Hotelling's T^2 control chart is better using residuals from VAR model when the small changes occurred in the process parameter. However, some authors such as Longnecker and Ryan (1990), Zhang (1997) suggested that for the univariate case, using X-chart based on residuals do not have the same properties as the X-charts for an independent process and show that when the process has mean shift, the detection capability of X-chart based on residuals and X-chart for an independent process are not equal.

2.6 TOWARDS IDENTIFICATION OF OUT OF CONTROL CHARACTERISTICS

Woodall and Montgomery (1999) issued that once an out of control signal is given by a multivariate chart, it is difficult to pinpoint the variable(s) that contributed to the signal. Several methods have been proposed in the interpretation; therefore more work is required on data reduction methods and graphical approaches.

In this chapter, we present the overview of previous approaches to the identification of out of control variable, when in fact, there is out of control signal.

When a univariate control chart gives an out of control, it is easy to identify and a solution is proffered, since univariate control chart is related to a single variable. In a multivariate case, controlling several characteristics at the same time, the use of p univariate control charts would be thought of as a solution to identify variable(s) contributing to the signal. However the correlation among the variables are unaccounted for, thereby leading to wrong control limits, false alarm rate and wrong identification of variable(s) (Mason,*et al*1997).

2.6.1 UNIVARIATE CONTROL CHARTS WITH BONFERRONI CONTROL LIMITS

A first method used to investigate variables responsible for the out of control signal was proposed by Alt (1985). Control limits for bonferroni is given

$$UCL = u_i + Z_{1-\alpha/2p}\frac{\sigma_i}{\sqrt{n}}$$
$$LCL = u_i - Z_{1-\alpha/2p}\frac{\sigma_i}{\sqrt{n}}$$

Thus p individual control chart is required, each with probability of the mean falling outside the control limits if we are in control, equal to α/p instead of α. Hayter and Tsui (1994) extended the idea of Benferroni type control limits by giving a procedure for exact simultaneous control intervals for each of the variable means, using simulation. The variables with confidence interval not contained in the u_{0i} is responsible for the signal

$$M = \max\left|x_i - u_{oi}\right| / \sigma_i > CR.\alpha$$

Where: u_{0i} is the standard value

α is the probability of mean falling outside the control limit if we are in control.

CR.α is the critical point.

R is the correlation matrix obtained from ∑

∑ is the variance-covariance matrix.

Simulated minimax is a similar control chart presented by Sepuldveda and Nachlas (1997).

2.6.2 ELLIPTICAL CONTROL REGION

Alt (1985) and Jackson (1991) proposed the second method which is based on elliptical control region. But the major setback of this method is its restriction to two quality characteristics (bivariate).

Considering the case where variable x_1 and x_2 are bivariate normal distribution with mean μ_i and standard deviation $\sigma_i = 1, 2$ and correlation coefficient ρ. The equation below as proposed by Jackson (1991) defines an elliptical region centered at (μ_1, μ_2) and it can be used in place of X^2-chart, since all points lying on the ellipse would have the same value of X^2. While a signal is given whenever the process is out of control. This elliptical region is useful in identifying variable responsible for out of control signal.

Thus, construction of 100(1 - α) % elliptical region is achieved by this equation

$$Q = \left\{ \frac{1}{1-\rho^2} \times \left[\left(\frac{x_1 - \mu_1}{\sigma_1} \right)^2 + \left(\frac{x_2 - \mu_2}{\sigma_2} \right)^2 - 2\rho \frac{x_1 - \mu_1 \quad x_2 - \mu_2}{\sigma_1 \sigma_2} \right] \right\} = \chi^2_{2,1-\alpha}$$

2.6.3 PRINCIPAL COMPONENT ANALYSIS APPROACH

The principal component analysis defines the multivariate relationship among the p variables, this procedure is targeted to simplify the complexity of the data by dimensionality reduction of the quality variable space. The principal component analysis is the set of linear combinations of the variables that redefines existing variable space. The first principal component explains the greatest amount of variation among the variables. The second principal component explains the next greatest amount of variation and so on to the last principal component. However, the first two principal components are independent. Therefore, all the principal components are independent of one another.

Jackson (1991) proposed the use of principal components for monitoring and identifying the p variables in a multivariate control chart. If a T^2 control chart signal, the practice involve using the first k most significant principal components, for further investigation. Since the principal component are independent. A direction to the signal can be interpreted by looking at the behavior of the first principal component and the second principal component. Tracy *et al* (1992) also proposed bivariate setting in which principal components has meaningful interpretation. When monitoring process with paired measurements on a single sample the principal components of the correlation matrix actually represent the characteristics of interest for process control. The correlation coefficient between the original variables is the only additional information needed to describe the condition of the process. Kourti and MacGregor (1996) further contributed to the principal component analysis in which T^2 statistic is expressed in terms of normalized principal components scores of the multinomial variables. And when an out-of-control signal is observed, the normalized scores with high values are detected, and contribution plot are used to verify the variables responsible for the signal.

2.7 THE DECOMPOSITION APPROACH

The major setback of most multivariate control chart procedures is that they hardly give precise and direct information on the variable causing out of control signal. At this end, Mason *et al* (1995) proposed decomposition method. This method is based on decomposing T^2, into independent parts, each of which indicates the contribution of each and every variable.

2.7.1 MASON, YOUNG, AND TRACY'S T^2 DECOMPOSITION

Mason *et al*. (1995, 1997, 1999.) presented the following interpretation method of an out-of control signal. They decomposed the T^2 statistic into p orthogonal components. This method is applied whenever an abnormal measurement is detected in the multivariate control chart and the approach is known as MYT decomposition. The T^2 statistic is divided into conditional and unconditional parts. The decomposition term carries information about the residuals which are generated by all possible linear regression of the other subset of variables. One form of the MYT decomposition is given by

$$T^2 = T_1^2 + (T_{2.1}^2 + T_{3.1,2}^2 + \ldots + T_{p.1,2\ldots,p-1}^2) = T_1^2 + \sum_{j=2}^{p-1} T_{j.1,2,\ldots,j-1}^2$$

Where T_1^2 and $\sum_{j=2}^{5} T_{j.1,2,\ldots,j-1}^2$ are the series of unconditional term and conditioner term of the T^2 respectively, the unconditional term of the T^2 for the first variable of the observation vector x,

$$T_1^2 = \frac{(x_1 - \overline{x}_1)^2}{s_1^2}$$

Where $\overline{x}_1$ and s_1 are mean and standard deviation of variable x_1 respectively.

the general form of j^{th} unconditional T^2 is given as

$$T_j^2 = \frac{(x_j - \overline{x}_j)^2}{s_j^2}$$

For $j = 1,2,\ldots,p$

x_j, is the j^{th} component of the observation vector that gives out control signal and $\bar{x}_j$ and s_j^2 are its corresponding mean and variance from the historical data set, and this follows *F* distribution

$$T_j^2 \sim \frac{n+1}{n} F(1, n-1)$$

andalso the general form of the conditional T^2 is given as

$$T_{j.1,2,\ldots,j-1}^2 = \frac{\left(X_j - \bar{X}_{j.1,2,\ldots,j-1}\right)^2}{s_{j.1,2,\ldots,j-1}^2} \quad \text{for } j = 1, 2, \ldots, p$$

Where $\bar{x}_{j.1,2,\ldots,j-1} = \bar{x}_j + B_j\left(X^{(j-1)} - \bar{X}^{(j-1)}\right)$,

$\bar{x}_j$ is the sample mean of the j^{th} variable, and $X^{(j-1)}$ is the $(j-1)^{th}$ vector with the exception of j^{th} variable, $\bar{x}^{(j-1)}$ is the sample mean vector with the exception of j^{th} variable and $B_j = \left[S_{xx}^2 S_{Xx}\right]$ is a $(j-1)^{th}$ dimensional vector estimating the regression coefficients of the j^{th} variable regressed on the first *(j – 1)* variables.

$$S_{j.1,2,\ldots,j-1}^2 = S_X^2 - S_{Xx}^t S_{xx}^{-1} S_{Xx}$$

And $S = \begin{bmatrix} S_{xx} & S_{Xx} \\ S_{Xx} & S_X^2 \end{bmatrix}$

Similar to this covariance expression $S = \begin{bmatrix} S_x^2 & S_{yx} \\ S_{xy}^{'} & S_y^2 \end{bmatrix}$

Therefore $T_{j.1,2,\ldots,j-1}^2$ follow *F* distribution as given below

$$T^2_{j.1,2,...,j-1} \sim \frac{(n+1)(n-1)}{n(n-k-1)} F_{1,n-k-1}$$

Where k is the number of conditioned variables, the conditional T^2 term has the form of a squared standardized residual.

Thus, this statistic can be used to check whether the j^{th} variable is conforming to the relationship with other variables as established by the historical data set, since the adjusted observation is more sensitive to changes in the covariance structure.

Mason, *et al* (1995) revealed that each p components of T^2 is not unique. There are $p!$ different components that can be formed, and each and every components gives same overall T^2 statistic. For instance, by selecting anyone of the p variables, we can choose any of the (p – 1) remaining variables given that the first variable is selected, subsequently we can choose any of the remaining (p – 2) variables given that the first two variables are selected. Going by the similar procedure will generate all the decomposition equations which consist the same overall T^2 statistic, this is known as the invariant property of T^2. They later presented two variables for illustration. Masoud and Javad (2010), presented MYT decomposition to interpret signal using three variables. Also seen in the work of Sani and Abubakar (2013), they demonstrate the invariant property of MYT decomposition. Mesut and Ibrahim (2013) applied the MYT decomposition in pharmaceutical industry using three variables. Nathan *et al* (2014) discussed decomposing hotelling's T^2 statistic using four variables

This thesis, will investigate the presence of autocorrelation and failure of normality assumption. In order to deal with these problems two general monitoring approaches are recommended. Firstly, the study proposed to use traditional Hotelling's T^2 control charts to monitor the original observations embedded with autocorrelation and non-normality.Secondly, the study proposed to fit time series model to the data, and then apply traditional Hotelling's T^2 control chart, then construct Hotelling's T^2 using the normalized residuals of the fitted data Likewise, we will proceed to reveal the contribution of each and every variable along with their respective joint contributions by simplifying the MYT decomposition for larger variables (5 variables)

The following is a sequential procedure that has the potential of further reducing the computations to a reasonable number when the overall T^2 signals, as proposed by Mason *et al.*

Step 0

Conduct T^2test with a specified nominal confidence level if an out of control condition is signaled then, continue with step 1.

Step 1

Compute the individual T^2statistic for every component of the *X*vector. Remove variables whose observation produces a significant T_i^2. With significant variables removed a reduced set of variables is achieved. Check the subvector of the remaining *k*variables, if there is any signal.

Step 2

Optional but useful for a very large *p*: examine the correlation structure of the reduced set of variables. Remove any variable having a very weak correlation (0.3 or less) with all the other variables. The contribution of a variable that falls in this category is measured byT_i^2 component.

Step 3

If a signal remains in the subvector of *k* variables not deleted, compute all$T_{i.j}^2$terms. Remove from the scheme all pairs of variables (x_i,x_j) that have a significant $T_{i.j}^2$ terms. This indicates that there is problem with the bivariate relationship. When this occurs it will further reduce the set of variables under consideration. Examine all removed variables for cause of the signal. Compute the T^2 terms for the remaining subvector. If there is no signal then the cause is from the bivariate relationship.

Step 4

Continue computing the higher order terms in this way until there are no variable left in the reduced set

CHAPTER THREE

METHODOLOGY

3.1 MONITORING MULTIVARIATE TIME SERIES

Nowadays, statistical process control (SPC) applications, involve more than one quality characteristic to be monitored. Monitoring these quality characteristics individually might not be correct enough since there might be correlation among the characteristics therefore, simultaneous monitoring of such characteristics is important since correlation among the characteristics will be considered in the process. In multivariate statistical process control applications, several variables are of interest, there are three main multivariate control charts usually used which comprises the Hotelling's T^2 control chart, Multivariate Exponentially-Weighted Moving Average (MEWMA) and Multivariate Cumulative Sum (MCUSUM) control charts. For the purpose of this research, we consider Hotelling's T^2 control chart technique for monitoring simultaneously several correlated process characteristics. Hotelling's T^2 chart is the multivariate extension of univariate control chart.

However, in real life situation, quality or process characteristics are collected or measured overtime it is of importance to consider the effect of autocorrelation and partial autocorrelation. We suggest vector autoregressive model Var(1), when there is existence of autocorrelation in the data. We are going to confirm the correlation among the process characteristics and then the existence of autocorrelation will be investigated among the process characteristics. By the means of correlogram, we confirmed the existence of autocorrelation in the original data, since the correlogram is visual verification of autocorrelation which gives you an idea as to the order of autocorrelation as well as whether there is existence of autocorrelation in the regression equation.

For the implementation of multivariate statistical control of the process mean vector, when there is significant autocorrelation between two or more characteristics it is suggested that we subject the process to a VAR(p) model.

Also, the normality assumption was verified with the aid of Quantile-Quantile plot (QQ plot). The Q-Q plot is a graphical tool for comparing a dataset with a theoretical distribution and this is achieved by plotting the quantiles against a reference line from a normal distribution, and when the points fall approximately over the line there is evidence that both come from an identical distribution. Meanwhile, QQ plot can only give a simple visual inspection allowing verification of the presence of non-normality.

Another approach is the Royston multivariate normality test which is the appropriate test for multivariate normality, was adopted in this work, this tool is rarely applied in multivariate statistical process control (MSPC) publications. This is due to the fact that, this method lack simplicity and the software availability is limited. Royston (1983) is a multivariate extension of the Shapiro and Wilks normality test. Details on Royston multivariate statistic is given below.

$$H = \frac{e\sum_{j=1}^{p} R_j^{'}}{p} \qquad (3.1)$$

where

$$R_j = \left\{ \Phi^{-1}\left[\frac{\Phi \ -Z_j}{2} \right] \right\}^2$$

Z_j Can be computed For $4 \leq n \leq 11$ and For $12 \leq n \leq 2000$

Then For $4 \leq n \leq 11$:

$$Z_j = \frac{\log\ \gamma - \left[\log\ 1 - W_j\ \right] - \mu}{\sigma}$$

Then For $12 \leq n \leq 2000$

$$Z_j = \frac{\log\ 1 - W_j\ - \mu}{\sigma}$$

W_j is the statistics of the univariate Shapiro-Wilks test

and γ, μ and σ

can be obtained for given n from the Royston approximation (1982)

e in *H* statistic is given by

$$e = \frac{m}{1 +\ m - 1\ \bar{c}}$$

Where $\bar{c}$ is an estimate of the average correlation among the R_j's

Royston's *H* statistics follows approximately χ^2 distribution with *e* degree of freedom.

Whenever Royston's statistic is significant at a specified significance level, it implies that the observations were sampled from non-normal distribution. Therefore, the observations required some adjustment. There are so many transformation methods which include some preliminary ones used in practice such as: logarithm transformation, square root transformation, arcsine transformation and inverse transformation.

Another method of transformation is the well-known Box-Cox Transformation (BCT) that is probably the most used approach for practitioners and professionals of quality control. The BCT is widely used to improve the normality in some practical situations but the disadvantage is that it does not allow negative data values. This approach may not be appropriate since data obtained from the VAR(p) model is not restricted to positive value alone. Finally, another type of transformation, although not so well known is the Johnson's system of distributions recognized as the Johnson Transformation. Johnson Transformation is composed by three distributions named Unbounded (SU), Lognormal (SL), and Bounded (SB). This approach allows transformation of observation into a normal distribution through selecting one of the three of them. The transformations includes

For SU

$$Z = \gamma + \eta \sinh^{-1}\left(\frac{x-\varepsilon}{\lambda}\right) \tag{3.2}$$

Where

$$\eta, \lambda > 0, -\infty < \gamma < \infty, -\infty < \varepsilon < \infty, and -\infty < x < \infty$$

For SL

$$Z = \gamma + \eta \text{In}^{-1} \quad x - \varepsilon \tag{3.3}$$

Where

$$\eta > 0, -\infty < \gamma < \infty, -\infty < \varepsilon < \infty \, and \, \varepsilon < x$$

For SB

$$Z = \gamma + \eta \text{In}\left(\frac{x-\varepsilon}{\lambda+\varepsilon-x}\right) \tag{3.4}$$

Where

Where: x is the transformed value. γ is the shape 1 parameter, η is the shape 2 parameter, ε is the location parameter and λ is the scale parameter

The simplified method is given below;

In the first approach, the study applied Hotelling's T^2, control charts to the multivariate data (original data embedded with autocorrelation, partial autocorrelation and non-normality),in the second approach, the study used a multivariate time series model, that is, Vector Autoregressive, VAR(p) model to handle the autocorrelation in the data, that is, removing autocorrelation. After obtaining a new data set represented by blank noise residuals originated from the VAR(p) model and meeting the assumptions of control charts for independence and normality, Then monitored the residuals of the vector autoregressive model by applying Hotelling's T^2 control chart. The study then compare these approaches, that is, the Hotelling's T^2, one based on original data and Hotelling's T^2 based on the residuals from a VAR (1) model in detecting a shift in the mean of the process using the data collected for verification, the study proceeded to check if the VAR(1) model will give a stable process, otherwise, the out of control points will be removed so that the control chart for phase I can be established. For the phase II, new observation will be taken and subjected to the same procedures so as to have autocorrelated and partially autocorrelated free observations for proper phase II monitoring.

3.2 TIME SERIES MODELLING

3.2.1 VECTOR AUTOREGRESSIVE MODEL (VAR)

Monitoring many quality characteristics at the same time (multivariate process), it is assumed that the characteristics monitored should be independently distributed. But when the observations in the multivariate are auto-correlated, then it is required to remove the autocorrelation from the multivariate process through time series models. The appropriate time series model will generate uncorrelated residuals which can be used to monitor the process, thereby satisfying the assumption of the independency of multivariate observations. The VAR(*p*) model is given as

$$\begin{pmatrix} x_{1t} \\ x_{2t} \\ \vdots \\ x_{kt} \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_k \end{pmatrix} + \begin{pmatrix} \phi_{11} & \phi_{12} & \cdots & \phi_{1k} \\ \phi_{21} & \phi_{22} & \cdots & \phi_{2k} \\ \vdots & \vdots & \vdots & \vdots \\ \phi_{k1} & \phi_{k2} & \cdots & \phi_{kk} \end{pmatrix} \begin{pmatrix} x_{1t-1} \\ x_{2t-1} \\ \vdots \\ x_{kt-1} \end{pmatrix} + \cdots$$

$$\begin{pmatrix} \phi_{11} & \phi_{12} & \cdots & \phi_{1k} \\ \phi_{21} & \phi_{22} & \cdots & \phi_{2k} \\ \vdots & \vdots & \vdots & \vdots \\ \phi_{k1} & \phi_{k2} & \cdots & \phi_{kk} \end{pmatrix} \begin{pmatrix} x_{1t-p} \\ x_{2t-p} \\ \vdots \\ x_{kt-p} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \vdots \\ \varepsilon_{kt} \end{pmatrix} \tag{3.5}$$

for simplicity the VAR(1) model can be expressed in matrix form as

$$\mathbf{x_t} = \mathbf{c} + \mathbf{\Phi x_{t-1}} + \boldsymbol{\varepsilon}_\mathbf{t} \tag{3.6}$$

$\mathbf{X_t}$ denotes, observation vector at time t of *kx1* dimension, $\mathbf{C}$ is constants vector of kx1, $\mathbf{\Phi}$ is autoregressive parameter matrices of *kxk* dimension, $\boldsymbol{\varepsilon_t}$ is an error terms vector of *kx1* dimension, assumed to be normally distributed with zero mean vector and variance covariance matrix $\sum_{k\times k}$.

3.3 ASSUMPTIONS ABOUT THE ERROR TERMS

1. The expected residuals are zero $\quad \mathrm{E}(\varepsilon_{i,t}) = 0$

2. The error terms are not autocorrelated: $\mathrm{E}[\varepsilon_{i,t}.\varepsilon_{j,\tau}] = 0$ with $t \neq \tau$

In this research work we are adopting data from cement factory with five process characteristics (IV Temperature, Kiln Speed, Fuel Burner, Fan Speed and Kiln Feed) to verify the Vector Autoregressive Model (VAR)

The matrix form of the VAR(1) model is given as

$$\begin{pmatrix} X_{1t} \\ X_{2t} \\ X_{3t} \\ X_{4t} \\ X_{5t} \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{pmatrix} + \begin{pmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \phi_{15} \\ \phi_{21} & \phi_{22} & \phi_{23} & \phi_{24} & \phi_{25} \\ \phi_{31} & \phi_{32} & \phi_{33} & \phi_{34} & \phi_{35} \\ \phi_{41} & \phi_{42} & \phi_{43} & \phi_{44} & \phi_{45} \\ \phi_{51} & \phi_{52} & \phi_{53} & \phi_{54} & \phi_{55} \end{pmatrix} \begin{pmatrix} X_{1t-1} \\ X_{2t-1} \\ X_{3t-1} \\ X_{4t-1} \\ X_{5t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \\ \varepsilon_{5t} \end{pmatrix} \qquad (3.7)$$

where the constant vector

$$\mathbf{c} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{pmatrix}$$

And the autoregressive matrix

$$\Phi = \begin{pmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \phi_{15} \\ \phi_{21} & \phi_{22} & \phi_{23} & \phi_{24} & \phi_{25} \\ \phi_{31} & \phi_{32} & \phi_{33} & \phi_{34} & \phi_{35} \\ \phi_{41} & \phi_{42} & \phi_{43} & \phi_{44} & \phi_{45} \\ \phi_{51} & \phi_{52} & \phi_{53} & \phi_{54} & \phi_{55} \end{pmatrix}$$

Also, the vector error

$$\boldsymbol{\varepsilon}_t = \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \\ \varepsilon_{5t} \end{pmatrix}$$

It is assumed that the $\boldsymbol{\varepsilon}_t$ is multivariate normal distribution with mean vector zero and variance covariance matrix $\Sigma_{5\times 5}$.

The model (equation 3.7) can rewritten as

$$x_{1,t} = c_1 + \phi_{11}x_{1,t-1} + \phi_{12}x_{2,t-1} + \phi_{13}x_{3,t-1} + \phi_{14}x_{4,t-1} + \phi_{15}x_{5,t-1} + \varepsilon_{1,t}$$
$$x_{2,t} = c_2 + \phi_{21}x_{1,t-1} + \phi_{22}x_{2,t-1} + \phi_{23}x_{3,t-1} + \phi_{24}x_{4,t-1} + \phi_{25}x_{5,t-1} + \varepsilon_{2,t}$$
$$x_{3,t} = c_3 + \phi_{31}x_{1,t-1} + \phi_{32}x_{2,t-1} + \phi_{33}x_{3,t-1} + \phi_{34}x_{4,t-1} + \phi_{35}x_{5,t-1} + \varepsilon_{3,t}$$
$$x_{4,t} = c_4 + \phi_{41}x_{1,t-1} + \phi_{42}x_{2,t-1} + \phi_{43}x_{3,t-1} + \phi_{44}x_{4,t-1} + \phi_{45}x_{5,t-1} + \varepsilon_{4,t}$$
$$x_{5,t} = c_5 + \phi_{51}x_{1,t-1} + \phi_{52}x_{2,t-1} + \phi_{53}x_{3,t-1} + \phi_{54}x_{4,t-1} + \phi_{55}x_{5,t-1} + \varepsilon_{5,t}$$

This model can be estimated using Ordinary Least Square Regression. That is,

$$\begin{bmatrix} \hat{\phi}_1 \\ \hat{\phi}_2 \\ \vdots \\ \hat{\phi}_5 \end{bmatrix} = \hat{\mathbf{\Phi}} = \left[\mathbf{X}'\mathbf{X}\right]\mathbf{X}'y$$

In this equation (3.7), it is observed that 35 parameters are to be estimated, that is, the constant term vector (5x1), autoregressive parameter matrix (5x5), error term vector (5x1).

In the stationarity of the process, all eigenvalues (λ_i) of autocorrelation coefficient matrix $\boldsymbol{\varphi}$ in a VAR (1) model should be within the unit circle or absolute value of λ_i should be less than one, that is ($|\lambda_i| < 1$). It assumed that all absolute eigenvalues of autocorrelation coefficient matrix $\boldsymbol{\varphi}$ less than one, and process variables have finite mean and finite variance. Therefore the expected value and the covariance matrix estimate of a stationary first order vector autoregressive model VAR(1) is as given below

$$x_t = c + \boldsymbol{\varphi} x_{t-1} + \varepsilon_t$$

Then $E(x_t) = E(c) + \boldsymbol{\varphi} E(x_{t-1}) + E(\varepsilon_t)$

Therefore $\mu = c + \boldsymbol{\varphi}\mu$

Hence $\mu = (I - \boldsymbol{\varphi})^{-1} c$ (3.8)

And the covariance matrix of the stationary first order autoregressive model (VAR(1)) can be estimated using the following

$$\operatorname{cov}(x_t) = \operatorname{cov}(c) + \boldsymbol{\varphi} \operatorname{cov}(x_{t-1}) + \operatorname{cov}(\varepsilon_t)$$

$$\Gamma(0) = \boldsymbol{\varphi}^1 \Gamma(0) \boldsymbol{\varphi} + \sum \text{ (3.9)}$$

where, μ is the expected values of each variable in the vector, I is the identity matrix, $\boldsymbol{\varphi}$ is the matrix of autocorrelation coefficients, c is the constant vector and the constant vector term is assumed to be zero. Also the $\Gamma(0)$ is the covariance matrix of the data which have first order vector autoregressive structure, $\boldsymbol{\varphi}$ is the matrix of autocorrelation coefficients, and $\sum$ is the covariance matrix of the errors. it is observed vividly from the equation (3.9) that covariance of the first order vector autoregressive process is a function of the autocorrelation coefficients and the covariance matrix of the error terms. It is assumed that the multivariate autocorrelated time series is well modeled and control limits are constructed by taking autocorrelation into account. In addition, for multivariate time series, we know that if the parameters are unknown, Hotelling's T^2, statistics will be dependent on sample mean vector and sample variance covariance matrix

3.4 TOWARDS SEARCHING FOR THE INFLUENTIAL VARIABLE

In many industrial settings, process is monitored to ascertain conformability of products produced, when the process gives an out of control alert, indicating production of nonconformable products are about to be produced or has been produced, the operator who is having direct interface and saddled with the responsibility of maintaining a specified quality may not be able to pin point which of the variables in the process is responsible for such ill situation. Therefore, it is important to address such ill situation in this research.

Many approaches have been explored towards identification of variable(s) contributing to out of control situation which includes analytical method, neural network and the graphical approach. An efficient graphical display is very important for practical multivariate process monitoring. The importance of efficient graphical display for successful implementation of multivariate process monitoring methods was well identified by various authors. In this research work, MYT decomposition model was studied, extended and simplified for five variables. Graphical approach will also be adopted because we are given consideration on the operator who is at the first interface to sense the alert and expected to give a responsive measure exposing the influential variable in the process immediately. The graphical methods consist of drawing some type of chart that helps the operator in deciding the variables that might have caused the signal. For example, Blazek*et al* (1987), Subrmanyam and Houshmand (1995), Fuchs and Benjamin(1994), Iglewicz and Hoaglin (1987), Atienza, *et al* (1988) have made a lot of contribution towards different graphical work. The major advantage of the graphical approach is that it helps the operator to easily visualized what the situation of the process like and be able to interpret the results, more work is needed on graphical methods as mentioned by Woodall and Montgomery (1999). Here we consider the procedure developed by Mason, Tracy and Young, known as MYT Decomposition approach, The MTY method consists of decomposing the T^2 value into independent components, each of which reflects the contribution of an individual variable to the out-of-control situation. Therefore, we will adopt the MYT approach and augment it with the proposed graphical approach since a graphical comparison of the set of reduced Hotelling's T^2 statistics to the overall chart will help the operator to identify the variable causing the alarm.

Suppose a Hotelling's T^2 chart gives out of control alarm, one might think of which of the variable(s) could be the cause of such alarm, the decomposition of the T^2 statistic will be required.

3.5 GRAPHICAL APPROACH TO IDENTIFICATION OF INFLUENTIAL VARIABLE USING MYT DECCOMPOSITION

FOR THE CASE OF TWO VARIABLE (p=2)

Consider Hotelling's T^2 chart signal when two variables are being monitored. That is $T_s^2 > h$

The number of unique term required to construct the bar chart can be generated as $T_1^2, T_2^2, T_{2.1}^2$ and $T_{1.2}^2$. T_1^2 and T_2^2 are the unconditional term of the decomposition indicating the contribution of variable one and variable two respectively, while the $T_{2.1}^2$ and $T_{1.2}^2$ are the conditional term indicating the bivariate relationship between variable one and two.

FOR THE CASE OF THREE VARIABLE (p=3)

The number of unique term of the decomposition for three variables is generated as given below

$T_1^2, T_2^2, T_3^2, T_{1.2}^2, T_{1.3}^2, T_{2.1}^2, T_{2.3}^2, T_{3.1}^2, T_{3.2}^2, T_{1.2,3}^2, T_{2.1,3}^2$ and $T_{3.1,3}^2$

That is, we have twelve number of unique term, similarly, the T_1^2, T_2^2, T_3^2 are the unconditional term of the decomposition indicating the contribution of the variable one, variable two variable three independently, also the conditional term $T_{1.2}^2, T_{1.3}^2, T_{2.1}^2, T_{3.1}^2, T_{2.3}^2, T_{3.2}^2$ indicating the bivariate relationship between the variable one, two and three, likewise, $T_{1.2,3}^2, T_{2.1,3}^2, T_{3.1,2}^2$ indicate the multivariate relationship that exist between variable one, two and three.

FOR THE CASE OF FOUR VARIABLES(p=4)

The number of unique term of the decomposition for four variables is given as below

$T_1^2, T_2^2, T_3^2, T_4^2, T_{1.2}^2, T_{1.3}^2, T_{1.4}^2, T_{2.1}^2, T_{3.1}^2, T_{4.1}^2, T_{2.3}^2, T_{2.4}^2, T_{3.2}^2, T_{3.4}^2, T_{4.2}^2, T_{4.3}^2, T_{1.2,3}^2, T_{1.2,4}^2, T_{1.3,4}^2,$
$T_{2.1,3}^2, T_{2.1,4}^2 T_{3.1,2}^2, T_{2.3,4}^2, T_{3.1,2}^2, T_{3.1,4}^2, T_{3.2,4}^2, T_{4.1,2}^2, T_{4.1,3}^2, T_{4.2,3}^2, T_{1.2,3,4}^2, T_{2.1,3,4}^2, T_{3.1,2,4}^2, T_{4.1,2,3}^2,$

That is, we have thirty two numbers of unique terms, where $T_1^2, T_2^2, T_3^2, T_4^2$ are the unconditional terms of the decomposition, indicating the contribution of the variable one, variable two, variable three and variable four independently, also the conditional terms

$T_{1.2}^2, T_{1.3}^2, T_{1.4}^2, T_{2.1}^2, T_{3.1}^2, T_{4.1}^2, T_{2.3}^2, T_{2.4}^2, T_{3.2}^2, T_{3.4}^2, T_{4.2}^2, T_{4.3}^2$ also indicating the bivariate relationship between the variable one, two, three and four, likewise,

$T_{1.2,3}^2, T_{1.2,4}^2, T_{1.3,4}^2, T_{2.1,3}^2, T_{2.1,4}^2 T_{3.1,2}^2,, T_{2.3,4}^2, T_{3.1,2}^2, T_{3.1,4}^2, T_{3.2,4}^2, T_{4.1,2}^2, T_{4.1,3}^2, T_{4.2,3}^2, T_{1.2,3,4}^2, T_{2.1,3,4}^2, T_{3.1,2,4}^2, T_{4.1,2,3}^2$
indicating the multivariate relationship that exist between variable one, two, three and four.

FOR THE CASE OF FIVE VARIABLES(p=5)

The number of unique term of the decomposition for five variables is given as below

$T_1^2, T_2^2, T_3^2, T_4^2, T_5^2, T_{1.2}^2, T_{1.3}^2, T_{1.4}^2, T_{1.5}^2, T_{2.1}^2, T_{2.3}^2, T_{2.4}^2, T_{2.5}^2, T_{3.1}^2, T_{3.2}^2, T_{3.4}^2, T_{3.5}^2, T_{4.1}^2, T_{4.2}^2, T_{4.3}^2, T_{4.5}^2, T_{5.1}^2, T_{5.2}^2, T_{5.3}^2, T_{5.4}^2, T_{1.2,3}^2,$
$T_{1.2,4}^2, T_{1.2,5}^2, T_{1.3,4}^2, T_{1.3,5}^2, T_{1.4,5}^2 T_{2.1,3}^2, T_{2.1,4}^2, T_{2.1,5}^2, T_{2.3,4}^2, T_{2.3,5}^2, T_{2.4,5}^2, T_{3.1,2}^2, T_{3.1,4}^2, T_{3.1,5}^2, T_{3.2,4}^2, T_{3.2,5}^2, T_{3.4,5}^2, T_{4.1,2}^2, T_{4.1,3}^2, T_{4.1,5}^2,$
$T_{4.2,3}^2, T_{4.2,5}^2, T_{4.3,5}^2, T_{5.1,2}^2, T_{5.1,3}^2, T_{5.1,4}^2, T_{5.2,3}^2, T_{5.2,4}^2, T_{5.3,4}^2, T_{1.2,3,4}^2, T_{1.2,3,5}^2, T_{1.2,4,5}^2, T_{1.3,4,5}^2, T_{2.1,3,4}^2, T_{2.1,3,5}^2, T_{2.1,4,5}^2, T_{2.3,4,5}^2, T_{3.1,2,4}^2,$
$T_{3.1,2,5}^2, T_{3.1,4,5}^2, T_{3.2,4,5}^2, T_{4.1,2,3}^2, T_{4.1,2,5}^2, T_{4.1,3,5}^2, T_{4.2,3,5}^2, T_{5.1,2,3}^2, T_{5.1,2,4}^2, T_{5.1,3,4}^2, T_{5.2,3,4}^2, T_{1.2,3,4,5}^2, T_{2.1,3,4,5}^2, T_{3.1,2,4,5}^2, T_{4.1,2,3,5}^2, T_{5.1,2,3,4}^2$
That is, we have eighty numbers of unique terms, where $T_1^2, T_2^2, T_3^2, T_4^2, T_5^2$ are the unconditional terms of the decomposition, indicating the contribution of the variable one, variable two, variable three and variable four, and variable five independently, and the conditional terms

$T_{1.2}^2, T_{1.3}^2, T_{1.4}^2, T_{1.5}^2, T_{2.1}^2, T_{2.3}^2, T_{2.4}^2, T_{2.5}^2, T_{3.1}^2, T_{3.2}^2, T_{3.4}^2, T_{3.5}^2, T_{4.1}^2, T_{4.2}^2, T_{4.3}^2, T_{4.5}^2, T_{5.1}^2, T_{5.2}^2, T_{5.3}^2, T_{5.4}^2$

indicating the bivariate relationship between the variable one, two, three, four, and five likewise,

$T^2_{1.2,3}, T^2_{1.2,4}, T^2_{1.2,5}, T^2_{1.3,4}, T^2_{1.3,5}, T^2_{1.4,5} T^2_{2.1,3}, T^2_{2.1,4}, T^2_{2.1,5}, T^2_{2.3,4}, T^2_{2.3,5}, T^2_{2.4,5}, T^2_{3.1,2}, T^2_{3.1,4}, T^2_{3.1,5}, T^2_{3.2,4}, T^2_{3.2,5}, T^2_{3.4,5}, T^2_{4.1,2}, T^2_{4.1,3}, T^2_{4.1,5},$
$T^2_{4.2,3}, T^2_{4.2,5}, T^2_{4.3,5}, T^2_{5.1,2}, T^2_{5.1,3}, T^2_{5.1,4}, T^2_{5.2,3}, T^2_{5.2,4}, T^2_{5.3,4}, T^2_{1.2,3,4}, T^2_{1.2,3,5}, T^2_{1.2,4,5}, T^2_{1.3,4,5}, T^2_{2.1,3,4}, T^2_{2.1,3,5}, T^2_{2.1,4,5}, T^2_{2.3,4,5}, T^2_{3.1,2,4},$
$T^2_{3.1,2,5}, T^2_{3.1,4,5}, T^2_{3.2,4,5}, T^2_{4.1,2,3}, T^2_{4.1,2,5}, T^2_{4.1,3,5}, T^2_{4.2,3,5}, T^2_{5.1,2,3}, T^2_{5.1,2,4}, T^2_{5.1,3,4}, T^2_{5.2,3,4}, T^2_{1.2,3,4,5}, T^2_{2.1,3,4,5}, T^2_{3.1,2,4,5}, T^2_{4.1,2,3,5}, T^2_{5.1,2,3,4}$
indicating the multivariate relationship that exist between variable one, two, three, four and five.

It is obvious that the more the number of variables, the bigger the number of decomposition partition and the unique terms to be evaluated. Generally, there is $p!$ different decomposition partitions that can generate the same overall T^2 statistic (Mason, Young & Tracy, 1995) and the number of unique term required to construct the decomposition chart is given as $p \times 2^{(p-1)}$, while the number of possible term is given as $p! \times p$.

3.6 ILLUSTRATION OF THE INVARIANT PROPERTY OF THE DECOMPOSITION FOR THE FIVE PROCESS VARIABLES

$$T^2 = T_1^2 + T_{2.1}^2 + T_{3.1,2}^2 + T_{4.1,2,3}^2 + T_{5.1,2,3,4}^2$$
$$T^2 = T_1^2 + T_{3.1}^2 + T_{4.1,3}^2 + T_{5.1,3,4}^2 + T_{2.1,3,4,5}^2$$
$$T^2 = T_1^2 + T_{4.1}^2 + T_{5.1,4}^2 + T_{2.1,4,5}^2 + T_{3.1,2,4,5}^2$$
$$T^2 = T_1^2 + T_{5.1}^2 + T_{2.1,5}^2 + T_{3.1,2,5}^2 + T_{4.1,2,3,5}^2$$
$$T^2 = T_1^2 + T_{2.1}^2 + T_{4.2,1}^2 + T_{5.1,2,4}^2 + T_{3.1,2,4,5}^2$$
$$T^2 = T_1^2 + T_{3.1}^2 + T_{2.1,3}^2 + T_{4.1,2,3}^2 + T_{5.1,2,3,4}^2$$
$$T^2 = T_1^2 + T_{4.1}^2 + T_{3.1,4}^2 + T_{5.1,3,4}^2 + T_{2.1,3,4,5}^2$$
$$T^2 = T_1^2 + T_{5.1}^2 + T_{3.1,5}^2 + T_{4.1,2,5}^2 + T_{2.1,3,4,5}^2$$
$$T^2 = T_1^2 + T_{2.1}^2 + T_{3.1,2}^2 + T_{5.1,2,3}^2 + T_{4.1,2,3,5}^2$$
$$T^2 = T_1^2 + T_{3.1}^2 + T_{4.1,3}^2 + T_{2.1,3,4}^2 + T_{5.1,2,3,4}^2$$
$$T^2 = T_1^2 + T_{4.1}^2 + T_{5.1,4}^2 + T_{3.1,4,5}^2 + T_{2.1,3,4,5}^2$$
$$T^2 = T_1^2 + T_{5.1}^2 + T_{2.1,5}^2 + T_{4.1,2,5}^2 + T_{3.1,2,4,5}^2$$
$$T^2 = T_1^2 + T_{2.1}^2 + T_{4.1,2}^2 + T_{5.1,2,4}^2 + T_{3.1,2,4,5}^2$$
$$T^2 = T_1^2 + T_{3.1}^2 + T_{2.1,3}^2 + T_{5.1,2,3}^2 + T_{4.1,2,3,4}^2$$
$$T^2 = T_1^2 + T_{4.1}^2 + T_{3.1,4}^2 + T_{2.1,3,4}^2 + T_{5.1,2,3,4}^2$$
$$T^2 = T_1^2 + T_{5.1}^2 + T_{3.1,5}^2 + T_{2.1,3,5}^2 + T_{4.1,2,3,5}^2$$
$$T^2 = T_1^2 + T_{2.1}^2 + T_{5.1,2}^2 + T_{4.1,2,5}^2 + T_{3.1,2,4,5}^2$$
$$T^2 = T_1^2 + T_{3.1}^2 + T_{5.1,3}^2 + T_{4.1,2,5}^2 + T_{2.1,3,4,5}^2$$
$$T^2 = T_1^2 + T_{4.1}^2 + T_{2.1,4}^2 + T_{5.1,2,4}^2 + T_{3.1,2,4,5}^2$$
$$T^2 = T_1^2 + T_{5.1}^2 + T_{4.1,3}^2 + T_{3.1,4,5}^2 + T_{2.1,3,4,5}^2$$
$$T^2 = T_1^2 + T_{2.1}^2 + T_{5.1,2}^2 + T_{3.1,2,5}^2 + T_{4.1,2,3,5}^2$$
$$T^2 = T_1^2 + T_{3.1}^2 + T_{5.1,3}^2 + T_{2.1,3,5}^2 + T_{4.1,2,3,5}^2$$
$$T^2 = T_1^2 + T_{4.1}^2 + T_{2.1,4}^2 + T_{3.1,2,4}^2 + T_{5.1,2,3,4}^2$$
$$T^2 = T_1^2 + T_{5.1}^2 + T_{4.1,5}^2 + T_{2.1,4,5}^2 + T_{3.1,2,4,5}^2$$

$$T^2 = T_2^2 + T_{1.2}^2 + T_{3.1,2}^2 + T_{4.1,2,3}^2 + T_{5.1,2,3,4}^2$$
$$T^2 = T_2^2 + T_{3.2}^2 + T_{4.2,3}^2 + T_{5.2,3,4}^2 + T_{1.2,3,4,5}^2$$
$$T^2 = T_2^2 + T_{4.2}^2 + T_{1.2,4}^2 + T_{3.1,2,4}^2 + T_{5.1,2,3,4}^2$$
$$T^2 = T_2^2 + T_{5.2}^2 + T_{3.2,5}^2 + T_{4.2,3,5}^2 + T_{1.2,3,4,5}^2$$
$$T^2 = T_2^2 + T_{1.2}^2 + T_{3.1,2}^2 + T_{5.1,2,3}^2 + T_{4.1,2,3,5}^2$$
$$T^2 = T_2^2 + T_{3.2}^2 + T_{5.2,3}^2 + T_{1.2,3,5}^2 + T_{4.1,2,3,5}^2$$
$$T^2 = T_2^2 + T_{4.2}^2 + T_{5.2,4}^2 + T_{1.2,4,5}^2 + T_{3.1,2,3,4}^2$$
$$T^2 = T_2^2 + T_{5.2}^2 + T_{1.2,5}^2 + T_{3.1,2,5}^2 + T_{4.1,2,3,5}^2$$
$$T^2 = T_2^2 + T_{1.2}^2 + T_{5.1,2}^2 + T_{4.1,2,5}^2 + T_{3.1,2,4,5}^2$$
$$T^2 = T_2^2 + T_{3.2}^2 + T_{1.2,3}^2 + T_{4.1,2,3}^2 + T_{5.1,2,3,4}^2$$
$$T^2 = T_2^2 + T_{4.2}^2 + T_{3.2,4}^2 + T_{1.2,3,4}^2 + T_{5.1,2,3,4,}^2$$
$$T^2 = T_2^2 + T_{5.2}^2 + T_{1.2,5}^2 + T_{4.1,2,5}^2 + T_{3.1,2,4,5}^2$$
$$T^2 = T_2^2 + T_{1.2}^2 + T_{4.1,2}^2 + T_{3.1,2,4}^2 + T_{5.1,2,3,4}^2$$
$$T^2 = T_2^2 + T_{3.2}^2 + T_{4.2,3}^2 + T_{1.2,3,4}^2 + T_{5.1,2,3,4}^2$$
$$T^2 = T_2^2 + T_{4.2}^2 + T_{3.2,4}^2 + T_{5.2,3,4}^2 + T_{1.2,3,4,5}^2$$
$$T^2 = T_2^2 + T_{5.2}^2 + T_{3.2,5}^2 + T_{4.2,3,5}^2 + T_{1.2,3,4,5}^2$$
$$T^2 = T_2^2 + T_{1.2}^2 + T_{4.1,2}^2 + T_{5.1,2,4}^2 + T_{3.1,2,4,5}^2$$
$$T^2 = T_2^2 + T_{3.2}^2 + T_{1.2,3}^2 + T_{5.1,2,3}^2 + T_{4.1,2,3,5}^2$$
$$T^2 = T_2^2 + T_{4.2}^2 + T_{5.2,4}^2 + T_{3.2,4,5}^2 + T_{1.2,3,4,5}^2$$
$$T^2 = T_2^2 + T_{5.2}^2 + T_{4.2,5}^2 + T_{3.2,4,5}^2 + T_{1.2,3,4,5}^2$$
$$T^2 = T_2^2 + T_{1.2}^2 + T_{5.1,2}^2 + T_{3.1,2,5}^2 + T_{4.1,2,3,5}^2$$
$$T^2 = T_2^2 + T_{3.2}^2 + T_{5.2,3}^2 + T_{4.2,3,5}^2 + T_{1.2,3,4,5}^2$$
$$T^2 = T_2^2 + T_{4.2}^2 + T_{1.2,4}^2 + T_{5.1,2,4}^2 + T_{3.1,2,4,5}^2$$
$$T^2 = T_2^2 + T_{5.2}^2 + T_{4.2,5}^2 + T_{3.2,4,5}^2 + T_{1.2,3,4,5}^2$$

$$T^2 = T_3^2 + T_{1.3}^2 + T_{2.1,3}^2 + T_{4.1,2,3}^2 + T_{5.1,2,3,4}^2$$
$$T^2 = T_3^2 + T_{2.3}^2 + T_{1.2,3}^2 + T_{4.1,2,3}^2 + T_{5.1,2,3,4}^2$$
$$T^2 = T_3^2 + T_{4.3}^2 + T_{1.3,4}^2 + T_{2.1,3,4}^2 + T_{5.1,2,3,4}^2$$
$$T^2 = T_3^2 + T_{5.3}^2 + T_{4.3,5}^2 + T_{1.3,4,5}^2 + T_{2.1,3,4,5}^2$$
$$T^2 = T_3^2 + T_{1.3}^2 + T_{2.1,3}^2 + T_{5.1,2,3}^2 + T_{4.1,2,3,5}^2$$
$$T^2 = T_3^2 + T_{2.3}^2 + T_{1.2,3}^2 + T_{5.1,2,3}^2 + T_{4.1,2,3,5}^2$$
$$T^2 = T_3^2 + T_{4.3}^2 + T_{1.3,4}^2 + T_{5.1,3,4}^2 + T_{2.1,3,4,5}^2$$
$$T^2 = T_3^2 + T_{5.3}^2 + T_{4.3,5}^2 + T_{2.3,4,5}^2 + T_{1.2,3,4,5}^2$$
$$T^2 = T_3^2 + T_{1.3}^2 + T_{4.1,3}^2 + T_{5.1,3,4}^2 + T_{2.1,3,4,5}^2$$
$$T^2 = T_3^2 + T_{2.3}^2 + T_{4.2,3}^2 + T_{1.2,3,4}^2 + T_{5.1,2,3,4}^2$$
$$T^2 = T_3^2 + T_{4.3}^2 + T_{2.3,4}^2 + T_{1.2,3,4}^2 + T_{5.1,2,3,4}^2$$
$$T^2 = T_3^2 + T_{5.3}^2 + T_{1.3,5}^2 + T_{2.1,3,5}^2 + T_{4.1,2,3,5}^2$$
$$T^2 = T_3^2 + T_{1.2}^2 + T_{4.1,3}^2 + T_{2.1,3,4}^2 + T_{5.1,2,3,4}^2$$
$$T^2 = T_3^2 + T_{2.3}^2 + T_{4.2,3}^2 + T_{5.2,3,4}^2 + T_{1.2,3,4,5}^2$$
$$T^2 = T_3^2 + T_{4.3}^2 + T_{2.3,4}^2 + T_{5.2,3,4}^2 + T_{1.2,3,4,5}^2$$
$$T^2 = T_3^2 + T_{5.3}^2 + T_{1.3,5}^2 + T_{4.1,3,5}^2 + T_{2.1,3,4,5}^2$$
$$T^2 = T_3^2 + T_{1.3}^2 + T_{5.1,3}^2 + T_{2.1,3,5}^2 + T_{4.1,2,3,5}^2$$
$$T^2 = T_3^2 + T_{2.3}^2 + T_{5.2,3}^2 + T_{4.2,3,5}^2 + T_{1.2,3,4,5}^2$$
$$T^2 = T_3^2 + T_{4.3}^2 + T_{5.3,4}^2 + T_{1.3,4,5}^2 + T_{2.1,3,4,5}^2$$
$$T^2 = T_3^2 + T_{5.3}^2 + T_{2.3,5}^2 + T_{1.2,3,5}^2 + T_{4.1,2,3,5}^2$$
$$T^2 = T_3^2 + T_{1.3}^2 + T_{5.1,3}^2 + T_{4.1,3,5}^2 + T_{2.1,3,4,5}^2$$
$$T^2 = T_3^2 + T_{2.3}^2 + T_{5.2,3}^2 + T_{1.2,3,5}^2 + T_{4.1,2,3,5}^2$$
$$T^2 = T_3^2 + T_{4.3}^2 + T_{5.3,4}^2 + T_{2.3,4,5}^2 + T_{1.2,3,4,5}^2$$
$$T^2 = T_3^2 + T_{5.3}^2 + T_{2.3,5}^2 + T_{4.2,3,5}^2 + T_{1.2,3,4,5}^2$$

$$T^2 = T_4^2 + T_{1.4}^2 + T_{2.1,4}^2 + T_{3.1,2,4}^2 + T_{5.1,2,3,4}^2$$
$$T^2 = T_4^2 + T_{2.4}^2 + T_{1.2,4}^2 + T_{3.1,2,4}^2 + T_{5.1,2,3,4}^2$$
$$T^2 = T_4^2 + T_{3.4}^2 + T_{1.3,4}^2 + T_{2.1,3,4}^2 + T_{5.1,2,3,4}^2$$
$$T^2 = T_4^2 + T_{5.4}^2 + T_{1.4,5}^2 + T_{2.1,4,5}^2 + T_{3.1,2,4,5}^2$$
$$T^2 = T_4^2 + T_{1.4}^2 + T_{2.1,4}^2 + T_{5.1,2,4}^2 + T_{3.1,2,4,5}^2$$
$$T^2 = T_4^2 + T_{2.4}^2 + T_{1.2,4}^2 + T_{5.1,2,4}^2 + T_{3.1,2,4,5}^2$$
$$T^2 = T_4^2 + T_{3.4}^2 + T_{1.3,4}^2 + T_{5.1,3,4}^2 + T_{2.1,3,4,5}^2$$
$$T^2 = T_4^2 + T_{5.4}^2 + T_{1.4,5}^2 + T_{3.1,4,5}^2 + T_{2.1,3,4,5}^2$$
$$T^2 = T_4^2 + T_{1.4}^2 + T_{3.1,4}^2 + T_{2.1,3,4}^2 + T_{5.1,2,3,4}^2$$
$$T^2 = T_4^2 + T_{2.4}^2 + T_{3.2,4}^2 + T_{1.2,3,4}^2 + T_{5.1,2,3,4}^2$$
$$T^2 = T_4^2 + T_{3.4}^2 + T_{2.3,4}^2 + T_{1.2,3,4}^2 + T_{5.1,2,3,4}^2$$
$$T^2 = T_4^2 + T_{5.4}^2 + T_{2.4,5}^2 + T_{1.2,4,5}^2 + T_{3.1,2,4,5}^2$$
$$T^2 = T_4^2 + T_{1.4}^2 + T_{3.1,4}^2 + T_{5.1,3,4}^2 + T_{2.1,3,4,5}^2$$
$$T^2 = T_4^2 + T_{2.4}^2 + T_{3.2,4}^2 + T_{5.2,3,4}^2 + T_{1.2,3,4,5}^2$$
$$T^2 = T_4^2 + T_{3.4}^2 + T_{2.3,4}^2 + T_{5.2,3,4}^2 + T_{1.2,3,4,5}^2$$
$$T^2 = T_4^2 + T_{5.4}^2 + T_{2.4,5}^2 + T_{3.2,4,5}^2 + T_{1.2,3,4,5}^2$$
$$T^2 = T_4^2 + T_{1.4}^2 + T_{5.1,4}^2 + T_{2.1,4,5}^2 + T_{3.1,2,4,5}^2$$
$$T^2 = T_4^2 + T_{2.4}^2 + T_{5.2,4}^2 + T_{3.2,4,5}^2 + T_{1.2,3,4,5}^2$$
$$T^2 = T_4^2 + T_{3.4}^2 + T_{5.3,4}^2 + T_{2.3,4,5}^2 + T_{1.2,3,4,5}^2$$
$$T^2 = T_4^2 + T_{5.4}^2 + T_{3.4,5}^2 + T_{1.3,4,5}^2 + T_{2.1,3,4,5}^2$$
$$T^2 = T_4^2 + T_{1.4}^2 + T_{5.1,4}^2 + T_{3.1,4,5}^2 + T_{2.1,3,4,5}^2$$
$$T^2 = T_4^2 + T_{2.4}^2 + T_{5.2,4}^2 + T_{1.2,4,5}^2 + T_{3.1,2,4,5}^2$$
$$T^2 = T_4^2 + T_{3.4}^2 + T_{5.3,4}^2 + T_{1.3,4,5}^2 + T_{2.1,3,4,5}^2$$
$$T^2 = T_4^2 + T_{5.4}^2 + T_{3.4,5}^2 + T_{2.3,4,5}^2 + T_{1.2,3,4,5}^2$$

$$
\begin{aligned}
T^2 &= T_5^2 + T_{1.5}^2 + T_{2.1,5}^2 + T_{3.1,2,5}^2 + T_{4.1,2,3,5}^2 \\
T^2 &= T_5^2 + T_{2.5}^2 + T_{1.2,5}^2 + T_{3.1,2,5}^2 + T_{4.2,3,4,5}^2 \\
T^2 &= T_5^2 + T_{3.5}^2 + T_{1.3,5}^2 + T_{2.1,3,5}^2 + T_{4.1,2,3,5}^2 \\
T^2 &= T_5^2 + T_{4.5}^2 + T_{1.4,5}^2 + T_{2.1,4,5}^2 + T_{3.1,2,4,5}^2 \\
T^2 &= T_5^2 + T_{1.5}^2 + T_{2.1,5}^2 + T_{4.1,2,5}^2 + T_{3.1,2,3,5}^2 \\
T^2 &= T_5^2 + T_{2.5}^2 + T_{1.2,5}^2 + T_{4.1,2,5}^2 + T_{3.1,2,4,5}^2 \\
T^2 &= T_5^2 + T_{3.5}^2 + T_{1.3,5}^2 + T_{4.1,3,5}^2 + T_{2.1,3,4,5}^2 \\
T^2 &= T_5^2 + T_{4.5}^2 + T_{1.4,5}^2 + T_{3.1,4,5}^2 + T_{2.1,3,4,5}^2 \\
T^2 &= T_5^2 + T_{1.5}^2 + T_{3.1,5}^2 + T_{2.1,3,5}^2 + T_{4.1,2,3,5}^2 \\
T^2 &= T_5^2 + T_{2.5}^2 + T_{3.2,5}^2 + T_{1.2,3,5}^2 + T_{4.1,2,3,5}^2 \\
T^2 &= T_5^2 + T_{3.5}^2 + T_{2.3,5}^2 + T_{1.2,3,5}^2 + T_{4.1,2,3,5}^2 \\
T^2 &= T_5^2 + T_{4.5}^2 + T_{2.4,5}^2 + T_{1.2,4,5}^2 + T_{3.1,2,4,5}^2 \\
T^2 &= T_5^2 + T_{1.5}^2 + T_{3.1,5}^2 + T_{4.1,3,5}^2 + T_{2.1,3,4,5}^2 \\
T^2 &= T_5^2 + T_{2.5}^2 + T_{3.2,5}^2 + T_{4.2,3,5}^2 + T_{1.2,3,4,5}^2 \\
T^2 &= T_5^2 + T_{3.5}^2 + T_{2.3,5}^2 + T_{4.2,3,5}^2 + T_{1.2,3,4,5}^2 \\
T^2 &= T_5^2 + T_{4.5}^2 + T_{2.4,5}^2 + T_{3.2,4,5}^2 + T_{1.2,3,4,5}^2 \\
T^2 &= T_5^2 + T_{1.5}^2 + T_{4.1,5}^2 + T_{2.1,4,5}^2 + T_{3.1,2,4,5}^2 \\
T^2 &= T_5^2 + T_{2.5}^2 + T_{4.2,5}^2 + T_{1.2,4,5}^2 + T_{3.1,2,4,5}^2 \\
T^2 &= T_5^2 + T_{3.5}^2 + T_{4.3,5}^2 + T_{1.3,4,5}^2 + T_{2.1,3,4,5}^2 \\
T^2 &= T_5^2 + T_{4.5}^2 + T_{3.4,5}^2 + T_{1.3,4,5}^2 + T_{2.1,3,4,5}^2 \\
T^2 &= T_5^2 + T_{1.5}^2 + T_{4.1,5}^2 + T_{3.1,4,5}^2 + T_{2.1,3,4,5}^2 \\
T^2 &= T_5^2 + T_{2.5}^2 + T_{4.2,5}^2 + T_{3.2,4,5}^2 + T_{1.2,3,4,5}^2 \\
T^2 &= T_5^2 + T_{3.5}^2 + T_{4.3,5}^2 + T_{2.3,4,5}^2 + T_{1.2,3,4,5}^2 \\
T^2 &= T_5^2 + T_{4.5}^2 + T_{3.4,5}^2 + T_{2.3,4,5}^2 + T_{1.2,3,4,5}^2
\end{aligned}
\tag{3.10}
$$

It is obvious that the larger the number of variables, the larger the number of terms, which makes the computation become burdensome. Despite the complexity, it is obviously shown that the number of decomposition partition required for five variables is 120 decomposition partitions

3.7 ESTIMATION OF THE TERMS OF DECOMPOSITION

In this work, there is need to obtain the estimation of the terms, the computation procedure is as given. Consider five process characteristics that is $p = 5$, then the overall T^2 containing the five variables will be denoted by $T^2_{(x_1, x_2, x_3, x_4, x_5)}$

Now, to obtain any of the unconditional unique terms T_j^2

$$T_j^2 = \frac{(x_j - \bar{x}_j)^2}{s_j^2}$$

$$j = 1, 2, \ldots p$$

That is T_j^2 is the square of the univariate t statistics of the j^{th} observation.

Thus estimate of $T_{x_1}^2 = \dfrac{(x_1 - \bar{x}_1)^2}{s_1^2}$,

Where $\bar{x} = \dfrac{\sum x_j}{n}$ and $s_j^2 = \dfrac{\sum \left(x_j - \bar{x}\right)^2}{n-1}$.

Now, Given mean and covariance of $T^2_{(x_1, x_2)}$ as

$\bar{X}^{(2)} = \begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \end{pmatrix} \quad S_{22} = \begin{pmatrix} S_1^2 & S_{12} \\ S_{21} & S_2^2 \end{pmatrix}$ respectively, Then the estimation of $T^2_{(x_1,x_2)}$ will be obtained using

$$T^2_{(x_1,x_2)} = (X^2 - \bar{X}^{(2)})' S_{22}^{-1} (X^2 - \bar{X}^{(2)})$$

Similarly, given the mean and variance-covariance matrix of the $T^2_{(x_1,x_2,x_3)}$ as

$$\bar{X}^{(3)} = \begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \end{pmatrix}, \quad S_{33} = \begin{pmatrix} S_1^2 & S_{12} & S_{13} \\ S_{21} & S_2^2 & S_{23} \\ S_{31} & S_{32} & S_3^2 \end{pmatrix} \text{respectively,}$$

then the estimation of $T^2_{(x_1,x_2,x_3)}$ will be obtained using

$$T^2_{(x_1,x_2,x_3,)} = (X^3 - \bar{X}^{(3)})' S_{33} (X^3 - \bar{X}^{(3)})$$

Also, given the mean and variance-covariance matrix of the $T^2_{(x_1,x_2,x_3,x_4)}$ as

$$\bar{X}^{(4)} = \begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \\ \bar{x}_4 \end{pmatrix}, \quad S_{44} = \begin{pmatrix} S_1^2 & S_{12} & S_{13} & S_{14} \\ S_{21} & S_2^2 & S_{23} & S_{24} \\ S_{31} & S_{32} & S_3^2 & S_{34} \\ S_{41} & S_{42} & S_{43} & S_4^2 \end{pmatrix} \text{ respectively.}$$

Then the estimation of $T^2_{(x_1,x_2,x_3,x_4)}$ will be obtained using

$$T^2_{(x_1,x_2,x_3,x_4)} = (X^4 - \bar{X}^{(4)})' S_{44} (X^4 - \bar{X}^{(4)}) \text{ matrix,}$$

Also, given the mean and variance-covariance matrix of the $T^2_{(x_1,x_2,x_3,x_4,x_5)}$ as

$$\bar{X}^{(5)} = \begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \\ \bar{x}_4 \\ \bar{x}_5 \end{pmatrix}, \quad S_{55} = \begin{pmatrix} S_1^2 & S_{12} & S_{13} & S_{14} & S_{15} \\ S_{21} & S_2^2 & S_{23} & S_{24} & S_{25} \\ S_{31} & S_{32} & S_3^2 & S_{34} & S_{35} \\ S_{41} & S_{42} & S_{43} & S_4^2 & S_{45} \\ S_{51} & S_{52} & S_{53} & S_{54}^2 & S_5 \end{pmatrix}$$ respectively.

Then the estimation of $T^2_{(x_1,x_2,x_3,x_4,x_5)}$ will be obtained using

$T^2_{(x_1,x_2,x_3,x_4,x_5)} = (X^5 - \bar{X}^{(5)})' S_{55} (X^5 - \bar{X}^{(5)})$ matrix,

Now, to obtain any of the conditional unique terms

Say $T^2_{i.j} = T^2_{(x_i,x_j)} - T^2_{x_i}$

$i = 1, 2, \ldots i$, $j = 1, 2, \ldots j$

Such that $i \neq j$

Likewise $T^2_{i.j,k} = T^2_{(x_i,x_j,x_k)} - T^2_{(x_j,x_k)}$

For $i = 1, 2, \ldots i$, $j = 1, 2, \ldots j$, $k = 1, 2, \ldots, k$,

Such that $i \neq j \neq k$

Similarly, $T^2_{i.j,k,l} = T^2_{(x_i,x_j,x_k,x_l)} - T^2_{(x_j,x_k,x_l)}$

For $i = 1, 2, \ldots i$, $j = 1, 2, \ldots j$, $k = 1, 2, \ldots, k$, $l = 1, 2, \ldots l$

Such that $i \neq j \neq k \neq l$

Finally, $T^2_{i.j,k,l,m} = T^2_{(x_i,x_j,x_k,x_l,x_m)} - T^2_{(x_j,x_k,x_l,x_m)}$

For $i = 1, 2, \ldots i$, $j = 1, 2, \ldots j$, $k = 1, 2, \ldots, k$, $l = 1, 2, \ldots l$ $m = 1, 2, \ldots m$.

One of the disadvantages of MYT decomposition approach is it complex calculation involved when the process characteristics is large, despite it computational complexity, all the unique term has to be computed.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 RESULTS

The data for this research was collected from Ashaka Cement factory in Gombe State; the data contain 84 data point, embedded with autocorrelation and partial autocorrelation and non-normality.

Below are the results of various approaches to the Hotelling's T^2 control chart

HOTELLING'S T^2 CONTROL CHART CONSTRUCTED WITH ORIGINAL DATA

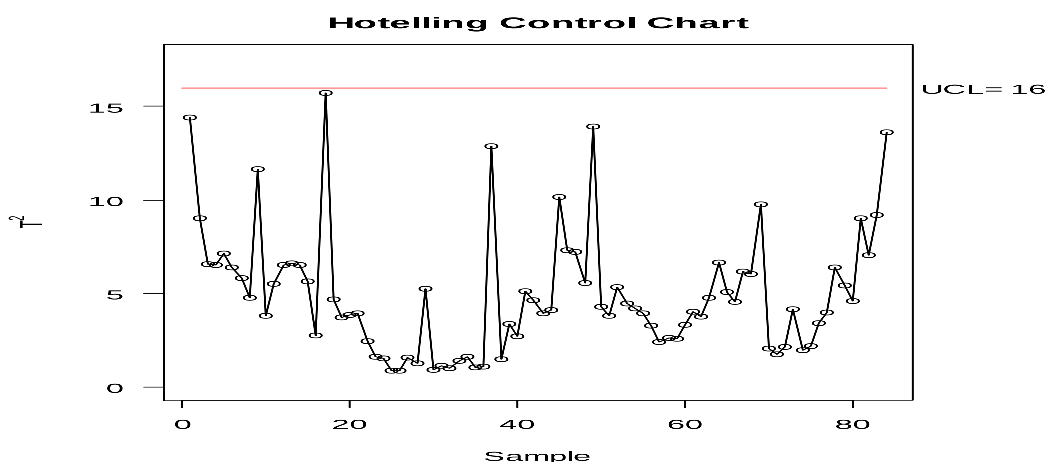

Figure 4.1: The Hotelling's T^2 control chart for original data

The control chart in figure 4.1 is the Hotelling's T^2 control chart constructed from the original data, the original data is embedded with autocorrelation and partial autocorrelation and non- normality, the test for the evidence of autocorrelation and partial autocorrelation can be seen in the appendix 6, likewise the test for the normality can be seen in the appendix 9 and appendix 10. From these appendices, it is obvious that the original data contained autocorrelation and partial autocorrelation and non-normality. The Hotelling's T^2 above shows that the process is in control, since there is no single point falling outside the control limit.

HOTELLING'S T^2 CONTROL CHART CONSTRUCTED FROM RESIDUALSDATA

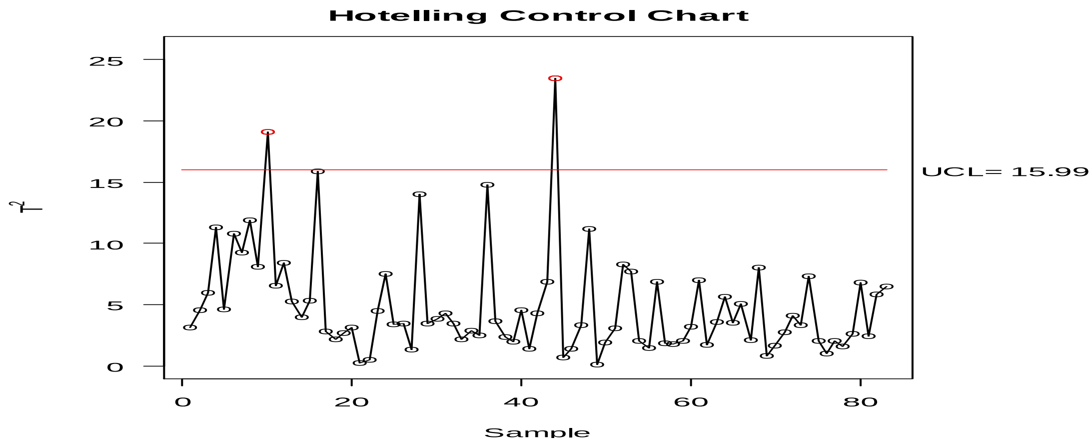

Figure 4.2: The Hotelling's T^2 control chart for residual data after the removal of autocorrelation

The control chart in figure 4.2 is the Hotelling's T^2 control chart constructed from the residuals of the VAR(1) model, after removal of the autocorrelation and partial autocorrelation, the test for the evidence of removal of autocorrelation and partial autocorrelation can be seen in the appendix 7. From this appendix, it is obvious that the autocorrelation and partial autocorrelation has been removed. The Hotelling's T^2 above shows that the process is not in control, since there is two points falling outside the control limit.

HOTELLING'S T^2 CONTROL CHART CONSTRUCTED FROM THE RESIDUALS OF THE VAR(1) AND NORMAL TRANSFORMED DATA

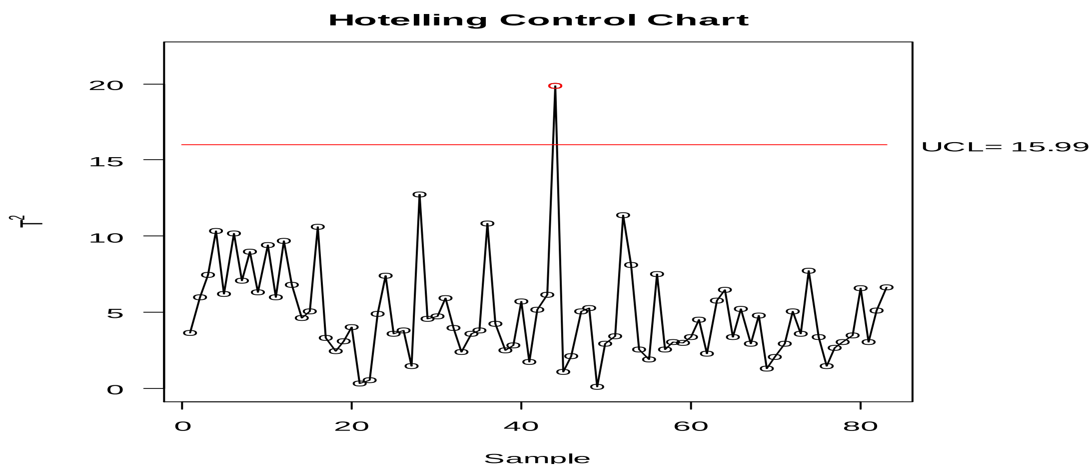

Figure 4.3: The Hotelling's T^2 control chart for residual after the removal of autocorrelation and transformed to normal data

The control chart in figure 4.3 is the Hotelling's T^2 control chart constructed from the residuals of the VAR(1) model, after removal of the autocorrelation and partial autocorrelation, and transformed to normality, the test for the evidence of removal of autocorrelation and partial autocorrelation can be seen in the appendix 7 and evidence of normality of the data after the normality transformation can be seen in the appendix 10. From these appendices, it is obvious that the autocorrelation and partial autocorrelation has been removed, and the data is normal. The Hotelling's T^2 above shows that the process is not in control, since there is one points falling outside the control limit.

HOTELLING'S T^2 CONTROL CHART CONSTRUCTED FROM RESIDUALSDATA

(PHASE I)

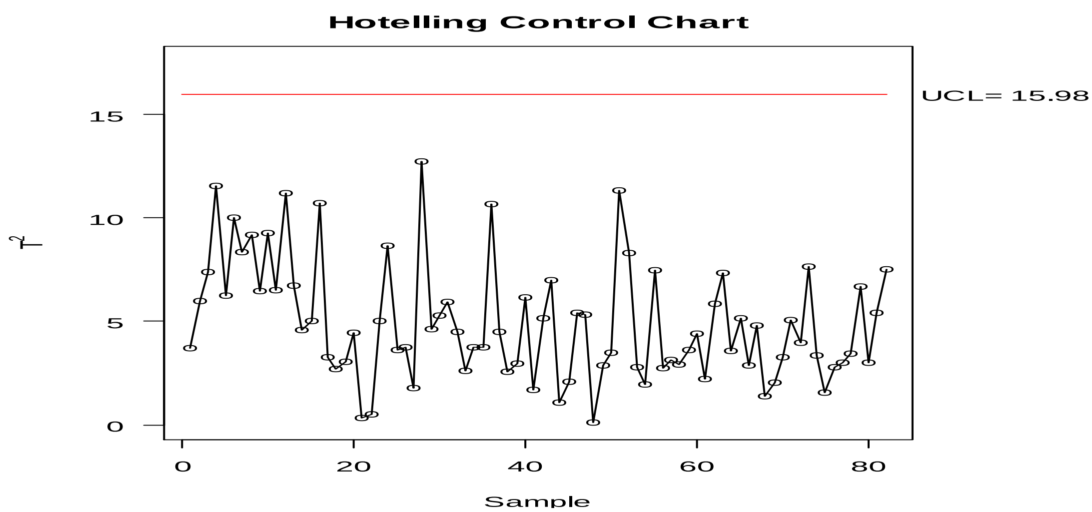

Figure 4.4: The Hotelling's T^2 control chart for phase I

The control chart in figure 4.4 is the Hotelling's T^2 control chart obtained after removal of the point falling outside the control limit. This is obtained since the study intended to establish a stable process which will be used as a reference point for future or subsequence monitoring.

HOTELLING'S T^2 CONTROL CHART CONSTRUCTED FROM NEW OBSERVATIONS DATA (PHASE II)

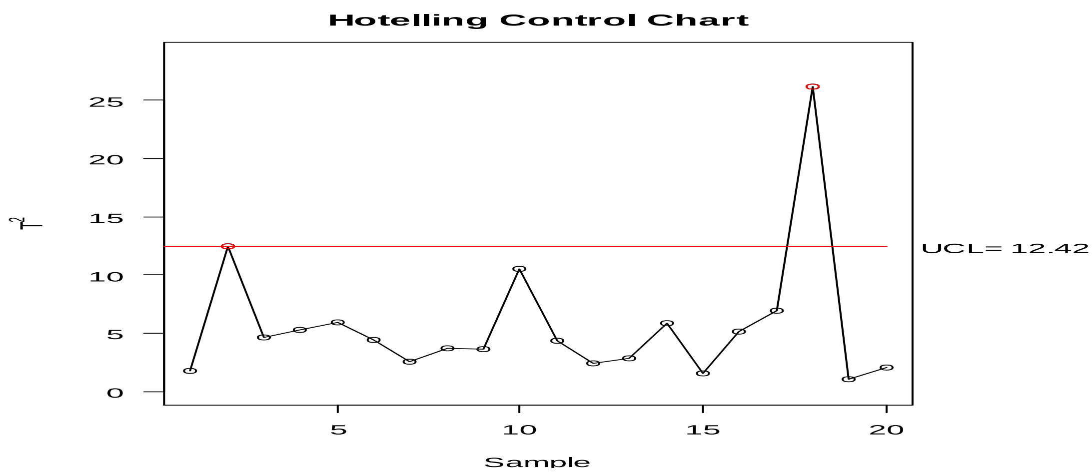

Figure 4.5:The Hotelling's T^2 control chart for Phase II

The control chart in figure 4.5 is the Hotelling's T^2 control chart constructed to monitor the future process known as phase II (note that the phase II data is also subjected to VAR(1) and transformed to normal after we have confirmed the existed of autocorrelation, partial autocorrelation and non normality in the data). The result revealed that the process is out of control since points 2 and 18 are out of the control limit.

Below is the estimation of the parameter of the VAR(1) model

$$\begin{pmatrix} X_{1t} \\ X_{2t} \\ X_{3t} \\ X_{4t} \\ X_{5t} \end{pmatrix} = \begin{pmatrix} 29.148160941 \\ -297.60436136 \\ 258.87430698 \\ 158.87107977 \\ -18.730301418 \end{pmatrix}$$

$$+ \begin{pmatrix} 0.961213614 & -0.002155292 & 0.155851564 & -0.033777061 & 0.165513626 \\ 0.19746079 & 1.26092199 & 0.70369319 & -0.07552637 & -0.24184330 \\ -0.12302751 & -0.24714886 & 0.39631883 & 0.06535194 & -0.01419113 \\ -0.22246859 & -0.04619501 & -0.35087362 & 0.94719917 & -0.05320175 \\ -0.007253834 & 0.028848783 & 0.225363111 & -0.013505563 & -0.182543914 \end{pmatrix} \begin{pmatrix} X_{1t-1} \\ X_{2t-1} \\ X_{3t-1} \\ X_{4t-1} \\ X_{5t-1} \end{pmatrix}$$

$$+ \begin{pmatrix} 0.046559357 \\ -0.04128626 \\ 0.03447976 \\ 0.07128361 \\ 0.003263530 \end{pmatrix}$$

Where

$$\mathbf{C} = \begin{pmatrix} 29.148160941 \\ -297.60436136 \\ 258.87430698 \\ 158.87107977 \\ -18.730301418 \end{pmatrix}$$

$$\mathbf{\Phi} = \begin{pmatrix} 0.961213614 & -0.002155292 & 0.155851564 & -0.033777061 & 0.165513626 \\ 0.19746079 & 1.26092199 & 0.70369319 & -0.07552637 & -0.24184330 \\ -0.12302751 & -0.24714886 & 0.39631883 & 0.06535194 & -0.01419113 \\ -0.22246859 & -0.04619501 & -0.35087362 & 0.94719917 & -0.05320175 \\ -0.007253834 & 0.028848783 & 0.225363111 & -0.013505563 & -0.182543914 \end{pmatrix}$$

$$\boldsymbol{\varepsilon}_t = \begin{pmatrix} 0.046559357 \\ -0.04128626 \\ 0.03447976 \\ 0.07128361 \\ 0.003263530 \end{pmatrix}$$

Where X_1, X_2, X_3, X_4 and X_5 represent, respectively, the quality characteristics the Stage IV Temperature, Kiln Speed, Fuel Burner, Fan Speed and Kiln Feed. $\mathbf{C}_i$ is the vector of coefficients defining the mean of the process, $\boldsymbol{\Phi}_i$ are the matrices of autoregressive coefficients and $\boldsymbol{\varepsilon}_t$ is the vector of residues (blank noise)

4.2 DISCUSSION

We verified the autocorrelation and the normality assumption through correlogram, QQ Plot for Univariate Normality Test and Royston Multivariate Normality Test respectively of this model. The results of the verification are presented in the appendices.

The control chart of the residual of VAR(1) model shows that two points were out of control, and after normalizing the observation, the point dropped to one point. This show vividly the effect of autocorrelation and non-normality of observation in a process, thereby those points are expected to be removed from the observations since we are to establish an in control process in the Phase I, whose parameters will be used as a reference, to monitor future observations.

Below is the Hotelling's T^2 constructed for an in-control process.

HOTELLING'S T^2 CONSTRUCTED FOR PHASE I

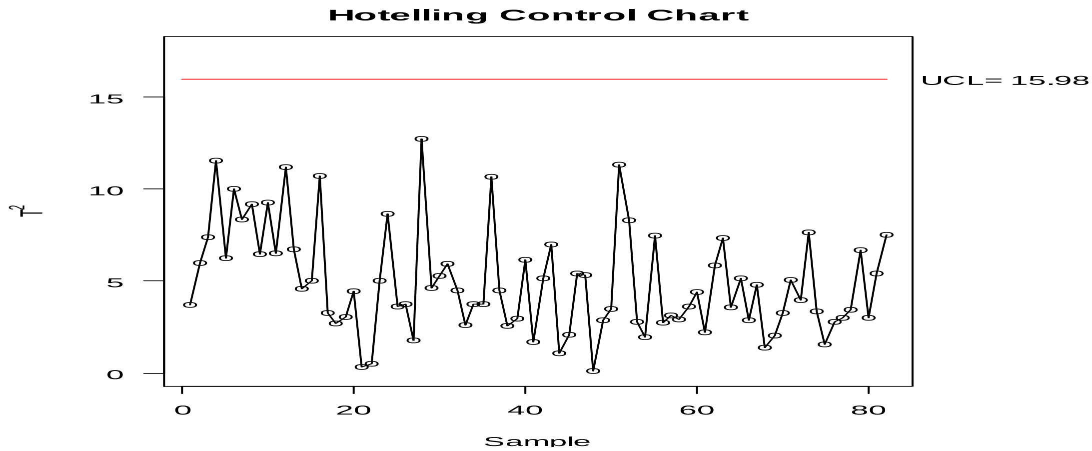

Figure 4.6: Phase I. Hotelling's T^2 control chart for in-control process

Figure 4.6 depicts the Hotelling's T^2 chart after removing the out of control sample point, this form the phase I which will be used for reference purpose known as the retrospective phase. The control chart in the figure 4.6 exhibits statistical-in- control, we then proceeded to the phase II.

HOTELLING'S T^2 CONSTRUCTED FOR PHASE II

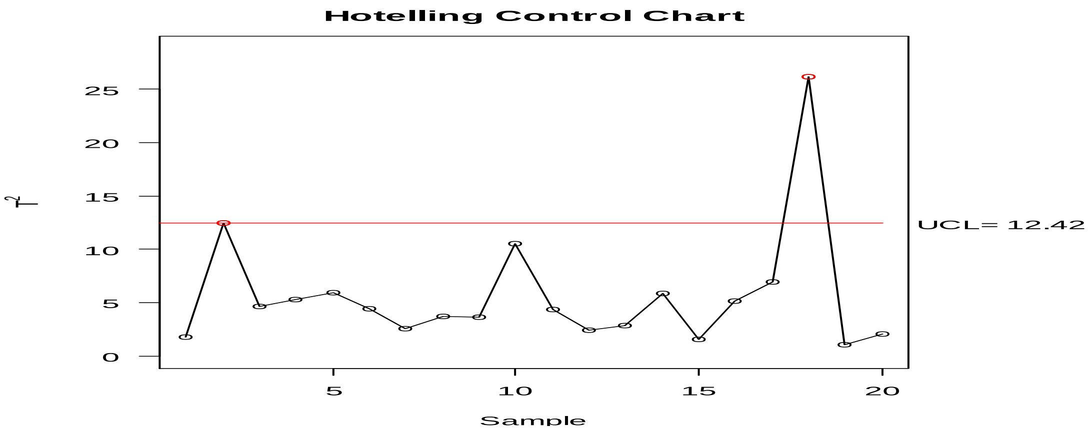

Figure 4.7: Phase II. Hotelling's T^2 control chart for in-control process

For the phase II analysis, 20 sample were drawn and subjected toVAR(1) model, and the residuals variable were obtained.

The control charts in Figure 4.7 exhibit statistical out-of-control, we would proceed to identify the influential variables in the model. It means that the statistic of some of the variables in the new 20 observations deviated from the phase I statistic. It can be seen here that observation point 2 and 18 are out of control, with T^2 value of 12.44 and 26.18 respectively.

Table 4.1: MYT DECOMPOSITION OF THE POINT 2 AND 18 FROM FIVE VARIABLES MODEL

Components	Point 2 values	Point 18 Values	Critical value
T(var1)	*5.9768	*11.1147	4.0071
T(var2)	1.5344	3.8946	4.0071
T(var3)	0.2062	3.9377	4.0071
T(var4)	0.1244	*5.4464	4.0071
T(var5)	3.1971	1.8107	4.0071
T(var1.var2)	5.4128	*9.8746	6.3761
T(var1.var3)	5.8630	*10.3533	6.3761
T(var1.var4)	6.3213	*13.527	6.3761
T(var1.var5)	*6.4333	*10.7011	6.3761
T(var2.var1)	0.9704	2.6545	6.3761
T(var2.var3)	1.7095	0.6291	6.3761
T(var2.var4)	1.4355	2.6463	6.3761
T(var2.var5)	0.9552	4.8601	6.3761
T(var3.var1)	0.0924	3.1763	6.3761
T(var3.var2)	0.3813	0.6722	6.3761
T(var3.var4)	0.1647	2.7329	6.3761
T(var3.var5)	0.0924	*7.7169	6.3761
T(var4.var1)	0.4689	*7.8587	6.3761
T(var4.var2)	0.0255	4.1981	6.3761

T(var4.var3)	0.0829	4.2416	6.3761
T(var4.var5)	0.0107	*6.4878	6.3761
T(var5.var1)	3.6536	1.3971	6.3761
T(var5.var2)	2.6179	2.7762	6.3761
T(var5.var3)	3.0833	5.5899	6.3761
T(var5.var4)	3.0834	2.8521	6.3761
T(var1.var2,var3)	5.3567	*9.984	8.4694
T(var1.var2,var4)	5.6592	*12.225	8.4694
T(var1.var2,var5)	5.9544	*9.2283	8.4694
T(var1.var3,var4)	6.1954	*12.6204	8.4694
T(var1.var3,var5)	6.6262	*9.2043	8.4694
T(var1.var4,var5)	6.6086	*13.1521	8.4694
T(var2.var1,var3)	1.2032	0.2598	8.4694
T(var2.var1,var4)	0.7734	1.3443	8.4694
T(var2.var1,var5)	0.4763	3.3873	8.4694
T(var2.var3,var4)	1.6642	0.4171	8.4694
T(var2.var3,var5)	3.0107	0.0791	8.4694
T(var2.var4,var5)	0.9455	3.4930	8.4694
T(var3.var1,var2)	0.3252	0.7816	8.4694
T(var3.var1,var4)	0.0388	1.8263	8.4694
T(var3.var1,var5)	0.2853	6.2201	8.4694
T(var3.var2,var4)	0.3934	0.5037	8.4694
T(var3.var2,var5)	2.1479	2.9359	8.4694

T(var3.var4,var5)	0.1000	6.4009	8.4694
T(var4.var1,var2)	0.2719	6.5485	8.4694
T(var4.var1,var3)	0.4153	6.5087	8.4694
T(var4.var1,var5)	0.1860	*8.9388	8.4694
T(var4.var2,var3)	0.0376	4.0296	8.4694
T(var4.var2,var5)	0.0010	5.1207	8.4694
T(var4.var3,var5)	0.0183	5.1718	8.4694
T(var5.var1,var2)	3.1595	2.1299	8.4694
T(var5.var1,var3)	3.8465	4.4409	8.4694
T(var5.var1,var4)	3.3707	2.4772	8.4694
T(var5.var2,var3)	4.3845	5.0399	8.4694
T(var5.var2,var4)	2.5934	3.6988	8.4694
T(var5.var3,var4)	3.0187	6.5201	8.4694
T(var1.var2,var3,var4)	5.6235	*12.2876	10.4645
T(var1.var2,var3,var5)	6.0242	9.1289	10.4645
T(var1.var2,var4,var5)	6.0608	*11.5952	10.4645
T(var1.var3,var4,var5)	6.8541	*11.4308	10.4645
T(var2.var1,var3,var4)	1.0923	0.0843	10.4645
T(var2.var1,var3,var5)	2.4087	0.0037	10.4645
T(var2.var1,var4,var5)	0.3977	1.9361	10.4645
T(var2.var3,var4,var5)	2.9932	0.0045	10.4645
T(var3.var1,var2,var4)	0.3577	0.5663	10.4645
T(var3.var1,var2,var5)	2.2177	2.8365	10.4645

T(var3.var1,var4,var5)	0.3455	4.6796	10.4645
T(var3.var2,var4,var5)	2.1477	2.9124	10.4645
T(var4.var1,var2,var3)	0.3044	6.3332	10.4645
T(var4.var1,var2,var5)	0.1074	7.4876	10.4645
T(var4.var1,var3,var5)	0.2462	7.3983	10.4645
T(var4.var2,var3,var5)	0.0008	5.0972	10.4645
T(var5.var1,var2var3)	5.0520	4.1848	10.4645
T(var5.var1,var2,var4)	2.9950	3.0690	10.4645
T(var5.var1,var3,var4)	3.6774	5.3305	10.4645
T(var5.var2,var3,var4)	4.3477	6.1075	10.4645
T(var1.var2,var3,var4,var5)	6.1352	11.4781	12.4223
T(var2.var1,var3,var4,var5)	2.2743	0.0518	12.4223
T(var3.var1,var2,var4,var5)	2.2221	2.7933	12.4223
T(var4.var1,var2,var3,var5)	0.1118	7.4464	12.4223
T(var5.var1,var2,var3,var4)	4.8594	5.298	12.4223

GRAPHICAL PRESENTATION OF THE INFLUENTIAL VARIABLE AT POINT 2

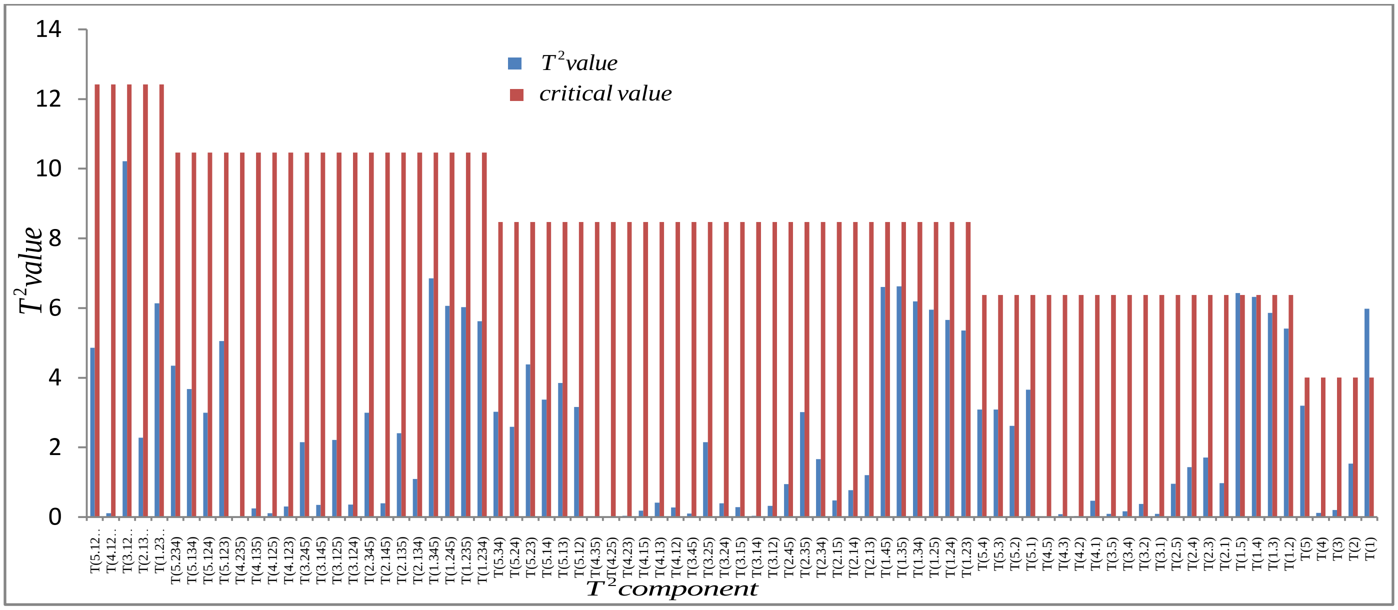

Figure 4.8The Hotelling's T^2 Decomposition chart for point 2

GRAPHICAL PRESENTATION OF THE INFLUENTIAL VARIABLE AT POINT 18

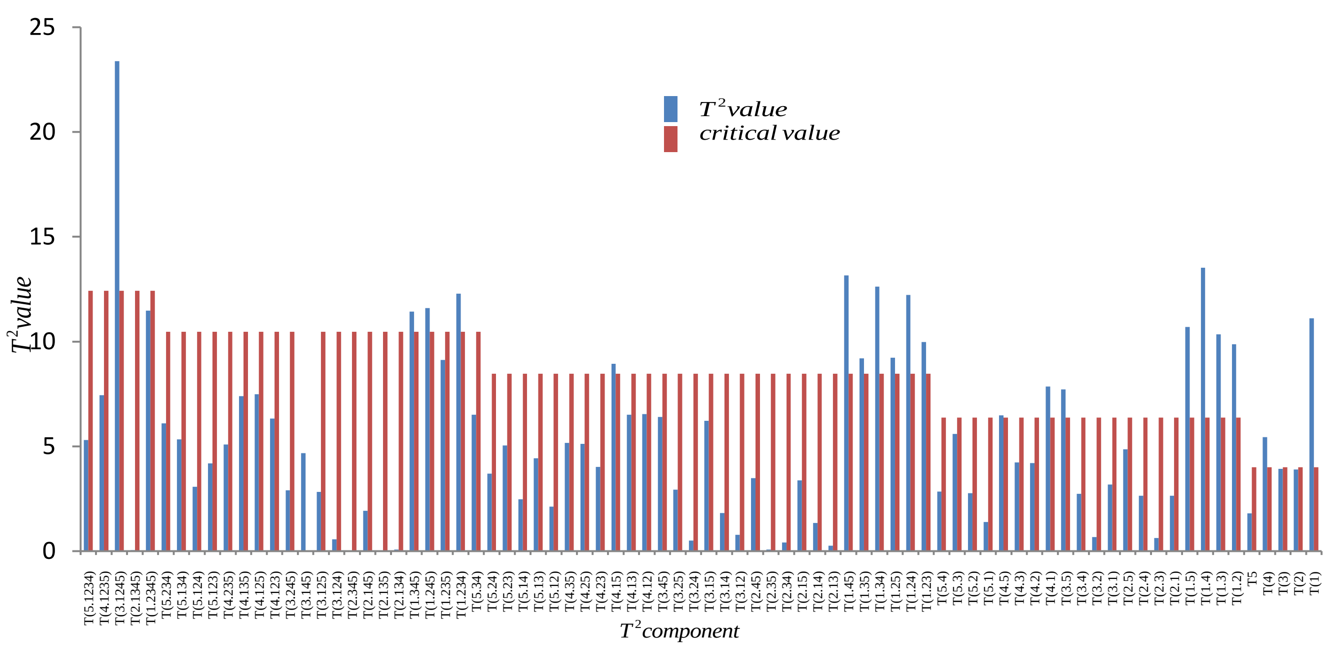

Figure 4.9The Decomposition Chart for point 18

From the T^2 Decomposition chart in figure 4.8, only one out of the five unconditional terms of observation point 2 is significant, that is, only variable one is significant. It is obvious at this point that the influential variable here is variable one.

By verification $T^2 - T_1^2 = 12.44 - 5.98 = 6.46 < 12.42$

By implication, this result revealed that removal of the contribution of variable one at the sample point 2 will bring the process back to in-control.

From the T^2 contribution chart in figure 4.9, only two out of the five unconditional terms of observation point 18 are significant, that is variable one and variable four are significant. It is obvious at this point that the influential variable here is variable one and variable four. By verification

$T^2 - T_1^2 = 26.18 - 11.1147 = 15.06 > 12.42$, indicating that the contribution of variable one alone is not enough to trigger the process.

$T^2 - T_4^2 = 26.18 - 5.4464 = 20.73 > 2.42$, indicating that the contribution of variable four alone is not enough to trigger the process.

$T^2 - T_1^2 - T_4^2 = 26.18 - 11.1147 - 5.4464 = 9.6189 < 12.42$

Therefore, by removing the contribution of the two variables, the process will be brought back to in-control. There is no need to check the bivariate relationship among the variables. Hence, variable one and variable four are the influential variables.

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATION

5.1 SUMMARY

The summary will be categorized into two stages: Stage I focusing on the identification of out-of-control, and stage II focusing on the interpretation of out-of-control

Stage I:The data used for quality control are increasingly multivariate and sampled in time at a high sampling rate. Hence the data are typically autocorrelated especially if sampled quickly relative to the dynamics of the system being monitored, however, ignoring such autocorrelation can greatly impair the performance of a (multivariate) control chart.

We investigated the autocorrelation, partial autocorrelation and normality assumption in data, and we adopted different approach in applying Hotelling's T^2;

(1) Hotelling's T^2 control chart was constructed using the original data, and then the control chart showed that the process is in control, knowing well that the data is autocorrelated and failed the normality assumption.

(2) Hotelling's T^2 control chart was constructed using residuals from the VAR(1) model, then the control chart showed that the process is out of control, knowing well that the data from the residuals is free from autocorrelation and partial autocorrelation.

(3) Hotelling's T^2 control chart was constructed using residuals from the VAR(1) model and transformed to normal, then control chart showed that the process is out of control, knowing well that the data from the residuals is free from autocorrelation, partial autocorrelation and satisfied the normality assumption.

By comparing the Hotelling's T^2 control chart using the original data with the Hotelling's T^2 control chart using the residuals, it is revealed that the effect of ignoring the verification of auto-correlation and normality can impair the performance of the Hotelling's T^2 control chart. This is an undesirable situation, leaving the process unchecked, bearing in mind that the process is in-control when in fact, some points in the process have actually gone out-of-control as seen in this work will definitely lead to production of deformed products, likewise the investigation of possible cause of the problem may not even arise, and production of substandard product will recur. But if a process is been tested for failure of some statistical assumption, and appropriate model or transformation is applied, then a better industrial decision will be achieved.

Stage II: MYT decomposition method which partitions the overall Hotelling's T^2 into orthogonal component gives important information on how to detect influential variable. However, even with the reduced computation scheme, the numerous computations involved in the process of identification especially when the number of variable is large, is discouraging. Despite this disadvantage the MYT approach has a wonderful way of evaluating the contribution of individual variables and their joint contributions. This work illustrate the decomposing of T^2 statistic for five variables which involved 120 decomposition and 80 number of unique term required to construct the decomposition chart are evaluated. And this aided the identification of the outrageous variables in the process.

5.2 CONCLUSIONS

From this research we were able to deduce the effect of ignoring two of the most important assumption of the multivariate statistical process control. The control chart for residuals detected two out of control points , likewise, the normality transformation effect brought the out of control points to just only one point, but the control chart using the original data could not detect this points, the likely reason for this insensitivity could be attributed to presence of autocorrelation and failure of normality in the original data, since it was confirmed (see the appendix) that the original data contain autocorrelation, in the same vein, the reason for control chart for residuals to be able to identify the two out of control can be justify by the ability of the fitted model to remove the autocorrelation through a time series model, as it can be vividly seen (see the appendices) that there is no autocorrelation and autocorrelation in the residual data. Thereby the assumption of independence is achieved in the fitted model alongside the normality assumption mentioned earlier. This research lies in the application of control charts of fitted model (VAR) in detriment of control charts for traditional multivariate control chart, wherein some mistakes of observing a process as a stable process, when in fact there are assignable causes of variation in the process. (False alarm), and also seeing a process as an unstable process, when in fact it is stable (False alarm). This can be attributed to inappropriate approach to parameters estimation and incorrect analysis about the process capability. Wastage of resources (time, energy) that may be required to recoup the process or products as a result of non-interference of the process on time, when in fact the process needed to be adjusted.

More so, we proceeded to identify the influential variables, in this research we adopted MYT approached and we integrated it with a graphical modification for easy and vivid identification of the influential variables. The MYT decomposition is a very reliable method of indentifying influential variable in a process, it easy and simple to estimate the decomposition term more especially when the number of variable are small say two or three, but tends to be complex when variable increase say four, five, and so on. We were able to illustrate the invariant property for five variables and show how variables that deviate from the characteristics of the underline process could be detected, the graphical approach aid easy and vivid understanding and identification of any variable which may have contributed to the out of control points. In order to indulge in any corrective action in the process, appropriate identification of combination of process characteristic responsible for the signal of the control chart is paramount. Therefore the proposed contribution chart is easy, vivid unambiguous approach to see the contribution of each and every variable alongside their interrelationship with other variables.

5.3 RECOMMENDATION

Whenever a multivariate statistical process is to be monitored, it is important to check the underlying assumptions of independent and normality of the observations so as not to drive towards wrong decision

5.4 CONTRIBUTION TO KNOWLEDGE

This research work succeeded in comparing Hotelling's T^2 control chart with and without autocorrelation, partial autocorrelation and non normality, which will enhance industrial decision. We also derived and simplified MYT decomposition for five variables, as this derivation and simplification will help quality engineer and others to develop decomposition model for larger variables. Finally this derived decomposition will be applicable in some other related studies that require interpretation of out of control process.

5.5 FUTURE WORK

Area such as non parametric applications in statistical process control can be adopted in the plight of failure of normality assumption. And the result in this thesis can be compared. Likewise A computer program can be written to simplify the decomposition of Larger variables

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APPENDICES

Appendix 1

ORIGINAL DATA CONTAINING AUTOCORRELATION AND DEVIATE FROM NORMAL DISTRIBUTION.

	X1	X2	X3	X4	X5
1	405.3665	929.6105	7.53	386.1361	1.846245
2	404.6398	929.8040	7.70	388.1358	1.313627
3	403.8149	930.3184	7.47	390.5401	1.274388
4	404.2158	931.4277	7.27	393.9638	1.240268
5	405.0467	932.6620	7.37	396.7647	1.573284
6	404.4167	933.5509	7.13	400.0217	1.216384
7	402.8191	933.5315	7.40	400.7515	1.262446
8	401.9773	933.0769	8.33	405.7335	1.421105
9	402.0897	932.1238	8.83	409.0504	1.064056
10	401.3067	930.6359	10.43	411.3984	1.779367
11	401.6302	929.0971	12.20	413.0194	2.081331
12	401.5638	928.5633	12.77	415.1670	2.178573
13	402.8157	929.0694	12.43	414.6621	2.205687
14	403.1421	930.2655	12.23	415.7319	2.086449
15	403.0786	931.6770	11.70	416.2315	1.996030
16	403.7188	932.1390	11.20	418.1439	1.910730
17	404.8668	932.2767	11.27	419.7352	1.226717

18	405.6362	932.8328	11.47	420.4842	1.956792
19	405.1363	933.7334	11.30	420.9309	1.927790
20	406.0246	934.1772	11.17	422.1124	1.905612
21	406.4123	934.5928	11.00	423.6278	1.766095
22	406.3009	935.6067	10.63	423.9887	1.813487
23	406.3354	936.5111	10.27	424.1902	1.752071
24	406.7737	937.4201	10.20	426.1270	1.740129
25	405.1525	938.4159	9.67	426.8578	1.497104
26	404.9298	938.9992	9.60	426.7457	1.637768
27	404.5765	939.2354	9.60	426.8858	1.637768
28	404.1995	939.6795	9.50	428.8403	1.620708
29	405.9499	940.2497	9.50	430.1223	1.207082
30	405.8221	941.4358	9.03	430.2307	1.540526
31	406.4463	942.2981	8.70	430.3930	1.484228
32	407.0512	943.5322	8.13	432.0284	1.386985
33	407.9460	944.3490	7.87	433.3886	1.426288
34	408.1796	944.8215	7.67	433.9641	1.308509
35	408.5998	945.0671	7.80	434.4844	1.330687
36	409.0906	945.8067	7.73	436.1569	1.318745
37	408.7042	946.8697	7.57	438.2651	1.914485
38	408.9803	946.8766	7.57	438.7636	1.291449
39	408.3287	947.2497	7.33	439.9498	1.250504
40	407.8857	947.6513	7.57	441.8359	1.291449

41	407.2605	948.1840	7.63	443.1769	1.016846
42	406.7752	948.3492	7.60	444.3592	1.296567
43	406.1794	948.0322	8.17	444.5236	1.393809
44	405.4398	947.1065	9.20	446.9694	1.569528
45	403.2800	946.0796	10.17	450.1586	1.350108
46	403.3649	946.1838	10.33	451.5464	1.762307
47	403.3807	946.2258	10.40	452.2984	1.774249
48	404.0032	945.9978	10.37	453.1201	1.769131
49	404.4774	945.5183	10.60	453.9991	1.083692
50	404.7868	945.3514	11.00	454.9552	1.876610
51	405.2710	945.2918	11.40	455.4824	1.944850
52	405.3830	945.4008	11.73	456.1009	2.001148
53	405.1564	945.9058	11.07	457.2027	1.885516
54	406.4700	945.9035	11.67	457.3886	1.990912
55	406.2293	946.3190	11.47	457.7799	1.956792
56	406.7265	946.5796	11.30	457.5535	1.927790
57	408.5785	946.7800	10.97	458.8024	1.714915
58	409.6767	947.6283	10.63	459.0564	1.813487
59	410.3858	948.6221	10.10	459.1578	1.723069
60	410.5395	949.3992	9.67	459.7037	1.649710
61	410.4453	949.9481	9.53	459.7037	1.258262
62	410.6256	949.7945	9.47	460.0258	1.615590
63	410.8672	949.9534	9.50	461.0257	1.620708

64	411.2359	950.2502	9.27	461.3039	1.581470
65	410.6637	950.5380	9.50	461.4031	1.207082
66	410.8085	950.7871	9.43	462.9277	1.608766
67	412.1160	950.8695	9.70	464.6888	1.654828
68	412.9994	950.9281	9.90	465.0717	1.688949
69	412.9551	951.8457	9.43	464.2851	1.087662
70	412.8241	952.6005	9.30	464.0344	1.586588
71	413.0489	953.5976	8.87	463.4535	1.513230
72	413.6110	954.1434	8.77	465.0717	1.496170
73	413.6048	954.5426	8.60	466.0889	1.671674
74	412.9684	955.2631	8.33	466.6171	1.421105
75	412.2659	956.0561	8.17	465.7478	1.393809
76	412.9106	956.7966	8.03	465.8995	1.369925
77	413.8294	957.3865	7.90	466.4099	1.477468
78	414.2242	958.0634	7.87	466.9552	1.342629
79	415.1678	958.7166	7.53	467.6281	1.284625
80	415.7016	959.4881	6.93	467.7026	1.182264
81	416.8674	960.3625	6.80	469.1348	1.600859
82	417.6104	960.7834	6.70	469.3364	1.143026
83	418.0030	961.0290	6.93	470.0117	1.182264
84	417.2667	961.7657	6.87	469.6472	1.172028

Appendix 2

RESIDUALS OF THE ORIGINAL DATA AFTER REMOVAL OF THE AUTOCORRELATION

	X1	X2	X3	X4	X5
1	-0.678294674	-0.40822444	0.1873206916	-0.760657370	0.0147528669
2	-0.721618214	-0.05043980	-0.3243567950	-0.443172071	-0.1471333738
3	0.550286725	0.94836158	-0.5997177531	0.389353256	-0.1281983306
4	1.104127884	1.13711158	-0.3556794300	-0.055514352	0.2575428837
5	-0.344541169	0.56858170	-0.4408300543	0.771863899	-0.0561196918
6	-1.174732107	-0.07740332	-0.1859260893	-1.856937228	-0.0106096636
7	-0.552551757	-0.27452999	0.3542044544	2.103387683	0.0911732529
8	0.318533878	-0.68669783	-0.0881020783	0.756536875	-0.3754715182
9	-0.527914298	-1.14140447	0.8356966741	0.028906514	0.2318246726
10	0.210002120	-1.38376068	1.3297481645	-0.288873647	0.3694747184
11	-0.488310136	-1.04992707	0.7216205199	0.890027878	0.1883142205
12	0.747313862	-0.03173650	-0.1578279094	-1.554620054	0.1453815650
13	-0.143650342	0.52797585	0.0549219028	0.106170428	0.0921139016
14	-0.477831200	0.60082173	-0.1661287945	-0.427477694	0.0040480713
15	0.294330537	-0.27432136	-0.1834541684	0.800722063	-0.0160120774
16	0.938037386	-0.32864158	0.1169809131	0.493106301	-0.5890335454
17	0.713754755	-0.22606644	0.3163253681	-0.086529858	0.0229860243
18	-0.697772364	-0.04491100	0.2260906798	-0.114693303	0.0785723328

19	0.672803250	-0.45034894	0.2604616512	0.441608925	0.0625737837
20	0.224893438	-0.55311339	0.2489452967	0.937936202	-0.0453602413
21	-0.204060287	0.10181751	-0.0387616557	-0.169454353	0.0229012190
22	-0.044844010	0.09013750	-0.0726374460	-0.486329913	0.0250731621
23	0.388761887	0.14692851	0.1792834746	1.108197827	0.0566679069
24	-1.620032642	0.14393950	-0.2056158010	0.047481619	-0.1729119655
25	-0.181316662	0.20240112	-0.1045959491	-1.341672602	0.0089511934
26	-0.382015285	-0.13682307	0.0147524628	-1.206350928	0.0271836647
27	-0.460736626	0.13107756	-0.1139721589	0.476475973	-0.0006246842
28	1.690864813	0.47089082	-0.1734150692	-0.263458857	-0.3872419955
29	-0.053010966	0.63045612	-0.4112741953	-1.046908027	-0.1190051231
30	0.671751369	0.48326227	-0.3144159575	-1.179378414	-0.0454587446
31	0.698192148	0.77896179	-0.5096058169	0.290922944	-0.1000283596
32	1.127823876	0.46260369	-0.3070146338	0.017198891	0.0375896455
33	0.536490798	0.06497733	-0.2148292408	-0.619304203	-0.0163870600
34	0.756713427	-0.13432899	0.0661903678	-0.718045924	0.0219361251
35	0.791224171	0.20708169	-0.0111027958	0.541963020	-0.0155284457
36	-0.042526457	0.45456456	-0.0441376259	1.112845719	0.5953559473
37	0.558260035	-0.34541889	0.0706543127	-0.518125070	0.1088657426
38	-0.285316565	-0.10727932	-0.2095717631	0.153208200	-0.0405373415
39	-0.063497443	0.24239606	0.0250099928	0.630343793	0.0442865874
40	-0.289047889	0.38093718	-0.1225117415	0.119934132	-0.2695188135
41	-0.137413960	0.03183126	-0.2475651064	-0.147279763	-0.0585024497

42	-0.314627699	-0.17830490	0.2076723926	-1.270014122	0.1009802212
43	-0.628156613	-0.91053037	0.8162810451	1.006763558	0.1697731786
44	-2.232596582	-1.08045178	0.8664719126	1.971468101	-0.1985856597
45	-0.127577258	0.29157213	-0.1234833354	0.067925459	0.0087237192
46	-0.286004064	0.31861804	-0.2000220110	-0.464110569	0.0729413965
47	0.287354092	0.08625111	-0.3288951871	-0.395336268	0.0600235189
48	0.149425115	-0.10545870	-0.1550230134	-0.248777701	-0.6006621279
49	0.062718521	0.01867309	-0.0079958514	-0.068970929	0.0411800502
50	0.041314867	0.13220821	0.1445818411	-0.275015765	0.1807258160
51	-0.414620644	0.03687861	0.2929292688	0.021783243	0.1684240138
52	-0.835059036	0.25171844	-0.5312386318	0.615192148	-0.0085437434
53	0.810097372	0.21837124	0.3191384537	-0.578626842	0.2198888239
54	-0.844493463	0.03598697	-0.0027453292	0.073565153	0.0786323913
55	-0.111558549	0.02352933	-0.0809401082	-0.701101627	0.0767629667
56	1.240176244	-0.06604348	-0.2380852822	0.752418981	-0.1133269519
57	0.640930080	0.48021789	-0.2890432142	0.046334016	0.0420125555
58	0.294948108	0.51109103	-0.3892099111	0.005311241	0.0298719543
59	-0.176374084	0.29510224	-0.3186951641	0.396769281	0.0340303995
60	-0.365600216	0.20115299	-0.1485057771	-0.276277237	-0.2910968540
61	-0.053521022	-0.58083204	-0.0689851675	-0.091008964	0.0065433793
62	-0.071030178	-0.06960438	-0.0814449722	0.563488155	0.0972368542
63	0.047474765	0.07606063	-0.3540916679	-0.104812610	0.0595811642
64	-0.873307266	0.13147283	0.0325581892	-0.327460652	-0.2755299897

65	-0.194968783	-0.07294761	-0.1741376981	0.978674825	-0.0083988311
66	0.923246952	-0.03039873	0.0745688095	1.264921129	0.1379549653
67	0.513259323	-0.33844025	0.1998688805	0.300295671	0.1472638448
68	-0.450494270	0.26855126	-0.2852482656	-0.649044747	-0.4862421119
69	-0.437291775	0.04227255	0.0007484003	-0.390328420	-0.0318362343
70	-0.202291674	0.34798994	-0.2183144840	-0.818394386	-0.0141960951
71	0.158854079	-0.12560473	0.1286334793	1.220043545	0.0140154606
72	-0.359960220	-0.29586218	0.0618386058	0.747470079	0.2158652307
73	-1.004295301	0.20268620	-0.1413532777	0.207645865	0.0345580445
74	-1.038691849	0.42343393	-0.1671256606	-1.449610440	0.0008382906
75	0.236502238	0.38435971	-0.1122091019	-0.723037320	-0.0349472959
76	0.521540804	0.05872845	0.0308725184	-0.300371119	0.0818862989
77	0.007590407	0.00770728	0.2449155365	-0.118042682	-0.0107479847
78	0.572014787	-0.19959678	0.0606417345	0.068466098	-0.0991683911
79	0.238980717	-0.12071933	-0.2063616885	-0.447974852	-0.1416687266
80	0.959757731	0.11984847	0.1169759564	0.780798459	0.3728165573
81	0.536853194	-0.44983192	0.3058933737	-0.169060598	0.0200029680
82	0.267791498	-0.86550950	0.7168077796	0.369293202	-0.0090908139
83	-0.911441652	-0.57608576	0.5965997247	-0.524658871	-0.0623783157

Appendix 3

DATA OBTAINED AFTER REMOVAL OF AUTOCORRELATION EFFECT

AND NORMALITY TRANSFORMATION

	[Y1]	[Y2]	[Y3]	[Y4]	[Y5]
[1,]	-1.10368438	-1.11726667	0.78355981	-1.124921986	-0.059118328
[2,]	-1.17061759	-0.30076794	-1.30841723	-0.673680379	-1.220961341
[3,]	0.82694650	2.08997309	-2.49710872	0.543845320	-1.138490174
[4,]	1.59969722	2.31953791	-1.46575887	-0.094763136	1.719524251
[5,]	-0.57693662	1.45719404	-1.86945766	1.029753978	-0.726815585
[6,]	-1.84065213	-0.37631040	-0.60690869	-2.349040113	-0.337078612
[7,]	-0.90724399	-0.85642303	1.18270079	2.251857046	0.801216329
[8,]	0.47594684	-1.54192285	-0.15769871	1.011595167	-1.852942170
[9,]	-0.86842145	-2.02871962	1.97216088	0.030627322	1.626536019
[10,]	0.30719625	-2.22664049	2.50269592	-0.444498346	2.034037187
[11,]	-0.80581927	-1.94485433	1.81806283	1.166090450	1.441316848
[12,]	1.11331562	-0.24701893	-0.47187883	-2.062903316	1.209299751
[13,]	-0.25491325	1.36986154	0.38554967	0.144097888	0.809801039
[14,]	-0.78921766	1.52322653	-0.51131583	-0.650571807	-0.181460153
[15,]	0.43852757	-0.85597895	-0.59486042	1.063646441	-0.390611457
[16,]	1.37864054	-0.96752605	0.58290658	0.682288895	-2.185325701
[17,]	1.06539423	-0.74982495	1.10013022	-0.141097726	0.038768690
[18,]	-1.13382820	-0.28499213	0.88506959	-0.183258426	0.680200335

[19,]	1.00643121	-1.19063250	0.97027496	0.614167733	0.510379420
[20,]	0.33048980	-1.35480560	0.94220187	1.219546419	-0.646367641
[21,]	-0.35182470	0.16270675	0.04487408	-0.265388532	0.037748039
[22,]	-0.09665019	0.12556313	-0.09241843	-0.736918208	0.063950076
[23,]	0.58376026	0.30698891	0.76175011	1.401322634	0.443451976
[24,]	-2.43513641	0.29741450	-0.70391493	0.058035456	-1.321723747
[25,]	-0.31533489	0.48378628	-0.22912226	-1.839374616	-0.126233545
[26,]	-0.63679804	-0.53434075	0.24620405	-1.686954995	0.089518480
[27,]	-0.76210352	0.25621513	-0.27054400	0.660420386	-0.232671371
[28,]	2.30636569	1.23868904	-0.54625376	-0.406438259	-1.875304727
[29,]	-0.10971101	1.58148822	-1.73412433	-1.496501023	-1.095400931
[30,]	1.00490973	1.26798240	-1.25786285	-1.655571160	-0.647133718
[31,]	1.04304921	1.84258580	-2.16180414	0.408262171	-0.999091960
[32,]	1.63046698	1.21879186	-1.22008677	0.013314949	0.216261364
[33,]	0.80645653	0.04613054	-0.74988150	-0.928354879	-0.394249604
[34,]	1.12667290	-0.52794424	0.42294124	-1.066503456	0.026148176
[35,]	1.17546817	0.49855960	0.15125308	0.745805222	-0.385904335
[36,]	-0.09294477	1.19927752	0.02360600	1.406110472	2.439801258
[37,]	0.83876352	-1.00038813	0.43755602	-0.783189036	0.952813670
[38,]	-0.48213141	-0.45723664	-0.72361036	0.212407124	-0.608155354
[39,]	-0.12648798	0.60883173	0.28271122	0.857896698	0.297392420
[40,]	-0.48811009	1.01062176	-0.30877968	0.164151535	-1.617797587
[41,]	-0.24491199	-0.05685429	-0.91548380	-0.232114173	-0.743788346

[42,]	-0.52907772	-0.63770775	0.83759123	-1.759699458	0.887766764
[43,]	-1.02572230	-1.80504169	1.94700275	1.294547807	1.348434136
[44,]	-3.14517374	-1.97347703	2.01120947	2.156263559	-1.411310029
[45,]	-0.22913918	0.75781703	-0.31316044	0.088111067	-0.128827564
[46,]	-0.48323304	0.83695611	-0.67617715	-0.704419655	0.622457548
[47,]	0.42771799	0.11323727	-1.33141386	-0.603079147	0.481733572
[48,]	0.21203067	-0.45238982	-0.45864378	-0.384430426	-2.200014241
[49,]	0.07481630	-0.09708314	0.16288902	-0.114852104	0.259872600
[50,]	0.04079159	0.25983606	0.66435091	-0.423752046	1.404486037
[51,]	-0.68877917	-0.04131722	1.04697651	0.020097278	1.341276461
[52,]	-1.34377340	0.63752779	-2.24710994	0.838939389	-0.316054615
[53,]	1.20198835	0.53405482	1.10640741	-0.870398946	1.579609215
[54,]	-1.35802693	-0.04406600	0.18241134	0.096390322	0.680804003
[55,]	-0.20346074	-0.08228374	-0.12726267	-1.043066627	0.661892695
[56,]	1.77375173	-0.34476949	-0.86722452	1.006697735	-1.067676533
[57,]	0.96017615	1.26081916	-1.12803450	0.056344326	0.269950561
[58,]	0.43948395	1.33211278	-1.62959866	-0.004291535	0.122199995
[59,]	-0.30740527	0.76826743	-1.27965333	0.553897365	0.172886180
[60,]	-0.61059073	0.47984154	-0.42807360	-0.425641258	-1.671519382
[61,]	-0.11052687	-1.39587428	-0.07724018	-0.147798027	-0.153536867
[62,]	-0.13854372	-0.35470154	-0.12939668	0.773440249	0.855491830
[63,]	0.05058929	0.08100976	-1.45787045	-0.168459039	0.476724569
[64,]	-1.40141104	0.25748093	0.30916485	-0.502160975	-1.633122374

[65,]	-0.33723829	-0.36398904	-0.54973469	1.264192617	-0.314568462
[66,]	1.35849997	-0.24313347	0.45028093	1.557895064	1.162363779
[67,]	0.77182983	-0.98680663	0.81707282	0.421340835	1.220828424
[68,]	-0.74583995	0.68882975	-1.10856126	-0.970355392	-2.042273429
[69,]	-0.72485719	-0.02465200	0.19530362	-0.595660638	-0.535575465
[70,]	-0.34898715	0.92036908	-0.76735418	-1.202847710	-0.372850167
[71,]	0.22688442	-0.50540602	0.61773850	1.514068693	-0.067745285
[72,]	-0.60158098	-0.90118229	0.40858629	1.000801478	1.563183827
[73,]	-1.59568449	0.48468698	-0.39482238	0.290613316	0.179320380
[74,]	-1.64585697	1.12171019	-0.51607838	-1.955094336	-0.216793290
[75,]	0.34861945	1.01979151	-0.26271029	-1.073385229	-0.562080936
[76,]	0.78419100	0.02655420	0.30328710	-0.461697068	0.713123446
[77,]	-0.01292247	-0.13028599	0.93226714	-0.188276887	-0.338475341
[78,]	0.85910546	-0.68858308	0.40461950	0.088905079	-0.994468152
[79,]	0.35248675	-0.49267412	-0.70762356	-0.680740608	-1.197966780
[80,]	1.40808337	0.22027489	0.58289162	1.040288646	2.041795486
[81,]	0.80699547	-1.18975488	1.07664083	-0.264797553	0.003003969
[82,]	0.39735180	-1.75632639	1.81120936	0.516537668	-0.321651612
[83,]	-1.45847819	-1.38893127	1.62936951	-0.792661594	-0.770782168

Appendix 4

PHASE I DATA

	[Y1]	[Y2]	[Y3]	[Y4]	[Y5]
1	-1.10368438	-1.11726667	0.78355981	-1.124921986	-0.059118328
2	-1.17061759	-0.30076794	-1.30841723	-0.673680379	-1.220961341
3	0.82694650	2.08997309	-2.49710872	0.543845320	-1.138490174
4	1.59969722	2.31953791	-1.46575887	-0.094763136	1.719524251
5	-0.57693662	1.45719404	-1.86945766	1.029753978	-0.726815585
6	-1.84065213	-0.37631040	-0.60690869	-2.349040113	-0.337078612
7	-0.90724399	-0.85642303	1.18270079	2.251857046	0.801216329
8	0.47594684	-1.54192285	-0.15769871	1.011595167	-1.852942170
9	-0.86842145	-2.02871962	1.97216088	0.030627322	1.626536019
10	0.30719625	-2.22664049	2.50269592	-0.444498346	2.034037187
11	-0.80581927	-1.94485433	1.81806283	1.166090450	1.441316848
12	1.11331562	-0.24701893	-0.47187883	-2.062903316	1.209299751
13	-0.25491325	1.36986154	0.38554967	0.144097888	0.809801039
14	-0.78921766	1.52322653	-0.51131583	-0.650571807	-0.181460153
15	0.43852757	-0.85597895	-0.59486042	1.063646441	-0.390611457
16	1.37864054	-0.96752605	0.58290658	0.682288895	-2.185325701
17	1.06539423	-0.74982495	1.10013022	-0.141097726	0.038768690
18	-1.13382820	-0.28499213	0.88506959	-0.183258426	0.680200335
19	1.00643121	-1.19063250	0.97027496	0.614167733	0.510379420

20	0.33048980	-1.35480560	0.94220187	1.219546419	-0.646367641
21	-0.35182470	0.16270675	0.04487408	-0.265388532	0.037748039
22	-0.09665019	0.12556313	-0.09241843	-0.736918208	0.063950076
23	0.58376026	0.30698891	0.76175011	1.401322634	0.443451976
24	-2.43513641	0.29741450	-0.70391493	0.058035456	-1.321723747
25	-0.31533489	0.48378628	-0.22912226	-1.839374616	-0.126233545
26	-0.63679804	-0.53434075	0.24620405	-1.686954995	0.089518480
27	-0.76210352	0.25621513	-0.27054400	0.660420386	-0.232671371
28	2.30636569	1.23868904	-0.54625376	-0.406438259	-1.875304727
29	-0.10971101	1.58148822	-1.73412433	-1.496501023	-1.095400931
30	1.00490973	1.26798240	-1.25786285	-1.655571160	-0.647133718
31	1.04304921	1.84258580	-2.16180414	0.408262171	-0.999091960
32	1.63046698	1.21879186	-1.22008677	0.013314949	0.216261364
33	0.80645653	0.04613054	-0.74988150	-0.928354879	-0.394249604
34	1.12667290	-0.52794424	0.42294124	-1.066503456	0.026148176
35	1.17546817	0.49855960	0.15125308	0.745805222	-0.385904335
36	-0.09294477	1.19927752	0.02360600	1.406110472	2.439801258
37	0.83876352	-1.00038813	0.43755602	-0.783189036	0.952813670
38	-0.48213141	-0.45723664	-0.72361036	0.212407124	-0.608155354
39	-0.12648798	0.60883173	0.28271122	0.857896698	0.297392420
40	-0.48811009	1.01062176	-0.30877968	0.164151535	-1.617797587
41	-0.24491199	-0.05685429	-0.91548380	-0.232114173	-0.743788346
42	-0.52907772	-0.63770775	0.83759123	-1.759699458	0.887766764

43	-1.02572230	-1.80504169	1.94700275	1.294547807	1.348434136
44	-0.22913918	0.75781703	-0.31316044	0.088111067	-0.128827564
45	-0.48323304	0.83695611	-0.67617715	-0.704419655	0.622457548
46	0.42771799	0.11323727	-1.33141386	-0.603079147	0.481733572
47	0.21203067	-0.45238982	-0.45864378	-0.384430426	-2.200014241
48	0.07481630	-0.09708314	0.16288902	-0.114852104	0.259872600
49	0.04079159	0.25983606	0.66435091	-0.423752046	1.404486037
50	-0.68877917	-0.04131722	1.04697651	0.020097278	1.341276461
51	-1.34377340	0.63752779	-2.24710994	0.838939389	-0.316054615
52	1.20198835	0.53405482	1.10640741	-0.870398946	1.579609215
53	-1.35802693	-0.04406600	0.18241134	0.096390322	0.680804003
54	-0.20346074	-0.08228374	-0.12726267	-1.043066627	0.661892695
55	1.77375173	-0.34476949	-0.86722452	1.006697735	-1.067676533
56	0.96017615	1.26081916	-1.12803450	0.056344326	0.269950561
57	0.43948395	1.33211278	-1.62959866	-0.004291535	0.122199995
58	-0.30740527	0.76826743	-1.27965333	0.553897365	0.172886180
59	-0.61059073	0.47984154	-0.42807360	-0.425641258	-1.671519382
60	-0.11052687	-1.39587428	-0.07724018	-0.147798027	-0.153536867
61	-0.13854372	-0.35470154	-0.12939668	0.773440249	0.855491830
62	0.05058929	0.08100976	-1.45787045	-0.168459039	0.476724569
63	-1.40141104	0.25748093	0.30916485	-0.502160975	-1.633122374
64	-0.33723829	-0.36398904	-0.54973469	1.264192617	-0.314568462
65	1.35849997	-0.24313347	0.45028093	1.557895064	1.162363779

66	0.77182983	-0.98680663	0.81707282	0.421340835	1.220828424
67	-0.74583995	0.68882975	-1.10856126	-0.970355392	-2.042273429
68	-0.72485719	-0.02465200	0.19530362	-0.595660638	-0.535575465
69	-0.34898715	0.92036908	-0.76735418	-1.202847710	-0.372850167
70	0.22688442	-0.50540602	0.61773850	1.514068693	-0.067745285
71	-0.60158098	-0.90118229	0.40858629	1.000801478	1.563183827
72	-1.59568449	0.48468698	-0.39482238	0.290613316	0.179320380
73	-1.64585697	1.12171019	-0.51607838	-1.955094336	-0.216793290
74	0.34861945	1.01979151	-0.26271029	-1.073385229	-0.562080936
75	0.78419100	0.02655420	0.30328710	-0.461697068	0.713123446
76	-0.01292247	-0.13028599	0.93226714	-0.188276887	-0.338475341
77	0.85910546	-0.68858308	0.40461950	0.088905079	-0.994468152
78	0.35248675	-0.49267412	-0.70762356	-0.680740608	-1.197966780
79	1.40808337	0.22027489	0.58289162	1.040288646	2.041795486
80	0.80699547	-1.18975488	1.07664083	-0.264797553	0.003003969
81	0.39735180	-1.75632639	1.81120936	0.516537668	-0.321651612
82	-1.45847819	-1.38893127	1.62936951	-0.792661594	-0.770782168

Appendix 5

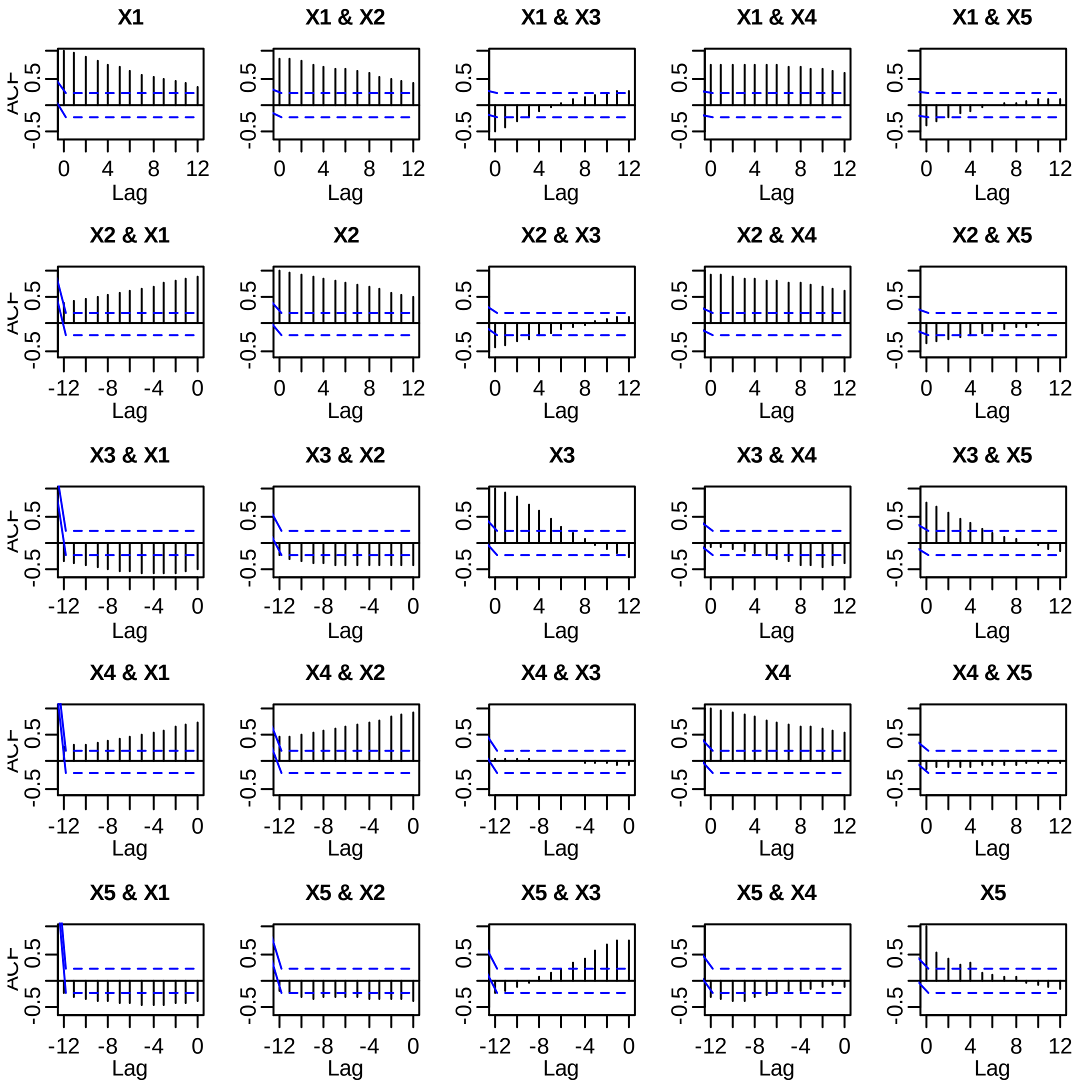
PHASE II DATA

1	-0.76210352	0.25621513	-0.27054400	0.660420386	-0.232671371
2	2.30636569	1.23868904	-0.54625376	-0.406438259	-1.875304727
3	-0.10971101	1.58148822	-1.73412433	-1.496501023	-1.095400931
4	1.00490973	1.26798240	-1.25786285	-1.655571160	-0.647133718
5	1.04304921	1.84258580	-2.16180414	0.408262171	-0.999091960
6	1.63046698	1.21879186	-1.22008677	0.013314949	0.216261364
7	0.80645653	0.04613054	-0.74988150	-0.928354879	-0.394249604
8	1.12667290	-0.52794424	0.42294124	-1.066503456	0.026148176
9	1.17546817	0.49855960	0.15125308	0.745805222	-0.385904335
10	-0.09294477	1.19927752	0.02360600	1.406110472	2.439801258
11	0.83876352	-1.00038813	0.43755602	-0.783189036	0.952813670
12	-0.48213141	-0.45723664	-0.72361036	0.212407124	-0.608155354
13	-0.12648798	0.60883173	0.28271122	0.857896698	0.297392420
14	-0.48811009	1.01062176	-0.30877968	0.164151535	-1.617797587
15	-0.24491199	-0.05685429	-0.91548380	-0.232114173	-0.743788346
16	-0.52907772	-0.63770775	0.83759123	-1.759699458	0.887766764
17	-1.02572230	-1.80504169	1.94700275	1.294547807	1.348434136
18	-3.14517374	-1.97347703	2.01120947	2.156263559	-1.411310029
19	-0.22913918	0.75781703	-0.31316044	0.088111067	-0.128827564
20	-0.48323304	0.83695611	-0.67617715	-0.704419655	0.622457548

Appendix 6

CORRELOGRAM

TEST FOR PRESENCE OF AUTOCORRELATION IN THE ORIGINAL DATA

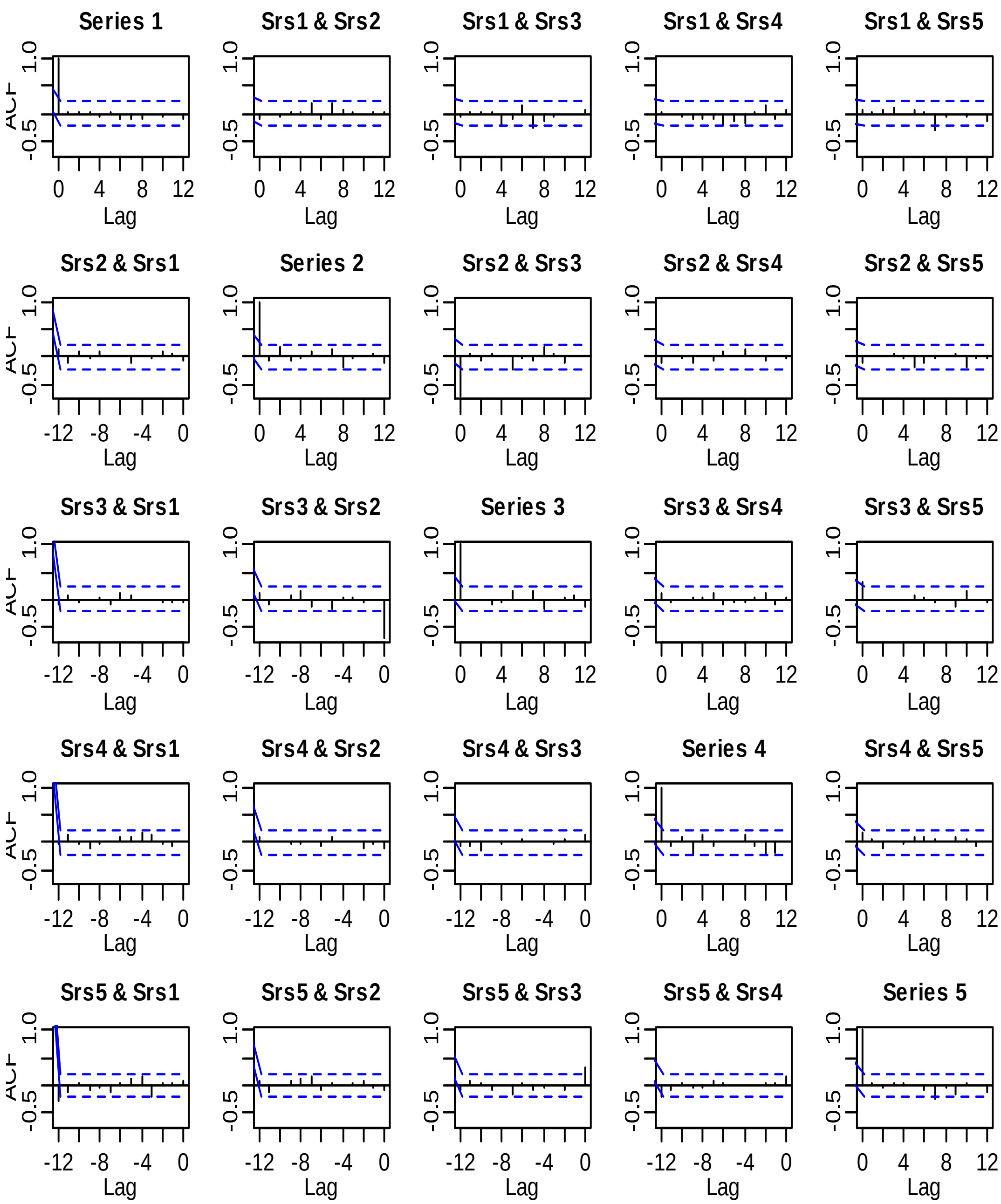


Appendix 7

CORRELOGRAM

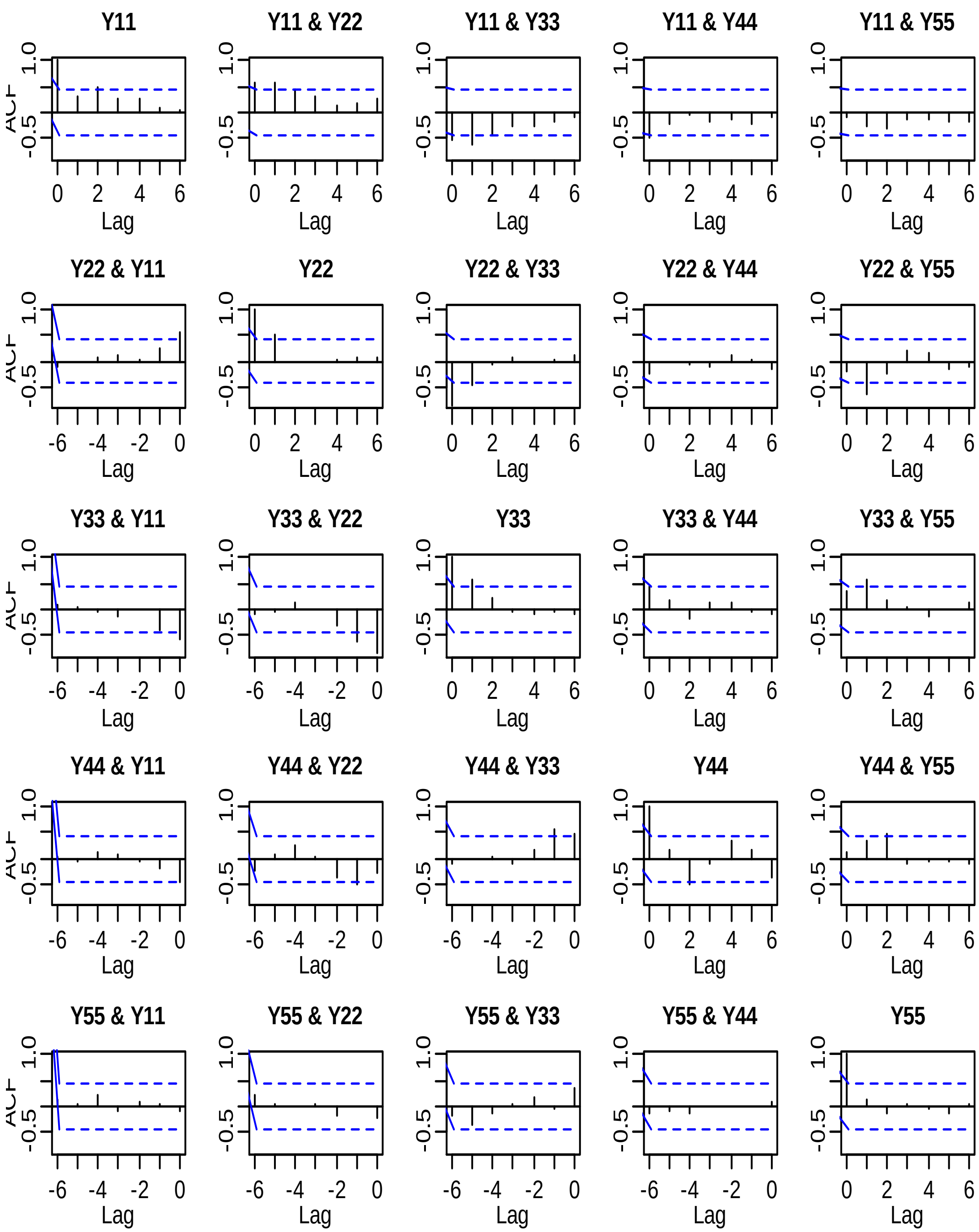
TEST FOR PRESENCE OF AUTOCORRELATION IN THE RESIDUALS (PHASE I)

DATA



Appendix 8

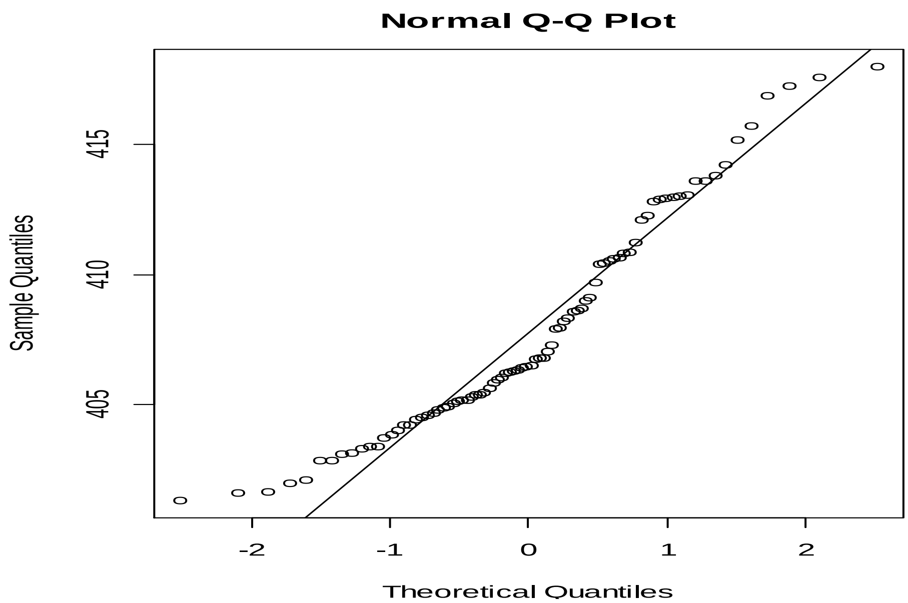
TEST FOR AUTOCORRELATION IN PHASE II DATA

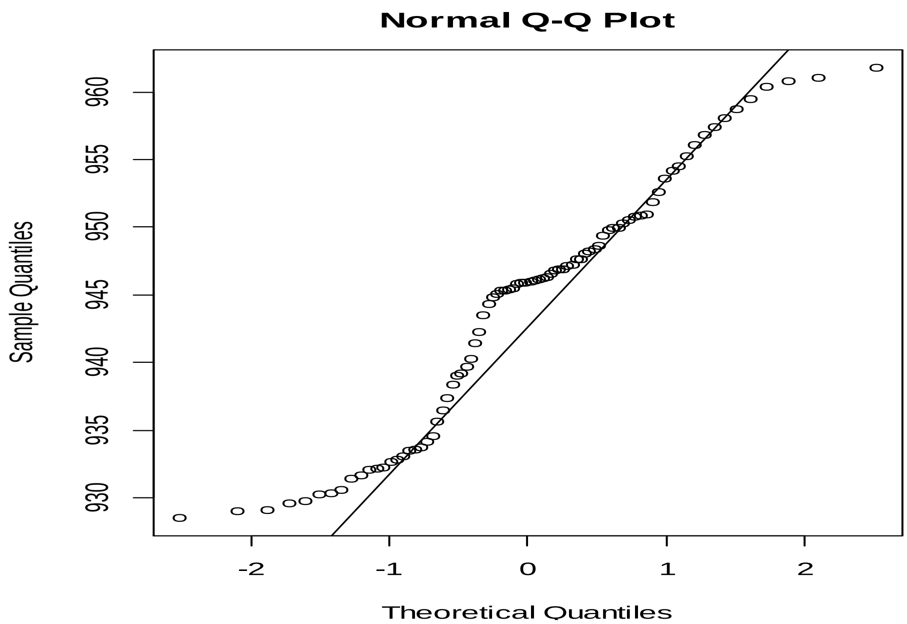


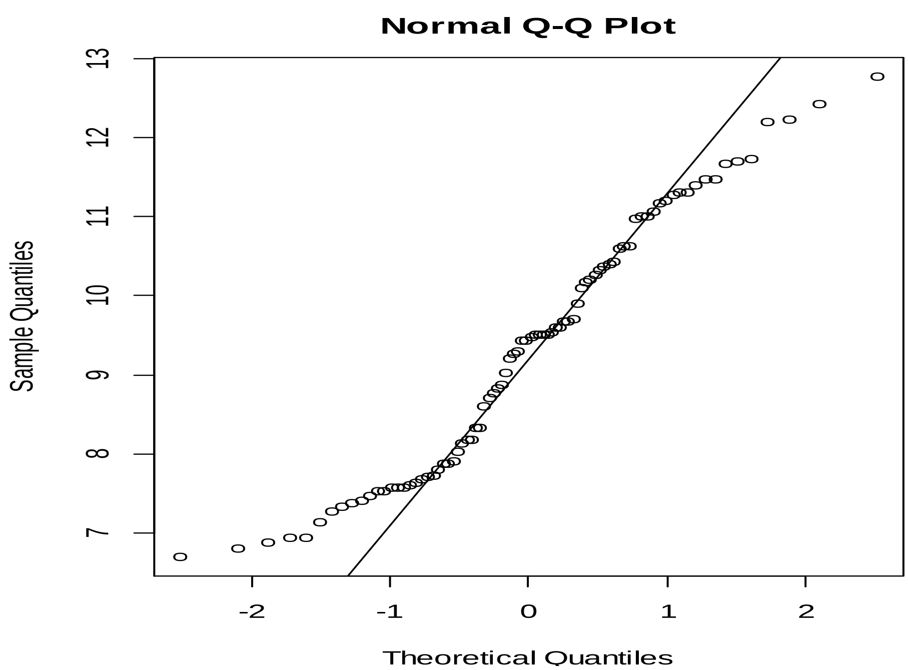
Appendix 9

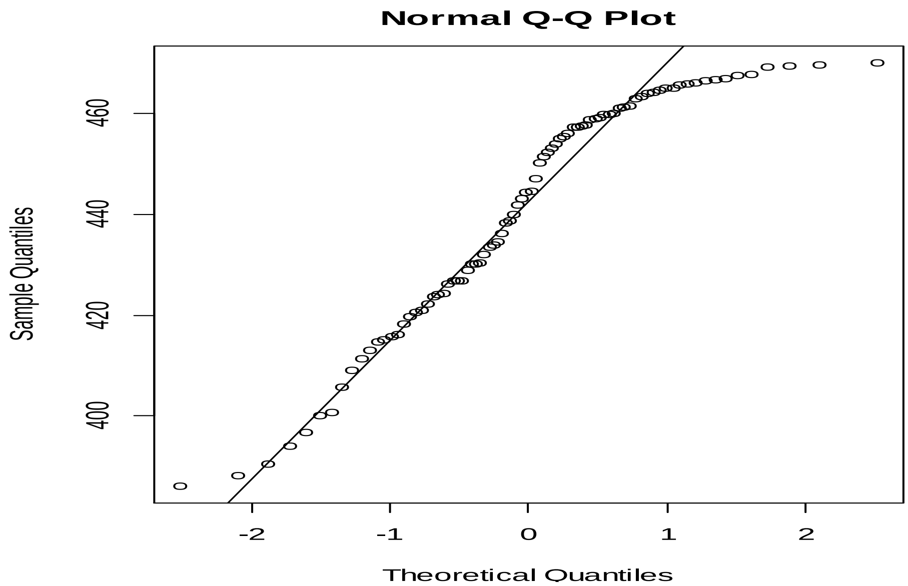
UNIVARIATE NORMALITY TEST FOR ORIGINAL DATA

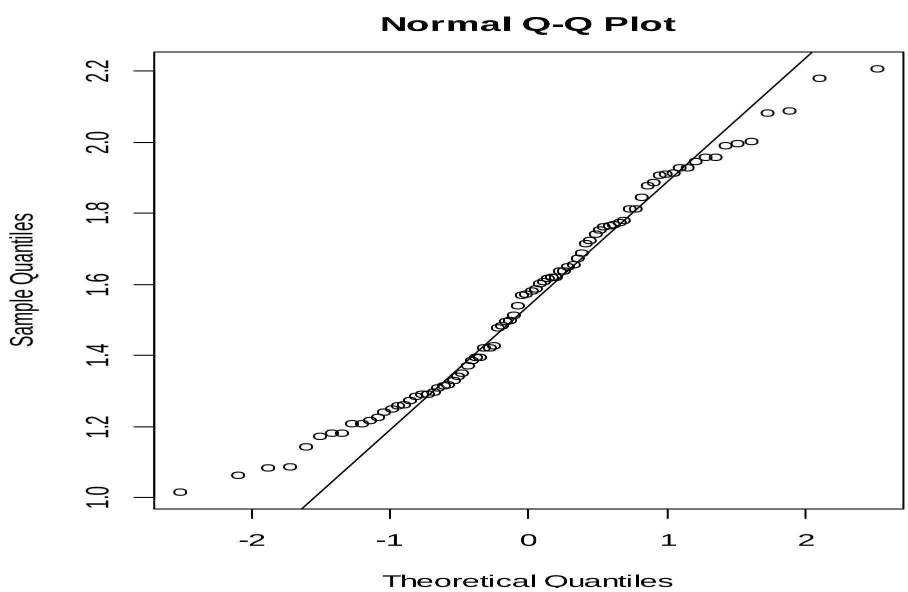
NORMAL PLOT OF X1, X2, X3, X4 AND X5 RESPECTIVELY FOR THE ORIGINAL DATA




Normal Q-Q Plot
Sample Quantiles
930
935
940
945
950
955
960
-2
-1
0
1
2
Theoretical Quantiles


Normal Q-Q Plot
Sample Quantiles
7
8
9
10
11
12
13
-2
-1
0
1
2
Theoretical Quantiles

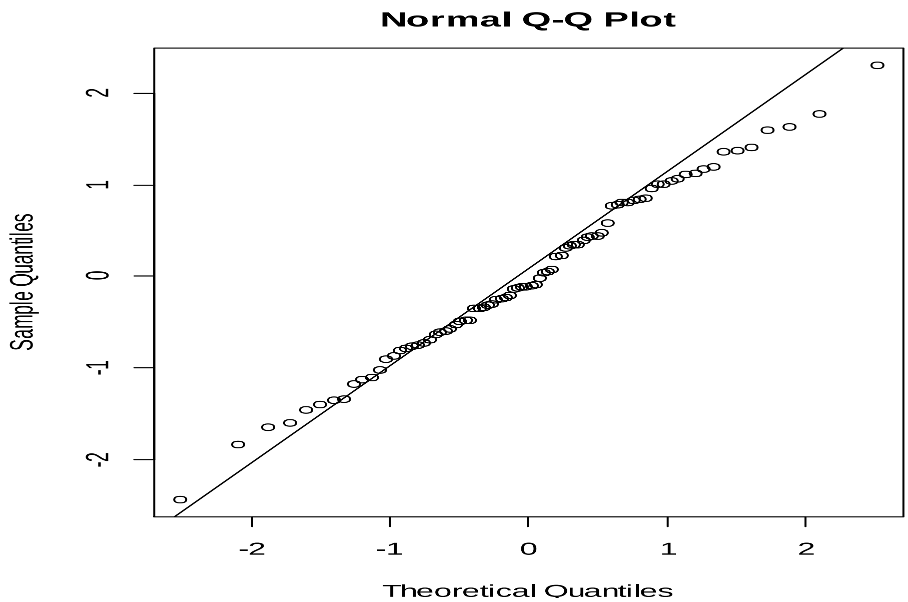

Normal Q-Q Plot
Sample Quantiles
400
420
440
460
-2
-1
0
1
2
Theoretical Quantiles

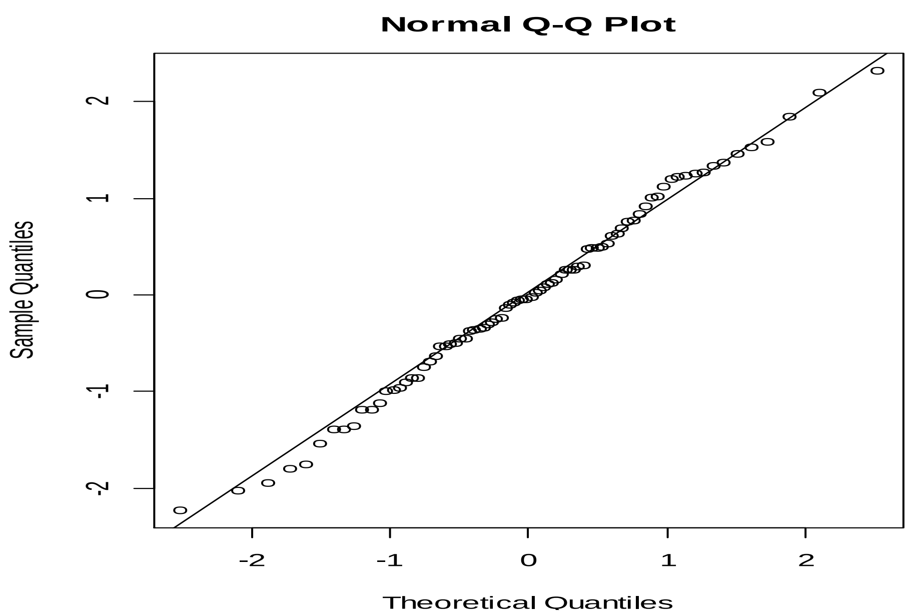

Normal Q-Q Plot
Sample Quantiles
1.0
1.2
1.4
1.6
1.8
2.0
2.2
-2
-1
0
1
2
Theoretical Quantiles

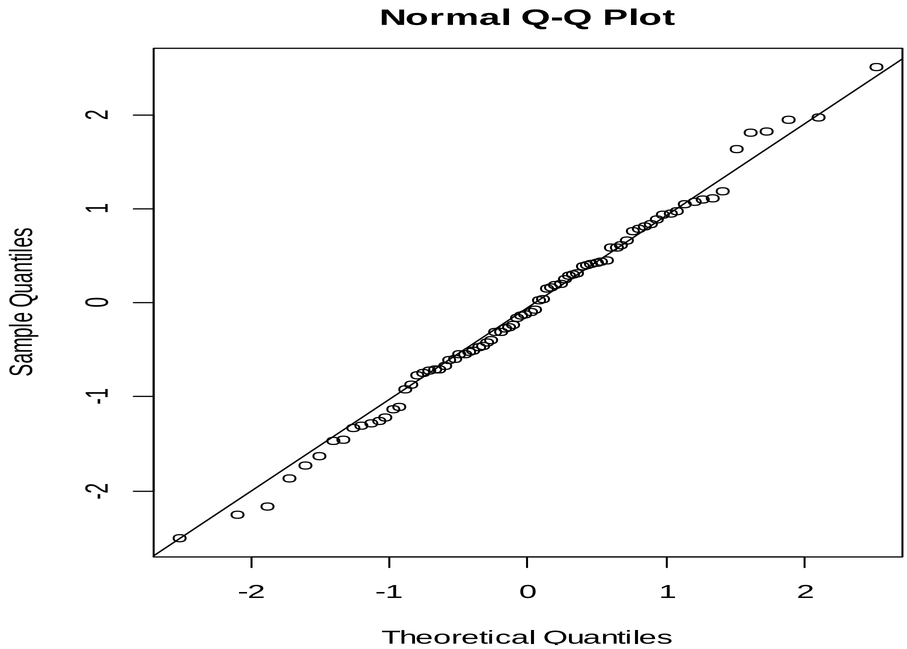
Appendix 10

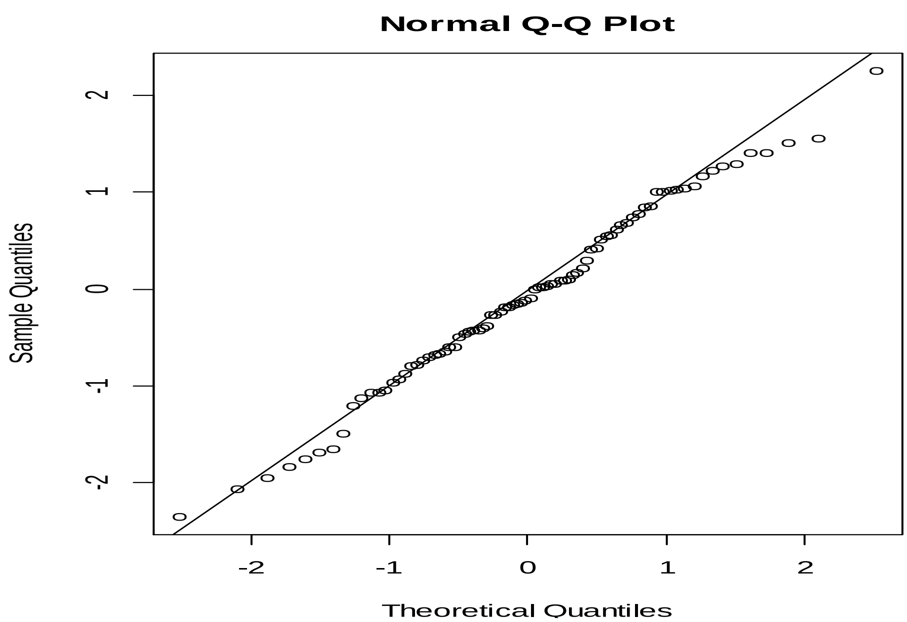
UNIVARIATE NORMALITY TEST FOR THE TRANSFORMED DATA

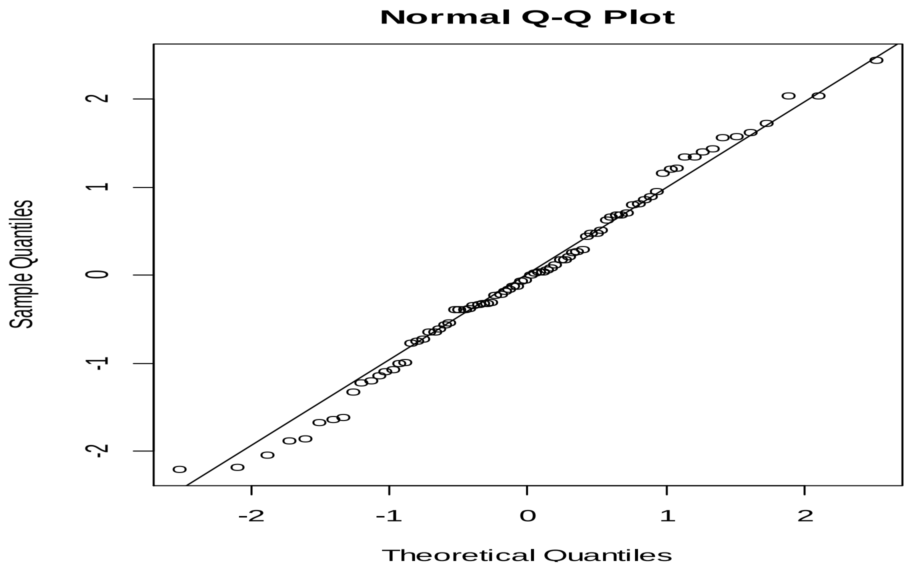
NORMAL PLOT OF X1, X2, X3, X4 AND X5 RESPECTIVELY






Normal Q-Q Plot
Sample Quantiles
Theoretical Quantiles
-2
-1
0
1
2


Normal Q-Q Plot
Sample Quantiles
Theoretical Quantiles
-2
-1
0
1
2


Normal Q-Q Plot
Sample Quantiles
Theoretical Quantiles
-2
-1
0
1
2
-2
-1
0
1
2

Appendix 11

MULTIVARIATE NORMALITY TEST

ROYSTON NORMALITY TEST FOR THE ORIGINAL DATA

test.statistic	p.value	level of significant
3.371754e+01	7.280865e-07	$\alpha = 0.05$

at $\alpha = 0.05$ the Royston multivariate normality test is significant, meaning that the five process characteristics from the original data are not normally distributed. Since the p-value = 7.280865e-07 is less than the $\alpha = 0.05$

ROYSTON NORMALITY TEST FOR THE RESIDUALS (PHASE I DATA)

test.statistic	p.value	level of significant
0.251522	0.994929	$\alpha = 0.05$

at $\alpha = 0.05$ the Royston multivariate normality test is not significant, meaning that the five process characteristics from the residual data are normally distributed. Since the p-value = 0.994929is greater than the $\alpha = 0.05$

ROYSTON NORMALITY TEST FOR THE PHASE II DATA

test.statistic	p.value	level of significant
2.8725082	0.4736851	$\alpha = 0.05$

at $\alpha = 0.05$ the Royston multivariate normality test is not significant, meaning that the five process characteristics from the phase data are normally distributed. Since the p-value = 0.4736851is greater than the $\alpha = 0.05$

Appendix 12

CORRELATION BETWEEN THE VARIABLES IN THE ORIGINAL DATA

	X1	X2	X3	X4	X5
X1	1.0000000	0.8724331	-0.49722655	0.74783420	-0.3738596
X2	0.8724331	1.0000000	-0.41975053	0.92730669	-0.3687147
X3	-0.4972265	-0.4197505	1.00000000	-0.05856638	0.7598920
X4	0.7478342	0.9273067	-0.05856638	1.00000000	-0.1167717
X5	-0.3738596	-0.3687147	0.75989200	-0.11677166	1.0000000

COVARIANCE BETWEEN THE VARIABLES IN THE ORIGINAL DATA

	X1	X2	X3	X4	X5
X1	17.7787211	33.687562	-3.3695906	73.377191	-0.46447729
X2	33.6875617	83.863765	-6.1780442	197.613257	-0.99490898
X3	-3.3695906	-6.178044	2.5831234	-2.190415	0.35985719
X4	73.3771915	197.613257	-2.1904152	541.515645	-0.80066101
X5	-0.4644773	-0.994909	0.3598572	-0.800661	0.08681834

Appendix 13

CORRELATION BETWEEN THE VARIABLES IN THE RESIDUALS

	X1	X2	X3	X4	X5
X1	1.00000000	0.1720488	-0.1329043	0.03527236	0.09733684
X2	0.17204884	1.0000000	-0.7422238	-0.20721339	-0.12119986
X3	-0.13290429	-0.7422238	1.0000000	0.19902152	0.37691870
X4	0.03527236	-0.2072134	0.1990215	1.00000000	0.09653781
X5	0.09733684	-0.1211999	0.3769187	0.09653781	1.00000000

COVARIANCE BETWEEN THE VARIABLES IN THE RESIDUALS

	[X1]	[X2]	[X3]	[X4]	[X5]
[X1,]	0.99649605	0.1744398	-0.1390507	0.03445385	0.10175192
[X2,]	0.17443984	1.0316022	-0.7901099	-0.20593935	-0.12890977
[X3,]	-0.13905072	-0.7901099	1.0984820	0.20410888	0.41368687
[X4,]	0.03445385	-0.2059394	0.2041089	0.95748240	0.09892144
[X5,]	0.10175192	-0.1289098	0.4136869	0.09892144	1.09661752

Appendix 14

CORRELATION OF THE NEW SET OF OBSERVATION (PHASE II DATA)

	Y11	Y22	Y33	Y44	Y55
Y11	1.00000000	0.5591834	-0.5483303	-0.4734196	-0.06810814
Y22	0.55918344	1.0000000	-0.8359776	-0.2623354	-0.21505900
Y33	-0.54833029	-0.8359776	1.0000000	0.4494462	0.34739688
Y44	-0.47341963	-0.2623354	0.4494462	1.0000000	0.10211784
Y55	-0.06810814	-0.2150590	0.3473969	0.1021178	1.00000000

COVARIANCE BETWEEN VARIABLES IN THE PHASE II

	Y11	Y22	Y33	Y44	Y55
Y11	1.3944170	0.7146061	-0.6899647	-0.5964754	-0.0854947
Y22	0.7146061	1.1712031	-0.9640489	-0.3029165	-0.2474102
Y33	-0.6899647	-0.9640489	1.1354725	0.5109942	0.3935121
Y44	-0.5964754	-0.3029165	0.5109942	1.1384130	0.1158232
Y55	-0.0854947	-0.2474102	0.3935121	0.1158232	1.1300241