STATE-SPACE DESIGN AND CONSTRUCTION OF A MAGNETIC SUSPENSION SYSTEM

BY

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(M.Sc /ENG/09415/2006-07)

THESIS SUBMITTED TO THE POSTGRADUATE SCHOOL, AHMADU BELLO UNIVERSITY ZARIA, IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE AWARD OF MASTER OF SCIENCE (M.Sc) DEGREE IN ELECTRICAL ENGINEERING.

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DECLARATION

I, Raheem Azeez Olalekan (M.Sc/ENG/09415/2006-07) hereby declare that this thesis titled **"State-Space Design and Construction of a Magnetic Suspension System"** presented to the Department of Electrical Engineering, Ahmadu Bello University, Zaria is the effort of my own research work and has never been presented for the award of any degree.

______________________________ ______________________________

Raheem Azeez Olalekan Date

CERTIFICATION

This thesis titled " **State-Space Design and Construction of a Magnetic Suspension System"** by Raheem Azeez Olalekan (M.Sc/ENG/09415/2006-07) meets the requirement governing the award of degree of Master of Science in Electrical Engineering, Ahmadu Bello University, Zaria and is approved for its contribution to knowledge and literary presentation

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DEDICATION

This work is dedicated to my late father, Mallam AbdulRaheem Adebayo Akani.

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Bismillahi –Rahmani –Raheem. All praise is due to Allah for His grace and Mercy from beginning to the end of this program.

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SYMBOLS AND ABBREVIATION

a	Decay constant for the coil inductance
B	Magnetic flux density
d	Airgap distance at equilibrium position
g	Acceleration due to gravity
K_n	Gain of non-inverting amplifier
K_p	Gain of power amplifier
K	Boltzmann's constant
L(x)	Inductance at position
L_1	inductance at $x = \infty$
L_o	Incremental inductance due to sphere at $x = 0$
L_d	Incremental inductance per turn at equilibrium position
M	Mass of sphere
M_c	Controllability Matrix
M_o	Observability Matrix
N	Number of turns
SSE	Steady State Error
U_{ss}	Steady state value
X	Airgap distance
T	Temperature
t_s	Settling time
Wn	Natural frequency

ABSTRACT

The state space is applied in the design of Magnetic Suspension System, where a controller is designed with the aim of making the system stable and providing the performance specifications of settling time less than or equal to 0.5 secs, Maximum overshot less than or equal to 5 percent and Steady State error less or equal to 1 percent. A reference input was introduced which help to make steady state error equal to zero from a value of about 100%. Also an observer is designed which estimate the state variable that can not be measured. The designed controller had a gain of 9.6, which was used to determine the gain of the power amplifier used in the system. The designed controller caused the overshoot and settling time to reduce from undefined values to 4.1% and 0.22 seconds respectively which conformed with the performance specifications. The observer also made it possible to observe the convergence of the actual and estimated values of the state variables in less than 0.5 sec. Prototype of magnetic suspension system was constructed and the system was able to suspend a ball of mass of 28g at a distance of 1.2cm below the coil.

CHAPTER ONE

GENERAL INTRODUCTION

1.1 Introduction

The chapter one gives an overview of the magnetic suspension system, motivation and statement of the problems, aim and objectives and significance of study.

1.2 Background Information

Magnetic suspension is a means by -which metallic object is suspended with no support other than magnetic fields. The electromagnetic force is used to counteract the effects of the gravitational force (Dolga and Dolga, 2007). There are several practical applications of the magnetic suspension system and this includes; the Magnetic Levitation (MagLev) train, which is a high-speed train that runs using electromagnetic principle of levitation. The train floats above the guide rail and the polarity of the magnet is used to move the train. Because the MagLev train is not in contact with the track, there is no friction and thus the MagLev train can travel faster than conventional trains. Another application is frictionless bearing, in which the use of magnetic suspension reduces wear and tear on the bearing since there is no contact with other metallic parts (Glavin, 2005). The magnetic suspension system is an unstable non-linear system. Therefore, it is always a challenging effort to design a feedback controller to control the position of the suspended object. In recent years, many approaches have been reported. Hurley and Wolfe (1997) described the linearization of the plant by examining perturbation around the operating point. Compensation is achieved by implementing proportional plus derivative (PD) control. Glavin (2005) carried out work on state space control of a magnetic suspension system in which a controller was designed and system behaviour was simulated using MATLAB/simulink and Pspice

This research work deals with the design of magnetic suspension system using state-space concept. In addition, a prototype model will be constructed. The dynamic compensation of the system will be illustrated by working with state variables of the system. A simplified magnetic suspension system is shown in Figure 1.1

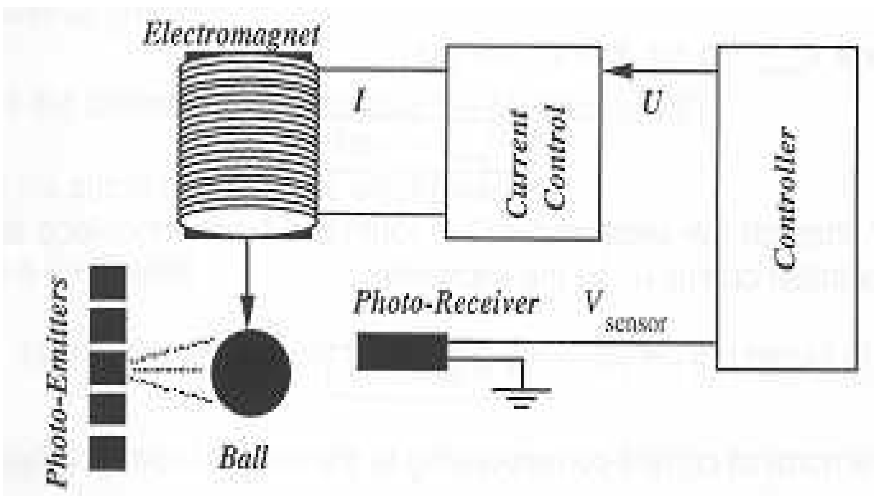

Figure 1.1 Simplified magnetic suspension system.

Coil acts as electromagnetic actuator, while an optoelectronic sensor determine the position of the ferromagnetic ball. By regulating the electric current in the circuit through a controller, the electromagnetic force can be adjusted to be equal to the weight of the steel ball, thus the ball will levitation in an equilibrium state.

1.3 Motivation

A control system provides the means by which any quantity of interest in a machine, mechanism can be maintained or altered in accordance with a desired manner (Nagrath and Gopal, 2005). The methods of achieving this objective usually involve the use of certain control strategies such as discrete PID controller and state feedback control techniques. More so, control systems play a vital role in the advancement of engineering technology such as in magnetic suspension system. However, research is still on going in this area because it allows the study of various control strategies. Therefore, the design of magnetic suspension system based on classical methods (Bode plot, polar plot, root locus etc) do not permit the control engineer the ability to specify all poles in the control system for the systems higher than second order and also it is applicable to only linear system. Hence, the need to design and analyse the system using state-space technique and MATLAB, which will allow the specifications of all the poles of the system, even for higher order systems irrespective of whether the system is linear or non-linear.

Linear system; A system is called linear if the principle of superposition applies. The principle of superposition state that the response produced by the simultaneous application of the two different forcing functions is the sum of the two individual responses while Non-linear system the response to two inputs cannot be calculated by treating one input at a time and adding the results.

1.4 Statement of Problem

The magnetic suspension system is an unstable non-linear system. Therefore, it is always a challenging effort to design a feedback controller to control the position of the suspended object. This work involves the application of state space concepts in the design and analysis of a magnetic suspension system. The mathematical model would be developed, representation of the model in the state space form, verification of controllability and observability of the system using Kalma's method. Controller would be designed by using a pole placement technique so as make the system stable; observer would also be designed in order to estimate the state variables that can be measured. The following components of feedback control would be designed. And these are reference input, position sensor, comparator, and control mechanism.

1.5 Aim and Objectives

This work aimed at construction of prototype non-linear dynamic model of magnetic suspension system where a non-linear state-space transformation is used to linearized the system. The prototype system suspended a mass of 28 gram at a distance of 1.2cm below the coil. Critical issues in the construction of magnetic suspension system considered includes weight of the ball, power requirement to energize the coils in order to produce the required field to levitate the ball, the time and the temperature at which the field will become weak such that the ball drops.

1.6 Significance of the Studies

The Magnetic Suspension System is a classical non-linear control problem just like other non-linear systems such as Ball -and –Beam system and Inverted pendulum. These systems are generally regarded as unstable. It is very important that different approaches are adopted in modelling these systems and designing controller that will ensure system stability. The State-Space approach is adopted in modelling the magnetic suspension system and designing the controller and observer.

1.7 Thesis Outline

This work comprises of five chapters. The first chapter one gives an overview of the Magnetic Suspension system, motivation and statement of the problems. Chapter two covers related works that have been done and theoretical background. Chapter three focuses on modelling and analysis of Magnetic Suspension System, development of mathematical model, representation of the model in the state space form, performance specification, verification of controllability and obervability of the system using Kalma's method whether a solution exist to the problem stated, achieving the set specifications with aid of a controller, the introduction of reference input to remove steady state error and design of an observer that estimate the state variables that are not available for control and measurement purpose. The design of position sensor, reference input, comparator, control mechanism, and implementation of controller. Chapter four highlights the result, analysis and experimental measurements. Chapter five center on the significance of the study, limitation and recommendation for further work.

CHAPTER TWO

LITERATURE REVIEW AND THEORETICAL BACKGROUND

2.1 Introduction

This section reviews previous works that have been done on magnetic suspension system using different methods. The concept of magnetically levitated trains was first identified at the turn of the century by two Americans, Robert Goddard and Emile Bachelet. By the 1930s, Hermann Kemper was developing a concept and demonstrating the use of magnetic field to combine the advantages of trains and airplanes. In 1968, James and Gordon were granted a patent on their design for a magnetic levitation System.

2.2 Review of Past Works in the Area

Walter and John (1996) described linear and nonlinear state-space controller for magnetic levitation. The problem associated with magnetic levitation is its inability to control the height of a steel ball above the ground by levitating it against the force of gravity using electromagnetism. Experimental results were used to compare effectiveness of the two controllers in terms of their ability to respond to sinusoidal reference trajectory. The nonlinear controller was found to provide superior tracking performance compared to the linear one.

Ying (2001) designed and implemented a controller for a magnetic levitation system. In his work, it was concluded that a virtual pole cancellation method with a phase–lead compensated controller was able to maintain better stability in a levitated ball.

Milica and Boban (2003) described a magnetic levitation system in control engineering education. This paper deal with the magnetic levitation control of metallic sphere, which is interesting and visually impressive equipment for demonstrating many intricate problems. The paper also presented some initial outcome in creating a laboratory environment for remote monitoring of the magnetic levitation equipment.

Liliankamp and Lundberg (2004) worked on low cost magnetic levitation project kits for teaching feedback system. In this work magnetic suspension system was designed and constructed with the aim to provide students with an open-ended design problem.

Glavin (2005) carried out work on state space control of a magnetic suspension system in which a controller was designed and system behaviour simulated using MATLAB/simulink and Pspice.

Toru and Hideto (2006) applied Generalized Internal Model Control (GIMC) concept to the magnetic suspension system. From the experiment it was showed that GIMC can achieve both higher performance and robustness.

Dolga and Dolga (2007) carried out work on modelling and simulation of magnetic levitation system in which a controller was designed using MALTAB/Simulink.

Hurley and Wolfle (2007) described the linearization of the plant by examining perturbation around the operating point. Compensation was achieved by implementing proportional plus derivative (PD) control.

Jinxin (2007) worked on magnetic levitation control system using analogue and digital phase-lead controller. In this work phase-lead controllers were designed to stabilize the magnetic levitation system with the aid of MATLAB. Analog controllers were implemented using resistors, capacitors, and operational amplifiers while the digital controller was implemented using the TMS320C6711 DSK.

Stephen and Won-Jong (2005) worked on design, fabrication, and control of a single actuator Maglev Test Bed. A controller was designed and implemented using a dSPACE 1104.

This research will be focused on state-space design method and construction of magnetic suspension system. It is a technique in which a dynamic compensation of the system will be illustrated by working with the state variable describing the system.

2.3 Theoretical Background

The time domain analysis and design of control system utilize the concept of the State-space of a system.

2.3.1 State

The state of a dynamic system is a minimal set of variables (known as state variables) such that the knowledge of these variables at $t = t_0$ together with the knowledge of the inputs for $t \geq t_o$, completely determines the behaviour of the system for $t > t_o$.

2.3.2 State Variables

State variables are the smallest set of variable that describe the behaviour of the system. The output can be determined given the initial state and inputs to the system. "The state variables of a dynamic system are the variables making up the smallest set of variables that determine the state of dynamic system. if at least n variables x_1, x_2…….,x_n are needed to completely describe the behaviour of a dynamic system (so that once the input is given $t \geq t_o$ and the initial state at $t = t_o$ is specified, the future state of the system is completely determined), then such n variables are a set of state variables.(Katsushiko Ogata, 2005)

2.3.3 State Vector

"If n state variables are needed to completely describe the behaviour of a given system, then these n state variables can be considered the n components of a vector x. such a vector is called a state vector. A state vector is thus a vector that determines uniquely the system state x(t) for any time $t \geq t_o$, once the state at $t = t_o$ is given and the input u (t) for $t \geq t_o$ is specified."

2.3.4 State Space

The n dimensional space whose coordinate axes consists of the x_1 axis, x_2 axis, x_3 axis … x_n axis wherex$_1$ x_2, x_3 ……x_n are state variables is called a state space. Any state can be represented by a point in the state space."

2.3.5 State-Space Equation

There are three different variables involved in state-space design these are input variables, output variables, and state variables. The state –space representation for a given system is not unique, except that the number of the state variables is the same for any of the different state-space representation of the same system.

2.3.6 The Mathematical Relationship between Magnetic flux density (B) and Temperature

The mathematical relationship between magnetic flux density and temperature is given (Lovell and Avery, 1981):

$$B = \mu_o H \qquad (2.1)$$

Where: B is magnetic field intensity

H is magnetic field strength

μ_o is the permeability of vacuum

When a magnetic material is in a magnetic field H with magnetization (or dipole moment per unit volume) M magnetic induction in the material B is

$$\begin{aligned} B &= \mu_o (H + M) \\ &= \mu_o \mu_r H \end{aligned} \qquad (2.2)$$

Where μ_r is the relative permeability of the material (=1 for vacuum) (Lovell and Avery, 1981)

$$\mu_r = 1 + \frac{M}{H} \quad (2.3)$$

$$= 1 + x \quad (2.4)$$

Where x is the magnetic susceptibility of the material. B, M and H are vectors; the unit of B is Tesla or Weber m^{-2}, while M and H have the same units of ampere per metre. Hence x has no unit

Similarly,

$$x = \frac{C}{T} \quad \text{(Curie's law) (Lovell and Avery, 1981) :} \quad (2.5)$$

$$C = \frac{N\mu^2}{3k} \text{(Curies constant)} \quad (2.6)$$

N is the number per unit volume

K is Boltzmann's constant

T is temperature

Substituting all the parameters:

$$B = \mu_o \left(1 + \frac{C}{T}\right) H \quad (2.7)$$

From equation (2.7) it can be concluded that magnetic flux density is inversely proportion to temperature.

2.3.7 The Magnetic flux density on the Axis of a Circular Coil

Figure 2.1 shows a circular one-turn coil carrying a current of I. the magnetizing force at the axis point P due to a small element dl is given by (Theraja, 2005) :

$$dH = \frac{Idl}{4\Pi(r^2 + x^2)} \quad (2.8)$$

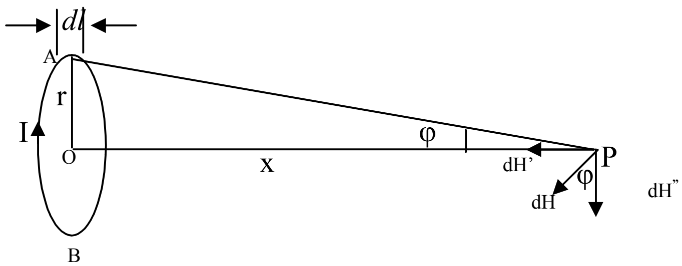

Figure 2.1 Circular one- Turn coil.

The direction of dl is at right angle to the AP joining point P to the element dl. Now dH can be resolved into two components

(a) The axial component dH=dHsinφ

(b) The vertical component dH= dHcosφ

Now, the vertical component dHcosφ will be cancelled by an equal and opposite vertical component dH due to element dl at point B. the same applied to all other diametrically opposite pairs of dl's taken around the coil.

Hence, the resultant magnetizing force at point at P will be equal to the sum of all axis components

$$H = \sum dH \int dl \tag{2.9}$$

$$= \sum dH \sin\theta \int dl \tag{2.10}$$

$$= \sum \frac{Idlr}{4\Pi(r^2 + x^2)^{\frac{3}{2}}} \tag{2.11}$$

$$\sin\theta = \frac{r}{\sqrt{(r^2 + x^2)}} \tag{2.12}$$

$$H = \frac{Ir}{(r^2 + x^2)^{\frac{3}{2}}} \int_0^{2\Pi r} dl \tag{2.13}$$

$$= \frac{Ir 2\Pi r}{4\Pi(r^2 + x^2)} \tag{2.14}$$

$$= \frac{Ir^2}{2(r^2 + x^2)^{\frac{3}{2}}} \tag{2.15}$$

$$B = \mu_o H$$

$$= \frac{\mu_o I r^2}{2(r^2 + x^2)^{\frac{3}{2}}} \quad (2.16)$$

If the number turns is N, then B become

$$B = \mu_o H = \frac{\mu_o N I r^2}{2(r^2 + x^2)^{\frac{3}{2}}} \quad (2.17)$$

From the equation (2.17), it shows that magnetic flux density (B) reduces with distance (x) away from the coil because of their inverse relationship, if other parameters remain constant. (Theraja, 2005)

2.3.8 Determination of Step Response

The response of the system to a step input is shown in Figure 2.2.

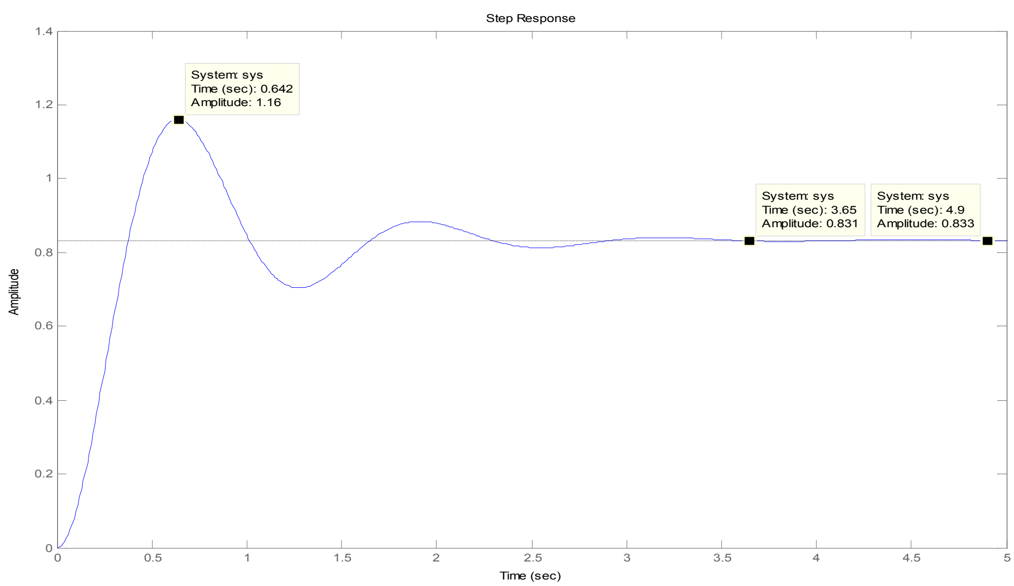

Figure 2.2 A typical response of the system to a step input

2.3.8.1 Maximum Overshoot

Let $U(t)$ be the unit step response of the system, $U_{\max}$ be the maximum value of $U(t)$ and U_{ss}, the steady state value of $U(t)$. The maximum overshoot is given as:

$$\text{Maximum overshoot} = U_{\max} - U_{ss}$$
$$= 1.16 - 0.833$$
$$= 0.327.$$

The maximum overshoot is given as a percentage of the final value of the step response; that is:

$$\text{The percentage maximum overshoot} = \frac{Maximum\ overshoot}{Steady\ state\ value} \times 100\%$$

$$= \frac{0.327}{0.833} \times 100\%$$
$$M_p = 39.3\%$$

2.3.8.2 Settling Time

The settling time t_s is defined as the time required for the step response to decrease and stay within a specified percentage of its final value. From figure 2.2 the settling time;

$$t_s = 4.93\text{seconds}$$

2.3.8.3 Steady State Error (SSE)

One of the objectives of control systems is that the system output response should follow a specified reference in the steady state. The difference between the output and the reference input when the steady state is reached is the steady state error. The steady state error from figure 2.2

SSE = Reference input – steady state output

= 1 – 0.833

= 0.167

CHAPTER THREE

METHODOLOGY

3.1 Introduction

Chapter three focuses on modelling and analysis of Magnetic Suspension System, development of mathematical model, representation of the model in the state space form, performance specification, verification of controllability and observability of the system using Kalma's method whether a solution exist to the problem stated, achieving the set specifications with aid of a controller, the introduction of reference input to remove steady state error and design of an observer that estimate the state variables that are not available for control and measurement purpose. The design of position sensor, reference input, comparator, control mechanism, and implementation of controller.

The realization of the magnetic suspension system will be tested on the methodology to be adopted in achieving the set objective of designing, analyzing and developing a magnetic suspension system involves the following:

i. Development of mathematical model
ii. Representation of mathematical model in state space

After deriving the mathematical model, the next step is to find the state matrices A, B, C and D.

$$\dot{X} = Ax + Bu \tag{3.1}$$

$$Y = Cx + Du \tag{3.2}$$

A is the system matrix, B is the input matrix, C is the output matrix, D is the transmission matrix, u is the input vector, x is the state vector and y is the output

The linearization state equation is expressed in the form of equation (3.1) and (3.2) with the coefficient matrices A, B, C and D evaluate as

$$A=\begin{bmatrix} 0 & 1 & 0 \\ \frac{x^2{}_{03}}{Mx^2{}_{01}} & 0 & \frac{-2x_{03}}{Mx_{01}} \\ 0 & 0 & -\frac{R}{L} \end{bmatrix}$$

$$=\begin{bmatrix} 0 & 1 & 0 \\ \frac{g}{x_{01}} & 0 & -2\left(\frac{g}{Mx_{01}}\right)^{\frac{1}{2}} \\ 0 & 0 & -\frac{R}{L} \end{bmatrix} \qquad (3.3)$$

$$B=\begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} \qquad (3.4)$$

$$C=\begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \qquad (3.5)$$

$$D=0 \qquad (3.6)$$

The characteristic equation is given by $|sI - A|$ (3.7)

The transfer function can be determined by

$$\frac{Y(s)}{V(s)} = C(sI - A)^{-1} B = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} (sI - A)^{-1} B \qquad (3.8)$$

Where the ball position $y(t)$ is the output and $V(t)$ is the input.

iii Determination of Controllability

Controllability of the system will be determined using Kalman's Method to verify if a solution exist to the control problem and feedback will be obtained

$$M_c = [B \ : AB : ..A^{n-1}B] \qquad (3.9)$$

The system is said to be controllable if Mc is a full rank (having n linearly independent column) i.e Mc ≠0, where Mc is the composite matrix.

vi Determination of Observability

Observability of the system will be determined using Kalman's Method to verify if a solution exists to the control problem and observer gain will be obtained.

$$Mo = \begin{bmatrix} C \\ --- \\ CA \\ --- \\ . \\ --- \\ CA^{n-1} \end{bmatrix} \tag{3.10}$$

Where Mo is the observability matrix, the system is said to be observable if Mo is a full rank (having n linearly independent rows) i.e $Mo \neq 0$.

v. The Controller and Estimator will be combined.

vi The reference input will be introduced.

vii The system will be simulated using Matlab

The entire system will be simulated using MATLAB and graphs of the system response will be obtained.

viii Analysis of the System

Analysis of the system will be carried out based on the results obtained

viii Design of Position Sensor

The photo-emitter/detector sensor is used for detecting the vertical position of the ball. It generates an analogue signal corresponding to the actual ball position. This section is going to be designed with infrared emitter, photo transistor, operational amplifier and resistors. A block diagram for this subsystem is shown in Figure 3.1

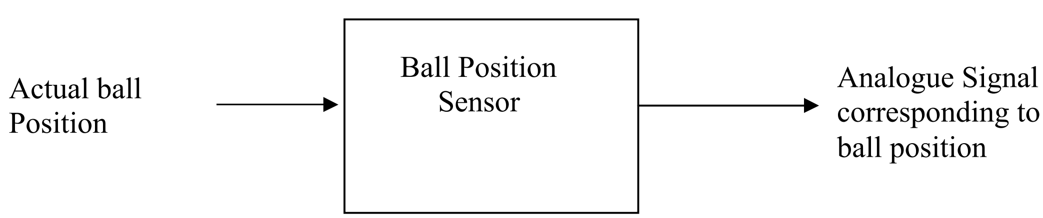

Figure 3.1 Block diagram of position sensor

xi Design of Reference Input

This measures the relative brightness of the infrared LED, along with the total ambient light. The section will be designed by using phototransistor, resistors and operational amplifier. A block diagram for this subsystem is shown in Figure 3.2

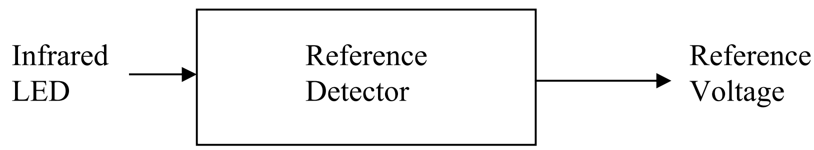

Figure 3.2 Block diagram reference input

x Design of Comparator

The comparator is an electrical device that compares the two inputs. (the reference input and the output from ball position sensor) It finds the difference between two input voltages and amplifiers it. This sector will be designed by using operational amplifier and resistors. A block diagram for this subsystem is Figure 3.3

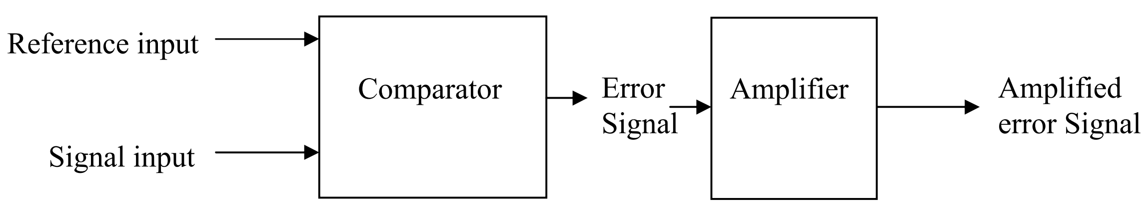

Figure 3.3 Block diagram Comparator

ix Design of Control Mechanism.

The control mechanism is the electromagnet lifting coils. This consist of coil driver and electromagnetic. The control signal produced by the comparator is amplified and fed to a coil driver to produce a current. This current is fed to an electromagnet, which produces the required force to suspend the ball at the desired position. To design this section, power transistor, 24 standard wire gauge, a carriage bolt (4inches long) and 3/8 inches thickness will be used The subsystem block diagram is shown in Figure 3.4

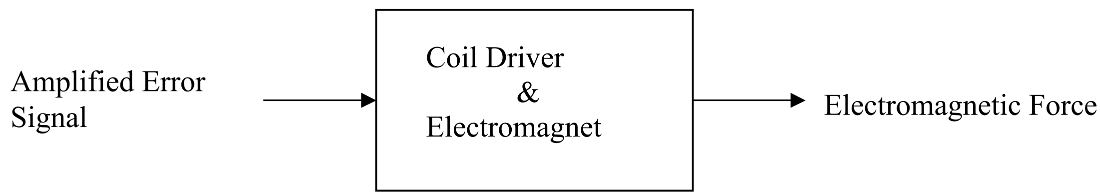

Figure 3.4 Block diagram control mechanism

xii Overall System

After designing the different sub-sections they will be developed and coupled as Magnetic Suspension System. The block diagram of the system is shown in Figure 3.5

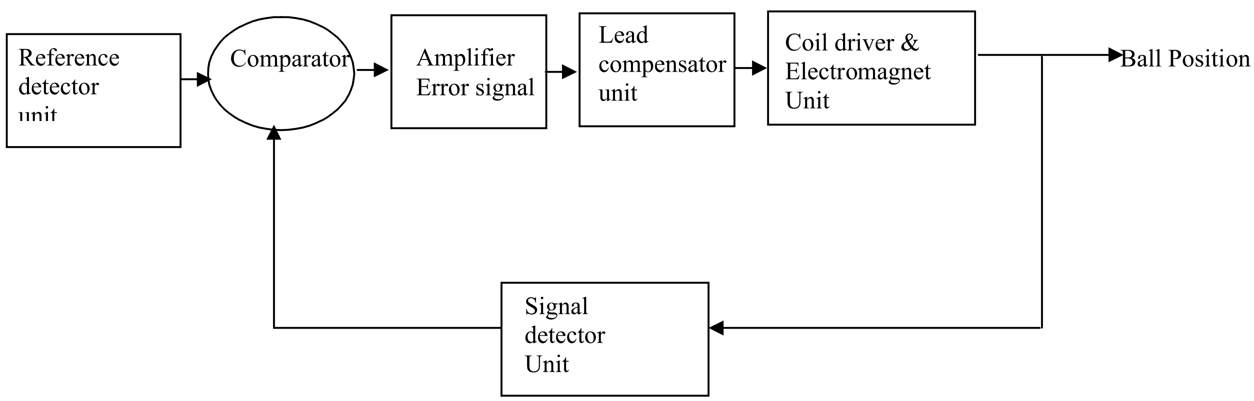

Figure 3.5 Block diagram of whole system

3.2 Development of Mathematical Model

The inductance of the coil varies with the position of the ball as illustrated in Figure 3.5. When the ball is in contact with the magnetic the coil inductance is (Glavin, 2005, Hurley and Wolfle,1997).

$$L_0 + L_1(x = 0) \quad (3.11)$$

When the ball is removed, the inductance of the coil is $L_1(x = \infty)$,D is the diameter of the ball.

$$a = \frac{D}{9} \tag{3.12}$$

Where "a" is decay constant for coil inductance.

The varying inductance between these two extremes is given by

$$L(x) = L_1 + L_o e^{-x/a} \tag{3.13}$$

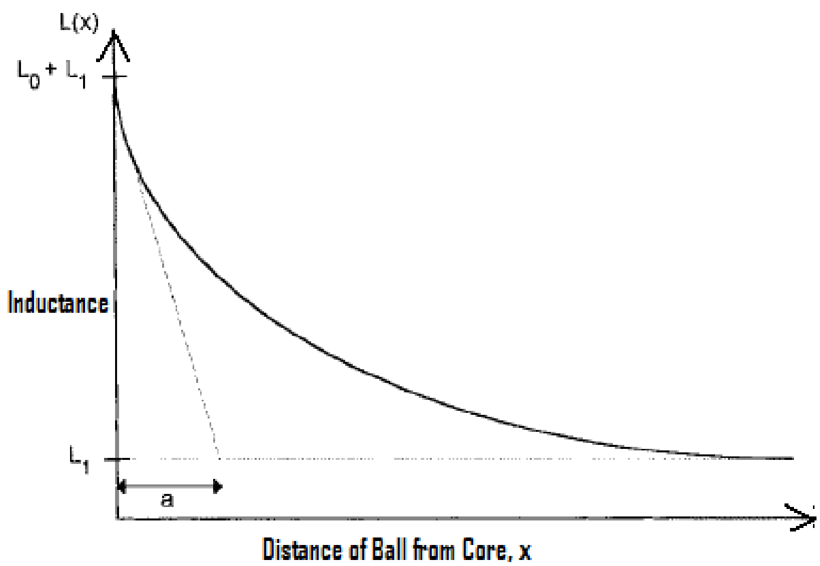

Figure 3.6 Coil inductance as a Function of Distance

The magnetic co-energy of the system is a function of the coil current i and distance x

$$W'(i,x) = \frac{1}{2}L(x)i^2 \tag{3.14}$$

The magnetic force originally acting on the ball is given by (Glavin, 2005) :

$$f = \frac{\partial W}{\partial x} = -\frac{L_0}{2a} i^2 e^{-x/a} \tag{3.15}$$

When the ball is in its equilibrium position, the gravitational force is equal to magnetic force acting on the ball.

At

$x = d$

and

$i = I$

$L_d = \frac{L_o}{N^2} e^{-d/a}$

(3.16)

$$Mg = \frac{L_o}{2a} e^{-d/a} I^2 = \frac{N^2 L_d}{2a} \quad I^2 \tag{3.17}$$

Where N = the number of turns in the coil

L_d = the incremental inductance at $x = d$ due to a single turn coil.

Rearranging equation 3.17 to obtain an equation for I

$$I = \sqrt{2Mg \frac{a}{N^2 L_d}} \tag{3.18}$$

The magnetic force originally acting on the ball is given by (Glavin, 2005) :

$$f(i, x) = -\frac{1}{2a} L_o e^{-x/a} i^2 \tag{3.19}$$

Consider a perturbation about the equilibrium point at

$x = d$

$i = I$

$x = d + x'$

$i = I + I'$

By Taylor's series expansion

$$f(i, x) = f(I, d) = \frac{\partial f}{\partial x}\Big|_{I,d} x' + \frac{\partial f}{\partial i}\Big|_{I,d} i' \tag{3.20}$$

This now becomes

$$f' = \frac{1}{2a} L_o e^{-dla} I^2 x' - \frac{I}{a} L_o e^{-d/a} I i' \tag{3.21}$$

The mechanical force is

$$f^m = Mg + M \frac{d^2 x'}{dt^2} \tag{3.22}$$

At equilibrium

$Mg = f(I, d)$

And the incremental equation of motion becomes

$$M\frac{d^2x'}{dt^2}-\frac{N^2L_dI^2}{2a^2}x'+\frac{N^2L_dI}{a}i'=0 \tag{3.23}$$

The Laplace transform of eqn 3.23 is given by (Glavin, 2005)

$$\frac{X(s)}{I(s)}=\frac{\frac{N^2L_d}{a}I}{Ms^2-\frac{N^2L_dI}{2a^2}} \tag{3.24}$$

Using equation 3.18 gives

$$\frac{X(s)}{I(s)}=\frac{-\frac{2g}{I}}{s^2-w^2{}_n} \tag{3.25}$$

Where $w^2{}_n=\sqrt{\frac{g}{a}}$

Equation 3.25 is the plant transfer function which relates the coil current and the ball position.

Equation 3.23 can also be written as

$$M\frac{d^2x(t)}{dt^2}=Mg-\frac{i^2(t)}{x(t)} \tag{3.26}$$

$$v(t)=Ri(t)+L\frac{di(t)}{dt} \tag{3.27}$$

Equation (3.26) is a nonlinear differential equation of the system.

Where

$v(t)$ = input voltage

$y(t)=x(t)$ = ball position

$i(t)$ = winding current

R= winding resistance

L = winding inductance

M = mass of ball

g = gravitational acceleration

The state variables are defined as

$$x_1=x(t) \tag{3.28}$$

$$x_2 = \frac{dx(t)}{dt} \tag{3.29}$$

$$x_3 = i(t) \tag{3.30}$$

The state equations are

$$\frac{dx_1(t)}{dt} = x_2 \tag{3.31}$$

$$\frac{dx_2}{dt} = g - \frac{1x_3^{\,2}(t)}{Mx_1(t)} \tag{3.32}$$

$$\frac{dx_3(t)}{dt} = -\frac{R}{L}x_3 + \frac{1}{L}v(t) \tag{3.33}$$

The linearization of the system about the equilibrium point

$y_o(t) = x_{01} =$ Constant

Then,

$$x_{02}(t) = \frac{dx_{01}}{dt} = 0 \tag{3.34}$$

$$\frac{d^2 y_o(t)}{dt^2} = 0 \tag{3.35}$$

The nominal value of $i(t)$ is determined by substituting equation (3.25) into equation (3.26) thus

$$i_o(t) = x_{03} = \sqrt{Mgx_{01}} \tag{3.36}$$

3.3 States-Space Model

After deriving the mathematical model, the next step is to find the state matrices A, B, C and D.

$$\dot{X} = Ax + Bu \tag{3.37}$$

$$Y = Cx + Du \tag{3.38}$$

A is the system matrix, B is the input matrix, C is the output matrix, D is the transmission matrix, u is the input vector, x is the state vector and y is the output

The coefficient matrices A , B, C and D evaluated as (Benjamin and Farid, 2003)

$$A=\begin{bmatrix} 0 & 1 & 0 \\ \frac{x^2_{03}}{Mx^2_{01}} & 0 & \frac{-2x_{03}}{Mx_{01}} \\ 0 & 0 & -\frac{R}{L} \end{bmatrix} \tag{3.29}$$

Substituting equation (3.26) into (3.29) matrix A become

$$=\begin{bmatrix} 0 & 1 & 0 \\ \frac{g}{x_{01}} & 0 & -2\left(\frac{g}{Mx_{01}}\right)^{1/2} \\ 0 & 0 & -\frac{R}{L} \end{bmatrix} \tag{3.30}$$

$$B=\begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} \tag{3.31}$$

$$C=\begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \tag{3.32}$$

$$D = 0 \tag{3.33}$$

The characteristic equation is given by

$$|sI - A| \tag{3.34}$$

The transfer function can be determined by

$$\frac{Y(s)}{V(s)} = C(sI - A)^{-1}B = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}(sI - A)^{-1}B \tag{3.35}$$

Where the ball position y(s) is the output and V(s) is the input.

Parameters of magnetic suspension system are:

The inductance of coil $L = 0.5H$

Resistance of the coil $R = 8\Omega$

Mass of ball to be suspended by system $M = 28g$

Acceleration due gravity $a = 9.81 m/s^2$

The distance below the coil where the ball to be suspended is $x = 1.2 cm$

In order to establish the inherent instability of the system it is important to determine the poles of the system, substituting the values of inductance, resistance, mass of the ball, distance and acceleration due to gravity into equations (3.30), (3.31) and (3.32) gives

$$A=\begin{bmatrix} 0 & 1 & 0 \\ 817.5 & 0 & -4.204 \\ 0 & 0 & -16 \end{bmatrix} \quad (3.36)$$

$$B=\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} \quad (3.37)$$

$$C=\begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \quad (3.38)$$

D = 0

The poles of the system were found to be

$$P_1 = 28.592$$
$$P_2 = -28.592$$
$$P_3 = -16.000$$

It can be seen that one of the pole $(P_1 = 28.592)$ is on the right hand side and as such the system typically is an unstable system (Benjamin and Farid, 2003) :

The step response at this poles location is shown in the Figure 3.7.

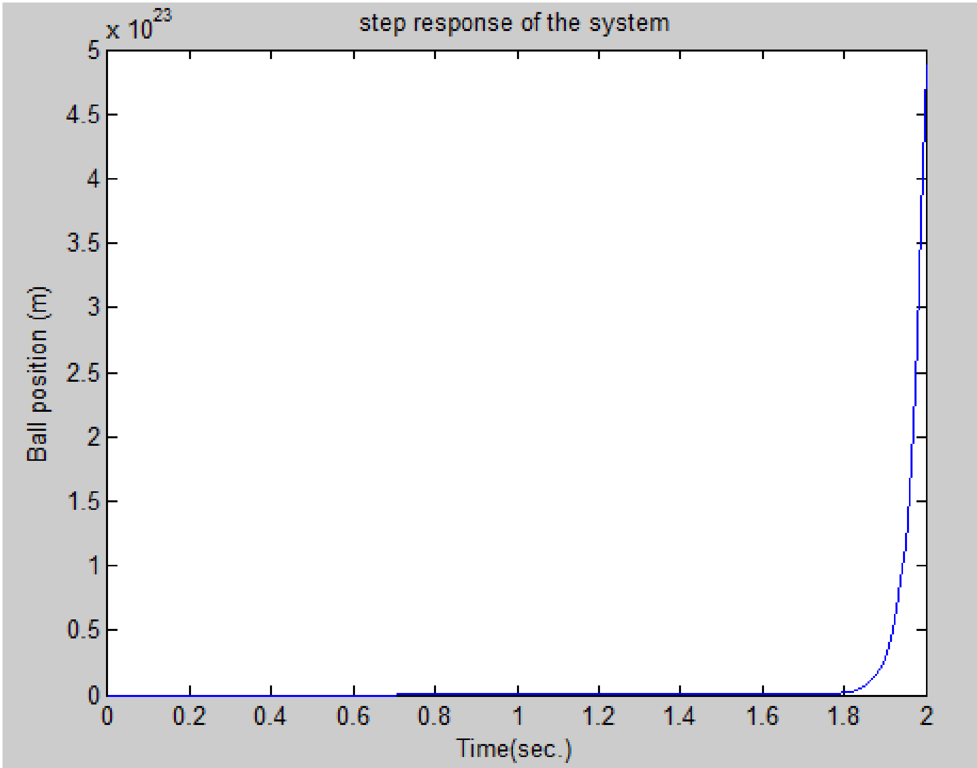

Figure 3.7 Step response of the system

```
The response parameters at this location using "stepinfo"
command are as follows:
```

S=

```
    Rise Time = NaN
     Settling Time= NaN
      Settling Min= NaN
      Settling Max= NaN
        Overshoot=  NaN
       Undershoot= NaN
             Peak    = Inf
         Peak Time=Inf
NaN= Not a number
```

Since the settling time and over shoot are undefined it means that the system is unstable. This leads to test of controllability and observability of the system to determine if a control solution exist to the problem.

3.4 Controllability and Observability of the Magnetic Suspension System

The concept of controllability and observability in the design of controllers for dynamic systems mainly involves verifying from the outset whether a control solution exist for the problem. This gives the designer the privilege of placing the poles anywhere in the left hand side of the s-plane (Nagrath and Gopal, 2005). If any one of the state variables is independent of the control signal, there will be no way of driving the system from an initial state to a desired state in a finite time by the application of a control vector. Hence, the system is said to be uncontrollable. The observability of a system relates to the condition of observing or estimating the state variables from the output variables. If any one of the state variables cannot be observed from the measurement of the output, the system is said to be unobservable (Nagrath and Gopal, 2005) :

3.4.1 Controllability

A system is said to be completely state controllable if it is possible to transfer the system from any initial state $X(t_o)$ to any desired state $X(t)$ in specified finite time by a control vector $U(t)$.The controllability of the magnetic suspension system is found using the Kalman controllability matrix (Nagrath and Gopal, 2005) :

$$M_c = \left[A \vdots AB \vdots \cdots \vdots A^{n-1}B\right] \tag{3.39}$$

Where M_c is the controllability matrix, a system is said to be controllable if it has n linearly independent columns.Since the determinant of $M_c = -141.9081$, is not equal to zero, the system is controllable. The Matlab program used to verify the controllability of magnetic suspension system is shown in Appendix A1.

3.4.2 Observability

A system is said to be completely observable if every state $X(t_o)$ can be completely identified by measurement of the output $Y(t)$ over a finite time interval.

The observability of the system is verified using the Kalman observability matrix (Nagrath and Gopal,2005).

$$Mo = \begin{bmatrix} C \\ CA \\ CA^2 \\ -- \\ -- \\ CA^{n-1} \end{bmatrix} \tag{3.40}$$

Where M_o is the observability matrix, a system is said to be observable if it has n linearly independent row. Since the determinant of $M_0 = -4.2043$ is not equal to zero, the system is said to be observable. The Matlab program used to verify the observability of the system is shown in Appendix A1.

It was discovered that the system is controllable because the determinant of Kalman controllability matrix is Mc= -141.905 with a rank of 141.905. Similarly, the observabillity of the system was verified using Kalman Observability matrix which shows that the determinant of the matrix is equal to -4.2043 with a rank of -4.2043

Since the system is controllable it means that the system can be stabilized and since also it is observable it means that the poles can be placed anywhere in the s-plane in order to ensure that the system meets up with its design specifications. This will entail designing a controller.

3.5 Controller Design using Pole Placement

The State-Space controller will be designed on the basis of State-Space matrices and the state vectors as shown in the equations (3.37) and (3.38).

$$\dot{x} = Ax + Bu \tag{3.41}$$

$$y = Cx + Du \tag{3.42}$$

Where A is the system matrix, B is the input Matrix, C is the Output matrix and D is the transmission matrix which equal to 0. X is the state of the system. y is the output of the system, u is the input of the system.

$$x = \begin{bmatrix} x_i \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x \\ \dot{x} \\ i \end{bmatrix} \tag{3.43}$$

x is the distance of ball below the coil, $\dot{x}$ denote the velocity of the ball and i is the current applied to the coil. The block diagram showing the structure of state feedback and the controller K is shown Figure 3.8

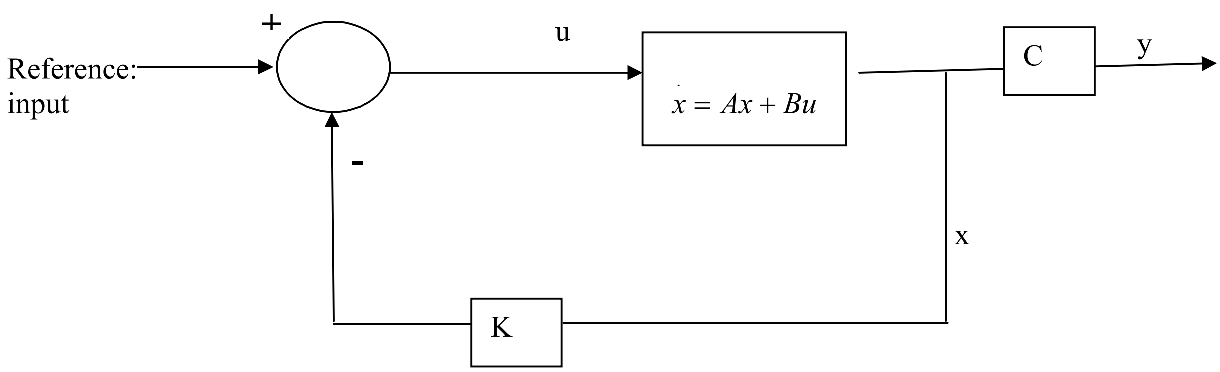

Figure 3.8 Structure of state feedback and the controller K

In order to design the controller, it is necessary to first of all give the basic design specification of the system.

3.5.1 Performance Specification and System Response

The performance specifications for magnetic Suspension System are as follows;

Settling Time $\leq$ 0.5secs

Maximum Overshoot $\leq$ 5%

Steady State Error $\leq$ 1% (Dolga and Dolga, 2007, Toru and Hideto, 2006, Lundberg and Lilienkamp, 2004) :

In order to meet the above specifications for the stability of the system, the poles of the system are then placed arbitrarily in the s-plane using the Matlab function, ‘place’.

The following value of the poles (p_1,p_2, and p_3) were arbitrarily chosen

[-3.5+3.5i, -3.5-3.5i, -16]

[-5+16i,-5-16i,-57],

[-8+7i,-8-7i, -40]

[-10+310i, -10-10i, -50]

[-20+20i, -20-20i, -100],

A typical Matlab script for determining the controller gain for the different values of the chosen poles is as follows;

```
MATLAB SCRIPT
p1=-3.5+3.5i;
p2=-3.5-3.5i;
p3=-16;
K=place(A,B,[p1,p2,p3]);
sys_cl=ss(A-B*K,B,C,0);
lsim(sys_cl,u,t,x0);
title('Response with k Controller')
sys = ss(A-B*K,B,C,0);
S = stepinfo(sys)
```

The step response of the system using the different values of the poles is as shown in Figures 3.9 to 3.13.

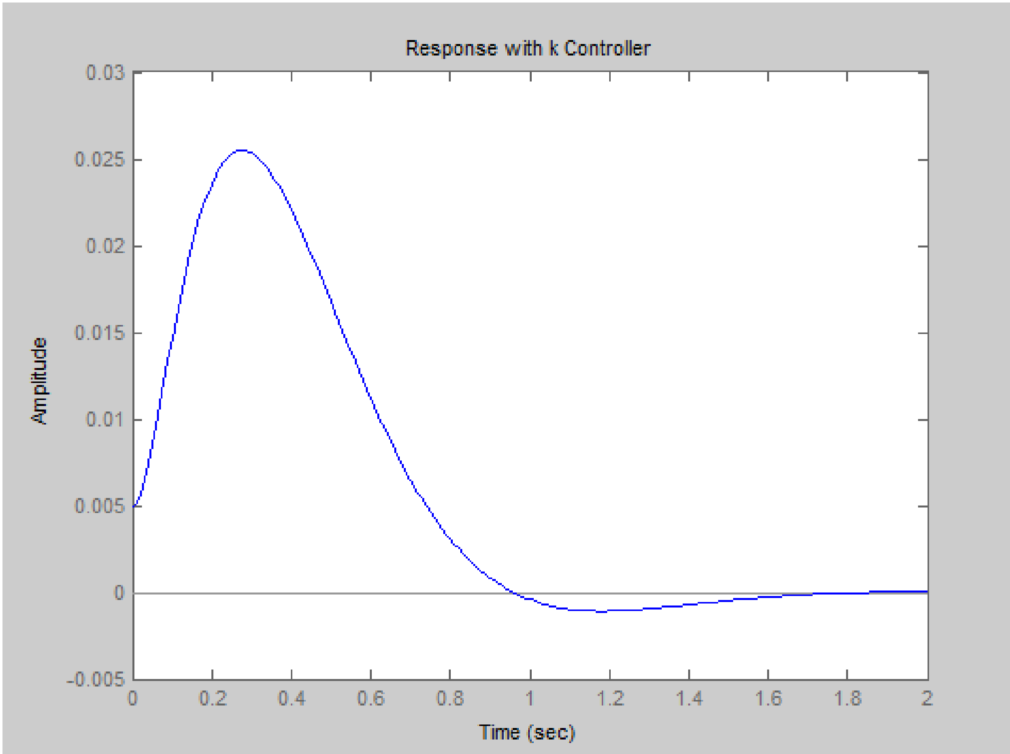

Figure 3.9 System Response with poles located at [-3.5+3.5i, -3.5-3.5i, -16]

The response parameters at this location using "stepinfo" command are as follows:

```
        Rise Time= 0.4572
    Settling Time= 1.2683
     Settling Min= -0.0223
     Settling Max =-0.0199
       Overshoot =4.0516
      Undershoot =0
            Peak =0.0223
        Peak Time= 0.9688
```

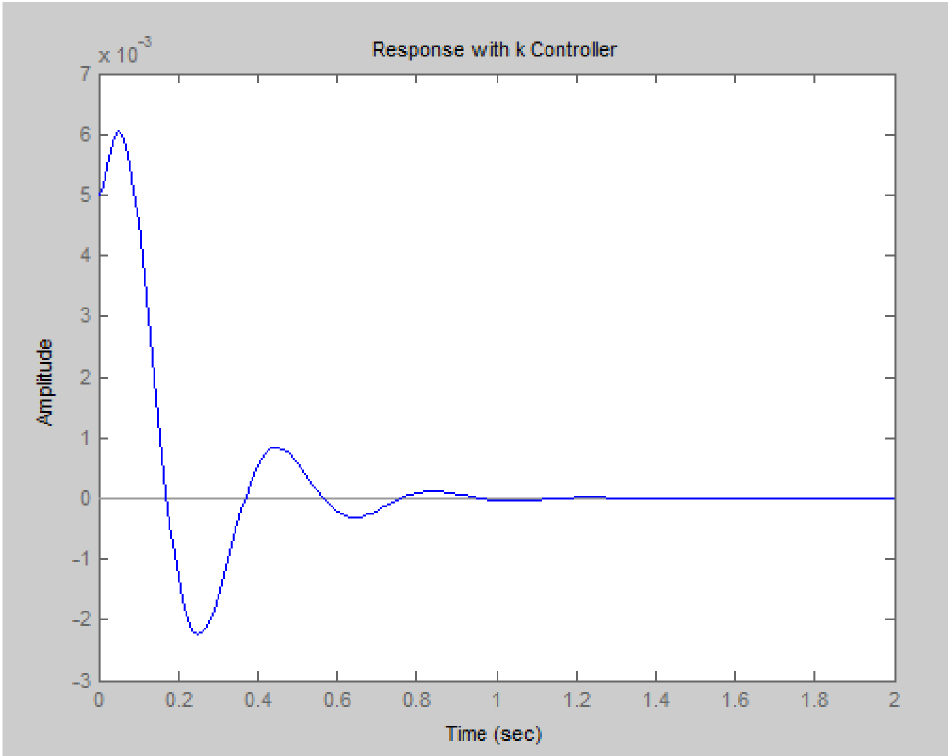

Figure 3.10 System Response with poles located at [-5+16i,-5-16i,-57]

The response parameters at this location using "stepinfo" command are as follows:

```
  Rise Time = 0.0847
Settling Time = 0.6872
 Settling Min = -7.1101e-004
 Settling Max = -4.5471e-004
   Overshoot = 35.4371
  Undershoot= 0
        Peak = 7.1101e-004
    Peak Time= 0.2231
```

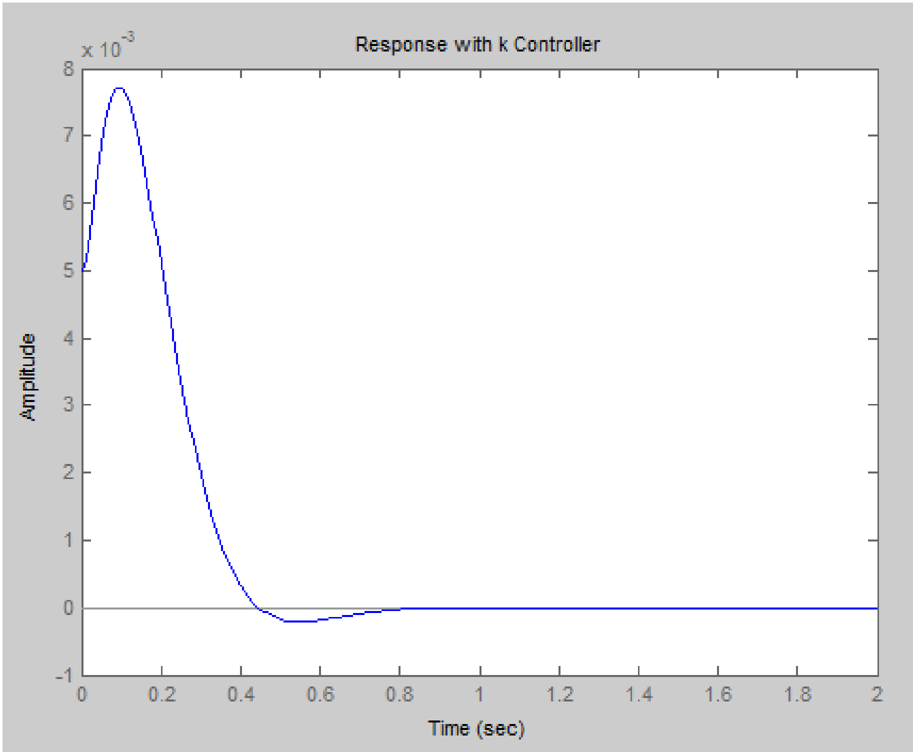

Figure 3.11 System Response with poles located at [-8+7i,-8-7i,-40]

The response parameters at this location using "stepinfo" command are as follows:

```
    Rise Time = 0.2242
Settling Time = 0.5612
 Settling Min = -0.0019
 Settling Max = -0.0017
    Overshoot = 2.6327
   Undershoot = 0
         Peak = 0.0019
    Peak Time = 0.4768
```

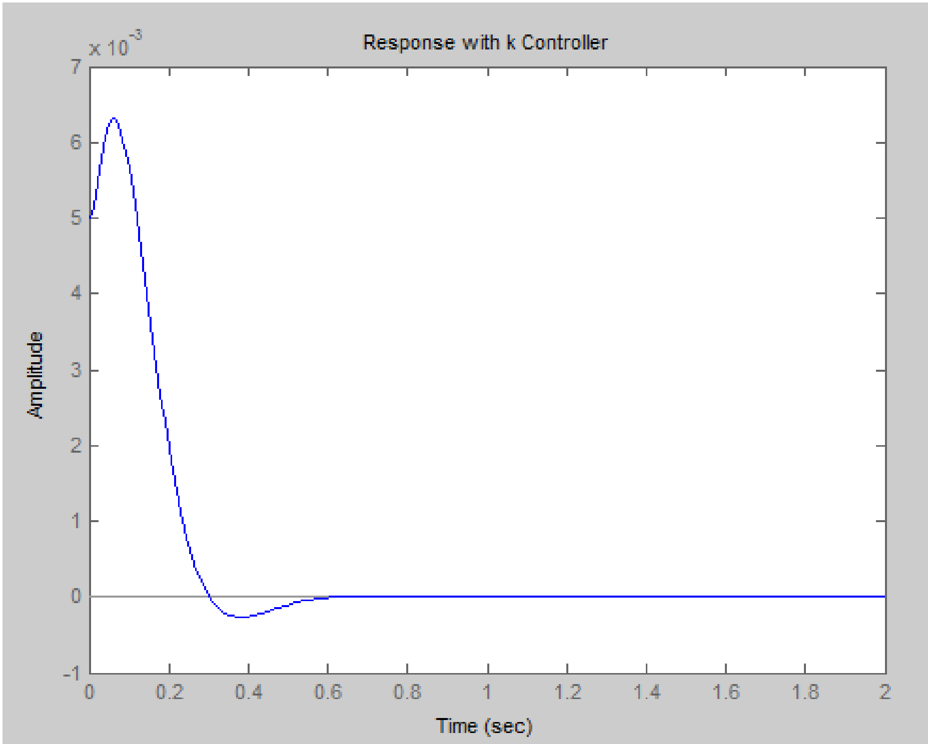

Figure 3.12 System Response with poles at located at [-10+10i, -10-10i, -50]

The response parameters at this location using "stepinfo" command are as follows:

```
        Rise Time = 12.8700
    Settling Time = 49.6951
     Settling Min = -5.3562e-004
     Settling Max = 7.3095e-004
       Overshoot = 5.5195e+022
      Undershoot = 1.3020e+024
            Peak = 0.0126
        Peak Time = 7
```

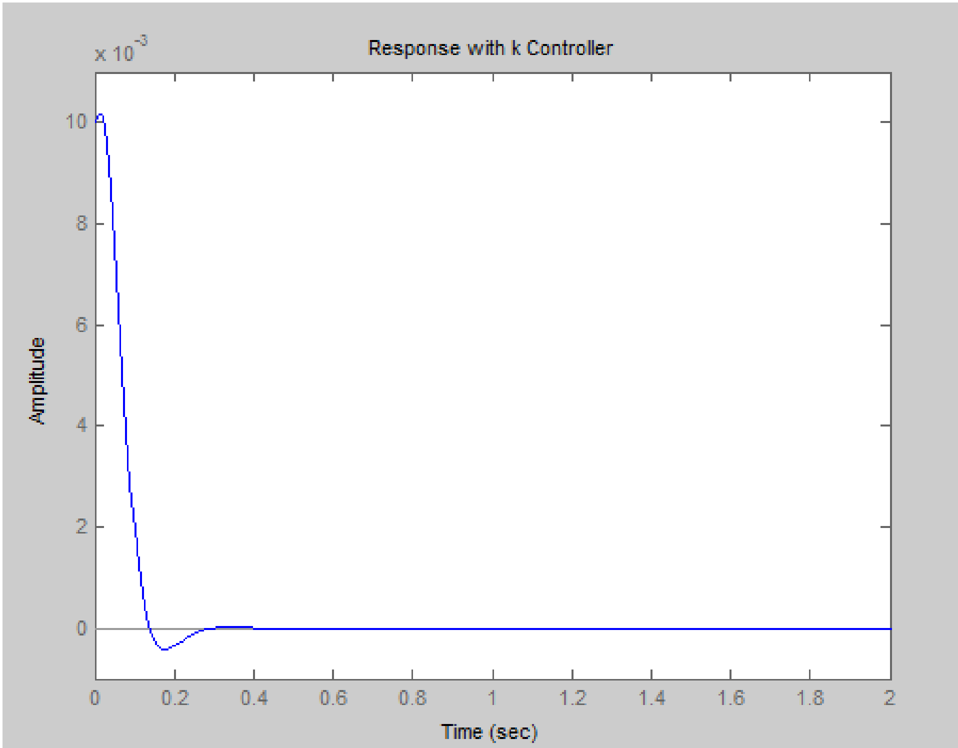

Figure 3.13 System Response with poles located at [-20+20i, -20-20i, -100]

```
The response parameters at this location using "stepinfo"
command are as follows:
        Rise Time = 0.0795
    Settling Time = 0.2210
     Settling Min = -1.0942e-004
        Settling Max = -9.5073e-005
       Overshoot = 4.1017
      Undershoot =   0
        Peak = 1.0942e-004
        Peak Time = 0.1695
```

From the results of the response parameters at different locations using "stepinfo" command, the poles location at [-20+20i, -20-20i, -100] was chosen because the settling time is 0.2210 seconds which is less than the performance specification of 0.5 second and the overshoot is 4.1017% also less than the performance specification of 5% . The system settling time and the overshoot have been met.

In order to determine gain K of the controller at this location using "place" command

$k = 1.0x10^4 x2.3125x0.0668x0.0062 = 9.6$

The system has a large steady state error because in the design of the controller the reference input was taken to be zero as shown in Figure 3.8

From Figure 3.13, the steady state error SSE is:

SSE= Reference input – steady state value

=1-0

=1

=100%

Therefore in order to solve this problem, a reference input is introduced into the system.

3.6 The Reference Input

The reference input was taken as zero when the feedback gain K was determined. This caused the output signal to deviate from the input as seen in Figure 3.13.

A block diagram showing the feedback gain K and the reference input is shown in Figure 3.14.

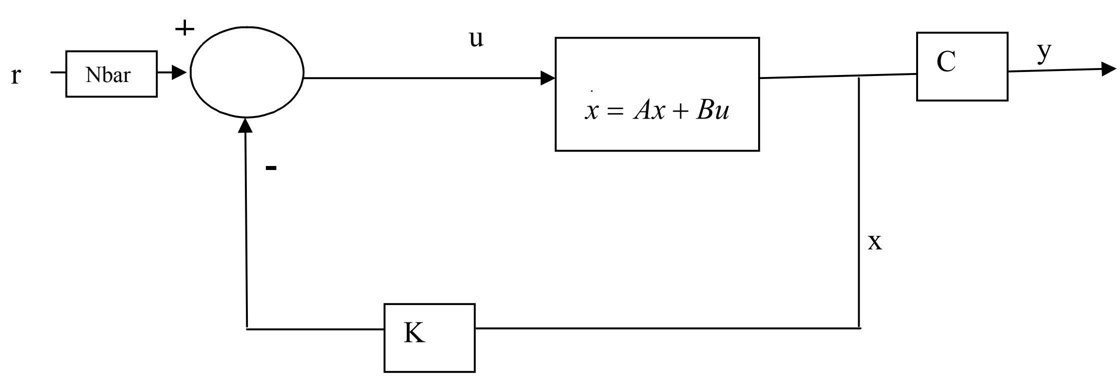

Figure 3.14 Block diagram of the feedback gain K and the reference input

Where A is the system matrix, B is the input Matrix, C is the Output matrix and D is the transmission matrix which equal to 0. x is the state of the system. y is the output of the system, u is the input of the system. rNbar is the reference input

Figure 3.15 shows the response of the system with step input function without the scale factor.

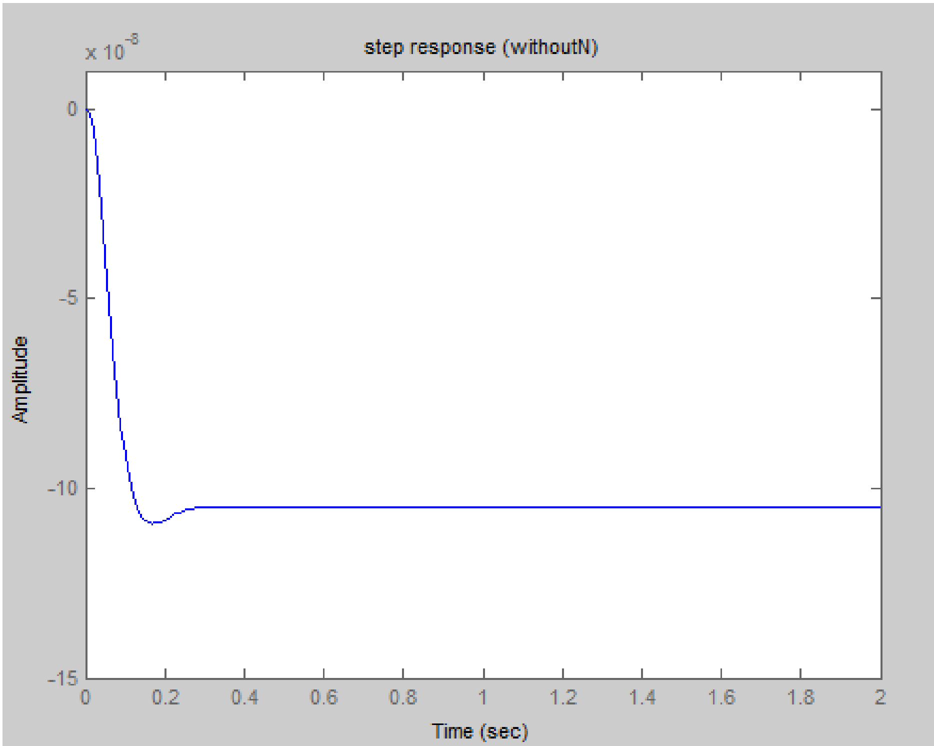

Figure 3.15 Step Response of the system without Nbar

By the application of a step input, the system does not track the step response as expected as shown in Figure 3. 15 .In order to eliminate this problem, the reference input must be scaled and made equal to K*x which is the steady state value of the output. This scale factor is called Nbar.

The Nbar can be determined from Matlab by using the function 'rscale'.

Function Nbar = rscale (A, B, C, D, K)

This function 'rscale' finds the scale factor N which eliminates the steady error.

The value of Nbar when run on the command window is;

Nbar = 9.514

3.7 Observer Design (State Variables Estimator)

An observer needs to be designed and implemented since not all state variables of the system are known. The state variables that are known are the current i and the position x of the ball, but the velocity $\dot{x}$ of the ball is not known. Hence there is need to design an observer that will estimate the states.

An observer compares the actual measured output y to the estimated output $\hat{y}$ of the system. It will cause the estimated state $\hat{x}$ to approach the values of the actual states x. The dynamics of the observer are given by the poles of (A-L*C) where L is the observer gain.

A block diagram showing the schematic of a state feedback system with the full observer is shown in 3.16

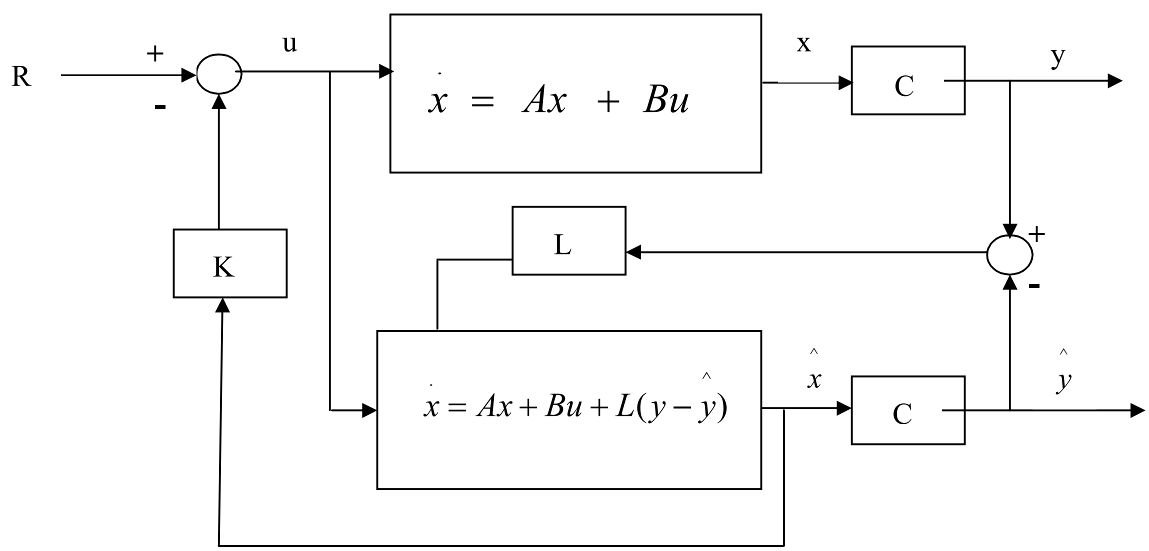

Figure 3.16 Block Diagram of State Feedbacks with Observer

The observer model is represented thus:

$$\dot{x} = Ax + Bu + L(y - \hat{y}) \quad (3.44)$$

$$\hat{y} = C\hat{x} \quad (3.45)$$

Subtracting equation 3.44 from equation 3.41 and equation 3.45 from equation 3.42 gives equation 3.46 and 3.47 respectively.

$$(\dot{x}-\dot{\hat{x}}) = A(x-\hat{x}) - L(y-\hat{y}) \quad (3.46)$$

$$(y-\hat{y}) = C(x-\hat{x}) \quad (3.47)$$

Rearranging equation (3.46) and representing $(\dot{x}-\dot{\hat{x}}) = e$ as the error gives:

$$\dot{e} = (A-LC)e \quad (3.48)$$

$$(y-\hat{y}) = Ce \quad (3.49)$$

From equation (3.48), the dynamics of the observer is governed by the expression (A-LC). The observer poles can be arbitrarily assigned like the poles of the controller. This is done using the Matlab command “place” as used earlier in finding the controller gain. The observer poles must be placed at least two or five times farther to the left of the s-plane than the dominant poles of the system; this is to ensure that the observation error converges to zero as quickly as possible (Toru and Hideto, 2006, Abbas, 2008) :

The observer poles were chosen to be:

```
op1=-100;
op2=-101;
op3=-102;
```

The observer feedback gain L at this location of the pole using the Matlab function “place” is

L= [0.0029,0.2683,1.4605]

3.8 System Block Diagram

The block diagram of magnetic suspension system is shown in Figure 3.17

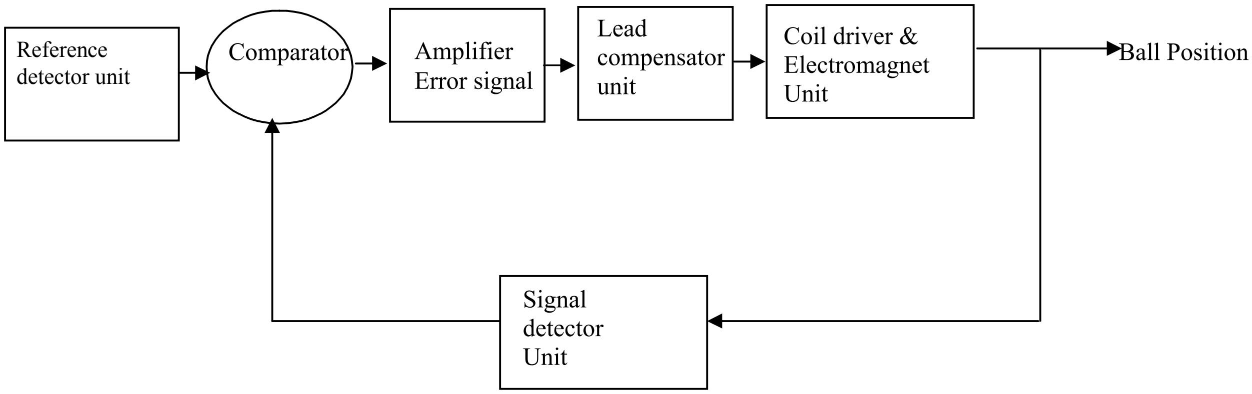

Figure 3.17 Block Diagram of Magnetic Suspension System

3.9 Design of Infrared Emitter

The infrared emitter is designed and implemented as shown in the circuit diagram of Figure 3.18.

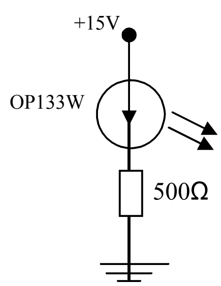

Figure 3.18 Infrared Emitter Diagram

The infrared emitter circuit converts electrical energy into light energy.

For the LED,the maximum forward voltage $V_f = 1.75V$ and maximum forward current $I_f = 100mA$, $I_f = 30mA$ is chosen for safe operation of Light Emitting Diode (LED) and with a supply voltage $\mathrm{V} = 15\mathrm{V}$ (OP505A Datasheet)

$$R = \frac{V - V_f}{I_f} \tag{3.50}$$

$= 442\Omega$

A standard value of 500Ω is used.

3.10 Design of Signal Detector

The signal detector is designed and implemented as shown in Figure 3.19

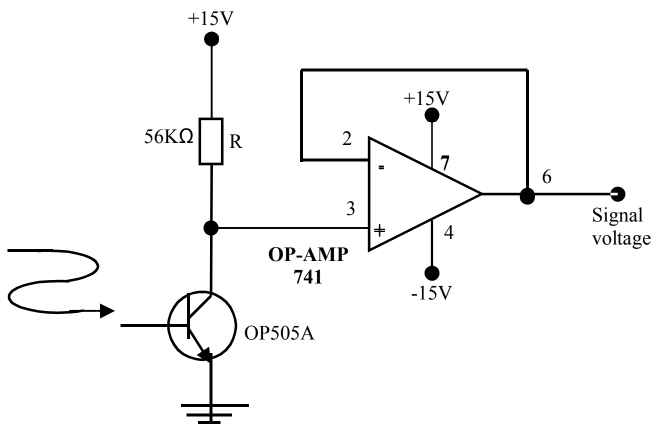

Figure 3.19 Signal Detector Circuit

The signal detector circuit converts the received light energy into electrical energy. The phototransistor (OP505A) has a light-sensitive collector base P-N junction which is exposed to incident light through a lens. When light strikes the P-N junction, a base current I_B is produced which is proportional to the light intensity. This action produces collection current Ic whose value is given by

$$I_C = \beta I_B . \qquad (3.51)$$

The current is converted to a voltage by the 56kΩ resistor connected from its collector to the positive supply voltage. The op-amp is wired as a voltage follower (the input voltage to pin 3 will be the same as output pin 6) to isolate the photodetector circuit from the next stage.

$V_{CE} = 5V$

Maximum power dissipation $P_T = 100mW$ (OP505A Datasheet)

$$I_C = \frac{P_T}{V_{CE}} \qquad (3.52)$$

$$= 20mA$$

The maximum value of collector current I_C is 20mA

For safe operation of the phototransistor collector current of 0.130mA is chosen.

The reference voltage is set halfway between 0V and 15V which is 7.5V (Mehts and Mehta, 2008).

The supply voltage V_{CC} = 15V

$$R = \frac{Vcc - Vref}{I} \tag{3.53}$$
$$= 57.7K\Omega$$

A standard value of 57KΩ is used

The design principle of the signal detector is the same as that of the of reference detector.

3.11 Design of Comparator

The comparator is designed and implemented as shown in Figure 3.20

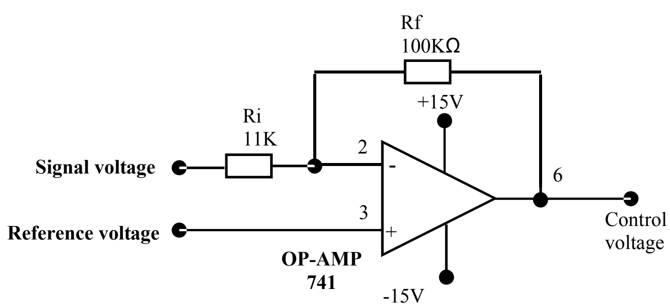

Figure 3.20 Comparator Circuit

The circuit creates a control signal from the two optodetectors. It finds the difference between the two input voltages and amplifies it, in order to obtain the required signal to sustain the ball at particular position.

$$Gain = \frac{Rf}{Ri} \tag{3.54}$$
$$= 9$$

So the output in pin 6 is 9 times the difference between the two input signals.

3.12 Design of Lead Compensator

The compensator is designed and implemented as shown Figure 3.21

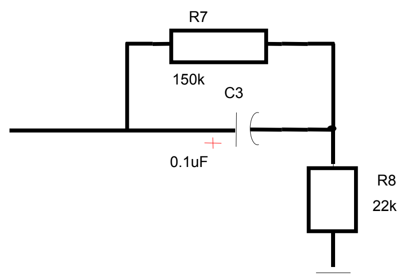

Figure 3.21 Lead Compensator Circuit

One of the main reasons for the system being unstable is the slight negative phase margin introduced by the coil. A lead compensator is used to force the phase margin positive and increase the crossover frequency.

The phase margin is a measure of the relative stability of a system and gives an indication of the tolerance allowable on the phase before the system become unstable. It is measured at the gain cross-over frequency and usually a negative value of the phase margin is an indication of instability.

The lead compensator takes on the form as follows:

$$\frac{\alpha(1+ST)}{(1+S\alpha T)} \tag{3.55}$$

Where α is attenuation of the lead compensator

$$\alpha = \frac{R_8}{(R_7 + R_8)} \tag{3.56}$$

For the optimal design the value of α lies between: (0< α<1) (Benjamin and Farid,2003). In this control problem the value α is chosen to be

$$\frac{22k}{22k+150k} = 0.128 \ .$$

This is a drawback as it means that signal reduce by a factor of eight. To negate this effect, a non-inverting amplifier with a gain of 247 (in order to reduce power dissipation in the final output transistor) is added to the output of the lead network. Another section of the LM741 is used along with two more resistors to achieve the desired gain (output amplifier)

3.13 Design of Output Amplifier

The output amplifier is designed and implemented as shown in Figure 3.22

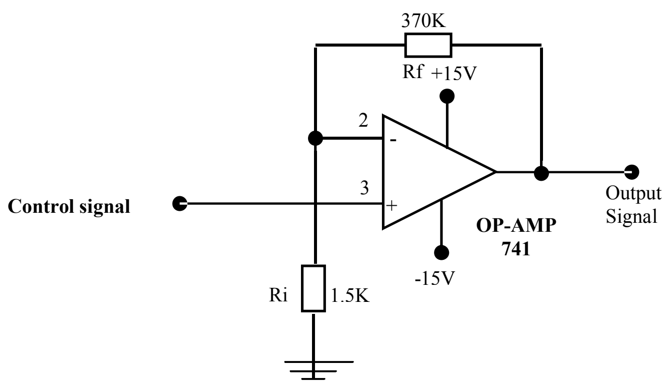

Figure 3.22 Output Amplifier

The output of comparator is reduced by one – ninth by the lead compensator network. The 741 Op-amp is wired as a standard non-inverting amplifier. From Figure 3.17 the gain is determined as

$$Gain = \frac{R_f + R_i}{R_i} \qquad (3.57)$$

$= 247$

This gain is very high therefore the power dissipation in the final output transistor is reduced.

3.14 Design of Coil Driver and Electromagnet

The coil driver is designed and implemented as shown in Figure 3.23

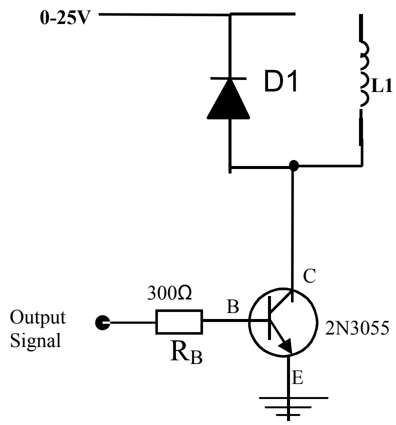

Figure 3.23 Coil Driver and Electromagnet

This circuit controls the current in the electromagnetic coil. It is driven by the 741 Op-amp and power transistor 2N3055.The diode D1 is connected to the input terminal of coil to protect the collector of the transistor from the effect of the back electromotive force (back emf) from the self induction of the coil which can damage the transistor.

The base resistor provides some protection as it limits the output current demand on the Op-amp. Supply voltage $V_{CC} = 15V$, $V_{BE} = 0.7V$ and the base current $I_B = 50mA$ (2N3055 Datasheet). The value of the resistor R_B which supplies the base current to the transistor is obtained as follows;

$$R_B = \frac{V_{CC} - V_{BE}}{I_B} \tag{3.58}$$

$$R_B = 300\Omega$$

The lifting coil is designed with 24 standard wire gauge, because of its current rating of 3.5A (Standard wire gauge Datasheet)

For the optimum coil design (Hurley and Wolfle, 1997)

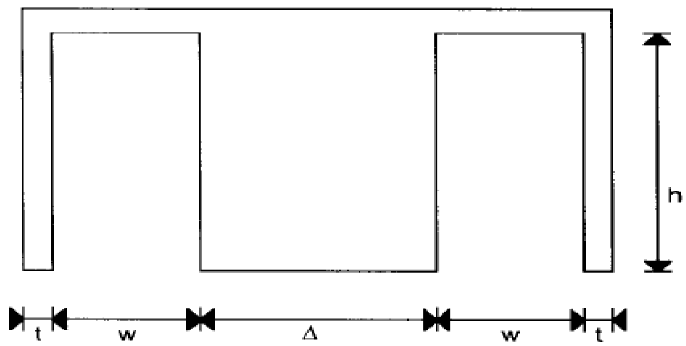

Figure 3.24 Dimension for laminated plate for coil design

Core diameter $\Delta = .8D$

Window width $w = 0.5D$

Window height $h = 2w$

Jacket thickness $t = 0.1D$

Where D is the diameter of ball

1250 turns on 24 layers wounded on a carriage bolt.

3.15 Design D.C Power Supply Unit

The D.C power supply is designed and implemented as shown in Figure 3.25

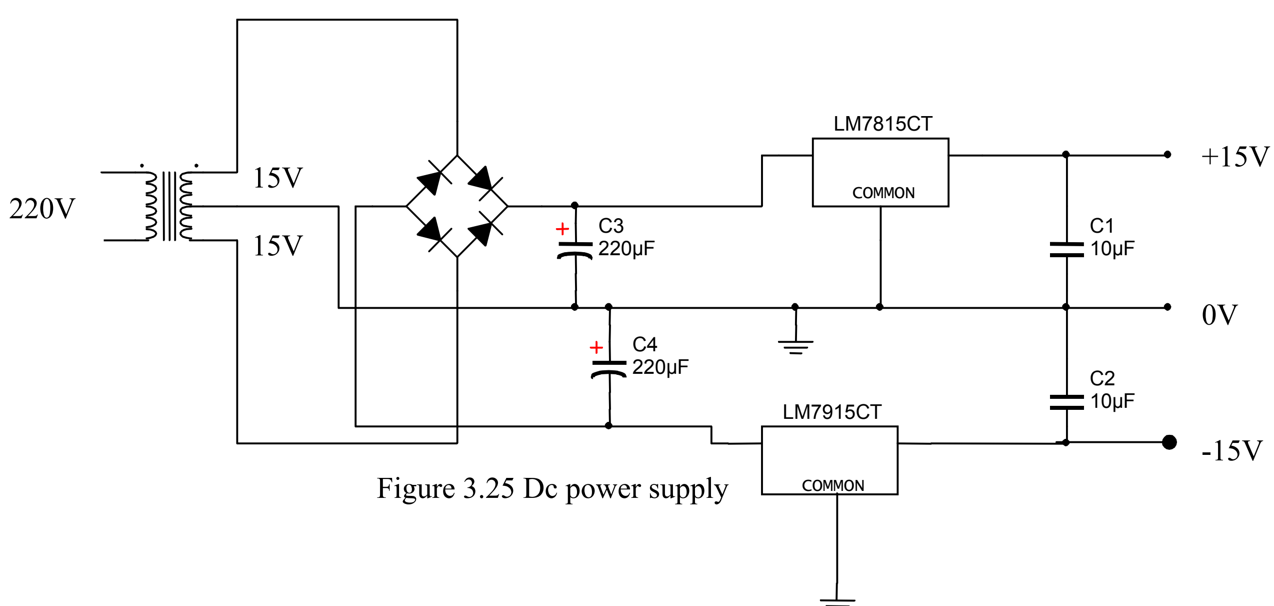

Figure 3.25 Dc power supply

A centre tap transformer that uses 220-230V from the main source and produce stepped down voltage of two 15V centre tap and 30V is used. For two centre tapped 15V output, a bridge rectifier is used to rectify the steeped down voltage and the output is passed through filter capacitors to remove the ripple component to make the output smoother. The value of this filter capacitor is calculated using the relation:

$$C = \frac{V_{\max}}{\Delta V f p R_L} \tag{3.58}$$

Where the V_{max} is the maximum voltage response, ΔV is the ripple factor. The f_p is the peak to peak frequency of the full wave rectification which is twice the maximum, frequency in the input (50Hz). R_L is the resistance at the output of the circuit. The filtered outputs are then connected to integrated circuit regulator to produce the required voltage. In the power supply system, the voltage regulators IC 7815 and IC7915 give +15V and – 15V respectively. These voltages are required to supply the different parts of the circuit. C_2 act as a line filter to improve transient response. C_1 is used to prevent unwanted oscillation.

3.16 Design of Adjustable D.C Power Supply

The Adjustable D.C power supply is designed and implemented as shown in Figure 3.26

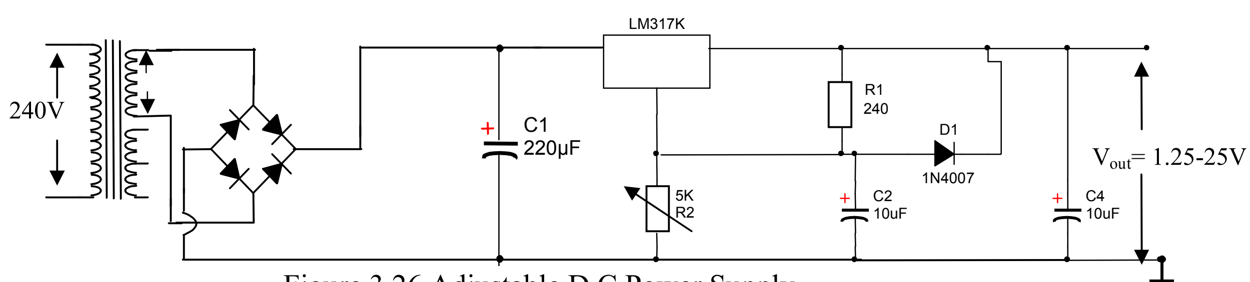

Figure 3.26 Adjustable D.C Power Supply

The auto transformer is used which has multiple has outputs that takes 220V-240V from the mains and produce two 15V centre tap, and 30V. For 30V a bridge rectifier is used to rectify the stepped down voltage to dc voltage. The output of voltage is passed through filter capacitor C1 to remove the ripple components to make the output smoother while C_2 and C_4 act as line filters to improve transient response. LM 317K voltage regulator is used to provide any dc output voltage that is within its two specified limits. The voltage divider is used to change the dc output voltage of the regulator. By changing R2 a wide range of output voltage can be obtained. LM 317 is an IC with three – terminal positive adjustable voltage regulation and supply 1.5A of load current over an adjustable output of 1.25V to 25V which is the voltage required by the coil. The output voltage

$$V_{out} = 1.25\left(\frac{R_2}{R_1} + 1\right) \quad (3.59)$$

After the design of the controller and observer there is a need to implement it, on the Magnetic Suspension System.

3.17 Implementation of Designed Controller

The overall block diagram of the magnetic suspension is shown in Figure 3.27. The set point or reference point determines the steady –state position of the ball. The position of ball is sensed optically, and a voltage is generated by phototransistor.

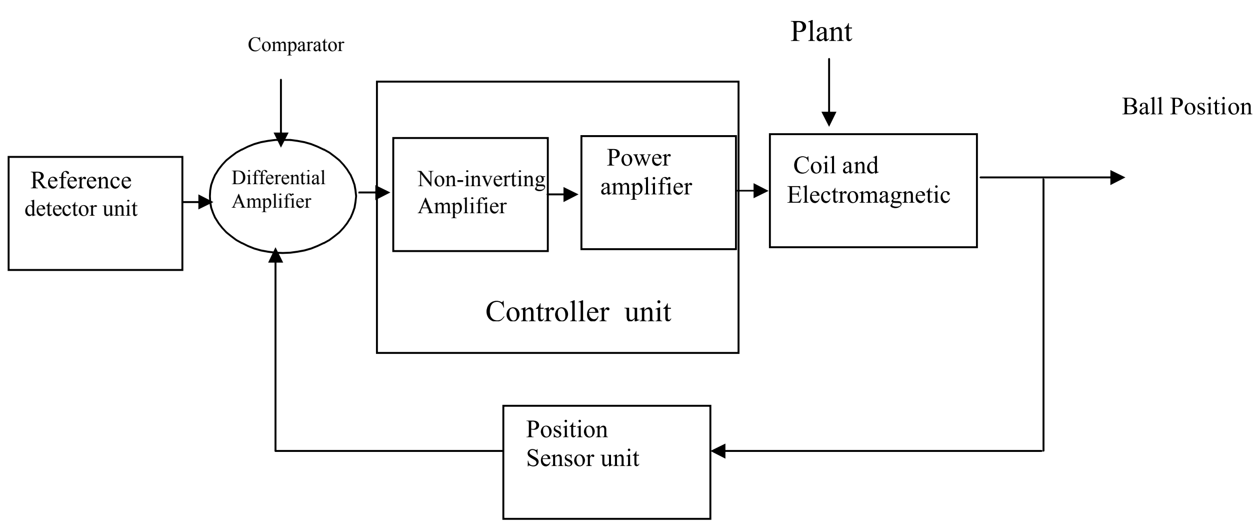

Figure 3.27 Block Diagram showing the implementation of controller

The comparator unit:-The circuit creates a control signal for the two opto-detectors i.e (Reference detector and position detector unit). It finds the difference between the two input voltage and amplifier it.

The controller unit –This unit consists of Non- inverting amplifier and the power amplifier unit. The overall gain of the controller is equal to the product of Non-inverting gain and power amplifier gain.

The controller designed has gain K=9.6

The gain of non-inverting amplifier k_n=0.2 (LM 741 Datasheet)

$$K = k_n x k_p$$

where k_p is the gain of power amplifier

$$k_p = \frac{9.6}{0.2} = 48$$

The power transistor of gain 48 is used in this system. Therefore it can be concluded that the controller designed is used to determine the gain of the power amplifier.

3.18 Temporary Construction of the Circuit on Breadboard

The circuit is constructed on a project board to assess its workability with respect to the design specifications. The components were inserted into the slot on the board and connected accordingly by means of jumper wire. After all the connections were made carefully into the correct position, the circuit was connected to the main supply. The device worked perfectly. The following precautions were taken during the connection.

- The device was not power until all the components were inserted into the slot of the breadboard.
- The device was power when the continuity tested was ensured

- ➢ Calculated value or equivalents value of the components were used during the construction

The list of components used for the construction is shown in Appendix A5

3.19 Permanent Construction of the Circuit on Veroboard

The components were assembled on the veroboard, jumper wire were used to make connections. The board was then soldered using soldering iron and lead.

3.20 Design of the Casing

The case for the system was made of a plastic material and the dimensions were such that the veroboard sits properly and room was also given for proper ventilations.

The circuit diagram of magnetic suspension system showing the implementation of the controller is shown in Figure 3.28

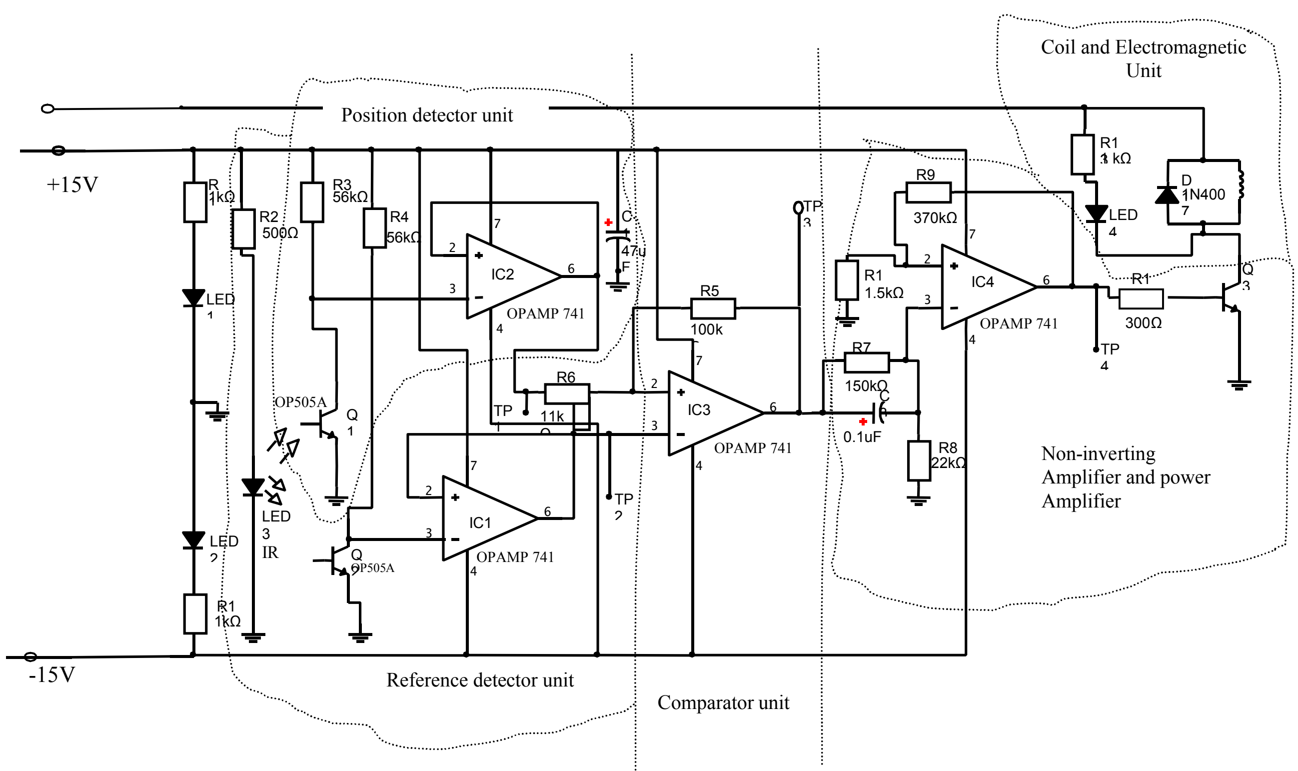

Figure 3.28 The circuit diagram of Magnetic Suspension System

CHAPTER FOUR

PERFORMANCE EVALUATION, RESULTS AND ANALYSIS

4.1 Introduction

Chapter four highlights the Performance Evaluation, result, analysis and experimental measurements.

4.2 Performance Evaluation

The constructed magnetic suspension system suspends a mass of 28 gram at a distance of 1.2 cm below the coil. By regulating the electric current in the circuit through a controller, the electromagnetic force can be adjusted to be equal to the weight of the steel ball, thus the ball would be suspended. The current of 1.3A produced electromagnetic force that suspended the ball. Below this current, force of gravity acting the ball will be greater than electromagnetic force, and therefore the ball will drop. Similarly, if the current is above 1.3A electromagnetic force will be greater than the force of the gravity acting on the ball, and hence the ball gets attracted to the electromagnet.

When the ball is being suspended, while the thermometer and magnetometer were used to measure the temperature and magnetic flux density respectively, at intervals of two (2) minutes till the field can no longer suspend the ball. It was found that as temperature increases, the magnetic flux decreases. At an average temperature of $46^{O}C$, the field was no longer strong enough to hold the ball. The total time taken for this operation was twenty two (22) minutes.

4.3 Analysis of design of Controller and Observer

The poles of the system were placed arbitrarily in the s-plane using the Matlab function, ‘place’.

The following value of the poles (p_1, p_2, and p_3) were arbitrarily chosen

[-3.5+3.5i, -3.5-3.5i, -16]

[-5+16i,-5-16i,-57],

[-8+7i,-8-7i, -40]

[-10+310i, -10-10i, -50]

[-20+20i, -20-20i, -100],

From the results of the response parameters at different locations using "stepinfo" command, the poles location at [-20+20i, -20-20i, -100] was chosen because the settling time is 0.2210 seconds which is less than performance specification of 0.5 second and overshoot is 4.1017% is also less than performance specification of 5% . The system settling time and the overshoot has been met.

The response of the system with the inclusion of Nbar is shown in Figure 4.1

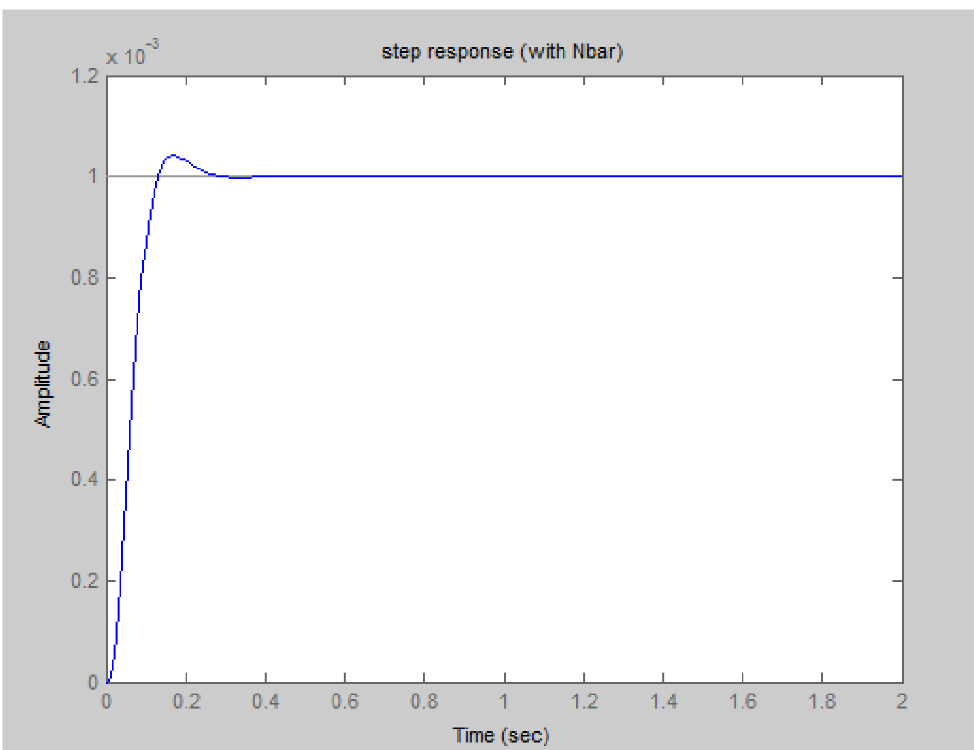

Figure 4.1 Response of System with Controller K and Reference input with Nbar

From the Figure 4.1 the steady state error (SSE)

SSE = Reference input –steady state value

SSE= 1-1

= 0%

It means that by the application of reference input with Nbar, the steady state value has reduced from 100% to 0%. It implies that the remaining design specification (steady state error value) has also being met.

In the design of the controller, not all states variables of the system were known. In order to determine the unknown state, an observer needs to be designed to estimate the effect of the state variables that may not be available for control and measurement purpose.

The response of all state both the actual states and the estimated states is shown in Figure 4.2 The Matlab program used in plotting Figure 4.2 is shown in Appendix A4

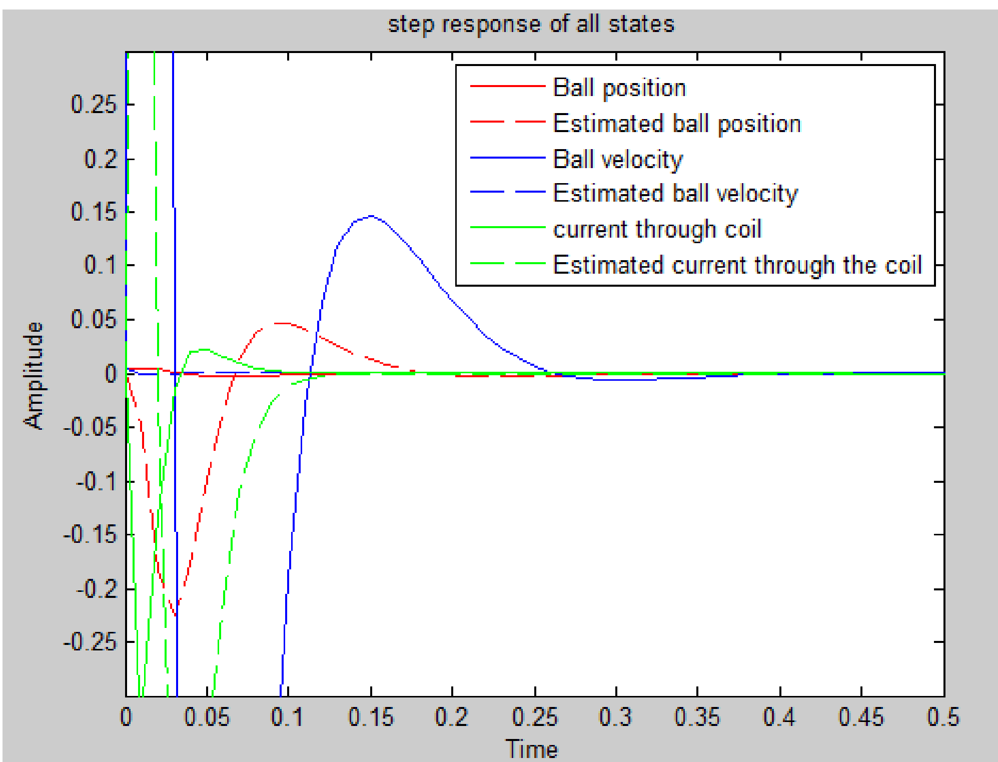

Figure 4.2 Step Response of all states variables

The red solid line is the response of the ball position, the red dotted line is the estimated state, The Blue solid line is the response of the ball velocity, the Blue dotted is the estimated state. The green solid line is the response of the current, the green dotted is the estimated state.

It can be seen that the estimator estimates the states quickly and tracks the states reasonable well in the steady state, the actual state and the estimated state quickly converge together and goes to zero less than five seconds as shown in Figure 4.2

4.4 Complete Circuit Diagram

The complete circuit diagram of magnetic suspension system is as shown in Figure 4.3

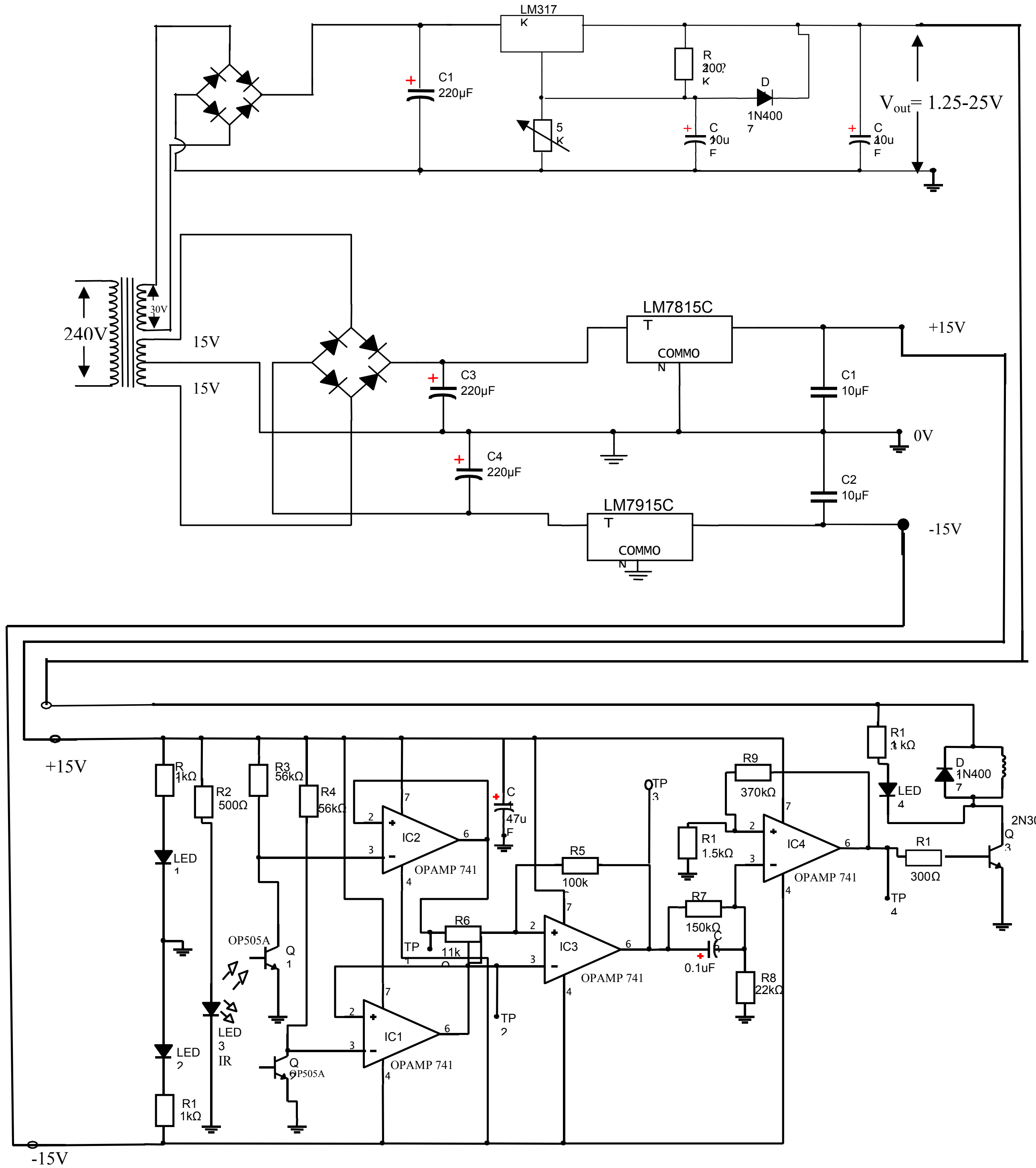

Figure 4.3 Complete Circuit Diagram of the Magnetic suspension system

The list of the components used in the construction of the magnetic suspension system is shown in Appendix A5, the photograph of the constructed system is shown in Appendix A6. The experimental set- up is shown in Appendix A7, the ball is being suspended, while the thermometer and magnetometer were used to measure the temperature and magnetic flux density respectively, at intervals of two (2) minutes till the field can no longer suspend the ball.

It was found that as temperature increases, the magnetic flux decreases. At an average temperature of $46^{O}C$, the field was no longer strong enough to hold the ball. The total time taken for this operation was twenty two (22) minutes

The experimental measurements are shown in Table 4.1, the plot of magnetic flux density against the temperature is shown in Figure 4.4 and the plot of magnetic flux density against time is shown in Figure 4.5

Table 4.1 Measurement of Temperature and Magnetic Flux Density against Time

Time (Mins)	Temperature (T) ^{O}C				Magnetic Flux Density (B) (Tesla)			
	T_1	T_2	T_3	Average	B1	B2	B3	Average
0	31	32	30	31	0.250	0.240	0.213	0.234
2	34	33	32	33	0.250	0.240	0.213	0.234
4	36	35	35	34.7	0.250	0.240	0.213	0.234
6	37	36	34	35.7	0.240	0.240	0.213	0.231
8	38.5	38	36	37.5	0.240	0.240	0.213	0.231
10	40	39	38	39	0.240	0.240	0.213	0.187
12	41	40	39	40	0.190	0.195	0.176	0.187
14	42	41	40	41	0.190	0.195	0.176	0.187
16	43.5	43	42	42.8	0.180	0.195	0.176	0.187
18	44	44	43	43.7	0.180	0.190	0.176	0.182
20	45	45.5	44	44.8	0.180	0.190	0.176	0.182
22	46	47	45	46	0.180	0.145	0.141	0.145

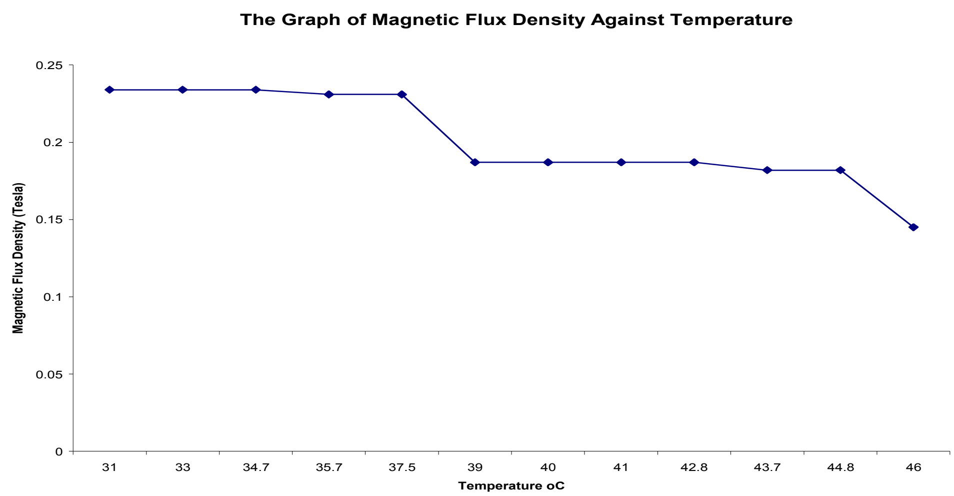

Figure 4.4 shows that as temperature increases the Magnetic flux decreases. It can be deduced that as magnetic material is heated, its molecules become more volatile. As a consequence, individual molecules get out of alignment as the temperature increases, thereby reducing the magnetic flux density of the magnetized substance.

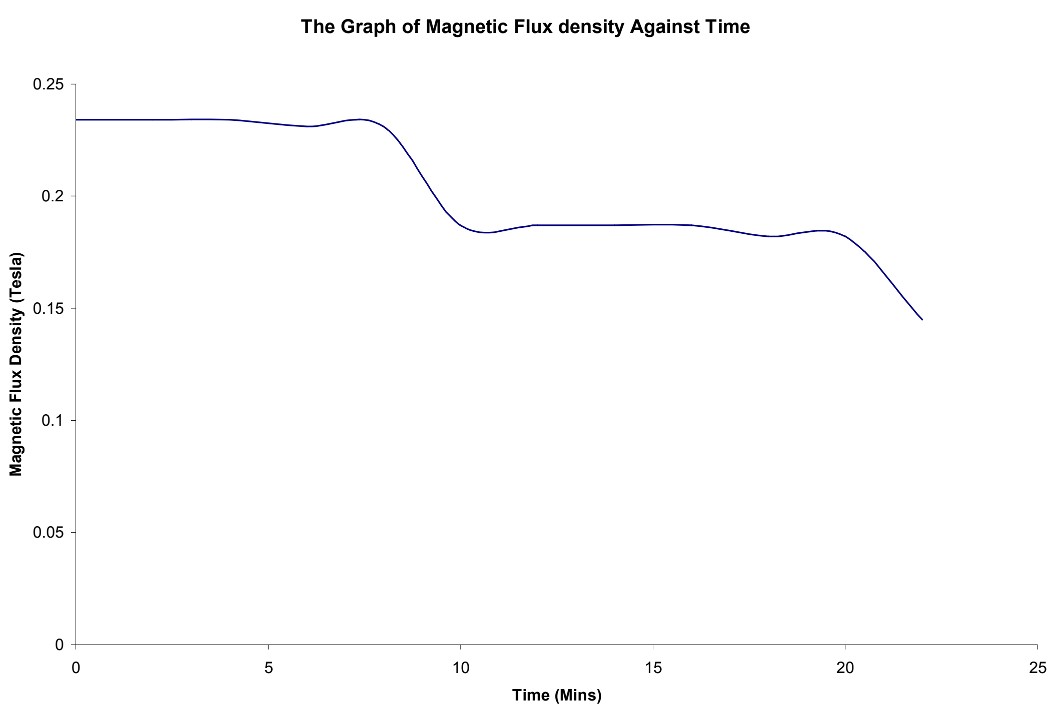

Figure 4.5 Plot of Magnetic Flux density against Time

Figure 4.5 shows that as time increases the Magnetic flux decreases

b) Effect of Distance on Magnetic flux density.

Table 4.2 shows the measurement of the magnetic flux density against the distance below the coil.

Table 4.2 Magnetic field against Distance.

Distance (m)	Magnetic Flux Density (B) (Telsa)			
	B1	B2	B3	Average
0.001	0.680	0.670	0.660	0.672
0.002	0.670	0.680	0.642	0.654
0.004	0.590	0.580	0.588	0.586
0.006	0.460	0.448	0.439	0.449
0.008	0.409	0.400	0.400	0.403
0.010	0.330	0.320	0.319	0.320
0.012	0.260	0.280	0.249.5	0.252
0.014	0.250	0.190	0.204	0.198
0.016	0.160	0.150	0.158	0.156
0.018	0.130	0.120	0.122	0.124
0.02	0.100	0.099	0.098	0.099

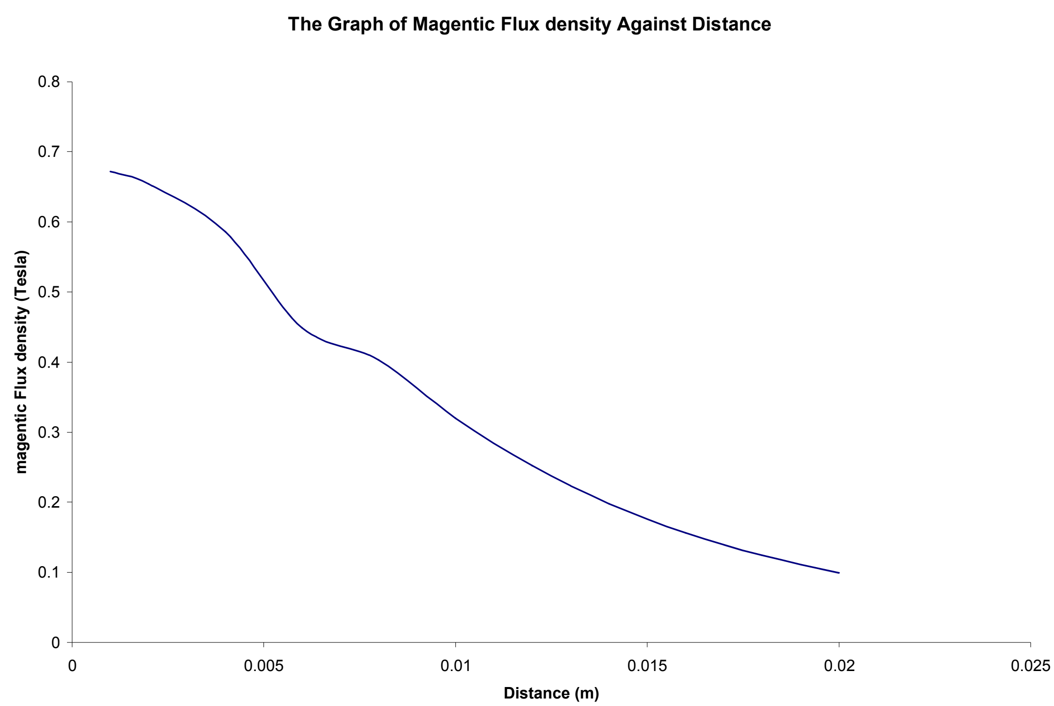

Figure 4.6 plot of Magnetic field against Distance.

Figure 4.6 shows that the magnetic field B decreases with increase in the distance.

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 Summary of the work

The work involves state-space design and construction of magnetic suspension system. The mathematical model of system was developed and transformed into state space, the stability of the system is determined. The test of controllability and observability was carried out using kalman test. The controller and observer was then designed in order to stabilized the system, the system is then constructed and experimental measurement was then carried out.

5.2 Limitation (s)

The constructed magnetic suspension system can only suspend the metallic ball of mass 28g at distance 1.2 cm below the coil. If the mass of the ball is greater than 28 gram, the force of gravity acting the ball will be greater than electromagnetic force, and therefore the ball will drop. Similarly, if the mass is less than 28 gram the electromagnetic force will be greater than the force of the gravity acting on the ball, and hence the ball gets attracted to the electromagnet. The controller designed can not be used in a situation where the suspended mass is of variable values.

5.3 Conclusion

From the analysis it can be concluded that the controller of gain has caused the peak overshoot to reduce from an undefined value to 4.1% and the settling time from an undefined to 0.221 second. A reference input with Nbar has caused steady state error to be zero. The introduction of the observer L [1.4605, 0.2683, 0.029] helps to estimate the state variables and caused the actual state variables and estimated state variables to converge in less than 0.5 seconds. The gain of the controller which is the overall gain of the system is used to determine the gain of the power amplifier. This is estimated to be 48. The State- Space method eliminates the need to measure the system parameters after it has been built.

5.4 Suggestions for Further Work

Future studies can be carried out with respect to the following

i) Designing a programmable controller which automatically regulates the required parameters for different masses of the metallic ball.

ii) Designing an optimal controller that will track a reference trajectory.

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2N3055 Data sheet www.national.com

OP130W Data Sheet www.optekin.com

APPENDIX
PROGRAM LISTING AND RESULTS

A1: Matlab script for Controllability and Observability

```
MATLAB SCRIPT
g=9.81
x1=0.012;
m=0.028;
r=8;
l=0.5
A=[0,1,0;g/x1,0,-2*(g/m*x1)^1/2;0,0,-r/l]
B=[0;0;1/l]
C=[1,0,0]
D=0
pole=eig(A)
C0=ctrb(A,B)
y=det(C0)
OB=obsv(A,C)
Z=det(OB)
```

The result of MATLAB script shown above is as follows

```
g =

    9.8100


l =

    0.5000


A =

         0    1.0000         0
  817.5000         0   -4.2043
         0         0  -16.0000


B =

     0
     0
     2


C =

     1     0     0
```

```
D =

     0


pole =

   28.5920
  -28.5920
  -16.0000


C0 =

         0         0   -8.4086
         0   -8.4086  134.5371
    2.0000  -32.0000  512.0000


y =

 -141.4081


OB =

    1.0000         0         0
         0    1.0000         0
  817.5000         0   -4.2043


Z =

   -4.2043
```

A2: Matlab Script for System Response

```
MATLAB SCRIPT

p1=-3.5+3.5i;
p2=-3.5-3.5i;
p3=-16;
K=place(A,B,[p1,p2,p3]);
sys_cl=ss(A-B*K,B,C,0);
lsim(sys_cl,u,t,x0);
title('Response with k Controller')
sys = ss(A-B*K,B,C,0);
```

```
S = stepinfo(sys)


p1=-5+16i;
p2=-5-16i;
p3=-57;
K=place(A,B,[p1,p2,p3]);
sys_cl=ss(A-B*K,B,C,0);
lsim(sys_cl,u,t,x0);
title('Response with k Controller')
sys = ss(A-B*K,B,C,0);
S = stepinfo(sys)

p1=-10+10i;
p2=-10-10i;
p3=-50;
K=place(A,B,[p1,p2,p3]);
lsim(A-B*K,B,C,0,u,t,x0);
title('Response with k Controller')
sys = ss(A-B*K,B,C,0);
S = stepinfo(sys)
p1=-8+7i;
p2=-8-7i;
p3=-40;
K=place(A,B,[p1,p2,p3]);
lsim(A-B*K,B,C,0,u,t,x0);
title('Response with k Controller')
sys = ss(A-B*K,B,C,0);
S = stepinfo(sys)



p1=-20+20i;
p2=-20-20i;
p3=-100;
K=place(A,B,[p1,p2,p3]);
lsim(A-B*K,B,C,0,u,t,x0);
K=place(A,B,[p1,p2,p3])
axis([0 2 -0.0010 5.5e-3])
title('Response with k Controller')
sys = ss(A-B*K,B,C,0);
S = stepinfo(sys)



        RiseTime: 0.4572
    SettlingTime: 1.2683
     SettlingMin: -0.0223
     SettlingMax: -0.0199
       Overshoot: 4.0516
      Undershoot: 0
```

```
            Peak: 0.0223
        PeakTime: 0.9688

S =

        RiseTime: 0.0847
    SettlingTime: 0.6872
     SettlingMin: -7.1101e-004
     SettlingMax: -4.5471e-004
       Overshoot: 35.4371
      Undershoot: 0
            Peak: 7.1101e-004
        PeakTime: 0.2231
S =

RiseTime: 0.2242
    SettlingTime: 0.5612
     SettlingMin: -0.0019
     SettlingMax: -0.0017
       Overshoot: 2.6327
      Undershoot: 0
            Peak: 0.0019
        PeakTime: 0.4768



S =

RiseTime: 0.2242
    SettlingTime: 0.5612
     SettlingMin: -0.0019
     SettlingMax: -0.0017
       Overshoot: 2.6327
      Undershoot: 0
            Peak: 0.0019
        PeakTime: 0.4768

S =

RiseTime: 12.8700
    SettlingTime: 49.6951
     SettlingMin: -5.3562e-004
     SettlingMax: 7.3095e-004
       Overshoot: 5.5195e+022
      Undershoot: 1.3020e+024
            Peak: 0.0126
        PeakTime: 7

S =

        RiseTime: 0.0795
```

```
     SettlingTime: 0.2210
      SettlingMin: -1.0942e-004
      SettlingMax: -9.5073e-005
        Overshoot: 4.1017
       Undershoot: 0
             Peak: 1.0942e-004
         PeakTime: 0.1695
```

A3: Matlab script for Controller Design

```
t=0:0.01:2;
u=0.001*ones(size(t));
lsim(A-B*K,B,C,0,u,t)
axis([0 2 -0.00015e-3 0.00001e-
3]),xlabel('Time'),ylabel('Amplitude'),title('step response
(withoutN)')

Nbar=rscale(A,B,C,D,K)
lsim(A-B*K,B*Nbar,C,D,u,t)
xlabel('Time'),ylabel('Amplitude'),title('step response
(with Nbar)')
 K =

   1.0e+004 *

      2.3125    0.0668    0.0062


K =

   1.0e+004 *

      2.3125    0.0668    0.0062

Nbar =

      9.5141
```

A4: Matlab script for Observer Design

```
op1=-100;
op2=-101;
op3=-102;
L=place(A',C',[op1,op2,op3])'
At=[A-B*K B*K
    zeros(size(A)) A-L*C];
Bt=[B*Nbar
    zeros(size(B))];
Ct=[C,zeros(size(C))];
lsim(At,Bt,Ct,0,zeros(size(t)),t,[x0,x0])
```

```
sys=ss(At,Bt,Ct,0);
lsim(sys,zeros(size(t)),t,[x0 x0])


[y,t,x]=lsim(sys,zeros(size(t)),t,[x0 x0]);
plot(t,x(:,1),'r',t,x(:,2),'r--',t,x(:,3),'b',t,x(:,4 ),'b-
-',t,x(:,5),'g',t,x(:,6),'g--'),legend('Ball
position','Estimated ball position','Ball
velocity','Estimated ball velocity','current through
coil','Estimated current through the coil')
axis([0,.5,-
.2,.2]),xlabel('Time'),ylabel('Amplitude'),title('step
response of all states')
```

```
L =

  1.0e+005 *

    0.0029
    0.2683
    1.4605
```

A5: Lists of Components

Components	Calculated value	Standard
Resistor	442Ω	500Ω
	57.7K Ω	57KΩ
	300 Ω	300Ω
	1K Ω	1KΩ
	1.5K Ω	1.5KΩ
	10K Ω	10KΩ
	22K Ω	22KΩ
	100K Ω	100KΩ
	149K Ω	150KΩ
	50 K Ω linear taper	50 KΩ linear taper
	370K Ω	370KΩ
	56K Ω	56KΩ
Semiconductors	OP505A Infrared Photo Detector	OP505A Infrared Photo Detector
	2N3055 NPN Power Transistor	2N3055 NPN Power Transistor
	LM741 Op-Amp	LM741 Op-Amp
	IN4001Silicon Diode	IN4001Silicon Diode
	Red- light-emitting Diode	Red- light-emitting Diode
	Infrared Led Emitter	Infrared Led Emitter
	LM 317K Voltage Regulator	LM 317K Voltage Regulator
	LM7815	LM7815
	LM7915	LM7915
Capacitors	47 μF Electrolytic	47 μF Electrolytic
	0.1 μF Ceramic	0.1 μF Ceramic
	Tantalum	Tantalum
Magnetic wire	24-standard wire gauge	24-standard wire gauge
Auto-Transformer	240-15 step-down	240-15 step-down

A6: Photograph of Constructed System

Photograph of the Suspension System in Operation

A7: System Experimental Set-up

Photograph of Experimental Setup