ANALYSIS OF SINGLE AND MULTI-TELLER MODELS OR USING SIMULATION FOR REDUCING WAITING TIME: A CASE STUDY OF FIRST BANK AND ECO BANK, NIG. LTD

BY

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DECLARATION

I declare that the work in this project report entitled Analysis of Single and Multi-Sever Models or Systems using Simulation: A case study of First Bank and ECO Bank has been performed by me in the Department of Computer Science under the supervision of Assoc, Prof. **S. E Abdullahi** and Professor **S. B Junaidu**. The information derived from the literature has been duly acknowledged in the text and list of references provided. No part of this project report was previously presented for another degree or diploma at any university.

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CERTIFICATION

This project report entitled **Analysis of Single and Multi – server models or systems using Simulation: Case studies of First and ECO Banks** by MUHAMMAD, Adamu meets the regulation governing the award of the degree of M.Sc. Computer Science of Ahmadu Bello University, and is approved for its contribution to knowledge and literary presentation.

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DEDICATION

This Dissertation is dedicated to my beloved parents in the name of late Alhaji Mohammed Baba and Hajia Rakiya Mohammed for the support, time and resources they used in bringing me up and training me in many aspects of life and in the pursuit of my education with the help of the ALMIGHTY ALLAH. This Dissertation is also dedicated to my entire family and those who contributed to my success, no matter how little.

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ABSTRACT

Queue causes customer dissatisfaction and defection, low service level and abandonment, decreased contact center efficiency, Increased Telecom costs, employee dissatisfaction, (Virtual Hold Technology (VHT), 2013), a satisfied customer becomes angry and eventually becomes non-customer, all of which calls for reduction of waiting time. This research carried out analysis of Single Server-Single queue and Multi Server-Single queue systems for reducing waiting time, case studies of First and ECO banks using Microsoft Excel, then it used Window version of Quantitative Systems for Business (Win QSB) to validate the results obtained from Microsoft Excel analysis which showed that the result of the analysis was accurate. A Java program was developed which determined the performance measures, the total cost of waiting and the facility costs for both case study Banks at the same time when the program is ran and after the necessary data are given. The result of the analysis revealed that the case study banks have varied waiting problems which can be reduced. The case study Banks and similar service systems should study the discussion of results of this dissertation and other gray areas uncovered from the analysis and use it to improve their waiting problems and achieve increased efficiency. The Recommendations which will help the case study banks to reduce their waiting problems were outlined.

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LIST OF ABBREVIATION

ATM: Automated Teller Machine.

Cw: expected waiting cost/unit/unit time or the opportunity cost of waiting by Bank customers.

Ce: cost of servicing one unit

C_S: Service cost of each Teller.

DDA: Disability Decree Act

E(SC): Expected Service Cost

E(WC): Expected waiting cost in the system.

FCFS: First Come, First Served

FSS**:** full-system simulation

GPSS/H: General Purpose Simulation Software is a discrete-event simulation language.

ISS**:** Instruction Set Simulator

L: the average number of customers in the service system

LCFS: Last Come, First Served

Ls: number of customer receiving service (excluding those in the queue).

L*s*: expected number of units in the system/unit time

L_Q: the average number of customers waiting in the queue (excluding that receiving service)

MATLAB: **mat**rix **lab**oratory

M.Sc: Masters of Science Degree

M/M/1: M means Arrival process which is Markovian or Poisson process or (Random) arrival process, M means Service time distribution which is Exponential, 1 means the number of servers. M/M/1 queueing system:

M - Poisson arrival distribution means Infinite possible queue length

M - Exponential service time distribution means Infinite input population

1- Single server means Only one queue

2- M/M/2: M means Arrival process which is Markovian or Poisson process or (Random) arrival process, M means Service time distribution which is Exponential, 2 means two servers.

M/M/S: M means Arrival process which is Markovian or Poisson process or (Random) arrival process, M means Service time distribution which is Exponential, S means Multi-servers.

M/G/1: an **M/G/1 queue** is a queue model where arrivals are **M**arkovian (modulated by a Poisson process), service times have a **G**eneral distribution and there is a single server

NNPC: Nigeria National Petroleum Cooperation

n: Number of customers in the system

P: Utilization or the average utilization of the system

Plc: Public liability Company

PD: Priority discipline

Pn: Probability of number of customers in the service system

QOS: Quality of Service

S: Number of Tellers

SC: service cost

s: the number of servers in the system

SIRO: Service in Random Order

W: average time spent waiting in the system, including service

WinQSB: The Window of Quantitative Systems for Business is developed and maintained by Yih-

Long Chang. The WinQSB is the Windows version of the QSB software package runs under the CD-ROM Windows.

W_Q (or Tw): the average time each customer waits in the queue.

W_S: the average time an arrival spends in the system.

VHT: Virtual Hold Technology

CHAPTER ONE

GENERAL INTRODUCTION

This chapter discusses the introductory part of the thesis. It includes the background of the study, research motivations and goals, the research questions for which the thesis should provide answers to, the methodology that is used to answer those questions and finally the contribution to knowledge.

1.1 Background of the Study

In every bank, there is banker-customer relationship. The corporate objective of any bank which is maximization of shareholders' wealth can only be achieved if customers are retained and satisfied. This is according to the Kotler P (1999) perception that the key to successful marketing of financial services is identification and packaging of customers' need to their satisfaction.

The competition in Nigerian banking sector of which First Bank Samaru is part of, is getting more intense, partly due to regulatory imperatives of universal banking and also due to customers' awareness of their rights. Bank customers have become increasingly demanding, because they require high quality, low priced and immediate service delivery. They want additional improvement of value from their chosen banks (Kasum A. S et al, 2006). Service delivery in banks is personal, customers are either served immediately or join a queue (waiting line) if the system is busy. A queue occurs where facilities are limited and cannot satisfy demand made against them at a particular period. However, most customers are not comfortable with waiting or queuing, (Kasum A. S et al, 2006). The danger of keeping customers in a queue is that customers waiting time may amount to or could become a cost to them. Customers are prepared not to spend more cost of waiting or queuing. The time wasted on the queue would have been judiciously utilized elsewhere (the opportunity cost of time spent in queuing), (Elegalam, 1978).

Waiting is one of the important issues to the service industry because of its impact towards the operations capabilities and customer satisfaction of the organization. The determination of how long a customer should wait for a product or service has long been a major concern for service management specialists who bear the trade-off between minimizing operation costs incurred in optimizing the configuration of a queue system, as well as, minimizing the cost of waiting of the customers.

ATM is the abbreviation of Automated Teller Machine which acts as a teller in a bank who takes and gives money over the counter. ATMs are placed not only near or inside the premises of banks, but also in locations such as shopping centers/malls, airports, grocery stores, petrol/gas stations, restaurants, or any place where large numbers of people may gather. ATM services includes function such as cash withdrawal, balance enquiry, bill payment, cash and cheque deposit, saving and credit account on a 24-hour basis. Thus with the introduction of ATMs, some limitation of time and geographic location has been resolved, (Lukwiya, 2011).

As the current economies progressively changes into a service dominated one, it is essential to thoroughly understand the know-how to effectively deal with waiting lines so as to improve customer satisfaction of service, (Mandia, 2012). In view of this, there is need to find out inadequacies in queue management, explore the psychological and social aspect of queues, and finally formulate a general framework based on the consumer behavior in queues, which would serve to aid the design of queuing systems.

1.2 Research Motivations and Goals

The major goal or purpose of the dissertation is to analyze single and multi-teller models of a Bank so that management of the Bank or similar service system can take decision on how to improve on waiting in the two case study Banks and achieve increased efficiency.

One of the motivating factors is that, McKinsey consultants write in “Is Simulation Better than Experience?”, “Simulations can be better than experience because they compress time and remove extraneous details. Unlike life, simulations are optimized for learning.” Indeed, the true beauty of simulation is that it provides an immersive learning experience, where skills, process, and knowledge can all be enhanced in a way reality cannot. And the ability to explore, experiment, and repeatedly apply this knowledge to unlimited model situations is what makes simulation the most versatile form of learning available. And now, thanks to the development of new technology, computer simulation is making this type of learning more effective than ever, (Access, 2012).

Customers are prepared not to spend more cost of waiting / queuing. The time wasted on the queue would have been judiciously utilized elsewhere (the opportunity cost of time spent in queuing), (Elegalam, 1978).

As one of the customer of First bank in Samaru who used to join long queue in the bank or in the ATM premises, which lasted for one hour or more especially during the last two weeks of the month, it is always boring and precious time is wasted and this always give rise to most of customers complaining in the queue. The same thing happens in the first Bank in Bida and all the Nigerian queue areas such as petrol stations, and hospitals. Therefore, there is need for reducing the waiting time.

"Statistic was produced during the 1980s by a consulting firm, Priority Management Pittsburgh, which says that average Americans spend five years of their lives waiting in lines and six months sitting at traffic lights (compared, incidentally, with four years doing housework). Even if the time spent in queues has been more heavily curtailed in recent years by the prevalence of online retail transactions, the statistic retains a certain force" (Kevin, 2012).

Similarly, Nigerians have not developed like the Americans but they are already spending a lot of time in queues (such as in banks, filling stations, hospitals, traffic, etc.), and this will worsen when Nigeria become developed, therefore, it is necessary to pay attention to queue management in order to reduce wait time and achieve efficiency.

1.3 Problem Statement

Most customers are not comfortable with waiting or queuing (Kasum A. S et al, 2006). The danger of keeping customers in a queue is that customers waiting time may amount to or could become a cost to them. Customers are prepared not to spend more cost of waiting or queuing. The time wasted on the queue would have been judiciously utilized elsewhere (the opportunity cost of time spent in queuing), (Elegalam, 1978). Bank customers have become increasingly demanding, because they require high quality, low priced and immediate service delivery. They want additional improvement of value from their chosen banks (Kasum A. S et al, 2006).

The aim of the research is to analyze Single Server – Single queue and Multi-server – Single queue systems, case studies of First and ECO banks using Simulation which will help to control and reduce waiting problems in the case study Banks and achieve increased efficiency.

1.4 Aim and Research Objectives

The aim of the research is to analyze Single Server – Single queue and Multi-server – Single queue systems, case studies of First and ECO banks using Simulation in order to help controlling waiting in the case study Banks and achieve increased efficiency.

The objectives are to:

(i). use direct observation to collect data necessary from the First and ECO Banks for analyzing them using electronic device.

(ii). fit that data in to the Queuing formula for M/M/1 (M means Arrival process which is Markovian or Poisson process or (Random) arrival process, M means Service time distribution which is Exponential, 1 means the number of servers) and M/M/S (M means Arrival process which is Markovian or Poisson process or (Random) arrival process, M means Service time distribution which is Exponential, S means Multi-servers) models in order to determine the performance measures or the operating characteristics of the case study Banks.

(iii). use Microsoft Excel to analyze the set of real data collected from four Tellers (Cashiers) of the First Bank and the three set of data collected from the three set of Multi-tellers of the ECO bank for getting the real performance measures of the case study Banks (iv). use WinQSB Simulation software to validate the results obtain from the Microsoft Excel analysis performed.

(v). develop Java program that will determine the performance measures, the total waiting cost, and the facility costs of both case study Banks after given the necessary inputs, so that the case study Banks can take decision on how to reduce waiting or extended hold.

(vi). present the result to show how queue can be reduced in the First and ECO Banks and similar service systems should use the discussion of results of this paper and other gray areas uncovered from the analysis to improve their waiting problems and achieve increased efficiency.

1.5 Methodology

1. Review of literature was carried out in chapter two to understand queuing system and characteristics, why it is necessary to Simulate and how to do it.

2. Direct observation was used to collect the input data manually and with the assistance of electronic device. Data for this study were collected from First Bank, Plc, Samaru Zaria and ECO Bank, Plc, Minna for two weeks and one week respectively. The methods employed during data collection were direct observation of customers where their arrival time, the time service starts and the time service ends were collected directly in real time. Data were collected for three weeks, two weeks for first Bank and one week for ECO Bank. Data that fits properly were used to calculate the performance or operating characteristics and in the analysis made using Microsoft Excel.

3. The Models Used:

(a) The M means the Arrival process which is Markovian or Poisson process or (Random) arrival process, the second M means the Service time distribution which is Exponential, 1 means the number of servers (M/M/1 Model)

The first model adopted in this work is the easiest waiting line model, which involves a single-server, single-line, single-phase system.

The following conditions were used for queuing system at the two banks. They are:

(i) Arrivals follow a Poisson probability distribution at an average rate of λ customers per unit of time.

(ii) Service times are distributed exponentially, with an average of μ customers per unit of time. That is, service time varies from one customer to the next and is independent of one another but their average rate is known.

(iii) The queue discipline is First-Come, First-Served (FCFS) basis by any of the Teller. There is no priority classification for any arrival or service.

(iv) There is no limit to the number of the queue (that is, arrivals come from an infinite or large population).

(v) The service providers were working at their full capacity.

(vi) The average arrival rate (λ) is greater than average service rate (μ).

(vii) The systems studied will have $p < 1$, if not the number of customers in the system will grow without bound. p is the utilization factor (also called traffic intensity).

(viii) Servers here represent only Cashiers or Tellers that pay or receive money from customers but not, other Bank personnel.

(ix) Service rate is independent of line length; that is, service providers do not go faster because the line is longer.

(x) The customers are patient (that is, no balking, reneging or jockeying) and came from a population that is infinite.

The above conditions were met, and a series of equation that define the queue's operating characteristics were developed.

A Poisson arrival rate/exponential service time/single server means M/M/1 queue in terms of the standard notation.

Figure 1.1: A Single Server model (Conceptual model) - the most elementary of queuing model, M/M/1 queuing model, (Sturgul, 2000).

(b) The M means Arrival process which is Markovian or Poisson process or (Random) arrival process, the second M means Service time distribution which is Exponential, S means the number of servers (M/M/S Model)

The second model adopted in this work is the (M/M/S) :(∞ /FCFS) - Multi-Server Queuing Model. For this queuing system: All of the conditions listed earlier for the single-channel model apply to Multi-server system as well in addition to the following three:

(i) The service times are distributed exponentially, with an average of μ customers per unit of time and number of Tellers s. If there are n customers in the queuing system at any point in time, then the following two cases may arise:

(a) If $n < s$, (number of customers in the system is less than the number of Tellers), then there will be no queue. However, $(S - n)$ number of Tellers will not be busy. The combined service rate will then be $\mu n = n\mu$; $n < s$.

(b) If $n > s$, (number of customers in the system is more than the number of servers) then all servers will be busy and the maximum number of customers in the queue will be $(n – s)$. The combined service rate will be $\mu n= n\mu$; $n \geq s$.

(ii) In this multiple-channel queuing system, two or more servers or channels are available to handle arriving Bank customers. Customers awaiting service form one single line and then proceed to the first available Teller. The arrival process is Poisson with rate λ. Arrivals will join a single queue and enter the first available service channel.

(iii) Each of these Tellers or channels has an independent and identical exponential service time distribution with mean $1/\mu$. In multi-server system, all Tellers are assumed to perform at the same rate.

Figure 1.2: Multi-server model

4. Data was analyzed which led to the identification of the theoretical distribution that represents the input data.

5. The input data was used in the model which led to the specification of theoretical distribution in the simulation program code. This process is represented in Figure 1.3.

Figure 1.3: Role of theoretical probability distributions in simulation, (Delia et al, 2005)

6. Two types of data were observed:

(i) Deterministic data - means that the event involving the data occurs in the same manner or in a predictable manner each time. This means it never varies in value, such as time the Bank or ATM opens and closes. Banks always open at 8.00 am and closes at 4.00 pm daily. Also, the Queue discipline is deterministic because it never varies, for example, Din Kendall Notation A/B/C is constant or deterministic (that is, it is always FIFO).

(ii) A probabilistic process or stochastic model does not occur with the same type of regularity, such as arrival and service times. In this case, the process will follow some probabilistic distribution. Thus, it is not known with the same type of certainty that the process will follow an exactly known behavior, for example, an M/M/1 queue is a stochastic process where the value correspond to the number of customers in the System (Bank). This M/M/1 queue represents the queue length in a system (Bank) having a single server, where arrivals are determined by a Poisson process and job service times have an exponential distribution. Sturgul (2000), this is the most elementary of queuing model. Figure 1.4 Figure 1.5 below is a flowchart which shows the logic of a single server model: Figure 1.4 mean that when a customer enters a service system, he goes for service directly if the Sever is idle or else joins the queue and wait for service, Figure 1.5 show that waiting Customers depart or leave the service system after the Sever attended to them and if there is nobody to attend to, the queue becomes empty and the server becomes idle.

Flow chart of an arrival event

Idle Busy

Figure 1.4: The logic of a single server model. (Raid, 2010)

Flow Chart of Departure Event

No Yes

Figure 1.5: The logic of a single server model. (Raid, 2010)

Any time there are events which occur individually at random moments, but which tend to occur at an average rate when viewed as a group, there is a Poisson process.

7. Microsoft Excel analysis software was used to calculate the performance and generate the graph of the probability of 15 customers in the system and the graphs of performance characteristics for the M/M/1 and M/M/S models, which are shown in the appendices A and B respectively. Analysis of Queuing situations - Queuing situations is the idea of uncertainty in, for example, inter-arrival times and service times. This means that probability and statistics are needed to analyze queuing situations. In the analysis of queuing situations, the questions of interest would be answered using a Microsoft Excel. In order to get answers to the questions, two basic approaches used in this research work are:

(i). Analytic methods or queuing theory (formula based); and (ii). Simulation (computer based, using Simulation Software like Java Programming language and WinQSB).

The reason for there being two approaches (instead of just one) is that analytic methods are only available for relatively simple queuing systems. Complex queuing systems are almost always analyzed using Simulation (more technically known as discrete-event simulation).

8. Turnaround and response time were determined

9. The performance measures were calculated to allow managers or investors choose the scenario that is the most likely to give them their desired results.

10. The Cost Analysis was made by determining the Unhappiness Cost to customers as well as the facility cost.

11. Simulation software (WinQSB) was used to get the operational characteristics of the two models used. Java program was written to simulate the operational characteristics of Single and multi-server models server, calculate the waiting cost and facility cost in order to help the management to make a trade-off between the increased costs of providing better service and the decreased waiting time costs of customers derived from providing that service.

1.6 Contribution to Knowledge

This dissertation contributes to Queue management in service organizations in Nigerian Banks, which is still a great problem in all the queue areas in the country. The results of the analysis will help reduce customer waiting time as a trade-off or opportunity cost. Economic analysis of these costs will help the management to make a trade-off between the increased costs of providing better service and the decreased waiting time costs of customers derived from providing that service. In other words, the result of analysis will always show whether the Bank analysed is operating at lost or not and whether the customer is experiencing long wait or not, but the banks analysed are on one hand profiting and the customers on the other are experience long wait especially in some periods of the month, therefore there is need for the analysed banks to use some of this profit to reduce the long waiting time experienced by the customers by employing more staff and acquire more facilities that will help reduce the waiting time of the customers in order not to loose their customers to other banks that customer feel will give them less wait time.

CHAPTER TWO

LITERATURE REVIEW

This chapter discusses some related literature such as Performance measures, methodologies, and analysis methods.

2.1 Performance Measures, Queuing Models, Methodologies and Analysis Methods

Queue or waiting in lines is something everyone does and waiting lines are encountered everywhere. When queue management works, your contact center is a healthy organization that represents your company in a positive light; but when queue management fails, your contact center is infected with the “contact center disease” – hold time. When implementing an intelligent Queue Management Strategy, contracting the “contact center disease” or hold time will be avoided, which will increase customer satisfaction and reduce your contact center costs

Extended hold time leads to customer dissatisfaction and defection (grumble), Low service levels and high abandonment, decreased contact center efficiency, increased telecom expenses, employee dissatisfaction and turnover, (VHT, 2013).

Queuing situations is the idea of uncertainty, for example, inter-arrival times and service times. This means that probability and statistics are needed to analyze queuing situations, (Beasley J. E, 2013). A long wait on hold might turn an otherwise satisfied customer into an angry non-customer, (Wallace, 2013).

Dimitri (2002) said that Measurement of system performance can be used to describe and analyse system and compare alternatives. He added that three methodologies used to carry out performance measure are:

(i) Analytical result (formula) - Analyses Data and uncover causes and effects of queue. It gives the basis to make decision and as well give quick answers and insights.

(ii) Explicit Simulation –the use of hybrid computer to simulate and is accurate.

(iii) Hybrid Simulation - Combines both advantages and disadvantage of Analysis and simulation.

The five steps which can be used to analyse queuing models or systems are:

(i) Carry out fact analyses

(ii) Let the system run some n number of times, collect the real data and analyse.

(iii) Predict simple trends/projections based on experience.

(iv) Develop analytical model based on queuing theory.

(v) Run simulation (not real system) and collect data to analyse.

In queuing theory, Kendall's notation (or sometimes Kendall notation) is the standard system used to describe and classify the queuing model that a queuing system corresponds to. It was first suggested by D. G. Kendall in 1953, as a three-factor A/B/C notation system for characterising queues, it has since been extended to include K and D by Lee, Lee, Alec Miller (1966) and N by Taha, Taha, Hamdy A. (1968), Sen, Rathindra P. (2010), where A is the arrival process, B is the service time distribution, C is the number of servers, K is the number of places in the system, N is the calling population, D is the queue's discipline, is a constant, ().

Daniel and Julie (2008), calculated the performance measure of single server only using arbitrary data and used average number of customers per unit time (arrival rate) and average service time (Service) rate and queuing formula to determine the performance measures of single model. Only seven operating characteristics were determined: λ, μ, P, L, Lq, W, and Wq and, gave six types of disciplines that can be used but used FCFS discipline. Moreover, they use exponential customer arrival and service time distributions

Raid (2010), calculated the performance measure of single server system only using queuing formula like Daniel and Julie (2008). Computed P, L, Lq, W, and Wq. Arrival were Poisson distribution and exponential service. He used FCFS service discipline.

Vasumathi and Dhanavanthan (2010) explained the application of simulation in queuing model for ATM facility. Explained the introduction of costs into the model in order to evaluate and determine the optimum number of Bank Tellers in the System. Emphasized that two opposing costs namely service costs and waiting time cost of customers must be consider in making these decisions. Emphasized that economic analysis of these cost will help the management to make a trade-off between the cost of providing better service and decreased waiting time costs of customers derived from providing that service. They gave the formula for computing service costs and waiting costs of customers.

Munir et al, (2010), Carried out a Novel (long and complex) approach to improve Quality of Service (QOS) of a multiple-server queue. Found that the existing models of server work on M/G/1 model which is some way predictable and compare this with various other server queuing model. Used some mathematical analysis to reduce the mean service time of a multiple sever model. They derived the distribution of the mean service time using little's law and provided a C++ simulation code to enable a test run so that the QOS of the multi-server system can be improved by reducing the mean service time.

Mathias and Erwin (2011), derived the Arrival rate, service rate, utilization rate, waiting time in queue and the probability of potential customers to balk based on the data using Little's theorem and M/M/1 model. They also measured p, L, Lq, W, Wq and Pn and used Poisson and exponential distribution because conversations and telephone hold time fit into these distributions. They used FCFS discipline.

Jayshkumar et al, (2012), carried out the analysis of check out operation in Big bazaar using empirical data (used data base on observation and experiment not data based on theory) for a Big bazaar (Street market) service unit in order to review (look at it critically) the efficiency of the model used in terms of utilization and waiting length and reduce the queue length when servers are too busy. They estimated the waiting time and queue lengths used queuing simulation to obtain a sample performance result and tried to obtain estimated solution for multiple queuing models. Some conditions used is that arrival of customers follow a Poisson process but they allowed jockeying. FIFO service discipline was used and as well used confidence interval to show the number of customers (in range) that arrived in 2 server systems per hour. Finally, empirical data was used to find expected length of queue and ran simulation for 100 hours and found that the server was 99% busy and the number of customers in queue in overall 2 servers on weekday, Lq = 67.5812. This Lq is the estimate they made which is long but they said it can be reduced by decreasing service time or utilization.

Dan and Nada (2012), calculated performance measure for both single and multi-teller server systems using MS Excel only but they did not use real data or case study and used average number of customers per unit time and average service time to determine the performance measure of both multi-server and single server system. Unlike the previous authors, they included the probability of number of customers in the service system: Pn = (1-P)Pn. They listed some types of disciplines which can be adopted: (i) Best customers first (Reward Royalty). (ii) Highest Profit customers first (iii) Quickest service requirement s first (iv) Largest service requirements first (v) Earliest reservation first and emergencies first.

Supplement (2012), calculated the performance measure of both multi-server and single server models using MS Excel only just like was done by R Dan & Nada (2012), but, the used fast food restaurant as a case study and used Poisson and exponential distribution. They used FCFS service discipline.

Arnika (2013) carried out simulation of queuing model and cconsidered a sales checkout service with five waiting lines in form of parallel cash counters. Customers were served on FIFO basis. He considered three out of five servers to be idle and collected data for two out of five servers from questionnaires. He tabulated the data in a spreadsheet and calculated confidence interval to estimate service rate and customer arrival rates. Then, he did analysis for the model involving one queue and parallel servers. Finally, he carried out queuing simulation for the model involving two queues for each corresponding servers.

Mohammad et al, (2014) computed some performance measures and used multiple channels with Poisson and exponential service times. They gave theoretical method of calculating waiting cost and mmeasured the performance measures for M/M/2 model only.

Hamid and MusatyaBere (2014), first collected data and then used the data to build a model of the system that would behave in a steady state situation at peak time to benefit management decision. They used GPSS/H simulation software package to build the model and as well analyse the data. They used M/M/1 model, Poisson and exponential service and used FCFS discipline.

Ezeliora et al, (2014) carried out analysis of queuing system using single line – multi- teller only using a case study of Shoprite plaza in Enugu state. They used queuing theory to compute the performance characteristics of the Shoprite plaza, used FCFS service discipline and Poisson arrival and exponential service times. They finally used the result of the analysis to show that the Shoprite plaza needs five servers to handle Shoprite queuing system and suggested that to the Shoprite plaza management

Ogunoh et al, (2014) simulated existing single line multiple channel waiting line using only MATLAB software programming in order to reveal waiting line problem in the sector (NNPC ENUGU and Owerri petroleum mega stations. They collected experimental observation of arrival and service rate of customers at the NNPC mega petroleum stations Oweri from 9.0 am to 5.0 pm from Monday to Saturday and used MATLAB software to write program that will show the performance evaluation solution for the two mega stations. They showed the result of experimental observation of arrival and service rate of customers at the NNPC mega petroleum stations Owerri from the data generated and found that the two case studies should have 8 servers each.

In designing queuing systems, we need to aim for a balance between service to customers (short queues implying many servers) and economic considerations (not too many servers), Beasley (2013).

The central problem in virtually every waiting line situation is a trade-off decision. The manager must weigh the added cost of providing more rapid service (more traffic lanes, additional landing strips, more checkout stands) against the inherent cost of waiting. Frequently, the cost trade-off decision is straightforward. For example, if we find that the total time our employees spend in the line waiting to use a copying machine would otherwise be spent in productive activities; we could compare the cost of installing one additional machine to the value of employee time saved. The decision could then be reduced to dollar or Naira terms and the choice easily made. On the other hand, suppose that our waiting line problem centers on demand for beds in a hospital. We can compute the cost of additional beds by summing the costs for building construction, additional equipment required, and increased maintenance. But what is on the other side of the scale? Here we are confronted with the problem of trying to place a dollar or Naira figure on a patient's need for a hospital bed that is unavailable. While we can estimate lost hospital income, what about the human cost arising from this lack of adequate hospital care? (Helmut, 2000).

The cost of waiting customers includes either the indirect cost of lost business or direct cost of idle equipment and persons. The cost of idle facilities is the payment to be made to the servers for the period, for which they remain idle, (MEN, 2010).

A single Server Simulation program and output using C++ programming language can be obtained on the web site, (Tracy, 2003).

Cost Involved in Queuing System includes:

1. FACILITY COST

Facility cost includes cost of (acquiring) services facilities, Construction (capital investment) expressed by interest and amortization, cost of operation: labor, energy and materials cost of maintenance and repair, Other costs such as insurance, taxes, rental of space.

2. WAITING COST

Cost of waiting may include ill-will due to poor service, opportunity loss of customers who get impatient and leave or a possible loss of repeat business due to dissatisfaction, (Oppres3, 2014)

Queuing model is used to overcome the congestion of traffic. This traffic can be of any form and this model is mainly used in a situation where customers are involved, because of this, when the queuing model is being coupled with simulation, it is very much conducive to get solution to solve the problem related to customers, (Vasumathi and Dhanavanthan, 2010). Multi-channel, single phase.

To evaluate and determine the optimum number of Bank Tellers in the system; two opposing costs must be considered in making these decisions:

(i) Service Costs

(ii) Waiting time costs of customers.

The service cost detail was not given by the case study Banks because it is a vital document that is not to be released to outsiders, but service cost is an advantage to them, i.e., they are profiting from the services they are providing because the banks were in operation (not stressed) which is an evidence that the service cost is at their advantage. On the other hand, there is cost of waiting for the customers and this cost must be known and compared in order to evaluate and determine the optimum number of Bank Tellers in the system.

Economic analysis of these costs will help the management to make a trade-off between the increased costs of providing better service and the decreased waiting time costs of customers derived from providing that service.

Costs can be assigned to factors such as customer waiting time and server idle time then we can investigate how to design a system at minimum total cost. In order for management to evaluate alternatives in an attempt to control/improve the situation, four problems that are often investigated in practice are: Is it worthwhile to invest effort in reducing the service time? How many servers should be employed? Should priorities for certain types of customers be introduced? Is the waiting area for customers adequate? We can get answers to the above questions and others through any or all of the following basic approaches:

1. Analytic methods or queuing theory (formula based); and

2. Simulation (computer based).

“The reason for there being two approaches (instead of just one) is that analytic methods are only available for relatively simple queuing systems. Complex queuing systems are always analyzed using simulation (more technically known as discrete-event simulation), (Beasley, 2013) but, Complex queuing systems is the future work of this dissertation.

The ATM queuing resembles the typical simulation model coupled with queuing theory in ‘Operations Research’ literature. In order to solve the queuing model with simulation, the service facility must be manipulated so that an optimum balance is obtained between the cost of waiting time and the cost of idle time. The cost of waiting generally includes either the indirect cost or loss of customers. By increasing the investment in labor and service facility waiting time and the losses associated with it can be decreased. If we consider Cw= expected waiting cost/unit time, Ls= expected number of units in the system/unit time and Ce=cost of servicing one unit, then:

(1) The expected waiting cost per unit time=Cw*Ls

(2) Expected service cost per unit time=Ce*A

(3) Total cost=Cw*Ls+ Ce*A

In designing queuing systems, we need to aim for a balance between service to customers (short queues implying many servers) and economic considerations (not too many servers), (Beasley, 2013). This is one of the reasons for introducing costs into the Model. In order to evaluate and determine the optimum number of Bank Tellers in the system, two opposing costs were considered in making these decisions:

(i) Service Costs

(ii) Waiting Time Costs of Customers.

Economic analysis of these costs will help the management to make a trade-off between the increased costs of providing better service and the decreased waiting time costs of customers derived from providing that service.

(1) Expected Service Costs is given by:

$E(SC) = SCS$

S = number of Tellers

CS = Service cost of each Teller.

(2) Expected waiting cost in the system is:

$E(WC) = (\lambda WS)CW$

λ = the number of arrivals

WS = the average time an arrival spends in the system.

CW= the opportunity cost of waiting by Bank customers.

By adding (1) and (2) above, we have:

(3) Expected total costs are:

$$E(TC) = E(SC) + E(WC)$$

$$= SCS + (\lambda \; WS) \; CW$$

The cost model is designed to set m number of customer to effectively use the system to minimum total cost, each starting and ending of the process, such that each period of time whether it is a busy or free period the total cost occurred to bank must be less, and customer need not to stay for long in queue and get the best out of the service.

The central problem in virtually every waiting line situation is a trade-off decision. The manager must weigh the added cost of providing more rapid service (more traffic lanes, additional landing strips, more checkout stands) against the inherent cost of waiting. Frequently, the cost trade-off decision is straightforward. For example, if we find that the total time our employees spend in the line waiting to use a copying machine would otherwise be spent in productive activities; we could compare the cost of installing one additional machine to the value of employee time saved. The decision could then be reduced to dollar or Naira terms and the choice easily made. On the other hand, suppose that our waiting line problem centers on demand for beds in a hospital. We can compute the cost of additional beds by summing the costs for building construction, additional equipment required, and increased maintenance. But what is on the other side of the scale? Here we are confronted with the problem of trying to place a dollar or Naira figure on a patient's need for a hospital bed that is unavailable. While we can estimate lost hospital income, what about the human cost arising from this lack of adequate hospital care? (Helmut Hutter, 2000).

The cost of waiting customers includes either the indirect cost of lost business or direct cost of idle equipment and persons. The cost of idle facilities is the payment to be made to the servers for the period, for which they remain idle, (MEN, 2010).

Seven Types of Simulation that can be performed namely: (i) Performance (ii) Proof (iii) Discovery. (iv) Entertainment, (v) Training (vi) Education (vii) Prediction, (Raid, 2010).

Moreover, there are twelve objectives or benefits of Simulating service systems namely (1) Optimize asset management (2) Increase productivity (3) Analyze and optimize supply chain and logistics (4) Forecast demand and predict performance (5) Increase upstream profitability (6) Design, plan, and manage operations (7) Optimize resource reallocations (8) Identify and resolve bottlenecks (9) Analyze alternative work processes (10) Test the effect of alternative layouts (11) Facilitate and support decision making(12) Address risks and vulnerabilities.

He added that an important issue for simulation input data concerning time intervals is the time unit that should be used. It is usually less labor intensive to collect the data correctly in the first place using a relative (changing), inter-arrival time approach. A second time collection issue is what types of units to use. The simulation practitioner should know that unbiased data is needed in order not to disrupt the process. If the data are biased in either manner, it can lead to a model that may yield inaccurate results, (Raid, 2010).

With the flexibility of simulation software, however, many of service systems characteristics can be captured in a computer model that behaves almost similar to a real-world service system. In the context of service systems, simulation is used to study the service system behavior, quantify the provided service, compare proposed alternatives for providing services, improve service level, better utilize resources, reduce service time and cost, and setup/configure the service system to provide the best performance possible within given business constraints, (Raid, 2010).

Industries which can benefit from service system simulation: (1) Healthcare and hospital management (2) Hospitality and hotel management (3) Banking and finance (4) Supply chain and logistics (5) Warehousing and storage systems (6) Airports and aviation (7) Traffic and transportation systems (8) Restaurants and food services (9) Postal services (10) IT systems, communication networks, and data flow, (Raid, 2010).

Summary of literature

The differences and similarities in the literature are shown below in Table 2.1:

Table 2.1: Summary of Literature

Author Year Model used Discipline used Software used or needed Distribution used Case study used Results

1 Prof Dimitri P. Bertsekas 2000 Non Not stated Analytic, hybrid and Explicit Nonnon Non

2 Daniel B Mclaughlin and Julie M. Hays 2008 Single sever Not stated NonNot stated Non Evaluatesλ, μ, P, L, Lq, W, and Wq

3 Raid Al-Aomar 2010 Single server FCFS Non Not stated Used arbitrary data Computed P, L, Lq, W, and Wq

4 Vasumathi A and Dhanavanthan P 2010 Not stated Not stated NonNot stated Non Gave formula for computing service costs and waiting costs of customers.

5 Munir B. Sayyadi et al 2010 Multiserver Not stated C++ code Not stated Non Stated use of litle's law

6 Mathias Dharmawirya and Erwin Adi 2011 M/M/1 FCFS NonPoison & Exponential conversations and telephone hold time measured p, L, Lq, W, Wq and Pn

7 Jayshkumar J. et al 2012 analysis of check out operation FIFO ran simulation for 100 hours and found that the server was 99% busy Not stated Big Bazzare & Used empirical data used confidence interval to show the number of customers (in range) that arrived in 2 server systems per hour

8 R Dan Reid & Nada R Sanders (2012) 2012 Single & Multi-server systems Stated five types of discipline but used FCFS MS Excel Not stated did not use real data or case study Computed P, L, Lq, W, and Wq and Pn

9 Supplement D 2012 multi-server and single server FCFS MS Excel Poison & Exponential used restaurant Computed P, L, Lq, W, and Wq and Pn

10 ArnikaTripathi 2013 one queue and parallel servers FCFS MS Excel carried out queuing simulation for the model involving two queues for each corresponding servers Non tabulated the data in a spreadsheet and calculated confidence interval to estimate service rate and customer arrival rates

11 Mohammad ShyfurRahmanChowdhury et al 2014 Multiple Channel Not Stated Non Poison And Exponential service times

Nongave theoretical method of calculating waiting cost and mmeasured the performance measures for M/M/2 model

12 Hamid Khan and MusatyaBere 2014 M/M/1 FCFS GPSS/H Poison and Exponential service Non analyse data

13 Ezeliora C. D et al 2014 Single line-Multi teller FCFS service discipline

NonPoisson arrival and exponent

tial service times Nonperformance characteristics of the shoprite plaza usingqueuingtheory

14 OgunohArinze V et al software programming 2014 single line multiple channel waiting line Not Stated MATLAB Not stated NNPC ENUGU and Owerri petroleum mega stations. show the performance evaluation solution for the two mega stations

15 Adamu Muhammad 2015 M/M/1 & M/M/2 FCFS MS Excel, WINQSB, JAVA Poison and exponential First and ECO Banks Determined Performance measures &Costs, Management makes trade- off between cost of providing better service and decreased waiting cost

CHAPTER THREE

REDUCED WAITING LINE, WAITING COST AND ENHANCE EFFICIENCY

This chapter describes how the analysis of Single -line, Single-server and single-line, multi-server models were carried out using simulation. Feasibility study was carried out. System requirements are defined where the deficiencies in the existing system were addressed by considering the reasons why queue in these service systems should be managed, the rules of queue management must be used, and data should be collected to enable performance measures to be carried for the existing system using at least a simple simulation so that we can study what is wrong with the system and solve the deficiencies, the proposed system is designed, the new system is developed, the system should be put into use and once the new system is up and running for a while, it should be exhaustively evaluated to ensure that it is working accordingly.

3.1 Data collection

In order to evaluate the system, real data collected through observation from the existing system; from the case study banks using data collection forms for some weeks. It was found that there were waiting problems in the case study banks. There were long queues during salary periods and during student registrations. Customers form long queue in the Bank and around the ATMs outside the Banks, which results to customers waiting for service, reneging, jockeying, balking behaviors and waiting cost, which leads to customer - service provider dissatisfaction and some service organization inefficiency.

Many of Nigerian Banks have queues that affect customers in the Banks in particular, therefore, Real data was collected from two banks which were analyzed in order to determine the characteristics or performance measures of the two Banks. The two Banks used two different queuing disciplines. First Bank uses Multiple lines, multiple servers while the ECO bank uses Single line, Multiple servers. Some detail about the discipline and structures, and models used are described in sections 3.4 and 3.5 3.2 Random Arrival Customers come at random times, i.e., sometimes arrivals are apart, and sometimes customers arrive almost at the same time. Queue was formed when customers arrived at short time than they can be serviced. Where arrival rate was not random, queue was not formed.

3.2 Use of Exponential Function and Relationship between Exponential and Poison Function

The exponential function is widely used for times between independent events such as Inter-arrival times and life time for devices with a constant hazard rate (when describing the time to failure of a system's component). It is related to the Poisson function in that, if the number of arrivals in a given time period is Poisson distributed with a mean ofλ, then the inter-arrival times (i.e., the time between arrivals) are exponentially distributed with a mean of $1/\lambda$. The above explanation similarly applies to multi server model (M/M/S Model).

3.3 Queue Discipline and Structure of Queue

Discipline means (a) Order of Service (or queuing discipline) (b) Structure of the Queue (the physical setup of the queue). There are many queuing disciplines which service systems can adopt such as First Come, First Served (FCFS), Last Come, First Served (LCFS), Service in Random Order (SIRO) and Priority discipline (PD) but FCFS was used in this research because it is the type of discipline acceptable to the customers and it is the type of discipline commonly used by most service systems including my case studies.

There are many structures of the queue such as Single line and single server, Single line and multiple servers, Multiple line and single server and Multiple lines and multiple servers but the two queuing structures used from the case study are Single line and single server (or multiple line multiple servers), Single line and multiple servers as shown in Figures 3.1, 3.2 and 3.3 respectively.

Figure 3.1: A Single Server model (Conceptual model) - the most elementary of queuing model.

Figure 3.2: Queuing system structure and parameters for single server

Figure 3.3: Multi-server model.

3.4 Queuing Models Used (M/M/1 and M/M/s)

In order to change to a new system, the following models and formulas are use so that the deficiencies can be revealed and addressed accordingly.

The simplest queuing model used is M/M/1 where both arrival and service times are exponentially distributed. M/M/1 corresponds to arrival distribution/service distribution/Number of server. M means Arrival process which is Markovian or Poisson process or (Random) arrival process, M means Service time distribution which is Exponential, 1 means the number of servers.

This M/M/1 queue is the queue length in a system (Bank) having a single server, where arrivals are determined by a Poisson process and job service times have an exponential distribution. Since the various arrival and service times are given by the exponential distribution, this is known as the M/M/1 model. An M/M/1 queue is stochastic process.

The easiest waiting line model involves a single-server, single-line (M/M/1), single-phase system. The following conditions were met when we model this environment:

1. The customers are patient, that is, there was no balking, reneging or jockeying and come from a population that can be considered infinite.

2. Customer arrivals are described by a Poisson distribution with a Mean arrival rate λ(lambda). This means that the time between successive customer arrivals follows an exponential distribution with an average of $1/\lambda$.

3. The customer service rate is described by a Poisson distribution with a Mean service rate μ(mu), μ is also called service speed or speed of the Teller. This means that the service time for one customer follows an exponential distribution with an average of $1/\mu$.

4. The waiting line priority rule used is first-come, first-served.

5. For multi-server model, the total service rate is greater than the arrival rate, that is, $s\mu > \lambda$. If $s\mu$ λ, the waiting line would eventually grow infinitely large and will give an unstable graph, therefore, $s\mu$ λ was not used in the analysis, s is the number of Tellers or servers.

6. Since there were n customers in the queuing system at any point in time, then the following two cases occurred:

(i). When $n < s$, (number of customers in the system is less than the number of Tellers), then there will be no queue. However, $(s - n)$ number of Tellers will not be busy. The combined service rate will then be $\mu n = n\mu$; $n < s$. This did not occur.

(ii). When $n > s$ (number of customers in the system is more than or equal to the number of servers) all servers were busy and the maximum number of customers in the queue will be $(n - s)$. The combined service rate will be $\mu n = n\mu$; $n \geq s$. This condition occurred.

Using the above conditions, we can calculate the operating characteristics of a waiting line system using the following formula for a single server and multi-server models:

λ = mean arrival rate of customers (average number of customers arriving per unit of time)

μ = mean service rate (average number of customers that can be served per unit of time)

(i) The arrival rate (λ) – is the average rate new customers arrive measured in arrivals per time period. The units are access/minute.

(ii) The Inter-arrival time (a) is the average time between customer arrivals. It is measured in time per customer.

$a = 1/\lambda$ ………………………...…… 3.1

The unit is minutes/access.

(iii) Service time (μ) is the average time required to service one customer measured in service per time period. The units are access/minute.

(iv) The Inter-service time (s) is the average time between customer service times. It is measured in time per customer.

$s = 1/\mu$ ....(3.2)

The unit is minutes/service.

(v) Utilization (p) is the fraction of time the Server is busy is given respectively for M/M/1 and M/M/s:

$p = \lambda/\mu$..(3.3)

s = the number of servers in the system

$p = \lambda / s\mu$ = the average utilization of the system…………………………….. (3.4)

Utilization is always between $0 \leq p \leq 100$.

(vi) Number (of customers) waiting in line (LQ) is the average number of customers waiting in the queue (excluding that receiving service) for single and multi-tellers respectively are:

$LQ = PL = \lambda 2/ \mu (\mu - \lambda)$ = the average number of customers waiting in line…………... (3.5)

= the average number of customers waiting in line……. (3.6)

(vii) The corresponding time customers wait in the queue (Tw) or WQ is the average time each customer waits in the queue.

$WQ = PW = \lambda/ \mu (\mu - \lambda)$ = the average time spent waiting in line……………...….. (3.7)

=the average time spent waiting in line……………………………………...….. (3.8)

(viii) Number of customers in the System (L) is the average total number of customers in the system (i.e., number in queue plus number in service)

$L = \lambda/ (\mu - \lambda)$ = the average number of customers in the service system……….……… (3.9)

$L = \lambda W$ = the average number of customers in the service system……………………………….... (3.10)

Number of customer in service (Ls) is the number of customer receiving service (excluding those in the queue).

$L= Lq + Ls$ and $Ls = L – Lq$ …………………………………………………... (3.11)

(xi) Time in the System (Tq) or W is the average total each customer spends in the system (i.e., time in queue plus time in service).

$W = 1/ (\mu - \lambda)$ = the average time spent waiting in the system, including service…..... (3.12)

= the average time spent in the system, including service………………… (3.13)

(x) The probability that n customers are in the service system at a given time for M/M/1.

$Pn = (1- P)Pn$ = (Harry, 2012)……….……………………………………………..... (3.14)

…………………………..………. (3.15)

(xi) The probability that no customers are in the system.

…………………………... (3.16)

(xii) The probability that n customers are in the system at a given time for M/M/s

………………………………………… (3.17)

In these models, the service rate must be greater than the arrival rate, that is, $\mu > \lambda$ because if $\mu < \lambda$ or $\mu = \lambda$, the waiting line will eventually grow infinitely large.

The above equations were used in Microsoft Excel software to make queuing analysis, and by writing code in Java programming language statements which is a simulation modeling language. The simulation model or analysis will include:

(i). Model inputs that are uncertain numbers (or uncertain variables, such as arrivals and service times)

(ii). Intermediate calculations required, such as the performance measures would be calculated using Microsoft Excel, in order to get the model outputs.

(iii). Model outputs that depend on the inputs (hence called uncertain functions)

In other words, an Excel spreadsheet is a simple tool for analyzing the model. The model was simulated using Java programming language and a simulation software called Win QSB for a better solution.

3.5 The Cost of Waiting or Economics of waiting line

In order to solve the queuing model with simulation, the service facility must be manipulated so that an optimum balance is obtained between the cost of waiting time and the cost of idle time. The cost of waiting generally includes either the indirect cost or loss of customers. By increasing the investment in labor and service facility waiting time and the losses associated with it can be decreased.

The Cost of unhappiness of customer waiting was calculated as follows:

(i) Average wait per hour (or per minute) = Average time a customer spent waiting in line (WQ) (was calculated in the spreadsheet analysis).

(ii) Number of Arrivals per hour (was converted to Number of arrivals per minute) (customers/minute) = Y customers per minute……………………………..…………….… 3.16

(iii) Total waiting time per customer = WQ x Number of Arrivals per hour…………....…. 3.17

(iv) Total Cost for waiting = Total waiting time per customer x cost per hour…………….... 3.18

Let the cost of waiting per hour = ₦y ……………………………………………………... 3.19

The cost of waiting per minute = Cost of waiting per minute = ₦y/60 = Z customers/minute... 3.20

(v) We now 3.4 into 3.3 (and this give cost of unhappiness of the customer in the Bank).

(vi) If the customer waited in the Bank for two hours, the unhappiness cost for two hours = unhappiness cost x hours waited in the Bank = K…………………………………………... 3.21

The salary cost was calculated as follows:

If the customer that encounter this delay earns ₦T per hour in his place of work, then the Salary cost = hour(s) waited in the bank x amount earn in the office per hour = 2 hours x amount earn per hour = M………………………………………………………………………….….…….3.22

Total cost = Salary cost + Waiting cost = M + K for the two hours waited in the Bank……. 3.23

3.6 Simulation Analysis Steps Used

The following Seven-Step Approach for Conducting a Successful Simulation Study was used:

(1) The problem was formulated

(2) Information/Data was collected and a Conceptual Model was constructed.

(3) It was found that the Conceptual Model was Valid.

(4) The model was programmed

(5) The Programmed Model was Valid

(6) Simulation experiments were designed, made, and Analyzed

(7) Simulation results were documented and presented.

Figure 3.4 illustrates the above seven simulation analysis steps:

s

Figure 3.4: Simulation Analysis Steps

Some details of the simulation analysis are:

(1) A Problem Statement was written – performance measures were determined using two different models and compared to get the best

(2) Input Variables and Entities were selected: Entities used are (customers and Tellers) and variables (such as arrival, time service starts and time service ends) for the simulation.

(3) Constraints on Decision Variables were made: Real data collected were assigned to the variables that will enable decision to be made so that the simulation can be customized to the real world system.

(4) Output Variables were determined: The variables that the simulation will output are P, Po, W, Wq, L, Lq, and the probability of specific number of customers are the variables that the Simulation should output.

(5) Data was collected from Real-Life System: Information was gathered from the system to input into the simulation, that is, from the two case studies directly electronic timing.

(6) Model Development: A logical diagram/flowchart was develop as shown in Chapter one which help me to visually see the progression of the simulation and to better model the system.

(7) Simulation Software was selected: MS Excel was chosen for the analysis and the simulation software (WinQSB) was used to run the model. Java programming language, which is also simulation software, will be used to show Bank Simulation program.

(8) Model Verification: The Simulation was run and the results were compared to the actual system. It was confirmed the data found was comparable to real data and the model is consistent with the initial logic. The model was built correctly and the simulation was reworked until it resembles the real world data.

(9) Model Experimentation & Optimization: The Simulation was tested to find the best possible solution to the problem, using the different data collected from different Tellers in

the case study Banks, and all findings were graphed to see all possible solutions. The one that have utilization less than one were considered as the best solution.

(10) Implementation of Simulation Results: the results were applied to the real-world system after determining the best solution based on the data input by giving appropriate recommendation.

Figure 3.5 shows the steps and decision for conducting simulation

Figure 3.5: Steps and Decisions for Conducting a Simulation Study

The following Solution Process was taken in calculating the performance of the system:

Solution

1. Determine what quantities you need to know:

(i) Mean arrival rate of customers = average number of customers arriving the bank per minute (λ).

(ii) Mean service rate = average number of customers that can be served per unit of time (μ).

(iii) The average utilization of the system (P)

(iv) The average number of customers in the service system (L) customers.

(v) The average number of customers waiting in line (LQ) customers.

(vi) The average time spent waiting in the system, including service (W) minutes

(vii) Average time a customer spent waiting in line (WQ) minutes.

(viii) The probability that there are more than five customers in the system equals one minus the

Probability that there are four or fewer customers equal (P):

2. Identify the server

– The Teller is the server. Single and multiple servers were used

3. Identify the queued items

– Customer requests

4. Identify the queuing model

– M/M/1 and M/M/2

5. Determine the service time (μ)

6. Determine the arrival rate (λ)

7. Calculate ρ

8. Calculate the desired values such as p, W, Tw, Tq, etc.

CHAPTER FOUR

ANALYSIS OF SINGLE AND MULTI-TELLER MODELS

4.1 Stable and Unstable Queue

It is only stable queue data that is required for analysis, that is, when the average time between arrivals is greater than Average time between service. This means that the queue in the bank is stable, which in turn means that there are idle and busy periods continuously alternating and also the graph of probability of n customers in the system will be normal, or resulted in positive exponential graph, therefore, this was verified before each analysis was carried out for each of the case study Banks as was done in Table 4.1, Figure 4.1 and Figure 4.2, after which, the analysis was done as in Figure 4.3.

Table 4.1: Teller One data for the First Bank showing the mean arrival and mean service time

S/NO	ARRIVAL TIME	INTER-ARRIVAL TIME	TIME SERVICE STARTS	TIME SERVICE ENDS	SERVICE TIME
45	9.55	1	9.57	9.58	1
46	9.56	2	9.58	10.00	2
47	9.58	1	10.00	10.04	4
48	9.59	0	10.04	10.05	1
49	9.59	0	10.05	10.06	1
50	9.59	6	10.06	10.07	1
51	10.05	1	10.07	10.09	2
52	10.06	5	10.09	10.10	1
53	10.11	1	10.11	10.13	2
54	10.12	0	10.13	10.14	1
55	10.12	2	10.14	10.18	4
56	10.14	1	10.18	10.19	1
57	10.15	2	10.19	10.21	2
58	10.17	0	10.21	10.25	4
59	10.17	3	10.25	10.26	1
60	10.20	1	10.26	10.28	2
61	10.21	2	10.28	10.30	2
62	10.23	1	10.30	10.31	1

63	10.24	1	10.31	10.32	1
64	10.25	3	10.32	10.33	1
65	10.28	1	10.33	10.34	1
66	10.29	2	10.34	10.37	3
67	10.31	1	10.37	10.39	2
68	10.32	0	10.39	10.40	1
69	10.32	2	10.40	10.42	2
70	10.34	7	10.42	10.44	2
71	10.41	1	10.44	10.47	3
72	10.42	1	10.47	10.49	2
73	10.43	1	10.49	10.51	2
74	10.43		10.51	10.53	2
SUM		49			55

MEAN

1.689655172

1.8333333

(= 1.69)

(= 1.83)

The following is the probability of number of 15 customers in the System calculated from the table which showed a stable Queue :

Figure 4.1: Probabilities of 15 customers in the System of Teller One data of the First Bank

Figure 4.2: The corresponding Stable queue graph for Figure 4.1

The following is analysis for Teller One data of the First Bank, after data validation

Figure 4.3: Analysis of Teller One data of the First Bank

From Figure 4.3, the average time between arrivals is greater than Average time between service (0.59172 > 0.54645). This means that the queue in the bank is stable, which in turn means that there are idle and busy periods continuously alternating and also the graph of probability of n customers in the system will be normal (Figure 4.2, otherwise the queue will be unstable (Figure 4.4), which was not used for the analysis as it will not give a valid result for the analysis.

Figure 4.4: Unstable queue graph

Mean arrival rate and mean services rate are important values required for the analysis of the queuing models and are therefore explained briefly in units 4.2 and 4.3

4.2 Mean Arrival Rates (λ) (or Arrival Rates)

Mean arrival rate is the mean number of arrivals per unit time, for example, what is the number of customers that arrived in the First Bank in one minute or in one hour for Teller One.

Mean arrival rate = Sum of arrival time/ (total number of customers - 1) = 49/ (30-1) = 1.69 customers arrived/minute (Figure 4.3).

4.3 Mean Service Rates (μ) (or Service Rates)

Mean service rate is the mean number Tellers served customers per unit time, for example, what is the number of customers that Tellers served in the First Bank in one minute or in one hour for, for instance, From Table 4.1, Sum of service time is 55 and Total Number of Customers is 30, so,

Service rate = Sum of service time/Total number of customers = 55/30 = 1.83 customers per minute (Figure 4.3).

4.4 Turnaround and Response times for multi-tellers, multi-lines of the First Bank and Multi-tellers, single line of the ECO Bank.

4.4.1 Turnaround and Response times for multi-tellers, multi-lines of the First Bank

(i) From Table 4.1

The number of customers that the Teller one served in the system from 9.55 to 10.43 am = 30 customers in 48 minutes (5+43 minutes).

Turnaround = Number of customers' teller one served per minute

= number of customer/ time

= 30/48

= 0.6977 customers per minute

Response time = time / number of customers (is the opposite of Turnaround time)

= 48 / 30

= 1.4333 minutes per customer

Table 4.2: Teller 2 data of the First Bank (8.28 TO 9.13 AM)

S/NO ARRIVAL INTER-ARRIVAL TIME
SERVICE TIME SERVICE SERVICE

TIME TIME STARTS ENDS
TIME

1	8.28	1	8.28	8.31	3
2	8.29	0	8.29	8.32	3
3	8.29	2	8.29	8.34	5
4	8.31	1	8.34	8.35	1
5	8.32	1	8.35	8.36	1
6	8.33	3	8.36	8.37	1
7	8.36	1	8.37	8.38	1
8	8.37	1	8.38	8.39	1
9	8.38	1	8.39	8.40	1
10	8.39	1	8.40	8.41	1
11	8.40	4	8.41	8.42	1
12	8.44	2	8.44	8.48	4
13	8.46	7	8.48	8.49	1
14	8.53	2	8.53	8.54	1
15	8.55	3	8.55	8.56	1
16	8.58	2	8.58	8.59	1
17	9.00	1	9.00	9.01	1
18	9.01	6	9.01	9.05	4
19	9.07	4	9.07	9.10	3
20	9.11	1	9.11	9.15	4
21	9.12	0	9.15	9.19	4
22	9.12	1	9.19	9.21	2
23	9.13		9.20	9.23	3

SUM	45	48
MEAN	2.045454545 (= 2.05)	2.086957 (=2.09)

(ii) From Table 4.2

The number of customers that the Teller two served in the system from 8.28 to 9.13 am = 23 customers in 45 minutes (32+13 minutes).

Turnaround = Number of customers' teller one served per minute

= number of customer/ time

= 23/45

= 0.5111 customers per minute

Response time = time / number of customers (is the opposite of Turnaround time)

= 45 / 23

= 1.9565 minutes per customer

Table 4.3: Teller three data of the First Bank (9.16 TO 10.18 AM)

S/NO	ARRIVAL TIME	INTER-ARRIVAL TIME	SERVICE STARTS	SERVICE ENDS	SERVICE TIME
37	9.16	5	9.18	9.22	4
38	9.21	2	9.22	9.23	1
39	9.23	1	9.23	9.24	1
40	9.24	3	9.24	9.26	2
41	9.27	0	9.27	9.28	1

42	9.27	4	9.28	9.30	2
43	9.31	3	9.31	9.32	1
44	9.34	2	9.34	9.36	2
45	9.36	0	9.36	9.38	2
46	9.36	4	9.38	9.39	1
47	9.40	1	9.40	9.43	3
48	9.41	2	9.43	9.45	2
49	9.43	2	9.45	9.46	1
50	9.45	1	9.46	9.48	2
51	9.46	2	9.48	9.49	1
52	9.48	2	9.49	9.50	1
53	9.50	4	9.51	9.54	3
54	9.54	0	9.54	9.55	1
55	9.54	6	9.55	9.56	1
56	10.00	1	10.01	10.03	2
57	10.01	1	10.03	10.04	1
58	10.02	1	10.04	10.05	1
59	10.03	6	10.05	10.09	4
60	10.09	1	10.10	10.15	5
61	10.10	4	10.12	10.14	2
62	10.14	0	10.15	10.25	10
63	10.14	1	10.16	10.18	2
64	10.15	1	10.18	10.19	1
65	10.16	2	10.19	10.21	2
66	10.18		10.21	10.24	3
SUM		62			65

MEAN 2.137931034 2.17

(= 2.14) (= 2.17)

Table 4.4: Teller three data of the First Bank (8.38 TO 9.06 AM)

S/NO	ARRIVAL TIME	INTER-ARRIVAL TIME	TIME SERVICE STARTS	SERVICE ENDS	SERVICE TIME
1	8.38	2	8.38	8.42	4
2	8.40	1	8.42	8.43	1
3	8.41	0	8.43	8.46	3
4	8.41	0	8.46	8.48	2
5	8.41	1	8.48	8.50	2
6	8.42	1	8.50	8.51	1
7	8.43	1	8.51	8.54	3
8	8.44	0	8.54	8.55	1
9	8.44	1	8.55	8.56	1
10	8.45	0	8.57	8.59	2
11	8.45	0	8.59	9.02	3
12	8.45	1	9.02	9.03	1
13	8.46	0	9.03	9.06	3
14	8.46	1	9.06	9.08	2
15	8.47	2	9.09	9.11	2
16	8.49	0	9.11	9.12	1
17	8.49	2	9.15	9.17	2

18	8.51	1	9.17	9.18	1
19	8.52	2	9.18	9.19	1
20	8.54	1	9.19	9.24	5
21	8.55	0	9.24	9.26	2
22	8.55	1	9.26	9.27	1
23	8.56	1	9.27	9.29	2
24	8.57	2	9.29	9.30	1
25	8.59	2	9.31	9.32	1
26	9.01	2	9.32	9.33	1
27	9.03	0	9.33	9.35	2
28	9.03	1	9.36	9.38	2
29	9.04	2	9.39	9.41	2
30	9.06		9.41	9.42	1
SUM		28			56
MEAN	1.87	0.965517241			
	(= 1.87)	(= 0.97)			

(iii) From Table 4.3

The number of customers that the Teller two served in the system from 9.16 to 10.18 am = 30 customers in 62 minutes (44+18 minutes).

Turnaround = Number of customers' teller one served per minute

= number of customer/ time

= 30/ 62

= 0.4839 customers per minute

Response time = time / number of customers (is the opposite of Turnaround time)

= 62 / 30

= 2.0667 minutes per customer

Table 4.5: Teller Four data of the First Bank (9.42 TO 10.26 AM)

S/NO	ARRIVAL TIME	INTER-ARRIVAL TIME	TIME SERVICE STARTS	TIME SERVICE ENDS	SERVICE TIME
61	9.42	1	10.07	10.11	4
62	9.43	1	10.11	10.12	1
63	9.44	1	10.12	10.13	1
64	9.45	1	10.13	10.14	1
65	9.46	0	10.14	10.15	1
66	9.46	1	10.15	10.17	2
67	9.47	1	10.17	10.20	3
68	9.48	5	10.20	10.21	1
69	9.53	2	10.21	10.22	1
70	9.55	2	10.22	10.23	1
71	9.57	4	10.23	10.26	3
72	10.01	2	10.26	10.27	1
73	10.03	1	10.27	10.31	4
74	10.04	2	10.31	10.34	3
75	10.06	1	10.34	10.38	4
76	10.07	1	10.38	10.40	2
77	10.08	0	10.40	10.42	2
78	10.08	0	10.42	10.44	2
79	10.08	1	10.44	10.45	1

80	10.09	2	10.45	10.47	2
81	10.11	5	10.47	10.50	3
82	10.16	1	10.50	10.55	5
83	10.17	2	10.55	10.56	1
84	10.19	0	10.56	10.58	2
85	10.19	1	10.58	11.02	4
86	10.20	1	11.02	11.03	1
87	10.21	2	11.03	11.04	1
88	10.23	2	11.04	11.05	1
89	10.25	1	11.05	11.07	2
90	10.26		11.07	11.10	3
SUM		44			63
MEAN	2.10 (= 2.10)	1.517241379 (= 1.52)			

(iv) From Table 4.5

The number of customers that the Teller two served in the system from 9.42 to 10.26 am = 30 customers in 44 minutes (18+26 minutes).

Turnaround = Number of customers' teller one served per minute

= number of customer/ time

= 30/44

= 0.6818 customers per minute

Response time = time / number of customers (is the opposite of Turnaround time)

= 44 / 30

= 1.4667 minutes per customer

The following table is a summary of Turnaround and Response time from unit 4.4.1:

Table 4.6: Summary of Turnaround and response times for the four single Tellers of the First bank

Single Tellers	Turnaround	Response time
Teller one	0.6977	1.4333
Teller two	0.5111	1.9565
Teller three	0.4839	2.0667
Teller four	0.6818	1.4667

The corresponding graphs of Turnaround and Response time for Table 4.6 is given below:

Figure 4.5: Graph of Turnaround and response times of Table 4.6

From Figure 4.3, the less the turnaround time, the more the response time, but most of the single Tellers of the first Bank have approximately turnaround time of one customer per minute.

Turnaround in terms of cost

From Table 4.6:

The average Turnaround = 0.6977 + 0.5111 + 0.4839 + 0.6818 = 2.3745/4 = 0.5936 minutes

Let the cost of Turnaround for 1 minute = N100

Cost of Turnaround for 0.5936 = 0.5936 x N100 = N59.36

4.4.2 Turnaround and Response times for multi-tellers, single line of the ECO Bank

Table 4.7: Multi-Teller One Data of ECO Bank (8.02 TO 9.13 AM)

S/NO ARRIVAL INTER-ARRIVAL TIME
SERVICE TIME SERVICE SERVICE
TIME TIME STARTS ENDS
TIME

1	8.02	1	8.16	8.19	3
2	8.03	5	8.18	8.21	3
3	8.08	1	8.20	8.22	2
4	8.09	3	8.22	8.24	2
5	8.12	3	8.22	8.31	9
6	8.15	2	8.24	8.29	5
7	8.17	3	8.29	8.30	1
8	8.20	1	8.30	8.33	3
9	8.21	3	8.32	8.37	5
10	8.24	2	8.33	8.35	2
11	8.26	12	8.35	8.36	1
12	8.38	0	8.38	8.39	1
13	8.38	2	8.38	8.44	6
14	8.40	1	8.41	8.43	2
15	8.41	6	8.43	8.48	5
16	8.47	0	8.47	9.00	13
17	8.47	1	8.48	9.00	12
18	8.48	0	8.49	8.50	1
19	8.48	0	8.51	8.52	1
20	8.48	1	8.52	8.58	6
21	8.49	1	8.58	9.02	4

22	8.50	4	9.01	9.10	9
23	8.54	4	9.06	9.12	6
24	8.58	5	9.01	9.07	6
25	9.03	5	9.08	9.15	7
26	9.08	1	9.10	9.14	4
27	9.09	1	9.14	9.19	5
28	9.10	1	9.15	9.17	2
29	9.11	2	9.17	9.19	2
30	9.13		9.19	9.27	8
SUM		71			136
MEAN	4.5333333	2.448275862			
	(μ=4.53)	(ʎ=2.45)			

(i) From Table 4.7:

The number of customers that the Teller two served in the system from 8.02 to 9.13 am = 30 customers in 71 minutes (58+13 minutes).

Turnaround time = Number of customers' teller one served per minute

= number of customer/ time

= 30/71

= 0.4225 customers per minute

Response time = time / number of customers (is the opposite of Turnaround time)

= 71 / 30

= 2.3667 minutes per customer

Table 4.8: Multi Teller Two data of ECO Bank (8.42 TO 10.59 AM)

S/NO	ARRIVAL TIME	INTER-ARRIVAL TIME	TIME SERVICE STARTS	TIME SERVICE ENDS	SERVICE TIME
31	8.42	11	9.43	9.51	8
32	8.53	3	9.44	9.50	6
33	8.56	3	9.50	9.55	5
34	8.59	6	9.51	9.56	5
35	9.05	3	9.55	10.00	5
36	9.08	4	9.56	10.01	5
37	9.12	5	10.00	10.05	5
38	9.17	4	10.01	10.06	5
39	9.21	8	10.05	10.12	7
40	9.29	3	10.06	10.11	5
41	9.32	8	10.11	10.19	8
42	9.40	5	10.12	10.18	6
43	9.45	1	10.18	10.24	6
44	9.46	7	10.19	10.39	2
45	9.53	4	10.24	10.29	5
46	9.57	3	10.29	10.48	19
47	10.00	3	10.39	10.46	7
48	10.03	5	10.46	10.52	6
49	10.08	4	10.48	10.59	11
50	10.12	5	10.55	11.12	7

51	10.17	0	10.59	11.06	7
52	10.17	5	11.06	11.13	7
53	10.22	6	11.12	11.17	5
54	10.28	4	11.14	11.20	6
55	10.32	1	11.17	11.24	7
56	10.42	4	11.20	11.31	11
57	10.46	3	11.24	11.25	1
58	10.49	6	11.31	11.44	13
59	10.53	6	11.35	11.43	8
60	10.59		11.43	11.46	3
SUM		130			201
MEAN	6.7	4.482758621			
	(= 6.70)	(= 4.48)			

(ii) From Table 4.8:

The number of customers that the Teller two served in the system from 8.42 to 10.59 am = 30 customers in 77 minutes (18 + 59 minutes).

Turnaround time = Number of customers' teller one served per minute

= number of customer/ time

= 30/77

= 0.3896 customers per minute

Response time = time / number of customers (is the opposite of Turnaround time)

= 77 / 30

= 2.5667 minutes per customer

Table 4.9: Multi-Teller Three data of the ECO Bank (8.07 TO 9.54 AM)

S/NO	ARRIVAL TIME	INTER-ARRIVAL TIME	SERVICE STARTS	SERVICE ENDS	SERVICE TIME
1	8.07	1	8.09	8.15	6
2	8.08	1	8.09	8.12	3
3	8.09	1	8.12	8.17	5
4	8.10	8	8.15	8.29	14
5	8.18	1	8.21	8.24	3
6	8.19	1	8.24	8.31	7
7	8.20	3	8.29	8.37	8
8	8.23	1	8.31	8.34	3
9	8.24	2	8.34	8.38	4
10	8.26	4	8.38	8.43	5
11	8.30	2	8.37	8.51	14
12	8.32	3	8.45	8.47	2
13	8.35	5	8.47	8.55	8
14	8.40	31	8.51	8.59	8
15	9.11	1	9.12	9.16	4
16	9.12	1	9.16	9.22	6
17	9.13	2	9.16	9.18	2
18	9.15	2	9.17	9.22	5
19	9.17	0	9.22	9.27	5
20	9.17	6	9.23	9.26	3
21	9.23	2	9.27	9.33	6

22	9.25	1	9.33	9.44	11
23	9.26	1	9.34	9.44	1
24	9.27	3	9.44	9.55	11
25	9.30	1	9.45	10.01	16
26	9.31	4	9.56	9.57	1
27	9.35	5	10.01	10.03	2
28	9.40	11	10.03	10.04	1
29	9.51	3	10.04	10.06	2
30	9.54		10.06	10.07	1
SUM		107			167
MEAN		3.689655172			
	5.5666667				

(iii) From Table 4.9:

The number of customers that the Teller two served in the system from 8.07 to 9.54 am = 30 customers in 107 minutes (53 + 54 minutes).

Turnaround time = Number of customers' teller one served per minute

= number of customer/ time

= 30/107

= 0.2804 customers per minute

Response time = time / number of customers (is the opposite of Turnaround time)

= 107 / 30

= 3.5667 minutes per customer

The following table is a summary of Turnaround and Response times for unit 4.4.2:

Table 4.10: Turnaround and response times for Multi-Tellers

Multi-tellers	Turnaround time	Response time
Multi-teller 1	0.4225	2.3667
Multi-teller 2	0.3896	2.5667
Multi-teller 3	0.2804	3.5667

Figure 4.6: Graph of Turnaround and response times of Table 4.10

From Figures 4.4, just like in Figure 4.3, the less the turnaround time, the more the response time, but the turnaround for multi-teller is approximately three customers per minute, which is better than that of the single servers. However, the response time of the multi-tellers is more than that of single servers, i.e., single servers gave low turnaround time (that is serve less customers per minute than the multi-tellers because turnaround is the opposite of response times, i.e. the less the turnaround time the less the response time and vice versa. Nigeria Banks want rapid turnaround, i.e., they want 8 hours in each day of a week operation. Multi-server model provides better turnaround than single servers.

Turnaround in terms of cost

From Table 4.2:

The turnaround for Teller 1 = 0.4225 minutes

Let Cost of turnaround for 1 minute = N100

Cost of 0.4225 minutes Turnaround = 0.4225 x 100 = N42.25

In terms of cost, the total Turnaround earlier calculated in unit 4.4.1 for the four Tellers of the

First Bank is N59.36 while that of the Tellers of the ECO Bank is N42.25 which is an advantage for the ECO bank over the First Bank.

4.5 The Operating Characteristics of a Single Teller or Server When $p < 1$ or $p > 1$

Why do we need $p < 1$ implies $\mu > \lambda$, that is, Service rate of the Teller is larger than the arrival rate? If this is not so, that is, if $p > 1$, the number of customers in the system will grow forever. In other words, $p < 1$ is required to ensure the stability of the queuing system. The stability and instability of the queuing system is shown in the analysis in appendix 1 for Tellers One to four, some of which are stable while some were unstable as shown respectively by Graphs in Figures on pages 62 and 64.

4.5.1 The Operating Characteristics of a Single Teller or Server When $p < 1$

Some real data analyzed in the Excel sheets for Teller one to four in appendix 1 satisfies $p < 1$, for the single server model, which was used to get the real operating characteristics of the Bank as follows, which help to know whether the queue is stable or unstable.

$\lambda = 1.63$ = mean arrival rate of customers = average number of customers arriving the bank per

Minute.

$\mu = 1.87$ = mean service rate = average number of customers that can be served per unit of time.

From equations (3.1), (3.3), (3.5), (3.7), (3.10) and (3.13): -

(1) The average utilization of the system: $p = \lambda / \mu = 1.63/1.87 = 0.87 = (0.87 \times 100) = 87\ \%$

(2) The average number of customers in the service system:

$$L = \lambda / (\mu - \lambda) = 1.63/ (1.87 - 1.63) = 1.63/ (0.24) = 6.79 \text{ customers}$$

(3) The average number of customers waiting in line:

$$LQ = pL = 0.87 \times 6.79 = 5.91 \text{ customers.}$$

(4) The average time spent waiting in the system, including service:

$$W = 1/ (1.87 - 1.63) = 1/ (0.24) = 4.17 \text{ minutes}$$

(5) Average time a customer spent waiting in line: $WQ = pW = 0.87 \times 4.17 = 3.63$ minutes.

(6) The probability that there are more than five customers in the system equals one minus the

Probability that there are four or fewer Customers equals:

$$= 1 - (1 - 0.87) (0.870 + 0.871 + 0.872 + 0.873 + 0.874 + 0.875)$$

= 1 - 0.13(1 + 0.87 + 0.872 + 0.873 + 0.874 + 0.875)

= 1 - 0.13 (1 + 0.87 + 0.76 + 0.66 + 0.57 + 0.50)

= 1 - 0.13 x 4.36

= 1 - 0.57

= 0.43

This means that there is a 0.43 (43.00 %) chance of having more than five customers in the system.

Figure below shows a spreadsheet solution of the problem and the spreadsheet formulas are a direct implementation of the single server formulas for performance measures.

Probability of Number of customers in the system when $p < 1$

The above analysis resulted in a Stable queue because it resulted to a normal exponential graph in Figure on page 62 and as well resulted in positive wait values and probability of number of customers in this section because the Teller was not over utilized and there were no significant surges in demand.

4.5.2 The Operating Characteristics of a Single Teller or Server When $p > 1$

The conditions that can be used to model this environment has been given in section 1.5. Using these conditions, the operating characteristics of the waiting line in the Bank Queue for four different Tellers was simulated with the corresponding graphs of probability of number of customers in the System when $p > 1$. Refer to units 4.5.3 and 4.7 for Tellers One to Four analysis and Multi-Tellers One to Three of the ECO Bank. However, an example of how the performance measures were carried out is given below with the corresponding graph of probability of number of customers in the System (which resulted in an unstable queue with negative wait times and inverted graph because of $p > 1$.

From equations (3.1), (3.3), (3.5), (3.7), (3.10) and (3.13):-

(1) The average utilization of the system: $p = \lambda / \mu = 2.32/1.80 = 1.29$ (= 1.29 x 100 = 129%)

(2) The average number of customers in the service system:

$L = \lambda / (\mu - \lambda) = 2.32 / (1.80 - 2.32) = 2.32 / (-0.52) = -4.46$ customers

(3) The average number of customers waiting in line:

$LQ = pL = 1.29 \times (-4.46) = -5.75$ customers.

(4) The average time spent waiting in the system, including service:

$W = 1 / (1.80 - 2.32) = 1 / (-0.52) = -1.92$ minutes

(5) Average time a customer spent waiting in line: WQ = pW = 1.29 x -1.92 = - 2.48 minutes.

(6) The probability that there are more than five customers in the system equals one minus the probability that there are four or fewer customers' equals:

= 1 - (1 - 1.29) (1.290 + 1.291 + 1.292 + 1.293 + 1.294 + 1.295)

= 1 - (-0.29) (1 + 1.29 + 1.66 + 2.15 + 2.77 + 3.57)

= 1 - (-0.29) x 12.44

= 1 - (-3.61)

= 1 + 3.61

= 4.61

This means that there is a 4.61 (461.00 percent) chance of having more than five customers in the system. (This is too high)

The following figure shows a spreadsheet solution of the problem and the spreadsheet formulas in used for the analysis are a direct implementation of the single server formulas for performance measures.

Probability of Number of customers in the system when P > 1

The above analysis resulted in an unstable queue because it resulted to an inverted exponential graph above as well as the server efficiency for the Teller was greater than 1 which makes the values of the average waiting time in the system and in line to be negative. This means that customers arrived faster than they received service; consequently, the queue was not stable or not moving which is an indication that there will be much waiting in these cases and will result to inverse graph shown in the Figure above because p > 1. In addition, the Teller was over utilized (more than 100% in this case) because there were significant surges in demand.

4.5.3 The Operating Characteristics of Multi-line, Multi-Server of the First Bank When P < 1.

The following are the Analysis for the four Multi-line, Multi-Server of the First Bank for each of the four Tellers of the First Bank, but a stable queue validation of data which must give a positive probability result for certain number of cutomers in the system and corresponding positive exponetial graph was obtained first before each analysis is carried out as shown below.

(1) Teller One Data validation and Analysis

Data validation for Table 4.1 when λ =1.69 and μ = 1.83, the following is the probability of number of 15 customers in the System which showed a stable Queue for the analysis because the probabilities of number of customers in the system is positive with a positive exponential graph, which showed a staable queue for the analysis:

Figure 4.7: Probabilities of 15 customers in the System of Teller One data of the First Bank

Figure 4.8: The corresponding graph for Figure 4.7

The following is the analysis for Teller One data of the First Bank after data validation

Figure 4.9: Analysis of Teller One data of the First Bank

(2) Teller Two Data validation and Analysis

Data validation for Table 4.2 when λ =2.05 and μ = 2.09, the following is the probability of number of 15 customers in the System which showed a stable Queue for the analysis because the probabilities of number of customers in the system is positive with a positive exponential graph, which showed a staable queue for the analysis:

Figure 4.10: Probabilities of 15 customers in the System of Teller Two data of the First Bank

Figure 4.11: The corresponding graph for Figure 4.10

The following is analysis for Teller Two data of the First Bank after data validation

Figure 4.12: Analysis of Teller Two data of the First Bank

(3) Teller Three Data validation and Analysis

Data validation for Table 4.3 when λ = 0.97 and μ = 1.87, the following is the probability of number of 15 customers in the System which showed a stable Queue for the analysis because the probabilities of number of customers in the system is positive with a positive exponential graph, which showed a staable queue for the analysis:

Figure 4.13: Probabilities of 15 customers in the System of Teller Two data of the First Bank

Figure 4.14: The corresponding graph for Figure 4.13

The following is analysis for Teller Three data of the First Bank after data validation

Figure 4.15: Analysis of Teller Three data of the First Bank

(4) Teller Four Data validation and Analysis

Data validation for Table 4.4 when λ =1.52 and μ = 2.10, the following is the probability of number of 15 customers in the System which showed a stable Queue for the analysis because the probabilities of number of customers in the system is positive with a positive exponential graph, which showed a staable queue for the analysis:

Figure 4.16: Probabilities of 15 customers in the System for Teller Four data of the First Bank

Figure 4.17: The corresponding graph for Figure 4.16

The following is analysis for Teller Four data of the First Bank after data validation

Figure 4.18: Analysis of Teller Four data of the First Bank

The following table is a summary of the Analysis for the four Multi-line, Multi Server of the First Bank when p < 1 and the corresponding graph (Figure 4.9):

Table 4.11: Summary of Performance measures for First Bank Analysis, when p < 1

Tellers	λ	μ	p(rho)	P0	L	Lq	W	Wq	Pn
1	1.69	1.83	0.9235	0.0765	12.0714	11.1479	7.14289	6.59641	0.02319
2	2.05	2.09	0.98086	0.01914	51.25	50.2691	25	24.5215	0.01432
3	0.97	1.87	0.51872	0.48128	1.07778	0.55906	1.1111	0.57635	2.50E-05
4	1.52	2.1	0.72381	0.27619	2.62069	1.89688	1.72414	1.24795	0.00217

Figure 4.19: The corresponding Graph of Performance measures for Table 4.11

The following is one of the data entered into a simulation software which was used to verify the analysis results in unit 4.5.3 before developing the software for Simulating performance measure of the Two banks at a run after given the necessary data (Figure 4.22).

Figure 4.20: Data entered for M/M/1 analysis and simulation (Source of WinQSB: WinQSB free trial software) [Yih-Long (2015), Microsoft (2015)]

The corresponding output for the data entered in Figure 1.2 is given below:

Figure 4.21: The corresponding Performance measures of First Bank and cost from Simulation (Yih-Long, 2015), (Microsoft, 2015).

Figure 4.22: Performance measures and facility cost from Java software developed for the banks

From Table 4.9, all the four single tellers have server efficiency less than 1 which resulted in a stable queue in the Bank (or is an indication that customers did not arrived faster than they received service) and a good performance from all tellers. Despite the fact that tellers 1 to 2 and Teller 4 have higher efficiency than Teller 3, the customers waited for longer time in the System and in the queue than customers of Teller 3 as shown in Table 4.9 and Figure 4.19. This is because Teller 1 to 2 and Teller 4 were attending to larger amount of deposits and savings than Teller 3, which led to longer customer waits.

4.5.4 Cost Analysis or Unhappiness Cost for the First Bank customers from each Teller

The following show Cost Analysis or Unhappiness Cost for the First Bank customer for each Teller

ECONOMICS OR COSTS OF WAITING PROBLEM FOR THE M/M/1 MODEL

TELLER ONE

WQ = 3.36 6.5964 minutes (= Average time a customer spent waiting in line)

NumberOfArrivalPerHour = 38 customers/hour

NumberOfArrivalPerMinute = 0.633333333 Customers/minute

TotalWaitingTimePerCustomer = 4.17772 minutes

TotalCostForWaiting = TotalWaitingTimePerCustomer x cost per minute (1)

Let the cost of waiting per hour = 500 Naira

then, the cost of waiting per minute = 8.33333 Naira per minute

from (1), TotalCostForWaiting = 34.8143 Naira per minute

This means that the cost of unhapiness of a cutomer in the queue per minute is 34.8143 (or)

2088.86 Naira per hour.

If the customer spent two hours in the Bank, TotalCostForWaiting for 2 hours = 4177.72

Naira is the cost of unhapinnes the customer waited inte Bank for the two hours

If the customer that encounters the above delay in the queue earns 400

Naira per hour in his place of work, then the

Salary Cost = hours waited in the Bank x Total cost for waiting

= 2 hour x amount earn in place of work per hour (if a customer waited in the bank for 2 hours during working hours) is:

Single Teller cost analysis

800 Naira for the two hours

Total Cost or Loss on the Customer's side = 4977.72 (₦4977.72)

TELLER TWO

WQ 3.36 24.5215 minutes

NumberOfArrivalPerHour = 31

NumberOfArrivalPerMinute 0.516666667 Customers/minute

TotalWaitingTimePerCustomer = 12.66944167 minutes

TotalCostForWaiting = TotalWaitingTimePerCustomer x cost per minute (1)

Let the cost of waiting per hour = 500 Naira

then, the cost of waiting per minute = 8.33333 Naira per minute from (1), TotalCostForWaiting = 105.579 Naira per minute

This means that the cost of unhapiness of a cutomer in thequeue per minute is ₦102.579 (or)

6334.72 Naira per hour

If the customer spent two hours in the Bank, TotalCostForWaiting for 2 hours = 12669.4

If the customer that encounters the above delay in the queue earns 400

Naira per hour in his place of work, then the

Salary Cost = hours waited in the Bank x Total cost for waiting

= 2 hour x amount earn in place of work per hour (if a customer waited in the bank for 2 hours during working hours) is:

Single Teller cost analysis

800 Naira for the two hours

Total Cost or Loss on the Customer's side = 13469.4 (₦13469.4)

TELLER THREE

WQ 3.36 0.57635 minutes

NumberOfArrivalPerHour = 35

NumberOfArrivalPerMinute 0.583333333 Customers/minute

TotalWaitingTimePerCustomer = 0.336204167 minutes

TotalCostForWaiting = TotalWaitingTimePerCustomer x cost per minute (1)

Let the cost of waiting per hour = 500 Naira

then, the cost of waiting per minute = 8.33333 Naira per minute

from (1), TotalCostForWaiting = 2.8017 Naira per minute

This means that the cost of unhapiness of a cutomer in thequeue per minute is ₦159.797 (or)

168.102 Naira per hour

If the customer spent two hours in the Bank, TotalCostForWaiting for 2 hours = 336.204

If the customer that encounters the above delay in the queue earns 400

Naira per hour in his place of work, then the

Single Teller cost Analysis

Salary Cost = hours waited in the Bank x Total cost for waiting

= 2 hour x amount earn in place of work per hour (if a customer waited in the

bank for 2 hours during working hours) is:

800 Naira for the three hours

Total Cost or Loss on the Customer's side = 1136.2 (₦1136.2)

TELLER FOUR

WQ 3.36 1.24795 minutes

NumberOfArrivalPerHour = 29

NumberOfArrivalPerMinute 0.483333333 Customers/minute

TotalWaitingTimePerCustomer = 0.603175833 minutes

TotalCostForWaiting = TotalWaitingTimePerCustomer x cost per minute (1)

Let the cost of waiting per hour = 500 Naira

then, the cost of waiting per minute = 8.33333 Naira per minute

from (1), TotalCostForWaiting = 5.02647 Naira per minute

This means that the cost of unhapiness of a cutomer in thequeue per minute is ₦5.02647 (or)

301.588 Naira per hour

110 Single Teller cost analysis

If the customer spent three hours in the Bank, TotalCostForWaiting for 2 hours = 603.176

If the customer that encounters the above delay in the queue earns 400

Naira per hour in his place of work, then the

Salary Cost = hours waited in the Bank x Total cost for waiting

= 2 hour x amount earn in place of work per hour (if a customer waited in the

bank for 2 hours during working hours) is:

800 Naira for the two hours

Total Cost or Loss on the Customer's side = 1403.18 (₦1403.18)

Figure 4.23: Cost Analysis or Unhappiness Cost for the First Bank customer for each Teller

The following table is a summary of the cost of waiting for the customers from the above cost Analysis for the First Bank and the corresponding graph.

Table 4.12: Summary of Wq, Unhappiness cost salary and total cost for the First Bank

Tellers Wq

(minutes) Unhappiness Cost for 2 hours (N) Salary

Cost (N) Total

Cost (N)

Teller One	6.5964	4177.72	800.00	4977.72
Teller Two	24.5215	12669.40	800.00	13469.40
Teller Three	0.57635	336.24	800.00	1136.20
Teller Four	1.24795	603.176	800.00	1403.18

Figure 4.24: The corresponding Graph of cost of waiting for First Bank Customers

From Table 4.10, Teller Three and Teller Four has very high total cost compared to tellers One and Two, because their customers average waiting time was very high while Teller. This highest very high total cost can be reduced by enforcing the factors that will reduce the waiting time and the total cost on the concerned Tellers.

4.6 Discussion of Results for the First Bank

The spreadsheet analysis and from simulation (Table 4.9, Figures 4.19, 4.22 and 4.23) of the First Bank revealed that Tellers 1, 3 and 4 had moderate waiting time with high values of turnaround and low values of total cost while Teller 2 had highest value 24 minutes of wait because of the long queue and consequently resulted in low values and highest value of total cost (Figure 4.5). higher customer unhappiness ((Table 4.9 and Figure 4.19, Table 4. and Figure 4.8). This type of Teller 2's higher waiting time is also evident during salary payment time and requires injecting all possible waiting reduction measures on the concerned Teller. The Bank Tellers (Servers) were utilized between 50 to 98%. Therefore, this type of higher waiting time and over utilization of the servers can be reduce by bringing more Teller (s) (Servers) into the Bank which will incur more cost for the service provider. The First Bank management should use the Java program developed (Figure 4.23) to check trade-offs between the increased costs of providing better service and the decreased waiting time costs of customers derived from providing that service before introducing new Teller in the system or other cost effective measures. Also, there here is need for First Bank to turn multi Tellers multi server system to single queue multi-tellers by calling customers in any of their single queue to one of the multi-tellers that his queue has become empty. This will help reduce customer wait time further for the Bank.

4.7 The Operating Characteristics of a Multi-Server or Servers When $p \leq 1$ or $p > 1$

It should be noted that the total service rate must be greater than the arrival rate, that is, $s\mu > \lambda$. For instance, in this case, $s = 3$, $\mu = 18$ and $\lambda = 45$, therefore, $s\mu = 3 \times 18 = 54$, implies $s\mu > \lambda$ or $54 > 45$, which means that the queue will not eventually grow infinitely large, and this is why we are able to obtain the graph as in Figure 4.24 which is a normal exponential graph. However, if s μ λ, the waiting line would eventually grow infinitely large giving (an inverted exponential graph) as in Figure 4.4. Therefore, before using the formulas, it was checked to be sure that $s\mu > \lambda$ and this is the condition used in the analysis of multi-server as shown in Data validation before the analysis is carried out for Multi-Tellers One to Three.

4.7.1 The Operating Characteristics of a Multi-Server or Servers When $p \leq 1$

The Multi server model

In the single-line, multi-server, single-phase model, customers form a single line and are served

by the first server available. In this model, there are s identical servers, the service time distribution for each server is exponential, and the mean service time is $1/\mu$. Using these conditions, the operating characteristics can be described with the following formulas:

s = the number of servers in the system

$p = \lambda / s\mu$ = the average utilization of the system

= the probability that no customers are in the system.

= the average number of customers waiting in line.

= the average time spent waiting in line

= the average time spent in the system, including service.

$L = \lambda W$ = the average number of customers in the service system

= the probability that n customers are in the system at a given time.

Instead of a single teller working in the bank, the ECO bank uses two identical tellers. It expects that customers will arrive at a rate of λ per hour or per minute, according to a Poisson distribution. The service rate for each of the two servers is μ customers per hour or per minute, with exponential service times. The following operating characteristics of the service system or ECO Bank can be calculated:

(a) The average utilization of the Teller

(b) The probability that there are no customers in the system

(c) The average number of customers waiting in line

(d) The average time a customer spends waiting in line

(e) The average time a customer spends in the system

(f) The average number of customers in the system.

The Solution is given below using equations (3.2), (3.13), (3.4), (3.6), (3.11), (3.8), and (3.15):-

(a) The Average utilization of the system: $p = \lambda / s\mu = 1.28/ (2x5.23) = 0.12237094$ 0r 12.243%

(b) The probability that there are no customers in the system (or in the Bank):

= [(1.28/5.23)0/0! + (1.28/5.23)1/1! + (1.28/5.23)2/2! ((1/(1- 0.12237094))]-1

= 0.78217722 = 78.217722% of having no customers in the bank

(c) The average number of customers waiting in line:

= (0.78217722 (1.28/5.23)2x 0.12237094)/ 2! x(1 - 0.12237094)2 = 0.0037217571

Customers.

(d) The average time a customer spends waiting in line:

= 0.0037217571/1.28 = 0.00290762 minutes

(e) The average time a customer spends in the system:

= 0.00290762 + (1/5.23) = 0.19411220 minutes.

(f) The average number of customers in the system:

L = λ W = 1.28 (0.19411220 = 0.24846361 customers.

(g) Pn= as calculated in 4.7.3, where n is the number of customers in the system.

The following figure shows a spreadsheet solution of this problem. The spreadsheet formulas are a direct implementation of the multiple-server formulas for performance measures. Because of the complexity of the P0 calculation, the columns N to Q break this computation down. Then, the formula in cell H23 of the spreadsheet looks up the value from column Q corresponding to the number of servers. The spreadsheet shown in Figure 4.15 will work for up to a 100-server system. Key formulas were used in the Spreadsheet to obtain the result in the spreadsheet.

Figure 4.25: Probabilities of certain numbers of customers in the system when $p < 1$ for multi-server

The above analysis resulted in a Stable queue because it resulted to a normal exponential graph in Figure 4.24 and as well resulted in positive wait values and probability of number of customers in this section because the Teller was not over utilized and there were no significant surges in demand. Unlike in section Figure 4.6 which resulted in unstable queue.

4.7.2 The Operating Characteristics of a Multi-Server or Servers of the ECO Bank When $p > 1$

The data collected did not satisfy the condition $p > 1$, it only satisfied $p < 1$ and the analysis for this condition is done as follows for the three Multi-Tellers of the ECO bank.

4.7.3 The Operating Characteristics of the Three Multi-server or Servers of the ECO Bank

When $p < 1$

Data validation for Table 4.7, the following is the probability of number of 15 customers in the System which showed a stable Queue for the analysis because the probabilities of number of customers in the system is positive with a positive exponential graph, which showed a stable queue for the analysis:

To get the probabilities of number of customers in the system for the multi-tellers of the ECO bank, first calculate Po using MS Excel as shown below:

Figure 4.26: Calculation of Po used to get the result for Figure 4.27

The result obtained in Figure 4.25 was used to get the following probabilities of number of customers in the system

Figure 4.27: Probability of number of 15 customers in the system used to validate the stability of the queue before analysis

Figure 4.28: The corresponding positive graph for Figure 4.27

From Table 4.7, the following is the Analysis for Multi-Server One of the ECO bank when p < 1:

Figure 4.29: Analysis of Multi-Teller One data of the ECO Bank

Data validation for Table 4.8, the following is the probability of number of 15 customers in the System which showed a stable Queue for the analysis because the probabilities of number of customers in the system is positive with a positive exponential graph, which showed a staable queue for the analysis:

To get the probabilities of number of customers in the system for the multi-tellers of the ECO bank, we

first calculate Po using Excel as shown below:

Figure 4.30: Calculation of Po used to get the result for Figure 4.31

The result obtained in Figure 4.30 was used to get the following probabilities of number of customers in the system

Figure 4.31: Probability of number of 15 customers in the system used to validate the stability of the queue before analysis.

Figure 4.32: The corresponding graph for Figure 4.31

From Table 4.8, the following is the Analysis for Multi-Server Two of the ECO Bank when p < 1:

Figure 4.33: Analysis of Multi-Teller Two data of the ECO Bank

Data validation for Table 4.9, the following is the probability of number of 15 customers in the System which showed a stable Queue for the analysis because the probabilities of number of customers in the system is positive with a positive exponential graph, which showed a staable queue for the analysis:

To get the probabilities of number of customers in the system for the multi-tellers of the ECO bank, we

first calculate Po using Excel as shown below:

Figure 4.34: Calculation of Po used to get the result for Figure 4.35

The result obtained in Figure 4.33 was used to get the following probabilities of number of customers in the system

Figure 4.35: Probability of number of 15 customers in the system used to validate the stability of the queue before analysis

Figure 4.36: The corresponding graph for Figure 4.35

From Table 4.8, the following is Analysis for Multi-Server Three of the ECO Bank when $p < 1$:

Figure 4.37: Analysis of Multi-Teller Three data of the ECO Bank

Table 4.13: Summary of Performance Measures for the three Multi-servers of the ECO Bank when $p < 1$ (Unit 4.7.3)

Multi-Tellers	λ	μ	P	P0	Lq	Wq	W	P15
1	2.45	4.53	0.27042	0.57428	0.04267	0.01742	0.23817	2.18E-08
2	4.48	6.70	0.33432	0.49888	0.08414	0.01878	0.16804	2.42E-08
3	3.69	5.56	0.33183	0.50168	0.08212	0.02225	0.20211	2.18E-08

Figure 4.38: The corresponding Graph of Performance measures for Table 4.13

The following is one of the data entered into the simulation software which was used to verify the analysis results in unit 4.7.3 before developing the custom software for Simulating performance measure of the Two banks at a run after given the necessary data (Figure 4.41).

Figure 4.39: Data entered for M/M/2 analysis and simulation [Yih-Long (2015), Microsoft (2015)]

Figure 4.40: Performance of ECO bank for M/M/2 analysis, cost and from simulation (Yih-Long, 2015), (Microsoft, 2015)

Figure 4.41: One of the Java program outputs for viewing the performance measure of both the case study Banks and the opportunity cost between service provider and customers.

Findings and Discussion of results for the multi-server system analysis

One observational insight provided by comparing queuing models is that single queue with multiple (identical) servers performs better than each server having their own queue and that a single large pool of servers performs better than two or more smaller pools, even though there are the same number of servers in the system. This is the reason why the waiting time of Second Generation Bank (SGB) or ECO Bank that uses single queue with multiple (identical) servers performs better than each server having their own queue. For example, customer waiting times are smaller for SGB that uses single queue with multiple (identical) servers was lower than that of First Generation Bank (FGB) or First Bank where each server has their own queue and is an advantage to the customer. However, the server utilization SGB was found to be lower than that of FGB because of the network problem, which is a disadvantage to service organization.

4.7.4 Costs or Economics of Waiting Problem for the M/M/S Model

From (5) above, Average wait per hour (or per minute) = Average time a customer spent waiting in line, WQ = 0.0174164 minutes.

Number of Arrivals per hour = 25 customers per hour = 25/60 =0.416666667 customers/minute

Total waiting time per customer = WQ x Number of Arrivals per hour

= 0.0174164 x 0.416666667 = 0.007151666672388 hours

Total Cost for waiting = Total waiting time per customer x cost per hour

Let the cost of waiting per hour = ₦500.00, then,

The cost of waiting per minute = 500/60 = 8.3333333 Naira per minute

Total Cost for waiting per minute = 0.007151666672388 x 8.3333333 hours = ₦0.0595972220315111109204

= the cost of unhappiness of the customer in the Bank.

This means that unhappiness cost of customer per minute = 0.05960 = 0.05960 x 60 = ₦3.576 per hour

If the customer waited in the Bank for two hours, the unhappiness cost for two hours = 3.576 x 2 = ₦7.152

If the customer that encounter this delay earns ₦400.00 per hour in his place of work, then the Salary cost = hour(s) waited in the bank x Total Cost for waiting = 2 hours x 400 = ₦800.00

Total cost (Unhappiness cost) = Salary cost + Waiting cost (or unhappiness cost) = ₦800.00 +

₦7.152 = ₦807.152 for two hours.

The following is Cost Analysis or Unhappiness Cost for the ECO Bank customers for each Multi-Teller:

MULTI-TELLER ONE

WQ = 3.36 0.0174164 minutes (= Average time a customer spent waiting in line)

NumberOfArrivalPerHour = 25 customers/hour

NumberOfArrivalPerMinute = 0.416666667 Customers/minute

TotalWaitingTimePerCustomer = 0.007256833 minutes

TotalCostForWaiting = TotalWaitingTimePerCustomer x cost per minute (1)

Let the cost of waiting per hour = 500 Naira

then, the cost of waiting per minute = 8.333333333 Naira per minute

from (1), TotalCostForWaiting = 0.060473611 Naira per minute

This means that the cost of unhapiness of a cutomer in the queue per minute is 0.06047 or

3.628416667 Naira per hour.

If the customer spent two hours in the Bank, TotalCostForWaiting for 2 hours = 7.25683333

Naira is the cost of unhapinnes the customer waited in the Bank for the two hours

If the customer that encounters the above delay in the queue earns 400

Naira per hour in his place of work, then the

Salary Cost = hours waited in the Bank x Total cost for waiting

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= 2 hours x amount earn in place of work per hour (if a customer waited in the

bank for 2 hours during working hours) is:

800 Naira for the two hours

Total Cost or Loss on the Customer's side = 807.2568333 (₦807.257)

MULTI-TELLER TWO

WQ 3.36 0.0029067 minutes

NumberOfArrivalPerHour = 34

NumberOfArrivalPerMinute 0.566666667 Customers/minute

TotalWaitingTimePerCustomer = 0.00164713 minutes

TotalCostForWaiting = TotalWaitingTimePerCustomer x cost per minute (1)

Let the cost of waiting per hour = 500 Naira

then, the cost of waiting per minute = 8.333333333 Naira per minute

from (1), TotalCostForWaiting = 0.013726083 Naira per minute

This means that the cost of unhapiness of a cutomer in thequeue per minute is ₦0.01373 or

0.823565 Naira per hour

If the customer spent two hours in the Bank, TotalCostForWaiting for 2 hours = 1.64713

If the customer that encounters the above delay in the queue earns 400

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Naira per hour in his place of work, then the

Salary Cost = hours waited in the Bank x Total cost for waiting

= 2 hour x amount earn in place of work per hour (if a customer waited in the

bank for 2 hours during working hours) is:

800 Naira for the two hours

Total Cost or Loss on the Customer's side = 801.64713 (₦801.647)

MULTI-TELLER THREE

WQ 3.36 0.022553 minutes

NumberOfArrivalPerHour = 15

NumberOfArrivalPerMinute 0.25 Customers/minute

TotalWaitingTimePerCustomer = 0.00563825 minutes

TotalCostForWaiting = TotalWaitingTimePerCustomer x cost per minute (1)

Let the cost of waiting per hour = 500 Naira

then, the cost of waiting per minute = 8.333333333 Naira per minute

from (1), TotalCostForWaiting = 0.046985417 Naira per minute

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This means that the cost of unhapiness of a cutomer in thequeue per minute is ₦0.04699 or

2.819125 Naira per hour

If the customer spent three hours in the Bank, TotalCostForWaiting for 2 hours = 5.63825

If the customer that encounters the above delay in the queue earns 400

Naira per hour in his place of work, then the

Salary Cost = hours waited in the Bank x Total cost for waiting

= 2 hour x amount earn in place of work per hour (if a customer waited in the

bank for 2 hours during working hours) is:

800 Naira for the three hours

Total Cost or Loss on the Customer's side = 805.63825 (₦805.638)

Figure 4.42: Cost Analysis or Unhappiness Cost for the ECO Bank customer for each Multi-Teller

From the unit 4.7.4, the following Table is a summary of the time a customer waited in the queue (Wq), Unhappiness cost for two hours, the salary cost and Total Cost for Multi-server system and their corresponding graph patterns: (Figure 4.43)

Table 4.14: Summary of customers waiting time in the queue, unhappiness and total cost for Multi – Tellers of the ECO Bank.

Tellers	Wq	Unhappiness Cost for 2 hours (₦)	Salary Cost (₦)	TotalCost (₦)
Multi-Teller One	0.01742	7.25683	800	807.257
Multi-Teller Two	0.00291	1.64713	800	801.647
Multi-Teller-Three	0.02255	5.63825	800	805.638

Figure 4.43: The corresponding Figure for Table 4.14

4.8 Discussion of Results for the ECO Bank

The spreadsheet analysis (Table 4.13, Figure 4.38) and from simulation (Figure 4.40 and Figure 4.41) of the ECO Bank revealed that customers waited for a very short time for all the three multi-tellers compared to that of the single queue single Tellers of the First Bank and consequently resulted to lower turnarounds time values (Figure 4.6) and lower customer unhappiness cost (Table 4.14 and Figure 4.43), but the servers were underutilized (27 to 33%). The ECO Bank should ask their multi-tellers to improve on their turnaround in order to have a further reduction of waiting time and customer unhappiness cost. They do not need to increase server for now, but the management can also use the Java program developed (Figure 4.41) to carry out performance check trade-offs between the increased costs of providing better service and decreased waiting time costs of customers derived from providing that service before introducing a new Teller in the system. This will help both service provider and customers and as well improve efficiency and save cost.

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATION

5.1 Summary

Data was collected from case study banks for two weeks and one week respectively. The data was analyzed using spreadsheet (MS-Excel) and the appropriate queuing formula and results were obtained and also performance measures and cost analysis were also obtained from simulation using simulation program such as Win SQB. The analysis revealed how long customers waited in the two cases studies and Java program was developed to carry out performance check trade-offs between the increased costs of providing better service and decreased waiting time costs of customers derived from providing that service before introducing a new Teller in the system.This will help reduce wait time and will help both service provider and customers and as well improve efficiency and save cost if the gray areas of the discussion of results are considered judiciously in addition to the recommendations given.

5.2 Conclusion

The case study Banks and similar service systems should study the discussion of results of this paper and other gray areas uncovered from the analysis and use it to improve their waiting problems and achieve increased efficiency.

5.3 Recommendation

(1) Banks should adopt multi-teller – single line system in order to have reduced customer waiting time, cost of waiting and get quick service to customers, good utilization of servers as well as prevent or reduce customer reneging, jockeying and balking.

(2) Use complaint barriers or railings to control people in a clean single line and at least one security guard to help prevent customers from reneging. Use rope, or rail that do not exceed a height of 27” from the floor surface when measured and 12” or more away from a vertical post surface. This will help ensure that blind and visually impaired people who employ a cane or guided to detect the protruding object without a collision.

(3) The causes of long waiting line such as poor server efficiency, inadequate system capacity, and/or significant surges in demand should be avoided, for example:

(i) When there is long wait, the management should provide adequate system capacity such as many cashiers open, that is, additional tellers should be added so that they can be more than 4 Teller operating per day, in the case of First Bank and more than two in the case of ECO bank. Organization should have excess capacity (such as many Tellers) on standby.

(ii) Significant surges in demand can be avoided by ensuring that there is relatively constant customer arrival (no major surges in demand). These significant surges in demand can be controlled or reduced by giving bonus for transactions performed on non-busy days and weekends (Saturdays and Sundays).

(iii) The management of each Bank should make sure that their servers are well utilized between 80 – 90% and not allowed to exceed 100% and as well work honestly to give reasonable response time and turnarounds.

(4) The management of each Bank shouldcreate enough lowered section of the counter or service desk for customers to write cheques, sign documents, etc.

(5) Customers should be thoroughly enlightened about all queue management rules and obey them judiciously when they are in the Bank for service. They should not renege, balk or jokey unless officially ordered to do so by the Bank officials when necessary. Moreover, they should be enlightened on all the aspects that can cause delay in their service.

(6) Banks should adopt other priority rules, like SJF, LJF, etc., for instance, deposit up to certain amount should go to bulk room or special deposit collector ATMs.

(7) System Performance should be changed by Changing Server efficiency, by adding resources to each phase (e.g., bagger helping a checker at the grocery store) and by using technology (e.g. price scanners) to improve efficiency, especially for products service centers. This price scanner (or barcode reader) allows the cashier to move items across the scanning area quickly, without having to worry about which way the barcode is facing. Once the UPC is read, the price of the item is retrieved from the store's database and inventory figures are adjusted.

System performance can also be changed by changing the number of lines, for example:

(i) Reduce multiple lines to single queue to avoid jockeying.

(ii) Dedicate specific servers to specific transactions.

(8) Provide circulation space and suitably designed seats for people with mobility impairments to sit and rest if customers have to wait to receive a service, or to walk considerable distances within the premises, but if service system have small premises where there is not enough space to provide seating adjacent to queuing areas, it is important to make alternative provision for customers who cannot stand and wait. Any such arrangements should be made publicly known to avoid customers getting upset if they think other customers are queue jumping.

(9) The management of the Bank should comply with Disability Discrimination Act (DDA), a requirement which is not properly paid attention to in the Nigerian constitution. Banks or service centers should provide an alternative pathway for people in wheelchairs so that they can have equal or superior access, and in this case the queue can have a narrower pathway than is normally required, but it is still important to use compliant posts, even if there is on offer an alternative pathway to people in wheelchairs.

(10) The management can use the thesis as a tool by which it can evaluate system performance and make decisions as to how to improve the performance while weighing performance against the costs to achieve that performance.

(11) Banks Tellers should not use the policy of collecting money first from depositing customers for paying the withdrawing customer instead they should start the business with enough money to pay any amount customers want to withdraw at any time.

(12) Tellers should not collect cheques from customers who were not legally in the queue.

5.4 Future Work

This thesis has carried out performance Simulation for banks and finance; however, any of the six types of simulation namely Proof, Discovery, Entertainment, Training, Education and Prediction or a combination of them or explicit simulation methodology can be carried out in future for the analysis of Multi-lines – multi-servers and Single line – Multi-servers for other industries that can benefit from simulation such as Healthcare and hospital management, Hospitality and hotel management, Supply chain and logistics, Warehousing and storage systems, Airports and aviation, Traffic and transportation systems, Restaurants and food services, Postal services, and IT systems, communication networks.

REFERENCES

Access Technology Group inc (2012), Learning Through Simulation, Simulation White Paper. Retrieved on September 2, 2012 from http:// http://www.atghome.com/resources/simulation_white_paper.pdf. pp.2.

ArnikaTripathi (2013), Simulation of queuing model, International Journal of Engineering science and innovative Technology (IJESIT), Volume 2, Issue 6, November 2013. ISSN2319-5967. Retrieved on 12 January, 2014 from, http://www.ijesit.com/Volume%202/Issue%206/IJESIT201306_24.pdf

Beasley J. E (2013), Queuing Theory, OR-Notes, Retrieved on May 9, 2013 from, http://people.brunel.ac.uk/~mastjjb/jeb/or/queue.html , pp.1.

Daniel B Mclaughlin and Julie M. Hays (2008),Healthcare Operations Management, An integrated approach to improving quality and efficiency, Chapter 10. pp. 14, 18-21,. Retrieved on May 5, 2013 from

Delia UNGUREANU, Francisc SISAK, Dominic Mircea KRISTALY, Sorin MORARU (2005), Simulation Modeling. Input Data Collection AndAnalysis, Sozopol, Bulgaria. Retrieved on September 11, 2012 from http://www.tu-sofia.bg/meetings/Elec/2005/pdf/Paper051-D_Ungureanu2.pdf. pp.1-3, 7.

Elegalam (1978) "Customer retention versus cost reduction technique" a paper presented at the Bankers forum held at Lagos, pg.9-10.

Ezeliora C. D,OgunohArinze Victor; UmehMaryroseNgozi, MbeledeoguNjide N (2014),

Analysis of queuing system using single line multi teller system: A case study of Shoprite plaza, Enugu State, Nigeria, International journal of scientific and Technology research, volume 3, Issue 3, March 2014, Retrieved on 28th December, 2014. fromhttp://www.ijstr.org/final-print/mar2014/Analysis-Of-Queuing-System-Using-Single-line-Multiple-Servers-System-a-Case-Study-Of-Shoprite-Plaza-Enugu-State-Nigeria.pdf

Gustavo Ceballos (2014), Queue Analysis at toll and Parking Exit plaza: a comparison between

multi-server queuing models and traffic simulation Retrieved in January 2015 from, http://data.ptvamerica.com/docs/VISSIMQueueAnalysis.pdf.

Hamid Khan and MusatyaBere (2014), Waiting lines: system simulation fosters company

performance

http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.90.8849&rep=rep1&type=pdf

Harry Perros (2012), Queuing Theory - A Primer. Retrieved on May 26, 2012) from, http://www4.ncsu.edu/~hp/SSME_QueueingTheory .pdf.pp. 23.

.

Helmut Hutter (2000), Queue theory, http://orms.pef.czu.cz/text/QueTeory/QueuingModels.html, pp.2, 3.

Jayshkumar J, Rajeshkumar M. Chaudhary,Dr. PrashantMakwana (2012),Analysis

of the Sales Checkout Operation in Big Bazar Using Queuing Simulation, International conference on management, humanity and economics (ICEMHE'2012).Phuket (Thailand). http://psrcentre.org/images/extraimages/812007.pdf

Kasum, A. S.Abdulraheem, A. and Olaniyi, T. A. (2006),Queue Efficiency In Nigeria Banks: A Comparative Analysis Of Old And New Generation Banks. Retrieved on August, 2012 from, http://www.unilorin.edu.ng/publications/askasum/QUEUE%20EFFICIENCY%20IN%20 NIGERIA%20BANKS%202.htm.pp.2-5.

Kendall, David G. (1953). "Stochastic Processes Occurring in the Theory of Queues and their

Analysis by the Method of the Imbedded Markov Chain". Annals of Mathematical Statistics 24 (3): 338–354. doi:10.1214/aoms/1177728975. JSTOR 2236285.

Kevin Gray (2012) The Legal Order of the Queue. Retrieved on June 20, 2012 from http://www.lse.ac.uk/collections/law/projects/techniquesofownership/tech-gray.pdf.pp.3, 4, 5.

Kotler, P. (1999).Marketing Management, the New Millennium Edition.

Lee, Alec Miller (1966). "A Problem of Standards of Service (Chapter 15)". Applied Queueing

Theory. New York: MacMillan. ISBN 0-333-04079-1.

LukwiyaBenard Osteen (2011), The effect of automated teller machine use on customer satisfaction in banking (a case study of centenary rural development bank, gulubank branch). p2.

Mandia S. O (2012), Design of Queuing System. Retrieved on October 1, 2012 from http://www.mie.utoronto.ca/undergrad/thesis-catalog/files/117.pdf.pp.1, 9-12.

Mathias Dharmawirya and Erwin Adi (2011),International conference on management and

artificial intelligence IPEDR Vol. 6 (2011) IACSIT Press, Bali, Indonesia. http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2014470

Microsoft (2015), Microsoft Corporation, 11 Times Square New York, NY 10036, Retrieved in

January 2015 fromhttp://www.microsoft.com/about/companyinformation/usaoffices/nymetro/en/us/newyork.aspx

MEN 170 (2010), Systems Modeling and Simulation.QUT, School of Mechanical, Manufacturing, and Medical Engineering. Retrieved on June 20, 2012 from http://www.cs.utexas.edu/users/browne/cs380ns2003/Papers/SimpleQueuingModelspdf.pdf. pp.1-2, 4, 5.

Mohammad ShyfurRahmanChowdhury, Mohammad ToufiqurRahman and Mohammad RokibulKabir(2014),

Solving Of Waiting Lines Models in the Bank Using Queuing Theory Model the Practice Case: Islami Bank Bangladesh Limited, Chawkbazar Branch, Chittagong from http://www.iosrjournals.org/iosr-jbm/papers/Vol10-issue1/D01012229.pdf

Munir B. Sayyadi et al (2010),Novel Approach to Improve QoS of a Multiple Server Queue

fromhttp://www.scirp.org/journal/PaperInformation.aspx?PaperID=1179#.VN7IyGe_gxo

OgunohArinze V., Ezeliora C. Daniel, Okoye Patrick U., UmehMaryrose N (2014),

Simulation of waiting line system using single line multiple

channelmodel: A case study of NNPC mega stations in Owerri and Enugu states, Nigeria. International journal of scientific Engineering and Research (IJSER) retrieved in January 2015 from, http://www.ijser.in/archives/v2i7/SjIwMTMyNjM=.pdf. www.isjer, ISSN(Online):2347-3878, Volume 2, issue 7, July 2004.

Oppres3 (2014), Queuing theory, Retrived on in January, 2015.

fromhttp://www.slideshare.net/benghuid/queueing-theory-7085316 pp.66.

Prof Dimitri P. Bertsekas (2002), Traffic behaviour and queuing in a QoS environment, OPNET

Technologies, Inc. Retrived on 1/1/2015 from, http://web.mit.edu/dimitrib/www/OPNET_Full_Presentation.ppt .pp.6.

Raid Al-Aomar (2010), Simulating service systems, Industrial Engineering Department

Jordan University of Science & Technology Irbid 22110, JORDAN.Retrieved on June 20, 2013 from, http://www.intechopen.com/books/discrete-event-simulations/simulating-service-systems .pp.16.

R. Dan Reid & Nada R. Sanders (2012), Operations Management, 4th Edition © Wiley 2010,

Retrieved on September 7 & 22 2012). http://courses.csusm.edu/ba662jh/docs/QueuingModels.pdf. pp.6,

Sen, Rathindra P. (2010). Operations Research: Algorithms and Applications. Prentice-Hall of

India. p. 518.ISBN 81-203-3930-4.

Sturgul, John R. (2000), Mine design: examples using simulation.SME.pvi. ISBN 087335-181-9.

Retrieved on August 8, 2013 from, http://www.amazon.com/Mine-Design-Examples-Using Simulation/dp/0873351819#reader_0873351819 . pp.6. 13-14, 19-20, 23-24.

Sturgul, John R. (2000). Mine design: examples using simulation. SME.p. vi. ISBN 0-87335-181-9

Supplement D (2012), Waiting line model. Retrieved on September 7 & 22, 2012 from, http://courses.csusm.edu/ba662jh/docs/QueuingModels.pdf .pp.D6 – D13.

Syed Shujauddin Sameer (2014), Simulation: Analysis of Single Server Queuing Model,

Department of Computer Engineering, King Khaled University, Abha, Saudi Arabia, International Journal on Information Theory (IJIT), Vol. 3, No. 3, 2014 Retrieved in January 2015 from, http://airccse.org/journal/ijit/papers/3314ijit05.pdf .

Taha, Hamdy A. (1968). Operations research: an introduction (Preliminary ed.).

Tracy Bradley Maples (Spring 2003), CECS 340 Discrete Event Systems Modeling and Simulation, Chapter 3, Single Server Queue exemple. Retrieved on December 8, 2011 fromhttp:// p.4-8.

Vasumathi.AandDhanavanthan P (2010),

Application of Simulation Technique in Queuing Model for ATM Facility, International Journal Of Applied Engineering Research, Dindigul Volume 1, No 3, 2010. ISSN 09764259.Retrieved on October 20, 2012 from, http://www.ipublishing.co.in/jarvol1no12010/EIJAER2015.pdf. pp.471, 475 -476 .

VHT (2013), Queue Management Strategies, www.virtual hold.com/white papers/VHT_Queue Management - Strategies. 3875 Embassy Pkwy. • Suite 350 Akron, OH 44333, www.virtualhold.com. Retrieved on June 18, 2013 from, http://www.virtualhold.com/WhitePapers/VHT_Queue-Management-Strategies.pdf.pp.1.

Wallace J. Hopp (2013), Single Server Queuing Models, Retrieved from

http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.132.863&rep=rep1&type=pdf , pp.1. 11/8/2013.

Yih-Long (2015), WinQSB Free download software (free trial), Retrieved in January 2015 from,

https://winqsb.jaleco.com/

APPENDIX

```
/*
 * To change this template, choose Tools | Templates
 * and open the template in the editor.
 */

/*
 * NewJFrame.java
 *
 * Created on Jan 26, 2015, 10:24:00 AM
 */

package mal_adamu_1;

importjava.awt.event.ActionEvent;
importjavax.swing.JOptionPane;

/**
 *
 * @author User
 */
public class NewJFrame extends javax.swing.JFrame {

    /** Creates new form NewJFrame */
```

```
publicNewJFrame() {
initComponents();
jTextField1.setText("1");
jTextField2.setText("2");
jTextField3.setText("15");
jTextField4.setText("2.45");
jTextField5.setText("4.53");
jTextField6.setText("2");
jTextField7.setText("400");
jTextField8.setText("25");
jTextField9.setText("500");
jTextField10.setText("23000000");
jTextField11.setText("345000");
jTextField12.setText("120000");
jTextField13.setText("155000");
jTextField14.setText("23000");
jTextField15.setText("0");
    }

    /** This method is called from within the constructor to
     * initialize the form.
     * WARNING: Do NOT modify this code. The content of this method is
     * always regenerated by the Form Editor.
     */
@SuppressWarnings("unchecked")
    // <editor-fold defaultstate="collapsed" desc="Generated Code">
private void initComponents() {
```

```
jLabel1 = new javax.swing.JLabel();
jTextField1 = new javax.swing.JTextField();
jLabel2 = new javax.swing.JLabel();
jTextField2 = new javax.swing.JTextField();
jLabel3 = new javax.swing.JLabel();
jTextField3 = new javax.swing.JTextField();
jButton1 = new javax.swing.JButton();
jTextField4 = new javax.swing.JTextField();
jLabel4 = new javax.swing.JLabel();
jLabel5 = new javax.swing.JLabel();
jTextField5 = new javax.swing.JTextField();
jScrollPane1 = new javax.swing.JScrollPane();
jTable1 = new javax.swing.JTable();
jLabel6 = new javax.swing.JLabel();
jTextField6 = new javax.swing.JTextField();
jLabel7 = new javax.swing.JLabel();
jTextField7 = new javax.swing.JTextField();
jLabel8 = new javax.swing.JLabel();
jLabel9 = new javax.swing.JLabel();
jTextField8 = new javax.swing.JTextField();
jTextField9 = new javax.swing.JTextField();
jLabel10 = new javax.swing.JLabel();
jTextField10 = new javax.swing.JTextField();
jLabel11 = new javax.swing.JLabel();
jTextField11 = new javax.swing.JTextField();
jLabel12 = new javax.swing.JLabel();
```

```
        jTextField12 = new javax.swing.JTextField();
        jLabel13 = new javax.swing.JLabel();
        jTextField13 = new javax.swing.JTextField();
        jLabel14 = new javax.swing.JLabel();
        jTextField14 = new javax.swing.JTextField();
        jLabel15 = new javax.swing.JLabel();
        jTextField15 = new javax.swing.JTextField();

setDefaultCloseOperation(javax.swing.WindowConstants.EXIT_ON_CLOSE);

jLabel1.setText("Number of Servers for Single Queue:");

jTextField1.setText("jTextField1");

jLabel2.setText("Number of Servers for Multiple Queues:");

jTextField2.setText("jTextField2");

jLabel3.setText("Enter Mean number of customers:");

jTextField3.setText("jTextField3");

jButton1.setText("COMPUTE");
jButton1.addActionListener(new java.awt.event.ActionListener() {
public void actionPerformed(java.awt.event.ActionEventevt) {
jButton1ActionPerformed(evt);
            }
```

```
        });

jTextField4.setText("jTextField4");
jTextField4.addActionListener(new java.awt.event.ActionListener() {
public void actionPerformed(java.awt.event.ActionEventevt) {
jTextField4ActionPerformed(evt);
            }
        });

jLabel4.setText("Enter Mean arrival rate:");

jLabel5.setText("Enter mean service rate:");

jTextField5.setText("jTextField5");

jTable1.setModel(new javax.swing.table.DefaultTableModel(
new Object [][] {
                {null, null, null},
                {null, null, null},
                {null, null, null},
                {null, null, null},
                {null, null, null},
                {null, null, null},
                {null, null, null},
                {null, null, null},
                {null, null, null}
            },
```

```
new String [] {
        "Performance Measures", "Single Server", "Multiple Server"
    }
  ));
jScrollPane1.setViewportView(jTable1);

jLabel6.setText("Enter time spent waiting:");

jTextField6.setText("jTextField4");
jTextField6.addActionListener(new java.awt.event.ActionListener() {
public void actionPerformed(java.awt.event.ActionEventevt) {
jTextField6ActionPerformed(evt);
    }
  });

jLabel7.setText("Enter customer earn per hour:");

jTextField7.setText("jTextField5");

jLabel8.setText("Enter customers per hour:");

jLabel9.setText("Enter cost of waiting per hour:");

jTextField8.setText("jTextField4");
jTextField8.addActionListener(new java.awt.event.ActionListener() {
public void actionPerformed(java.awt.event.ActionEventevt) {
jTextField8ActionPerformed(evt);
```

```
        }
    });

jTextField9.setText("jTextField5");

jLabel10.setText("Construction expressed by interest:");

jTextField10.setText("jTextField1");

jLabel11.setText("Cost of operation: labor& materials:");

jTextField11.setText("jTextField1");

jLabel12.setText("Cost of maintenance & repair:");

jTextField12.setText("jTextField2");

jLabel13.setText("Other Costs such as insurance e.t.c.");

jTextField13.setText("jTextField2");

jLabel14.setText("annual profit after tax:");

jTextField14.setText("jTextField2");

jLabel15.setText("Value of d:");
```

```
jTextField15.setText("jTextField2");

javax.swing.GroupLayout layout = new javax.swing.GroupLayout(getContentPane());
getContentPane().setLayout(layout);
layout.setHorizontalGroup(
layout.createParallelGroup(javax.swing.GroupLayout.Alignment.LEADING)
        .addGroup(layout.createSequentialGroup()
            .addContainerGap()

.addGroup(layout.createParallelGroup(javax.swing.GroupLayout.Alignment.LEADING)
                .addGroup(layout.createSequentialGroup()
                    .addComponent(jScrollPane1, javax.swing.GroupLayout.DEFAULT_SIZE,
531, Short.MAX_VALUE)
                    .addContainerGap())
                .addGroup(layout.createSequentialGroup()

.addGroup(layout.createParallelGroup(javax.swing.GroupLayout.Alignment.LEADING)

.addGroup(layout.createParallelGroup(javax.swing.GroupLayout.Alignment.LEADING,
false)
                            .addComponent(jLabel1, javax.swing.GroupLayout.DEFAULT_SIZE,
javax.swing.GroupLayout.DEFAULT_SIZE, Short.MAX_VALUE)
                            .addComponent(jLabel2, javax.swing.GroupLayout.DEFAULT_SIZE,
javax.swing.GroupLayout.DEFAULT_SIZE, Short.MAX_VALUE))
                        .addComponent(jLabel3)
                        .addComponent(jLabel4)
                        .addComponent(jLabel5)
                        .addComponent(jLabel6)
                        .addComponent(jLabel7)
                        .addComponent(jLabel8)
```

```
.addComponent(jLabel9))
.addPreferredGap(javax.swing.LayoutStyle.ComponentPlacement.RELATED)

.addGroup(layout.createParallelGroup(javax.swing.GroupLayout.Alignment.LEADING,
false)
.addComponent(jTextField9)
.addComponent(jTextField8)
.addComponent(jTextField7)
.addComponent(jTextField6)
.addComponent(jTextField5)
```