PROBABILISTIC ASSESSMENT OF BS 449 DESIGN CRITERIA OF STEEL GIRDERS

BY

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DECLARATION

I hereby declare that this dissertation has been composed by myself and that it is a record by my own research work. It has not been accepted in any previous application for higher degree. All sources of information are specifically acknowledged by means of references.

A. Yusuf

CERTIFICATION

This project thesis entitled probabilistic Assessment of BS 449 Design Criteria of steel Girders by Abdulkadir Yusuf meets the regulation governing the award of the degree of M.Sc Structural Enginnering Ahmadu Bello University and is approved for its contribution to knowledge and literary presentation.

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DEDICATION

To all the good people of the world.

ACKNOWLEDGEMENT

I wish to express my sincere gratitude to my friends, lecturers and fellow compatriots who have helped in no small way up to the successful completion of this work.

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ABSTRACT

The BS 449 design requirements for plate girders under uniformly distributed load (UDL) and point load were reviewed. The design equations were compared with test results obtained from literature. The comparison showed that BS 449 gives a reasonable prediction of the capacity of plate girders although there are cases of under and over estimation of this capacity. Since there are observable scatter in experimental results, the study employed a probabilistic technique to examine the consistency of the design provisions. The First Order Reliability Method (FORM) was reviewed as a basis for the study in which the inherent uncertainties in material and geometric properties together with loading conditions were explicitly considered.

A well coded algorithm in a computer package called FORM5 was followed to compute the reliability levels implicit in the design provisions in respect of an I-section plate girder in bending and shear. The statistical values for the design variables were based on information contained in literature.

The results obtained showed that with all the basic variables assuming normal distribution, the implict reliability levels, ß, decrease from 4.96 to 2.08 for a span range of 11 to 14m in girders subjected to UDL in bending. For shear

condition β values ranged from 4.89 to 2.00 for span range of 14 to 23m. With the girder subjected to point load, β ranged from 4.90 to 2.15 for span varying from 12 to 19m while under the point load the shear condition showed a constant reliability level of about 4.69 for all the basic variables assuming normal distribution.

The sensitivity analysis conducted to examine the influences of the values of the basic design variables on the implied reliability levels showed that the most significant variables are the yield strength of steel and depth of the I-section. The implication of this is that during design of I-section plate girders, these two variables need be considered very seriously.

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NOTATION

B_f	:	Flange width
f[C]	:	Covariance Matrix
CDF, $F_R(r)$	:	Cumulative Distribution Function
COV,	:	Coefficient of Variation
D	:	Overall depth of section
E[X]	:	Expected value of a random variables
f_y	:	Yield Strength of steel
F_v	:	Shear stress
F	:	Failure region
g	:	Uniformity distributed dead load
$g(x_i)$	:	State function of basic variables
G	:	Point dead load
G_i	:	Gradient vector.
I	:	Moment of inertia of section
L	:	Span Length
M	:	safety margin
M_{max}	:	maximum bending moment
M^{Test}	:	Experimental moment capacity
$M_{Pred.}$	:	Predicted moment capacity
Pdf, $f_R(r)$	:	Probability density function
P_f	:	Probability of failure
P_u^{exp}	:	Experimental ultimate load capacity
P_u^{Th}	:	Predicted ultimate load capacity
q	:	Uniformly distributed live load
Q	:	Point live load

R	:	Resistance Capacity
S	:	Load effect
T_f	:	Thickness of flange
t_w	:	Thickness of web
T	:	Orthogonal transformation matrix
V	:	shear force
V_u^{exp}	:	Experimental ultimate shear capacity
V_u^{Th}	:	Predicted ultimate shear capacity
X_i	:	Basic Variables
X_i'	:	Reduced basic variate
Y_i	:	Uncorrelated variate
Z	:	Section Modulus
Z_i	:	Uncorrelated normalized variate
μ	:	Mean value
σ	:	Standard deviation
σ_{all}	:	Allowable bending stress
τ_{all}	:	Allowable shear stress
ρ	:	Correlation coefficient
$\in$	:	Belong to
Σ	:	Summation
α_i	:	direction cosine
λ	:	Value of Lagrange's multiplier
$[\lambda]$	:	Diagonal matrix of eigenvalues of [C']
β	:	Reliability Index
Φ	:	Normal distribution function
ϕ	:	Normal density function

CHAPTER ONE

INTRODUCTION

1.1 General

Quantitative methods of modeling and evaluation are tools of modern engineering. some of these methods have become quite elaborate and included sophisticated mathematical modeling and analysis, computer simulation and optimization techniques. However, irrespective of the level of sophistication in the models including experimental models, they are based on idealized assumptions or conditions. Hence, information derived from such quantitative models may or may not reflect the reality closely [1].

Structural problems are often nondeterministic, although structures are designed and built in spite of imperfect information and knowledge. In short, problems of structural design must be resolved in the face of uncertainty and as a consequence, risk (i.e the probability of adverse event or failure) is virtually unavoidable. Failure should be interpreted with respect to some predefined limit states, it may be excessive deflection, major cracking in concrete beam, total collapse of structure etc.

Therefore depending on the limit state under

consideration, the concept of failure is applicable to both safety and performance of structures. Properly therefore, the analysis of safety and performance must include consideration of risk as measured by 'probability of failure'. Thus modern structural reliability analysis is based on probability considerations.

1.2. Probability in Engineering

In the development of engineering designs, decisions are often required irrespective of the state of completeness and quality of information at our disposal. Thus the design must be formulated under conditions of uncertainty in the sense that the consequence of a given decision cannot be determined with complete confidence.

Aside from the fact that information must be inferred from similar (or even different) circumstances, or derived through modeling and thus may be in various degrees of imperfection, many problems in engineering involve natural processes and phenomena that are inherently random. The state of some of these phenomena are naturally indeterminate and thus cannot be described with definiteness. For these reasons decisions required in the process of engineering planning and design invariably must be made and are made under conditions of uncertainty.

The effect of inherent uncertainty on design and planning is important. However, quantification of such uncertainty and evaluation of its effect on the performance and design of engineering system should include concepts and methods of probability. In structural design, uncertainties result from random nature of the loading and the structural resistance, as well as imperfections in the load and resistance prediction models. The effect of such uncertainties is included in design through the use of safety factors which are based on engineering judgements and past experience with similar structures. Recognizing that safety involves a consideration of random variables (design parameters), it is generally accepted that the rational approach to the analysis of structural safety is through the use of probabilistic methods [2].

Thus, under conditions of uncertainty the design and planning of engineering systems involve risks and the formulation of the related decisions requires risk-benefit trade-off which are within the realm of applied probability. It is therefore glaring that the role of probability is quite pervasive in engineering. It ranges from the description of information to the development of bases for design and decision making.

1.3 Structural Code Objectives

The general aim of structural design is to produce safe and economic structure that fulfil its required purpose. Theoretical knowledge alone is not sufficient since theoretical analysis is based on idealized structure, examples are, representing members by their centre lines and assuming perfect pinned and fixed joints. The calculated forces and deflections are only approximations of the actual ones that cannot be always accurately estimated.

The designer uses his knowledge of structural mechanics, the codes of practice and practical experience to produce a safe solution of a given problem. BS 449, a code for the design of steel structure was drawn up by British Standard Institutions, including engineers from universities, colleges, professional bodies, government authorities and industries [3]. This code under consideration gives allowable stresses for safe design and construction. From the analysis of structural frames, the shears, axial loads and moments on the structural elements are determined. The sizes of elements are so chosen so that the allowable stresses, deflections e.t.c., given in the code are not exceeded and relevant requirements such as minimum thickness of material or maximum width/thickness are satisfied.

By and large, codes are compiled to meet up the following

requirements [4]:

a. The material selected must be suitable and adequate.

b. Proper load and service condition must be considered.

c. Computations and design must be made so that the structure and its detail possess the required strength and rigidity.

d. The workmanship must be good.

It follows then that code's specifications for structural design represent a compromise between theoretical considerations and practical requirements. Therefore, the specifications are not entirely rational and, for some structures or loading conditions, they may lead to more conservative results than for other. also the values of loads and allowable stresses are based on past experience and test data which have to be revised periodically to agree with latest findings.

1.4 Purpose and Scope

In BS 449 the influence of the inherent uncertainties associated with engineering problems are supposedly taken care of by the introduction of factors of safety as an allowance to neutralise their undesirable effects. As the derivation of these safety factors are not probabilistic in nature, it was conjectured that the consistency of its provisions are questionable and therefore investigated.

The term 'factor of safety' of structure acquires a rational meaning possibly by its correlation with probability of failure or survival (reliability) of the structure. This principle neither implies nor specifically advocates for a reduction of safety factor in relation to the convectional approach in which any risk of failure is considered inadmissible, although, as it is well known, failures of structures have occurred. The principle only attempts to place the concept of structural safety in the realm of physical reality in which absolutes, such as minimum strength values, do not exist and knowledge is not perfect. The risk or probability of failure is introduced as a measure in term of which the safety and reliability of various parts of a structural system can be defined, compared and uniform reliability or a safety of a complete system assured [5].

BS 449, the code on which this investigation is focussed is based on allowable stress design philosophy and as such it possesses the following drawbacks [6]:

a. In the design procedures, internal stress resultants based on elastic theory is compared with the specified allowable stress in the code without regards to uncertainties and consequence of failure.

b. The designer in his computation always compares his maximum stress, say for example, in a simply supported beam carrying a uniformly distributed load,

occurring at the midspan, with the allowable stress value in the code. This is a highly localized point. Most structures do not fail because of high local stress. The stresses may be redistributed by plastic flow to less highly stressed parts.

c. The allowable stress method does not give accurate picture of collapse conditions and cannot give reliable estimate of them. It only gives safe estimate and has a virtue of being simple, straight forward and easy to use.

d. Another disadvantage of this method is that it is not logically complete. It does not provide a framework for logical reasoning through which all the limiting conditions on a structure can be examined.

e. It is obvious that effects other than stresses have to be checked in a design, e.g deflections, crack control etc. Clauses relating to these effects are ambiguous without unifying philosophy.

f. There is too much emphasis on elastic stress and too little emphasis on limiting conditions controlling the structure in use.

A single span steel girder with a symmetrical I-section, supporting a uniformly distributed load and a point load were respectively investigated. The bending and shear conditions with strict compliance to provision of BS 449 were assessed

such that the failure probabilities (as a measure of safety indices) implicit in the design code for varying design parameters were computed. Sensitivity of the design equations to some relevant design parameters to ascertain their influence and relative importance was also investigated.

Digital computations became necessary to arrive at the objective of the study. Consequently, a well coded algorithm based on probability technique (i.e FORM 5) was adopted.

CHAPTER TWO

LITERATURE REVIEW

2.1 Structural Steel Failures

Only a negligibly small percentage of structures which are designed and built fail before the expiration of their projected life span. When failure occurs, a lot of time and efforts are spent on investigations to establish the causes of the accident. However, experience and judgement, which play such an important role in structural design receive little attention in technical literatures. If experience and judgement are to be of real benefits, the designer must learn from lessons of past failures.

Unfortunately, technical literature on the failures of the past is scarce. Understandably people do not wish to discuss their mistakes. Yet full discussions could be just as useful to engineering profession as discussions of great achievements. Thus some of the past failures reported in literature are hereby reviewed.

The failure of Tacoma Narrows suspension bridge on November 7, 1940 has been recorded as probably the greatest bridge failure in history [4]. The bridge was in service for about four months before its collapse and, from the first day

of operation, excessive vibration of the bridge was observed. Effort to arrest the alarming situation failed. The collapse occurred when a moderate 64km/hr wind caused vibrations which were gradually increasing in violence. Traffic flow over the bridge was stopped. About 1.5 hours later a predominantly twisting motion of the bridge broke the suspenders and the main girder of the centre buckled. The floor system fell from the bridge, followed shortly by the centre span girder.

The failure of the Point Pleasant bridge took 46 lives in 1968 as a result of brittle fracture [7]. This bridge was 39 years at that time and the cause was attributed to stress corrosion or corrosion fatigue of an eye-bar that was usually susceptible to fracture.

The Milford Haven box girder bridge collapse occurred during construction in 1970. The collapse occurred when a side span which was being cantilevered out from a completed anchor span hinged over its supporting concrete pier. It was also recorded that, on 10th October 1970, one of the side span of West Gate bridge in Melbourne over River Yarra failed killing 35 men.

These two collapses and two others involving similar box girder bridges in various parts of the world focused attention on many aspects of steel box girder bridge building practice.

It became clear that there was a need for reappraisal of contractual procedures, design values, research methods and recommendations of code practice. The Morrison Committee which was set up then looked into these matters and set a pace for a far reaching stride in bridge design practice. Encouragement was given thereby for research into loading, safety factors, aerodynamic effects and strength of welded members and connections. The results were incorporated into the latest code for steel bridges, BS 5400, with reliability method applications [7]. This shows that panic resulting from these collapses paid some dividends and it shows also that failures represent instances when engineer's conjectures are falsified and are central to growth of engineering knowledge.

The central roof beam of a single storey dance hall in Deventer, central Holland broke at midspan in September 1975 leading to the death of a teenager whilst one hundred and twenty people narrowly escaped serious injury [8]. The roof consisted of three 14-metre long I-section joist at 7.5 metre centres, overlaid with precast concrete planks. Each joist was fabricated from two 7-metre long beam joined together by welded plates. Report claimed that the fractured midspan weld was only forty-seven percent of the strength of the beam and that whilst the bolts in the midspan of the beam were of good quality, the nuts were not.

Also, a single length of beam had been proposed in the original planning application but its use would have been expensive and tortuous because of the site inaccessibility. Hence a suggestion that site welding which enables shorter beams to be transported was agreed to. However the design change and potential hazard was not communicated to the municipality's building inspector.

The failure of Dyan Ryan in Chicago in January 4, 1978 could have been a real tragedy [7]. It was averted however, because a crack extending almost completely through a steel bent that was supporting the overhead superstructure was observed from a train passing underneath. Prompt investigation by going to the site showed major cracks in three steel bents. A view of one of the cracks is shown in fig (2.1).

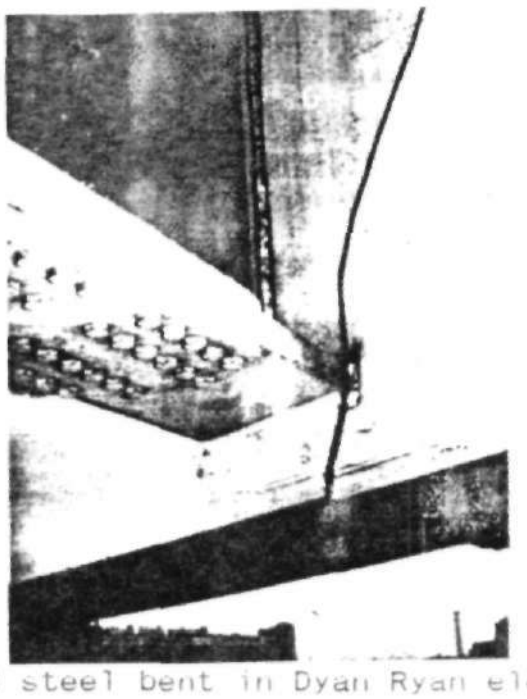


Fig. 2.1: Cracked steel bent in Dyan Ryan elevated structure

Chicago's Rosemond Horizon stadium collapse on 13th August 1979 during erection killing five men [8]. The United State Occupational Safety and Health Administration (OSHA) which investigated the accident, whilst accepting that other factors might have been involved, considered the erection procedure suspect. The body started its investigation by trying to establish the exact location of the one Hundred and twenty-five workers during the time of collapse in order to form the picture of the roof loading.

The report showed that adverse weather forced a section of the structure which will normally be completed in a day to be left over the weekend following the collapse. By the time the 30-metre long section of the fifteenth main beam was lifted and bolted into position on a Friday, rain was falling and the contractor had to fix only the main lateral beams. Thus the 30-metre long section without purlin was left for the weekend. It was felt that the beam was secured enough without purlin and so the crane supports were removed. At resumption of work on Monday morning the roof failed.

The collapse of "Sky bridge" during a tea dance in the Hyath Regency Hotel in Kansas city in July 17, 1981 claimed 114 lives [7]. It was a shock to the engineering profession as well as the general public. The collapse occurred after the structure was in use for about two years when people were

watching tea dance and was probably the maximum in service loading that the bridge had experienced. Investigation showed that a connection failed under 28 percent of the code strength. A view of the joint where the collapse initiated is shown in fig (2.2).



Fig. 2.2: Joint where collapse initiated in the Skybridge in the Hyatt Regency Hotel

The failure of a "blocked" expansion joint in the segmental post-tensioned Zilwanke bridge at Segwano, Michigan on August 28, 1982 caused a long delay in construction [7].

The cantilevered and partially completed structure twisted and deflected about 1.5metre when the failure of the 'blocked' joint occurred in the adjacent span as shown in fig (2.3).

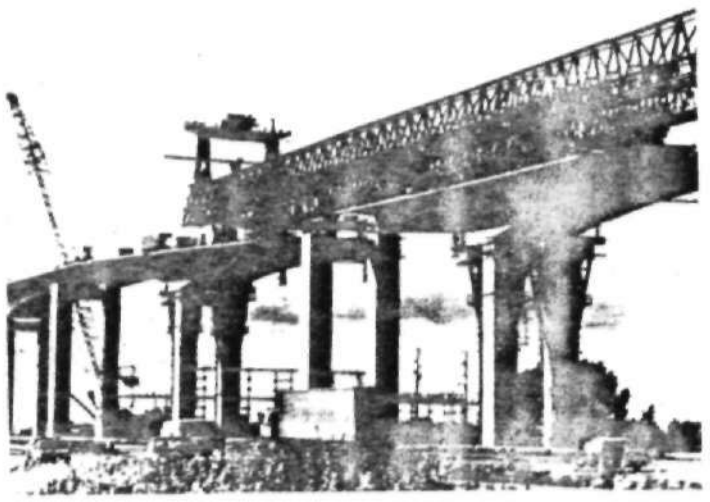


Fig. 2.3: Deflected Cantilever of the Zilwanke bridge

From some of the past failures stated above, it is clear that structural failures result into situations where our designed and built structures no longer serve their intended purpose. It also brought the inadequencies of the safety factor design concept to the fore.

2.2 Uncertainties in engineering

In engineering practice, variability, scatter, dispersion or generally, uncertainty in the quantity of interest is the rule rather than exception. Repeated samples of concrete strength specimens from the same source yield a set of numbers of which no two are the same. The total demand on an urban water supply system varies from day to day throughout the same month. The possible examples are endless. It is difficult in fact to think of quantities about which some level of uncertainty does not exist. Even the same physical quality e.g modulus of elasticity of a particular bar, when observed n times by n repetitions of n measuring conditions, the n observations will not be exactly equal. Therefore even a physical quantity must be considered as uncertain as long as it can only be evaluated by imperfect measurements.

Nevertheless this inherent uncertain qualities of engineering remain inside the traditional system of engineering. The system is customarily presented as being deterministic, to each quantity, it is presumed, a unique value can be assigned (e.g the yield strength of steel is 260 N/mm^2; 100 vehicles per hour make a left turn at that intersection e.t.c.). In turn structures, pipeworks, earthworks, e.t.c., are conventionally modelled to behave in a unique mechanistic way for a given set of model parameter values (dimension etc). The conclusion is that their responses are predicted with

certainty. Yet all of these assumptions of determinisms are inconsistent with the uncertainty observed in reality [9].

There are numerous sources of uncertainties in a structural design. The absolute safety of a structure cannot be guaranteed due to unpredictability of future loading conditions, our inability to obtain and express the in place material properties accurately, the use of simplified assumptions in predicting the behavior of the structure due to the loading under consideration, limitation of numerical method used, and human factors, eg errors, omission, etc. Consequently, uncertainties with regard to their source and nature are grouped into three classes [10].

a. Physical uncertainty: These are uncertainties associated with the design parameters (i.e the model parameters and input quantities). In this category, the inherent variability observed in these parameters could be described to some extent by probability distribution function based on observation of data collected over considerable length of time. Since physical variability can be quantified only by examining sample data and sample sizes that are limited by practical and economic considerations, some uncertainty must remain. This practical limit gives rise to statistical uncertainties.

b. Statistical uncertainty: This concerns inferences that

can be drawn from sample observations. Very large samples are required to be able to make reliable estimate of the parameter of any type of probability distribution considered appropriate. For a given set of data therefore, the distribution parameters may be considered as random variables, the uncertainty of which is dependent on the amount of data and any prior knowledge. This uncertainty evolves due to lack of information.

c. Model uncertainty: In structural design and analysis, mathematical model are used to predict desired consequences e.g., maximum deflections, stresses, shears, etc. These models are generally deterministic in form. They may be based on mechanics of the system or might be highly empirical. However accurate prediction of these consequences (i.e deflection, shear, etc) is not practical even if the basic variables are known exactly.

2.3 Applications of Reliability Methods.

Reliability method are virtually applicable to all fields of endeavours in which defined objectives or targets are set. On one side of these objectives, a favourable situation is realized while on the other side an undesirable situation prevails. In addition these objectives are dependents on inputs which cannot be predicted with certainty. Thus for any given realization of the set objectives is an associated measurable degree of desirability or unpleasantness such

situations are inexhaustible and are common in areas like, hydrology, mathematical statistics, control theory, classical hardware reliability, econometrics, engineering etc. attention will be focussed on engineering applications with emphasis on structural engineering.

In the case of hydrologic and water resources systems, the available supply of water (from all sources) relative to maximum demand or usage is of concern in planning and design of such a system. The available supply from different sources may be highly variable, whereas the actual usage may also fluctuate significantly, such that predictions in either cases may be subjected to significant uncertainties. Consequently in the design of a water supply system, the question of adequacy of water is a matter of the probability of available supply relatively to the demand.

In foundation designs, properties of soil material are inherently heterogenous, and natural earth stratification is not regular. Various material (e.g., clay, silt, sand, gravel or combination thereof) deposit with wide ranges of density, moisture content and other properties that affect the strength and compressibility of the deposit, consequently there is a likelihood of a risk of overestimating the actual bearing capacity of the soil deposit at a site. In view of this fact, the safety of a foundation design on the basis of such estimate

may not be assured with complete confidence, unless sufficient margin of safety is provided. On the other hand, an excessively large safety margin may yield an unnecessarily costly support system. Therefore the required safety margin for the design may be viewed as a problem involving trade-off between cost and tolerable risk or probability of failure.

In construction planning and management, reliability methods are applied because many factors in the construction industry are subjected to variability and uncertainty, some of which cannot be controlled. For example, the required durations of various activities in a construction project will depend on the availability of resources, including labour and equipment and their respective productivity, on weather conditions and on availability of material. None of these factors are completely predictable and thus duration of individual activities as well as the project duration cannot be estimated with precision or certainty. such durations must be described as random variables. Therefore in preparing a bid for a project, if conservative (or pessimistic) time estimate are used, the bid price may be too high, thus reducing the chances of winning the bid. Conversely, if the bid is based on optimistic estimate of the project time, the contractor may loose money in a successful bid. What degree of conservativeness the contractor should exercise to maximize his profit potential is based on a consideration of probability. The bid price may be based on a

target time corresponding to a specified probability of completion.

In design and planning of airport pavement, among the factors that have bearing on the design, the thickness of pavement system (consisting of several layers of subgrade base material and finished pavement), is one of the principal decision variables. In general useable life of a pavement will depend on the thickness of the system, the thicker the pavement system is, the longer its useful life will be. Indeed, thickness of pavement has its cost implication with respect to initial cost and subsequent maintenance. Hence, the thickness of a pavement system may be determined on the basis of a trade-off between high initial cost with low maintenance, versus low initial cost but high maintenance cost. For the purpose of such trade-off analysis, the relation between life of a pavement system and its thickness is required. However, the pavement life is also a function of other variables including drainage and moisture content, temperature ranges, density and degree of compaction of the subgrade. Since these factors are random, the life of a pavement cannot be determined with certainty. Hence the total cost associated with a given pavement thickness cannot be estimated with complete confidence and, any meaningful trade-off analysis, therefore would properly require concepts of probability as in reliability methods.

In structural engineering design, magnitudes of loads (e.g wind pressures) may be described with random variables strengths of structural material components are random and thus the resulting structural resistance will likewise be random. Consideration of risk or probability of failure is a better realistic measure of performance or nonperformance of structural engineering structures. To be specific, consider for example, the design of an offshore drilling tower, which is subjected to occasional hurricane forces. In such a case, we recognize that apart from the fact that the maximum wind effect during a hurricane is random, the occurrence of hurricane in a given coastal region is unpredictable. Hence, in determining the safety level for the design of the tower, the probability of occurrence of strong hurricane within the specified life of the structure must be considered as well as the survival probability of the structure during the hurricane.

As uncertainties exist in most areas of civil and structural Engineering and rational decisions cannot be made without modeling them and taking them into account. Many structural engineers are shielded from having to think about such problems at least when designing simple structures, because of the prescriptive and essentially deterministic nature of most codes of practice. This is an undesirable situation.

Finally in all the areas of applications of reliability

methods discussed above, the state of the system considered is a function of many variables (e.g design parameters), some of which are uncertain.

CHAPTER THREE

THEORETICAL FORMULATION OF RELIABILITY METHOD

3.1 Safety Index Concept

The problems of reliability of engineering systems can be essentially reduced to problem of supply to meet up a demand. In case of a structure, we are concerned with ensuring that the strength of the structure (supply) is enough to withstand the life time maximum applied load (demand). In reality of course, the determination of available supply as well as the determination of maximum demand are not simple problems. Estimation and predictions are invariably necessary for these purposes, implying that uncertainties are unavoidable for the simple reason that engineering information is invariably incomplete. In the light of such uncertainties the available supply and actual demand cannot be determined precisely. They may be described as belonging to the respective ranges (or populations) of possible supply and demand. In order to explicitly represent or reflect the significance of uncertainty, the available supply and required demand may be modeled as random variables [11].

Let a structural system with resistance capacity R (i.e the supply) be acted upon by the load effect S (i.e the demand). The objective of the reliability analysis is to

ensure the event (R > S) throughout the useful life of the system. Conversely, the probability of the complimentary event (R < S) is the corresponding measure of unreliability or failure.

Assume also that the necessary probability distributions of R and S are available (though rarely possible), that is, the cumulative probability function (CDF) or probability density function (PDF), $F_R(r)$ or $f_R(r)$ and $F_S(s)$ or $f_S(s)$ respectively.

The required failure probability is given by [12],

$$P_f = \int_0^{\infty} F_R(s)\, f_S(s)\, ds \qquad (3.1)$$

This is diagrammatically represented in Fig (3.1)

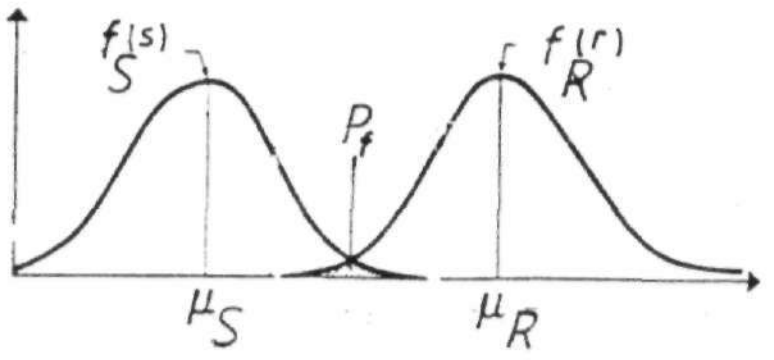


Fig. 3.1: Basic Resistance/Load Effect problem: $f_R(r)$, $f_S(s)$ representation

As portrayed in fig 3.1, the overlapping (shaded region) of the curves $f_R(r)$ and $f_S(s)$ is dictated by the positions of their respective means μ_R and μ_S and also the degree of dispersions of $f_R(r)$ and $f_S(s)$ called their respective standard deviations σ_R and σ_S. The R and S relationship could be formulated in terms of safety margin M, such that

$$M = R - S \tag{3.2}$$

As R and S are random variables, M is also random variable with corresponding PDF $f_M(m)$ and CDF $F_M(m)$. In this case failure is clearly the event $(M < 0)$ and thus

$$P_f = \int_{-\infty}^{0} f_M(m)\, dm = F_M(0) \tag{3.3}$$

Graphically, this is represented by the area under $f_M(m)$ less than zero as shown in fig 3.2.

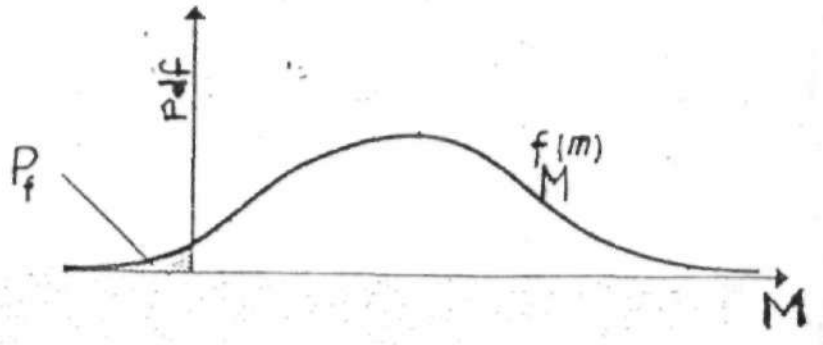


Fig 3.2: Distribution of Safety Margin

Now consider an idealized case where R is a normal random variable [12], $N(\mu_R, \sigma_R)$ and similarly S a normal random variable $N(\mu_S, \sigma_S)$ and therefore the mean of M

$$\mu_M = \mu_R - \mu_S \tag{3.4}$$

and for statistically independent R and S the variance is

$$\sigma_M^2 = \sigma_R^2 + \sigma_S^2 \tag{3.5}$$

If M is transformed into standard normal distribution i.e $(M - \mu_M)/\sigma_M$ is $N(0,1)$ then,

$$P_f = F_M(0) = \Phi(M - \mu_M)/\sigma_M$$
$$= 1 - \Phi(\mu_M/\sigma_M) \tag{3.6}$$

It is now shown that reliability is a function of the ratio $\beta = \mu_M/\sigma_M$ and is called reliability index or safety index, that is,

$$\beta = \frac{\mu_M}{\sigma_M} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \tag{3.7}$$

So that

$$P_f = \Phi(-\beta) \tag{3.8}$$

In which Φ represents the normal distribution function common in statistical tables.

It may be recognised from the outset that practical application and incomplete information will restrict most conventional design procedures to dealing with mean values and coefficient of variation [13]. On this basis, a design scheme can be based simply on requiring that some minimum safety

margin be maintained. In place of requiring that a calculated risk of failure must be below a specified probability P_f, it may be required that the average safety margin $\mu_M = \mu_R - \mu_S$, must lie at least β_1 (a specified number) standard deviations (of M = R-S) above zero. Thus the mean design resistance from equation (3.7) is

$$\mu_R = \mu_S + \beta_1 (\sigma_R^2 + \sigma_S^2)^{1/2} \qquad (3.9)$$

in which $\sigma_R = \Omega_R \mu_R$ and $\sigma_S = \Omega_S \mu_S$ where Ω's are the respective coefficients of variation.

However, R and S are functions of other variables (e.g material properties, dimensions, loading etc) and these design parameters are termed the basic variables.

3.2 Second Moment Formulation

The calculation of P_f requires the knowledge of the distributions of $F_R(r)$ and $F_S(s)$. In practice, this information is often unavailable or difficult to obtain for the reasons of insufficient data. Furthermore, even when the required distribution can be specified the exact evaluation of the probabilities, generally requiring numerical integration of equation (3.1) may be impractical. Therefore alternative equivalent normal distribution may be resorted to in approximation.

Most often information on first and second moments, namely

mean values and variance of respective random variables are available. Practical measures of safety or reliability, must often be limited to functions of these first two moments. Under this condition, the implementation of reliability concepts must be necessarily be limited to a formulation based on first and second moment i.e restricted to second-moment formulation [14]. Thus with the second moment approach, the reliability may be measured entirely with a function of first and second moments of design variables, namely the reliability index β when there is no information on the probability distributions.

Recall the safety margin M = R - S, therefore M > 0 implies a safe state, while M < 0 denotes a failure state.

The boundary separating the safe and failure state is the limit-state defined by

$$M = R - S = 0 \tag{3.10}$$

Introducing the reduced variate

$$R' = (R - \mu_R)/\sigma_R$$

then

$$R = \sigma_R R' + \mu_R \tag{3.11}$$

Similarly,

$$S' = (S - \mu_S)/\sigma_S$$

and,

$$S = \sigma_S S' + \mu_S \tag{3.12}$$

Substituting equations (3.11) and (3.12) into the limit state equation (3.10) gives

$$\sigma_R R' - \sigma_S S' + \mu_R - \mu_S = 0 \qquad (3.13)$$

Equation (3.13) is a linear equation of the reduced variables and the graphical representation is shown in fig 3.3.

The straight line graph (fig 3.3) is a separation between realization of safe and failure conditions.

In the popular x-y coordinate, the graphical representation of equation (3.13). becomes.

$$y = \sigma_R/\sigma_S x + (\mu_R - \mu_S)/\sigma_S \qquad (3.14)$$

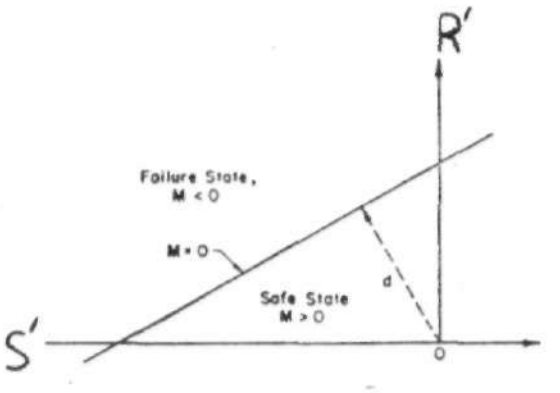


Fig. 3.3 Limit state boundary.

The perpendicular to line M = 0 and passing through the origin is

$$y = - \sigma_S x/\sigma_R \tag{3.15}$$

Solving for x and y from equations (3.14) and (3.15) gives

$$x = \sigma_R(\mu_R - \mu_S)/(\sigma_R^2 + \sigma_S^2) \tag{3.16}$$

and

$$y = - \sigma_S(\mu_R - \mu_S)/(\sigma_R^2 + \sigma_S^2) \tag{3.17}$$

Therefore the line distance, d, from the origin to M = 0 in a general form is

$$d = \sqrt{(x-0)^2 - (y-0)^2} = \sqrt{x^2 + y^2} \tag{3.18}$$

Substituting equations (3.10) and (3.11) in (3.18) we have

$$d = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \tag{3.19}$$

It was earlier shown for R and S described as normal random variables in equations (3.7) and now the distance given by equation (3.19) equals β. Therefore

$$d = \beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \tag{3.20}$$

3.3 Generalization as a function of Basic Variables.

R and S are functions of the basic variables. Thus to

investigate the effect of these variables on the performance of a structural system, a limit state equation of interest was developed in term of the basic design variables. This is referred to as the performance or state function, given by

$$g(X_i) = g(X_1, X_2, \ldots\ldots X_n) = R - S \qquad (3.21)$$

where X_i for $i = 1, 2, \ldots\ldots n$ represent the basic design variables and the limit state of the system is

$$g(X_i) = 0 \qquad (3.22)$$

Consequently, a graphical representation of equation (3.22) with the basic variables, X_i as the coordinate axis gives a failure surface. It follows, therefore, that $[g(X_i) > 0]$ implies safe state and $[g(X_i) < 0]$ corresponds to failure state.

Introducing the set of uncorrelated reduced variates

$$X_i' = (X_i - \mu_{Xi})/\sigma_{Xi} \ , \quad i = 1, 2, \ldots\ldots n \qquad (3.23)$$

Obviously, the safe state and failure state may also be represented graphically in the space of the reduced variate (equation (3.23)), separated by the appropriate limit state equation. In terms of the reduced variate the limit state equation is,

$$g(\sigma_{X1}X'_1 + \mu_{X1}, \ \sigma_{X2}X_2' + \mu_{X2}, \ \ldots\ldots\sigma_{Xn}X'_n + \mu_{Xn}) = 0 \qquad (3.24)$$

Recall, the position of the failure surface relative to the origin (equation (3.20)) is β and represented the minimum distance from the surface $g(X_i) = 0$ to the origin of the reduced variate axis [15]. Indeed the point on the failure surface

with the minimum distance to the origin is the most probable failure point [16]. The distance from a point $X_i' = (X_1', X_2', \ldots, X_n')$ on the failure surface $g(X'_i) = 0$ to the origin of X'_i space is of course

$$D = \sqrt{X_1'^2 + X_2'^2 + \ldots\ldots\ldots + X_n'^2} \qquad (3.25)$$

In a matrix form, we have

$$\left\{ \left[(X'_1 X'_2 \ldots\ldots X'_n) \begin{Bmatrix} X'_1 \\ X'_2 \\ \cdot \\ \cdot \\ \cdot \\ X'_n \end{Bmatrix} \right]^{1/2} \right\} = (X_i X'_i)^{1/2} \qquad (3.26)$$

The point on the failure surface $(x'^*_1, x'^*_2, \ldots\ldots x'^*_n)$, having the minimum distance to the origin may be determined by minimizing the function D (equation (3.25) subject to the constraint $g(X_i) = 0$, that is

minimise D

subject to $g(X_i) = 0$

For this purpose, the method of langrange's multiplier [17] may be used.

Let $L = D + \lambda g(X_i)$ (3.27)

Substituting equation (3.26) in (3.27) gives

$$L = (X'_i X'_i)^{1/2} + \lambda g(X_i) \qquad (3.28)$$

where λ is the value of the multiplier.

In scalar notation,

$$L=\sqrt{X'^{n}_{1}+X'^{n}_{2}+\ldots\ldots\ldots+X'^{n}_{n}}+\lambda g(X_1, X_2, \ldots\ldots X_n) \qquad (3.29)$$

in which $X_i = \sigma_{Xi} X'_i + \mu_{Xi}$

Minimizing L, we obtain n + 1 equations with n + 1 unknown as

$$\frac{\partial L}{\partial X_i}=\frac{X'_i}{\sqrt{X'^{n}_{1}+\ldots\ldots\ldots+X'^{n}_{n}}}+\frac{\lambda \partial g}{\partial X'_i}=0$$

and

$$\frac{\partial L}{\partial \lambda}=g(X_1, X_2, \ldots\ldots, X_n)=0$$

The solution of the above set of equations should yield the most probable failure point $(x_1'^{*}, x_2'^{*}, \ldots\ldots x_n'^{*})$.

Introducing the gradient vector,

$$G=\frac{\partial g}{\partial X'_1}, \frac{\partial g}{\partial X'_2}, \ldots\ldots\ldots, \frac{\partial g}{\partial X'_n}$$

in which

$$\frac{\partial g}{\partial X'_i}=\frac{\partial g}{\partial X'_i}\cdot\frac{\partial X_i}{\partial X'_i}=\sigma_{X_i}\frac{\partial g}{\partial X_i}$$

therefore in vector form we have

$$\frac{X'}{(X'^{t}X')^{1/2}} + \lambda G = 0$$

from which

$$X' = -\lambda DG \tag{3.30}$$

From equation (3.26)

$$D = [(\lambda DG^{t})(\lambda DG)]^{1/2} = \lambda D(G^{t}G)^{1/2}$$

and

$$\lambda = (G^{t}G)^{-1/2} \tag{3.31}$$

Substituting equation (3.31) in (3.30) gives

$$X' = \frac{-GD}{(G^{t}G)^{1/2}} \tag{3.32}$$

Multiply both sides of Eq. (3.32) by G^{t} we have,

$$G^{t}X' = \frac{-G^{t}GD}{(G^{t}G)^{1/2}}$$

implying that,

$$G^{t}X' = -(G^{t}G)^{1/2} D$$

and

$$D = \frac{-G^{t}X'}{(G^{t}G)^{1/2}} \tag{3.33}$$

Recall that at the failure point, the minimum distance

from the origin to the point equals β and therefore equation (3.33) becomes

$$\beta = \frac{-G^{*t}X'^{*}}{(G^{*t}G^{*})^{1/2}} \tag{3.34}$$

where G^{*} is the gradient vector at the most probable failure point ($X_1'^{*}$, $X_2'^{*}$.....$X_n'^{*}$). In scalar form equation (3.34) is

$$\beta = \frac{\sum_i x'^{*}_i \left(\frac{\partial g}{\partial X'_i}\right)_*}{\sqrt{\sum_i \left(\frac{\partial g}{\partial X'_i}\right)^2_*}} \tag{3.35}$$

where the derivative

$$\left(\frac{\partial g}{\partial X'_i}\right)_*$$

are performed at ($x_1'^{*}$, $x_2'^{*}$,.....$x_n'^{*}$) and the most probable point on the failure surface for minimum β from equation (3.32) is

$$X'^{*} = \frac{-G^{*}\beta}{(G^{*t}G^{*})^{1/2}} \tag{3.36}$$

and in scalar form is

$$x''_i = \alpha^{*}_i \beta \; ; i = 1, 2, \ldots, n \tag{3.37}$$

in which

$$\alpha_i = \frac{\left(\frac{\partial g}{\partial x'_i}\right)^*}{\sqrt{\sum_i \left(\frac{\partial g}{\partial x'_i}\right)^{2*}}}$$

are direction cosines along the axes X_i

3.3.1 First Order Interpretation

Consider again the generalized state function, g(x). When expanded in a Taylor series [17] at a point X^* which is on the failure surface

$$g(x^*) = 0 \qquad (3.38)$$

gives,

$$g(X_1, X_2, \ldots\ldots\ldots\ldots X_n) = g(X_1^*, X_2^*, \ldots\ldots\ldots X_n^*) + \sum_{i=1}^{n} (x_i - x^*_i)\left(\frac{\partial g}{\partial x_i}\right)^* + \frac{\sum_{j=1}^{n}\sum_{i=1}^{n} (x_i - x^*_i)(x_j - x^*_j)}{\left(\frac{\partial^2 g}{\partial x_i \partial x_j}\right)^*} + \ldots\ldots\ldots \qquad (3.39)$$

where the derivative are evaluated at $(x_1^*, x_2^*, \ldots\ldots\ldots x_n^*)$. But $g(x_1^*, x_2^*, \ldots\ldots\ldots x_n^*) = 0$ on the failure surface. Therefore

$$g(X_1, X_2, \ldots\ldots\ldots, X_n) = \sum_{i=1}^{n} (x_i - x^*_i)\left(\frac{\partial g}{\partial x_i}\right)^*$$

$$+\frac{\sum_{i=1}^{n}\sum_{j=1}^{n}(x_i-x^*_i)(x_j-x^*_j)}{\left(\frac{\partial^2 g}{\partial x_i \partial x_j}\right)_*}+\cdots\cdots \qquad (3.40)$$

Introducing the concept of reduced variate we have,

$$x_i - X_i^* = (\sigma_{x_i} x'_i + \mu_{x_i}) - (\sigma_{x_i} x'^*_i + \mu_{x_i})$$
$$= \sigma_{x_i}(x'_i - x'^*_i)$$

and

$$\frac{\partial g}{\partial x_i} = \frac{\partial g}{\partial x'_i}\left(\frac{\partial x'_i}{\partial x_i}\right) = \frac{1}{\sigma_{x_i}}\left(\frac{\partial g}{\partial x'_i}\right)$$

Truncating equation (3.40) at the first order linear term and substituting the reduced variate into it gives

$$g(X_i) = \sum_{i=1}^{n}(x'_i - x'^*_i)\left(\frac{\partial g}{\partial x'_i}\right)_* \qquad (3.41)$$

Applying the properties of expectation and variance [18], the mean value of equation (3.41) is

$$\mu_g = \sum_{i=1}^{n} x'^*_i\left(\frac{\partial g}{\partial x'_i}\right)_*$$

and the corresponding variance is

$$\sigma_g^2 = \sum_{i=1}^{n} \sigma_{x'_i}^2 \left(\frac{\partial g}{\partial x'_i}\right)^2 = \sum_{i=1}^{n} \left(\frac{\partial g}{\partial x'_i}\right)^2$$

thus the ratio

$$\frac{\mu_g}{\sigma_g} = -\frac{\sum_{i=1}^{n} x'^*_i \left(\frac{\partial g}{\partial x'_i}\right)}{\sqrt{\sum_{i=1}^{n} \left(\frac{\partial g}{\partial x'_i}\right)^2}} \qquad (3.42)$$

Equation (3.42) obtained by this method conforms to equation (3.35) derived earlier, therefore

$$\beta = \frac{\mu_g}{\sigma_g} \qquad (3.43)$$

3.4 Transformation of Basic Variables

The prescribed probability distributions of the basic variables may be nonnormal and correlation between their pairs may exist. In this section, we highlight practicable methods of their transformation into forms suitable for safety index, whereby equation (3.8) for finding probability of failure is applicable.

3.4.1 Non-normal Variate

In a situation where the original PDF, $f_i(x_i)$ and CDF, $F_i(x_i)$ are not normal, it is necessary to transform the

distribution into an equivalent normal distribution at design point (x_i^*).

Equating the CDF with the standard cumulative probability gives

$$\Phi\left(\frac{x_i^* - \mu_{X_i}^N}{\sigma_{X_i}^N}\right) = F_{X_i}(x_i^*) \qquad (3.44)$$

where μ_{xi}^N , σ_{xi}^N are respectively the mean and standard deviation of equivalent normal distribution of the basic variable X_i. Thus, the equivalent mean is

$$\mu_{xi}^N = x_i^* - \sigma_{xi}^N \Phi^{-1}[F_{xi}(x_i^*)] \qquad (3.45)$$

whereas equating the probability densities gives

$$\frac{1}{\sigma_{X_i}^N}\varphi\left(\frac{x_i^* - \mu_{X_i}^N}{\sigma_{X_i}^N}\right) = f_{X_i}(x_i^*) \qquad (3.46)$$

Substituting equation (3.45) in (3.46) gives

$$\frac{1}{\sigma_{X_i}^N}\varphi\left(\frac{x_i^* - x_i^* + \sigma_{X_i}^N \Phi^{-1}(F_{X_i}(x_i^*))}{\sigma_{X_i}^N}\right) = f_{X_i}(x_i^*)$$

therefore

$$\sigma_{X_i}^N = \frac{\varphi\{\Phi^{-1}[F_{X_i}(x_i^*)]\}}{f_{X_i}(x_i^*)} \qquad (3.47)$$

In the above equations μ_{xi}^N , σ_{xi}^N and the design points x_i^*, are unknowns and thus iterative method is employed until convergence is achieved.

3.4.2 Correlated Variate

The procedures described so far were for cases where the basic variates X_1, X_2.....X_n are uncorrelated or statistically independent. For correlated variate, transformation into a set of uncorrelated variate must be carried out.

Consider a system which is a function of two variates X_1 and X_2 with expected values $E[X_1] = \mu_{x1}$ and $E[X_2] = \mu_{x2}$. The covariance of X_1 and X_2 is denoted $COV[X_1, X_2]$ and is defined by [10]

$$COV[X_1, X_2] = E[(X_1 - \mu_{x1})(X_2 - \mu_{x2})]$$
$$= E[X_1 X_2] - E[X_1]E[X_2] \qquad (3.48)$$

The correlation coefficient is given by

$$\rho_{x_1 x_2} = \frac{cov[X_1 X_2]}{\sigma_{x_1} \sigma_{x_2}} \qquad (3.49)$$

where σ_{x1} and σ_{x2} are standard deviations of X_1 and X_2 respectively. A pair of variates is said to be uncorrelated if

$$COV[X_1, X_2] = 0$$

and correlation coefficient satisfies the inequality

$$-1 \le \rho_{x_1 x_2} \le 1$$

The required transformation is necessarily dependent on the covariance or covariance matrix of the original variate and may be obtained as follows.

Suppose the covariance matrix of the original variate X_1, X_2,.....X_n is

$$|C| = \begin{vmatrix} \sigma^2_{X_1} & COV[X_1, X_2] & \cdots & COV[X_1, X_n] \\ COV[X_2, X_1] & \sigma^2_{X_2} & \cdots & COV[X_2, X_n] \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ COV[X_n, X_1] & \cdot & \cdots & \sigma^2_{X_n} \end{vmatrix} \qquad (3.50)$$

The corresponding covariance between pairs of the reduced variate X_i' and X_j' is

$$COV[X_i', X_j'] = E[(X_i' - \mu_{X_i'})(X_j' - \mu_{X_j'})]$$

$$= E\left[\left(\frac{X_i - \mu_{X_i}}{\sigma_{X_i}}\right)\left(\frac{X_j - \mu_{X_j}}{\sigma_{X_j}}\right)\right]$$

$$= \frac{E[(X_i - \mu_{X_i})(X_j - \mu_{X_j})]}{\sigma_{X_i}\sigma_{X_j}}$$

$$= \rho_{X_i X_j} \qquad (3.51)$$

Equation (3.51) shows that the covariance between a pair of reduced variate X_i' and X_j', is equal to the correlation coefficient between the corresponding pair of the original variate X_i and X_j [as in equation (3.49)]. Therefore, the

covariance matrix of the reduced variate (X_1', X_2',.....X_n') is the corresponding correlation matrix of the original variate (X_1, X_2, ..., X_n). Thus, the covariance matrix of X_1', X_2'....., X_n' is

$$|C'| = \begin{vmatrix} 1 & \rho_{12} & \rho_{13} & \cdots & \rho_{1n} \\ \rho_{21} & 1 & \rho_{23} & \cdots & \rho_{2n} \\ . & . & . & . & . \\ . & . & . & . & . \\ \rho_{n1} & \rho_{n2} & \rho_{n3} & \cdots & 1 \end{vmatrix} \qquad (3.52)$$

The required set of (uncorrelated) transformed variate can be obtained from X' through the following orthogonal transformation,

$$Y = T^t X' \qquad (3.53)$$

in which

$$X' = \{X_1', X_2', ...X_n'\}$$

and $$Y = \{Y_1, Y_2, ...Y_n\}$$

are the required set of uncorrelated transformed variate.

T^t = the transpose of an orthogonal transformation matrix which is composed of the eigenvectors corresponding to the eigenvalues of the correlation matrix [C]. Specifically, T, is such that

$$T^t[C']T = [\lambda] \qquad (3.54)$$

in which $[\lambda]$ is a diagonal matrix of eigenvalues of [C']. It may be emphasized that the matrix [C'] is real and symmetric as $\rho_{ij} = \rho_{ji}$.

With the orthogonal transformation the safety index becomes

$$\beta = -\frac{G^{*T}X^{*}}{(G^{*T}[C']G^{*})^{1/2}} \qquad (3.55)$$

Summarily, when the basic variables X_i are correlated, the first step is to obtain a set of uncorrelated variables Y_i. The next step is to normalize the uncorrelated variables and thereby a set of normalized and uncorrelated variables $Z_i = (Z_1, Z_2.....Z_n)$ is obtained. Finally, the safety index can be defined in the Z coordinate system as previously explained. A flow chart of the numerical algorithm for the computation of ß is shown in Fig. 3.4.

3.5 FORM 5

A computer package coded as FORM5 [19] using the algorithm depicted in fig 3.4 is used in this study to compute the implied safety levels in the BS 449 design criteria of girders.

FORM5 (written in FORTRAN 77) requires the user to specify by easy to use input parameters of the state function as a subroutine and the stochastic model in term of the type of distribution together with its distribution parameters. The state function must be written as a subroutine (DOUBLE PRECISION) in FORTRAN 77.

flow chart of the numerical algorithm for the computation of β is shown in Fig. 3.4.

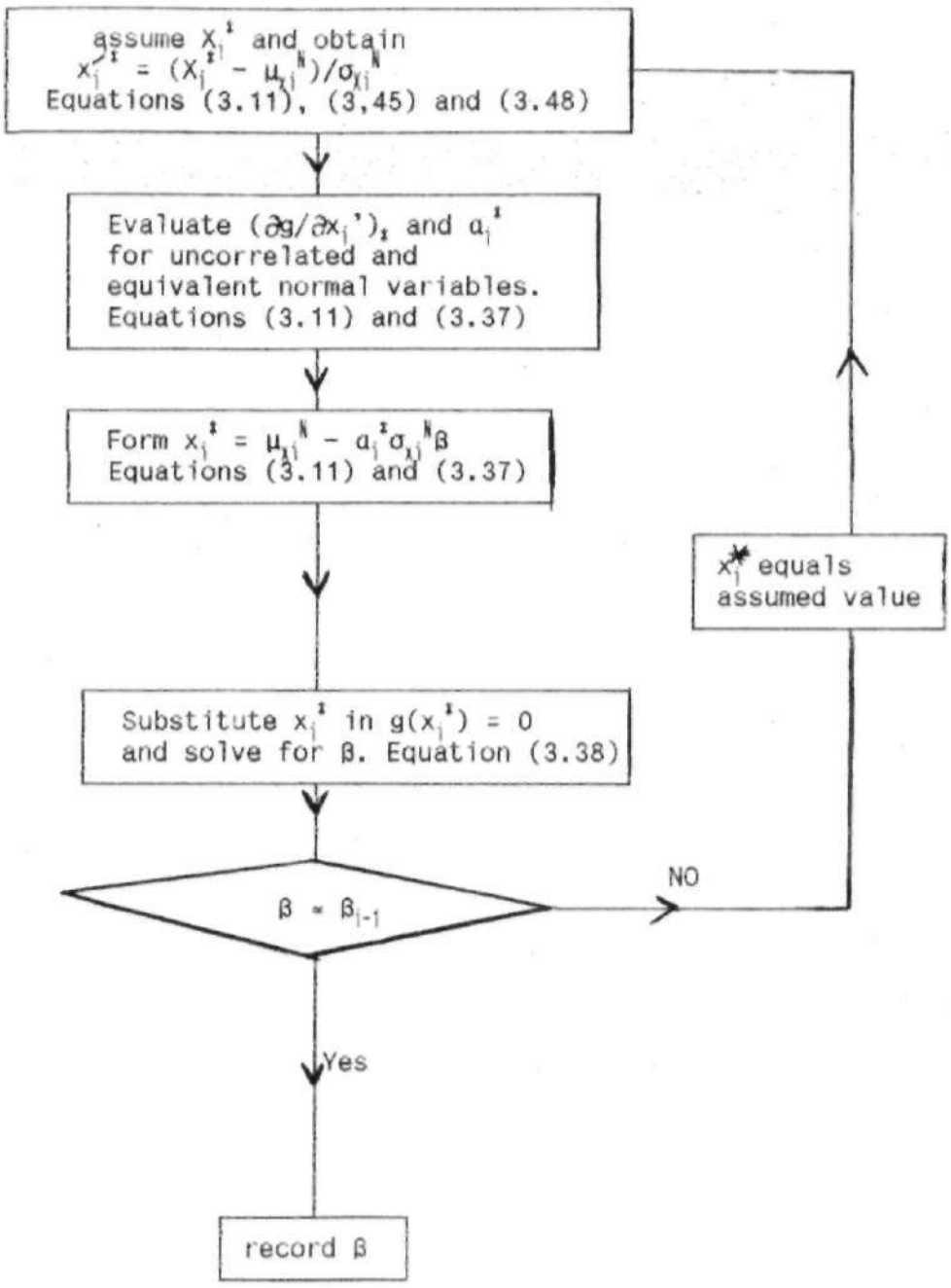


Fig 3.4: Flow Chart for Computation of β

CHAPTER FOUR

ALLOWABLE STRESS DESIGN CONSIDERATION OF GIRDERS

4.1 Bending

For the determination of the resistance to pure bending (flexural and no axial load) in most flexural members a single calculation will suffice in the form:

$$Z = \frac{M_{max}}{\sigma_{all}} \qquad (4.1)$$

where

Z = The required section modulus (mm^3)

M_{max} = The maximum bending moment (Nmm)

σ_{all} = the allowable bending stress (N/mm^2)

The principal assumptions for justification of this procedure include the following condition [20]:

(a) the material is homogenous

(b) Section plane before bending remains plane.

(c) The cross-section has an axis of symmetry.

(d) Stress is proportional to strain

(e) The moment is applied in the plane of symmetry

(f) The member is straight in the plane of symmetry

(g) The member is stable laterally to the plane of bending

Usually the maximum moment, M_s resisted by the member is obtained by the simple method of structural mechanics and

section modulus Z for the trial section is computed. The ratio M_{max}/Z is such that

$$M_{max}/Z \leq \sigma_{all} \text{ (prescribed in the code)} \qquad (4.2)$$

4.2 Shear

The elastic formula for analysis of shear stress on a section is too complex for routine use with variety of shapes available or possible for steel members. For a member that possesses an axis of symmetry in the plane of loading, two simplifying assumptions that result in a negligible loss of (theoretical) accuracy are permitted [20]:

a. The contribution of flanges to shear capacity may be neglected.

b. The use of an average value of shear stress on the gross area of the web (overall depth times web thickness).

This assumption has been justified by an approximate reduction of the allowable shear stress. With these assumptions, derivation and application of accepted shear capacity formula becomes simple. Neglecting the flanges all symmetrical rolled shapes, box shapes and built-up sections reduce to an equivalent rectangular section with dimensions $\Sigma t_w D$ and shear stress becomes

$$f_v = \frac{V}{\sum t_w D} \qquad (4.3)$$

where,

f_v = the shear stress at any point in the depth D

V = the vertical load at that point

t_w = web thickness

D = overall depth

BS 449 prescribes the average shear stress on the gross section for plate girder as being carried by the depth of the web multiplied by its thickness. Thus the shear requirement is given by

$$\frac{V_{max}}{(D-2t_f)\,t_w} \leq \tau_{all} \qquad (4.4)$$

where,

V_{max} = maximum shear force

t_f = thickness of flange

τ_{all} = allowable shear stress (prescribed in the code)

D = overall depth

4.3. Conditions for Reappraisal

For allowable stress (elastic) design, the term 'safety factor' is usually expressed and accepted as the ratio of the specified minimum yield stress to the allowable stress. It ignores the overload capacity of the structure between first yielding to final overall instability. It also disregards the entire approach of probabilities of overload, the ratio of live load to dead load, the actual strength versus minimum specified yield strength and variation of the actual section strength, etc.

Although adequate data relative to strength of a given material may not be available, it is a well-known fact that the strength varies from specimen to specimen and the nature of that variation has been investigated by several investigators. Previous studies [e.g., 21, 22] on plate girders showed the relationship between the ultimate stresses obtained by theoretical analysis based on elastic method as in allowable stress design concept with values obtained experimentally. A plot for pure bending is shown in fig 4.1 while a similar plot for pure shear in fig 4.2.

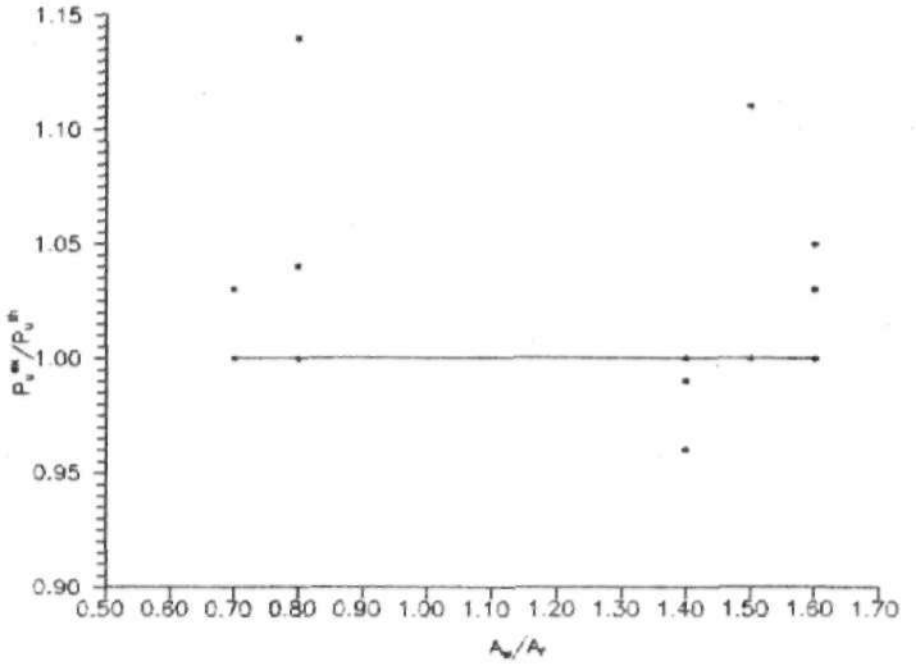


Fig. 4.1: Ultimate Beam Strength in Bending Compared with Test Results

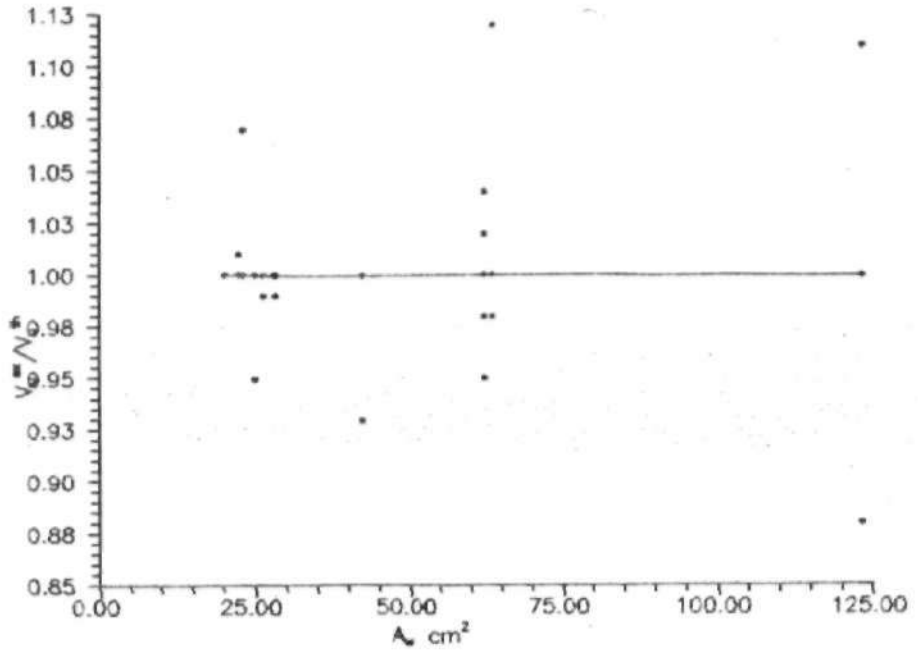


Fig. 4.2: Ultimate Beam Strength in Shear Compared with Test Results

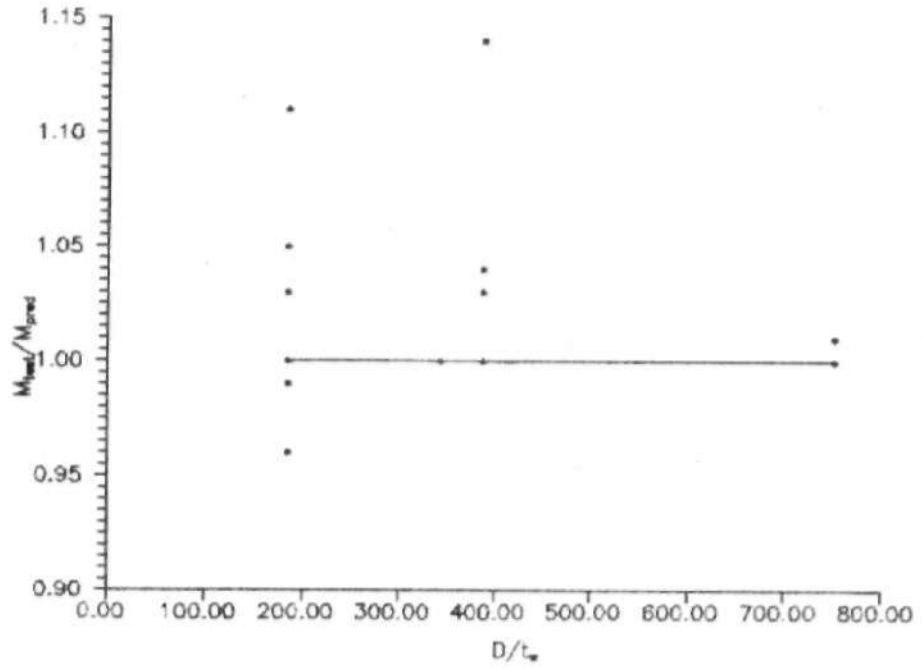


Fig. 4.3: Variations in Comparison between Predicted and Experimental Capacity of Plate Girders in Bending

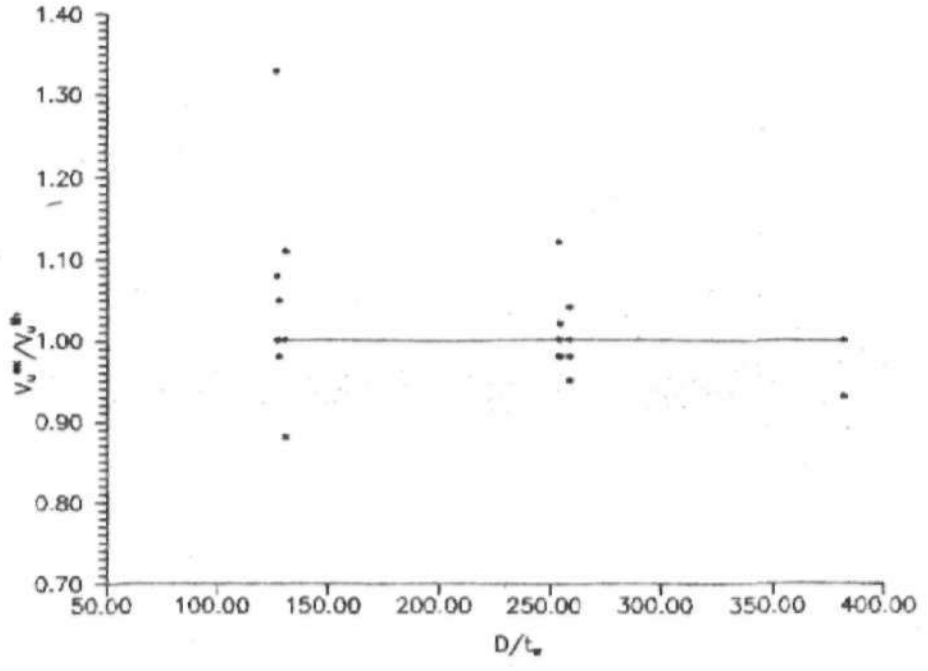


Fig. 4.4: Variations in Comparison between Predicted and Experimental Capacity of Plate Girders in Shear

Similar plots for research work carried out on transversely stiffened A36 steel plate girder [23] are depicted in figs 4.3 and 4.4.

The formulas used to predict the ultimate strength of plate girder can be considered to be quite reasonable. The assumptions that bending moment is taken by the flanges and shear force taken by the web [24] was also proved adequate through research work [25]. The plots in fig 4.1 to 4.4 buttresses this assertions and therefore these formulas (used in computing load resistance capacity) can be considered to produce the actual ultimate strength in the simplest form

conceivable.

Nevertheless, the conventional approach has shortcomings that are well recognised, principally the approach is limited to the extent that all variables and factor must be treated as though they were deterministic. In spite of the steps taken to include statistical aspects of engineering information in the formulation of safety factors, the procedure essentially remains deterministic in the sense that any risk associated with randomness (e.g shown by the scatter in figs 4.1 to 4.4) is not determined.

Therefore in the presence of statistical variabilities, such as evidenced in loads and material strength, proper evaluation of structural design requires explicit consideration of probability. Clearly, the conventional design approach is not adequate from reliability stand point. Hence another design methodology that considers the probabilistic nature of design is needed so that component reliability becomes intrinsic part of design. However the direction in this study is to show the safety levels implicit in the BS 449 requirements for design of girders when the decision variables are explicitly considered random.

CHAPTER FIVE

COMPUTATION OF SAFETY INDICES

5.1 Statistical Characters of Relevant Design Parameters.

The probability distributions of the basic variables (design parameters) can be obtained either by direct inference from observed data or by subjective assessment or combination of these techniques. In practice, direct inference is rarely always possible since there is seldom sufficient data available to identify unambiguously only one distribution as appropriate. This further implies that past observations and experience for similar structures can be validly used for the structure under assessment.

A physical reasoning may be used to suggest an appropriate distribution. Thus where the basic variable consists of the sum of many other variables (which are not explicitly considered), the central limit theorem may be invoked to suppose that normal distribution is appropriate [10]. Central limit theorem stated loosely says that the sum of large number of individual random components none of which is dominant, tends to a normal distribution as the number of components (regardless of their initial distributions) increase without limit [1]. Therefore if a physical process is the result of totality of a large number of individual effects, then according to the central

limit theorem, the process would tend to be normally distributed.

Consequently, a preferable approach is to make use of physical reasoning about the nature of each particular random variable to guide the choice of its distribution. Imposing central limit theorem, the basic variables were assumed normally distributed. In addition yield strength was modeled by other appropriate distributions. Logarithmic normal (or lognormal) distribution was used because it has a theoretical advantage of precluding negative values. Weibull distribution was also used because it characterizes the strength of structural component whose strength in governed by the size of its largest defect. Gumbel distribution was also tried for its similarity with weibull as an extreme-value distribution. This subjective judgement in the choice of the distribution should not however be seen as a limitation, since the aim is not to produce a perfect image of the reality (an impossible task), but to develop a mathematical model of the phenomenon which embodies its salient features and which can be used to make optimal design decisions using the available data [10].

With limited information the probability distributions of the design variables were prescribed. However, it is important also to recognize that regardless the distribution of the individual variate, it is the distribution of the safety margin

(R-S) that is important in the calculation of probability of failure, P_f [13]. It was also observed that if the risk level is high (e.g., $P_f > 10^{-3}$ or $\beta < 3.1$) the calculated failure probability is not very much different regardless of the type of distribution. However, for a very small risk (e.g., $P_f \leq 10^{-5}$ or $\beta \geq 4.25$) the failure probability could be sensitive to this distribution.

As explained previously in second moment reliability method, the parameters of interest are first and second moments (mean and variance) for the computation of safety indices [14]. It is also important to note that it is the coefficient of variation (ratio of standard deviation to mean values that are mostly published in literature [e.g 26, 27]. The values of these relevant parameters for the basic variables used in this study are shown in Table 5.1.

Table 5.1 Statistical values of design parameters

S/No	Basic Variables	Mean	Coeff. of Variation
1	Yield strength in bending	1.05 fy	0.10
2	Yield strength in shear	0.64 fy	0.10
3	Live load	Q,q	0.2-0.35
4	Dead load	G,g	0.05-0.10
5	Width of flange	B	0.002
6	Overall depth of section	D	0.002
7	Thickness of flange	t_f	0.05
8	Thickness of web	t_w	0.05

In Table 5.1, f_y represents the specified yield strength of steel while the other mean values are practical values in the design of an adequate I-section girder with strict compliance to BS 449 provisions. Computed safety indices are shown in Tables 5.2 to 5.46.

5.2 Bending Limitations

In the design of plate girders in bending, the maximum stress is assumed to be taken by the flanges [24]. The maximum stress on the extreme fibre of the flange given by equation (4.1) is

$$F_b = M_{max}/Z$$
$$= M_{max}D/2I \qquad (5.1)$$

where

M_{max} = maximum bending moment

D = overall depth of I-section

I = Moment of area about axis of bending

The moment M_{max} in equation (5.1) depends on the type of loading on the girder. With a uniformly distributed load (UDL) w kN/m on a simply supported single span length L,

$$M_{max} = wL^2/8$$

where w is sum of the live load and dead load, q and g respectively. Therefore

$$M_{max} = (q + g)L^2/8 \qquad (5.2)$$

For a point load P kN with live load, Q and dead load G

components,

$$M_{max} = PL/4$$
$$= (Q + G)L/4 \qquad (5.3)$$

The moment of area I (in equation (5.1) for a symmetrical I-section (fig. 5.1) about the axis of bending x-x, is computed

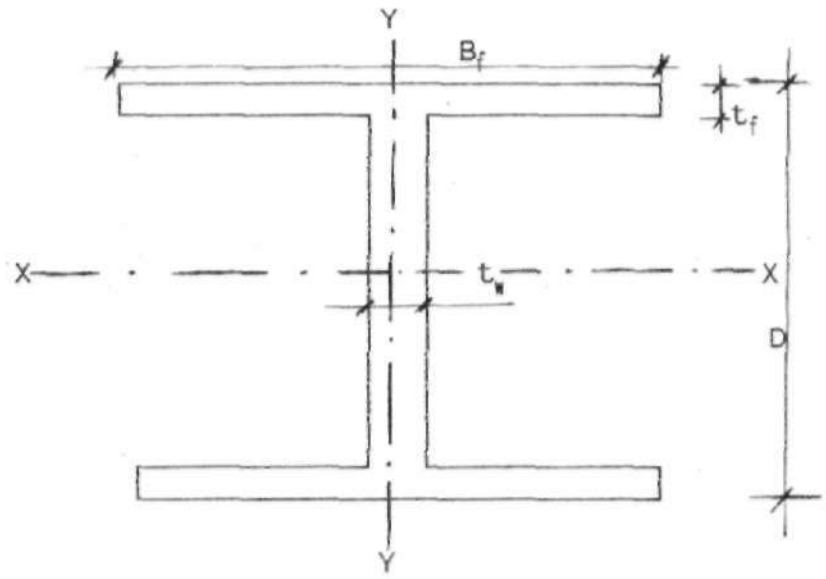


Fig 5.1 symmetrical I-section of girder

in terms of the basic variables as

$$I_{xx} = t_w(D-2t_f)^3/12+B_f t_w^3/6+2B_f t_f[(D-t_f)/2]^2 \qquad (5.4)$$

The limit state equation for bending consideration is therefore

$$FX = 1.05f_y - F_b$$
$$= 1.05f_y - M_{max}D/2I \qquad (5.5)$$

Equations (5.2) or (5.3) and (5.4) substituted in equation (5.5) gives the limit state equation for bending consideration in terms of the basic design variables in a form suitable for FORM5.

5.3 Shear Limitation

The maximum shear stress assumed to be taken by the web [24] is given by equation (4.4). i.e

$$f_v = V/t_w(D-2t_f) \qquad (5.6)$$

where

V = Maximum shear force

t_w = web thickness

D = overall depth of section

t_f = flange thickness

for UDL,

$$V = (q + g)L/2$$

implying that,

$$F_v = (q + g)L/2t_w(D - 2t_f) \qquad (5.7)$$

Similarly for the point load,

$$V = (Q + G)/2$$

and

$$f_v = (Q + G)/2t_w(D - 2t_f) \qquad (5.8)$$

Consequently, the limit state equation for shear consideration is

$$FX = \tau_y - f_v = 0.64f_y - f_v \qquad (5.9)$$

Therefore substituting equations (5.7) or (5.8) in equation

(5.9) gives the limit state equation for shear consideration in terms of the basic design variables suitable for FORM5. The computed reliability levels for both uniformly distributed and point loads are depicted on Tables 5.2 to 5.46. Since the aim is to show the implicit reliability levels in the BS 449 design equations for plates girders, recorded values of the computed reliability levels are in the range: $2 \leq \beta < 6$. Also the prescribed probability distributions function (pdf) for yield strength, f_y, of the steel plate girder are coded as follows:

a. Normal distribution 2

b. lognormal " 3

c. Gumbel " 7

d. Weibull " 9

All other basic variables were prescribed normal. For bending consideration under uniformly distributed load configuration, Tables 5.2 to 5.14 show the implied reliability levels, while Tables 5.15 to 5.24 give the associated reliability levels for shear condition for the load configuration. Tables 5.25 to 5.37 represents the computed reliability levels for bending requirement under point load configuration. In Tables 5.38 to 5.46, the implied reliability levels for shear requirements when point loads are applied are shown.

TABLE 5.2: Variation of β with span, L.

span, L (m)	β for pdf of f_y			
	2	3	7	9
8			5.99	
9		5.93	5.11	
10		4.99	4.34	
11	4.96	4.13	3.66	5.52
12	3.87	3.35	3.04	4.26
13	2.92	2.63	2.46	3.15
14	2.08			2.18

Table 5.3: Variation of β with yield strength, f_y.

f_y (N/mm^2)	β for pdf of f_y			
	2	3	7	9
231	2.23	2.18	2.03	2.34
257	2.92	2.63	2.46	3.15
283	3.60	3.12	2.84	3.98
308	4.28	3.57	3.19	4.83
334	4.94	3.99	3.51	5.69
360	5.59	4.37	3.80	
386		4.73	4.07	
411		5.07	4.33	
437		5.59	4.57	
463		5.69	4.81	
488		5.97	5.03	

Table 5.4: Variation of β with live load, q.

q kN/m	β for pdf of f_y			
	2	3	7	9
10		2.89	4.07	2.92
20		3.01	3.92	3.51
30	5.69	3.17	3.76	4.17
40	5.29	3.29	3.60	4.59
50	4.89	3.34	3.43	4.71
60	4.49	3.31	3.26	4.60
70	4.09	3.22	3.07	4.33
80	3.70	3.07	2.88	3.97
90	3.31	2.87	2.68	3.57
100	2.92	2.63	2.46	3.15
110	2.54	2.36	2.23	2.71

Table 5.5: Variation of β with dead load, g.

g kN/m	β for pdf of f_y			
	2	3	7	9
1	3.07	2.74	2.55	3.34
2	3.04	2.71	2.53	3.29
3	3.00	2.69	2.51	2.24
4	2.96	2.66	2.48	3.19
5	2.92	2.63	2.46	3.15
6	2.88	2.60	2.44	3.10
7	2.84	2.57	2.42	3.05
8	2.80	2.55	2.37	3.01

Table 5.6: Variation of β with flange width, B_f.

B_f (mm)	β for pdf of f_y			
	2	3	7	9
315	2.39	2.21	2.11	2.53
350	2.92	2.63	2.46	3.15
385	3.42	3.01	2.77	2.73
420	3.89	3.37	3.05	4.28
455	4.34	3.69	3.31	4.80
525	5.16	4.28	3.78	5.75
560	5.54	4.55	3.99	
595	5.91	4.80	4.20	
630		5.04	4.39	

Table 5.7 Variation of β with Overall depth, D.

D (mm)	β for pdf of f_y			
	2	3	7	9
900	2.20	2.06		2.31
1000	2.92	2.63	2.46	3.15
1100	3.60	3.15	2.88	3.94
1200	4.25	3.63	3.26	4.70
1300	4.86	4.06	3.61	5.41
1400	5.45	4.47	3.93	
1500		4.85	4.23	
1600		5.21	4.52	
1700		5.55	4.80	
1800		5.87	5.06	

Table 5.8: Variation of β with flange thickness, t_f.

t_f (mm)	β for pdf of f_y			
	2	3	7	9
40	2.92	2.63	2.46	3.15
50	4.02	3.45	3.11	4.45
60	4.97	4.11	3.64	5.58
70	5.80	4.67	4.07	
80		5.14	4.45	
90		5.55	4.78	
100		5.91	5.07	

Table 5.9: Variation of β with web thickness, t_w

t_w (mm)	β for pdf of f_y			
	2	3	7	9
3	2.42	2.24	2.14	2.57
6	2.59	2.37	2.25	2.76
9	2.76	2.57	2.36	2.96
12	2.92	2.63	2.46	3.15
15	3.08	2.75	2.56	3.34
18	3.24	2.87	2.66	3.52
24	3.55	3.10	2.84	3.89
36	4.14	3.53	3.18	4.59
48	4.70	3.93	3.49	5.26
60	5.23	4.29	3.77	5.89
72	5.73	4.62	4.04	

Table 5.10: Variation of β with Ratio B_f/t_f.

B_f/t_f	β for pdf of f_y			
	2	3	7	9
8	2.27	2.15	2.08	2.36
9	2.80	2.59	2.46	2.95
10	3.29	2.99	2.79	3.49
11	3.74	3.35	3.09	3.99
12	4.16	3.69	3.36	4.45
13	4.54	3.99	3.61	4.87
14	4.91	4.28	3.84	5.27
15	5.25	4.54	4.06	5.64
16	5.57	4.79	4.26	5.99
17	5.87	5.02	4.46	
18		5.24	4.69	
19		5.45	4.81	
20		5.65	4.98	
21		5.84	5.14	
22			5.29	
23			5.44	
24			5.58	
25			5.71	
26			5.84	
27			5.97	

Table 5.11: Variation of β with Ratio g/q.

g/q	β for pdf of f_y			
	2	3	7	9
0.05	3.80	3.29	2.99	4.19
0.10	3.53	3.09	2.82	3.87
0.15	3.27	2.89	2.67	3.56
0.20	3.02	2.70	2.51	3.27
0.25	2.78	2.52	2.37	3.00
0.30	2.56	2.35	2.22	2.74
0.35	2.35	2.18	2.08	2.50

Table 5.12: Variation of β with Ratio B_f/D.

B_f/D	β for pdf of f_y			
	2	3	7	9
0.30	2.15	2.02		2.21
0.35	2.92	2.63	2.46	3.15
0.40	3.63	3.17	2.90	3.97
0.45	4.28	3.65	3.28	4.72
0.50	4.88	4.08	3.62	5.42
0.55	5.44	4.47	3.93	
0.60	5.96	4.83	4.22	
0.65		5.70	4.49	
0.70		5.48	4.74	
0.75		5.77	4.98	
0.80			5.20	
0.85			5.42	
0.90			5.62	
0.95			5.81	

Table 5.13: Variation of β with Ratio D/t_w.

D/t_w	β for pdf of f_y			
	2	3	7	9
80	2.47	2.31	2.21	2.58
90	3.22	2.92	2.72	3.43
100	3.91	3.47	3.17	4.21
110	4.55	3.96	3.57	4.92
130	5.70	4.83	4.27	
140		5.21	4.59	
150		5.57	4.89	
160		5.91	4.17	

Table 5.14: Variation of β with Ratio D/t_f.

D/t_f	β for pdf of f_y			
	2	3	7	9
22.50	2.57	2.42	2.32	2.00
25.5	3.13	2.90	2.73	3.29
27.5	3.66	3.33	3.09	2.63
30.0	4.15	3.73	3.42	3.86
32.5	4.61	4.10	3.72	4.40
35.0	5.04	4.45	4.01	4.90
37.5	5.45	4.77	4.27	5.37
40.0	5.84	5.07	4.53	5.80
42.5		5.36	4.77	
45.0		5.64	5.00	
47.5		5.90	5.22	

TABLE 5.15 Variation of β with span, L.

S (m)	β for pdf of f_y			
	2	3	7	9
12	5.85	4.80	4.20	
13	5.35	4.44	3.92	5.93
14	4.89	4.11	3.66	5.40
15	4.47	3.81	3.41	4.91
16	4.08	3.52	3.18	4.46
17	3.71	3.25	2.97	4.05
18	3;38	3.00	2.76	3.66
19	3.07	2.76	2.57	3.30
20	2.77	2.53	2.38	2.96
21'	2.50	2.31	2.20	2.65

Table 5.16: Variation of β with yield strength, f_y

f_y (N/mm^2)	β for pdf of f_y			
	2	3	7	9
180	2.75	2.58	2.47	2.86
206	3.63	3.26	3.01	3.87
231	4.50	3.88	3.49	4.90
257	5.35	4.44	3.92	5.93
283		4.95	4.31	
308		5.42	4.67	
334		5.85	5.00	
360			5.31	

Table 5.17: Variation of β with live load, q.

q (kN/mm)	β for pdf of f_y			
	2	3	7	9
90	5.70	4.54	4.07	
100	5.35	4.44	3.92	5.93
110	5.00	4.30	3.76	5.54
120	4.66	4.12	3.61	5.14
130	4.33	3.92	3.44	4.74
140	4.00	3.69	3.27	4.34
150	3.67	3.44	3.09	3.95
160	3.34	3.19	2.90	3.57
170	3.02	2.92	2.70	3.20
180	2.71	2.64	2.49	2.82

Table 5.18: Variation of β with dead load, g.

g (kN/m)	β for pdf of f_y			
	2	3	7	9
5	5.35	4.44	3.92	5.93
10	5.18	4.34	3.84	5.71
20	4.83	4.13	3.69	5.27
30	4.50	3.91	3.52	4.85
40	4.16	3.68	3.35	4.45
50	3.83	3.45	3.18	4.06
60	3.15	3.21	2.99	3.68
70	3.18	2.97	2.80	3.31
80	2.87	2.71	2.60	2.96

Table 5.19: Variation of β with overall depth, D.

D (mm)	β for pdf of f_y			
	2	3	7	9
700	2.98	2.69	2.51	3.20
800	3.85	3.35	3.05	4.20
900	4.64	3.93	3.51	5.11
1000	5.36	4.44	3.92	5.93
1100		4,90	4.29	
1200		5.31	4.62	

Table 5.20: Variation β with flange thickness, t_f.

t_f (mm)	β for pdf of f_y			
	2	3	7	9
20	5.62	4.63	4.07	
30	5.48	4.54	4.00	
40	5.35	4.44	3.92	5.93
50	5.35	4.35	3.84	5.77
60	5.21	4.25	3.76	5.61
70	5.07	4.14	3.60	5.45
80	4.93	4.04	3.51	5.28
90	4.78	3.93	3.42	5.11
100	4.64	3.82	3.24	4.94
120	4.49	3.59	3.05	4.58
130	3.17	3.35	2.85	4.20
140	3.85	3.10	2.63	3.81
160	3.51	2.54	2.39	3.41

Table 5.21: Variation of β with web thickness, t_w.

t_w (mm)	β for pdf of f_y			
	2	3	7	9
9	3.47	3.10	2.86	3.72
12	5.35	4.44	3.92	5.93
15		5.48	4.74	
18			5.43	

Table 5.22: Variation of β with Ratio g/q.

g/q	β for pdf of f_y			
	2	3	7	9
0.05			4.41	
0.10	5.97	5.08	4.25	
0.15	5.69	4.87	4.09	
0.20	5.43	4.68	3.95	
0.30	4.93	4.49	3.67	
0.40	4.48	4.14	3.41	5.48
0.50	4.07	3.81	3.17	4.96
0.60	3.69	3.23	2.94	4.48
0.70	3.34	2.96	2.73	4.04
0.80	3.01	2.71	2.53	3.63
0.90	2.71	2.47	2.33	3.25
1.00	2.43	2.25	2.14	2.90
1.10	2.16	2.03		2.58
1.20				2.00

Table 5.23: Variation of β with Ratio D/t_f.

$D\backslash t_f$	β for pdf of f_y			
	2	3	7	9
17.5	2.86	2.62	2.47	3.04
20.0	3.69	3.27	3.00	3.97
22.5	4.42	3.84	3.46	4.80
25.0	5.09	4.33	3.86	5.56
27.5	5.70	4.78	4.22	
30.0		5.18	4.54	
32.5		5.55	4.54	
35.0		5.89	5.13	

Table 5.24: Variation of β with Ratio D/t_w.

D/t_w	β for pdf of f_y			
	2	3	7	9
60	2.76	2.59	2.48	2.87
70	3.61	3.30	3.07	3.79
80	4.34	3.90	3.56	4.59
90	4.99	4.42	3.99	5.28
100	5.56	4.88	4.37	5.90
110		5.29	4.72	

Table 5.25: Variation of β with span length, L.

L (m)	β for pdf of f_y			
	2	3	7	9
11	5.45	4.48	3.94	5.62
12	4.90	4.09	3.63	5.01
13	4.40	3.73	3.34	4.46
14	3.94	3.40	3.08	3.95
15	3.52	3.09	2.83	3.50
16	3.14	2.80	2.60	3.08
17	2.28	2.52	2.37	2.70
18	2.45	2.26	2.16	2.35
19	2.15	2.02		2.03

Table 5.26: Variation of β with yield strength, f_y.

f_y (N/mm^2)	β for pdf of f_y			
	2	3	7	9
231	2.80	2.56	2.41	2.72
257	3.52	3.09	2.83	3.50
283	4.24	3.58	3.21	4.30
308	4.94	4.03	3.55	5.10
334	5.63	4.44	3.86	5.91
360		4.83	4.16	
411		5.18	4.43	
437		5.52	4.69	
463		5.83	4.94	
			5.17	

Table 5.27: Variation in β with live load, Q.

Q (kN)	β for pdf of f_y			
	2	3	7	9
43.2		3.34	4.03	4.44
57.6	5.84	3.51	3.88	4,81
72.0	5.45	3.60	3.72	4.90
86.4	5.06	3.62	3.56	4.79
100.8	4.67	3.56	3.39	4.55
115.2	4.28	3.45	3.21	4.24
129.6	3.90	3.29	3.03	3.88
144.0	3.52	3.09	2.83	3.50
258.4	3.15	2.85	2.63	3.11
172.8	2.77	2.58	2.41	2.71

Table 5. 28: Variation of β with dead load, G.

G (kN)	β for pdf of f_y			
	2	3	7	9
4	3.54	3.10	2.84	3.52
5	3.52	2.09	2.83	3.50
10	3.42	3.02	2.73	3.39
15	3.33	2.95	2.67	3.28
20	3.23	2.88	2.62	3.18
25	3.13	2.81	2.56	3.07
30	3.03	2.74	2.50	2.97
35	2.94	2.67	2.02	2.86
75	2.17	2.07		2.06

Table 5.29: Variation of β with flange width, B_f.

B_f (mm)	β for pdf of f_y			
	2	3	7	9
700	3.52	3.09	2.83	3.50
875	4.82	4.04	3.59	4.92
1050	5.93	4.81	4.21	
1225		5.47	4.75	
1400			5.21	
1575			5.63	

Table 5.30: Variation of β with overall depth, D.

D (mm)	β for pdf of f_y			
	2	3	7	9
960	2.07			
1080	2.82	2.55	2.40	2.75
1200	3.52	3.09	2.82	3.50
1320	4.17	3.57	3.21	4.21
1440	4.78	4.01	3.56	4.88
1560	5.35	4.41	3.88	5.51
4.78	5.89	4.78	4.18	
1800		5.13	4.46	
1920		5.45	4.72	
2040		5.76	4.97	
2160			5.21	
2280			5.43	

Table 5.31: Variation of β with flange thickness, t_f.

t_f (mm)	β for pdf of f_y			
	2	3	7	9
40	2.37	2.21	2.12	2.27
45	2.97	2.67	2.50	2.90
50	5.52	3.09	2.83	3.50
55	4.04	3.46	3.13	4.07
80	4.52	3,80	3.40	4.60
65	4.97	4.12	3.64	5.11
70	5.40	4.40	3.87	5.58
75	5.80	4.67	4.07	
80		4.91	4.27	
85		5.14	4.45	
90		5.35	4.62	
95		5.55	4.78	
100		5.74	4.93	
105		5.92	5.08	
115			5.21	
120			5.34	
125			5.47	
130			5.58	
135			5.70	
140			5.81	
			5.91	

Table 5.32: Variation of β with web thickness, t_w.

t_w (mm)	β for pdf of f_y			
	2	3	7	9
5	3.38	2.98	2.75	3.34
10	3.57	3,09	2.83	3.50
15	3.66	3.19	2.91	3.65
20	3.80	3.29	2.99	3.81
25	3.94	3.39	3.07	3.96
30	4.0	3.49	3.15	4.11
35	4.21	3.59	3.22	4.26
40	4.34	3.68	3.30	4.41
45	4.47	3.77	3.37	4.55
50	4.60	3.86	3.44	4.69
55	4.73	3.95	3.51	4.84
60	4.86	4.03	3.58	4.98
80	5.34	4.36	3.83	5.52
100	5.81	4.67	4.08	
120		4.96	4.30	
140		5.23	4.52	
160		5.48	4.72	
180		5.73	4.92	
200		5.95	5.11	
220			5.29	
240			5.46	
260			5.62	
280			5.78	
300			5.94	

Table 5.33 Variation of β with Ratio G/Q.

G/Q	β for pdf of f_y			
	2	3	7	9
0.02	3.50	3.07	2.81	3.48
0.04	3.39	2.98	2.74	3.36
0.06	3.28	2.90	2.68	3.24
0.08	3.17	2.82	2.61	3.12
0.10	3.07	2.74	2.55	3.01
0.15	2.81	2.54	2.39	2.74
0.20	2.58	2.36	2.23	2.25
0.25	2.35	2.18	2.08	2.02

Table 5.34:Variation of β with Ratio B_f/t_f.

B_f/t_f	β for pdf of f_y			
	2	3	7	9
12	2.34	2.22	2.14	2.24
14	3.08	2.83	2.66	3.01
16	3.73	3.36	3.10	3.69
18	4.32	3.83	3.49	4.31
20	4.85	4.25	3.83	4.88
22	5.33	4.63	4.14	5.39
24	5.78	4.98	4.43	4.86
26		5.30	4.69	
28		5.59	4.94	
30		5.87	5.17	

Table 5.35:Variation of β with Ratio B_f/D.

B_f/D	β for pdf of f_y			
	2	3	7	9
0.50	2.57	2.36	2.24	2.48
0.60	3.58	3.14	2.87	3.56
0.70	4.48	3.80	3.40	4.54
0.80	5.28	4.38	3.86	5.43
0.90		4.88	4.27	
0.00		5.34	4.63	
1.10		5.75	4.97	
1.20			5.28	
1.30			5.57	
1.40			5.84	

Table 5.36: Variation of β with Ratio D/t_f.

D/t_f	β for pdf of f_y			
	2	3	7	9
20	2.06			
25	3.30	3.03	2.84	3.23
30	4.33	3.87	3.53	4.31
35	5.21	4.58	4.11	5.24
40	5.98	5.19	4.62	
45		5.73	5,07	
50			5.48	
55			5.86	

Table 5:37: Variation of β with Ratio D/t_w

D/t_w	β for pdf of f_y			
	2	3	7	9
100	2.11	2.01		2.00
110	2.69	2.48	2.36	2.60
120	3.22	2.91	2.71	3.16
130	3.73	3.31	3.04	3.70
140	4.20	3.67	3.33	4.20
150	4.64	4.01	3.60	4.68
160	5.06	4.32	3.85	5.13
170	5.46	4.62	4.09	5.56
180	5.83	4.90	4.32	5.97
190		5.16	4.53	
200		5.41	4.74	
210		5.64	4.93	
220		5.87	5.12	
230			5.30	
240			5.47	
250			5.64	
260			5.80	
270			5.96	

Table 5.38: Variation of β, with yield strength, f_y

f_y (N/mm^2)	β for pdf of f_y			
	2	3	7	9
180	2.24	2.15	2.09	2.20
206	3.06	2.81	2.64	2.99
231	3.88	3.42	3.12	3.87
257	4.69	3.97	3.54	4.76
283	5.47	4.47	3.92	5.65
308		4.93	4.28	
334		5.36	4.60	

Table 5.39: Variation of β in live load, Q.

Q (kN)	β for pdf of f_y			
	2	3	7	9
576		3.93	4.44	5.71
720		4.11	4.30	5.91
864		4.21	4.16	5.88
1008	5.77	4.23	4.01	5.70
1152	5.40	4.20	3.86	5.43
1296	5.04	4.20	3.86	5.43
1440	4.69	3.97	3.54	4.76
1584	4.33	3.79	3.37	4.39
1728	3.98	3.58	3.20	4.02
1872	3.64	3.35	3.01	3.65
2016	3.30	3.10	2.82	3.28
2304	2.63	2.54	2.40	2.56
2448	2.30	2.25	2.17	2.22

Table 5.40 Variation of β with dead load, G.

G (kN)	β for pdf of f_y			
	2	3	7	9
15	4.83	4.06	3.61	4.93
30	4.80	4.04	3.59	4.89
45	4.75	4.01	3.58	4.84
60	4.72	3.99	3.56	4.80
75	4.69	3.97	3.54	4.76
90	4.65	3.95	3.52	4.72
105	4.61	3.92	3.51	4.67
120	4.57	3.90	3.49	4.63
135	4.54	3.88	3.47	4.59
150	4.50	3.85	3.45	4.55
165	4.46	3.83	3.44	4.51
180	4.43	3.80	3.42	4.47
195	4.39	3.78	3.40	4.43
210	4.35	3.76	3.38	4,39
225	4.32	3.73	3.37	4.34
300	4.14	3.61	3.27	4.14
375	3.95	3.49	3.18	3,94
450	3.78	3.36	3.09	3.75
525	3.60	3.24	2.99	3.55
600	3.42	3.11	2.89	3.36
675	3.24	2.98	2.78	3.18
750	3.07	2.84	2.68	2.99
1120	2.20	2.14	2.10	2.11

Table 5.41: Variation of β with overall depth, D.

D (mm)	β for pdf of f_y			
	2	3	7	9
840	2.36	2.20	2.11	2.26
960	3.22	2.87	2.66	3.16
1080	3.98	3.45	3.13	3.99
1200	4.69	3.97	3.54	4.76
1320	5.33	4.43	3.91	5.46
1440	5.92	4.84	4.24	
1560		5.22	5.55	

Table 5.42: Variation of β with flange thickness, t_f.

t_f (mm)	β for pdf of f_y			
	2	3	7	9
25	4.96	4.17	3.70	5.06
50	4.69	3.97	3.54	4.76
75	4.40	3.74	3.38	4.45
100	4.11	3.54	3.20	4.13
125	3.80	3.32	3.02	3.79
150	3.48	3.07	2.83	3.45
175	3.15	2.82	2.62	3.04
200	2.80	2.55	2.40	2.72
225	2.44	2.26	2.16	
250	2.06			

Table 43 : Variation of β with web thickness, t_w.

t_w (mm)	β for pdf of f_y			
	2	3	7	9
7	2.48	2.31	2.22	2.38
8	3.26	2.93	2.72	3.21
9	4.00	3.48	3.16	4.00
10	4.59	3.97	3.54	4.76
11	5.33	4.41	3.89	5.47
12	5.94	4.82	4.21	
13		5.19	4.50	
14		5.53	4.78	
15		5.53	5.04	

Table 5.44: Variation of β with Ratio G/Q.

G/Q	β for pdf of f_y			
	2	3	7	9
0.005	4.84	4.06	3.16	4.94
0.010	4.81	4.04	3.59	4.91
0.015	4.78	4.02	3.58	4.87
0.020	4.75	4.00	3.56	4.82
0.040	4.63	3.91	3.49	4.71
0.060	4.52	3.83	3.43	4.59
0.080	4.41	3.75	3.36	4.46
0.10	4.30	3.67	3.30	4.34
0.20	3.78	3.29	2.99	3.78

Table 5.45 Variation of β with Ratio D/t_f

D/t_f	β for pdf of f_y			
	2	3	7	9
30	2.19	2.06	2.00	2.08
35	3.05	2.76	2.58	2.99
40	3.82	3.36	3.07	3.81
45	4.52	3.89	3.49	4.56
50	5.15	4.36	3.87	5.25
60		5.16	4.52	
65		5.51	4.81	

Table 5.46: Variation of β with Ratio D/t_w.

D/t_w	β for pdf of f_y			
	2	3	7	9
150	3.03			
160	2.36			
170	2.66	2.50	2.39	2.25
180	3.96	2.74	2.59	2.57
190	3.32	2.97	2.79	2.88
200	3.50	3.19	2.97	3.17
225	4.11	3.69	3.38	4.09
250	4.66	4.13	3.74	4.67
275	5.16	4.53	4.07	5.20
300	5.67	4.89	4.37	5.68
325		5.22	4.64	

5.4 Discussion of Results.

Appropriate computer programmes (shown in appendix A) were employed in this study to compute the safety levels, β, implicit in BS 449 provision for design of girders subjected to UDL and point load for bending and shear considerations. All basic variables were prescribed normal except for yield strength, f_y. For f_y the distributions prescribed and their coded numbers are: normal = 2; lognormal = 3; gumbel= 7; and weibull = 9. Therefore subsequent references to these distributions relate to f_y only. The β values in Tables 5.2 to 5.46 were studied with the following deductions.

With respect to varying span for UDL in bending condition (Table 5.2), for 12m span the following reliability levels in descending order (with respect to choice of distribution for f_y) were obtained: weibull, β = 5.22; normal, β = 4.96; lognormal, β = 3.35; and gumbel, β = 3.04. Shear condition (Table 5.16) also gave a similar order with respect to choice of distributions except that higher reliability levels were obtained (i.e weibull, β > 6; normal, β = 5.85; lognormal β=4.80; and gumbel, β = 4.20). With basic variables prescribed normal, computed β ranged from 4.96 to 2.08 for a corresponding span of 11-14m in bending condition and β ranged from 5.85 to 2.00 for span range of 12m to 23m in shear condition. This implies that the rate of change of β is higher for bending condition and also under UDL consideration shear requirement

gives higher reliability levels.

In girders subjected to point load in bending (Table 5.25), 12m span gave the following reliability levels in descending order with respect to the prescribed distributions: weibull, $\beta = 5.01$; normal, $\beta = 4.90$; lognormal, $\beta = 4.90$; and gumbel, $\beta = 3.63$. The trend was similar to what was obtained for UDL consideration but variation in span does not influence β value for shear condition in girders subjected to point load.

At a yield stress, $f_y = 257N/mm^2$ under a UDL in bending condition (Table 5.3), obtained β values for the prescribed distributions are: $\beta = 3.15$ for weibull; $\beta = 2.92$ for normal, $\beta = 2.63$ for lognormal and $\beta = 2.46$ for gumbel. Similarly for shear condition (Table 5.16), $\beta = 5.93$ for weibull; $\beta = 5.35$ for normal; $\beta = 4.44$ for lognormal; and $\beta = 3.92$ for gumbel. The lower reliability in bending condition shows that it controls the design.

Under varying live load values the computed reliability levels are dependent on the types of distributions assumed by the material yield strength. This is clearly shown in Table 5.4. At various levels of f_y different order of the prescribed distributions were obtained.

The variations of load ratios are shown in Table 5.11 and

5.22, considerations are given to bending and shear under UDL respectively. The values of β are higher for shear than bending. Similarly Tables 5.33 and 5.44 depict the variation of load ratios for bending and shear requirements under point load. In all cases the probability of failure levels are higher for bending criterion than shear.

Varying aspect ratio B_f/t_f with all distributions prescribed normal showed that, under UDL in bending condition (Table 5.10), reliability levels increase from 2.27 to 5.87 for a ratio of 8 to 17 while for point load in bending (Table 5.34) gave β range of 2.34 to 5.78 for a ratio 12 to 24. Higher reliability was obtained for UDL than point load.

In appendix B plots for β values obtained for varying design parameters to show their trend with respect to prescribed probability distributions are given. Significant increases or decreases are observed with changing design paramaters.

5.5 Significance of Design Parameters.

The parameters considered here are those involved in the fabrication of the steel section in consideration. They are the specified yield strength (f_y), flange width (B_f), overall depth (D), flange thickness (t_f) and web thickness (t_w).

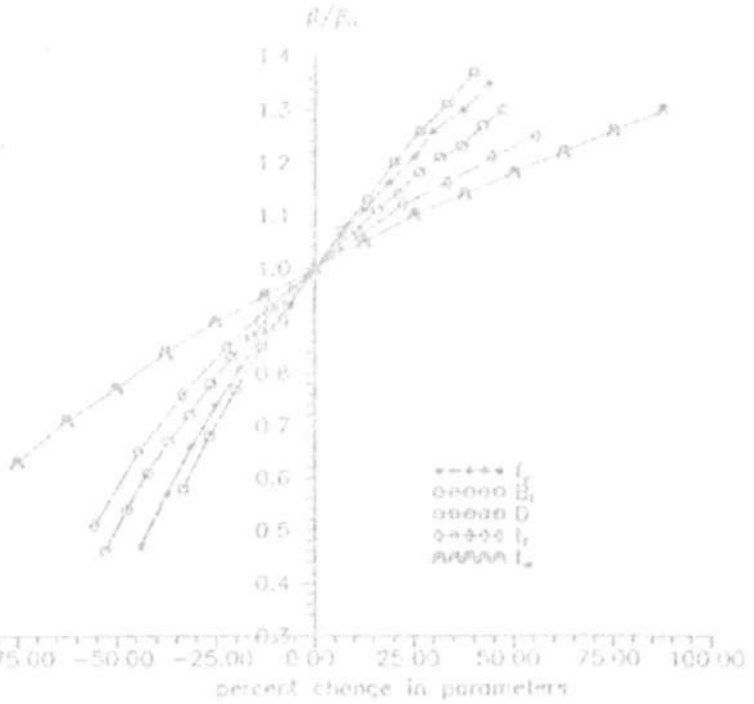


Fig 5.2: Sensitivity Plots for Bending Condition Under Uniformly Distributed Load.

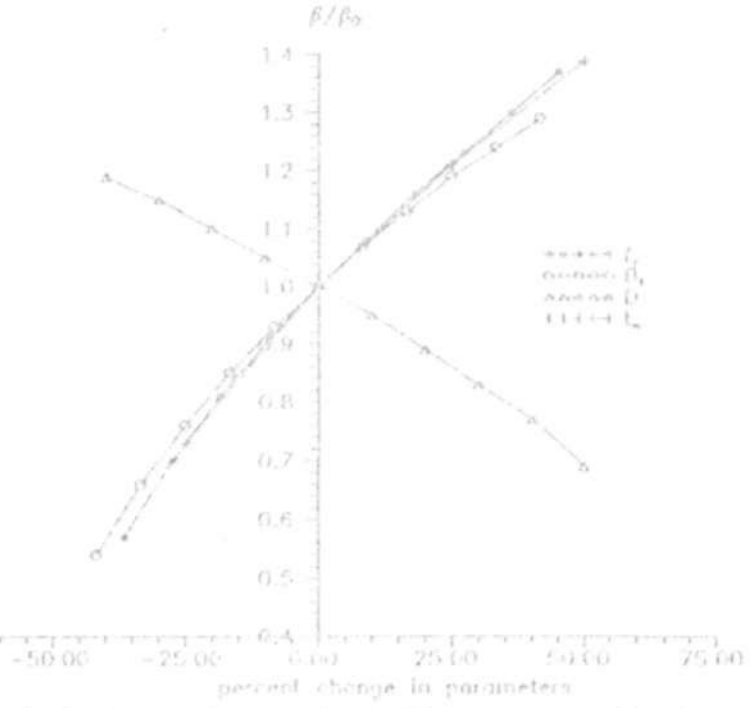


Fig 5.3: Sensitivity Plots for Shear Condition Under Uniformly Distributed Load.

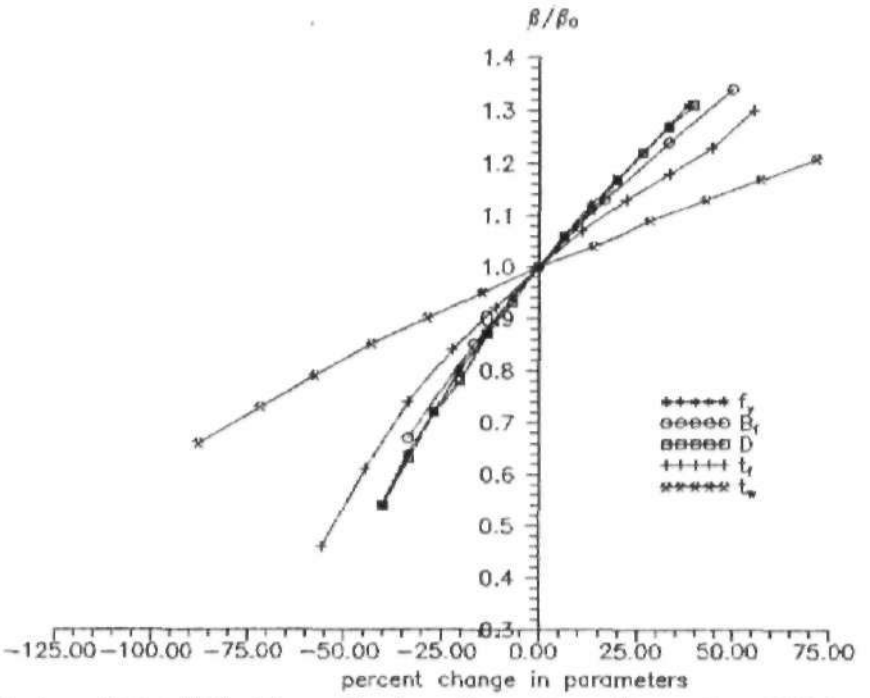


Fig 5.4: Sensitivity PLots for Bending Condition Under Point Load.

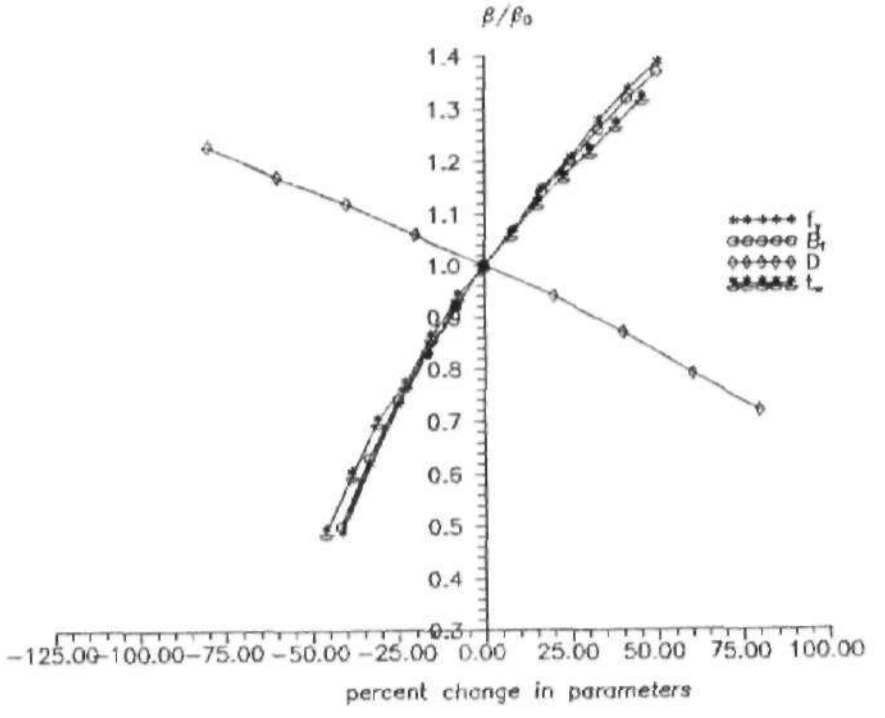


Fig 5.5: Sensitivity Plots for Shear Condition Under Point Load.

Graphical plots for the various sensitivity studies are shown in figs 5.2 to 5.5. For each of these parameters considered, a central reliability level is taken as βo and the corresponding value of the parameter is noted (reference point). For an increase or decrease above or below the reference point of a design parameter the reliability ratio β/β_o is computed for the varying β values. Comparisons made in the plot for the different design parameters follows.

Considering UDL in bending (fig 5.2) a 10% increase in t_w showed about 4% change in reliability, an equal increase showed about 6% increase for t_f and corresponding 7.8% increase was observed B_f, f_y and D have about 8.2% increase in reliability level. A 10% decrease gave about 3.8% decrease for t_w, 5% decrease for t_f, 8.8% decrease for B_f and 10% decrease for f_y and D. These indicate that f_y and D influence reliability most and the effect is more with decrease in values of these parameters.

In shear condition also for UDL (fig 5.3) a 10% increase in the values of the parameters showed about 7.9% increase in reliability level for D, 8% increase for t_w and 9% increase for f_y. t_f decreased reliability by 5%. A 10% decrease in the value of the parameters showed 8% decrease for D, 10% decrease for f_y and t_w and a 5% increase for t_f. This implies that t_f influences reliability inversely in shear condition.

Sensitivity study of girders subjected to point load in bending is shown in fig 5.4. A 10% increase in the parameters showed a 4% increase in reliability for t_w, 6% increase in reliability for t_f, 8% increase for B_f and 8.2% increase for both f_y and D. A corresponding 10% decrease showed 3.8% decrease for t_w, 6.8% decrease for t_f, 8.2% decrease for B_f and 10% decrease for f_y and D. It therefore indicates that decrease in the parameter is more critical to reliability level and f_y and D have the highest effect.

With girders subjected to point load in shear (fig 5.5). a 10% increase in the parameters gave 8% increase in reliability for t_w, and about 8.2% increase for D and f_y. At 20% increase in reliability increased by 15% for t_w, 16% for D and 17% for f_y.With up to 20% decrease in the values of the parameters, t_w and D caused a decrease of reliability by 19.8% and 20.8% for f_y. t_f like in the case for UDL influences reliability level inversely by the same margin. It shows here also that a decrease in the value of the parameters is more critical.

Tables for sensitivity plots are given in appendix C

CHAPTER SIX

CONCLUSIONS AND RECOMMENDATIONS

In this thesis the adequacy of BS 449 provisions with regards to the design criteria for plate girders subjected to uniformly distributed load and point load was assessed. Prediction models use to compute the ultimate strength of girders are compared with experimental results as shown in Figs 4.1 to 4.4. There are observable scatters in test results. The model overestimates the ultimate strength of the girders in some cases while in other cases underestimated values were arrived at. Thus it is expected that these deviations will result in varying reliability levels with regards to the design criteria studied.

The reliability levels associated with the bending and shear limitations were studied. With the basic variate assuming gaussian distribution, varying reliability levels (β) were computed for different values of the basic variables. It was found that β values decrease with increasing values of some of these parameters and decrease with others. For example, for varying span when UDL is considered in bending (Table 5.2), the value of β ranges from 4.96 to 2.92 within a span range of 11m to 13m and for varying yield strength (Table 53) β value ranges from 2.23 to 3.60 within a strength range of

231 N/mm^2 to 283 N/mm^2.

For UDL in shear, the variation in span (Table 5.15) gave a range of β from 5.85 to 4.89 for span range of 12 to 14m and for variation in yield strength (Table 5.16), β ranges from 3.63 to 5.35 for strength range of 206 N/mm^2 to 257 N/mm^2.

For point load in bending the variation in yield strength (Table 5.26) gave β ranging from 2.80 to 4.24 for strength within the range of 231 N/mm^2 to 283 N/mm^2 and for variation of live load (Table 5.27), β ranges form 4.67 to 3,90 for load within the range 100.8 kN to 129.6kN. For shear consideration, varying yield strength (Table 5.38) gave β's ranging form 3.88 to 5.47 for strength within the range of 231 N/mm^2 to 283 N/mm^2 and for varying load (Table 5.39), β ranges from 5.77 to 5.04 for loads within the range of 100.8 to 129.6 KN.

Consequently it is observed that the minimum information required for assessing the implied safety levels in design are expected values (e.g mean) of the design variable plus a measure of the uncertainty (e.g coefficient of variation). These information may be obtained from observed data coupled generally with professional judgement.

In probability method of design, unforeseen changes could be simulated and their influences on safety and performance of

a structure predicted. For instance, there are instances where the usage of a structure is altered with a corresponding drastic change in the loading condition. Gross errors occur in geometric dimensions of sections as induced by the steel rolling procedures, fabrication and erection processes on site. This method gives the associated measure of risk and will also allow for introduction of precautionary signs or measure as to the limitations in the usage of a structure.

The appreciable influence of specified yield strength f_y steel and the overall depth of the I-section girder considered calls for high quality control measures. Laboratory tests to assess the steel grade and good quality workmanship should be ensured.

In conclusion the average of the computed safety index in the study was 3.98 (i.e the probability of failure, $P_f = 3.5 \times 10^{-5}$) which is quite reasonable and within the range 2 to 4 established for code safety index [28]. However, the findings provide other useful information as follows:

(a) The BS 449 design criteria for plate girders are not consistent because the implied reliability levels change significantly with changes in the values of the basic design variables. Also, depending on the probability distribution of the steel grade, different levels of reliability are associated with the design equations for

both bending and shear conditions.

(b) In the design of plate girders bending controls the design.

(c) Shear is more critical with respect to girders subjected to point load than uniformly distributed load.

(d) The overall depth of section and grade of steel are the most influential design parameters as far as steel girders are concerned.

It is important to remark here that the probabilistic approach employed in this study provides an extremely useful procedure for evaluating those aspects of safety that are direct functions of material properties and assumed loading conditions when the basic decision variables are random in nature.

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APPENDIX A: COMPUTER PROGRAMMES FOR THE COMPUTATION OF SAFETY INDICES

```
PROGRAM AI

      PROGRAM  GIRDER
C
C     ASSESSMENT OF CURRENT DESIGN PRACTICE -BS 449
C     CASE 1-UNIFORMLY DISTRIBUTED LOAD
C     BENDING CONSIDERATION
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
C     DECLARE EXTERNAL
      EXTERNAL GGIRDER
      DIMENSION X(6),EX(6),SX(6),VP(10,6),COV(6,6),ZES(3),
     1UU(6),EIVEC(6,6),IV(2,6)
      CHARACTER*10 PRT
      COMMON/GGIRDER/S,RM
C     MEAN,STANDARD DEVIATION AND PARAMETER IN DATA BLOCK
C
      DATA EX/2.57D2,8.50D1,3.5D2,1.0D3,4.0D1,1.2D1/,
     1SX/2.57D1,1.70D1,0.7D0,2.0D0,2.0D0,0.6D0/,
     2N/6/,NC/6/,NE/6/,IRHO/0/
      WRITE(*,*)'RM  ='
      READ(*,*)RM
C      EX(2)=RM*EX(2)
      S=13000.D0
C
C
C     RESULT WRITTEN IN FILE NO.7
      NAUS=7
C
C     PRINT TO SCREEN
      ICRT=0
      OPEN(7,FILE='YUSUF.RES',STATUS='OLD',ERR=10)
      GOTO 20
 10   OPEN(7,FILE='YUSUF.RES',STATUS='NEW')
C
C     PRESETING VARIABLES COV AND IV-YNIT
 20   CALL YINIT(N,IV,VP,IRHO,COV,NC)
      IV(1,1)=3
      IV(1,2)=9
                              IV(1,2)=9
      DO 100 I=1,N
 100  X(I)=EX(I)
      V1=1.0D0
      BETA=1.D0
C
      WRITE(NAUS,5000)
 5000 FORMAT(////,5X,70('*'),/,30X,'FORM',/,5X,70('*'),/,
     120X,'GGIRDER,7 VARIABLES:')
C     MODEL IS PRINTED USING ''YKOPF''
C
```

```
      CALL YKOPF(NAUS,N,IV,EX,SX,VP,IRHO)
C     PRINT ALSO TO SCREEN
      WRITE(ICRT,*)'START OF FORM5'
      WRITE(ICRT,*)'STOCHASTIC MODEL'
C
      CALL YKOPF(ICRT,N,IV,EX,SX,VP,IRHO)
      PRT = ' COV'
      CALL YMAUS(NAUS,NC,N,COV,PRT)
C     THE MAIN SUBROUTINE IN FORM CALLED
      CALL FORM5(N,IV,EX,SX,VP,GGIRDER,IRHO,COV,NC,EIVEC,
     1NE,V1,NAUS,BETA,X,UU,ZES,IER)
C     THE COORDINATES OF THE BETA-POINT ARE
C     PRINTED WITH THE TITLE VECTOR UU
      PRT='UU'
      CALL YFAUS(NAUS,N,UU,PRT)
C     THE VECTOR ZES IS PRINTED-TITLE 'VECTOR ZES'
      PRT='ZES'
      CALL YFAUS(NAUS,3,ZES,PRT)
      WRITE(ICRT,*)'END OF FORM5:IER=',IER
      WRITE(ICRT,*)'RESULTS,SEE FILE YUSUF.RES'
      STOP
      END

      SUBROUTINE GGIRDER(N,X,FX,IER)
C     GIRDER SUBJECTED TO U D L
C     LIMIT STATE FUNCTION
C     BENDING REQUIREMENT
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION X(N)
      COMMON/GGIRDER/S,RM
C
      AMOM=0.125*(X(2)+X(2)*RM)*S**2
      TERM1=2.0*X(3)*X(5)*(0.5*(X(4)-X(5)))**2
      TERM2=X(3)*X(5)**3/6.0
      TERM3=X(6)*(X(4)-2.0*X(5))**3/12.0
      A=TERM1+TERM2+TERM3
C
      FX=1.05*X(1)-AMOM*X(4)/(2.0*A)
      IER=0
      RETURN
      END
```

```
PROGRAM AII

      PROGRAM SHEAR
C     ASSESSEMENT OF CURRENT DESIGN PRACTICE USING BS 449
C
C
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
C     LIMIT STATE FUNTION DECLARED EXTERNAL
      EXTERNAL GSHEAR
C
C     SHEAR CONSIDERATION
      DIMENSION X(5),EX(5),SX(5),VP(10,5),COV(5,5),ZES(3),UU(5)
     1,EIVEC(5,5),IV(2,5)
      CHARACTER*10PRT
C
C     LENGTH S AND LIMIT STATE FUNCTION SET
      COMMON/GSHEAR/S,RM
C
C     MEAN STANDARD DEVIATION AND PARAMETER IN DATA BLOCK
      DATA EX/257.0D0,8.50D1,1.0D3,4.0D1,1.2D1/,
     1SX/2.57D1,1.70D1,2.0D0,2.0D0,0.6D0/,N/5/,NC/5/,
     2NE/5/,IRHO/0/
      WRITE(*,*)'RM    ='
      READ(*,*) RM
C      EX(6)=RM*EX(6)
C
      S=13000.D0
C
C     RESULT WRITTEN IN FILE NO 7
      NAUS=7
C     PRINT TO SCREEN
      ICRT=0
      OPEN(7,FILE='ABDUL.RES',STATUS='OLD',ERR=10)
      GOTO 20
 10   OPEN(7,FILE='ABDUL.RES',STATUS='NEW')
C     PRESETING VARIABLES VP,COV AND IV-YINIT
 20   CALL YINIT(N,IV,VP,IRHO,COV,NC)
      IV(1,1)=3
      IV(1,2)=9
      DO 100 I=1,N
 100  X(I)=EX(I)
      V1=1.0D0
      BETA=1.0D0
C
      WRITE(NAUS,5000)
 5000 FORMAT(////,5X,70('*'),/,30X,'FORM5',/,5X,70('*'),/,20X,
     *       'GSHEAR,6 VARIABLES:')
C
C     MODEL IS PRINTED USING "YKOPF"
      CALL YKOPF(NAUS,N,IV,EX,SX,VP,IRHO)
C
C     PRINT ALSO TO SCREEN
```

```
      WRITE(ICRT,*)'STOCHASTIC MODEL:'
      CALL YKOPF(ICRT,N,IV,EX,SX,VP,IRHO)
C
C     MATRIX COV IS PRINTED USING "YMAUS"
      PRT='COV'
      CALL YMAUS(NAUS,NC,N,COV,PRT)
C
C     THE MAIN SUBROUTINE IN FORM CALLED
      CALLFORM5(N,IV,EX,SX,VP,GSHEAR,IRHO,COV,NC,EIVEC,NE,V1,
     1          NAUS,BETA,X,UU,ZES,IER)
C     THE COORDINATE OF BETA - POINT PRINTED , TITLE VECTOR UU
      PRT='UU'
      CALL YFAUS(NAUS,N,UU,PRT)
C     THE VECTOR ZES IS PRINTED
      PRT='ZES'
      CALL YFAUS(NAUS,3,ZES,PRT)
      WRITE(ICRT,*)'END OF FORM :IER=',IER
      WRITE(ICRT,*)'RESULT SEE ABDUL.RES'
      STOP
      END


      SUBROUTINE GSHEAR(N,X,FX,IER)
C     GIRDER SUBJECTED TO U D L
C     LIMIT STATE FUNCTION
C     SHEAR REQUIREMENT
C
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION X(N)
      COMMON/GSHEAR/S,RM
C
      FX=0.64*X(1)-(0.5*(X(2)+X(2)*RM)*S)/((X(3)-2.0*X(4))
     1  *X(5))
      IER=0
      RETURN
      END
```


456078

PROGRAM AIII

```
      PROGRAM TRIAL
C
C     ASSESSMENT OF CURRENT DESIGN PRACTICE USING BS 449
C
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
C
C     LIMIT STATE FUNCTION DECLARED EXTERNAL
      EXTERNAL GTRIAL
C
C     BENDING CONSIDERATION FOR POINT LOAD
      DIMENSION X(6),EX(6),SX(6),VP(10,6),COV(6,6),ZES(3),
     1UU(6),EIVEC(6,6),IV(2,6)
      CHARACTER*10 PRT
C
C     LENGTH S,LOAD LOCATION AL,LIMIT STATE SET
      COMMON/ GTRIAL/S,AL,RM
C
C     MEAN STANDARD DEVIATION AND PARAMETER IN DATA BLOCK
      DATA EX/257.00D0,1.44D6,7.50D4,7.0D2,5.0D1,
     11.0D1/,SX/25.7D0,290.88D3,9.0D3,1.4D0,2.50

     2D0,0.50D0/,N/6/,NC/6/,NE/6/,IRHO/0/
      WRITE(*,*)'RM     ='
      READ(*,*)RM
C      EX(7)=RM*EX(7)
C
C
      S=15000.D0
      AL=S/2.0
C
C     RESULT WRITTEN IN FILE NO 7
      NAUS=7
C     PRINT TO SCREEN
      ICRT=0
      OPEN(7,FILE='UNYAH.RES',STATUS='OLD',ERR=10)
      GOTO 20
 10   OPEN(7,FILE='UNYAH.RES',STATUS='NEW')
C
 20   CALL YINIT(N,IV,VP,IRHO,COV,NC)
      IV(1,1)=3
      IV(1,2)=9
      DO 100 I=1,N
 100  X(I)=EX(I)
      V1=1.0D0
      BETA=1.0D0
C
      WRITE(NAUS,5000)
 5000 FORMAT(////,5X,70('*'),/,30X,'FORM5',/,5X,70('*'),/,
     120X,'GTRIAL,7 VARIABLE:')
C
```

```
C      MODEL IS PRINTED USING "YKOPF"
       CALL YKOPF(NAUS,N,IV,EX,SX,VP,IRHO)
C
C      PRINT ALSO TO SCREEN
       WRITE(ICRT,*)'START OF FORM5'
       WRITE(ICRT,*)'STOCHASTIC MODEL:'
       CALL YKOPF (ICRT,N,IV,EX,SX,VP,IRHO)
C
C      MATRIX COV IS PRINTED USING "YMAUS"
       PRT='COV'
       CALL YMAUS(NAUS,NC,N,COV,PRT)
C      THE MAIN SUBROUTINE IN FORM IS CALLED
       CALL FORM5(N,IV,EX,SX,VP,GTRIAL,IRHO,COV,NC,EIVEC,NE,
      1V1,NAUS,BETA,X,UU,ZES,IER)
C      THE COORDINATE OF BETA-POINT PRINTED TITLE VECTOR UU
       PRT='UU'
       CALL YFAUS(NAUS,N,UU,PRT)
C      VECTOR ZES IS PRINTED
       PRT='ZES'
       CALL YFAUS(NAUS,3,ZES,PRT)
       WRITE(ICRT,*)'END OF FORM5:IER=',IER
       WRITE(ICRT,*)'RESULT SEE FILE UNYAH.RES'
       STOP
       END
```

```
       SUBROUTINE GTRIAL(N,X,FX,IER)
C      GIRDER UNDER POINT LOAD
C      BENDING REQUIREMENT
C
       IMPLICIT DOUBLE PRECISION(A-H,O-Z)
       DIMENSION X(N)
       COMMON/GTRIAL/S,AL,RM
C
       AMOM=X(2)*(S-AL)*AL/S+0.5*X(3)*AL
       TERM1=2.0*X(4)*X(5)*(0.5*(RM*X(6)-X(5)))**2
       TERM2=X(4)*X(5)**3/6.0
       TERM3=X(6)*(RM*X(6)-2.0*X(5))**3/12.0
       A=TERM1+TERM2+TERM3
C
       FX=1.05*X(1)-RM*X(6)*AMOM/(2.0*A)
       IER=0
       RETURN
       END
```

PROGRAM AIV

```
      PROGRAM TEST
C
C     ASSESSMENT OF CURRENT DESIGN PRACTICE USING BS 449
C
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
C     LIMIT STATE FUNCTION DECLARED EXTERNAL
      EXTERNAL GTEST
C
C     SHEAR CONSIDERATION FOR POINT LOAD
      DIMENSION X(5),EX(5),SX(5),VP(10,5),COV(5,5),ZES(2),
     1          UU(5),
     1EIVEC(5,5),IV(2,5)
      CHARACTER*10PRT
C
C     LENGTH S,LOAD LOCATION AL,LIMIT STATE SET
      COMMON/GTEST/S,AL,RM
C
C     MEAN STANDARD DEVIATION AND PARAMETER IN DATA BLOCK
      DATA EX/257.00D0,1.44D6,7.50D4,5.0D1,1.0D1/,
     1SX/25.7D0,290.88D3,9.0D3,2.50D0,0.5D0/,N/5/,
     2NC/5/,NE/5/,IRHO/0/
C
      WRITE(*,*)'RM     ='
      READ(*,*)RM
C      EX(6)=RM*EX(6)
C
      S=15000.D0
      AL=S/2.0
C
C     RESULT WRITTEN IN FILE NO 7
      NAUS=7
C     PRINT TO SCREEN
      ICRT=0
      OPEN(7,FILE='SHEHU.RES',STATUS='OLD',ERR=10)
      GOTO 20
 10   OPEN(7,FILE='SHEHU.RES',STATUS='NEW')
C
 20   CALL YINIT(N,IV,VP,IRHO,COV,NC)
      DO 100 I=1,N
      IV(1,1)=3
      IV(1,2)=9
 100  X(I)=EX(I)
      V1=1.0D0
      BETA=1.0D0
C
      WRITE(NAUS,5000)
 5000 FORMAT(////,5X,70('*'),/,30X,'F O R M 5',/,5X,70('*'),
     1/,20X,'GTEST, 6 VARIABLES:')
C
C     MODEL IS PRINTED USING "YKOPF"
```

```
      CALL YKOPF(NAUS,N,IV,EX,SX,VP,IRHO)
C     PRINT ALSO TO SCREEN
      WRITE(ICRT,*)'STOCHASTIC MODEL'
      CALL YKOPF(ICRT,N,IV,EX,SX,VP,IRHO)
C
C     MATRIX COV IS PRINTED USING"YMAUS"
      PRT='COV'
      CALL YMAUS(NAUS,NC,N,COV,PRT)
C     THE MAIN SUBROUTINE IN FORM IS CALLED
      CALL FORM5(N,IV,EX,SX,VP,GTEST,IRHO,COV,NC,EIVEC,
     1NE,V1,NAUS,BETA,X,UU,ZES,IER)
C
C     COORDINATE OF BETA - POINT PRINTED TITLE VECTOR UU
      PRT='UU'
      CALL YFAUS(NAUS,N,UU,PRT)
      PRT='ZES'
      CALL YFAUS(NAUS,3,ZES,PRT)
      WRITE(ICRT,*)'END OF FORM:IER=',IER
      WRITE(ICRT,*)'RESULT SEE SHEHU.RES'
      STOP
      END


      SUBROUTINE GTEST(N,X,FX,IER)
C     GIRDER UNDER POINT LOAD
C     SHEAR REQUIREMENT
C
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION X(N)
      COMMON/GTEST/S,AL,RM
C
      VMAX=(X(2)+X(2))*AL/S
C
      FX = 0.64*X(1) - VMAX/((RM*X(5)-2.0*X(4))*X(5))
      IER=0
      RETURN
      END
```

APPENDIX B: SELECTED PLOTS OF VARIATION OF SAFETY INDICES, β

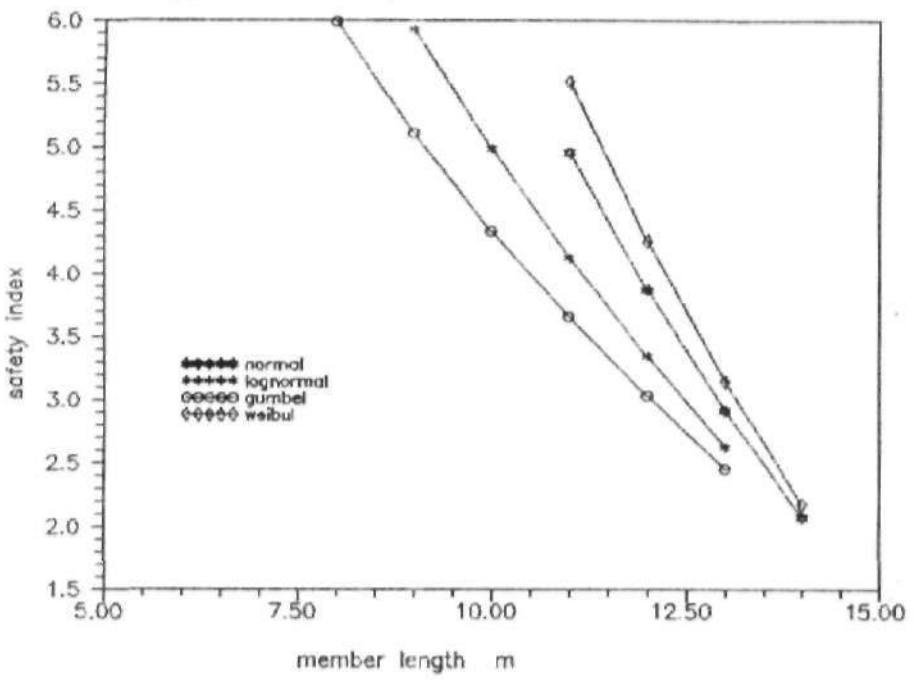


Fig. B1: Variation of β with the span, L (UDL in bending)

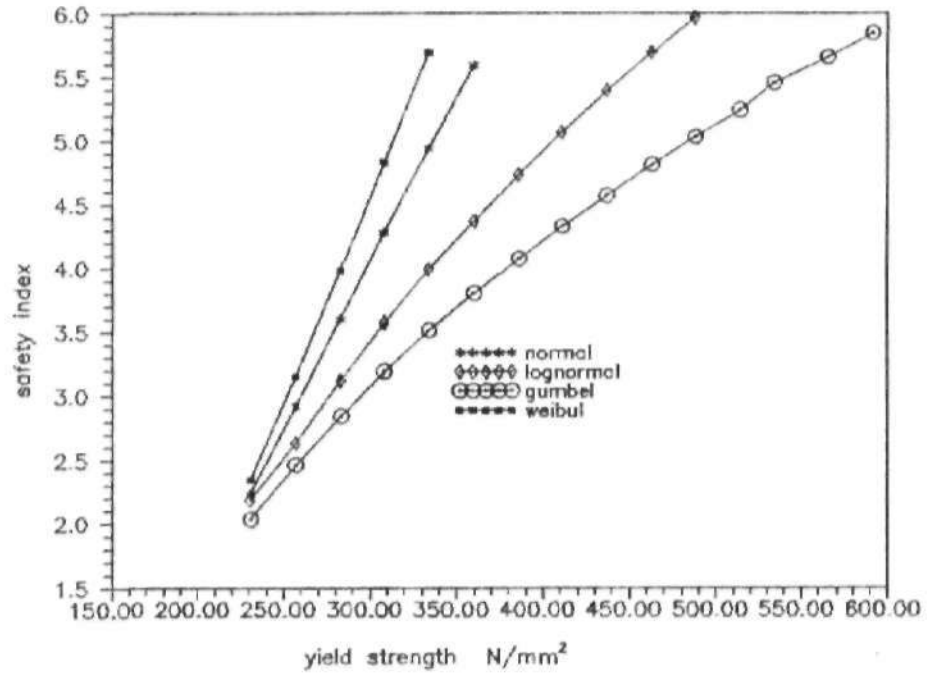


Fig. B2: Variation of β with yield strength, f_y (UDL in bending)

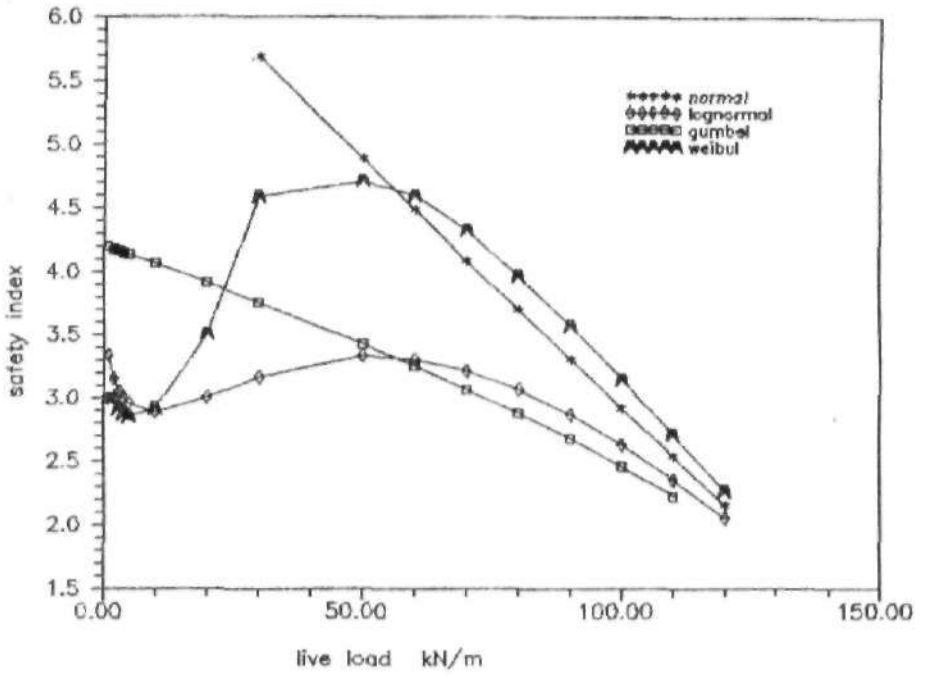


Fig. B3: Variation of ß with the live load, q (UDL in bending)

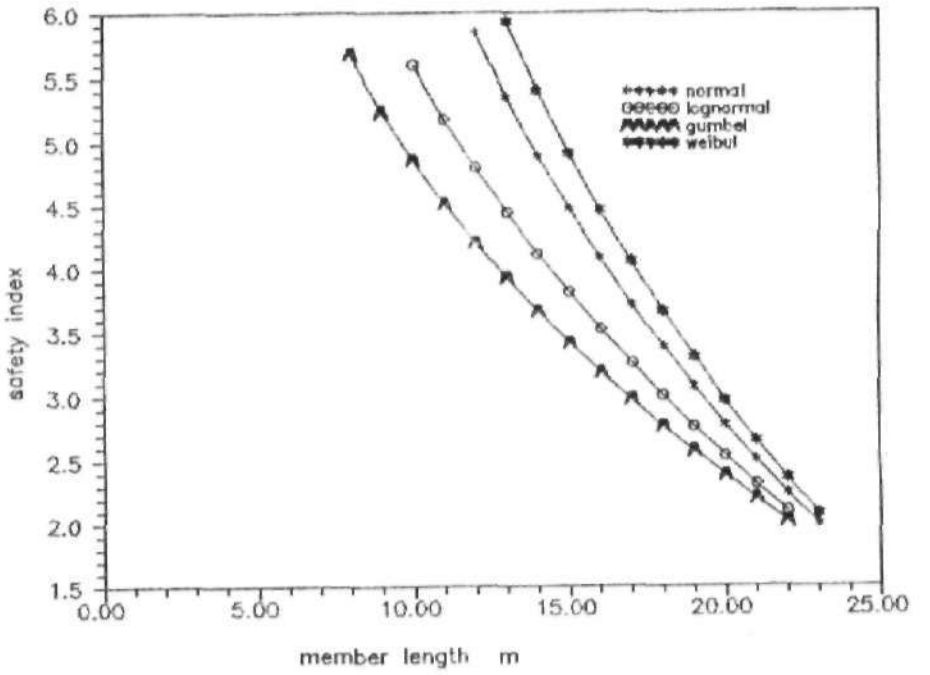


Fig. B4: Variation of ß with the span, L (UDL in shear)

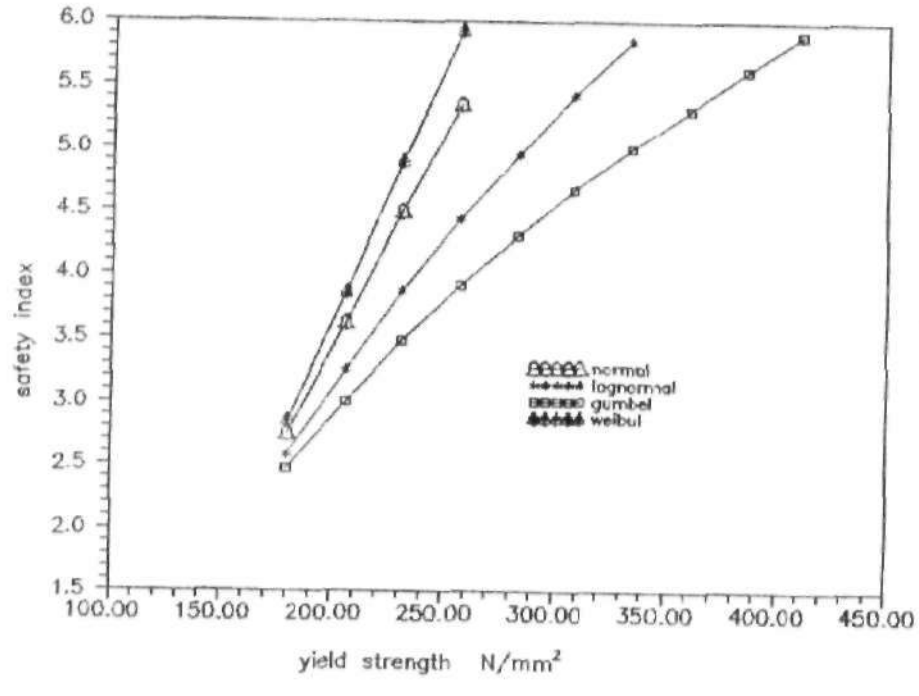


Fig. B5: Variation of β with yield strength, f_y (UDL in shear)

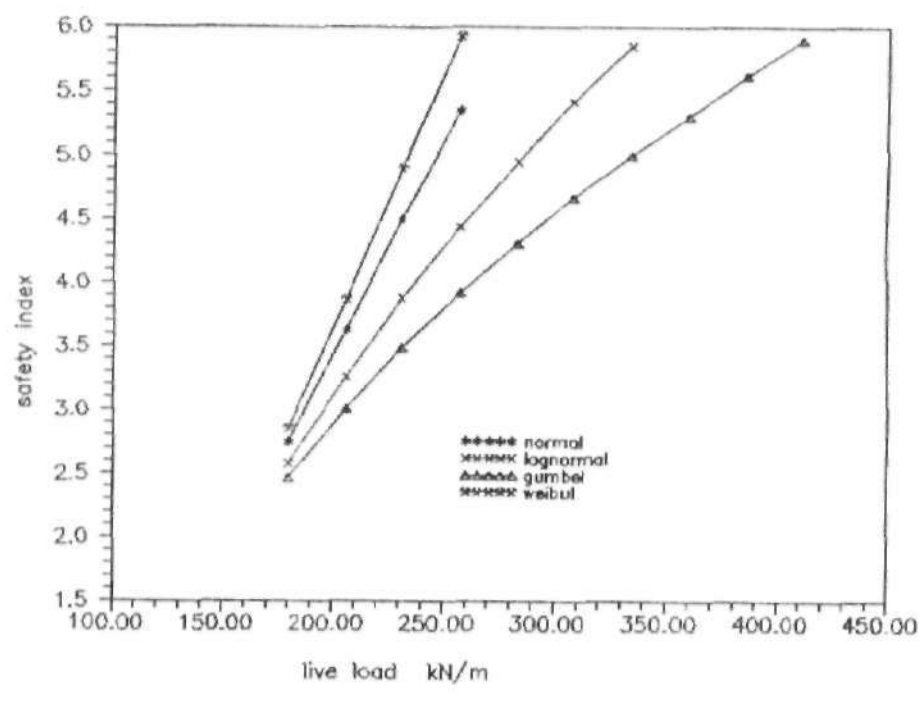


Fig. B6: Variation of β with live load, q (UDL in shear)

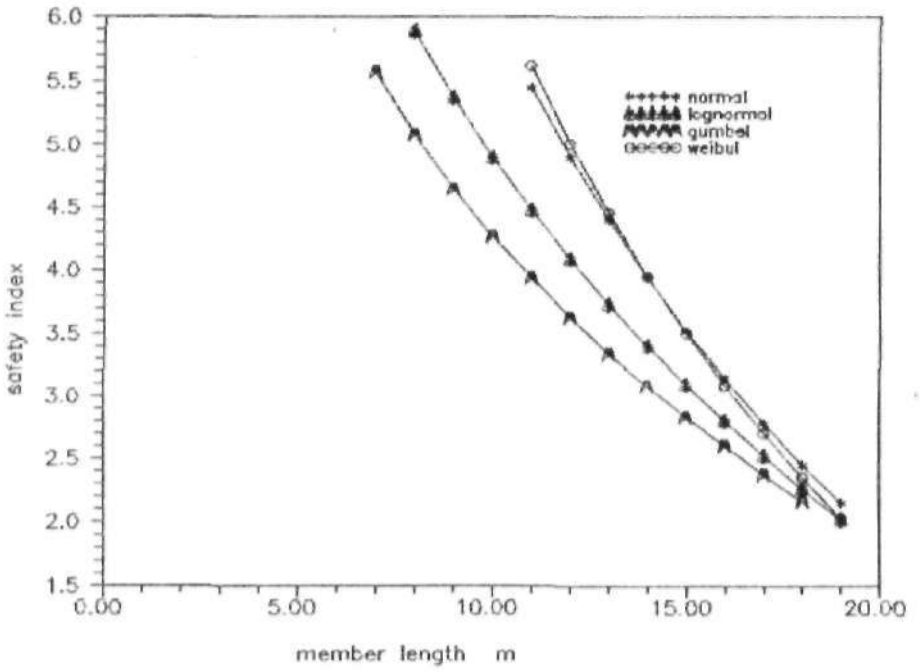


Fig. B7: Variation of β with the span, L (Point load in bending)

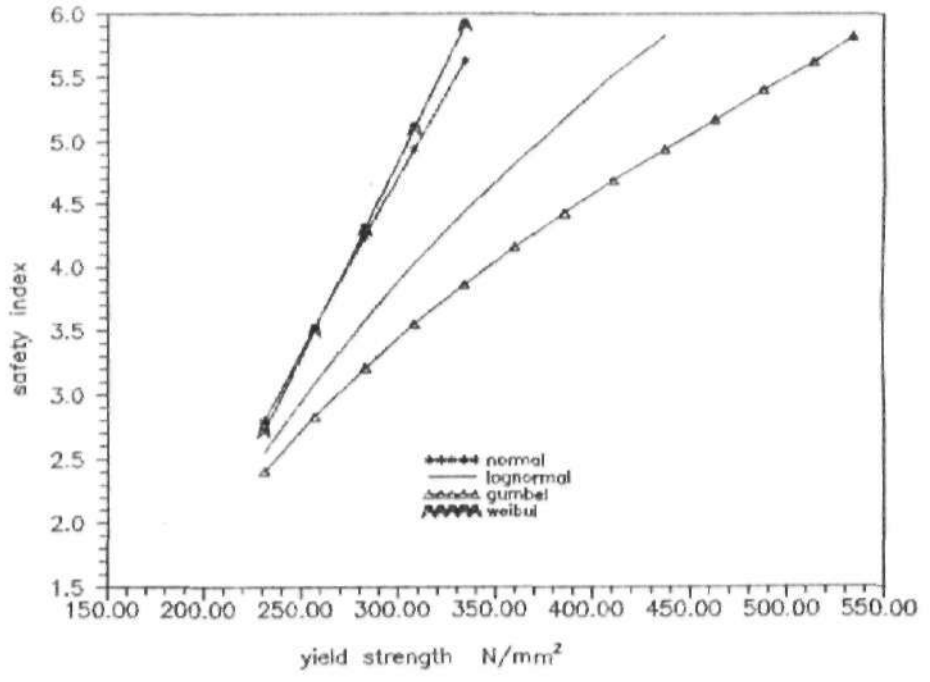


Fig. B8: Variation of β with yield strenth, f_y (Point load in bending)

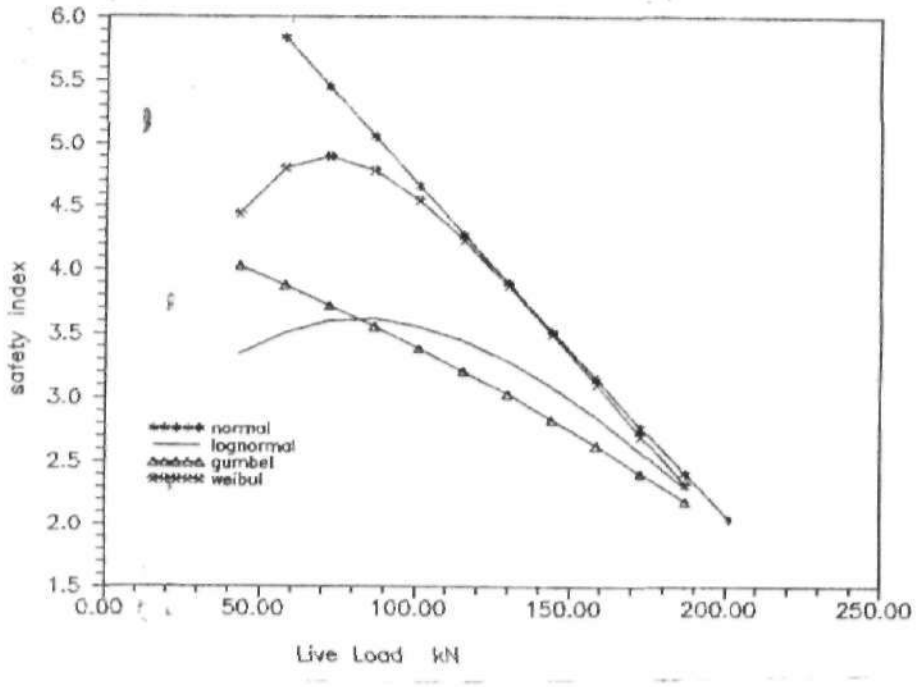


Fig. B9: Variation of β with live load, Q (Point load in bending)

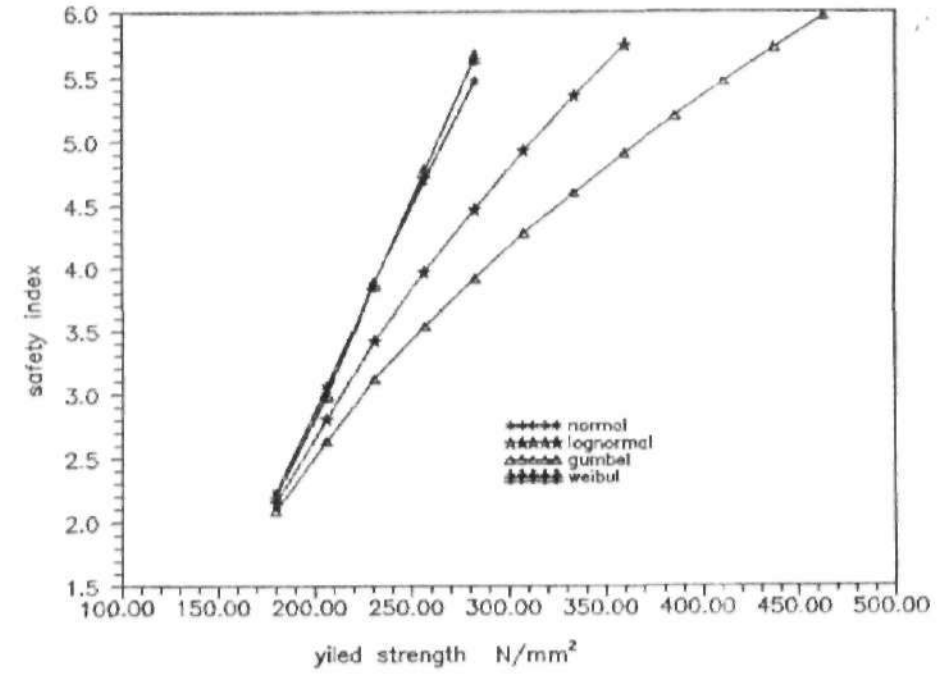


Fig. B10: Variation of β with yield strength, f_y (Point load in shear)

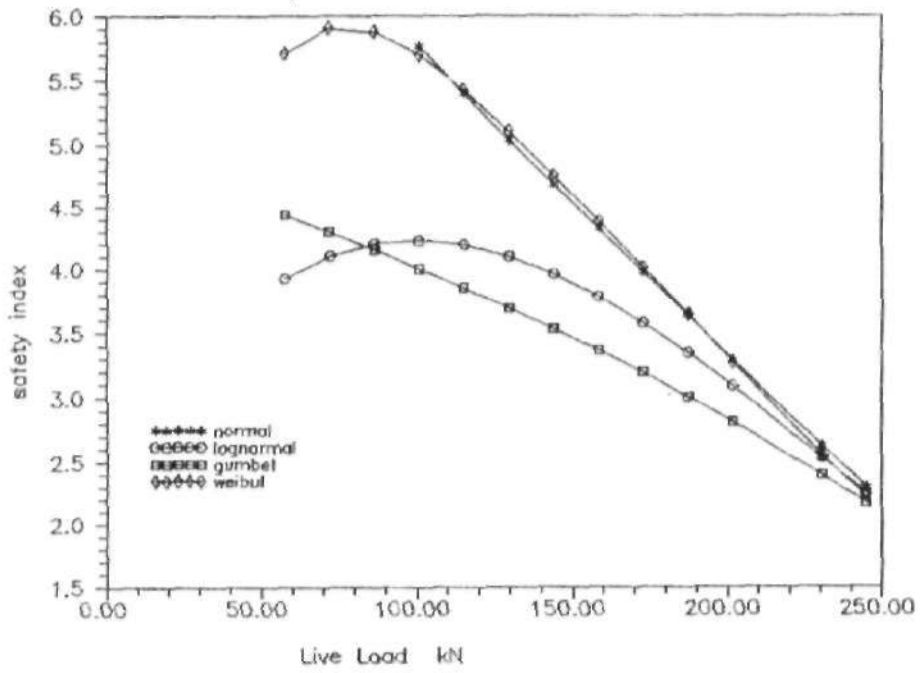


Fig. B11: Variation of β with live load, Q (Point load in shear)

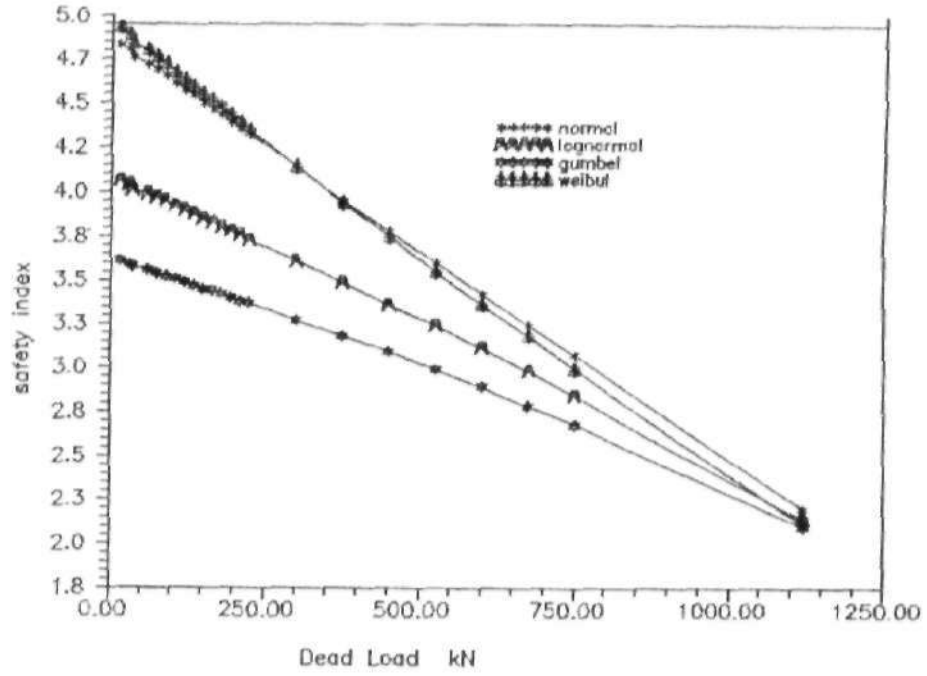


Fig. B12: Variation of β with dead load, G (Point load in shear)

APPENDIX C: TABLES OF VALUES FOR SENSITIVITY PLOT

CI: BENDING CONSIDERATION FOR UNIFORMLY DISTRIBUTED LOAD

TABLE: 1(I)

% change in f_y	β/β_o
-43.80	0.47
-37.47	0.57
-31.14	0.66
-25.06	0.74
-18.73	0.81
-12.41	0.88
- 6.08	0.93
- 0.00	1.00
6.33	1.06
12.65	1.11
18.73	1.16
25.06	1.21
29.93	1.26
37.47	1.30
43.80	1.35

Table: 1(II)

% change in B_f	β/β_o
-52.63	0.46
-47.37	0.54
-42.11	0.61
-36.84	0.67
-31.58	0.72
-26.32	0.78
-21.05	0.83
-15.79	0.87
-10.53	0.92
- 5.26	0.96
0.00	1.00
5.26	1.04
10.53	1.07
15.79	1.11
21.05	1.14
26.32	1.18
31.58	1.21
36.84	1.23
42.11	1.27
47.37	1.30

Table: 1(III)

% change in D	β/β_o
-33.33	0.58
-26.67	0.68
-20.00	0.77
-13.33	0.85
- 6.67	0.93
- 0.00	1.00
- 6.67	1.07
13.33	1.13
20.00	1.20
26.67	1.25
33.33	1.31
40.00	1.37

Table: 1(IV)

% change in t_f	β/β_o
-55.56	0.51
-44.44	0.65
-33.33	0.76
-22.22	0.85
-11.11	0.93
0.00	1.00
11.11	1.06
22.22	1.12
33.33	1.16
44.44	1.21
55.56	1.24

Table: 1(V)

% change in t_w	β/β_o
-75.0	0.63
-62.5	0.71
-50.0	0.77
-37.5	0.84
-25.0	0.90
-12.5	0.95
0.00	1.00
12.5	1.05
25.0	1.10
37.5	1.14
50.0	1.18
62.5	1.22
75.0	1.26
87.5	1.30

TABLE: 2(I)

% change in f_y	β/β_o
-36.40	0.57
-27.21	0.70
-18.37	0.81
-9.19	0.91
0.00	1.00
8.83	1.08
18.02	1.16
27.21	1.23
36.40	1.30
45.23	1.37

Table: 2(II)

% change in D	β/β_o
-41.67	0.54
-33.33	0.66
-25.00	0.76
-16.67	0.85
-8.33	0.93
0.00	1.00
8.33	1.07
16.67	1.13
25.00	1.19
33.33	1.24
41.67	1.29

Table: 2(III)

% change in t_f	β/β_o
-40.00	1.19
-30.00	1.15
-10.00	1.10
0.00	1.00
10.00	0.95
20.00	0.89
30.00	0.83
40.00	0.77
50.00	0.69

Table: 2(IV)

% change in t_w	β/β_o
-25	0.73
0.00	1.00
25	1.21
50	1.39

CIII: BENDING CONSIDERATION FOR POINT LOAD

Table: 3(I)

change in f_y	β/β_o
-40.16	0.54
-33.42	0.64
-26.68	0.72
-20.21	0.80
-13.47	0.87
- 6.74	0.94
0.00	1.00
6.48	1.06
13.21	1.12
19.95	1.17
33.16	1.27
38.34	1.31

Table: 3(II)

% change in B_f	β/β_o
33.33	0.67
16.67	0.85
0.00	1.00
16.67	1.13
33.33	1.24
50.00	1.34

Table: 3(III)

% change in d	β/β_o
-40.00	0.54
-33.33	0.63
-26.67	0.72
-20.00	0.78
-13.33	0.87
- 6.67	0.93
0.00	1.00
6.67	1.06
13.33	1.11
20.00	1.17
26.67	1.22
33.33	1.27
40.00	1.31

Table: 3(IV)

% change in t_f	β/β_o
-55.56	0.44
-44.44	0.61
-33.33	0.74
-22.22	0.84
-11.11	0.92
0.00	1.00
11.11	1.07
22.22	1.13
33.33	1.18
44.44	1.23
55.56	1.30

Table:3(V)

% change in t_w	β/β_o
-87.51	0.66
71.42	0.73
-57.14	0.79
-42.86	0.85
-28.57	0.90
-14.29	0.95
0.00	1.00
14.29	1.04
28.57	1.09
42.86	1.13
57.14	1.17
71.42	1.21

CIV: SHEAR CONSIDERATION FOR POINT LOAD

Table:4(I)

% change in f_y	β/β_o
41.56	0.49
-33.12	0.62
-25.00	0.73
-16.56	0.83
- 8.12	0.92
-0.00	1.00
- 8.44	1.07
16.88	1.15
25.32	1.21
33.44	1.28
41.88	1.34
50.23	3.39

Table:4(II)

% change in D	β/β_o
41.67	0.50
-33.33	0.63
-25.00	0.74
-16.67	0.83
- 8.33	0.92
0.00	1.00
16.67	1.14
25.00	1.20
33.33	1.26
41.67	1.32
50.00	1.37

Table:4(III)

% change in t_f	β/β_o
-80	1.23
-60	1.17
-40	1.12
-20	1.06
0.00	1.00
20	0.94
40	0.87
60	0.79
80	0.72

Table:4(IV)

% Change in t_w	β/β_o
-46.15	0.49
-38.46	0.60
-30.77	0.70
-23.08	0.77
-15.38	0.86
- 7.69	0.94
0.00	1.00
7.69	1.06
15.38	1.06
23.08	1.12
30.77	1.17
38.46	1.22
38.46	1.27
46.15	1.32