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CRITICAL LOADS OF IMPERFECT COLUMNS WITH CROOKEDNESS AND EDGE CRACKS

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B.ENG. (CIVIL)

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DECLARATION

I hereby declare that this thesis has been composed by myself and that it is a record of my own research work. It has not been accepted in any previous publication for a higher degree.

All the sources of information which are not my own originals are specifically acknowledged by means of numbers corresponding to the reference numbers in the bibliography.

AFOLAYAN, J. Olasehinde

July, 1984

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ABSTRACT

The primary purpose of this investigation is to examine the proposition that the combined effect of crookedness and crack on the critical loads of columns may not follow a simple superposition law even within a linear constitutive theory.

The solution techniques employed are the Method of Undetermined Coefficients and the Global Energy Balance Concept to obtain the equilibrium paths under practical magnitudes of these imperfections. Thus, the sensitivity of structural members to imperfections in terms of load sustaining capabilities is examined.

The boundary conditions considered are: both ends fixed, one end fixed and the other free, and both ends pinned. A comparison of the load carrying capacities is presented for various magnitudes of crookedness and crack depth to least dimension ratios. The response of the models show some nonlinear geometric interaction. The fixed-fixed model shows a peak response to the deflection in the positive direction and for all magnitudes of initial crookedness the model changes from one deflection mode to another at the same capacity. An asymmetric bifurcation is exhibited by this model under the influence of the combined imperfections. The load sustaining capabilities of all models are governed by the crookedness since the critical loads for the perfect systems are nearly attained when the imperfection is modelled as crack-like alone.

The application of this analysis to uniformly tapered and rectangular columns is presented in the thesis and results show that crack-like imperfection (provided the crack depth to least dimension ratio is small) has little effect on their critical buckling loads.

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NOTATIONS

A = cross sectional area

b = radius of the solid cylindrical column

C = crack size

E = Young's Modulus

F = Work done by an external force

$f(...)$ = function of (----)

G = Strain Energy Release Rate

I = Moment of Inertia

K = Spring stiffness

K_1 = Stress intensity factor for mode I

k = Column stiffness

$\bar{K}$ = Dimensionless stress intensity

$\hat{k}$ = dimensionless column stiffness

L = Length of member

P = Axial load,

Q = Driving force for crack growth

r = radius of gyration

U = Elastic Energy

v = crack opening

$\hat{v}$ = Poisson's ratio

W = Energy Required for crack growth

W_m = mid-height deflection

$W(x)$ = deflection mode

$\hat{W}_m$ = W_m/L

λ = Loading parameter

$\mu, \bar{\lambda}$ = Lamés constants

2λ = ratio of load sustained by an imperfect system to critical load of a perfect system.

ϵ, ξ = measures of imperfection

δ = displacement

σ = stress

e_{ij} = Strain components

δ_{ij} = kronecker's delta

$$= \begin{cases} 1 \; ; \; i = j \\ 0 \; ; \; i \neq j \end{cases}$$

γ = Local compliance.

R = Non dimensional load of cracked column.

CHAPTER 1

INTRODUCTION

1.1 Purpose and Scope

Columns as structural elements are seldom perfect. In spite of all manufacturing controls, imperfections may still appear in the form of initial crookedness, twist and cross-sectional deformities. In practical columns, they (imperfections) may be modelled as eccentric loading and non-uniformity in the cross-sectional areas. The behaviour of columns is generally affected by other factors such as material properties, residual stresses and cracks /30/*

Structures with cracks and initial imperfections (geometric, structural) have their load carrying capabilities reduced. The effects of cracks and crookedness have been separately studied especially for columns, and cylindrical and spherical shells. This thesis examines the proposition that the combined effect of these imperfections may not follow a simple superposition law even within a linear constitutive theory.

The Method of Undetermined Coefficients and the Global Energy Balance (i.e. the strain Energy Release Rate) technique are used to find the response characteristics that are valid for small imperfection magnitudes. The additional boundary condition [illegible].

* Numbers in brackets refer to the references listed in the bibliography.

introduced by the surface crack is also determined.

Numerical experiments are carried out on pinned-pinned, fixed-fixed and fixed-free column models at various crack depth to least dimension ratios. The models considered are of circular and rectangular cross-sections. Cracked tapered columns are also studied to quantify the effects of cracks and crookedness on their critical loads.

1.2 Literature Review

a. General

The earliest scientific study of the strength of cast iron columns - of circular section, hollow and solid - was made by Eaton Hodgkinson in 1840. His experiment, of course, provided a basis for the development of the Rankine Column formula, which for cast iron was given in 1866 as /25/;

$$\sigma = \frac{80,000}{1 + 0.000625(l/r)^2} \qquad (1.1)$$

for pin-ended columns, where

σ = average compressive stress at which failure occurs;

and

l/r = slenderness ratio.

But generally, numerous theoretical and experimental studies have been on axially loaded columns and struts based on Euler's classic elastic buckling analysis. These studies point out the importance of these elements and highlight the role played by instability in their failure /31/. The theories of Euler and others like Engesser and Shanley apply only to hypothetical columns that are generally perfect and for which the load is perfectly concentric with the longitudinal centroidal axis /14/. Olsen /22/ pointed out that a theoretically perfect column with an infinite slenderness ratio which is loaded with a perfectly axial compressive load will not bend no matter how great the load becomes. It rather crushes. Thus, buckling loads of columns studied through the bending characteristics of crooked columns is of interest to researchers.

b. Imperfection - sensitivity

In 1900, Moncrieff gave three factors affecting safety when deciding upon a strut formula for use in design. They are as follows /25/:

(i) accidental increases of end load over that adopted as the maximum working load (accidental overloading) shall not cause undue lateral deflection or elastic instability. Hence, the working load should not exceed a third of Euler's load.

(ii) allowance to be made for accidental geometrical imperfections. This is done by applying a reasonable safety factor to the adopted accentricity.

(iii) provision to be made for imperfections of the column material itself by using one third of the yield stress.

Moncrieff's approach has not actually furnished us with the behaviour of such struts at these conditions. And for the imperfections, their exact quantitative measures pose some difficulties /22/. The consequence of the uncertainty in the magnitudes and shapes of these imperfections in practical columns is behavioural uncertainty. Thus, series of studies in buckling analysis of imperfect columns have been conducted.

The influence of small imperfections is shown to be closely related to the initial post buckling behaviour of the perfect structural system /17, 36/. Chapius and Galanbus /6/ derived the strength of aluminium pinned crooked columns as a basis of a design column curve. The maximum of a load-deformation curve is found by assuming a sinusoidal shape for both the initial imperfection and the deflected shape. The equilibrium condition at mid-span is expressed, and for a given length of the column the thrust is found for different values of the deflection by using the moment-thrust-curvature relations. The maximum of the thrust-deflection curve is taken as the searched strength.

Amazigo et al /1/ studied the buckling of imperfect columns on nonlinear elastic foundations for various kinds of deterministic and stochastic initial imperfections. In all the techniques employed, asymptotic results are sought for small imperfection magnitudes. Though they obtained perturbation methods for harmonic imperfections (deterministic) and two-timing for dimples, they were unable to invent a perturbation scheme for the random imperfections. It was observed that the degradations of the buckling strength, as a measure of the degree of imperfection parameter, are greater for the harmonic imperfection, less for the random and least for the dimple. A reliability study on the buckling of a stochastically imperfect finite column using a probabilistic approach has been conducted and reported /9, 10/. The Monte Carlo method was used to generate initial imperfection functions for each realisation of which the buckling load was determined numerically. However, there is the difficulty of compiling extensive experimental information on imperfections, classified in accordance with the manufacturing process so that statistical measures maybe found for them.

To obtain results of general validity, imperfections in columns should be considered as random in nature. This was the stand of Singh and Ang /30/. The [illegible] load-carrying capacity of a column with initial displacement depends on physical criteria established with respect to its serviceability. Thus, Singh and Ang monitored these criteria in the form of a limit on lateral deflection, or end-shortening, or bending moment or stress at some critical section in the column. With these, they deduced that a smaller load is required to produce a given axial strain in an imperfect column than in a perfect one. Therefore, the product of the mean imperfection factor and the critical strain, "ε_c", is the reduction in capacity. But keener /15/ gave a general iteration procedure for calculating the buckling load of imperfect structures, of which the form of the imperfections was not specified. However, he maintained that the iterates converged for small imperfections. Moreso, they gave asymptotically valid estimates of the buckling load as a function of the imperfection amplitude. Keener's method is also valid for problems which are imperfection-insensitive by determining the imperfection sensitivity of certain "unbuckling" loads.

The snap-through phenomenon exhibited by columns was also investigated by Galoussis and Vassilas /11/

Extensive experimental studies on cylindrical shells subjected to axial compression have shown that actual buckling loads fall short of theoretical values. Thus, Roorda and Hansen /26/ statistically investigated the imperfection-sensitivity of axially loaded cylindrical shells of infinite length. The analysis showed that the mean, variance, and all other statistical measures of the critical load distribution are functions of all the statistical measures of the imperfections distribution. Koiter's work showed that by assuming a very special initial imperfection amplitude in the form of the axisymmetric buckling mode of a perfect shell, an imperfection amplitude of only 0.5 of the wall thickness would reduce the buckling load to almost 0.3 of the classical value. However, the measurement of imperfections has not been very successful while carrying out buckling tests /2/ with thin shells. This is because an accurate determination of imperfections of the order of a few percent of the wall thickness proves to be a very formidable task.

c. Crack-like Imperfections

The presence of a crack in structures reduces their strength. This awareness has led to finding:

i) the residual strength as a function of crack size

ii) the crack size that can be tolerated at the expected service load (i.e. the critical size)

iii) the size of pre-existing flaw that can be permitted at the moment the structure starts its life and

iv) the frequency of inspection of structures for cracks.

All these are questions in fracture mechanics /4/ and have been the subjects of many investigations.

Dimarogonas /8/ studied the decrease in the critical load due to surface crack (longitudinal) on rings and tubes. It was found that the presence of this crack could reduce the critical pressure up to 50%. This reduction increases with decreasing mean radius (R) to thickness (h) ratios. For low R/h ratios, internal and external surface cracks are equally damaging. In crack-like imperfection problems, a slight perturbation of the load from ideal condition of having it placed perfectly normal to the crack plane can be considered /29/ This is analogue to small eccentricity in columns.

Benthem and Koiter /3/ used asymptotic approach to evade the difficult problem of obtaining a complete solution. By knowing the asymptotic solutions for both ends of the range of the parameter, the solution for intermediate values of the parameter are found by interpolation. The procedure of interpolation between the asymptotic solutions at the ends of the interval is often facilitated, if the variable parameter varies from zero to infinity. Also, the results presented must be of order unity throughout.

Chuang et al /7/ showed load-displacement curves for several values of maximum velocity of crack propagation for precracked specimens under bending. They observed that there are no differences in the linear parts of the curves because linear elastic behaviour governs. Thus, it was concluded that the velocity plays no role within the region.

However, the load decreases with increasing velocity at the non-linear region. The non-linearity is associated with the stress-strain non-linearity of the material concerned /5/. Some area around crack appear unloaded and thus the strain energy released by such region is stored in the highly stressed region (i.e. crack tip) /16/. Therefore this has led to series of studies on the stress-distribution in the vicinity of an external crack /20, 21, 23, 24, 35/.

1.3 Concept of Stability

The approach to stability problems differ from case to case, and hence the concepts and quantities used in stability theory are to a large extent not invariant. They are however, chosen and defined in accordance with the researcher's particular intent and purpose of investigation /19/.

a Principles

The behaviour of an elastic system is described, by one or more characteristics which exhibit at the onset of instability a certain property suitable for the formulation of a stability criterion.

In a general state, the system possesses a degree of stability. The system also depends on a variety of parameters, for example, load parameters, structural parameters and possibly time. The parameters control the system's behaviour as the degree of stability depends on the parameter values /19/. At critical values, the degree of stability vanishes and a perturbation of arbitrary smallness can destabilize the system /37/.

Stability can be illustrated as shown in Fig. 1. Fig. 1 (a) shows a bar subjected to an axial compressive load (i.e. load and controlling parameter) which when displaced lightly from the vertical will accelerate towards a horizontal position. Thus, an unstable equilibrium position. If the bar is displaced by an amount "δ" in either direction, there are displacing moment '$P\delta$' and a restoring moment $2k\delta L$ (see fig. 1 (b). Therefore we obtain

$$P\delta < 2k\delta L \quad \text{(stable)} \qquad (1.2)$$

$$P\delta > 2k\delta L \quad \text{(unstable)} \qquad (1.3)$$

The critical condition is when

$$P\delta = 2k\delta L \qquad (1.4)$$

or

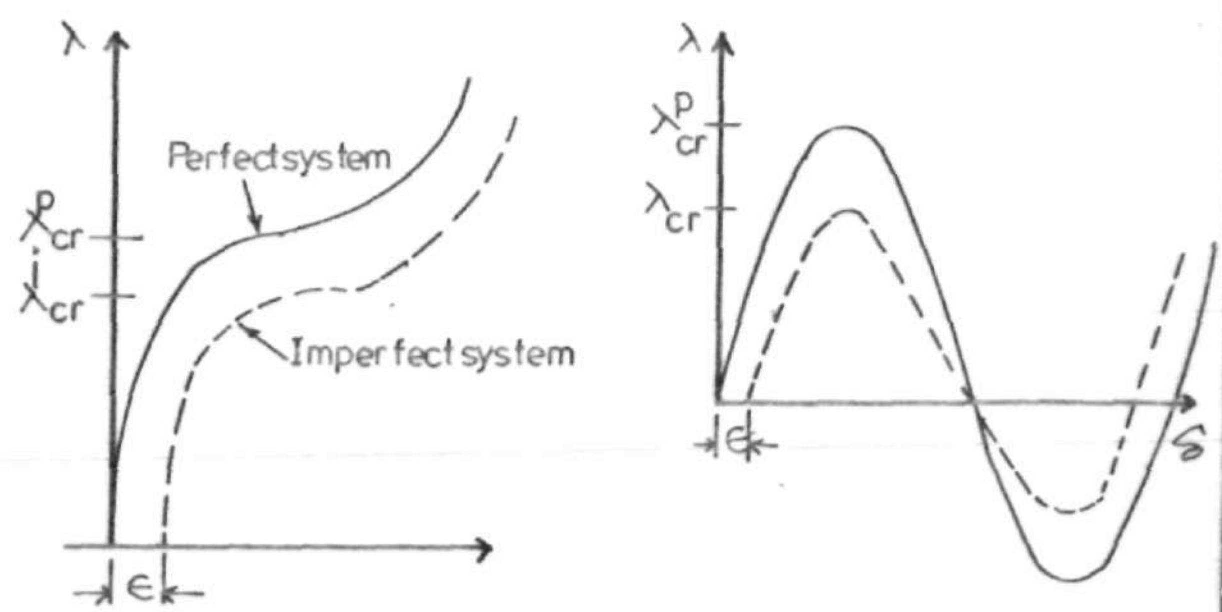
$P = 2kL$ and P is termed the critical load, being the borderline between stable and unstable equilibrium.

b. Equilibrium Paths

Under a gradual increase in applied load, the load - deformation characteristics similar to those in Fig. 2 (a) and (b) are obtained. The curves show the equilibrium paths for perfect and imperfect

(a) Unstable Equilibrium (b) Stable or unstable equilibrium

FIG. 1: STABLE AND UNSTABLE EQUILIBRIUM CONFIGURATIONS



(a) Imperfection - Insensitive Systems (b) Imperfection - Sensitive Systems

FIG. 2: RESPONSE TO IMPERFECTIONS

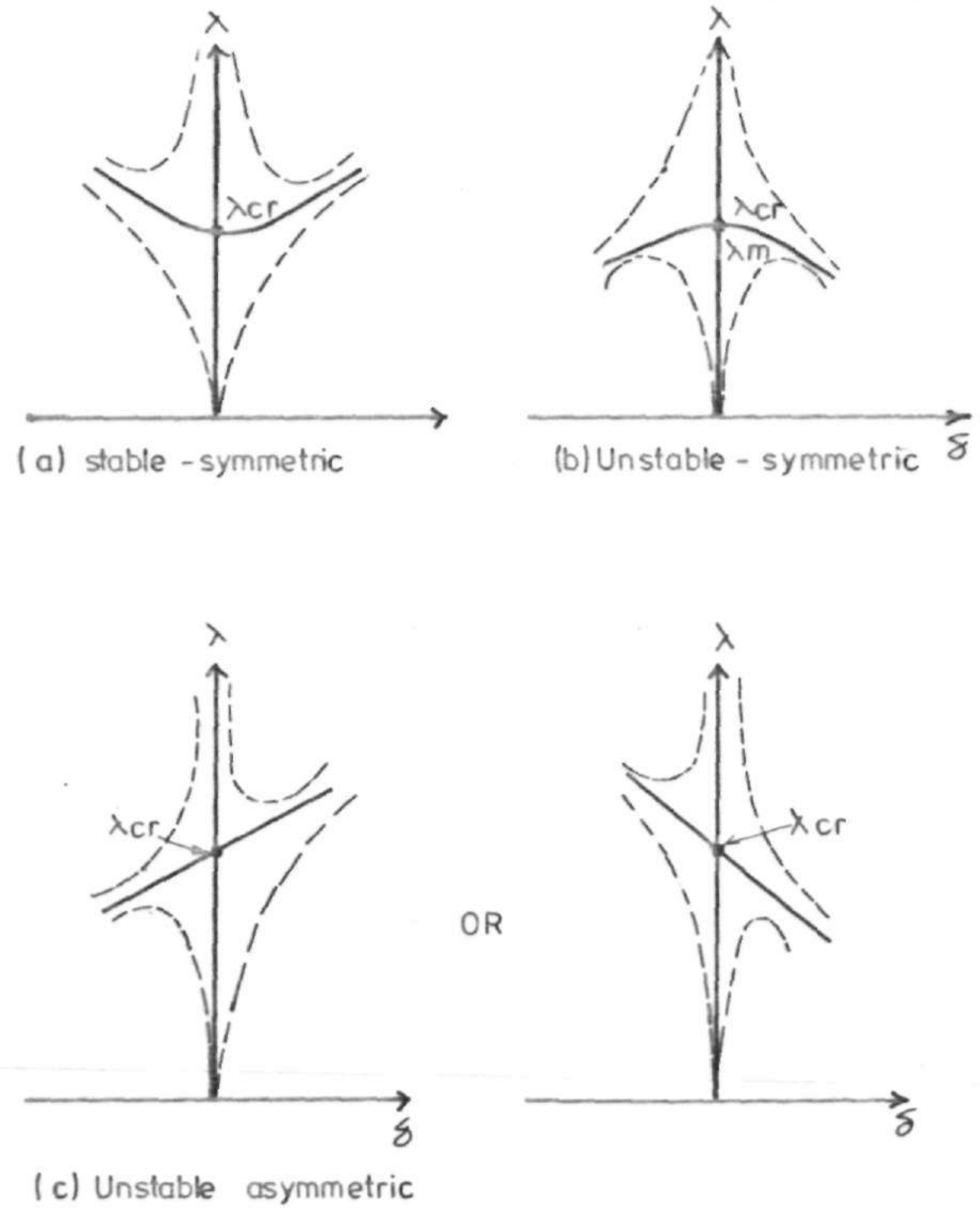


FIG. 3. EQUILIBRIUM PATHS

systems for structures that are insensitive (Fig. 2(a)) and those that are sensitive (Fig. 2 (b)) to imperfections. In Fig. 2, if $\lambda^{i}_{cr} < \lambda^{p}_{cr}$, it implies imperfection sensitivity and thus the buckling load is unstable. But if $\lambda^{i}_{cr} \leqq \lambda^{p}_{cr}$, the structure maybe said to be imperfection - insensitive and buckling is stable.

Unlike the cases in Fig. 2 there may be more than one equilibrium paths for a given system. Thus, the system is said to exhibit bifurcation phenomenon (i.e. the point of equilibrium state changes). Refer to Fig. 3. Fig. 3 (a) depicts the imperfection - insensitive systems whereas (b) and (c) depict imperfection - sensitive systems.

CHAPTER 2

THE COLUMN MODEL

A uniform column with initial crookedness and circumferential edge crack is considered under various boundary conditions that are kinematically admissible. The governing equilibrium differential equation is presented for solution to buckling of such columns, in section 2.1. The additional boundary condition introduced by the crack is also provided; and the solution techniques for the governing differential equation and for determining the local flexibility introduced by crack are presented.

2.1 Problem Formulation

Consider a uniform column with an initial imperfection and a mid-way circumferential surface crack (see Fig. 4). The deflection is non restrained and the crack is quasi-static (i.e. at each instant, equilibrium is preserved). Buckling occurs before any failure by fracture so that the crack depth, c, is much less than the smallest dimension of the column. For small deflections, the governing non-dimensional differential equation is /15/:

$$W^{iv} + 2\lambda W^{ii} = - 2\lambda\zeta W_o^{ii} \qquad (2.1)$$

The different boundary conditions can be applied to solve the equation and this is treated in Chapter 3 . Equation (2.1) is based on references /1,15/

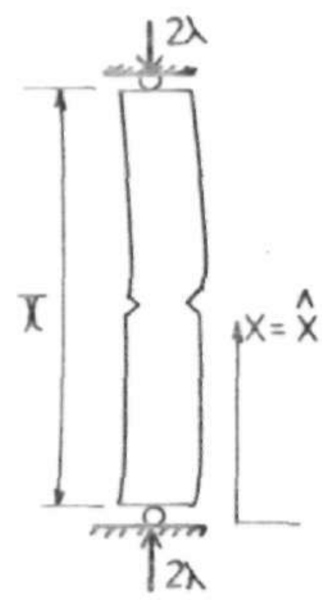


FIG. 4 IMPERFECT COLUMN MODEL

where the equation for an imperfect uncracked column on a "softening" non-linear elastic foundation is given by

$$w^{iv} + 2\lambda w'' + w - w^3 = -2\lambda \zeta w_0'' \qquad (2.2)$$

The presence of a crack in the column is seen as only introducing an additional boundary condition to the problem of buckling. This is, of course, through the member's stiffness. Thus, this additional boundary condition is discussed in the following section.

2.2 Additional Boundary Condition

At a distance $x = \hat{x}$ as shown in Fig. 4, there is a vee - grooved edge crack round the column. A quasi - static crack is assumed so that if an indefinitely small change in the external conditions is made, the process becomes reversible.

This additional boundary condition (B.C) can be established since crack faces are stress-free The generalised Hooke's law for an isotropic elastic body gives /28, 32/:

$$\sigma_{ij} = 2\mu e_{ij} + \bar{\lambda} \delta_{ij} e_{kk} \qquad (2.3)$$

Thus, for a 3-dimensional stress

$$\sigma_{11} = 2\mu e_{11} + \bar{\lambda}(e_{11} + e_{22} + e_{33}) \qquad (2.4)$$

and

$$e_{ij} = \frac{1}{2}\left\{\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right\} \qquad (2.5)$$

For a unaxial loading

$$e_{ii} = \frac{\partial U}{\partial x} \qquad (2.6)$$

and for an initially crooked member

$$e_{11} = \frac{\partial}{\partial x}\left(w(x) - w_o(x)\right) \qquad (2.7)$$

If the lateral strain is in the direction normal to the axis of stress induced by compressive load /18/

$$e_{22} = -\nu e_{11}$$

For plane strain

$$e_{33} = 0.$$

Thus,

$$\begin{aligned}\sigma_{rr} &= 2\mu e_{11} + \bar{\lambda}(e_{11} - \nu e_{11}) \\ &= e_{11}(2\mu + \bar{\lambda} - \bar{\lambda}\nu) \\ &= e_{11}(2\mu + \bar{\lambda}(1-\nu)) \qquad (2.8)\end{aligned}$$

Equations (2.7) and (2.8) imply that

$$\sigma_{11} = (2\mu + \bar{\lambda}(1-\nu))\frac{\partial}{\partial x}\left(w(x) - w_o(x)\right) \qquad (2.9)$$

Let $w(x) = b \sin x$ (2.10a)

and $w_o(x) = a_o \sin x$ (2.10b)

With equations (2.10a) and (2.10b) in(2.9) and equating to zero since $\sigma_{11} = 0$ at column surface where crack exists, we obtain

$$(2\mu + \bar{\lambda}(1-\nu))(b - a_o)\cos x = 0 \qquad (2.11)$$

This implies

$$\cos x = 0$$

since $(2\mu + \bar{\lambda}(1 - \nu))\ (b - a_0) \neq 0.$ (2.12)

Therefore from

$$w(x) = b\sin x$$

then $w'(x) = b\cos x$ (2.13)

By comparing eqns (2.12) and (2.13)

$$b \cos x = 0$$

and therefore $x = \pi/2$ since $b \neq 0$.

Hence, the only additional boundary condition introduced is

$$w'(\pi/2) = 0 \qquad (2.14)$$

2.3 Solution Techniques

The Method of Undetermined Coefficients is used as an explicit method, to solve the governing differential equation formulated in this study. Surface cracks introduce a local flexibility (compliance) to members containing them. The compliance is derived in relation to strain Energy Release Rate, G, by virtual work arguments.

a. Method of Undetermined Coefficients

This is one of the explicit methods for solving higher order differential equations.

A nonhomogenous differential equation can be given by

$$a_0 y^n + a_1 y^{n-1} + \ldots\ldots + a_{n-1} y^1 + a_n y = F(x),$$

where the coefficients a_i, $i = 0, 1, ---, n$ are constants and F is a nonconstant function of x. The general solution to this maybe written as

$$Y = Y_c + Y_p,$$

where

Y_c =complementary function

and

Y_p = a particular integral.

The particular integral is to be determined by Undetermined Coefficients Method. This method applies when the nonhomogenous function F is a finite linear combination of UC functions.

Definitions

A UC function is a function defined either

(1) by one of the followings /27/:

(i) X^n, where n is a positive integer or zero.

(ii) e^{ax}, where a is a constant $\neq 0$

(iii) sin(bx + c), where b and c are constants, $b \neq 0$

(iv) cos (bx + c), where b and c are constants, $b \neq 0$.

or (2) as a finite product of two or more functions of the above four types.

UC function f:

The set of functions consisting of f itself and all linearly independent UC functions of which the successive derivatives of f are either constant multiples or linear combinations will be called the UC set of f.

Let a UC function

$$f = f(x) = x^3 \text{ for all real } x.$$

The derivatives of f are given by

$f'(x) = 3x^2$, $f''(x) = 6x$, $f^{iii}(x) = 6.1$,

$f^n(x) = 0$ for $n > 3$.

The linearly independent UC functions of which the successive derivatives of f are either constant multiples or linear combinations are those given by

x^2, x, 1.

Thus the UC set of x^3 is

$$S = \{x^2, x, 1\}.$$

Outline of the method

Let $a_0y^n + a_1 y^{n-1} + ---- + a_{n-1}y^1 + a_ny = F(x)$

represent a nonhomogeneous differential equation.

F is a finite linear combination

$$F = A_1U_1 + A_2U_2 + ---- + A_mU_m$$

of UC functions U_1, U_2, ----, U_n and A_i; = 1, ---,m are known constants.

Assuming the complementary solution has been found, the particular solution is obtained as f llows /27/

(i) obtain the respective sets

S_1, S_2 ------, S_m

for the UC functions

$U_1, U_2, \text{-------}, U_m$

(ii) If one of the sets, say, S_j, is identical and completely included in another S_k, omit the (identical or smaller) set S_j and retain S_k in further consideration

(iii) Each of the UC sets remaining after step (ii) is considered. If set S_1 in the UC sets includes one or more members of Y_c (complementary solution), then each member of S_1 should be multiplied by the lowest positive integral power of x. Thus, S_1 is replaced by this revised set.

(iV) Two categories of UC sets are now remaining viz:

* certain of the original UC sets, which were neither omitted in step (ii) nor needed revision in step (iii), and
* revised sets resulting from step (iii).

Therefore a linear combination of all the elements of these categories with unknown constant coefficients (undetermined coefficients) can be formed.

(v) The unknown coefficients in step (iv) are determined by substituting the linear combination in which they are found into the differential equation. Then the requirement is that the linear combination identically satisfy the differential equation (that is, it is a particular solution).

This method is employed in Chapter 3 for solving the governing differential equation (2.1).

b. Energy Release Rate Concept

It was realised from Griffith's achievement that it was possible to derive a thermodynamic criterion for fracture by considering the total change in energy of a cracked body as the crack length was increased. The energy criterion for fracture states /4/:

"crack growth can occur if the energy required to form an additional crack size of "dc" can just be delivered by the system".

Thus, the work done to extend a crack equals that to close it /13/.

That is, if U = Elastic Energy;

W = Energy required for crack growth or closure;

F = work done by the external force,

then the condition for crack growth is

$$\frac{d}{dc}(U-F) = \frac{dW}{dc} \qquad (2.15)$$

where

$$\frac{d}{dc}(U-F) = G \qquad (2.16)$$

is the energy release rate.

Consider a cracked plate of unit thickness under a load λ shown in Fig. 5.

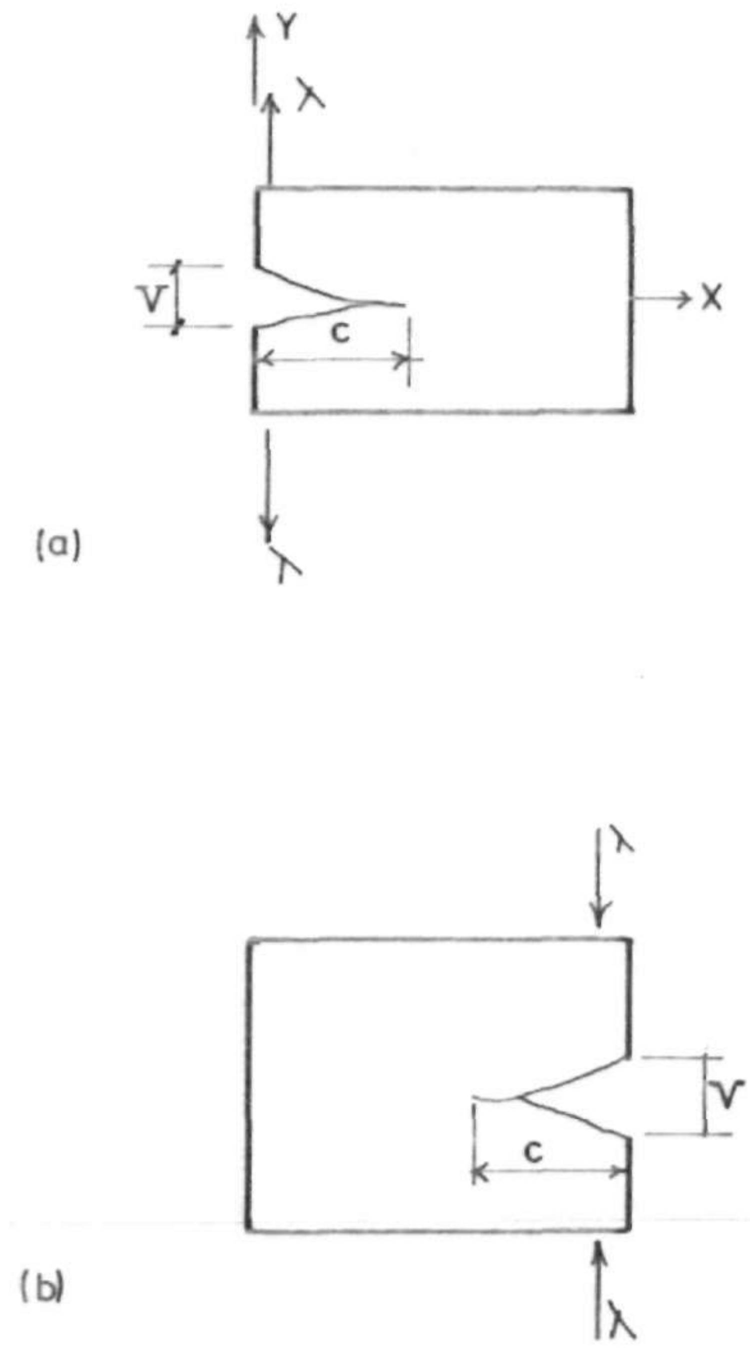


FIG. 5: CRACKED PLATE

When the crack increases in size by an amount ",dc", the displacement will increase by an amount, "dv". Hence, the work done by the external load (force) is

$$\lambda dv$$

Therefore

$$G = \frac{d}{dc} \quad (F - U)$$

becomes

$$G = \frac{d}{dc}(\lambda dv - U)$$

$$= \lambda \frac{dv}{dc} - \frac{dU}{dc} \qquad (2.17)$$

If there is no crack growth, the displacement is proportional to load, i.e.

$$v = \gamma\lambda \qquad (2.18)$$

where

γ = the local compliance (inverse of stiffness).

The elastic energy contained in the cracked plate is

$$U = \tfrac{1}{2}\lambda v = \tfrac{1}{2}\gamma\lambda^2 \qquad (2.19)$$

Putting (2.19) in (2.17) gives

$$G = \frac{\lambda^2 \partial\gamma}{\partial c} + \gamma\lambda\frac{d\lambda}{dc} - \frac{1}{2}\frac{\lambda^2\partial\gamma}{\partial c} - \gamma\lambda\frac{d\lambda}{dc}$$

$$= \frac{\lambda^2 \partial\gamma}{2\,\partial c} \qquad (2,20)$$

And this function is related to the stress intensity factor K_1 by the same argument as

$$G = \frac{K_1^2}{E} \quad \text{(plane stress)}$$

$$= (1 - \nu^2)\frac{K_1^2}{E} \quad \text{(plane strain)} \qquad (2.21)$$

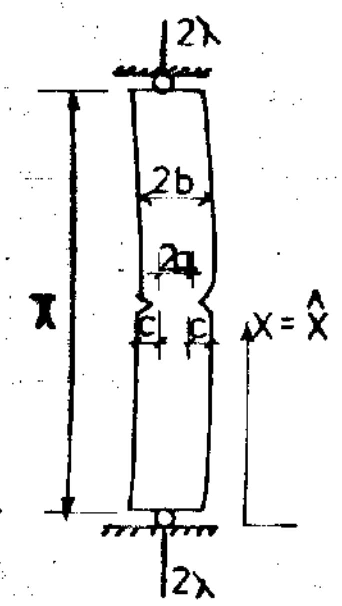


FIG. 6 - CRACKED CROOKED COLUMN

Thus, the applied loads that can cause failure or allowable crack size that will not cause premature failure can be shown by the values of G_{1c} and K_{1c} (i.e. critical values of K_1 and G_1 under normal stress)/29/.

The compliance coefficient, γ, (inverse of the stiffness), for a plane strain problem, is given by /3,8/

$$\tfrac{1}{2}Q^2 \frac{d\gamma}{dA} = \frac{(1-\nu^2)K_1^{\,2}}{E} \qquad (2.22)$$

This is also obtained by equating equations (2.20) and (2.21)

Let A = crack surface area

$= \pi(2b - c) \cdot c$

for a solid cylind rical column.

Therefore

$$dA = 2\pi(b-c)dc$$

and

$$d\gamma = \frac{2(1-\nu^2)}{EQ^2} 2\pi(b-c)\ dc.K_1^{\,2}$$

where

$$K_1 = K \frac{N}{\pi a^2} \left\{\frac{ac}{b}\right\}^{\frac{1}{2}}$$

Hence

$$d\gamma = \frac{4\pi(1-\nu^2)}{EQ^2} (b-c) \left\{ K \frac{N}{\pi a^2} \left(\frac{ac}{b}\right)^{\frac{1}{2}} \right\}^2 dc$$

$$= \frac{4\pi(1-\nu^2)}{E\ Q^2} N^2 \left(\frac{K^2}{\pi^2 a^4}\right) \left(\frac{ac}{b}\right) (b-c)dc \qquad (2,24)$$

From Fig. 6 , a = b-c which implies that

$$d\gamma = \frac{4(1-\nu^2)}{\pi E} \quad K^2 \left(\frac{c}{b}\right) (b-c)^2 (b-c)^{-4} dc \qquad (2.25)$$

for Q = N = the driving force

$$d\gamma = \frac{4(1-\nu^2)}{\pi E} K^2 (c/_b) \quad (b-c)^{-2} dc$$

$$= \frac{4(1-\nu^2)}{E} K^2 (c/_b) (1-c/_b)^{-2} b^{-2} dc$$

$$= \frac{4(1-\nu^2)}{\pi Eb} K^2 (1 - 2(c/_b) + 3(c/_b)^2) \ (c/_b) \ d(c/_b)$$

$$= \frac{4(1-\nu^2)}{\pi Eb} K^2 (c/_b - 2(c/_b)^2 + 3(c/_b)^3) \ d(c/_b) \qquad (2.26)$$

But from /3/

$$K^2 = K^2 (c/_b)$$

$$= \tfrac{1}{4}\{1 + (1 - c/_b) + (1-c/_b)^2 - 0.351 \ (1-c/_b)^3 + 1.24 \ (1 - c/_b)^4 + \text{higher terms}\}.$$

Thus,

$K^2(c/_b - 2(c/_b)^2 + 3(c/_b)^3)$ becomes

$\tfrac{1}{4}(3.889(c/_b) - 14.685 (c/_b)^2 + 32.868 (c/_b)^3 - 40.104 (c/_b)^4 + 32.529(c/_b)^5 - 16.307(c/_b)^6 + 3.72 (c/_b)^7 + \text{higher terms})$

Let $(c/_b) = \eta$

and

$$Y(\eta) = 3.889\eta - 14.685\eta^2 + 32.868\eta^3 - 40.104\eta^3 + + 32.529\eta^5 - 16.307\eta^6 + 3.72\eta^7 + ---)$$

Therefore

$$\gamma = \frac{4(1-\nu^2)}{\pi Eb} \tfrac{1}{4} \int_0^n Y(\eta) \, d\eta. \qquad (2.27)$$

$$= \frac{4(1-\nu^2)}{\pi E b} \tfrac{1}{2} \left| \frac{3.889}{2}\eta^2 - \frac{14.685}{3}\eta^3 + \frac{32.868}{4}\eta^4 - \right.$$

$$\left. - \frac{40.104}{5}\eta^5 + \frac{32.529}{6}\eta^6 - \frac{16.307}{7}\eta^7 + \frac{3.72}{8}\eta^8 \right|_0^\eta$$

$$= \frac{4(1-\nu^2)}{\pi E b} (0.486\eta^2 - 1.224\eta^3 + 2.054\eta^4 - 2.005\eta^5 +$$

$$+ 1.355\eta^6 - 0.582\eta^7 + 0.116\eta^8)$$

$$= \frac{4(1-\nu^2)}{\pi E b} J(\eta) \qquad (2.28)$$

where

$$J(\eta) = 0.486\eta^2 - 1.224\eta^3 + 2.054\eta^4 - 2.005\eta^5 + 1.355\eta^6 - 0.582\eta^7 + 0.116\eta^8.$$

Since load is proportional to displacement when there is no crack growth, eq (2.18) can be expressed as

$$v = \gamma_o \lambda \qquad (2.29)$$

But $v = \frac{2\sigma}{E}\sqrt{c^2 - x^2}$ (see fig. 5). At the edge of the element i.e. at x = 0

$$v = \frac{2\sigma c}{E}$$

Hence, e.q. (2.29) becomes

$$\frac{2\sigma c}{E} = \gamma_o \lambda$$

That is, $\gamma_o = \frac{2\sigma c}{E\lambda} = \frac{2c}{E\lambda}\left(\frac{\lambda}{\pi b^2}\right) = \frac{2c}{\pi E b^2}$ (2.30)

Therefore, the dimensionless stiffness for the cracked column is given as

$$\hat{k} = \gamma_o/\gamma = \frac{2c}{\pi E b^2} \cdot \frac{\pi E b}{4(1-\nu^2)J(\eta)}$$

$$= \frac{c/b}{2(1-\nu^2)J(\eta)}$$

Since $\eta = c/b$ then

$$\hat{R} = \frac{\eta \; J(\eta)^{-1}}{2(1-\nu^2)} \qquad (2.31)$$

CHAPTER 3

CRITICAL LOADS DERIVATIONS

The solution techniques for solving the governing equilibrium differential equation were discussed in Chapter 2. This Chapter deals with the solution of the differential equation for three different end restraints viz; pinned-pinned, clamped-clamped and clamped-free.

3.1 Pinned-Pinned Model

Equation (2.1) is a basis for obtaining the deflection mode of this model. Let

$$W_o(x) = a_o \sin x.$$

Therefore eq (2.1) becomes

$$w^{iv} + 2\lambda w'' = -2\lambda\tau a_o \sin x \qquad (3.1)$$

The boundary conditions are

$$w(o) = w(\pi) = 0 \qquad (3.2)$$

and $w''(o) = W''(\pi) = 0$ (3.3)

Equation (3.1) has a complementary solution

$$w_c = C_1 + C_2 x + C_3 \sin\sqrt{2\lambda}\, x + C_4 \cos\sqrt{2\lambda}\, x$$

The particular solution is obtained by the method of Undetermined Coefficients. The UC function is

Sinx

with the UC set $S = \{\sin x, \cos x\}$

Thus a linear combination is formed as:

A sin x + Bsinx;

giving a particular solution

$$W_p = A\sin x + B\cos x; \qquad (3.4)$$

where A and B are the undetermined coefficients.

From eqns (3.4) and (3.1), we obtain

$$B = 0 \quad \text{and} \quad A = \frac{2\lambda\zeta a_o}{1 - 2\lambda}$$

That is,

$$W_p = \frac{2\lambda\zeta a_o}{1 - 2\lambda} \sin x$$

The general solution for equation (3.1) then becomes

$$W(x) = C_1 + C_2 x + C_3 \sin\sqrt{2\lambda}x + C_4 \cos\sqrt{2\lambda}x + $$

$$+ \frac{2\lambda\zeta a_o \sin x.}{1 - 2\lambda} \qquad (3.5)$$

The constants C_i, i = 1, 2, 3, 4, are obtained from Eqns (3.2), (3.3) and (3.5), i.e.

$$C_1 = C_2 = C_3 = C_4 = 0$$

Therefore

$$W(x) = \frac{2\lambda\zeta a_o \sin x}{1 - 2\lambda} \qquad (3.6)$$

At the mid-height of the column, $x = \pi/2$, then

$$W_m = \frac{2\lambda\xi}{1 - 2\lambda} ; \xi = \zeta a_o \qquad (3.7)$$

Thus, the transmitted load through the neck, 2a, (see fig. 6) can be expressed as

$$2\lambda = \hat{k}\, w(\hat{x},c) \qquad (3.8)$$

Combining equations (2.31), (3.7) and (3.8), we obtain

$$2\lambda = \frac{\eta J(\eta)^{-1}}{2(1-\nu^2)} \frac{2\lambda\hat{\xi}}{1 - 2\lambda} ; \quad \hat{\xi} = \xi/L \cdot$$

Thus,

$$2\lambda = 1 - \frac{\eta\hat{\xi} J(\eta)^{-1}}{2(1 - \nu^2)} \qquad (3.9)$$

When $\nu = 0.3$, Equation (3.9) becomes

$$2\lambda = 1 - 0.549\,\eta\,\hat{\xi}\,J^{-1}(\eta) \qquad (3.10)$$

3.2 Clamped-Free (Cantilever) Model

A similar approach to the pinned model holds except that the boundary conditions are not the same. With reference to eqn (3.1), the general solution given by eqn (3.5) holds. But the boundary conditions are

$$W(o) = 0 \qquad (3.11)$$

$$W'(o) = 0 \qquad (3.12)$$

$$W''(\pi) = 0 \qquad (3.13)$$

$$EI\,W'''(\pi) - 2\,w^1(\pi) = 0 \qquad (3.14)$$

Thus, with these conditions and eqn (3.5) the general solution is

$$W(x) = -\frac{2\beta}{3\alpha}\tan\alpha\pi - \frac{5\beta x}{3} + \frac{2\beta}{3\alpha}\sin\alpha x + \frac{2\beta}{3\alpha}\tan\alpha\pi\,\cos\alpha x + \beta\sin x \qquad (3.15)$$

where

$$\alpha^2 = 2\lambda \text{ and } \beta = \frac{2\lambda\tau\,a_o}{1 - 2\lambda}$$

At mid - height, $x = \pi/2$

$$W_m = \beta\left\{(1 - \frac{5\pi}{6} + \frac{2}{3\alpha}\sin\frac{\alpha\pi}{2}) + \frac{2}{3\alpha}\tan\alpha\pi(\cos\frac{\alpha\pi}{2} - 1)\right\} \qquad (3.16)$$

Hence equation (3.8) implies

$$2\lambda = \hat{k}\,\frac{2\lambda\,\hat{\xi}}{1 - 2\lambda}\left\{1 - \frac{5\pi}{6} + \frac{2}{3\alpha}(\sin\frac{\alpha\pi}{2} + \tan\alpha\pi\,(\cos\frac{\alpha\pi}{2} - 1)\right\} \qquad (3.17)$$

Therefore

$$2\lambda = 1 - \frac{\eta \hat{\xi} J(\eta)^{-1}}{2(1 - v^2)} \left\{ 1 - \frac{5\pi}{6} + \frac{2}{3\alpha}\left(\sin\frac{\alpha\pi}{2} + \right.\right.$$

$$\left.\left. + \tan\alpha\pi\left(\cos\frac{\alpha\pi}{2} - 1\right)\right\} \qquad (3.18)$$

By trigonometric simplification

$$2\lambda = 1 - \frac{\eta \hat{\xi} J(\eta)^{-1}}{2(1 - \nu^2)} \left(1 - \frac{5\pi}{6} - \pi/_3\right)$$

$$= 1 - \frac{\eta \hat{\xi} J(\eta)^{-1}}{12(1 - \nu^2)} (6 - 7\pi)$$

$$= 1 + \frac{(16)}{(12)} \frac{\eta \hat{\xi} J(\eta)^{-1}}{(1 - \nu^2)}$$

For $v = 0.3$,

$$2\lambda = 1 + 1.465\,\eta \hat{\xi} J(\eta)^{-1} \qquad (3.19)$$

3.3 Clamped-Clamped Model

By the Undetermined Coefficients Method and triangular decomposition method for solving linear simultaneous equations, the general solution to equation(3.1)is

$$W(x) = C_1^* + C_2^* x + C_3^* \sin\alpha x + C_4^* \cos\alpha x + \beta \sin x \qquad (3.20)$$

where $\alpha^2 = 2\lambda$ and $\beta = \frac{2\lambda \hat{\xi}}{1 - 2\lambda}$;

$$C_1^* = \frac{\beta\varphi\pi}{\alpha\pi\sin\alpha\pi + (\cos\alpha\pi - 1)\varphi}$$

and $\varphi = \frac{1 + \alpha(\pi\cos\alpha\pi - \sin\alpha\pi)}{\alpha\pi - \sin\alpha\pi}$

$$C_2^* = \frac{\beta\left((\alpha\pi - (1 - 2\pi)\sin\alpha\pi)(\cos\alpha\pi - 1)\varphi + \alpha\pi^2\sin^2\alpha\pi\right)}{(\alpha\pi\sin\alpha\pi + (\cos\alpha\pi - 1)\varphi)(\alpha\pi - \sin\alpha\pi)}$$

$$C_3^* = - \frac{\beta\pi(\alpha\pi\sin\alpha\pi + 2\varphi(\cos\alpha\pi - 1))}{(\alpha\pi\sin\alpha\pi + (\cos\alpha\pi - 1)\varphi)\ (\alpha\pi - \sin\alpha\pi)}$$

and

$$C_4^* = - \frac{\beta\varphi\pi}{\alpha\pi\sin\alpha\pi + (\cos\alpha\pi - 1)\varphi}$$

Therefore at $x = \frac{\pi}{2}$

$$W(\pi/2) = C_1^* + \frac{\pi}{2} C_2^* + C_3^* \sin\frac{\alpha\pi}{2} + C_4^* \cos\frac{\alpha\pi}{2} + \beta \tag{3.21}$$

$$= \frac{\beta}{\alpha\pi\sin\alpha\pi + (\cos\alpha\pi - 1)\varphi}\left\{(1-\cos\frac{\alpha\pi}{2}) + \frac{\alpha\pi(1+11\alpha)\ (\cos\alpha\pi - 1)\varphi}{2(\alpha\pi - \sin\alpha\pi)}\right\} \tag{3.22}$$

Equation (3.8) can then be expressed as

$$2\lambda = \frac{2\lambda\hat{\xi}}{1-2\lambda}\left(\frac{1}{\alpha\pi\sin\alpha\pi + (\cos\alpha\pi - 1)\varphi}\right)\left\{(1-\cos\frac{\alpha\pi}{2}) + \frac{\alpha\pi(1 + 11\alpha)(\cos\alpha\pi - 1)\varphi}{2(\alpha\pi - \sin\alpha\pi)}\right\} \tag{3.23}$$

In a simpler form, eqn. (3.23) can be given by

$$2\lambda = 1 - \frac{\eta\hat{\xi} J(\eta)^{-1}\Phi(\alpha)}{2(1 - \nu^2)} \tag{3.24}$$

for clamped - clamped model; where

$$\Phi(\alpha) = \frac{\Phi_2(\alpha)\Phi_4(\alpha) + \Phi_3(\alpha)}{\Phi_1(\alpha)\Phi_4(\alpha)};$$

$$\Phi_1(\alpha) = \alpha\pi\sin\alpha\pi + (\cos\alpha\pi - 1)\varphi$$

$$\Phi_2(\alpha) = (1 - \cos\frac{\alpha\pi}{2});$$

$$\Phi_3(\alpha) = \alpha\pi(1 + 11\alpha)(\cos\alpha\pi - 1)\varphi ,$$

$$\Phi_4(\alpha) = 2(\alpha\pi - \sin\alpha\pi) .$$

CHAPTER 4

NUMERICAL EXPERIMENTS

In Chapter 3, the buckling load derivations were discussed for various boundary conditions. This chapter is devoted to analytical experiments based on these derivations. In Section 4.1, the experiments are performed on solid cylindrical cross-sections. Uniform tapered and rectangular columns are also assessed for applications of the analysis. The results of the numerical experiments are discussed in Section 4.2.

4.1 Critical loads of Solid Cylindrical Columns

The pinned-pinned, ~~clamped-free~~ and clamped-clamped models are considered for the analytical experiments.

a. Pinned-Pinned Model

The non dimensional critical load expression was given in Chapter 2 (see eq. 3.10) as

$$2\lambda = 1 - 0.549\eta\hat{\xi}J(\eta)^{-1}$$

The Variations of 2λ with η and $\hat{\xi}$ are represented in Fig. 7.

b. Clamped-Free Model

Fig. 8 shows the variation of 2λ with η and $\hat{\xi}$ using the expression (3.19) of Chapter 3.

c. Clamped-Clamped Model

For given values of $\hat{\xi}$ and η the corresponding critical buckling loads are obtained for this model. The plots in Fig. 9 are obtained via equation (3.24).

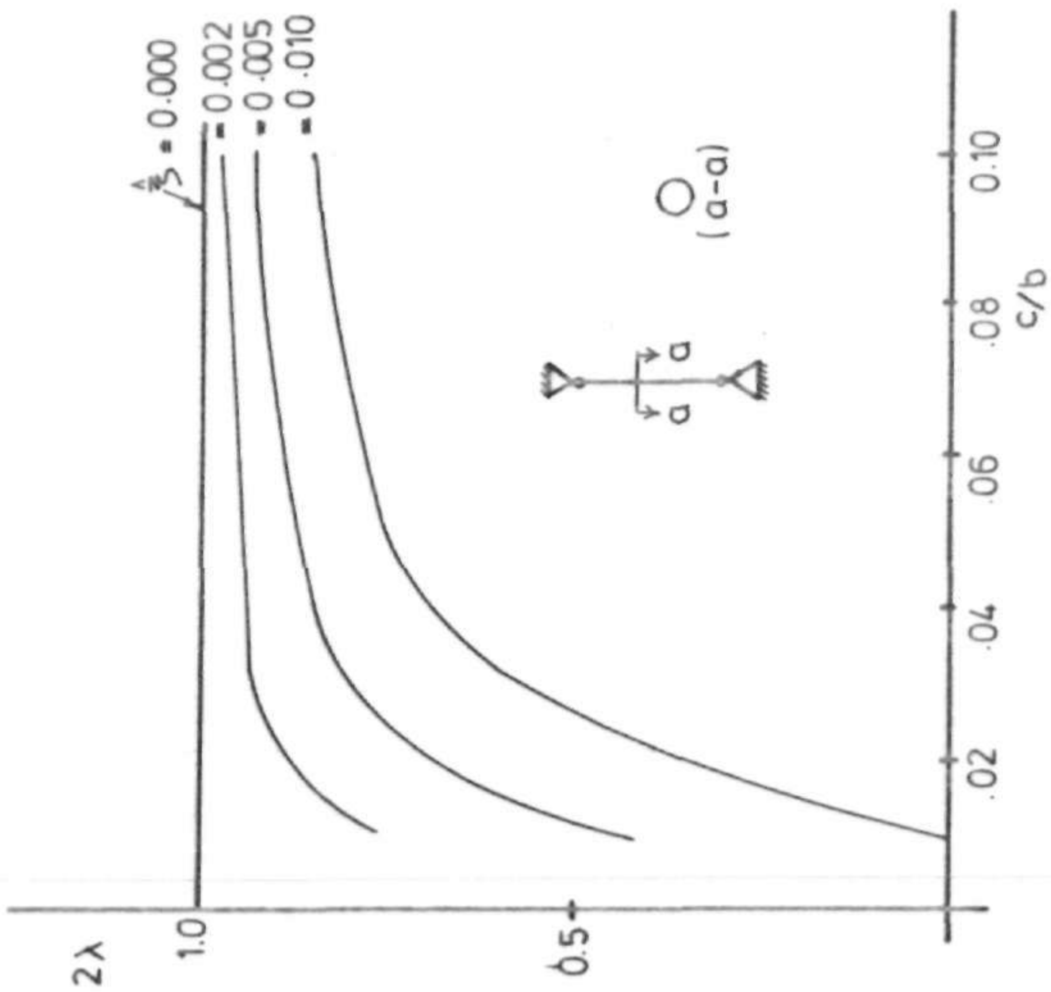


FIG. 7: LOAD AGAINST c/b: (PINNED COLUMN)

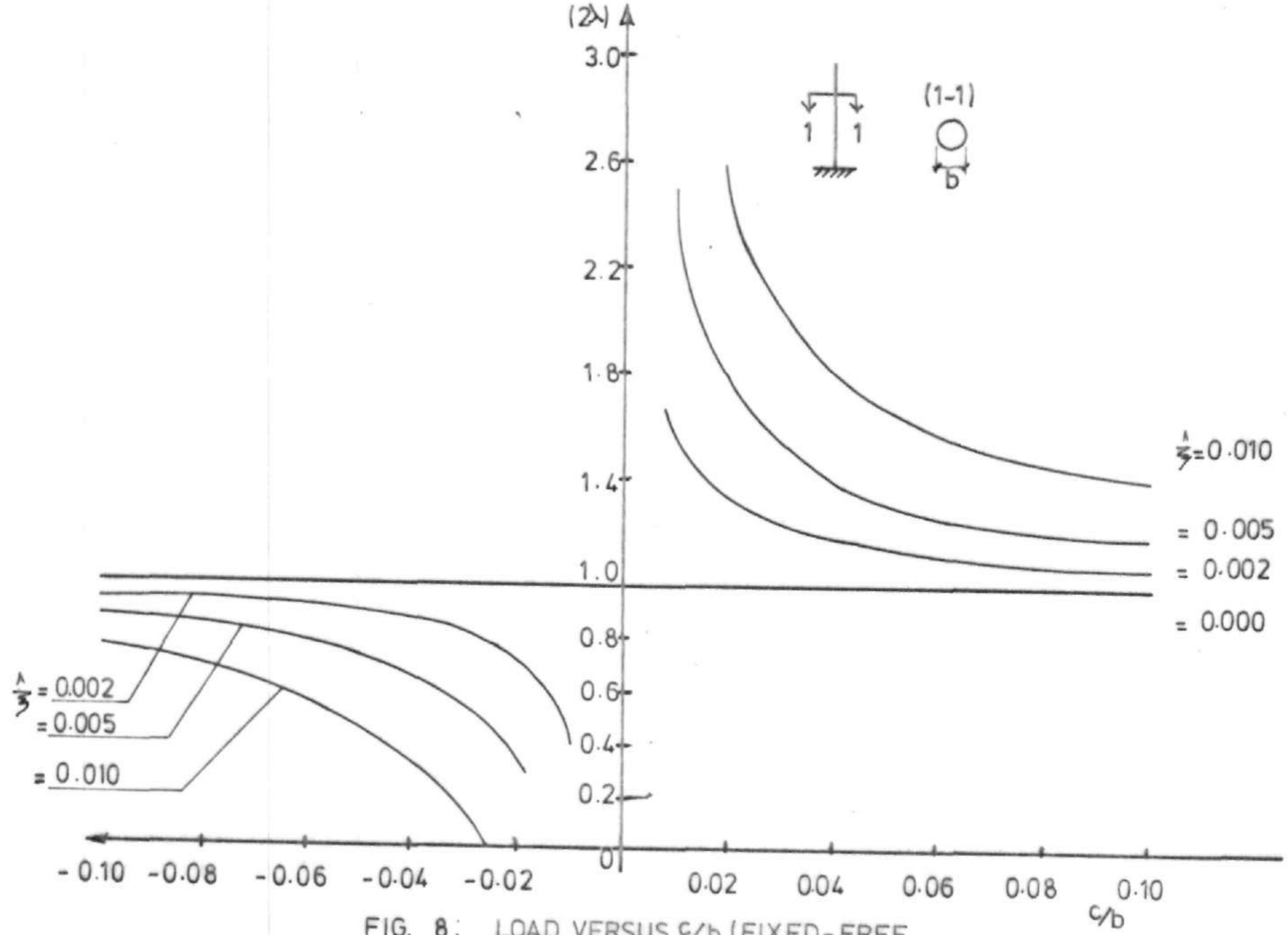


FIG. 8: LOAD VERSUS c/b (FIXED-FREE CYLINDRICAL COLUMN)

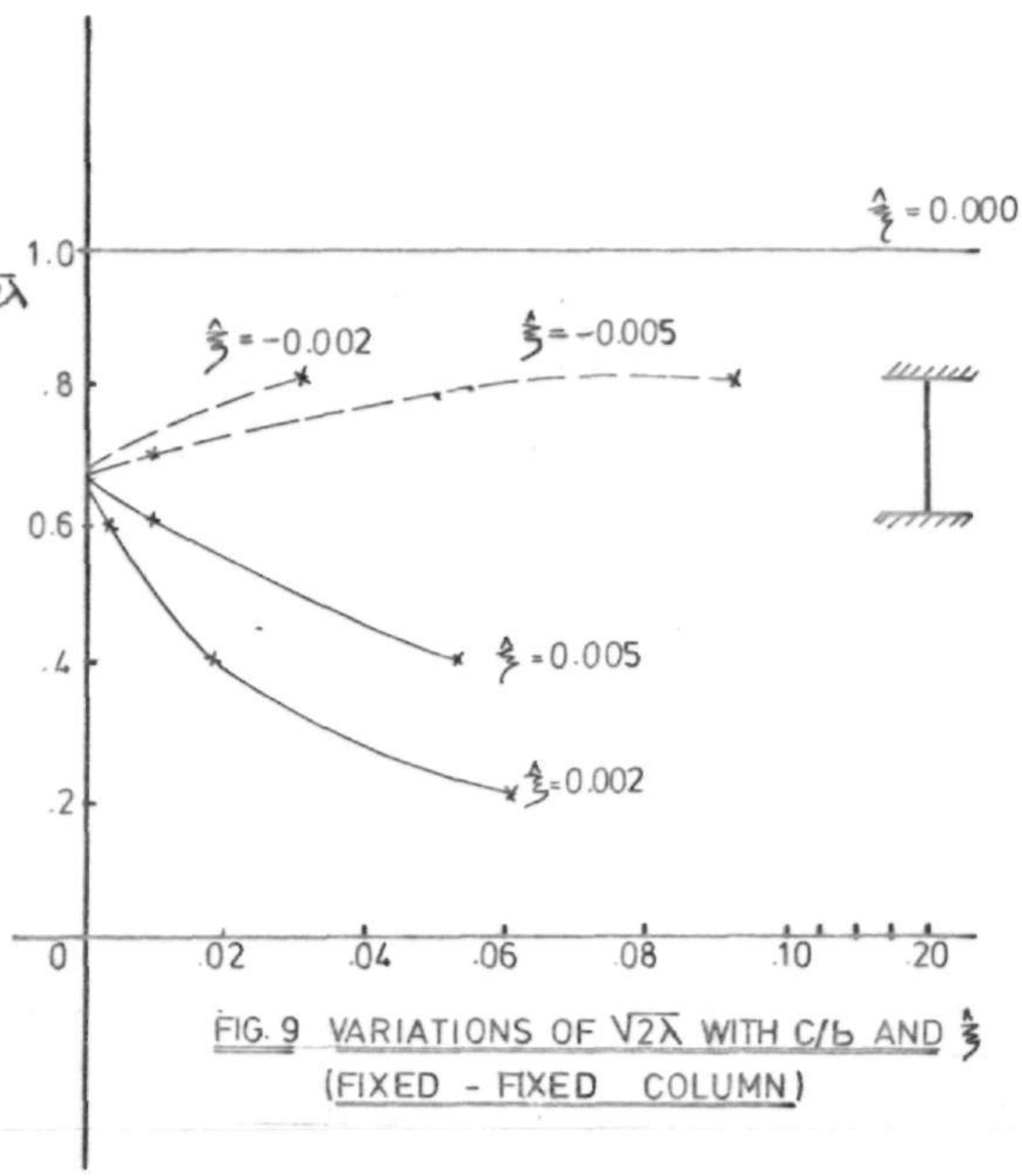


FIG. 9 VARIATIONS OF $\sqrt{2\lambda}$ WITH C/b AND $\hat{\xi}$
(FIXED - FIXED COLUMN)

4.2 Applications of the Asymptotic Analysis

In section 2.3 (b) of chapter 2, the asymptotic expression valid for small crack depth was derived. The derivation was based on a uniform solid cylindrical section. However, the expression also holds for solid, circular tapered columns. Thus, this section deals with the reduction in the critical elastic buckling load of a pinned, cracked tapered column. A similar expression to Eq. (2.31) was given in /8/ for a rectangular bar. Hence, the reduction in the critical load of a cracked crooked rectangular column is assessed following the application to the tapered column.

a. Tapered Column

Information regarding the critical elastic buckling load for a nonuniform column is desirable since a uniform section may not be the most economical under a compressive load. Gere and Carter /12/ worked on tapered columns and their results showed that the critical buckling loads increase with the increasing ratio of larger to smaller dimension. Though the stability of a uniform member increases by removing a portion of the material from the ends and increasing the cross-section over the middle portion, crack presence could still have a noticeable effect on its critical load. A uniform column which is initially straight and perfectly elastic is subjected to a compressive load. (see Fig. 10). At any distance x from end A, the dimension

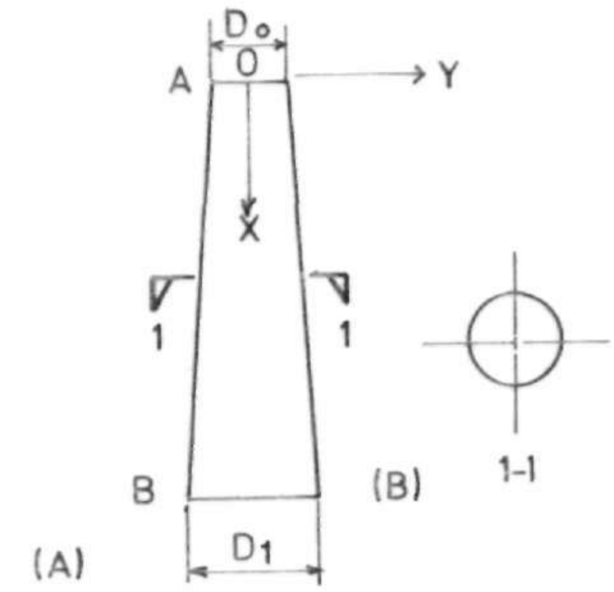


FIG 10 - TAPERED COLUMN

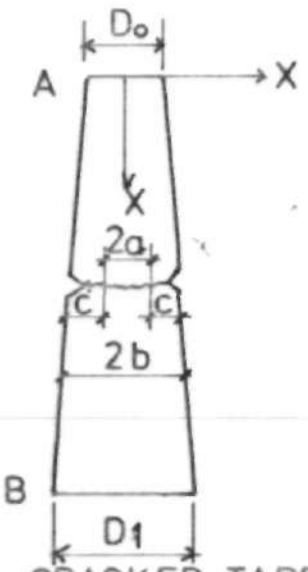


FIG.11- CRACKED TAPERED COLUMN

$$D_x = D_o \left\{ 1 + \left(\frac{D_1}{D_o} - 1 \right) \frac{x}{L} \right\} \qquad (4.1)$$

and the moment of inertia, I_x, about the axis of buckling can be expressed as

$$I_x = I_A \left\{ \frac{D_x}{D_o} \right\}^n \qquad (4.2)$$

In eq (4.2), n depends on the cross-sectional shape and dimensions of the column, n = 4 for a solid circular section /12, 34/.

The buckling load of a pinned tapered column can be given as /12/

$$P_{cr} = \frac{\pi^2 E I_A S^2}{L^2} \qquad (4.3)$$

where

$$S = D_1/D_o$$

Similarly

$$P^* = \frac{\pi^2 E I_c S^2}{L^2} \qquad (4.4)$$

is the load a cracked tapered column (see Fig. 11) can sustain. Due to this surface crack, the strength of the column is expected to reduce. From eqs (4.3) and (4.4) this reduction can be shown to be

$$1 - \frac{P^*}{P_{cr}} = \frac{(I_A - I_c)}{I_A} \qquad (4.5)$$

Let

$$\lambda = \frac{P^*}{P_{cr}} \quad \text{and} \quad \hat{\rho} = \frac{I_A - I_c}{I_A} = \frac{\Delta I}{I_A}$$

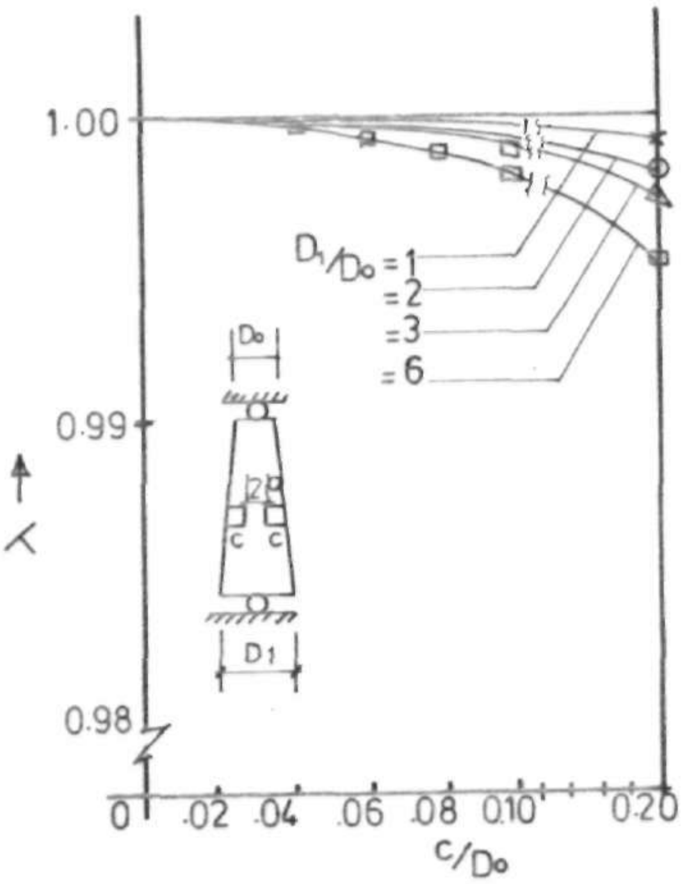


FIG.12 CRITICAL LOAD VS C/D_o
(PINNED TAPERED COLUMN)

Equation (4.5) can then be expressed as

$$\lambda = 1 - \hat{\beta} \qquad (4.6)$$

In terms of the stiffness ratio, $\hat{\beta}$ can be represented as

$$\hat{\beta} = \frac{EI_1}{D^3_1 k_c}, \qquad (4.7)$$

where

$$K_c = \frac{1}{\gamma} = \frac{\pi E D_0}{4(1-\nu^2)J(\eta)}$$

from equation (2.28); and

$$I_1 = \frac{\pi D_1^4}{64}.$$

Therefore,

$$\lambda = 1 - \frac{(1-\nu^2)}{16} (D_1/D_0)\ J(\eta)$$

That is,

$$\lambda = 1 - 0.0625(1 - \nu^2)S\ J(\eta). \qquad (4.8)$$

The variations of λ with η for given values of s are plotted in Fig. 12.

b. Crooked Rectangular Column

Dimarogonas /8/ derived a similar asymptotic expression for the dimensionless stiffness of a cracked rectangular bar as

$$\frac{EI}{hK_T} = 6(1 - \nu)J(\beta) \qquad (4.9)$$

Thus, from equations (2.31) and (4.9) and for a rectangular bar

$$\hat{R} = 6(1 - \nu)J(\beta) \qquad (4.10)$$

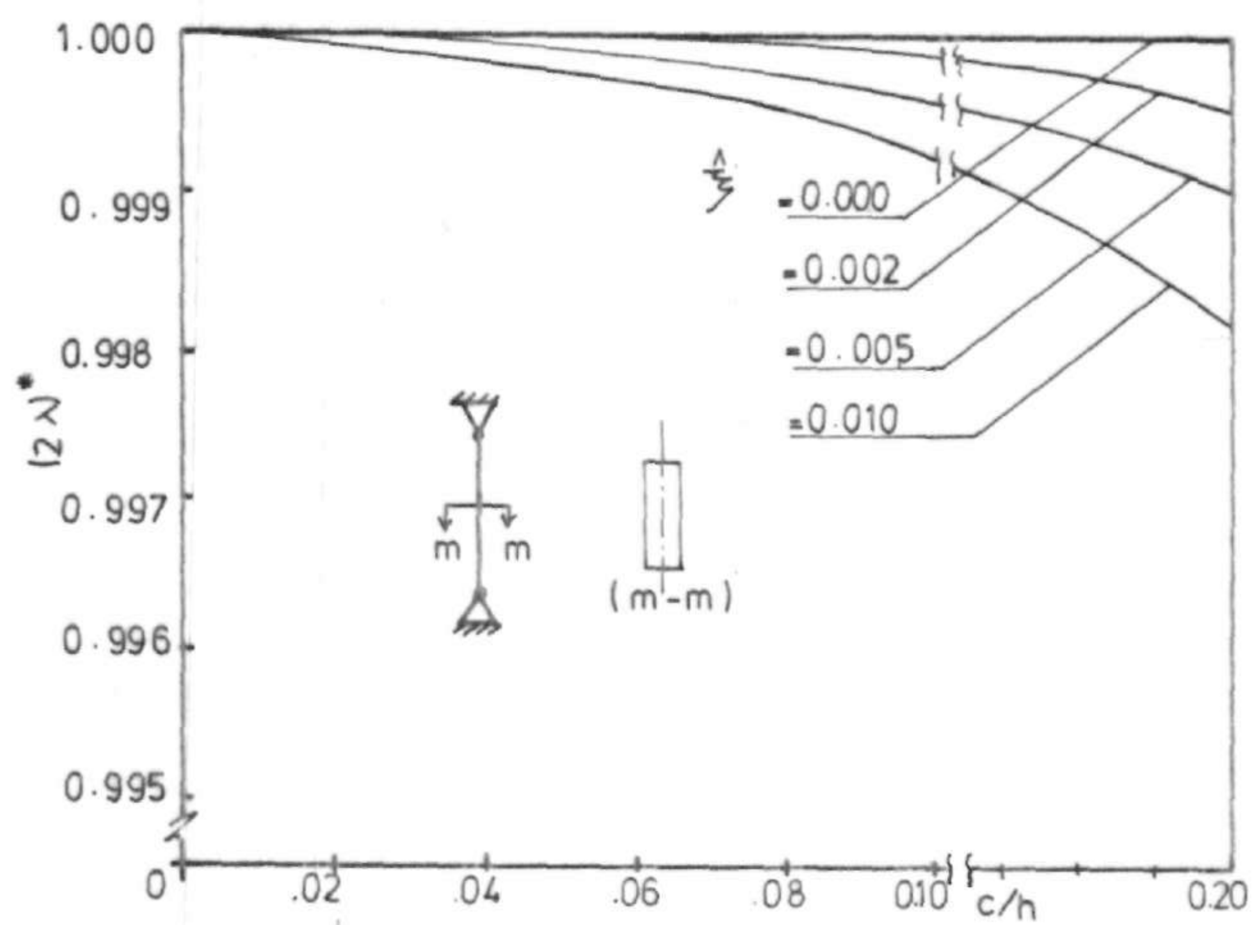


FIG. 13: VARIATION OF LOAD WITH $\hat{\xi}$ AND c/h FOR SOLID RECTANGULAR COLUMN

where

$$J(\beta) = 1.862\beta^2 - 3.95\beta^3 + 16.375\beta^4 - 37.226\beta^5 + 76.81\beta^6 - 126.9\beta^7 + 172.5\beta^8 - 143.97\beta^9 + 66.56\beta^{10}.$$

and

$\beta = c/_h$ = crack depth/thickness of bar.

Therefore the critical load expression in nondimensional form, employing equations (3.7), (3.8) and (4.10), is

$$(2\lambda)^* = 6(1-\nu)\, J(\beta)\, \frac{(2\lambda)^*\,\hat{\xi}}{1-(2\lambda)^*}$$

That is,

$$(2\lambda)^* = 1 - 6\,(1-\nu)\,\hat{\xi}\, J(\beta) \qquad (4.11)$$

The plots in Fig. 13 show the decreases in the critical buckling loads for given $c/_h$ and $\hat{\xi}$ values.

CHAPTER 5

RESULTS

This Chapter discusses the results obtained on the basis of the asymptotic analysis. The effects of crookedness and cracks have been shown in the previous chapters. It is important also to consider the effect of cracks alone on columns. This is to allow us compare the influence of the individual imperfections. The effect of crookedness only is easily shown using the solution to the governing differential equation discussed in Chapter 2.

The first presentation here is to formulate for the effect of an edge crack on the buckling loads of solid cylindrical columns with the boundary conditions employed throughout this study.

5.1 Cracked Uniform Solid Cylindrical Columns

The influence of edge cracks on buckling of uniform columns is assessed through their stiffnesses. Generally, the buckling modes of compressive members depend on their boundary conidtions. Thus, the effective length of columns used in design is always taken as a buckling factor times the theoretical length.

Let the buckling load of a cracked column be expressed in nondimensional form:

$$R = 1 - f\ (k^*/\bar{\lambda})$$

where

$f(----)$ = function of $(-----)$

$$K^* = \frac{EI}{b^3 K_c} \; ; \; K_c = f\,(c/_b)$$

$^c/_b$ = ratio of crack depth to least dimension of column

$\bar{\alpha}$ = buckling factor for varying boundary conditions

= 1.0 for pinned ends

= 0.5 for fixed ends

= 2.0 for one end fixed and the other free.

In Chapter 2

$$\gamma = 4\left(\frac{1 - \nu^2}{\pi Eb}\right) J\,(\eta)$$

$$\text{Let } K_c = \frac{1}{\gamma} = \frac{\pi Eb}{4(1 - \nu^2)\, J\,(\eta)}$$

and $I = \frac{\pi (2b)^4}{64} = \frac{\pi b^4}{4}$ for circular section.

Therefore

$$K^* = \left(\frac{E\pi b^4}{4b^3}\right)\left(4\left(\frac{1 - \nu^2}{\pi Eb}\right) J\,(\eta)\right)$$

$$= (1 - \nu^2)\, J\,(\eta)$$

That is,

$$R = 1 - \frac{(1 - \nu^2)}{\bar{\alpha}}\, J\,(\eta)$$

The plot in Fig. 14 shows the variation of load with c/b, for pinned, cantilever and fixed - fixed models.

5.2 General Observations

The individual and combined effects of crack

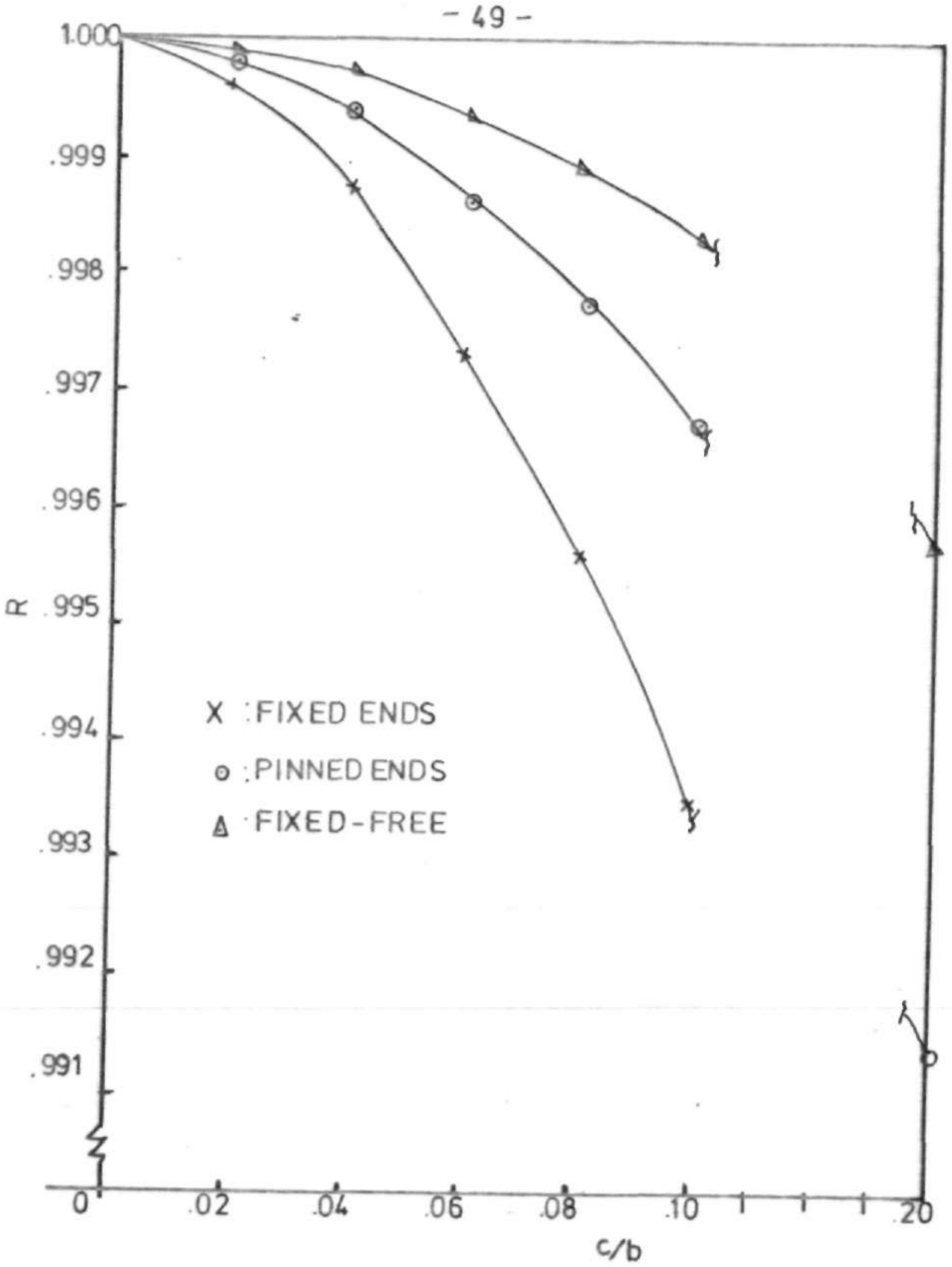


FIG.14 : VARIATIONS OF CRITICAL LOADS WITH c/b

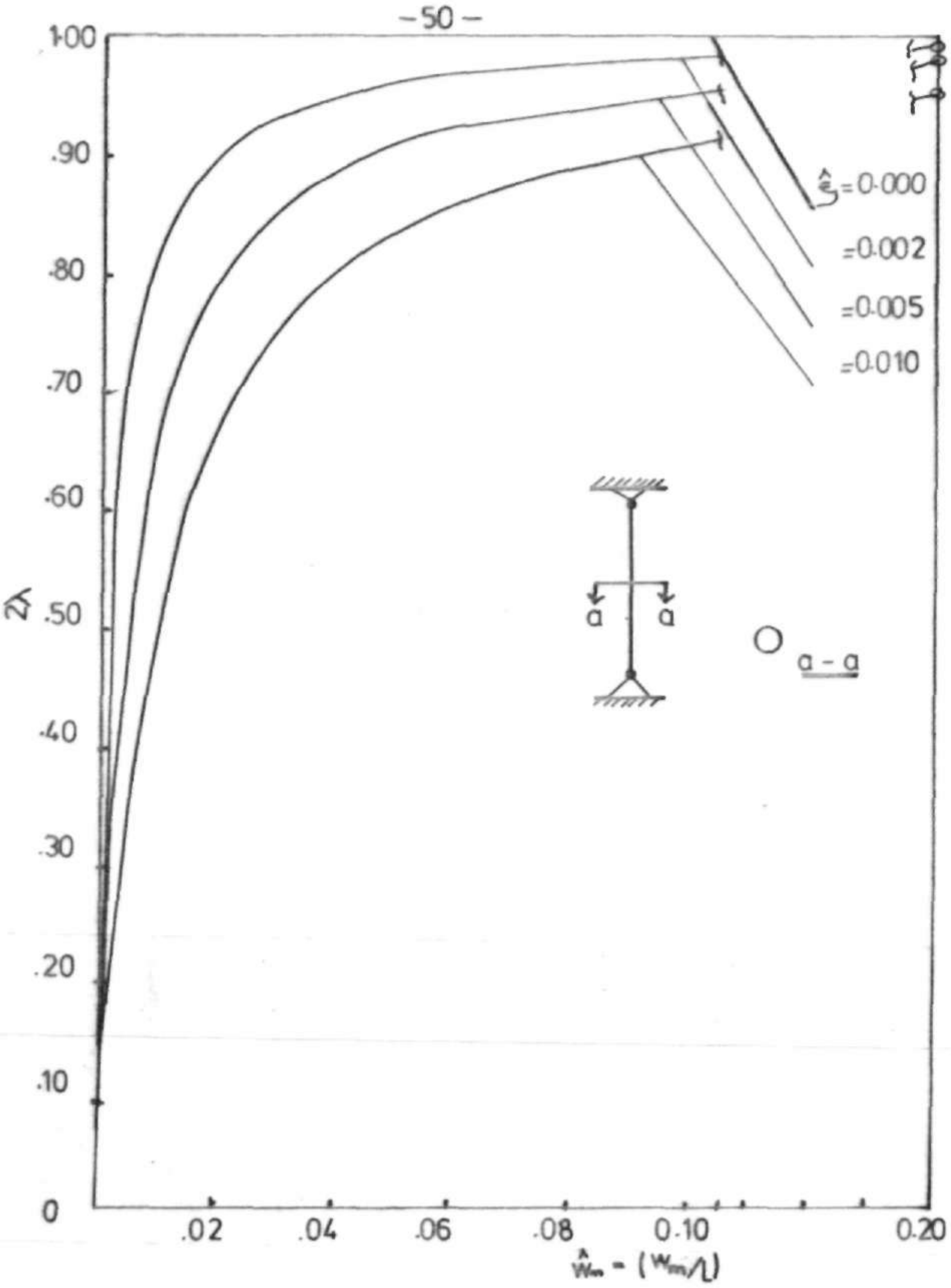


FIG. 15 : LOAD VERSUS DISPLACEMENT (PINNED SOLID CYLINDRICAL COLUMN)

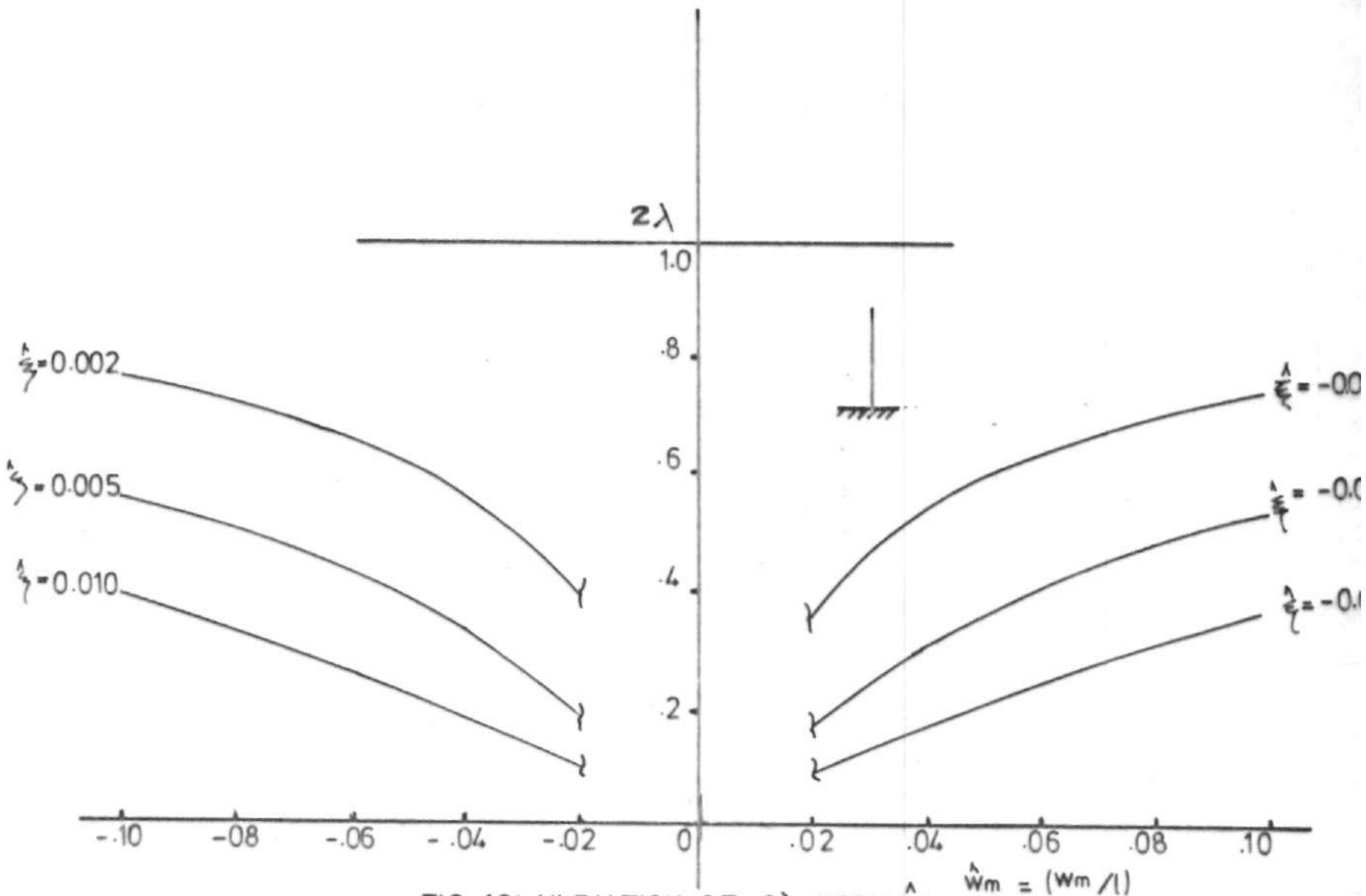


FIG. 16: VARIATION OF 2λ WITH $\hat{W}_m$
(FIXED - FREE COLUMN)

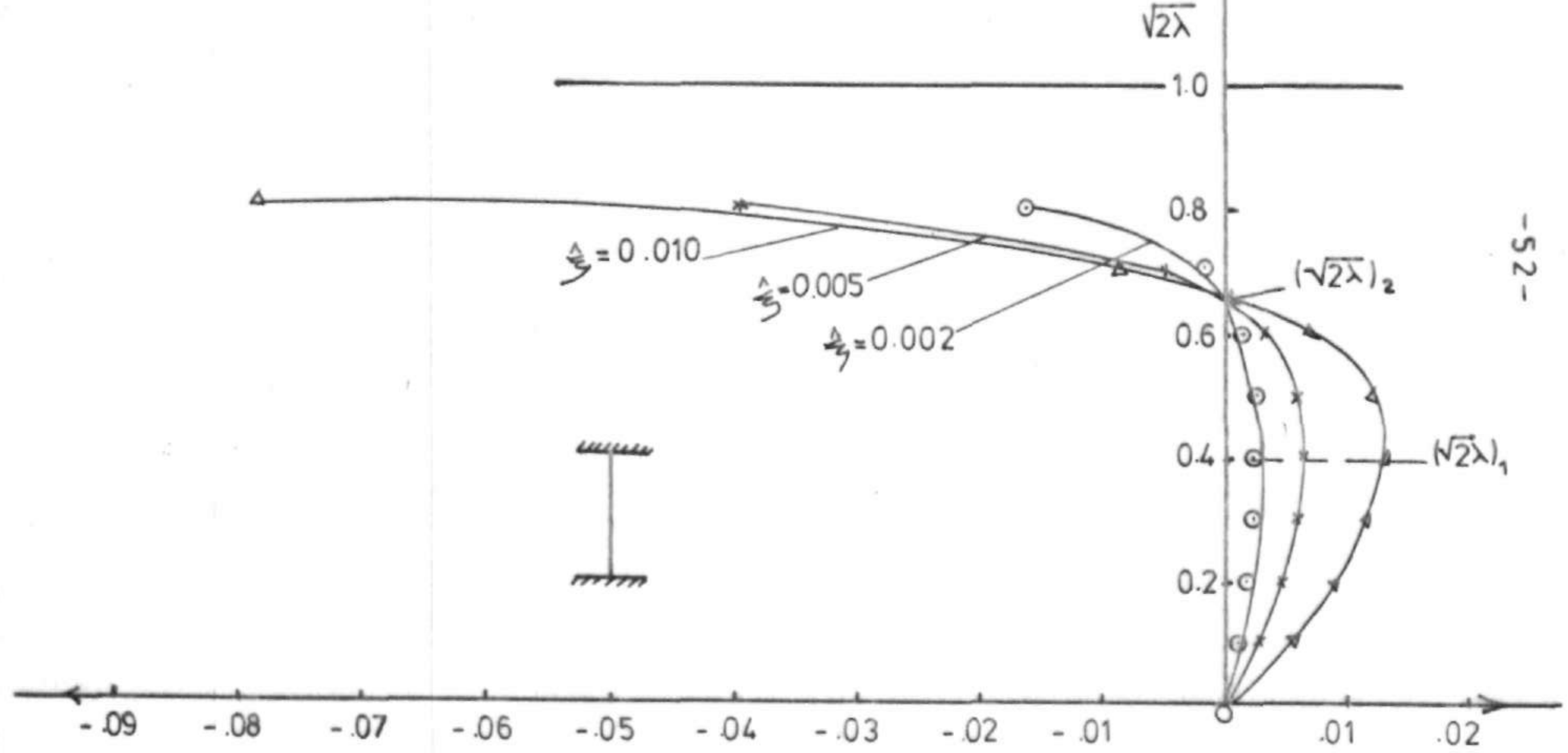


FIG.17 VARIATIONS OF LOAD WITH DEFLECTION
(FIXED - FIXED COLUMN)

and crookedness on the critical loads of the model assessed are discussed.

a. <u>crack effect</u>

Figure 14 shows the effect of crack alone. The reductions in the critical loads of the models do not exceed 1% within $0 \leq c/b \leq 0.2$. For example, at $c/b = 0.08$, the reduction for each model is as follows:

0.45% for fixed-fixed

0.22% for pinned-pinned

and 0.11% for fixed-free.

Thus, the fixed-free model sustains more load than either of the fixed-fixed or the pinned-pinned model. This implies that the effect of edge crack on the critical buckling loads of columns depends on their end restraints. This is in turn a dependence on the effective lengths of the columns. Similarly for a pinned tapered column, the reduction is less than 1%. This reduction increases as ratio D_1/D_o is augmented. However, the reductions become significant when c/D_o is large (see Fig. 12).

b. <u>crookedness effect</u>

The critical load of perfect system is approached with increasing deflection. (see Fig. 15). The plots are non-linear, but the degree of non-linearity diminishes with decreasing imperfection magnitude and consequently with decreasing deflection.

The cantilever model exhibits a similar non-linearity as depicted in Fig. 16. However, eventhough its load sustaining capability under crookedness increases with increasing deflection, its response to imperfection is quite damaging.

For the fixed ended model, the response differs. The model exhibits bifurcation - the system follows more than one equilibrium path. Figure 17 shows that $\sqrt{2\lambda}$ increases with $\hat{W}_m$ until $\hat{W}_m$ becomes critical corresponding to $(\sqrt{2\lambda})_1$ and decreases to zero at $(\sqrt{2\lambda})_2$. Thus, at $(\sqrt{2\lambda})_2$ the model maintains response to negative deflection, $\hat{W}_m$. The load sustaining capability thus increases with increasing negative deflection but never attains the critical load of the perfect system.

c. crookedness and crack effect

For all cases, the plots exhibit nonlinearity. The pinned model responds to both imperfections when they take positive values (see Fig. 7). However, the cantilever model shows a similar response for positive and negative e/b (Fig. 8). For both models the degree of non-linearity diminishes as $c/b \to 0$.

The fixed-fixed model shows an asymmetric bifurcation (Fig. 9), a common case in the buckling of rigid-jointed frames /33/.

5.3 Comparison of Results

A quantitative comparison of the results obtained

TABLE 5.1

Loads sustained by the Imperfect models in % of critical loads of the perfect models.
(crack-like imperfection)

MODEL $\hat{\xi} = 0$	Load sustained c/b = 0.02	Load sustained C/b = 0.10
(pinned-pinned)	99.98	99.67
(clamped-free)	99.99	99.83
(clamped-clamped)	99.96	99.34

TABLE 5.2

Loads sustained by the Imperfect models in % of critical loads of the Perfect systems.
(crookedness imperfection)

MODEL c/b = 0	$\hat{W}_m$	load sustained $\hat{\xi}$ = 0.002	-0.002	0.005	-0.005
	0.02	90	-	80	-
	0.02	36	36	18	18
	-0.02	80	-	76	-

is made with respect to the analytical perfect systems. This is on the basis of load sustainance of the imperfect models.

Table 5.1 depicts the loads sustained by the columns when the defect is crack-like only. At c/b = 0.02, the clamped-free column can sustain as much as 99.99% of the load of the perfect system. But the pinned-pinned and clamped-clamped sustain 99.98% and 99.96% respectively. Similarly, at c/b = 0.1, the corresponding loads sustained are:

99.83% for clamped-free,

99.34% for clamped-clamped

and 99.67% for pinned-pinned columns.

Thus, the % reductions can be said to be very small except when c/b becomes large (Fig. 14).

From Table 5.2, it is clear that a pinned column could have its critical load reduced by 10% when an initial croodedness, $\hat{\xi} = 0.002$, is present and the deflection is in the positive direction. Unlike this model, the fixed-fixed and fixed-free models have properties that depend on the signs of the imperfections and deformation - asymmetric characteristics. For positive deflection and at $\hat{\xi} = -0.002$, the reduction in the critical load of the fixed-free model is 64%. Similarly, for a fixed-fixed column, when the crookedness is assigned a positive value, say 0.002, and the deflection is negative, the reduction is 20%.

TABLE 5.3

Loads sustained by the Imperfect models in % of critcal loads of the perfect systems. (crack-like and crookedness Imperfections)

Model	$c/_b$	Load sustained $\hat{\xi}$			
		0.002	-0.002	0.005	-0.005
	0.02	85	-	68	-
	-0.02	70	-	32	-
	0.02	-	61	-	53

The combined effect of both imperfections (crack and crookedness) is depicted on Table 5.3. Under these imperfections, asymmetric characteristics are exhibited by the fixed-fixed and fixed-free models. The pinned model responds to both imperfections when they are positive. At $c/_b = 0.02$ and $\hat{\xi} = 0.002$, the reduction is 15% for the pinned model. For fixed-free model the reduction is 30% at $c/_b = -0.02$ and $\hat{\xi} = 0.002$. Similarly for $c/_b = 0.02$ and $\hat{\xi} = -0.002$, the reduction is 39% for the fixed-fixed model.

In all cases the negative values of $c/_b$ are employed so that the behaviour of the columns can be realised. Practically, the values have no significance. However, for negative values of $\hat{\xi}$, they maybe realised by manoeuvring the system practically. For example, the cantilever model can exhibit the behaviour shown in Fig. 16 if there exists a lateral load.

From these observations (i.e. from the Tables and the corresponding characteristic curves) it could be inferred that the load carrying capacities of these models are dependent on the signs of the imperfections. This is, of course, a dependence upon the boundary conditions of the columns. The % reductions due to both imperfections on the models are more than those due to the sum of the individuals except for the cantilever model.

The application to the tapered columns shows that the net reduction in the critical load of imperfect columns is greater for tapered columns than for uniform columns (Fig. 12).

Chapter 6

CONCLUSIONS

The significance of imperfections (crack and crookedness) measured in terms of reduction in column capacity of cylindrical columns under three different end restraints were examined.

A comparison of the load sustaining capabilities for the models is presented. For all models, these capabilities increase as the deflection and the ratio of crack depth to least dimension are augmented. The analysis shows that:

1) When the defect is modelled as an existing edge-crack, the reduction in the critical load for each model is less than 1% for very small crack depth to radius ratios.
2) With crookedness only, the reductions are significant. The cantilever model has the least load sustaining capability while the pinned model sustains most. The fixed-fixed model exhibits some buckling modes. The response to positive deflection is maximised whereas it is infinite for negative values.
3) Under the combined imperfections, the pinned model can sustain as much as 85% of the critical load of the perfect column; the cantilever and fixed-fixed

models can only sustain 70% and 61% respectively of the perfect critical loads. Eventhough the sustaining capability increases as ratio of crack depth to least dimension is augmented, it decreases with increasing crookedness.

4) The fixed-fixed model exhibits an asymmetric bifurcation under the influence of both imperfections.

The application of the analysis to cracked tapered and cracked crooked columns shows that the critical loads of the members are reduced significantly when the imperfections become large.

Quantitatively, the total effect of the individual imperfections of crookedness or crack is less than the combined effect for pinned and fixed-fixed models. The cantilever model, however, gives a reverse response. In this case, the combined effect is less than the sum of the individual effects. Imperfections in the form of initial crookedness mainly governs the variation in the load sustainance of all the models.

This investigation has shown that there is some definite deflection for every value of the axial load, which, however, increases more rapidly as the axial load approaches the critical value for the perfect system. Therefore the load-carrying capacity of a column with initial crookedness depends on some physical criteria established with respect to its serviceability. A limit on the lateral deflection can thus be a criterion (as seen in the response of the fixed-fixed model in positive sense of the deflection) for practical application.

It should be mentioned that the findings in this thesis are relevant to steel structures. This is because of the limitation on $c/b \leq 2\%$. However, for higher values of c/b, the cantilever model approximates the behaviour of concrete cross sections under the combined imperfections.

This thesis did not consider eccentricity in loading of columns directly. The problem was treated indirectly via the assumed initial deflection patterns (i.e. initial crookedness). The erratic nature of imperfections appear to limit the effectiveness of a deterministic approach to column stability studies. Therefore, a statistical treatment of imperfections /9, 30/ is required to fully quantify response of columns to imperfections. This area of research is recommended for further study.

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