RELIABILITY ASSESSMENT OF THE STRENGTH CAPACITY OF SOLID TIMBER COLUMNS IN FRP LAMINATES AND SPRAYS

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DECLARATION

I hereby sincerely declare that this project work titled RELIABILITY ASSESSMENT OF THE STRENGTH CAPACITY OF SOLID TIMBER COLUMNS IN FRP LAMINATES AND SPRAYShas been prepared by me and that it is a record of my research work. It has not been submitted to any previous publications for the award of any certificate, degree or higher degree. All quotations are indicated and the sources of the information are appropriately acknowledged by means of references.

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CERTIFICATION

This research dissertation entitledRELIABILITY ASSESSMENT OF THE STRENGTH CAPACITY OF SOLID TIMBER COLUMNS IN FRP LAMINATE AND SPRAYSBy **Samuel IliyaBajahry** meets the regulation governing the award of the degree of **Masters of Science Civil Engineering (M.Sc. (Structures))** of Civil Engineering Department of Ahmadu Bello University, Zaria and it is approved for its contribution to knowledge and literary presentation.

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DEDICATION

I dedicate this dissertation to God Almighty the author of my life, for his divine wisdom, knowledge and understanding. To my lovely family members, especially my parents, who through their struggles made my education a success. Once again I dedicate this dissertation to my Late father NdeIliyaBajahry.

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ABSTRACT

Carbon Fiber-Reinforced Polymer (CFRP) composite materials have provenvaluable properties and suitability to be used in the construction of new buildings and in upgrading theexisting ones. One of the objectives of this study is to laminate solid timber columns with varying thicknesses of CFRP laminates to check the patterns of the stresses, displacements and reactions of the solid timber columns. An analytical model for CFRP strengthened timber columnswith length of 3700mm, width of 150mm, depth of 200mm and varying thicknesses of 0.2mm, 0.4mm, 0.6mm, 0.8mm and 1 mm with coded specified axial applied load for three different timber species of *Abura*, *Afromosia* and *Confusa*. The secondlayer of CFRP is consideredmost effective wrapping scheme due to the cost effectivenessof increase in axial applied load. This research findings have shown that CFRP laminates and sprays are not only effective in restoring the lost capacity of damaged timber columns sections, but are also quite effective in strengthening of timber columns sections to sustain higher loads,extend their fatigue life and reduce crack propagation. The reliability assessment of composite timber columnswith varioustimber species was also executed using MATLAB programming. Adequate safety indices were obtained for different load ratio, MOE of CFRP, thickness of CFRP and strength classes of timber species..

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SYMBOLS AND NOTATIONS

A	Area
B	Width of column
D	Depth
$E_{0.05}$	Fifth- Percentile of modulus of elasticity
$E_{m,g}$	Global modulus of elasticity in bending
$E_{0,mean}$	Mean modulus of elasticity parallel to grain
$E_{90,mean}$	Mean modulus of elasticity perpendicular to the grain
F_{max}	Maximum Load (N)
f_m	Bending strength (N/mm^2)
$f_{m,d}$	Characteristic bending strength
$f_{c,0,k}$	Characteristic compression parallel to grain
$f_{c,90,k}$	Characteristic compression perpendicular to grain
f_{05}	Mean (in N/mm^2) of the adjusted 5-percentile values
G_k	Dead load
G_{mean}	Mean shear modulus
h_1	Height of column
h_2	Height of rafter
H	Height of the portal frame
I	Second moment of area
I_Y	Second moment of area about y-y axis
I_Z	Second moment of area about z-z axis
k_s	Factor for adjusting number and size of samples

K_c, K_z	Instability factor
K_{mod}	Strength modification factor
K_{cri}	Takes account of the effect of axial instability reducing the design Compression strength of a member
K_{cz} , K_{cy}	Takes account of the effect of lateral torsional instability, reducing the design bending strength of a member
L	Span of portal frame
l_e	Effective length
M	Applied bending Moment
M_B	Moment at B
M_D	Moment at D
Q_k	Imposed load
R_{EH}	Horizontal force acting at right fixed end
R_{AH}	Horizontal force at left fixed end
V	Design load
V_m	Applied Force at joint
Z	Section Modulus
α	Load ratio
β	Safety Index
γ_g	Material factor of safety for dead load
γ_q	Material factor of safety for imposed load
$\sigma_{m,o,d}$	Bending stress parallel to grain
$\sigma_{c,0,d}$	Compressive stress parallel to grain
$\sigma_{m,cri}$	Critical bending stress

ρ_k	Characteristic density
ρ_{05}	Fifth-percentile value of density
ρ_{mean}	Mean (in N/mm^2) of the adjusted 5-percentile values
grain	
μ	Measured moisture content
$\lambda_{rel,y}$	Relative slenderness of column about y-y axis
$\lambda_{rel,z}$	Relative slenderness about z-z axis
λ_z	Slenderness ratio about y-y axis
λ_y	Slenderness ratio about z-z axis

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CHAPTER ONE

INTRODUCTION

1.1General

Timber have for ages remained among the major structural materials for building construction worldwide due to their renewable nature, availability in various sizes, shapes and colours, affordability, relatively high fatigue resistance and specific strength, ease of joining, durability, and aesthetic appeal. In Europe, timber, have been successfully utilized in both simple and complex structures(Ratief and Holicky, 2005). In Nigeria however, the only area where timberreceived wide acceptance is in roof framings. The utilization of the material in the engineered design of residential and commercial building receives little or no attention.

NCP 2 (1973), which is the timber design code in Nigeria since 1973 is based on permissible stress design philosophy. The code made references to the CP 112(1967). CP 112 (1967) had passed through several revisions since its inception and up to the current BS 5268 (2002). EC5 (2004) is a limit state design code for timber structures which currently co-exist with BS5268 (2002), which was fully replaced in 2010 (Tord, 2001; TRADA, 2009). BS 5268 (2002) is fully deterministic, while the EC5 (2004) is semi-probabilistic (Chanakya, 2009). The use of limit state design, instead of permissible stresses, enables differentiation of partial safety factors for permanent and variable loads, and makes it possible to reach a more even nominal safety level in all structures (Alpo, 2004). As a recently developed and formulated structural design standard, EC5 (2004) provides a wide range of consistent and up to date models and procedures that can be considered for the development of a local code. Revision of theEC5 (2004) design requirements based on the data on the properties of Nigerian timber species will go a long way in providing a background for the adaptation of advances in technology in the developed countries of Europe to local design practice of timber structures in Nigeria, just as it is being done in other developing countries like South Africa (Ratief and Holicky, 2005).

These work is governed by structural economics, reliability, and so these costs must be considered as vital parts of engineering management. The concept of rehabilitation to increase structures stability should be considered as an alternative to complete reconstruction which can be uneconomical and timely due to jacking up of the structure, while repairs occur. The replacement of structural components would normally require the use of hardwood which is generally stronger than softwood, due to its superior density. This is typically an expensive process (Emerson, 2004), also creating sustainability issues due to the difficulty of obtaining old growth mature forest sawn timber (Walter, 1996). For softwood to be used in an outside environment, it generally has to be treated with toxic preservatives to protect it against fungi and insect pests. Although considered weaker, softwood timber forests are sustainable due to the speed at which the trees grow. If the softwood timber were able to be sufficiently protected, it could make for a viable option environmentally. Due to these concerns, column strengthening using alternative construction methods must be considered.

Carbon Fibre Reinforced Polymers (CFRP) have the capability to strengthen timber components in compression. Limited research has been conducted on layers of CFRP as a protective method from environmental degradation, but some promising results have been demonstrated.

The benefits of confining timber have mostly been documented from experimental research. North American and European standards establish that CFRP confinement of steel and reinforced concrete has structural benefits such as increased strength and ductility, comparable to the limited timber research conducted. The advantages of using CFRP confinement for reinforcement are that it is durable, corrosion resistant, easy to use and transport due to its high strength to weight ratio and is more flexible than steel (Micheal, 2006). In a circular or rectangular timber column, the axial capacity may be increased, enhancing the compressive strength. Comparably to steel plate bonding, an alternative strengthening system in CFRP confinement, requires less manpower and scaffolding as it is a light material, saving time and money (Heslehurst, 2008).

1.2 Background of Study and Statement of the Problem

1.2.1 Background of Study

CFRP confinement as a strengthening system has been found to increase the load capacity of structures. The uncertainty of the mechanical properties of timber can be dissipated through the use of CFRP wraps. The laminates are carbon fibre reinforced polymers. The fibres, which are highly anisotropic (Pearce, 1970) provide the stiffness and strength of the system, while the polymer matrix holds the fibres in place. As timber does not have equal properties in all directions, confinement with CFRP can help to mitigate the random character of wood (Kasal and Heiduschke, 2010). Fibres are used from a mass of materials, a significant increase in strength and decrease in brittleness occurring when the materials undergoes an extrusion-like process (Heslehurst, 2008). The bond of the confinement of timber with CFRP is of great significance. The epoxy matrix displays excellent adhesion and strength, forcing the individual and flexible fibres to cooperate in the same direction, transfer loads between the fibres and protect the fibres from environmental factors (Pearce, 1970). The epoxy matrix is cured by the addition of a hardener, the mix forming a chemical bond. When cured, the once flexible and workable fibres become very stiff. This is important for compressive loads and the avoidance of buckling. Strengthening of structures is a requirement for rehabilitation of structures in order to increase its structural capacity. Compressive strength, ductility and environmental protection are three of the factors considered to be benefited from confinement. The benefits vary depending on wrap number and orientation, and produce differing failure mode from the reference samples (Zhang et al., 2012; Heiduschke and Haller 2012; Kasal and Heiduschke 2010). Compressive forces are jointly transferred to the timber and the carbon fibre reinforcement polymers, thereby increasing the strength of the columns. Tests have shown that with the combination of different orientations and types of wraps, as well as number of layers, different benefits can be found including increase in strength, ductility and stiffness (Zhang et al., 2012; Heiduschke and Haller, 2012). Dissimilar failure modes were also observed in comparison to the reference specimens. The notion of CFRP for rehabilitation of timber structures has not been examined to satisfied extent. The benefit of confining timber has mostly been documented from experimental research.

There are so many advantages derived from wrapping timber with CFRP. Some of the main advantages of CFRP wrapping of timber are (Najm et al., 2007; Heiduschke and Haller, 2012; Zhang et al., 2012 Jimenez et al., 2011; Tsakania and Mouzakis, 2010; Webber and Yao, 2001): (i) Increase in compressive strength; (ii) Protection from environmental degradation; (iii) Restoration of historical building or structures; (v) Can be designed or manufactured to meet specific mechanical properties. Emerson (2004) found that transverse reinforcement increased the strength of the column so that it exceeded that of the design value of the column. Additional compressive and bending strength was provided by longitudinal reinforcement. In both of these cases, full wraps were utilized, which has shown to produce the most promising results. Longitudinal cracks in timber are a common concern and reduce compressive strength in columns. Cai et al.,(2012) found that with wraps 50 mm wide and 115 mm apart that eccentric load capacity could be increased, and that the CFRP could contain further crack opening and confine local ruptures. Najm et al.,(2007) compared the behavior of full wraps and spirals, finding that the full wrapping had greater benefits than those with spiral reinforcement. This coincides with research conducted by Zhang et al.,(2012), a main variable being CFRP spacing and width along the length of the timber columns. The benefits increased with smaller spacing between wraps and an increase in the width of the sheets. The number of layers also had an effect with Zhang et al.,(2012) finding that results became stable after three layers of CFRP were applied. The use of three layers produced the best results Najm et al.,(2007), they found that tests fully confined with CFRP showed an increase in load capacity as more layers were added. A favorable failure mode occurs when deformation is observed before failure. A sudden failure can cause sudden collapse without warning, potentially resulting in the loss of lives. The failure modes observed by Heidushke and Haller (2012) of reinforced tubes demonstrated ductile behavior, the wood fibres crushing parallel to the grain, in conjunction with local buckling. Song et al.,(2010) found that two types of failure modes were prevalent, significant changes in the columns occurring in both types around 70% of the maximum loads. For the first types of failure, compression wrinkles become apparent at mid-height of the specimens which then propagated until maximum load, while in the second type, significant crushing deformation occurred near the ends of the cylinders, the deformations propagating until maximum load.

1.2.2 Statement of the Problem

Timber of historical structures may experience significant damages and cracks during their service life due to fungal decay, moisture changes or external loading. These damages and cracks may significantly affect the compressive strength or behaviour of timber column and it carrying capacity. According to Sousa et al.,(2011), the material properties of a timber element vary both in different parts of the same cross-section as well as along the element itself. As such, timber structures are better analysed using probabilistic models. EC5 (2004) provides a wide range of consistent and up to date models and procedures that can be considered for the development of a local code.

EC5 (2004) does not contain any data on the material properties of Nigerian timber species.

According to standards such as probabilistic model code, JCSS (2006) and EN 408 (2003), the basic material properties of any timber species are determined through laboratory experiments, while other material properties can be determined using the JCSS (2006) and EN 408 (2000). This research work therefore has to come up with data through laboratory experiments that was used to develop the strength classes for the three (3) selected Nigerian timber species based on the recommendations of EN 338 (2009), which are: *Mitragynaciliata* (Abura), *Afromosia elata* (Afromosia) and *Berlinia/confusa grandiflora* (Berlinia).

CFRP is a strong and light fiber reinforced polymer which contains carbon fibers. It has high strength to weight ratio, recyclable, environmentally friendly and with good rigidity when compared to steel. It can also be used as an alternative instead of steel, on the onset of thisstudy. The safety of the timber columns in CFRP and thus of the whole structure depends mainly on the evaluation of thickness of timber column and laminates or sprays since the two materials involved are anisotropic.

1.3 Justification of Study

Timber is an organic material and thus is subject to deterioration with time. Trees are of immense importance in the ecosystem and felling of trees could exacerbate the problems of ozone layers depletion.

Timber benefits from its natural growth characteristics such as grain patterns, colors and availability in many species, sizes and shapes that make it remarkable, versatile and aesthetically pleasing materials. Timber can easily be shaped and connected using nails, screws, bolts and dowels or adhesively bonded together (Jack and Abdy, 2007). The limitations in maximum cross-sectional dimensions and length of solid sawn timber, due to available log sizes and natural defects, are overcome by the recent development in composite and engineered wood products.

Timber structures can be highly durable when properly treated, detailed and built, but solid timber is rapidly becoming scarce and expensive due to logging and the long period of time it takes for most trees to grow to maturity. Timber is an excellent choice for any sort of wood work but good quality timber with minimum flaws comes with a bit of extra cost due to the reasons above. It is for these reasons that it becomes necessary to study the behavior of timber columns laminated with CFRP in economical way and how to maximally use, while it meets up with its design life structurally in service.

1.4 Scope and Limitation

1.4.1 Scope

This research work considers a reliability analysis using EC 5 (2004) of axially loaded timber columns of three different species of timber columns,fixed-free end and fixed-fixed end laminated with CFRP and sprays and to use Finite Element Analysis (FEA) as coded in ABAQUS software.

1.4.2 Limitations

The limitations of this thesis include the following (Micheal, 2006).

(i.) There have been attempts to improve the ductility of CFRP with little or no success.

(ii.) Structural limitation of CFRP is that, it lacks fatigue endurance limit.

(iii.) CFRP columns when loaded in tension, exhibit a linear stress-strain behavior up to rupture.

(iv.) Many researches have been carried out using CFRP both for retrofitting and as an alternative to steel as reinforcement or pre-stressing materials. Cost remains an issue and long term durability questions still remain.

(v.) Only three species of Nigerian timber are catalogued in BS 5268 (2000).

(vi.) 1.5 Aim and Objectives

1.5.1 Aim

The aim of this work is to check the reliability assessment of the strength capacity of solid timber columns in Carbon Fibre Reinforced Plastics (CFRP) laminates and sprays.

1.5.2 Objectives

The objectives of this study include:

(i.) To evaluate the structural reliability assessment of three timber species mentioned in BS 5268 (2000) and EC5 (2004) laminated with CFRP under compression.

(ii.) Use Finite Element Method as coded in ABAQUS software for structural safety assessment of the timber columns with CFRP laminates and sprays and

(iii.) Perform reliability analysis of the designed timber columns in FRP laminates and sprays using First Order Reliability Method in view to generating structural safety indices for varying thicknesses of FRP laminates and sprays in the designed structure.

CHAPTER 2

LITERATURE REVIEW

2.1 General

Timber is an efficient building material, not only in regard to its mechanical properties but also because it is a highly sustainable material. Timber is considered a sustainable building material because it is derived from a renewable source and has a low embodied energy (Chanakya, 2009; Porteous and Kermani, 2007; Robert, 2009). Embodied energy reflect the minimal non-renewable energy used in the production of timber and its application in construction. Timber has sound thermal properties, meaning that timber structures rely less on carbon-emitting heating and cooling appliances than building construction of other material. Wood is also durable, since many products, particularly hardwoods have a service life of greater than 50 years, and often require little energy in maintenance (Robert, 2009). Wood can also be recycled, which is important in terms of storing carbon through the life of a product and its transformation. Wood is an inexpensive material. Forest is a wood factory which produces wood using only solar energy (Dorina *et al*, 2012).

Timber is a widely available natural resource in many countries; with proper management, there is a potential for a continuous and sustainable supply of raw timber material in the future. Due to the low energy use and low level of pollution associated with the manufacturing of timber structures, the environmental impact of timber structures is much smaller than for structures built using other building materials (Porteous and Kermani, 2007).

The concept of environmental performance becomes clearer when wood is compared with steel and concrete.

Recent research studies confirmed that houses made from wood present significantly lower risk for the environment. Wood-based building materials require less energy to be produced, emit less pollution to the air and water, contribute with lower amount of CO_2 to the atmosphere, are easily disposed of or recycled, and are derived from a renewable resource. It can be easily concluded that the environmental advantages of wood compared to steel and concrete are obvious. Life-cycle analysis results for the steel-framed versus wood-framed home (Dorina *et al*, 2012) showed that the steel-framed home use 17% more energy; had 26% more global warming potential; had 14% more air emissions; had over 300% more water emission and had about the same level of solid waste production than timber. Analysis results for the concrete-framed home used 165 more energy; had 31% more global warming potential; had 23% more air emission; had roughly the same level of water emission and produced 51% more solid waste.

Timber is an advantageous building material due to its material properties. Timber is a light material and, compared to its weight, the strength is high; the strength to weight ratio is even higher than for steel (Porteous and Kermani, 2007). However, timber is still not utilized to its full potential in the building and construction sector considering its beneficial properties. Many building owners, architects and structural engineers, do not consider timber as a competitive building material compared to concrete, steel or masonry. Attributes such as high performance in regard to reliability, serviceability and durability are generally not associated with timber as a building material. One of the main reasons for this is that timber is a highly complex material; it actually requires a significant amount of expertise to fully appreciate the potential of timber as a structural building material. In addition to this there are still a number of issues which need to be further researched before timber materials can achieve the same recognition as a high quality building material such as for example, steel and concrete.

Timber is an abundant, renewable and recyclable material, which has been used by humans for thousands of years. Its use in construction is still widespread, ranging from structural frames to floors, paneling, doors, interior and exterior woodwork, and furniture, among its multiple uses in an average dwelling (FPL 1999). Three polymeric materials make up the wood cells: cellulose, hemi-cellulose and lignin (Kollmann and Cote, 1968). Cellulose makes up the cell walls, and provides the tensile strength of the wood matrix. Hemicellulose is similar to cellulose, and grows around the cellulose fibres. Lignin gives rigidity to the wood, allowing trees to grow upright; it cements the cells together, thus accounting for the compressive and shear strengths of wood (Kollmann and Cote 1968; Moraes 2003). These three polymers form an inhomogeneous and anisotropic material, which exhibits great variability among different species.

In general, tree species can be divided into two major groups, hardwoods and softwoods. Hardwoods are porous, and present greater hardness than softwoods (although some exceptions exist) (FPL, 1999). Timber structures have traditionally been built using heavy timber frames, with the walls being constructed of various materials such as interwoven branches and split logs in the very early versions of these types of structures (as early as 6500 BC), and later using plastered panels and bricks (Foliente, 2000).

FRP wrapping has been proved to be an effective method for retrofitting timber columns; however, it is quite construction intensive and may significantly affect the architectural appearance of the timber members and the historical timber buildings (Xiobin et al, 2012).

In the field of structural engineering, various researches on the use of fiber reinforced polymer (FRP) have been conducted and several others are ongoing in order to explore its effectiveness in structural engineering. Some of the broad applications of the composites of FRP include;

rebar, grating into concrete, and wrapping around columns and piers, etc. (Saadatmanesh, 1994).

The FRP composites may contain other fibers such as Kevlar, aluminum or glass fibers as well as carbon fibers. The properties of the final CFRP product can also be affected by the type of additives introduced to the binding matrix (or resin). Composite plastics refer to those types of plastics that result from bonding two or more homogeneous materials with different material properties to derive a final product with certain desired material and mechanical properties. Fibre-reinforced plastics (FRP) are a category of composite plastics that specifically use fibre materials to mechanically enhance the strength and elasticity of plastics.

Carbon fiber reinforced polymers or carbon fiber reinforced plastic (CFRP) is very strong and light fiber reinforcement. The polymer is most often epoxy but other polymers such as polyester, vinylester or nylon are sometimes used. They are used wherever strength-to-weight ratio and rigidity are required.

2.2 Timber

CFRP confinement as a strengthening system has been found to increase the load capacity and reduce strain in columns. The compressive forces aretransferred from the timber to the carbon fibre reinforced polymers, and thereby increase the strength of the columns. Tests have shown that with the combination of different orientations and types of wraps, as well as number of layers, different benefits can be found including an increase in strength, ductility and stiffness (Sweeney, 2012).

2.2.1 Failure Modes of Timber

A favourable failure mode occurs when deformation is observed before failure. A sudden failure can cause sudden collapse without warning, potentially resulting in the loss of lives.

The failure modes observed by Heiduschke and Haller (2012) of reinforced tubes demonstrated ductile behavior, the wood fibres crushing parallel to the grain, in conjunction with local buckling. Song, Zhang and Gu (2010) found that two types of failure modes were prevalent, significant changes in the columns occurring in both types around 70% of the maximum loads. For the first type of failure, compression wrinkles became apparent at mid-height of the specimens which then propagated until maximum load, while in the second type, significant crushing deformation occurred near the ends of the column, the deformations propagating until maximum load. Favorable results were observed from the stress-strain diagrams; the failure modes demonstrating significant deformation before failure (Jobin and Olga, 2007).

The identification of different failure modes is important for both modeling and experimentation. Different failure modes will be identified based on material properties, cross sectional distribution and loading configurations.

2.2.2 Tensile Failure in Timber

This is the most common failure mode in timber structures. This failure mode is brittle as the timber does not have plastic behavior under tensile loads. This can even introduce cracks along the fiber direction which will result in catastrophic destruction of the cross section. Timber limit state is considered to be attained when maximum tensile stress is equal to its tensile strength (Jobin and Olga, 2007). That is; $f_{tapplied} \leq f_{tpar}$.

2.2.3 Compressive Failure in Timber

This failure mode is not very common in un-reinforced (un-strengthened) sections. But when the column is reinforced in the tension side, this failure mode can occur. A detailed study of this phenomenon can be seen in the modeling part. The ultimate state of compression is considered to have been attained when the maximum strain in the compression zone reaches the value of the ultimate compressive strain (Jobin and Olga, 2007). That is; $f_{capplied} \leq f_{cpar}$.

2.2.4 Structural Behavior of Timber

Timber is a unique engineering material because it is a defect-filled natural composite. A distinction must be made between timber and wood. Madsen (1992) defines timber as a useful construction material produce from logs of trees and wood as defect free wood.

Timber and wood, in the sense of clear wood, are two very different materials. Failure in clear wood beams in bending is initiated by wrinkles in the compression zone, while failure in timber is initiated by cracking in the tension zone. The cracking in the tension zone is created by tension perpendicular to the grain stresses where the fibres have been disturbed in the vicinity of defects such as knots or other localized slope of grain. The presence of defects in timber causes different behavior in bending and tension than in compression (Christopher, 2000).

2.2.5 Stress-Strain Behaviour

The general stress-strain relationship for wood and timber is similar, but with different relative values of compression and tension strength. When wood or timber laminates is tested to failure in axial tension, the stress-strain relationship is fairly linear up to maximum load, and the timber fails in brittle tension. In axial compression, timber is a much more ductile material, exhibiting a linear stress-strain relationship up to a proportional limit, beyond which ductile yielding takes place. For modeling purposes, the shape of the entire stress-strain relationship must be known (Christopher, 2000).

2.3 Fibre Reinforced Polymers (FRP)

Fiber reinforced plastic (FRP) (also *fiber-reinforced polymer*) is a composite material made of a polymer matrix reinforced with fibers. The fibers are usually fiberglass, carbon, or aramid, while the polymer is usually an epoxy, vinylester or polyester thermosetting plastic. FRPs are commonly used in the aerospace, automotive, marine, and construction industries (Lofgren, 2005).

In recent years, the fiber reinforced polymer (FRP) composites are becoming a popular material for a wide range of structural rehabilitation due to their superior material properties including; corrosion and weather resistance, high mechanical strength and low weight, ease of handling, good fatigue resistance, and versatility of size, shape and quality (Ede, 2012). Unlike most of the traditional building materials, the FRP composites can be specifically designed by blending the best combination of material properties in response to specific necessities (Ede, 2012). The use of fiber reinforced polymer (FRP) composites in various engineering fields, for example, aerospace, automotive, and marine engineering applications has attained an advanced level, while the use in civil structural applications is constantly increasing (Bakis et. al., 2002). The fiber reinforced polymer (FRP) is gradually taking the place of steel in some field of structural rehabilitation. In fact, FRP sheets may be wrapped around structural elements, resulting in considerable increases in strength and ductility without excessive stiffness change (Ede, 2012). The most common FRP products for civil structural applications are internal reinforcing bars, pre-stressing tendons/anchor systems, and externally bonded plates, sheets, shells and tapes.

FRP composites for structural strengthening are produced by embedding continuous fibers in a resin matrix which binds them together. The fibers are the load carrying elements and have highly oriented-defect free micro structures. The resin matrix binds the fibers together, protects fibers from the environment, provides stability to the fibers and acts as a medium to transfer stresses between adjacent fibers (Karbhari and Zhao, 2000). The main material properties include anisotropy, linear elasticity to failure, high tensile strength/modulus in the direction of the fibers, and generally limited compressive properties (Nanni, 1993).

2.3.1Types of Carbon Fibers

The three most common fibres used in structural applications are glass, aramid, and carbon. Glass fibres have high tensile strength combined with good mechanical properties, high chemical resistance, and excellent insulating properties. Some disadvantages of glass are low tensile modulus, sensitivity to abrasion, and reduced tensile strength in the presence of water and sustained loads due to creep rupture characteristics. The types of glass fibre commonly found in structural applications are E (Electrical) and S (high strength) (Christopher, 2000). Aramid fibre has the lowest specific gravity and the highest tensile strength to weight ratio of the main fibre types. In bending, aramid fibres exhibit a high degree of yielding on the compression side that is not observed in other fibres. The non-catastrophic failure mode gives aramid fibre composites superior damage tolerance to dynamic and impact loading. Some disadvantages of aramid are low compressive strength, difficulty in machining, and sensitivity to ultra-violet radiation. Aramid fibres are sold commercially under the trade names Kevlar and Twaron (Christopher, 2000).

Carbon fibres have very high tensile strength, high elastic modulus, and high fatigue strength. The main advantages of carbon fibres are their high strength-to-weight ratios, excellent durability, and low relaxation under sustained load. Some disadvantages include low impact resistance and high cost. Due to their high cost, carbon fibres have been used for very specific applications such as pre-stressing of structural members and flexural reinforcement of timber and concrete structures (Christopher, 2000).

2.3.2 Applications

In the past, the high cost of FRP materials had restricted their applications to areas where weight reduction has more importance than cost, such as the aerospace and sporting goods industries. More recently, the need to upgrade and repair infrastructures has led to increased research and use of FRP materials in structural engineering applications. Neale and Labossiere (1997) reviewed the state of the art in Canada on strengthening existing structures using FRP materials and Triantafillou (1998) reviewed the basic concepts and application of FRP laminate for the strengthening of civil engineering structure, with emphasis on reinforced concrete structures. For the repair of existing structures, FRP laminates (approximately 1 to 2-mm thickness) have been bonded to the tension surface of columns.

FRP sheets and fabrics have been wrapped around circular columns to improve ductility and strength and have been wrapped around the webs of beams toenhance the shear strength.

2.3.3 Strengthening of Structures

Strengthening of structures is a requirement for rehabilitation of structures and increasing their capability. Compressive strength, ductility and environmental protection are three of the factors considered to be benefited from confinement. The benefits vary depending on wrap number and orientation, and produce different failure modes from the reference samples. (Jobin and Olga, 2007).

Timber fibres have axial orientation which makes them bear axial loads efficiently. But for slender timber columns, there is need for increasing load bearing capacity against buckling. Thus, timber is associated with CFRP composite material in form of strips obtained by successively impregnating the glass thread with polyester resin with reinforcement ratio of 50% at environment temperature of more than 15^{o}C (Decher et al, 2012)

The wood planks are also impregnated with the same type of resin and then successively glued to the CFRP strips, until the number of strips resulted from the design computations are reached.

The procedure of obtaining such a hybrid cross section follows some steps presented here:

(a) Wood surface preparation by clearing away dirt or de-fibrated zones resulted from the cutting off process, ensuring a healthy surface

(b) Impregnation of wood surface with polyester resin, which constitutes the primer, and the start of gel formation;

(c) Application of the second layer of resin and of successively impregnated glass thread strips until the desired thickness of CFRP strip is reached;

(d) Positioning the second timber element previously prepared as the first one;

(e) Finally, the whole packet is pressed, for obtaining the optimal contact between materials.

This packet forms a module with different strength properties, superior to wood. For a certain structural element this module will repeat until reaching the necessary strength, without extremely increasing the cross-section`s dimensions.

2.3.4Manufacture of Carbon Fiber Reinforced Polymer (CFRP)

The process by which most carbon fiber-reinforced polymer is made varies, depending on the piece being created, the finish (outside gloss) required, and how many of this particular piece are going to be produced. In addition, the choice of matrix can have a profound effect on the properties of the finished composite (Nanni, 1993). Some of the methods of manufacturing polymer composites include; continuous reinforcing process, filament winding, pultrusion, hand lay-up processes, molding processes, matched-dye Molding, vacuum bagging, autoclave molding, resin injection process, resin transfer molding, reaction injection molding, and integrated manufacturing systems.

2.3.5Properties of Carbon Fiber Reinforced Polymer (CFRP)

The properties of carbon fiber reinforced polymer (CFRP) are as follow (Gopal, 2011):

(i.) Carbon fiber reinforced polymer composite (CFRP) has low thermal conductivity.

(ii.) Carbon fiber reinforced polymer (CFRP) is alkali resistant.

(iii.) CFRP has high strength-to-weight ratio and hence it eliminates requirements of heavy construction equipment and supporting structures.

(iv.) CFRP is available in rolls of very long length. Therefore, they need very few joints, avoiding laps and splices, and its transportation is also very easy.

(v.) CFRP has a short curing time. Therefore, the application takes a shorter time. This reduces the project duration and down time of the structure to a great extent.

(vi.) Carbon fiber reinforced polymer (CFRP) are resistant to corrosion, hence they are used for corrosion control and rehabilitation of reinforced concrete structures.

(vii.) CFRP is a bad conductor of electricity and is non-magnetic.

(viii.) CFRP possess high ultimate strain; therefore, they offer ductility to the structure and are suitable for earthquake resistant applications.

(ix.) Due to the lightweight of prefabricated components in CFRP, they can easily be transported. This encourages prefabricated construction, reduce site erection, labor cost and capital investment requirements.

(x.) Application of CFRP does not require bulky and dusty materials in a large quantity; therefore, the site remains tidier.

2.3.6 Failure Modes

Structural failure can occur in FRP materials when (Erdhard et al.,2006):

(i.) Tensile forces stretch the matrix more than the fibres, causing the material to shear at the interface between matrix and fibres.

(ii.) Tensile forces near the end of the fibres exceed the tolerances of the matrix, separating the fibres from the matrix.

(iii.) Tensile forces can also exceed the tolerances of the fibres causing the fibres themselves to fracture leading to material failure.

2.4 Composite Timber Structures

Composite or hybrid timber structures are those timber structures which incorporate reinforcing elements within them. Timber structures are now combined with different reinforcing materials to enhance their strength and stiffness properties, which allow them to be used as structural members in more massive constructions. The composite action allows better utilization of the cross section, as the reinforcement prevents premature failure of weak zones. The high strain capacity of the reinforcement allows the fibres in compression to fail plastically upon their ultimate compressive strains and the fibres in tension to reach their ultimate tensile capacity.

More over the variability in strength properties of reinforced timber seems to be very less compared to un-reinforced ones (Steiger, 2004).

Earlier investigations considered the use of metallic reinforcements in timber structures.

Steel and aluminum were the most important metallic reinforcements. The results were quite satisfactory, with an average increase of 40-50% in stiffness as well as strength values for reinforcement ratios as low as 1% (Sliker, 1962). Various adhesive systems and mechanical fasteners were also investigated for connecting the metal reinforcements to timber elements.

Recent studies about reinforcing timber bring out the possibility of using polymer-based fibre composites as reinforcement in timber structures. Different types of fibre-matrix combinations were used with different material as well as different loading configurations. Different fibres such as carbon fibres, glass fibres, aramid fibres and even different natural fibres such as jute, cotton and others, were used as primary reinforcing elements in combination with appropriate polymer matrices (Sliker, 1962).

2.5 Reinforcing With Fibre Reinforced Plastics (FRP)

The principal disadvantage of using metallic reinforcements in timber structures was the incompatibility between the timbers and the reinforcing materials (Jobin and Olga, 2007).

A possible method for avoiding such problems is to use high strength fibre reinforced plastic (FRP) to reinforce glue laminated timber (Glulam) members. A new process of reinforcing Glulam with plastic fibres is rapidly altering the structural timber market by creating stronger, stiffer, lighter and smaller structural members using lower grade lumber at significant cost savings over conventional Glulam (Jobin and Olga 2007).

2.6 CFRP Strengthening of Channel Columns

Silvestre (2007) investigated non-linear behavior and load-carrying capacity of CFRP-strengthened lipped channel steel columns, with the aim of assessing how the CFRP strengthening influences (enhances) the nonlinear behaviour and load-carrying capacity of cold formed steel lipped channel columns. He also proposed design methods to calculate the ultimate load capacity. However, the results might vary it surface preparation and adhesive material change. Thus, unique design procedure should be developed in order to standardize the design equations.

2.7 Design of Circular Hollow Steel Sections with CFRP

Haedir et al. (2011) investigated the design and experimental evaluation of externally bonded CFRP sheets for strengthening circular steel tubular short columns and proposed design methods for cylindrical CFRP-reinforced steel tubular columns under compression.

The concept of modular ratio was applied for calculating the axial section capacities of CFRP reinforced steel tubular short columns. The thickness of each carbon fiber layer was assumed to be uniform and the bond between the CFRP and steel to be adequate. The strengthening influence of the CFRP can be gauged by using the effective area of the supplanted and steel cross-sections given in the existing local buckling design procedure. The test results suggest that the use of a combination of hoop and longitudinal CFRP in a slender tube can promote the attainment of the yield capacity of the bare tube.

2.8 Design of Square Hollow Sections with CFRP

Bambach et al. (2009) investigated axial capacityand design of thin-walled steel Square Hollow Section[SHS] strengthened with externally bonded CFRP. TheSHS were fabricated by spot-welding and had platewidth-to-thickness ratios between 42 and 120, resultingin plate slenderness ratios between 1.1 and 3.2. Twodifferent matrix layouts of the CFRP were investigated.It is shown that the application of CFRP to slender sections delays local buckling and subsequently resultsin a significant increase in elastic buckling stress, axialcapacity and strength-to-weight ratio of thecompression members.A design method was developed whereby thetheoretical elastic buckling stress of the composite steel–CFRP sections was used to determine the axial capacity.

2.9 Increases in Compressive Strength of Timber Column

One of the benefits observed from confinement of timber with FRP is an increase in compressive strength. This has been demonstrated by Najm et al, (2007), who found that confinement increased strength by up to 35%. Stiffness and ductility were also found to increase, whereas the variability of the behavior of the columns under axial load reduced. The toughness of the specimens was increased, several of the samples being able to sustain higher loads at larger deflections compared to the controls. Heiduschke and Haller (2012) conducted two similar experiments, using hollow timber tubes. They made similar conclusion to Najm, et al (2007), finding that the FRP-confined samples had significantly higher loads and ductility to the reference samples. On longitudinally cracked timber columns, up to 20% of the maximum load wasable to be recovered by wrapping columns with FRP, as concluded by Zhang et al, (2012). Due to the ease and efficiency of utilizing this type of strengthening system, it is able to be used in-situ. Emerson (2004) repaired heavily degraded timber columns using glass fibre finding that the strength of a repaired column could exceed that of the design value for strength. Another case in Canada found out that for CFRP-confinement of the piers (Christopher, 2000)

2.10 Wrap Orientation

Haedir and Zhao (2006) found that longitudinal layers increased moment capacity contribution by the CFRP in the tension zones, corresponding with examples from timber and concrete, and that transverse layers restrained or delayed buckling. It was concluded that for best results, the layers should be applied together, with the transverse layer applied first.

2.11 Fiber Reinforced Polymer Spray for Structural Strengthening

The majority of projects using FRP strengthening have involved the use of laminated plates or wraps bonded to the concrete surface and timber surface. An entirely new method of repair using Sprayed Fiber Reinforced Polymer coating has been developed at the University of British Columbia.

The technique consists of spraying polymer and short, randomly distributed fibers concurrently on the surface of concrete to be repaired such that a 2-dimensional random distribution of fibers is obtained on the application of the surface. There are three basic ingredients of the final composite which are handled simultaneously by the pumping and spraying equipment. The resin and catalyst are fed separately into a spray gun, where they are mixed and then sprayed as a single compound.

The glass fiber is implemented in a roving format. Two strands of roving are fed into a chopper units mounted on top of the spray gun, wherein they pass between a pair of roller. One of these rollers has a series of blades mounted at equal intervals around its circumference. The blades break the fiber into short lengths with that length dependent entirely upon the spacing of blades on the wheel. This allows production of fibers of consistent length, adjustable from 8 to 48 mm. as shown in Plate 1, the gun sprays the resin/catalyst mixture from the lower spray nozzle while, at the same time, spraying the chopped fibers from the top-mounted chopper units. These two streams combine and continue on to the spraying surface together. This approach allows the operator to build up the FRP material to whatever thickness is required. After spraying, a ribbed aluminium roller is used to force out any entrapped air voids and to work the material into a consistent thickness. In both the laboratory trials and field application so far, the spray consisted of a polyester resin and chopped, randomly distributed fibers. The polyester resin developed an approximate tensile strength of 75 MPa and a modulus of 6 GPa when fully hardened.

Before applying the spray, the outer surface of the structure is coated with a layer of a suitable secondary bonding agent (for example, a two-component elastomer modified vinyl-ester primer system with a methyl ethyl keytone peroxide catalyst). After this bonding agent had dried sufficiently, the spray is applied. With a 20% by volume of 48mm long fiber, a composite with a density of 1400 kg/m^3, a secant tensile modulus of 12 GPa, a tensile strength of 110 MPa and a tensile elongation capacity of 1.2% can be obtained.

Plate 1: The Spray Operation

Table 2.1: Results of Load Test on Safe Bridge

Test		Before Application of Sprays	After Application of Spray	Percentage Reduction
Static Tests	Max. Rebar (micro strain)	101.76	65.55	36%
	Max. Deflection (mm)	1.55	1.08	30%
	Max. % of Yield Capacity Reached	8.0	5.2	36%

2.12 Protection from Corrosion Damage

Mullins and Sen (2007), conducted research on repairing corrosion damage on timber laminate using FRP in marine environments. It was found that CFRP is effective in limiting corrosion, comparatively to the controls, as the overall metal loss was close to a 14% reduction comparatively.

2.13 Deterministic and Probabilistic Design Approaches

Deterministic design approach includes the factor of safety method, the load factor method, and the partial factor method (limit state method). The Eurocode 5(2004) employs the limit state approach, in which the material strength are divided by factors of safety and the load action is multiplied by further factor of safety. This is with the aim of achieving low probability of failure (Melchers, 1999).

Design of engineering involves the consideration of many dependent and independent variables, of which values are associated with varying degrees of uncertainties; for example, both the dead and the live loads on structures cannot be estimated with certainty. However, the uncertainties associated with the live load are higher than that of dead load. This is because of high variability associated with the former, even though, this can change. For timber structures owing to the fact that timber is a live material, it is an isotropic material. Its properties therefore change with change in environmental conditions and load duration. The properties not only vary from specie to specie, but even with a particular specie (Blass et al,1995).Generally, the uncertainties associated with the variable that defines resistance and loading in structures invalidates the current deterministic approach.

The Eurocode 5 (2004) uses partial probabilistic approach, being a limit state design code(Stehn and Johanssen 2002). It recognized that there is likelihood; in other words, finite probability that a particular limit state may be exceeded. However, there was no attempt to compute such probability (Afolayan, 1992). This therefore signifies the need for the computation of the probability of failure of a structure designed based on Eurocode 5 (2004) requirements. This can be achieved through reliability-based analysis (that is, fully probabilistic design approach). In probabilistic assessment, any uncertainty about a variable is taken into account (Melchers, 1999).

2.14 Behaviour of Columns

Under a concentric load, that is, a load that is applied exactly through the centroid of the cross-section of a column, only compressive stresses are generated. A column will experience a very small shortening in length due to this compression.

2.15 Eccentricity of loading

A perfectly formed column with a load applied concentrically through the centroid could be made very slender and infinitely long. In practice, however, loads are never perfectly concentrically applied and in addition, columns are never perfectly straight. This results in loads being applied *eccentrically* or not precisely through the centroid of the cross-section. Columns may also be designed to support loads that are intentionally eccentric.

2.16 Design Criteria of Timber Columns using Eurocode 5 (2004)

Compression failure

The design compression resistance, to resist the compression failure is given by the EC5 (2004) as:

$$F_{c,o,d} = \frac{F_{c,o,k}\, K_{mod}}{\gamma_m} \qquad (2.1)$$

Where K_{mod} is a modification factor to take care of variation in density and moisturecontent$F_{c,o,k}$is the characteristic compressive strength parallel to grain, γ_m is the timbermaterial partial safety factor for strength.

Bending Failure

Bending strength is given by EC5 (2004):

$$f_{m,d} = \left(\frac{k_{mod}\, f_{m,k}}{\gamma_m}\right) \quad (2.2)$$

Applied Bending Stress = $\frac{M}{Z}$ (2.3)

Therefore

G = R – S (2.4)

The limit state function is given by Equation (3.44)

$$G = \frac{k_{mod}\, f_{m,k}}{\gamma_m} - \frac{M}{Z} \quad (2.5)$$

Where:

M = $M_B - M_D$ moment at B which is maximum and Z = $\frac{I}{y} = \frac{BH^2}{6}$, B = Breadth of section,

D = Depth of section

From Equation (2.5) the limit state equation is given by:

$$G = \frac{K_{mod}\, f_{m,k}}{\gamma_m} - \frac{6M}{BH} \quad (2.6)$$

Flexural Buckling failure

The design buckling resistance of a timber member according to EC 5 (2004) is given by:

$$\frac{\sigma_{c,o,d}}{k_c f_{c,o,d}} + \frac{\sigma_{m,y,d}}{k_{m,y,d}} + \frac{\sigma_{m,z,d}}{k_{m,z,d}} \leq 1 \quad (2.7)$$

$$\frac{\sigma_{c,o,d}}{k_{c,z} f_{c,o,d}} + \frac{\sigma_{m,y,d}}{f_{m,y,d}} + \frac{\sigma_{m,z,d}}{f_{m,z,d}} \leq 1 \quad (2.8)$$

Where

$$k_{c,y} = \frac{1}{k_y + \sqrt{k_y^2 - \lambda_{rely}{}^2}} \; and \quad k_{c,z} = \frac{1}{k_z + \sqrt{k_y^2 - \lambda_{rely}{}^2}} \tag{2.9}$$

$$k_y = 0.5\,(1 + \beta_c(\lambda_{rel,y} - 0.3) + \lambda_{rely}{}^2) \tag{2.10}$$

$$k_z = 0.5\,(1 + \beta_c(\lambda_{rel,z} - 0.3) + \lambda_{relz}{}^2) \tag{2.11}$$

$$\boldsymbol{\lambda_{rel}} = \sqrt{\frac{(f_{c,o,k})}{(\sigma_{c,crit})}} \tag{2.12}$$

$$\boldsymbol{\sigma_{c,crit}} = \frac{\boldsymbol{\pi^2 E_{0.05}}}{\boldsymbol{\lambda^2}} \tag{2.13}$$

Where $\beta_c$is 0.2 for solid timber, β_c is 0.1 for glue-laminated timber,$\sigma_{c,o,d}$ is design compressive stress,$f_{c,o,d}$ is design compressive strength,$\sigma_{m,d}$ is design bending stress,$f_{m,d}$ is design bending strength,$\boldsymbol{\lambda_{rel}}$is slenderness ratio,$\boldsymbol{E_{0.05}}$ is the fifth percentile of the modulus of elasticity parallel to grain, $\beta_c$is the factor for members within the straightness limits and $k_{c,}$ is a factor which take into account lateral instability, $\boldsymbol{\lambda}$= bending about either y, x and z axis.

Lateral Torsional Buckling

The lateral torsional buckling is expressed in EC5 (2004) as:

$$\left(\frac{\boldsymbol{\sigma_{m,y}}}{\boldsymbol{k_{crit} f_{m,d}}}\right)^{\mathbf{2}} + \frac{\boldsymbol{\sigma_c}}{\boldsymbol{k_c f_{c,o,d}}} \leq \mathbf{1} \tag{2.14a}$$

Where

$$k_c = \frac{1}{k_y + \sqrt{k_y^2 - \lambda_{rely}{}^2}} \tag{2.14b}$$

Where k_{crit} is a factor which takes into account the reduced bending strength due to lateral buckling.

All these design criteria for structural safety of the columns will be evaluated in conjunction with the properties of CFRP in laminates and sprays using the Finite Element Method.

The procedure employed is clearly explained in Chapter three.

CHAPTER THREE

MATERIALS AND METHODS

3.1 General

The methodology adopted is in conformity with the safety of the structural members in construction and in service to sustain the expected design loads needed to be resisted by the resisting moment of a solid timber column. The risk of failure of any structure is often measured in terms of the probability of failure. The failure mode considered is that due to buckling, that is, deformation caused by buckling due to axial load. The safety margin is obtained by equating the resistance of the column and the load effects computed by simple structural analysis. The basic variables are analyzed, while their statistical behavior is obtained from literature. The safety of the member may however change during the lifetime as degradation might occur. For example, buckling of the timber columns will considerably reduce the axial load carrying capacity of the timber columns and consequently deter the safety of the timber columns while in service, and will affect the safety and behavior of the structure negatively, as well as a decrease in effective height caused by compression due to axial loads on the solid timber columns.

3.2 Timber

Three timber species will be used in this work, namely, *Mitragynaciliata* (Abura), *Afromosia elata* (Afromosia) and *Berlinia/confusa grandiflora* (Berlinia).The demand for alternative and cheaper source of timber had increased, due to the increase in the demand for timber, especially in construction of timber roof trusses, brought about by the increase in construction activities and available knowledge. As the world engineering community is shifting from the outdated permissible stress design criteria in the design of timber structures to the limit state design method, there is need to provide adequate properties of local timber species to the construction industry.

The basic reliability problem is to consider only one load effect which is resisted by one resistance. R and S are described by a known probability density function, F_S (S) and F_R (R) respectively.

A structure is considered to have failed if its resistance is less than the load S (Melchers; 1987; 1999). The probability of failure P_f can be stated in any of the following ways (Melchers; 1987; 1999).

$$P_f = P(R \leq S) \qquad (3.1)$$

$$P_f = (R - S \leq 0) \qquad (3.2)$$

$$= \left(\frac{R}{S} \leq 0\right) \qquad (3.3)$$

$$P_f = P(\ln R - \ln S \leq 0) \qquad (3.4)$$

$$P_f = [G(R;S) < 0] \qquad (3.5)$$

Where G is the “limit state function” and the probability of failure is identical with the limit state violation for any random variable x, the cumulative distribution function$F_R(x)$, the ultimate strength, R and $F_S(x)$ the load effect.

Equation (3.2) can be replaced by:

$$P_f = P(R - S) < 0 = \int_{-\infty}^{+\infty} F_R . F_S \, dx \qquad (3.6)$$

Under the condition that R and S are statically independent. The integral is known as a “convolution integral” F_R (X) is the probability that R < X or the probability that the actual resistance R of the member is less than some value X. Assuming that this represents failure, F_S is the probability that the load effect S, acting on the member has some value between X and X + dx in the limit as dx→0. The total failure probability is obtained by taking the overall integral x.

Equation (3.6) could be written as:

$$\mathrm{P_f} = \int_{-\infty}^{+\infty} 1 - \mathrm{F_S F_R dx} \qquad (3.7)$$

Which is the sum overall in the cases of resistance for which the load exceed the resistance. The convolution integral may be integrated for a few distributions of R and S. This is very easy when R and S are normal random variables with mean and variance respectively. The safety margin will be;

$$\mathrm{M = R - S} \qquad (3.8)$$

Therefore, the mean and variance will be given by:

$$\mu_M = E(M) = \mu_R - \mu_S \qquad (3.9)$$

$$\sigma_M^2 + Var(\mu) = d_R^2 + \sigma_S^2 \qquad (3.10)$$

$$= \sqrt{\sigma_\mathrm{R}^2 + \sigma_\mathrm{S}^2} \qquad (3.11)$$

M is a linear function of R and S and is normally distributed, since R and S are normal.

Therefore,

$$P_f = \emptyset\left[0 - \frac{\mu_\mathrm{M}}{\sigma_\mathrm{M}}\right] = \emptyset\left[\mu_\mathrm{S} - \mu_\mathrm{R}/\sigma_\mathrm{R}^2 + \sigma_\mathrm{S}^2)^{\frac{1}{2}}\right] \qquad (3.12)$$

Where $\emptyset$ is the standard normal distribution function. The reliability index,β, may now be defined as the ratio $\frac{\mu_M}{\sigma_M}$ or the number of standard deviation by which it exceeds zero. Then,

$$P_f = \emptyset\left[\frac{\mu_M}{\sigma_M}\right] \tag{3.13}$$

$P_f$increases if resistance is reduced or if standard deviation is increased.

3.3 First Order Reliability Method (FORM)

The First-Order Reliability Method (FORM) is designed to provide approximations of probability integrals occurring in structural reliability. Gollwitzer et al.,(1988) referred the FORM to be designed for the approximate computations of general probability functions over given domains with locally smooth boundaries, especially for probability integrals occurring in structural reliability. Reliability estimations are carried out by representing each variable by only its first two moments (mean and standard deviation), while ignoring higher moments. FORM is essentially a level II method of reliability analysis and it would be used for reliability analysis throughout this study.

To be able to evaluate the reliability of a structure, it is necessary to select the variables of relevance: these variables are selected such that the performance of the structure or system depends upon them, that is, it is a function of the design variables (Atim, 2006).

FORM can be easily extended to non-linear limit states and has a reasonable balanced ease of use and accuracy. First Order Reliability Method (FORM) is one of most common basic techniques and it is applicable to all probabilistic problems. It is usually a preferred method, because it does not depend on the number of simulations to be carried out (Webster and Banmister, 2001).

If X is a vector of all relevant basic variables, for a given structure, each variable X_i, (i = 1,….,n) is considered a realization of a random variable X_i, such that the set of variables X = $(X_{1,} \dots \dots , X_n)$ is a realization of the random vector.

X = $(X_1 , X_2, \dots . , X_n)$ (3.14)

Alternatively, the variable x, is a point in an n-dimensional basic variable space, that is, X = x, defines a particular "point" x. X = $(X_1 ,\ X_2, \ldots ., X_n)$ is chosen in a manner that a limit state surface (or failure surface) can be defined in the "n" dimensional variable space. This failure can be represented as:

G = F $(X_1 ,\ X_2, \ldots ., X_n)$ = 0 (3.15)

Where G is the function expressing the relationship between the limit state and basic variables: then G(X) is the locally sufficiently smooth limit state function which must be at least once differentiable. FORM assumes a set of basic random variables in such a way that G(X) $>$ 0 indicates the safe or acceptable region, G(X) $<$ 0 indicates an unsafe set of variables or the failure region and G(X) = 0 is the limit state itself or failure boundary.

Structural reliability is usually measured by the reliability index, β. FORMassumes that if the probability density functions of the resistance and load variables are normal and independent, they may be combined to define β as:

$$\beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \qquad (3.16)$$

Let G be the safety margin. Thus$G = R - S$, such that

$$\beta = \frac{\mu_G}{\sigma_G} \qquad (3.17)$$

Where μ_R and μ_S are the mean values of the resistance and load variables and σ_R and σ_S are the corresponding standard deviations. $\beta,$ which is referred to in other literature as the safety index, is related to the probability of failure (P_f) using: (Gollwitzeret al..., 1988)

$$P_f = \emptyset(-\beta) \tag{3.18}$$

Where $\emptyset$ is the standard normal distribution function. β is a measure in standard deviation unit by which the mean μ_G exceeds zero. Recall that the point "R – S = 0" represents the origin. The higher the value of β, the safer the structure and the lower the nominal probability of failure will be.

3.4 Computation of Reliability Index

In FORM, the performance function is linearized at some point of the failure surface using the Taylor's Series expansion rather than at the mean. The linearization point is known as checking or design point (Afolayan; 1994). For non-linear functions determining the reliability index β, on the basis of linearization will depend on the choice of linearization points.

When G(X) is linear and uncorrelated, the reliability index, β, can be calculated by the following (Gollwitzer et al.,(1988)):

(1). Generating a reduced, normally distributed variable expression for each variable.

Let the variance be:

$$X_i = X_1, X_2, X_3 \ldots X_n \tag{3.18}$$

Where $X_i = X_1 - \frac{\mu X_i}{\delta X_i}$, i = 1,2 … n. (3.19)

Therefore;

$$X_i = \partial X_i + \mu X_i \tag{3.20}$$

$$G(X) = G(\partial X_1 + \mu X_1 + \partial X_2 + \mu X_2 + \ldots\ldots.. + \partial X_n + \mu X_n) = 0 \qquad (3.21)$$

(3). Determine an expression to define the distance from the failure surface to the origin of the reduced variable space. The distance will be denoted as D:

$$D = \sqrt{(x_1^1)^2 + (x_2^1)^2 + \ldots\ldots.. + (x_n^1)^2} \qquad (3.22)$$

In matrix form:

$$D = [X_1^t, X_i^1]^{1/2} \qquad (3.23)$$

(4). Reducing the function, D, with constraint G to determine the minimum distance from the failure surface to the origin. The Lagrange method is used for this purpose.

$$L^1 = D + LG(X_i) \qquad (3.24)$$

Where L = Lagrange multiplier

$$\text{Then,} \frac{\partial L^1}{\partial X^1} = \frac{X_1}{D} + L\emptyset \frac{G(X_1)}{\partial X_1^1} = 0 \qquad (3.25)$$

$$\frac{\partial L^1}{\partial X^1} = G = 0 \qquad (3.26)$$

The solution to the Equations (3.22) and (3.23) gives the minimum distance X^1 from the origin.

(5)The minimum distance is obtained by introducing the gradient vector

$$G^1 = \frac{\partial G}{\partial X_i} \qquad (3.27)$$

and together with matrix representation of D in Equations (3.20) and (3.21) can be expressed for D as:

$$D = \frac{G^{lt} X^{l}}{\left(G^{lt} G^{l}\right)^{\frac{1}{2}}} \tag{3.28}$$

Where G^{lt} is the transpose of the gradient reception of G^{l} which represents the reliability index. Hence,

$$\beta = \frac{-G^{lt*} G^{l*}}{\left(G^{lt*} G^{l*}\right)^{\frac{1}{2}}} \tag{3.29}$$

The reliability index can also be defined in terms of the multivariate performance function G, that is:

$$\beta = \frac{\mu_g}{\partial_g} \tag{3.30}$$

Where μ_g and ∂_g are the mean and standard deviation of the joint probability distribution of the variable of the performance function G.

Hence, the probability of failure P_f is given as:

$$P_f = \emptyset(-\beta) \tag{3.31}$$

The performance function, G(X), is non-linear in the basic variable, X; hence FORM involves the linearization of G(X) to aim at the reliability index, β. This is obtained by expanding G(X) as a first order Taylor Series about some point, X*, on the failure surface and retaining the linear terms, Baker, (1982).

$$G(X_1, X_2, \dots, X_n) = G(X_1^*, X_2^*, \dots, X_n^*) + \sum (X_i - X_1^*)\, \partial G(X_i)/\partial X_i \tag{3.32}$$

But $G(X_1^*) = 0$ at the failure surface, hence;

$$G(X_1, X_2, \dots, X_n) = \sum (X_i - X_1^*)\, \partial G(X_i)/\partial X_i \tag{3.33}$$

Introducing the reduced variable;

$$X_i - X_1^* = (\partial X_i - \mu X_i) - (\partial X_1^* - \mu X_i) = \partial X_i (X_i^1 - X_i^1) \quad (3.34)$$

Also,

$$\frac{\partial \mathrm{G}}{\partial \mathrm{X_i}} = \frac{\partial \mathrm{G}}{\partial \mathrm{X_i}\left[\frac{\partial \mathrm{X_i^1}}{\partial \mathrm{X_i}}\right]} = \frac{1}{\partial \mathrm{X_i}\left[\frac{\partial \mathrm{G}}{\partial \mathrm{X_i}}\right]} \qquad (3.35)$$

$$\mathrm{G(X_i)} = \frac{\sum(\mathrm{X_i} - \mathrm{X_1^*})\,\partial \mathrm{G}}{\partial \mathrm{X_i^1}} \qquad (3.36)$$

Where parameter of X_i are N(0,1).

$$\mu_{\mathrm{G}} = \frac{\sum \mathrm{X_i^1}\,\partial \mathrm{G}}{\partial \mathrm{X_i}} \qquad (3.37)$$

$$\partial \mathrm{G} = \sum \partial^2 \mathrm{X_i} (\partial \mathrm{G}/\partial \mathrm{X_i})^2 = \sum(\partial \mathrm{G}/\partial \mathrm{X_i})^2 \qquad (3.38)$$

The basic variable in Equation (3.34) and (3.35) are assumed to be normally distributed and uncorrelated, Taylor's expression of G(X) about the mean yield accurate estimation of safety index, if $G(X)_R$ contain non-normally distributed and correlated variables, and accuracy of the safety index will depend on the choice of point of linearization of FORM5 (Gollwitzer et al, 1988). Gollwitzer et al, (1988) solves this problem by introducing appropriate transformation, converting all variables into uncorrelated normal variables, subsequently linearizing the non-linear function G(X) about a suitable point selected through an optimization procedure and hence is obtained using the standard normal integral.

3.5 Derivation of Limit State Equations

3.5.1 Compressive Resistance of Solid Timber Columns

The compressive resistance of solid timber columns is given by EC5 (2004) while the limit state function is given as:

$$G = R - S \tag{3.39}$$

$$G = \frac{K_{mod}\ F_{c,o,d}}{\gamma_m} - \sigma_{c,o,d} \tag{3.40}$$

$$G = \frac{K_{mod}\ F_{c,o,d}}{\gamma_m} - \frac{R_{AV}}{BH} \tag{3.41}$$

But $R_{AV} = V_L - R_{EV}$ (3.42)

Where:

$$V_L = (1.35g_k + 1.5q_k)L \tag{3.42a}$$

and

$$R_{EV} = \frac{VL}{2} = \frac{L}{2}(1.35g_k + 1.5q_k) \tag{3.42b}$$

Multiply Equation (3.42b) through by $\frac{q_k}{q_k} = 1$; to establish the load ratio $\alpha = \frac{g_k}{q_k}$ thereby simplifying the equations and we obtained:

$$R_{AV} = \left[(1.35g_k + 1.5q_k)L - \frac{L}{2}(1.35g_k + 1.5q_k)\right]\frac{q_k}{q_k} \tag{3.43}$$

$$R_{AV} = g_k\frac{L}{2}[2(1.35\alpha + 1.5) - (1.35\alpha + 1.5)] \tag{3.44}$$

$$R_{AV} = q_k\frac{L}{2}(1.35\alpha + 1.5) \tag{3.45}$$

Therefore;

$$G = \frac{K_{mod}\ F_{c,o,k}}{\gamma_m} - \frac{q_k L}{2BH}[(1.35\alpha + 1.5)] \qquad (3.46)$$

3.6Compressive Resistance of Composite Solid Timber Columns Laminated in CFRP.

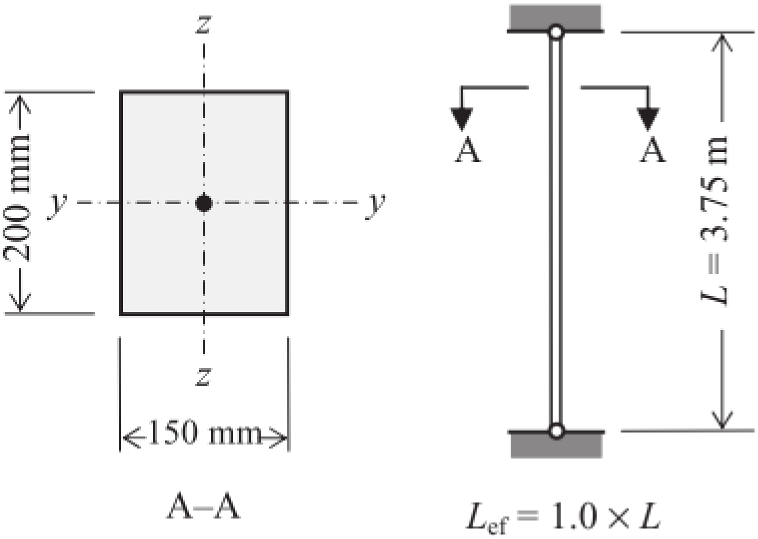


Figure 3.1: Solid timber column section laminated with CFRP(Sources; Structural Timber Design to Eurocode 5, Jack Porteous and Abdy Kermani).

3.6.1Column Geometric Properties

Column length, L $L = 3.7\ m$

Effective length about the y-y axis, $L_{e,y} = 1.0L$ that is, $L_{e,y} = 3.75\ m$

Effective length about the z-z axis, $L_{e,z} = 1.0L$ that is $L_{e,z} = 3.75\ m$

Width of the member, b $b = 150\ mm$

Depth of the member, h $b = 200\ mm$

Cross-sectional area of the column, A $A = b.hA = 3 \times 10^4 mm^2$

Second moment of area about the y-y $I_y = \frac{b.h^3}{12}$ $I_y = 1 \times 10^8 mm^4$

Radius of gyration about the y-y axis, i_y $i_y = \sqrt{\frac{I_y}{A}}$ $i_y = 57.74\ mm$Slenderness ratio about the y-y axis, λ_y $\lambda_y = \frac{L_{e,y}}{i_y}$ $\lambda_y = 64.95$

Second moment of area about the z-z axis, I_z $I_z = \frac{h.b^3}{12}$ $I_z = 5.63 \times 10^7 mm^4$

Radius of gyration about the z-z axis, i_z $i_z = \sqrt{\frac{i_z}{A}}$ $i_z = 43.3\ mm$

Slenderness ratio about the z-z axis, λ_z $\lambda_z = \frac{L_{e,z}}{i_z}$ $\lambda_z = 86.6$

3.6.2 Timber Properties

The strength class of C18 and D30 (BS EN 338:2003(E), Table 1)

Characteristic compression strength parallel to the grain,$f_{c,0,k}$ $f_{c,0,k} = 18\ N/mm^2$

Fifth-percentile modulus of elasticity parallel to the grain $E_{0,05}$ $E_{0,05} = 6.0\ KN/mm^2$

3.6.3 Partial Safety Factors

The UKNA to BS EN 1990:2002, Table NA.A1.2 (B) for the ULS

Permanent actions, γ_G $\gamma_G = 1.35$

Variable action, γ_Q $\gamma_Q = 1.5$

Table 2.6 (UKNA to EC5, Table NA.3)

Material factor for solid timber, γ_m $\gamma_m = 1.3$

3.6.4 Actions

The actions of load on the column can be represented as:

Characteristic permanent compressive action actions, G_K $G_k = 30KN$

Characteristic medium-term compressive variable action, Q_k $Q_k = 50KN$

Design compressive action for the critical load combination, $N_d N_d = \gamma_G . G_k + \gamma_Q . Q_k$

$$N_d = 1.16 \times 10^5 N$$

3.6.5 Modification Factors

Factor for medium duration loading and service class 2, $k_{mod\,.med}$ $k_{mod\,.med} = 0.8$

(EC5, Table 3.1)

System strength factor, k_{sys} not relevant $k_{sys} = 1.0$

3.6.6 Compression Strength of Column

The critical design load case at the ULS will be due to the combination of permanent and unfavorable medium-duration variable action:

Design compression stress, $\sigma_{c,0,d}$ $\sigma_{c,0,d} = \frac{N_d}{A}$ $\sigma_{c,0,d} = 3.85\ N/mm^2$

Design compression strength, $f_{c,0,d} f_{c,0,d} = \frac{k_{mod\ .me\ d}.k_{sys} \cdot f_{c,0,k}}{\gamma_m} f_{c,0,d} = 11.08\ N/mm^2$

Buckling resistance condition (5.3.1 (EC5, 6.3.2)):

Relative slenderness about the y-y axis, $\lambda_{rel\,.y}$ (Equation (5.3): EC5, Equation (6.21))

$$\lambda_{rel\,.y} = \frac{\lambda_y}{\pi} \sqrt{\frac{f_{c,0,k}}{E_{0.05}}} \lambda_{rel\,.y} = 1.13$$

Relative slenderness about the z-z axis, $\lambda_{rel\,.z}$ (Equation (5.3): EC5, Equation (6.22))

$$\lambda_{rel.z} = \frac{\lambda_z}{\pi}\sqrt{\frac{f_{c,0,k}}{E_{0.05}}}\lambda_{rel.z} = 1.51$$

As both relative slenderness ratios are greater than 0.3, the condition in 5.3.1(b) apply. (EC5, 6.3.2(3)):

Maximum relative slenderness ratio of the column $\lambda_{rel.z} = 1.51$

Factor β_c for solid timber (Equation (5.6); (EC5, Equation) (6.29)) $\beta_c = 0.2$

Factor k_z (Equation (5.5b); EC5, Equation (6.28)) $k_z = 0.5.[1 + \beta_c(\lambda_{relz} - 0.3) + \lambda^2_{rel.z}]$

$k_z = 1.76$

Instability factor, $k_{c,z}$ (Equation (5.4b); (EC5) Equation (6.26))

$$k_{c,z} = \frac{1}{k_z + \sqrt{k_z^2 - \lambda^2_{rel.z}}} k_{c,z} = 0.37$$

Design buckling strength, $k_{c,z}f_{c,0,d}$ $(Equation\ (5.7b))k_{c,z}.f_{c,o,d} = 4.15\ N/mm^2$

Design stress/design buckling strength ratio (Equation (5.11b)) $\frac{\sigma_{c,0,d}}{k_{c,z}f_{c,0,d}} = 0.93$

The design stress is less than the design buckling strength: therefore the 150 mm by 200 mm timber section in strength class C18 meets the ULS requirement of EC5.

3.6.7 Composite Solid Timber Columns in CFRP Laminates

The limit state equations for composite solid timber columns in CFRP laminates are as follows:

$$G = R - S \qquad 3.47$$

$$G = \left(\frac{f_{c,k}.k_{mod}}{\gamma_m}\left[1 + \frac{\gamma_m\,\alpha_{pc}.2N_b.E_{frp}.\varepsilon_{frp}.t_{frp}.(b+h)}{bh.f_{c,k}.k_{mod}}\right]\right) - \left(q_k\left[g_k\alpha_{pc} + q_k\right] \times 10^3\right) \qquad 3.48$$

$$G = \frac{f_{c,k}.k_{mod}}{\gamma_m}\left[1 + \frac{\gamma_m\,\alpha_{pc}\,2.N_b.E_{frp}.\varepsilon_{frp}.t_{frp}.(b+h)}{bh.f_{c,k}.k_{mod}}\right] bh - q_k\left[g_k\alpha_{pc} + q_k\right] \times 10^3 \qquad 3.49$$

$$= \left[\frac{f_{c,k}.k_{mod}.bh}{\gamma_m} + 2\alpha_{pc}.N_b.E_{frp}.\varepsilon_{frp}.t_{frp}.(b+h)\right] - \left[q_k\left(g_k\alpha_{pc} + q_k\right) \times 10^3\right] \qquad 3.50$$

$$\frac{\partial G(X)}{\partial f_{c,k}} = \frac{k_{mod}.bh}{\gamma_m} \qquad 3.51$$

$$\frac{\partial G(X)}{\partial k_{mod}} = \frac{f_{c,k}.bh}{\gamma_m} \qquad 3.52$$

$$\frac{\partial G(X)}{\partial b} = \frac{f_{c,k}k_{mod}.h}{\gamma_m} + 2\alpha_{pc}.N_{pc}.E_{frp}.\varepsilon_{frp}.t_{frp} \qquad 3.53$$

$$\frac{\partial G(X)}{\partial h} = \frac{f_{c,k}k_{mod}.b}{\gamma_m} + 2\alpha_{pc}.N_{pc}.E_{frp}.\varepsilon_{frp}.t_{frp} \qquad 3.54$$

$$\frac{\partial G(X)}{\partial E_{frp}} = 2\alpha_{pc}.N_b.\varepsilon_{frp}.t_{frp}(b+h) \qquad 3.55$$

$$\frac{\partial G(X)}{\partial t_{frp}} = 2\alpha_{pc}.N_b.\varepsilon_{frp}.E_{frp}(b+h) \qquad 3.56$$

$$\frac{\partial G(X)}{\partial q} = -(g_k\alpha_{pc} + q_k) \times 10^3 \qquad 3.57$$

3.6 Finite-Element Method

This approach discretizes the structure into small divisions (or elements) where each element is defined by a specified number of nodes. The behaviour of each element (ultimately the structure) is assumed to be a function of its nodal quantities (displacements and/or stresses), that serve as the primary unknowns in this formulation. This is one of the most general and accurate methods to use, because it does not put any limitations on the geometry, loads, or boundary conditions, and can be applied to open/closed girders and static/dynamic analysis. Additionally, the structure’s response can always be improved by refining the mesh and increasing the number of nodes (or degrees of freedom) for each element. However, the rather involved modeling and analysis efforts required by this method may in some cases make it impractical for preliminary analysis (Beneditti and Tralli, 1989).

In analyzing a system using finite element approach, the following key steps are to be considered:

1. Selection of appropriate probability models to describe the uncertainties in the system parameters and boundary conditions.
2. Discretization of random fields (that is, the replacement of this random field by an equivalent set of random variables.
3. Formulation of finite element equations
4. Estimation of system response statistics, which involves the inversion of random matrix operators.
5. Using these system results to make a safety assessment of the structure.

In order to apply the finite element technique, the region of interest is discretized by a finite element mesh. The basic idea of the mean-based, second- moment analysis as used in stochastic finite element analysis is to expand via Taylor series, the entire vector and the matrix stochastic field variables, to retain only the second order methods terms and to use it in the analysis, only in the first tier statistical moments. (Stefanou, 2003).

3.7 Finite Element Modeling

The finite element method originated from the need for solving complex elasticity and structural analysis problems in civil and aeronautical engineering. The Finite Element Method (FEM)(andits practical application often known as finite element analysis (FEA)) is a numerical technique for finding approximate solutions of Partial Differential Equations (PDE) as well as integral equations. In a structural simulation, FEM helps tremendously in producing stiffness and strength visualizations and also in minimizing weight, materials, and costs.

The FEM allows detailed visualization of where structures bend or twist, and indicates the distribution of stresses and displacements. FEM software provides a wide range of simulation options for controlling the complexity of both modeling and analysis of a system. Similarly, the desired level of accuracy required and associated computational time requirements can be managed simultaneously to address most engineering applications. The FEM allows entire designs to be constructed, refined, and optimized before the design is manufactured. This powerful design tool has significantly improved both the standard of engineering designs and the methodology of the design process in many industrial applications.

Tables 3.1 to 3.6; show the material properties of CFRP and stochastic parameters obtained from the literature (Abubakar Mamman Msc thesis Civil engineering Ahmadu Bello University Zaria).

Table 3.1: Material properties used for CFRP.

Material	Young Modulus	Poison Ratio	Density(kN/m^3)	Yield Stress	Strain
CFRP	310000Mpa	0.34	1400	2250Mpa	0.019

Table 3.2: Characteristic Values of Other Material Properties

Other Material Properties	Timber species		
	Mitragyna ciliata (Abura)	*Afromosia elata* (Afromosia)	*Confusa grandiflora* (Berlinia)
Tension Parallel $f_{t,k,0}$ (N/mm^2)	48.12	78.72	53.52
Tension Perpendicular $f_{t,k,90}$(N/mm^2)	0.6	0.6	0.6
Compression Parallel $f_{c,k,0}$ (N/mm^2)	35.96	57.27	47.22
Compression Perpendicular $f_{c,k,0}$ (N/mm^2)	3.30	3.36	3.2
Shear Strength $f_{v,k}$ (N/mm^2)	3.8	3.8	3.8
5% MOE Parallel $E_{0.05}$(KN/mm^2)	5.3	6.5	6.7
Mean MOE Perpendicular $E_{0.90}$(kN/mm^2)	0.27	0.32	0.33
Mean Shear Modulus G_{mean} (kN/mm^2)	0.5	0.61	0.62
Mean Density ρ_{mean} (kg /m)	545	544	526

Table 3.3: Stochastic parameters for *Mitragyna ciliata* Timber Species

S/N	Variable	Meaning	Distribution	Mean E(X)	Coefficient of Variation (COV)	Standard Deviation S(X)
1	E	Modulus of elasticity *Mitragyna ciliata* (Abura)	Log-Normal	7973kPa	0.0126	104.54
2	E_{CFRP}	Modulus of elasticity of CFRP	Log-Normal	120kPa	0.0833	10000
3	B	Breadth	Deterministic	150mm	-	-
4	H	Depth	Deterministic	200mm	-	-
5	T	Thickness of CFRP	Normal	0.2mm	0.0052	1.93
6	L	Length	Deterministic	3700mm	-	-
7	Q_k	Live Load	Deterministic	1.5	-	-
8	G_k	-	Deterministic	1.35		
9	K_{mod}	-	Log-Normal	0.8	0.16	0.105
10	ρ_{mean}	Mean Density	Deterministic	544.73	0.062	33.68

Table 3.4: Stochastic parameters for *Afromosia elata* Timber Specie

S/N	Variable	Meaning	Distribution	Mean E(X)	Coefficient of Variation (COV)	Standard Deviation S(X)
1	E	Modulus of elasticity *Afromosia elata*	Log-Normal	9695kPa	0.00672	64.06
2	E_{CFRP}	Modulus of elasticity of CFRP	Log-Normal	120kPa	0.0833	10000
3	B	Breadth	Deterministic	150mm	-	-
4	H	Depth	Deterministic	200mm	-	-
5	T	Thickness of CFRP	Normal	0.2mm	0.0052	1.93
6	L	Length	Deterministic	3700mm	-	-
7	Q_k	Live Load	Deterministic	1.5	-	-
8	G_k	-	Deterministic	1.35		
9	K_{mod}	-	Log-Normal	0.8	0.16	0.105
	ρ_{mean}	Mean Density		544.27	0.047	24.84

Table 3.5: Stochastic parameters for *Confusa grandiflora* Timber Specie

S/N	Variable	Meaning	Distribution	Mean E(X)	Coefficient of Variation (COV)	Standard Deviation S(X)
1	E	Modulus of elasticity *Confusa grandiflora*	Log-Normal	9958kPa	0.00561	54.63
2	E_{CFRP}	Modulus of elasticity of CFRP	Log-Normal	120kPa	0.0833	10000
3	B	Breadth	Deterministic	150mm	-	-
4	H	Depth	Deterministic	200mm	-	-
5	T	Thickness of CFRP	Normal	0.2mm	0.0052	1.93
6	L	Length	Deterministic	3700mm	-	-
7	Q_k	Live Load	Deterministic	1.5	-	-
8	G_k	-	Deterministic	1.35		
9	K_{mod}	-	Log-Normal	0.7	0.15	0.105
10	ρ_{mean}	Mean Density		525.71	0.029	15.28

Table 3.6: Properties of timber in strength Classes

Strength Properties (N/mm^2)	Strength classes and timber species		
	Mitragyna ciliata (Abura)	*Afromosia elata* (Afromosia)	*Confusa grandiflora* (Berlinia)
	C18	D30	D30
Bending Strength, $f_{m,k}$	18	30	30
Tension Strength Parallel to grain, $f_{c,0,k}$	11	18	18
Tension Strength Perpendicular to grain, $f_{t,90,k}$	0.5	0.6	0.6
Compression Strength Parallel to grain, $f_{c,0,k}$	18	23	25
Compression Strength Perpendicular to grain, $f_{c,90,k}$	2.2	8	8
Shear Strength. $f_{v,k}$	2.0	3	3
Stiffness Properties (kN/mm^2)			
Mean Modulus of Elasticity Parallel to grain, $E_{0,mean}$	9	10	10
5% Fractile value of Modulus of Elasticity in bending, $E_{o,05}$	6.0	8	8.0
Mean Modulus of Elasticity Perpendicular to grain, $E_{90,mean}$	0.30	0.64	0.64
Mean Shear Modulus, G_{mean}	0.56	0.6	0.6
Density (kg/m^3)			
Characteristic Density, ρ_k	320	530	530
Mean density, ρ_{mean}	380	640	640

CHAPTER FOUR

RESULTS AND DISCUSION

4.1 General

This chapter covers the discussion of analytical results obtained from structural models in finite element method using ABAQUS (a finite element software) as well as the outcome of reliability-based using a FORM for First Order Reliability Method computing the safety indices. In this design, the strength limit states functions are considered. The timber columns carry both permanent ($\boldsymbol{g_k}$) and imposed loads (q_k) which causes resistance by applied loads. The typical CFRP thicknesses by stress, displacement and reactions after loadings are discussed herein.

4.2 Analytical Results

The use of non-linear finite element analysis is a powerful tool in determining the internal stresses, displacement and reactions distribution in timber column structures. Almost all the structures exhibit a certain degree of nonlinearity at various load stages. This may be due to material nonlinearity or geometric nonlinearity. With the aid of nonlinear finite element analysis it is possible to study the behavior of structural elements. The stresses, displacements and reactions relationships can be used to realistically predict the behavior of the structures.

Tables 4.1, 4.2 and 4.3 indicate axial load acting on unstrengthen timber columns for Abura, Afromosia and Confusa timber species

Table 4.1: *Abura* Timber Species for Unstrengthen Timber Column (Fixed-Free)

Parameters	Axial load on the Timber Column(kN)		
	31kN	36kN	41kN
Stress (N/mm^2)	3.446E+01	4.002E+01	4.558E+01
Displacement (mm)	2.156E+01	2.502E+01	2.848E+01
Reactions (kN)	1.453E+05	1.688E+05	1.924E+05

Table 4.2: *Afromosia* Timber Species for Unstrengthen Timber Column (Fixed-Free)

Parameters	Axial load on the Timber Column(kN)		
	53kN	58kN	63kN
Stress (N/mm^2)	5.892E+01	6.448E+01	7.004E+01
Displacement (mm)	3.002E+01	3.284E+01	3.565E+01
Reactions (kN)	2.487E+05	2.723E+05	2.959E+05

Table 4.3: *Confusa* Timber Species for Unstrengthen Timber Column Fixed-Free

Parameters	Axial load on the Timber Column(kN)		
	44kN	48kN	54kN
Stress (N/mm^2)	4.780E+01	5.336E+01	5.892E+01
Displacement (mm)	2.365E+01	2.639E+01	2.912E+01
Reactions (kN)	2.016E+05	2.252E+05	2.487E+05

4.3 Performance Model of Bonded CFRP Timber Columns

The column prototype is modeled for performance in service loads. The column is bonded with 0.2mm, 0.4mm, 0.6mm, 0.8 and1.0mm thickness of CFRP-laminates, which improved strength capacity of the timber columns.The lengths of the columns are 3700mm; width is 150mm and the depth 200mm.

Results in Tables 4.4 to 4.6show variation in stresses, deformations, and reactions with varying thicknesses of CFRP laminates for strengthened timber columns and it has been found that there are changes in the parameters with changes in applied loads and thicknesses.

Table 4.4:*Mitragyna ciliate*(Abura) Timber Column Species Laminated with Various Thicknesses of CFRP Laminate with Axial load of 36 kN(Fixed-Free)

Parameters	Thickness of CFRP (mm)				
	0.2	0.4	0.6	0.8	1.0
Stress (N/mm^2)	1.833E+03	1.506E+03	1.278E+03	1.109E+03	9.782E+02
Displacement (mm)	1.997E+01	1.663E+01	1.429E+01	1.256E+01	1.123E+01
Reaction (kN)	1.257E+05	1.026E+05	1.033E+05	1.044E+05	1.052E+05

The test results from Table 4.4 on *Mitragyna ciliate* species of timber columns with axial loadof 36kN(fixed-free)was modelled with varying thicknesses of CFRP laminates. From the result obtained there are decreases in stress as thicknesses of CFRP laminates increase which makes it accommodate more carrying capacity compared to unstrengthened timber columns. Also from Table 4.4 the result of the displacements show little decreases as thicknesses of CFRP increase, which show that CFRP laminates are good retrofitting materials to reduce deformations on timber columns. From Table 4.4 thereactions decreaseas CFRP thicknesseincreases when compared to displacement patterns for unstrengthened timber columns. When 0.2mm CFRP thickness is used as reinforcement in timber columnswith axial load of 36kN the stress obtained is 1.833E+03 N/mm^2 as compared to 9.782E+02N/mm^2 for1mm CFRP laminates with the same axial load acting on the timber columns species. The differences in stress are 854.8 N/mm^{2}which is about 46% reduction from 0.2 mm thick laminate. The displacements obtained for 0.2mm CFRP laminates are 1.997E+01 mm while for 1mm CFRP laminates are 1.123E+1 mm. The differences in displacement are 0.251E+01 mm which is about 43% reduction from 0.2 mm thick laminate. The reaction obtained for 0.2mm CFRP laminates are 1.257E+05kN while for 1mm CFRP laminates are 1.052E+05kN. The differences in reactions are0.205E+05kN or 20500N which is about 16% decreases from the 0.2 mm thick CFRP laminate.

This shows that the strength capacity of the timber column has been increased, because the CFRP reinforcement offered some degree of resistance to the timber column, thus making it to accommodate more loadings prior to its failure. The distributions patterns of stress, displacement and reaction are showed in Figures 4.1 to 4.3.

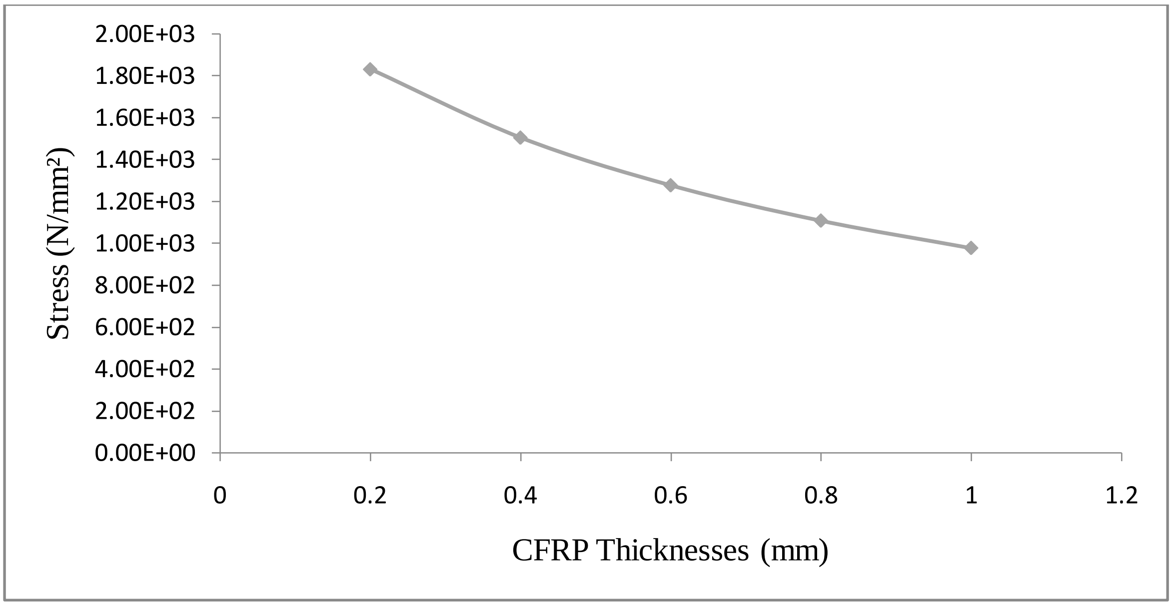


Figure 4.1: Relationship of stress and thicknesses of CFRP laminate with Abura Timber Column species

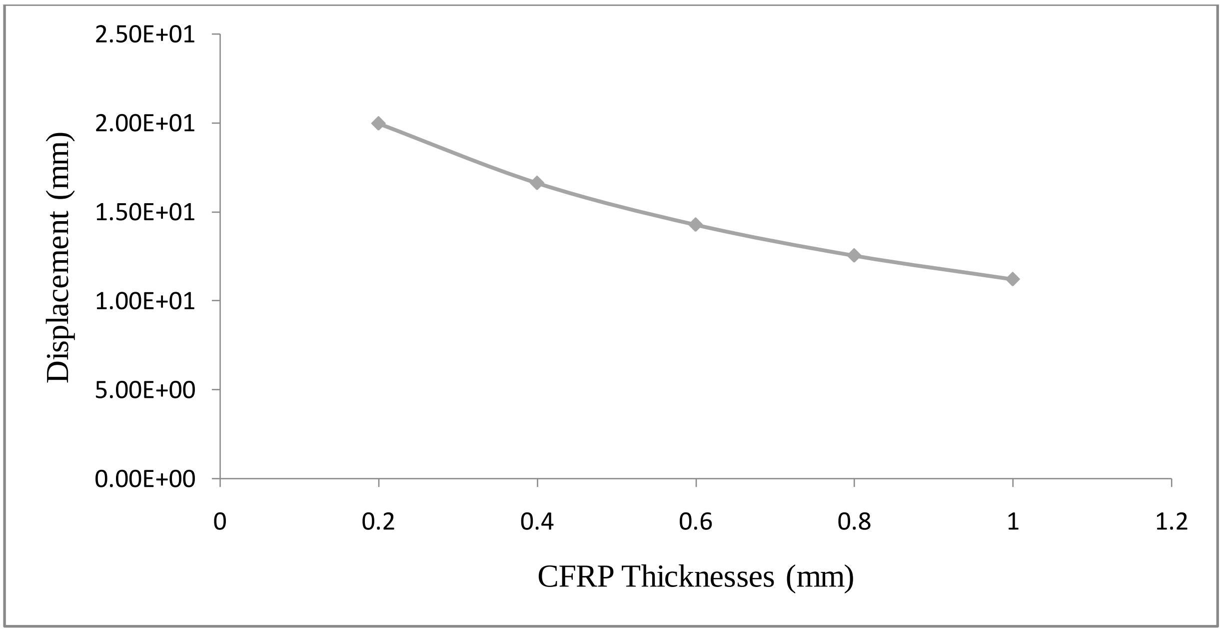


Figure 4.2: Relationship of displacement and thicknesses of CFRP laminate with Abura timbercolumn species

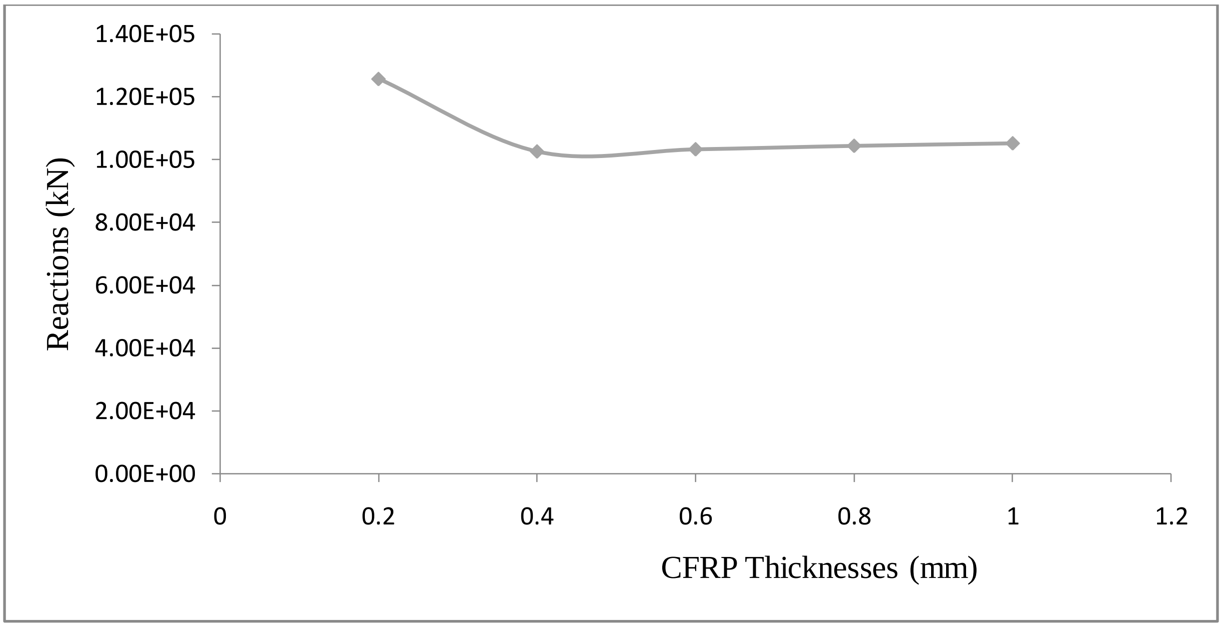


Figure 4.3: Relationship of reactions and thicknesses of CFRP laminate with Abura timber column species

Table 4.5:*Afromosia elata* (Afromosia) Timber Species for Various Thicknesses of CFRP with Axial load of 58kN(Fixed-Free)

Parameters	Thickness of CFRP (mm)				
	0.2	0.4	0.6	0.8	1.0
Stress (N/mm^2)	2.509E+03	2.114E+03	1.828E+03	1.609E+03	1.435E+03
Displacement (mm)	2.721E+01	2.323E+01	2.032E+01	1.809E+01	1.632E+01
Reactions (kN)	2.115E+05	1.771E+05	1.653E+05	1.670E+05	1.683E+05

The test results from Table 4.5 on *Afromosia elata* species of timber columns with axial load of 58kN(fixed-free) was modeled with varying thicknesses of CFRP laminates. From the result obtained there are decreases in stresses as thicknesses of CFRP increases. Also from Table 4.5 the displacement patterns of the results obtained show little decreases as thicknesses of CFRP increases which shows that, CFRP has great significant in controlling the deformation in timber column sections while reactions distribution patterns from Table 4.5 show more decreases as CFRP thicknesses increase been compared to the displacement patterns. When 0.2mm CFRP thickness is used as reinforcement in timber columns with axial load of 58kNon the timber columns, the stress obtained is 2.509E+03 N/mm^2 as compared to 1.435E+03 N/mm^2 for 1mm CFRP laminate with the same axial compression acting on the timber column species. The difference in stress is 1.071E+03 (N/mm^2) which is about 42% reduction of stress from 0.2 mm thick CFRP laminate. The displacements result obtained for 0.2mm CFRP laminates is 2.721E+01 while for 1mm CFRP laminates are 1.632E+01. The differences in displacement are 1.089E+01 which is about 40% reduction of displacement from 0.2 mm thick CFRP laminate. The reaction result obtained for 0.2mm CFRP laminates is 2.115E+05 while for 1mm CFRP laminates is 1.683E+05. The differences in reactions is 0.432E+05 which is about 20% reduction of reaction from 0.2 mm thick CFRP laminate.

This shows that the tensile capacity of the timber columns has been increased, because the CFRP reinforcement offered some degree of resistance to deformation on the timber columns, thus making it to accommodate more loadings prior to deformations at the same time.

The distributions patterns of stress, displacements and reactions are showed in Figure 4.4 to Figure 4.6 above.

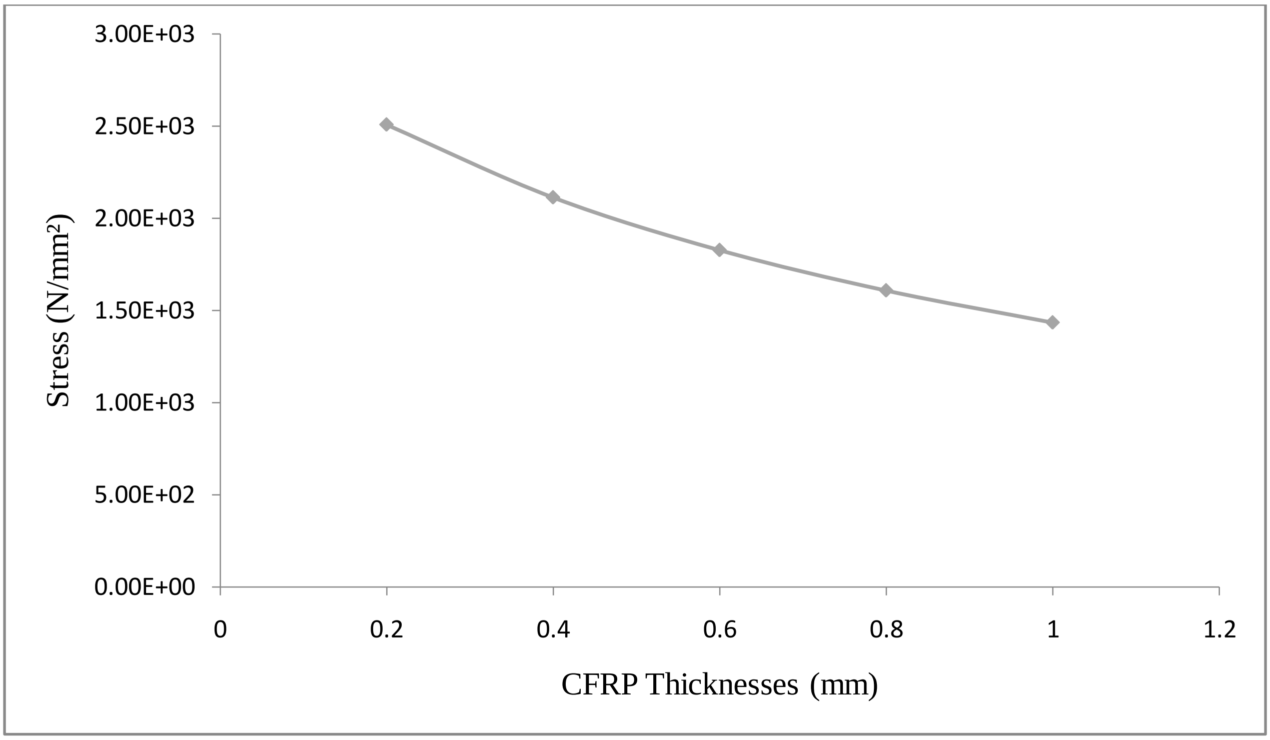


Figure 4.4: Relationship of stress and thicknesses of CFRP laminate with Afromosia timber column species.

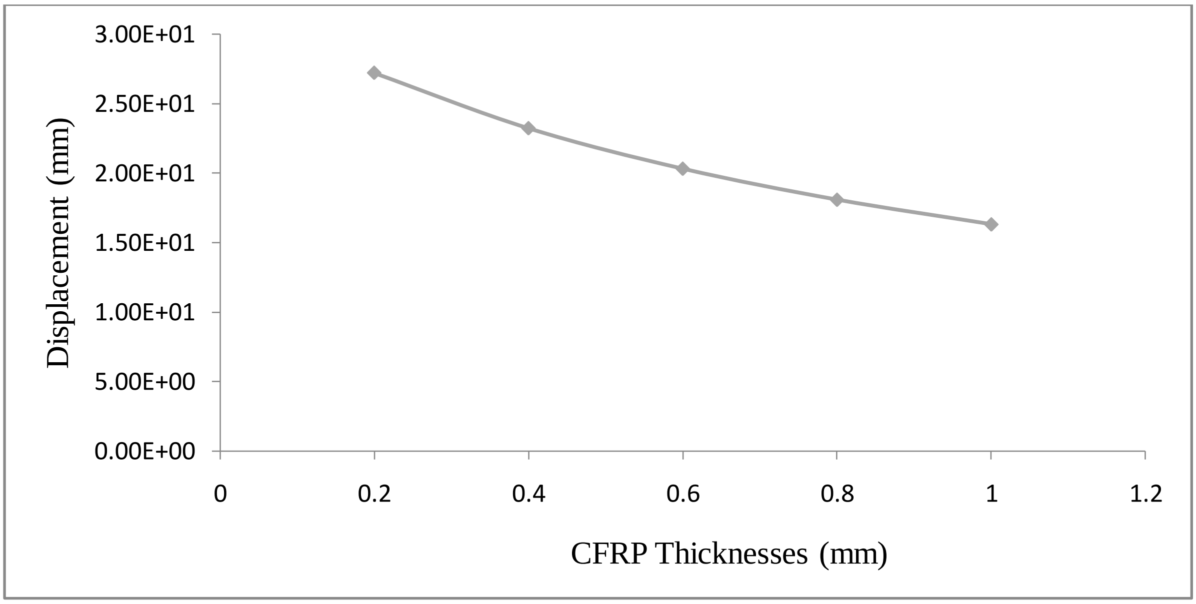


Figure 4.5: Relationship of displacements and thicknesses of CFRP laminate with Afromosia timber column species

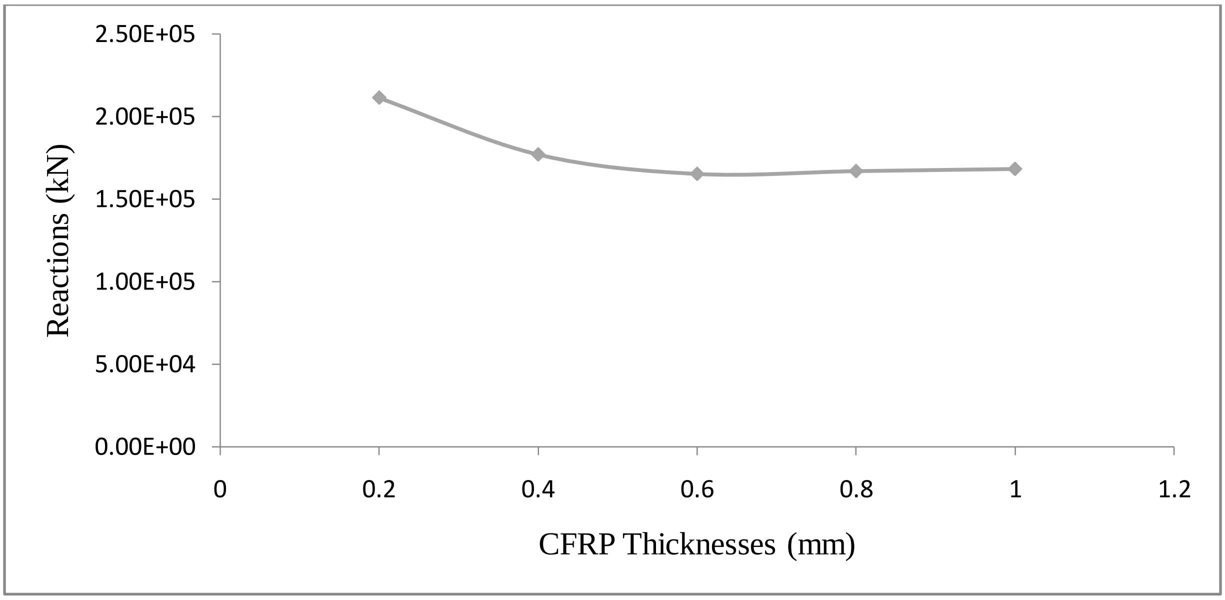


Figure 4.6: Relationship of reactions and thicknesses of CFRP laminate with Afromosia timber column species

Table 4.6: *Confusa grandiflora* (Berliania) Timber Species for Various Thicknesses of CFRP with Axial load of 49kN(Fixed-Free)

Parameters	Thickness of CFRP (mm)				
	0.2	0.4	0.6	0.8	1.0
Stress (N/mm^2)	2.027E+03	1.715E+03	1.487E+03	1.312E+03	1.173E+03
Displacement (mm)	2.199E+01	1.884E+01	1.652E+01	1.473E+01	1.332E+01
Reaction (kN)	1.761E+05	1.481E+05	1.367E+05	1.381E+05	1.392E+05

The test results from Table 4.6 on *Confusa grandiflora* (Berliania) species of timber column with axial load of 49kN(fixed free) was modeled with varying thicknesses of CFRP laminates. From the result obtained there are decreases in stresses as thicknesses of CFRP increases. The displacement results obtained from Table 4.6 shows little decreases as thicknesses of CFRP increases which show that, the effect of increasing thickness of CFRP laminates has no much significant effect when compared to stress distribution patterns while Table 4.6 shows reactions distribution patterns which indicate decreases in reactions as CFRP thicknesses increases when compared to displacement patterns. When 0.2mm CFRP thickness is used as reinforcement in timber columns with axial load of 49kN on the timber columns, the stress obtained is 2.027E+03 N/mm^2 when compared to 1.173E+03 N/mm^2 for1mm CFRP laminates with the same axial compression acting on the timber columns. The differences in stress are 0.854E+01 N/mm^{2}which is about 42% reduction of stress from 0.2 mm thick CFRP laminate. The displacements result obtained for 0.2mm CFRP laminates is 2.199E+01 mmwhile for 1mm CFRP laminates is 1.332E+01 mm. The differences in displacement are 0.867E+01 mm or about 39% reduction of displacements from 0.2 mm thick CFRP laminate. The reaction result obtained for 0.2mm CFRP laminate is 1.761E+05kN while for 1mm CFRP laminates is 1.392E+05kN. The difference in reactions is 0.369E+05kN which is about 20% reduction of reactions from 0.2 mm thick CFRP laminate.

This shows that the tensile capacity of the timber columns has been increased, because the CFRP reinforcement offered some degree of resistance to the timber columns, thus making it to accommodate more loadings prior to its failure.

The distributions patterns of stress, displacement and reactions are showed in Figure 4.7 to Figure 4.9.

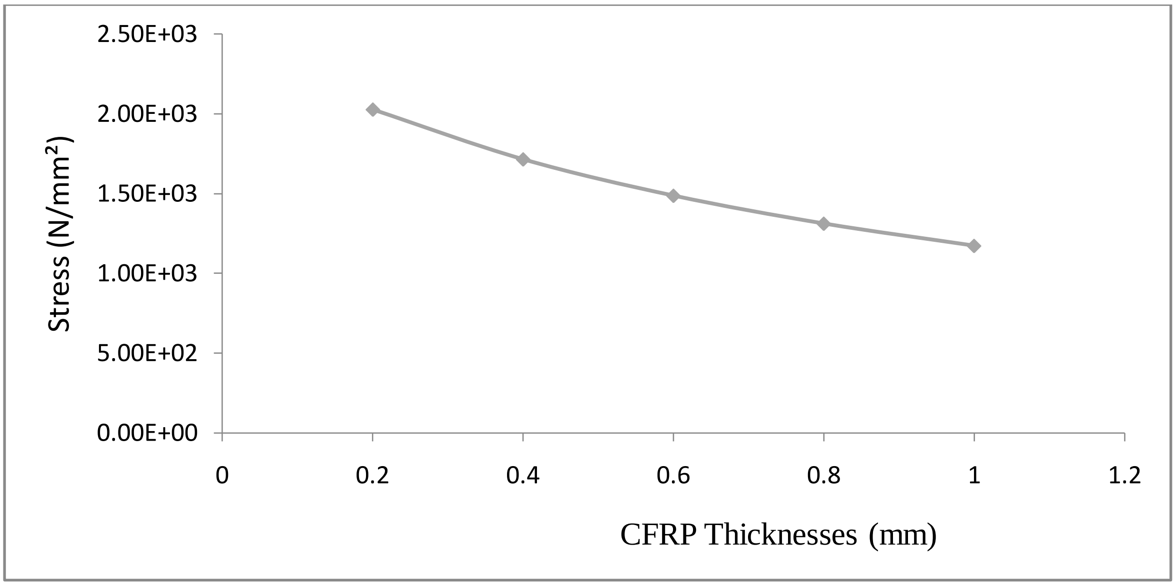


Figure 4.7: Relationship of stress and thicknesses of CFRP laminate with Confusa timber column species

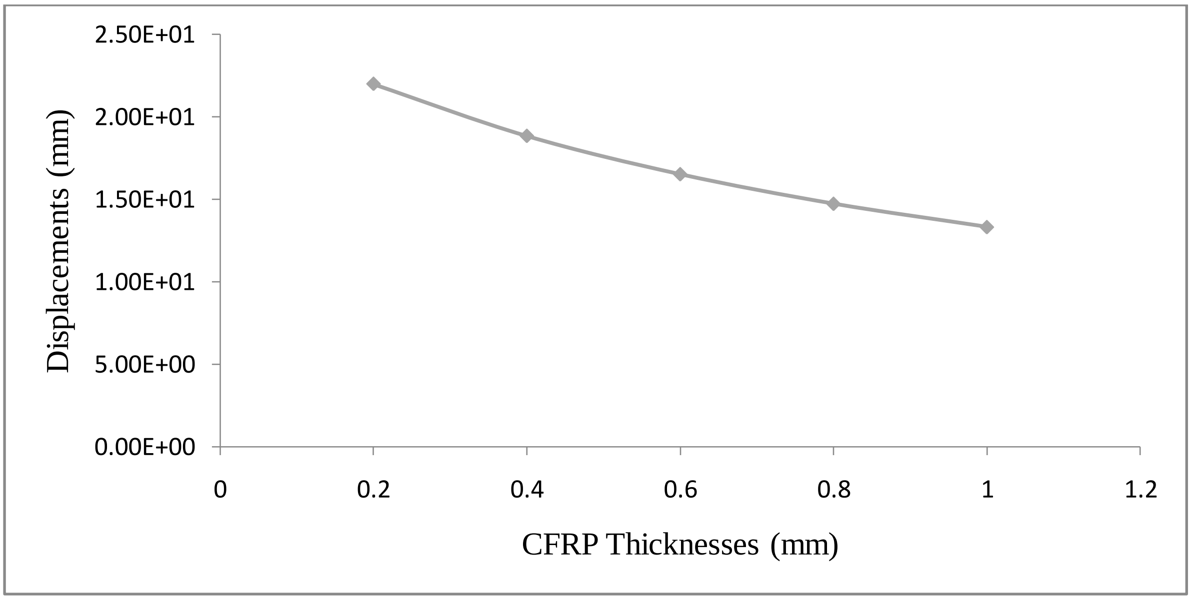


Figure 4.8: Relationship of displacements and thicknesses of CFRP laminate with Confusa timber column species

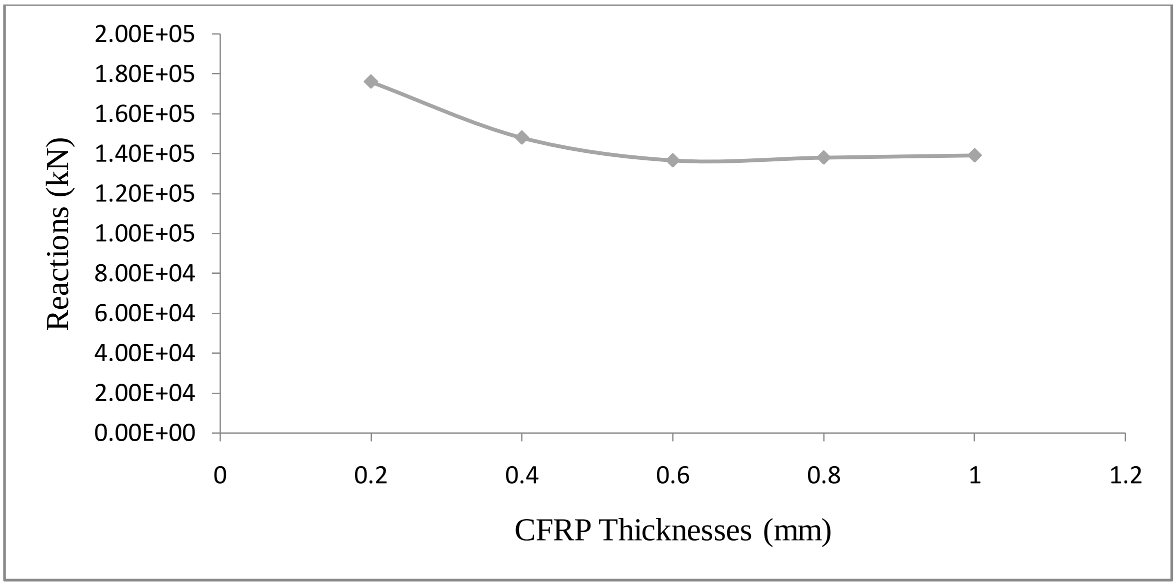


Figure 4.9: Relationship of reactions and thicknesses of CFRP laminate with Confusa timber column species

4.3.1 Stress Patterns in solid timber column sections

The stress patterns obtained after the analysis for CFRP and timber for various end conditions can be seen in the deformed diagram of the model shown in Plate 2 to Plate 4 for fixed-free ended timber columns laminated with CFRP. For fixed-fixed ended strengthen timber columns and fixed-free ended unstrengthen timber columns whichare shown in Appendix using the same axial applied load of 36kNfor *Mitragyna ciliate* (Abura) species of timber column, while 58kN for *Afromosia elata*(Afromosia) timber column species and 49kN for*Confusa* timber column species. The analysis shows that there is low stresses withinCFRP laminates sections than the unstrengthen timber sections which make it accommodate more load when compared to unstrengthen timber column.

The analysis shows that there are no stresses within the timber columns and the contour blue indicates low stresses at the timber columns structure. The contour blue indicates zero stresses and hence it is an indication that the bonded timber columns are stable after the deformation caused by the applied loads.

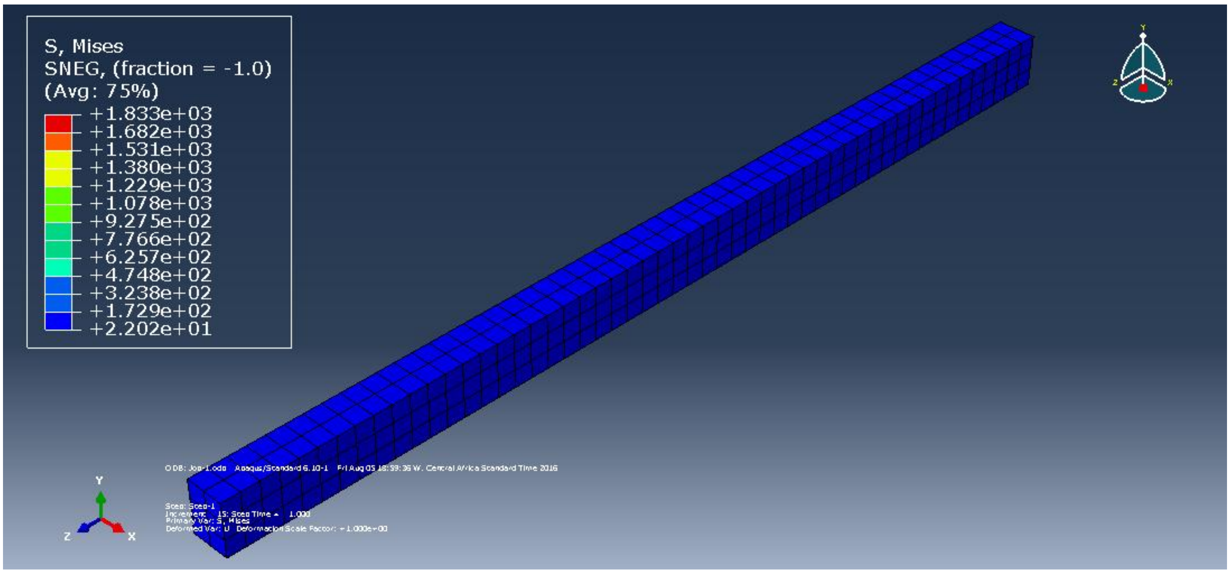


Plate 2: Stress Distribution Model for Abura Timber Species for Strengthen Timber Column (Fixed-Free), 0.2mm CFRP Laminate using (ABAQUS/CAE, 6.10)

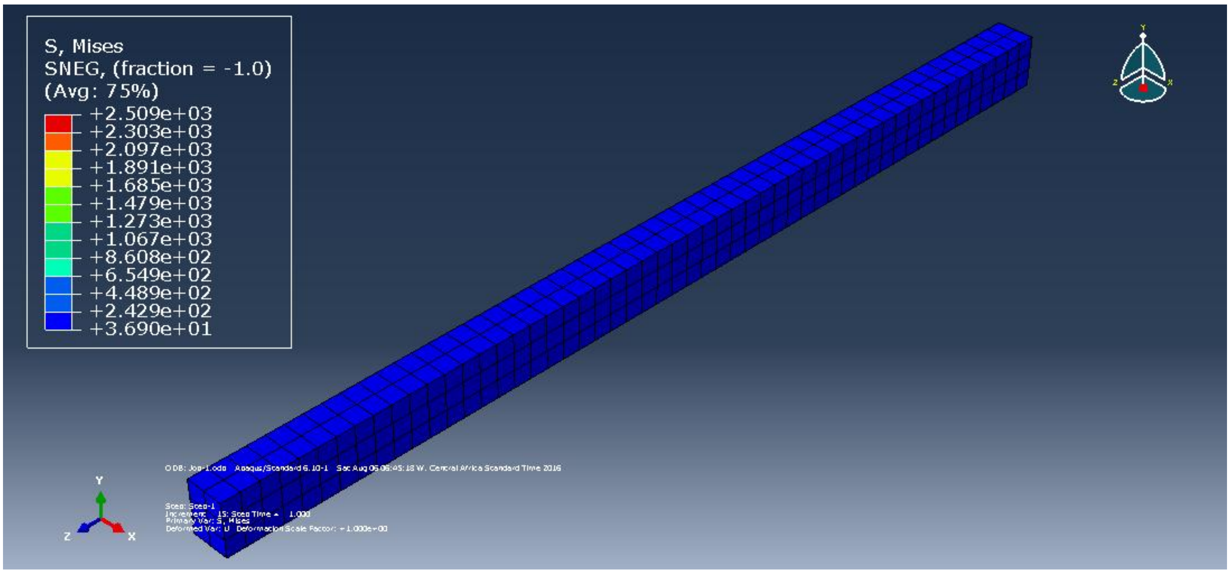


Plate 3: Stress Distribution Model for Afromosia Timber Species for Strengthen Timber Column (Fixed-Free), 0.2mm CFRP Laminate using (ABAQUS/CAE, 6.10)

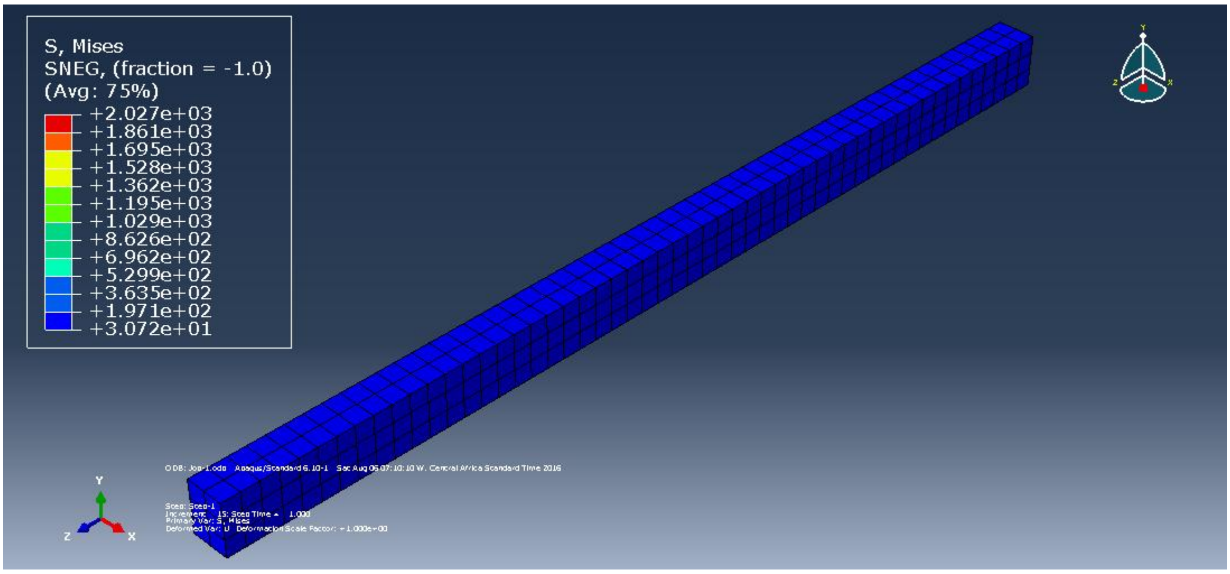


Plate 4: Stress Distribution Model for Confusa Timber Species for Strengthen Timber Column (Fixed-Free), 0.2mm CFRP Laminate using (ABAQUS/CAE, 6.10)

4.3.2 Axial Displacementof solid timber columns sections

The axial displacements pattern of CFRP laminates with various end conditions obtained after the analysis can be seen in the deformed model as shown in Plate 5 to Plate 7 for fixed-free ended strengthen timber column with CFRP laminate. For both ends fixed-fixed for strengthen timber columns with CFRP laminates and fixed-free ends and for unstrengthened timber columns sections are shown in Appendix. For axially applied load of 36kNfor *Mitragyna ciliate* (Abura) species of timber columns, while 58kN for *Afromosia elata* (Afromosia) timber column species and 49kN for *Confusa* timber columns species.The analysis shows that CFRP laminates can sustain higher loads before failure can take place when compared to unstrengthened timber column withoutCFRP laminates. The analysis is shownin deformed plates as red contour. However, the deformation obtained for strengthenedtimber column before failure was considerably lower compared to unstrengthened timber columns.

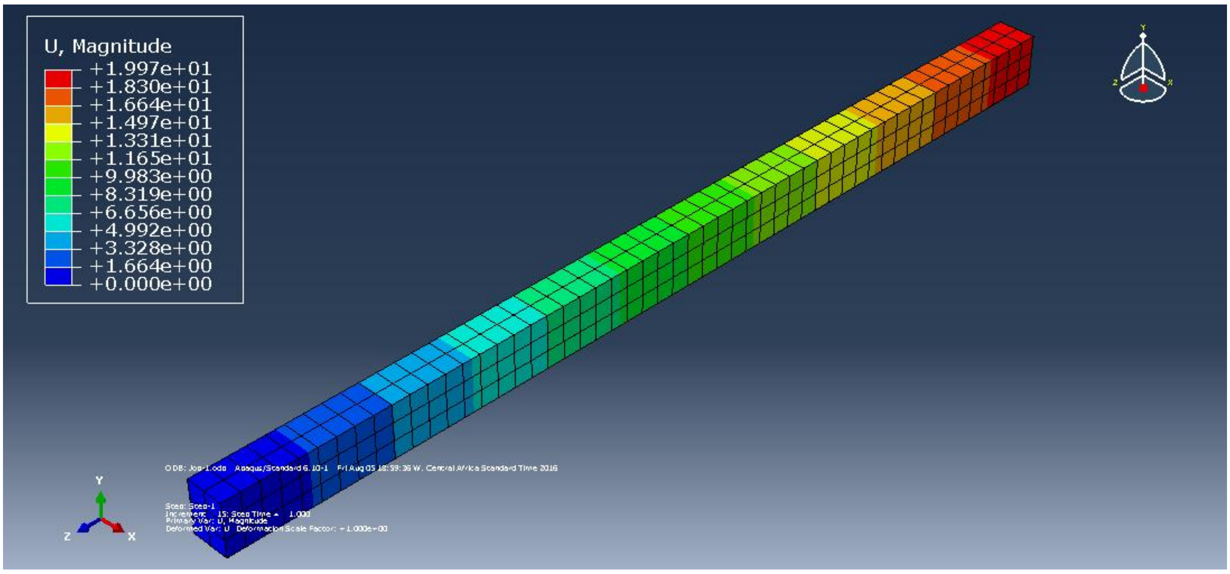


Plate 5 Displacement Distribution Model for Abura Timber Species for Strengthen Timber Column (Fixed-Free), 0.2mm CFRP Laminate using (ABAQUS/CAE, 6.10)

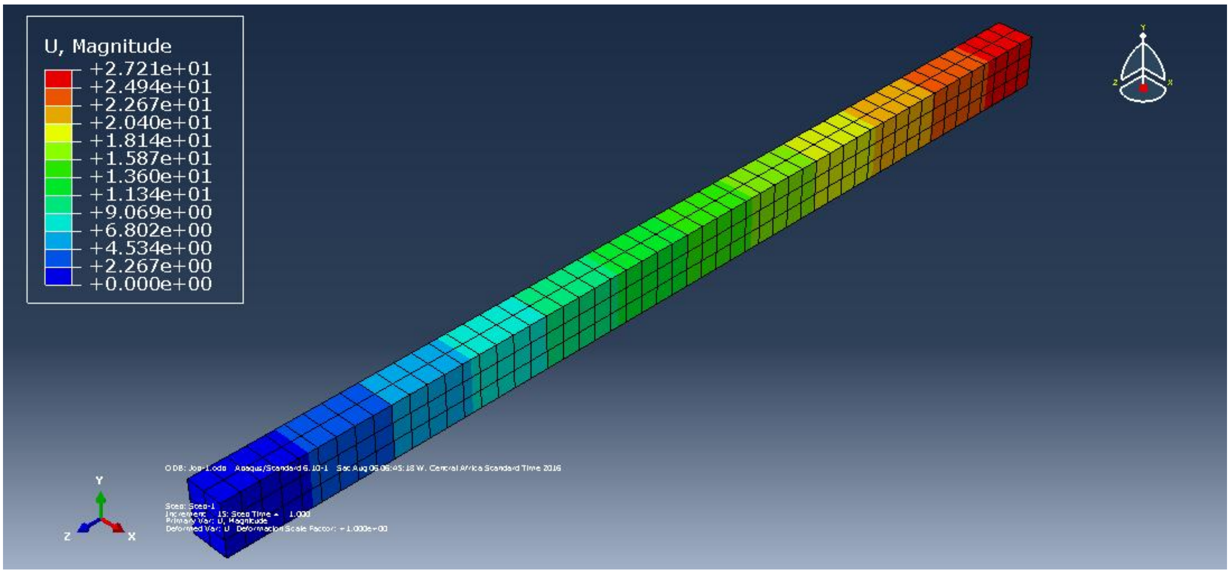
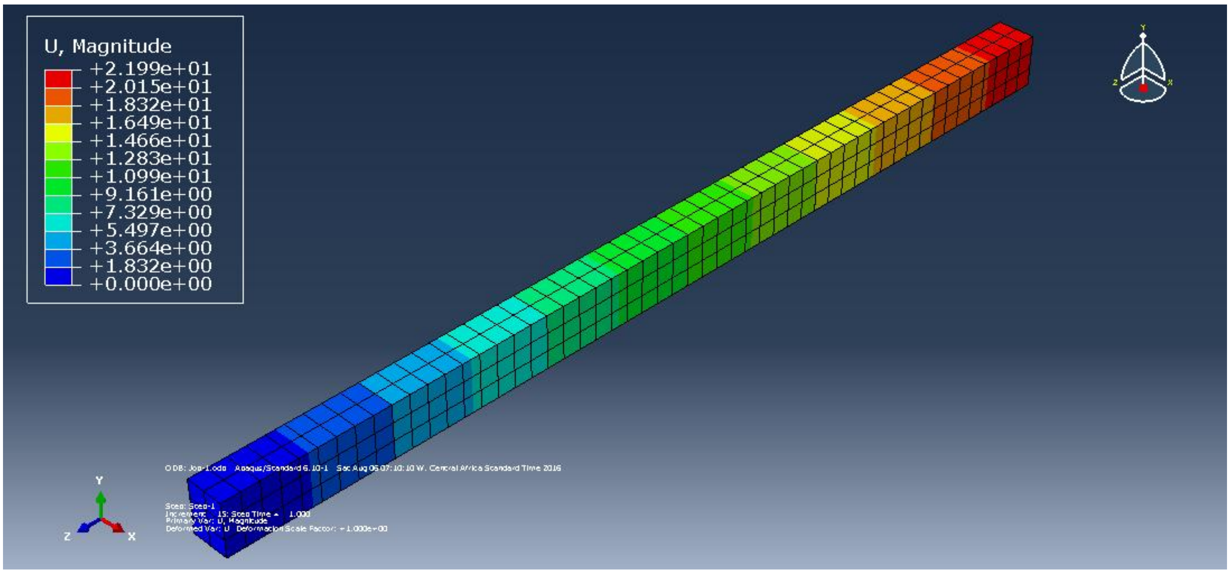


Plate 6: Displacement Distribution Model for Afromosia Timber Species for Strengthen Timber Column (Fixed-Free), 0.2mm CFRP Laminate using (ABAQUS/CAE, 6.10)



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ODB: Job-1.odb    Abaqus/Standard 6.10-1    Sat Aug 06 07:10:10 W. Central Africa Standard Time 2016

Step: Step-1
Increment     15: Step Time =    1.000
Primary Var: U, Magnitude
Deformed Var: U   Deformation Scale Factor: +1.000e+00
```

Plate 7: Displacement Distribution Model for Confusa Timber Species for Strengthen Timber Column (Fixed-Free), 0.2mm CFRP Laminate using (ABAQUS/CAE, 6.10)

4.3.3 Reactions pattern of solid timber columns sections

The reactions pattern obtained in the analysis of bonded CFRP laminate and sprays totimber for various end conditions can be seen in the reaction diagram of the model shown in Plate 8 to Plate 10 for fixed-free ended strengthened timber columns. For fixed-fixed ended strengthened timber columns and fixed- free ended unstrengthened timber columnsare shown in Appendix using the same applied load of 36kNfor *Mitragynaciliate* (Abura) species of timber column, while 58kN for *Afromosia elata* (Afromosia) timber column species and 49kN for *Confusa* timber column species. The analysis shows that, with increases in CFRP laminates and sprays thicknesses the reaction of the timber columns also decreases drastically, while for unstrengthened timber columns the reactions pattern increases with increases in applied load on the timber column sections.

The reactions pattern for unstrengthened and strengthened timber columnswhich is mostly represented by the light green after load is been applied on the timber columns. The analysis shows that there are high reactionswithin the base of unstrengthened and strengthened timber columns.

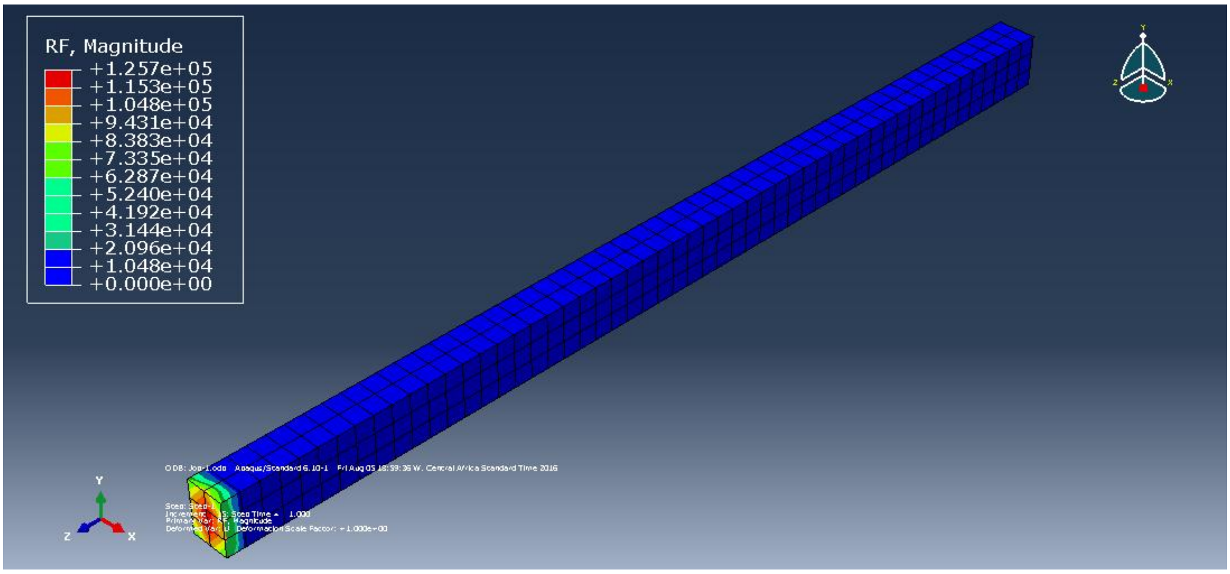

Plate 8: Reactions Distribution Model for Abura Timber Species for Strengthen Timber Column (Fixed-Free), 0.2mm CFRP Laminate using (ABAQUS/CAE, 6.10)

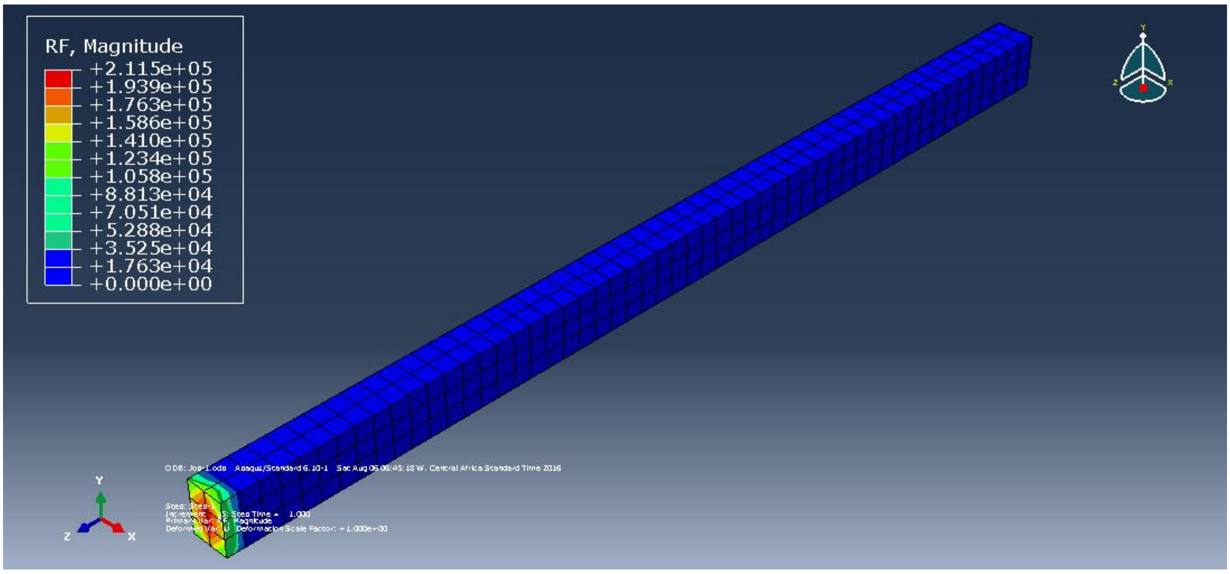

Plate 9: Reactions Distribution Model for Afromosia Timber Species for Strengthen Timber Column (Fixed-Free), 0.2mm CFRP Laminate using (ABAQUS/CAE, 6.10)

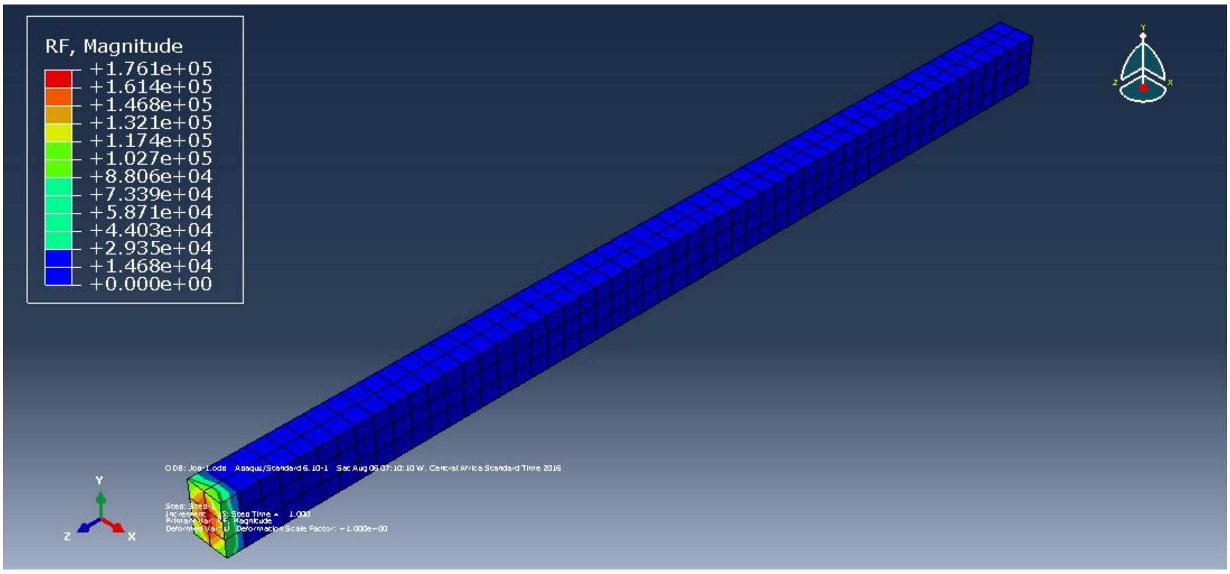

Plate 10: Reactions Distribution Model for Confusa Timber Species for Strengthen Timber Column (Fixed-Free), 0.2mm CFRP Laminate using (ABAQUS/CAE, 6.10)

4.4 Program Development

A reliability analysis program called *Samuel Iliya Bajahry_2016.* was developed to carry out the analysis of solid timber column laminated with CFRP in accordance with EC5(2004) design requirements for timber and First Order Reliability Method (FORM) as earlier explained. The program starts by requesting the user to input the following values: Imposed load coefficient(γ_q),dead load coefficient(γ_g), imposed load (Q_k), dead to live-load ratio (α), length (L), depth (h), width (b), Modulus of Elasticity (MOE) and thickness of CFRP. The program then proceeds to compute the structural safety index (β) and the probability of failure for several numbers of iterations and stops when the stopping criteria are met. Lastly, the***Elitism*** step then selects the global minimal value of safety index (β) and its probability of failure amongst others

4.5Reliability Analysis of Solid Timber Column Laminates With CFRP

The solid timber structures analyzed in this work are strengthened with CFRP laminates and sprays. The performance function for solid timber columns is obtained from Eurocode-5(2004), EN 338(2009) and analyzed. The program was developed in*MATLAB* environment by employing the First Order Reliability Method (FORM) to compute the safety indices and their corresponding probability of failure. In the analysis, dead to live ratio (α_{pc}), Modulus Of Elasticity (MOE) of CFRP (E_{frp}), thickness of CFRP (t_{frp}), different strength classes of timber modeled as uncertainty and resistance are considered as random variables, while others as deterministic for simplicity.

Also, for reliability analysis, the limit state equations used are non-linear; thus, Rackwitz-Fiessler algorithm was used to deal with the nonlinearity in the limit state function and to transform non-normal variables into independent standard normal ones.Thesafety index, β, associated with the minimum distance between the failure surface and the origin of the limit state functionas obtained from the reliability program developed in *MATLAB* is computed on the basis of the input variables.

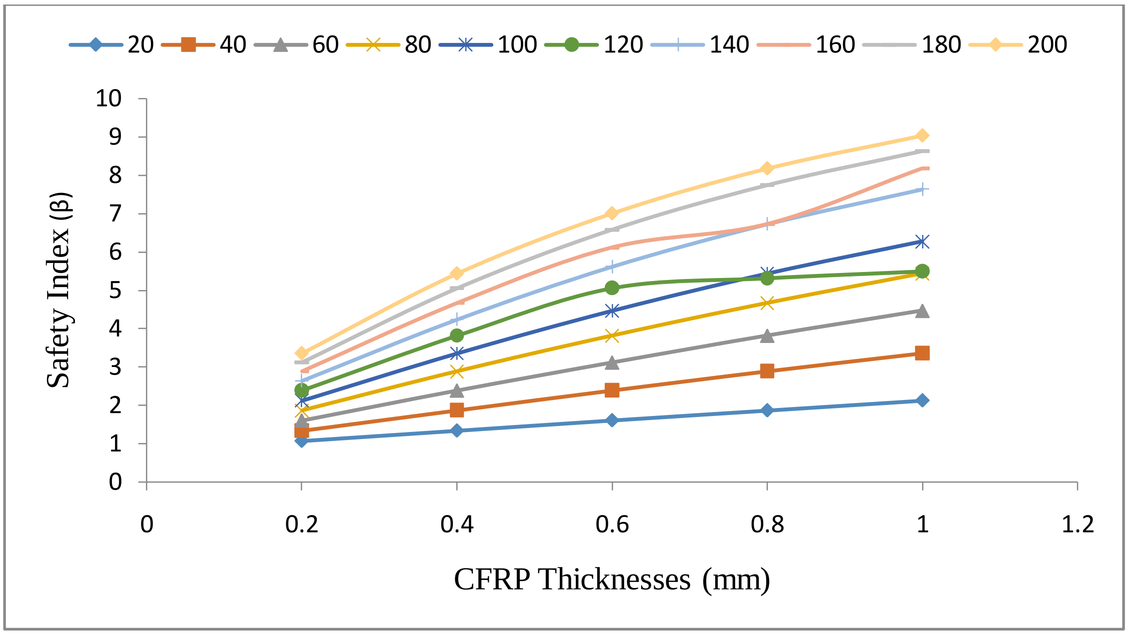

Figure 4.10: Relationship of Safety Index to CFRP thicknesses at varying MOE of CFRP (kPa) for D30 strength class of timber.

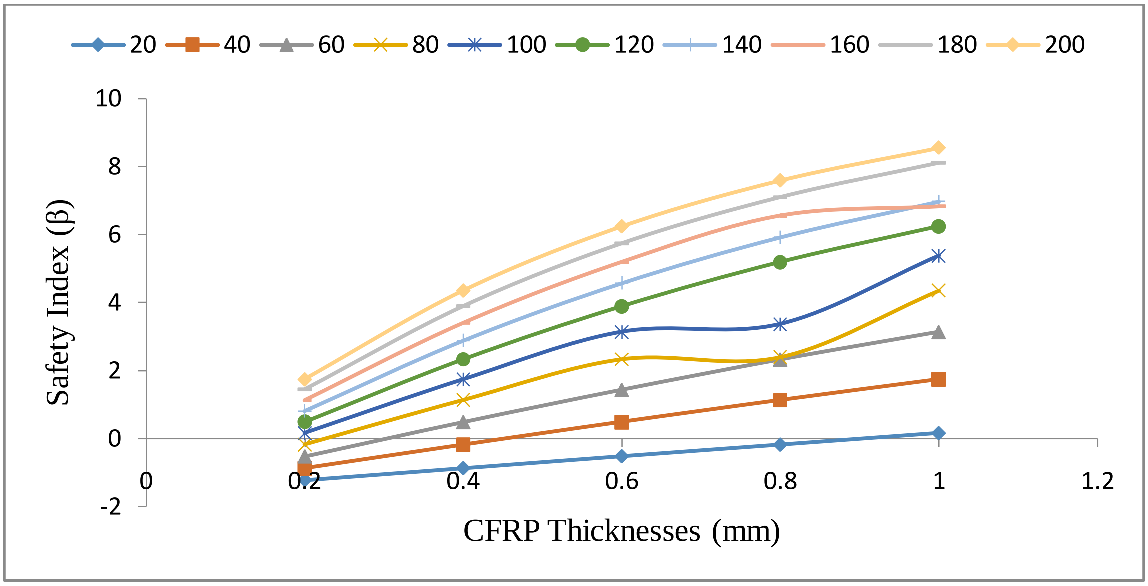

Figure 4.11: Relationship of Safety Index to CFRP thicknesses at varying MOE of CFRP (kPa) for C18 strength class of timber.

Figure 4.10 and Figure 4.11 show the relationship between safety indices to thickness of CFRP (mm) at various MOE of CFRP for D30 and C18 of hard and soft wood timber species. The distribution pattern from the figures 4.10 and 4.11 for both D30 and C18 shows that, the safetyindices of the timber columns species increases as MOE of CFRP increases. The safety indices for D30 timber species show that as thicknesses of CFRP laminates at a regular MOE increases, it has high tendency to accommodate more loads while for C18 timber species, the safety index shows ahigher probabilty of failure but as CFRP thickness to MOE increases adequately with good safety margins was observed which has ability to accommodate more loadings.

This result of CFRP bonded composite timber column gave safety indices value of 9.03 and 8.55 for D30 and C18 when varying CFRP thicknesses at 200 kPa MOE of CFRP. This shows that the influence of variation in CFRP thicknesses to MOE plays a vital role in improving the structural performance of composite solid timber column. Also these safety indices obtained are much higher than the JCSS (2006) value.

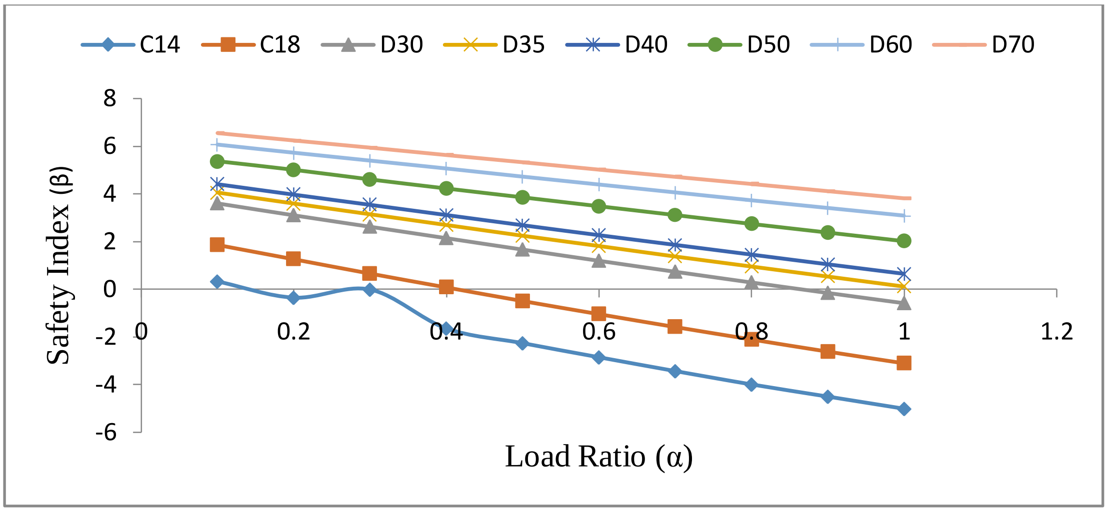

Figure 4.12: Relationship of Safety Index to load ratio at various strength classes of timber species.

Figure 4.12, shows relationship of safety indices to load ratios; it was observed that the safety indices decrease with increases in load ratios with different strength classes of solid timber. From the distribution pattern above it is clearthat soft wood has a higherprobability of failure as compared to hard wood. Also from the distribution pattern above C14 has a safety index of -5.02 at a 1.0 load ratio, while at D70 timber species, the safety index at 1.0 load ratio is 3.80.

The result shows that using D70 species of timber column have more ability to accommodate and resist more stress compared to C14 species of timber column.

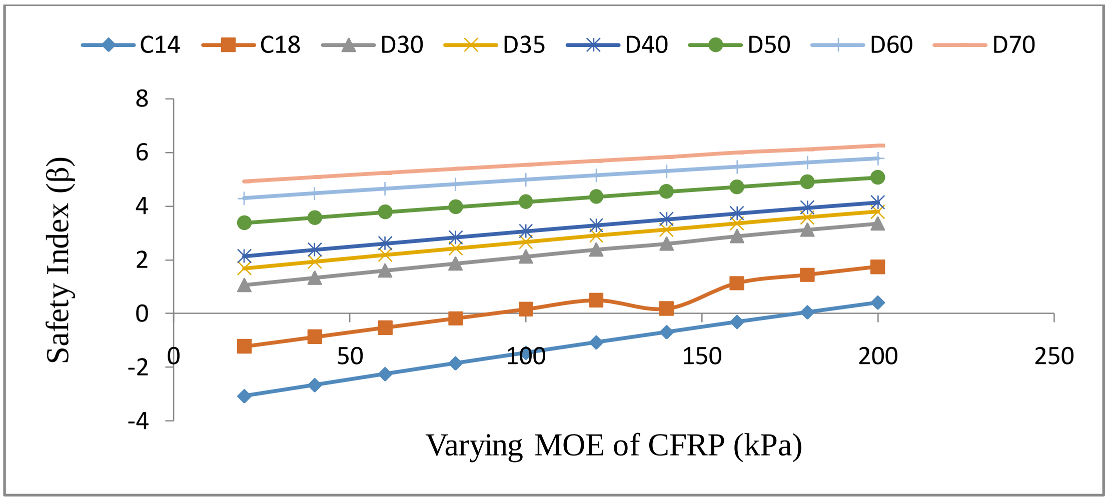

Figure 4.13: Relationship of Safety Index to varying MOE of CFRP at various strength classes of timber species.

Figure 4.13, shows relationship between safety indices at MOE of CFRP. In the result it is observed that the safety indices increase with an increases in MOE of CFRP with different strength classes of solid timber. From the distribution pattern above, we find that as MOE of CFRP increases, different strength class of timber also increases. But still soft wood has higher probability of failure from the distribution pattern when compared to hard wood. The safety indices for C14 timber specieswas obtained as 0.41 at 200 kP_a, while for D70 timber species, the safety index at 200 kP_a was obtained as 6.26. The result shows that using D70 species of timber there is ability to accommodate and resist more axial stresses compared to C14 species of solid timber columns.

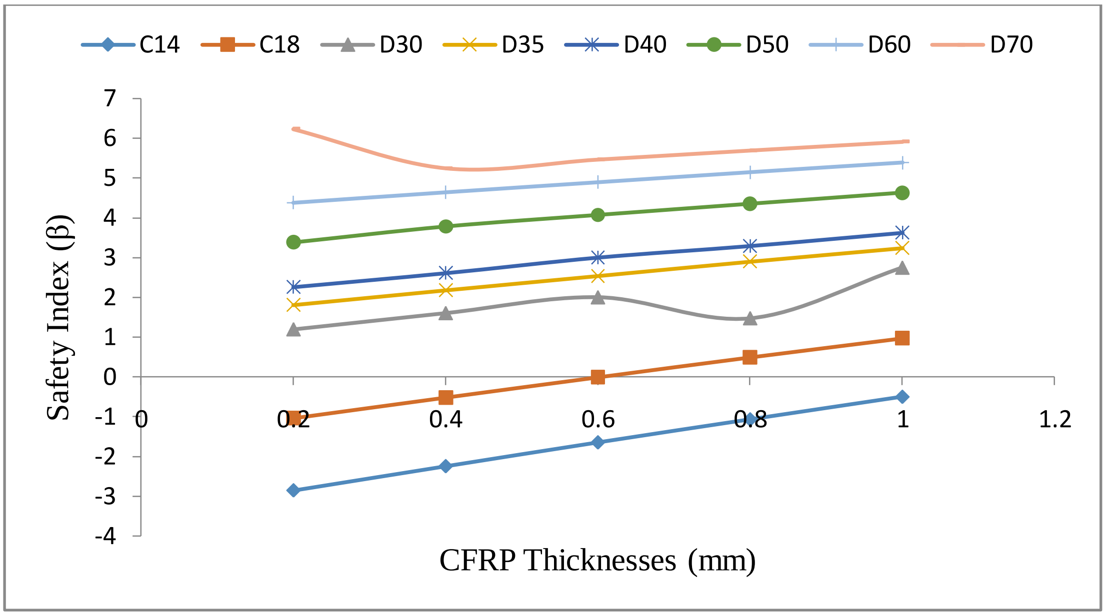

Figure 4.14: Relationship of Safety Indices to varying CFRP thicknesses at various strength classes of timber species.

Figure 4.14, shows relationship between safety indices to varying CFRP thickness; in the result it was observed that the safety indices increase with increases in CFRP thicknesses and with different strength classes of timber. From the distribution pattern above, the result shows that as CFRP thickness increases the safety indices also increase which implies that with a 0.2 mm CFRP laminates, timber strength increase rapidly which has higher tendency to accommodate more load and to resist tendency of failure. But still soft wood has a higher probability of failure from the distribution pattern when compared to hard wood. The safety indices for C14 timber species was obtained as -0.50 at 1.0 mm while for D70 timber species, the safety index at 1.0 mm CFRP laminates the result was obtained as 5.91. The result shows that using D70 species of timber has ability to accommodate and resist more stresses compared to C14 species of timber column and with adequate structural safety.

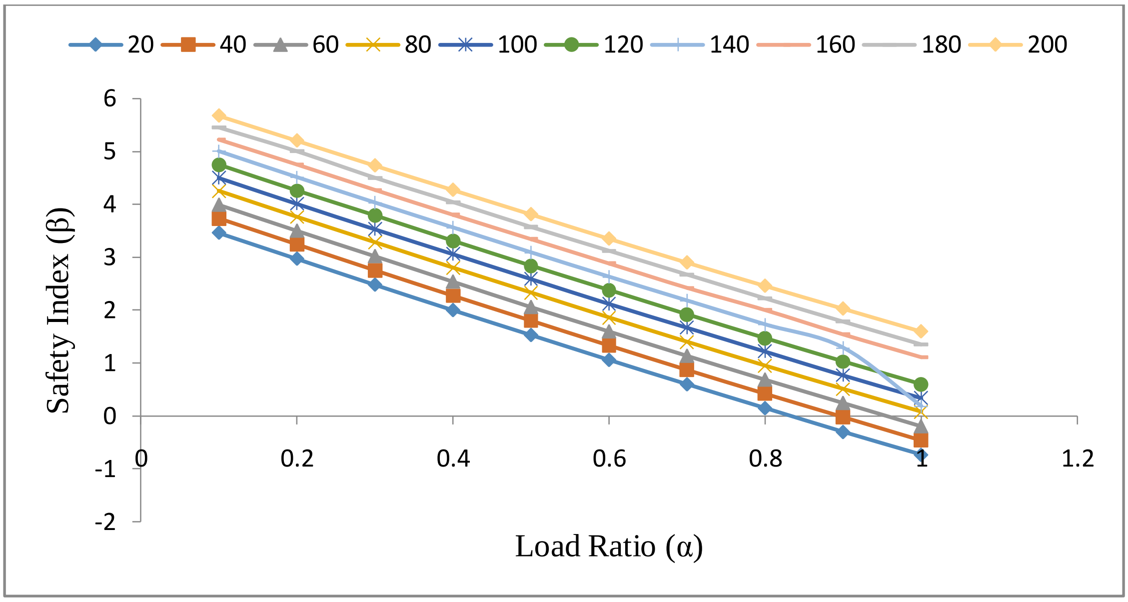

Figure 4.15: Relationship of Safety Indices to load ratios at varying MOE of CFRP (kPa) for D30 strength class of timber.

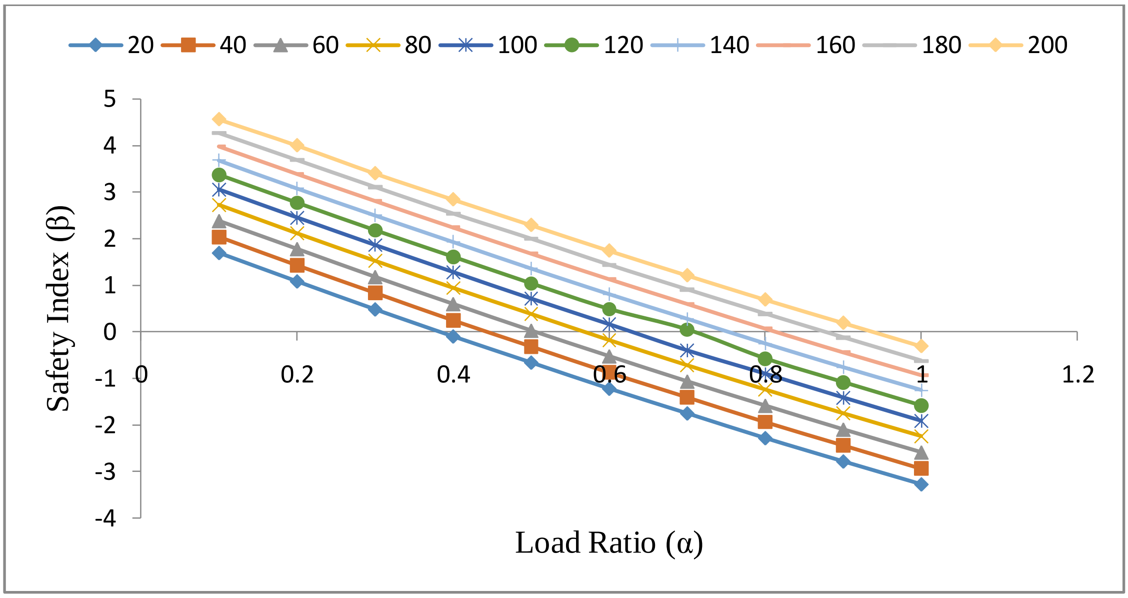

Figure 4.16: Relationship of Safety Indices to load ratios at varying MOE of CFRP (kPa) for C18 strength class of timber.

Figure 4.15 and Figure 4.16, show relationships of safety indices to load ratios with varying MOE of CFRP. The result showsthat the safety indices decrease with increases in load ratios for both D30 and C18 strength classes of solid timber columns. For D30 solid timber columns, the result above shows adequate structural safety of the solid timber columns when compared with C18 solid timber column species. From the distribution patterns, soft woods have a higherprobability of failure when compared to hard woods.

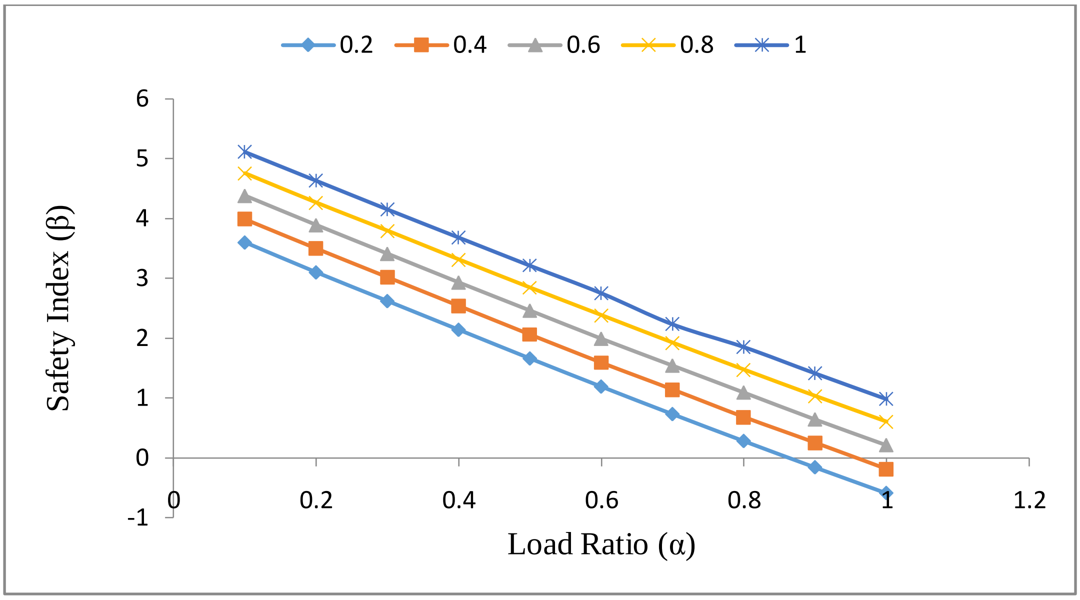

Figure 4.17: Relationship of Safety Indices to load ratios at varying CFRP-thicknesses (mm) for D30 strength class of solid timber.

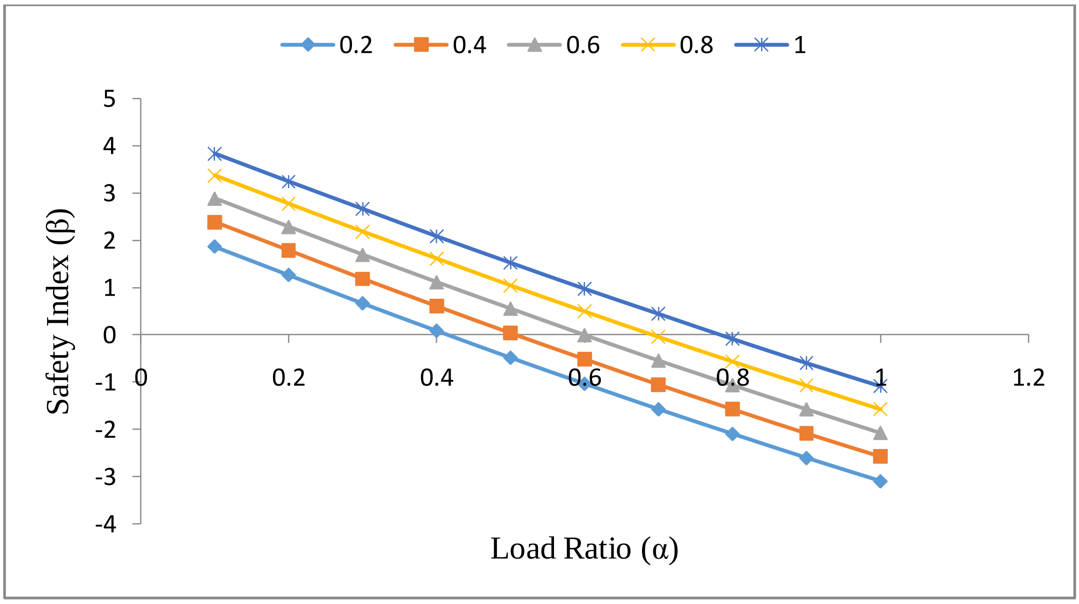

Figure 4.18: Relationship of Safety Indices to load ratios at varying CFRP-thicknesses (mm) for C18 strength class of solid timber.

Figures4.17 and4.18 showthe relationship between safety indices to load ratios at varying thickness of CFRP to solid timber column.The distribution pattern from the figures for both D30 and C18 show that, the structural safety indices of the timber column species decrease as load ratio increases with varying thickness of CFRP.

The result of CFRP bonded composite solid timber columns for both D30 and C18 gave a good safety margin as they are well over the target safety of JCSS (2006). The influence of variation in CFRP thicknesses to load ratios plays a vital role in improving the structural performance of composite solid timber columns.

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 Summary

The study resulted in checking the structural reliability of CFRP laminates and sprays as reinforcement for retrofitting different species of solid timber columns. The analysis for fixed–free ended both strengthened and unstrenthened timber columns andfixed–fixed ended for strengthened timber column with axial applied loadof 36kNfor *Mitragyna ciliate* (Abura) species of solid timber column for both strengthened and unstrengthened timber column while axial applied load of 58kN for *Afromosia elata* (Afromosia) timber column species for both strengthened and unstrengthened timber column, and axial applied load of 49kN for *Confusa* timber columns species for both strengthened and unstrengthened timber columns.

Considering the analysis of three timber columns species with fixed-free ended laminated with 0.2mm –1mm thicknesses of CFRP has the following stresses, displacements and reactions. For 0.2mm and 1mm CFRP laminates, the results obtained for *Mitragyna ciliate* (Abura) species solid timber column with axial applied load of 36 kNare 1.833E+03 (N/mm²) and 9.782E+02 (N/mm²),1.997E+01 (mm) and 1.123E+01 (mm), and 1.257E+05 (kN) and 1.052E+05 (kN) , while for *Afromosia elata* (Afromosia) species of timber column with axial applied load of 58 kN, the results obtained are 2.509E+03 (N/mm²) and 1.435E+03 (N/mm²), 2.721E+01 (mm) and 1.632E+01 (mm), and 2.115E+05 (kN) and 1.683E+05 (kN) and for *Confusa* species of timber column with axial applied load of 49 kN, the result obtained are 2.027E+03 (N/mm²) and 1.173E+03 (N/mm²), 2.199E+01 (mm) and 1.332E+01 (mm), and 1.761E+05 (kN) and 1.392E+05 (kN). Also from the obtained shows that about average of 41% reduction of stress from range of 0.2 mm to 1.0 mm thick CFRP laminate while about 40% reduction of displacements from range of 0.2 mm to 1.0 mm thick CFRP laminate and about 20% reduction of reactions from 0.2 mm to 1.0 mm thick CFRP laminate.

Generally, the resistances of engineering structures as well as the load applied to them are functions of several variables. Most of these variables possess some element of randomness in them. To adequately accommodate such uncertainties, probabilistic method is used herein.

The provision of EC5(2004) is based on limit state design approach, which applies the use of partial safety coefficients to both the resistance and loadings so as to reduce the risk of failure. More so, when these columns are subjected to both dead and imposed load, the stresses developed at thetop and bottom of the solid timber columns are responsible for loss in strength and stiffness of the timber columns, thereby resulting in the loss of structural integrity of the timber columns.

In conclusion, structural reliability of solid timber columns using MATLAB programming was adequate. Good safety measures was checked and adequate safety margins observed for the timber columns, this proves that laminates and sprays of solid timber with CFRP has good bonding relationship which can enable the solid timber columns to withstand more load and to resist more stresses.

5.2 Conclusions

From the finite element and structural reliability analysis of the typical solid timber column sections laminated with CFRP, the following conclusions are obtained.

(1) There was decrease in stresses as CFRP laminates and sprays are increasing in thickness with the same axial applied load acting on the timber columns.

(2) Bond characters are studied and empirical load-carrying capacity based on stress-based approach for solid timber column strengthened with CFRP is found to be in good agreement with the test results reviewed from literature.

(3) External strengthening of timber columns using normal modulus CFRP strips is quite an effective technique in increasing the load carrying capacity and stiffness of the solid timber columns sections.

(4) Safety indices also decrease with decreases in load ratios, and different strength classes of timber columns. This implied that strengthened timber columns offer high resistanceto strength as compared to unstrengthened timber columns.

(5) From the reliability analysis of solid timber columns laminated with CFRP, various parameter are obtained for safely index. From the result obtained shows that C18 strength class of timber have 97% chances of failing because their safety index values falls within on safe zones.

5.3 Recommendation

(1) From reliability assessment of strength capacity of solid timber column laminates it recommended that the moisture content of timber changes with time which also affect the strength capacity of timber columns.

(2) The reliability analysis used in this work is implemented using probabilistic methods; thus the uncertainties in variables were accommodated. This program addresses the issue of safety, it is then recommended that a higher safety index value should not be the target safety index in the reliability, the efficiency or effectiveness of the solid timber column element should also be considered.

REFERENCES

Afolayan J. O. (1992)."Reliability-Based Analysis and Design", Departmental Seminar Civil Engineering Ahmadu Bello University Zaria.

Alpo. R. (2004) “Theoretical and Practical Aspects of the Reliability Analysis of Timber Structures”, WCTE 2004 conference in Lahti, Finland.

Atim, I.T. (2006)“Reliability-based Design Charts for Short Symmetrically Reinforced Concrete Columns”, An M.Sc. Thesis Submitted to the Postgraduate School, Ahmadu Bello University Zaria.

Bakis C.E, (2002),“Fiber Reinforced Polymer Composites for Construction State of the Art” Review Journal of Composites for Construction Vol.6 No.2 pp73-87.

Bambach, M.R., Jama, H.H., and Elchalakani, M. (2009). "Axial Capacity and Design of Thin-Walled Steel SHS Strengthened with CFRP". Thin-Walled Structures, 47,1112-1121.

Benedetti, A., and Tralli, A. (1989), “New hybrid FE model for arbitrarily curved beam I: Linear analysis”.

Blass H. J., Aune P., Choo B. S., Goerlacher R, Griffths D. R and Hilson B. O (1995) “Timber Engineering Step1”, First Edition, Centrum Hout, the Netherlands

BS 5268: Part 2 (2002) Code of Practice for Permissible Stress Design, Material and Workmanship.

Chanakya A. (2009), “Design of Structural Elements”, Concrete, Steelwork, Masonry and Timber Design to British Standards and Euro codes. 3rd Ed. Taylor and Francis, USA.

Christopher J.G (2000), “Flexural Strengthening of Timber Bridge Beams using FRP”, A Civil Engineering Thesis Ahmadu Bello University Zaria.

Dorina, I, Oana, S and Iulian, A. (2012). Analysis of Wood Bending Properties on Standardized Samples and Structural Size Beams Tests. BULETINUL Institutului Politehnic Din Iaşi Publicat de Universitatea Tehnică “Gheorghe Asachi” Din Iaşi Tomul lviii (lxii), Fasc. Vol. 1, 2012. Secţia Construcţii. Arhitectură

Emerson, R. (2004), “In Situ Repair Technique for Decayed Timber Piles”, School of Civil and Environmental Engineering, Oklahoma State University.

EN 338 (2003) “Structural Timber: Strength Classes”, European Committee for Standardization, CEN, Brussels, Belgium

EN 408 (2003): “Timber structures – Structural timber and Glued-laminated Timber: Determination of some Physical and Mechanical Properties”, European Committee for Standardisation, CEN, Brussels, Belgium

Eurocode 5 (2004), “Design of Timber Structures; Part 1-1: General Rules andRules for Buildings European committee for standardization. Brussels, 2004.

Foliente, G. (2000). History of Timber Construction, in ‘Wood Structures: A Global Forum on the Treatment, Conservation, and Repair of Cultural Heritage, ASTM STP 1351’, American Society for Testing and Materials, Washington DC pp. 3–22.

FPL (1999). Wood Handbook - Wood as an Engineering Material, Technical Report FPLGTR 113, USDA Forest Service, Forest Products Laboratory

Gollwitzer S., Abdo T and Rackwiz K. (1988) **“**First Order Reliability Method (FORM)”, Users Manual, RCP-GM BH, Munich, West Germany.

Haedir, J., Bambach, M.R., Zhao, X.L., and Grzebieta, R. (2006),"Bending Strength of CFRP—Strengthened Circular Hollow Steel Sections". In: Third International Conference on FRP Composites in Civil Engineering (CICE). Miami, FL, USA.

Hasal and Heiduscke (2010), “Fiber-Reinforced Plastic-Confined Wood Profiles Under Axial load”, Structural Engineering International (3): 246-253.

Heslehurst, R. (2008), “Composite Laboratory Safety”, Australian Defence Force Academy, School of Engineering Sydney.

Jack P., Abdi K. (2007)“Structural Timber Design to Eurocode 5”, 2nd Edition.

JCSS (2006) “Probabilistic Model Code: Part 3: Resistance Models: Properties of Timber”, Joint Committee on Structural Safety, 12th Draft, Internet version: Retrieved from http://www.jcss.ethz.ch

Jobin J, Olga L, Garzon B. (2007)., “Flexural Strengthening of Glued Laminated Timber Beams with Steel and Carbon Fibre Reinforced Polymers”. Master’s Thesis inthe international Masters Programmed for Structural EngineersBritish Columbia, Canada..

Kollmann, F. and Cote, W. (1968). Principles of Wood Science and Technology, George Allan & Unwin Ltd.

Kasal, B. Heiduschke, A. (2010),“The use of High-Strength Composites in the Reinforcement of Timber” Advanced Materials Research 133-134.

Lofgren, I., (2005),“Fiber Reinforced Concrete for Industrial Construction – A Fracture Mechanics Approach to Material Testing and Structural Analysis”. Unpublished PhD Thesis, Chalmers University of Technology, Goteborg.

Madsen, B. (1992),” Structural Behaviour of Timber” Timber Engineering Ltd., North Vancouver, British Columbia, Canada.

Melchers, R. E. (1999). “Structural Reliability Analysis and Prediction”. Ellis Horwood Limited, Great Britain.

Michael, A. P. (2006), “Using Carbon Fiber Reinforced Polymer Grid as Confinement Reinforcement for Concrete”, PhD Thesis, University of Florida.

Moraes, P. (2003). Influence De La Temperature Sur Les Assemblages Bois, PhD thesis, Universite Henri Poincar´e, Nancy 1.

N. Banthia, “Fiber Reinforced Polymers in Concrete Construction and Advanced Repair Technologies”Department of Civil Engineering University of British Columbia 2324 Main Mall Vancouver, BC, Canada V6T 1Z4

Najm H., Secaras J., Balaguru P.,(2007),“Compression tests of circular timber column confined with Carbonfibers using inorganic matrix.Civil. Engineering.,ASCE, 19(2): 198-204.

Nanni, A. and Dolan, C. W., (1993), ”Fiber – Reinforced – Plastic Reinforcement forConcrete Structures”, SP – 138, American Concrete Institute, Detroit, Michigan.

NCP 2 (1973), “Timber Design Manual” Nigerian Industrial Standard, Lagos.

Neale, K.W and Labossiere, P. (1997),“State of the art report on Retrofitting and Strengthening by Continuous Fibre in Canada” Third International Symposium onNon-Metallic Reinforcement for Concrete Institute,British Columbia, Canada.

Pearce, D. (1970),“Understanding CFRP as a Design Materials” Fibre Science and Technology, 3:129-146.

Porteous J. and Kermani, A. (2007). Structural Timber Design to Eurocodes. Blackwell Publishing. USA.

Retief, V. And Holicky, M., (2005)“Reliability assessment of alternative Eurocode and South African load combination schemes for structural design”, Journal of the South African Institution of Civil Engineering. Vol. 47 (1) pp 15-21.

Saadatmanesh, H, (1994): “Fiber Composite for New and Existing Structures”, ACI Structural Journal, Vol. 9, No 3.

Salisu D, (2007). “Reliability- based Analysis and Design of Mechanically Jointed Timber beam under gravity loads”. A Thesis Submitted to the Postgraduate School Ahmadu Bello University Zaria, for the Award of Master of Science Degree, Department of Civil Engineering, Ahmadu Bello University Zaria

Silvestre (2007), “Axially Loaded Steel Columns Strengthened with CFRP”Jordan Journal of Civil Engineering, Volume 8, No. 1, 2014

Sliker, A. (1962), “Reinforced Wood Laminated Beam”. Forest Products Journal, 12(1).

Sousa H. S., Sørensen J. D., Kirkegaard P. H., Lourenço P. B. and Branco J. M. (2011) “Reliability based robustness of Timber Structures through NDT Data updating”, SHATIS' 11th International Conference on Structural Health Assessment of Timber Structures - Lisbon, Portugal

Stefanou, I.,Sulem., Vardoulakis, I. (2003), Three- dimensional cosserat homogenization of masonry structures:elasticity.ActaGeotechnica, Vol.3, pp. 71-83, 2008.

Stehn L, and Johanssen H. (2002). Ductility Aspects in Nailed Glue-Laminated Timber Connection Design. Journal of Structural Division. ASCE Vol. 128. Pp. 382-389.

Steiger R. (2004), "Bonding of carbon fiber reinforced plastics (CFRP) with wood". In: Proceedings of COST E34 conference "Innovations in Wood Adhesives", Biel, Switzerland, 33 - 50.

Sweeney S.R (2012),“Behaviour of Timber Confined with CFRP under Compression” A Final year Project Report British Columbia University.

Tord. I. (2001) System Effect. Nordic Wood-Reliability of Timber Structures.

TRADA (2009) “Embracing Eurocode 5”, Timber Research and Development Association, Version 2 - Construction Briefings, UK

Triantafillou, T.C. (1998),“Strengthening of Structure with Advanced FRPs”. Progress in Structural and Materials, 1(2), pp. 126-134.

Walter G. (1996),“Timber Bridges in Australia” National Conference on Wood Transportation Structure. Madison, Wisconsin: 17-21.

APPENDIX

Table A. Abura Timber Species for Various Thicknesses of CFRP with Axially Load of 36 kN(Fixed-Fixed)

Parameters	Thickness of CFRP (mm)				
	0.2	0.4	0.6	0.8	1.0
Stress (N/mm^2)	1.131E-16	2.997E-16	4.984E-16	6.714E-16	8.635E-16
Displacement (mm)	4.002E-17	1.040E-16	1.705E-16	2.388E-16	2.979E-16
Reaction (kN)	1.800E+05	1.800E+05	1.800E+05	1.800E+05	1.800E+05

Table B. Afromosia Timber Species for Various Thicknesses of CFRP with Axial loadof 58kN(Fixed-Fixed)

Parameters	Thickness of CFRP (mm)				
	0.2	0.4	0.6	0.8	1.0
Stress (N/mm^2)	1.081E-16	2.823E-16	4.788E-16	6.672E-16	8.390E-16
Displacement (mm)	2.758E-17	8.040E-17	1.319E-16	1.920E-16	2.451E-16
Reaction (kN)	2.900E+05	2.900E+05	2.900E+05	2.900E+05	2.900E+05

Table C. Confusa Timber Species for Various Thicknesses of CFRP with Axial load of 48kN(Fixed-Fixed)

Parameters	Thickness of CFRP (mm)				
	0.2	0.4	0.6	0.8	1.0
Stress (N/mm^2)	1.077E-16	2.797E-16	4.753E-16	6.654E-16	8.388E-16
Displacement (mm)	2.600E-17	7.756E-17	1.270E-16	1.270E-16	2.378E-16
Reaction (kN)	2.400E+05	2.400E+05	2.400E+05	2.400E+05	2.400E+05

Table D. Safety Indexagainst different strength classes of timber species for different dead to live-load range of values (MATLAB Program)

	Strength Classes of a Timber							
α	C14	C18	D30	D35	D40	D50	D60	D70
0.1	0.32	1.86	3.60	4.05	4.41	5.36	6.05	6.53
0.2	-0.36	1.26	3.10	3.59	3.97	5.00	5.71	6.22
0.3	-0.02	0.66	2.62	3.14	3.54	4.60	5.38	5.92
0.4	-1.65	0.08	2.14	2.69	3.11	4.22	5.05	5.61
0.5	-2.27	-0.49	1.66	2.24	2.68	3.85	4.71	5.31
0.6	-2.86	-1.04	1.19	1.81	2.26	3.48	4.38	5.00
0.7	-3.44	-1.58	0.73	1.37	1.85	3.11	4.05	4.70
0.8	-4.00	-2.10	0.28	0.95	1.44	2.74	3.72	4.40
0.9	-4.51	-2.61	-0.16	0.53	1.03	2.38	3.40	4.10
1.0	-5.02	-3.10	-0.59	0.11	0.63	2.02	3.08	3.80

Table E. Safety Indexagainst varying CFRP MOE values for different CFRP thicknesses for D30 timber strength class (MATLAB Program)

	Varying MOE of CFRP (kPa)									
t_{cfrp}	20	40	60	80	100	120	140	160	180	200
0.2	1.06	1.33	1.60	1.86	2.12	2.38	2.63	2.88	3.12	3.35
0.4	1.33	1.86	2.38	2.88	3.35	3.81	4.23	4.66	5.05	5.43
0.6	1.60	2.38	3.11	3.81	4.46	5.05	5.61	6.11	6.58	7.00
0.8	1.86	2.88	3.81	4.66	5.43	5.31	6.72	6.72	7.74	8.17
1.0	2.12	3.35	4.46	5.43	6.27	5.49	7.63	8.17	8.63	9.03

Table F. Safety Indexagainst varying CFRP MOE values for different CFRP thickness for C18 timber strength class (MATLAB Program)

	Varying MOE of CFRP (kPa)									
t_{cfrp}	20	40	60	80	100	120	140	160	180	200
0.2	-1.22	-0.87	-0.52	-0.18	0.16	0.49	0.81	1.13	1.44	1.74
0.4	-0.87	-0.18	0.49	1.13	1.74	2.33	2.88	3.40	3.89	4.35
0.6	-0.52	0.49	1.44	2.33	3.14	3.89	4.57	5.19	5.74	6.24
0.8	-0.18	1.13	2.33	2.39	3.37	5.19	5.92	6.55	7.10	7.59
1.0	0.16	1.74	3.14	4.35	5.38	6.24	6.97	6.82	8.11	8.55

Table G. Safety Indexagainst varying CFRP MOE values for different timber strength classes (MATLAB Program)

E_{frp}	Strength Classes of a Timber							
	C14	C18	D30	D35	D40	D50	D60	D70
20	-3.07	-1.22	1.06	1.68	2.14	3.38	4.29	4.92
40	-2.66	-0.87	1.33	1.93	2.38	3.58	4.47	5.08
60	-2.25	-0.52	1.60	2.18	2.61	3.78	4.64	5.24
80	-1.85	-0.18	1.86	2.42	2.84	3.97	4.81	5.39
100	-1.46	0.16	2.12	2.66	3.07	4.16	4.98	5.54
120	-1.07	0.49	2.38	2.90	3.29	4.35	5.14	5.69
140	-0.69	0.18	2.60	3.12	3.51	4.54	5.30	5.83
160	-0.31	1.13	2.88	3.35	3.73	4.72	5.46	6.00
180	0.05	1.44	3.12	3.58	3.94	4.90	5.62	6.12
200	0.41	1.74	3.35	3.79	4.14	5.07	5.77	6.26

Table H. Safety Indexagainst varying CFRP thicknesses for different timber strength classes (MATLAB Program)

t_{cfrp}	Strength Classes of a Timber							
	C14	C18	D30	D35	D40	D50	D60	D70
0.2	-2.86	-1.04	1.19	1.81	2.26	3.48	4.38	6.23
0.4	-2.25	-0.52	1.60	2.18	2.61	3.78	4.64	5.24
0.6	-1.65	-0.01	2.00	2.54	3.00	4.07	4.89	5.46
0.8	-1.07	0.49	1.47	2.90	3.29	4.35	5.14	5.69
1.0	-0.50	0.97	2.75	3.24	3.62	4.63	5.38	5.91

Table I. Safety Indexagainst varying CFRP MOE values for different dead to live-load range of values for D30 timber strength class (MATLAB Program)

α	Varying MOE of CFRP (kPa)									
	20	40	60	80	100	120	140	160	180	200
0.1	3.46	3.73	3.99	4.25	4.50	4.75	5.00	5.22	5.45	5.67
0.2	2.97	3.24	3.50	3.76	4.01	4.26	4.51	4.75	5.00	5.20
0.3	2.48	2.75	3.02	3.28	3.53	3.79	4.03	4.27	4.50	4.73
0.4	2.00	2.27	2.54	2.80	3.06	3.31	3.56	3.80	4.04	4.27
0.5	1.53	1.80	2.06	2.33	2.59	2.84	3.09	3.34	3.57	3.81
0.6	1.06	1.33	1.60	1.86	2.12	2.38	2.63	2.88	3.12	3.35
0.7	0.60	0.87	1.14	1.40	1.67	1.92	2.18	2.42	2.67	2.90
0.8	0.15	0.42	0.69	0.95	1.22	1.47	1.73	2.00	2.22	2.46
0.9	-0.30	-0.02	0.25	0.51	0.77	1.03	1.29	1.54	1.78	2.03
1.0	-0.73	-0.46	-0.19	0.08	0.34	0.60	0.19	1.11	1.35	1.60

Table J. Safety Indexagainst varying CFRP MOE values for different dead to live-load range of values for C18 timber strength class (MATLAB Program)

	Varying MOE of CFRP (kPa)									
α	20	40	60	80	100	120	140	160	180	200
0.1	1.69	2.04	2.38	2.72	3.05	3.37	3.68	3.98	4.27	4.56
0.2	1.08	1.43	1.78	2.11	2.45	2.77	3.08	3.39	3.69	4.00
0.3	0.48	0.84	1.18	1.52	1.86	2.18	2.50	2.81	3.11	3.40
0.4	-0.10	0.25	0.60	0.94	1.28	1.61	1.93	2.24	2.54	2.84
0.5	-0.66	-0.31	0.03	0.38	0.71	1.04	1.36	1.68	2.00	2.29
0.6	-1.22	-0.87	-0.52	-0.18	0.16	0.49	0.81	1.13	1.44	1.74
0.7	-1.75	-1.40	-1.06	-0.72	-0.40	-0.05	0.28	0.60	0.91	1.21
0.8	-2.28	-1.93	-1.58	-1.24	-0.90	-0.57	-0.25	0.07	0.39	0.69
0.9	-2.78	-2.43	-2.09	-1.75	-1.41	-1.08	-0.76	-0.44	-0.12	0.19
1.0	-3.27	-2.93	-2.58	-2.24	-1.91	-1.58	-1.25	-0.93	-0.62	-0.31

Table K. Safety Indexagainst CFRP laminate thickness values for different dead to live-load range of values for D30 timber strength class (MATLAB Program)

	Thicknesses of CFRP laminate (mm)				
α	0.2	0.4	0.6	0.8	1.0
0.1	3.60	3.99	4.38	4.75	5.11
0.2	3.10	3.50	3.89	4.26	4.63
0.3	2.62	3.02	3.41	3.79	4.15
0.4	2.14	2.54	2.93	3.31	3.68
0.5	1.66	2.06	2.46	2.84	3.21
0.6	1.19	1.59	1.99	2.38	2.75
0.7	0.73	1.14	1.54	1.92	2.23
0.8	0.28	0.68	1.09	1.47	1.85
0.9	-0.16	0.25	0.64	1.03	1.41
1.0	-0.59	-0.19	0.21	0.60	0.98

Table L. Safety Indexagainst CFRP laminate thickness values for different dead to live-load range of values for C18 timber strength class (MATLAB Program)

	Thicknesses of CFRP laminate (mm)				
α	0.2	0.4	0.6	0.8	1.0
0.1	1.86	2.38	2.88	3.37	3.83
0.2	1.26	1.78	2.28	2.77	3.24
0.3	0.66	1.18	1.69	2.18	2.66
0.4	0.08	0.60	1.11	1.61	2.08
0.5	-0.49	0.03	0.55	1.04	1.52
0.6	-1.04	-0.52	-0.01	0.49	0.97
0.7	-1.58	-1.06	-0.55	-0.05	0.44
0.8	-2.10	-1.58	-1.07	-0.57	-0.09
0.9	-2.61	-2.09	-1.58	-1.08	-0.60
1.0	-3.10	-2.58	-2.08	-1.58	-1.09

Fixed - Free Stresses for Three Different Timber Species for Unstrengthen Columns.

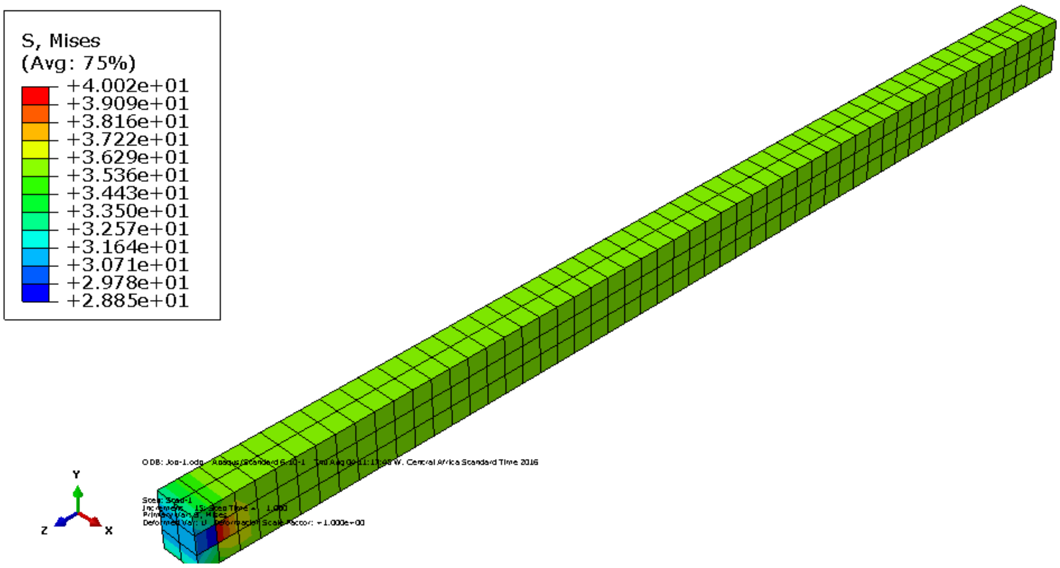

Figure 1: Stress Distribution Model for Abura Timber Column Species for Unstrengthen Timber Column (Fixed-Free) with Axial Applied Load of 36kN using (ABAQUS/CAE, 6.10)

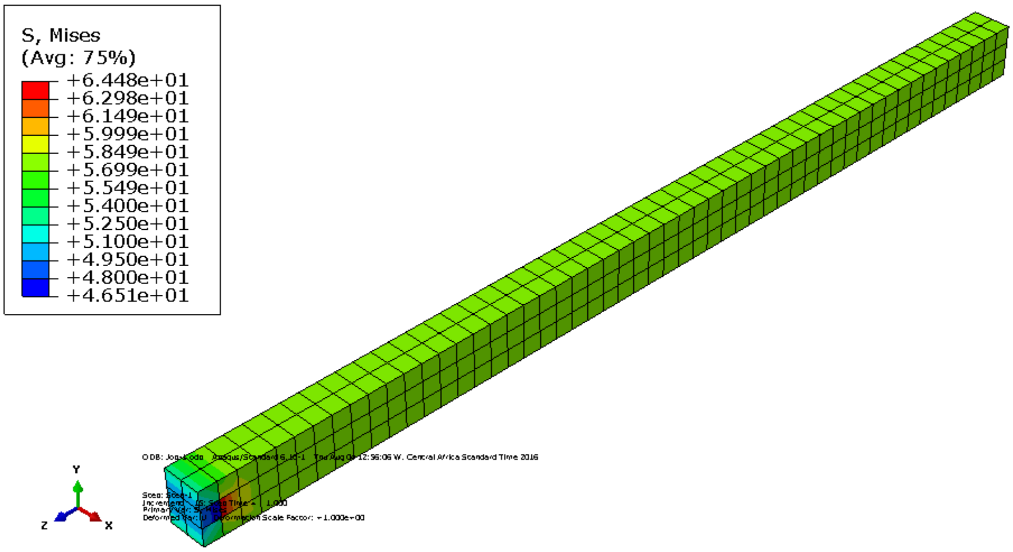

Figure 2: Stress Distribution Model for Afromosia Timber Species for Unstrengthen Timber Column (Fixed-Free) with Axial Applied Load of 58kN using (ABAQUS/CAE, 6.10)

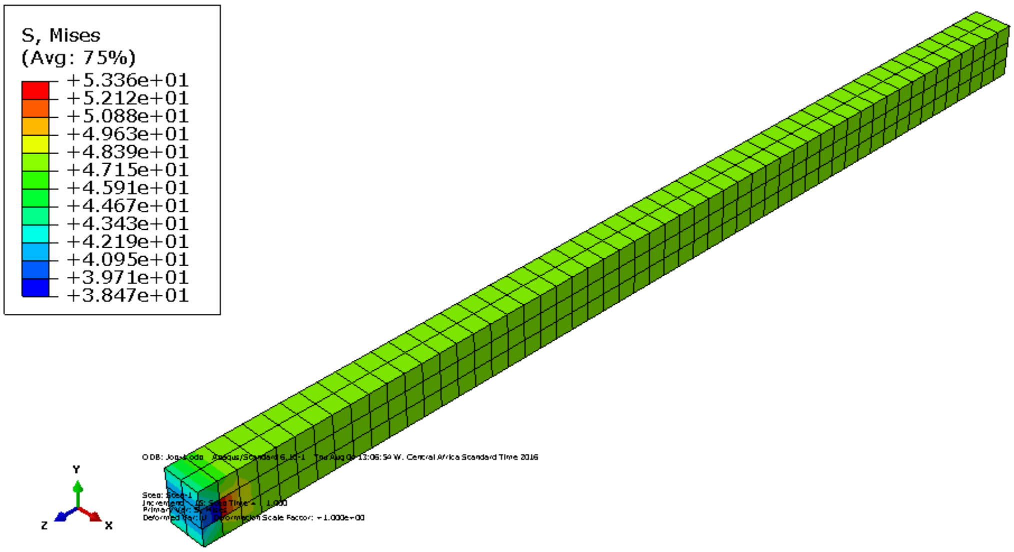

Figure 3: Stress Distribution Model for Confusa Timber Species for Unstrengthen Timber Column (Fixed-Free) with Axial Applied Load of 49kN using (ABAQUS/CAE, 6.10)

Fixed - Free Displacements for Three Different Timber Species for Unstrengthen Timber Columns.

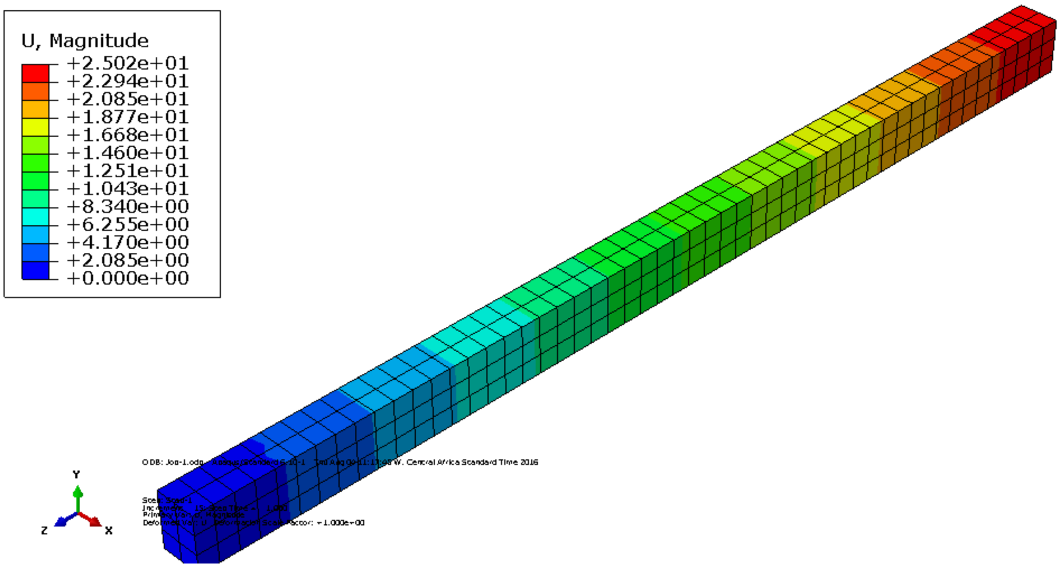

Figure 4: Displacement Distribution Model for Abura Timber Species for Unstrengthen Timber Column (Fixed-Free) with Axial Applied Load of 36kN using (ABAQUS/CAE, 6.10)

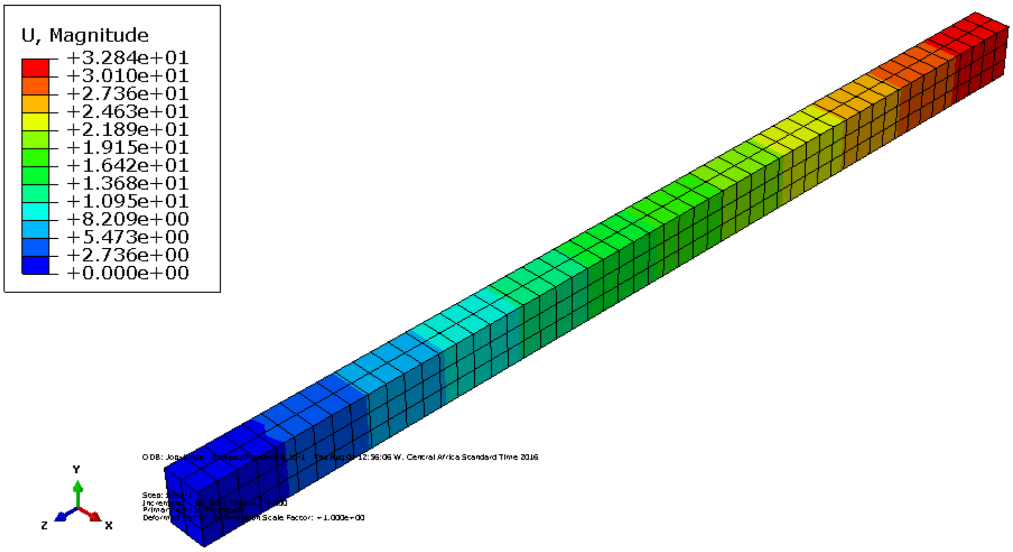

Figure 5: Displacement Distribution Model for Afromosia Timber Species for Unstrengthen Timber Column Fixed-Free with Axial Applied Load of 58kN using (ABAQUS/CAE, 6.10)

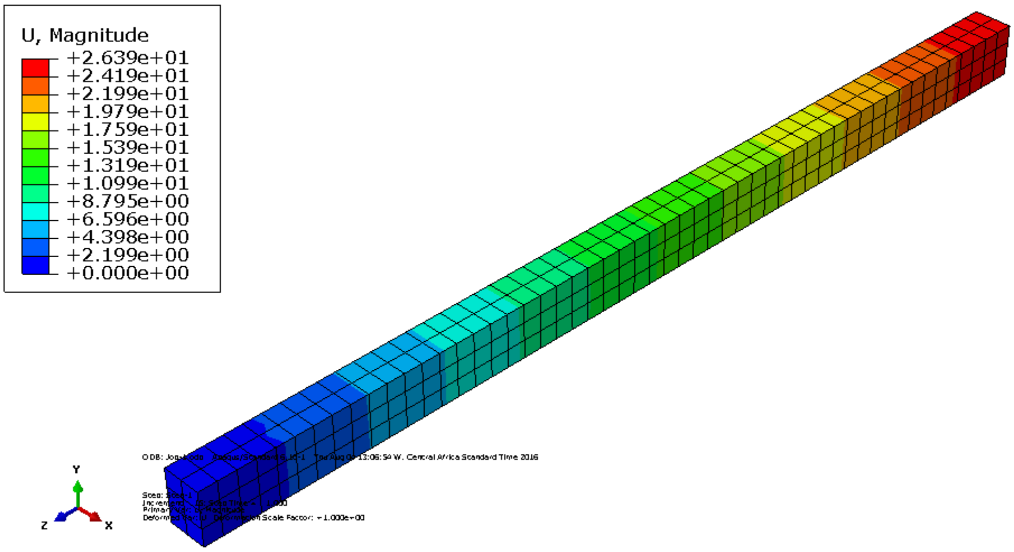

Figure 6: Displacement Distribution Model for Confusa Timber Species for Unstrengthen Timber Column (Fixed-Free)with Axial Applied Load of 48kN using (ABAQUS/CAE, 6.10)

Fixed - Free Reactions for Three Different Timber Species for Unstrengthen Timber Columns

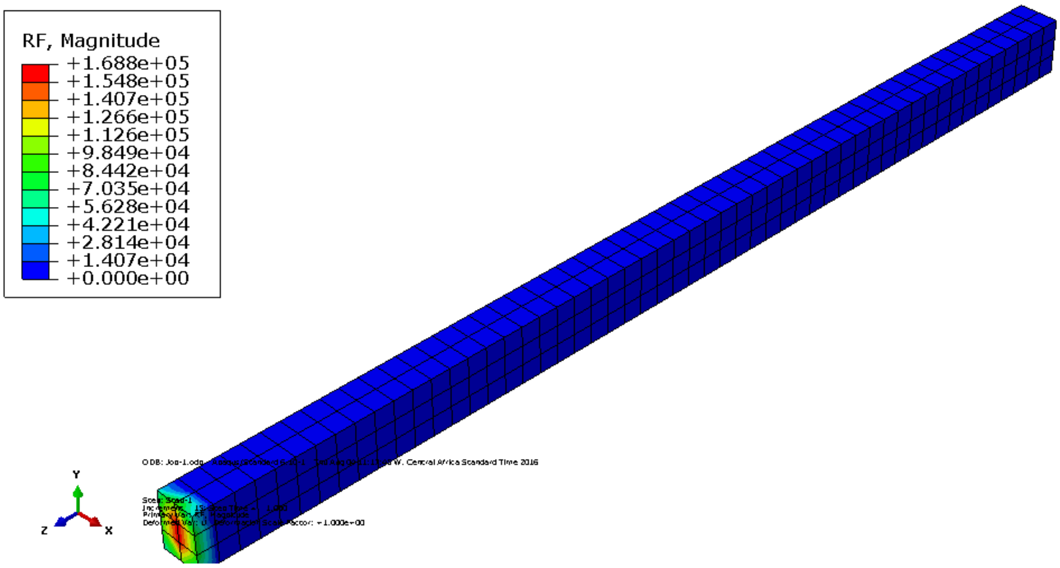

Figure 7: Reaction Distribution Model for Abura Timber Species for Unstrengthen Timber Column (Fixed-Free)with Axial Applied Load of 36kNusing (ABAQUS/CAE, 6.10)

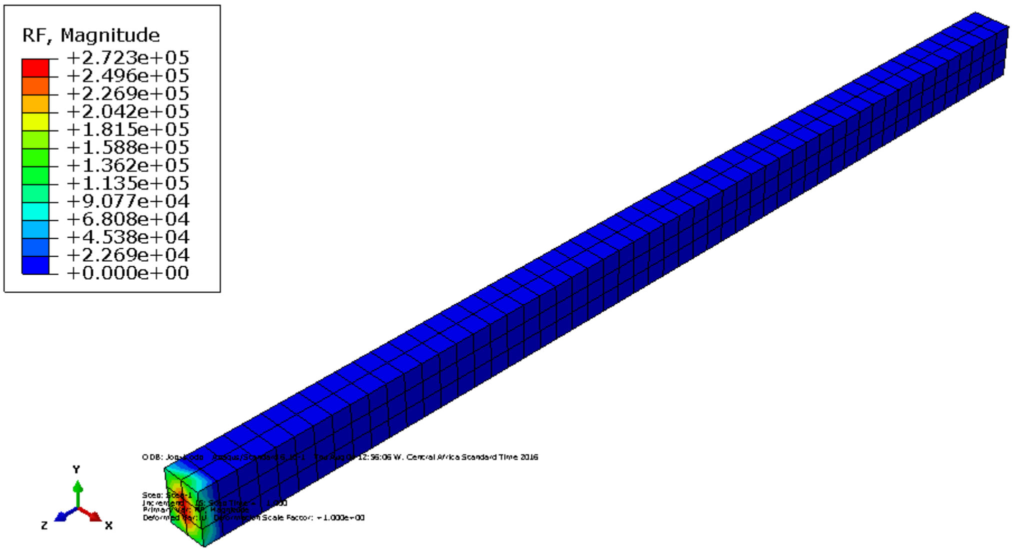

Figure 8: Reaction Distribution Model for Afromosia Timber Species for Unstrengthen Timber Column (Fixed-Free) with Axial Applied Load of 58kN using (ABAQUS/CAE, 6.10)

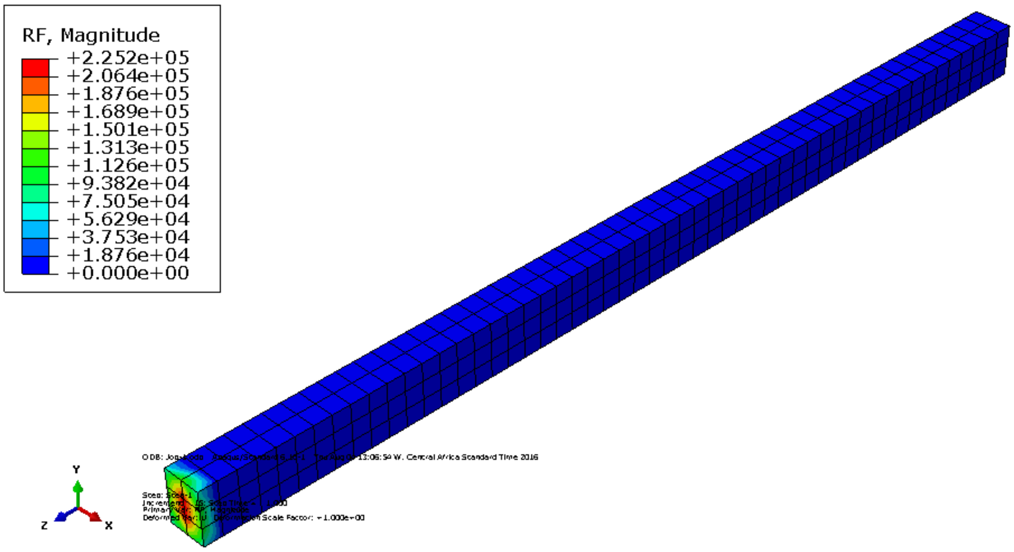

Figure 9: Reaction Distribution Model for Confusa Timber Species for Unstrengthen Timber Column (Fixed-Free) with Axial Applied Load of 48kN using (ABAQUS/CAE, 6.10)

Fixed - Fixed Stresses for Three Different Timber Species for Strengthen Column

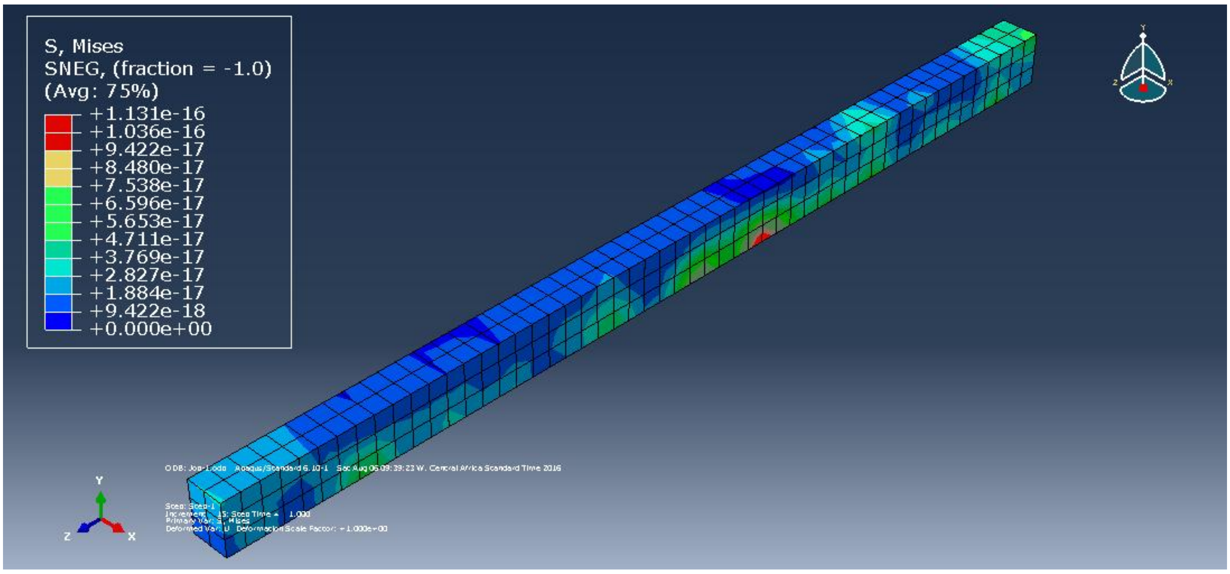

Figure 10: Stress Distribution Model for Abura Timber Species for Strengthen Timber Column (Fixed-Fixed) with Axial Applied Load of 36kNand 0.2mmCFRP Laminates using (ABAQUS/CAE, 6.10)

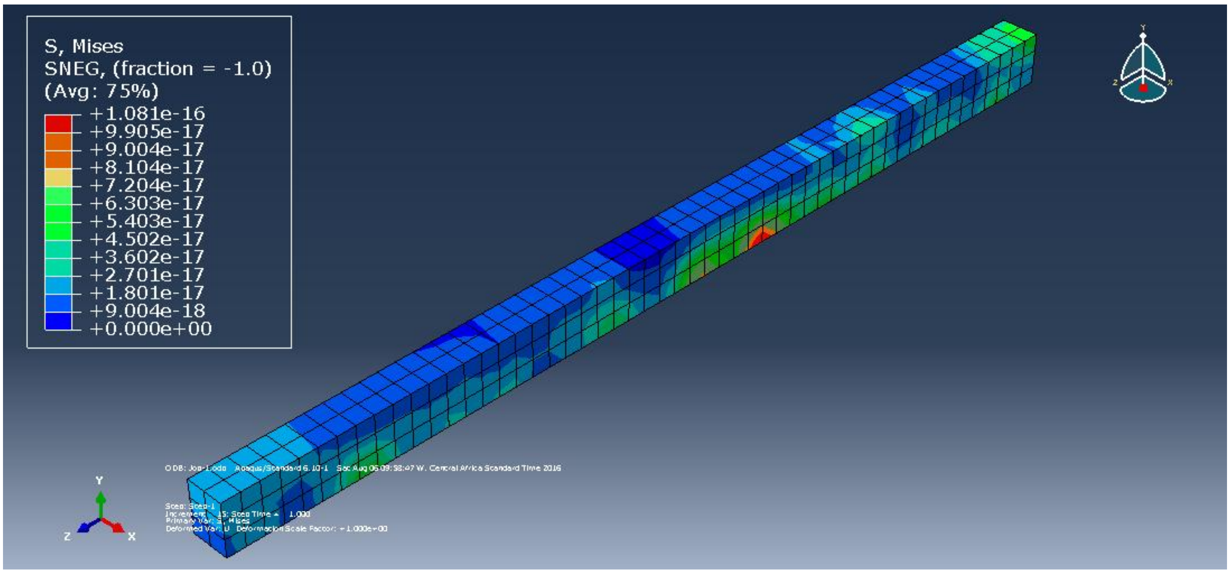

Figure 11: Stress Distribution Model for Afromosia Timber Species for Strengthen Timber Column (Fixed-Fixed) with Axial Applied Load of 58kN and 0.2mm CFRP Laminate using (ABAQUS/CAE, 6.10)

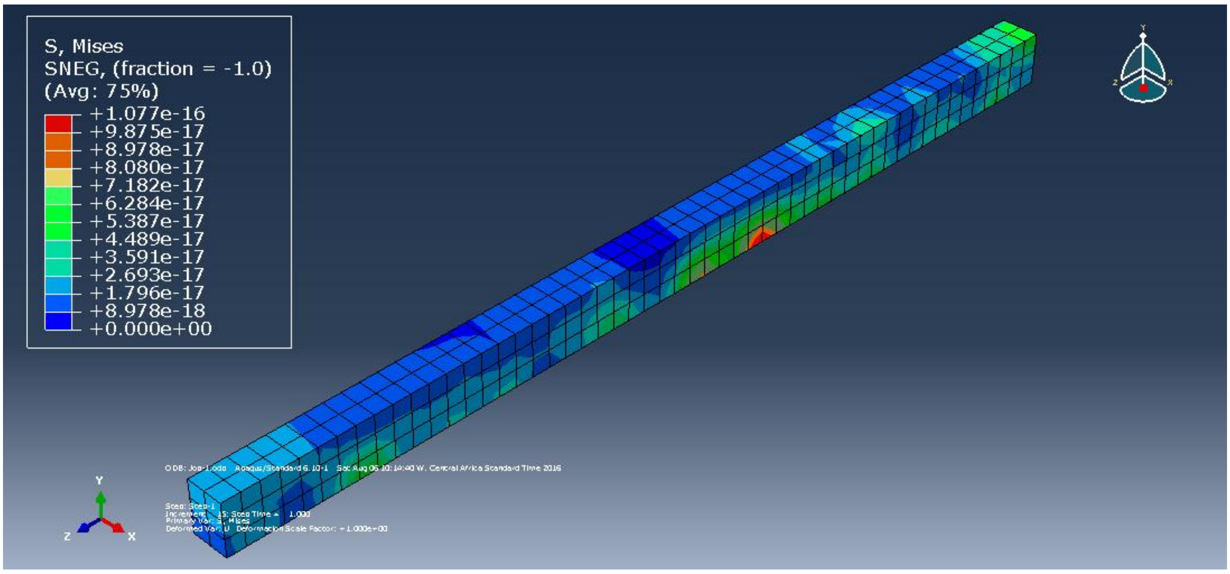

Figure 12: Stress Distribution Model for Confusa Timber Species for Strengthen Timber Column (Fixed-Fixed) with Axial Applied Load of 49kN and 0.2mm CFRP Laminate using (ABAQUS/CAE, 6.10)

Fixed - Fixed Displacements for Three Different Timber Species for Strengthen Timber Columns

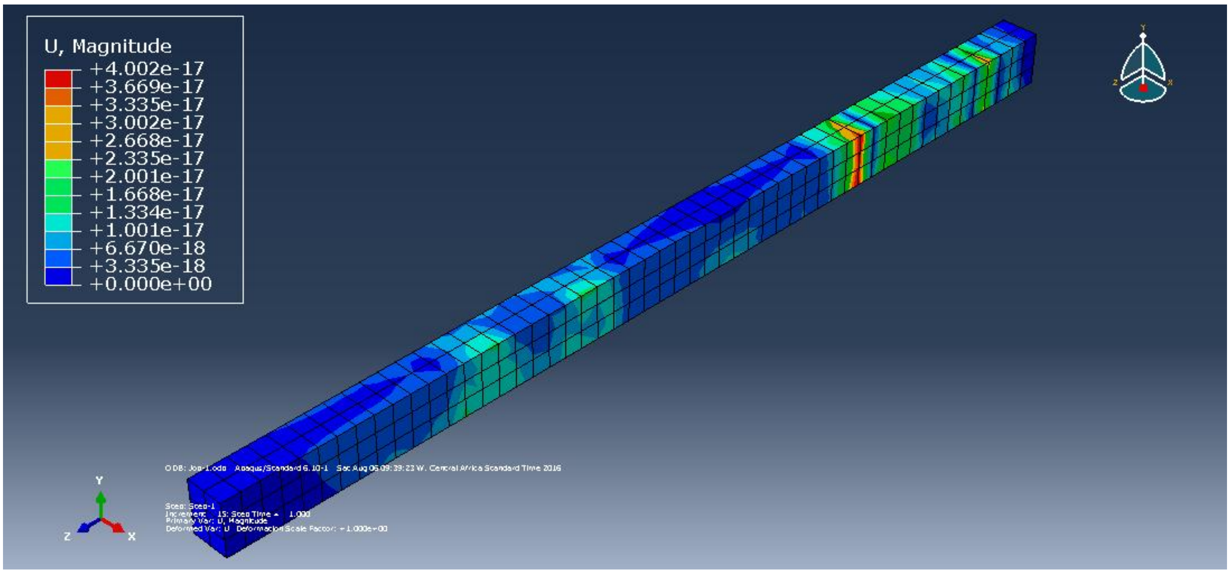

Figure 13: Displacement Distribution Model for Abura Timber Species for Strengthen Timber Column (Fixed-Fixed) with Axial Applied Load of 36kNand 0.2mm CFRP Laminate using (ABAQUS/CAE, 6.10)

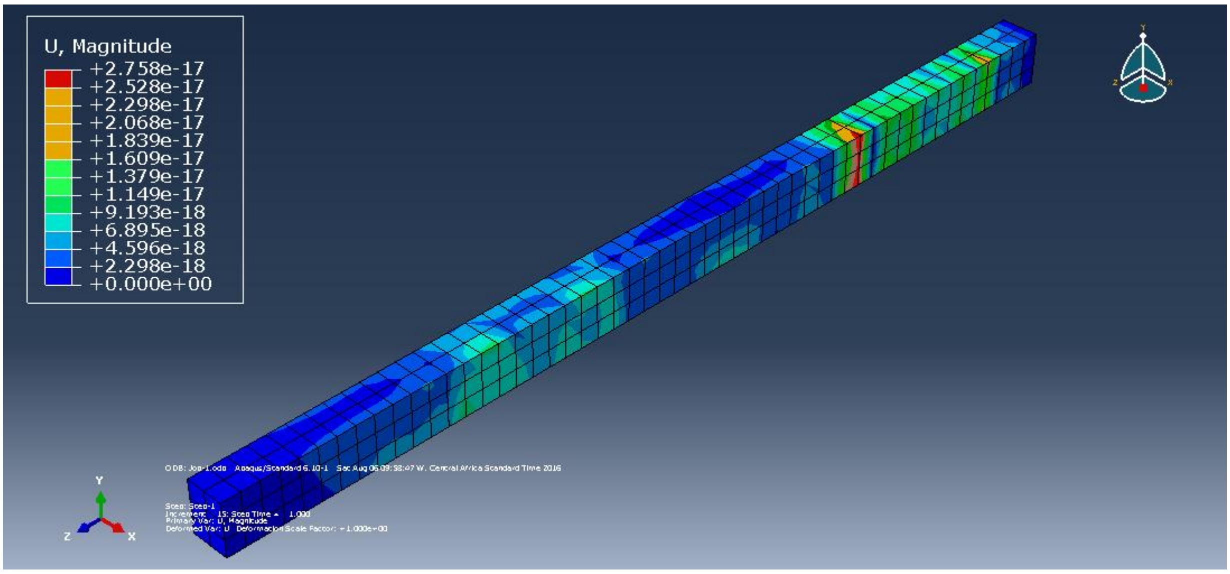

Figure 14: Displacement Distribution Model for Afromosia Timber Species for Strengthen Timber Column (Fixed-Fixed) with Axial Applied Load of 58kNand 0.2mm CFRP Laminate using (ABAQUS/CAE, 6.10)

\

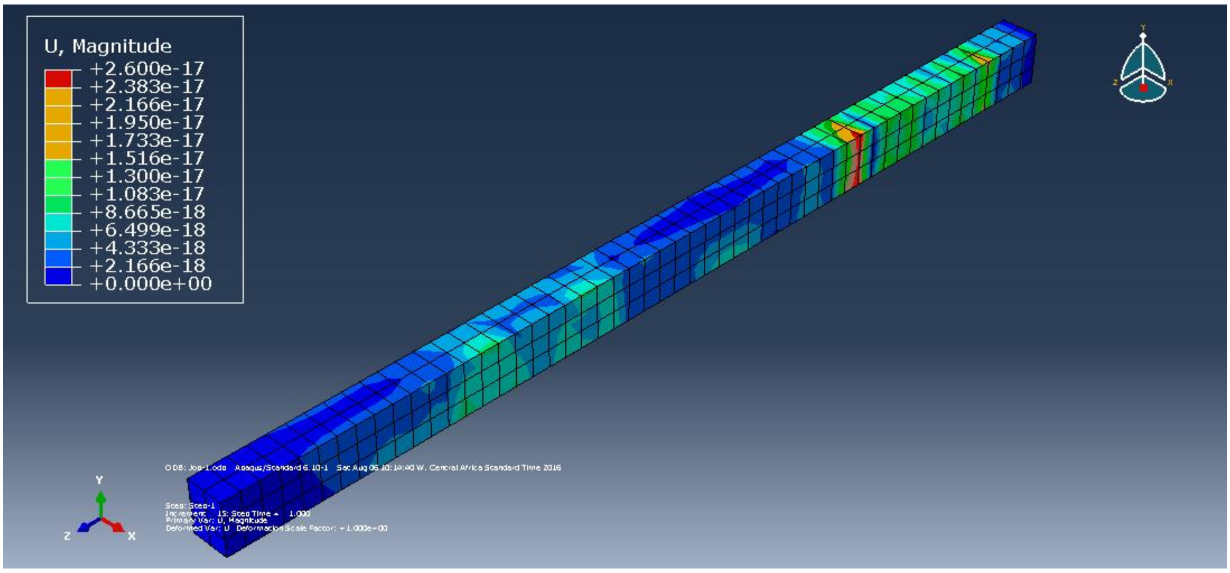

Figure 15: Displacement Distribution Model for Confusa Timber Species for Strengthen Timber Column (Fixed-Fixed) with Axial Applied Load of 58kNand 0.2mm CFRP Laminate using (ABAQUS/CAE, 6.10)

Fixed - Fixed Reactions for Three Different Timber Species for Strengthen Timber Columns.

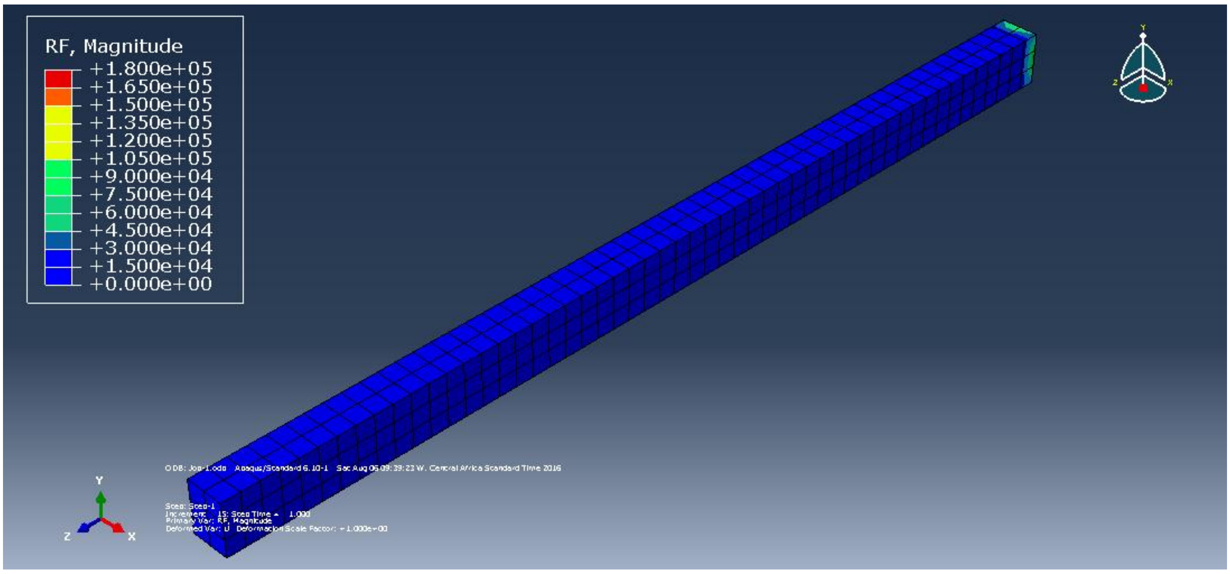

Figure 16: Reactions Distribution Model for Abura Timber Species for Strengthen Timber Column (Fixed-Free) with Axial Applied Load of 36kNand 0.2mm CFRP Laminate using (ABAQUS/CAE, 6.10)

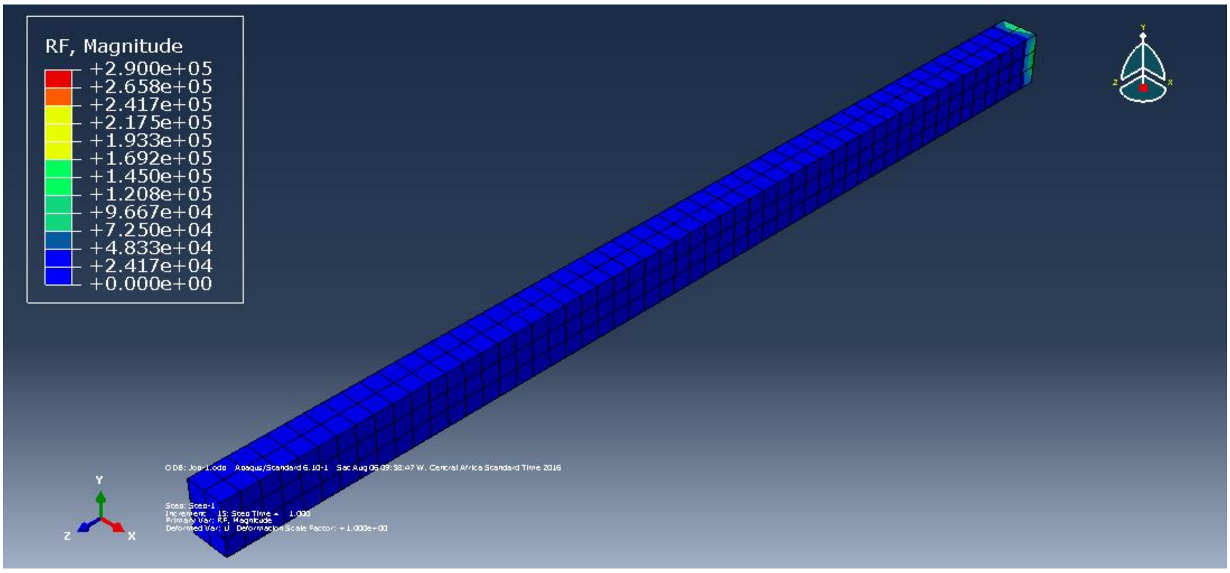

Figure 17: Reactions Distribution Model for Afromosia Timber Species for Strengthen Timber Column (Fixed-Fixed) with Axial Applied Load of 58kNand 0.2mm CFRP Laminate using (ABAQUS/CAE, 6.10)

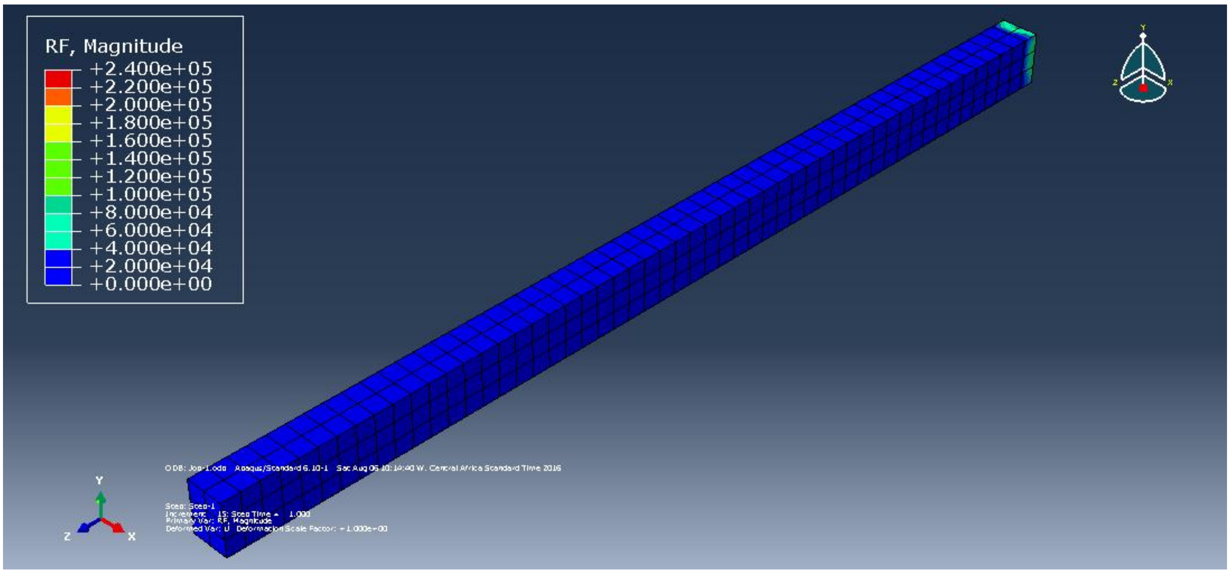

Figure 18: Reactions Distribution Model for Confusa Timber Species for Strengthen Timber Column (Fixed-Fixed) with Axial Applied Load of 49kNand 0.2mm CFRP Laminate using (ABAQUS/CAE, 6.10)

APPENDIX II

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
%
%                            THE MAIN PROGRAM
%
%
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
%
%  CODE DEVELOPED BY: ENGINEER Samuel Iliya Bajahry
%
%
%
%  SUPERVISORS
%
%  PROF O. S. Abejide
%
%  DR. J. M. Kaura
%
%
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%
%
disp('      ')
%
    clc
disp('MATERIAL PROPERTIES FOR SOLID TIMBER (SOFTWOOD):')
disp('ENTER 1 FOR STRENGTH CLASS C14 ACCORDING TO EN338, 2008')
disp('ENTER 2 FOR STRENGTH CLASS C16 ACCORDING TO EN338, 2008')
disp('ENTER 3 FOR STRENGTH CLASS C18 ACCORDING TO EN338, 2008')
disp('ENTER 4 FOR STRENGTH CLASS C20 ACCORDING TO EN338, 2008')
disp('ENTER 5 FOR STRENGTH CLASS C22 ACCORDING TO EN338, 2008')
disp('ENTER 6 FOR STRENGTH CLASS C24 ACCORDING TO EN338, 2008')
disp('ENTER 7 FOR STRENGTH CLASS C27 ACCORDING TO EN338, 2008')
disp('ENTER 8 FOR STRENGTH CLASS C30 ACCORDING TO EN338, 2008')
disp('ENTER 9 FOR STRENGTH CLASS C35 ACCORDING TO EN338, 2008')
disp('ENTER 10 FOR STRENGTH CLASS C40 ACCORDING TO EN338, 2008')
disp('ENTER 11 FOR STRENGTH CLASS C45 ACCORDING TO EN338, 2008')
disp('ENTER 12 FOR STRENGTH CLASS C50 ACCORDING TO EN338, 2008')
disp('        ')
disp('MATERIAL PROPERTIES FOR SOLID TIMBER (HARDWOOD):')
disp('ENTER 13 FOR STRENGTH CLASS D30 ACCORDING TO EN338, 2008')
disp('ENTER 14 FOR STRENGTH CLASS D35 ACCORDING TO EN338, 2008')
disp('ENTER 15 FOR STRENGTH CLASS D40 ACCORDING TO EN338, 2008')
disp('ENTER 16 FOR STRENGTH CLASS D50 ACCORDING TO EN338, 2008')
disp('ENTER 17 FOR STRENGTH CLASS C60 ACCORDING TO EN338, 2008')
disp('ENTER 18 FOR STRENGTH CLASS C70 ACCORDING TO EN338, 2008')
disp('        ')
class_2 = input('   SELECT OPTION FROM ABOVE');
[fmk,fcprk,fcpdk,ftprk,ftpdk,fvk,emean,ecen,eper,gmean,...
```

```
    rhok] = material_properties(class_2);
%
[q,gammag,gammaq,kmod,gammam,b,h,alphapc,Nb,strain]...
         = input_mode;
[cov1,cov2,cov3,cov4,cov5,cov6,cov7]...
    = covariance_mode;
disp('   ')
Efrp1 = input('   Input the elastic modulus of FRP in kPa');
Efrp = Efrp1*1000.0;
disp('   ')
tfrp = input('   Input thickness of FRP in mm');
disp('   ')
length = input('   Input column height in m');
disp('   ')
%
disp('    ')
alpha = input('   Input dead to live load ratio');
disp('    ')
%
disp('    ')
area = b*h; inertia = b*(h^3)/12; gyration = sqrt(inertia/area);
lambda = (length*1000.0)/gyration; lambdarel =
(lambda/3.142)*(sqrt(fcprk/ecen));
betanot = 0.2;
kz = 0.5*(1 + (betanot*(lambdarel - 0.3)) + lambdarel^2);
kc = 1.0/(kz + sqrt((kz^2) - (lambdarel^2)));
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%
beta_mode_1 = strengthened_column(alpha,q,gammag,gammaq,fcprk,kmod,...
              gammam,b,h,alphapc,Nb,Efrp,strain,tfrp,kc,...
         cov1,cov2,cov3,cov4,cov5,cov6,cov7);
pf1 = probability_of_failure(beta_mode_1);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%
disp('this is the safety index for failure mode 1')
disp('   ')
disp(beta_mode_1)
disp('   ')
disp('this is the probability of failure for failure mode 1')
disp('   ')
disp(pf1)
```

```
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%
function beta_mode_1 = strengthened_column(alpha,q,gammag,...
    gammaq,fcprk,kmod,...
              gammam,b,h,alphapc,Nb,Efrp,strain,tfrp,kc,...
         cov1,cov2,cov3,cov4,cov5,cov6,cov7)
%
cv(1) = cov1; cv(2) = cov2; cv(3) = cov3; cv(4) = cov4;
cv(5) = cov5; cv(6) = cov6;  cv(7) = cov7;
%
[DISTYPE1,DISTYPE2,DISTYPE3,DISTYPE4,DISTYPE5,DISTYPE6,DISTYPE7] ...
         = distribution;
%
rfmu(1) = fcprk;  rmfx1 = fcprk*cv(1);
rfmu(2) = kmod;   rmfx2 = kmod*cv(2);
rfmu(3) = b;      rmfx3 = b*cv(3);
rfmu(4) = h;      rmfx4 = h*cv(4);
rfmu(5) = Efrp;   rmfx5 = Efrp*cv(5);
rfmu(6) = tfrp;   rmfx6 = tfrp*cv(6);
rfmu(7) = q;      rmfx7 = q*cv(7);
%
if(DISTYPE1 == 1)
    %
    % DOING NORMAL
    %
    mean(1) = rfmu(1);
    sd(1) = rmfx1;
elseif(DISTYPE1 == 2)
    %
    % DOING LOGNORMAL TRANSFORMATION
    %
 [mean(1) sd(1)] = lognormal(rfmu(1),rmfx1);
elseif(DISTYPE1 == 3)
    %
    % DOING GUMBEL TRANSFORMATION
    %
 [mean(1) sd(1)] = gumbel(rfmu(1),rmfx1);
elseif(DISTYPE1 == 4)
    %
    % DOING WEIBULL TRANSFORMATION
    %
 [mean(1) sd(1)] = weibull(rfmu(1),rmfx1);
elseif(DISTYPE1 == 5)
    %
    % DOING FRECHET TRANSFORMATION
    %
 [mean(1) sd(1)] = frechet(rfmu(1),rmfx1);
end
if(DISTYPE2 == 1)
    %
    % DOING NORMAL
    %
    mean(2) = rfmu(2);
    sd(2) = rmfx2;
elseif(DISTYPE2 == 2)
    %
    % DOING LOGNORMAL TRANSFORMATION
```

```
    %
 [mean(2) sd(2)] = lognormal(rfmu(2),rmfx2);
elseif(DISTYPE2 == 3)
    %
    % DOING GUMBEL TRANSFORMATION
    %
 [mean(2) sd(2)] = gumbel(rfmu(2),rmfx2);
elseif(DISTYPE2 == 4)
    %
    % DOING WEIBULL TRANSFORMATION
    %
 [mean(2) sd(2)] = weibull(rfmu(2),rmfx2);
elseif(DISTYPE2 == 5)
    %
    % DOING FRECHET TRANSFORMATION
    %
 [mean(2) sd(2)] = frechet(rfmu(2),rmfx2);
end
if(DISTYPE3 == 1)
    %
    % DOING NORMAL
    %
    mean(3) = rfmu(3);
    sd(3) = rmfx3;
elseif(DISTYPE3 == 2)
    %
    % DOING LOGNORMAL TRANSFORMATION
    %
 [mean(3) sd(3)] = lognormal(rfmu(3),rmfx3);
elseif(DISTYPE3 == 3)
    %
    % DOING GUMBEL TRANSFORMATION
    %
 [mean(3) sd(3)] = gumbel(rfmu(3),rmfx3);
elseif(DISTYPE3 == 4)
    %
    % DOING WEIBULL TRANSFORMATION
    %
 [mean(3) sd(3)] = weibull(rfmu(3),rmfx3);
elseif(DISTYPE3 == 5)
    %
    % DOING FRECHET TRANSFORMATION
    %
 [mean(3) sd(3)] = frechet(rfmu(3),rmfx3);
end
if(DISTYPE4 == 1)
    %
    % DOING NORMAL
    %
    mean(4) = rfmu(4);
    sd(4) = rmfx4;
elseif(DISTYPE4 == 2)
    %
    % DOING LOGNORMAL TRANSFORMATION
    %
 [mean(4) sd(4)] = lognormal(rfmu(4),rmfx4);
elseif(DISTYPE4 == 3)
    %
```

```
    % DOING GUMBEL TRANSFORMATION
    %
 [mean(4) sd(4)] = gumbel(rfmu(4),rmfx4);
elseif(DISTYPE4 == 4)
    %
    % DOING WEIBULL TRANSFORMATION
    %
 [mean(4) sd(4)] = weibull(rfmu(4),rmfx4);
elseif(DISTYPE4 == 5)
    %
    % DOING FRECHET TRANSFORMATION
    %
 [mean(4) sd(4)] = frechet(rfmu(4),rmfx4);
end
if(DISTYPE5 == 1)
    %
    % DOING NORMAL
    %
    mean(5) = rfmu(5);
    sd(5) = rmfx5;
elseif(DISTYPE5 == 2)
    %
    % DOING LOGNORMAL TRANSFORMATION
    %
 [mean(5) sd(5)] = lognormal(rfmu(5),rmfx5);
elseif(DISTYPE5 == 3)
    %
    % DOING GUMBEL TRANSFORMATION
    %
 [mean(5) sd(5)] = gumbel(rfmu(5),rmfx5);
 elseif(DISTYPE5 == 4)
    %
    % DOING WEIBULL TRANSFORMATION
    %
 [mean(5) sd(5)] = weibull(rfmu(5),rmfx5);
elseif(DISTYPE5 == 5)
    %
    % DOING FRECHET TRANSFORMATION
    %
 [mean(5) sd(5)] = frechet(rfmu(5),rmfx5);
end
if(DISTYPE6 == 1)
    %
    % DOING NORMAL
    %
    mean(6) = rfmu(6);
    sd(6) = rmfx6;
elseif(DISTYPE6 == 2)
    %
    % DOING LOGNORMAL TRANSFORMATION
    %
 [mean(6) sd(6)] = lognormal(rfmu(6),rmfx6);
elseif(DISTYPE6 == 3)
    %
    % DOING GUMBEL TRANSFORMATION
    %
 [mean(6) sd(6)] = gumbel(rfmu(6),rmfx6);
 elseif(DISTYPE6 == 4)
```

```
    %
    % DOING WEIBULL TRANSFORMATION
    %
 [mean(6) sd(6)] = weibull(rfmu(6),rmfx6);
elseif(DISTYPE6 == 5)
    %
    % DOING FRECHET TRANSFORMATION
    %
 [mean(6) sd(6)] = frechet(rfmu(6),rmfx6);
end
if(DISTYPE7 == 1)
    %
    % DOING NORMAL
    %
    mean(7) = rfmu(7);
    sd(7) = rmfx7;
elseif(DISTYPE7 == 2)
    %
    % DOING LOGNORMAL TRANSFORMATION
    %
 [mean(7) sd(7)] = lognormal(rfmu(7),rmfx7);
elseif(DISTYPE5 == 3)
    %
    % DOING GUMBEL TRANSFORMATION
    %
 [mean(7) sd(7)] = gumbel(rfmu(7),rmfx7);
 elseif(DISTYPE7 == 4)
    %
    % DOING WEIBULL TRANSFORMATION
    %
 [mean(7) sd(7)] = weibull(rfmu(7),rmfx7);
elseif(DISTYPE7 == 7)
    %
    % DOING FRECHET TRANSFORMATION
    %
 [mean(7) sd(7)] = frechet(rfmu(7),rmfx7);
end
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%
sev1 = sd(1); sev2 = sd(2); sev3 = sd(3); sev4 = sd(4);
sev5 = sd(5); sev6 = sd(6); sev7 = sd(7);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%
dev1 = mean(1); dev2 = mean(2); dev3 = mean(3); dev4 = mean(4);
dev5 = mean(5); dev6 = mean(6); dev7 = mean(7);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%
nume = ((kc*dev1*dev2*dev3*dev4/gammam)+  ...
        2.0*alphapc*Nb*dev5*strain*dev6*(dev3 + dev4)) - ...
```

```
        (dev7*(gammag*alpha + gammaq)*1000.0);
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
ca1 = 1; ca2 = 1; ca3 = 1; ca4 = 1; ca5 = 1; ca6 = 1; ca7 = 1;
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
if (ca1 == 1)
part1 = dev2*dev3*dev4/gammam;
else
    part1 = 0;
end
%
if(ca2 == 1)
part2 = kc*dev1*dev3*dev4/gammam;
else
    part2 = 0;
end
%
if(ca3 == 1)
part3 = (kc*dev1*dev2*dev4/gammam) + (2.0*alphapc*Nb*dev5*strain*dev6);
else
    part3 = 0;
end
%
if(ca4 == 1)
part4 = (kc*dev1*dev2*dev3/gammam) + (2.0*alphapc*Nb*dev5*strain*dev6);
else
    part4 = 0;
end
%
if(ca5 == 1)
part5 = (2.0*alphapc*Nb*strain*dev6)*(dev3 + dev4);
else
    part5 = 0;
end
%
%
if(ca6 == 1)
part6 = (2.0*alphapc*Nb*strain*dev5)*(dev3 + dev4);
else
    part6 = 0;
end
%
if(ca7 == 1)
part7 = -(gammag*alpha + gammaq)*1000.0;
else
    part7 = 0;
end
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%
deno = sqrt(((part1^2)*(sev1^2)) + ((part2^2)*(sev2^2)) + ...
    ((part3^2)*(sev3^2)) + ((part4^2)*(sev4^2)) + ...
    ((part5^2)*(sev5^2)) + ((part6^2)*(sev6^2)) + ...
    ((part7^2)*(sev7^2)));
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%
%
beta_mode_1 = nume/deno;
%
```

```
%
%  SUBROUTINE NUMBER 1
%
function pf = probability_of_failure(beta)
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%   REFERENCE: Melchers R. E. Structural Reliability analysis
%              and prediction. 1999. Ellis Horwood Limited
%              John Wlley and Sons. pp 372 to 374
%
% COMPLEMENTARY STANDARD NORMAL ROUTINE
%
if(beta <= 0)
    pf = 0.5;
elseif(beta > 0) && (beta <= 0.01)
    pf = 0.4960;
elseif(beta > 0.01) && (beta <= 0.02)
    pf = 0.4920;
elseif(beta > 0.02) && (beta <= 0.03)
    pf = 0.4880;
elseif(beta > 0.03) && (beta <= 0.04)
    pf = 0.4841;
elseif(beta > 0.04) && (beta <= 0.05)
    pf = 0.4801;
elseif(beta > 0.05) && (beta <= 0.06)
    pf = 0.4761;
elseif(beta > 0.06) && (beta <= 0.07)
    pf = 0.4721;
elseif(beta > 0.07) && (beta <= 0.08)
    pf = 0.4681;
elseif(beta > 0.08) && (beta <= 0.09)
    pf = 0.4642;
elseif(beta > 0.09) && (beta <= 0.10)
    pf = 0.4602;
elseif(beta > 0.10) && (beta <= 0.11)
    pf = 0.4562;
elseif(beta > 0.11) && (beta <= 0.12)
    pf = 0.4522;
elseif(beta > 0.12) && (beta <= 0.13)
    pf = 0.4483;
elseif(beta > 0.13) && (beta <= 0.14)
    pf = 0.4443;
elseif(beta > 0.14) && (beta <= 0.15)
    pf = 0.4404;
elseif(beta > 0.15) && (beta <= 0.16)
    pf = 0.4364;
elseif(beta > 0.16) && (beta <= 0.17)
    pf = 0.4325;
elseif(beta > 0.17) && (beta <= 0.18)
    pf = 0.4286;
elseif(beta > 0.18) && (beta <= 0.19)
    pf = 0.4247;
elseif(beta > 0.19) && (beta <= 0.20)
    pf = 0.4207;
elseif(beta > 0.20) && (beta <= 0.21)
    pf = 0.4168;
elseif(beta > 0.21) && (beta <= 0.22)
```

```
        pf = 0.4129;
    elseif(beta > 0.22) && (beta <= 0.23)
        pf = 0.4091;
    elseif(beta > 0.23) && (beta <= 0.24)
        pf = 0.4052;
    elseif(beta > 0.24) && (beta <= 0.25)
        pf = 0.4013;
    elseif(beta > 0.25) && (beta <= 0.26)
        pf = 0.3974;
    elseif(beta > 0.26) && (beta <= 0.27)
        pf = 0.3936;
    elseif(beta > 0.27) && (beta <= 0.28)
        pf = 0.3897;
    elseif(beta > 0.28) && (beta <= 0.29)
        pf = 0.3859;
    elseif(beta > 0.29) && (beta <= 0.30)
        pf = 0.3821;
    elseif(beta > 0.30) && (beta <= 0.31)
        pf = 0.3783;
    elseif(beta > 0.31) && (beta <= 0.32)
        pf = 0.3745;
    elseif(beta > 0.32) && (beta <= 0.33)
        pf = 0.3707;
    elseif(beta > 0.33) && (beta <= 0.34)
        pf = 0.3669;
    elseif(beta > 0.34) && (beta <= 0.35)
        pf = 0.3632;
    elseif(beta > 0.35) && (beta <= 0.36)
        pf = 0.3594;
    elseif(beta > 0.36) && (beta <= 0.37)
        pf = 0.3557;
    elseif(beta > 0.37) && (beta <= 0.38)
        pf = 0.3520;
    elseif(beta > 0.38) && (beta <= 0.39)
        pf = 0.3483;
    elseif(beta > 0.39) && (beta <= 0.40)
        pf = 0.3446;
    elseif(beta > 0.40) && (beta <= 0.41)
        pf = 0.3409;
    elseif(beta > 0.41) && (beta <= 0.42)
        pf = 0.3372;
    elseif(beta > 0.42) && (beta <= 0.43)
        pf = 0.3338;
    elseif(beta > 0.43) && (beta <= 0.44)
        pf = 0.3300;
    elseif(beta > 0.44) && (beta <= 0.45)
        pf = 0.3264;
    elseif(beta > 0.45) && (beta <= 0.46)
        pf = 0.3228;
    elseif(beta > 0.46) && (beta <= 0.47)
        pf = 0.3192;
    elseif(beta > 0.47) && (beta <= 0.48)
        pf = 0.3156;
    elseif(beta > 0.48) && (beta <= 0.49)
        pf = 0.3121;
    elseif(beta > 0.49) && (beta <= 0.50)
        pf = 0.3085;
    elseif(beta > 0.50) && (beta <= 0.51)
```

```
    pf = 0.3050;
elseif(beta > 0.51) && (beta <= 0.52)
    pf = 0.3015;
elseif(beta > 0.52) && (beta <= 0.53)
    pf = 0.2981;
elseif(beta > 0.53) && (beta <= 0.54)
    pf = 0.2948;
elseif(beta > 0.54) && (beta <= 0.55)
    pf = 0.2912;
elseif(beta > 0.55) && (beta <= 0.56)
    pf = 0.2877;
elseif(beta > 0.56) && (beta <= 0.57)
    pf = 0.2843;
elseif(beta > 0.57) && (beta <= 0.58)
    pf = 0.2810;
elseif(beta > 0.58) && (beta <= 0.59)
    pf = 0.2776;
elseif(beta > 0.59) && (beta <= 0.60)
    pf = 0.2743;
elseif(beta > 0.60) && (beta <= 0.61)
    pf = 0.2709;
elseif(beta > 0.61) && (beta <= 0.62)
    pf = 0.2676;
elseif(beta > 0.62) && (beta <= 0.63)
    pf = 0.2644;
elseif(beta > 0.63) && (beta <= 0.64)
    pf = 0.2611;
elseif(beta > 0.64) && (beta <= 0.65)
    pf = 0.2579;
elseif(beta > 0.65) && (beta <= 0.66)
    pf = 0.2546;
elseif(beta > 0.66) && (beta <= 0.67)
    pf = 0.2514;
elseif(beta > 0.67) && (beta <= 0.68)
    pf = 0.2483;
elseif(beta > 0.68) && (beta <= 0.69)
    pf = 0.2451;
elseif(beta > 0.69) && (beta <= 0.70)
    pf = 0.2420;
elseif(beta > 0.70) && (beta <= 0.71)
    pf = 0.2389;
elseif(beta > 0.71) && (beta <= 0.72)
    pf = 0.2358;
elseif(beta > 0.72) && (beta <= 0.73)
    pf = 0.2327;
elseif(beta > 0.73) && (beta <= 0.74)
    pf = 0.2297;
elseif(beta > 0.74) && (beta <= 0.75)
    pf = 0.2266;
elseif(beta > 0.75) && (beta <= 0.76)
    pf = 0.2236;
elseif(beta > 0.76) && (beta <= 0.77)
    pf = 0.2207;
elseif(beta > 0.77) && (beta <= 0.78)
    pf = 0.2177;
elseif(beta > 0.78) && (beta <= 0.79)
    pf = 0.2148;
elseif(beta > 0.79) && (beta <= 0.80)
```

```
        pf = 0.2119;
    elseif(beta > 0.80) && (beta <= 0.81)
        pf = 0.2090;
    elseif(beta > 0.81) && (beta <= 0.82)
        pf = 0.2061;
    elseif(beta > 0.82) && (beta <= 0.83)
        pf = 0.2033;
    elseif(beta > 0.83) && (beta <= 0.84)
        pf = 0.2005;
    elseif(beta > 0.84) && (beta <= 0.85)
        pf = 0.1977;
    elseif(beta > 0.85) && (beta <= 0.86)
        pf = 0.1949;
    elseif(beta > 0.86) && (beta <= 0.87)
        pf = 0.1922;
    elseif(beta > 0.87) && (beta <= 0.88)
        pf = 0.1894;
    elseif(beta > 0.88) && (beta <= 0.89)
        pf = 0.1867;
    elseif(beta > 0.89) && (beta <= 0.90)
        pf = 0.1841;
    elseif(beta > 0.90) && (beta <= 0.91)
        pf = 0.1814;
    elseif(beta > 0.91) && (beta <= 0.92)
        pf = 0.1788;
    elseif(beta > 0.92) && (beta <= 0.93)
        pf = 0.1762;
    elseif(beta > 0.93) && (beta <= 0.94)
        pf = 0.1736;
    elseif(beta > 0.94) && (beta <= 0.95)
        pf = 0.1711;
    elseif(beta > 0.95) && (beta <= 0.96)
        pf = 0.1685;
    elseif(beta > 0.96) && (beta <= 0.97)
        pf = 0.1660;
    elseif(beta > 0.97) && (beta <= 0.98)
        pf = 0.1635;
    elseif(beta > 0.98) && (beta <= 0.99)
        pf = 0.1611;
    elseif(beta > 0.99) && (beta <= 1.00)
        pf = 0.1587;
    elseif(beta > 1.00) && (beta <= 1.01)
        pf = 0.1563;
    elseif(beta > 1.01) && (beta <= 1.02)
        pf = 0.1539;
    elseif(beta > 1.02) && (beta <= 1.03)
        pf = 0.1515;
    elseif(beta > 1.03) && (beta <= 1.04)
        pf = 0.1492;
    elseif(beta > 1.04) && (beta <= 1.05)
        pf = 0.1469;
    elseif(beta > 1.05) && (beta <= 1.06)
        pf = 0.1446;
    elseif(beta > 1.06) && (beta <= 1.07)
        pf = 0.1423;
    elseif(beta > 1.07) && (beta <= 1.08)
        pf = 0.1401;
    elseif(beta > 1.08) && (beta <= 1.09)
```

```
    pf = 0.1379;
elseif(beta > 1.09) && (beta <= 1.10)
    pf = 0.1357;
elseif(beta > 1.10) && (beta <= 1.11)
    pf = 0.1335;
elseif(beta > 1.11) && (beta <= 1.12)
    pf = 0.1314;
elseif(beta > 1.12) && (beta <= 1.13)
    pf = 0.1292;
elseif(beta > 1.13) && (beta <= 1.14)
    pf = 0.1271;
elseif(beta > 1.14) && (beta <= 1.15)
    pf = 0.1251;
elseif(beta > 1.15) && (beta <= 1.16)
    pf = 0.1230;
elseif(beta > 1.16) && (beta <= 1.17)
    pf = 0.1210;
elseif(beta > 1.17) && (beta <= 1.18)
    pf = 0.1190;
elseif(beta > 1.18) && (beta <= 1.19)
    pf = 0.1170;
elseif(beta > 1.19) && (beta <= 1.20)
    pf = 0.1151;
elseif(beta > 1.20) && (beta <= 1.21)
    pf = 0.1131;
elseif(beta > 1.21) && (beta <= 1.22)
    pf = 0.1112;
elseif(beta > 1.22) && (beta <= 1.23)
    pf = 0.1094;
elseif(beta > 1.23) && (beta <= 1.24)
    pf = 0.1075;
elseif(beta > 1.24) && (beta <= 1.25)
    pf = 0.1057;
elseif(beta > 1.25) && (beta <= 1.26)
    pf = 0.1038;
elseif(beta > 1.26) && (beta <= 1.27)
    pf = 0.1020;
elseif(beta > 1.27) && (beta <= 1.28)
    pf = 0.1003;
elseif(beta > 1.28) && (beta <= 1.29)
    pf = 0.9853e-01;
elseif(beta > 1.29) && (beta <= 1.30)
    pf = 0.968e-01;
elseif(beta > 1.30) && (beta <= 1.31)
    pf = 0.9510e-01;
elseif(beta > 1.31) && (beta <= 1.32)
    pf = 0.9342e-01;
elseif(beta > 1.32) && (beta <= 1.33)
    pf = 0.9176e-01;
elseif(beta > 1.33) && (beta <= 1.34)
    pf = 0.9013e-01;
elseif(beta > 1.34) && (beta <= 1.35)
    pf = 0.8851e-01;
elseif(beta > 1.35) && (beta <= 1.36)
    pf = 0.8692e-01;
elseif(beta > 1.36) && (beta <= 1.37)
    pf = 0.8535e-01;
elseif(beta > 1.37) && (beta <= 1.38)
```

```
        pf = 0.8380e-01;
    elseif(beta > 1.38) && (beta <= 1.39)
        pf = 0.8227e-01;
    elseif(beta > 1.39) && (beta <= 1.40)
        pf = 0.8076e-01;
    elseif(beta > 1.40) && (beta <= 1.41)
        pf = 0.7927e-01;
    elseif(beta > 1.41) && (beta <= 1.42)
        pf = 0.7781e-01;
    elseif(beta > 1.42) && (beta <= 1.43)
        pf = 0.7636e-01;
    elseif(beta > 1.43) && (beta <= 1.44)
        pf = 0.7494e-01;
    elseif(beta > 1.44) && (beta <= 1.45)
        pf = 0.7353e-01;
    elseif(beta > 1.45) && (beta <= 1.46)
        pf = 0.7216e-01;
    elseif(beta > 1.46) && (beta <= 1.47)
        pf = 0.7078e-01;
    elseif(beta > 1.47) && (beta <= 1.48)
        pf = 0.6944e-01;
    elseif(beta > 1.48) && (beta <= 1.49)
        pf = 0.6811e-01;
    elseif(beta > 1.49) && (beta <= 1.50)
        pf = 0.6681e-01;
    elseif(beta > 1.50) && (beta <= 1.51)
        pf = 0.6552e-01;
    elseif(beta > 1.51) && (beta <= 1.52)
        pf = 0.6426e-01;
    elseif(beta > 1.52) && (beta <= 1.53)
        pf = 0.6301e-01;
    elseif(beta > 1.53) && (beta <= 1.54)
        pf = 0.6178e-01;
    elseif(beta > 1.54) && (beta <= 1.55)
        pf = 0.6057e-01;
    elseif(beta > 1.55) && (beta <= 1.56)
        pf = 0.5938e-01;
    elseif(beta > 1.56) && (beta <= 1.57)
        pf = 0.5821e-01;
    elseif(beta > 1.57) && (beta <= 1.58)
        pf = 0.5706e-01;
    elseif(beta > 1.58) && (beta <= 1.59)
        pf = 0.5592e-01;
    elseif(beta > 1.59) && (beta <= 1.60)
        pf = 0.5480e-01;
    elseif(beta > 1.60) && (beta <= 1.61)
        pf = 0.5370e-01;
    elseif(beta > 1.61) && (beta <= 1.62)
        pf = 0.5282e-01;
    elseif(beta > 1.62) && (beta <= 1.63)
        pf = 0.5155e-01;
    elseif(beta > 1.63) && (beta <= 1.64)
        pf = 0.5050e-01;
    elseif(beta > 1.64) && (beta <= 1.65)
        pf = 0.4947e-01;
    elseif(beta > 1.65) && (beta <= 1.66)
        pf = 0.4846e-01;
    elseif(beta > 1.66) && (beta <= 1.67)
```

```
    pf = 0.4746e-01;
elseif(beta > 1.67) && (beta <= 1.68)
    pf = 0.4648e-01;
elseif(beta > 1.68) && (beta <= 1.69)
    pf = 0.4552e-01;
elseif(beta > 1.69) && (beta <= 1.70)
    pf = 0.4457e-01;
elseif(beta > 1.70) && (beta <= 1.71)
    pf = 0.4363e-01;
elseif(beta > 1.71) && (beta <= 1.72)
    pf = 0.4272e-01;
elseif(beta > 1.72) && (beta <= 1.73)
    pf = 0.4182e-01;
elseif(beta > 1.73) && (beta <= 1.74)
    pf = 0.4093e-01;
elseif(beta > 1.74) && (beta <= 1.75)
    pf = 0.4006e-01;
elseif(beta > 1.75) && (beta <= 1.76)
    pf = 0.3921e-01;
elseif(beta > 1.76) && (beta <= 1.77)
    pf = 0.3836e-01;
elseif(beta > 1.77) && (beta <= 1.78)
    pf = 0.3754e-01;
elseif(beta > 1.78) && (beta <= 1.79)
    pf = 0.3673e-01;
elseif(beta > 1.79) && (beta <= 1.80)
    pf = 0.3593e-01;
elseif(beta > 1.80) && (beta <= 1.81)
    pf = 0.3515e-01;
elseif(beta > 1.81) && (beta <= 1.82)
    pf = 0.3438e-01;
elseif(beta > 1.82) && (beta <= 1.83)
    pf = 0.3363e-01;
elseif(beta > 1.83) && (beta <= 1.84)
    pf = 0.3289e-01;
elseif(beta > 1.84) && (beta <= 1.85)
    pf = 0.3216e-01;
elseif(beta > 1.85) && (beta <= 1.86)
    pf = 0.3144e-01;
elseif(beta > 1.86) && (beta <= 1.87)
    pf = 0.3074e-01;
elseif(beta > 1.87) && (beta <= 1.88)
    pf = 0.3005e-01;
elseif(beta > 1.88) && (beta <= 1.89)
    pf = 0.2938e-01;
elseif(beta > 1.89) && (beta <= 1.90)
    pf = 0.2872e-01;
elseif(beta > 1.90) && (beta <= 1.91)
    pf = 0.2807e-01;
elseif(beta > 1.91) && (beta <= 1.92)
    pf = 0.2743e-01;
elseif(beta > 1.92) && (beta <= 1.93)
    pf = 0.2680e-01;
elseif(beta > 1.93) && (beta <= 1.94)
    pf = 0.2619e-01;
elseif(beta > 1.94) && (beta <= 1.95)
    pf = 0.2553e-01;
elseif(beta > 1.95) && (beta <= 1.96)
```

```
        pf = 0.2500e-01;
    elseif(beta > 1.96) && (beta <= 1.97)
        pf = 0.2442e-01;
    elseif(beta > 1.97) && (beta <= 1.98)
        pf = 0.2385e-01;
    elseif(beta > 1.98) && (beta <= 1.99)
        pf = 0.2330e-01;
    elseif(beta > 1.99) && (beta <= 2.00)
        pf = 0.2275e-01;
    elseif(beta > 2.00) && (beta <= 2.01)
        pf = 0.2222e-01;
    elseif(beta > 2.01) && (beta <= 2.02)
        pf = 0.2169e-01;
    elseif(beta > 2.02) && (beta <= 2.03)
        pf = 0.2118e-01;
    elseif(beta > 2.03) && (beta <= 2.04)
        pf = 0.2068e-01;
    elseif(beta > 2.04) && (beta <= 2.05)
        pf = 0.2018e-01;
    elseif(beta > 2.05) && (beta <= 2.06)
        pf = 0.1970e-01;
    elseif(beta > 2.06) && (beta <= 2.07)
        pf = 0.1923e-01;
    elseif(beta > 2.07) && (beta <= 2.08)
        pf = 0.1876e-01;
    elseif(beta > 2.08) && (beta <= 2.09)
        pf = 0.1831e-01;
    elseif(beta > 2.09) && (beta <= 2.10)
        pf = 0.1788e-01;
    elseif(beta > 2.10) && (beta <= 2.11)
        pf = 0.1743e-01;
    elseif(beta > 2.11) && (beta <= 2.12)
        pf = 0.1700e-01;
    elseif(beta > 2.12) && (beta <= 2.13)
        pf = 0.1659e-01;
    elseif(beta > 2.13) && (beta <= 2.14)
        pf = 0.1618e-01;
    elseif(beta > 2.14) && (beta <= 2.15)
        pf = 0.1578e-01;
    elseif(beta > 2.15) && (beta <= 2.16)
        pf = 0.1539e-01;
    elseif(beta > 2.16) && (beta <= 2.17)
        pf = 0.1500e-01;
    elseif(beta > 2.17) && (beta <= 2.18)
        pf = 0.1483e-01;
    elseif(beta > 2.18) && (beta <= 2.19)
        pf = 0.1426e-01;
    elseif(beta > 2.19) && (beta <= 2.20)
        pf = 0.1390e-01;
    elseif(beta > 2.20) && (beta <= 2.21)
        pf = 0.1355e-01;
    elseif(beta > 2.21) && (beta <= 2.22)
        pf = 0.1321e-01;
    elseif(beta > 2.22) && (beta <= 2.23)
        pf = 0.1287e-01;
    elseif(beta > 2.23) && (beta <= 2.24)
        pf = 0.1253e-01;
    elseif(beta > 2.24) && (beta <= 2.25)
```

```
        pf = 0.1222e-01;
    elseif(beta > 2.25) && (beta <= 2.26)
        pf = 0.11191e-01;
    elseif(beta > 2.26) && (beta <= 2.27)
        pf = 0.1160e-01;
    elseif(beta > 2.27) && (beta <= 2.28)
        pf = 0.1130e-01;
    elseif(beta > 2.28) && (beta <= 2.29)
        pf = 0.1101e-01;
    elseif(beta > 2.29) && (beta <= 2.30)
        pf = 0.1072e-01;
    elseif(beta > 2.30) && (beta <= 2.31)
        pf = 0.1044e-01;
    elseif(beta > 2.31) && (beta <= 2.32)
        pf = 0.1017e-01;
    elseif(beta > 2.32) && (beta <= 2.33)
        pf = 0.9903e-02;
    elseif(beta > 2.33) && (beta <= 2.34)
        pf = 0.9642e-02;
    elseif(beta > 2.34) && (beta <= 2.35)
        pf = 0.9387e-02;
    elseif(beta > 2.35) && (beta <= 2.36)
        pf = 0.9138e-02;
    elseif(beta > 2.36) && (beta <= 2.37)
        pf = 0.8894e-02;
    elseif(beta > 2.37) && (beta <= 2.38)
        pf = 0.8657e-02;
    elseif(beta > 2.38) && (beta <= 2.39)
        pf = 0.8424e-02;
    elseif(beta > 2.39) && (beta <= 2.40)
        pf = 0.8198e-02;
    elseif(beta > 2.40) && (beta <= 2.41)
        pf = 0.7976e-02;
    elseif(beta > 2.41) && (beta <= 2.42)
        pf = 0.7760e-02;
    elseif(beta > 2.42) && (beta <= 2.43)
        pf = 0.7550e-02;
    elseif(beta > 2.43) && (beta <= 2.44)
        pf = 0.7344e-02;
    elseif(beta > 2.44) && (beta <= 2.45)
        pf = 0.7143e-02;
    elseif(beta > 2.45) && (beta <= 2.46)
        pf = 0.6947e-02;
    elseif(beta > 2.46) && (beta <= 2.47)
        pf = 0.6758e-02;
    elseif(beta > 2.47) && (beta <= 2.48)
        pf = 0.6589e-02;
    elseif(beta > 2.48) && (beta <= 2.49)
        pf = 0.6387e-02;
    elseif(beta > 2.49) && (beta <= 2.50)
        pf = 0.6210e-02;
    elseif(beta > 2.50) && (beta <= 2.51)
        pf = 0.6037e-02;
    elseif(beta > 2.51) && (beta <= 2.52)
        pf = 0.5868e-02;
    elseif(beta > 2.52) && (beta <= 2.53)
        pf = 0.5703e-02;
    elseif(beta > 2.53) && (beta <= 2.54)
```

```
        pf = 0.5543e-02;
    elseif(beta > 2.54) && (beta <= 2.55)
        pf = 0.5386e-02;
    elseif(beta > 2.55) && (beta <= 2.56)
        pf = 0.5234e-02;
    elseif(beta > 2.56) && (beta <= 2.57)
        pf = 0.5085e-02;
    elseif(beta > 2.57) && (beta <= 2.58)
        pf = 0.4940e-02;
    elseif(beta > 2.58) && (beta <= 2.59)
        pf = 0.4799e-02;
    elseif(beta > 2.59) && (beta <= 2.60)
        pf = 0.4661e-02;
    elseif(beta > 2.60) && (beta <= 2.61)
        pf = 0.4527e-02;
    elseif(beta > 2.61) && (beta <= 2.62)
        pf = 0.4397e-02;
    elseif(beta > 2.62) && (beta <= 2.63)
        pf = 0.4269e-02;
    elseif(beta > 2.63) && (beta <= 2.64)
        pf = 0.4145e-02;
    elseif(beta > 2.64) && (beta <= 2.65)
        pf = 0.4025e-02;
    elseif(beta > 2.65) && (beta <= 2.66)
        pf = 0.3907e-02;
    elseif(beta > 2.66) && (beta <= 2.67)
        pf = 0.3793e-02;
    elseif(beta > 2.67) && (beta <= 2.68)
        pf = 0.3681e-02;
    elseif(beta > 2.68) && (beta <= 2.69)
        pf = 0.3573e-02;
    elseif(beta > 2.69) && (beta <= 2.70)
        pf = 0.3467e-02;
    elseif(beta > 2.70) && (beta <= 2.71)
        pf = 0.3364e-02;
    elseif(beta > 2.71) && (beta <= 2.72)
        pf = 0.3264e-02;
    elseif(beta > 2.72) && (beta <= 2.73)
        pf = 0.3167e-02;
    elseif(beta > 2.73) && (beta <= 2.74)
        pf = 0.3072e-02;
    elseif(beta > 2.74) && (beta <= 2.75)
        pf = 0.2980e-02;
    elseif(beta > 2.75) && (beta <= 2.76)
        pf = 0.2890e-02;
    elseif(beta > 2.76) && (beta <= 2.77)
        pf = 0.2803e-02;
    elseif(beta > 2.77) && (beta <= 2.78)
        pf = 0.2718e-02;
    elseif(beta > 2.78) && (beta <= 2.79)
        pf = 0.2635e-02;
    elseif(beta > 2.79) && (beta <= 2.80)
        pf = 0.2555e-02;
    elseif(beta > 2.80) && (beta <= 2.81)
        pf = 0.2477e-02;
    elseif(beta > 2.81) && (beta <= 2.82)
        pf = 0.2401e-02;
    elseif(beta > 2.82) && (beta <= 2.83)
```

```
        pf = 0.2327e-02;
    elseif(beta > 2.83) && (beta <= 2.84)
        pf = 0.2256e-02;
    elseif(beta > 2.84) && (beta <= 2.85)
        pf = 0.2286e-02;
    elseif(beta > 2.85) && (beta <= 2.86)
        pf = 0.2118e-02;
    elseif(beta > 2.86) && (beta <= 2.87)
        pf = 0.2052e-02;
    elseif(beta > 2.87) && (beta <= 2.88)
        pf = 0.1988e-02;
    elseif(beta > 2.88) && (beta <= 2.89)
        pf = 0.1926e-02;
    elseif(beta > 2.89) && (beta <= 2.90)
        pf = 0.1866e-02;
    elseif(beta > 2.90) && (beta <= 2.91)
        pf = 0.1807e-02;
    elseif(beta > 2.91) && (beta <= 2.92)
        pf = 0.1750e-02;
    elseif(beta > 2.92) && (beta <= 2.93)
        pf = 0.1695e-02;
    elseif(beta > 2.93) && (beta <= 2.94)
        pf = 0.1641e-02;
    elseif(beta > 2.94) && (beta <= 2.95)
        pf = 0.1589e-02;
    elseif(beta > 2.95) && (beta <= 2.96)
        pf = 0.1538e-02;
    elseif(beta > 2.96) && (beta <= 2.97)
        pf = 0.1489e-02;
    elseif(beta > 2.97) && (beta <= 2.98)
        pf = 0.1441e-02;
    elseif(beta > 2.98) && (beta <= 2.99)
        pf = 0.1395e-02;
    elseif(beta > 2.99) && (beta <= 3.00)
        pf = 0.1350e-02;
    elseif(beta > 3.00) && (beta <= 3.01)
        pf = 0.1306e-02;
    elseif(beta > 3.01) && (beta <= 3.02)
        pf = 0.14264e-02;
    elseif(beta > 3.02) && (beta <= 3.03)
        pf = 0.1223e-02;
    elseif(beta > 3.03) && (beta <= 3.04)
        pf = 0.1183e-02;
    elseif(beta > 3.04) && (beta <= 3.05)
        pf = 0.1144e-02;
    elseif(beta > 3.05) && (beta <= 3.06)
        pf = 0.1107e-02;
    elseif(beta > 3.06) && (beta <= 3.07)
        pf = 0.1070e-02;
    elseif(beta > 3.07) && (beta <= 3.08)
        pf = 0.1035e-02;
    elseif(beta > 3.08) && (beta <= 3.09)
        pf = 0.1001e-02;
    elseif(beta > 3.09) && (beta <= 3.10)
        pf = 0.9676e-03;
    elseif(beta > 3.10) && (beta <= 3.11)
        pf = 0.9354e-03;
    elseif(beta > 3.11) && (beta <= 3.12)
```

```
        pf = 0.9042e-03;
    elseif(beta > 3.12) && (beta <= 3.13)
        pf = 0.8740e-03;
    elseif(beta > 3.13) && (beta <= 3.14)
        pf = 0.8447e-03;
    elseif(beta > 3.14) && (beta <= 3.15)
        pf = 0.8163e-03;
    elseif(beta > 3.15) && (beta <= 3.16)
        pf = 0.7888e-03;
    elseif(beta > 3.16) && (beta <= 3.17)
        pf = 0.7622e-03;
    elseif(beta > 3.17) && (beta <= 3.18)
        pf = 0.7363e-03;
    elseif(beta > 3.18) && (beta <= 3.19)
        pf = 0.7113e-03;
    elseif(beta > 3.09) && (beta <= 3.20)
        pf = 0.6871e-03;
    elseif(beta > 3.20) && (beta <= 3.21)
        pf = 0.6636e-03;
    elseif(beta > 3.21) && (beta <= 3.22)
        pf = 0.6409e-03;
    elseif(beta > 3.22) && (beta <= 3.23)
        pf = 0.6189e-03;
    elseif(beta > 3.23) && (beta <= 3.24)
        pf = 0.5976e-03;
    elseif(beta > 3.24) && (beta <= 3.25)
        pf = 0.5770e-03;
    elseif(beta > 3.25) && (beta <= 3.26)
        pf = 0.5570e-03;
    elseif(beta > 3.26) && (beta <= 3.27)
        pf = 0.5377e-03;
    elseif(beta > 3.27) && (beta <= 3.28)
        pf = 0.5190e-03;
    elseif(beta > 3.28) && (beta <= 3.29)
        pf = 0.5009e-03;
    elseif(beta > 3.29) && (beta <= 3.30)
        pf = 0.4834e-03;
    elseif(beta > 3.30) && (beta <= 3.31)
        pf = 0.4664e-03;
    elseif(beta > 3.31) && (beta <= 3.32)
        pf = 0.4500e-03;
    elseif(beta > 3.32) && (beta <= 3.33)
        pf = 0.4342e-03;
    elseif(beta > 3.33) && (beta <= 3.34)
        pf = 0.4189e-03;
    elseif(beta > 3.34) && (beta <= 3.35)
        pf = 0.4040e-03;
    elseif(beta > 3.35) && (beta <= 3.36)
        pf = 0.3897e-03;
    elseif(beta > 3.36) && (beta <= 3.37)
        pf = 0.3758e-03;
    elseif(beta > 3.37) && (beta <= 3.38)
        pf = 0.3624e-03;
    elseif(beta > 3.38) && (beta <= 3.39)
        pf = 0.3494e-03;
    elseif(beta > 3.39) && (beta <= 3.40)
        pf = 0.3369e-03;
    elseif(beta > 3.40) && (beta <= 3.41)
```

```
    pf = 0.3248e-03;
elseif(beta > 3.41) && (beta <= 3.42)
    pf = 0.3131e-03;
elseif(beta > 3.42) && (beta <= 3.43)
    pf = 0.3017e-03;
elseif(beta > 3.43) && (beta <= 3.44)
    pf = 0.2908e-03;
elseif(beta > 3.44) && (beta <= 3.45)
    pf = 0.2802e-03;
elseif(beta > 3.45) && (beta <= 3.46)
    pf = 0.2700e-03;
elseif(beta > 3.46) && (beta <= 3.47)
    pf = 0.2602e-03;
elseif(beta > 3.47) && (beta <= 3.48)
    pf = 0.2507e-03;
elseif(beta > 3.48) && (beta <= 3.49)
    pf = 0.2415e-03;
elseif(beta > 3.49) && (beta <= 3.50)
    pf = 0.2326e-03;
elseif(beta > 3.50) && (beta <= 3.51)
    pf = 0.2240e-03;
elseif(beta > 3.51) && (beta <= 3.52)
    pf = 0.2157e-03;
elseif(beta > 3.52) && (beta <= 3.53)
    pf = 0.2077e-03;
elseif(beta > 3.53) && (beta <= 3.54)
    pf = 0.2000e-03;
elseif(beta > 3.54) && (beta <= 3.55)
    pf = 0.1926e-03;
elseif(beta > 3.55) && (beta <= 3.56)
    pf = 0.1854e-03;
elseif(beta > 3.56) && (beta <= 3.57)
    pf = 0.1784e-03;
elseif(beta > 3.57) && (beta <= 3.58)
    pf = 0.1717e-03;
elseif(beta > 3.58) && (beta <= 3.59)
    pf = 0.1653e-03;
elseif(beta > 3.59) && (beta <= 3.60)
    pf = 0.1591e-03;
elseif(beta > 3.60) && (beta <= 3.61)
    pf = 0.1531e-03;
elseif(beta > 3.61) && (beta <= 3.62)
    pf = 0.1473e-03;
elseif(beta > 3.62) && (beta <= 3.63)
    pf = 0.1417e-03;
elseif(beta > 3.63) && (beta <= 3.64)
    pf = 0.1363e-03;
elseif(beta > 3.64) && (beta <= 3.65)
    pf = 0.1311e-03;
elseif(beta > 3.65) && (beta <= 3.66)
    pf = 0.1261e-03;
elseif(beta > 3.66) && (beta <= 3.67)
    pf = 0.1212e-03;
elseif(beta > 3.67) && (beta <= 3.68)
    pf = 0.1166e-03;
elseif(beta > 3.68) && (beta <= 3.69)
    pf = 0.1121e-03;
elseif(beta > 3.69) && (beta <= 3.70)
```

```
    pf = 0.1077e-03;
elseif(beta > 3.70) && (beta <= 3.71)
    pf = 0.1036e-03;
elseif(beta > 3.71) && (beta <= 3.72)
    pf = 0.9956e-04;
elseif(beta > 3.72) && (beta <= 3.73)
    pf = 0.9569e-04;
elseif(beta > 3.73) && (beta <= 3.74)
    pf = 0.9196e-04;
elseif(beta > 3.74) && (beta <= 3.75)
    pf = 0.8837e-04;
elseif(beta > 3.75) && (beta <= 3.76)
    pf = 0.8491e-04;
elseif(beta > 3.76) && (beta <= 3.77)
    pf = 0.8157e-04;
elseif(beta > 3.77) && (beta <= 3.78)
    pf = 0.7836e-04;
elseif(beta > 3.78) && (beta <= 3.79)
    pf = 0.7527e-04;
elseif(beta > 3.79) && (beta <= 3.80)
    pf = 0.7230e-04;
elseif(beta > 3.80) && (beta <= 3.81)
    pf = 0.6943e-04;
elseif(beta > 3.81) && (beta <= 3.82)
    pf = 0.6667e-04;
elseif(beta > 3.82) && (beta <= 3.83)
    pf = 0.6402e-04;
elseif(beta > 3.83) && (beta <= 3.84)
    pf = 0.6147e-04;
elseif(beta > 3.84) && (beta <= 3.85)
    pf = 0.5901e-04;
elseif(beta > 3.85) && (beta <= 3.86)
    pf = 0.5664e-04;
elseif(beta > 3.86) && (beta <= 3.87)
    pf = 0.5437e-04;
elseif(beta > 3.87) && (beta <= 3.88)
    pf = 0.5218e-04;
elseif(beta > 3.88) && (beta <= 3.89)
    pf = 0.5007e-04;
elseif(beta > 3.89) && (beta <= 3.90)
    pf = 0.4804e-04;
elseif(beta > 3.90) && (beta <= 3.91)
    pf = 0.4610e-04;
elseif(beta > 3.91) && (beta <= 3.92)
    pf = 0.4422e-04;
elseif(beta > 3.92) && (beta <= 3.93)
    pf = 0.4242e-04;
elseif(beta > 3.93) && (beta <= 3.94)
    pf = 0.4069e-04;
elseif(beta > 3.94) && (beta <= 3.95)
    pf = 0.3902e-04;
elseif(beta > 3.95) && (beta <= 3.96)
    pf = 0.3742e-04;
elseif(beta > 3.96) && (beta <= 3.97)
    pf = 0.3588e-04;
elseif(beta > 3.97) && (beta <= 3.98)
    pf = 0.3441e-04;
elseif(beta > 3.98) && (beta <= 3.99)
```

```
    pf = 0.3298e-04;
elseif(beta > 3.99) && (beta <= 4.00)
    pf = 0.3162e-04;
elseif(beta > 4.00) && (beta <= 4.05)
    pf = 0.2557e-04;
elseif(beta > 4.05) && (beta <= 4.10)
    pf = 0.2062e-04;
elseif(beta > 4.10) && (beta <= 4.15)
    pf = 0.1659e-04;
elseif(beta > 4.15) && (beta <= 4.20)
    pf = 0.1332e-04;
elseif(beta > 4.20) && (beta <= 4.25)
    pf = 0.1067e-04;
elseif(beta > 4.25) && (beta <= 4.30)
    pf = 0.8524e-05;
elseif(beta > 4.30) && (beta <= 4.35)
    pf = 0.6794e-05;
elseif(beta > 4.35) && (beta <= 4.40)
    pf = 0.5402e-05;
elseif(beta > 4.40) && (beta <= 4.45)
    pf = 0.4285e-05;
elseif(beta > 4.45) && (beta <= 4.50)
    pf = 0.3391e-05;
elseif(beta > 4.50) && (beta <= 4.55)
    pf = 0.2677e-05;
elseif(beta > 4.55) && (beta <= 4.60)
    pf = 0.2108e-05;
elseif(beta > 4.60) && (beta <= 4.65)
    pf = 0.1656e-05;
elseif(beta > 4.65) && (beta <= 4.70)
    pf = 0.1298e-05;
elseif(beta > 4.70) && (beta <= 4.75)
    pf = 0.1015e-05;
elseif(beta > 4.75) && (beta <= 4.80)
    pf = 0.7914e-06;
elseif(beta > 4.80) && (beta <= 4.85)
    pf = 0.6158e-06;
elseif(beta > 4.85) && (beta <= 4.90)
    pf = 0.4780e-06;
elseif(beta > 4.90) && (beta <= 4.95)
    pf = 0.3701e-06;
elseif(beta > 4.95) && (beta <= 5.00)
    pf = 0.2859e-06;
elseif(beta > 5.00) && (beta <= 5.05)
    pf = 0.2203e-06;
elseif(beta > 5.05) && (beta <= 5.10)
    pf = 0.1694e-06;
elseif(beta > 5.10) && (beta <= 5.15)
    pf = 0.1299e-06;
elseif(beta > 5.15) && (beta <= 5.20)
    pf = 0.9935e-07;
elseif(beta > 5.20) && (beta <= 5.25)
    pf = 0.7582e-07;
elseif(beta > 5.25) && (beta <= 5.30)
    pf = 0.5772e-07;
elseif(beta > 5.30) && (beta <= 5.35)
    pf = 0.4384e-07;
elseif(beta > 5.35) && (beta <= 5.40)
```

```
    pf = 0.3321e-07;
elseif(beta > 5.40) && (beta <= 5.45)
    pf = 0.2510e-07;
elseif(beta > 5.45) && (beta <= 5.50)
    pf = 0.1892e-07;
elseif(beta > 5.50) && (beta <= 5.55)
    pf = 0.1423e-07;
elseif(beta > 5.55) && (beta <= 5.60)
    pf = 0.1067e-07;
elseif(beta > 5.60) && (beta <= 5.65)
    pf = 0.7985e-08;
elseif(beta > 5.65) && (beta <= 5.70)
    pf = 0.5959e-08;
elseif(beta > 5.70) && (beta <= 5.75)
    pf = 0.4436e-08;
elseif(beta > 5.75) && (beta <= 5.80)
    pf = 0.3293e-08;
elseif(beta > 5.80) && (beta <= 5.85)
    pf = 0.2438e-08;
elseif(beta > 5.85) && (beta <= 5.90)
    pf = 0.1800e-08;
elseif(beta > 5.90) && (beta <= 5.95)
    pf = 0.1325e-08;
elseif(beta > 5.95) && (beta <= 6.00)
    pf = 0.9716e-09;
elseif(beta > 6.00) && (beta <= 6.10)
    pf = 0.5220e-09;
elseif(beta > 6.10) && (beta <= 6.20)
    pf = 0.2778e-09;
elseif(beta > 6.20) && (beta <= 6.30)
    pf = 0.1483e-09;
elseif(beta > 6.30) && (beta <= 6.40)
    pf = 0.7636e-10;
elseif(beta > 6.40) && (beta <= 6.50)
    pf = 0.3945e-10;
elseif(beta > 6.50) && (beta <= 6.60)
    pf = 0.2010e-10;
elseif(beta > 6.60) && (beta <= 6.70)
    pf = 0.1023e-10;
elseif(beta > 6.70) && (beta <= 6.80)
    pf = 0.5130e-11;
elseif(beta > 6.80) && (beta <= 6.90)
    pf = 0.2549e-11;
elseif(beta > 6.90) && (beta <= 7.00)
    pf = 0.1254e-11;
elseif(beta > 7.00) && (beta <= 7.10)
    pf = 0.6107e-12;
elseif(beta > 7.10) && (beta <= 7.20)
    pf = 0.2946e-12;
elseif(beta > 7.20) && (beta <= 7.30)
    pf = 0.1407e-12;
elseif(beta > 7.30) && (beta <= 7.40)
    pf = 0.6654e-13;
elseif(beta > 7.40) && (beta <= 7.50)
    pf = 0.3116e-13;
elseif(beta > 7.50) && (beta <= 7.70)
    pf = 0.1445e-13;
elseif(beta > 7.60) && (beta <= 7.80)
```

```
    pf = 0.6636e-14;
elseif(beta > 7.70) && (beta <= 7.80)
    pf = 0.3017e-14
elseif(beta > 7.80) && (beta <= 7.90)
    pf = 0.1359e-14;
elseif(beta > 7.90) && (beta <= 8.00)
    pf = 0.6056e-15;
elseif(beta > 8.00) && (beta <= 8.10)
    pf = 0.2673e-15;
elseif(beta > 8.10) && (beta <= 8.20)
    pf = 0.1169e-15;
elseif(beta > 8.20) && (beta <= 8.30)
    pf = 0.5058e-16;
elseif(beta > 8.30) && (beta <= 8.40)
    pf = 0.2167e-16;
elseif(beta > 8.40) && (beta <= 8.50)
    pf = 0.9197e-17;
elseif(beta > 8.50) && (beta <= 8.60)
    pf = 0.3864e-17;
elseif(beta > 8.60) && (beta <= 8.70)
    pf = 0.1608e-17;
elseif(beta > 8.70) && (beta <= 8.80)
    pf = 0.6623e-18;
elseif(beta > 8.80) && (beta <= 8.90)
    pf = 0.2701e-18;
elseif(beta > 8.90) && (beta <= 9.00)
    pf = 0.1091e-18;
elseif(beta > 9.00) && (beta <= 9.10)
    pf = 0.4363e-19;
elseif(beta > 9.10) && (beta <= 9.20)
    pf = 0.1728e-19;
elseif(beta > 9.20) && (beta <= 9.30)
    pf = 0.6773e-20;
elseif(beta > 9.30) && (beta <= 9.40)
    pf = 0.2629e-20;
elseif(beta > 9.40) && (beta <= 9.50)
    pf = 0.1011e-20;
elseif(beta > 9.50) && (beta <= 9.60)
    pf = 0.3847e-21;
elseif(beta > 9.60) && (beta <= 9.70)
    pf = 0.1450e-21;
elseif(beta > 9.70) && (beta <= 9.80)
    pf = 0.5408e-22;
elseif(beta > 9.80) && (beta <= 9.90)
    pf = 0.1998e-22;
elseif(beta > 9.90)
    pf = 0.1998e-22;
end
if(pf <= 0)
    pf = 0;
end
```