MODAL ANALYSIS OF VERTICALLY CURVED CONCRETE FLY-OVER BRIDGES

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A DISSERTATION SUBMITTED TO THE SCHOOL OF POSTGRADUATE STUDIES,
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AUGUST 2016

DECLARATION

I hereby declare that the work in this dissertation titled "Modal Analysis of Vertically Curved Concrete Flyover Bridges" was performed by me in the Department of Civil Engineering, under the supervision of Prof. I. Abubakar and Prof. O.S Abejide.

The information derived from literature have been duly acknowledged in the text and a list of references provided. No part of this work has been presented for another degree at any institution.

Idris Ahmed JA'E	_____	_____
Name of Student	Signature	Date

CERTIFICATION

This thesis titled "Modal Analysis of Vertically Curved Concrete Flyover Bridges" meets the regulations governing the award of the degree of Master of Science in Civil engineering of the Ahmadu Bello University, and is approved for its contribution to knowledge and literary presentation.

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Chairman, Supervisory Committee	Signature	Date
Prof. O.S Abejide		
Member, Supervisory Committee	Signature	Date
Dr. Y.D Amartey		
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Prof. Kabir Bala		
Dean of Postgraduate School	Signature	Date

DEDICATION

To Teeya, Shishishim and Fauzi

ACKNOWLEDGEMENT

My sincere appreciation goes to my kids Yusra and AbdulMuhsin, my wife Fauziyyah Abdulkadir for their understanding and encouragement throughout the period of this study. Although yusra have so many time disrupted the process of the analysis anytime I stepped out.

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To my parent,I will remain eternally grateful for their continuous prayer all the time.

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Finally, I will not forget to register my profound gratitude to Engr. Ashiru Muhammad for providing the CSiBridge software with the complete tutorial package.

ABSTRACT

The effectiveness and sensitivity of vertically curved concrete flyover bridge (VCCFB) profile is presented. Modal analysis was conducted on three VCCFB models; one for a profile achieved using horizontal beams, profile achieved using slightly curved beams and a profile achieved using combination of straight and curved beams. Twelve vibration mode shapes were recorded with corresponding natural frequencies for each model. In each mode, a model with a profile using straight beams were found to have higher natural frequencies with major differences in modes 2, 6 and 8 corresponding to 2.26, 3.01 and 2.87% differences from the model with curved beams respectively. Also, major differences were recorded in modes 2, 3 and 5 corresponding to 6.31, 5.51 and 4.24% differences in natural frequencies from model with combine beams when using CSiBridge (2015) software. Three vehicles were simulated each passing the bridge for a period of 10 seconds per lane at the same speed of 10, 20, 30, 40, 50km/h and at varying vehicular speeds of 45, 50, 60km/h and 60, 65,70km/h respectively. The response spectrum from time history analysis conducted for the three (3) vehicles moving on each of the model were plotted. The vertical component of acceleration was compared with the Irwin (1979) base curve for human perceptibility threshold. The profile achieved using a combination of straight and curved beams were found to induce less vibration compared to profiles achieved using straight and slightly curved beams. These result thus indicate that profiles of VCCFB achieved using combination of straight and slightly curved beams at the cusp induced less vibration compared to profiles achieved with straight or slightly curved beams. As such the use of combine geometry precast beams should be encouraged in achieving vertical profiles for this class of bridges.

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CHAPTER ONE
INTRODUCTION

1.1 General

The use of Vertically Curved Concrete Flyover Bridge (VCCFB) at interchanges of highway systems is becoming increasingly popular because of increasing demand for curved roadway alignment for the passage of congested traffic as well as modern emphasis on aesthetic consideration. As such the health and performance of such bridges is very important considering the strategic roles they play in our cities.

In advanced countries, Structural Health Monitoring (SHM) systems are Implemented more and more frequently with the aim to safeguarding the safety and service lives of structures especially bridges, because changes in the integrity of the material and / or structural properties of these structures are known to adversely affect their performance which can also be observed from the structures dynamic response such as the modal parameters (Netti *et al.*, 2015).

It is well known that structures can resonate, which means that small forces can result in important deformations, and possibly, damages can be induced in the structure. The Tacoma Narrow suspension bridge disaster (1940) is a typical example of this, it collapsed due to wind-induced vibration (that is flutter). It was situated on the Tacoma Narrows in Puget Sound, near the city of Tacoma, Washington, and the bridge had only been open for traffic a few months after completion (Patrick, 2014).

Vibration is a mechanical phenomenon whereby oscillations occur about a point such as repetitive periodic change in displacement with respect to some reference point. The oscillations may be periodic such as the motion of a pendulum, or random such as the movement of a tyre on a gravel road. Vertical vibrations caused by vehicles are the major causes of significant motions for bridge decks with local vibrations in the neighbourhood of the expansion joints. Bridge vibrations can be stimulated in different ways as a result of pulsating loads that oscillate at the same natural frequency of the structures. This phenomenon can make the bridge to resonate thereby leading to permanent structural damages (Akiije and Omotoso, 2012).

According to David and Joanna (2014) Vibration generated by vehicles traveling at speeds can be a significant issue in considering the design life of highway bridges. Dynamic effects can potentially become more serious if the bridge is old and has been subjected to increases in both magnitude and frequency of loadings during its working life.

Bridges are therefore subjected to dynamic loads, in the form of vehicular traffic, which cause them to vibrate. A moving vehicle on a bridge generates deflections and stresses that are generally greater than those caused by the same vehicular loads applied statically. This is due to the dynamic interaction between the bridge and the vehicle. This interaction is a problem of considerable complexity and its solution is governed by both vehicle and bridge dynamic characteristics. Dynamic behaviours due to vehicles moving across rough surface decks have long been recognized as one of the primary concerns in designing and rating of bridges (Proenca and Fernando, 2005). In spite of its important role, the bridge-vehicle interaction dynamic analysis is hardly taken into consideration in bridge designs because of its considerable complexity. This complex dynamic phenomenon depends on so many parameters including types of bridge, dynamic properties of the bridge, vehicle characteristics, vehicle speeds, vehicle's moving paths, number of vehicles, road surface roughness, etc., that prevent the interaction analysis (Senthilvasa *et al.*, 2002).

Determination of the dynamic response of structures, especially bridges, has been the topic of numerous studies in recent years; however, the related question of user comfort on these vibrating bridges has received relatively little attention (Awall *et al.*, 2012). A bridge vibration due to moving traffic is important for two reasons: (1) the stresses are increased above those due to static and dynamic load applications. This is normally accounted for by the "impact factor" or "dynamic amplification factor" in the design; (2) Excessive vibration may be noticeable to persons on the bridge. The human body, however, is primarily sensitive to dynamic effects such as acceleration and change of acceleration. Although not related to issues of safety, this may have the psychological effect of impairing public confidence in the structure and, therefore, demands consideration at the design stage (Awall *et al.*, 2012).

The basic idea behind modal analysis is that modal parameters (notably modal frequencies, modal shapes, and modal damping) are functions of the physical properties of the structure (mass, damping, and stiffness). Therefore, changes in the physical properties will cause detectable changes in the modal parameters (Zhang and Lynch, 2013).

1.2 Problem Statement and Justification

1.2.1 Problem Statement of Research

Mostly, profiles of Vertically Curved Concrete Flyover Bridges (VCCFB) are constructed using straight precast beams which in most cases provide less smooth profile for the bridges. That is, provide polygonal profile instead of the desired smooth parabolic profile. This type of bridges are mostly associated with problematic expansion joints especially at the cusp of the curve, with conspicuous gapping of the parapet at such locations. By observation at some of these type of bridges, the disparity in width of expansion gap at the cusp is due to the geometry of both the beam (straight) and the bridge as a whole (suppose parabolic). In some of such bridges vehicles moving at certain speed could bounce at each transition of the polygonal end formed by the straight beams. Also, vibrations or motion occurrences that often affects the comfort of bridge users are quite alarming. People without engineering training could easily spot pronounce gaping of parapet at varying elevations and ubiquitous expansion joints deterioration, resulting in unpleasant noise that lowers public confidence.

A typical example of interest is the Kawo flyover bridge in Kaduna state, which based on verbal discussion with an Engineer in the design and implementation department of the Kaduna State Ministry of works and transport reveals that, many efforts have been made to provide a lasting solution to the continuous increase in the width of expansion joints which increasingly affects the comfort of users due to increasing vibration effect as a result of impact of tyres on the damage expansion joint, but all effort were not successful (Anonymous, 2015a). Hence, investigating the modal properties as well as dynamic response of different profiles of VCCFB with respect to human perceptibility to vehicular induced vibration is a step to finding best method of achieving profile for vertically curved bridges.

1.2.2 Justification of study

Understanding the dynamic properties of different profiles of VCCFB achieved using beams of different shape is timely. As such, models of VCCFB achieved using straight beams, slightly curved beam and combination of both (straight and curved beams) with the curved beams placed at the cusp of the curve will be studied with respect modal behaviour and human perceptibility to vibration threshold. The outcome of this study will serve as a guide to providing a suitable means of achieving the best profile for VCCFB.

1.3 Aim and Objectives

1.3.1 Aim

The aim of this work is to carryout modal analysis on the effect of vibrations induced by vehicular traffic on different profiles of vertically curved concrete flyover bridges.

1.3.2 Objectives

Objectives of the study are to:

(i) Conduct Modal analysis on Vertically Curved Concrete Flyover Bridge

(ii) Compare modal parameters of the bridge model with the profile achieved using horizontal beams, and a model with the profile achieved using slightly curved beams.

(iii) Analyse bridge –vehicle response of the three models at varying vehicular velocities

(iv) Identify human perceptibility to vibration for the models at varying vehicular speeds.

(v) Establish a suitable deck profile for such type of bridges

1.4 Scope and Limitations

1.4.1 Scope

The study will involve modal analysis of VCCFB models, one with the profile achieved using straight beams as it is commonly practiced, a model with the profile achieved with slightly curved beams and a model comprising straight and curved beam (). The analysis will be conducted using the finite element method as packaged in CSiBridge (2015) software. The bridge models will be simulated for all modal parameters at various damping ratios and vehicular velocities.

1.4.2 Limitation

The analysis is limited to models of Vertically Curved Concrete Flyover Bridges with profiles achieved using straight, curved and combination of both beams.

CHAPTER TWO
LITERATURE REVIEW

2.1 General

All bridges undergo some form of dynamic loading, which causes them to vibrate. From as early as the 1920's, bridge engineers have increased the static live load on bridges by a factor called the impact factor to account for the dynamic behaviour. In 1922, the American Railway Engineering Association (AREA) and the American Association of State Highway Officials (AASHO) adopted simple empirical span length dependent relationships for the impact factor. In 1927 a joint committee of AREA and AASHO agreed and adopted the relationship for impact factor in equation 2.1 (Memory *et al.*, 1995) :

$$I = \frac{50}{L+125} \qquad (2.1)$$

The term impact factor was in exclusive usage for several years, until in 1979 the term Dynamic Load Allowance (DLA) was introduced by the Ontario Ministry of Transportation to specify design provisions for the dynamic effects. The term impact factor was abandoned by the Ontario Highway Bridge Design Code, and subsequently the of draft National Association of Australian State Road Authorities (NAASRA) Bridge Design Specification, as it incorrectly implies some form of impact (Memory *et al.*, 1995).

The Ontario Ministry of Transportation (1979) published the first limit state bridge design code in which the Dynamic Load Allowance (DLA) - first flexural frequency of the bridge superstructure relationship, was introduced. This seemed reasonable as usual vehicular traffic on a symmetrical bridge will excite the first flexural mode of vibration. The second edition of the Ontario Highway Bridge Design Code (OHBDC) (1983) was also published. Though the DLA provision was modified slightly, the concept of *quasi* resonance remained. As a result of the research undertaken in Canada by the National Association of Australian Road Authorities (NAASRA), now known as AUSTROADS 2, the current OHBDC (1983) dynamic load allowance provision in its draft Bridge Design Specifications was adopted. Accordingly, bridge designers in Australia will need to calculate the first flexural frequency of proposed bridges in order to evaluate the DLA.

Most bridges are still designed using static analysis, and bridge designers have proved reluctant to accept proposals to link bridge design and dynamic response. This has, in part,

been due to the apparently complex processes required to accurately estimate levels of vibration (Memory *et al.*, 1995).

In July, 2012 the repairs of Lagos Third Mainland Bridge (TMB) commenced. The repairs which involves the removal and replacement of eight (8) of defective expansion joints, was executed by M/S Borini Prono and Co. (Nig.) Ltd. The remedy was majorly to offset the hazardous effect of vibration and undue motion of the bridge. Lagos TMB used to be subjected to impose heavy dynamic traffic volume from over-loaded vehicles, static potholes, and dilapidated expansion joints. These phenomenon led to the earlier panicking effect of vibrations and the bridge motion enforced on the Lagos TMB users (Akiije and Omotoso, 2012).

Modal analysis is well suited for these studies because modal analysis represent the global characteristics of the structure, and in theory should remain fixed for an invariant system. However, the subtle variation in structural system's behaviour due to environment and time can be efficiently described by modal parameters. Modal parameters extracted are also used to calibrate finite element models of instrumented bridges (Hsieh *et al.*, 2006).

Updated finite element models are essential tools used by engineers to monitor the health of a structure after extreme loading event such as strong motion. Some example of long term bridge performance that adopted modal analysis are the long-term structural monitoring system of the Tamar bridge in the UK, and Ting Kau bridge in Hong Kong (Zhang and Lynch, 2013).

Vibration problems generally require vibration characterization in terms of peak values, preferred vibration indicators, frequency content and identification of the predominant sources and vibration transmission paths (Proenca and Fernando, 2005).

The goal of modal analysis in structural mechanics is to determine the natural mode shapes and frequencies of an object or structure during free vibration. It is common to use the Finite Element Method (FEM) to perform this analysis because, like other calculations using the FEM, the object being analysed can have arbitrary shape and the results of the calculations are acceptable. The types of equations which may arise from modal analysis are those seen in Eigen systems (Wikipedia, 2015). The physical interpretation of the eigenvalues and eigenvectors which come from solving the system are that they represent the frequencies and corresponding mode shapes. Sometimes, the only desired modes are the lowest frequencies

because they can be the most prominent modes at which the object will vibrate, dominating all the higher frequency modes. It is also possible to test a physical object to determine its natural frequencies and mode shapes. This is called an Experimental Modal Analysis. The results of the physical test can be used to calibrate a finite element model to determine if the underlying assumptions made were correct. For example, correct material properties and boundary conditions were used (Wikipedia, 2015).

2.2 The "Modal" Model

Modes are inherent properties of a structure, and are determined by the material properties of mass, damping, and stiffness; and boundary conditions of the structure. Each mode is defined by a natural (modal or resonant) frequency, modal damping, and a mode shape (that is the so-called "modal parameters"). If either the material properties or the boundary conditions of a structure change, its modes will change (Patrick, 2014). For instance, if mass is added to a structure, it will vibrate differently.

To have an indebt understanding of the concept, we will make use of the concept of single and multiple degree of freedom systems.

2.2.1 Structural Dynamics of a Single Degree of Freedom (SDOF) System

Although most physical structures are continuous, their behaviour can usually be represented by a discrete parameter model as illustrated in Figure 2.1 (where **m** can only move along vertical axis).

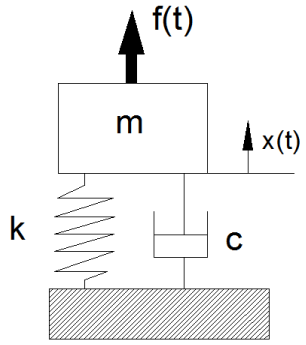


Figure 2-1: Single Degree of Freedom System, Patrick, (2014).

The idealized elements are called mass, spring, damper and excitation. The first three elements describe the physical system. Energy is stored by the system in the mass and the spring in the form of kinetic and potential energies, respectively. Energy enters the system

through excitation and is dissipated through damping. The idealized elements of the physical system can be described by the equation of motion shown in Equation 2.2

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = f(t) \quad (2.2)$$

This equation relates the effects of the mass, stiffness and damping in a way that leads to the calculation of natural frequency and damping factor of the system. This computation is often facilitated by the use of the modal definitions that lead directly to the natural frequency and damping factor (Agilent, 1987).

Equation 2.2 states that the sum of all forces acting on the mass ***m*** should be equal to zero. With ***f (t)*** an externally applied force, where – $\boldsymbol{m\ddot{x}(t)}$ is the inertial force, $\boldsymbol{-\ c\dot{x}(t)}$ the (viscous) damping force, and $\boldsymbol{kx(t)}$ the restoring force. The variable ***x(t)*** stands for the position of the mass ***m*** with respect to its equilibrium point, that is the position of the mass when $\boldsymbol{f(t) = 0}$ (Anonymous, 1982).

If there is no damping and no applied force the dynamic equation of motion in matrix form reduces to:

$$[m][\ddot{x}] + [k][x] = 0 \quad (2.3)$$

Equation (2.3) is the undamped equation of free vibration.

where:

$[m]$= Mass matrix

$[k]$= Stiffness matrix

$[\ddot{x}] =$ Time history of acceleration of the system

$[\dot{x}]$= Time history of velocity of the system

To solve Equation (2.3), we have to assume a harmonic solution of the form,

$$[\dot{x}] = [\phi] \sin \omega t \quad (2.4)$$

where:

$[\phi]$ = eigenvector or mode shape

ω = circular natural frequency

The natural frequency, ω, is in units of radians per second (rad/s). The typical units used in the software CSiBridge (2015) is in Hertz (Hz).

Aside from this harmonic form being the key to the numerical solution, this form also has a physical importance. The harmonic form of the solution means that all the degrees of

freedom of the vibrating structure move in a synchronous manner. The structural configuration does not change its basic shape during motion; only its amplitude changes.

If differentiation of the assumed harmonic equation (2.4) is performed and substituted into the equation of motion, the following is obtained:

$$-\omega^2[m][\phi]\sin\omega t + [k][\phi]\sin\omega t = 0 \quad (2.5)$$

Which after simplifying becomes:

$$([k] - \omega^2[M])[\emptyset] = 0 \quad (2.6)$$

This Equation (2.6) is called the Eigen equation, which is a set of homogenous algebraic equation for the components of the eigenvector and forms the basis of eigenvalue problems. An eigenvalue problem is a specific equation form that has many applications in linear matrix algebra.

The basic form of an eigenvalue problem is:

$$[A - \lambda I]x = 0 \quad (2.7)$$

where:

A = square matrix

λ = Eigenvalues

I = identity matrix

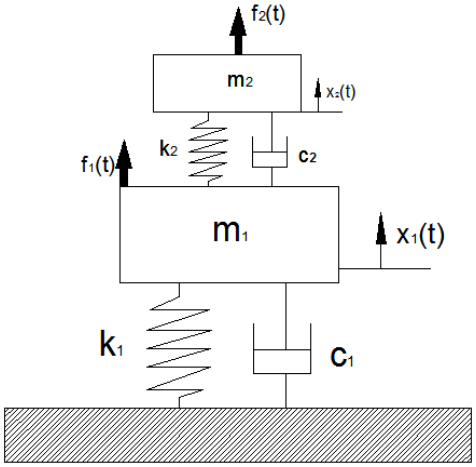
x = eigenvector

2.3.2 Structural Dynamics of a Multiple Degree of Freedom System

The extension of SDOF concepts to a more general MDOF system, with ***n*** degrees of freedom, is a straightforward process. The physical system simply comprised of an interconnection of idealized SDOF models, as illustrated in Figure 2.2, and is described by the matrix equations of motion, that is Equation (2.6). The solution of the equation (2.6) with no excitation again leads to the modal parameters (roots of the equation) of the system. For the MDOF case, however, a unique displacement vector called the mode shape exists for each distinct frequency and damping.

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = \{f(t)\} \quad 2.15$$

In Figure (2.2), the different matrices are defined for a **Two-DOF** system with both DOF along the vertical x-axis.



$$M = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \quad K = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}$$

$$C = \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \quad f(t) = \begin{Bmatrix} f_1(t) \\ f_2(t) \end{Bmatrix}$$

$$x(t) = \begin{Bmatrix} x_1(t) \\ x_2(t) \end{Bmatrix}$$

Figure 2-2: Two-DOF System. Patrick, (2014).

Bridge decks have the highest potential to wave down and up under vehicular traffic dynamic loads. Bridge decks excessive vibration prevention is to ensure that the frequency of applied periodic traffic loading will not be equal to the bridge modal frequency that can lead to the bridge sudden collapse (Clough and Penzien, 1975) . Recommended generally acceptable bridge motion magnitudes for people in normal conditions and in storm conditions after which, bridge horizontal, torsion and vertical modes of vibration intermingle are presented in Figure (2.3), curves 1 and 6 respectively (Awall *et al.*, 2012).

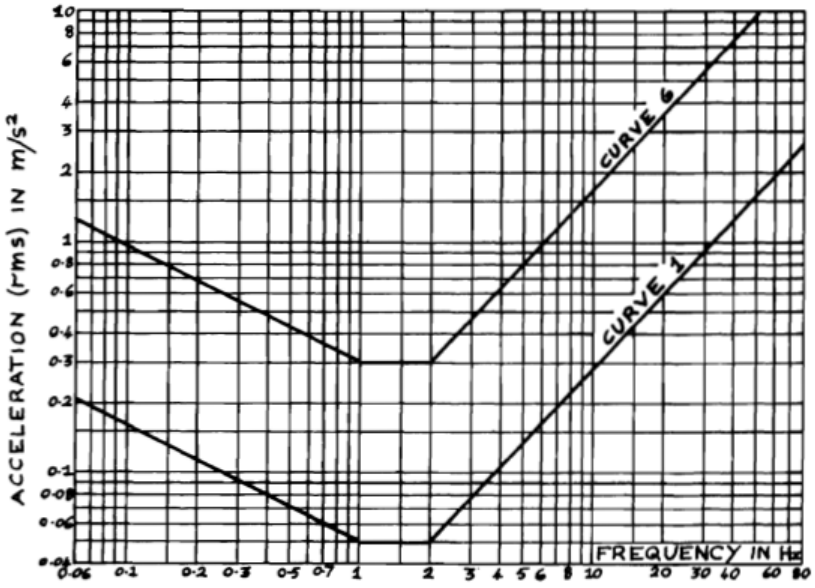


Figure 2-3: Suggested Maximum Magnitude of Vertical Vibrations of Bridges. (Irwin ,1979)

2.3 Modal Analysis

The modes of a structure or system can be obtained from two very different approaches (Anonymous, 1982):

(i) Mathematical Model, and

(ii) Experimental Modal Analysis

In their most basic form, mathematical model "discretise" a structure by breaking it up into hundreds or even thousands of masses and springs. This process can be done by the simple lumped mass and lumped spring approach or it can be done by using finite element approach. Each element in a FE system can be conceptualized as a mass/spring system. This modelling process simply reduces the complicated structure into many mass/spring system. Often this representation is further simplified by selecting a subset of a total problem (called master degree of freedom) and using matrix condensation technique such as Guyan reduction to reduce the size of the problem (for example to 100 degree of freedom). The analytical program then solves an eigenvalue problem to get frequency, mass and mode shape for each mode for the assumed mass and stiffness distribution. Another analysis is then run to define how the structure will response to various dynamic inputs. The common practice is to input variable frequency, unit force at one point while monitoring the response as a function of frequency at several important locations (Anonymous, 1982).

Experimental modal analysis, on the other hand, starts out with the unit forced response data (that is, frequency response function FRF's) and extract the modes of vibration directly from the FRF's without having to make any assumption of mass and stiffness distribution and without having to solve any eigenvalue problem. The experimental modal analysis will end up with a set of modes defined by frequency, mode, damping and residue. Certain assumptions and post-processing of the data is necessary to calculate modal mass or stiffness with even further assumptions and post-processing to obtain physical stiffness or mass representations (Anonymous, 1982).

There are two types of modal analysis to choose from when defining a modal Load Case

2.3.1 Eigenvector Analysis

Eigen vector analysis determines the undamped free-vibration mode shapes and frequencies of the system. These natural modes provide an excellent insight into the behaviour of the

structure. They can also be used as the basis for response- spectrum or time- history analyses (CSI, 2013).

Eigen vector analysis involves the solution of the generalized eigenvalue problem given in equation (2.10).

Each eigenvalue- eigenvector pair is called a natural Vibration Mode of the structure. The Modes are identified by numbers from 1 to *n* in the order in which the modes are found by the program.

2.3.2 Ritz-Vector Analysis

This analysis seeks to find modes that are excited by a particular loading. Ritz-vectors can provide a better basis than eigenvectors when used for response-spectrum or time-history analyses that are based on modal super position (CSI, 2013).

Research has indicated that the natural free- vibration mode shapes are not the best basis for a mode- superposition analysis of structures subjected to dynamic loads. It has been demonstrated by (Wilson *et al*., 1982) that dynamic analyses based on a special set of load-dependent Ritz vectors yield more accurate results than the use of the same number of natural mode shapes. The algorithm is detailed in (Wilson *et al.*, 1982)

The reason why Ritz vectors yield excellent results is that they are generated by taking into account the spatial distribution of the dynamic loading, whereas the direct use of the natural mode shapes neglects this very important information. In addition, the Ritz-vector algorithm automatically includes the advantages of the proven numerical techniques of static condensation, Guyan reduction, and static correction due to higher-mode truncation (Wilson *et al.*, 1982).

2.4 Number of Modes

The maximum and minimum number of modes to be found can be specified. This number includes any static correction modes requested. The CSiBridge (2015) a finite element software may compute fewer modes if there are fewer mass degrees of freedom, all dynamic participation targets have been met, or all modes within the cut off frequency range have been found (CSI, 2013).

Only the modes that are actually found will be available for use by any subsequent response-spectrum or modal time-history Load Cases.

2.6 Natural Frequencies of Bridges

There are a number of computer programs available that use advanced analytical methods to calculate the natural frequencies of bridges. A simple relationship between fundamental flexural frequency and maximum span length, based on experimental investigations of 224 bridges, was developed by (Cantieni, 1983).

Tilly (1986) proposed a similar equation based on testing performed on more than 200 European bridges. An advantage of these functions is that they require a minimum calculating time and therefore reduce the complex mathematical calculations required to find the frequency of bridges with complex geometry. Natural frequency, however, depends on stiffness and mass distributions of the structure and the suggested simple formulae may not be suitable, in many cases.

The Australian Bridge Design Code (1992) also provides a simple frequency equation. This equation is based on the first mode of beam vibration and thus can only be used to estimate the frequency of single span simply supported narrow bridges. The Australian code refers to specialist literature or the use of a computer program for other bridges, especially for those with complicated geometry (Senthilvasa *et al.*, 2002).

Karl and Haiyong (2007) presented that based on the Finite Element Analysis (FEA) natural frequency data, multiple variable nonlinear regression analysis was conducted to develop a set of more rational natural frequency equations. It was desired that the proposed equations should be simple enough to be used in practical situations, as well as for predicting the natural frequencies with acceptable accuracy. As a result, the following equation is suggested to predict the first bending natural frequency, f, of continuous-span bridges.

$$f = \lambda^2 f_{sb} \tag{2.26}$$

where:

$\lambda^2 = \frac{I^c}{L_{max}^b}$ (Natural frequency coefficient)

f_{sb} = natural frequency from simple beam equation, Hz

L_{max} = maximum span length, m

I = average moment of inertia of the composite girder section, m^4

a = 1.44; b = 0.046 and c = 0.032 for two-span bridges

a = 1.49; b = −0.033 and c = 0.033 for three or more-span bridges.

2.7 Causes of Dynamic Motion of Civil Engineering Structures

According to Irwin (1979), Vibration of civil engineering structures can be induced in many ways, resulting in a range of motion intensity and duration and can cause varied degrees of public reaction. The most common sources of time varying forces to which structures may be exposed to include: wind, waves, earthquakes, climatic, mechanical services, activities on adjacent construction sites, neighbouring factory operations, road, rail, sea and air traffic of various forms, and blasting.

Bridges are normally rigid in their horizontal plane, except for light footbridges. Vertical vibrations caused by road or rail vehicles are usually of most significance and can be perceptible both as vibration of complete bridge decks and as local vibrations in the neighbourhood of rail track joints and expansion joints (Irwin, 1979).

2.8 Overview of Code Provisions for Vibration

2.8.1 General Design Codes

The guidance provided in the Australian Standards AS2670-1 (2001) , British Standards BS8110-1(1997), BS5950-4 (1994) and Structural Euro codes, EN1992; EN1993; and EN1994 covering concrete, steel, composite and bridge structures is generic and limited to isolating the vibration source, increasing the damping and limiting frequencies to control the effects of floor vibration induced by human activity (Anonymous, 2015b)

2.8.2 Australian Standard

The Australian Standard that relates directly to vibration is AS2670 (2001). It provides guidance on the evaluation of human exposure to whole-body vibration: Part 1 gives general requirements, while Part 2 treats continuous and shock induced vibration in buildings and presents base curves for acceleration limits.

2.8.3 International Organisation for Standardisation Codes

International Standardisation Organisation (ISO) Codes provide guidelines for occupancy comfort and operating criteria for structures subjected to vibration. Currently there are three ISO publications which are:

(i) General requirements for the evaluation of human exposure to whole-body vibrations in ISO2631-1 (1997).

(ii) Evaluation of human exposure to vibrations in buildings (1-80Hz range) in ISO2631-2 (2003).

(iii) Bases for the serviceability design of building structures and walkways subjected to vibrations in ISO10137 (2007).

ISO2631-1 (1997) suggests the use of frequency-weighting functions to evaluate vibration for human perception/discomfort in both the vertical and horizontal directions. It describes the frequency weighting method and the method of determining the RMS acceleration. This ISO code also refers to the Vibration Dose Value (VDV), which can be considered as the cumulative measurement of the vibration level received over a period of time, and explains the method of determining VDV. ISO2631-2 (2003) has extended requirements compared to ISO2631-1(1997), but it does not provide any guidance on vibration assessment based on acceptable limits.

The latest edition, ISO10137 (2007) provides base curves with acceptable Root Mean Square (RMS) acceleration limits for vibration assessment. As the acceptable vibration level varies with the frequency of the motion, the acceleration needs to be frequency weighted. If the ratio of peak to RMS value of weighted acceleration is greater than 6, use of VDV is suggested as the acceptance criteria for residential building floors. This ISO code also provides damping ratios for various types of floor structures and walkways. It suggests the use of the simplified methods in the practice guides (discussed below) or numerical techniques such as the finite element and boundary element methods for determining the vibration response and acceleration of structures.

2.8.4 British Standards Codes

Currently there are two relevant British Standards. BS6841 (1987) provides general requirements for the measurement and evaluation of human exposure to whole-body vibration and repeated shocks. BS6472-1 (2008) gives guidance on the evaluation of human exposure to building vibrations (1- 80-Hz range). It does not have the base curves and associated multiplying factors given in the previous British Standard publications of BS 6841 (1987). It is recommended that VDV is the only method to evaluate vibration. Acceptable VDV limits for various occupancy classes are also given.

BS 6472-1 (2008) refers to existing methods to calculate vibration response of simple structures such as rectangular plates under harmonic or impulsive loads. It suggests the use

of finite element techniques for other floor structures. Excitation functions for human activities are provided in this standard which also highlights the use of realistic damping based on previous experience with similar floor structures. This standard requires the acceleration to be frequency-weighted using the charts provided. VDV can then be calculated using the formulae in the standard and used to determine the acceptability of the vibration by referring to the base curves in BS6841 (1987).

2.9 Human Perceptibility to Traffic-Induced Bridge Vibrations

People have a high sensitivity to vibrations, so human response to vibrations due to dynamic loads should be considered as a serviceability limit state. Several factors influence human sensitivity and perception of vibration namely: position of the human body, health condition, age, type of activity, excitation source characteristics, exposure time, direction of motion, floor and deck system characteristics, level of expectancy (Murray *et al.*, 1997; Naeim, 1991).

The acceptable level of vibration also depends on the kind of structure: for instance a higher level is acceptable on bridges, with respect to residential buildings, due to the awareness of the presence of wind and traffic and for the exposure limited in time (Wiss and Parmelee, 1974).

A survey in the United States of America Smith (1988) indicated that pedestrians are more sensitive than drivers or passengers to disturbing vibrations, because people inside the vehicles seldom notice the oscillations of the bridges, perhaps because their vehicle's normal vibration obscures these. Moreover pedestrians are less susceptible to the vertical component of vibration when walking than when standing and accept a higher threshold after a period of acclimatization, even with an upper limit (Adriana *et al.*, 2015).

The first analyses on Human Response to Vibration (HRV) were made by Mallock (1902); (Netti *et al.*, 2015) where acceleration was underlined as the main parameter that influences comfort and fixed 1% of gravity acceleration as a threshold. Various studies were carried out that usually resulted in sensitivity curves, usually relating acceleration or displacement with frequency, with different levels of comfort. Smith (1988), presents the work of Irwin (1979), who suggested a base curve for acceptable human response to the vibration of a bridge under frequent forms of loading. Bachmann *et al*. (1995), illustrate the main factors affecting

human sensitivity to vibrations and for vertical vibrations propose the human perceptibility thresholds reported in Table 2.1.

Table 2.1: Human perceptibility thresholds according to Bachmann

Description	Just perceptible	Clearly perceptible	Disturbing	Intolerable
Threshold (g)	0.0034	0.01	0.055	0.18

(Adriana *et al.*, 2015)

British standard for bridge design, BS5400-3 (2000), contains recommendations on this topic valid exclusively in the case of footbridges. The provision of the code is reasonably close to Irwin (1979) suggested curve for storm conditions. Also EN (1990) furnishes indications exclusively for footbridges. The EN (1990) states that the pedestrian comfort criteria for serviceability has to be defined in terms of maximum acceptable acceleration of any part of the deck. The acceptable values of acceleration are 0.7 and 0.2 m/s^2 in the vertical and horizontal directions respectively (Eurocode, 2002).

ISO2631/2 (1989) proposes similar base curves just for residential and office places under transition vibration, without any indications for bridges.

The Spanish standard, similarly according to Bachmann *et al.* (1995) study, fixes more thresholds for each level of comfort, as shown in Table 2.2.

Table 2.2: Human perceptibility thresholds according to Spanish standard.

Description	Easily perceptible	Clearly perceptible/disturbing	Very disturbing
Threshold (g)	0.025	0.075	0.125

(Adriana *et al.*, 2015)

In majority of the standards previously described, it has been generally accepted that the acceleration is the vibration parameter which should be used to describe the exposure condition.

The main researches and standard previously exposed are compared in Figure 2.4.

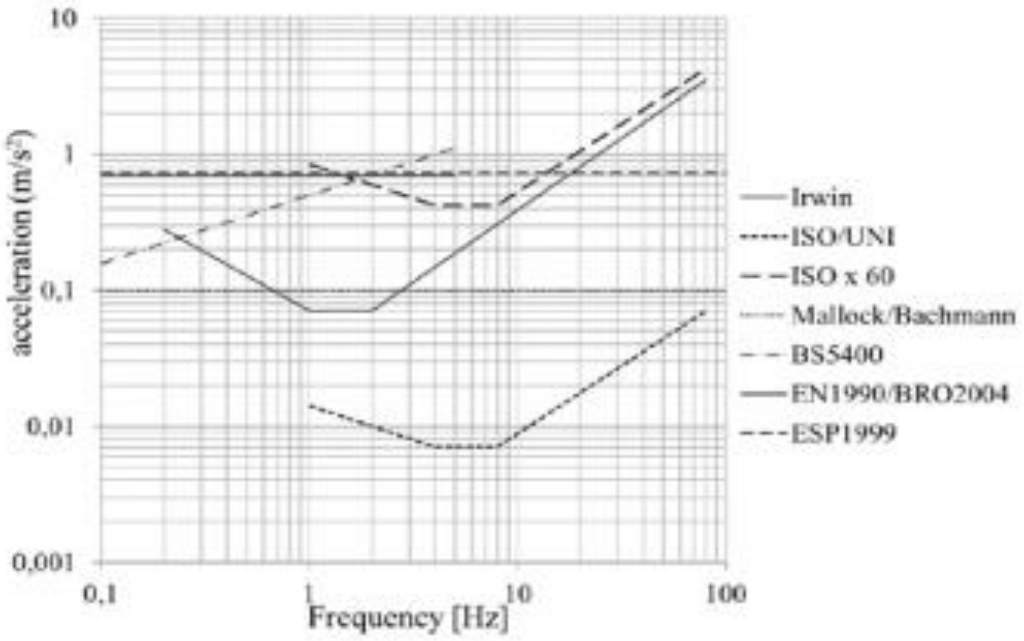


Figure 2.4: Comparison of the various literature and standard thresholds (Adriana *et al.*, 2015)

2.10 Finite Element Method: The Big Picture

Application of physical principles, such as mass balance, energy conservation, and equilibrium, naturally leads many engineering analysis situations into differential equations. Methods have been developed for obtaining exact solutions for various classes of the differential equations. However, these methods do not apply to many practical problems because either their governing differential equations do not fall into classes or they involve complex geometries. Finding analytical solutions that also satisfy boundary conditions specified over arbitrary two – and three – dimension regions becomes a very difficult task. Numerical methods are therefore widely used for solution of practical problems in all branches of engineering (Bhatti, 2005).

The finite element method is one of the numerical methods for obtaining approximate solutions of ordinary and partial differential equations. It is especially powerful when dealing with boundary and conditions defined over complex geometries that are common in practical applications. As such, because of its versatility in handling arbitrary domains and availability of sophisticated commercial finite element software, over the last few decades, the finite element method has become the preferred method for solution of many practical problems.

CHAPTER THREE

MATERIALS AND METHODS

3.1 Description of Bridge Model

The proposed bridge model is made up of 16 spans of 18m each. The total length of the bridge is 288m. The superstructure consists of eight (8) precast concrete T- beams of section 1160 mm deep x 350 mm web width x 17950 mm long. The carriageway is 8m width, flanked by 0.7m safety Kerb with a precast parapet wall on both sides. It is overlain with 50mm layer of asphaltic concrete surfacing. Figure 3.1 shows section of the bridge superstructure.

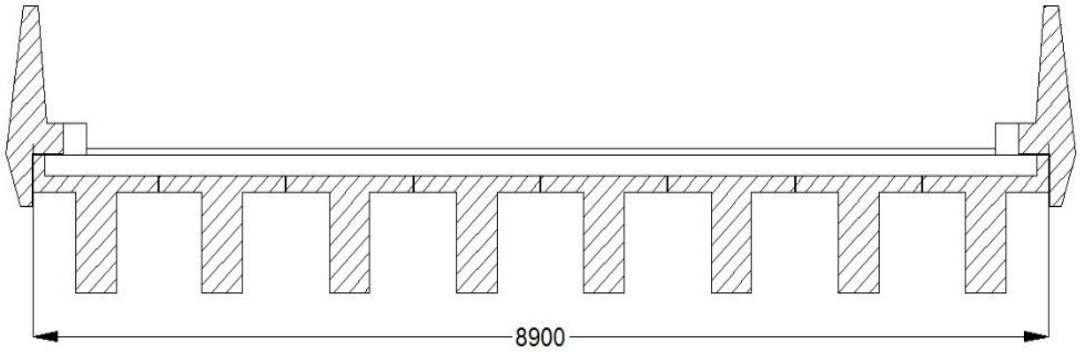


Figure 3.1: Bridge Super structure

The pier substructures are of the hammerhead type with vertical grooved surfaces, while the footing is supported on diameter 600mm rotary bored piles. The abutment, which is of the bank seat type, occupies the extremities of the flyover. The retaining walls on both ends ramped the bridge to the ground levels.

3.2 Material

3.2.1 CSiBridge (2015)

The software is an ultimate, easy-to-use, integrated finite element software program for modelling, analysis, and design of bridge structures. Considering the flexibility of the package, the software will be used to model and conduct analysis of the bridge.

A **CSiBridge** model is defined parametrically. The modelling process begins by specifying the basic layout of the bridge (that is, the length, bearing, alignment, spans, number of lane, lane widths and slopes). Bridge components are then defined: deck sections, abutments, diaphragms, bents, foundation springs, and so on. These components are located relative to the layout to assemble a complete bridge object. Applied loading, load cases, and

superstructure and substructure design, and load ratings are defined for the assembled bridge object. Editable templates and defaults based on bridge-related codes are used to speed model building, analysis, and design. A detailed model of fundamental geometric objects is generated automatically from the bridge object. For unique needs, Advanced modelling options can be used to modify those fundamental geometric objects using Draw, Edit and Assign commands (that is, the fundamental point, line, and area objects are selected so that materials, properties, loads and other parameters can be assigned to them).

In order to analyse the modal parameters of the bridge as built, information provided in the bridge approved drawing obtained from design and implementation office of Kaduna State Ministry of Works and Transport shall be used to model and analyse the bridge. Secondly, the bridge will be re modelled with curved beams. Modal analysis will be conducted on the two models and their modal parameters analysed. Modal parameters such as: modal frequency, modal damping and modal stiffness will be studied in order to understand the best section that offers low vibration.

3.3 Methods

3.3.1 Finite element analysis

Finite Element Analysis (FEA) is a computerized method for predicting how a product reacts to real-world forces, vibration, heat, fluid flow, and other physical effects. FEA shows whether a product will break, wear out, or work the way it was designed. It is called analysis, but in the product development process, it is used to predict what is going to happen when the product is used (Saeed, 1999). It also refers to the explicit treatment of uncertainties in any quantity by entering the corresponding deterministic analysis at a specific period of time. The exact values of these quantities are usually unknown because they cannot be precisely measured (Klieber and Hein, 1992). It is used to describe the merging of advanced reliability methods with the finite element methods which are generally applicable to both linear and non-linear problems.

3.4 Steps in Creating Bridge Object Model in CSiBridge Software

The CSiBridge software operates on the concept of object-based modelling which largely eliminates the need for meshing physical models into smaller finite elements for analysis purposes. While running analysis, CSiBridge automatically converts object-based model into

an element-based model that is used for analysis (CSI, 2015). This element-based model is called the analysis model, and it consists of traditional finite elements and joints (nodes). Results of the analysis are reported back on the object-based model.

The term “element” is used more often than “object”, since what is described herein is the finite-element analysis portion of the program that operates on the element-based analysis model. However, it should be clear that the properties described in the CSiBridge Software for elements are actually assigned in the interface to the objects, and the conversion to analysis elements is automatic.

3.4.1 Layout line

The first step involves defining the layout line. They are reference lines used for defining the horizontal and vertical alignment of the bridge and the vehicle lanes. They are defined using stations for distance, bearings for horizontal alignment and grades for vertical alignment.

Figures 3.2, 3.3 and 3.4 show the general layout data form, profile data for curve and straigth beams respectively. Elevations were used for models with straigth beams while parabolic profiles were used to represent the curved beams according to CSI (2013).

Initial station for the model is 0m while the end station is 288m according to the adopted drawing of the the bridge.

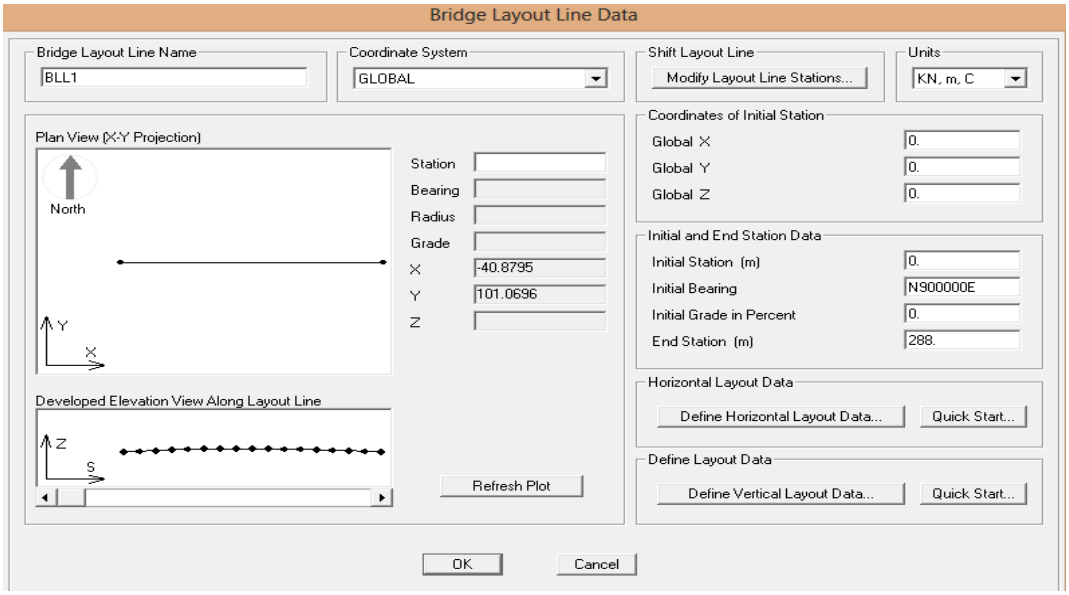


Figure 3-2: Data Form Defining Span of Bridge

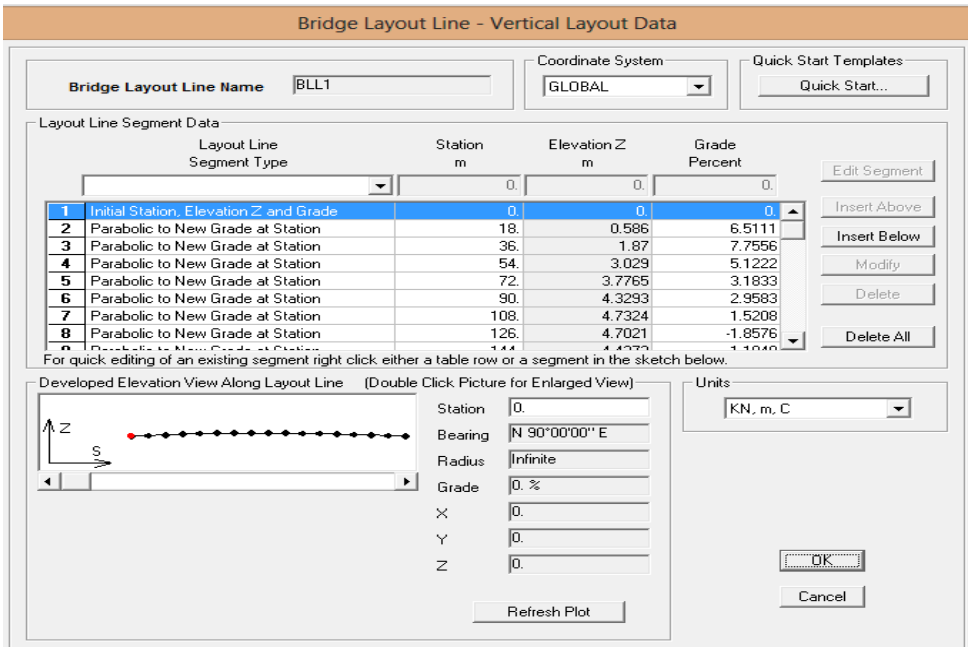


Figure 3-3: Data Form for Curved Beam Profile

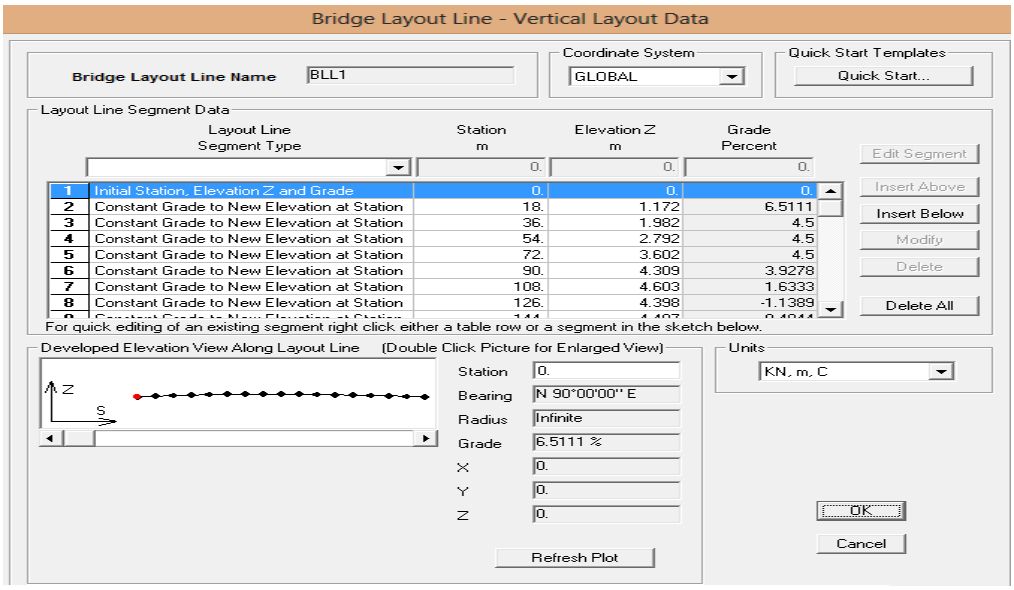


Figure 3-4: Data Form of Straight Beam Profile

The bridge lane data form presented in Figure 3.4 is used for defining the lane widths in both directions of the model. The layout tab, also containing the width of lanes are defined including the descritization criteria, which is 3.04 along and across the lane.

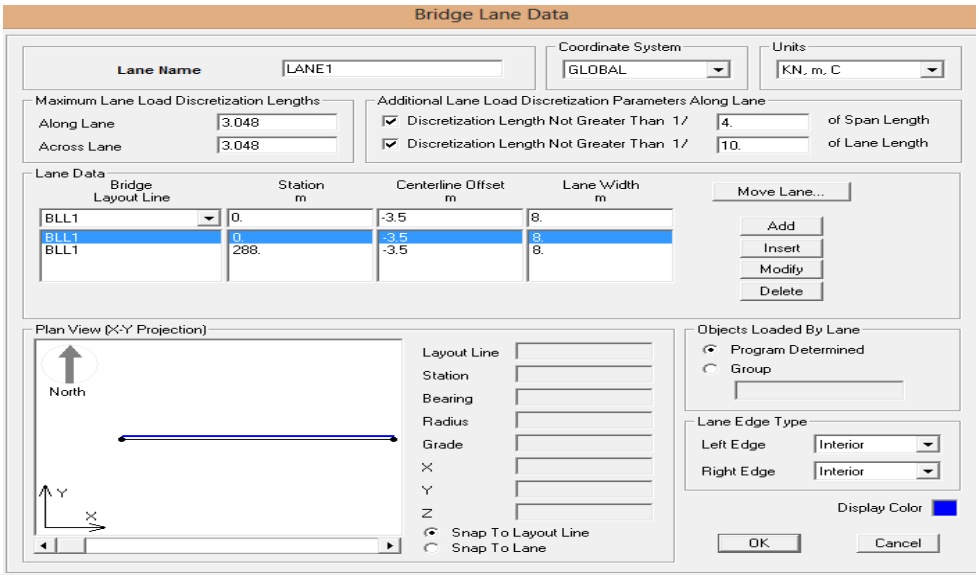


Figure 3.5: Lane Data Form

3.5 Basic Properties

3.5.1 Definition of Basic properties

Material properties were predefined through the bridge wizard. These properties include grade of concrete for precast girder, deck, abutment, pier, and bent cap. Several default material properties are automatically defined when a model is first created.

3.5.2 Frame section

Dimensions of various structural elements such as pier, bent cap and abutment were specified through the component tab as shown in Figure 3.6.

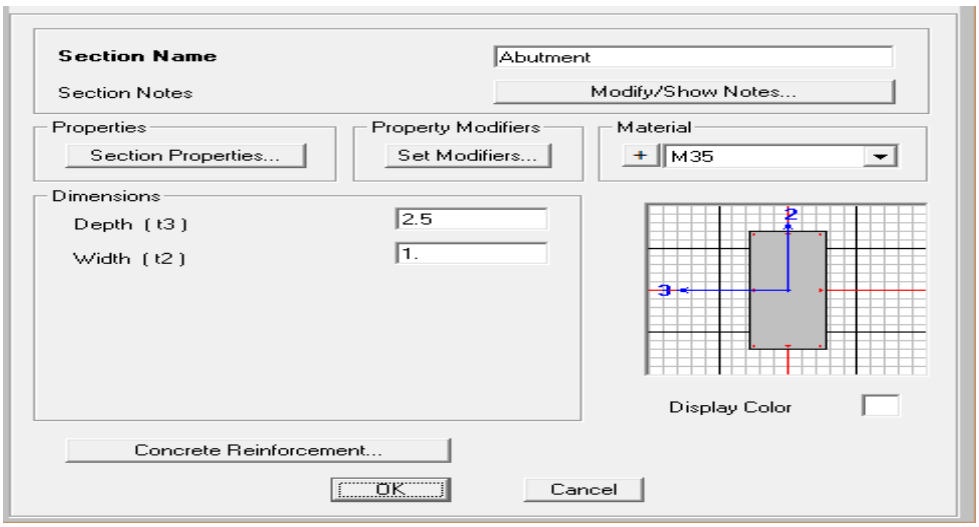


Figure 3.6: Data Form Defining Structural Element

Frame section properties can be defined at the time they are needed, or, they can be predefined.

Dimensions of structural element used are as given below:

Pier = 2800x1200mm

Bent cap =1200x1500mm

Abutment = 2500 x 1000mm

3.6 Bridge Component Properties

3.6.1 Deck section

The bridge deck section is defined using the item tab, in component section. Figure 3.7 shows the bridge section data form from which the type of bridge deck is selected and dimensioned accordingly.

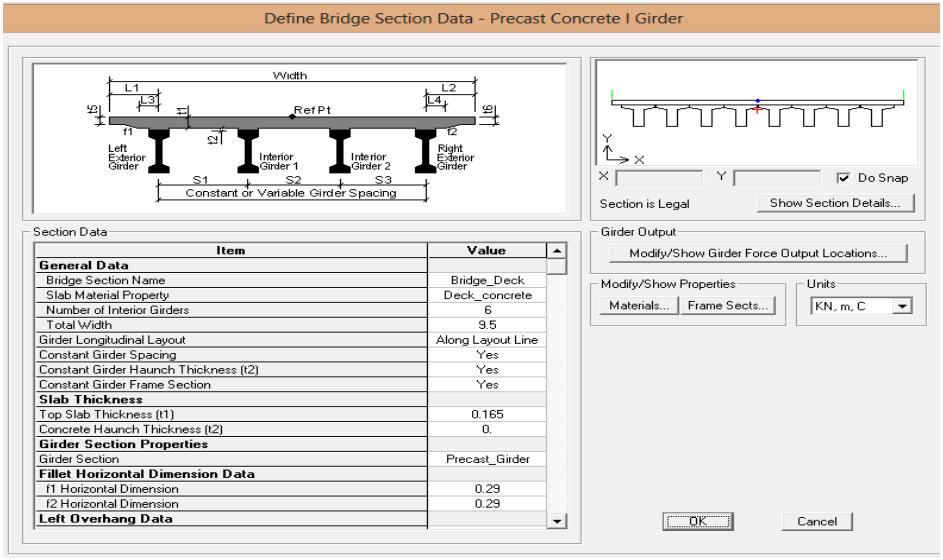


Figure 3-7: Bridge Deck Type and Dimension

Various parametric deck sections are available. Precast concrete I girder is selected for this work.

3.7 Lane and Vehicle Definitions

In this section the lane, vehicle and vehicle classes are defined.

Vehicle type $HS_n - 44$ was selected as prescribed for the adopted bridge model.

3.7.1 Load patterns

Load pattern related to the model were assigned as shown in Figure 3.8

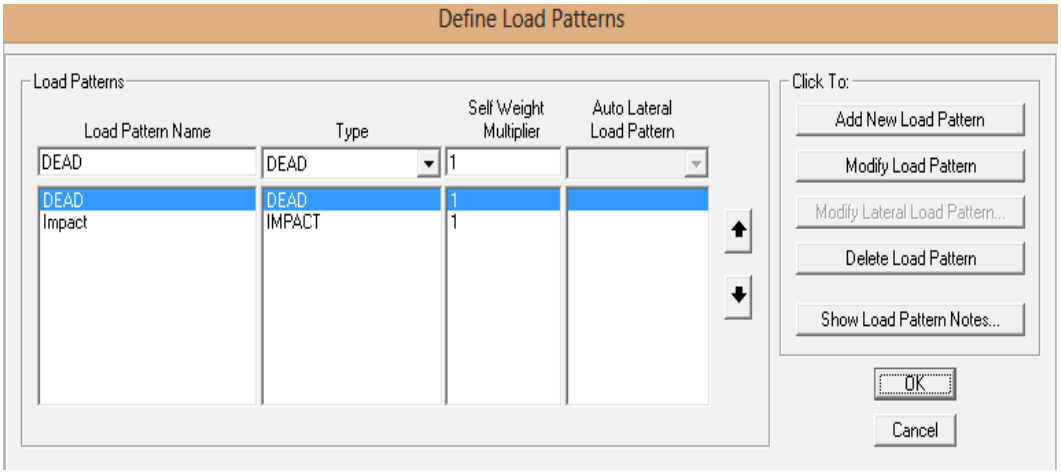


Figure 3-8: Load Pattern Form

3.7.2 Bridge object definition

The bridge object definition is the main component of the bridge modeller.

In this section, deck section properties were assigned to each span, abutment assign to ends of the bridge, bents assign to each bent location and finally bridge span were clearly defined as shown in Figure 3.9.

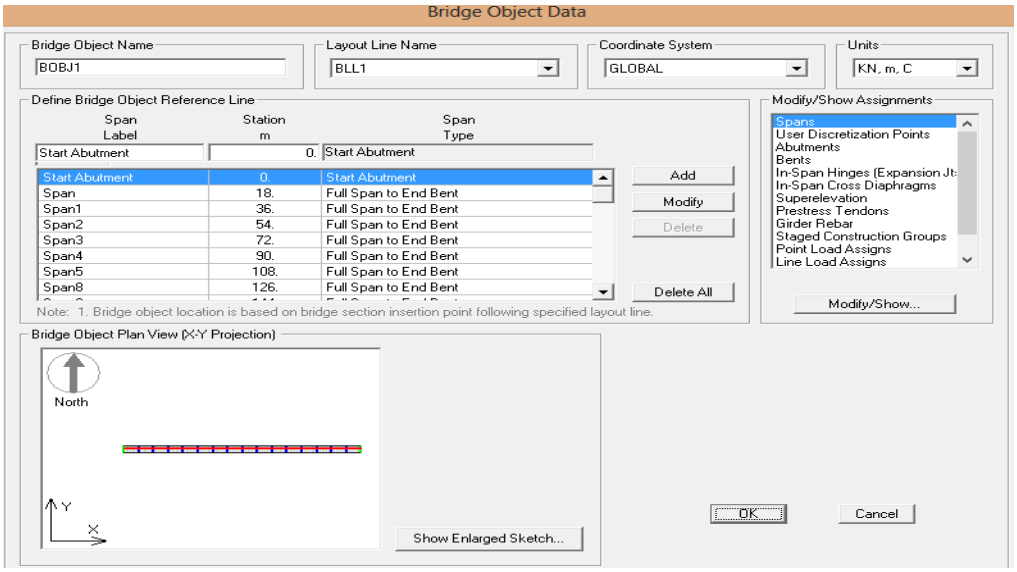


Figure 3.9: Bridge Object Data Form

3.8 Updated Linked Model

After defining bridge object as stated earlier the update and link tab is used to link all bridge components earlier defined to create a model as shown in Figure 3.10.

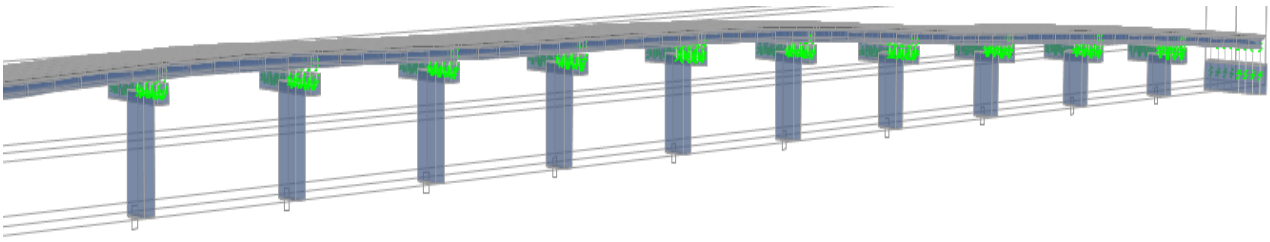


Figure 3.10: 3D View of Updated Bridge Model

3.9 Analysis of Model

In this section, the load case analysis type is selected, which include, modal, time history and response spectrum analysis.

3.9.1 Modal analysis

Eigen vector type of modal analysis is adopted in this work. It involves the solution of generalised eigenvalue problem as shown in equation (3.1)

$$([k] - \omega^2[M])[\emptyset] = 0 \quad (3.1)$$

3.9.2 Time History Analysis

This a step by step type of analysis use for the determination of the dynamic response of a structure under specified loading as presented in equation (3.2) (CSI, 2013).

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = F(t) \quad (3.2)$$

3.9.3 Response Spectrum Analysis

Is a statistical type of analysis use for the determination of the likely response of a structure under seismic loading as represented in equation (3.3) which is dynamic equilibrium equations associated with the response of a structure to ground motion (CSI, 2013).

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = m_x\ddot{x}_{gx}(t) + m_y\ddot{x}_{gy}(t) + m_z\ddot{x}_{gz}(t) \quad (3.3)$$

Where:

m_x, m_y , and m_z are the unit acceleration Loads

$\ddot{x}_{gx}$, $\ddot{x}_{gy}$ and $\ddot{x}_{gz}$ are the components of uniform ground motion

Having identified vertical component of acceleration as the influencing parameter in vibration analysis, only the component $m_z\ddot{x}_{gz}(t)$ was considered in the analysis.

3.10 Bridge Response to Varying Vehicular Speed

Vehicle model $H_n s - 44$ was adopted to study the response of the bridge at various vehicular speed because it was the same vehicle class specified in the structural drawing adopted. The Finite element model of the design vehicle is shown in Figure 3.11.

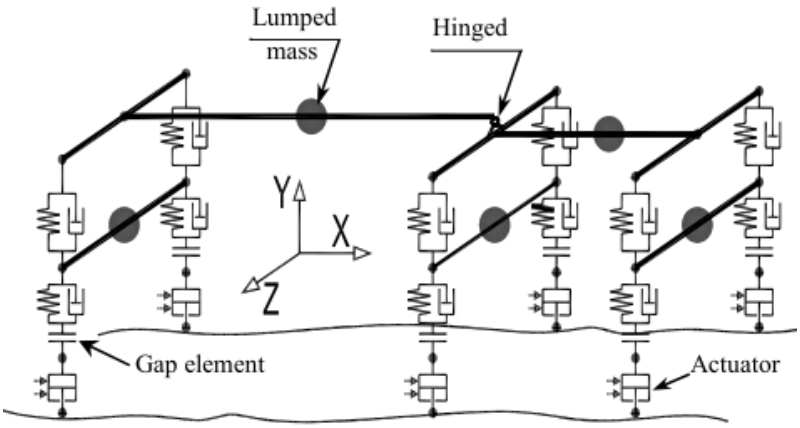


Figure 3.11: Finite Element Model of HS20-44 Design Truck (Awall *et al.*, 2012)

The bridge was simulated for various speeds, and in each case three vehicles were allowed to pass in the following sequence:

(i) First vehicle accesses the bridge on lane 1 at 10km/sec at starting distance of 0m,

(ii) Second vehicle accesses the bridge on the same lane with the same speed after 2sec,

(iii)Third vehicle accesses the bridge with a starting distance 288m on lane 2 at 10km/sec.

The time history analysis was run using a linear direct history analysis load case, the response of the following speeds combination for three vehicles were recorded.

(i) 20, 20, 20km/h, (ii) 30, 30, 30km/h (iii) 40, 40, 40km/h (iv) 50, 50, 50km/h

(v) 45, 50, 60km/h (vi) 60, 65, 70km/h. This is represented pictorially in Figure 3.11a.

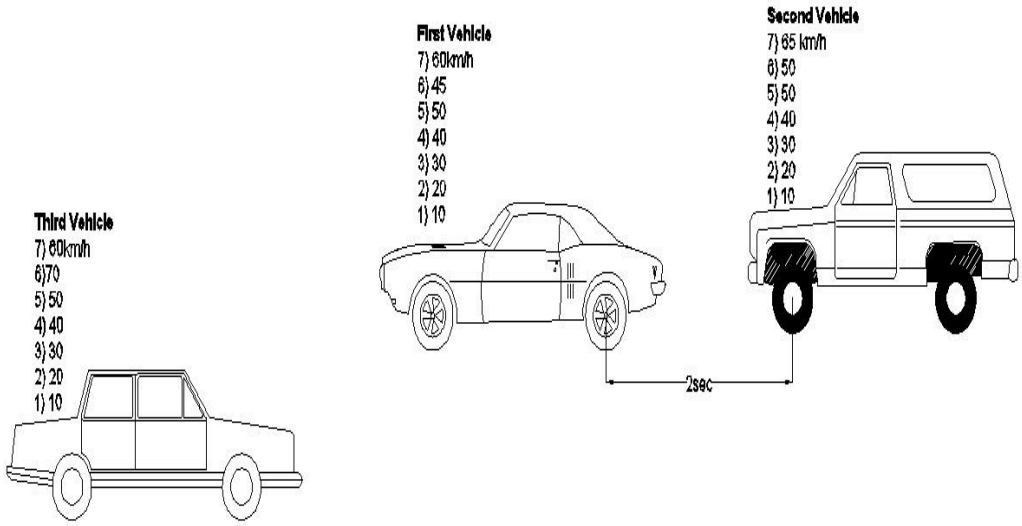


Figure 3.11a: Arrangement of vehicles passage sequence with speed

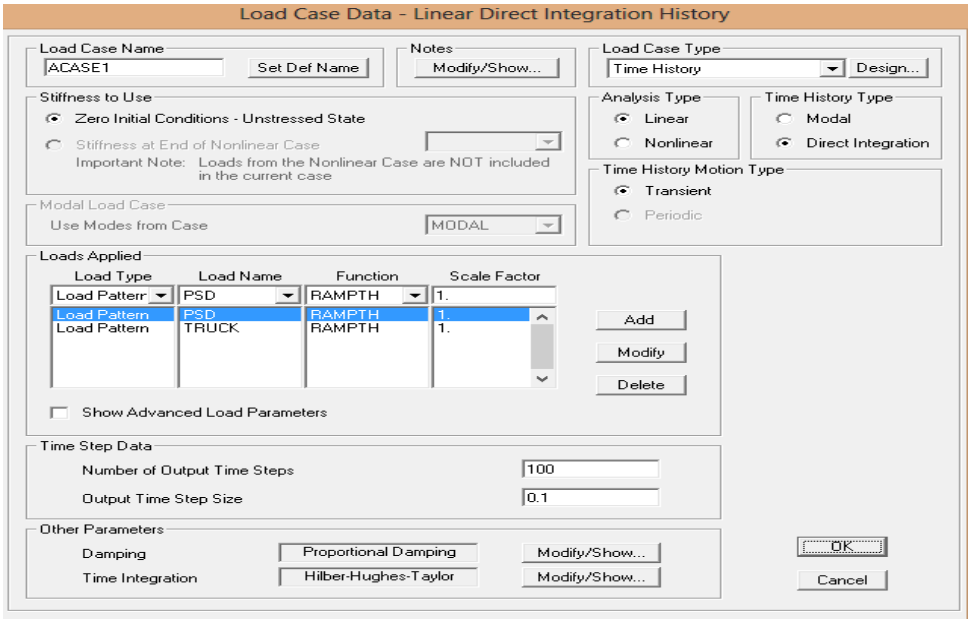


Figure 3.12: Time History Data Form

The model was simulated for loading duration of 24sec and discretised for 0.1 sec, with 100 number of output time steps having output time step size of 0.1sec as shown in Figure 3.12.

CHAPTER FOUR

RESULTS AND DISCUSSIONS

4.1 Results of Modal, Time history and Response Spectrum analysis

This section presents the results obtained from the analysis carried out on three models of VCCFB using the CSiBridge (2015) software. Twelve modes were recorded for each of the models of the three models, vertical component of acceleration at varying vehicular speed and damping ratios were also recorded. Also, bridge- vehicle interaction were recorded for a period of 10 seconds at varying speed.

4.1.1 Modal Analysis Result

Appendixes 1, 2 and 3 present record of natural frequencies and periods of different mode for model profiles achieved using curved, straight and combination of straight and curve beams respectively.

Relationship for the natural frequencies of the profiles are presented in Figure 4.1

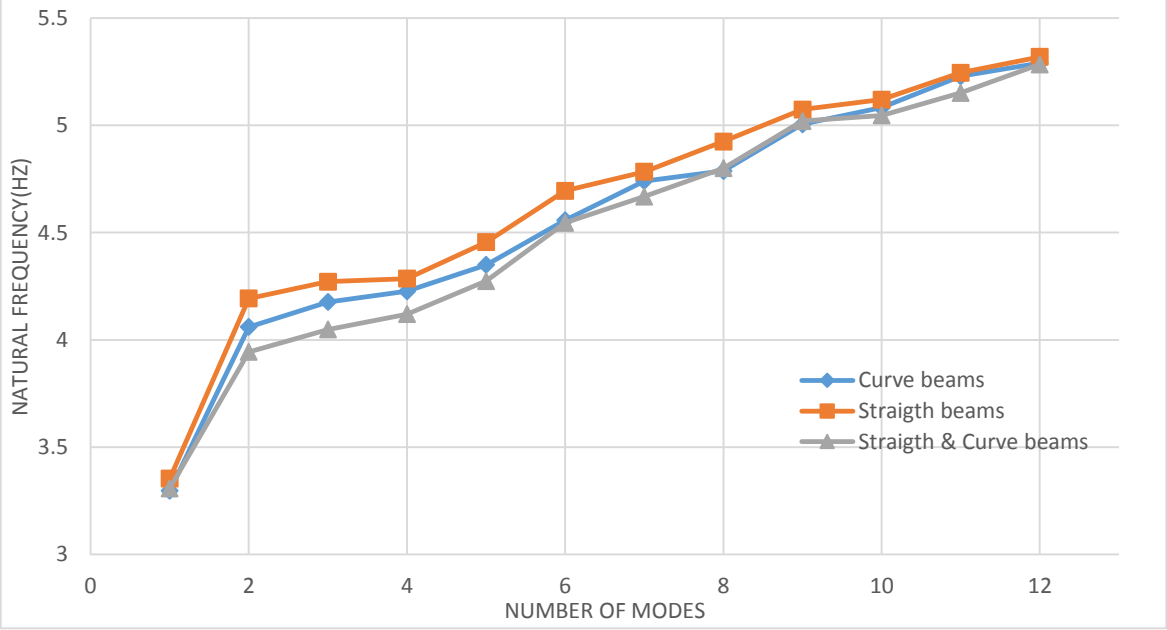


Figure 4-1: Relationship of Natural frequency to Number of mode for curved, straight and Straight-Curve beams

Variations in natural frequencies are presented in Tables 4.1 and 4.2 for models containing straight-curve beams and straight- straight/curve respectively.

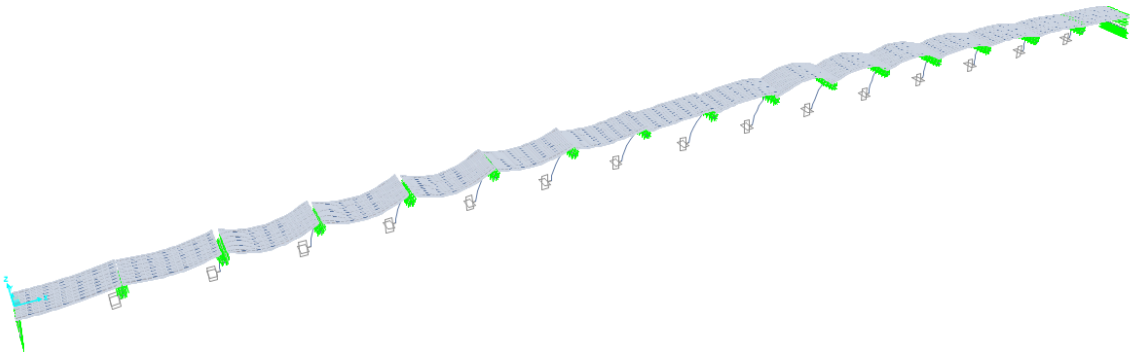
Table 4-1: Differences in frequency between models with curved beams and straight beam

Mode No	Frequency of curved beam (Hz)	Frequency of straight beam (Hz)	Difference in percentage (%)
1	3.296259	3.353384	1.733025
2	4.060484	4.193029	3.264266
3	4.176786	4.271131	2.258794
4	4.226323	4.28547	1.399491
5	4.349716	4.455593	2.434113
6	4.557598	4.694582	3.005618
7	4.740383	4.783542	0.910454
8	4.787311	4.924665	2.869126
9	5.005267	5.074043	1.374073
10	5.08293	5.120001	0.729323
11	5.228857	5.245227	0.31307
12	5.291043	5.319221	0.53256

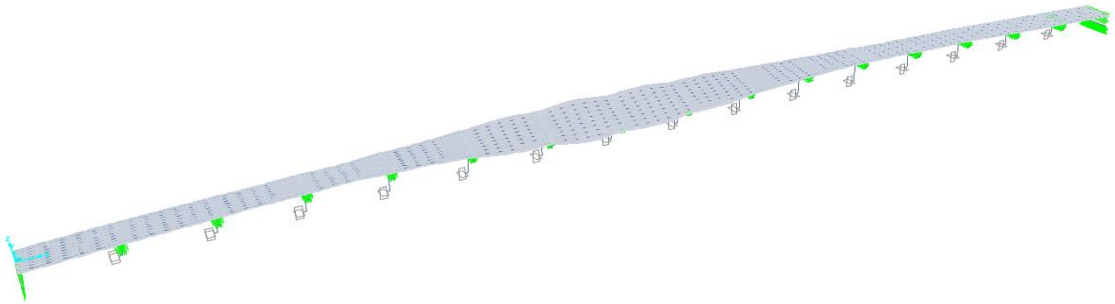
Table 4.2: Differences in frequency between models with combine curved and straight beams

Mode No	Frequency of combine curve and straight beam (Hz)	Frequency of straight beam (Hz)	Difference in percentage (%)
1	3.3074	3.353384	1.390337
2	3.944	4.193029	6.314123
3	4.048	4.271131	5.512129
4	4.1197	4.28547	4.023837
5	4.2743	4.455593	4.241466
6	4.5459	4.694582	3.270683
7	4.668	4.783542	2.475193
8	4.8002	4.924665	2.592913
9	5.0202	5.074043	1.072527
10	5.0461	5.120001	1.464517
11	5.1511	5.245227	1.827318
12	5.2843	5.319221	0.660844

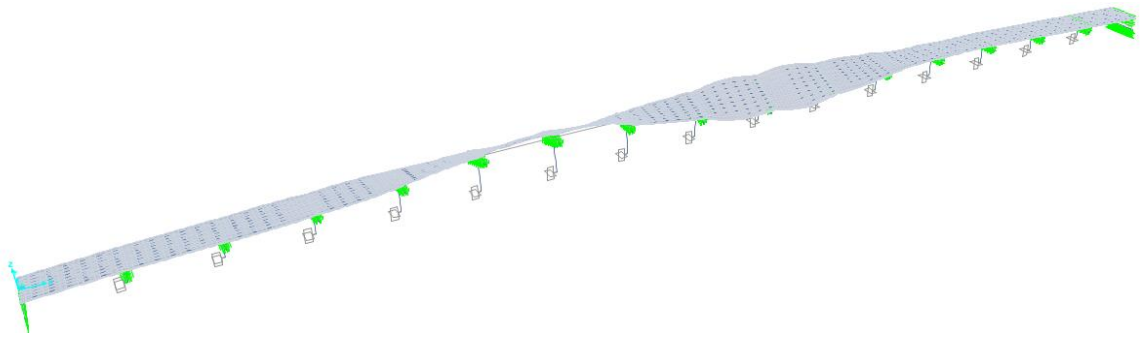
Pictorial representation of mode shapes from mode 1 to 12 for the profiles considered are presented in Figures 4.2, 4.3, and 4.4 each with their respective natural frequencies.



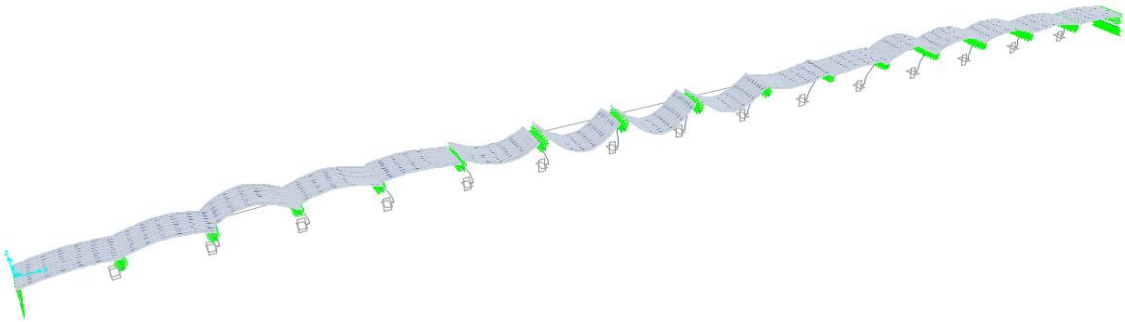
Mode 1; Frequency 3.35Hz



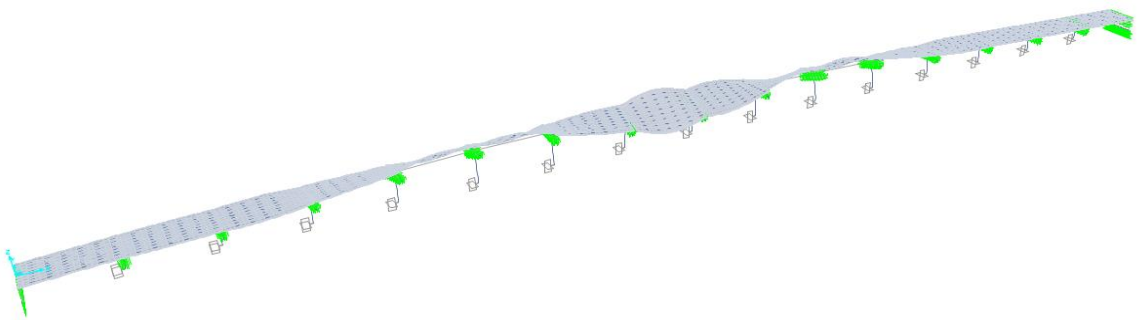
Mode 2 ; Frequency 4.19Hz



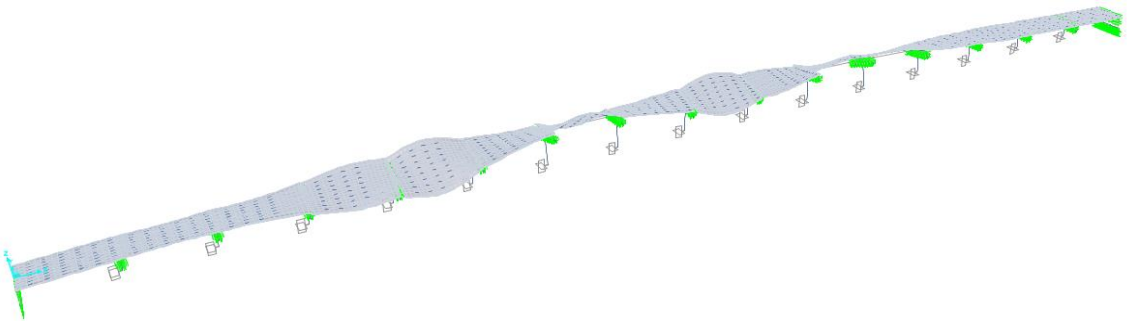
Mode 3; Frequency 4.27Hz



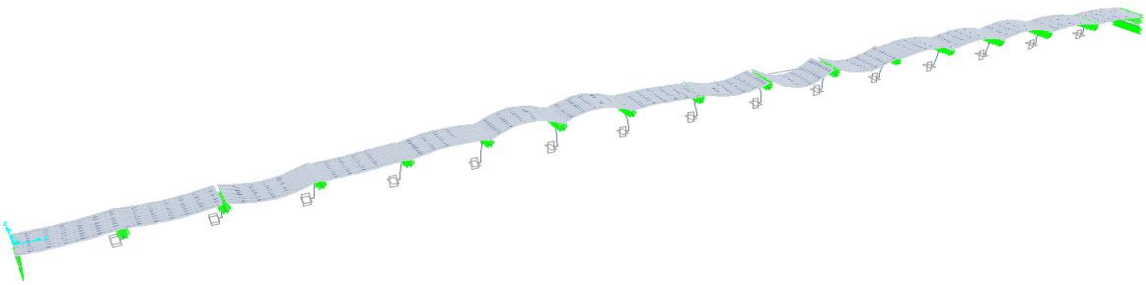
Mode 4; Frequency 4.29Hz



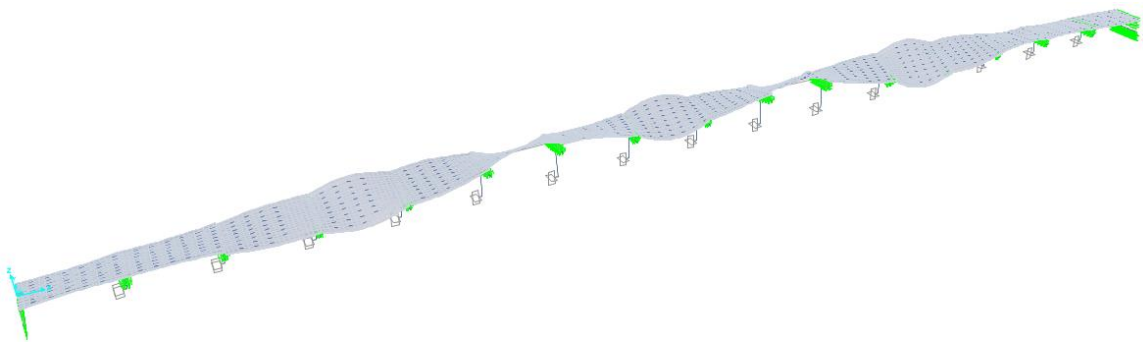
Mode 5; Frequency 4.46Hz



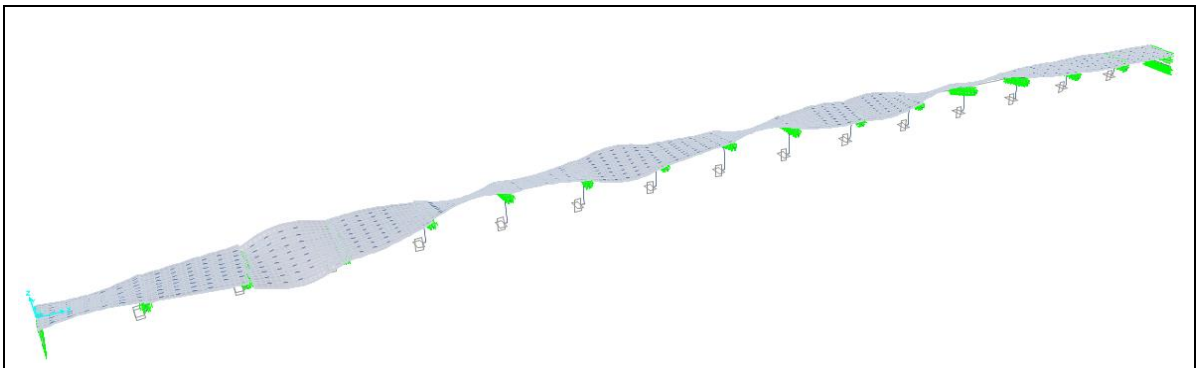
Mode 6; Frequency 4.69Hz



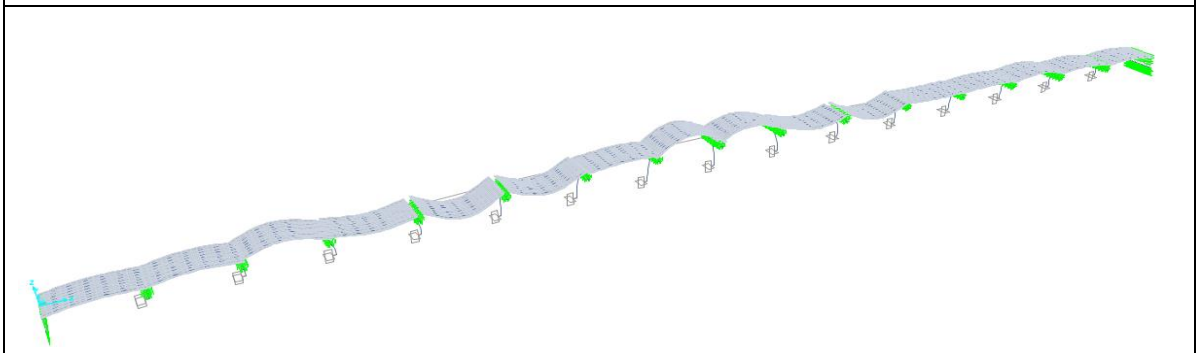
Mode 7; Frequency 4.78Hz



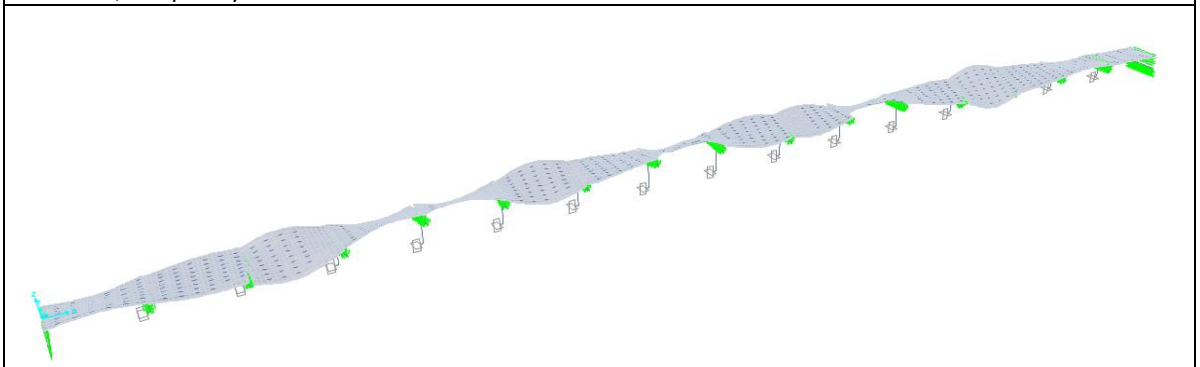
Mode 8;Frequency 4.92Hz



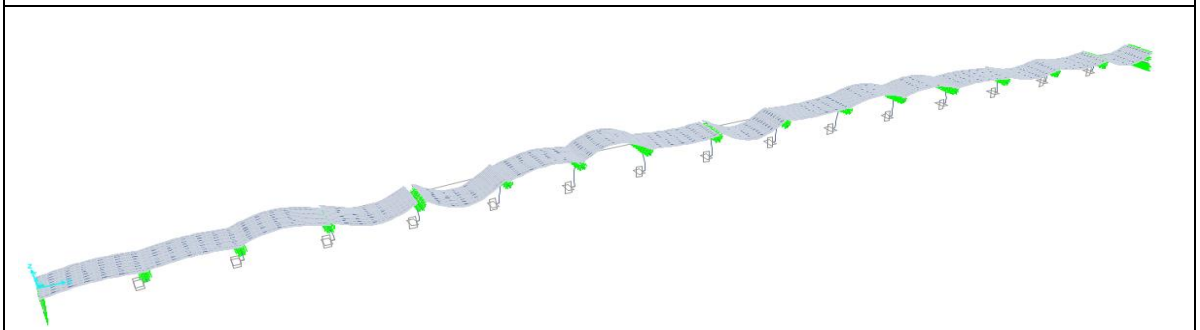
Mode 9; Frequency 5.07Hz



Mode 10; Frequency 5.12Hz

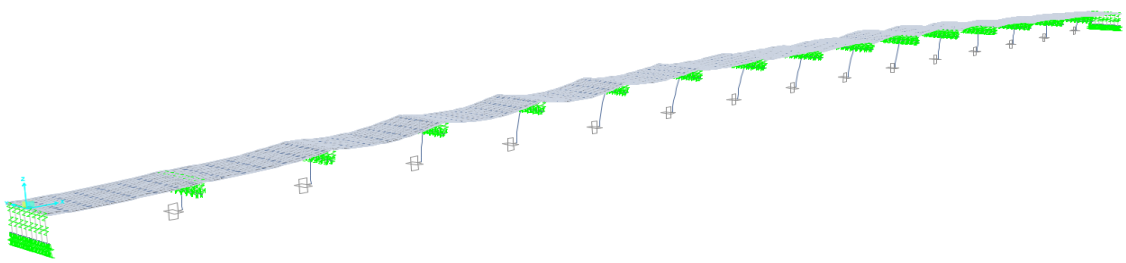


Mode 11; Frequency 5.25Hz

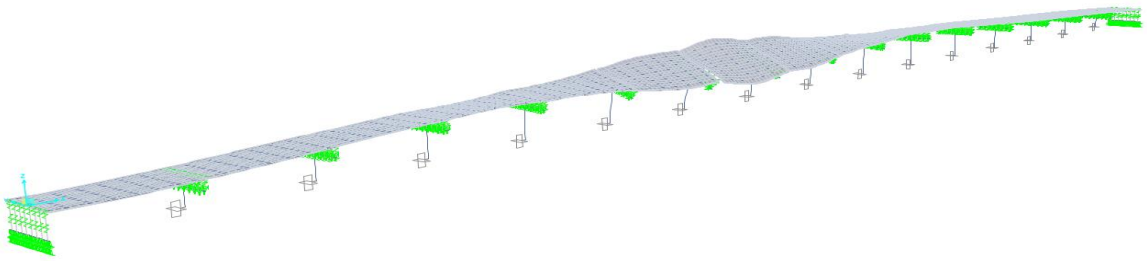


Mode 12; Frequency 5.32Hz

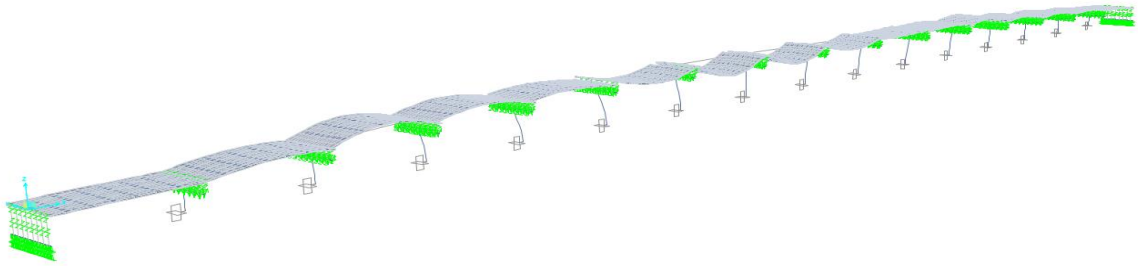
Figure 4-2: Mode Shapes for Model with Straight Beams



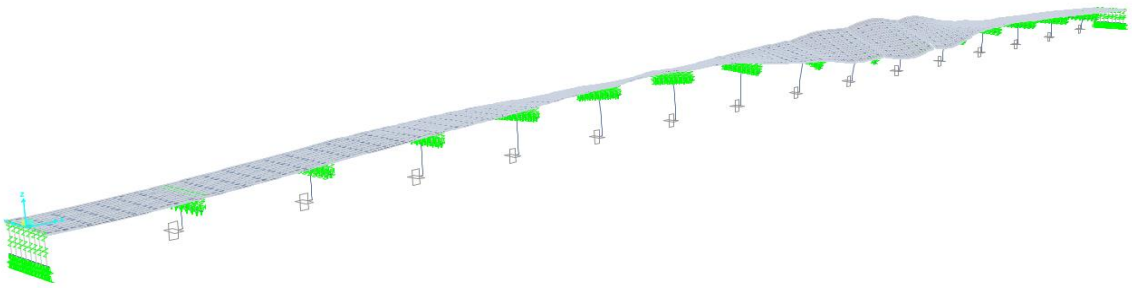
Mode 1, Frequency 3.30Hz



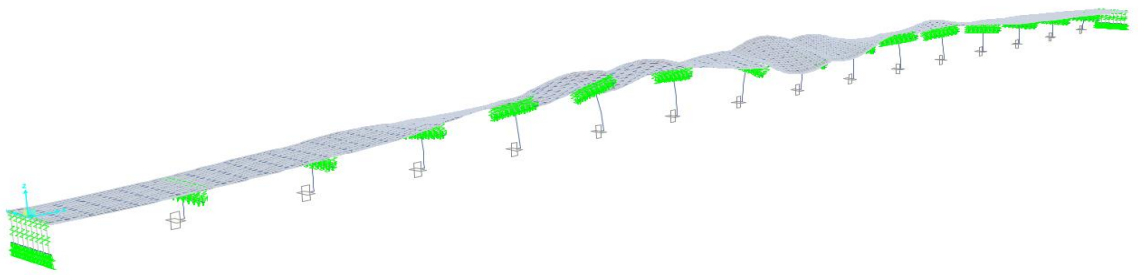
Mode 2 ; Frequency 4.06Hz



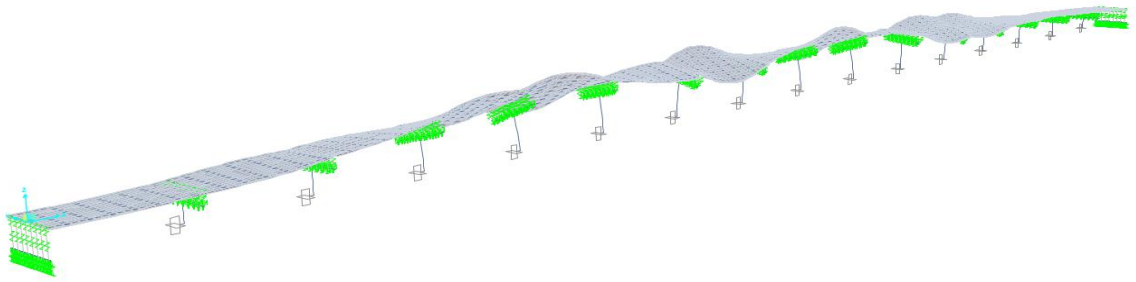
Mode 3; Frequency 4.18Hz



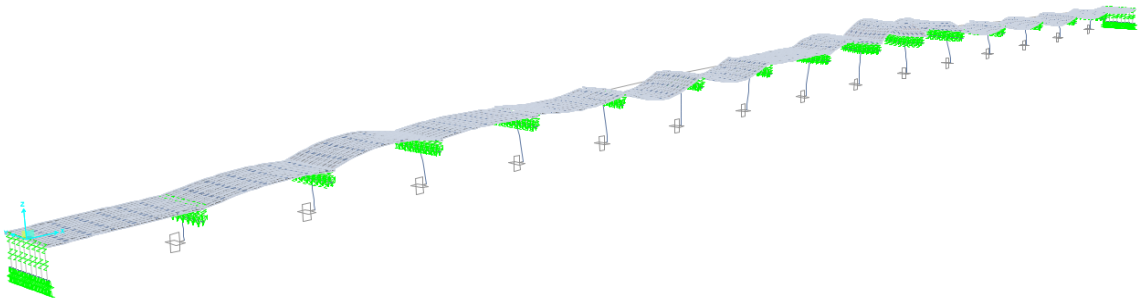
Mode 4, Frequency 4.23Hz



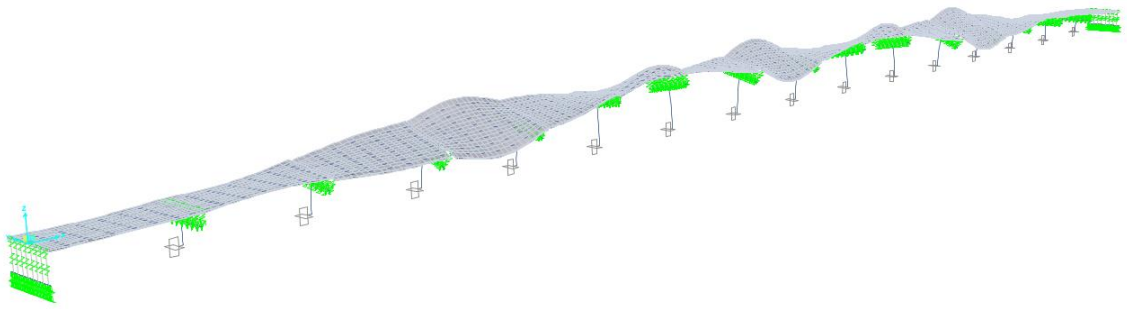
Mode5; Frequency 4.35Hz



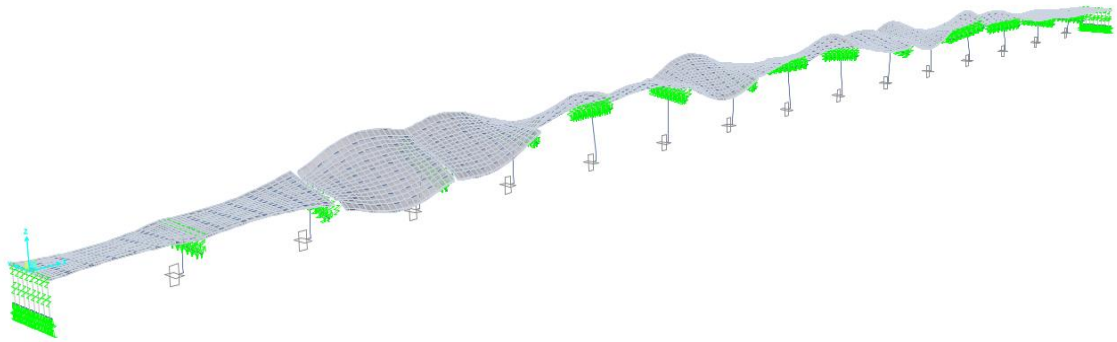
Mode 6; Frequency 4.56Hz



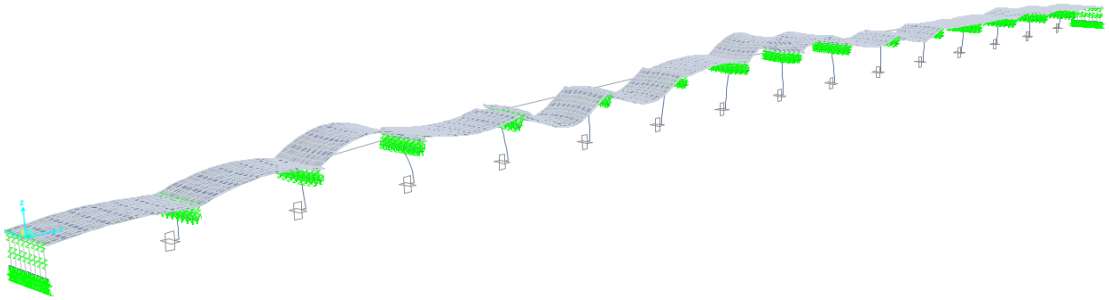
Mode 7; Frequency 4.74Hz



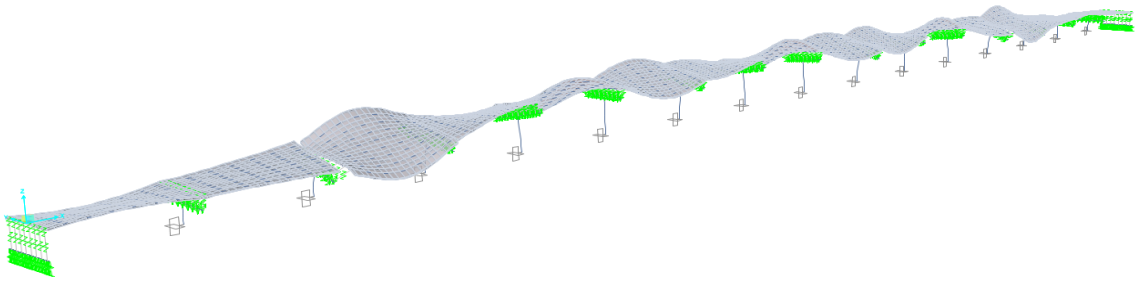
Mode 8 ;Frequency 4.79Hz



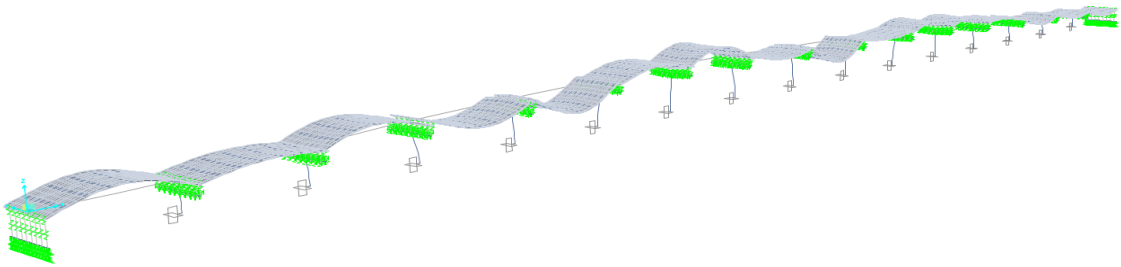
Mode 9; Frequency 5.01Hz



Mode 10 ; Frequency 5.08Hz

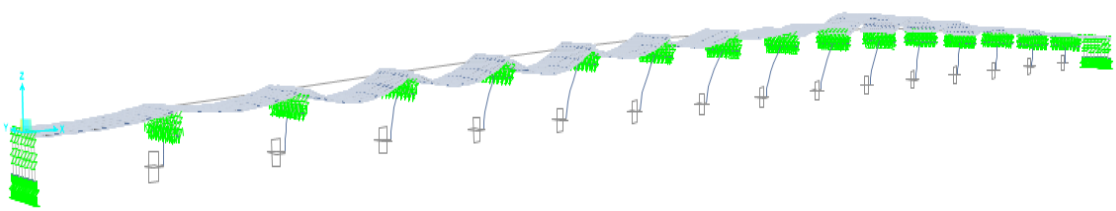


Mode11 ; Frequency 5.23Hz

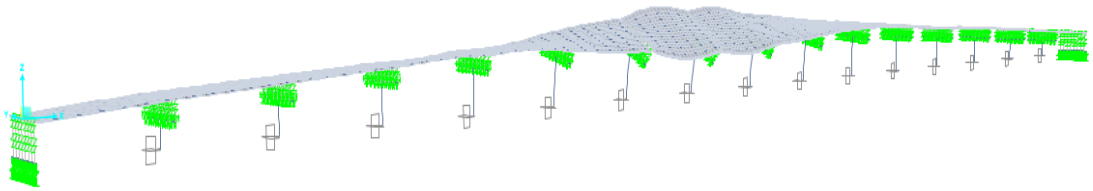


Mode 12 ; Frequency 5.29Hz

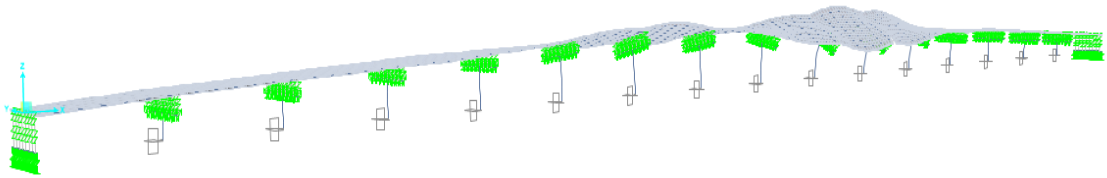
Figure 4.3: Mode Shapes for Model with Curved Beams



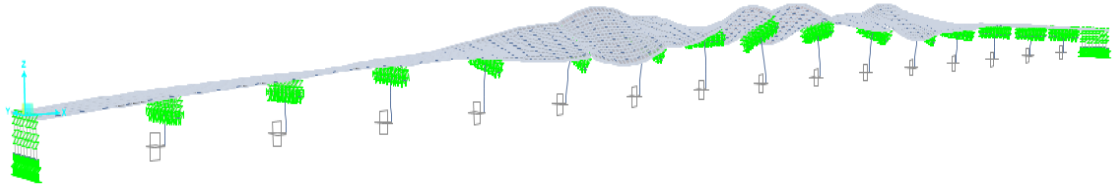
Mode 1,f=3.31Hz



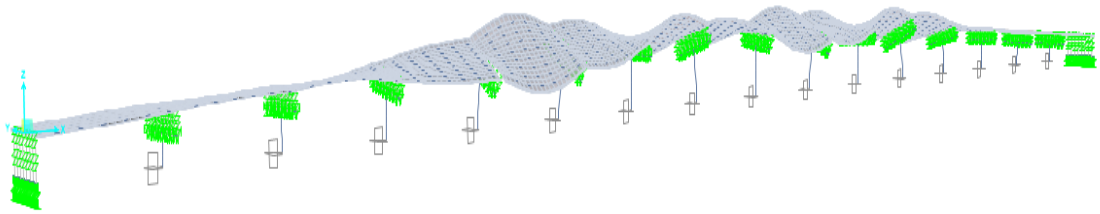
Mode 2, f=4.94Hz



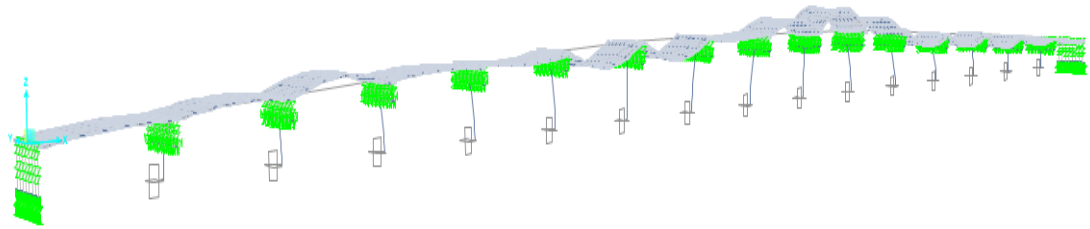
Mode 3, f=4.05Hz



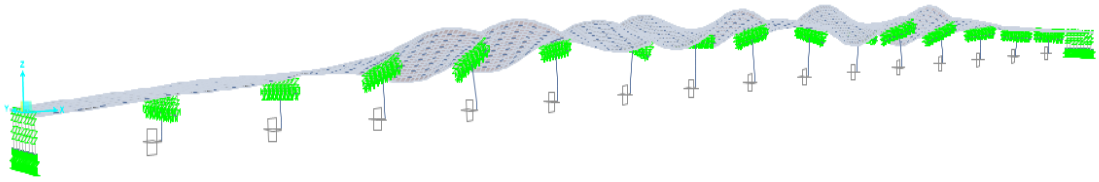
Mode 5, f=4.27Hz



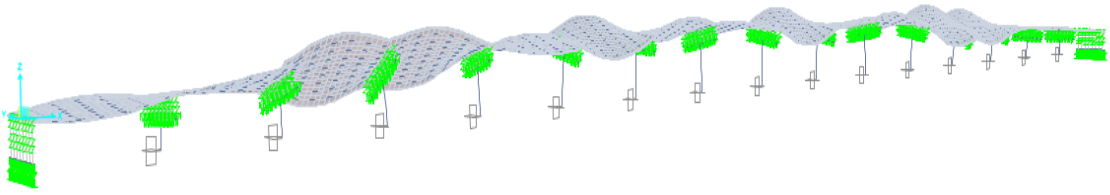
Mode 6, f=4.55Hz



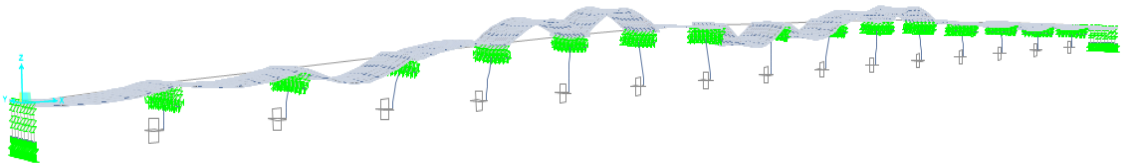
Mode 7, f=4.67Hz



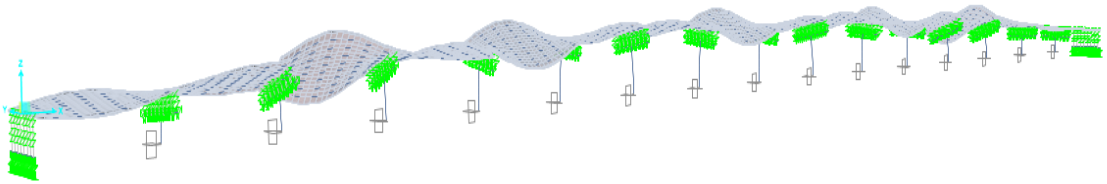
Mode 8, f=4.80Hz



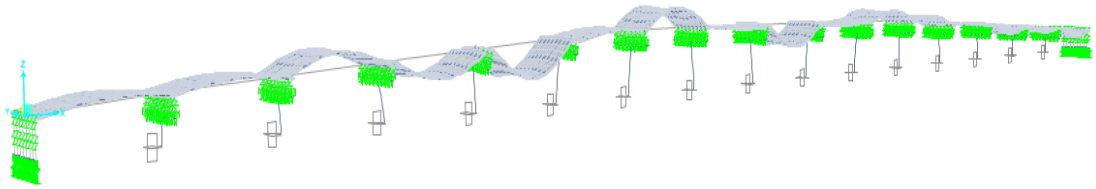
Mode 9, f=5.02Hz



Mode 10, f=5.05Hz



Mode 11, f=5.15Hz



Mode 12, f= 5.28Hz

Figure 4.4: Mode Shapes for Model with Curved Beams and Straight Beams

4.1.2 Bridge-Vehicle response to varying vehicular speed

Response of the models based of vertical accelerations components at varying speed as well as response based on time of entrance into the bridge. This are presented in the response spectrum curves and plot functions for curve, straight and straight-curve beams model respectively.

4.1.2.1 Response spectrum for curved beams

Response spectrum curves are plotted for a specified point (point 6511) on the model as a representation for the time history result. Vertical component of Pseudo-Spectral acceleration (PSA) and vertical response from FE simulation at varying vehicular speeds and damping ratios are presented in Tables 4.6 to 4.12 (Appendix 2) and Figures 4.3 to 4.22 respectively.

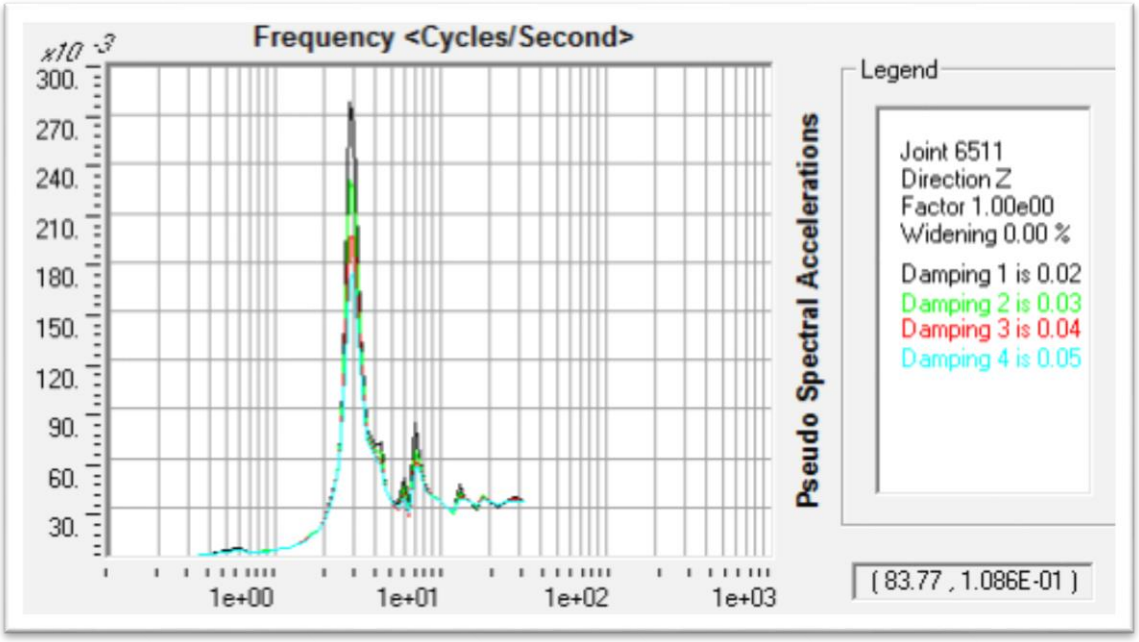


Figure 4.5: PSA spectrum from time history at 10km/h

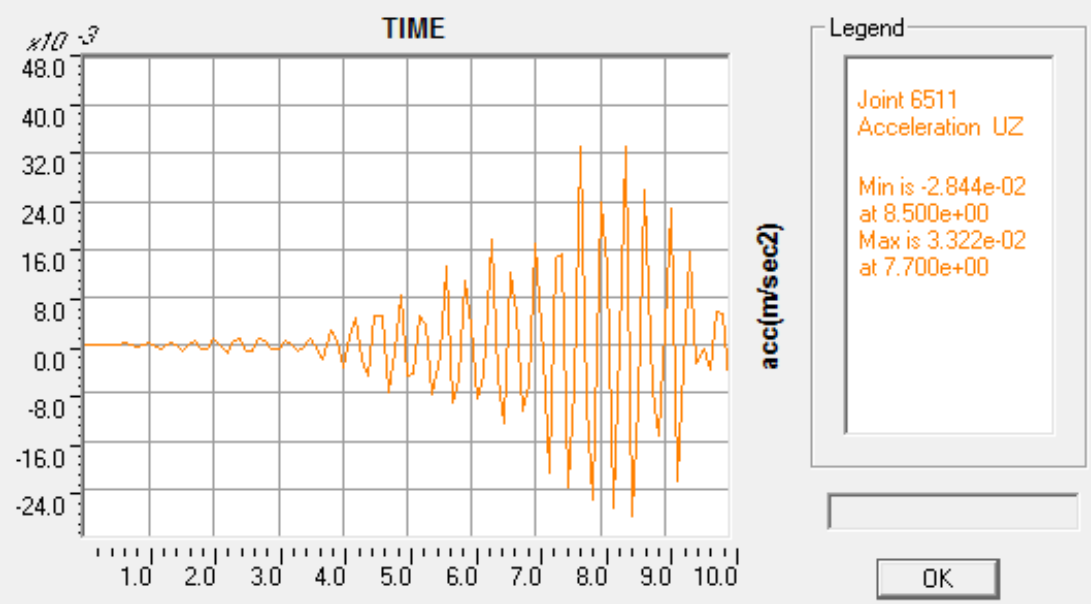


Figure 4.6: Vertical response of vehicles from FE simulation at 10km/h

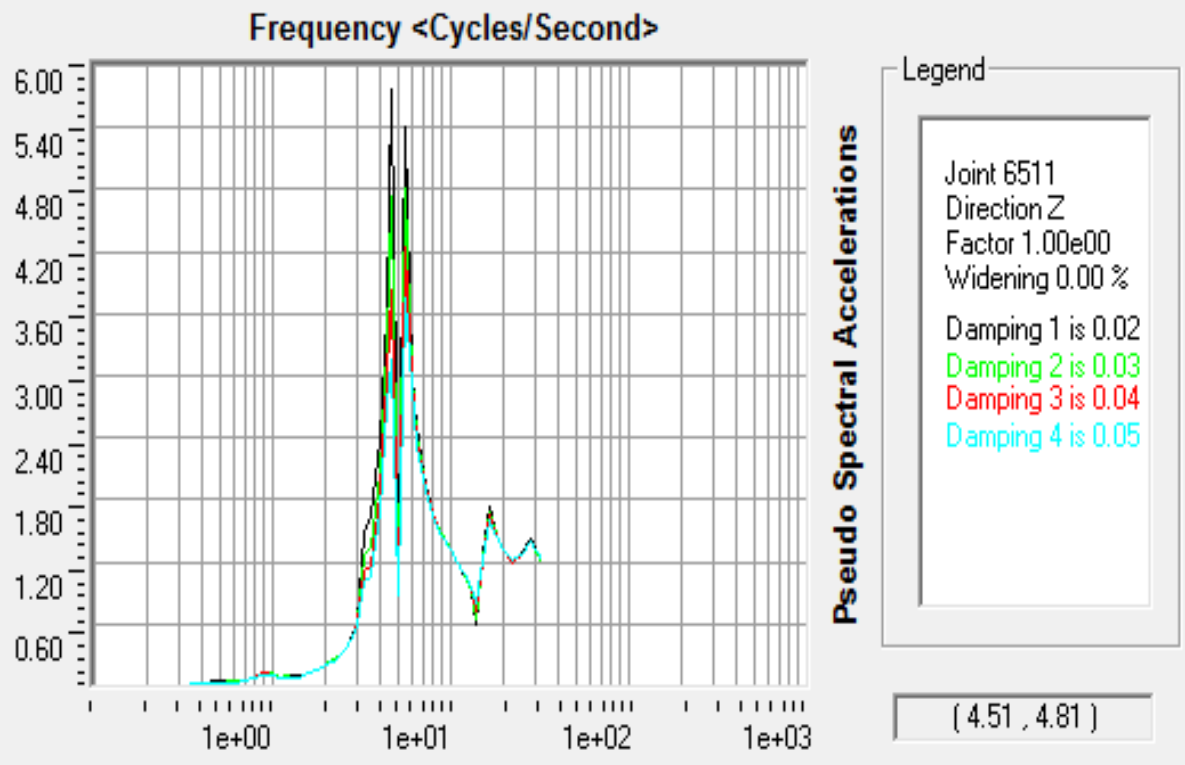


Figure 4.7: PSA spectrum from time history at 20km/h

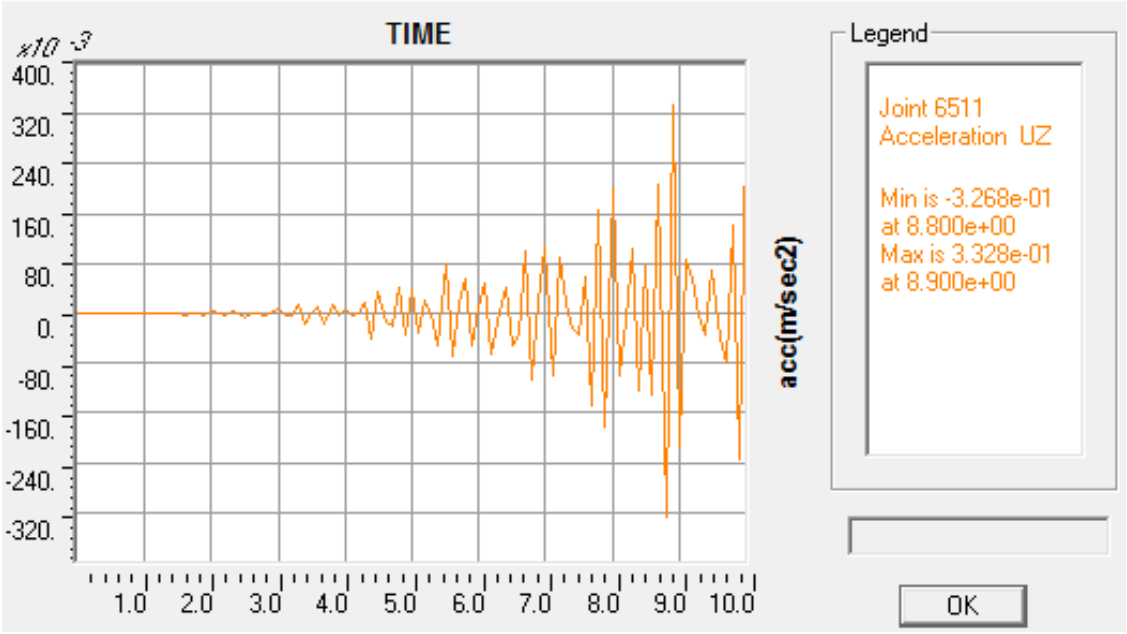


Figure 4.8: vertical response of vehicles from FE simulation at 20km/h

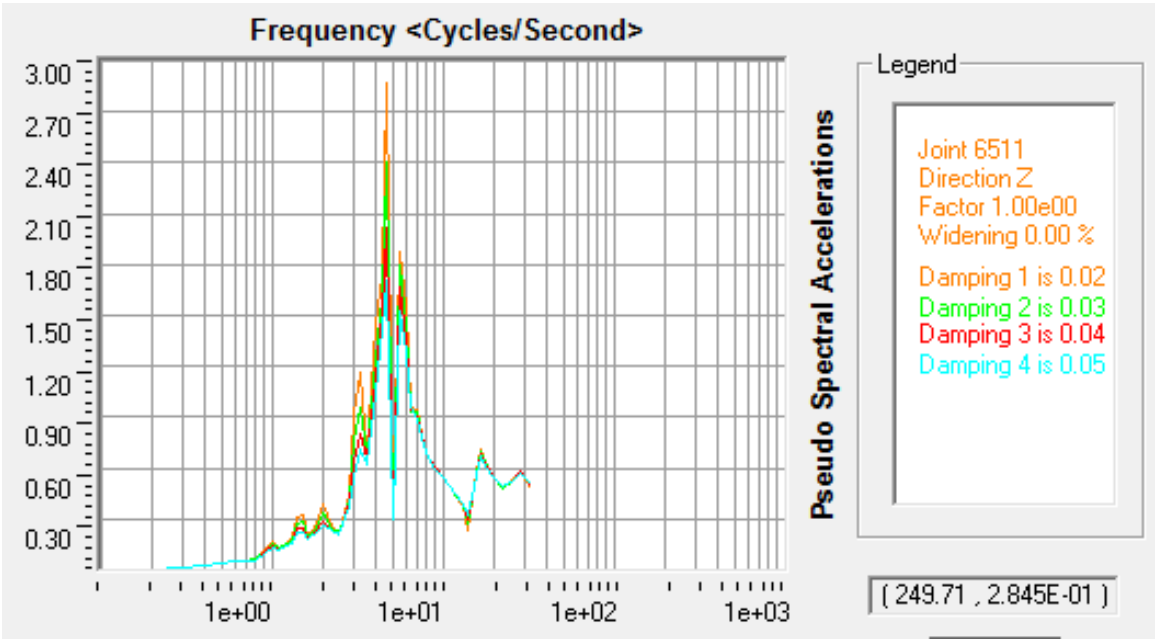


Figure 4.9: PSA spectrum from time history at 30km/h

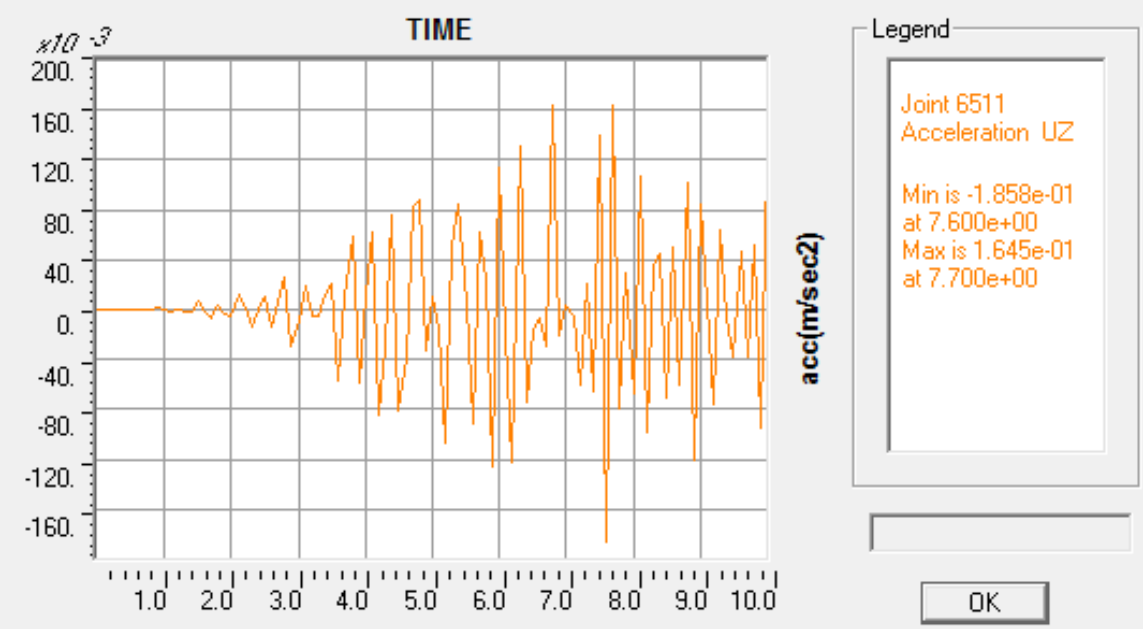


Figure 4.10: vertical response of vehicles from FE simulation at 30km/h

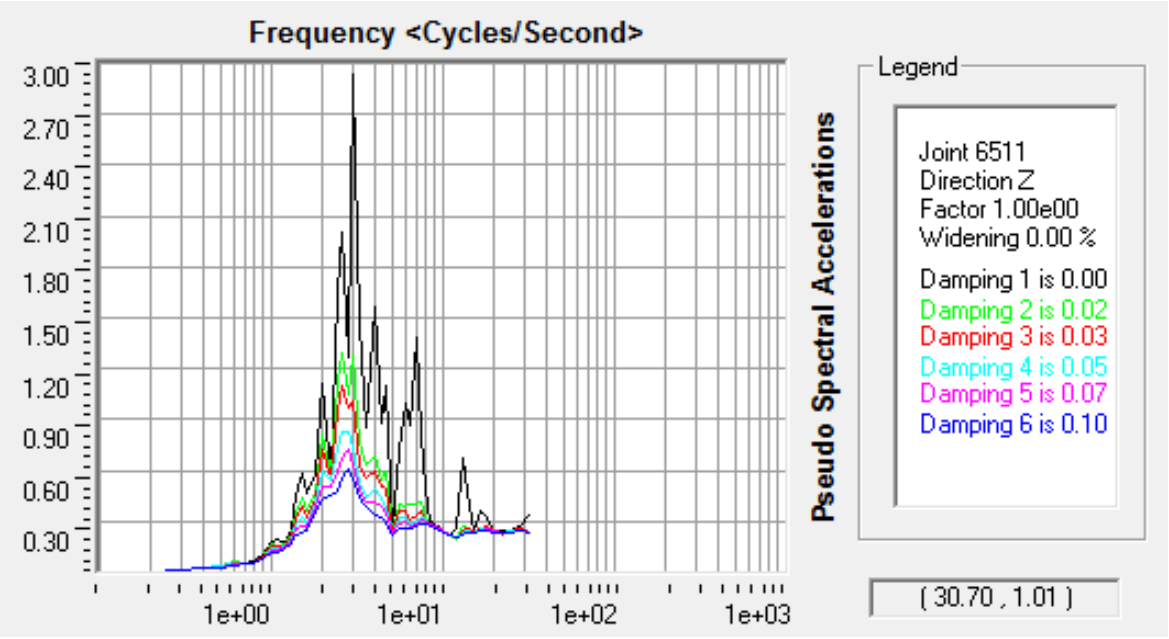


Figure 4.11: PSA spectrum from time history at 40km/h

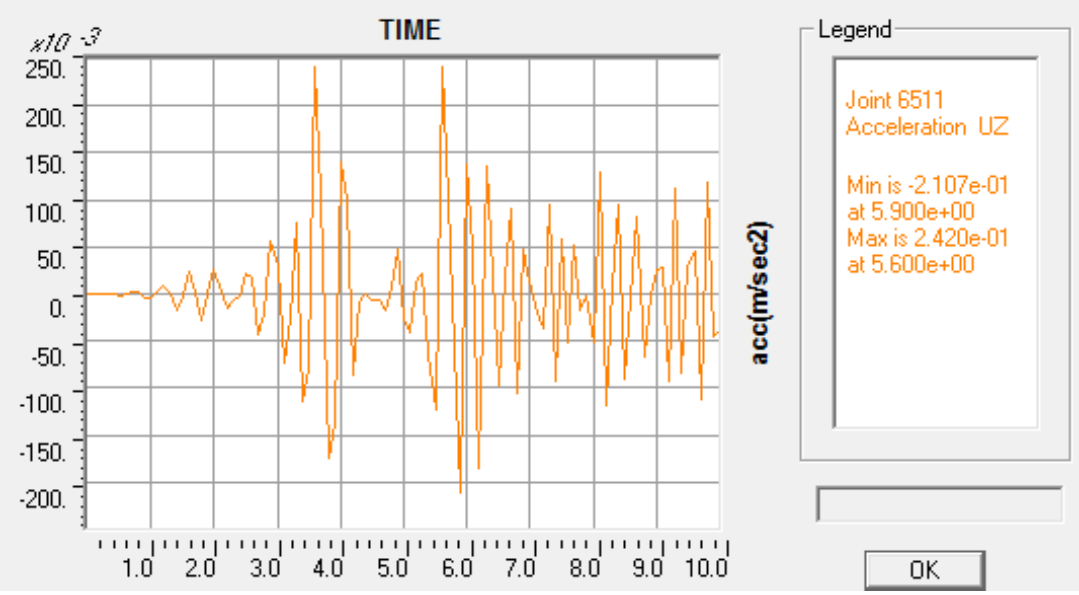


Figure 4.12: Response of vehicles from FE simulation at 40km/h

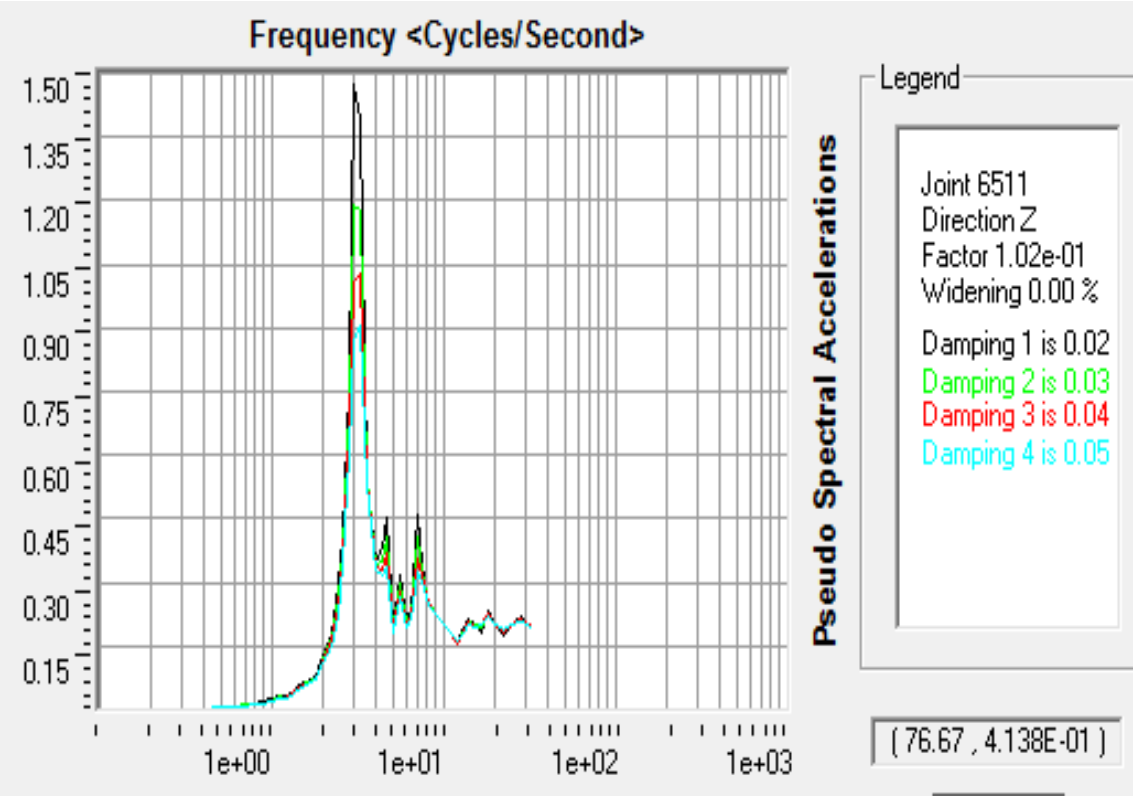


Figure 4.13: PSA Spectrum from time history at 50km/h

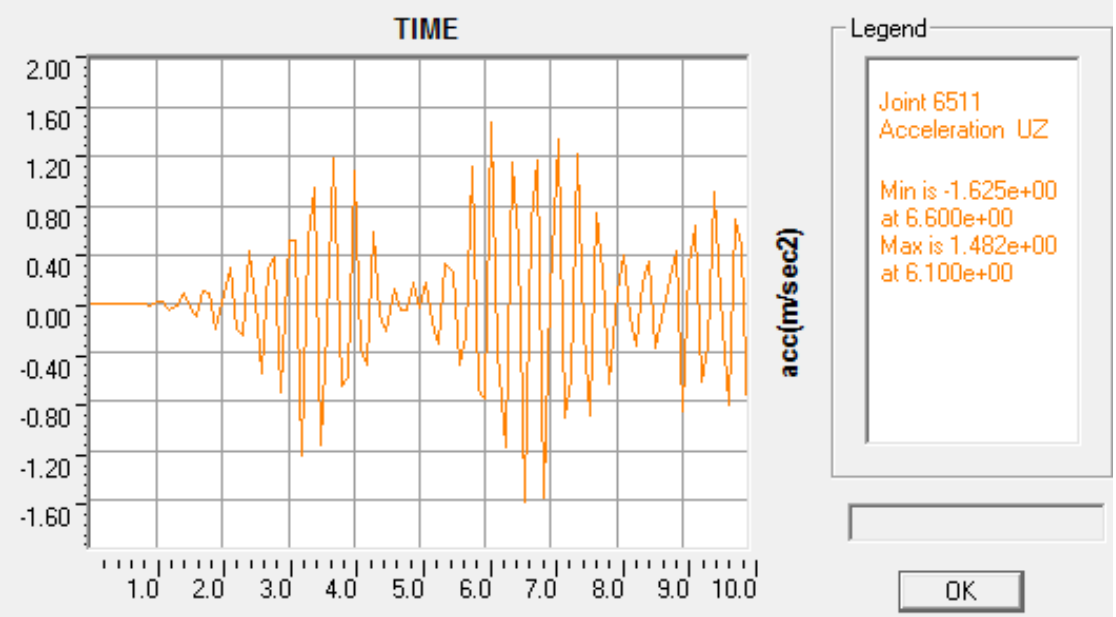


Figure 4-14: Vertical response of vehicles from FE simulation at 50km/h

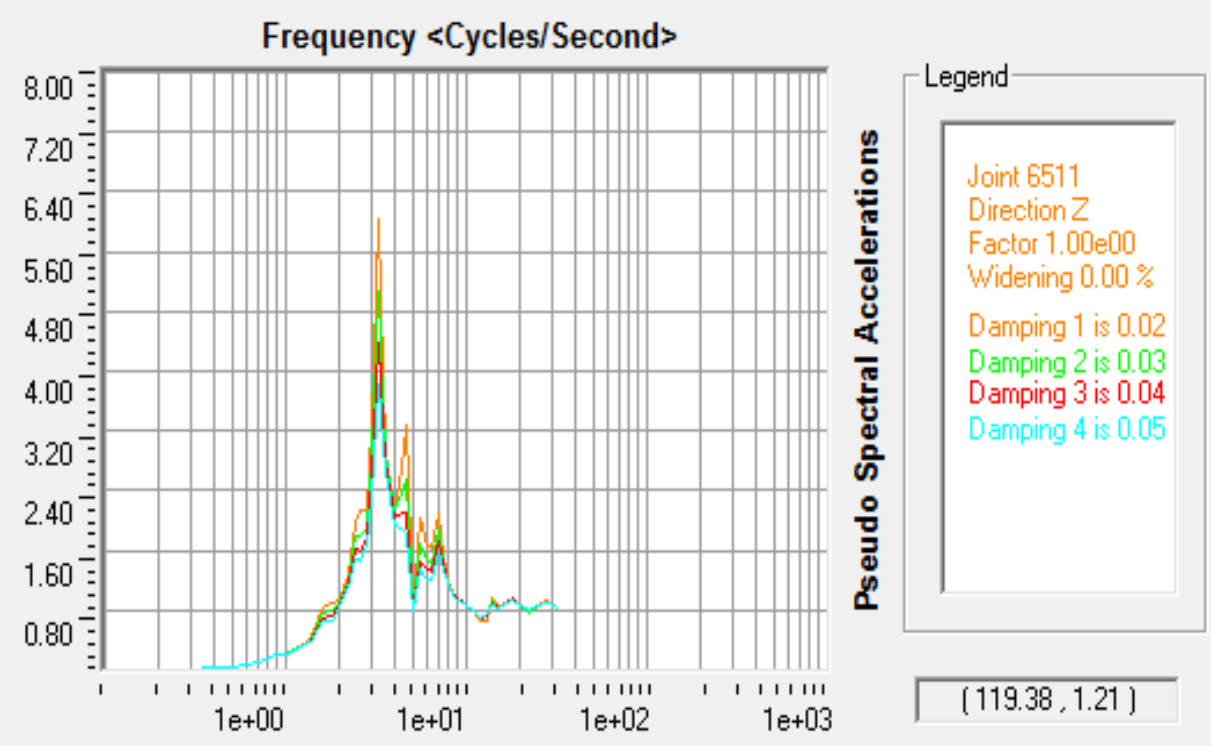


Figure 4-15: PSA Spectrum from time history at 45, 50, 60km/h

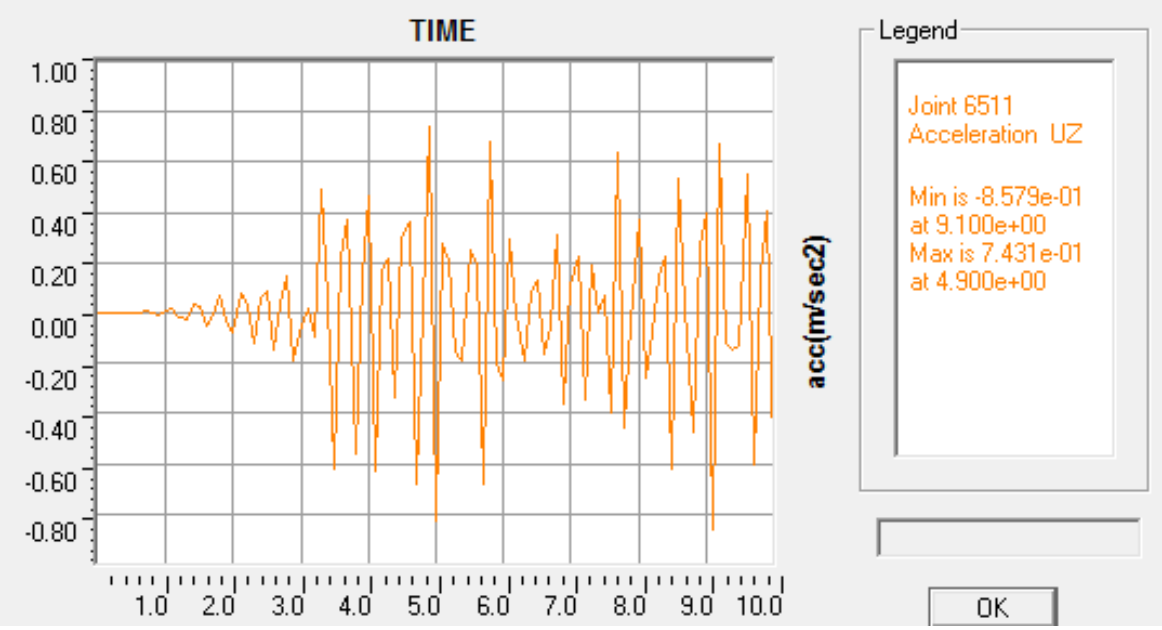


Figure 4-16: Vertical response of vehicles from FE simulation at 45, 50, 60 km/h

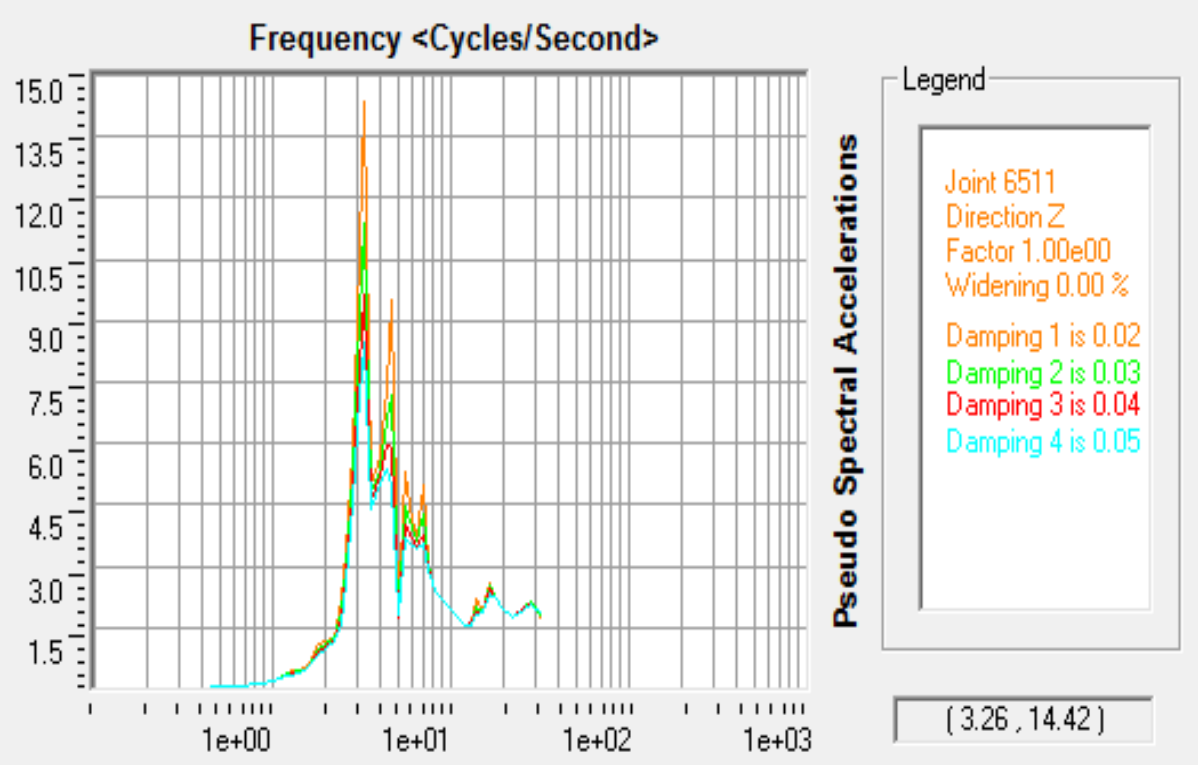


Figure 4-17: PSA Spectrum from time history at 55, 60, 65 km/h

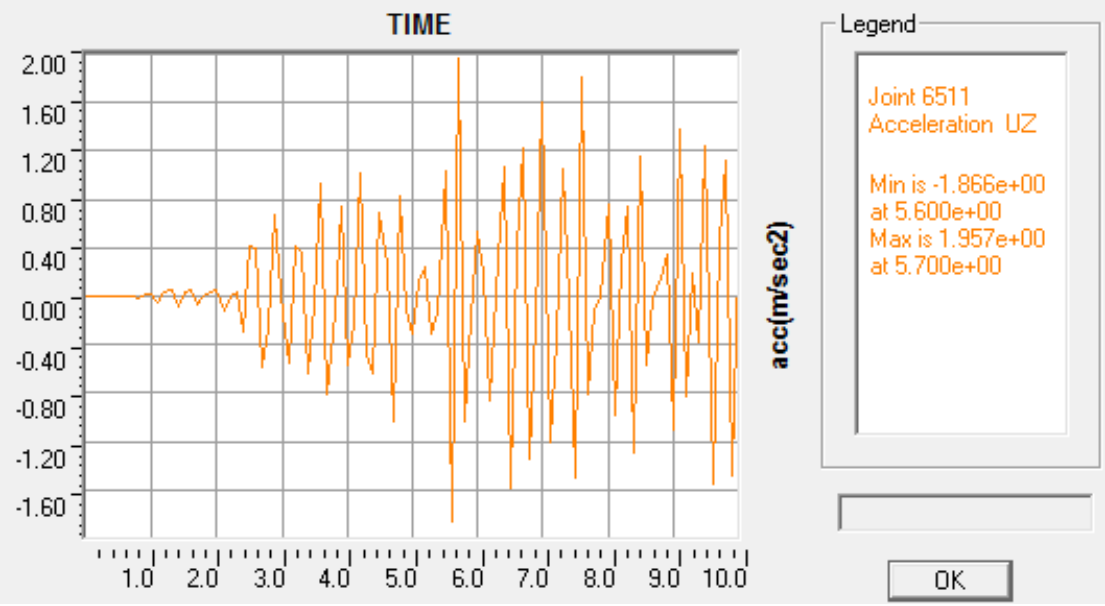


Figure 4.18: Vertical response of vehicles from FE simulation at 55, 60, 65km/h

4.1.2.2 Response Spectrum for straight beams

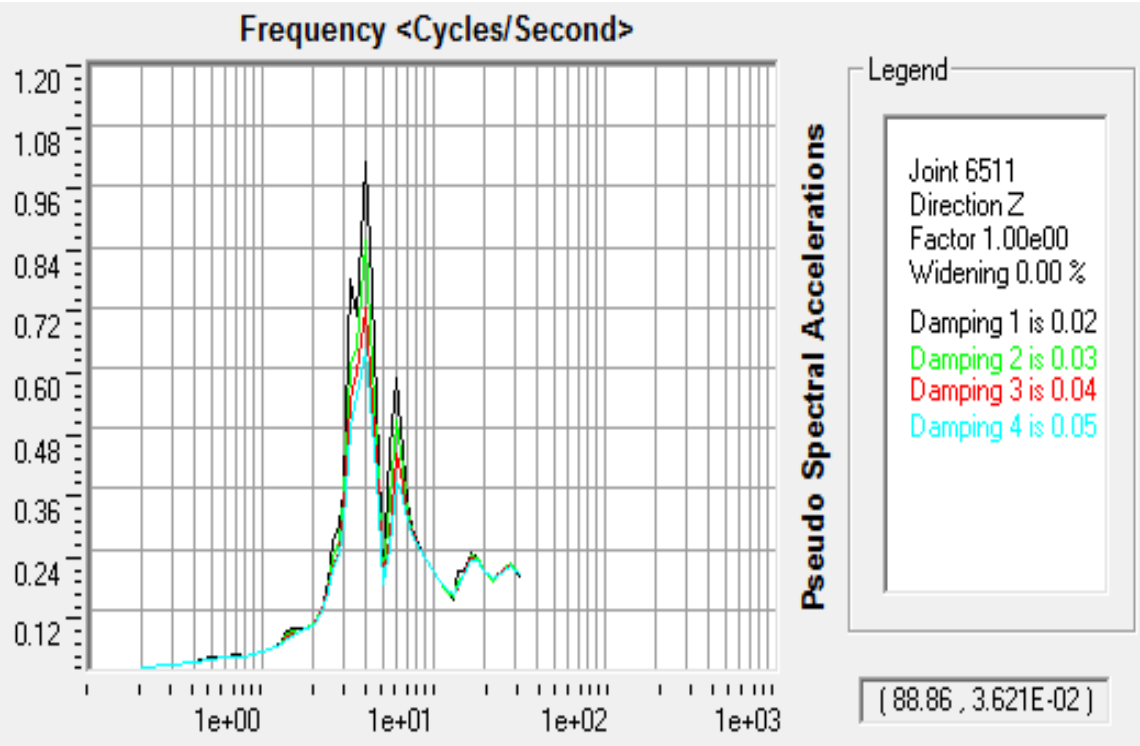


Figure 4-19: PSA spectrum from time history at 10km/h

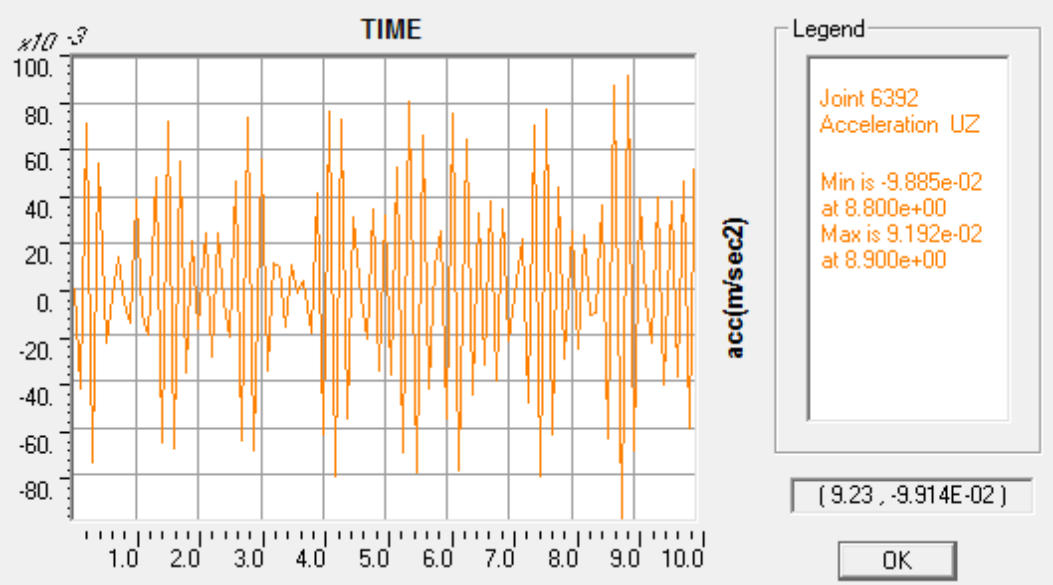


Figure 4-20: vertical response of vehicles from FE simulation at 10km/h

For speed combination of 20km/h, 30km/h, 40km/h, 45, 50, 60km/h and 60, 65, 70km/h response of bridge model is surprisingly the same. So only 20km/h case is presented.

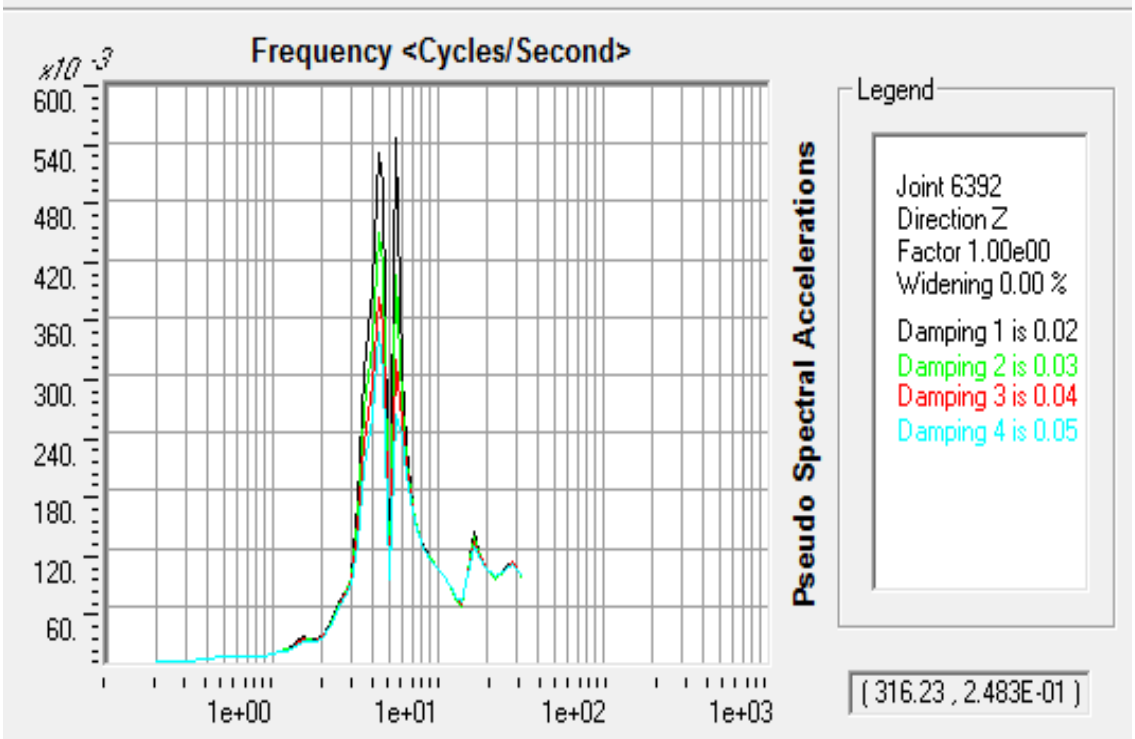


Figure 4.21: PSA spectrum from time history at 20km/h

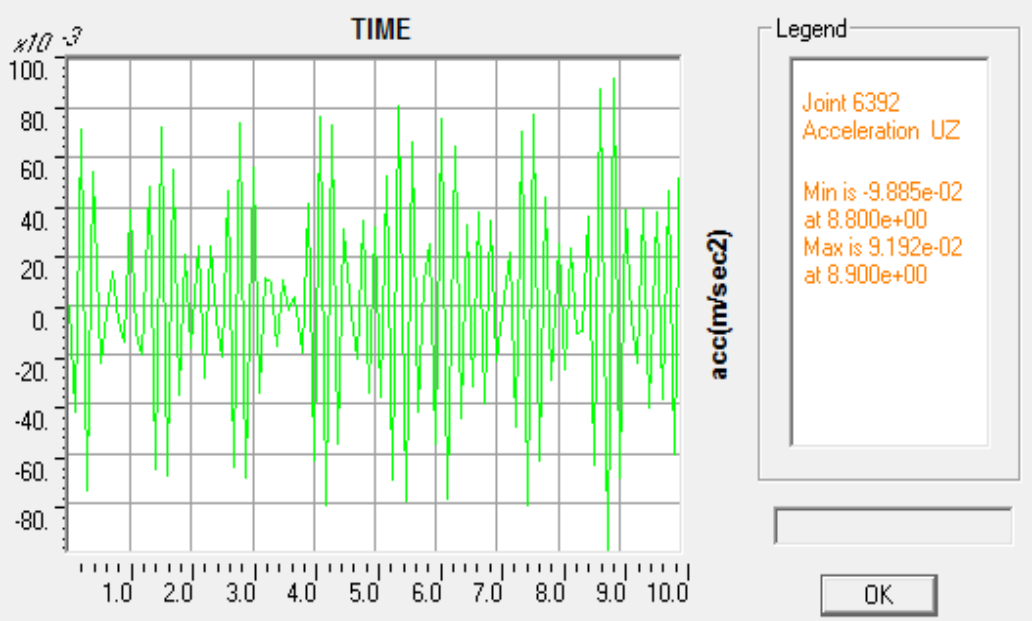


Figure 4.22: Vertical response of vehicles from FE simulation at 20km/h

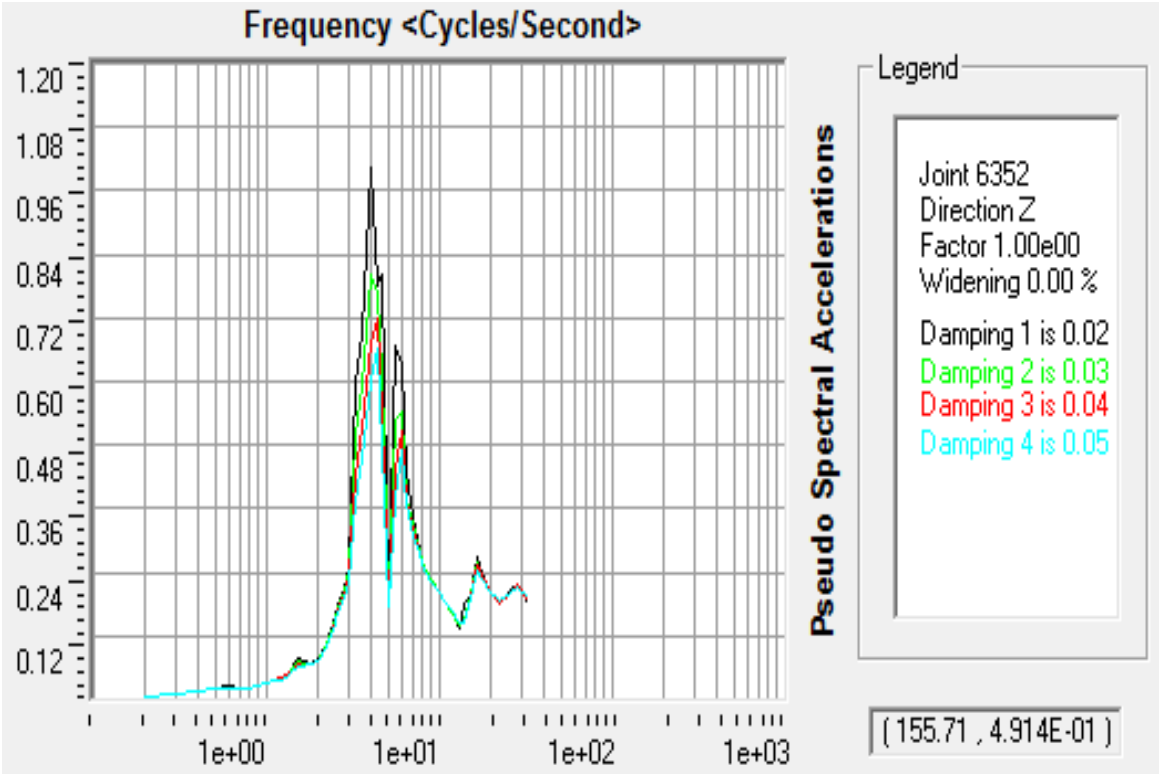


Figure 4.23: PSA spectrum from time history at 50km/h

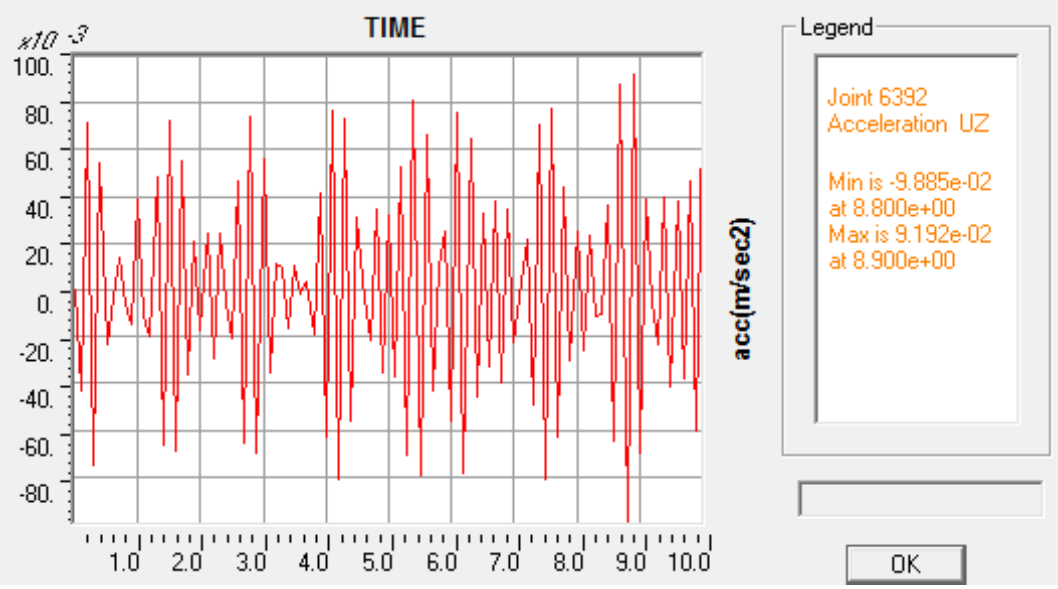


Figure 4.24: Vertical response of vehicles from FE simulation at 50km/h

4.1.2.3 Response Spectrum for combination of straight and curved beams

Response of VCCFB model with profile achieved using a combination of both vertical and horizontal and curved beams are presented.

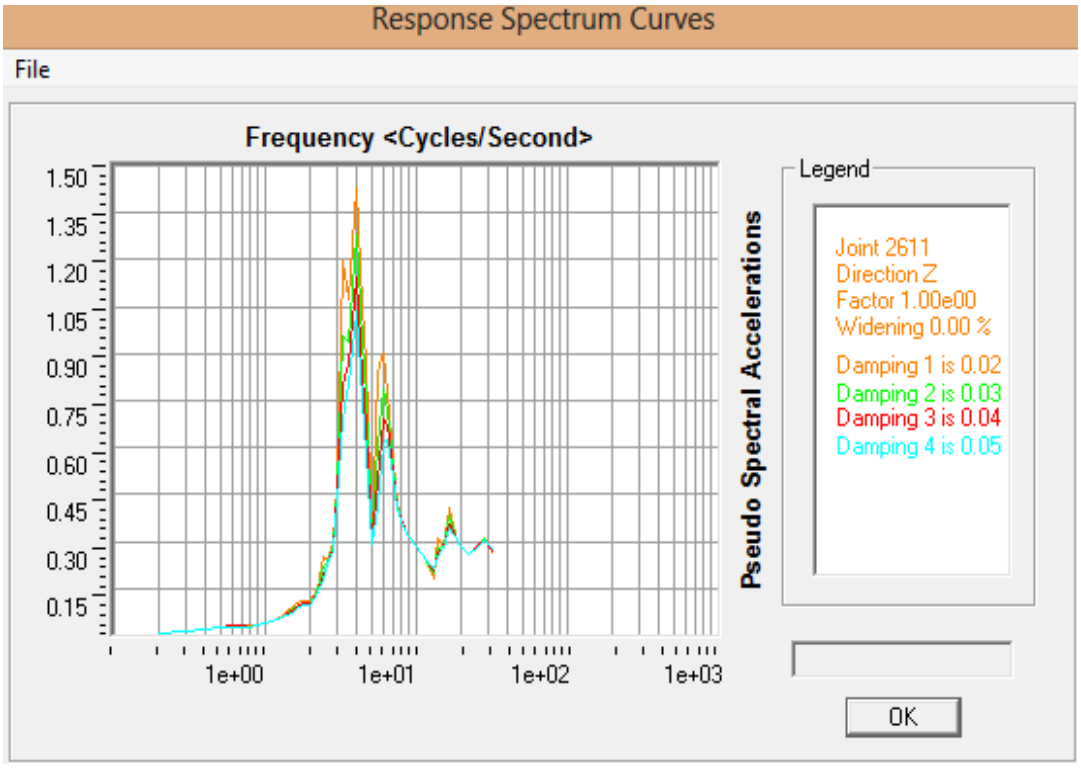


Figure 4.25: PSA spectrum for time history at 10km/h

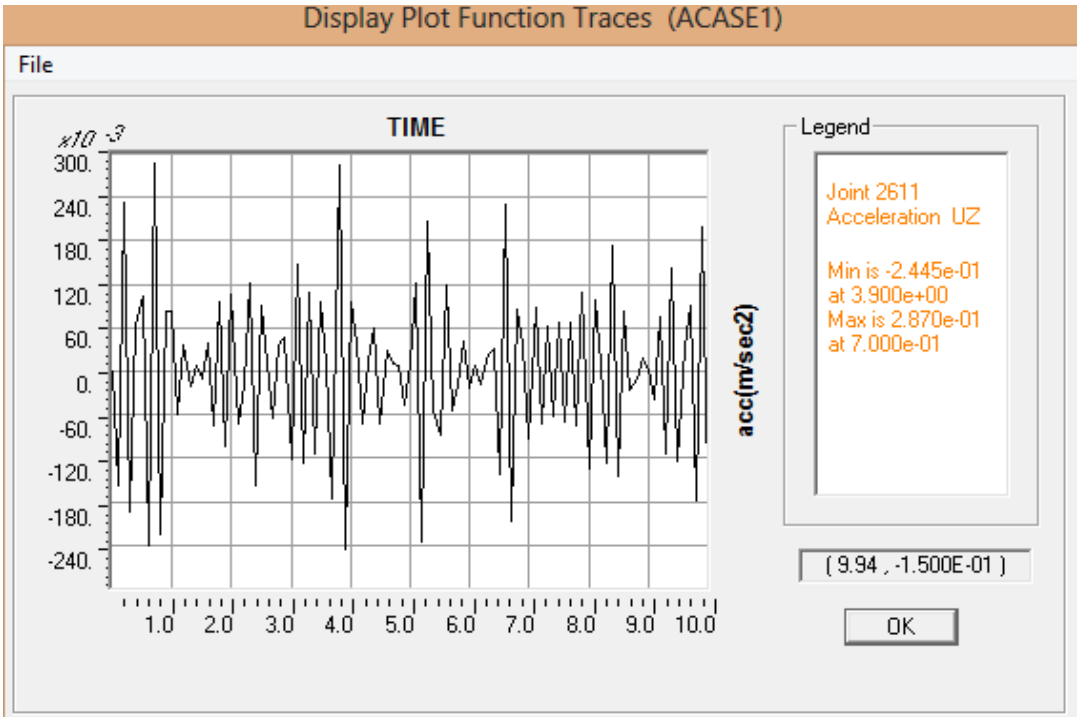


Figure 4.26: Vertical response of vehicles from FE simulation at 10km/h

4.1.3 Human perceptibility to vehicular vibration

Irwin (1979) base curve for human perceptibility to vibration is used as a basis in this work. The points recorded from the analysis are plotted in the curve as presented in Figure 4.25, 4.26 and 4.27 for curve, straight and combination of straight-curve beams respectively.

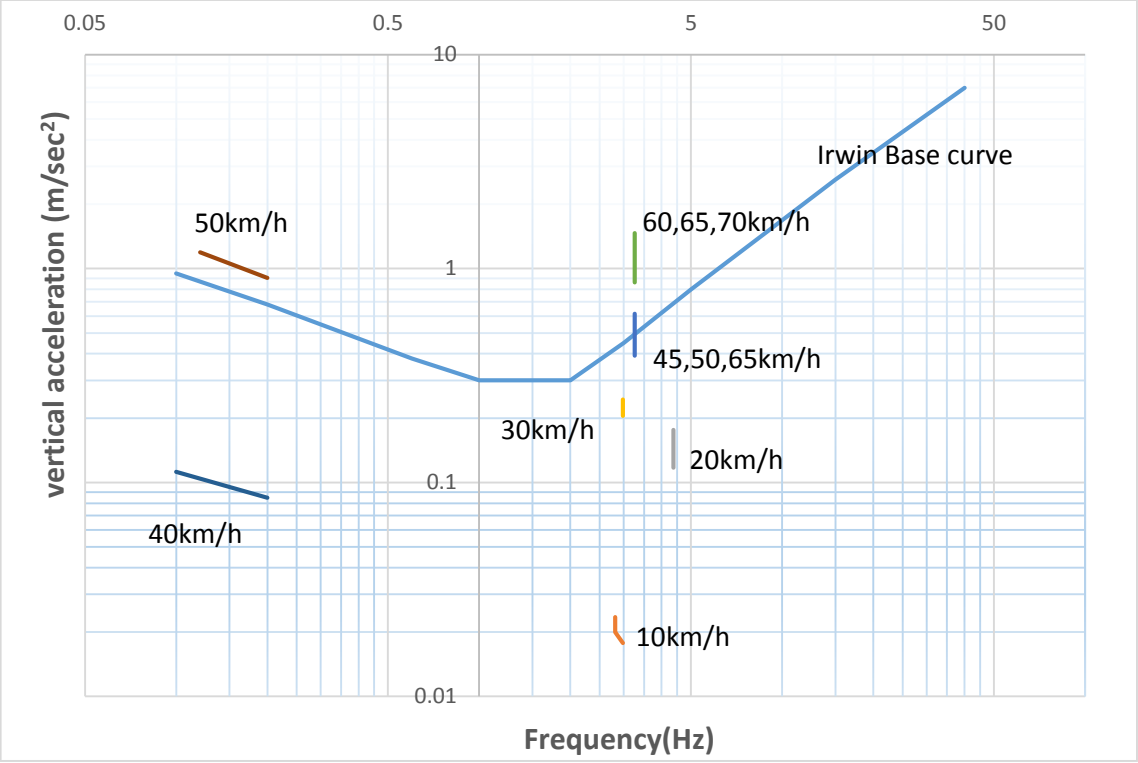


Figure 4.27: Human response to the vertical component of vibration for bridge with curved beams

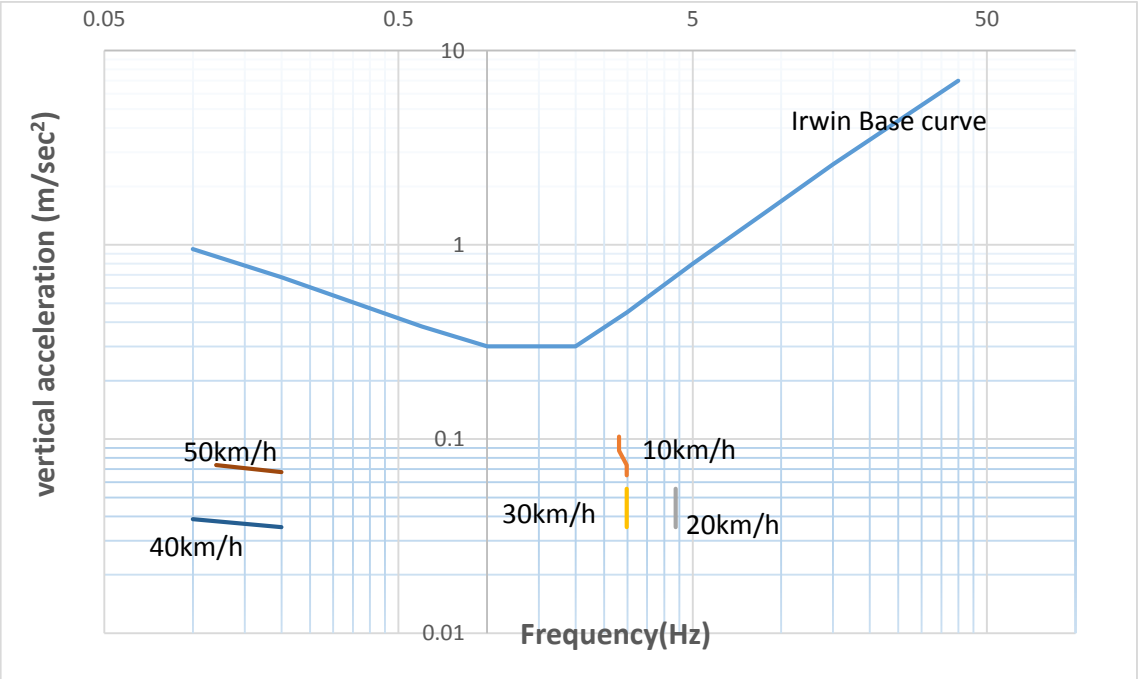


Figure 4.28: Human response to the vertical component of vibration for bridge with straight beams

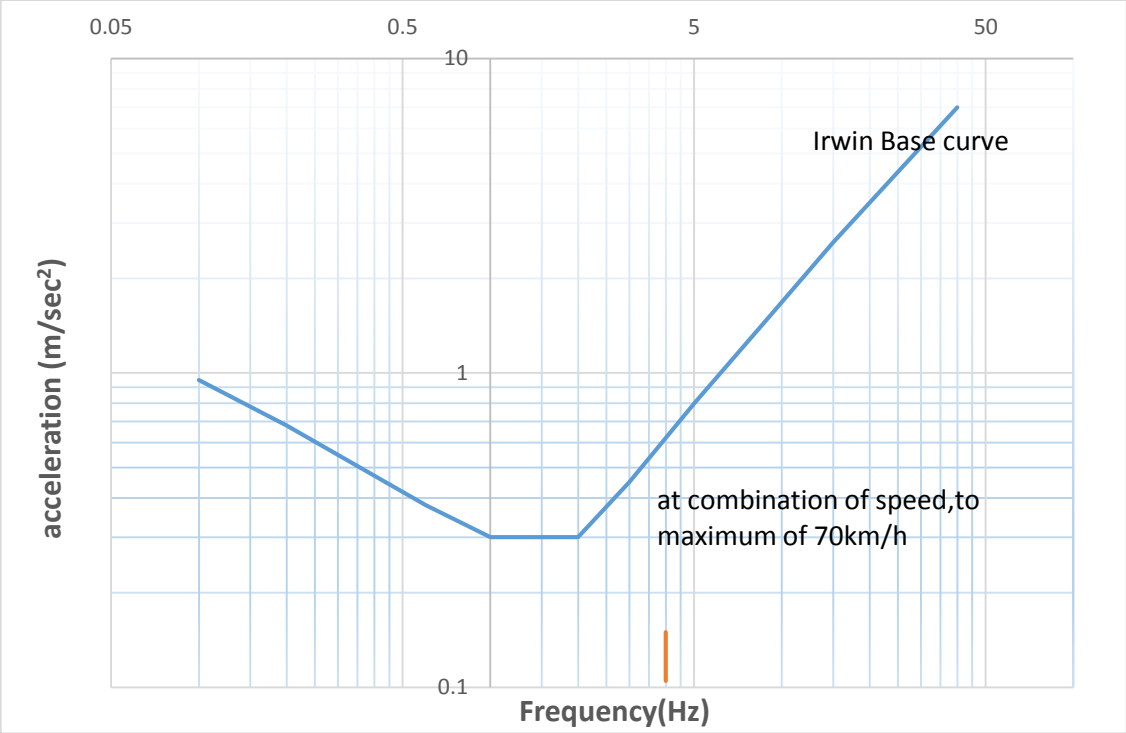


Figure 4-29: Human response to the vertical component of vibration for bridge profile achieved using combination of curve and straight beams

4.2 Discussion of Results

4.2.1 Modal analysis

Result of modal analysis for the three models (curved, straight and combination of both) are presented in Figure 4.1. Twelve mode shapes are presented for each model, with the natural frequencies ranging from 3.34 to 5.32 Hz for straight beams, 3.30 and 5.29 Hz for curved beams and 3.31 and 5.28Hz for combination of straight-curve beam respectively with increasing number of modes. For each mode, the natural frequency of the model with straight beams is found to be higher than the curve beams and combination of straight-curve beams. The relationship between the natural frequencies are presented in Tables 4.1 and 4.2. It is clear from Table 4.1 and 4.2 that for each mode, the natural frequencies of model with straight beams are higher with major difference in modes 2, 6 and 8 representing 2.26, 3.01 and 2.87% differences in natural frequencies between straight and curved beams respectively. While for major difference between straight and combination of straight-curve beams is more pronounced in modes 2, 3 and 5 representing 6.31, 5.51 and 4.24%

respectively. The mode shapes for all models are majorly couple, consisting of longitudinal and torsional modes of vibration.

4.2.2 Bridge- vehicle response to vehicular speed

The results present the vertical component of the pseudo-spectral acceleration (PSA) at various frequencies, which are later converted to actual vertical acceleration and compared with the Irwin (1979) base curve for bridge vibrations.

4.2.3 Response spectrum curves

For bridge models with curved beams, the vehicle response during the first few seconds after entering the bridge were only noticeable after some time ranging from 4.2 , 4.5, 2.8, 1.2, 1.4, 1.2 and 1.1 seconds respectively for vehicular velocities considered in the analysis. Also, the maximum accelerations recorded are 0.033m/sec^2 at 7.7 sec for 10km/h, 0.332m/sec^2 at 8.9sec for 20km/h, 0.165m/sec^2 at 7.7sec for 30km/h, 0.242m/sec^2 at 5.6sec for 40km/h, 1.482m/sec^2 at 6.1sec for 50km/h, 0.743m/sec^2 at 4.9sec for 45,50,65km/h and 1.957m/sec^2 at 5.7sec for 55,60,65km/h. This implies that, profiles of VCCFB achieved using curved beams will provide smoother entrance into the bridge.

For models with straight beams, it is clear from data presented that at all speed vehicle response at the first few seconds after entering the bridge is highly noticeable. The implication of this is that there will be impact of wheel at the entrance which may be disturbing.

Also, for models with combine straight beams and curve beams at cusp of the bridge, the property of the straight beam still dominates, that is, response of vehicles at all speed are very noticeable at few seconds after entering the bridge. Which implies wheel impact at the entrance but considerably reduced vibration afterwards.

4.2.4 Human perceptibility to vibration due vehicular speed

Irwin (1979), suggested a base curve for acceptable human response to the vibration of bridges under various forms of loading as shown in Figure 2.3. In this study, Irwin's (1979) curve for vertical component of bridge motions in stormy wind conditions was selected as a base curve for the human-perceptible vertical component of vibrations on the footway of the bridge deck because of it wide range of frequency; 0.2 to 80Hz. Vibration magnitude lower than the curve are considered as acceptable limit according to his recommendation.

For bridge models with curved beams, as presented in Figure 4.25 vehicular velocities from 10 to 40km/h are acceptable, while combine speeds of 45, 50 and 60km/h will only be allowable at damping ratio of 5 percent. But velocity of 50km/h and combine speed of 60, 65, 70km/h will induce more vibration since it is above the base curve considered and as noticed in corresponding vertical component of acceleration of 0.9 to 1.5m/sec^2 for 50km/h and combination of 60,65,75km/h respectively.

For bridge models with straight beams as presented in Figure 4.26, vehicular speeds ranging from 10 to 50km/h, falls below the base curve, which makes it within the acceptable limit specified by Irwin (1979).

While for model with combination of straight and curve beams at the cusp as presented in Figure 4.27, at all speed considered, the vibration threshold is quite lower than the base curve considered.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

Based on the results and analysis presented for modal and time history analysis for different profiles of VCCFB and vehicles passing at different speed, the following conclusions are drawn as follows:

(i) Modal Analysis on three profiles of VCCFB were conducted.

(ii) Natural Frequencies and Mode shapes for the profiles considered were compared and it was established that Mode Shapes are majorly couple, consisting longitudinal and torsional modes of vibrations for all profiles.

(iii) Profiles with straight beams are considered more stable as a result of their higher natural frequency.

(iv) Bridge–Vehicle response at varying vehicular speed were simulated using Time History and Response Spectrum Analysis.

(v) Vertical component of vibrations which is the major contributing parameter in structural vibration reduces as damping ratio increases from 2 to 5% for all vehicular velocities considered.

(vi) Using Straight beams at entrance of VCCFB induce higher Vehicle response.

(vii) Bridge-Vehicle response results were compared with Irwin (1979) base Curve for acceptable human response to vibrations of bridges.

(viii) Profiles achieved with combination of straight and curved beams at the cusp are more suitable for VCCFB because of their lesser vehicular induced vibrations recorded at varying speed.

5.2 Recommendations

Considering the difference in response at varying speed, and vehicle response at entrance of each bridge model, it is clear that each method have its peculiar advantage ranging from stability to lesser induced vibration. Model consisting combination of straight and curve beams at the cusp may have added advantage of providing a more stable expansion joint, as such should be encouraged.

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APPENDICES

Appendix 1: Natural Frequencies of bridge model with curved beams

Mode Number	Frequency(Hz)	Period(sec)
1	3.296259	0.303374
2	4.060484	0.246276
3	4.176786	0.239419
4	4.226323	0.236612
5	4.349716	0.229900
6	4.557598	0.219414
7	4.740383	0.210953
8	4.787311	0.208886
9	5.005267	0.199790
10	5.082930	0.196737
11	5.228857	0.191246
12	5.291043	0.188999

Appendix 2: Natural Frequencies of bridge model with curved beams

Mode Number	Frequency(Hz)	Period(sec)
1	3.353384	0.29821
2	4.193029	0.238491
3	4.271131	0.234130
4	4.285470	0.233347
5	4.455593	0.224437
6	4.694582	0.213012
7	4.783542	0.209050
8	4.924665	0.203060
9	5.074043	0.197082
10	5.120001	0.195312
11	5.245227	0.190650
12	5.319221	0.187997

Appendix 3: Natural Frequencies of bridge model with combine straight and curved beams

Mode Number	Frequency (Hz)	Period (sec)
1	0.302357	3.3074
2	0.253549	3.944
3	0.247038	4.048
4	0.242738	4.1197
5	0.233958	4.2743
6	0.219979	4.5459
7	0.214226	4.668
8	0.208326	4.8002
9	0.199194	5.0202
10	0.198172	5.0461
11	0.194132	5.1511
12	0.189239	5.2843

Appendix 4: Spectral response of bridge to 3 vehicle moving at 10km/h

Damping ratio (%)	Freq. (Hz)	PSA(g)	Acc.(m/sec2)
2	2.81	0.2767	0.028205912
3	2.81	0.2306	0.023506626
4	2.98	0.1957	0.019949032
5	2.98	0.174	0.017737003

Appendix 5: Spectral response of model to 3 vehicles moving at 20km/h

Damping ratio (%)	Freq.(Hz)	PSA (g)	Acc. (m/sec^2)
2	4.38	1.73	0.176
3	4.38	1.50	0.153
4	4.38	1.31	0.133
5	4.38	1.15	0.117

Appendix 6: Spectral response of model to 3 vehicles moving at 30km/h

Damping ratio (%)	Freq.(Hz)	PSA (g)	Acc.(m/sec^2)
2	2.98	0.9181	0.093588175
3	2.98	0.7552	0.076982671
4	2.98	0.6379	0.065025484
5	2.98	0.5431	0.055361876

Appendix 7: Spectral response of model to 3 vehicles moving at 40km/h

Damping ratio (%)	Freq.(Hz)	PSA (g)	Acc. (m/sec^2)
2	3	2.94	0.29969419
3	2.54	1.29	0.131498471
4	2.59	1.1	0.112130479
5	2.79	0.832	0.084811417

Appendix 8: Spectral response of model to 3 vehicles moving at 50km/h

Damping ratio (%)	Freq. (Hz)	PSA (g)	Acc.(m/sec^2)
2	3	14.52	1.480122324
3	3	11.7	1.19266055
4	3.2	10.01	1.02
5	3.31	8.87	0.904179409

Appendix 9: Spectral response of model to 3 vehicles moving at 45, 50, 60km/h

Damping ratio (%)	Freq. (Hz)	PSA (g)	Acc.(m/sec^2)
2	3.26	6.03	0.6147
3	3.26	5.07	0.5168
4	3.26	4.38	0.4465
5	3.26	3.84	0.3914

Appendix 10: Spectral response of model to 3 vehicles moving at 60, 65, 70km/h

Damping ratio (%)	Freq.(Hz)	PSA (g)	Acc. (m/sec^2)
2	3.26	14.35	1.4628
3	3.26	11.38	1.1600
4	3.26	9.66	0.9847
5	3.26	8.45	0.8614

Appendix 11: Spectral response of model to 3 vehicles moving at 10km/h

Damping ratio (%)	Freq.(Hz)	PSA (g)	Acc. (m/sec^2)
2	4	1.01	0.103
3	4	0.854	0.087
4	4	0.720	0.073
5	4	0.638	0.065

Appendix 12: Spectral response of model to 3 vehicles moving at 20km/h

Damping ratio (%)	Freq.(Hz)	PSA (g)	Acc. (m/sec^2)
2	5.5	0.546	0.056
3	4.41	0.447	0.046
4	4.41	0.379	0.039
5	4.41	0.345	0.035

Appendix 13: Spectral response of model to 3 vehicles moving at 50km/h

Damping ratio (%)	Freq.(Hz)	PSA (g)	Acc. (m/sec^2)
2	4	1.0	0.102
3	4	0.799	0.081
4	4.41	0.720	0.073
5	4.41	0.664	0.068

Appendix 14: Spectral response of model to 3 vehicles moving at varying Speeds,10,20,30,40,50,455060 and 556070km/h

Damping ratio (%)	Frequency (Hz)	PSA (g)	Acceleration (m/sec^2)
2	4	1.41	0.1497
3	4	1.28	0.1303
4	4	1.15	0.1171
5	4	1.03	0.1049