QUEUE MODELLING AND APPLICATIONS AT THE KADUNA TOLL BOOTHS

BY

CHIKEZIE, CLARKSON UKA

B.ENG (CIVIL) UNILORIN

MSc./ENG./00646/93-94

A THESIS SUBMITTED TO THE POSTGRADUATE SCHOOL,
AHMADU BELLO UNIVERSITY, ZARIA
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR
THE AWARD OF THE DEGREE OF MASTER OF SCIENCE IN
HIGHWAYS AND TRANSPORTATION ENGINEERING

DEPARTMENT OF CIVIL ENGINEERING
FACULTY OF ENGINEERING
AHMADU BELLO UNIVERSITY
ZARIA, NIGERIA

APRIL, 1998

APPROVAL PAGE

This thesis entitled "Queue Modelling and Applications" by Chikezie Clarkson Uka, meets the regulations governing award of degree of Master of Engineering of Ahmadu Bello Univeristy, Zaria, and is approved for its contribution to knowledge and literary presentation.

Prof. O. Adebisi
Chairman, Supervisory Committee

DIGITIZED

3/7/98
Date

Dr. J.O. Afolayan
Member Supervisory Committee

3-7-98
Date

Dr. K.J. Osinubi
Head of Department

3.7.98
Date

Prof. J.Y. Olayemi
Dean, Postgraduate School

2/11/98
Date

DECLARATION

I hereby declare that this thesis has been composed by myself and that it is the record of my own research work. It has not been accepted in any previous application for a higher degree.

All sources of information are specifically acknowledged by means of references.



CHIKEZIE CLARKSON UKA

1998

ABSTRACT

The toll booth on the Kaduna - Zaria highway has been modelled as a queue system by adapting the makino model. Data collected on several days at the site was used to calibrate the model. Field conditions wherein a police check-point is encountered just before the toll booth justified the idealization of the system as a two-stage queue. Allowance for U-turning in-between the police check-point and the toll booth was made by incorporating a fixed probability, deduced from field observations into the model. A wide range of vehicle arrival rates was simulated for each of the vehicles categories served.

Results from the simulation correlated highly with actual observed simulations and the trends obtained conformed with theoretical values. For the Car channels, the expected waiting time ranges from 5.76 to 29.16 seconds; number in queue ranges from

0.4 to 9 cars; customer loss ratio ranges from 0.8585 to 9.3899 while utilization factor ranges from 0.4612 to 0.9541 and blocking probability ranges from 0.0000 to 0.4100.

A number of options for improving the performance of the system was then investigated. Of all the options available for cars, the option which has one policeman in series with two collectors is adopted. Here, the two police channels are replaced by the super service channel, the three toll booths normally used for cars are reduced to just two channels.

ACKNOWLEDGEMENT

I am particularly indebted to Professor O. Adebisi, my first supervisor who painstakingly guided me during this study with his limited time. In addition the willing Co-operation of Dr. J.O Afolayan, my second supervisor for his open - door discussion and regular assistance is gratefully acknowledged.

Considerable credit is due to Mr. M.W Shebe for his assistance and making his Departmental Computer available during the research work. Appreciation is due to the management and staff of the Zaria - Kaduna toll booth for allowing the author to carry out data collection on the site.

Thanks are also extended to those who assisted me in collecting the data.

DEDICATION

This thesis is dedicated to

the glory of our Lord Jesus Christ

and

to my mother.

TABLE OF CONTENTS **PAGE**

APPROVAL PAGE ii

DECLARATION iii

ABSTRACT iv

ACKNOWLEDGEMENT vi

DEDICATION vii

TABLE OF CONTENTS viii

LIST OF SYMBOLS xii

LIST OF TABLES xvi

LIST OF FIGURES xx

LIST OF PLATES xxii

CHAPTER ONE - INTRODUCTION 1

1.1 PREAMBLE 1

1.2 NEED FOR THE STUDY 2

1.3 AIMS AND OBJECTIVES OF STUDY 4

1.4 THE CASE STUDY 5

1 5 DESCRIPTION OF THE TOLL GATE ENVIRONMENT 6

1.4.2 SYSTEM OPERATION AND MANAGEMENT POLICIES 7

1.4.3 PRELIMINARY OBSERVATION ON THE CASE STUDY 12

1.5 ELEMENTS OF QUEUING THEORY 14
1.5.1 DEFINITION, BACKGROUND AND APPLICATION 14
1.5.2 DESCRIPTION OF QUEUEING PROBLEMS 15
1.5.3 ARRIVAL PATTERN OF CUSTOMERS 15
1.5.4 SERVICE PATTERN OF SERVERS 17
1.5.5 QUEUE DISCIPLINE 17
1.5.6 SYSTEM CAPACITY 18
1.5.7 NUMBER OF SERVICE CHANNELS AND STAGES 18
1.5.8 CLASSIFICATION OF QUEUES 19
1.6 COMPUTER METHODS 20
1.7 SCOPE AND LIMITATION OF THE STUDY 21
CHAPTER TWO - REVIEW OF RELATED LITERATURE 22
2.1 PREAMBLE 22
2.2 CLASSES OF QUEUEING MODELS 22
2.3 MAIN FEATURES OF EXISTING QUEUEING MODELS 23
2.3.1 SINGLE CHANNEL QUEUEING MODEL 23
2.3.2 THE MULTIPLE CHANNEL QUEUEING MODELS 26
2.3.3 MODEL FOR SERIAL AND CYCLIC QUEUEING PROCESSES . 30
2.4 SIMULATION MODELS 35

2.5 SELECTION OF POTENTIAL CANDIDATE MODEL 36

CHAPTER THREE - DATA COLLECTION AND ANALYSIS 43

3.1 PREAMBLE 43

3.2 ORGANIZATION OF DATA COLLECTION 43

3.3 DATA COLLECTION METHODS AND TECHNIQUES 45

3.3.1 PRELIMINARY SURVEY AND ANALYSIS 46

3.4 DETAILED DATA COLLECTION 60

3.4.1 PREAMBLE 60

3.5 ANALYSIS OF RESULTS 61

3.5..1 PREAMBLE 61

3.5.2 GOODNESS OF - FIT TESTS 61

3.6 RESULTS OF GOODNESS - OF -FIT TESTS 63

3.7 OBSERVATION ON THE RESULTS OF DATA ANALYSIS 68

3.8 MODEL SPECIFICATIONS FOR THE SERVICE AND

ARRIVAL PATTERN 69

CHAPTER FOUR - MODEL CALIBRATION 71

4.1 PREAMBLE 71

4.2.1 ADAPTATION OF MAKINO MODEL TO THE SYSTEM UNDER

STUDY 73

4.2.2 THEORETICAL ASPECTS AND DERIVATIONS 74

4.3 MODEL SOLUTION 80

4.3.1 PRIMARY OBJECTIVES OF MODEL SOLUTION 80

4.3.2 COMPUTERIZATION OF THE MODEL SOLUTION 83

4.3.3 THE INPUT TO THE PROGRAM 84

4.4 GENERATION OF INPUT DATA FOR MODEL 92

4.4.1 CHOICE OF SYSTEM EFFECTIVENESS MEASURES 95

4.5 EVALUATION OF THE SIMULATION MODEL 95

4.6 APPLICATIONS OF RESULTS 106

4.6.1. PREDICTING THE ARRIVAL RATES OF VEHICLE 107

CHAPTER FIVE - DISCUSSIONS, COMMENTS, RECOMMENDATIONS

AND CONCLUSION 108

5.1 DISCUSSIONS AND COMMENTS 108

5.2 RECOMMENDATIONS 111

5.3 SUGGESTION FOR FURTHER RESEARCH 114

5.4 CONCLUSION 115

BIBLIOGRAPHY 116

APPENDICES 121

LIST OF SYMBOLS

The following is a list of the essential symbols used in this thesis together with their mathematical and operational meanings:

X,Y,Z: Usually used to denote general random variable.

M : Refers to Poisson -distributed events or to exponentially distributed interval.

G : General distribution.

GI : General distribution with independence assumption.

D : Constant distribution.

EK : Erlang type K (K= 1,2,..).

C: Number of parallel channels; also number of parallel channels in the first stage queue (1,2,..∞).

S : Number of parallel channels in the 2nd stage queue (1,2,...∞).

N : Maximum queuing length restriction for 2nd stage queue.

n : Number of observations in a data set.

L : General restriction on system capacity (mostly in chapter 2) also random variable that defines the number of 1st stage servers that are busy (mostly in chapter 4).

FIFO: First In, First OUT.

LIFO: Last In, First OUT.

SIRO: Service In Random Order

PSPO: Preemptive service priority order

NPPS: Non-Preemptive priority service.

GD: General discipline

C.V: Co-efficient of Variation

δ: Standard deviation

T : Observed inter-arrival\service time variable.

M : Observed absolute frequency.

C1: Observed Cumulative frequency

FT: Expected Cumulative frequency.

CDIFF: K - S statistic

$\bar{T}$: Mean inter - arrival\service time variable.

ρ: Traffic intensity or utilization factor, usually the ratio of the input rate to service rate.

λ: Arrival rate.

μ: Service rate.

P_n: steady - state probability of having n units in the system.

K: Random variables that defines the number of vehicles in the second stage queue.

LL: Mean number of vehicle in system.

Lq Mean number of vehicles in queue.

W: Mean waiting time in system

Wq: Mean waiting time in queue.

P(i,j): state probabilities of having i vehicle in the 2nd stage queue of having i of the 1st stage service channels being blocked while second stage channels are busy.

h: Random variable that defines the number of 2nd stage servers that are busy.

Poo: Probability of no vehicle in the system.

a: Total traffic offered per service time.

a′: Total traffic carried per service time.

ρ': Channels utilization factor.

LR: Customer loss Ratio .

Poh: State probability of second stage servers being engaged with no vehicle in queue.

Pbls: First stage blocking probability.

PKs: State probability of K - vehicles in the queue while all 2nd stage servers are busy.

μ_1: Rate of service at first stage channel.

μ_2: Rate of service at second stage channel.

β: Fixed probability with which a vehicle leaves stage 1 and gets to stage 2.

α: Percentage level of significance.

SPSS: Statistical package for social sciences.

K -S: Kolmogorov - Smirnov statistics.

χ^2: Chi - square statistics.

D_n^{∞}: Critical K _S statistics for n observed data points.

LIST OF TABLES

TABLE **PAGE**

1.1 Queueing Notation 20

3.1 STATISTICAL SUMMARIES OF INTER-ARRIVAL AND SERVICE TIMES FOR LANE A AT THE POLICE CHECK POINTS (FRIDAY) 51

3.2 STATISTICAL SUMMARIES OF INTER-ARRIVAL AND SERVICE TIMES FOR LANE A AT THE POLICE CHECK POINT (SATURDAY 1) 52

3.3 STATISTICAL SUMMARIES OF INTER - ARRIVAL AND SERVICE TIMES FOR LANE A AT THE POLICE CHECK POINT (SUNDAY 1) 53

3.4 STATISTICAL SUMMARIES OF INTER-ARRIVAL AND SERVICE TIMES FOR LANE B AT THE POLICE CHECK POINT (FRIDAY 2) 54

3.5 STATISTICAL SUMMARIES OF INTER - ARRIVAL AND SERVICE TIMES FOR LANE B AT THE POLICE CHECK POINT (SATURDAY 2) 55

3.6 STATISTICAL SUMMARIES OF INTER - ARRIVAL AND

SERVICE TIMES FOR LANE B AT THE POLICE CHECK POINT (SUNDAY 2) 56
3.7 STATISTICAL SUMMARIES OF INTER - ARRIVAL AND SERVICE TIMES FOR CAR LANES (TOLL BOOTH QUEUE SYSTEM) 57
3.8 STATISTICAL SUMMARIES OF INTER - ARRIVAL AND SERVICE TIMES FOR BUS LANES (TOLL BOOTH QUEUE SYSTEM) 58
3.9 STATISTICAL SUMMARIES OF INTER - ARRIVAL AND SERVICE TIMES FOR TRUCK LANE (TOLL BOOTH QUEUE SYSTEM) 59
3.10 GOODNESS - OF - FIT TEST FOR CARS 64
3.11 GOODNESS - OF - FIT TEST FOR BUSES 65
3.12 GOODNESS - OF - FIT TEST FOR TRUCKS 65
3.13 K - S GOODNESS -OF - FIT TEST RESULTS 68
4.1 NOTATIONS OF SPECIAL RELEVANCE TO THE CHAPTER . . 82
4.2 STATISTICAL REDUCTIONS OF COMPUTED SERVICE TIME DATA FOR TOLL BOOTH CHANNELS 83
4.3 NUMBER OF VEHICLES SERVED AT THE TOLL BOOTHS

BASED ON ACTUAL COUNTING (RAW DATA) 89

4.4 PROPORTION OF EACH VEHICLE CLASS SERVED AT THE POLICE QUEUE (COMPUTED) 89

4.5 NUMBER OF VEHICLES SERVED AT THE TOLL BOOTH BASED ON COUNTING DISTRIBUTION (RAW DATA) 90

4.6 PROPORTION OF EACH VEHICLE CLASS SERVED AT THE TOLL BOOTHS (COMPUTED) 90

4.7 NUMBER OF VEHICLE SERVED AT THE TOLL BOOTHS INCLUDING THOSE MAKING U- TURNING (RAW DATA) 91

4.8 PROPORTION OF EACH VEHICLE CLASS SERVED AT THE TOLL BOOTHS INCLUDING THOSE MAKING U - TURN (COMPUTED) 92

4.9 VALUES OF UTILIZATION AGAINST SOME SYSTEM MEASURES 97

4.10 OBSERVED AND PREDICTED MEASURES OF EFFECTIVENESS 100

4.11 VALUES OF UTILIZATION AGAINST SOME SYSTEM MEASURES FOR VARIED PARAMETERS 102

4.12 COMPARATIVE TABLE FOR THE DIFFERENT VALUE CLASSES USING SOME SYSTEM EFFECTIVENESS MEASURES FOR THE EXISTING FACILITY ORGANIZATION 102

4.13 COMPARATIVE TABLE ON VARIED PARAMETERS FOR DIFFERENT VEHICLE CLASSES USING SOME SYSTEM FFECTIVENESS MEASURES FOR THE EXISTING FACILITY ORGANIZATION FOR RELIABILITY OF SIMULATOR 103

4.14(a) SUGGESTED OPTIONS AND SOME OF THEIR PREDICTED SYSTEM MEASURES 105

4.14(b) BRIEF DESCRIPTION OF THE SUGGESTED OPTIONS 106

LIST OF FIGURES

FIGURES **PAGE**

1.1 DIAGRAMMATIC SKETCH OF THE ZARIA - KADUNA EXPRESSWAY TOLL BOOTH SYSTEM 9

1.2 DIAGRAMMATIC SKETCH WITH A DESCRIPTION OF THE ENTIRE EXPRESSWAY SYSTEM AND ITS MODELS OF SERVICE 10

1.3 EVENTS FLOW CHART FOR THE SERIAL QUEUES SYSTEM . 13

3.1(a),b,c) HOURLY TRAFFIC VOLUME PATTERN AT THE POLICE CHECK POINTS 48

3.2 HISTOGRAM OF INTER - ARRIVAL TIMES (POLICE CHECK POINT) 49

3.3 HISTOGRAM OF SERVICE TIMES (TOLL BOOTHS) 50

3.4(a,b,c) OBSERVED CUMULATIVE FREQUENCY VERSUS EXPECTED DISTRIBUTION FUNCTIONS FOR CARS, BUS AND TRUCK LANES AT 5% LEVEL OF SIGNIFICANCE 66

3.5 PROGRAM FLOW CHART FOR THE K-S TEST 67

4.1 MODEL STRUCTURE UNDER STUDY 71

4.2 MAKINO MODEL STRUCTURE ADAPTED TO THE SYSTEM UNDER

INVESTIGATION 72
4.3 PROGRAM FLOWCHART OF MODEL CALIBRATION 81
4.4(a) SYSTEM EFFECTIVENESS MEASURES VERSUS UTILIZATION FOR CARS 98
4.4(b) SYSTEM EFFECTIVENESS MEASURES VERSUS UTILIZATION FOR CARS 98
4.5(a) SYSTEM EFFECTIVENESS MEASURES VERSUS UTILIZATION FOR BUSES 99
4.5(b) SYSTEM EFFECTIVENESS MEASURES VERSUS UTILIZATION FOR BUSES 99
4.6 SYSTEM EFFECTIVENESS MEASURES VERSUS UTILIZATION FOR TRUCKS 99
4.7 COMPARISON OF OBSERVED VERSUS PREDICTED UTILIZATION 104
4.8 COMPARISON OF OBSERVED VERSUS PREDICTED WAITING TIME IN QUEUE 104
4.9 COMPARISON OF OBSERVED VERSUS PREDICTED CUSTOMER LOS RATIO 104
4.10(a)(b) SYSTEM EFFECTIVENESS MEASURES VERSUS UTILIZATION FOR VARIED PARAMETERS (CAR LANES) . . . 105

LIST OF PLATES

PLATE		PAGE
I	TYPICAL OFF PEAK SITUATION TO SHOW TOLL BOOTH ARRANGEMENTS	11
II	NEAR - PEAK TRAFFIC SITUATION IN THE VICINITY OF THE POLICE CHECK POINT	11
III	TYPICAL PARKING FACILITY LOCATED AFTER THE TOLL BOOTHS	11

APPENDICES

APPENDIX		PAGE
A1	Service Time Data Analysis for Lane A	121
A2	Service Time Data Analysis For Lane B	123
B1	Kolmogorov - Smirnov (K-S) Test Program	126
C1	Queue Model Program	128
C2	Queue Model Results	131
C3	Queue Model Results for Varied Parameters	140

CHAPTER ONE

INTRODUCTION

1.1 PREAMBLE

In many real life situations, one is faced with the problem of providing services for randomly originating demands. A simple case is that of motorists arriving at a toll booth. If the arrival rate is heavy and the rate of servicing is not able to cope up with it, it is inevitable that a queue builds up. Delay occurs as a consequence.

Inter - arrival times in a traffic stream are the intervals in time between successive vehicles as they pass a given point on the road way. Service times are the intervals between the instant at which a customer begins to receive service and that at which this service ends. Inter - arrival and service times affects the individual road users to a great extent and are directly related to the levels of service. The inter - arrival and service times give the driver travelling within the queue his sense of freedom of movement or congestion and continuously affect his choice of speed . Inter - vehicle spacing is an index of the level of stream congestion. It is also apparent that, at any point in the traffic lane, the vehicular arrival rate is regulated by the sequence of times between successive vehicles in the stream.

This thesis is concerned with the analysis of queueing process with random

clearing of the queues. Also, it aims at identifying some of the factors which can lengthen or reduce waiting times in the toll booth. The development of mathematical models for replicating inter - arrival time (or arrival rate) and service time (or mean service rate) and their distribution in the traffic stream is one of the primary tasks undertaken in the work.

A model, in a general sense, is a small physical or mathematical idealization of an object or a system. From this general definition, Queue modelling can be defined as the representation of a real queueing situation in mathematical terms so that a more precise understanding of its significant properties may be gained and which, hopefully, will allow some form of prediction of future events.

1.2 NEED FOR THE STUDY

The compound nature of the study of events at a toll booth could be seen in the erratic levels of queue and congestion and consequent delays at the Kaduna - Zaria toll booth.

The importance of delay could be looked at in terms of the patrons or road users costs. These costs are in the form of times wasted: by the road users, additional fuel consumed by motor vehicles slowing down and stopping and accelerating again. These have cumulative effects on individuals.

Wasting the time of any businessman is equivalent to robbing his wallet.

Keeping him waiting is bad business practice and bad manners. Waiting is painful because it causes us to renounce more productive or rewarding ways of using time. Waiting may also be costly in itself as shelters or holding bays may have to be provided.

While attorneys and other professionals are fortunate enough to claim a fee for doing nothing in a professional way, others are often denied this luxury.

The passenger waiting time could be translated to an annual basis of passenger delays lost and it could be quantified in monetary terms. Delay and congestion are relevant to the analysis of social systems because they undermine the efficiency with which these systems conducts their businesses.

A gross estimate of the dimension of social problem is furnished by Orlov (1969), who reports that the soviet (Russian) population wasted about thirty billion hours a year waiting during shopping hours alone. This is equivalent to a year's work for no less than fifteen million people (New York Times May 13, 1969). If figures like these were aggregated for the entire service sector of the labour force, social inefficiency occasioned by client's waiting would stand out even more dramatically.

Queueing is, therefore, of fundamental interest to all organizations which process people. Congestion is inefficient in the use of resources and is detrimental to the environment adjacent to the facility. It is, therefore, always the ultimate

goal in the planning, design and operation of queueing systems to move in the direction of automatic methods of keeping the service force in correct balance with demand requirements. This is in order to achieve optimum conditions within the limitations of the non- controllable variables of the system.

It is, however, the central aim of this thesis to analyze the queueing system and determine some system effectiveness measures which provide indices to a better understanding of the factors contributing to unsatisfactory toll booth service. The toll gate between Kaduna and Zaria is taken as a case study.

Virtually, all of the previous works reported in the literature on "Queue modelling" were undertaken in developed countries, hence the need for researchers to be conducted in this area to reflect the Nigerian "Local" conditions. It is against this background that this study was carried out.

1.3 AIMS AND OBJECTIVES OF STUDY

The study aims to systematically investigate the system performance levels of the Zaria - Kaduna toll booth as a case study of queue system. In pursuance of the main aim of the thesis, the following are the specific objectives:-

(i) To undertake an articulation of queue system definition generally and within the narrow confines of the case study.

(ii) To undertake a study of queue system evaluation procedures.

(iii) To undertake an evaluation of existing level of performance as obtaining in the case study.

(iv) To collect reasonable representative arrival\service time data from the case study.

(v) To analyses the data in (iv) and fit the data obtained to suitable mathematical models.

(vi) To incorporate the models in (v) into appropriate queue model which is then to be used to evaluate system effectiveness measures.

(vii) To use results in (vi) to draw practical inferences on system operations and evaluate various operational polices.

(viii) To illustrate a few applications of the model in other specific areas of traffic engineering practice.

1.4 THE CASE STUDY

The chosen study site is located in the outskirts of Kaduna. It is about 12 kilometres away from Kaduna metropolis, and 2 kilometres away from Maraba town. It lies on the Kaduna - Zaria expressway and houses twelve toll collection booths. The expressway is a dual carriage - way with two lanes in each direction. The two lanes in each direction empty into six booth stations (via six traffic lanes) where motorists have to pay tolls.

Just before entering the service lanes, motorists are forced to go through police checks. Thus, a queue builds up even before the toll gate is reached. Two modes of operation at the booths are, therefore, possible for the whole expressway, depending on a vehicle's origin and destination.

1.4.1 DESCRIPTION OF THE TOLL GATE ENVIRONMENT

(1) Mandatary checks are conducted by two policemen at a time in each direction of the expressway just before vehicles enter the service lanes of the toll booths.

(2) Effectively, therefore two queues in series are identifiable for this system under study consisting of the police and the toll plaza.

(3) At both stages, vehicles are served in the order of arrival and at the booths one collector is always available .

(4) Initial preliminary observations indicated that traffic is heaviest at weekends but more so toward the month-ends and during special events or public holidays.

(5) Weather conditions such as rainfall and extremely hot daily temperatures seem to affect performance especially at the police check points. This is because, when it rains or is extremely hot the policemen will enter their booths for shelter and leave the motorists unserved. To this end vehicles are

not checked.

(6) In addition to police check points, a junk yard for accidented vehicles and parking facilities are provided for vehicles with problems, both before and after the toll booths.

(7) Provisions, for U - turning vehicles immediately after the police check point is available through median openings in each direction.

(8) At peak periods, there are always long queues behind the police check points, and motorists sometimes jump the police check points to directly queues in front of the toll booths.

(9) The queue lengths in front of the toll booths are limited to approximate distance of 150 meters away from the police check points.

(10) The booth stations are designated for different types of vehicles classes. There is one lane for heavy trucks, two lanes for buses and three lanes for cars. However, the system operators do not operate some booths and thus there is one lane for trucks, one for buses and one for cars.

1.4.2 SYSTEM OPERATION AND MANAGEMENT POLICIES

It was observed that the problems of organization and management of the toll booth operations are fairly enormous. The traffic manager and his supervisors employ managerial principles in the face of conflicting objectives, from time to

time, in arriving at an optimum level of operations. They utilize available human resources to meet the fluctuating levels of traffic flow, patron complaints and ever increasing costs. Unfortunately, even with the overwhelming demands, no attempt at applying scientific solution has been made. Thus, the findings from this thesis would be a valuable guide. Currently, some of the strategies that are being employed are:-

(i) When traffic is not heavy, usually during weekdays, some service points are closed to motorists leaving three in operation. At weekends, such as Fridays, Saturdays and Sundays, when demand is heaviest, the supervisor opens a lane each for trucks, buses and cars. The effect of this management process could be seen in reduced waiting times.

(ii) Operational problems that cause delays to users are often due to motorists demanding for change from the toll collections when they do not have the correct fare as stipulated by the no - change rule.

The general disposition and layout is shown on the sketches in figures 1.1 and 1.2, while events in the vicinity of the toll booth is shown in figure 1.3. The photographs shown in plates I, II and III further illustrate these events.

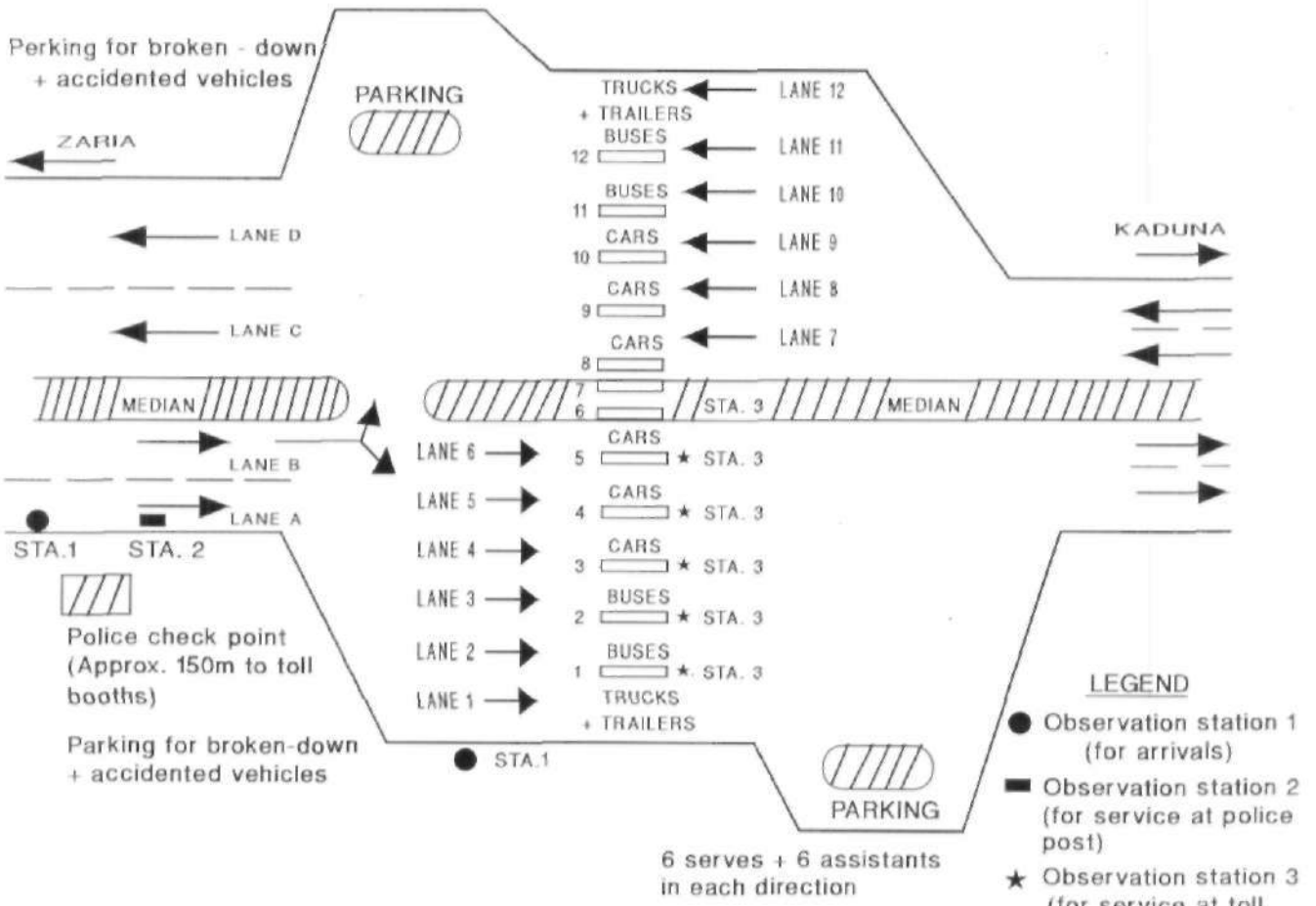
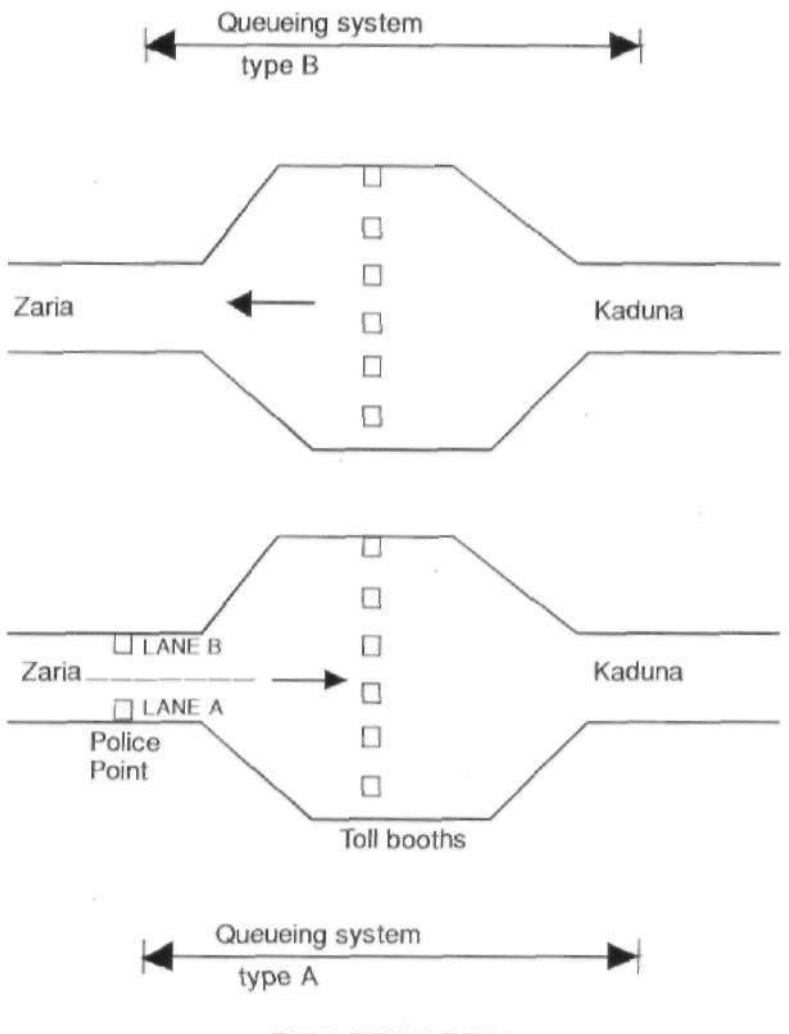


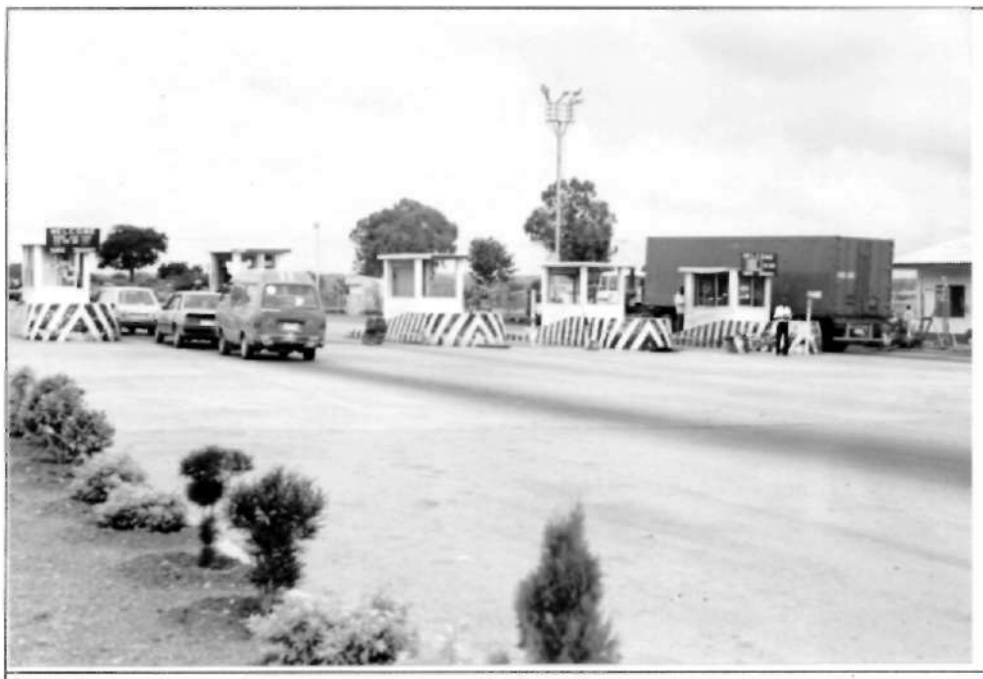
Fig. 1.1 Diagramatic Sketch of the Zaria-Kaduna Expressway Toll booth Station



DESCRIPTION

TYPE A	TYPE B
Has the following features:	Has the following features:
1. 2 queues in series	1. No Police constraint
2. 2 servers in aprallel (Police constraint)	2. A single point service system with 6 toll booth servers in parallel
3. 6 Toll booths in parallel	3. (a) Zaria end for North bound traffic
4. Kaduna end for North bound traffic	3. (b) Kaduna end for South bound traffic
5. Mode of service - motorists make payment and receive tickets	4. Mode of service - motorists make payment and receive tickets

FIG. 1.2 Diagramatic Sketch with a description of the entire expressway system and its modes of service



TYPICAL OFF - PEAK SITUATION TO SHOW THE ARRANGEMENT OF TOLL BOOTHS

PLATE I



NEAR - PEAK TRAFFIC SITUATION IN THE VICINITY OF THE POLICE POST

PLATE II



TYPICAL PARKING FACILITY LOCATED AFTER THE TOLL BOOTHS

PLATE III

1.4.3 PRELIMINARY OBSERVATIONS ON THE CASE STUDY

A preliminary survey of the case study site was undertaken to isolate the important features. It was found that the critical periods of delay are Fridays, Saturdays and Sundays and it is for such periods that the factors capable of affecting the system performance can best be captured. The preliminary observation also indicated the likelihood of U - turnings by motorists before the toll booth and characteristics of the police check points. The findings guided the model formulation as will later be explained.

On the basis of findings from the preliminary observation, a flow- chart depicting events, as well as their probabilities of occurring, is given in figure 1.3.

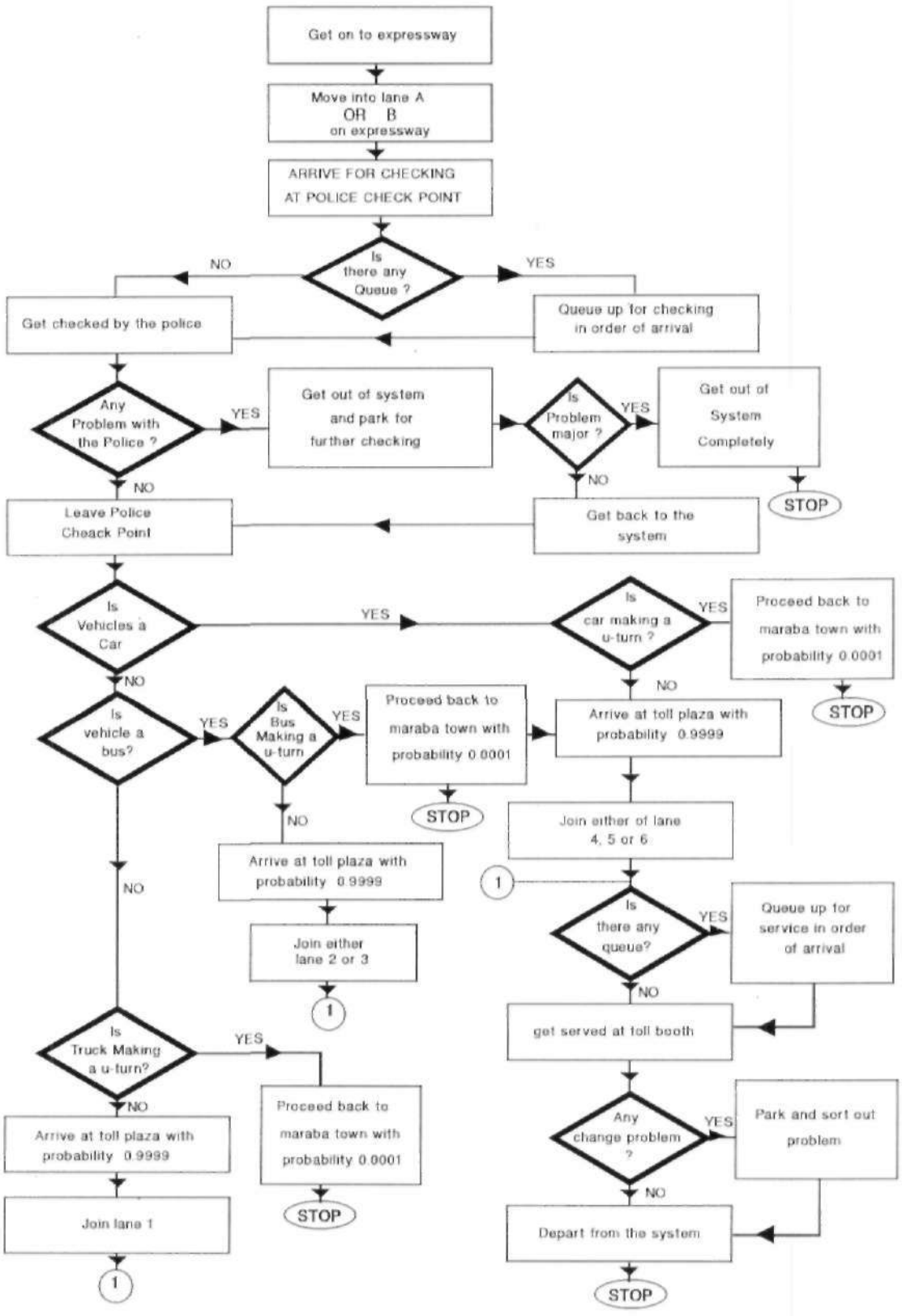


Fig. 1.3 EVENTS FLOWCHART FOR THE SERIAL QUEUE SYSTEM

1.5 ELEMENTS OF QUEUEING THEORY

1.5.1 DEFINITION, BACKGROUND AND APPLICATIONS

Queueing theory is mostly a mathematically descriptive theory which was developed to provide tools which predict the behaviour of systems that attempt to provide service for randomly arising demands, Unlike most other techniques of operations research, it does not yield the optimum solution directly. Rather, it is a tool for analysis and the ultimate goal is to use the resulting measures of performance in some cost model so as to determine optimal procedures for the operation of the system.

Queueing theory has been successfully, applied to problems of telephone conversations, many areas of air service, machine repair and so many other areas. However, it is recognized by several authors notably Lee (1966) and salty (1961), that a wide gap exists between theory and applications of queue theory and that there seems to be a lot of what will pass for queueing theory today in dozen of journals and texts while very little attention is paid to application in these same volumes.

This appearance of inactivity in the application of queueing theory is regrettable because queueing theory originated as a very practical subject whereas recent literature has been of little direct practical value.

1.5.2 DESCRIPTION OF QUEUEING PROBLEMS

A queueing system can be described as one where customers arriving for service, waiting for service if it is not immediately available and eventually leaving the system after being served.

These systems have some features that are common to all and from a consideration of these common features of queueing processes, the following primary factors could be said to pertain to any queueing problem:

(i) The probability distribution of customer arrivals

(2) The probability distribution of service times

(3) The order in which customers receive service

(4) The number of servers or channels

1.5.3 ARRIVAL PATTERN OF CUSTOMERS

The arrival pattern or input to a queueing system is often measured in terms of the average number of arrival per some unit of time (mean arrival rate) or by average time between successive arrivals (mean inter - arrival time). Since these quantities are clearly related, either of these measures suffices in describing the system input. In the event that the system of input is deterministic (Completely known and thus void of uncertainty), then arrival pattern is fully determined by either the mean arrival rate or the mean inter- arrival time. On the other hand, if

there is uncertainty in the arrival pattern (often referred to as random, probabilistic or stochastic) then these mean values provide only measures of central tendency for the input process and further characterization is required in the form of probability distribution associated with this random process.

Another factor of interest concerning the input process is the possibility that arrivals come in batches instead of one at a time. In the event that more than one arrival can enter the system simultaneously (entering the system does not necessarily mean entering into service, but may instead require joining the line when immediate service is not available), the input is said to occur in bulk or batches. In the bulk arrival situation, not only may the time between successive arrivals of the batches be probabilistic, but also the number of customers in a batch (the batch size). It is also necessary to know the reaction of a customer upon entering the system. A customer may decide to wait no matter how long the queue becomes, or if the queue is too long to suit him may decide not to enter it. If a customer decides not to enter a queue upon arrival, he is said to have BALKED.

On the other hand, customer may enter the queue, but after a time lose patience and decides to leave. In this case, he is said to have RENEGED. In the event that, there are two or more parallel waiting lines, customers may switch from one to another, that is JOCKEY for position. These three situations are all examples of queues with impatient customers.

1.5.4 SERVICE PATTERN OF SERVERS

The interval between the instant at which a customer begins to receive service and that at which this service ends, is known as his service time. Much of the discussion about arrival pattern is also appropriate in discussing service pattern.

1.5.5 QUEUE DISCIPLINE

This refers to the manner by which customers are selected for service when a queue is formed. Most common of this is the first In, First out (FIFO) discipline. Others in common use are

(i) The last In, First Out (LIFO)

(ii) Selection of service in random order independent of the time of arrival to the queue (SIRO)

(iii) A variety of priority schemes where customers are given priorities upon entering the system.

There are two general situations in priority schemes called the pre-emptive service priority order (PSPO) and the non-pre-emptive priority service (NPPS). The queue discipline is known to affect some aspects of system performance measures.

1.5.6 SYSTEM CAPACITY

This refers to the physical limitation to the amount of waiting room in a queueing system, such that when the line reaches a certain length, no further customers are allowed to enter until space becomes available by a service completion. Such queues with limited waiting room can be viewed as ones with forced balking as customer are forced to balk if they arrive at a time when the queue size is at its limit. The limited waiting space may also be of an excessively long dimension when it is then referred to as infinite.

1.5.7 NUMBER OF SERVICE CHANNELS AND STAGES

The number of service channels refers to the number of parallel service stations which can serve customers simultenously and it is generally assumed that the service mechanism of parallel channels operate independently of one another when customers may, in fact, select or be directed to any of the channels, the channels are normally referred to as providing full availability, while the case of restricted availability occurs when the channels are fully available with respect to some customer but not to others.

Another form of multi-channel arrangement is the serial type where a customer passes through many channels in turn and may have to wait to be served at each stage of the process. For some of this type of system, recycling or

feedback may occur as in manufacturing processes where quality control inspections are performed after certain stages.

1.5.8 CLASSIFICATION OF QUEUES

Based on the characteristic mentioned above for queues, they can be divided into different classes and this classification is now rather standardized throughout the queueing literature. A queueing process is normally described by a series of symbols and slashes such as A/B/X: (Y/Z) where A indicates, in some way ,the inter-arrival distribution, B, the service pattern as described by the probability distribution for service time, X the number of parallel service channels, Y the restriction on service capacity, and Z the queue discipline. This classification was due, for the most part, to Kendal (1953) and Lee (1966). Other researchers have also made some modifications to this system of classification. In many situations, however, only the first three symbol are in use.

The letters are normally replaced with their standard symbols shown in the notation table 1.1.

Table 1.1 QUEUEING NOTATION

CHARACTERISTICS	SYMBOLS	EXPLANATIONS
INTER -ARRIVAL TIME DISTRIBUTION (A)	M D E_K GI	EXPONENTIAL DETERMINISTIC ERLANG TYPE K GENERAL INDEPENDENT
SERVICE - TIME DISTRIBUTION (B)	M D E_K G	EXPONENTIAL DETERMINISTIC ERLANG TYPE K GENERAL
NUMBER OF PARALLEL SERVERS (X)	1,2,... ∞	
RESTRICTION ON SYSTEM CAPACITY (Y)	1,2,... ∞	
QUEUE DISCIPLINE (Z)	FIFO LIFO SIRO PRI GD	FIRST IN, FIRST OUT LAST IN, FIRST OUT SERVICE IN RANDOM ORDER PRIORITY GENERAL DISCIPLINE

SOURCE: (Ikem 1982)

1.6 COMPUTER METHODS

Digital Computer methods are normally employed in queueing system

analysis in the specific area of simulation as an alternative technique to the analytical method. Similarly, standard computer programs exist and such programs can also be developed to yield solution to analytical models of queueing systems which would otherwise be very tedious or time consuming to solve manually. Most of these Programs are written in fortran iv language except in the area of simulation where a choice is normally made from a number of specific computer language applicable only to simulation.

Computer Programs are thus ready tools in solving queueing problems and they are normally designed to yield all the system effectiveness measures as may be desired.

1.7 SCOPE AND LIMITATION OF THE STUDY

Toll booth queueing system is not the only area queueing system performance could be measured. Other areas are: Signalized intersections, cars parks, Bottleneck situations. This study however, focuses on the toll booth modelling so as to allow for a concentrated and thorough investigation of this seemingly critical aspect of the system.

The study is limited to the Zaria - Kaduna toll booth and not to all toll booths in the country, due to financial and time constraints.

CHAPTER TWO

REVIEW OF RELATED LITERATURE

2.1 PREAMBLE

It is possible, for most real-life cases, to find an existing model in the literature that can be applied directly. It is then only necessary to search for suitable models, evaluate these models against what is required of the system under study and finally modify this base model to reflect a complete reality of the system.

The outlined procedure is followed for model selection in this work. Adhering to the first part of the procedure involves a review of existing models with a view to identifying candidate models which are subjected to further consideration.

The review of existing literature, presented herein, is thus intended to provide a basis from which the study framework could be developed.

2.2 CLASSES OF QUEUEING MODELS

In queueing literature, most of the idealized situations fall into the following categories:

(a) Single server queueing models for single stage service

(b) Multiple server queueing models for single stage service

(c) Serial and cycle queueing models.

Imposing limited waiting room on any of these models normally means further classifications. Similarly, balking and reneging by patrons could be additional bases for finer classifications. Several other types of classifications are also possible depending on the researcher and the purpose of the study. For example, the number of channel, input type and service characteristics are valid bases for categorization. In this review, the queueing models will be discussed according to whether they are single, multiple or serial server but the other cases will be mentioned.

2.3 MAIN FEATURES OF EXISTING QUEUEING MODELS

2.3.1 SINGLE - CHANNEL QUEUEING MODEL

This class of queueing models have been well scrutinized by researchers and have been widely applied. Interestingly, the queueing literature continues to grow as researchers have continued to explore new aspects of this near saturated area. This is because models in this group are more fundamental and form the basic building blocks upon which the relatively more complex models are based. Most common queueing situations are, however, not single server queues and where they are, they are soon modified to the two-channel system when the patronage level appreciates.

Of all the single server queueing models currently known, the M/m/1, (∞/FIFO) model remains the simplest and it is quite easy to treat mathematically. Hence it, alone, has attracted the greatest number of research efforts. Many authors such as Saaty (1961), Cox and Smith (1961), Lee (1966), Prabu (1965), Teha 1971) and Giffin (1978) present excellent approaches for the formulation and solution of this class of queueing problems. Gaver (1953) derived the relationship between the distribution of customer - service times and that of customer - waiting times for this class of models and showed, mathematically, that this holds for a large class of servicing time distributions. He showed further that if the right sort of alteration is made in service-time distributions, considerable reduction in customer waiting times could be made. New researches continue to yield results for this class of models and recently Labetoulle, et al, (1981) has analysed the M/M/1 model with feedback and obtained the distribution of inter-input, inter-output and inter-feedback times.

Generally, because it aids in the understanding of all other queueing models, the mathematics for this model is usually set out in considerable details in most of the existing literature. Cox and Smith, (1961) and Morse, (1958) presented excellent methods for solving for the transient state conditions, while Giffin, (1978) suggests the use of the system balance arguments and a transition rate of flow diagram to quickly construct steady state equations. Typical mathematical

relationships derived for this class of queues include the following:

$$P_o^1(t) = -\lambda P_o(t) + \mu P_1(t) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.1$$

Where: J = number of customers = 0,1,2,.....

$$P_j^1(t) = \lambda P_{j-1}(t) - (\lambda + \mu) P_j(t) + \mu P_{j+1}(t), j \geq 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots 2.2$$

Where: $P_j(t)$ = probability of state S_j at some time t

Under the assumption of steady state

$dP_j(t)/dt = 0$ for j = 0,1,2,.....

Then the system of equations become a set of simple difference equations of the form:

$$P_1 = \frac{\lambda}{\mu} P_o$$

$$P_{j+1} = \frac{\lambda+\mu}{\mu} P_j - \frac{\lambda}{\mu} P_{j-1}, j \geq 1 \quad \ldots\ldots\ldots\ldots 2.4$$

Expected number of customer in the system

$$L = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda} \quad \ldots\ldots\ldots\ldots 2.5$$

ρ = traffic intensity

μ = Service Rate

λ = Arrival Rate

Expected number in queue

$$L_q = \frac{\rho^2}{1-\rho} = \frac{\lambda^2}{\mu(\mu-\lambda)} \quad \text{.....2.6}$$

Expected time in the system

$$W = \frac{L}{\lambda} = \frac{1}{\mu-\lambda} \quad \text{.....2.7}$$

Expected time in queue

$$W_q = \frac{\lambda}{\mu(\mu-\lambda} \quad \text{.....2.8}$$

2.3.2 THE MULTIPLE CHANNEL QUEUEING MODELS

These models typify situations that are characterized by many parallel servers attending to customers who are all served from a single line. The principal difference between the multiple and single server queueing processes is that the former exhibits a greater variety of queue disciplines: The possibilities of unusual customer behaviour are often realised and, except for a few recent attempts, classical multiple-channel models assume FIFO rule.

Of all the existing multiple channel models, the simplest is the m/m/c: (∞/FIFO) and it is a logical extension of the m/m/1: (∞/FIFO) model. It has attracted more research materials than any other model in this class, possibly because of its mathematical simplicity and its steady state equations are easily set out using the balance argument of Giffin, (1978). the main assumptions for this

class of models are:

i. Customers arrive in poisson fashion at mean rate λ.

ii. There are C identical servers or channels in parallel.

iii. The service-time distribution for each channel is exponential with mean $1/\mu$

iv. Servers are non-cooperative in the sense that one customer is never attended by more than a single server.

v. Arriving customers enter the first available open channel. When all are full, they form a single queue with unlimited waiting space from which they are served in FIFO fashion. The possibility of disintegration (i.e. the single queue would breakdown into several queues), and the associated phenomena of jockeying, preferential queueing and son on - are not modelled for.

vi. If there are two or more free channels, the arriving customer goes to any one of the channels and exercises no preference for any channel or group of channels.

vii. As soon as a channel falls free, the customer who has been queueing longest goes into it instantaneously.

Typical mathematical relationship derived include:

$$P_o = \left[\sum_{j=o}^{c-1} \frac{1}{j!}\left(\frac{\lambda}{\mu}\right)^j + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c \left(\frac{c\mu}{c\mu-\lambda}\right)\right]^{-1} \quad \text{.....2.9}$$

Expected number in the system

$$L = \frac{\lambda}{\mu} + \left[\frac{\left(\frac{\lambda}{\mu}\right)^c \lambda\mu}{(c-1)!\ \ (c\mu-\lambda)^2}\right] P_o \quad \text{.....2.10}$$

Expected number in queue

$$L_q = \left[\frac{\left(\frac{\lambda}{\mu}\right)^c \lambda\mu}{(c-1)!\ \ (c\mu-\lambda)^2}\right] P_o \quad \text{.....2.11}$$

Expected total waiting time in the system

$$W = \frac{1}{\mu} + \left[\frac{\left(\frac{\lambda}{\mu}\right)\mu}{(c-1)!\ (c\mu-\lambda)^2}\right] P_o \quad \text{.....2.12}$$

Expected time spent in the queue

$$W_q = \left[\frac{\left(\frac{\lambda}{\mu}\right)^c \mu}{(c-1)!\ \ (c\mu-\lambda)^2}\right] P_o \quad \text{.....2.13}$$

The truncated m/m/c : (L/FIFO) model follows directly from the m/m/c: (∞/FIFO) model on making the limited waiting room assumption and is by far more general and more useful. Saaty, (1961) and Lee, (1966) show how the waiting - time distribution can be determined for this class of models.

Among relevant mathematical relationships are:

$$P_n = \begin{cases} \dfrac{(1-\rho)\rho^n}{1-\rho^{k+1}} & , \quad P \neq 1 \\ \dfrac{1}{k+1} & , \quad P = 1, n = 0,1,2,\ldots\ldots K \end{cases} \quad \ldots\ldots 2.14$$

Expected number in the system

$$L = \begin{cases} \dfrac{k}{2} & , \quad \rho = 1 \\ \rho \dfrac{[1-(k+1)\rho k + k\rho k+1]}{(1-\rho^{k+1})(k-\rho)} & , \quad \rho \neq 1 \end{cases} \quad \ldots\ldots 2.15$$

Expected number in queue

$$L_q = L-(1-P_o) \ldots\ldots 2.16$$

Expected time in the system

$$W = \frac{L}{\lambda^l} = \frac{L}{\lambda(1-P_k)} \ldots\ldots 2.17$$

Expected time in the queue

$$W_q = W-\frac{1}{\mu} = \frac{L_q}{\lambda^l} = \frac{L_q}{\lambda(1-P_k)} \ldots\ldots 2.18$$

2.3.3 MODEL FOR SERIAL AND CYCLIC QUEUEING PROCESSES

Many real world problems exist in which two or more queues are arranged in series with the output from one queueing process, or part of it, being the input into another. When the ends are joined to form a loop, so that customers rotate through the system, the system is called a cyclic queueing system.

Two important features of these systems are the Phenomenon of blocking and the associated concept of maximum utilization as well as the input - output relationships within the system.

(a) Blocking

The blocking of a facility is that situation that occurs when a unit has completed service in one stage but cannot proceed because the next stage queue is at its maximum length. The phenomenon is therefore a feature of serial queues which occurs only when a limited queue is allowed at the second or other stages.

For such blocked situations, the maximum possible utilization which is unity for a single stage of service will be reduced and the first stage is then said to become blocked, or not available for service on incoming units. The blocking phenomenon results in an effective service rate for the first stage that is lower than the actual service rate of the stage. The effective service rate is given by the product of the original service rate and a factor equal to the fraction of time that the first stage is unblocked. The relevant mathematical relationships are:

$$\mu^{1} = M\,\mu \qquad \text{.....2.19}$$

where M = number of channel

μ^{1} = Effective service rate

Facility utilization factor becomes

$$\rho = \frac{\lambda}{M\mu}$$

define $r = \frac{\lambda}{\mu}$

Mean number in system

$$L = L_q + r \quad \text{.....2.20}$$

Mean waiting time

$$W_q = \frac{L_q}{\lambda} \quad \text{.....2.21}$$

Mean time in system, including service

$$W = W_q + \frac{1}{\mu} = \frac{L}{\lambda} \quad \text{.....2.22}$$

For steady - state operation, therefore, the maximum allowable arrival rate is less than this effective service rate, and this explains the reduction in maximum possible utilization as earlier pointed out. The situation becomes worse if more than two stages are operated in sequence. Blocking can occur in all stages except the last stage, and the blocking in each stage will have an effect on all preceding stages. The maximum possible utilization will then decrease further from

$$\rho = \frac{\lambda}{\mu} \quad to$$

$$\rho = \frac{\lambda}{M\mu} \quad \text{.....2.23}$$

In the general N - stage problem, blocking occurs more frequently in the first stage than in any succeeding stages and it determines maximum possible

utilization for the entire system.

Hence, the first stage maximum utilization is also the maximum utilization of the system.

(b) Output-input Relationships

This is an important area to address in the analysis of tandem queues because the output from one channel constitutes the input into the subsequent channels.

Burke (1956), Reich (1957) and others have studied this problem. Their studies have shown that the distribution of outputs of a steady state queue is independent of the type of service discipline used. Burke's work (1957), established for steady state, the independence of time intervals between departures for the m/m/c. For this system, the departure satisfy a poisson process with the same parameter as that of the input distribution - a fact previously speculated upon from intuitive reasoning and inferred from steady state conditions. He also established the independence of the inter-departure intervals and the state of the system at the end of the interval. the output distribution at any time t, depends only on the state of the system at t and not on any previous state.

Reich, using different methods, showed that, it is not generally reasonable to expect, the outputs to match the inputs even in the steady state. He also gave the partial converse to Burke's results that for a single - channel, poisson arrivals and departures imply an exponential service time distribution or a step function at zero.

Hunt (1956), by assuming poisson inputs, exponential service times, no leaving of system until all channels are entered, beginning at the first channel, has

studied utilization. Four particular cases of service facilities in series were treated involving infinite storage space between stages, no storage space between stages, finite storage between stages and the case of Un-paced belt-production line. In the last case the probability of completion of service time t after the line moves is one of the results computed.

A comparison is given for considering a two-stage case, and the maximum possible utilization is obtained for all cases discussed above. Any, or all, of the stages may be in operation simultaneously. The mean number of Units in the system, as a measure of effectiveness for the comparison purposes, were determined. The maximum possible utilization was also obtained, in order to show the effects of inter stage storage space. When no queues are allowed before subsequent channels, blocking was considered to have occurred and effective service rate is then the product of the given service rate and the fraction of time during which the channel is unblocked. The basic mathematical relationships are:

$$\mu^{1} = M\mu \quad \text{.....2.24}$$

where M = number of channel.

Facility utilization factor becomes:

$$\rho = \frac{\lambda}{M\mu}$$

define $r = \frac{\lambda}{\mu}$

Mean number in system

$$L = L_q + r \text{.....2.25}$$

Expected number in queue

$$L_q = \left[\frac{\left(\frac{\lambda}{\mu}\right)^c \lambda\mu}{(c-1)!(c\mu-\lambda)^2}\right] P_o \text{.....2.26}$$

Expected total waitingtime in the system

$$W = \frac{1}{\mu} + \left[\frac{\left(\frac{\lambda}{\mu}\right)^c \mu}{(c-1)!(c\mu-\lambda)^2}\right] P_o \text{.....2.27}$$

Expected time in the queue

$$W_q = \left[\frac{\left(\frac{\lambda}{\mu}\right)^c \mu}{(c-1)!(c\mu-\lambda)^2}\right] P_o \text{.....2.28}$$

Application of the above idealization to model serial queueing process with capacity restrictions are:

a. In all case, distribution of arrival to the first stage is assumed to be poisson and service is exponential at all stages.

b. Queues Operate on a first - come, first served basis with no defection.

c. Service commences instantaneously when a unit arrives at an empty stage and units are transferred instantaneously from one stage to the next on completion of service.

The results obtained, when each station is treated independently include the steady state probabilities:

$$P_{ni} = (1-\rho)\rho_i^{ni} \quad , \quad ni = 0,1,2,\ldots\ldots\ldots\ldots\ldots 2.29$$

where ni = number in the system consisting of station i only.

2.4 SIMULATION MODELS

Surprising, not much of simulation techniques applied to queueing processes are reported in the literature on queues. Many text books, Lee (1961), Gross and Harris (1974) and many journal authors recommend that simulations should be used whenever they can be justified.

Simulation models offer means of studying queueing situations which are too complex to be handled analytically, either because of the complication of the system or its mathematical intractability. Simulation models also come into use when the characteristics of the input and service mechanism do not permit stability or when the nature of the queue discipline is not easy to model. A combination of any of the factors described above will also justify the adoption of simulations.

Gross and Harris (1974), report a case of the application of Simulation to tollbooth traffic and it is suggested that when serial queues are fairly complicated or when the distributions are not exponential as normally assumed, then the only course of action is simulation. Given the particular system under study in this

work with its application in character of customers, other operating conditions and configuration, one is bound to include the simulation modelling technique as one of the approaches to solving this queueing problem.

2.5 SELECTION OF POTENTIAL CANDIDATE MODELS

From the foregoing discussion of existing queueing models, it is the objective here to consider queueing models further in brief detail and in relation to the current case study.

First, the single server queueing models are seen to be unfit for further consideration since the queueing problem is definitely too far from that of a simple single server model. To model the system in this way would just amount to unjust over simplification in the face of other more suitable techniques. such a model will also not be flexible for all the manipulations that are vital in studying the system.

The multiple-channel queues have far more practical applicability than the single servers queues and this is well demonstrated in subsequent parts of this study. The system under study entails, essentially, the police checkpoint, which is just upstream of the toll booth, and the queue formed at the toll booth. Thus, the check point serves as a kind of filter from which cleared vehicles emerge as arrivals to the queue at the toll booth. The two queues involved in the system under study constitute a serial arrangement and it is thus better modelled as multiple-channel queues after making valid assumption, necessary to make them flexible enough for analytical manipulations.

Given the assumptions normally made for this class of models and the fact

that evidence exists that the arrivals on all lanes and services in all the channels could be reasonably represented by poisson and negative exponential distributions respectively, it is then an easy matter to propose an m/m/2: (∞/FIFO) model for the queues at the police check point. Also, depending on the idealizations made about the toll booth queue concerning the non-homogeneity of service at the channels, vehicle class differences and customer behaviours this queueing situation can be idealized by a M/M//C: (∞/FIFO) or a M/MC: (L/FIFO) model. The multiple channel model is therefore suitable model for further consideration either on its own or a serial queueing arrangement.

Of all the serial and cyclic queueing models proposed, or mentioned in the literature, three appear to be particularly appealing and Warrant further consideration. The first is the two- stage model proposed by Makino. Makino model constitutes C servers in stage 1 known (for this purpose of this study) as the register stage. Units leaving these first stage service points, either queue up in the limited waiting room provided or go straight for service in any of the S channels of service in stage 2, known as the marker stage. The model is restricted to only those states of the system in which all of the first stations are in service or in blocking.

ASSUMPTIONS

The assumptions upon which the Makino model is based are as follows:

(i) Customers arrive in poisson fashion at a constant rate in both queues.

(ii) There are C identical servers in stage 1 and S identical servers in stage 2.

(iii) The service times for each channel in both stages are independent with

exponential distribution but the service rates have a common value μ_1 (service completions per unit time) for all first stage servers and μ_2 for all second stages servers.

(iv) servers are non-cooperative in the sense such that one customers is never attended to by more than a single server at a time.

(v) At both stages, arriving customers enter the first available open channel and, when all channels are full, they form a single queue with unlimited waiting space in the second queue. From these single lines of queue, customers are drawn and served in a FIFO fashion with no defections.

(vi) If there are two or more free channels in any of the stages, the customer goes to any of them and exercises no preference for any channel or group of channels.

Using Makino notation with the states (i,j) for the system:

Where i = number of units in stage 1 and

j = number of units in stage 2.

Thus, by defining P_{ij} as the probabilities that the system is in state i,j where

$$i = \begin{cases} 1,2,.....,K(K \leq N) = \text{No of units in the limited waiting room for the second stage} \\ \\ 1,2,.....,c(c > N) = \text{No of blocked first stage channels} \end{cases}$$

and

j = 1,2,....., S = No of second stage channel busy

Makino obtained the following steady state results:

$$\mu_2 P_{o,1} - c\mu_1 P_{o,o} = 0$$

$$c\mu_1 P_{o,h-2} + h\mu_2 P_{o,h} - \{c\mu_1 + (h-1)\mu_2\} P_{o,h-1} = 0$$

$$(for\ 2 \le h \le s)$$

Where h = random variable that defines the no of 2nd stage servers that are busy.

$$C\mu_1 p_{O,S-1} + s\mu_2 p_{1,S} - (C\mu_1 + S\mu_2) p_{O,S} = 0$$

$$C\mu_1 p_{K-2,S} + s\mu_2 p_{K,S} - (C\mu_1 + s\mu_2) P_{k-1,s} = 0$$

$$(for\ 2 \le k \le N)$$

$$c\mu_1 P_{N-1,s} + s\mu_2 P_{B1,s} - (c\mu_1 + s\mu_2) P_{N,s} = 0$$

$$c\mu_1 P_{N,s} + s\mu_2 P_{B2,s} - \{(c-1)\mu_1 + s\mu_2\} P_{B1,s} = 0$$

$$\{c-(L-1)\}\mu_1 P_{BL-1,s} + s\mu_2 P_{BL+1,s} - \{(c-L)\mu_1 + s\mu_2\} P_{BL,s} = 0$$

$$(\text{for } 2 \le L \le c-1)$$

$$\mu_1 P_{BL-1,s-1} - s\mu_2 P_{BL,s} = 0 \quad \text{.....2.30}$$

Where μ_1 = rate of service at the first stage service station

μ_2 = rate of service at the second stage service station

S = Number of identical servers in stage 2

C = Number of identical server in stage 1.

From these equations the following results follow.

$$P_{o,h} = \frac{1}{h!}\left(\frac{c\mu_1}{\mu_2}\right)^h P_{oo} \text{(for } o \le h \le s\text{).....2.31}$$

$$P_{k,s} = \frac{1}{s!}\left(\frac{c\mu_1}{\mu_2}\right)^s \left(\frac{c\mu_1}{s\mu_2}\right)^k P_{oo} \quad \text{(for } 0 \le k \le N\text{).....2.32}$$

$$P_{BL,s} = \left\{\frac{1}{s!}\left(\frac{c\mu_1}{\mu_2}\right)^s \left(\frac{c\mu_1}{s\mu_2}\right)^N P_{oo}\right\} \left(\frac{\mu_1}{s\mu_2}\right)^L \{c(c-1).....(c-L+1)\}$$

(for $1 \le L \le c$).....2.33

Considering that:

$$\sum_{h=0}^{s} P_{o,h} + \sum_{k=1}^{N} P_{k,s} + \sum_{L=1}^{c} P_{BL,s} = 1$$

$$P_{oo} = \frac{1}{\sum_{L=1}^{s} \frac{1}{h!}\left(\frac{c\mu_1}{\mu_2}\right)^h + \sum_{k=1}^{N}\left\{\frac{1}{s!}\left(\frac{c\mu_1}{\mu_2}\right)^s \left(\frac{c\mu_1}{s\mu_2}\right)^k\right\}}$$

$$+ \frac{1}{\sum_{L=1}^{c}\left[\frac{1}{S!}\left(\frac{c\mu_1}{\mu_2}\right)^s \left(\frac{c\mu_1}{s\mu_2}\right)^N \left(\frac{c\mu_1}{s\mu_2}\right)^L \{c(c-1)....(c-L+1)\}\right]} \text{.....2.34}$$

Equations (2.30) To (2.34) Represent the basic equation of state probabilities of the Makino model.

The Makino two - stage model represent the exact configuration of the system without too much over simplifying assumptions. The model appears to posses the further attribute of being easy to manipulate.

The model can also be used following the assumptions implicit in its

formulation for fairly varying traffic conditions. It can be used to study the organization of the entire system by manipulating the system variables.

The second candidate model in this class is the cyclic queue model of Koenigsber, (1960). It is proposed here to replicate the general machine interference model with standby generators, operators and service men to the traffic situation in the vicinity of the toll booth. It includes the Police as repairmen and the vehicles as the breakdown machines, the assumption being made is for the peak traffic operations. This system, like that idealized by the machine - interference model, has a limited number of vehicles always present in between the police and the toll booths.

Assumptions have to be made, however, about output of the system Vis-a-Vis the input to the Police queue in terms of equality. This is the main draw back of the models but aside from this it has most of the attributes of the Makino model and the system under study satisfies fairly well all the assumptions made about the model.

The third type of model in this class is the serial queueing model of Jackson, (1958), where a M/M/C: (∞/FIFO) model feeds into m/m/c: (∞/FIFO) model. This is a model with sufficient merit also to justify it for further consideration. The model can be applied to the system in studying the different vehicle classes separately and since the characteristics of the system are such that the Markovian assumption are fairly fulfilled and service discipline can be accepted to be FIFO type, not much error would be introduced by applying this model. It also has the advantage that, at steady state and for independence of the queues, excellent charts exist as in Lee (1961), for reading off some effectiveness measures. Its major

drawback is that it does not consider the effect of inter-stage blocking due to the limited capacity between the service stations. However, from the work of Burke, Reich and Finch, and the fact that authors such as Lee (1961), suggest independence at a probability equal to or less than 0.01 of a customer being rejected, this model could be applied without much error of assumption.

The model proposed by Hunt (details) is rather too limited in scope for the system under investigation and would require a lot of modifications before it could be applicable. Sincerely, the model developed by Hiller and Boiling (1968), does not replicate the system much and the mathematical methods of evaluation and analysis may not be flexible enough in terms of the modified system.

Sharma's model, (1973) has a lot of potentials as it is a fairly good representation of the system like the Makino and the Koenigsberg models but unlike them, it does not have the near mathematical simplicity that is so vital for practical application. Herein lies its major draw back and reason for it not being considered further.

Having considered the essential attributes of the Makino model and that at least one can modify the model from any stage of its development for practical applications so far stated, it is finally selected for this study.

CHAPTER THREE

DATA COLLECTION AND ANALYSIS

3.1 PREAMBLE

This work is pursued through various activities, spanning collection of field data, development of model and calibration of the model. This section of the write -up is devoted to a description of the field work.

3.2 ORGANIZATION OF DATA COLLECTION

For the purpose of data collection, the study site was divided into four zones namely

(a) Area before the police checking area

(b) The police checking area

(c) Waiting area, where vehicles queue just before the toll plaza (i.e covering the toll booth lanes).

(d) The toll booths.

Data collection from each zone was designed to capture queue characteristics and other activities taking place at the particular zone. Detail of the data collection techniques adopted are described later.

In the different zone, data collection was designed to facilitate the estimation of parameters needed in assessing system effectiveness measures. Such parameters provide indices of service delivery and efficiency factors and include the following:

(i) Waiting time characteristics (i.e mean, variance, e.t.c)

(ii) Number in queue and in system

(iii) Customer loss Ratio

(iv) Facility Utilization, as measured by times in which the system is busy.

(v) Fixed probability with which a vehicles leaves stage 1 and get to stage 2.

(vi) Blocking probability (i.e relative frequency of instance when the system is blocked).

In order to obtain the listed parameters, the following activities were undertaken at each zone:

(i) Collection of data on the volumes of traffic on hourly basis, spanning 7 a.m to 7 p.m on carefully selected days. The duration of observation agrees with the recommendation by pignataro (1973).

(ii) Collection of data on inter -arrival and service time distributions within the same period in each zone.

(iii) Collection of data relating to number of vehicles making U - turn

(iv) Collection of data on the different vehicle types so as to determine the proportion(s) of vehicle types (e.g cars, buses and trucks) in the traffic stream.

The data collectors were positioned so as not to interrupt the normal traffic flow in any way. The study was carried out in two phases, namely:

(a) Preliminary survey and analysis

(b) Detailed data collection and analysis

The preliminary survey was designed to provide a basic understanding of the system and thus have a basis on how to organize the detailed data collection exercise(s).

3.3 DATA COLLECTION METHODS AND TECHNIQUES

The full outline of the methods and techniques employed for the data collection exercises are outlined in this sub-section.

The data was collected within the months of November and December 1996 as well as January, February and March 1997. The recorded data was confined to only normal traffic and it excluded such occasions as queue breakdown or off-peak period.

(1) The following types of data were collected throughout the period of study.

(a) Inter - arrival time data, by timing the arrival of consecutive vehicles at a specified point on the lanes.

(b) Service time data, by timing a vehicle as it arrived for service and as it departed from the system.

(c) The vehicle classification data were recorded instantaneously as well as the vehicle service times by indicating on a side column on the field data sheet with the appropriate symbol for that type of vehicle.

(d) The vehicle distribution study was carried out in order to obtain the pattern of distribution of vehicles into the toll booth lanes as the vehicles left the police point. This was undertaken by two collectors with each assigned to one of the police lanes. Each then counted the vehicles as they arrived into the lanes for normal periods of study. Also recorded were the U -turning vehicles after the police check point.

(ii) Synchronized electronic digital watches were used for all timings.

(iii) All the data was collected manually, using human data collectors who were first trained on what to do in the field . The collection for a

particular day was designed to give representative data and was undertaken in two stages; at the police queue stage and the toll booth queue stage in sequence. The collectors were allocated in such a way that all data necessary for each zone of the study site where the collector is situated were collected. All relevant data about a stage was normally collected before they all moved together to the next stage.

(iv) A duration of 20 or 30 minutes was selected for most of the data collected on inter - arrival and service times.

3.3.1 PRELIMINARY SURVEY AND ANALYSIS

The purpose of this survey was to identify period of near constant vehicular flow to the system as well as the peak periods of operation within the defined 12 hours observation period.

The first involved a visit to the study site to collect traffic data to indicate the range of hourly volumes obtainable at the site. Typical of the results obtained are figures 3.1 (a) 3.1(b) and 3.1(c) which show periods of high traffic intensity and thus indicates the peak period(s). Interestingly, the weekend data gave the peak volumes.

The second stage of the preliminary survey involved collecting sets of representative data on inter - arrival times and service times for both police and toll booth queues in the manner discussed.

The typical histograms of the observed inter -arrival and service times are plotted in figures 3.2 to 3.3. These were then compared to the shape of known standard distribution to see how reasonable the exponential assumption is for the system. Another quantity which was considered important at this stage of the study is the co-efficient of variation (c.v) which is the ratio of the standard deviation to the sample mean time of the distribution. This value is known to be equal to unity for the exponential, much less than one or tend to zero for the constant time distribution and to lie between zero and unity for the gamma distribution.

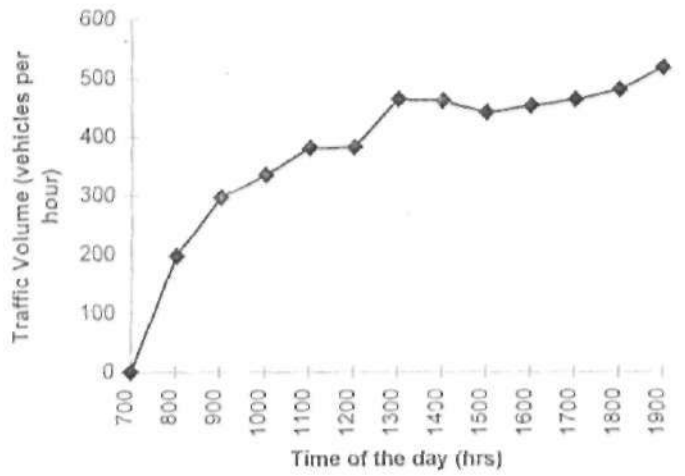


Fig. 3.1a Hourly Traffic Volume Pattern at the Police check point on Friday 18/11/96 (Both lanes combined)

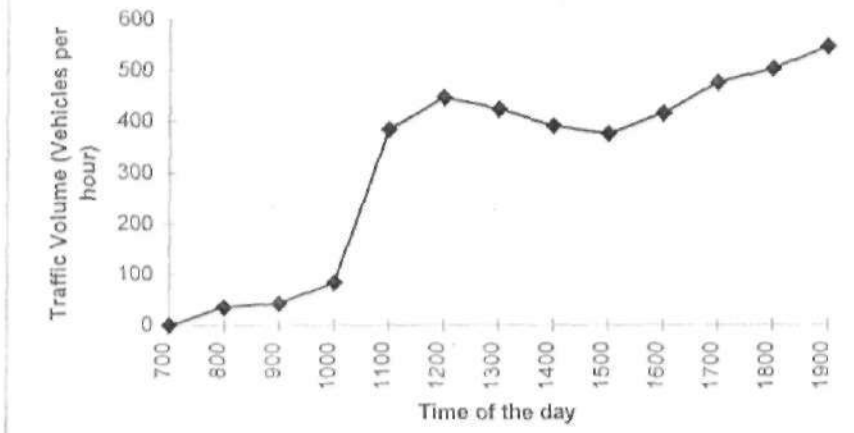


Fig. 3.1b Houly Traffic Volume Pattern at the police check point on Saturday 19/11/96 {Both lanes combined)

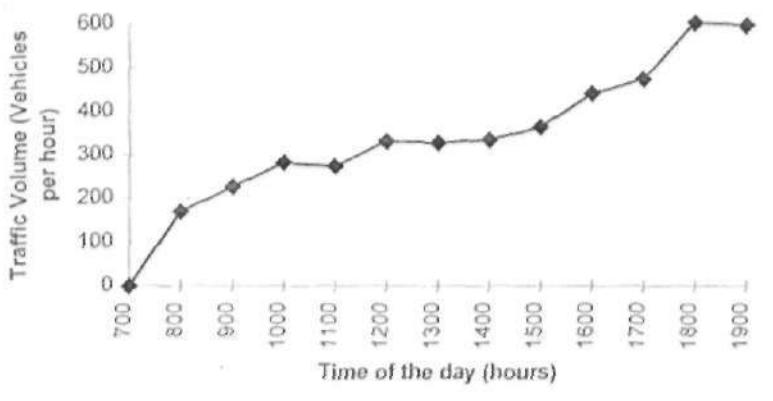


Fig. 3.1c Houlry Traffic Volume pattern at the poice check point on Sunday 20/11/96 (Both lanes combined)

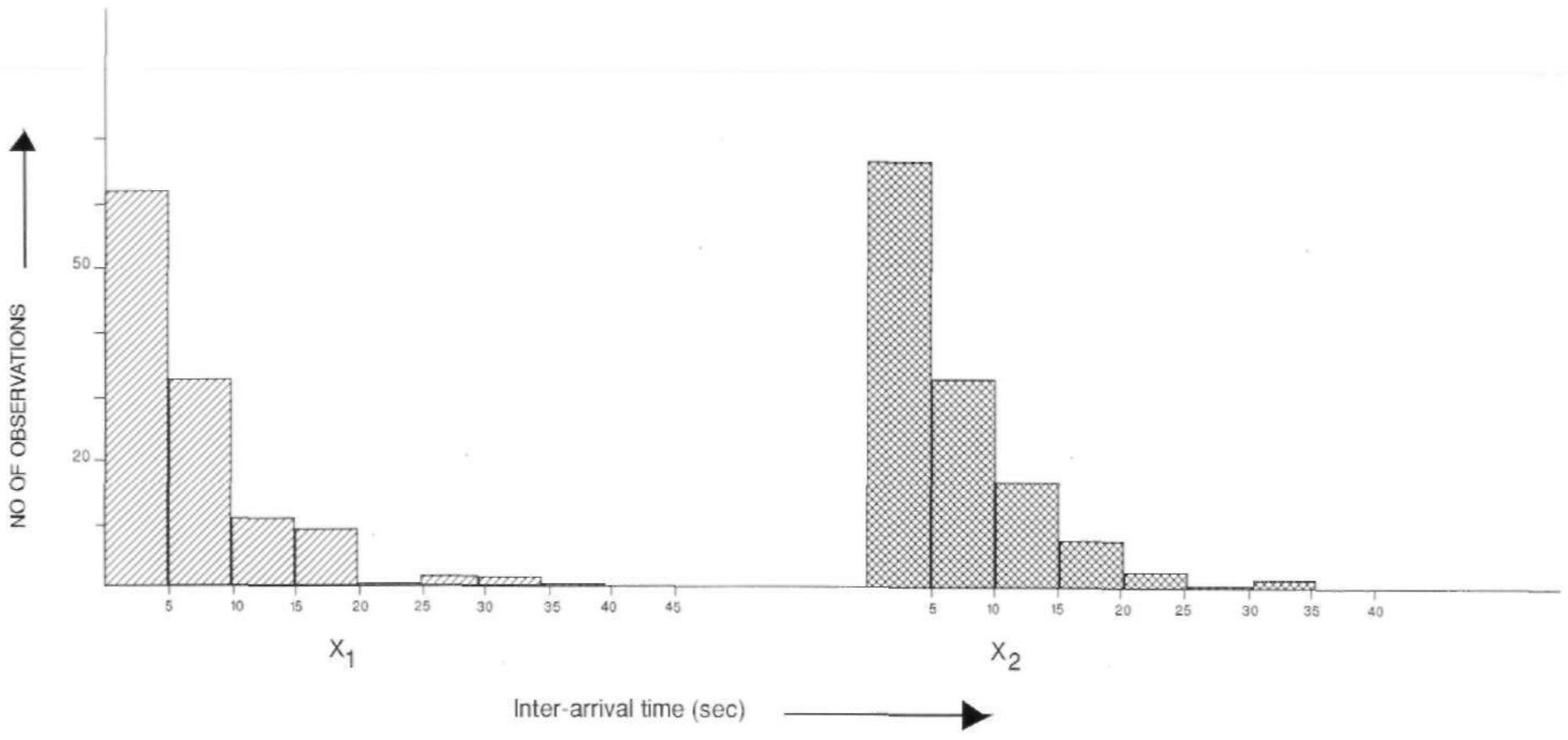


FIG. 3.2 HISTOGRAM OF INTER-ARRIVAL TIMES (POLICE CHECK POINT)

LEGEND: X_1 = LANE A
X_2 = LANE B

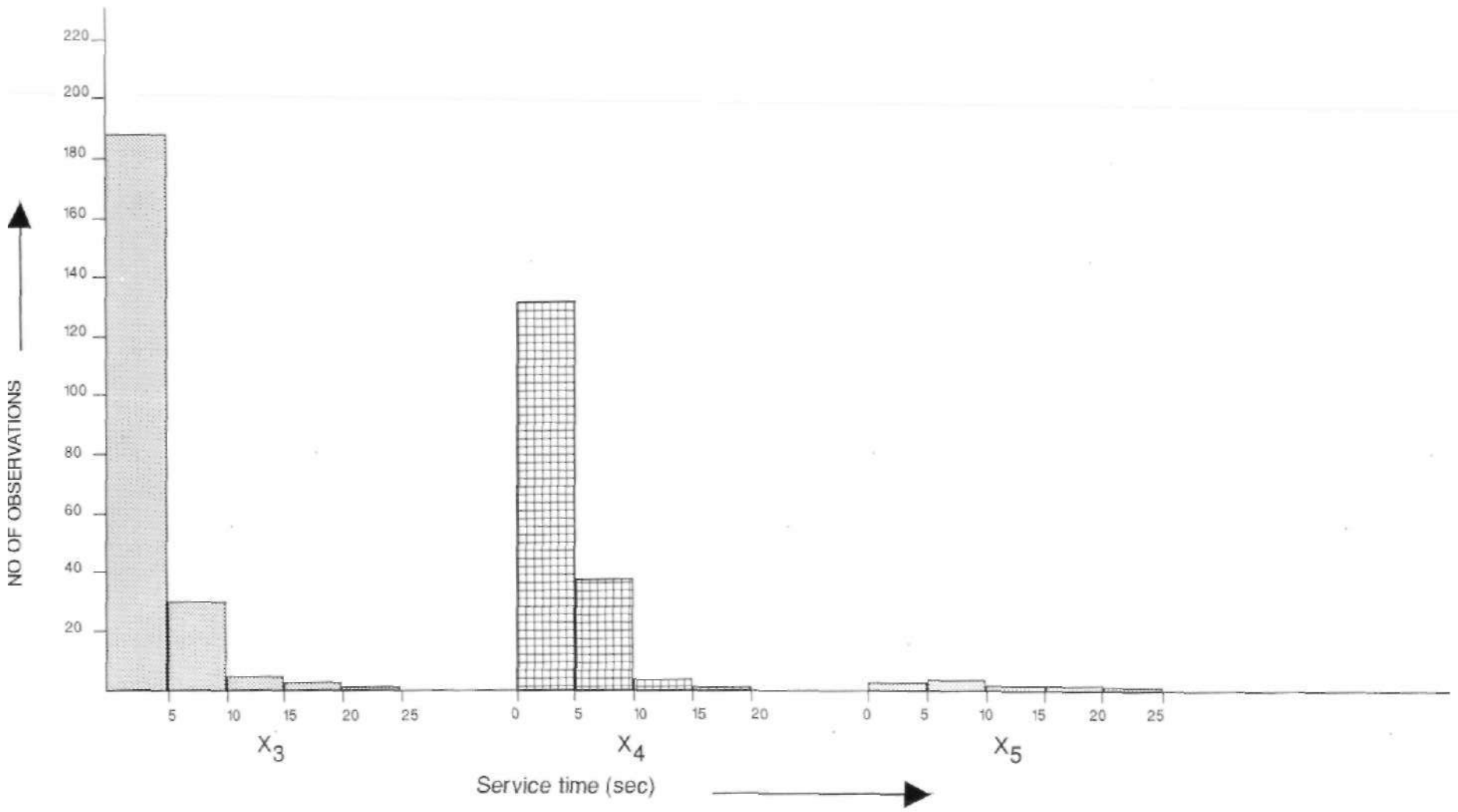


FIG 3.3 HISTOGRAM OF SERVICE TIME (TOIL BOOTHS)

X_3 = CAR LANE

X_4 = BUS LANE

X_5 = TRUCK LANE

TABLE 3.1 STATISTICAL SUMMARIES OF INTER - ARRIVAL AND SERVICE TIMES FOR LANE A AT THE POLICE CHECKPOINT (FRIDAY 1).

Date	Starting time	Duration	Mean Inter - Arrival Time (Sec)				Mean Service Time (Sec).				Coefficient of variation (CV)		Utilization Factor $\rho = ts \backslash ta$
		(mins)	Ta	Tc	Tb	Tt	Tsa	Tsc	Tsb	Tst	inter-arrival time	Service time	
13\12\96	1740	33\30	6.82	-	-	-	4.31	-	-	-	0.95	0.71	0.63
13\12\96	1740	30\30	-	8.56	-	-	-	3.82	-	-	0.98	1.26	0.45
13\12\96	1740	30\30	-	-	30.13	-	-	-	4.22	-	1.06	0.57	0.14
13\12\96	1740	30\30	-	-	-	46.00	-	-	-	7.88	1.00	0.57	0.17
7\1\97	1312	30\30	7.73	-	-	-	3.32	-	-	-	1.07	1.57	0.42
7\1\97	1312	30\30	-	12.36	-	-	-	2.46	-	-	1.13	0.90	0.20
7\1\97	1312	30\30	-	-	23.43	-	-	-	2.30	-	1.02	0.49	0.10
7\1\97	1312	30\30	-	-	-	121.13	-	-	-	10.47	1.04	1.08	0.09
18\3\97	1210	20\20	9.29	-	-	-	3.75	-	-	-	1.04	1.80	0.40
18\3\97	1210	20\20	-	13.84	-	-	-	1.90	-	-	0.99	0.54	0.14
18\3\97	1210	20\20	-	-	25.93	-	-	-	2.27	-	1.17	0.52	0.09
18\3\97	1210	20\20	-	-	-	200.50	-	-	-	15.14	0.93	0.91	0.08

Legend

Ta = Mean inter - arrival time for all vehicles Tc = Mean inter -arrival time for cars
Tb = Mean inter - arrival time for buses.
Tt = Mean inter - arrival time for truck Tsa = mean service time for all vehicles
Tsc = mean service time for cars Tsb = mean service time for buses
Tst = mean service time for trucks
ts = mean service time for class i vehicle
ta = mean inter - arrival time for class i vehicle

Table 3.2 **STATISTICAL SUMMARIES OF INTER - ARRIVAL AND SERVICE TIMES FOR LANE A AT THE POLICE CHECK POINT (SATURDAY).**

DATE	STARTING TIME	DURATION	MEAN INTER - ARRIVAL TIME (Sec)				MEAN SERVICE TIME (Sec)				COEFFICIENT OF VARIATION (CV)		UTILIZATIO N ρ = ts\ta
		(Mins)	Ta	Tc	Tb	Tt	Tsa	Tsc	Tsb	Tst	Inter - arrival Time	Service Time	
16\11\96	1710	30\30	6.55	-	-	-	5.04	-	-	-	1.03	1.31	0.76
"	1710	30\30	-	11.24	-	-	-	3.98	-	-	1.09	0.91	0.35
"	1710	30\30	-	-	17.04	-	-	-	3.59	-	1.01	0.71	0.21
"	1710	30\30	-	-	-	62.36	-	-	-	10.52	1.06	1.24	0.71
14\12\96	1500	20\30	5.65	-	-	-	1.31	-	-	-	0.97	0.52	0.23
"	1500	20\30	-	8.07	-	-	-	1.17	-	-	0.89	0.32	0.14
"	1500	20\30	-	-	19.71	-	-	-	1.11	-	0.89	0.45	0.06
"	1500	20\30	-	-	-	11.00	-	-	-	2.38	1.05	0.61	0.22
18\1\97	1230	30\30	7.48	-	-	-	2.25	-	-	-	0.96	0.25	0.30
18\1\97	1230	30\30	-	12.14	-	-	-	2.05	-	-	0.82	0.70	0.17
18\1\97	1230	30\30	-	-	21.38	-	-	-	2.21	-	0.90	0.23	0.10
18\1\97	1230	30\30	-	-	-	64.80	-	-	-	11.33	1.04	1.47	0.17

Table 3.3 **STATISTICAL SUMMARIES OF INTER -ARRIVAL AND SERVICE TIMES FOR LANE A AT THE POLICE CHECK POINT (SUNDAY).**

DATE	STARTING TIME	DURATION	MEAN INTER - ARRIVAL TIME (Sec)				MEAN SERVICE TIME (Sec)				COEFFICIENT OF VARIATION (CV)		UTILIZATION factor $\rho = ts/ta$
		(Mins)	Ta	Tc	Tb	Tt	Tsa	Tsc	Tsb	Tst	Inter - arrival Time	Service Time	
17/11/96	1708	30/30	6.51	-	-	-	1.12	-	-	-	0.84	0.62	0.17
17/11/96	1708	30/30	-	9.16	-	-	-	1.06	-	-	0.85	0.23	0.12
17/11/96	1708	30/30	-	-	25.56	-	-	-	1.01	-	0.88	0.20	0.03
17/11/96	1708	30/30	-	-	-	120.3	-	-	-	2.23	0.62	0.73	0.02
15/12/96	1823	30/30	6.74	-	-	-	3.19	-	-	-	0.96	1.05	0.47
15/12/96	1823	30/30	-	1.58	-	-	-	2.63	-	-	0.70	0.48	0.23
15/12/96	1823	30/30	-	-	29.06	-	-	-	5.07	-	1.20	1.22	0.17
15/12/96	1823	30/30	-	-	-	126.0	-	-	-	5.14	1.15	0.66	0.04

Table 3.4 **STATISTICAL SUMMARIES OF INTER -ARRIVAL AND SERVICE TIMES FOR LANE B AT THE POLICE CHECK POINT (FRIDAY 2)**

DATE	STARTING TIME	DURATION	MEAN INTER - ARRIVAL TIME (Sec)				MEAN SERVICE TIME(Sec)				COEFFICIENT OF VARIATION (CV)		Utilization factor $\rho = \lambda/\mu$
		(Mins)N	Ta	Tc	Tb	Tt	Tsa	Tsc	Tsb	Tst	Inter - arrival Time	Service Time	
13/11/96	1740	30/30	6.88	-	-	-	3.02	-	-	-	0.88	0.51	0.44
13/11/96	1740	30/30	-	11.01	-	-	-	2.89	-	-	0.93	0.51	0.26
13/11/96	1740	30/30	-	-	21.09	-	-	-	2.68	-	0.94	0.47	0.13
13/11/96	1740	30/30	-	-	-	110.62	-	-	-	3.54	1.02	0.65	0.03
7/1/97	1740	30/30	6.91	-	-	-	3.02	-	-	-	0.86	1.43	0.43
7/1/97	1312	30/30	-	13.69	-	-	-	2.80	-	-	0.71	1.29	0.20
7/1/97	1312	30/30	-	-	17.12	-	-	-	2.43	-	0.84	1.20	0.14
7/1/97	1312	30/30	-	-	-	100.1	-	-	-	6.57	1.39	1.20	0.07
18/3/97	1312	20/20	7.37	-	-	-	2.84	-	-	-	0.86	1.96	0.39
18/3/97	1210	20/20	-	13.74	-	-	-	1.70	-	-	0.94	0.31	0.12
18/3/97	1210	20/20	-	-	19.55	-	-	-	1.89	-	0.99	0.38	0.10
18/3/97	1210	20/20	-	-	-	48.47	-	-	-	17.25	0.86	0.81	0.36

ABU BELLO UNIVERSITY
ZARIA IBRAHIM LIBRARY

Table 3.5 STATISTIC SUMMARIES OF INTER- ARRIVAL AND SERVICE TIMES FOR LANE B AT THE POLICE CHECK POINT (SATURDAY 2)

DATE	STARTING TIME	DURATION	MEAN INTER- ARRIVAL TIME (Sec)O				MEAN SERVICE TIME (Sec)				COEFFICIENT OF VARIATION (CV)		UTILIZATION ρ = b\ta
		(Mins)	Ta	Tc	Tb	Tt	Tsa	Tsc	Tsb	Tst	INTER-ARRIVAL TIME	SERVE TIME	
16\11\96	1500	30\30	7.13	-	-	-	1.73	-	-	-	0.86	1.10	0.24
"	1500	30\30	-	11.19	-	-	-	1.54	-	-	0.86	0.75	0.14
"	1500	30\30	-	-	22.24	-	-	-	1.29	-	0.85	0.45	0.06
"	1500	30\30	-	-	-	60.46	-	-	-	5.40	0.79	0.89	0.09
14\12\96	1730	30\20	6.14	-	-	-	3.64	-	-	-	0.98	0.57	0.57
"	1730	30\20	-	11.55	-	-	-	3.58	-	-	0.87	0.54	0.31
"	1730	30\20	-	-	20.85	-	-	-	3.83	-	0.95	0.12	0.18
"	1730	30\20	-	-	-	52.13	-	-	-	7.14	1.22	0.46	0.14
8\3\97	1232	20\20	7.73	-	-	-	1.94	-	-	-	0.96	0.57	0.25
8\3\97	1232	20\20	-	12.43	-	-	-	1.72	-	-	1.02	0.39	0.14
8\3\97	1232	20\20	-	-	19.43	-	-	-	1.90	-	0.89	0.35	0.10
8\3\97	1232	20\20	-	-	-	25.33	-	-	-	5.40	0.75	0.38	0.21

TABLE 3.6 STATISTICAL SUMMARIES OF INTER-ARRIVAL AND SERVICE TIMES FOR LANE B AT THE POLICE CHECK POINT (SUNDAY 2)

DATE	STARTING TIME	DURATION	MEAN INTER - ARRIVAL TIME (Sec)				MEAN SERVICE TIME (Sec)				COEFFICIENT OF VARIATION (CV)		UTILIZATION FACTOR $\rho = ts/ta$
		(Mins)	Ta	Tc	Tb	Tt	Tsa	Tsc	Tsb	Tst	INTER-ARRIVAL TIME	SERVICE TIME	
17\11\96	1708	30\30	8.31	-	-	-	4.25	-	-	-	0.98	1.09	0.51
"	1708	30\30	-	13.79	-	-	-	3.66	-	-	0.98	1.31	0.26
"	1708	30\30	-	-	27.32	-	-	-	4.22	-	1.14	0.87	0.15
"	1708	30\30	-	-	-	77.87	-	-	-	7.50	1.06	0.72	0.10
15\12\96	1842	30\30	8.06	-	-	-	3.16	-	-	-	1.12	1.32	0.39
"	1842	30\30	-	11.91	-	-	-	2.87	-	-	1.27	0.81	0.24
"	1842	30\30	-	-	26.73	-	-	-	2.60	-	1.08	0.58	0.10
15\12\96	1842	30\30	-	-	-	156.25	-	-	-	6.53	1.13	1.62	0.04

TABLE 3.7 STATISTICAL SUMMARIES OF INTER -ARRIVAL AND SERVICE TIMES FOR CAR LANE (TOLL BOOTH QUEUE SYSTEM)

DATE	STARTING TIME	DURATION	MEAN INTER - ARRIVAL TIME (Car) (Sec)	MEAN SERVICE TIME (Car) (Sec)	COEFFICIENT VARIATION OF (CV)		UTILIZATION FACTOR ρ = ts\fa
		(Mins)	Tc	Tsc	INTER - ARRIVAL TIME	SERVICE TIME	
13\12\96 8\3\97	1700 1633	30\30 4.87	FRIDAY CASE 4.97 3.68	4.36	30\30	0.81 1.57	0.51 0.75	0.98 O.84
14\12\96 14\12\96 7\1\97 7\1\97 8\3\97	1304 1600 1700 1630 1730	30\30 30\30 30\30 30\30 30\30	**SATURDAY CASE** 5.23 5.24 4.67 5.54 5.78	4.40 4.11 3.38 3.72 3.32	0.85 0.87 0.87 0.86 0.95	0.54 0.62 0.72 0.89 0.89	0.84 0.78 0.72 0.67 0.57
17\11\96 15\12\96 15\12\96	1747 1450 1718	30\30 30\30 30\30	**SUNDAY CASE** 4.49 5.89 5.42	4.00 3.27 3.13	1.12 1.05 0.91	0.75 0.67 0.61	0.89 0.56 0.58

LEGEND

Tc= MEAN INTER -ARRIVAL TIME FOR CARS Tsc = MEAN SERVICE TIME FOR CARS
ts = MEAN SERVICE TIME fa = MEAN INTER - ARRIVAL TIME

TABLE 3.8 **STATISTICAL SUMMARIES OF INTER - ARRIVAL AND SERVICE TIMES FOR BUSES LANE (TOLL BOOTH QUEUE SYSTEM)**

DATE	STARTING TIME	DURATION	MEAN INTER - ARRIVAL TIME (BUS) (Sec)	MEAN SERVICE TIME (Sec)	COEFFICIENT OF VARIATION (CV)		UTILIZATION FACTOR ρ = ts\fa
		(Mins)	Tb	Tsb	INTER - ARRIVAL TIME	SERVIC E TIME	
			FRIDAY CASE				
13\12\96	1700	30\30	6.14	4.86	1.04	0.66	0.79
8\3\97	1633	30\30	5.94	3.36	1.03	1.06	0.57
			SATURDAY CASE			0.57	0.59
14\12\96	1304	30\30	5.93	3.52	1.00	0.75	0.47
14\12\96	1600	30\30	6.37	6.37	1.19	0.70	0.58
7\1\97	1700	30\30	6.48	6.48	1.06	0.95	0.61
7\1\97	1700	30\30	6.70	6.70	1.08	0.66	0.51
8\3\97	1730	30\30	6.44	6.44	1.11		
			SUNDAY CASE				
17\11\96	1747	3030	6.81	4.58	0.99	0.56	0.67
15\12\96	1450	30\30	7.41	3.17	1.04	0.72	0.43
15\12\96	1718	30\30	7.19	2.73	1.17	0.52	0.38

LEGEND

Tb =MEAN INTER - ARRIVAL TIME FOR BUSES
Tsb = MEAN SERVICE TIME FOR BUSES.

TABLE 3.9 STATISTICAL SUMMARIES OF INTER - ARRIVAL AND SERVICE TIMES FOR TRUCK LANE (TOLL BOOTH QUEUE SYSTEM)

DATE	STARTING TIME	DURATION	MEAN INTER - ARRIVAL TIME (BUS) (Sec)	MEAN SERVICE TIME (Sec)	COEFFICIENT OF VARIATION (CV)		UTILIZATION FACTOR ρ = ts\ta
		(Mins)	Tt	Tst	INTER - ARRIVAL TIME	SERVICE TIME	
			FRIDAY CASE				
13\12\96	1700	30\30	96.21	12.96	1.23	0.71	0.13
8\3\97	1633	30\30	107.07	22.70	0.73	0.70	0.21
			SATURDAY CASE				
14\12\96	1304	30\30	101.12	9.45	1.00	0.62	0.09
14\12\96	1600	30\30	111.07	10.08	1.19	0.65	0.09
7\1\97	1700	30\30	102.92	21.00	1.06	0.58	0.20
7\1\97	1630	30\30	106.57	16.30	1.08	0.88	0.15
8\3\97	1730	30\30	114.56	25.80	1.11	0.51	0.23
			SUNDAY CASE				
17\11\96	1747	30/30	112.13	14.97	0.82	0.79	0.13
15\12\96	1450	30/30	72.09	6.21	1.15	0.65	0.09
15\12\96	1718	30/30	81.86	6.74	0.67	0.61	0.08

LEGEND

Tt =MEAN INTER - ARRIVAL TIME FOR TRUCKS
Tst = MEAN SERVICE TIME FOR TRUCKS

The coefficient of variation was, thus, determined for the inter - arrival time and service time at both stages of queueing in the system. The values obtained are shown in Tables 3.1 to 3.9 along with other relevant statistics. The utilization factors (ratio of average service time to mean inter - arrival time) at both stages of the queuing system were also calculated so as to give an indication of how saturated the system is. The results obtained are also indicated on the tables. Tables 3.1 to 3.9

3.4 DETAILED DATA COLLECTION

3.4.1 PREAMBLE

This section describes the more detailed data collection exercise. The data obtained was subsequently used for the estimation of the statistical properties of the input and service times as well as fitting their underlying processes into standard distributions.

For this stage of data collection, more accurate and varied data than was used in the preliminary study was needed. Thus, days and times of observations were varied as much as necessary to enable changes of performance during the selected period of study to be captured. The data collection exercises spanned, as earlier enumerated, from November, 1996 to March, 1997. typical results got from the field work are summarized in figures 3.2 and 3.3.

3.5 ANALYSIS OF RESULTS

3.5.1 PREAMBLE

When a theoretical distribution has been assumed, perhaps determined on the basis of the general shape of histogram, or on the basis of the data plotted on a given probability paper, the validity of the assumed distribution may be verified statistically by goodness- of -fit tests.

3.5.2 GOODNESS - OF -FIT TESTS

The main objectives of applying goodness -of -fit tests is to test the null hypothesis that the samples of observations, X1,X2, X3, ...Xn are from specified distribution as against the alternative hypothesis that they are not from that distribution. Two such tests, frequently used are the Chi- square (X^2) and the Kolmogorov - Smirnov (K - S) tests. The conclusion from these tests, however, depends on a specified significance level and, the choice of which is arbitrary, and may not provide absolute information on the validity of a specified distribution. Conceivably, a distribution that is acceptable at one Significance level maybe unacceptable at another significance level.

The chi-square goodness of fit tests compares the observed frequencies say, O_1, O_2, O_3, ... O_n in n intervals of a variate with the corresponding frequencies e_1, e_2, e_3,...., e_n from an assumed theoretical distribution. The basis for appraising the goodness of this comparison is the distribution of the quantity.

$$\left(\frac{o_i - e_i}{e_i}\right)^2$$

which approaches the χ_α^2, f distribution with f = (n- r) degree of freedom, where r is the number of parameters estimated from data (or constraints) and n is the number of variate intervals. On this basis, if an assumed distribution yields;

$$\sum_{i=1}^{n}\left(\frac{o_i - e_i}{e_i}\right)^2 \quad < \quad C_1 - \alpha, f \text{.....(1)}$$

where $C_1 - \alpha$, f is the value of the appropriate x_f^2 distribution (1 - α) the assumed theoretical distribution is acceptable model, otherwise, the assumed distribution is not substantiated by the data at the α level of significance. In applying the x^2 test for goodness of fit, it is generally necessary to have $n \geq 5$ and $e_i \geq 5$ for satisfactory results. The x^2 test utilizes only the number of observations in intervals rather than the observations themselves. Thus, since the test requires grouping of data, sample information is lost.

The Kolmogorov - Smirnov (K- S) test involves specifying the Cumulative frequency distribution which would occur under a theoretical distribution F (X), and comparing it with the observed cumulative frequency distribution S (X). Now under null hypothesis H_o, that the sample, has been drawn from the specified theoretical distribution, it is expected that for every value of X, S (X) should be fairly close to F(X). That is under H_0, we would expect the difference between S (X) and F(X) to be small and within the limits of random errors. The K - S test focusses on the largest of the deviations. The maximum absolute deviation between F (X) and S (X), Dn is obtained and compared with standard published values that correspond to a desired level of significance Dn. If Dn = maxF(X) - S(X) ,.....(2) is greater than Dn, the null hypothesis is rejected, otherwise it is accepted.

The (K- S) test is non-parametric and it treats individual observations separately and thus, unlike the X^2 test, needs not lose information through groupings. The (K-S) test is applicable to both small and large samples . It is exact, conceptually sound and operationally simple and it is generally considered to be more powerful than the X^2 test Siegel, (1956) Conover, 1971). In consideration of all the above and the fact that a lot of data collected for this thesis could be called small sample sizes, there was no hesitation in adopting the K - S technique.

3.6 RESULTS OF GOODNESS -OF -FIT TEST

The Kolmogorov - Smirnov test was used in evaluating the goodness of fit of the negative exponential model. The observed data were analyzed with the aid of computer and the values of the negative exponential cumulative distribution function F (X) were obtained and compared with the corresponding values of observed relative cumulative frequency S_n (X). The maximum absolute deviation between F (X) and S_n (X), Dn were obtained and compared with the standard published values that correspond to 0.05 level of significance, Dn 0.05. When Dn was greater than Dn 0.05, the null hypothesis H_o, that the distribution function of the observed random variable is negative exponential was rejected, otherwise it was not rejected. Here, only few of the observed random variables were rejected at 5% level of significance, but few were accepted at 10% level of significance. figure 3.5 gives the programm flow chart for the K -S test.

Appendix B gives the computer program documentation, Tables 3.10 to 3.14 and figures 3.4(a) to 3.4(c) summarize the results obtained.

TABLE 3.10 GOODNESS-OF-FIT TEST FOR CARS
DATE: 8 3 1997
ARRIVAL RATE= 492 VEHICLES PER HOUR
STANDARD DEV= 8.28 COEFF OF VAR=1.13

TIME	OBS FREQ.	OBS. CUM. FREQ.	EXPTD. CUM. FREQ.	K-S
1	8	.0672	.1278	.0606
2	24	.2689	.2393	.0296
3	17	.4118	.3366	.0752
4	11	.5042	.4214	.0828
5	4	.5378	.4954	.0425
6	7	.5966	.5599	.0368
7	8	.6639	.6161	.0477
8	9	.7395	.6652	.0743
9	3	.7647	.7080	.0567
10	4	.7983	.7453	.0530
11	7	.8571	.7779	.0792
12	1	.8655	.8063	.0593
13	4	.8992	.8311	.0681
14	1	.9076	.8526	.0549
15	1	.9160	.8715	.0445
16	1	.9244	.8879	.0365
18	1	.9328	.9147	.0180
20	1	.9412	.9351	.0060
24	1	.9496	.9625	.0129
25	1	.9580	.9673	.0093
26	1	.9664	.9715	.0051
27	1	.9748	.9751	.0003
32	1	.9832	.9874	.0042
49	1	.9916	.9988	.0072
55	1	1.0000	.9995	.0005

MAXIMUM K-S = .0828 AT 4 SEC.
THIS IS LESS THAN THE CRITICAL K-S OF 0.1247
THEREFORE ACCEPT THE NULL HYPOTHESIS
AT 5% LEVEL OF SIGNIFICANCE

TABLE 3.11 GOODNESS-OF-FIT TEST FOR BUSES

DATE: 14 12 1996
SERVICE RATE=1186 VEHICLES PER HOUR
STANDARD DEV= 2.33 COEFF OF VAR= .77

TIME	OBS FREQ.	OBS. CUM. FREQ.	EXPTD. CUM. FREQ.	K-S
1	14	.1591	.2808	.1217
2	35	.5568	.4827	.0741
3	18	.7614	.6280	.1334
4	4	.8068	.7324	.0744
5	7	.8864	.8076	.0788
6	6	.9545	.8616	.0930
7	1	.9659	.9005	.0655
8	1	.9773	.9284	.0489
9	1	.9886	.9485	.0401
18	1	1.0000	.9973	.0027

MAXIMUM K-S = .1334 AT 3 SEC.
THIS IS LESS THAN THE CRITICAL K-S OF 0.1449
THEREFORE ACCEPT THE NULL HYPOTHESIS
AT 5% LEVEL OF SIGNIFICANCE

Table 3.12 GOODNESS-OF-FIT TEST FOR TRUCKS

DATE: 14 12 1996
SERVICE RATE= 354 VEHICLES PER HOUR
STANDARD DEV= 6.53 COEFF OF VAR= .64

TIME	OBS FREQ.	OBS. CUM. FREQ.	EXPTD. CUM. FREQ.	K-S
1	1	.0833	.0937	.0103
5	2	.2500	.3885	.1385
6	1	.3333	.4458	.1124
7	2	.5000	.4977	.0023
8	1	.5833	.5447	.0386
11	1	.6667	.6611	.0056
12	1	.7500	.6928	.0572
17	1	.8333	.8122	.0212
18	1	.9167	.8298	.0869
25	1	1.0000	.9145	.0855

MAXIMUM K-S = .1385 AT 5 SEC.
THIS IS LESS THAN THE CRITICAL K-S OF 0.4900
THEREFORE ACCEPT THE NULL HYPOTHESIS
AT 5% LEVEL OF SIGNIFICANCE

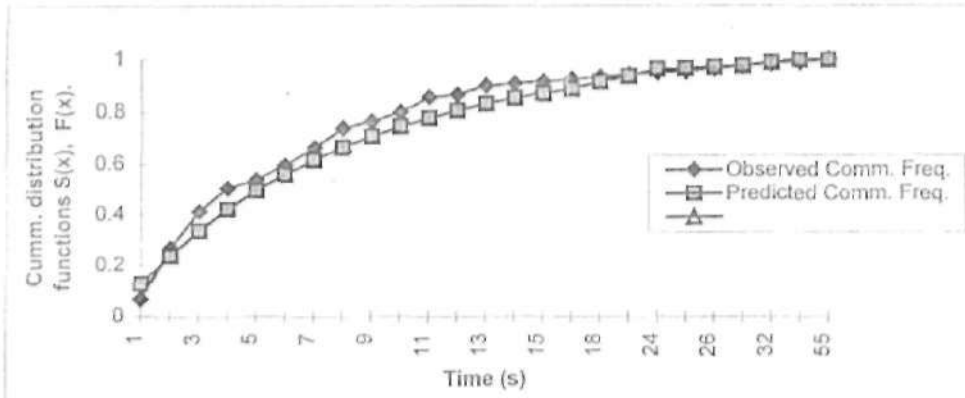


Fig. 3.4a Observed Cumm. Freq. Versus Expected distribution functions for Car lanes at 5% level of significance

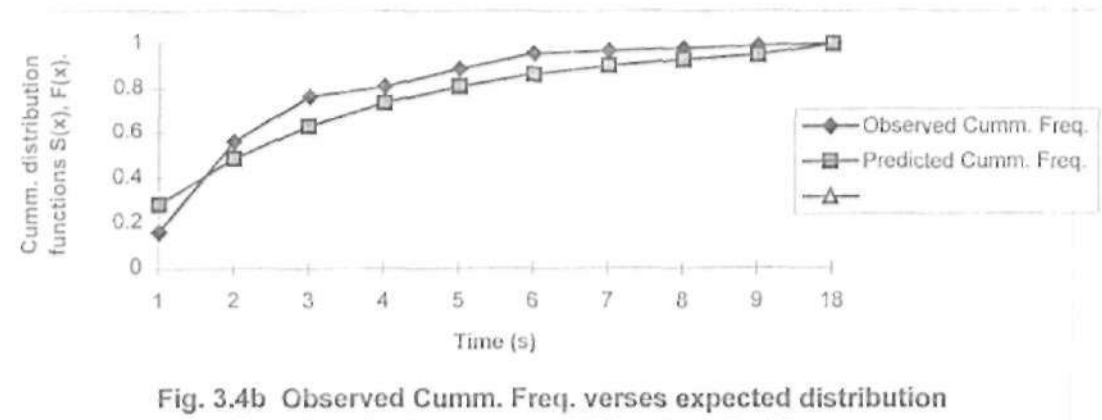


Fig. 3.4b Observed Cumm. Freq. verses expected distribution functions for bus lanes at 5% level of significance

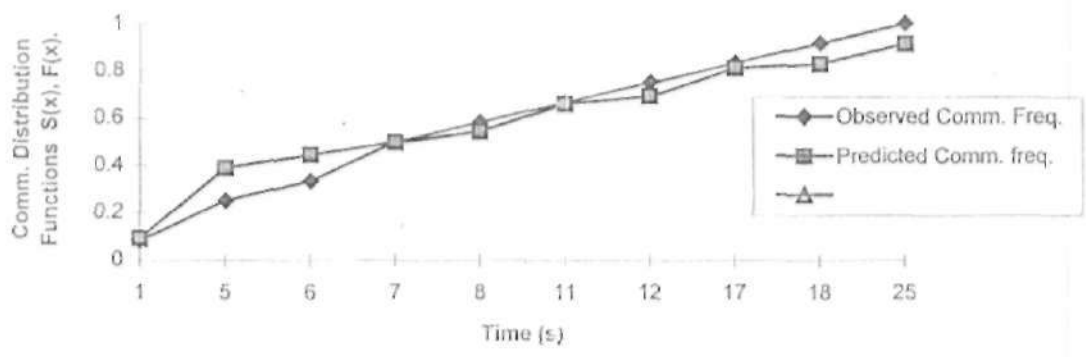


Fig. 3.4c Observed Comm. Freq. Versus Expected Distribution functions for truck lanes at 5% level of significance

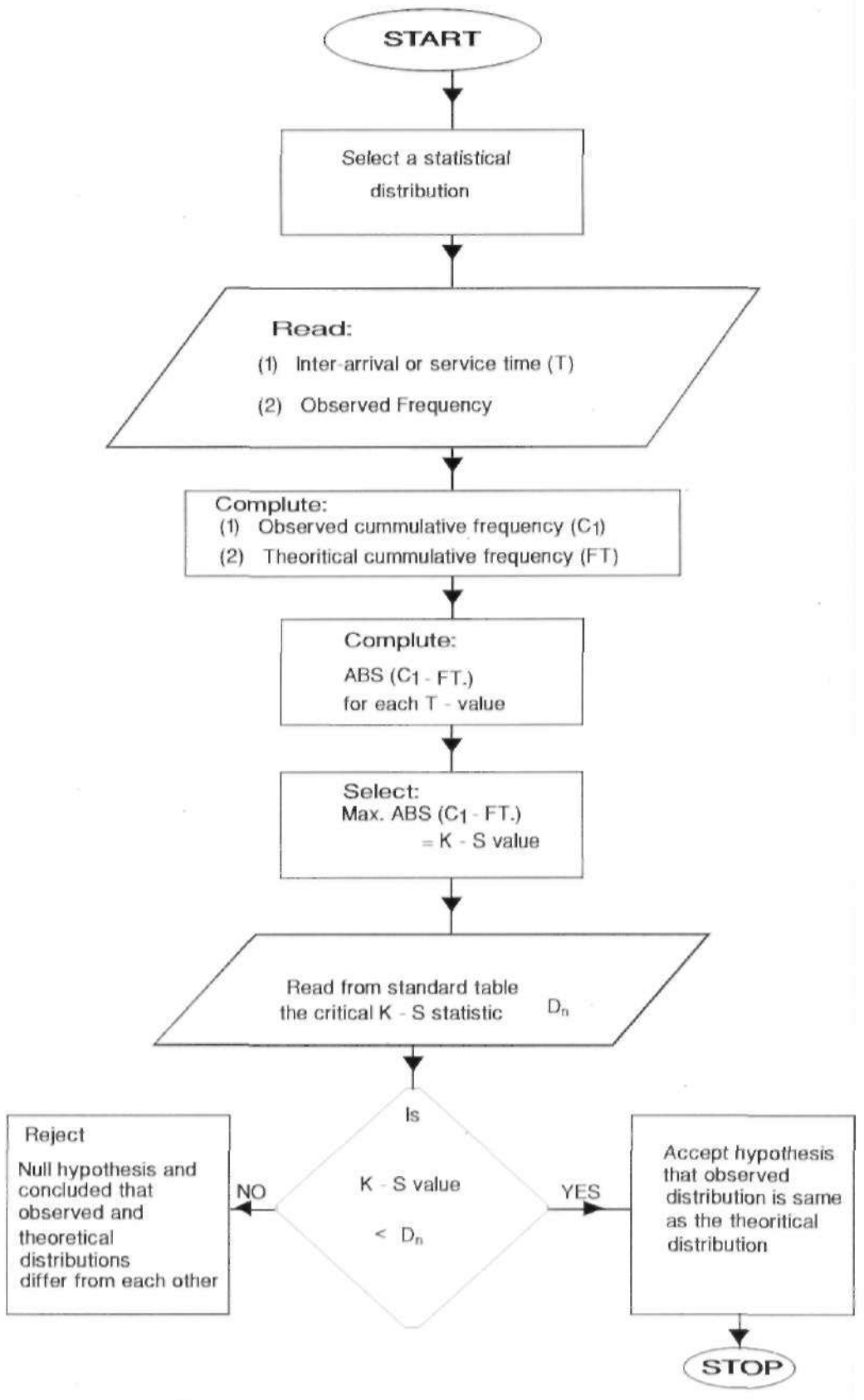


FIG. 3.5 PROGRAM FLOWCHART FOR THE K-S TEST

Table 3.13 K-S GOODNESS -OF-FIT TEST RESULTS

PART OF SYSTEM	TEST FOR ARRIVAL TIME DISTRIBUTION	TEST FOR SERVICE DISTRIBUTION	COMMENT
LANE A (POLICE CHECK POINT)	7\3\97 7\3\97 8\3\97	7\3\97 7\3\97 8\3\97	Accept null hypothesis at 5% level of significance Accept null hypothesis at 10% level of significance Accept null hypothesis at 5% level of significance
LANE B POLICE CHECK POINT	7\3\97 8\3\97	7\3\97 8\3\97	Accept null hypothesis at 5% level of significance Accept null hypothesis at 5% level of significance
CAR LANE	17\11\96 7\3\97 8\3\97	17\11\96 7\3\97 8\3\97	Accept nu hypothesis at 5% level of significance Accept null hypothesis at 10% level of significance Accept null hypothesis at 5% level of significance
BUS LANE	7\3\97 8\3\97	13\12\96 7\3\97 8\3\97	Accept null hypothesis at 5% level of significance Accept null hypothesis at 5% level of significance Accept null hypothesis at 5% level of significance
TRUCK LANE	14\12\96 7\3\97	13\12\96 7\3\97	Accept null hypothesis at 5% level of significance Accept null hypothesis at 5% level of significance Accept null hypothesis at 5% level of significance

3.7 OBSERVATIONS ON THE RESULTS OF DATA ANALYSIS

Tables 3.1 to 3.9, that it is observed from the mean inter- arrival and service times for cars is the least followed by that of buses and then by trucks. This then means that the arrival and service rates for cars is the highest followed by buses and then the trucks. Likewise, the traffic intensity for car is the highest followed by buses and then by trucks.

Although other distributions notably the Erlang and the Lognormal can be fitted to the observed sets of data, the approximate tests using graphical methods and coefficient of variation

criteria indicate that the negative exponential distribution is acceptable for further confirmation using statistical tests. The negative exponential distribution might not be the best fitting of all the possible distribution models but the mathematical tractability of the system models resulting from it made its case a very strong one, at least for approximate solutions.

Except in a few cases, it was found that the variation in mean input and output are not significant and assumption of the distribution patterns for both arrival and departure for cars, buses and trucks channels is acceptable at 5% level of significance in both cases and for both stages of the queueing system . Table 3.13 shows the results of twenty four sets of data analyzed for exponential validation via the K- S test technique.

3.8 MODEL SPECIFICATION FOR THE SERVICE AND ARRIVAL PATTERNS

In finally selecting and justifying the exponential assumption for the queueing system of the Zaria - Kaduna expressway, it is appreciated that :

(i) The system involves a serial queueing process, the analysis of which becomes less mathematically attractive even for moderate system complexities.

(ii) Most of the models contained in queueing literature to be examined for the selection of an adequate queueing models for the system are based on the exponential assumption.

(iii) An operational solution is what is being sought for the system.

(iv) Unless the distribution of the variable is either exponential or deterministic, it might not be possible to give strong theoretical justification for the choice of a particular distribution.

Based on the results of the analysis done so far, the distributions in most cases fitted the exponential distribution, but seem to be far from deterministic assumption not in all cases. However, considering the arguments raised in this sub- section enough justification could be shown for selecting

the negative exponential to described the arrival and service process for the Zaria - Kaduna expressway toll booth queues.

Having selected the negative exponential to describe the arrival and service distribution patterns for the queueing system, the next step will be to calibrate suitable model for the queueing system. This very important stage in this study is carried out in chapter four.

CHAPTER FOUR

MODEL CALIBRATION

4.1 PREAMBLE:

Inspite of best efforts to incorporate all the needed inputs to the model, it is quite likely that certain essential features have been either missed or have been erroneously determined. The initial run of the model can detect such errors and give the modeller an opportunity to calibrate the model to eliminate any inaccuracies.

The actual system under study is schematically illustrated in the diagrammatic sketch of figure 4.1.

4.3 Calibration of Model Parameters

The actual system under study is schematically illustrated in Figure 4.1

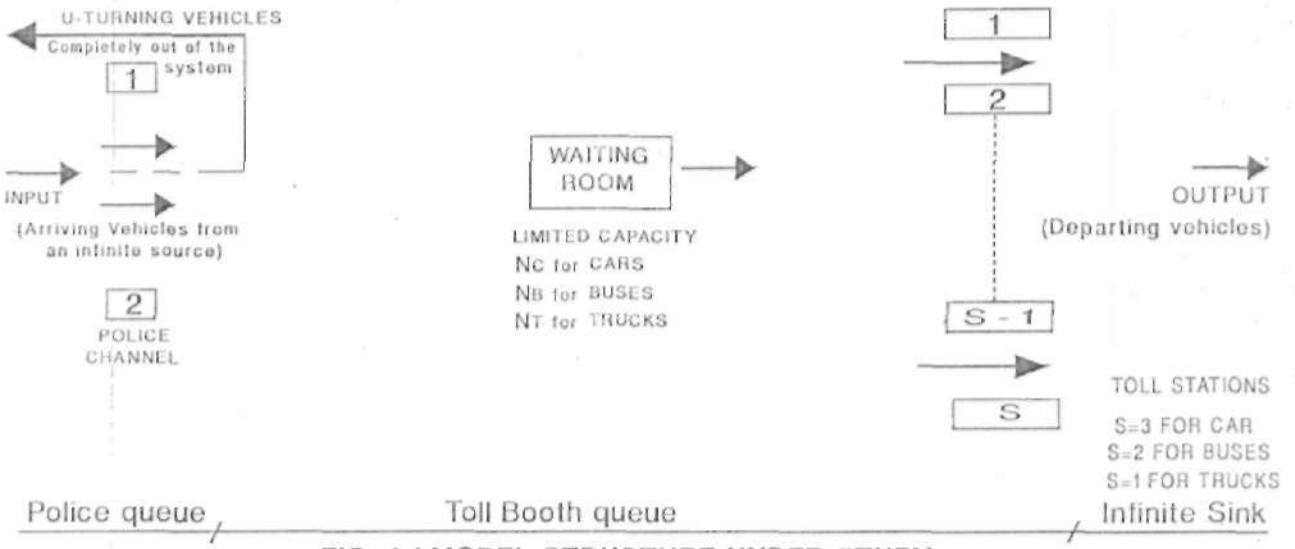


FIG. 4.1 MODEL STRUCTURE UNDER STUDY

There are two servers in stage 1 which is the police checkpoint.

Units leaving this first stage service proceed to the toll booth where they either queue up in the limited waiting room provided or go straight for service in any of the six channels of service available. It is also relevant to note that a U-turn is constructed immediately after the first stage service for U-turning vehicles to move completely out of the system. This facility enables motorists whose destinations are on the opposite sides of the road and not beyond the toll booth to complete their journey since there is a median that limits access to the other side of the road.

The focus of the work presented in this section is the calibration of models used for the analysis. The structure of the Makino model selected for this study when adapted to the system under investigation, take the form represented by the diagrammatic sketch of figure 4.2.

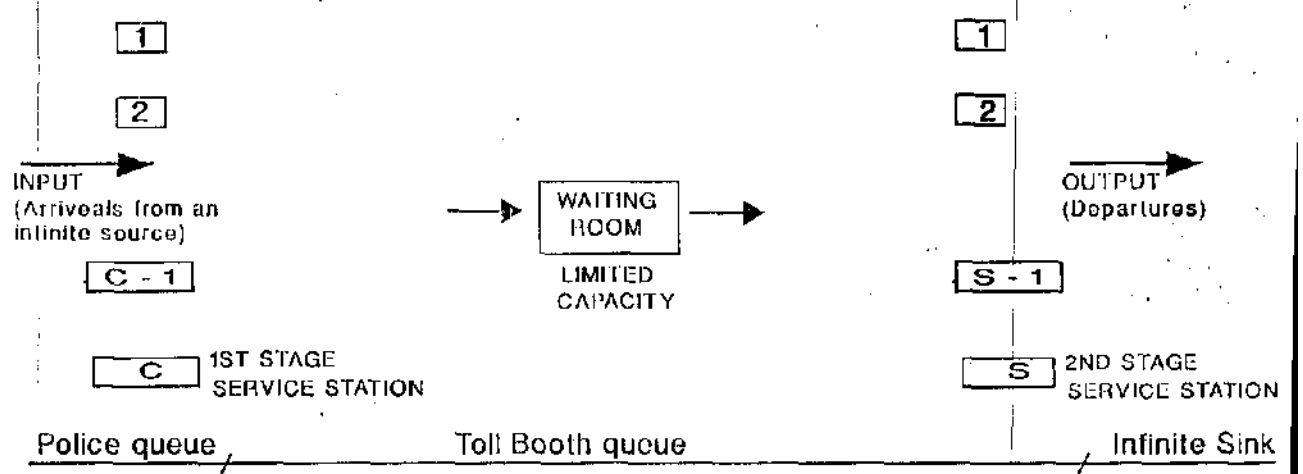


Fig. 4.2 MAKINO MODEL STRUCTURE ADAPTED TO THE SYSTEM UNDER INVESTIGATION

In the original form, Mankino model assumes that there are C servers in stage 1. Units leaving these first stage service points either queue up in the limited waiting room provided or go straight for service in any of the S channels of service in stage 2.

Basically, the only difference between the actual system under study and the idealised Makino model is the U-turn channel for vehicles making U-turning movements that exists in the model under investigation.

4.2.1 ADAPTATION OF MAKINO MODEL TO THE SYSTEM UNDER STUDY

In view of the factors stated in preceding chapters, it was necessary to either simplify the system or adjust the base model.

The factors considered are as follows:

(a) U - Turning Vehicles:

At present, the number of U-turning vehicles is low. However, as economic activities along both sides of the road intensify over time, the number of turnings is likely to increase. One of the assumptions of the Makino model is that every unit passing through the first stage must get to the second state.

In light of the above explanation, Makino model is modified by introducing a factor probability of a given type of vehicle leaving stage 1 and getting to stage 2 without making a U-turn. This arrangement is an approximation but is suitable for the level of exactness expected in this solution and follows the line of discussion in Saaty, (1961)

(b) Heterogenous Servers:

The heterogenous set of servers can be approximated by a homogenous set only if the extent of dishomogeneity is small [Gumbel, 1959]. hence, to further reduce the dishomogeneity

between the servers at the toll booths, the different vehicle classes were considered separately.

Finally, it is necessary to modify the assumption that on completion of service at stage 1 a customer moves instantaneously to stage 2 as contained in the Makino model to read: Upon completion of a service at stage 1, the customer engages a Bernoulli switch and with fixed probability β, he moves instantaneously from stage 1 to stage 2; while with probability 1-β he leaves the system completely.

4.2.2 THEORETICAL ASPECTS AND DERIVATIONS:

The modified model of the system to be studied was shown in the diagrammatic sketch of figure 4:1. Basically, the only difference between this model and the Makino model represented in Figure 4.2 that affects the mathematical formulations is the number of U-turning vehicles. It is therefore the purpose of this section to introduce the effect of this turning in the Makino equations (2.30 to 2.33) and to obtain some measures of system effectiveness therefrom.

From the technique in the literature earlier reviewed, [Saaty, (1961), Lee (1966), Jackson (1954), Koenigsber (1950) and Hunt (1956), among others], it is a trivial matter to show that it is only necessary to multiply the first stage rate of service by the probability factor to get the desired effect.

Mathematically, this implies that the values μ_1, in all the equations should be replaced with $\beta\mu$ with this substitution only the new equivalents of equations 4.2 to 4.5 will be restated here. These are the set of equations required for computing the system effectiveness measures.

Thus the equation of steady state probabilities are:

$$\text{Po, h} = \frac{1}{h!}\left(\frac{C\beta\mu_1}{\mu_2}\right)^h . Poo \quad (for\ 0 \leq h \leq s).....4.6$$

$$\text{PK,S} = \frac{1}{S!}\left(\frac{C\beta\mu_1}{\mu_2}\right)^S \cdot \left(\frac{C\beta\mu_1}{S\mu_2}\right)^K .Poo \qquad (For\ 0 \leq K \leq N).....4.7$$

$$\text{PBL,S} = \left[\frac{1}{S!}\left(\frac{C\beta\mu_1}{\mu_2}\right)^S \left(\frac{C\beta\mu_1}{S\mu_2}\right)^N .Poo\right]\left(\frac{\beta\mu_1}{S\mu_2}\right)^L [C(C-1).....(C-L+1)]$$

$(for\ 1 \leq L \leq C).....4.8$

and

$$P_{00} = \frac{1}{\sum_{h=0}^{S}\frac{1}{h!}\left(\frac{C\beta\mu_1}{\mu_2}\right)^h + \sum_{k=1}^{N}\left[\frac{1}{S!}\left(\frac{C\beta\mu_1}{\mu_2}\right)^S + \left(\frac{C\beta\mu_1}{\mu_2}\right)^K\right]}$$

$$+ \frac{1}{\sum_{L=1}^{C}\left[\frac{1}{S!}\left(\frac{C\beta\mu_1}{\mu_2}\right)^S \cdot \left(\frac{C\beta\mu_1}{S\mu_2}\right)^N \left(\frac{\beta\mu_1}{S\mu_2}\right)^L [\,C(c-1).....(C-L+1)]\right]} .....4.9$$

Define

(*i*) $\alpha_C = \frac{C!}{C^c}$

(*ii*) $\alpha_S = \frac{S!}{S^s}$

(*iii*) $X = \frac{C\mu_1}{S\mu_2}$

(*iv*) $\rho = X\beta$

Then, equation 4.6; implies that:

$$P_{oh} = \frac{(S\beta X)^h}{h!} P_{oo} \text{.....4.10}$$

(for $0 \leq h \leq s$)

similarly by substitution

$$P_{k,s} = \frac{1}{\alpha_s}(\beta X)^{s+k} P_{oo} \text{.....4.11}$$

(For $1 \leq K \leq N$)

and

$$P_{BL,S} = \left\{\frac{S^{s}}{S!}(X\beta)^{s}.(\beta X)^{N}.P_{oo}\right\}\left\{\frac{(X\beta)^{L}}{C^{L}}\right\}\{C!\}$$

$$= \frac{\alpha_C}{\alpha_S}(X\beta)^{S+N+L}.C^{C-L}.Poo.....4.12$$

(For $1 \leq L \leq C$)

Now let $R = \sum_{h=0}^{s} \frac{(SX\beta)^{h}}{h!}$

and

$$S = \sum_{K=1}^{N} \frac{1}{\alpha_S}(X\beta)^{S+K}$$

= second term in the denominator for equation 4.9 similarly let the third term in the denominator for equation 4.9 be defined as

$$T = \sum_{L=1}^{c} \frac{\alpha_c}{\alpha_s}(X\beta)^{S+N+L}.C^{C-L}$$

Then equation 4.9 can be written as

$$P_{oo} = \frac{1}{R+S+T}$$

Now for this truncated system, the effective arrival rate representing those calling customers who actually arrived at toll plaza without being blocked, defined by λe, is calculated as:

$$\lambda e = \lambda (1 - P_{BL,S}) = C \beta\mu_1 (1- P_{BL,S})$$

Where $\lambda e \leq c\mu$ under steady state and $P_{BL,S}$ is the blocking probability.

Hence,

$$\text{Offered load } a = \frac{C\beta\mu_1}{\mu_2} = S\beta X \text{.....4.14}$$

$$\text{Carried load } a' = \left(\frac{C\beta\mu_1}{\mu_2}\right)(1-P_{BL,S})$$

$$= S\beta X(1-P_{BL,S}) \text{.....4.15}$$

$$\text{Traffic intensity} = \rho = \beta X \text{.....4.16}$$

$$\text{and Utilization factor} = \rho = \beta X(1-P_{BL,S}) \text{.....4.17}$$

The expected queue length is calculated from the definition of expected values and the technique of sum calculus as follows:

Probability of K units being in the queue for $0 \le K \le N$ is given by equation 4.7

Hence

Let Lq = mean number in queue

$$Lq = \sum_{K=1}^{N} KP_{K,S}$$

$$= \frac{1}{S!}\left(\frac{C\beta\mu_1}{\mu_2}\right)^s .P_{oo} \sum_{K=1}^{N} K\left(\frac{C\beta\mu_1}{S\mu_2}\right)^k$$

$$= \frac{1}{S!}\left(\frac{C\beta\mu_1}{\mu_2}\right)^s .P_{oo}\left[\sum_{K=0}^{N} K\rho^k\right]$$

Now from the technique of sum calculus we know that the following definite sum holds

$$\sum_{M}^{N} nP^n = \left[\frac{P^n[n(P-1)-P]}{(P-1)^2}\right]_{n=m}^{n=N+1}$$

Therefore, by substitution:

$$Lq = \frac{1}{S!}\left(\frac{C\beta\mu_1}{\mu_2}\right)^s Poo.\left\{\left[\rho^K \frac{[K(\rho-1)-\rho]}{(\rho-1)^2}\right]_{K=0}^{K=N+1}\right\}$$

Which, when simplified further gives:

$$Lq = \frac{1}{S!}\left(\frac{C\beta\mu_1}{\mu_2}\right)^s \frac{P_{oo}}{(\rho-1)^2}[N\rho^{N+2}-(N+1)\rho^{N+1}+\rho]$$

and relevant substitution ⇒

$$Lq = \frac{(\beta X)^s}{\alpha_s(\beta X-1)^2}[N\ (\beta X)^{N+2}-(N+1)(\beta X)^{N+1}+\beta X].P_{oo} \ldots\ldots\ldots\ldots 4.18$$

Under steady state condition, it is known that the carried load (a′) is equivalent to the mean number of servers. Hence, the expected number LL, in the system ⇒

$$LL = Lq + a' \ldots\ldots\ldots\ldots 4.19$$

With the effective arrival rate, expected number in the system and the expected number in the queue now known; the mean waiting times are easily computed using Little's formulae (Giffin 1978).

Mean waiting time in queue ⇒

$$Wq = \frac{Lq}{\lambda e} = \frac{Lq}{C\ \beta\mu_1(1-P_{BL,S})} \ldots\ldots\ldots\ldots 4.20$$

and

mean waiting time in system ⇒

$$W = \frac{LL}{\lambda e} = Wq + \frac{1}{\mu_2} \ldots\ldots\ldots\ldots 4.21$$

The customer Loss Ratio represents the ratio of the time lost by customer in the queue to the time usefully spent there.

It is given by $LR = Wq\ \mu_2$ 4.22

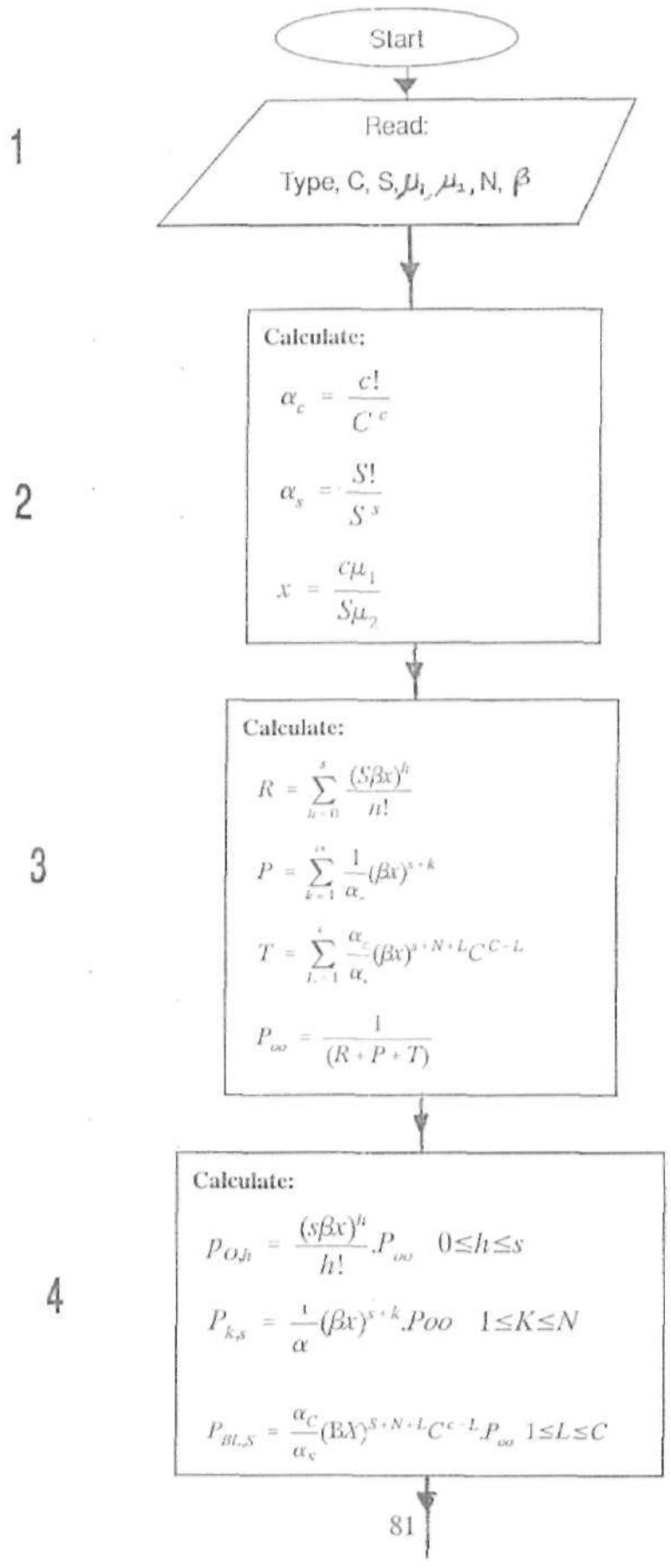
The customer Loss ratio (LR) is particularly important to customers requiring little service as in the case under study.

These equations for the evaluation of relevant system performance measures, together with the equations of steady state probabilities, constitute the full set of model equations.

4.3 MODEL SOLUTION

4.3.1 PRIMARY OBJECTIVES OF MODEL SOLUTION

The primary objective of model solution is to enable us compare the observed with the theoretical queue distributions. This is done by fitting in the mathematical distributions to measured (or observed) queue distributions.


Start
1
Read:
Type, C, S, μ_1, μ_2, N, β
Calculate:
$\alpha_c = \frac{c!}{C^c}$
2
$\alpha_s = \frac{S!}{S^s}$
$x = \frac{c\mu_1}{S\mu_2}$
Calculate:
$R = \sum_{h=0}^{s} \frac{(S\beta x)^h}{n!}$
3
$P = \sum_{k=1}^{n} \frac{1}{\alpha_s}(\beta x)^{s+k}$
$T = \sum_{L=1}^{c} \frac{\alpha_c}{\alpha_s}(\beta x)^{s+N+L} C^{C-L}$
$P_{oo} = \frac{1}{(R+P+T)}$
Calculate:
$P_{OJ_h} = \frac{(s\beta x)^h}{h!} P_{oo} \quad 0 \le h \le s$
4
$P_{k,s} = \frac{1}{\alpha}(\beta x)^{s+k} Poo \quad 1 \le K \le N$
$P_{BL,S} = \frac{\alpha_C}{\alpha_s}(\beta x)^{S+N+L} C^{c-L} P_{oo} \quad 1 \le L \le C$
81

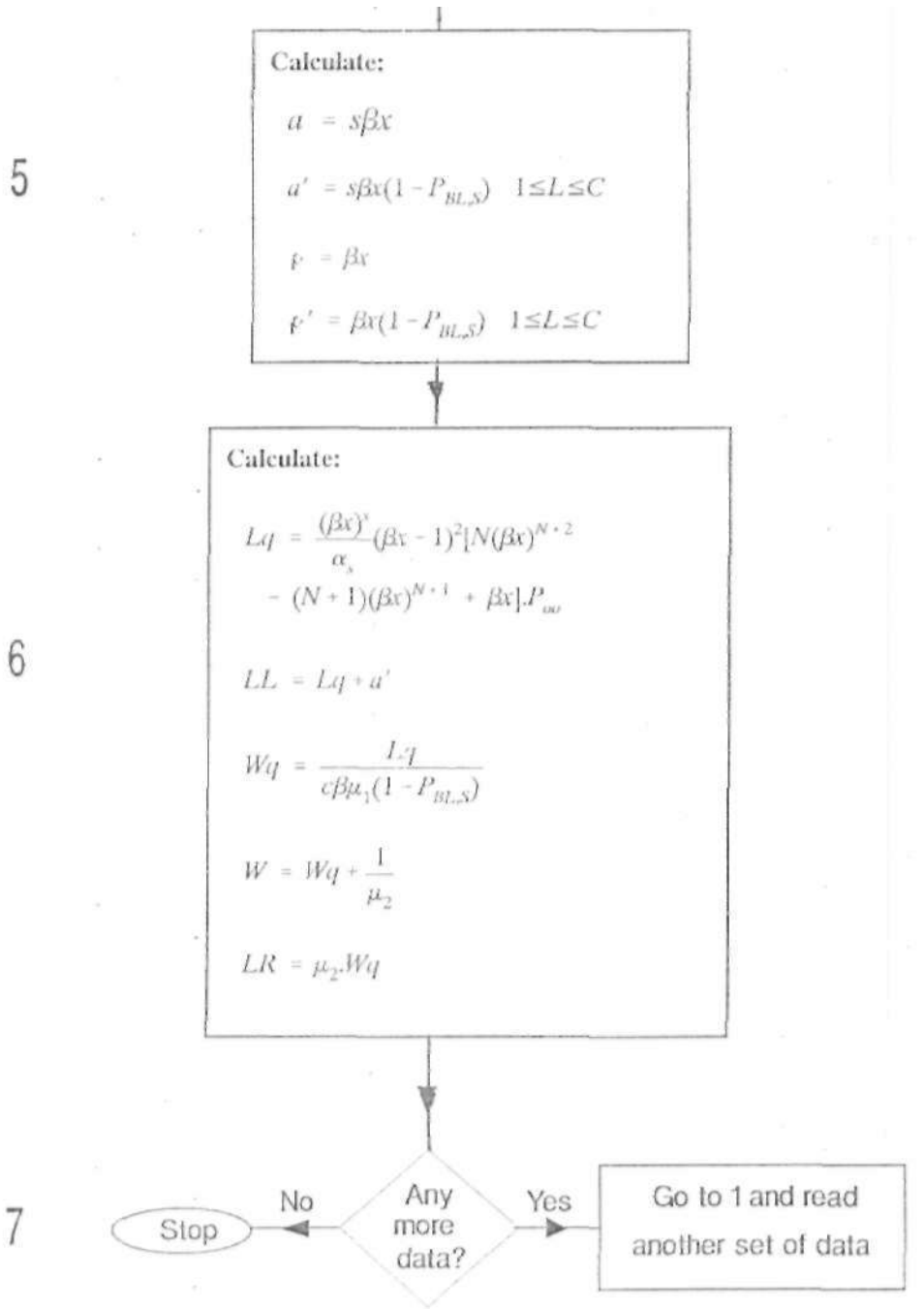


Fig. 4.3 PROGRAM FLOCHART OF MODEL CALIBRATION

TABLE 4.1 NOTATIONS OF SPECIAL RELEVANCE TO THIS CHAPTER

Define:

α_1 = Proportion of all vehicles that are trucks

α_2 = Proportion of all vehicles that are buses

α_3 = Proportion of all vehicles that are cars

α'_1 = Proportion of trucks in the toll booth queues

α'_2 = Proportion of buses in the toll booth queues

α'_3 = Proportion of cars in the toll booth queues

α = Proportion of all vehicles that get to the toll booth queues

α_u = Proportion of all vehicles that make a U-turn back to Maraba town

K = Total number of all vehicle classes arriving into the Police queues

K′ = Total number of all vehicle classes arriving into the toll booth queues

α''_1 = Proportion of U-turning vehicles that are trucks

α''_2 = Proportion of U-turning vehicles that are buses

α''_3 = Proportion of U-turning vehicles that are cars.

TABLE 4.2 STATISTICAL REDUCTIONS OF COMPUTED SERVICE TIME DATA FOR TOLLBOOTH CHANNELS

Day of the week	Period	Mean service Time (s)		
		Truck	Bus	Car
SATURDAY 14/12/96	1423-1455	10.08	3.03	4.11
SATURDAY 14/12/96	1745-1816	9.45	3.52	4.40
SATURDAY 18/1/97	1105-1125	21.00	3.76	3.38
SATURDAY 18/1/97	1300-1319	16.30	4.07	3.72
SATURDAY 18/1/97	1443-1502	25.80	3.26	3.32
	MEAN	16.53	3.53	3.79

4.3.2 COMPUTERISATION OF THE MODEL SOLUTION

The manual computations of the different measures of effectiveness would be very tedious. It is quite evident that in a practical situation the inputs could be so unwieldy as to render manual calculations either impossible or completely inaccurate and time wasting. therefore, a computer approach to the problem has been adopted and the model has been programmed for execution on a digital computer. The program flowchart is shown in Figure 4.3 while the program documentation is given in Appendix C 1, Appendices C2 and C3 respectively.

4.3.3 THE INPUTS TO THE PROGRAM

(a) Estimation of The Maximum Waiting Room For Each Vehicle Type

From the Federal Ministry of Works and Housing Highway Design Manual, (1972) the overall design length specified for different vehicle types are as follows:

i. Cars = 19ft = 5.8m

ii. Buses = 30ft = 9.2m

iii. Trucks = 50ft = 15.3m

But the distance between the Police check point and toll booths was measured and found to be approximately 150m

Thus,

$N_{car} = \frac{150}{5.8}$ = 26 cars, say 20 cars after allowing for inter-vehicle spacing.

$N_{Bus} = \frac{150}{9.2}$ = 16 Buses, say 12 Buses

$N_{truck} = \frac{150}{15.3}$ = 10 Trucks, say 8 Trucks

Perhaps, the above figures could have been directly estimated from actual observations in the field. However, there were no instances where the storage spaces were completely occupied.

(b) Estimation of Probability of Vehicle leaving stage 1 and getting to stage 2

These probabilities are deduced from direct field observations, the relevant results are given in Tables 4.1 and 4.6

Thus;

$\overline{\alpha_1}$ = 0.0670 = Proportion of all vehicles that are trucks

$\overline{\alpha_2}$ = 0.3519 = Proportion of all vehicles that are buses

$\overline{\alpha_3}$ = 0.5811 = Proportion of all vehicles that are cars

Where αi, i = 1,2,3 indicate the proportion of vehicle i in the traffic stream leaving stage 1.

Also, from Tables 4.1. and 4.4

$\overline{\alpha'_1}$ = 0.0722 = Proportion of trucks in the toll booth queue

$\overline{\alpha'_2}$ = 0.3184 = Proportion of buses in the toll booth queue

$\overline{\alpha'_3}$ = 0.5888 = Proportion of cars in the toll booth queue

$\overline{\alpha'_4}$ = 0.0206 = Proportion of autocycles in the toll booth queue

Where:

α', i = 1,2,3 is proportion of vehicle i in the traffic stream arriving at stage 2 service.

From Tables 4.1 and 4.8.

$\overline{\alpha_u}$ = 0.0079 = proportion of all vehicles turning back (i.e U-turning).

The proportion of U-turning vehicles that is of type i was not determined in the field and so it had to be estimated approximately. Hence from approximate estimates indicated in Table 4.1, the proportion of U-turning vehicles that are of class i are calculated thus.

For Trucks

$$\bar{\alpha}_1 = \frac{\bar{\alpha}_1 . \bar{\alpha}_u}{1 - \bar{\alpha}_u}$$

$$= \frac{0.0670x0.0079}{1 - 0.0079}$$

$$\bar{\alpha}_1 = \underline{0.0005}$$

For Buses

$$\bar{\alpha}_2 = \frac{\bar{\alpha}_2 \bar{\alpha}_u}{1 - \bar{\alpha}_u}$$

$$= \frac{0.3519x0.0079}{1 - 0.0079}$$

$$\underline{\bar{\alpha}_2^{11} = 0.0028}$$

For Cars

$$\bar{\alpha}_3 = \frac{\bar{\alpha}_3 \alpha_u}{1 - \bar{\alpha}_u}$$

$$= \frac{0.5811x0.0079}{1 - 0.0079}$$

$$\bar{\alpha}_3 = 0.0046$$

Thus, for cars using Table 4.6.

$$P\ (u/c) = \frac{0.0046 \times 0.0079}{0.5811}$$

$$= 0.000063$$

Hence, the probability of getting to stage 2 from 1

$$= 1 - P\ (u/c)$$

$$= 1 - 0.000063$$

$$= 0.9999$$

$$\beta_{car} = 0.9999$$

Similarly for

ß for Buses

$$ß_{Bus} = 0.9999$$

and for trucks

$$\beta_{truck} = 0.9999$$

(c) Estimation of first stage service Rate

Typical values of service times in each of the Police lanes from Appendices A1 and A2

For lane A, $\frac{1}{\mu_A}$ = 3.02sec as derived from 121 observations

For lane B, $\frac{1}{\mu_B}$ = 3.32sec as derived from 132 observations

Thus weighted mean

$$\left(\frac{1}{\mu}\right)_{av} = \frac{3.02x121 + 3.32x132}{121 + 132} = \frac{365.5 + 438.24}{253}$$

$$\left(\frac{1}{\mu}\right)_{av} = 3.2\text{sec } \textit{for both lanes}$$

This implies that

$$\mu_1 = \frac{3600}{3.2}$$

μ_t = 1125 vehicles/hour for the mixed population of vehicles.

Applying the decomposition technique for exponential random variable (Giffin 1978).

$\Rightarrow \mu_{1\ cars}$ for cars = α_3 x 1125 veh/hr

$$= 0.5811 \times 1125$$

$$\mu_{1\ cars} = 654 \text{ veh/hr}$$

Similarly

For Buses

$$\mu_{1buses} = \alpha_2 \text{ x } 1125$$

$$= 0.3519 \text{ x } 1125$$

$$\mu_{1buses} = 396 \text{ veh/hr}$$

and for trucks

$$\mu_{1trucks} = \alpha_1 \text{ x } 1125$$

$$= 0.0670 \text{ x } 1125$$

$$\mu_{1truck} = 75 \text{ veh/hr}$$

(d) Estimation for Second Stage Service Rate

Following from Table 4.2

(i.) Mean service time for Trucks = 16.53sec

$$\Rightarrow \mu_2 = \frac{3600}{16.53}$$

$$\overline{\mu_{2Trucks}} = 218 veh/hr$$

(ii) Mean Service time for buses = 3.53sec

$$\mu_2 = \frac{3600}{3.53}$$

$$\overline{\mu_{2\ buses}} = 1020\ veh/hr$$

iii) Mean service time for cars = 3.79sec

$$\mu_2 = \frac{3600}{3.79}$$

$$\overline{\mu_{2Cars}} = 950\ veh/hr$$

TABLE 4.3 NUMBER OF VEHICLES SERVED AT THE TOLL BOOTHS ON ACTUAL COUNTING (RAW DATA)

Day of the week	Period	LANE				
		CARS	BUSES	TRUCKS	AUTO CYCLES	TOTAL
FRIDAY	1210-1240	118	77	10	5	210
FRIDAY	1240-1310	112	55	7	3	177
FRIDAY	1310-1340	127	79	18	3	227
FRIDAY	1340-1410	121	49	19	5	194
FRIDAY	1410-1440	97	56	12	6	171
FRIDAY	1440-1510	131	68	22	2	223

TABLE 4.4 PROPORTION OF EACH VEHICLE CLASS SERVED AT THE POLICE QUEUE (COMPUTED)

Day of the week	Period	PROPORTION OF			
		Auto Cycle (α_4')	CARS (α_3')	BUSES (α_2')	TRUCKS (α_1')
FRIDAY	1210-1240	0.0238	0.5619	0.3667	0.0476
FRIDAY	1240-1310	0.0169	0.6328	0.3107	0.0395
FRIDAY	1310-1340	0.132	0.5595	0.3480	0.0793
FRIDAY	1340-1410	0.0258	0.6237	0.2526	0.0979
FRIDAY	1410-1440	0.0351	0.5673	0.3275	0.0702
FRIDAY	1440-1510	0.0089	0.5874	0.3184	0.0987
	MEAN	0.0206	0.5888	0.3184	0.0722

TABLE 4.5 NUMBER OF VEHICLES SERVED AT THE TOLL BOOTHS BASED ON COUNTING DISTRIBUTIONS (RAW DATA)

Day of the week	Period	LANE			
		CAR	BUS	TRUCK	TOTAL
FRIDAY 15/11/96	1100-1120	73	55	13	141
FRIDAY 15/11/96	1120-1140	87	38	10	135
FRIDAY 15/11/96	1500-1520	77	58	8	143
FRIDAY 15/11/96	1520-1540	95	48	8	151
FRIDAY 15/11/96	1540-1600	99	54	9	162
FRIDAY 15/11/96	16-1620	84	59	11	154

TABLE 4.6 PROPORTION OF EACH VEHICLE CLASS SERVED AT THE TOLL BOOTH (COMPUTED)

Day of the week	Period	LANE		
		PROPORTION OF CARS (α_3)	PROPORTION OF BUSES (α_2)	PROPORTION OF TRUCKS (α_1)
FRIDAY 15/11/96	1100-1120	0.5177	0.3901	0.0922
FRIDAY 15/11/96	1120-1140	0.6444	0.2815	0.0741
FRIDAY 15/11/96	1150-1520	0.5385	0.4056	0.0559
FRIDAY 15/11/96	1520-1540	0.6291	0.3179	0.0530
FRIDAY 15/11/96	1540-1600	0.6111	0.3333	0.0556
FRIDAY 15/11/96	1600-1620	0.5455	0.3831	0.0714
MEAN		0.5811	0.3519	0.0670

TABLE 4.7 NUMBER OF VEHICLES SERVED AT THE TOLL BOOTHS INCLUDING THOSE MAKING U-TURNING (RAW DATA)

Day of the week	Period	LANE				
		CAR	BUS	TRUCK	U-TURN	TOTAL
SATURDAY 16/11/96	1117-1137	85	37	9	3	134
SATURDAY 16/11/96	1138-1158	86	41	8	1	136
SATURDAY 16/11/96	1200-1220	85	48	9	0	142
SATURDAY 16/11/96	1220-1240	71	44	5	2	140
SATURDAY 16/11/96	1600-1620	84	43	8	1	136
SATURDAY 16/11/96	1620-1640	89	54	7	1	151
SATURDAY 16/11/96	1640-1700	96	49	12	1	158
SATURDAY 16/11/96	1700-1720	89	32	9	1	131
SATURDAY 16/11/96	1720-1740	92	37	12	1	142
SATURDAY 16/11/96	1740-1800	93	44	13	0	150

TABLE 4.8 PROPORTION OF EACH VEHICLE CLASS SERVED AT THE TOLL BOOTHS INCLUDING THOSE MAKING U-TURN (COMPUTED)

Day of the week	Period	LANE			
		CAR (α_C)	TRUCK (α_B)	TRUCK (α_T)	U-TURN VEH (α_U)
SATURDAY 16/11/96	1117-1137	0.6343	0.2761	0.0672	0.0224
SATURDAY 16/11/96	1138-1158	0.6324	0.3015	0.0588	0.0074
SATURDAY 16/11/96	1200-1220	0.5986	0.3380	0.0634	0.0000
SATURDAY 16/11/96	1220-1240	0.5071	0.3143	0.0357	0.0143
SATURDAY 16/11/96	1600-1620	0.6176	0.3102	0.0588	0.0074
SATURDAY 16/11/96	1620-1640	0.5894	0.3576	0.0464	0.0066
SATURDAY 16/11/96	1640-1700	0.6076	0.3101	0.0760	0.0063
SATURDAY 16/11/96	1700-1720	0.6794	0.2443	0.0887	0.0076
SATURDAY 16/11/96	1720-1740	0.6479	0.2606	0.0845	0.0070
SATURDAY 16/11/96	1740-1800	0.6200	0.2933	0.0867	0.0000
	MEAN	0.6134	0.3006	0.0646	0.0079

4.4 GENERATION OF INPUT DATA FOR MODEL

For the inputs obtained it was needful to generate a table of data for use with the program. This data was generated with variations of the inputs in order to simulate the system under varied conditions of inputs. Considering the fact that results generated depend on the variations of input to the computer and that each set of data will give a different

system response, it was necessary to exercise due care.

A parametric variation of the key variables was undertaken wherein for each value chosen the system of equations developed were solved. Thus, the following sets of data were generated for each vehicle type in this study:-

(a) **MODEL VALIDATION DATA**

Validation is an essential part of model building and application of existing models to real life problems. Briefly the process seeks to justify the model with regards to its prediction of the system behaviour. Often this is done either by comparing the results obtained from the model in question with those of a known model that work well for the system or by using simulation results as a basis of comparison. Another popular method of achieving this is to compare the system effectiveness measures predicted by the model with observed or measured.

For this thesis, the last option is adopted and the comparison with the observed values is carried out by means of tables and graphs. Here, inputs are selected such that the result obtained can be used to show how reasonably close the model approximates the realities of the system. Table 4.10 and figures 4.7 to 4.9 show a direct comparison of some measures of effectiveness predicted by the model and those actually observed.

(b) **WORKING TABLES OR CURVES DATA**

The data here are so selected that some of the predicted system measures can be linked to others either through graphs or tables. By so doing, data which could serve as a

working tool are prepared in the form of tables and graphs as a practical guide in the study of the system. For this study utilization factor of table 4.9 results has been selected as the independent variable and its relationship with some other measures like the mean waiting time in queue, customer loss ratio and the mean queue length is plotted in the graphs of figures 4.4 (a),to 4.6. This method could be used to produce extensive practical results such that once a value of system measure is known through field measurements, other system measures such as waiting time in queue, customer loss ratio and mean queue length can then be determined.

(c) **DATA USEFUL FOR DESIGN OF OPTIMUM SOLUTIONS**

Here, so many options of the facility organization are suggested and enough data is generated to see how these different options compare with each other. The aim here is to use the results generated by these options to build up tables or graphs that are useful in the determination of optimum solution. For this study, the options considered are shown in the results of Table 4.14(a) and 4.14(b). Here, (a) the various combinations of numbers of servers at both stages have been studied; (b) the maximum number of waiting customers has also produced a set of options - this will show in particular, what effect waiting room has on blocking, (c) with everything fixed, the fixed probability of vehicles getting to stage 2 has also been varied to produce the last set of options in order to study particularly the effect of removing the U - turning point (at least to another place).

All results are presented graphically or in a tabular form throughout and should be useful decision tools in thinking about improvement. The comparative table 4.12 is an

example of such a table of results showing measures that describe the system performance as it relates to cars, buses, and trucks in the present organized state of the facility.

4.4.1 CHOICE OF SYSTEM EFFECTIVENESS MEASURES.

All measures determined here are important in evaluating the system, but the customer loss ratio, the mean queue length and the mean waiting time could be considered as more important for this system. The customer loss ratio is normally very useful under situations of the type studied where customers require only short service. Similarly, a motorist joining the queue will depend, for most of his decisions on the mean queue length in terms of the number of vehicles he is actually observing. This can lead him to very many unpredictable behaviour which could even result in a breakdown of the queue.

Likewise, the mean waiting time is a very good decision parameter to a patron who might be waiting to use the facility. At least he can appreciate off-hand the average time he might have to waste while queueing and if possible make decisions that could balance this effect.

It is on the above bases that the system was simulated to give results tabulated in Table 4.12 and also presented in Figure 4.7 to 4.9

4.5 EVALUATION OF THE SIMULATION MODEL

Once a simulator has been designed and coded, the analyst must evaluate the validity of the model. The ability of a simulation model to accurately predict the performance of the real system depends on both the validity of the model and the reliability of the performance

measures produced.

The variable-parameter validity and reliability tests were performed by varying the parameters and variables to determine their effect on the simulator and the subsequent output. This was undertaken to know if the impact of the variables or parameters is large when compared with the initial estimate of their impact. If the impact is large then the validity and reliability of the model must be questioned.

To this end the system performance was simulated using arbitrary variables to see the system response. From the results of the output sheets in Appendix C3 displayed in Table 4.11 and 4.13 and figures 4.10(a) and 4.10(b) show the system response whose impacts are not significantly different from the results earlier predicted in Tables 4.9 and 4.12 and Figures 4.4(a) and 4.4(b).

Based on the response of the system some options are therefore suggested in Table 4.14(a).

Table 4.9 **VALUES OF UTILIZATION AGAINST SOME SYSTEM MEASURES**

Type of vehicle	Mean Waiting Time in Queue (WQ) (Sec)	Customer Loss Ratio (LR) (Sec)	Mean Number In Queue (LQ)	Blocking Probability (P_BL,S)	Utilization Factor (ρ')
CARS	6.12 5.76 16.20 25.92 24.84 29.16 21.96 18.72	0.8559 0.8585 2.9415 5.2408 5.6118 9.0711 9.2596 9.3899	0.3947 0.3965 2.2038 4.4929 4.8781 8.6060 8.8141 8.9584	0.0000 0.0000 0.0004 0.0054 0.0070 0.0365 0.0391 0.041	0.4612 0.4619 0.7492 0.8573 0.8693 0.9519 0.9541
BUSES	1.80 1.80 2.16 2.16 2.52 2.16 2.52 2.88 5.40 9.00	0.3907 0.4527 0.5452 0.5844 0.8206 0.8208 0.9996 1.0616 2.2942 3.9339	0.1098 0.1411 0.1924 0.2156 0.3699 0.3701 0.5001 0.5472 1.6267 3.3219	0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0024 0.0191	0.2810 0.316 0.3528 0.3889 0.4508 0.4509 0.5003 0.5155 0.7091 0.8444
TRUCKS	2.52 2.88 2.52 2.52 2.16 2.16 2.16 2.52 3.6	0.2242 0.2756 0.3916 0.4003 0.4190 0.4309 0.5507 0.6641 0.9887	0.0411 0.0595 0.1102 0.1144 0.1238 0.1298 0.1958 0.2656 0.4961	0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0001 0.0005	0.1831 0.2160 0.2815 0.2859 0.2954 0.3012 0.3555 0.3999 0.5017

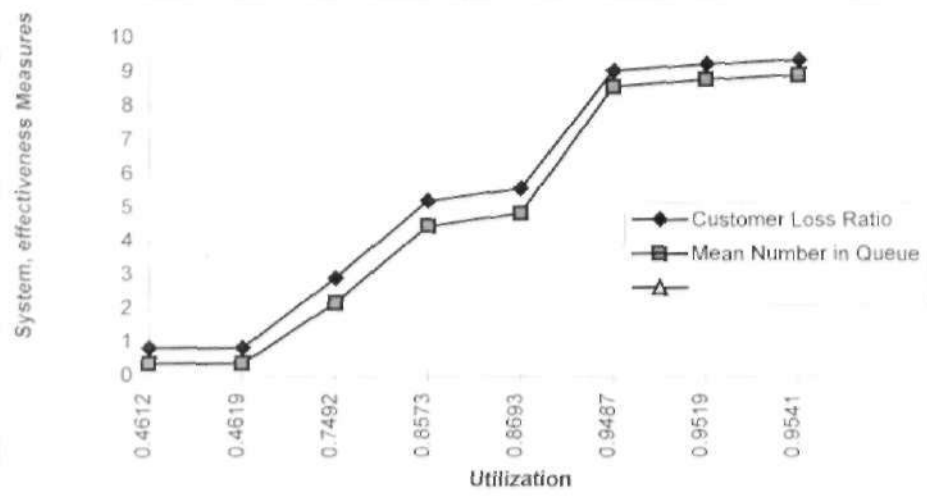


Fig. 4.4a System effectiveness measures Vs Utilization
(FOR CARS)

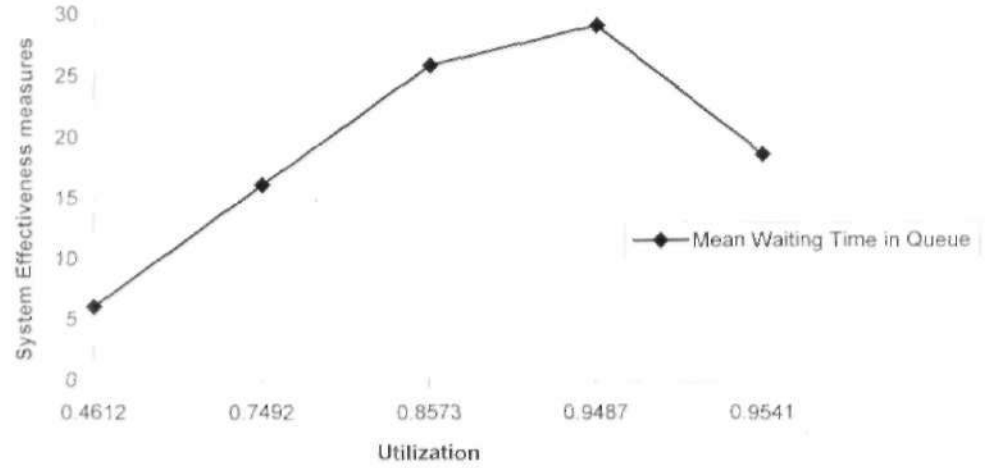


Fig. 4.4b System effectiveness measures Vs Utilization (FOR CARS)

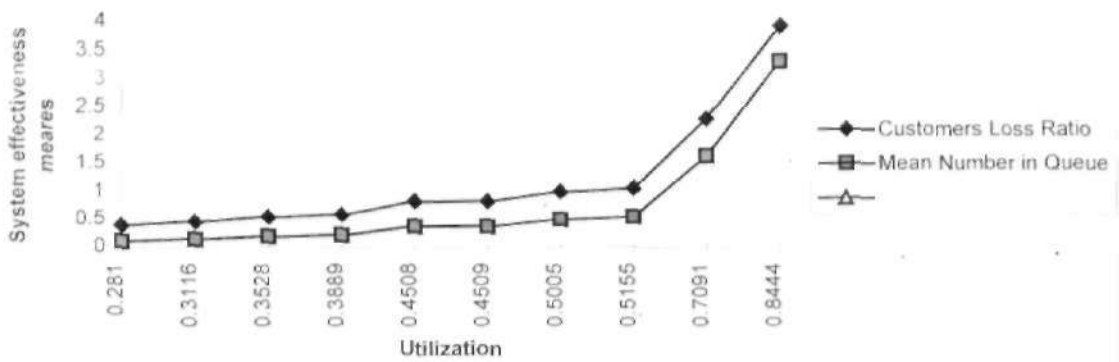


Fig. 4.5a System effectiveness measure Vs Utilization (FOR BUSES)

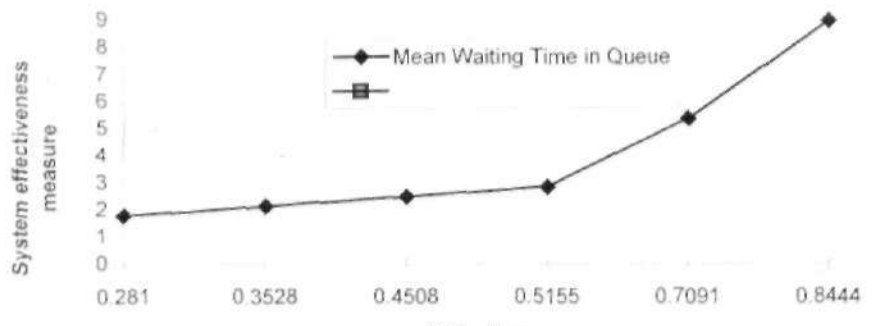


Fig 4.5b System effectiveness measures Vs Utilization (FOR BUSES)

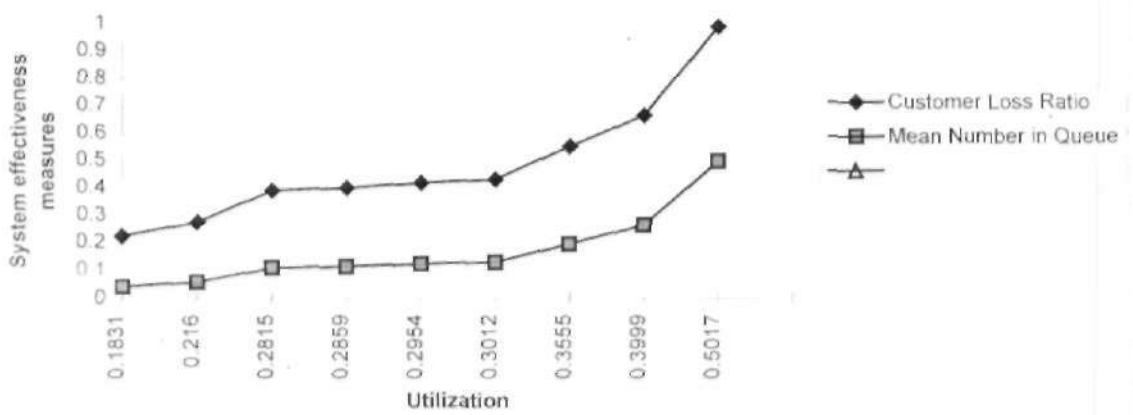


Fig. 4.6 System effectiveness measures Vs Utilization (FOR TRUCKS)

Table 4.10 OBSERVED AND PREDICTED MEASURES OF EFFECTIVENESS

Type of Vehicle	Observed Utilization ρobs	Predicted Utilization Spread ρpred	Observed Waiting Time in Queue WQ	Predicted Waiting Time in Queue WQ Pred.	Observed Customer Loss LRobs.(SEC)	PREDICTED CUSTOMER LOSS RATIO LRpred (sec)
CARS	0.9800	0.9541	19.50	18.72	10.1000	9.3899
BUSES	0.7800	0.8444	8.00	9.00	3.5000	3.9339
TRUCKS	0.3500	0.5017	0.5017	3.60	0.8500	0.9887

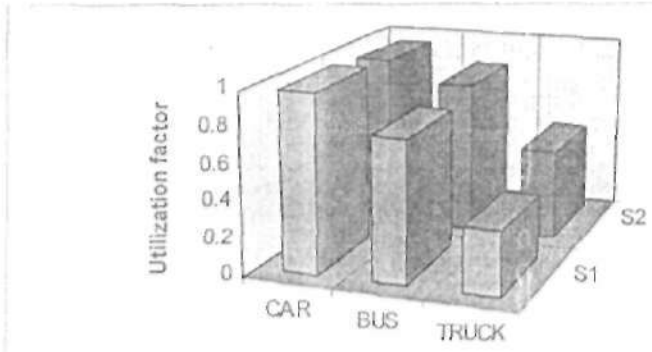


Fig. 4.7 Comparison of Observed Versus Predicted Utilization

LEGEND

S1 = Observed Utilization
S2 = Predicted Utilization

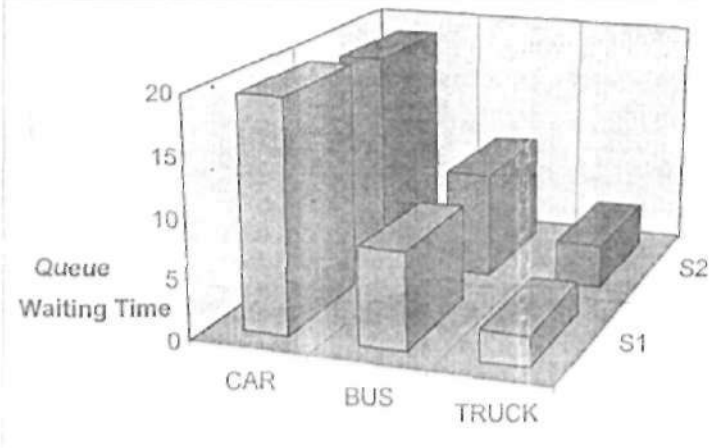


Fig. 4.8 Comparison of Observed Vesus Predicted Waiting Time in Queue

LEGEND

S1 = Observed Waiting Time in Queue
S2 = Predicted Waiting Time in Queue

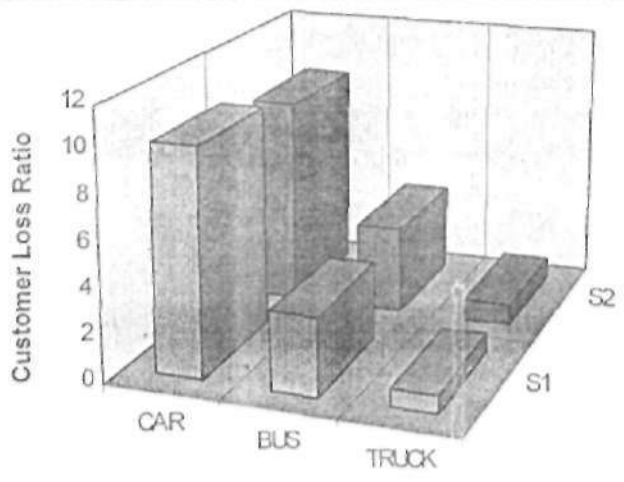


Fig. 4.9 Comparison of Observed Versus Predicted Customer Loss Ratio

LEGEND

S1 = Observed Customer Loss Ratio
S2 = Predicted Customer Loss Ratio

463476
KASHIM IBRAHIM LIBRARY
RECEIVED
28 MAY 19
ZARIA NIGERIA
Ahmadu Bello University

TABLE 4.11 VALUES OF UTILIZATION AGAINST SOME SYSTEM MEASURES FOR VARIED PARAMETERS.

Type of Vehicle	Mean Waiting Time in Queue WQ Sec	Customer Loss Ratio LR (Sec).	Mean Number in Queue (LQ)	Blocking Probability (P_{BLS})	UTILIZATION FACTOR (ρ^1)
CARS	1.80	0.2500	0.0500	0.0000	0.2000
	9.00	0.9998	0.4998	0.0000	0.4999
	11.88	1.9951	1.3299	0.0000	0.6666
	27.00	4.5147	3.7504	0.0031	0.8307
	44.00	8.0726	7.5098	0.0246	0.9303
BUSES	1.80	0.2500	0.0500	0.0000	0.2000
	4.68	0.6665	0.2666	0.0000	0.4000
	6.48	0.7995	0.3553	0.0000	0.4444
	7.20	0.9989	0.4990	0.0000	0.4999
	23.04	3.2055	2.5408	0.0091	0.7926
TRUCKS	0.72	0.1666	0.0238	0.0000	0.1428
	2.88	0.3333	0.0833	0.0000	0.2500
	5.40	0.5985	0.2244	0.0000	0.3749
	5.04	0.6344	0.2467	0.0000	0.3888
	6.84	0.9451	0.4629	0.0004	0.4898

TABLE 4.12 COMPARATIVE TABLE FOR THE DIFFERENT VEHICLE CLASSES USING SOME SYSTEM EFFECTIVENESS MEASURES FOR THE EXISTING FACILITY ORGANIZATION.

TYPE OF VEHICLE	PROBABILITY OF AN EMPTY SYSTEM (POO)	MEAN QUEUE LENGTH (LQ)	FACILITY UTILIZATION (ρ)	MEAN WAITING TIME IN QUEUE (WQ) (Sec)	BLOCKING PROBABILITY (P_BLS)	CUSTOMER LOSS RATIO (LR) (Sec)
CARS	0.0459	8.9584	0.9541	18.72	0.0411	9.3899
BUS	0.1556	3.3219	0.8444	9.00	0.0191	3.9339
TRUCK	0.4983	0.4961	0.5017	3.60	0.0005	0.9887

TABLE 4.13 COMPARATIVE TABLE ON VARIED PARAMETERS FOR DIFFERENT VEHICLES CLASSES USING SOME SYSTEM EFFECTIVENESS MEASURES FOR THE EXISTING FACILITY ORGANIZATION (FOR RELIABILITY OF SIMULATOR)

Type of Vehicle	Probability of an empty system (Poo)	Mean Queue Length (LQ)	Facility Utilization (ρ^1)	Mean Waiting Time in Queue WQ (sec)	Blocking Probability (P_{BLS}	Customer Loss Ratio (LR) (sec)
CARS	0.0697	7.5098	0.9303	44.64	0.0246	8.00726
BUSES	0.2074	2.5408	0.7926	23.04	0.0091	3.2055
TRUCKS	0.5102	0.4629	0.4898	6.84	0.0004	0.9451

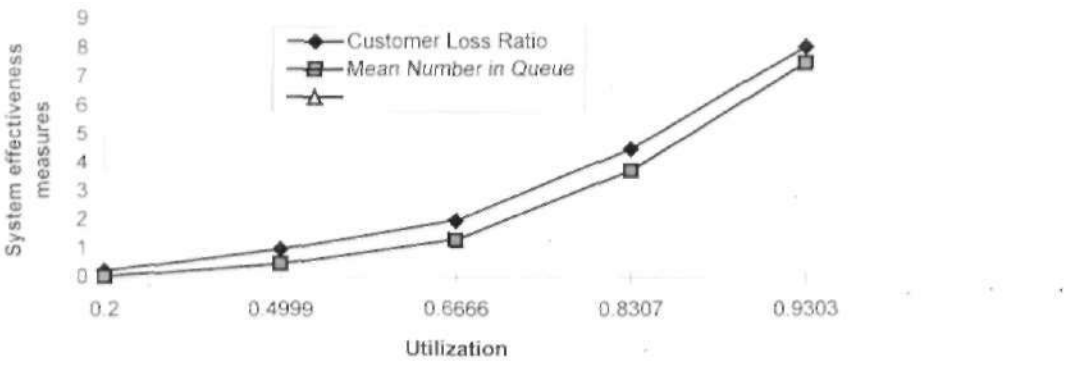


Fig. 4.10a System effectiveness measures versus utilization for varied parameters (CAR LANES)

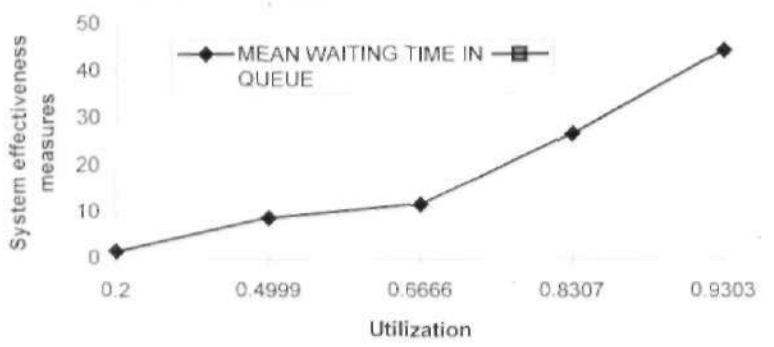


Fig. 4.10b System effectiveness Vs Utilization for varied parameters (CAR LANES)

TABLE 4.14a Suggested Options and some of their Predicted system measures

OPTION NO.	COMMENT	C	S	Mean Queue Length Lq	Mean Waiting Time in Queue Wq (Sec)	Customer Loss Ratio LR	Blocking Probability P_{BLS}	Probability of empty system P_{oo}
1.	For these first 7	1	1	8.9584	18.72	9.38.99	0.0411	0.0459
2.	Options, all variables	1	2	0.3272	0.72	0.3289	0.0000	0.3356
3.	are fixed while	1	3	0.446	0.00	0.448	0.0000	0.3656
4.	different combination	2	1	6.3836	9.36	4.9758	0.3311	0.0000
5.	of C & S are tried	2	2	8.5847	9.00	4.4004	0.0196	0.0221
6.	Existing Arrangement	2	3	0.8644	0.72	0.4344	0.0000	0.1126
7.	1 more channel added to existing No. at each stage	3	4	1.4539	1.08	0.4872	0.0001	0.0386
		Waiting Room N						
8.	Every variable is	60		27.5206	55.44	28.0392	0.0135	0.0185
9.	fixed for these next	20		8.9584	18.72	9.3899	0.0411	0.0459
10.	four options while	10		4.1893	9.00	4.559	0.0746	0.0793
11.	N is varied	8		3.2450	7.20	3.5786	0.0886	0.0932
		Fixed Proba-bility						
12.	for these last four	0.9950		8.7933	18.36	9.2407	0.0389	0.0481
13.	options, all variables	0.7500		2.1594	5.76	2.8947	0.0004	0.2540
14.	are fixed except the probability of	0.5500		0.6615	2.52	1.2088	0.0000	0.4527
15.	Leaving stage 1 and getting to stage 2	0.2500		0.0824	0.72	0.3311	0.0000	0.7512

TABLE 4.14(b) BRIEF DESCRIPTION OF THE SUGGESTED OPTIONS

OPTION NO	DESCRIPTION
1	One policeman in series with one toll collector. This implies collapsing the service at each of the channels into one super service facility at a single channel for both stages of the queuing system.
2	One policeman in series with two collectors. Here, while the two police channels are replaced by one super service channel, the three toll booths normally used for cars are reduced to just two channels.
3	One police in series with three toll collectors. This is the case of one super service channel replacing the two police channels while the three cars lanes are retained.
4	The case of two policeman in series with one toll collector. Here, the three toll lanes constitute one super channels while the police lanes remain unchanged.
5	Two policemen in series with two toll collectors.
6	The existing arrangement of two policemen in series with three toll collectors.
7	Three policemen in series with four toll collectors, implying an addition of one extra channel at each stage.
	All other conditions being fixed:
8	No. of Cars allowed in waiting room is 28
9	No. of Cars allowed in waiting room is 9
10	No. of cars allowed in waiting room is 4
11	No. of cars allowed in waiting room is 3
12	Probability of getting to stage 2 is 0.9950
13	Probability of getting to stage 2 is 0.7500
14	Probability of getting to stage 2 is 0.5500
15	Probability of getting to stage 2 is 0.2500

4.6 APPLICATIONS OF RESULTS

Apart from the comparison of observed with expected arrival times and service time distributions, the results obtained in this study can also be useful in:

* Predicting the arrival rates at a point
* Testing the randomness of traffic flow.

4.6.1 PREDICTING THE ARRIVAL RATES OF VEHICLES

From the queue data, it was possible to estimates the rates at which vehicles arrive at the point of observation. This was done by using the understanding that:

$$\text{Arrival Rate (Veh\hr)} = \frac{3600}{\bar{t}} \text{ ----------(1)}$$

$$\text{or Arrival rate (Veh\hr)} = \frac{1}{\bar{t}}$$

Where $\bar{t}$ is average arrival time

Perhaps a more useful application is that the results can be used for simulating arrival rates on the computer. Such an application requires a random number generator and a simple illustration is given below:

Let us take the case when observed arrival time could be described by the Negative exponential model, we have

$$F_H(h) = \lambda e^{-\lambda h} \ldots\ldots\ldots\ldots\ldots (2)$$

$$F_H(h) = 1 - e^{-\lambda h} \ldots\ldots\ldots\ldots\ldots (3)$$

and the inverse is;

$$h = -\frac{1}{\lambda} \ln (1 - F) \ldots\ldots\ldots\ldots\ldots (4)$$

If F, (1-F) are uniform (0, 1) we have the rule to find an exponential variate, therefore the inverse cumulative distribution function becomes.

$$h = -\frac{1}{\lambda} \ln (u.\ r.v)$$

where λ is a parameter corresponding to flow in V.P.S and u.r.v is a uniform random variate. Hence using appropriate values of the uniform random variate and λ, arrival times can be simulated on the computer. The arrival rates (λ) simulated can be useful in estimating delays at vehicular points on the queue lanes.

CHAPTER FIVE

DISCUSSIONS, COMMENTS, RECOMMENDATION AND CONCLUSION.

5.1 DISCUSSION AND COMMENTS.

The result of this study are presented in tables and graphs of previous chapters. Thus discussion of these results with comments and recommendations follow in this chapter.

Looking at the traffic flow patterns of Figure 3.1(a), 3.1(b), and 3.1(c) it is clearly discernible that the number of vehicles arriving at both stages of the queue does not remain constant throughout the observation period. The peak periods are 1300 to 1800 hours for FRIDAY, 1100 to 1400 hours and 1600 to 1900 hours for Saturdays and 1200 hours to 1800 hours for Sundays. On the average the peak volumes are not substantial too different for the three days of the weekends.

From the coefficient of variation and K-S Statistical tests undertaken in the samples, it is evident that the negative exponential distribution is quite appropriate to describe the arrival and service times characteristics in the system. The results in table 3.13 for the K-S test show these clearly as a good number of the data sets are accepted at 5% level of significance, that is, the model output compares favourably with the actual observation.

An inspection of table 4.12 shows that the utilization factors portrays the car lanes to be the most utilized followed by the bus lanes and then the truck lane. Blocking is therefore most unlikely to occur in truck lane than all the other lanes.

From Table 4.10 and figures 4.7 to 4.9, the predicted values appear to be in agreement with the observed effectiveness measures. This enables us to conclude that the Makino model is adequate for the system. The mean waiting time for cars (18.72 seconds) are more than that of buses (9.00 seconds) and then followed by the trucks (3.60 Seconds). Also the customers loss ratio for car is the greatest (9.39 seconds) followed by that of the buses (3.93 seconds) and lastly by that of the trucks (0.99 seconds). Overall prediction of these effectiveness measures appear to compare favourably for cars, buses and trucks with the actual observation.

From the comparative results in table 4.12 showing some values of the effectiveness measures for the system as currently organised, it could be summarised as follows:-

(a) **Car lane**, the probability of being in an empty state (i.e. idle servers) in 0.0459, which is at acceptable level, but the queue length 9 cars, waiting time 18.72 seconds and loss ratio of about 9.4 seconds are high enough to warrant concern for a patron requiring only a small service time. The toll collectors are operating at 95% utilization. The blocking probability is not negligible for a simpler analytical approach.

(b) **Bus lane**, the probability of being in an empty state is 0.1556. This mean queue length is 3, resulting in a wait of 9.00 seconds. The customers loss ratio of 3.9 is unacceptable for motorists requiring small service time. The toll collectors are operating at 84% Utilization. The blocking probability is not negligible for a simpler analytical approach.

(c) **Truck lane**, the probability of being in the empty state (i.e. idle server) is high enough to warrant concern since this is also an unhealthy queueing situation where a server is idle for most of the time. There is nothing much to worry about on the customers side

since the queue length and waiting time are low enough to be considered reasonable. The toll collectors are operating at 50% Utilization, while the loss ratio of 0.9887 seconds is acceptable. The blocking probability is for this case negligible for practical purposes.

What all the above observations mean is that, from the customers point of view the truck lane arrangement is currently okay. And so if the customers point of view remains the criterium for evaluation, it is also evident that some improvements must be carried out to reduce the waiting time of the queue and the customer loss ratio. This will result in effective utilization of the facilities.

Table 4.9 and Figures 4.2(a) to 4.4 together suggest the blocking in the system becomes pronounced at a certain critical value of the utilization factor reached at the second stage channels. This critical value for the range of utilization factors used in this experiment, is equal to 0.0411 for cars, equals to 0.0191 for buses and equals to 0.0005 for trucks. These asymptotic properties are better appreciated from the graph under discussion.

Table 4.13 of the varied parameters when evaluated shows no significant difference from that of Table 4.12. This shows that the simulator is validated and reliable experimental device.

Table 4.14(a) of available options when considering improvements, as illustrated with the case of cars shows that depending on the particular measure of effectiveness of interest, different options can be substitutes for the present system. Thus, if it is the general overall aim to minimize mean waiting time and loss ratio, then option 1,4, and 5 are definitely not okay. Rather, one would consider option number 2 very seriously for an improved performance of the system. This is because, excluding option number 3 which involves

unrealistic zero waiting time in queue, it gives the lowest values of mean waiting time and loss ratio.

Also for a wide range of values of the maximum waiting room, it can be seen from option 8 to 11 that while the effectiveness measures appear to increase with increasing size of waiting room, the blocking probability decreases rather gradually with increasing waiting room.

The options 12 to 15 which represents the effects of either reducing or increasing the intensity of turning vehicles show that as the intensity of turning increases, the blocking effects and other measures reduce steadily to zero while the probability of the system being empty increases. This implies that the effect of removing the U-turning point (at least from its present location) is to further increase all other system measures including the blocking probability while the probability of the empty state becomes zero.

5.2 RECOMMENDATIONS

From the model output the level of performance at the toll booths is not too low as envisaged, however, it is pertinent to make useful recommendations to improve the system performance for the customer satisfaction.

The service mechanisms can be modified as follows:-

(a) Reduce the mean service time;

(b) Reduce the coefficient of variation of service time;

(c) Arrange for service times to be reduced during peak periods;

(d) Change the capacity of the system by providing more servers especially during the

peak periods.

The above could be achieved through the following measures:

i. Introduction of incentives and better spaced relief for toll collectors;

ii. Reduction in shift hours;

iii. Encouraging motorists to change to faster lanes whenever a particular service channel runs into problems;

iv. Intensifying the campaign for the need of correct-fare (to avoid change of currency problems).

v. Better management appreciation and approach to the problem before it becomes critical.

vi. Closure of a service point for cars and buses respectively to motorists, when traffic is not heavy, usually during weekdays.

Normally, a decrease in mean service time and in the coefficient of variation is always desirable as the decrease in mean service time means more efficient use of the facility while a decrease in coefficient of variation results in more rate of services has been shown to yield lower waiting times and improved services to customers (Gavar, 1953).

However, for this system, what is important is the reduced coefficient of variation and the ratio of the rate of service at both stages, and it is therefore, realistic to think in terms of increasing server availability for both stages rather than to think of providing more servers and more facilities which means more capital and operating costs.

Service capacity can also be increase or decrease temporarily depending on the traffic situation.

Operation at the critical periods therefore calls for innovative action in the form of temporary measures, some of which are as suggested above. However, any long term measure must consider the use of automation at the service channels and the choice of other system variables such that facility cost and customers satisfaction are balanced. For this purpose it is important to refer again to the list of options in Table 4.14(a) which use car as an example. Of all the possible options into which the existing system can be organised, it is clear that although option 3 provides the best results in terms of customers service, as can be seen from the values of mean waiting time and loss ratio. The use of option 2 seems to provide a more balanced design in terms of customer satisfaction and facility than option 3 and should be adopted if ever the present arrangement for cars is to be re-organised. Lower values of mean number in queue, mean waiting time in queue and the customers loss ratio can also be achieved by decreasing the waiting room and the choice of any of these options must be backed up by a thorough cost appraisal.

Another area to be considered is the police check point. The imposition of policemen on the system clearly means that the rate of arrival at the toll booths sometimes depends on the rate at which the policemen release the vehicles. Thus the poor performance of the queueing system facilities can be due sometimes to the police holding back motorists even when there are no vehicles in the space between the police point and the toll booths. This is reflected in the high coefficient of variation for the police queue. Likewise, this accounts for the sometimes observed high frequency of zero-back-up on the individual lanes even though patrons are delayed unnecessarily too long behind the police. The police post therefore, constitutes a very big delay factor for motorists approaching the toll booths and so motorists

suffer terrible hold-ups before getting into the toll booth queues. Their complete removal from the system or a change in their location may then be further justified.

The most important recommendation in connection with data collection is that which has to do with the method used for the data gathering. It is maintained that, for such system as the one studied here where the hazard of data collection is very much, the use of human collectors and timing devices are grossly inadequate without being supplanted with mechanical or electrical devices.

It would be best however, to collect such data by means of a second time lapse film taken from a vantage point on the study, but the financial aspects of such ventures are definitely beyond the financial resources available for this thesis.

5.3 SUGGESTION FOR FURTHER RESEARCH

Further research pursuits are suggested on the following which this study does not centre on:

(a) An attempt should be made to analyse the system using one of the proposed alternative models as a means of comparism.

(b) It must be mentioned quite clearly that the aim of this research is essentially to study the toll booth operations and not just a serial queue. For example, if the police were not there, it would have been a straight forward case. But now that they are there, it would be enough just to evaluate their effect on the toll booth operations. This police effect is of course accommodated in themodel and so specifically, only level of service at this facility and not behind the

police is covered. Further research is therefore, encouraged in order to obtain operational values of the measures of effectiveness for the police queues;

(c) A study dealing solely with "Application of Queue models" is hereby suggested as a worthwhile area of future research, since only few applications could be considered in this study.

5.4 CONCLUSION

Considering the picture at which the toll booths patrons paint the situation at the toll booths, it was therefore necessary to determine the indices which arguments or diminish the toll booth level of performance and use it as an input to the model. Thus the results show the level of dissatisfaction in the analysis carried out for cars and buses, the level of such indices of dissatisfaction are low enough to be considered as a level of tolerance. The truck lane does not appear to do otherwise about the customers satisfaction for it infact, provide the best indices of satisfaction for the customers by judging from the output model.

Motorists get to the toll booth queue already fuming with frustration and exhaustion as a result of the hold-up already suffered behind the police check point and so this calls for a change of police check point for healthy level of performance at the toll booths.

The negative exponential is apparently a sound model for describing the queue. Likewise the Makino model is justified for practical applications. The information derived from the queue data and their distributions are useful in:

* Predicting arrival rates of vehicles at a point;
* Testing the randomness of traffic flow.

REFERENCES

1. ANG, A.H.S. and TANG, W.H. (1975), Probability concepts In Engineering planning and Design. John Wiley and Sons, Inc; New York pp. 275-281.

2. Bald, W.G. and UNDERWOOD, J.R. (1977), "An Investigation into the Effect of Toll Booths On Morning Peak Traffic Flow", The chartered Municipal Engineer, Vol. 104, pp 205-208.

3. BARRY, S. (1975), Queueing And Waiting Studies in Social organisation of access and delay, U.S.A. pp 1-9, 14-62 and 91-166; The University of Chicago.

4. BURKE, P.J. (1956), "The Output of A Queueing System", Operations Research, Vol. 4, pp 699 - 704.

5. CHIEJINA, E.E. (1982), Analysis of Intra-Urban Bus Travel Times Unpublished M. ENG. Thesis, Dept. of Civil Engineering, Ahmadu Bello University, Zaria, Nigeria.

6. CHURCHMAN, C.W; ACKOFF, R.L. AND ARNOFF, E.L. (1957), Introduction To Operations Research, John Wiley and Sons Inc. New York.

7. CONOVER, W.J. (1971), Practical parametric Statistics John Wiley and Sons Inc; New York.

8. COX, D.R. AND SMITH, W.L. (1965), QUEUES Mathenn and Company Limited, London.

9. CRUON, R. (1961), Queueing Theory Recent development and Applications,

London pp 10-34, The English University Press.

10. DEBAVN, R.M. AND KATZ, S. (1959), "An Approximation To Distributions of Summed Waiting Times", Operations Research, Vol. 7, No 6.

11. EVERETT, J.L. (1953), "State probabilities In Congestion problems Characterised by Constant Holding Times", Journal of Operations Research Society of America, Vol 1, pp 279 -285.

12. FREUD, J.E. AND WALYYOLE, R.E. (1980), Mathematical Statistics Prentice - Hall Inc; Englewood Cliffs, New York.

13. GAVER, O.P. (1953), "The Influence of Servicing Times In Queueing Processes", Journal of Operations Research, Vol 2, pp 139-149.

14. GIFFIN, W.C. (1978), Queueing - Basic Theory And Applications, Grid Inc; Columbus, Ohio.

15. GORDOM, W.J. and NEWELL, G.F. (1956), "Cyclic Queueing Systems with Restricted length Queues", Operations Research, Vol. 5, pp 267-277.

16. GROSS, D. AND HARRIS, C.M. (1974), Fundamentals of Queueing theory, John Wiley and Sons Inc, New York.

17. GUMBEL, H. (1959), "Waiting Lines with Heterogeneous Servers", Operations Research, Vol. 8 No. 4 pp. 504 - 511.

18. HANN, G.J. AND SHAPIRD, S.S. (1967), Statistical model In Engineering, John Wiley and son Inc, New York.

19. HUNT, G.C. (1956), "Sequential Arrays of Waiting Lines", Operations Research, vol 4, pp 674 - 683.

20. HUNTER, J.F. (1971), "Further studies on Two Queues In Parallel," The Australian Journal of Statistics, vol. 13, No. 2, pp. 83 - 93.

21. IKEM, N.C. (1982), Toll Booth Congestion studies at the Lagos-Ibadan Expressway, Unpublished M. ENG. Thesis, Dept of civil Engineering, Ahmadu Bello University, Zaria, Nigeria.

22. JACKSON, J.R.P. (1954), "Queueing Systems with Phase Type service", Operations Research Quarterly, Vol. 5, pp 109-120.

23. JACKSON, J.R. (1957), "Networks of Waiting Lines", Operations Research, Vol 5, No.4 pp 518-521.

24. JOSEPH, A.P. (1969), Queueing Theory, a study of Waiting Lines for Business, Economics and Science, New Jersey, pp 57, 98, 99, and 121, prentice-Hall Inc.

25. KADIYALI, L.R. (1991), Traffic Engineering and Transport Planning, 4th edition, Delhi, pp. 565-575, Khana Publishers.

26. KOENIGSBER, E. (1960), "Finite Queues and Cyclic Queues", operations Research, Vol 8, No 2, pp 246-253.

27. LABETOULLE, J. PUJOLLT, G. AND SOULA, C. (1981), "Stationary Distributions of Flows In Jackson Networks", Mathematics of Operations Research, Vol. 6, No.2, pp 173-185.

28. LEE, A.M. (1961), Applied Queueing Theory, Macmillan and Company Limited, London.

29. LEROY, S.F. and SHOUP, T.E. (1978), Introduction To Engineering Including Fortran Programming, Prentice - Hall, Inc, Englewood Cliffs, New york.

30. LESLIE, E.C. (1954), "Traffic delays At Toll Booths", Journal of Operations Research, Vol. 2, No. 2 pp 107-138.

31. LITTLECHILD, S.C. (1977) Operations Research for Managers, John Wiley and Sons, Inc, New York.

32. MCCRACKEN, D.D. (1967), Fortran With Engineering Applications, John Wiley and Sons, Inc, New York.

33. MAKINO, T. (1964), "On The Mean Passage Time Concerning Some Queueing Problems Of The Tandem Type", Journal of the Operations Research of Japan, vol. 7, No. 1 pp 17-47.

34. MARTINS, F.F. (1968), Computer Modelling and Simulations, John Wiley and Sons Inc, New York.

35. MORRISON, J.A. (1980), "An Over Flow System In Which Queueing Takes Precedence," The bell system Technical Journal, Vol 60, No.1, pp 1-12.

36. MORSE, P.M. (1958), Queues, Inventories And Maintenance, John Wiley and Sons Inc, New York.

37. NELSON, R.T. (1958), "Waiting Time Distributions For Applications To A Series of Service Centres", Operations Research, Vol 6. pp 856 - 862.

38. PIGNATARO, L.J. (1973), Traffic Engineering Theory And Practice, Prentice-Hall-Inc; Englewood Cliffs, New Jersey.

39. REICH, E. (1957), "Waiting Times When Queues Are In tandem," Ann. Math. Statist, Vol. 28, No. 3, pp 768.

40. ROSS, S.M. (1972), Introduction To Probability Models, Academic Press, Inc; New York.

41. SAATY, T.L. (1961), Elements Of Queueing Theory With Applications, McGraw-Hill Book Company, Inc,; New York.

42. SHARMA, O.P. (1973), "A Model For Queues In Series," Journal Of Applied Probability, Vol. 10, pp 691-701.

43. SMITH, D.R. and WHITT, W. (1980), "Resource Sharing For Efficiency In Traffic Systems," The Bell System Technical Journal, Vol. 60, No. 1 pp 39-55.

44. STOPHER, P.R. and MEYBURG, A.H. (1975), Urban Transportation Modelling And Planning, D.C. Health and Company, Lexingstong, Massachusetts.

45. TAHA, H.A. (1971), Operations Research - An Introduction, Macmillan Publishing Company, Inc, New York.

APPENDIX A1 SERVICE TIME DATA ANALYSIS FOR LANE A

Date: 17/1/97
Starting Time: 16 50 06
Position: At Police Check Point
Day of The week: Friday

Duration: 20 minutes
Lane : A
Direction of traffic flow: Zaria to Kaduna

S/N	ARRIVAL TIME	DEPARTURE TIME	TYPE OF VEH. C=Car B=Bus T=Truck	SERVICE TIME FOR ALL VEH. T_{sa}	SERVICE TIME FOR CARS T_{sc}	SERVICE TIME FOR BUSES T_{sb}	SERVICE TIME FOR TRUCKS T_{st}
1	16 50 06	16 50 08	T	2	-	-	2
2	16 50 16	16 50 40	T	24	-	-	24
3	16 50 44	16 50 46	C	2	2	-	-
4	16 50 51	16 50 58	T	7	-	-	7
5	16 51 03	16 51 06	T	3	-	-	3
6	16 51 10	16 51 12	C	2	2	-	-
7	16 51 15	16 51 16	C	1	1	-	-
8	16 51 19	16 51 21	C	2	2	-	-
9	16 51 24	16 51 26	B	2	-	2	-
10	16 51 30	16 51 49	B	19	-	19	-
11	16 52 04	16 52 10	B	6	-	6	-
12	16 52 18	16 52 20	B	2	-	2	-
13	16 52 24	16 52 25	B	1	-	1	-
14	16 52 28	16 52 30	C	2	2	-	-
15	16 52 30	16 52 32	T	2	-	-	2
16	16 52 37	16 52 38	B	1	-	1	-
17	16 52 42	16 53 05	C	23	23	-	-
18	16 53 11	16 53 12	C	1	1	-	-
19	16 53 15	16 53 16	C	1	1	-	-
20	16 53 20	16 53 23	B	3	-	3	-
21	16 53 29	16 53 30	C	1	1	-	-
22	16 53 32	16 53 34	C	2	2	-	-
23	16 53 37	16 53 38	B	1	-	1	-
24	16 53 58	16 53 59	C	1	1	-	-
25	16 54 07	16 54 10	C	3	3	-	-
26	16 54 12	16 54 14	C	2	2	-	-
27	16 54 17	16 54 18	C	1	1	-	-
28	16 54 25	16 54 28	B	3	-	3	-
29	16 54 31	16 54 35	C	4	4	-	-
30	16 54 38	16 54 41	B	3	-	3	-
31	16 54 45	16 54 48	C	3	3	-	-
32	16 54 54	16 54 56	C	2	2	-	-
33	16 54 57	16 54 59	C	2	2	-	-
34	16 55 06	16 55 08	C	2	2	-	-
35	16 55 10	16 55 14	C	4	4	-	-
36	16 55 26	16 55 28	C	2	2	-	-
37	16 55 32	16 55 34	B	2	-	2	-
38	16 55 43	16 55 46	C	3	3	-	-
39	16 55 52	16 55 54	B	2	-	2	-
40	16 55 57	16 55 59	C	2	2	-	-
41	16 56 02	16 56 04	C	2	2	-	-
42	16 56 16	16 56 18	C	2	2	-	-
43	16 56 31	16 56 32	C	1	1	-	-
44	16 56 45	16 56 47	B	2	-	2	-
45	16 56 56	16 56 58	C	2	2	-	-
46	16 57 04	16 57 06	C	2	2	-	-
47	16 57 23	16 57 25	C	2	2	-	-
48	16 57 31	16 57 33	C	2	2	-	-
49	16 57 42	16 57 44	B	2	-	2	-
50	16 57 57	16 57 59	C	2	2	-	-
51	16 58 04	16 58 08	C	4	4	-	-
52	16 58 19	16 58 22	B	3	-	3	-
53	16 58 30	16 58 31	C	1	1	-	-
54	16 58 57	16 58 59	C	2	2	-	-
55	16 59 12	16 59 13	C	1	1	-	-
56	16 59 16	16 59 17	C	1	1	-	-

S/N	ARRIVAL TIME	DEPARTURE TIME	TYPE OF VEH. C=Car B=Bus T=Truck	SERVICE TIME FOR ALL VEH. T_{sa}	SERVICE TIME FOR CARS T_{sc}	SERVICE TIME FOR BUSES T_{sb}	SERVICE TIME FOR TRUCKS T_{st}
57	16 59 23	16 59 25	C	2	2	-	-
58	16 59 28	16 59 30	C	2	2	-	-
59	16 59 40	16 59 48	C	8	8	-	-
60	16 59 49	16 00 05	C	16	16	-	-
61	16 00 16	17 00 17	C	1	1	-	-
62	17 00 18	17 00 19	C	1	1	-	-
63	17 00 22	17 00 24	C	2	2	-	-
64	17 00 28	17 00 30	C	2	2	-	-
65	17 00 46	17 00 48	B	2	-	2	-
66	17 00 54	17 00 56	C	2	2	-	-
67	17 01 01	17 01 03	T	2	-	-	2
68	17 01 08	17 01 10	C	2	2	-	-
69	17 01 15	17 01 16	B	1	-	1	-
70	17 01 19	17 01 20	B	1	-	1	-
71	17 01 31	17 01 33	B	2	-	2	-
72	17 01 41	17 01 43	B	2	-	2	-
73	17 01 45	17 01 48	C	3	3	-	-
74	17 01 52	17 01 54	B	2	-	2	-
75	17 01 57	17 01 59	B	2	-	2	-
76	17 02 05	17 02 06	B	1	-	1	-
77	17 02 08	17 02 09	C	1	1	-	-
78	17 02 13	17 02 14	B	1	-	1	-
79	17 02 43	17 02 45	B	2	-	2	-
80	17 02 49	17 02 55	C	6	6	-	-
81	17 02 56	17 03 13	C	17	17	-	-
82	17 03 20	17 03 22	T	2	-	-	2
83	17 03 24	17 03 26	C	2	2	-	-
84	17 03 30	17 03 31	C	1	1	-	-
85	17 03 33	17 03 36	C	3	3	-	-
86	17 03 43	17 03 49	T	6	-	-	6
87	17 03 53	17 03 57	T	4	-	-	4
88	17 04 25	17 04 52	T	27	-	-	27
89	17 05 02	17 05 03	C	1	1	-	-
90	17 05 04	17 05 06	C	2	2	-	-
91	17 05 10	17 05 11	C	1	1	-	-
92	17 05 14	17 05 16	C	2	2	-	-
93	17 05 19	17 05 20	B	1	-	1	-
94	17 05 27	17 05 29	C	2	2	-	-
95	17 06 08	17 06 11	B	3	-	3	-
96	17 06 14	17 06 15	C	1	1	-	-
97	17 06 16	17 06 18	C	2	2	-	-
98	17 06 21	17 06 22	B	1	-	1	-
99	17 06 26	17 06 28	B	2	-	2	-
100	17 06 35	17 06 38	B	3	-	3	-
101	17 06 42	17 06 45	C	3	3	-	-
102	17 06 52	17 06 53	C	1	1	-	-
103	17 06 53	17 06 59	C	6	6	-	-
104	17 07 02	17 07 07	T	5	-	-	5
105	17 07 10	17 07 12	B	2	-	2	-
106	17 07 14	17 07 16	C	2	2	-	-
107	17 07 18	17 07 20	C	2	2	-	-
108	17 07 39	17 07 41	C	2	2	-	-
109	17 08 01	17 08 02	B	1	-	1	-
110	17 08 05	17 08 06	B	1	-	1	-
111	17 08 10	17 08 12	C	2	2	-	-
112	17 08 30	17 08 32	B	2	-	2	-
113	17 08 40	17 08 42	B	2	-	2	-
114	17 08 46	17 08 48	B	2	-	2	-
115	17 08 50	17 08 52	T	2	-	-	2
116	17 09 08	17 09 11	T	3	-	-	3
117	17 09 14	17 09 16	C	2	2	-	-
118	17 09 17	17 09 18	C	1	1	-	-
119	17 09 24	17 09 26	C	2	2	-	-
120	17 09 29	17 09 30	C	1	1	-	-
121	17 09 57	17 10 00	T	4	-	-	3

APPENDIX A2 SERVICE TIME DATA ANALYSIS FOR LANE B

Date: 17/1/97
Starting Time: 16 10 12
Position: At Police Check Point
Day of The week: Friday

Duration: 20 minutes
Lane : B
Direction of traffic flow: Zaria to Kaduna

S/N	ARRIVAL TIME	DEPARTURE TIME	TYPE OF VEH. C=Car B=Bus T=Truck	SERVICE TIME FOR ALL VEH. T_a	SERVICE TIME FOR CARS T_a	SERVICE TIME FOR BUSES T_a	SERVICE TIME FOR TRUCKS T_a
1	16 10 12	16 10 25	T	13	-	-	13
2	16 10 31	16 10 33	B	2	-	2	-
3	16 10 41	16 10 43	B	2	-	2	-
4	16 10 44	16 10 47	C	3	3	-	-
5	16 10 52	16 11 11	C	19	19	-	-
6	16 11 14	16 11 16	C	2	2	-	-
7	16 11 21	16 11 24	B	3	-	3	-
8	16 11 38	16 11 44	T	6	-	-	6
9	16 11 57	16 11 59	C	2	2	-	-
10	16 12 05	16 12 07	C	2	2	-	-
11	16 12 34	16 12 49	T	15	-	-	15
12	16 12 56	16 12 58	C	2	2	-	-
13	16 13 01	16 13 02	B	1	-	1	-
14	16 13 05	16 13 06	B	1	-	1	-
15	16 13 08	16 13 09	C	1	1	-	-
16	16 13 11	16 13 12	C	1	1	-	-
17	16 13 15	16 13 16	B	1	-	1	-
18	16 13 18	16 13 23	C	5	5	-	-
19	16 13 35	16 13 38	C	3	5	-	-
20	16 13 40	16 13 42	C	2	2	-	-
21	16 13 46	16 13 50	C	4	4	-	-
22	16 13 58	16 14 00	B	2	-	2	-
23	16 14 04	16 14 06	C	2	2	-	-
24	16 14 10	16 14 12	B	2	-	2	-
25	16 14 14	16 14 18	B	4	-	4	-
26	16 14 20	16 14 28	C	8	8	-	-
27	16 14 32	16 14 34	C	2	2	-	-
28	16 14 37	16 14 39	C	2	2	-	-
29	16 14 41	16 14 44	B	3	-	3	-
30	16 14 49	16 14 55	C	6	6	-	-
31	16 14 01	16 15 04	C	3	3	-	-
32	16 14 07	16 15 09	C	2	2	-	-
33	16 14 10	16 15 11	C	1	1	-	-
34	16 15 20	16 15 22	C	2	2	-	-
35	6 15 25	16 15 31	C	6	6	-	-
36	16 15 35	16 15 50	T	15	-	-	15
37	16 15 01	16 15 03	C	2	2	-	-
38	16 15 04	16 15 05	C	1	1	-	-
39	16 15 06	16 15 08	C	2	2	-	-
40	16 15 13	16 15 15	C	2	2	-	-
41	16 15 32	16 15 17	T	45	-	-	45
42	16 16 25	16 16 26	C	2	2	-	-
43	16 16 37	16 16 39	B	2	-	2	-
44	16 16 47	16 16 52	T	5	-	-	5
45	16 16 55	16 16 58	C	3	3	-	-
46	16 17 01	16 17 05	T	4	-	-	4
47	16 17 11	16 17 13	C	2	2	-	-
48	16 17 17	16 17 18	C	1	1	-	1
49	16 17 20	16 17 21	T	1	-	-	-

S/N	ARRIVAL TIME	DEPARTURE TIME	TYPE OF VEH. C=Car B=Bus T=Truck	SERVICE TIME FOR ALL VEH. T_{sa}	SERVICE TIME FOR CARS T_{sc}	SERVICE TIME FOR BUSES T_{sb}	SERV2E TIME FOR TRUCKS T_{st}
50	16 17 25	16 17 28	C	3	3	-	-
51	16 17 32	16 17 33	B	1	-	1	-
52	16 17 40	16 17 42	C	2	2	-	-
53	16 17 46	16 17 47	B	1	-	1	-
54	16 17 50	16 17 53	C	3	3	-	-
55	16 18 02	16 18 05	C	3	3	-	-
56	16 18 06	16 18 08	C	2	2	-	-
57	16 18 14	16 18 16	B	2	-	2	-
58	16 18 18	16 18 20	C	2	2	-	-
59	16 18 22	16 18 23	C	1	1	-	-
60	16 18 34	16 18 36	C	2	2	-	-
61	16 18 40	16 18 41	C	1	1	-	-
62	16 18 48	16 18 50	C	2	2	-	-
63	16 19 56	16 19 57	C	1	1	-	-
64	16 20 01	16 20 03	C	2	2	-	-
65	16 20 06	16 20 08	C	2	2	-	-
66	16 20 12	16 20 14	C	2	2	-	-
67	16 20 16	16 20 18	C	2	2	-	-
68	16 20 20	16 20 23	C	3	3	-	-
69	16 20 29	16 20 30	C	1	1	-	-
70	16 20 36	16 20 38	B	2	-	2	-
71	16 20 44	16 20 46	B	2	-	2	-
72	16 21 01	16 21 03	B	2	-	2	-
73	16 21 10	16 21 12	C	2	2	-	-
74	16 21 15	16 21 18	C	3	3	-	-
75	16 21 19	16 21 25	B	6	-	6	-
76	16 21 27	16 21 33	C	6	6	-	-
77	16 21 40	16 21 42	C	2	2	-	-
78	16 21 45	16 21 47	B	2	-	2	-
79	16 21 57	16 21 59	C	2	2	-	18
80	16 22 04	16 22 20	T	18	-	-	-
81	16 22 45	16 22 47	C	2	2	-	-
82	16 22 56	16 22 59	C	3	3	-	-
83	16 23 06	16 23 09	B	3	-	3	-
84	16 23 20	16 23 21	C	1	1	-	-
85	16 23 27	16 23 29	C	2	2	-	-
86	16 23 31	16 23 33	C	2	2	-	-
87	16 23 36	16 23 37	C	1	1	-	-
88	16 23 47	16 23 50	B	3	-	3	-
89	16 23 53	16 23 55	B	2	-	2	-
90	16 23 58	16 23 59	C	1	1	-	-
91	16 24 04	16 24 06	C	2	2	-	-
92	16 24 08	16 24 10	C	2	2	-	-
93	16 24 13	16 24 15	T	2	-	-	2
94	16 24 19	16 24 21	C	2	2	-	-
95	16 24 24	16 24 26	C	2	2	-	-
96	16 24 29	16 24 30	C	1	1	-	-
97	16 24 33	16 24 36	C	3	3	-	-
98	16 25 02	16 25 04	B	2	-	2	-
99	16 25 14	16 25 16	C	2	2	-	-
100	16 25 24	16 25 26	C	2	2	-	-
101	16 25 32	16 25 33	T	1	-	-	1
102	16 25 36	16 25 38	C	2	2	-	-
103	16 25 43	16 25 45	T	2	-	-	2
104	16 25 55	16 25 58	B	3	-	3	-
105	16 25 01	16 25 02	C	1	1	-	-

S/N	ARRIVAL TIME	DEPARTURE TIME	TYPE OF VEH. C=Car B=Bus T=Truck	SERVICE TIME FOR ALL VEH. T_{sa}	SERVICE TIME FOR CARS T_{sc}	SERVICE TIME FOR BUSES T_{sb}	SERVICE TIME FOR TRUCKS T_{st}
106	16 26 13	16 26 14	C	1	1	-	-
107	16 26 29	16 26 44	T	15	-	-	15
108	16 26 49	16 26 55	C	6	6	-	-
109	16 26 57	16 26 59	C	2	2	-	-
110	16 26 01	16 26 03	C	2	2	-	-
111	16 27 05	16 27 07	C	2	2	-	-
112	16 27 08	16 27 09	C	1	1	-	-
113	16 27 12	16 27 14	B	2	-	2	-
114	16 27 16	16 27 17	C	1	1	-	-
115	16 27 25	16 27 27	B	2	-	2	-
116	16 27 31	16 27 58	T	27	-	-	27
117	16 28 02	16 28 04	C	2	2	-	-
118	16 28 08	16 28 10	C	2	2	-	-
119	16 28 17	16 28 20	B	3	-	3	-
120	16 28 24	16 28 26	T	2	-	-	2
121	16 28 31	16 28 33	C	2	2	-	-
122	16 28 38	16 28 39	C	1	1	-	-
123	16 28 44	16 28 48	T	4	-	-	4
124	16 29 01	16 29 02	B	1	-	1	-
125	16 29 04	16 29 06	C	2	2	-	-
126	16 29 23	16 29 26	T	3	-	-	3
127	16 29 37	16 29 39	B	2	-	2	-
128	16 29 40	16 29 43	C	3	3	-	-
129	16 29 44	16 29 49	B	5	-	5	-
130	16 29 50	16 29 52	C	2	2	-	-
131	16 29 57	16 29 59	C	2	2	-	-
132	16 30 05	16 30 08	C	3	3	-	-

APPENDIX B KOLMOGOROV - SMIRNOV (K-S) TEST PROGRAM

```
C     PROGRAM TO FIT NEGATIVE EXPONENTIAL DISTRIBUTION
C     TO OBSERVED DATA USING KOLMOGOROV-SMIRNOV (K-S)
C     GOODNESS OF FIT TECHNIQUES.
C
C     T =OBSERVED INTER ARRIVAL/SERVICE TIME
C     F =OBSERVED ABSOLUTE FREQUENCY
C     EF=EXPECTED CUMMULATIVE FREQUENCY
C
      INTEGER DAY,YEAR,T,F,FLAG,CUMFX,CUMTF,TT
      REAL       KS,KSMAX
      CHARACTER*1  V
      CHARACTER*12 VEH,RATE
C
      DIMENSION T(50),F(50)
C
      OPEN(UNIT=5, FILE='KOL.DAT', STATUS='OLD')
      OPEN(UNIT=6, FILE='KOL.RES', STATUS='NEW')
C
      K3   = 0
      KD   = 0
      FLAG = 1
      GOTO 3
C
C     READ IN THE INPUT DATA
C
   7  KD   = 0
      READ (5,110) RATE,V,DAY,MONTH,YEAR
      N    = 0
      CUMTF= 0
      TT   = 0
      KSMAX= 0.0
C
   3  KD   = KD + 1
      READ (5,100,END=6) T(KD), F(KD)
      IF (T(KD).EQ.999) GOTO 4
      N    = N + F(KD)
      CUMTF= CUMTF + T(KD) * F(KD)
      TT   = TT + F(KD) * T(KD)**2
      K3 = K3 + 1
      GO TO 3
C
   6  IF (K3.EQ.0) GOTO 13
      FLAG = 0
C
   4  IF (K3.EQ.0) GOTO 7
C
      IF (V.EQ.'C') THEN
         VEH = 'CARS'
      ELSE IF (V.EQ.'B') THEN
         VEH = 'BUSES'
      ELSE IF (V.EQ.'T') THEN
         VEH = 'TRUCKS'
      ELSE
         VEH = 'ALL VEHICLES'
      END IF
C
      IF (RATE.EQ.'A') THEN
         RATE = 'ARRIVAL RATE'
      ELSE
         RATE = 'SERVICE RATE'
      END IF
C
C     ********** MAIN COMPUTATIONS **********
      KD    = KD - 1
      CUMFX = 0
C
C     *** CALCULATION OF MEAN,SIGMA,COEFF,RATE
      XMEAN = FLOAT(CUMTF)/FLOAT(N)
        N2 = N * TT - CUMTF**2
      SIGMA = SQRT(FLOAT(N2)/FLOAT(N**2))
      COEFF = SIGMA/XMEAN
      LAMU  = IFIX(3600.0/XMEAN)
C
```

```
C      PRINT ALL RELEVANT STATISTICS
C
C      *** PRINT HEADING
       WRITE(6,120) VEH
       WRITE(6,130) DAY,MONTH,YEAR
       WRITE(6,140) RATE,LAMU
       WRITE(6,150) SIGMA,COEFF
       WRITE(6,160)
       WRITE(6,170)
C
       DO 90 J = 1, KD
C
C      *** CALCULATION OF OBSERVED CUMULATIVE FREQUENCIES
       CUMFX = CUMFX + F(J)
       FX    = FLOAT(CUMFX)/FLOAT(N)
C
C      *** CALCULATION OF EXPECTED FREQUENCIES
       EF    = 1 - EXP(-FLOAT(T(J))/XMEAN)
C
C      *** CALCULATION OF K-S STATISTICS
       KS    = ABS(FX - EF)
       IF (KS.GT.KSMAX) KSMAX = KS
C
C      *** PRINT COMPUTATION RESULTS
       WRITE(6,180)T(J),F(J),FX,EF,KS
C
   90  CONTINUE
       WRITE(6,190) KSMAX
C
       IF (FLAG.EQ.1) GOTO 7
       GOTO 116
C
   13  WRITE(*,*) 'DATA FILE IS EMPTY'
       GOTO 116
C
C      ***** FORMAT STATEMENTS ARE AS FOLLOWS *****
  100  FORMAT(I3,I4)
  110  FORMAT(A1,1X,A1,2I3,I5)
  120  FORMAT(//8X,'GOODNESS-OF-FIT TEST FOR ',A12/)
  130  FORMAT(8X,'DATE:',2I3,I5)
  140  FORMAT(8X,A12,'=',I4,' VEHICLES PER HOUR')
  150  FORMAT(8X,'STANDARD DEV=',F6.2,5X,'COEFF OF VAR=',F4.2)
  160  FORMAT(/8X,'TIME      OBS     OBS.     EXPTD.    K-S'/
     +  18X,'FREQ.   CUM.     CUM.'/26X,'FREQ.    FREQ.')
  170  FORMAT(8X,'-------------------------------------')
  180  FORMAT(8X,I4,5X,I4,3F9.4)
  190  FORMAT(/8X,'MAXIMUM K-S = ',F6.4)
C
  116  CONTINUE
       STOP
       END
```

APPENDIX C 1 QUEUE MODEL PROGRAM

```
C     THIS PROGRAM CALCULATES THE PROBABILITY AND THE RELATED
C     MEASURES OF EFFECTIVENESS FOR A TRUNCATED TWO STAGE MULTI
C     SERVER SERIAL QUEUE WITH POISSON ARRIVAL AND EXPONENTIAL
C     SERVICE TIME
C
C     STATISTICAL TESTS ARE USED TO JUSTIFY THE APPROXIMATION OF
C     MEAN ARRIVAL RATES, MEAN SERVICE RATES AND PROBABILITY
C     DISTRIBUTIONS OVER TIME
C
C     THE QUEUE STATISTICS CALCULATED IN RELATION TO STAGE2 QUEUE
C     ARE:
C
C     1. RHO    = TRAFIC INTENSITY OR RATIO OF UTILIZATION OF
C                 STAGE2 TO STAGE1
C     2. POO    = PROBABILITY OF NO VEHICLE IN SYSTEM
C     3. LQ     = MEAN NUMBER IN THE QUEUE
C     4. A      = TOTAL TRAFFIC OFFERED PER SERVICE TIME
C     5. ADASH  = TOTAL TRAFFIC CARRIED PER SERVICE TIME
C     6. RODASH = CHANNEL UTILIZATION FACTOR
C     7. WQ     = MEAN WAITING TIME IN QUEUE
C     8. W      = MEAN WAITING TIME IN SYSTEM
C     9. LL     = MEAN NUMBER OF VEHICLES IN SYSTEM
C    10. LR     = CUSTOMER LOSS RATIO
C    11. POH    = STATE PROBABILITY OF SECOND STAGE SERVER BEING
C                 ENGAED
C    12. PBLS   = FIRST STAGE BLOCKING PROBABILTY
C    13. PKS    = STATE PROBABILITY OF K-VEHICLES IN THE QUEUE
C                 WHILE ALL SECOND STAGE SERVERS ARE BUSY
C
C     OTHER VARIABLES USED ARE:
C      1. TYPE   = VEHICLE TYPE
C      2. C      = NUMBER OF FIRST STAGE SERVERS
C      3. S      = NUMBER OF SECOND STAGE SERVERS
C      4. MU1    = RATE OF SERVICE AT THE FIRST STAGE CHANNELS
C      5. MU2    = RATE OF SERVICE AT THE SECOND STAGE CHANNELS
C      6. BETA   = FIXED PROBABILITY WITH WHICH A VEHICLE LEAVES
C                  STAGE1 FOR STAGE2
C      7. R      = COMPUTED VARIABLE FOR DETERMINATION OF POO
C      8. P      = COMPUTED VARIABLE FOR DETERMINATION OF POO
C      9. T      = COMPUTED VARIABLE FOR DETERMINATION OF POO
C
C
      INTEGER TYPE, C, S, H, H1
      REAL LL(100),LR(100), LQ
      DIMENSION POH(100),PBLS(100),PKS(100),ADASH(100),
     + RODASH(100),WQ(100),W(100)
C
      OPEN(UNIT=1, FILE= 'QMODEL.DAT', STATUS= 'OLD', IOSTAT=IERR)
      OPEN(UNIT=2, FILE= 'QMODEL.RES',STATUS='NEW')

C
    9 READ(1,10,END=999) TYPE,C,S,MU1,MU2,N,BETA
C
C      CALCULATION OF TRAFFIC INTENSITY
C
C
      XC=FLOAT(IFACT(C))/FLOAT(C**C)
      XS=FLOAT(IFACT(S))/FLOAT(S**S)
      X=FLOAT(C*MU1)/FLOAT(S*MU2)
C
C      INITIALISATION OF VARIABLES
C
      R=1.0
      T=0.0
      P=0.0
      POO=0.0
      DO 102 JJ=1,100
      POH(JJ)=0.0
      PBLS(JJ)=0.0
      PKS(JJ)=0.0
C
      LL(JJ)=0.0
      LR(JJ)=0.0
```

```
      ADASH(JJ)=0.0
      RODASH(JJ)=0.0
      WQ(JJ)=0.0
      W(JJ)=0.0
  102  CONTINUE
C
      DO 105 H=1,S
  105  R=R+((S*BETA*X)**H)/FLOAT(IFACT(H))
      DO 106 K=1,N
  106  P=P+1/XS*(BETA*X)**(S+K)
      DO 107 L=1,C
  107  T=T+XC/XS*(BETA*X)**(S+N+L)*C**(C-L)
      POO=1/(R+P+T)
      H1=0
      K=0
  109  POH1=((S*BETA*X)**H1)/FLOAT(IFACT(H1))*POO
      K=K+1
      POH(K)=POH1
      H1=H1+1
      IF(H1.LE.S) GO TO 109
      DO 110 K=1,N
  110  PKS(K)=1/XS*(BETA*X)*(S+K)*POO
      DO 111 L=1,C
  111  PBLS(L)=XC/XS*(BETA*X)**(S+N+L)*C**(C-L)*POO
      A=S*BETA*X
      DO 112 IK=1,C
  112  ADASH(IK)=S*BETA*X*(1-PBLS(IK))
      RHO=X*BETA
      DO 113 IK=1,C
      RODASH(IK)=BETA*X*(1-PBLS(IK))
  113  CONTINUE
      LQ=(BETA*X)**S/(XS*(BETA*X-1)**2)*(N*(BETA*X)**(N+2))
     +  -(N+1)*(BETA*X)**(N+1)+BETA*X)*POO
      DO 114 IK=1,C
      LL(IK)=ADASH(IK)+LQ
      WQ(IK)=LQ/((C*BETA*MU1)*(1-PBLS(IK)))
      W(IK)=WQ(IK)+1/FLOAT(MU2)
      LR(IK)=WQ(IK)*FLOAT(MU2)
  114  CONTINUE
      WRITE(2,117)
      KK=TYPE-2
      IF(KK)151,152,153
  151  WRITE(2,141)
      GO TO 142
  152  WRITE(2,143)
      GO TO 142
  153  WRITE(2,144)
  142  WRITE(2,145)C,S,MU1,MU2,N,BETA
C
C      PRINT SYSTEM EFFECTIVENESS MEASURES
C
      WRITE(2,115)RHO,POO,LQ,A
      WRITE(2,131)(ADASH(I),I=1,C)
      WRITE(2,132)(RODASH(I),I=1,C)
      WRITE(2,133)(WQ(I),I=1,C)
      WRITE(2,134)(W(I),I=1,C)
      WRITE(2,135)(LL(I),I=1,C)
      WRITE(2,136)(LR(I),I=1,C)
      WRITE(2,137)(POH(I),I=1,S+1)
      WRITE(2,138)(PBLS(I),I=1,C)
      WRITE(2,139)(PKS(J),J=1,N)
      GO TO 9
C
  117  FORMAT(//'RESULTS TRUNCATED TOLL BOOTH QUEUEING SYSTEM')
  141  FORMAT(16X,'FOR CARS')
  143  FORMAT(16X,'FOR BUSES')
  144  FORMAT(16X,'FOR TRUCKS')
  145  FORMAT(/'C=',I1,3X,'S=',I1,3X,'MU1=',I5,3X,'MU2=',I5,3X,
     +  'N=',I2,3X,'BETA=',F7.4)
  115  FORMAT(/'RHO=',F9.4/'POO=',F9.4/'LQ  =',F9.4/'A   =',F9.4)
  131  FORMAT('ADASH =',10F9.4)
  132  FORMAT('RODASH=',10F9.4)
  133  FORMAT('WQ    =',10F9.4)
  134  FORMAT('W     =',10F9.4,)
  135  FORMAT('LL    =',10F9.4)
  136  FORMAT('LR    =',10F9.4)
  137  FORMAT('POH   =',10F9.4)
  138  FORMAT('PBLS  =',10F9.4)
```

```
  139  FORMAT('PKS    =',5F9.4)
   10  FORMAT(I1,2I2,2I6,I4,F7.4)
C
C     THIS PROGRAM IS COMPLETELY GENERAL WITH
C     THE FOLLOWING EXCEPTIONS:
C        1.  TYPE MUST NOT BE GREATER THAN 3
C        2.  N MUST NOT BE GREATER THAN  100
C
  999  STOP
       END
C
       FUNCTION IFACT(N)
       IFACT=1
       IF(N.EQ.0.OR.N.EQ.1)GO TO 12
       DO 11 K=2,N
   11  IFACT=IFACT*K
   12  CONTINUE
       END
```

APPENDIX C 2 QUEUE MODEL RESULTS

RESULTS TRUNCATED TOLL BOOTH QUEUEING SYSTEM FOR CARS

C=1 S=1 MU1= 232 MU2= 503 N=20 BETA= .9999
RHO= .4612
POO= .5388
LQ = .3947
A = .4612
ADASH = .4612
RODASH= .4612
WQ = .0017
W = .0037
LL = .8559
LR = .8559
POH = .5388 .2485
PBLS = .0000
PKS = .4970 .7455 .9940 1.2425 1.4910
PKS = 1.7395 1.9879 2.2364 2.4849 2.7334
PKS = 2.9819 3.2304 3.4789 3.7274 3.9759
PKS = 4.2244 4.4729 4.7214 4.9699 5.2184

RESULTS TRUNCATED TOLL BOOTH QUEUEING SYSTEM FOR CARS

C=1 S=1 MU1= 249 MU2= 539 N=20 BETA= .9999

RHO= .4619
POO= .5381
LQ = .3965
A = .4619
ADASH = .4619
RODASH= .4619
WQ = .0016
W = .0034
LL = .8585
LR = .8585
POH = .5381 .2485
PBLS = .0000
PKS = .4971 .7456 .9942 1.2427 1.4913
PKS = 1.7398 1.9884 2.2369 2.4855 2.7340
PKS = 2.9826 3.2311 3.4797 3.7282 3.9768
PKS = 4.2253 4.4739 4.7224 4.9710 5.2195

RESULTS TRUNCATED TOLL BOOTH QUEUEING SYSTEM FOR CARS

C=1 S=1 MU1= 491 MU2= 655 N=20 BETA= .9999

RHO= .7495
POO= .2508
LQ = 2.2038
A = .7495
ADASH = .7492
RODASH= .7492
WQ = .0045
W = .0060
LL = 2.9531
LR = 2.9415
POH = .2508 .1880
PBLS = .0004
PKS = .3760 .5639 .7519 .9399 1.1279
PKS = 1.3158 1.5038 1.6918 1.8798 2.0677
PKS = 2.2557 2.4437 2.6317 2.8196 3.0076
PKS = 3.1956 3.3836 3.5715 3.7595 3.9475

RESULTS TRUNCATED TOLL BOOTH QUEUEING SYSTEM FOR CARS

C=1 S=1 MU1= 1502 MU2= 1516 N=20 BETA= .9999

RHO= .9907
POO= .0481
LQ = 8.8141
A = .9907
ADASH = .9519
RODASH= .9519
WQ = .0061
W = .0068
LL = 9.7660
LR = 9.2596
POH = .0481 .0477
PBLS = .0391
PKS = .0953 .1430 .1906 .2383 .2860
PKS = .3336 .3813 .4289 .4766 .5243
PKS = .5719 .6196 .6672 .7149 .7626
PKS = .8102 .8579 .9056 .9532 1.0009

RESULTS TRUNCATED TOLL BOOTH QUEUEING SYSTEM FOR CARS

C=1 S=1 MU1= 1806 MU2= 1815 N=20 BETA= .9999

RHO= .9949
POO= .0459
LQ = 8.9584
A = .9949
ADASH = .9541
RODASH= .9541
WQ = .0052
W = .0057
LL = 9.9125
LR = 9.3899
POH = .0459 .0457
PBLS = .0411
PKS = .0914 .1371 .1829 .2286 .2743
PKS = .3200 .3657 .4114 .4571 .5029
PKS = .5486 .5943 .6400 .6857 .7314
PKS = .7771 .8229 .8686 .9143 .9600

RESULTS TRUNCATED TOLL BOOTH QUEUEING SYSTEM FOR BUSES

C=1 S=1 MU1= 229 MU2= 815 N=12 BETA= .9999

RHO= .2810
POO= .7190
LQ = .1098
A = .2810
ADASH = .2810
RODASH= .2810
WQ = .0005
W = .0017
LL = .3907
LR = .3907
POH = .7190 .2020
PBLS = .0000
PKS = .4040 .6061 .8081 1.0101 1.2121
PKS = 1.4141 1.6161 1.8182 2.0202 2.2222
PKS = 2.4242 2.6262

RESULTS TRUNCATED TOLL BOOTH QUEUEING SYSTEM FOR BUSES

C=1 S=1 MU1= 729 MU2= 1414 N=12 BETA= .9999

RHO= .5155
POO= .4845
LQ = .5472
A = .5155
ADASH = .5155
RODASH= .5155
WQ = .0008
W = .0015
LL = 1.0627
LR = 1.0616
POH = .4845 .2498
PBLS = .0000
PKS = .4995 .7493 .9991 1.2489 1.4986
PKS = 1.7484 1.9982 2.2479 2.4977 2.7475
PKS = 2.9973 3.2470

RESULTS TRUNCATED TOLL BOOTH QUEUEING SYSTEM FOR BUSES

C=1 S=1 MU1= 1067 MU2= 1501 N=12 BETA= .9999

RHO= .7108
POO= .2909
LQ = 1.6267
A = .7108
ADASH = .7091
RODASH= .7091
WQ = .0015
W = .0022
LL = 2.3358
LR = 2.2942
POH = .2909 .2068
PBLS = .0024
PKS = .4136 .6204 .8272 1.0340 1.2408
PKS = 1.4476 1.6544 1.8612 2.0680 2.2748
PKS = 2.4816 2.6884

RESULTS TRUNCATED TOLL BOOTH QUEUEING SYSTEM FOR BUSES

C=1 S=1 MU1= 1356 MU2= 1575 N=12 BETA= .9999

RHO= .8609
POO= .1556
LQ = 3.3219
A = .8609
ADASH = .8444
RODASH= .8444
WQ = .0025
W = .0031
LL = 4.1663
LR = 3.9339
POH = .1556 .1339
PBLS = .0191
PKS = .2679 .4018 .5357 .6697 .8036
PKS = .9375 1.0714 1.2054 1.3393 1.4732
PKS = 1.6072 1.7411

RESULTS TRUNCATED TOLL BOOTH QUEUEING SYSTEM FOR TRUCKS

C=1 S=1 MU1= 63 MU2= 344 N= 8 BETA= .9999

RHO= .1831
POO= .8169
LQ = .0411
A = .1831
ADASH = .1831
RODASH= .1831
WQ = .0007
W = .0036
LL = .2242
LR = .2242
POH = .8169 .1496
PBLS = .0000
PKS = .2992 .4488 .5984 .7479 .8975
PKS = 1.0471 1.1967 1.3463

RESULTS TRUNCATED TOLL BOOTH QUEUEING SYSTEM FOR TRUCKS

C=1 S=1 MU1= 78 MU2= 361 N= 8 BETA= .9999

RHO= .2160
POO= .7840
LQ = .0595
A = .2160
ADASH = .2160
RODASH= .2160
WQ = .0008
W = .0035
LL = .2756
LR = .2756
POH = .7840 .1694
PBLS = .0000
PKS = .3387 .5081 .6775 .8468 1.0162
PKS = 1.1856 1.3550 1.5243

RESULTS TRUNCATED TOLL BOOTH QUEUEING SYSTEM FOR TRUCKS

C=1 S=1 MU1= 152 MU2= 540 N= 8 BETA= .9999

RHO= .2815
POO= .7185
LQ = .1102
A = .2815
ADASH = .2815
RODASH= .2815
WQ = .0007
W = .0026
LL = .3917
LR = .3916
POH = .7185 .2022
PBLS = .0000
PKS = .4045 .6067 .8090 1.0112 1.2134
PKS = 1.4157 1.6179 1.8201

RESULTS TRUNCATED TOLL BOOTH QUEUEING SYSTEM FOR TRUCKS

C=1 S=1 MU1= 175 MU2= 612 N= 8 BETA= .9999

RHO= .2859
POO= .7141
LQ = .1144
A = .2859
ADASH = .2859
RODASH= .2859
WQ = .0007
W = .0023
LL = .4004
LR = .4003
POH = .7141 .2042
PBLS = .0000
PKS = .4083 .6125 .8167 1.0208 1.2250
PKS = 1.4292 1.6334 1.8375

RESULTS TRUNCATED TOLL BOOTH QUEUEING SYSTEM FOR TRUCKS

C=1 S=1 MU1= 192 MU2= 650 N= 8 BETA= .9999

RHO= .2954
POO= .7046
LQ = .1238
A = .2954
ADASH = .2954
RODASH= .2954
WQ = .0006
W = .0022
LL = .4191
LR = .4190
POH = .7046 .2081
PBLS = .0000
PKS = .4162 .6244 .8325 1.0406 1.2487
PKS = 1.4568 1.6650 1.8731

RESULTS TRUNCATED TOLL BOOTH QUEUEING SYSTEM FOR TRUCKS

C=1 S=1 MU1= 216 MU2= 717 N= 8 BETA= .9999

RHO= .3012
POO= .6988
LQ = .1298
A = .3012
ADASH = .3012
RODASH= .3012
WQ = .0006
W = .0020
LL = .4310
LR = .4309
POH = .6988 .2105
PBLS = .0000
PKS = .4210 .6315 .8420 1.0524 1.2629
PKS = 1.4734 1.6839 1.8944

RESULTS TRUNCATED TOLL BOOTH QUEUEING SYSTEM FOR CARS

C=1 S=1 MU1= 1806 MU2= 1815 N=60 BETA= .9999
RHO= .9949
POO= .0185
LQ = 27.5206
A = .9949
ADASH = .9815
RODASH= .9815
WQ = .0154
W = .0160
LL = 28.5021
LR = 28.0392
POH = .0185 .0184
PBLS = .0135
PKS = .0368 .0552 .0736 .0920 .1104
PKS = .1288 .1472 .1656 .1840 .2024
PKS = .2208 .2392 .2576 .2760 .2944
PKS = .3128 .3313 .3497 .3681 .3865
PKS = .4049 .4233 .4417 .4601 .4785
PKS = .4969 .5153 .5337 .5521 .5705
PKS = .5889 .6073 .6257 .6441 .6625
PKS = .6809 .6993 .7177 .7361 .7545
PKS = .7729 .7913 .8097 .8281 .8465
PKS = .8649 .8833 .9017 .9201 .9385
PKS = .9569 .9753 .9938 1.0122 1.0306
PKS = 1.0490 1.0674 1.0858 1.1042 1.1226

RESULTS TRUNCATED TOLL BOOTH QUEUEING SYSTEM FOR CARS

C=1 S=1 MU1= 1806 MU2= 1815 N=20 BETA= .9999
RHO= .9949
POO= .0459
LQ = 8.9584
A = .9949
ADASH = .9541
RODASH= .9541
WQ = .0052
W = .0057
LL = 9.9125
LR = 9.3899
POH = .0459 .0457
PBLS = .0411
PKS = .0914 .1371 .1829 .2286 .2743
PKS = .3200 .3657 .4114 .4571 .5029
PKS = .5486 .5943 .6400 .6857 .7314
PKS = .7771 .8229 .8686 .9143 .9600

RESULTS TRUNCATED TOLL BOOTH QUEUEING SYSTEM FOR CARS

C=1 S=1 MU1= 1806 MU2= 1815 N=10 BETA= .9999
RHO= .9949
POO= .0793
LQ = 4.1893
A = .9949
ADASH = .9207
RODASH= .9207
WQ = .0025
W = .0031
LL = 5.1100
LR = 4.5501
POH = .0793 .0789
PBLS = .0746
PKS = .1578 .2367 .3155 .3944 .4733
PKS = .5522 .6311 .7100 .7888 .8677

RESULTS TRUNCATED TOLL BOOTH QUEUEING SYSTEM FOR CARS

C=1 S=1 MU1= 1806 MU2= 1815 N= 8 BETA= .9999
RHO= .9949
POO= .0932
LQ = 3.2450
A = .9949
ADASH = .9068
RODASH= .9068
WQ = .0020
W = .0025
LL = 4.1517
LR = 3.5786
POH = .0932 .0928
PBLS = .0886
PKS = .1855 .2783 .3710 .4638 .5566
PKS = .6493 .7421 .8348

RESULTS TRUNCATED TOLL BOOTH QUEUEING SYSTEM FOR CARS

C=1 S=1 MU1= 1806 MU2= 1815 N=20 BETA= .9950

RHO= .9901
POO= .0484
LQ = 8.7933
A = .9901
ADASH = .9516
RODASH= .9516
WQ = .0051
W = .0056
LL = 9.7448
LR = 9.2407
POH = .0484 .0479
PBLS = .0389
PKS = .0959 .1438 .1918 .2397 .2876
PKS = .3356 .3835 .4314 .4794 .5273
PKS = .5753 .6232 .6711 .7191 .7670
PKS = .8149 .8629 .9108 .9588 1.0067

RESULTS TRUNCATED TOLL BOOTH QUEUEING SYSTEM FOR CARS

C=1 S=1 MU1= 1806 MU2= 1815 N=20 BETA= .7500

RHO= .7463
POO= .2540
LQ = 2.1594
A = .7463
ADASH = .7460
RODASH= .7460
WQ = .0016
W = .0021
LL = 2.9054
LR = 2.8947
POH = .2540 .1896
PBLS = .0004
PKS = .3791 .5687 .7583 .9479 1.1374
PKS = 1.3270 1.5166 1.7061 1.8957 2.0853
PKS = 2.2749 2.4644 2.6540 2.8436 3.0332
PKS = 3.2227 3.4123 3.6019 3.7914 3.9810

RESULTS TRUNCATED TOLL BOOTH QUEUEING SYSTEM FOR CARS

C=1 S=2 MU1= 1806 MU2= 1815 N=20 BETA= .9999

RHO= .4975
POO= .3356
LQ = .3272
A = .9949
ADASH = .9949
RODASH= .4975
WQ = .0002
W = .0007
LL = 1.3221
LR = .3289
POH = .3356 .3339 .1661
PBLS = .0000
PKS = 1.0017 1.3356 1.6694 2.0033 2.3372
PKS = 2.6711 3.0050 3.3389 3.6728 4.0067
PKS = 4.3405 4.6744 5.0083 5.3422 5.6761
PKS = 6.0100 6.3439 6.6778 7.0116 7.3455

RESULTS TRUNCATED TOLL BOOTH QUEUEING SYSTEM FOR CARS

C=1 S=3 MU1= 1806 MU2= 1815 N=20 BETA= .9999

RHO= .3316
POO= .3656
LQ = .0446
A = .9949
ADASH = .9949
RODASH= .3316
WQ = .0000
W = .0006
LL = 1.0395
LR = .0448
POH = .3656 .3637 .1809 .0600
PBLS = .0000
PKS = 2.1823 2.7279 3.2734 3.8190 4.3646
PKS = 4.9101 5.4557 6.0013 6.5469 7.0924
PKS = 7.6380 8.1836 8.7292 9.2747 9.8203
PKS = 10.3659 10.9114 11.4570 12.0026 12.5482

RESULTS TRUNCATED TOLL BOOTH QUEUEING SYSTEM FOR CARS

C=2 S=1 MU1= 1806 MU2= 1815 N=20 BETA= .9999

RHO= 1.9899
POO= .0000
LQ = 6.3836
A = 1.9899
ADASH = 1.3277 1.3311
RODASH= 1.3277 1.3311
WQ = .0026 .0026
W = .0032 .0032
LL = 7.7114 7.7147
LR = 4.8079 4.7958
POH = .0000 .0000
PBLS = .3328 .3311
PKS = .0000 .0000 .0000 .0000 .0000
PKS = .0000 .0000 .0000 .0000 .0000
PKS = .0000 .0000 .0000 .0000 .0000
PKS = .0000 .0000 .0000 .0000 .0000

RESULTS TRUNCATED TOLL BOOTH QUEUEING SYSTEM FOR CARS

C=2 S=2 MU1= 1806 MU2= 1815 N=20 BETA= .9999

RHO= .9949
POO= .0221
LQ = 8.5847
A = 1.9899
ADASH = 1.9115 1.9509
RODASH= .9558 .9754
WQ = .0025 .0024
W = .0030 .0030
LL = 10.4962 10.5356
LR = 4.4910 4.4004
POH = .0221 .0440 .0438
PBLS = .0394 .0196
PKS = .1321 .1761 .2201 .2642 .3082
PKS = .3522 .3963 .4403 .4843 .5284
PKS = .5724 .6164 .6604 .7045 .7485
PKS = .7925 .8366 .8806 .9246 .9687

*RESULTS TRUNCATED TOLL BOOTH QUEUEING SYSTEM FOR CARS

C=2 S=3 MU1= 1806 MU2= 1815 N=20 BETA= .9999

RHO= .6633
POO= .1127
LQ = .8644
A = 1.9899
ADASH = 1.9898 1.9899
RODASH= .6633 .6633
WQ = .0002 .0002
W = .0008 .0008
LL = 2.8543 2.8543
LR = .4344 .4344
POH = .1127 .2244 .2232 .1481
PBLS = .0000 .0000
PKS = 1.3461 1.6826 2.0192 2.3557 2.6922
PKS = 3.0288 3.3653 3.7018 4.0383 4.3749
PKS = 4.7114 5.0479 5.3844 5.7210 6.0575
PKS = 6.3940 6.7306 7.0671 7.4036 7.7401

RESULTS TRUNCATED TOLL BOOTH QUEUEING SYSTEM FOR CARS

C=3 S=4 MU1= 1806 MU2= 1815 N=20 BETA= .9999

RHO= .7462
POO= .0386
LQ = 1.4539
A = 2.9848
ADASH = 2.9832 2.9844 2.9847
RODASH= .7458 .7461 .7462
WQ = .0003 .0003 .0003
W = .0008 .0008 .0008
LL = 4.4371 4.4384 4.4387
LR = .4874 .4872 .4871
POH = .0386 .1153 .1720 .1712 .1277
PBLS = .0005 .0001 .0000
PKS = 1.5370 1.8444 2.1518 2.4592 2.7666
PKS = 3.0740 3.3814 3.6888 3.9962 4.3036
PKS = 4.6110 4.9184 5.2258 5.5331 5.8405
PKS = 6.1479 6.4553 6.7627 7.0701 7.3775

APPENDIX C 3 QUEUE MODEL RESULTS FOR VARIED PARAMETERS

RESULTS TRUNCATED TOLL BOOTH QUEUEING SYSTEM
FOR CARS

C=1 S=1 MU1= 100 MU2= 500 N=20 BETA= .9999

RHO= .2000
POO= .8000
LQ = .0500
A = .2000
ADAS= .2000
RODASH= .2000
WQ = .0005
W = .0025
LL = .2500
LR = .2500
POH = .8000 .1600
PBLS = .0000
PKS = .3200 .4800 .6400 .7999 .9599
PKS = 1.1199 1.2799 1.4399 1.5999 1.7599
PKS = 1.9199 2.0798 2.2398 2.3998 2.5598
PKS = 2.7198 2.8798 3.0398 3.1998 3.3597

RESULTS TRUNCATED TOLL BOOTH QUEUEING SYSTEM
FOR CARS

C=1 S=1 MU1= 200 MU2= 400 N=20 BETA= .9999

RHO= .4999
POO= .5001
LQ = .4998
A = .4999
ADASH = .4999
RODASH= .4999
WQ = .0025
W = .0050
LL = .9998
LR = .9998
POH = .5001 .2500
PBLS = .0000
PKS = .5000 .7500 1.0000 1.2500 1.5000
PKS = 1.7500 2.0000 2.2500 2.5000 2.7500
PKS = 3.0000 3.2500 3.5000 3.7500 4.0000
PKS = 4.2500 4.5000 4.7500 5.0000 5.2500

RESULTS TRUNCATED TOLL BOOTH QUEUEING SYSTEM
FOR CARS

C=1 S=1 MU1= 400 MU2= 600 N=20 BETA= .9999

RHO= .6666
POO= .3334
LQ = 1.3299
A = .6666
ADASH = .6666
RODASH= .6666
WQ = .0033
W = .0050
LL = 1.9964
LR = 1.9951
POH = .3334 .2223
PBLS = .0000
PKS = .4445 .6668 .8891 1.1113 1.3336
PKS = 1.5558 1.7781 2.0004 2.2226 2.4449
PKS = 2.6672 2.8894 3.1117 3.3340 3.5562
PKS = 3.7785 4.0008 4.2230 4.4453 4.6675

RESULTS TRUNCATED TOLL BOOTH QUEUEING SYSTEM
FOR CARS

C=1 S=1 MU1= 500 MU2= 600 N=20 BETA= .9999

RHO= .8332
POO= .1693
LQ = 3.7504
A = .8332
ADASH = .8307
RODASH= .8307
WQ = .0075
W = .0092
LL = 4.5811
LR = 4.5147
POH = .1693 .1411
PBLS = .0031
PKS = .2821 .4232 .5643 .7053 .8464
PKS = .9875 1.1286 1.2696 1.4107 1.5518
PKS = 1.6928 1.8339 1.9750 2.1160 2.2571
PKS = 2.3982 2.5392 2.6803 2.8214 2.9624

RESULTS TRUNCATED TOLL BOOTH QUEUEING SYSTEM
FOR CARS

C=1 S=1 MU1= 620 MU2= 650 N=20 BETA= .9999

RHO= .9538
POO= .0697
LQ = 7.5098
A = .9538
ADASH = .9303
RODASH= .9303
WQ = .0124
W = .0140
LL = 8.4401
LR = 8.0726
POH = .0697 .0665
PBLS = .0246
PKS = .1330 .1994 .2659 .3324 .3989
PKS = .4654 .5319 .5983 .6648 .7313
PKS = .7978 .8643 .9308 .9972 1.0637
PKS = 1.1302 1.1967 1.2632 1.3297 1.3961