RELIABILITY ANALYSIS OF A PRESTRESSED CONCRETE BEAM

BY

Hadiza Ibrahim HAMIDU

DEPARTMENT OF CIVIL ENGINEERING,

FACULTY OF ENGINEERING

AHMADU BELLO UNIVERSITY,

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Hadiza Ibrahim HAMIDU,

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DECLARATION

I hereby declare that this thesis is a product of my effort under the supervision of Dr.I.Abubakar and Prof. S.P Ejeh. It has not been presented in any previous application for a higher degree. All quotations and their sources are specifically acknowledged by means of references

Hadiza IbrahimHAMIDU **_____**

(M.Sc./ENG/10240/2008-09) **SIGNATURE**

CERTIFICATION

The thesis titled **Reliability Analysis of aPrestressed Concrete Beam**by **HadizaIbrahim HAMIDU**meets the regulations governing the award of the degree

of Master of Science in Civil Engineering, Department of Civil Engineering, Ahmadu Bello University, Zaria and is approved for its contribution to knowledge and literary presentation.

_____ _____

Dr. I. Abubakar **Date**

Chairman, Supervisory Committee

_____ _____

Prof. S. P. Ejeh **Date**

Member, Supervisory Committee

_____ _____

Dr. Y. D.Amartey **Date**

Head of Department

_____ _____

Prof. A. Z. Hassan **Date**

Dean, School of Postgraduate Studies

DEDICATION

This thesis is dedicated to my little angels Michelle M. Ali and Ibrahim O. Ali.

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I acknowledge God Almighty for all His infinite mercies and His kind correction, His grace over my life, to Him is the glory and honour forevermore.

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strength reduction factor

β Safety index

β_1 tension stiffening constant

β_2 tension stiffening constant

λ ratio of actual moment

σ_{pu} stress in the prestressing steel

σ_R standard deviation of resistance variables

σ_S standard deviations of load variables

Φ standard normal distribution function.

γ ratio of the depth of the idealized rectangular compressive stress block tothe depth of the neutral axis at the ultimate strength in bending

or combined bending and compression

$\gamma_p$environmental constant.

$\varkappa_{sh}^{*}$curvature induced by shrinkage

A_{pt} area of prestressed steel

A_{sc}area of compressive reinforcement

A_{st} area of reinforcement steel

bwidth of the compressive zone of the cross section of the Prestressed beam

$C_c$compressive force carried by the concrete

$C_s$compressive force in the non-prestressed steel

d_n, depth from the extreme compressive fiber to the neutral axis

$d_o$depth to the bottom layer of tensile reinforcement

$d_p$depth to prestressed steel

E section elastic modulus

$f_p$ultimate strength of the prestressing steel

$f_x$joint density function

$f_y$yield stress of non-prestressed steel.

f_c'characteristic compressive strength of concrete at 28 days

G safety margin,

G_k dead load

I second moment of area about the centriod axis

Lspan

$l_c$lever arm distance of compressive force in concrete above the non-prestressed tensile steel.

$l_p$lever arm distance of the tension force in prestressed steel above the non-prestressed tensile steel.

$l_s$lever arm distance of compressive forces in the steel above the nonprestressed tensile steel

M* applied bending moment

M_U ultimate bending moment

*N*number of iterations

P prestressing force

$P_e$effectiveprestressing force after time-dependent losses

P_f probability of failure

$P_V$vertical component of prestressing force

Q_k imposed load

R variable associated with resistance

S variable associated with loading

T_p tension in the prestressed steel.

$\mu_R$mean values of the resistance variables

$\mu_S$mean value of load variables

V corresponding coefficient ofvariation.

V* factored design shear force for the strength limit state

$\nu_i$deflection at transfer

ν_{sh}deflection due shrinkage

V_{uc}contribution of concrete to the shear strength

Wuniformly distributed load

w uniformly distributed load on the beam

w_b transverse load exerted on the beam by the tendons

$w_G$maximum gravity load

w_{sus} sustained gravity load

X basic random variables

Z_b bottom section modulus

ABSTRACT

The Reliability Analysis of a Prestressed Concrete Beam (PCB) was presented using First Order Reliability Method and Eurocode 2 procedures to carry out the analysis. The results show that the safety of the PCBin bending decreasedfrom 2.9 to 1.0 and 3.1 to 2.6 as prestress force and the depth from the extreme compressive fiber to the neutral axis of the beam increased from 20kN to 100kN and 150mm to 350mm respectively, therefore the PCB is safer at low prestress force and depth to the bottom layer of the beam. The safety of the PCB increased from 1.52 to 2.5, 1.52 to 2.5 and 0.1 to 2.0 respectively as depth to the neutral axis, area of the compressive reinforcement and eccentricity increased from 150mm to 350mm, 800mm^2 to 1200mm^2 and 100mm to 300mm respectively. The safety of the prestressed concrete beam remained constant at 0.1 with eccentricity of 100mm, 0.8 with eccentricity of 150mm, 1.3 with eccentricity of 200mm, 1.7 with eccentricity of 250mm and 1.95 with eccentricity of 300mmas effective widths and load ratios of the beam increased from 100mm to 300mm and 0.2 to 1.0 respectively in shear. Increase in dead load and span resulted to a corresponding increase in the safety of the beam in deflection. The best value of eccentricity for a reliable prestressed beam is within the range of 200mm to 300mm. It was also observed that the variation of depth to the bottom layer and prestressing force of the prestressed beam in bending are at equilibrium at 60kN and the safety index is 1.83. Also, the target safety indices considering bending, shear and deflection failure criteria of the prestressed beam were obtained to be 2.01, 1.526 and 4.716 respectively.

CHAPTER ONE

INTRODUCTION

1.1 PREAMBLE

Prestressed concrete beam is one of the most widely used construction element in bridge projects around the world. It has rent itself to an enormous array of structural applications, including buildings, bridges, nuclear power vessels, Television towers and offshore drilling platform (Antonie,2004).

Prestressing involves inducing compressive stresses in the zone which will tend to become tensile under external loads. Thesecompressive stresses neutralize the tensile stresses so that no resultant tension exists (or in only very small values). Cracking is therefore eliminated under working load and all of the concrete may be assumed effective in carrying load. Therefore lighter sections may be used to carry a given bending moment and over much longer spans than reinforced concrete.

The prestressing force also reduces the magnitude of principal tensile stress in the web so that thin webbed I sections may be used without the risk of diagonal tension failures and with further savings in self weight.

The prestressing force has to be produced by high tensile steel and it is necessary to use high quality concrete to resist the higher compressive stresses that are developed(Mosley and Bungey, 2002).

In the design and construction of the prestressed concrete, there are many sources of uncertainties which are put into consideration during the construction in order for the structure to serve its intended purpose. Consequently, structures must be designed to serve their functions with a finite probability of failure (Nowak and Colins, 2002).

In order to treat these uncertainties in the construction processes of the prestressed concrete, reliability analysis comes into play. Reliability is the probability that the structure will perform its required function without failure under a specified limit state during a specified reference period (Thoft-Christensen and Baker, 1982). The probability of structural failure from all possible causes is inevitable. However, the uncertainties and their significance on the structural safety and performance can be analyzed systematically only through methods of probability.

The development of structural reliability methods during the last three to four decades have provided a more rational basis for the design of structures in the sense that the method facilitate a consistent basis of comparison between

the reliability of a well-tested structural design and the reliability of new types of structures. For this reason the methods of structural reliability have been applied increasingly in connection with the development of new design codes (Sorenson et,al., 2002). In recent years design codes have been continuously revised to include limit states based on probabilistic methods. In fact the limit state design approach has been used nearly in all the recent advances in codified design (Ellingwood, 1996). The use of structural reliability methods for design can lead to structures that have a more consistent level of risk (Zimmerman et, al., 1992). Therefore, to account for the system effect in structural assessment and design, the safety importance of structural members must be quantified (Gharaibeh, et, al., 2001)

1.2 PROBLEM STATEMENT AND JUSTIFICATION OF STUDY

The way in which civil engineering systems fail, the occurrence and frequency of failure, its economics and social consequences, demonstrate considerable differences between hypothetical and actual systems. Induced loading, site characterization, material properties, developed formulations and procedures and adequacy of predicted sizes and shapes of the system and its elements are far from certain. All these factors are subject to complex inter relationships, materials defects, structural deficiencies, human errors and hence to varying degrees of randomness. Reliability analysis of structures is required

as a result of these problems. Failure of a system is assessed by its inability to perform its intended function adequately on demand for a period of time and under specified condition, while its antithesis, the measure of success, is called reliability (Milton 1987).

1.3 AIM AND OBJECTIVES

1.3.1 Aim

The aim of this research is to carry out a reliability analysisof a prestressed concrete beam using First Order Reliability Method (FORM) and Eurocode 2 (2004) design procedure.

1.3.2 Objectives

The objectives of this research are to:

- identify the various modes of failure due to ultimate limit state of a prestressed concrete beam.
- compute the probabilities of failure associated with the various modes of failure of theprestressed concrete beam.
- determine the implied safety indices related to the probabilities of failure.
- compute the mean safety index for each mode of failure.
- establish a target safety indices associated with the modes of failure of the Prestressed concrete beam.

- design a typical Prestressed concrete beam to meet a target safety level.

1.4 SCOPE OF RESEARCH

The scope of this research was limited to the reliability analysis of a simply supported Prestressed concrete beam in accordance with the recommendation of Eurocode 2 (2004).

CHAPTER TWO

LITERATURE REVIEW

2.1 Prestressed Concrete

Reinforced concrete was the most widely used materials before the introduction of prestressed concrete. Because the tensile strength of concrete is low, steel bars are embedded in the concrete to carry all internal tensile forces. These tensile forces could be as a result of the applied imposed loads or deformations, or by load independent effects such as temperature changes or shrinkage. As a result of this, the low tensional resistance of the concrete cracks in tension is ignored in design (Mosley and Bungey, 2002).

Although the steel reinforcement provides the cracked concrete beam with flexural strength, it does not prevent cracking and does not prevent the loss of stiffness caused by cracking. Crack widths are approximately proportional to the strain and hence stress, in the reinforcement. Steel stresses must therefore be limited to some appropriately low value in order to avoid excessively wide cracks. Similarly, large steel strain is the result of large curvature, which in turn is associated with large deflection (Gilbert andMickleborough, 1990).

Prestressed concrete is a particular form of reinforced concrete. Prestressing involves the application of an initial compressive load on a structure to reduce or eliminate the internal tensile forces and thereby control or eliminate cracking. The initial compressive load is imposed and sustained by highly tensional steel reinforcement in the concrete. With cracking reduced or eliminated, a prestressed section is considerably stiffer than the equivalent reinforced section. With service load behavior improved, the use of high strength steel reinforcement and high-strength concrete become both economical and structurally efficient (Kong and Evans, 1999).

2.2 METHOD OF PRESTRESSING

There are two basic techniques commonly employed in the construction of prestressed concrete, their main differences being whether the steel tensioning process is performed before or after the hardening of concrete.

2.2.1 Pre-tensioning Prestressed Concrete

In this method of prestressing, steel wires or strands are stretched to the required tension and anchored to the ends of the mould for the concrete. The concrete is cast around the tensioned steel, and when it has reached sufficient strength the anchors are released and the force in the steel is transferred to the bond.

2.2.2 Post-tensioning Prestressed Concrete

This method of prestressing involves the stressing against the hardened concrete of tendons or steel bars which are not bonded to the concrete. The tendons are passed through a flexible sheathing, which is cast into the concrete in the correct position. They are tensioned by jacking against the concrete, and anchored mechanically by means of steel thrust plates or anchorage blocks at each end of the member.

2.3 ADVANTAGES OF PRESTRESSED CONCRETE OVER REINFORCED CONCRETE

According to Kong and Evans (1999), prestressed concrete is advantageous over reinforced concrete in the following ways:

i. The entire concrete section is effective in resisting the applied moment, whereas only the portion of the section above the neutral axis is fully effective in reinforced concrete, this lead to greatly reduced deflection under service conditions.

ii. The use of curved tendon profiles (in post-tensioning) enables part of the shear force to be carried by the tendon.

iii. The precompression in the concrete tends to reduce the diagonal tension.

iv. The same applied load can be carried by a lighter section in prestressedconcrete; this yield more clearance where it is required

v. Enable longer spans to be used.

vi. The absence of cracks under service loading.

2.4 LIMIT STATE DESIGN

A structure and its components must simultaneously satisfy a number of different limit states or design requirements. It must possess adequate strength, be stable, and perform satisfactorily under service loads. If a structure becomes unfit for service in any way, it is said to have entered a limit state. Limit state are the undesirable consequences associated with each possible mode of failure (Gilbert and Mickleborough, 1990).

The purpose of the limit state design is to achieve acceptable probabilities that the prestressed beam will not become unfit for its intended use. The two type of limit state are ultimate limit state and serviceability limit state.

Ultimate limit state requires that, the structure must be able to withstand, an adequate factor of safety against collapse, the loads for which it is designed.

Serviceability limit state ensures that the structure behaves satisfactorily and can perform its intended functions at service load. Deflection must not be

excessive, cracks must be adequately controlled, and no portion of the structure should suffer excessive vibration.

In the course of this research, only failures due to ultimate limit state will be considered.

2.5 STRUCTURAL RELIABILITY

Structural reliability has become a discipline of international interest, addressing issues, such as the safety of the structures. The reliability basis for designing the range of structures, structural materials, safety and functional requirements and the actions to which these facilities are exposed form a vital framework for a coherent suite of structural design standard (Holicky and Retief, 2005).

Structural reliability is the ability of a structure to fulfill its design purpose for some specified time; in order words it is the probability that a structure will not attain each specified limit state during a specified reference period (Thoft-Christensen and Baker, 1992). Structural reliability analysis is concerned with the rational treatment of uncertainties (Melchers, 1999).

According to Melchers (1999), the study of structural reliability is concerned with the calculation and prediction of the probability of limit state violation for engineered structures at any stage during its life. In particular, the

study of structural safety is concerned with the violation of the ultimate or limit states for the structures.

The probability of occurrence of an event such as limit state violation is a numerical measure of the chance of its occurring. This measure mayeither be obtained from measurements of long-term frequency of occurrence of the event for generally similar structures, or may be simply a subjective estimation of the numerical value. However in practice, it is not usually possible to observe for a sufficiently long period of time. So a combination of subjective estimation and frequency observations for structural components and properties is used to predict the probability of limit state violation for the structure as a whole.According to Yang and Dewolf (2002) the risk of structural failure can never be totally eliminated and that the goal in design is to define safety margins so that the risk of failure is small. It is as a result of this desire to minimize the risk that the over conservatism of the deterministic method which most researchers have complained about has become an issue (Vederaine 1994).

In practice, most of the problems encountered are as a result of uncertainties in various stages of construction. The stages of construction include planning, design, construction and maintenance. Each of these stages involves various uncertainties which should be considered when making decisions in the process

of engineering projects,(Moris and Ellingwood, 1993). The effect of such uncertainties is included in design through the use of safety factors which are based on engineering judgments and past experience with similar structures.Recognizing that safety involves a consideration of random variables, it is generally accepted that the rational approach to the analysis of safety factor is through the use of probabilistic methods (Moris and Ellingwood, 1993).

According to JCSS (2001), failure is a probabilistic event and its probability of occurrence, P_f, is given by:

$$P_f = \text{Prob}\{g(X) \leq 0\} = \text{Prob}\{G \leq 0\} \quad (2.1)$$

Where X is a basic random variables

G is a random variable called the safety margin.

If the limit state function is expressed as

$$r(r) - s(s) = 0 \quad (2.2)$$

Then equation (1) can be written as

$$P_f = \text{Prob}\{r(R) \leq s(S)\} = \text{Prob}\{R \leq S\} \quad (2.3)$$

Where R = r(r) and S = s(s) are random variables associated with resistance and loading respectively.

2.5.1 Methods of Structural Reliability Analysis

The regular methods by which analysis of structural reliability could be undertaken are classified broadly into two (Thoft-Christensen and Baker, 1982):

Level 3: Methods in which calculations are made to determine the exact probability of failure for a structure or structural component, making use of a full probabilistic description of the joint occurrence of various quantities which affect the response of the structure and taking into account the true nature of the failure domain.

Level 2: Methods involving certain approximate iterative calculations procedures to obtain an approximation of the failure probability of a structure or structural system generally requiring an idealization of the failure domain often associated with a simplified representation of joint probability distribution of variables.

These methods can be used for checking the safety of a design or used directly, provided a target reliability or reliability index has been specified (Ocholi,2000).

2.5.2 Reliability-Based Design

The methods being developed for analyzing uncertainties systematically, where models are formulated for evaluating risk associated with various design are derived from the design criteria which may be formulated most rationally within probabilistic framework (Ellingwood and Ang ,1974).

The probabilistic design is concerned with the probability that the structure will realize the functions assigned to it. The major advantage of reliability based-design is in the elimination of deterministic design restrictions on the individual members making up a structure.(Afolayan and Abubakar, 2003).

2.6 PREVIOUS WORKS ON RELIABILITY OF STRUCTURES

The research on System Reliability Models for Bridge Structures was conducted by Nowak(2004), and the reliabilityindex for bridge girders was determined. The reliability index was compared with the reliabilityindices determined for individual girders. The statistical parameters of component resistance for various types of structures are as shown in Table 2.1

TYPES OF STRUCTURES	λ	V
Non-composite steel and girders		
Moment	1.12	0.10
Shear	1.14	0.105
Composite steel girders		
Moment	1.12	0.10
Shear	1.14	0.105
Reinforced concrete T-beams		
Moment	1.14	0.13
Shear	1.20	0.155
Prestressed concrete girders		
Moment	1.05	0.075
Shear	1.15	0.14

Table 2.1 – Bias Factor λ &V For Bridge Structures (Nowak 2004).

Statistical Parameters of Component Resistance

In Table 2.1,$\boldsymbol{\lambda}$ is the bias factor (ratio of actual moment) and $\boldsymbol{V}$ is the corresponding coefficient ofvariation.

Nowak (2004) carried out a research on comparison of bridge system reliability index and girder reliability index and concluded that that ratio of system reliability/girder reliabilitydecreased with increase in girder reliability.

Biondini (2003) carried out a research on reliability analysis of framed structures understatic loads using Monte Carlo Simulation. The work revolved how the number of iterations *(N)* used in the Monte Carlo simulations affected theoverall reliability.

The work showed that the number of iterations or sampling size plays an integral part in obtaining a realistic reliability value and the values of reliability progressively increases as the limit state of collapse moves towards the tails of probability distributions.

Hsin and Lance (2004) carried out work on reliability-based design of steel bridges under Fatigue Loading by determining the fatique reliability of steel bridges over their service life. The study showed that the reliability index of the steel bridges decreased with the life span of the bridge as shown in Figure 2.1.

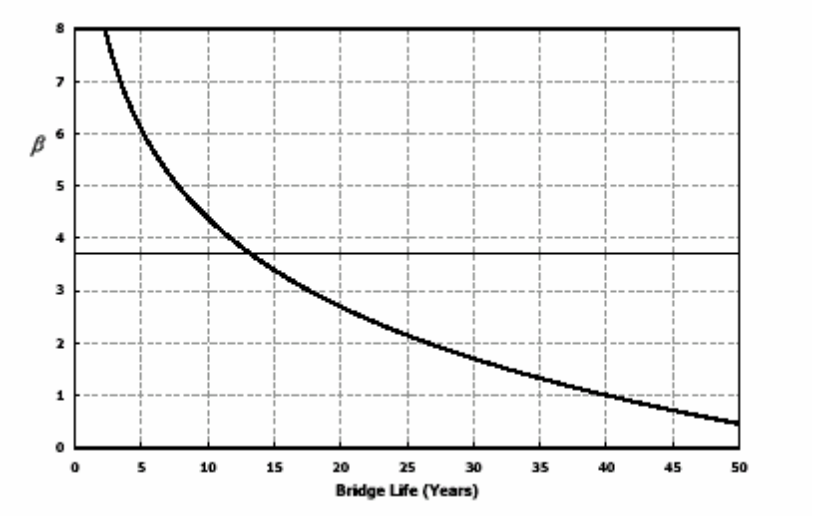

Figure 2.1 – Fatigue Reliability of Bridges in 50 years (Hsin and Lance,2004)

Chan et al.,(2000), showed that the load and resistance factors given in the deterministic codes are basically conservative. The values can be reduced to obtain a cost effective bridge. The results obtained are as depicted in Figure 2.2

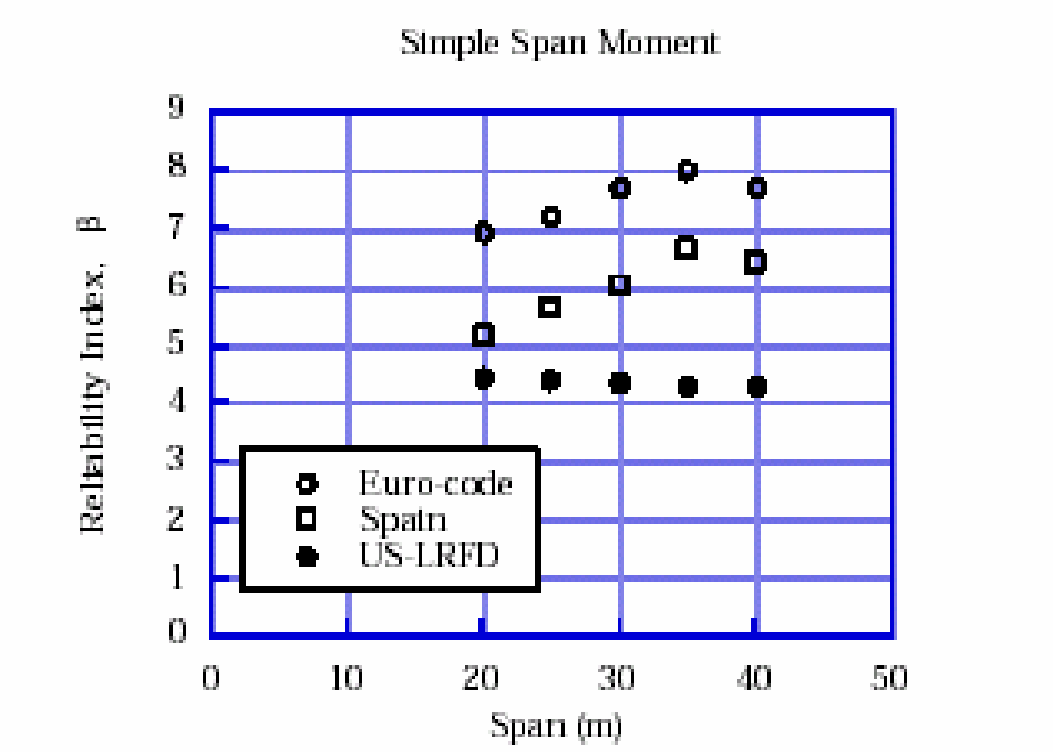

Figure 2.2: Reliability Indexes For Various Code of Practice.(Chan and Nowak, 2000)

Sigitand Mark (2007), considered the development of probabilistic models to predict the spatial distribution of maximum depths of pitting for prestressing strands subjected to pitting corrosion. He concluded that the spatial variability of pitting corrosion in the reliability analysis of a pretensionedprestressed concrete AASHTO bridge girder increased the probability of strength failure by only 10% when compared with a mid-section analysis. However, the effect of the spatial variability of pitting corrosion is more significant on the serviceability limit state. It was also shown that approximately 45% of

collapses have not been preceded by excessive deflection. It was noted that the high variability of predictions of deterioration processes also significantly affects the structural reliability of a corroding structure. The probabilistic approach developed in the study also allowed for a more realistic representation of service life prediction.

Nowak et al. (2000), carried out a research on reliability analysis of prestressed concrete bridge girders: comparison of Eurocode, Spanish Norma IAP andAASHTO LRFD.They were able to come to a conclusion that, the Eurocode is more conservativethan the other two codes, while AASHTO LRFD is the most permissive code.

Guna (2005) carried out a research on reliability analysis of prestressed concrete bridge girder beam. The authorconductedan investigation approach to formulatethe various load factor models from various codes which can be used to analyze maximum mid-span moment effects on a standard prestressed bridge girder beam.The study also assessed the reliability of a standard bridge girder based on the load models and load factors given by various codes and specifications namely: JKR, BS 5400 Part 2, AASHTO, eurocode, austroads,Australian Highway Code, and Canadian Code (OHBDC). Assuming the maximum mid-span moment was to govern the highway bridge design, it was foundout that the load factorsgiven in the codes can be reduced if

reliability method is utilized. It was also found out that AASHTO and austroads are the most conservative codes for all bridge spans. BS5400 Part 2 and eurocode are the most permissive codes for all bridge spans.

Hosseinnezhadet, al. (2007), carried out a research workon the application of the first- order second – moment level 2 reliability of a prestressed concrete bridge. It was concluded that:

a. Variation in the threshold level has a significant effect on the reliability index β.

b. Sensitivity of the reliability index,β to the variation of the random variables in high threshold level is much more than that of low threshold level.

c. The structural damping ratio has a significant effect on β. The reliability index increases with increase in value of the structural damping ratio.

d. The reliability index β decreases with increase in value of the coefficient of variation (COV) of ultimate moment**.**

Prabir, et.al, (2007), carried out a research on Reliability of Prestressed Concrete Containment Structure Against Internal Pressure. The following findings were drawn:

a. A simple methodology was developed for reliability assessment of prestressed concrete containment structure against internal pressure loading. Both

epistemic and aleatory uncertainties of basic variables are treated separately in the method.

b. Failure probability of a prestressed concrete containment structure against pressure loading is higher than that of RCC containment structure.
c. COV for the ultimate strength of prestressing cable has significant bearing on the failure probability of prestressed concrete containment structure. Influence of co-efficient of variations of other basic variables is generally insignificant, so also that of probability density function of pressure load.
d. The results indicate that it is very important to rationally assess the prestressing loss for safety of prestressedconcrete containment structure during the entire span of its operating life.

Jinsheng andXila (2002) carried out a research onunbondedprestressed concretestructural reliability. It was concluded thatunbondedprestressing (including inside and outside of the member) is widely adopted in bridge engineering. The main difficulty in the analysis of unbondedprestressedconcrete members is the coupling behaviour of flexural strength and structural deformation. By use of unbondedprestressing tendons, the numbers of possible failure sections in structure are increased, and the reliability of structure is affected. According to the theory of extreme values of independent and identically distributed random variables, the reliability of

unbondedprestressed concrete members in bending was analyzed. It was shown that decrease in reliability exists when longitudinal probability distributions of cross sections ofunbondedprestress tendons was considered, and the safety index reduced with the increase in the number of independent and identically distributed random variables. It was suggested in practice to increase the safety margin of unbonded members by 15% in comparison with the bonded prestressed concrete bending members. It was also shown that the lower COV of cross-section area of prestressing tendons, the lower was the reduction of index of safety in unbondedprestressed concrete bending members. Therefore, the quality of prestressed tendons must be strictly controlled in practice.

Christopher and Elin(2012) carried out research on Reliability Analysis of a Prestressed Concrete Beam Exposed to fire. The research involved subjecting the beam to fire, identifying relevant load combination, specifying critical load and resistance random variable. He was able to find out that reliability decrease non-linearly as a function of time, and the most significant parameters were concrete cover, load ratio, fire type, end restraints and proportion of end strands to total strands.

In applying the First Order Method to the evaluation of BS449 (1969) design criteria for plate girders, Afolayan (1997) was able to accomplish the task of choosing structural dimensions and appropriate loading that are commensurate

with known levels of failure probability and consequent potential loss with known levels of performance.

Ellingwood and Reinhold (1980) carried out a research on the Reliability Analysis of Steel Beam-Columns compared with measures of reliability calculated from single resistance variable formulations of the safety problem for hot-rolled steel beam-columns subjected to dead and live loads considered the implications for reliability-based design.

2.7 FIRST ORDER RELIABILITY METHOD(FORM)

FORM is a 'level 2'method of reliability analysis. According to Gollwitzeret.al.(1988), FORM is designed for the approximate computation of general probability functions over given domains with locally smooth boundaries but especially for probability integrals occurring in structural reliability. The reliability estimations are carried out by representing each variable by its mean and standard deviation.

FORM produces an approximation to the probability of failure as;

$$P_f = P\{G(X) \leq 0\} = \int \dots \int_{G(X)\leq 0}^{G(X)\geq 0} f_x(X)dx \quad (2.4)$$

Where $f_x(X)$, is the joint density function for the 'n' vector X basic variables.

2.7.1 Failure Surface

According to Atim (2006), to evaluate the reliability of a structure, it is necessary to select relevant parameter as the variables so that the state of the structure is a function of the variables.

If X is a vector of all relevant basic variables, for a given structure, each variable is considered a realization of a random variable. Then the failure surface can be represented as:

$G(X) = f(X1, X2 \ldots\ldots Xn) = 0$ (2.5)

Where G(X) is the function expressing the relationship between the limit state and basic variables;

Then G(X) is the locally sufficiently smooth limit state function which must be at least once differentiable. FORM assumes a set of basic random variables in such a way that:

(i) $G(X) > 0$ indicates the safe or acceptable region

(ii) $G(X) \leq 0$ indicates an unsafe set of variables or the failure region

(iii) $G(X) = 0$ is the limit state or failure boundary.

2.7.2 Reliability Index

Structural reliability is usually measured by the reliability index, β. FORM assumes that if the probability density function of both the resistance and load variables are normal and independent, they may combine to define β as:

$$\beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \qquad (2.6)$$

Let G be the safety margin, G = R – S, such that

$$\beta = \frac{\mu_G}{\sigma_G} \qquad (2.7)$$

where μ_R and μ_S are the mean values of the resistance and load variables respectively and σ_R and σ_S are their corresponding standard deviations. β can also be related to the probability of failure P_f using:

$$P_f = \Phi(-\beta) \quad (2.8)$$

Where Φ is the standard normal distribution function.

It should be noted that the higher the safety index, the safer the structure and the lower the nominal probability.

2.8 SOLUTION APPROACH

In this study, the procedure to be followed in achieving the aim and objective is that the FORM will be used to determine and ascertain the safety levels of the prestressed concrete beam. The results obtained from FORM will be used to determine the reliability or safety level of the prestressed concrete beam.

After obtaining the safety level, the mean safety index will be computed.

CHAPTER THREE

MATERIALS AND METHODS

3.1 PREAMBLE

The safety evaluation of a structural member requires that the effects of the loads applied on the member should not exceed the capacities of the individual member capacity(Gilbert and Mickleborough, 1990). The determination of structural member capacities and the effects of theloads applied on the members, particularly beams are associated with high levels of uncertainties (Nowak and Colins, 2002). This chapter will discuss on how to model the load due to the self weight of the beam and the effects of the loads applied to it in order to account for their uncertainties during reliability analysis. Due to the fact that the self weight of the beam is the most typical load encountered in an unloaded simply supported beam design, only this load is discussed in this chapter. The third level approach under the reliability method namely Monte Carlo simulation will be conducted utilizing computer software FORM 5 to pick random variables from a specified probability distribution and the probability of failure will be estimated.

3.2 APPLICATION OF FIRST ORDER RELIABILITY METHOD

The First Order Second Moment (FOSM) method in particular has proved very popular with those wishing to obtain probability statement about particular problems with relative ease. The procedure to determine the safety index β is straight forward even for non linear limit state functions. Once experience has been gained with the FOSM method, its extension to non-normal random variables, known as the First Order Reliability Method is a natural step to incorporate distributional probability information. The application of FORM is mainly for problems with relatively simple or with only a few limit state function (Melchers, 1999).

3.3LIMIT STATE FUNCTION FOR PRESTRESSED BEAM

FORM require the use of derivatives of limit state function. However when the limit state function is a complex one dependent on structural behavior or analysis, resort may have to be made to numerical procedure (Melchers 1999).This limit state function for the prestressed beam was derived using Eurocode 2(2004).

3.3.1 Failure Due to Bending Moment

Assuming a rectangular prestressed concrete beam with a single tendon, breath (b) , depth(d), the regular beam has a span of L and it is to beanalyzed using the Eurocode 2 (2004) procedures.

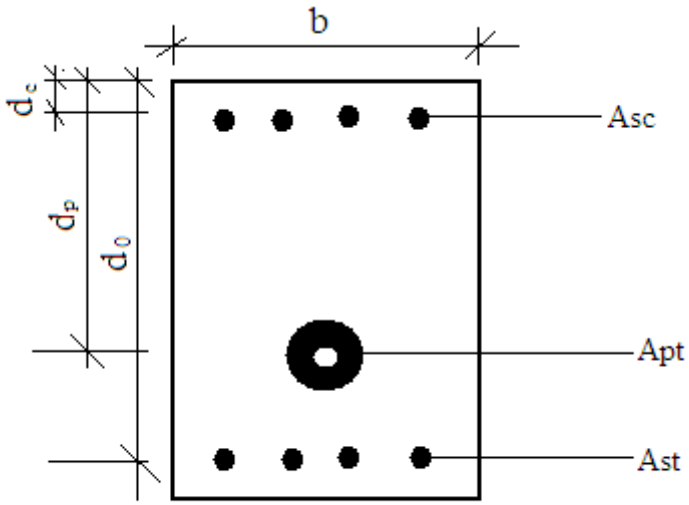

Fig 3.1 Typical cross section of the prestressed concrete beam

Failure due to bending occurs when the strength of the beam due to bending moment is less or equal to the design bending moment strength i.e

$$M_u \leq M^* \quad (3.1)$$

Where $M_u = \sigma_{pu}A_p(d_p - \frac{d_n}{2}) + f_yA_{st}(d_o - \frac{d_n}{2})$ (3.2)

$$M^* = \frac{wl^2}{8} \quad (3.3)$$

$$W = 1.35G_k + 1.5Q_k + \gamma_p p \quad (3.4)$$

In equation (3.1) to (3.4)

M_U and M* are ultimate bending moment and applied bending moment respectively, $\sigma_{pu}$is the stress in the prestressing steel,A_p and A_{st} are area of prestressed steel and area of reinforcement steel respectively.d_p, d_o, and d_n are depth to prestressed steel, depth to the bottom layer of tensile reinforcement and depth from the extreme compressive fiber to the neutral axis respectively.f_y is yield stress of non-prestressed steel, w is the uniformly distributed load on the beam. G_k, Q_k and P are dead load, imposed load and prestressing force respectively and $\gamma_p$is an environmental constant. It can be taken as γ_p,fav = 0.9 when beneficialor $\gamma_p$unfav = 1.1 when unfavourable.

Dividing equation (3.4) through by Q_k

Let a = $\frac{G_k}{Q_k}$ (3.5)

It implies that $w = Q_{ki}(1.35a + 1.5) + \gamma_p p$ (3.6)

Therefore M* = $\frac{(Q_k(1.35a+1.5)+\gamma_p P)l^2}{8}$ (3.7)

The condition for failure is given as:

$$G(X) = M_u - M^* \leq 0 \quad (3.8)$$

Therefore safety margin is given by:

$$G(X)=(\sigma_{pu}A_p(d_p-\frac{d_n}{2})+f_yA_{st}(d_o-\frac{d_n}{2})-\frac{(Q_k(1.35a+1.5)+\gamma_pP)l^2}{8})\leq 0 \quad (3.9)$$

Definition of terms in equation (3.9):

C_c compressive force carried by the concrete

l_c lever arm distance of compressive force in concrete above the non-prestressed tensile steel.

C_s compressive force in the non-prestressed steel

l_s lever arm distance of compressive forces in the steel above the non prestressed tensile steel

T_p Tension in the prestressed steel.

l_p lever arm distance of the tension force in prestressed steel above the non-prestressed tensile steel.

f_c' characteristic compressive strength of concrete at 28 days

b width of the compressive zone of the cross section of the beam

d_n depth from the extreme compressive fiber to the neutral axis

d_o depth to the bottom layer of tensile reinforcement

□ ratio of the depth of the idealized rectangular compressive stress block to the depth of the neutral axis at the ultimate strength in bending or combined bending and compression

f_y yield stress of non-prestressed steel.

A_{sc} area of compressive reinforcement

d_c depth to the top layer of non presstressed steel

σ_{pu} stress in the prestressing steel

A_p area of prestressed steel

d_p depth to the prestressed steel

f_p ultimate strength of the prestressing steel

Table 3.1 Parameters of Stochastic Model for Bending Failure Criterion

S/N	Variables	Distribution Type	Expected Values	Units	Coefficient of Variation	Standard Deviation
1	f_c'	Log-normal	35	N/mm^2	0.15	5.25
2	b	Log-normal	350	mm	0.01	3.5
3	d_n	Normal	239	mm	0.01	2.39
4	□	Gamma	1.15	-	0.3	0.345
5	d_o	Normal	690	mm	0.01	6.9
6	f_y	Log-normal	410	N/mm^2	0.15	61.5
7	A_{sc}	Log-normal	820	mm^2	0.15	123
8	d_c	Normal	60	mm	0.01	0.6
9	σ_{pu}	Log-normal	70	N/mm^2	0.15	10.5
10	A_p	Log-normal	1200	mm^2	0.15	180
11	d_p	Normal	650	mm	0.01	97.5
12	Q_k	Gumbel	5	kN/m^2	0.3	1.5
13	a_o	Gamma	1.5	-	0.3	0.45
14	γ_p	Gamma	0.9	-	0.3	0.27
15	P	Log-normal	100	kN	0.15	150
16	L	Normal	10000	mm	0.01	100

Source: Kong and Evans, 1999

3.3.2 Failure due to shear

The condition for failure is:

$P_f = \square V_{uc} \leq V^*$ (3.10)

Where $V^* = \frac{Wl}{2}$ (3.11)

$V_{uc} = \beta_1\beta_2 b_v d_o \left(\frac{A_{st}+A_{pt}}{b_{vd_o}} f_c'\right)^{1/3} + V_o + P_V$ (3.12)

$G(X) = \left(\beta_1\beta_2 b_v d_o \left(\frac{A_{st}+A_{pt}}{b_{vd_o}} f_c'\right)^{1/3} + Z_b\left(\frac{P_e}{A} + \frac{P_e e}{Z_b}\right)\left(\frac{4}{l}\right) + P_V\right) - \frac{Wl}{2} \leq 0$ (3.13)

$G(X) = \left(\beta_1\beta_2 b_v d_o \left(\frac{A_{st}+A_{pt}}{b_{vd_o}} f_c'\right)^{1/3} + Z_b\left(\frac{P_e}{A} + \frac{P_e e}{Z_b}\right)\left(\frac{4}{l}\right) + P_V\right) -$

$\left(\frac{(Q_k(1.35a+1.5)+\gamma_p P)l}{2}\right) \leq 0$ (3.14)

Definition of terms in equation (3.14)

V_{uc} contribution of concrete to the shear strength

V^* factored design shear force for the strength limit state

W uniformly distributed load

strength reduction factor

β_1 tension stiffening constant

β_2 tension stiffening constant

b_v effective width of web for shear calculation

d_o depth to the bottom layer of tensile reinforcement

A_{st} area of non-prestressed tensile

A_{pt} area of prestressed steel in the tensile zone

f_c' characteristic compressive strength at 28 days

Z_b bottom section modulus

P_e effective prestressing force after time-dependent losses

P_V vertical component of prestressing force

Q_k live load

γ ratio of the depth of the idealized rectangular compressive stress block to the depth of the neutral axis at the ultimate strength in bending or combined bending and compression

P prestressing force

L span

Table 3.2 Parameters of Stochastic Model for Shear Failure Criterion

S/N	Variables	Distribution Type	Expected Values	SI Units	Coefficient of Variation	Standard Deviation
1		Normal	0.7	-	0.01	0.007
2	β_1	Normal	1.1	-	0.01	0.011
3	β_2	Normal	1.0	-	0.01	0.001
4	b_v	Normal	260	mm	0.01	2.6
5	d_o	Normal	690	mm	0.01	6.9
6	A_{st}	Lognormal	620	mm^2	0.15	2.6
7	A_{pt}	Lognormal	1800	mm^2	0.15	270
8	f_c'	Lognormal	32	N/mm^2	0.15	4.8
9	Z_b	Normal	666	mm^3	0.01	6.66
10	P_e	Gumbel	70	kN	0.3	21
11	A	Lognormal	2.42E5	mm^2	0.15	3.63E4
12	e	Normal	261	mm	0.3	78.3
13	P_V	Gumbel	20.4	kN	0.3	6.12
14	Q_k	Gumbel	3	kN/m^2	0.3	0.9
15	a_o	Gamma	0.6	-	0.3	0.18
16	P	Gumbel	100	kN	0.3	300
17	l	Normal	10000	mm	0.01	100

Source: Kong and Evans, 1999

3.3.3 Failure due to Deflection

Failure due to deflection occurs when the maximum deflection is less than the span divided by 500. The deflection model is shown below

$\nu_i = \frac{5w_G - w_b l^4}{384EI}$ (3.15)

$w_b = \frac{8P_e e}{l^2}$ (3.16)

where

w_G is the maximum gravity load

ν_i is the deflection at transfer

w_b is the upward transverse force exerted by the prestresse on the member.

E is the section elastic modulus

I is thesecond moment of area (moment of inertial) about the centriod axis

Deflection under sustained load is given by:

$\nu_{sus} = \frac{5(w_{sus} - w_b)L^4}{384EI}$ (3.17)

Where: w_{sus} is the sustained gravity load, w_b is the transverse load exerted on the beam by the tendons, L is the span and E is the section elastic modulus.

Deflection induced by creep

$$\nu_{cr}=\frac{\emptyset^*}{\propto}\nu_{sus} \quad (3.18)$$

where ν_{cr} is the deflection due to creep, $\emptyset^*$ is the final creep coefficient at time infinity, $\propto$ is a parameter to account for the effect of cracking and reinforcement quantity on the restraint to creep and ν_{sus} is the deflection due to sustained load

The deflection due to shrinkage is given by

$$\nu_{sh}=\beta\varkappa_{sh}^{*}l^2 \quad (3.19)$$

whereν_{sh} is the deflection due shrinkage, $\varkappa_{sh}^{*}$ is the curvature induced by shrinkage and L is span.

Final deflection due sustained load and shrinkage is:

$$\nu_{max} = \nu_{sus}+ \nu_{cr}+ \nu_{sh} (3.20)$$

$$\nu_{max} = \frac{5(G_k b-\frac{8p_e e}{l^2})l^4}{384EI} + \frac{5(G_k b-\frac{8p_e e}{l^2})l^4\emptyset^*}{384EI^{\circ}\propto}+ \beta\frac{\varepsilon_{sh}^{*}}{D}\left(1-\frac{A_{sc}}{A_s}\right)l^2 \quad (3.21)$$

For a design to be satisfactory:

$$\frac{span}{500} \geq \nu_{max} \quad (3.22)$$

Therefore, the safety margin is given as:

$$G(X)= \frac{span}{500} - \nu_{max} \leq 0 \quad (3.23)$$

Which implies:

$$G(X) = \frac{5(G_k bl^4 - 8P_e el^2)}{384EI}\left(1 + \frac{\emptyset^*}{\propto}\right) + \beta\frac{\varepsilon^*_{sh}}{D}\left(1 - \frac{A_{sc}}{A_s}\right)L^2 - \frac{L}{500} \leq 0 \quad (3.24)$$

G_k is dead load, b is the width of the beam, P_e is the effective prestressed force e is eccentricity, D is the depth of the beam, A_{sc} is the area of the compression steel, A_s is the area of the tensile steel.

Table 3.3Parameters of Stochastic Model for Deflection Failure Criterion

S/N	Variables	Distribution Type	S.I Units	Expected Values	Coefficient of Variation	Standard Deviation
1	G_k	Gumbel	kN/m^2	2	0.3	0.6
2	B	Normal	mm	350	0.01	3.50
3	L	Normal	mm	10000	0.01	100
4	E	Normal	mm	261	0.01	2.61
5	P_e	Gumbel	kN	70	0.3	21
6	E	Normal	mm^3	28600 $x10^6$	0.01	$286x10^6$
7	I	Normal	mm^4	$62500x10^6$	0.01	$625x10^6$
8	$\emptyset^*$	Normal	-	3.3	0.01	0.033

9	$\propto$	Normal	-	3	0.01	0.03
10	β	Normal	-	0.125	0.01	0.00125
11	ε_{sh}^{*}	Normal	-	0.00056	0.01	0.0000056
12	D	Normal	mm	1150	0.01	11.5
13	A_{sc}	Lognormal	mm^2	900	0.01	9.0
14	A_s	Lognormal	mm^2	11500	0.01	115

Source: Kong and Evans, 1999

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 INTRODUCTION

The results were obtained using FORM (Gowittzer et al., 1988) and using the limit state functions derived in the previous chapter. The derived limit state functions were derived considering the Eurocode 2 failure criteria of prestressed concrete beam.

4.2FAILURE DUE TO BENDING MOMENT

The results obtained from the running of the computer program considering failure due to bending moment criterion are presented in Figs.4. 1 to 4.14

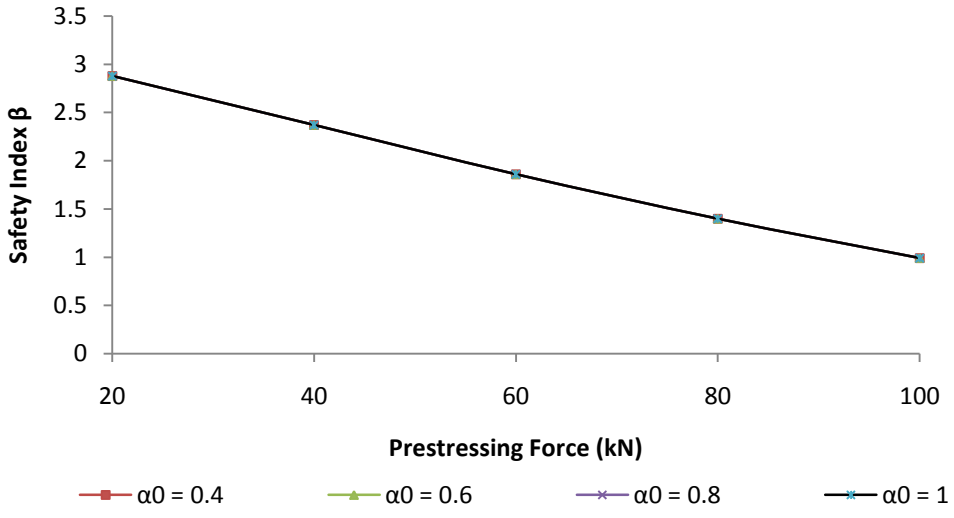

Fig.4.1: Variation of safety index with Prestressing force and load ratio for bending failure

The variation of load ratio with prestressing force in reliability based design of a prestressed concrete beam is shown in Fig. 4.1 and Fig.4.2. Fig 4.1 shows that as the prestressing force increased the safety index decreased. This indicates that as the prestressing force increased the safety index of the prestressed beam tends to failure. The figure also indicates that though the prestressed beam tends to failure with increase in prestressing force still the beam is reliable when the value of prestress force is within 80kN and 100kN. Within this range of prestress force the safety index is within 1.5 and 1 respectively, which is safe and economical.

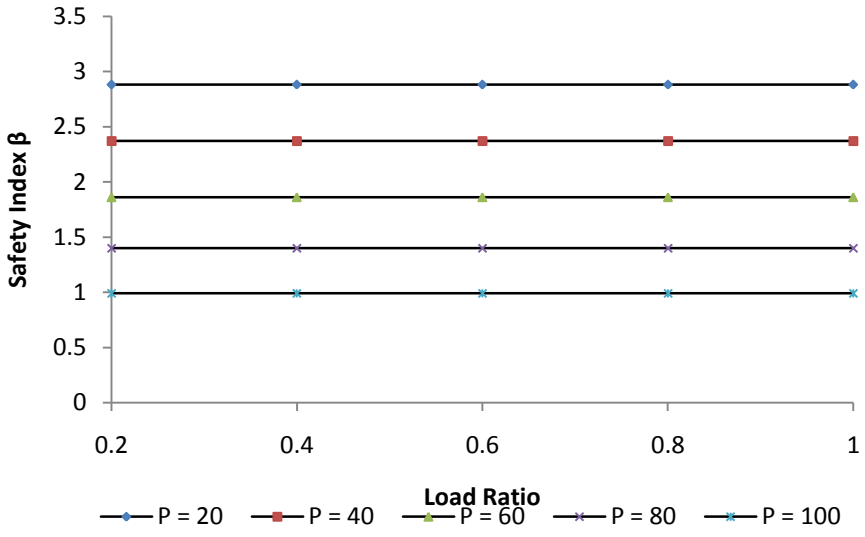

Fig.4.2: Variation of safety index with load ratio andprestressing force for bending failure

Fig.4.2: shows that as the load ratio increased the safety index remained constant. This indicates that the load ratio does not have effect on failure due to bending of the beam. This is because of the compressive force that was induced in the concrete which neutralizes all tensile stress in the beam and made the concrete effective in carrying load (Mosley and Bungey, 2007).

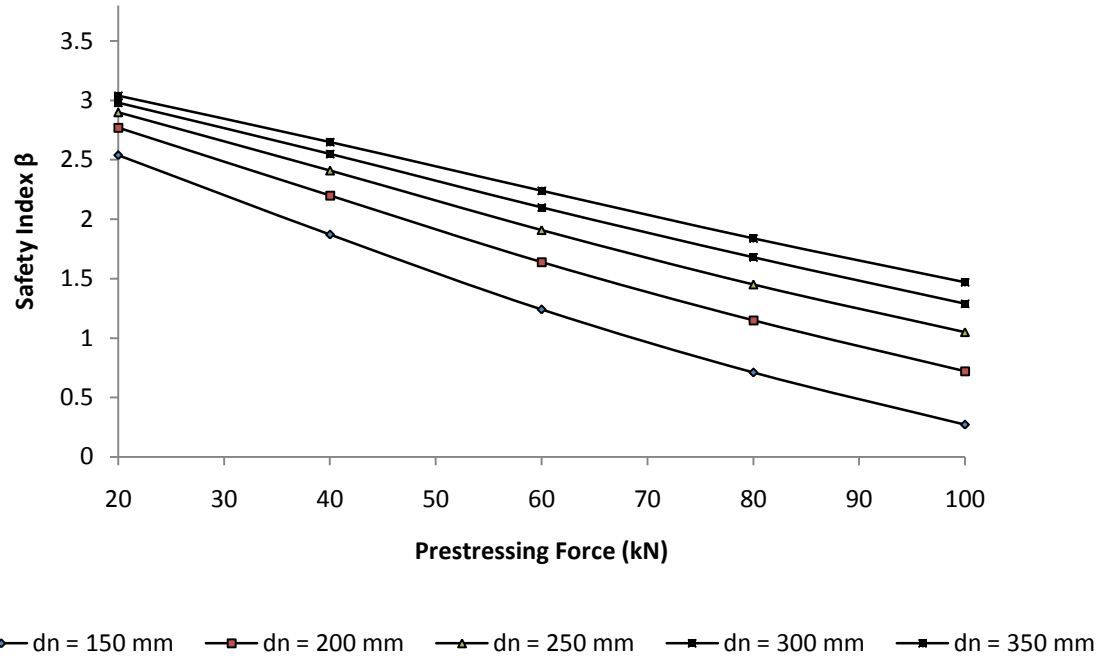

Fig.4.3: Variation of safety index with prestressing force and depth from extreme compressive fiber to the neutral axis for bending failure

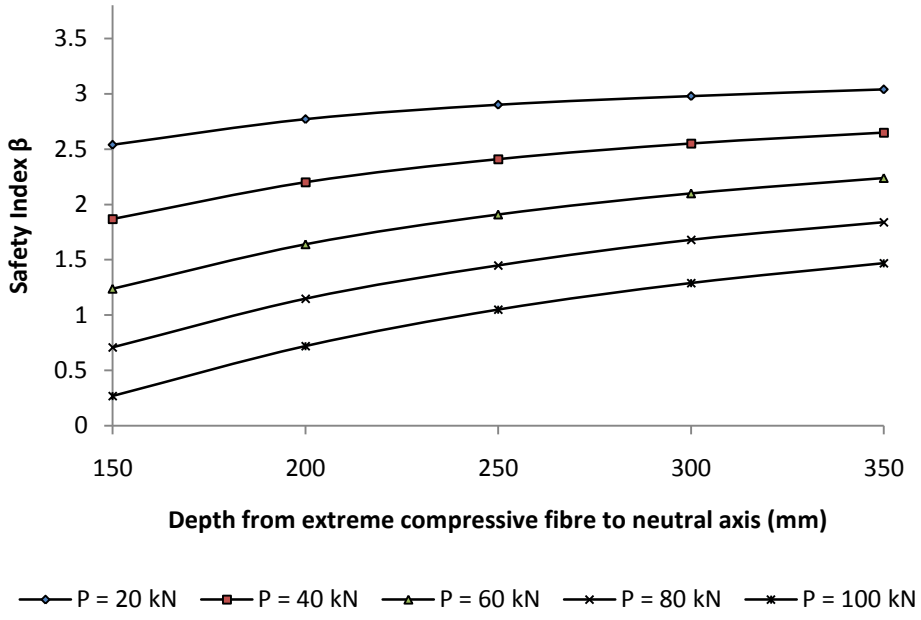

Fig.4.4: Variation of safety index with depth from the extreme compressive fiber to neutral axis d_n andprestress force for bending failure

The effect of the variation of the prestress force and the depth from the compressive fiber to the neutral axis is shown on Fig 4.3 and Fig 4.4. Fig.4.3 shows that as the prestressing force increases the safety index reduces. This means that at increased prestressing force the beam tend to fail by bending while on the other hand Fig.4.4 shows that as the depth from the compressive fiber to the neutral axis increases the safety index increases. This means that at increased depth from the compressive fiber to the neutral axis the beam is saved from failure due to bending. This implies that the greater the thickness of the beam, the more rigid and reliable the beam tend to be.

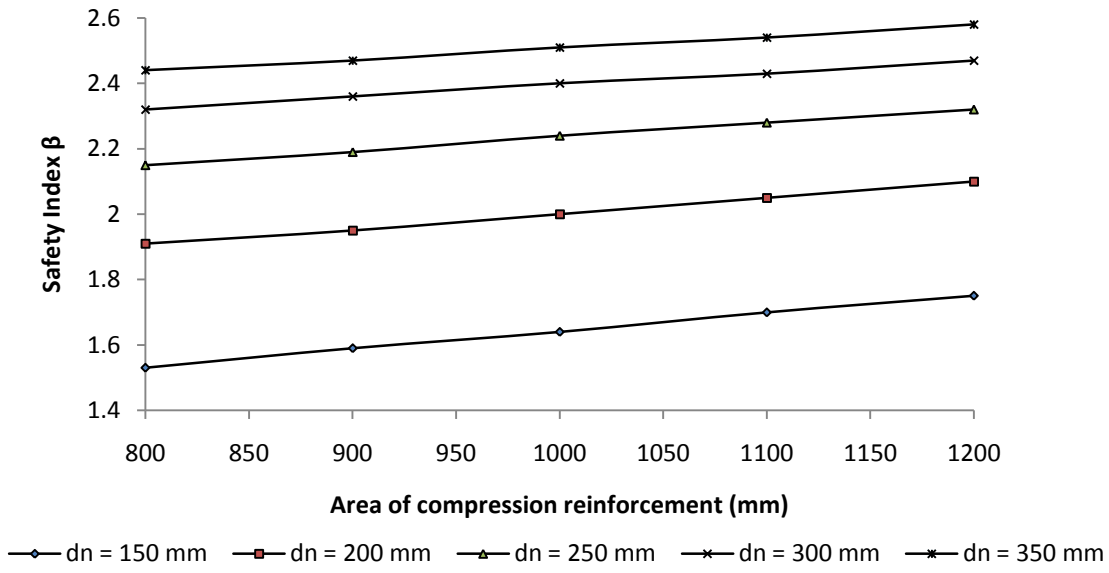

Fig.4.5: Variation of safety index with Area of compression reinforcement anddepth to neutral axis for bending failure

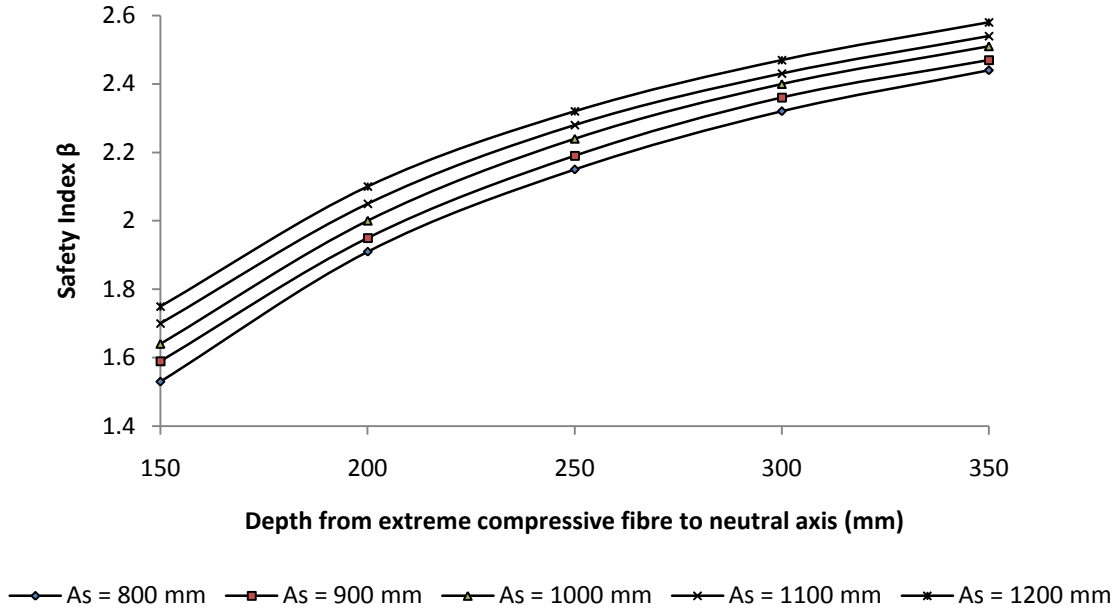

Fig.4.6: Variation of safety index with Depth from extreme compressive fibre to the neutral axis for bending failure

The effect of the variation of area of compressive reinforcement with depth from extreme compressive fiber to the neutral axis is shown in Fig.4.5 and Fig.4.6. The figures show that as both the area of the compressive reinforcement and the depth from extreme compressive fiber to the neutral axis increases the safety index increases. This means that at the increase of the both variables the beam is saved from failure due to bending. This is because the compressive reinforcement provides the cracked concrete beam with flexural strength (Gilbert and Mickleborough,1990). While the depth from the extreme compressive fiber to the neutral axis increases the safety through rigidity.

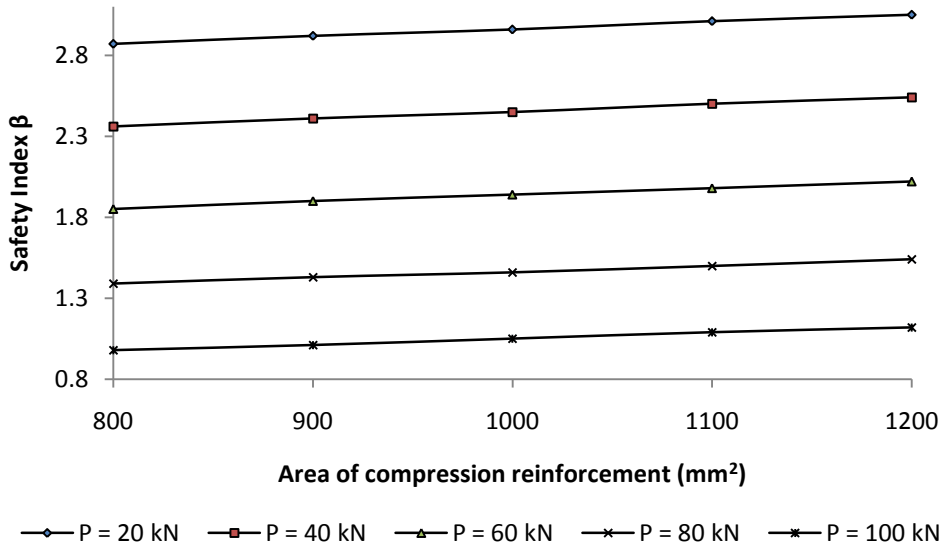

Fig.4.7: Variation of safety index with area of compression reinforcement and prestressing force for bending failure

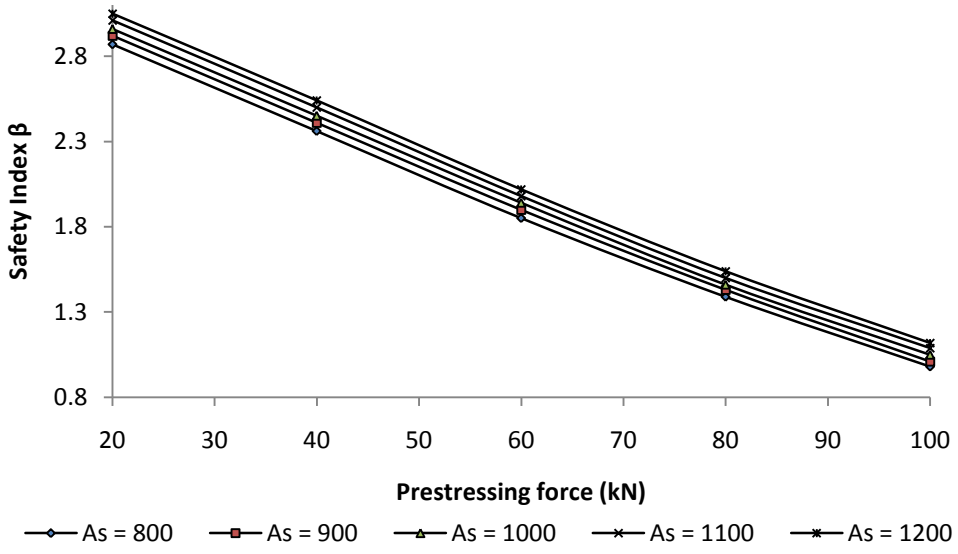

Fig4.8: Variation of safety index with prestressing force and area of compression reinforcement for bending failure

The effect of the variation of area of compressive reinforcement and prestressing force is shown in Fig.4.7 and Fig.4.8. Fig.4.7 shows that as the area of the compressive reinforcement increases the safety index increases. This shows that with increase in the area of compressive reinforcement the beam is safe. While on the other hand the safety index decreases as the prestressing force increases. This means that the beam tends to failure with an increase in prestressing force.

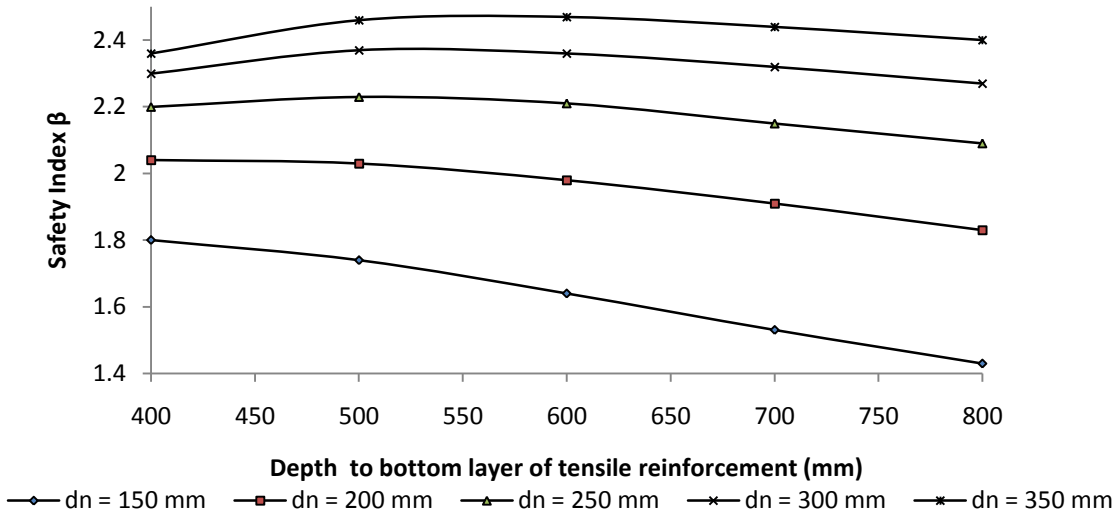

Fig.4.9: Variation of safety index with depth to bottom layer of tensile reinforcement and depth from extreme compressive fiber to the neutral axis for bending failure

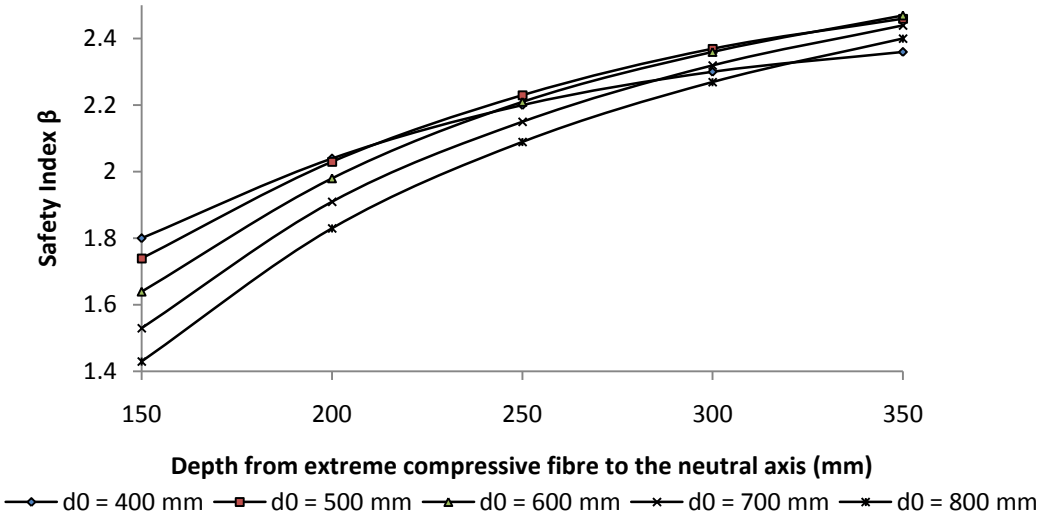

Fig.4.10: Variation of safety index with depth to the neutral axis and depth to the bottom layer of tensile reinforcement for bending failure

The effect of variation of depth to the bottom layer of tensile reinforcement and depth to the extreme compressive fiber to the neutral axis is shown in Fig.4.9 and Fig.4.10. The figure shows that as the depth to the bottom layer of tensile reinforcement increases the safety index increases. This means that the beam is saved with increased in depth to the bottom layer of tensile reinforcement. While on the other hand the safety index decreases with increases in depth to the extreme compressive fiber to the neutral axis indicating that the beam is not saved with increased in the depth to the extreme compressive fiber to the neutral axis.

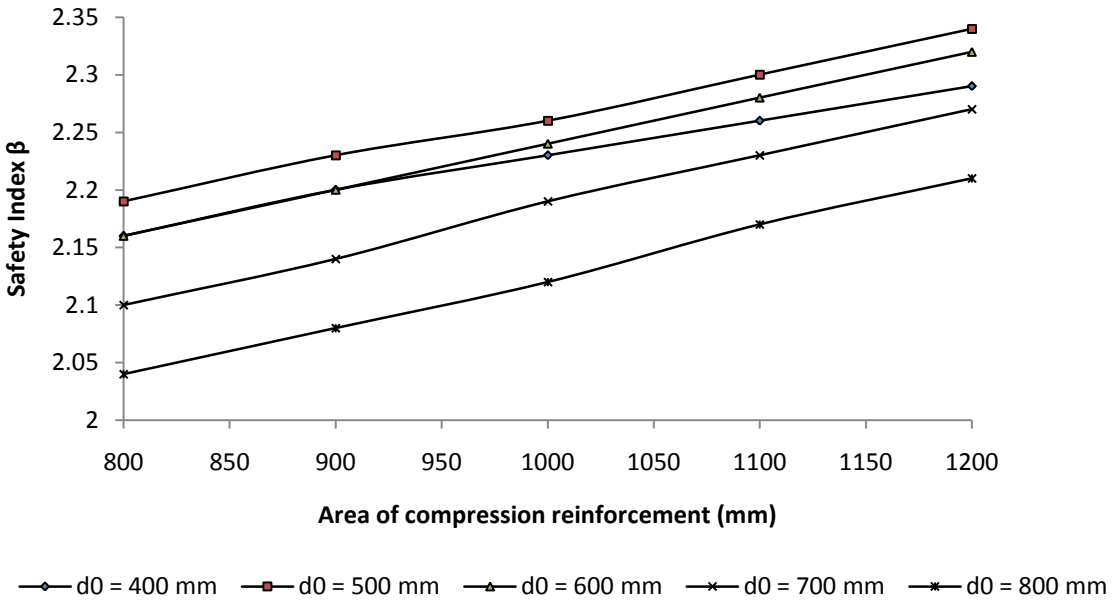

Fig.4.11: Variation of safety index with area of compression reinforcement and depth to the tensile reinforcement for bending failure

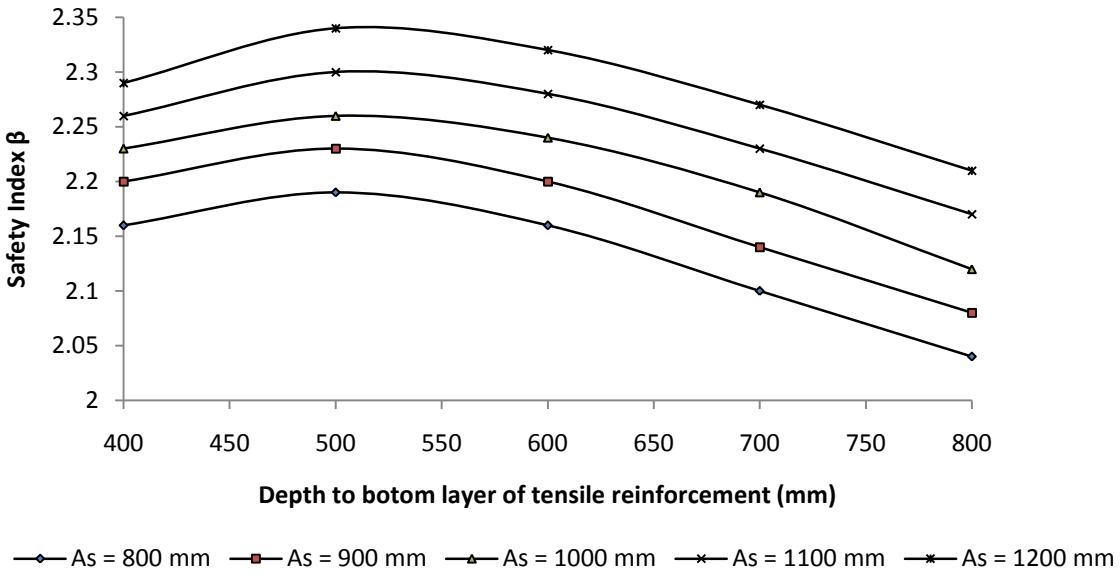

Fig.4.12: Variation of safety index with depth to bottom layer of tensile reinforcement and area of compression reinforcement for bending failure

The effect of area of compressive reinforcement and depth to the bottom layer of tensile reinforcement is shown in Fig.4.11 and Fig.4.12. Fig.4.11 shows that as the area of compressive reinforcement increases the safety index increases. On the other hand as the as the depth to the bottom layer increases the safety index reduces. The maximum safety index occurs when the depth is at 500m. At this point the safety index varies from 2.19 to 2.34 with the varied area of compressive reinforcement from 800mm^2 to 1200mm^2 respectively.

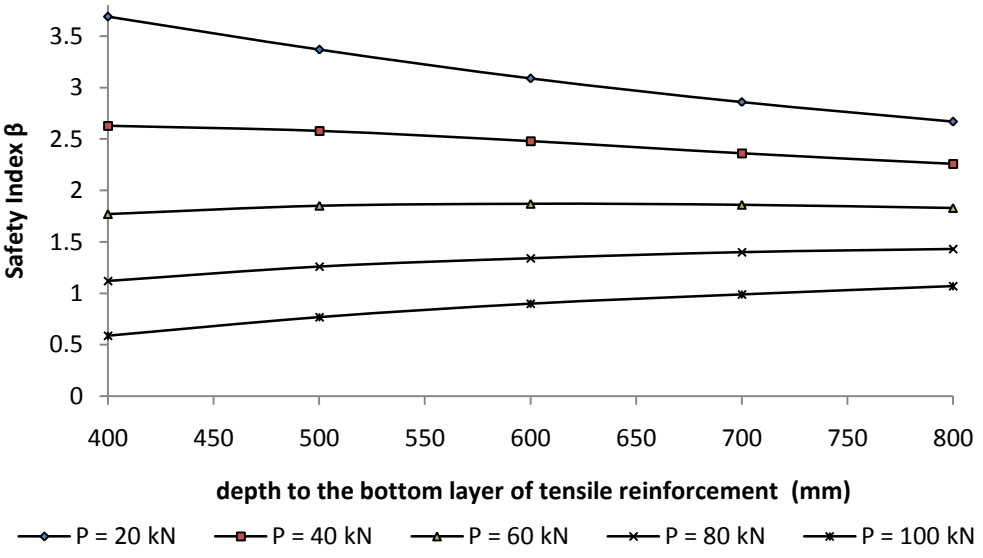

Fig.4.13: Variation of safety index with depth to the bottom layer of tensile reinforcement and prestress force for bending failure

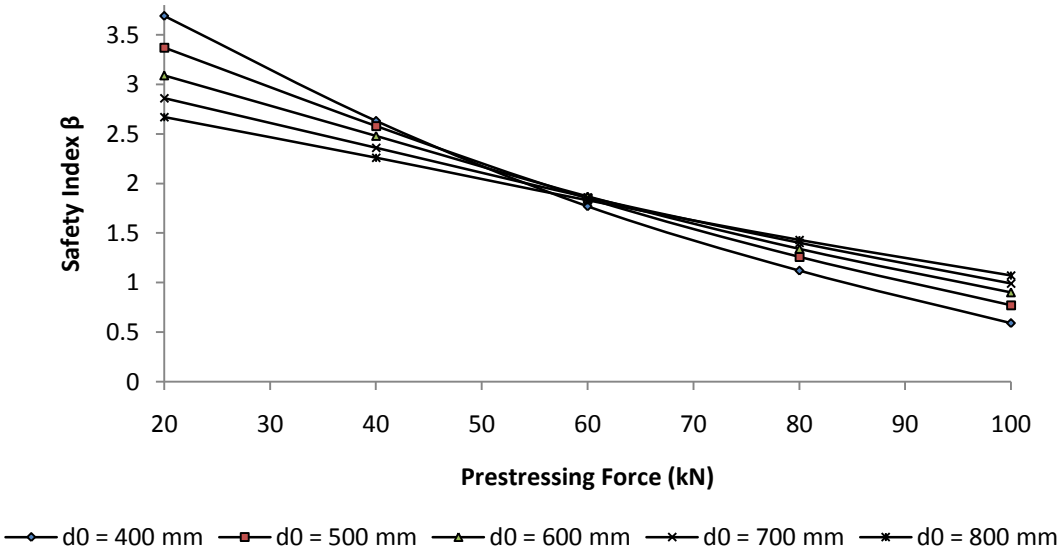

Fig4.14: Variation of safety index with prestress force and depth to the bottom layer of tensile reinforcement for bending failure

The effect of variation of depth to the bottom layer of tensile reinforcement with prestressing force is shown in Fig.4.13 and Fig.4.14. the figure shows that as the depth to the bottom layer of tensile reinforcement increases the safety index tend to converge thereby it becomes constant meaning the depth to the bottom layer of tensile reinforcement will not have effect on the failure due to bending as it increases. While Fig.4.14 shows that as the prestress force increases the safety index decreases. It also shows that when prestress force is 60kN the safety of the beam is at equilibrium. That is at 60kN the safety index for all depth is 1.83 which is safe and economical.

4.3 FAILURE DUE TO SHEAR

The results obtained from the running of the computer program considering shear failure criterion are presented in figs. 4.15 to 4.24

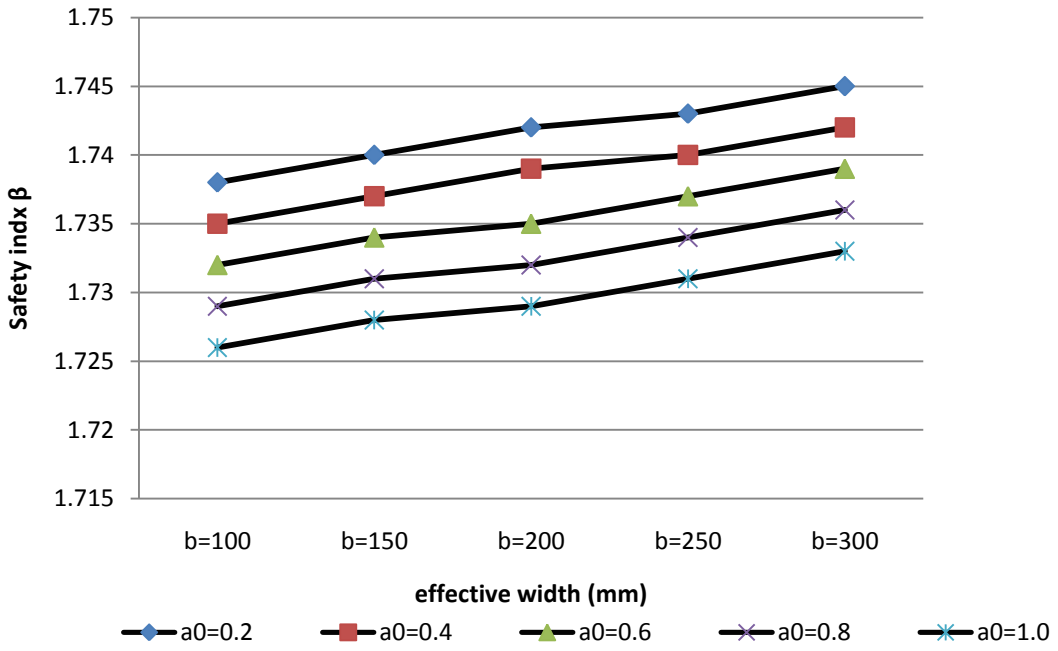

Fig 4.15: Variation of Safety Index with effective width and load ratio for shear failure

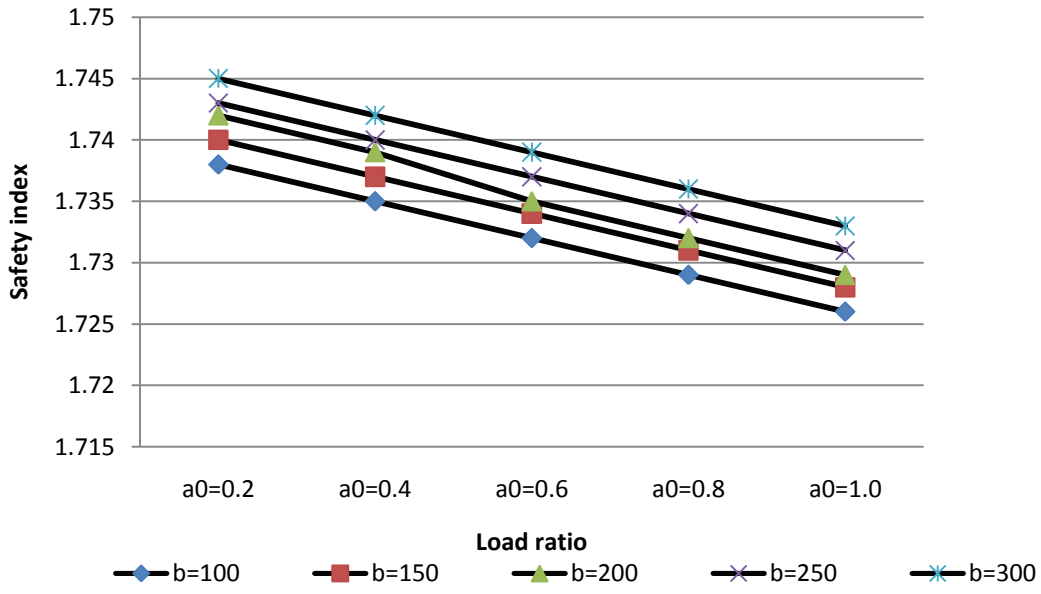

Fig.4.16: Variation of safety index with load ratio and effective width for shear failure.

The effect of the variation of effective width of beam with load ratio on a prestress concrete beam in shear is shown in Fig.4.15 and Fig.4.16. Fig 4.15 shows that as the effective width of web increases the safety index increases with a constant value of load ratio for all values of load ratio which signify that. While fig.4.16 shows that as the load ratio increases the safety index decreases with a constant effective width of beam.

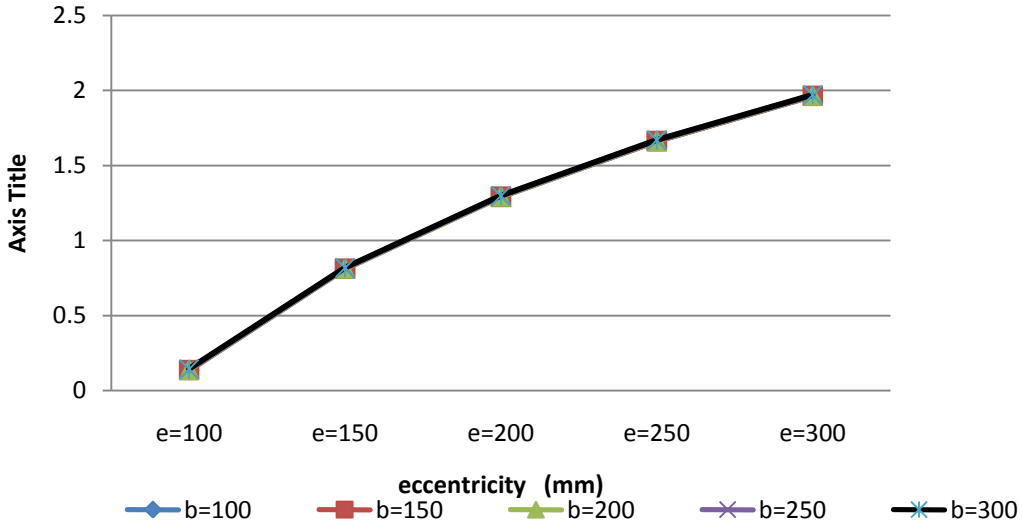

Fig.4.17: Variation of safety index with eccentricity and effective width of web for shear failure

Fig.4.17 shows the effect of eccentricity in the presstress beam. The safety index of the beam increases as eccentricity increases. This implies that the prestressed concrete beam is safe at increase in eccentricity

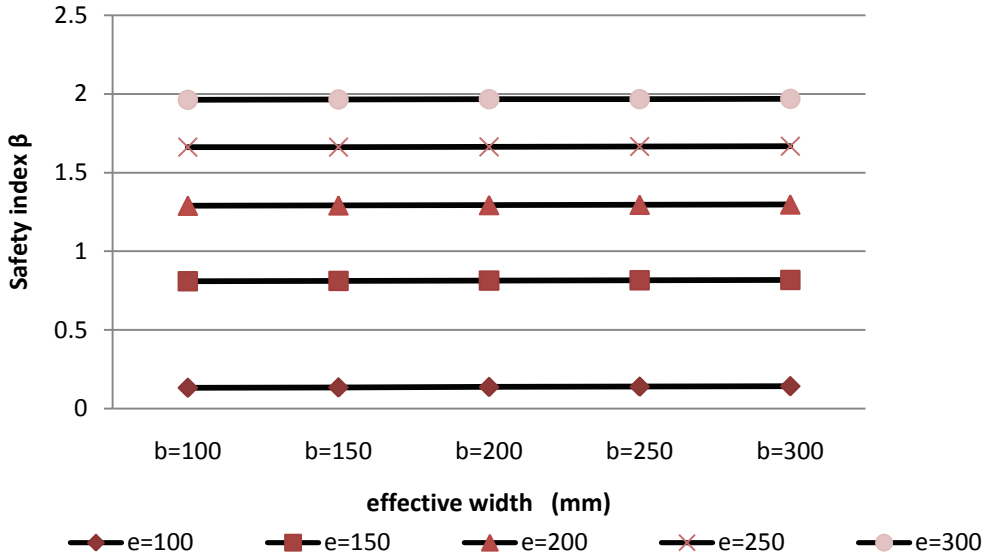

Fig.4.18: Variation of safety index with effective width of web and eccentricity for shear failure

Fig.4.18 shows that the safety of the prestressed concrete beam remains constant with increase in the effective width of the beam. This indicates that the effective width does not have significant effect on the failure due to shear.

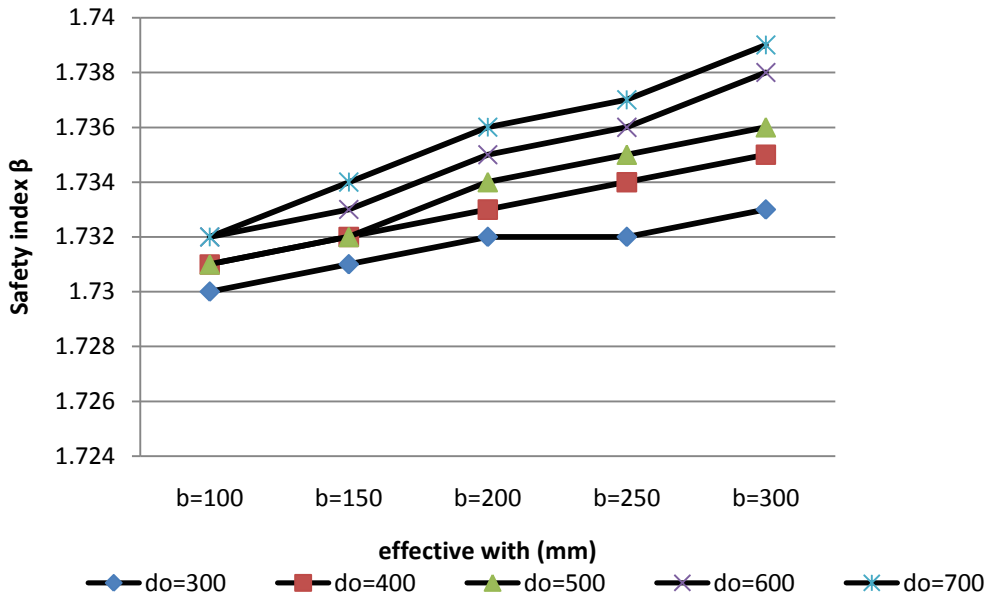

Fig.4.19: Variation of safety index with depth to the bottom layer of tensile reinforcement and effective width of web for shear failure

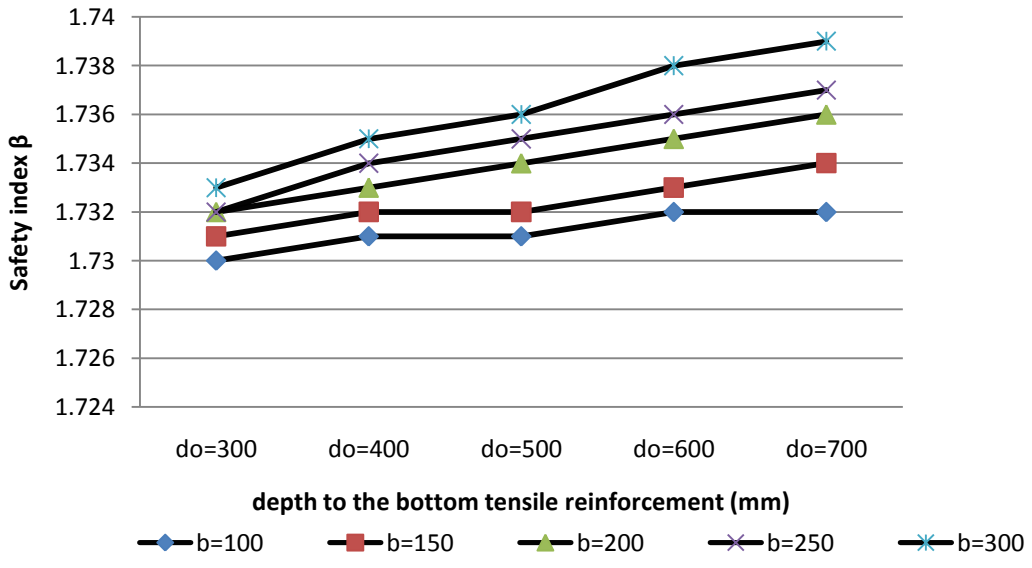

Fig.4.20: Variation of safety index with effective width of web and depth from extreme fiber to neutral axis for shear failure

The effect of variation of effective width of web and depth to the bottom layer of the prestressed concrete beam considering shear failure is shown in Fig.4.19 and Fig.4.20. These figures show that as both the effective width of web and the depth to the bottom tensile reinforcement of the beam increases the safety index increases. This signifies that the depth to the bottom tensile reinforcement contributes significantly to the reliability of the prestressed concrete beam considering failure due shear criteria.

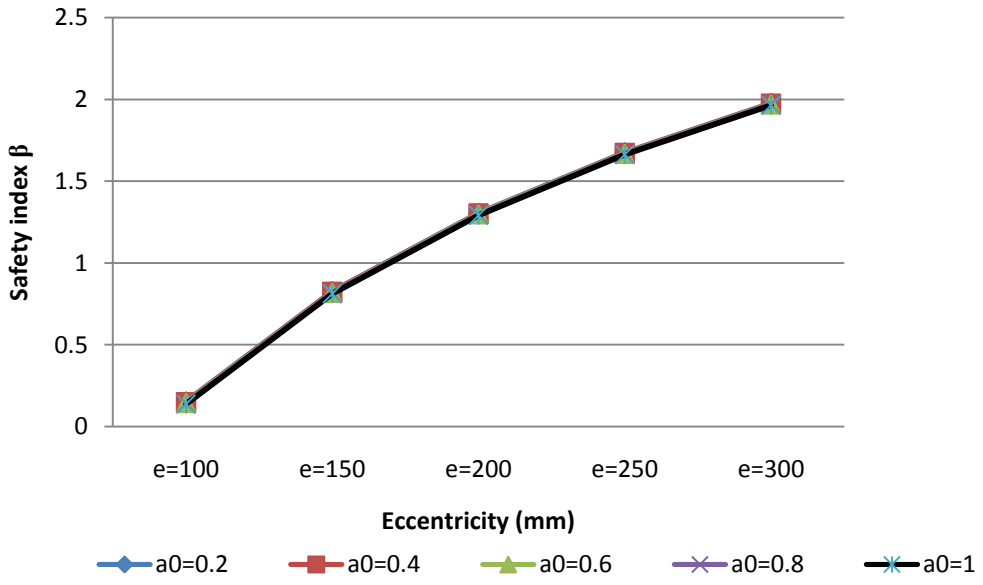

Fig4.21: Variation of safety index with eccentricity and load ratio for shear failure

The effect of variation of eccentricity and load ratio on reliability analysis of theprestressed concrete beam is shown in Fig.4.21 and Fig.4.22. fig.4.21 shows that as eccentricity increases the safety index of the prestressed beam increases. This implies that eccentricity has a significant effect on the reliability of the prestressed concrete beam. Hence, as eccentricity increases the higher the reliability of the prestressed beam. The best values for eccentricity have safety indices between 1.292 and 2.0 as shown in fig.4.21. The values are between the range of 200mm and 300mm as shown above.

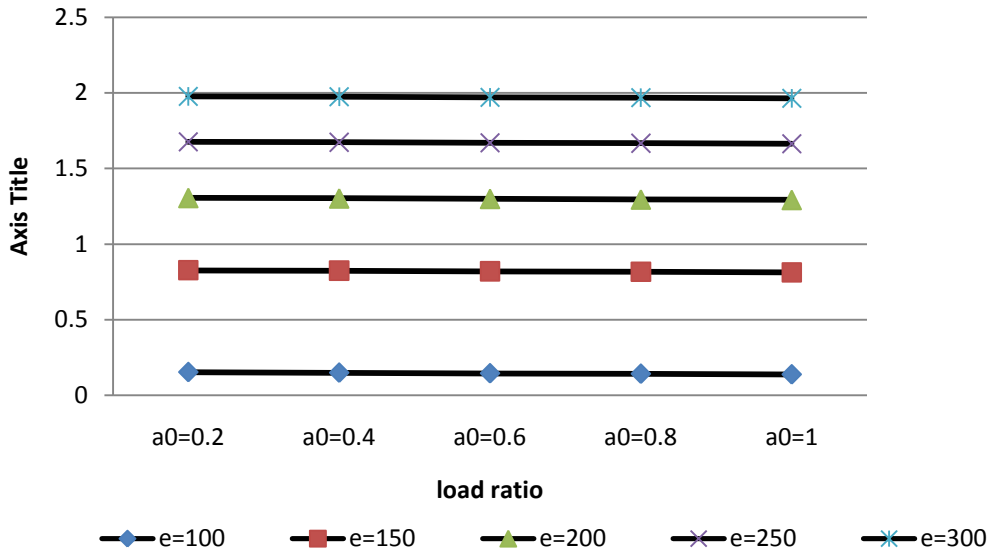

Fig4.22: Variation of safety index with load ratio and eccentricity for shear failure

Fig.4.22 shows that as the load ratio increases with a constant eccentricity the safety index remains constant. This indicates that the load ratio does not have any effect on the shear failure of a prestressed concrete beam with a constant eccentrity.

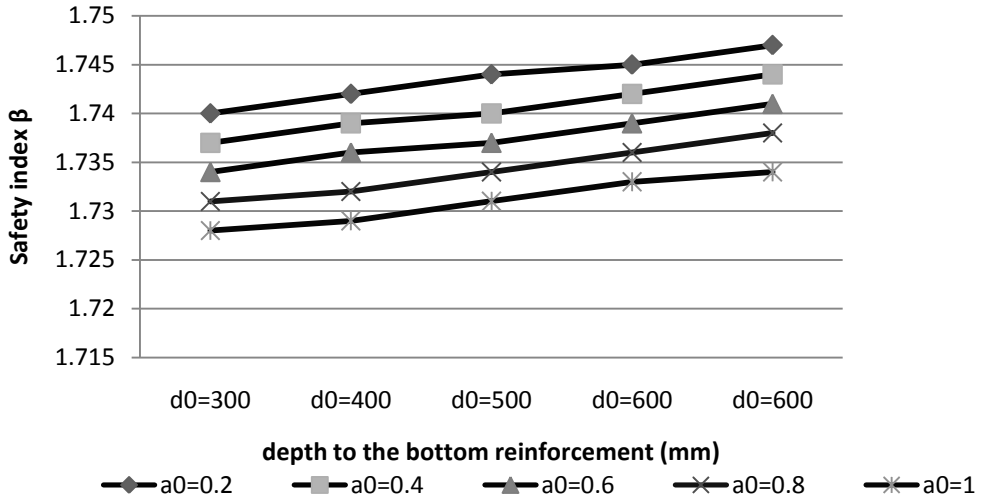

Fig.4.23: Variation of safety index with depth to the bottom layer of tensile reinforcement and load ratio for shear failure

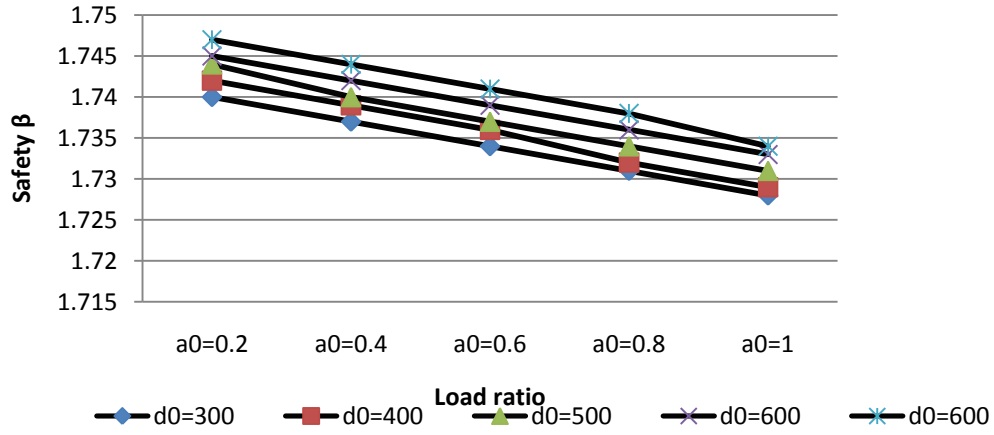

Fig.4.24: Variation of Safety Index with load ratio and depth bottom reinforcement for shear failure

The effect on the variation of depth to the bottom layer of the beam and load ratio on theprestressed concrete beam is shown in Fig.4.23 and Fig.4.24. Fig 4.23 shows that as the depth to the bottom reinforcement increases with a constant load ratio the safety index of the prestressed concrete beam increases. This results to a high reliability of the prestressed concrete beam. While Fig.4.24 shows that as the load ratio increases with a constant depth to the bottom reinforcement there is a corresponding decrease in the safety index of the beam whichmay tend to failure due to shear, in this case a higher load ratio of the beam results to a lower reliability of the same beam.

4.4FAILURE DUE TO DEFLECTION

The results obtained from the running of the computer program considering deflection failure criterion are presented in Fig.4.25 to 4.28

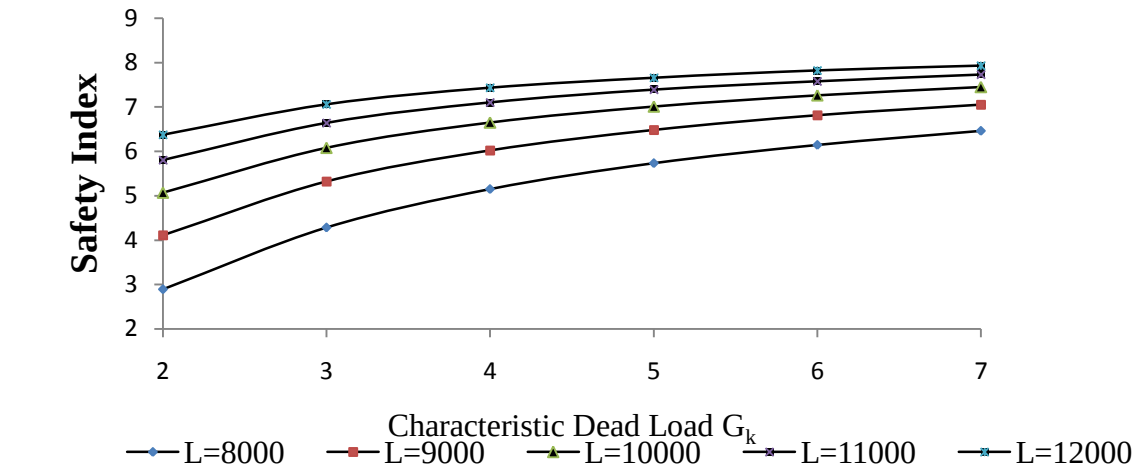

Fig.4.25: Variation of safety index with dead load $G_k$and span for deflection failure

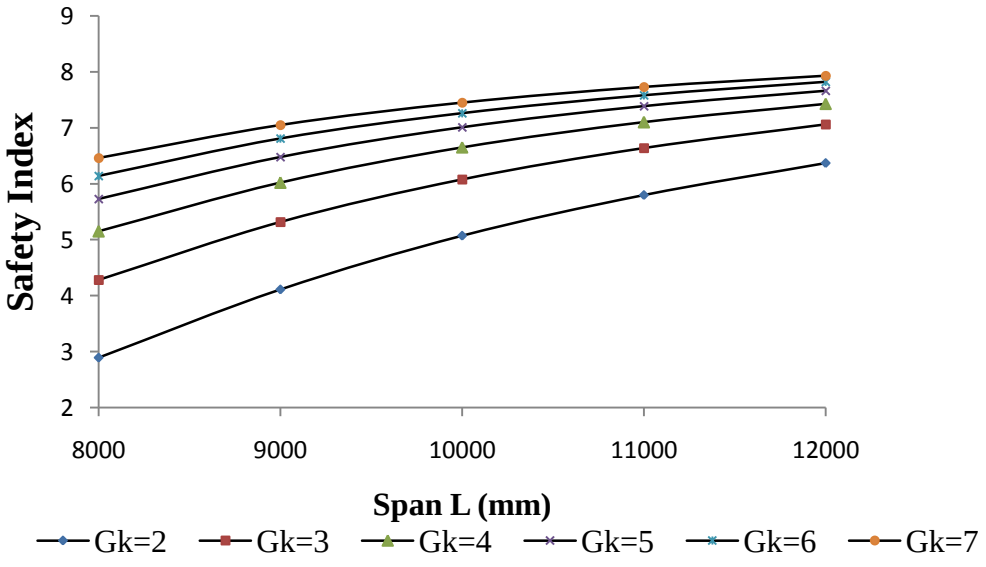

Fig.4.26: Variation of safety index with span and dead load for deflection failure

The effect of the variation of dead load and span on the reliability analysis of a reinforced prestress concrete beam in deflection is shown in Fig.4.25 and

Fig.4.26. This graph shows a reverse in the trend indicating that an increase in the characteristic dead load leads to a corresponding increase in the safety indices and also an increase in the span shows a corresponding increase in the safety indices. This shows that the prestressed beam is much more reliable with a long span of prestressed concrete beam as well as with a short span of prestressed concrete beam unregard of the weight of the loadings. This verifies the statement that "prestressed concrete Enable longer spans to be used" (Kong and Evans (1999). Also the ability to control deflection in prestressed beam permits longer spans to be achieved.

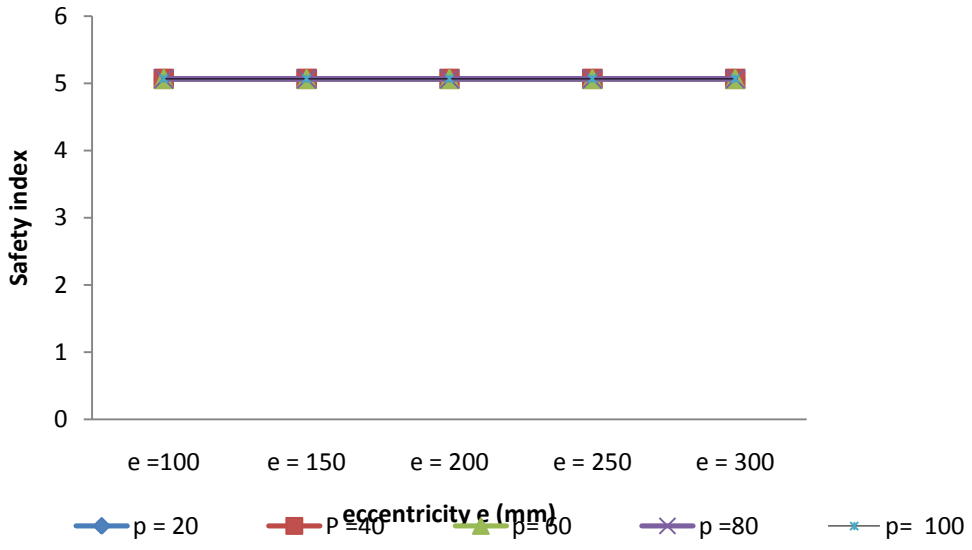

Fig.4.27: Variation of safety index with eccentricity andprestressing force for deflection failure

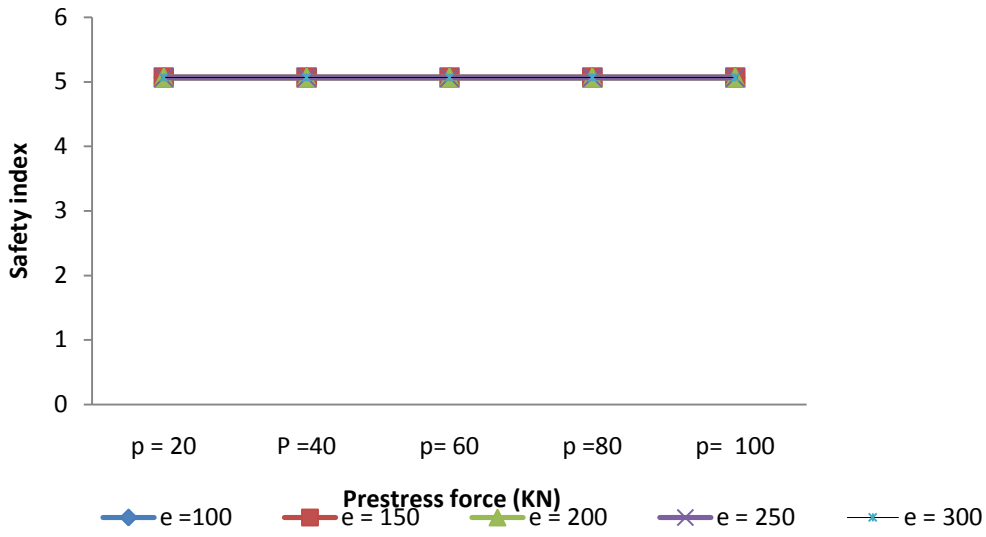

Fig.4.28: Variation of safety index with prestress force and eccentricity for deflection failure

Fig.4.27 and Fig.4.28 shows the effect of the variation of prestress force with eccentricity and variation of eccentricity with prestress force respectively. Fig.4.27 shows that as eccentricity increase the safety index remain constant. Likewise Fig.4.28 shows that as the pretress force increases the safety index remain constant. This means that eccentricity and the prestress force do not have any effect on the deflection of the prestress beam. Therefore the beam cannot be able to undergo failure due to deflection as a result of increase in eccentricity or increase in the prestress force.

4.5 TARGET SAFETY INDEX

The mean safety indices of the prestressed concrete beam in bending, shear and deflection were obtained by deriving the mean of all safety indices obtained in bending, shear and deflection respectively.

Hence the mean safety indices of the beam in bending was 2.01, in shear it was 4.716 and in deflection the mean safety index was 1.526. The safety indices obtained indicate that the beam is safe.

The target safety indicesconsidering bending, shear and deflection failure criteria are therefore 2.01, 4.716 and 1.526 respectively.

4.6Design Example

It is required to check the suitability of the prestressedbeam using deterministic design method and to compare it with the reliability results presented at a target safety level of 2.

4.6.1 Deterministic Design

Analyse a simple pre-tensioned prestressedconcrete beam designed with the parameters shown in Tables 3.1, 3.2, and 3.3 using Eurocode 2 procedures.

Solution:

In order to analyze the safety of the pre-tensioned prestressed beam the various modes of failures are checked to ensure the safety of the beam. Failures due to ultimate limit state are considered for this analysis.

(a) Failure due to Bending Moment

Considering equation (3.1)

$\square M_u \leq M^*$

Where $M_u = \sigma_{pu}A_p(d_p - \frac{d_n}{2}) + f_yA_{st}(d_o - \frac{d_n}{2})$ (3.2)

Where: $\sigma_{pu} = 0.7P$

Substituting the various values from table (3.1)

$M_u = 0.7x100000x\ 1200(650 - \frac{1.15X239}{2}) + 500\ x\ 820\ (690- \frac{1.15X239}{2})$

$M_u = 84000000(650-137.425)+410000(690- 137.425) = 4558.856KN$

From equation (3.3)

$M^* = \frac{wl^2}{8}$

$W = 1.35G_k + 1.5Q_k + \gamma_p p$

Therefore M* = $\frac{(Q_k(1.35a+1.5)+\gamma_p P)l^2}{8}$

Substituting the values in the above equation

M* =$\frac{(5(1.35X1.5+1.5)+0.9X100)10^2}{8}$ = 1345.313KNm

$M_u \geq M^*$

This implies that the beam is safe in the mode of failure due to the bending moment.

(b) Failure due to Shear

The prestressed concrete beam will satisfy the condition of safety if

$V_{uc} \geq V^*$ obtained from equation (3.10)

Where $V_{uc}=\left(\beta_1\beta_2 b_v d_o \left(\frac{A_{st+A_{pt}}}{b_{vd_o}} f_c'\right)^{1/3} + Z_b\left(\frac{P_e}{A} + \frac{P_e e}{Z_b}\right)\left(\frac{4}{l}\right) + P_V\right)$

Substituting the various values from table 3.2

V_{uc} = 1.1x1.0x 350x690$\left(\frac{820+1200}{350X690}35\right)^{1/3}$+ 666$\left(\frac{70000}{241500} + \frac{70000X261}{666}\right)\left(\frac{4}{1}\right)$ +24

V_{uc} =265650+666(0.289855+27432.4324)4+24

=73346446.087772 = 73346.446KN

$\square V_{uc}$ = 0.7x 73346.446KN = 51342.512KN

From equation (3.11)

$V^* = \frac{Wl}{2}$

$W = 1.35G_k + 1.5Q_k + \gamma_p p$

Therefore $V^* = \frac{(Q_k(1.35a+1.5)+\gamma_p P)l}{2}$

$V^* = \frac{(5(1.35X1.5+1.5)+0.9X100)10}{2}$=538.125KN

$\square V_{uc} \geq V^*$

This implies that the beam is safe.

(c) Failure due to Deflection

For the prestressed beam to satisfy the requirement for deflection $\frac{span}{500} > \nu_{max}$

From equation (3.20) and (3.21)

$\nu_{max} = \nu_{sus} + \nu_{cr} + \nu_{sh}$

$\nu_{max} = \frac{5(G_k b - \frac{8p_e e}{l^2})l^4}{384EI} + \frac{5(G_k b - \frac{8p_e e}{l^2})l^4 \emptyset^*}{384EI^\circ \propto} + \beta \frac{\varepsilon^*_{sh}}{D}\left(1 - \frac{A_{sc}}{A_s}\right) l^2$

$$\nu_{max} = \frac{5\left(2^{-3}X350-\frac{8X70000X261}{10000^2}\right)10000^4}{384X28600X10^6X62500X10^6} + \frac{5\left(2^{-3}X350-\frac{8X70000X261}{10000^2}\right)10000^4X3.3}{384X28600X10^6X62500X10^63} +$$

$$0.125\frac{0.00056}{690}\left(1-\frac{620}{820}\right)10000^2$$

$$= -5.5478 \text{ x}10^{-8}+2.474$$

ν_{max} = 2.474 mm

span =L = 10,000

therefore$\frac{L}{500}$= $\frac{10000}{500}$= 20mm

$\frac{L}{500} \geq \nu_{max}$

The beam is safe in deflection.

4.5.2 Reliability- based Design

Parameters for the design of the beam are obtained from the graphs as follow:

From Fig 4.2 P=37KN, dn = 150mm

Fig.4.5 d_n = 200mm ,A_{sc} =1100mm^2

Fig.4.9 d_o = 500m, d_n = 200mm

Fig.4.17 e = 300mm, b =300mm

Fig 4.21 e = 300mm, a_0 =1

(a) Failure due to Bending Moment

(i) Prestress force and depth to the neutral axis

With prestress force of 37KN and depth from extreme compressive fibre to the neutral axis of 150mm from Fig 4.2 the safety of the section is 2.0

(ii) Depth to the neutral axis and area of the compressive reinforcement

With value of depth to the neutral axis of 200mm and area of the compressive reinforcement of 1100mm^2 from Fig. 4.5 the safety of the beam is 2.0

(iii) Depth to the bottom layer of tensile reinforcement and depth to the neutral axis

With the value of depth to the bottom layer of tensile reinforcement of 500mm and depth to the neutral axis of 200mm from Fig. 4.10 the safety of the section is 2.0

(b)Failure due to Shear

(i) Eccentricity and effective width

With eccentricity value of 300mm and effective width of the beam of 300mm from Fig 4.17 the safety of the section is 2.0

(ii) Load ratio and eccentricity

With eccentricity value of 300mm and load ratio 1 from Fig.4.21 the safety of the section is 2.

(b) Failure due to Deflection

The safety of the section in deflection is greater than safety target adopted.

The reliability based design section of the beam with G_k = 2kN, Q_k = 5kN and L= 10m is 300mm x 550mm. The reliability based design Prestressed force for the beam at a safety target of 2 = 37kN and the eccentricity is 300mm.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATION

5.1 CONCLUSION

The study concludes that:

i. The safety of the prestressedconcrete beam in bending decreased as prestress force and the depth to the bottom layer of the beam increased. It increased as depth to the neutral axis, area of the compressive reinforcement and eccentricity increases. Also the safety of the prestressedconcrete beam remains constant in bending with increase in the effective width and load ratio of the beam.

ii. The safety of the prestressed concrete beam increases in shear with increase in effective width, depth to the bottom tensile reinforcement and eccentricity.

iii. The safety of the prestressed concrete beam increases in deflectionwith increase in dead load and span. It alsoremains constant in deflection with increase in eccentricity and prestressing force.

5.3 RECOMMENDATIONS

Based on the outcome of the study and observation its recommended that:

(i) The safety of prestressed concrete beam depends on bending, shears and deflection conditions as revealed in this study.

(ii) Eccentricity value of between 200 to 300mm should be adopted when prestressing a beam for bridge construction.

(iii) When designing a prestressed concrete beam a section of width which is one half the overall thickness should be adopted.

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APPENDIX I

Table 1, Variation of Safety Index withPrestress Force and Load Ratio (Bending Failure)

	$a_0 = 0.2$	$a_0 = 0.4$	$a_0 = 0.6$	$a_0 = 0.8$	$a_0 = 1$
P = 20	2.88	2.88	2.88	2.88	2.88
P = 40	2.37	2.37	2.37	2.37	2.37
P = 60	1.86	1.86	1.86	1.86	1.86
P = 80	1.4	1.4	1.4	1.4	1.4
P = 100	0.986	0.986	0.986	0.986	0.986

Table 2, Variation of Safety Index withPrestress Force and Area of Steel Reinforcement (Bending Failure)

	$A_S = 800$	$A_S = 900$	$A_S = 1000$	$A_S = 1100$	$A_S = 1200$
P = 20	2.87	2.92	2.96	3.01	3.05
P = 40	2.36	2.41	2.45	2.5	2.54
P = 60	1.85	1.9	1.94	1.98	2.02
P = 80	1.39	1.43	1.46	1.5	1.54
P = 100	0.98	1.01	1.05	1.09	1.12

Table 3, Variation of Safety Index with Area of Non-prestressed and Depth to the bottom for (Bending Failure)

	d0 = 400	d0 = 500	d0 = 600	d0 = 700	d0 = 800
As = 800	2.16	2.19	2.16	2.1	2.04
As = 900	2.2	2.23	2.2	2.14	2.08
As = 1000	2.23	2.26	2.24	2.19	2.12
As = 1100	2.26	2.3	2.28	2.23	2.17
As= 1200	2.29	2.34	2.3	2.27	2.21

Table 4, Variation of Safety Index with Area of Non-prestress and Depth to the Neutral Axis (Bending Failure)

	dn = 150	dn = 200	dn = 250	dn = 300	dn = 350
As = 800	1.53	1.91	2.15	2.32	2.44
As = 900	1.59	1.95	2.19	2.36	2.47
As = 1000	1.64	2	2.24	2.4	2.51
As = 1100	1.7	2.05	2.28	2.43	2.54
As = 1200	1.75	2.1	2.32	2.47	2.58

Table 5, Variation of Safety Index with Depth to the Bottom layer and Depth to the Neutral Axis (Bending Failure)

	dn = 150	dn = 200	dn = 250	dn = 300	dn = 350
d0 = 400	1.8	2.04	2.2	2.3	2.36
d0 = 500	1.74	2.03	2.23	2.37	2.46
d0 = 600	1.64	1.98	2.21	2.36	2.47
d0 = 700	1.53	1.91	2.15	2.32	2.44
d0 = 800	1.43	1.83	2.09	2.27	2.4

Table 6, Variation of Safety Index withPrestress Force and Depth to theNeutral Axis(Bending Failure)

	dn = 150	dn = 200	dn = 250	dn = 300	dn = 350
P = 20	2.54	2.27	2.9	2.98	3.04
P = 40	1.87	2.2	2.41	2.55	2.65
P = 60	1.24	1.64	1.91	2.1	2.24
P = 80	0.71	1.15	1.45	1.68	1.84
P = 100	0.27	0.72	1.05	1.29	1.47

Table 7, Variation of Safety Index withPrestress Force and Depth to the Bottom (Bending Failure)

	d0 = 400	d = 500	d = 600	d = 700	d = 800
P = 20	3.69	3.37	3.09	2.86	2.67
P = 40	2.63	2.58	2.48	2.36	2.26
P = 60	1.77	1.85	1.87	1.86	1.83
P = 80	1.12	1.26	1.34	1.4	1.43
P = 100	0.59	0.77	0.9	0.99	1.07

Table 8, Variation of Safety Index with Effective Width of Web and Load Ratio (Shear Failure)

	a_0=0.2	a_0=0.4	a_0=0.6	a_0=0.8	a_0=1.0
b=100	1.738	1.735	1.732	1.729	1.726
b=150	1.74	1.737	1.734	1.731	1.728
b=200	1.742	1.739	1.735	1.732	1.729
b=250	1.743	1.74	1.737	1.734	1.731
b=300	1.745	1.742	1.739	1.736	1.733

Table 9, Variation of Safety Index with Effective Width of Weband Eccentricity (Shear Failure)

	e = 100	e = 150	e = 200	e = 250	e = 300
b = 100	0.133	0.809	1.289	1.661	1.962
b = 150	0.135	0.811	1.291	1.662	1.964
b = 200	0.138	0.813	1.293	1.664	1.966
b = 250	0.140	0.815	1.295	1.666	1.967
b = 300	0.143	0.817	1.297	1.668	1.969

Table 10, Variation of Safety Index with Effective Width of Web and Depth to the Bottom Layer of Tensile Reinforcement (Shear failure)

	do= 300	do =400	do = 500	do = 600	do = 700
b = 100	1.730	1.731	1.731	1.732	1.732
b = 150	1.731	1.732	1.732	1.733	1.734
b = 200	1.732	1.733	1.734	1.735	1.736
b = 250	1.732	1.734	1.735	1.736	1.737
b = 300	1.733	1.735	1.738	1.738	1.739

Table 11, Variation of Safety Index with Load Ratio and eccentricity (Shear Failure)

	e = 100	e = 150	e = 200	e = 250	e = 300
$a_0 = 0.2$	0.153	0.826	1.305	1.676	1.977
$a_0 = 0.4$	0.149	0.823	1.302	1.672	1.974
$a_0 = 0.6$	0.145	0.819	1.299	1.669	1.971
$a_0 = 0.8$	0.142	0.816	1.295	1.666	1.968
$a_0 = 1$	0.138	0.812	1.292	1.663	1.964

Table 12, Variation of Safety Index with Load ratio and Depth to the Neutral Axis (Shear Failure)

	d0 = 900	d0 = 1000	d0 = 1100	d0 = 1200	d0 = 1300
a0 = 0.2	1.740	1.742	1.744	1.745	1.747
a0 = 0.4	1.737	1.739	1.740	1.742	1.744
a0 = 0.6	1.734	1.736	1.737	1.739	1.741
a0 = 0.8	1.731	1.732	1.734	1.736	1.738
a0= 1	1.738	1.729	1.731	1.733	1.734

Table 13, Variation of Safety Index withPrestress Force and Eccentricity (Deflection Failure)

	e =100	e = 150	e = 200	e = 250	e = 300
p = 20	5.068	5.068	5.068	5.068	5.068
P =40	5.068	5.068	5.068	5.068	5.068
p= 60	5.068	5.068	5.068	5.068	5.068
p =80	5.068	5.068	5.068	5.068	5.068
p= 100	5.068	5.068	5.068	5.068	5.068

Table 14, Variation of Safety Index withPrestress Force and Span (Deflection Failure)

	L = 6000	L =8000	L =10000	L = 12000	L = 14000
p = 20	-0.255	2.887	5.068	6.371	7.136
P =40	-0.255	2.887	5.068	6.371	7.136
p= 60	-0.255	2.887	5.068	6.371	7.136
p =80	-0.255	2.887	5.068	6.371	7.136
p= 100	-0.255	2.887	5.068	6.371	7.136

APPENDIX II

```
PROGRAM  MOS1
C     RELIABILITY BASE DESIGN OF A REINFORCED CONCRETE PRESTRESSED BEAM
c     considering bending and shear
c     x(1)=characteristic compressive strength
c     x(2)=width of compressive zone
c     x(3)=ratio of the depth of the idealized rectang. compre. stress
c     x(4)=depth from the extrem compressive fibre to the neutral axis
c       x(5)=depth to the bottom layer of tensile reinforcement
c       x(6)=yield stress of non prestress steel
c       x(7)=area of compressive reinforcement
c       x(8)=depth to the top layer of non prestress steel
c       x(9)=ultimate strength of the prestressing steel
c       x(10)=area of prestress steel
c       x(11)=depth to the prestress steel
c       x(12)=imposed load
c       x(13)=ratio of the dead load to live load
c       x(14)=environmental constant
c       x(15)=prestressing force
c       x(16)=span
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C     THE SUBROUTINE WITH THE LIMIT STATE FUNCTION IS
C     DECLARED EXTERNAL
        EXTERNAL GBEND
        DIMENSION X(16),EX(16),SX(16),VP(10,16),COV(16,16),ZES(3),
     +           UU(16),EIVEC(16,16),IV(2,16)
        CHARACTER*10 PRT
c        common/cbend/load
       DATA EX/3.5D1,3.5D2,1.15D0,2.39D2,6.9D2,4.1D2,8.2D2,6.0D1,1.77D3,
     +       1.2D3,6.5D2,5.0D-3,6.D-1,9.0D-1,1.0D5,1.0D4/,
     +     SX/5.25D0,3.5D0,3.45D-1,2.39D0,6.9D0,6.15D1,1.5D2,6.0D-1,
     +      2.655D2,1.8D2,9.75D1,1.5D-3,2.7D-1,2.7D-1,1.5D4,1.0D2/,
     +     N/16/,NC/16/,NE/16/,IRHO/1/
WRITE(*,*)'ENTER DEPTH TO NEUTRAL AXIS dn....>'
READ(*,*)EX(4)
WRITE(*,*)'ENTER DEPTH TO BOTTOM OF PRESTRESS STEEL d0....>'
READ(*,*)EX(5)
WRITE(*,*)'ENTER AREA OF COMPRESSIVE REINFORCEMENT As....>'
READ(*,*)EX(7)
WRITE(*,*)'ENTER LOAD RATIO ao....>'
READ(*,*)EX(13)
      WRITE(*,*)'ENTER PRESTRESS FORCE P....>'
      READ(*,*)EX(15)

SX(4) = 0.01*EX(4)
SX(5) = 0.01*EX(5)
SX(7) = 0.15*EX(7)
SX(13) = 0.3*EX(13)
SX(15)= 0.15*EX(15)
C       THE RESULTS ARE WRITTEN TO NAUS 7
        NAUS=7
C       PRINT TO SCREEN
        ICRT=0
OPEN(7,FILE='hadiz1.RES',STATUS='OLD',ERR=10)
        GOTO 20
```

```
10      OPEN(7,FILE='hadiz1.RES',STATUS='NEW')
C       PRESETTING VARIABLES VP,COV,AND IV IS DONE USING YINT
20      CALL YINIT (N,IV,VP,IRHO,COV,NC)
IV(1,1)=3
IV(1,2)=3
IV(1,4)=5
IV(1,6)=3
IV(1,7)=3
IV(1,9)=3
IV(1,12)=5
IV(1,13)=7
IV(1,14)=3
        DO 100 I=1,N
100      X(I) = EX(I)
        V1=1.0
        BETA=1.D0
C       INITIAL SOLUTION ESTIMATE
        WRITE (NAUS,5000)
5000    FORMAT (////,5X,70('*'),/,30X,'F O R M 5',/,5X,70('*'),/,
     + 'ECCENTRICALLY LOADED CONNECTION OF STEEL, 16 VARIABLES:')
        CALL YKOPF (NAUS,N,IV,EX,SX,VP,IRHO)
        WRITE (ICRT,*) ' START OF FORM5'
        WRITE (ICRT,*) ' STOCHASTIC MODEL :'
        CALL YKOPF (ICRT,N,IV,EX,SX,VP,IRHO)
        PRT='  COV  '
        CALL YMAUS (NAUS,NC,N,COV,PRT)
        CALL FORM5(N,IV,EX,SX,VP,GBEND,IRHO,COV,NC,
     +            EIVEC,NE,V1,NAUS,BETA,X,UU,ZES,IER)
C       THE CORDINATE OF THE BETA POINT ARE PRINTED WITH THE
C       THE TITLE VECTOR UU
        PRT='UU'
        CALL YFAUS (NAUS,N,UU,PRT)
C       THE VECTOR ZES IS PRINTED WITH THE TITLE VECTOR ZES
        PRT='ZES'
        CALL YFAUS (NAUS,3,ZES,PRT)
        WRITE (ICRT,*) ' END OF FORM5 : IER =',IER
        WRITE (ICRT,*) ' RESULTS SEE FILE hadiz1.RES'
        STOP
        END

        SUBROUTINE GBEND (N, X, FX, IER)
        IMPLICIT DOUBLE PRECISION (A-H,O-Z)
        DIMENSION X(N)
c        common/cbend/load
        A=(X(5)-X(4)/2)
        B=0.85*X(1)*X(2)*X(3)*X(4)*A
        C=X(6)*X(7)*(X(5)-X(8))
        D=0.9*X(9)*X(10)*(X(5)-X(11))
        FS=B+C-D
        E=(1.35*X(13)+1.5)
        AG=X(12)*E
        AH=AG+(x(14)*x(15))
        PT=(AH*(X(16)**2))/8
C       CHECK FOR ERRORS
IF(PT.GT.0)THEN
        FX=FS-PT
        IER=0
```

```
ELSE
FX=1.D+20
IER=1
ENDIF
RETURN
END
```

```
PROGRAM  MOS1
C     RELIABILITY BASE DESIGN OF A REINFORCED CONCRETE
C     PRESTRESSED BEAM
c     considering bending and shear
c     x(1)=strenght reduction factor
c     x(2)=tension stiffening constant
c     x(3)=tension stiffening constant
c     x(4)=effective width of web for shear calculation
c       x(5)=depth to the bottom layer of tensile reinforcement
c       x(6)=area of non-prestressed tensile
c       x(7)=area of prestressed steel in tensile zone
c       x(8)=characteristic compressive strenght at 28 days
c       x(9)=bottom section muduli
c       x(10)=effective prestressing force after time-dependent losses
c       x(11)=cross sectional area of beam
c       x(12)=eccentricity
c       x(13)=vertical component of prestressing force
c       x(14)=live load
c       x(15)=ratio of dead to live load
c       x(16)=presstressing force
c       x(17)=span
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C     THE SUBROUTINE WITH THE LIMIT STATE FUNCTION IS
C     DECLARED EXTERNAL
        EXTERNAL GSHEAR
        DIMENSION  X(17),EX(17),SX(17),VP(10,17),COV(17,17),ZES(3),
     +           UU(17),EIVEC(17,17),IV(2,17)
        CHARACTER*10 PRT
common/cshear/ALPHA
        DATA EX/7.0D-1,1.1D0,1.0D0,2.6D2,1.1D3,6.2D2,1.8D3,3.2D1,6.66D2,
     +         1.79D9,2.42D5,2.61D2,2.04D5,3.0D-3,6.0D-1,1.0D6,1.0D3/,
     +      SX/7.0D-3,1.1D-1,1.0D-3,2.6D0,1.1D1,9.3D1,2.7D2,4.8D0,6.7D0,
     +         5.37D8,2.42D3,7.83D1,6.12D4,9.0D-4,1.8D-1,3.0D5,1.0D1/,
     +      N/17/,NC/17/,NE/17/,IRHO/1/
WRITE(*,*)'ENTER WIDTH OF WEB bV....>'
READ(*,*)EX(4)
WRITE(*,*)'ENTER DEPTH TO THE BOTTOM LAYER d0....>'
READ(*,*)EX(5)
WRITE(*,*)'ENTER ECCENTRICITY e....>'
READ(*,*)EX(12)
WRITE(*,*)'ENTER LOAD RATIO a....>'
        READ (*,*)EX(15)
WRITE(*,*)'ENTER PRESSTRESSING FORCE P....>'
READ(*,*)EX(16)
SX(4) = 0.01*EX(4)
SX(5) = 0.01*EX(5)
SX(12) = 0.3*EX(12)
SX(15) = 0.3*EX(15)
SX(16) = 0.3*EX(16)
C       THE RESULTS ARE WRITTEN TO NAUS 7
        NAUS=7
C       PRINT TO SCREEN
        ICRT=0
OPEN(7,FILE='HADIZ2.RES',STATUS='OLD',ERR=10)
        GOTO 20
10      OPEN(7,FILE='HADIZ2.RES',STATUS='NEW')
C       PRESETTING VARIABLES VP,COV,AND IV IS DONE USING YINT
```

```
20      CALL YINIT (N,IV,VP,IRHO,COV,NC)
IV(1,6)=3
IV(1,7)=3
IV(1,8)=3
IV(1,10)=5
IV(1,12)=5
IV(1,16)=5
IV(1,17)=3
        DO 100 I=1,N
X(I) = EX(I)
100     CONTINUE
        V1=1.0
        BETA=1.D0
C       INITIAL SOLUTION ESTIMATE
        WRITE (NAUS,5000)
5000    FORMAT (////,5X,70('*'),/,30X,'F O R M 5',/,5X,70('*'),/,
     + 'ECCENTRICALLY LOADED CONNECTION OF STEEL, 17 VARIABLES:')
        CALL YKOPF (NAUS, N, IV, EX, SX, VP, IRHO)
        WRITE (ICRT,*) ' START OF FORM5'
        WRITE (ICRT,*) ' STOCHASTIC MODEL :'
        CALL YKOPF (ICRT, N, IV, EX, SX, VP, IRHO)
        PRT='  COV  '
        CALL YMAUS (NAUS,NC,N,COV,PRT)
        CALL FORM5 (N, IV, EX, SX, VP, GSHEAR, IRHO, COV, NC,
     +            EIVEC, NE, V1, NAUS, BETA, X, UU, ZES, IER)
C       THE CORDINATE OF THE BETA POINT ARE PRINTED WITH THE
C       THE TITLE VECTOR UU
        PRT='UU'
        CALL YFAUS (NAUS,N,UU,PRT)
C       THE VECTOR ZES IS PRINTED WITH THE TITLE VECTOR ZES
        PRT='ZES'
        CALL YFAUS (NAUS,3,ZES,PRT)
        WRITE (ICRT,*) ' END OF FORM5 : IER =',IER
        WRITE (ICRT,*) ' RESULTS SEE FILE HADIZ2.RES'
        STOP
        END

        SUBROUTINE GSHEAR (N, X, FX, IER)
        IMPLICIT DOUBLE PRECISION (A-H,O-Z)
        DIMENSION X(N)
common/cshear/ALPHA
        A=(X(6)+X(7))
        B=(X(4)*X(5))
        C=((A/B)*X(8))**(1/3)
        D=X(1)*X(2)*X(3)*X(4)*X(5)*C
        E=((X(10)/X(11))+(X(10)*X(12)/X(9)))
        AG=4/X(14)
        AH=X(9)*E*AG
        FS=D+AH+X(13)
        O=X(14)*((1.35*X(15)+1.5)+(ALPHA*X(16)))
        PT=O*X(17)/2
C       CHECK FOR ERRORS
IF(PT.GT.0)THEN
        FX=FS-PT
        IER=0
        ELSE
        FX=1.D+20
```

```
IER=1
ENDIF
RETURN
END
```

```
      PROGRAM MOS1
C     RELIABILITY BASE DESIGN OF A REINFORCED CONCRETE PRESTRESSED BEAM
c     considering bending and shear and deflection
c     x(1)=dead load
c     x(2)=width of compressive zone of cross section of the beam
c     x(3)=span
c     x(4)=effective prestressing force after time dependent losses
c      x(5)=eccentricity of prestress
c      x(6)=elastic modulus
c      x(7)=second moment of inertia about centriodal axis
c      x(8)=final creep coeficient at time infinity
c      x(9)=parameter to account for the effect of cracking and
reinforcement
c      x(10)=deflection coefficient
c      x(11)=final shrinkage strain at time infinity
c      x(12)=overall depth of the cross section
c      x(13)=compressive reinforcement
c      x(14)=area of the non-prestress
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C     THE SUBROUTINE WITH THE LIMIT STATE FUNCTION IS
C     DECLARED EXTERNAL
      EXTERNAL GDEFLECTION
      DIMENSION X(14),EX(14),SX(14),VP(10,14),COV(14,14),ZES(3),
     +   UU(14),EIVEC(14,14),IV(2,14)
      CHARACTER*10 PRT
common/cdeflection/load
C     THE MEAN AND STANDARD DEVIATION OF THE VARIABLES AS WELL
C     PARAMETERS OF FORM5 ARE GIVEN IN THIS DATA BLOCK
       DATA EX/2.D3,3.D2,1.D4,2.61D2,1.79D3,2.86D4,6.25D10,3.3D0,3.D0,
     +       1.25D-1,5.6D-4,1.15D3,9.D2,1.15D4/,
     +     SX/6.D-1,3.D0,1.D2,2.61D0,5.37D2,2.86D2,6.25D8,3.3D-2,3.D-2,
     +       1.25D-3,5.6D-6,1.15D1,9.D0,1.15D2/,
     +     N/14/,NC/14/,NE/14/,IRHO/1/

WRITE(*,*)'DEAD LOAD GK....>'
READ(*,*)EX(1)
WRITE(*,*)'SPAN L....>'
READ(*,*)EX(3)
WRITE(*,*)'EFFECTIVE PRESTRESSING FORCE Pe....>'
READ(*,*)EX(4)
WRITE(*,*)'ECCENTRICITY e....>'
READ(*,*)EX(5)
WRITE(*,*)'OVERALL DEPTH OF CROSS SECTION D...>'
READ(*,*)EX(12)
WRITE(*,*)'COMPRESSIVE REINFORCEMENT ASC....>'
READ(*,*)EX(13)
WRITE(*,*)'AREA OF NON-PRESTRESS AS....>'
READ(*,*)EX(14)
SX(1) = 0.3*EX(1)
SX(3) = 0.01*EX(3)
SX(4) = 0.3*EX(4)
SX(5) = 0.01*EX(5)
SX(12) = 0.01*EX(12)
SX(13) = 0.01*EX(13)
SX(14) = 0.01*EX(14)
C       THE RESULTS ARE WRITTEN TO NAUS 7
        NAUS=7
```

```
C       PRINT TO SCREEN
        ICRT=0
OPEN(7,FILE='HADIZ3.RES',STATUS='OLD',ERR=10)
        GOTO 20
10      OPEN(7,FILE='HADIZ3.RES',STATUS='NEW')
C       PRESETTING VARIABLES VP,COV,AND IV IS DONE USING YINT
20      CALL YINIT (N,IV,VP,IRHO,COV,NC)
IV(1,1)=7
IV(1,2)=2
IV(1,4)=2
IV(1,5)=7
IV(1,7)=2
IV(1,13)=3
        DO 100 I=1,N
100     X(I) = EX(I)
        V1=1.0
        BETA=1.D0
C       INITIAL SOLUTION ESTIMATE
        WRITE (NAUS,5000)
5000    FORMAT (////,5X,70('*'),/,30X,'F O R M 5',/,5X,70('*'),/,
     + 'ECCENTRICALLY LOADED CONNECTION OF STEEL, 14 VARIABLES:')
        CALL YKOPF (NAUS,N,IV,EX,SX,VP,IRHO)
        WRITE (ICRT,*) ' START OF FORM5'
        WRITE (ICRT,*) ' STOCHASTIC MODEL :'
        CALL YKOPF (ICRT,N,IV,EX,SX,VP,IRHO)
        PRT='  COV  '
        CALL YMAUS (NAUS,NC,N,COV,PRT)
        CALL FORM5 (N,IV,EX,SX,VP,GDEFLECTION,IRHO,COV,NC,
     +            EIVEC,NE,V1,NAUS,BETA,X,UU,ZES,IER)
C       THE CORDINATE OF THE BETA POINT ARE PRINTED WITH THE
C       THE TITLE VECTOR UU
        PRT='UU'
        CALL YFAUS (NAUS,N,UU,PRT)
C       THE VECTOR ZES IS PRINTED WITH THE TITLE VECTOR ZES
        PRT='ZES'
        CALL YFAUS (NAUS,3,ZES,PRT)
        WRITE (ICRT,*) ' END OF FORM5 : IER =',IER
        WRITE (ICRT,*) ' RESULTS SEE FILE HADIZ3.RES'
        STOP
        END

        SUBROUTINE GDEFLECTION (N, X, FX, IER)
        IMPLICIT DOUBLE PRECISION (A-H,O-Z)
        DIMENSION X(N)
common/cdeflection/loading

        A=5*((X(1)*X(2)*X(3)**4)-(8*X(4)*X(5)*X(3)**2))
        B=384*X(6)*X(7)
        C=A/B
        D=1+(X(8)/X(9))
        E=X(10)*(X(11)/X(12))
        AG=(1-(X(13)/X(14))*X(3)**2)
        AH=E*AG
        FS=(C*D)+AH
        PT=X(3)/500
        FX=FS-PT
```

```
C       CHECK FOR ERRORS
IF(PT.GT.0)THEN
        FX=FS-PT
        IER=0
        ELSE
        FX=1.D+20
        IER=1
        ENDIF
        RETURN
        END
```