THE BEHAVIOUR AND DESIGN OF REINFORCED CONCRETE SHORT CANTILEVERS UNDER PURE TORSION

BY

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June, 1988.

DECLARATION

I hereby declare that this thesis has been prepared by me and that it is a record of my own research work. It has not been presented in any previous application for a higher degree.

All quotations are indicated and the sources of information are specifically acknowledged by means of refrences.

Sign: _____

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CERTIFICATION

This thesis entitled:

'THE BEHAVIOUR AND DESIGN OF REINFORCED CONCRETE SHORT CANTILEVER UNDER PURE TORSION' by Adebayo Johnson Fasuba meets the regulations governing the award of the degree of M.Sc. Engineering of Ahmadu Bello University and is approved for its contribution to knowledge and literary presentation.

Chairman, Supervisory Committee Date: _____

Member, Supervisory Committee Date: _____

Member, Supervisory Committee Date: _____

Head of Department Date: _____

Dean, Postgraduate School Date: 31-1-89

DEDICATION

This piece of work is dedicated to my parents for their encouragement over the years.

ABSTRACT

A theory for the design of reinforced concrete short cantilever under pure torsion is presented. It is based on the shear-friction theory. The failure plane assumed is supported by observations on test-specimens. Based on the failure surface, the failure torque was found to be primarily a function of percentage of longitudinal reinforcement, dimension of the cross section and the characteristic strength of steel. The torsional capacities of reinforced specimens tested agreed well with the theoretically predicted values. Torsional reinforcement perpendicular to the assumed crack at the root of the cantilever is preferred to reinforcement inclined at 45 to the axis of the short cantilever.

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LIST OF SYMBOLS USED

T	=	Torque or Torsional Moments
X_1	=	Smaller dimension of links
Y_1	=	Longer dimension of Links
S	=	Spacing of links
f_y	=	Characteristics Strength of steel
As	=	Area of Cross-section of links
Tr	=	Torque of reinforced Concrete Section
T_p	=	Torque of Plain Concrete
p	=	Percentage of reinforcement
A_1	=	Area of Longitudinal reinforcement
A_o	=	Area enclosed between the centres of longitudinal bars provided.
P_o	=	Perimeter formed by the longitudinal bars
T_{s1}	=	Torque contribution of stirrup in y-direction
T_{s2}	=	Torque contribution of stirrup in X-direction
αt	=	A constant factor taking care of the non-uniform stress distribution along the stirrups
Tc		Torque resistance of concrete
K_1 and K_2	=	Constant
m	=	Effective volumetric ratio of the longitudinal and transverse tension members
V_t	=	Maximum torsional shear stress
X,y	=	Overall dimensions of a rectangular section $x < y$

ψ_t = A stress fuction which is a function of y/x.

V = Volume of heap of sand

F = Frictional Force

μ = Coefficient of friction (1.4 for monolithic concrete)

N = Normal force

a,b = Distance between the centre line of the short element and points of fixity from both ends respectively

M_o = Concentrated Moment

R = Reaction

EI = Flexural rigidity

θ = Rotation

V = Deflection.

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CHAPTER ONE

INTRODUCTION

1.1 Background

Torsion in reinforced concrete structures arises from the monolithic nature and assymetry in loading of the structures. This has been considered a secondary problem in the design of reinforced concrete structures. Recently, greater attention has been paid to this aspect of design as there is a growing tendency among engineers and architects to use structural systems and shapes in which torsion is a major type of loading like helical stairs spandrel beams and girders. Furthermore, design methods have become more realistic, safety factors have been reduced, torsion must now be accounted for in the design of concrete structures. Numerous aspects of torsion in reinforced concrete elements had been and are currently being examined in various parts of the world, however the study of short cantilevers under torsion has received no attention from investigators.

A short cantilever can be defined as an element in which the shear span/depth ratio is less than or equal to unity. In the design of an industrial building, a structural engineer proposed a cantilever slender beam to be casted monolithically with a short cantilever. In this situation the short cantilever will be under torsion, and shear, when the concentrated load from the crane is at the end of the slender cantilever beam. While shear and flexure in short cantilever have been

taken care of in text books on reinforced structures, no information exists on the design procedure of short cantilevers under torsion. Also if the line of action of a prestressed beam or girders supported on a short cantilever is off its axis due to construction errors, the short cantilever will be subjected to twisting moments. The above reasons therefore prompted a detailed investigation into the behaviour and design of short cantilevers under torsion.

Thetraditional model (space truss model) on which the design philosophy of codes of practice on torsion is based will not be applicable to short cantilevers. In developing, the space truss model, investigators assumed a crack pattern inclined at 45^{o} to the horizontal axis whereas it is reasonable to assume a crack pattern perpendicular to the horizontal axis of the short cantilever under torsion. The porverty in the space truss model in accommodating the crack pattern of a short cantilever under torsion can be seen below.

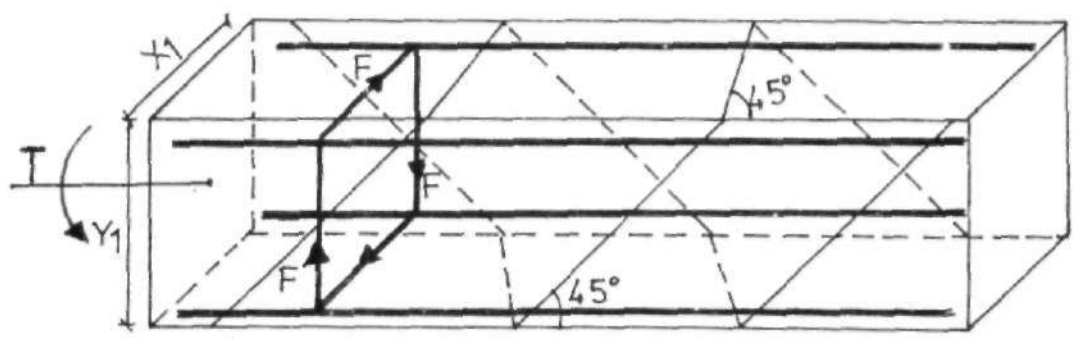


Fig. 1.1 Crack pattern assumed in the space truss model.

Based on equilibrium of moments in the above figure.

$$T = \frac{A_s f_y X_1 Y_1}{S \tan 45^o}$$

T = Torsional Moment

A_s = Area of two legs of stirrup

X_1 = Smaller dimension of links

Y_1 = Larger dimension of links

S = Spacing of stirrups

45^o = Crack propagation angle

fy = Characteristic strength of the links.

If the crack angle of 45^o is replaced by 90^o we obtain

$$T = \frac{A_s f_y X_1 Y_1}{S \tan 90^o}$$

Since $\tan 90 \rightarrow \infty$, $T \rightarrow 0$.

It can therefore be seen that on the basis of the current design philosophy for torsion, the torsional capacity of a reinforced concrete short cantilever approaches zero. This is not so in practice as short cantilevers can resist twisting moments. A rational model that can be used to predict the ultimate torsional resistance of reinforced concrete short cantilever is

therefore desirable.

1.2 Objectives of the Report

The report is designed to

i. study the behaviour and rupture criteria of reinforced concrete short cantilevers under pure torsion.

ii. study the ultimate capacity of short cantilevers under torsion.

iii. derive an expression suitable for ultimate strength design method establishing a relation between material properties, geometrical properties and Torque.

1.3 Scope of Exposition

In the expression that will be arrived at later on, many parameters are involved. To make the tests as wide perspective as possible, the following parametric variations are chosen.

1. Percentage reinforcement.
2. Detailing of reinforcement.

Pure Torsion is desirable because it is usually necessary to know the strength of concrete elements in pure torsion and later see how other actions-shear and bending affect the strength obtained under pure torsion. Also previous investigators have been conservative by using the expressions derived on the basis of pure

torsion in design despite the presence of other actions.

The percentage reinforcement used are 0.5%, 1.0% and 2.0%. Two types of detailings were used. In the type A detailing, the short cantilever reinforcements for torsion intercept the proposed crack at 90^{o} and in the type B detailing the reinforcement intercept the proposed crack at 45^{o}. The essence of the type B detailing is to take care of a tapered short cantilever under torsion.

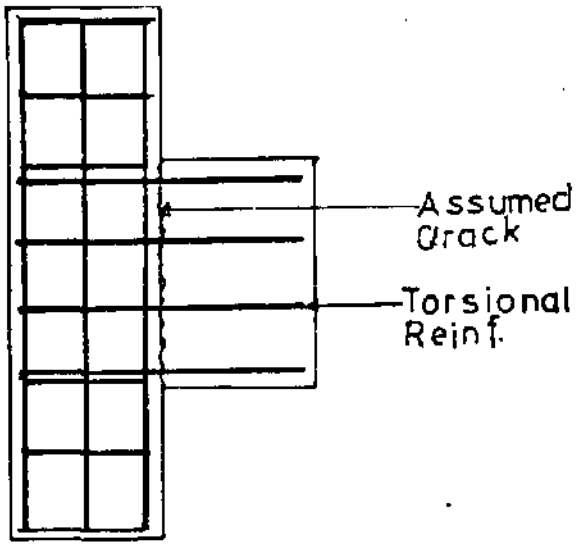


Fig 1.2 Arrangement of Reinforcement in Type A Detailing

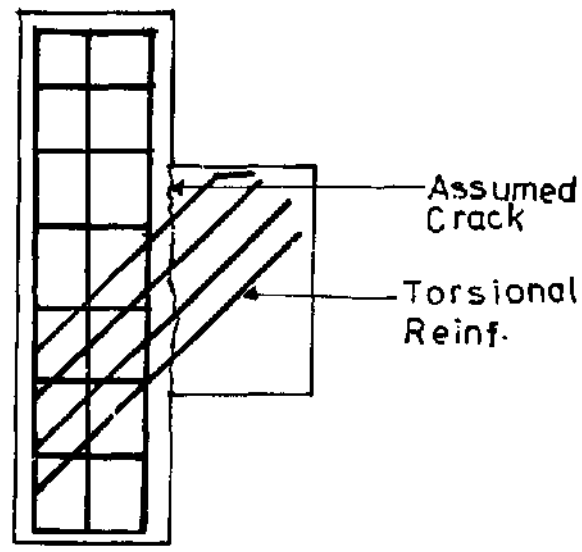


Fig 1.3 Arrangement of Reinforcement in Type B Detailing.

CHAPTER TWO

2.0 REVIEW OF PREVIOUS WORK

Evidence of experiments on torsion performed earlier on is available in literatures but the first detailed experiment on torsion reported in published literature was carried out by Bach and Graf (1913). They tested a variety of plain and reinforced concrete specimens - circular (hollow and full), square and rectangular sections in pure torsion with subsidiary tension and compression tests. The parametric variations used were ratio of longitudinal reinforcement, spiral reinforcement at 45^{o} and inclined reinforcement. At failure, they observed large angle of twist and fractures in form of a continous helix at 45^{o} to the axis of the beams. They concluded that the ultimate torque resistance can be enhanced by using spiral reinforcement at 45^{o} to the axis of the beams.

Graf and Morsch (1922) tested circular specimens 400mm diameter under pure torsion with the following parametric variations - ratio of longitudinal reinforcement, rings, longitudinal bars with rings and longitudinal bars with 45^{o} spirals. They observed that specimens with longitudinal bars alone and rings alone produced no significant increase in ultimate torque. Also specimens with longitudinal reinforcement and rings 0.62% each

produced an increase of 62% in ultimate torque with respect to plain concrete of the same section.specimens with longitudinal reinforcement and 45^{o} spirals produced an increase of 200% in ultimate torque. The above results were confirmed by Young et al (1922). They stated that the increase in ultimate torque due to increase in percentage of longitudinal reinforcement was nil and 80% in beams with 0.79% longitudinal reinforcement and 1.88% spiral reinforcement. Miyamoto (1927) carried out torsional tests under repeated loading. He tested circular beams (300mm diameter) with the following parametric variations - spiral reinforcement at 30^{o} and 60^{o}; spirals with longitudinal reinforcement, spirals with lateral rings; longitudinal bars with lateral rings; two layers of 45^{o} spirals and two layers of 45^{o} spirals with longitudinal reinforcement. He concluded that the ultimate torque for plain concrete was tha. of first crack; that fatique caused a reduction of 18% in the ultimate torque of the sections considered and increment in ultimate resistance ranged from 12% for specimens with longitudinal bars and lateral rings to 118% for specimens with two layers of 45^{o} spirals and longitudinal reinforcement. Failure was along a 45^{o} helix. He observed that the reinforced specimens were not exhausted at maximum torque as continued twisting was possible though with reduced

resistance. This was accompanied by further cracking and spalling of concrete. In no case was the reinforcement stressed to the yield point indicating torque is resisted partly by the steel reinforcement and partly by the concrete.

In 1934 Turner and Davies included T-Sections amongst thespecimens tested under pure torsion. They drew attention to the fact that if the behaviour of concrete is to some degree plastic, then the true ultimate shear stress in a torsion test will be less than the apparent ultimate value based on the elastic theory. They used a ratio of 1:1.2 to correct the elastic ultimate shears stress values. For the T-section the elastic theory predicts infinite stresses in torsion at the re-entrant angle. They showed that this was not so in practice plastic redistribution taking place. They also confirmed the ineffectiveness of the longitudinal reinforcement and that 45^{o} spiral reinforcements are by far the most efficient.

Anderson (1935) performed torsional tests on circular cross-section and square specimens. Various combinations of longitudinal reinforcement, rings and spirals were employed. Hollow plain concrete cylinders were tested at constant load over a period of 110 days to investigate creep. He concluded that with time the shear modulus of concrete is considerably lowered

causing higher final stresses in torsional reinforcement

Marshall and Tembe (1941) published the results of their research into the torsional strength of plain circular, plain and reinforced rectangular, T and L sections. For plain concrete circular specimens, the authors observed that the specimens behaved elastically up to 80% of the failure load. They concluded that the ultimate torque agreed well with calculations based on the mean of elastic and plastic values of torsional stresses. For rectangular sections the tests showed increase in strength over circular sections of 50-80% The suggested the following empirical formula.

$$T_r = T_p (1.33 + 0.1p) \quad \ldots\ldots\ldots\ldots \quad 2.1$$

T_r = Torque of reinforced concrete section

T_p = Torque of plain concrete

p = Percentage reinforcement.

Nylander (1945) reported the results of 60 beams in pure torsion, combined bending and torsion, and combined torsion and shear. He concluded that the behaviour of concrete at failure is plastic - the maximum torsional shearing stress being constant over the whole cross-section and equal to the tensile strength. He finally concluded that additional shearing forces would reduce the torsional strength and compressive forces would increase it. He also noted that

bending improves the torsional strength slightly.

Cowan and Armstrong (1951) published the results of experiments on rectangular reinforced concrete beams under pure torsion and combined bending and torsion. Size of the sections tested were 225x150x1200mm^3. The specimens consisted of plain, reinforced, prestressed concrete and encased joist. They concluded that the ultimate strength of reinforced concrete sections depends on the strength of concrete in tension and prestressing can enhance the torsional strength considerably. For the encased joist the torsional strength was approximately that of concrete. Fo- combined bending and torsion test a dual criterion of failure was observed - primary torsion failure and primary bending failure.

Rausch (1929) proposed the concept of truss analogy which was later modified by other investigators- Anderson, Cowan, Lampert, Thurmalin, Kuyt and Hsu. The theory was developed on the assumption that after crackingthe behaviour of the solid reinforced concrete section may be modelled by a thin walled hollow section in which only the reinforcement cage provides the total resisting torque. The truss analogy assumes that in each shear wall, the concrete between the consecutive diagonal cracks provides the compressive diagonal of the space truss while the longitudinal bars act as a

stringer and stirrups as ties. Expression for the torsional capacity of a reinforced beam have been derived by considering the equilibrium of this space truss model. The ultimate torsional capacity for a section provided with a symmetrical arrangement of longitudinal steel has been shown to be

$$T_{ur} = 2X_1Y_1\sqrt{\frac{A_i f_{iy}}{2(X_1+Y_1)}\frac{A_s f_{sy}}{S}} \quad \ldots\ldots\ldots \quad 2.2$$

Al = total area of longitudinal bars provided to resist torsion

A_s = cross sectional area of one leg of stirrup.

f_{1y} and f_{sy} = yield strength of longitudinal steel and stirrups.

X_1, Y_1 = smaller andlarger centre to centre dimensions of stirrups.

Lampert (1970) showed that equation 2.2 is valid for any compact non symmetrical section in the form

$$T_{ur} = 2A_o\sqrt{\frac{A_1 f_{1y}}{P_o}\quad\frac{A_s f_{sy}}{S}} \quad \ldots\ldots\ldots \quad 2.3$$

A_o = Area enclosed between the centres of longitudinal bars provided.

P_o = Perimeter formed by the longitudinal bars.

Kuyt (1971) used the corresponding outer dimensions of the stirrups which did not affect the results obtained using equation 2.2 adversely. The space truss analogy discussed above suffers from a number of shortcomings amongst them are the om ission of the contribution of concrete to the ultimate torque; effect of increasing the spacing of stirrups and the effect of increasing Al and As respectively.

Hsu (1968) published the results obtained from experiments on torsion. In the ensue analysis, he assumed that all cracks have an inclination of 45^{o} in each face of the rectangular section. The number of stirrups crossing a 45^{o} crack in the y - direction is $y_1/\text{Stan}45^{o}$. Taking torsional moments with respect to the longitudinal axis the contribution ofthe stirrups in the y direction is

$$T_{s1} = K_1 A_s f_{sy} \frac{y_1 X_1}{S} \quad \ldots\ldots\ldots 2.4$$

similarly for the X_1 - direction

$$T_{s2} = K_2 A_s f_{sy} \frac{X_1 Y_1}{S} \quad \ldots\ldots\ldots 2.5$$

K_1 and K_2 were introduced to take into account the fact that steel stresses are not uniform along the stirrup as been observed experimentally. The total contribution of the transverse steel is obtained by adding T_{s1} and T_{s2}

$$\text{ie} \quad T_s = \frac{A_s f_{sy} X_1 Y_1}{S} (K_1 + K_2)$$

$$\text{if} \quad (K_1 + K_2) = \alpha t$$

$$T_s = \alpha t \frac{A_s f_{sy} X_1 Y_1}{S}$$

Hsu proceeded to determine the contribution of concrete T_c to the ultimate torque and αt experimentaliy in which the principal variable is $A_s f_{sy} X_1 Y_1 / S$. If the results are presented graphically the intercept on the y-axis represents T_c and the slope of the line represent αt. T_c obtained by this approach is not measurable but it represents about 40% of the strength of an equivalent plain section. The value αt is not constant and depends principally on the ratio of stirrup Legs Y_1/X_1. Hsu suggested the following relation $\alpha t = 0.66 + 0.33\ Y_1/X_1 \leq 1.5$.

Ernst (1957) and Victor et al (1973) probed the effect of increasing thespacing of stirrups, A_1 and A_s independently. They reported that when the stirrup spacing is progressively increased while keeping A_1 and A_s constant, the ultimate torsional capacity converges to a value equal to that of plain concrete. Also the torsional capacity is not in any way enhanced

by increasing A_l and A_s separately.

Pandit (1970) also demonstrated that the torsional strength of a beam with symmetrical arrangement of longitudinal steel is greater than that obtained with the steel arranged unsymmetrically. Thus equation 2.2 cannot be extended easily to an unsymmetrical distribution of longitudinal steel.

On account of the lapses outlined above Victor et al (1973), Chakraborty (1979) and Pandit (1970) suggested an empirical approach taking into consideration theeffectiveness of the reinforcement as well as the concrete to the ultimate torsional strength.

Pandit and Venkappa (1980) summarised that the various published theories and semi-empirical methods for computing the ultimate torque can be expressed by the general form given below

$$T_u = K_1 T_c + K_2 T_r.$$

T_u = Ultimate torsional capacity

T_c = Torsional capacity of plain concrete

T_r = Torsional capacity of reinforcing steel

K_1 and K_2 are constants which respectively account for the contribution of concrete and steel to the ultimate torsional capacity.

They show that K_1 varies from 0 to 1 and K_2 from 1 to $2\sqrt{m}$ where m is the effective volumetric ratio of the longitudinal and transverse tension members of the space truss.

$$m = \frac{A_l}{2(X_1+Y_1)} \cdot \frac{S}{A_s}$$

In 1985, Narayanan and Kareem - Palanjian carried out series of torsional tests on reinforced concrete beams containing steel fibres. They concluded that the ultimate torque is the sum of the contributions from concrete, reinforcing steel and fibres.

In all the review works attention has been confined to the behaviour and design of reinforced concrete slender beams of various cross-sections under torsion. No mention has been made of reinforced concrete short element of any cross-section under torsion. Reinforced concrete short cantilever can be under torsion when a cantilever slender beam loaded by concentrated load from cranes is casted monolithically with it. Also it may be under torsion when concentrated loads from girders or prestressed concrete beam is off the longitudinal axis of the short element due to construction errors.

CHAPTER THREE

3.0 PROPOSED MODEL FOR SHORT CANTILEVER UNDER PURE TORSION

3.1 Torsional Shear Stress

Many theories have been proposed as regards the distribution of torsional shear stress in a rectangular section. These theories can be classified into elastic and plastic theories.

3.1.1 Elastic Theory

a. St Venant Approach. St Venant postulated that the maximum torsional shear stress is generated at the middle of the longer

$$\nu_t = \psi_t T/X^2Y \quad \ldots\ldots\ldots\ldots \quad 3.1$$

T = Torsional Moment at the section

X,Y = Overall dimensions of the rectangular section $x < y$.

ψ_t = a stress factor which is a function of y/X.

b. Prandtl's Membrane Analogy. In this approach a soap film is streched across a hole cut on the top of a box to the shape of the cross-section. A small inside air pressure is then used to raise a bubble over the hole. The volume between the bubble

and the original plane is proportional to the torque resistance of the section. The slope of the membrane which is under uniform tension measures the unit torsional shear stress. The membrane analogy is very useful in visualizing the torsional shear stress distribution over compound cross-section. The assumptions associated with linear elastic behaviour are seldom encountered in reinforced concrete.

3.1.2 Plastic Theory (Nadal Sand Heap Analogy)

According to this theory, the volume of sand placed over a given cross-section is proportional to the plastic torque sustained by the section. The heap over the rectangular section has a height XV_t

X = smaller dimension of the cross section.

For a rectangular section of dimension X and y where X <y, the volume of the heap of sand is

$$V = \frac{X^2}{3} xV_t + (y-x)x.\frac{xV_t}{2}$$

$$V = T = \frac{x^2 V_t}{2} (y-\frac{X}{3}) \quad \ldots\ldots\ldots\ldots 3.2$$

Neither of the above analyses gives a true picture of the maximum torsional stress distribution because concrete is not ductile enough particularly in tension to permit plastic distribution of shear stress

and also, concrete will be cracked at failure. To be on the safer side researchers prefer the lower bound values given by the plastic analysis and this forms the basis for calculating the ultimate torsional stress in the British Code of Practice CP 110. The ACI 318-71 went a step further by comparing equations 3.1 and 3.2

$$\text{ie } V_t = \psi_t T/X^2 y$$

and

$$V_t = \frac{2T}{X^2(y-\frac{X}{3})} = \frac{2}{1-\frac{1}{3}\frac{X}{y}} \quad \frac{T}{X^2 y}$$

$$\text{ie } \psi_t = \frac{2}{1-\frac{1X}{3y}}$$

$$\text{when } \frac{X}{y} = 1, \ \psi_t = 3$$

$$\text{when } \frac{X}{y} = 0 \ \ \psi_t = 2$$

Thus $\psi_t = 3$ is the maximum for the plastic theory, hence the ultimate torsional stress corresponding to this value is

$$V_t = \frac{3T}{X^2 y} \quad \ldots\ldots\ldots\ldots\ldots \quad 3.3$$

The ultimate torsional shear stress induced by torsion is therefore based on equation 3.3

3.2 Design Model

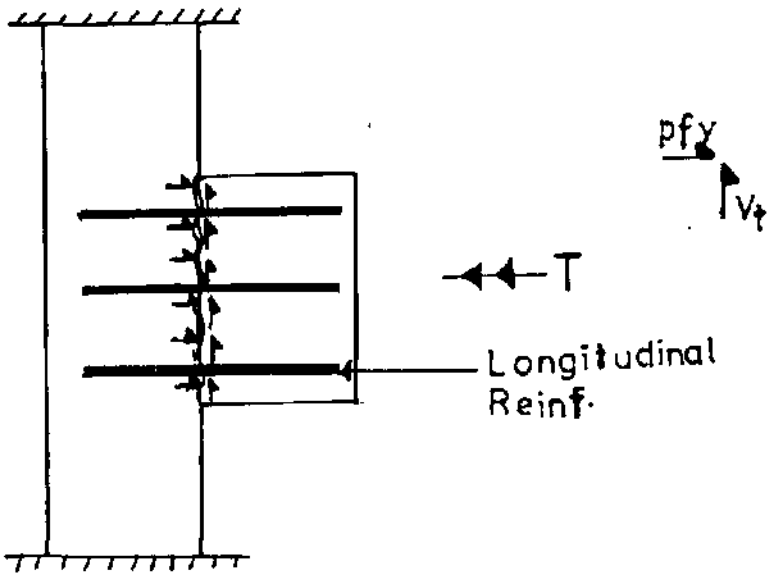


Fig 3.1 Shear friction concept

Assuming the crack pattern shown in Fig 3.1 on slippage due to the applied torque, a relative displacement between the short canti-lever and the vertical member takes place. This displacement gives rise to tensile forces in the longitudinal steel which produces reacting compressive forces on the contact surface between the short canti-lever and the vertical member. Through the presence of these compressive forces, friction forces are generated. This can be estimated by equation of the form.

$$F = \mu N \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 3.4$$

F = friction force = shear resistance due to the applied torque at the

interface between the vertical member and the short cantilever.

μ = Coefficient of friction (1.4 for monolithic concrete)

N = Normal force i.e. tension developed in the longitudinal steel. At yield $N=A_s f_y$.

Dividing both sides of equation 3.4 by the cross-sectional area Xy and substituting $N = A_s f_y$ we obtain ie $\frac{F}{XY} = \frac{\mu A_s}{XY} f_y$

$$V_{tmax} = \mu p f_y \quad \ldots\ldots\ldots\ldots\ldots 3.5$$

V_{tmax} = maximum torsional shear stress due to the torque applied and equal to $3T/X^2Y$ from equation 3.3

p = ratio of longitudinal reinforcement

f_y = characteristic strength of longitudinal steel.

$$\text{i.e } \frac{3T}{X^2Y} = \mu p f_y$$

$$\text{i.e } T = \frac{1}{3} \mu p X^2 Y f_y \quad \ldots\ldots\ldots\ldots 3.6$$

Equation 3.6 is the proposed design equation.

3.3 Structural Analysis Scheme

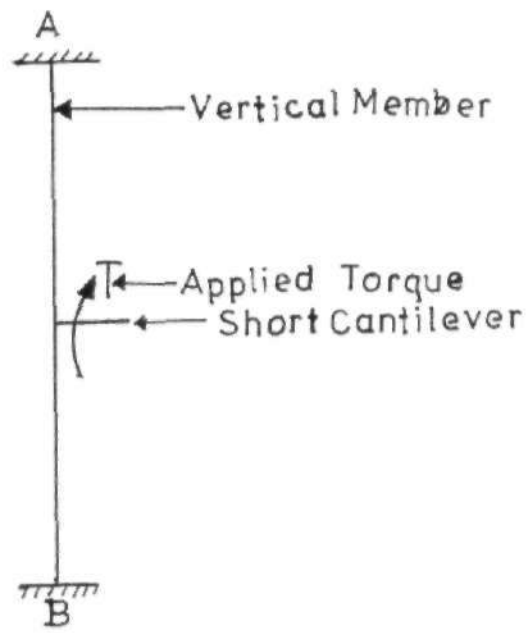


Fig 3.2 Analysis scheme of member AB

The effect of the applied torque on the short element is a moment on member AB. Member AB can thus be analysed as a beam subjected to concentrated moment.

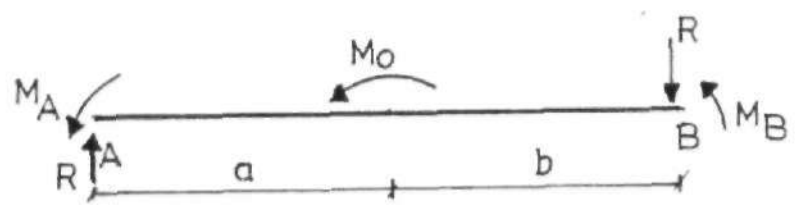


Fig 3.2a Forces acting on member AB

Take the reaction and moment at end B as redundants.

The flexibility approach employing the moment-area for the determination of rotation θ and deflection V of end B is used to determine the unknown moment and reactions.

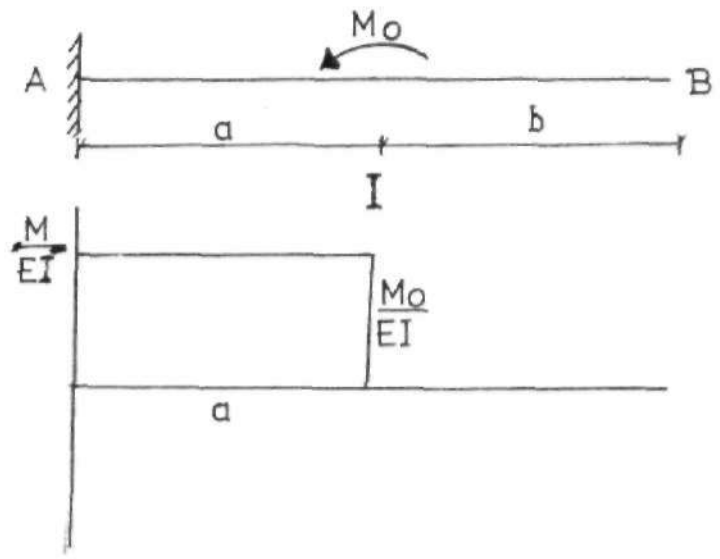


Fig 3.2b Pry structure, $\frac{M}{EI}$ diagram due to concentrated moment M_o.

$$\theta_{BI} = \frac{M_{oa}}{EI} \quad , \quad V_{BI} = (\frac{a}{2} + b)a\frac{M_o}{EI}$$

Removing the applied load M_o and inserting one redundant.

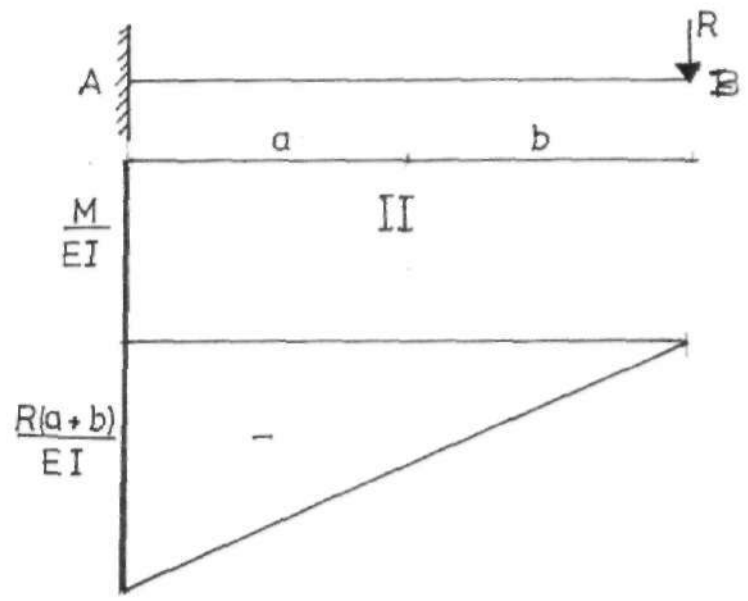


Fig 3.2c Sub-structure and $\frac{M}{EI}$ diagram due to reaction R.

$$\theta_{B¶} = -\frac{R(a+b)^2}{2EI} \quad , \quad V_{B¶} = -\frac{2}{3}(a+b)^3\tfrac{1}{2}\frac{R}{EI}$$

Removing the applied load M_o and inserting the second redundant.

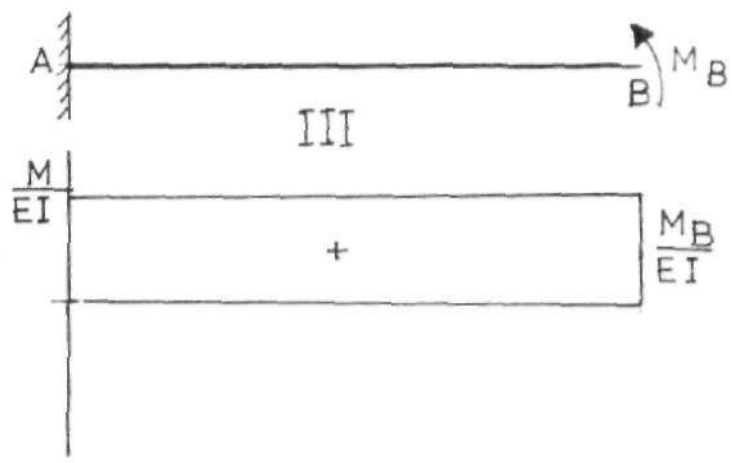


Fig 3.2d Sub-structure and $\frac{M}{EI}$ diagram due to moment MB.

$$\theta_{BIII} = \frac{M_B(a+b)}{EI} \quad , \quad V_{BIII} = \frac{(a+b)^2}{2EI} M_B$$

Imposing Compatibility

$$\theta_{BI} + \theta_{BII} + \theta_{BIII} = 0 \text{ ie } -\frac{R(a+b)^2}{2EI} + \frac{M_B(a+b)}{EI} = -\frac{M_{oa}}{EI} \quad \ldots 1$$

$$V_{BI} + V_{BII} + V_{BIII} = 0 \text{ ie } -\frac{R(a+b)^3}{3EI} + \frac{M_B(a+b)^2}{2EI} = -M_o(a+2b)\frac{a}{2EI}$$

$\ldots\ldots\ldots$ 2

Solving the above equations simultaneously multiply equation 1 by $\frac{a+b}{2}$ and subtract equation 2 from equation 1 we obtain

$$-\frac{R}{4}(a+b)^3 + \frac{R}{3}(a+b)^3 = -\frac{M_o(a+b)a}{2} + M_o(a+2b)\frac{a}{2}$$

$$\text{i.e } \frac{R}{12}(a+b)^3 = \frac{M_o ab}{2}$$

$$\text{i.e } R = \frac{6M_o ab}{(a+b)^3}$$

Substitute R into equation 1.

$$M_B(a+b) = \frac{3M_o ab}{a+b} - M_o a.$$

$$\text{i.e } M_{B(a+b)} = M_o a\left(\frac{3b}{a+b} - 1\right) = M_o\left(\frac{2b}{a+b} - a\right)a$$

$$\text{i.e } M_B = \frac{M_o(2b - a)a}{(a+b)^2}$$

$$\Sigma M \text{ @ } A = 0$$

$$\text{i.e } M_A + M_o + M_B = R(a+b)$$

$$M_A = R(a+b) - M_o - M_B \quad \text{substituting R and } M_B$$

$$M_A = \frac{M_o b(2a+b)}{(a+b)^2}$$

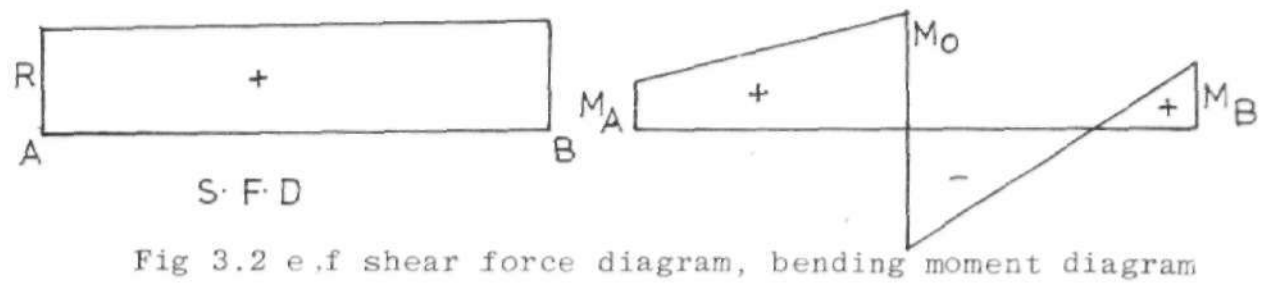


Fig 3.2 e.f shear force diagram, bending moment diagram of member AB of member AB

3.4 A Typical Example (Sepecimen A2)

Data:

Type A. Detailing

$$p = 1.0\%$$

$$f_y = 250N/mm^2$$

$$X,Y = 200mm,\ 300mm$$

The predicted failure Torque is given by equation 3.6

i.e. $T_{pf} = \frac{1}{3}(1.4)x0.01x(200)^2x300x250$

$$T_{pf} = 14 \times 10^6 Nmm$$

This torque is transferred as a concentrated moment to the vertical member AB shown in fig 3.2.

For specimen A2, a,b defined in Fig 3.2 a are 0.296m and 0.293m respectively: $M_o = 14KNm$.

The shear force along member AB $= R = \frac{6M_{oab}}{(a+b)^3}$

i.e $R = \frac{6x14 \times 0.296 \times 0.293}{(0.589)^3} = 35.65KN$

$$+M_{max} = \frac{M_o}{2} = 7KNm.$$

$$-M_{max} = \frac{M_o}{2} = 7KNm.$$

3.5 Theoretical Results

Table 3.1 Theoretically predicted failure torque for specimens obtained according to equation 3.6

Specimen	p%	T_P KNm
A1	0.5	7.0
A2	1.0	14.0
A3	2.0	28.0
B1	0.5	7.0
B2	1.0	14.0

Values predicted for Type A Detailing and Type B detailing with the same p were the same because the same area of reinforcement crosses the section in both type detailing.

CHAPTER FOUR

4.0 EXPERIMENTAL INVESTIGATION

4.1 Materials

4.1.1. Aggregates

The coarse aggregate used in the test was selected from a 19.05mm sized batch of crushed rock. In order to ensure effective compaction in a condition of congested reinforcement, it was decided that aggregates with a maximum nominal size of 15mm should be used. For the purpose of mix design, the aggregate was grouped into two - coarse aggregate and medium coarse aggregate. Aggregates retained on 9.52mm size sieve are coarse aggregate and those retained on 4.76mm size sieve are medium coarse aggregates. The coarse aggregate and medium coarse aggregate were sieved separately and stored in a dry cubicle within the concrete laboratory. The fine aggregate used was sieved excluding the portion with nominal size more than 4.76mm. A sample of the sieved fines was further subjected to sieve analysis in order to determine its classification. The following were obtained in a randomly selected 1000g sample.

Table 4.1 Reduced sample of fine aggregate.

Sieve size(mm)	Mass retained(g)	% retained	Cumulative % Passing	Cumulative % retained
9.52	30	03	97	03
4.76	50	05	92	08
1.18	260	26	66	34
600μ	250	25	41	59
300μ	310	31	10	90
150μ	90	09	01	99
<150μ	10	01	00	100

With this result, the fine aggregate used belongs to the zone 2 of BSS 882 fine aggregate classification.

It is important to use combined aggregate with a grading such that a reasonable workability and minimum segregation are obtained. To this end, the coarse, medium coarse and fine aggregates were combined in a certain proportion. The aim of this proportioning was to adjust the combined aggregate to be in zone B or C of the Road Research Note No. 4 on the Design of Concrete Mixes prepared for 19.05mm maximum size of aggregate. The combined aggregate in either zone has been shown on the basis of experience to give the desired properties stated above. Various proportions were tried, but the proportion finally selected was 40: 25: 35 (Fine Agg: Medium Coarse: Coarse).

The result of thesieve analysis carried out on the combined aggregate is given in Table 4.2

Table 4.2 Reduced Sample of Combined Aggregate

Sieve size(mm)	Mass retained(g)	% retained	cumulative % Passing
9.52	300	30	70
4.76	270	27	43
1.18	180	18	25
600μ	130	13	12
300μ	110	11	01
150μ	20	00	00
<150μ	00	00	00

The sieve analysis revealed that the combined aggregate belongs to zone B of theRoad Research Note No. 4 on the Design of Concrete Mixes prepared for 19.05mm maximum size of aggregate.

4 1.2 Cement

The cement used was a locally produced brand of ordinary portland cement property stored in a dry environment for it to maintain its integrity. It was deemed unnecessary to conduct a separate quality test on the cement since the cube strength test will be performed and is the main characteristic needed. The concrete strength desired is 30MPa.

4.1.3 Reinforcement

Mild steel bars were used throughout the experiment. Test was conducted to determine the tensile strength in order to check the characteristic strength used in computations. The test was performed on 500 length of bar cut from the same steel used as reinforcement for the various specimens. Gauge length Lo were set on each test specimen such that $Lo = 5.65\sqrt{Ax}$ where Ax is the effective cross-sectional area of the bar. The tensile strength and ultimate strength for each bar obtained from the yield load and ultimate failure load observed from tests using the universal testing machine. The results are tabulated in Table 4.3

Nominal diameter (mm)	Effective cross sectional Area (mm^2)	Guage Length (mm)	Yield Load (KN)	Yield Stress N/mm^2	Ult. Load (KN)	Ult. Stress N/mm^2
8	50	40	13.1	262.0	17.6	352.0
8	50	40	12.6	252.0	15.9	318.7
8	50	40	12.7	254.0	17.1	341.0
10	76	50	19.8	260.5	25.9	341.0
10	76	50	19.5	256.6	26.6	350.0
10	76	50	19.1	251.3	26.0	344.0
16	201	80	53.8	267.7	76.1	378.4
16	201	80	51.5	256.2	73.1	363.5
16	201	80	52.5	261.2	74.4	370.3
Average Yield Stress						
8				256		
10				256		
16				261.7		

4.2 Details and Design of Test Specimens

4.2.1 Details of Specimen

a. Shape and size: The elements chosen for the test programme are single rectangular brackets. Portability was a major consideration in the selection of the dimension of the sections. The section chosen are as follows:

Short cantilever: 200 x 200 x 300mm^3
Vertical member AB: 200 x 200 x 700mm^3
Loading Arm : 1500 x 200 x 300mm^3.

The sections for each specimen are the same. The dimension of the loading arm was such that there will be no shear failure in it by ensuring that the ratio of the length between the applied forces to the effective depth of theloading arm lies between 3 and 6

$$\text{i.e} \quad \frac{1300}{275} = 4.7$$

The scale of the specimen used can thus be regarded as a full scale. The test specimen components were casted monolithically producing a tee shape.

b. Identification of specimen: Specimens to be casted will be labelled by an alphabet and a number. The alphabet represent the type of detailing while the number represent certain percentage reinforcement. 'A' represent type A

detailing where the torsional reinforcement is perpendicular to the assumed crack at the root of the short cantilever 'B' represent type B detailing where the torsional reinforcement is inclined at 45^{o} to the crack assumed.

Label	p%
A1	0.5
A2	1.0
A3	2.0
B1	0.5
B2	1.0

c. Preparation of the reinforcement cages and formwork.

The various bar sizes required for the specimens were cut to the desired lengths by measurements taken on the reinforcement detailing sketches provided. The torsional bars were bent as shown in a detailed drawing using the bending machine available in the concrete laboratory. The shear links for the components were also formed into their respective shapes and sizes. The various reinforcement cages for the five specimens intended for the test programme were then assembled and tied by means of mild steel binding wire.

A pair of suitable formwork was prepared from plywood for the specimens. The formworks were prepared to be re-useable. The internal surfaces were carefully prepared and smoothed by using wood sand paper. They were then coated with an oil based paint to avoid any surface roughness in the finished elements and also prevent water absorption from the concrete mix.

d. Attachment of strain gauges: The points on the torsional bars where the strain gauges are desired were cleaned. This was accomplished by the use of files to scrape the outer rust. The points chosen were very close to the face of the vertical member AB on the torsional reinforcement. These are the points where the

maximum twisting is expected. In selecting the strain gauges, stability of measurements was a major consideration. This was because measurements will be taken under the action of a simulated static loading. Each stage of loading will extend will extend to 15 minutes. The strain gauges employed were of 5mm gauge length (pl-5-11) type with a gauge factor K=20 The wires used in connecting the gauges were seven strand light electrical wires. They were long enough to give ample access during measurements. The strain gauges were carefully mounted over a thin film of quick drying polymide resin. After drying for about five minutes, they were protected by a film of the same resin layer and left untouched for 24 hours. The electrical wires were then connected to the strain gauges by soldering. The earth resistance of each strain gauge was checked using the multipurpose avometer. This was done to ensure that the continuity of the connections was alright by ensuring that the avometer reading was 120 ohms. Each strain gauge was then identified by a label as in Fig 4.1.

AR = right hand side first level of reinforcement

AL = left hand side first level of reinforcement

BR = right hand side second level of reinforcement

etc.

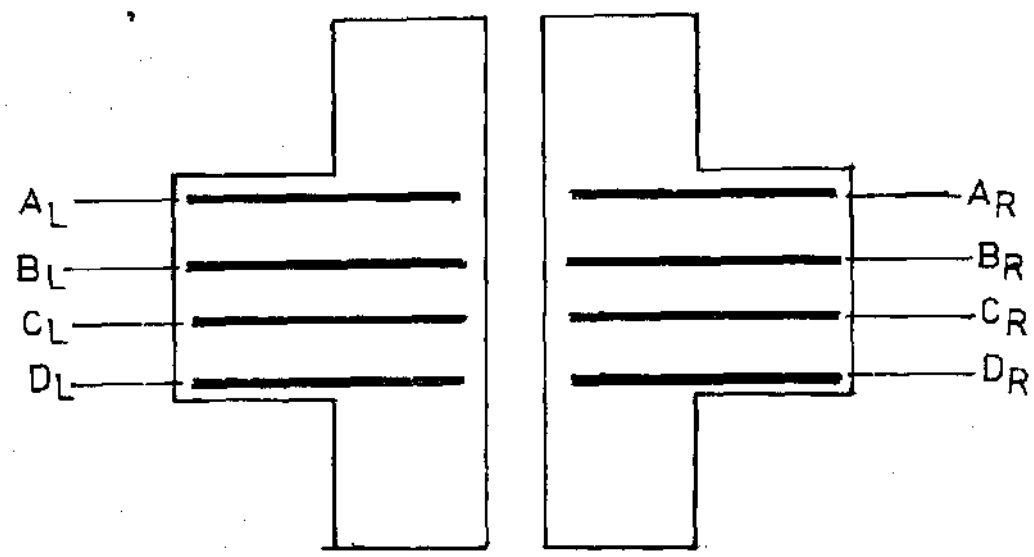


Fig 4.1 Identification of Strain Gauges at each reinforcement levels.

4.2.2 Design of the Test Specimen Reinforcement

The design of the reinforcement is carried out using the results obtained in section 3.4. For specimen A2 considered, the following data were obtained. Analysis and Design of Specimens A1 and A3 are given in the Appendix. Data:

T_{pf} = 14KNm.

Max S.F on AB = 35.65KN

+ B.M = 7KNm.

- B.M = 7KNm.

a. Selection of Torsion Reinforcement.

p = 1%, X = 200mm, Y = 300mm

A_s = 1/100 x 200x300 = $600mm^2$

Use 8-10mm ϕ bars ($628mm^2$)

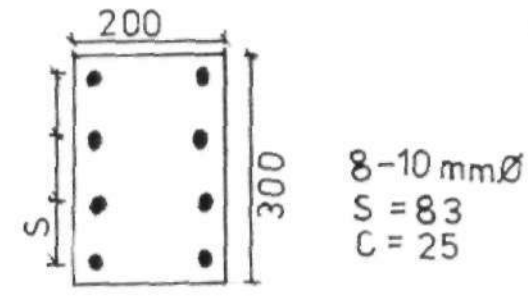


Fig 4.2 Section showing Torsional Reinforcement for Specimen A2.

b. Design of the loading Arm.

Assuming the distance between the forces constituting the couple is 1.30m. Also the loading arm is assumed to act as a cantilever (fixed) on both sides of the short cantilever.

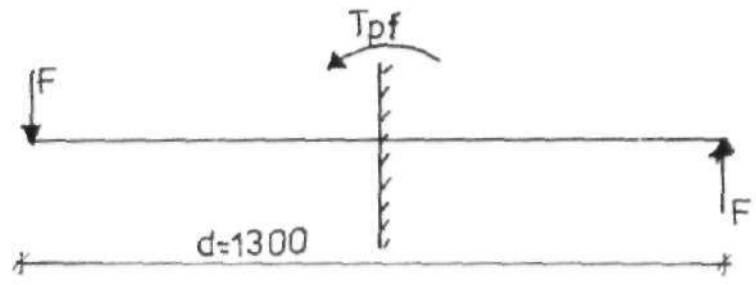


Fig 4.3 Analysis Scheme for the Loading Arm of Specimen A2.

$T_{pf} = Fd$ i.e. $F = \frac{T_{pf}}{d} = \frac{14.00}{1.3} = 10.77KN$

Design force $F_1 = 1.6 \times 10.77 = 17.23KN$

$M_{max} = F_1.d/2 = 17.23 \times 0.65 = 11.2KNm.$

Assuming a singly reinforced section

$M/bd^2 = \frac{11.2 \times 10^6}{200x275^2} = 0.74$

$$p = 0.36, f_{cu} = 30MP_a$$

$$\text{i.e } A_s = \frac{0.36 \times 200 \times 275}{400} = 198mm^2$$

Use 3 - 10mm ϕ bars ($236mm^2$) top and bottom

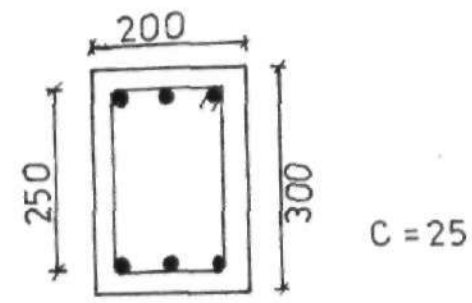


Fig 4.4 Section Showing Reinforcement for the Loading Arm of A2.

Shear: Maximum shear force = 17.23KN

$$\text{Shear stress } v = \frac{17.23 \times 10^3}{200 \times 300} = 0.287$$

Ultimate shear stress $V_c = 0.45MP_a$

$p = 0.36$, $f_{cu} = 30MP_a$

$V < V_c$ Provide minimum area of links

i.e $A_{sv}/S_v = 0.002b_t$, Min $S_v = 100mm$.

i.e $A_{sv} = 0.002 \times 200 \times 100 = 40mm^2$

Use 6mm ϕ bars ($A_{sv} = 57mm^2$) at 100mm spacing.

c. Design of Vertical Member AB.

Max + B.M = 7KNm, Max - B.M = 7KNm.

Assuming a singly reinforced section

$$M/bd^2 = \frac{7 \times 10^6}{200 \times 175^2} = 1.14, \quad p = 0.55$$

$$A_s = 0.55 \times 200 \times 175/100 = 192.5mm^2$$

Use 3-10mm ϕ bars ($A_s = 236mm^2$)

Shear: Maximum S.F = 35.65KN

$$V = \frac{35.65}{200 \times 175 \times 10^3} = 1.02$$

$$V_c = 0.57 \quad p = 0.55, \; f_{cu} = 30MPa$$

$$S_v = \frac{57 \times 0.87 \times 250}{200(1.02-0.57} = 110.2mm$$

Provide 6mm ϕ links at 100mm spacing

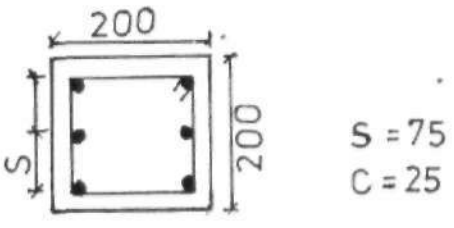


Fig 4.5 Section chosen for vertical member AB of specimen A2.

4.3 Concrete Mix Design

In order to achieve the desired concrete strength of 30MPa, the coarse aggregate, fine aggregate, cement

and water were combined in a certain proportion determined by process of mix design. Based on the mix design a trial mix was done. It was necessary to add more water to achieve the desired slump. Data:

Data:

Desired Workability - medium

Slump - 50 - 100

Minimum strength - 30 mPa

Mean strength - 40 mPa

Agg: cement - 4.5

$e_c = 3.15$, e_{agg} - 2.65

W:C = 0.60

Mix proportion

1 part of cement to (0.4:0.25:0.35)x4.5 for the three aggregates respectively

i.e 1: 1.8 : 1.125:1.575

From

$$\frac{W}{1000} + \frac{C}{1000e_c} + \frac{A_1}{1000e_1} + \frac{A_2}{1000e_2} = 1$$

$$\text{i.e} \quad \left(.6 + \frac{1}{3.15} + \frac{1.8}{2.65} + \frac{(1.125+1.575)}{2.65} \right) C = 1000$$

$$C = 382.41 \text{ kg per } m^3.$$

Water = 0.6 x 382.41 = 229.45 kg/m^3

Fine Agg. = 1.80x382.41 = 688.34 kg/m^3

Medium Agg = 1.125x382.41 = 430.21 kg/m^3

Coarse Agg = 1.575x382.41 = 602.30 kg/m^3

Six cubes were casted based on the preceeding calculations. The results of the crushing strength obtained at 7 days and 28 days are shown in Table 4.4

Table 4.4 Trial Cubes Strength

Cube No.	1	2	3	4	5	6
Mass (kg)	8.0	7.9	8.0	8.0	8.0	8.0
Crushing(KN) Load at 7 days	730	700	740	-	-	-
Crushing(KN) Load at 28days	-	-	-	840	850	880
F_{cu}(mPa)	32.44	31.11	32.89	37.33	37.78	39.11

The trial cubes strength is satisfactory at 28 days.

4.4 Method of Casting

After checking the connections of strain gauges, each reinforcement cage was carefully placed in a form work (prepared with mould oil) at the time of casting. Adequate alignment and spacing of reinforcement for concrete cover were adhered strictly to. The form-work and three standard steel cubes were then placed on the vibrating table. This was done in a careful manner to ensure an even distribution of the concrete mass and hence effect a sound vibration. The mixing and batching of concrete for a specimen was done in two stages. This was because the capacity of the mixing drum was smaller than the volume of the specimen. The casting of the specimens was done in the

order A2, A1, A3, B1 and B2. With each specimen three cubes were casted along to determine the strength of concrete produced. The aggregates and cement were first weighed on the weighing balance. The batch was then transfered to the mixing machine where the contents were dry mixed, using an open rotating drum driven by an electric motor. The required quantity of water based on the water: cement ratio was added, and mixing was repeated till a uniform mix was obtained. Each mass of mixed concrete was then transfered to theform works and cubes effecting an even distribution each time. After pouring each concrete batch the vibrating table was set on for about a minute to ensure adequate compaction and reduction of air voids. This was done for all the specimens to ensure an effective control of concrete strength.

4.5 Curing Conditions

After casting, the specimen was left in the mould for 48 hours before removing the forms. The specimen was then transfered to the curing room. The test specimen and the accompanying cubes were cured under the same condition to ensure that properties obtained from cubes were typically representative of the concrete of the specimen. This was achieved by sprinkling water on the specimen twice a day for a period of 28 days.

4.6 Test Procedure

The test specimens were carefully painted using white paint. This was done to facilitate easy monitoring of crack appearance and propagation. Loading points on the loading were then marked and the speimen consequently mounted on the testing frame. The test set consisted of two loading cell 250KN and 500KN maximum capacity respectively. The two cells and specimen were supported on rectangular frames of heavy channel steel sections. Before loading all the strain gauges were connected to the strain measuring system. This consisted of a Wheat stone bridge and a strain meter. After the connection was accomplished for the specimen under test, a dummy concrete with a strain gauge was connected to the wheatstone bridge to complete the circuit. Balancing in process was performed on the strain gauges to eliminate errors due to resistance of the lead wires. This was accomplished by tuning some channels on the wheat stone bridge until a deflection is registered on the strain meter. The wheatstone bridge was then switched to balancing out. Before applying pressure, the initial readings were recorded. Pressure was then applied through the jacks to the loadings cells in pre-determined stages. The strains were read at the end of each loading stage. During the reading

of the strains, careful visual observation was carried on the specimen under test to monitor the appearance and propagation of cracks. The profiles of the cracks seen were traced out on the face of the specimen using a dark pencil. Application of pressure was stopped when the pressure meter can no longer register the strokes of pumps through the jacks.

4.7 Test Results

The values of the strains obtained at each loading stage are presented for each specimen in a tabular form and are given in Tables 4.5 - 4.9. The crushing load and stress of cubes casted along with specimens are reported in Table 4.10.

Table 4.5 Load Vs Strains ($x10^{-6}$) for Specimen A1

Load(KN)	AR	AL	BR	BL	CR	CL
0.0	10540	10350	10330	16530	8580	8730
5.0	12130	11790	11220	17660	10060	10250
10.0	-	12090	-	-	10170	-

Table 4.6 Load Vs Strains ($x10^{-6}$) for Specimen A2

Load (KN)	AR	AL	BR	BL	CR	CL	DR	DL
0.0	9480	11420	9130	10540	3520	12045	8680	4845
5.0	9860	12120	9380	10620	3870	12200	9380	5580
10.0	10350	12850	9550	10680	4270	12440	10160	5750
15.0	-	13060	10555	-	4855	13420	10255	6370

Table 4.7 Load Vs Strains ($x10^{-6}$) for Specimen A3

Load (KN)	AR	AL	BR	BL	CR	CL
0.0	14000	12050	12580	12890	12190	12420
5.0	-	-	-	-	-	-
10.0	14460	12140	12810	13060	12405	12450
15.0	14920	-	13030	13230	12620	12500
20.0	-	-	14160	14575	13825	-

Table 4.8 Load Vs Strains ($x10^{-6}$) for Specimen B1

Load (KN)	AR	AL	BR	BL	CR	CL
0.0	7924	12065	18410	9965	12588	18695
5.0	8335	13515	18640	10065	12765	19700
10.0	9210	-	-	11680	14040	-

Table 4.9 Load Vs Strains ($x1o^{-6}$) for Specimen B2

Load (KN)	AR	AL	BR	BL	CR	CL	DR	DL
0.0	7870	14200	14170	12825	-	15225	15210	14255
5.0	8050	14800	14720	13150	-	15450	15600	14410
10.0	-	15445	15065	14140	-	15965	16080	15150
15.0	-	15800	15890	-	-	-	16765	-

'___' Implies reading cannot be taken.

Table 4.10 Crushing Load of Cubes Casted with Specimens

Specimen	Cube No.	Cube Mass(kg)	Crushing Load(KN)	Crushing Stress N/mm^2	Average crushing Stress N/mm^2
	1	7.90	830	36.89	
A1	2	7.95	840	37.33	37.33
	3	8.15	850	37.78	
	1	7.60	920	40.89	
A2	2	7.70	940	41.78	40.15
	3	8.00	850	37.78	
	1	7.85	740	32.89	
A3	2	8.20	780	34.66	33.78
	3	7.60	760	33.78	
	1	7.75	750	33.33	
B1	2	7.85	780	34.67	34.52
	3	7.90	800	35.56	
	1	7.75	730	32.44	
B2	2	7.70	760	33.78	34.96
	3	7.90	870	38.67	

CHAPTER FIVE

5.0 ANALYSIS AND DISCUSSION OF TEST RESULTS

5.1 Tensile Strength of Reinforcing Steel

The yield stress and the ultimate tress of the various sizes of reinforcement used are reported in Table 4.3. The values are obtained from the yield load and ultimate failure load observed on test specimens. The specimens are tested using the universal testing machine. The yield stress values in column 5 are obtained by dividing the yield load of column 4 by the effective cross-sectional area in column 2. The ultimate stress values in the last column are obtained by dividing the ultimate load in column 6 by the effective cross-sectional area. The average yield stress values for the sizes of reinforcement used are given at the lower part of Table 4.3. The characteristic strength recommended by CP110 for mild steel used in reinforced concrete element is 250mPa. All the yield stress values of the specimens tested are above this value; hence the average yield stress values given at the lower part of Table 4.3 are acceptable for our test programme.

5.2 Crushing Stress of Concrete Cubes

The crushing forces and stressed of cubes casted with each specimen are reported in Table 4.10.

These values are obtained by crushing the cubes on the universal testing machine. The cubes are cured and tested after 28 days.according to standard procedures. An average crushing stress of 30mPa is desirable as concrete of grade 25-30 is recommended by CP110 for reinforced concrete elements used in the construction of industrial buildings. The average crushing stress for each specimen reported in the last column of Table 4.10 is thus satisfactory for our test programme.

5.3 Failure Torque of Specimens

Table 5.1 Failure Torque of Specimens.

Specimen	p%	T test (KNm)
A1	0.5	6.5
A2	1.0	13.0
A3	2.0	26.0
B1	0.5	6.5
B2	1.0	13.0

The failure torque of each specimen are given in Table 5.1. The values were obtained from an analysis based on the characteristic strength of steel - 250mPa (for mild steel used).

The force (KN) recorded in the first column of Table 4.5 - 4.9 were converted to their torque values by multiplying them by 1.30m (the length between the forces constituting the couple). The values obtained were then recorded in the first column of Tables 5.2 - 5.6. The values of strains at zero torque for each level of reinforcement were subtracted from the strain values at other torque values for each level of reinforcement. The values obtained were then multiplied by the Young Modulus of steel Es = 210 x 10^3mPa.

The resulting values of stress for each level of reinforcement were recorded as shown in Tables 5.2 - 5.6. Graphs showing the variation of stresses at the reinforcement levels were then plotted from Tables 5.2 - 5.6 for each specimen. The graphs are shown in figures 5.1 - 5.5. Failure is deemed to have occured when the stress value in any steel have exceeded the characteristic strength of steel used. This failure stress is indicated by a straight horizontal line on the plotted graphs. This condition is termed 'first yield'. The torque at which the stress in any reinforcement exceeds the failure stress of 250mPa is taken as the failure torque.

Table 5.2 Torque and stress values at various reinforcement levels for specimen A1.

Torque (KNm)	AR	AL	BR	BL	CR	CL
0.0	0.0	0.0	0.0	0.0	0.0	0.0
6.5	333.9	302.4	186.9	237.7	310.5	318.66
13.0	-	365.5	-	-	334	-

Table 5.3 Torque and stress values at various reinforcement level for specimen A2.

Torque (KNm)	AR	AL	BR	BL	CR	CL	DR	DL
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6.5	79.8	147.0	52.5	16.8	72.5	32.6	147.0	154.4
13.0	182.7	300.3	88.2	29.4	156.5	83.0	310.0	190.0
19.5	-	344.4	298.8	-	279.1	288.7	330.6	320.5

Table 5.4 Torque and stress values at various reinforcement levels for specimen A3,

Torque (KNm)	AR	AL	BR	BL	CR	CL
0.0	0.0	0.0	0.0	0.0	0.0	0.0
13.0	96.6	75.6	47.3	35.7	45.2	6.3
19.5	193.2	-	94.5	71.4	90.3	16.8
26.0	-	-	332.9	354.0	343.0	-

Table 5.5 Torque and stress values at various reinforcement levels for specimen B1

Torque (KNm)	AR	AL	BR	BL	CR	CL
0.0	0.0	0.0	0.0	0.0	0.0	0.0
6.5	85.9	304.5	48.3	21.0	37.1	211.0
13.0	270.0	-	-	360.5	305.3	-

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Table 5.6 Torque and stress values at various reinforcement levels for specimen B2.

Torque (KNm)	AR	AL	BR	BL	CR	CL	DR	DL
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6.5	37.8	126.0	115.5	68.3	-	47.3	81.9	32.5
13.0	-	261.4	187.9	276.1	-	155.4	182.7	187.9
19.5	-	336.5	360.9	-	-	-	326.3	-

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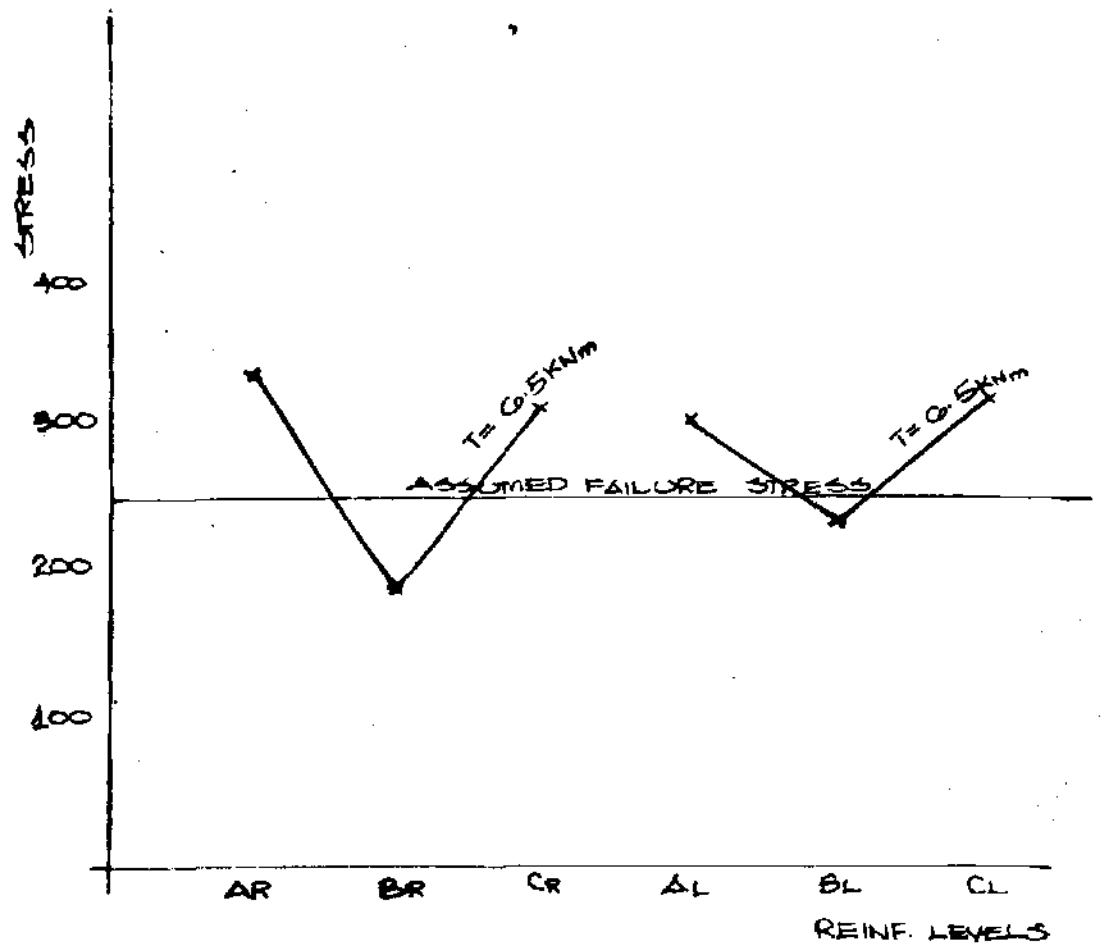


FIG. 5.1 STRESS VALUES AT EACH LEVEL OF REINFORCEMENT FOR SPECIMEN A1

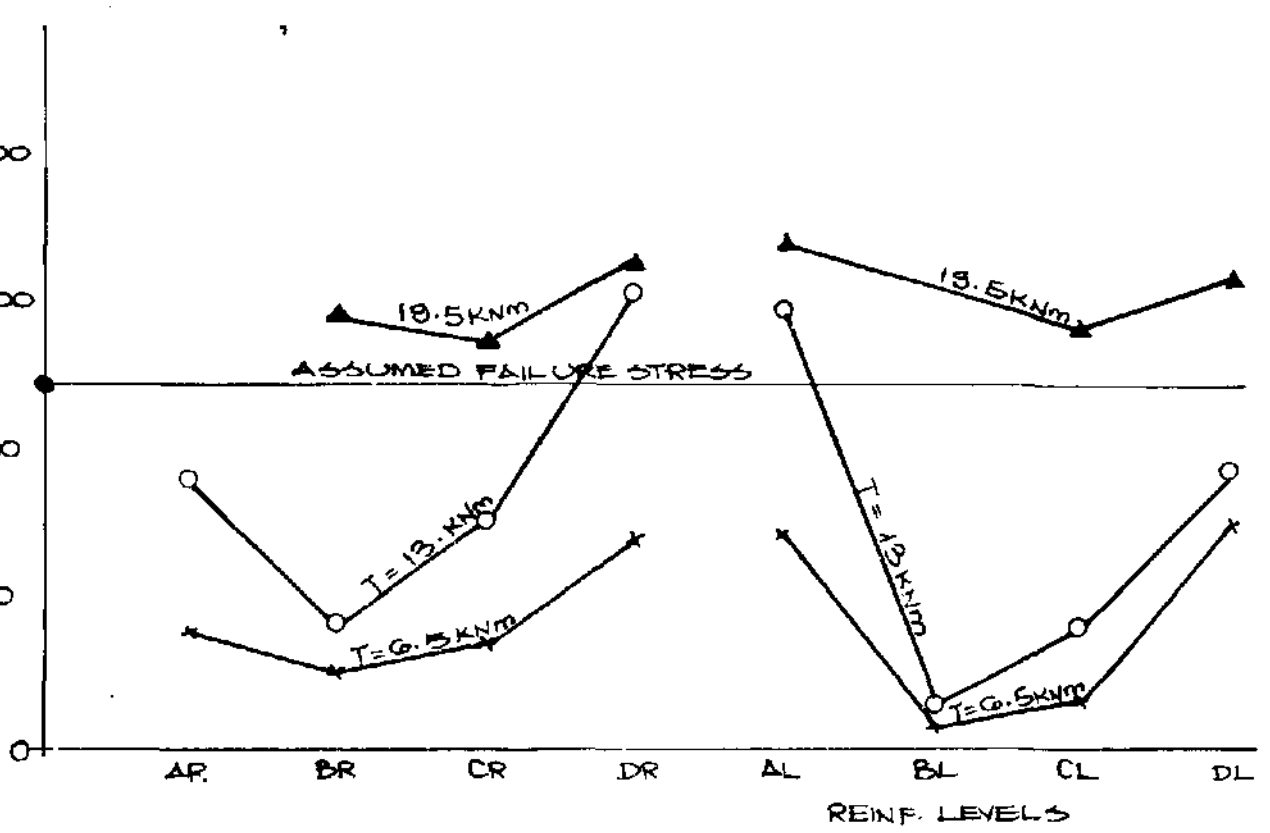


FIG. 5.2 STRESS VALUES AT EACH LEVEL OF REINFORCEMENT FOR SPECIMEN A2

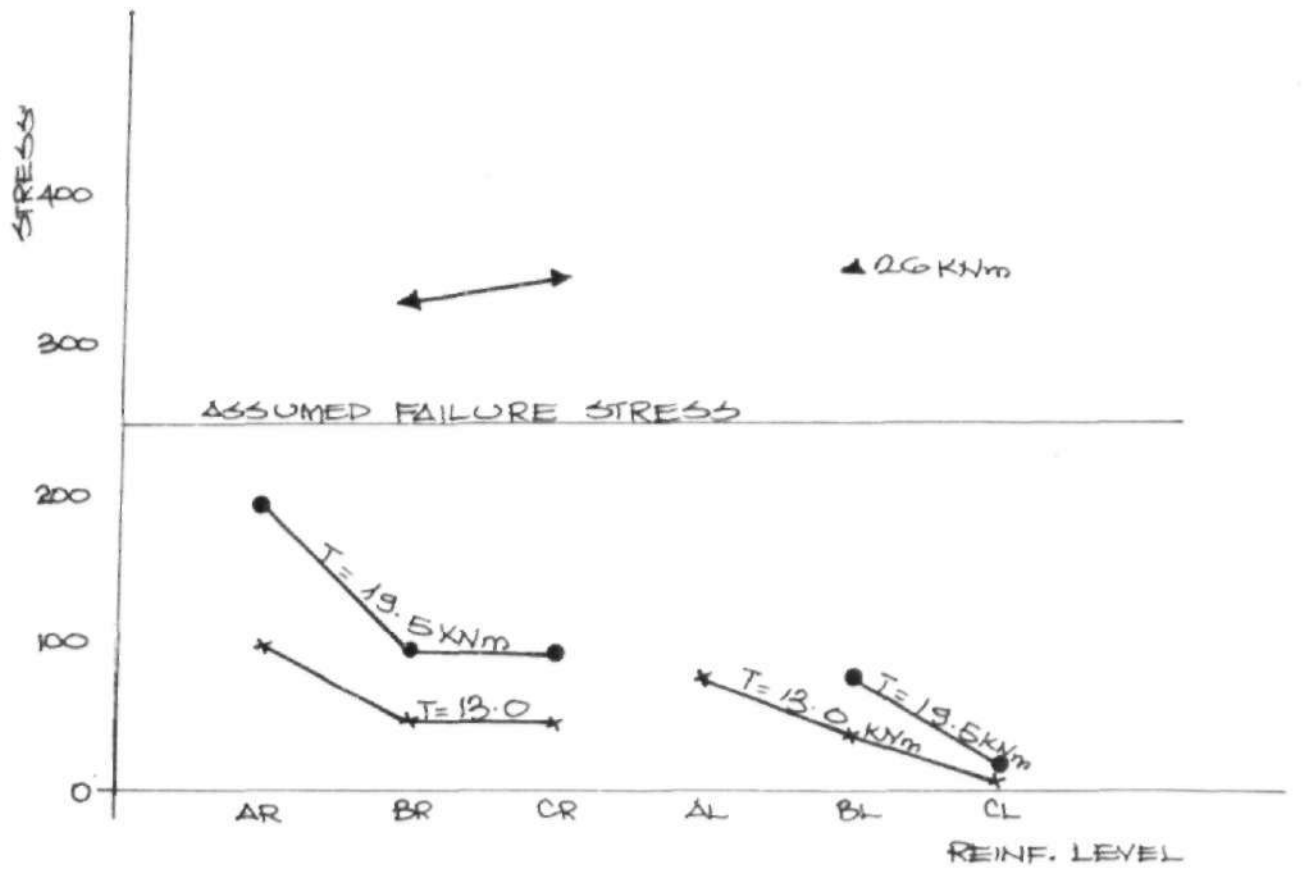


FIG. 5.3 STRESS VALUES AT EACH LEVEL OF REINFOR-CEMENT FOR SPECIMEN A3

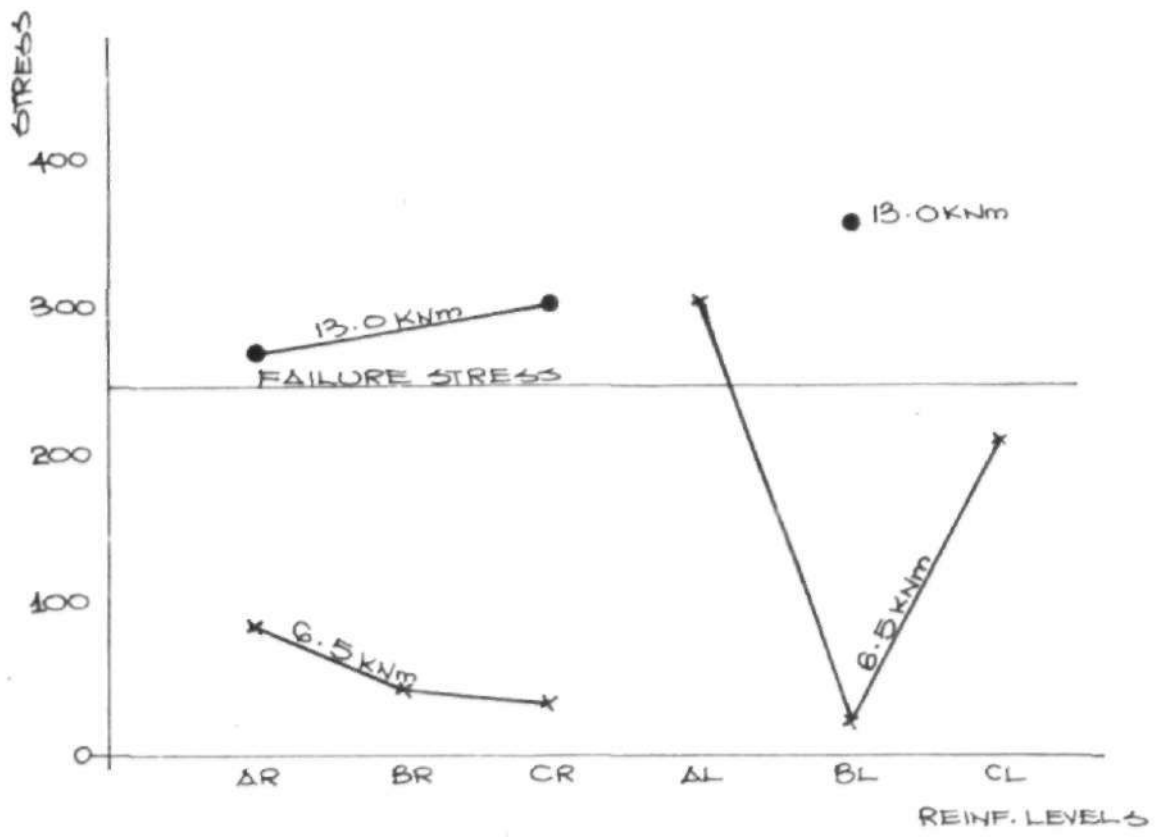


FIG. 5.4 STRESS VALUES AT EACH LEVEL OF REINF. FOR SPECIMEN B1

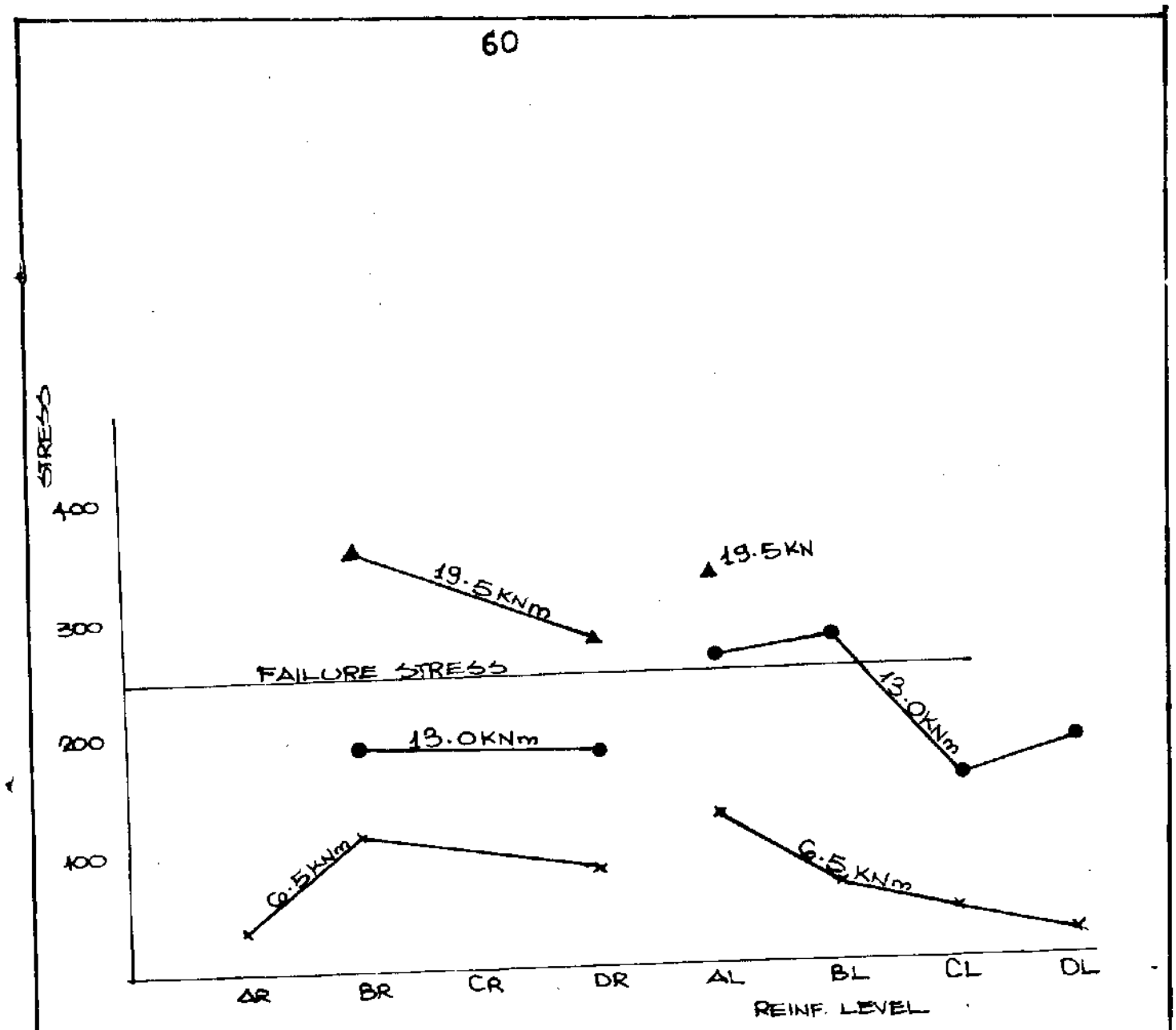


FIG. 5.5 STRESS VALUES AT EACH LEVEL OF REINFOR-CEMENT FOR SPECIMEN B2

The following comments can be made concerning the failure torque of the specimens tested.

a. Specimen A1

The failure torque for specimen A1 shown on Fig 5.1 is 6.5KNm. This is the torque at which the stresses on reinforcement AR CR, AL and CL have exceeded the failure stress of 250mPa. The value obtained compared favourably well with that obtained from the model based on the assumptions of the shear friction theory given by equation 3.6.

$$\text{i.e} \quad T_{pf} = \frac{1}{3} \mu p x^2 Y_{fy}$$

$$T_{pf} = \frac{1}{3}(1.4)(0.005(200)^2 x300x250$$

$$T_{pf} = 7.00\text{KNm}$$

$$\frac{Ttest}{T_{pf}} = \frac{6.5}{7.0} = 0.928.$$

b. Specimen A2

The failure torque for specimen A2 on Fig. 5.2 is 13.0 KNm. This is the torque at which the stresses on reinforcement DR and AL have exceeded the failure stress of 250mPa indicated by the straight line on the graph. The value obtained compared favourably well with that obtained from equation 3.6.

$$\text{i.e } T_{pf} = \frac{1}{3} \mu_p X^2 Y f_y$$

$$T_{pf} = \frac{1}{3} (1.4)(0.01)x200^2x300x250$$

$$T_{pf} = 14.0\text{KNm}$$

$$\frac{Ttest}{T_{pf}} = \frac{13.00}{14.00} = 0.93$$

c. Specimen A3

The failure torque for specimen A3 on Fig 5.6 is26.0KNm. This is the torque at which the stresses on all the torsional reinforcements provided exceeded the failure stress of 250 mPa indicated by a straight line on the graph. The value obtained compared well with that obtained from equation 3.6.

i.e $T_{pf} = \frac{1}{3} \mu \, pX^2Y \, f_y$

$T_{pf} = \frac{1}{3} (1.4)(0.02)x200^2x300x250$

$T_{pf} = 28KNm$

$\frac{Ttest}{T_{pf}} = 0.928$

d. Specimen B1

The failure torque for specimen B1 on Fig 5.4 is 6.5 KNm. This is thetorque at which the stress on reinforcement AL have exceeded the failure stress of 250mPa. The value obtained compared well with that obtained from equation 3.6.

$T_{pf} = \frac{1}{3} \mu \, pX^2Y \, f_y$

$T_{pf} = \frac{1}{3} (1.4)x0.005x200^2x300x250$

$T_{pf} = 7.0KNm$

$\frac{Ttest}{T_{pf}} = \frac{6.5}{7.0} = 0.93$

e. Specimen B2

The failure torque for specimen B2 on Fig.5.5 is 13.0KNm. This is the torque at which the stresses on reinforcements AL and BL have exceeded the failure stress of 250mPa.indicated by a straight line on the graph. The value obtained compared favourably well with that obtained from equation 3.6.

i.e. $T_{pf} = \frac{1}{3} \mu p X^2 Y f_y$

$T_{pf} = \frac{1}{3}$ x1.4x0.01x200^2x300x250

$T_{pf} = 14.00$KNm

$\frac{Ttest}{T_{pf}} = \frac{13.0}{14.0} = 0.93$

5.4 General Behaviour and Mode of Rupture of Specimens

a. Specimen A1

In specimen A1, cracks appeared at a torque of 6.5KNm. The cracks started along the root of the short cantilever and extend to the faces of the vertical member AB as shown in plate 1.0. At this torque value, the steel stresses in reinforcement AR, AL, CR and CL have exceeded 250mPa. This can be seen from

the second row of table 5.2. In effect it can be said that cracking of concrete and yielding or steel took place simultaneously. This implies that the short cantilever behaved in a ductile manner at failure. It does satisfy the essential requirement of a balanced failure of CP110.

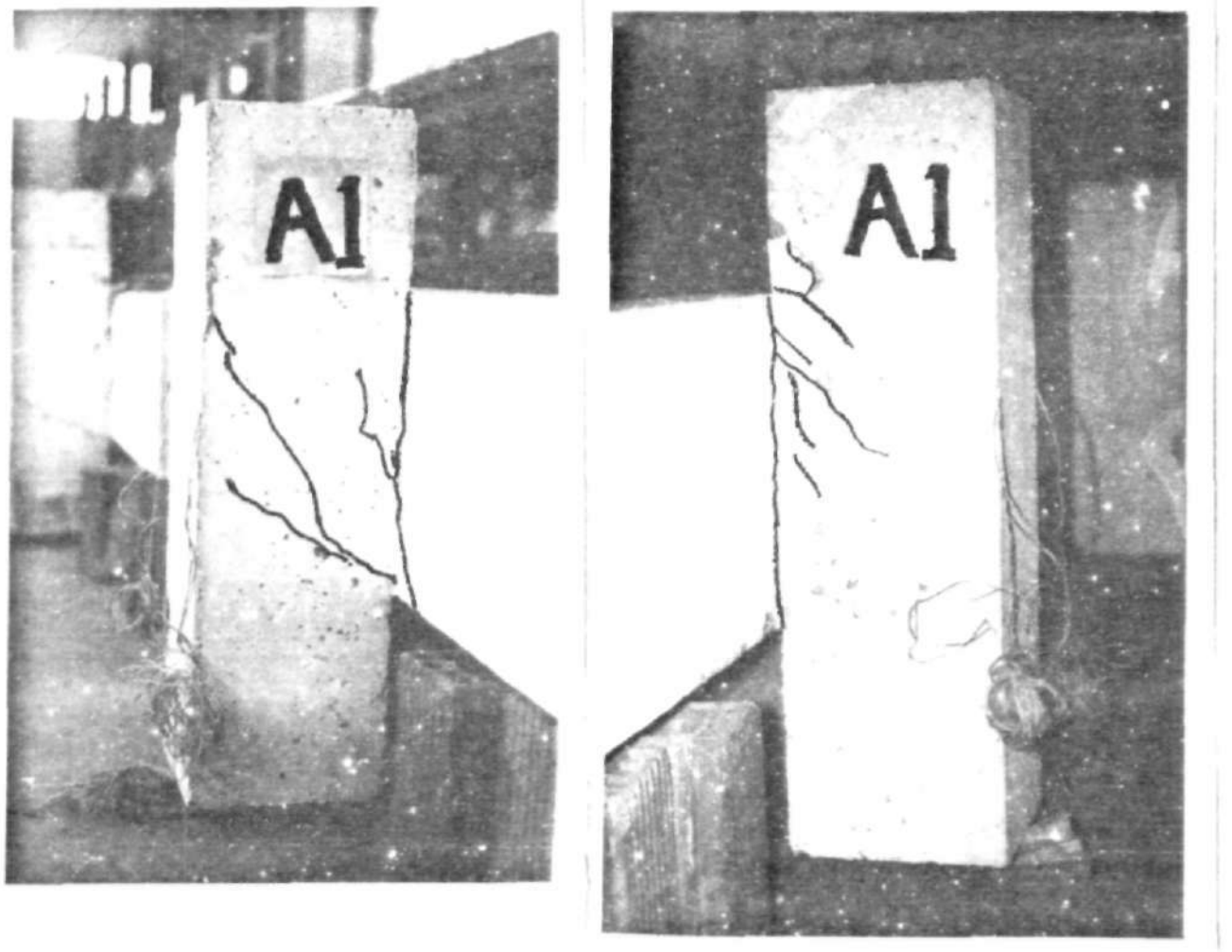


Plate 1. Faces of Specimen A1 at Failure.

b. Specimen A2

In specimen A2, cracks appeared at a torque of 13.00KNm. The crack started propagation at the root of the short cantilever and extend to the faces of the vertical member AB as shown in Plate 2.0. At this torque, the value of steel stresses in reinforcement AL and DR have exceeded the failure stress of 250mPa. This can be seen from the third row Table 5.3. In the same vein as specimen A1, it can be said that cracking of concrete and yielding of steel took place at the same time. This implies also that the short cantilever of specimen A2 behaved in a eductile manner at failure. It thus satisfy the balanced failure condition of ultimate limit state in CP110.

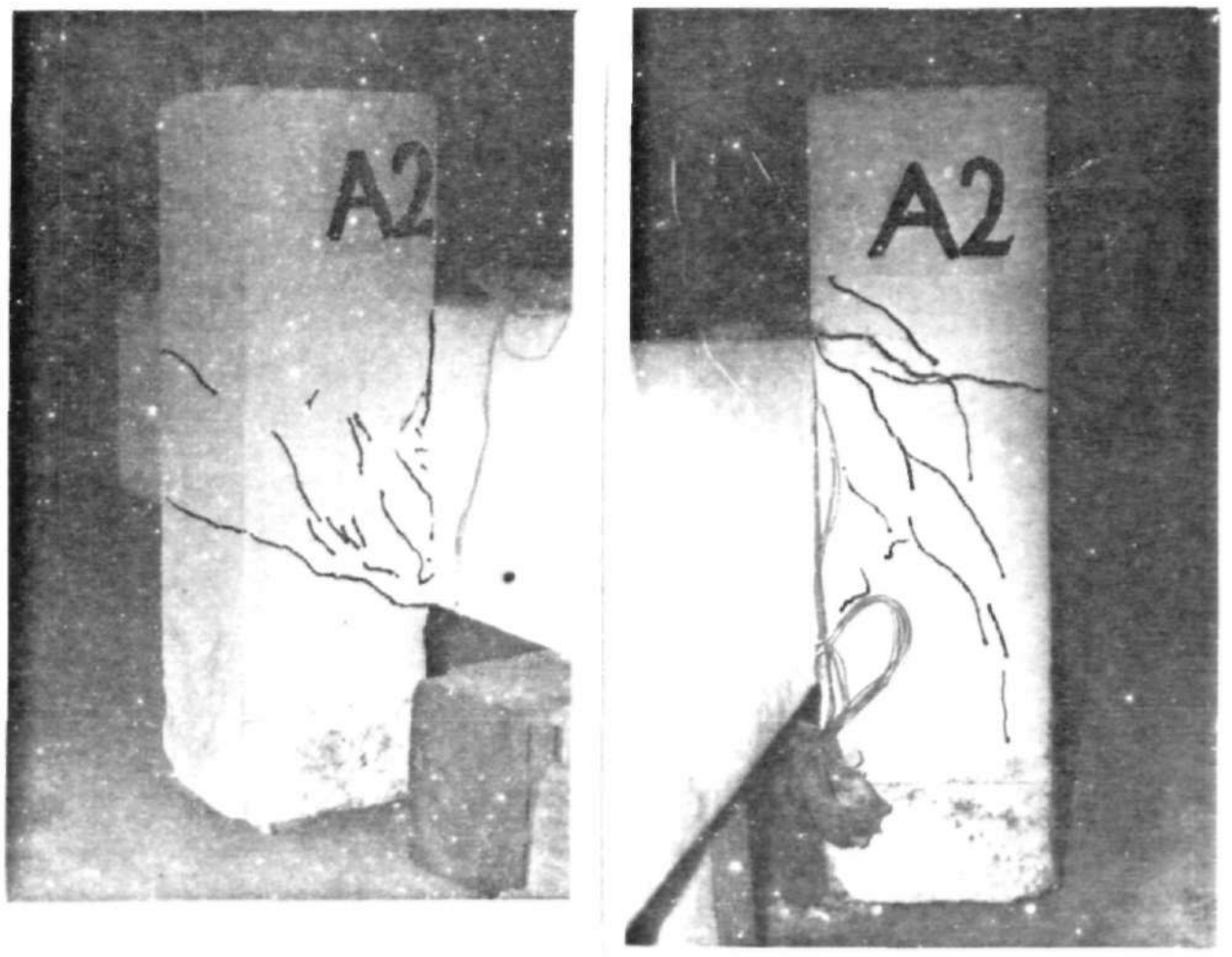


Plate 2. Faces of Specimen A2 at Failure.

c. Specimen A3

In specimen A3. the cracks commenced propagation at a torque of 13.0KNm. The cracks commenced along the root of the short contilever and extend to the faces of the vertical member AB as shown in Plate 3.0. At this torque value the steel stresses in all the reinforcements were below

the failure stress of 250mPa. This can be seen from the last row of Table 5.4. The cracking of concrete and yielding of the steel did not occur simultaneously. The short cantilever did not behave in ductile manner as required by ultimate limit state design approach of CP110.

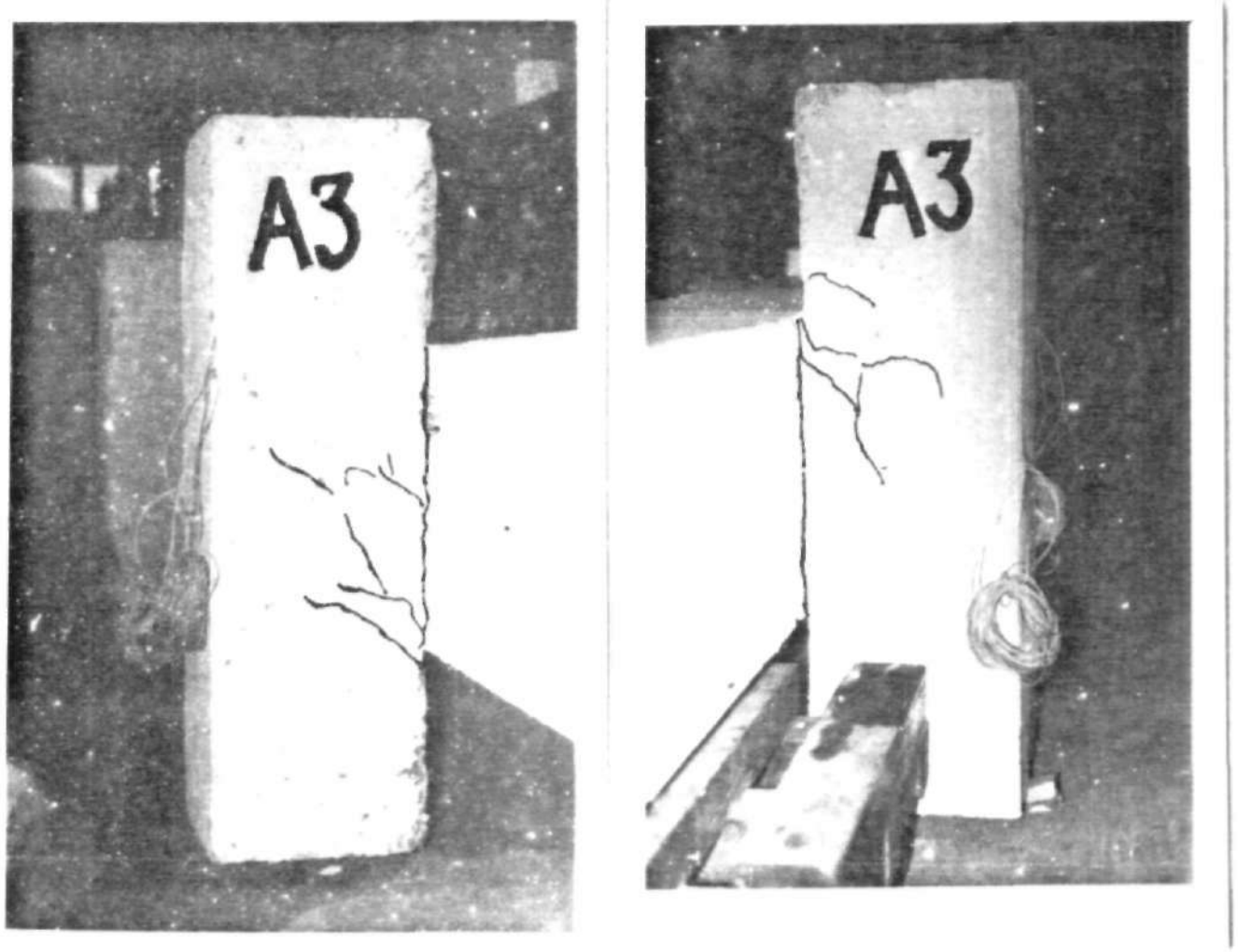


Plate 3. Faces of Specimen A3 at Failure.

d. Specimen B1

In specimen B1, cracks appeared at a torque of 6.5KNm. The cracks started propagation along the root of the short cantilever and extend to the faces of the vertical member as shown in Plate 4.0. At this torque value, the steel stress in reinforcement AL has exceeded the failure stress of 250mPa. This can be seen from the second row of Table 5.5. The cracking of concrete and yielding of steel took place at the same time. It can therefore be said that the short cantilever behaved in a ductile manner at failure. It therefore satisfies thebbalanced failure criterion of ultimate strength design methods of CP110.

It must also be mentioned here that the cracks on the faces of the vertical member AB was inclined more to the vertical for specimen B1 than that on the faces of the vertical member in specimens A1, A2 and A3. The implication of this is that significant shear force is transferred to the vertical member of specimen B1.

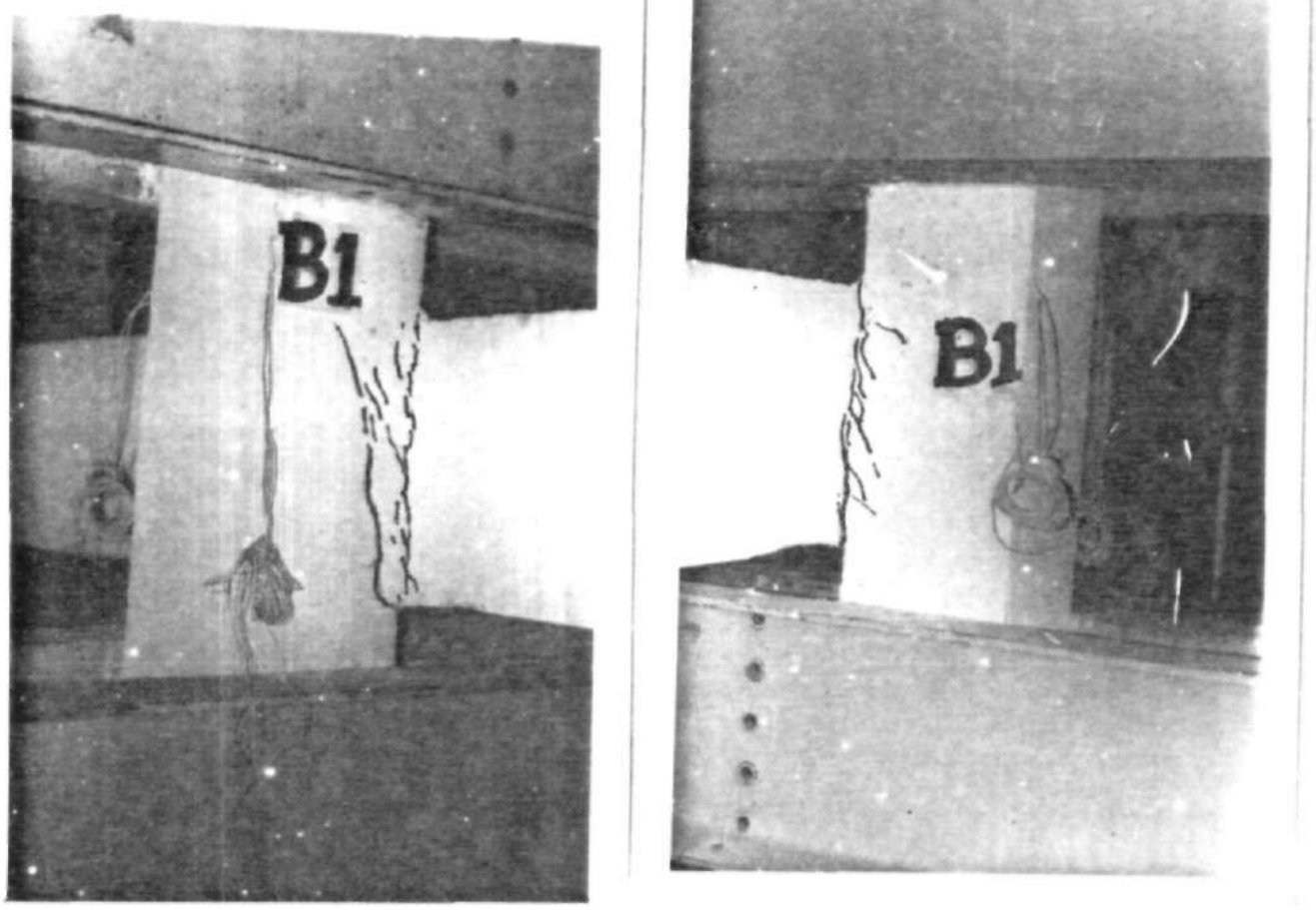


Plate 4. Faces of Specimen B1 at Failure.

e. Specimen B2

In specimen B2, cracks appeared at a torque of 13.0KNm. The cracks commenced propagation at the root of the short cantilever and extend to the faces of the vertical member AB as shown in Plate 5.0. At this torque, the value of steel stresses in reinforcement AL and BL have exceeded the

failure stress of 250mPa. This can be seen from the third row of Table 5.6. TIt can therefore be said that cracking of concrete and yielding of steel took place simultaneously. This implies that specimen B2 behaved in a ductile manner at failure. It thus satisfy the balanced failure criterion of the ultimate limit state design approach of CP110. Also it must be mentioned that the cracks on the faces of the vertical which implies that significant shear force is transferred to the vertical member.

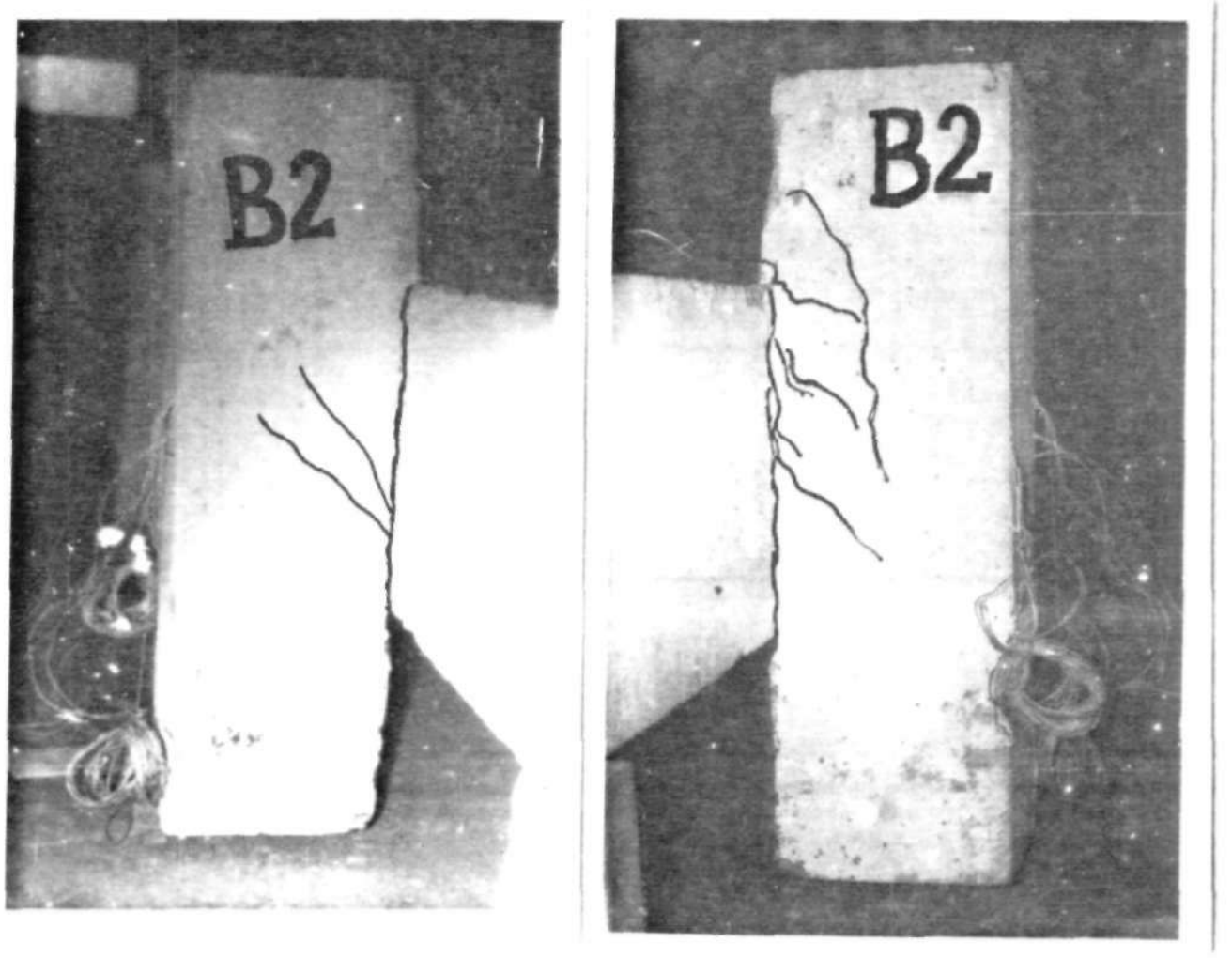


Plate 5. Faces of Specimen B2 at Failure.

CHAPTER SIX

6.0 CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

1. Torsional theories based on the traditional truss model are not applicable to reinforced concrete short cantilevers under torsion. The crack pattern assumed in the space truss model was essentially different from the one obtained on specimens of reinforced concrete short cantilever tested. The truss model predicts zero torsional capacity for a reinforced concrete short cantilever under torsion as arrived at in chapter 1.

2. Based on the shear-friction theory and test reports, the failure torque was found to be primarily a function of percentage of longitudinal reinforcement, dimension of the cross-section and the characteristic strength of steel.

3. 'Type A' detailing is prefered to 'Type B' detailing on account of simplicity and because limited shear force is transfered to the vertical member casted along with it.

4. For lower ratios of longitudinal steel, in specimens A1, A2, B1 and B2, the rup ture mode conforms with the balanced failure criterion which is an essential requirement of ultimate strength design approach in CP110 and other Codes of Practice on Concrete.

6.2 Recommendations

1. The torsional reinforcement provided to resist torsion should be anchored by being bent into the concrete within the vertical member casted with the short cantilever and hooked on the reinforcement there for it to be fully effective on either side of any potential vertical crack.

2. Torsional reinforcement must be of the closed type and placed across the cracks occuring on the longer dimension of the shott element where we have the maximum torsional shear stress.

3. Torsional reinforcement should extend a distance greater or equal to 12 ϕ beyond the point thecretically required.

4. The spacing of torsional reinforcement shall not exceed 300mm in order for reinforcement to adequately intercept the growing cracks on concrete.

CHAPTER SEVEN

7.0 LIST OF REFRENCES

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CHAPTER EIGHT

8.0 APPENDIX

Analysis and Design of Specimen A1

Data:

Type A Detailing

p = 0.5%

f_y = 250MPa

X,Y = 200mm, 300mm

Predicted failure Torque T_{pf} = 7KNm.

a,b =)0.296m, 0.293m, M_o = 7KNm

$$R = \frac{6M_{oab}}{(a+b)^3} = \frac{6x7x0.296x0.293}{(0.589)^3} = 17.83KN$$

$+M_{max}$ = 3.5KNm, $-M_{max}$ = 3.5KNm.

a. Selection of Torsion Reinforcement

p = 0.5%, x = 200, Y = 300mm

$$As = \frac{0.5}{100} \times 200 \times 300 = 300mm^2$$

Use 6 - 8mm ϕ bars ($302mm^2$)

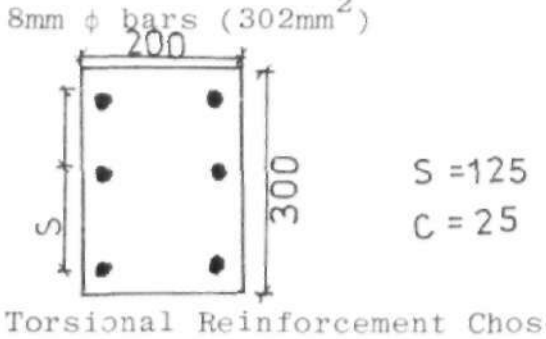


Fig 8.1 Torsional Reinforcement Chosen for Specimen A1.

b. Design of the Loading Arm

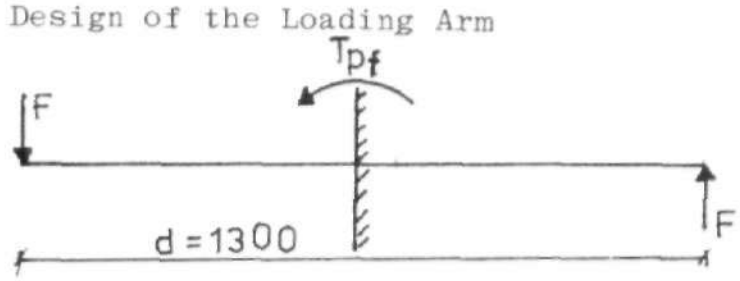


Fig. 8.2 Analysis Scheme for the Loading Arm of Specimen A1.

$$F = \frac{T_{pf}}{d} = \frac{7.0}{1.3} = 5.4KN$$

Design Force $F_1 = 1.6x5.4 = 8.64KN$

$$M_{max} = F_1 x d/2 = 8.64x0.65 = 5.62KNm.$$

$$M/bd^2 = \frac{5.62x10^{-6}}{200x275^2} = 0.37$$

p = 0.25 minimum % reinforcement for mild steel

$$As = \frac{0.25x200x275}{100} = 137.5mm^2.$$

Use 3 - 8mm ϕ bars ($151mm^2$)

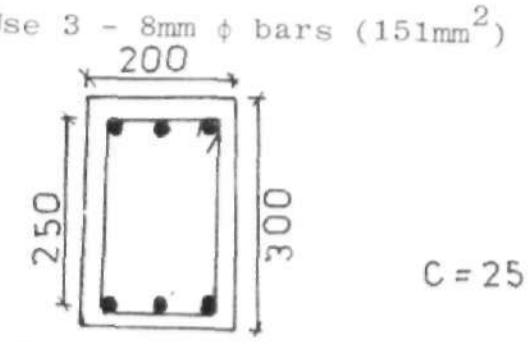


Fig. 8.3 Arrangement of Reinforcement for the Loading Arm of Specimen A1.

Shear: Maximum Shear Force = 8.64KN

$$\text{shear stress } V = \frac{8.64 \times 10^3}{200 \times 300} = 0.144$$

for p = 0.25, f_{cu} = 30MPa, Vc = 0.35Mpa.

V < Vc → Provide minimum area of links

Use 6mm ϕ bars, Asv = 57mm^2 at 100mm ϕ.

c. Design of Vertical Member AB for Specimen A1

Max + B.M = 3.5KNm, Max - B.M = 3.5KNm

Assuming a singly reinforced section

$M/bd^2 = 3.5 \times 10^6 / 200 \times 175^2 = 0.57$, p = 0.28

i.e. As = 0.28x200x175/100 = 98.0mm^2.

Provide 2 - 8mm ϕ bars (As = 101mm^2)

Shear: Maximum S.F = 17.83KN

Shear stress = $17.83 \times 10^3 / 200 \times 200 = 0.445$

p =).28, f_{cu} = 30MPa, Vc = 0.40

Since V < Vc Provide minimum area of

links i.e 6mm ϕ links at 100mm spacing.

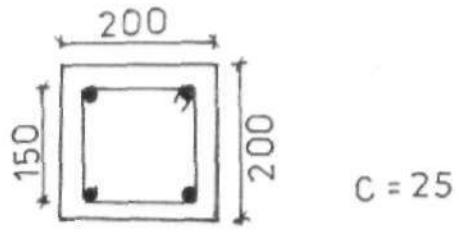


Fig. 8.4 Arrangement of Reinforcement for vertical Member AB of Specimen A1

Analysis and Design of Specimen A3.

Data:

Type A. Detailing

$p = 2.0\%$

$f_y = 250$ MPa

$X_1 Y = 200mm, 300mm$

Predicted failure Torque $T_{pf} = 28KNm$

$a,b = 200mm, 300mm, M_o = 28KNm$

$$R = \frac{6^{M}oab}{(a+b)^3} = \frac{6x28x0.296x0.293}{(0.589)^3} = 71.30KN$$

$+M_{max} = 14KNm, \quad -M_{max} = 14KNm$

a. Selection of Torsion Reinforcement

$p = 2\%, X = 200mm, Y = 300mm$

$As = 2/100 \times 200 \times 300 = 1200mm^2$

Provide 6 - 16mm ϕ bars ($1206mm^2$)

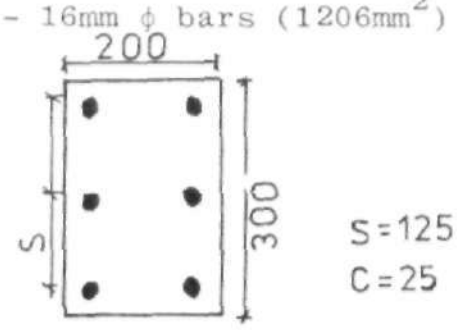


Fig 8.5 Torsional Reinforcement for Specimen A3

b. Design of the Loading Arm.

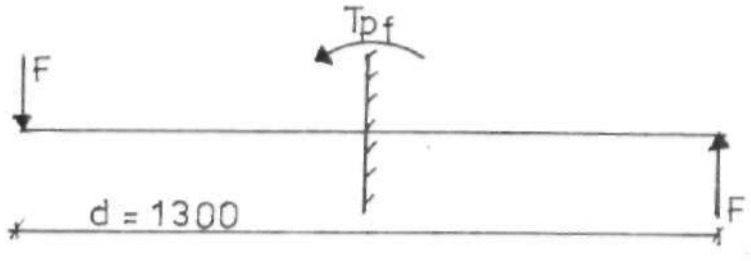


Fig. 8.6 Analysis Scheme for theLoading Arm of Specimen A3.

$$F = \frac{T_{pf}}{d} = \frac{2.8}{1.3} = 21.54KN$$

Design Force F_1 = 1.6x21.54 = 34.5KN

Design M_{max} = 34.5x0.65 = 22.43KNm

Assuming a singly reinforced section

$M/bd^2 = 22.43x10^6/200x275^2 = 1.48$

p = 0.73

As = $0.73x200x275/100 = 401.5mm^2$

Provide 4 - 12mm ϕ bars ($452mm^2$)

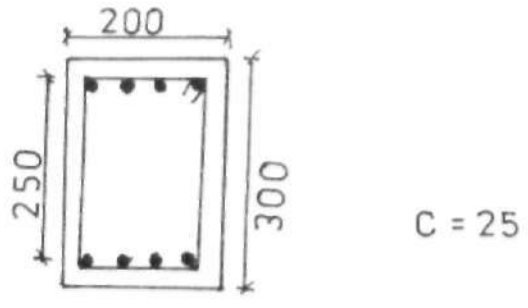


Fig 8.7 Loading Arm Reinforcement for Specimen A3.

Shear: Max Shear Force = 34.5KN

Shear stress V = $34.5x10^3/200x300$ = 0.575

p = 0.73, f_{cu} = 30MPa, Vc = 0.625MPa

Since Vc < V Provide 6mm links at 100mm spacing.

Design of Vertical Member AB for Specimen A3

Max + B.M = 14KNm, Max - B.M = 14KNm

Assuming a singly reinforced section

$M/bd^2 = 14x10^6/200x175^2 = 2.28.$

p = 1.18

As = $1.18x200x175/100 = 413mm^2$

Use 3 - 16mm bars

Shear: Maximum S.F = 71.30KN

$$\text{shear stress} = \frac{71.30x10^3}{200 \times 200} = 1.78$$

p = 1.18, f_{cu} = 30MPa, Vc = 0.72

$$Sv = \frac{101x0.87x250}{200(1.78-0.72)} = 103.4mm$$

Provide 8mm ϕ links at 100mm

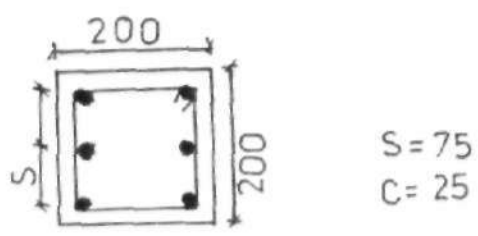


Fig 8.8 Vertical Member Reinforcement for Specimen A3