THE APPLICATION OF REDUCTION TO THE POLE TECHNIQUE FOR THE INTERPRETATION OF AEROMAGNETIC DATA OF ZARIA AREA, NORTH CENTRAL NIGERIA

BY

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CERTIFICATION

This thesis entitled The Application of Reduction to the Pole Technique for the Interpretation of the Aeromagnetic Data of Zaria Area in the North Central Nigeria, by Kolawole Muideen Lawal meets the regulations governing the award of the degree of Master of Science, Applied Geophysics of Ahmadu Bello University, and is approved for its contribution to knowledge and literary presentations.

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APSTHACT

The principles and applications of reduction to the pole of magnetic anomalies is analysed and demonstrated by applying the technique to the AEROMAGNETIC data of Zaria area, north central part of Nigeria. The data was obtained by digitizing six AEROMAGNETIC maps which covered the area and using the method of Silva(1986) a computer software is developed to perform the general three dimensional case. The effectiveness of the method is assessed using theoretical noise-free and noise-corrupted data. In both cases the method showed good results regardless of the latitude. After reducing the data to the pole, comparison made between the total field and the reduced to the pole maps indicates that the technique eliminates the dipolar nature of the anomalies. Also the presence of positive anomalies in the reduced to the pole map existing in most part of the area validates the assumptions of induced magnetization within the area. Interpretation of some prominent anomalies in the area indicates that the sources appear to be vertical intrusives and are granitic in nature. All these results shows that reducing magnetic fields to the earth's magnetic pole makes interpretation of magnetic anomalies easier, because the anomalies are not shifted as a result of the obliquity of the normal field.

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CHAPTER 1

INTRODUCTION

1.1 The Study Area

The area in which the reduction-to-the-pole technique is applied is the Zaria area of north central part of Nigeria as shown in Figure 1.1. The area covered lies between geographic latitudes 10^0 30' N and 11^0 30' N, and longitude 7^0 00' E and 8^0 30' E within the Northern Nigerian Precambrian belt. It is located within topographic and aeromagnetic map sheet 101 (Maska), 102 (Zaria), 103 (Ikara), 123 (Kaduna), 124 (Igabi) and 125 (Dutse Wai) within an area of approximately 19,800 sq km. The survey for each sheet was completed in 1974 and each sheet has a mean terrain clearance of 152.4 m (500 ft) with the flight lines at intervals of 2 km from each other.

1.2 An Over-view of Magnetic Interpretation Methods

The high cost of geophysical data acquisition has forced mineral and petroleum exploration companies to rely increasingly on the aeromagnetic technique as an economical method for both reconnaissance and detailed determination of subsurface geological structure. This interest has generated an active search for better techniques of processing and interpreting the mass of data that is being accumulated, as is exemplified by the rapidly expanding literature on the subject.

In the pertinent literature, the different methods of treatment of the magnetic data are mainly concerned with two types of information. The first type is the vertical derivative map of the total field. Usually, the first and second derivatives are evaluated. Second

derivative maps are found to be of much use in approximately delineating the boundary of the body causing magnetic anomalies. The second type of information is the continuation of the total field above or below the level of observation. Such continuation of the field is important in either reducing or accentuating the effects of bodies close to the earth's surface.

Interpretation of magnetic anomalies is complicated by three factors. First, the magnetic field has both attractive and repulsive forces. Second, a magnetic anomaly may be generated both by induction in the direction of the earth's field and by remanence magnetism, which may be oriented differently. Numerous authors such as Hays and Scharon (1963) have shown that remanence can be a significant or even dominant (Park, 1968) contributor to the anomalous field. However, because reliable information in remanence is seldom available, anomalies usually are assumed to be caused entirely by induction.

The third complication is the inclination and declination of the earth's inducing field. This causes distortion of the magnetic anomaly, with the degree of distortion being a function of the magnetic latitude. Baranov (1957) and Baranov and Naudy (1964) developed a method which mathematically transformed the total intensity field to an equivalent vertical field. In essence, this willed if the geological source was physically moved to the magnetic pole. The transformation therefore is called "reduction to the pole".

1.3 Previous Geophysical Work in the Study Area

Adeniyi (1987) conducted a gravity survey over the Older Granite plutons in the Zaria area (sheet 102) and Igabi area (sheet 124), in order to investigate the extent of the Older Granite suite within the Zaria basement and also to determine the shape of the plutons. His

result confirmed the existence of three separate batholiths as proposed by McCurry (1970). The batholiths dipping inwardly with elliptical shape, extends to depths between 6 km and 8 km.

Onugba (1979) conducted direct-current resistivity and seismic refraction measurements around Masari/Kafur area situated at latitude 7^0 42' N and longitude 11^0 38' E. He measured the location, extent and possible physical properties of suspected Kimberlite pipes within the area. He deduced that the Kimberlite pipe is between 560 m and 660 m long with a maximum width of 140 m and is oriented in a direction north-east of the area.

Olatinwo (1994) conducted geoelectrical investigation within the Northern part of Kaduna located within latitude 10^0 40' N and 11^0 20' N and longitude 7^0 00' E and 8^0 00' E. The geophysical investigation confirmed limited lateral extent of basement aquifers. The aquifers were found to be heterogenous in their occurrences in relation to their host rocks and they are of variable thicknesses. Attempts were made towards lithological differentiation between the gneiss, granite and the metasediments using the parameters involving coefficient of electrical anisotropy. But the three rock types (metasediments, granites and gneiss) show significant overlap in coefficient of anisotropy values. However structural trend maps obtained from his interpretation delineate a series of bedrock ridges (most especially around Zaria) and depressions within the survey area.

Shemang (1990) conducted a geoelectrical survey over the Kubani basin which occupies the south-eastern part of Zaria bounded by latitudes 11^0 4' N and 11^0 10' N and longitudes 7^0 36' E and 7^0 44' E. The survey was carried out with the aim of determining the subsurface

layerings, the different components of aquifer in the basin and their influences on the flows of rivers within the basin. Some of his results confirmed that the boundary between the Zaria granitic batholith and the Gneiss unit is sharp and not gradational. Of all the above geophysical survey none has applied the reduction to the pole technique around the study area. The present work is therefore aimed at achieving this purpose.

1.4 Present Work

1.4.1 The Aim and Objective

The aim of the work is to develop a computer program that will reduce the data to the pole and to investigate further the interpretation of the so transformed data at low magnetic latitude. The objectives are:

(i) To see how the obliquity in the total field has been removed and to check how it has affected the correlation of the field with the geology of the area.

(ii) When the reduction to the pole technique is applied, induced magnetization is assumed. Therefore the presence of negative anomalies on the reduced to the pole map is an indication of remanence magnetization (Leao and Silva, 1989). An objective of the work, therefore is to investigate the presence of negative anomalies so as to validate the assumption often made when doing magnetic interpretation. Also an indication of remanence magnetization will lead to valuable information for Paleomagnetic studies such as the age or time of formation of the rock units within the area.

1.4.2 Methodology

Traditionally, reduction to the pole has been accomplished either by space domain filtering (Bhattacharyya, 1965; Ervin, 1976) or wavenumber domain filtering (Kanasewich

and Agarwal, 1970; Gunn, 1975). In the two dimensional case, this procedure is stable regardless of the latitude, as long as the source strike is not parallel to the horizontal projection of the geomagnetic field. In the three-dimensional case, however, reduction-to-the-pole filtering is stable only at high magnetic latitudes. At latitudes lower than 15 degrees, it is of no practical use due to a sharply increasing instability towards the magnetic equator (Silva, 1986). However, Silva (1986) has shown that the three dimensional case can be solved by formulating the problem in the context of a general linear inverse problem from which stable solutions are found by using well-known stabilizing procedures developed for inverse linear problems. Also by windowing the data (Leao and Silva, 1989), C.P.U. time and the memory necessary for the transformation can be drastically reduced.

The aeromagnetic map of the area is digitized to obtain a 38 by 55 data matrix at intervals of 3 km in the x and y directions corresponding to north-south and east-west directions respectively. Since reduction to the pole involves centralization of the anomaly with respect to its source, the regional component of the anomaly which can create a fictitious trend has to be removed. Therefore, the digitized fields were separated into regional and residual components before applying the reduction to the pole technique.

CHAPTER 2

PSEUDOGRAVITY

In applied geophysics, the first face of interpretation is a detailed analysis of the data obtained by each of the methods applied. In gravity work, the main problem of the preliminary step is the elimination of the regional anomaly while in magnetic work, the preliminary phase becomes very important because in gravity there is a simple connection between the residual anomalies and the tectonic features, in magnetism the anomalies do not take place at the apex of the disturbing bodies. The magnetic picture of the tectonic occurrences undergoes a distortion, due to the inclination of the magnetizing vector.

An additional difficulty arises at the final stage of the interpretation, when one tries to check the geological assumptions and to compare them, for instance with a simple typical case. In gravity the computation of the influence is quite immediate; in magnetism, the computation is much more complicated and a greater number of variable factors must be taken into account such as the inclination of the norm al field, the orientation of the structure with respect to the magnetic meridian and so on.

These considerations show that, when faced with the problem of magnetic interpretation, it is advantageous to transform the isoanomally map in order to suppress the distortion and to make the map as clear as the map of the gravity anomalies. This transformation is possible because of the relation that exists between magnetic potential V and gravity potential U called the Poisson relation given by given by

$$V = \frac{J}{G\rho}\frac{\partial U}{\partial s} \qquad (2.1)$$

Where,

J = magnetization

ρ = density contrast

G = gravitational constant.

s = direction of magnetization.

Equation (2.1) can be used for the theoretical transformation of magnetic anomalies into gravity anomalies called " pseudo-gravity" anomalies.

The term `pseudo" is used because the ratio J/ρ is assumed to be constant, which is never true due to the following reasons: firstly, the true density ρ is unknown and secondly only the magnetized masses contribute to the potential U. Thus, the pseudogravity anomaly is not the true gravity anomaly due to the body, but a fictitious anomaly completely assuming the ratio J/ρ to be a constant throughout the body. From equation (2.1), letting Gρ/J = k (a constant) we obtain

$$k\frac{\partial U}{\partial s} = V \qquad (2.2)$$

To obtain the vertical component Z of the magnetic anomaly we differentiate equation (2.2) with respect to z (z being the vertical axis), thus:

$$k\frac{\partial}{\partial z}\left(\frac{\partial U}{\partial s}\right) = k\frac{\partial}{\partial s}\left(\frac{\partial U}{\partial z}\right) = \frac{\partial V}{\partial z} = Z$$

$$but \quad g = \frac{\partial U}{\partial z}$$

$$\frac{\partial g}{\partial s} = Z \qquad \text{.....(2.3)}$$

where g is the pseudogravity.

In the region of the earth's magnetic pole, the direction s is vertical. Therefore the derivative with respect to s and z are the same. That is

$$dg/ds = dg/dz \qquad (2.4)$$

And also at the poles the total field T is identical to Z

$$\therefore \quad Z = T_{pole} \qquad (2.5)$$

From equation (2.4) and (2.5) we have

$$k(dg/dz) = T_{pole}$$

That is, the pseudo-gravity vertical gradient g' is equal to the magnetic anomaly reduced to the pole.

CHAPTER 3

REDUCTION TO THE POLE BY FILTERING

3.1 Analysis in space domain

Although the discovery of Baranov (1957), was of considerable theoretical significance, its practical application was not generally as satisfactory. First, it required a rather unusual trigonal grid with one axis oriented along the magnetic north. A grid of this nature does not lend itself easily to indexing for computer processing. Second, the transformation required numerical evaluation of a double integral with the consequent loss in accuracy. Third, the process could not be used in the lower magnetic latitudes. Finally it did not include a method of compensation for effects of remanent magnetism in those situations where the orientation of the remanent magnetism is known.

Bhattacharyya (1965) proposed an alternative formulation that eliminated many of the restrictions of Baranov's method. He first expressed the total intensity field as a double Fourier Series on a rectangular grid. The double integration can then be made analytically, and remanent magnetism, if known can easily be included. Furthermore the method is applicable to data from any magnetic latitude except at 0^0 and 180^0 which have to be treated as special cases. Bhattacharyya (1965) expressed the total field T as a double Fourier series with (x, y) in the datum plane as:

$$T(x,y) = \sum_{m=0}^{M} \sum_{n=0}^{N} [A_{mn} \cos K_m x . \cos K_n y + B_{mn} Sin K_m x . \cos K_n y + C_{mn} \cos K_m x . \sin K_n y + D_{mn} \sin K_m x . \sin K_n y] e^{-(K_m^2 + K_n^2)^{1/2} z} \quad (3.1)$$

where

K_m = $2\pi m/L_x$ and $K_n = 2\pi n/L_y$

T(x,y) = magnetic field at the point (x,y)

L_x = number of rows in the grid (x - direction)

L_y = number of columns in the grid (y -direction)

M = maximum number of harmonics in the x-direction

N = maximum number of harmonics in the y-direction

A_{mn}, B_{mn}, C_{mn}, D_{mn} = Series coefficients

The coefficients are given by (James, 1966)

$$A_{mn} = \frac{w}{L_x L_y} \sum_{x=0}^{L_x-1} \sum_{y=0}^{L_y-1} T(x,y) \cos K_m x \cos K_n y$$

$$B_{mn} = \frac{w}{L_x L_y} \sum_{x=0}^{L_x-1} \sum_{y=0}^{L_y-1} T(x,y) \sin K_m x \cos K_n y$$

$$C_{mn} = \frac{w}{L_x L_y} \sum_{x=0}^{L_x-1} \sum_{y=0}^{L_y-1} T(x,y) \cos K_m x \sin K_n y$$

$$D_{mn} = \frac{w}{L_x L_y} \sum_{x=0}^{L_x-1} \sum_{y=0}^{L_y-1} T(x,y) \sin K_m x \sin K_n y$$

where,

w = 1, if m = 0, M and n= 0, N

w = 2, if m = 0, M or n = 0, N

w = 4, if m ≠ 0, M and n ≠ 0, N

The equation for the field reduced to the pole is given by:

$$T(x,y) = \frac{1}{2\sin I_{os}\sin I_s}\sum_{m=0}^{M}\sum_{n=0}^{N} e^{-P_{mn}Z}$$
$$[E_{mn}\cos(K_m x + K_n y) + F_{mn}Sin(K_m x + K_n y) \qquad (3.2)$$
$$G_{mn}\cos(K_{mx} - K_{ny}) + H_{mn}\sin(K_{mx} - K_{ny})]$$

where

I_{os} = $180^0 - I_o$

I_s = $180^0 - I$

I and I_o = inclination of polarizing field and earth's normal total field respectively.

P_{mn} = $(K^2_m + K^2_n)^{1/2}$

$$E_{mn} = \frac{P_{mn}}{P^2_{mn} + \psi^2_1}(A_{mn} - D_{mn}) + \frac{\psi_1}{P^2_{mn} + \psi^2_1}(B_{mn} + C_{mn})$$

$$F_{mn} = \frac{P_{mn}}{P^2_{mn} + \psi^2_1}(B_{mn} + C_{mn}) - \frac{\psi_1}{P^2_{mn} + \psi^2_1}(A_{mn} - D_{mn})$$

$$G_{mn} = \frac{P_{mn}}{P^2_{mn} + \psi^2_2}(A_{mn} + D_{mn}) + \frac{\psi_2}{P^2_{mn} + \psi^2_2}(B_{mn} - C_{mn})$$

$$H_{mn} = \frac{P_{mn}}{P^2_{mn} + \psi^2_2}(B_{mn} - C_{mn}) - \frac{\psi_2}{P^2_{mn} + \psi^2_2}(A_{mn} + D_{mn})$$

$$\psi_1 = \cot I_{os}(K_m \cos D_o + K_n \sin D_o)$$

$$\psi_2 = \cot I_{os}(K_m \cos D - K_n \sin D)$$

D and D_o = declination of polarizing field and declination of earth's normal total field respectively.

3.2 Analysis in wavenumber domain

Kanasewich and Agarwal (1970) modified Bhattacharyya's method to take advantage of the fast Fourier transform algorithm to reduce the total magnetic field to the pole for arbitrary values of inclination and declination.

The spectral decomposition of the potential field data t(x,y) is carried out with the following complex Fourier transform relations in the digital form:

$$T(K_1,K_2)=\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} t(x,y).\exp\left[\frac{2\pi i}{N}(K_1x+K_2y)\right]$$

where

$$t(x,y)=\frac{1}{N^2}\sum_{K_1=0}^{N-1}\sum_{K_2=0}^{N-1} T(K_1,K_2).\exp\left[-\frac{2\pi i}{N}(K_1x+K_2y)\right] \quad (3.3)$$

for K_1, K_2 = 0, 1, 2 ..... (N-1)

x, y = 0, 1,2, ...(N-1)

The modified formula (Kanasewich and Agarwal,1970) used for the reduction of total magnetic field to the pole is:

$$f(x,y) = \frac{1}{\sin I_{os} \sin I_s} \sum_{K_1=0}^{N-1} \sum_{K_2=0}^{N-1} T(K_1,K_2).\exp(P_{12}Z)$$

$$\cdot \frac{P_{12}^4 - \psi_1 \psi_2 P_{12}^2 - 2(\psi_1 + \psi_2)P_{12}^3}{(P_{12}^2 - \psi_1 \psi_2)^2 + (\psi_1 + \psi_2)^2 P_{12}^2} \qquad (3.4)$$

$$\cdot \exp\left\{\frac{2\pi i}{N}(K_1 x + K_2 y)\right\}$$

where

$P_{12} = 2\pi/N(K_1^2 + K_2^2)^{1/2}$

$I_{os} = 180^0 - I_0$

$I_s = 180^0 - I$

$\psi_1 = 2\pi/N(K_1 \cos D_o + K_2 \text{ Sin } D_o) \cot I_{os}$

$\psi_2 = 2\pi/N(K_1 \cos D + K_2 \text{ Sin } D) \cot I_s$

Like equation (3.2), equation (3.4) is not applicable for inclinations of 0° and 180°

3.3 Instability of Reduction-to-the pole filtering

Gunn (1975) has shown that reduction to the pole is a linear transformation applied to the original field either in the space or wavenumber domain. In this respect, reduction to the pole can be viewed as a filtering operation. A better insight to the reduction to the pole process is possible by investigating the properties of the reduction to the pole filter in the wavenumber domain. However, the problem of numerical instability at low magnetic latitudes was not addressed in Gunn's paper.

Pearson and Skinner (1982) suggested a reduction to the pole procedure in the wavenumber domain which operated on the spectrum of the data previously smoothed. The

aim of this smoothing is to attenuate the noise content in the original data so that noise distortion in the reduced field is minimized. This approach is an attempt to deal with the instability of the reduction to the pole procedure at low magnetic latitudes.

However, applying the remedy to the data and not to the method presents some disadvantages. First, *a priori* information about the noise is needed. Secondly, noise elimination by spectral techniques is always imperfect due to Gibbs effect. As a result, part of the noise remains in the filtered data, and is therefore enhanced (since the method of reducing the anomaly to the pole is still unstable at low latitudes.) Gunn (1975), showed that the expressions for the spectra of a magnetic field in terms of equivalent layers is given by:

$$M_f(u,v,h) = 2\pi D_1(u,v).D_2(u,v).I(u,v) \\ M_s(u,v,h).H(u,v,h)$$

where

u,v = angular wavenumber coordinates

D_1 = factor responsible for direction of magnetization

D_2 = factor responsible for direction of measurement

H = depth factor

M_s = equivalent layer factor

I = factor distinguishing magnetic fields from gravity fields.

It is possible to remove the effect of any of these factors from the spectrum by dividing the spectrum by the factor it is desired to remove. This is equivalent to convolving in the space domain by an operator (or filter) whose frequency response is the inverse of that of the factor it is desired to cancel.

By convolving the magnetic field with a filter whose frequency response is given by

$$K(u,v) = (u^2+v^2)/[(iu\cos I_o\cos D_o + iv\cos I_o\sin D_o + (u^2+v^2)^{1/2}\sin I_o)$$

$$.(iu\cos I\cos D + iv\cos I\sin D + (u^2+v^2)^{1/2}\sin I)] \quad \ldots\ldots \quad (3.6)$$

where

i = $\sqrt{-1}$

I, I_0 = inclinations of the magnetization and the geomagnetic field.

D, D_0 = azimuths of the magnetization vector and geomagnetic field.

The magnetic field is reduced to what the magnetic field would be at the north magnetic pole. It can easily be seen from equation (3.6) that at u=v=0, the filter operator is undefined. That is, the base level value of the reduced to the pole anomaly obtained by filtering is arbitrary.

Silva (1986), demonstrates this instability by putting equation (3.6) in the polar form. Using the transformations $u = r\cos\theta$ and $v = r\sin\theta$, and after some manipulations, equation (3.6) becomes:

$$K(r,\theta) = \frac{1}{[i\cos I_0\cos(D_0-\theta) + \sin I_0][i\cos I\cos(D-\theta)\sin I]} \quad (3.7)$$

Equation (3.7) shows that close to the magnetic equator ($I_o \approx 0$ degrees) and for values of θ close to $D_o \pm 90$ degrees, the imaginary and real parts of the complex filter weights are very high. In practice, this filter operates on the discrete spectrum of the original data and this spectrum contains imperfections due to aliasing and Gibbs' phenomenon. Therefore, a poor performance of this filter can be expected at low magnetic

latitudes even in the absence of noise because the imperfections in the spectrum will be enhanced by the large values of the filter weights along the direction $D_o \pm 90$ degrees. Recall that the direction $D_o \pm 90$ degrees in the wavelength domain corresponds to the direction D_o in space domain; a fictitious trend along D_o, due to filter enhancement of imperfections in this direction, is expected to show up when reducing low-latitude anomalies to the pole.

In two-dimensional (2-D) problems this instability occurs only when the 2-D structure strike is close to the direction of the geomagnetic field or the magnetization. In three-dimensional (3-D) problems, however, the instability always occurs because there will always be a few points (u,v) in the 2-D data spectrum such that $\tan^{-1}(v/u)$ is close to the direction $D_o \pm 90$ degrees.

CHAPTER 4

REDUCTION TO THE POLE AS AN INVERSE PROBLEM

4.1 Introduction

Silva (1986) showed that by formulating the problem of reduction to the pole as an inverse problem, stable solutions in the presence of noise could be found using well-known methods, such as that of Freund and Minton (1979), and Hoerl and Kennard (1970 a,b). Consequently, meaningful results at low magnetic latitudes, where the instability is more pronounced, can be obtained.

Initially, the magnetization of an equivalent layer of doublets is computed from the observed data. All magnetic doublets are assumed parallel to the magnetization vector whose direction is supposedly constant throughout the sources. Once, strengths of the doublets are found, the inclinations of the field are changed to 90 degrees and the total magnetic field is recalculated. The first stage involves the solution of an inverse problem, while the second is a forward problem. Only the first stage presents instability so that a stabilizing procedure must be applied only there.

4.2 Theory of Method Used

Bott and Ingles (1972) and Emilia (1973) were the first to reduce magnetic anomalies to the pole with the approach based on equivalent sources. It has been extensively employed in processing satellite magnetic data (Mayhew, 1979; Mayhew *et al.*, 1980; von Frese *et al.*, 1982; Ridgway and Hinze, 1986). Most of these authors used two-dimensional

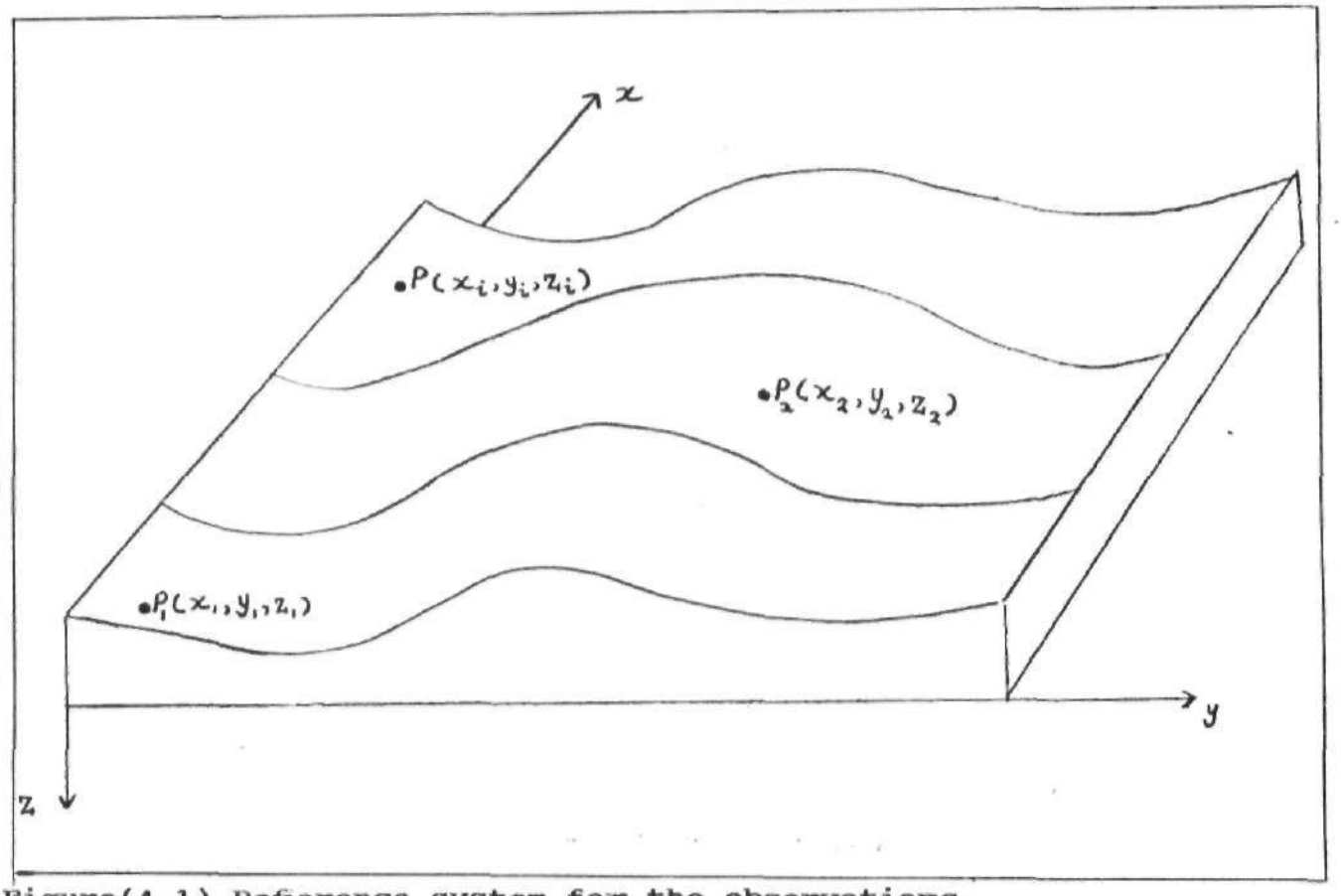
sources and analyzed the magnetic field along profiles perpendicular to the source strike. Unless the two-dimensional structure is in the direction of the horizontal projection of the geomagnetic field or the magnetization, the 2-D reduction to the pole is a stable transformation. However Silva (1986) has shown how this can be done in three dimensions.

Let h_i be the total magnetic field observed at point $P(x_i, y_i, z_i)$ referred to a right-handed Cartesian system of coordinates (figure (4.1)). The magnetic field due to arbitrary sources can be exactly fitted by the magnetic field due to a horizontal equivalent layer of dipoles at depth $z = d$. For each depth there is a unique distribution of magnetization. The equivalent layer of dipoles can be approximated by an equivalent layer of doublets at depth d. Each doublet has length b and is parallel to the unit magnetization vector α. The magnetization direction α is assumed constant throughout the sources (figure (4.2)).

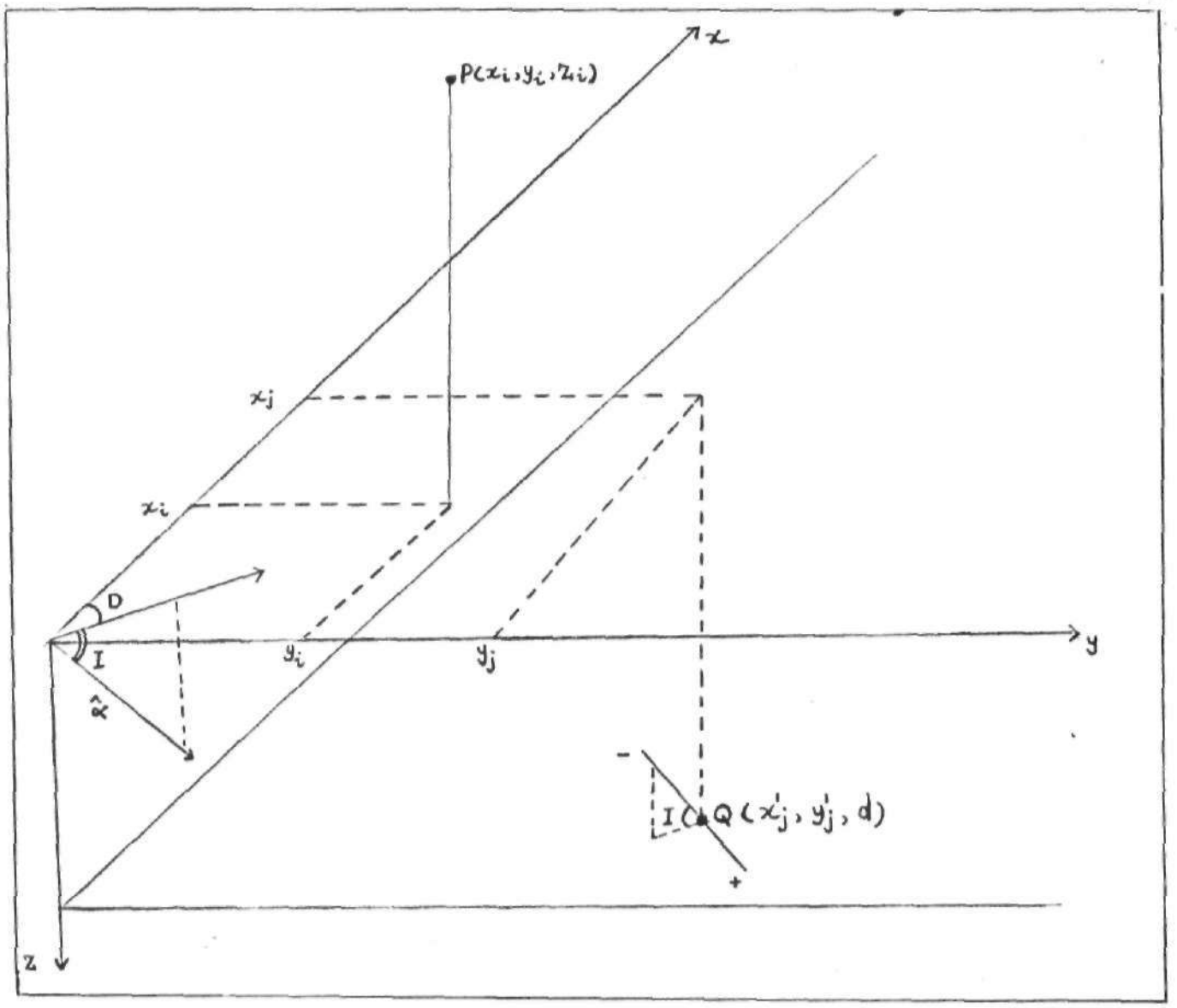
The magnetic field at point $P(x_i, y_i, z_i)$ due to the doublet centred at point $Q(x'_i, y'_i, d)$ is

$$h_{ij} = a_{ij} P_j \qquad (4.1)$$

where P_j = strength of magnetization of the j^{th} doublet.



Figure(4.1) Reference system for the observations.



Figure(4.2) Position of doublets with respect to the observations.

$$a_{ij}=\left[\frac{x_i-x_j'-(b/2)\cos I\cos D}{F_1}-\frac{x_i-x_j'+(b/2)\cos I\cos D}{F_2}\right]\cos I_o\cos D_o$$

$$+\left[\frac{y_i-y_j'-(b/2)\cos I\sin D}{F_1}-\frac{y_i-y_j'+(b/2)\cos I\sin D}{F_2}\right]\cos I_o\sin D_o$$

$$+\left[\frac{z_i-d-(b/2)\sin I}{F_1}-\frac{z_i-d+(b/2)\sin I}{F_2}\right]\sin I_o$$

where

$F_1 = [\,(x_i - x_j' - (b/2)\cos I\cos D)^2 + (y_i - y_j' - (b/2)\cos I\sin D)^2 + (z_i - d - (b/2)\sin I)^2]^{3/2}$

$F_2 = [\,(x_i - x_j' + (b/2)\cos I\cos D)^2 + (y_i - y_j' + (b/2)\cos I\sin D)^2 + (z_i - d + (b/2)\sin I)^2]^{3/2}$

The total magnetic field h_i at point $P(x_i, y_i, z_i)$ due to all doublets is the sum of all the magnetic fields at point $P(x_i, y_i, Z_i)$ due to each doublet. That is,

$$h_i=\sum_{j=1}^{M} a_{ij}P_j \qquad for \quad i = 1, 2, \ldots, N \tag{4.2}$$

where

N = number of observations

M = number of doublets

The above equation can be written in matrix notation as:

$$\mathbf{H = AP} \tag{4.3}$$

where

H = vector containing the computed magnetic field at the observation points due to a layer of doublets.

A = matrix whose a_{ij} elements are given by equation (4.1)

P = vector containing doublet strengths.

The smaller the doublet length b relative to the sampling interval, the better the approximation by the equivalent sources. However, if the doublet length is too small relative to the sampling interval, the matrix A becomes ill-conditioned. That is, large changes in the solution may result from small changes in the vector of observations. This is because the coefficients a_{ij} in equation (4.1) tend to very small values. Silva (1986) showed that a value of b between one-half and one-fifth of the sampling interval produced good results.

From equation (4.3), the solution for P is given by

$$P = A^{-1}H \tag{4.4}$$

where A^{-1} is a suitable inverse. Silva (1986), showed that A^{-1} can be expressed as

$$A^{-1} = D(DA^{T}AD + \lambda I)^{-1}DA^{T} \tag{4.5}$$

where D is a diagonal scaling matrix given by

$$d_{ii} = \frac{1}{\left(\sum_{j=1}^{N} a_{ji}^{2}\right)^{1/2}}$$

A^{T} is the transpose of A and λ is a real positive number usually between 0 and 1. I is an identity matrix. The criterion for selection of the best value of λ is given in Hoerl and Kennard (1970b), and Marquardt (1963).

Once the vector P of strengths is estimated, it is trivial to obtain the magnetic field reduced to the pole. The inclinations of the doublets and the geomagnetic field in equation (4.1) are changed to 90 degrees and we have,

$$b_{ij} = \frac{z_i - d - b/2}{\left\{(x_i - x_j')^2 + (y_i - y_i')^2 + (z_i - d - b/2)^2\right\}^{3/2}} - \frac{z_i - d + b/2}{\left\{(x_i - x_j')^2 + (y_i - y_j')^2 + (z_i - d - b/2)^2\right\}^{3/2}} \quad (4.6)$$

and the total field at the pole is computed by

$$H_{pole} = B.P \quad (4.7)$$

where B is a matrix whose b_{ij} elements is given by equation (4.6).

Therefore

$$H_{pole} = BD(DA^TAD + \lambda I)^{-1}DA^TH \quad (4.8)$$

4.3 Applications to noise free data

To analyse the behaviour of the proposed technique to noise free (synthetic) data, the methods of Bhattacharyya (1964) and Rao and Babu(1991) were used to generate the magnetic field due to a vertical prism uniformly magnetized in the direction of the geomagnetic field. In terms of grid units the prism is 2 units long (x-direction, assumed parallel to the north-south direction) 2 units wide (y-direction) and 5 units thick (z-direction). Its top is located 3 units below the plane of measurements and the magnetic susceptibility is 0.002 SI units. The field is computed at discrete points on the plane z = 0 using a 10 x 10 square grid with the magnetization vector and the geomagnetic field having inclinations of 45 degrees and azimuth of 5 degrees with respect to the x-axis. The magnetic field produced by this prism is shown in figure (4.3) and the field computed with an inclination of 90 degrees (at the pole) is shown in figure (4.4).

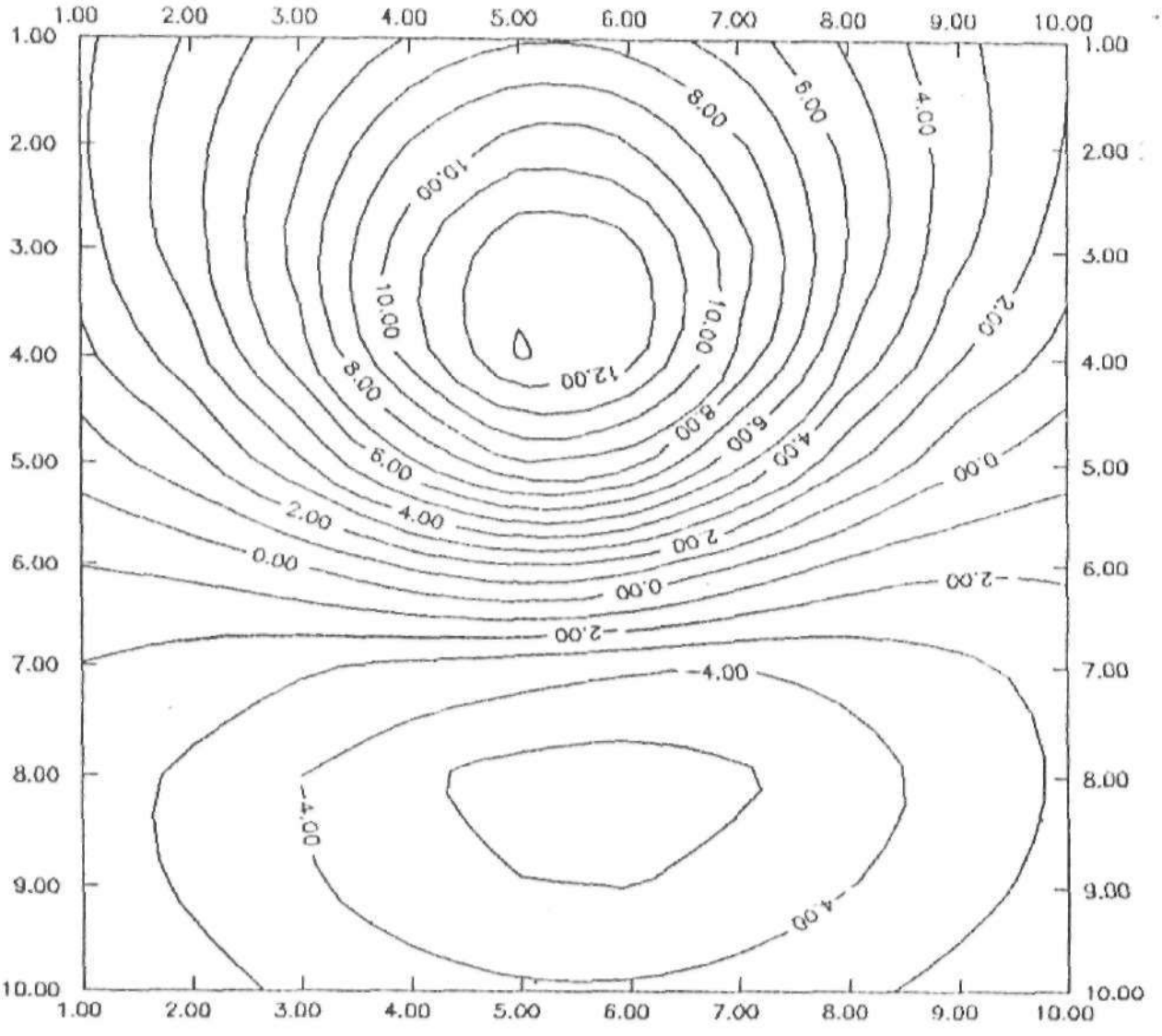


Figure (4.3) Total field produced by prism.
I = 45 degrees and D= 5 degrees.
contour interval is 1 gamma.

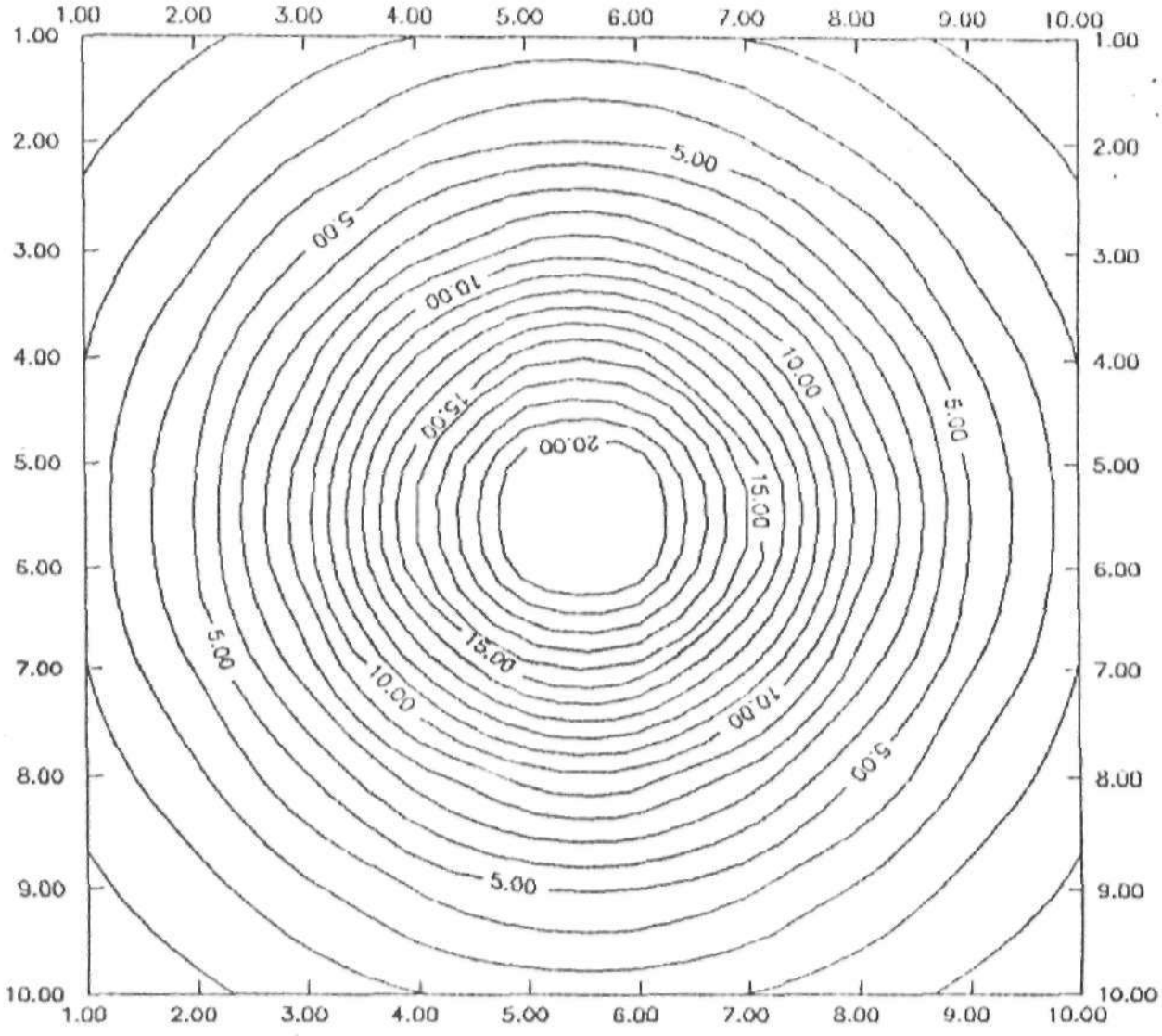


Figure (4.4) Theoretical total field at pole for the prism. Contour interval is 1 gamma. I = 45 degrees and D = 5 degrees.

A FORTRAN program REDPOL, (Appendix 1) was developed, based on equation (4.8) to reduce the magnetic field to the pole. Using the same number of doublets as observation points and doublets located directly below the observation points, the field due to the basic prism shown in figure (4.3) is used as an input to the program and the output or result (that is the field now reduced to the pole) is shown in figure (4.5).

The field in figure (4.5) which is now symmetric is comparable to that of figure (4.4), indicating the good performance of the technique. The parameter λ in equation (4.8) is a parameter which depends on the level of noise in the observed data. Because the data used in this case is noise free, $\lambda = 0$ was assumed. The technique was also tested for the same prism situated at a magnetic inclination of zero degrees (figure (4.6)), the result obtained is shown in figure (4.7).

4.4 Application to noise - corrupted data

In order to simulate the real practical situations, the basic prism used in figure (4.3) was corrupted with noise. Since noise is the effect produced by smaller bodies at shallower depths, four smaller prisms with the coordinates for their dimensions given in table (4.1), were at a depth of 1.2 grid units and above the plane of measurements and situated randomly around the prism as shown in figure (4.8); and the field produced by these bodies were computed and added to that produced by the major prism.

This noise corrupted field is shown in figure (4.9).

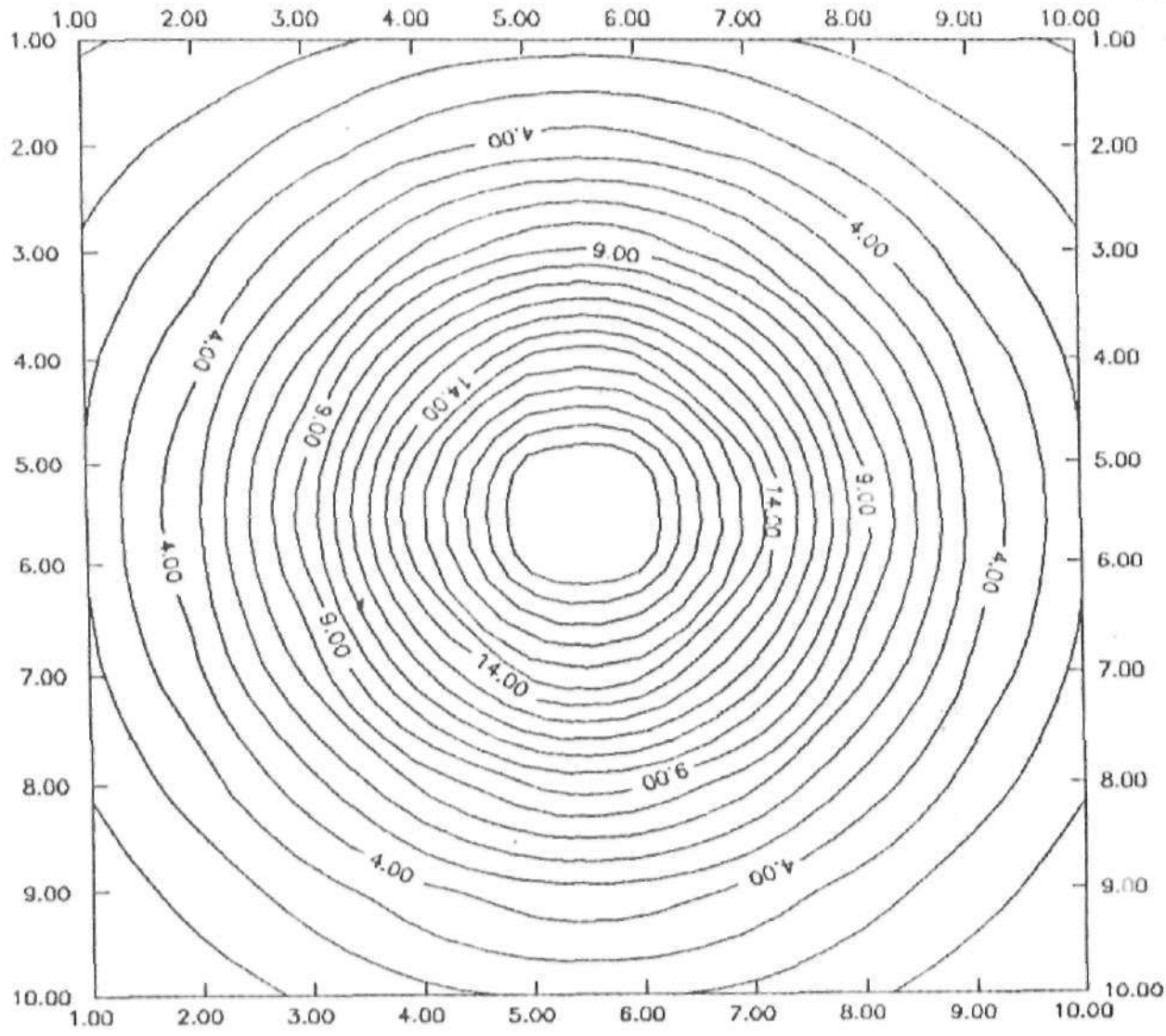


Figure (4.5) Total field reduced to the pole obtained by the application of the proposed technique to the megnetic field shown in figure (4.3) with λ = 0. I = 45 degrees and D = 5 degrees. Contour interval is 1 gamma.

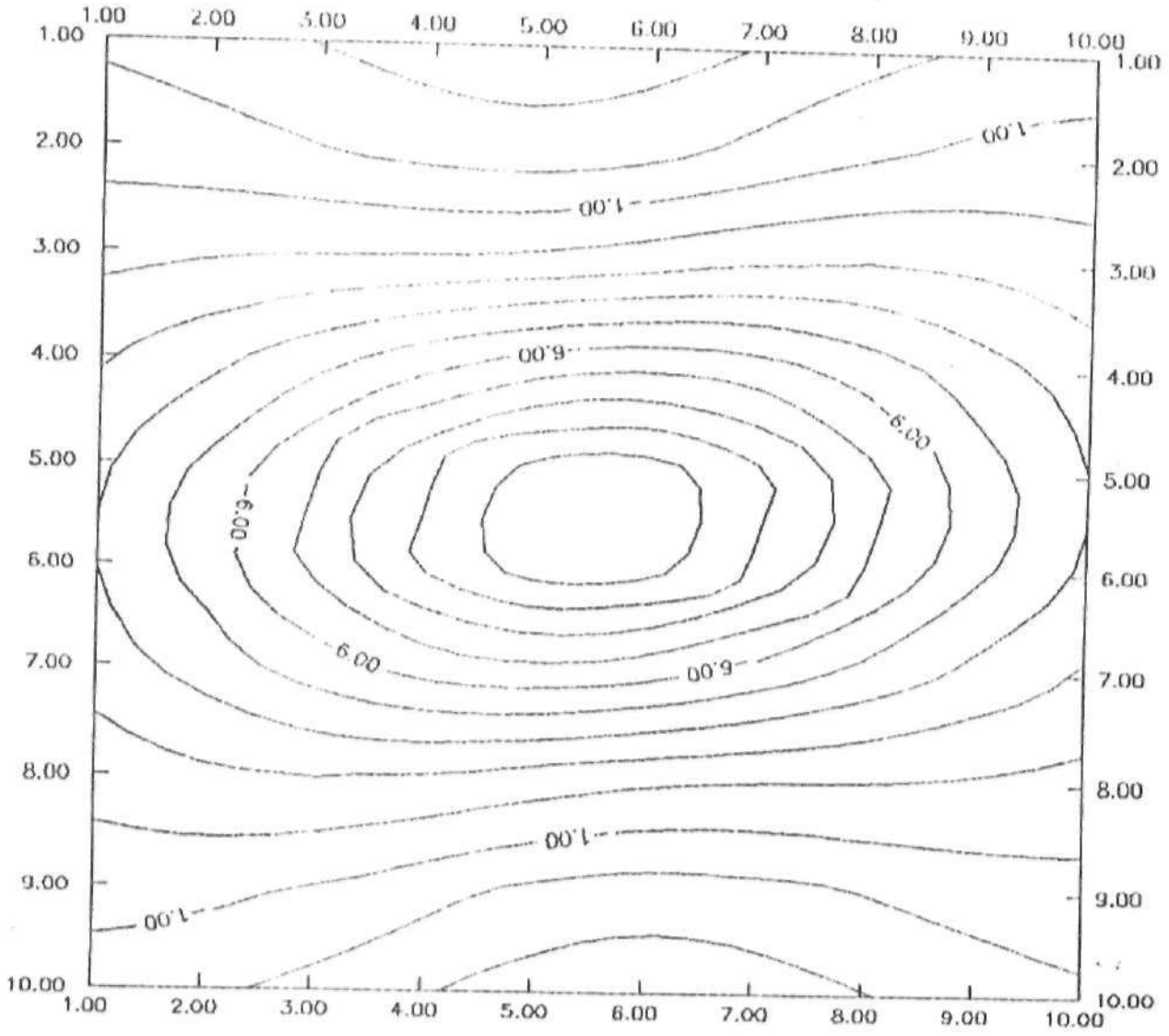


Figure (4.6) Theoretical field at I = 0 degrees and D = 5 degrees. Contour interval is 1 gamma.

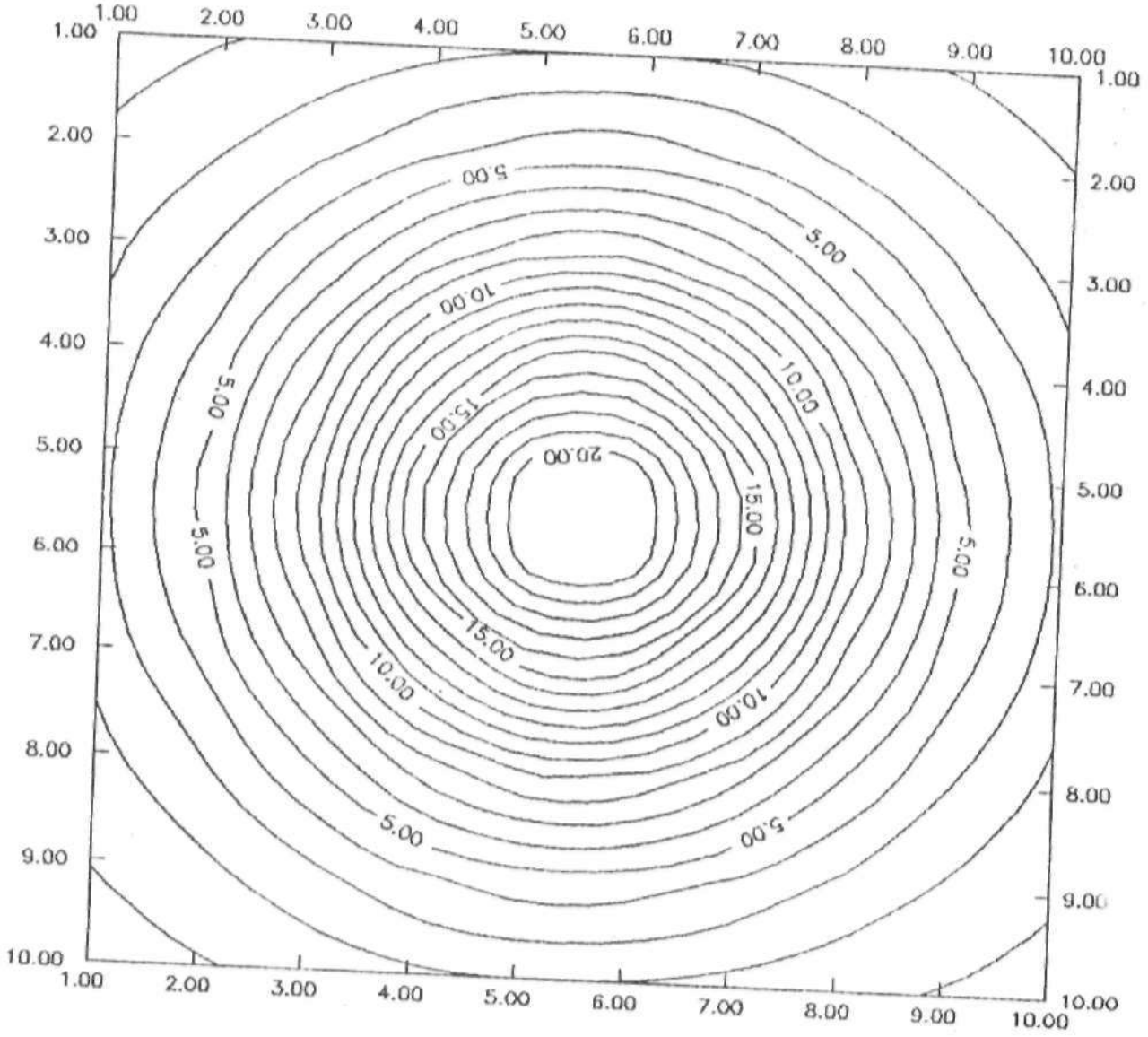


Figure (4.7) Total field of (4.6) reduced to the pole.
I = 0 degrees, D = 5 degrees and λ = 0.

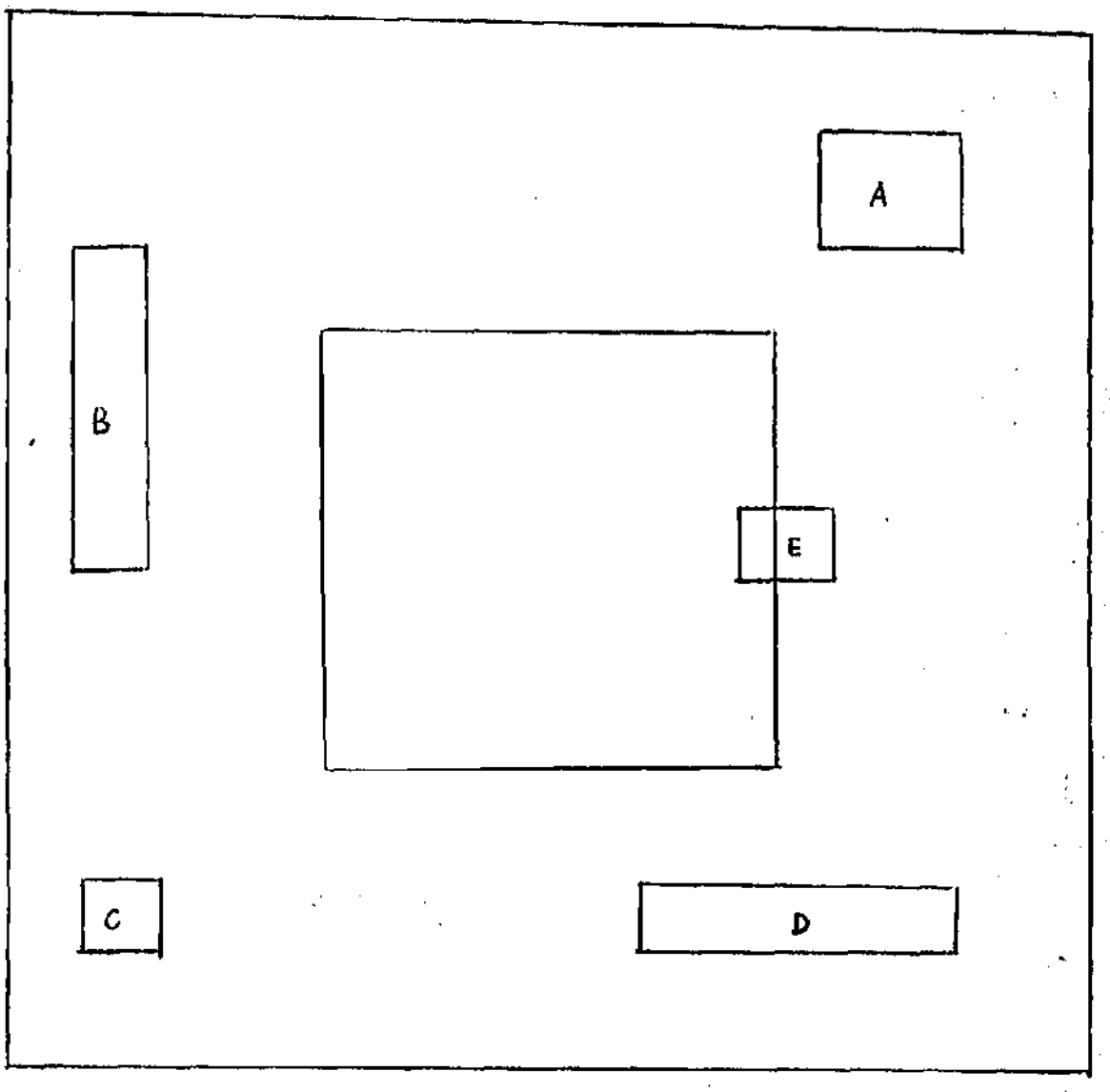


Figure (4.8) Plan view of prisms indicating the positions of noise bodies.

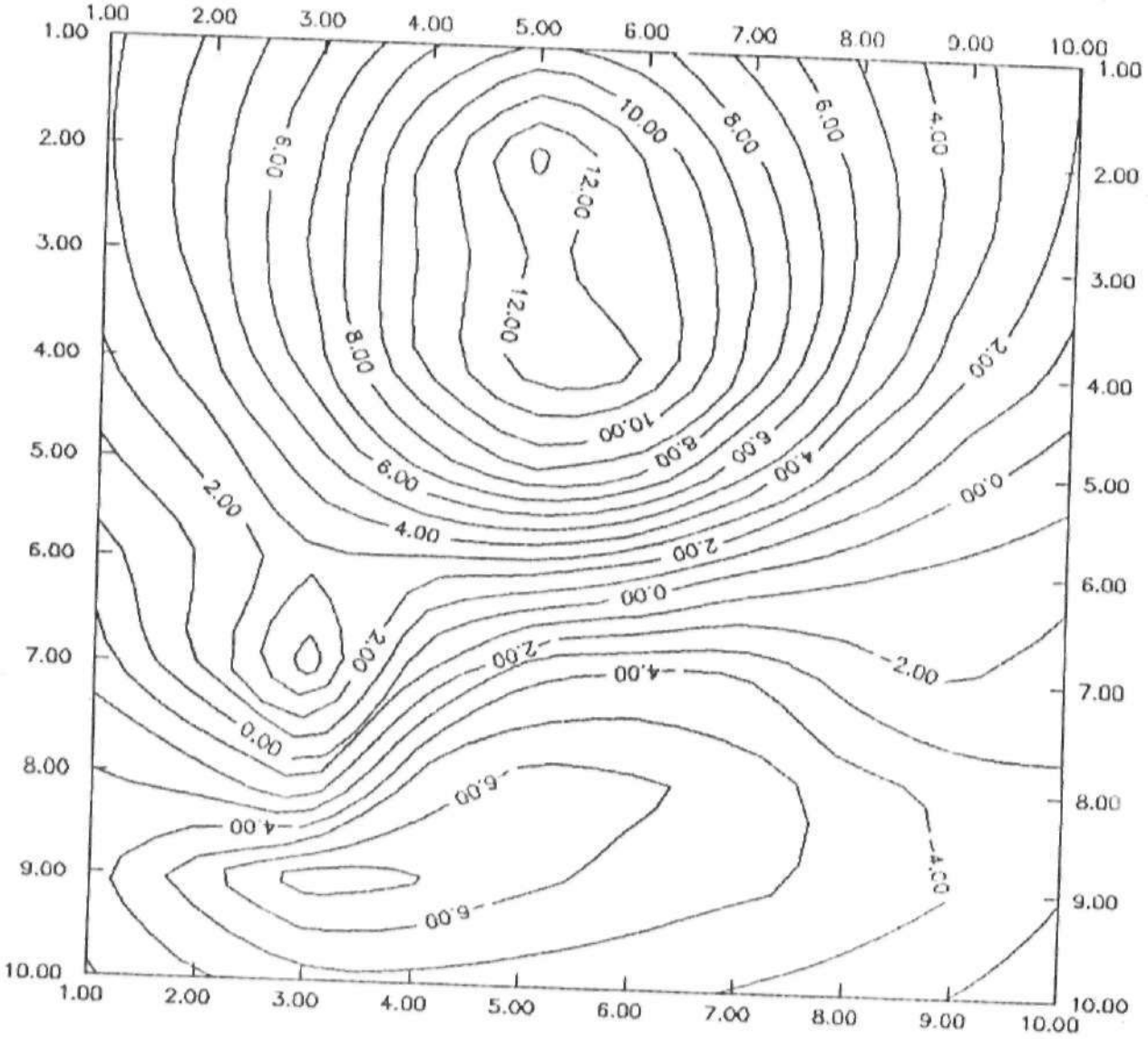


Figure (4.9) Total field due to the prism corrupted with noise. I = 45 degrees and D = 5 degrees.

Table 4.1. Dimensions and coordinates of prisms which produce noise

Body	Thickness (grid unit)		Vertical thickness (grid unit)
	x-direction	y-direction	
A	1.0	1.0	0.3
B	0.3	1.0	0.3
C	0.2	0.2	0.3
D	1.0	0.2	0.3
E	0.2	0.2	0.3

A criterion based on the analysis of the "ridge trace" as proposed by Hoerl and Kennard (1970 b) was employed to find the best value of λ. The Ridge trace is found in the methods of ridge regression analysis, which is a method that controls the general instability associated with least squares estimates. The ridge regression estimate of equation (4.4) is given by

$$P^* = [A^T A + \lambda I]^{-1} A^T H \qquad (4.9)$$

Hoerl and Kennard (1970 b), and Freund and Minton (1979) suggested that if the estimate P^* in equation (4.9) is plotted against values of λ between 0 and 1 , the best value of λ occurs where all the coefficients seem to settle down. Such a plot is what is called the ridge trace (Hoerl and Kennard, 1970 b). For the analysis of the proposed technique on noise corrupted data, the field in figure (4.9) was reduced to the pole with $\lambda = 0$. The result produced (figure. 4.10) is distorted due to the noise effect. However applying the stabilizing technique (ridge regression analysis) by plotting the ridge trace (figure. 4.11) for the noise

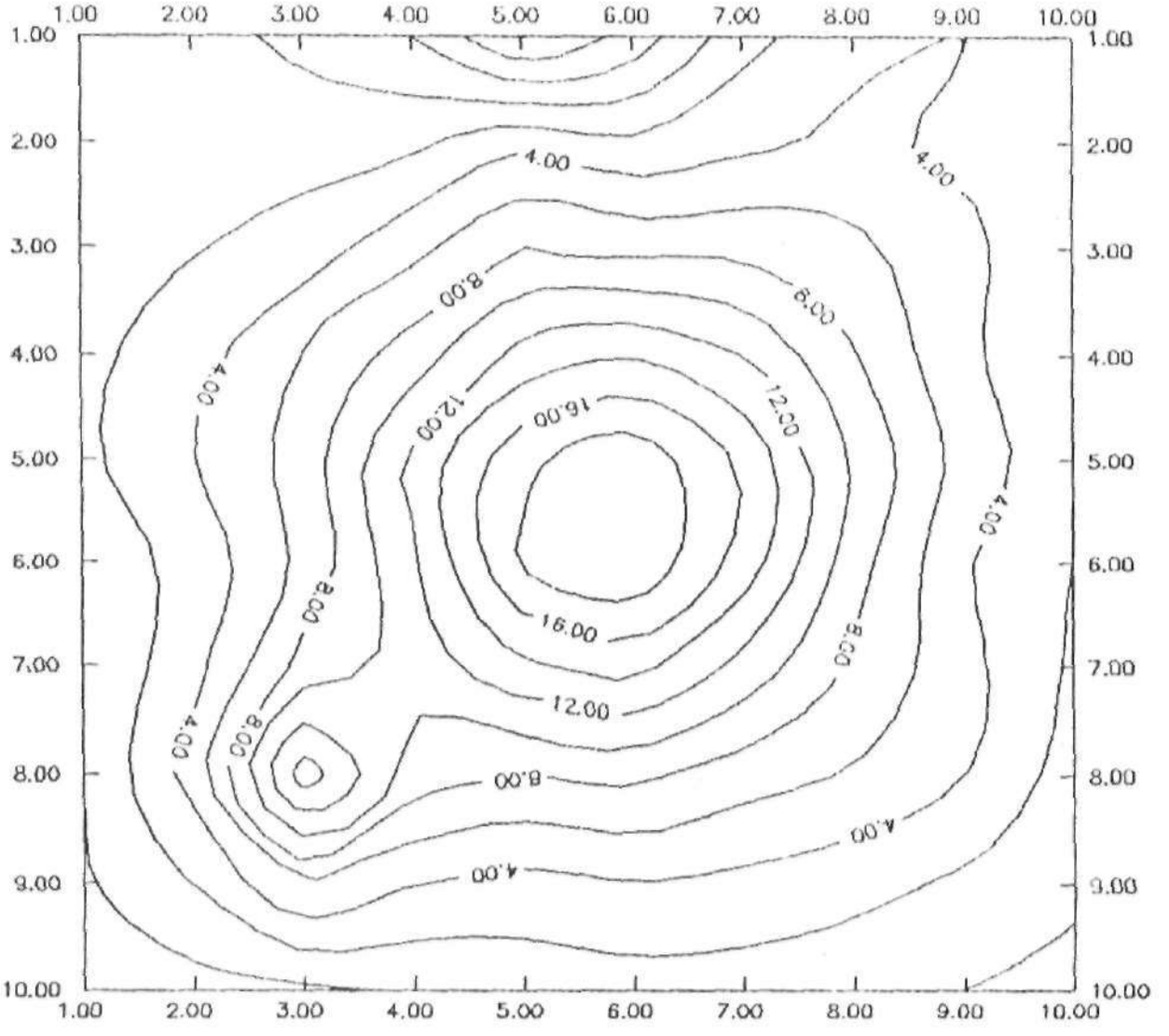
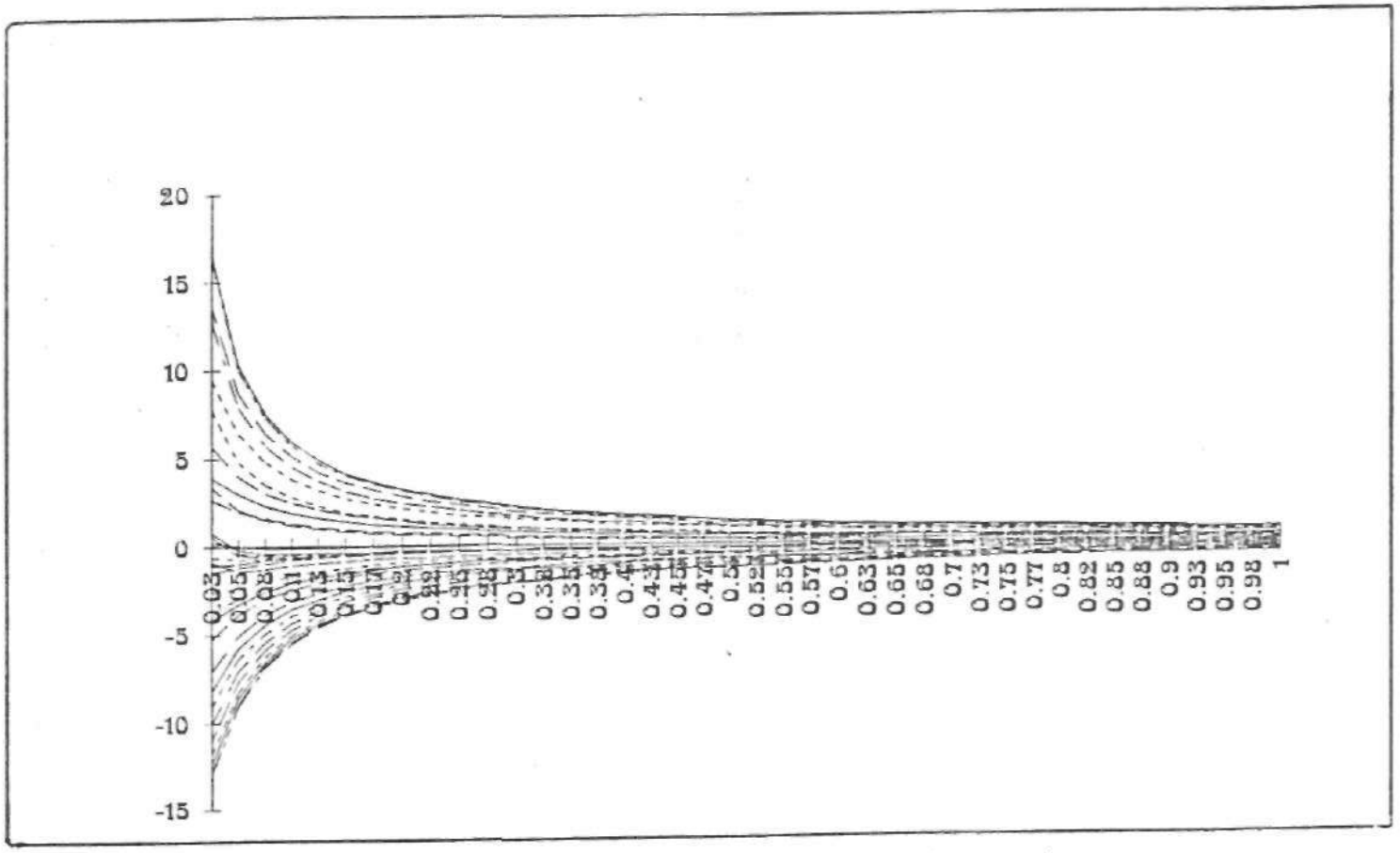


Figure (4.10) Noise corrupted field reduced to the pole with $\lambda = 0$.



Figure(4.11) Ridge trace

corrupted body, stable results can be obtained as shown in figure (4.12). A value of λ = 0.55 (from figure 4.11) was used. The above results indicate how λ can be used as a controlling factor for the presence of noise in the observed data.

4.5 Restrictions on the depth of equivalent layer

Dampney (1969) investigated the conditions under which the approximation of the continuous layer by a discrete layer remains valid for the gravity case. He concluded that the depth of the equivalent layer must be between 2.5 and 6 times the grid spacing. Silva (1986) indicated the limits on the depth of the equivalent layer to be about the same as those found by Dampney (1969). The lower limit follows from the fact that the approximation of the discrete doublets to a continuous distribution of magnetization produced a valid representation sufficiently far from the surface of measurement relative to the average doublet spacing. The upper limit follows from the fact that the elements of matrix **A** (equation (4.3)) approach a common value when d is large relative to the doublet spacing, causing the system to be ill-conditioned (equation (4.4)). Also to improve the performance of the proposed technique, the average spacing employed for the observations must be at most one-half the depth to the top of the sources yields a poor representation of the magnetic field.

In summary, the restriction on the use of the proposed technique are (1) the discrete nature of both the equivalent sources and the observations, (2) the sampling intervals have definite and fixed values in order to produce a valid representation of the observed magnetic field. Also, ill-conditioning should be avoided by a suitable choice of the depth of the equivalent layer.

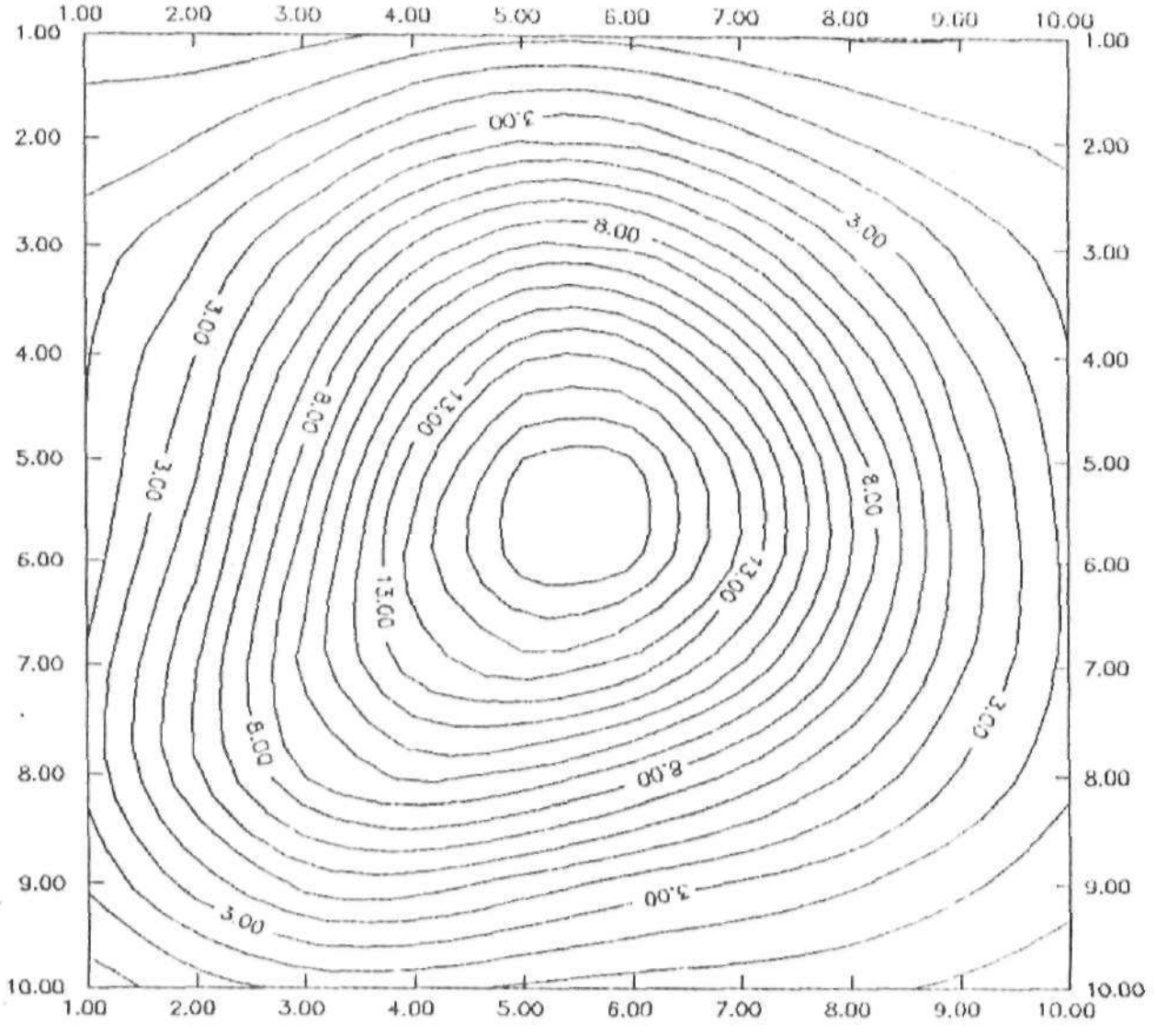


Figure (4.12) Total field reduced to the pole of the magnetic field shown in figure (4.9) with $\lambda = 0.55$

4.6 Windowing the data

In the use of inverse methods to formulate the reduction to the pole problem, the computation of the equivalent sources is formulated either as an exactly determined problem or as an over-determined least squares problem. As a result, the number of entries in the matrix to be inverted increases with the square of the total number of observations. A 30 x 30 area (in grid units), for example requires the inversion of a 900 x 900 matrix. Attempts to divide a large area into smaller subareas and process them individually incur undesirable edge effects, despite the adoption of some strategies such as overlapping adjacent subareas.

Von Frese *et al.* (1988) presented an iterative procedure to process large scale equivalent source inversions. A small equivalent layer is computed using all observations. The residuals between the observations and the field produced by the equivalent layer located at a different place within the area of investigation. Further iterations use the residuals between the observation and the field produced by the equivalent layer are used to calculate another small equivalent layer located at a different place, also within the area of investigation. Further iterations use the residuals between the observations and the field produced by all previously computed equivalent layers. This method produced good results, but it required large amount of computer time.

Leao and Silva (1989) have presented a method which can be used to process areas with large amount of data. An N x N data window is inverted using an M x M equivalent layer, with M greater than N (an under-determined least squares problem) so that the equivalent layer extends beyond the data window. Only the transformed field at the centre of the data window is computed by pre-multiplying the equivalent source matrix by the row of the matrix B in equation (4.7) corresponding to the data window. Since the inversion and

the multiplication by the row of matrix B are independent of the data, they are performed beforehand and just once for given values of N, M and the depth of the equivalent layer. As a result, a grid operator for the desired transformation is obtained which is applied to the data by a procedure similar to discrete convolution.

Since the problem is underdetermined, equation (4.5) is modified to:

$$A^{-1} = A'D(DAA'D+\lambda I)^{-1}D \qquad (4.10)$$

where D is the normalizing diagonal matrix with elements

$$d_{ii} = [\sum_{j=1}^{M} a_{ij}^2]^{-1/2}, \qquad i = 1, \ldots\ldots, N \qquad (4.11)$$

Equation (4.10) has the advantage that an increase in the number M of discrete equivalent layers does not increase appreciably either the memory or the processing time required to obtain the inverse because the matrix to be inverted is N x N instead of M x M. Therefore, the discrete equivalent layer may be conveniently extended beyond the data window limits in order to take the interfering sources into account.

The transformed field at the central point of the data window is given by:

$$t = b^T A'D[AA'D + \lambda I]^{-1}DH \qquad \ldots (4.12)$$

where b^T is the central row of matrix **B**.

The damping factor λ is selected by producing transformed fields for increasing values of λ and choosing the one for which there is no qualitative change in the transformed field with respect to that for the previous value. Equation(4.12) was tested on synthetic

total field anomalies (Figure 4.13) produced by vertical prisms whose plan views are also displayed. The magnetic field is computed at points on a 38 by 55 grid. All prisms have a magnetization intensities of 6.6 A/m, thickness of 3.0 grid unit, depth to top of 2.0 grid units, and the geomagnetic field has an inclination of 45 degrees and declination of 5 degrees. Figure(4.14) shows the theoretical anomaly at the pole. Using an equivalent layer of 81 (9 by 9) point sources having unit grid spacing and grid points located directly below the observations with data window of 25 (5 by 5) points, the anomalies shown in figure(4.13) were reduced to the pole and the result is shown in figure(4.15). Comparing this map with the theoretical anomaly at the pole (figure 4.14) we see that the apex of the anomalies are shifted to the centres of the bodies, thereby indicating the good performance of the method.

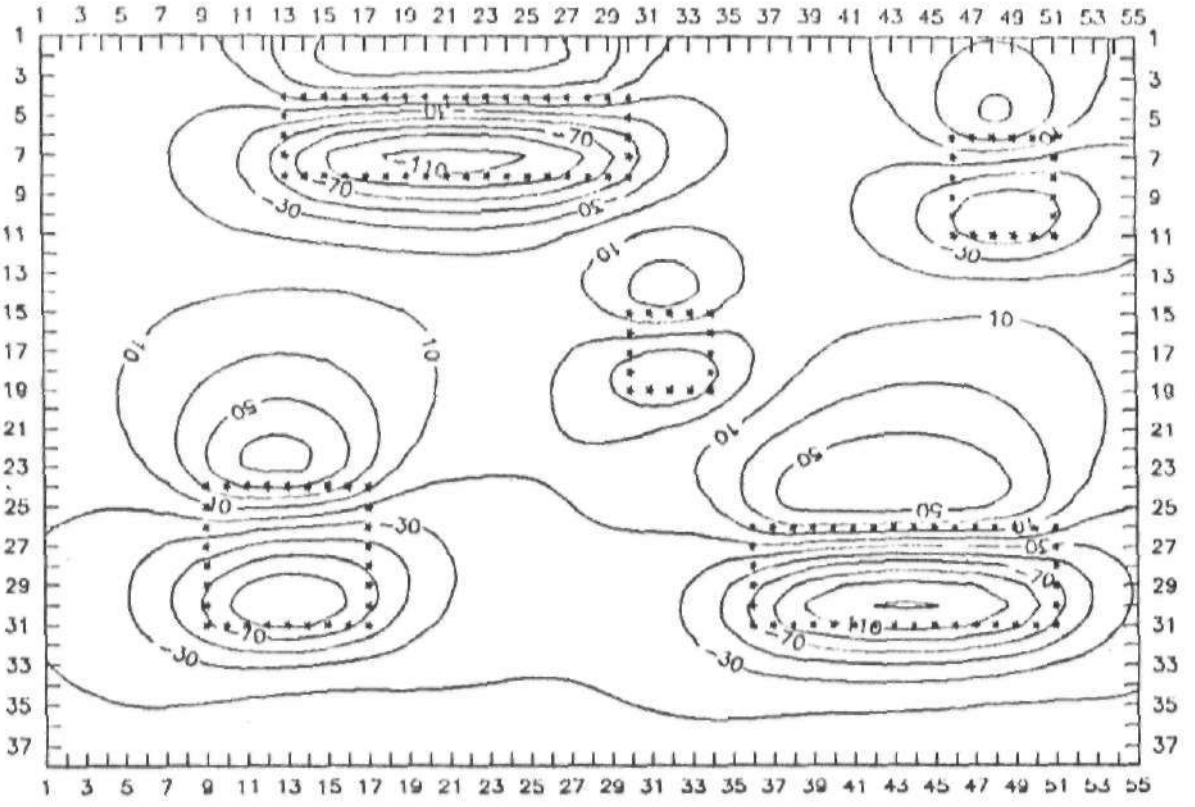


Figure (4.13) Total field at I = 45 degrees and D = 5 degrees.

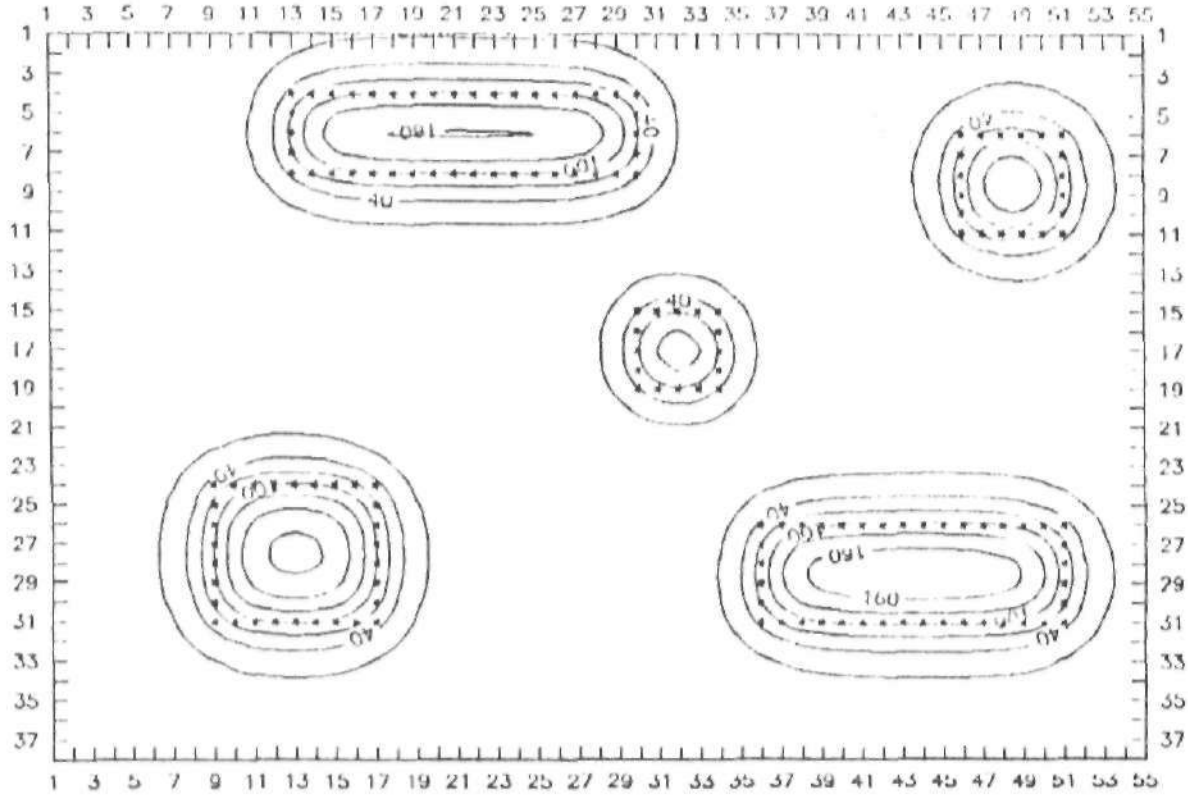


Figure (4.14) Total field at I = 90 degrees (at the pole) and D = 5 degrees.

CHAPTER 5

REDUCTION TO THE MAGNETIC POLE OF THE AEROMAGNETIC DATA OF ZARIA AREA

5.1 Brief Geology of the Area

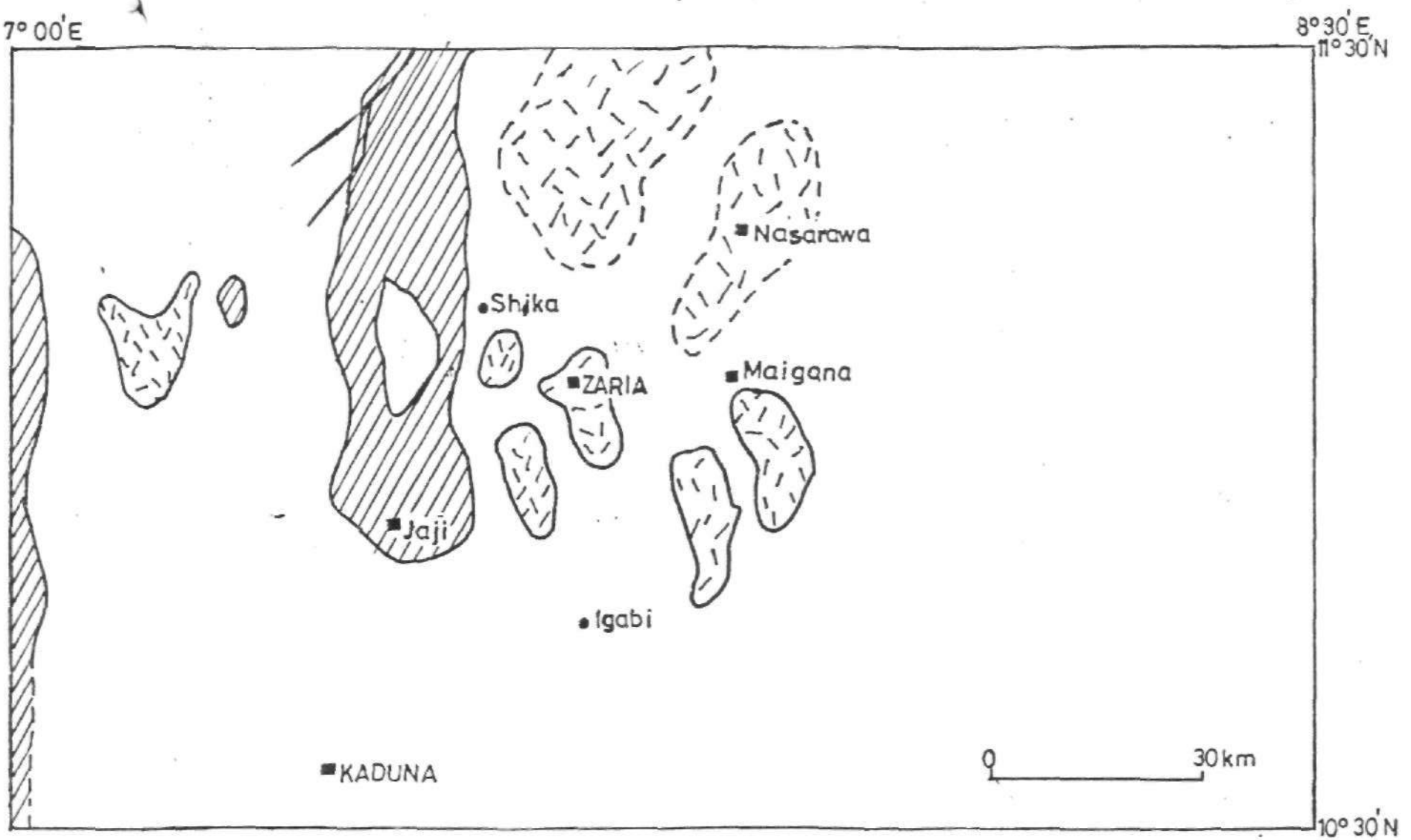
Based on the work of McCurry (1970), the principal rock units in the area are:

i. Meta-sediments, which is believed to have been laid down and then metamorphosed during the Pan-African Orogeny (between 850 and 650 million years ago) and occupy linear north-south trending isoclinal belt to the west of the project area Figure (5.1).

ii. A crystalline complex composed of gneiss, considered to have been metamorphosed more than 2,000 million years ago.

iii. Older granite which included the gneissic crystalline complex from 647 to 618 million years ago.

However, Webb (1972) mapped the extent of the granite plutons and metasediments to coincide more or less with the extent of granite inselbergs and quartzite ridges respectively. She considered the more or less featureless ground around Samaru and Shika to coincide with the extent of the gneiss. Webb (1972) mapped such ground in detail, and found much of it to be underlain by weathered granite and schists. Also orthogneisses outcropped around Maigana (Figure 5.1). These are considered to be intrusions into the gneissic unit, but older than the older granites (Webb, 1972). Furthermore, the metasedimentary belt is wider than before and extends beyond Jaji (Figure 5.1) (Webb, 1972). However, the various principal rock units in the area have variable petrographic compositions, the details of which are given by McCurry (1970) and also summarized in McCurry (1971, 1973, 1976).



Figure(5.1) Geology of the project area

5.2 Data Acquisition and Reduction

The data used in the survey were obtained from the Nigerian aeromagnetic survey sheets 101, 102, 103, 123, 124 and 125. The survey for each sheet was completed in 1974, and each sheet has a mean terrain clearance of 152.4 m (500 ft) with the flight lines at intervals of 2 km from each other.

The maps were each digitized on a grid of 3 km in the x and y directions corresponding to the north-south and east-west directions respectively. The choice of 3 km digitizing interval was guided by both the flight lines separation and the sizes of magnetic anomalies observed on the maps. Digitizing at interval more than twice the flight lines separation introduces aliasing. In this manner, all the sheets combined together were digitized to obtain a 55 x 38 data matrix, which was contoured as shown in figure (5.2).

Next, the digitized field data were separated into "regional" and "residual" components by polynomial fitting. The regional components represent the effect of deep seated structures and can be represented by a first order surface. The first order surface representing the whole area is given by:

$$T(x,y) = 0.00324x - 0.29759y + 32,873.64 \qquad (5.1)$$

where T(x, y) is the total magnetic field and "x" and "y" are coordinates of the maps measured from the top left corner of the map. This regional plot is shown in figure (5.3). The "residual" components, which are left after the removal of the regional components, represent effects of near surface or shallow rocks such as the granites and gneiss complex. Figure (5.4 a) shows the residual map of the area. This field was reduced to the pole by using a 5 x 5 data window and 9 x 9 equivalent layer with $\lambda = 0.13$. This is shown in Figure (5.4 b).

5.2 Data Acquisition and Reduction

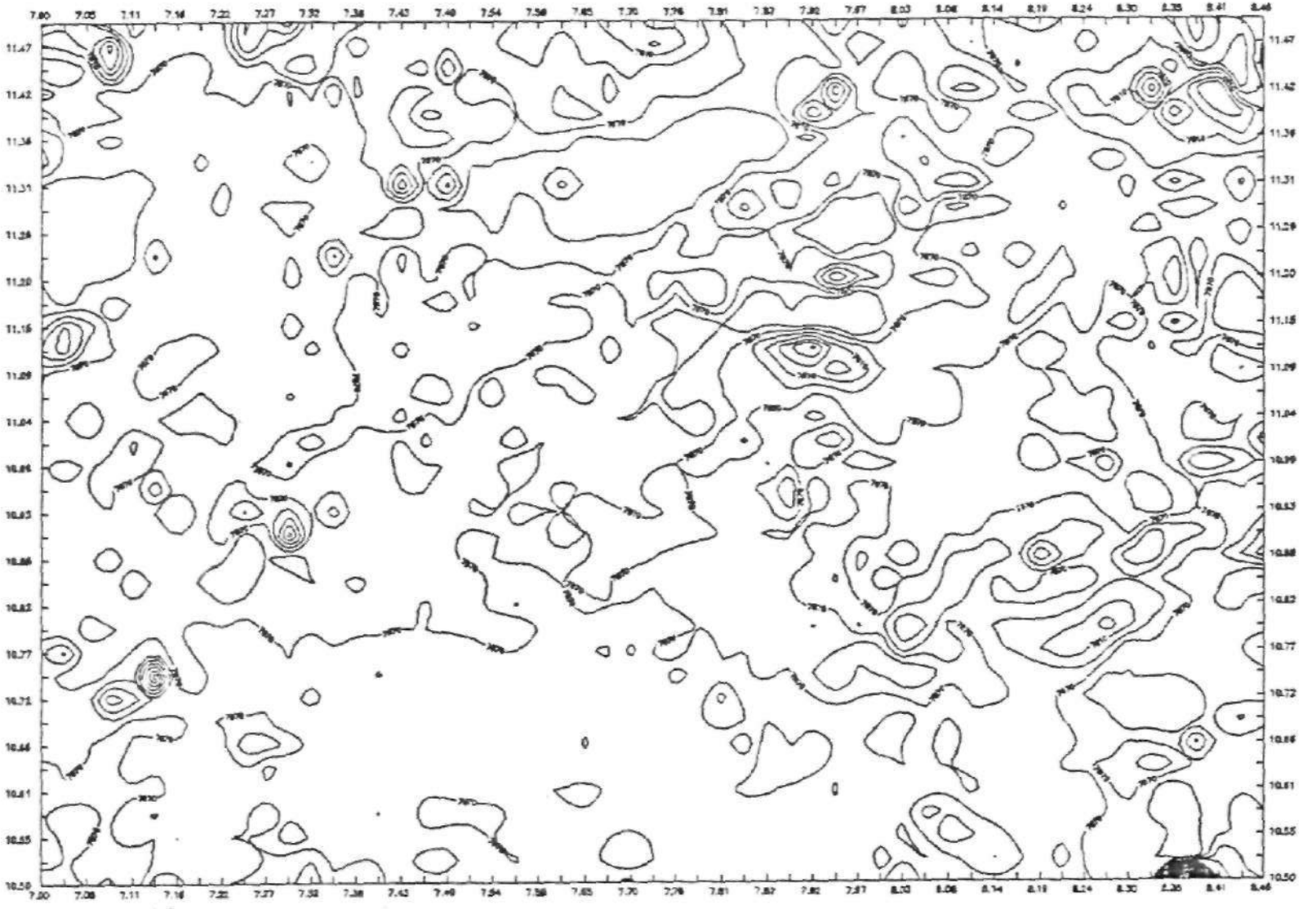
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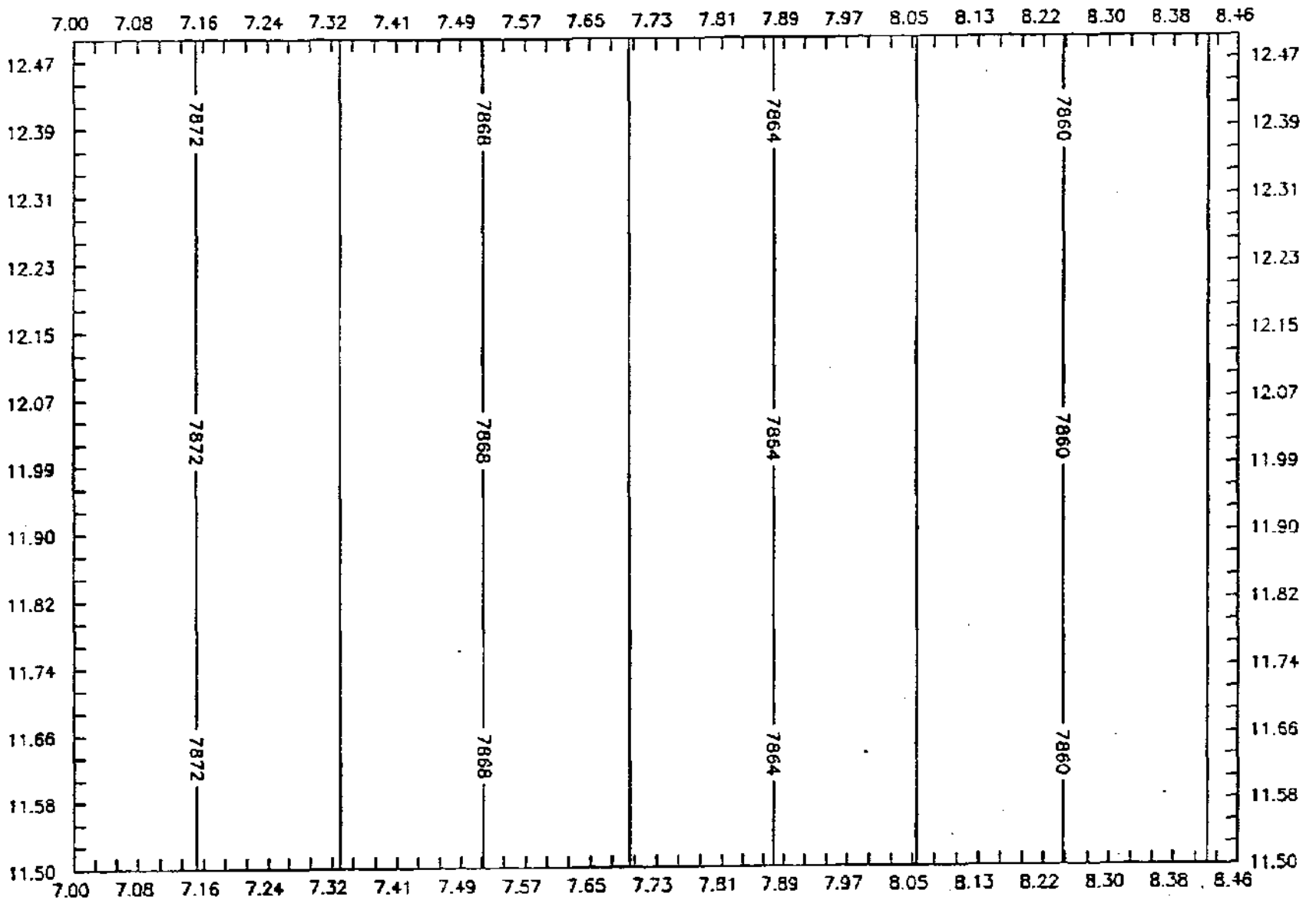
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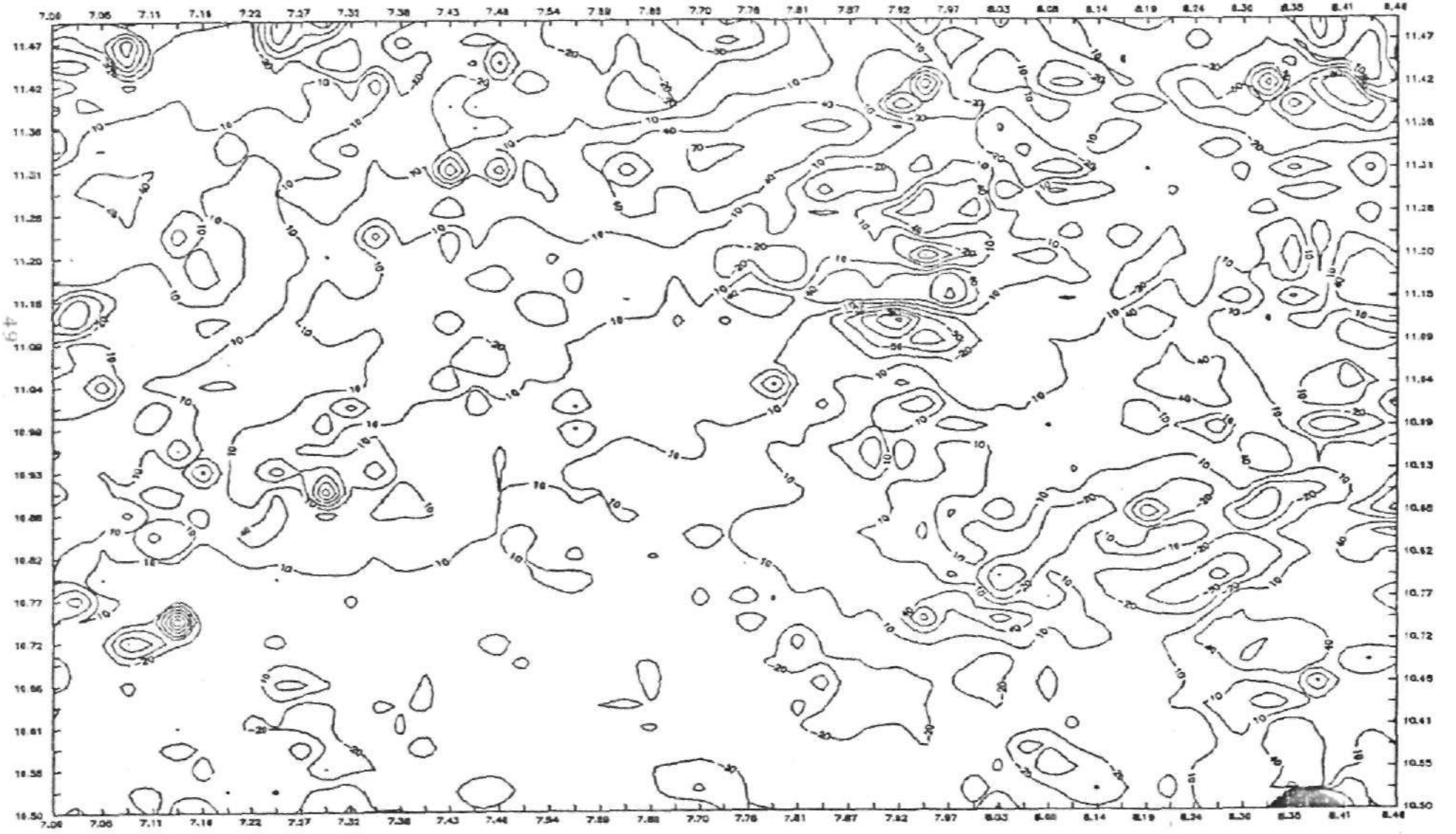
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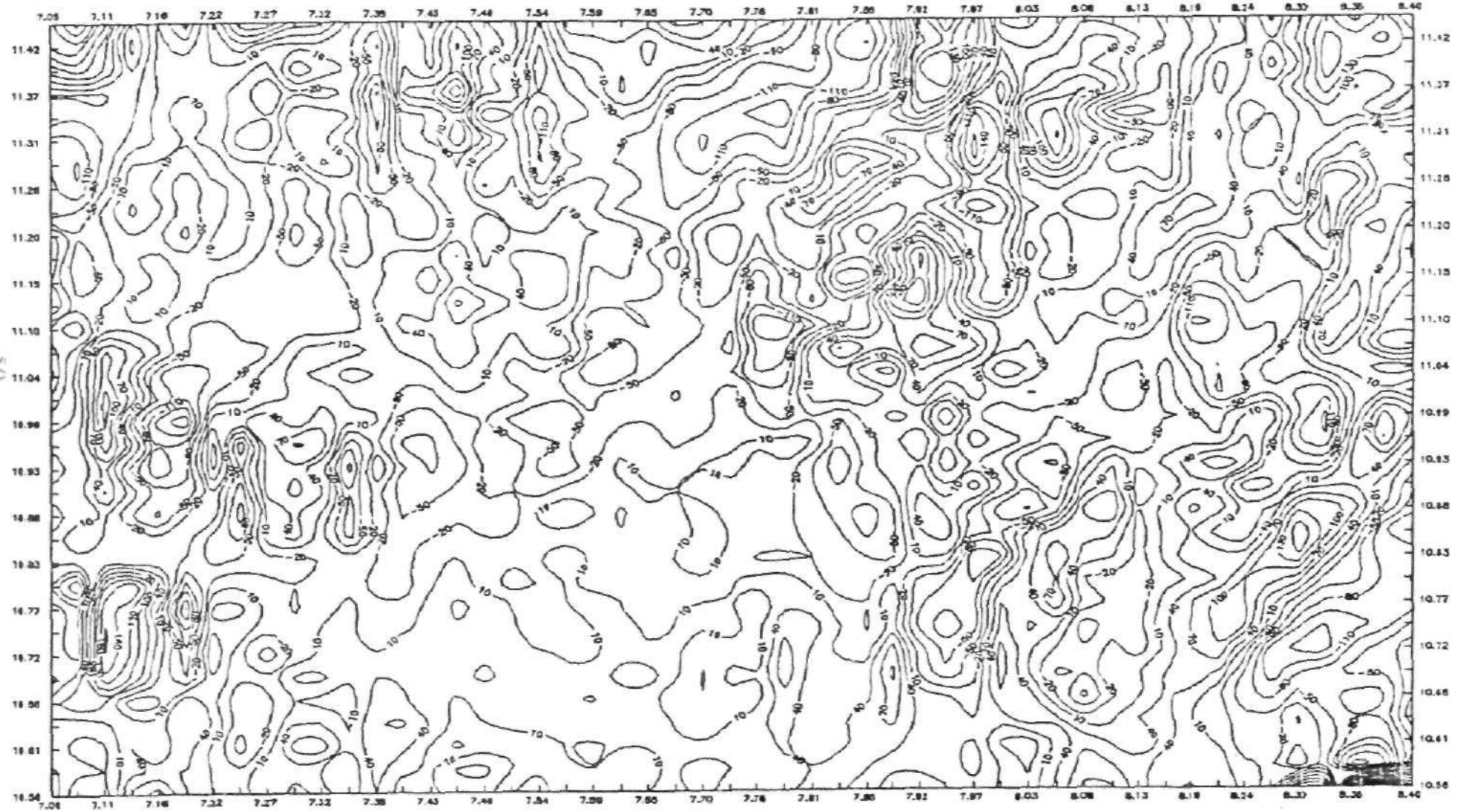
Figure(5.2) Total field map of Zaria area.(25,000 gamma is to be added to obtain original field)



Figure(5.3) Regional field map of Zaria area.(25,000 gamma is to be added to obtain original field)



Figure(5.4a) Residual field map of Zaria area



Figure(5.4b) The reduced to the pole field of the residual field. I = 2 degrees, D = 4 degrees and λ = 0.13

5.3 Comparison of the Total Field and the Reduced to the Pole Map.

i. The observed anomalies on the reduced to the pole map appears stronger than those observed on the total field map.

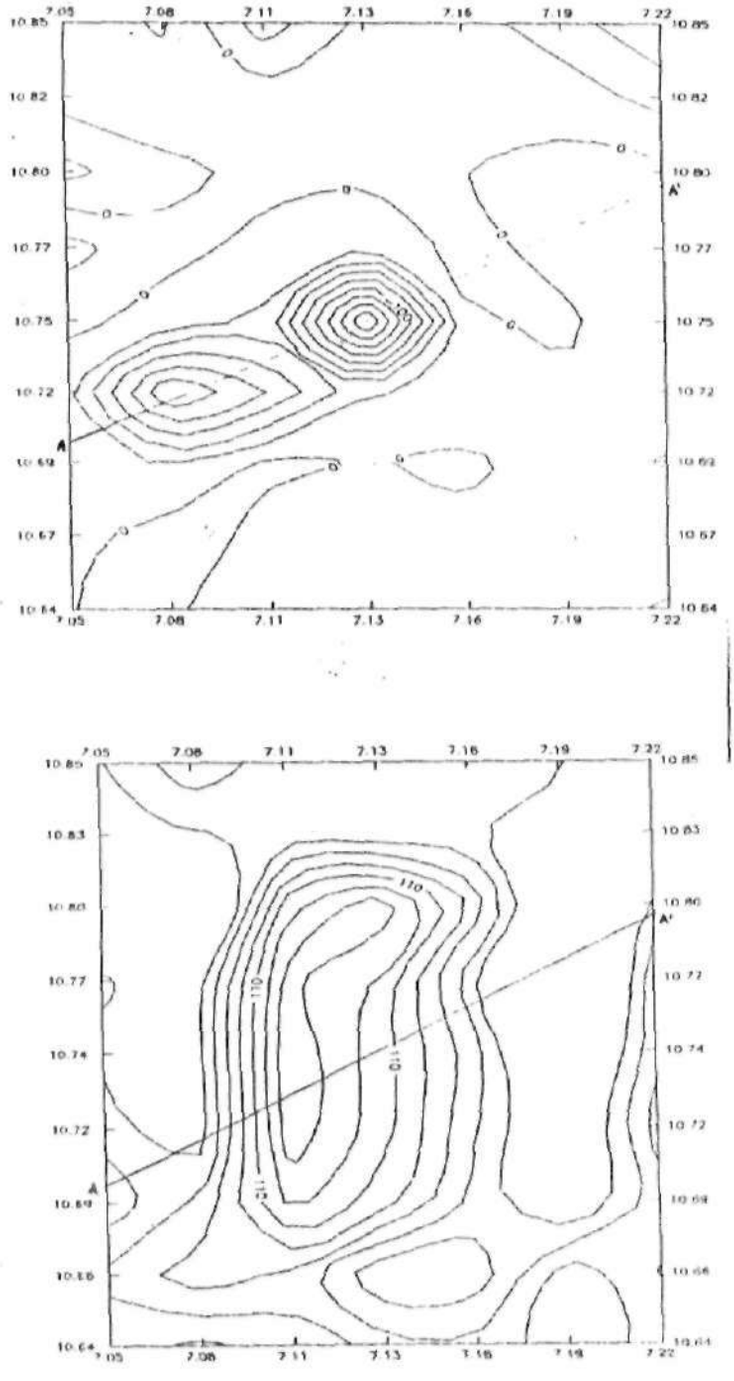
ii. On the reduced to the pole map there are more positive anomalies than there are negative anomalies which indicates that the assumption of induced magnetization is a good approximation.

iii. The anomalies on the reduced to the pole map appears to be more symmetrical than those on the total field map. This is confirmed by looking at figure (5.5) and (5.6). Figures (5.5 a) and (5.6 a) are selected anomalies within the area, while figure (5.5 b) and figure (5.6 b) show their corresponding reduced to the pole field. Figure (5.7) shows the profile AA' taken across the anomalies in figures (5.5a) and figure (5.5b). From these figures, one can observe that the curve from the reduced to the pole field appear to be more symmetrical than that for the total field.

iv. Also in figure (5.6), the anomaly (Figure 5.6 a) seems to have three poles; a negative pole at the point marked X, a positive pole at the point marked Y and another negative pole at the point marked Z. Naturally, at first sight one would imagine that the anomaly is caused by two or three sources. But from Figure (5.6 b), it can be observed that:

(a) the three poles are now reduced to one positive pole;

(b) the anomaly is displaced towards the left hand side of the map.



Figure(5.5) (a) Selected anomaly "P" on the residual map (Figure 5.4a)
(b) Its corresponding reduced to the pole anomaly.

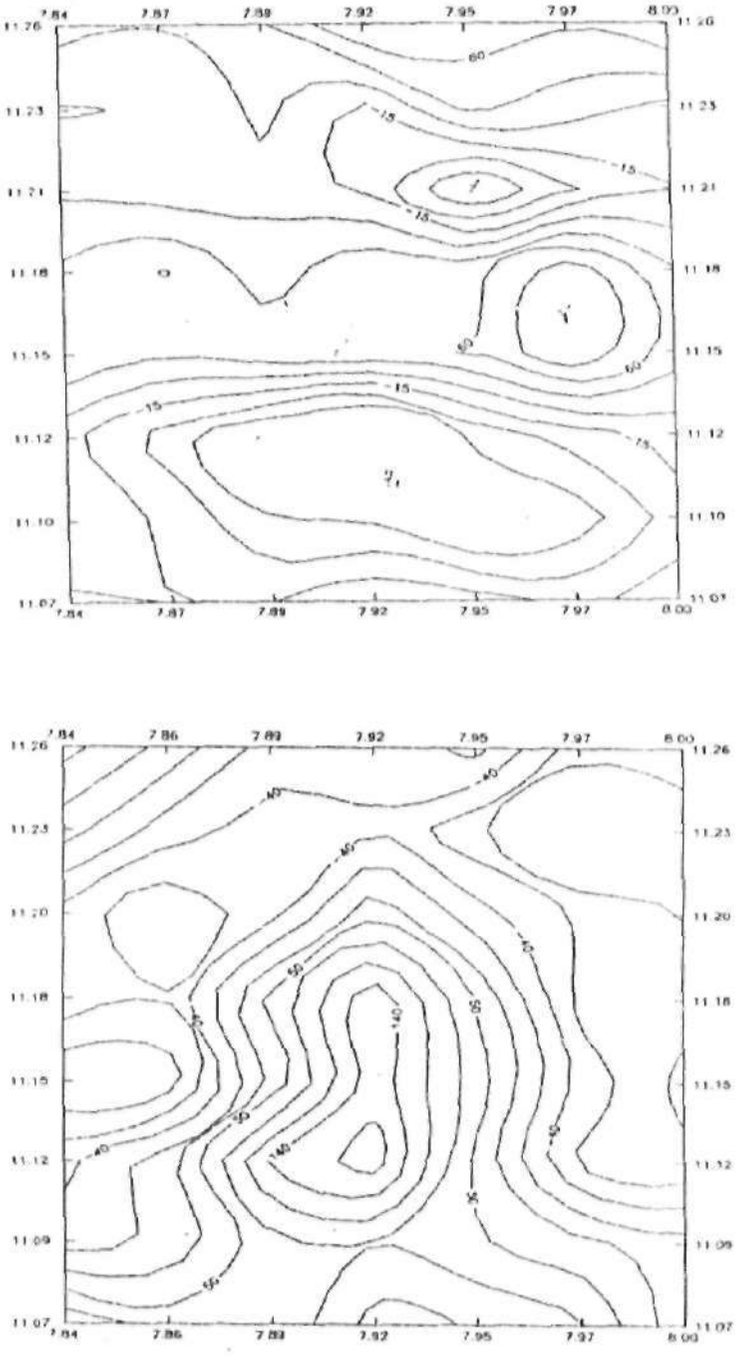
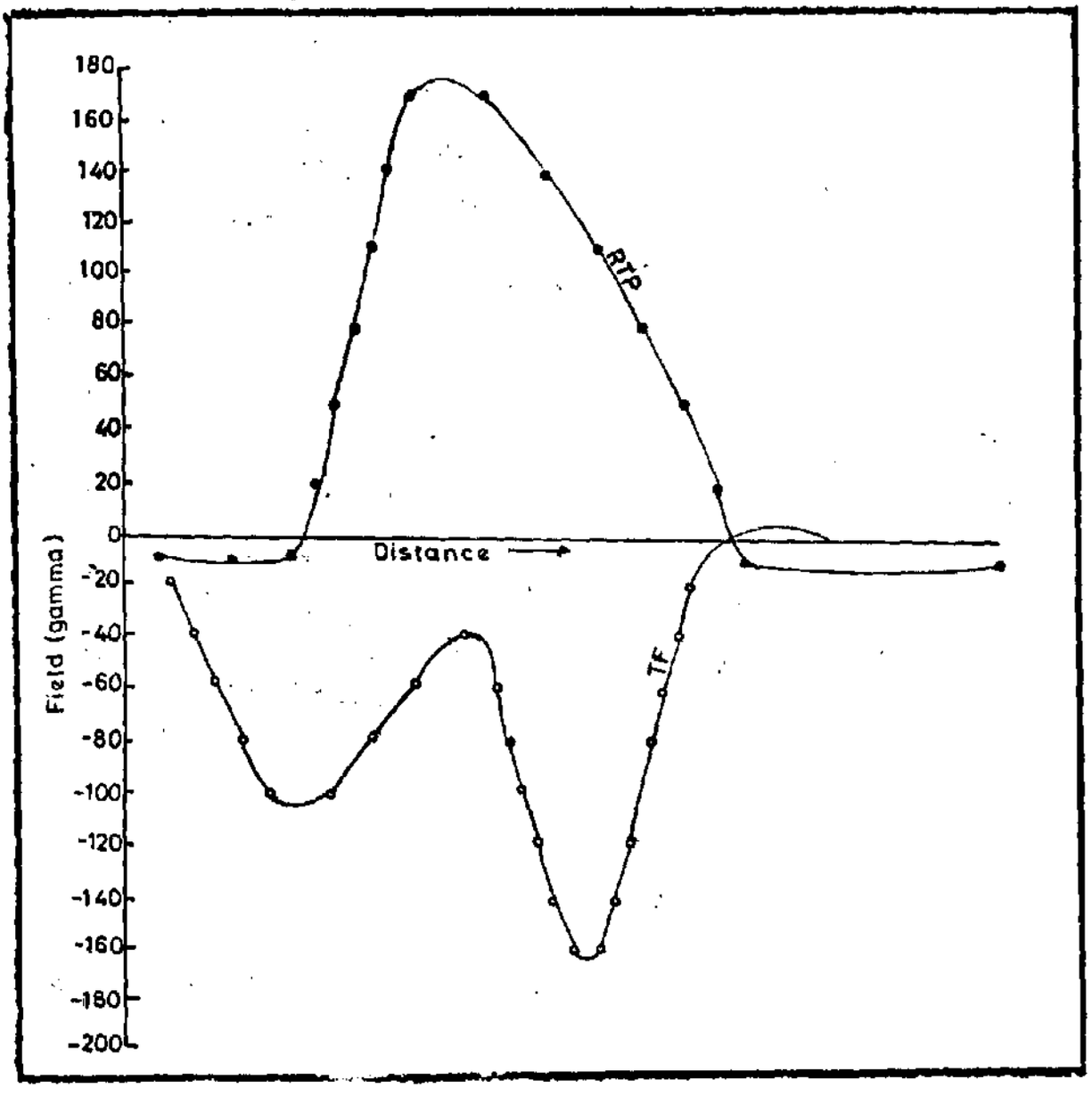


Figure (5.6) (a) Selected anomaly "Q" on the residual map (Figure 5.4a),
(b) Its corresponding reduced to the pole anomaly.



Figure(5.7) Curves of profile taken across profile AA in figure(5.5).
RTP = reduced to the pole.
TF = total field.

5.4 Comparison of the Total Field and Reduced to the Pole Maps with the Geology of the Area

When the two maps (Figure (5.2) and figure (5.4 b)) re compared with the geologic map of the area, the following could be observed:

i. The fault which occurs at the upper part of the schist belt (the younger metasediment) corresponds with a magnetic anomaly on the reduced to the pole map. This anomaly seems to have a trend (north-south) in the direction of the fault.

ii. Generally, the granitic intrusions (the older granite) do not have good correlations with the anomalies observed on the two maps. But on the reduced to the pole map there is little correlation around the Nasarawa area and south-eastern part of Maigana.

iii. Despite the poor magnetic nature of gneiss, some prominent anomalies in the south-western and eastern part of the magnetic maps fall on areas that are predominantly gneiss (i.e. the older matasediments).

5.5 Interpretation of anomalies

Interpretation of magnetic maps is usually done to determine the depth, dip, size and magnetic susceptibility of causative sources. Interpretation may be accomplished by assuming that the sources have simple geometric shapes. Among many simple models available for magnetic interpretation, the thick dike is the most widely used because of its suitability in many geologic situations. For these reasons, the dike model is assumed for interpretation. The parameters of the dike may be found from its magnetic anomaly by using either:

1. the curve matching techniques (Hutchinson, 1958; Gay, 1963).
2. the method of characteristics (Bruckshaw and Kunaratum, 1963; Am, 1972; Atchuta Rao and Babu, 1981).
3. the computer techniques (e.g. McGrath and Hood, 1970).

The method of Ram Babu et al., (1986) is employed because it is quick and reasonably accurate. When this was applied to the anomalies of figures (5.5 b) and 5.6 b), the sources are vertical intrusions with depth to the top of 1.8 km and 4.3 km respectively, and both have minimum susceptibility of 0.001972 and 0.002545 SI units respectively.

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

The principles and applications of reduction to the pole of magnetic anomalies have been analysed and demonstrated. From this, it becomes obvious to see that reducing magnetic fields to the earth's magnetic pole makes interpretation of magnetic anomalies easier, as the anomalies are not shifted as a result of the obliquity of the normal field.

In the comparison made between the total field and reduced to the pole maps with the geology of the area, the poor correlations between the granitic intrusions (the older granite) with the anomalies observed on the two maps is because the granite within the area are mostly acidic (McCurry, 1970). And the little correlation obtained around the Nasarawa area and South-Eastern part of Maigana occurs because the older granite series which were initially basic and intermediate intrusions represented by small irregular bodies of quartz and pyroxene diorite and gabbro, were later intruded by acid rocks (granitization) to form an acid-basic complex. Therefore, the anomaly at Nasarawa could be part of these basic rocks that have not been greatly affected by granitization.

Also, the prominent magnetic anomalies that fall on the areas that are predominantly gneiss (the older metasediments) is possible because tremolite (a ferromagnesian mineral) and biotite - plagioclase schist (a basic schist) have been reported to occur within the older metasediment in the area (McCurry, 1970).

The method of Silva (1986), used in reducing the magnetic data to the pole has advantage over the traditional filtering techniques because the filtering technique requires a priori information about the noise present in the data. And also elimination of noise by spectral techniques is not perfect since it may cause elimination of some part of the signal as well. And even when the data is noise free, the filtering techniques would be affected by Gibbs effect and aliasing. On the other hand, formulating the problem of reduction to the pole as an inverse problem, the stabilization is applied to the method itself, leaving the data unaltered.

It is recommended that pseudogravity field of the area be computed from the reduced to the pole data so that more comparison can be made with gravity and geological maps of the area, and also a different stabilizing procedure could be tested on the ill-conditioned equation (4.3). For example, the method of generalized inverses (Jackson, 1972 and Wiggins, 1972), obtained from singular value decompositions of a matrix.

REFERENCES

Adeniyi, O. O. (1987): Gravity survey of the older granite plutons in the Zaria area of Kaduna State, Nigeria. Unpublished M.sc. Thesis, Ahmadu Bello University, Zaria.

Am, K. (1972): The arbitrary magnetized dike: Interpretation by characteristics: Geoexploration, v.10, pp. 63-90.

Atchuta, Rao, D. and Ram Babu, H.V. (1981): Nomogram for rapid evaluation of magnetic anomalies over long tabular bodies. Pageoph, v.119, pp. 1037-1050.

Baranov, V. (1957): A new method for interpretation of aeromagnetic maps: Pseudo gravimetric anomalies: Geophysics v.22, pp 359-383.

Baranov, V. and Naudy, H. (1964): Numerical calculation of the formula of reduction to the magnetic pole: Geophysics, v.29, pp. 67-79.

Bhatacharyya, B.K. (1964): Magnetic anomalies due to prism-shaped bodies with arbitrary polarization. Geophysics, v.29, pp. 517-531.

Bhattacharyya, B.K. (1965): Two-dimensional harmonic analysis as a tool for magnetic interpretation: Geophysics, v.30, pp. 829-857.

Bott, M.P.H. and Ingles, A. (1972): Matrix method for joint interpretation of two-dimensional gravity and magnetic anomalies with application to the Ice Land Faeroe Ridge: Geophysical Journal of the Royal Astronomical Society, v. 30, pp. 55-67.

Bruckshaw, J.M. and Kunaratum, K. (1963): The interpretation of magnetic anomalies due to dike. Geophysical prospecting, v.11, pp. 509-522.

Dampney, C.N.G. (1969): The equivalent source technique: Geophysics, v. 34, pp. 55-67.

Emilia, D.A. (1973): Equivalent sources used as an analytical base for processing total magnetic field profiles: Geophysics, v. 38, pp. 339-348.

Ervin, C.P. (1976): Reduction to the magnetic pole using a Fast Fourier Series algorithm: Computers and Geosciences, v.2, pp. 211-217.

Freund, R.J. and Minton, P.D. (1979): Regression Methods: A tool for data analysis: Statistics textbooks and monographs, v.30, 261 p.

Gay, S.P. (1963): Standard curves for the interpretation of magnetic anomalies over long tabular bodies: Geophysics, v.28, pp. 161-200.

Gunn, P.J. (1975): Linear transformation of gravity and magnetic fields: Geophysical prospecting, v.23, pp. 300-312.

Hays, W.W. and Scharon, L. (1963): An example of the influence of remanent magnetism on magnetic intensity measurements: Geophysics, v.38, pp. 1037-1048.

Hoerl, A.E. and Kennard, R.W. (1970a): Ridge regression: Biased estimation for nonorthogonal problems: Technometrics, V. 12, pp. 55-67.

Hoerl, A.E. and Kennard, R.W. (1970b): Ridge regression: Applications to non-orthogonal problems: Technometrics, V. 12, pp. 69-82.

Hutchinson, R.D. (1958): Magnetic analysis by logarithmic curves: Geophysics, v. 23, pp. 749-769.

Jackson, D.D. (1972): Interpretation of inaccurate, insufficient and inconsistent data: Geophysical Journal of the Royal Astronomical Society, v.28, pp. 97-109.

James, W.R. (1966): The Fourier model in map analysis: Technical Report No. 1 - ONR Contract NONR-1228(36), office of Naval Research, Geography Branch, 37p.

Kanasewich, E.R. and Agarwal, R.G. (1970): Analysis of the combined gravity and magnetic fields in wavenumber domain: Journal of Geophysical Research, v.75, 5702-5712.

Leao, J.W.D. and Silva, J.B.C. (1989): Discrete linear transformations of potential field data: Geophysics, v.54, pp. 497-507.

Marquardt, D.W. (1963): An algorithm for least-squares estimation of non-linear parameters, J. Soc. Indust. Appl. Math., v.11, pp. 431-441.

Mayhew, M.A. (1979): Inversion of Satellite anomaly data, Journal of Geophysics, v. 45, pp. 119-128.

Mayhew, M.A.; Johansen, B.D. and Langel, R.A. (1980): An equivalent source model of the Satellite -altitude magnetic anomaly field over Australia: Earth and Planetary Science. Letters., v.51, pp. 189-198.

McCurry, P. (1970): The geology of degree sheet 21 (Zaria). M.Sc. Thesis, Geol. Dept., A.B.U., Zaria.

McCurry, P. (1971): Pan African Orogeny in Nigeria. Geol. Soc.Am. Bull., v. 82, pp. 3251-3262.

McCurry, P. (1973): Geology of degree sheet 21 Zaria, Nigeria. Overseas Geol. Min. Res., v. 45, HMSO, London.

McCurry, P. (1976): The geology of the Precambrian to Lower Paleozoic of Northern Nigeria - a review. In: Proc. 1973 Conf. African Geology., Univ. of Ife Press, Ife.

McGrath, P.H. and Hood, P.J. (1970): The dipping dike case: A Computer curve-matching method of magnetic interpretation: Geophysics, v. 35, pp. 831-845.

Olatinwo M. O. (1994): Geoelectrical investigation in the Northern part of Kaduna State, Nigeria. Unpublished Ph.D. Thesis, Ahmadu Bello University, Zaria.

Onugba, A. (1979): Detailed resistivity and seismic refraction survey of Kimberlite pipe area. Near Malumfashi, Kaduna State. M.sc. Thesis, Physics Dept., A.B.U., Zaria.

Park, F.B. (1968): Remanent magnetism and the anomaly of Comouer Mountain, Madison County, Missouri: Geophysics, v. 33, pp 613-620.

Pearson, W.C. and Skinner, C.M. (1982): Reduction to the pole of low-latitude magnetic anomalies: Presented at the 52nd Annual International Mtg. and Expes., Soc. Explor. Geophysics.

Ram Bau, H.V., Vijaykumar, V., and Rao A.D. (1986): A simple method for the analysis of magnetic anomalies over dike-like bodies: Geophysics, v. 51, pp. 1119-1126.

Rao, D.B. and Babu, N.R. (1991): A rapid method for three-dimensional modelling of magnetic anomalies: Geophysics, v. 56, pp. 1729-1739.

Ridgway, J.R. and Hinze, W.J. (1986): MAGSAT Scalar anomaly map of South America: Geophysics, v. 51, 1472-1479.

Shemang, E. M. (1990): Electrical depth soundings at selected well sites within the Kubanni River Basin, Zaria, Nigeria.Unpublished M.sc. Thesis, Ahmadu Bello University, Zaria.

Silva, J.B.C. (1986): Reduction to the pole as an inverse problem and its application to low latitude anomalies: Geophysics, v. 51, pp. 369-382.

Von Frese, R.R.B.; Ravart, D.N.; Hinze, W.J. and Braille, L.W.(1982): Regional North America gravity and magnetic anomaly corrections: Geophysical Journal of the Royal Astronomical Society, v. 69, pp. 745-761.

Von Frese, R.R. B., Ravart, D.N.; Hinze, W.J. and McGue, C.A. (1988): Improved inversion of geopotential field anomalies for lithospheric investigations: Geophysics, V. 53, PP. 375-385.

Webb, P.K. (1972): Notes on: Recent research on the geology between Zaria and Kaduna, Savannah, v. 1, No. 2.

Wiggins, R.A. (1972): The general linear inverse problem: Implication of surface wave and free oscillation for Earth structure: Review of Geophysics and Space Physics, v. 10, pp. 251-285.

```
                    PROGRAM REDPOLE
C     PROGRAM TRANSFORMS TOTAL MAGNETIC FIELD TO THE
C     EARTH'S MAGNETIC POLE
C                    INPUT DATA
C                    ----------
C          I1=INCLINATION OF EARTH'S TOTAL FIELD
C          I2=INCLINATION OF MAGNETIZATION
C          D1=DECLINATION OF EARTH'S TOTAL FIELD
C          D2=DECLINATION OF MAGNETIZATION
C          N=DIMENSION FOR DATA (NORTH-DIRECTION)
C          M=DIMENSION FOR DATA (EAST-DIRECTION)
C          DEPTH=DEPTH OF EQUIVALENT LAYER
C          DLEN=DOUBLET LENGTH
C          ZERO=VERY SMALL NUMBER
C          LAMDA=PARAMETER OBTAINED FROM RIDGE TRACE
C          XI,YI,ZI=DATA CORDINATE
C          XJ,YJ=DOUBLET CORDINATE
C          HO=OBSERVED FIELD
C                    OUTPUT
C                    ------
C          HP=FIELD REDUCED TO THE POLE
C
C
      DIMENSION A(100,100),D(100,100),HO(100),R1(100,100)
      DIMENSION P(100),AT(100,100),XI(100),YI(100),ZI(100)
      DIMENSION XJ(100),YJ(100),HP(100)
      OPEN(UNIT=5,FILE='REDIN.DAT',STATUS='OLD')
      OPEN(UNIT=6,FILE='REDOUT.DAT',STATUS='NEW')
      REAL I1,I2,LAMDA
      READ(5,1)I1,I2,D1,D2,N,M,DEPTH,DLEN,ZERO,LAMDA
1     FORMAT(1X,4(1X,F5.1),2(1X,I3),4(1X,F6.2))
      NM=N*M
      DO 333 I=1,NM
      READ(5,2)XI(I),YI(I),HO(I)
2     FORMAT(2X,F6.2,2X,F6.2,2X,F10.3)
      ZI(I)=0.0
      XJ(I)=XI(I)
333   YJ(I)=YI(I)
C
C     CONVERT ANGLES TO RADIAN
C
      RAD=3.14159/180.0
      I1=I1*RAD
      I2=I2*RAD
      D1=D1*RAD
      D2=D2*RAD
C
C     COMPUTE CONSTANTS
C
      BCC=DLEN*COS(I2)*COS(D2)/2.0
```

```
      BCS=DLEN*COS(I2)*SIN(D2)/2.0
      CC=COS(I1)*COS(D1)
      CS=COS(I1)*SIN(D1)
      BS=SIN(I2)*DLEN/2.0
      S1=SIN(I1)
C
C     CONPUTE ELEMENT OF MATRIX A
C
      DO 10 I=1,NM
      DO 10 J=1,NM
      F11=(XI(I)-XJ(J)-BCC)
      F12=(YI(I)-YJ(J)-BCS)
      F13=(ZI(I)-DEPTH-BS)
      F1=((F11**2.0)+(F12**2.0)+(F13**2.0))**(1.5)
      F21=(XI(I)-XJ(J)+BCC)
      F22=(YI(I)-YJ(J)+BCS)
      F23=(ZI(I)-DEPTH+BS)
      F2=((F21**2.0)+(F22**2.0)+(F23**2.0))**(1.5)
      A11=(F11/F1)-(F21/F2)
      A22=(F12/F1)-(F22/F2)
      A33=(F13/F1)-(F23/F2)
      A(I,J)=(A11*CC)+(A22*CS)+(A33*S1)
10    CONTINUE
C
C     COMPUTE DIAGONAL SCALING MATRIX
C
      DO 20 J=1,NM
      SUMA=0.0
      DO 30 I=1,NM
      ASQ=A(I,J)**2.0
      SUMA=SUMA+ASQ
30    CONTINUE
      D(J,J)=1.0/(SUMA**0.5)
20    CONTINUE
C
C     FIND TRANSPOSE OF MATRIX A
C
      DO 40 J=1,NM
      DO 40 I=1,NM
      AT(I,J)=A(J,I)
40    CONTINUE
C
C     START COMPUTATION OF DOUBLET STRENGTH P
C
      DO 100 I=1,NM
      DO 100 J=1,NM
      R1(I,J)=0.0
      DO 100 K=1,NM
      R1(I,J)=R1(I,J)+AT(I,K)*A(K,J)
100   CONTINUE
      DO 200 I=1,NM
      DO 200 J=1,NM
      A(I,J)=0.0
      DO 200 K=1,NM
```

```
      A(I,J)=A(I,J)+D(I,K)*R1(K,J)
200   CONTINUE
      DO 300 I=1,NM
      DO 300 J=1,NM
      R1(I,J)=0.0
      DO 300 K=1,NM
      R1(I,J)=R1(I,J)+A(I,K)*D(K,J)
300   CONTINUE
      DO 500 I=1,NM
      R1(I,I)=R1(I,I)+LAMDA
500   CONTINUE
C
C     FIND INVERSE OF MATRIX R1
C
      DO 60 I=1,NM
      DO 70 J=1,NM
      A(I,J)=0.0
70    CONTINUE
      A(I,I)=1.0
60    CONTINUE
      DO 80 I=1,NM
      DIV=R1(I,I)
      IF(ABS(DIV)-ZERO)99,99,81
81    DO 90 J=1,NM
      R1(I,J)=R1(I,J)/DIV
      A(I,J)=A(I,J)/DIV
90    CONTINUE
      DO 111 J=1,NM
      IF(I-J)112,111,112
112   RATIO=R1(J,I)
      DO 222 K=1,NM
      R1(J,K)=R1(J,K)-RATIO*R1(I,K)
      A(J,K)=A(J,K)-RATIO*A(I,K)
222   CONTINUE
111   CONTINUE
80    CONTINUE
C
      DO 1000 I=1,NM
      DO 1000 J=1,NM
      R1(I,J)=0.0
      DO 1000 K=1,NM
      R1(I,J)=R1(I,J)+A(I,K)*D(K,J)
1000  CONTINUE
      DO 2000 I=1,NM
      DO 2000 J=1,NM
      A(I,J)=0.0
      DO 2000 K=1,NM
      A(I,J)=A(I,J)+R1(I,K)*AT(K,J)
2000  CONTINUE
      DO 2201 I=1,NM
      DO 2201 J=1,NM
      R1(I,J)=0.0
      DO 2201 K=1,NM
      R1(I,J)=R1(I,J)+D(I,K)*A(K,J)
```

```
2201  CONTINUE
      DO 3000 I=1,NM
      P(I)=0.0
      DO 3000 K=1,NM
      P(I)=P(I)+R1(I,K)*HO(K)
3000  CONTINUE
C
C     COMPUTE ELEMENTS OF MATRIX B
C
      DO 4000 I=1,NM
      DO 4000 J=1,NM
      ZDB1=ZI(I)-DEPTH-(DLEN/2.0)
      ZDB2=ZI(I)-DEPTH+(DLEN/2.0)
      XY=((XI(I)-XJ(J))**2.0)+((YI(I)-YJ(J))**2.0)
      B1=ZDB1/((XY+(ZDB1**2.0))**1.5)
      B2=ZDB2/((XY+(ZDB2**2.0))**1.5)
      R1(I,J)=B1-B2
4000  CONTINUE
C
C     CALCULATE FIELD REDUCED TO POLE HP
C
      DO 5000 I=1,NM
      HP(I)=0.0
      DO 5000 K=1,NM
      HP(I)=HP(I)+R1(I,K)*P(K)
5000  CONTINUE
C
C     WRITE OUTPUT
C
      DO 555 I=1,NM
555   WRITE(6,2)XI(I),YI(I),HP(I)
      GO TO 88
99    WRITE(*,*)'MATRIX IS SINGULAR'
88    STOP
      END
```

```
2201  CONTINUE
      DO 3000 I=1,NM
      P(I)=0.0
      DO 3000 K=1,NM
      P(I)=P(I)+R1(I,K)*HO(K)
3000  CONTINUE
C
C     COMPUTE ELEMENTS OF MATRIX B
C
      DO 4000 I=1,NM
      DO 4000 J=1,NM
      ZDB1=ZI(I)-DEPTH-(DLEN/2.0)
      ZDB2=ZI(I)-DEPTH+(DLEN/2.0)
      XY=((XI(I)-XJ(J))**2.0)+((YI(I)-YJ(J))**2.0)
      B1=ZDB1/((XY+(ZDB1**2.0))**1.5)
      B2=ZDB2/((XY+(ZDB2**2.0))**1.5)
      R1(I,J)=B1-B2
4000  CONTINUE
C
C     CALCULATE FIELD REDUCED TO POLE HP
C
      DO 5000 I=1,NM
      HP(I)=0.0
      DO 5000 K=1,NM
      HP(I)=HP(I)+R1(I,K)*P(K)
5000  CONTINUE
C
C     WRITE OUTPUT
C
      DO 555 I=1,NM
555   WRITE(6,2)XI(I),YI(I),HP(I)
      GO TO 88
99    WRITE(*,*)'MATRIX IS SINGULAR'
88    STOP
      END
```