MONTE CARLO STUDY OF VARIABILITY IN STRENGTH OF REINFORCED CONCRETE BEAMS AND COLUMNS

BY

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for the Degree of Master of Science in Engineering
(M.Sc .)

August, 1988.

DECLARATION

I hereby declare that this disertation has been composed by myself and that it is a record of my own research work. It has not been accepted in any previous application for a higher degree. All sources of information are specifically acknowledged by menas of references.

J A. SADEEQ

August, 1988

CERTIFICATION

This project thesis entitled:
Monte Carlo Study of Variability in Strength of Reinforced Concrete Beams and Columns by Ja'afar Abubakar Sadeeq meets the regulations governing the aware of the degree of *MSc Structural Engineering* of Ahmadu Bello University and is approved for its contribution to knowledge and litarary presentation.

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DEDICATION

This work is dedicated to my mother

Hauwa'u.

ACKNOWLEDGEMENT

I wish to express my sincere gratitude to the various people and organisation that helped in one way or the other to the success of this work.

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ABSTRACT

The variability in strength of reinforced concrete beams and columns sections due to the variations ofconcrete and steel strengths are studied by Monte Carlo Simulation.

The study aims at

(i) establishing the probability distribution of the resistance of reinforced concrete elements.

(ii) finding the probability of failure of the reinforced concrete elements

(iii) establishing the safety factor for the reinforced concrete elements.

The study gives short discussion on the ' probability and statistical concepts. A brief explanation on the Monte Carlo Method is also provided. Some earlier works on thevariabilities of material stregth and member strength are then reviewed.

The distributions of strengths of concrete and steel are established. The data analysed are collected from the result of control tests on the materials, conducted over many years on live projects, by the Department of Civil Engineering, Ahmadu Bello University Zaria, Nigeria. Earlier studies suggest lower and upper bounds for the probability distributions of materials strengths. However, it has been found that only normal distribution fits the analysed data.

From the eatablished distribttions of concrete

and steel strengths, the resistances of reinforced concrete.beams and columns are simulated. The simulated values are analysed and their probability distributions established. The probability of failure for the various sections are found. The adequacy of CP110 and ACI recommended safety factors are tested.

The study shows that normal probability distribution fits the distribution of resistances of reinforced concrete sections irrespective of the possibility ofthe negative values admitted by normal distribution. It has also been established that the coefficient of variation of reinforced concrete elements can be onsidered as 0.13. The safety factors recommended by CP110 are found to be adequate for design.

Finally, the study concludes with the recommendation on the improvement of concrete quality which is expected to give higher contributions to the variation of the member strength. It also recommends further research in the area.

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NOTATION

a_l, b_l	Interval limits of X
A_m	Measured area of reinforcement
A_n	Nominal area of reinforcement
A_s	Area of tension steel
b,d	Cross sectional d mensions of beams
B(q,r)	Beta function
$B_x(q,r)$	Incomplete Beta function
CV	Coefficient of variation
D	Kolmogorov statistic
$\bar{E}_{cistrR}$	Mean value of initial tangent modulus for in-situ concrete at a given rate of loading R (psi/sec)
E(X)	Expected value of X
ϵ	Error
ϵ'_s	Steel strain in compression
ϵ_s	Steel strain in tension
ϵ_c	Ultimate compressive strain in concrete
d'	depth from compression surface of beam to the centre of compression steel.

δ	The ratio of d^1 to d
f(x)	Probability density function of X
F(x)	Cummulative Distribution Function of X
f_c^1	Concrete Cylinder strength
f_{cu}	Concrete cube strength
f_u	Steel ultimate strength.
$\bar{f}_{cstrR}$	Mean in-site concrete compressive strength at a given rate of loading R(psi/sec)
f_{spstrR}	Splitting tensile strength of concrete at a given rate of loading R (psi/sec)
$\bar{f}_{rstrR}$	Mean strength of in-situ concrete in flexural tension at a given rate of loading R (psi/sec)
H_o	Null hyphothesis
K,λ	Gamma parameters
K-S	Kolmogrov-Smiruov
M_r^1	r^{th} sample moment
M_r	r^{th} sample central moment.
P	Proportion
$P_r(A)$	Probability of event A

$P(x)$	Probability mass function of X
$P_r(X=x)$	Probability that variable X takes a value x
ρ	Tensile steel ratio
ρ^1	Compression steel ratio
M_u	Ultimate moment of resistance
A_s^1	Area of compression steel
q,r	Beta parameters
R	Rate of loading
S^2	Sample variance
μ	Population means
$\mu_r^1(X)$	r^{th} moment of X about origion
$\mu_r(X)$	r^{th} central moment of X
X_N^1	Normal variate realization
$\bar{X}$	Sample mean
$\overset{0}{X}$	Mode value of X
X_m	Median value of X
X^2	Chi-square
V_{cstrR}	Variation of in-situe concrete compressive strength at a given rate of loading R(psi/sec)

V_{ccyl}	Variation of cylinder strength
V_{cistrR}	Variation of initial tangent modulus for in-situ concrete at a given rate of loading
V_{rstrR}	Variation of in-situ concrete flexural tension strength at a given rate of loading, R(psi/sec)
V_{spstrR}	Variation of splitting tensile strength of concrete at a given rate of loading, R(psi/sec)
σ^2	Population variance
γ_1	Coefficient of Skewness
γ_2	Coefficient of Kutosis
	Gamma function
α	Significant level
Z_N^1	Standardized normal random number
$Z_{\alpha/2}$	Two-tailed standardized normal statistics
k_1, k_2, k_3	Stress block parameters
P_u	Ultimate load capacity of column
b,h	Cross-sectional dimensions of column
e	eccentricity

M_{un} Ultimate moment of resistance based on the nominal material strength

M_{ud} Design ultimate moment of resistance

γ_m partial safety factor

ε_y steel yield strain

ρ_b balanced steel ratio.

CHAPTER ONE

1. INTRODUCTION

1.1 Introduction

The capacity of structures to support the expected loads without distress or collapse, is far more predictable than it was 60 years ago; but the strength of a particular structure still cannot be predicted with absolute confidence. One of the factors contributing to this uncertainty is the variability of the strength of constituent materials. Other sources of uncertainty include variations in dimensions, variations in stress-strain relationships and variations in field practice. Some previous studies (4, 15, 31) have emphasized the variability which actually occurs in the magnitude and distribution of structural loadings, in the properties of structural materials, and in the response of real structures. Because of these unavoidable variabilities, the concept of structural safety is inevitably bound with probabilities of overload and understrength, and with notion of reliabilities.

The two areas of uncertainty, i.e. the magnitude and distribution of loads, and resistance of structure can be reasonably assumed to be unrelated, so are their effects. Thus, the load

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on a structure is taken to be independent of it's resistance. Then the safety of a structure can be assessed by the probability that its resistance exceeds its load. This work is devoted to a study of the variability in the resistance of reinforced concrete sections induced by variability in the strengths of the constituent materials. Thus it can be seen that this constributes only a small part ofthe total problem.

The difficulty with pure probabilities approach in the assessment of safety of structure is the large number of factors to be considerdd. There is also general lack of adequate data on which to base the analysis. Besides the many sources of uncertainty of strength of a structure, the resistance of structure itself is of different forms. The resistance could, for example, be in form of moment, shear or torsion. All these forms of resistances interact in complex ways in a particular structure. Until these problems could be solved, codes such as CP110 (11) and ACI (1) have adopted semi-empirical approach in the assessment of structural safety.

However, it is not out of place to start tackling the problems mentioned above from

somewhere. In the light of this it is hoped that this study will help towards understanding the variability of strength of structures in this country.

1.2 Problem Definition and Methodology

The study considers the effect of variations in steel and concrete strengths on the strength of reinforced concrete members. The data is collected from the results of the test conducted in concrete laboratory of Civil Engineering Department, Ahmadu Bello University, Zaria.

After establishing the probability distributions of the yield strength of reinforcing bars, and compressive strength of concrete cubes, the resultant member strengths were derived using Monte Carlo Technique. The sections considered are singly and doubly reinforced concrete beams, and axially loaded short columns. The result of the simulation was analysed and probability distributions of resistances obtained.

The probability distributions of the materials as well as the distributions of the resistances of the section are all found to be normal distributions. The probabilities of

resistances calculated from nominal values of yield strength and concrete strength not being exceeded are also obtained.

There are arguments against theuse of normal distribution for the strengths of materials when assessing the probability of failure. This is mainly due to possibility of negative values given by normal distribution, However, about 99.9 per cent of the area under normal curve is contained within plus or minus three times standard deviation about the sample mean. The lower limit of this range is above zero value for the samples used in this study. Due to inadequacy of the data used (which will be explained later), the limits offered by the candidate models, such as Beta distribution will not be reliable. Because of these reasons and easy manipulation, it has been decided to use normal distribution in fitting the data.

The Monte Carlo Technique of analysis to be used in this work requires understanding of

probability, therefore, some probability and statistical concepts will be given in the next section. However, the description given here is not in any way exhaustive. Excellent treatment of the topics are given in the references (3, 7, 17, 18).

1.3 PROBABILITY AND STATISTICAL CONCEPTS

1.3.1 Basic Probability Theory

It is appropriate to start with the definitions of some terms that will help in understanding probability concepts. The theory of probability is concerned with experiment and their outcomes. The term experiment refers to any process that results in an observation. The collection of all possible outcomes of an experiment is called its sample space. This space consists of a set S of points called sample points, each of which is associated with one and only one distinguishable outcome. An event is a collection of sample points in the sample space S of an experiment.

If two events contain no sample points in common, they are said to be mutually exclussive or disjoint. When a pair of events A and B

are not mutually exclussive, the set of points which they have in common is called their intersection, denoted A∩B. The union of two events A and C is the event which is the collection of all sample points which occur at least once in either A or C, denoted AUC.

With few definitions given above the interpretation of probability can be given as follows: The relative frequency interpretation of probability is that if an experiment is repeated M times and the simple event associated with a sample point is observed n times, then probability measure Pr of sample point is the ratio of n to M i.e

$$P_r = n/M \quad \ldots\ldots\ldots\ldots 1.3.1$$

Another interpretation is that, the probability of an event is a subjective measure of the degree of belief one holds in a specified proposition

Denoting the probability of an event A by $P_r(A)$ (A), the following axioms must hold on the probabilities assigned to events in the sample space.

1. The probability of an event is a number greater than or equal to zero but less than or equal to unity:

$$0 \leq P_r(A) \leq 1 \quad \ldots\ldots\ldots 1.3.2$$

2. The probability of the certain event S is unity:

$$P_r(S) = 1 \quad \ldots\ldots\ldots\ldots 1.3.3$$

in which S is the event associated with all the points in the sample space.
The probability of occurrence of either one event or another or both is the sum of their individual probabilities minus the probability of their joint occurrence.

$$P_r(AUB) = P_r(A) + P_r(B) - P_r(AnB)$$

If events A and B are mutually exclussive

$$P_r(AnB) = 0 \quad \ldots\ldots\ldots\ldots 1.3.4$$

3. Hence the probability of an event which is the union of two mutually exclussive events is thesum of the probabilities of these two events:

$$P_r(AUB) = P_r(A) + P_r(B) \ldots\ldots 1.3.5$$

The conditional probability of the even A given that event B has occured, denoted $P_r(A/B)$ is defined as the ratio of the probability of intersection of A and B to the probability of the even D

$$P_r(A/B) = P_r(AnB)/P_r(B) \ldots\ldots\ldots 1.3.6$$

If events are independent the probability of their joint occurrence is the product of their individual probability of occurrence.

1.3.2 Random Variables

A random variable is a function defined on a sample space of the experiment. It assigns a numerical value to every possible outcome. Every compound event corresponds to one or more or a range of random variables. A random variable that can take only a countable number of values is called discrete. A continous random variable may take on any value in one or more intervals.

The behaviour of a random variable is described by its probability law. The usual probability law is probability distribution. This is the list of values the variable can take on and their respective probabilities. The probability law of discrete random variables is usually represented in the form of a probability mass function or PMF. This function P(x) of the random variable X is the mathematical form of probability that X will have a value x, i.e.

$$P(x) = P_r(X = x) \quad \ldots\ldots\ldots \quad 1.3.7$$

To satisfy the three axioms of probability theory, the PMF must fulfill three conditions:

i. $0 \leq P(x) \leq 1$ for all x 1.3.8

ii. $\Sigma_{allxi} P(x_i) = 1$ 1.3.9

iii. $P_r(a \leq X \leq b) = \sum_{x_i \geq a}^{x_i \leq b} P(x_i)$ ... 1.3.10

Another form of description of the probability distribution of a random variable is through the use of cummulative distribution function or CDF. The value of this function F(x) is the probability of the event that random variable takes on value equal to or less than the argument i.e.

$$F(x) = P_r(X \leq x) \text{ ..... } 1.3.11$$

For discrete random variable this function is the sum of the values of the probability mass function over those values less than or equal to x that the random variable X can take on. Thus

$$F(x) = \sum_{allx_i < x} P(x_i) \text{ ..... } 1.3.12$$

F(x) is a non-decreasing function of x with minimum value of 0 and maximum value of 1.

The probability distribution of continous random variable can be desribed either by probability density function or cummulative distribution function. The probability density

function can be explained as follows:

If the x axis is separated into a large number of short intervals each of infinitesimal length dx, the function f(x) such that the probability of X is in interval x to x + dx is called probability density function or PDF of continous random variable. The area under PDF in an interval represents the probability that the random variable will take on a value in that interval

$$P\ (a < X < b) = \int_a^b f(x)dx \quad \ldots\ldots \quad 1.3.13$$

The value of f(x) is not itself a probability, it is a measure of the density or intensity of probability at the point. Therefore f(x) need not be restricted to values less than 1 but two conditions must hold:

$$f(x) \geq 0 \quad \ldots\ldots\ldots\ldots \quad 1.3.14$$

$$\int_{-\infty}^{\infty} f(x)\ dx = 1 \quad \ldots\ldots\ldots \quad 1.3.15$$

The cummulative distribution function or CDF of continous random variable is given as

$$F(x) = P_r(-\infty \leq X \leq x)$$

$$= \int_{\alpha}^{x} f(u)du \quad \ldots\ldots\ldots\ldots 1.3.16$$

in which U is a dummy variable of integration.

The cummulative distribution function of any type of random variable has the following properties:

$$0 \leq F(x) \leq 1 \quad \ldots\ldots\ldots\ldots 1.3.17$$

$$F(-\alpha) = 0 \quad \ldots\ldots\ldots\ldots 1.3.18$$

$$F(\alpha) = 1 \quad \ldots\ldots\ldots\ldots 1.3.19$$

$F_x(x+\ ) \geq F_x(x)$ for any possitive t_1

$\ldots\ldots\ldots\ldots$ 1.3.20

$$F(b)-F(a) = P(a \leq X \leq b) \quad \ldots\ldots\ldots 1.3.21$$

1.3.3 Moments and Expectations

The information about the distribution of a random variable can be summarized by a few descriptive values. One or more simple numbers are used in place of a whole probability density function. These numbers usually take the form of weighted averages of certain functions of the

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random variable. The weights used are PMF or PDF of the variable and the average is called the expectation of the function.

Mean:-

This is a measure of thecentral tendency of the variable and often of the value about which scatter can be expected if repeated observations of the phenomenon are to be made. The population mean μ or the expected value $E(x)$ of a discrete random variable X is defined as

$$\mu = E(X)$$

$$= \sum_{all\,x_i} x_i P(x_i) \quad \ldots\ldots \quad 1.3.22$$

For continous random variable

$$\mu = E(X)$$

$$= \int_{-\infty}^{\infty} xf(x)dx \quad \ldots\ldots \quad 1.3.23$$

Median :

This is the value of the random variable that has exactly one half of the area under probability density function to its left and one half to its right. For a continous probability density function f(x) the median is

the point Z such that

$$\int_{-\infty}^{Z} f(x)dx = 0.5 \quad \ldots\ldots\ldots \text{1.3.24}$$

Mode:

For a discrete random variable the mode is that value ofthe varaible that has the highest probability. The mode of a continous variate is the value associated with the maximum of the probability density function.

Variance:

This is a description which will give indication of the scatter or dispersion in the random variable's behaviour. It is the weighted average of the squared deviation from the mean i.e.

$$\sigma^2 = V_{ar}(X) = \sum_{all x_i} (x_i-\mu)^2 P(x_i) \text{ for}$$

discrete 1.3.25a

or

$$\sigma^2 = \int_{-\alpha}^{\alpha} (x - \mu)^2 f(x)dx \text{ for continous}$$

..... 1.3.25b

Standard Deviation:

The positive square root of the variance is given the name standard deviation

$$\sigma = \sqrt{\sigma^2} = +\left[\int_{-\alpha}^{\alpha} (x - \mu)^2 f(x)dx\right]^{\frac{1}{2}}$$

..... 1.3.26

The standard deviation has the same unit as the variable X itself and can be compared easily and quickly with the mean of the variable to gain some feeling for the degree and gravity of the uncertainty associated with random variable.

Coefficient of Variation:

The comparison of standard deviation with mean is made through coefficient of variation. Coefficient of variation is the ratio of standard deviation to the mean of variable

$$CV = \frac{\sigma}{\mu} \quad \text{..... 1.3.27}$$

Moments:

The r^{th} moment of X is defined as

$$\mu_r^1(X) = E(X^r) = \int_{-\alpha}^{\alpha} x^r f(x)dx \quad \text{..... 1.3.28}$$

when $r = 1$,

$\mu^1(X)$ = mean of variable $= \mu$

Moments with respect to the mean are called central moments:

$$\mu_r(X) = E\ (x - \mu^1(X)^r$$

$$= \int_{-\alpha}^{\alpha} (x - \mu^1(X)^r\ f_x(x)dx \quad .. \quad 1.3.29$$

when $r = 2$, $\mu_2(X) = \sigma^2$, the variance

Skewness

The third central moment is called Skewness.

$$\mu_3(X) = E\ (x - \mu^1(X)^3$$

$$= \int_{-\alpha} (x - \mu^1(X)^3 f(x)dx \quad ... \quad 1.3.30$$

The quantity

$$\gamma_1 = \frac{\mu_3(X)}{(\mu_2(X))^{3/2}} \quad ..... \quad 1.3.31$$

is called coefficient of Skewness. If a distribution is symmetrical, this coefficient is zero. Positive value of γ_1 usually corresponds to PDF with dominant tail on the right; negative value to long tail on the left. See Fig (1.3-1)

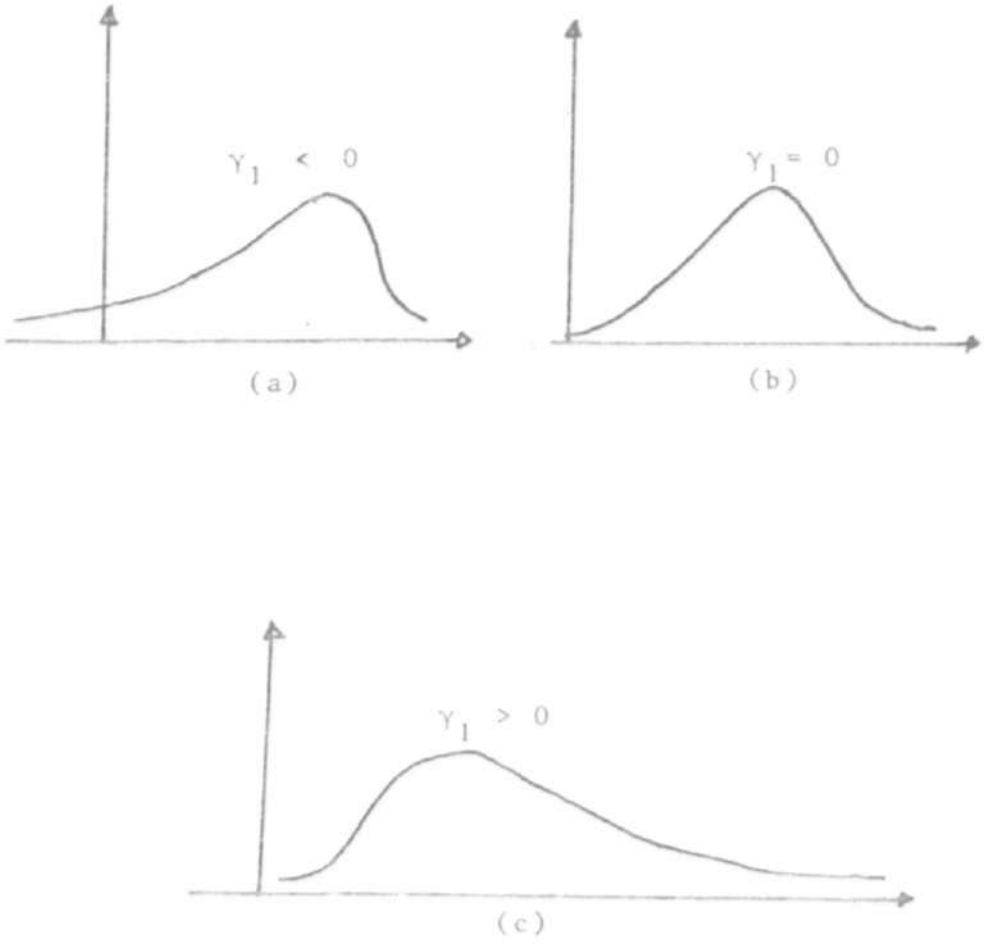


Fig (1.3.1) Variation of shape of PDF with Coefficient of skewness γ_1 (a) Negative skewness; (b) zero skewness; (c) positive skewness.

Kutosis:

The fourth moment about the mean is related to the peakedness also called Kutosis - of the distribution and is defined as

$$M_r = E((x - \mu)^4)$$

$$= \int_{-\infty}^{\infty} (x - \mu)^4 f(x)dx \quad \ldots\ldots \quad 1.3.32$$

The quantity

$$\gamma_2 = \frac{M_4}{M_2^2} \quad \ldots\ldots\ldots\ldots\ldots \quad 1.3.33$$

is called the coefficient of kutosis (flatness). It is often comparred to a standard value of 3. (γ_2 - 3) is called the coefficient of excess. The value of 3 is chosen only because it is the value of the kutosis coefficient of a common distribution called normal distribution to be described in the next section.

1.4 PROBABILITY MODELS AND GOODNESS-OF-FIT

A number of probability distributions have been used to model the variability of material strength as well as member strength. This section describes the properties of some continous invariate

probability distributions suitable for modelling material strength and member strength variations.

A complete description of a'random variable would be accomplished by specifying its probability distribution and the associated parameters. Based on sampled observational data, information about the probability distribution may be inferred and its parameter estimated statistically. The validity of an assumed probability distribution may be disproved statistically, based on finite samples of a population by GOODNESS-OF-FIT tests. The most common tests are CHI-SQUARE (X^2) and KOLMOGOROV-SMIRNOV (K-s) tests.

1.4.1 The Normal (Gaussian) Distribution

Normal distribution is perhaps the best-known and mostwidely used probability distribution. It is continous and unlimited in range. Thus a random variable that has the normal distribution can assume positive and negative values. The distribution is symmetrical about its mean and it is the only standard distribution whose mode, mean and median are essentially equal. See figure (1.4.1).

The normal distribution has often been found to be a most appropriate mode for processes resulting from ADDITIVE effects.

$$f(x) = \begin{cases} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] & -\infty < x < \infty \\ 0, \text{ otherwise} \end{cases} \quad \ldots\ldots\ldots 1.4.1$$

$\sigma > 0, \quad \mu < \infty$

μ and σ which are the parameters of a Normal distribution are the mean and standard deviation respectively. The coefficient of variation (CV) is thus

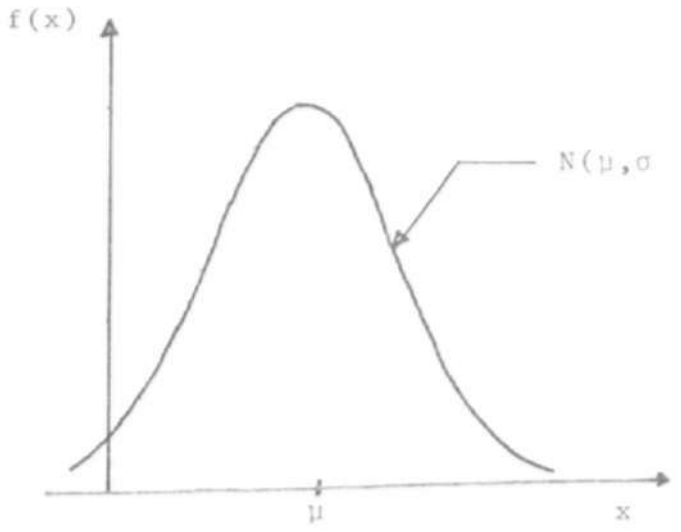
$$CV = \frac{\sigma}{\mu} \quad \ldots\ldots\ldots 1.4.2$$

The Normal Cummulative Distribution Function (CDF) is:

$$F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{1}{\sigma} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] dx$$

$\ldots\ldots\ldots\ldots 1.4.3$

As mentioned above, the limits on the argument of a normal density function are theoretically plus and minus infinily. However, random



Fig(1.4.1) Normal Probability Distribution with mean value equal to μ and σ standard deviation, μ(μ,σ).

variables in engineering work are typically positive and their frequency distributions are, in most instances, skewed to the right. This therefore calls for caution in using the Normal distribution in cases where tails of the distribution are important in drawing conclusions.

The strength of materials can be seen as additive to the strength of individual fibres forming the material. Similarly, the strength of a reinforced concrete member can be additive to the strengths of the constituent materials. This has made normal distribution as candidate model to thepresent work.

1.4.2 Other Distributions

Other possible candidate statistical models are log-normal, Gama, and Beta distributions. The log-normal distribution is the model for a random variable whose logarithm follows the normal distribution. It can be derived as the model for a process whose value results from the MULTIPLE of many small errors in a manner similar to that for the normal distribution for the addition of errors. The probability density function of a log-normally distributed random variable X is expressed as

$$f(x) = \begin{cases} \dfrac{1}{x\sigma\sqrt{2\pi}} \exp\left[-\dfrac{1}{2\sigma^2}(\ln x-\mu)^2\right] \\ \\ 0, \text{ elsewhere} \end{cases} \quad \ldots\ldots\ldots\ldots 1.4.4$$

$x > 0, -\infty < \mu, < \infty, \sigma > 0$

The log-normal distribution may be generalized to cover an interval other than $(0, \infty)$ by introducing the LOCATION parameter a. Thus

$$f(x) = \frac{1}{\sigma(x-a)\sqrt{2\pi}} \exp\left\{\frac{1}{2\sigma^2}\left[\ln(x-a)-\mu\right]^2\right\} \quad \ldots\ldots\ldots 1.4.5$$

$x \geq a, \quad \infty < \mu < \infty, \sigma > 0, \; -\infty < a < \infty$

The Gamma distribution is continous, unimodal and takes on positive values only. The Gamma model allows representation of a wide diversity of distributional shapes over the interval from zero or some arbitrary origin to infinity. The Gamma model is skewed to the right, hence it is suitable for representing a random variable whose values are essentially possitive and freqency distribution positively skewed.

The probability density function of a random variable X having a Gamma distribution is of the form

$$f(x) = \begin{cases} \frac{\lambda^K}{\Gamma(K)} x^{K-1} e^{-\lambda x}, & x > 0 \quad \ldots\ldots 1.4.6 \\ 0, \text{ elsewhere} \end{cases}$$

$$\lambda > 0, \; K > 0$$

where λ is SCALE parameter, K is SHAPE parameter and $\Gamma(K)$ is the Gamma function which is defined as

$$\Gamma(K) = \int_0^\infty x^{K-1} e^{x} dx \quad \ldots\ldots\ldots 1.4.7$$

$\Gamma(K)$ is equal to the factorial $(K-1)!$. When K is a positive integer. The shifted Gamma distribution has a density dfunction of the form

$$f(x) = \frac{\lambda^K}{\Gamma(K)} (x-a)^{K-1} e^{-\lambda(x-a)}, \quad x > 0$$

$$0, \text{ elsewhere} \qquad \ldots\ldots\ldots 1.4.8$$

$$x > 0, \quad \lambda > 0, \; k > 0$$

where a, k, and λ are the parameters. Thus a is a LOCATION parameter.

The Beta distribution is a useful model for variates whose values are limited to a finite interval. The Beta probability density function defined over the interval (0.1) is

$$f(x) = \begin{cases} \frac{1}{B(q,r)} x^{q-1}(1-x)^{r-1} \\ 0, \quad \text{elsewhere} \end{cases} \qquad \ldots\ldots 1.4.9$$

$$0 < x < 1, \; q > 0, \; r > 0$$

where q and r are SHAPE parameters and B(q,r) is the BETA FUNCTION, which is defined as

$$B(q,r) = \int_0^1 x^{q-1}(1-x)^{r-1} dx \qquad \ldots\ldots\ldots 1.4.10$$

which is related to the Gamma function as follows

$$B(q,r) = \Gamma(q)\ \Gamma(r)\ /\Gamma(q+r) \quad \ldots\ldots\ldots\ 1.4.11$$

and $\Gamma(.)$ is a Gamma function. The density function of a Beta distribution with limits a and b is

$$f(x) = \begin{cases} \dfrac{1}{B(q,r)} \dfrac{(x-a)^{q-1}(b-x)^{r-1}}{(b-a)^{q+r-1}} \\ 0, \text{ elsewhere} \end{cases} \quad \ldots\ldots\ 1.4.12$$

$$a < x < b, \quad 0 < a < b, \quad q > 0, \ r > 0$$

where q, r, a, and b are parameters. The Beta distribution model is very flexible and can be applied to different skewness situation. However, the estimation of Beta model parameters is quite difficult.

1.4.3 Parameter Structure

A complete description of a random variable would be accomplished by specifying its probability distribution as well as the associated values of its parameters. The univariate, continous distributions discussed in the previous sections feature basically three types of parameter:

1) LOCATION parameters (μ, a)

2) SCALE parameters (σ)

3) SHAPE Parameters (K)

A location parameter μ or a locates the probability distribution function f on the measurementaxis of x without changing the appearance of the distribution. That is μ relates f to the origin ofthe measurement scale. Thus μ is of the same measurement dimension as x. A location parameter μ is related to x in the form $(x-\mu)$. If the relationship is in any other form, the parameter μ is not location parameter.

A scale parameter σ is a factor that "Scales" the measurement variable x. It relates to the units of measurement on x. A scale parameter affects the dispersion of x in the probability density function. If the distribution model is not symmetrical a scale parameter σ also affects the value of certain certrality measures of x such as the expected value E(x). In a distribution model a scale parameter σ is related to x in theform x/σ. If a parameter σ is related to x in any other form, it is not a scale parameter.

A SHAPE parameter, K affects the basic 'shape' of a probability density functions. However, the specificeffect of K on the shape of a density function

depends on the density function in question. A shape parameter K is not associated with x in any standard form. It neither relates to the origin nor to the scale of measurement on x. A shape parameter K, is an intrinsic distingushing property of a density function. It is usually the most difficult to estimate.

1.4.4 Parameter Estimation Techniques

The techniques of deriving probabilities information and of estimating parameter values from sampled empirical data are embodied in the methods of STATISTICAL INFERENCE in which information obtained from sample data is used to make generalization about the population from which the samples were drawn. A set of sample values on which basis parameters can be estimated is usually assumed to constitute a RANDOM SAMPLE, implying that successive sample values are independent and theunderlying population remains the same from one sample value to another.

Classical methods of estimation of parameters are POINT and INTERVAL estimators. While point estimation is concerned with the computation of a single value from an empirical data set, to

represent the parameters of the underlying population, interval estimation goes further to establish a statement of confidence in the estimated quantity, resulting in the determination of an interval that contains the parameter value with a prescribed level of confidence. The common methods of point estimations are the METHOD OF MOMENTS and the METHOD OF MAXIMUM LIKELIHOOD. Another, but less commonly used method is the METHOD OF QUANTILES:

i. The Method of Moments:-

The method of moments evaluates a parameter via estimating the corresponding sample moments; usually the mean and variance, and higher moments, if necessary of the variate. Moment estimates are often simple to compute. However, these estimates are usually biased and relatively inefficient.

The sample mean and variance can be written as

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} x_i \quad \ldots\ldots\ldots\ldots \quad 1.4.13$$

$$S^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2 \ldots\ldots 1.4.14$$

The third moment about the mean is thus

$$M_3 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^3 \ldots\ldots 1.4.15$$

In general the r^{th} moment about orign is

$$M_r^1 = \frac{1}{n} \sum_{i=1}^{n} x_i^r \ldots\ldots\ldots 1.4.16$$

and about mean $\bar{x}$ is

$$M_r = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^r \ldots\ldots 1.4.17$$

ii. <u>The Mothod of Maximum Likelihood</u>:-

Consider a random variable x with density function $f(x_i \theta)$ in which θ is the parameter. On the basis of the sample values, x_i, x_2, -- x_n, the likelihood of obtaining a particular sample value x_i can be assumed to be proportional to the value of the probability density function random sampling, the LIKELIHOOD FUNCTION (LF) of observing the set x_1, x_2...., x_n

is $L(x_1,x_2,\ldots,x_n;\theta) = f(x_1;\theta)f(x_2;\theta)$

$\ldots\ldots f(x_n;\theta) \ldots\ldots\ldots$ 1.4.18

The MAXIMUM LIKELIHOOD ESTIMATOR θ is then the value of θ that maximizes the likelihood function $L(x_1,x_2 \ldots\ldots x_n;\theta)$. This estimator may be obtained as a solution to the following equation:

$$\frac{\partial L(x_1 x_2 \ldots,x_n;\theta)}{\partial \theta} = 0 \quad \ldots\ldots \text{1.4.19}$$

It is frequently move convenient, because of the multiplicative nature of the likelihood function, to maximize the logarithm of the likelihood function instead.

i.e $\partial \log L(x_1,x_2 \ldots\ldots x_n;\theta)/\partial\theta = 0 \ldots\ldots$ 1.4.20

The solution for θ from each of the two equations above is the same.

For density functions with two or more parameters, the likelihood function becomes

$$L(x_1,x_2 \ldots\ldots x_n;\theta_1,\theta_2,\ldots \theta_m) = \prod_{i=1}^{\wedge} f(x_i;\theta_1 \ldots \theta_m)$$

$\ldots\ldots\ldots$ 1.4.21

where $\theta_1, \ldots, \theta_m$ are the m parameters to be estimated. In this case, the maximum likelihood

estimators would be obtained from the solution to the following set of simultaneous equations.

$$\frac{\partial \text{Log } L(x_1,.....,x_n;\theta_1....\theta_m)}{\partial \theta_j} = 0$$

$$j = 1,....., m ..... 1.4.22$$

iii. The Method of Quantiles

The method of quantiles provides a simple and rapid method of estimation of a parameter. This method though convenient, is relatively inefficient. The method of quantiles is suitable when trading efficiency of estimation for computational convenience. The method of quantiles also applies directly to censored samples provided the chosen order statistics is available (7).
Consider a distribution model having a single parameter θ. The expression

$$F(x_q;\theta) = q ..... 1.4.23$$

defines the quantile x_q, implying that 100q percent of all possible values of the random variable X lie below x_q.

Suppose that the r^{th} observation from an ordered sample of size n is chosen so that its propertion r/n is equal to or just greater than q (i.e., r/n > q). With x(r) replacing xq, the equation above may then serve as an estimating equation for the parameter θ, and the order q may be chosen to minimize the variance of the corresponding estimator θ. That is

$$F(x_{(r)};\theta) = q \quad \ldots\ldots\ldots \quad 1.4.24$$

where q remains to be chosen.

For density function with two or more parameters $(\theta_1,\ldots,\theta_m)$ then m values of q and the corresponding SAMPLE approximations xq give a set of m simultaneous equations of the form shown above for determining the m estimators of the parameter components θ_i.

1.4.5 Goodness-of-fit Tests

From the knowledge of the physical phenomenon of a variableand frequency distribution of the data, subjective assessment of the possible probability distribution of the variable can be made. The validity of assumed probability distribution may be verified or disproved statistically in the

light of the available data, by Goodness-of-fit tests. Two such tests frequently used are the CHI-SQURE (X^2) and the KOLMOGOROV-SMIRNO (K-s) tests. These tests, however, depend on a specified significance level α, the choice of which is arbitrary, andmay not provide absolute information on the validity of a specific distribution usually, the null hypothesis H_o, that the available data and a specified theoretical distribution are not statistically different. The null hypothesis H_o, is NOT REJECTED if the probability of obtaining the observed event is greater than the chosen significance level α; *otherwise, H_o is rejected.*

The goodness-of-fit tests sometimes called SIGNIFICANCE TESTS are one-sided tests in that these check whether observational data disagree "SIGNIFICANTLY" with the given hypothesis H_o.

Although the X^2 and K-s tests are dependent on a specified significance level, the choice of which is largely arbitrary, these tests remain useful for delineating the relative degree of validity of the different distributions.

i. Chi-Squre (X^2) Test

The X^2 goodness-of-fit test is applicable to composite hypothesis (i.e. when parameters need to be estimated from data) and to discrete distributions. The X^2 test procedure is based on limiting process, hence it is required that the expected frequency andnumber of class intervals should not be too small (at least five). The X^2 test utilizes only the number of observations in intervals rather than the observations themselves. Thus, since the test requires grouping of data, sample information is lost.

ii. KOLMOGOROV -Smirnov (K-S) Test

Like the chi-square Test, the Kolmogorov-Smirnov test can be used to test the degree of agreement between the distribution of a set of empirical or sample data and some specified theoretical distribution.
It applies only to continous data. The K-S test is computationally simple and does not require grouping of data. Thus it can be used for small sample.

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The test is conducted by developing or specifying the cummulative probability distribution that would result from the theoretical distribution and comparing it to the cummulative probability distribution of the empirical or sample data. The process is based upon the class in which the theoretical and observed distributions have the largest obsolute deviation. This deviation is then compared to the critical values to determine if a deviation of this magnitude could occur owning to random variation.

1.5 MONTE CARLO METHOD

1.5.1 Simulation and Random Numbers

Many engineering situations involve complex systems whose performance fluctuates because of variations in their components. From a knowledge of these variations and an understanding of the system structure it is desired to draw conclusion about system performance.

Because of their complexity, some problems can rarely be solved by direct analytical methods, such as the transformation of variables. Monte Carlo simulation is one of the approximate methods for obtaining information about system performance from component data. It consists of building many

systems by computer calculations and evaluating the performance of such synthesized methods.

If the distribution for each component variable is known it is possible to obtain synthetic measurements on these components by drawing a number of random values from each distribution. When the relationship between the component variables and system performance is established, these random values can be used to calculate the performance of the corresponding number of synthetic systems. Theprocedure can be shown graphically as in figure (1.5.1)

An essential feature of Monte Carlo Simulation is the substitution, at some point, for a random variable a corresponding set of actual values, having the statistical properties of the random variable. The values substituted are called random numbers on the ground that they could well have been produced by chance by a suitable random process.

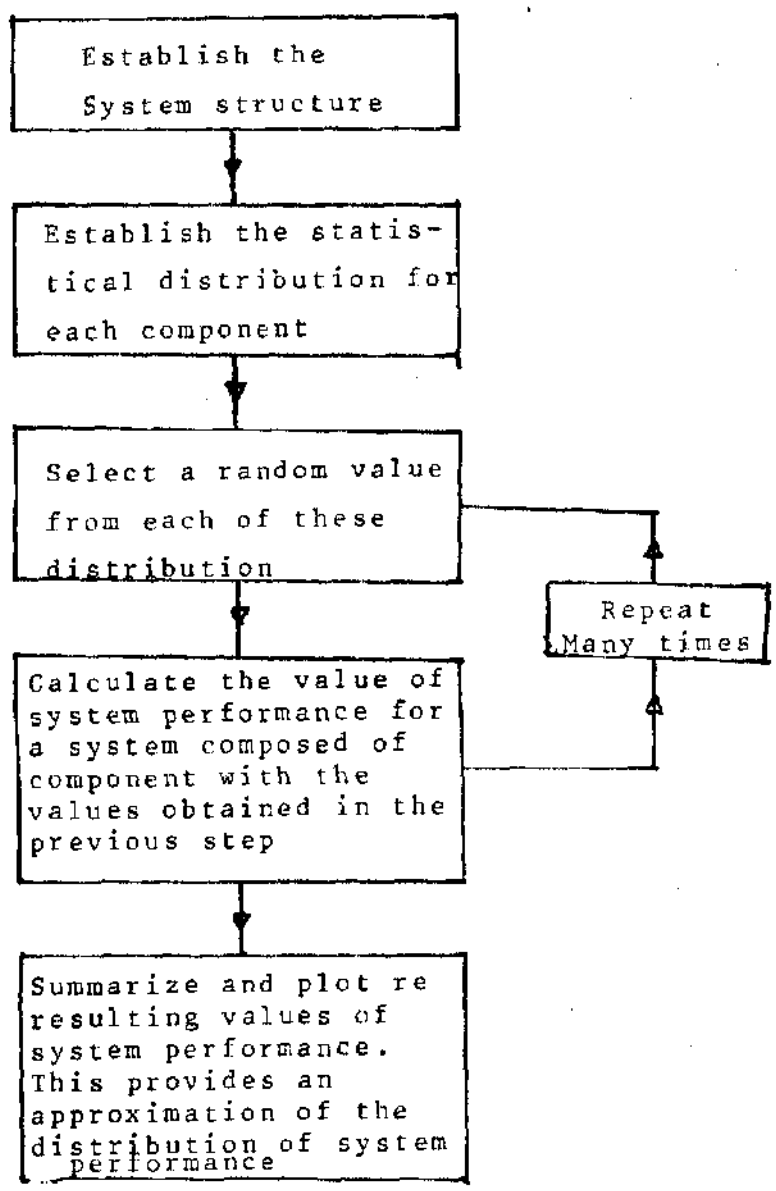
A sequence of random numbers from a uniform distribution is often used as a basic sequance to construct a sequence of independent observations of any other random variable whose distribution is known (23)

Uniform random numbers have the property that for a selected range of values (0.1) the generated random number is equally likely to occur anywhere in the range. Many mechanical or electronic devices have been constructed to generate random numbers. A lot of table of random numbers generated by these physical processes are published. The best known tables are cited by reference (17)

It is most convenient for electronic digital computers to generate a sequence of numbers by a rule that permits reproducibility in such a way that a reasonable statistical test will show no significant departure from randomness. Such a generated sequence is referred to as being pseudo-random.

Sometimes it is possible to use a non-random sequence having only the particular statistical properties that concern us. Such a sequence is called quasi random. This sequence is used in a Monte Carlo Work where it is known that the violation of some statistical test will not invalidate the result (22).

A number of techniques have been suggested for producing psaudo-random sequence. A relatively old technique is theMid-square method. In this



Fig(1.5.1): Flow Chart for Monte Carlo Method

method each number is generated by squaring its predecessor and taking the middle digits of the result. Most starting values in this method lead to disappointingly short period (22). Another disadvantage of this method is the slowness of generation as compared with other methods. Although the method is simple it is not recommened because of its shortcoming. A much more satisfactory method is Lehmer Multiplicative Congruentical Method.

The method consits of the following steps:-

1. Determining the largest prime m_1 less than the register capacity of a particular computer being used.
2. Find a positive primitive root X of m_1 of at least four digits.
3. Specify a starting integer $u_o < m_1$
4. Generate the sequence of pseido random integer log the recursive relationship.

$$u_{n+1} = X\, u_n \ (\text{Mod } m_1) \ldots\ldots 1.5.1.$$

The sequence of random number becomes cyclic after $m_1 - 1$ numbers have been generated.

For the binary computer, x is selected as

$$x = 8t \pm 3 \quad \ldots\ldots\ldots\ldots 1.5.2$$

where t may be any positive integer.

The starting value or seed (u_o) is selected as any positive but odd integer.

In the decimal computer x is selected based upon.

$$x = 200\ t \pm \theta \quad \ldots\ldots\ldots\ldots\ldots 1.5.3$$

where t may be any positive integer and θ is one of the following values: $\pm$(3, 11, 13, 19, 21, 27, 29, 29, 53, 61, 67, 69, 83, or 91). The seed u_o for the decimal case may be any odd but positve integer that is not divisible by 2 or 5.

Most of the computer systems have an in-built program generating sequence of random numbers. IYA ABUBAKAR COMPUTER CENTRE of Ahmadu Bello University has an in-built function RANF(o) which generates uniform random numbers. In this study this function is used to generate the needed random numbers. The function is contained in the main program RESISTJ.

1.5.2 Random Sampling

A sample is a subset of a population. It should always be representative ofthe population by containing important characteristics of the population e.g. mean, standard deviation etc. To achieve this, the selection of the sample must be accomplished in a random fashion from the

total population. The random selection of a sample from a population is achieved when all possible items in the population have equal chance of being selected. Distributions containing unequally likely events can always be transformed into equivalent distribution comprising equally likely, simpler events by employing some artificial samplingdevice.

When sampling from a continous distribution the distribution must be approximated with a discrete distribution which can usually be represented by a histogram. The cummulative distribution is then calculated.

Simulated sampling involves randomly selecting values of random variables whose distributions are known, so that their occurences as physical random processes are simulated. This could be achieved by sampling from a uniform (or rectangular) distribution and transforming the results to any other known distribution as is subsequently illustrated.

Suppose that x is random variable with a density function f(x) and a cummulative distribution function F(x). Let u be a uniformly or rectangularly distributed random variable on the interval

(0,1) with a cummulative distribution G(u).

Since $u = G(u)$,

it follows that

$$u = F(x)$$

both u and F(x) having been defined on the interval (0,1).

From ordinary inverse function we have

$$X = F^{-1}(u) \quad \ldots\ldots\ldots\ldots\ldots 1.5.4$$

If x_1 and u_1 are values such that $u_1 = (F(x_1)$ then

$$P_r(x \leq x_1) = F(x_1)$$

$$P_r(u \leq u_1) = G(u_1) = u_1$$

$$P_r(u \leq u_1) = P_r(x \leq x_1)$$

In some cases equation (1.5.4) can be solved explicitly. Numberical techniques may be required to aid in the calculation.

For example a sequence of standard normal random number ($\mu=0, \sigma=1$) can be generated from uniform random numbers. Using such standardized normal random numbers, Z_N^1 equivalent realizations X_N^1 of a general normal variable can be obtained as follows:

$$X_N^1 = \mu + \sigma Z_N^1 \quad \ldots\ldots\ldots 1.5.5.$$

Where μ and σ are mean and standard deviation of a particular Normal distribution. Procedures for obtaining random values for other distributions from the standard uniform and Normal variates are summarized in Table 7.2 of reference. Reference (2) also gives computer method of obtaining random variate from other distributions.

1.5.3 Sample Size and Error Band

Because Monte Carlo simulation involves random values, theresults are subject to statistical fluctuation. Thus any estimate will not be exact but, will have an associated error band. The larger the number of trials in the simulation, the more precise will be the final answer and we can obtain as small an error as desired by conducting sufficient trials. In practice, the allowable error is generally specified, and this information is used to determine the required trials. The required number of observations to obtain a desired degree of precision can be determined by confidence limit approach used on estimate of parameters. These parameters are population mean, proportion and population standard deviation. The sample size estimation methods are now presented.

i. Estimating the Population Mean

Suppose it is desired to determine an estimate $\bar{X}$ of the true population mean μ such that

$$P_r(\mu - \epsilon \leq \bar{X} \leq \mu + \epsilon) = 1 - \alpha \quad \ldots\ldots 1.5.6$$

where $\bar{X}$ is the sample mean, μ is the true population mean, and $1 - \alpha$ is the probability that the interval $\mu \pm \epsilon$ contains $\bar{X}$.
Assuming normal distribution,

$$\epsilon = \frac{\sigma Z_{\alpha/2}}{\sqrt{n}} \quad \ldots\ldots\ldots 1.5.7$$

where n = sample size

$Z_{\alpha/2}$ = the two tailed standardized normal statistic for the probability sought.

σ = Population standard deviation

ϵ = Maximum allowable error

Thus

$$n = \frac{(\sigma Z_{\alpha/2})^2}{\epsilon^2} \quad \ldots\ldots\ldots\ldots 1.5.8$$

ii. Estimating Proportions

Supposing it is desired to make an estimate of the proportion, P of the population between two limits above or below some

specified values based on the results of n trials. This problem is the same as estimating the parameter P of a binomial distribution. In doing so specification must be made of ϵ, the maximum allowable error in estimating P; $1-\alpha$, the desired probability or confidence level that the estimated proportion P does not differ from P by more than $\pm$ and P^1 an initial estimate of P. Based on the normal distribution approximation to the binomial distribution, the number of trial may beestimated from

$$n = \frac{P^1(1-P^1)}{\epsilon^2} Z^2_{1-\alpha/2} \quad \ldots\ldots\ldots 1.5.9$$

where ϵ and P^1 have been previously defined and $Z_{1-\alpha/2}$ designate the $(1-\alpha/2)100$ percent point of a standard normal distribution.

iii. Comparison of two distribution

An alternate procedure for determining sample size is by use of Kolmogorov statistic. This statistic is used to find whether the difference between two distributions is significant given the number of observations. In the sample size determination the Kolmogorov statistic is fixed from which the sample size is derived.

Kolmogorov statistic is given by

$$D_n = /F^*(x) - F_o(x)/ \quad \ldots\ldots\ 1.5.10$$

where $F^*(x)$ is the empirical cummulative distribution of data, $F_o(x)$ is the theoretical distribution of the population.

D_n is the maximum deviation of $F^*(x)$ from $F_o(x)$. It is used to construct a $(1-\alpha)$ confidence interval on $F_o(x)$:

$$F^*(x) \pm (d_n)_\alpha$$

where $(d_n)_\alpha$ is the asymptotic value of D_n such that

$$P_r(D_n \geq (d_n)_\alpha) = \alpha \quad \ldots\ldots\ 1.5.11$$

By specifying $\epsilon = (d_n)_\alpha$, the required sample size can be determined such that $F^*(x)$ approximates $F_o(x)$ to an accuracy of $\pm\epsilon$ at the $(1-\alpha)$ confidence level.

CHAPTER TWO

LITERATURE REVIEW

The development of limit state has brought about wide interest in the probabilitic nature of strength of concrete structures. In an attempt to describe the variation of strength in reinforced concrete sections two basic approaches have been adopted. One approach (29, 32) has been to apply the techniques of error statistics (regression analysis) to the results of full scale test. Another approach is to treat the material strengths as random variable and the ultimate resistance of a section as a random variable which is a function of the material strengths with the functional relationship derived from structural theory.

The strength of a structural member is derived from the strength of constituent materials. Other factors influencing the member strength is the geometry and workmanship. Therefore, knowledge of the variations in those factors can lead to the understanding of the variability of structural member strength. In the light of this, effort has been intensified to study the variability

of all the factors contributing to the strength of structural member. An attempt is made herein to give summary of some of the earlier works in this area.

2.1 VARIABILITY OF MATERIAL STRENGTH

As a contribution toward a research to introduce"limit state design" for concrete structures in North America, Mirza and MacGragor (21) reviewed published and unpublished data on thevariability of reinforcing bars. The authors identified the following sources of variations in steel yield strength and ultimate strength:

(i) Variation in the strength of material itself;

(ii) Variation in the area of cross-section of the bar;

(iii) Effect of rate of loading; and

(iv) Effect of strain at which yield is defined.

The authors considered the effect of these factors on the mechanical properties of deformed bars of Grades 40 and 60. The mechanical properties are yield strength, ultimate strength, and modulus of elasticity.

For yield strength, Beta distributions were

found to fit the data for mill test yield strength of grade 40 and grade 60 bars. The mean values and coefficients of variation for the data were found to be 48.8Ksi ($337MN/m^2$) and 10.7% for grade 40 and 7Ksi ($490\ MN/m^2$) and 9.3% for grade 60 bars. Based on the Beta distribution theprobability density function (PDF) forthe mill test yield strength of grade 40 reinforcement was modelled as:

$$PDF = 3.721\left[\frac{fy-36}{32}\right]^{2.21}\left[\frac{68-fy}{32}\right]^{3.82} \quad \ldots\ldots\ldots\ldots 2.1.1$$

in which $36 \leq fy(Ksi) \leq 68$

Similarly, theprobability density function for the mill test yield strength of grade 60 steel bars was modelled as:

$$PDF = 7.141\left[\frac{f_y-57}{51}\right]^{2.02}\left[\frac{108-f_y}{51}\right]^{6.95} \quad \ldots\ldots\ldots\ldots 2.1.2$$

in which $57 \leq f_y\ (Ksi) \leq 108$.

The measured crossectional areas of bars

were found to differ from the nominal value specified in the design. Mirza and MacGregor (21) approximated the distribution of measured to nominal areas, A_m/A_n to normal distribution truncated at 0.94 with a mean value of 0.99 and a coefficient of variation 2.4%. Similarly, for the evaluation of static yield strength from mill test, the distribution curve for the difference in their strengths, df_{ys} was approximated as the normal distributions with a mean value of 3.5 Ksi ($24.5MN/m^2$) and a coefficient of variation 13.4%. The distribution of A_m/A_n and df_{ys} were found to be the same for both grade 40 and grade 60 steel.

The probability distribution of static yield strength based on the nominal area of cross-section of reinforcing bars is represented by Beta distribution with PDF calculated as follows:

for Grade 40,

$$PDF = 4.106 \left[\frac{f_{ys}-33}{29}\right]^{2.21} \left[\frac{62-f_{ys}}{29}\right]^{3.82} \quad \ldots\ldots\ldots 2.1.3$$

in which $33 \leq f_{ys}(\text{Ksi}) \leq 62$;

and for Grade 60,

$$\text{PDF} = 7.587 \left[\frac{f_{ys} - 54}{48}\right]^{2.02} \left[\frac{102 - f_{ys}}{48}\right]^{6.95} \quad \ldots\ldots\ldots \text{2.1.4}$$

in which $54 \leq f_{ys}(\text{Ksi}) \leq 102$.

The mean value and standard deviation of the static yield strength refered to the nominal area of grade 60 bars are found to be 66.8 Ksi and 5.52 Ksi ($461 MN/m^2$) and($38 MN/m^2$)respectively.

The authors also recommend Beta distribution with the following PDF's for mill test ultimate strength of bars:

for Grade 40,

$$\text{PDF} = 2.381 \left[\frac{f_u - 55}{50}\right]^{2.21} \left[\frac{105 - f_u}{50}\right]^{3.82} \quad \ldots\ldots\ldots \text{2.1.5}$$

in which $55 \leq f_u(\text{Ksi}) \leq 105$; $f_u \geq f_y$

ăndfor Grade 60,

$$PDF = 4.61 \left[\frac{f_u - 88}{79}\right]^{2.02} \left[\frac{167 - f_u}{79}\right]^{6.95} \quad \ldots\ldots\ldots 2.1.6$$

in which $88 \leq f_u(\text{Ksi}) \leq 167$; $f_u \geq f_y$

The mean value and standard deviation of the mill test ultimate strength of grade 60 bars are found to be 110.8 Ksi and8.73 Ksi (764 MN/m^2 and 60.2 MN/m^2) respectively. The data is insufficient for the ultimate strength of grade 40 bars. However, the PDF for mill test ultimate strength of 40 bars was selected due to acceptable degree of correlation demonstrated by grade 60 reinforcement with the corresponding Beta distribution equation.

The distribution of the difference between mill test and static ultimate strength, df_{us} can/-be assumed to be normal with a mean value 4.5 Ksi (31MN/m^2) and a coefficient of variation 13.4%. The probability density function of Beta distribution for static ultimate strength referred to the nominal area of cross-section may be calculated for grade 40 as:

$$PDF = 2.646 \left[\frac{f_{us}-51}{45} \right]^{2.21} \left[\frac{96-f_{us}}{45} \right]^{3.82}$$

..... 2.1.7

in which $51 \leq f_{us}(Ksi) \leq 96$; $f_{us} \geq f_y$

and for Grade 60 as

$$PDF = 4.922 \left[\frac{f_{us}-84}{74} \right]^{2.02} \left[\frac{158-f_{us}}{74} \right]^{6.95}$$

..... 2.1.8

in which $84 \leq f_{us}(Ksi) \leq 158$; $f_{us} \geq f_{ys}$

Finally, the authors recommend normal distribution for Modulus of elasticity of grade 40 and grade 60 bars. The mean value is 29200 Ksi (201,000 MN/m^2) and a coefficient of variation is 3.3%.

In a similar treatment, Mirza, Hatzinikolas, and MacGregor (1979) reviewed the variability of strength and stiffness of normal weight structural concrete. In their treatment, the authors considered the following mechanical properties of concrete.

(i) Concrete strength in compression;

(ii) Concrete strength in tension;

(iii) Modulus of Elasticity of concrete in compression; and

(iv) Modulus of Elasticity in tension.

The probability distribution of the strength of in-situ normal weight concrete in compression was fitted by the normal distribution with the mean value computed from:

$$\bar{f}_{cstrR} = \bar{f}_{cstr35}(0.89(1+0.08LogR))Psi \quad \ldots\ldots\ldots 2.19a$$

where:

$\bar{f}_{cstr35}$ = the mean 28-day strength of concrete in a structure under the loads applied at 35 psi/sec.

$$= 0.675f_c^1 + 1100$$

$$< 1.15\ f_c^1 psi \quad \ldots\ldots\ldots 2.19b$$

f_c^1 = design compressive strength of concrete.

R = a given rate of leading (psi/sec).

Similarly, thecoefficient of variation of the in-situ compressive strength of concrete at a given rate of loading R(psi/sec) is found to be

$$V^2_{cstrR} = V^2_{ccyl} + 0.0084 \quad \text{..... 2.1.10}$$

where V_{ccyl} = variation of control cylinder.

The variation due to rate of loading was considered negligible.

For the splitting tensile strength of concrete in a structure, the probability model is also found to be cnormal distribution, with mean value obtained as follows:

$$\bar{f}_{spstrR} = 6.4 \ \bar{f}^{\frac{1}{2}}_{cstr35}\left[0.96(1+0.11LogR)\right] \text{ psi} \quad \text{..... 2.1.11}$$

in which

$\bar{f}_{cstr35}$ is calculated from equation (2.1-9) and R = rate of loading.

The coefficient of variation is given by

$$V^2_{spstrR} = \frac{V^2_{ccyl}}{4} + 0.019$$

$$\geq V^2_{cstrR} \quad \text{..... 2.1.12}$$

The authors also assume normal distribution for the probability model of the strength of in-situ concrete in flexural tension. Calculation of the mean and dispersion is by use of the following

formulae:

for mean

$$\bar{f}_{rstrR} = 8.3\bar{f}_{cstr35}\left[0.96(1+0.11\text{Log}R)\right] \text{ psi}$$

..... 2.1.13

and for dispersion;

$$V^2_{rstrR} = \frac{V^2_{ccyl}}{4} + 0.042 \text{ ..... } 2.1.14$$

$$\geq V^2_{cstrR}$$

The value of V_{cstr35} is calculated for equation (2.1-10).

Normal distribution is also found to fit the model for initial tangent modulus of elasticity of in-situ concrete. The mean value and coefficient of variation of the initial tangent modulus can be calculated from the following equation. Mean value is given by

$$\bar{E}_{cistrR} = 60{,}400\ \bar{f}^{\frac{1}{2}}_{cistr35}\ (1.16-0.08\text{Log } t)$$

..... 2.1.15

The coefficient of variation is given by

$$V^2_{cistrR} = \frac{V^2_{ccyl}}{4} + 0.0085 \text{ ..... } 2.1.16$$

V_{ccyl} is as previously defined.

The secant modulus at any stress level can be estimated from the initial tangent modulus assuming that the initial portion of the stress-strain curve is a parabola with a tangent at maximum stress. The coefficient of variation of the secant modulus can be calculated from

$$V^2_{cistrR} = \frac{V^2_{ccyl}}{4} + 0.0165 \quad \ldots\ldots 2.1.17$$

Finally, it is suggested by the authors that the mean value and coefficient of variation of the modulus of elasticity of concrete in compresion and in tension be the same.

In an earlier study, Benjamin (4) found the distribution of yield strength of reinforcing bars to be replicated Beta distribution.

The distribution of yield strength of reinforcing bar was established in another study by Zsutty(32) as log-normal distribution. The coefficient of variation of thedata analysed by Zsutty is 0.144.

In his study ofthe variability of ultimate moment in reinforced concrete beam Castello (8) uses Beta distribution to fit both steel and

concrete strength data. The probability density function are as follows:

For steel,

$$PDF = \frac{(\alpha + \beta + 1)!}{\alpha\,!\,\beta\,!} (Y_1)^{\alpha} (Y_2)^{\beta} \frac{1}{f_{yu} - f_{yl}} \quad \ldots\ldots\ldots 2.1.18$$

where

$$Y_1 = \frac{f_y - f_{yl}}{f_{yu} - f_{yl}}$$

and

$$Y_2 = \frac{f_{yu} - f_y}{f_{yu} - f_{yl}}$$

α , β are Beta parameters whose numerical values are 1 and 5 respectively.

f_{yl}, f_{yu} are lower and upper values of yield strength of steel with actual values as 37.702 Ksi and 77.535 Ksi respectively.

f_y is yield strength of steel

For concrete, the probability density function is

$$PDF = \frac{\alpha + \beta + 1}{\alpha!\,\beta\,!} (Y_3)^{\alpha} (Y_4)^{\beta} \frac{1}{f_{cu}^{1} - f_{c}^{1}} \quad \ldots\ldots\ldots 2.1.19$$

in which $Y_3 = \frac{f_c^1 - f_{c1}^1}{f_{cu}^1 - f_{c1}}$

and $Y_4 = \frac{f_{cu}^1 - f_c^1}{f_{cu}^1 - f_{c1}^1}$

α , β are Beta parameters whose numerical values are 2 for each.

f_{c1}^1, f_{cu}^1 are lower and upper limits of concrete strength with actual values as 2.456 Ksi and 4.787 Ksi respectively is concrete cylinder strength.

f_c^1 is concrete cylinder strength.

Majority of the works cited above seem to suggest limits to the tails of distributions of concrete and steel strengths. This is contrary to the usually assumed distribution which has no limits at the two tails.

If the strength of an element is viewed as the sum of the strengths of individual fibres then by central limit theorem it might be expected to tend towards normality. The assumption of normality however, admits the possibility of negative strength. To rectify this difficulty

some researchers suggest a Log-normal distribution for material strength. But this still leaves the possibility of an infinitely large strength, which seems contrary to experience and intuition. Thus themost practical model should have lower and upper limit.

About 99.9 per cent of the area under normal probability curve is contained within 3 times standard deviation from the mean value. For the strengths of steel and concrete the lower limit of this range is above zero. Thus the error in calculating the failure probability with normal distribution may be negligible.

2.2 VARIABILITY OF MEMBER STRENGTH

It was pointed out earlier that some previous studies have examined ways in which the variability ofthe strength of structures could be assessed. Among such studies is that by Shah and Zsutty (29,32)who applied the method of regression analysis to the results of full scale tests. Another viewpoint is to treat the material strength as random variables and the ultimate resistance of a section as a random variable which is a function of the material

strengths with the functional relationships derived from structural theory.

The works of Zsutty (32) and Shah (29) are on the full scale test results. Zsutty analised data from 109 specified beam tests. In the analysis, the ultimate moment of under-reinforced concrete beams is expressed as

$$\frac{M_u}{bd^2 f'_{cu}} = 0.431 \left[\frac{f_y}{f'_c}\right]^{0.902} (\rho)^{0.752} \quad \ldots\ldots 2.2.1$$

in which

M_u = Ultimate moment of resitance of beam section

b = Breadth of beam

d = Effective depth of beam

f'_c = Cylinder strength of concrete

f_y = Yield strength of steel

A_s = Area of reinforcing steel

= Steel ratio defined as:

ρ = A_s/bd.

The ratio of calculated resistance moment using the above formula to the test result was found to have 6.4% coefficient of variation against 12%

given by ACI 318-56 formula. The ACI 318-56 prediction equation is

$$\frac{M_u}{bd^2 f'_c} = q_1\left(1 - \frac{k_2}{k_1 k_3} q_1\right) \quad \ldots\ldots \quad 2.2.2$$

where

k_1, k_2 and k_3 are stress block parameters.

$$q_1 = f_y / f'_c$$

ρ = the steel ratio.

The other symbols are as previously defined.

In the final analysis Zsutty recommends Log-normal probability distribution for in-place ultimate moment of beams. The median value can be obtained using equation (2.2-1). Coefficient of variation is found to be 0.144. This variation is partly due to the variation of laboratory test beam results of ultimate moment about the central predicted value and in part due to workmanship errors.

Using the error statistics of the in-place of ultimate moment of resistance, Zsutty arrives at safety factor of 0.715 with 99% certainty.

The safety factor obtained is subject to the inaccuracy caused by sampling errors, differences in laboratory measurement and test procedures, and the estimated coefficient of variation values of in-place beam properties. Thus the safety factor is unstable and therefore not suitable for practical application.

In a similar treatment Shah (29) studied a number of columns with different mode of failures using multiple regression analysis. Eighty-three (83) columns failing in compression were studied with resultant prediction formular as

$$\frac{P_u(100)(1+\frac{e}{d})}{bd\ f_c^1} = 5.393\left[\frac{f_y}{f_c^1}\right]^{0.492}(\rho+\rho)^{0.453} \times(1+\frac{e}{d})^{-2.026} \quad \ldots\ldots\ldots 2.2.3$$

in whib

P_u = Ultimate load capacity of column

b,d = Cross-sectional dimensions of columns

f_c^1 = Concrete cylinder strength

f_y = Yield strength of steel in tension and compression

ρ, ρ^1 = Steel ratios in tension andcompression

e = Eccentricity of load.

The factor of 100 was introduced for better decimal control and forease of finding logarithms. Other constants are regression coefficients.

Equation (2.2.3) was obtained with correlation coefficient of R = 0.947 and coefficient of variation 0.175.

For tension failure, 82 columns were considered and the prediction formular was found as

$$\frac{P_u(100)(1 + \frac{e}{d})b}{bd\ f_c^1} = 6.0\left[\left(\frac{f_y}{f_c^1}\right)\right]^{0.947}(\rho + \rho^1)^{0.737} \times (1 + \frac{e}{d})^{-2.213} \quad \ldots\ldots\ldots 2.2.4$$

with coefficient of correlation = 0.973

and coefficient of variation = 0.176

The two formulae above when combined give the following prediction equation for balance failure.

$$\left[\frac{P_u(1 + \frac{e}{d})}{bd\ f_c^1}\right] b = 2.4\left[\left(\frac{f_y}{f_c^1}\right)\right]^{-0.236}(1 + \frac{e}{d})^{-1.765} \quad \ldots\ldots\ldots 2.2.5$$

The results of the prediction formulae above were shown to be in good agreementwith formulae given by ASCE-ACI joint committee Report on Ultimate Strength Design.

In the final analysis Shah obtained the distribution function of ultimate column strength to be log-normal distribution. The median values are obtained using the above formulae for the corresponding mode of failure. This makes the prediction equations given by the author to have added advantage over ASCE-ACI formulae by allowing the probability statement to be made about the predicted values.

In a separate study, Benjamin (4) found Beta distribution to fit all the data for reinforced concrete beams and column strengths.

Generally, the strength of a member would be expected to depend on the constituent materials, geometrical properties and workmanship. Therefore, some authors found it more appropriate to study the effects of these factors on the variability of concrete member strengths. Warner and Kobaila (31), Castello (8), and Mirza and others (16) are among the authors that follow this approach. Frendenthal, Garrets and Shinozuka (15) have also

recommended a similar approach.

Warner and Kabaila (31) have in their paper shown that with knowledge of the distribution of the constituent material, Monte Carlo technique can be used to determine the distribution of concrete member strength. Although no particular distribution has been recommended by the authors, it has been suggested that the distribution of strength of reinforced concrete streucture should have lower and upper limits.

To assess the effects of variability in steel and concrete strength on the ultimate moment in beams, Castello (8) assumed a deterministic functional relationship between the resistance of the sect on and strength of the materials. He used quasi-emperical model given in ACI Building Code. He found that Beta distribution fit the data for both steel and concrete. The probability density functions for strengths of concrete and steel were already shown. By transformation of variables the probability of resistance less than specified values were obtained. Tables (2.2.1) shows the probabilities of failure of singly and doubly reinforced concrete respectively, for specified steel ratios. The tables show that the probability

Table (2.2-1)

Failure Probabilities of Beam Sections

From Reference (8)

Tension steel P_1 as a percentage (1)	Compressive steel, P^1, as a percentage (2)	$\rho - \rho^1$ (3)	$P_r\left[\frac{M_u}{bd^2} < \frac{M_u}{bd^2}\right]^a$ (4)	$P_r\left[\frac{M_u}{bd^2} < \frac{M_{ud}}{bd^2}\right]^b$ (5)
4.0	3.0	1.0	0.0530	
	2.5	1.5	0.0474	0
	2.0	2.0	0.0398	0
	1.5	2.5	0.0313	0
	2.0	1.0	0.0515	0
3.0	1.5	1.5	0.0442	0
	1.0	2.0	0.0347	0
	0.5	2.5	0.0262	0
	1.5	1.0	0.0508	0
2.5	1.0	1.5	0.0423	0
	0.5	2.0	0.0316	0
	0	2.5	0.0242	0
	1.0	1.0	0.0490	0
2.0	0.5	1.5	0.0386	0
	0	2.0	0.0272	0
1.5	0.5	1.0	0.0462	0
	0	1.5	0.0330	0
1.0	0	1.0	0.0407	0

[a] M_{un} = Ultimate moment of resistance based on the nominal material strength.

[b] M_{ud} = Design Ultimate moment of resistance
= $0.9M_{un}$ (ACI Code)

of ultimate resisting moment less than the design moment specified by ACI code to be zero for all ratios of steel. The results show that the under-strength factor of 0.9 specified by ACI Code (1) reduces the failure probability to zero if only theeffect of variability in material strength is considered. Thus the failure probabilities derived by Cabtello are conditional probabilities, since sources of uncertainty other than concrete and steel strengths have not been considered.

CHAPTER THREE
PROBABILITY DISTRIBUTION OF MATERIAL STRENGTH

3.1 DISTRIBUTION OF CONCRETE STRENGTH

Tests results on the strength of nominally identical speimens always show some scatter around a central value. The major sources of variations in concrete strength are; the variations in material strength properties and proportions of the concrete mix, the variations in maxing, transporting, placing and curing methods, the variations in testing procedures and variations due to concrete being in structure rather than in control specimens.

Reference (13) has given the procedure for making, curing and testing cubes. It is worth while to outline some of the limit state requirements for quality control of concrete.

During the period of construction the sampling be carried out as follows:

(i) at least one sample per day of casting

(ii) one sample from one batch selected randomly to represent an average volume of not more than $20m^3$ or 20 batches.

One or more cubes are cast from a sample of concrete.

The results of tests comply with the specified characteristic strength of concrete if:

(i) the average strength determined from any group of four consecutive tests exceeds the specified characteristic strength by not less than 0.5 times current margin and

(ii) each individual test result is greater than 85% of specified characteristic strength.

If the average of four cubes test complies with (i) above, all the batches of concrete being judged, including those not actually tested, must be judged as complying. If the average of four tests fails, all the concrete in the batches mentioned above does not comply with the strength requirement.

If a batch fails to comply with (ii) above (one cube only) then that batch only does not comply with the requirement. If more than one cube test in a group of four fail, then all concrete in intervening batches, should be deemed not to comply with the strength requirements as in (i) for theaverage of four tests.

Mention is made above, of the current margin. This is determined on the ground of cube tests previously carried out by the contractor.who manufactured concrete of nominally similar proportions of similar materials, produced over a period specified below, by the same plant under similar supervision:

(i) From statistical analysis of the 5% defectives we get the current margin as 1.64 times standard deviation of cube strengths of at least 100 separate batches of concrete produced within 12 months. The standard deviation may fluctuate during the progress of work and therefore the lowest strength for the current margin is determined as follows: 1/6 of the characteristic strength for concrete of grade 7, 10 and 15 or 3.75 N/mm^2 for concrete of grade 20 or above.

(ii) 1.64 times standard deviation of cube tests on at least 40 separate batches of concrete produced over a period exceeding 5 days but not exceeding 6

months and not less than 1/3 of the characteristic strength for concrete of grade 7, 10 and 15 or 7.5 N/mm^2 for concrete of grade 20 or above.

If there is insufficient data to satisfy (i) or (ii) above, and, laboratory test were not carried out earlier until sufficient data is available to satisfy (i) or (ii) the curreent margin should be as follows:

(iii) 2/3 of the characteristic strength for concrete of grade 7, 10, and 15 or 15 N/mm^2 for concrete grade 20 or above.

If trial mixes were manufactured and tested for strength in the laboratory, the current margin may be assumed as follows:

(iv) Three cubes from each of three separate batches of concrete to be casted using materials similar to those which will be used on site. The cubes should be tested at 28 days and the average strength of the nine cubes should exceed the specified characteristic

strength by the current margin (iii) minus 3.5 N/mm^2.

Several buildings, lecture threatres, stores and tpther structures were designed in limit states in the Zaria region. The Department of Civil Engineering of Ahmadu Bello University carried out control of concrete strength for these buildings.

Florek (13) has analised the test results of cube tests on some of these buildings. He found the normal distribution to fit all the data. The mean values vary from 32.3 N/mm^2 to 34.7 N/mm^2. The standard deviation is between 6.01 to 7.3 N/mm^2 coefficient of variation has values of between 0.18 and 0.23.

The data to be analyzed in this study is shown in Table (3.1.1). It includes the data analyzed by Floreck (13). The types and sites of some buildings are notspecified, but only the strengths of the concrete used in structural element are included. Also the results of thetests carried out in less than 28 days are not included. The specified characteristic strength is 20 MN/m^2.

TABLE (3.1.1)

CONCRETE CUBE COMPRESSIVE STRENGTH
RESULTS OF QUALITY CONTROL TEST ON LIVE PROJECTS
CONDUCTED OVER MANY YEARS BY THE
DEPARTMENT OF CIVIL ENGINEERING
AHMADU BELLO UNIVERSITY, ZARIA, NIGERIA

S/NO	STRENGTH (N/mm^2)		
1.	31.10	39.00	40.80
2.	29.30	32.00	40.00
3.	35.60	37.30	38.20
4.	39.10	40.90	40.90
5.	28.40	28.90	31.10
6.	34.70	39.10	41.80
7.	29.30	30.20	30.70
8.	25.80	26.20	27.60
9.	18.70	19.60	21.30
10.	27.70	29.30	32.00
11.	32.00	37.30	38.20
12.	29.30	32.00	34.70
13.	24.40	26.70	29.30
14.	26.70	27.70	34.70
15.	28.40	33.30	34.70
16.	11.10	22.70	23.10
17.	15.10	16.00	20.40
18.	14.20	14.20	14.20
19.	38.20	44.40	50.70
20.	10.70	10.70	11.60
21.	31.10	31.60	31.60

Table (3.1.1.) Continued

S/NO	STRENGTH (N/mm^2)		
22.	40.00	41.80	42.70
23.	44.40	46.20	50.20
24.	43.10	43.60	43.60
25.	13.30	13.30	14.70
26.	42.70	47.60	51.10
27.	52.40	56.00	59.60
28.	38.20	44.40	52.40
29.	32.90	36.40	40.90
30.	40.00	40.40	40.90
31.	29.90	33.10	34.00
32.	37.50	43.90	50.90
33.	25.70	28.00	31.80
34.	34.80	44.10	45.20
35.	37.70	38.50	42.00
36.	29.80	38.20	44.00
37.	32.00	32.00	40.90
38.	28.00	29.30	48.90
39.	33.80	37.70	42.70
40.	26.70	38.20	38.70
41.	30.70	34.20	49.80
42.	36.90	44.40	45.30
43.	34.20	37.30	39.10
44.	34.70	40.90	41.80
45.	38.20	48.90	53.30

TABLE (3.1.1.) Continued

S/NO	STRENGTH (N/mm^2)		
46.	40.00	44.40	50.70
47.	34.70	35.60	38.20
48.	36.40	36.90	37.30
49.	36.40	40.00	40.00
50.	32.90	34.70	36.40
51.	45.30	48.90	54.20
52.	37.30	42.70	43.60
53.	34.70	37.30	40.90
54.	31.10	44.40	46.20
55.	18.20	22.21	23.10
56.	36.40	37.30	41.80
57.	10.20	10.70	11.60
58.	15.10	16.00	16.00
59.	32.90	34.70	40.90
60.	25.80	26.70	27.60
61.	30.70	32.00	34.70
62.	34.70	38.20	39.10
63.	22.20	23.10	24.00
64.	26.70	28.40	32.00
65.	34.70	35.60	39.10
66.	44.00	44.90	47.10
67.	16.90	16.90	17.80
68.	16.00	18.70	20.00
69.	45.30	45.80	49.80
70.	22.20	55.60	59.10
71.	21.30	22.20	22.20

TABLE (3.1.1.) Continued

S/NO	STRENGTH (N/mm^2)		
72.	41.80	41.80	42.70
73.	28.00	33.30	33.80
74.	30.20	31.10	34.40
75.	22.90	23.50	24.50
76.	28.40	30.20	33.30
77.	29.80	36.00	37.80
78.	24.00	27.60	30.20
79.	26.70	38.70	39.60
80.	31.60	35.60	36.00
81.	16.90	22.20	25.80
82.	17.80	17.80	18.70
83.	30.20	32.40	32.90
84.	15.10	19.60	31.10
85.	24.00	36.40	37.30
86.	44.90	50.70	52.40
87.	17.80	21.30	23.60
88.	57.80	59.60	61.30
89.	20.90	22.20	22.70
90.	17.80	21.30	24.40
91.	20.90	21.80	22.70
92.	32.90	33.80	37.80
93.	31.10	31.10	31.10
94.	32.00	32.40	34.40
95.	36.40	36.40	44.40

The sample mean and standard deviation are estimated to be 33.36MN/m^2 and10.56MN/m^2 respectively. The Kutosis and Skewness coefficients are respectively estimated as 2.75 and 0.00.

The large coefficient of variation may be due to differences in workmanship. Also most of thecontractors have no experience in providing quality of concrete in accordance with limit states requirement. The contractors also use proportional batching of cement, fine and coarse aggregate instead of designed concrete mix. The recommendation given on page (1.3.1) may be helpful in improving the concrete quality.

The values of Kutosis and Skewness coefficients strongly suggest normal distribution* for the data. By method of moment it has been found that the parameters of normal distribution are the mean and standard deviation of the population. The estimated values of mean and standard deviation of the sample is used to represent the population parameters. A computer program, TESKS, shown in appendix is used to carry out K–S test. The result shows that at 0.05 level of significant normal distribution could not be rejected.

*The Kutosis and Skewness coefficients for normal distribution are 3 and 0 respectively.

The K-S test is usually conducted on one-by-one bases. However due to lack of space another computer program ANORMJ, (see the appendix) is used to summarize the result in class interials. The result is shown on table (3.1.2) Actual and theoretical frequency curves are shown on figures (3.1.1 and 3.1.2).

The parameters of Beta distributions are also estimated by the method of matching moments. This method leads to the following equations:

$$q = \frac{1-\bar{x}}{s^2}\left[\bar{x}\,(1-\bar{x}) - s^2\right] \quad \ldots\ldots\ldots 3.1.1$$

$$r = \frac{\bar{x}\,q}{1-\bar{x}} \quad \ldots\ldots\ldots\ldots\ldots 3.1.2$$

where $\bar{x}$ and s^2 are detained from equation (1.4.13) and (1.4.14).

The estimation of Beta distribution parameters involves iteration. The lower and upper limits are fixed and the other parameters are estimated A number of trials were made, but the K-S test would not accept the distribution at 0.05 level of significant.

3.2 DISTRIBUTION OF STEEL YIELD STRENGTH

As with the case of concrete compressive strength data, the data for tensile strength was

Table (3.1.2)

Frequency Table For Compressive Strength of Concrete Cube Crushed at 28 days.

Characteristic Strength = $20N/mm^2$.

Class Interval	Class Frequency		Cummulative Relative Frequency		
	Actual	Theoretical	Actual	Theoretical	Difference
10 - 13	7	3.84	0.0245	0.0135	0.0111
13 - 16	13	6.61	0.0702	0.0367	0.0335
16 - 19	12	10.51	0.1125	0.0735	0.0388
19 - 22	11	15.42	0.1519	0.1276	0.0233
22 - 25	22	20.88	0.2281	0.2009	0.0272
25 - 28	19	26.11	0.2947	0.2925	0.0022
28 - 31	22	30.12	0.3719	0.3982	0.0263
31 - 34	37	32.08	0.5018	0.5108	0.0090
34 - 37	33	31.54	0.6175	0.6214	0.0039
37 - 40	39	28.61	0.7544	0.7218	0.0326
40 - 43	21	23.96	0.8281	0.8058	0.0222
43 - 46	22	18.52	0.9053	0.8709	0.0344
46 - 49	7	13.21	0.9298	0.9122	0.0126
49 - 52	8	8.70	0.9579	0.9477	0.0102
52 - 55	5	5.29	0.9754	0.9663	0.0092
55 - 58	3	2.96	0.9860	0.9767	0.0093
58 - 61	3	1.54	0.9965	0.9821	0.0144
61 - 64	1	0.73	1.0000	0.9846	0.0154

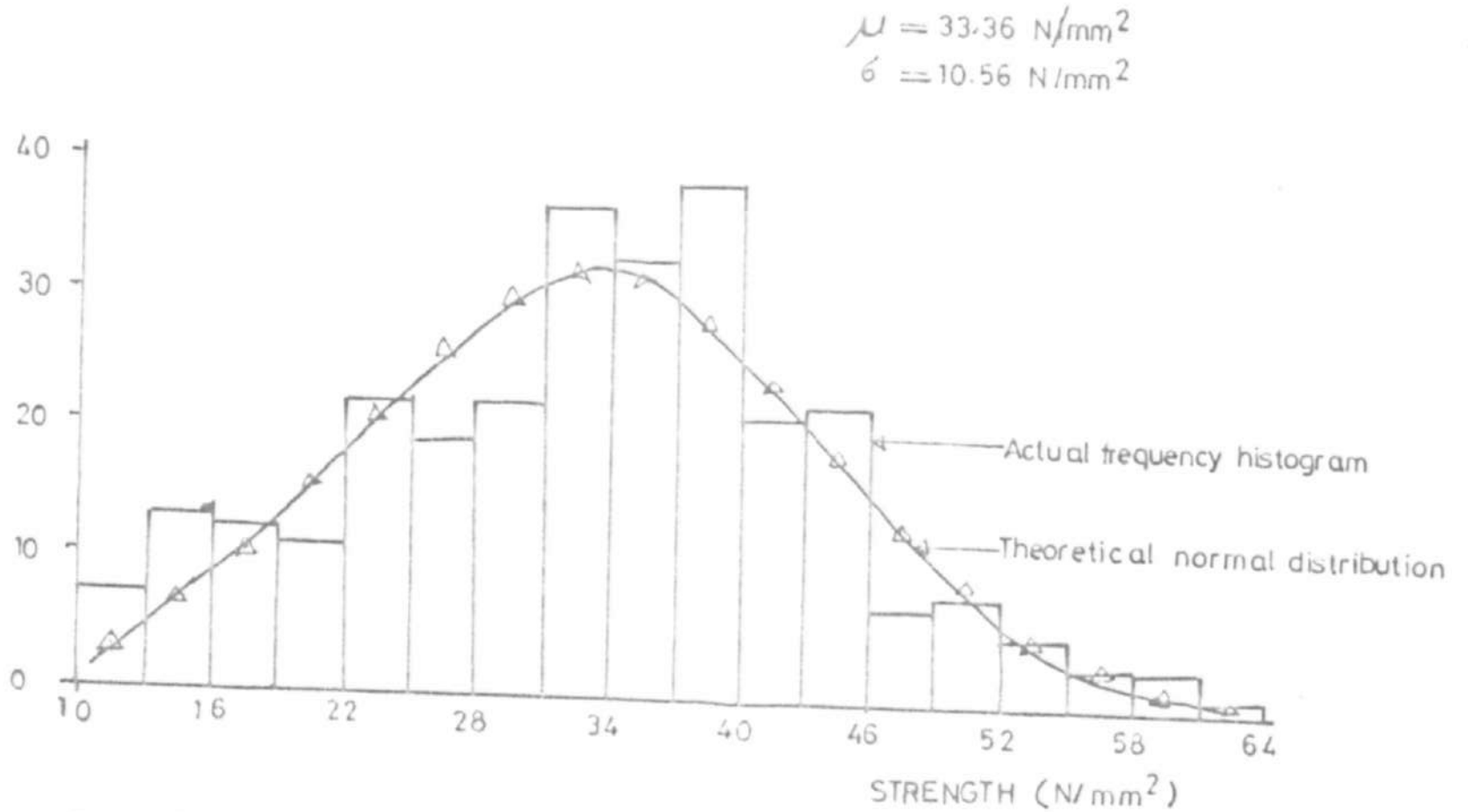


FIGURE (3.1.1) : Distribution of Concrete Compressive Strength After 28 Days

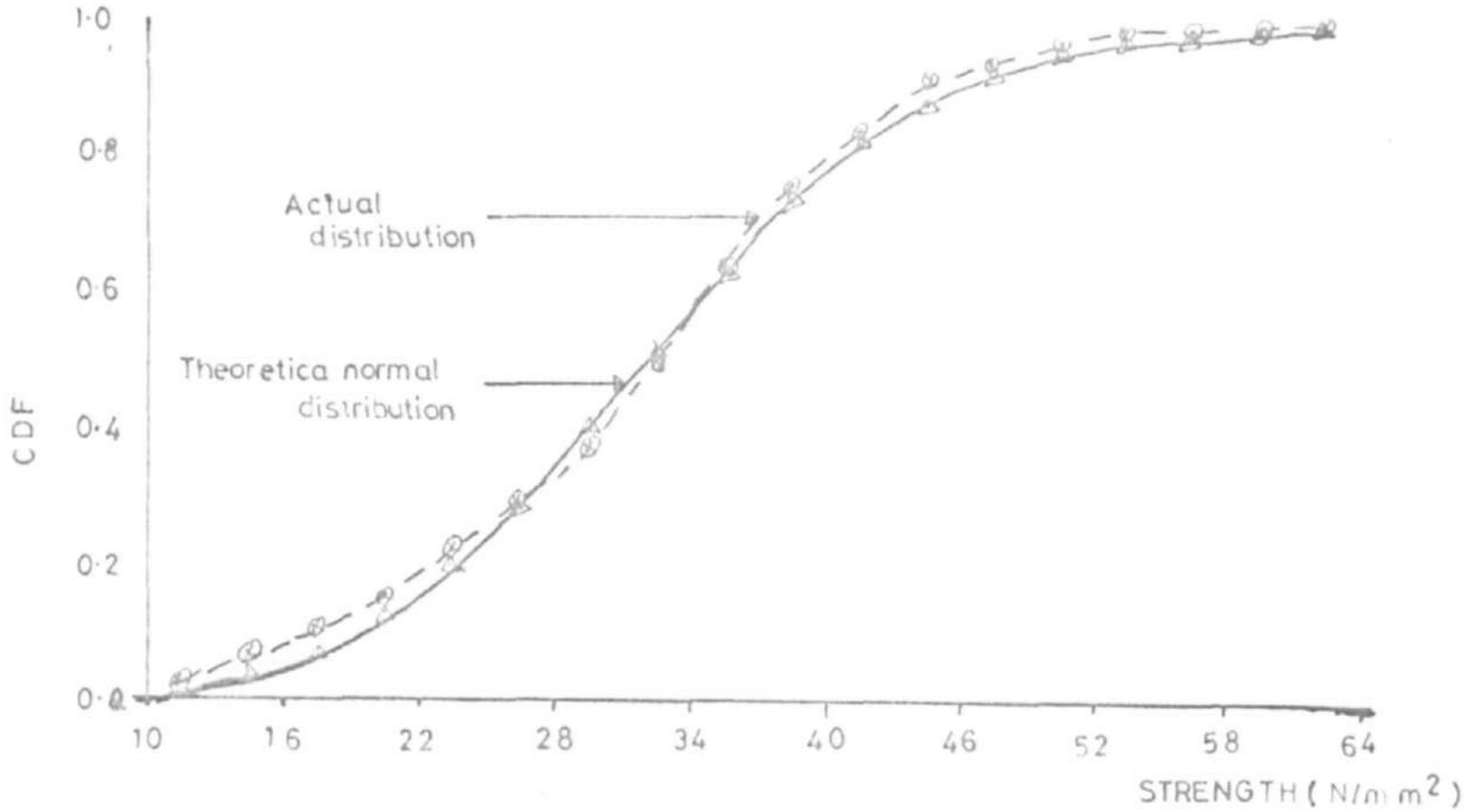


FIGURE (3·1·2): Cummulative Distribution of concre compressive strength after 28 days

obtained from test result carried out over many years in the Department of Civil Engineering, Ahmadu Bello University, Zaria. The site and the type of building in which the reinforcement was used is not known. However, it is assumed that the reinforcement is obtained from a common market. Thus the overall result is used in the analysis.

The type of steel used in the analysis is hot-rolled high yield steel characteristic strength of 410 N/mm^2. The tests were conducted according to BS 18, "Methods for tensile testing of metals".

The specification for hot-rolled steel bar of reinforcement given in BS4449 is that not less than ten different bars of one size and type should be tested; if any one tensile test gives a result less than 0.93 times the specified characteristic strength that bar should be rejected and if any other test gives a result less than 0.93 times the pecified characteristic strength then the whole delivery shall be rejected.

It has been observed that the sampling was conducted on site by the contractors. Only the test piecies were brought to the department for testing. The sampling procedure of some tests have not met with BS4449 specification. Such results are not

included in this analysis. However, all the tests that conform with the sampling procedure have been included irrespective of their failure to meet with strength specification.

Table (3.2.2) shows the actual class frequency andthe histogram is shown in figure (3.2-1). The mean value is obtained to be 449.44 MN/m^2 while standard deviation is found as 59.71 MN/m^2 with coefficient of variation being 0.13. The large coefficient of variation may be due to different practice in the manufacture of steel. The recommended overall coefficient of variation is 4 - 6% (26).

The Kutosis and Skewness coefficient are calculated to be 3.05 and 0.13 respectively. These values strongly suggest normal distribution for the data. The theoretical normal frequency is shown in table (3.2.2). The probability density frequency curve and the distribution curve are shown in figure (3.2.1) and (3.2.2). The result for the K-S test is summarized in table (3.2.2). The test was conducted at 0.05 level of significance. The result suggests normal distribution for the steel yield strength.

TABLE (3.2.1)

YIELD STRENGTH OF STEEL
RESULTS OF QUALITY CONTROL TEST ON LIVE PROJECTS
CONDUCTED OVER MANY YEARS BY THE
DEPARTMENT OF CIVIL ENGINEERING
AHMADU BELLO UNIVERSITY, ZARIA

S/No.	STRENGTH (N/mm^2)									
1,	530.60	561.80	553.80	521.90	504.60	537.80	550.90	532.70	533.90	536.10
2.	528.36	535.97	517.25	539.73	522.45	498.05	539.74	535.66	545.85	527.87
3.	416.00	411.77	415.53	435.65	441.36	429.95	418.28	424.40	417.50	417.64
4.	314.63	314.52	330.46	302.23	303.64	295.50	324.52	299.19	319.19	297.96
5.	439.75	441.76	462.50	433.41	441.76	451.22	437.65	430.29	442.64	438.38
6.	457.58	475.14	473.37	473.16	468.38	478.26	475.14	436.00	476.19	436.57
7.	446.90	475.90	464.70	459.90	456.60	459.80	470.70	479.10	464.40	470.70
8.	470.24	483.87	481.25	493.55	500.00	586.58	581.63	509.87	513.79	480.89
9.	504.76	519.05	514.29	528.57	524.51	514.29	530.00	514.28	524.51	504.67
10.	417.73	390.24	394.45	408.76	394.16	403.89	409.76	407.32	406.30	416.67
11.	380.60	395.00	395.20	401.40	396.60	399.90	397.40	394.20	391.40	391.40
12.	381.81	393.95	392.07	380.00	393.16	379.33	390.88	375.00	375.00	303.92
13.	434.21	472.73	481.13	446.43	460.53	442.98	497.61	472.22	450.00	464.99
14.	400.00	408.40	408.40	408.40	405.04	408.40	406.72	406.40	394.96	406.72
15.	418.18	422.73	424.96	418.18	422.73	422.73	413.64	414.47	406.87	400.00
16.	482.99	481.31	506.38	479.40	444.22	451.81	469.38	472.69	472.72	470.34
17.	478.00	438.75	475.30	450.36	484.21	478.58	474.40	470.00	470.00	420.00
18.	500.00	472.00	496.00	500.00	498.00	500.00	500.00	500.00	500.00	480.70

Table (3.2.2)

Frequency Table For Tensile Strength
High Yield Reinforcing SteelCCharacteristic
St·ength = 410 N/mm^2

Class Interval	Class Frequency		Cummulative Relative Frequency		
	Actual	Theore-tical	Actual	Theoretical	Difference
290-310	6	1.07	0.0333	0.0060	0.0274
310-330	4	2.33	0.0556	0.0189	0.0366
330-350	1	4.53	0.0611	0.0441	0.0170
350-370	0	7.88	0.0611	0.0879	0.0268
370-390	6	12.25	0.0944	0.1559	0.0615*
390-410	33	17.05	0.2778	0.2507	0.0271
410-430	19	21.23	0.3833	0.3686	0.0148
430-450	19	23.65	0.4889	0.5000	0.0111
450-470	16	23.57	0.5778	0.6309	0.0531
470-490	30	21.03	0.7444	0.7477	0.0033
490-510	17	16.78	0.8389	0.8410	0.0021
510-530	14	11.99	0.9167	0.9076	0.0091
530-550	10	7.66	0.9722	0.9501	0.0221
550-570	3	4.38	0.9889	0.9745	0.0144
570-590	2	2.24	1,0000	0.9869	0.0131

$\mu = 449.44$ N/mm^2

$\sigma' = 59.71$ N/mm^2

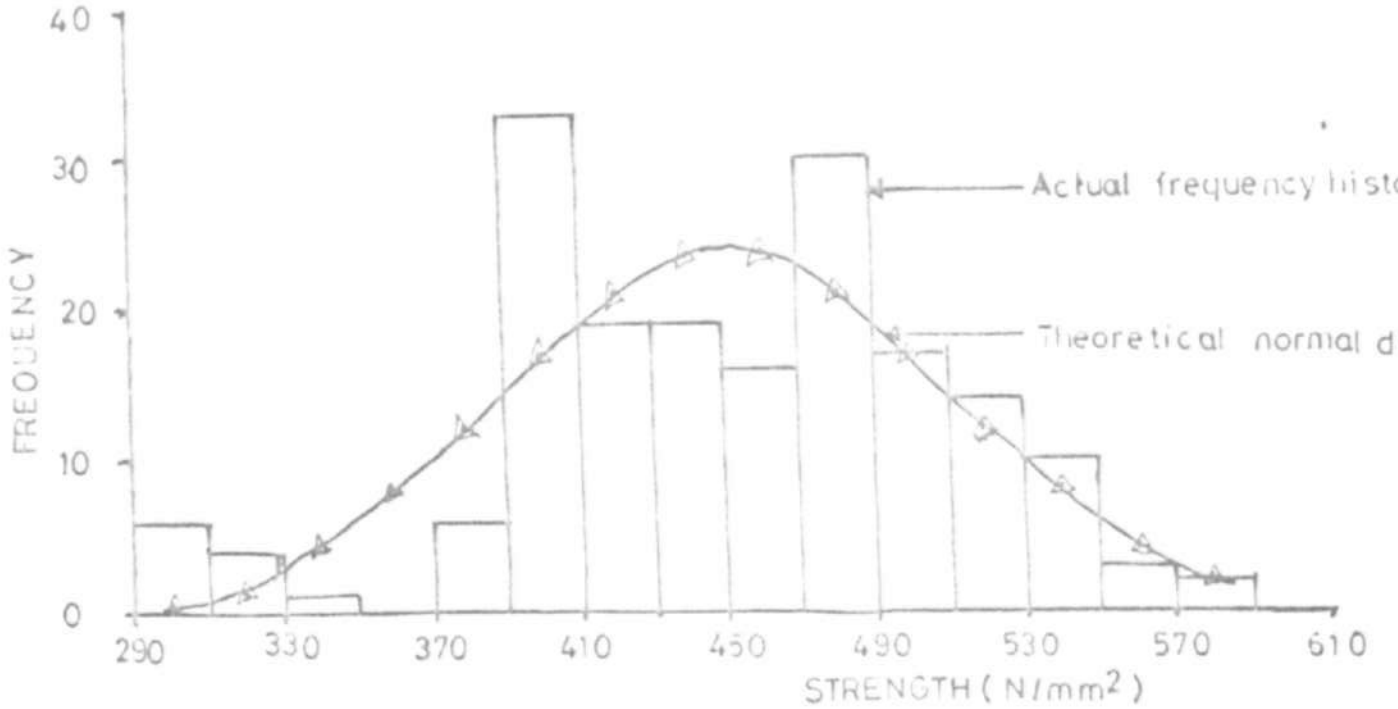


FIGURE (3·31): Distribution of steel yield strength; Characteristic strength = 410 N/mm^2

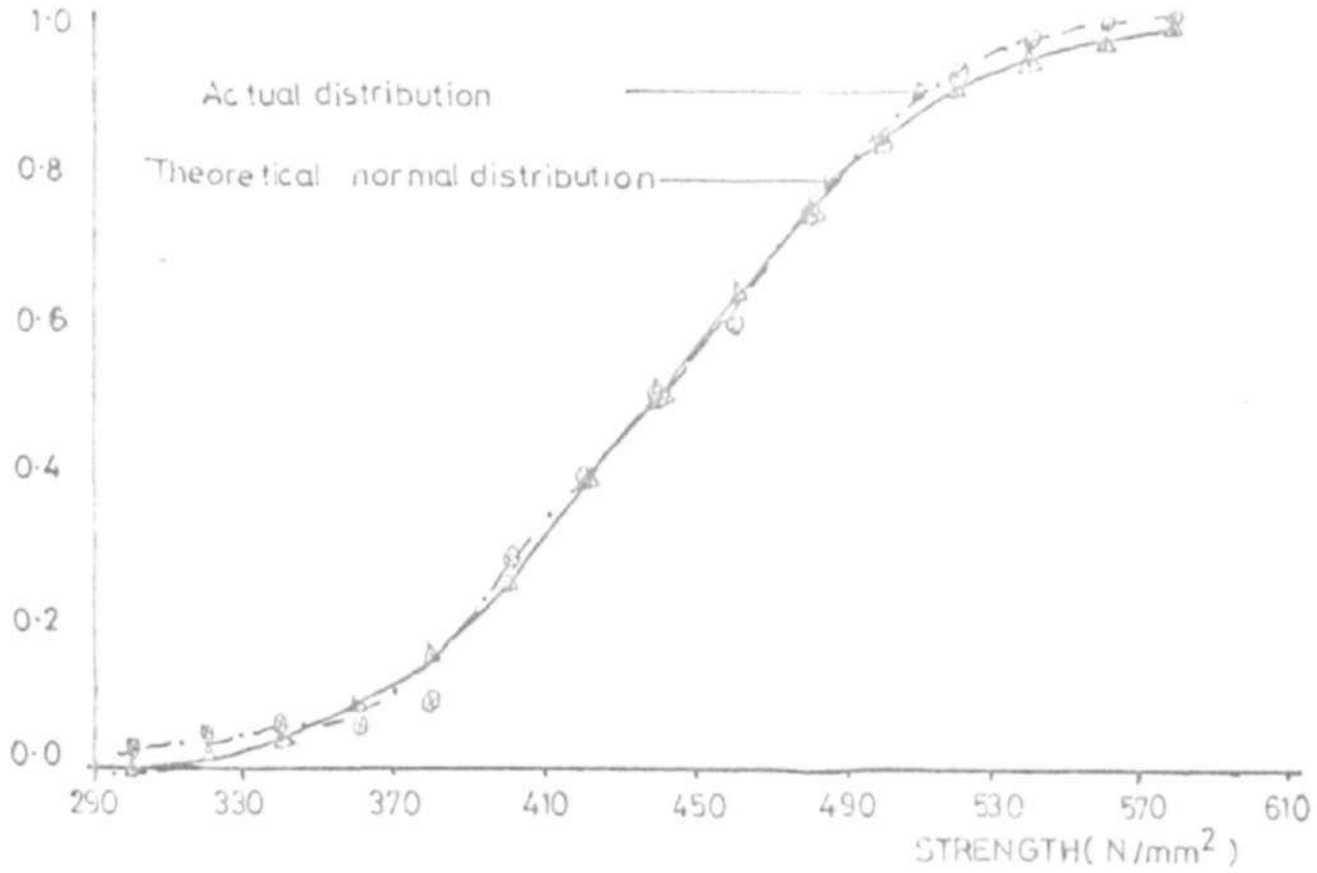


FIGURE (3·2·2) Cummulative distribution of steel yield strength
Characteristic strength =410 N/mm^2

CHAPTER FOUR

4. PROBABILITY DISTRIBUTION OF MEMBER STRENGTH

4.1 INTRODUCTION

In order to assess the effects of variability in steel and concrete strength on the ultimate strength in a concrete member, a deterministic functional relationship between the resistance of the section and strength of the components (steel and concrete) will be assumed. These deterministic models form the bases for the study of the influence of the observed variation in components strengths on the resistance of singly and doubly reinforced beams as well as the axial compressive resistance of columns. Some simplifying assumptions about the stress-strain behaviour of concrete, made in CP110 have also been adopted in this study.

The assumptions made by CP110 in deriving the general theory for ultimate flexural strengths are as follows:

(i) the strains in the concrete and the reinforcing steel are directly proportional to the distances from the

neutral axis, at which the strain is zero.

(ii) the ultimate limit state of collapse is reached when the concrete strain at the extreme compression fibre reaches a specified value ε_{cu}

(iii) at failure, the distribution of concrete compressive stresses is defined by the idealized stress-strain curve shown in figure (4.1.)

(iv) the tensile strength of concrete is ignored

(v) the stresses is the reinforcement are derived from the appropriate stress-strain curve, figure (4.1.2)

Based on the listed assumptions the strain and stress distributions at failure are as illustrated in figure (4.1.3). The figure shows a beam cross-section having an area A_s of longitudinal tension reinforcement and an area A_s^1 of longitudinal compression reinforcement; the distance d from the top face to the centroid of the tension reinforcement is called the effective dapth of the beam; x denotes the neutral axis depth of the beam;

The stress distribution diagram, figure (4.1.3c) is generally referred to as the stress block. The relevant characteristics of stress block are the ratio k_1 of the average compressive stress to the characteristic concrete strength f_{cu} and the ratio k_2 of the depth of the centroid of the stress block to the neutral axis depth.

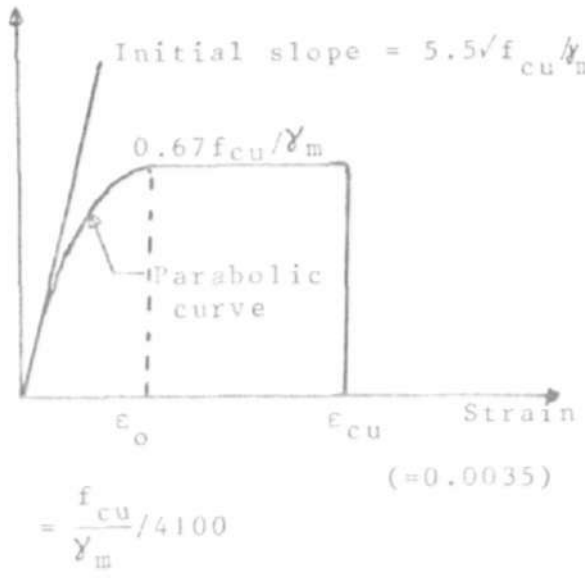


Fig (4.1.1) Stress/strain curves for concrete in compression (CP110)

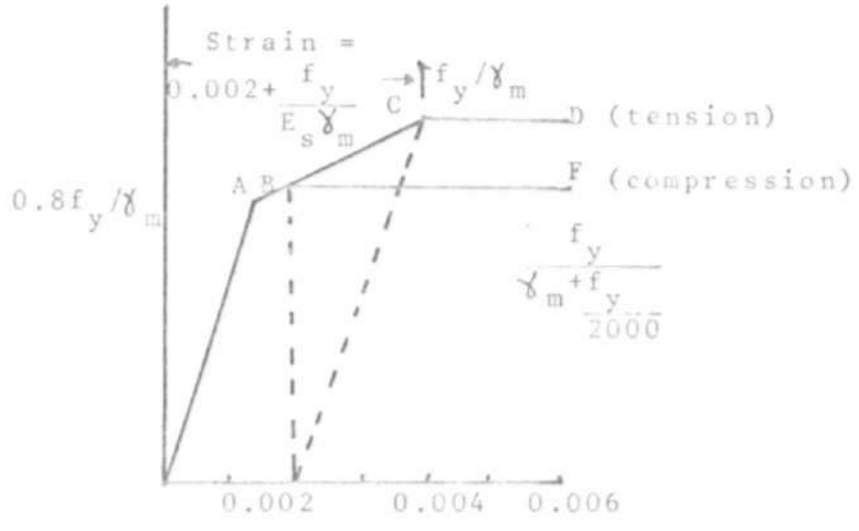


Fig (4.1.2) Stress/strain curve for reinforcement (CP110)

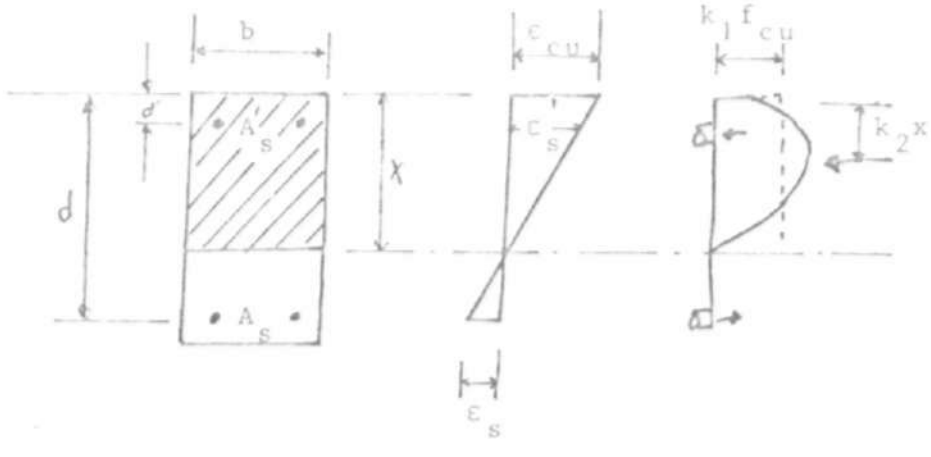
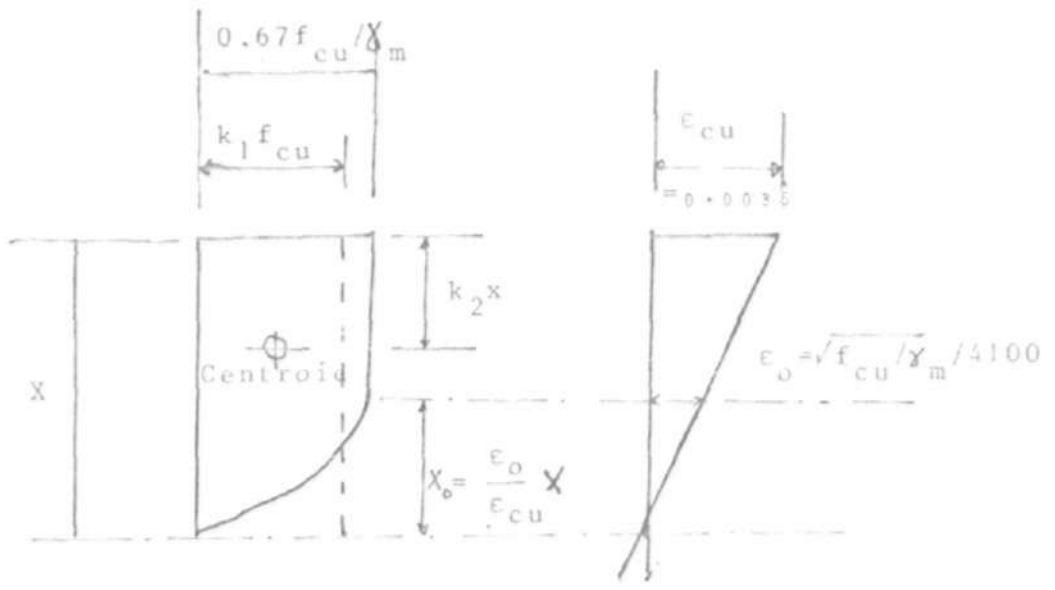


Fig (4.1.3) Strain and stress distribution at failure



Stress block (a)

Strain distribution (b)

Fig (4.1.4) Design stress block for ultimate limit state (CP110)

CP110 (11) adopts an idealized stress block as shown in Fig (4.1.4). It is assumed that ultimate concrete strain is constant at ε_{cu}=0.0035 and thatthe parabolic part of the stress block ends at a strain $\varepsilon_o = f_{cu}/5000$. A partial safety factor of 1.5 is also assumed for concrete.

From Fig (4.1.4) it can be found that

$$k_1 = 0.45\left[1 - \sqrt{f_{cu}}/52.5\right] \quad \ldots\ldots 4.1.1$$

$$k_2 = \frac{\left[2 - \frac{\sqrt{\bar{f}_{cu}}}{17.5}\right]^2 + 2}{4\left[3 - \frac{\sqrt{\bar{f}_{cu}}}{17.5}\right]} \quad \ldots\ldots\ldots\ldots 4.1.2$$

In the present work the partial safety factor is not included in the formulation of the models. Thus k_1 and k_2 become

$$k_1 = 0.67\ (1 - \sqrt{\bar{f}_{cu}}/43.05) \quad \ldots\ldots 4.1.3$$

$$k_2 = \frac{(2.0 - \sqrt{\bar{f}_{cu}}/14.35)^2 + 2.0}{4X(3 - \sqrt{f_{cu}}/14.35)} \quad \ldots\ldots 4.1.4$$

4.2 SINGLY REINFORCED BEAMS

4.2.1 Strength Model for Singly Reinforced Concrete Beams

The deterministic theory of reinforced concrete beam distingushes between tensile and compressive failures. The criterion used is based on whether or not the tensile steel has yielded before the concrete in the outmost compressive fiber reaches its ultimate strain value. The condition under which both events occur simultaneously is referred to as one of balanced design.

Applying the equilibrium condition in figure (4.1.4), then:

$$k_1 f_{cu} bx = A_s f_s - A_s^1 f_s^1 \quad \ldots\ldots 4.2.1$$

where f_s and f_s^1 are tensile stress and compressive stress respectively.

For singly reinforced beam, there is no compression reinforcement. Therefore, equation (4.2.1) becomes

$$k_1 f_{cu} bx = A_s f_s \quad \ldots\ldots\ldots 4.2.2$$

From figure (4.3b),

$$x = \frac{\varepsilon_{cu}}{\varepsilon_{cu}+\varepsilon_s} d \quad \ldots\ldots\ldots 4.2.3$$

Substitueting x from equation (4.2.3) in equation (4.2.2), it follows that

$$k_1 f_{cu} bd \frac{\varepsilon_{cu}}{\varepsilon_{cu}+\varepsilon_s} = A_s f_s \quad \ldots\ldots 4.2.4$$

$$\rightarrow \frac{A_s}{bd} = \frac{k_1 f_{cu}}{f_s} \cdot \frac{\varepsilon_{cu}}{\varepsilon_{cu}+\varepsilon_s} \quad \ldots\ldots\ldots 4.2.5$$

$\rho = \frac{A_s}{bd}$ is called steel ratio

therefore,

$$= \frac{k_1 f_{cu}}{f_s} \cdot \frac{\varepsilon_{cu}}{\varepsilon_{cu}+\varepsilon_s} \quad \ldots\ldots 4.2.6$$

A section is said to be balanced if the concrete reaches ε_{cu} simultaneously as the steel strain reaches ε_y (yield strain). Thus balance reinforcement can be given as

$$\rho_b = \frac{k_1 f_{cu}}{f_y} \cdot \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_y} \quad \ldots\ldots 4.2.7$$

where

f_y = the yield stress

ρ_b = the balanced reinforcement

$$\varepsilon_y = 0.002 + \frac{f_y}{E_s} \quad \ldots\ldots\ldots 4.2.8$$

E_s = Young Modulus of steel = 2100 N/mm^2

ε_{cu} = 0.0035

Thus equation (4.2.7) can be written

$$\rho_b = \frac{k_1 f_{cu}}{f_y} \left[\frac{\varepsilon_{cu} E_s}{(0.002+\varepsilon_{cu})E_s + f_y} \right]$$

$$= \frac{k_1 f_{cu}}{f_y} \left[\frac{0.0035 \times 2100}{0.0037 \times 2100 + f_y} \right] \ldots 4.2.9$$

If $\rho < \rho_b$, failure of the section will be due to yielding of the reinforcement. For $\rho > \rho_b$ failure will be brought about by compressive failure of the concrete. The crushing of concrete gives a sudden failure, without warning. With under-reinforced section, the yielding of reinforcement at failure gives large deflection than compression failure. Thus there is practical interest in assuming a tensile failure.

For the purpose of this study, all the specified steel ratio are less than or equal to balance reinforcement.

The resistance in a tensile failure mode is given by

$$M_u = A_s f_y (1 - \rho \frac{k_2 f_y}{k_1 f_{cu}}) d \quad \ldots\ldots\ldots 4.2.10$$

Dividing by bd^2,

$$\frac{M_u}{bd^2} = \frac{A_s}{bd} f_y \left(1 - \rho \frac{k_2 f_y}{k_1 f_{cu}}\right) \quad \ldots\ldots\ldots 4.2.11$$

Denoting $\frac{M_u}{bd^2}$ by R_{st}

and $\frac{A_s}{bd}$ by ρ = steel ratio ,

equation (4.1.11) becomes

$$R_{st} = \rho f_y \left(1 - \rho \frac{k_2 f_y}{k_1 f_{cu}}\right)$$

$$= \phi_s (f_{cu}, f_y) \quad \ldots\ldots\ldots 4.2.12$$

where

ϕ_s denotes "function of"

For a specified cvalue of $\rho < \rho_b$ the distribution of R_{st} can be obtained from that of f_y and f_{cu} using Monte Carlo Method. From the distribution of R_{st}, the probability of having resistance less than a specified value R_{sp} (i.e. $P_r(R_{st} < R_{sp})$ can be obtained.

4.2.2 Simulation

The nominal values of steel strength and concrete strength used in this study are 410 MN/m^2 and 20 MN/m^2 respectively. Substituting these values into equation (4.2.9) and equation (4.1.1), the balanced reinforcement is given as

$$\rho_b = 0.014$$

CP110 recommends minimum reinforcement for singly reinforced concrete as $\rho = 0.0015$. The specified steel ratio used in this study are 0.0025, 0.004, 0.006, 0.008, 0.01, 0.012.

The method of distribution comparison (see section (1.5.3(iii)can be used to determine the required sample size for the simulation. At 5% level of significance,

$$(d_n)_\alpha = \frac{1.36}{\sqrt{n}} = \epsilon$$

$$\text{Thus } n = \frac{(1.36)^2}{(\epsilon)^2}$$

Assuming $\epsilon = 0.05$, it follows that

$$n = (\frac{1.36}{0.05})^2 = 740.$$

A sample size of 800 will be simulated.

The computer program, RESISTJ, (appendix)

is used to simulate the ultimate moment of resistane for the singly and doubly reinforced concrete beams. The program contains a subroutine, NORMAL, which generates two normal random variates at a time.

4.2.3 Analysis of Results

In order to reduce the volume of work only the results for beam with steel ratio of 0.010 are shown in table (4.2.1). However, the analysis was carried out on the results of all the beams. The choice ofsteel ratio 0.010 is arbitrary. Figure (4.2.1) shows the histogram of thesimulated values, while the cummulative relative frequence curve is shown in figure (4.2.2).

The mean values and standard deviations are shown in table (4.2.2). The same table also contains the coefficients of variation. The coefficient of variation varies from 13% to 14% This is in fair agreement with recommended value of 12% (27).

Since the steel and concrete strengths are fitted with normal distribution, the resulting member strength is expected to have normal distribution. Figure (4.2.1) and (4.2.2) shows good agreement between the theoretical normal distribution and distribution of the simulated values. A computer programe TESTKS (see appendix) is used to conduct K-S test on one-by-one bases. The result shows that, at 0.05 level

Table (4.2.1)

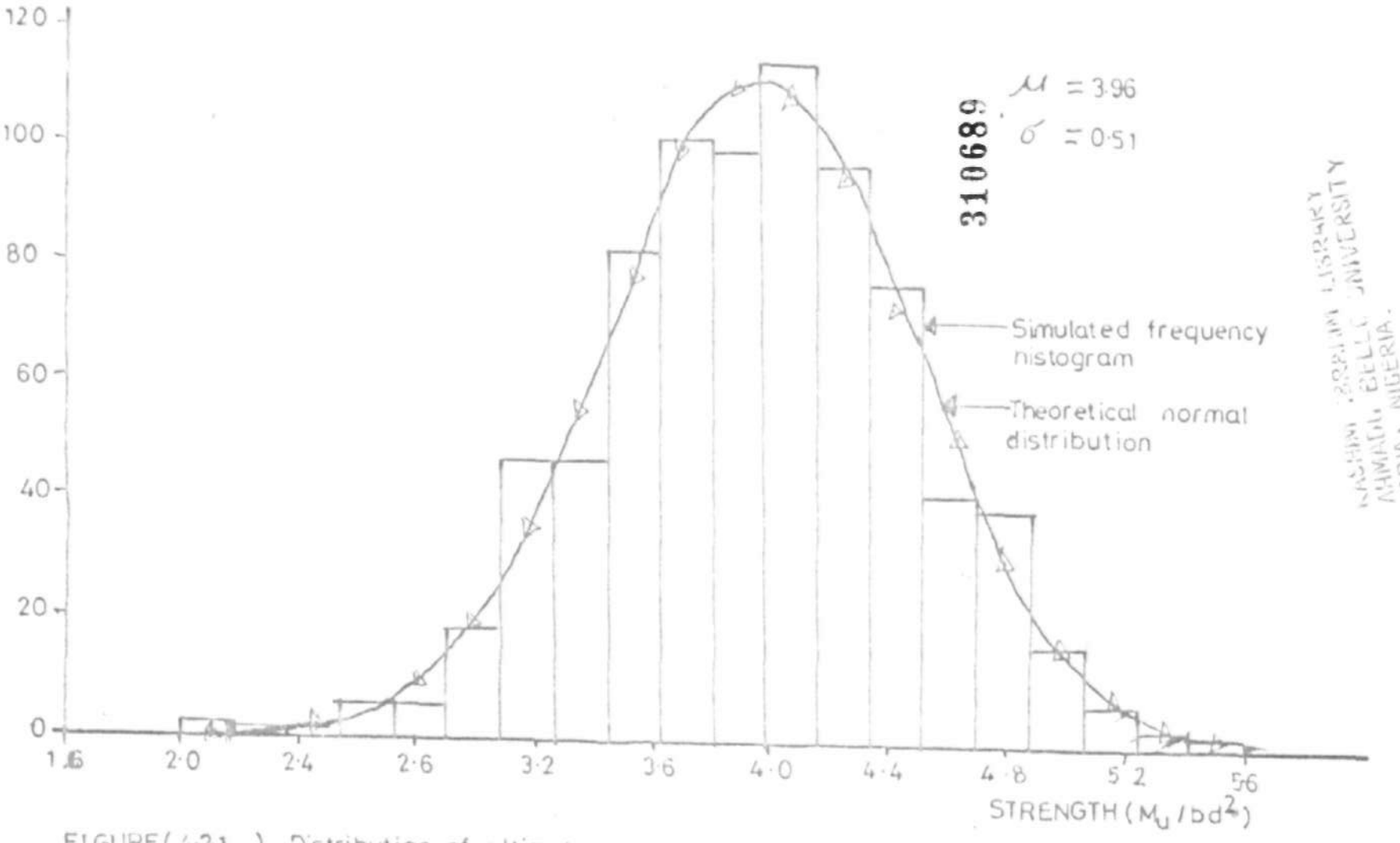
Frequency Table For Strength

of Singly Reinforced Beam

F_y = 410 Nmm^2; F_{cu} = 20 N/mm^2

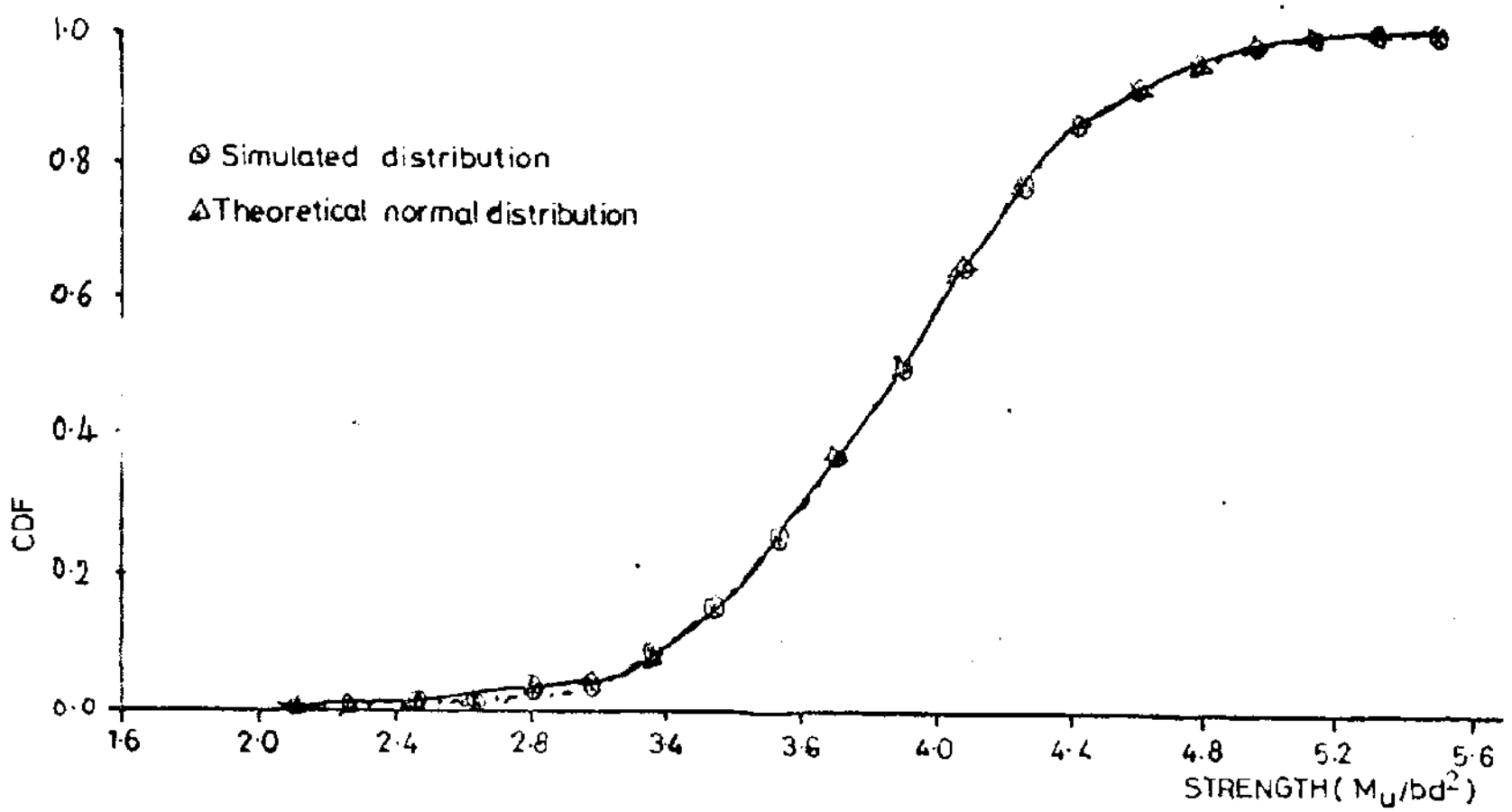
ρ = 0.010

Class Interval	Class Frequency		Cummulative Relative Frequency		
	Actual	Theoretical	Actual	Theoretical	Difference
2.000 - 2.180	2	0.16	0.003	0.000	0.003
2.180 - 2.360	1	0.53	0.004	0.001	0.003
2.360 - 2.540	1	1.56	0.005	0.003	0.002
2.540 - 2.720	5	4.06	0.011	0.008	0.003
2.720 - 2.900	5	9.37	0.018	0.020	0.002
2.900 - 3.080	18	19.14	0.040	0.044	0.004
3.080 - 3.260	46	34.64	0.098	0.087	0.011
3.260 - 3.440	46	55.49	0.155	0.156	0.001
3.440 - 3.620	82	78.74	0.258	0.255	0.003
3.620 - 3.810	101	98.95	0.384	0.379	0.005
3.810 - 3.980	99	110.13	0.508	0.517	0.009
3.980 - 4.160	114	108.54	0.651	0.653	0.002
4.160 - 4.340	97	94.74	0.772	0.771	0.001
4.340 - 4.52	77	73.23	0.869	0.863	0.006
4.520 - 4.710	41	50.13	0.920	0.925	0.005
4.710 - 4.880	39	30.39	0.969	0.964	0.005
4.880 - 5.060	16	16.32	0.989	0.984	0.005
5.060 - 5.24	6	7.76	0.996	0.994	0.002
5.240 - 5.420	2	3.27	0.999	0.998	0.001
5.420 - 5.600	1	1.22	1.000	0.999	0.001



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FIGURE(4·2·1) Distribution of ultimate moment of resistance;
Singly reinforced concrete beam;
Steel ratio = 0·01



FIGURE(4·2·2): Cummulativite distribution of ultimate moment of resistance;
Singly reinforced concrete beam;
Steel ratio=0·01

significant, normal distribution could not be rejected. Due to lack of space another computer programm, ANORMJ(appendix ( )) is used conduct K-S test on class intervals. The result is shown in table (4.2.1).

To obtain the probability of failure, nominal values of f_y and f_{cu} are substituted in the equation (4.2.12). For a particular steel ratio the probability of failure of ultimate moment of resistance to reach this value is obtained The results are tabulated in table (4.2.2). The result shows that the probability of s failure is well above that specified by CP110 (i.e. 0.05). Introducing partial safety factors of 1.5 for concrete and 1.15 for steel as stipulated by CP110 reduces the probability of failure to be less than 0.05. The American Code ACI (1) recommends that nominal moment be multiplied by 0.9 for design. Introducing this factor gives the results shown in table (4.2.2). The probability of failure of safety of 0.60 has practically reduces the failure of probability to almost zero.

Table (4.2.2)

Failure Probabilities for
Singly Reinforced Concrete Beam

F_y = 410 N/mm^2; F_{cu} = 20N/mm^2

CP110 Partial safety factors; 1.15 for steel
1.50 for Contrete

ACI Safely factor = 0.9

Steel Ratio	Mean Value	Standard Deviation	Coefficient of variation	Failure without	Probabilities With safely Factor		
				Safely factor	CP110	ACI	0.60
0.0025	1.0871	0.1427	0.1313	0.24	0.05	0.08	0.0003
0.0040	1.7084	0.2213	0.1295	0.22	0.04	0.07	0.0000
0.0060	2.5002	0.3208	0.1283	0.20	0.03	0.06	0.0000
0.0080	3.2501	0.4172	0.1284	0.18	0.02	0.06	0.0000
0.0100	3.9585	0.5138	0.1298	0.17	0.02	0.05	0.0000
0.0120	4.6249	0.6144	0.1328	0.16	0.01	0.05	0.0000
0.0140	5.2502	0.7227	0.1377	0.15	0.01	0.05	0.0000

4.3 DOUBLY REINFORCED BEAMS

4.3.1 Compressive Yield and Balanced Reinforcement of Doubly Reinforced Beams

It is desired that a doubly reinforced section fails in tension with yield in the compressive reinforcement. Ensuring this type of failure requires restrictions on the renforcement ratio.

From strain diagram of figure (4.1.3b) strain at compressive reinforcement is given by

$$\varepsilon_s^1 = 0.0035 \left(\frac{x-d^1}{x}\right) \quad \ldots\ldots\ldots 4.3.1$$

For compressive reinforcement to yield it can be seen from figure (4.1.1) that

$$\varepsilon_s^1 \geq 0.002 \quad \ldots\ldots\ldots\ldots 4.3.2$$

therefore,

$$0.0035 \left(\frac{x-d^1}{x}\right) \geq 0.002 \quad \ldots\ldots\ldots 4.3.3$$

It follows

$$x \geq \frac{0.0035d^1}{0.0035-0.002} = 2.33d^1 \quad \ldots 4.3.4$$

Referring to fig (4.1.3c), the equilibrium equation for the internal forces can be written

$$k_1 f_{cu} bx = (A_s - A_s^1) fy \quad \ldots\ldots\ldots 4.3.5$$

assuming that both tension and compressive reinforcements yield

Thus, from equation (4.3.5)

$$x = \frac{(A-A_s^1)f_s}{k_1 f_{cu} b} y = \frac{(\rho-\rho^1)}{k_1 f_{cu}} f_y d \quad \ldots\ldots 4.3.6$$

where $\rho = \frac{A}{bd}$ and $\rho^1 = \frac{A}{bd}$

Combining equation (4.3.4) and equation (4.3.5) it follows that

$$\frac{(\rho-\rho^1)}{k_1 f_{cu}} f_y d \geq 2.33 d^1 \quad \ldots\ldots\ldots 4.3.$$

This gives

$$\rho - \rho^1 \quad 2.33 \frac{k_1 f_{cu}}{f_y} \delta = \rho_y \quad \ldots\ldots\ldots 4.3.8$$

where $\delta = \frac{d^1}{d}$

The restriction to tensile mode of failure can be derived as follows:

From strain diagram of figure (4.1.3b) tensile yield strain isobtained as

$$\varepsilon_s = 0.0035 \left(\frac{d-x}{x}\right) \quad \ldots\ldots\ldots 4.3.9$$

From strain-stress diagram of steel the tensile steel yield if strain is

$$\varepsilon_s \geq 0.002 + \frac{f_y}{E_s} \quad \ldots\ldots\ldots 4.3.10$$

This gives

$$X \leq \frac{0.0035E_s d}{0.0037E_s + f_y} \quad \ldots\ldots 4.3.11$$

Relating equation (4.3.6) and equation (4.3.11) follows that

$$(\rho - \rho^1) \frac{f_y d}{k_1 f_{cu}} \leq \frac{0.0035E_s d}{0.0037E_s + f_y} \quad \ldots 4.3.12$$

This gives

$$(\rho - \rho^1) \leq \frac{k_1 f_{cu}}{f_y} \left(\frac{0.0035E_s}{0.0037E_s + f_y}\right) = b \quad \ldots 4.3.13$$

Therefore,,the restrictions on the reinforcement ratio to ensure tension failure with yield in compressive reinforcement can generally be written as

$$2.33\ \frac{k_1 f_{cu}}{f_y}\ \delta \leq (\rho - \rho^1) \leq \frac{k_1 f_{cu}}{f_y}\ \left(\frac{0.0035E_s}{0.0037E_s + f_y}\right)$$

..... 4.3.14

4.3.2 Strength Model for Doubly Reinforced Beams

The ultimate resistance of doubly reinforced beam is given from figure (4.144) by

$$M_u = A_s f_s (d - k_2 x) + A_s^1 f_s^1 (k_2 x - d^1) \quad .... \ 4.3.15$$

If both tension steel and compressive steel yield,

$$f_s = f_s^1 = f_y$$

Thus the ultimate resisting moment of a doubly reinforced section failing in tension with yeild in the compressive reinforcement can be given aa

$$M_u = f_y (A_s (d - k_2 x) + A_s^1 (k_2 x - d^1)) .... \ 4.3.16$$

Dividing equation (4.3.16) by bd^2

$$\frac{M_u}{bd^2} = f_y \left(\rho \left(1 - k_2 \frac{x}{d}\right) + \rho^1 \left(k_2 \frac{x}{d} - \varepsilon\right)\right) .. \ 4.3.17$$

where

$$\rho = \frac{A_s}{bd}, \quad \rho^1 = \frac{A'_s}{bd} \text{ and } \sigma = \frac{d^1}{d}$$

It follow that

$$\frac{M_u}{bd^2} = f_y\left[\rho - \rho^1\delta - (\rho - \rho^1)k_2\frac{x}{2d}\right] \quad \ldots\ldots 4.3.18$$

Substituting for $X = (\rho - \rho^1)\frac{f_y d}{k_1 f_{cu}}$

From equation (4.3.6)

$$\frac{M_u}{bd^2} = f_y\ \rho - \rho^1\ \delta - (\rho - \rho^1)^2\ \frac{k_2 f_y}{k_1 f_{cu}} \quad \ldots\ 4.3.19$$

Denoting $\frac{M_u}{bd^2}$ by R_d, $\rho - \rho^1\delta = \lambda_1$ and $(\rho - \rho^1)^2$

$$= \lambda_2,$$

it follows that

$$R_d = \lambda_1 f_y - \lambda_2\ \frac{k_2 f_y}{k_1 f_{cu}}$$

$$= \emptyset_d\ (f_y,\ f_{cu}) \quad \ldots\ldots 4.3.20$$

where $\emptyset_d$ is the functional relationship.
Three parameters, namely, ρ, ρ^1, and δ need to be specified to obtain the distribution of the resistance.

4.3.3 Simulation

It is assumed that the type of materials used and workmanship are the same as for singly reinforcement ratio is the same with that of singly reinforced beam. Thus

$$\rho_b = 0.014$$

To obtain the value of ρ_y, nominal values of f_y and f_{cu} are substituted in the equation (4.3.8). The values of ρ_y are calculated at specified δ. The values of δ chosen are 0.05, 0.1 and 0.15 The selection of ρ and ρ^1, and hence $(\rho - \rho^1)$ is restricted by the condition set by equation (4.3.14).

By similar treatment to singly reinforced beam, the required simulation sample size is obtained as 800.

The computer program RESISTJ is also used to simulate the ultimate moment of resistance for the doubly reinforced beams.

4.3.4 Analysis of Results

Ultimate moment resistanes for fourty-one beams are simulated and the result analysed. The result of three beams are summarised in Tables (4.3.1 to 4.3.3). Figures (4.3.1 to 4.3.6) show the histogram ofthe simulated values and cummulative relative requency curves.

The mean values and standard deviations are shown in table (4.3.4). Thetable also contains the coefficietns of variation.

By similar argument to singly reinforced concrete beams the resultant ultimate moment of resistance for doubly reinforced beams is expected to be normally distributed. The theoretical normal distribution is found to be in good agreement with thedistribution of the simulated values for all the beams (see figures (4.3.1 to 4.3.6). The K-s test conducted with program TESTKS shows a good of normal distribution to the simulated sample for all the beam. Some of the results aresummarized in tables (4.3.1 to 4.3.3).

Probabilities of failure obtained for various safety factors are shown in table (4.3.4). The provision of1.5 and 1.15 by CP110 for concrete and steel strengths respectively is

Table (4.3.1)

Frequency Table For Strength of Doybly Reinforced Beam

$f_y = 410/N/mm^2$; $f_{cu} = 20N/mm^2$

$\rho 1 = 0.03$; $\rho 2 = 0.002$; $\delta = 0.05$

Class Interval	Class Frequency		Cummulative Relative Frequency		
	Actual	Theore-tical	Actual	Theoretical	Difference
7.715-8.250	3	2.09	0.004	0.003	0.001
8.250-8.785	4	4.71	0.009	0.009	0.000
8.785-9.320	7	9.67	0.018	0.021	0.003
9.320-9.855	23	18.02	0.046	0.043	0.003
9.855-10.390	32	30.48	0.086	0.081	0.005
10.390-10.925	52	46.81	0.151	0.140	0.011
10.925-11.460	68	65.28	0.236	0.221	0.015
11.460-11.995	74	82.64	0.329	0.325	0.004
11.995-12.530	78	95.00	0.426	0.443	0.017
12.530-13.065	113	99.16	0.568	0.567	0.001
13.065-13.600	96	93.97	0.688	0.685	0.003
13.600-14.135	77	80.85	0.784	0.786	0.002
14.135-14.670	71	63.17	0.873	0.865	0.008
14.670-15.205	40	44.81	0.923	0.9 1	0.002
15.205-15.740	29	28.86	0.959	0.957	0.002
15.740-16.275	16	16.87	0.979	0.978	0.001
16.275-16.810	8	8.96	0.989	0.989	0.000
16.810-17.345	7	4.32	0.998	0.885	0.003
17.345-17.880	0	1.89	0.998	0.997	0.001
17.880-18.415	2	0.75	1.000	0.998	0.002

Table (4.3.2)

Frequency Table For Strength of Doubly Reinforced Beam

$f_y = 410\ N/mm^2$; $f_{cu} = 20\ N/mm^2$

$\rho 1 = 0.02$; $\rho 2 = 0.012$, $d^1/d = 0.1$

Class Interval	Class Frequency		Cummulative Relative Frequency		
	Actual	Theoretical	Actual	Theoretical	Difference
4.925-5.250	3	1.74	0.004	0.002	0.002
5.250-5.575	3	3.97	0.008	0.007	0.001
5.575-5.900	8	8.26	0.015	0.018	0.003
5.900-6.225	17	15.61	0.036	0.037	0.001
6.225-6.550	33	26.86	0.078	0.071	0.007
6.550-6.875	48	42.03	0.138	0.123	0.015*
6.875-7.200	53	59.83	0.204	0.198	0.006
7.200-7.525	80	77.48	0.304	0.295	0.005
7.525-7.850	80	91.28	0.404	0.409	0.005
7.850-8.175	94	97.82	0.522	0.532	0.010
8.175-8.500	100	95.36	0.6 7	0.651	0.004
8.500-8.825	95	84.57	0.766	0.757	0.009
8.825-9.150	64	68.23	0.846	0.842	0.004
9.150-9.475	45	50.07	0.902	0.905	0.003
9.475-9.800	36	33.43	0.947	0.947	0.000
9.800-10.125	22	20.30	0.975	0.972	0.003
10.125-10.450	11	11.22	0.989	0.987	0.002
10.450-10.775	4	5.64	0.994	0.993	0.001
10.775-11.100	3	2.58	0.998	0.997	0.001
11.100-11.425	2	1.07	1.000	0.998	0.002

Table (4.3.3)

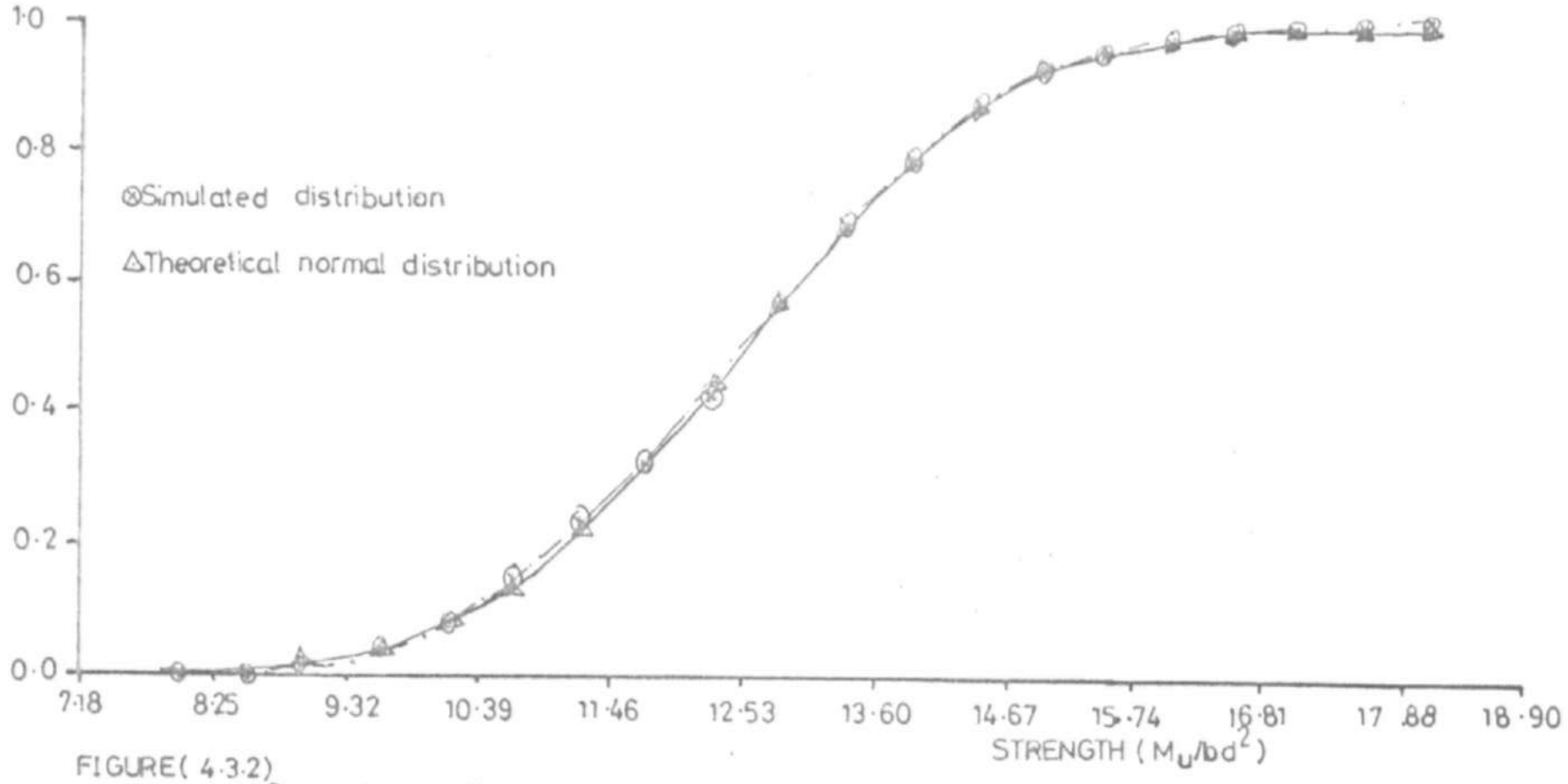
Frequency Table For Strength of

Doubly Reinforced Beam

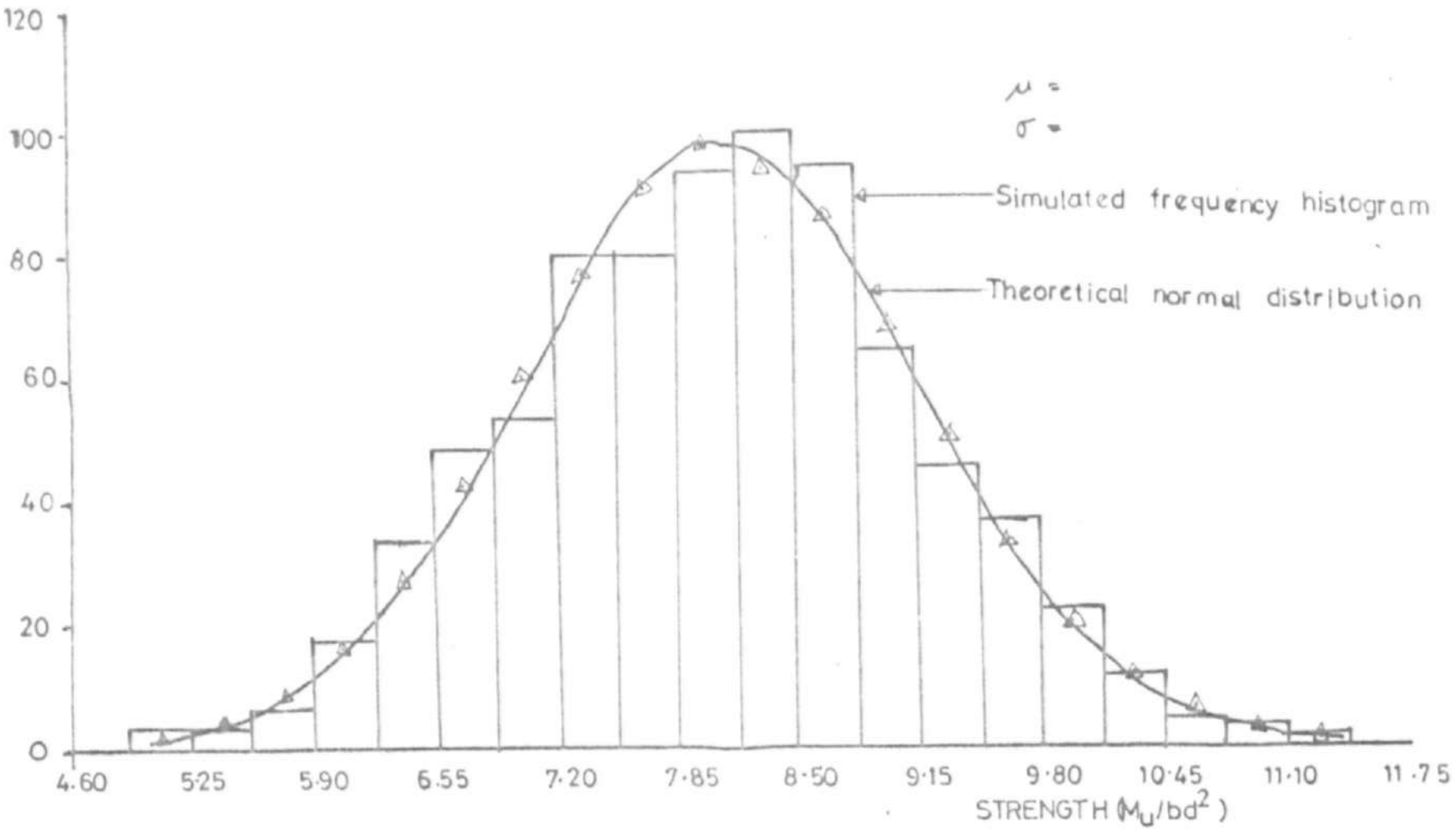
$F_y = 410\ N/mm^2$; $F_{cu} = 20N/mm^2$

$\rho_1 = 0.015$; $\rho_2 = 0.003$; 0.15

Class Interval	Class Frequency		Cummulative Relative Frequency		
	Actual	Theore-tical	Actual	Theoretical	Difference
2.935 - 3.185	2	0.16	0.003	0.000	0.003
3.185 - 3.435	1	0.50	0.004	0.001	0.003
3.435 - 3.685	1	1.42	0.005	0.003	0.002
3.685 - 3.935	3	3.56	0.009	0.007	0.002
3.935 - 4.185	7	8.02	0.018	0.017	0.002
4.185 - 4.435	10	16.16	0.030	0.037	0.007
4.435 - 4.685	37	29.16	0.076	0.074	0.002
4.685 - 4.935	41	47.16	0.128	0.133	0.005
4.935 - 5.185	71	68.09	0.217	0.218	0.001
5.185 - 5.435	97	88.12	0.338	0.328	0.010
5.435 - 5.685	83	102.10	0.442	0.456	0.014
5.685 - 5.935	103	105.89	0.571	0.589	0.018
5.935 - 6.185	108	98.32	0.706	0.712	0.006
6.185 - 6.435	97	81.72	0.827	0.814	0.013
6.435 - 6.685	53	60.80	0.894	0.890	0.004
6.685 - 6.935	33	40.50	0.935	0.941	0.009
6.935 - 7.185	30	24.15	0.973	0.971	0.002
7.185 - 7.435	13	12.89	0.989	0.987	0.002
7.435 - 7.685	7	6.16	0.998	0.995	0.003
7.685 - 7.935	2	2.63	1.000	0.998	0.002



FIGURE(4·3·2)
Cummulateve distribution of ultimate moment of resistance
Doubly reinforced concrete beam
Tensile steel ratio = 0·03 ; Compressive steel ratio=0·026 ; d'/d = 0·05



FIGURE(4·2·3): Distribution of ultimate moment of resistance
Doubly reinforced concrete beam
Tensile steel ratio = 0·02 ; Compressive steel ratio = 0.012 $d'/d = 0.1$

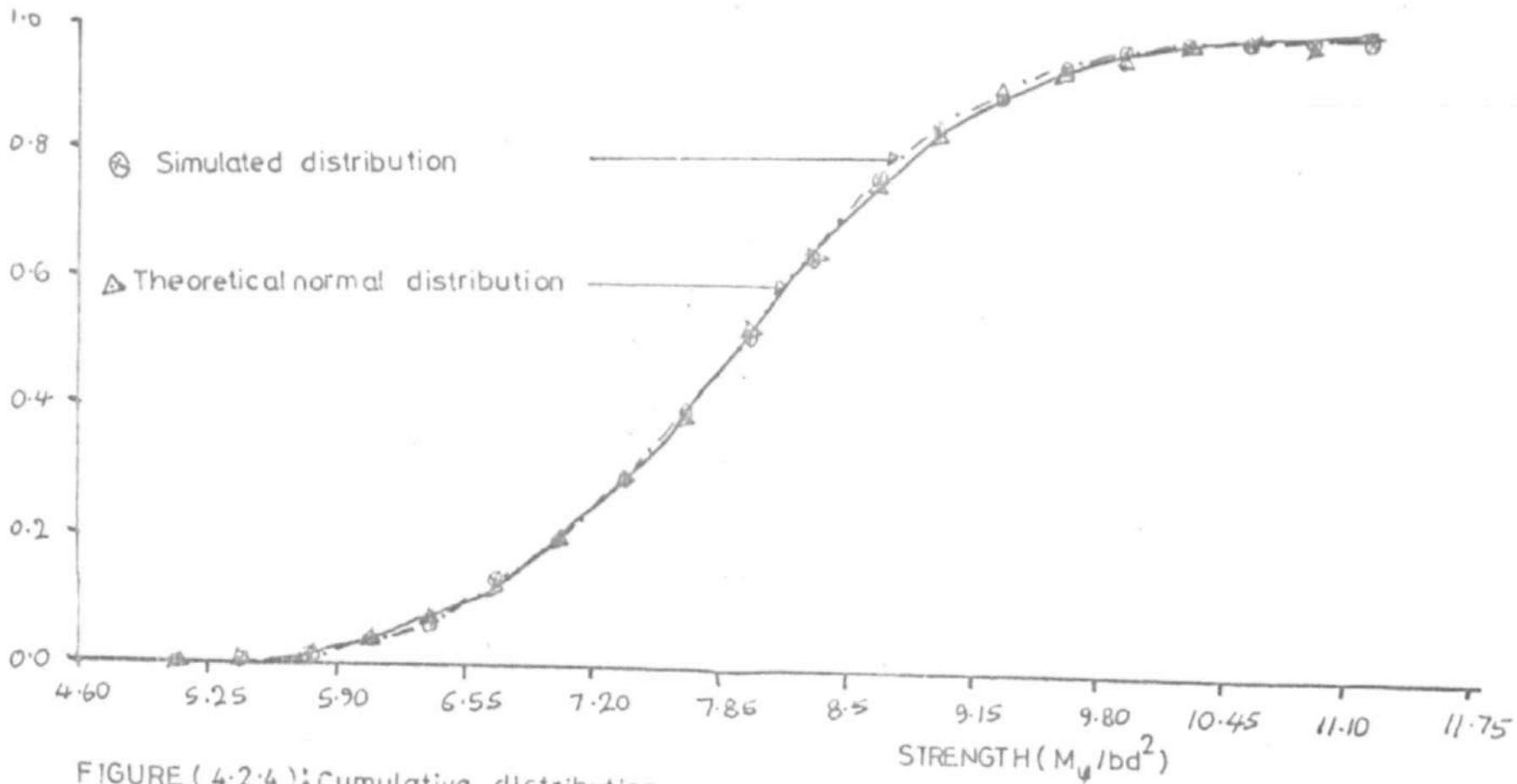
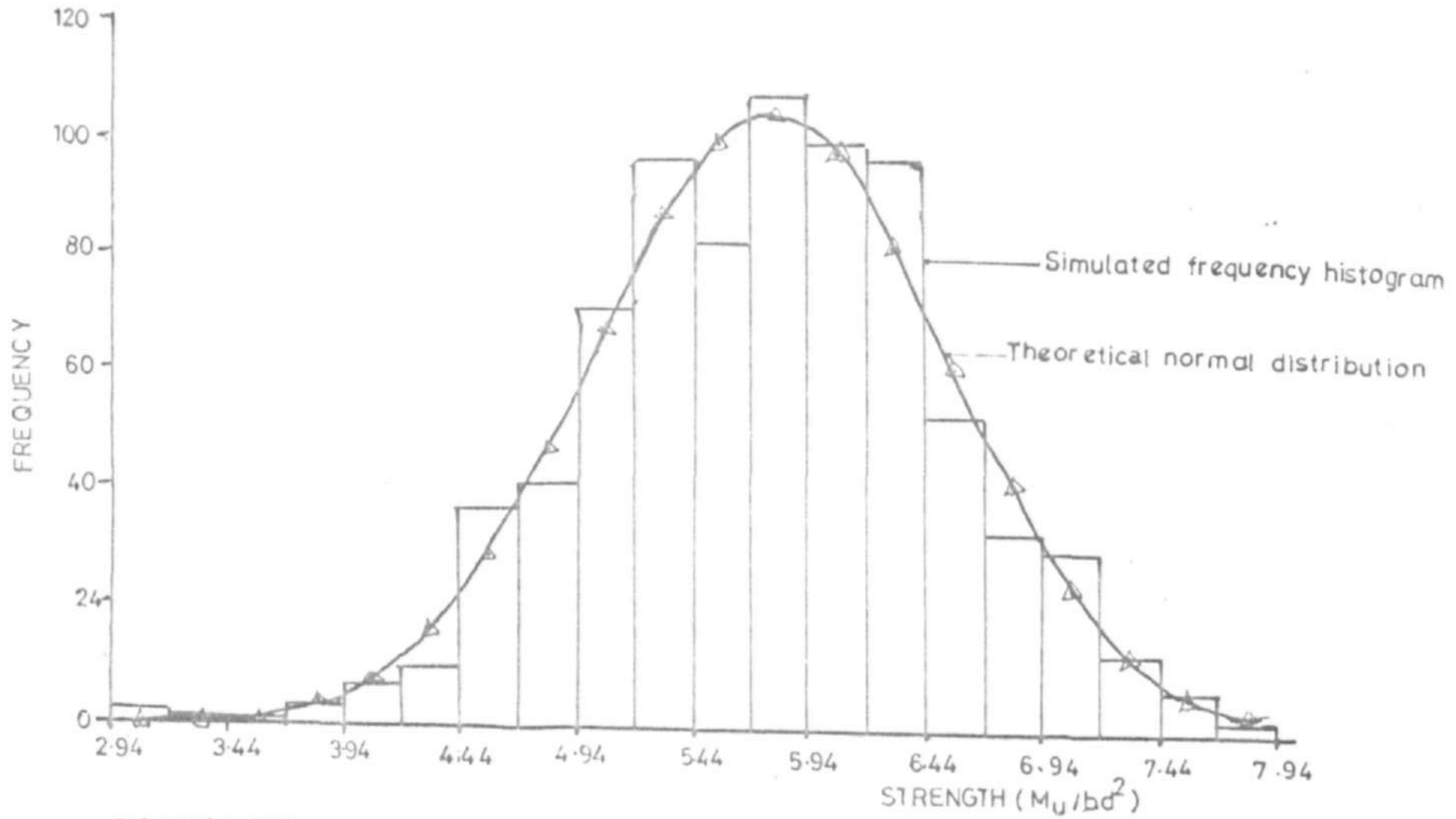
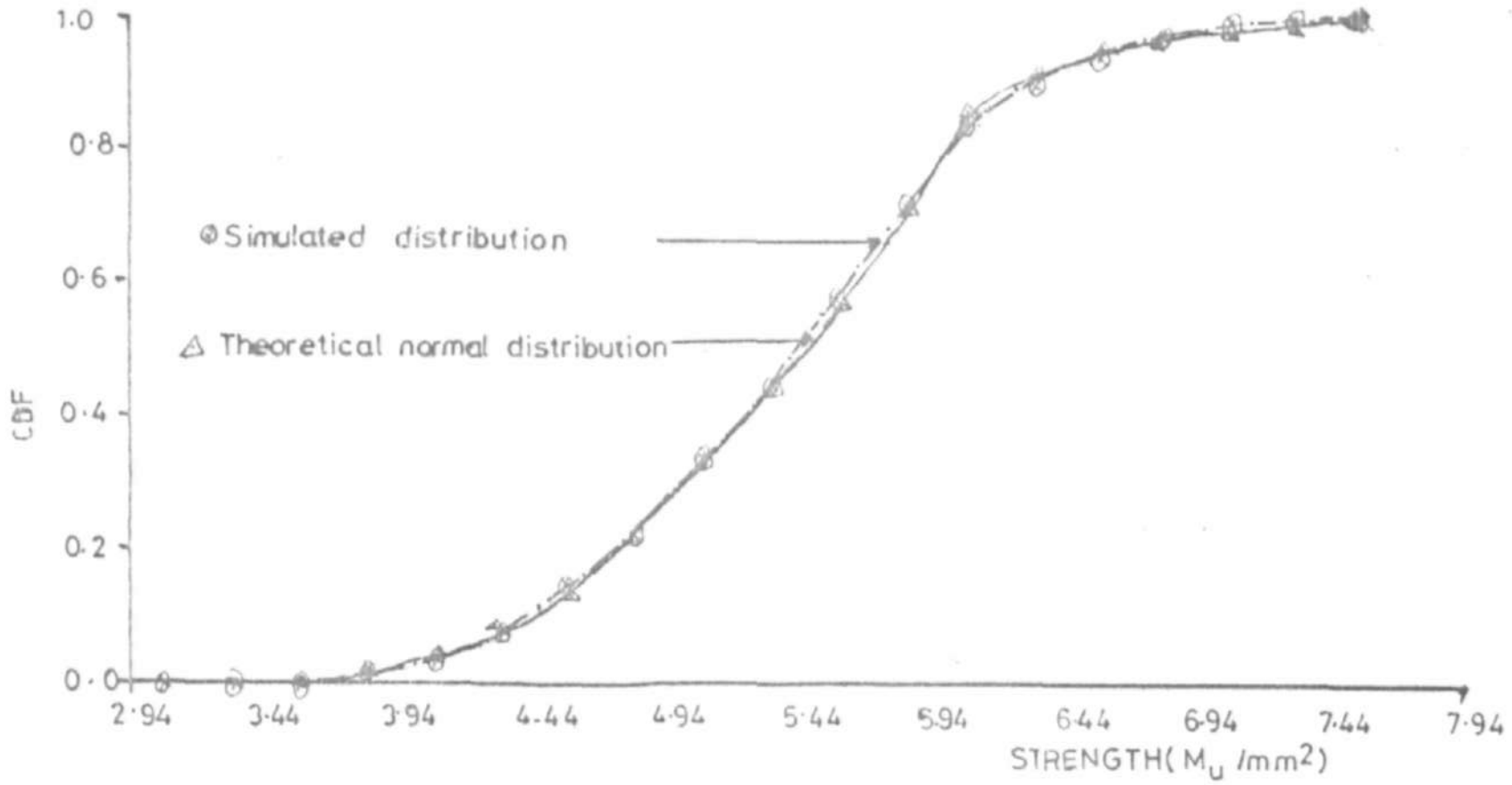


FIGURE (4·3·4): Cumulative distribution of ultimate moment of resistance
Doubly reinforced concrete beam
Tensile steel ratio = 0.02 ; Compressive steel ratio = 0.012 ; $d'/d = 0.1$



FIGURE(4·3·5): Distribution of ultimate moment of resistance
Doubly reinforced concrete beam
Tensile steel ratio = 0.015; Compressive steel ratio = 0.003; $\phi'_d = 0.15$



FIGURE(4·2.6): Cumulative distribution of ultimate moment of resistance
Doubly reinforced concrete beam
Tensile steel ratio =0.015 ; Compressive steel ratio = 0.003; d'/d = 0.15

Table (4.3.4)

Failure Probabilities for
Doubly Reinforced Concrete Beam

CP110 Partial Safety Factors; 1.15 for steel
1.50 for concrete

ACI Safely factor = 0.9

LAMIDA d'/d	Tension Steel Ratio	Compression Steel Ratio		Mean Value	Standard Devalue	Coefficient of	Failure without safely factor	Probability with safely factor CP110	ACI	0.60
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
0.05	0.030	0.026	0.004	12.7671	1.7148	0.1343	0.26	0.06	0.09	0.0004
		0.024	0.006	12.7008	1.7214	0.1355	0.26	0.06	0.09	0.0004
		0.022	0.008	12.6115	1.6579	0.1315	0.24	0.05	0.08	0.0003
		0.020	0.010	12.4690	1.6216	0.1301	0.23	0.04	0.07	0
		0.018	0.012	12.2848	1.5824	0.1288	0.21	0.03	0.07	0
		0.016	0.014	12.0587	1.5457	0.1282	0.19	0.02	0.06	0
	0.020	0.016	0.004	8.5117	1.1457	0.1346	0.26	0.06	0.09	0.0004

Table (4.3.4) Continued

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10	(11)
		0.014	0.006					0.08	0.003	
		0.012	0.008	8.3564	1.0880	0.1302	0.23	0.04	0.07	0
		0.010	0.010	8.2137	1.0573	0.1287	0.21	0.03	0.07	0
		0.008	0.012	8.0295	1.0292	0.1282	0.19	0.02	0.06	0
		0.006	0.014	7.8036	1.0123	0.1297	0.17	0.02	0.05	0
	0.010	0.006	0.004	4.2616	0.5635	0.1322	0.25	0.05	0.09	0.0003
		0.004	0.006	4.2022	0.5456	0.1298	0.23	0.04	0.07	0
		0.002	0.008	4.1010	0.5258	0.1282	0.20	0.03	0.06	0
		0	0.010	3.9565	0.5138	0.1298	0.17	0.02	0.05	0
	0.008	0.004	0.004	3.4104	0.4492	0.1317	0.24	0.04	0.08	0.0003
		0.002	0.006	3.3511	0.4325	0.1291	0.22	0.03	0.07	0
		0	0.008	3.2501	0.4172	0.1284	0.18	0.02	0.06	0
	0.006	0.002	0.004	2.5545	0.334	0.1308	0.24	0.04	0.08	0.0003
		0	0.006	2.5002	0.3208	0.1283	0.20	0.03	0.06	0
	0.004	0	0.004						0.07	0

Table (4.3.4) Continued

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
0.01	0.030	0.022	0.008	12.1189	1.5919	0.1314	0.24	0.05	0.08	0.003
		0.020	0.010	12.0211	1.5616	0.1299	0.23	0.04	0.07	0
		0.018	0.012	11.8816	1.5294	0.1287	0.21	0.03	0.07	0
		0.016	0.014	11.7004	1.4996	0.1282	0.19	0.02	0.06	0
	0.020	0.012	0.008	8.0875	1.0522	0.1301			0.07	0
		0.010	0.010	7.9896	1.0279	0.1287	0.21	0.03	0.07	0
		0.008	0.012	7.8503	1.0065	0.1282	0.19	0.02	0.06	0
		0.006	0.014	7.6688	0.9963	0.1299	0.17	0.02	0.05	0
	0.010	0.008	0.002	4.0989	0.5506	0.1343	0.26	0.06	0.09	0
		0	0.010	3.9585	0.5138	0.1298	0.17	0.02	0.05	0

Table (4.3.4) Continued

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10	(11)
0.15	0.030	0.018	0.012	11.4784	1.4765	0.1286	0.21	0.03	0.07	0
		0.016	0.014	11.3420	1.4537	0.1282	0.19	0.02	0.06	0
	0.020	0.008	0.012	7.6710	0.9838	0.1282	0.19	0.02	0.06	0
		0.006	0.014	7.5344	0.9803	0.1301	0.17	0.02	0.05	0
	0.015	0.003	0.012	5.7674	0.7478	0.1297	0.17	0.02	0.05	0
		0.001	0.014	5.6309	0.7629	0.1355	0.15	0.01	0.05	0
	0.014	0.002	0.012	5.3865	0.7024	0.1304	0.17	0.02	0.05	0
		0	0.014	5.2502	0.7227	0.1377	0.15	0.01	0.05	0
	0.012	0	0.012	4.6249	0.6144	0.1328			0.05	0

shown given probability of failure less than or equal to 0.05. The ACI recommended value of safety factor of 0.9 gives failure probability a little bit more than 0.05. Introducing a safety factor of 0.60 reduces the probability of failure to practically zero.

4.4 AXIALLY LOADED COLUMNS

4.4.1 Ultimate Axial Load Model

CP110 gives the formula for predicting the ultimate strength of axially loaded tied column as

$$P_u = 0.67 \frac{f_{cu}}{m} A_c + \frac{f_y}{m \frac{+f_y}{2000}} A_{sc} \quad \ldots 4.4.1$$

where P_u = collapse load

f_{cu} = concrete st ength

f_y = steel strength

A_c = gross Area of column cross section

A_{sc} = Area of steel

γ_m = partial factor of safety.

The coeeficient 0.67 is due to the concrete strength in place rather than test cubes. The partial safety factors recommended by CP110 are 1.5 and 1.15 for concrete and steel respectively. The safety factors

take into account of the variations in member strength. However, since this study is on the effect of strength variation, the factors will be given value 1 for both materials.

To allow for eccentricity of loading due to construction tolerances, CP110 limits the ultimate axial load at about 90% of that given by equation (4.4.1).

Thus the ultimate axial load can be written as

$$P_u = 0.603\, f_{cu} A_c + \left[\frac{0.9 f_y}{1 + \frac{f_y}{2000}}\right] A_{sc} \quad \ldots\ldots\ 4.4.2$$

Dividing equation (4.4.2) by A_c if follows that

$$\frac{P_u}{A_c} = 0.603\, f_{cu} + \left[\frac{0.9 f_y}{1 + \frac{f_y}{2000}}\right] \frac{A_{sc}}{A_c} \quad \ldots\ 4.4.3$$

Representing $\frac{P_u}{A_c} = R_c$

and $\frac{A_{sc}}{A_c} = \rho$

then

$$R_c = 0.603 f_{cu} + \rho \frac{1800\, f_y}{2000 + f_y}$$

$$= \phi_c\, (f_y, f_{cu}) \quad \ldots\ldots\ 4.4.4$$

$\emptyset_c$ denote functional relationship.
Specifying the value of ρ, the distribution of R_c can be obtained from that of f_{cu} and f_y

4.4.2 Simulation

The construction materials and workmanship are the same with that of singly reinforced beams. Thus the random sampling is done from the same distribution of concrete and steel as for simply and doubly reinforced beams.

CP110 recommend the reinforcement ratio of 1% and 4% as minimum and maximum values respectively, for column reinforcement. The reinforcement ratios used in the study are 0.01, 0.02, 0.03 and 0.04.

The simulation sample size is obtained as 800 by procedure shown in section 4.1.2. The simulation is performed by computer program COLUMNS (appendix). The program contains also the subprogram NORMAL.

4.4.3 Analysis of Results

As with beam sections, only one column simulation results are shown (table (4.4.1)). Figures (4.4.1) and (4.4.2) show the histogram and the cummulative relative frequency curve

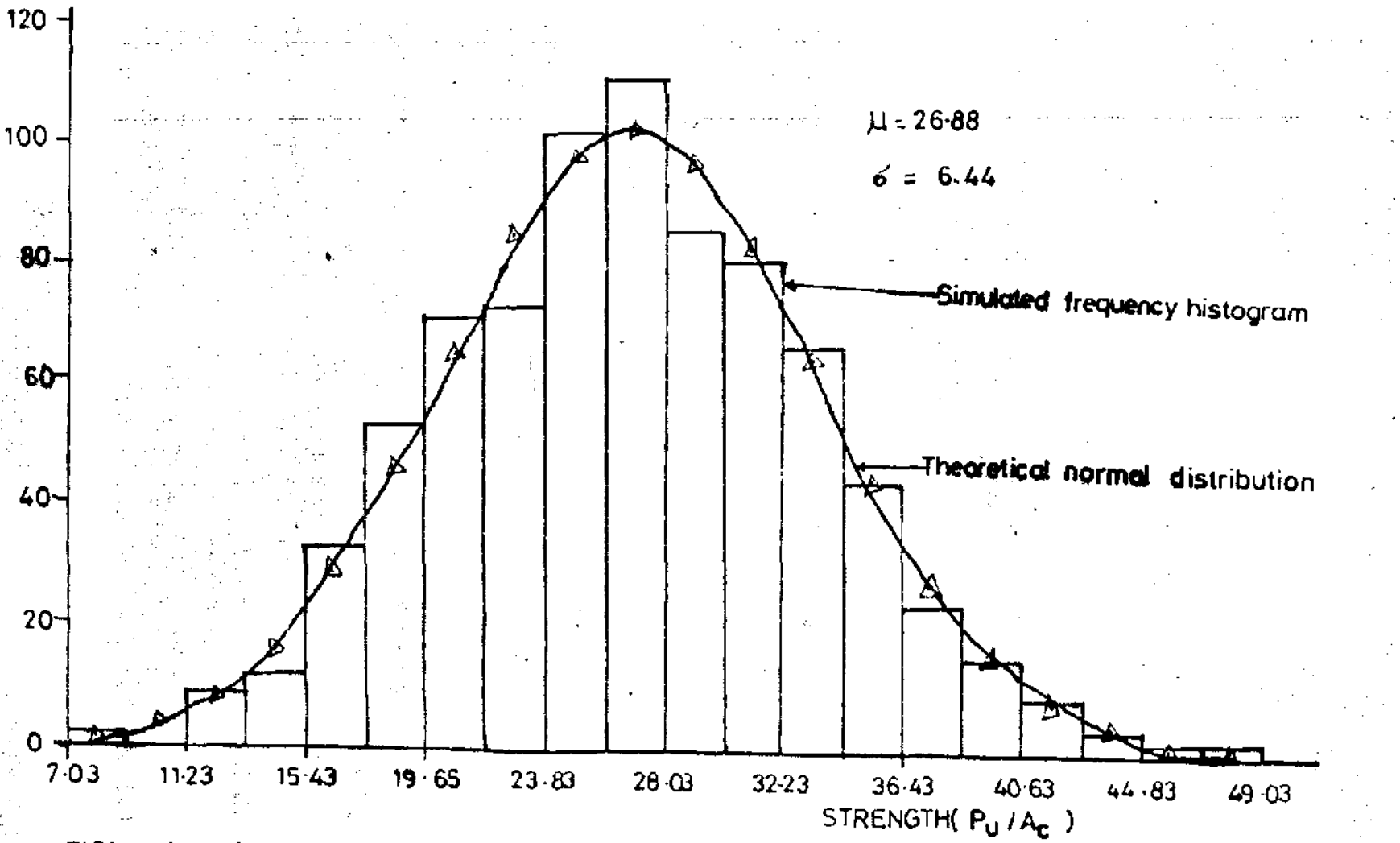
Table (4.4.1)

Frequency Table For Strength of Reinforced Concrete column

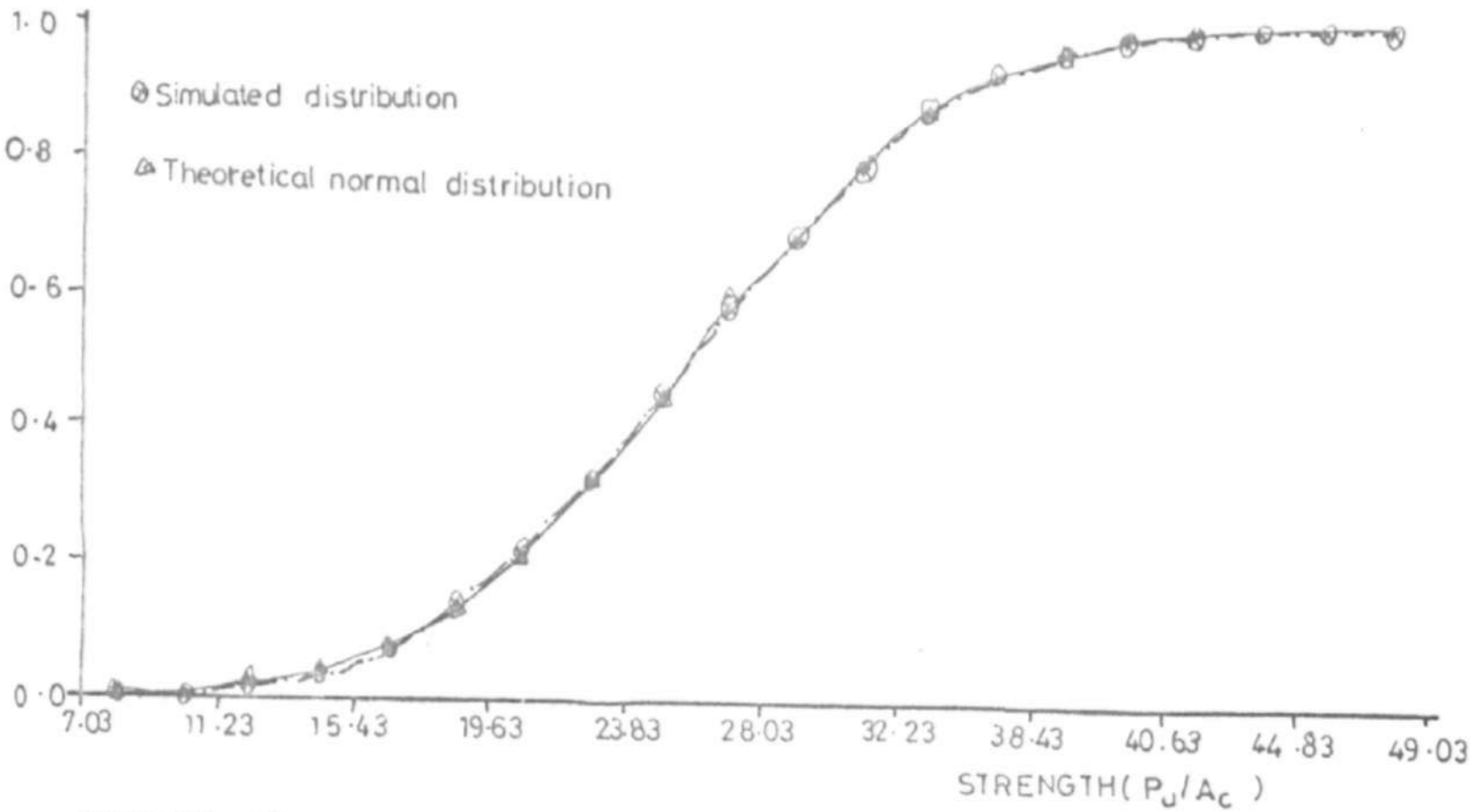
f_y ; 410 N/mm^2; f_{cu} = 20 N/mm^2

ρ = 0.02

Class Interval	Class Frequency		Cummulative Relative Frequency		
	Actual	Theoretical	Actual	Theoretical	Difference
7.025-9.125	2	1.56	0.003	0.002	0.001
9.125-11.225	1	3.77	0.004	0.007	0.003
11.225-13.325	9	8.23	0.015	0.017	0.002
13.325-15.425	12	16.16	0.030	0.037	0.007
15.425-17.525	33	28.60	0.071	0.073	0.002
17.525-19.625	53	45.57	0.138	0.130	0.008
19.625-21.725	71	65.41	0.226	0.212	0.014
21.725-23.825	73	84.55	0.318	0.317	0.001
23-825-25.925	102	98.44	0.445	0.440	0.005
25.925-28.025	111	103.22	0.584	0.569	0.015
28.025-30.125	86	97.48	0.691	0.691	0.000
30.125-32.225	81	82.92	0.793	0.795	0.002
32.225-34.325	66	63.63	0.875	0.874	0.001
34.325-36.425	44	43.84	0.930	0.929	0.001
36.425-38.525	24	27.24	0.960	0.963	0.003
38.525-40.625	15	15.25	0.979	0.982	0.003
40.625-42.725	9	7.69	0.990	0.992	0.002
42.725-44.825	4	3.49	0.995	0.996	0.001
44.825-46.925	2	1.43	0.998	0.998	0.000
46.925-49.025	2	0.53	1.000	0.999	0.001



FIGURE(44·1) Distribution of column ultimate axial load
Steel ration 0·02



FIGURE(4·4·2) Cummulative distribution of column ultimate axial load
Steel ratio =0·02

of the simulated values respectively.

The mean values and standard deviation of all the columns are shown in table (4.4.2). The table contains the coefficients of variation.

As with beam sections, the strength of column section is expected to follow normal distribution. Figure (4.4.1) and (4.4.2) show theoretical normal distribution to be in good agreement with the frequency curves of the simulated values. The computer program TESTKS is used to conduct K-S test on one-by-one bases. The result shows that at 0.05 level of significant, normal distribution could not be rejected. The summary of result for column with steel ratio of 0.02 is shown in table (4.4.1).

The probabilities of failure for the columns factors, the values range from 0.08 to 0.09 (see table (4.4.2). These values are higher than 0.05 recommended by CP110 as limit. However, introducing the partial safety factors of 1.5 and 1.15 for concrete and steel respectively reduces the failure probabilities to range from 0.01 to 0.02. Table (4.4.2) also show the probabilities of failure allowing for the strength recommended by ACI. Unlike the

Table (4.4.2)

Failure Probabilities For

Axially Loaded Short Reinforced Concrete Column

F_y = 410 N/mm^2; F_{cu} = 20 N/mm^2

CP110 Partial Safely Factors; 1.15 for Steel

1.50 for Concrete

ACI Safely Factor 0.75

Proposed Safety Factor = 0.60

Steel Ratio	Mean Value	Standard Deviation	Coefficiecy of variation	Failure Probability With Safety Factor			
ρ	μ	σ	$CV=\frac{\sigma}{\mu}$	Safety factor 1	CP110	ACI	0.60
0.01	23.59	6.45	0.2734	0.0951	0.0233	0.045	0.0122
0.02	26.88	6.44	0.2396	0.0885	0.0188	0.034	0.0066
0.03	30.16	6.50	0.2155	0.0853	0.0158	0.028	0.0037
0.04	033.45	6.55	0.1958	0.0808	0.0132	0.023	0.0020

case of beam sections, the probability of failure is less than 0.05 for column strength. The safety factor of 0.60 reduces the failure probability to practically almost zero.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATION

The study has considered only the effect of variations in material strength on the variability of reinforced concrete members. Other sources of variations notconsidered are:

(i) Variations in stress-strain behaviour of the materials;

(ii) Variations in the relationship between cube strength and concrete strength in structure;

(iii) Variations in overall dimensions and steel location; and

(iv) Variations in field pactice (workmanship, control, placement, curing etc.).

There are also uncertainties in the application of statistical theory such as:

(1) uncertaintity in statistical estimate due to sample size; and

(2) uncertainty in the choice of the distribution function.

These uncertainties have therefore made the conclusion derived from this study to be of limited application. Nonetheless the result

has demonstrated some variable nature of the strength of reinforced concrete structures as practiced in Zaria environ.

All the probability distributions of materials and meber strength are found to be normal distributions.

The results show the coefficient of variation for concrete to be 0.32. This value is large compared to 0.20 recommended in reference (25). This could be due to poor handling by the contractors. The coefficient of variation for steel tensile strength is found as 0.13. This is also larger than the recommended value of 0.06 (25). The coefficient of variation for the strength of reinforced concrete beams and columns should be taken as 0.13.

In obtaining the probability of failure comparison is made between ACI and CP110 recommended values of safety factors. CP110 recommends the values of partial safety factors of 1.5 and 1.15 for concrete and steel strengths respectively The ACI Code recommendation is 0.9 for resisting moment of reinforced concrete beams. It has been found that the probability of failure is less than or equal to 0.05 when CP110 is adopted, while ACI recommended value gives probability of failure to

be mostly greater than 0.05. Applying the safety factor of 0.60 to the resistances of beam and column reduces the failure probability to practically zero. While considering only the effects of material strength variation the factor of safety of 0.60 can be adopted for the moment resistanceof beam and axial load resistance of columns. However, because ofthe uncertainties mentioned earlier this factor is conditional. Since the CP110 recommended safety factors give failure probability not greater than 0.05, the factors can be accepted.

The Monte Carlo Method used in this study has demonstrated the ease with which a large number of members can be 'built' and tested to 'failure' by simulation. The accuracy of the simulation result, among other things depends on the accuracy of the distribution of the random variables, which in this case are the concrete and steel strength. This in turn depends on the adequacy ofthe data used. Thus, there is need for careful and adequate data collection. Unfortunately, not much attention is currently paid in this country to the material testing. It is therefore hoped that material testing would

be intensified both on site and laboratory. Proper record of the test result should also be kept.

The high coefficient of variation for concrete cube strength have clearly shown that contractors have to improve their methods of concrete manufacture andtheir attention should be drawn to the basic recommendation concerning concrete:

(1) use should be made of clean and properly graded aggregate with a greater ratio of coarse to fine aggregate.

(2) the batching of cement and aggregate should be by weight

(3) batching of water is by measured volume

(4) an appropriate concrete mixer should be used to produce a homogenous and cohessive concrete mix.

Besides the above measures adequate methods of handling, compacting and curing should be strickly observed, and rightly supervised by clerk of works. Attention should be paid to imporving production of crushed-stone aggregate as well as providing delivery of clean good graded sand.

The variation of steel strength could be due to different practice of manufacture. However, the poor sampling procedure could also have contributed to the variations. A lot of samples brought for testing have not met the BS4449 requirement on sample size and type of steel reinforcement. Though it was observed that no certificate is issued on these test, the contractors still proceed to use the reinforcement for building. It is therefore recommended that appropriate authorities should insist on thecontractors to issue certificate on material test.

Finally, it is recommended that other sources of variations should be included in further work in this area.

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