AN ANALYSIS OF E2-M1 MULTIPQLE MIXING RATIOS OF ($2'^{+}$ -> 2^{+}) GAMMA TRANSITIONS IN EVEN-EVEN NUCLEI

ADAMU NCHAMA BABA-KUTIGI

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Department of Physics
Faculty of Science
Ahmadu Bello University
Zaria - NIGERIA.

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DECLARATION

I hereby declare that the work presented in this thesis is the result of the original research by the author. It has not been accepted in any previous application for a higher degree. All the sources of information are specially acknowledged by means of references.

.....

Signature Date.....

CERTIFICATION

This thesis entitled "An Analysis of E2/M1 Multipole Mixing Ratios of ($2^{'+} \rightarrow 2^{+}$) Gamma Transitions in Even-Even Nuclei" by Adamu Nchama Baba Kutigi meets the regulations governing the award of the degree of Master of Science of Ahmadu Bello University, and is approved for its contribution to knowledge and literary presentation.

.....
DR. I.V.S. RATHORE
Chairman, Supervisory Committee Date:.....

.....
DR. S.B. ELEGBA
Member, Supervisory Committee Date:.....

.....
DR. J. Adetunji
Head of Department Date:.....

.....
Professor J.O. Osuide
Dean, Postgraduate School Date:.....

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ABSTRACT

A systematic survey is presented to analyse E2/M1 mixing ratios '6' of Gamma transition between the $2'^{+}$ and 2^{+} levels for the cascade $2^{*+} \rightarrow 2^{+} + 0^{+}$ of even-even nuclei of mass range 58SAS200. The experimental data for transition in even-even nuclei within this range of mass number have been critically surveyed to provide the most accurate result for comparison with theoretical calculati Particular attention is given to the variation in the phase of the mixing ratios '<5' which are deduced from the literature. Theroretical approaches to the calculations of these mixing ratios '<5' on the bases of different models are considered and compared with experimental value. The experimental values of the angular correlation coefficient A_{2} and A_{4} for the cascade $2'^{+} \rightarrow 2^{+} \rightarrow 0^{+}$ have ^eu collected and analysed. The multipole mixing ratios £or the ($2^{*} \rightarrow 2$) gamma-transition are analysed by two methods (i) Arns and Wiedenbeck using A values or A values and (ii) that of Coleman using A and A values. These mixing ratios are compared with the values obtained from the literatures by the different methods. The phases and values common are taken from the analysis. The plot of $\left|\frac{\delta}{0.835E_{\gamma}}\right|^{2}$; Mass number is given along with values obtained by the Weisskopf estimate and the Davydov and Filippov estimate. Many classiffications are obtained in the values of mixing ratios.

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ABBREVIATIONS

A	mass number
A_ℓ	Angular Correlation Coefficient
ARM	Asymmetric Rotor Model
B	mass parameter
B E(M)L	Reduced Transition Probability
C	Clebsch-Gordan Coefficient
$C_{\ell\tau}$	Racah radiation parameter
$D^L_{\mu M}$	Rotation matrix
EL	Electric multipole radiation
E2	Electric quadrupole radiation
g_R	Nuclear gyromagnetic ratio
J	Total angular momentum
j_i	initial angular momenta
j_f	Final angular momenta
k	direction of propagation
L	multipolarities
LM	quantum numbers
ML	magnetic multipole radiation
M1	magnetic dipole radiation
TE(M)L	Transition Probability of gamma-ray emission or absorption
Tr	Trace or Sum of the diagonal element
W(θ)	Directional Correlation function
Z	Atomic number
β	Beta
γ	Gamma
Π	Parity

μ_N	Nuclear magneton
δ	mixing ratio
δ_W	mixing ratio estimated by Weisskopf
δ_{DF}	mixing ratio estimated by Davydov-Filippov
ρ	Density matrix
σ	spin

Chapter 1

INTRODUCTION

Gamma-radiation is an electromagnetic radiation which occurs as a result of the nucleus decayed from its excited state to the ground state. This is usually followed by more than one multipole radiation. Each component of the radiation is related to a definite multipole moment in the source that is, the multipole field with quantum numbers (LM) is caused by a specific oscillating 2^L Pole of charge density. The two possible classes of radiation electric (EL) and magnetic (ML) Multipoles differ with respect to parity. The radiations are specified by the selection rules on the nuclear spins and parities. While any multiple order between the sum and difference of the initial and final nuclear spins is allowed, the strong dependence on the transition probabilities on the angular momentum carried off restricts the observable orders to the lowest ones - transition probability diminishes rapidly with increasing order L. However, when the selection rules allow, the E2 radiation often dominates the M1 component. The domination of E2 radiation occurs because nuclear strucuture effect over ride the angular momentum dependence of the transition probabilities. Thus experimental determination of the E2/M1 mixing ratio δ(E2/M1) radiations and nuclear transition particularly in even-even nuclei have provided a test of nuclear models. The importance of this has led to periodic surveys of E2/M1 mixing ratios, with theoretical studies to predict or explain the data in even-even. nuclei.

The survey on E2/M1 mixing ratios have been done by several authors. Among them are Arns and Wiedenbeck (1958), Davydov and Fillipov (1958), Kumar (1969), Hamilton (1971), Krane and Steffen (1971), Rathore and Singh (1978); Lange, Kumar and Hamilton (1982).

The aim of these project is to analyse the systematics on E2/M1 multipole mixing ratios of gamma transitions in even-even nuclei of the mass range $58 \leq A \leq 200$. The E2/M1 mixing ratios are used in explaining and predicting the nuclear structure and properties such as the nuclear collectivity, shape as well as shell closure.

To achieve these objectives the mixing ratios of $(2^{+} \rightarrow 2^{+})$ transition in even-even nuclei are obtained by several methods. These methods include single particle shell model in which the nucleus is assumed to be spherical and that only the nucleon (a proton) makes the nuclear transition, collective model and angular correlation methods. The thesis is therefore divided into five chapters.

The first Chapter deals with the general consideration of the complexity of E2/M1 multipole mixing ratios. It stresses attempts made by various authors to explain and predict E2/M1 mising ratios. [The procedure of achieving this, is therefore discussed in this chapter.]

In Chapter two, the theory of multipole radiation is treated. The multipolarity classifying the type of allowed radiations during electric (EL) or magnetic (ML) transitions are discussed. This is closely followed by selection rules on the possible multipolarities of gamma transition between two states of specified angular momenta (J_i, J_f) and parities (Π_i, Π_f)

$$|J_i - J_f| \leq L \leq J_i + J_f \quad \ldots\ldots (1.1)$$

$$\Pi_i = (-1)^L \Pi_f \text{ for EL radiation} \quad \ldots\ldots (1.2)$$

$$\Pi_i = (-1)^{L+1} \Pi_f \text{ for ML radiation} \quad \ldots\ldots (1.2)$$

An additional requirement, a consequence of the transverse nature of electromagnetic waves, is that no radiation of multipole order 0 occur.

The multipole mixing ratios are thus discussed in terms of transition probabilities and reduced transition probabilities B(E2), B(M1). But the reduced transition probability is much easier in determining E2/M1 mixing ratio as it does not depend on energy as the transition probability but does depend on the square of transition matrix element. Another important factor discussed is the ambiguities in the sign of δ(E2/M1) from nucleus and from one state to another in the same nucleus.

A sign convention to control the signs of δ(E2/M1) is therefore established and the one employed in this work is that of Biedeharn and Rose (1953) which is compared with those of Rose and Brink (1967), and Krane and Steffen (1970).

The third Chapter gives brief formulations of E2/M1 mixing ratio by single particle shell model as estimated by Weisskopf (1951). In determining E2/M1 mixing ratios of gamma transitions in even-even nuclei, Weisskopf (1951) estimated the radial integral in the reduced transition probability the radial integral to be $[3/(L+3)]R^L$.

(R being the radius of the nucleus) and statistical factor to be unity and therefore simplified its work.

$$\delta_W(E2/M1) = 1.52 \times 10^{-3} E_\gamma \text{ (in MeV)} A^{2/3} \quad \ldots (1.3)$$

Although this model gives a good account of the nuclear collectivity and shell closure, every nucleon in each energy state of the nucleus has definite angular momentum and magnetic moment thereby breaking the picture of closely packed nucleons in the nucleus. This is overcome by, the asymmetric rotor model (ARM) that explain certain parameters in the energy level formula, deviations in branching ratios of E2/M1 mixing ratio. The dependence of E2/M1 mixing ratio on the mass parameter deformation parameter and the asymmetry angle parameter are treated and mass parameter independent is computed.

Chapter four deals with the theory of gamma-gamma angular correlation which proceed through cascade transition by successive emission of two gamma-rays of different multipolarities. The directional correlation function $W(\theta)$ is thus measured from the coincidence counting rate of two successive gamma radiations as a function of the angle between the propagation directions on these radiations.

Other methods of angular correlation measurement include Gamma-Gamma polarisation correlation in which both the angular momentum of each gamma-ray and the parity are determined, and directional correlation involving conversion electrons.

The fifth Chapter is based on the experimental values of δ(E2/M1) mixing ratios of $(2'^{+} \rightarrow 2^{+})$ gamma transition in even-even nuclei obtained by various authors are compared with theoretical values estimated by Weisskopf and Davydov- Fillipov. *Finally, the discussion based on their success and failure are made and conclusion is drawn on their suitability and predicting power.*

Chapter 2

Theory of Multipole Radiation

2.1 Multipolarity of Gamma-Radiation

Multipole radiation is the radiation in free space that may be expanded into electric and magnetic multipole fields as well as into plane waves. This means that electric multipole radiation has a pure transverse magnetic field while magnetic multipole radiation has pure transverese electric field.

Basically, gamma ray is an Electromagnetic (E.M) radiation, and each component of the radiation is related to a definite multipole moment.

Multipolarity is the term used to specify the kind of radiation as to both class (electric or magnetic) and multipole order (L = 1,2,3...). These radiations are classified by multipole order L, according to the angular momentum L (in units of $\hbar$) carried off by each quantum. For each multipole order, these are two possible classes of radiation: electric (EL) and magnetic (ML) which differ with respect to parity.

2.2 Selection Rules

Slection rule classified the transition of a given parity. According to the law of conservation of angular momentum L, during transition a gamma *ray* carries an angular momentum $[L(L+1)]^{\frac{1}{2}}\hbar$ out of nucleus, which is equal to the difference in angular momenta J of the nucleus in the final (f) and initial (i) states.

$$J_i - J_f = L \quad \ldots\ldots\ldots \quad 2.1$$

The conservation of angular momentum and parity for the system of nucleus plus gamma (r)-rays imposes selec*tion* rule on the possible values of L of γ-transition between two states of specified angular momenta (J_i, J_f) and parities (Π_i,Π_f) given by the relation....

$$|J_i - J_f| \leq L \leq J_i + J_f \quad \ldots \quad 2.2$$

The parity conservation law is an Electromagnetic inferaction, on the other hand is given by the relation:-

$$\Pi_i = (-1)^L \Pi_f \quad \text{for EL radiation} \quad \ldots \quad 2.3$$

$$\Pi_i = (-1)^{L+1} \Pi_f \quad \text{for ML radiation} \quad \ldots \quad 2.4$$

The possible radiation that occurs in the transition $2^{'+} \rightarrow 2^+$ are

M1 + E2 + M3 + E4.

Only transitions involving the lowest order L(M1+E2) are considered. Since the transition probability diminishes rapidly with increasing order L. The decrease with increasing order arises from the large ratio of the wave length to the nuclear dimensions, or because the nuclear dimension (specifically, the nuclear radius R) is much smaller than the reduced compton wave length of the radiation.

$$\frac{2\pi R}{\lambda} \ll 1$$

$$\lambda \gg R$$

In general, when the electric component is of some order as the magnetic one, the radiation should be overwhel mingly of electric character. In the reverse situation, the magnetic component should predominate, especially at low energies, but the contributions may be comparable (e.g. M1 + E2 mixing ratio). Photon is estabish when equations 2.2 - 2.4 are used.

2.3 The Electric and Magnetic Gamma-Ray Transition Probabilities

The probability that a transition will occur between an excited state and lower energy state is the ratio of such transition per unit time to the total number of nuclei in the excited energy state. Thus, the transition probability of an excited nuclear state for gamma-radiation depends on the multipolarity, the energy of the gamma-rays, and the wavefunctions of the nuclear state involved.

Both the electric and magnetic gamma-ray transition probabilities of an electromagnetic-radiation in which gamma photon is either emitted or absorbed, are defined mathematically by both Weisskopf (1951), Bohr and Mottleson (1969) as follows:- the transition probability for gamma-ray emission or absorption of multiplicity E (M)L is expressed by

$$T[E(M)L, J_i \to j_f] = \frac{8\pi(L+1)}{L[(2L+1)!\,!]^2} \frac{1}{\hbar} q^{(2L+1)} \times |\langle j_f m_f | M[E(M)L] | j_i m_i \rangle|^2 \qquad \ldots \quad (2.5)$$

where $q = E_\gamma / \hbar c$ is the wave number for the photon matrix element (or transition amplitude) and M[E(M) L] is the multipole moment operator.

The equation 2.5 gives transition probability for one sub-process i.e. $m_i - m_f$. In order to obtain the total transition probability that determines the lifetime of the initial state, the summation has to be done over all the possible substates μ and m_f. The total transition probability is

$$T\left[E(M)\ L; j_i \to j_f\right] = \sum_{\mu m_f} T(L, \mu: j_i m_i \to j_f m_f)$$

$$= \frac{8\pi(L+1)}{L[(2L+1)!!]^2} \frac{1}{\hbar} q^{(2L+1)} B(L, j_i \to j_f) \qquad \ldots \quad (2.6)$$

where the quantity

$$B(L,j_i \to j_f) = \sum_{\mu m_f} |\langle j_f m_f | M(L,\mu) | j_i m_i \rangle| \quad \ldots\ldots (2.7)$$

is called "reduced transition Probability".

The reduced transition probability does not depend on energy $E^{(2L+1)}$ as the transition probability but does depend on the squared transition matrix. It is therefore convenient to convert T(L) into B(L).

According to the Wigner-Eckart theorem matrix element of an operator (M(Lμ) is factorised as

$$\langle j_f m_f | L\mu | j_i m_i \rangle = \langle j_i m_i | L\mu | j_f m_f \rangle \langle j_f || L || j_i \rangle \ldots (2.8)$$

Using orthorgonality of Clebsh-Gordan Coefficients

$$\langle j_i m_i | L\mu | j_f m_f \rangle = \sqrt{\frac{1}{2J+1}} \quad \ldots (2.9)$$

Hence, the reduced transition probability can be expressed as

$$B(L,j_i \to j_f) = \frac{1}{2J+1} |\langle j_f || M(L) || j_i \rangle|^2 \quad \ldots\ldots (2.10)$$

This shows that the lifetime of a state does not depend on its orientation (rotational invariance).

2.4 The Multipole Mixing Ratios

The multipole mixing ratios according to Blatt and Weisskopf (1952) and Roy and Nigam (1967) is defined as the square root of the ratio of electric gamma-ray transition probability to the magnetic gamma-ray transition probability. In other words it is the square root of the ratio of electric gamma-ray intensity to magnetic gamma-ray intensity, and also as the ratio of the reduced matrix element of the electric to magnetic gamma-ray transition. This is expressed as

$$\delta^2(EL/ML;j_i \to j_f) = \frac{\text{No of EL Transition/Sec}}{\text{No of ML Transition/Sec}}$$

$$= \frac{T[(EL);\ j_i \to j_f]}{T[(ML);\ j_i \to j_f]} \quad \ldots\ (2.11)$$

When equation 2.6 is used for the respective transition, equation 2.11 one obtains:

$$\delta^2 = \frac{\frac{8\pi(L+1)}{L[(2L+1)!!]^2}\frac{1}{\hbar}\ q^{(2L+1)}B(EL, j_i \to j_f)}{\frac{8\pi(L+1)}{L[(2L+1)!!]^2}\frac{1}{\hbar}\ q^{(L+1)}B(ML, j_i \to j_f)} \quad \ldots\ (2.12)$$

For the case of the cascade transition for the $2^+ \to 2^+$ level considered in this work, EL corresponds to E2 and ML to M1. Therefore equation 2.12 now becomes

$$\delta^2\ (E2/M1;j_i \to j_f) = \frac{3}{100}\ q^2\ \frac{B(E2;\ j_i \to j_f)}{B(M1;\ j_i \to j_f)}$$

$$= \frac{3}{100}\ q^2\ \frac{|\langle j_f||M(E2)||j_i\rangle|^2}{|\langle j_f||M(M1)||j_i\rangle|^2} \quad \ldots\ (2.13)$$

The mixing ratio 'δ' is expressed as the square root of equation 2.13 which is

$$\delta = \sqrt{\frac{3}{10}}\ q\ \frac{\langle j_f||M(E2)||j_i\rangle}{\langle j_f||M(M1)||j_i\rangle}, \text{ where } q = \frac{E\gamma}{Ec} \quad \ldots\ (2.14)$$

The multiple mixing ratio is therefore proportional to the energy E_γ of the transition.

2.5 The Sign Conventions

The sign of the mixing ratio δ is a physical quantity which indicates the relative phase between M1 and E2. The mixing ratio is the ratios of reduced matrix elements of different multipoles contributing to a gamma transition between a well defined nuclear states (in addition to the definition of 2.4). The mixing ratio is usually obtained from an angular distribution measurement of the gamma rays with respect to a fixed axis of quantization. The sign and magnitude of a mixing ratio is needed to fit the angular distribution measurement of the gamma rays with respect to a fixed axis of quantization. The sign and magnitude of a mixing ratio is needed to fit the angular distribution. Different authors, Biedeharn and Rose (1953), Rose and Brink (1967), Krane and Steffen (1970) have their definition of mixing ratio differing in sign convention.

It is important, while comparing the experimental values of the mixing ratio with its theoretical values to have the same sign convention.

The sign conventions employed for defining the sign of theoretical δ depends on

(a) the E(M)L operators, and

(b) the reduced matrix element

and the sign of experimental δ depends on the sign convention used to define (a) the axis of alignment with respect to which way the γ-ray angular distribution is measured. (b) the geometrical factors such as Clebsch-Gordan Coefficient and Racah Coefficients that entered the expression used for the expansion of the angular distribution probability in terms of various polynormials (see chapter four)

It is important to keep the definition of δ fixed and make the changes required by different experimental situations in the angular correlation relations. The sign convention of Biedenharn and Rose (1953) emoloyed in this work is compared to others; to ease reference

$$\delta(\gamma_1)_{BR} = -\delta(\gamma_1)_{KS} = \delta(\gamma_1)_{RB}$$

$$\delta(\gamma_2)_{BR} = \delta(\gamma_2) = -\delta(\gamma_2)_{RB}$$

Chapter 3.

NUCLEAR MODELS

Although there are several models developed to predict the mixing ratios - single particle shell model, Asymmetric Rotor Model, Rotation-vibration model and microscopic model, only two models are used in this work for predicting the mixing ratios. The single particle shell model and symmetric rotor model are discussed and used in obtaining the theoretical mixing ratio which are then compared with experimental values.

3.1 The Single-Particle Shell Model Weisskopf Unit

It is difficult to carry the general expressions for the transitions rates further than equation 2.6 without a specific model for the nuclear states. A simple model used here for predicting the mixing ratios is the single particle model. The assumption made is that only the valence nucleon, (a proton) makes the nuclear transition that generates the rediation field of the emitted photon. All other nucleous are assumed to be inert so that only one transition operator is considered for the proton that changes its orbit from $|i\rangle$ to $|f\rangle$. A single particle wave function consists of the radial part and the angular parts as follows:

$$|\ell j M\rangle = R_\ell(r) \sum_s \langle \ell\, m_\ell\, \tfrac{1}{2}\, s | j M\rangle \; Y_{\ell m}(\hat{r})\, X_{\frac{1}{2}s}. \qquad - 3.1$$

The electric Moment operator for the single-particle model is

$$M(EL, M) = e r^L \; Y_{LM}(\theta, \phi) \qquad - 3.2$$

and its reduced transition probability is given by

$$B(EL, J_i' \to j_f) = \frac{e^2}{2j_i+1} \; |\langle l_f j_f || r^L \; Y_{LM} || l_i j_i \rangle|^2 \qquad - 3.3$$

This is conviniently written as the product of radial and angular parts by MosZKowski (1968) as

$$B(EL) = \frac{e^2}{4\pi} \; |\int R_f^*(r) \; r^L \; R_i(r) \; r^2 dr|^2 \; S(j_i, L, j_f) \qquad - 3.4$$

Weisskopf (1951) assumed the statistical factor $S(j_i, L, j_f)$ to be unity and the radial part is integrated on the assumptions that

$$R_i(r) = R_f^*(r) = (4\pi)^{\frac{1}{2}} r^{\gamma} \quad 0<r<R_o$$
$$= 0 \quad , \quad r>R_o$$

For density model nuclei $\rho = (\frac{4}{3}\pi R_o^3)^{-L}$

and normalisation requires

$$\int_o^{R_o} |R(r)|^2 \, r^2 \, dr = |R(R)|^2 \, \frac{R_o}{3} = 1$$

The radial integral factor J_L

$$J_L = \frac{3}{R_o^3} \int_o^{R_o} r^{L+2} \, dr = \frac{3}{R_o^3} \left[\frac{r^{L+3}}{L+3} \right]_o^{R_o}$$

$$= [3/(L+3)] \, R_o^L \qquad - 3.5$$

where R_o is the nuclear radius and chosed as $1.2A^{\frac{1}{3}}$. With Weisskopf estimation of the statistical factor and radial integral, equation 3.4 now becomes

$$B^W(EL) = \tfrac{1}{4}\pi (\frac{3}{L+3})^2 \, e^2 \, R_o^{2L}$$
$$= \tfrac{1}{4}\pi (\frac{3}{L+3})^2 \, (1.2)^{2L} A^{2L/3} \text{ in } e^2 \text{ fm}^{2L} \qquad - 3.6$$

using this value, the $T^W(EL)$ is obtained as

$$T^W(EL) = \frac{4.38(L+1)}{L[(2L+1)!!]^2} \{\frac{3}{L+3}\}^2 [\frac{E}{197 \text{ Mev}}]^{2L+1} \times$$
$$\times (1.2)^{2L} \, A^{2L/3} \times 10^{21} \text{Sec}^{-1} \qquad -3.7$$

On the case of magnetic transition, it takes place when the intrinsic spin is flipped, i.e., $j_i = \ell_i \pm \frac{1}{2} \rightarrow j_f = \ell_f \mp \frac{1}{2}$, respectively. If $\ell_f = j_i - j_f = L$ and $\ell_i - \ell_f = L-1$, the reduced matrix element becomes

$$\langle \ell_f j_f || M(ML) || \ell_i j_i \rangle = (Lg_s - \frac{2L}{L+1} g_\ell)\ \mu_N \left\{\frac{(2L+1)\ (2j_i+1)}{4\pi}\right\}^{\frac{1}{2}}$$

$$x \langle j_i \tfrac{1}{2}\ L0 | j_f \tfrac{1}{2} \rangle J_{L-1}$$

and thus

$$B(ML, j_i \to j_f) = S(j_i, L, j_f)\ (Lg_s - \frac{2L}{L+1} g_\ell)^2\ \frac{\mu^2 N}{4\pi}\ J^2_{L-1}$$

The Weisskopf (1951) estimate for magnetic multipole transitions is obtained by the approximations

$$(Lg_s - \frac{2L}{L+1} g_\ell)^2 \to 40 \qquad - 3.10$$

and statistical and radial factors as in electric multipole transitions.

Hence equation 3.9 becomes

$$B^W(ML) = \frac{10}{\pi} \left[\frac{3}{L+3}\right]^2\ \mu^2 N\ R^{2L-3}$$

$$= \frac{10}{\pi} \left[\frac{3}{L+3}\right]^2\ (1.2)^{2L-2}\ A^{(2L-2)/3} \text{in } \mu^2 N(\text{fm})^{2L-2}$$

$$-3.11$$

and

$$T^W(ML) = \frac{1.94\ (L+1)}{L[(2L+1)!!]^2} \left[\frac{3}{L+3}\right]^2 \left(\frac{E}{197\text{Mev}}\right)^{2L+1}$$

$$x(1.2)^{2L-2} A^{(2L-2)/3} x10^{21} \text{Sec}^{-1} \qquad - 3.12$$

The single-particle estimates constitute convinient "units", in which the experimental observed properties can be expressed and thus give a qualitative picture of the nuclear properties.

3.1.1 The E2/M1 mixing ratios

The multipole mixing ratios has already been defined in section 2.4 as the ratio of electric quadropole gamma-ray transition probability to the magnetic dipole gamma-ray transition probability. Hence, equation 2.11 becomes

$$\delta^2 (E2\text{-}M1, j_i \to j_f) = \frac{\text{No of E2 Transitions/sec}}{\text{No of M1 Transitions/sec}} \qquad - 3.13$$

When the Weisskopf (1951) estimated values are substituted in 3.13

$$\delta_W^2(E2/M1) = \frac{\frac{4.38(3)}{2[(5)!!]^2}\left(\frac{3}{5}\right)^2\left(\frac{E}{197}\right)^5 (1.2)^4 A^{4/3} \times 10^{21}\,\text{sec}^{-1}}{\frac{1.94\ (2)}{[(3)!!]^2}\left(\frac{3}{4}\right)^2\left(\frac{E}{197}\right)^3 \times 10^{21}\,\text{sec}^{-1}} \qquad - 3.14(a)$$

$$|\delta_W(E2\text{-}M1)| = 1.521 \times 10^{-3} E_\gamma \text{ (in Mev)} A^{2/3} \qquad - 3.14(b)$$

The single-particle model relation 3.14(b) indicates that the M1 mode of r-ray emussion predominates over the E2 mode. Although this model gives a good account of the nuclear collectivity and shell closure, every nucleon in its energy state has definite angular momentum and magnetic moment thereby breaking away the picture of closely packed nucleons in the nucleus.

3.2 Asymmetric Rotor Model (ARM) (Davydov - Fillipov Estimate).

Davydov and Fillipov (1958) have extended the rotation- vibrational model of Bohr and Mottelson (1952, 1953) which was unable to explain certain parameters in the energy level formula and sometimes very large deviations in the branching ratios and applied it to the problem of E2/M1 mixing ratios. The low -lying states of an even-even nucleus are described in terms of rotation without change of shape (i.e. nuclei that do not possess axial symmetry).

According to them the nucleonsin the nucleus move nearly independent of each other in a constant varying potential energy field. The variation in the potential is slower than the motion of individual nucleons. It is this collective motion of the entire atomic nucleus that accounts for the rotational and vibrational energy state of the nucleus. This

rotational effect can be either rigid, in which case the nucleons actually move in circles around the axis of rotation, or wave-like, in which case they perform oscillatory motions thereby changing the shape of the nucleus as schematically illustrated in figure 3.1.

The deformed nuclei are far from a spherical shape in that they are not rigid and nucleons outside the close shell can set up tension in the closed shell core, thereby establishing polarisation of the nucleus to form a prolate spheroid as illustrated in figure 3.2(b). On the other hand, if the forces are ather hand, if the forces are attractive, polarisation is accomplished by pulling out the "equator" plane to form an oblate as shown in figure 3.2(c).

The nuclear shape is defined in the intrinsic (rotating) frame by two variables β and γ, where β gives the magnitude of nuclear deformation and γ gives deviations from axial symmetry ($\gamma=0^{\circ},120^{\circ}$..... for prolate shapes; $\gamma=60^{\circ},180^{\circ}$..... for oblate shapes). The Parameter γ in other words, called a gamma-vibrational band is a temporary change in the magnitude of its rotational band in deformed even-even nuclei. These bands besides the ground state rotational band in even-even nuclei, are formed from the coupling of nuclear rotation to a variety of vibrational oscillation. Similarly.

Lange et al (1982) used shrödinger equation to determined the energy levels and wave-functions of an asymmetric top.

$$H_{rot}\ \psi_{nJm}(\emptyset 0\psi) = E_{nJ}\ \psi_{nJM}(\emptyset\theta\psi) \qquad .. \quad 3.15$$

where ψ_{nJM} is a wave function in the space of the three Euler's angles ($\emptyset\theta\psi$) which connect the intrinsic frame axes to the lab frame axes; J denotes the nuclear angular momentum; M is the projection of J on the lab Z-axis; n=1,2,3....denote the first (lowest), second,third. .....state with the same J, and the Hamiltonian H_{rot} is given by

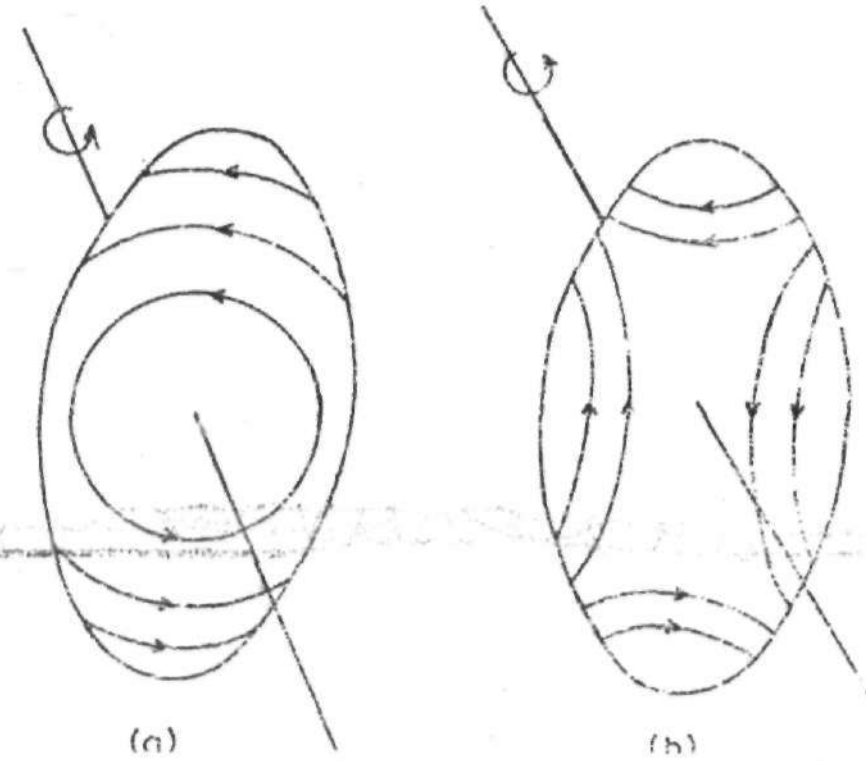


Fig 3.1 PICTORIAL ILLUSTRATION OF (a.) RIGID (b) WAVE-LIKE ROTATIONAL MOTION OF DEFORMED NUCLEUS.

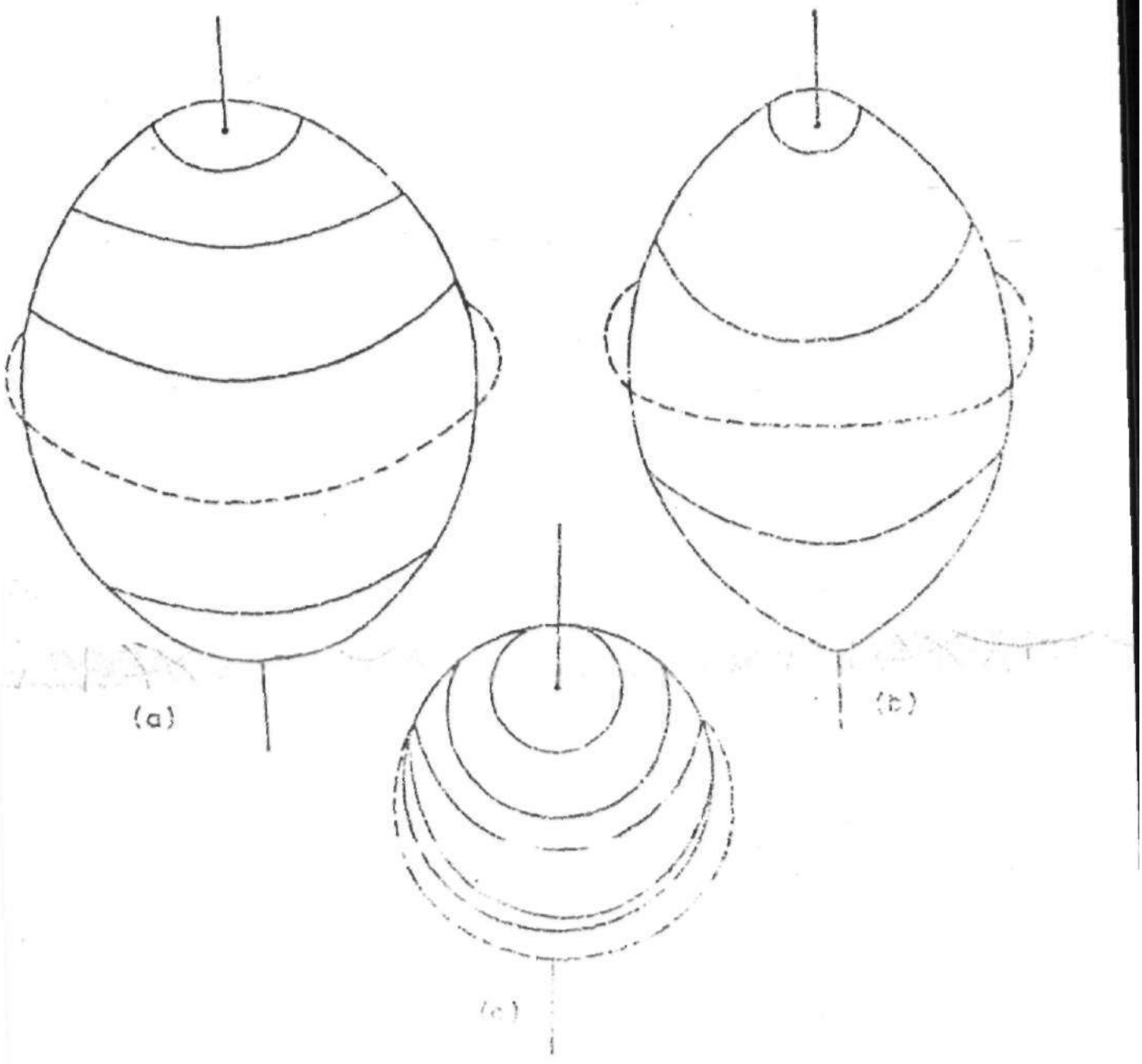


Fig. 3.2 THE NUCLEONS OUTSIDE CLOSED SHELLS ARE BELIEVED TO INTERACT WITH THE NUCLEAR CORE IN SUCH A WAY THAT THEY CAUSE THE CORE TO DEFORM FROM (a) SPHERICAL INTO (b) PROLATE (c) OBLATE SPHEROIDAL SHAPE [Adapted from C. S Cook structure of atomic nuclei D. Van Nastrand Co. Inc. Princeton, New Jersey (1964)]

$$H_{rot} = \sum_{k=1}^{3} J_k^2/(2I_k) \qquad - \quad 3.16$$

Where J_k is the projection of J on the intrinsic axis k and I_k is the k th component of the nuclear moment of inertia. The three moments of inertia are given by

$$I_k = 4B\beta^2 \sin^2(\gamma - \frac{2}{3}\Pi\, k) \qquad - \quad 3.17$$

The theory of Electormagnetic operation is needed to calculate the elctromagnetic moment. In collective model the contribution of individual nucleous are replaced by a volume integral over a uniform distribution of electric charges and magnetic moments.

$$\rho = \rho_o \qquad r \leq R(\Omega)$$
$$= 0 \qquad r > R(\Omega)$$

The nuclear surface is defined by Kumar et al (1982) as

$$R(\Omega) = R_o[1 + \sum_{\mu} a_{\mu}\, Y_2^{*\mu}(\Omega)] \qquad - \quad 3.18$$

Where a_μ ($\mu = -2,-1,0,1,2$) denotes the quadrupole deformation tensor in the lab system, R_o is the radius of the spherical nucleus and Zero outside and $R(\Omega)$ defines the location of points on the surface.

On the basis of 3.18, Kumar (1982) derive the general formulae for E2, M1 moments. In the lab system, the E2 operator is written as

$$M(E2,\mu) = \frac{3}{4\Pi} Ze\, R_o^2\, a_\mu \qquad - \quad 3.19$$

and the M1 operator as

$$M(M1,\mu) = \mu_N g_R [\{\frac{3}{4\Pi}\}^{\frac{1}{2}} J_\mu + \frac{5\sqrt{6}}{7\Pi} \sum C^2_{\mu-\nu\,\nu''\mu}\, d_{\mu-\nu}\, J_\nu] \qquad - \quad 3.20$$

Where Z is the nuclear charge, μ_N the nuclear magneton, g_R the nuclear gyromagnetic ratio, J the total angular momentum, and C a clebsch - Gordon Coefficient.

3.2.1 E2/M1 mixing ratios.

The Davydov-Fillipov theory has predicted successfully many rotational levels including those of higher excitation. Through this the ratios of the various reduced transition probabilities has been determined. Actually, this transition probabilities depend on three parameters (i.e. mass-parameter B, nuclear deformation parameter β and the asymmetry angle parameter γ.

The wave functions of the rigid rotor model defined by equation 3.15 - 3.16, and the electromagnetic operator of equation (3.19) and (3.20), the E2/M1 transition probabilities and the mixing ratios can be obtained.

The reduced transition probability for an electric quadrupole transition is described by Eisenberg and Greiner (1975) *is given as*

$$B(E2;\ j_i \to j_f) = \left\{\frac{3ZeR_0^2}{4\pi}\right\}^2 \frac{2j_f+1}{2j_i+1} |\langle j_i||Q_2||j_f\rangle|^2 \qquad - 3.21(a)$$

$$B(E2;\ 2'^+ \to 2+) = \frac{9Z^2e^2R_0^4}{16\pi^2} \cdot \frac{2}{7} \frac{\beta_0^2 \sin^2(3\gamma_0)}{[9-8\sin^2(3\gamma_0)]}$$

$$= \frac{9Z^2e^2R_0^4 \beta_0^2 \sin^2(3\gamma_0)}{56\ \pi^2[9-8\ \sin^2(3\gamma_0)]} \qquad - 3.21(b)$$

and

$$B(M1;2'^+ \to 2^+) = \frac{90\mu_N^2 g_R^2 \beta_0 \sin^2(3\gamma_0)}{49\pi^2[9-8\ \sin^2(3\gamma_0]} \qquad - 3.22$$

and the square of the mixing ratio by

$$\delta^2_{DF}\ (E2/M1;2'^{+}\rightarrow 2^{+}) \quad = \quad \frac{3q^2}{100} \cdot \frac{7}{80} \left[\frac{ZeR_o^2}{g_R \eta_N}\right] \qquad - \ 3.23$$

With $R_o = 1.2A^{1/3}$ fm and $g_R = Z/A$, the gyromagnetic ratio for a uniformly charged nucleus (Bohr and Mottelson, 1969). Equation (3.23) becomes

$$\delta_{DF}(E2/M1;2'^{+}\rightarrow 2^{+}) \quad = \quad 3.58 \times 10^{-3}\ E_\gamma \text{ (in Mev) } A^{5/3} \qquad - \ 3.24$$

In this model, $|S|$ is independent of nuclear shape. Hence the result is valid for the most asymmetric shape ($r=30^o$) as well as for an axially symmetric shape ($r=0^o$ or 60^o). It is however, true that the interpretation of the $2'^{+}$ state is quite different in the asymmetric rotor model of Davydov and Fillipov (1958) and in the axially symmetric rotor model of Bohr and Mottelson (1969).

In anxially symmetric model, two 2^+ states are determined by diagonalizing Hamiltonian H(3.14) in a basis of K=0 and 2 states: the lower 2^+ state mostly is K=0, the upper one mostly is K=2. In Davydov-Filippov model, the $2'^{+}$ state is generally associated with γ vibrational band (mostly K=2) (see figure 3.3).

The Bohr and Moltleson (1969) model allows the interpretation of large number of observed states which *are not* quite differnt from that of Daydov-Filippov method that predicts a useful, parameter-independent estimate for δ (equation 3.24).

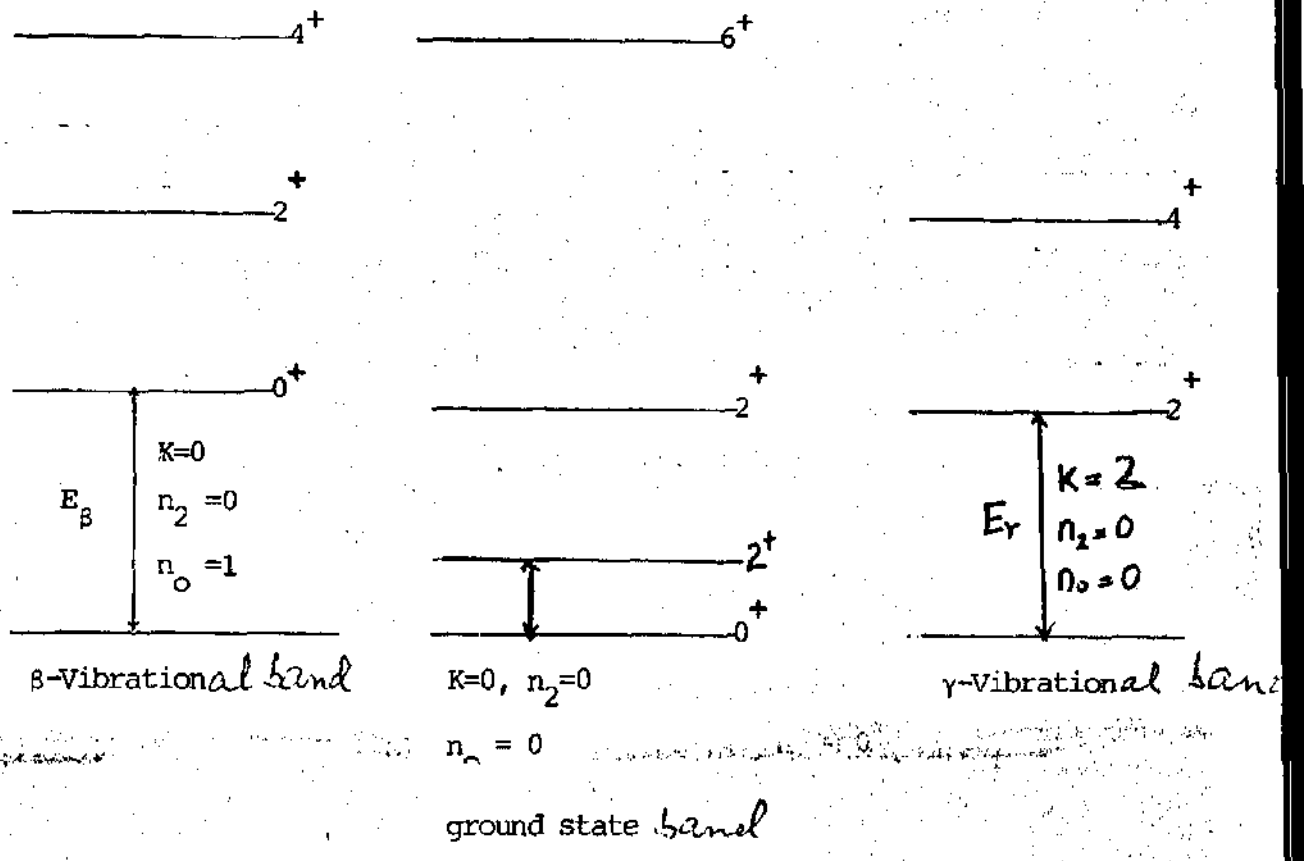


Figure 3.3 Typical band structure for a deformed even-even nucleus.

Chapter 4

ANGULAR CORRELATION

The coincidence observation of two radiations emitted by a nucleus yields a correlation in their relative propagation direction. The simplest type of angular correlation process consists of the measurement of coincidence rate of two successive nuclear radiations as a function of the angle between the propagation directions of these radiations. In measuring the angular correlation, the angular distribution of one radiation is observed when it is known that the other has a fixed direction. The fixed direction gives information about the substates of the nuclear level formed after the emission of this radiation. In particular it asserts that these sublevels are not uniformly populated - thus radiation emitted are not isotropic. Also an isolated nucleus has to be considered so that there is no appreciable interaction depending on the orientation of the nuclear spin in the intermediate state under the influence of possible spin in the intermediate state under the influence of possible spin couplings.

4.1 Theory of γ-γ Angular Correlation

The directional correlation function $W(\theta)[W(\vec{K}_1\vec{K}_2)]$ is the angle between the two gamma-rays of a nucleus that decays through a cascade $j_i \to j \to j_f$ by successive emission of two gamma-rays of multipolarity L_1 and L_2.

A nucleus decays from the initial level i with spin j_i, described by the density matrix ρ_i, to the intermediate level with spin j; the γ_1-radiation emitted is observed *in the direction of* K_1,

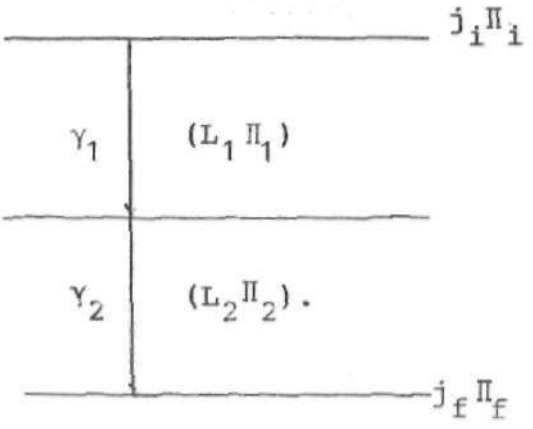


Figure 4.1 Quantum numbers involved in a nuclear radiation.

. Through the density matrix $\rho(\vec{K}_1)$ the intermediate state j is fully described; the argument $\vec{K}_1$ indicates that the density matrix depends on the direction $\vec{K}_1$ in which the γ_1-radiation has been observed. Similarly, the second transition $j \rightarrow j_f$, could be expressed with the final density matrix as $\rho_f(\vec{K}_1, \vec{K}_2)$.

In order to derive the correlation function $W(K_1, K_2)$, the first transition $j_i \rightarrow j$ is first considered and from equation A.30 (of the Appendix) $a = m_i, b = m$ which yields for the density matrix $\rho(K)$ in the intermediate state.

$$\langle m|\rho(K_1)|m'\rangle = S_1 \sum \langle m|H_1|m_i\rangle\langle m_i|\rho_i|m_i\rangle\langle m'|H_1|m_1'\rangle^* \quad \ldots 4.1$$

In equation 4.1 and others to follow, angle independent factors have been set equal to one. Where S_1 stands for summation over unmeasured radiation properties, such as spin and polarization. The matrix element $\langle m|H_1|m_i\rangle$ stands for $\langle j, m, k_1, \sigma_1|H_1|j_i m_i\rangle$; the polarization σ_1 is given by the component of the spin of σ_1 in the direction

polarization σ_1.

When A.30 is used again for the second transition and $a=m, b = m_f$, the density matrix in the final state is

$$\langle m_f|\rho_f(K_1,K_2)|m_f'\rangle = S_1S_2 \sum_{mm',\, m_i} \sum_{m_i} \langle m_f|H_2|m\rangle\langle m|H_1|m_i\rangle$$

$$\times \langle m_i|\rho_i|m_i'\rangle\langle m'|H_1|m_i'\rangle^* \langle m_f'|H_2|m'\rangle$$

.... 4.2

The probability $P_f(m_f)$ of finding the nucleus in the final state m_f while the two radiations γ_1 and γ_2 are observed in the direction K_1 and K_2 is given by the diagonal element of ρ_f:

$$P_f(m_f) = \langle m_f|\rho_f(K_1,K_2)|m_f\rangle \quad \ldots. \quad 4.3$$

The desired function $w(\vec{K}_1,\vec{K}_2)$ is identical to the probability (equation 4.3).

From equation A.24 and 4.2, the directional correlation function (as shown in the appendix)now becomes

$$W(K_1K_2) = S_1S_2 \sum_{m_f mm' m_i} \langle m_f|H_2|m\rangle\langle m'|H_1|m_i\rangle\langle m'|H_1|m_i\rangle.$$

$$\times \langle m_f|H_2|m'\rangle^* \quad \ldots. 4.4$$

Equation 4.4 can be rewritten by using density matrices for their respective transition.

$$W(K_1,K_2) = \sum_{mm'} \langle m|\rho(K_1)|m'\rangle\langle m'|\rho(K_2)|m\rangle \quad \ldots. 4.5$$

A transformation matrix is needed to relate the matrix element with that of the multipole expansion, which gives the properties of the photon in terms of angular momentum and parity. The connection between the 'observed' mastric element $\langle jmk\sigma |H| j_i m_i\rangle$ the 'desired' matrix element $\langle jmL M\Pi|H|j_i m_i\rangle$ is given by

$$\langle jmk\sigma |H| j_i m_i\rangle = \sum_{LM\Pi} \langle k\sigma |LM\Pi\rangle \langle jmLM\Pi|H|j_i m_i\rangle \quad \ldots \quad 4.6$$

There is no sum over Π since parity is conserved. Using the arguments AA.1 and equation (A.1) and (A.2),

$$\langle K\sigma|LM\Pi\rangle = \Sigma \langle 0\sigma|L\mu\Pi\rangle D^{L*}_{M\mu} \quad (2\rightarrow K) \quad \ldots \quad 4.7$$

The term $\langle jmLM\Pi|H|j_i m_i\rangle$ of equation 4.6 can be expressed as the angular part of the state $\langle jmLM\Pi| \equiv \langle jm|\langle LM\Pi$ in terms of states $\langle j_i' m_i'|$ by Clebsch-Gordan Coefficients

$$\langle jmLM\Pi|H|j_i m_i\rangle = \sum_{j_i' m_i'} \langle jmLM| j_i' m_i'\rangle \langle j_i' m_i' jL\Pi|H| j_i m_i\rangle \quad \ldots \quad 4.8(a)$$

Since H for physical reasons is invariant under rotations the matrix element $j_i' m_i' jL\Pi|H|j_i m_i\rangle$ is non zero when $j_i = j_i'$, $m_i' = m_i$, and is independent of m_i (magnetic quantum number).

The resulting "reduced" matrix element $\langle j||L\Pi||j_i\rangle$ which comes from equation 4.8a is $\langle jmLM\Pi|H|j_i m_i\rangle = \langle jm|j_i m_i\rangle \langle j||L\Pi||j_j\rangle$ .. 4.8(b) and can be chosen to be real.

When equations (4.6), (4.7) and (4.8b) are combined the matrix element $\langle m|H|m_i\rangle$ takes the form

$$\langle m'|\rho(K_2)|m\rangle = \sum_{L_2L_2'} \sum_{\ell_2 m_2 m_2'} (-1)^{j-m-\ell_2} (2j+1)\langle j_f\|L_2\Pi_2\|j\rangle$$

$$\times \langle j_f\|L_2'\Pi_2'\|j\rangle\langle jmj-m'|\ell_2 m_2'\rangle$$

$$\times W(jj_fL_2';L_2j)C_{\ell_2\tau_2}(L_2'L_2)\, D^{\ell_2}_{\tau_2'\tau_2}$$

... 4.12(b)

These are now inserted into equation 4.5 for $W(\theta)$, using the relation (A.12) and (A.4),

$$W(\theta) = \sum_{L_1L_1'L_2L_2'\ell\tau_1\tau_2} (-1)^{\ell}|\langle j\|L_1\Pi_1\|j_i\rangle\langle j_i\|L_1'\Pi_1'\|j_i\rangle|^2$$

$$\times \langle j_f\|L_2\Pi_2\|j\rangle\langle j_f\|L_2'\Pi_2'\|j\rangle W(jj_f\ell L_2';L_2j)$$

$$\times C_{\ell\tau_1}(L_1L_1)C_{\ell\tau_2}(L_2L_2')D^{\ell}_{\tau_1\tau_2}(\theta)W(jj_i\ell L_1';L_1j)$$

... 4.13

The factors $\langle j_f\|L\Pi\|j_i\rangle$ are physical parameters (that is, reduced matrix element) characteristic of the nuclear transition, involved and, in particular, of the state emitting or absorbing the radiation, and are thus independent of orientation and of the directions of motion. There are physical parameters in the sense that they are dependent on a physical model. The factors $C_{\ell\tau}(LL')$ are characteristic of the particles emitted and upon the multipolarity and parity of the transitions considered through the parameters $L' - \mu' \langle O\sigma|\mu\Pi\rangle$, but are in addition, dependent on the information specified as defined by the physical experiment under discussion.

If the gammas are of pure multipole radiations,

$$\langle m|H|m_i\rangle \equiv \sum_{L\,M\mu} \langle O\sigma|L\mu\Pi\rangle D^{L}_{M\mu} \langle jLmM|j_i m_i\rangle \langle j||L\Pi||j_i\rangle$$

... 4.9

When this expression is used for respective density matrices, equation (4.5) is reduced by using the relations (A.3), (A.5), (A.13) and (A.15). In the final expression, the factors that depend only on the properties of radiation are singled out and Racah introduced it as 'radiation parameter '.

$$C_{\ell\tau}(LL') = \sum_{\mu\mu'} (-1)^{L'-\mu'} \langle L\mu L' - \mu'|\ell\tau\rangle \langle O\sigma|L\mu\Pi\rangle^{*} \langle O\sigma'|L'\mu'\underline{\Pi}\rangle$$

... 4.10

These parameters satisfy the relation

$$C_{\ell\tau}(LL') = (-1)^{L-L'-\tau} C^{*}_{\ell-\tau}(L'L). \qquad \text{... 4.11}$$

They are characteristic of the emitted radiation through the transformation matrices $\langle O\sigma|L\mu\Pi\rangle$ and also eigenfunctions of the total angular momentum operator with eigenvalues ℓ,τ.

Thus, the matrices $\rho(K_1)$ and $\rho(K_2)$.

$$\langle m|\rho(K_1)|m'\rangle = \sum_{L_1 L_1'} \sum_{\ell\tau_1\tau_1'} (-1)^{j-m-\ell_1} (2j_i+1) \langle j||L_1\Pi_1||j_i\rangle$$

$$\times \langle j_i||L_1'\Pi'||j_i\rangle \langle jm\ j-m'|\ell_1\tau_1'\rangle$$

$$\times W(jj_i\ell_1 L_1';L_1 j_i)\ C_{\ell_1\tau_1}(L_1'L_1) D^{\ell 1}_{\tau_1'\tau_1} \qquad \text{... 4.12(a)}$$

(i.e. $L_1 = L_1'$ and $L_2 = L_2'$), the reduced matrix element becomes constants and are factored out. Hence

$$W(\theta) = \sum_{\ell\tau_1\tau_2} (-1)^{\ell} W(jj_i \ell L_1; L_1 j) W(jj_f \ell L_2; L_2 j)$$

$$X \; C_{\ell\tau} (L_1 L_1) \; C_{\ell\tau_2} (L_2 L_2) \; D^{\ell}_{\tau_1\tau_2}$$

.... 4.14

The correlation function for the two cascades $j_i(L_1)\, j\, (L_2)\, j_f$ and $j_f\, (L_2)\, j(L_1) j_i$ of the relation (4.13) are the same. The Racah Coefficients $W(jj\, L'L;\, \ell j_i)$ vanishes unless each of the triplets $(jj\ell)$, $(L'L\ell)$, $(jL'\, j_i)$, and (jLj_i) forms a triangle This leads to the selection rule for ℓ.

$$0 \leq \ell \leq \text{Min } (2j,\, 2L_2,\, 2L_2) \qquad \ldots \quad 4.15$$

and ℓ is always even for linearly polarized.

Experimentally, only the directions are observed not the polarisations of the two radiations γ_1 and γ_2. The correlation functions is independent of the *euler* angles, which denote relations about the directions of propagation $\vec{k}_1$ and $\vec{k}_2$. Independence of *euler* angles means $\gamma=0$.

With $\gamma_1 = \gamma_2 = 0$, the representation D^L become legendre Polynomials, and equation 4.14 is expressed as

$$W(\theta) = \sum_{\ell=\text{even}} A_{\ell\ell}\, P_\ell\, (\cos\theta) \qquad \ldots \quad 4.16$$

Franenfelder and steffer (1968) brake up $A_{\ell\ell}$ into two factors, each factor depending on only one transition of the cascade

$$A_{\ell\ell} \quad F_\ell(L_1 L_1 j_i j)\; F_\ell\, (L_2 L_2 j_f j) \qquad \ldots. \quad 4.17$$

From equations (4.14) and (4.15),

$$A_\ell = W(jj_i\ \ell L;\ Lj)\ \ C_{\ell 0}\ (LL) \qquad \ldots\ 4.18$$

Where $C_{\ell 0} \propto (-1)^{L-1}(2L+1)\ \ \langle L\ L-1 | \ell 0\rangle$

$$X\ W(jj_i\ \ell L; Lj) \qquad \ldots\ 4.19$$

Thus, the coefficient F_ℓ can be calculated numerically from Clebsch-Gordan and Racah coefficients. The F_ℓ coefficient have been calculated by Frauenfelder and Steffen (1968)

4.2 Gamma-Gamma Angular Correlation Measurments

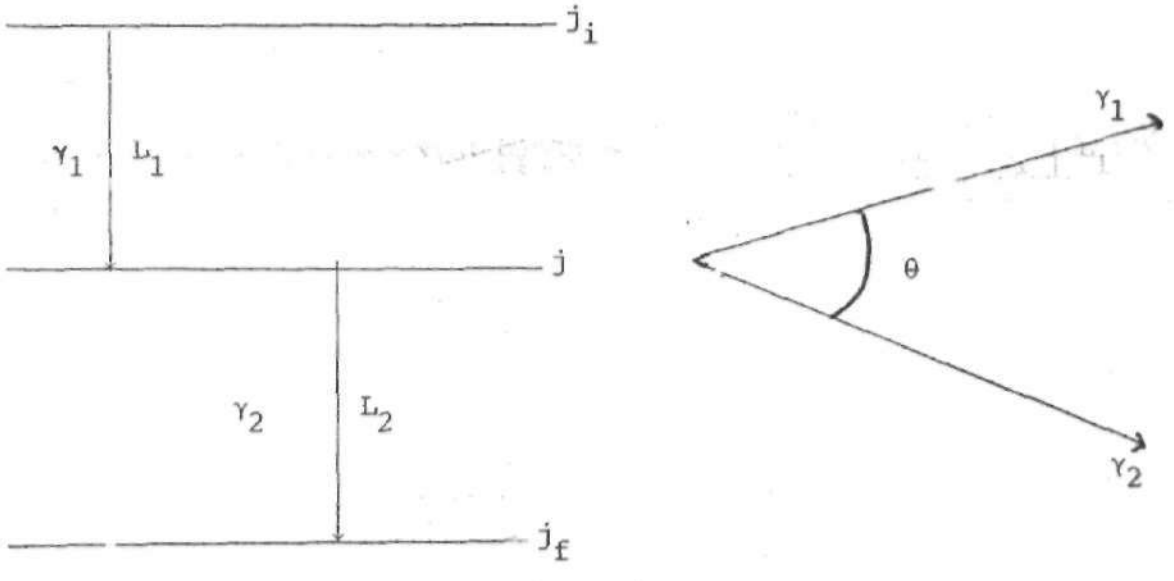


Figure 4.2 Directional Correlation of two gamma-rays in cascade

The measurement of angular correlation of nuclear radiation is a powerfful technique for determining the spins and parities of nuclear excited states. The origin of these gamma rays is frequently the cascade decay of a nucleus and should be an anisotropy (Figure4.2). The nuclear excited state (spin j_i) de-excites by a cascade of gamma

rays γ_1 and γ_2 through an intermediate state (spin j) to the final state (spin j_f). The multipolarities of the gamma rays γ_1 and γ_2 are L_1 and L_2 (both pure multipoles) respectively.

In studying the correlation function of the angle θ between the directions of the emission of γ_1 and γ_2, the Correlation function describing the coincidence counting rate N_{12}, normalised to $\theta = 90^{\circ}$, is given

$$W(\theta) = 1 + A_{22}P(\text{Cos }\theta) + A_{44}P_4(\text{Cos }\theta) + \ldots\ldots$$
$$= 1 + A_{\ell\ell}\, P_{\ell}\ (\text{Cos }\theta) \qquad \ldots\ 4.20$$

Where $A_{\ell\ell}$' are certain coefficients depending on L_1L_2, j_i,j and j_f and summation over ℓ is carried up to ℓmax which is as expressed in equation 4.15. In gamma-gamma directional correlation, measurements are limited to $\ell = 4$ because as ℓ increases, the $A_{\ell\ell}$ coefficients decreases rapidily and thus generally

$$W(\theta) + 1 + A_{22}P_2(\text{Cos }\theta) + A_{44}P_4(\text{Cos }\theta) \qquad \ldots\ 4.21$$

These coefficient can be broken up as already described in equations (4.16) - (4.19).

The directional correlation function for a double gamma cascade in which both the gamma rays have a mixture of multipole as shown in Figure 4.3 having components L_1, L_1' for γ_1 and L_2,L_2' for γ_2 is still given by equation (4.21).
However, the $A_{\ell\ell}$ coefficients are described by Frauenfelder and steffen (1968) as

$$A_{\ell\ell} = A_{\ell}\ (L_1L_1'\ j_i j)A_{\ell}\ (L_2L_2'j_f j)$$

$\ldots\ldots$ 4.22

with

$$A_\ell(L_1L_1'j_ij) = F_\ell(L_1L_1j_ij) + 2\delta_1(\gamma_1)F_\ell(L_1L_1'j_ij) + \delta_1^2(\gamma_1)\ F_\ell(L_1'L_1'j_ij)\Big] / (1+\delta_1^2(\gamma)) \quad \ldots 4.23$$

where the $\delta_1(\gamma_1)$ is the mixing ratio defined as the ratio of reduced matrix elements for the emission of L' to that of L radiation

$$\delta_1(\gamma_1) = \langle j||L_1'\Pi_1'||j_i\rangle / \langle j||L_1\Pi_1||j_i\rangle \quad \ldots 4.24$$

The ratio δ^2 is thus equal to the ratio of the total intensity of L'-Pole to that of the L-Pole gamma transition.

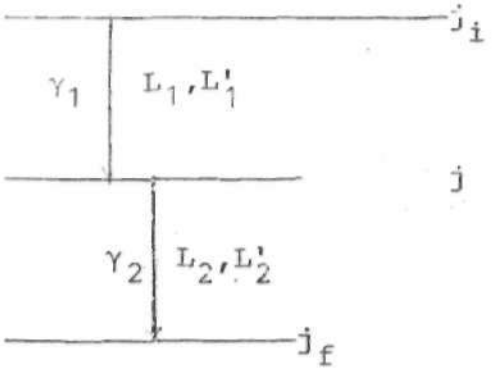


Figure 4.3 Gamma-ray cascade involving gamma rays having mixture of multipoles.

4.3 Theoretical Plot of Angular Correlation Coefficients Versus the Mixing Ratios

There are various methods of interpreting angular correlation data when one transition is pure and the other mixed. The two methods considered here are as described by Arns and Wiedenbeck (1958), and Coleman (1958).

4.3.1 Arns and Wiedenbeck method

Arns and Wiedenbeck (1958) suggested the method of determing the multipole mixing ratio if one transition of gamma-gamma cascade is mixed. It was suggested that the equation 4.23 be used for $\ell = 2$ and defining $Q = \delta^2/(1+\delta^2)$ When $\delta = \pm\sqrt{Q/(1-Q)}$ Equation 4.24 now takes the form

$$A_\ell = (1-Q)F_\ell(11j_ij) \mp 2\sqrt{Q(1-Q)}F_\ell(12j_ij) + QF_\ell(22j_ij) \quad \ldots. 4.25$$

and

$$A_{\ell\ell} = \left\{(1-Q)F_\ell(11j_ij) \mp 2\sqrt{Q(1-Q)}F_\ell(12j_ij) + QF_\ell(22j_ij)\right\} \times F_2(22\ j_fj) \quad \ldots. 4.26(a)$$

$$A_{22} = \left\{(1-Q)F_2(1122) \mp 2\sqrt{Q(1-Q)}F_2(1222) + QF_2(2222)\right\} \times F_2(2202) \quad \ldots. 4.26(b)$$

The value of Q runs from zero to one i.e. $0 \leq Q \leq 1$ and the F_2 coefficient are either calculated by the equation (4.19) or taken from the Table.

$$F_2(1122) = -0.4183,\ F_2(1222) = -0.5124$$
$$F_2(2222) = 0.1281 \quad F_2(2202) = -0.5976$$

When this values are put into equation (4.26(b) the value of A_{22} is obtained and a theoretical plot of A_2 versus Q gives an ellipse.

4.3.2 Coleman method (1958)

Coleman introduces the use of both A_2 and A_4 values for evaluating the mixing ratio 'δ' if only one transition is mixed in gamma-gamma directional correlation experiment.

The same equation (4.23) is used, when $\ell = 2$ and 4 for A_2 and A_4 coefficient respectively. One draws the curve $A_2 = A_2(\delta)$, $A_4 = A_4(\delta)$ in an A_2 A_4-Plane. This curve is labelled δ, or, more conveniently with the values of the parameter $\delta' = |\delta/(1-\delta)|$.

Chapter 5

RESULTS AND DISCUSSION

5.1 General Methods and Format of the Table

Table 5.1 provides averaged data on the magnitude and sign of the E2/M1 mixing ratios of $2'^{+} \rightarrow 2^{+}$ gamma transitions which were obtained from the quoted angular correlation coefficients (see appendix G)

The first column of the Table gives the nucleus and next is the mean value of δ taken from Coleman (1958) method and the next four column gives δ as explained by Arns and Wiedenbeck, when the values δ of the two are compared with the one taken from literature in magnitude and sign is taken. The sign Convention of the mixing ratio δ is that of Biedeharn and Rose (1951). The last three columns gives the mean value of the δ taken and δ value taken from literature and

$\Delta = \delta / 0.835 E_\gamma$

$_{54}Xe_{80}^{134}$	+0.05	0,003	-0.055	0.860	+2.473	-0.05	-0.04	-0.078
$_{56}Ba_{76}^{132}$	-5.67	0.300	-0.655	0.975	-6.24	-6.24	$-(9.6^{+7.6}_{-2.9})$	-13.180
$_{56}Ba_{78}^{134}$	-7.00	0.275	-0.616	0.990	-9.950	-9.950	$+(7.6^{+0.8}_{-0.6})$	-21.166
$_{56}Ba_{80}^{136}$	-0.18	0.035	-0.190	0.950	+4.359	-0.19	$-0.21 \leq \delta \leq -0.19$	-0.214
$_{56}Ba_{84}^{140}$	+0.31	0.105	+0.343	0.540	+1.08	+1.08	$+(1.1^{+0.14}_{-0.10})$	+1.425
$_{58}Ce_{82}^{140}$	-0.43	0.125	-0.380	0.995	+14.11	-0.380	-0.34	-0.605
$_{58}Ce_{84}^{142}$	+0.27	0.075	+0.285	0.590	+1.200	+0.27	+0.10	+0.361
$_{60}Nd_{84}^{144}$	+0.32	0.015	+0.123	0.720	+1.60	+1.60	+1.60	+2.215
$_{62}Sm_{90}^{152}$	-9.00	0.275	-0.616	0.990	-9.950	-9.950	-8^{+9}_{-3}	-17.320
$_{64}Gd_{88}^{152}$	+0.39	0.003	+0.055	0.800	+2.00	+2.00	2.0 ± 5	+4.087
$_{64}Gd_{90}^{154}$	-9.00	0.250	-0.577	0.990	-9.950	-9.950	-11 ± 3	-17.195
$_{64}Gd_{92}^{156}$	+0.46	0.060	-0.253	0.970	+5.690	+5.690	$+(5.9^{+1.4}_{-2.8})$	6.552
$_{64}Gd_{94}^{158}$	+0.17	0.050	-0.230	0.965	+5.251	-0.23	not deduced	-0.291
$_{66}Dy_{94}^{160}$	+0.47	0.100	-0.333	0.990	+9.950	+9.950	$+(9.7^{+2.5}_{-1.7})$	+13.557
$_{70}Yb_{104}^{174}$	-0.25	0.06	-0.250	0.965	+5.251	-0.25	0.26 ± 0.04	-0.241
$_{72}Hf_{102}^{174}$	-0.48	0.155	-0.430	0.997	+18.23	-0.43	$0 \geq \delta^{-1} \leq -0.24$	-0.637
$_{72}Hf_{106}^{178}$	-0.42	0.145	-0.410	0.997	+18.23	-0.41	$+(0.41^{+0.036}_{-0.035})$	-0.415
$_{72}Hf_{108}^{180}$	-0.17	0.040	-0.200	0.955	+4.607	-0.20	-0.20	-1.114
$_{74}W_{108}^{182}$	-9.00	0.230	-0.547	0.995	-14.11	-14.11	-12^{+2}_{-1}	-15.074

$^{184}_{74}W_{110}$	+0.49	0.120	-0.369	0.995	+14.11	+14.11	+15.00	+21.309
$^{186}_{76}Os_{110}$	+0.47	0.100	-0.333	0.990	+9.950	+9.950	$+(50^{+\infty}_{-4})$	+18.915
$^{188}_{76}Os_{112}$	+0.47	-.105	-0.343	0.995	+14.11	+14.11	$+(23^{+9}_{-5})$	+35.352
$^{190}_{76}Os_{114}$	+0.47	0.105	-0.343	0.995	+14.11	+14.11	$+(9^{+8}_{-3})$	+45.548
$^{192}_{78}Pt_{114}$	-9.00	0.255	-0.585	0.990	-9.950	-9.950	-8.84	-40.257
$^{194}_{78}Pt_{116}$	-39.00	0.180	-0.469	0.995	-14.11	-14.11	$-(19^{+5}_{-3})$	-57.477
$^{196}_{78}Pt_{118}$	+0.44	0.032	-0.182	0.942	+4.03	+4.03	+4.8	+14.493
$^{198}_{80}Hg_{110}$	-0.17	0.035	-0.190	0.950	+4.36	-0.17	-1.14	-0.320
$^{200}_{80}Hg_{120}$	-0.17	0.035	-0.190	0.950	+4.36	-0.19	-0.19 ± 0.01	-0.393

*Mean value of δ

(a) From the plot of A_2 versus A_4 - Figure 5.2

(b) From the plot of A_2 versus Q - Figure 5.1

(c) Sign convention of Biedenharn and Rose

5.2 Analysis of Experimental Data and Comparison with The Theoretical Calculations

Usually, in determining the mixing ratio δ from the measurements in radioactive decays, the expansion coefficients A_ℓ are calculated first and the values of δ is extracted from it. Two of the four possible solutions for A_2 and A_4 have to overlap, and can therefore check the validity of the corrections made for admixtures from interfering cascades, attenuations of the angular correlation, etc. The analysis of the angular correlation

data has been done by two methods which are asfollows:-

5.2.1 Analysis of the multipole mixing ratios by the method of Arns and Wiedenbeck (1958)

The method of determining mixing ratio has been briefly described in Section 4.3.1. The procedure expresses the mixing ratios in terms of $Q = \delta^2/(1+\delta^2)$. This quantity is positive and ranges between 0 and 1 and is the fraction of L+1 radiation present in the transition. The plot of A_2 versus Q is shown in the Figure 5.1 . The values of the mixing ratio (δ^b) obtained by this method are given in third column of Table 5.1.

5.2.2 Analysis of the multipole mixing ratios by the method of Coleman (1958)

In Section 4.3.2, the Coleman method of determining the multipole mixing ratios was briefly presented. The angular correlation coefficients A_2 and A_4 values are both used for a cascade $2'^+ \rightarrow 2^+ \rightarrow 0^+$. The plot of A_2 versus A_4 is shown in Figure 5.2. The curve labelled with the value of $\delta' = |\frac{\delta}{1-\delta}|$. The measured experimental values with errors for both A_2 and A_4 are depicted in the plot which gives the best fit value of 'δ' with phases as shown in Figure 5.2. The values of the mixing ratios (δ^a) obtained occupies the second column of Table 5.1.

5.3 Systematic of δ(E2/M1) for $(2'^+ \rightarrow 2^+)$ Transition

The Figure 5.3 illustrates E2/M1 mixing ratios of transition $2'^+ \rightarrow 2^+$ in even-even nuclei as in $|\Delta|$ (where $\Delta = \delta/(E_\gamma \times 0.835)$ versus mass number A. It shows clearly the single particle is reached at or near some shell closures.

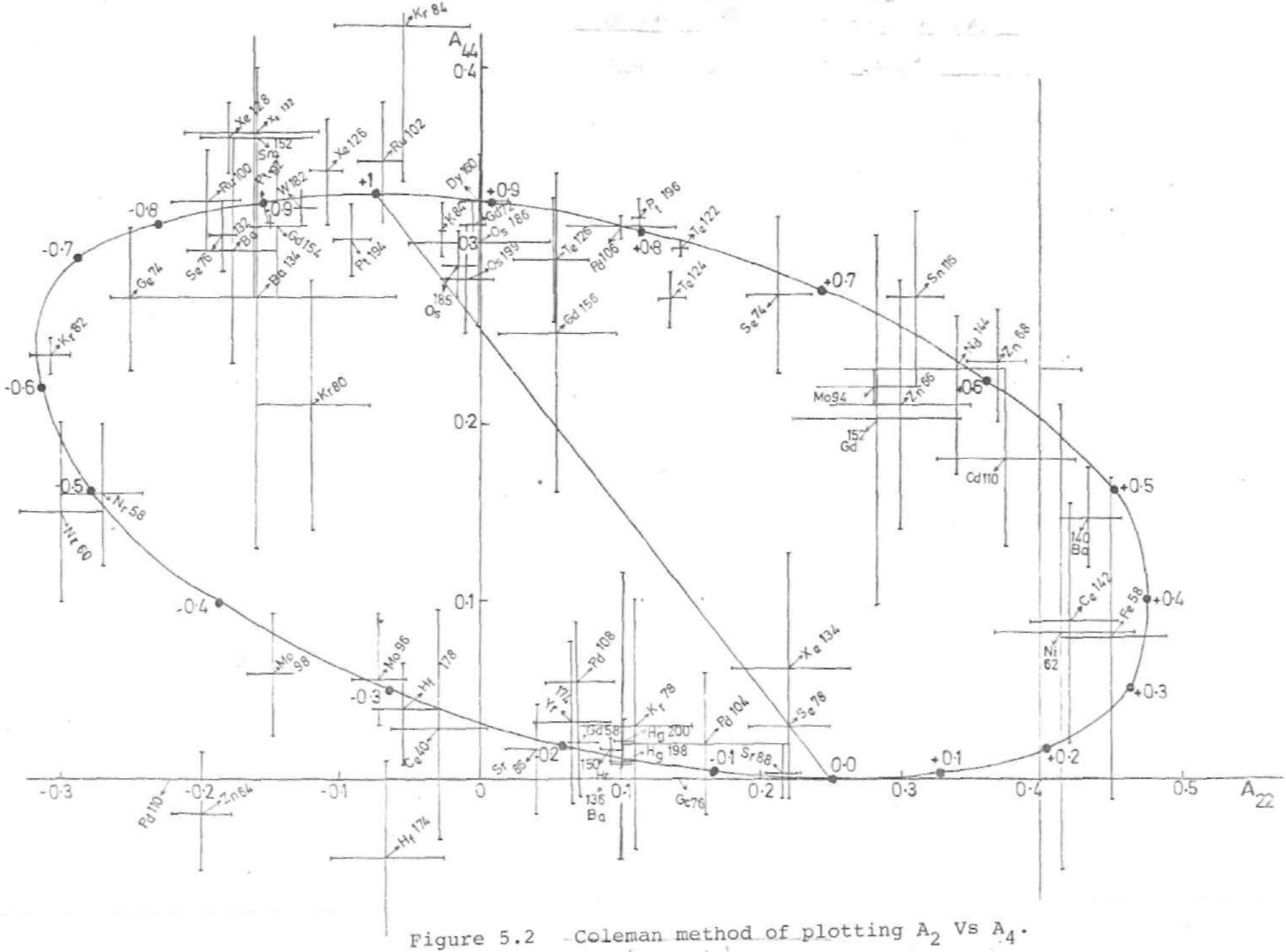


Figure 5.2 Coleman method of plotting A_2 Vs A_4.

For the sequence $2^+ \rightarrow 2^+ \rightarrow 0^+$

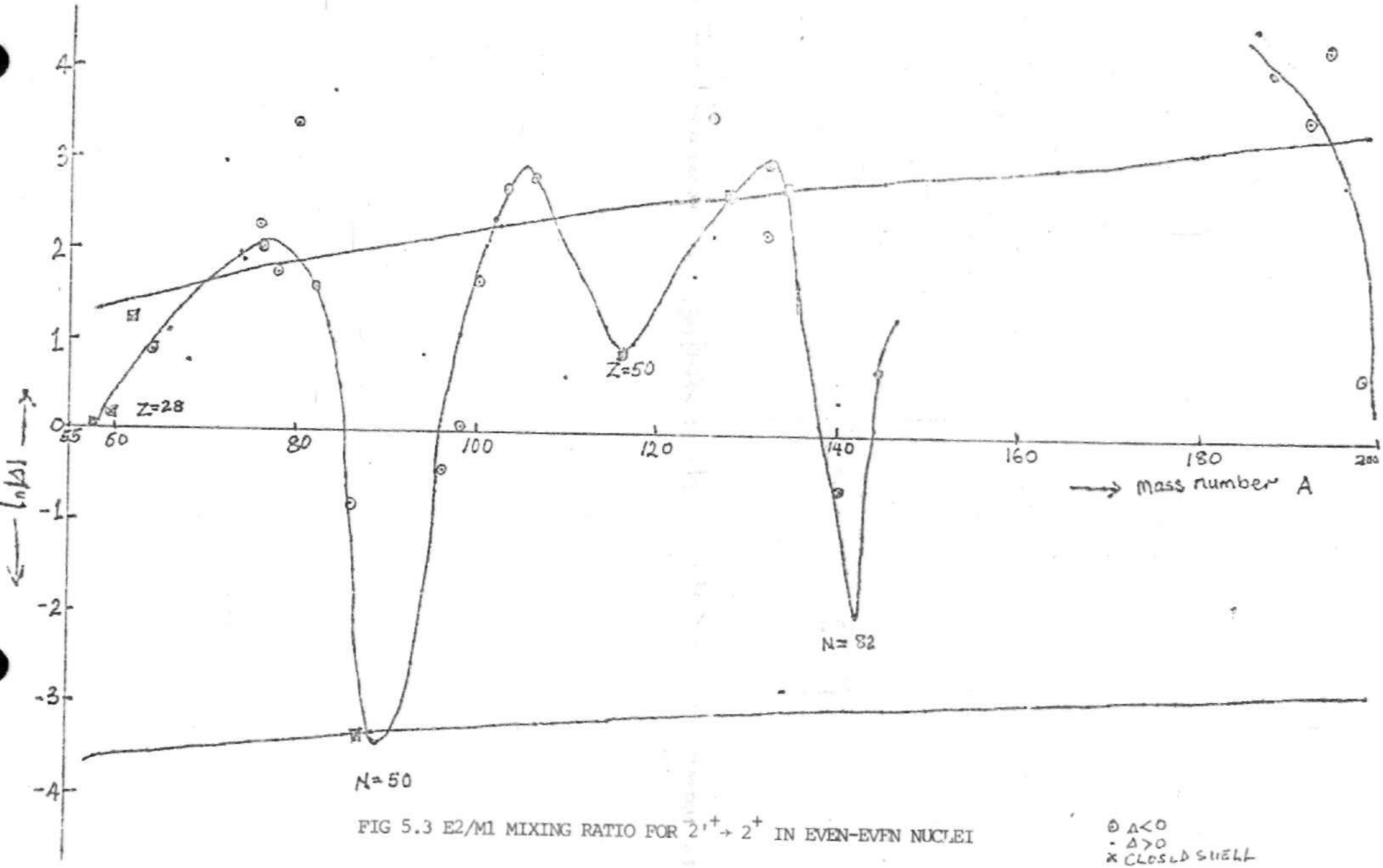


FIG 5.3 E2/M1 MIXING RATIO FOR $2'^{+} \rightarrow 2^{+}$ IN EVEN-EVEN NUCLEI

⊖ Δ<0
· Δ>0
✱ CLOSED SHELL

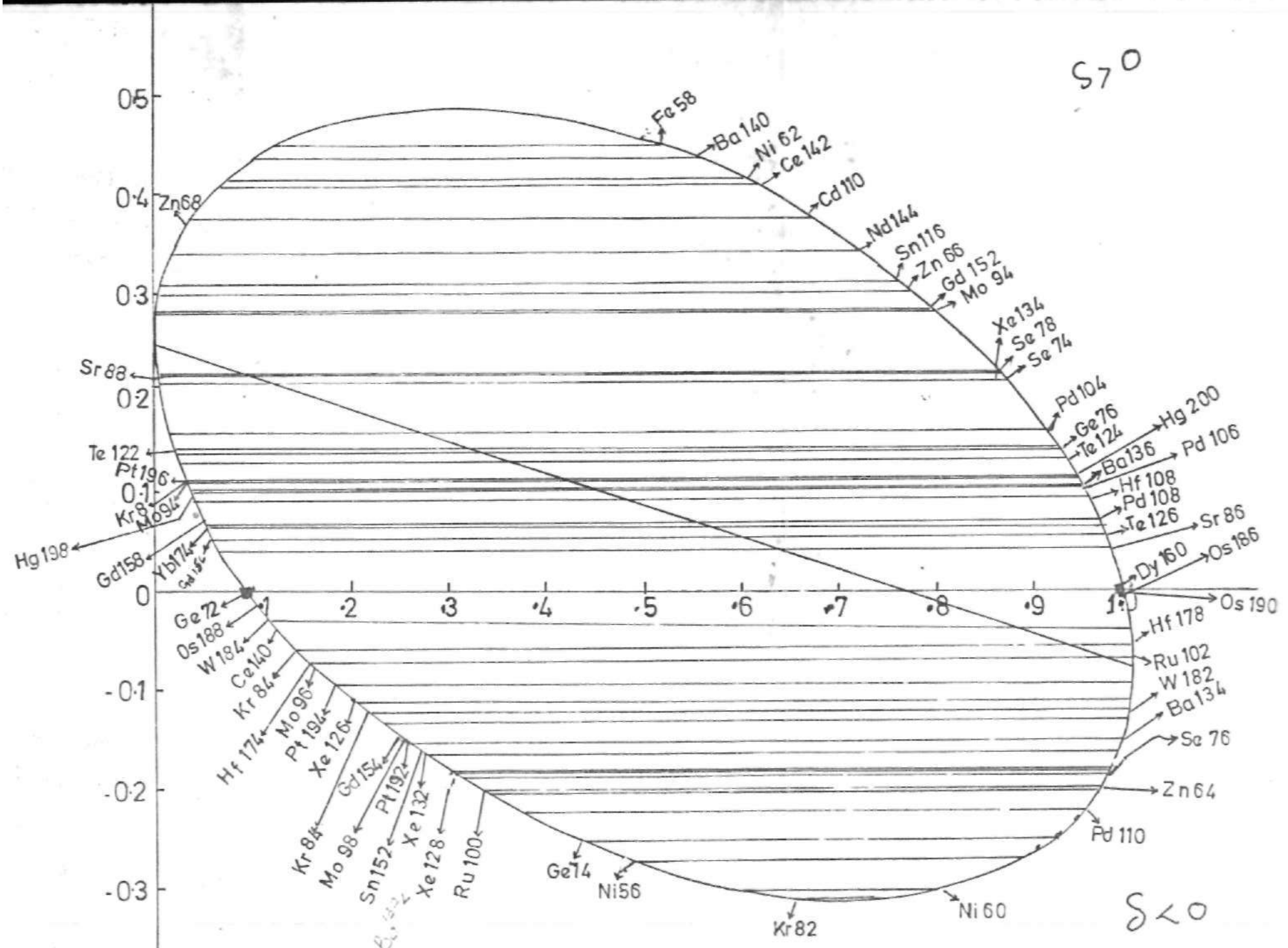


Figure 5.1 Theoretical plot of A_2 Vs Q.

The δ values for neutron closed-shell nuclei are lower than those for proton closed-shell nuclei. This is due to neutron excess which stabilizes the collectivity of the latter species. The experimental values of B(E2)/B(M1) are 5 orders of magnitude larger than the single particle values. For an E2 enhancement of 100, the M1 transition is only $\frac{1}{1000}$ of the shell model strength. This agrees with the collective picture that proton and neutron pairs contribute to the collective motion so that the magnetic moment is aligned with that of the mass to yield a vanishing M1 component. From Figure 5.3, the collective effect are the dominant feature responsible for the experimental mixing ratios in most even-even nuclei. Also as the mass number A increases the gap between the single particle estimate and Davydov-Filippov (1958) estimate increases, likewise the number of unfilled shells increases. This is due to greater collective effect of B(E2).

In general, the phase of mixing ratios show systematic variations in sign (i.e. positive or negative), as the closed shell is approached one phase begins to dominate.

5.4 Discussion and Conclusion

5.4.1 Discussion:- It has already been shown in the preceeding chapters that there exists an agreement of at least a qualitative nature between experimental values of $2^{'+} \rightarrow 2^{+}$ E2/M1 mixing ratios and theoretical values computed.

The dependence of the angular correlation coefficient on the E2/M1 mixing ratio $2'^{+} \rightarrow 2^{+}$ is shown in Figure 5.1. The mixing ratio 'δ' becomes infinite if the transition is pure E2. The coefficient A_{22} exhibits the shape of an ellipse and this coefficient is extremely sensitive to small M1 admixtures. The sign of the mixing ratio 'δ' is a physical quantity which indicates the relative phases between M1 and E2.

Most of $2'^{+} \rightarrow 2^{+}$ transitions are found to involve only small amounts of M1 component indicating large hindrance of the M1 strength. Empirical values of B(M1) are as low as $\frac{1}{100}$, and only in the closed-region do they become high. In the deformed region, the β-vibrational band - the k selection rules forbids M1 transition and in γ-vibration the k selection rules allowed M1 transition, the observed M1 strengths are small because these states are sufficiently collective. Nuclei far from the closed shell region exhibit collective properties such that (1) the energy of the lowest 2^{+} state is considerably lower than typical particle excitation energy and (2) the $2^{+} \rightarrow 0^{+}$ E2 transition is greatly enhanced over the single particle estimate.

Although the single-particle estimate gives good account of the nucleus collectivity and shell closure, it is less reliable between N = 82 and N = 126 (i.e. rare earth region $A \simeq 170$) where large deformation of the nuclei occur, and also every nucleus has definite angular momentum and magnetic moment thereby breaking away the picture of

closely packed nucleons in the nucleus. This necessitates the use of collective model as described by Davydov-Filippov as rotation without change of shape. Unfortunately, the experimental values are usually in between the collective estimate and single particle estimate (Figure 5.3), and this indicates the need for including both collective and single-particle aspects in the same model.

However, most of the experimental values of the E2/M1 mixing ratios of certain nuclei agrees with the theoretical values either obtained by Arns and Wiedenbeck or Coleman method. This is systematically shown on Table 5.1.

5.4.2 Conclusion: The survey of the E2/M1 mixing ratio shows its importance for the understanding of the atomic nuclei. In general, the magnitudes of the E2/M1 mixing ratios are large in agreement with the expected forbidance of M1 transitions between collective states. The phases seem to show little systematic variations and indeed one seems to find nearly equal frequencies for the occurence of positive or negative phases. As a closed shell is approached one phase begins to dominate.

The single particle model tells about nuclear collectivity as well as shell closure and indicates the predominance of M1 mode of γ-ray emission over E2 mode. The asymmetric rotor model of Davydov-Filippov used the mixing ratio to explain shape, collectivity as well as the first few excited energy levels of almost all even-even nuclei as compared to the experimental values.

In any event, it is apparent that both the phase (sign) and the magnitude of the E2/M1 mixing ratio can be successfully employed as a a probe of the nuclear structure.

APPENDIX - A

Rotations of eigenfunctions and the three-dimensional Rotation Group

Assume that a radiation R is omitted in the direction K with spin σ and denote with $\langle K\sigma | LM\Pi\rangle$ its eigenfunctions corresponding to the eigenvalues L,M, and Π of the operators of the angular momentum, Z-component of angular momentum, and parity. In this coordinate system Z, with the direction K of emission of the radiation R is characterized by the polar angles θ and ψ. If the angle ϑ is zero, that its Z-axis coincide with K, then the corresponding eigenfunctions are $\langle O\sigma | L\mu\Pi\rangle$.

The eigenfunctions in the two systems are related by

$$\langle K\sigma | LM\Pi\rangle = \sum_{\mu} \langle O\sigma | L\mu\Pi\rangle D^{L}_{\mu M}(K\to Z) \qquad \ldots A.1$$

where, $D^{L}_{\mu M} \equiv \langle \mu | D^{L} | M\rangle$ are the elements of a unitary matrix, called the rotation matrix. The argument $(Z\to K)$ of the rotation matrix D^{L} stands for the Euler angles (ψ,θ,γ) describing the roation that carries the arbit ary quantization system Z over into the coordinate system of the radiation. When $\theta = 0$; $(K\to Z)$ denotes the inverse rotation.

The properties are:-

$$D^{L}_{\mu M}(K\to Z) = D^{L*}_{M\mu}(Z\to K) \qquad \ldots A.2$$

$$D^{L*}_{\mu M}(K\to Z) = (-1)^{\mu-M} D^{L}_{-\mu-M}(K\to Z) \qquad \ldots A.3$$

$$\mu' D^{L}_{\mu M'}(K\to Z)\, D^{L}_{\mu' M}(Z'\to Z) = D^{L}_{\mu M}(K\to Z) \qquad \ldots A.4$$

The product of two matrix elements of the rotation group can be expressed in terms of a sum over matrix elements (Clebsch-Gordan series):

$$D^{L}_{\mu M} D^{L'}_{\mu' M'} = \sum_{K} \langle L\mu L'\mu' K\tau \rangle \langle LML'M' | KN \rangle D^{K}_{\tau N} \qquad \ldots \text{A.5}$$

E Vector Addition Coefficients and Racah Algebra

In nuclear spectroscopy, it is frequently necessary to form a composite angular momentum state $j_3 = j_1 + j_2$ of two noninteracting particles in angular momentum states j_1 and j_2 with z components m_1 and m_2. The product wave function is defined as

$$|j_1 j_2 m_1 m_2\rangle = |j_1 m_1\rangle |j_2 m_2\rangle \qquad \ldots \text{A.6}$$

The Clebsch-Gordan Coefficients for the vector addition $j_1 + j_2 = j_3$, $m_1 + m_2 = m_3$ are defined by the transformation

$$|j_1 j_2 j_3 m_3\rangle = \sum_{m_1 m_2} j_1 m_1 j_2 m_2 \rangle \langle j_1 m_1 j_2 m_2 | j_3 m_3 \rangle$$

$$\ldots \text{A.7}$$

Clebsch-Gordan Coefficients are often being replaced by Wigner 3-j symbols. They are related as follows:-

$$\begin{pmatrix} j_1 j_2 j_3 \\ m_1 m_2 m_3 \end{pmatrix} = (-1)^{j_1 - j_2 - m_3} (2j_3 + 1)^{-\frac{1}{2}} \langle j_1 m_1 j_2 m_2 | j_3 - m_3 \rangle$$

These 3-j symbols possess simple symmetry properties. An even permutation of the column leaves the coefficient unchanged while an odd permutation or an interchange of all m_1 to $-m_1$ is equivalent to multiplication by $(-1)^{j_1 + j_2 + j_3}$

$$\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} = \begin{pmatrix} j_2 & j_3 & j_1 \\ m_2 & m_3 & m_1 \end{pmatrix} = \begin{pmatrix} j_3 & j_1 & j_2 \\ m_3 & m_1 & m_2 \end{pmatrix} \qquad \ldots \text{ A.9}$$

$$(-1)^{j_1+j_2+j_3} \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} = \begin{pmatrix} j_2 & j_1 & j_3 \\ m_2 & m_1 & m_3 \end{pmatrix} = \ldots\ldots \qquad \ldots \text{ A.10}$$

$$\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} = (-1)^{j_1+j_2+j_3} \begin{pmatrix} j_1 & j_2 & j_3 \\ -m_1 & -m_2 & -m_3 \end{pmatrix} \qquad \ldots \text{ A.11}$$

The 3-j symbols satisfy the following orthogonality relations:-

$$\sum_{j_3 m_3} (2j_3+1) \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1' & m_2' & m_3' \end{pmatrix} = \delta_{m_1 m_1'} \delta_{m_2 m_2'} \qquad \ldots \text{ A.12}$$

$$\sum_{m_1 m_2} (2j_3+1) \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3' \end{pmatrix} = \delta_{j_3 j_3'} \delta_{m_3 m_3'}$$

Sums over products of 3-j symbols are related to the Wigner 6-j symbol or Racah Coefficient:-

$$\sum_{m_4 m_5 m_6} (-1)^{j_4+j_5+j_6+m_4+m_5+m_6} \begin{pmatrix} j_1 & j_5 & j_6 \\ m_1 & m_5 & -m_6 \end{pmatrix} \begin{pmatrix} j_4 & j_2 & j_6 \\ -m_4 & m_2 & m_6 \end{pmatrix} \begin{pmatrix} j_4 & j_5 & j_3 \\ m_4 & -m_5 & m_3 \end{pmatrix}$$

$$= \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix} \qquad \ldots \text{ A. 3}$$

The Wigner 6-j symbol, { }; is independent of magnetic quantum numbers. The symbol has simple symmetry properties

It is invariant against any permutation of the columns and against interchange of the upper and lower arguments in each of any two colums:

$$\begin{Bmatrix} j_1 j_2 j_3 \\ j_4 j_5 j_6 \end{Bmatrix} = \begin{Bmatrix} j_2 j_3 j_1 \\ j_5 j_6 j_4 \end{Bmatrix} = \begin{Bmatrix} j_1 j_5 j_6 \\ j_4 j_2 j_3 \end{Bmatrix} \qquad \ldots \text{ A.14}$$

The Wigner 6-j symbol is related to the Racah coefficient W by the relation

$$\begin{Bmatrix} j_1 j_2 j_3 \\ j_4 j_5 j_6 \end{Bmatrix} = (-1)^{j_1 + j_2 + j_4 + j_5} W(j_1 j_2 j_5 j_4 : j_3 j_6) \qquad \ldots \text{ A.15}$$

C- The Wigner-Eckart Theorem

It is common to take the matrix elements of an irreducible tensor operator T^{λ}_{q} of rank λ between the initial and final angular momentum states $|j_i m_i\rangle$ and $|j_f m_f\rangle$. The Wigner-Eckart theorem enables us to separate the magnetic quantum number dependence of the matrix element in the form of the Clebsch-Gordan coefficient. Thus we have

$$\langle j_f m_f | T^{(\lambda)}_{q} | j_i m_i \rangle = \langle j_i \lambda m_i q | j_f m_f \rangle \langle j_f || T^{(\lambda)} || j_i \rangle$$

Where $\langle j_f || T^{(\lambda)} || j_i \rangle$ is independent of magnetic quantum number and called the reduced matrix element.

D- The Density Matrix

Assume that the eigenstates of a certain operator form a complete orthonormal set $|m\rangle$. Assume that the system under discussion be in a pure state $|n\rangle$.

This state $|n\rangle$ can be expanded in terms of the states $|m\rangle$:

$$|n\rangle = \sum_m a_{nm} |m\rangle \qquad \ldots \text{A.17}$$

The expectation value of an operator F in the state $|n\rangle$ is given by

$$\langle n|F|n\rangle = \sum_{m'm} a^*_{nm'} a_{nm} \langle m'|F|m\rangle \qquad \ldots \text{A.18}$$

The system in a mixed state cannot be described by a single state $|n\rangle$ but as an incoherent sum of pure states $|n\rangle$ with weights g_n. The expectation value of F for the ensemble becomes

$$\langle F\rangle = \sum_n g_n \langle n|F|n\rangle = \sum_{nm'm} g_n a^*_{nm'} a_{nm} \langle m'|F|m\rangle \qquad \ldots \text{A.19}$$

The matrix elements of a density operator ρ is defined as

$$\langle m|\rho|m'\rangle = \sum_n g_n a^*_{nm'} a_{nm} \qquad \ldots \text{A.20}$$

Therefore

$$\langle F\rangle = \sum_{mm'} \langle m|\rho|m'\rangle\langle m'|F|m\rangle \qquad \ldots \text{A.21}$$

or

$$\langle F\rangle = \text{Tr}\ (\rho F) = \text{Tr}(F\rho) \qquad \ldots \text{A.22}$$

Where, Tr denotes the trace or sum of the diagonal elements of the matrix ρF, while $\langle m|\rho|m'\rangle$ is called the density matrix. The density matrix is Hermitean

$$\langle m|\rho|m'\rangle = \langle m'|\rho|m\rangle^* \qquad \ldots \text{A.23}$$

Consider the transitions from a level A to a level B. The letters a,a'...;b,b'... stand for the set of quantun numbers describing eigenstates of level A; b,b'... similarly characterize the states of level B. The operator H inducing the transitions from A to B is assumed to be linear, but it does not have to be Hermitean; H^+ denotes the adjoint of H:

$$\langle f|H|i\rangle^* = \langle i|H^*|f\rangle \qquad \text{A.26}$$

The system is assumed initially to be in an eigenstate $|a\rangle$ of level A. The transition then results in a state

$$|f\rangle = H|i\rangle \quad , \qquad \langle f| = \langle i|H^+ \qquad \text{A.27}$$

that is not necessarily an eigenstate $|b\rangle$ of level B. The probability $P_B(b)$ of finding the system after the transition in a certain eigenstate $|b\rangle$ is given by the absolute square of the corresponding expansion coefficient:

$$P_B(b) = |\langle b|f\rangle|^2 \qquad |\langle b|H|\rangle|^2 \qquad \text{A.28}$$

the density operator is described as

$$\rho_A = \sum_n |n\rangle g_n \langle |$$

and

$$\rho_B = \sum_n H|n\rangle g_n \langle n|H^+$$

Since each state $|n\rangle$ changes according to equation A.27. The matrix element now becomes.

If 2j + 1 states $|n\rangle$ exist with equal weight and equally populated. The states $|m\rangle$ is written such that the density matrix is diagonal

$$\langle m'|\rho|m\rangle = (2j+1)^{-1}\delta mm' \qquad \ldots\ \text{A.24}$$

The probability $\rho(m)$ of finding the ensemble in the state $|m\rangle$ of a diagonal elements of the density matrix is given as

$$\rho(m) = \langle m|\rho|m\rangle \qquad \ldots \quad \text{A.25}$$

$$\langle b|\rho_B|b'\rangle = \sum_n \langle b|H|n\rangle g_n \langle n|H^+|b'\rangle$$

The matrix element is expressed in terms of eigenstates $|a\rangle$ not eigenstates $|n\rangle$.

$$\langle b|\rho_B|b'\rangle = \sum_{aa'n} \langle b|H|a\rangle\langle a|n\rangle g_n \langle n|a'\rangle\langle a'|H^+|b'\rangle \qquad \ldots \text{A.29}$$

introducing the density matrix ρ_A

$$\langle b|\rho_B|b'\rangle = \sum_{aa'} \langle b|H|a\rangle\langle a|\rho_A|a'\rangle\langle b'|H|a'\rangle^* \qquad \ldots \text{A.30}$$

E. Weisskopf Estimate of Single Particle Model

The wavefunction of the single-particle model is written as

$$|\ell jM\rangle = R_\ell(r) \sum_s \langle \ell m \tfrac{1}{2} | jM\rangle Y_{\ell m}(\hat{r}) \chi_{\frac{1}{2}s}, \qquad \ldots \text{A.31}$$

where ℓ is the orbital angular momentum and $j = \ell \pm \frac{1}{2}$ is the total angular momentum. The intrinsic spin of the nucleon is represented by the spin or wavefunction $\chi_{\frac{1}{2}s}$ and $R_\ell(r)$ is the radial wavefunction.

The electric moment for the single-particle model is

$$M(EL,M) = er^L Y_{LM}(\theta,\phi) \qquad \ldots \text{A.32}$$

and the reduced transition probabilities is given

by

$$B(EL, j_i \to j_f) = \frac{e^2}{2j_i+1} \left| \langle \ell_f j_f || r^L Y_{LM} || \ell_i j_i \rangle \right|^2$$

... A.33

This is separated into radial and angular parts

$$B(EL,\ j_i \to j_f) = \frac{e^2}{4\pi} \left| \int_0^\infty R_f^*(r) r^{L+2} R_i(r) dr \right|^2 S(j_i, j_f\ L)$$

... A.34

where

$$S(j_i, j_f, L) = \frac{4\pi}{2j_{i+1}} \left| \langle \ell_f \tfrac{1}{2}\ j_f || Y_L || \ell_i \tfrac{1}{2} j_i \rangle \right|^2$$

... A.35

The spin dependent factor (statistical factor) is computed using Racah algebra technique

$$S(j_i, j_f, L) = \frac{4\pi}{2j_{i+1}} \left[(-1)^{\ell_f + \frac{1}{2} + j_i + L} \left[(2j_i+1)(2j_f+1) \right] \right]^{\frac{1}{2}} \begin{Bmatrix} \ell_f & j_i & \ell \\ j_i & \ell_i & L \end{Bmatrix} \langle \ell_f ||$$

$$= (2j_f+1)(2\ell_i+1)(2L+1) \begin{pmatrix} \ell_f & L & \ell_i \\ 0 & 0 & 0 \end{pmatrix}^2 \begin{Bmatrix} \ell_f & j_f & \frac{1}{2} \\ j_i & \ell_i & L \end{Bmatrix}^2$$

$$= (2L+1) \left| \langle j_i \tfrac{1}{2} L 0 || j_f \tfrac{1}{2} \rangle \right|^2$$

... A.36(a)

For special case that $|j_i - j_f| = L$, and $j_f = j_i - L$, $\ell_f = \ell_i - L$

$$S(j_i, L, j_f) = \frac{(j_i - \frac{1}{2})!}{(2j_i)!!} \times \frac{(2L+1)!!}{L!} \times \frac{(2j_f)!!}{(j_f - \frac{1}{2})!}$$

... A.36(b)

In particular,

$$S(L+\tfrac{1}{2},\ L, \tfrac{1}{2}) = \underline{1}$$

The radial integral is evaluated on the assumption for constant density model for both $R_f^*(r)$ and $R_i(r)$.

$$R_i(r) = R_f^*(r) = (4\pi)^{\frac{1}{2}} r, \quad 0<r<R$$

$$= 0, \quad r>R \qquad \ldots \text{ A.37}$$

This is normalised to unity when $\rho = (\frac{4}{3}\pi R^3)^{-1}$

$$\int_0^{R_0} |R(r)|^2 r^2 \, dr = |R(R)|^2 \frac{R_0^3}{3} = 1 \qquad \ldots \text{ A.38}$$

The radial integral J_L

$$J_L = \int_0^{\infty} R^*(r) r^L R(r) r^2 dr = |R(R)|^2 \int_0^{R_0} r^{L+2} \, dr$$

$$= [3/(L+3)] R_0^L \qquad \ldots \text{ A.39}$$

Therefore, the final single-particle estimate for the reduced electric multipole transition probability is

$$B^W(EL, j_i \rightarrow j_f) = \frac{e^2}{4\pi}\left(\frac{3}{L+3}\right)^2 R_0^{2L} \qquad \ldots \text{ A.40}$$

The magnetic multipole operator

$$\mathcal{M}(ML,M) = (\mu_s + \frac{2}{L+1}\mu_\ell) \nabla . r^L Y_{LM}(\hat{r}) \qquad \ldots\ldots \text{ A.41}$$

where

$$\nabla r^L Y_{LM}(\hat{r}) = \sqrt{L(2L+1)}\, r^{L-1} Y_{L-1,L,M}(\hat{r}) \qquad \ldots \text{ A.42}$$

Hence equation A.41, becomes

$$\mathcal{M}(ML,M) = \mu_N \sqrt{L(2L+1)}\, r^{L-1}$$

$$x[(g_s - \frac{2g_\ell}{L+1}) Y_{L-1,LM}(r) S + \frac{2g_\ell}{L+1} Y_{L-1,L,M}(\hat{r}) j]$$

... A.43

Since the magnetic transition takes place when the intrinsic spin is flipped, i.e. $j_i = \ell_i \pm \frac{1}{2} \rightarrow j_f = \ell_f \mp \frac{1}{2}$, respectively. If $\ell_f = j_i - j_f = L$ and $\ell_i - \ell_f = L - 1$, the reduced matrix element

$$<\ell_f j_f||M(LM)||\ell_i j_i> = (Lg_s - \frac{2L}{L+1}g_\ell)\mu_N\{\frac{(2L+1)(2j+1)}{4\pi}\}^{\frac{1}{2}}$$

$$\times <j_i \tfrac{1}{2} L0 | j_f \tfrac{1}{2}>J_{L-1} \qquad \ldots \text{A.44}$$

$$B(ML\ j_i \rightarrow j_f) = \frac{\mu^2_N}{4\pi}(Lg_s - \frac{2L}{L+1}g_\ell)^2(2L+1)(2j+1)|<j_i \tfrac{1}{2} L0|j_f \tfrac{1}{2}>|^2 J^2_{L-1}$$

$$= (Lg_s - \frac{2L}{L+1}g_\ell)^2 \frac{\mu^2_N}{4\pi} S(j_i, L, j_f) J^2_{L-1} \qquad \ldots \text{A.45}$$

The statistical factor $S(j_i, L, j_f)$ is assumed as in A.36 and radial integral J_L as in A.39.

The g factor is assumed as follows

$$g_s = \begin{cases} 5.586 \text{ for proton} \\ -3.826 \text{ for neutron} \end{cases}$$

$$g_\ell = \begin{cases} 1 \text{ for proton} \\ 0 \text{ for neutron} \end{cases}$$

Weisskopf estimated the term

$$(Lg_s - \frac{2L}{L+1}g_\ell)^2 \rightarrow 40 \qquad \ldots \text{A.46}$$

$$B(ML\ j_i \rightarrow j_f) = \frac{10}{\pi}(\frac{3}{L+2})^2 1.2^{2L-2} A^{(2L-2)/3}$$

$$\text{in } \mu^2_N (\text{fm})^{2L-2} \qquad \ldots \text{A.47}$$

F. The Directional Correlation Function $W(\vec{K}_1, \vec{K}_2)$

The directional correlation function $W(\vec{K}_1, \vec{K}_2)d\Omega_1 d\Omega_2$ is defined as the probability that a nucleus decaying through the cascade $j_i \to j \to j_f$ emits the two radiation R_1 and R_2 in the direction $\vec{K}_1$ and $\vec{K}_2$ into the solid angles $d\Omega_1$ and $d\Omega_2$.

In deriving the correlation function, the first transition $j_i \to j_f$ is considered. This is done by introducing the density matrix $\rho(K)$ transition of equation (A.30) of the appendix where $a = m_i$, $b = m$

$$\langle m|\rho(K_1)\ m'\rangle = S_1 \sum_{m_i m_i} \langle m|H|m_i\rangle\langle m_i|\rho_i|m_i'\rangle\langle m'|H_1|m_i'\rangle^* \qquad \ldots \text{A.48}$$

where S_1 is the summation over all unmeasured radiation properties such as spin and polarisation.

For the second transition, the same equation (A.30) of the appendix is set as $a = m$, $b = m_f$ and using equation (A.48) yields for the density matrix in the final state.

$$\langle m_f|\rho_f(K_1,K_2)|m_f'\rangle = S_1 S_2 \sum_{\substack{mm' \\ m_i m_i'}} \langle m_f|H_2|m\rangle\langle m|H_1|m_i\rangle\langle m_i|\rho_i|m_i'\rangle \times \langle m'|H|m_i'\rangle^*\langle m_f'|H_2|m'\rangle^* \qquad \ldots \text{A.49}$$

The probability of finding the nucleus in the final state m_f while the two radiations R_1 and R_2 are observed in the directions $\vec{K}_1$ and $\vec{K}_2$ is given by the diagonal element ρ_f (see A.25).

$$P_f(m_f) = \langle m_f|\rho_f(K_1,K_2)|m_f\rangle \qquad \ldots \text{A.50(a)}$$

and the correlation function is

$$W(K_1,K_2) = \sum_{m_f}\langle m_f|\rho_f(K_1,K_2)|m_f\rangle \qquad \ldots \text{A.50(b)}$$

The relation A.48 now takes the form

$$W(K_1,K_2) = S_1S_2\sum_{m_f m m' m_i}\langle m_f|H_2|m\rangle\langle m|H_1|m_i\rangle\langle m'|H_1|m_i\rangle^*\langle m_i|H_2|m'\rangle^* \qquad \ldots \text{A.51}$$

where

$$\langle m|\rho(K_1)|m'\rangle = S_1\sum_{m_i}\langle m|H_1|m_i\rangle\langle m'|H_1|m'\rangle^* \qquad \ldots \text{A.52}$$

$$\langle m'|\rho(K_2)|m\rangle = S_2\sum_{m_f}\langle m_f|H_2|m\rangle\langle m_f|H_2|m'\rangle^* \qquad \ldots \text{A.53}$$

$$W(K_1,K_2) = \sum_{mm'}\langle m|\rho(K_1)|m'\rangle\langle m'|\rho(K_2)|m\rangle \qquad \ldots \text{A.54}$$

The matrix element $\langle m|H|m_i\rangle \equiv \langle j_m K\sigma|H|j_i m_i\rangle$. This matrix element is represented in terms of the nuclear spins and the direction K and polarisation σ of the emitted radiation.

$$\langle jmK\sigma|H|j_i m_i\rangle = \sum_{LM}\langle K\sigma|LM\pi\rangle\langle jmLM\pi|H|j_i m_i\rangle \qquad \ldots \text{A.55}$$

where

$$\langle K\sigma|LM\pi\rangle = \sum_{\mu}\langle O\sigma|L\mu\pi\rangle\, D^{L^*}_{M\mu}(Z\to K) \qquad \ldots \text{A.56}$$

and

$$\langle jmLM\pi|H|j_i m_i\rangle = \sum_{j_i' m_i'} \langle jmLM|j_i' m_i'\rangle\langle j_i' m_i' jL\pi|H|j_i m_i\rangle \qquad \ldots \text{A.57}$$

Since H is invarinant under rotation, the matrix element $\langle j_i' m_i' jL\pi|H|j_i m_i\rangle$ is different from zero only if $j_i = j_i'$ and $m_i = m_i'$.

The resulting reduced matrix element $\langle j||L\pi||j_i\rangle$ comes from

$$\langle jmLM\pi|j_i m_i\rangle = \langle jmLM|j_i m_i\rangle\langle j||L\pi||j_i\rangle \qquad \ldots \text{A.58}$$

Combining equations (A.55), (A.56) and (A.58), the matrix element

$$\langle m|H_1|m_i\rangle = \sum_{L\pi M\mu} (-1)^{-j+L-m_i} \begin{pmatrix} j & L & j_i \\ m & m & m_i \end{pmatrix} \langle 0\sigma|L\mu\pi\rangle\langle j||L\pi||j_i\rangle^* D^{L^*}_{M\mu}(Z\to K) \qquad \ldots \text{A.59}$$

The relation (A.59) is inserted in (A.52) and simplifying with Clebsch-Gordan series (A.5) and replacing Clebsch-Gordan coefficient by 3-j symbols (A.8) gives

$$\langle m|\rho(K)|m'\rangle = S \sum_{m_i} \sum_{LM\mu} \sum_{\ell N\tau} \sum_{L'M'\mu'} (-1)^{M'-\mu'} (2\ell+1)\langle 0\sigma|L\mu\pi\rangle \times \langle 0\sigma'|L'\mu'\pi'\rangle \begin{pmatrix} j & L & j_i \\ m & M & -m_i \end{pmatrix} \begin{pmatrix} j & L' & j_i \\ m'm' & & -m_i \end{pmatrix} \begin{pmatrix} L & L' & \ell \\ M-M' & & N \end{pmatrix} \times \begin{pmatrix} L & L' & \ell \\ \mu & -\mu' & \tau \end{pmatrix} \langle j||L\pi||j_i\rangle\langle j||L'\pi'||j_i\rangle^* D^{\ell}_{N\tau}(Z\to K)$$

$$= S\Sigma_{\substack{L\mu \\ L'\mu'}} \Sigma_{\ell\mu\tau} (-1)^{2j-j_i-m-\mu'} (2\ell+1) \langle O\sigma | L\mu\pi \rangle \times$$

$$\times \langle O\sigma' | L'\mu'\pi' \rangle \begin{pmatrix} L & L' & K \\ \mu & -\mu' & \tau \end{pmatrix} \begin{pmatrix} j & j & \ell \\ m' & -m & N \end{pmatrix} \begin{Bmatrix} j & j & \ell \\ L & L' & j_i \end{Bmatrix}$$

$$\times \langle j || L\pi || j_i \rangle \langle j || L'\pi' || j_i \rangle D^{\ell}_{N\tau}(Z \to K) \qquad \ldots \text{A.60}$$

The expression (A.60) contains factors that depend on the properties of the radiation which are singled out and introduced as Racah radiation parameter $C_{\ell\tau}$

$$C_{\ell\tau}(LL') = S\Sigma_{\mu\mu'} (-1)^{L-\mu} (2\ell+1)^{\frac{1}{2}} \begin{pmatrix} L & L' & \ell \\ \mu & \mu' & -\tau \end{pmatrix} \langle O\sigma | L\mu\pi \rangle \langle O\sigma' | L'\mu'\pi' \rangle . \qquad \ldots \text{A.61}$$

These parameters satisfy the relation

$$C_{\ell\tau}(LL') = (-1)^{L-L'-\tau} C^{*}_{\ell\tau}(L'L) \qquad \ldots \text{A.62}$$

Introducing the relation (A.61= into (A.60) and using (A.62)

$$\langle m | \rho(k) | m' \rangle = \Sigma_{LL'} \Sigma_{\ell N\tau} (-1)^{2j-2j_i-m-L'} (2\ell+1)^{\frac{1}{2}} C_{\ell\tau}(LL') \times$$

$$\times \begin{pmatrix} j & j & \ell \\ m' & -m & N \end{pmatrix} \begin{Bmatrix} j & j & \ell \\ L & L' & j_i \end{Bmatrix} \langle j || L\pi || j_i \rangle \langle j || L'\pi' || j_i \rangle D^{\ell}_{N\tau}(Z \to K)$$

$$\ldots \text{A.63}$$

This expression (A.63) also applies to the matrix element of the second transiton $\langle m' | \rho(K_2) | m \rangle$.

The final expression for angular correlation between

two successive radiation is

$$W(K_1K_2) = (-1)^{2j-j_i-j_f} \sum_{\ell L_1 L_1'} \sum_{L_2 L_2'} \sum_{\tau_1 \tau_2} (-1)^{\ell - L_1' - L_2} \begin{Bmatrix} j & j & \ell \\ L_1 & L_1' & j_i \end{Bmatrix}$$

$$x \begin{Bmatrix} j & j & \ell \\ L_2 L_2' & j_f \end{Bmatrix} \langle j_f || L_2 \pi_2 || j \rangle \langle j_f || L_2' \pi_2' || j \rangle^* x$$

$$x \langle j || L_1 \pi_1 || j_i \rangle \langle j || L_1' \pi_1' || j_i \rangle^* C_{\ell\tau}(L_1' L_1) \; x$$

$$C^*_{\ell\tau_2}(L_2 L_2') \; D^{\ell}_{\tau_2 \tau_1} \; (K_2 \to K_1) .$$

... A.64

Appendix G: ANGULAR CORRELATION COEFFICIENTS FOR $2^{+} \rightarrow 2^{+} \rightarrow 0^{+}$ CASCADE

NUCLEUS	Transition Energy E_γ (Mev)	A_2 (EXpt)	A_4 (Expt)	Reference
$_{26}Fe^{58}_{32}$	0.864	0.08±0.04	0.08±0.09	R.A.Fex etal,P.R. CS, 853 (1972)
$_{28}Ni^{58}_{30}$	1.321	-0.27+0.03	0.16±0.04	D.F.H Start etal, N.P. A162, 49 (1971
$_{28}Ni^{60}_{32}$	0.827	-0.30±0.03	0.15±0.05	S.M.Shaforth etal, P.R. 149, 827 (1966)
$_{28}Ni^{62}_{34}$	1.129	0.415±0.053	0.082±0.131	D.M. Van Patter etal., N.P.A 178 (355 (1972)
$_{30}Zn^{64}_{34}$	0.807	-0.200±0.022	-0.02±0.032	A.K. Sen Gunta etal, P.L.3,355 (1963)
$_{30}Zn^{66}_{36}$	0.823	0.30±0.05	0.21±0.07	T.Hayashi etal., J.P.S(J) 27, 1375(1963)
$_{30}Zn^{68}_{38}$	0.806	0.369±0.025	0.234±0.034	J.Lange etal;,P.R.C7, 177(1973
$_{32}Ge^{72}_{40}$	0.630	-0.002±0.009	0.311±0.102	H.Chen etal., N.P. A219, 365(19
$_{32}Ge^{74}_{42}$	0.608	-0.251±0.015	0.27+0.04	M.C. Cambiagio etal; Z.PA 275, 183(1975).
$_{32}Ge^{76}_{44}$	0.545	0.14±0.04		K.C. Chung etal., P.R. C2 139 (1970)
$_{34}Se^{74}_{40}$	0.635	0.213±0.022	0.272±0.044	M.C.Cambiaggio etal., Z.P.A275, 183(1979)
$_{34}Se^{76}_{42}$	0.657	-0.185±0.012	0.305±0.021	T.Nagahara etal., J.P.S(J)34, 579(1973)
$_{34}Se^{78}_{44}$	0.695	0.22±0.03	0.03±0.04	R.M. Lieder etal.,P.R. C2,531 (1970)
$_{36}Kr^{78}_{42}$	0.664	0.11±0.04	0.03±007	N.E.Andernson etal.,J.P.A(GB)7, 1156(1974)
$_{36}Kr^{80}_{44}$	0.639	=0.12±0.04	0.21±0.07	T.Hayashi etal., J.P.S(J) 27, 1375(1969)
$_{36}Kr^{82}_{46}$	0.698	-0.307±0.0150	0.239±0.010	J.Kotch etal,N.P.103 300(1967).
$_{36}Kr^{84}_{48}$	1.012	-0.056±0.050	0.426±0.089	J.P.Roalsving etal., P.R. 138 (B1378(1965)

NUCLEUS	Transition Energy E_γ (Mev)	A_2(Expt)	A_4 (Expt)	Reference
$_{56}Ba^{136}_{80}$	1.065	0.10±0.008	0.00±0.018	Z.W.Grabowski etal.,N.P.20,1 (1960
$_{56}Ba^{140}_{84}$	0.908	0.436±0.023	0.147±0.028	L.J.Alquist etal.P.RC13,1277
$_{58}Ce^{140}_{82}$	.752	-0.030±0.036	0.029±0.064	R.N.Saxena etal,P.R.C7,395(1
$_{58}Ce^{142}_{84}$	0.895	0.417±0.034	0.089±0.036	G.J. Basinger etal.,P.R.C11 (1975)
$_{60}Nd^{144}_{84}$	0.865	$0.34^{+0.09}_{-0.08}$	$0.23^{+0.03}_{-0.06}$	M.Behar etal,N.P. A219,516(1
$_{62}Sm^{152}_{90}$	0.688	-0.16±0.04	0.36±0.104	C.A.Kalfas etal, J.P.A.(GB)6; (1973)
$_{64}Gd^{152}_{88}$	0.586	0.282±0.061	0.202±0.104	Barrette etal, C.J.P.48,2011
$_{46}Gd^{157}_{90}$	0.693	-0.145±0.022	0.311±0.040	L.C.Whitlock etal.,P.R.C3,313
$_{64}Gd^{156}_{92}$	1.040	0.0561±0.0183	0.25±0.09	J.H. Hammilton etal,P.R.C5,89 (1972)
$_{64}Gd^{158}_{94}$	0.946	0.070±0.015	0.02±0.03	F.Schina etal, N.P.63,305(196
$_{66}Dy^{160}_{94}$	0.879	0.002±0.017	0.324±0.025	K.S.Krane etal,N.P.A164,439(1
$_{70}Yb_{104}$	1.242	+0.064±0.023	-0.037±0.046	H.J.Prask etal, N.P.29,100(19
$_{72}Hf^{174}_{102}$	0.808	-0.067±0.04	-0.044±0.053	H.Ejiri etal.,N.P.A161,449(19
$_{72}Hf^{178}_{106}$	1.183	-0.054±0.023	0.041±0.033	P.E.Little etal.,P.R.C5,252(1
$_{72}Hf^{180}_{108}$	0.215	0.093±0.007	0.014±0.009	S.D.Koicki etal; B.I.N.S.B.Ki 13,1(1962)
$_{74}W^{182}_{108}$	1.121	-0.127±0.011	0.320±0.011	L.M. Quinones etal N.P.A. 242, 243 (1975)
$_{74}W^{184}_{110}$	0.793	-0.027±0.003	0.307±0.015	H.J.Komer etal.,P.A.C.Suppl. uppsalla May (1963)
$_{76}Os^{184}_{110}$	0.630	-0.001±0.052	0.302±0.049	K.S. Krane etal.,P.R. C3,240(
$_{76}Os^{188}_{112}$	0.478	-0.015±0.014	0.288±0.021	K.S.Krane etal.,P.R.C3.240(19

NUCLEUS	Transition Energy E_γ (Mev)	A_2 (Expt)	A_4 (Expt)	References
$_{76}Os^{190}_{114}$	0.371	-0.011±0.02	0.28±0.03	H.Helppi etal.,N.P.A223 13(1974).
$_{78}Pt^{192}_{114}$	0.296	-0.15±0.003	0.312±0.004	H.Helppi etal.,N.P.A223,13(1974)
$_{78}Pt^{194}_{116}$	0.294	-0.092±0.013	0.303±0.020	H.A Doubt etal.,N.P.A177,418(1971
$_{78}Pt^{196}_{118}$	0.333	0.113±0.005	0.315±0.010	H.A.Doubt etal.,N.P.A(177,418 (1971).
$_{80}Hg^{198}_{118}$	0.676	0.10±0.01	0.01±0.02	R.Berand etal.,PrivaEopnm.April (1971)
$_{80}Hg^{200}_{120}$	0.579	0.102±0.009	0.021±0.014	Z.Itattula etal,Z.P.241,117(1971)

The following are the abbreviations for journals used

P.R.: Physical Review

N.P.: Nuclear Physics

Z.P: Z. Physics

C.J.P: Canadian journal of physics

J.P.S: Journal of Physical Society

P.L.: Physics letters.

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