RELIABILITY-BASED DESIGN RULES FOR THIN-WALLED STRUCTURAL STEEL BEAM-COLUMN ELEMENTS

BY

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in Structural Engineering.

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DECLARATION

I hereby declare that this dissertation has been composed by myself and that it is a record of my research work. It has not been accepted in any previous application for a higher degree. All sources of information are specifically acknowledged by means of references.

AFOLAYAN, J. O.

February, 1994

CERTIFICATION

This dissertation entitled:

"RELIABILITY-BASED DESIGN RULES FOR THIN-WALLED STRUCTURAL STEEL BEAM-COLUMN ELEMENTS"

by *AFOLAYAN, Joseph Olasehinde* meets the regulations governing the award of the *Degree of Doctor of Philosophy (PhD)* in Structural Engineering of the **Ahmadu Bello University, Zaria** and it is approved for its literary presentation.

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DEDICATION

To the **HEROES** of Faith

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ABSTRACT

The problem considered in this study is to deterministically and probabilistically evaluate the adequacy of the design methods of cold-formed structural steel beam-columns having doubly-symmetric cross-sections in accordance with the AISI Specification of 1968 and ECCS Recommendation of 1987 with a view to proposing equivalent optimal criteria. The validity of the design formulae are verified using test results drawn from literature and those observed at the Heavy Structural Laboratory at the University of Dortmund, Dortmund, West Germany. Depending on the ratio of width to thickness of wall members, varying degrees of incongruities exist between the predictions given by the design formulae and test results. Both cases of good and over-estimation of member strength are noted and generally the predictions of ECCS (1987) show more conservatism than AISI (1968). Due to scatter in test results, the influence of the statistics of the design variables on the predicted strength is analyzed. The sensitivity is compared giving recognition to the parameter, Q_w, which represents the local buckling phenomenon. The design formulae are found sensitive to this parameter.

Safety factors are applied either to loading or strength parameters in the two design equations studied. The

dependability of such factors is measured by computing the reliability levels implicit in the design equations under uncertain dead and maximum life-time live loads. Within the range of practical data points considered, significant inconsistency is noted. Although the reliability levels computed assume random values, their distributions show that a target level is in focus. Thus, employing a socio-economic criterion, a target reliability level, $8_{2,}$ of 4.265 is obtained for each design equation. This value is equivalent to a failure probability level of 1 x 10-^{5}.

Since an ultra safe design method can be unduly expensive and that it is not economically feasible to design a structure that cannot fail, resistance and load factored Limit State Design Formats which trade-off safety and economy are derived as alternatives to the corresponding formulae in AISI (1968) and ECCS (1987). These formats are optimal having a minimum total cost implication as far as the design of thin-walled structural steel beam-column members of doubly-symmetric shapes or shapes not subject to torsional or torsional-flexural buckling is concerned. In addition, they supercede the current formats by providing uniform reliability level at minimum cost. This objective became realistic as a consequence of rigorous combination of the Advanced First-Order Reliability Method (FORM) and a Nonlinear Quadratic Programming Technique.

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NOTATIONS

A	=	cross sectional area.
A_g	=	gross cross-sectional area.
b,b*	=	effective width of plate
b_p	=	initial plate width.
C_{Fj}	=	cost of partial failure
C_{ij}	=	the non failure cost
C_m	=	a coefficient to conform to the boundary requirements of beam.
Cov( )	=	coefficient of variation of
C_{ij}	=	anticipated cost
C_{Tj}	=	expected total cost
d_N	=	ratio of N_D/N_{DN}
d_Q	=	ratio of Q_D/Q_{DN}
$d\lambda_S$	=	infinitesimal value of λ_S
$d\lambda_R$	=	infinitesimal value of λ_R
E	=	Young modulus
E()	=	expectation of ()
E_u	=	matrix of eigenvalues of matrix G_u
e_o	=	eccentricity
F_a	=	applied axial stress
F_{al}	=	allowable compressive stress under concentric loading in accordance with AISI (1968)

F_{ao} = allowable compressive stress under concentric loading in accordance with AISI (1968) when L = 0

F_b = allowable stress in bending

F_{bl} = allowable bending stress in compression according to AISI, 1968

F_{bw}, F_c = allowable bending stress of members with stiffened flanges or webs

F_e = factored Euler stress

$F_{\underline{Y}}$ = joint distribution function of vector $\underline{Y}$

f_a = applied bending stress

f_b = applied bending stress

f_{ty} = design value of yield stress

F_y = yield point of materials

$f_{\lambda R}(\ )$ = p.d.f of λ_R

$f_{\lambda S}(\ )$ = p.d.f.of λ_S

G_u = matrix of second and mix derivatives

I = moment of inertia

I_{ex} = moment of inertia of reduced cross section

I_{cg} = moment of inertia of the gross cross section

I_w = weighting function

I_{xy} = moment of inertia of gross cross section

i_g = radius of gyration for gross section

K = effective length factor in plane of bending

L = length of member

L_b = actual unbraced length in the place of bending.

l_N	=	ratio of N_L/N_{LN}
l_Q	=	ratio of Q_L/Q_{LN}
M	=	safety margins
M_D	=	moment effect due to Q_{DN} and N_{DN}
M_L	=	moment effect due to Q_{LN} and N_{NL}
M_{max}	=	maximum induced moment by P and q
M_o	=	moment induced by q
M_u	=	nominal beam strength in the plane of bending
M_x	=	calculated moment
M_{xd}	=	moment capacity
M_y	=	yield moment
m_i	=	mean value of variable
N	=	total calculated design axial load
N_D	=	dead Load component of the total axial loads
N_{DN}	=	nominal value of N_D
N_L	=	live load component of the total axial load
N_{LN}	=	nominal value of N_L
N_{max}	=	maximum axial load-carrying capacity
n	=	sample point
P	=	axial load
P_e	=	Euler load
P_f	=	probability of failure
P_{FC}, P^*_{FC}	=	probability of collapse and its allowable value
P_{Fj}	=	probability of occurrence of partial failure
P_{FL}	=	Hasofer-Lind probability failure estimate

P_{FS}, P^{*}_{FS}	=	unserviceability probability and its allowable value
P_{Fv}	=	Veneziano's estimate of probability of failure
P_s	=	probability of survival
P_{Tj}	=	total failure cost
P_u	=	nominal column strength
P_y	=	yield load
Q	=	uniformly distributed total lateral load.
Q_D	=	dead load component of the total lateral load
Q_{DN}	=	nominal value of Q_D
Q_L	=	live load component of the total lateral load
Q_{LN}	=	Nominal value of Q_L
Q_{max}	=	lateral load carrying capacity
Q_n	=	nominal load effect
Q_w	=	form factor
q	=	uniformly distributed load (udl)
q(.)	=	quadratic state function
R_S	=	reliability
R	=	member strength
R_m	=	mean resistance
R_n	=	nominal resistance
R_x	=	ratio I_{ex}/I_{cg}
r_b	=	radius of gyration about axis of bending
$S_{y,x}$	=	standard error estimate
t	=	thickness of plate
t_v	=	ratio λ_R/λ_S

U_i	=	standardized variable
v	=	lateral defection
Var()	=	variance of ()
w	=	initial plate width
W_{eff}	=	section modulus of effective section
w_i	=	frequency of given design.
X	=	mean value of x
Y′	=	predicted value of y
Y	=	mean value of y
Y_i	=	design variable
Y_i^*	=	characteristic values of the design variables
y^*	=	design point for
Z	=	elastic section modulus
Z_M	=	safety margins or limit state function
α	=	coefficient-dependent on cross section and plane of buckling
α_D	=	factor of safety applied to the nominal value of N_D
α_i	=	direction cosine
α_k	=	load factor
α_L	=	factor of safety applied to normal value of N_L
β	=	safety/reliability index
β'	=	moment coefficient accounting for moment distribution
β_{min}	=	minimum safety index
β_T	=	target safety index

ϵ_y = yield strain

φ = amplification factor

$\varphi()$ = p.d.f corresponding to $\Phi( )$

γ_i = partial safety factors

γ_N = ratio of N_{DN} to N_{LN}

γ_Q = ratio of Q_{DN} to Q_{LN}

λ = slenderness ratio

λ_R = strength or resistance parameter

λ'_R = reduced variate of λ_R

λ_R = mean of λ_R

λ_S = stress or load parameter

λ'_S = reduced variate of λ_S

λ_S = mean of λ_S

λ_{so} = particular stress value

λ_p, λ' = slenderness parameter

μ = mean value

$\mu_{Z'}$ = expectation of Z_M

= linear or quadratic approximation of F_Y

Π, Π^* = total cost or weight and its permissible value of a structure

ρ = product of the ratio of the mean to the nominal yield stress and the gross cross sectional area.

σ = standard deviation

$\sigma_{Z'M}$ = standard deviation of Z_M

$\sigma_{\lambda R}$ = standard deviation of λ_R

$\sigma_{\lambda S}$ = standard deviation of λ_S

σ_M	=	standard deviation of M
ν	=	product of L and Q_{LN}/N_{LN}
ϕ	=	resistance factor
ϕ_B	=	beam resistance factor
ϕ_C	=	column resistance factor
ϕ_{BC}	=	beam-column resistance factor
$\Phi()$	=	p.d.f of a standard nominal variate
$\Omega_{\lambda R}$	=	coefficient of variation of λ_R
$\Omega_{\lambda S}$	=	coefficient of variation of λ_S
Ω_T	=	total uncertainty
ψ	=	magnification parameter due to axial load
$\chi^2_n()$	=	cumulative distribution function for chi-square with n degrees.
χ	=	strength reduction coefficient.

CHAPTER 1

INTRODUCTION

1.1 General

The term 'thin-walled structures' encompasses a wide and diverse range of construction. The knowledge of their behaviour is constantly advancing to fill the requirements of engineers and designers. Although the applications of thin-walled structures are diverse and the forms of construction are varied, there are phenomena which are found common to many different types of these structures. Some of these phenomena, for example, local buckling (Rhodes, 1983), are peculiar to thin-walled structures and arise purely because of the thinness of the walls relative to the other structural dimensions. See some examples of cold-formed sections in Fig. 1.1. In other cases effects which are present in all structures are sometimes altered due to the thinness of the walls. Thus, in some circumstances simplification of analysis has been done, while in others there has been an increased complexity.

Exact stress analysis of thin-walled members for design use has been described (Winter, 1968) as too time-consuming. But instead, approximate design methods have been developed. Some of such developments with regards to beam-column design are investigated in this study.

Beam-column, as used herein, implies a structural member which is subjected simultaneously to axial compressive force and bending moment. Bending moment may result from transverse forces and/or from a known eccentricity of the axial force at

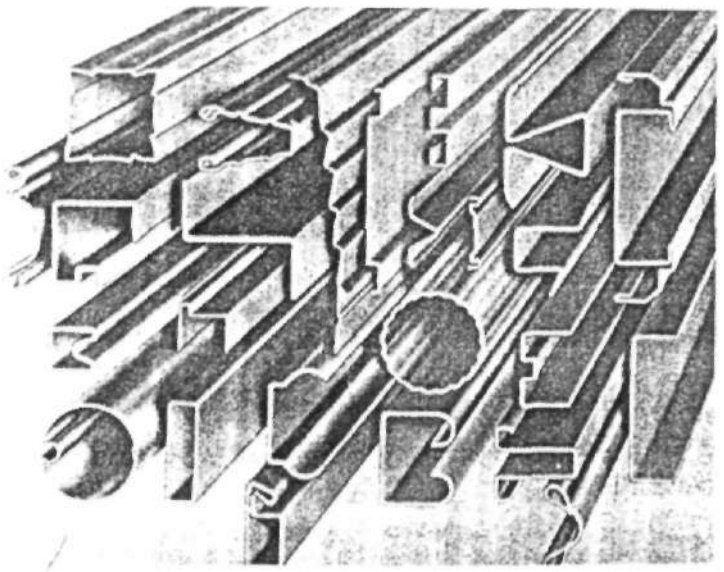


Fig. 1.1: Various Shapes of Cold-Formed Sections (Yu, 1985)

one or both ends. Since compression members in actual structures such as frames and trusses have unavoidable bending moment along the member due to eccentricity of axial force, initial deflection and/or rotation of adjacent members, then in reality, there are no pure columns. Although when the magnitude of one or the other is relatively small its effect is usually ignored, for many situations neither effect can be properly neglected. Therefore the behaviour under the combined loading is usually considered in design.

The design criteria of thin-walled beam-columns being examined are based on the Allowable Stress Concept, ASC, (AISI, 1968) and the Ultimate Capacity Concept, UCC, (ECCS, 1987). The ASC requires the design of structures for a particular load by using an allowable stress obtained as the appropriate strength divided by a specific single-valued factor of safety. This safety factor is considered as accounting for any imperfections in strength and loading. The implication of this singular value of the safety factor for any particular strength has been described as: the ability to predict loads and strength is invariant (Viest, 1981). But generally some loads and some strengths can be better predicted than the others.

The response of a loaded engineering structure depends on the type and the magnitude of the load, and the strength and the stiffness of the structure. The question of a satisfactory

response depends on the requirements which must be satisfied. Such requirements for thin-walled beam-column elements are stated in different forms for design purpose (e.g AISI, 1968; ECCS, 1987; CSAS, 1963), but the same objectives are meant. One cannot say exactly if these criteria will yield the same level of performance unless they are subjected to a critical evaluation as attempted in this study for AISI and ECCS design methods.

The local buckling phenomenon plays a major role in the prediction of strengths of thin-walled elements. Usually, observed stresses especially in the prediction of ultimate capacity do not always correspond to the stresses calculated by linear elastic structural analysis (Viest, 1981). Factors such as stress redistribution, stress concentration, changes due to boundary effects and physical sizes of members are contributors to the discrepancies.

The assurance of system performance within the constraint of economy is one of the principal aims of engineering design. Most planning and design of engineering systems must be accomplished without the benefit of complete information; consequently, the assurance of performance can seldom be perfect (Ang and Tang, 1984). In the case of a structure, its safety is a function of the maximum load (or combination of loads) that may be imposed over the useful life of the

structure. Structural safety also depends on the strength or load-carrying capacity of a structure or its components. As the lifetime maximum load and the actual capacity of a structure are difficult to predict exactly, and any prediction is subject to uncertainties, the absolute assurance of the safety of a structure is not possible (Ang and Tang, 1984). Then, it is only realistic to measure safety in terms of the probability that the available strength is adequate to withstand the lifetime maximum load.

In cases where sufficient statistical data are available, the problem of safety evaluation can be formulated with the application of the theory of probability and statistics. But in the great majority of cases, the design is used and built only once. Therefore, it is not economically feasible to collect sufficient data and, in fact, the structural behaviour near failure has been described as unknown for most civil engineering structures because it is not practical to conduct even a single full-scale destructive test of such large and massive structures (Yao, 1985).

Commonly, in civil engineering practice, a given structure can be studied either experimentally or analytically. The experimental studies can be field surveys and/or laboratory tests. Field surveys include the determination of exact locations of failed elements and other signs of distress, the

conduct of various nondestructive testing techniques to the existing structure, the discovery of poor workmanship and defective details, and proof-load and other load testing of a portion of a large structure. On the other hand, specimens can be collected from the field and tested in the laboratory for the evaluation of material strengths and other mechanical properties.

Analytically, studies may consist of the examination of the original design calculations and drawings, the review of the project specifications, the performance of additional analyses using newly acquired field observations and test data, and the possible explanation and description of the event under consideration.

Guedelhoefer, in 1981, classified into three types, the methods applicable for the strength evaluation of distressed structures, viz:

* analytical evaluation,
* in-situ load testing,
* model load testing.

Of interest, in this investigation, is the analytical evaluation of the predicted strength of thin-walled beam-columns as given by two different design criteria considering the presence of inherent uncertainties in the basic variables

of the resistance and loading.

1.2 Purpose and Scope

In this investigation, the advantages or disadvantages, measured by sensitivity to inherent uncertainties, of two design methods for thin-walled beam-columns having doubly-symmetric cross-sections are studied and consequently reliability-based modifications are proposed to foster uniform and economical satisfactory performance for all practical values of the parameters involved in the design equations. The study is essentially useful for the allowable stress and empirically based design interaction equations. The factor of safety used in the interaction equation by AISI depends, in part, on the expressions chosen to define the allowable stresses. It has been shown that the actual factor of safety of beam-columns cannot be determined directly from the factor of safety of the individual terms in the interaction equation (CRC Guide, 1976). And even then, predictions always have associated uncertainties so that the adequacy of any design criterion needs to be examined and evaluated in a probabilistic manner. The reasons are now obvious with due regards to the list of sources and types of uncertainties (see Sec. 5.2). However, errors in analysis may be decisive in causing structural malfunctions, and the control of allowable approximations founded on well defined rational criteria. There is yet one major obstacle found in the judgement of

errors in calculations. This is the difficulty of getting bounds on local mistakes in stresses, displacements, etc. Only comparisons are possible on the basis of some global measure of structural response or performance. As a possible solution, the probabilistic theory of structural safety (reliability analysis) has been suggested (Augusti et al, 1984) for an engineering evaluation of errors introduced by approximations. Such analysis of the propagation of numerical errors on the safety indices or on the failure probability can give useful criteria for judging the admissibility of approximations, or even to calibrate increments of the safety coefficients necessary to neutralize error effects on structural safety. According to Augusti et al (1984), it is unfortunate and somewhat surprising that there is not much work on the subject at the present stage, despite the interest that the matter should have in practical applications. The work in this study is a step in this direction with regards to thin-walled structures.

Kolakowski (1987) found that the interaction between local and overall buckling of compressed thin plates could result in an imperfection-sensitive structure. This means the elements whose design criteria are being evaluated can be sensitive to imperfections which may themselves be random in nature. Ordinary deterministic evaluation cannot be sufficient in quantifying the degree of such sensitivity for reasons that are

obvious.

Comparison of the design equations in consideration shows that there is no unique format for design criterion. However, when two or more design criteria yield the same level of probability of failure, they can be said to have equal level of performance. Such requirements appear to be possible only when the criteria are developed in a probabilistic manner.

Two design specifications (i.e. AISI, 1968 and ECCS, 1987) for the interaction of thin-walled beam-columns are rated using a probability-based technique. The interaction strength as a whole and some key parameters, especially those that represent the effect of the local buckling phenomenon, are rated. On the basis of the rating coupled with optimization task, Limit State Design Format (LSDF) based on socio-economic criterion is proposed for each specification in order to provide economical designs which yield uniform reliability level over all practical values of the slenderness parameter.

In order to achieve the stated objective, an evaluation model based on the First-Order Reliability Method (FORM) is employed taking into consideration possible correlation among parameters of the design equations and their associated probability distributions. The method is fully discussed in Chapter 4, and chosen among others due to its merits. The

interaction strength of a thin-walled beam-column is a function of so many parameters. Rather than considering the strength as a single random variable, it is treated as a function of the associated random parameters. Thus, the model is generalized so as to permit any load combination (as permitted by the specifications) and to study the influence of the statistics of the local buckling effect on the interaction strength. As well, the model permits the rating of the design criteria when different grades of steel from which thin-walled elements may be cold-formed are available. Of the identifiable uncertainties, only the measurable uncertainties are easily quantified and they are the ones considered in this study.

Since the proposed transitory formats are forged around safety and economy, a nonlinear programming technique is also employed.

CHAPTER 2

LITERATURE REVIEW

2.1 General Remarks

A great deal of research has been carried out on the behaviour of cold-formed steel structures. These research efforts have been concentrated on improving the design methods. But then there exists the indication that more research is needed to fully uncover the intricacies of the problems. For instance, with respect to doubly-symmetric cross-sections, a survey according to Yu, (1985) shows the following research activities and locations:

(A) USA

Research activities into thin-walled structural behaviour in USA include:

(1) Limit design of cold-formed box sections at the Georgia Institute of Technology.

(2) Local buckling of tubular columns at Lehigh University.

(3) Inelastic response of restrained steel tubes at University of Michigan.

(4) Ultimate design of box girders at New Mexico State University.

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(5) Local buckling of tubular beam-columns at University of Wisconsin-Milwankee.

(B) BELGIUM

Similarly research activities in Belgium include studies on the buckling of rectangular hollow sections at University of Liege.

(C) CANADA

As far as behaviour of thin-walled structural steel members are concerned, the following investigations are underway in Canada:

(1) Behaviour of stiffened box girders at McGill University.
(2) High-strength steel hollow section columns at University of Toronto.
(3) Local buckling of tubes at University of Alberta.
(4) Plastic behaviour of hollow sections at McMaster University.
(5) Behaviour of box girders at Queen's University at Kingston.

(D) JAPAN

At the University of Tokyo, the major work in cold-formed structural steel addresses circular and rectangular tubular members.

(E) *UNITED KINGDOM*

In United Kingdom, we have major activities at:

(1) University of Glasgow which investigates the behaviour of thin-walled rectangular hollow beams.

(2) University of Southampton studies postbuckled columns and box beams.

In this Chapter, the need for the present study is justified and the evolution of the probability-based technique employed is briefly discussed.

2.2 Developments and problems in the Design of Cold-Formed Steel Sections

There are two main families of structural members in use in steel construction. One of them is the familiar group of hot-rolled shapes and members built up of plates. The latter is composed of sections cold-formed from steel strips, plates, or flats in roll-forming machines (see example of such machine in Fig. 2.1) or by press brake or bending brake processes.

Cold-formed steel sections are used, to mention a few, in car bodies, railway coaches, storage racks, grain bins, highway products, transmission towers and poles, drainage facilities, and bridge and building construction. A few examples of applications of some cold-formed steel sections are shown in

Figs. 2.2 to 2.4. From the list of these uses, it can be said that the thickness of the material from which cold-formed sections are manufactured can be very small, and thus, permitting economical use of material.

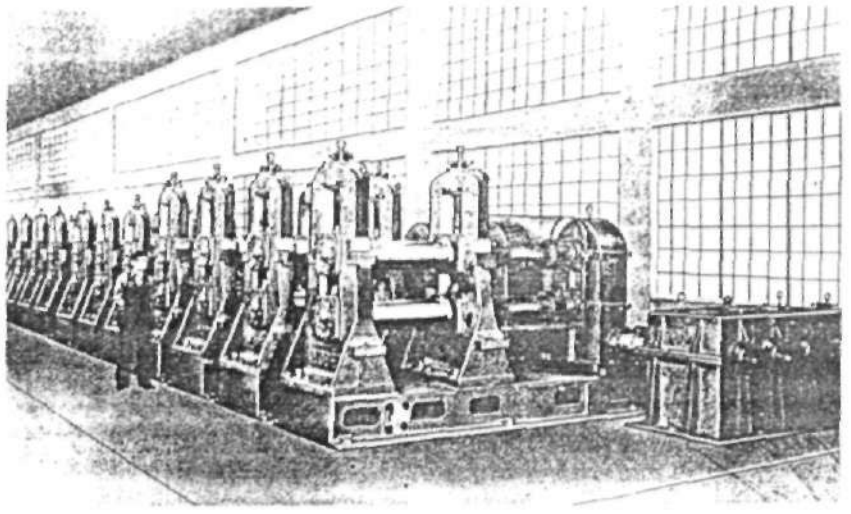


Fig. 2.1: Cold Roll Forming Machine (adopted from Yu, 1985)

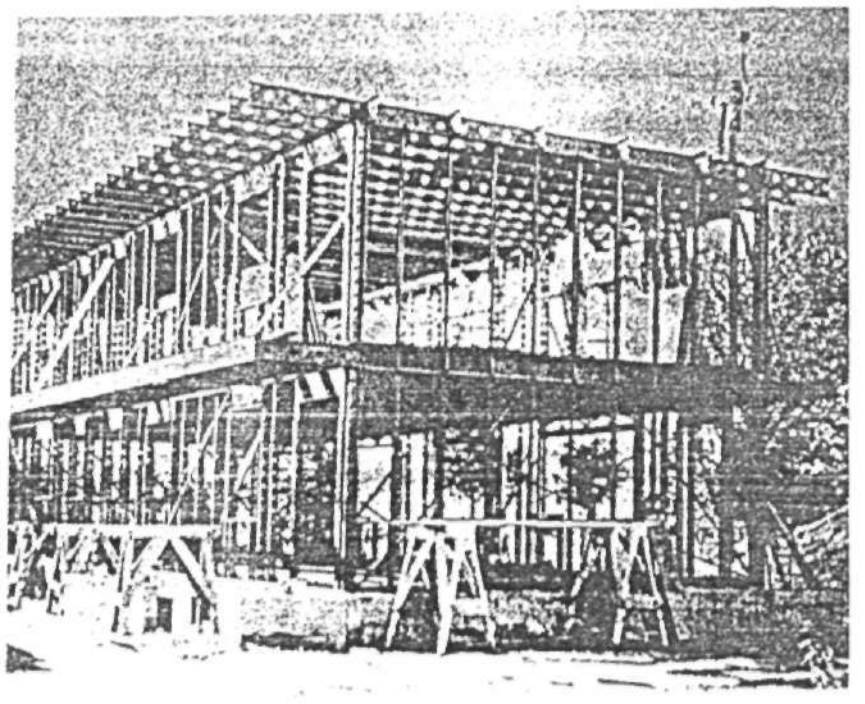


Fig. 2.2: Building Composed entirely of Cold-Formed Steel Sections (adopted from Yu, 1985)

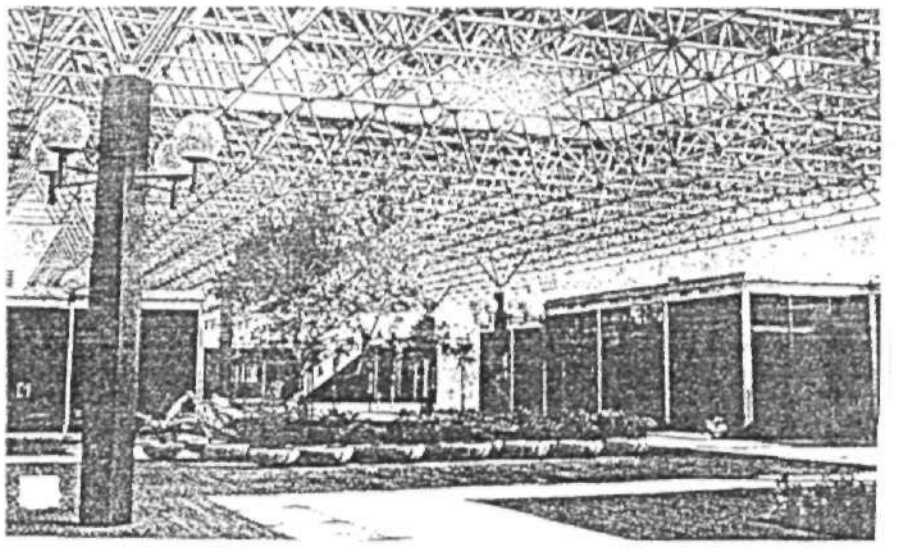
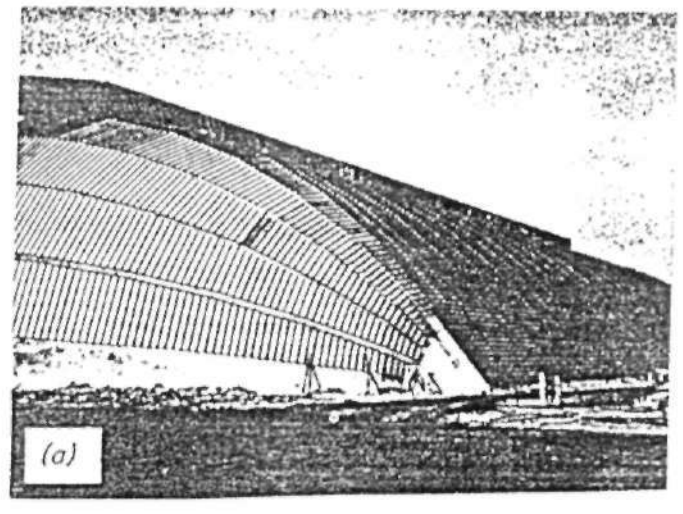


Fig. 2.3: Cold-Formed Steel Sections used in Space Frames (adopted from Yu, 1985)



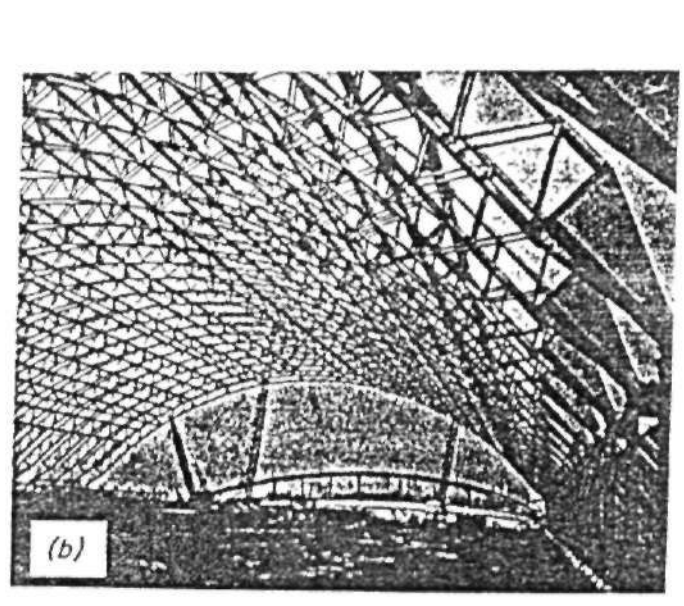


Fig. 2.4: Hanger-Type Arch Structures using Cold-Formed Steel Section (adopted from Yu, 1985)

Although the relative thinness of cold-formed sections is beneficial to economical usage, it is also the source of many design aspects that must be accounted for, if advantage is to be taken of the potential economies. In this section, a brief review of the investigations on these aspects is reported.

Rhodes and Walker (1985) remarked: *As the major producer of cold-formed sections, and having had the longest established research background in the field, the USA has probably the most comprehensive and useful specification for the design of cold-formed sections of any country*. The specification (i.e., AISI, 1968) referred to by these men is one of the two being evaluated in this study with particular attention to the design of beam-columns. The design specifications of Canada (CSA, 1974) and Australia (SAA, 1974) have been based very closely on the AISI (Rhodes and Walker, 1985), and the AISI has been translated into German (Eidamshaus et al, 1976) and used in Germany.

A bulk of research is done in the USA. Yu, (1985) gave a long list of research activities at different Universities in the USA as well as in other countries, especially, in Europe. Rhodes and Walker (1985) reported on some of the major areas of research with regards to buckling problems and areas which require further investigation are pinpointed.

Local buckling of cold-formed steel elements is one of the major design criteria. But it is well known that such elements will not necessarily fail when their buckling stress is reached and that they often will continue to carry loads in excess of that at which local buckling first appears. A number of pictures are shown by Yu, (1985) to illustrate why the postbuckling strength of compression elements is utilized in design. The incorporation of this aspect into the design of beam-columns is of major concern since it is done empirically in the two design methods under evaluation.

Other problematic aspect of cold-formed sections due to the relative thinness of the walls is that in some sections the centroid and the shear centre do not coincide. Thus, torsional-flexural buckling becomes a critical factor for compression members. This phenomenon is of no consequence in the sections (i.e. box and doubly-symmetric sections) that are considered in this study. Such sections are said to have sufficient torsional rigidity (Sherman, 1975).

The simple end conditions (i.e., pin-ended) assumed and generally connections of thin-walled elements have been found to influence their behaviour (e.g., Yu, 1985). Although at the design stage provisions are made for this, such provisions are only approximate and are critically deterministic. This is

because the true behaviour is hidden in the real construction, and can be influenced by many other factors.

With specific reference to the behaviour of thin-walled box sections, reports of investigations documented include (Little, 1978; Chiew et al, 1987; Usami and Fokumoto, 1982; Usami and Fukumoto, 1984; Hasegawa et all, 1988; Shanmugam et al, 1987; Nakai et al, 1985a; Nakai et al, 1985b; Murray, 1987; Hancock et al, 1987). These reports are a combination of experimental and analytical investigations. On the basis of the conclusions drawn by the individual investigators, the prediction of the ultimate load-carrying capacity of box sections is diversified and discrepancies exist among them due, in part, to the appraisal methods.

In order to retain the shape of thin-walled box columns, they are often stiffened by diaphragms or traverse ribs. Krolak (1989) studied the influence of flexural rigidity of such diaphragms on the stability of thin-walled box columns under uniform compression, using an energy method. On the basis of the analysis it was found advisable to take into account the flexural rigidity of the diaphragms during calculations of the local stability of such elements with densely spaced diaphragms. Also the critical loads of uniformly compressed members with densely spaced diaphragms whose thicknesses are equal to that of the walls are fairly

significantly greater than those with perfectly flexible diaphragms. By a way of generalization there is an indication that a unique design format for thin-walled sections may not necessarily exist. It is good then to evaluate critically some of the existing design equations in order to bring forward the role of inherent uncertainties in influencing the resultant performance of components designed in accordance with those requirements and to suggest possible modifications that will make any design criteria yield uniform level of reliability. To do so will require a probability-based technique. In the subsequent section the evolution of such a technique is briefly reviewed.

2.3. Evolution of Structural Reliability Concepts

In a qualitative and very broad sense, the purpose of design can be conceived to be the attainment of equilibrium between safety and economy. Safety calculations have, for many years, been simply formulated on a deterministic approach according to allowable stress criteria. This is to say, that, structures were regarded safe if within an elastic linear model, appropriate combinations of the stresses induced at any point by the nominal values of the applied loads did not exceed predefined proportion of the material strengths which were equally viewed as deterministic. The application of this allowable stress method has revealed some shortcomings, among

which are (Geschwindner, 1988):

(a) Reliability is not uniform over the many different conditions of design. Thus, the exact factor of safety for many individual components or for a system as a whole is not known.

(b) The basis of the method is hidden within arbitrary allowable stresses and factors of safety.

(c) The format of Allowable Stress Design (ASD) is not consistent with the one used for reinforced concrete, nor is it consistent with those of the international community.

Modern developments in design, analysis, building material and techniques have also led to the refinement of structures to suit more precisely the loading and environmental conditions assumed in design. Therefore, the accuracy and adequacy of the design assumptions have assumed much greater importance since a precisely designed structure may be sensitive to a greater or different loading condition. And worse still, the reserve strength previously available may be inadequate or even completely absent. If structures are to be designed with both precision and safety, a conscious assessment of all conditions and hazards that might arise should be made. But according to

Rodin and Chanon (1969), this does not mean that we must design against all hazards. It simply means the combination of risk and consequence of the hazard should be considered so that appropriate action, if any, can be determined to achieve an acceptable and uniform standard of safety. Hence attempts to improve the method called for the need to take into account the random character of the most important design parameters. These include the actions, the strength of the materials, and even at times, the geometric dimensions. It is this consideration that led to the introduction of probabilistic concepts.

One of the first proposals in this direction was put forward by Mayer in 1926. In later years, the problem has been the computation of the probability of failure and consequently the reliability of a structural element or a system. Shinozuka (1972) described reliability analysis as that analysis which combines all available probabilistic and statistical techniques as well as the modern science of decision making in such a manner that the reliability estimate will not only reflect the quality and quantity of the available information but will be consistent with the present state of

* structural or stress analysis
* failure analysis, and
* environmental and load analysis.

The consequence is, therefore, that the accuracy of the reliability estimate cannot be better than the accuracy of the information and the state of art of the engineering science pertinent to the reliability problem in hand. The essential benefit that one can receive by performing reliability analysis has been listed as follows (Shinozuka,. 1972): A definite and better understanding will be achieved on

(1) the effect of increasing or decreasing safety factors on the structural reliability,

(2) the sensitivity of the structural reliability to the other design parameters,

(3) the additional information needed to improve accuracy of the reliability estimate, and

(4) from this understanding, a consistent and systematic way of making decisions will hopefully emerge.

These items constitute part of the objectives of this present investigation.

Reliability analysis has also been grouped under three basic levels, which are levels 1, 2, and 3, depending on the degree of sophistication applied to the treatment of the various problems. The difference among them are operational rather than conceptual. Some comments are made on these levels

without showing any mathematical details which can be found elsewhere (e.g., Leporati, 1979; Madsen et al, 1986; Thoft-Christensen and Baker, 1982).

2.3.1 Level 1 Methods

In level 1 methods, the probabilistic aspect of the problem is taken into consideration by introducing in the limit state equations suitable nominal values of the basic variables, in association with partial safety factors. To ensure appropriate levels of structural reliability, these nominal values are deduced from probabilistic considerations. There are, in progress, continuous revisions of the operational methods used (e.g., CEB, 1976) under this level. At the same time a number of alternative proposals have been recently formulated by various research groups (e.g., NCBR, 1974; CECM-ECCS, 1975).

Broadly, if the failure criterion in respect of a predetermined limit state is expressed in the form:

$$g(Y_1, Y_2, \ldots\ldots, Y_n) < 0 \qquad (2.1)$$

by means of level 1 methods, safety analysis is effected by checking that the following inequality is satisfied

$$g(Y_1^*/\gamma_1, \quad Y_2^*/\gamma_2, \ldots\ldots, Y_n^*/\gamma_n) \geq 0 \qquad (2.2)$$

In the above equations,

Y_i = the design variables,

Y_i^* = the characteristic values of the design variables and

γ_i = the partial safety factors.

Level 1 schemes are said to have great operational simplicity due to the use of fixed and constant partial safety factors for a given class of design situation (Leporati, 1979). But the shortcomings are that the reliability of individual designs will necessarily vary from case to case. In order words, the objective of obtaining constant reliability cannot be reached for all the design situations covered in a given specification.

2.3.2 Level 2 methods

With level 2 methods, safety checks are made at finite number of points of the safety domain boundary which is projected into the space of noncorrelated and standardized basic random variables. In this way, direct multidimensional integration or the use of simulation techniques are avoided. The parameters that require determination are the distances of appropriately selected points of the safety domain boundary from the origin of the system of coordinates, in case of multiple checks. On the other hand, when only one check is needed, the minimum distance is sought. It is possible to associate a precise meaning in terms of reliability with this

distance if joint normal distribution is assumed for the random variables involved and suitable geometries for safety domain boundary are chosen.

The Mean First Order-Second Moment Method (MFOSMM) and the Advanced Mean First Order-Second Moment Method (AMFOSMM) are in this level. The basic concept of second moment methods is to set out all random quantities in terms of their first two moments. These operational procedures were introduced by Cornell (1969) for codification purposes.

Rather than checking safety in terms of probability of failure, it is checked using the reliability index. That is, let a limit state function be:

$$Z_M = \lambda_R - \lambda_S \qquad (2.3)$$

where λ_R and λ_S are respectively strength and stress variables which are usually functions of other variables.

In terms of the first two moments of the strength and stress employing Taylor series the reliability index, ß, is expressed as

$$\beta = \frac{\overline{\lambda}_R - \overline{\lambda}_S}{\sqrt{\sigma_{\lambda_R}^2 + \sigma_{\lambda_S}^2}} \qquad (2.4)$$

where

$\overline{\lambda}_R$ = mean value of λ_R

$\overline{\lambda}_S$ = mean value of λ_S

$\sigma_{\lambda R}$ = standard deviation of λ_R

$\sigma_{\lambda S}$ = standard deviation of λ_S

Since one of the procedures of level 2 is employed in this investigation, the detail of Equation (2.4) is fully discussed in Chapter 4. But it should be pointed out here that the condition to be checked for safety is

$$\overline{\lambda}_R - \overline{\lambda}_S - \beta\sqrt{\sigma_{\lambda_R}^2 + \sigma_{\lambda_S}^2} \geq 0 \qquad (2.5)$$

The so-called AMFOSMM which is also described in Chapter 4 requires the determination of ß by iteration (Smith, 1986). This is because the value of ß depends on the minimum distance from the origin to the failure surface in the space of standardized variables (See Fig. 2.5).

Generally, in the level 2 procedures, it is not necessary to insist on the relative frequencies of real non-failures, but only on establishing the significance of the compared measures of risk within standardized operational procedures (Leporati, 1979).

2.3.3 Level 3 Methods

These are the most complete methods of reliability analysis (Leporati, 1979). In the procedure all the random variables involved are represented by their distribution functions and joint probabilistic behaviour is expressed in terms of the covariance matrix. The knowledge required is the appropriate safety domain for each limit state.

Level 3 methods are not easily applicable to practical safety checking and design because of the analytical and numerical difficulties involved. However, they are the only available means to check the approximations and also the validity of the assumptions made in the operational methods of levels 1 and 2. Studies conducted on level 3 methods include (Freudenthal et al, 1966; Tichy and Volicek, 1972) and using a computer (Paloheimo, 1969).

The fundamental formulation in the level 3 methods is of the form:

$$P_F = \int\int_{DF} f_{\lambda_R \lambda_S}(\lambda_R, \lambda_S)\, d\lambda_R d\lambda_S \qquad (2.6)$$

where DF is the failure domain; and λ_R and λ_S are as earlier

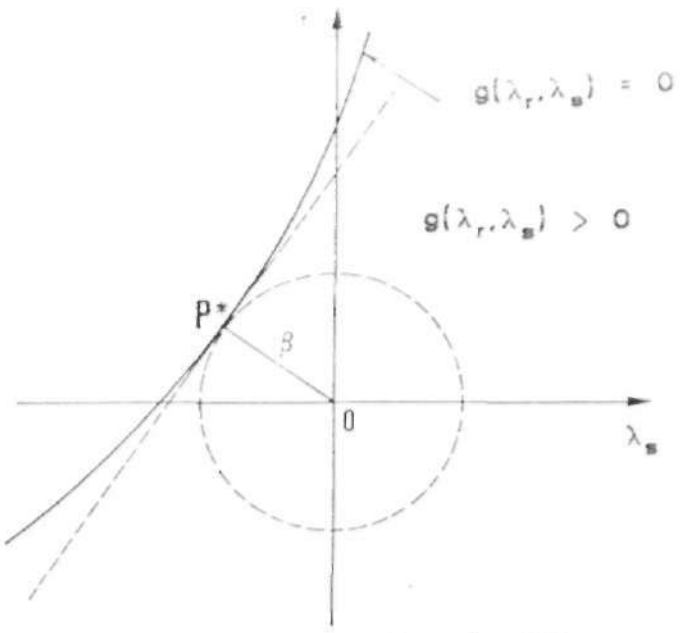


Fig. 2.5: The Geometric Significance of ß

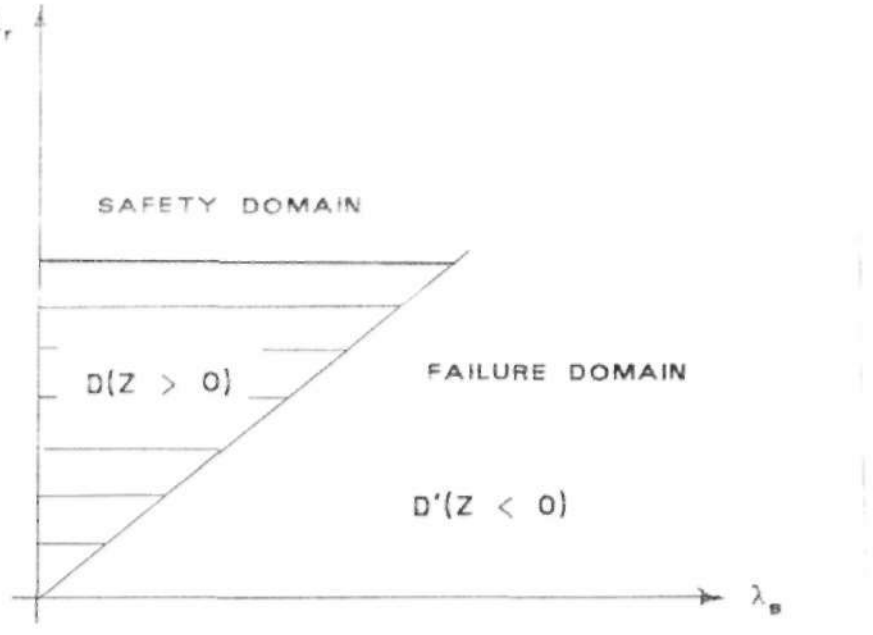


Fig. 2.6: Limit State Function for Safety Analysis

defined. From Eq. (2.6), the reliability of a component is, by definition,

$$R = 1 - P_F = 1 - \iint_{DF} f_{\lambda_R \lambda_S}(\lambda_R, \lambda_S)\, d\lambda_R d\lambda_S \tag{2.7}$$

Generally, the criterion for safety has been formulated in terms of the resistance, λ_R, and load applied, λ_S. Mathematically, the criterion is of the form:

$$\lambda_R - \lambda_S > 0 \tag{2.8}$$

or

$$\frac{\lambda_R}{\lambda_S} > 1 \tag{2.9}$$

On the basis of any of the above requirements, the conclusion is that safe structures are characterized by a relatively low probability of failure. The distribution of λ_R and λ_S are needed, in a simple form, in safety analysis. According to Borges and Castanheta (1969), each distribution measures the probability of it being smaller than a given value and these probabilities must refer to the same variable that measures both loads and resistances, taking for reference the same interval of time.

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A more generalized concept of failure takes into account the concept of limit state (Leporati, 1979). That is, a state beyond which a structure, or part of it, can no longer fulfil the functions or satisfy the conditions for which it was designed. For instance, let Eq. (2.8) be expressed as

$$Z_M = \lambda_R - \lambda_S \tag{2.10}$$

If λ_R and λ_S take positive arguemental values, the condition of probability of survival (i.e. reliability) is the shaded area in Fig. 2.6. That is, Z_M takes a positive value. To establish a criterion for safety the requirement is that $Z_M > 0$. And this is the key assumption on which the present work is conducted.

2.4 Significance of the Reliability Methods in Structural Safety

In the practice of planning, design, analysis and construction, civil engineers have always been aware of the uncertainties (natural and man-made) that must be negotiated. The awareness has always been resolved through the so-called safety factors. The associated problems with the use of the allowable stresses through safety factors have been pointed out in the previous section. To ascertain a more meaningful

quantitative measure of safety, the reliability technique which is probability-based has been sought. More recently, some of the existing design codes (e.g. CEB, 1976; CIRIA, 1977; NBS, 1980; ECCS, 1981) have been reexamined and modified on the basis of these probabilistic concepts.

Reliability analysis methodology has made and can further make genuine contributions towards enhancing the structural safety and integrity of constructed facilities (Shinozuka, 1986). Beyond the applicability of reliability concepts to traditional engineering structures, reliability analysis has become an integral part of the risk assessment and management procedures for a wide variety of structures (e.g., Rodin and Chanon, 1969; Committee, 1982a; 1982b; 1982c; 1982d; Matsuo and Asaoka, 1983, McAnally, 1983; Cazzulo, 1987; Cheieleskwi, 1987; Costa, 1987; Ditlevsen et al, 1987; Grigoriu, 1987; Guenard and Lebas, 1987; Nielsen et al, 1987; Nowak and Lind, 1987; Schmidt and Schomburg, 1987; Black, 1972, Konishi, 1972; Chang, 1983) including such risk-sensitive structural systems as nuclear power plants and aircrafts (e.g., Gürpinar and Giuliani, 1987; Jaeger, 1972; Kececioglu and Koherecheck, 1983; Thomas and Bosinoff, 1983).

As far back as 1969, Ang showed that availability of a minimum acceptable risk permitted the formulation of an extended reliability basis for safe design and evaluation of

safety factors. Lightenberg (1969) also pointed out some factors that in reality have a great influence on the probability of failure (and consequently the reliability) of a structure. Some of his findings have been answered in reliability studies, attesting to its importance.

The interrelationship among the probability of structural failure, the expected cost of failure and the proof-load testing has been established and used for a general reliability analysis (Shinozuka, 1969).

In 1969, Ott and Shah studied the feasibility of developing the mathematical tools to investigate the reliability of structural systems under probabilistic and stochastic loading. They found that derivation of the reliability distributions was fairly simple for the case where only one parameter was a random variable. But it was not possible to find a useable analytical solution for the distributions chosen. Therefore approximations were suggested.

In the context of structural engineering the resistance of a structure or element is usually a function of a number of variables. Each of these variables is random, and an analysis could be made if the relevant statistical parameters of each were known. Inclusion of all these effects would be a formidable task, although analyses with some of the variables

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have been made (Augusti and Baratta, 1970; Chung and Lee, 1971; Bjorhovde, 1972; Bjorhovde and Tall, 1971) especially for column elements. In order to circumvent the problem, the mean resistance has been expressed as (Galambos, 1972):

$$R_m = [BiasFactor]\,[NominalFormula] \qquad (2.11)$$

where

$$[BiasFactor] = \left[\frac{TestCapacity}{TheoreticalPrediction}\right]\left[\frac{TheoreticalPrediction}{NominalStrength}\right]_m$$

and the [Nominal Formula] is the resistance equation which is to be used in the new code for the particular type of element. This idea has been used to evaluate reliability of structural steel elements (e.g. Chapius and Galambos, 1982; Galambos et al, 1979; Shin et al, 1988). However, such an approach obscures the possibility of monitoring the contribution of the statistics of the key parameters to the variation in the strength, and particularly for cold-formed elements.

In cold-formed elements, interaction of local and overall buckling phenomenon makes a key difference between the behaviour of hot-rolled and thin-walled elements. As previously reviewed, it is clear that the phenomenon is yet a question of further verifications. Bearing this in mind, it is then preferred in this study to treat the resistance of the

beam-column as a function of random parameters rather than a single random variable. The quantification of random parameters allows the opportunity to examine the statistical contributions of such parameters to the statistics of the resistance and consequently their influence on the reliability of the design criteria.

Although the applications discussed above are numerous and diversified, a number of major issues that have not really been resolved in the structural reliability analysis have been pointed out and some of them are (Shinozuka, 1986):

(1) It is not easy to obtain a consensus on the target levels of reliability even for traditional civil engineering structures.

(2) Not everything is always probabilistic, although the reliability analysis methodology developed so far presumes that we can somehow formulate everything in terms of probability.

(3) Even if, it is agreed that everything can be interpreted as probabilistic, the casual fashion in which the source of the variability is often divided into "uncertainty" could give the false impression that such a division is easy, while certainly it is not.

(4) The reliability methodology has so far been unable to properly incorporate human factors, managerial as well as technical.

These problems are however not ignoring the many advantages a Reliability-Based Design (RBD) criterion has, at least, over the traditional Allowable Stress Design (ASD). Some of the advantages and the disadvantages of both RBD and ASD were listed by Geschwindner in 1988. It is clear from the comparison that RBD has the largest list of advantages.

Within the scope of literature availability, no similar course of analysis as undertaken herein for cold-formed beam-columns has been reported.

2.5 Optimization in Structural Engineering

A vast quantity of literature has been published in the area of structural optimization in the last two decades (e.g. Rozvany and Cohn, 1970; Weiji, 1986; Mroz and Lekszycki, 1981; Ovunc, 1989). However, most of the activities are centered around a deterministic design philosophy characterized essentially by a well defined deterministic structural code for checking constraints, and usually an element optimization procedure for finding the best design variables which result in the minimization of material weight or cost.

Optimization formulations which do not incorporate the inherent nature of both structural loading and strength present a significant drawback. This is because structural problems are generally nondeterministic and, consequently, engineering optimum design must cope with uncertainties.

Code calibration using the reliability concepts discussed in the previous section has shown that safety level does not remain uniform within the context of deterministic code applications due to inherent uncertainties and the fact that the mathematical models upon which such codes are built cannot fully replicate the true nature of state. It is then logical to say that methods of optimization developed on deterministic safety concepts, while minimizing the weight or cost of an element, may change the elemental safety level.

Since the proper tool for assessing and analyzing uncertainties requires methods and concepts of reliability (e.g. Ang and Tang, 1984), it may be welcoming to affirm that the combination of reliability-based design procedures and optimization techniques is the only means of providing a powerful tool to obtain a practical optimum design solution (Frangopol, 1985). In this direction various studies have also been carried out with various criteria and constraints (e.g., Hilton and Feigen, 1960; Kalaba, 1962; Frangopol, 1985a; Moses and Kinser, 1967; Surahman and Rojiani, 1983; Parimi and Cohn,

1978; Frangopol, 1985b), and a large number of applications is found, for examples, in Thoft-Christensen and Murotsu (1986) and Brebbia abd Hernandez (1989).

In general, for example, the problem of probabilistic optimization with collapse and loss of serviceability as the criteria can be stated in mathematical-programming form as follows (Frangopol, 1986):

$$\pi = \pi(Z') \leq \pi^* \tag{2.12}$$

as cost or weight constraint, and

$$P_{FC} = P_{FC}(Z') \leq P^*_{FC} \tag{2.13}$$

as reliability constraint against collapse and

$$P_{FS} = P_{FS}(Z') \leq P^*_{FS} \tag{2.14}$$

as reliability constraint against unserviceability in which Z' = the vector of design variables; π = the total cost or weight of the structure and π^* the permissible value; P_{FC} and P^*_{FC} = the probabilities of plastic collapse and the allowable value, respectively; and P_{FS} and P^*_{FS} = the probabilities of unserviceability and allowable value.

Equations (2.12) to (2.14) provide nine possible combinations for formulating an optimization problem under uncertainty particularly the optimization of several conflicting objectives as is the case in this investigation.

CHAPTER 3

BEAM-COLUMN INTERACTION STRENGTH

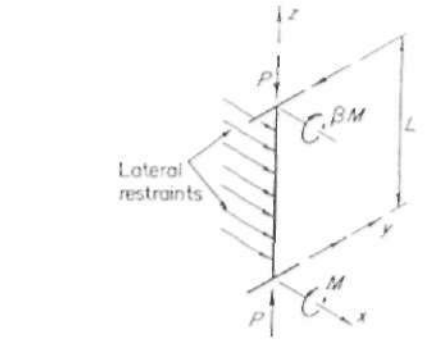
3.1 Introduction

Beam-columns are structural members which combine the beam function of transmitting transverse forces or moments with the compression (or tension) member function of transmitting axial forces. The behaviour of isolated beam-columns can be conveniently discussed under three distinct headings, viz (Trahair, 1977): In-Plane Behaviour, Flexural-Torsional Buckling and Biaxial Bending. These different behaviours are schematically shown in Fig. 3.1. However, in an expanded manner, a number of categories of combined and axial load along with the likely mode of failure may be summarized as follows (Salmon and Johnson, 1980):

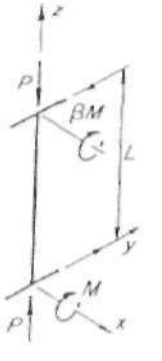
(1) Axial tension and bending in which failure is usually by yielding.

(2) Axial compression and bending about one axis, failure is by instability in the plane of bending, without twisting.

(3) Axial compression and bending about the strong axis, failure is by lateral-torsional buckling.



(a)



(b)

(c)

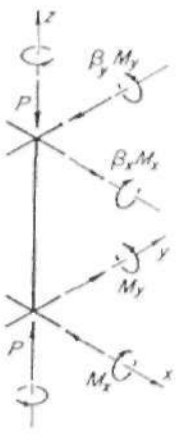


Fig. 3.1: Beam-Column Behaviour (a) In-Plane Behaviour, (b) Flexural-Torsional Buckling and (c) Biaxial Bending.

(4) Axial compression and biaxial bending (for torsionally stiff sections), failure is by instability in the plane of principal directions.

(5) Axial compression and biaxial bending (thin-walled open sections), failure is by combined twisting and bending on these torsionally weak sections.

(6) Axial compression, biaxial bending, and torsion; failure is by combined twisting and bending when the plane of bending does not contain the shear centre.

The in-plane behaviour is the focus in this investigation and subsequent discussions are on the behaviour of beam-columns not subject to flexural-torsional buckling and are under uniaxial bending. Therefore in this Chapter, the general review of the exact analysis of the in-plane behaviour is done in Section 3.2 while the rest of the Chapter concentrates on the design rules with the view to examining their adequacy considering different assumptions adopted and the consequence of unavoidable uncertainties. Two different design criteria are discussed but to up-date information, comments on the relevant research investigations are included for comparison. This is to show that for the number of failure modes there is no simple design procedure which will likely account for such varied behaviour. Design procedures have been categorized viz

(Salmon and Johnson, 1980):

(1) limitation on combined stress;

(2) semi-empirical interaction formulae based on working stress procedure, and

(3) semi-empirical interaction procedures based on ultimate strength.

According to Salmon and Johnson (1980), design procedure based on (1) cannot provide a proper criterion unless instability is prevented, or large safety factors are used. But generally the interaction equations are closer to describing the true behaviour since they account for the stability situations commonly encountered.

3.2 General Analysis

In this section, only the in-plane behaviour is discussed assuming failure by instability in the plane of bending. For a simple illustration of the exact analysis, a beam-column element with uniformly distributed lateral load (See Fig. 3.2) is considered.

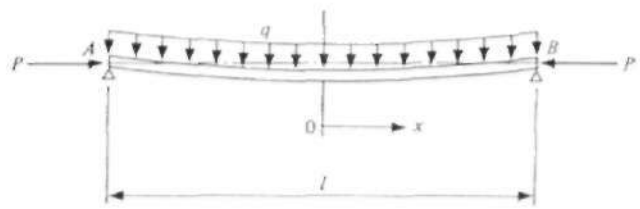


Fig. 3.2: In-Plane Beam-Column with Uniform Load

The elastic limit state of the element is attained when the maximum stress reaches the elastic limit or the yield point, f_y, of the material. Thus, the governing differential equation for elastic beam-columns with uniformly distributed load can be given as:

$$\frac{d^4v}{dx^4} + k\frac{d^2v}{dx^2} = \frac{q}{EI} \tag{3.1}$$

in which

$$k = \sqrt{\frac{P}{EI}}$$

and v is the lateral deflection of the element. Since the system is symmetric about the mid-span, it is enough to take the coordinate x from the mid-span. Then the general solution to Eq. (3.1) can be expressed as:

$$v = c_1 \cos kx + c_4 + \frac{q}{2k^2EI}x^2 \tag{3.3}$$

When the boundary conditions are applied, that is, at x = +L/2 or -L/2, v = 0 and

$$\frac{d^2v}{dx^2} = 0 \tag{3.4}$$

the constants c_1 and c_4 are obtained and shown as

$$c_1 = \frac{q}{EIk^4 \cos(kL/2)} \tag{3.5}$$

and

$$c_4 = -(q/EIk^4)(1 + kL/8) \tag{3.6}$$

It then follows by substitution in Eq. (3.3) that

$$v = -\frac{q}{EIk^4}\left[1 + \frac{k^2L^2}{8} - \frac{k^2x^2}{2} - \frac{\cos kx}{\cos(kL/2)}\right] \tag{3.7}$$

From Eq. (3.7) the moment distribution along the member can be determined as

$$M = -EI\frac{d^2v}{dx^2} = \frac{q}{k^2}\left[\frac{\cos kx}{\cos(kL/2)} - 1\right] \tag{3.8}$$

the maximum bending moment occurs at the mid-span where x = 0, that is,

$$M_{max} = M_o(8/k^2L^2)\left[\frac{1}{\cos(kL/2)} - 1\right] \tag{3.9}$$

in which M_o is the maximum bending moment induced by q only, and the expression in the square bracket represents the magnification factor due to axial compression.

When P = 0,

$$M_o = qL^2/8 \tag{3.10}$$

Using power series expansion and further simplifications, Eq. (3.9) is reduced to:

$$M_{max} = M_o\,[1 + 1.0281(P/p_c)][1 + P/P_c + (P/P_c)^2 + \ldots]$$

$$= M_o\left(1 + \frac{1.0281(P/P_e)}{1 - P/P_e}\right) \tag{3.11}$$

so that

$$M_{max} = M_o\left(\frac{1 + 0.0281(P/P_e)}{1 - P/P_e}\right) \tag{3.12}$$

Since the axial thrust P is essentially a constant value along the element, the elastic limit stress occurs on the cross-section of the maximum bending moment. Therefore for a beam-column, the criterion for the elastic limit state can be given as (Trahair, 1977; Chen and Atsuta, 1976):

$$\frac{M_{max}}{Z} + \frac{P}{A} = f_y$$

or

$$\frac{M_{max}}{M_y} + \frac{P}{P_y} = 1 \qquad (3.13)$$

where

$$M_y = Zf_y \qquad (3.14)$$

and

$$P_y = Af_y \qquad (3.15)$$

are the initial yield moment in pure bending of a section

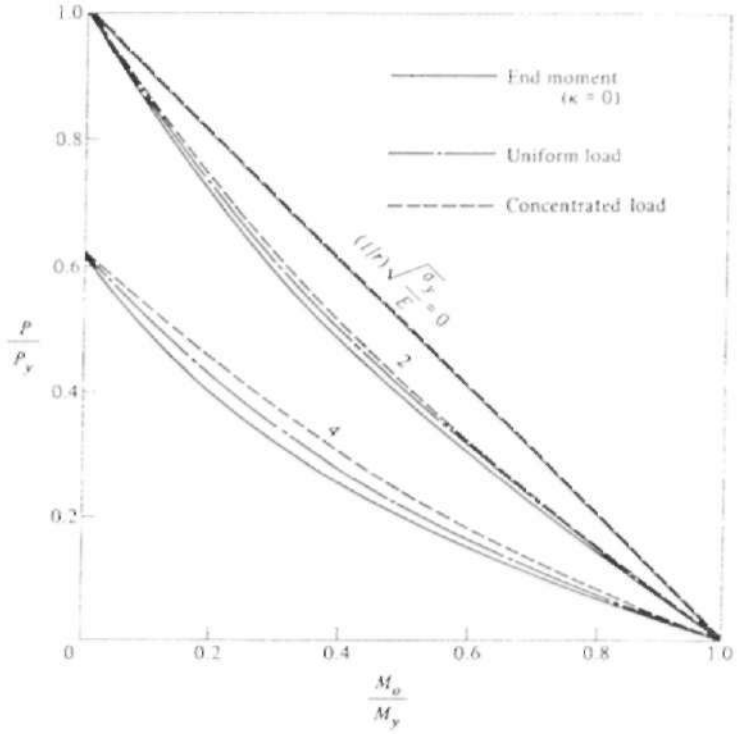


Fig. 3.3: Elastic Limit Interaction Curves

having an elastic section modulus Z and the yield thrust in pure compression of the cross-section of area A, respectively. When Eq. (3.12) is substituted into Eq. (3.13) the interaction equation yields:

$$\frac{P}{P_y}+\frac{1+0.0281P/P_e}{1-P/P_e}\frac{M_o}{M_y}=1 \qquad (3.16)$$

Similar derivation can be obtained if concentrated loads were applied. For a comparison, Fig. 3.3 has been taken from Chen and Atsuta, (1976) to show the interaction curves for uniformly distributed and concentrated loads at a given slenderness parameter.

For design purposes, either for cold-formed or hot-rolled beam-column sections, Eq. (3.16) is generally taken as (Yu, 1985; AASHTO, 1977)

$$\frac{f_a}{F_a}+\varphi\frac{f_B}{F_b}\leq 1.0 \qquad (3.17)$$

where

F_a = allowable/ultimate stress for the design of compression members

F_b = allowable / ultimate stress for the design of beams, and

φ = amplification factor due to the presence of the axial load.

Eq. (3.17) is further examined as its presentation has taken different forms especially in the design specifications for thin-walled members. A number of analytical investigations exist in literature on the interaction of local and global buckling in thin-walled beam-columns (e.g, Davids and Hancock, 1987; Benito and Sridharan, 1984; Benito and Sridharan, 1985; Sridharan, 1983; Sridharan and Ali, 1985). It can be said, gathering from these works especially from Davids and Hancock (1987) that the ultimate strength of slender beam-columns is sensitive to initial local and global geometric imperfections while it is not for short length axially compressed thin-walled beam-columns. However, these observations are based on I-sections.

Dawe and Kulak (1986) developed a computer-aided analytical technique to study the local buckling behaviour of beam-columns. Their work showed that the plate slenderness, as well as residual stresses are notable among those parameters affecting local buckling behaviour. The examination of the criteria specified by AISI and ECCS is focussed herein and therefore both are subsequently reviewed.

3.3 Design Formulations

On the basis of the discussion in the previous section, the design requirements prescribed for doubly-symmetric shapes subject to both axial compression and bending are reviewed in accordance with AISI, 1968 and ECCS, 1987. The objective is to study the two interaction equations so that the manner in which the local buckling effect is taken into consideration can be assessed and then to be able to establish the key parameters that can be probabilistically treated as random variables in the reliability computations which occupy the rest of the Chapters of this thesis.

3.3.1 AISI Specification

AISI specifies that when doubly-symmetric and torsionally stable shapes are under the combined action of axial compression and bending, the member shall be proportioned to meet the requirements below.

When $f_a/F_{a1} > 0.15$

$$\frac{f_a}{F_{a1}} + \frac{C_{mx} f_{bx}}{(1 - f_a/F_{ex}') F_{bx}} + \frac{C_{my} f_{by}}{(1 - f_a/F_{ey}') F_{by}} \leq 1.0 \qquad (3.18)$$

$$\frac{f_a}{F_{ao}} + \frac{f_{bx}}{F_{b1x}} + \frac{f_{by}}{F_{b1y}} \leq 1.0 \qquad (3.19)$$

Instead of Eqs. (3.18) and (3.19), the following formula is recommended when $f_a/F_{a1} \leq 0.15$

$$\frac{f_a}{F_{a1}} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0 \qquad (3.20)$$

where

f_a = axial stress, that is, axial load divided by full cross-sectional area of member (= P/A, [KNm^{-2}])

f_b = maximum compressive bending stress, noting that for members having stiffened compression elements, the section modulus is based upon the effective design widths of such elements, [kNm^{-2}]

F_{a1} = allowable compressive stress under concentric loading according to Section 3.6.1.1 of the Specification for buckling in the plane of symmetry, [kNm^{-2}]

F_{ao} = allowable compression stress under concentric loading determined by Section 3.6.1.1 of the Specification for L = 0, [kNm^{-2}]

F_b = allowable bending stress in compression where bending stress only exists (Section 3.1, 3.2, and 3.3 of the Specification), [kNm^{-2}]

F_{b1} = allowable bending stress in compression where bending stress only exists and the possibility of lateral buckling is excluded (Section 3.1 and 3.2 of the

Specification), [kNm^{-2}]

F_e' = $12\pi^2E/[23(Kl_b/r_b^2)]$, [$kNm^{-2}$] which may be increased one-third in the presence of Wind, Earthquake, and Combined Forces.

C_m = a coefficient whose value shall be taken as in the Specification, but taken as unity as one of the possibilities given and to conform to the boundary requirements of the beam-column elements being studied.

L_b = actual unbraced length in the plane of bending, [m]

r_b = radius of gyration about axis of bending [m]

k = effective length factor in the plane of bending.

The subscripts x and y in Eqs. (3.18) to (3.20) indicate the axis of bending about which a particular stress or design property applies. The values of C_m can be found as summarized in a Table obtainable in (Yu, 1985) and any other relevant text.

Equations (3.18) to (3.20) are cases of biaxial bending. To present a general formulation for a uniaxial bending as investigated herein, we have respectively:

$$\frac{f_a}{F_{a1}} + \frac{C_m f_b}{(1 - f_a/F_e')F_b} \leq 1.0 \qquad (3.21)$$

and

$$\frac{f_a}{F_{ao}} + \frac{f_b}{F_{b1}} \leq 1.0 \qquad (3.22)$$

when $f_a/F_{a1} > 0.15$,

and

$$\frac{f_a}{F_{a1}} + \frac{f_b}{F_b} \leq 1.0 \qquad (3.23)$$

when $f_a/F_{a1} \leq 0.15$.

The criteria represented by Eqs. (3.21) and (3.22) are subjected to reliability evaluation. As will be seen later, the expression for F_{a1} and F_{b1} are not the same for AISI and ECCS. Although in general the two design expressions are not the same, their adequacy can be examined on the basis of issuing a targeted uniform performance level.

The criteria specified by AISI is examined using the interaction curves. It is clear from the plots in Fig. 3.4 that the effect of the local buckling on the interaction strength produces different failure surfaces. The explanation cannot be far from the fact that the empirical incorporation of the form factor Q_w which takes care of the weakening effect of local buckling seems inadequate. The adequacy can only be effectively examined using any probabilistic models as discussed in Chapter 4.

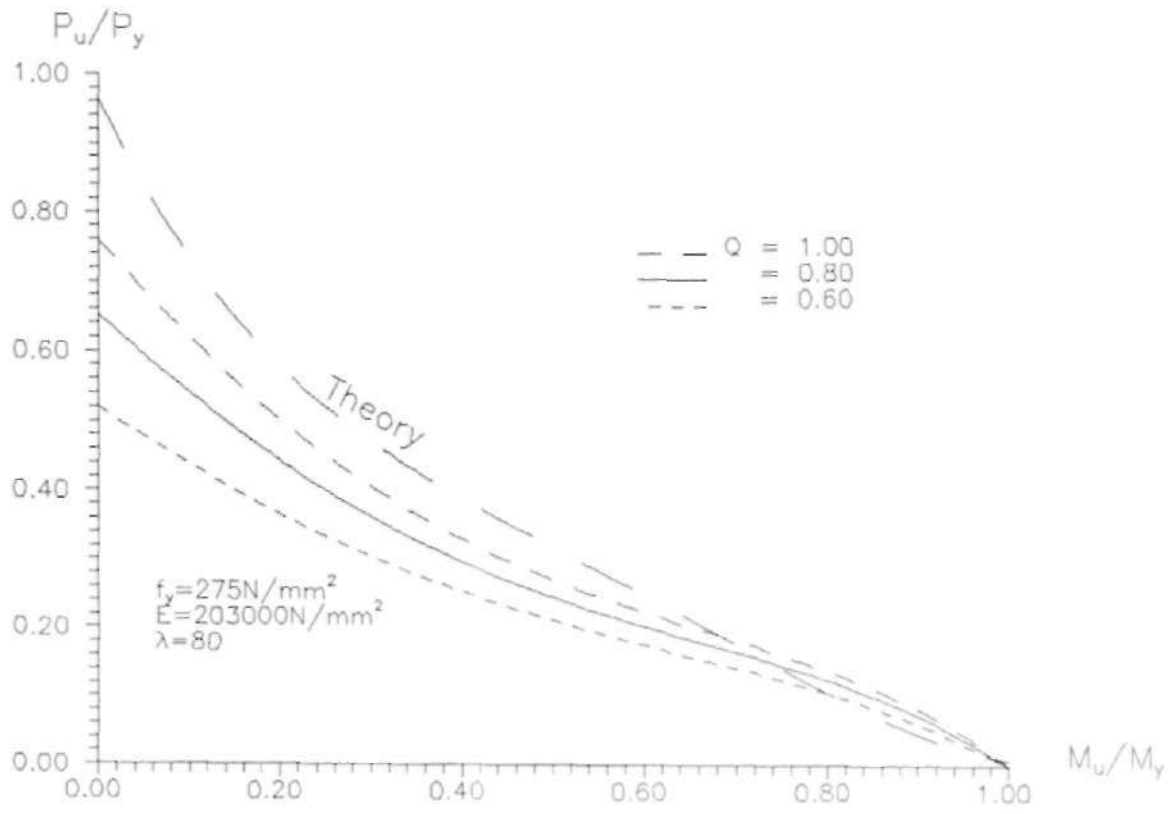


FIG. 3.4: AISI Design Curves for Isolated Beam-Columns

3.3.2 ECCS Recommendation

The ECCS Recommendation prescribes for members in flexure and compression, with mono or bisymmetric cross-sections, an interaction equation of the form:

$$\frac{N}{N_{xd}} + \frac{1}{1 - \frac{N(\chi\lambda'_x)^2}{Q_c N_{xd} R_x}} \frac{M_x}{M_{xd}} \leq 1.0 \qquad (3.24)$$

where

N = the calculated axial force due to design load

N_{xd} = design value of concentric axial compressive force according to Section R 5.1.2 of the Recommendation (Buckling about the x-axis)

χ = reduction coefficient with respect to buckling

λ' = slenderness parameter

M_x = calculated moment about the x-axis or an eccentric axial force, M_x is the product of the axial force and its eccentricity with respect to the centroid of the effective cross-section at uniform compression

R_x = I_{cx}/I_{xg}, where I_{cx} and I_{xg} are the moments of inertia of the reduced cross section in pure bending and the full cross-section, respectively, and

M_{xd} = design value of moment about the x-axis in pure flexure according to Section R 4.2.1 of the Recommendation.

It has been established earlier that in-plane behaviour is being considered for torsionally stable members. Therefore for the sake of comparison and uniformity, Eq. (3.24) is modified to reflect the influence of varying moment along the member as done for AISI. Thus, Eq. (3.24) becomes

$$\frac{N}{N_{xd}} + \frac{1}{1 - \frac{N(\chi\lambda_x')^2}{N_{xd}Q_wR_x}} \frac{M_{eq}}{M_{xd}} \leq 1.0 \qquad (3.25)$$

where

$$M_{eq} = \beta' M_{max}$$

and β'= moment coefficient taking account of the moment distribution along the member.

For instance, when a uniformly distributed load is considered and from Table 6.2.2 of the ECCS recommendation, β' = 1.0. Hence Eq. (3.25) is reduced to:

$$\frac{N}{N_{xd}} + \frac{1}{1 - \frac{N(\chi\lambda_x')^2}{N_{xd}Q_wR_x}} \frac{M_{max}}{M_{xd}} \leq 1.0 \qquad (3.26)$$

Similar to the criteria of AISI, the interaction curves expressed as a function of load ratio are also influenced by the local buckling phenomenon. See Fig. 3.5. Different failure surfaces are exhibited for varying values of the form factor Q_w.

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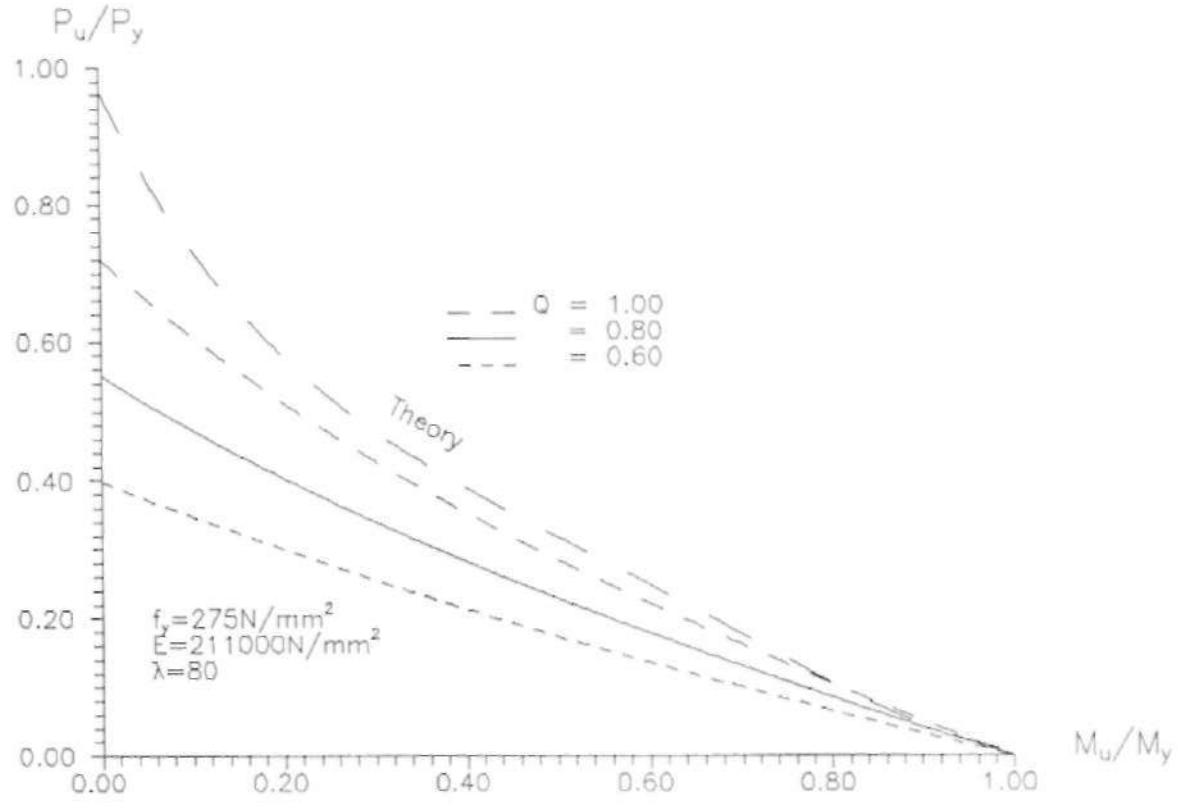


FIG. 3.5: ECCS Design Curves for Isolated Beam-Columns

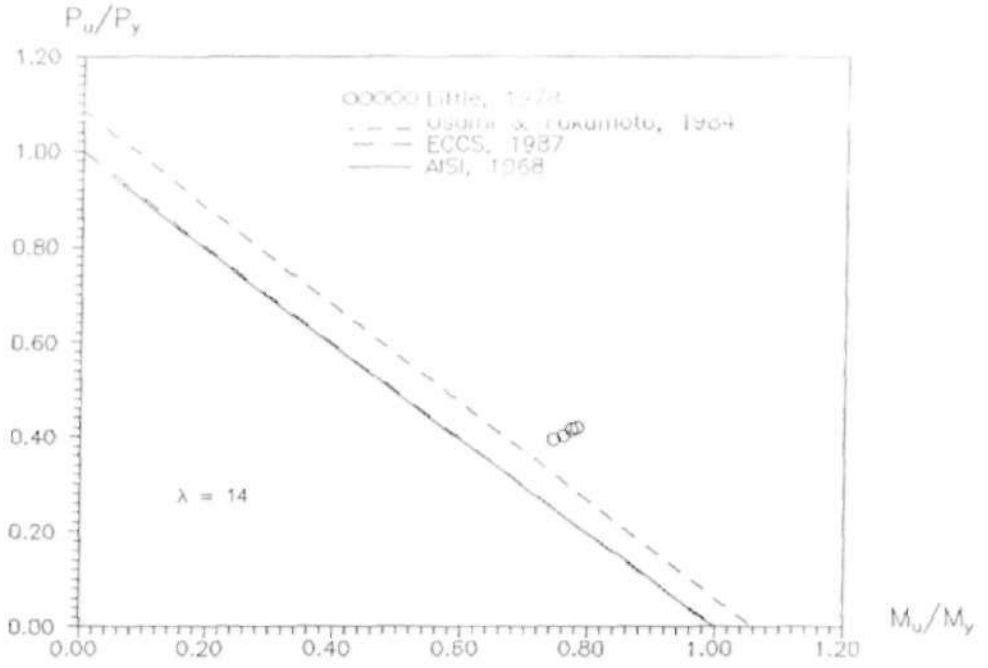


Fig. 3.6: Comparison of Interaction Equation and Test Results (λ = 14)

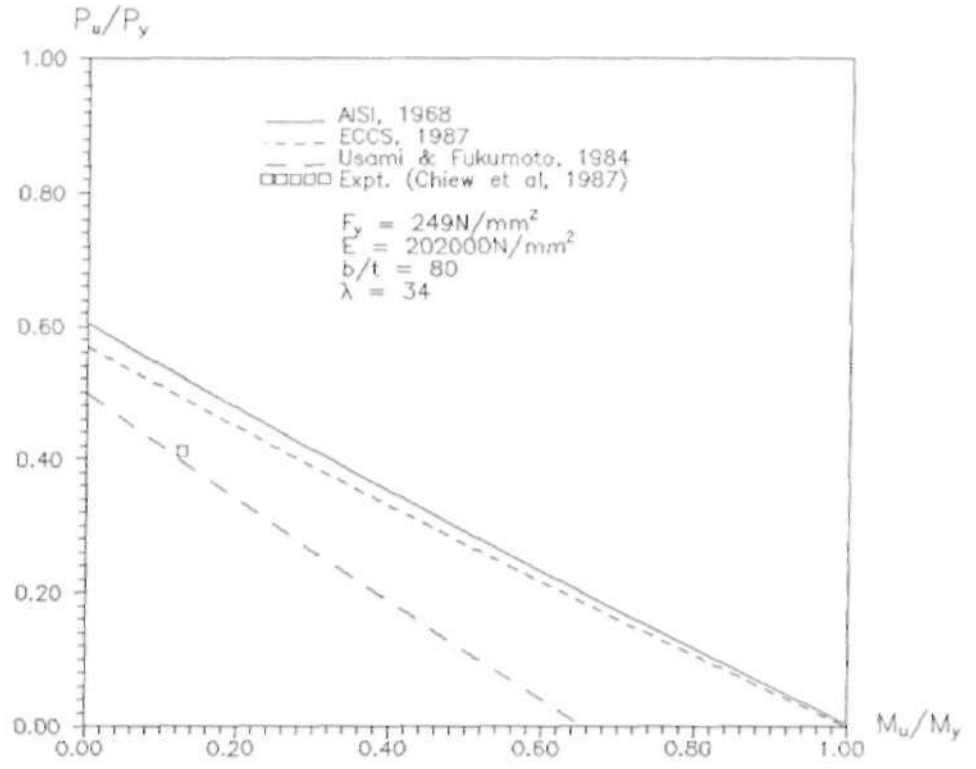


Fig. 3.7: Comparison of Interaction Equation and Test Results (λ = 34)

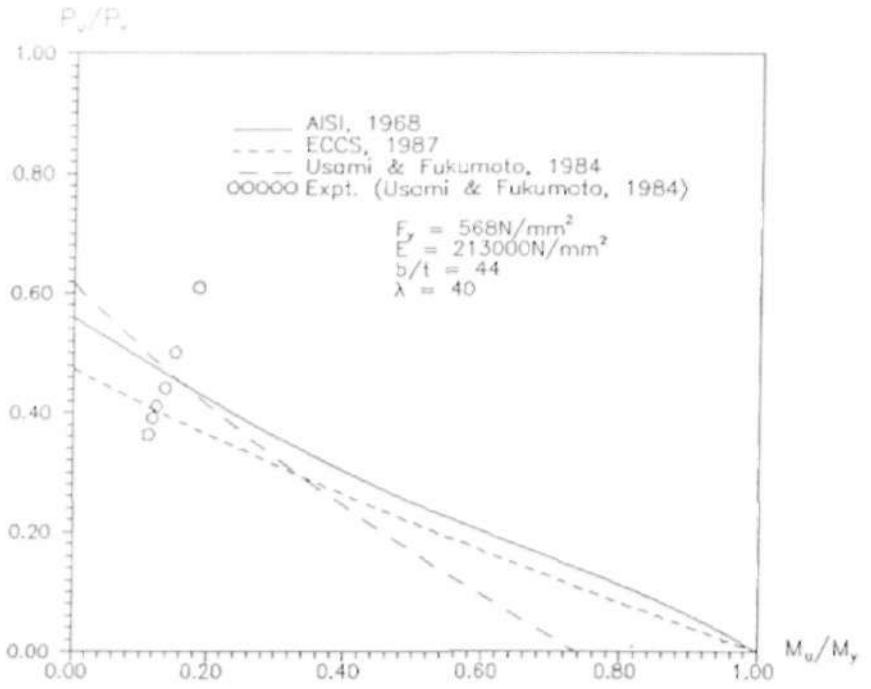


Fig. 3.8: Comparison of Interaction Equation and Test Results (λ = 40)

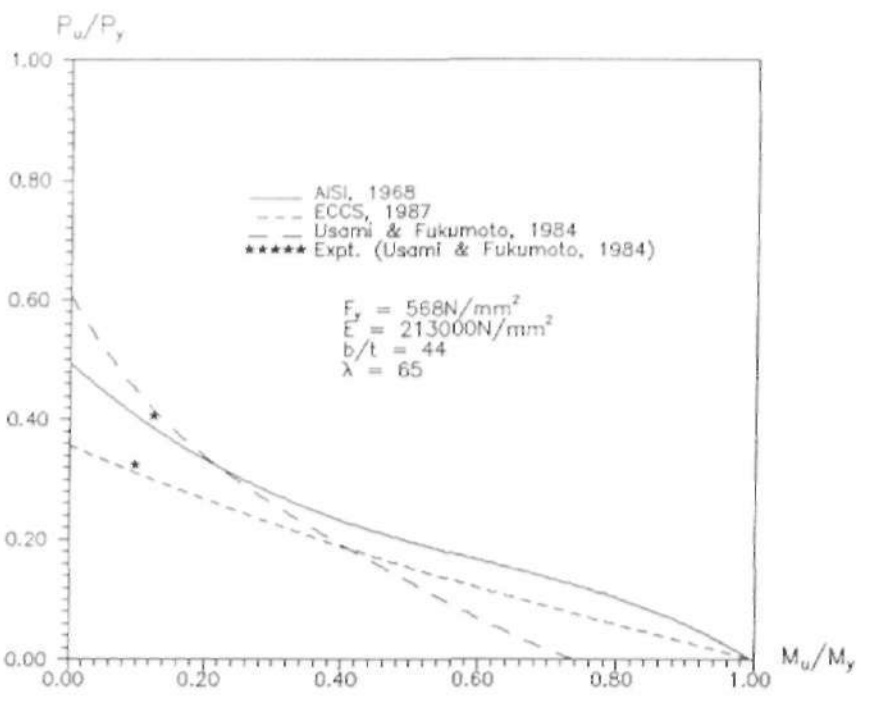


Fig. 3.9: Comparison of Interaction Equation and Test Results (λ = 65)

3.4 Justification of the Design Equations

Before sensitivity analysis, as intended, is carried out the two design equations under examination are contrasted with experimental results. On the basis of available data (e.g., Little, 1978; Usami and Fukumoto, 1984; Chiew et al, 1987), Figs. 3.6 to 3.9 show how well the load carrying capacity of beam-columns are predicted by AISI (1968) and ECCS (1987). At a low slenderness ratio of about $\lambda = 14$, both design equations give a conservative estimate of the strength. In the intermediate range of slenderness, cases of unconservative and conservative predictions are obtained depending on the b/t ratio. In general, the strength prediction implied by the use of ECCS (1987) equation is more conservative than AISI (1968) equation.

3.5 Allowable Stresses and Ultimate Capacities

The strength prediction of columns has a significant contribution in the interaction strength of beam-columns. The resolution of strength prediction for hot-rolled sections has a long list of research documentation (e.g., Tebedge et al, 1969; Young, 1969; Nylander, 1969). As used in the AISI Specification, the strength of a thin-walled column is derived on the basis of the formulation for the hot-rolled sections. But in general a number of reports on the strength prediction of thin-walled columns based on analytical and experimental

investigations is now available (e.g., Little, 1979; Hasegawa et al, 1988; Hancock et al, 1987). The different appraisal methods have led to various expressions denoting the strength as found in the cited references. In this section, the allowable axial compressive stresses and the ultimate capacities employed in the interaction design equations of thin-walled beam-columns represented by Eqs. (3.18) and (3.26) are hereby examined.

3.5.1 AISI Specifications

When residual stress induced by cold-forming of the section and the stress strain characteristics of the gradual-yielding steel sheets and strips are considered and the effective proportional limit is assumed to equal to one-half the yield point, the allowable compressive stresses for axially loaded columns not subjected to torsional-flexural buckling are expressed as:

$$F_{a1} = \frac{Q_w F_y}{n} - \frac{(Q_w F_y)^2}{4n\pi^2 E}\left(\frac{kL}{r}\right)^2 \qquad (3.27)$$

when $kL/r < C_c/\sqrt{Q_w}$ and

$$F_{a1} = \frac{\pi^2 E}{n(kL/r)^2} \qquad (3.28)$$

when $kL/r \geq C_c/\sqrt{Q_w}$

In Eqs. (3.27) and (3.28),

n = 23/12, a safety factor

C_c = $\sqrt{(2\pi^2 E/F_y)}$

E = Modulus of Elasticity

F_{al} = allowable average compression stress under concentric loading

F_y = yield point of steel

Q_w = a form factor which is less than unity, representing the weakening influence due to local buckling.

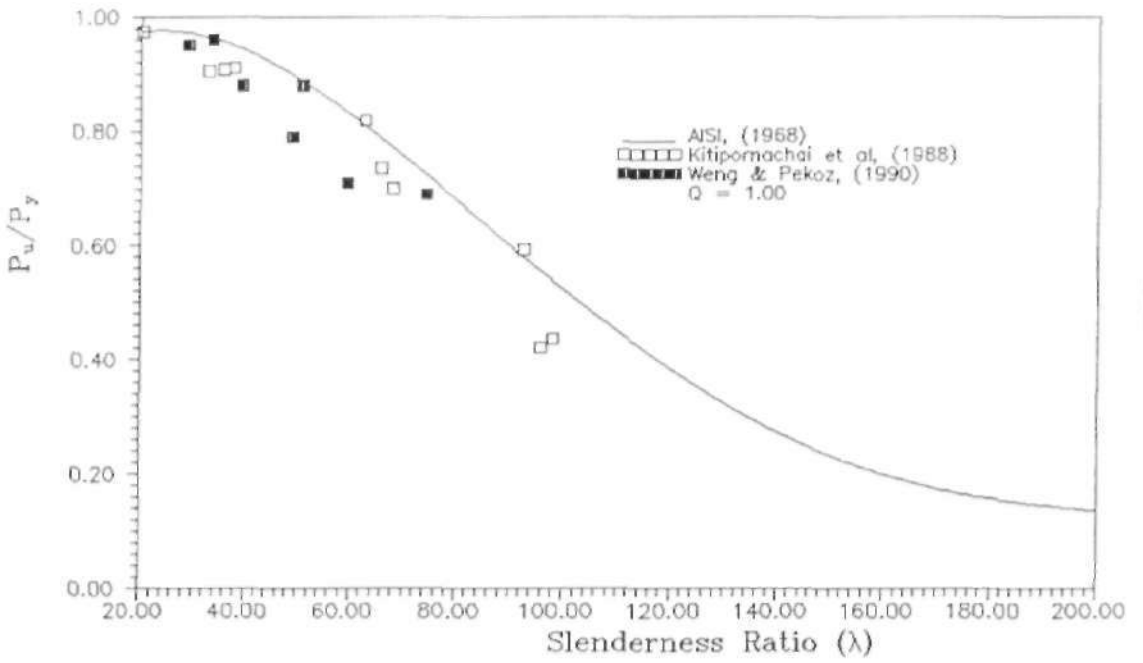


Fig. 3.10: Comparison of AISI Column Strength with Tests.

and others are as earlier defined. For members composed entirely of stiffened elements, Q_w is the ratio between the effective design area, as determined from the effective widths of such elements, and the full or gross area of the cross-section. The effective design area used in determining Q_w is based on the basic design stress F defined as (AISI, 1968):

$$F=F_y/n_o \tag{3.29}$$

where $n_o = 5/3$.

The use of a uniform safety factor of 23/12 for members not subject to torsional-flexural buckling is based on the fact that cold-formed steel members are usually noncompact sections, and connections are sometimes relatively flexible (Yu, 1985).

The interaction effect of the local and global column buckling is envisaged as reducing the overall column strength, the empirical manner in which the local buckling effect is accounted for requires some examinations. This is moreso in the face of uncertainties associated with strength prediction. It has only been said that accumulated experience has proved that the use of the form factor Q_w is a convenient and simple method for the design of cold-formed steel columns. The question of how satisfactory it is in the interaction equation seems not to have been answered. Even experimentally there are no sufficient data on thin-walled beam-column elements.

However, recognizable scatters exist among experimental observations on thin-walled columns. Figure 3.10 shows the comparison between the predicted capacity and experimental results on columns. The strength prediction given by AISI, (1968) is almost in all cases higher than the experimental values.

In the case of the allowable bending stress, the following specifications are made as relevant to the cross-sections in question:

$$F_b = F = (3/5) F_y \tag{3.30}$$

where members fail by yielding.

When the strength is governed by buckling instead of yielding, reduced design stresses are used. For beams having stiffened compression flanges or webs:

$$F_d = F_{bw} = [1.21 - 0.00034 (h/t)\sqrt{F_y}]\, 0.60 F_y \leq 0.60 F_y \tag{3.31}$$

where

h = width of web or flange

t = thickness of web or flange

F_{bw} = allowable bending stress.

Equation (3.31) is based on the computation of full web

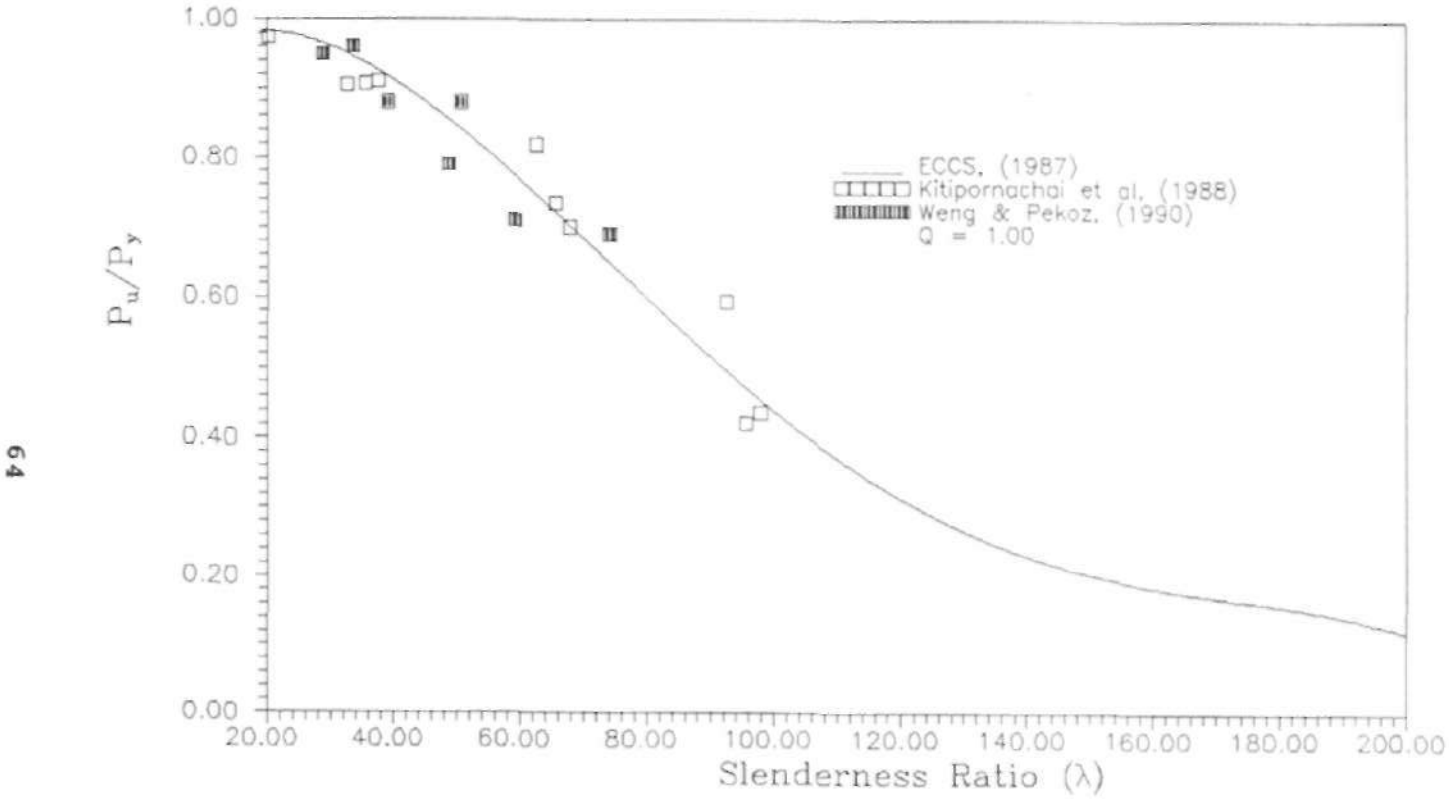


Fig. 3.11: Comparison of ECCS Column Strength and Tests.

area and effective width of compression flange determined on the basis of F_y. The mode of failure considered in this study is the yielding of a given cross-section. Thus, Eq. (3.30) is appropriately employed.

3.5.2 ECCS Recommendation

The flexural buckling is taken as the criterion for members with double or polar symmetrical cross-sections. Thus, the design strength with respect to flexural buckling is used in the interaction equation in place of the allowable stress in compression. According to Section R 5.1.2 of the Recommendation:

$$N_{xd}=\chi f_{ty} A_g \tag{3.32}$$

in which

$$\chi=F-\sqrt{F^2-Q_w/\lambda'^2} \geq Q_w \tag{3.33}$$

where

$$F=\frac{1}{2}\left[Q_w+\frac{1+\eta(\lambda'-0.2)}{\lambda'^2}\right] \tag{3.34}$$

In Eqs. (3.32) to (3.34),

f_{ty} = design value of yield stress

A_g = gross cross-sectional area

Q_w = A_{eff}/A_g ; A_{eff} = area of effective cross-section (at uniform compression)

λ' = $(L/i_g\pi)(f_{ly}/E)^{1/2}$

L = effective length of member

i_g = radius of gyration of gross cross-section

η = $\alpha(4 - Q_w) \leq 0.76$

α = coefficient depending on type of cross-section and plane of buckling (= 0.21 for hollow sections).

In Eq. (3.32), χ is a reduction coefficient which accounts for the interaction of local and global buckling. From Eqs. (3.33) and (3.34) it is clear that

$$\chi = Q_w \text{ when } \lambda' \leq 0.20 \qquad (3.35a)$$

$$\chi \rightarrow 1/\lambda'^2 \text{ when } \lambda' \rightarrow \infty \qquad (3.35b)$$

Equations (3.35) represent the behaviour at two extreme ends (i.e., very low and high slenderness parameter ranges). Figure 3.11 depicts the comparison between Eq. (3.32) and experimental results. There is a general fair prediction of the column strength at least for cases where Q_w = 1.00.

On the other hand, the design bending stress, M_{xd}, is expressed as

$$M_{xd} = f_{cy} W_{eff} \qquad (3.36)$$

when yielding first occurs at the compression edge of the section of members not subject to lateral buckling. In Eq. (3.36),

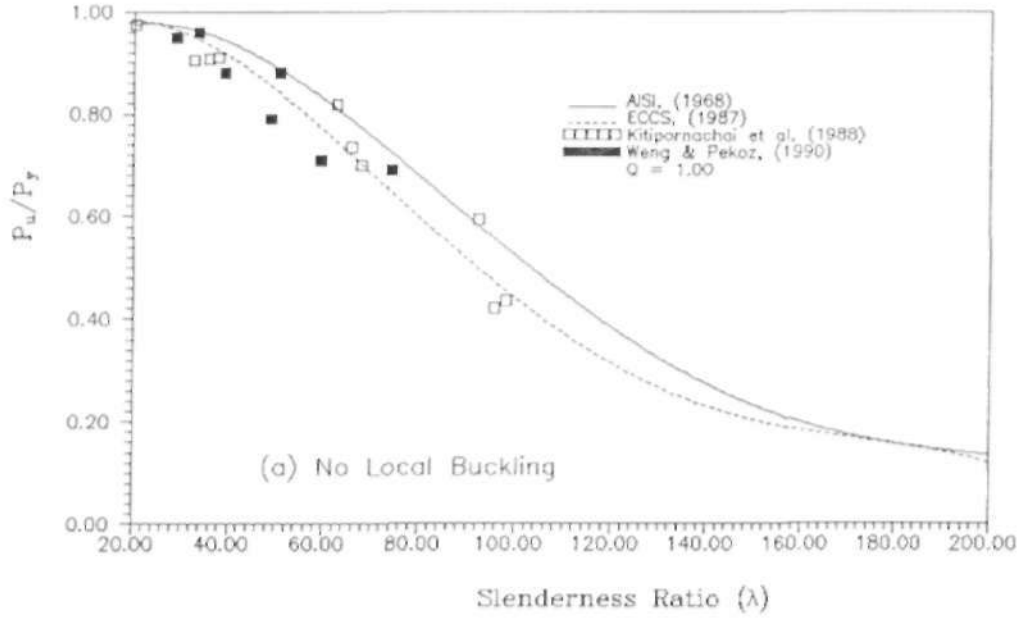
f_{ty} = design value of yield stress, and

W_{eff} = section modulus of the effective cross-section.

In general there are different appraisal methods for the strength prediction of thin-walled columns (e.g. Cescotto et al, 1980; Rondal and Maquoi, 1985; Pignatario et al, 1985; Gradzki and Kowal-Michalska, 1985; Loughlan, 1983) and beams (e.g. Boswell and Zhang, 1983; Boswell and Zhang, 1985).

3.5.3 Comparison of Ultimate Column Strength with Experimental Observations

Examination of the interaction design equations has shown already that they are built upon the knowledge of the individual behaviour of column and beam. The formulations for the strength of compression members are of major interest in Sec. 3.5.1 and 3.5.2 since they play a major role in the interaction behaviour. Thus, in this section, the adopted design equations are compared with experimental investigations on column elements. In Fig. 3.12 the strength predicted by AISI (1968) and ECCS (1987) are compared together with experimental tests. For cases of no local buckling the two predictions agree well with experimental observations while they both overestimate the capacity when local buckling is



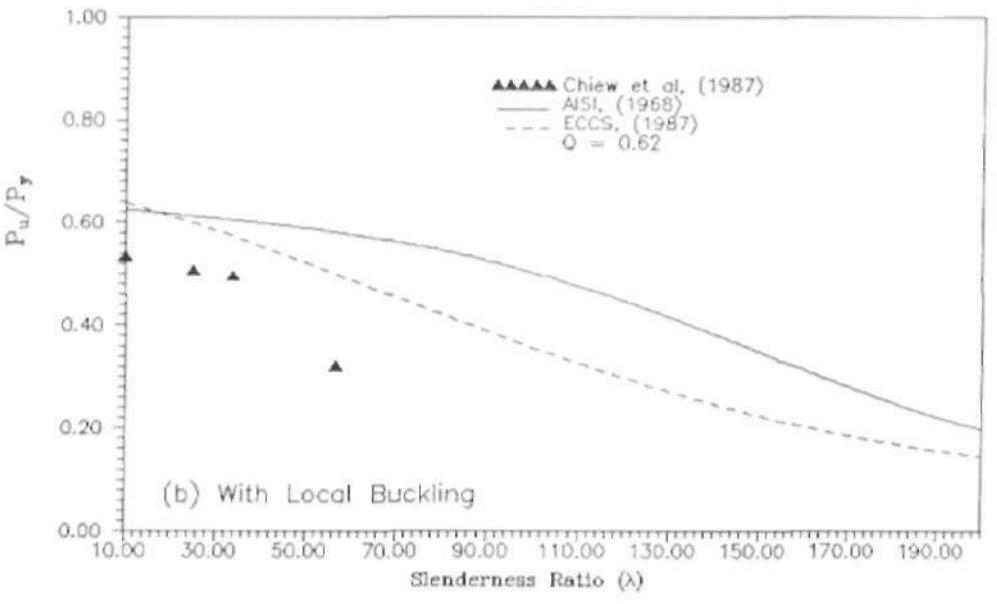


Fig. 3.12: Comparison of Column Strength Predictio and Tests.

considered. It is also noticeable that in either case (i.e., with and without local buckling) the AISI predictions are higher than those of ECCS indicating that the AISI has higher tendency to overestimate the column strength than recommended by ECCS.

3.6 Effective Width Parameters

The design equations discussed in Sec. 3.3 and 3.5 reflect a parameter Q_w which shows the effects of local buckling. The basis of its derivation using the well known concept of "effective width" is briefly here reviewed. Using this concept, the most severely buckled portions of an element are assumed to be ineffective in resisting load, and the applied compression stress is resisted by effective portions situated adjacent to the supported edges. This idea is diagrammatically presented in Fig. 3.13.

The effective width expression adopted by AISI (1968) for stiffened elements is of the form:

$$\frac{b}{t} = 1.9\sqrt{1/\varepsilon_y}\left[1 - \frac{0.415}{w/t}\sqrt{1/\varepsilon_y}\right] \qquad (3.37)$$

where

b = the effective width of plate after local buckling

t = thickness of plate

w = initial width of plate before local buckling

ε_y = yield strain.

Similarly, the recommendation of ECCS (1987) shows an expression of the form:

$$\frac{b^*}{b_p} = \frac{1-0.22/\bar{\lambda}_p}{\bar{\lambda}_p} \qquad (3.38)$$

where λ_p must be greater than 0.673 otherwise, b^*/b_p equals unity. In Eq. (3.38),

$$\bar{\lambda}_p = 0.526\,(b_p/t)\sqrt{\varepsilon_y}$$

Beside these two expressions there are other formulae which have been derived by various researchers on the basis of experiment or a combination of experiment and intuition. As shown in Fig. 3.14 they differ quite significantly from each other. Recent tests at the University of Strachclyde have indicated that the use of an effective width criterion gives accurate design analysis for many situations, so long as realistic effective widths are used (Rhodes and Walker, 1985).

It has been observed that if a conservative expression, such as found in (AISI, 1968 and ECCS, 1987) for the effective width, is used without modification, then under combinations of

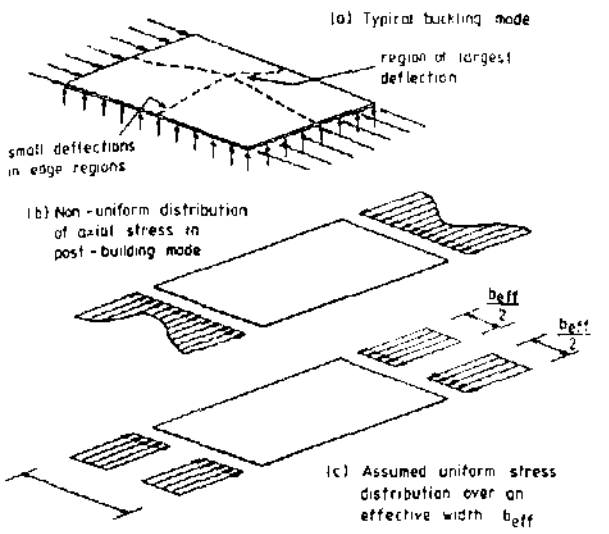


Fig. 3.13: The Effective Width of Stiffened Compression Element

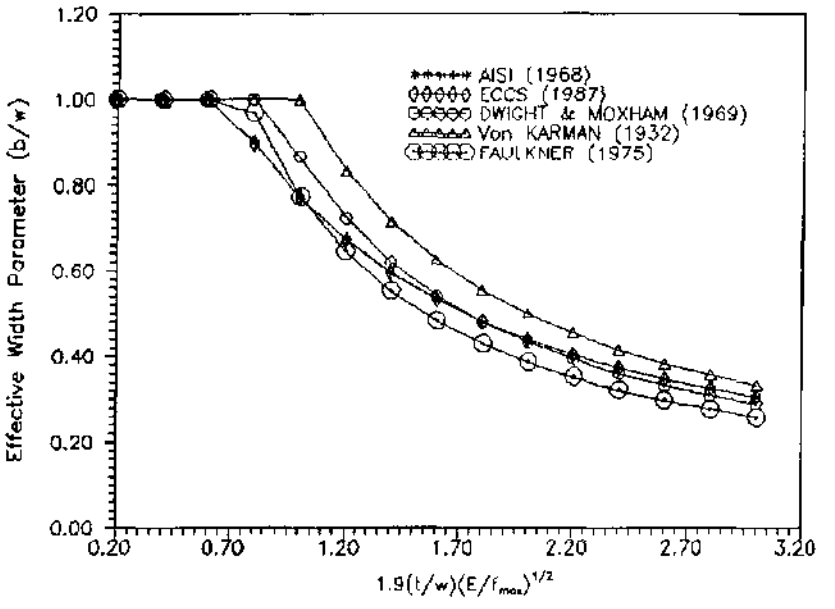


Fig. 3.14: Comparison of Different Effective Width Expressions.

bending and compression very conservative failure predictions can arise (Rhodes and Walker, 1985). But the fact still remains, the empirical incorporation of the expression into the equation of beam-column can influence this conservatism. The extent of this influence can be better shown using a probabilistic technique and not any other evaluation method based on deterministic procedure. This should not be contended since there is evident statistical nature of the associated strength parameters.

Narayanan and Chow (1983) presented an energy method for the analysis of axially loaded rectangular plates with initial imperfections under nine different combinations of boundary conditions. A number of different formulae for the effective widths of such plates were derived which were shown, by comparison with the test results carried out by other investigators, to adequately predict the ultimate capacity of plates. But in all cases, the initial imperfections have significant influence on the ultimate capacity. In reality, the exact magnitudes of these imperfections are not known and could be randomly distributed. According to Narayanan and Chow (1983), the actual imperfections in a plate follow no general pattern but their effects are mostly marked at or around the local buckling load, and are most detrimental to the load-carrying capacity if their pattern is similar in form to the deflected shape due to local plate buckling. Therefore any

estimate of the ultimate capacity of such plates include, to varying degrees, some amount of uncertainty.

Iyengar (1988) gave an approximate derivation for the effective width of a compressed plate. The expression was proved to satisfy experimental data fairly accurately, and it is by far different from the expressions found in AISI (1969) or ECCS (1987).

All foregoing shows clearly that the problem of local buckling has been tackled using different methods and any particular conclusion adopted for design purpose should be thoroughly evaluated using a method different from the deterministic procedures. And in fact, a rigorous analysis of the post-buckling action is considered complex (Ed. Dowling et al, 1988), and only a simple and satisfactory solution can be obtained by using an effective width approach which is itself an approximate solution.

3.7 Calibration Necessity and the Approach for Corrective Measures

3.7.1 The Necessity

The review of the AISI and ECCS design criteria for thin-walled beam-columns shows that:

(1) the two criteria consider the influence of local buckling phenomenon in their respective formulations. And this is clearly accounted for by the use of the effective width concept.

(2) The effect of local buckling is limited to a certain range of slenderness in AISI whereas it is considered for all practical values in ECCS.

(3) AISI utilizes the Allowable Stress Concept while ECCS recommends on the basis of Ultimate Carrying Capacity Concept.

(4) The influence of mode interaction (i.e., local and global buckling interaction) is considered in the two formulations which is to imply that a low probability of overestimating the load carrying capacity (Kolakowski, 1987) is aimed at.

(5) For the condition of small axial load, the moment magnification factor is ignored in AISI but not in ECCS.

Rhodes and Walker (1985) remarked that it is difficult to ascertain which of the approaches, ranging from the AISI Specification which implies that local buckling does not affect

long columns, to the ECCS Recommendation which implies significant local buckling effects, is more realistic. Eventhough experimental results are found to justify each approach, the topic is yet to be conclusively solved and further research is advocated. However, it has been noted from observations made on thin-walled square box-sections (Murray, 1987) that a number of interesting phenomena which can be explained in terms of the interaction of local and global elastic buckling followed by plastic mechanisms cannot be treated using simple empirical rules.

Whether the design equations are based on combined experimental and analytical judgement, it is clear that inaccuracies exist in physical models resulting from uncertainties in similitude. Conditions and mathematical models deviate from the prototypes because of their idealized nature (Uitto, 1979). With very few exceptions, it is rarely possible to make highly accurate predictions about the magnitude of the typical civil engineering structures to loading even when the governing input quantities are known exactly (Thoft-Christensen and Baker, 1982). This is to say that the response of typical structures and structural components/elements contains a component of uncertainty occurring as a result of simplifying assumptions, unknown boundary conditions and the unknown effects of the other variables and their interactions which are not included in

modelling (Thoft-Christensen and Baker, 1982). In 1981, Ballio and Campanini described the approximate procedures for the prediction of the interaction capacity of beam-columns as being erroneous on the side of safety and are sufficiently general to include all cases relevant for design. This implies that no design equation can necessarily include all variables affecting the behaviour which may not easily be quantified since it is clear that certain uncertainties are obviously unmeasurable.

The variation of the section modulus with the yield point of steel employed to cold-form a member, as pointed out by Yu, (1985) gives an indication that the question of adequacy of the design criteria requires a probability-based solution. Another factor of importance that can influence this property is the accuracy of the linear method of computing the properties of a given section which depends on the thickness of the steel sheet to be used and the configuration of the section. An error of less than 1% in the computed moment of inertia of two arbitrarily chosen channel sections when the material is 6.35mm or thinner was reported. This error seems small but it exists and its randomness can be of significant consequence.

In 1975, Freudenthal wrote: "..... it seems absurd to strive for more and more refinement of methods of stress analysis if, in order to determine the dimensions of structural elements, its results are subsequently compared with the so-

called working stress, derived in a rather crude manner by dividing the values of somewhat dubious material parameters obtained in conventional material tests by still more dubious empirical numbers called safety factors." Further still, the use of partial safety factors defined deterministically has also been regarded (Cornell, 1975) as inadequate for ensuring structural safety against the uncertainties inherent in loadings, material strengths and response. Iyengar and Gupta (1980) agreed that reliability-based design provides a more realistic treatment of the inherent uncertainty because of the limitations associated with the deterministic approaches.

The design interaction equation is an embodiment of a number of parameters. And so, the resulting predicted strength is subjected to the consequence of some factors of importance which may themselves be random in nature. Some of such influences were pointed out by Herzog in 1969.

In terms of material strength, investigations are numerous on the yield strength and residual stresses in steel components (e.g., Tall and Alpstein, 1969; Baker, 1969; Borges and Castanheta, 1975) and scatters have been observed.

Of consequence too, are the uncertainties associated with the applied loads (Gumbel, 1975) which of course, are usually accounted for by introducing load factors. The use of partial

safety factors is seen as covering a wide spectrum of various influences which are due to the properties of loads or to the properties of structures. But the character of the influences is very heterogenous and thus, the different methods of establishing the values of the partial safety factors have certain drawbacks. Some of these drawbacks have been extensively discussed (e.g., Tichy, 1969).

In 1969, Beer remarked that the causes of failures and collapses of steel structures have always been due to negligence on the part of the designer or constructor and more or less serious errors in the application of the principles of technology, and of structural design and of the rules of calculation. The causes might have been reduced but not completely. This is because both measurable and unmeasurable errors do exist.

The local buckling effect is accounted for in the interaction equations of thin-walled beam-columns empirically. This is clear from the basis of the derivation of the strength of thin-walled columns (e.g, Winter, 1970; Johnson, 1976; Yu, 1985). Different investigations on the interaction of local and global buckling of cold-formed components show that the problem is under constant pursuit for definite clarifications.

Since the question of satisfactory performance cannot be

effectively handled using a deterministic technique and from Yu's statement (Yu, 1985), any formulation based on the form factor, Q_w, must be thoroughly examined via a probabilistic methodology. The emphasis is that every prediction to model the state of the real world involves uncertainties.

The interaction of overall bending with the primary local mode has been shown to trigger a secondary local mode having the same wavelength as the primary. A three-mode analysis was proposed by Sridharan and Ali (1986) and their results demonstrate some of the intricacies in the behaviour of thin-walled elements. In 1988, Sridharan noted in a study on the behaviour of thin-walled columns that in order to account for the interaction of plate elements, the exact local critical stress should be used in the effective width formula. And that to obtain a reliable and safe estimate of the column strength, the collapse load given by the design methods should be multiplied by an appropriate factor which must itself be a function of the key parameters governing the problem. And moreso, such a factor should be statistically derived.

Since the thinness of the walls of cold-formed components leads to the consequence of the local buckling which affects significantly the ultimate capacities of such elements, the contribution of the statistics of the effect requires a clear clarification. In addition, the examination of the interaction

curves at varying values of the form factor, Q_w, shows that the beam-column capacity is a function of the factor. It is then meaningful to formulate to show the significance of its statistics on the statistics of the complete interaction capacity. And any evaluation of the interaction strength as a single variable is regarded as a "lumping" of contributions of the statistics of the important parameters making up the resistance. Therefore in this investigation, a generalized formulation is adopted to reflect the contributions of parameters that are considered important.

3.7.2 The Approach

As noted in the preceding Chapter and with regards to the different reasons pointed out in Section 3.7.1, the need for sensitivity study is justified. A deterministic model is seen as one in which the behaviour of each variable is known with certainty. But this is naturally unrealistic. It is then conjectured more appropriately to utilize a probabilistic model in which the behaviour of each variable is described by a probability distribution. Actually, any model in which at least one input variable is random lends itself to a probabilistic model (Rau, 1970).

This study involves defined stages. The first stage comprises the examination of some key parameters in the interaction equations so that the consequence of variability in

them can be established. Following this, is the second stage in which the complete interaction equations are evaluated under assumed practical values of possible inherent uncertainties. On the basis of the information obtained from these two stages, the third stage is established. At this stage, the implied safety levels of the current codes (i.e., AISI, 1968 and ECCS, 1987) are calibrated using the First-Order Reliability Method, FORM (see Chapter 4) and subsequently, based on cost-effectiveness analysis a target safety level appropriate for each code is derived. This target safety level provides the basis for transition from the current codes to their respective optimized Limit State Design Formats which offer uniform reliability levels within practical load combinations examined.

In the following Chapters the details of the development of the calibration models, sensitivity analysis and the actual calibration task are presented.

CHAPTER 4

UNCERTAINTY ANALYSIS METHODS

4.1 Introduction

The behaviour of thin-walled elements is very much different from the hot-rolled components. The difference arises from the thickness of the walls employed in cold-forming which leads to the local buckling consequence. The manner in which AISI and ECCS account for this effect in the interaction of beam-columns is based on approximation. Yu (1985) remarked that accumulated experience has proved that the use of the form factor Q_w in AISI formulation to account for the effect of local buckling in thin-walled columns is a convenient and simple method for design of such components. The question of whether or not a satisfactory performance on the basis of the approximation is achieved cannot be ordinarily answered using a deterministic approach. Thus, a probabilistic approach is offered as a more meaningful tool for evaluating these criteria and for proposing measures that will lead to uniform level of reliability.

Methods for evaluating uncertainty measures depend on the form of the data and information available (or even lack thereof). Some methods were described by Ang and Tang, (1984)

and they noted that depending upon the situation at hand, *ad hoc*, methods may sometimes have to be devised when necessary. To evaluate the two design criteria (i.e., AISI, 1968 and ECCS, 1987) and in order to propose modifications for the benefit of establishing uniform level of satisfactory performance, reliable techniques are required. Hence, in this chapter a number of such techniques are discussed and based on their merits, the relevant ones are adopted to achieve the objectives of this investigation.

4.2 Statistical Moment Generation Techniques

Generally, it is difficult to find the density function of random variables. However, for engineering purposes the knowledge of the moments of the transformed random variable is useful (Kapur and Lamberson, 1977). Both the exact and the approximate approaches for deriving the moments of a function of n random variables are discussed in Sections 4.2.1 and 4.2.2 respectively.

4.2.1 Exact Approach

Given a function $g = f(Y_1, Y_2, \ldots\ldots\ldots\ldots, Y_n)$ where Y_i are continuous, its mathematical expectation can be obtained as (Ang and Tang, 1975; Smith, 1986)

$$E(g) = E[f(y_i)] = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f(y_i)\, f_{Y_i}(y_i)\, dy_i \qquad (4.1)$$

where i = 1, 2, ... , n.

For example, let

$$G = aY + b \qquad (4.2)$$

with a and b as deterministic constants.

The expectation of G is

$$E(G) = E(aY+b) = \int_{-\infty}^{\infty} (ay+b)\, f_Y(y)\, dy \qquad (4.3)$$

That is,

$$E(G) = a\int_{-\infty}^{\infty} y f_Y(y)\, dy + b\int_{-\infty}^{\infty} f_Y(y)\, dy = aE(Y) + b \qquad (4.4)$$

whereas the variance of G is

$$var(G) = E[(G-\mu_G)^2]$$

$$= a^2 \int_{-\infty}^{\infty} (y-\mu_Y)\, f_Y(y)\, dy$$

$$= a^2 Var(Y) \qquad (4.5)$$

Generally, if

$$G = \sum_{i=1}^{n} a_i Y_i \tag{4.6}$$

then the expectation is

$$E(G) = \sum_{i=1}^{n} a_i E(Y_i) \tag{4.7}$$

and the variance is

$$Var(G) = \sum_{i=1}^{n} a_i^2 Var(Y_i) + \sum_{i \neq j}^{n} \sum a_i a_j Cov(Y_i, Y_j)$$

$$= \sum_{i=1}^{n} a_i^2 \sigma_{Y_i}^2 + \sum_{i \neq j}^{n} \sum a_i a_j \rho_{ij} \sigma_{Y_i} \sigma_{Y_j} \tag{4.8}$$

where ρ_{ij} is the correlation coefficient between Y_i and Y_j.

To obtain the mean and the variance of the function G using the above relations, information on $f_Y(y)$ is needed. But in many applications the density function $f_Y(y)$ may not be known and information may be limited to the mean and the variance of the original variate Y. And moreso, even when $f_Y(y)$ is known, the integrations involved may be difficult to perform. Therefore an approximate technique as in the next section is considered practically useful.

4.2.2 Approximate Approach

Let

$$g = f(y_1, y_2 \ldots\ldots\ldots, y_n) = g(y) \tag{4.9}$$

We shall denote the vectors of the expected values and the standard deviations of $y_1, \ldots\ldots\ldots, y_n$, respectively, as

$$\mu = (\mu_1 \ldots\ldots\ldots\ldots, \mu_n) \tag{4.10}$$

and

$$\sigma = (\sigma_1 \ldots\ldots\ldots\ldots, \sigma_n) \tag{4.11}$$

Then by Taylor series expansion, we obtain

$$g = f(y_1 \ldots\ldots, y_n) = f(\mu_1, \ldots\ldots\ldots, \mu_n) + \sum_{i=1}^{n} g_i'(y) \big|_{y=\mu} (y_i - \mu_i) +$$

$$+ \sum_{j=1}^{n} \sum_{i=1}^{n} g_{ij}'(y) \big|_{y=\mu} (y_i - \mu_i)\,(y_j - \mu_j) + \mathbb{R} \tag{4.12}$$

where

$$g_i' = \partial g(y) / \partial y_i$$

and

$$g_{ij}''(y) = \partial^2 g(y) / \partial y_i \partial y_j$$

Taking the expectation of Eq. (4.12), we have

$$E(g) = f(\mu_1, \ldots\ldots, \mu_n) + \sum_{i=1}^{n} g_i'(y) \Big|_{y=\mu} E(y_i - \mu_i) +$$

$$+ \sum_{j=1}^{n} \sum_{i=1}^{n} g_{ij}(y) \Big|_{y=\mu} E[(y_i - \mu_i)(y_j - \mu_j)] + \mathbb{R} \qquad (4.13)$$

Since y_i; i = 1, 2, ..., n, are independent random variables, and if the zero terms are ignored and the variables are uncorrelated, we obtain

$$E(g) = f(\mu_1, \ldots, \mu_n) + \frac{1}{2} \sum_{i=1}^{n} g_{ii}''(y) \Big|_{y=\mu} V(y_i) + E(\mathbb{R}) \qquad (4.14)$$

If Eq. (4.14) is approximated by dropping the remainder term, then it implies that

$$E(g) = f(\mu_1, \ldots\ldots, \mu_n) + \frac{1}{2} \sum_{i=1}^{n} g_{ii}''(y) \Big|_{y=\mu} V(y_i) \qquad (4.15)$$

which is the Second-Order First Moment. In many applications (e.g., Ang and Cornell, 1974), Eq. (4.15) is further approximated by ignoring the second term so that

$$E(g) \approx f(\mu_1, \ldots\ldots, \mu_n) \tag{4.16}$$

which is the First-Order First Moment. It has been demonstrated (Kapur and Lamberson, 1977) that Eq. (4.15) provides better approximations than Eq. (4.16) because of the higher order terms considered in the former.

Consideration of the first two terms of Eq. (4.15) leads to the approximation for the variance of g(y), that is,

$$V(g) \approx V(g(y)\,|_{y=\mu}) + V\left[\sum_{I=1}^{n} g_I'(y)\,|_{y=\mu}(y_I - \mu_I)\right]$$

$$\approx \sum_{I=1}^{n} [g'(y)\,|_{y=\mu}]^2 V(y_I) \tag{4.17}$$

Equations (4.15) and (4.17) are some of the fundamental approximations employed in this study.

If correlation among parameters are taken into consideration, the expectation and variance of g(y) will read, respectively

$$E(g) \approx f(\mu_1, \ldots\ldots, \mu_n) + \frac{1}{2}\sum_{I=1}^{n}\sum_{J=1}^{n}(g_{IJ}''(y)\,|_{y=\mu})cov(y_I, y_J) \tag{4.18}$$

and

$$V(g) \approx \sum_{i=1}^{n} [g_i(y)|_{y=\mu}]^2 + \sum_{i<j}^{n}\sum (g_{ij}''(y)|_{y=\mu}) cov(y_i, y_j) \qquad (4.19)$$

Eq. (4.18) and (4.19) can be used to examine the sensitivity of any parameter expressed as a function of other variables to uncertainty in an approximate manner.

4.3 Regression Analysis

In this study, due to a numerical experimentation, the development of a quantitative relationship between one dependent variable and the other independent variable(s) has become necessary. A straight line equation has been frequently a fairly good empirical statement of the relation between two variables even when the true relation is more complex than the straight can portray (Ezekiel and Fox, 1959). However, it may be just as important to know the exact or approximate form of the relationship as it is to have its empirical statement. It is of use then to consider other ways of expressing a relationship than the straight line. See examples of various curves in Fig. 4.1 that may be used to describe possible relationships. In this section, practical ways of fitting linear, simple parabolic equations are briefly discussed. Their applications are later demonstrated in the analysis of the numerical data in the relevant Chapters.

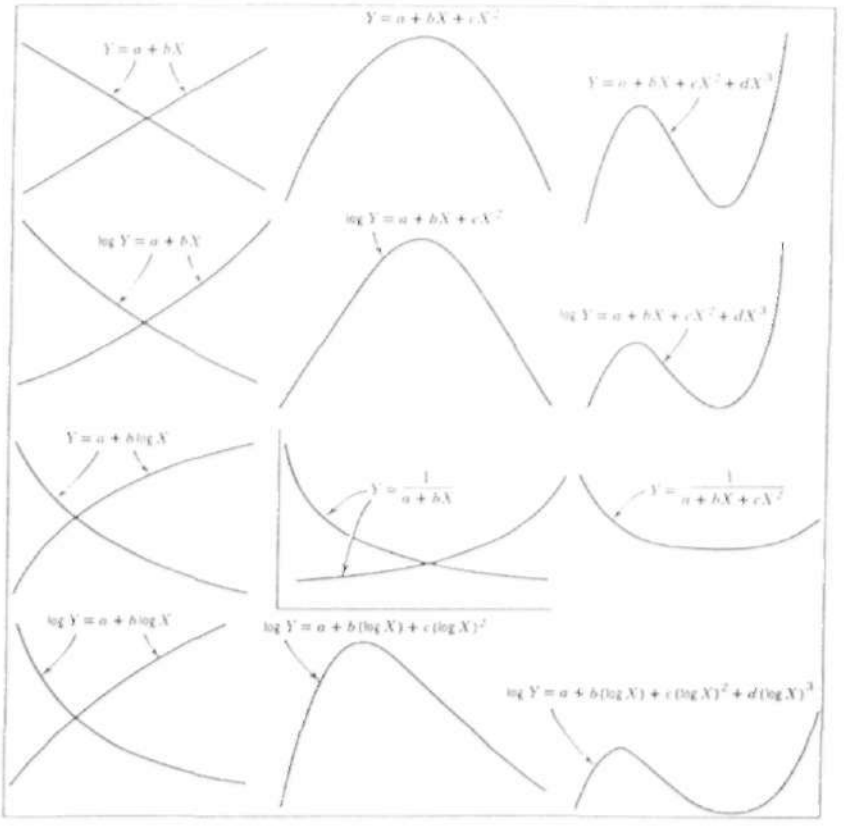


Fig. 4.1: Curves illustrating different mathematical functions

4.3.1 The Linear Equation

Ordinarily, there are several ways by which a straight line can be determined to show the functional relation between two variables. One way, for example, would be simply to place a ruler over the chart along the several group averages, or to stretch a black thread over them, and draw the line by eye so as to fall as nearly as possible along them. Although this method might give fairly satisfactory results where only a

rough measure was wanted, no two persons would draw their lines exactly the same. However, it is considered advantageous (Ezekiel and Fox, 1959) to determine a particular straight line that could be exactly duplicated by other persons and yet qualify in some sense as the best possible straight line for expressing the relationship in the particular set of data. Hence, the regression method (Gunst and Mason, 1980; Mosteller and Tukey, 1977) is considered a useful method for determining the straight line.

Put in a simple form, the equation of the regression line is

$$Y' = a_o + a_1 X \tag{4.20}$$

where

$$a_o = \bar{Y} - a_1 \bar{X}$$

and

$$a_1 = \frac{\sum (X - \bar{X})(Y - \bar{Y})}{\sum (X - \bar{X})^2}$$

In Eq. (4.20), Y′ is regarded as a prediction of the observed value of Y for the corresponding value of X, if the relationship between Y and X is not perfect (Edwards, 1976). Thus, an error of prediction exists and can be given as

$$Y - Y' = Y - (a_o + a_1 X) \tag{4.21}$$

Using the criterion of least squares method, it is required that the values of a_o and a_1 in Eq. (4.20) be determined in such a way that the residual sum of squares, i.e.

$$\sum (Y - Y')^2 = \sum [Y - (a_o + a_1 X)]^2 \tag{4.22}$$

be a minimum. Taking the partial derivative of the right hand side of Eq. (4.22) with respect to each constant (i.e., a_o and a_1) and setting equal zero yields

$$\sum Y = n a_o + a_1 \sum X \tag{4.23}$$

$$\sum XY = a_o \sum X + a_1 \sum X^2 \tag{4.24}$$

Dividing both sides of Eq. (4.23) by n and solving for a_o, we have

$$a_o = \overline{Y} - a_1 \overline{X} \tag{4.25}$$

where

$$\overline{Y} = \sum \frac{Y}{n} \; ; \; \overline{X} = \sum \frac{X}{n}$$

Then substituting Eq. (4.25) into Eq. (4.24), we obtain

$$\sum XY = (\bar{Y} - a_1\bar{X})\sum X + a_1\sum X^2 = n\bar{X}\bar{Y} + a_1(\sum X^2 - n\bar{X}^2) \qquad (4.26)$$

Solving for a_1 yields

$$a_1 = \frac{\sum XY - n\overline{XY}}{\sum X^2 - n\bar{X}^2} \qquad (4.27)$$

which is called the regression coefficient. It has been shown (Bethea et al, 1975) that a_o and a_1 are unbiased. Some additional insight into the nature of Eq. (4.22) can be gained if examined.

Let us recall, by definition

$$y' = a_o + a_1 X$$

Substituting an identity for a_o, as given by Eq. (4.25), we obtain

$$Y' = \bar{Y} - a_1\bar{X} + a_1 X \qquad (4.28)$$

Summing over the n values, we have

$$\sum Y' = n\bar{Y} - na_1\bar{X} + a_1\sum X$$

But

$$n\bar{X} = \sum X \; ; \; n\bar{Y} = \sum Y$$

Therefore

$$\sum Y' = \sum Y \tag{4.29}$$

Since $\Sigma Y' = \Sigma Y$, the mean of the predicted values must be equal to the mean of the observed values of Y (Edwards, 1976).

Equation (4.28) can be rearranged to obtain

$$Y' = \overline{Y} + a_1 (X - \overline{X})$$

Then it follows that an error of prediction will also be given by

$$Y - Y' = (Y - \overline{Y}) - a_1 (X - \overline{X}) \tag{4.30}$$

Let

$$y = Y - \overline{Y} \; ; \; x = X - \overline{X}$$

so that Eq. (4.30) yields

$$Y - Y' = y - a_1 x \tag{4.31}$$

Squaring both sides and summing yields again

$$\sum (Y - Y')^2 = \sum y^2 - 2a_1 \sum xy + a_1^2 \sum x^2 \tag{4.32}$$

But

$$a_1 = \frac{\sum xy}{\sum x^2}$$

and substituting into the preceding expression we have

$$\sum (Y - Y')^2 = \sum y^2 - \frac{(\sum xy)^2}{\sum x^2} \qquad (4.33)$$

It is now obvious from Eq. (4.33) that only if the regression coefficient a_1 equal to zero could the residual sum of squares, $\Sigma(Y-Y')^2$, be equal to $\Sigma(Y - \overline{Y})^2$, because $\Sigma y^2 = \Sigma(Y - \overline{Y})^2$ measures the variation of the Y values about its mean values. Thus, we know that there is no tendency for the Y values to be linearly related to the X values.

On the other hand, if a linear relationship between Y and X exists, regardless of whether it is positive or negative, then

$$\sum (Y - Y')^2 < \sum (\overline{Y} - Y)^2 \qquad (4.34)$$

The residual sum of squares becomes zero when the relationship is perfect. In such a case, there will be no errors of prediction. However, if there is no linear relationship, this residual sum will be exactly equal to the sum of squared deviations of the Y values from the mean values. Then the best

prediction that could be made for each Y value would be its mean values, since the prediction would minimize the sum of the squared errors of the prediction (Edwards, 1976).

The standard error of estimate is simply obtained by dividing the residual sum of squares by n - 2 (Edwards, 1976), that is,

$$s_{Y,X}=\sqrt{\frac{\sum (Y-Y')^2}{n-2}} \qquad (4.35)$$

In another form, the region of the measure of the variation of the Y values about the regression line is described by the confidence intervals. This aspect is not discussed in this study but well documented elsewhere (e.g., Ott, 1984; Chatterjee and Price, 1972).

4.3.2 Parabolic Equations

(a) The Simple Parabolic Equation

There are cases where quadratic equations best fit the regression line. In such cases, the following procedure was put forward by Ezekiel and Fox (1959) for obtaining the regression coefficients. To represent the relationship between two variables according to the expression

$$Y=a_o+a_1X+a_3X^3 \qquad (4.36)$$

the coefficients a_o, a_1 and a_2 have to be determined from the observations. Hence, to determine these constants, the following equations need to be solved

$$(\sum x^2)\, a_1 + (\sum xu)\, a_2 = \sum xy \qquad (4.37)$$

$$(\sum xu)\, a_1 + (\sum u^2)\, a_2 = \sum uy \qquad (4.38)$$

and

$$a_o = M_y - a_1 M_x - a_2 M_u \qquad (4.39)$$

Equations (4.37) to (4.39) are constructed as follows. Let U = X^2 values of Eq. (4.36), then

$$M_x = \frac{\sum X}{n} = \overline{X} \qquad (4.40)$$

$$M_u = \frac{\sum U}{n} \qquad (4.41)$$

$$M_y = \frac{\sum Y}{n} = \overline{Y} \qquad (4.42)$$

$$\sum x^2 = \sum X^2 - nM_x^2 \qquad (4.43)$$

$$\sum xu = \sum XU - nM_xM_u \qquad (4.44)$$

$$\sum u^2 = \sum U^2 - nM_u^2 \qquad (4.45)$$

$$\sum xy = \sum XY - nM_xM_y \qquad (4.46)$$

and

$$\sum uy = \sum UY - nM_uM_y \qquad (4.47)$$

the solution procedure requires that the values of Eqs. (4.40) to (4.47) be first computed, then Eqs. (4.37) and (4.38) are solved simultaneously to obtain the values for a_1 and a_2. Then these values are substituted into Eq. (4.39) to obtain the value of a_o. The Doolittle Method was recommended (Ezekiel and Fox, 1959) for solving the problem. But the advent of computer has simplified the solution technique.

(b) The Cubic Parabolic Equation

In an attempt to describe a particular relationship more closely, the cubic parabolic equation is recommended. The computational procedure follows the preceding section, although

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with some modifications. Let the equation of the cubic parabola be expressed as

$$Y = a_o + a_1 X + a_2 X^2 + a_3 X^3 \qquad (4.48)$$

The values to be initially computed are those in the previous section. The additional computations are

$$M_v = \frac{\sum V}{n} \qquad (4.49)$$

$$\sum uv = \sum UV - nM_u M_v \qquad (4.50)$$

$$\sum xv = \sum XV - nM_x M_v \qquad (4.51)$$

$$\sum v^2 = \sum V^2 - nM_v \qquad (4.52)$$

$$\sum vy = \sum VY - nM_v M_y \qquad (4.53)$$

where V represents the X^3 values and U as earlier defined.

Having computed the above values, the following equations are then solved simultaneously

$$(\sum x^2)\, a_1 + (\sum xu)\, a_2 + (\sum xv)\, a_3 = \sum xy \qquad (4.54)$$

$$(\sum xu)\, a_1 + (\sum u^2)\, a_2 + (\sum uv)\, a_3 = \sum uy \qquad (4.55)$$

$$(\sum xv)\, a_1 + (\sum uv)\, a_2 + (\sum v^2)\, a_3 = \sum vy \qquad (4.56)$$

Then the value of a_o is obtained from

$$a_o = M_y - a_1 M_x - a_2 M_u - a_3 M_v \qquad (4.57)$$

Parabolas of higher orders can be attempted using a similar approach but such situations are not expected in this investigation and such have also been described as rarely useful (Ezekiel and Fox, 1959).

4.3.3 Multiple Analysis

The nature of the observations for a specific problem determines the type of the regression function for fitting the appropriate relationship. The numerical experimentation conducted in the later part of this investigation shows that a regression analysis which relates a dependent variable to more than one other independent variable may be required. Very briefly the multiple regression analysis for linear models, as

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may be found applicable to the present problem, will be reviewed.

The general form of a linear regression function associated with one dependent variable and two independent variables (i.e., a plane) for a universe is (Parsons, 1974):

$$\hat{\vartheta}_{1.23}=a_{1.23}+b_{1.23}X_2+b_{13.2}X_3 \qquad (4.58)$$

where

ϑ is the conditional mean of X_1 associated with a specific value of X_2 and X_3.

$a_{1.23}$ is the conditional mean of X_1 associated with $X_2 = 0$ and $X_3 = 0$. $a_{1.23}$ is the X_1 intercept.

$b_{12.3}$ and $b_{13.2}$ are referred to as partial regression coefficients.

$b_{12.3}$ is the change in the value of $\vartheta_{1.23}$ associated with a one-unit change in the value of X_2. The value of all variables indicated to the right of the period in $b_{12.3}$ are presumed to remain constant. $b_{12.3}$ is the slope of the plane with respect to the X_2 axis.

$b_{13.2}$ is the change in the value of $\vartheta_{1.23}$ associated with a one-unit change in the value of X_3. The value of all variables indicated to the right of the period in $b_{13.2}$ are presumed to remain constant. $b_{13.2}$ is the slope of the plane with respect to the X_3 axis.

To determine the constants $a_{12.3}$, $b_{12.3}$ and $b_{13.2}$, certain

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assumptions are essential, viz:

1. The conditional means are linearly related. That is to say that the regression function is a plane.
2. All the conditional probability distributions of X_1 are normally distributed. There is a separate conditional probability distribution associated with each conceivable pair of X_2, X_3 values.
3. All the conditional standard deviations are equal. $\sigma_{1.23}$ is a measure of the variability in X_1 when the values of X_2 and X_3 are held constant. This measure of dispersion in the conditional probability distributions is presumed to be the same for all combinations of X_2 and X_3.

The typical unknown in the problem is the universe regression function which will be estimated from sample data. The sample estimates takes the form:

$$X_{1.23} = i_{1.23} + j_{12.3}X_2 + j_{13.2}X_3 \qquad (4.59)$$

The least-squares criterion is utilized to determine the coefficients of the regression function, i.e., Eq. (4.59). That is, the values of $i_{1.23}$, $j_{12.3}$ and $j_{13.2}$ which minimize the function:

$$F = \sum (X_1 - i_{1.23} - j_{12.3}X_2 - j_{13.2}X_3)^2 \qquad (4.60)$$

are determined. This requirement leads to the following equations that must be solved simultaneously:

$$\sum X_1 = ni_{1.23} + j_{12.3}\sum X_2 + j_{13.2}\sum X_3 \qquad (4.61)$$

$$\sum X_1X_2 = 1_{1.23}\sum X_2 + j_{12.3}\sum X_2^2 + j_{13.2}\sum X_2X_3 \qquad (4.62)$$

$$\sum X_1X_3 = 1_{1.23}\sum X_3 + j_{12.3}\sum X_2X_3 + j_{13.2}\sum X_3^2 \qquad (4.63)$$

Visualization of the regression fitting process can be facilitated by plotting the observations on a scatter diagram. With one dependent and two independent variables, the scatter diagram is three dimensional as shown in Fig. 4.2.

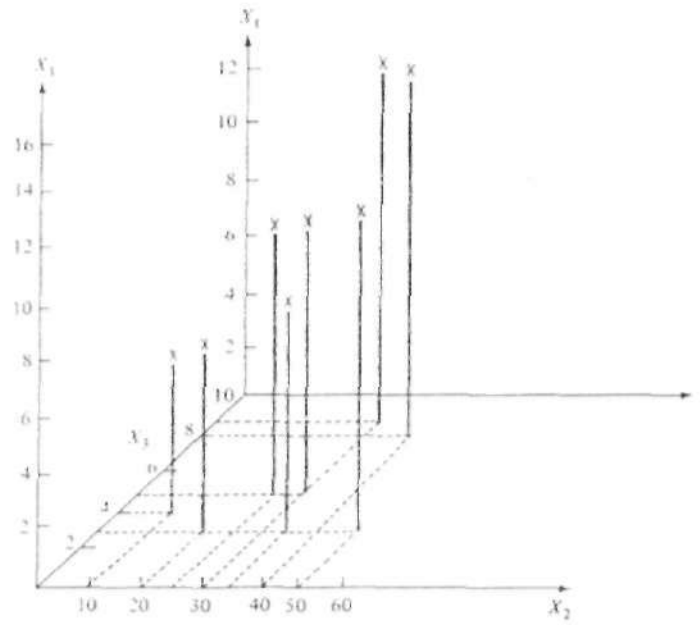


Fig. 4.2: Trivariate Observations in 3-Dimensional Space

4.4 Determination of Failure Probabilities

As a measure of rational belief on the occurrence of an accident during a reference time, probabilities can be assigned to given accident scenarios (Borges, 1977). For instance, the collapse of a building or a bridge, being a structural failure, the accident can be described as a limit state. Since different limit states can be defined based on structural requirement, the performance of an engineering system can be measured with regards to these states and the levels of reliability associated with them. Thus, reliability may be defined as (Frangopol and Dotreppe, 1978; Calibro, 1962): *the probability of a successful operation of a device in the manner and under the conditions of intended use*. However, as employed in this study the definition precisely is: *probability that the resistance of a structure exceeds the load effects* (Lloyd and Lipow, 1962; Dover and Bea, 1979). By inference the probability of failure is obvious.

During the last decades, methods for computing theoretical failure probability have been developed. These methods are based on the fact that the failure condition is expressed as a function of several random variables.

In general, let $\underline{Y}$ be the random vector of the uncertain variables such as the strength of materials, dimensions, actions or quantities expressing the uncertainty in the

physical model for a given limit state function, having a joint probability $F_{\underline{Y}}(\underline{y})$. The probability of survival of a structure for all realizations $\underline{Y}$ in the safe domain ΔS can be expressed as

$$P_S = R \approx \int_{\Delta S} dF_{\underline{Y}}(\underline{y}) \tag{4.64}$$

The safe domain ΔS is separated from the failure region ΔF by the limit safe function.

The determination of Eq. (4.64) for arbitrary type and dimension of the uncertainty vector $\underline{Y}$ and the typical complexity of ΔS in civil engineering problems has been a major difficulty for which several methods have been proposed. Once P_S is known the probability of failure P_F can easily be deduced using $P_F = 1 - P_S$. In this section various attempts to solving this relationship are reviewed.

4.4.1 Simple Exact Solutions

In order to compute the reliability of a system or a component, the nature of the stress, λ_S, and strength, λ_R, as random variables has to be known. In a general term, we assume that the density functions for the stress and the strength are known. The basis of reliability computation using the interference theory (Kapur and Lamberson, 1977) is that the probability of the strength exceeding the stress is tractable.

Let

$f_{\lambda S}(\lambda_S)$ = density function of the stress λ_S

and

$f_{\lambda R}(\lambda_R)$ = density function of the strength λ_R

By definition (Kapur and Lamberson, 1977; Lloyd and Lipow, 1962)

Reliability, $R = p(\lambda_R > \lambda_S) = p(\lambda_R - \lambda_S > 0)$ (4.65)

The situation is depicted on Figs. 4.3 and 4.4. The shaded portion represents the interference area which indicates the probability of failure. The region depends on the relative positions of $f_{\lambda R}(\lambda_R)$ and $f_{\lambda S}(\lambda_S)$.

As the two curves become further apart, P_F (the probability of failure) decreases, whereas P_F increases as they become closer. In a similar argument the overlap is dependent on the degree of dispersions in $f_{\lambda R}(\lambda_R)$ and $f_{\lambda S}(\lambda_S)$. The probability of a stress value, λ_{so}, lying in a small interval of width $d\lambda_S$ is equal to the area of the element $d\lambda_S$, that is,

$$P\left(\lambda_{so} - \frac{d\lambda_S}{2} \le \lambda_{so} + \frac{d\lambda_S}{2}\right) = f_{\lambda_S}(\lambda_{so})\, d\lambda_S \qquad (4.66)$$

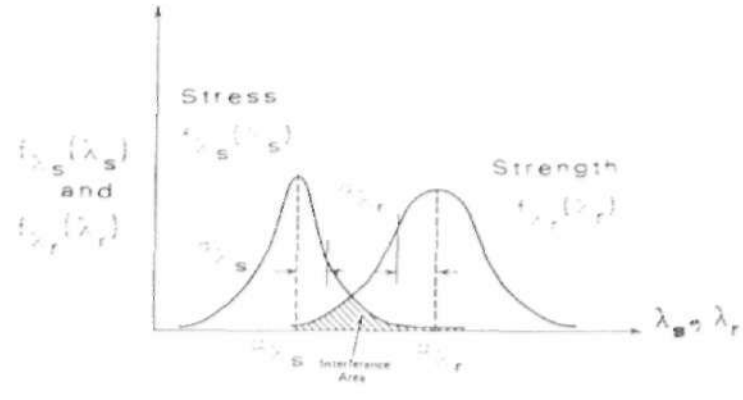


Fig. 4.3: Stress-Strength Inference

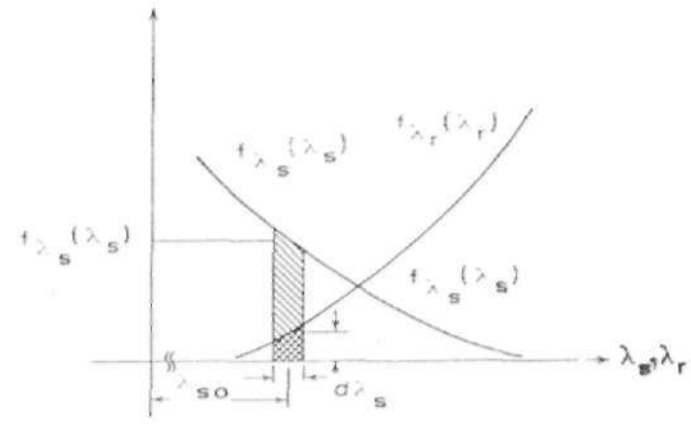


Fig. 4.4: Enlarged Portion of the Interference Diagram for the Interference Area

Hence,

$$p(\lambda_R > \lambda_{so}) = \int_{\lambda_{so}}^{\infty} f_{\lambda_R}(\lambda_R)\, d\lambda_R \qquad (4.67)$$

is the probability that the strength λ_R is greater than a certain stress λ_{so}. for the stress value lying in the interval $d\lambda_R$ and the strength λ_R exceeding the stress given by this interval,

$$f_{\lambda S}(\lambda_{so})\, d\lambda_S \int_{\lambda_{so}}^{\infty} f_{\lambda_R}(\lambda_R)\, d\lambda_R \qquad (4.68)$$

becomes the probability of this situation provided the stress and the strength random variables are independent. Thus,

$$p(\lambda_R > \lambda_S) = R = \int_{-\infty}^{\infty} f_{\lambda_S}(\lambda_S)\, [\int_{\lambda_S}^{\infty} f_{\lambda_R}(\lambda_R)\, d\lambda_R]\, d\lambda_S \qquad (4.69)$$

Equation (4.69) will now be examined for assumed distributions for λ_S and λ_R.

4.4.1.1 Normally Distributed Stress and Strength (Normal-Normal)

Assuming the stress and the strength random variables are normally distributed, that is,

$$f_{\lambda_S}(\lambda_S) = \frac{(2\pi)^{-1/2}}{\sigma_{\lambda_S}} \exp\left[-0.5\left(\frac{\lambda_S - \mu_{\lambda_S}}{\sigma_{\lambda_S}}\right)^2\right], \quad -\infty < \lambda_S < \infty \qquad (4.70)$$

for the stress λ_S and

$$f_{\lambda_R}(\lambda_R) = \frac{(2\pi)^{-1/2}}{\sigma_{\lambda_R}} \exp\left[-\frac{1}{2}\left(\frac{\lambda_R - \mu_{\lambda_R}}{\sigma_{\lambda_R}}\right)^2\right], \quad -\infty < \lambda_R < \infty \tag{4.71}$$

for the strength λ_R, where

$\mu_{\lambda S}$ = mean value of the stress, λ_S

$\sigma_{\lambda S}$ = standard deviation of the stress, λ_S

$\mu_{\lambda R}$ = mean value of the strength, λ_R

$\sigma_{\lambda R}$ = standard deviation of the strength, λ_R

Let the safety margin be formulated as

$$M = \lambda_R - \lambda_S \tag{4.72}$$

which measures the relative position between $f_{\lambda R}(\lambda_R)$ and $f_{\lambda S}(\lambda_S)$. Since λ_R and λ_S are assumed normally distributed, M is also a random variable whose distribution is Gaussian. See Fig. 4.5. Thus, it has a mean value of

$$\mu_M = \mu_{\lambda_R} - \mu_{\lambda_S} \tag{4.73}$$

and the standard deviation of

$$\sigma_M = \sqrt{\sigma_{\lambda_R}^2 + \sigma_{\lambda_S}^2} \tag{4.74}$$

The probability of the strength λ_R exceeding the stress λ_S is expressed as

$$R = p(M > 0)$$

That is,

$$R = \int_0^{\infty} \frac{1}{\sigma_M \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{M-\mu_M}{\sigma_M}\right)^2\right] dM \qquad (4.75)$$

If we make

$$z = \frac{M-\mu_M}{\sigma_M} \qquad (4.76)$$

then

$$\sigma_M dz = dM \qquad (4.77)$$

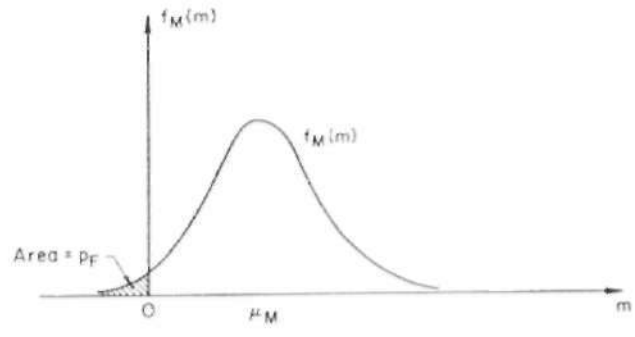


Fig. 4.5: PDF of Safety Margin

When M = 0, the lower limit of z in eq. (4.75) becomes

$$z = \frac{\mu_{\lambda_R} - \mu_{\lambda_S}}{\sqrt{\sigma^2_{\lambda_R} + \sigma^2_{\lambda_S}}} \tag{4.78}$$

and when M → + ∞, the upper limit of z → + ∞. Therefore Eq. (4.75) can be simplified as

$$R = \frac{1}{\sqrt{2\pi}} \int_{-z}^{\infty} \exp\left(\frac{-z^2}{2}\right) dz \tag{4.79}$$

In terms of the coupling equation (4.78), the reliability can be expressed as

$$R = 1 - \Phi\left[-\frac{\mu_{\lambda_R} - \mu_{\lambda_S}}{(\sigma^2_{\lambda_R} + \sigma^2_{\lambda_S})^{1/2}}\right] \tag{4.82}$$

while in terms of the reliability index, ß,

$$R = 1 - \Phi(-\beta) \tag{4.81}$$

so that

$$\beta = \frac{\mu_{\lambda_R} - \mu_{\lambda_S}}{(\sigma^2_{\lambda_R} + \sigma^2_{\lambda_S})^{1/2}} \tag{4.82}$$

Equation (4.82) represents the measure of safety of the system and has been considered a more meaningful measure of safety

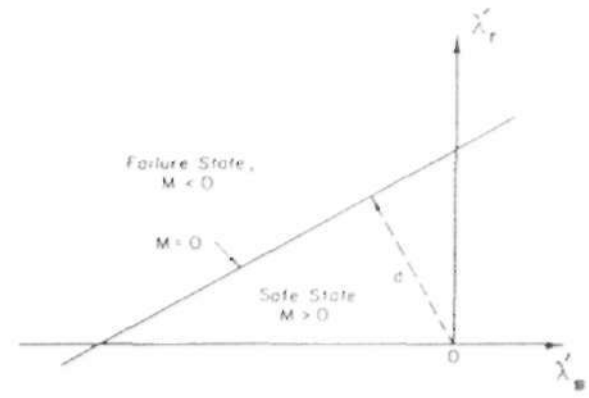


Fig. 4.6: Space of reduced variate λ_R' and λ_S'

than the traditional safety factor (Smith, 1986). Assuming the variables λ_R and λ_S are expressed in their reduced variates, respectively as

$$\lambda_R' = (\lambda_R - \mu_{\lambda_R}) / \sigma_{\lambda_R} \tag{4.83}$$

and

$$\lambda_S' = (\lambda_S - \mu_{\lambda_S}) / \sigma_{\lambda_S} \tag{4.84}$$

the implication of Eq. (4.82) is shown in Fig. 4.6. By definition, the determination of the failure probability becomes a matter of a simple mathematical relation (e.g.,

Woeste and Schaffer, 1979) of the type:

$$P_F = 1 - R = 1 - (1 - \Phi(-\beta)) = \Phi(-\beta) \qquad (4.85)$$

in accordance with Eq. (4.81).

4.4.1.2 Lognormally Distributed Stress and Strength (Lognormal-Lognormal)

The standard form of a lognormal density function is

$$f_t(t) = \frac{1}{t\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln t - \mu}{\sigma}\right)^2\right] ; \; t>0 \qquad (4.86)$$

where t is the random variable. The parameter μ and σ are the mean and the standard deviation, respectively, of the variable lnt which is normally distributed.

Suppose the strength, λ_R, and the stress, λ_S, are lognormally distributed, and we assume that

$$t = \lambda_R / \lambda_S \qquad (4.87)$$

It then implies that

$$Int = \ln\lambda_R - \ln\lambda_S \qquad (4.88)$$

Since both $\ln\lambda_R$ and $\ln\lambda_S$ are normally distributed, it follows that lnt is also normally distributed.

The log-normal density function is positively skewed and hence the median is a better and more convenient measure of its central tendency (Kapur and Lamberson, 1977). This means that the antilog of the mean of $\ln\lambda_R$ is the median of $f_{\lambda R}(.)$, and that of $\ln\lambda_S$ the median of $f_{\lambda S}(.)$. If $m\lambda_R$ denotes the median (i.e., the 50th percentile values) of λ_R,

$$\mu_{\ln\lambda_R} = \ln(m\lambda_R) \tag{4.89}$$

and similarly for the stress λ_S

$$\mu_{\ln\lambda_S} = \ln(m\lambda_S) \tag{4.90}$$

From Eq. (4.88) we can write

$$\mu_{lnt} = \ln(m\lambda_R) - \ln(m\lambda_S) \tag{4.91}$$

It is known that

$$\sigma_{lnt} = \sqrt{\sigma^2_{\ln\lambda_R} + \sigma^2_{\ln\lambda_S}} \tag{4.92}$$

Therefore from the definition of reliability, we have

$$R = p\left(\frac{\lambda_R}{\lambda_S} > 1\right) = \int_t^{\infty} f_t(t)\,dt \tag{4.93}$$

Let

$$z = \frac{lnt - \mu_{lnt}}{\sigma_{lnt}} \tag{4.94}$$

which is a normal variate. To find the limits of integration we have: when t = 1 and when t → ∞, z → ∞, and from Eqs. (4.91) to (4.92)

$$z = -\frac{\ln(m\lambda_R) - \ln(m\lambda_S)}{\sqrt{\sigma^2_{\ln\lambda_R} + \sigma^2_{\ln\lambda_S}}} \tag{4.95}$$

Thus, the reliability expression becomes

$$R = \int_z^{\infty} \Phi(z)\, dz \tag{4.96}$$

where $\Phi(z)$ is the pdf for the standard normal variate z. The reliability index, ß, in terms of the coupling equation (4.95), is

$$\beta = \frac{\ln(m\lambda_R) - \ln(m\lambda_S)}{\sqrt{\sigma^2_{\ln\lambda_R} + \sigma^2_{\ln\lambda_S}}} \tag{4.97}$$

Employing small variance approximates (Committee, 1982; Galambos, 1972), Eq. (4.97) is used in reliability computations (e.g., Ang and Cornell, 1974; Ellingwood and Ang, 1974), as

$$\beta = \frac{\ln(\lambda_R/\lambda_S)}{(\Omega^2_{\lambda_R} + \Omega^2_{\lambda_S})^{1/2}} \tag{4.98}$$

in which Ω_λ are the coefficients of variation of the random variables λ_i. Equation (4.98) is an equivalent of Eq. (4.82) and in many applications Eq. (4.98) has been much preferred

(e.g. Ang and Cornell, 1974; Leporati, 1979; Rosenblueth, 1972; Cornell, 1967; Galambos and Ravindra, 1972; Parimi and Lind, 1976; Ravindra and Galambos, 1978).

When the stress and strength assume distributions other than normal and lognormal, reliability computations can also be done (e.g., Kapur and Lamberson, 1977) analytically. However, in any case, the stress and strength are treated as the only random quantities and not as functions of any other variables of which they represent.

4.4.2 Approximations

In the previous section, the determination of P_S, from which P_F is computed, appears to be tractable and less cumbersome. This follows as far as the resistance and stress are represented as the only uncertainties not expressed in terms of other variables of the vector $\underline{Y}$. As a matter of fact, resistance and loading effects are functions of other quantities which may be random in nature and assume any probability distributions. Consequently, Eq. (4.64) turns out to be multidimensional and analytical solution presents a formidable task not easy to perform in routine design practice. On these grounds, approximate procedures have evolved. The present section deals with the review of the fundamental concepts of such methods.

4.4.2.1 The Mean First Order-Second Moment Method (MFOSMM)

This method involves the statistical character of the uncertain quantities in the safety analysis described by their means and variances, even when their probability distributions are unknown. For the joint probabilistic behaviour, the correlation coefficient is employed and moments of higher orders are not used.

Assuming a limit state function defined by

$$Z_M = \lambda_R - \lambda_S \tag{4.99}$$

where the variable λ_R and λ_S denote the effects of the material strengths and of the actions respectively. If each of these variables is expressed by means of their functional relations:

$$\lambda_R = g_R(y_1, y_2, \ldots, y_m) \tag{4.100}$$

and

$$\lambda_S = g_S(y_{m+1}, y_{m+2}, \ldots, y_n), \tag{4.101}$$

the limit state function (Eq. (4.99)) becomes

$$Z_M = (y_1, y_2, \ldots\ldots, y_n) \tag{4.102}$$

in the space of the basic variables Y_i.

Expanding Eq. (4.102) in power series in correspondence

with the mean values of the basic variables, neglecting nonlinear terms, we obtain

$$Z_M' = g(\mu_{Y_1}, \mu_{Y_2}, \ldots, \mu_{Y_n}) + \sum_{i=1}^{n} g'|_{\mu_{Y_i}} (y_i - \mu_{Y_i}) \qquad (4.103)$$

where

$$g'|_{\mu_{Y_i}}$$

indicates the partial derivatives of the limit state function w.r.t. each basic variable calculated at the mean values μ_{Y_i}.

If the variables Y_i are independent, then

$$\mu_{Z_M'} = g(\mu_{Y_1}, \mu_{Y_2}, \ldots, \mu_{Y_n}) \qquad (4.104)$$

and

$$\sigma_{Z_M'} = \sqrt{\sum (g'|_{\mu_{Y_i}} \sigma_{Y_i})^2} \qquad (4.105)$$

A measure of risk can be estimated by introducing a parameter ß as

$$\beta = \frac{\mu_{Z_M'}}{\sigma_{Z_M'}} \qquad (4.106)$$

Since no assumption is made about the statistical distribution of the random variables, the distribution of the safety margin Z_M is also undefined. The drawback consequently limits the information on the reliability associated with a given value of ß in Eq. (4.106). However, if the limit state function is approximately linear, the central limit theorem of probability calculation suggests that the distribution of $\underline{Y}$ should tend toward the normal distribution. It then follows that the probability of failure P_F can be estimated using

$$P_F = 1 - \Phi(\beta) \qquad (4.107)$$

in which $\Phi()$ = cumulative probability distribution function of the standard normal variate (Ang and Cornell, 1975; Benjamin and Cornell, 1970).

In practice, this method has some advantages owing to its operational simplicity. Its major drawback lies in variance consideration. This lack of variance can be explained considering a failure criterion, Z_M, involving two variables λ_R and λ_S:

$$Z_M = \lambda_R - \lambda_S < 0 \qquad (4.108)$$

$$Z_M = (\lambda_R - \lambda_S)^3 < 0 \tag{4.109}$$

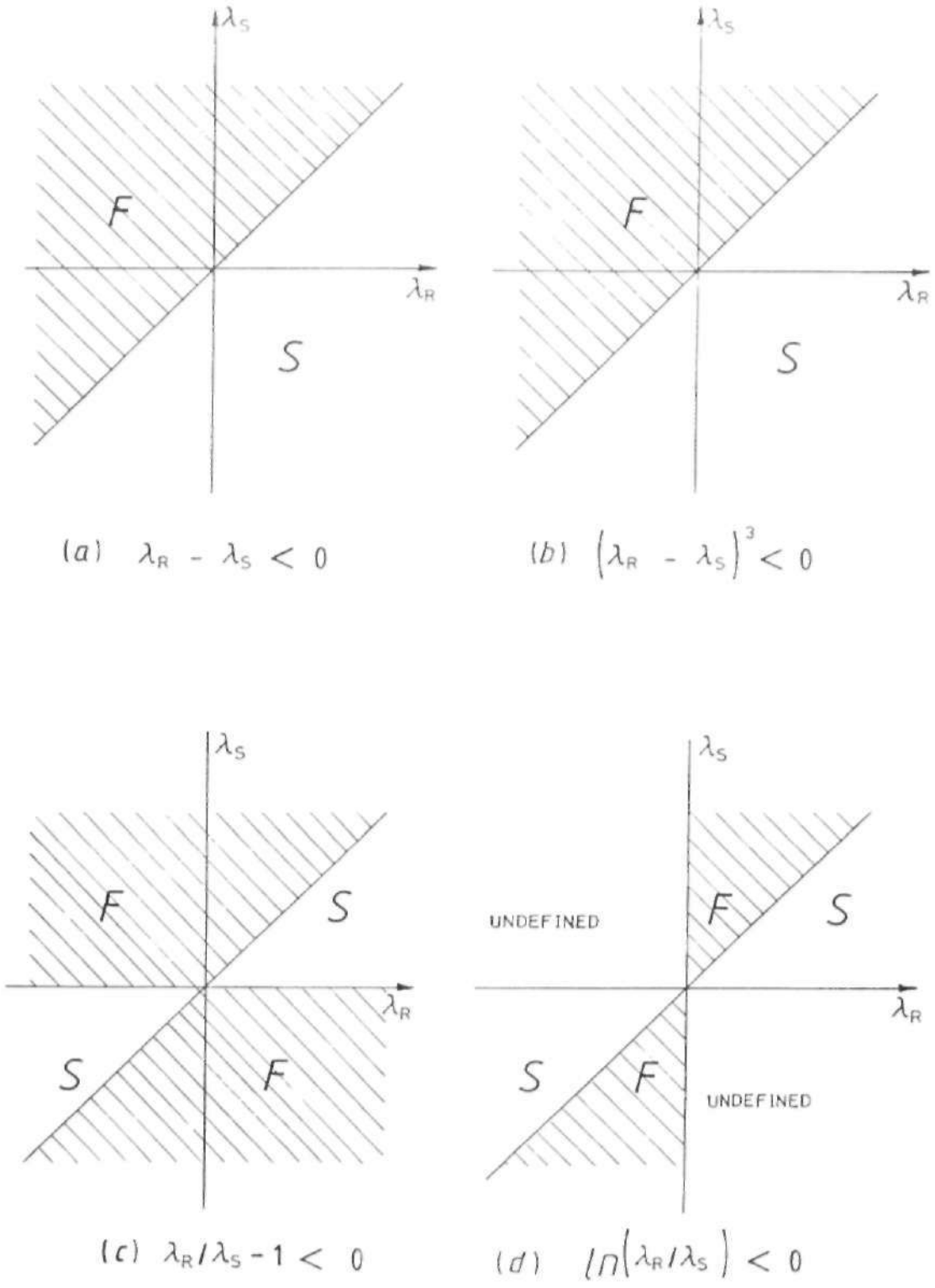
$$Z_M = \frac{\lambda_R}{\lambda_S} - 1 < 0 \tag{4.110}$$

$$Z_M = \ln(\lambda_R / \lambda_S) < 0 \tag{4.111}$$

All the four criteria are equivalent provided that λ_R and λ_S are positively defined as represented in Fig. 4.7. From the computation of the mean and the standard deviation of Z_M in accordance with the **MFOSMM** operational schemes, the values of the parameter ß obtained from the various criteria are not necessarily the same.

4.4.2.2 Hasofer and Lind Method (HLM)

This method rests on the assumption that most, or at least an appropriate fraction of the joint probability density of the variables involved are concentrated in the hypersphere whose radius is described by a parameter β and consequently they are associated with values of the variables belonging to the safety



(a) $\lambda_R - \lambda_S < 0$

(b) $\left(\lambda_R - \lambda_S\right)^3 < 0$

(c) $\lambda_R/\lambda_S - 1 < 0$

(d) $ln\left(\lambda_R/\lambda_S\right) < 0$

Fig. 4.7: Various Failure Criteria

domain. The task involved can be described as follows.

Let $\underline{Y}$ be the vector of the basic uncorrelated random variables involved in a given structural problem whose statistical character will be described by their means and variances. If correlation exists between the uncertain variables of $\underline{Y}$ and it is expressed in terms of a known covariance matrix, through orthogonal transformation these variables can be replaced by the corresponding uncorrelated ones (Hasofer and Lind, 1974).

If

$$z_M = g(y_1, y_2, \ldots\ldots\ldots\ldots, y_n) = 0 \qquad (4.112)$$

be the boundary of the safety domain, δS. That means, the values of $\underline{Y}$ belonging to the failure domain satisfy the inequality:

$$z_M = g(y) < 0 \qquad (4.113)$$

The task in the HLM consists of projecting the limit state function in the space of standardized variables, i.e.,

$$u_i = \frac{y_i - \mu_{Y_i}}{\sigma_{Y_i}} \qquad (4.114)$$

Then from the origin of the axes of the reduced coordinates,

the minimum distance β to the transformed surface:

$$g(u_1, u_2, \ldots\ldots\ldots, u_n) = 0 \qquad (4.115)$$

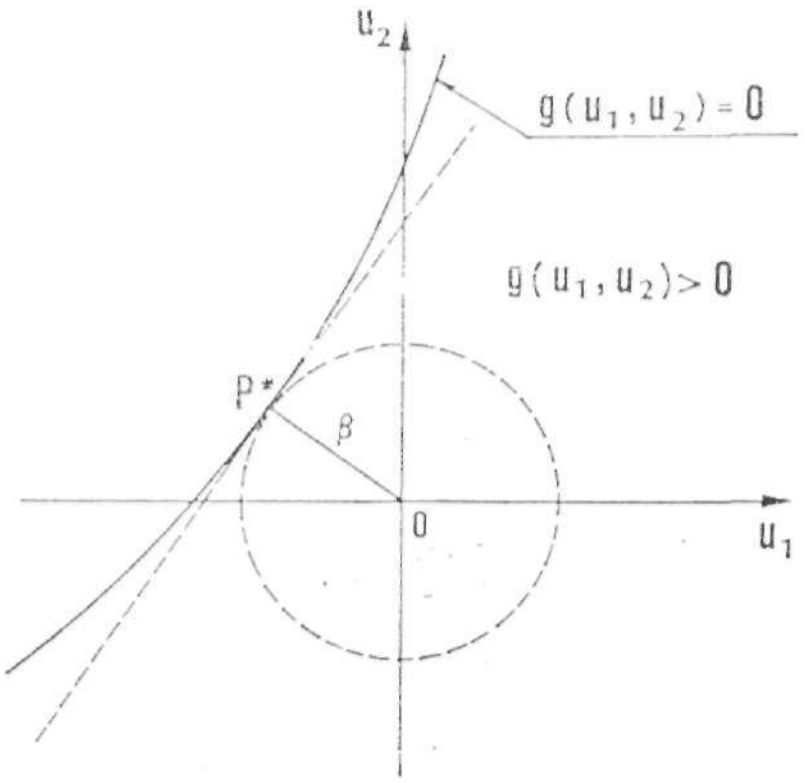


Fig. 4.8: The Minimum Distance β

is measured. In brief the operation involves the following optimization problem:

$$\beta = \min \sqrt{\sum_{i=1}^{n} u_i^2} \qquad (4.116)$$

subject to:

$$g(u_1, u_2, \ldots\ldots\ldots, u_n) = 0 \qquad (4.117)$$

Having found β the computation of probability of failure is resolved using the relation (4.107), that is,

$$P_{F_L} = 1 - \Phi(\beta) \qquad (4.118)$$

The concepts upon which this method is based have a substantial contribution to the evolution of the second moment theory and have more or less inspired other proposals discussed subsequently. However,in spite of the clear definition, the computation of β is by no means easy especially for safety domain boundaries of any given form presenting peaks and local minima (Leporati, 1979). The method also does not take full account of the geometry of these surfaces, since it considers only their point of contact with the hypersphere centered at the origin of the axes of the reduced coordinates (Fig.4.8). Consequently, the information contained in β is limited and since no assumption is made as to the distributions of the variables, the probability of attaining the limit state is left, in a broad sense, undetermined.

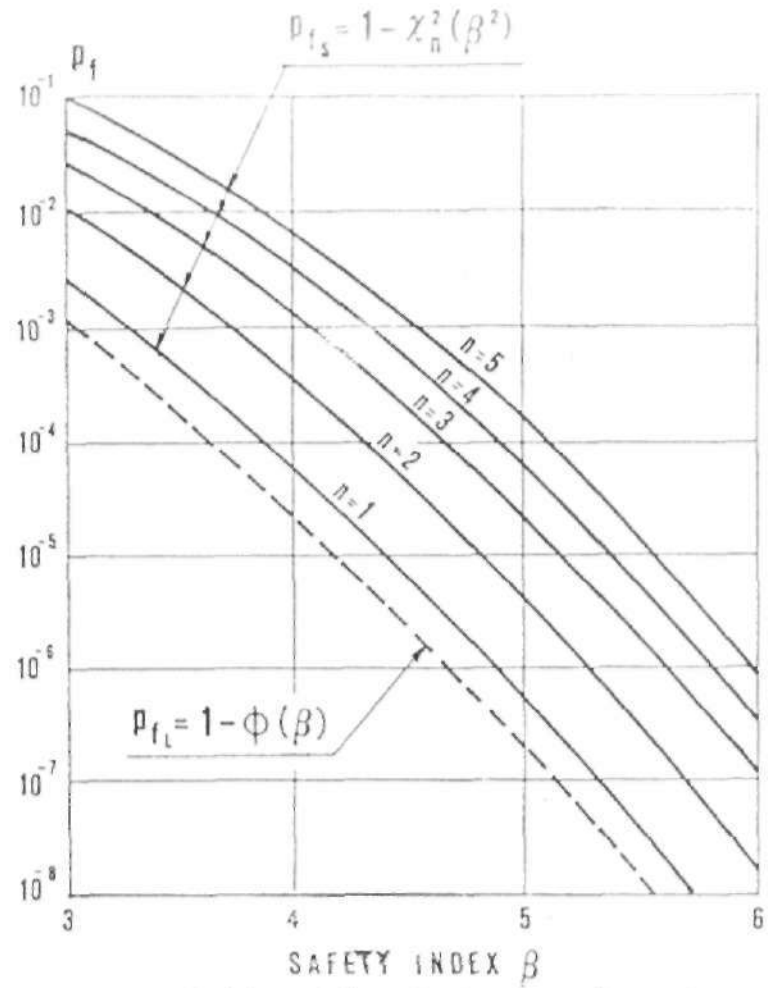


Fig. 4.9: Relationship between β and p_F for varying degrees of freedom

For instance, if the limit state surface is a hypersphere centered at the origin having a radius β, this surface in the space of the standardized variables, has equation.

$$g(u) = \beta^2 - v \qquad (4.119)$$

where v is the sum of the squares of n standardized independent normal variables having Chi-square distribution with n degrees

of freedom. Thus, the probability of failure becomes:

$$P_{FS} = P[(\beta^2 - v) < 0] = 1 - \chi_n^2(\beta^2) \qquad (4.120)$$

where $\chi^2_n(\ )$ is the cumulative distribution function for Chi-square with n degrees of freedom. The relationship between β and P_F for varying n is given in Fig. 4.9. The dotted curve represents the situation when the limit state surface is linear and it is independent of n. Then it implies that the minimum distance parameter β has a probability interpretation when the geometry of the limit state surface and the dimensions of the operating space are regarded. Hence, summarily, working in the space of standardized normal variables, the following relations are conclusive:

(a) For linear state surface (Fig 4. 10)

$$P_F = P_{F_L} = 1 - \Phi(\beta) \qquad (4.121)$$

(b) for concave safety domain (Fig. 4.11)

$$0 \leq P_F \leq 1 - \Phi(\beta) \qquad (4.122)$$

(c) for a hypersphere safety domain having radius β with centre at the origin (Fig. 4.12)

$$P_F \leq P_{F_S} = 1 - \chi_n^2(\beta^2) \qquad (4.123)$$

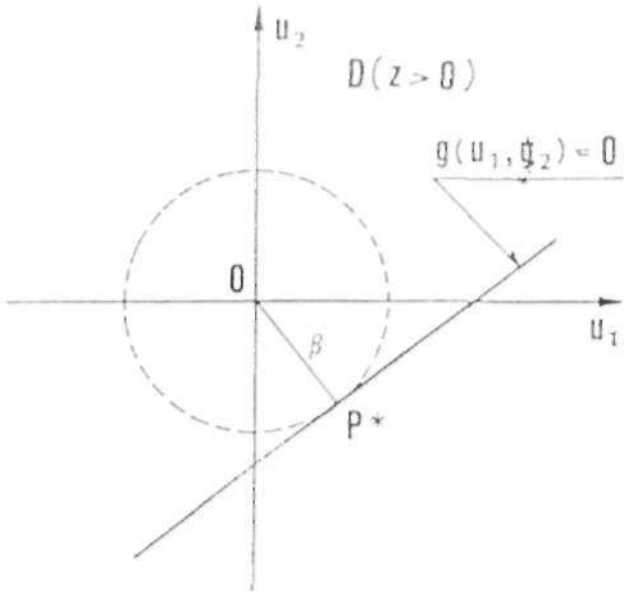


Fig 4.10: Linear Limit State

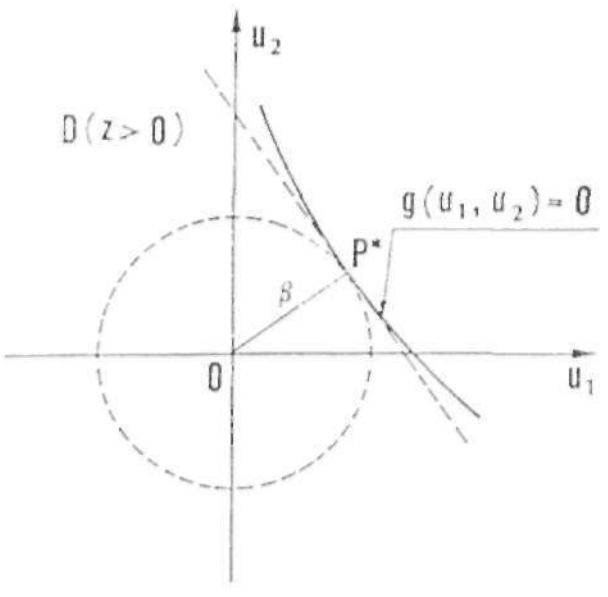


Fig. 4.11: Concave Limit State

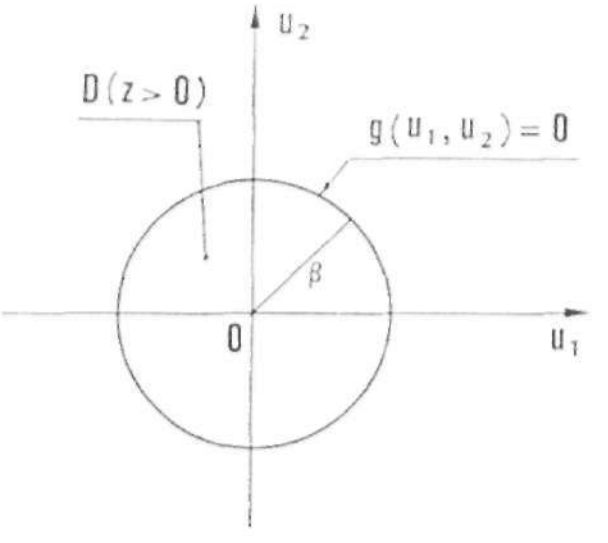


Fig. 4.12: Hypersphere Safety Domain

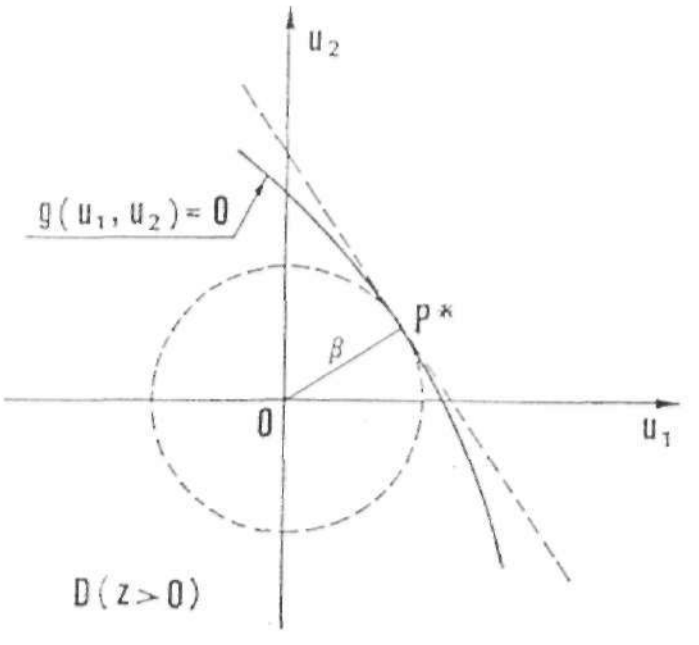


Fig. 4.13: Convex Safety Domain

d. For a convex safety domain (Fig. 4.13)

$$P_{F_L} \leq P_F \leq P_{F_S} \tag{4.124}$$

It becomes clear with reference to Figs 4.11 and 4.13 that the approximation for the evaluation of β is either on the conservative or unconservative side depending on whether the exact nonlinear failure surface is convex toward the origin (Shinozuka, 1983).

4.4.2.3 Veneziano's Multiple Checks Method (VMCM)

In many structural problems the limit state does not deviate much from linear behaviour and in that case the estimate given by HLM is acceptable. However, for markedly nonlinear limit state surfaces, discretization at several points of the safety domain boundary has been proposed (Veneziano, 1974). Conceptually, the method involves the determination of the intersections of the safety domain boundary expressed in the space of the uncorrelated standardized variables with a number of straight lines starting from the origin (Fig. 4.14). For instance, for 2n directions of the axes and 2^n directions of the bisecting lines, $2n + 2^n$ intersection points and the corresponding number of β_i distances of these points from the origin are defined. Consequently, the safety domain boundaries are approximately represented by spherical sectors.

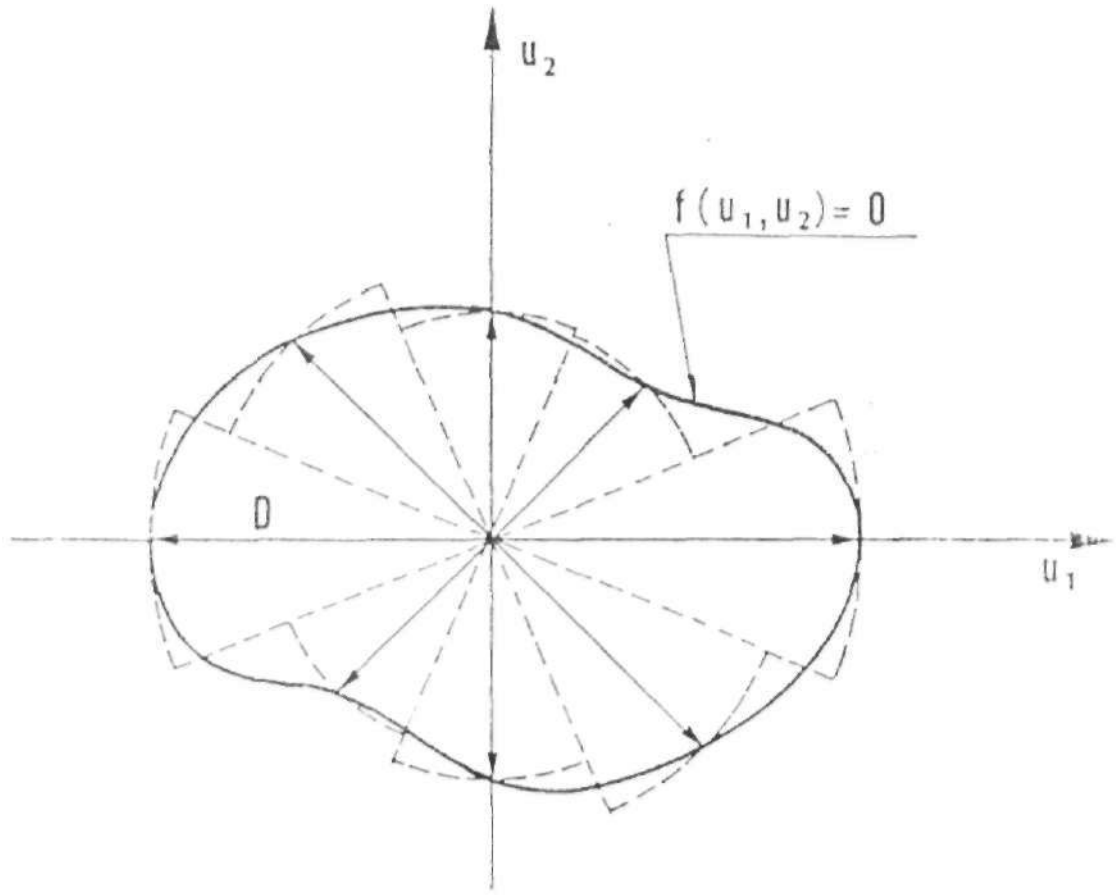


Fig. 4.14: Markedly Nonlinear Limit State Surface

According to Eq. (4.120) and if β_i have equal weight, P_F is estimated from the relation:

$$P_{F_V} = \frac{1}{N}\sum_{i=1}^{N} [1 - \chi_n^2(\beta^2)] \tag{4.125}$$

It follows that if β_i = constant = β, then

$$P_{F_V} = P_{F_S} = P_F \tag{4.126}$$

Equivalent to HLM, β_{min} is obtained without solving any minimization problem from the relations:

$$\beta_{min} \approx \min[\beta_i] \tag{4.127}$$

$$P_{F_L} \approx 1 - \Phi(\beta_{min}) \tag{4.128}$$

Operationally, this method is attractive since no minimization problem is required. It is also less sensitive to dimensional invariance and particularly useful when there exists discontinuities in the first derivatives of the limit state equation (leporati, 1979).

4.4.2.4 FORM

In the previous sections, various methods have been discussed with their shortcomings. To overcome them, the Advanced Second Moment Method, herein called First-Order

Reliability Method (FORM) was proposed (e.g Rackwitz, 1976; Rackwitz and Fiessler, 1978; Fiessler et al, 1979), in which the Taylor series expansion of the performance function g( ) is linearized at some point, say point (y^*_1, y^*_2, ..., y^*_m) on the failure surface rather than at the mean. The linearizing point is called the design or the checking point. Basically, the procedure is as follows:

With the limit state and its variables given by

$$Z_M = g(y_1, y_2, ..., y_m) \qquad (4.129)$$

the random variables Y_i are first transformed to reduced uncorrelated (for independent random variables Y_i) variables with zero mean and unit variance. That is,

$$U_i = \frac{Y_i - \mu_{Y_i}}{\sigma_{Y_i}} \qquad (4.130)$$

Now for any variable having non-normal distribution, it can be transformed into normal standardized variable in accordance with the equality of probabilities. Thus Y_i and U_i can be related using

$$F(y_i) = \Phi(U_i) \qquad (4.131)$$

so that

$$U_i = \Phi^{-1}[F(y_i)] \tag{4.132}$$

By a Taylor power series expansion of Eq. (4.132) in correspondence with the checking point vector $\underline{y}^*$ and considering linear terms only,

$$u_i \approx \Phi^{-1}[F(y_i)] + \left[\frac{\partial \Phi^{-1}[F(y_i)]}{\partial y_i}\right]_{y^*} (y_i - y_i^*) \tag{4.133}$$

where the notation:

$$\left[\frac{\partial}{\partial y_i}\Phi^{-1}[]\right]_{y^*}$$

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indicates that the derivatives are calculated at the point having coordinates y_i^*.

By the rules of derivative of the inverse function, Eq. (4.133) becomes:

$$u_i \approx \Phi^{-1}[F(y_i^*)] + \frac{f(y_i^*)\,(y_i - y_i^*)}{\varphi\,(\Phi^{-1}[F(y_i^*)])} \tag{4.134}$$

in which f( ) and φ( ) denote the probability densities corresponding to the cumulative distributions F( ) and Φ( ) respectively. Rearrangement of Eq. (4.134) yields:

$$u_i \approx \frac{y_i - \left[y_i^* - \dfrac{\Phi^{-1}[F(y_i^*)]\,\varphi(\Phi^{-1}[F(y_i^*)])}{f(y_i^*)} \right]}{\dfrac{\varphi(\Phi^{-1}[F(y_i^*)])}{f(y_i^*)}} \qquad (4.135)$$

Hence, by comparing Eqs. (4.130) and (4.135) it follows that

$$u_{Y_i} = y_i^* - \frac{\Phi^{-1}[F(y_i^*)]\,\varphi(\Phi^{-1}[F(y_i^*)])}{f(y_i^*)} \qquad (4.136)$$

and

$$\sigma_{Y_i} = \frac{\varphi(\Phi^{-1}[F(y_i^*)])}{f(y_i^*)} \qquad (4.137)$$

Finally, in terms of u_i the safety domain boundary can be expressed in the form:

$$g(u_1, u_2, \ldots\ldots\ldots\ldots, u_n) = 0 \qquad (4.138)$$

the parameter β defined as the minimum distance to this surface from the origin in the reduced coordinate system (See Fig. 4.8) can be determined iteratively, assuming the transformed limit state is capable of differentiation, by solving the following equations:

$$g(u_1^*, u_2^*, \ldots\ldots, u_n^*) = g(\alpha_1\beta, \alpha_2\beta, \ldots\ldots, \alpha_n\beta) = 0 \qquad (4.139)$$

in which

$$\alpha_i = -\frac{\left(\frac{\partial g}{\partial u_i}\right)_{u^*}}{\sqrt{\sum_{j=1}^{n}\left(\frac{\partial g}{\partial u_j}\right)^2_{u^*}}} \; ; (i = 1, 2, \ldots\ldots, n) \qquad (4.140)$$

Having obtained a satisfactory value of the minimum distance β corresponding to convergency of the iterative process, the operational probability of failure P_F can be computed from:

$$P_F = \Phi\{-\beta\} \qquad (4.141)$$

Thus far, the derivation has assumed independent random basic variables. If Y_i are correlated, they must be transformed to uncorrelated random variables using the method proposed in Hohenbichler and Rackwitz, (1981). Whatever the case, in the normalized space, if the original uncertainty vector has a distribution function that deviates significantly from the normal, the originally sufficiently smooth failure surfaces can become distinctively curved (Fiessler el, 1979). There is also associated with this approach the possibility of nonconvergence of the iteration process (Ayyub and White, 1987) either because of the existence of some numerical noise in the performance function or jumps in the distribution functions

(FORM, 1988). For practical applications an algorithm for solving these equations has been developed and coded (e.g., FORM, 1988). Conceptually the so-called FORM is based on the works of Rackwitz and Fiessler, (1978); Hasofer and Lind, (1974); and Rackwitz, (1976). A number of illustrative examples for simple cases can be found in the work of Ang and Tang (1984) and Smith (1986).

4.4.2.5 SORM

This is otherwise known as Second-Order Reliability Method which is an extension of FORM (Sec. 4.4.2.4) to include the second order term.

Assuming the transformed state function $g(\underline{u})$ is locally at least twice differentiable such that $g(\underline{u}) = 0$ defines the limit state and $g(\underline{u}) < 0$ the failure region. Expansion of the failure surface at the design point (see Fig. 4.15) to second order leads to the estimation of the probability of failure in the form (SORM, 1988):

$$P_F \approx P[q(\underline{u}) \leq 0] = 1 - F_N(\gamma) \qquad (4.142)$$

in which $q(\underline{u})$ is a quadratic approximation to $g(\underline{u})$ at $\underline{u}^*$ and it has the form, using matrix notation:

$$q(\underline{u}) = g'^T(\underline{u} - u^*) + \frac{1}{2}(\underline{u} - u^*)^T G_u(\underline{u} - u^*) \qquad (4.143)$$

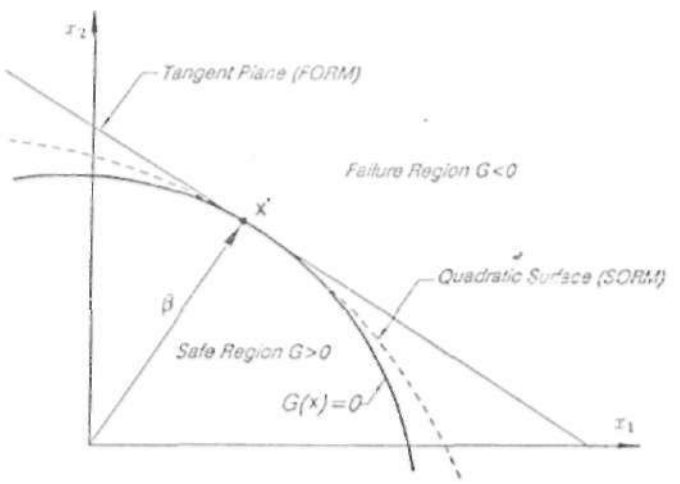


Fig. 4.15: Linear and Quadratic Approximations to Limit State Surface, g(X) = 0

where g'^{T} is the vector of gradients of $g(\underline{u})$ evaluated at $\underline{u}^{*}$ and G_u is the matrix of second and mixed derivatives.

Further transformation using $\underline{u} = \underline{T}\underline{v}$ and followed by translation with $\underline{z} = \underline{v} + \underline{\delta}$ yields:

$$q(z) = (z-\underline{\delta})^{T} E_u (z-\underline{\delta}) - \gamma \qquad (4.144)$$

in which E_u is the matrix of the eigenvalues of the matrix G_u and F_W is the distribution function of

$$W = \sum_{i=1}^{n} \lambda_i (z_i - \delta_i)^2 \qquad (4.145)$$

The task of the necessary computation lies in the resolution of F_W for which there are no sufficiently efficient numerical computation schemes (Fiessler, el al, 1979). Thus, in 1988, Hohenbichler and Rackwitz proposed an approximate technique which was coded for practical applications in SORM, (1988) in order to accurately compute probability integrals over simply connected domains with at least twice differentiable boundary in multidimensional uncertainty spaces involving arbitrarily distributed functions. However, the method as it is only approximate has its own flaws. The method is a single point checking approach. As soon as the location of the checking point and the curvature of the failure surface in it is not sufficiently representative of the entire shape of

the safe domain, the method fails to give accurate results (Fiessler et al., 1979).

4.4.2.6 Importance Sampling Method (ISM)

In an attempt to overcome the seemingly drawback of FORM or SORM, though with additional computational efforts, the importance sampling technique has been employed in FORM and SORM (Fujita and Rackwitz, 1988; Hohenbichler and Rackwitz, 1988) in order to obtain accurate probability estimates, since FORM has been found appropriate and elegantly adequate in this study, its improvement by importance sampling will only be briefly discussed.

Conceptually, the improvement can be stated as follows. Let the probability of failure be described by

$$P_F = P(F_{\underline{Y}}) = \int_{F_{\underline{Y}}} dF_{\underline{Y}}(\underline{y}) \qquad (4.146)$$

where $F_{\underline{Y}}(\underline{y})$ is the joint distribution function of the random vector $\underline{Y}$ and $F_{\underline{Y}} = \{\underline{y}: g(\underline{y}) \leq 0\}$ the failure domain. Employing approximation (linear or quadratic) to $F_{\underline{Y}}$ one may write

$$P_F = P(\Lambda)\,\frac{P(F)}{P(\Lambda)} = P(\Lambda)\,C_E \qquad (4.147)$$

in which

$$C_E \sim \frac{1}{N}\sum_{i=1}^{N} C_i \qquad (4.148)$$

where C_i is a probability ratio to be estimated by simulation and Λ is an approximation (linear or quadratic) to F_i. Following the procedure employed in the FORM (Fujita and Rackwitz. 1988):

$$C_E \sim \frac{1}{N}\sum_{i=1}^{N} \frac{\Phi(-\|\underline{u}^* + b_i\underline{\alpha}\|)}{\Phi(-\beta)} \qquad (4.149)$$

in which b_i corresponds to the root of $g(\underline{u} + b_i\underline{\alpha}) = 0$. It is assumed that $g(\underline{u})$ is a continuous at-least-twice differentiable function near the point $\underline{u}^*$ and the $\underline{u}$ is the transformed standardized normal vector.

The appropriateness of the improvement lies in the fact that the variance of the estimate C_E is very small and that as N ---> ∞, this variance vanishes and the probability for the linearized surface is rendered exact, i.e.,

$$P(\Lambda) = \Phi(-\beta) \qquad (4.150)$$

The work of Hohenbichler and Rackwitz, (1988) can be consulted for the corresponding discussion relevant to SORM.

4.5 Choice of Computation of Failure Probabilities

A number of techniques for computing failure probabilities has been discussed with respect to concepts, merits and demerits. Among them has FORM been adopted for the calibration required in this study. Basically, the choice is zeroed in on the following reasons:

1. It takes into account the available information on the type of distributions of the random variables.
2. It also considers the geometry of the safety domain so as to be able to deduce information about the reliability of the design consideration.
3. The failure surfaces in question are not complex and in general departure from linearity does not lead to cases of minima and maxima.
4. The relevant performance functions are at least twice differentiable.
5. There exists a standard algorithm coded for practical applications (e.g., FORM, 1988).

In the adopted form of FORM, rather than solving Eq (4.139) iteratively some optimization technique is proposed. First of all, the failure surface expressed in the reduced variable space, $g(\underline{u}) < 0$ is expanded at the design point $\underline{u}^*$ corresponding to β the first order estimate so that

$$P_p = P[g(\underline{u}) \leq 0] \approx P[l(\underline{u}) \leq 0] = \Phi(-\beta) \qquad (4.151)$$

In this way,

$$l(\underline{u}) = \underline{\alpha}^T(\underline{u} - \underline{u}^*) = \underline{\alpha}^T\underline{u} + \beta \tag{4.152}$$

and it is required that

$$\beta = \|u^*\| = -\underline{\alpha}^T\underline{u}^* = \min|\underline{u}| \tag{4.153}$$

subject to

$$g(\underline{u}) \leq 0$$

In Eq. (4.152)

$$\underline{\alpha} = \frac{g'(\underline{u}^*)}{\|g'(\underline{u}^*)\|} \tag{4.154}$$

is the normalized gradient of $g(\underline{u})$ at $\underline{u}^*$. The point $\underline{u} = \underline{u}^*$ was found using a modified projected gradient algorithm (See FORM, 1988).

It must be mentioned in passing however, that there are more other methods that have been proposed for the purpose of computing the failure probability (e.g., Nishino et al, 1984; Paloheimo and Hannus, 1974; Ayyub and Haldar, 1984), and it is predicted that more methods with perhaps additional computational efforts will still evolve.

CHAPTER 5

UNCERTAINTIES AND SENSITIVITIES OF THE BASIC RESISTANCE PARAMETERS

5.1. Introduction

This Chapter discusses the sources and types of uncertainties and also analyses the sensitivity of the structural resistance of beam-column sections to some quantified uncertain parameters. In Chapter 3, the interaction equations have shown that the strength of a beam-column consists of many basic parameters. It is conjectured that the statistics of these parameters will have significant influence on the statistics of the complete interaction strength. Therefore, it is intended in this Chapter to evaluate the effective width parameter, axial reduction and the moment magnification factors as expressed in the interaction design equations, and consequently the interaction design equations themselves.

A range of total uncertainty, i.e., $5\% \leq \Omega_T \leq 30\%$ is assumed for the evaluation purpose. This range is adopted throughout this study for numerical computations only and it is considered practical on the basis of the definition of coefficient of variation. In 1969, Borges and Castanheta

employed a similar range although in a different context. Such a range is necessary because there is usually a probability to exceed the adopted bounds for loading and material strength since the parameters making them have certain scatter about their mean values (Schmitt, 1977). It has already been shown that the survival of a component or structure under loading depends on these scatters. The influence of the mathematical simplification on the sensitivity of each parameter is taken into consideration by expressing the standard deviation as a function of the associated basic variables.

5.2. Sources and Types of Uncertainties

In engineering, uncertainties may be associated with physical phenomena that are inherently random or with predictions and estimations of reality performed under conditions of incomplete or inadequate information (Ang and Tang, 1984). It then follows that uncertainty may be associated with the inherent variability of the physical process or with the imperfection in modelling the physical process. Inherent uncertainty is essentially a state of nature and the resulting uncertainty may not be controlled or reduced. On the other hand, the uncertainty associated with prediction or modelling error may be reduced through the use of more accurate models or acquisition of more data. In a specific manner, some types of uncertainty identified are discussed in this section according to (Melchers, 1987).

5.2.1. Phenomenological Uncertainty

This uncertainty may arise whenever the form of construction or the design technique generates uncertainty about any aspect of the possible behaviour of a structure under construction, service, and extreme conditions. In such circumstances only subjective estimates of the effect of this type of uncertainty can be given.

5.2.2 Decision Uncertainty

This arises in connection with the decision as to whether a particular phenomenon has occurred. In terms of limit states it is concerned purely with the decision as to whether a limit state violation has occurred.

5.2.3. Modelling Uncertainty

This form of uncertainty is caused by using a simplified relationship between the basic variables to represent the real relationship of phenomenon in question. In a simple form, modelling uncertainty has to do with uncertainty of physical models.

5.2.4. Physical Uncertainty

This uncertainty is usually identified with the inherent random nature of a basic variable. It can be reduced but not eliminated with greater availability of data (Ang and Tang, 1984) or greater effort in quality control (Melchers, 1987).

In general, this uncertainty is not known a *priori* for any basic variable but can be estimated from observations or subjectively assessed.

5.2.5. Statistical Uncertainty

Usually, sample means and higher moments are used as statistical estimators. They can be determined from available data and then used to suggest an appropriate distribution functions. But observations of a variable do not generally perfectly represent it and as a result the recorded data may be biased. In addition, different sample data sets will normally produce different statistical estimators. This is what causes statistical uncertainty. It is then appropriate, in reliability analysis, to examine sensitivity to changing values of these estimators.

5.2.6. Uncertainties due to Human Factors

These uncertainties are the consequences of human involvement in the design, construction, use, etc., of structures. They have been classified into two categories (Melchers, 1987):

1. Human Errors
2. Human intervention

More information on these classes is well documented (e.g., Ed. Nowak, 1986; Regulinski and Askien, 1972; Lamb,

1972; Techner, 1972) and will be omitted.

It is now not surprising having the evolution of safety or reliability arising principally because of the presence of uncertainty.

5.3 Variability of the Effective Width Parameter

The local buckling phenomenon has been identified as one of the considerations in the design of thin-walled elements. The inclusion of this phenomenon in the design criteria of beam-columns is rather empirical in both the AISI specification and the ECCS Recommendations. Before the evaluation of the interactions equations, the effective width parameters as expressed by AISI and ECCS will be statistically examined in this section.

5.3.1 AISI Expression

For stiffened compression elements, the effective width to be employed in the design of such elements due to occurrence of local buckling has been given (Section 2.3.1.1 of the Specification) as

$$\frac{b}{t}=1.9\sqrt{\left(\frac{E}{f_{max}}\right)}\left[1-0.145\frac{t}{w}\sqrt{\left(\frac{E}{f_{max}}\right)}\right] \qquad (5.1)$$

where

b = the effective width of plate after local buckling

t = Thickness of plate

w = initial width of plate before local buckling

E = Young's Modulus of Elasticity

f_{max} = Maximum value of the stress = fy at failure

Equation (5.1) can further be simplified to present a non-dimensionalized expression of the form:

$$\frac{b}{w} = 1.9\frac{t}{w}\sqrt{\frac{E}{f_y}}\left[1-0.415\frac{t}{w}\sqrt{\frac{E}{f_y}}\right] \qquad (5.2)$$

It will be assumed that the ratios E/f_y and w/t are random variables and are uncorrelated so that their statistical influence on the effective width parameter can be examined. And in fact, available information (e.g., Ravindra et al, 1974) on these variables prove that their values are associated with some uncertainties. The first two moments of the effective width parameter are only required and are sufficient for estimating its sensitivity to inherent uncertainties in the associated basic variables.

Let Eq. (5.2) be rewritten in the form:

$$b/w = \omega = f(m_i;\ i= 1,2,.....,n) \qquad (5.3)$$

so that

$$\omega = a\frac{\sqrt{m_2}}{m_1} - \frac{abm_2}{m_1^2} \qquad (5.4)$$

in which a = 1.90 and b = 0.415, m_1 = mean value of w/t and m_2

= mean value of E/f_y.

Employing the Taylor truncation series, the expectation and the standard deviation of the parameter ω can be expressed as

$$\mu_\omega = a\frac{\sqrt{m_2}}{m_1}\left(1+\Omega_1^2-\frac{\Omega_2^2}{8}\right) - \frac{abm_2}{m_1^2}(1+3\Omega_1^2) \qquad (5.5)$$

It is here assumed that

$$\Omega_1 \approx \Omega_2 \approx \Omega \qquad (5.6)$$

Then, the total uncertainty in the effective width parameter expression is

$$\Omega_T^2 = 2\Omega^2 \qquad (5.7)$$

By virtue of Eqs. (5.7), Eq. (5.5) can be expressed as

$$\mu_\omega = \frac{am_2}{m_1}\left[\sqrt{\frac{1}{m_2}}(1+0.438\Omega_T^2) - b(1+1.5\Omega_T^2)\right] \qquad (5.8)$$

Similarly, the standard deviation of ω is

$$\sigma_\omega = \frac{m_2}{m_1}\Omega_T\sqrt{\frac{1.930}{m_2}+\frac{1.555}{m_1^2}-\frac{3.391}{m_1\sqrt{m_2}}} \qquad (5.9)$$

Since two criteria are being examined, the statistics of the effective width parameter as expressed by ECCS follows.

5.3.2 ECCS Expression

According to Section R 3.2.1 of the Recommendation, for the purpose of computing the load bearing capacity of stiffened elements under uniform compression, the effective width, b*, is given as

$$b^* = \omega' b_p \tag{5.10}$$

where ω' is a reduction coefficient given by

$$\omega' = 1; \quad \bar{\lambda}_p \leq 0.673 \tag{5.11a}$$

and

$$\omega' = \frac{1 - \frac{0.22}{\bar{\lambda}_p}}{\bar{\lambda}_p}; \quad \bar{\lambda}_p > 0.673 \tag{5.11b}$$

with

$$\bar{\lambda}_p = 0.526 \frac{b_p}{t} \sqrt{\frac{f_{ty}}{E}} \tag{5.12}$$

whence

b^* = effective width of plate

b_p = initial width of plate before local buckling

t = thickness of plate

f_{ty} = yield stress, and

E = Modulus of Elasticity.

Let us examine Eq. (5.10) for $\bar{\lambda}_p > 0.673$. Then from Eqs. (5.10) and (5.12)

$$\omega' = \frac{b^*}{b_p} = \frac{0.526 b_p/t\sqrt{f_{ty}/E} - 0.22}{0.277 (b_p/t)^2 f_{ty}/E} \tag{5.13}$$

Assuming

m_1 = mean value of b_p/t, and

m_2 = mean value of f_{ty}/E,

and that they are independent random quantities. Employing Taylor series as in Sections 5.3.1, the expectation and the standard deviation of ω' can be expressed respectively, as

$$\mu'_\omega = \frac{1.9}{m_1}\sqrt{\frac{1}{m_2}}(1+\Omega_1^2+0.375\Omega_2^2) + \frac{0.794}{m_1^2 m_2} C^* \tag{5.14}$$

where $C^* = 1 + 3\Omega_1^2 + \Omega_2^2$ and

$$\sigma'_\omega = \sqrt{(4\Omega_1^2+\Omega_2^2)\left[\frac{0.902}{m_1^2 m_2} - \frac{1.508}{m_1^3 m_2^{1.5}} + \frac{0.631}{m_1^4 m_2^2}\right]} \tag{5.15}$$

But from Eqs. (5.6) and (5.7), Eqs. (5.14) and (5.15) can be reduced, respectively, to

$$\mu'_\omega = 1.90\frac{1}{m_1\sqrt{m_2}}(1+0.688\Omega_T)^2 - \frac{0.794}{m_1^2 m_2}(1+2\Omega_T^2) \tag{5.16}$$

and

$$\sigma'_{\omega} = \frac{1.942\Omega_T}{m_1}\sqrt{\frac{0.598}{m_2} - \frac{1}{m_1 m_2^{1.5}} + \frac{0.421}{m_1^2 m_2^2}} \qquad (5.17)$$

5.3.3 Discussion of the Numerical Results.

In Figs 5.1 (a) to (c), the sensitivity of the effective width function to total variability in its parameters is depicted. At low w/t ratio, the expressions, utilized by ECCS and AISI, show similarity in sensitivity. However, as w/t increases the sensitivity also augments.

Figure 5.2 gives the plot of relative difference in sensitivity to total uncertainty for the expressions utilized by AISI and ECCS. For practical value of the total uncertainty, this relative difference remains uniform for a given w/t ratio, although there is a wider gap between w/t = 40 and 80.

5.4 Variability of Resistance Factors

The load carrying capacity of a thin-walled beam-column has been shown to be influenced by the interaction of local and global buckling. In an attempt to account for these phenomena, certain reduction coefficients are applied in the interaction equations. But the consideration of the local buckling effect is merely empirical. Evaluation is then required so that the adequacy of the said reduction factors can be ascertained. As

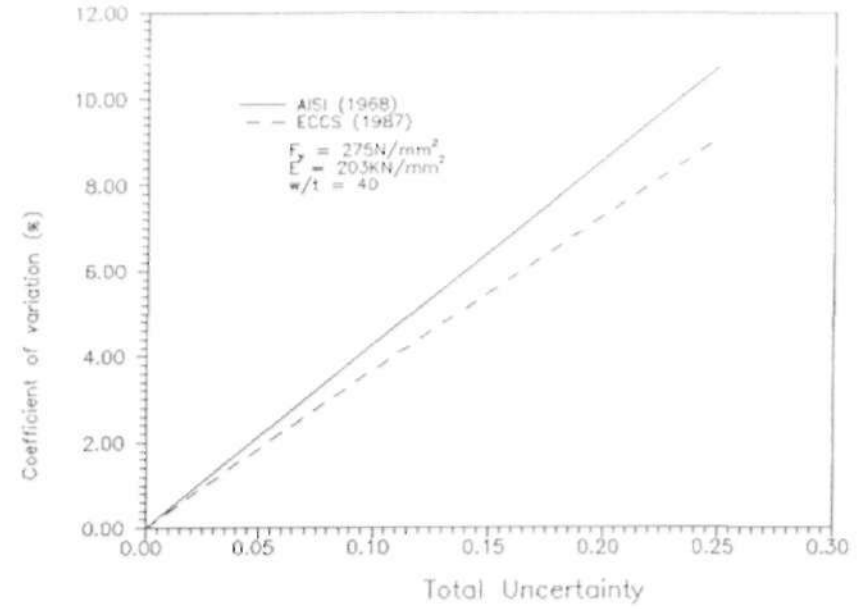


Fig. 5.1(a): Coefficient of Variation of Effective Width Parameter with Total Uncertainty (w/t = 40)

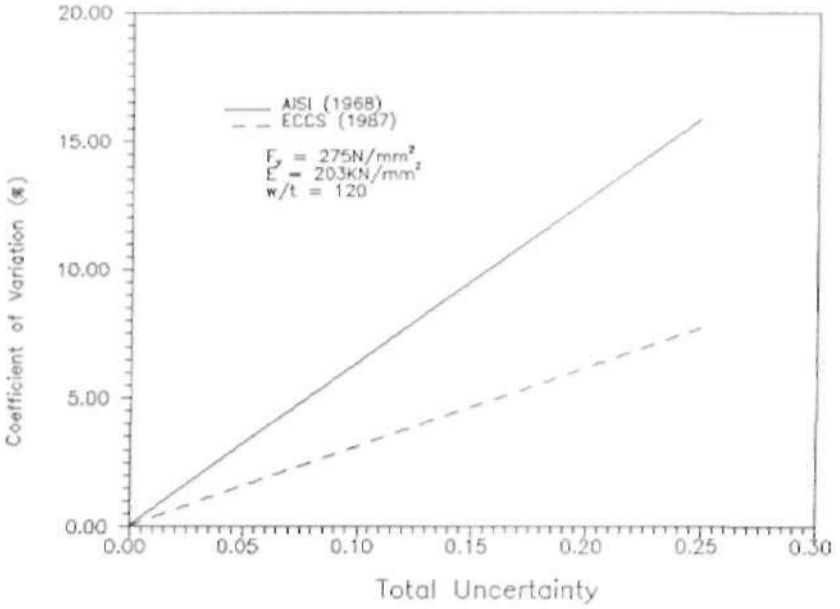


Fig. 5.1(b): Coefficient of Variation of Effective Width Parameter with Total Uncertainty (w/t = 120)

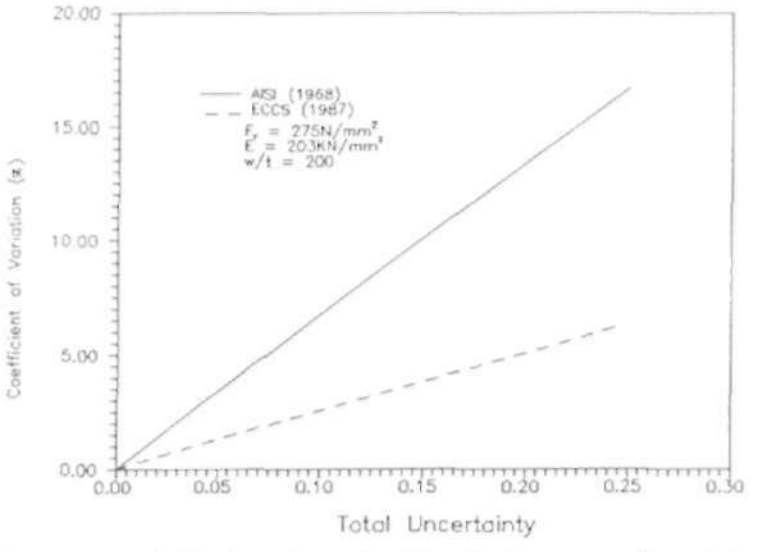


Fig. 5.1(c): Coefficient of Variation of Effective Width Parameter with Total Uncertainty (w/t = 200)

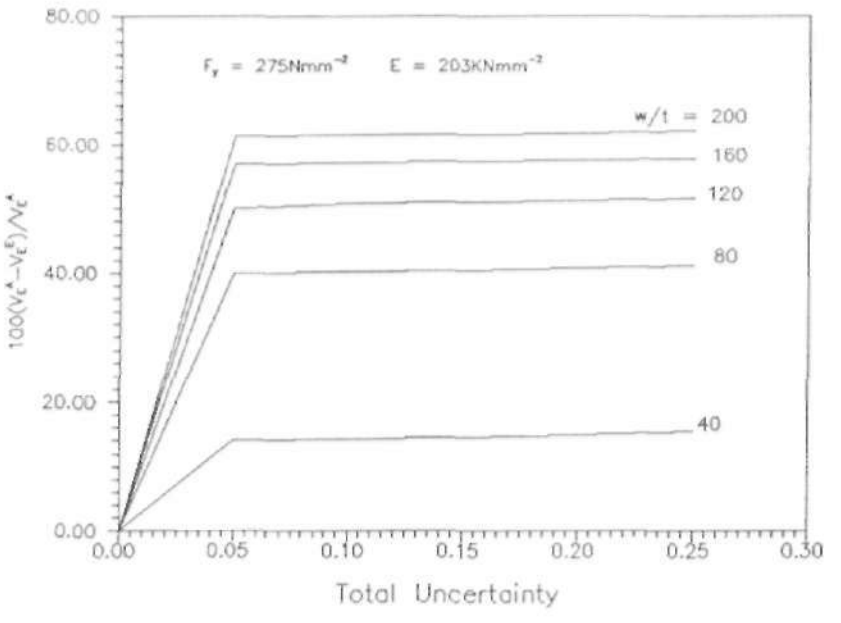


Fig. 5.2: Differential Sensitivity of AISI and ECCS Effective Width Equations

earlier examined in Chapter 3, the formats for these coefficients are not the same for the AISI and ECCS criteria. Therefore, each will be separately treated as follows.

5.4.1 AISI Axial Reduction Factor

According to Section 3.3.1, the influence of local and global buckling on the compressive strength of a beam-column has been limited to a range of slenderness parameter. As a consequence, in a dimensionless form, the applicable reduction coefficients can be expressed as

$$\xi = 0.522 Q_w - 0.13 (Q_w \bar{\lambda})^2 \qquad (5.18)$$

when $\bar{\lambda} < \sqrt{(2/Q_w)}$ and

$$\xi = \frac{0.522}{\bar{\lambda}^2} \qquad (5.19)$$

when $\bar{\lambda} > \sqrt{(2/Q_w)}$.

Equations (5.18) and (5.19) are modified versions of the expressions in AISI specification so that any value of the material strength (i.e., the yield strength) can be allowed. The quantities Q_w and $\bar{\lambda}$ are, respectively, the form factor and the slenderness parameter. The sensitivity of the reduction factor to inherent uncertainties in the associated basic variables will now be examined assuming Q_w and $\bar{\lambda}$ are independent random variables. The first two moments of Eqs

(5.18) and (5.19) are needed and will be obtained using an approximate method. That is, employing the Taylor truncation series, we have

$$\mu_\xi = 0.522 m_q - (0.13 m_q m_\lambda)^2 - 0.13 (m_q m_\lambda)^2 (\Omega_q^2 + \Omega_\lambda^2) \qquad (5.20)$$

when $\overline{\lambda} < \sqrt{(2/Q_w)}$, as the expectation of the reduction factor. From the principle of total uncertainty, Eq. (5.20) can be reduced to:

$$\mu_\xi = 0.522 m_q - (0.13 m_q m_\lambda)^2 (1 + \Omega_T^2) \qquad (5.21)$$

in which

m_q = mean value of Q_w

m_λ = mean value of $\overline{\lambda}$, and

Ω_T = total uncertainty resulting from Q_w and $\overline{\lambda}$.

The variation in ξ expressed by its variance is

$$\sigma_\xi^2 = m_q^2 [0.272 \Omega_q^2 (1 - m_q m_\lambda^2) + 0.068 m_q^2 m_\lambda^4 (\Omega_q^2 + \Omega_\lambda^2)] \qquad (5.22)$$

By definition, $\Omega_q >> \Omega_\lambda$ and it is assumed that

$$\Omega_q \approx 0.80 \Omega_T \qquad (5.23)$$

It then follows from Eq. (5.23) that

$$\sigma_\xi = 0.417 m_q \Omega_T \sqrt{1 - m_q m_\lambda^2 (1 - 0.391 m_q m_\lambda^2)} \qquad (5.24)$$

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On the other hand, when $\overline{\lambda} > \sqrt{(2/Q_w)}$, the expectation and the variance of the reduction factor yield, respectively

$$\mu_\xi = \frac{0.522}{m_\lambda^2}(1+3\Omega_T^2) \tag{5.25}$$

and

$$\sigma_\xi = \frac{1.044\Omega_T}{m_\lambda^2} \tag{5.26}$$

Equations (5.21), and (5.24) to (5.26) are numerically examined for the established range of Ω_T and some of the results are displayed in Fig. 5.3.

In Figure 5.3, the coefficient of variation of the reduction factor in relation to total inherent uncertainty of the basic varaibles is depicted. The statistics of this parameter shows remarkable sensitivity to the empirical consideration of the local buckling effect typified by the use of a form factor, Q_w.

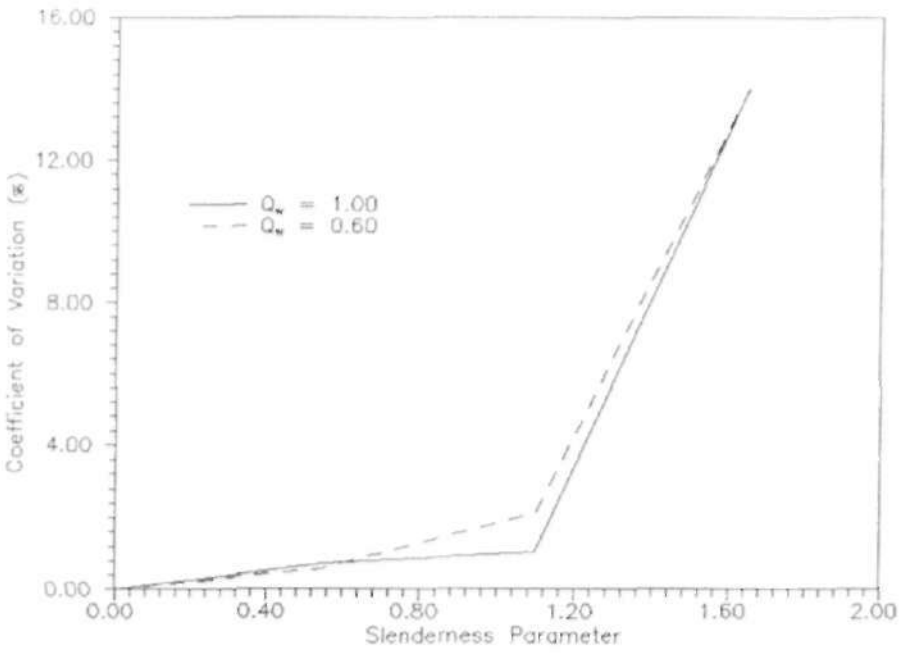


Fig. 5.3: Variability of AISI Column Strength Factor

5.4.2. ECCS Axial Reduction Factor

Recalling Eqs. (3.32) to (3.34), the expression representing the reduction factor is

$$\chi = F - \sqrt{F^2 - \frac{Q_w}{\bar{\lambda}^2}} \qquad (5.27)$$

in which the associated parameters are as previously defined. In Eq. (5.27), three basic parameters are identified and will be treated as independent random quantities, viz: F, Q_w, and $\overline{\lambda}$.

Application of Taylor series leads to

$$\mu_\chi = m_p - \sqrt{\Psi_o} + \frac{1}{2}\frac{1}{\Psi_o^{3/2}}\left[m_F^4\Omega_F^2 + \frac{m_q^2\Omega_q^2}{4m_\lambda^4} + \frac{m_q^2}{m_\lambda^4}\Omega_\lambda^2\right]$$

$$+ \frac{1}{2}\frac{1}{\sqrt{\Psi_o}}\left[\frac{3m_q}{m_\lambda^2}\Omega_\lambda^2 - m_F^2\Omega_F^2\right] \qquad (5.28)$$

Unlike in Section 5.4.1, and in accordance with parameteric definition, Ω_λ is taken to be approximately equal to zero, and that $\Omega_q = \Omega_F$. Therefore, Eq. (5.28) reduces to

$$\mu_\chi = m_F - \sqrt{\Psi_o} + \frac{\Omega_T^2}{2\Psi_o^{3/2}}\left(m_F^2(m_F^2 - \Psi_o) + \frac{m_q^2}{4m_\lambda^4}\right) \qquad (5.29)$$

Similarly, the standard deviation of the factor is

$$\sigma_\chi = \Omega_T\sqrt{\left(1 - \frac{m_F}{\sqrt{\Psi_o}}\right)^2 m_F^2 + \frac{m_q^2}{4m_\lambda^4\Psi_o}} \qquad (5.30)$$

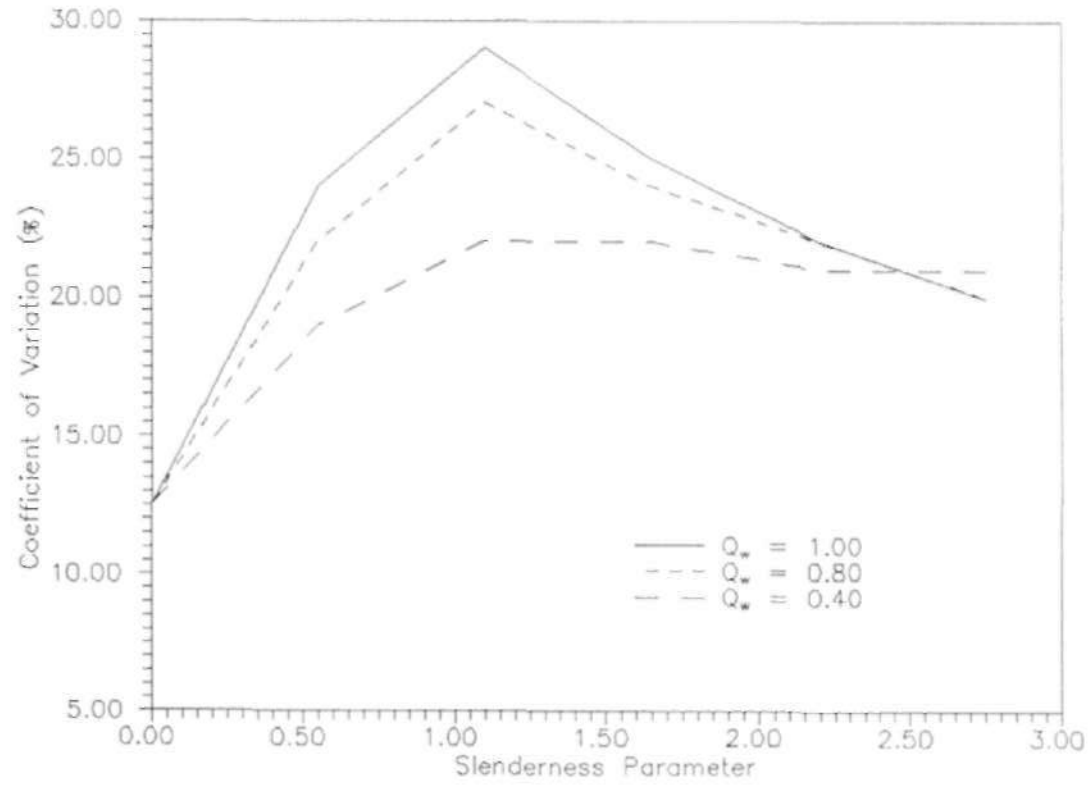


FIG. 5.4: Variability of ECCS Column Strength Factor

In Eqs. (5.29) and (5.30),

$$\Psi_o = m_p^2 - \frac{m_a}{m_\lambda^2} \tag{5.31}$$

The result of these statistics are plotted in Fig. 5.4. From this plot the dispersion of the axial reduction factor around the mean value is more significant when Q_w = 1.0 and mostly pronounced in the intermediate range of slenderness parameter. It appears however that this factor becomes less sensitive to inherent uncertainty when Q_w < 1.0 and when the member becomes very slender.

5.4.3 The Moment Magnification Factors

The presence of the axial force in a beam-column element is to reduce its moment resistance. Its influence is accounted for by modifying the moment side of the interaction equation using a magnification factor that may simply be expressed as

$$k_b = f(c_m, x_i; i=1,2,.....,n) \tag{5.32}$$

According to the problem in hand, Eq (5.32) is seen as

$$k_b = \frac{c_m}{1 - \prod_{i=1}^{5} x_i} \tag{5.33}$$

in which

c_m = a coefficient which takes into account the distribution along the beam-column length; and

x_i = are the quantified variables.

The expected value of the function $k_b(x)$ may be calculated on the basis of the Taylor series of the function. That is,

$$\mu_{k_b} = \frac{1}{\psi'}\left[1 + \left(\frac{(1-\psi')\,\Omega_T}{\psi'}\right)^2\right] \qquad (5.34)$$

where

$$\psi' = 1 - m_1 m_3 m_4 m_5$$

and m_i are the mean values of the quantified variables. Neglecting functional dependence, the standard deviation of $k_b(x)$ is

$$\sigma_{k_b} = \frac{1-\psi'}{\psi'^2}\,\Omega_T \qquad (5.35)$$

Equations (5.34) and (5.35) hold for the AISI and ECCS expressions except that the quantities x_i have different definitions.

The sensitivity of the magnification factors to total inherent uncertainty is depicted in Figs. 5.5 to 5.6 for AISI Specification and ECCS Recommendation respectively. When the

influence of local buckling is substantial, the sensitivity of these factors grow with decreasing value of Q_w and increasing value of slenderness parameter.

5.5 Variability of the Design Interaction Equations

The probabilistic treatment of the maximum strength of a beam-column interaction is essentially a study of a structure which exhibits a random non-linear behaviour. According to the different formulations in Chapter 3, the maximum strength, λ_R of a thin-walled beam-column may be expressed as

$$\lambda_R = f(x_i,\ i = 1,\ 2,\ .....,\ n) \qquad (5.36)$$

where x_i denote the parameter descriptors of the strength. Equation (5.36) represents a multidimensional probability density function, or a response surface, since the parameters involved may be treated as random. The random variation of the parameters constitutes the randomness of the predicted strength.

In a deterministic manner, the failure surfaces on the interaction diagrams are marked by the slenderness parameter. Each surface has associated with it some uncertainties. This is illustrated in Figs. 5.7(a) to 5.7(c).

The exact procedure for computing the expectation and variance of the strength is highly involving. Therefore the

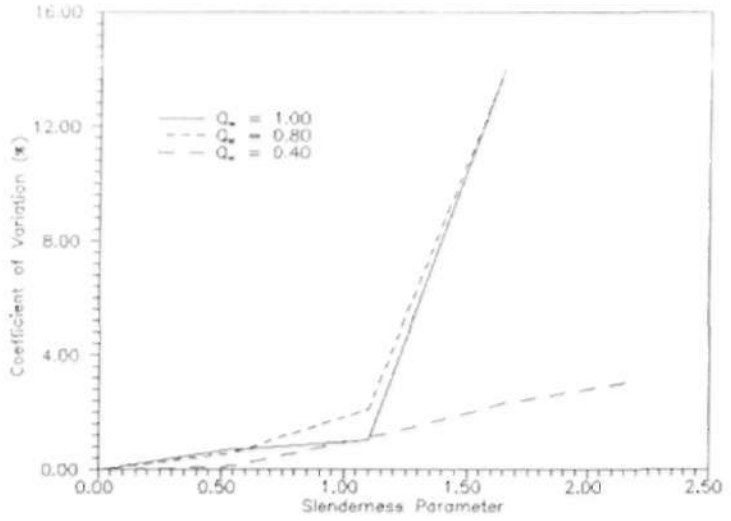


Fig. 5.5: Variability of AISI Magnification Factor

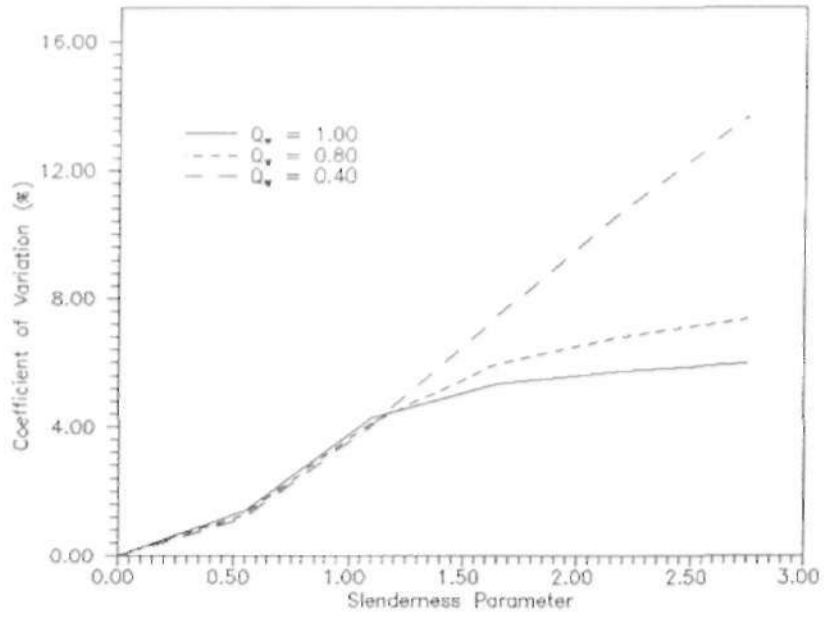


Fig. 5.6: Variability of ECCS Magnification Factor

approximate technique of Section 4.2.2 is adopted.

According to Fig 5.7(c), the interactive strength will be fully investigated when points at the intersections of the rays defined by ϵ_i and the corresponding failure surface are considered. This is so because such points represent a probabilistic description of the predicted strength. Should points be considered as in Figs 5.7(a) and (b) the implications are that: (1) either of the parameters of the strength is considered deterministic and the other probabilistic, and (2) working along any of the axes of the interaction coordinate system will be inadequate because (a) as the applied axial load approaches the ultimate compressive capacity (or the applied moment approaches the ultimate moment capacity), the ability of the beam-column to withstand moment (axial load) becomes small, and any variation in any of the associated variables will lead to significant error in the computation and also (b) the level of evaluation will vary from one failure surface to the other.

In the past, probabilistic analyses of safety of structural members have frequently relied on the formulation of the ultimate limit state equation in which load carrying capacity (resistance) is measured by a single variable, such as the full plastic moment for a beam, (e.g., Bjorhovde et al, 1978). But the feeling expressed herein is that alternative

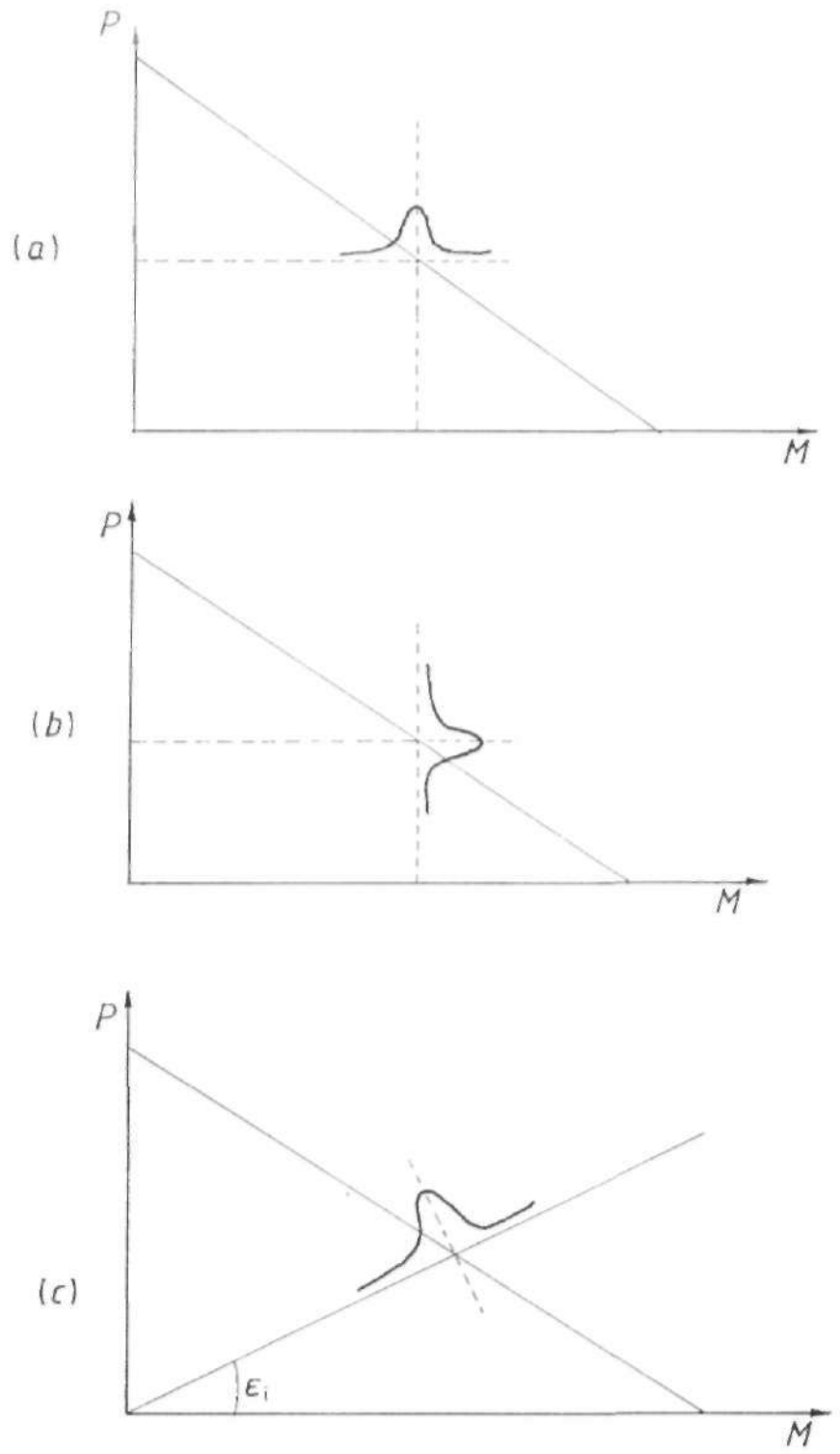


Fig. 5.7: Possible Variations in the Interaction Strength

formulations in terms of the basic parameters that determined the resistance may be more desirable. The idea of a singly expressed resistance obscures the possibility of adequately checking and controlling the parameters which significantly affect the resistance. As pointed out earlier, the local buckling effect is significant to the behaviour of thin-walled members. Therefore the effect, as introduced into the interaction equation, is fully quantified.

For the purpose of evaluation and simplicity, the resistance formulations (i.e., the interaction design equations) as given by AISI and ECCS can be expressed as in Eq. (5.36). Recalling from Chapter 3, Eq. (5.36) can be explicitly rewritten as

$$\lambda_R = X_1 + \frac{X_2}{1 - \frac{1}{X_2}\prod_{i=1}^{5} X_i} \qquad (5.37)$$

Employing Taylor series, the first two moments of the resistance are obtained as follows. Taking the partial derivative of the resistance function with respect to the quantified parameters we obtain the following derivatives:

$$f_1 = 1 + \frac{\prod_{i=2}^{5} X_i}{\left[1 - \frac{1}{X_2}\prod_{i=1}^{5} X_i\right]^2} \qquad (5.38)$$

$$f_{11} = \frac{2\prod_{i=1}^{5} X_i^2}{X_2\left[1 - \frac{1}{X_2}\prod_{i=1}^{5} X_i\right]^3} \qquad (5.39)$$

$$f_2 = \frac{1}{1 - \frac{1}{X_2}\prod_{i=1}^{5} X_i} \qquad (5.40)$$

$$f_{22} = 0 \qquad (5.41)$$

$$f_3 = \frac{\prod_{i=1}^{5} X_i}{X_3\left[1 - \frac{1}{X_2}\prod_{i=1}^{5} X_i\right]^2} \qquad (5.42)$$

$$f_{33} = \frac{2\prod_{i=1}^{5} X_i^2}{X_1 X_3\left[1 - \frac{1}{X_2}\prod_{i=1}^{5} X_i\right]^3} \qquad (5.43)$$

$$f_4 = \frac{\prod_{i=1}^{5} X_i}{X_4\left[1 - \frac{1}{X_2}\prod_{i=1}^{5} X_i\right]^2} \qquad (5.44)$$

$$f_{44} = \frac{2\prod_{i=1}^{5} X_i^2}{X_2 X_4\left[1 - \frac{1}{X_2}\prod_{i=1}^{5} X_i\right]^3} \qquad (5.45)$$

$$f_5 = (X_4/X_5)\, f_4 \qquad (5.46)$$

and

$$f_{55} = \frac{2\prod_{i=1}^{5} X_i^2}{X_2\left[1 - \frac{1}{X_2}\prod_{i=1}^{5} X_i\right]^3} \qquad (5.47)$$

in which

$$f_i = \frac{\partial f}{\partial X_i} \; ; \; f_{ii} = \frac{\partial^2 f}{\partial X_i^2}$$

Considering the second order term and assuming independence, the expectation of the interaction strength

becomes

$$\mu_{\lambda_R} = X_1 + \frac{X_2}{1 - \frac{1}{X_2}\prod_{I=1}^{5} X_I}$$

$$+ \frac{\prod_{I=1}^{5} X_I^2 \left(\sum_{i=1}^{5} \Omega_i^2 - \Omega_2^2 \right)}{X_2 \left[1 - X_2 \prod_{I=1}^{5} X_I^2 \right]^3} \qquad (5.48)$$

where Ω_i's are the coefficients of variation of the random parameters.

From the principle of total uncertainty, Eq. (5.48) shows that variation in x_2 does not affect the expectations. If ψ' = $(1 - x_1x_3x_4x_5)$ and Ω_T represents the total uncertainty, the expectation can be expressed as

$$\mu_{\lambda_R} = X_1 + \frac{X_2}{\psi'} \left(1 + \left[\frac{X_1 X_3 X_4 X_5 \Omega_T}{\psi'} \right]^2 \right) \qquad (5.49)$$

Similarly, the variance of the resistance is

$$\sigma_{\lambda_R}^2 = (X_1\Omega_1)^2\left(1+\frac{\prod_{i=2}^{5}X_i}{\psi'^2}\right)^2+\left(\frac{X_2\Omega_2}{\psi'}\right)^2+\left(\frac{\prod_{i=1}^{5}X_i}{\psi'^2}\right)^2\sum_{i=3}^{5}\Omega_i^2 \qquad (5.50)$$

By parametric definition, $\Omega_1 = \Omega(\Omega_i;\ i = 1, 3, 4, 5)$ and $\Omega_2 = \Omega_T$. Therefore Eq. (5.50) is further reduced to

$$\sigma_{\lambda_R}^2=\left(\frac{\Omega_T}{\psi'}\right)^2\left[X_1^2(\psi'^2+2\prod_{i=2}^{5}X_i)+X_2^2+\left(\frac{\prod_{i=1}^{5}X_i}{\psi'}\right)^2\right] \qquad (5.51)$$

In terms of the standard deviation, Eq. (5.51) implies

$$\sigma_{\lambda_R}=\frac{\Omega_T}{\psi'}\sqrt{X_1^2(\psi'^2+2\prod_{i=2}^{5}X_i)+X_2^2+\left(\frac{\prod_{i=1}^{5}X_i}{\psi'}\right)^2} \qquad (5.52)$$

Further simplifications can be made by substituting for ψ' so that the expectation and the standard deviation of the interaction resistance become, respectively

$$\mu_{\lambda_R}=X_1+\frac{X_2}{\psi'}\left(1+\left(\frac{(1-\psi')\,\Omega_T}{\psi'}\right)^2\right) \qquad (5.53)$$

and

$$\sigma_{\lambda_R} = \frac{\Omega_T}{\psi'}\sqrt{(X_1\psi')^2 + 2X_1X_2(1-\psi') + X_2^2 + \left(\frac{X_2(1-\psi')}{\psi'}\right)^2} \qquad (5.54)$$

Although the above derivations are done on a generalized basis, there are certain imposed conditions on the applicability of the design equations that must be observed. Such conditions include the range of slenderness parameter specifically imposed by AISI and the noninclusion of the effect of local buckling at high slenderness applied to the allowable stress (i,e., $f_a/F_{a1} \leq 0.15$), the condition imposed by AISI specification reduces Eqs (5.53) and (5.54), respectively to

$$\mu_{\lambda_R} = X_1 + X_2 \qquad (5.55)$$

and

$$\sigma_{\lambda_R} = \frac{\Omega_T}{2}\sqrt{2(X_1^2 + X_2^2)} \qquad (5.56)$$

Equations (5.53) to (5.56) hold if correlation among parameters is ignored. In the past, correlation among parameters or variables has usually been neglected in reliability studies. It is considered that accounting for it leads to complexity of the problem (Melchers, 1987). Nevertheless the avoidance of any envisaged complexity does not remove the possibility of the existence of functional dependence. And although correlation makes no statement about

cause of effect (Benjamin and Cornell, 1970; Freeman, 1963), its presence in the present problem may contribute to the sensitivity of the design strengths to inherent uncertainties.

The examination of the interaction equations shows that instability is one of the criteria considered in derivation. That means, possible correlations among certain parameters cannot be neglected. By definition, there will be functional dependence among x_1, x_2 and x_5.

It will now be assumed that the significance of correlation is paramount to the variance of the resistance. From Eq. (4.19), employing Taylor series, if functional dependence is assumed, the variation in strength can be expressed as

$$\sigma_{\lambda_z}^2 = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i} \sigma_{x_i} \right)^2 + \sum_{\substack{i=0 \\ i \neq j}} \sum_{\substack{i=0 \\ i \neq j}} \left(\frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} \right) \rho_{ij} \sigma_{x_i} \sigma_{x_j}$$

$$= A + B_k \qquad (5.57)$$

In Eq. (5.57) only B_k has to be formulated since A has been derived as in Eqs (5.54) and (5.56). Judging from the interaction equation and the established random parameters, four different cases will be considered in deriving B_k.

Case I

It is assumed that only x_1 and x_2 are correlated so that

$$B_o = \frac{\partial f}{\partial x_1}\frac{\partial f}{\partial x_2}\rho_{1,2}x_1x_2\Omega_1\Omega_2 \qquad (5.58)$$

in terms of the coefficients of variation (Ω_1 and Ω_2) of the parameters x_1 and x_2. From Eqs (5.38) and (5.40) we obtain

$$B_o = \frac{\rho_{1,2}x_1x_2\Omega_1\Omega_2}{(1+x_2x_3x_4x_5[1-x_1x_3x_4x_5])^2(1-x_1x_3x_4x_5)}$$

Since $\psi' = 1 - x_1x_3x_4x_5$, B_o can be reduced to

$$B_o = \left(\frac{x_1}{\psi'} + \frac{x_2(1-\psi')}{\psi'^3}\right)\rho_{1,2}x_2\Omega_1\Omega_2 \qquad (5.59)$$

Let

$$\Omega_2 = a_o\Omega_1 \qquad (5.60)$$

where $a_o < 1.0$. Therefore Eq (5.59) yields

$$B_o = \left(\frac{x_1}{\psi'} + \frac{x_2(1-\psi')}{\psi'^3}\right)\rho_{1,2}a_ox_2\Omega_1^2 \qquad (5.61)$$

Putting Eq. (5.61) into Eq. (5.58) and according to Eq. (5.57), the standard deviation of the resistance, considering functional parametric dependence between x_1 and x_2 becomes

$$\sigma_{\lambda_R} = \frac{\Omega_T}{\psi'}\sqrt{(X_1\psi')^2 + 2X_1X_2(1-\psi') + \left(\frac{X_2(1-\psi')}{\psi'}\right)^2 + \Xi} \quad (5.62)$$

where

$$\Xi = \left(\frac{X_1}{\psi'} + \frac{X_2(1-\psi')}{\psi'^3}\right)\rho_{1,2}X_2 e_o \Omega_1^2$$

and $\rho_{1,2}$ is the correlation coefficient when x_1 and x_2 are correlated and its value lies in the range $-1.0 < \rho_{1,2} < 1.0$.

Case II

If x_1 and x_5 are correlated and following a similar procedure as in case I, we obtain

$$B_1 = \left[1 + \frac{X_2}{X_1}\left(\frac{1-\psi'}{\psi'^2}\right)\right](1-\psi')X_1X_2\Omega_1\Omega_5 \quad (5.63)$$

Let $\Omega_1 = a_1\Omega_5$, so that

$$B_1 = \left[1 + \frac{X_2}{X_1}\left(\frac{1-\psi'}{\psi'^2}\right)\right]\frac{(1-\psi')}{a_1}X_1X_2\rho_{1,5}\Omega_1^2 \quad (5.64)$$

The standard deviation can then be expressed as

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$$\sigma_{\lambda_R} = \frac{\Omega_T}{\psi'}\sqrt{(x_1\psi')^2 + 2x_1x_2(1-\psi') + x_2^2 + \frac{x_2^2(1-\psi')^2}{\psi'^2} + \Xi^*} \qquad (5.65)$$

where

$$\Xi^* = \frac{\rho}{a_1 g}(1-\psi')(x_1\Omega_1)^2\left(1 + \frac{1-\psi'}{\psi'^2 g}\right)$$

in which $g = x_1/x_2$.

Case III

A functional dependence is assumed to exist between x_2 and x_5 so that

$$B_2 = \frac{x_2^2(1-\psi')}{\psi'^3}\rho_{2,5}\Omega_2\Omega_5 \qquad (5.66)$$

represents the contribution to the variance of the resistance due to this functional dependence.

Let $\Omega_2 = a_2\Omega_5$, and $g = x_1/x_2$ as earlier established. Then

$$\sigma_{\lambda_R} = \frac{\Omega_T}{\psi'}\sqrt{(x_1\psi')^2 + 2x_1x_2(1-\psi') + x_2^2 + \frac{x_2^2(1-\psi'^2)}{\psi'^2} + \Xi^{**}} \qquad (5.67)$$

where

$$\Xi^{**} = \rho_{2,5}\left(\frac{x_1\Omega_2}{g}\right)^2 \frac{1-\psi'}{a_2\psi'^2}$$

Case IV

In this case, all the conditions of Cases I to III are combined. That means the variance of the resistance in the presence of functional dependence among x_1, x_2, and x_5 can be given as

$$\sigma_{\lambda_R}^2 = A + \sum_{k=0}^{2} B_k \qquad (5.68)$$

That is,

$$\sigma_{\lambda_R} = \frac{\Omega_1^2}{\psi' g^2} \cdot \frac{x_1\Omega_T}{\psi'} \sqrt{(1-\psi'+g\psi'^2)\left(a_o\rho_{1,2} + \frac{\rho_{1,5}\psi'(1-\psi')}{a_1}\right) + \xi^{**}} \qquad (5.69)$$

where

$$\xi^{**} = \rho_{2,5}\frac{a_o^2}{a_2}(1-\psi')$$

According to the design criteria of AISI, Eq. (5.69) is only valid when the ratio of the applied to allowable stress is greater than 0.15 (i.e when $f_a/F_{a1} > 0.15$). Similar expression to Eq. (5.69) is obtained when $f_a/F_{a1} \leq 0.15$ and it is given as:

$$\sigma_{\lambda_R} = \frac{\Omega_T}{2}\sqrt{2(x_1^2 + x_2^2 + \rho_{12}x_1x_2)} \qquad (5.70)$$

Since $g = x_1/x_2$, Eq. (5.70) can be declared as

$$\sigma_{\lambda_R} = \frac{\Omega_T}{2} x_2 \sqrt{2\,(1 + \rho_{12} g + g^2)} \qquad (5.71)$$

In deriving Eq. (5.71), it has been assumed that the uncertainty in x_1 and x_2 are approximately equal. Such assumption is seen as the worst condition.

For numerical analysis, information obtained from the work of Rojiani and Woeste (1982) leads to the following inference. The values for the ratios a_o, a_1 and a_2 are taken as $a_o = 0.964$, $a_1 = 0.795$, $a_2 = 0.727$ and that $\Omega_1 = 0.60\Omega_T$.

The above values are substituted into the appropriate equations and the problem was solved using computer. In the program, the coefficient of correlation assumed extreme equal values of -1.0, 0 and 1.0. It is the outcome of these situations that dictates the importance of functional dependence being considered since actual values of these coefficients must lie within $-1.0 \leq \rho_{ij} \leq 1.0$

Based on the statistics of the interaction formulae, Figs 5.8 and 5.9 are plotted. Generally, there is a fairly uniform dispersion of the beam-column strength around the expected value as given by ECCS Recommendation and AISI Specification. Numerical experiment also shows that correlation among the

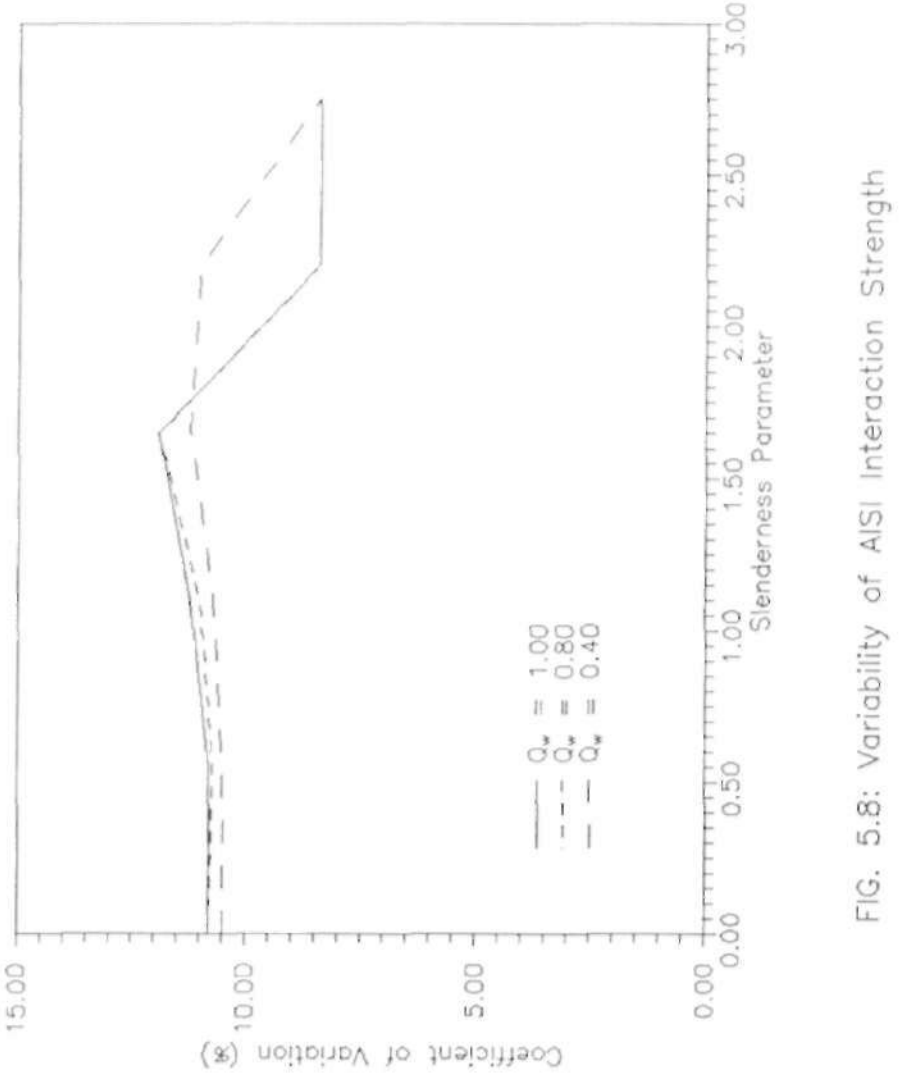


FIG. 5.8: Variability of AISI Interaction Strength

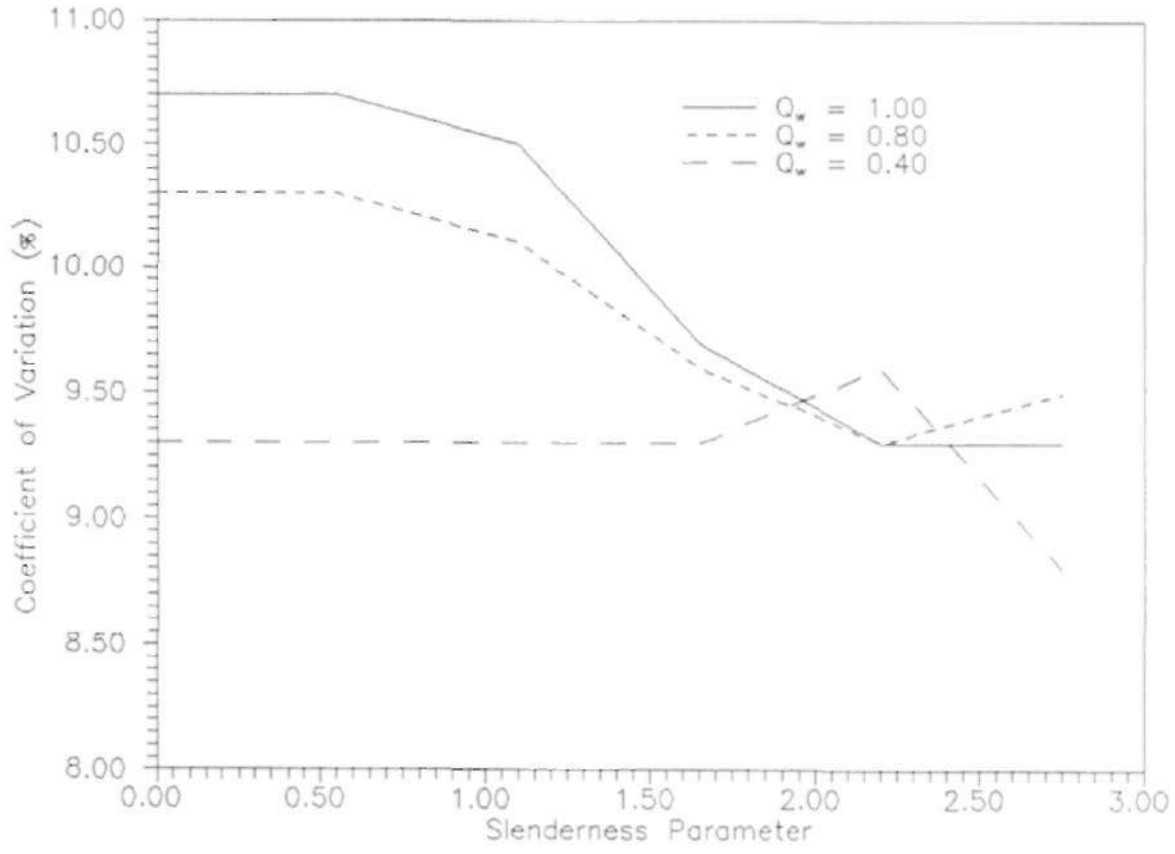


FIG. 5.9: Variability of ECCS Interaction Strength

variables considered is not significant and as such no further comments on it. This observation is only valid as far as the total uncertainty is based on variability in the resistance basic variables. Usually the interaction of a structure and its environment is crucial to structural safety analysis. Hence, bearing in mind that discrepancies exist between test results and the predicted strength of beam-columns as noted in Chapter 3, the computation of implied levels of risk in the different design equations, when extreme loadings are considered is desirable. It is this aspect that is focussed in the next Chapter with a view to proposing optimized design equations in terms of safety and economy.

CHAPTER 6

CALIBRATION TASK

6.1. Introduction

The proper performance of structures in the interest of public safety is the engineering objective of structural code provisions. By this, all provisions aim at reducing the probability of inadequate performance to a somewhat negligible level by prescribing acceptable routine designs.

Generally, structural code provisions have evolved as a consequence of some reference to theoretical, experimental and judgmental activities. Undoubtedly, the aims of structural codes can be regarded as serving satisfactorily since structural failures and cases of inadequate performance have been very few. Nonetheless because of the inability to predict the real nature, it is possible that the code provisions lead to designs of varying safety levels. Further still since safety and economy are conflicting aims, the possibility that the existing code provisions lead to overly expensive structures cannot be ruled out.

The purpose of this Chapter is to assess the degree of conservatism or nonconservatism in the existing codes (i.e., AISI, 1968 and ECCS, 1987) with respect to the design of

torsionally stable thin-walled beam-columns considering a set of all such structural components to be designed and load combinations. Subsequently, the two design criteria are compared and within the jurisdiction of the development in reliability and optimization techniques, design formats which are optima (having minimum total expected cost) with the provision of consistent safety levels are derived.

6.2. Calibration Procedure

The maximization of the total expected utility of a structure, given a prescribed reference data set and availability of materials and labour is a sound structural engineering design objective. The realization of which depends on some matters which are not known with any degree of certainty but can only be predicted. An attempt to account for such imperfect knowledge led to structured codes. In this section, the design criteria for thin-walled structural steel beam-columns as specified by the AISI, (1968) and recommended by the ECCS, (1987) are calibrated to determine their implied reliability levels. To achieve this, certain essential steps are required and according to Melchers (1987), the Flow Chart in Fig. 6.1 whose details depend on individual need and computational choice can be adopted. This flexibility has also been demonstrated in the works of other researchers (e.g., Allen, 1975; Hawrenek and Rackwitz, 1976; Ellingwood et al., 1980).

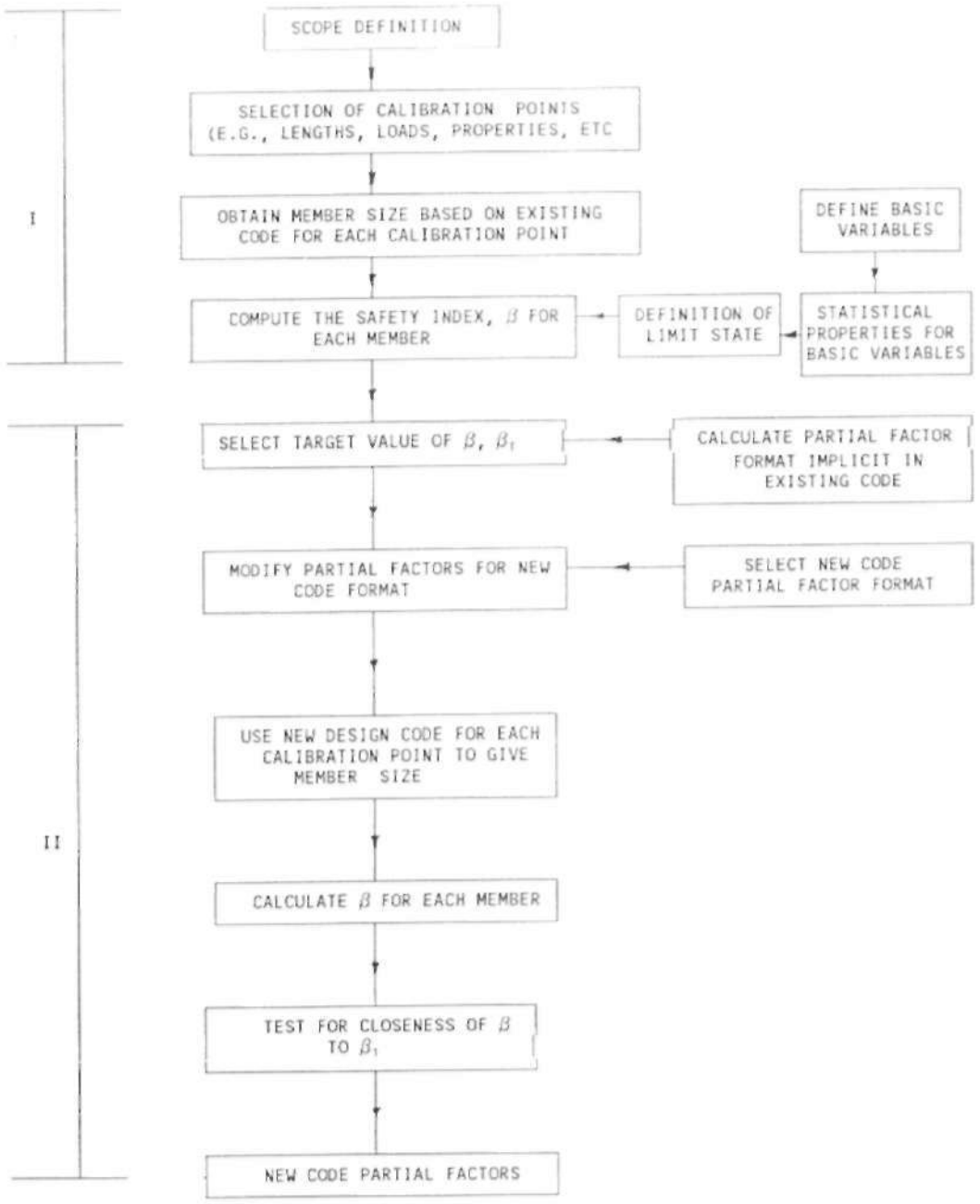


Fig. 6.1: Flow Chart for Calibration of Safety Checking Format

6.3 Calibration Model

For calibration purposes a beam-column under an axial load N applied at an eccentricity e_o and a lateral load Q uniformly distributed as shown in Fig. 6.2 is considered. It is assumed that the beam-column is laterally restrained and pinned at both ends. As discussed in Chapter 3, the mode of failure of such a member depends on its slenderness ratio, λ. The differences between experimental results and the design equations noted in Sec. 3.4 stimulate the need for probabilistic evaluation of the design criteria under investigation with the view to offer safe and economical design formats.

The safety of a beam-column will be considered under the influence of the randomness in the loading, and the material properties plus the uncertainties in axial load eccentricities. Generally when values for all the intervening variables are given, the safety and failure domains are separated by an interaction curve as schematically shown in Fig. 6.3. Thus, for instance, a limit state (load-carrying capacities N_{max} and Q_{max}) is reached at, say, point B, which satisfies the interaction equation. If proportional loading is considered, that is, a case in which N and Q vary along the path OA, a performance function G may be expressed as the difference between the lengths of the vectors OB and OA, in the form:

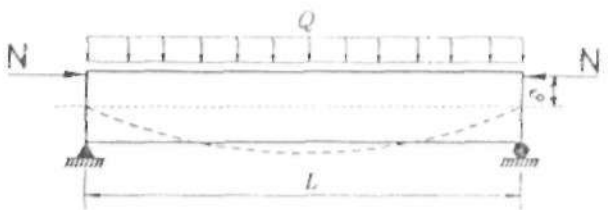
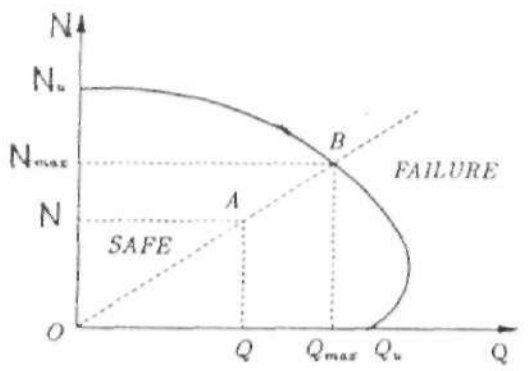


Fig. 6.2: Loaded Beam-Column



Figs. 6.3: Interaction Curve Used in Load Resistance Factor Design

$$G = [N_{max}^2 + Q_{max}^2] - [(N_D + N_L)^2 + (Q_D + Q_L)^2 L^2]$$

$$= R^2 - [(N_D + N_L)^2 + (Q_D + Q_L)^2 L^2] \qquad (6.1)$$

in which N_D and N_L refer, respectively, to the dead and live load components of the total axial load N; and Q_D and Q_L correspond to the dead and live load components of the lateral load Q and R is the beam-column strength.

In Eq. (6.1), the expression for R depends on the mode of failure in consideration. It is considered herein that a beam-column fails when the most stressed cross-section yields. However, for design purposes the requirement is of the form:

$$\frac{\alpha_D N_{DN} + \alpha_L N_{LN}}{P_U} + \psi \frac{[\alpha_D M_D(Q_{DN}, N_{DN}) + \alpha_L M_L(Q_{LN}, N_{LN})]}{M_U} = 1.0 \qquad (6.2)$$

where P_U and M_U are the expressions corresponding to the nominal strength of column and beam respectively, in the codes (i.e., AISI, 1968 and ECCS, 1987), ψ is the magnification parameter due to the presence of the axial load and the associated boundary conditions. The bending effects $M_D(Q_{DN}, N_{DN})$ and $M_L(Q_{LN}, N_{LN})$ are the unamplified maximum bending moments produced by the applied loads; and (N_{DN}, N_{LN}) and (Q_{DN}, Q_{LN}) are, respectively, the dead and live load components of the nominal

loads N_N and Q_N, with α_D and α_L denoting respectively the load factor applied to dead and live load components.

Now for a simply supported beam-column with uniformly distributed lateral load Q_{DN}, the induced bending moment due to this load and N_{DN} of the eccentrically applied axial load can be expressed as:

$$M_D(Q_{DN}, N_{DN}) = 0.125\,(Q_{DN}L^2 - 8N_{DN}e_n) \tag{6.3}$$

Let

$$\gamma_N = \frac{N_{DN}}{N_{LN}} \; ; \; \gamma_Q = \frac{Q_{DN}}{Q_{LN}} \tag{6.4}$$

then from Eq. (6.2) considering the smaller positive root of the resulting second degree equation in terms of N_{LN} the applied loads can be expressed in the form:

$$N = N_{LN}(\gamma_N d_N + l_N) \tag{6.5}$$

$$Q = N_{LN}(v/L)\,(\gamma_Q d_Q + l_Q) \tag{6.6}$$

where

$$d_N = N_D / N_{DN} \tag{6.7}$$

$$l_N = N_L / N_{LN} \tag{6.8}$$

$$d_Q = Q_D / Q_{DN} \tag{6.9}$$

$$l_Q = Q_L / Q_{LN} \tag{6.10}$$

$$\nu = L(Q_{LN} / N_{LN}) \tag{6.11}$$

and noting that ψ is defined according to the individual design equations. Employing Eq. (6.2) together with Eqs. (6.5) and (6.6), the performance function given by Eq. (6.1) is expressible in the form:

$$G = (\rho F_y)^2 - N_{LN}^2[(\gamma_N d_N + l_N)^2 + (\nu(\gamma_Q d_Q + l_Q))^2] \tag{6.12}$$

in which ρ is the product of ratio of the mean to the nominal yield stress and the gross cross-sectional area. For given values of ν, γ_N, γ_Q and the slenderness ratio, λ, Eq. (6.12) becomes the basis for the calibration of the current design equations in order to calculate their implied reliability levels.

6.3.1 Safety Levels of the Current Formulations

The reliability indices implicit in the Allowable Stress Design Specification of AISI (AISI, 1986) and the Ultimate

Carrying-Capacity of ECCS (ECCS, 1987) are computed based on the calibration mechanical model developed in the previous section. To this end, the Advanced First-Order Second Moment Reliability Analysis (i.e., FORM) discussed in Section 4.4.2.4 is adopted. Examining the performance function, G, of Eq. (6.12) and the related expressions from Eqs, (6.2) to (6.11) seven random variables are identified. According to the information available in the relevant literature (e.g., Corotis and Doshi, 1977; Chalk and Corotis, 1980; Foschi et al, 1989; Ravindra and Galambos, 1978; Galambos and Ravindra, 1978) data for the material strength and loading are collected with their distribution parameters in Table 6.1.

TABLE 6.1: STATISTICS OF THE BASIC RANDOM VARIABLES

Variable	Distribution	Mean	Coefficient of Variation
F_y	Lognormal	275kN/mm^2	0.05
E	Lognormal	203000kN/mm^2	0.02
e_o	Normal	varying	0.10
d_N	Normal	1.0	0.07
d_Q	Normal	1.0	0.07
l_N	Gumbel	1.0	0.25
l_Q	Gumbel	1.0	0.25

Since a beam-column action may be induced by eccentricity in loading or direct axial and lateral loading two cases are

considered for the determination of the safety indices, viz:

(a) axial load with varying eccentricity

(b) uniformly distributed lateral load plus eccentrically applied axial load.

In addition the parameters λ, γ_N and γ_Q are considered deterministic and varied arbitrarily within their practical and permissible ranges.

TABLE 6.2: SAFETY INDEX VALUES FOR ECCENTRIC AXIAL LOAD (AISI, 1968 EQUATION)

Length [m]	Load Ratio N_D/N_L	Eccentricity, e_o [mm]				
		50	100	150	200	250
1	0.25	4.859	4.677	4.486	4.286	4.074
	0.50	5.357	5.156	4.946	4.725	4.490
	1.00	6.234	6.002	5.757	5.498	5.222
2	0.25	4.894	4.704	4.504	4.291	4.064
	0.50	5.394	5.186	4.965	4.730	4.479
	1.00	6.278	6.036	5.778	5.503	5.207
3	0.25	4.951	4.747	4.528	4.291	4.031
	0.50	5.457	5.232	4.992	4.730	4.441
	1.00	6.350	6.089	5.808	5.500	5.158
4	0.25	5.030	4.802	4.550	4.263	3.916
	0.50	5.543	5.293	5.015	4.696	4.304
	1.00	6.449	6.158	5.833	5.455	4.969
5	0.25	5.130	4.861	4.997	-	-
	0.50	5.652	5.357	4.997	-	-
	1.00	6.573	6.231	5.804	-	-

TABLE 6.3: AISI SAFETY INDEX VALUES FOR ECCENTRIC AXIAL LOAD AND LATERAL UDL (N = 0.0)

Length [m]	γ_N	γ_Q	Eccentricity, e_o [mm]					
			0	50	100	150	200	250
3	0.25	0.25	5.143	4.952	4.751	4.538	4.311	4.065
	0.25	0.50	5.143	4.952	4.751	4.538	4.311	4.065
	0.25	1.00	5.143	4.952	4.751	4.538	4.311	4.065
	0.50	0.25	5.668	5.459	5.238	5.005	4.755	4.486
	0.50	0.50	5.668	5.459	5.238	5.005	4.755	4.486
	0.50	1.00	5.668	5.459	5.238	5.005	4.755	4.486
	1.00	0.25	6.594	6.353	6.098	5.829	5.540	5.229
	1.00	0.50	6.594	6.353	6.098	5.829	5.540	5.229
	1.00	1.00	6.594	6.353	6.098	5.829	5.540	5.229
5	0.25	0.25	5.374	5.136	4.870	4.562	4.174	-
	0.25	0.50	5.374	5.136	4.870	4.562	4.174	-
	0.25	1.00	5.374	5.136	4.870	4.562	4.174	-
	0.50	0.25	5.920	5.660	5.368	5.030	4.605	-
	0.50	0.50	5.920	5.660	5.368	5.030	4.605	-
	0.50	1.00	5.920	5.660	5.368	5.030	4.605	-
	1.00	0.25	6.885	6.584	6.248	5.858	5.365	-
	1.00	0.50	6.885	6.584	6.248	5.858	5.365	-
	1.00	1.00	6.885	6.584	6.248	5.858	5.365	-

TABLE 6.4: AISI SAFETY INDEX VALUES FOR ECCENTRIC AXIAL LOAD AND LATERAL UDL (N = 0.5)

Length [m]	γ_N	γ_Q	Eccentricity, e_o [mm]					
			0	50	100	150	200	250
3	0.25	0.25	5.758	5.592	5.419	5.239	5.051	4.854
	0.25	0.50	5.863	5.700	5.531	5.355	5.172	4.979
	0.25	1.00	6.059	5.901	5.738	5.569	5.393	5.209
	0.50	0.25	6.240	6.054	5.862	5.660	5.449	5.227
	0.50	0.50	6.340	6.158	5.968	5.771	5.565	5.348
	0.50	1.00	6.528	6.351	6.168	5.978	5.780	5.573
	1.00	0.25	7.102	6.883	6.654	6.414	6.162	5.896
	1.00	0.50	7.193	6.977	6.752	6.517	6.270	6.009
	1.00	1.00	7.367	7.157	6.938	6.711	6.473	6.222
5	0.25	0.25	6.515	6.351	6.179	6.000	5.810	5.608
	0.25	0.50	6.700	6.543	6.379	6.209	6.030	5.842
	0.25	1.00	7.043	6.897	6.746	6.590	6.428	6.259
	0.50	0.25	6.991	6.803	6.607	6.399	6.179	5.941
	0.50	0.50	7.168	6.988	6.800	6.603	6.396	6.174
	0.50	1.00	7.499	7.331	7.157	6.977	6.788	6.590
	1.00	0.25	7.847	7.618	7.375	7.117	6.838	6.533
	1.00	0.50	8.011	7.558	7.558	7.312	7.048	6.764
	1.00	1.00	8.321	7.898	7.898	7.670	7.431	7.176

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TABLE 6.5: AISI SAFETY INDEX VALUES FOR ECCENTRIC AXIAL LOAD AND LATERAL UDL (N = 1.0)

Length [m]	γ_N	γ_Q	Eccentricity, e_o [mm]					
			0	50	100	150	200	250
3	0.25	0.25	6.926	6.738	6.545	6.345	6.137	5.922
	0.25	0.50	6.250	6.099	5.942	5.781	5.613	5.440
	0.25	1.00	6.244	6.094	5.940	5.781	5.618	5.448
	0.50	0.25	6.711	6.539	6.361	6.177	5.986	5.787
	0.50	0.50	7.634	7.429	7.217	6.998	6.770	6.534
	0.50	1.00	6.872	6.706	6.536	6.360	6.178	5.989
	1.00	0.25	7.537	7.332	7.120	6.899	6.668	6.427
	1.00	0.50	7.693	7.492	7.284	7.069	6.844	6.610
	1.00	1.00	8.885	8.648	8.403	8.150	7.887	7.614
5	0.25	0.25	8.290	8.122	7.949	7.772	7.588	7.398
	0.25	0.50	7.507	7.375	7.240	7.101	6.958	6.811
	0.25	1.00	7.766	7.642	7.515	7.386	7.254	7.118
	0.50	0.25	7.810	7.652	7.489	7.321	7.146	6.964
	0.50	0.50	9.126	8.924	8.754	8.559	8.358	8.150
	0.50	1.00	8.347	8.206	8.061	7.913	7.761	7.604
	1.00	0.25	8.615	8.419	8.215	8.003	7.780	7.547
	1.00	0.50	8.886	8.699	8.506	8.095	8.096	7.878
	1.00	1.00	-	-	10.173	9.949	9.717	9.477

TABLE 6.6: AISI SAFETY INDEX VALUES FOR ECCENTRIC AXIAL LOAD AND LATERAL UDL (N = 1.5)

Length [m]	γ_S	γ_Q	Eccentricity, e_o [mm]					
			0	50	100	150	200	250
3	0.25	0.25	5.040	4.917	4.791	4.662	4.528	4.389
	0.25	0.50	5.036	4.914	4.790	4.661	4.529	4.392
	0.25	1.00	5.017	4.896	4.773	4.647	4.516	4.382
	0.50	0.25	5.552	5.417	5.277	5.133	4.984	4.829
	0.50	0.50	5.551	5.417	5.279	5.136	4.989	4.837
	0.50	1.00	5.538	5.405	5.269	5.129	4.985	4.835
	1.00	0.25	6.457	6.298	6.134	5.964	5.787	5.603
	1.00	0.50	6.457	6.302	6.139	5.971	5.797	5.617
	1.00	1.00	6.457	6.299	6.139	5.974	5.804	5.628
5	0.25	0.25	6.232	6.129	6.024	5.917	5.807	5.694
	0.25	0.50	6.384	6.285	6.185	6.082	5.977	5.870
	0.25	1.00	6.661	6.568	6.474	6.378	6.281	6.181
	0.50	0.25	6.709	6.592	6.472	6.349	6.222	6.092
	0.50	0.50	6.857	6.744	6.629	6.511	6.391	6.267
	0.50	1.00	7.125	7.020	6.912	6.802	6.691	6.576
	1.00	0.25	7.563	7.420	7.272	7.119	6.961	6.798
	1.00	0.50	7.705	7.566	7.424	7.278	7.128	6.973
	1.00	1.00	7.961	7.831	7.699	7.567	7.423	7.287

TABLE 6.7: AISI SAFETY INDEX VALUES FOR ECCENTRIC AXIAL LOAD AND LATERAL UDL (N = 2.0)

Length [m]	γ_N	γ_Q	Eccentricity, e_o [mm]					
			0	50	100	150	200	250
3	0.25	0.25	4.312	4.207	4.700	3.990	3.876	3.756
	0.25	0.50	4.303	4.199	4.093	3.983	3.871	3.755
	0.25	1.00	4.273	4.470	4.064	3.956	3.844	3.730
	0.50	0.25	4.759	4.643	4.525	4.402	4.276	4.146
	0.50	0.50	4.754	4.639	4.521	4.400	4.275	4.147
	0.50	1.00	4.731	4.617	4.501	4.321	4.258	4.131
	1.00	0.25	5.548	5.413	5.273	5.128	4.979	4.832
	1.00	0.50	5.547	5.413	5.274	5.132	4.985	4.824
	1.00	1.00	5.534	5.402	5.265	5.125	4.980	4.831
5	0.25	0.25	5.567	5.483	5.398	5.311	5.223	5.133
	0.25	0.50	5.726	5.646	5.564	5.482	5.397	5.311
	0.25	1.00	6.016	5.941	5.865	5.788	5.710	5.630
	0.50	0.25	5.974	5.879	5.782	5.682	5.581	5.477
	0.50	0.50	6.129	6.038	5.944	5.849	5.753	5.654
	0.50	1.00	6.411	6.325	6.238	6.149	6.059	5.968
	1.00	0.25	6.706	6.589	6.469	6.346	6.219	6.089
	1.00	0.50	6.854	6.742	6.627	6.509	6.388	6.264
	1.00	1.00	7.123	7.018	6.910	6.800	6.688	6.744

TABLE 6.8: SAFETY INDEX VALUES FOR ECCENTRIC AXIAL LOAD (ECCS, 1987 EQUATION)

Length [m]	Load Ratio N_D/N_L	Eccentricity, e_o [mm]				
		50	100	150	200	250
1	0.25	3.753	3.549	3.332	3.100	2.85
	0.50	4.066	3.549	3.603	3.346	3.067
	1.00	4.628	4.367	4.088	3.785	3.456
2	0.25	3.89	3.681	3.458	3.217	2.953
	0.50	4.215	3.986	3.740	3.473	3.180
	1.00	4.799	4.533	4.246	3.933	3.585
3	0.25	4.140	3.927	3.698	3.445	3.163
	0.50	4.488	4.255	4.002	3.724	3.411
	1.00	5.112	4.842	4.547	4.221	3.850
4	0.25	4.410	4.191	3.950	3.680	3.364
	0.50	4.781	4.542	4.277	3.9679	3.629
	1.00	5.445	5.168	4.861	4.512	4.096
5	0.25	4.710	4.486	4.234	3.942	3.576
	0.50	5.105	4.860	4.584	4.262	3.854
	1.00	5.810	5.527	5.207	4.830	4.341

TABLE 6.9: ECCS SAFETY INDEX VALUES FOR ECCENTRIC AXIAL LOAD AND LATERAL UDL (N = 0.0)

Length [m]	γ_S	γ_Q	Eccentricity, e_o [mm]					
			0	50	100	150	200	250
3	0.25	0.25	4.337	4.140	3.927	3.689	3.445	3.163
	0.25	0.50	4.325	4.140	3.927	3.689	3.445	3.163
	0.25	1.00	4.325	4.140	3.927	3.689	3.445	3.163
	0.50	0.25	4.678	4.488	4.255	4.092	3.724	3.411
	0.50	0.50	4.688	4.488	4.255	4.092	3.724	3.411
	0.50	1.00	4.688	4.488	4.255	4.092	3.724	3.411
	1.00	0.25	5.347	5.112	4.547	4.547	4.221	3.850
	1.00	0.50	5.360	5.112	4.547	4.547	4.221	3.850
	1.00	1.00	5.322	5.112	4.842	4.547	4.221	3.850
5	0.25	0.25	4.914	4.710	4.486	4.234	3.942	3.576
	0.25	0.50	4.891	4.710	4.710	4.710	4.710	3.576
	0.25	1.00	4.891	4.710	4.710	4.710	4.710	3.576
	0.50	0.25	5.298	5.105	4.860	4.710	4.262	3.854
	0.50	0.50	5.298	5.105	4.860	4.584	4.262	3.854
	0.50	1.00	5.298	5.105	4.860	4.584	4.262	3.854
	1.00	0.25	6.020	5.810	5.527	4.584	4.830	4.341
	1.00	0.50	6.020	5.810	5.527	5.707	4.830	4.341
	1.00	1.00	6.020	5.810	5.527	5.207	4.830	4.341

TABLE 6.10: ECCS SAFETY INDEX VALUES FOR ECCENTRIC AXIAL LOAD AND LATERAL UDL (N = 0.5)

Length [m]	γ_N	γ_Q	Eccentricity, e_o [mm]					
			0	50	100	150	200	250
3	0.25	0.25	4.990	4.826	4.631	4.424	4.202	3.962
	0.25	0.50	5.086	4.925	4.532	4.532	4.315	4.082
	0.25	1.00	5.263	5.107	4.726	4.726	4.518	4.295
	0.50	0.25	5.314	5.136	4.685	4.685	4.434	4.161
	0.50	0.50	5.408	5.233	4.792	4.792	4.547	4.281
	0.50	1.00	5.582	5.412	4.985	4.985	4.751	4.497
	1.00	0.25	5.899	5.697	5.157	5.157	4.852	4.513
	1.00	0.50	5.988	5.790	5.260	5.260	4.961	4.633
	1.00	1.00	6.155	5.962	5.715	5.450	5.163	4.850
5	0.25	0.25	6,017	5.857	5.685	5.504	5.311	5.103
	0.25	0.50	6.161	6.022	5.857	5.683	5.499	5.303
	0.25	1.00	6.476	6.328	6.174	6.012	5.842	5.662
	0.50	0.25	6.349	6.191	5.996	5.789	5.567	5.325
	0.50	0.50	6.504	6.351	6.163	5.965	5.753	5.524
	0.50	1.00	6.794	6.649	5.965	6.288	6.092	5.883
	1.00	0.25	6.977	6.796	6.288	6.303	6.024	5.714
	1.00	0.50	7.123	6.947	6.303	6.472	6.205	5.912
	1.00	1.00	7.411	7.231	6.6.78	6.785	6.538	6.271

TABLE 6.11: ECCS SAFETY INDEX VALUES FOR ECCENTRIC AXIAL LOAD AND LATERAL UDL (N = 1.0)

Length [m]	γ_N	γ_Q	Eccentricity, e_o [mm]					
			0	50	100	150	200	250
3	0.25	0.25	5.999	5.824	5.607	5.379	5.137	4.878
	0.25	0.50	5.468	5.319	5.143	4.958	4.762	4.555
	0.25	1.00	5.391	5.242	5.067	4.883	4.690	4.485
	0.50	0.25	5.814	5.651	5.453	5.244	5.021	4.782
	0.50	0.50	6.497	6.312	6.074	5.822	5.554	5.268
	0.50	1.00	5.890	5.731	5.538	5.335	5.120	4.892
	1.00	0.25	6.376	6.190	5.953	5.700	5.428	5.133
	1.00	0.50	6.518	6.336	6.103	5.856	5.590	5.303
	1.00	1.00	7.385	7.185	6.909	6.615	6.300	5.961
5	0.25	0.25	7.638	7.473	7.297	7.113	6.920	6.718
	0.25	0.50	6.931	6.812	6.673	6.529	6.379	6.222
	0.25	1.00	7.103	6.989	6.857	6.720	6.578	6.432
	0.50	0.25	7.160	7.025	6.861	6.690	6.511	6.321
	0.50	0.50	8.250	87.099	7.907	7.705	7.493	7.270
	0.50	1.00	7.577	7.452	7.302	7.1477	6.986	6.819
	00	0.25	7.746	7.588	7.387	7.173	6.947	6.705
	1.00	0.50	7.984	7.832	7.639	7.435	7.220	6.991
	1.00	1.00	9.375	9.214	8.991	8.757	8.510	8.248

TABLE 6.12: ECCS SAFETY INDEX VALUES FOR ECCENTRIC AXIAL LOAD AND LATERAL UDL (N = 1.5)

Length [m]	γ_N	γ_Q	Eccentricity, e_o [mm]					
			0	50	100	150	200	250
3	0.25	0.25	4.422	4.298	4.158	4.013	3.860	3.700
	0.25	0.50	4.375	4.257	4.111	3.966	3.814	3.654
	0.25	1.00	4.267	4.141	4.001	3.854	3.701	3.541
	0.50	0.25	4.827	4.692	4.538	4.377	4.207	4.027
	0.50	0.50	4.786	4.651	4.498	4.337	4.169	3.991
	0.50	1.00	4.692	4.557	4.403	4.242	4.074	3.897
	1.00	0.25	5.543	5.391	5.210	5.019	4.816	4.600
	1.00	0.50	5.512	5.360	5.180	4.991	4.791	4.577
	1.00	1.00	5.437	5.286	5.107	4.920	4.722	4.512
5	0.25	0.25	5.802	5.699	5.592	5.482	5.368	5.251
	0.25	0.50	5.896	5.802	5.699	5.592	5.482	5.369
	0.25	1.00	6.078	5.988	5.889	5.788	5.684	5.578
	0.50	0.25	6.185	6.079	5.958	5.833	5.704	5.570
	0.50	0.50	6.284	6.180	6.063	5.942	5.817	5.688
	0.50	1.00	6.462	6.363	6.251	6.137	6.019	5.898
	1.00	0.25	6.886	6.761	6.615	6.462	6.303	6.136
	1.00	0.50	6.981	6.860	6.718	6.570	6.417	6.257
	1.00	1.00	7.154	7.038	6.903	6.763	6.619	6.469

TABLE 6.13: ECCS SAFETY INDEX VALUES FOR ECCENTRIC AXIAL LOAD AND LATERAL UDL ($\bar{N}$ = 2.0)

Length [m]	γ_N	γ_Q	Eccentricity, e_o [mm]					
			0	50	100	150	200	250
3	0.25	0.25	3.766	3.658	3.658	3.416	3.288	3.154
	0.25	0.50	3.706	3.598	3.479	3.355	3.226	3.092
	0.25	1.00	3.572	3.460	3.339	3.213	3.081	2.944
	0.50	0.25	4.123	4.005	3.875	3.739	3.597	3.448
	0.50	0.50	4.071	3.953	3.822	3.686	3.544	3.396
	0.50	1.00	3.952	3.833	3.700	3.563	3.419	3.448
	1.00	0.25	4.754	4.620	4.467	4.307	4.139	3.396
	1.00	0.50	4.713	4.579	4.427	4.268	4.101	3.269
	1.00	1.00	4.618	4.483	4.331	4.172	4.005	3.961
5	0.25	0.25	5.185	5.101	5.014	4.925	4.834	4.740
	0.25	0.50	5.286	5.208	5.123	5.037	4.949	4.859
	0.25	1.00	5.475	5.400	5.320	5.238	5.155	5.070
	0.50	0.25	5.516	5.426	5.328	5.228	5.124	5.017
	0.50	0.50	5.618	5.531	5.436	5.339	5.239	5.136
	0.50	1.00	5.803	5.720	5.630	5.538	5.443	5.346
	1.00	0.25	6.120	6.014	5.895	5.772	5.645	5.513
	1.00	0.50	6.219	6.116	6.001	5.882	5.759	5.632
	1.00	1.00	6.399	6.301	6.191	6.078	5.962	5.843

Tables 6.2 to 6.7 show the allowable safety levels implicit in the AISI Specification of 1968. In this case, for the load combination, the dead and the live loads are unfactored as characterized by the ASD philosophy.

A general inconsistency in safety level is observed for the design equation specified by AISI (1968) as far as beam-column members are concerned. This point is established when consideration is given to the information in Tables 6.2 to 6.7. Whatever the cause of the beam-column action, the equation cannot give uniform reliability level. For clarity, Figs. 6.4 and 6.5 are plotted to show the variation of the implied allowable reliability level associated with AISI Specification. If the adjustment and safety factors empirically incorporated are adequately considered, there should have been a relatively uniform safety level for varying sets of data points. This is not the case however, and hence, a reliability-based modification as conjectured is necessary.

Similarly Tables 6.8 to 6.13 contain the information on the safety levels associated with the recommendation of ECCS, (1987). Unlike the unfactored load combination used for AISI, the values of α_D and α_L are taken as in Section R 1.3.8 of the Recommendation.

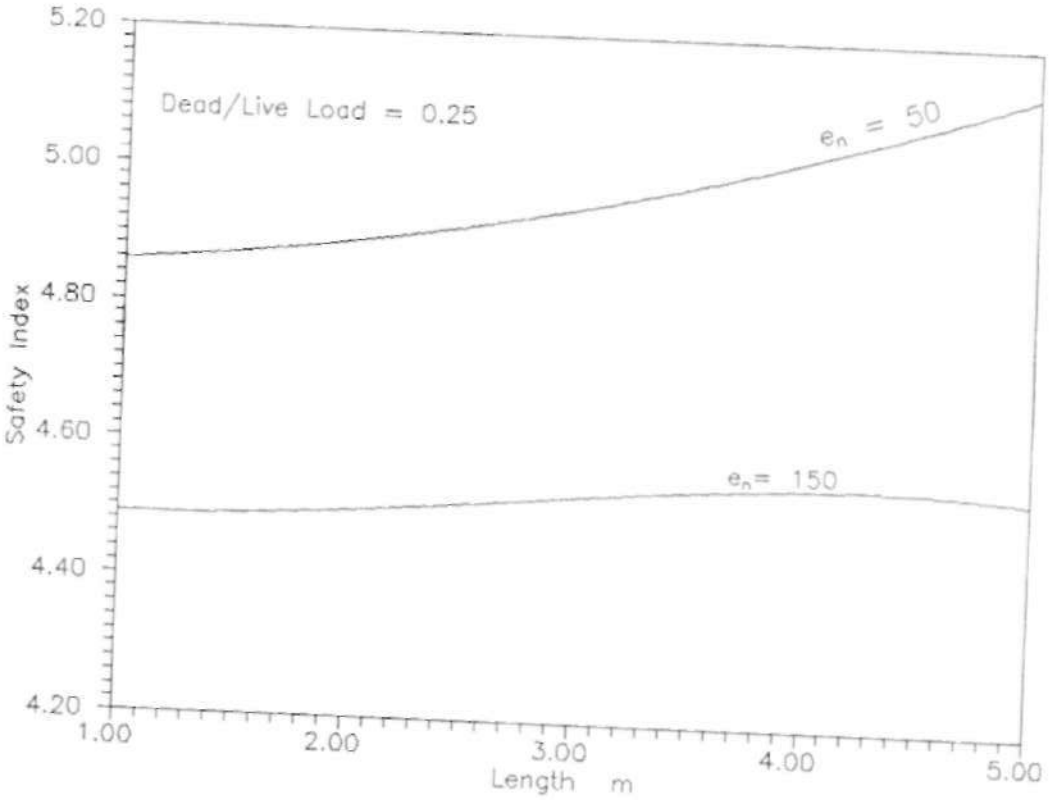


FIG. 6.4: Allowable Safety Index for Varying Member Length.

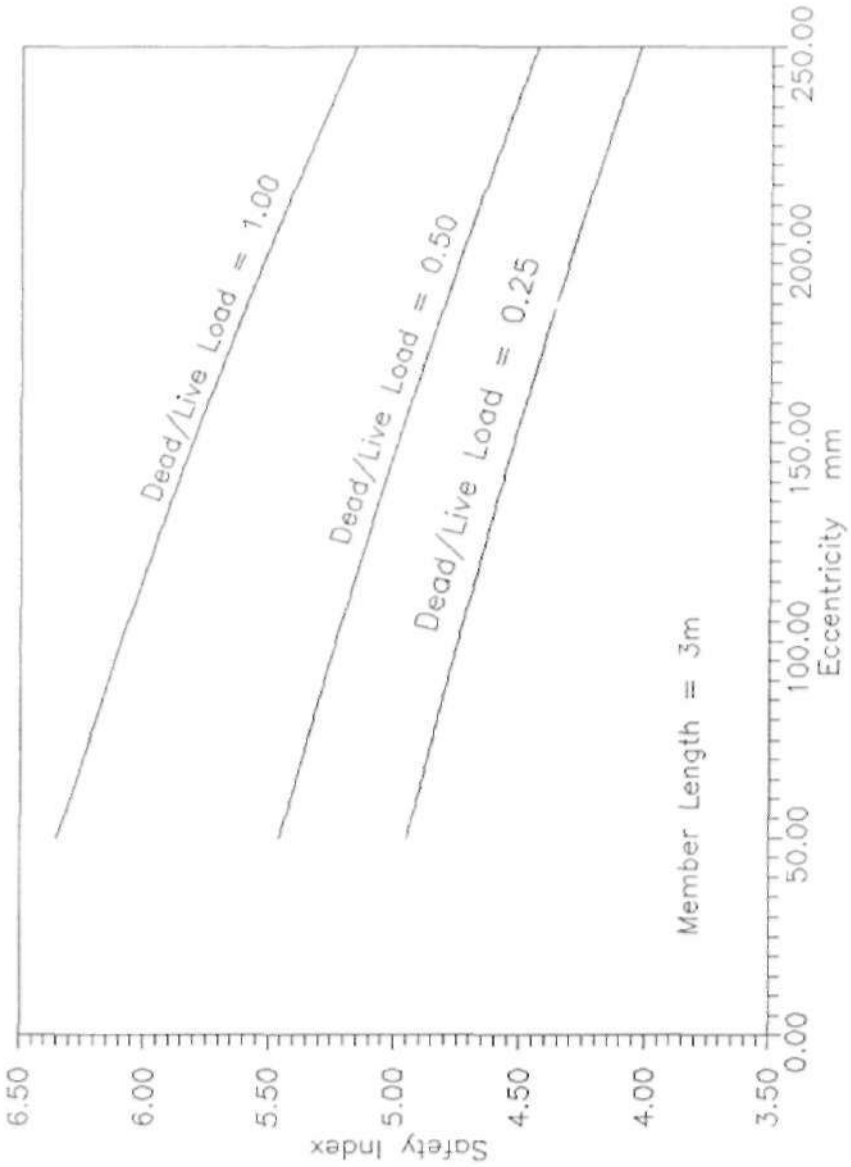


FIG. 6.5: Allowable Safety Index for Varying Eccentricity.

As noted already for the AISI Specification, there is a similar trend in the implied reliability level of the design equation recommended by ECCS. Tables 6.8 to 6.13 give attestation to this claim. Similarly, typical plots are shown in Figs. 6.6 and 6.7. With regards to member length and/or load ratio, the sensitivity of the equation can be read from these figures. The conclusion remains that the recommended design equation is not capable of yielding uniform safety level for varying practical data points.

Allowing for comparison, typical plots of the variation of the safety index, β, with the parameters of loading and strength are shown in Figs 6.8 and 6.9. In these plots the reliability levels implicit in AISI and ECCS design equations are compared. Although both equations are meant to design similar element, their reliability levels are not the same. Figure 6.8 shows that the safety level decreases with increasing eccentricity at varying ratio of total lateral to axial load. This variation also shows a notable change from column to beam behaviour. Depending on whether a beam-column acts more as a beam or a column than as a beam-column, the implied risk (or safety) can be read from Fig. 6.8. It is however safer to load a beam-column in such a way that it acts more as a column than as a beam. This decision is established using the ratio of applied lateral to axial loads.

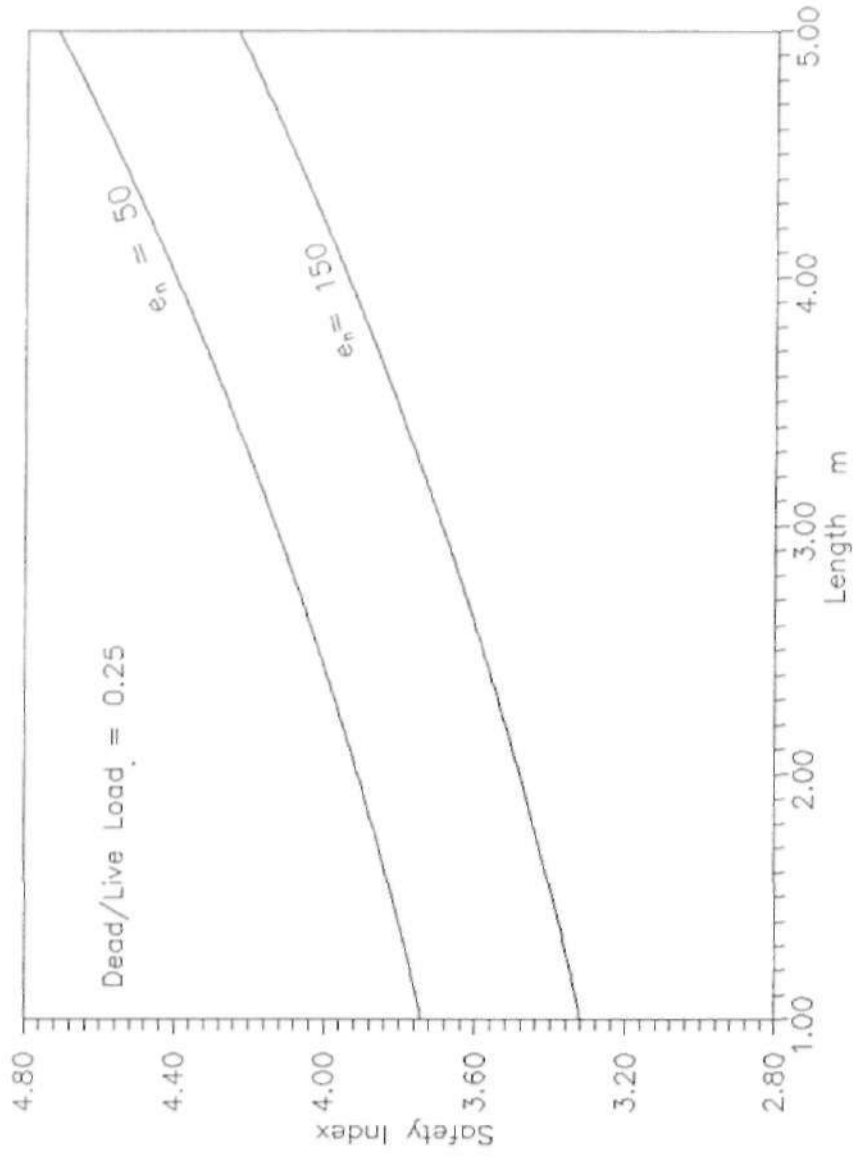


FIG. 6.6: Ultimate Safety Index for Varying Member Length.

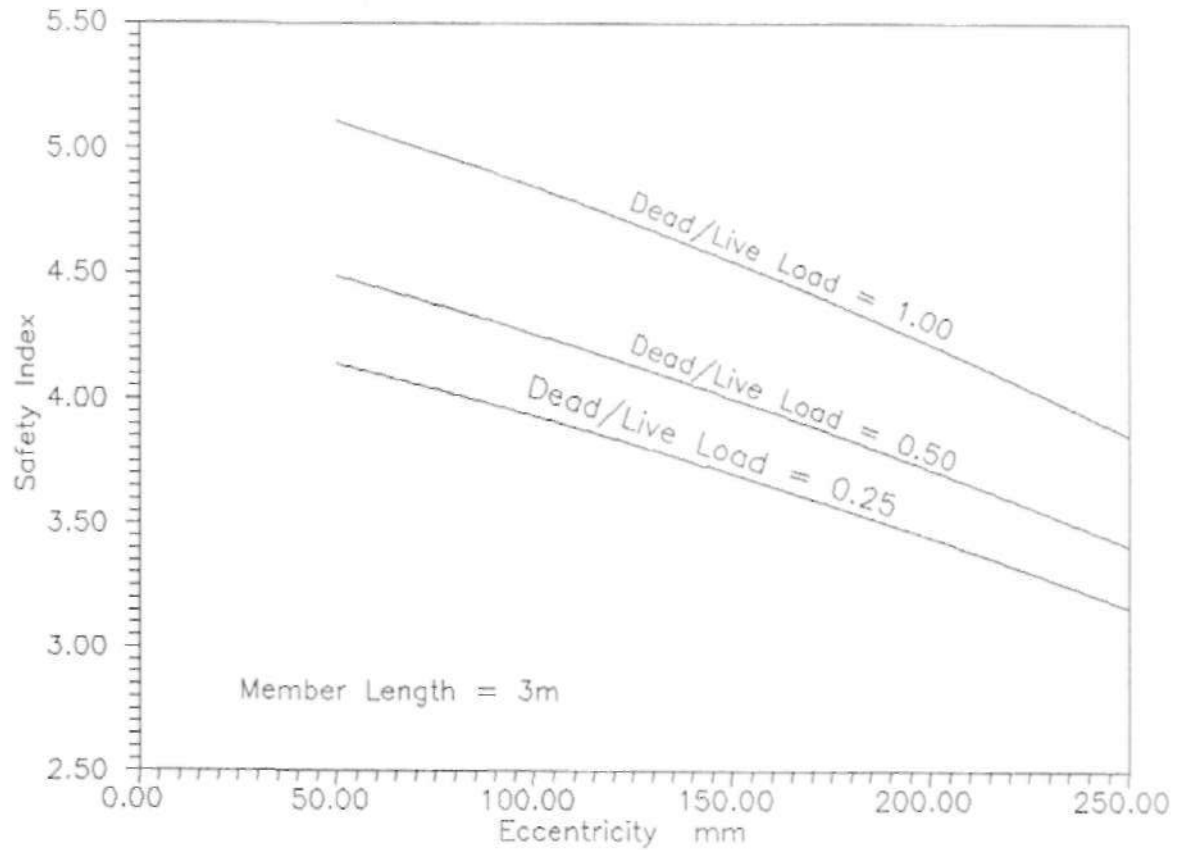


FIG. 6.7: Ultimate Safety Index for Varying Eccentricity.

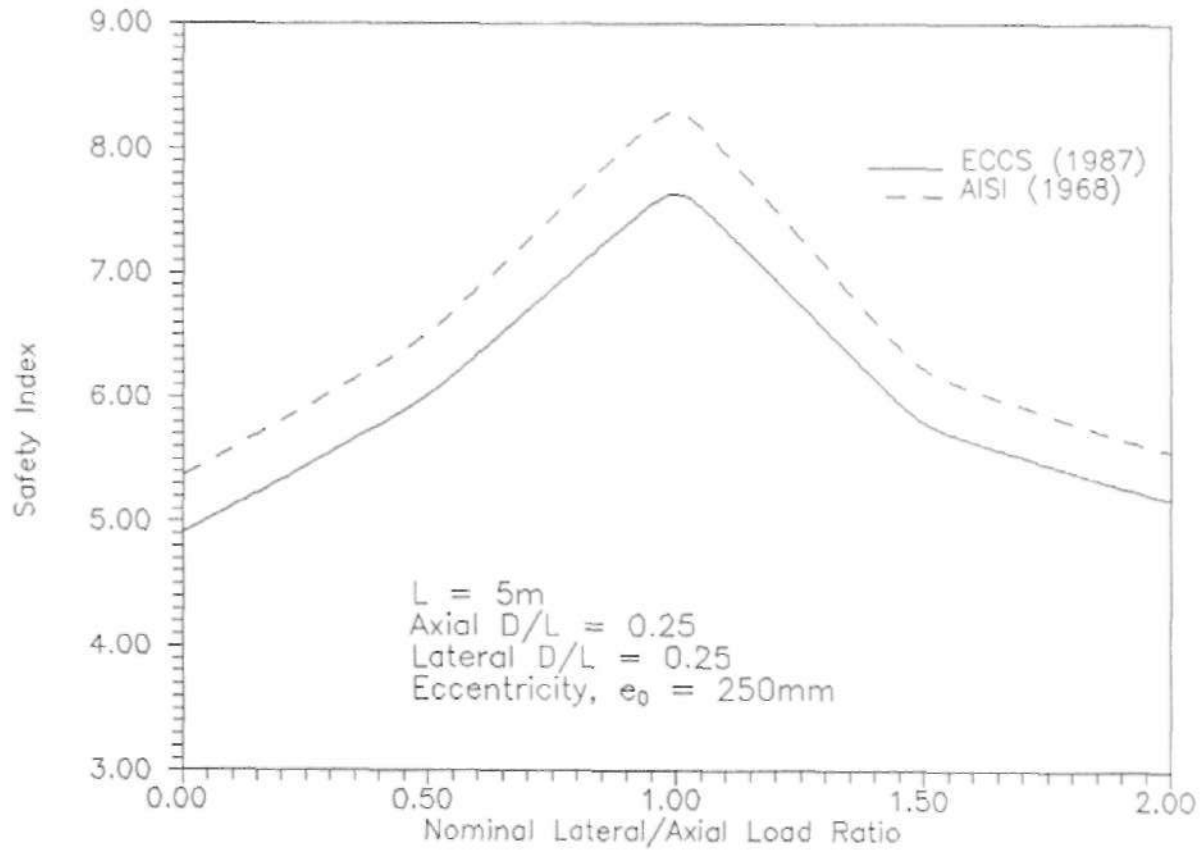


FIG. 6.8: Variation of Safety Indices with the Ratio of Total Lateral to Axial Load.

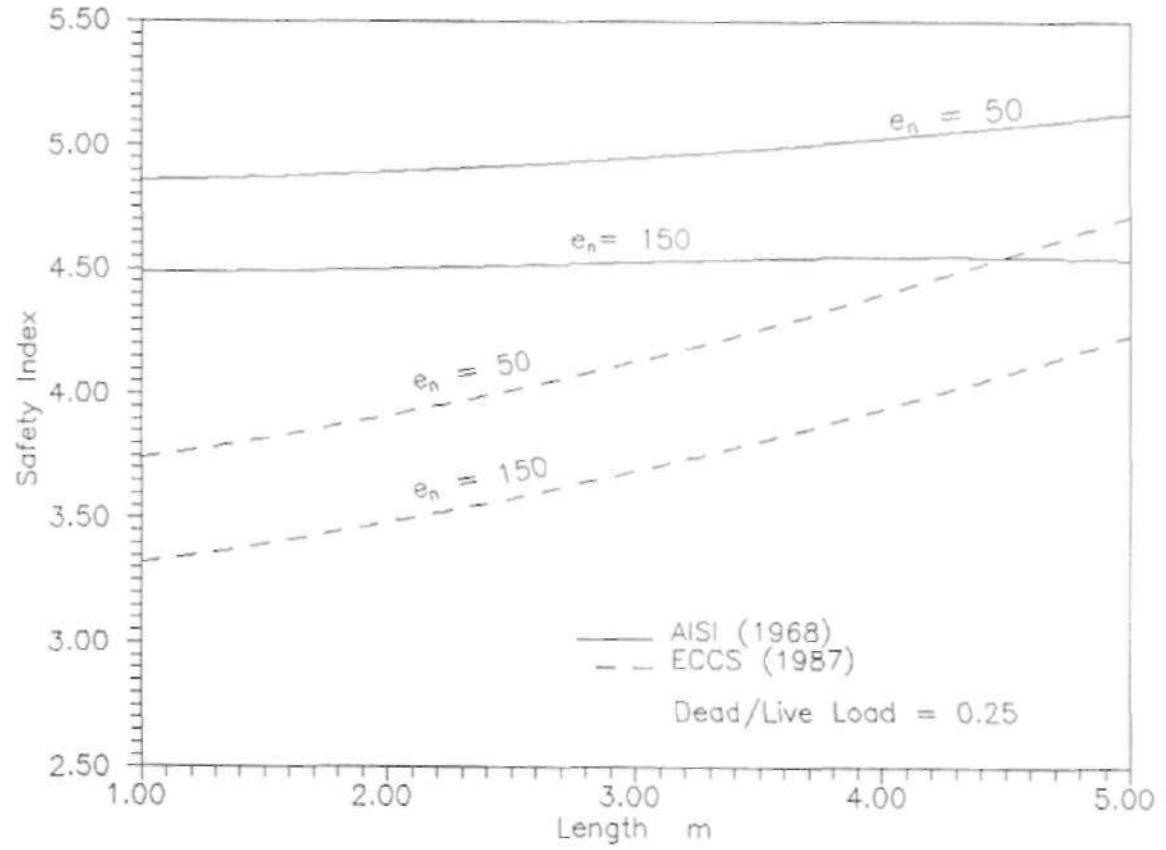


FIG. 6.9: Comparison of Implied Safety Indices for Varying Member Length.

Due to the wide range of safety levels exhibited by both design methods (AISI, 1968 and ECCS, 1987), statistical information on this range are depicted by the histograms plotted in Figs. 6.10 and 6.11. From these histograms, it follows that the distributions of the implied safety levels of both design equations assume gaussian with definite dispersions around the means. This observation again reflects one of the inherent drawbacks in the equations predicting the strength of torsionally stable thin-walled beam-columns.

6.3.2 Sensitivity of the Model Parameters

The influence of gross errors (human errors) are not quantified in this study. But in reality humans play an important role during design, production and operation phases of a system. The degree of this role varies from one system to another and from one system phase to another. Since most of the human errors occur because humans are capable of doing so many different things in many diverse ways (Rigby, 1970), the overall reliability of a system becomes affected (Flint and Smith, 1980). Although not all human errors necessarily cause system failures, some of them cause more significant failures than others (Dhillon, 1986). Besides, for decision-making processes and most especially in more complex systems, it is important to understand the implications of the value set for any design variable before usage (Rowe, 1977). This is because real values of design variables of an erected structure may

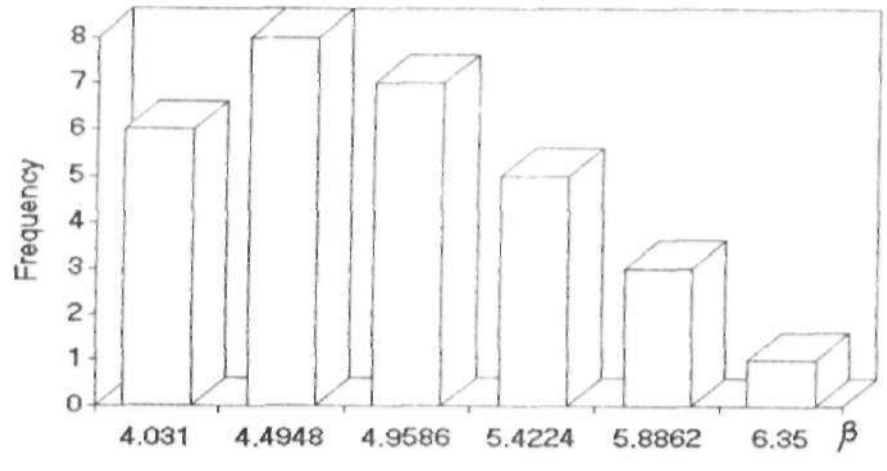


Fig. 6.10: Distribution of AISI Safety Levels

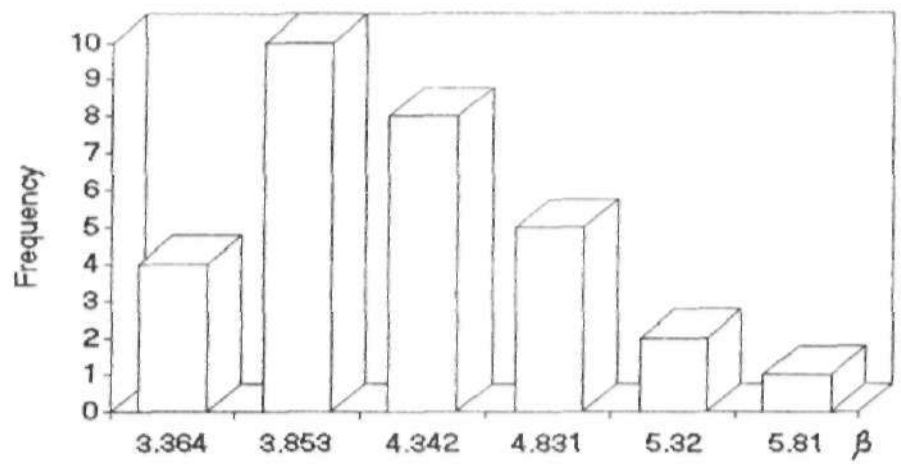


Fig. 6.11: Distribution of ECCS Safety Levels

differ by small values from the assumed values (Gutkowski, 1991).

Before the selection of the target safety level for each design method, the sensitivity of the calibration model to gross errors in the assignment of the values of the componential parameters is measured by the changes in the value of β. That is, a parameteric sensitivity analysis will be conducted. Each parameter of the design variables will be varied by a fixed amount while others are held constant and the result (i.e the value of β) is computed for each variation. Thus, the parameters which change the output β are noted as the most critical at the variation range used and their relative importance in the design process is established. For this purpose, datum values of the design variables involved are: Yield Stress, F_y = 275 N/mm^2, Young's Modulus of Elasticity, E = 203 KN/mm^2, thickness of member, t = 2.67mm and width of box section, w = 500mm. For these values, the corresponding reliability indices for AISI and ECCS are denoted by β_o. The results are presented in Figs 6.12 and 6.13. For both design methods, the most influential design variable is the thickness of the plate. This variable significantly affects the safety index in both positive and negative directions. The influence is more for ECCS than AISI. For example when its value is increased by 50%, this trend is about 22%. It is also coincidental that this parameter characterizes the notable

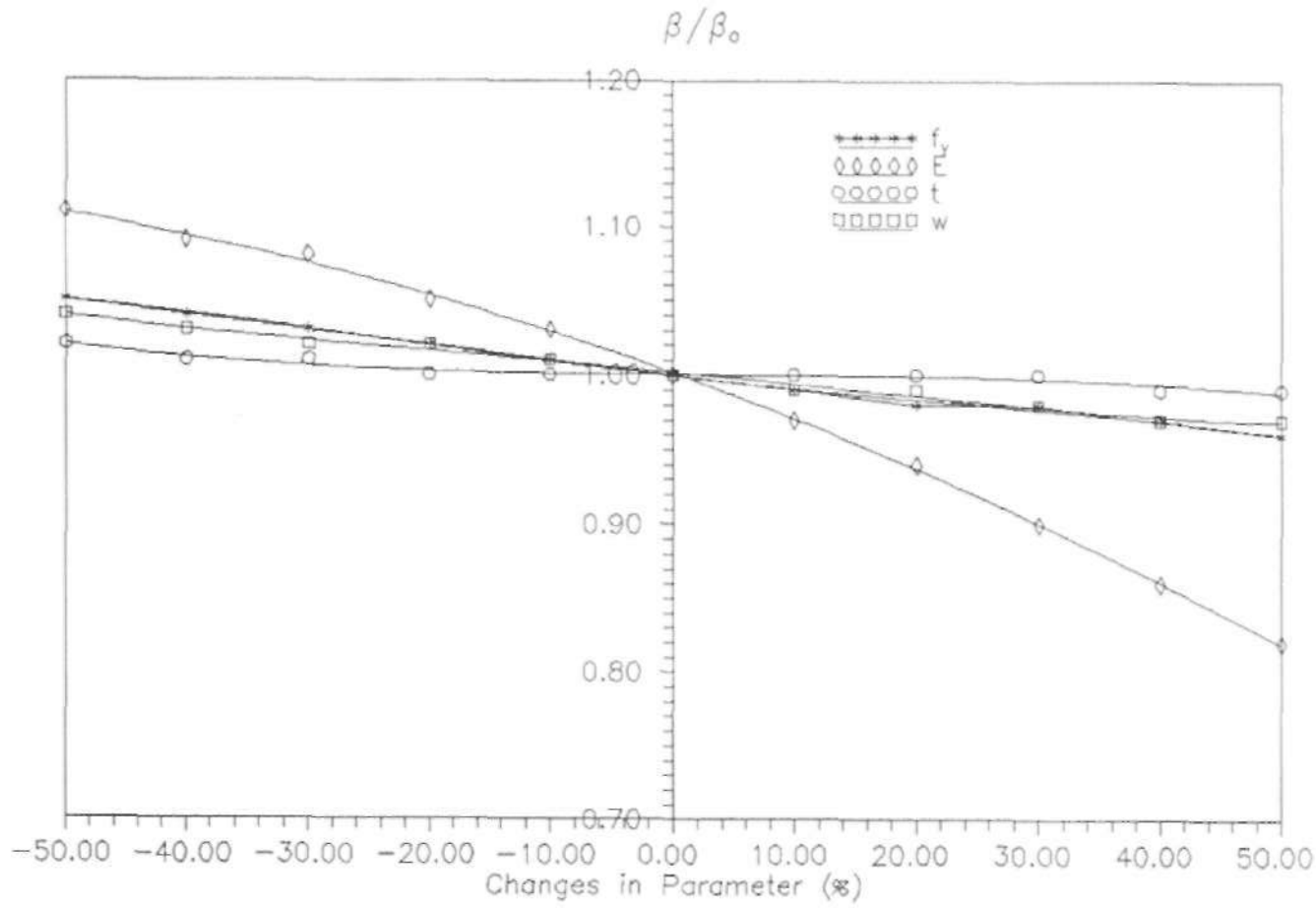


FIG. 6.12: Sensitivity of AISI Parameters to Gross Errors

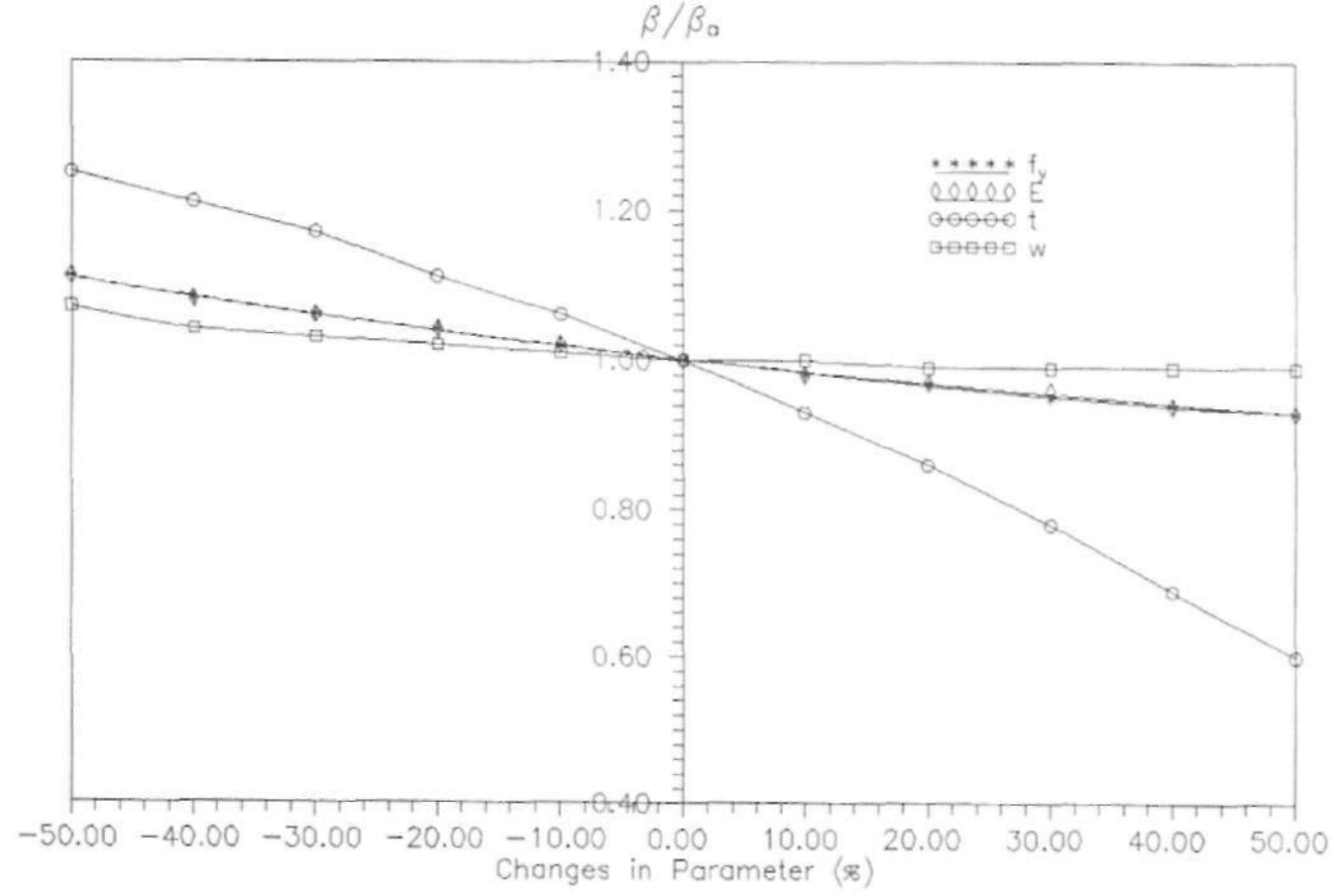


FIG. 6.13: Sensitivity of ECCS Parameters to Gross Errors

difference between the behaviour of hot-rolled and cold-formed steel members, thus, necessitating new design criteria. Other parameters show approximate minor linear effect on the safety of beam-column. It must be emphasized that these observations are strictly limited to torsionally stable thin-walled beam-column members.

6.3.3. Selection of the Target Safety Levels.

The safety levels implicit in the current codes under consideration have been determined in the previous section. The problem of nonuniformity in safety level is clearly evident. It is of more interest to establish an acceptable level upon which a new code can be developed to achieve the desired consistency. The level can be determined as a value agreed upon by the profession (Galambos, 1972). However, some criteria for making this decision are necessary and for complete information on this the work of Rowe (1977) can be used. As found appropriate to the intent of this investigation the decision based on socio-economic criterion will be followed.

According to Rosenblueth (1976), a structural code should lead to the establishment of safety levels that produce minimum total expected cost of an individual structure on behalf of the client. If failure consequences are to be accounted for, an acceptable optimum failure probability for the current codes can be prescribed using cost-benefit analysis. Consequently,

the problem lends itself to optimization requirements. However, should the entire spectrum of all possibilities be considered, such a multicriteria optimization becomes a complex problem.

Within the framework of probabilistic structural design coupled with realistic assessment of the risks and potential consequences of partial and total failures and the selection of decision rules compatible with the principles of the economics of uncertainty as adopted herein the basic equation for structural code optimization can then be expressed in a form similar to the work of Kupfer and Freudenthal, (1977). That is,

$$C_{Tj} = C_{Ij} + (1 - P_{Tj}) C_{Fj} P_{Fj} + P_{Tj} C_{Uj} \qquad (6.13)$$

in which

C_{Tj} = the expected total cost

C_{Ij} = the non-failure cost

P_{Tj} = total failure probability

C_{Uj} = the anticipated cost of failure of structure j itself which encompasses the cost of replacement and cost of associated losses.

C_{Fj} = the cost of partial failure

P_{Fj} = the probability of occurrence of partial failure.

As the total collapse of a structure under extreme loading conditions is very rare P_{Tj} may be considered very small so that P_{Fj} controls Eq. (6.13) and can then be simplified, as

earlier proposed by Turkstra (1970), to:

$$C_{Tj} = C_{ij} + P(\underline{X} \in D_F)C_{Fj} \qquad (6.14)$$

with D_f being the failure region.

The task now is the establishment of that particular value of P_{Fj}, the optimum probability of failure, at which the expected total cost C_{Tj}, is a minimum for all structures j in the data space $R \in \{\underline{X}: g(\underline{X}) \leq 0\}$, where $g(\underline{X})$ is a function corresponding to the violation of a specified limit state. As shown in Fig 6.14 from the variation of the components of Eq. (6.14) with the probability of failure it is clear that an ultra-conservative code which fulfills its purpose of safeguarding public life and property can be very expensive. Consequently the point corresponding to the minimum provides the needed trade-off between safety and economy. It is this point that an analyst and designer should resolve as a primary task (Polykov 1985). To this end, a suitable nonlinear programming algorithm, NLPQL, implemented by Schnittkowski (1989) together with FORM are used. The NLPQL algorithm is built upon an iterative process comprising two parts. First, a search direction is determined, and then a line search is performed. The search direction is determined by solving a quadratic approximation of the Lagrangian function of the nonlinear problem. With Eq (6.14) as the objective function subject to the fulfillment of the performance function expressed by Eq (6.1) being less or equal to zero (i.e., $g(\underline{X})$

≤ 0), a target safety index, $\beta_T = 4.265$ is arrived at for both design methods (i.e, AISI, 1968 and ECCS, 1987). To arrive at this value, a number of design points particularly different loading configurations and varying load ratios, are tested. This is found necessary since a code-writing body may wish to study the variation in the safety index values with reference

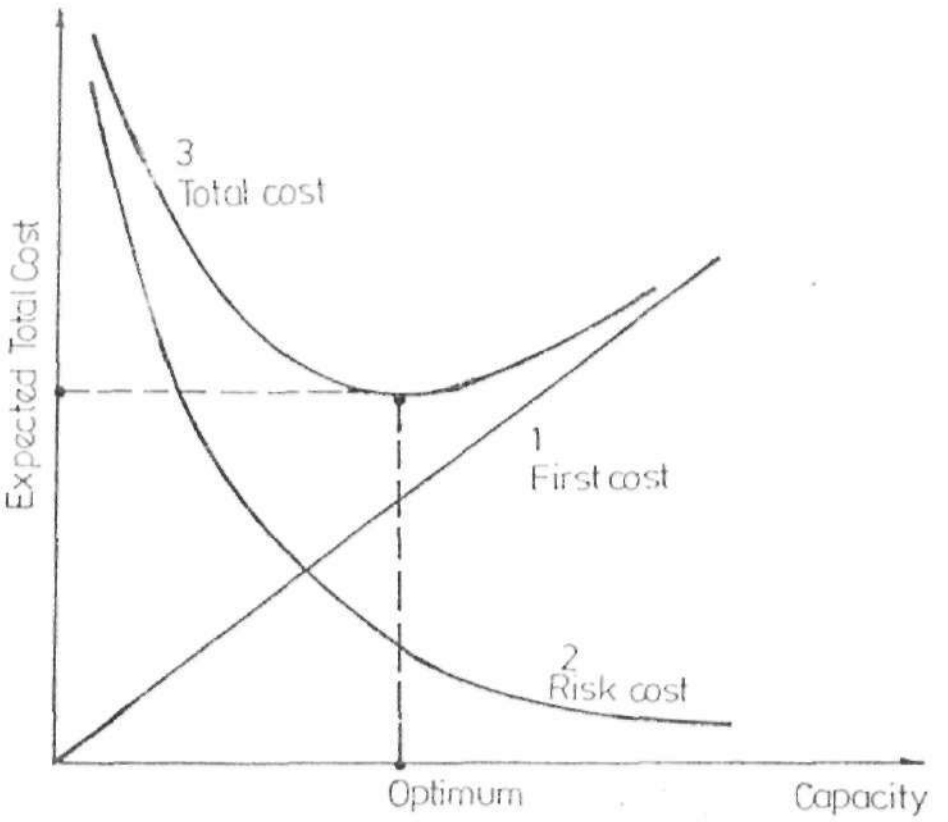
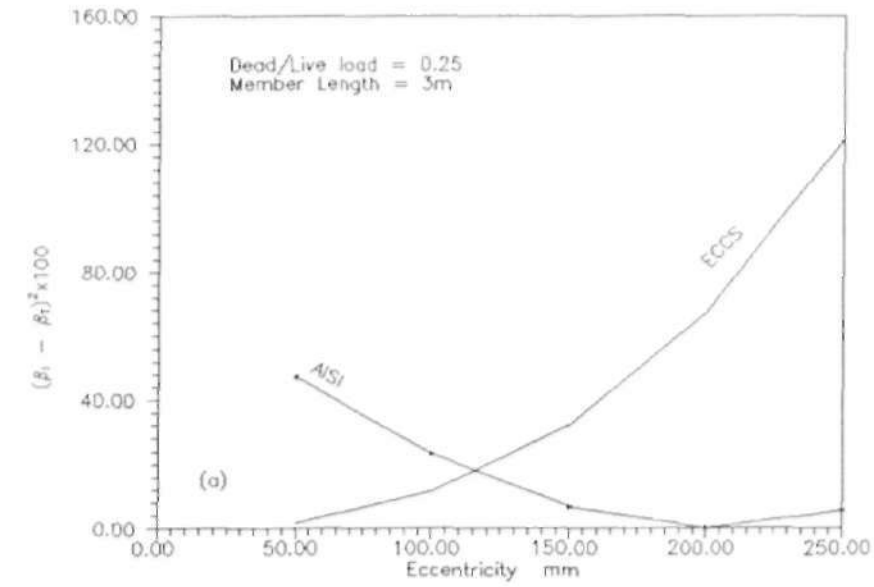


Fig. 6.14: Economics of Value Analysis



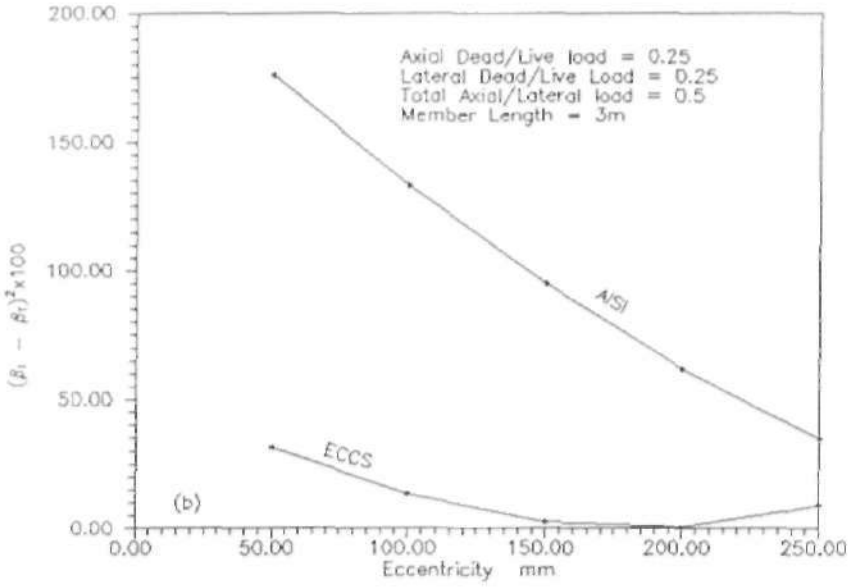


Fig. 6.15: Deviation from Target Safety Level (a) Eccentric Loading and (b) Eccentric Loading and UDL)

to other data points in order to arrive at a reasonable value. The difference between β_T and the existing safety indices are typically shown in Figs. 6.15(a) and (b). For the existing design equations there exists a wide range of safety levels which implies that different designs will have different reliabilities. Thus, the selection of a target safety level will lead to the prevention of over- and under-conservatism in the new code. Depending on the loading configuration in consideration, both AISI specification and ECCS Recommendation show different magnitudes of departure from what may be called a consistent safety level. For increasing eccentricity, departure from the target reliability is much less for AISI than ECCS especially at high eccentricity. At modearate eccentricity of about 120mm, AISI shows a higher departure (Fig 6.15(a)). On the other hand, when lateral load is considered, the deviation from the target reliability level decreases with increasing eccentricity for both design methods. The trend however shows a higher departure for AISI than ECCS. This characteristic may be connected with the observations noted in Section 3.4 where results are compared with the strength predictions. The information obtained from these figures also point to the characteristic nature of the cost-benefit of the current design methods. Since these departures vary with different design variables, the diversity in the level of failure probability will equally be reflected on the failure cost.

The selected β_T belongs to the range recommended for ultimate limit states where $\beta \geq 3.0$ (e.g., Leporati, 1979) and for ordinary buildings composed of hot-rolled structural steel sections $\beta = 3.0$ was adopted in a Load and Resistance Factor Design (LRFD) project (e.g., Ravindra and Galambos, 1978). In many other applications the selection of β_T is based on the importance of a structure (MacGregor, 1983) and the actions to which a structure is subjected (e.g., NKB, 1978).

6.4 Proposed Limit State Design Formats.

As noted in the preceding sections a major inconsistency in traditional (non-limit state design) codes regarding safety checking and load combination rules for safety checking is clearly reflected. Working by regulatory authorities and structural engineers, tendencies are to rate structural safety as a very important requirement to be met while some other parties (e.g., Rackwitz, 1991; Ravindra and Lind) including this work express the view that safety is necessary but not sufficient; other matters such as costs are also important. Actually absolute safety cannot be completely ensured and beyond a certain point, attempts to reduce risk of collapse will cause disproportionate increases in cost (Astill and Martin, 1975). Therefore the design criteria adopted by AISI, (1968) and the ECCS, (1987) require modifications to allow transition to reliability-based Limit State Design Formats (LSDF) that are centered around the balance of safety and

economy.

With reference to Section 6.3, a target reliability level appropriate for the older codes has been established to be β_T = 4.265 based on a plausible socio-economic criterion. The relevant formats must be developed to maintain this level consistently considering all practical associated design parameters. Thus, this attempt leads to Step II of the Flow-Chart in Fig. 6.1. The necessary details are given below.

To allow a direct transition into a probabilistic Limit State Design Format, Eq. (6.2) will be modified in order to satisfy the following requirement:

$$\phi R_n \geq \sum_{k=1}^{j} \alpha_k Q_{kn} \qquad (6.15)$$

In Eq. (6.15) the left side relates to the capacity of the beam-column while the right side characterizes the loading acting on it. R_n is the nominal resistance computed according to the formula in AISI, (AISI, 1968) or ECCS, (ECCS, 1987) and ϕ is the resistance factor which is always less than unity and it reflects the uncertainties associated with R. On the other hand, the loading side of the design criterion, Eq (6.15), is the sum of products αQ_n, in which Q_n is the nominal load effect and α the corresponding load factor. The load factors reflect

potential overloads and the uncertainties inherent in the calculation of the load effects. The combination of load effects from different load sources is denoted by the summation sign, for example, as considered in this study, for dead load and live load effects we have:

$$\sum_{k=1}^{j} \alpha_k Q_{kn} = \alpha_D Q_{Dn} + \alpha_L Q_{Ln} \qquad (6.16)$$

so that Q_{Dn} and Q_{Ln} are the nominal dead and live load effects, respectively; and α_D and α_L are the corresponding load factors.

Taking R_n as the beam-column capacity equation in AISI (AISI, 1968) or ECCS (ECCS, 1987), Eq. (6.15) can be expressed as in Eq. (6.2) in the form:

$$\frac{\alpha_D N_{DN} + \alpha_L N_{LN}}{\phi_C P_U} + \psi \frac{[\alpha_D M_D(Q_{DN}, N_{DN}) + \alpha_L M_L(Q_{LN}, N_{LN})]}{\phi_B M_U} = 1.0 \quad (6.17)$$

In Eq. (6.17), the loading as shown in Fig. 6.2 is considered and ϕ_C and ϕ_B denote resistance factors corresponding to column and beam capacities, respectively. Other quantities are as defined previously.

Solving Eq. (6.17) for N_{LN} and expressing the total applied loads as in Eqs. (6.5) and (6.6), performance function

corresponding to Eq. (6.1) is derived. Since the parameters ϕ, α_D and α_L must be chosen so as to provide a consistent safety level (i.e., β_T = 4.265) pertinent to the current codes, deviations from this target value of β associated with different design calibration points must be minimum. Thus, the criterion for establishing these values is as follows:

Minimize

$$I_w = \sum_i w_i (\beta_i - \beta_T)^2 \qquad (6.18)$$

or

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Minimize

$$I_w = \sum_i w_i (p_{f_i} - p_{f_T})^2 \qquad (6.19)$$

in which β_T and p_{fT} are the target safety measures, β_i and p_{fi} are safety levels associated with the i^{th} structure designed using the checking equation format and w_i is a weight function which describes the relative importance of the i^{th} structural design within the scope of the code. If the minimum value of I_w is too large, another format must be selected.

Tables 6.14 to 6.16 show the variation of ϕ with member length and other design parameters. To arrive at these values, the basic design variables are assumed while α_D and α_L are varied until I_w becomes a minimum. At this point the vector

containing α_D, α_L and ϕ is obtained. Since objectively the strength and load factors are sought, partial safety factors for other design variables are not computed. It is however possible on the basis of the advanced computational technique employed in this study to derive partial factor for each basic variable of the resistance model but doing so is considered as too many for practical design use (Thoft-Christensen and Baker, 1982). In any case, the technique supercedes that proposed by Ravindra et al (1974) in which the member resistance was considered as one-parameter random variable. The use of the separation function approach in their work posed a major limitation to the possibility of obtaining partial safety factors for the basic variables of the member resistance.

6.4.1 Revised AISI Equation

Based on the information contained in Tables 6.14 and 6.15 the improved version of AISI, (1968) can now be stated as follows:

For the specification to provide safe and economical torsionally stable beam-columns, it will be required to have:

$$\phi R_n \geq \alpha_D Q_{DN} + \alpha_L Q_{LN} \qquad (6.20)$$

which by comparison with the adopted design method in AISI

Specification implies that

$$\frac{f_a}{\phi_C F_v} + \frac{C_{mx} f_b}{\phi_B F_b \left(1 - \frac{f_a}{\phi_C F_e}\right)} \leq 1.0 \qquad (6.21)$$

with other relevant conditions accordingly modified and imposed. In Eq. (6.21) F_a and F_b are the relevant expressions in the specification without the safety factor 1.92 applied. Also the computations of f_a and f_b takes into consideration the factored applied loads.

TABLE 6.14: AISI INTERACTION STRENGTH REDUCTION FACTOR (N_D/N_L = 0.25)

Reduction Factor	Length [m]	Eccentricity, e_o [mm]				
		50	100	150	200	250
ϕ_C	1	0.77	0.74	0.70	0.67	0.64
	2	0.78	0.74	0.71	0.67	0.63
	3	0.79	0.75	0.71	0.67	0.63
	4	0.80	0.76	0.72	0.67	0.62
	5	0.83	0.77	0.72	0.65	-
ϕ_B	1	0.89	0.85	0.81	0.77	0.73
	2	0.89	0.85	0.81	0.77	0.73
	3	0.91	0.86	0.82	0.77	0.72
	4	0.93	0.87	0.82	0.77	0.71
	5	0.95	0.89	0.82	0.75	-

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TABLE 6.15: AISI INTERACTION STRENGTH REDUCTION FACTOR (N_L/N_L = 0.50)

Reduction Factor	Length [m]	Eccentricity, e_o [mm]				
		50	100	150	200	250
ϕ_C	1	0.83	0.80	0.76	0.72	0.69
	2	0.84	0.80	0.76	0.73	0.69
	3	0.85	0.81	0.77	0.73	0.68
	4	0.87	0.82	0.77	0.72	0.67
	5	0.89	0.83	0.77	0.70	-
ϕ_B	1	0.96	0.92	0.87	0.83	0.79
	2	0.97	0.92	0.88	0.84	0.79
	3	0.98	0.93	0.88	0.84	0.78
	4	1.00	0.95	0.89	0.83	0.77
	5	1.03	0.96	0.89	0.81	-

TABLE 6.16: ECCS INTERACTION STRENGTH REDUCTION FACTOR

Load Ratio N_D/N_L	Length [m]	Eccentricity, e_o [mm]				
		50	100	150	200	250
0.25	1	0.763	0.723	0.683	0.643	0.601
	2	0.791	0.749	0.706	0.662	0.617
	3	0.843	0.798	0.751	0.703	0.652
	4	0.904	0.855	0.803	0.748	0.687
	5	0.976	0.923	0.864	0.800	0.725
0.50	1	0.825	0.782	0.739	0.694	0.649
	2	0.855	0.810	0.763	0.715	0.665
	3	0.911	0.863	0.812	0.760	0.704
	4	0.977	0.924	0.868	0.809	0.741
	5	1.053	0.995	0.933	0.864	0.782

In Eq. (6.21), the parameters ϕ_c and ϕ_B are resistance factors corresponding to column and beam strength respectively. Using simple regression analysis on the result shown in Tables 6.14 and 6.15, we have:

$$\phi_C = 0.72 + 0.0008\lambda \tag{6.22}$$

and

$$\phi_B = 0.82 + 0.0008\lambda \tag{6.23}$$

Hence, for design purpose the relationship in Eq. (6.21) and the expressions for ϕ's must be used in conjunction with the appropriate load combination. In this case, dead and imposed live loads are of interest in this study and the result of the multicriteria optimization coupled with the advanced reliability analysis showed that a factor of 1.08 must be applied to the dead load while 1.32 should be applied to the live load.

6.4.2 Revised ECCS Equation

Similar to the derivation obtained for AISI specification, the basis of an economical and safe design of doubly-symmetric thin-walled beam-columns is as follows:

$$\frac{N}{\phi_{BC} N_{xd}} + \frac{M_x}{\phi_{BC} M_{xd}} \left(\frac{1}{1 - \dfrac{N(k\bar{\lambda}_x)^2}{\phi_{BC} Q_w N_{xd} R_x}} \right) \leq 1.0 \tag{6.24}$$

in which ϕ_{BC} is expressed as:

$$\phi_{BC}=0.72+0.003\lambda \qquad (6.25)$$

These expressions are obtained following the information contained in Table 6.16. The values of α_D and α_L are also respectively given as 1.08 and 1.32.

General Comments

It must be emphasized however, that ϕ_C, ϕ_B and ϕ_{BC} together with α_D and α_L herein derived can be improved if and when more data information are available. This possibility is reflected in the existence of varying safety factors in different codes across the globe for structural materials and systems. And in fact, as knowledge increases in understanding more of environmental loadings and behaviour of materials together with the advent of digital computers to handle complex analyses, design decisions accruing from intuition need give way to technique-based approach.

6.4.3 Comparison of Safety Levels.

Safety levels implicit in the current AISI specification and ECCS recommendation for torsionally stable cold-formed beam-columns are compared with the implied levels associated with the proposed formats. Figs. 16 and 17 give typical plots of the variation of the safety indices implicit in the proposed formats with other important design parameter. For both cases,

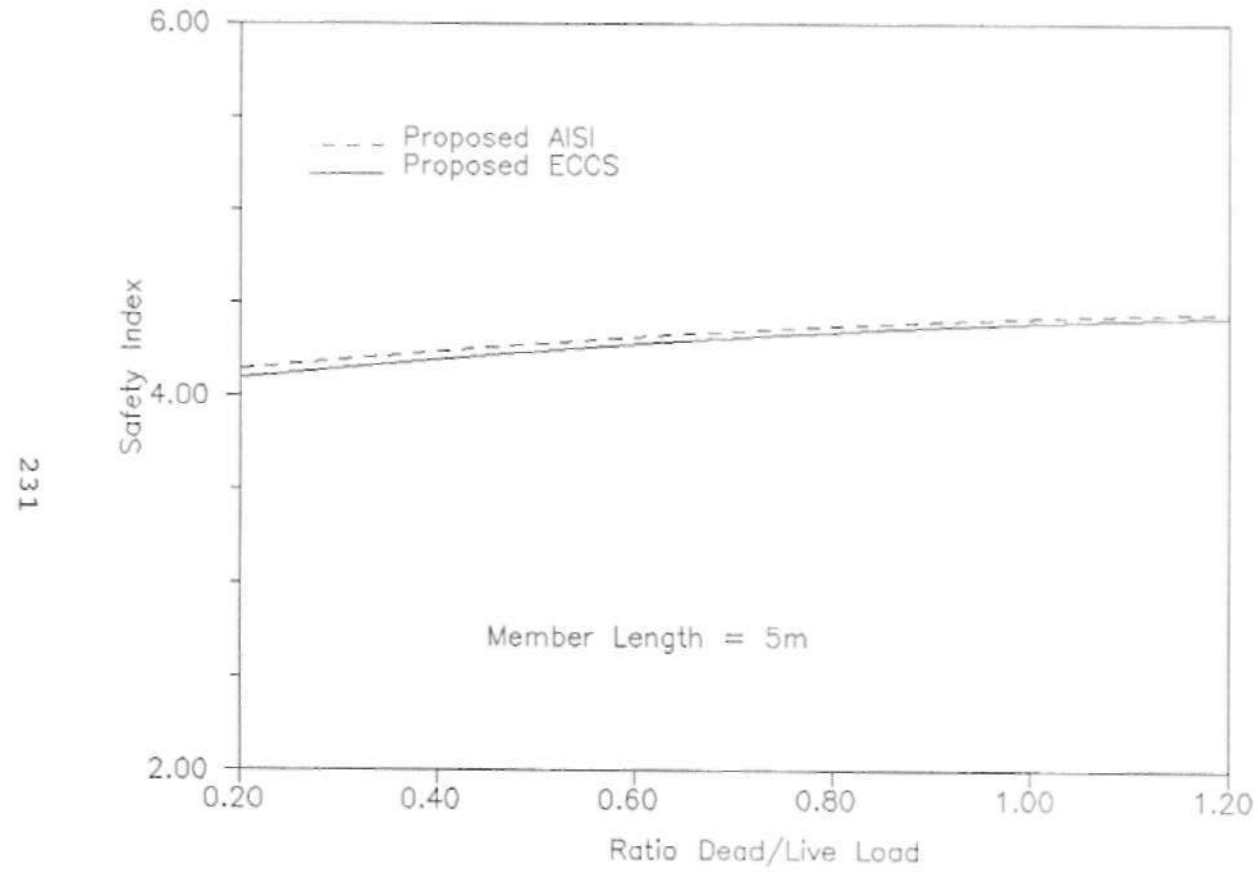


FIG. 6.16: Safety Levels of the Proposed Limit States.

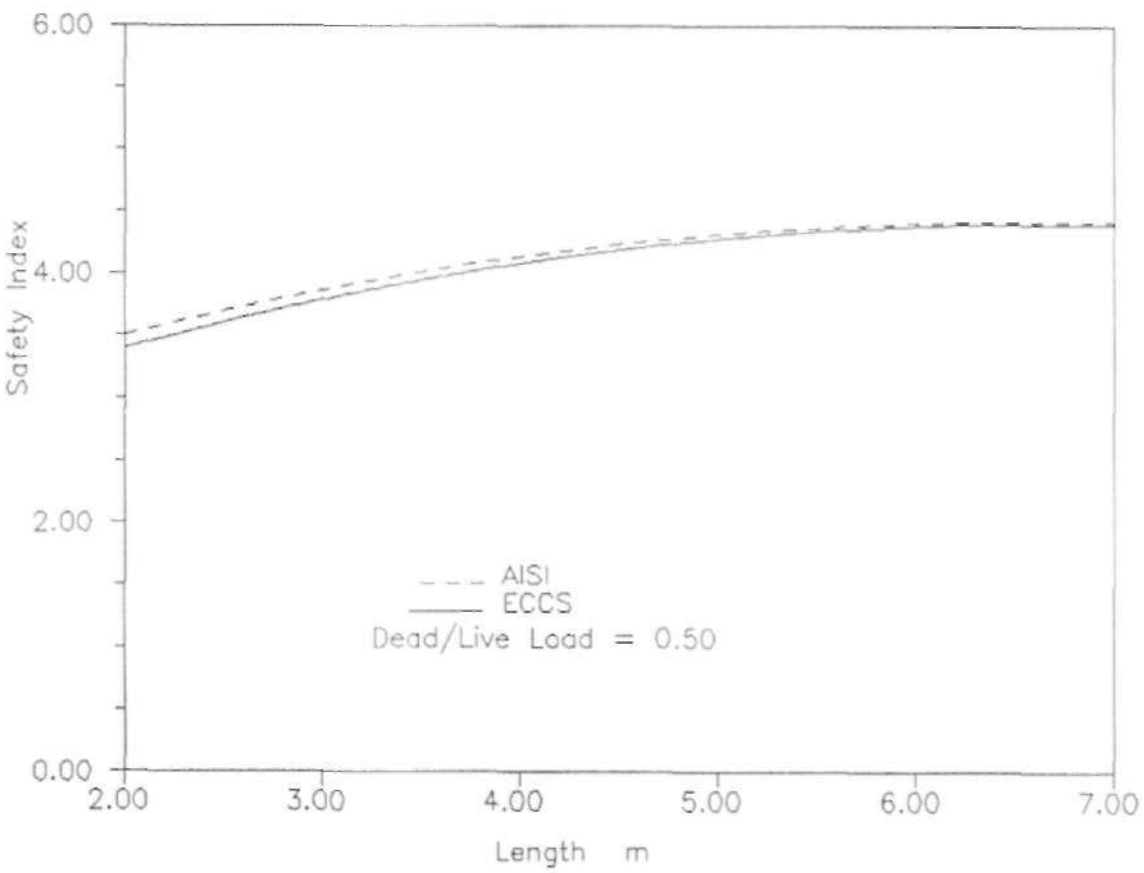


FIG. 6.17: Variation of Proposed Safety Indices with Member Length.

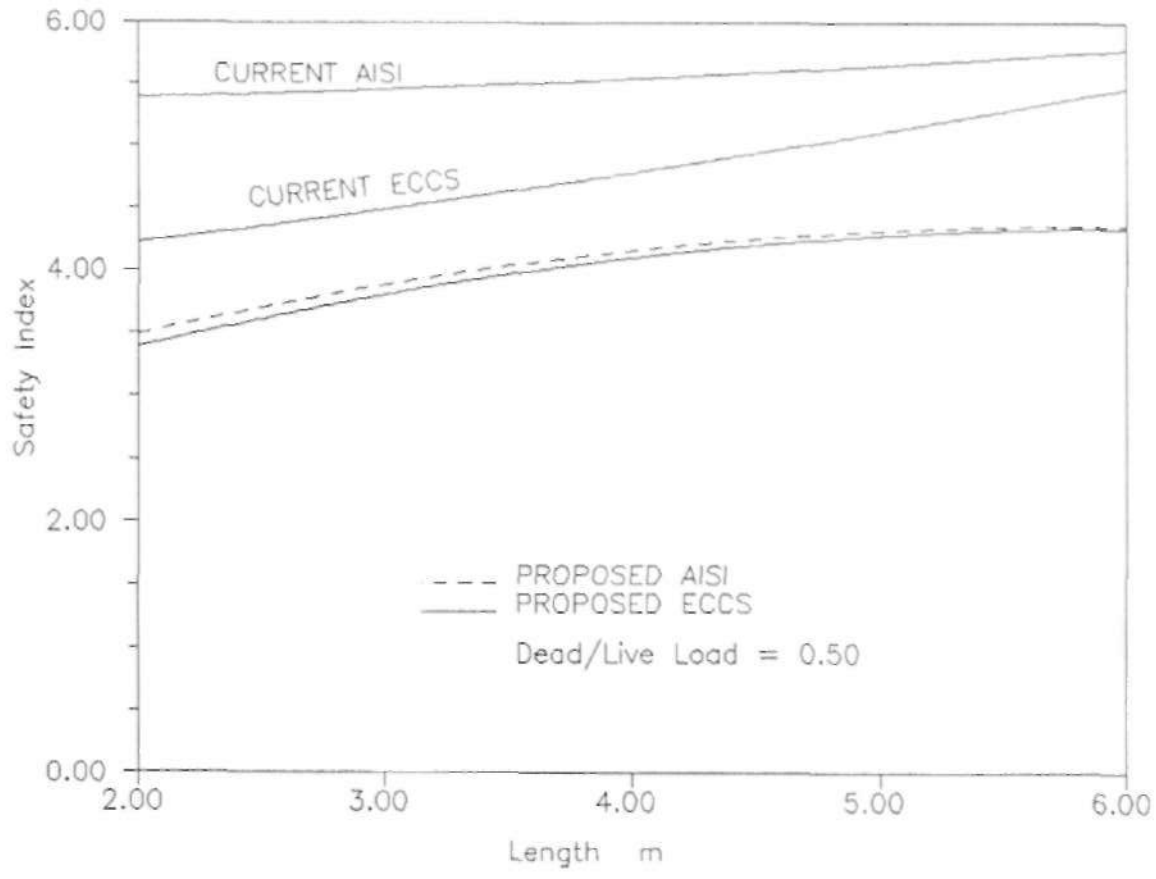


FIG. 6.18: Comparison of Safety Indices for Current and Proposed Interaction Equations

proposed methods tend to provide equal reliability level with significant uniformity objectively conjectured. On the other hand, Fig. 18 gives the clear difference which exists between the current design methods and the proposed formats. The difference between the safety levels implicit in AISI and ECCS is wide and both show distinct nonuniformity. Also, for both methods, the safety levels are higher than those obtained for the proposed formats. These higher values do not demonstrate better design methods since extraordinary effectiveness may not necessarily imply economical design. The target level of β_T = 4.265 was obtained based on risk balancing to trade off economy and safety. Considering various practical design inputs, the outcome safety indices are bounded on the predetermined target level. This is to say that the sensitivity of the safety level to other decision design parameters has been greatly minimized.

CHAPTER 7

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

The use of thin-walled structural steel in a light weight building system has become a solution to industrialize the construction of various building types. In this regard, the development of a flexible building technique such that buildings of different dimensions may be constructed rapidly and with reduced labour input has been the objective. This need has led to the development of new specifications for cold-formed steel construction. Basically the necessity was prompted not only because the performance of thin cold-formed structural members under load differs in several significant respect from that of heavy hot-rolled steel sections, but also the forms, shapes, connections, and fabrication practices which have been developed in cold-formed steel construction differ in many ways from those of heavy steel structures. Consequently, design specifications for heavy hot-rolled steel construction cannot possibly cover the design features of cold-formed steel construction completely.

In a measure, this work has reviewed varying intricacies associated with the structural behaviour of cold-formed steel structures in general and specially torsionally stable beam-columns giving situation report on research projects conducted

or being conducted at many individual companies and universities across the world. It is found that cold-formed steel sections are highly competitive for a range of structural applications. One major factor which gives such sections significant advantages over their hot-rolled counterparts is embedded in the wall thickness which can be used. The thickness of the material from which cold-formed sections are manufactured are usually small, thus permitting more economical use of materials. However, the relative thinness of the walls of cold-formed sections is also found to be the source of many design aspects that must be considered if advantage of its potential economies is to be given recognition.

Local buckling, characterized by short-wavelength local ripples along the walls of a section, is extremely important in the design of cold-formed section. It is found, using experimental and theoretical investigations, that locally buckled elements can still sustain additional loads. Thus, the concept of effective width is utilized in accounting for the effect of local buckling. The concept simply assumes that the most severely buckled portions of an element are ineffective in resisting load, and that the applied compression is resisted by effective portions situated adjacent to the supported edges. A number of expressions recommended for effective width for design purpose exist in different codes. Hence, this

investigation has critically studied two different design methods (as found in AISI, 1968 and ECCS, 1987) with regards to torsionally stable beam-columns under deterministic and probabilistic considerations. The design equations for predicting the strength of these members are compared with some existing test results. A lot of inconsistencies exist depending on member geometry being considered. There are cases where both formulations yield conservative predictions and for some others they show overestimation. All cases show that ECCS formulation leads to more conservative predictions than that of AISI.

Since it is generally recognized that structural problems are often nondeterministic a quantitative analysis of design uncertainties is carried out with its influence on the levels of risk implicit in the design equations computed. It is found that different degrees of uncertainty in design variables significantly affect the safety of beam-columns designed to the two codes. As well, the levels of safety associated with them are not consistent for all possible design points in the data space. It is found however, that both codes have equal reliability target of β_T = 4.265. This target is obtained considering the implied cost-effectiveness of the design equations using socio-economic criterion.

Following the objectives of the investigation, an attempt

is made to optimize the two codes so that conflicting aims of safety and economy can be traded off. Consequently resistance and load factors useful for transistion from the current codes to reliability-based formats are derived. The proposed formulations yield optimal code criteria. An optimal code criterion is defined as one with a minimum total cost implication. The total cost of a code is normally taken as the sum of initial costs of all structures in the data space designed in accordance with that particular code, and the expected value of costs of failure of the designed structures. Strictly speaking the applicability of this development is limited to the design of cold-formed structural steel beam-column members of doubly-symmetric shapes or shapes not subject to torsional or torsional-flexural buckling. Further modifications may also be necessary if and when refined information exists that may be relevant for such changes.

There is no doubt the development in this study elegantly supercedes the current AISI and ECCS methods for doubly-symmetric thin-walled structural steel beam-columns since the strength predictions have been shown to exhibit nonuniform reliability over many different conditions. And as such it is not really known what factor of safety should be for the many individual components or for a system as a whole. In addition, the basis of allowable stress design (typical of AISI, 1968) is lumped within arbitrary allowable stresses and factors of

safety.

The methodology followed in this study makes consistency of reliability and cost minimization its focus and as such for a wide range of design situations, relative uniform level of safety can be achieved. It is also possible to adjust structural reliability in order to reflect the consequences of failure for unique designs if there are wishes to control safety in a manner different from normal. All that is required for proper implementation of the employed concept is that the designer should have a better understanding of structural behaviour. Also since the statistical characters of design variables are significantly required, the derived formats can be consistently updated as new sets of data become available.

Finally, the practicality of studies like this is often grossly overlooked because of tradition, familiarity and convenience, particularly on the part of experienced engineers who may have developed a feel for the expected design under the existing conventional design philosophy. It will eventually register itself as time allows practicing engineers to become more familiar with probability-based design decisions, and the universities begin to integrate the fundamentals into their engineering curricula. At least for now, there are conscious efforts to establish standard codes based on this concept in North America and Europe.

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